In general:

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_1 \partial x_n} & \frac{\partial^2 f}{\partial x_2 \partial x_n} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

Function b.iii:

$$Q = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{5\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{5}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{5\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{4} & -\frac{5\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ -\frac{5\sqrt{3}}{4} + \frac{\sqrt{3}}{4} & \frac{5}{4} + \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 4 & -\sqrt{3} \\ -\sqrt{3} & 2 \end{bmatrix}$$

$$f(x) = x^T Q x = \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 4 & -\sqrt{3} \\ -\sqrt{3} & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4x_1 - \sqrt{3}x_2 & -\sqrt{3}x_1 + 2x_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 4x_1^2 - \sqrt{3}x_1x_2 - \sqrt{3}x_1x_2 + 2x_2^2 = 4x_1^2 - 2\sqrt{3}x_1x_2 + 2x_2^2$$

$$\nabla f(x) = \begin{bmatrix} 8x_1 - 2\sqrt{3}x_2 \\ 4x_2 - 2\sqrt{3}x_1 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} 8 & -2\sqrt{3} \\ -2\sqrt{3} & 4 \end{bmatrix}$$

Function c.Rosenbrock:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 =$$
$$100x_2^2 - 200x_1^2x_2 + 100x_1^4 + 1 - 2x_1 + x_1^2 =$$
$$100x_1^4 + x_1^2 - 2x_1 - 200x_1^2x_2 + 100x_2^2$$

$$\nabla f(x) = \begin{bmatrix} 400x_1^3 + 2x_1 - 2 - 400x_1x_2 \\ 200x_2 - 200x_1^2 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} 1200x_1^2 + 2 - 400x_2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}$$

Function qp (ex.2):

$f_0(x,y,z) = x^2 + y^2 + (z+1)^2$
$h_1(x,y,z) = x + y + z - 1 \ (= 0)$
$f_1(x,y,z) = -x$
$f_2(x,y,z) = -y$
$f_3(x,y,z) = -z$

Function for newton method:

From lecture 7+8, slide 61:
Let $f^{new}(x,y,z) = tf_0(x,y,z) + \phi(x,y,z)$

$f^{new}(x,y,z) = t(x^2 + y^2 + (z+1)^2) - log(x) - log(y) - log(z) =$

$t(x^2 + y^2 + z^2 + 2z + 1) - log(x) - log(y) - log(z)$

$$\nabla f^{new}(x,y,z) = \begin{bmatrix} 2tx - \frac{1}{x} \\ 2ty - \frac{1}{y} \\ 2tz + 2t - \frac{1}{z} \end{bmatrix}$$

$$\nabla^2 f^{new}(x,y,z) = \begin{bmatrix} 2t + \frac{1}{x^2} & 0 & 0 \\ 0 & 2t + \frac{1}{y^2} & 0 \\ 0 & 0 & 2t + \frac{1}{z^2} \end{bmatrix}$$

Let $\phi_i(x) = -log(-f_i(x))$

$\phi_1(x,y,z) = -log(x)$

$$\nabla \phi_1(x,y,z) = \begin{bmatrix} -\frac{1}{x} \\ 0 \\ 0 \end{bmatrix}$$

$$\nabla^2\phi_1(x,y,z) = diag(\frac{1}{x^2}, 0, 0)$$

$$\phi_2(x,y,z) = -log(y)$$

$$\nabla\phi_2(x,y,z) = \begin{bmatrix} 0 \\ -\frac{1}{y} \\ 0 \end{bmatrix}$$

$$\nabla^2\phi_2(x,y,z) = diag(0, \frac{1}{y^2}, 0)$$

$$\phi_3(x,y,z) = -log(z)$$

$$\nabla\phi_3(x,y,z) = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{z} \end{bmatrix}$$

$$\nabla^2\phi_3(x,y,z) = diag(0, 0, \frac{1}{z^2})$$

<u>**Function lp (ex.2)**</u>:
$f_0(x,y) = -x - y$
$f_1(x,y) = -x - y + 1$
$f_2(x,y) = y - 1$
$f_3(x,y) = x - 2$
$f_4(x,y) = -y$

<u>Function for newton method</u>:

From lecture 7+8, slide 61:
Let $f^{new}(x,y) = tf_0(x,y) + \phi(x,y)$

$$f^{new}(x,y) = -tx - ty - log(x+y-1) - log(1-y) - log(2-x) - log(y)$$

$$\nabla f^{new}(x,y) = \begin{bmatrix} -t - \frac{1}{x+y-1} + \frac{1}{2-x} \\ -t - \frac{1}{x+y-1} + \frac{1}{1-y} - \frac{1}{y} \end{bmatrix}$$

$$\nabla^2 f^{new}(x,y) = \begin{bmatrix} \frac{1}{(x+y-1)^2} + \frac{1}{(2-x)^2} & \frac{1}{(x+y-1)^2} \\ \frac{1}{(x+y-1)^2} & \frac{1}{(x+y-1)^2} + \frac{1}{(1-y)^2} + \frac{1}{y^2} \end{bmatrix}$$

Let $\phi_i(x) = -log(-f_i(x))$

$\phi_1(x, y) = -log(x + y - 1)$

$$\nabla\phi_1(x, y) = \begin{bmatrix} -\frac{1}{x+y-1} \\ -\frac{1}{x+y-1} \end{bmatrix}$$

$$\nabla^2\phi_1(x, y) = \begin{bmatrix} \frac{1}{(x+y-1)^2} & \frac{1}{(x+y-1)^2} \\ \frac{1}{(x+y-1)^2} & \frac{1}{(x+y-1)^2} \end{bmatrix}$$

$\phi_2(x, y) = -log(1 - y)$

$$\nabla\phi_2(x, y) = \begin{bmatrix} 0 \\ \frac{1}{1-y} \end{bmatrix}$$

$\nabla^2\phi_2(x, y) = diag(0, \frac{1}{(1-y)^2})$

$\phi_3(x, y) = -log(2 - x)$

$$\nabla\phi_3(x, y) = \begin{bmatrix} \frac{1}{2-x} \\ 0 \end{bmatrix}$$

$\nabla^2\phi_3(x, y) = diag(\frac{1}{(2-x)^2}, 0)$

$\phi_4(x, y) = -log(y)$

$$\nabla\phi_4(x, y) = \begin{bmatrix} 0 \\ -\frac{1}{y} \end{bmatrix}$$

$\nabla^2\phi_4(x, y) = diag(0, \frac{1}{y^2})$