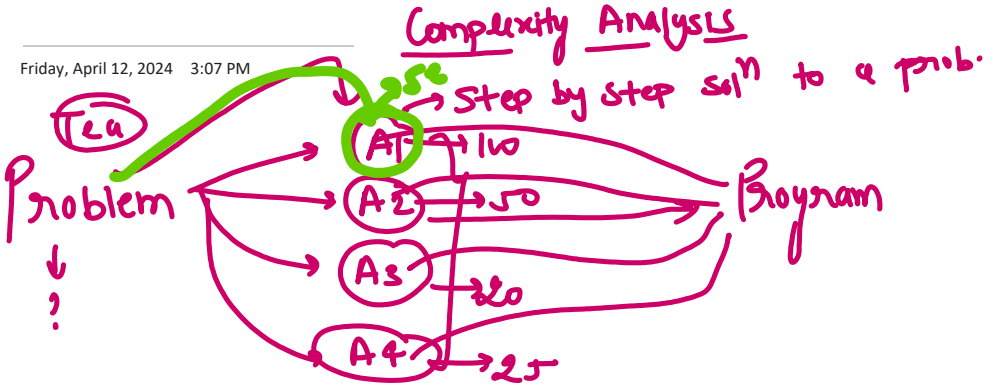
Friday, April 12, 2024 3:07 PM

# Complexity Analysis




Which Algo is good or bad?

Complexity

Time

Space

Complexity → No. of operations?

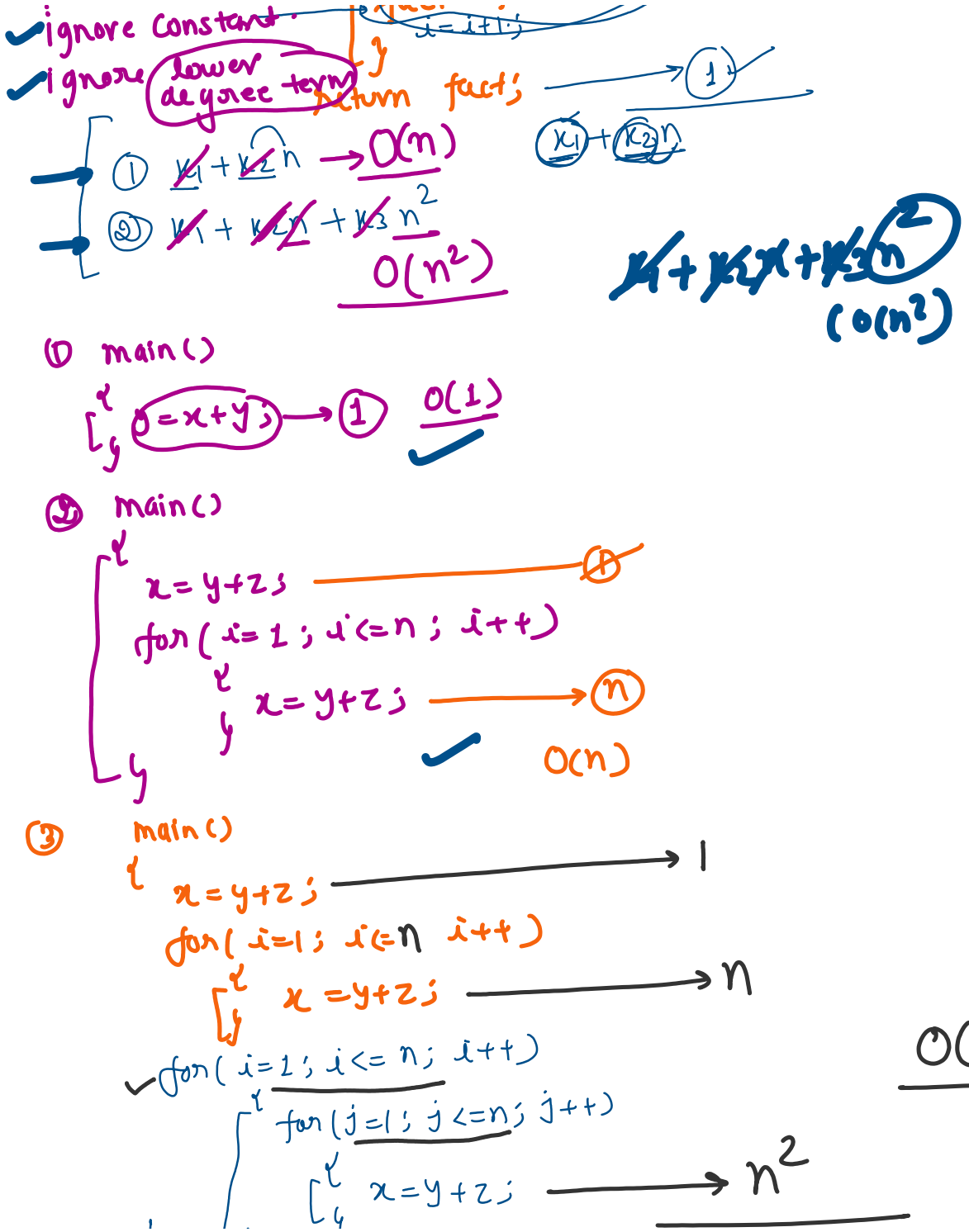
Time taken by Algorithm A for input size n.

T(A, n) | S(A, n)

n → input size

No. of operations → ? No. of operation

```
fact = 1;                          // 1 → constant
for(int i=1; i<=n; i++)            // n times
    fact = fact * i;               // 1
```


ignore constant
ignore lower degree term
return fact;
1
O(n)
O(n²)
O(1)
main()
x=y+z;
for(i=1; i<=n; i++)
O(n)
O(n²)

$1+1+n^2$

④
```
main()
{
    i=1;          -> 1
    while (i<=n)
    {
        x=y+z;    -> 2n
        i=i+1;
    }
}
```
$1+2n$ → $O(n)$

⑤
```
main()
{
    i=1;
    while (i<=n)
    {
        i=i+5;
        i=i+7;
    }
}
```
$\frac{n}{12}$ → $\frac{1}{12} * n \Rightarrow O(n)$

```
main()
{
    i=1;          -> 1
    while (i<=10)
    {
        i=i+2;    <- ⌈n/2⌉
    }
}
```
$\frac{10}{3}$

$i=7$ 7 8 9 10 $i=10$

```
main()
{
    i=n;
    while(i>=1)
    {
        i=i-2;
    }
}
```

$\lceil \frac{n}{3} \rceil$

$\frac{1}{3} * n$

```
main()
{
    i=n;
    while(i>=1)
    {
        i=i-5;
        i=i-7;
        i=i-50;
    }
}
```

$\lceil \frac{n}{62} \rceil \Rightarrow \frac{1}{62} * n$

$\Rightarrow O(n)$

i=11

```
main()
{
    int i=1;
    while(i<=n)
    {
        i=2*i;
    }
}
```

1
$2^1$ —
$2^2$ —
$2^3$ —
$2^4$ —
k times
$2^k$ — n

7 times

$\log_2 128$

$\log_2 2^7$ (7)

$\lfloor \rfloor$

$O(\log_2 n)$

$2^x = n$

$\log_2 2^x = \log_2 n$

$x \log_2 2 = \log_2 n$

$x = \log_2 n$

1

3

$3^2$

$3^3$

$3^x = n$

$\log_3 3^x \Rightarrow \log_3 n$

$x \log_3 3 = \log_3 n$

$x = O(\log_3 n)$

1

```
main()
{
  i=1;
  while(i<=n)
  {
    i=2*i;
    i=3*i;
  }
}
```

$O(\log_6 n)$

$6, 6^2, 6^3, \ldots, 6^k \Rightarrow n$

```
main()
{
  i=n;
  while(i>1)
  {
    i=i/5;
    i=i/7;
    i=i/2;
    i=i/2;
    i=1*i;
```

$i/5/7 \Rightarrow i/35/2 \Rightarrow i/70/2 \Rightarrow \ldots \Rightarrow i/140 \times 4$

$\Rightarrow i/35 \ldots$

$\Rightarrow O(\log_{35} n)$


$2$

$2^2 \rightarrow (2^2)^1 \Rightarrow 2^2$

$4^2 \rightarrow (2^2)^2 \rightarrow 2^4 \rightarrow 2^{2^2}$

$16^2 \rightarrow (2^4)^2 \Rightarrow 2^8 \rightarrow 2^{2^3}$

$256^2 \rightarrow (2^8)^2 \Rightarrow 2^{16} \rightarrow 2^{2^4}$

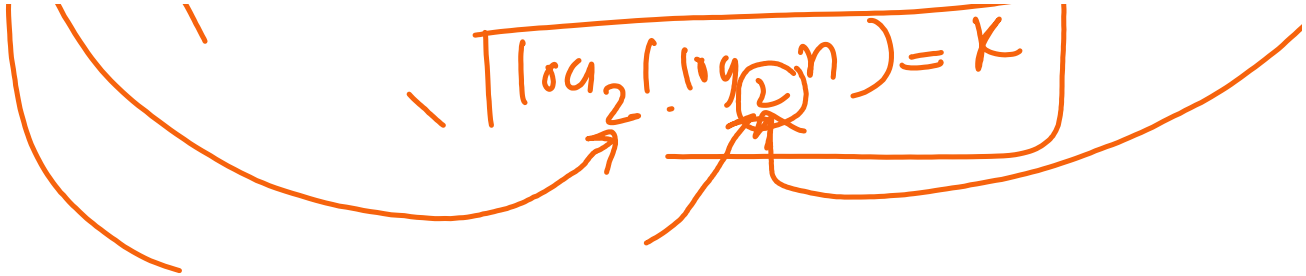
$2^{2^x}$

$n = 2^{2^x}$

$\log_2 n = 2^x \log_2 2$

$\log_2 n = 2^x$

$\log_2(\log_2 n) = x \log_2 2$




| $i = i+2$ | $i = i \times 2$ | $2^2$ |
|---|---|---|
| $i-2$ | $\log_2 3$ | $\log \log n$ |
| $n/2$ | / | |