SREE VIDYANIKETHAN ENGINEERING COLLEGE
(An Autonomous Institution, Affiliated to JNTUA, Ananthapuramu)
II B.Tech I Semester (SVEC-19) Regular Examinations February – 2021
SPECIAL FUNCTIONS AND COMPLEX ANALYSIS
[Electrical and Electronics Engineering, Electronics and Communication Engineering, Electronics and Instrumentation Engineering]

Time: 3 hours   Max. Marks: 60

Answer One Question from each Unit
All questions carry equal marks

UNIT-I

1. a) Define gamma function and express $\int_0^\infty x^{p-1} e^{-kx} \, dx \ (k > 0)$ in terms of gamma function. — 6 Marks — L1 — CO1 — PO1

   b) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \beta(m,m)$. — 6 Marks — L2 — CO1 — PO1

(OR)

2. a) Show that $\int_a^b (x-a)^{m-1} (b-x)^{n-1} \, dx = (b-a)^{m+n-1} \beta(m,n)$. — 6 Marks — L2 — CO1 — PO1

   b) Prove that $\int_0^1 x^3 \left(1 - \sqrt{x}\right)^5 \, dx = 2\beta(8,6)$. — 6 Marks — L2 — CO1 — PO1

UNIT-II

3. a) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. — 6 Marks — L3 — CO1 — PO1
   b) Show that $(n+1) P_{n+1}(x) = (2n+1) x P_n(x) - n P_{n-1}(x)$. — 6 Marks — L2 — CO1 — PO1

(OR)

4. a) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. — 6 Marks — L2 — CO2 — PO2

   b) Prove that $P_n(x) = \frac{1}{2^n \, n!} \frac{d^n}{dx^n} \left(x^2 - 1\right)^n$. — 6 Marks — L2 — CO1 — PO1

UNIT-III

5. a) Define analytic function and construct the analytic function $f(z) = u + iv$ where $u = e^{x^2 - y^2} \cos 2xy$. — 6 Marks — L3 — CO3 — PO2

   b) Discuss the transformation $W = \sqrt{z}$. Is it Conformal at the Origin? — 6 Marks — L2 — CO2 — PO1

(OR)

6. a) Find the analytic function $f(z) = u + iv$ if $2u + v = e^x (\cos y - \sin y)$. — 6 Marks — L3 — CO2 — PO2

   b) Find the bilinear transformation which maps the points $z = 1$, i , -1 on to the points $w = i$, $0, -i$. — 6 Marks — L2 — CO2 — PO2

UNIT-IV

7. Write Cauchy's integral formula and evaluate $\int_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} \, dz$ where C is the Circle $|z| = 1$. — 12 Marks — L2 — CO2 — PO2

(OR)

8.

a) Evaluate f (2) and f (3) where $f(a) = \int_C \dfrac{2z^2 - z - 2}{z - a}\, dz$ 6 Marks   L3   CO2   PO2

and C is the Circle |z| = 2.5.

b) Expand $f(z) = \dfrac{z-1}{z+1}$ in Taylor's series about the Point z =1 .   6 Marks   L2   CO2   PO1

UNIT-V

9.

a) Write Residue theorem and evaluate $\int_C \dfrac{1 - 2z}{z(z-1)(z-2)}\, dz$ 6 Marks   L2   CO2   PO1

where C is the circle |z| =1.5 by Residue theorem.

b) Evaluate $\int_0^\infty \dfrac{\cos ax}{x^2 + 1}\, dx$ by applying Calculus of   6 Marks   L3   CO2   PO2

Residues.

(OR)

10. By Integrating around a Unit circle, evaluate   12 Marks   L3   CO2   PO2
.
$\int_0^{2\pi} \dfrac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta$.

✾   ✾   ✾