Tutorial - 1

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Q1

Ans 1. (i) Big O(n)

$$f(n) = O(g(n))$$

if $f(n) \leq g(n) \times c \quad \forall \; n > n_0, \; c > 0$

→ g(n) is 'tight upper bound' of f(n)

(ii) Big Omega (Ω)

$$f(n) = \Omega(g(n))$$

if $f(n) \geq c \cdot g(n) \quad \forall \; n > n_0, \; c > 0$

→ g(n) is 'tight lower bound' of f(n)

(iii) Big Thetha (Θ)

$$f(n) = \Theta(g(n))$$

if $c_1 g(n) \leq f(n) \leq c_2 \cdot g(n)$

$\forall \; c_1, c_2 > 0, \; n_1, n_2 > n_0$

→ It gives both 'tight' upper and lower bound both.

(iv) Small o (o)

$f(n) = o\, g(n)$

if $f(n) < c \cdot g(n)$

$\forall \; n > n_0 \;, c > 0$

→ It gives upper bound.

(v) Small omega (ω)

$f(n) = \omega(g(n))$

if $f(n) > c \cdot (g(n))$

$\forall \; n > 0 \;, \; c > 0$

→ It gives lower bound.

<u>Q2</u>

<u>Ans 2</u> for $i = 1, 2, 4, 6, 8 \ldots\ldots n$ times (series is a GP)

$a = 1 \quad , r = 2/1$

k^{th} value of GP:

$t_k = a r^{k-1}$

$2n = 2^k$

$\log_2 n + 1 = k$

⇒ Time complexity $T(n) = O(\log_2 n)$ Ans

Q3

Ans 3 $T(n) = 3T(n-1) \quad -(1)$

$T(n) = 1$

put $n = (n-1)$ in (1)

$T(n-1) = 3T(n-2) \quad -(2)$

put (2) in (1)

$T(n) = 3 \times 3T(n-2)$

$T(n) = 9T(n-2) \quad -(3)$

put $n = n-2$ in (1)

$T(n-2) = 3T(n-3)$

put in (3)

$T(n) = 27T(n-3) \quad -(4)$

Generalising series:

$T(k) = 3^k T(n-k) \quad -(5)$

Base Case $\Rightarrow n-k = 1$

$k = n-1$

$T(n) = 3^{n-1}\ \cancel{T(n-n+1)}\ T(n-k+1)$

$T(n) = 3^{n-1}$

$T(n) = O(3^n)$

Q4

Ans 4 $T(n) = 2T(n-1) - 1 \quad -(1)$

put $n = n-1$

$T(n-1) = 2T(n-2) - 1 \quad -(2)$

put in (1)

$$T(n) = 2\ (2T(n-2) - 1) - 1$$

$$= 4T(n-2) - 3 \qquad -(3)$$

$$T(n-2) = 2T(n-3) - 1$$

put in (1)

$$T(n) = 8T(n-3) - 4 - 3 \qquad -(4)$$

Generalising Series :-

$$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} \cdots 2^0$$

k^{th} term : Let $n-k=1$

$k = n-1$

$$T(n) = 2^{n-1} T(1) - 2^k \left(\frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n} \right)$$

$$= 2^{n-1} - 2^{n-1} \left(\frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{k-1}} \right)$$

Series in GP

$$a = \frac{1}{2}, \quad r = \frac{1}{2}$$

$$T(n) = 2^{n-1} \left(1 - \left(\frac{1}{2} \frac{(1-(1/2)^{n-1})}{1-1/2} \right) \right)$$

$$T(n) = 2^{n-1} \left(1 - 1 + (1/2)^{n-1} \right)$$

$$T(n) = \frac{\cancel{2^{n-1}}}{\cancel{2^{n-1}}}$$

$$\underline{T(n) = O(1)} \text{ Ans}$$

Q5

Ans 5

$i = 1 . 2 \ 3 \ 4 \ \ldots$

$S = 1 + 3 + 6 + 10 + 15 + \ldots$

Sum of $S = 1 + 3 + 6 + 10 \ldots + n \quad -(1)$

Also $S = 1 + 3 + 6 + 10 + \ldots T_{n-1} + T_n \quad - (2)$

$0 = 1 + 2 + 3 + 4 + \ldots n - T_n$

$T_k = 1 + 2 + 3 + 4 + \ldots + k$

$T_k = \frac{1}{2} k (k+1)$

for k iterations :-

$1 + 2 + 3 + \ldots k \leq n$

$\frac{k(k+1)}{2} \leq n$

$\frac{k^2 + k}{2} \leq n$

$O(k^2) \leq n$

$k = O(\sqrt{n})$

$\boxed{T(n) = O(\sqrt{n})}$ Ans

Q6

Ans 6 As $i^2 = n$

$i = \sqrt{n}$

$i = 1, 2, 3, 4, \ldots \sqrt{n}$

$\sum_{i=1} 1 + 2 + 3 + 4 + \ldots + \sqrt{n}$

$T(n) = \frac{\sqrt{n} * (\sqrt{n} + 1)}{2}$

$$T(n) = \frac{n * \sqrt{n}}{2}$$

$$\underline{T(n) = O(n)}$$ Ans

Q 7

Ans 7 $k = k^2$

$k = 1, 2, 4, 8 \ldots n$

→ Series in GP

$a = 1$, $r = 2$

$$= \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1(2^k - 1)}{1}$$

$\Rightarrow$ $n = 2^k - 1$

$n + 1 = 2^k$

$\log_2(n) = k$

i	j	k
1	log(n)	log(n) * log(n)
2	log(n)	log(n) * log(n)
n	log(n)	log(n) * log(n)

$T(= O(n * \log n * \log n)$

$= \underline{O(n \log^2(n))}$ → Ans

Q8

Ans 8 for (i=1 to n)

we get j = n times every turn

$\therefore i * j = n^2$

k^{th}, $T(n) = n^2 + T(n-3);$

$T(n-3) = (n^2 . 3)^2 + T(n-6);$

$T(n-6) = (n^3 . 6)^2 + T(n-9);$

and $T(1) = 1;$

Now substitute each value in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

Let $n^n - 3k = 1$

$k = (n-1)/3$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

$T(n) \simeq kn^2$

$T(n) \simeq (k-1)/3 \quad - n^2$

So,

$\underline{T(n) = O(n^3)}$ Ans

Q9

Ans 9 for i = 1 $\quad j = 1 + 2 + \dots (n \geq j + i)$

i = 2 $\quad j = 1 + 3 + 5 \dots (n \geq j + i)$

i = 3 $\quad j = 1 + 4 + 7 \dots (n \geq j + i)$

n^{th} term of AP is

$T(n) = a + d * m$

$T(m) = 1 + d * m \longrightarrow m = (n-1)/d$

for $i=1$ $(n-1)/1$ times

$i=2$ $(n-1)/2$ times

$\vdots$

$i=n-1$

we get

$$T(n) = i_1 j_1 + i_2 j_2 + \cdots + i_{n-1} j_{n-1}$$

$$= \frac{n-1}{2} + \frac{n-2}{2} + \frac{n-3}{3} + \cdots + 1$$

$$= n\left[1 + 1/2 + 1/3 + \cdots 1/n-1\right] - n + 1$$

$$= n \log n - n + 1$$

Since, $\int \frac{1}{x} = \log x$

$$\underline{T(n) = O(n \log n)} \text{ - Ans}$$

Q16

Ans 10 As given n^k and c^n is :-

$$n^k = O(c^n)$$

$$n^k \leq a(c^n) \quad \forall\ n \geq n_0,\ a > 0$$

for $n_0 = 1$, $c = 2$

$$= 1^k < a \cdot 2$$

$$= \underline{n_0 = 1 \ \& \ c = 2} \text{ Ans}$$