NCERT Solutions for Class 9 Mathematics

Chapter 11 – Surface Areas and Volumes

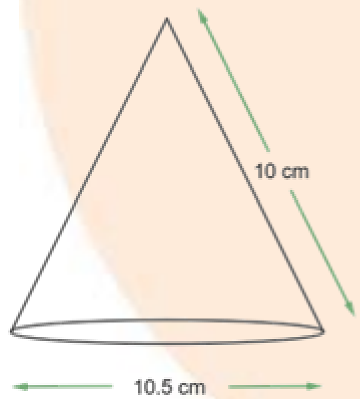
Exercise 11.1

1. Diameter of the base of a cone is 10.5 cm **and its slant height is** 10 cm**. Find its curved surface area.** $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: We are given the following:

The slant height (l) of the cone $= 10 \text{ cm}$

The diameter of the base of cone $= 10.5 \text{ cm}$



So, the radius (r) of the base of cone $= \dfrac{10.5}{2} \text{ cm} = 5.25 \text{ cm}$

The curved surface area of cone, $A = \pi rl$

$$\Rightarrow A = \left(\frac{22}{7} \times 5.25 \times 10\right) \text{cm}^2$$

$\Rightarrow A = (22 \times 0.75 \times 10)\ cm^2$

$\Rightarrow A = 165\ cm^2$

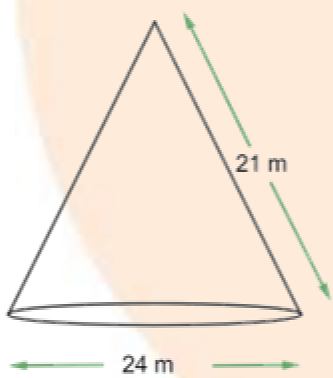
Therefore, the curved surface area of the cone is $165\ cm^2$.

2. Find the total surface area of a cone, if its slant height is $21\ m$ **and diameter of its base is** $24\ m$**.** $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: We are given the following:

The slant height (l) of the cone $= 21\ m$

The diameter of the base of cone $= 24\ m$



So, the radius (r) of the base of cone $= \frac{24}{2}\ m = 12\ m$

The total surface area of cone, $A = \pi r(l + r)$

$\Rightarrow A = \left(\frac{22}{7} \times 12 \times (21 + 12)\right)\ m^2$

$\Rightarrow A = \left(\frac{22}{7} \times 12 \times 33\right) m^2$

$\Rightarrow A = 1244.57\ m^2$

Therefore, the total surface area of the cone is $1244.57\ m^2$.

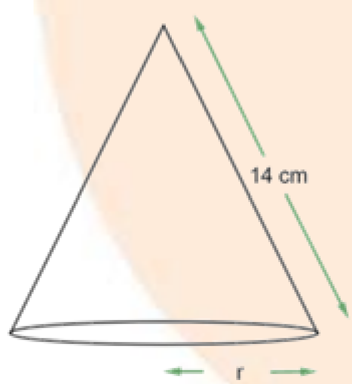
3. Curved surface area of a cone is $308\ cm^2$ and its slant height is $14\ cm$. Find

(i) Radius of the base

Ans: It is given that the slant height (l) of the cone $= 14\ cm$

The curved surface area of the cone $= 308\ cm^2$

Let us assume the radius of base of the cone be r.



We know that curved surface area of the cone $= \pi rl$

$\therefore\ \pi rl = 308\ cm^2$

$\Rightarrow \left(\frac{22}{7} \times r \times 14\right) cm = 308\ cm^2$

$\Rightarrow r = \dfrac{308}{44}$ cm

$\Rightarrow r = 7$ cm

Hence, the radius of the base is 7 cm.

(ii) Total surface area of the cone. $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: The total surface area of the cone is the sum of its curved surface area and the area of the base.

Total surface area of cone, $A = \pi rl + \pi r^2$

$\Rightarrow A = \left[308 + \dfrac{22}{7} \times (7)^2\right] \text{cm}^2$

$\Rightarrow A = [308 + 154]\ \text{cm}^2$

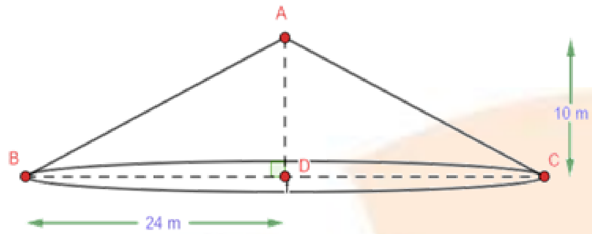
$\Rightarrow A = 462\ \text{cm}^2$

Hence, the total surface area of the cone is $462\ \text{cm}^2$.

4. A conical tent is 10 m **high and the radius of its base is** 24 m**. Find**

(i) slant height of the tent

Ans:



From the figure we can say that ABC is a conical tent.

It is given that the height (h) of conical tent $= 10\text{ m}$

The radius (r) of conical tent $= 24\text{ m}$

Let us assume the slant height as l.

In ΔABD, we will use Pythagorean Theorem.

$\therefore AB^2 = AD^2 + BD^2$

$\Rightarrow l^2 = h^2 + r^2$

$\Rightarrow l^2 = (10\text{ m})^2 + (24\text{ m})^2$

$\Rightarrow l^2 = 676\text{ m}^2$

$\Rightarrow l = 26\text{ m}$

The slant height of the tent is 26 m.

(ii) cost of canvas required to make the tent, if cost of 1 m^2 canvas is Rs. 70.
$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: The curved surface area of the tent, $A = \pi rl$

$$\Rightarrow A = \left(\frac{22}{7} \times 24 \times 26\right) m^2$$

$$\Rightarrow A = \left(\frac{13728}{7}\right) m^2$$

It is given that the cost of $1\ m^2$ of canvas $=$ Rs. 70

So, the cost of $\frac{13728}{7}\ m^2$ canvas $=$ Rs. $\left(\frac{13728}{7} \times 70\right) =$ Rs. 137280

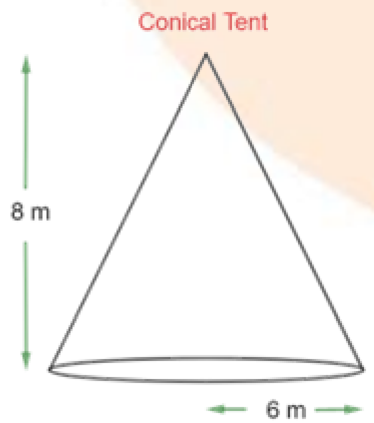
Hence, the cost of canvas required to make the tent is Rs. 137280.

5. What length of tarpaulin $3\ m$ wide will be required to make conical tent of height $8\ m$ and base radius $6\ m$? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately $20\ cm$. $[\text{Use } \pi = 3.14]$

Ans: We are given the following:

The base radius (r) of tent $= 6\ m$

The height (h) of tent $= 8\ m$



So, the slant height of the tent, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \left(\sqrt{6^2 + 8^2}\right)$ m

$\Rightarrow l = \left(\sqrt{100}\right)$ m

$\Rightarrow l = 10$ m

The curved surface area of the tent, $A = \pi rl$

$\Rightarrow A = (3.14 \times 6 \times 10)\ m^2$

$\Rightarrow A = 188.4\ m^2$

It is given the width of tarpaulin $= 3$ m

Let us assume the length of the tarpaulin sheet required be x.

It is given that there will be a wastage of 20 cm.

So, the new length of the sheet $= (x - 0.2)$ m

We know that the area of the rectangular sheet required will be the same as curved surface area of the tent.

$\therefore \left[(x - 0.2) \times 3\right]$ m $= 188.4\ m^2$

$\Rightarrow x - 0.2$ m $= 62.8$ m

$\Rightarrow x = 63$ m

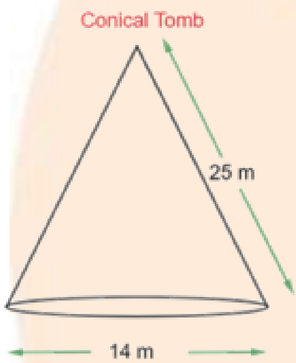
The length of tarpaulin sheet required is 63 m.

6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of $\text{Rs. }210$ per 100 m^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: We are given the following:

The base radius (r) of tomb $= 7\text{ m}$

The slant height (l) of tomb $= 25\text{ m}$



The curved surface area of the conical tomb, $A = \pi rl$

$$\Rightarrow A = \left(\frac{22}{7} \times 7 \times 25\right)\text{ m}^2$$

$$\Rightarrow A = 550\text{ m}^2$$

It is given that the cost of white-washing 1 m^2 area $= \text{Rs. }210$

So, the cost of white-washing 550 m^2 area $= \text{Rs. }\left(\frac{210}{100} \times 550\right) = \text{Rs. }1155$

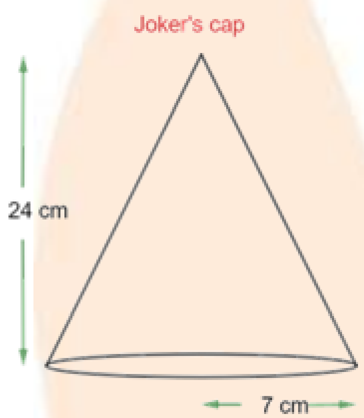
Hence, the cost of white-washing the curved surface area of a conical tomb is $\text{Rs. }1155$.

7. A joker's cap is in the form of right circular cone of base radius 7 cm and the height 24 cm. Find the area of sheet required to make 10 such caps. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: We are given the following:

The base radius (r) of conical cap $= 7\text{ cm}$

The height (h) of conical cap $= 24\text{ cm}$



So, the slant height of the tent, $l = \sqrt{r^2 + h^2}$

$$\Rightarrow l = \left(\sqrt{7^2 + 24^2}\right)\text{ cm}$$

$$\Rightarrow l = \left(\sqrt{625}\right)\text{ cm}$$

$$\Rightarrow l = 25\text{ cm}$$

The curved surface area of one conical cap, $A = \pi rl$

$$\Rightarrow A = \left(\frac{22}{7} \times 7 \times 25\right)\text{ cm}^2$$

$$\Rightarrow A = 550\text{ cm}^2$$

So, the curved surface area of 10 conical caps $= (550 \times 10)\ \text{cm}^2 = 5500\ \text{cm}^2$

Therefore, the total area of sheet required is $5500\ \text{cm}^2$.

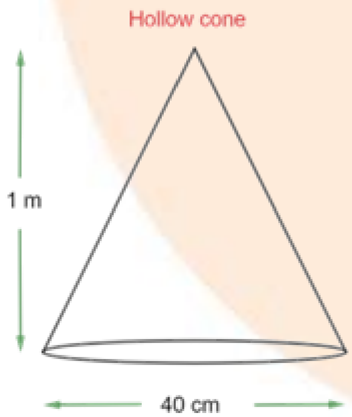
8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of $40\ \text{cm}$ and height $1\ \text{m}$. If the outer side of each of the cones is to be painted and the cost of painting is $\text{Rs. } 12$ per m^2, what will be the cost of painting all these cones?

$\left[\text{Use } \pi = 3.14 \text{ and take } \sqrt{1.02} = 1.02\right]$

Ans: We are given the following:

The base radius (r) of cone $= \dfrac{40}{2} = 20\ \text{cm} = 0.2\ \text{m}$

The height (h) of cone $= 1\ \text{m}$



So the slant height of the cone, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \left(\sqrt{(0.2)^2 + (1)^2}\right)\ \text{m}$

$\Rightarrow l = \left(\sqrt{1.04}\right)\ \text{m}$

$\Rightarrow l = 1.02\ \text{m}$

The curved surface area of one cone, $A = \pi rl$

$\Rightarrow A = (3.14 \times 0.2 \times 1.02)\ \text{m}^2$

$\Rightarrow A = 0.64056\ \text{cm}^2$

So, the curved surface area of 50 cones $= (50 \times 0.64056)\ \text{m}^2 = 32.028\ \text{m}^2$

It is given that the cost of painting $1\ \text{m}^2$ area $= \text{Rs. } 12$

So, the cost of painting $32.028\ \text{m}^2$ area $= \text{Rs. } (32.028 \times 12) = \text{Rs. } 384.336$

We can also write the cost approximately as $\text{Rs. } 384.34$.

Therefore, the cost of painting all the hollow cones is $\text{Rs. } 384.34$.

Exercise 11.2

1. Find the surface area of a sphere of radius: $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

(i) 10.5 cm

Ans: Given radius of the sphere $r = 10.5\ \text{cm}$

The surface area of the sphere $A = 4\pi r^2$

$\Rightarrow A = \left[4 \times \dfrac{22}{7} \times (10.5)^2\right]\ \text{cm}^2$

$\Rightarrow A = (88 \times 1.5 \times 1.5)\ \text{cm}^2$

$\Rightarrow A = 1386\ \text{cm}^2$

Hence, the surface area of the sphere is $1386\ \text{cm}^2$.

(ii) 5.6 cm

Ans: Given radius of the sphere $r = 5.6\ \text{cm}$

The surface area of the sphere $A = 4\pi r^2$

$$\Rightarrow A = \left[4 \times \frac{22}{7} \times (5.6)^2\right] \text{cm}^2$$

$$\Rightarrow A = (88 \times 0.8 \times 5.6)\ \text{cm}^2$$

$$\Rightarrow A = 394.24\ \text{cm}^2$$

Hence, the surface area of the sphere is $394.24\ \text{cm}^2$.

(iii) 14 cm

Ans: Given radius of the sphere $r = 14\ \text{cm}$

The surface area of the sphere $A = 4\pi r^2$

$$\Rightarrow A = \left[4 \times \frac{22}{7} \times (14)^2\right] \text{cm}^2$$

$$\Rightarrow A = (4 \times 44 \times 14)\ \text{cm}^2$$

$$\Rightarrow A = 2464\ \text{cm}^2$$

Hence, the surface area of the sphere is $2464\ \text{cm}^2$.

2. Find the surface area of a sphere of diameter: $\left[\text{Assume } \pi = \frac{22}{7}\right]$

(i) 14 cm

Ans: Given diameter of the sphere $= 14$ cm

So, the radius of the sphere $r = \frac{14}{2} = 7$ cm

The surface area of the sphere $A = 4\pi r^2$

$\Rightarrow A = \left[4 \times \frac{22}{7} \times (7)^2\right] \text{cm}^2$

$\Rightarrow A = (88 \times 7) \text{ cm}^2$

$\Rightarrow A = 616 \text{ cm}^2$

Hence, the surface area of the sphere is 616 cm^2.

(ii) 21 cm

Ans: Given diameter of the sphere $= 21$ cm

So, the radius of the sphere $r = \frac{21}{2} = 10.5$ cm

The surface area of the sphere $A = 4\pi r^2$

$\Rightarrow A = \left[4 \times \frac{22}{7} \times (10.5)^2\right] \text{cm}^2$

$\Rightarrow A = 1386 \text{ cm}^2$

Hence, the surface area of the sphere is 1386 cm^2.

(iii) 3.5 m

Ans: Given diameter of the sphere $= 3.5 \text{ m}$

So, the radius of the sphere $r = \frac{3.5}{2} = 1.75 \text{ m}$

The surface area of the sphere $A = 4\pi r^2$

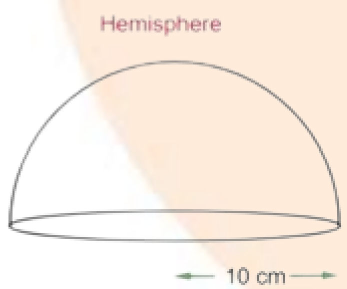
$$\Rightarrow A = \left[4 \times \frac{22}{7} \times (1.75)^2\right] \text{m}^2$$

$$\Rightarrow A = 38.5 \text{ m}^2$$

Hence, the surface area of the sphere is 38.5 m^2.

3. Find the total surface area of a hemisphere of radius 10 cm. $[\text{Use } \pi = 3.14]$

Ans:



Given the radius of hemisphere $r = 10 \text{ cm}$

The total surface area of the hemisphere is the sum of its curved surface area and the circular base.

Total surface area of hemisphere $A = 2\pi r^2 + \pi r^2$

$$\Rightarrow A = 3\pi r^2$$

$\Rightarrow A = \left[3 \times 3.14 \times (10)^2\right] \text{ cm}^2$

$\Rightarrow A = 942 \text{ cm}^2$

Hence, the total surface area of the hemisphere is 942 cm^2.

4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Ans: Given the initial radius of the balloon $r_1 = 10 \text{ cm}$

The final radius of the balloon $r_2 = 14 \text{ cm}$

We have to find the ratio of surface areas of the balloon in the two cases**.**

The required ratio $R = \dfrac{4\pi r_1^2}{4\pi r_2^2}$

$\Rightarrow R = \left(\dfrac{r_1}{r_2}\right)^2$

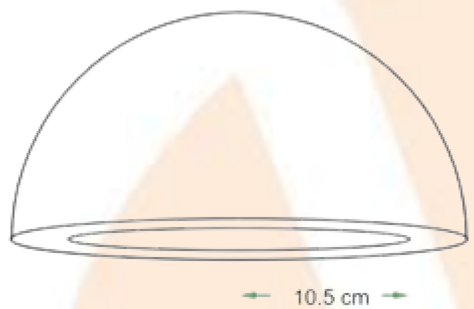
$\Rightarrow R = \left(\dfrac{7}{14}\right)^2$

$\Rightarrow R = \dfrac{1}{4}$

Hence, the ratio of the surface areas of the balloon in both case is $1:4$.

5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tinplating it on the inside at the rate of $\text{Rs. } 16$ per 100 cm^2. $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: Given the radius of inner hemispherical bowl $r = \frac{10.5}{2} = 5.25\text{ cm}$



The surface area of the hemispherical bowl $A = 2\pi r^2$

$\Rightarrow A = \left[2 \times \frac{22}{7} \times (5.25)^2\right]\text{cm}^2$

$\Rightarrow A = 173.25\text{ cm}^2$

It is given that the cost of tin-plating 100 cm^2 area $=$ Rs. 16

So, the cost of tin-plating 173.25 cm^2 area $=$ Rs. $\left(\frac{16}{100} \times 173.25\right) =$ Rs. 27.72

Hence, the cost of tin-plating the hemispherical bowl is Rs. 27.72.

6. Find the radius of a sphere whose surface area is 154 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: Let us assume the radius of sphere be r.

We are given the surface area of the sphere, $A = 154\text{ cm}^2$.

$\therefore 4\pi r^2 = 154\text{ cm}^2$

$$\Rightarrow r^2 = \left(\frac{154 \times 7}{2 \times 22}\right) cm^2$$

$$\Rightarrow r = \left(\frac{7}{2}\right) cm$$

$$\Rightarrow r = 3.5 \text{ cm}$$

Therefore, the radius of the sphere is 3.5 cm.

7. The diameter of the moon is approximately one-fourth of the diameter of the earth. Find the ratio of their surface area.

Ans: Let us assume the diameter of earth is d.

So, the diameter of the moon will be $\frac{d}{4}$.

The radius of the earth $r_1 = \frac{d}{2}$

The radius of the moon $r_2 = \frac{1}{2} \times \frac{d}{2} = \frac{d}{8}$

The ratio of surface area of moon and earth $R = \frac{4 \pi r_2^2}{4 \pi r_1^2}$

$$\Rightarrow R = \frac{4 \pi \left(\frac{d}{8}\right)^2}{4 \pi \left(\frac{d}{2}\right)^2}$$

$$\Rightarrow R = \frac{4}{64}$$

$\Rightarrow R = \frac{1}{16}$

Therefore, the ratio of surface area of moon and earth is $1 : 16$.

8. A hemispherical bowl is made of steel, 0.25 cm **thick. The inner radius of the bowl is** 5 cm**. Find the outer curved surface area of the bowl.** $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: Given the inner radius $= 5\text{ cm}$

The thickness of the bowl $= 0.25\text{ cm}$

So, the outer radius of the hemispherical bowl is $r = (5 + 0.25)\text{ cm} = 5.25\text{ cm}$

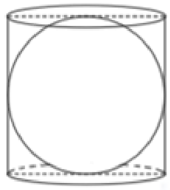
The outer curved surface area of the hemispherical bowl $A = 2\pi r^2$

$\Rightarrow A = \left[2 \times \frac{2}{7} \times (5.25)^2\right]\text{cm}^2$

$\Rightarrow A = 173.25\text{ cm}^2$

Therefore, the outer curved surface area of the hemispherical bowl is 173.25 cm^2.

9. A right circular cylinder just encloses a sphere of radius r (see the below figure). Find

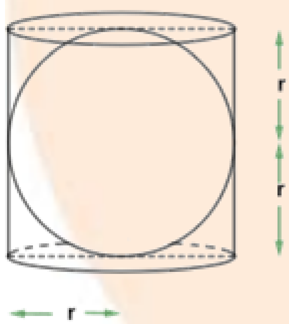


(i) surface area of the sphere,

Ans: The surface area of the sphere is $4\pi r^2$.

(ii) curved surface area of the cylinder,

Ans:



Given the radius of cylinder $= r$

The height of cylinder $= r + r = 2r$

The curved surface area of cylinder $A = 2\pi rh$

$\Rightarrow A = 2\pi r\left(2r\right)$

$\Rightarrow A = 4\pi r^2$

Therefore, the curved surface area of cylinder is $4\pi r^2$.

(iii) ratio of the areas obtained in (i) and (ii).

Ans: The ratio of surface area of the sphere and curved surface area of cylinder $R = \frac{4\pi r^2}{4\pi r^2}$

$R = \frac{1}{1}$

Therefore, the required ratio is $1:1$.

Exercise 11.3

1. Find the volume of the right circular cone with

(i) Radius 6 cm**, height** 7 cm $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given the radius of cone $r = 6\text{ cm}$

The height of the cone $h = 7\text{ cm}$

The volume of the cone $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7\right] \text{cm}^3$

$\Rightarrow V = (12 \times 22)\ \text{cm}^3$

$\Rightarrow V = 264\ \text{cm}^3$

The volume of the right circular cone is $264\ \text{cm}^3$.

(ii) Radius 3.5 cm**, height** 12 cm $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given the radius of cone $r = 3.5 \text{ cm}$

The height of the cone $h = 12 \text{ cm}$

The volume of the cone $V = \frac{1}{3} \pi r^2 h$

$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12\right] \text{cm}^3$

$\Rightarrow V = (1.75 \times 88) \text{ cm}^3$

$\Rightarrow V = 154 \text{ cm}^3$

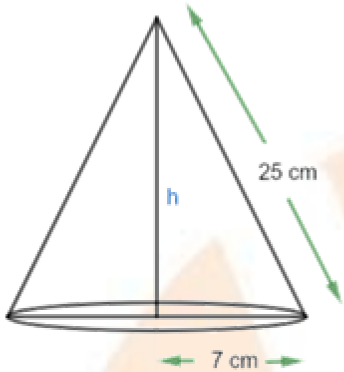
The volume of the right circular cone is 154 cm^3.

2. Find the capacity in litres of a conical vessel with $\left[\text{Assume } \pi = \frac{22}{7}\right]$

(i) Radius 7 cm**, slant height** 25 cm

Ans: It is given the radius of cone $r = 7 \text{ cm}$

The slant height of the cone $l = 25 \text{ cm}$



So, the height of the cone $h = \sqrt{l^2 - r^2}$

$\Rightarrow h = \sqrt{25^2 - 7^2}$ cm

$\Rightarrow h = 24$ cm

The volume of the cone $V = \frac{1}{3} \pi r^2 h$

$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24\right] \text{cm}^3$

$\Rightarrow V = (154 \times 8) \text{ cm}^3$

$\Rightarrow V = 1232 \text{ cm}^3$

We know that $1000 \text{ cm}^3 = 1$ litre

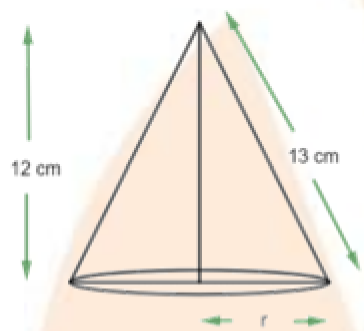
So, the capacity of the conical vessel $= \frac{1232}{1000} = 1.232$ litres

Therefore, the capacity of the conical vessel is 1.232 litres.

(ii) height 12 cm**, slant height** 13 cm

Ans: It is given the height of cone $h = 12\text{ cm}$

The slant height of the cone $l = 13\text{ cm}$



So, the radius of the cone $r = \sqrt{l^2 - h^2}$

$\Rightarrow r = \sqrt{13^2 - 12^2}\text{ cm}$

$\Rightarrow r = 5\text{ cm}$

The volume of the cone $V = \frac{1}{3}\pi r^2 h$

$$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12\right]\text{cm}^3$$

$$\Rightarrow V = \left(4 \times \frac{22}{7} \times 25\right)\text{cm}^3$$

$$\Rightarrow V = \frac{2200}{7}\text{ cm}^3$$

We know that $1000\text{ cm}^3 = 1\text{ litre}$

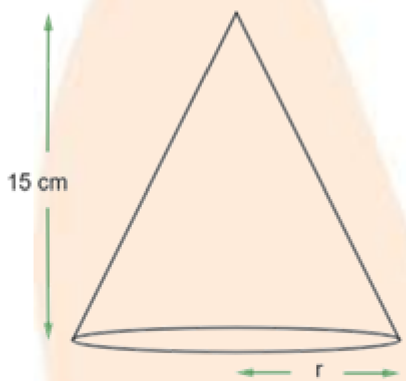
So, the capacity of the conical vessel $= \frac{2200}{7} \times \frac{1}{1000} = 0.314\text{ litres}$

Therefore, the capacity of the conical vessel is 0.314 litres .

3. The height of a cone is 15 cm**. It its volume is** 1570 cm^3**, find the diameter of its base.** $\left[\text{Use } \pi = 3.14\right]$

Ans: It is given the height of cone $h = 12\text{ cm}$

Let us assume the radius of the cone be r.



The volume of the cone is $V = 1570\text{ cm}^3$

We know the formula for the volume of the cone $= \frac{1}{3}\pi r^2 h$

$$\therefore \frac{1}{3}\pi r^2 h = 1570\text{ cm}^3$$

$$\Rightarrow \left[\frac{1}{3} \times \frac{22}{7} \times (r)^2 \times 12\right]\text{cm} = 1570\text{ cm}^3$$

$$\Rightarrow r^2 = 100\text{ cm}^2$$

$$\Rightarrow r = 10\text{ cm}$$

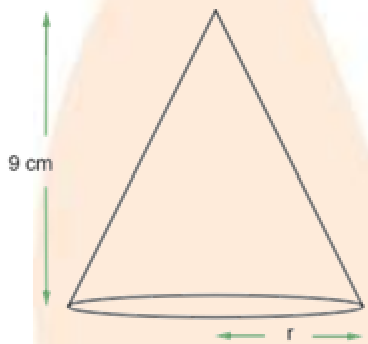
Diameter of base $= 2r = 20\text{ cm}$

Therefore, the diameter of the cone is 20 cm

4. If the volume of right circular cone of height 9 cm is $48\pi\text{ cm}^3$, find the diameter of its base.

Ans: It is given the height of cone $h = 9\text{ cm}$

Let us assume the radius of the cone be r.



The volume of the cone is $V = 48\pi\text{ cm}^3$

We know the formula for the volume of the cone $= \frac{1}{3}\pi r^2 h$

$\therefore \frac{1}{3}\pi r^2 h = 48\pi\text{ cm}^3$

$\Rightarrow \left[\frac{1}{3} \times \pi \times (r)^2 \times 9\right]\text{cm} = 48\pi\text{ cm}^3$

$\Rightarrow r^2 = 16\text{ cm}^2$

$\Rightarrow r = 4\text{ cm}$

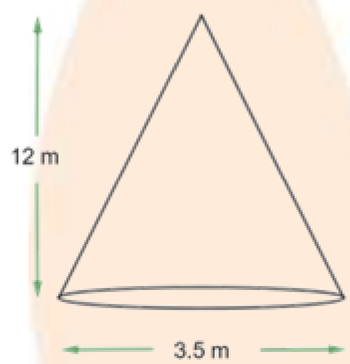
Diameter of base $= 2r = 8\text{ cm}$

Therefore, the diameter of the base of the cone is 8 cm

5. A conical pit of top diameter 3.5 m is 12 m deep. What is the capacity in kilolitres?
$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given the height of conical pit $h = 12\text{ m}$

The radius of conical pit $r = \frac{3.5}{2}\text{ m} = 1.75\text{ m}$



We know the volume of the conical pit $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12\right]\text{ m}^3$

$\Rightarrow V = 38.5\text{ m}^3$

We know that $1\text{ kilolitre} = 1\text{ m}^3$

So, the capacity of the pit $= (38.5 \times 1)\text{ kilolitres} = 38.5\text{ kilolitres}$

Therefore, the capacity of the conical pit is 38.5 kilolitres.

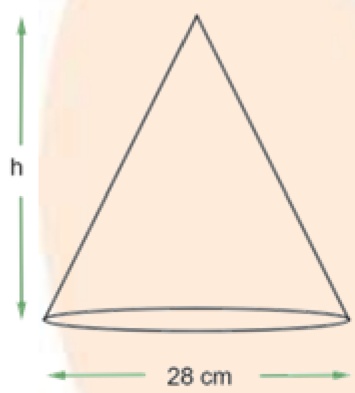
6. The volume of a right circular cone is $9856\ \text{cm}^3$. If the diameter of the base is $28\ \text{cm}$, find $\left[\text{Assume } \pi = \frac{22}{7}\right]$

(i) Height of the cone

Ans: It is given the diameter of base of cone $= 28\ \text{cm}$

So, the radius $r = \frac{28}{2} = 14\ \text{cm}$

Let us assume the height of the cone be h.



The volume of the cone is $V = 9856\ \text{cm}^3$

We know the formula for the volume of the cone $= \frac{1}{3}\pi r^2 h$

$$\therefore \frac{1}{3}\pi r^2 h = 9856\ \text{cm}^3$$

$$\Rightarrow \left[\frac{1}{3} \times \frac{22}{7} \times (14)^2 \times h\right]\text{cm}^2 = 9856\ \text{cm}^3$$

$$\Rightarrow h = \left(\frac{9856 \times 21}{22 \times 196}\right)\text{cm}$$

$\Rightarrow h = 48\text{ cm}$

Therefore, the height of the cone is 48 cm

(ii) Slant height of the cone

Ans: The slant height of the cone $l = \sqrt{h^2 + r^2}$

$\Rightarrow l = \sqrt{48^2 + 14^2}\text{ cm}$

$\Rightarrow l = \sqrt{2304 + 196}\text{ cm}$

$\Rightarrow l = 50\text{ cm}$

Therefore, the slant height of the cone is 50 cm.

(iii) Curved surface area of the cone.

Ans: The curved surface area of the cone $A = \pi rl$

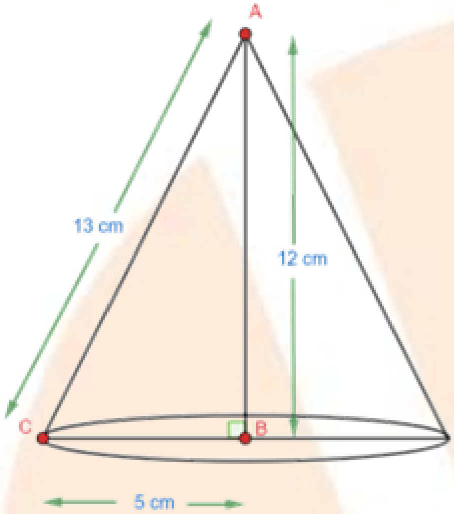
$\Rightarrow A = \left(\frac{22}{7} \times 14 \times 50\right)\text{cm}^2$

$\Rightarrow A = 2200\text{ cm}^2$

Therefore, the curved surface area of the cone is 2200 cm^2.

7. A right triangle ΔABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Ans: We will draw the given figure.



If the triangle is revolved about the side 12 cm, we will get a cone with:

Radius $r = 5\text{ cm}$

Slant height $l = 13\text{ cm}$

Height $h = 12\text{ cm}$

We know the volume of the cone $V = \frac{1}{3}\pi r^2 h$

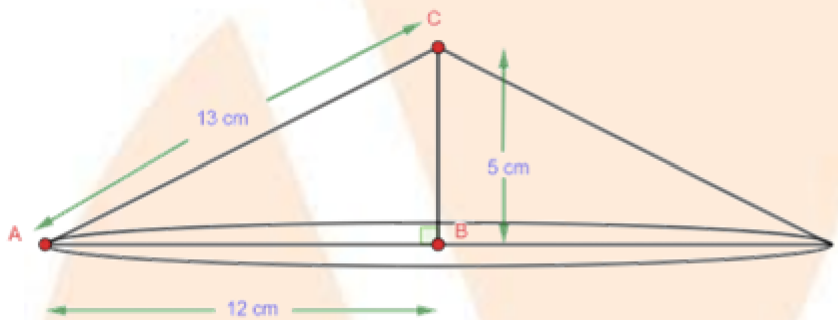
$$\Rightarrow V = \left[\frac{1}{3} \times \pi \times (5)^2 \times 12\right]\text{cm}^3$$

$$\Rightarrow V = 100\pi\ \text{cm}^3$$

Therefore, the volume of the cone will be $100\pi\ \text{cm}^3$.

8. If the triangle Δ ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Ans:



If the triangle is revolved about the side 5 cm, we will get a cone with:

Radius r = 12 cm

Slant height l = 13 cm

Height h = 5 cm

We know the volume of the cone $V = \frac{1}{3} \pi r^2 h$

$$\Rightarrow V = \left[\frac{1}{3} \times \pi \times (12)^2 \times 5\right] cm^3$$

$$\Rightarrow V = 240 \pi \ cm^3$$

Therefore, the volume of the cone will be $240 \pi \ cm^3$.

The ratio of volume of cone from previous question an the one we obtained above $= \frac{100 \pi}{240 \pi} = \frac{5}{12} = 5 : 12$

Therefore, the required ratio is 5 : 12.

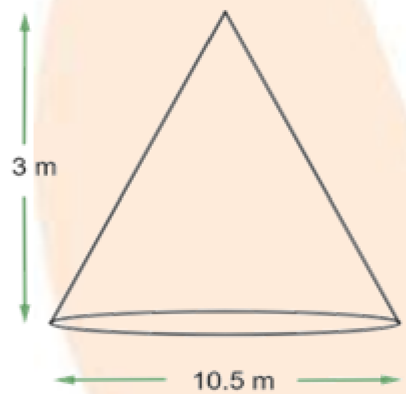
9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

$\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: It is given that diameter of the heap $= 10.5\text{ m}$

So, the radius of heap $r = \dfrac{10.5}{2} = 5.25\text{ m}$

Height of heap $h = 3\text{ m}$



We know the volume of the cone $V = \dfrac{1}{3}\pi r^2 h$

$$\Rightarrow V = \left[\frac{1}{3} \times \frac{22}{7} \times (5.25)^2 \times 3\right]\text{ m}^3$$

$$\Rightarrow V = 86.625\text{ m}^3$$

Hence, the volume of heap is 86.625 m^3.

The area of canvas required is same as curved surface area of the cone.

$\therefore A = \pi rl$

$$\Rightarrow A = \pi r\sqrt{h^2 + r^2}$$

$$\Rightarrow A = \frac{22}{7} \times 5.25 \times \sqrt{(3)^2 + (5.25)^2}\ m^2$$

$$\Rightarrow A = \left(\frac{22}{7} \times 5.25 \times 6.05\right) m^2$$

$$\Rightarrow A = 99.825\ m^2$$

Therefore, to protect the heap from the rain, the amount of canvas required is $99.825\ m^2$.

Exercise 11.4

1. Find the volume of the sphere whose radius is $\left[\text{Assume } \pi = \frac{22}{7}\right]$

(i) 7 cm

Ans: It is given the radius of sphere $r = 7$ cm

The volume of the sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (7)^3\right] cm^3$$

$$\Rightarrow V = \frac{4312}{3}\ cm^3$$

$$\Rightarrow V = 1437.33\ cm^3$$

Therefore, the volume of the sphere is $1437.33\ cm^3$.

(ii) 0.63 m

Ans: It is given the radius of sphere $r = 0.63\text{ m}$

The volume of the sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (0.63)^3\right] \text{m}^3$$

$$\Rightarrow V = 1.0478\text{ m}^3$$

Therefore, the volume of the sphere is 1.0478 m^3.

2. Find the amount of water displaced by a solid spherical ball of diameter $\left[\text{Assume } \pi = \frac{22}{7}\right]$

(i) 28 cm

Ans: It is given the diameter of ball $= 28\text{ cm}$

So, the radius of ball $r = \frac{28}{2} = 14\text{ cm}$

The volume of the ball $V = \frac{4}{3}\pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (14)^3\right] \text{cm}^3$$

$$\Rightarrow V = 11498\text{ cm}^3$$

Therefore, volume of the sphere is 11498 cm^3.

(ii) 0.21 m

Ans: It is given the diameter of ball $= 0.21 \text{ m}$

So, the radius of ball $r = \frac{0.21}{2} = 0.105 \text{ m}$

The volume of the sphere $V = \frac{4}{3} \pi r^3$

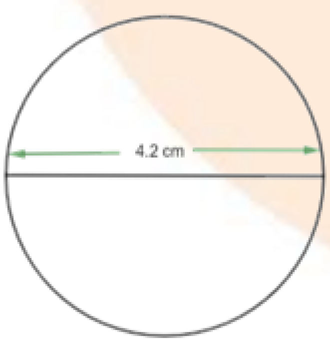
$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (0.105)^3 \right] \text{m}^3$$

$$\Rightarrow V = 0.004851 \text{ m}^3$$

Therefore, the volume of the sphere is 0.004851 m^3.

3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? $\left[\text{Assume } \pi = \frac{22}{7} \right]$

Ans: It is given the diameter of metallic ball $= 4.2 \text{ cm}$



So, the radius of ball $r = \frac{4.2}{2} = 2.1 \text{ cm}$

The volume of the sphere $V = \frac{4}{3}\pi r^3$

$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (2.1)^3\right] cm^3$

$\Rightarrow V = 38.808\ cm^3$

We know that $\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

$\Rightarrow \text{Mass} = \text{Density} \times \text{Volume}$

$\Rightarrow \text{Mass} = (8.9 \times 38.808)\ g$

$\Rightarrow \text{Mass} = 345.39\ g$

Therefore, the mass of the metallic ball is $345.39\ g$.

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Ans: Let us assume the diameter of earth be d.

So, the radius of earth will be $R = \frac{d}{2}$.

From the question, we can write the diameter of the moon as $\frac{d}{4}$.

So, the radius of moon will be $r = \frac{d}{8}$.

The volume of earth $V = \frac{4}{3}\pi R^3$

$$\Rightarrow V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$\Rightarrow V = \frac{1}{8} \times \frac{4}{3}\pi d^3$$

The volume of moon $V' = \frac{4}{3}\pi r^3$

$$\Rightarrow V' = \frac{4}{3}\pi\left(\frac{d}{8}\right)^3$$

$$\Rightarrow V' = \frac{1}{512} \times \frac{4}{3}\pi d^3$$

The ratio of volume of moon and that of earth $= \dfrac{\frac{1}{512} \times \frac{4}{3}\pi d^3}{\frac{1}{8} \times \frac{4}{3}\pi d^3} = \dfrac{1}{64}$

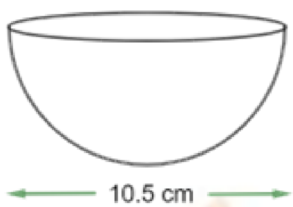
So, $\dfrac{\text{Volume of moon}}{\text{Volume of earth}} = \dfrac{1}{64}$

$\Rightarrow$ Volume of moon $= \left(\dfrac{1}{64}\right)$ Volume of earth

Therefore, the volume of moon is $\frac{1}{64}$ times the volume of earth.

5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm can hold? $\left[\text{Assume } \pi = \dfrac{22}{7}\right]$

Ans: It is given the diameter of the hemispherical bowl $= 10.5\text{ cm}$.



So, the radius of the bowl $r = \frac{10.5}{2} = 5.25 \text{ cm}$.

The volume of the hemispherical bowl $V = \frac{2}{3} \pi r^3$

$$\Rightarrow V = \left[\frac{2}{3} \times \frac{22}{7} \times (5.25)^3 \right] \text{cm}^3$$

$$\Rightarrow V = 303.1875 \text{ cm}^3$$

We know that $1000 \text{ cm}^3 = 1 \text{ litre}$

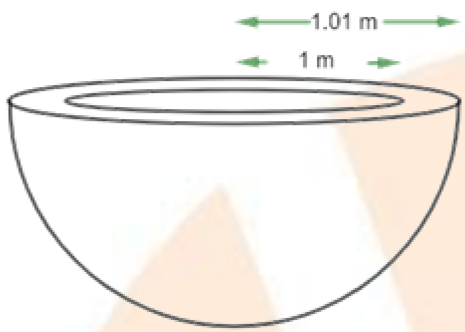
So, the capacity of the bowl $= \frac{303.1875}{1000} = 0.303 \text{ litre}$

Therefore, the volume of the hemispherical bowl is 0.303 litre.

6. A hemispherical tank is made up of an iron sheet 1 cm **thick. If the inner radius is** 1 m**, then find the volume of the iron used to make the tank.** $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: The inner radius of hemispherical tank $r = 1 \text{ m}$

The thickness of iron sheet $= 1 \text{ cm} = 0.01 \text{ m}$



So, the outer radius of the hemispherical tank $R = (1 + 0.01) = 1.01$ m

The volume of iron sheet required to make the tank $V = \frac{2}{3}\pi\left(R^3 - r^3\right)$

$$\Rightarrow V = \frac{2}{3}\times\frac{22}{7}\times\left((1.01)^3 - (1)^3\right)\ m^3$$

$$\Rightarrow V = \frac{44}{21} \times (1.030301 - 1)\ m^3$$

$$\Rightarrow V = 0.06348\ m^3$$

Therefore, the volume of iron sheet required to make the hemispherical tank is $0.06348\ m^3$.

7. Find the volume of a sphere whose surface area is $154\ cm^2$. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: Let us assume the radius of the sphere be r.

It is given the surface area of the sphere $= 154\ cm^2$.

$$\therefore 4\pi r^2 = 154\ cm^2$$

$$\Rightarrow r^2 = \left(\frac{154 \times 7}{4 \times 22}\right) \text{cm}^2$$

$$\Rightarrow r^2 = \left(\frac{49}{4}\right) \text{cm}^2$$

$$\Rightarrow r = \left(\frac{7}{2}\right) \text{cm}$$

The volume of the sphere $V = \frac{4}{3} \pi r^3$

$$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3\right] \text{cm}^3$$

$$\Rightarrow V = \left[\frac{49 \times 11}{3}\right] \text{cm}^3$$

$$\Rightarrow V = 179.67 \text{ cm}^3$$

Therefore, the volume of the sphere is 179.67 cm^3.

8. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of Rs. 498.96**. If the cost of white-washing is** Rs. 2.00 **per square meter, find the**

(i) Inside surface area of the dome,

Ans: It is given that it costs Rs. 2.00 to whitewash an area $= 1 \text{ m}^2$

So, it costs Rs. 498.96 to whitewash an area $= \frac{498.96}{2} \text{ m}^2 = 249.48 \text{ m}^2$.

Therefore, the inner surface area of the dome is 249.48 m^2.

(ii) Volume of the air inside the dome. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: Let us assume the radius of the hemispherical dome be r.

We obtained the curved surface area of the inner dome $= 249.48\ \text{m}^2$

$\therefore 2\pi\ ^{2} = 249.48\ \text{m}^2$

$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 249.48\ \text{m}^2$

$\Rightarrow r^2 = \left(\frac{249.48 \times 7}{2 \times 22}\right)\ \text{m}^2$

$\Rightarrow r^2 = 39.69\ \text{m}^2$

$\Rightarrow r = 6.3\ \text{m}$

Volume of hemispherical dome $V = \frac{2}{3}\pi r^3$

$\Rightarrow V = \left[\frac{2}{3} \times \frac{22}{7} \times (6.3)^3\right]\ \text{m}^3$

$\Rightarrow V = 523.908\ \text{m}^3$

$\Rightarrow V = 523.9\ \text{m}^3 \,(\text{approximately})$

Therefore, the volume of air inside the hemispherical dome is $523.9\ \text{m}^3$.

9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r' of the new sphere,

Ans: It is given the radius of one iron sphere $= r$.

The volume of one iron sphere $= \frac{4}{3}\pi r^3$

So, the volume of 27 iron spheres $= 27 \times \frac{4}{3}\pi r^3$

These spheres are melted to form one big sphere.

Let us assume the radius of this new sphere be r'.

The volume of new iron sphere $= \frac{4}{3}\pi r'^3$

We can now equate the volumes.

$\Rightarrow \frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$

$\Rightarrow r'^3 = 27r^3$

$\Rightarrow r' = 3r$

Therefore, the radius of the new sphere is $3r$.

(ii) ratio of S and S'.

Ans: The surface area of an iron sphere of r is $S = 4\pi r^2$.

The surface area of an iron sphere of r' is $S' = 4\pi r'^2$.

$\Rightarrow S' = 4\pi (3r)^2$

$\Rightarrow S' = 36\pi r^2$

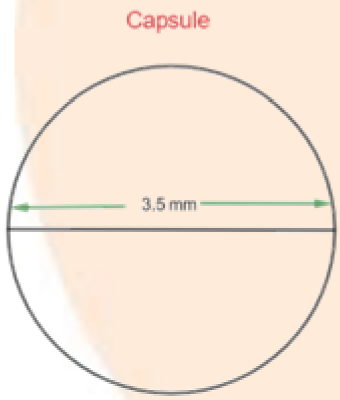
The ratio of $\frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1:9$

Therefore, the required ratio is $1:9$.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine $\left(\text{in mm}^3\right)$ is needed to fill this capsule? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Ans: It is given that the diameter of the capsule $= 3.5\text{ mm}$.

So, the radius will be $r = \left(\frac{3.5}{2}\right) = 1.75\text{ mm}$.



Volume of spherical capsule $V = \frac{4}{3}\pi r^3$

$\Rightarrow V = \left[\frac{4}{3} \times \frac{22}{7} \times (1.75)^3\right]\text{mm}^3$

$\Rightarrow V = 22.458\text{ mm}^3$

$\Rightarrow V = 22.46\text{ mm}^3\,(\text{approx})$

Hence, the amount of medicine required to fill the capsule is $22.46\ \text{mm}^3$.