$t, \kappa \in \mathbb{R}, x, \mu \in \mathbb{R}^n$ $(x, \mu$ *are a column vectors*$), G \in \mathbb{R}^{nXn}$

$$f(x) = -tx^T\mu + t\kappa x^T Gx - \sum_{i=1}^{n} log(x_i) =$$

$$-t\begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}\begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix} + t\kappa \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}\begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} - \sum_{i=1}^{n} log(x_i) =$$

$$-t(\mu_1 x_1 + \dots \mu_n x_n) + t\kappa \begin{bmatrix} g_{11}x_1 + ... + g_{n1}x_n & \cdots & g_{1n}x_1 + ... + g_{nn}x_n \end{bmatrix}\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} - \sum_{i=1}^{n} log(x_i) =$$

$$-t(\mu_1 x_1 + \dots \mu_n x_n) + t\kappa((x_1(g_{11}x_1 + ... + g_{n1}x_n) + \cdots + x_n(g_{1n}x_1 + ... + g_{nn}x_n)) - \sum_{i=1}^{n} log(x_i)$$

<u>First derivative</u>

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix} = \begin{bmatrix} -t\mu_1 + t\kappa((2g_{11}x_1 + ... + g_{n1}x_n) + g_{12}x_2 + \cdots + g_{1n}x_n) - \frac{1}{x_1} \\ \vdots \\ -t\mu_n + t\kappa(g_{n1}x_1 + g_{n1}x_2 + ... + (g_{1n}x_1 + ... + 2g_{nn}x_n)) - \frac{1}{x_n} \end{bmatrix} =$$

$$-t\mu + t\kappa \cdot 2Gx - \begin{bmatrix} \frac{1}{x_1} \\ \vdots \\ \frac{1}{x_n} \end{bmatrix}$$

<u>Second derivative</u>

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix} = \begin{bmatrix} 2t\kappa g_{11} + \frac{1}{x_1^2} & t\kappa g_{21} + g_{12} & \cdots & t\kappa g_{n1} + g_{1n} \\ t\kappa g_{21} + g_{12} & 2t\kappa g_{22} + \frac{1}{x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ t\kappa g_{n1} + g_{1n} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & 2t\kappa g_{nn} + \frac{1}{x_n^2} \end{bmatrix}$$

$$\overset{(g\ is\ symmetric)}{=} \begin{bmatrix} 2t\kappa g_{11} + \frac{1}{x_1^2} & \cdots & 2t\kappa g_{n1} \\ \vdots & \ddots & \vdots \\ 2t\kappa g_{n1} & \cdots & 2t\kappa g_{nn} + \frac{1}{x_n^2} \end{bmatrix} = 2t\kappa G + diag\left(\frac{1}{x_1^2} \cdots \frac{1}{x_n^2}\right)$$