SREE VIDYANIKETHAN ENGINEERING COLLEGE

(An Autonomous Institution, Affiliated to JNTUA, Ananthapuramu)

II B.Tech I Semester (SVEC-16) Regular/Supplementary Examinations November - 2019

SPECIAL FUNCTIONS AND COMPLEX ANALYSIS

[Electrical and Electronics Engineering, Electronics and Communication Engineering, Electronics and Instrumentation Engineering]

Time: 3 hours **Max. Marks: 70**

Answer One Question from each Unit
All questions carry equal marks

UNIT-I

1	a)	Establish the relation between Beta and Gamma functions.	CO1 CO4	7 Marks
	b)	Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$.	CO4	7 Marks
		(OR)		
2	a)	Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.	CO4	7 Marks
	b)	Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.	CO4	7 Marks

UNIT-II

3	a)	If f(z) is an analytic function with constant modulus, show that f(z) is constant.	CO4	7 Marks
	b)	Find the analytic function, whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.	CO4	7 Marks
		(OR)		
4	a)	Show that polar form of Cauchy Riemann equations are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. Hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.	CO1 CO4	7 Marks
	b)	If *f(z)* is a regular function of *z*, Prove that $\nabla^2 \lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$.	CO4	7 Marks

UNIT-III

5	a)	State and prove Cauchy's Integral formula.	CO4	7 Marks
	b)	Construct Laurent's series about $z = 0$ for $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ in the region $2 < \lvert z \rvert < 3$.	CO2 CO3	7 Marks
		(OR)		
6	a)	Evaluate $\oint_C \frac{\cos\pi z}{z^2 - 1} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$ using Cauchy's integral formula.	CO3 CO4	7 Marks
	b)	Expand $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region: (i) $\lvert z \rvert < 1$, (ii) $1 < \lvert z \rvert < 2$, (iii) $\lvert z \rvert > 2$.	CO3	7 Marks

UNIT-IV

7		Define the singularity of a function. Determine the poles of the function $f(z)=\dfrac{z^2-2z}{(z+1)^2(z^2+4)}$ and the residue at each pole, Hence evaluate $\oint_C f(z)dz$, where C is the circle $\lvert z \rvert = 10$.	CO1 CO4	14 Marks

(OR)

8		By integrating around a unit circle, evaluate $\int_0^{2\pi} \dfrac{\cos 3\theta}{5-4\cos\theta}\, d\theta$.	CO5	14 Marks

UNIT-V

9	a)	Show that $w=\dfrac{i-z}{i+z}$ maps the real axis of z – plane into the circle $\lvert w \rvert = 1$ and the half of the plane $y>0$ into the interior of the unit circle $\lvert w \rvert = 1$ in the w-plane.	CO1 CO4	7 Marks
	b)	Find the bilinear transformation which maps the points $z=1,\ i,\ -1$ onto the points $w=0, 1, \infty$.	CO1 CO4	7 Marks

(OR)

10		Determine the region of the w-plane into which the following regions are mapped by the transformation $w=z^2$. i) first quadrant of the z – plane. ii) region bounded by $x=1, y=1, x+y=1$. iii) the region $1 \le x \le 2$ and $1 \le y \le 2$. iv) circle $\lvert z-1 \rvert = 2$.	CO3 CO4	14 Marks

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