Instructor's Solutions Manual

ENGINEERING MECHANICS

STATICS

TENTH EDITION

R. C. Hibbeler

PEARSON
Prentice Hall

Pearson Education, Inc.
Upper Saddle River, New Jersey 07458

Executive Editor: *Eric Svendsen*
Associte Editor: *Dee Bernhard*
Executive Managing Editor: *Vince O'Brien*
Managing Editor: *David A. George*
Production Editor: *Barbara A. Till*
Director of Creative Services: *Paul Belfanti*
Manufacturing Manager: *Trudy Pisciotti*
Manufacturing Buyer: *Ilene Kahn*

About the cover: The forces within the members of this truss bridge must be determined if they are to be properly designed. *Cover Image:* R.C. Hibbeler.

Pearson Prentice Hall
Pearson Education, Inc.
Upper Saddle River, NJ 07458

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-141212-4

Pearson Education Ltd., *London*
Pearson Education Australia Pty. Ltd., *Sydney*
Pearson Education Singapore, Pte. Ltd.
Pearson Education North Asia Ltd., *Hong Kong*
Pearson Education Canada, Inc., *Toronto*
Pearson Educación de Mexico, S.A. de C.V.
Pearson Education—Japan, *Tokyo*
Pearson Education Malaysia, Pte. Ltd.
Pearson Education, Inc., *Upper Saddle River, New Jersey*

Contents

1-1. Round off the following numbers to three significant figures: (a) 4.65735 m, (b) 55.578 s, (c) 4555 N, (d) 2768 kg.

a) 4.66 m b) 55.6 s c) 4.56 kN d) 2.77 Mg **Ans**

1-2. Wood has a density of 4.70 slug/ft^3. What is its density expressed in SI units?

$$(4.70\ \text{slug/ft}^3)\left\{\frac{(1\ \text{ft}^3)(14.5938\ \text{kg})}{(0.3048\ \text{m})^3(1\ \text{slug})}\right\} = 2.42\ \text{Mg/m}^3 \quad \textbf{Ans}$$

1-3. Represent each of the following quantities in the correct SI form using an appropriate prefix: (a) 0.000431 kg, (b) 35.3(10^3) N, (c) 0.00532 km.

a) $0.000431\ \text{kg} = 0.000431(10^3)\ \text{g} = 0.431\ \text{g}$ **Ans**

b) $35.3(10^3)\ \text{N} = 35.3\ \text{kN}$ **Ans**

c) $0.00532\ \text{km} = 0.00532(10^3)\ \text{m} = 5.32\ \text{m}$ **Ans**

***1-4.** Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) m/ms, (b) μkm, (c) ks/mg, and (d) km $\cdot$ μN.

(a) $\text{m/ms} = \left(\frac{\text{m}}{(10)^{-3}\ \text{s}}\right) = \left(\frac{(10)^3\ \text{m}}{\text{s}}\right) = \text{km/s}$ **Ans**

(b) $\mu\text{km} = (10)^{-6}(10)^3\ \text{m} = (10)^{-3}\ \text{m} = \text{mm}$ **Ans**

(c) $\text{ks/mg} = \left(\frac{(10)^3\ \text{s}}{(10)^{-6}\ \text{kg}}\right) = \left(\frac{(10)^9\ \text{s}}{\text{kg}}\right) = \text{Gs/kg}$ **Ans**

(d) $\text{km}\cdot\mu\text{N} = \left[(10)^3\ \text{m}\right]\left[(10)^{-6}\ \text{N}\right] = (10)^{-3}\ \text{mN} = \text{mmN}$ **Ans**

1-5. If a car is traveling at 55 mi/h, determine its speed in kilometers per hour and meters per second.

$$55\ \text{mi/h} = \left(\frac{55\ \text{mi}}{1\ \text{h}}\right)\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)$$
$$= 88.5\ \text{km/h} \quad \textbf{Ans}$$

$$88.5\ \text{km/h} = \left(\frac{88.5\ \text{km}}{1\ \text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 24.6\ \text{m/s} \quad \textbf{Ans}$$

1-6. Evaluate each of the following and express with an appropriate prefix: (a) $(430\ \text{kg})^2$, (b) $(0.002\ \text{mg})^2$, and (c) $(230\ \text{m})^3$.

(a) $(430\ \text{kg})^2 = 0.185(10^6)\ \text{kg}^2 = 0.185\ \text{Mg}^2$ **Ans**

(b) $(0.002\ \text{mg})^2 = \left[2(10^{-6})\ \text{g}\right]^2 = 4\ \mu\text{g}^2$ **Ans**

(c) $(230\ \text{m})^3 = \left[0.23(10^3)\ \text{m}\right]^3 = 0.0122\ \text{km}^3$ **Ans**

1-7. A rocket has a mass of 250(10^3) slugs on earth. Specify (a) its mass in SI units, and (b) its weight in SI units. If the rocket is on the moon, where the acceleration due to gravity is g_m = 5.30 ft/s^2, determine to three significant figures (c) its weight in SI units, and (d) its mass in SI units.

Using Table 1 - 2 and applying Eq. 1 – 3, we have

a) $250(10^3)\ \text{slugs} = \left[250(10^3)\ \text{slugs}\right]\left(\frac{14.5938\ \text{kg}}{1\ \text{slugs}}\right)$
$= 3.64845(10^6)\ \text{kg}$
$= 3.65\ \text{Gg}$ **Ans**

b) $W_e = mg = \left[3.64845(10^6)\ \text{kg}\right](9.81\ \text{m/s}^2)$
$= 35.791(10^6)\ \text{kg}\cdot\text{m/s}^2$
$= 35.8\ \text{MN}$ **Ans**

c) $W_m = mg_m = \left[250(10^3)\ \text{slugs}\right](5.30\ \text{ft/s}^2)$
$= \left[1.325(10^6)\ \text{lb}\right]\left(\frac{4.4482\ \text{N}}{1\ \text{lb}}\right)$
$= 5.894(10^6)\ \text{N} = 5.89\ \text{MN}$ **Ans**

Or

$$W_m = W_e\left(\frac{g_m}{g}\right) = (35.791\ \text{MN})\left(\frac{5.30\ \text{ft/s}^2}{32.2\ \text{ft/s}^2}\right) = 5.89\ \text{MN}$$

d) Since the mass is independent of its location, then

$m_m = m_e = 3.65(10^6)\ \text{kg} = 3.65\ \text{Gg}$ **Ans**

*1-8. Represent each of the following combinations of units in the correct SI form: (a) kN/μs, (b) Mg/mN, and (c) MN/(kg · ms).

(a) $kN/\mu s = 10^3 N/(10^{-6}) s = GN/s$ **Ans**

(b) $Mg/mN = 10^6 g/10^{-3} N = Gg/N$ **Ans**

(c) $MN/(kg \cdot ms) = 10^6 N/kg(10^{-3} s) = GN/(kg \cdot s)$ **Ans**

1-9. The *pascal* (Pa) is actually a very small unit of pressure. To show this, convert 1 Pa = 1 N/m² to lb/ft². Atmospheric pressure at sea level is 14.7 lb/in². How many pascals is this?

Using Table 1-2, we have

$$1 \text{ Pa} = \frac{1 \text{ N}}{\text{m}^2}\left(\frac{1 \text{ lb}}{4.4482 \text{ N}}\right)\left(\frac{0.3048^2 \text{ m}^2}{1 \text{ ft}^2}\right) = 20.9(10^{-3}) \text{ lb/ft}^2 \quad \textbf{Ans}$$

$$1 \text{ ATM} = \frac{14.7 \text{ lb}}{\text{in}^2}\left(\frac{4.4482 \text{ N}}{1 \text{ lb}}\right)\left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right)\left(\frac{1 \text{ ft}^2}{0.3048^2 \text{ m}^2}\right)$$
$$= 101.3(10^3) \text{ N/m}^2$$
$$= 101 \text{ kPa} \quad \textbf{Ans}$$

1-10. What is the weight in newtons of an object that has a mass of: (a) 10 kg, (b) 0.5 g, (c) 4.50 Mg? Express the result to three significant figures. Use an appropriate prefix.

(a) $W = (9.81 \text{ m/s}^2)(10 \text{ kg}) = 98.1 \text{ N}$ **Ans**

(b) $W = (9.81 \text{ m/s}^2)(0.5 \text{ g})(10^{-3} \text{ kg/g}) = 4.90 \text{ mN}$ **Ans**

(c) $W = (9.81 \text{ m/s}^2)(4.5 \text{ Mg})(10^3 \text{ kg/Mg}) = 44.1 \text{ kN}$ **Ans**

1-11. Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) 354 mg(45 km)/(0.035 6 kN), (b) (.004 53 Mg)(201 ms), (c) 435 MN/23.2 mm.

a) $$(354 \text{ mg})(45 \text{ km})/0.0356 \text{ kN} = \frac{[354(10^{-3}) \text{ g}][45(10^3) \text{ m}]}{0.0356(10^3) \text{ N}}$$
$$= \frac{0.447(10^3) \text{ g} \cdot \text{m}}{\text{N}}$$
$$= 0.447 \text{ kg} \cdot \text{m/N} \quad \textbf{Ans}$$

b) $$(0.00453 \text{ Mg})(201 \text{ ms}) = [4.53(10^{-3})(10^3) \text{ kg}][201(10^{-3}) \text{ s}]$$
$$= 0.911 \text{ kg} \cdot \text{s} \quad \textbf{Ans}$$

c) $$435 \text{ MN}/23.2 \text{ mm} = \frac{435(10^6) \text{ N}}{23.2(10^{-3}) \text{ m}} = \frac{18.75(10^9) \text{ N}}{\text{m}} = 18.8 \text{ GN/m} \quad \textbf{Ans}$$

*1-12. Convert each of the following and express the answer using an appropriate prefix: (a) 175 lb/ft³ to kN/m³, (b) 6 ft/h to mm/s, and (c) 835 lb · ft to kN · m.

(a) $$175 \text{ lb/ft}^3 = \left(\frac{175 \text{ lb}}{\text{ft}^3}\right)\left(\frac{\text{ft}}{0.3048 \text{ m}}\right)^3\left(\frac{4.4482 \text{ N}}{\text{lb}}\right)$$
$$= \left(\frac{27.5(10)^3 \text{ N}}{\text{m}^3}\right) = 27.5 \text{ kN/m}^3 \quad \textbf{Ans}$$

(b) $$6 \text{ ft/h} = \left(\frac{6 \text{ ft}}{\text{h}}\right)\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$$
$$= 0.508(10)^{-3} \text{ m/s} = 0.508 \text{ mm/s} \quad \textbf{Ans}$$

(c) $$835 \text{ lb} \cdot \text{ft} = (835 \text{ lb} \cdot \text{ft})\left(\frac{4.4482 \text{ N}}{1 \text{ lb}}\right)\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right)$$
$$= 1.13(10)^3 \text{ N} \cdot \text{m} = 1.13 \text{ kN} \cdot \text{m} \quad \textbf{Ans}$$

1-13. Convert each of the following to three significant figures. (a) 20 lb · ft to N · m, (b) 450 lb/ft^3 to kN/m^3, and (c) 15 ft/h to mm/s.

Using Table 1 - 2, we have

a) $20\text{ lb}\cdot\text{ft} = (20\text{ lb}\cdot\text{ft})\left(\frac{4.4482\text{ N}}{1\text{ lb}}\right)\left(\frac{0.3048\text{ m}}{1\text{ ft}}\right)$
$= 27.1\text{ N}\cdot\text{m}$ **Ans**

b) $450\text{ lb/ft}^3 = \left(\frac{450\text{ lb}}{\text{ft}^3}\right)\left(\frac{4.4482\text{ N}}{1\text{ lb}}\right)\left(\frac{1\text{ kN}}{1000\text{ N}}\right)\left(\frac{1\text{ ft}^3}{0.3048^3\text{ m}^3}\right)$
$= 70.7\text{ kN/m}^3$ **Ans**

c) $15\text{ ft/h} = \left(\frac{15\text{ ft}}{\text{h}}\right)\left(\frac{304.8\text{ mm}}{1\text{ ft}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 1.27\text{ mm/s}$ **Ans**

1-14. If an object has a mass of 40 slugs, determine its mass in kilograms.

$40\text{ slugs }(14.5938\text{ kg/slug}) = 584\text{ kg}$ **Ans**

1-15. Water has a density of 1.94 slug/ft^3. What is the density expressed in SI units? Express the answer to three significant figures.

Using Table 1 - 2, we have

$$\rho_w = \left(\frac{1.94\text{ slug}}{\text{ft}^3}\right)\left(\frac{14.5938\text{ kg}}{1\text{ slug}}\right)\left(\frac{1\text{ ft}^3}{0.3048^3\text{ m}^3}\right)$$
$= 999.8\text{ kg/m}^3 = 1.00\text{ Mg/m}^3$ **Ans**

***1-16.** Two particles have a mass of 8 kg and 12 kg, respectively. If they are 800 mm apart, determine the force of gravity acting between them. Compare this result with the weight of each particle.

$$F = G\frac{m_1 m_2}{r^2}$$

Where $G = 6.673(10^{-11})\text{ m}^3/(\text{kg}\cdot\text{s}^2)$

$F = 6.673(10^{-11})\left[\frac{8(12)}{(0.8)^2}\right] = 10.0(10^{-9})\text{ N} = 10.0\text{ nN}$ **Ans**

$W_1 = 8(9.81) = 78.5\text{ N}$ **Ans**

$W_2 = 12(9.81) = 118\text{ N}$ **Ans**

1-17. Determine the mass of an object that has a weight of (a) 20 mN, (b) 150 kN, (c) 60 MN. Express the answer to three significant figures.

Applying Eq. 1 – 3, we have

a) $m = \frac{W}{g} = \frac{20(10^{-3})\text{ kg}\cdot\text{m/s}^2}{9.81\text{ m/s}^2} = 2.04\text{ g}$ **Ans**

b) $m = \frac{W}{g} = \frac{150(10^{3})\text{ kg}\cdot\text{m/s}^2}{9.81\text{ m/s}^2} = 15.3\text{ Mg}$ **Ans**

c) $m = \frac{W}{g} = \frac{60(10^{6})\text{ kg}\cdot\text{m/s}^2}{9.81\text{ m/s}^2} = 6.12\text{ Gg}$ **Ans**

1-18. If a man weighs 155 lb on earth, specify (a) his mass in slugs, (b) his mass in kilograms, and (c) his weight in newtons. If the man is on the moon, where the acceleration due to gravity is $g_m = 5.30\ \text{ft/s}^2$, determine (d) his weight in pounds, and (e) his mass in kilograms.

(a) $m = \dfrac{155}{32.2} = 4.81\ \text{slug}$ **Ans**

(b) $m = 155\left[\dfrac{14.5938\ \text{kg}}{32.2}\right] = 70.2\ \text{kg}$ **Ans**

(c) $W = 155\,(4.4482) = 689\ \text{N}$ **Ans**

(d) $W = 155\left[\dfrac{5.30}{32.2}\right] = 25.5\ \text{lb}$ **Ans**

(e) $m = 155\left[\dfrac{14.5938\ \text{kg}}{32.2}\right] = 70.2\ \text{kg}$ **Ans**

Also,

$$m = 25.5\left[\frac{14.5938\ \text{kg}}{5.30}\right] = 70.2\ \text{kg} \quad \textbf{Ans}$$

1-19. Using the base units of the SI system, show that Eq. 1-2 is a dimensionally homogeneous equation which gives F in newtons. Determine to three significant figures the gravitational force acting between two spheres that are touching each other. The mass of each sphere is 200 kg and the radius is 300 mm.

Using Eq. 1 – 2.

$$F = G\frac{m_1 m_2}{r^2}$$

$$\text{N} = \left(\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)\left(\frac{\text{kg}\cdot\text{kg}}{\text{m}^2}\right) = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} \qquad (Q.E.D.)$$

$$F = G\frac{m_1 m_2}{r^2}$$

$$= 66.73(10^{-12})\left[\frac{200(200)}{0.6^2}\right]$$

$$= 7.41(10^{-6})\ \text{N} = 7.41\ \mu\text{N} \quad \textbf{Ans}$$

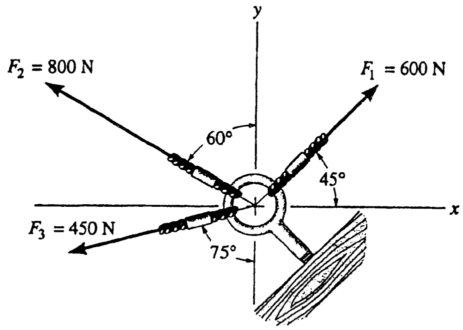
***1-20.** Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) (0.631 Mm)/(8.60 kg)^2, (b) (35 mm)^2(48 kg)^3.

(a) $0.631\ \text{Mm}/(8.60\ \text{kg})^2 = \left(\dfrac{0.631(10^6)\ \text{m}}{(8.60)^2\ \text{kg}^2}\right) = \dfrac{8532\ \text{m}}{\text{kg}^2}$

$= 8.53(10^3)\,\text{m/kg}^2 = 8.53\ \text{km/kg}^2$ **Ans**

(b) $(35\ \text{mm})^2(48\ \text{kg})^3 = \left[35(10^{-3})\ \text{m}\right]^2(48\ \text{kg})^3 = 135\ \text{m}^2\text{kg}^3$ **Ans**

2-1. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_3$ and its direction, measured counterclockwise from the positive x axis.

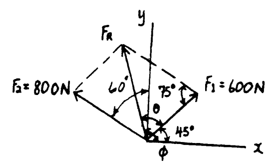


$$F_R = \sqrt{(600)^2 + (800)^2 - 2(600)(800)\cos 75^\circ} = 866.91 = 867 \text{ N} \quad \textbf{Ans}$$

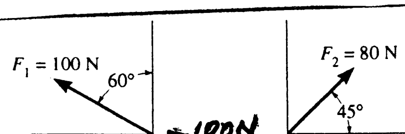
$$\frac{866.91}{\sin 75^\circ} = \frac{800}{\sin\theta}$$

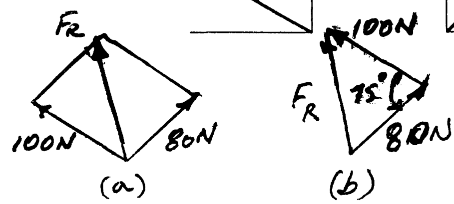
$$\theta = 63.05^\circ$$

$$\phi = 63.05^\circ + 45^\circ = 108^\circ \quad \textbf{Ans}$$



2-2. Determine the magnitude of the resultant force if: (a) $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$; (b) $\mathbf{F}'_R = \mathbf{F}_1 - \mathbf{F}_2$.



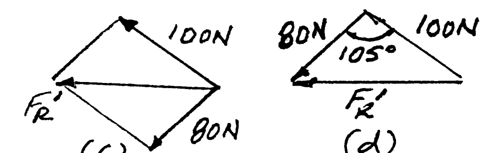


Parallelogram Law : The parallelogram law of addition is shown in Fig. (a) and (c).

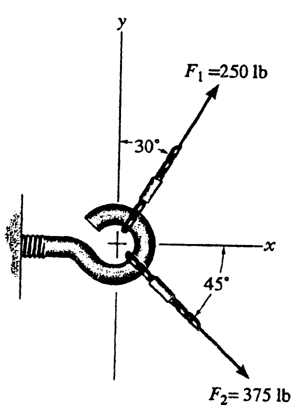
Trigonometry : Using law of cosines [Fig. (b) and (d)], we have

a) $F_R = \sqrt{100^2 + 80^2 - 2(100)(80)\cos 75^\circ}$
$= 111$ N **Ans**

b) $F_R' = \sqrt{100^2 + 80^2 - 2(100)(80)\cos 105^\circ}$
$= 143$ N **Ans**



2-3. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.

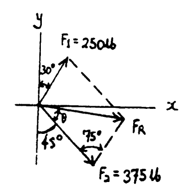


$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75^\circ} = 393.2 = 393 \text{ lb} \quad \textbf{Ans}$$

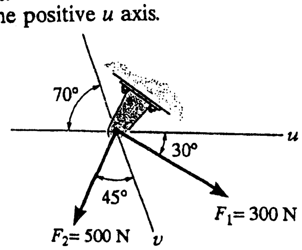
$$\frac{393.2}{\sin 75^\circ} = \frac{250}{\sin\theta}$$

$$\theta = 37.89^\circ$$

$$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ \quad \textbf{Ans}$$



***2-4.** Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.

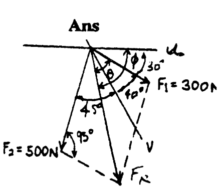


$$F_R = \sqrt{(300)^2 + (500)^2 - 2(300)(500)\cos 95^\circ} = 605.1 = 605 \text{ N} \quad \textbf{Ans}$$

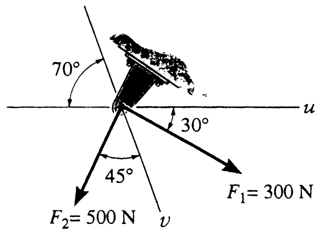
$$\frac{605.1}{\sin 95^\circ} = \frac{500}{\sin\theta}$$

$$\theta = 55.40^\circ$$

$$\phi = 55.40^\circ + 30^\circ = 85.4^\circ \quad \textbf{Ans}$$



2-5. Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.

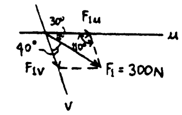


$$\frac{F_{1u}}{\sin 40^\circ} = \frac{300}{\sin 110^\circ}$$

$F_{1u} = 205$ N **Ans**

$$\frac{F_{1v}}{\sin 30^\circ} = \frac{300}{\sin 110^\circ}$$

$F_{1v} = 160$ N **Ans**



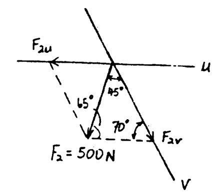
2-6. Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.

$$\frac{F_{2u}}{\sin 45^\circ} = \frac{500}{\sin 70^\circ}$$

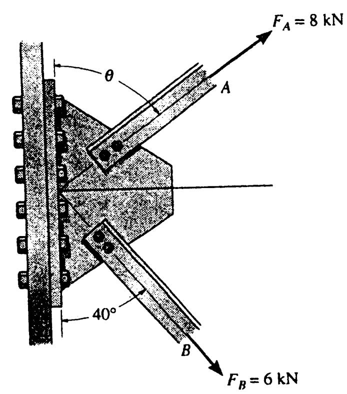
$F_{2u} = 376$ N **Ans**

$$\frac{F_{2v}}{\sin 65^\circ} = \frac{500}{\sin 70^\circ}$$

$F_{2v} = 482$ N **Ans**



2-7. The plate is subjected to the two forces at A and B as shown. If $\theta = 60^\circ$, determine the magnitude of the resultant of these two forces and its direction measured from the horizontal.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of cosines [Fig. (b)], we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100^\circ}$$
$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$

Ans

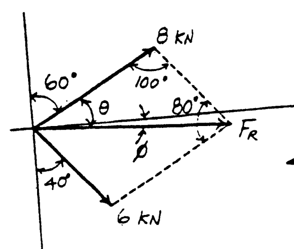
The angle θ can be determined using law of sines [Fig. (b)].

$$\frac{\sin\theta}{6} = \frac{\sin 100^\circ}{10.80}$$
$$\sin\theta = 0.5470$$
$$\theta = 33.16^\circ$$

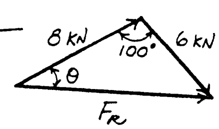
Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ$$

Ans

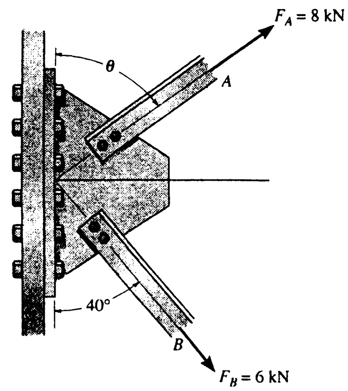


(a)



(b)

***2-8.** Determine the angle θ for connecting member A to the plate so that the resultant force of F_A and F_B is directed horizontally to the right. Also, what is the magnitude of the resultant force.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

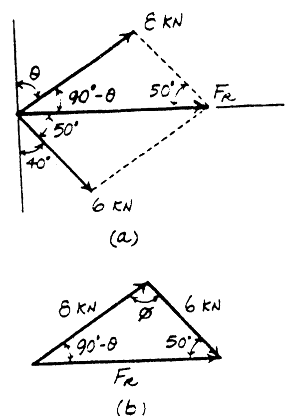
Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{\sin(90° - \theta)}{6} = \frac{\sin 50°}{8}$$
$$\sin(90° - \theta) = 0.5745$$

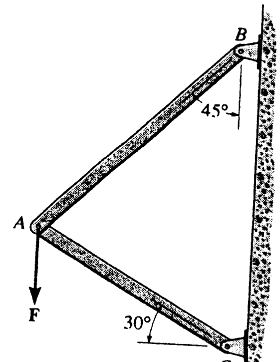
$$\theta = 54.93° = 54.9° \qquad \textbf{Ans}$$

From the triangle, $\phi = 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$
$$= 10.4 \text{ kN} \qquad \textbf{Ans}$$



2-9. The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

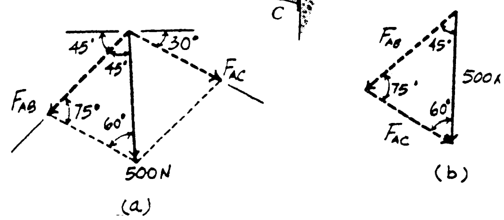
Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{F_{AB}}{\sin 60°} = \frac{500}{\sin 75°}$$

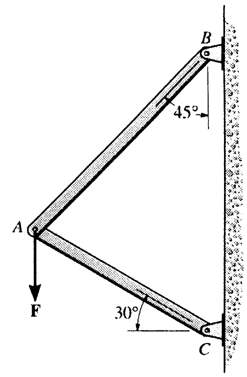
$$F_{AB} = 448 \text{ N} \qquad \textbf{Ans}$$

$$\frac{F_{AC}}{\sin 45°} = \frac{500}{\sin 75°}$$

$$F_{AC} = 366 \text{ N} \qquad \textbf{Ans}$$



2-10. Solve Prob. 2-9 with $F = 350$ lb.



Parallelogram Law: The parallelogram law of addition is shown in Fig. (a).

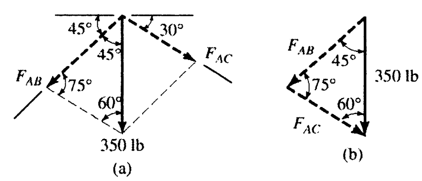
Trigonometry: Using law of sines [Fig. (b)], we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{350}{\sin 75^\circ}$$

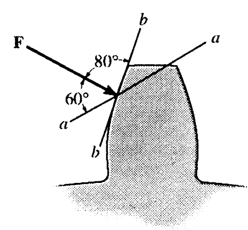
$F_{AB} = 314$ lb **Ans**

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{350}{\sin 75^\circ}$$

$F_{AC} = 256$ lb **Ans**

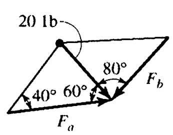


2-11. The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines aa and bb.

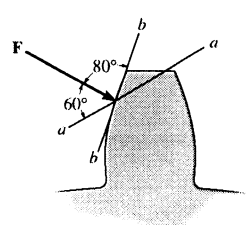


$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \qquad F_a = 30.6 \text{ lb} \quad \textbf{Ans}$$

$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.9 \text{ lb} \quad \textbf{Ans}$$

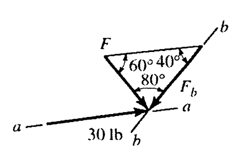


***2-12.** The component of force **F** acting along line aa is require to be 30 lb. Determine the magnitude of **F** and its component along line bb.

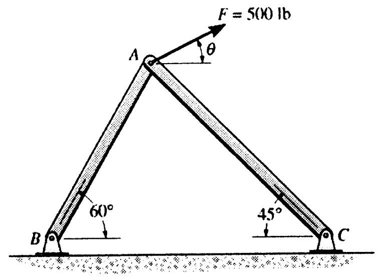


$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \qquad F = 19.6 \text{ lb} \quad \textbf{Ans}$$

$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.4 \text{ lb} \quad \textbf{Ans}$$



2-13. The 500-lb force acting on the frame is to be resolved into two components acting along the axis of the struts AB and AC. If the component of force along AC is required to be 300 lb, directed from A to C, determine the magnitude of force acting along AB and the angle θ of the 500-lb force.



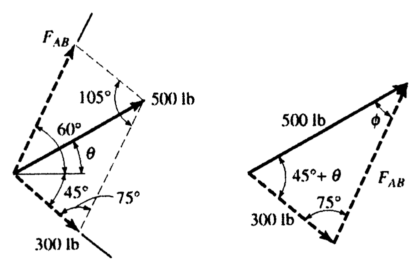
Parallelogram Law: The parallelogram law of addition is shown in Fig. (a).

Trigonometry: Using law of sines [Fig. (b)], we have

$$\frac{\sin\phi}{300} = \frac{\sin 75^\circ}{500}$$

$$\sin\phi = 0.5796$$

$$\phi = 35.42^\circ \qquad \textbf{Ans}$$



Thus,

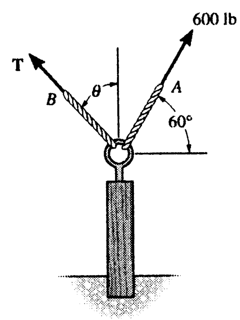
$$45^\circ + \theta + 75^\circ + 35.42^\circ = 180^\circ$$

$$\theta = 24.58^\circ = 24.6^\circ$$

$$\frac{F_{AB}}{\sin(45^\circ + 24.58^\circ)} = \frac{500}{\sin 75^\circ}$$

$$F_{AB} = 485 \text{ lb} \qquad \textbf{Ans}$$

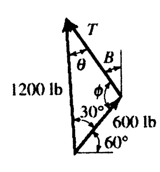
2-14. The post is to be pulled out of the ground using two ropes A and B. Rope A is subjected to a force of 600 lb and is directed at 60° from the horizontal. If the resultant force acting on the post is to be 1200 lb, vertically upward, determine the force T in rope B and the corresponding angle θ.



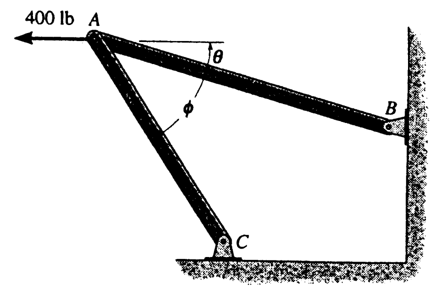
$$T = \sqrt{(600)^2 + (1200)^2 - 2(600)(1200)\cos 30^\circ}$$

$$T = 743.59 \text{ lb} = 744 \text{ lb} \qquad \textbf{Ans}$$

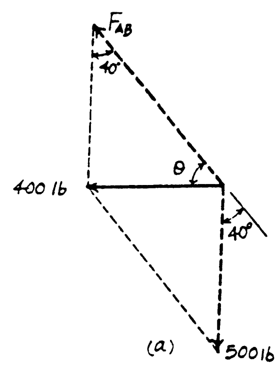
$$\frac{\sin\theta}{600} = \frac{\sin 30^\circ}{743.59}, \quad \theta = 23.8^\circ \qquad \textbf{Ans}$$



2-15. Determine the design angle θ ($0° \leq \theta \leq 90°$) for strut AB so that the 400-lb horizontal force has a component of 500-lb directed from A towards C. What is the component of force acting along member AB? Take $\phi = 40°$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).



Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{\sin\theta}{500} = \frac{\sin 40°}{400}$$
$$\sin\theta = 0.8035$$

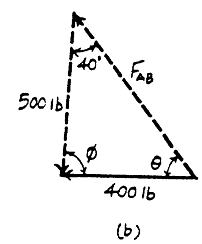
$$\theta = 53.46° = 53.5° \qquad \textbf{Ans}$$

Thus, $\phi = 180° - 40° - 53.46° = 86.54°$

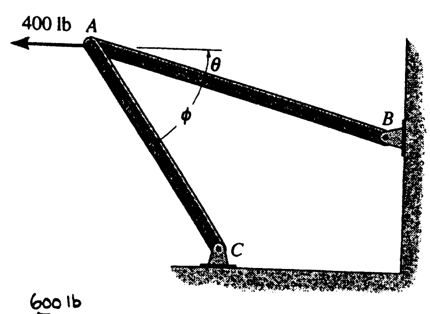
Using law of sines [Fig. (b)]

$$\frac{F_{AB}}{\sin 86.54°} = \frac{400}{\sin 40°}$$

$$F_{AB} = 621 \text{ lb} \qquad \textbf{Ans}$$



***2-16.** Determine the design angle ϕ ($0° \leq \phi \leq 90°$) between struts AB and AC so that the 400-lb horizontal force has a component of 600-lb which acts up to the left, in the same direction as from B towards A. Take $\theta = 30°$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

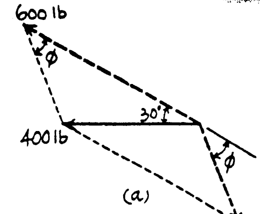
Trigonometry : Using law of cosines [Fig. (b)], we have

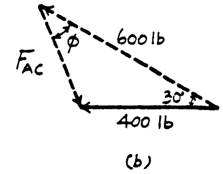
$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30°} = 322.97 \text{ lb}$$

The angle ϕ can be determined using law of sines [Fig. (b)].

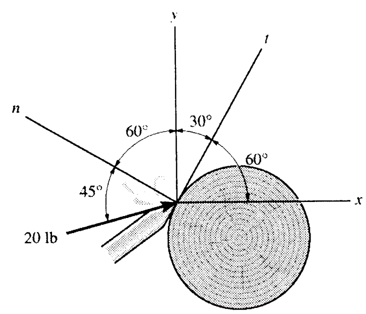
$$\frac{\sin\phi}{400} = \frac{\sin 30°}{322.97}$$
$$\sin\phi = 0.6193$$

$$\phi = 38.3° \qquad \textbf{Ans}$$





2-17. The chisel exerts a force of 20 lb on the wood dowel rod which is turning in a lathe. Resolve this force into components acting (a) along the n and y axes and (b) along the x and t axes.



(a) $\dfrac{F_y}{\sin 45^\circ} = \dfrac{20}{\sin 60^\circ}$

$F_y = 16.3$ lb **Ans**

$\dfrac{-F_n}{\sin 75^\circ} = \dfrac{20}{\sin 60^\circ}$

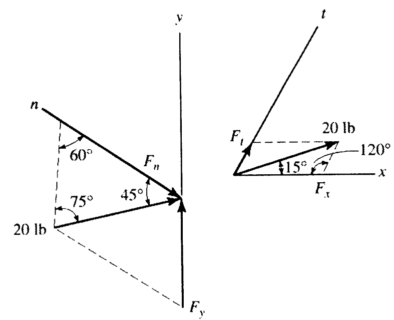
$F_n = -22.3$ lb **Ans**

(b) $\dfrac{F_t}{\sin 15^\circ} = \dfrac{20}{\sin 120^\circ}$

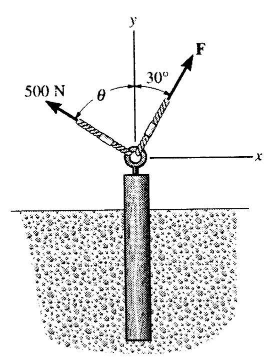
$F_t = 5.98$ lb **Ans**

$\dfrac{F_x}{\sin 45^\circ} = \dfrac{20}{\sin 120^\circ}$

$F_x = 16.3$ lb **Ans**



2-18. Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle $\theta\,(0^\circ \le \theta \le 90^\circ)$ and the magnitude of force **F** so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.



Parallelogram Law: The parallelogram law of addition is shown in Fig. (a).

Trigonometry: Using law of sines [Fig. (b)], we have

$\dfrac{\sin \phi}{750} = \dfrac{\sin 30^\circ}{500}$

$\sin \phi = 0.750$

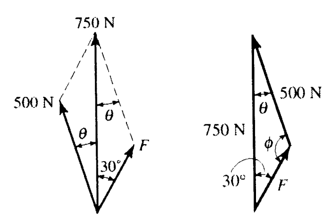
$\phi = 131.41^\circ$ (By observation, $\phi > 80^\circ$)

Thus,

$\theta = 180^\circ - 30^\circ - 131.41^\circ = 18.59^\circ = 18.6^\circ$ **Ans**

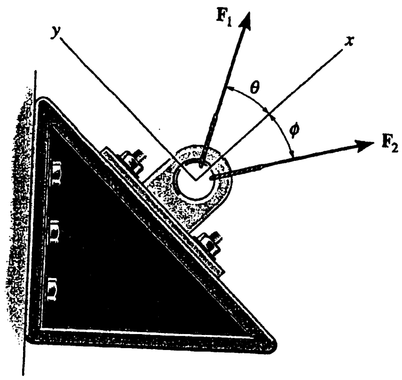
$\dfrac{F}{\sin 18.59^\circ} = \dfrac{500}{\sin 30^\circ}$

$F = 319$ N **Ans**



(a) (b)

2-19. If $F_1 = F_2 = 30$ lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 20$ lb.

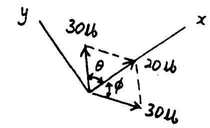


$$\frac{30}{\sin\phi} = \frac{30}{\sin\theta}$$

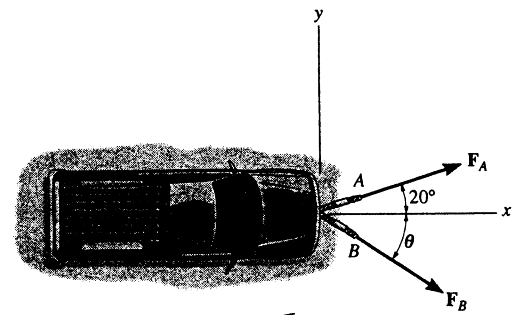
$$\phi = \theta$$

$$(30)^2 = (30)^2 + (20)^2 - 2(30)(20)\cos\theta$$

$$\phi = \theta = 70.5^\circ \qquad \textbf{Ans}$$



***2-20.** The truck is to be towed using two ropes. Determine the magnitude of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope in order to develop a resultant force of 950 N directed along the positive x axis. Set $\theta = 50^\circ$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

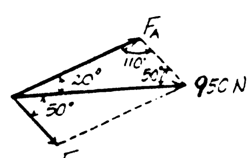
Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 50^\circ} = \frac{950}{\sin 110^\circ}$$

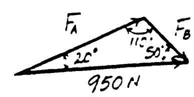
$$F_A = 774 \text{ N} \qquad \textbf{Ans}$$

$$\frac{F_B}{\sin 20^\circ} = \frac{950}{\sin 110^\circ}$$

$$F_B = 346 \text{ N} \qquad \textbf{Ans}$$

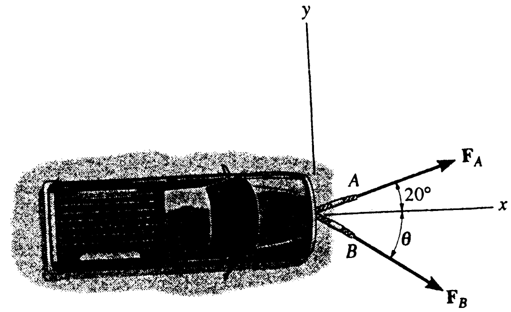


(a)



(b)

2-21. The truck is to be towed using two ropes. If the resultant force is to be 950 N, directed along the positive x axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope and the angle of θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 20° from the x axis as shown.



Parallelogram Law : In order to produce a *minimum* force $\mathbf{F}_B$, $\mathbf{F}_B$ has to act perpendicular to $\mathbf{F}_A$. The parallelogram law of addition is shown in Fig. (a).

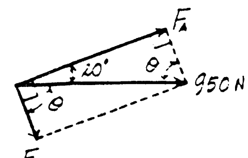
Trigonometry : Fig. (b).

$$F_B = 950 \sin 20° = 325 \text{ N} \qquad \textbf{Ans}$$

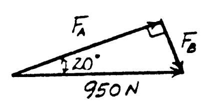
$$F_A = 950 \cos 20° = 893 \text{ N} \qquad \textbf{Ans}$$

The angle θ is

$$\theta = 90° - 20° = 70.0° \qquad \textbf{Ans}$$

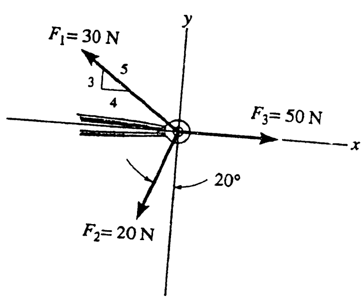


(a)



(b)

2-22. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.

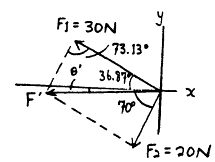


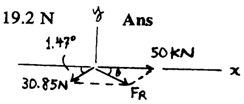
$$F' = \sqrt{(20)^2 + (30)^2 - 2(20)(30)\cos 73.13°} = 30.85 \text{ N}$$

$$\frac{30.85}{\sin 73.13°} = \frac{30}{\sin(70° - \theta')}; \qquad \theta' = 1.47°$$

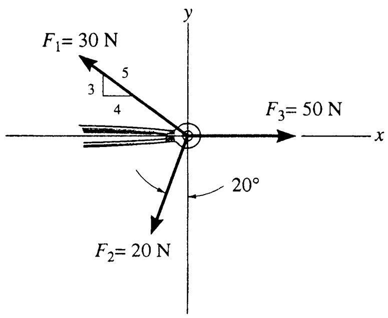
$$F_R = \sqrt{(30.85)^2 + (50)^2 - 2(30.85)(50)\cos 1.47°} = 19.18 = 19.2 \text{ N} \qquad \textbf{Ans}$$

$$\frac{19.18}{\sin 1.47°} = \frac{30.85}{\sin\theta}; \qquad \theta = 2.37° \qquad \textbf{Ans}$$

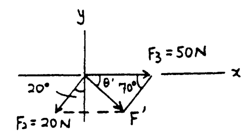




2-23. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.

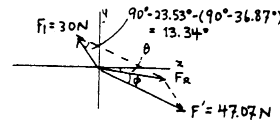


$$F' = \sqrt{(20)^2 + (50)^2 - 2(20)(50)\cos 70^\circ} = 47.07 \text{ N}$$



$$\frac{20}{\sin\theta'} = \frac{47.07}{\sin 70^\circ}; \qquad \theta' = 23.53^\circ$$

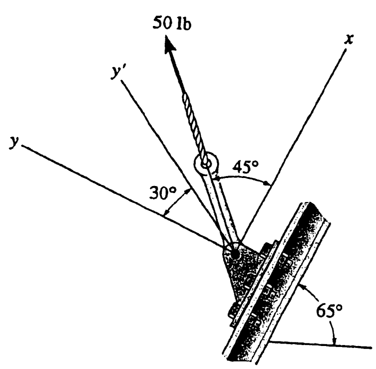
$$F_R = \sqrt{(47.07)^2 + (30)^2 - 2(47.07)(30)\cos 13.34^\circ} = 19.18 = 19.2 \text{ N} \qquad \textbf{Ans}$$



$$\frac{19.18}{\sin 13.34^\circ} = \frac{30}{\sin\phi}; \qquad \phi = 21.15^\circ$$

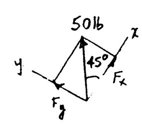
$$\theta = 23.53^\circ - 21.15^\circ = 2.37^\circ \qquad \textbf{Ans}$$

***2-24.** Resolve the 50-lb force into components acting along (a) the x and y axes, and (b) the x and y' axes.



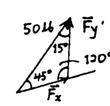
(a) $F_x = 50\cos 45^\circ = 35.4$ lb **Ans**

$F_y = 50\sin 45^\circ = 35.4$ lb **Ans**



(b) $$\frac{F_x}{\sin 15^\circ} = \frac{50}{\sin 120^\circ}$$

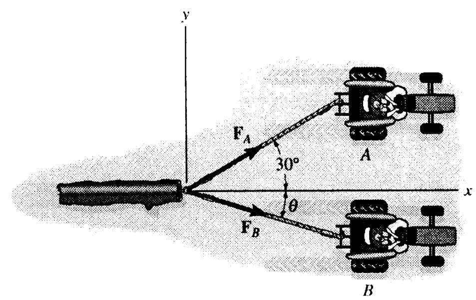
$F_x = 14.9$ lb **Ans**



$$\frac{F_{y'}}{\sin 45^\circ} = \frac{50}{\sin 120^\circ}$$

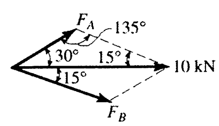
$F_{y'} = 40.8$ lb **Ans**

2-25. The log is being towed by two tractors A and B. Determine the magnitude of the two towing forces $\mathbf{F}_A$ and $\mathbf{F}_B$ if it is required that the resultant force have a magnitude $F_R = 10$ kN and be directed along the x axis. Set $\theta = 15°$.

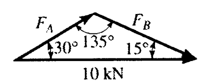


Parallelogram Law: The parallelogram law of addition is shown in Fig. (a).

Trigonometry: Using law of sines [Fig. (b)], we have



(a)



(b)

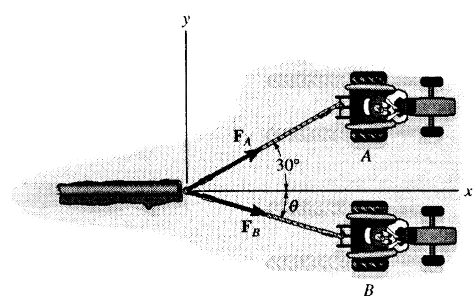
$$\frac{F_A}{\sin 15°} = \frac{10}{\sin 135°}$$

$F_A = 3.66$ kN **Ans**

$$\frac{F_B}{\sin 30°} = \frac{10}{\sin 135°}$$

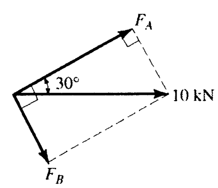
$F_B = 7.07$ kN **Ans**

2-26. If the resultant $\mathbf{F}_R$ of the two forces acting on the log is to be directed along the positive x axis and have a magnitude of 10 kN, determine the angle θ of the cable, attached to B such that the force $\mathbf{F}_B$ in this cable is minimum. What is the magnitude of the force in each cable for this situation?

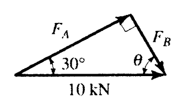


Parallelogram Law: In order to produce a *minimum* force $\mathbf{F}_B$, $\mathbf{F}_B$ has to act perpendicular to $\mathbf{F}_A$. The parallelogram law of addition is shown in Fig. (a).

Trigonometry: Fig. (b).



(a)



(b)

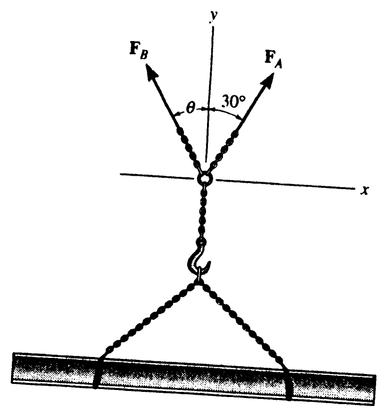
$F_B = 10 \sin 30° = 5.00$ kN **Ans**

$F_A = 10 \cos 30° = 8.66$ kN **Ans**

The angle θ is

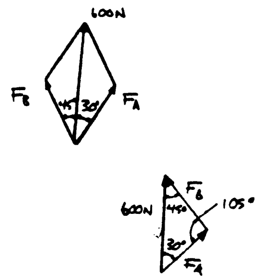
$\theta = 90° - 30° = 60.0°$ **Ans**

2-27. The beam is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain in order to develop a resultant force of 600 N directed along the positive y axis. Set $\theta = 45°$.

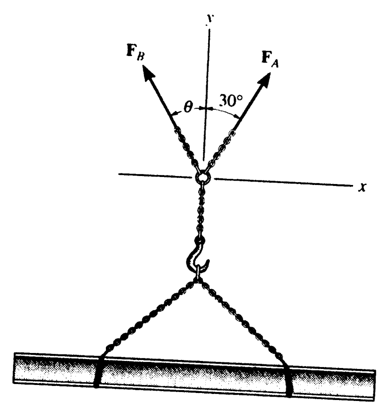


$$\frac{F_A}{\sin 45°} = \frac{600}{\sin 105°}; \qquad F_A = 439 \text{ N} \qquad \textbf{Ans}$$

$$\frac{F_B}{\sin 30°} = \frac{600}{\sin 105°}; \qquad F_B = 311 \text{ N} \qquad \textbf{Ans}$$



***2-28.** The beam is to be hoisted using two chains. If the resultant force is to be 600 N, directed along the positive y axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain and the orientation θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 30° from the y axis as shown.

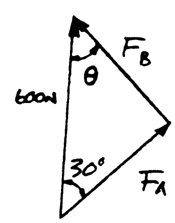


For minimum F_B, require

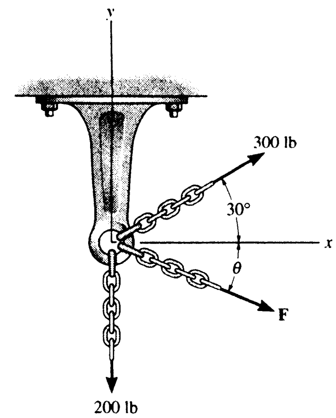
$\theta = 60°$ **Ans**

$F_A = 600 \cos 30° = 520 \text{ N}$ **Ans**

$F_B = 600 \sin 30° = 300 \text{ N}$ **Ans**

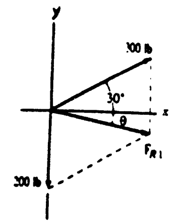


2-29. Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the orientation θ of the third chain, measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint:* First find the resultant of the two known forces. Force **F** acts in this direction.



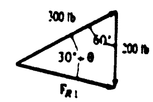
Cosine law :

$$F_{R1} = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 60^\circ} = 264.6 \text{ lb}$$



Sine law :

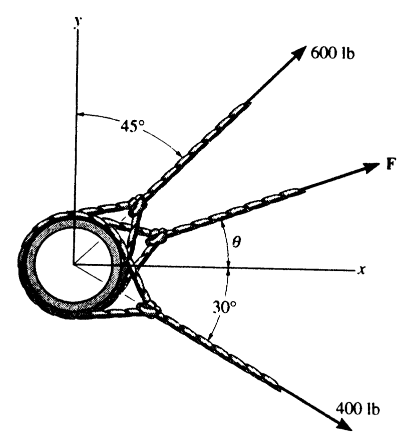
$$\frac{\sin(30^\circ + \theta)}{200} = \frac{\sin 60^\circ}{264.6} \qquad \theta = 10.9^\circ \qquad \textbf{Ans}$$



When **F** is directed along $\mathbf{F}_{R1}$, F will be minimum to create the resultant force.

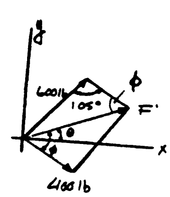
$$F_R = F_{R1} + F$$
$$500 = 264.6 + F_{min}$$
$$F_{min} = 235 \text{ lb} \qquad \textbf{Ans}$$

2-30. Three cables pull on the pipe such that they create a resultant force having a magnitude of 900 lb. If two of the cables are subjected to known forces, as shown in the figure, determine the direction θ of the third cable so that the magnitude of force **F** in this cable is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint:* First find the resultant of the two known forces.



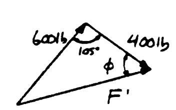
$$F' = \sqrt{(600)^2 + (400)^2 - 2(600)(400)\cos 105^\circ} = 802.64 \text{ lb}$$

$$F = 900 - 802.64 = 97.4 \text{ lb} \qquad \textbf{Ans}$$

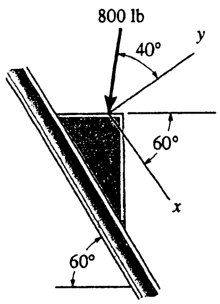


$$\frac{\sin\phi}{600} = \frac{\sin 105^\circ}{802.64}; \qquad \phi = 46.22^\circ$$

$$\theta = 46.22^\circ - 30^\circ = 16.2^\circ \qquad \textbf{Ans}$$

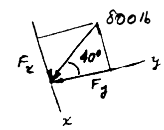


2-31. Determine the x and y components of the 800-lb force.

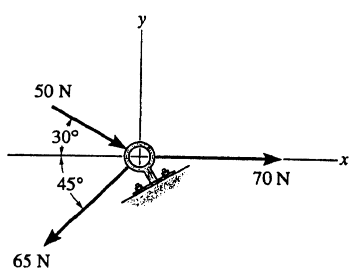


$F_x = 800\sin 40° = 514 \text{ lb}$ **Ans**

$F_y = -800\cos 40° = -613 \text{ lb}$ **Ans**



***2-32.** Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



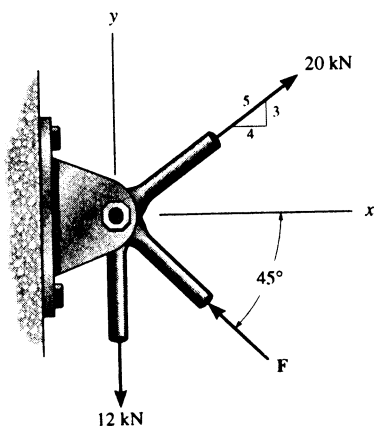
$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x;\quad F_{R_x} = 70 + 50\cos 30° - 65\cos 45° = 67.34 \text{ N}$

$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = -50\sin 30° - 65\sin 45° = -70.96 \text{ N}$

$F_R = \sqrt{(67.34)^2 + (-70.96)^2} = 97.8 \text{ N}$ **Ans**

$\theta = \tan^{-1}\dfrac{70.96}{67.34} = 46.5°$ **Ans**

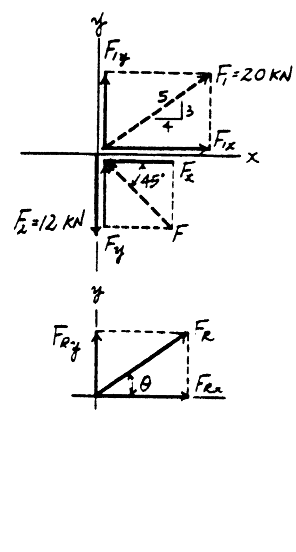
2-33. Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 20\left(\frac{4}{5}\right) - F\cos 45^\circ$$
$$= 16.0 - 0.7071F \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 20\left(\frac{3}{5}\right) - 12 + F\sin 45^\circ$$
$$= 0.7071F \uparrow$$



The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$
$$= \sqrt{(16.0 - 0.7071F)^2 + (0.7071F)^2}$$
$$= \sqrt{F^2 - 22.63F + 256} \qquad [1]$$

$$F_R^2 = F^2 - 22.63F + 256$$
$$2F_R\frac{dF_R}{dF} = 2F - 22.63 \qquad [2]$$
$$\left(F_R\frac{d^2F_R}{dF^2} + \frac{dF_R}{dF}\times\frac{dF_R}{dF}\right) = 1 \qquad [3]$$

In order to obtain the *minimum* resultant force $\mathbf{F}_R$, $\frac{dF_R}{dF} = 0$. From Eq. [2]

$$2F_R\frac{dF_R}{dF} = 2F - 22.63 = 0$$

$$F = 11.31 \text{ kN} = 11.3 \text{ kN} \qquad \textbf{Ans}$$

Substitute $F = 11.31$ kN into Eq. [1], we have

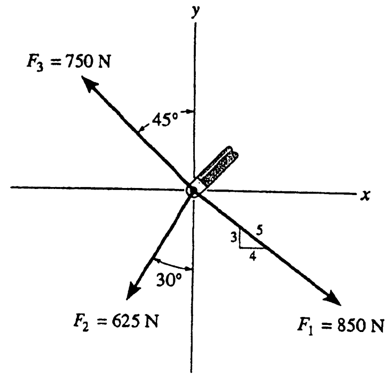
$$F_R = \sqrt{11.31^2 - 22.63(11.31) + 256} = \sqrt{128} \text{ kN}$$

Substitute $F_R = \sqrt{128}$ kN with $\frac{dF_R}{dF} = 0$ into Eq. [3], we have

$$\left(\sqrt{128}\frac{d^2F_R}{dF^2} + 0\right) = 1$$
$$\frac{d^2F_R}{dF^2} = 0.0884 > 0$$

Hence, $F = 11.3$ kN is indeed producing a minimum resultant force.

2-34. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = \frac{4}{5}(850) - 625\sin 30° - 750\sin 45° = -162.8$ N

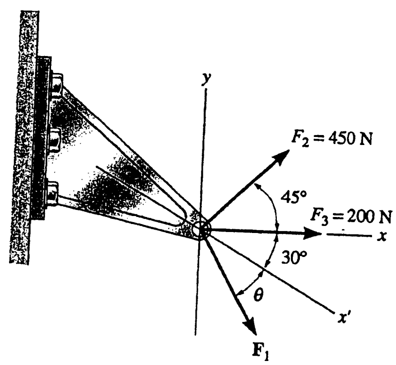
$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = -\frac{3}{5}(850) - 625\cos 30° + 750\cos 45° = -520.9$ N

$F_R = \sqrt{(-162.8)^2 + (-520.9)^2} = 546$ N **Ans**

$\phi = \tan^{-1}\left[\frac{-520.9}{-162.8}\right] = 72.64°$

$\theta = 180° + 72.64° = 253°$ **Ans**

2-35. Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 1 kN.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad 1000\cos 30° = 200 + 450\cos 45° + F_1\cos(\theta + 30°)$

$+\uparrow F_{Ry} = \Sigma F_y;\quad -1000\sin 30° = 450\sin 45° - F_1\sin(\theta + 30°)$

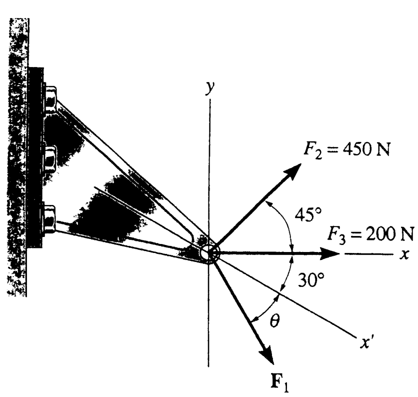
$F_1\sin(\theta + 30°) = 818.198$

$F_1\cos(\theta + 30°) = 347.827$

$\theta + 30° = 66.97°,\quad \theta = 37.0°$ **Ans**

$F_1 = 889$ N **Ans**

***2-36.** If $F_1 = 300$ N and $\theta = 20°$, determine the magnitude and direction, measured counterclockwise from the x' axis, of the resultant force of the three forces acting on the bracket.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 300\cos 50° + 200 + 450\cos 45° = 711.03$ N

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -300\sin 50° + 450\sin 45° = 88.38$ N

$F_R = \sqrt{(711.03)^2 + (88.38)^2} = 717$ N **Ans**

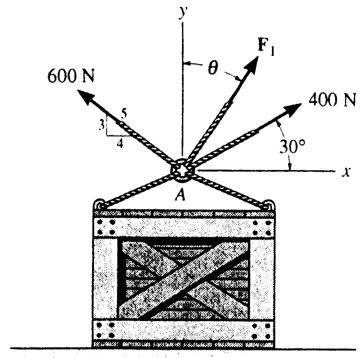
ϕ' (angle from x axis) $= \tan^{-1}\left[\frac{88.38}{711.03}\right]$

$\phi' = 7.10°$

ϕ (angle from x' axis) $= 30° + 7.10°$

$\phi = 37.1°$ **Ans**

2-37. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed vertically upward and has a magnitude of 800 N.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 0 = F_1 \sin\theta + 400\cos 30° - 600\left(\frac{4}{5}\right)$$

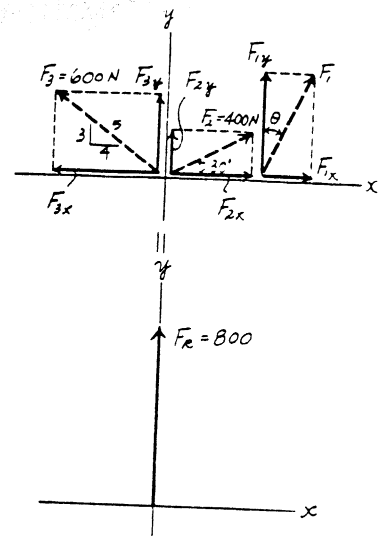
$$F_1 \sin\theta = 133.6 \qquad [1]$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 800 = F_1\cos\theta + 400\sin 30° + 600\left(\frac{3}{5}\right)$$

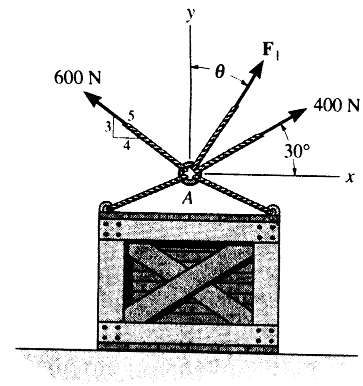
$$F_1\cos\theta = 240 \qquad [2]$$

Solving Eq. [1] and [2] yields

$\theta = 29.1°$ $\qquad F_1 = 275\text{ N}$ $\qquad$ **Ans**



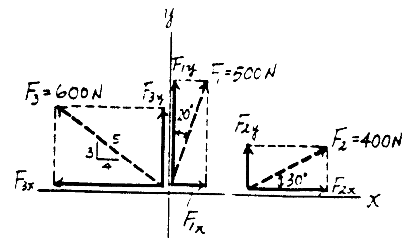
2-38. Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force of the three forces acting on the ring A. Take $F_1 = 500$ N and $\theta = 20°$.



Scalar Notation : Suming the force components algebraically, we have

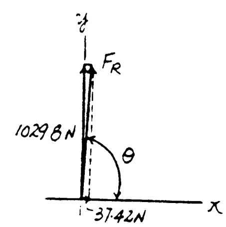
$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 500\sin 20° + 400\cos 30° - 600\left(\frac{4}{5}\right) = 37.42 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 500\cos 20° + 400\sin 30° + 600\left(\frac{3}{5}\right) = 1029.8 \text{ N} \uparrow$$



The magnitude of the resultant force $\mathbf{F}_R$ is

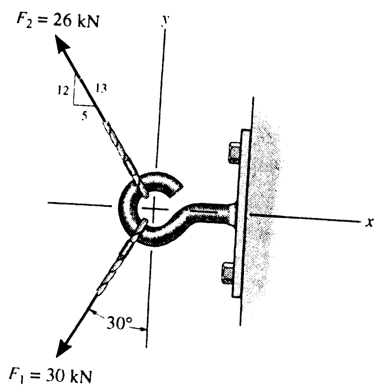
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{37.42^2 + 1029.8^2} = 1030.5 \text{ N} = 1.03 \text{ kN} \qquad \textbf{Ans}$$



The directional angle θ measured counterclockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{1029.8}{37.42}\right) = 87.9° \qquad \textbf{Ans}$$

2-39. Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.



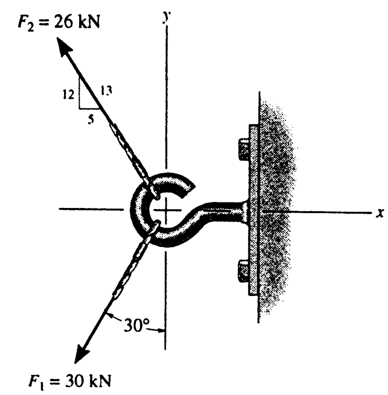
$$\mathbf{F}_1 = -30\sin 30°\,\mathbf{i} - 30\cos 30°\,\mathbf{j}$$

$$= \{-15.0\,\mathbf{i} - 26.0\,\mathbf{j}\} \text{ kN} \qquad \textbf{Ans}$$

$$\mathbf{F}_2 = -\frac{5}{13}(26)\,\mathbf{i} + \frac{12}{13}(26)\,\mathbf{j}$$

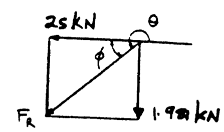
$$= \{-10.0\,\mathbf{i} + 24.0\,\mathbf{j}\} \text{ kN} \qquad \textbf{Ans}$$

***2-40.** Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = -30\sin 30° - \frac{5}{13}(26) = -25 \text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -30\cos 30° + \frac{12}{13}(26) = -1.981 \text{ kN}$

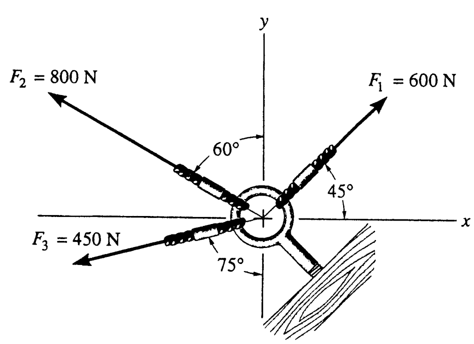


$F_R = \sqrt{(-25)^2 + (-1.981)^2} = 25.1 \text{ kN}$ **Ans**

$\phi = \tan^{-1}\left(\frac{1.981}{25}\right) = 4.53°$

$\theta = 180° + 4.53° = 185°$ **Ans**

2-41. Solve Prob. 2–1 by summing the rectangular or x, y components of the forces to obtain the resultant force.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 600\cos 45° - 800\sin 60° = -268.556 \text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 600\sin 45° + 800\cos 60° = 824.264 \text{N}$

$F_R = \sqrt{(824.264)^2 + (-268.556)^2} = 866.91 = 867 \text{ N}$ **Ans**

$\theta = 180° - \tan^{-1}\left(\frac{824.264}{268.556}\right)$

$= 180° - 71.95° = 108°$ **Ans**

2-42. Solve Prob. 2–22 by summing the rectangular or x, y components of the forces to obtain the resultant force.

$$F_x' = F_{1x} + F_{2x} = -30(\frac{4}{5}) - 20(\sin 20°) = -30.8404$$

$$F_y' = F_{1y} + F_{2y} = 30(\frac{3}{5}) - 20(\cos 20°) = -0.79385$$

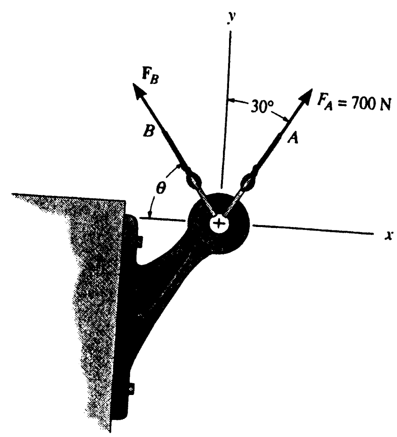
$$F_{Rx} = F_x' + F_{3x} = -30.8404 + 50 = 19.1596$$

$$F_{Ry} = F_y' + F_{3y} = -0.79385 + 0 = -0.79385$$

$$F_R = \sqrt{(19.1596)^2 + (-0.79385)^2} = 19.2 \text{ N} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}(\frac{-0.79385}{19.1596}) = -2.3726° = 2.37° \qquad \textbf{Ans}$$

2-43. Determine the magnitude and orientation θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



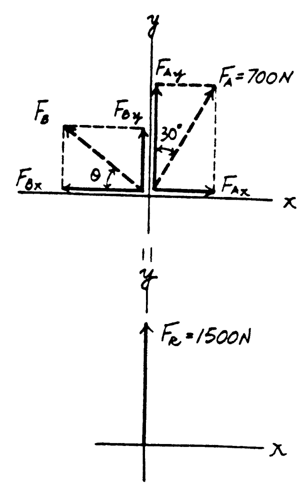
Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad 0 = 700\sin 30° - F_B\cos\theta$$
$$F_B\cos\theta = 350 \qquad [1]$$

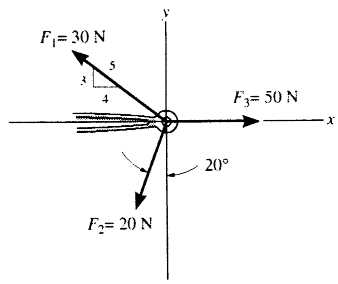
$$+\uparrow F_{R_y} = \Sigma F_y; \quad 1500 = 700\cos 30° + F_B\sin\theta$$
$$F_B\sin\theta = 893.8 \qquad [2]$$

Solving Eq. [1] and [2] yields

$$\theta = 68.6° \qquad F_B = 960 \text{ N} \qquad \textbf{Ans}$$



2-42. Solve Prob. 2-22 by summing the rectangular or x, y components of the forces to obtain the resultant force.



$$F'_x = F_{1x} + F_{2x} = -30\left(\frac{4}{5}\right) - 20(\sin 20^\circ) = -30.8404$$

$$F'_y = F_{1y} + F_{2y} = 30\left(\frac{3}{5}\right) - 20(\cos 20^\circ) = -0.79385$$

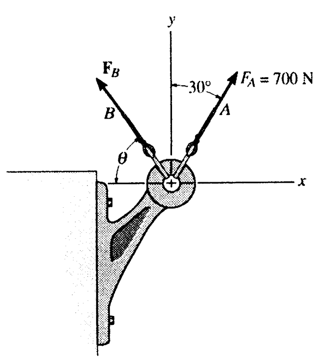
$$F_{Rx} = F'_x + F_{3x} = -30.8404 + 50 = 19.1596$$

$$F_{Ry} = F'_y + F_{3y} = -0.79385 + 0 = -0.79385$$

$$F_R = \sqrt{(19.1596)^2 + (-0.79385)^2} = 19.2 \text{ N} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{-0.79385}{19.1596}\right) = -2.3726^\circ = 2.37^\circ \ \theta \qquad \textbf{Ans}$$

2-43. Determine the magnitude and orientation θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \quad 0 = 700 \sin 30^\circ - F_B \cos\theta$$

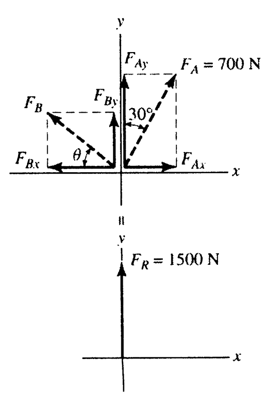
$$F_B \cos\theta = 350 \qquad [1]$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad 1500 = 700 \cos 30^\circ + F_B \sin\theta$$

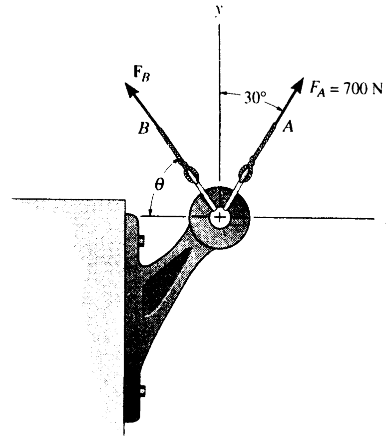
$$F_B \sin\theta = 893.8 \qquad [2]$$

Solving Eq. [1] and [2] yields

$\theta = 68.6^\circ \quad F_B = 960$ N*Ans*



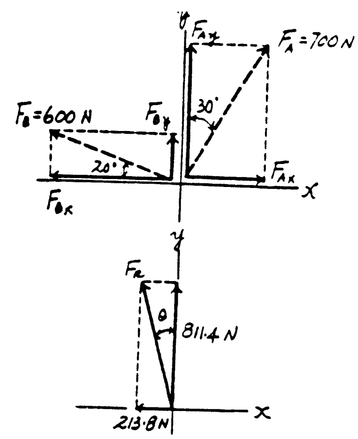
2-44. Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20°$.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x ; \quad F_{R_x} = 700\sin 30° - 600\cos 20° = -213.8\text{ N} = 213.8\text{ N} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y ; \quad F_{R_y} = 700\cos 30° + 600\sin 20° = 811.4\text{ N} \uparrow$$



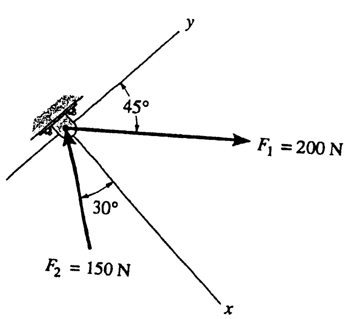
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839\text{ N} \qquad \textbf{Ans}$$

The directional angle θ measured counterclockwise from positive y axis is

$$\theta = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8° \qquad \textbf{Ans}$$

2-45. Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



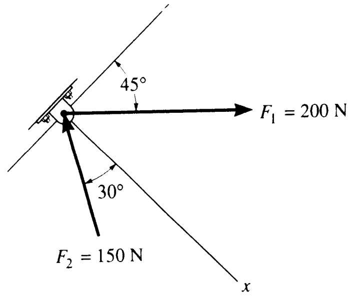
$F_{1x} = 200\sin 45° = 141$ N **Ans**

$F_{1y} = 200\cos 45° = 141$ N **Ans**

$F_{2x} = -150\cos 30° = -130$ N **Ans**

$F_{2y} = 150\sin 30° = 75$ N **Ans**

2-46. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



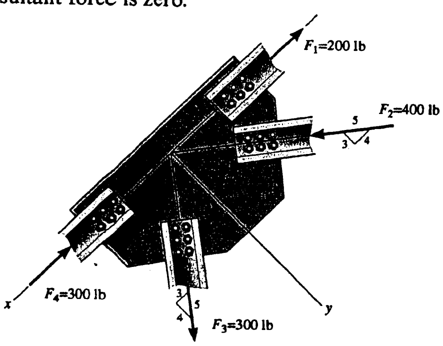
$+\searrow F_{Rx} = \Sigma F_x;\quad F_{Rx} = -150\cos 30° + 200\sin 45° = 11.518\text{ N}$

$\nearrow + F_{Ry} = \Sigma F_y;\quad F_{Ry} = 150\sin 30° + 200\cos 45° = 216.421\text{ N}$

$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217\text{ N}$ **Ans**

$\theta = \tan^{-1}(\frac{216.421}{11.518}) = 87.0°$ **Ans**

2-47. Determine the x and y components of each force acting on the *gusset plate* of the bridge truss. Show that the resultant force is zero.



$F_{1x} = -200\text{ lb}$ **Ans**

$F_{1y} = 0$ **Ans**

$F_{2x} = 400(\frac{4}{5}) = 320\text{ lb}$ **Ans**

$F_{2y} = -400(\frac{3}{5}) = -240\text{ lb}$ **Ans**

$F_{3x} = 300(\frac{3}{5}) = 180\text{ lb}$ **Ans**

$F_{3y} = 300(\frac{4}{5}) = 240\text{ lb}$ **Ans**

$F_{4x} = -300\text{ lb}$ **Ans**

$F_{4y} = 0$ **Ans**

$F_{Rx} = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

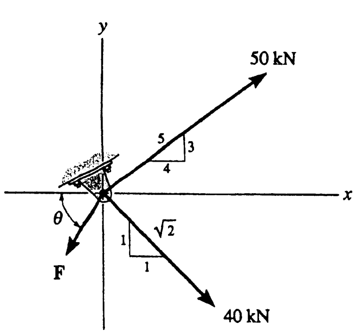
$F_{Rx} = -200 + 320 + 180 - 300 = 0$

$F_{Ry} = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$F_{Ry} = 0 - 240 + 240 + 0 = 0$

Thus, $F_R = 0$

***2-48.** If $\theta = 60°$ and $F = 20\text{ kN}$, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.



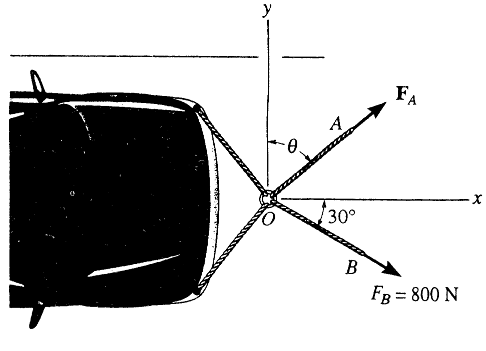
$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 50(\frac{4}{5}) + \frac{1}{\sqrt{2}}(40) - 20\cos 60° = 58.28\text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 50(\frac{3}{5}) - \frac{1}{\sqrt{2}}(40) - 20\sin 60° = -15.60\text{ kN}$

$F_R = \sqrt{(58.28)^2 + (-15.60)^2} = 60.3\text{ kN}$ **Ans**

$\theta = \tan^{-1}\left[\frac{15.60}{58.28}\right] = 15.0°$ **Ans**

2-49. Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



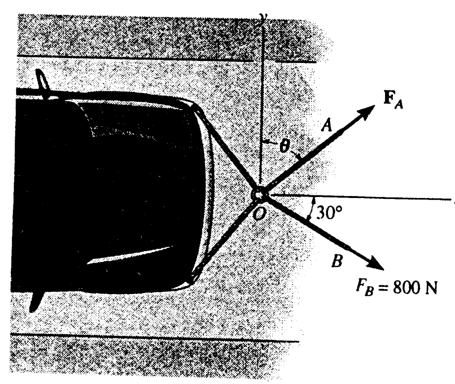
$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = F_A \sin\theta + 800\cos 30° = 1250$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = F_A \cos\theta - 800\sin 30° = 0$

$\theta = 54.3°$ **Ans**

$F_A = 686$ N **Ans**

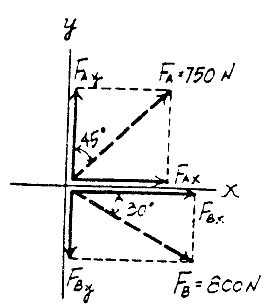
2-50. Determine the magnitude and direction, measured counterclockwise from the positive x axis, of the resultant force acting on the ring at O, if $F_A = 750$ N and $\theta = 45°$.



Scalar Notation : Suming the force components algebraically, we have

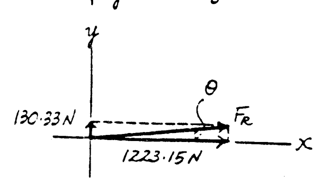
$\xrightarrow{+} F_{R_x} = \Sigma F_x;$ $\quad F_{R_x} = 750\sin 45° + 800\cos 30°$
$= 1223.15 \text{ N} \rightarrow$

$+\uparrow F_{R_y} = \Sigma F_y;$ $\quad F_{R_y} = 750\cos 45° - 800\sin 30°$
$= 130.33 \text{ N} \uparrow$



The magnitude of the resultant force $\mathbf{F}_R$ is

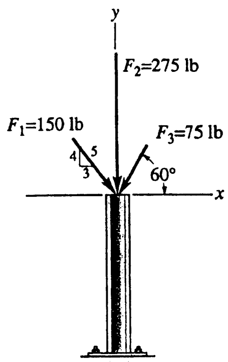
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$
$$= \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \quad \textbf{Ans}$$



The directional angle θ measured counterclockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08° \quad \textbf{Ans}$$

2-51. Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



$$\mathbf{F}_1 = 150\left(\frac{3}{5}\right)\mathbf{i} - 150\left(\frac{4}{5}\right)\mathbf{j}$$

$$\mathbf{F}_1 = \{90\mathbf{i} - 120\mathbf{j}\}\ \text{lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_2 = \{-275\mathbf{j}\}\ \text{lb} \qquad \textbf{Ans}$$

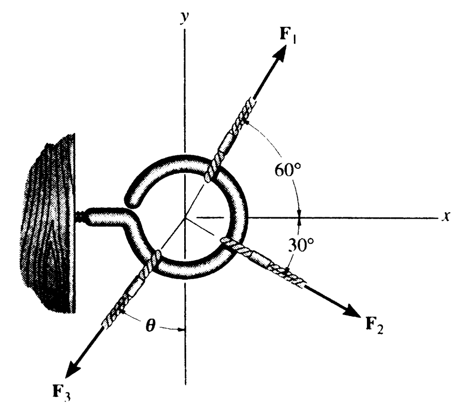
$$\mathbf{F}_3 = -75\cos 60°\mathbf{i} - 75\sin 60°\mathbf{j}$$

$$\mathbf{F}_3 = \{-37.5\mathbf{i} - 65.0\mathbf{j}\}\ \text{lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{52.5\mathbf{i} - 460\mathbf{j}\}\ \text{lb}$$

$$F_R = \sqrt{(52.5)^2 + (-460)^2} = 463\ \text{lb} \qquad \textbf{Ans}$$

***2-52.** The three concurrent forces acting on the screw eye produce a resultant force $\mathbf{F}_R = 0$. If $F_2 = \frac{2}{3}F_1$ and $\mathbf{F}_1$ is to be 90° from $\mathbf{F}_2$ as shown, determine the required magnitude of $\mathbf{F}_3$ expressed in terms of F_1 and the angle θ.



Cartesian Vector Notation :

$$\mathbf{F}_1 = F_1\cos 60°\mathbf{i} + F_1\sin 60°\mathbf{j}$$
$$= 0.50F_1\mathbf{i} + 0.8660F_1\mathbf{j}$$

$$\mathbf{F}_2 = \frac{2}{3}F_1\cos 30°\mathbf{i} - \frac{2}{3}F_1\sin 30°\mathbf{j}$$
$$= 0.5774F_1\mathbf{i} - 0.3333F_1\mathbf{j}$$

$$\mathbf{F}_3 = -F_3\sin\theta\mathbf{i} - F_3\cos\theta\mathbf{j}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{0} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$0 = (0.50F_1 + 0.5774F_1 - F_3\sin\theta)\mathbf{i}$$
$$+ (0.8660F_1 - 0.3333F_1 - F_3\cos\theta)\mathbf{j}$$

Equating **i** and **j** components, we have

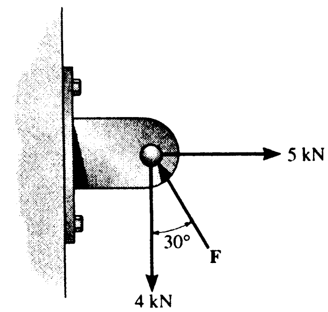
$$0.50F_1 + 0.5774F_1 - F_3\sin\theta = 0 \qquad [1]$$

$$0.8660F_1 - 0.3333F_1 - F_3\cos\theta = 0 \qquad [2]$$

Solving Eq. [1] and [2] yields

$$\theta = 63.7° \qquad F_3 = 1.20F_1 \qquad \textbf{Ans}$$

2-53. Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?



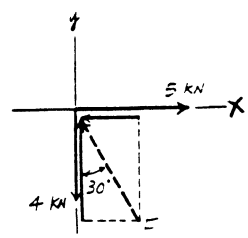
Scalar Notation : Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 5 - F\sin 30^\circ$$
$$= 5 - 0.50F \ \rightarrow$$

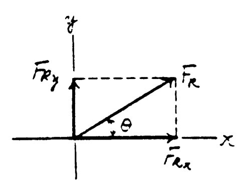
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = F\cos 30^\circ - 4$$
$$= 0.8660F - 4 \ \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$
$$= \sqrt{(5 - 0.50F)^2 + (0.8660F - 4)^2}$$
$$= \sqrt{F^2 - 11.93F + 41} \qquad [1]$$



$$F_R^2 = F^2 - 11.93F + 41$$
$$2F_R\frac{dF_R}{dF} = 2F - 11.93 \qquad [2]$$
$$\left(F_R\frac{d^2F_R}{dF^2} + \frac{dF_R}{dF} \times \frac{dF_R}{dF}\right) = 1 \qquad [3]$$



In order to obtain the *minimum* resultant force $\mathbf{F}_R$, $\frac{dF_R}{dF} = 0$. From Eq.[2]

$$2F_R\frac{dF_R}{dF} = 2F - 11.93 = 0$$

$$F = 5.964 \text{ kN} = 5.96 \text{ kN} \qquad \textbf{Ans}$$

Substituting $F = 5.964$ kN into Eq.[1], we have

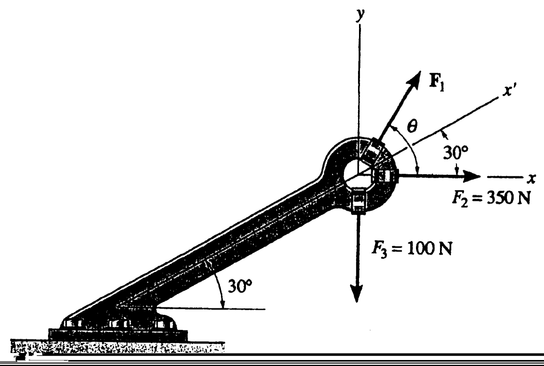
$$F_R = \sqrt{5.964^2 - 11.93(5.964) + 41}$$
$$= 2.330 \text{ kN} = 2.33 \text{ kN} \qquad \textbf{Ans}$$

Substituting $F_R = 2.330$ kN with $\frac{dF_R}{dF} = 0$ into Eq.[3], we have

$$\left[(2.330)\frac{d^2F_R}{dF^2} + 0\right] = 1$$
$$\frac{d^2F_R}{dF^2} = 0.429 > 0$$

Hence, $F = 5.96$ kN is indeed producing a minimum resultant force.

2-54. Express each of the three forces acting on the bracket in Cartesian vector form with respect to the x and y axes. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of $F_R = 600$ N.



$\mathbf{F}_1 = \{F_1 \cos\theta\,\mathbf{i} + F_1 \sin\theta\,\mathbf{j}\}$ N **Ans**

$\mathbf{F}_2 = \{350\mathbf{i}\}$ N **Ans**

$\mathbf{F}_3 = \{-100\mathbf{j}\}$ N **Ans**

Require,

$$\mathbf{F}_R = 600 \cos 30°\,\mathbf{i} + 600 \sin 30°\,\mathbf{j}$$

$$\mathbf{F}_R = \{519.6\mathbf{i} + 300\mathbf{j}\}\ \text{N}$$

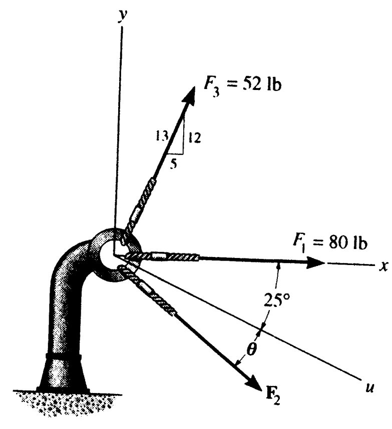
$$\mathbf{F}_R = \Sigma\mathbf{F}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields :

$$519.6 = F_1 \cos\theta + 350$$

$$F_1 \cos\theta = 169.6$$

*2-56. Three forces act on the bracket. Determine the magnitude and orientation θ of $\mathbf{F}_2$ so that the resultant force is directed along the positive u axis and has a magnitude of 50 lb.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad 50\cos 25^\circ = 80 + 52\left(\frac{5}{13}\right) + F_2\cos(25^\circ + \theta)$$

$$F_2\cos(25^\circ + \theta) = -54.684 \qquad [1]$$

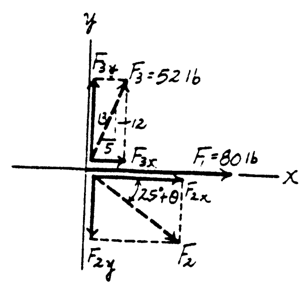
$$+\uparrow F_{R_y} = \Sigma F_y; \quad -50\sin 25^\circ = 52\left(\frac{12}{13}\right) - F_2\sin(25^\circ + \theta)$$

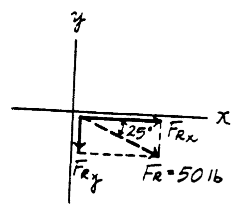
$$F_2\sin(25^\circ + \theta) = 69.131 \qquad [2]$$

Solving Eq.[1] and [2] yields

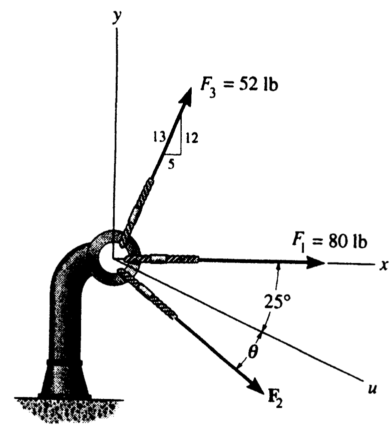
$25^\circ + \theta = 128.35^\circ \qquad \theta = 103^\circ$ **Ans**

$F_2 = 88.1$ lb **Ans**





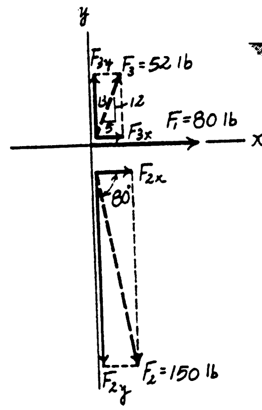
*2-57.** If $F_2 = 150$ lb and $\theta = 55°$, determine the magnitude and orientation, measured clockwise from the positive x axis, of the resultant force of the three forces acting on the bracket.

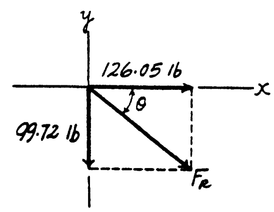


Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 80 + 52\left(\frac{5}{13}\right) + 150\cos 80°$$
$$= 126.05 \text{ lb} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 52\left(\frac{12}{13}\right) - 150\sin 80°$$
$$= -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow$$





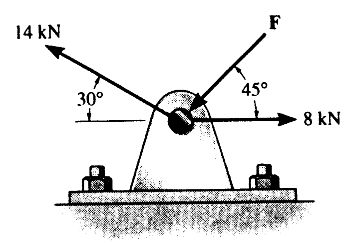
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb} \qquad \textbf{Ans}$$

The directional angle θ measured clockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{99.72}{126.05}\right) = 38.3° \qquad \textbf{Ans}$$

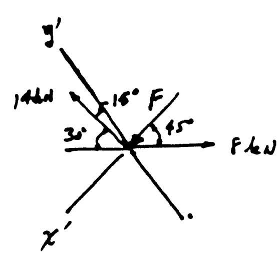
2-58. Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 8 - F\cos 45° - 14\cos 30°$$
$$= -4.1244 - F\cos 45°$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -F\sin 45° + 14\sin 30°$$
$$= 7 - F\sin 45°$$

$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad (1)$$



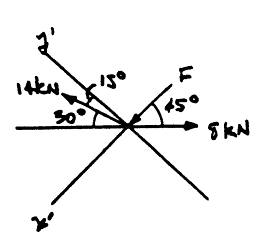
$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$

$$F = 2.03 \text{ kN} \qquad \textbf{Ans}$$

From Eq. (1); $\quad F_R = 7.87$ kN $\quad$ **Ans**

Also, from the figure require

$$(F_R)_{x'} = 0 = \Sigma F_{x'}; \quad F + 14\sin 15° - 8\cos 45° = 0$$



$$F = 2.03 \text{ kN} \qquad \textbf{Ans}$$

$$(F_R)_{y'} = \Sigma F_{y'}; \quad F_R = 14\cos 15° - 8\sin 45°$$

$$F_R = 7.87 \text{ kN} \qquad \textbf{Ans}$$

2-59. Determine the magnitude and coordinate direction angles of $\mathbf{F}_1 = \{60\mathbf{i} - 50\mathbf{j} + 40\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-40\mathbf{i} - 85\mathbf{j} + 30\mathbf{k}\}$ N. Sketch each force on an x, y, z reference.

$\mathbf{F}_1 = 60\,\mathbf{i} - 50\,\mathbf{j} + 40\,\mathbf{k}$

$F_1 = \sqrt{(60)^2 + (-50)^2 + (40)^2} = 87.750 = 87.7\text{ N}$ **Ans**

$\alpha_1 = \cos^{-1}\left(\frac{60}{87.750}\right) = 46.9°$ **Ans**

$\beta_1 = \cos^{-1}\left(\frac{-50}{87.750}\right) = 125°$ **Ans**

$\gamma_1 = \cos^{-1}\left(\frac{40}{87.750}\right) = 62.9°$ **Ans**

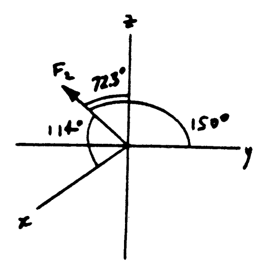
$\mathbf{F}_2 = -40\,\mathbf{i} - 85\,\mathbf{j} + 30\,\mathbf{k}$

$F_2 = \sqrt{(-40)^2 + (-85)^2 + (30)^2} = 98.615 = 98.6\text{ N}$ **Ans**

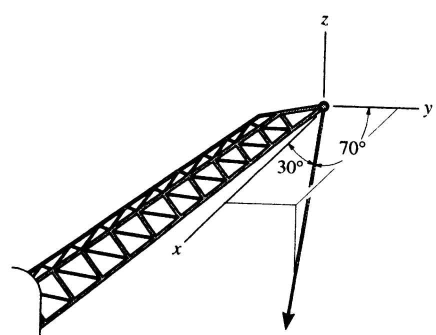
$\alpha_2 = \cos^{-1}\left(\frac{-40}{98.615}\right) = 114°$ **Ans**

$\beta_2 = \cos^{-1}\left(\frac{-85}{98.615}\right) = 150°$ **Ans**

$\gamma_2 = \cos^{-1}\left(\frac{30}{98.615}\right) = 72.3°$ **Ans**



***2-60.** The cable at the end of the crane boom exerts a force of 250 lb on the boom as shown. Express **F** as a Cartesian vector.



$F = 250$ lb

Cartesian Vector Notation : With $\alpha = 30°$ and $\beta = 70°$, the third coordinate direction angle γ can be determined using Eq. 2 – 10.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\cos^2 30° + \cos^2 70° + \cos^2\gamma = 1$$
$$\cos\gamma = \pm 0.3647$$

$$\gamma = 68.61° \text{ or } 111.39°$$

By inspection, $\gamma = 111.39°$ since the force **F** is directed in negative octant.

$$\mathbf{F} = 250\{\cos 30°\mathbf{i} + \cos 70°\mathbf{j} + \cos 111.39°\}\text{ lb}$$
$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\}\text{ lb}$$ **Ans**

2-61. Determine the magnitude and coordinate direction angles of the force **F** acting on the stake.

$$\frac{4}{5}F = 40, \qquad F = 50 \text{ N}$$

$$\mathbf{F} = \left(40\cos 70°\mathbf{i} + 40\sin 70°\mathbf{j} + \frac{3}{5}(50)\mathbf{k}\right)$$

$$\mathbf{F} = \{13.7\mathbf{i} + 37.6\mathbf{j} + 30.0\mathbf{k}\} \text{ N} \qquad \textbf{Ans}$$

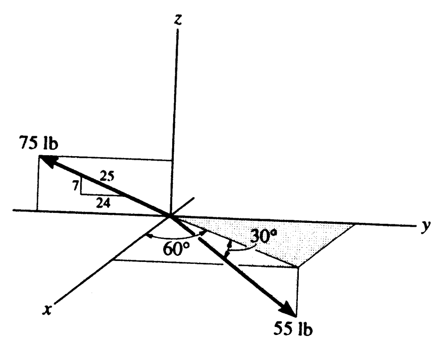
$$F = \sqrt{(13.68)^2 + (37.59)^2 + (30)^2} = 50 \text{ N} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{13.68}{50}\right) = 74.1° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{37.59}{50}\right) = 41.3° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{30}{50}\right) = 53.1° \qquad \textbf{Ans}$$

2-62. Determine the magnitude and the coordinate direction angles of the resultant force.



Cartesian Vector Notation :

$$\mathbf{F}_1 = 75\left\{-\frac{24}{25}\mathbf{j} + \frac{7}{25}\mathbf{k}\right\} \text{ lb} = \{-72.0\mathbf{j} + 21.0\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = 55\{\cos 30°\cos 60°\mathbf{i} + \cos 30°\sin 60°\mathbf{j} - \sin 30°\mathbf{k}\} \text{ lb}$$
$$= \{23.82\mathbf{i} + 41.25\mathbf{j} - 27.5\mathbf{k}\} \text{ lb}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= \{23.82\mathbf{i} + (-72.0 + 41.25)\mathbf{j} + (21.0 - 27.5)\mathbf{k}\} \text{ lb}$$
$$= \{23.82\mathbf{i} - 30.75\mathbf{j} - 6.50\mathbf{k}\} \text{ lb}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$
$$= \sqrt{23.82^2 + (-30.75)^2 + (-6.50)^2}$$
$$= 39.43 \text{ lb} = 39.4 \text{ lb} \qquad \textbf{Ans}$$

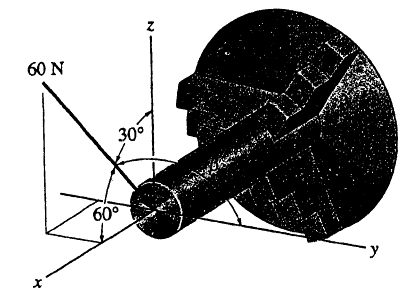
The coordinate direction angles are

$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{23.82}{39.43} \qquad \alpha = 52.8° \qquad \textbf{Ans}$$

$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{-30.75}{39.43} \qquad \beta = 141° \qquad \textbf{Ans}$$

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{-6.50}{39.43} \qquad \gamma = 99.5° \qquad \textbf{Ans}$$

2-63. The stock S mounted on the lathe is subjected to a force of 60 N, which is caused by the die D. Determine the coordinate direction angle β and express the force as a Cartesian vector.



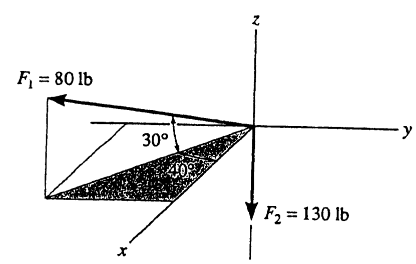
$\cos^2 60° + \cos^2\beta + \cos^2 30° = 1$

$\beta = 90°$ **Ans**

$\mathbf{F} = -60(\cos 60°\mathbf{i} + \cos 90°\mathbf{j} + \cos 45°\mathbf{k})$

$\mathbf{F} = \{-30\mathbf{i} - 52.0\mathbf{k}\}$ N **Ans**

***2-64.** Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



$\mathbf{F}_1 = (80\cos 30°\cos 40°\,\mathbf{i} - 80\cos 30°\sin 40°\mathbf{j} + 80\sin 30°\mathbf{k})$

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

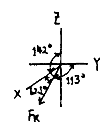
$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb

$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114$ lb **Ans**

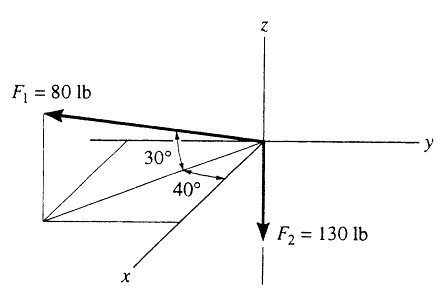
$\alpha = \cos^{-1}(\frac{53.1}{113.6}) = 62.1°$ **Ans**

$\beta = \cos^{-1}(\frac{-44.5}{113.6}) = 113°$ **Ans**

$\gamma = \cos^{-1}(\frac{-90.0}{113.6}) = 142°$ **Ans**



2-65. Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



$\mathbf{F}_1 = (80\cos 30°\cos 40°\,\mathbf{i} - 80\cos 30°\sin 40°\mathbf{j} + 80\sin 30°\mathbf{k})$

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb **Ans**

$\alpha_1 = \cos^{-1}(\frac{53.1}{80}) = 48.4°$ **Ans**

$\beta_1 = \cos^{-1}(\frac{-44.5}{80}) = 124°$ **Ans**

$\gamma_1 = \cos^{-1}(\frac{40}{80}) = 60°$ **Ans**

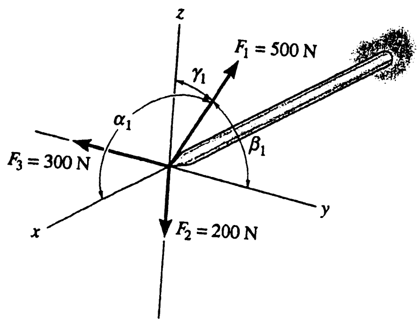
$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb **Ans**

$\alpha_2 = \cos^{-1}(\frac{0}{130}) = 90°$ **Ans**

$\beta_2 = \cos^{-1}(\frac{0}{130}) = 90°$ **Ans**

$\gamma_2 = \cos^{-1}(\frac{-130}{130}) = 180°$ **Ans**

2-66. The mast is subjected to the three forces shown. Determine the coordinate direction angles α_1, β_1, γ_1 of $\mathbf{F}_1$ so that the resultant force acting on the mast is $\mathbf{F}_R = \{350\mathbf{i}\}$ N.



$$\mathbf{F}_1 = 500\cos\alpha_1\,\mathbf{i} + 500\cos\beta_1\,\mathbf{j} + 500\cos\gamma_1\,\mathbf{k}$$

$$\mathbf{F}_R = \mathbf{F}_1 + (-300\mathbf{j}) + (-200\mathbf{k})$$

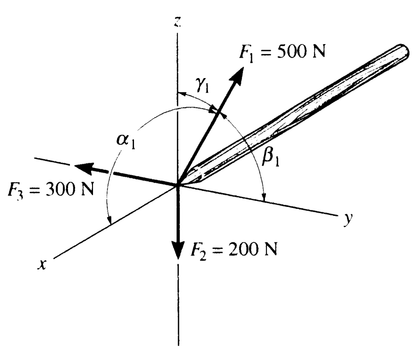
$$350\mathbf{i} = 500\cos\alpha_1\,\mathbf{i} + (500\cos\beta_1 - 300)\mathbf{j} + (500\cos\gamma_1 - 200)\mathbf{k}$$

$350 = 500\cos\alpha_1;$ $\qquad \alpha_1 = 45.6°$ **Ans**

$0 = 500\cos\beta_1 - 300;$ $\qquad \beta_1 = 53.1°$ **Ans**

$0 = 500\cos\gamma_1 - 200;$ $\qquad \gamma_1 = 66.4°$ **Ans**

2-67. The mast is subjected to the three forces shown. Determine the coordinate direction angles α_1, β_1, γ_1 of $\mathbf{F}_1$ so that the resultant force acting on the mast is zero.



$$\mathbf{F}_1 = \{500\cos\alpha_1\,\mathbf{i} + 500\cos\beta_1\,\mathbf{j} + 500\cos\gamma_1\,\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = \{-200\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{-300\mathbf{j}\}\text{ N}$$

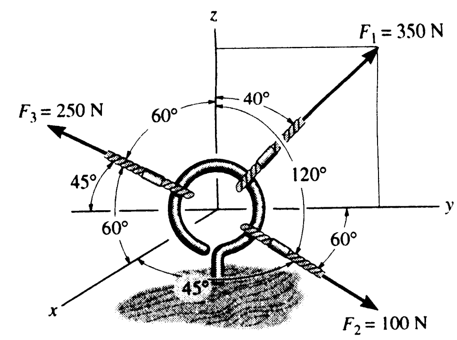
$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$$

$500\cos\alpha_1 = 0;$ $\qquad \alpha_1 = 90°$ **Ans**

$500\cos\beta_1 = 300;$ $\qquad \beta_1 = 53.1°$ **Ans**

$500\cos\gamma_1 = 200;$ $\qquad \gamma_1 = 66.4°$ **Ans**

***2-68.** The cables attached to the screw eye are subjected to the three forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



Cartesian Vector Notation :

$$\mathbf{F}_1 = 350\{\sin 40°\mathbf{j} + \cos 40°\mathbf{k}\} \text{ N}$$
$$= \{224.98\mathbf{j} + 268.12\mathbf{k}\} \text{ N}$$
$$= \{225\mathbf{j} + 268\mathbf{k}\} \text{ N} \qquad \textbf{Ans}$$

$$\mathbf{F}_2 = 100\{\cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}\} \text{ N}$$
$$= \{70.71\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\} \text{ N}$$
$$= \{70.7\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\} \text{ N} \qquad \textbf{Ans}$$

$$\mathbf{F}_3 = 250\{\cos 60°\mathbf{i} + \cos 135°\mathbf{j} + \cos 60°\mathbf{k}\} \text{ N}$$
$$= \{125.0\mathbf{i} - 176.78\mathbf{j} + 125.0\mathbf{k}\} \text{ N}$$
$$= \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\} \text{ N} \qquad \textbf{Ans}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= \{(70.71 + 125.0)\mathbf{i} + (224.98 + 50.0 - 176.78)\mathbf{j} + (268.12 - 50.0 + 125.0)\mathbf{k}\} \text{ N}$$
$$= \{195.71\mathbf{i} + 98.20\mathbf{j} + 343.12\mathbf{k}\} \text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$
$$= \sqrt{195.71^2 + 98.20^2 + 343.12^2}$$
$$= 407.03 \text{ N} = 407 \text{ N} \qquad \textbf{Ans}$$

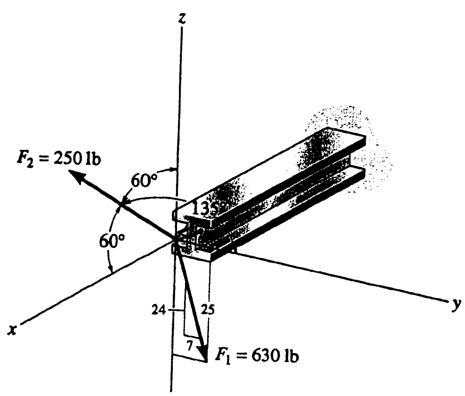
The coordinate direction angles are

$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{195.71}{407.03} \qquad \alpha = 61.3° \qquad \textbf{Ans}$$

$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{98.20}{407.03} \qquad \beta = 76.0° \qquad \textbf{Ans}$$

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{343.12}{407.03} \qquad \gamma = 32.5° \qquad \textbf{Ans}$$

2-69. The beam is subjected to the two forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{F}_1 = 630\left(\frac{7}{25}\right)\mathbf{j} - 630\left(\frac{24}{25}\right)\mathbf{k}$$

$$\mathbf{F}_1 = (176.4\,\mathbf{j} - 604.8\mathbf{k})$$

$$\mathbf{F}_1 = \{176\mathbf{j} - 605\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_2 = 250\cos 60°\mathbf{i} + 250\cos 135°\mathbf{j} + 250\cos 60°\mathbf{k}$$

$$\mathbf{F}_2 = (125\mathbf{i} - 176.777\mathbf{j} + 125\mathbf{k})$$

$$\mathbf{F}_2 = \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = 125\mathbf{i} - 0.3767\mathbf{j} - 479.8\mathbf{k}$$

$$\mathbf{F}_R = \{125\mathbf{i} - 0.377\mathbf{j} - 480\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$F_R = \sqrt{(125)^2 + (-0.3767)^2 + (-479.8)^2} = 495.82$$

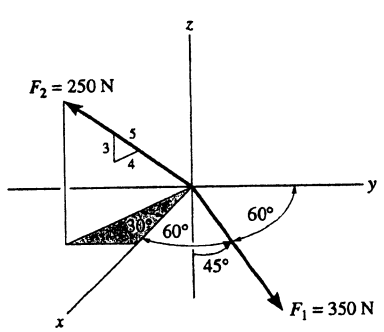
$$= 496\text{ lb} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{125}{495.82}\right) = 75.4° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-0.3767}{495.82}\right) = 90.0° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-479.8}{495.82}\right) = 165° \qquad \textbf{Ans}$$

2-70. Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = 350\cos 60°\mathbf{i} + 350\cos 60°\mathbf{j} - 350\cos 45°\mathbf{k} + 250\left(\frac{4}{5}\right)\cos 30°\mathbf{i} - 250\left(\frac{4}{5}\right)\sin 30°\mathbf{j} + 250\left(\frac{3}{5}\right)\mathbf{k}$$

$$\mathbf{F}_R = \{348.21\mathbf{i} + 75.0\mathbf{j} - 97.487\mathbf{k}\}\text{ N}$$

$$F_R = \sqrt{(348.21)^2 + (75.0)^2 - (97.487)^2}$$

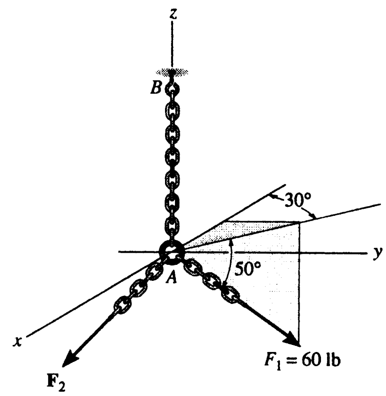
$$= 369.29 = 369\text{ N} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{348.21}{369.29}\right) = 19.5° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{75.0}{369.29}\right) = 78.3° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-97.487}{369.29}\right) = 105° \qquad \textbf{Ans}$$

2-71. The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at A have a resultant force of $\mathbf{F}_R = \{-100\mathbf{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$.



Cartesian Vector Notation :

$$\mathbf{F}_R = \{-100\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_1 = 60\{-\cos 50^\circ \cos 30^\circ \mathbf{i} + \cos 50^\circ \sin 30^\circ \mathbf{j} - \sin 50^\circ \mathbf{k}\} \text{ lb}$$
$$= \{-33.40\mathbf{i} + 19.28\mathbf{j} - 45.96\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = \{F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j} + F_{2_z}\mathbf{k}\} \text{ lb}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$-100\mathbf{k} = \{(F_{2_x} - 33.40)\mathbf{i} + (F_{2_y} + 19.28)\mathbf{j} + (F_{2_z} - 45.96)\mathbf{k}\}$$

Equating **i**, **j** and **k** components, we have

$$F_{2_x} - 33.40 = 0 \qquad F_{2_x} = 33.40 \text{ lb}$$
$$F_{2_y} + 19.28 = 0 \qquad F_{2_y} = -19.28 \text{ lb}$$
$$F_{2_z} - 45.96 = -100 \qquad F_{2_z} = -54.04 \text{ lb}$$

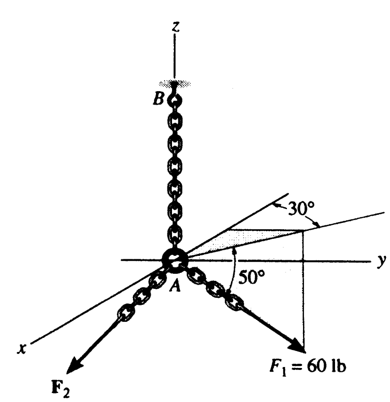
The magnitude of force $\mathbf{F}_2$ is

$$F_2 = \sqrt{F_{2_x}^2 + F_{2_y}^2 + F_{2_z}^2}$$
$$= \sqrt{33.40^2 + (-19.28)^2 + (-54.04)^2}$$
$$= 66.39 \text{ lb} = 66.4 \text{ lb} \qquad \textbf{Ans}$$

The coordinate direction angles for $\mathbf{F}_2$ are

$$\cos\alpha = \frac{F_{2_x}}{F_2} = \frac{33.40}{66.39} \qquad \alpha = 59.8^\circ \qquad \textbf{Ans}$$
$$\cos\beta = \frac{F_{2_y}}{F_2} = \frac{-19.28}{66.39} \qquad \beta = 107^\circ \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{F_{2_z}}{F_2} = \frac{-54.04}{66.39} \qquad \gamma = 144^\circ \qquad \textbf{Ans}$$

***2-72.** Determine the coordinate direction angles of the force $\mathbf{F}_1$ and indicate them on the figure.



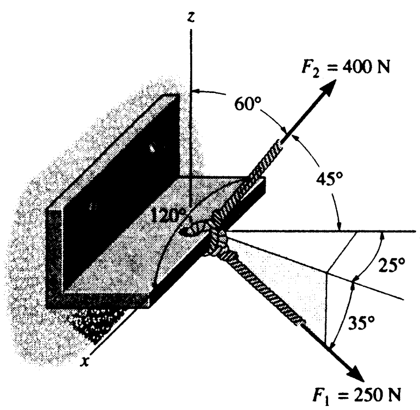
Unit Vector For Foce $\mathbf{F}_1$:

$$\mathbf{u}_{F_1} = -\cos 50^\circ \cos 30^\circ \mathbf{i} + \cos 50^\circ \sin 30^\circ \mathbf{j} - \sin 50^\circ \mathbf{k}$$
$$= -0.5567\mathbf{i} + 0.3214\mathbf{j} - 0.7660\mathbf{k}$$

Coordinate Direction Angles : From the unit vector obtained above, we have

$$\cos\alpha = -0.5567 \qquad \alpha = 124^\circ \qquad \textbf{Ans}$$
$$\cos\beta = 0.3214 \qquad \beta = 71.3^\circ \qquad \textbf{Ans}$$
$$\cos\gamma = -0.7660 \qquad \gamma = 140^\circ \qquad \textbf{Ans}$$

2-73. The bracket is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force $\mathbf{F}_R$. Find the magnitude and coordinate direction angles of the resultant force.



Cartesian Vector Notation :

$$\mathbf{F}_1 = 250\{\cos 35^\circ \sin 25^\circ \mathbf{i} + \cos 35^\circ \cos 25^\circ \mathbf{j} - \sin 35^\circ \mathbf{k}\}\ \text{N}$$
$$= \{86.55\mathbf{i} + 185.60\mathbf{j} - 143.39\mathbf{k}\}\ \text{N}$$
$$= \{86.5\mathbf{i} + 186\mathbf{j} - 143\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_2 = 400\{\cos 120^\circ \mathbf{i} + \cos 45^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}\}\ \text{N}$$
$$= \{-200.0\mathbf{i} + 282.84\mathbf{j} + 200.0\mathbf{k}\}\ \text{N}$$
$$= \{-200\mathbf{i} + 283\mathbf{j} + 200\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= \{(86.55 - 200.0)\mathbf{i} + (185.60 + 282.84)\mathbf{j} + (-143.39 + 200.0)\mathbf{k}\}$$
$$= \{-113.45\mathbf{i} + 468.44\mathbf{j} + 56.61\mathbf{k}\}\ \text{N}$$
$$= \{-113\mathbf{i} + 468\mathbf{j} + 56.6\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

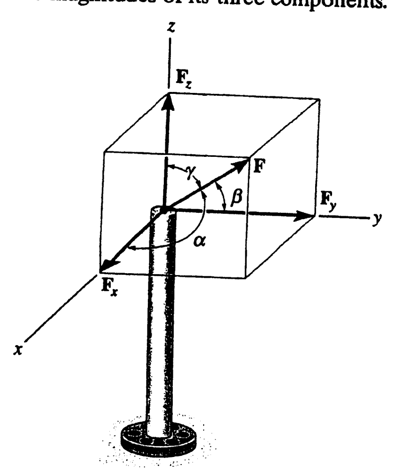
The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$
$$= \sqrt{(-113.45)^2 + 468.44^2 + 56.61^2}$$
$$= 485.30\ \text{N} = 485\ \text{N} \qquad \textbf{Ans}$$

The coordinate direction angles are

$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{-113.45}{485.30} \qquad \alpha = 104^\circ \qquad \textbf{Ans}$$
$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{468.44}{485.30} \qquad \beta = 15.1^\circ \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{56.61}{485.30} \qquad \gamma = 83.3^\circ \qquad \textbf{Ans}$$

2-74. The pole is subjected to the force **F**, which has components acting along the x, y, z axes as shown. If the magnitude of **F** is 3 kN, and $\beta = 30^\circ$ and $\gamma = 75^\circ$, determine the magnitudes of its three components.



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 30^\circ + \cos^2 75^\circ = 1$$

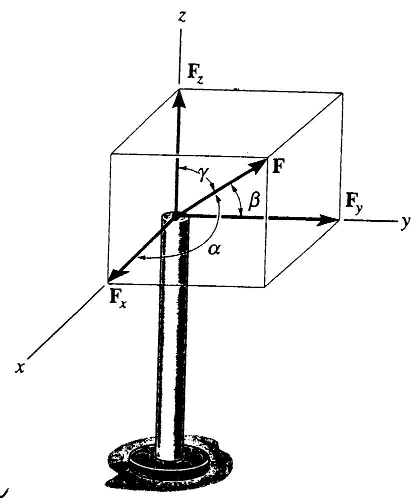
$$\alpha = 64.67^\circ$$

$$F_x = 3\cos 64.67^\circ = 1.28\ \text{kN} \qquad \textbf{Ans}$$

$$F_y = 3\cos 30^\circ = 2.60\ \text{kN} \qquad \textbf{Ans}$$

$$F_z = 3\cos 75^\circ = 0.776\ \text{kN} \qquad \textbf{Ans}$$

2-75. The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.



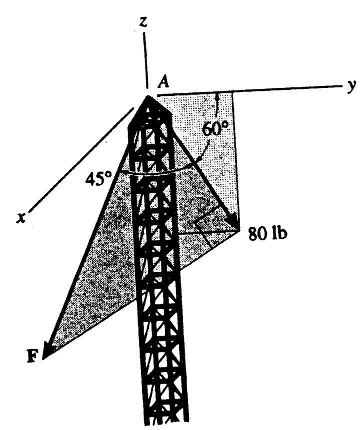
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

$F = 2.02$ kN **Ans**

$F_y = 2.02 \cos 75° = 0.523$ kN **Ans**

***2-76.** A force **F** is applied at the top of the tower at A. If it acts in the direction shown such that one of its components lying in the shaded y-z plane has a magnitude of 80 lb, determine its magnitude F and coordinate direction angles α, β, γ.



Cartesian Vector Notation : The magnitude of force **F** is

$F\cos 45° = 80$ $\quad F = 113.14$ lb $= 113$ lb **Ans**

Thus,

$$\mathbf{F} = \{113.14\sin 45°\mathbf{i} + 80\cos 60°\mathbf{j} - 80\sin 60°\mathbf{k}\}\ \text{lb}$$
$$= \{80.0\mathbf{i} + 40.0\mathbf{j} - 69.28\mathbf{k}\}\ \text{lb}$$

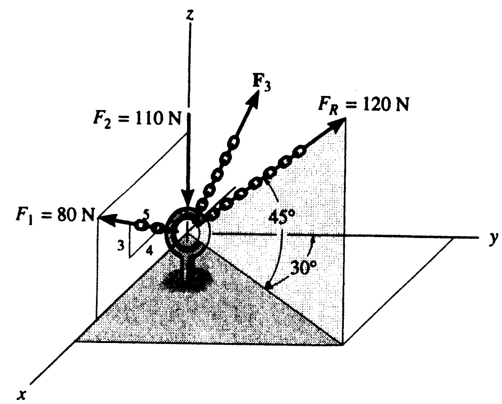
The coordinate direction angles are

$\cos\alpha = \frac{F_x}{F} = \frac{80.0}{113.14}$ $\quad \alpha = 45.0°$ **Ans**

$\cos\beta = \frac{F_y}{F} = \frac{40.0}{113.14}$ $\quad \beta = 69.3°$ **Ans**

$\cos\gamma = \frac{F_z}{F} = \frac{-69.28}{113.14}$ $\quad \gamma = 128°$ **Ans**

2-77. Three forces act on the hook. If the resultant force $\mathbf{F}_R$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F}_3$.



Cartesian Vector Notation :

$$\mathbf{F}_R = 120\{\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}\}\ \text{N}$$
$$= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\}\text{N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\}\ \text{N}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}$$
$$= \left\{\left(64.0 + F_{3_x}\right)\mathbf{i} + F_{3_y}\mathbf{j} + \left(48.0 - 110 + F_{3_z}\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57\ \text{N}$$
$$F_{3_y} = 73.48\ \text{N}$$
$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85\ \text{N}$$

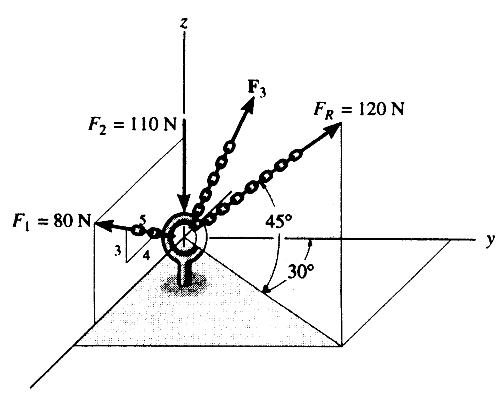
The magnitude of force $\mathbf{F}_3$ is

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2}$$
$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$
$$= 165.62\ \text{N} = 166\ \text{N} \qquad \textbf{Ans}$$

The coordinate direction angles for $\mathbf{F}_3$ are

$$\cos\alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5° \qquad \textbf{Ans}$$
$$\cos\beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7° \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5° \qquad \textbf{Ans}$$

2-78. Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.



Unit Vector of $\mathbf{F}_1$ and $\mathbf{F}_R$:

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

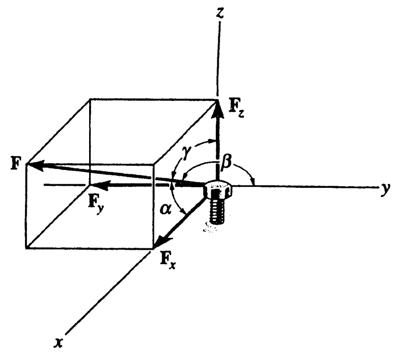
$$\mathbf{u}_R = \cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}$$
$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

$$\cos\alpha_{F_1} = 0.8 \qquad \alpha_{F_1} = 36.9° \qquad \textbf{Ans}$$
$$\cos\beta_{F_1} = 0 \qquad \beta_{F_1} = 90.0° \qquad \textbf{Ans}$$
$$\cos\gamma_{F_1} = 0.6 \qquad \gamma_{F_1} = 53.1° \qquad \textbf{Ans}$$

$$\cos\alpha_R = 0.3536 \qquad \alpha_R = 69.3° \qquad \textbf{Ans}$$
$$\cos\beta_R = 0.6124 \qquad \beta_R = 52.2° \qquad \textbf{Ans}$$
$$\cos\gamma_R = 0.7071 \qquad \gamma_R = 45.0° \qquad \textbf{Ans}$$

2-79. The bolt is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 80 N, and $\alpha = 60^\circ$ and $\gamma = 45^\circ$, determine the magnitudes of its components.



$$\cos\beta = \sqrt{1 - \cos^2\alpha - \cos^2\gamma}$$

$$= \sqrt{1 - \cos^2 60^\circ - \cos^2 45^\circ}$$

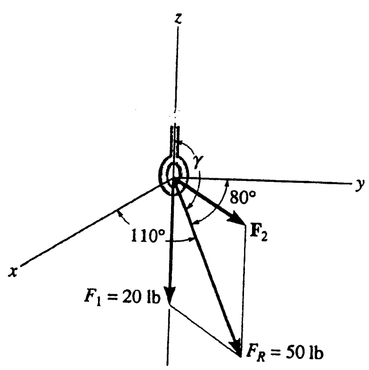
$\beta = 120^\circ$

$F_x = |80\cos 60^\circ| = 40\text{ N}$ **Ans**

$F_y = |80\cos 120^\circ| = 40\text{ N}$ **Ans**

$F_z = |80\cos 45^\circ| = 56.6\text{ N}$ **Ans**

***2-80.** Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bolt. If the resultant force $\mathbf{F}_R$ has a magnitude of 50 lb and coordinate direction angles $\alpha = 110^\circ$ and $\beta = 80^\circ$, as shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



$(1)^2 = \cos^2 110^\circ + \cos^2 80^\circ + \cos^2\gamma$

$\gamma = 157.44^\circ$

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$50\cos 110^\circ = (F_2)_x$

$50\cos 80^\circ = (F_2)_y$

$50\cos 157.44^\circ = (F_2)_z - 20$

$(F_2)_x = -17.10$

$(F_2)_y = 8.68$

$(F_2)_z = -26.17$

$F_2 = \sqrt{(-17.10)^2 + (8.68)^2 + (-26.17)^2} = 32.4\text{ lb}$ **Ans**

$\alpha_2 = \cos^{-1}\left(\frac{-17.10}{32.4}\right) = 122^\circ$ **Ans**

$\beta_2 = \cos^{-1}\left(\frac{8.68}{32.4}\right) = 74.5^\circ$ **Ans**

$\gamma_2 = \cos^{-1}\left(\frac{-26.17}{32.4}\right) = 144^\circ$ **Ans**

2-81. If $\mathbf{r}_1 = \{3\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m, $\mathbf{r}_2 = \{4\mathbf{i} - 5\mathbf{k}\}$ m, $\mathbf{r}_3 = \{3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}\}$ m, determine the magnitude and direction of $\mathbf{r} = 2\mathbf{r}_1 - \mathbf{r}_2 + 3\mathbf{r}_3$.

$\mathbf{r} = 2\mathbf{r}_1 - \mathbf{r}_2 + 3\mathbf{r}_3$

$= 6\mathbf{i} - 8\mathbf{j} + 6\mathbf{k} - 4\mathbf{i} + 5\mathbf{k} + 9\mathbf{i} - 6\mathbf{j} + 15\mathbf{k}$

$= 11\mathbf{i} - 14\mathbf{j} + 26\mathbf{k}$

$r = \sqrt{(11)^2 + (-14)^2 + (26)^2} = 31.51\text{ m} = 31.5\text{ m}$ **Ans**

$$\mathbf{u}_r = \frac{11}{31.51}\mathbf{i} - \frac{14}{31.51}\mathbf{j} + \frac{26}{31.51}\mathbf{k}$$

$\alpha = \cos^{-1}\left(\frac{11}{31.51}\right) = 69.6^\circ$ **Ans**

$\beta = \cos^{-1}\left(\frac{-14}{31.51}\right) = 116^\circ$ **Ans**

$\gamma = \cos^{-1}\left(\frac{26}{31.51}\right) = 34.4^\circ$ **Ans**

2-82. Represent the position vector **r** acting from point $A(3\text{ m}, 5\text{ m}, 6\text{ m})$ to point $B(5\text{ m}, -2\text{ m}, 1\text{ m})$ in Cartesian vector form. Determine its coordinate direction angles and find the distance between points A and B.

Position Vector : This can be established from the coordinates of two points.

$$\mathbf{r}_{AB} = \{(5-3)\mathbf{i} + (-2-5)\mathbf{j} + (1-6)\mathbf{k}\}\text{ ft}$$
$$= \{2\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}\}\text{ ft} \qquad \textbf{Ans}$$

The distance between point A and B is

$$r_{AB} = \sqrt{2^2 + (-7)^2 + (-5)^2} = \sqrt{78}\text{ ft} = 8.83\text{ ft} \qquad \textbf{Ans}$$

The coordinate direction angles are

$$\cos\alpha = \frac{2}{\sqrt{78}} \qquad \alpha = 76.9° \qquad \textbf{Ans}$$
$$\cos\beta = \frac{-7}{\sqrt{78}} \qquad \beta = 142° \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{-5}{\sqrt{78}} \qquad \gamma = 124° \qquad \textbf{Ans}$$

2-83. A position vector extends from the origin to point A (2 m, 3 m, 6 m). Determine the angles α, β, γ which the tail of the vector makes with the x, y, z axes, respectively.

Position Vector : This can be established from the coordinates of two points.

$$\mathbf{r} = \{2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}\}\text{ ft} \qquad \textbf{Ans}$$

The distance between point A and B is

$$r_{AB} = \sqrt{2^2 + (3)^2 + (6)^2} = 7\text{ft} \qquad \textbf{Ans}$$

The coordinate direction angles are

$$\cos\alpha = \frac{2}{7} \qquad \alpha = 73.4° \qquad \textbf{Ans}$$
$$\cos\beta = \frac{3}{7} \qquad \beta = 64.6° \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{6}{7} \qquad \gamma = 31.0° \qquad \textbf{Ans}$$

2-82. Represent the position vector r acting from point $A(3\text{ m}, 5\text{ m}, 6\text{ m})$ to point $B(5\text{ m}, -2\text{ m}, 1\text{ m})$ in Cartesian vector form. Determine its coordinate direction angles and find the distance between points A and B.

Position Vector: This can be established from the coordinates of two points.

$$\mathbf{r}_{AB} = \{(5-3)\mathbf{i} + (-2-5)\mathbf{j} + (1-6)\mathbf{k}\}\text{ft}$$

$$= \{2\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}\}\text{ft} \qquad \textbf{Ans}$$

The distance between point A and B is

$$r_{AB} = \sqrt{2^2 + (-7)^2 + (-5)^2} = \sqrt{78}\text{ ft} = 8.83\text{ ft} \qquad \textbf{Ans}$$

The coordinate direction angles are

$$\cos\alpha = \frac{2}{\sqrt{78}} \qquad \alpha = 76.9^\circ \qquad \textbf{Ans}$$

$$\cos\beta = \frac{-7}{\sqrt{78}} \qquad \beta = 142^\circ \qquad \textbf{Ans}$$

$$\cos\gamma = \frac{-5}{\sqrt{78}} \qquad \gamma = 124^\circ \qquad \textbf{Ans}$$

2-83. A position vector extends from the origin to point $A(2\text{ m}, 3\text{ m}, 6\text{ m})$. Determine the angles α, β, γ which the tail of the vector makes with the x, y, z axes, respectively.

Position Vector: This can be established from the coordinates of two points.

$$\mathbf{r} = \{2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}\}\text{ ft} \qquad \textbf{Ans}$$

The distance between point A and B is

$$r_{AB} = \sqrt{2^2 + (3)^2 + (6)^2} = 7\text{ ft} \qquad \textbf{Ans}$$

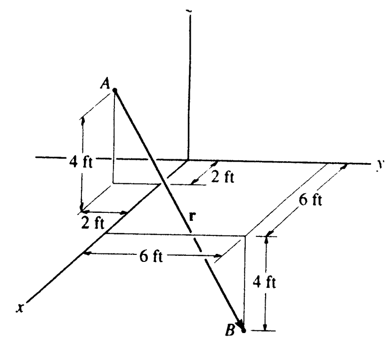
The coordinate direction angles are

$$\cos\alpha = \frac{2}{7} \qquad \alpha = 73.4^\circ \qquad \textbf{Ans}$$

$$\cos\beta = \frac{3}{7} \qquad \beta = 64.6^\circ \qquad \textbf{Ans}$$

$$\cos\gamma = \frac{6}{7} \qquad \gamma = 31.0^\circ \qquad \textbf{Ans}$$

*2-84. Express the position vector **r** in Cartesian vector form; then determine its magnitude and coordinate direction angles.



Position Vector:

$$\mathbf{r} = \{(6-2)\mathbf{i} + [6-(-2)]\mathbf{j} + (-4-4)\mathbf{k}\}\text{ ft}$$
$$= \{4\mathbf{i} + 8\mathbf{j} - 8\mathbf{k}\}\text{ ft} \qquad \textbf{Ans}$$

The magnitude of **r** is

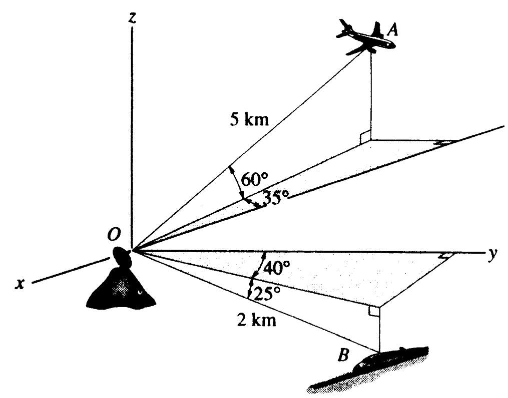
$$r = \sqrt{4^2 + 8^2 + (-8)^2} = 12.0\text{ ft} \qquad \textbf{Ans}$$

The coordinate direction angles are

$$\cos\alpha = \frac{4}{12.0} \qquad \alpha = 70.5° \qquad \textbf{Ans}$$
$$\cos\beta = \frac{8}{12.0} \qquad \beta = 48.2° \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{-8}{12.0} \qquad \gamma = 132° \qquad \textbf{Ans}$$

2-86. Express force **F** as a Cartesian

***2-88.** At a given instant, the position of a plane at A and a train at B are measured relative to a radar antenna at O. Determine the distance d between A and B at this instant. To solve the problem, formulate a position vector, directed from A to B, and then determine its magnitude.



Position Vector : The coordinates of points A and B are

$$A(-5\cos 60^\circ\cos 35^\circ,\ -5\cos 60^\circ\sin 35^\circ,\ 5\sin 60^\circ)\ \text{km}$$
$$= A(-2.048,\ -1.434,\ 4.330)\ \text{km}$$

$$B(2\cos 25^\circ\sin 40^\circ,\ 2\cos 25^\circ\cos 40^\circ,\ -2\sin 25^\circ)\ \text{km}$$
$$= B(1.165,\ 1.389,\ -0.845)\ \text{km}$$

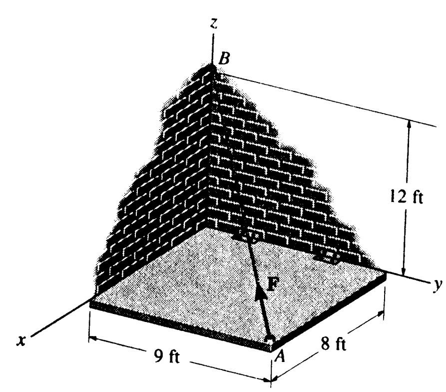
The position vector $\mathbf{r}_{AB}$ can be established from the coordinates of points A and B.

$$\mathbf{r}_{AB} = \{[1.165-(-2.048)]\mathbf{i} + [1.389-(-1.434)]\mathbf{j} + (-0.845-4.330)\mathbf{k}\}\ \text{km}$$
$$= \{3.213\mathbf{i} + 2.822\mathbf{j} - 5.175\mathbf{k}\}\ \text{km}$$

The distance between points A and B is

$$d = r_{AB} = \sqrt{3.213^2 + 2.822^2 + (-5.175)^2} = 6.71\ \text{km} \qquad \textbf{Ans}$$

2-89. The hinged plate is supported by the cord AB. If the force in the cord is $F = 340$ lb, express this force, directed from A toward B, as a Cartesian vector. What is the length of the cord?



Unit Vector :

$$\mathbf{r}_{AB} = \{(0-8)\mathbf{i} + (0-9)\mathbf{j} + (12-0)\mathbf{k}\}\ \text{ft}$$
$$= \{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}\}\ \text{ft}$$

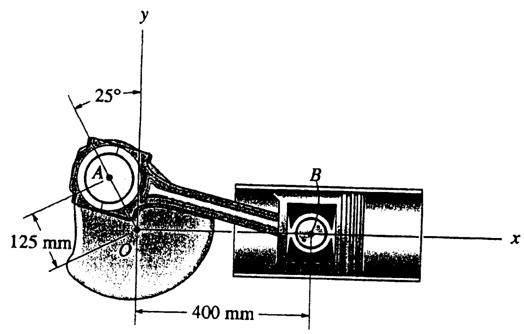
$$r_{AB} = \sqrt{(-8)^2 + (-9)^2 + 12^2} = 17.0\ \text{ft} \qquad \textbf{Ans}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}}{17} = -\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}$$

Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 340\left\{-\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}\right\}\ \text{lb}$$
$$= \{-160\mathbf{i} - 180\mathbf{j} + 240\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

2-90. Determine the length of the crankshaft AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.

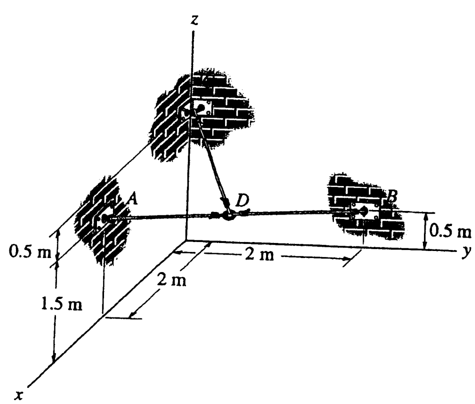


$$\mathbf{r}_{AB} = ((400 + 125\sin 25°)\mathbf{i} - 125\cos 25°\mathbf{j})$$

$$\mathbf{r}_{AB} = \{452.83\mathbf{i} - 113.3\mathbf{j}\}\text{ mm}$$

$$r_{AB} = \sqrt{(452.83)^2 + (-113.3)^2} = 467\text{ mm} \quad \textbf{Ans}$$

2-91. Determine the lengths of wires AD, BD, and CD. The ring at D is midway between A and B.



$$D\left(\frac{2+0}{2}, \frac{0+2}{2}, \frac{1.5+0.5}{2}\right)\text{ m} = D(1, 1, 1)\text{ m}$$

$$\mathbf{r}_{AD} = (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1.5)\mathbf{k}$$

$$= -1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{r}_{BD} = (1-0)\mathbf{i} + (1-2)\mathbf{j} + (1-0.5)\mathbf{k}$$

$$= 1\mathbf{i} - 1\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{r}_{CD} = (1-0)\mathbf{i} + (1-0)\mathbf{j} + (1-2)\mathbf{k}$$

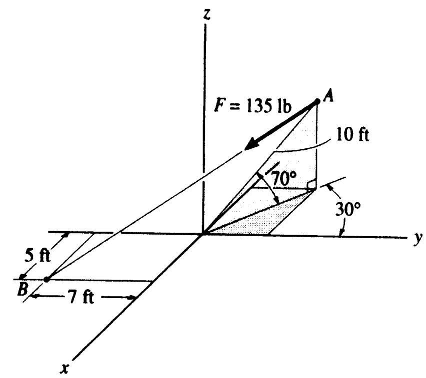
$$= 1\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$$

$$r_{AD} = \sqrt{(-1)^2 + 1^2 + (-0.5)^2} = 1.50\text{ m} \quad \textbf{Ans}$$

$$r_{BD} = \sqrt{1^2 + (-1)^2 + 0.5^2} = 1.50\text{ m} \quad \textbf{Ans}$$

$$r_{CD} = \sqrt{1^2 + 1^2 + (-1)^2} = 1.73\text{ m} \quad \textbf{Ans}$$

***2-92.** Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



Unit Vector : The coordinates of point A are

$$A(-10\cos 70°\sin 30°,\ 10\cos 70°\cos 30°,\ 10\sin 70°)\ \text{ft}$$
$$= A(-1.710,\ 2.962,\ 9.397)\ \text{ft}$$

Then

$$\mathbf{r}_{AB} = \{[5-(-1.710)]\mathbf{i} + (-7-2.962)\mathbf{j} + (0-9.397)\mathbf{k}\}\ \text{ft}$$
$$= \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{6.710^2 + (-9.962)^2 + (-9.397)^2} = 15.250\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}}{15.250}$$
$$= 0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k}$$

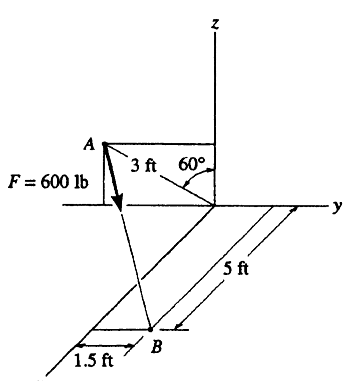
Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 135\{0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k}\}\ \text{lb}$$
$$= \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

Coordinate Direction Angles : From the unit vector $\mathbf{u}_{AB}$ obtained above, we have

$\cos\alpha = 0.4400$	$\alpha = 63.9°$	**Ans**
$\cos\beta = -0.6532$	$\beta = 131°$	**Ans**
$\cos\gamma = -0.6162$	$\gamma = 128°$	**Ans**

2-93. Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



$$\mathbf{r} = (5\mathbf{i} + (1.5 + 3\sin 60°)\mathbf{j} + (0 - 3\cos 60°)\mathbf{k})$$

$$\mathbf{r} = \{5\mathbf{i} + 4.098\mathbf{j} - 1.5\mathbf{k}\}\ \text{ft}$$

$$\mathbf{u} = \frac{\mathbf{r}}{r} = (0.7534\mathbf{i} + 0.6175\mathbf{j} - 0.226\mathbf{k})$$

$$\mathbf{F} = 600\mathbf{u} = (452.04\mathbf{i} + 370.49\mathbf{j} - 135.61\mathbf{k})$$

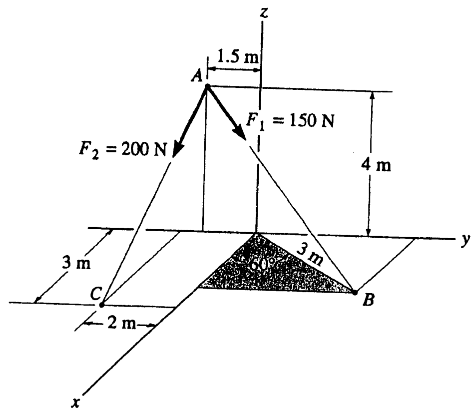
$$\mathbf{F} = \{452\mathbf{i} + 370\mathbf{j} - 136\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{452.04}{600}\right) = 41.1° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{370.49}{600}\right) = 51.9° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-135.61}{600}\right) = 103° \qquad \textbf{Ans}$$

2-94. Determine the magnitude and coordinate direction angles of the resultant force acting at point A.



$$\mathbf{r}_{AC} = \{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$|\mathbf{r}_{AC}| = \sqrt{3^2 + (-0.5)^2 + (-4)^2} = \sqrt{25.25} = 5.02494$$

$$\mathbf{F}_2 = 200\left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.02494}\right) = (119.4044\mathbf{i} - 19.9007\mathbf{j} - 159.2059\mathbf{k})$$

$$\mathbf{r}_{AB} = (3\cos 60°\mathbf{i} + (1.5 + 3\sin 60°)\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_{AB} = (1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k})$$

$$|\mathbf{r}_{AB}| = \sqrt{(1.5)^2 + (4.0981)^2 + (-4)^2} = 5.9198$$

$$\mathbf{F}_1 = 150\left(\frac{1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k}}{5.9198}\right) = (38.0080\mathbf{i} + 103.8405\mathbf{j} - 101.3548\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (157.4124\mathbf{i} + 83.9398\mathbf{j} - 260.5607\mathbf{k})$$

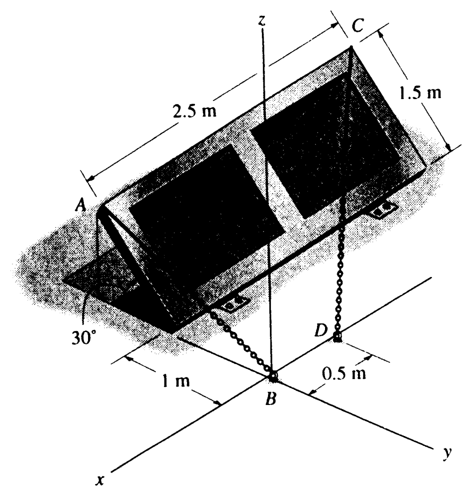
$$F_R = \sqrt{(157.4124)^2 + (83.9398)^2 + (-260.5607)^2} = 315.7791 = 316\text{ N} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{157.4124}{315.7791}\right) = 60.099° = 60.1° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{83.9398}{315.7791}\right) = 74.584° = 74.6° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-260.5607}{315.7791}\right) = 145.60° = 146° \qquad \textbf{Ans}$$

2-95. The door is held opened by means of two chains. If the tension in AB and CD is $F_A = 300$ N and $F_C = 250$ N, respectively, express each of these forces in Cartesian vector form.



Unit Vector : First determine the position vector $\mathbf{r}_{AB}$ and $\mathbf{r}_{CD}$. The coordinates of points A and C are

$$A[0,\ -(1+1.5\cos 30^\circ),\ 1.5\sin 30^\circ]\ \text{m} = A(0,\ -2.299,\ 0.750)\ \text{m}$$
$$C[-2.50,\ -(1+1.5\cos 30^\circ),\ 1.5\sin 30^\circ]\ \text{m} = C(-2.50,\ -2.299,\ 0.750)\ \text{m}$$

Then

$$\mathbf{r}_{AB} = \{(0-0)\mathbf{i} + [0-(-2.299)]\mathbf{j} + (0-0.750)\mathbf{k}\}\ \text{m}$$
$$= \{2.299\mathbf{j} - 0.750\mathbf{k}\}\ \text{m}$$
$$r_{AB} = \sqrt{2.299^2 + (-0.750)^2} = 2.418\ \text{m}$$
$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.299\mathbf{j} - 0.750\mathbf{k}}{2.418} = 0.9507\mathbf{j} - 0.3101\mathbf{k}$$

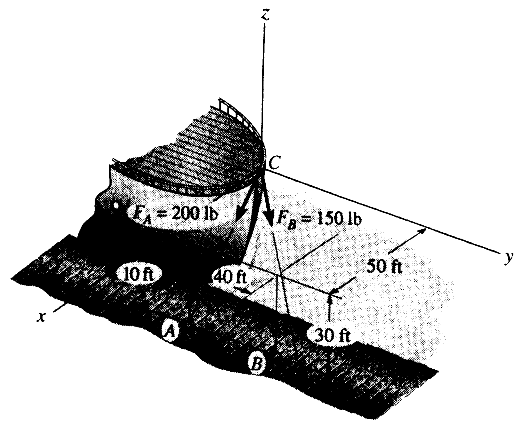
$$\mathbf{r}_{CD} = \{[-0.5-(-2.5)]\mathbf{i} + [0-(-2.299)]\mathbf{j} + (0-0.750)\mathbf{k}\}\ \text{m}$$
$$= \{2.00\mathbf{i} + 2.299\mathbf{j} - 0.750\mathbf{k}\}\ \text{m}$$
$$r_{CD} = \sqrt{2.00^2 + 2.299^2 + (-0.750)^2} = 3.138\ \text{m}$$
$$\mathbf{u}_{CD} = \frac{\mathbf{r}_{CD}}{r_{CD}} = \frac{2.00\mathbf{i} + 2.299\mathbf{j} - 0.750\mathbf{k}}{3.138} = 0.6373\mathbf{i} + 0.7326\mathbf{j} - 0.2390\mathbf{k}$$

Force Vector :

$$\mathbf{F}_A = F_A\mathbf{u}_{AB} = 300\{0.9507\mathbf{j} - 0.3101\mathbf{k}\}\ \text{N}$$
$$= \{285.21\mathbf{j} - 93.04\mathbf{k}\}\ \text{N}$$
$$= \{285\mathbf{j} - 93.0\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_C = F_C\mathbf{u}_{CD} = 250\{0.6373\mathbf{i} + 0.7326\mathbf{j} - 0.2390\mathbf{k}\}\ \text{N}$$
$$= \{159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}\}\ \text{N}$$
$$= \{159\mathbf{i} + 183\mathbf{j} - 59.7\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

*2-96. The two mooring cables exert forces on the stern of a ship as shown. Represent each force as as Cartesian vector and determine the magnitude and direction of the resultant.



Unit Vector :

$$\mathbf{r}_{CA} = \{(50-0)\mathbf{i}+(10-0)\mathbf{j}+(-30-0)\mathbf{k}\}\text{ ft} = \{50\mathbf{i}+10\mathbf{j}-30\mathbf{k}\}\text{ ft}$$
$$r_{CA} = \sqrt{50^2+10^2+(-30)^2} = 59.16\text{ ft}$$
$$\mathbf{u}_{CA} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{50\mathbf{i}+10\mathbf{j}-30\mathbf{k}}{59.16} = 0.8452\mathbf{i}+0.1690\mathbf{j}-0.5071\mathbf{k}$$

$$\mathbf{r}_{CB} = \{(50-0)\mathbf{i}+(50-0)\mathbf{j}+(-30-0)\mathbf{k}\}\text{ ft} = \{50\mathbf{i}+50\mathbf{j}-30\mathbf{k}\}\text{ ft}$$
$$r_{CB} = \sqrt{50^2+50^2+(-30)^2} = 76.81\text{ ft}$$
$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{50\mathbf{i}+50\mathbf{j}-30\mathbf{k}}{76.81} = 0.6509\mathbf{i}+0.6509\mathbf{j}-0.3906\mathbf{k}$$

Force Vector :

$$\mathbf{F}_A = F_A\mathbf{u}_{CA} = 200\{0.8452\mathbf{i}+0.1690\mathbf{j}-0.5071\mathbf{k}\}\text{ lb}$$
$$= \{169.03\mathbf{i}+33.81\mathbf{j}-101.42\mathbf{k}\}\text{ lb}$$
$$= \{169\mathbf{i}+33.8\mathbf{j}-101\mathbf{k}\}\text{ lb}$$ **Ans**

$$\mathbf{F}_B = F_B\mathbf{u}_{CB} = 150\{0.6509\mathbf{i}+0.6509\mathbf{j}-0.3906\mathbf{k}\}\text{ lb}$$
$$= \{97.64\mathbf{i}+97.64\mathbf{j}-58.59\mathbf{k}\}\text{ lb}$$
$$= \{97.6\mathbf{i}+97.6\mathbf{j}-58.6\mathbf{k}\}\text{ lb}$$ **Ans**

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_A+\mathbf{F}_B$$
$$= \{(169.03+97.64)\mathbf{i}+(33.81+97.64)\mathbf{j}+(-101.42-58.59)\mathbf{k}\}\text{ lb}$$
$$= \{266.67\mathbf{i}+131.45\mathbf{j}-160.00\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{266.67^2+131.45^2+(-160.00)^2}$$
$$= 337.63\text{ lb} = 338\text{ lb}$$ **Ans**

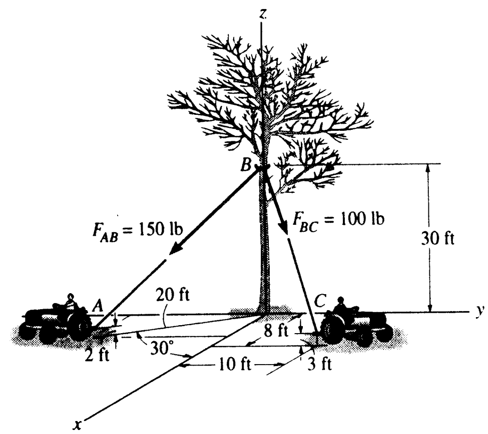
The coordinate direction angles of $\mathbf{F}_R$ are

$$\cos\alpha = \frac{266.67}{337.63} \qquad \alpha = 37.8^\circ$$ **Ans**

$$\cos\beta = \frac{131.45}{337.63} \qquad \beta = 67.1^\circ$$ **Ans**

$$\cos\gamma = -\frac{160.00}{337.63} \qquad \gamma = 118^\circ$$ **Ans**

2-97. Two tractors pull on the tree with the forces shown. Represent each force as a Cartesian vector and then determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{r}_{BA} = \{(20\cos 30° - 0)\mathbf{i} + (-20\sin 30° - 0)\mathbf{j} + (2 - 30)\mathbf{k}\} \text{ ft}$$
$$= \{17.32\mathbf{i} - 10.0\mathbf{j} - 28.0\mathbf{k}\} \text{ ft}$$
$$r_{BA} = \sqrt{17.32^2 + (-10.0)^2 + (-28.0)^2} = 34.41 \text{ ft}$$
$$\mathbf{u}_{BA} = \frac{\mathbf{r}_{BA}}{r_{BA}} = \frac{17.32\mathbf{i} - 10.0\mathbf{j} - 28.0\mathbf{k}}{34.41} = 0.5034\mathbf{i} - 0.2906\mathbf{j} - 0.8137\mathbf{k}$$

$$\mathbf{r}_{BC} = \{(8-0)\mathbf{i} + (10-0)\mathbf{j} + (3-30)\mathbf{k}\} \text{ ft} = \{8\mathbf{i} + 10\mathbf{j} - 27\mathbf{k}\} \text{ ft}$$
$$r_{BC} = \sqrt{8^2 + 10^2 + (-27)^2} = 29.88 \text{ ft}$$
$$\mathbf{u}_{BC} = \frac{\mathbf{r}_{BC}}{r_{BC}} = \frac{8\mathbf{i} + 10\mathbf{j} - 27\mathbf{k}}{29.88} = 0.2677\mathbf{i} + 0.3346\mathbf{j} - 0.9035\mathbf{k}$$

Force Vector:

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{BA} = 150\{0.5034\mathbf{i} - 0.2906\mathbf{j} - 0.8137\mathbf{k}\} \text{ lb}$$
$$= \{75.51\mathbf{i} - 43.59\mathbf{j} - 122.06\mathbf{k}\} \text{ lb}$$
$$= \{75.5\mathbf{i} - 43.6\mathbf{j} - 122\mathbf{k}\} \text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{u}_{BC} = 100\{0.2677\mathbf{i} + 0.3346\mathbf{j} - 0.9035\mathbf{k}\} \text{ lb}$$
$$= \{26.77\mathbf{i} + 33.46\mathbf{j} - 90.35\mathbf{k}\} \text{ lb}$$
$$= \{26.8\mathbf{i} + 33.5\mathbf{j} - 90.4\mathbf{k}\} \text{ lb} \qquad \textbf{Ans}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{BC}$$
$$= \{(75.51 + 26.77)\mathbf{i} + (-43.59 + 33.46)\mathbf{j} + (-122.06 - 90.35)\mathbf{k}\} \text{ lb}$$
$$= \{102.28\mathbf{i} - 10.13\mathbf{j} - 212.41\mathbf{k}\} \text{ lb}$$

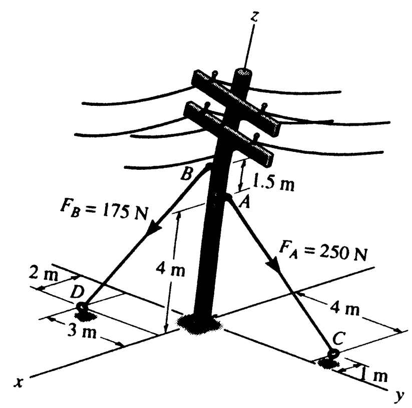
The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{102.28^2 + (-10.13)^2 + (-212.41)^2}$$
$$= 235.97 \text{ lb} = 236 \text{ lb} \qquad \textbf{Ans}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\cos\alpha = \frac{102.28}{235.97} \qquad \alpha = 64.3° \qquad \textbf{Ans}$$
$$\cos\beta = -\frac{10.13}{235.97} \qquad \beta = 92.5° \qquad \textbf{Ans}$$
$$\cos\gamma = -\frac{212.41}{235.97} \qquad \gamma = 154° \qquad \textbf{Ans}$$

2-98. The guy wires are used to support the telephone pole. Represent the force in each wire in Cartesian vector form.



Unit Vector:

$$\mathbf{r}_{AC} = \{(-1-0)\mathbf{i}+(4-0)\mathbf{j}+(0-4)\mathbf{k}\}\text{ m} = \{-1\mathbf{i}+4\mathbf{j}-4\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(-1)^2+4^2+(-4)^2} = 5.745\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-1\mathbf{i}+4\mathbf{j}-4\mathbf{k}}{5.745} = -0.1741\mathbf{i}+0.6963\mathbf{j}-0.6963\mathbf{k}$$

$$\mathbf{r}_{BD} = \{(2-0)\mathbf{i}+(-3-0)\mathbf{j}+(0-5.5)\mathbf{k}\}\text{ m} = \{2\mathbf{i}-3\mathbf{j}-5.5\mathbf{k}\}\text{ m}$$

$$r_{BD} = \sqrt{2^2+(-3)^2+(-5.5)^2} = 6.576\text{ m}$$

$$\mathbf{u}_{BD} = \frac{\mathbf{r}_{BD}}{r_{BD}} = \frac{2\mathbf{i}-3\mathbf{j}-5.5\mathbf{k}}{6.576} = 0.3041\mathbf{i}-0.4562\mathbf{j}-0.8363\mathbf{k}$$

Force Vector:

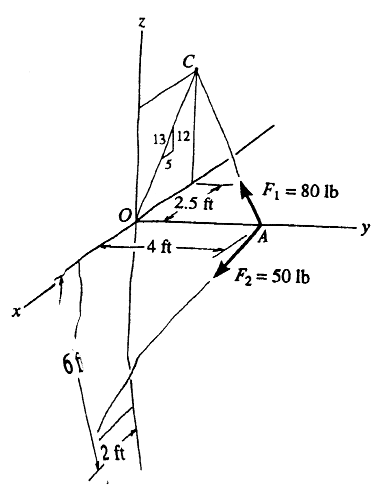
$$\mathbf{F}_A = F_A\mathbf{u}_{AC} = 250\{-0.1741\mathbf{i}+0.6963\mathbf{j}-0.6963\mathbf{k}\}\text{ N}$$
$$= \{-43.52\mathbf{i}+174.08\mathbf{j}-174.08\mathbf{k}\}\text{ N}$$
$$= \{-43.5\mathbf{i}+174\mathbf{j}-174\mathbf{k}\}\text{ N}$$

Ans

$$\mathbf{F}_B = F_B\mathbf{u}_{BD} = 175\{0.3041\mathbf{i}-0.4562\mathbf{j}-0.8363\mathbf{k}\}\text{ N}$$
$$= \{53.22\mathbf{i}-79.83\mathbf{j}-146.36\mathbf{k}\}\text{ N}$$
$$= \{53.2\mathbf{i}-79.8\mathbf{j}-146\mathbf{k}\}\text{ N}$$

Ans

2-99. Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



$\mathbf{r}_{AC} = (-2.5\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$; $r_{AC} = 7.6322$

$\mathbf{F}_1 = 80\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = \{-26.2\mathbf{i} - 41.9\mathbf{j} + 62.9\mathbf{k}\}$ lb **Ans**

$\mathbf{r}_{AB} = \{2\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}\}$; $r_{AB} = 7.48$

$\mathbf{F}_2 = 50\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = \{13.4\mathbf{i} - 26.7\mathbf{j} - 40.1\mathbf{k}\}$ lb **Ans**

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= \{-12.8\mathbf{i} - 68.7\mathbf{j} + 22.8\mathbf{k}\}$ lb

$F_R = \sqrt{(-12.8)^2 + (-68.7)^2 + (22.8)^2} = 73.47$ lb

$= 73.5$ lb

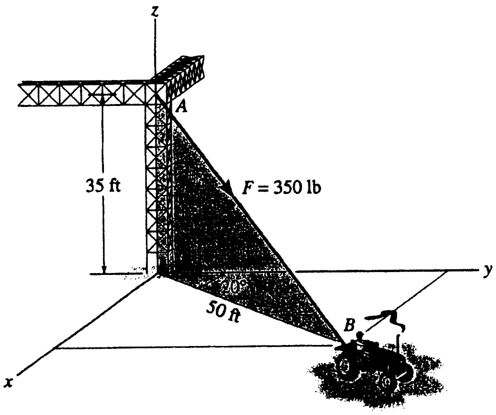
Ans

$\alpha = \cos^{-1}\left(\frac{-12.8}{73.47}\right) = 100°$ **Ans**

$\beta = \cos^{-1}\left(\frac{-68.7}{73.47}\right) = 159°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{22.8}{73.47}\right) = 71.9°$ **Ans**

*2-100. The cable attached to the tractor at B exerts a force of 350 lb on the framework. Express this force as a Cartesian vector.



$$\mathbf{r} = 50\sin 20°\mathbf{i} + 50\cos 20°\mathbf{j} - 35\mathbf{k}$$

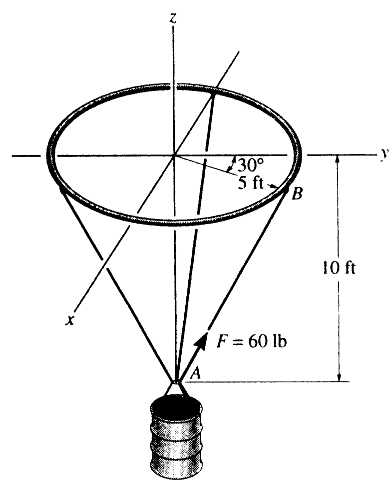
$$\mathbf{r} = \{17.10\mathbf{i} + 46.98\mathbf{j} - 35\mathbf{k}\}\ \text{ft}$$

$$r = \sqrt{(17.10)^2 + (46.98)^2 + (-35)^2} = 61.03\ \text{ft}$$

$$\mathbf{u} = \frac{\mathbf{r}}{r} = (0.280\mathbf{i} + 0.770\mathbf{j} - 0.573\mathbf{k})$$

$$\mathbf{F} = F\mathbf{u} = \{98.1\mathbf{i} + 269\mathbf{j} - 201\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

2-101. The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.



Unit Vector : First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

$$B(5\sin 30°,\ 5\cos 30°,\ 0)\ \text{ft} = B(2.50,\ 4.330,\ 0)\ \text{ft}$$

Then

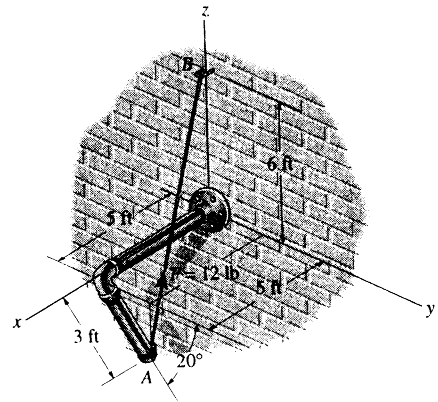
$$\mathbf{r}_{AB} = \{(2.50-0)\mathbf{i} + (4.330-0)\mathbf{j} + [0-(-10)]\mathbf{k}\}\ \text{ft}$$
$$= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10.0\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10.0\mathbf{k}}{11.180}$$
$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 60\{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\}\ \text{lb}$$
$$= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

2-102. The pipe is supported at its ends by a cord AB. If the cord exerts a force of $F = 12$ lb on the pipe at A, express this force as a Cartesian vector.



Unit Vector: The coordinates of point A are

$$A(5,\ 3\cos 20°,\ -3\sin 20°)\ \text{ft} = A(5.00,\ 2.819,\ -1.026)\ \text{ft}$$

Then

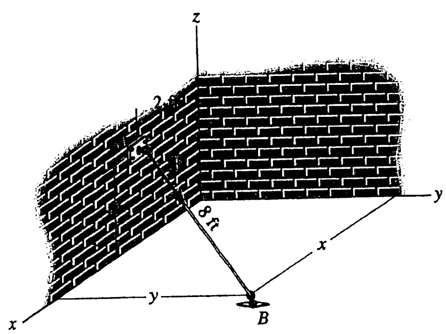
$$\mathbf{r}_{AB} = \{(0-5.00)\mathbf{i} + (0-2.819)\mathbf{j} + [6-(-1.026)]\mathbf{k}\}\ \text{ft}$$
$$= \{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{(-5.00)^2 + (-2.819)^2 + 7.026^2} = 9.073\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}}{9.073}$$
$$= -0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 12\{-0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}\}\ \text{lb}$$
$$= \{-6.61\mathbf{i} - 3.73\mathbf{j} + 9.29\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

2-103. The cord exerts a force of $\mathbf{F} = \{12\mathbf{i} + 9\mathbf{j} - 8\mathbf{k}\}$ lb on the hook. If the cord is 8 ft long, determine the location x, y of the point of attachment B, and the height z of the hook.



$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{\{12\mathbf{i} + 9\mathbf{j} - 8\mathbf{k}\}}{\sqrt{(12)^2 + (9)^2 + (-8)^2}} = (0.706\mathbf{i} + 0.529\mathbf{j} - 0.471\mathbf{k})$$

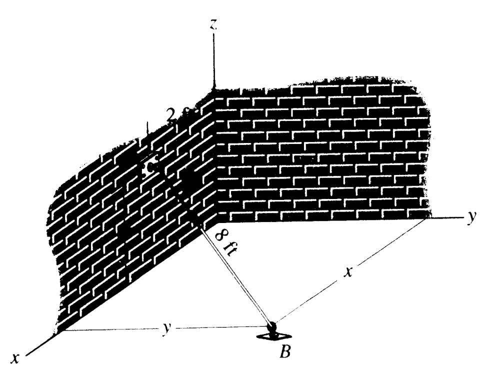
$$\mathbf{r} = r\mathbf{u} = 8\mathbf{u} = \{5.65\mathbf{i} + 4.24\mathbf{j} - 3.76\mathbf{k}\}\ \text{ft}$$

$x - 2 = 5.65;\quad x = 7.65$ ft **Ans**

$y - 0 = 4.24;\quad y = 4.24$ ft **Ans**

$0 - z = -3.76;\quad z = 3.76$ ft **Ans**

***2-104.** The cord exerts a force of $F = 30$ lb on the hook. If the cord is 8 ft long, $z = 4$ ft, and the x component of the force is $F_x = 25$ lb, determine the location x, y of the point of attachment B of the cord to the ground.



$$u_x = \frac{25}{30} = 0.833$$

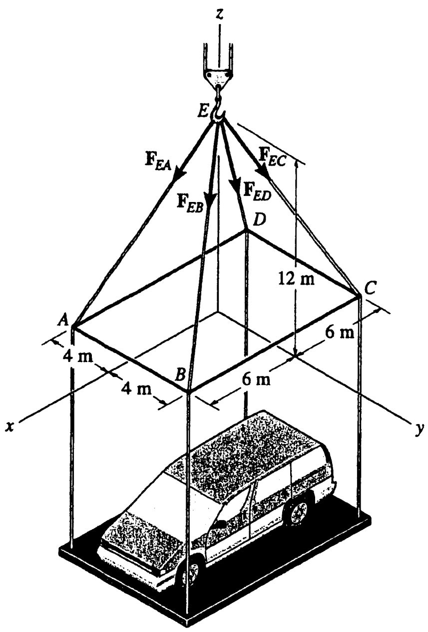
$$r_x = ru_x = 8(0.833) = 6.67\ \text{ft}$$

$x - 2 = 6.67;\quad x = 8.67$ ft **Ans**

$$r = \sqrt{(6.67)^2 + y^2 + 4^2} = 8$$

$y = 1.89$ ft **Ans**

2-105. Each of the four forces acting at E has a magnitude of 28 kN. Express each force as a Cartesian vector and determine the resultant force.



$$\mathbf{F}_{EA} = 28\left(\frac{6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$\mathbf{F}_{EA} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}$ kN **Ans**

$$\mathbf{F}_{EB} = 28\left(\frac{6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$\mathbf{F}_{EB} = \{12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}$ kN **Ans**

$$\mathbf{F}_{EC} = 28\left(\frac{-6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$\mathbf{F}_{EC} = \{-12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}$ kN **Ans**

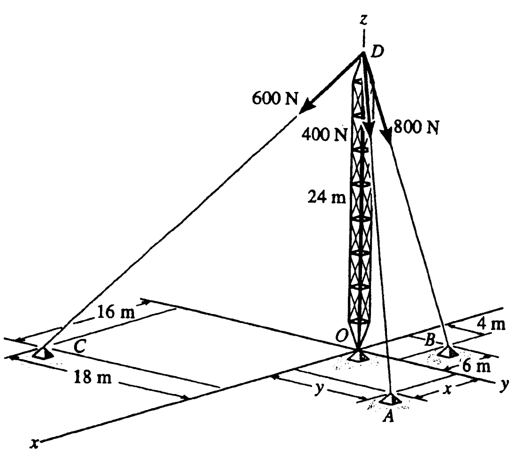
$$\mathbf{F}_{ED} = 28\left(\frac{-6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$\mathbf{F}_{ED} = \{-12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}$ kN **Ans**

$\mathbf{F}_R = \mathbf{F}_{EA} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED}$

$= \{-96\mathbf{k}\}$ kN **Ans**

2-106. The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\mathbf{i} + \frac{15}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right)\text{N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right)\text{ N}$$

$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$

$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\}$ N

$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$

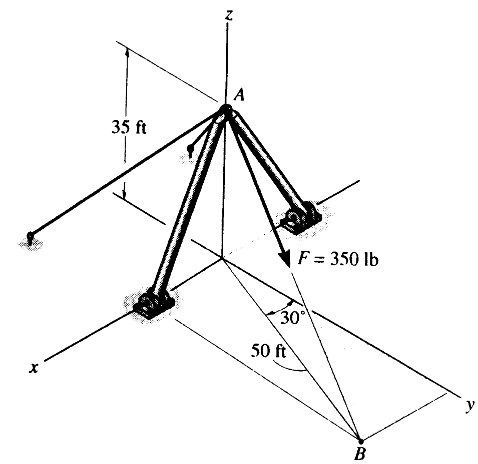
$= 1501.66$ N $= 1.50$ kN **Ans**

$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6°$ **Ans**

$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168°$ **Ans**

2-107. The cable, attached to the shear-leg derrick, exerts a force on the derrick of $F = 350$ lb. Express this force as a Cartesian vector.



Unit Vector : The coordinates of point B are

$$B(50\sin 30°,\ 50\cos 30°,\ 0)\ \text{ft} = B(25.0,\ 43.301,\ 0)\ \text{ft}$$

Then

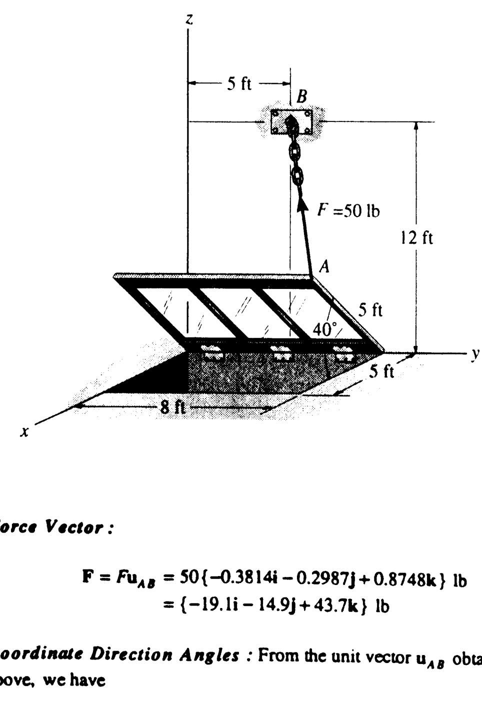
$$\mathbf{r}_{AB} = \{(25.0-0)\mathbf{i} + (43.301-0)\mathbf{j} + (0-35)\mathbf{k}\}\ \text{ft}$$
$$= \{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{25.0^2 + 43.301^2 + (-35.0)^2} = 61.033\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}}{61.033}$$
$$= 0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}$$

Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 350\{0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}\}\ \text{lb}$$
$$= \{143\mathbf{i} + 248\mathbf{j} - 201\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

***2-108.** The window is held open by chain AB. Determine the length of the chain, and express the 50-lb force acting at A along the chain as a Cartesian vector and determine its coordinate direction angles.



Unit Vector : The coordinates of point A are

$$A(5\cos 40°,\ 8,\ 5\sin 40°)\ \text{ft} = A(3.830,\ 8.00,\ 3.214)\ \text{ft}$$

Then

$$\mathbf{r}_{AB} = \{(0-3.830)\mathbf{i} + (5-8.00)\mathbf{j} + (12-3.214)\mathbf{k}\}\ \text{ft}$$
$$= \{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{(-3.830)^2 + (-3.00)^2 + 8.786^2} = 10.043\ \text{ft} = 10.0\ \text{ft} \qquad \textbf{Ans}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}}{10.043}$$
$$= -0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}$$

Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 50\{-0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}\}\ \text{lb}$$
$$= \{-19.1\mathbf{i} - 14.9\mathbf{j} + 43.7\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

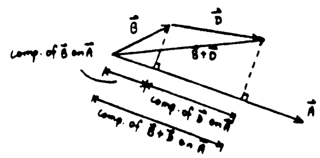
Coordinate Direction Angles : From the unit vector $\mathbf{u}_{AB}$ obtained above, we have

$\cos\alpha = -0.3814$ $\qquad \alpha = 112°$ $\qquad$ **Ans**

$\cos\beta = -0.2987$ $\qquad \beta = 107°$ $\qquad$ **Ans**

$\cos\gamma = 0.8748$ $\qquad \gamma = 29.0°$ $\qquad$ **Ans**

2-109. Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.



Since the component of $(\mathbf{B}+\mathbf{D})$ is equal to the sum of the components of **B** and **D**, then

$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D}$ (QED)

Also,

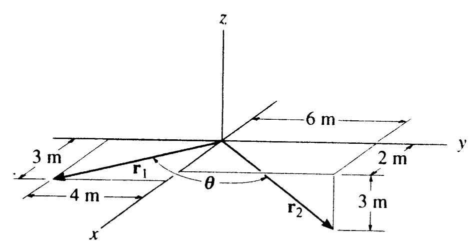
$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \text{(QED)}$$

2-110. Determine the angle θ between the tails of the two vectors.



Position Vectors :

$$\mathbf{r}_1 = \{(3-0)\mathbf{i} + (-4-0)\mathbf{j} + (0-0)\mathbf{k}\}\ \text{m} = \{3\mathbf{i} - 4\mathbf{j}\}\ \text{m}$$

$$\mathbf{r}_2 = \{(2-0)\mathbf{i} + (6-0)\mathbf{j} + (-3-0)\mathbf{k}\}\ \text{m} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}\ \text{m}$$

The magnitude of postion vectors are

$$r_1 = \sqrt{3^2 + (-4)^2} = 5.00\ \text{m} \qquad r_2 = \sqrt{2^2 + 6^2 + (-3)^2} = 7.00\ \text{m}$$

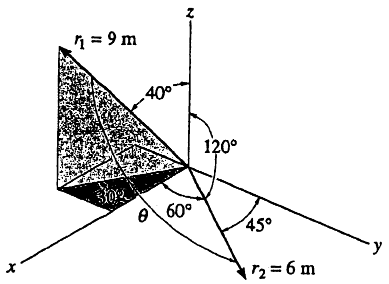
Angle Between Two Vectors θ:

$$\mathbf{r}_1\cdot\mathbf{r}_2 = (3\mathbf{i} - 4\mathbf{j})\cdot(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 3(2) + (-4)(6) + 0(-3) = -18.0\ \text{m}^2$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) = \cos^{-1}\left[\frac{-18.0}{5.00(7.00)}\right] = 121^\circ \qquad \textbf{Ans}$$

2-111. Determine the angle θ between the tails of the two vectors.



$$\mathbf{r}_1 = 9(\sin 40°\cos 30°\mathbf{i} - \sin 40°\sin 30°\mathbf{j} + \cos 40°\mathbf{k})$$

$$\mathbf{r}_1 = \{5.010\mathbf{i} - 2.8925\mathbf{j} + 6.894\mathbf{k}\}\ \text{m}$$

$$\mathbf{r}_2 = 6(\cos 60°\mathbf{i} + \cos 45°\mathbf{j} + \cos 120°\mathbf{k})$$

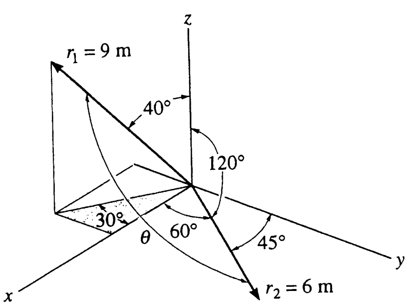
$$\mathbf{r}_2 = \{3\mathbf{i} + 4.2426\mathbf{j} - 3\mathbf{k}\}\ \text{m}$$

$$\mathbf{r}_1 \cdot \mathbf{r}_2 = 5.010(3) + (-2.8925)(4.2426) + (6.894)(-3) = -17.93$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2}\right)$$

$$= \cos^{-1}\left(\frac{-17.93}{9(6)}\right) = 109° \qquad \textbf{Ans}$$

***2-112.** Determine the magnitude of the projected component of $\mathbf{r}_1$ along $\mathbf{r}_2$, and the projection of $\mathbf{r}_2$ along $\mathbf{r}_1$.



$$\mathbf{r}_1 = 9(\sin 40°\cos 30°\mathbf{i} - \sin 40°\sin 30°\mathbf{j} + \cos 40°\mathbf{k})$$

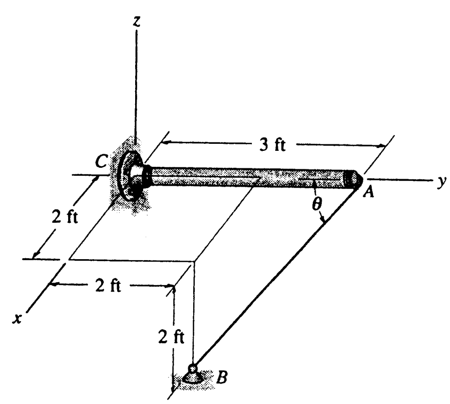
$$\mathbf{r}_1 = 5.010\mathbf{i} - 2.8925\mathbf{j} + 6.894\mathbf{k}$$

$$\mathbf{r}_1 \cdot \mathbf{r}_2 = 5.010(3) + (-2.8925)(4.2426) + (6.894)(-3) = -17.93$$

$$\text{Proj.}\mathbf{r}_1 = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_2} = \frac{-17.93}{6} = |\,2.99\ \text{m}| \qquad \textbf{Ans}$$

$$\text{Proj.}\mathbf{r}_2 = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1} = \frac{-17.93}{9} = |\,1.99\ \text{m}| \qquad \textbf{Ans}$$

2-113. Determine the angle θ between the y axis of the pole and the wire AB.



Position Vector :

$$\mathbf{r}_{AC} = \{-3\mathbf{j}\}\ \text{ft}$$

$$\mathbf{r}_{AB} = \{(2-0)\mathbf{i} + (2-3)\mathbf{j} + (-2-0)\mathbf{k}\}\ \text{ft}$$
$$= \{2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

The magnitudes of the postion vectors are

$$r_{AC} = 3.00\ \text{ft} \qquad r_{AB} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.00\ \text{ft}$$

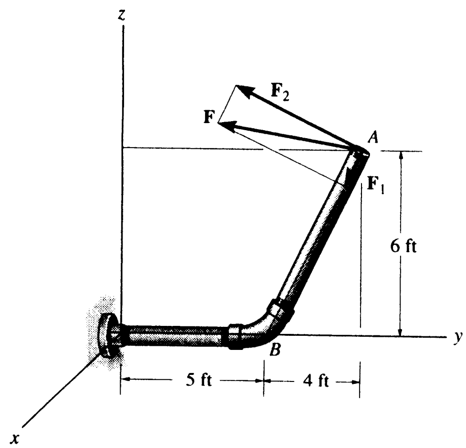
The Angles Between Two Vectors θ**:** The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = (-3\mathbf{j}) \cdot (2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k})$$
$$= 0(2) + (-3)(-1) + 0(-2)$$
$$= 3$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5° \qquad \textbf{Ans}$$

2-114. The force $\mathbf{F} = \{25\mathbf{i} - 50\mathbf{j} + 10\mathbf{k}\}$ N acts at the end A of the pipe assembly. Determine the magnitude of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the axis of AB and perpendicular to it.



Unit Vector : The unit vector along Aa axis is

$$\mathbf{u}_{AB} = \frac{(0-0)\mathbf{i}+(5-9)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(0-0)^2+(5-9)^2+(0-6)^2}} = -0.5547\mathbf{j} - 0.8321\mathbf{k}$$

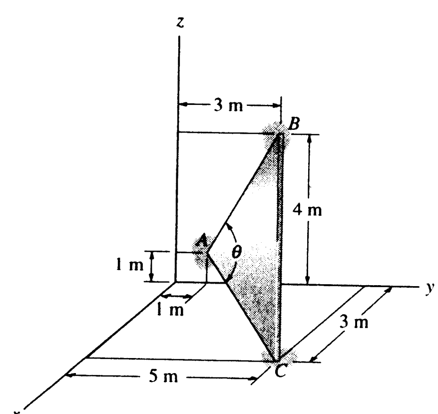
Projected Component of F Along AB Axis :

$$\begin{aligned} F_1 = \mathbf{F}\cdot\mathbf{u}_{AB} &= (25\mathbf{i}-50\mathbf{j}+10\mathbf{k})\cdot(-0.5547\mathbf{j}-0.8321\mathbf{k}) \\ &= (25)(0)+(-50)(-0.5547)+(10)(-0.8321) \\ &= 19.414\text{ N} = 19.4\text{ N} \qquad \textbf{Ans} \end{aligned}$$

Component of F Perpendicular to AB Axis : The magnitude of force **F** is $F = \sqrt{25^2+(-50)^2+10^2} = 56.789$ N.

$$F_2 = \sqrt{F^2 - F_1^2} = \sqrt{56.789^2 - 19.414^2} = 53.4\text{ N} \qquad \textbf{Ans}$$

2-115. Determine the angle θ between the sides of the triangular plate.



$$\mathbf{r}_{AC} = \{3\mathbf{i} + 4\mathbf{j} - 1\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990\text{ m}$$

$$\mathbf{r}_{AB} = \{2\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

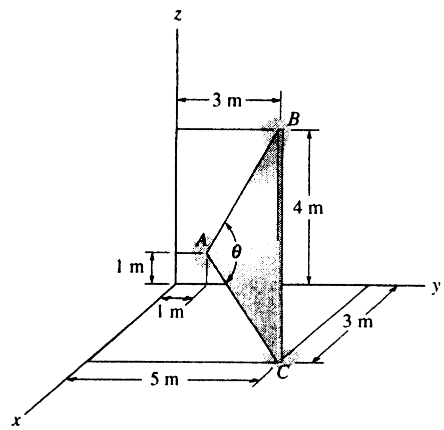
$$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056\text{ m}$$

$$\mathbf{r}_{AC}\cdot\mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC}\cdot\mathbf{r}_{AB}}{r_{AC}\,r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$\theta = 74.219° = 74.2°$ **Ans**

***2-116.** Determine the length of side BC of the triangular plate. Solve the problem by finding the magnitude of $\mathbf{r}_{BC}$; then check the result by first finding θ, r_{AB}, and r_{AC} and then use the cosine law.



$\mathbf{r}_{BC} = \{3\mathbf{i}+2\mathbf{j}-4\mathbf{k}\}$ m

$r_{BC} = \sqrt{(3)^2+(2)^2+(-4)^2} = 5.39$ m **Ans**

Also,

$\mathbf{r}_{AC} = \{3\mathbf{i}+4\mathbf{j}-1\mathbf{k}\}$ m

$r_{AC} = \sqrt{(3)^2+(4)^2+(-1)^2} = 5.0990$ m

$\mathbf{r}_{AB} = \{2\mathbf{j}+3\mathbf{k}\}$ m

$r_{AB} = \sqrt{(2)^2+(3)^2} = 3.6056$ m

$\mathbf{r}_{AC}\cdot\mathbf{r}_{AB} = 0+4(2)+(-1)(3) = 5$

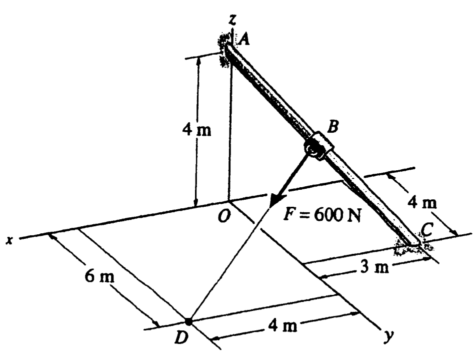
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC}\cdot\mathbf{r}_{AB}}{r_{AC}r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$\theta = 74.219°$

$r_{BC} = \sqrt{(5.0990)^2+(3.6056)^2-2(5.0990)(3.6056)\cos 74.219°}$

$r_{BC} = 5.39$ m **Ans**

2-117. Determine the components of **F** that act along rod AC and perpendicular to it. Point B is located at the midpoint of the rod.



$\mathbf{r}_{AC} = (-3\mathbf{i}+4\mathbf{j}-4\mathbf{k})$, $\quad r_{AC} = \sqrt{(-3)^2+4^2+(-4)^2} = \sqrt{41}$ m

$$\mathbf{r}_{AB} = \frac{\mathbf{r}_{AC}}{2} = \frac{-3\mathbf{i}+4\mathbf{j}-4\mathbf{k}}{2} = -1.5\mathbf{i}+2\mathbf{j}-2\mathbf{k}$$

$\mathbf{r}_{AD} = \mathbf{r}_{AB}+\mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{AD}-\mathbf{r}_{AB}$

$= (4\mathbf{i}+6\mathbf{j}-4\mathbf{k})-(-1.5\mathbf{i}+2\mathbf{j}-2\mathbf{k})$

$= \{5.5\mathbf{i}+4\mathbf{j}-2\mathbf{k}\}$ m

$r_{BD} = \sqrt{(5.5)^2+(4)^2+(-2)^2} = 7.0887$ m

$\mathbf{F} = 600(\frac{\mathbf{r}_{BD}}{r_{BD}}) = 465.528\mathbf{i}+338.5659\mathbf{j}-169.2829\mathbf{k}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \frac{(465.528\mathbf{i}+338.5659\mathbf{j}-169.2829\mathbf{k})\cdot(-3\mathbf{i}+4\mathbf{j}-4\mathbf{k})}{\sqrt{41}}$$

$F_{||} = 99.1408 = 99.1$ N **Ans**

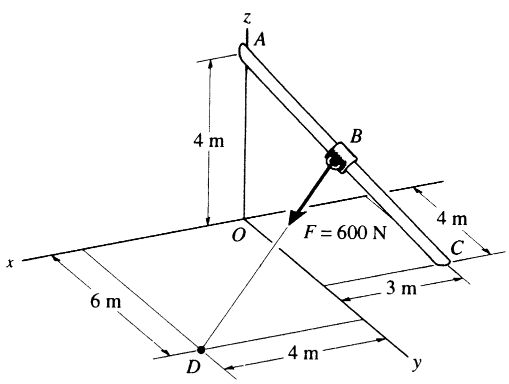
Component of F perpendicular to $\mathbf{r}_{AC}$ is $F_{\perp}$

$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

$F_{\perp}^2 = 600^2 - 99.1408^2$

$F_{\perp} = 591.75 = 592$ N **Ans**

2-118. Determine the components of **F** that act along rod AC and perpendicular to it. Point B is located 3 m along the rod from end C.



$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

$r_{CA} = 6.403124$

$\mathbf{r}_{CB} = \dfrac{3}{6.403124}(\mathbf{r}_{CA}) = 1.40556\mathbf{i} - 1.874085\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$

$= -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$

$= -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$

$= 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$

$r_{BD} = \sqrt{(5.5944)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$

$\mathbf{F} = 600(\dfrac{\mathbf{r}_{BD}}{r_{BD}}) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \quad r_{AC} = \sqrt{41}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \frac{(475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k})\cdot(-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$F_{||} = 82.4351 = 82.4$ N **Ans**

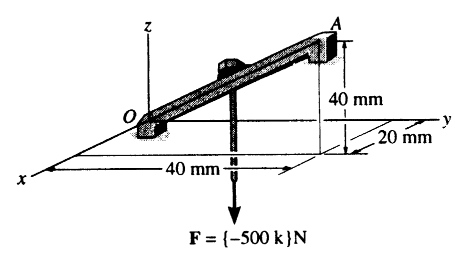
Component of **F** perpendicular to $\mathbf{r}_{AC}$ is $\mathbf{F}_\perp$

$F_\perp^2 + F_{||}^2 = F^2 = 600^2$

$F_\perp^2 = 600^2 - 82.4351^2$

$F_\perp = 594$ N **Ans**

2-119. The clamp is used on a jig. If the vertical force acting on the bolt is $\mathbf{F} = \{-500\mathbf{k}\}$ N, determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the OA axis and perpendicular to it.



Unit Vector : The unit vector along OA axis is

$$\mathbf{u}_{AO} = \frac{(0-20)\mathbf{i}+(0-40)\mathbf{j}+(0-40)\mathbf{k}}{\sqrt{(0-20)^2+(0-40)^2+(0-40)^2}} = -\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

Projected Component of F Along OA Axis :

$$F_1 = \mathbf{F}\cdot\mathbf{u}_{AO} = (-500\mathbf{k})\cdot\left(-\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right)$$

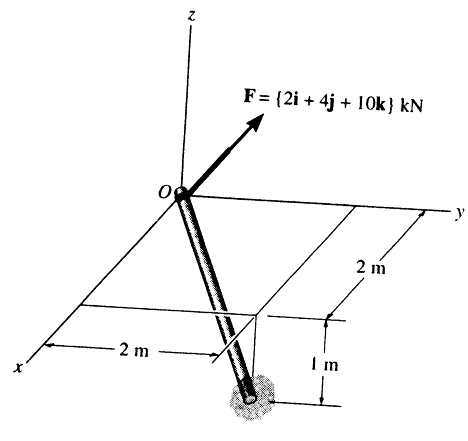
$$= (0)\left(-\frac{1}{3}\right)+(0)\left(-\frac{2}{3}\right)+(-500)\left(-\frac{2}{3}\right)$$

$= 333.33$ N $= 333$ N **Ans**

Component of F Perpendicular to OA Axis : Since the magnitude of force **F** is $F = 500$ N so that

$F_2 = \sqrt{F^2 - F_1^2} = \sqrt{500^2 - 333.33^2} = 373$ N **Ans**

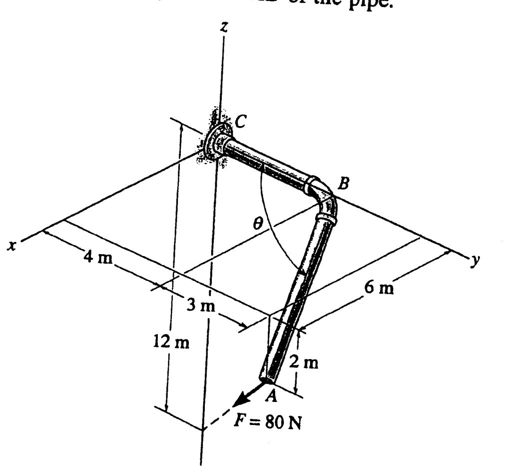
***2-120.** Determine the projection of the force **F** along the pole.



$$\text{Proj } F = \mathbf{F}\cdot\mathbf{u}_a = (2\mathbf{i}+4\mathbf{j}+10\mathbf{k})\cdot\left(\frac{2}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}\right)$$

$$\text{Proj } F = 0.667\text{ kN}$$ **Ans**

2-121. Determine the projected component of the 80-N force acting along the axis AB of the pipe.



$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \{-\frac{6}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\}$$

$$= \{-0.857\mathbf{i} - 0.429\mathbf{j} + 0.286\mathbf{k}\}$$

$$\mathbf{F} = 80\left[\frac{-6\mathbf{i} - 7\mathbf{j} - 10\mathbf{k}}{\sqrt{(6)^2 + (7)^2 + (10)^2}}\right]$$

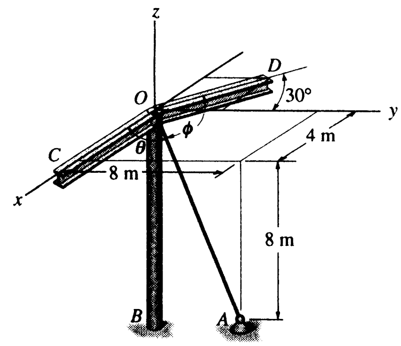
$$= \{-35.29\mathbf{i} - 41.17\mathbf{j} - 58.82\mathbf{k}\}\text{ N}$$

$$\text{Proj. } F = F\cos\theta = \mathbf{F}\cdot\mathbf{u}_{AB}$$

$$= (-35.29)(-0.857) + (-41.17)(-0.425) + (-58.82)(0.286)$$

$$= 31.1\text{ N}$$ **Ans**

2-122. Cable OA is used to support column OB. Determine the angle θ it makes with beam OC.



Unit Vector:

$$\mathbf{u}_{OC} = 1\mathbf{i}$$

$$\mathbf{u}_{OA} = \frac{(4-0)\mathbf{i}+(8-0)\mathbf{j}+(-8-0)\mathbf{k}}{\sqrt{(4-0)^2+(8-0)^2+(-8-0)^2}} = \frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

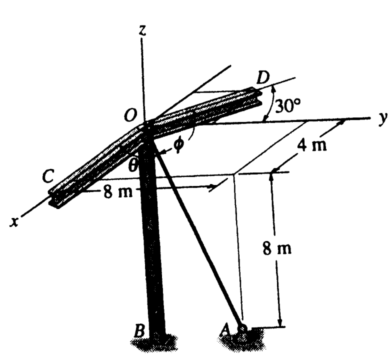
The Angles Between Two Vectors θ:

$$\mathbf{u}_{OC}\cdot\mathbf{u}_{OA} = (1\mathbf{i})\cdot\left(\frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\right) = 1\left(\frac{1}{3}\right)+(0)\left(\frac{2}{3}\right)+0\left(-\frac{2}{3}\right) = \frac{1}{3}$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{OC}\cdot\mathbf{u}_{OA}) = \cos^{-1}\frac{1}{3} = 70.5° \qquad \textbf{Ans}$$

2-123. Cable OA is used to support column OB. Determine the angle ϕ it makes with beam OD.



Unit Vector:

$$\mathbf{u}_{OD} = -\sin 30°\mathbf{i}+\cos 30°\mathbf{j} = -0.5\mathbf{i}+0.8660\mathbf{j}$$

$$\mathbf{u}_{OA} = \frac{(4-0)\mathbf{i}+(8-0)\mathbf{j}+(-8-0)\mathbf{k}}{\sqrt{(4-0)^2+(8-0)^2+(-8-0)^2}} = \frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

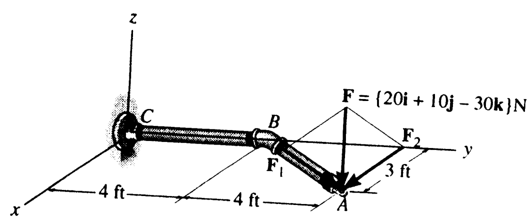
The Angles Between Two Vectors ϕ:

$$\mathbf{u}_{OD}\cdot\mathbf{u}_{OA} = (-0.5\mathbf{i}+0.8660\mathbf{j})\cdot\left(\frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\right) = (-0.5)\left(\frac{1}{3}\right)+(0.8660)\left(\frac{2}{3}\right)+0\left(-\frac{2}{3}\right) = 0.4107$$

Then,

$$\phi = \cos^{-1}(\mathbf{u}_{OD}\cdot\mathbf{u}_{OA}) = \cos^{-1}0.4107 = 65.8° \qquad \textbf{Ans}$$

*2-124. The force **F** acts at the end A of the pipe assembly. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the axis of AB and perpendicular to it.



Unit Vector : The unit vector along AB axis is

$$\mathbf{u}_{BA} = \frac{(3-0)\mathbf{i}+(8-4)\mathbf{j}+(0-0)\mathbf{k}}{\sqrt{(3-0)^2+(8-4)^2+(0-0)^2}} = \frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}$$

Projected Component of F Along AB Axis :

$$\begin{aligned} F_1 = \mathbf{F}\cdot\mathbf{u}_{BA} &= (20\mathbf{i}+10\mathbf{j}-30\mathbf{k})\cdot\left(\frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}\right) \\ &= (20)\left(\frac{3}{5}\right)+(10)\left(\frac{4}{5}\right)+(-30)(0) \\ &= 20.0\text{ N} \end{aligned}$$

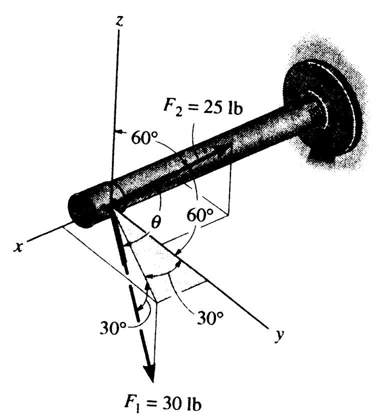
Ans

Component of F Perpendicular to AB Axis : The magnitude of force **F** is $F = \sqrt{20^2+10^2+(-30)^2} = 37.417$ N.

$$F_2 = \sqrt{F^2 - F_1^2} = \sqrt{37.417^2 - 20.0^2} = 31.6\text{ N}$$

Ans

2-125. Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



Force Vector :

$$\begin{aligned} \mathbf{u}_{F_1} &= \cos 30^\circ \sin 30^\circ \mathbf{i} + \cos 30^\circ \cos 30^\circ \mathbf{j} - \sin 30^\circ \mathbf{k} \\ &= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_1 = F_1\mathbf{u}_{F_1} &= 30(0.4330\mathbf{i}+0.75\mathbf{j}-0.5\mathbf{k})\text{ lb} \\ &= \{12.990\mathbf{i}+22.5\mathbf{j}-15.0\mathbf{k}\}\text{ lb} \end{aligned}$$

Unit Vector : One can obtain the angle $\alpha = 135^\circ$ for $\mathbf{F}_2$ using Eq. 2 – 10, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1$, with $\beta = 60^\circ$ and $\gamma = 60^\circ$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135^\circ\mathbf{i}+\cos 60^\circ\mathbf{j}+\cos 60^\circ\mathbf{k} = -0.7071\mathbf{i}+0.5\mathbf{j}+0.5\mathbf{k}$$

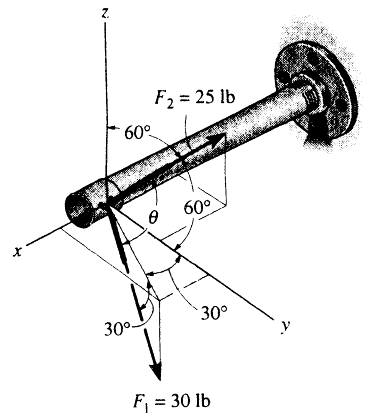
Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:

$$\begin{aligned} (F_1)_{F_2} = \mathbf{F}_1\cdot\mathbf{u}_{F_2} &= (12.990\mathbf{i}+22.5\mathbf{j}-15.0\mathbf{k})\cdot(-0.7071\mathbf{i}+0.5\mathbf{j}+0.5\mathbf{k}) \\ &= (12.990)(-0.7071)+(22.5)(0.5)+(-15.0)(0.5) \\ &= -5.44\text{ lb} \end{aligned}$$

Negative sign indicates that the projected component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(\mathbf{F}_1)_{F_2} = 5.44$ lb. **Ans**

2-126. Determine the angle θ between the two cables attached to the pipe.



Unit Vector:

$$\mathbf{u}_{F_1} = \cos 30°\sin 30°\mathbf{i} + \cos 30°\cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$
$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k}$$
$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

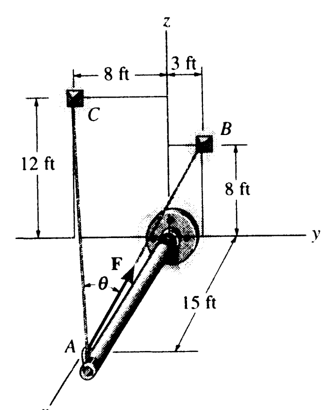
The Angles Between Two Vectors θ:

$$\mathbf{u}_{F_1}\cdot\mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k})\cdot(-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$
$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$
$$= -0.1812$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{F_1}\cdot\mathbf{u}_{F_2}) = \cos^{-1}(-0.1812) = 100° \qquad \textbf{Ans}$$

2-127. Determine the angle θ between cables AB and AC.



Position Vector:

$$\mathbf{r}_{AB} = \{(0-15)\mathbf{i} + (3-0)\mathbf{j} + (8-0)\mathbf{k}\}\text{ ft}$$
$$= \{-15\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AC} = \{(0-15)\mathbf{i} + (-8-0)\mathbf{j} + (12-0)\mathbf{k}\}\text{ ft}$$
$$= \{-15\mathbf{i} - 8\mathbf{j} + 12\mathbf{k}\}\text{ ft}$$

The magnitudes of the postion vectors are

$$r_{AB} = \sqrt{(-15)^2 + 3^2 + 8^2} = 17.263\text{ ft}$$
$$r_{AC} = \sqrt{(-15)^2 + (-8)^2 + 12^2} = 20.809\text{ ft}$$

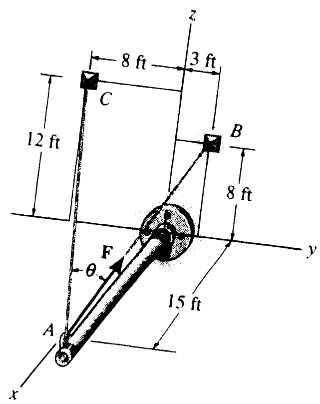
The Angles Between Two Vectors θ:

$$\mathbf{r}_{AB}\cdot\mathbf{r}_{AC} = (-15\mathbf{i} + 3\mathbf{j} + 8\mathbf{k})\cdot(-15\mathbf{i} - 8\mathbf{j} + 12\mathbf{k})$$
$$= (-15)(-15) + (3)(-8) + 8(12)$$
$$= 297\text{ ft}^2$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB}\cdot\mathbf{r}_{AC}}{r_{AB}r_{AC}}\right) = \cos^{-1}\left[\frac{297}{17.263(20.809)}\right] = 34.2° \qquad \textbf{Ans}$$

2-128. If **F** has a magnitude of 55 lb, determine the magnitude of its projected component acting along the x axis and along cable AC.



Force Vector :

$$\mathbf{u}_{AB} = \frac{(0-15)\mathbf{i}+(3-0)\mathbf{j}+(8-0)\mathbf{k}}{\sqrt{(0-15)^2+(3-0)^2+(8-0)^2}}$$
$$= -0.8689\mathbf{i}+0.1738\mathbf{j}+0.4634\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 55(-0.8689\mathbf{i}+0.1738\mathbf{j}+0.4634\mathbf{k})\ \text{lb}$$
$$= \{-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k}\}\ \text{lb}$$

Unit Vector : The unit vector along negative x axis and AC are

$$\mathbf{u}_x = -1\mathbf{i}$$

$$\mathbf{u}_{AC} = \frac{(0-15)\mathbf{i}+(-8-0)\mathbf{j}+(12-0)\mathbf{k}}{\sqrt{(0-15)^2+(-8-0)^2+(12-0)^2}}$$
$$= -0.7209\mathbf{i}-0.3845\mathbf{j}+0.5767\mathbf{k}$$

Projected Component of F :

$$F_x = \mathbf{F}\cdot\mathbf{u}_x = (-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k})\cdot(-1\mathbf{i})$$
$$= (-47.791)(-1)+9.558(0)+25.489(0)$$
$$= 47.8\ \text{lb}$$

Ans

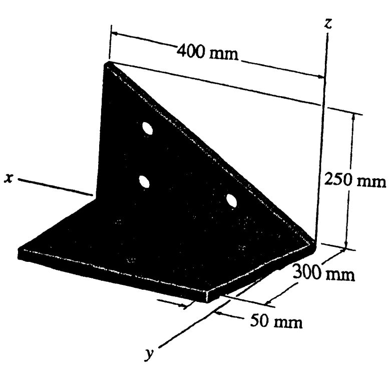
$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k})\cdot(-0.7209\mathbf{i}-0.3845\mathbf{j}+0.5767\mathbf{k})$$
$$= (-47.791)(-0.7209)+(9.558)(-0.3845)+(25.489)(0.5767)$$
$$= 45.5\ \text{lb}$$

Ans

The projected component acts along cable AC, F_{AC}, can also be determined using $F_{AC} = F\cos\theta$. From the solution of Prob. 2–137, $\theta = 34.2°$. Then

$$F_{AC} = 55\cos 34.2° = 45.5\ \text{lb}$$

2-129. Determine the angle θ between the edges of the sheet-metal bracket.



$\mathbf{r}_1 = \{400\mathbf{i} + 250\mathbf{k}\}$ mm ; $\quad r_1 = 471.70$ mm

$\mathbf{r}_2 = \{50\mathbf{i} + 300\mathbf{j}\}$ mm ; $\quad r_2 = 304.14$ mm

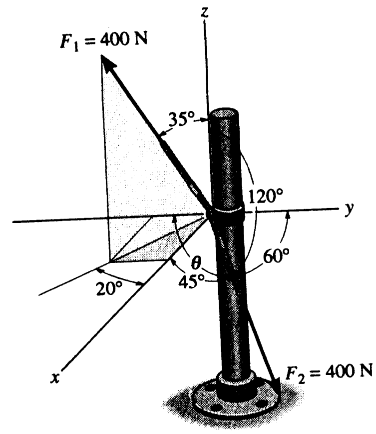
$$\mathbf{r}_1\cdot\mathbf{r}_2 = (400)(50) + 0(300) + 250(0) = 20000$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right)$$

$$= \cos^{-1}\left(\frac{20000}{(471.70)(304.14)}\right) = 82.0°$$

Ans

2-130. The cables each exert a force of 400 N on the post. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



Force Vector:

$$\mathbf{u}_{F_1} = \sin 35°\cos 20°\mathbf{i} - \sin 35°\sin 20°\mathbf{j} + \cos 35°\mathbf{k}$$
$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{F}_1 = F_1\mathbf{u}_{F_1} = 400(0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \text{ N}$$
$$= \{215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}\} \text{ N}$$

Unit Vector: The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}$$
$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

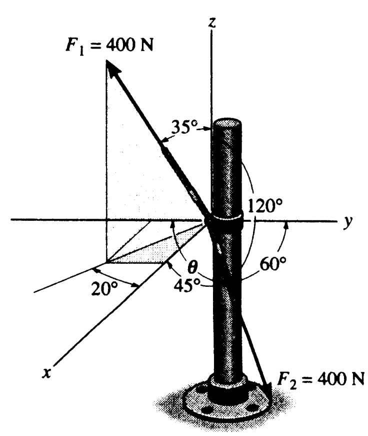
Projected Component of $\mathbf{F}_1$ Along Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$
$$= (215.59)(0.7071) + (-78.47)(0.5) + (327.66)(-0.5)$$
$$= -50.6 \text{ N}$$

Negative sign indicates that the force component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

thus the magnitude is $(\mathbf{F}_1)_{F_2} = 50.6$ N **Ans**

2-131. Determine the angle θ between the two cables attached to the post.



Unit Vector:

$$\mathbf{u}_{F_1} = \sin 35°\cos 20°\mathbf{i} - \sin 35°\sin 20°\mathbf{j} + \cos 35°\mathbf{k}$$
$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}$$
$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

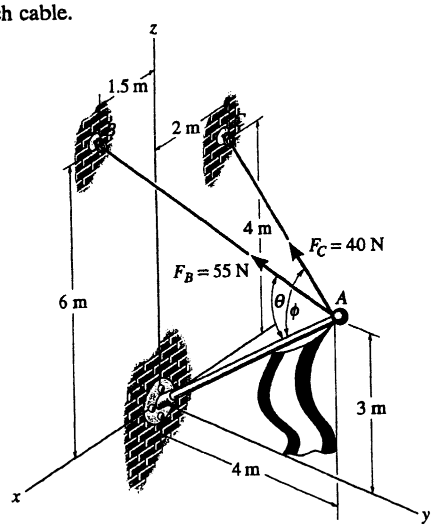
The Angles Between Two Vectors θ: The dot product of two unit vectors must be determined first.

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$
$$= 0.5390(0.7071) + (-0.1962)(0.5) + 0.8192(-0.5)$$
$$= -0.1265$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1265) = 97.3° \qquad \textbf{Ans}$$

*2-132. Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.



$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m ; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$ m

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB} r_{AO}}\right)$$

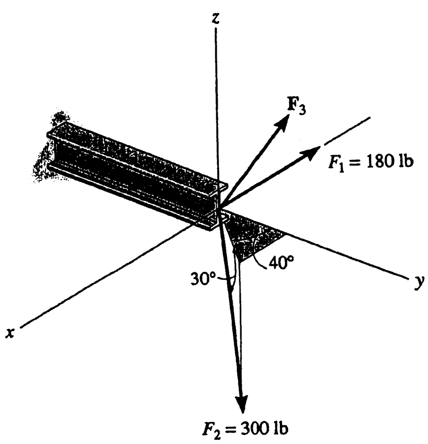
$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.44° = 74.4° \qquad \textbf{Ans}$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC} r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4° \qquad \textbf{Ans}$$

2-133. Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces acts along the positive y axis and has a magnitude of 600 lb.



$F_{Rx} = \Sigma F_x$; $\quad 0 = -180 + 300\cos 30° \sin 40° + F_3 \cos\alpha$

$F_{Ry} = \Sigma F_y$; $\quad 600 = 300\cos 30° \cos 40° + F_3 \cos\beta$

$F_{Rz} = \Sigma F_z$; $\quad 0 = -300 \sin 30° + F_3 \cos\gamma$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Solving :

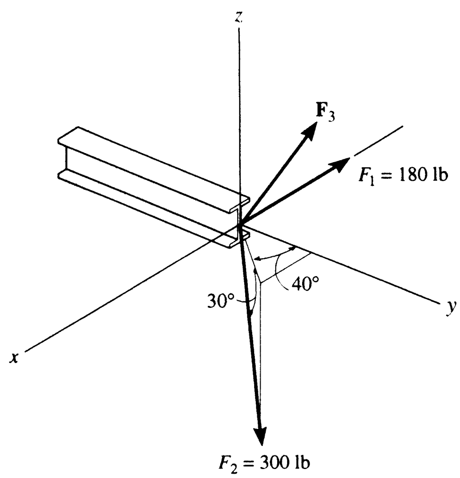
$F_3 = 428$ lb **Ans**

$\alpha = 88.3°$ **Ans**

$\beta = 20.6°$ **Ans**

$\gamma = 69.5°$ **Ans**

2-134. Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.



$F_{Rx} = \Sigma F_x;\quad 0 = -180 + 300\cos 30^\circ \sin 40^\circ + F_3 \cos\alpha$

$F_{Ry} = \Sigma F_y;\quad 0 = 300\cos 30^\circ \cos 40^\circ + F_3 \cos\beta$

$F_{Rz} = \Sigma F_z;\quad 0 = -300\sin 30^\circ + F_3 \cos\gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Solving :

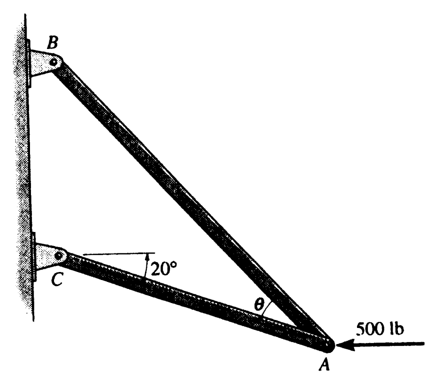
$F_3 = 250$ lb **Ans**

$\alpha = 87.0^\circ$ **Ans**

$\beta = 143^\circ$ **Ans**

$\gamma = 53.1^\circ$ **Ans**

2-135. Determine the design angle $\theta (\theta < 90^\circ)$ between the two struts so that the 500-lb horizontal force has a component of 600-lb directed from A toward C. What is the component of force acting along member BA?



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

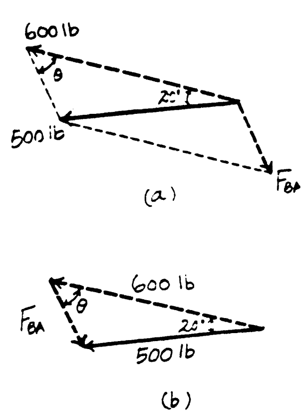
Trigonometry : Using law of cosines [Fig. (b)], we have

$$F_{BA} = \sqrt{600^2 + 500^2 - 2(600)(500)\cos 20^\circ}$$
$$= 214.91 \text{ lb} = 215 \text{ lb} \qquad \textbf{Ans}$$

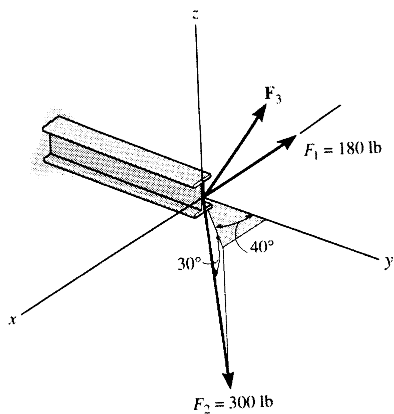
The design angle θ $(\theta < 90^\circ)$ can be determined using law of sines [Fig. (b)].

$$\frac{\sin\theta}{500} = \frac{\sin 20^\circ}{214.91}$$
$$\sin\theta = 0.7957$$

$$\theta = 52.7^\circ \qquad \textbf{Ans}$$



2-134. Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.



$F_{Rx} = \Sigma F_x;\quad 0 = -180 + 300\cos 30^\circ \sin 40^\circ + F_3 \cos\alpha$

$F_{Ry} = \Sigma F_y;\quad 0 = 300\cos 30^\circ \cos 40^\circ + F_3 \cos\beta$

$F_{Rz} = \Sigma F_z;\quad 0 = -300\sin 30^\circ + F_3\cos\gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Solving:

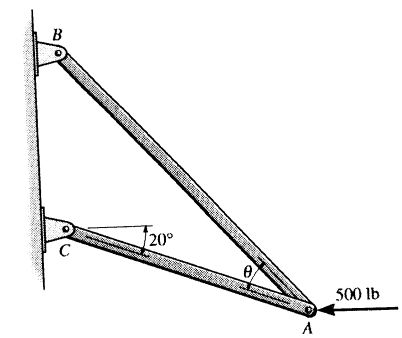
$F_3 = 250\text{ lb}$ **Ans**

$\alpha = 87.0^\circ$ **Ans**

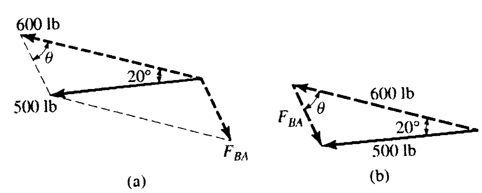
$\beta = 143^\circ$ **Ans**

$\gamma = 53.1^\circ$ **Ans**

2-135. Determine the design angle $\theta (\theta < 90^\circ)$ between the two struts so that the 500-lb horizontal force has a component of 600-lb directed from A toward C. What is the component of force acting along member BA?



Parallelogram Law: The parallelogram law of addition is shown in Fig. (a).



Trigonometry: Using law of cosines [Fig. (b)], we have

$F_{BA} = \sqrt{600^2 + 500^2 - 2(600)(500)\cos 20^\circ}$

$= 214.91\text{ lb} = 215\text{ lb}$ **Ans**

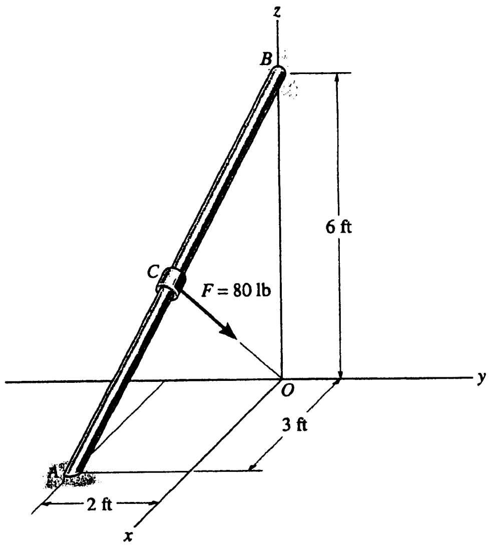
The design angle θ ($\theta < 90^\circ$) can be determined using law of sines [Fig. (b)].

$$\frac{\sin\theta}{500} = \frac{\sin 20^\circ}{214.91}$$

$\sin\theta = 0.7957$

$\theta = 52.7^\circ$ **Ans**

***2-136.** The force **F** has a magnitude of 80 lb and acts at the midpoint C of the thin rod. Express the force as a Cartesian vector.



$\mathbf{r}_{AB} = (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$

$\mathbf{r}_{CB} = \frac{1}{2}\mathbf{r}_{AB} = (-1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k})$

$\mathbf{r}_{CO} = \mathbf{r}_{BO} + \mathbf{r}_{CB}$

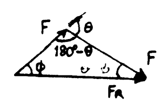
$= -6\mathbf{k} - 1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k}$

$= -1.5\mathbf{i} + 1\mathbf{j} - 3\mathbf{k}$

$r_{CO} = 3.5$

$F = 80(\frac{\mathbf{r}_{CO}}{r_{CO}}) = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}$ lb **Ans**

***2-137.** Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the hook. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.

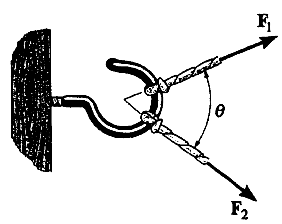


$\frac{F}{\sin\phi} = \frac{F}{\sin(\theta-\phi)}$

$\sin(\theta-\phi) = \sin\phi$

$\theta - \phi = \phi$

$\phi = \frac{\theta}{2}$ **Ans**



$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180^\circ - \theta)}$

Since $\cos(180^\circ - \theta) = -\cos\theta$

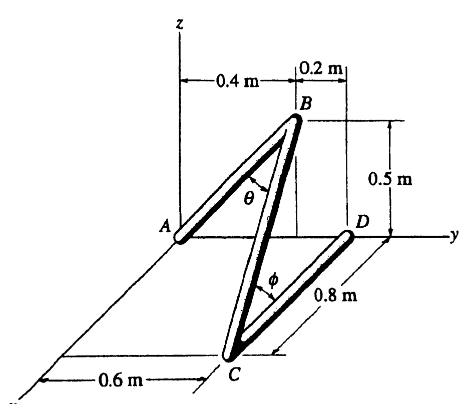
$F_R = F(\sqrt{2})\sqrt{1 + \cos\theta}$

Since $\cos(\frac{\theta}{2}) = \sqrt{\frac{1 + \cos\theta}{2}}$

Thus

$F_R = 2F\cos(\frac{\theta}{2})$ **Ans**

2-138. Determine the angles θ and ϕ between the wire segments.



$\mathbf{r}_{BA} = \{-0.4\mathbf{j} - 0.5\mathbf{k}\}$ m ; $r_{BA} = 0.640$ m

$\mathbf{r}_{BC} = \{0.8\mathbf{i} + 0.2\mathbf{j} - 0.5\mathbf{k}\}$ m; $r_{BC} = 0.964$ m

$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = 0 + (-0.4)(0.2) + (-0.5)(-0.5) = 0.170$ m^2

$\theta = \cos^{-1}(\frac{0.170}{(0.640)(0.964)}) = 74.0^\circ$ **Ans**

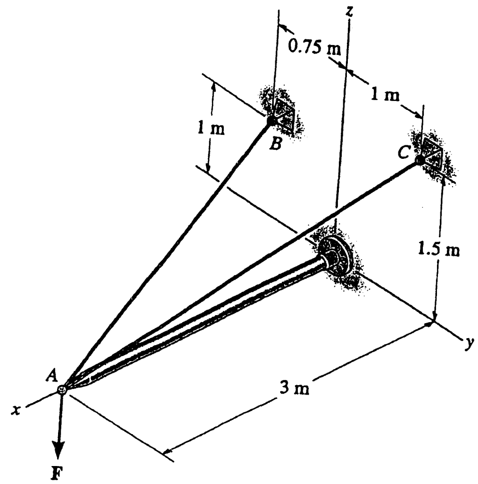
$\mathbf{r}_{CB} = \{-0.8\mathbf{i} - 0.2\mathbf{j} + 0.5\mathbf{k}\}$ m ; $r_{CB} = 0.964$ m

$\mathbf{r}_{CD} = \{-0.8\mathbf{i}\}$ m; $r_{CD} = 0.800$ m

$\mathbf{r}_{CB} \cdot \mathbf{r}_{CD} = (-0.8)(-0.8) = 0.640$ m^2

$\phi = \cos^{-1}(\frac{0.640}{(0.964)(0.800)}) = 33.9^\circ$ **Ans**

2-139. Determine the magnitudes of the projected components of the force $\mathbf{F} = \{60\mathbf{i} + 12\mathbf{j} - 40\mathbf{k}\}$ N in the direction of the cables AB and AC.



$$\mathbf{F} = \{60\mathbf{i} + 12\mathbf{j} - 40\mathbf{k}\}\ \text{N}$$

$$\mathbf{u}_{AB} = \frac{(-3\mathbf{i} - 0.75\mathbf{j} + \mathbf{k})}{\sqrt{(-3)^2 + (-0.75)^2 + 1^2}}$$

$$= (-0.9231\mathbf{i} - 0.2308\mathbf{j} + 0.3077\mathbf{k})$$

$$\mathbf{u}_{AC} = \frac{(-3\mathbf{i} + \mathbf{j} + 1.5\mathbf{k})}{\sqrt{(-3)^2 + (1)^2 + (1.5)^2}}$$

$$= (-0.8571\mathbf{i} + 0.2857\mathbf{j} + 0.4286\mathbf{k})$$

$$\text{Proj } F_{AB} = \mathbf{F} \cdot \mathbf{u}_{AB}$$

$$= 60(-0.9231) + 12(-0.2308) + (-40)(0.3077) = -70.46\ \text{N}$$

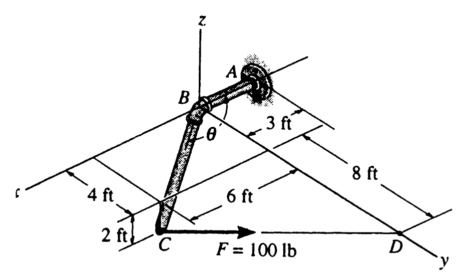
$\text{Proj } F_{AB} = 70.5\ \text{N}$ **Ans**

$$\text{Proj } F_{AC} = \mathbf{F} \cdot \mathbf{u}_{AC}$$

$$= 60(-0.8571) + 12(\ 0.2857) + (-40)(0.4286) = -65.14\ \text{N}$$

$\text{Proj } F_{AC} = 65.1\ \text{N}$ **Ans**

***2-140.** Determine the magnitude of the projected component of the 100-lb force acting along the axis BC of the pipe.



$$\mathbf{u}_{CD} = \frac{(0-6)\mathbf{i} + (12-4)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-6)^2 + (12-4)^2 + [0-(-2)]^2}}$$
$$= -0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{CD} = 100(-0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k})$$
$$= \{-58.835\mathbf{i} + 78.446\mathbf{j} + 19.612\mathbf{k}\}\ \text{lb}$$

Unit Vector : The unit vector along CB is

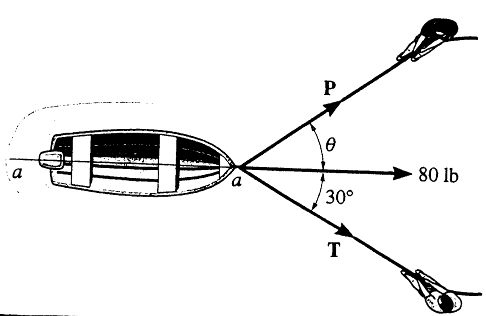
$$\mathbf{u}_{CB} = \frac{(0-6)\mathbf{i} + (0-4)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-6)^2 + (0-4)^2 + [0-(-2)]^2}}$$
$$= -0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k}$$

Projected Component of F Along CB :

$$F_{CB} = \mathbf{F} \cdot \mathbf{u}_{CB} = (-58.835\mathbf{i} + 78.446\mathbf{j} + 19.612\mathbf{k}) \cdot (-0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k})$$
$$= (-58.835)(-0.8018) + (78.446)(-0.5345) + (19.612)(0.2673)$$
$$= 10.5\ \text{lb}$$

Ans

2-141. The boat is to be pulled onto the shore using two ropes. If the resultant force is to be 80 lb, directed along the keel aa, as shown, determine the magnitudes of forces **T** and **P** acting in each rope and the angle θ of **P** is a *minimum*. **T** acts at 30° from the keel as shown.

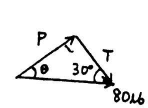


From the figure P is minimum, when

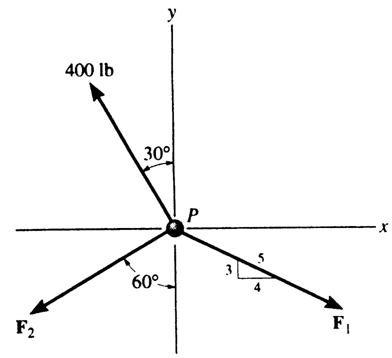
$\theta + 30° = 90°$; $\theta = 60°$ **Ans**

$\dfrac{P}{\sin 30°} = \dfrac{80}{\sin 90°}$; $P = 40\ \text{lb}$ **Ans**

$\dfrac{T}{\sin 60°} = \dfrac{80}{\sin 90°}$; $T = 69.3\ \text{lb}$ **Ans**



3-1. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ so that particle P is in equilibrium.



Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_1\left(\frac{4}{5}\right) - 400\sin 30^\circ - F_2 \sin 60^\circ = 0$$

$$0.8F_1 - 0.8660F_2 = 200.0 \qquad [1]$$

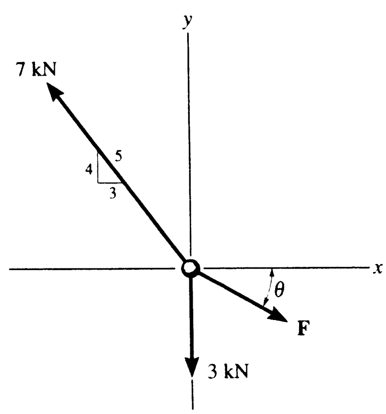
$$+\uparrow \Sigma F_y = 0; \quad 400\cos 30^\circ - F_1\left(\frac{3}{5}\right) - F_2\cos 60^\circ = 0$$

$$0.6F_1 + 0.5F_2 = 346.41 \qquad [2]$$

Solving Eqs. [1] and [2] yields

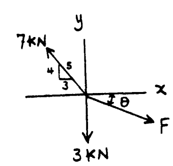
$F_1 = 435$ lb $\quad F_2 = 171$ lb **Ans**

3-2. Determine the magnitude and direction θ of $\mathbf{F}$ so that the particle is in equilibrium.



$$\xrightarrow{+} \Sigma F_x = 0; \quad -7\left(\frac{3}{5}\right) + F\cos\theta = 0$$

$$+\uparrow \Sigma F_y = 0; \quad 7\left(\frac{4}{5}\right) - 3 - F\sin\theta = 0$$

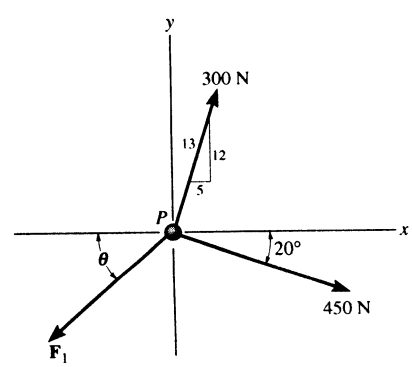


Solving,

$\theta = 31.8^\circ$ **Ans**

$F = 4.94$ kN **Ans**

3-3. Determine the magnitude and angle θ of $\mathbf{F}_1$ so that particle P is in equilibrium.



Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \quad 300\left(\frac{5}{13}\right) + 450\cos 20^\circ - F_1\cos\theta = 0$$

$$F_1\cos\theta = 538.25 \qquad [1]$$

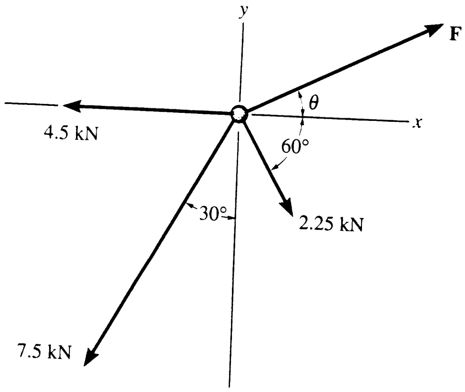
$$+\uparrow \Sigma F_y = 0; \quad 300\left(\frac{12}{13}\right) - 450\sin 20^\circ - F_1\sin\theta = 0$$

$$F_1\sin\theta = 123.01 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$\theta = 12.9^\circ \quad F_1 = 552$ N **Ans**

***3-4.** Determine the magnitude and angle θ of $\mathbf{F}$ so that the particle is in equilibrium.



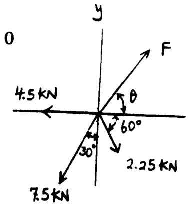
$\xrightarrow{+} \Sigma F_x = 0;\quad F\cos\theta + 2.25\cos 60° - 4.5 - 7.5\sin 30° = 0$

$+\uparrow \Sigma F_y = 0;\quad F\sin\theta - 2.25\sin 60° - 7.5\cos 30° = 0$

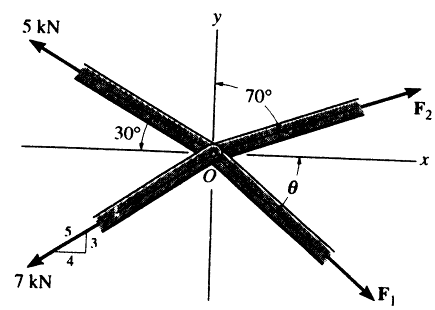
$\tan\theta = \dfrac{8.444}{7.125} = 1.185$

$\theta = 49.8°$ **Ans**

$F = 11.0\ \text{kN}$ **Ans**



3-5. The members of a truss are pin-connected at joint O. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ for equilibrium. Set $\theta = 60°$



Equations of Equilibrium :

$\xrightarrow{+} \Sigma F_x = 0;\quad F_1\cos 60° + F_2\sin 70° - 5\cos 30° - 7\left(\frac{4}{5}\right) = 0$

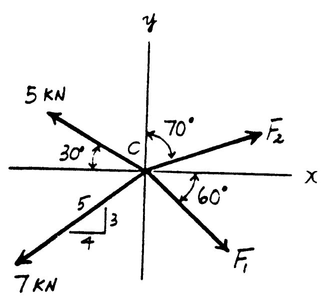
$$0.5F_1 + 0.9397F_2 = 9.9301 \qquad [1]$$

$+\uparrow \Sigma F_y = 0;\quad F_2\cos 70° - F_1\sin 60° + 5\sin 30° - 7\left(\frac{3}{5}\right) = 0$

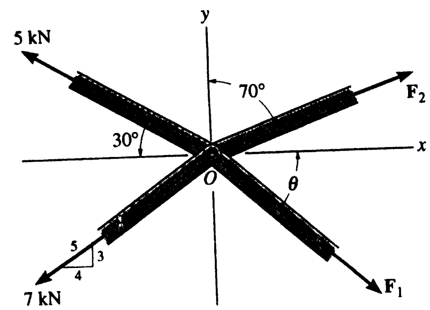
$$0.3420F_2 - 0.8660F_1 = 1.70 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$F_1 = 1.83\ \text{kN} \qquad F_2 = 9.60\ \text{kN}$ **Ans**



3-6. The members of a truss are pin-connected at joint O. Determine the magnitude of $\mathbf{F}_1$ and its angle θ for equilibrium. Set $F_2 = 6$ kN.



Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_1 \cos\theta + 6\sin 70^\circ - 5\cos 30^\circ - 7\left(\frac{4}{5}\right) = 0$$

$$F_1 \cos\theta = 4.2920 \qquad [1]$$

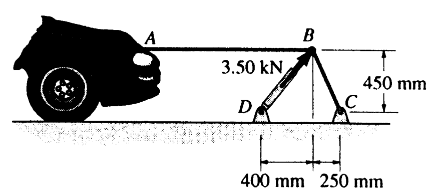
$$+\uparrow \Sigma F_y = 0; \quad 6\cos 70^\circ - F_1 \sin\theta + 5\sin 30^\circ - 7\left(\frac{3}{5}\right) = 0$$

$$F_1 \sin\theta = 0.3521 \qquad [2]$$

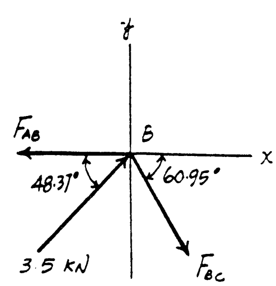
Solving Eqs. [1] and [2] yields

$$\theta = 4.69^\circ \qquad F_1 = 4.31 \text{ kN} \qquad \textbf{Ans}$$

3-7. The device shown is used to straighten the frames of wrecked autos. Determine the tension of each segment of the chain, i.e., AB and BC, if the force which the hydraulic cylinder DB exerts on point B is 3.50 kN, as shown.



Equations of Equilibrium : A direct solution for F_{BC} can be obtained by summing forces along the y axis.



$$+\uparrow \Sigma F_y = 0; \quad 3.5\sin 48.37^\circ - F_{BC}\sin 60.95^\circ = 0$$

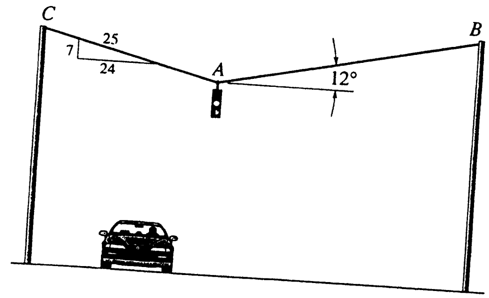
$$F_{BC} = 2.993 \text{ kN} = 2.99 \text{ kN} \qquad \textbf{Ans}$$

Using the result $F_{BC} = 2.993$ kN and summing forces along x axis, we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad 3.5\cos 48.37^\circ + 2.993\cos 60.95^\circ - F_{AB} = 0$$

$$F_{AB} = 3.78 \text{ kN} \qquad \textbf{Ans}$$

***3-8.** Determine the force in cables AB and AC necessary to support the 12-kg traffic light.



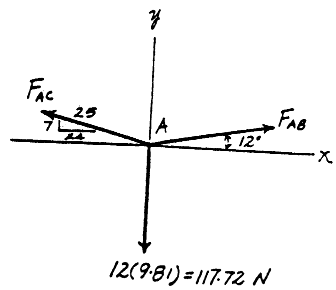
Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB}\cos 12° - F_{AC}\left(\frac{24}{25}\right) = 0$$

$$F_{AB} = 0.9814F_{AC} \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\sin 12° + F_{AC}\left(\frac{7}{25}\right) - 117.72 = 0$$

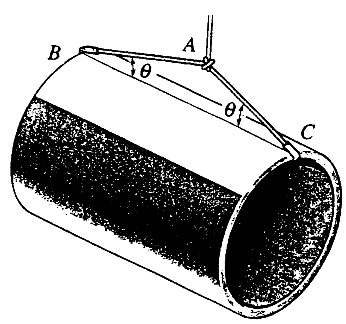
$$0.2079F_{AB} + 0.28F_{AC} = 117.72 \qquad [2]$$



Solving Eqs.[1] and [2] yields

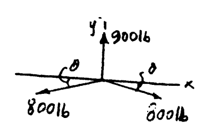
$F_{AB} = 239$ N $\quad F_{AC} = 243$ N **Ans**

3-9. Cords AB and AC can each sustain a maximum tension of 800 lb. If the drum has a weight of 900 lb, determine the smallest angle θ at which they can be attached to the drum.

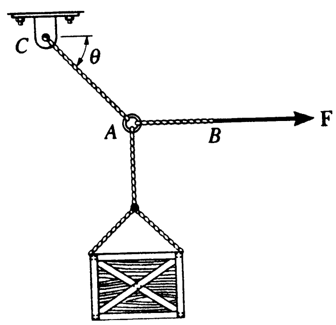


$$+\uparrow \Sigma F_y = 0; \quad 900 - 2(800)\sin\theta = 0$$

$\theta = 34.2°$ **Ans**

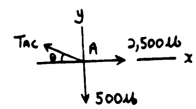


3-10. The 500-lb crate is hoisted using the ropes AB and AC. Each rope can withstand a maximum tension of 2500 lb before it breaks. If AB always remains horizontal, determine the smallest angle θ to which the crate can be hoisted.



Case 1 : Assume $T_{AB} = 2500$ lb

$\xrightarrow{+}\Sigma F_x = 0;\quad 2500 - T_{AC}\cos\theta = 0$



$+\uparrow \Sigma F_y = 0;\quad T_{AC}\sin\theta - 500 = 0$

Solving,

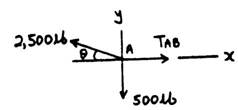
$\theta = 11.31°$

$T_{AC} = 2549.5\text{ lb} > 2500\text{ lb}$ (N.G!)

Case 2 : Assume $T_{AC} = 2500$ lb

$+\uparrow \Sigma F_y = 0;\quad 2500\sin\theta - 500 = 0$

$\theta = 11.54°$

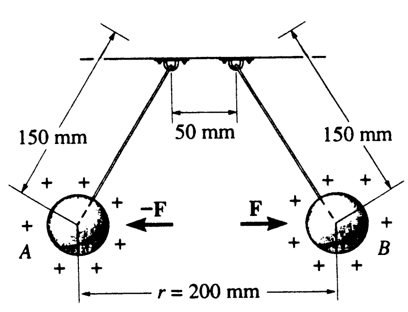


$\xrightarrow{+}\Sigma F_x = 0;\quad T_{AB} - 2500\cos 11.54° = 0$

$T_{AB} = 2449.49\text{ lb} < 2500\text{ lb}$

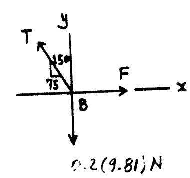
Thus, the smallest angle is $\theta = 11.5°$ **Ans**

3-11. Two electrically charged pith balls, each having a mass of 0.2 g, are suspended from light threads of equal length. Determine the resultant horizontal force of repulsion, F, acting on each ball if the measured distance between them is $r = 200$ mm.



$\xrightarrow{+}\Sigma F_x = 0;\quad F - T\left(\frac{75}{150}\right) = 0$

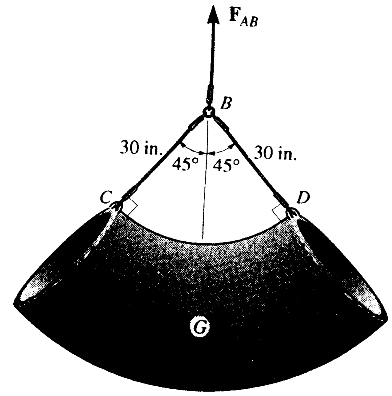
$+\uparrow \Sigma F_y = 0;\quad T\left[\frac{\sqrt{150^2 - 75^2}}{150}\right] - 0.2(9.81)(10^{-3}) = 0$



$T = 2.266(10^{-3})$ N

$F = 1.13$ mN **Ans**

*3-12. The concrete pipe elbow has a weight of 400 lb and the center of gravity is located at point G. Determine the force in the cables AB and CD needed to support it.



Free Body Diagram : By observation, cable AB has to support the entire weight of the concrete pipe. Thus,

$$F_{AB} = 400 \text{ lb} \qquad \textbf{Ans}$$

The tension force in cable CD is the same throughout the cable, that is $F_{BC} = F_{BD}$.

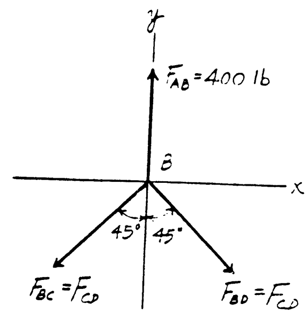
Equations of Equilibrium :

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BD} \sin 45° - F_{BC} \sin 45° = 0$

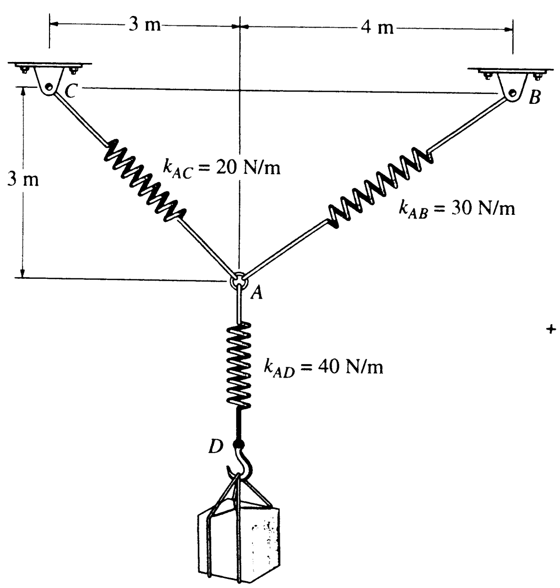
$F_{BC} = F_{Bc} = F$

$+\uparrow \Sigma F_y = 0; \quad 400 - 2F\cos 45° = 0$

$$F = F_{BD} = F_{CB} = 283 \text{ lb} \qquad \textbf{Ans}$$



3-13. Determine the stretch in each spring for equilibrium of the 2-kg block. The springs are shown in the equilibrium position.

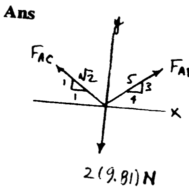


$F_{AD} = 2(9.81) = x_{AD}(40)$

$x_{AD} = 0.4905$ m **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB}\left(\frac{4}{5}\right) - F_{AC}\left(\frac{1}{\sqrt{2}}\right) = 0$

$+\uparrow \Sigma F_y = 0; \quad F_{AC}\left(\frac{1}{\sqrt{2}}\right) + F_{AB}\left(\frac{3}{5}\right) - 2(9.81) = 0$



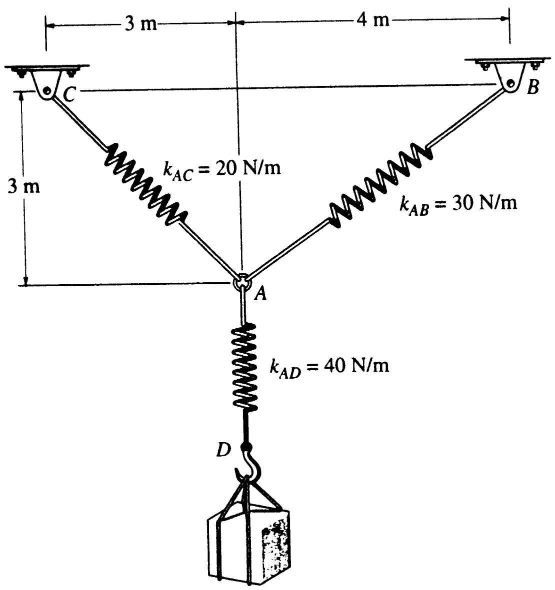
$F_{AC} = 15.86$ N

$x_{AC} = \frac{15.86}{20} = 0.793$ m **Ans**

$F_{AB} = 14.01$ N

$x_{AB} = \frac{14.01}{30} = 0.467$ m **Ans**

3-14. The unstretched length of spring AB is 2 m. If the block is held in the equilibrium position shown, determine the mass of the block at D.



$F = kx = 30(5-2) = 90$ N

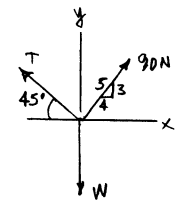
$\xrightarrow{+} \Sigma F_x = 0; \quad T\cos 45° - 90(\frac{4}{5}) = 0$

$T = 101.82$ N

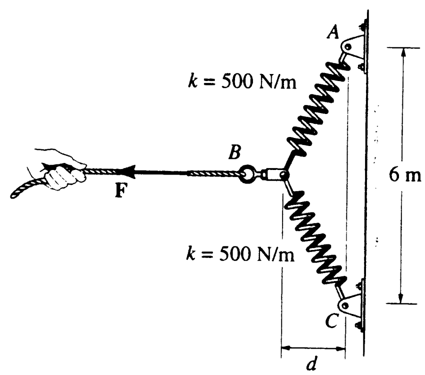
$+\uparrow \Sigma F_y = 0; \quad -W + 101.82 \sin 45° + 90(\frac{3}{5}) = 0$

$W = 126.0$ N

$m = \dfrac{126.0}{9.81} = 12.8$ kg **Ans**



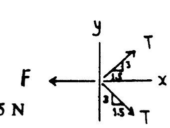
3-15. The spring ABC has a stiffness of 500 N/m and an unstretched length of 6 m. Determine the horizontal force **F** applied to the cord which is attached to the *small* pulley B so that the displacement of the pulley from the wall is $d = 1.5$ m.



$\xrightarrow{+} \Sigma F_x = 0; \quad \dfrac{1.5}{\sqrt{11.25}}(T)(2) - F = 0$

$T = ks = 500(\sqrt{3^2 + (1.5)^2} - 3) = 177.05$ N

$F = 158$ N **Ans**

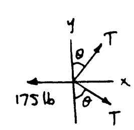


***3-16.** The spring ABC has a stiffness of 500 N/m and an unstretched length of 6 m. Determine the displacement d of the cord from the wall when a force $F = 175$ N is applied to the cord.

$\xrightarrow{+} \Sigma F_x = 0; \quad 175 = 2T\sin\theta$

$T\sin\theta = 87.5$

$T\left[\dfrac{d}{\sqrt{3^2 + d^2}}\right] = 87.5$

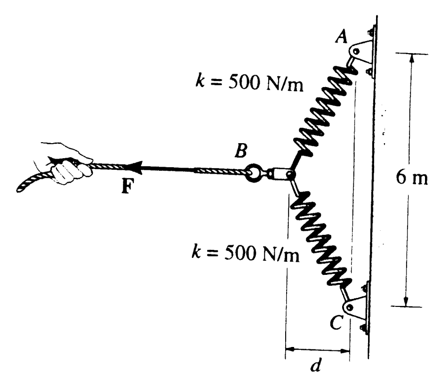


$T = ks = 500(\sqrt{3^2 + d^2} - 3)$

$d(1 - \dfrac{3}{\sqrt{9 + d^2}}) = 0.175$

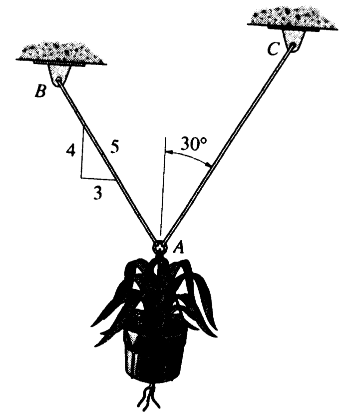
By trial and error :

$d = 1.56$ m **Ans**



Probs. 3–15/16

3-17. Determine the maximum weight of the flowerpot that can be supported without exceeding a cable tension of 50 lb in either cable AB or AC.



Equations of Equilibrium :

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{AC}\sin 30° - F_{AB}\left(\frac{3}{5}\right) = 0$

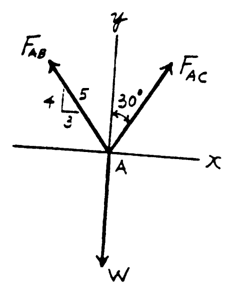
$$F_{AC} = 1.20F_{AB} \qquad [1]$$

$+\uparrow \Sigma F_y = 0;\qquad F_{AC}\cos 30° + F_{AB}\left(\frac{4}{5}\right) - W = 0$

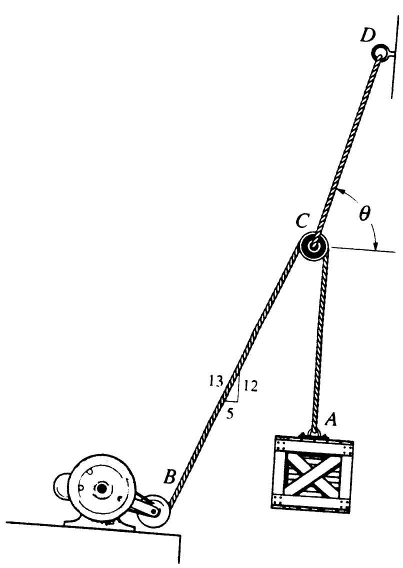
$$0.8660F_{AC} + 0.8F_{AB} = W \qquad [2]$$

Since $F_{AC} > F_{AB}$, failure will occur first at cable AC with $F_{AC} = 50$ lb. Then solving Eq.[1] and [2] yields

$$F_{AB} = 41.67 \text{ lb}$$
$$W = 76.6 \text{ lb} \qquad \textbf{Ans}$$



3-18. The motor at B winds up the cord attached to the 65-lb crate with a constant speed. Determine the force in cord CD supporting the pulley and the angle θ for equilibrium. Neglect the size of the pulley at C.

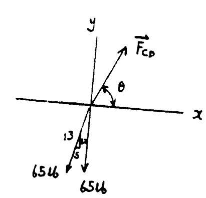


$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{CD}\cos\theta - 65\left(\frac{5}{13}\right) = 0$

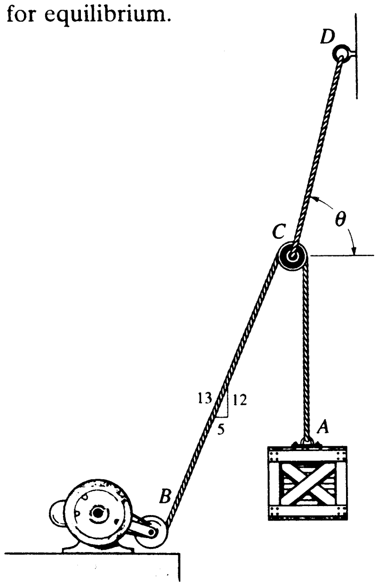
$+\uparrow \Sigma F_y = 0;\qquad F_{CD}\sin\theta - 65 - 65\left(\frac{12}{13}\right) = 0$

$$\theta = \tan^{-1}(5) = 78.7° \qquad \textbf{Ans}$$

$$F_{CD} = 127 \text{ lb} \qquad \textbf{Ans}$$



3-19. The cords BCA and CD can each support a maximum load of 100 lb. Determine the maximum weight of the crate that can be hoisted at constant velocity, and the angle θ for equilibrium.

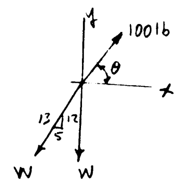


$\xrightarrow{+}\Sigma F_x = 0; \qquad 100\cos\theta = W(\frac{5}{13})$

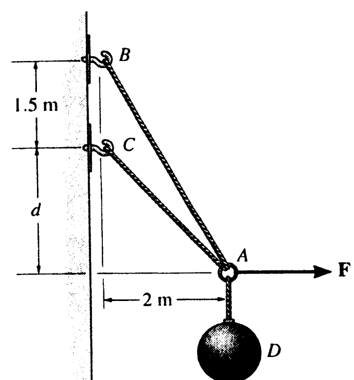
$+\uparrow \Sigma F_y = 0; \qquad 100\sin\theta = W(\frac{12}{13}) + W$

$\theta = 78.7°$ **Ans**

$W = 51.0$ lb **Ans**



***3-20.** Determine the forces in cables AC and AB needed to hold the 20-kg ball D in equilibrium. Take F = 300 N and d = 1 m.



Equations of Equilibrium :

$$\xrightarrow{+}\Sigma F_x = 0; \quad 300 - F_{AB}\left(\frac{4}{\sqrt{41}}\right) - F_{AC}\left(\frac{2}{\sqrt{5}}\right) = 0$$

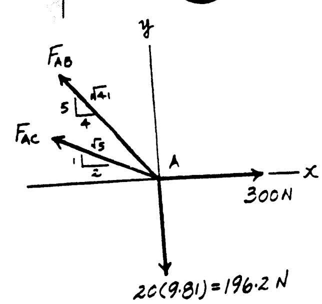
$$06247F_{AB} + 0.8944F_{AC} = 300 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\left(\frac{5}{\sqrt{41}}\right) + F_{AC}\left(\frac{1}{\sqrt{5}}\right) - 196.2 = 0$$

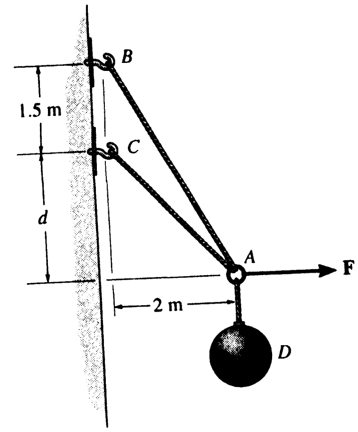
$$0.7809F_{AB} + 0.4472F_{AC} = 196.2 \qquad [2]$$

Solving Eqs. [1] and [2] yields

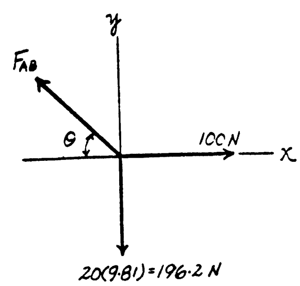
$F_{AB} = 98.6$ N $\quad F_{AC} = 267$ N **Ans**



3-21. The ball D has a mass of 20 kg. If a force of F = 100 N is applied horizontally to the ring at A, determine the largest dimension d so that the force in cable AC is zero.



Equations of Equilibrium :



$\xrightarrow{+} \Sigma F_x = 0;\quad 100 - F_{AB}\cos\theta = 0 \quad F_{AB}\cos\theta = 100$ [1]

$+\uparrow \Sigma F_y = 0;\quad F_{AB}\sin\theta - 196.2 = 0 \quad F_{AB}\sin\theta = 196.2$ [2]

Solving Eqs. [1] and [2] yields

$$\theta = 62.99^\circ \qquad F_{AB} = 220.21 \text{ N}$$

From the geometry,

$$d + 1.5 = 2\tan 62.99^\circ$$
$$d = 2.42 \text{ m} \qquad \textbf{Ans}$$