

In general:

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_1 \partial x_n} & \frac{\partial^2 f}{\partial x_2 \partial x_n} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

Function b.iii:

$$Q = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{5\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{5}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{5\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{4} & -\frac{5\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ -\frac{5\sqrt{3}}{4} + \frac{\sqrt{3}}{4} & \frac{5}{4} + \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 4 & -\sqrt{3} \\ -\sqrt{3} & 2 \end{bmatrix}$$

$$f(x) = x^T Q x = \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 4 & -\sqrt{3} \\ -\sqrt{3} & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = (4x_1 - \sqrt{3}x_2 \quad -\sqrt{3}x_1 + 2x_2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 4x_1^2 - \sqrt{3}x_1x_2 - \sqrt{3}x_1x_2 + 2x_2^2 = 4x_1^2 - 2\sqrt{3}x_1x_2 + 2x_2^2$$

$$\nabla f(x) = \begin{bmatrix} 8x_1 - 2\sqrt{3}x_2 \\ 4x_2 - 2\sqrt{3}x_1 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} 8 & -2\sqrt{3} \\ -2\sqrt{3} & 4 \end{bmatrix}$$

Function c.Rosenbrock:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 = 100x_2^2 - 200x_1^2x_2 + 100x_1^4 + 1 - 2x_1 + x_1^2 = 100x_1^4 + x_1^2 - 2x_1 - 200x_1^2x_2 + 100x_2^2$$

$$\begin{aligned}\nabla f(x) &= \begin{bmatrix} 400x_1^3 + 2x_1 - 2 - 400x_1x_2 \\ 200x_2 - 200x_1^2 \end{bmatrix} \\ \nabla^2 f(x) &= \begin{bmatrix} 1200x_1^2 + 2 - 400x_2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}\end{aligned}$$