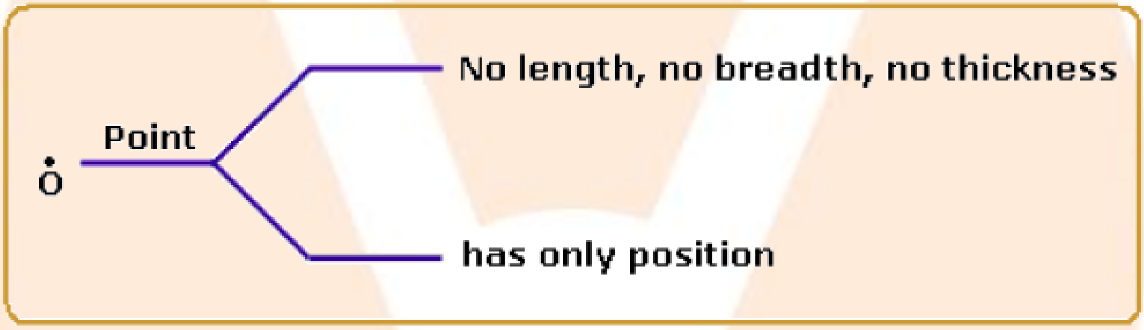
Revision Notes

Class 9 Maths

Chapter 6 - Lines and Angles

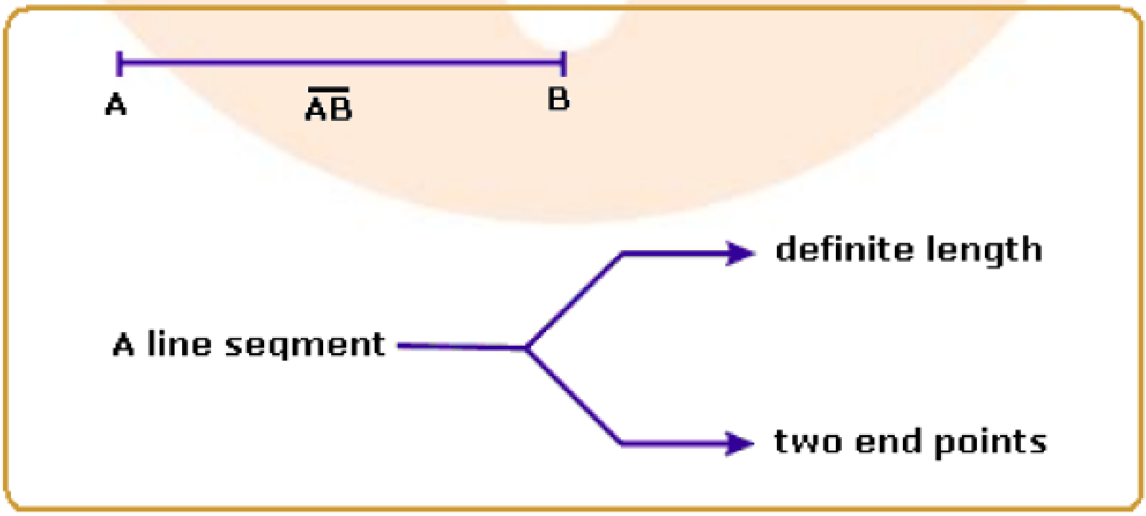
Geometrical Concepts
Point:

- It is a **precise position**.
- It is a **small dot** with **no length, width, or thickness**, but it does have location, i.e. **no magnitude**.
- It is denoted by capital letters A, B, C, O etc.



Line Segment:

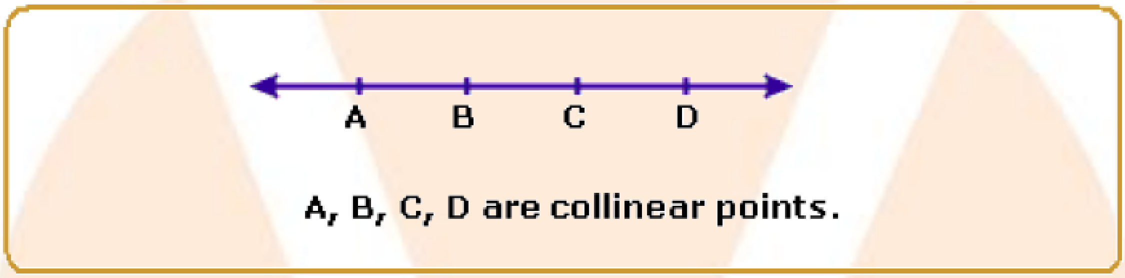
- A line segment $\overline{AB}$ is a straight path that connects two points A and B.
- It has a **defined length** and **end points**. (There is no width or thickness)



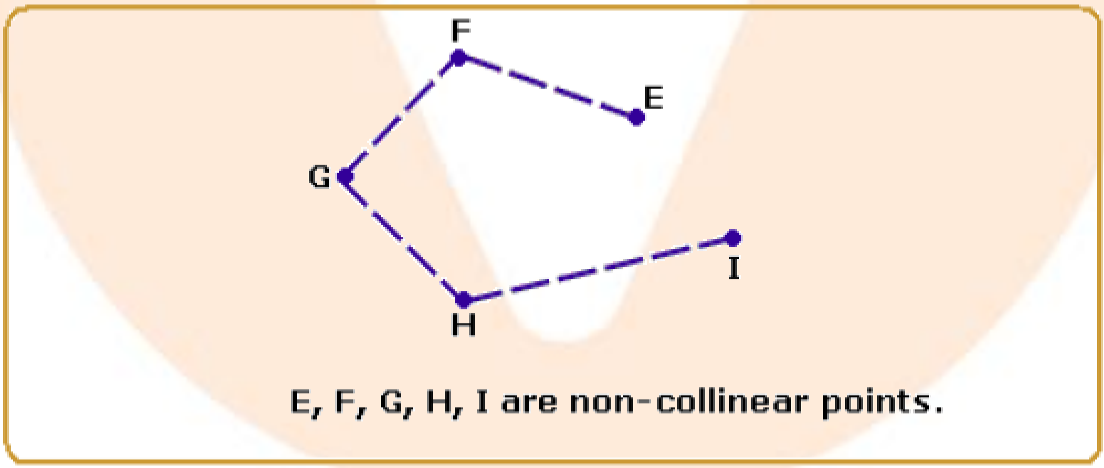
Ray:
A ray is a **line segment** that can only be **extended in one direction**.

Line:
A line is formed when a **line segment** is **stretched in both directions indefinitely**.

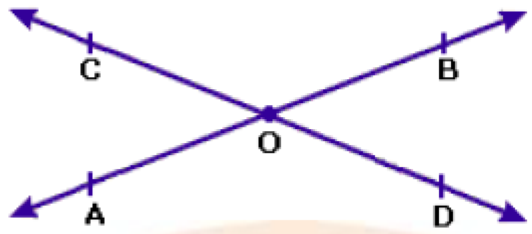
Collinear Points:
Collinear points are defined as two or more points that are on the same line.



Non-collinear Points:
- Non-collinear points are those that do not lie on the same line.
- Example: A, B, C, D, E

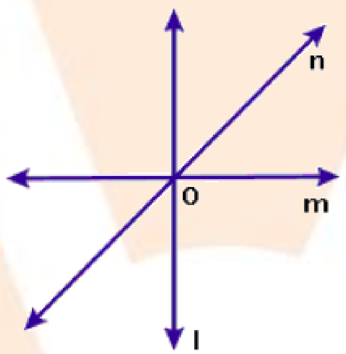


Intersecting Lines:
- Intersecting lines are two lines that have a common point.
- The common point is called as the point of intersection.



O is the point of intersection

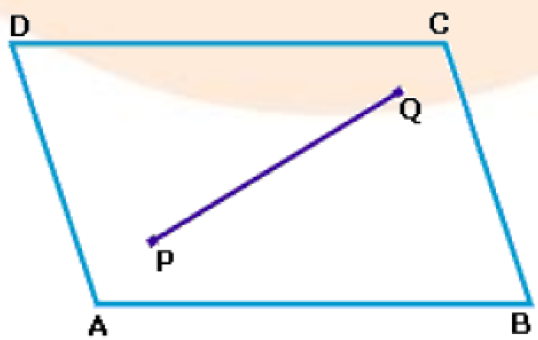
Concurrent Lines:
Concurrent lines are defined as two or more lines intersecting at the same point.



l, m, n are concurrent lines

Plane:

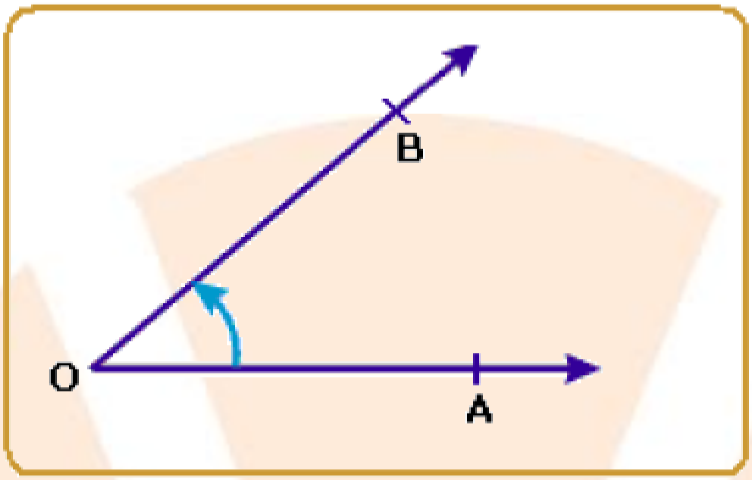
- A plane is a surface on which every point of a line connecting any two points lies on that line.
- Surface of a smooth wall, surface of a paper.



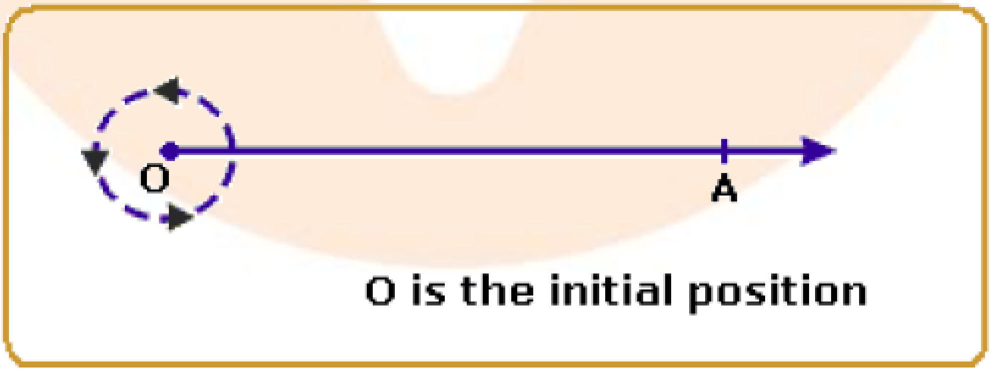
ABCD is a plane

Angles:

- An angle is formed when two straight lines intersect at a point.



- It can be represented as $\angle AOB$ or AOB.
- OA and OB are the arms of $\angle AOB$.
- The vertex of the angle (O) is defined as the place where the arms meet.
- The amount of turning from one arm OA to other OB is called the measure of the angle $\angle AOB$ and written as $m\angle AOB$.
- Degrees, minutes, and seconds are used to calculate an angle.
- If a ray spins in an anticlockwise direction around its initial position and returns to its original position after one complete rotation, it has turned 360°.



1 complete rotation is divided into 360 equal parts.

Each part is 1°.

Each part $\left(1^{\circ}\right)$ is divided into 60 equal parts where, each part measures 1 minute $\left(1'\right)$.

$1'$ is divided into 60 equal parts where, each part measures 1 second $\left(1''\right)$

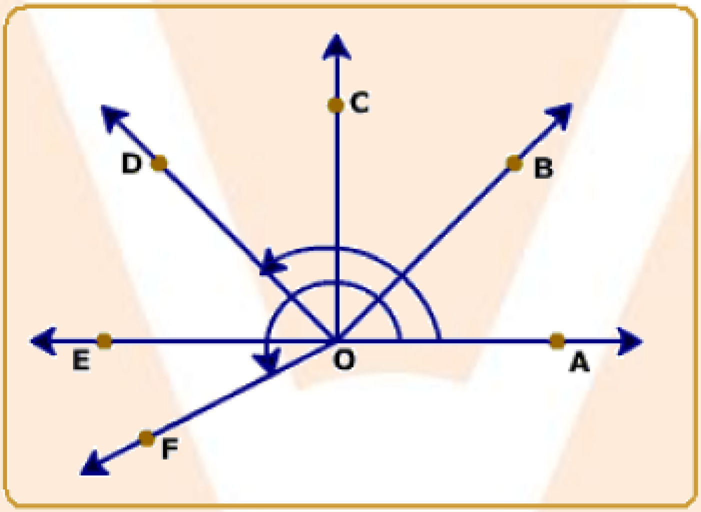
Degrees $\rightarrow$ minutes $\rightarrow$ seconds

$1^{\circ} = 60'$

$1' = 60''$

By recalling that the **union of two rays forms an angle.**

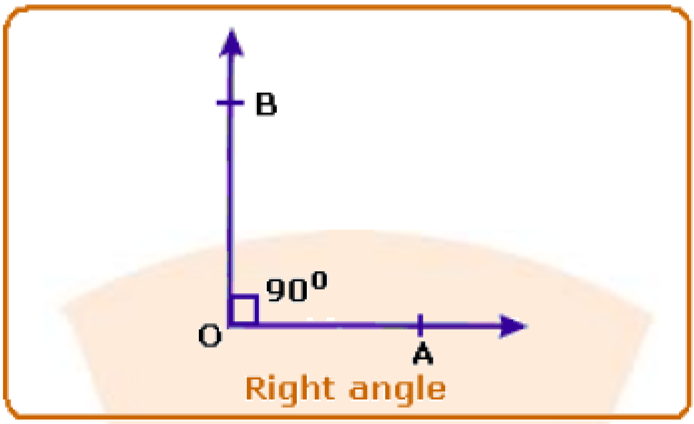
By observing the different type of angles in below figure, we conclude that



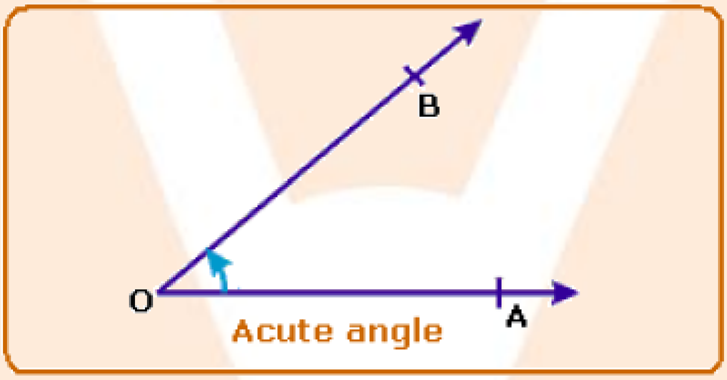
- AOB is an **acute** angle $\left(0^{\circ} < AOB < 90^{\circ}\right)$
- AOC is a **right** angle $\left(\text{an angle equal to } 90^{\circ}\right)$
- AOD is an **obtuse** angle $\left(90^{\circ} < AOD < 180^{\circ}\right)$
- AOE is a **straight** angle $\left(\text{an angle equal to } 180^{\circ}\right)$
- AOF (measured in anti-clock wise direction) is a **reflex** angle $\left(180^{\circ} < AOF < 360^{\circ}\right)$

Right Angle:

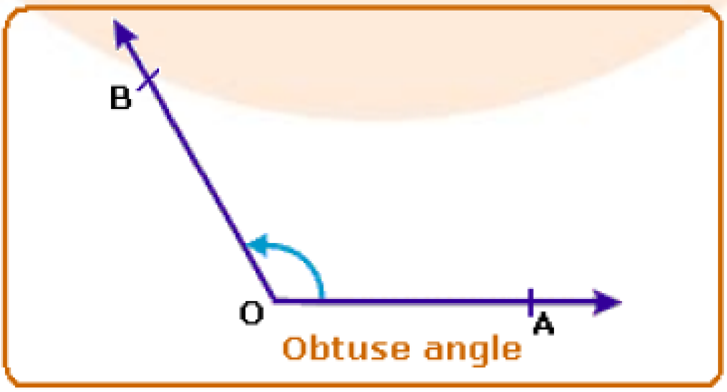
An angle whose measure is 90° known as a **right** angle.



Acute Angle:
An angle whose measure is less than one right angle (that is, less than 90°), known as an **acute** angle.

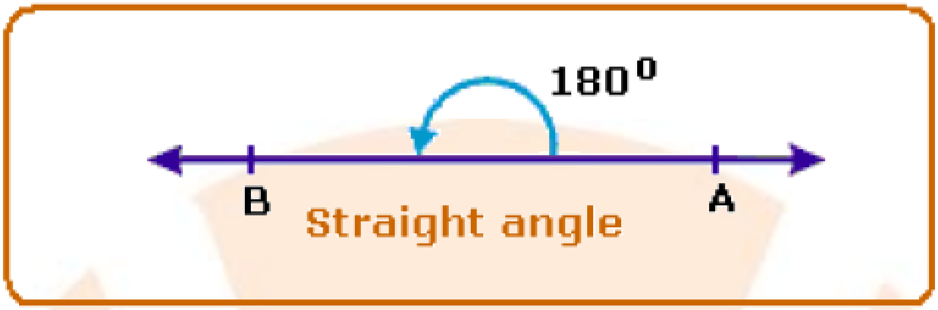


Obtuse Angle:
An angle whose measure is more than one right angle and less than two right angles (that is less than 180° and more than 90°) known as an **obtuse** angle.



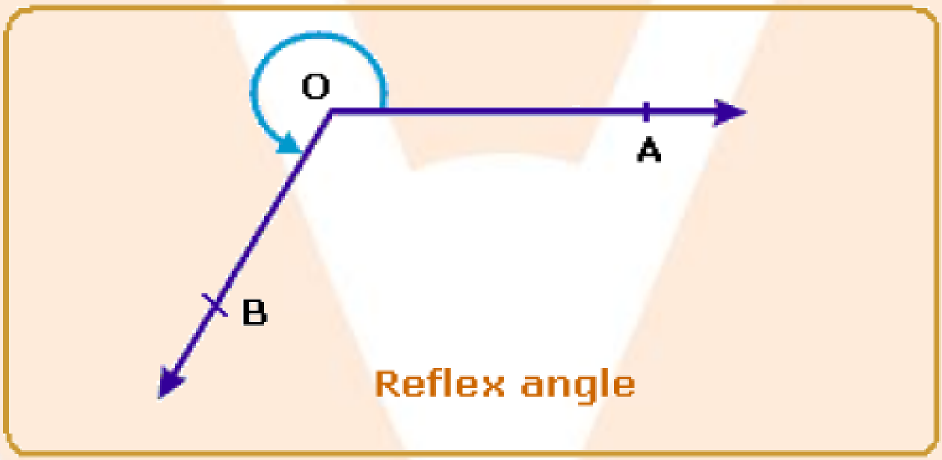
Straight Angle:

An angle whose measure is 180° known as a **straight** angle.



Reflex Angle:

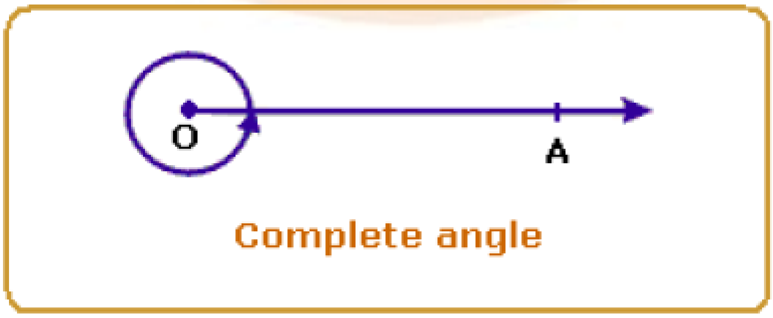
An angle whose measure is more than 180° and less than 360° is called a **reflex** angle.



It can be written as ref. $\angle AOB$.

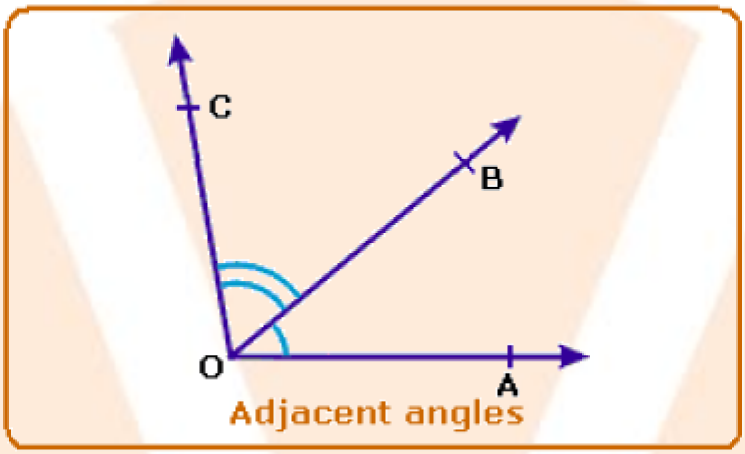
Complete Angle:

An angle whose measure is 360° called a **complete** angle.



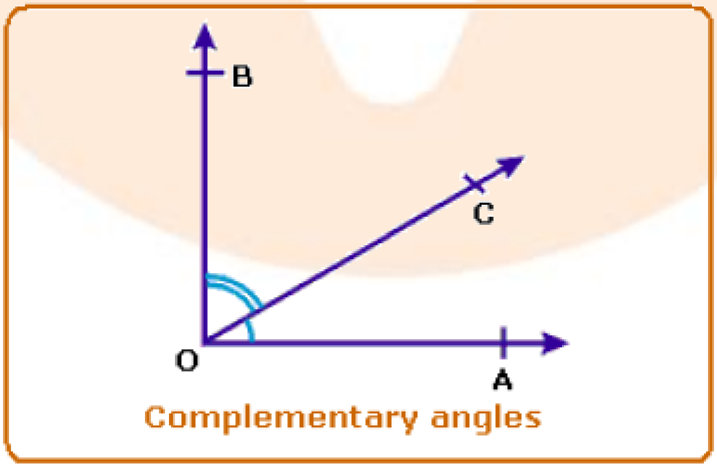
Equal Angles:
When two angles have the same measure, they are said to be **equal**.

Adjacent angles:
Adjacent angles are two angles that share a common vertex and a common arm and have their other arms on opposite sides of the common arm.



O is the common vertex.
AOB and BOC are adjacent angles.
Arm BO separates the two angles.

Complementary Angles:
Complementary angles are those in which the total of the two angles is one right angle (that is 90°).



If the measure of

$AOC = a^{\circ}$

$COB = b^{\circ}$, then

$a^{\circ} + b^{\circ} = 90^{\circ}$

Therefore, AOC and COB are complementary angles.

AOC is **complement** of COB.

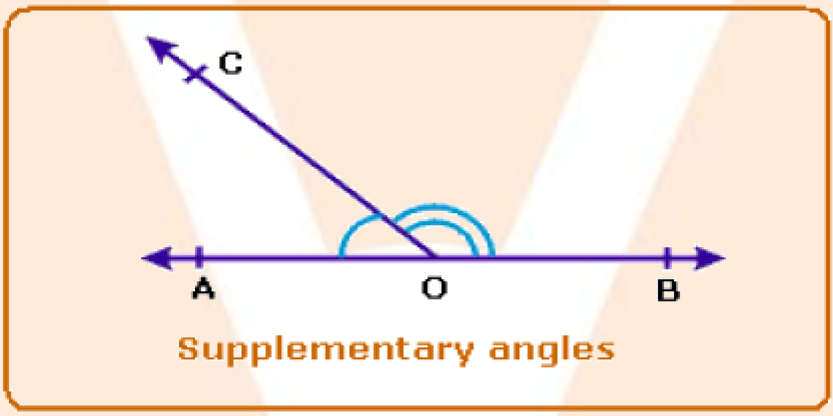
Supplementary Angles:

If the sum of two angles' measurements is 180°, they are said to be **supplementary**.

Example:

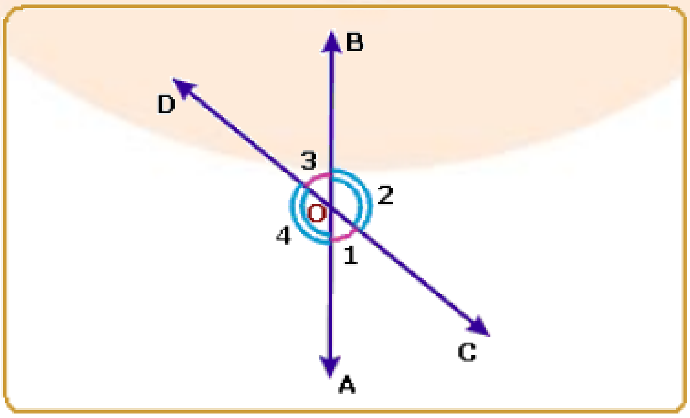
Angles measuring 130° and 50° are supplementary angles.

Two supplementary angles are mutually beneficial.



Vertically Opposite Angles:

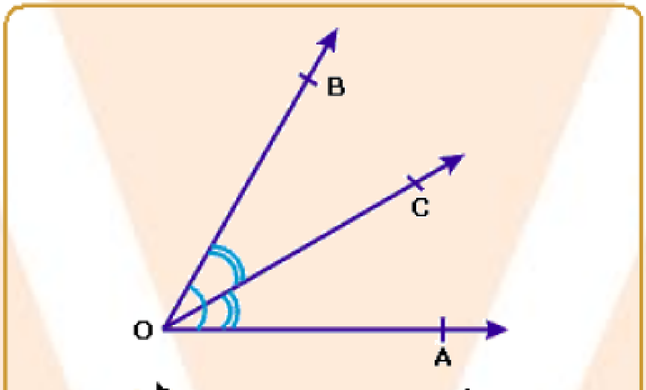
Vertically opposing angles are generated when two straight lines intersects each other at a point and form pairs of opposite angles.



Angles $\angle 1$ and $\angle 3$, and angles $\angle 2$ and $\angle 4$ are vertically opposite angles. **Vertically opposite angles** are always **equal**.

Bisector of an Angle

- A ray or a straight line passing through the vertex of an angle is known as the **Bisector of that angle** if it divides the angle into two equal-sized angles.



$\overrightarrow{OC}$ is the bisector of $A\hat{O}B$

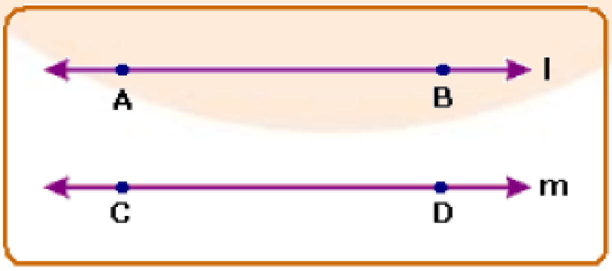
- $BOC = COA$ and,

  $BOC + COA = AOB$ and.

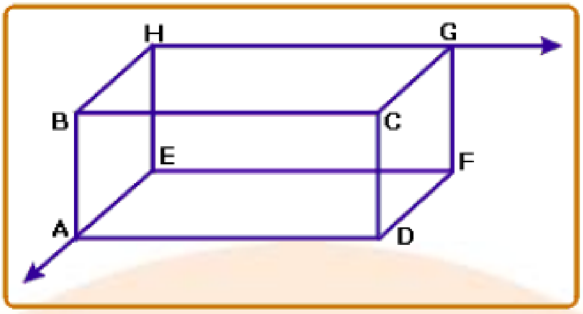
  $AOB = 2BOC = 2COA$

Parallel Lines

- Even if they are extended on each side, two lines are **parallel** if they are coplanar and do not overlap.
- There are, however, lines that don't intersect yet aren't **parallel**.

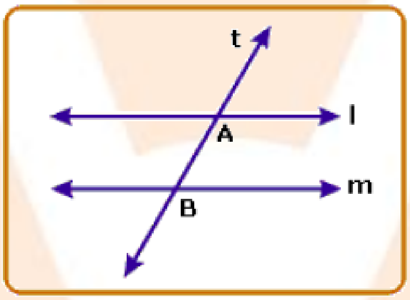


- They're **skew lines**, as the name implies. Lines that are not **coplanar** and do not intersect are referred to as skew lines.
- The lines AE and HG are **skewed**.



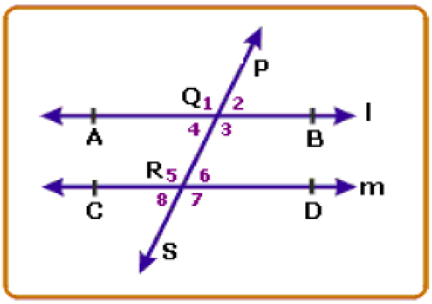
Transversal

- Observe the three lines 'l', 'm' and 't'.
- In the diagram 'l' and 'm' are two parallel lines. 't' intersects 'l' at two distinct points 'A' and 'B' and 'm' at 'C' and 'D'. Line t is known as **transversal.**
- A **transversal** is a line that at different points intersects (or slices) two or more parallel lines.



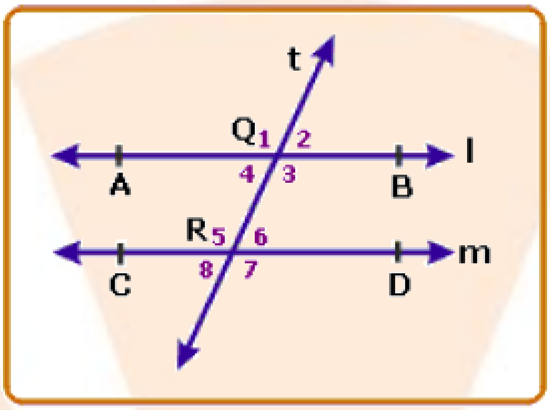
Angles Formed by a Transversal

- In the diagram $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two parallel lines.
  PQRS is a transversal intersecting $\overleftrightarrow{AB}$ at Q and $\overleftrightarrow{CD}$ at R.
  There are total eight angles formed.
- Some of the angles can be grouped together due to their placements. Special names are given to the **paired angles** (apart from adjacent angles and vertical angles).



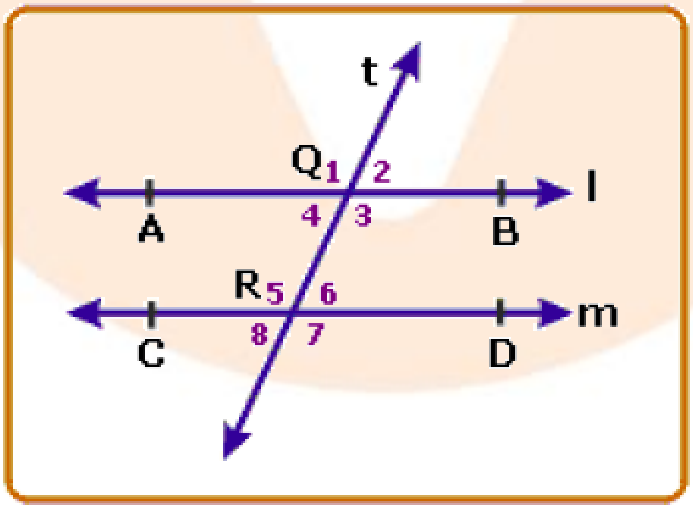
Interior Angles which are on the same side of the Transversal

- From the below figure,
  $AQR\ (\angle 4)$ and $QRC\ (\angle 5)$ and $BQR\ (\angle 3)$ and $QRD\ (\angle 6)$ form two **pairs of interior angles** on the same side of the **transversal**.



Alternate Angles

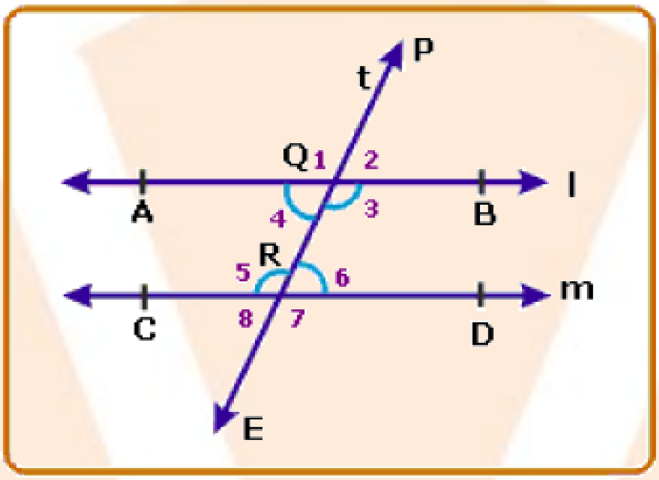
- A pair of angles are said to be alternate angles if
  1) both angles are internal angles,
  2) they're on opposing sides of the transversal axis, and
  3) they are not adjacent angles, they are said to be alternate angles.
- **Alternate interior angles** are another name for **alternate angles**.



- In the above figure,
  AQR and $QRD\ (\angle 4 \text{ and } \angle 6)$
  BQR and $QRC\ (\angle 3 \text{ and } \angle 5)$ are the two pairs of **alternate** angles.

Corresponding Angles

- A pair of angles are said to be corresponding angles if
- One is an interior angle and the other is an exterior angle
- They are in the same transverse plane and
- They are not adjacent angles.



The four pairs of corresponding angles are given as follows;

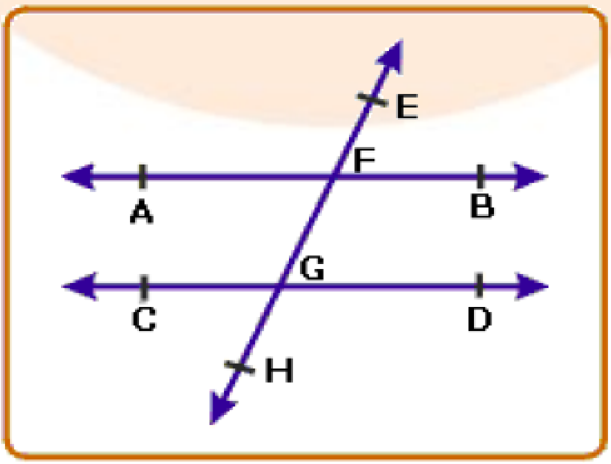
AQP and CRQ $(\angle 1 \text{ and } \angle 5)$

AQR and CRE $(\angle 4 \text{ and } \angle 8)$

BQR and DRE $(\angle 3 \text{ and } \angle 7)$

Parallel Lines - Theorem 1

- **Statement:**
  Each pair of alternating angles is equal when a transversal intersects two parallel lines.



- **Given:**

  ΔABC, side BC is produced to D and ACD is the exterior angle formed.
  $AB \parallel CD$ and $EFGH$ is a transversal.

- **To prove:**

  $AFD = FGD$ (One pair of interior alternate angles)

  $BFG = FGC$ (Another pair of interior alternate angles)

- **Proof:**

  $AFG = EFB$ (Vertically opposite angles)
  But

  $EFB = FGD$ (Corresponding angles)

  $\therefore AFG = FGD$
  Now,

  $BFG + AFG = 180^{\circ}.....(i)(\text{Linear Pair})$

  $FGC + FGD = 180^{\circ}.....(ii)(\text{Linear Pair})$

  From (i) and (ii),

  $BFG + AFG = FGC + FGD$
  But

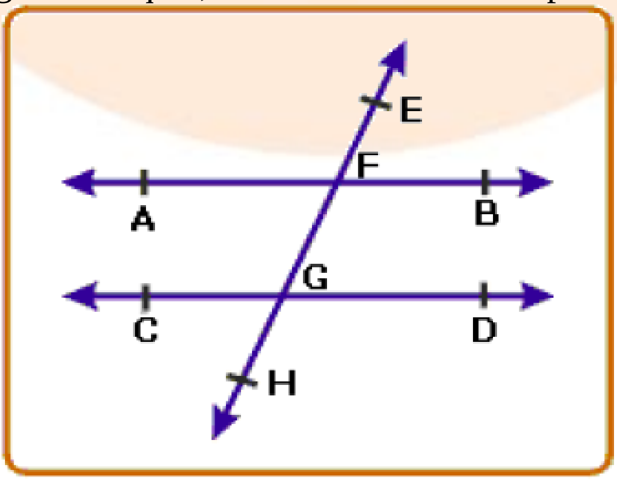
  $AFG = FGD$ $\qquad (\text{Proved})$

  $\therefore BFG = FGC$

Converse of Theorem 1

- **Statement:**

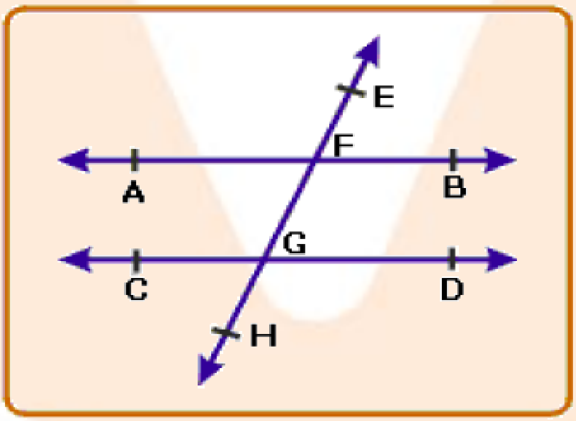
  If a transversal intersects two lines in such a way that a pair of alternate interior angles are equal, then the two lines are parallel.



- **Given:**
  Transversal EFGH intersects lines AB and CD such that a pair of alternate angles are equal.
  $\left(AFD = FGD\right)$
- **To prove:**
  $AB \parallel CD$
- **Proof:**
  $AFG = FGD$ (Given)
  But
  $AFG = EFB$ (Vertically opposite angles)
  $\therefore EFB = FGD$ (Corresponding angles)
  Therefore,
  $AB \parallel CD$ (Corresponding angles axiom)

Parallel Lines - Theorem 2

- **Statement:**
  Each set of consecutive interior angles is additional or supplementary when a transversal connects two parallel lines.



- **Given:**
  $AB \parallel CD$ and EFGH is a transversal.
- **To prove:**
  $BFG + FGD = 180^\circ$
  $AFG + FGC = 180^\circ$
- **Proof:**
  $EFB + BFG = 180^\circ$ (Linear Pair)

But

$EFB = FGD$ (Corresponding angles axiom)

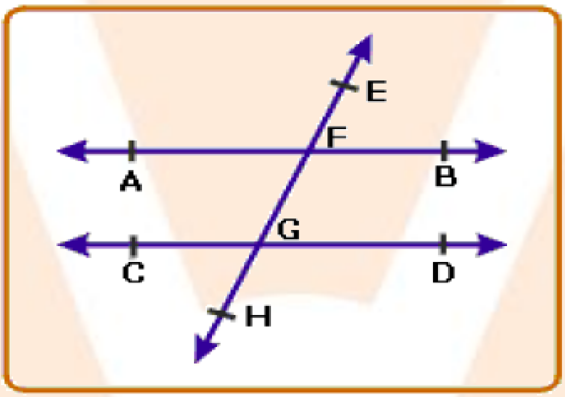
$\therefore BFG + FGD = 180^\circ$ (Substitute FGD for EFB)

Similarly, we can prove that

$AFG + FGC = 180^\circ$

Converse of Theorem 2

- **Statement:**
  If a transversal intersects two lines in such a way that a pair of consecutive interior angles are supplementary, then the two lines are parallel.



- **Given:**
  Transversal $EFGH$ intersects lines AB and CD at F and G such that BFG and FGD are supplementary.

  That is $\left(BFG + FGD = 180^\circ\right)$
- **To prove:**
  $AB \parallel CD$
- **Proof:**
  $EFB + BFG = 180^\circ ......(i)$ [Linear pair (ray FB stands on $EFGH$)]

  (Corresponding angles postulate)

  $BFG + FGD = 180^\circ .....(ii)$ (Given)

  $EFB + BFG = BFG + FGD$

  Therefore,

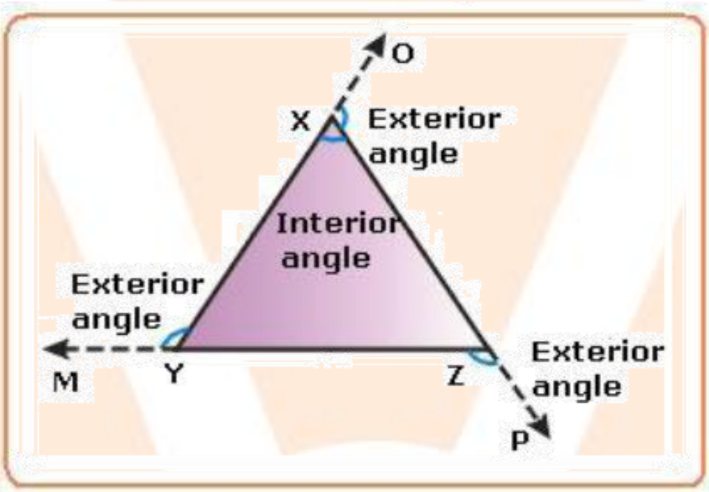
  $EFB = FGD$ (Subtract BFG from both sides)

  Since these are corresponding angles,

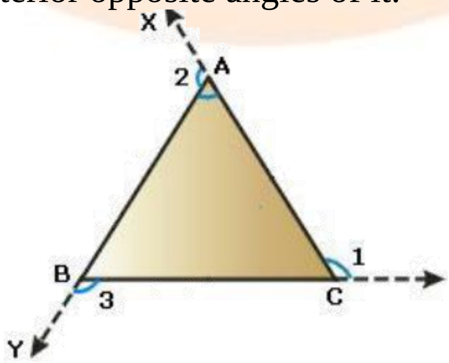
Therefore,
AB || CD

Interior and Exterior Angles of a Triangle

- When we talk about an angle in a triangle, we're talking about the angle formed by the two sides.
- The three angles are located in the triangle's interior. These angles are known as the **triangle's inner angles**.



- Now look at the triangle that the sides are formed in.
- In the fig, is extended to O. An angle OXZ is formed.
  $\overline{XZ}$ is produced to P forming an angle YZP.
- Similarly, $\overline{ZY}$ is formed to M forming angle MYX.
- These angles OXZ, YZP and MYX X are called **exterior angles** of ABC
- There can be three external angles because the triangle has three sides.
- **The interior angles opposite to the vertices where the exterior angles are formed, are called the interior opposite angles.**
- From the figure, at X, the exterior angle OXZ is formed and angles XYZ and XZY are interior opposite angles of it.



From above figure,

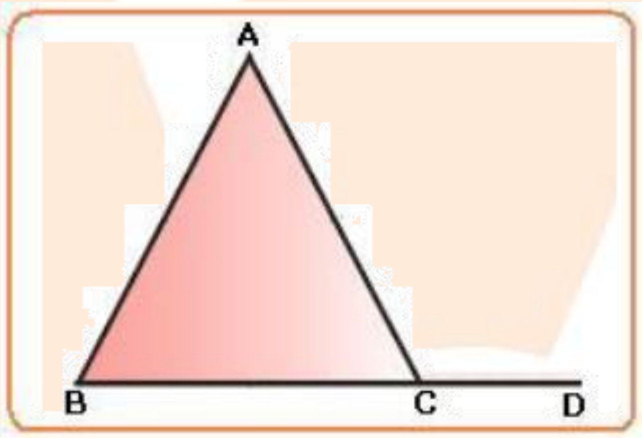
For exterior angle 1; the interior opposite angles are BAC and ABC.

For exterior angle 2; the interior opposite angles are ABC and ACB.

For exterior angle 3; the interior opposite angles are BAC and ACB.

Triangles - Theorem 1:

- **Statement:**
  If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the interior opposite angles.



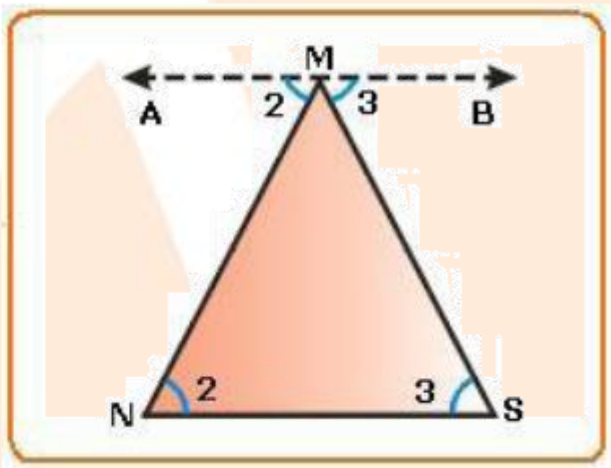
- **Given:**
  In triangle, ΔABC, side BC is produced to D and ACD is the exterior angle formed.
  ABC and BAC are the interior opposite angles.
- **To prove:**
  $ACD = ABC + BAC$
- **Proof:**
  $ABC + BAC + ACB = 180^\circ .....(i)(\text{Theorem})$
  $ACB + ACD = 180^\circ .....(ii)(\text{Linear pair})$
  From (i) and (ii), we get
  $ABC + BAC + ACB = ACB + ACD$
  Subtract ACB from both sides, we get
  $ABC + BAC = ACD$

Angle Sum Property

Three line segments unite three non-collinear points to make a triangle, which is a plane closed geometric figure.

Triangles - Theorem 2:

- **Statement:**
  The sum of the three angles of a triangle is 180°.



- **Given:**
  A triangle MNS.
- **To prove:**
  $M + N + \hat{S} = 180^\circ$
- **Construction:**
  By using a scale through the vertex M, draw a line $\overleftrightarrow{AB}$ parallel to the base $\overline{NS}$.
- **Proof:**
  $\overline{NS} \parallel \overleftrightarrow{AB}$
  MN is a **transversal.**
  Therefore,
  $AMN = MNS$ .....(1) Alternate angles
  Similarly, $\overleftrightarrow{AB} \parallel \overline{NS}$ and MN
  Therefore,
  $BMS = M\hat{S}N$ .....(2) Alternate angles
  From the figure,
  $AMN + NMS + BMS = 180^\circ$
  Since, $\overleftrightarrow{AB}$ is a **straight line** and sum of the angles at $M = 180^\circ$

From (1) and (2),

$MNS + NMS + M\hat{S}N = 180^\circ$ - By substituting MNS and $M\hat{S}N$

Thus it is proved that **sum of the measures of the three angles of a triangle is equal to** 180° **or two right angles.**