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Assignment

Q1) Find Rank

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \longleftrightarrow R_2 - 2R_1$
$R_3 \longleftrightarrow R_3 - 3R_1$
$R_4 \longleftrightarrow R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \longleftrightarrow R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \longleftrightarrow -R_2/4$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

Aman Roy Gupta

Page No.:

Date:

$R_1 \longleftrightarrow R_1 - 2R_2$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \longleftrightarrow R_4 + 4R_2$

$$A_2 \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 \longleftrightarrow -R_3/3$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_1 \longleftrightarrow R_1 + R_3$

$\cancel{R_4 \longleftrightarrow R_4 + 3R_3}$

$R_2 = R_2 - 2R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

Aman Raj Gupta

$R_4 \longleftrightarrow R_4 + 3R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ The Rank of Matrix $= 3$

Q2) Let W be the Vector space of all Symmetric 2×2 matrices and let $T: W \to P_2$ be, the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

Find Rank and nullity of T.

Soln Since, the maximum degree of polynomial $T \Rightarrow 2$. So, ~~dimension~~ $\dim(P_2) \Rightarrow 3$

Kernel

So, a subset of kernel T is $T(A) = 0$

$$(a-b) + (b-c)x + (c-a)x^2 = 0$$

Let $a = b = c = d$

Aman Raj Gupta

new matrix $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

dimension of Kernel is 1, because there's only one independent parameter as 't'.

Acc. To Rank Nullity Theorem:

$$\text{rank}(T) + \text{nullity}(T) = \dim(W)$$
$$\text{rank}(T) + 1 = 4$$

So, Rank of $T = 3$ & nullity $= 1$

Q3) Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. find eigen values and eigen vectors of A^{-1} & $A + 4I$

$$A - \lambda I = 0$$

Let $\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$

$$(2-\lambda)^2 - 1 = 0$$
$$(2-\lambda) = \pm 1$$
$$\boxed{\lambda = 1, 3}$$

Aman Raj Gupta

Page No.

Date

for $\lambda = 1$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x - y = 0$$
$$x = y$$

Let $x, y = t$

Eigen vector $v_1 = t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda = 3$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x - y = 0$$
$$x = -y$$

Let ~~x, y~~ = $x = t$

$y = -t$

So, eigen value $v_2 = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Aman Roy Gupta

Page No.
Date

Now, find for A^{-1}

eigen values of A^{-1} will be

$\frac{1}{\lambda_1}$ & $\frac{1}{\lambda_2}$ = $1, \frac{1}{3}$

& eigen vectors are same as of A,

$v_1 = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $v_2 = t\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Now, for $A + 4I$

- eigen values for $A + 4I$ will be
$\lambda_1 + 4$, $\lambda_2 + 4$ = $\underline{5, 7}$

- and eigen ~~value~~ vectors are same as of A

$v_1 = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $v_2 = t\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

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Q4) Solve by Gauss - Siedel Method (Take 3 iterations)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x - 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

with initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

Equ $= x^{k+1} = \dfrac{7.85 + 0.1y^k + 0.2z^k}{3}$

$$y^{k+1} = \frac{-19.3 - 0.1x^{k+1} - 0.3z^k}{-7}$$

$$z^{k+1} = \frac{71.4 - 0.3x^{k+1} + 0.2y^{k+1}}{10}$$

As $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

Iteration 1

$$x(1) = \frac{7.85 + 0.1(0) + 0.2(0)}{3} = 2.6167$$

$$y(1) = \frac{-19.3 - 0.1(2.6167) - 0.3(0)}{-7} = 2.793$$

$$z = \frac{71.4 - 0.3(2.6167) - 0.3(2.7956)}{10} = 7.1373$$

Aman Raj Gupta

Page No.

Date:

Iteration (2)

$$x(2) = \frac{7.85 + 0.1(2.7956) + 0.2(7.13\ldots)}{3}$$

$$= 3$$

$$y(2) = \frac{-19.3 - 0.1(3) - 0.3(7.1373)}{-7} = 3$$

$$z(2) = \frac{71.4 - 0.3(3) - 0.2(3)}{10} = 3$$

Iteration (3)

$x(3) = (7.85 + 0.1(3) + 0.2(3))/3 = 3$

$y(3) = (-19.3 - 0.1(3) - 0.3(3))/-7 = 3$

$z(3) = (71.4 - 0.3(3) + 0.2(3))/10 = 3$

So, value of $x = 3, y = 3, z = 3$

Q5) Define consistent & inconsitent system of equations

Aman Raj Gupta

Page No.:
Date:

→ **Consistent** (at least one sol^n)

- **Dependent** (infinite sol^n.)
- **Independent** (unique sol^n.)

→ **Inconsistent** (No sol^n)

Ex.

$$A = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_2 - 2R_1$

$R_3 \leftrightarrow R_3 - 3R_1$

$R_4 \leftrightarrow R_4 - R_1$

$$= \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_3 \leftrightarrow R_3 - 2R_2$

$R_4 \leftrightarrow R_4 + 2R_2$

$$= \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Aman Raj Gupta

Page No.:
Date:

$\rho(A) = 2$
$\rho(A:B) = 2$
$n = 3$

$\rho(A) = \rho(A:B) \neq n$

$\therefore$ Consitent, but infinite Soln.

Q6) Determine, whether fn $T: P_2 \to P$ is linear transformation or not.

$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1$

Soln
1.) Additive

$T(u+v) = T(u) + T(v)$

$u = a_1 + b_1 x + c_1 x^2$
$v = a_2 + b_2 x + c_2 x^2$

$T(u+v) = T\big((a_1+a_2) + (b_1+b_2)x + (c_1+c$

$= (a_1+a_2+1) + (b_1+b_2+1)x + (c_1+c_2+1)x^2$

$= (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1)$
$+ (b_2+1)x + (c_2+1)x^2$

$= T(u) + T(v)$

= Hence Proved

Aman Raj Gupta

Page No.
Date:

Q7) Determine whether set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis, determine the dimⁿ & basis of subspace spanned by S.

$$a(1,2,3) + b(3,1,0) + c(-2,1,3) = (0,0,0)$$

$$a + 3b - 2c = 0$$
$$2a + b + c = 0$$
$$3a + 3c = 0$$

$c = +a$, $b = -a$

only one solⁿ is possible $a = b = c = 0$

So, linearly independent.

Since, dim = 3
and S contain 3 vectors
So, it spans $V_3(R)$ making it a basis for $V_3(R)$.

Q8) Using Jacobi's Method (Perform 3 iterations)

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

$x_0 = 1$, $y_0 = 1$, $z_0 = 1$

Aman Raj Gupta

first eqn $x = \frac{1}{3}(23 + 6y - 2z)$

second eqn $y = 1(-15 + 4x + z)$

Third eqn $z = \frac{1}{7}(16 - x + 3y)$

$x(0) = 1, \quad y(0) = 1, \quad z(0) = 1$

Iteration 1

$x(1) = (23+6-2)/3 = 9$

$y(1) = (-15 + 4 + 1) = -10$

$z(1) = (16 - 1 + 3)/7 = 18/7$

Q9) Affine Transformation

Rotation

→ Suppose we have a 2-D image represented as grid or pixels we can use AT matrix to rotate around centre.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ rotation of image by θ. to rotate it around centre.

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(i) Translatn to origin
(ii) Rotations
(iii) Translatn Back

Q10) Briefly Description of linear transformatn for Computer Vision for rotating 2-D image.

Sol:- Linear transformation for rotation of 2-D images involves applying a rotation matrix to each pixel coordinate. This matrix rotates points counterclockwise by an angle θ around the origin. It preserves geometric properties like parallel lines and distances.