Tutorial - 4

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Sem - 4

Ans 1 $T(n) = 3T(n/2) + n^2$

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a = 3, \quad b = 2$

$C = \log_2 3 = 1.58$

$n^C = n^{1.58}$

$f(n) = n^2$

By case 3: $f(n) > n^C$

$T(n) = \theta(f(n)) = \underline{\theta(n^2)}$

Ans 2 $T(n) = 4T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a = 4, \quad b = 2$

$C = \log_2 4 = 2$

$n^C = n^2$

$f(n) = n^2$

By case 2: $f(n) = n^C$

$T(n) = \theta(n^C \log n) = \underline{\theta(n^2 \log n)}$

Ans 3 $T(n) = T(n/2) + 2^n$

$T(n) = aT(n/b) + f(n)$

$a = 1, \quad b = 2$

$C = \log_2 1 = 0$

$n^C = n^0 = 1 \; ; \; f(n) = 2^n$

By Case 1 $\quad f(n) > n^C$

$T(n) = \theta(f(n)) = \underline{\theta(2^n)}$

Ans 4 $T(n) = 2^n T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a = 2^n, \quad b = 2$

$C = \log_2 2^n = n$

$n^C = n^n, \quad f(n) = n^n$

$f(n) = n^C$

$T(n) = \theta(n^C \log n)$

$\underline{T(n) = \theta(n^n \log n)}$

Ans 5 $T(n) = 16T(n/4) + n$

$T(n) = aT(n/b) + f(n)$

$a = 16, b = 4$

$C = \log_4 16 = 2$

$n^C = n^2, \quad f(n) = n$

$f(n) = n^C$

$\underline{T(n) = \theta(n^2)}$

Ans 6 $T(n) = 2T(n/2) + n\log n$

$T(n) = aT(n/b) + f(n)$

$a = 2 \quad b = 2 \quad c = \log_2 2 = 1$

$n^c = n$, $f(n) = n\log n$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n\log n)$

Ans 7 $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2, \quad c = \log_2 2 = 1$

$n^c = n^1$, $f(n) = n/\log n$

$n^c > f(n)$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n)$

Ans 8 $T(n) = 2T(n/4) + n^{0.51}$

$T(n) = aT(n/b) + f(n)$

$a = 2 \quad b = 4 \quad c = \log_4 2 = 0.5$

$n^c = n^{0.5}$, $f(n) = n^{0.51}$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n^{0.51})$

Ans 9 $T(n) = 0.5\,T(n/2) + 1/n$

$T(n) = a\,T(n/b) + f(n)$

$a = 0.5, \quad b = 2, \quad c = \log_2 0.5 = -1$

$n^c = n^{-1} = 1/n, \quad f(n) = 1/n$

$f(n) = n^c$

$T(n) = \Theta(n^c \log n)$

$= \Theta(\log n / n)$

Ans 16 $T(n) = 16\,T(n/4) + n!$

$T(n) = a\,T(n/b) + f(n)$

$a = 16, \quad b = 4, \quad c = \log_4 16 = 2$

$n^c = n^2, \quad f(n) = n!$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n!)$

Ans 11 $T(n) = 4\,T(n/2) + \log n$

$a = 4, \quad b = 2, \quad c = \log_2 4 = 2$

$n^c = n^2, \quad f(n) = \log n$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^2)$

Ans 12 $T(n) = \text{Sqrt}(n)\, T(n/2) + \log n$

$T(n) = aT(n/b) + f(n)$

$a = n^{1/2}$, $b = 2$, $f(n) = \log n$

$c = \log_2 n^{1/2} = \frac{1}{2} \log n$

$n^c = n^{1/2} \log n$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

Ans 13 $T(n) = 3T(n/2) + n$

$a = 3 \quad b = 2 \quad c = \log_2 3 = 1.58$

$n^c = n^{1.58}$, $f(n) = n$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^{1.58})$

Ans 14 $T(n) = 3T(n/3) + \text{Sqrt}(n)$

$a = 3$, $b = 3$, $c = \log_3 3 = 1$

$n^c = n^1$; $f(n) = n^{1/2}$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n)$

Ans 15 $T(n) = 4T(n/2) + cn$

$a = 4, \quad b = 2, \quad c = \log_2 4 = 2$

$n^c = n^2 \quad , \quad f(n) = cn = n$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$T(n) = \underline{\theta(n^2)}$

Ans 16 $T(n) = 3T(n/4) + n \log n$

$a = 3, \quad b = 4, \quad c = \log_4 3 = 0.79$

$n^c = n^{0.79} \quad , \quad f(n) = n \log n.$

$f(n) > n^c$

$T(n) = \theta(f(n))$

$T(n) = \underline{\theta(n \log n)}$

Ans 17 $T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3, \quad c = \log_3 3 = 1$

$n^c = n^1 \quad , \quad f(n) = n/2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$T(n) = \underline{\theta(n)}$

Ans 18 $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3, c = \log_3 6 = 1.63$

$n^c = n^{1.63}$, $f(n) = n^2 \log n$

$f(n) > n^c$

$T(n) = \theta(f(n))$

$T(n) = \theta(n^2 \log n)$

Ans 19 $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2, c = \log_2 4 = 2$

$n^c = n^2$, $f(n) = n/\log n$

$f(n) < n^c$

$T(n) = \theta(n^c) = \underline{\theta(n^2)}$

Ans 20 $T(n) = 64T(n/8) - n^2 \log n$

$a = 64, b = 8, c = \log_8 64 = 2$

$n^c = n^2$, $f(n) = -n^2 \log n = n^2 \log n^{-1}$

$= n^2 \log 1/n$

$f(n) > n^c$

$T(n) = \theta(f(n)) = \underline{\theta(n^2 \log 1/n)}$

Ans 21 $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, c = \log_3 7 = 1.77$

$n^c = n^{1.77}$, $f(n) = n^2$

$f(n) > n^c$

$T(n) = \theta(f(n)) = \underline{\theta(n^2)}$

Ans 22 $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2, \quad c = \log_2 1 = 0$

$n^c = n^0 = 1, \quad f(n) = n(2 - \cos n)$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \underline{\Theta(n(2 - \cos n))}$