$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$$

$$ds^2 = e^{\nu} c^2 dt^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) - e^{\lambda} dr^2$$

$g^{0\sigma} = 0$ for $\sigma \in [1,3]$

$$\begin{array}{l|l} g_{00} = e^{\nu} & g^{00} = e^{-\nu} \\ g_{11} = -e^{\lambda} & g^{11} = -e^{-\lambda} \\ g_{22} = -r^2 & g^{22} = -r^{-2} \\ g_{33} = -r^2\sin^2\theta & g^{33} = -r^{-2}\sin^{-2}\theta \end{array}$$

$$\Gamma^0_{00} = \sum_\sigma \frac{1}{2} g^{0\sigma}(\partial_0 g_{\sigma 0} + \partial_0 g_{\sigma 0} - \partial_\sigma g_{00})$$

$\because \nu \,\&\, \lambda \to F(r,t)$
No θ & ϕ dependence.

$$= \frac{1}{2} g^{00}(\partial_0 g_{00} + \cancel{\partial_0 g_{00}} - \cancel{\partial_0 g_{00}})$$

$$= \frac{1}{2} e^{-\nu}\left(\frac{\partial}{\partial t} e^{+\nu}\right) = \frac{1}{2} \cancel{e^{-\nu}}\dot{\nu}\cancel{e^{\nu}} = \frac{\dot{\nu}}{2}$$

$\cancel{\boxed{\Gamma^0_{00} = 0}}$ $\boxed{\Gamma^0_{00} = \frac{\dot{\nu}}{2}}$

$$\Gamma^0_{01} = \frac{1}{2} g^{0\sigma}[\partial_0 g_{\sigma 1} + \partial_1 g_{\sigma 0} - \partial_\sigma g_{01}]$$

$\sigma = 0$, $\because g^{0\sigma} = 0 \,|_{\sigma \in [1,3]}$

$$= \frac{1}{2} g^{00}[\cancel{\partial_0 g_{01}}^{0} + \partial_1 g_{00} - \cancel{\partial_0 g_{01}}^{0}]$$

$$= \frac{1}{2} e^{-\nu}[\cancel{\partial_t}\, \partial_r e^{\nu}]$$

$$= \frac{1}{2} \cancel{e^{-\nu}}\, \cancel{e^{\nu}}\, \nu'$$

$$\boxed{\Gamma^0_{01} = \frac{\nu'}{2} = \Gamma^0_{10}}$$

$\sigma = 0$, $\sigma \neq 1,2,3$

$$\Gamma^0_{11} = \frac{1}{2}\sum_\sigma g^{0\sigma}(\partial_1 g_{\sigma 1} + \partial_1 g_{\sigma 1} - \partial_\sigma g_{11})$$

$$= \frac{1}{2} g^{00}(\cancel{\partial_1 g_{01}}^{0} + \cancel{\partial_1 g_{01}}^{0} - \partial_0 g_{11})$$

$$= \frac{1}{2} e^{-\nu}(+\partial_t(+e^{\lambda}))$$

$$\Gamma^0_{11} = \frac{1}{2} e^{-\nu} e^{\lambda} \dot{\lambda}$$

$$\boxed{\Gamma^0_{11} = \frac{\dot{\lambda}}{2} e^{(\lambda-\nu)}}$$ * wrong in thesis (haven't put derivatives)

$\sigma = 1$

$$\Gamma^1_{11} = \frac{1}{2} g^{1\sigma}[\partial_1 g_{\sigma 1} + \partial_1 g_{\sigma 1} - \partial_\sigma g_{11})$$

$$= \frac{1}{2} g^{11}[\partial_1 g_{11} + \cancel{\partial_1 g_{11}} - \cancel{\partial_1 g_{11}})$$

$$= \frac{1}{2} (+e^{-\lambda})\left[\frac{\partial}{\partial r}(+e^{\lambda})\right]$$

$$= \frac{1}{2} \cancel{e^{-\lambda}}\, \cancel{e^{\lambda}}\, \lambda'$$

$$\boxed{\Gamma^1_{11} = \frac{\lambda'}{2}}$$

$$\Gamma^1_{01} = \frac{1}{2} g^{1\sigma}[\partial_0 g_{\sigma 1} + \partial_1 g_{\sigma 0} - \partial_\sigma g_{01}]$$

$$= \frac{1}{2} g^{11}[\partial_0 g_{11} + \cancel{\partial_1 g_{10}}^{\,0} - \cancel{\partial_1 g_{01}}]$$

$$= \frac{1}{2} + e^{-\lambda}\left[\frac{\partial}{\partial t} + e^{+\lambda}\right]$$

$$= \frac{1}{2} \cancel{e^{-\lambda}}\, \cancel{e^{\lambda}}\, \dot{\lambda}$$ * wrong in Thesis

$$\boxed{\Gamma^1_{01} = \frac{\dot{\lambda}}{2} = \Gamma^1_{10}}$$

$\sigma = 1$

$$\Gamma^1_{00} = \frac{1}{2} g^{1\sigma}(\partial_0 g_{\sigma 0} + \partial_0 g_{\sigma 0} - \partial_\sigma g_{00})$$

$$= \frac{1}{2} g^{11}(\partial_0 \cancel{g_{10}}^{\,0} + \partial_0 \cancel{g_{10}}^{\,0} - \partial_1 g_{00})$$

$$= \frac{1}{2} + e^{-\lambda}[+\partial_r e^{\nu}]$$

$$= \frac{1}{2} e^{-\lambda} e^{\nu} \nu'$$

$$\boxed{\Gamma^1_{00} = \frac{\nu'}{2} e^{(\nu-\lambda)}}$$

$\cancel{\Gamma^{r}_{20} = \frac{1}{2} g^{r\sigma}[\partial}$ $\quad \Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$

$\sigma = 0$

$$\Gamma^{0}_{20} = \frac{1}{2} g^{00}[\partial_2 g_{00} + \partial_0 \cancel{g_{02}}^{0} - \partial_0 \cancel{g_{20}}^{0}]$$

$$= \frac{1}{2} g^{00}[\partial_2 g_{00}]$$

$$\boxed{\Gamma^{0}_{20} = \frac{1}{2} g^{00}[\partial_\theta e^{\nu}] = 0 = \Gamma^{0}_{02}}$$

$\sigma = 0$

$$\boxed{\Gamma^{0}_{22} = \frac{1}{2} g^{00}[\partial_2 \cancel{g_{02}}^{0} + \partial_2 \cancel{g_{02}}^{0} - \partial_0 \cancel{g_{22}}^{0}] =}$$

$\sigma = 0$

$$\Gamma^{0}_{30} = \frac{1}{2} g^{00}[\partial_3 g_{00} + \partial_0 \cancel{g_{03}}^{0} - \partial_0 \cancel{g_{30}}^{0}]$$

$$= \frac{1}{2} g^{00}[\partial_\phi g_{00}]$$

$$= \frac{1}{2} g^{00}[\partial_\phi e^{\nu}]$$

$$\boxed{\Gamma^{0}_{30} = \Gamma^{0}_{03} = 0}$$

$\sigma = 0$

$$\Gamma^{0}_{33} = \frac{1}{2} g^{00}[\partial_3 \cancel{g_{03}}^{0} + \partial_3 \cancel{g_{03}}^{0} - \partial_0 g_{33}]$$

$$= \frac{1}{2} g^{00}[\cancel{\partial_\phi} - \partial_t g_{33}]$$

$$= \frac{1}{2} e^{-\nu}[-\partial_t r^2 \sin^2\theta]$$

$$\boxed{\Gamma^{0}_{33} = 0}$$

$\sigma = 2$

$$\Gamma^{2}_{12} = \frac{1}{2} g^{2\sigma}(\partial_1 g_{\sigma 2} + \partial_2 \cancel{g_{\sigma 1}}^{0} - \partial_\sigma \cancel{g_{12}}^{0})$$

$$= \frac{1}{2} g^{22}(\partial_1 g_{22})\ \cancel{r g_2}$$

$$= \frac{1}{2} \cancel{r}\frac{1}{r^2}(\partial_r(\cancel{r}\, r^2))$$

$$\boxed{\Gamma^{2}_{12} = \frac{1}{\cancel{2}} \frac{1}{r^{\cancel{2}}} 2r = \frac{1}{r}}$$

$$\Gamma^3_{13} = \frac{1}{2} g^{3\sigma}\left(\partial_1 g_{\sigma 3} + \cancel{\partial_3 g_{\sigma 1}}^{\,0} - \cancel{\partial_\sigma g_{13}}^{\,0}\right) \qquad \sigma = 3$$

$$= \frac{1}{2} g^{33}\left(\partial_1 g_{33}\right)$$

$$= \frac{1}{2} \frac{1}{r^2 \sin^2\theta} \frac{\partial}{\partial r}\left(r^2 \sin^2\theta\right)$$

$$= \frac{1}{2} \frac{1}{r^2 \sin^2\theta} \sin^2\theta \; 2r$$

$$\boxed{\Gamma^3_{13} = \frac{1}{r}}$$

$\sigma = 1$

$$\Gamma^1_{22} = \frac{1}{2} g^{1\sigma}\left[\cancel{\partial_2 g_{\sigma 2}}^{\,0} + \cancel{\partial_2 g_{\sigma 2}}^{\,0} - \partial_\sigma g_{22}\right]$$

$$= \frac{1}{2} g^{11}\left(-\partial_1 g_{22}\right)$$

$$= \frac{1}{2}\left(e^{-\lambda}\right)\left(\partial_r\left(-r^2\right)\right)$$

$$= \frac{1}{2} e^{-\lambda} \, -r \, 2$$

$$\boxed{\Gamma^1_{22} = -r e^{-\lambda}}$$

$\sigma = 2$

$$\Gamma^2_{33} = \frac{1}{2} g^{2\sigma}\left(\cancel{\partial_3 g_{\sigma 3}}^{\,0} + \cancel{\partial_3 g_{\sigma 3}}^{\,0} - \partial_\sigma g_{33}\right)$$

$$= \frac{1}{2} g^{22}\left(-\partial_2 g_{33}\right)$$

$$= \frac{1}{2}\left(\frac{1}{r^2}\right)\left(\partial_\theta\left(-r^2 \sin^2\theta\right)\right)$$

$$= \frac{1}{2}\frac{1}{r^2}\left(-r^2\right) 2\sin\theta\cos\theta$$

$$\boxed{\Gamma^2_{33} = -\sin\theta\cos\theta}$$

$\sigma = 3$

$$\Gamma^3_{23} = \frac{1}{2} g^{3\sigma}\left(\partial_2 g_{\sigma 3} + \cancel{\partial_3 g_{\sigma 2}}^{\,0} - \cancel{\partial_\sigma g_{23}}^{\,0}\right)$$

$$= \frac{1}{2} g^{33}\left(\partial_\theta g_{33}\right)$$

$$= \frac{1}{2}\left(\frac{1}{r^2\sin^2\theta}\right)\left(\frac{\partial}{\partial\theta}\left(r^2 \sin^2\theta\right)\right)$$

$$= \frac{1}{2}\frac{1}{\sin^2\theta} \, 2\sin\theta\cos\theta$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$\boxed{\Gamma^3_{23} = \cot\theta}$$

$\sigma = 1$

$$\Gamma^1_{33} = \frac{1}{2} g^{1\sigma}\left(\partial_3 g_{\sigma 3} + \partial_3 g_{\sigma 3} - \partial_\sigma g_{33}\right)$$

$$= \frac{1}{2} g^{11}\left(\cancel{\partial_3 g_{13}}^{\,0} + \cancel{\partial_3 g_{13}}^{\,0} - \partial_1 g_{33}\right)$$

$$= \frac{1}{2} \, -e^{-\lambda}\left(\partial_r \, r^2 \sin^2\theta\right)$$

$$= -\frac{1}{2} e^{-\lambda} \, 2r \sin^2\theta$$

$$\boxed{\Gamma^1_{33} = -r\sin^2\theta \, e^{-\lambda}}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Ricci tensor   Ricci scalar

$$R_{\mu\nu} = R^{\alpha}_{\ \mu\alpha\nu} = \partial_\alpha \Gamma^{\alpha}_{\mu\nu} - \partial_\nu \Gamma^{\alpha}_{\mu\alpha} + \Gamma^{\alpha}_{\mu\nu} \Gamma^{\beta}_{\alpha\beta} - \Gamma^{\alpha}_{\mu\beta} \Gamma^{\beta}_{\nu\alpha}$$

$$\frac{8\pi K}{c^4} T^1_1 = -e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2}$$

For free space, stress tensor has zero value

$$e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = 0 \quad -(3.7) \qquad \lambda = 0 \quad -(3.9)$$

$$e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = 0 \quad -(3.8)$$

$$\frac{\nu'}{r} + \frac{1}{r^2} = \frac{e^\lambda}{r^2}$$

$$\boxed{\nu' = \frac{e^\lambda}{r} - \frac{1}{r}}$$

$$\boxed{\nu' = \frac{e^\lambda - 1}{r}} \quad -(1)$$

$$\frac{\lambda'}{r} - \frac{1}{r^2} = -\frac{e^\lambda}{r^2}$$

$$\lambda' = \frac{1}{r} - \frac{e^{\lambda}}{r}$$

$$\boxed{\lambda' = \frac{1-e^\lambda}{r}} \quad -(2)$$

$$\frac{\partial \lambda}{\partial r} = \frac{(1-e^\lambda)}{r}$$

at

$$\int \frac{d\lambda}{(1-e^\lambda)} = \int \frac{dr}{r}$$

$$\lambda - \ln(|e^\lambda - 1|) = \ln(r)$$

$$e^{-\lambda}\left[\frac{\nu'}{r} + \cancel{\frac{1}{r^2}} + \frac{\lambda'}{r} - \cancel{\frac{1}{r^2}}\right] = 0$$

$$\frac{\nu'}{\cancel{r}} = -\frac{\lambda'}{\cancel{r}} \Rightarrow \boxed{\nu' = -\lambda'} \Rightarrow$$

$$\frac{\partial(\gamma)}{\partial r} = -\frac{\partial(\lambda)}{\partial r}$$

Is this valid.

$$\boxed{\gamma = -\lambda}$$

$$\boxed{e^{\gamma} = e^{-\lambda}}$$

$$\lambda = \ln|r| + \ln(|e^{\lambda}-1|)$$

$$\lambda = \ln\left(\frac{e^{\lambda}-1}{r}\right)$$

$$e^{\lambda} = \frac{e^{\lambda}-1}{r}$$

$$-re^{\lambda} + e^{\lambda} = +1$$

$$e^{\lambda}(1-r) = 1$$

$$e^{\lambda} = \frac{1}{1-r}$$

$$\boxed{\lambda = \ln\left(\frac{1}{1-r}\right)}$$

$$ds^2 = \left(1 - \frac{r_s}{r}\right)c^2dt^2 - r^2(\sin^2\theta\, d\phi^2 + d\theta^2) - \frac{dr^2}{1-\frac{r_s}{r}}$$

$$e^{-\lambda}\left(\frac{\gamma' r_h}{r_h^2 R} + \frac{1}{r_h^2 R^2}\right) - \frac{1}{r_h^2 R^2} = 0$$

$R = \frac{r}{r_h}$

$$e^{-\lambda}\left(\frac{\gamma' r_h}{R} + \frac{1}{R^2}\right) - \frac{1}{R^2} = 0$$

~~This is still not dim less $\because r_h$ has units of length.~~

$\boxed{r = R r_h}$

$\gamma' \cancel{\to} \gamma'$
with parameter

$$\gamma' \Rightarrow \frac{\partial \gamma}{\partial r}$$

$$\gamma' \Rightarrow \frac{\partial \gamma}{r_h \partial h}$$

$$\boxed{\gamma' \Rightarrow \frac{\gamma'}{r_h}}$$

$$\int_{r=r_h}^{\infty} (\{ \}) \, dR$$

$$e^{-\lambda}\left(\frac{\nu'}{R}+\frac{1}{R^2}\right)-\frac{1}{R^2}=0 \Rightarrow e^{-\lambda}\left(\frac{\partial \nu}{\partial R}\frac{1}{R}+\frac{1}{R^2}\right)-\frac{1}{R^2}=0$$

$$e^{-\lambda}\left(\frac{\lambda'}{R}-\frac{1}{R^2}\right)+\frac{1}{R^2}=0 \Rightarrow e^{-\lambda}\left(\frac{\partial \lambda}{\partial R}\frac{1}{R}-\frac{1}{R^2}\right)+\frac{1}{R^2}=0$$

$$\boxed{\begin{aligned}\nu' &= \frac{1}{R}(1-e^{-\lambda})\\ \lambda' &= \frac{1}{R}(e^{-\lambda}-1)\end{aligned}}$$

$$\frac{\nu'}{R}+\frac{1}{R^2}=\frac{e^{\lambda}}{R^2} \qquad\qquad \frac{\lambda'}{R}-\frac{1}{R^2}=-\frac{e^{\lambda}}{R^2}$$

$$\nu'=\frac{e^{\lambda}-1}{R} \qquad\qquad \boxed{\lambda'=\frac{1-e^{\lambda}}{R}}$$

$\therefore$ λ & ν are $-$vely co-related, their initial conditions will be opposite.