Multifield coupled quintessence

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We study models of quintessence consisting of a number of scalar fields coupled to several dark matter components. In the case of exponential potentials the scaling solutions can be described in terms of a single field. The corresponding effective logarithmic slope and effective coupling can be written in a simple form in terms of the individual slopes and couplings of the original fields. We also investigate solutions where the scalar potential is negligible, in particular those leading to transient matter dominated solutions. Finally, we compute the evolution equations for the linear perturbations which will allow these models to be tested against current and future observational data.

I. INTRODUCTION

In the past decade, our understanding of the evolution of the Universe, its components and respective abundances has increased to an unprecedented level. Results from various independent observations of different scales have provided us with evidence for dark matter. Supernovae combined with other independent observations suggest that the Universe is currently undergoing accelerated expansion, possibly due to a negative pressure component usually dubbed dark energy. We have also learned from analysis of the cosmic microwave background radiation that the Universe is close to flat and that the large scale structure developed through gravitational instability from a spectrum of adiabatic, nearly Gaussian and nearly scale invariant density perturbations. These conclusions are consistent with the predictions of the simplest inflationary paradigm, a short period of accelerated expansion in the early universe, introduced to explain the flatness, homogeneity and isotropy of the Universe.

Scalar fields are the most popular building blocks to construct candidate models of early universe inflation and of the present day cosmological acceleration. They are appealing because such fields are ubiquitous in theories of high energy physics beyond the standard model. Models are usually constructed using a single field; however, there is also the interesting possibility that a cosmological behavior arises from the presence of multiple scalar fields. The idea that our Universe contains a vast number of light

scalar fields is also based on expectations from landscape models; see e.g. [1]. Assisted inflation is an example of a model where many fields can cooperate to sustain inflation even if none is able to fuel it if evolving in isolation. Assisted inflation for exponential potentials was proposed in [2] and extended in [3,4], and the inclusion of a background fluid was evaluated in Refs. [5,6]. It was studied for quadratic and quartic potentials in the context of higher dimensional reduction in Refs. [7–9], for Bianchi models [10,11], for an ensemble of tachyon fields [12–15], in braneworld models [16,17], in particular string theory realizations [18–20] and for taking into account loop quantum cosmology corrections [21]. A multifield dynamics was also studied in the context of logamediate inflation [22] and k inflation [23].

A set of several fields working together could also give rise to the accelerated evolution of the Universe we currently observe. This is usually known as assisted dark energy or assisted quintessence [24–29].

These fields could, of course, interact with the rest of the world, and new forces between matter particles would arise. For common particles, these forces are tightly constrained by solar system and gravitational experiments on Earth. Limits on these forces are not so strong for interactions involving neutrinos or dark matter, and current bounds come from cosmological observations. Coupled quintessence, where a scalar field interacts with dark matter was introduced in Refs. [30–32], and its dynamics,

properties and possible couplings were studied in Refs. [33–41].

There might be, however, more than one dark matter species, a suggestion made in Refs. [42–44]. The possibility that a scalar field is indeed coupled to more than a single dark matter component has been raised recently in Refs. [45,46], and the phenomenology of such a setup was investigated and compared to observations in Refs. [47–49].

A natural extension of these works is, therefore, to investigate the cosmological dynamics considering a group of several scalar fields coupled to various dark matter components. Such a system has been addressed in Ref. [26]. Here we extend that work with an explicit analysis of the background and of linear perturbations. In Sec. II we introduce the general equations for n scalar fields coupled to m matter components. We then apply these to scalar potentials consisting of a sum of exponential terms in Sec. III, and an exponential of a sum of terms in Sec. IV. We follow this in Sec. V with the solutions where the scalar potential is negligible. In Sec. VI we extend our analysis to the linear perturbations. Finally we conclude in Sec. VII.

II. GENERAL EQUATIONS

We consider an ensemble of n scalar fields ϕ_i cross coupled to an ensemble of m dark matter components ρ_α. The cross couplings are described by the matrix $C_{i\alpha}$ where latin indices i, j identify scalar field indices and greek indices α, β identify the dark matter components. The equation of motion for the fields and the various dark matter components in a spatially flat Friedmann-Robertson-Walker metric with scale factor $a(t)$ is then written as

$$\ddot{\phi}_i + 3H\dot{\phi}_i + V_{,\phi_i} = \kappa \sum_\alpha C_{i\alpha}\rho_\alpha, \qquad (1)$$

$$\dot{\rho}_\alpha + 3H\rho_\alpha = -\kappa \sum_i C_{i\alpha}\dot{\phi}_i \rho_\alpha. \qquad (2)$$

The solution for the dark matter component evolution can be given immediately in terms of the values of the fields as

$$\rho_\alpha = \rho_{\alpha 0} \exp\left(-3N - \kappa \sum_i C_{i\alpha}(\phi_i - \phi_{i0})\right). \qquad (3)$$

The rate of change of the Hubble function is

$$\dot{H} = -\frac{\kappa^2}{2}\left(\sum_\alpha \rho_\alpha + \sum_i \dot{\phi}_i^2\right), \qquad (4)$$

subject to the Friedmann constraint

$$H^2 = \frac{\kappa^2}{3}\left(\sum_\alpha \rho_\alpha + \sum_i \rho_{\phi_i}\right), \qquad (5)$$

where $\rho_{\phi_i} = \sum_i \dot{\phi}_i^2/2 + V(\phi_1, ..., \phi_n)$. In the next two sections we consider two possible forms for $V(\phi_1, ..., \phi_n)$, both leading to scaling solutions: a sum of exponential terms and an exponential of a sum of terms.

The dark matter field can in principle include the baryon component. However, in this case the coupling will have to satisfy the strong solar system constraints and be in practice negligible.

III. SUM OF EXPONENTIAL TERMS: $V(\phi_1,...,\phi_n) = M^4\sum_i e^{-\kappa\lambda_i\phi_i}$

With the aim of finding the critical points of the evolution, we will rewrite the equations of motion as a system of first order differential equations. To do this we define the new variables

$$x_i \equiv \frac{\kappa\dot{\phi}_i}{\sqrt{6}H}, \qquad y_i^2 \equiv \frac{\kappa^2 V_i}{3H^2}, \qquad z_\alpha^2 \equiv \frac{\kappa^2 \rho_\alpha}{3H^2}, \qquad (6)$$

where $V_i = M^4 e^{-\kappa\lambda_i\phi_i}$. The evolution is now described by

$$x_i' = -\left(3 + \frac{H'}{H}\right)x_i + \sqrt{\frac{3}{2}}\left(\lambda_i y_i^2 + \sum_\alpha C_{i\alpha}z_\alpha^2\right), \qquad (7)$$

$$y_i' = -\sqrt{\frac{3}{2}}\left(\lambda_i x_i + \sqrt{\frac{2}{3}}\frac{H'}{H}\right)y_i, \qquad (8)$$

$$z_\alpha' = -\sqrt{\frac{3}{2}}\left(\sum_i C_{i\alpha}x_i + \sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}}\frac{H'}{H}\right)z_\alpha, \qquad (9)$$

$$\frac{H'}{H} = -\frac{3}{2}\left(1 + \sum_i (x_i^2 - y_i^2)\right), \qquad (10)$$

where a prime means differentiation with respect to $N = \ln a$ and the Friedmann equation now reads

$$\sum_i (x_i^2 + y_i^2) + \sum_\alpha z_\alpha^2 = 1. \qquad (11)$$

Equation (10) also defines the effective equation of state parameter w_{eff}, such that

$$\frac{H'}{H} = -\frac{3}{2}(1 + w_{\text{eff}}), \qquad (12)$$

and then,

$$w_{\text{eff}} = \sum_i (x_i^2 - y_i^2). \qquad (13)$$

A. Scalar field dominated solution

We will start by reviewing the case in which we can neglect the matter contributions ($z_\alpha = 0$) studied in the context of assisted inflation. Using Eqs. (8), (10) and (11), we obtain that

$$x_i = \frac{1}{\sqrt{6}} \frac{1}{\lambda_i \sum_j 1/\lambda_j^2}. \qquad (14)$$

By using the Friedmann equation the effective equation of state is obtained as

$$w_{\text{eff}} = -1 + \frac{1}{3}\lambda_{\text{eff}}^2, \qquad (15)$$

where the effective slope, λ_{eff}, given by

$$\frac{1}{\lambda_{\text{eff}}^2} = \sum_i \frac{1}{\lambda_i^2}, \qquad (16)$$

describes the corresponding logarithmic slope in the potential needed to replicate the dynamics with only one field. This means that increasing the number of fields makes λ_{eff} smaller and, therefore, easier to obtain an accelerated expansion even if the individual slopes are too large to fuel it if acting in isolation. This effective slope λ_{eff} is similar to the result obtained in the assisted inflation scenarios that were studied in [2–4].

B. Scaling solution

The system of equations has many fixed points, but we are particularly interested in the case when all the variables x_i, y_i and z_α are nonvanishing since these are the most interesting in cosmology. Using Eqs. (8) and (9) and considering for a moment only two fields, the fixed points of the system are at

$$x_1 = \sqrt{\frac{3}{2}} \frac{1}{\lambda_1 - \gamma_1}, \qquad (17)$$

$$x_2 = \sqrt{\frac{3}{2}} \frac{1}{\lambda_2 - \gamma_2}, \qquad (18)$$

where

$$\gamma_1 = C_{11} + C_{21}\frac{\lambda_1}{\lambda_2}, \qquad (19)$$

$$\gamma_2 = C_{22} + C_{12}\frac{\lambda_2}{\lambda_1}. \qquad (20)$$

Equation (9) also provides the constraint

$$\frac{x_2}{x_1} = \frac{\lambda_1}{\lambda_2} = \frac{C_{12} - C_{11}}{C_{21} - C_{22}} \qquad (21)$$

and $\gamma_1/\lambda_1 = \gamma_2/\lambda_2$.

We can now compute the effective equation of state parameter of the Universe knowing that

$$\frac{H'}{H} = -\sqrt{\frac{3}{2}}\lambda_i x_i = -\frac{3}{2}(1 + w_{\text{eff}}); \qquad (22)$$

it is obtained that

$$w_{\text{eff}} = -1 + \frac{1}{2}\sqrt{\frac{2}{3}}(\lambda_1 x_1 + \lambda_2 x_2). \qquad (23)$$

Substituting for x_1 and x_2 as found earlier, we can write

$$w_{\text{eff}} = \frac{\gamma_i}{\lambda_i - \gamma_i}, \qquad (24)$$

where $i = 1, 2$. Making use of the property that $\gamma_1/\lambda_1 = \gamma_2/\lambda_2$ we can define an effective coupling of the system such that

$$C_{\text{eff}} \equiv \lambda_{\text{eff}}\frac{\gamma_i}{\lambda_i}, \qquad (25)$$

to write

$$w_{\text{eff}} = \frac{C_{\text{eff}}}{\lambda_{\text{eff}} - C_{\text{eff}}}. \qquad (26)$$

Moreover, $\Omega_\phi = x_1^2 + x_2^2 + y_1^2 + y_2^2$ and given the equality $w_{\text{eff}} = x_1^2 + x_2^2 - y_1^2 - y_2^2$, it results that the total contribution of the fields to the total energy budget is

$$\Omega_\phi = 1 + 2(x_1^2 + x_2^2) - \frac{1}{2}\sqrt{\frac{2}{3}}(\lambda_1 x_1 + \lambda_2 x_2). \qquad (27)$$

Substituting now for x_1 and x_2 we obtain that

$$\Omega_\phi = \frac{3 - \lambda_{\text{eff}}C_{\text{eff}} + C_{\text{eff}}^2}{(\lambda_{\text{eff}} - C_{\text{eff}})^2}, \qquad (28)$$

where the effective λ_{eff} is given by

$$\frac{1}{\lambda_{\text{eff}}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}. \qquad (29)$$

A typical evolution of the energy densities is illustrated in Fig. 1.

The generalization to more than two fields is fairly trivial for this potential. Equations (17) and (18) are still valid for n fields $\times$ m dark matter components, and in general we have

$$\frac{\gamma_i}{\lambda_i} = \sum_j^n \frac{C_{j1}}{\lambda_j} = \cdots = \sum_j^n \frac{C_{jm}}{\lambda_j} \qquad (30)$$

for every $i = 1, \ldots, n$. It is also instructive to consider how these quantities simplify when we consider that all the fields are just replicas of, say, the field ϕ_1. By this we mean that the matrix C is diagonal with entries $C_{ii} = C_{11}$ and $\lambda_i = \lambda_1$ for all indexes i. It is easy to verify that in this case $\lambda_{\text{eff}} = \lambda_1/\sqrt{n}$ and $C_{\text{eff}} = C_{11}/\sqrt{n}$ and consequently

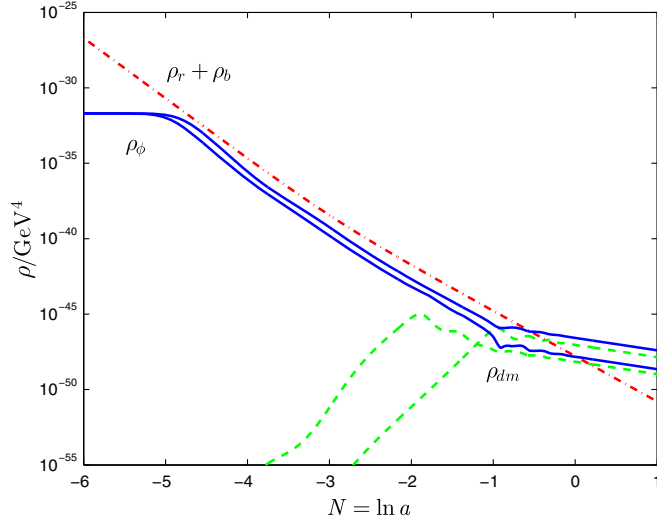$$w_{\text{eff}} = \frac{C_{11}}{\lambda_1 - C_{11}} \qquad (31)$$

FIG. 1 (color online). Evolution of energy densities for the sum of exponentials potential with $\lambda_1 = 10$, $\lambda_2 = 5.4$, $C_{11} = 90$, $C_{12} = -8$, $C_{21} = -63$, $C_{22} = -10$.
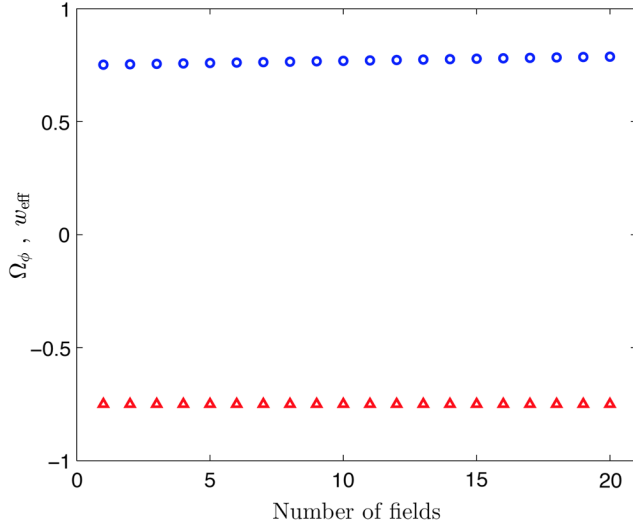

FIG. 2 (color online). Dependence of w_{eff} (triangles) and Ω_ϕ (circles) with the number of fields in the case when all fields are replicas of the same field and for the sum of exponentials potential. We used $\lambda_1 = 10$ and $C_{11} = -30$.

takes the same value as for a single field system; however,

$$\Omega_\phi = \frac{3n - \lambda_1 C_{11} + C_{11}^2}{(\lambda_1 - C_{11})^2} \qquad (32)$$

has a slight dependence on the number of fields n as illustrated in Fig. 2.

IV. EXPONENTIAL OF A SUM OF TERMS: $V(\phi_1,...,\phi_n) = M^4 e^{-\sum_i \kappa \lambda_i \phi_i}$

For this case we only need to define one single y such that

$$x_i \equiv \frac{\kappa\dot{\phi}_i}{\sqrt{6}H}, \qquad y^2 \equiv \frac{\kappa^2 V}{3H^2}, \qquad z_\alpha^2 \equiv \frac{\kappa^2\rho_\alpha}{3H^2}. \qquad (33)$$

The evolution is now described by

$$x_i' = -\left(3 + \frac{H'}{H}\right)x_i + \sqrt{\frac{3}{2}}\left(\lambda_i y^2 + \sum_\alpha C_{i\alpha} z_\alpha^2\right), \qquad (34)$$

$$y' = -\sqrt{\frac{3}{2}}\left(\sum_i \lambda_i x_i + \sqrt{\frac{2}{3}}\frac{H'}{H}\right)y, \qquad (35)$$

$$z_\alpha' = -\sqrt{\frac{3}{2}}\left(\sum_i C_{i\alpha} x_i + \sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}}\frac{H'}{H}\right)z_\alpha, \qquad (36)$$

$$\frac{H'}{H} = -\frac{3}{2}\left(1 + \sum_i x_i^2 - y^2\right), \qquad (37)$$

where the Friedmann equation now reads

$$\sum_i x_i^2 + y^2 + \sum_\alpha z_\alpha^2 = 1. \qquad (38)$$

A. Scalar field dominated solution

When the matter components are negligible ($z_\alpha = 0$), we can use Eqs. (35), (10) and (38) to obtain

$$x_i = \frac{\lambda_i}{\sqrt{6}}, \qquad (39)$$

and then the equation of state parameter reads

$$w_{\text{eff}} = -1 + \frac{1}{3}\lambda_{\text{eff}}, \qquad (40)$$

where the effective slope λ_{eff} is now

$$\lambda_{\text{eff}}^2 = \sum_i \lambda_i^2. \qquad (41)$$

For this case, increasing the number of fields increases the value of λ_{eff} and an accelerated expansion becomes more difficult to be attained. Again, this mimics previous results obtained in an assisted inflation setting in [3,4].

B. Scaling solution

In order to find the critical points corresponding to the scaling solution, when all the variables x_i, y and z_α are nonvanishing, we could proceed as for the previous potential. However, it is considerably easier to first start with a redefinition of variables. Let us observe that the evolution equations are invariant under an orthogonal transformation of x_i, λ_i and C_{ij}. That is,

preserve inner product. i.e angle and lenght of vector

$$\hat{x}_i = Q_{ij}x_j, \tag{42}$$

$$\hat{\lambda}_i = Q_{ij}\lambda_j, \tag{43}$$

$$\hat{C}_{ij} = Q_{il}C_{lj}, \tag{44}$$

where Q_{ij} is an orthogonal matrix, i.e., $Q_{il}Q_{lj}^T = Q_{il}Q_{jl} = \delta_{ij}$.

We will now give a working example for two fields and two dark matter components. We will show that, in the case of this potential it is always possible to rotate the fields such that $\hat{x}_2 = 0$. Looking for the case where all the variables $\hat{x}_i$, $\hat{y}$ and $\hat{z}_i$ are nonvanishing, from Eqs. (35) and (36), we obtain the conditions

$$(\hat{\lambda}_1 - \hat{C}_{11})\hat{x}_1 + (\hat{\lambda}_2 - \hat{C}_{21})\hat{x}_2 = \sqrt{\frac{3}{2}}, \tag{45}$$

$$(\hat{\lambda}_1 - \hat{C}_{12})\hat{x}_1 + (\hat{\lambda}_2 - \hat{C}_{22})\hat{x}_2 = \sqrt{\frac{3}{2}}. \tag{46}$$

It is easy to show that when $\hat{C}_{11} = \hat{C}_{12}$ the solution yields $\hat{x}_2 = 0$. Using the orthogonal transformation (43) with the condition $\hat{C}_{11} = \hat{C}_{12}$,

$$Q_{11}C_{11} + Q_{12}C_{21} = Q_{11}C_{12} + Q_{12}C_{22} \tag{47}$$

together with the constraint that the rows of Q are unit vectors, $Q_{11}^2 + Q_{12}^2 = 1$, gives us the result

$$Q_{11} = \frac{C_{22} - C_{21}}{\sqrt{(C_{11} - C_{12})^2 + (C_{22} - C_{21})^2}}, \tag{48}$$

$$Q_{12} = \frac{C_{11} - C_{12}}{\sqrt{(C_{11} - C_{12})^2 + (C_{22} - C_{21})^2}}, \tag{49}$$

where we have chosen the positive roots. Since $\hat{x}_2 = 0$ we do not need to compute the rest of the matrix Q. We can now obtain the effective coupling,

$$C_{\text{eff}} = \hat{C}_{11} = Q_{11}C_{11} + Q_{12}C_{21} \tag{50}$$

$$= \frac{C_{22}C_{11} - C_{21}C_{12}}{\sqrt{(C_{11} - C_{12})^2 + (C_{22} - C_{21})^2}}, \tag{51}$$

and the effective λ_{eff},

$$\lambda_{\text{eff}} = \hat{\lambda}_1 = Q_{11}\lambda_1 + Q_{12}\lambda_2 \tag{52}$$

$$= \frac{(C_{22} - C_{21})\lambda_1 + (C_{11} - C_{12})\lambda_2}{\sqrt{(C_{11} - C_{12})^2 + (C_{22} - C_{21})^2}}. \tag{53}$$

The scaling solution can then be written as

$$\hat{x}_1 = \sqrt{\frac{3}{2}}\frac{1}{\lambda_{\text{eff}} - C_{\text{eff}}}, \tag{54}$$

$$\hat{x}_2 = 0. \tag{55}$$

We can then compute the effective equation of state parameter and the total scalar field contribution from the expressions

$$w_{\text{eff}} = -1 + \sqrt{\frac{2}{3}}(\hat{\lambda}_1\hat{x}_1 + \hat{\lambda}_2\hat{x}_2) = \frac{C_{\text{eff}}}{\lambda_{\text{eff}} - C_{\text{eff}}}, \tag{56}$$

$$\Omega_\phi = 1 + 2(\hat{x}_1^2 + \hat{x}_2^2) - \sqrt{\frac{2}{3}}(\hat{\lambda}_1\hat{x}_1 + \hat{\lambda}_2\hat{x}_2)$$

$$= \frac{3 - \lambda_{\text{eff}}C_{\text{eff}} + C_{\text{eff}}^2}{(\lambda_{\text{eff}} - C_{\text{eff}})^2}. \tag{57}$$

It is straightforward to extend these results to the case of n scalar fields and n dark matter components. We can have $\hat{x}_i = 0$ for $i \geq 2$ provided that

$$\hat{C}_{ii} = \hat{C}_{ij} \quad \text{for } j \geq i. \tag{58}$$

This can always be achieved through the orthogonal transformation Eqs. (42)–(44). Again we only really need to obtain the first row Q_{1j}, of the Q matrix to obtain the value of the effective coupling C_{eff}. This comes through the solution to the equation

$$C_{\text{eff}} = \hat{C}_{1i} = \sum_j Q_{1j}C_{j1}, \tag{59}$$

for any i, together with the unitarity condition, $Q_{1i}^2 = 1$. Once we have Q_{1i} we can obtain the effective slope λ_{eff}

$$\lambda_{\text{eff}} = \hat{\lambda}_1 = \sum_j Q_{1j}\lambda_j, \tag{60}$$

and $\hat{x}_1$, Ω_ϕ and w_{eff} will be given by the same Eqs. (54)–(57).

To see how this would work, let us look at the simple case of a diagonal coupling matrix C. In this case, we get for Q_{1i}

$$Q_{1i} = \frac{1}{C_{ii}\sqrt{\sum_l 1/C_{ll}^2}}, \tag{61}$$

such that

$$\frac{1}{C_{\text{eff}}^2} = \sum_i \frac{1}{C_{ii}^2}, \tag{62}$$

$$\lambda_{\text{eff}} = C_{\text{eff}}\sum_i \frac{\lambda_i}{C_{ii}}. \tag{63}$$

When all the fields are a copy of field ϕ_1, then the expressions become fairly simple and give $C_{\text{eff}} = C_{11}/\sqrt{n}$
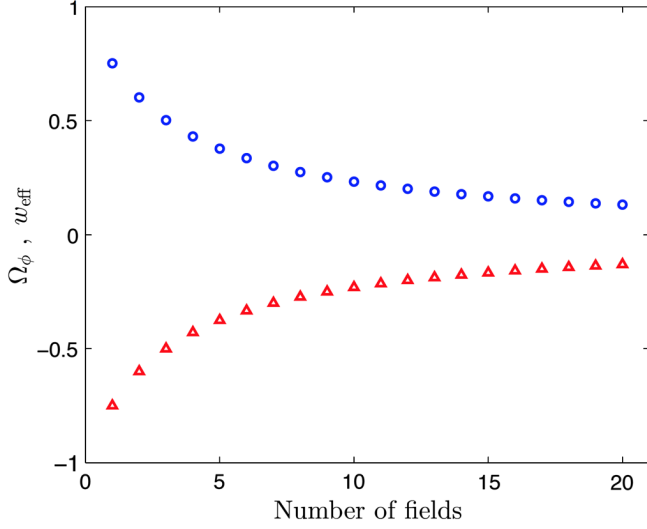
FIG. 3 (color online). Dependence of w_{eff} (triangles) and Ω_ϕ (circles) with the number of fields in the case when all fields are replicas of the same field and for the exponential of the sum of fields potential. We used $\lambda_1 = 10$ and $C_{11} = -30$.

and $\lambda_{\text{eff}} = \sqrt{n}\lambda_1$. For this potential both w_{eff} and Ω_ϕ have a strong dependence on the number of fields,

$$w_{\text{eff}} = \frac{C_{11}}{n\lambda_1 - C_{11}}, \tag{64}$$

$$\Omega_\phi = \frac{3n - C_{11}\lambda_1 n + C_{11}^2}{(n\lambda_1 - C_{11})^2}, \tag{65}$$

which is illustrated in Fig. 3 for $\lambda_1 = 10$ and $C_{11} = -30$.

V. SCALAR POTENTIAL INDEPENDENT SOLUTIONS

We can find fixed point solutions where the scalar potential energy density is negligible, so these solutions are independent of the type of scalar potential used. These can be divided into three different types.

A. Subdominant potential solution

This critical point corresponds to the case where the scalar potential is negligible and the total energy density is composed of the scalar fields kinetic energies and the dark matter components. We can use either Eq. (9) or Eq. (36) and the Friedmann constraint to obtain

$$x_i = \sqrt{\frac{2}{3}} C_{i\alpha}, \tag{66}$$

for any α. This immediately gives the simple expression for the equation of state parameter and the total scalar field contribution

$$w_{\text{eff}} = \Omega_\phi = \sum_i x_i^2 = \frac{2}{3}\sum_i C_{i\alpha}^2. \tag{67}$$

B. Kinetic dominated solution

This critical point occurs when only the kinetic energy of the fields is nonvanishing. It is immediately obtained that $\sum_i x_i^2 = 1$ and

$$w_{\text{eff}} = \Omega_\phi = 1. \tag{68}$$

C. Matter dominated solution

The matter dominated solution is the fixed point characterized by x_i, $y_i = 0$ (or $y = 0$) and

$$\sum_\alpha C_{i\alpha} z_\alpha^2 = 0, \tag{69}$$

which essentially means that the partial derivative with respect to the field ϕ_i of the sum of all the dark matter contributions must vanish. This solution was discussed in a number of recent publications for a single field system and two dark matter components with symmetric couplings [46–49]. For the simple system of two fields and two dark matter components we obtain that the field must settle at the bottom of a valley defined by the flat direction in the ϕ_1–ϕ_2 plane,

$$(C_{11} - C_{12})(\phi_1 - \phi_{10}) + (C_{21} - C_{22})(\phi_2 - \phi_{20})$$
$$= \frac{1}{\kappa}\ln\left(-\frac{C_{11}\rho_{10}}{C_{12}\rho_{20}}\right)$$
$$= \frac{1}{\kappa}\ln\left(-\frac{C_{21}\rho_{10}}{C_{22}\rho_{20}}\right). \tag{70}$$
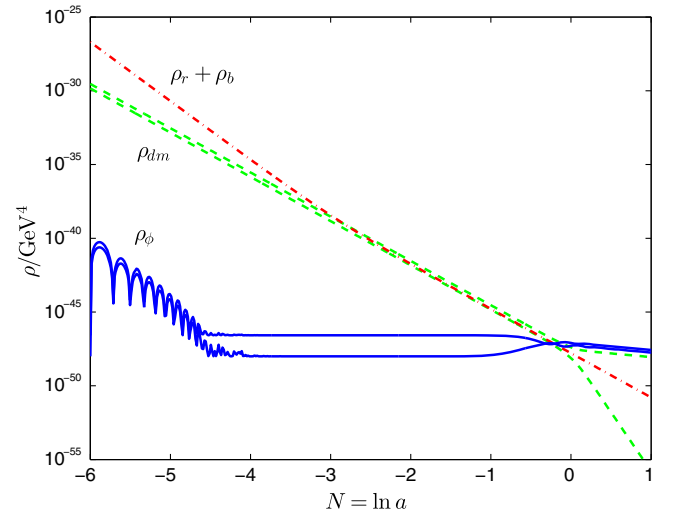

FIG. 4 (color online). Example of a matter dominated solution followed by a scaling solution by only one of the dark matter components. Evolution of energy densities for the sum of exponentials potential with $\lambda_1 = 10$, $\lambda_2 = 10$, $C_{11} = -20$, $C_{12} = 40$, $C_{21} = -30$, $C_{22} = 60$. The initial condition for the scalar fields is very near the bottom of the effective potential which causes the initial oscillations before $N = -4$.

For consistency, we observe that the couplings must satisfy the simple relation

$$\frac{C_{11}}{C_{12}} = \frac{C_{21}}{C_{22}};$$

(71)

otherwise the flat direction is nonexistent and the field will keep on evolving.

This solution is necessarily unstable. As the dark matter contribution decays with e^{-3N}, eventually the scaling solution or the scalar field dominated solution becomes more important. Such an example is illustrated in Fig. 4. This critical point allows the dark matter to have a substantial contribution before the scaling regime and before the Universe accelerates.

VI. LINEAR DENSITY PERTURBATIONS

We now extend our analysis to linear perturbations. This would allow us to test the multicoupled model with current and future data on galaxy clustering. We perturb the flat-space metric as

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)\eta_{ij}dx^i dx^j,$$

(72)

where Φ, Ψ are functions of space and time, and we expand the fields as $\phi_i \to \phi_i + \delta\phi_i$.

The energy-momentum tensor for the various fields can be written as

$$T^\phi_{\mu\nu} = \sum_i \partial_\mu \phi_i \partial_\nu \phi_i$$
$$+ g_{\mu\nu}\left(-\frac{1}{2}g^{\rho\sigma}\sum_i \partial_\rho \phi_i \partial_\sigma \phi_i - V(\phi_1,...,\phi_n)\right),$$

(73)

and for the component α of dark matter as

$$T^{\mathrm{dm}(\alpha)}_{\mu\nu} = \sum_\alpha \rho_\alpha u_\mu^{(\alpha)} u_\nu^{(\alpha)},$$

(74)

where $u_\mu^{(\alpha)}$ is the four-velocity of component α of the matter fluid. The conservation equations then read

$$\nabla_\mu T^{\phi\,\mu}_{\ \ \nu} = -\kappa \sum_i \left(\sum_\alpha C_{i\alpha}\rho_\alpha\right)\nabla_\nu \phi_i,$$

(75)

$$\nabla_\mu T^{\mathrm{dm}(\alpha)\,\mu}_{\ \ \ \ \nu} = \kappa\left(\sum_i C_{i\alpha}\nabla_\nu \phi_i\right)\rho_\alpha.$$

(76)

We want to obtain the equations for the linear perturbations around the background values for the fields $\bar{\phi}_i$ and for the matter components $\bar{\rho}_\alpha$. We define the field perturbations as $\delta\phi_i = \phi_i - \bar{\phi}_i$ and the matter density contrasts as $\delta_\alpha = (\rho_\alpha - \bar{\rho}_\alpha)/\bar{\rho}_\alpha$.

Going to Fourier space, we obtain the following equations of motion for the ϕ_i field perturbations of wave number k:

$$\delta\ddot{\phi}_i + 3H\delta\dot{\phi}_i + \sum_j V_{,\phi_i\phi_j}\delta\phi_j + \frac{k^2}{a^2}\delta\phi_i + 2V_{,\phi_i}\Phi - 4\dot{\phi}_i\dot{\Phi}$$
$$- 2\kappa\sum_\alpha C_{i\alpha}\rho_\alpha\Phi - \kappa\sum_\alpha C_{i\alpha}\rho_\alpha\delta_\alpha = 0,$$

(77)

and for the density contrast of the matter component α,

$$\dot{\delta}_\alpha + \frac{\theta_\alpha}{a} - 3\dot{\Phi} + \kappa\sum_i C_{i\alpha}\delta\dot{\phi}_i = 0,$$

(78)

where $\theta_\alpha = \vec{\nabla}\cdot\vec{v}_\alpha$ and $\vec{v}_\alpha$ is the velocity of the α component in the reference frame of the homogeneous dark matter fluid.

By differentiation we then get

$$\ddot{\delta}_\alpha + 3\ddot{\Phi} + \kappa\sum_i C_{i\alpha}\delta\ddot{\phi}_i + \frac{k^2}{a^2}\left(\Phi - \sum_i C_{i\alpha}\delta\phi_i\right)$$
$$+ \left(2H - \kappa\sum_i C_{i\alpha}\dot{\phi}_i\right)\left(\dot{\delta}_\alpha - 3\dot{\Phi} - \kappa\sum_i C_{i\alpha}\delta\dot{\phi}_i\right) = 0.$$

(79)

The ij component of Einstein's equations for $i \neq j$ yields

$$\Psi = \Phi,$$

(80)

and using this equality from now on, the 00 component gives

$$3H(\dot{\Phi} + H\Phi) + \frac{1}{2}\kappa^2\sum_i (\dot{\phi}_i\delta\dot{\phi}_i + V_{,\phi_i}\delta\phi_i - \dot{\phi}_i^2\Phi)$$
$$+ \frac{1}{2}\kappa^2\sum_\alpha \rho_\alpha\delta_\alpha + \frac{k^2}{a^2}\Phi = 0.$$

(81)

We now consider the limit of subhorizon scales, such that $(k/a)^2 \gg H^2$. However, they have to be large enough to be evolving in the linear regime. The regime of validity is therefore approximately from horizon scales down to some tens of Mpc at the present. Combining Eqs. (79) and (81), it turns out that the equation of motion for the linear matter perturbations is

$$\ddot{\delta}_\alpha + 2H\left(1 - \frac{1}{2}\kappa\sum_i C_{i\alpha}\frac{\dot{\phi}_i}{H}\right)\dot{\delta}_\alpha$$
$$- \frac{1}{2}\kappa^2\sum_\beta\left(1 + \sum_i C_{i\alpha}C_{i\beta}\right)\rho_\beta\delta_\beta = 0.$$

(82)

Using a time variable, $N = \ln a$, we can rewrite these equations of motion in the form

$$\delta_\alpha'' + \left(2 - \frac{3}{2}\sum_\beta \Omega_\beta - \sum_i \left(\frac{1}{2}\kappa^2 \phi_i'^2 + C_{i\alpha}\kappa\phi_i'\right)\right)\delta_\alpha'$$

$$- \frac{3}{2}\sum_\beta \left(1 + \sum_i C_{i\alpha}C_{i\beta}\right)\Omega_\beta \delta_\beta = 0, \tag{83}$$

which can also be written, using the definition of x_i, as

$$\delta_\alpha'' + \left(2 - \frac{3}{2}\sum_\beta \Omega_\beta - \sum_i (3x_i^2 + \sqrt{6}C_{i\alpha}x_i)\right)\delta_\alpha'$$

$$- \frac{3}{2}\sum_\beta \left(1 + \sum_i C_{i\alpha}C_{i\beta}\right)\Omega_\beta \delta_\beta = 0. \tag{84}$$

It should be clear that for a given α component a too large positive or negative $\sum_i C_{i\alpha}C_{i\beta}$ might imply, respectively, a very strong growth or damping of δ_α, a situation that must be avoided.

For the case of two scalar fields coupling with two components of dark matter, the equations for the matter linear perturbations become

$$\delta_1'' + \left(2 - \frac{3}{2}(\Omega_1 + \Omega_2) - 3(x_1^2 + x_2^2) - \sqrt{6}(C_{11}x_1 + C_{21}x_2)\right)\delta_1'$$

$$- \frac{3}{2}(1 + C_{11}^2 + C_{12}^2)\Omega_1 \delta_1$$

$$- \frac{3}{2}(1 + C_{11}C_{12} + C_{21}C_{22})\Omega_2 \delta_2 = 0 \tag{85}$$

and

$$\delta_2'' + \left(2 - \frac{3}{2}(\Omega_1 + \Omega_2) - 3(x_1^2 + x_2^2) - \sqrt{6}(C_{12}x_1 + C_{22}x_2)\right)\delta_1'$$

$$- \frac{3}{2}(1 + C_{12}C_{11} + C_{22}C_{21})\Omega_1 \delta_1$$

$$- \frac{3}{2}(1 + C_{12}^2 + C_{22}^2)\Omega_2 \delta_2 = 0. \tag{86}$$

That is, for a scaling solution they are of the form

$$\delta_1'' + A_1\delta_1' - B_1\delta_1 - B_2\delta_2 = 0, \tag{87}$$

$$\delta_2'' + A_2\delta_2' - C_1\delta_1 - C_2\delta_2 = 0, \tag{88}$$

where A_j, B_j and C_j are all constants.

The solutions for these coupled equations can be written as $\delta_1 \propto e^{\xi N} \propto a^\xi$ and $\delta_2 = b\delta_1$; however, the analytical relations between ξ and b in terms of the A_j, B_j, C_j are too complicated to be of any practical use. In the cases when $b \ll B_1/B_2$ and $b \ll C_1/C_2$ we can substitute the solutions in the above equations and obtain a set of equations to estimate ξ and b given by

$$\xi^2 + A_1\xi - B_1 \approx 0, \tag{89}$$

$$\xi^2 + A_2\xi - C_1/b \approx 0, \tag{90}$$

which give the analytical results

$$\xi \approx -\frac{1}{2}\left(A_1 \pm \sqrt{A_1^2 + 4B_1}\right), \tag{91}$$

$$b \approx \frac{1}{2}\frac{C_1}{B_1^2 + B_1 A_2(A_1 - A_2)}$$

$$\times \left[2B_1 + (A_1 - A_2)(A_1 \pm \sqrt{A_1^2 + 4B_1})\right]. \tag{92}$$

Of course this approximation is only reliable when the assumptions $b \ll B_1/B_2, C_1/C_2$ are indeed verified. This happens for instance when $\Omega_2 \ll \Omega_1$, i.e. when one of the dark matter components is much smaller than the other.

The growth rate $f \equiv d\log\delta/d\log a = \xi$ can then be directly compared to observations. This will be performed in future work.

VII. CONCLUSIONS

We have studied a cosmological system composed of a set of scalar fields coupled to an ensemble of dark matter components, thus generalizing previous work. The obtained solution can either be applied to construct early Universe inflationary solutions or provide a mechanism to explain the current accelerated expansion of the Universe and the ratio of abundances between dark energy and dark matter. More specifically, we have investigated two representative types of potential leading to analytical solutions. We have seen that the scalar field dominated solution allows for an inflationary evolution which is easier to attain with the sum of exponentials potential. The effective coupling of the scaling solution has an explicit dependence on the value of the coupling $C_{i\alpha}$ and the potential parameters, λ_i, for the sum of exponentials potential. For the exponential of a sum potential, however, the effective coupling can be written solely in terms of the individual couplings.

The parameter values required to give a realistic dynamics of the Universe are comparable to the interval allowed in the original coupled quintessence model. This means $\lambda_i = \mathcal{O}(10)$ and $C_{i\alpha} = \mathcal{O}(-50)$. Density perturbations, however, may constrain the $C_{i\alpha}$ couplings to be small as discussed in Sec. VI. We believe this qualifies the value of the parameters to be considered as natural given that they are not many orders of magnitude away from unity. The coincidence problem is neither ameliorated nor aggravated in this multifield scenario. However, because of the use of scaling solutions, i.e., the presence of a stable attractor to which the solutions converge at late times, the initial conditions problem is alleviated when compared to the cosmological constant scenario.

In this work we have ignored any loop effects arising from the coupling of the scalar fields to the dark matter components. These could spoil the flatness of the potentials, and thus, the potentials here studied must be viewed

instead as effective potentials already taking into account quantum corrections.

When studying solutions independent of the form of the scalar potential, we found a relation between the value of the couplings which provides an early dust like dominated behavior. Essentially, the fields must settle at the bottom of the effective potential which has a flat direction. Finally, we observed that the equations of motion for the matter density contrasts possess a source or damping term which might lead to an unacceptable growth or damping of the density contrasts. It would be interesting to carry out a numerical analysis to test the range of parameter space for which these models are compatible with current and forecasted future large scale structure data.

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