Assignment #1: the value of time

qin

February 2026

1. One sentence each.

(a) Why do governments issue bonds and not simply print more money?
Governments issue bonds to raise funds through borrowing and to help maintain a benchmark interest-rate structure in financial markets; directly printing money can expand the money supply too quickly, fuel inflation, and undermine price stability.

(b) Give a hypothetical example of why the long-term part of a yield curve might flatten.
Suppose the central bank raises short-term rates to curb inflation, but markets expect future growth and inflation to weaken and anticipate eventual rate cuts; investors then buy long-term bonds, pushing down long-term yields and causing the long end of the yield curve to flatten.

(c) Explain what quantitative easing is and how the (US) Fed has employed this since the beginning of the COVID-19 pandemic.
Quantitative easing refers to a policy used when the policy rate is near zero, in which a central bank purchases longer-term securities (such as Treasuries and agency MBS) to inject liquidity, lower long-term interest rates, and restore market functioning; during the early COVID-19 period, the Fed announced in mid-March 2020 that it would increase its holdings of Treasuries and agency MBS by at least \$500 billion and \$200 billion respectively, then on March 23, 2020 stated it would continue purchases "in the amounts needed," later maintained large monthly purchase plans (e.g., \$80B Treasuries and \$40B MBS), began tapering in November 2021, and net purchases ended in early March 2022[1].

2. Selected bonds.

We selected bonds using the following screening criteria:
(1) Keep only bonds with a remaining maturity between 0 and 5 years.
(2) Using 0.5-year nodes (0.5, 1.0, . . . , 5.0), automatically select 10 bonds, one per node, by choosing the bond whose maturity date is closest to, and not earlier than, the corresponding node.
We use 0.5-year nodes because Government of Canada bonds typically pay

coupons semi-annually; therefore, the 0–5 year horizon corresponds to roughly 10 semi-annual cash-flow nodes, and selecting 10 bonds to cover these nodes aligns well with the bootstrapping procedure.
The screened results are shown in Figure 1.

3. Principal Component Analysis

When we treat the yield at each maturity (each unique point on a stochastic curve) as a set of highly correlated stochastic processes and perform an eigen-decomposition (PCA) of their covariance matrix:
The eigenvectors provide a set of mutually orthogonal principal-component directions. They describe the typical shapes in which the curve moves together and the weights (loadings) of each maturity under a given mode, i.e., which points tend to rise and fall together and which move in opposite directions.
The eigenvalues quantify the amount of variability explained by each mode. A larger eigenvalue means that mode contributes more variance and has higher explanatory power, i.e., it is a more important driver of movements.
In the standard empirical interpretation of yield-curve PCA, the first three principal components are usually associated with level (a parallel shift up/down), slope (relative steepening/flattening between short and long maturities), and curvature (a "hump" or twist in the middle maturities). Therefore, the largest eigenvalue and its corresponding eigenvector typically represent the curve's dominant overall movement pattern, most often a level/parallel-shift factor.

4. Empirical Questions 1

(a) Five-year yield curve.
The results are shown in Figure 2–4.

(b) Pseudo code of the spot curve.
Step 1: For a given date, take the 10 chosen coupon bonds intended to span 0.5, 1.0, . . . , 5.0 year.
Step 2: Sort the bonds by maturity (shortest to longest). Set face value $F = 100$.
Step 3: Initialize an empty map for discount factors $DF(t)$ at each node t.
Step 4: For the shortest maturity (0.5 y, one semi-annual period), solve:

$$P = (F + (C/2)F)\, DF(0.5) \quad \Rightarrow \quad DF(0.5) = \frac{P}{F + (C/2)F}.$$

Step 5: For each next maturity t (with $n = 2t$ semi-annual periods), solve iteratively:

$$P = \sum_{k=1}^{n-1} (C/2)\, F \cdot DF(0.5k) + \big(F + (C/2)F\big)\, DF(t),$$

$$DF(t) = \frac{P - \sum_{k=1}^{n-1} (C/2)\, F \cdot DF(0.5k)}{F + (C/2)F}.$$

Step 6: Convert $DF(t)$ to the spot rate $S(t)$ under semi-annual compounding:

$$DF(t) = \frac{1}{\left(1 + S(t)/2\right)^{2t}} \quad \Rightarrow \quad S(t) = 2\left(DF(t)^{-1/(2t)} - 1\right).$$

Step 7: Plot the spot curve for maturities 1–5 years for each day.

(c) Pseudo code of the 1-year forward curve.
Step 1: From bootstrapping, obtain discount factors $DF(t)$ for $t = 1, 2, 3, 4, 5$ years (semi-annual compounding).
Step 2: For each $n = 1, 2, 3, 4$, compute the 1-year forward rate starting at $t = 1$ and ending at $t = 1 + n$ using:

$$DF(1{+}n) = \frac{DF(1)}{\left(1 + f_{(1,1+n)}/2\right)^{2n}} \quad \Rightarrow \quad f_{(1,1+n)} = 2\left(\left(\frac{DF(1)}{DF(1+n)}\right)^{1/(2n)} - 1\right).$$

Step 3: Plot the forward curve points {1y-1y, 1y-2y, 1y-3y, 1y-4y} for each day.

5. Covariance matrix.

The results are shown in Figure 5–8.
These tables show the 9-day log-returns of the 1Y–5Y yields and 1y-1y. . . 1y-4y forward rates, and their covariance matrices indicate that maturities/tenors co-move positively (diagonals = each series' variance; off-diagonals = pairwise co-variation), consistent with common factors driving rate changes.

6. Eigenvalues and Eigenvectors.

The results are shown in Figure 9–12.
PC1 explains 82.96% of yield variance and 79.71% of forward variance, so the largest eigenvalue indicates one dominant common movement. The PC1 eigenvector loadings are mostly same-sign across maturities/tenors, implying a level-type (parallel-shift) factor where rates move together. PC2 adds a smaller share (≈9.57% for yields; 18.89% for forwards), capturing additional shape changes beyond the common move (e.g., slope/curvature).

GitHub Repository

https://github.com/Amandahu1414/HW

References

[1] https://www.federalreserve.gov/newsevents/pressreleases/monetary20200315a1.htm?