Revision Notes

Class 9 Maths

Chapter 5 - Introduction to Euclid Geometry

Introduction to Euclid Geometry:

- **Geometry**'s importance has been recognised in various parts of the world since ancient times.
- This branch of mathematics sprang from the practical issues that ancient civilizations faced.
- Let's look at a few examples below;
- The demarcations of land owners on river-side land were used to wipe out with the floods in the river.
- The concept of **area** was established in order to rediscover the borders. Geometry could be used to determine the volume of granaries.
- **Egyptian pyramids** show that geometry has been used from ancient times.
- There was a geometrical construction guidebook called as **Sulbasutra**'s throughout the **Vedic period**.
- **Altars** of various geometrical shapes were built to fulfil various Vedic ceremonies.
- **Geometry** is derived from the **green words 'Geo' (earth)** and **metrein (measurement) (to measure).**
- Geometry has been developed and implemented in various parts of the world since ancient times, but it has never been presented in a systematic fashion. Later, approximately 300 BC, the **Egyptian mathematician Euclid** gathered all known work and organised it into a systematic framework.
- Euclid's 'Elements' is a famous treatise on geometry written by him.
- This was the book that had the most impact.
- For several years, the 'element' was utilised as a textbook in Western Europe.
- The 'elements' began with 28 definitions, five postulates, and five common conceptions and created the rest of plane and solid geometry in a systematic manner.
- The Euclid method refers to Euclid's geometrical approach.
- The Euclid method entails making a small set of assumptions and then using these assumptions to prove a large number of other propositions.

- The assumptions that were made were self-evident universal truths.
- Axioms and postulates were the two types of assumptions made.

Euclid's Definitions:

- Euclid listed 23 definitions in book 1 of the 'elements'.
- We list a few of them are as follows:
  1. A **point** is that which has no part
  2. A **line** is a breadth less length
  3. The ends of a line are points
  4. A **straight line** is a line which lies evenly with the points on itself.
  5. A **surface** is that which has length and breadth only.
  6. The edges of a surface are lines
  7. A **plane surface** is surface which lies evenly with straight lines on its self. Euclid made some assumptions, known as **axioms and postulates**, based on these definitions.

Euclid's Axioms:

- **Axioms** are assumptions that are employed in all areas of mathematics but are not directly related to geometry.
- Only a few of **Euclid's axioms** are true, they are as follows:
  1) Things which are equal to the same thing are equal to one another.
  2) It equals are added to equals; the wholes are equal.
  3) If equals are subtracted from equals, the remainders are equal.
  4) Things which coincide one another are equal to one another.
  5) The whole is greater than the part
  6) Things which are double of the same thing are equal to one another.
  7) Things which are half of the same things are equal to one another.
- All these axioms refer to magnitude of same kind.
- **Axiom** - 1 can be written as follows:
  If $x = Z$ and $y = Z$, then $x = y$
- **Axiom** - 2 explains the following:
  If $x = y$, then $x + Z = y + Z$
- According to **Axiom** - 3 ,
  If $x = y$, then $x - Z = y - Z$
- **Axiom** - 4 justifies the **principle of superposition** that everything equals itself.
- **Axiom** - 5 , gives us the concept of **comparison**.
  If x is a part of y , then there is a quantity Z such that $x = y + Z$ or $x > y$

Note that **magnitudes of the same kind can be added, subtracted or compared.**

Euclid's Postulates:

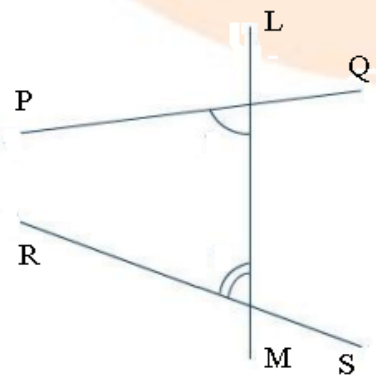
- The term "**postulate**" was coined by **Euclid** to describe the assumptions that were unique to geometry.
- The following are **Euclid's five postulates**:
- **Postulate** 1 **:**
  - A straight line may be drawn from any one point to any other point.
  - Same may be stated as axiom 5.1
  - Given two distinct points, there is a unique line that passes through them.

- **Postulate** 2 **:**
  A terminated line can be produced indefinitely.

- **Postulate** 3 **:**
  A circle can be drawn with any centre and any radius.

- **Postulate** 4 **:**
  All right angles are equal to one another

- **Postulate** 5 **:** If a straight line falling on two straight lines makes the interior angle on the same side of it taken together less than two right angles, then two straight lines, if produced indefinitely, meet on that side on which the sum of the angles is less than two right angles.
- Postulates 1 to 4 are exceedingly basic and obvious, hence they are referred regarded as "**self-evident truths**."
- Postulate 5 is complicated and it should to be discussed.



- If the line LM intersects two lines PQ and RS at a place where Sum of Angles $= 180°$, the lines LM and PQ will intersect at that point.

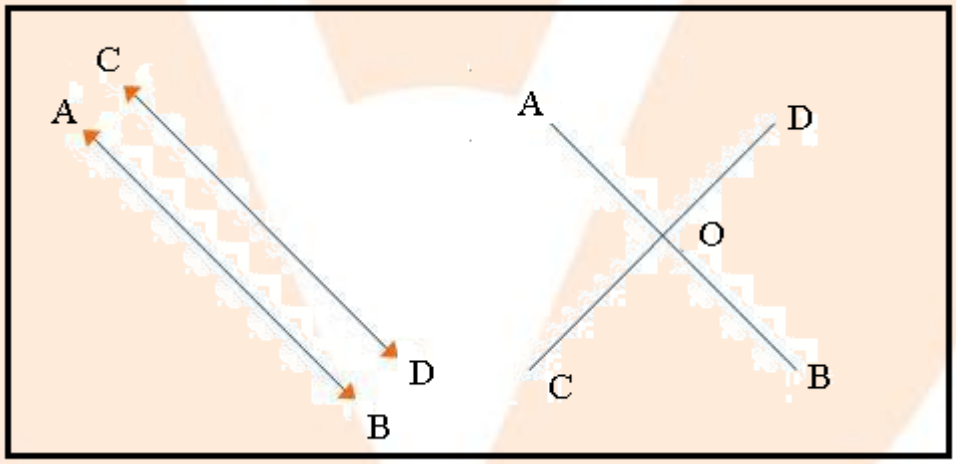
Note: The terms **axiom** and **postulate** are often used interchangeably in mathematics, however according to Euclid, they have different meanings.

System of Consistent Axioms: If it is impossible to deduce a statement from these axioms that contradicts any of the given axioms or propositions, the system is said to be **consistent**.

Proposition or Theorem:

Propositions or **Theorems** are **statements or results that have been proven using Euclid's axioms and postulates.**

Theorem: Two distinct lines cannot have more than one point in common.



Proof: Given: AB and CD are two lines.

To prove: They intersect at one point or they do not intersect.

Proof:

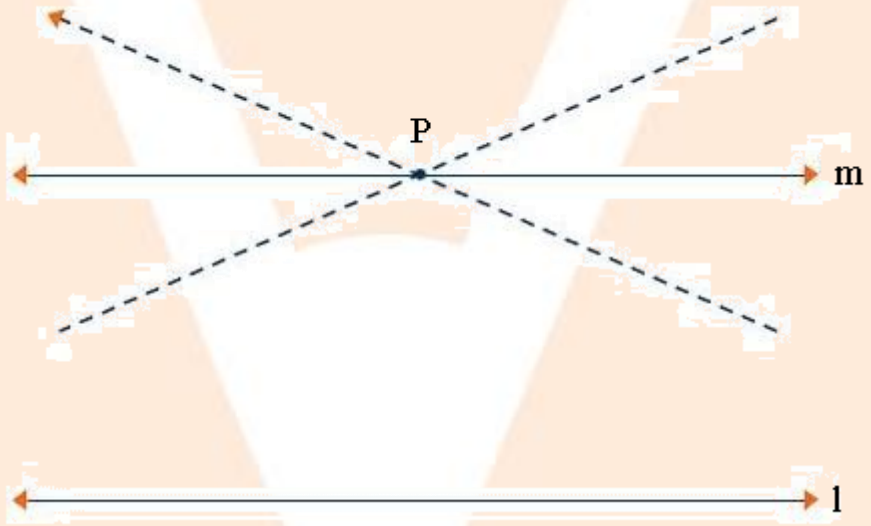
- Assume that the lines AB and CD cross at positions P and Q.
- The line AB must therefore pass through the points P and Q.
- Also, the CD line also runs through the P and Q points.
- This means that there are two lines passing through two distinct points P and Q.

- However, we know that only one line can cross through two separate places.
- This axiom goes against our belief that two separate lines can share more than one point.
- The lines AB and CD are unable to travel via the points P and Q.

Equivalent Versions of Euclid's Fifth Postulate:

- The two different version of fifth postulate

  a) For every line l and for every point P not lying on l, there exist a unique line m passing through P and parallel to l.



  b) Two distinct intersecting lines cannot be parallel to the same line.