Important Solutions for Class 9

Mathematics

Chapter 6 – Lines and Angles

Very Short Answer Type Questions **1 Mark**

1. Measurement of reflex angle is

(i) 90°

(ii) between 0° **and** 90°

(iii) between 90° **and** 180°

(iv) between 180° **and** 360°

Ans: (iv) between 180° and 360°

2. The sum of angle of a triangle is

(i) 0°

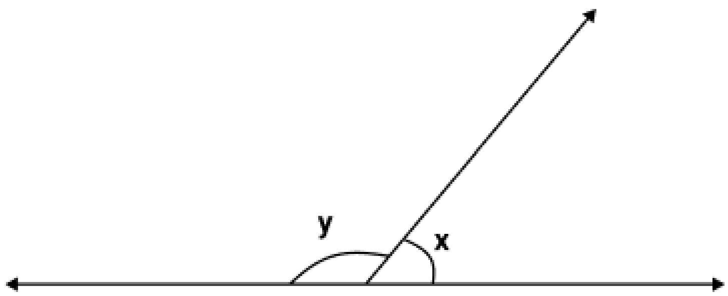
(ii) 90°

(iii) 180°

(iv) none of these

Ans: (iii) 180°

3. In fig $x = 30^{\circ}$ **then** $y =$



(i) 90°

(ii) 180°

(iii) 150°

(iv) 210°

Ans: (iii) 150°

4. If two lines intersect each other then

(i) Vertically opposite angles are equal

(ii) Corresponding angle are equal

(iii) Alternate interior angle are equal

(iv) None of these

Ans: (i) Vertically opposite angles are equal

5. The measure of Complementary angle of 63° **is**

(a) 30°

(b) 36°

(c) 27°

(d) None of there

Ans: (c) 27°

6. If two angles of a triangle is 30° and 45° what is measure of third angle

(a) 95°

(b) 90°

(c) 60°

(d) 105°

Ans: (d) 105°

7. The measurement of complete angle is

(a) 0°

(b) 90°

(c) 180°

(d) 360°

Ans: (d) 360°

8. The measurement of sum of linear pair is

(a) 180°

(b) 90°

(c) 270°

(d) 360°

Ans: (a) 180°

9. The difference of two complementary angles is 40°. The angles are

(a) $65^\circ, 35^\circ$

(b) $70^\circ, 30^\circ$

(c) $25^\circ, 65^\circ$

(d) $70^\circ, 110^\circ$

Ans: (c) $25^\circ, 65^\circ$

10. Given two distinct points P and Q in the interior of $\angle ABC$, then $\overrightarrow{AB}$ will be

(a) In the interior of $\angle ABC$

(b) In the interior of $\angle ABC$

(c) On the $\angle ABC$

(d) On the both sides of $\overrightarrow{BA}$

Ans: (c) On the $\angle ABC$

11. The complement of $(90-a)^0$ is

(a) $-a^0$

(b) $(90+2a)^0$

(c) $(90-a)^0$

(d) a^0

Ans: (d) a^0

12. The number of angles formed by a transversal with a pair of lines is

(a) 6

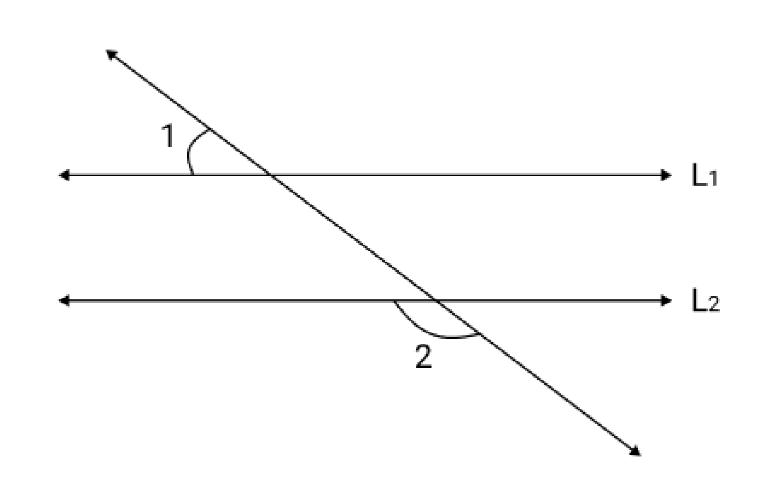
(b) 3

(c) 8

(d) 4

Ans: (c) 8

13. In fig $L_1 \parallel L_2$ and $\angle 1 = 52^\circ$ the measure of $\angle 2$ is.



(A) 38°

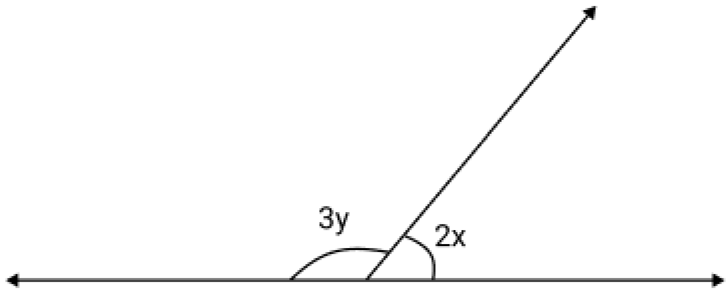
(B) 128°

(C) 52°

(D) 48°

Ans: (B) 128°

14. In fig $x = 30^\circ$ the value of Y is



(A) 10°

(B) 40°

(C) 36°

(D) 45°

Ans: (B) 40°

15. Which of the following pairs of angles are complementary angle?

(A) $25^{\circ}, 65^{\circ}$

(B) $70^{\circ}, 110^{\circ}$

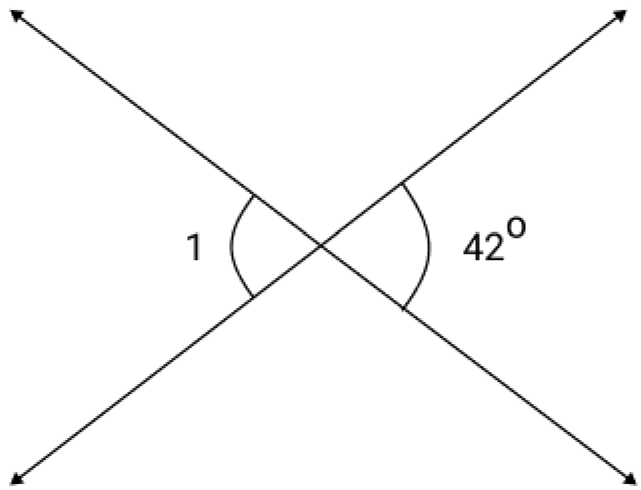
(C) $30^{\circ}, 70^{\circ}$

(D) $32.1^{\circ}, 47.9^{\circ}$

Ans:

(A) $25^{\circ}, 65^{\circ}$

16. In fig the measures of $\angle 1$ is.



(A) 158°

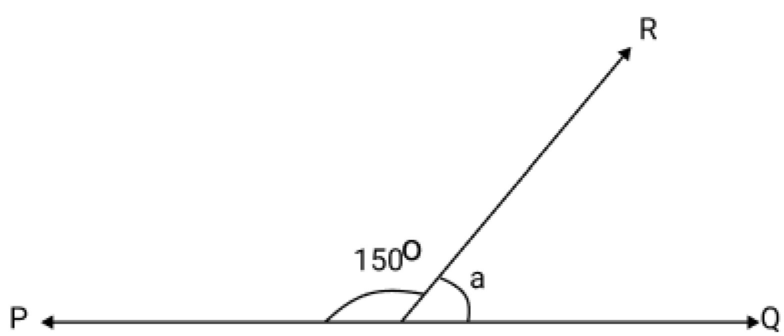
(B) 138°

(C) 42°

(D) 48°

Ans: (C) 42°

17. In figure the measure of $\angle a$ is



(a) 30°

(b) 150^0

(c) 15°

(d) 50°

Ans: (a) 30°

18. The correct statement is-

(a) A line segment has one end point only.

(b) The ray AB **is the same as the ray** BA**.**

(c) Three points are collinear if all of them lie on a line.

(d) Two lines are coincident if they have only one point in common.

Ans: (c) Three points are collinear if all of them lie on a line.

19. One angle is five times its supplement. The angles are-

(a) $15^{\circ}, 75^{\circ}$

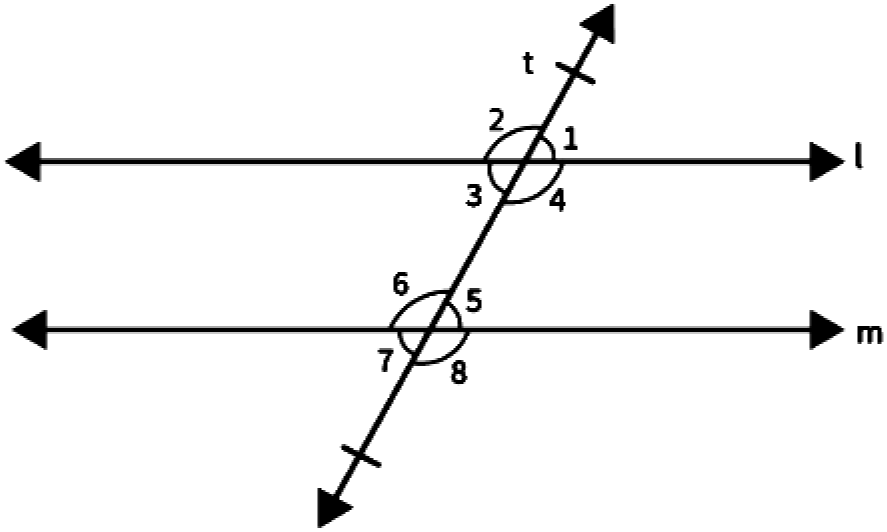
(b) $30^{\circ}, 150^{\circ}$

(c) $36^{\circ}, 144^{0}$

(d) $160^{\circ}, 40^{\circ}$

Ans: (b) $30^{\circ}, 150^{\circ}$

20. In figure if $m \parallel n$ **and** $\angle 1 : \angle 2 = 1 : 2.$ **the measure of** $\angle 8$ **is**



(a) 120°

(b) 60°

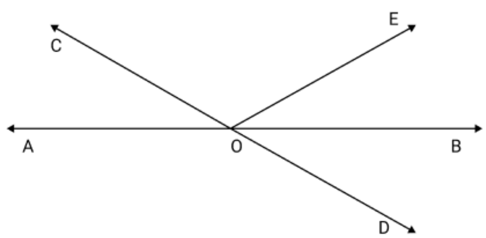
(c) 30°

(d) 45°

Ans: (b) 60°

Short Answer Type Questions **2 Marks**

1. In Fig. 6.13, lines AB **and** CD **intersect at O. If** $\angle AOC + \angle BOE = 70^\circ$ **and** $\angle BOD = 40^\circ$**, find** $\angle BOE$ **and reflex** $\angle COE$.



Ans: According to the question given that, $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$.

We need to find $\angle BOE$ and reflex $\angle COE$.

According to the given figure, we can conclude that $\angle COB$ and $\angle COE$ form a linear pair.

As we also know that sum of the angles of a linear pair is 180°.

So, $\angle COB + \angle COE = 180^\circ$

Because, $\angle COB = \angle AOC + \angle BOE$, or

So, $\angle AOC + \angle BOE + \angle COE = 180^\circ$

$\Rightarrow 70^\circ + \angle COE = 180^\circ$

$\Rightarrow \angle COE = 180^\circ - 70^\circ$

$= 110^\circ$.

Reflex $\angle COE = 360^\circ - \angle COE$

$= 360^\circ - 110^\circ$

$= 250^\circ$.

$\angle AOC = \angle BOD$ (Vertically opposite angles), or

$\angle BOD + \angle BOE = 70^\circ$

But, according to the question given that $\angle BOD = 40^\circ$.

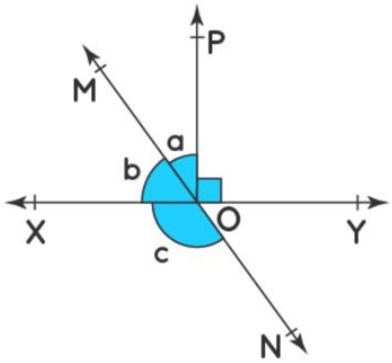
$40^\circ + \angle BOE = 70^\circ$

$\angle BOE = 70^\circ - 40^\circ$

$= 30^\circ$.

Hence, we can conclude that Reflex $\angle COE = 250^\circ$ and $\angle BOE = 30^\circ$.

2. In Fig. 6.14, lines XY and MN intersect at 0. If $\angle POY = 90^\circ$ and $a : b = 2 : 3$, find c.



Ans: According to the question given that $\angle POY = 90^\circ$ and $a : b = 2 : 3$.

We need to find the value of c in the given figure.

Suppose a be equal to 2x and b be equal to 3x.

Because, $a + b = 90^\circ \Rightarrow 2x + 3x = 90^\circ \Rightarrow 5x = 90^\circ$

$\Rightarrow x = 18^\circ$

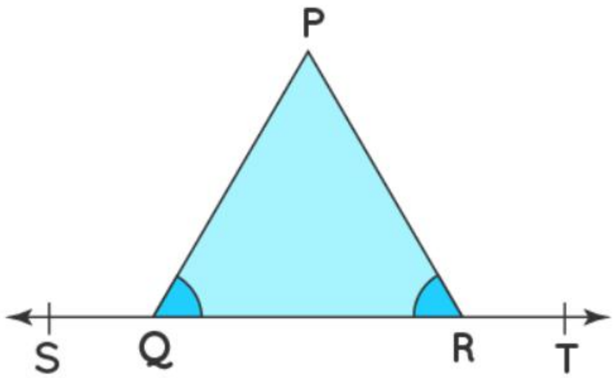
Hence, $b = 3 \times 18^\circ = 54^\circ$

Now $b + c = 180^\circ$ [Linear pair]

$\Rightarrow 54^\circ + c = 180^\circ$

$\Rightarrow c = 180^\circ - 54^\circ = 126^\circ$

3. In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Ans: According to the question we need to prove that $\angle PQS = \angle PRT$.

According to the question given that $\angle PQR = \angle PRQ$.

According to the given figure, we can conclude that $\angle PQS$ and $\angle PQR$, and $\angle PRS$ and $\angle PRT$ form a linear pair.

As we also know that sum of the angles of a linear pair is 180° .

So, $\angle PQS + \angle PQR = 180^{\circ}$, and(i)

$\angle PRQ + \angle PRT = 180^{\circ}$ ..(ii)

According to the equations (i) and (ii), we can conclude that

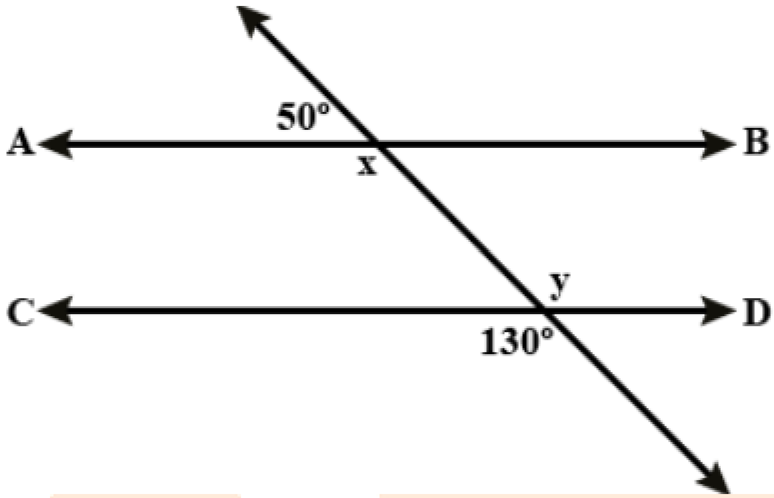
$\angle PQS + \angle PQR = \angle PRQ + \angle PRT$

But, $\angle PQR = \angle PRQ.$

So, $\angle PQS = \angle PRT$

Hence, the desired result is proved.

4. In the given figure, find the values of x and y and then show that $AB \parallel CD$.



Ans: According to the question we need to find the value of x and y in the figure given below and then prove that $AB \parallel CD$

According to the figure, we can conclude that $y = 130^\circ$ (Vertically opposite angles), and x and 50° form a pair of linear pair.

As we also know that the sum of linear pair of angles is 180°.

$x + 50^\circ = 180^\circ$

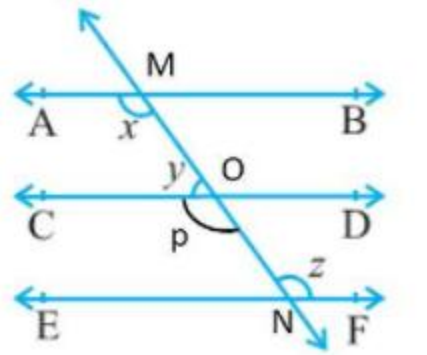
$x = 130^\circ$

$x = y = 130^\circ$

According to the given figure, we can conclude that x and y form a pair of alternate interior angles parallel to the lines AB and CD.

Hence, we can conclude that $x = 130^\circ, y = 130^\circ$ and $AB \parallel CD$.

5. In the given figure, if $AB \parallel CD, CD \parallel EF$ and $y:z = 3:7$, find x.



Ans: According to the question given that, $AB \parallel CD, CD \parallel EF$ and $y:z=3:7$.

We need to find the value of x in the figure given below.

As we also know that the lines parallel to the same line are also parallel to each other.

We can determine that $AB \parallel CD \parallel EF$.

Assume that, $y=3a$ and $z=7a$.

We know that angles on same side of a transversal are supplementary.

So, $x+y=180^{\circ}$.

$x=z$ (Alternate interior angles)

$z+y=180^{\circ}$, or $7a+3a=180^{\circ}$

$\Rightarrow 10a=180^{\circ}$

$a=18^{\circ}$.

$z=7a=126^{\circ}$

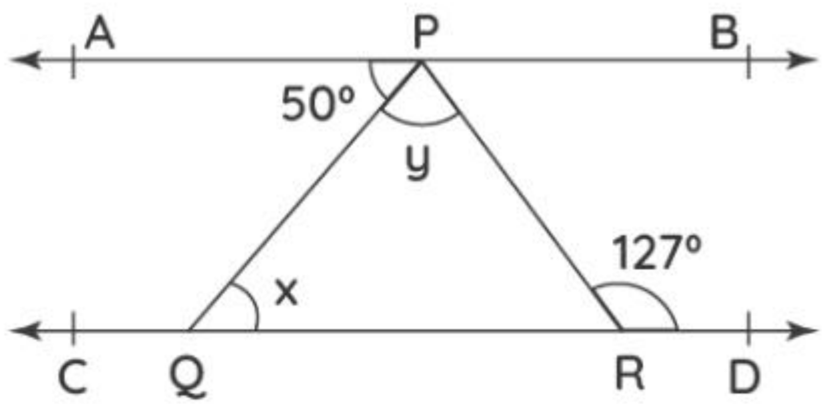
$y=3a=54^{\circ}$

Now, $x+54^{\circ}=180^{\circ}$

$x=126^{\circ}$

Hence, we can determine that $x=126^{\circ}$.

6. In the given figure, if $AB \parallel CD, \angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y.



Ans: According to the question given that, $AB \parallel CD, \angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$.

As we need to find the value of x and y in the figure.

$\angle APQ = x = 50^\circ$. (Alternate interior angles)

$\angle PRD = \angle APR = 127^\circ$. (Alternate interior angles)

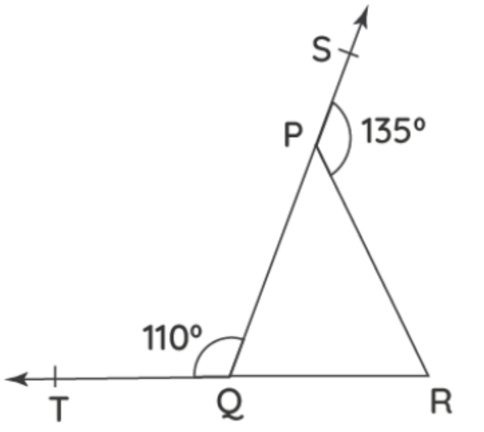
$\angle APR = \angle QPR + \angle APQ$

$127^\circ = y + 50^\circ$

$\Rightarrow y = 77^\circ$

Hence, we can determine that $x = 50^\circ$ and $y = 77^\circ$.

7. In the given figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.



Ans: According to the question given that, $\angle SPR = 135^{\circ}$ and $\angle PQT = 110^{\circ}$.

As we need to find the value of $\angle PRQ$ in the figure given below.

According to the given figure, we can determine that $\angle SPR$ and $\angle RPQ$, and $\angle SPR$ and $\angle RPQ$ form a linear pair.

As we also know that the sum of angles of a linear pair is 180°.

$\angle SPR + \angle RPQ = 180^{\circ}$, and

$\angle PQT + \angle PQR = 180^{\circ}$

$135^{\circ} + \angle RPQ = 180^{\circ}$, and

$110^{\circ} + \angle PQR = 180^{\circ}$, or

$\angle RPQ = 45^{\circ}$, and

$\angle PQR = 70^{\circ}$.

According to the figure, we can determine that

$\angle PQR + \angle RPQ + \angle PRQ = 180^{\circ}$. (Angle sum property)

$\Rightarrow 70^{\circ} + 45^{\circ} + \angle PRQ = 180^{\circ}$

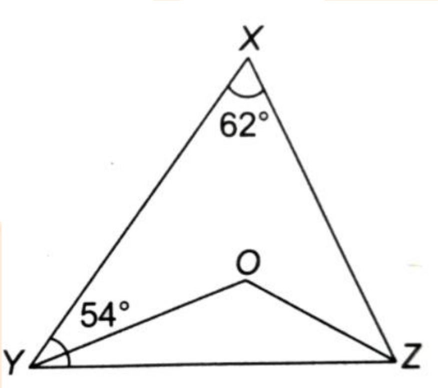
$\Rightarrow 115^{\circ} + \angle PRQ = 180^{\circ}$

$\Rightarrow 115^{\circ} + \angle PRQ = 180^{\circ}$

$\Rightarrow \angle PRQ = 65^\circ$.

Hence, we can determine that $\angle PRQ = 65^\circ$.

8. In the given figure, $\angle X = 62^\circ, \angle XYZ = 54^\circ$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.



Ans: According to the question given that, $\angle X = 62^\circ, \angle XYZ = 54^\circ$ and YO and ZO are bisectors of $\angle XYZ$ and $\angle XZY$, respectively.

As we need to find the value of $\angle OZY$ and $\angle YOZ$ in the figure.

According to the given figure, we can determine that in ΔXYZ

$\angle X + \angle XYZ + \angle XZY = 180^\circ$ (Angle sum property)

$\Rightarrow 62^\circ + 54^\circ + \angle XZY = 180^\circ$

$\Rightarrow 116^\circ + \angle XZY = 180^\circ$

$\Rightarrow \angle XZY = 64^\circ$.

According to the question given that, OY and OZ are the bisectors of $\angle XYZ$ and $\angle XZY$, respectively.

$\angle OYZ = \angle XYO = \frac{54^\circ}{2} = 27'$, and

$\angle OZY = \angle XZO = \frac{64^\circ}{2} = 32^\circ$

According to the figure, we can determine that in ΔOYZ

$\angle OYZ + \angle OZY + \angle YOZ = 180^\circ$. (Angle sum property)

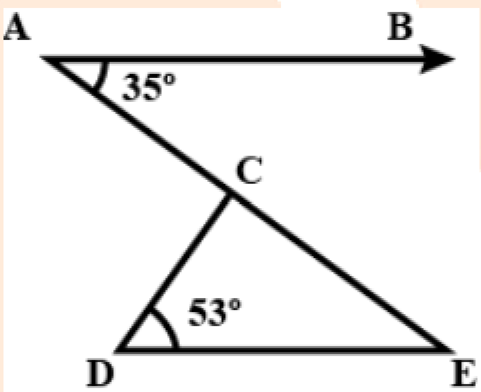
$27^\circ + 32^\circ + \angle YOZ = 180^\circ$

$\Rightarrow 59^\circ + \angle YOZ = 180^\circ$

$\Rightarrow \angle YOZ = 121^\circ$.

Hence, we can determine that $\angle YOZ = 121^\circ$ and $\angle OZY = 32^\circ$.

9. In the given figure, if $AB \parallel DE, \angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.



Ans: According to the question given that, $AB \parallel DE, \angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$.

As we need to find the value of $\angle DCE$ in the figure given below.

According to the given figure, we can determine that

$\angle BAC = \angle CED = 35^\circ$ (Alternate interior angles)

According to the figure, we can determine that in ΔDCE

$\angle DCE + \angle CED + \angle CDE = 180^\circ$. (Angle sum property)

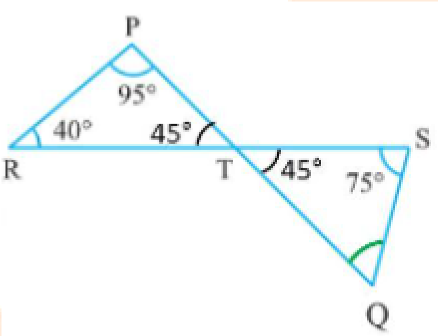
$\angle DCE + 35^\circ + 53^\circ = 180^\circ$

$\Rightarrow \angle DCE + 88^\circ = 180^\circ$

$\Rightarrow \angle DCE = 92^\circ$.

Hence, we can determine that $\angle DCE = 92^{\circ}$.

10. In the given figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40^{\circ}$, $\angle RPT = 95^{\circ}$ and $\angle TSQ = 75^{\circ}$, find $\angle SQT$.



Ans: According to the question given that, $\angle PRT = 40^{\circ}, \angle RPT = 95^{\circ}$ and $\angle TSQ = 75^{\circ}$.

As we need to find the value of $\angle SQT$ in the figure.

According to the given figure, we can determine that in ΔRTP

$\angle PRT + \angle RTP + \angle RPT = 180^{\circ}$ (Angle sum property)

$40^{\circ} + \angle RTP + 95^{\circ} = 180^{\circ}$

$\Rightarrow \angle RTP + 135^{\circ} = 180^{\circ}$

$\Rightarrow \angle RTP = 45^{\circ}$.

According to the figure, we can determine that

$\angle RTP = \angle STQ = 45^{\circ}$. (Vertically opposite angles)

According to the figure, we can determine that in ΔSTQ

$\angle SQT + \angle STQ + \angle TSQ = 180^{\circ}$. (Angle sum property)

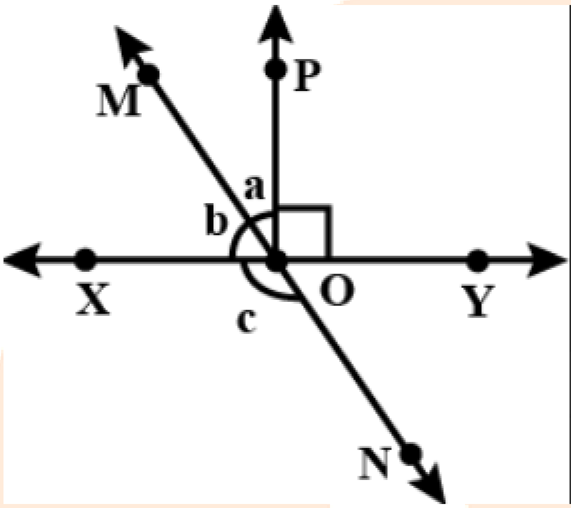
$\angle SQT + 45^{\circ} + 75^{\circ} = 180^{\circ}$

$\Rightarrow \angle SQT + 120^{\circ} = 180^{\circ}$

$\Rightarrow \angle SQT = 60^{\circ}$.

Hence, we can determine that $\angle SQT = 60^{\circ}$.

11. In fig lines xy **and** mn **intersect at 0 If** $\angle$ **poy** $= 90^{\circ}$ **and** $ab = 2:3$ **find** c



Ans: According to the given figure $\angle POY = 90^{\circ}$

a: b: 2: 3

Assume that, $a = 2x$ and $b = 3x$

$a + b + \angle POY = 180^{\circ} (\because XOY$ is a line $)$

$2x + 3x + 90^{\circ} = 180^{\circ}$

$5x = 180^{\circ} - 90^{\circ}$

$5x = 90^{\circ}$

$x = \frac{90^{\circ}}{5} = 18^{\circ}$

So, $a = 36^{\circ}, \quad b = 54^{\circ}$

MoN is a line.

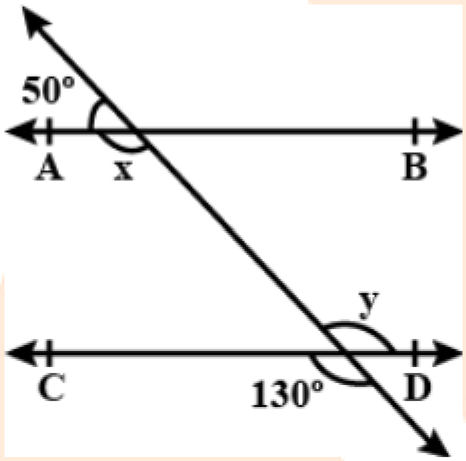
$b + C = 180^{\circ}$

$54^{\circ} + C^{\circ} = 180^{\circ}$

$C = 180^\circ - 54^\circ = 126^\circ$

Hence, the value of $C = 126^\circ$.

12. In fig find the volume of x and y then Show that $AB \parallel CD$



Ans: According to the given figure, $50^\circ + x = 180^\circ$

(by linear pair)

$x = 180^\circ - 50^\circ$

So, $x = 130^\circ$

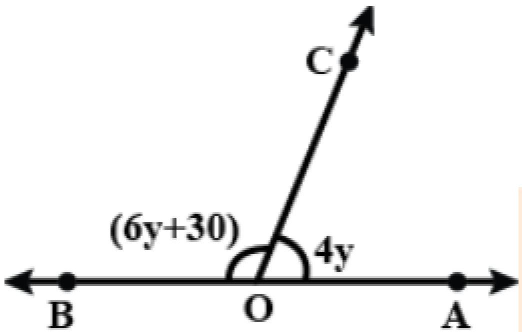
$y = 130^\circ$ (Because vertically opposite angles are equal)

$x = y$ as they are corresponding angles.

So, $AB \parallel CD$

Hence proved.

13. What value of x would make AOB a line if $\angle AOC = 4x$ and $\angle BOC = 6x + 30^\circ$



Ans: According to the question given that, $\angle AOC = 4x$ and $\angle BOC = 6x + 30^{\circ}$

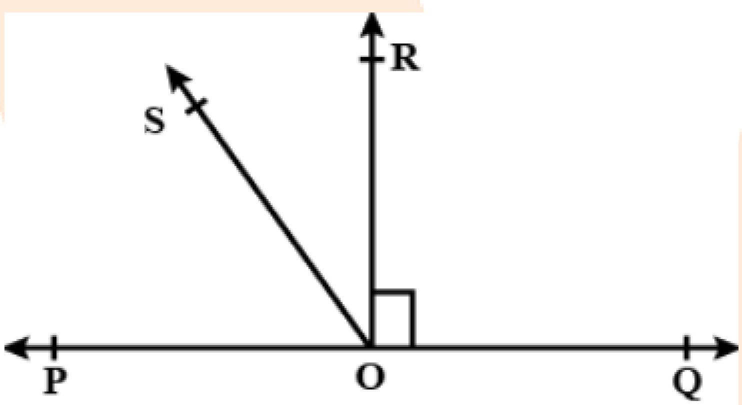
$\angle AOC + \angle BOC = 180^{\circ}$ (By linear pair)

$4x + 6x + 30^{\circ} = 180^{\circ}$

$10x = 180^{\circ} - 30^{\circ}$

$10x = 150^{\circ} = x = 15^{\circ}$

14. In fig POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$



Ans: According to the question,

$R.H.S = \frac{1}{2}(\angle QOS - \angle POS)$

$= \frac{1}{2}(\angle ROS + \angle QOR - \angle POS)$

$= \frac{1}{2}\left(\angle ROS + 90^{\circ} - \angle POS\right)$........ (1)

Because, $\angle POS + \angle ROS = 90^{\circ}$

So, by equation 1

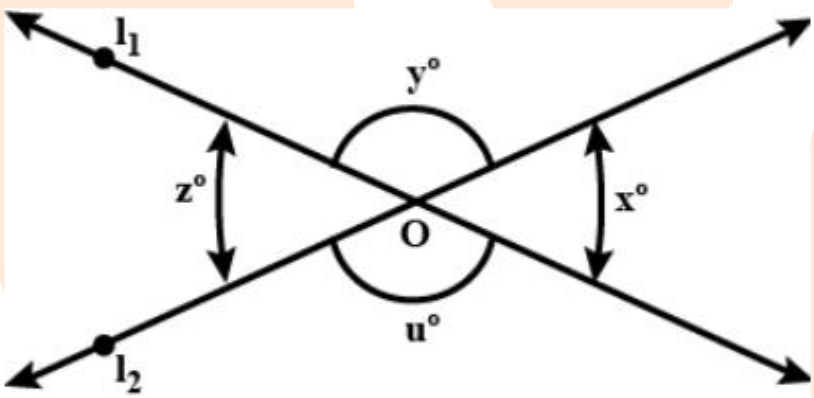
$= \frac{1}{2}(ROS + \angle POS + \angle ROS - \angle POS)$ (by equation 1)

$= \frac{1}{2} \times 2\angle ROS = \angle ROS$

= L.H.S

Hence proved.

15. In fig lines P and R intersected at 0 , if $x = 45^{\circ}$ find x, y and u



Ans: According to the question given that,

$X = 45^{\circ}$

So, $Z = 45^{\circ}$ (Because vertically opposite angles are equal)

$X + y = 180^{\circ}$

$45^{\circ} + y = 180^{\circ}$ (By linear pair)

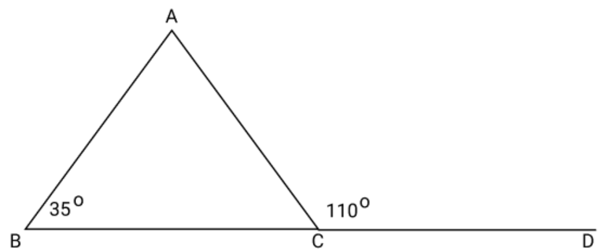
$y = 180^{\circ} - 45$

$y = 135^{\circ}$

$\mathrm{y} = \mathrm{u}$

Hence, the value of $\mathrm{u} = 135^{\circ}$ (Vertically opposite angles)

16. The exterior angle of a triangle is 110° and one of the interior opposite angle is 35°. Find the other two angles of the triangle.



Ans: As we all know that the exterior angle of a triangle is equal to the sum of interior opposite angles.

So, $\angle \mathrm{ACD} = \angle \mathrm{A} + \angle \mathrm{B}$

$110 = \angle \mathrm{A} + 35^{\circ}$

$\angle \mathrm{A} = 110^{\circ} - 35^{\circ}$

$\angle \mathrm{A} = 75^{\circ}$

$\angle \mathrm{C} = 180 - (\angle \mathrm{A} + \angle \mathrm{B})$

$\angle \mathrm{C} = 180 - \left(75^{\circ} + 35^{\circ}\right)$

$\angle \mathrm{C} = 70^{\circ}$

17. Of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Find the angles.

Ans: Assume that the smallest angle be x°

Then the other two angles are $2x^{\circ}$ and $3x^{\circ}$

$x^{\circ}+2x^{\circ}+3x^{\circ}=180^{\circ}$ [As we know that the sum of three angle of a triangle is 180°]

$6x^{\circ}=180^{\circ}$

$\mathrm{x}=\frac{180}{6}$

$=30^{\circ}$

Hence, angles are $30^{\circ}, 60^{\circ}$ and 90°.

18. Prove that if one angle of a triangle is equal to the sum of other two angles, the triangle is right angled.

Ans: According to the question given that in ΔABC, $\angle \mathrm{B}=\angle \mathrm{A}+\angle \mathrm{C}$

To prove: ΔABC is right angled.

Proof: $\angle A+\angle B+\angle C=180^{\circ}$….. (1) [As we know that the sum of three angles of a $\Delta \mathrm{ABC}$ is 180°]

$\angle A+\angle C=\angle B$….. (2)

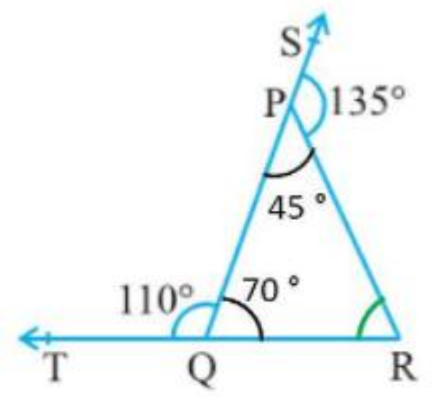
From equations (1) and (2),

$\angle B+\angle B=180^{\circ}$

$2\angle \mathrm{B}=180^{\circ}$

$\angle \mathrm{B}=90^{\circ}$

19. In fig. sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR=135^{\circ}$ and $\angle \mathrm{PQT}=110^{\circ}$, find $\angle \mathrm{PRQ}$.



Ans: According to the given figure,

$\angle PQT + \angle PQR = 180^\circ$

$110^\circ + \angle PQR = 180^\circ$

$\angle PQR = 180^\circ - 110^\circ$

$\angle \text{PQR} = 70^\circ$

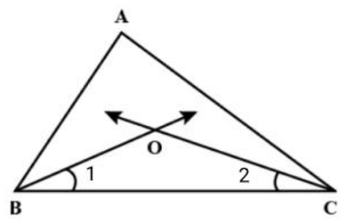
Also, $\angle \text{SPR} = \angle \text{PQR} + \angle \text{PRQ}$ [According to the Interior angle theorem]

$135^\circ = 70^\circ + \angle PRQ$

$\angle \text{PRQ} = 135^\circ - 70^\circ$

Hence, the value of $\angle \text{PRQ} = 65^\circ$.

20. In fig the bisector of $\angle ABC$ and $\angle \text{BCA}$ intersect each other at point O prove that $\angle BOC = 90^\circ + \frac{1}{2}\angle A$



Ans: According to the question given that in $\triangle ABC$ such that the bisectors of $\angle ABC$ and $\angle \mathrm{BCA}$ meet at a point O.

To Prove $\angle BOC = 90^{\circ} + \frac{1}{2}\angle A$

Proof: In $\triangle BOC$

$\angle 1 + \angle 2 + \angle BOC = 180^{\circ}$ (1)

In $\triangle ABC$

$\angle A + \angle B + \angle C = 180^{\circ}$

$\angle A + 2\angle 1 + 2\angle 2 = 180^{\circ}$

[BO and CO bisects $\angle B$ and $\angle \mathrm{C}$]

$\Rightarrow \frac{\angle A}{2} + \angle 1 + \angle 2 = 90^{\circ}$

$\angle 1 + \angle 2 = 90^{\circ} - \frac{\angle A}{2}$

[Divide forth side by 2]

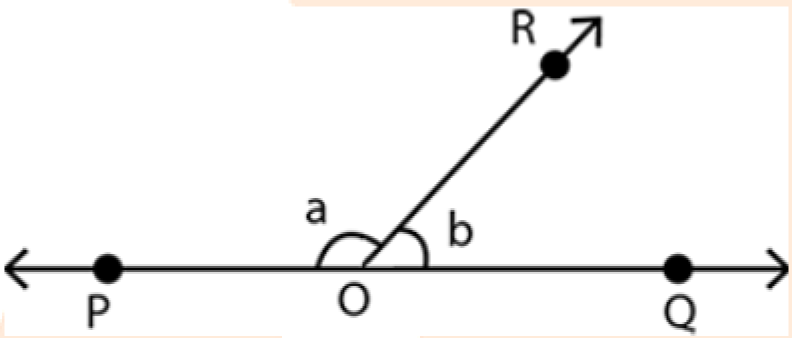
$\angle 1 + \angle 2 = 90^{\circ} - \frac{\angle A}{2}$ in (i)

Substituting, $90^{\circ} - \frac{\angle A}{2} + \angle BOC = 180^{\circ}$

$\Rightarrow \angle BOC = 90^{\circ} + \frac{\angle A}{2}$

Hence proved.

21. In the given figure $\angle POR$ and $\angle QOR$ form a linear pair if $a-b=80^{\circ}$. Find the value of 'a' and 'b'.



Ans: $a+b=180^{\circ} \rightarrow (1)$ [By line as pair]

$a-b=80^{0} \rightarrow (2)$

$2a=260^{\circ}$ [Adding equations (1) and (2)]

$a=130^{\circ}$

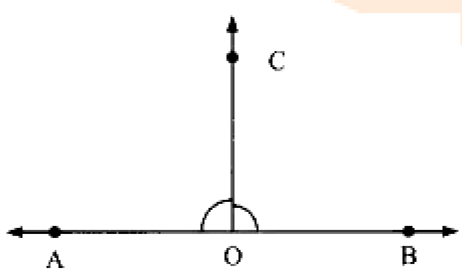
Put $a=130^{\circ}$ in equation (1)

$130^{\circ}+b=180^{\circ}$

$b=180^{\circ}-130^{\circ}=50^{\circ}$

Hence the value of $a=130^{\circ}$ and $b=50^{\circ}$.

22. If ray OC stands on a line AB such that $\angle AOC=\angle BOC$, then show that $\angle AOC=90^{\circ}$



Ans: According to the question given that,

$\angle AOC = \angle BOC$

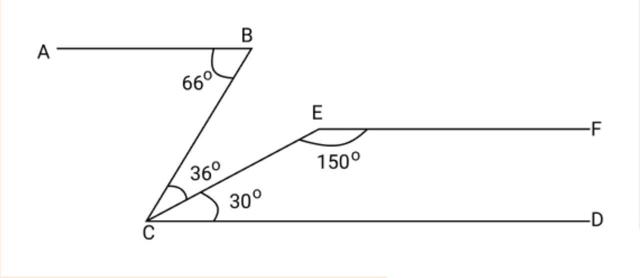
$\angle \mathrm{AOC} + \angle \mathrm{BOC} = 180^{\circ}$ [By lines pair]

$\angle \mathrm{AOC} + \angle \mathrm{AOC} = 180^{\circ}$

$2\angle \mathrm{AOC} = 180^{\circ}$

$\angle \mathrm{AOC} = 90^{\circ} = \angle B\mathrm{OC}$

23. In the given figure show that $\mathrm{AB} \parallel \mathrm{EF}$



Ans: $\angle \mathrm{BCD} = \angle \mathrm{BCE} + \angle \mathrm{ECD}$

$= 36^{\circ} + 30^{\circ} = 66^{\circ} = \angle ABC$

So, $\mathrm{AB} \parallel \mathrm{CD}$ [Alternate interior angles are equal]

Again, $\angle \mathrm{ECD} = 30^{\circ}$ and $\angle \mathrm{FEC} = 150^{\circ}$

So, $\angle \mathrm{ECD} + \angle \mathrm{FEC} = 30^{\circ} + 150^{\circ} = 180^{\circ}$

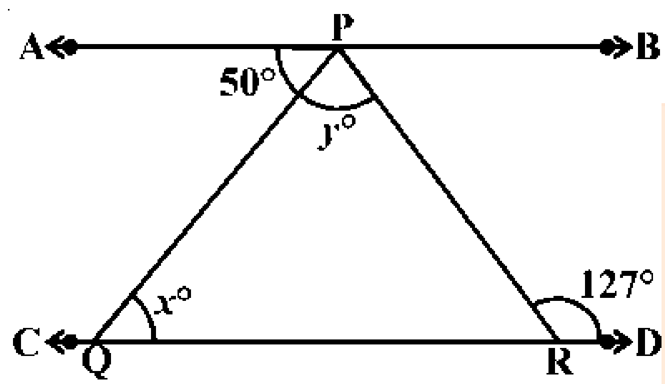
Therefore, $\mathrm{EF} \parallel \mathrm{CD}$ [We know that the sum of consecutive interior angle is 180°]

$AB \parallel CD$ and $\mathrm{CD} \parallel \mathrm{EF}$

Then $\mathrm{AB} \parallel \mathrm{EF}$

Hence proved.

24. In figure if $AB \parallel CD, \angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$ Find x and y.



Ans: $AB \parallel CD$ and PQ is a transversal

$\angle APQ = \angle PQD$ [Pair of alternate angles]

$50^\circ = x$

Also $AB \parallel CD$ and PR is a transversal

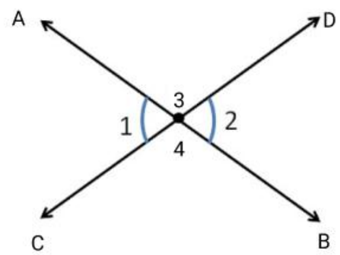
$\angle APR = \angle PRD$

$50^\circ + Y = 127^\circ$

$Y = 127^\circ - 50^\circ = 77^\circ$

Hence the value of $x = 50^\circ$ and $Y = 77^\circ$.

25. Prove that if two lines intersect each other then vertically opposite angler are equal.



Ans: According to the given figure: AB and CD are two lines intersect each other at O.

To prove: (i) $\angle 1 = \angle 2$ and (ii) $\angle 3 = \angle 4$

Proof:

$\angle 1 + \angle 4 = 180^\circ \rightarrow (i)$ [By linear pair]

$\angle 4 + \angle 2 = 180^\circ \quad \rightarrow (ii)$

$\angle 1 + \angle 4 = \angle 4 + \angle 2$ [By equations (i) and (ii)]

$\angle 1 = \angle 2$

Similarly,

$\angle 3 = \angle 4$

Hence proved.

26. The measure of an angle is twice the measure of supplementary angle. Find measure of angles.

Ans: Assume that the measure be x^0.

Then its complement is $180^\circ - x^0$.

According to question

$x^0 = 2\left(180^\circ - x^0\right)$

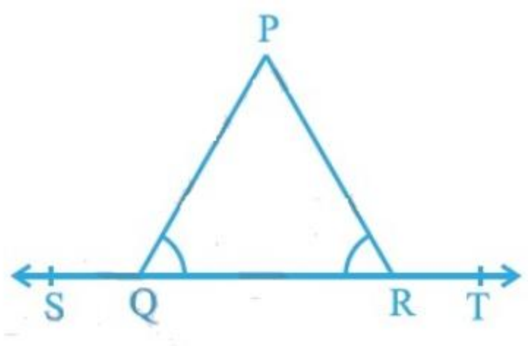
$x^0 = 360^\circ - 2x^0$

$3x = 360^\circ$

$x = 120^\circ$

The measure of the angles are 120° and 60°.

27. In fig $\angle PQR = \angle PRQ$. Then prove that $\angle PQS = \angle PRT$.



Ans: $\angle PQS + \angle PQR = \angle PRQ + \angle PRT$ [By linear pair]

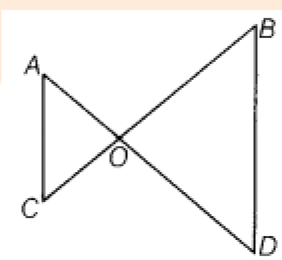
But,

$\angle PQR = \angle PRQ$ [Accordign to the question]

So, $\angle PQS = \angle PRT$

Hence proved.

28. In the given fig $\angle \mathrm{AOC} = \angle \mathrm{ACO}$ and $\angle \mathrm{BOD} = \angle \mathrm{BDO}$ prove that $\mathrm{AC} \parallel \mathrm{DB}$



Ans: According to the question given that,

$\angle AOC = \angle ACO$ and $\angle BOD = \angle BDO$

But,

$\angle AOC = \angle BOD$ [Vertically opposite angles]

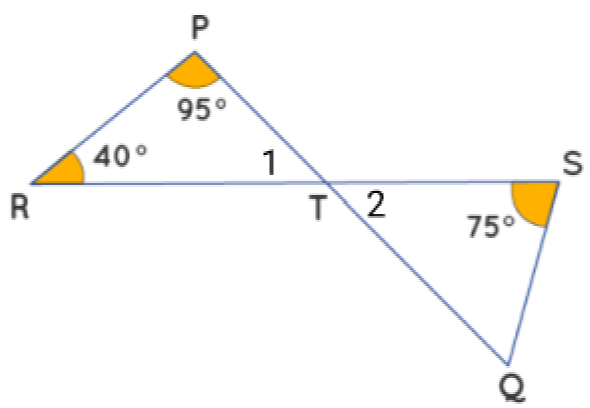
$\angle AOC = \angle BOD$ and $\angle BOD = \angle BDO$

$\Rightarrow \angle ACO = \angle BDO$

So, $AC \parallel BD$ [By alternate interior angle property]

Hence proved.

29. In figure if lines PQ **and** RS **intersect at point** T**. Such that** $\angle PRT = 40^{\circ}$**,** $\angle RPT = 95^{\circ}$ **and** $\angle TSQ = 75^{\circ}$**, find** $\angle SQT$ **.**



Ans: According to the Δ PRT

$\angle P + \angle R + \angle 1 = 180^{\circ}$ [By angle sum property]

$95^{\circ} + 40^{\circ} + \angle 1 = 180^{\circ}$

$\angle 1 = 180^{\circ} - 135^{\circ}$

$\angle 1 = 45^{\circ}$

$\angle 1 = \angle 2$ [Vertically opposite angle]

$\angle 2 = \angle 45^{\circ}$

According to the ΔTQS $\angle 2 + \angle Q + \angle S = 180^{\circ}$

$45^{\circ} + \angle Q + 75^{\circ} = 180^{\circ}$

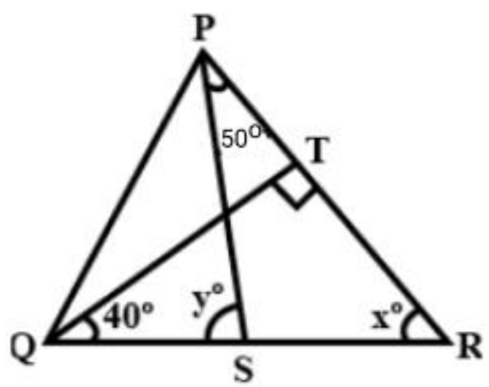
$\angle Q + 120^{\circ} = 180^{\circ}$

$\angle Q = 180^{\circ} - 120^{\circ}$

$\angle Q = 60^\circ$

Hence, the value of $\angle SQT = 60^\circ$.

30. In figure, if $QT \perp PR, \angle TQR = 40^\circ$ and $\angle SPR = 50^\circ$ find x and y.



Ans: According to the ΔTQR

$90^\circ + 40^\circ + x = 180^\circ$ [Angle sum property of triangle]

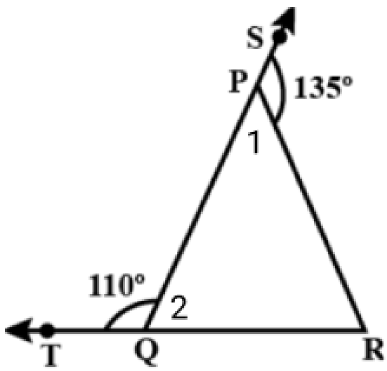
So, $x = 50^\circ$

Now, $y = \angle SPR + x$

So, $y = 30^\circ + 50^\circ = 80^\circ$.

Hence, the value of $x = 50^\circ$ and $y = 80^\circ$.

31. In figure sides QP and RQ of ΔPQR are produced to points S and T respectively if $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.



Ans: According to the given figure,

$110^{\circ}+\angle 2=180^{\circ}$ [By linear pair]

$\angle 2=180^{\circ}-110^{\circ}$

$\angle 2=70^{\circ}$

$\angle 1+135^{\circ}=180^{\circ}$

$\angle 1=180^{\circ}-135^{\circ}$

$\angle 1=45^{\circ}$

$\angle 1+\angle 2+\angle \mathrm{R}=180^{\circ}$ [By angle sum property]

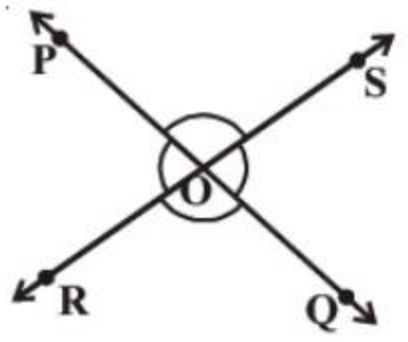
$45^{\circ}+70^{\circ}+\angle R=180^{\circ}$

$\angle \mathrm{R}=180^{\circ}-115^{\circ}$

$\angle \mathrm{R}=65^{\circ}$

Hence, the value of $\angle \mathrm{PRQ}=65^{\circ}$.

32. In figure lines PQ and RS intersect each other at point O. If $\angle POR:\angle ROQ=5:7$. Find all the angles.



Ans: $\angle POR + \angle ROQ = 180^{\circ}$ [Linear pair of angle]

But, $\angle \text{POR} : \angle \text{ROQ} = 5:7$ [According to the question]

So, $\angle \text{POR} = \frac{5}{12} \times 180^{\circ} = 75^{\circ}$

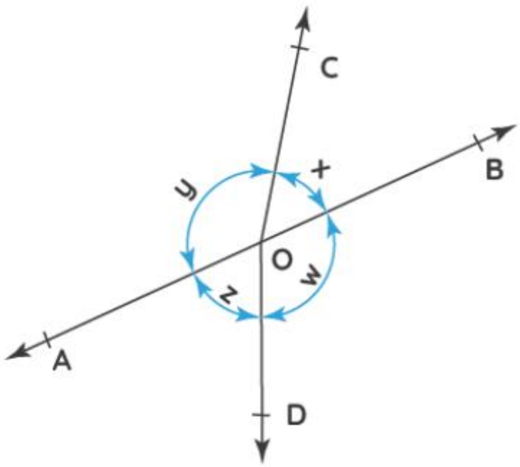
Similarly, $\angle \text{ROQ} = \frac{7}{12} \times 180^{\circ} = 105^{\circ}$

Now, $\angle \text{POS} = \angle \text{ROQ} = 105^{\circ}$ [Vertically opposite angle]

And $\angle \text{SOQ} = \angle \text{POR} = 75^{\circ}$ [Vertically app angle]

Short Answer Type Questions **3 Mark**

1. In Fig. 6.16, if $x + y = w + z$, then prove that AOB is a line.



Ans: As we need to prove that AOB is a line.

According to the question, given that $x+y=w+z$.

As we know that the sum of all the angles around a fixed point is 360°.

Hence, we can determine that $\angle AOC+\angle BOC+\angle AOD+\angle BOD=360^\circ$, or

$y+x+z+w=360^\circ$

But, $x+y=w+z$ (According to the question).

$2(y+x)=360^\circ$

$y+x=180^\circ$

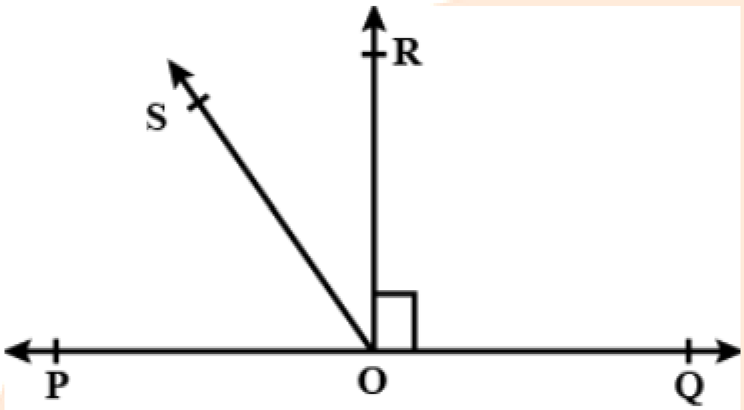
According to the given figure, we can determine that y and x form a linear pair.

As we also know that if a ray stands on a straight line, then the sum of the angles of linear pair formed by the ray with respect to the line is 180°.

$y+x=180^\circ$.

Hence, we can determine that AOB is a line.

2. In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Ans: As we need to prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.

According to the question, given that OR is perpendicular to PQ, or $\angle QOR = 90^\circ$.

According to the given figure, we can determine that $\angle POR$ and $\angle QOR$ form a linear pair.

As we also know that sum of the angles of a linear pair is 180°.

So, $\angle POR + \angle QOR = 180^\circ$, or $\angle POR = 90^\circ$

According to the figure, we can determine that $\angle POR = \angle POS + \angle ROS$.

$\Rightarrow \angle POS + \angle ROS = 90^\circ$, or

$\angle ROS = 90^\circ - \angle POS \cdot (i)$

According to the given figure, we can determine that $\angle QOS$ and $\angle POS$ form a linear pair.

As we also know that sum of the angles of a linear pair is 180°.

$\angle QOS + \angle POS = 180^\circ$, or

$\frac{1}{2}(\angle QOS + \angle POS) = 90^\circ . (ii)$

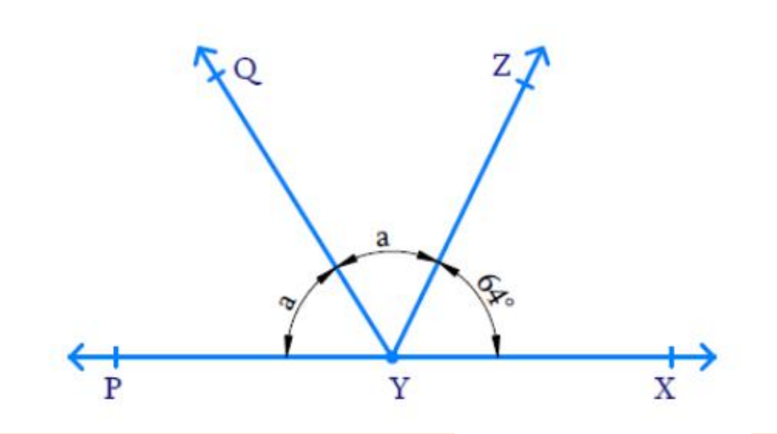
Substitute tha value of equation (ii) in equation (i), to get

$$\angle ROS = \frac{1}{2}(\angle QOS + \angle POS) - \angle POS$$

$$= \frac{1}{2}(\angle QOS - \angle POS)$$

Hence proved.

3. It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.



Ans: According to the question, given that $\angle XYZ = 64^\circ, XY$ is produced to P and YQ bisects $\angle ZYP$.

As we can determine the given below figure for the given situation:

As we need to find $\angle XYQ$ and reflex $\angle QYP$.

According to the given figure, we can determine that $\angle XYZ$ and $\angle ZYP$ form a linear pair.

As we also know that sum of the angles of a linear pair is 180°.

$$\angle XYZ + \angle ZYP = 180^\circ$$

But, $\angle XYZ = 64^\circ$.

$\Rightarrow 64^\circ + \angle ZYP = 180^\circ$

$\Rightarrow \angle ZYP = 116^\circ$.

Ray YQ bisects $\angle ZYP$, or

$\angle QYZ = \angle QYP = \frac{116'}{2} = 58^\circ$

$\angle XYQ = \angle QYZ + \angle XYZ$

$= 58^\circ + 64^\circ = 122^\circ$.

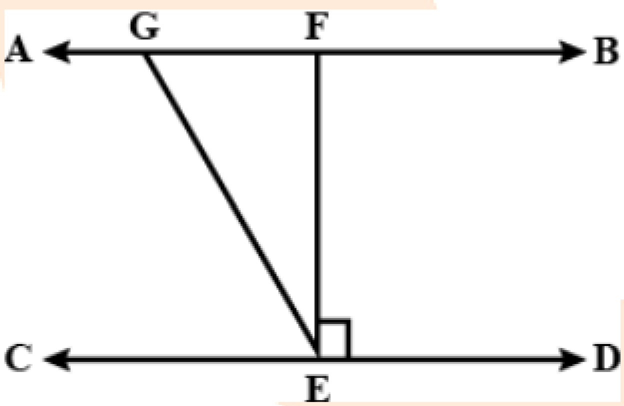
Reflex $\angle QYP = 360^\circ - \angle QYP$

$= 360^\circ - 58^\circ$

$= 302^\circ$

Hence, we can determine that $\angle XYQ = 122^\circ$ and reflex $\angle QYP = 302^\circ$.

4. In the given figure, If $AB \parallel CD$ **,** $EF \perp CD$ **and** $\angle GED = 126^\circ$**, find** $\angle AGE, \angle GEF$ **and** $\angle FGE$ **.**



Ans: According to the question, given that $AB \parallel CD, EF \perp CD$ and $\angle GED = 126^\circ$.

As we need to find the value of $\angle AGE, \angle GEF$ and $\angle FGE$ in the figure given below.

$\angle GED = 126^\circ$

$\angle GED = \angle FED + \angle GEF$

But, $\angle FED = 90^\circ$.

$126^\circ = 90^\circ + \angle GEF \Rightarrow \angle GEF = 36^\circ$

Because, $\angle AGE = \angle GED$ (Alternate angles)

$\angle AGE = 126^\circ$.

According to the given figure, we can determine that $\angle FED$ and $\angle FEC$ form a linear pair.

As we know that sum of the angles of a linear pair is 180°.

$\angle FED + \angle FEC = 180$

$\Rightarrow 90^\circ + \angle FEC = 180^\circ$

$\Rightarrow \angle FEC = 90^\circ$

But $\angle FEC = \angle GEF + \angle GEC$

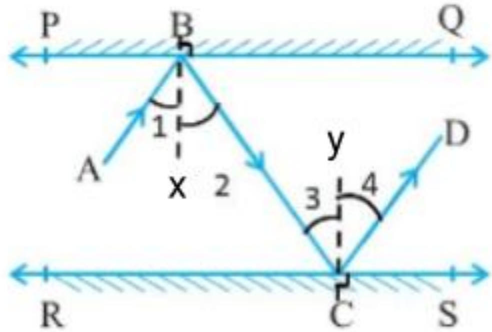
So, $90^\circ = 36^\circ + \angle GEC$

$\Rightarrow \angle GEC = 54^\circ$.

$\angle GEC = \angle FGE = 54^\circ$ (Alternate interior angles)

Hence, we can determine that $\angle AGE = 126^\circ, \angle GEF = 36^\circ$ and $\angle FGE = 54^\circ$.

5. In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $\mathrm{AB} \parallel \mathrm{CD}$.



Ans: According to the question, given that PQ and RS are two mirrors that are parallel to each other.

As we need to prove that $AB \parallel CD$ in the given figure.

Now we draw lines BX and CY that are parallel to each other, to get

As we also know that according to the laws of reflection

$\angle ABX = \angle CBX$ and $\angle BCY = \angle DCY$.

$\angle BCY = \angle CBX$ (Alternate interior angles)

As we can determine that $\angle ABX = \angle CBX = \angle BCY = \angle DCY$.

According to the figure, we can determine that

$\angle ABC = \angle ABX + \angle CBX$, and

$\angle DCB = \angle BCY + \angle DCY$.

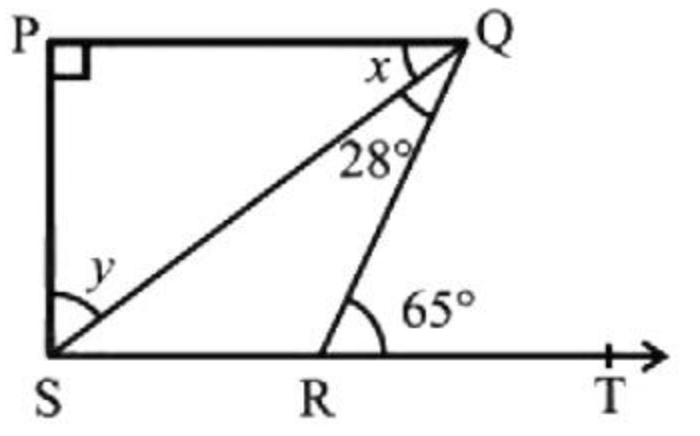
Hence, we can determine that $\angle ABC = \angle DCB$.

According to the figure, we can determine that $\angle ABC$ and $\angle DCB$ form a pair of alternate interior angles corresponding to the lines AB and CD, and transversal BC.

Hence, we can determine that $AB \parallel CD$.

6. In the given figure, if $PQ \perp PS, PQ \parallel$ SR, $\angle SQR = 28^{\circ}$ and $\angle QRT = 65^{\circ}$, then find

the values of x and y.



Ans: According to the question, given that $PQ \perp PS, PQ \parallel SR, \angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$.

As we need to find the values of x and y in the figure.

As we know that "If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles."

According to the figure, we can determine that

$\angle SQR + \angle QSR = \angle QRT$, or

$28^\circ + \angle QSR = 65^\circ$

$\Rightarrow \angle QSR = 37^\circ$

According to the figure, we can determine that

$x = \angle QSR = 37^\circ$ (Alternate interior angles)

According to the figure, we can determine that ΔPQS

$\angle PQS + \angle QSP + \angle QPS = 180^\circ$. (Angle sum property)

$\angle QPS = 90^\circ \quad (PQ \perp PS)$

$x + y + 90^\circ = 180^\circ$

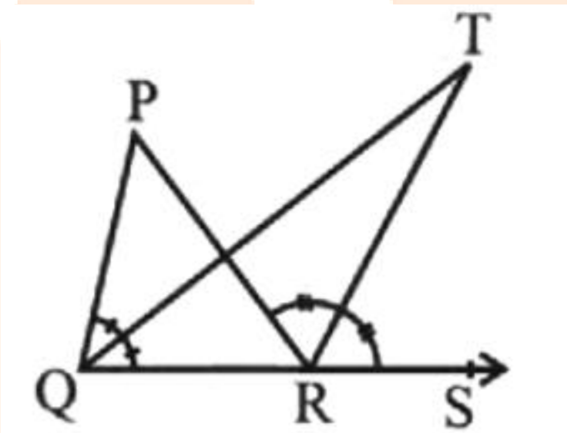
$\Rightarrow x + 37^\circ + 90^\circ = 180^\circ$

$\Rightarrow x + 127^\circ = 180^\circ$

$\Rightarrow x = 53^\circ$

Hence, we can determine that $x = 53^\circ$ and $y = 37^\circ$.

7. In the given figure, the side QR of Δ PQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.



Ans: As we need to prove that $\angle QTR = \frac{1}{2}\angle QPR$ in the figure given below.

As we also know that "If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles."

According to the figure, we can determine that in $\Delta QTR, \angle TRS$ is an exterior angle

$\angle QTR + \angle TQR = \angle TRS$, or

$\angle QTR = \angle TRS - \angle TQR$ ……….(i)

According to the figure, we can determine that in $\Delta QTR, \angle TRS$ is an exterior angle

$\angle QPR + \angle PQR = \angle PRS$

According to the question given that QT and RT are angle bisectors of $\angle PQR$ and $\angle PRS$.

$\angle QPR + 2\angle TQR = 2\angle TRS$

$\angle QPR = 2(\angle TRS - \angle TQR)$

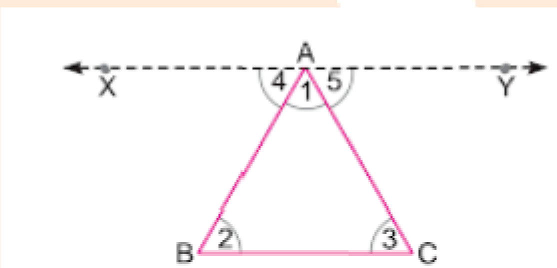
As we need to substitute the value of equation (i) in the above equation, to get

$\angle QPR = 2\angle QTR$, or

$\angle QTR = \frac{1}{2}\angle QPR$

Hence, we can determine that the desired result is proved.

8. Prove that sum of three angles of a triangle is 180°



Ans: According to the question given that, $\triangle ABC$

To prove that, $\angle A + \angle B + \angle C = 180^{\circ}$

Now we draw $XY \parallel BC$ through point A.

Proof: Because, $XY \parallel BC$

So, $\angle 2 = \angle 4 \rightarrow (1)$

Because, Altemate interior angle

And $\angle 3 = \angle 5 \rightarrow (2)$

Now we adding the equation (1) and equation (2)

$\angle 2+\angle 3=\angle 4+\angle 5$

Adding both sides $\angle 1$,

$\angle 1+\angle 2+\angle 3=\angle 1+\angle 4+\angle 5$

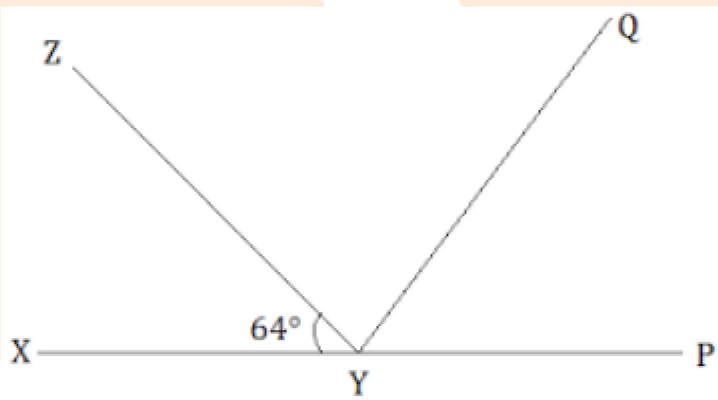
$\angle 1+\angle 2+\angle 3=180^{\circ}$ (Because, $\angle 1, \angle 4$, and $\angle 5$ forms a line)

$\angle A+\angle B+\angle C=180^{\circ}$

9. It is given that $\angle XYZ=64^{\circ}$ and XY is produced to point P, draw a fig from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Ans: As we know that,

$\angle XYZ+\angle PYZ=180^{\circ}$ (linear pair)



$\Rightarrow 64^{\circ}+\angle PYZ=180^{\circ}$ (According to the question given that, $\angle XYZ=64^{\circ}$)

$\angle PYZ=180^{\circ}-64^{\circ}$

$\angle PYZ=116^{\circ}$

$\angle ZYQ=\frac{1}{2}\angle ZYP=\frac{116}{2}=58^{\circ}$

$\angle XYQ=\angle XYZ+\angle ZYQ$

$=64^{\circ}+58^{\circ}=122^{\circ}$

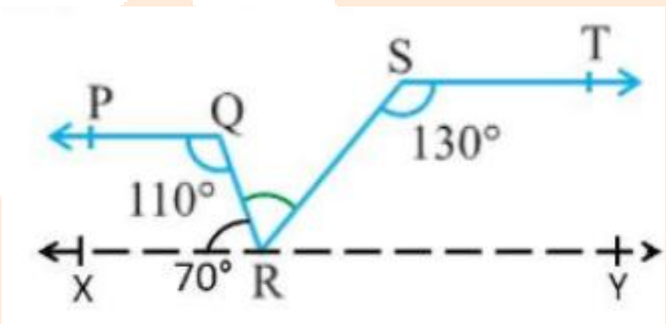
Also reflex $\angle QYP=\angle XYQ+$ straight $\angle XYP$

$=122^\circ+180^\circ$

$=302^\circ$

Hence, the value of $\angle XYQ=122^\circ$ and $\angle QYP=302^\circ$.

10. In fig if $PQ \parallel ST, \angle PQR=110^\circ$ **and** $\angle RST=130^\circ$ **find** $\angle QRS$**.**



Ans: Through point R Draw line Kleist

Because, $PQ \parallel ST$

$ST \parallel KL$, So, $PQ \parallel KL$

Because, $PQ \parallel KL$

So, $\angle PQR+\angle 1=180^\circ$

(As we know that the sum of interior angle on the same side of transversal is 180°)
$110^\circ+\angle 1=180^\circ$

$\angle 1=70^\circ$

Similarly $\angle 2+\angle RST=180^\circ$

$\angle 2+130^\circ=180^\circ$

$\angle 2=50^\circ$

$\angle 1+\angle 2+\angle 3=180^\circ$

$70^\circ+50^\circ+\angle 3=180^\circ$

$\angle 3 = 180^\circ - 120^\circ$

$\angle 3 = 60^\circ$

Hence, the value of $\angle QRS = 60^\circ$.

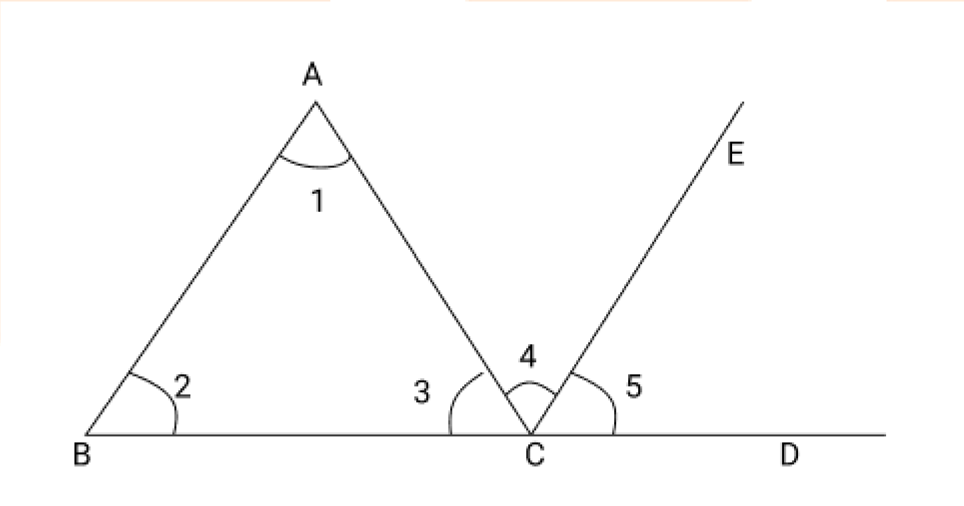
11. The side BC of $\triangle ABC$ is produced from ray BD. CE is drawn parallel to AB, show that $\angle ACD = \angle A + \angle B$. Also prove that $\angle A + \angle B + \angle C = 180^\circ$.

Ans: Because, $AB \parallel CE$ and AC intersect them

$\angle 1 = \angle 4$ ………. (1) [Alternate interior angles]

Also $AB \parallel CE$ and BD intersect them

$\angle 2 = \angle 5$ …………. (2) [Corresponding angles]



Now adding equation (1) and equation (2)

$\angle 1 + \angle 2 = \angle 4 + \angle 5$

$\angle A + \angle B = \angle ACD$

Adding $\angle C$ on both sides, we get

$\angle A + \angle B + \angle C = \angle C + \angle ACD$

$\angle A + \angle B + \angle C = 180^\circ$

Hence, proved.

12. Prove that if a transversal intersect two parallel lines, then each pair of alternate interior angles is equal.

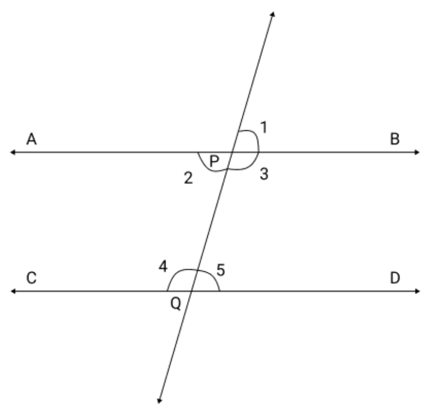
Ans: According to the question given that, line $AB \parallel CD$ intersected by transversal PQ

To Prove: (i) $\angle 2 = \angle 5$ (ii) $\angle 3 = \angle 4$

Proof:

$\angle 1 = \angle 2$ ………… (i) [Vertically Opposite angle]

$\angle 1 = \angle 5$ …………... (ii) [Corresponding angles]



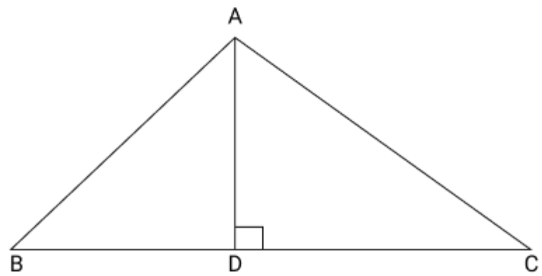
By equations (i) and (ii)

$\angle 2 = \angle 5$

Similarly, $\angle 3 = \angle 4$

Hence Proved.

13. In the given figure ΔABC is right angled at A. AD is drawn perpendicular to BC. Prove that $\angle BAD = \angle ACB$



Ans: According to the figure,

$AD \perp BC$

So, $\angle ADB = \angle ADC = 90^\circ$

From $\triangle ABD$

$\angle ABD + \angle BAD + \angle ADB = 180^\circ$

$\angle ABD + \angle BAD + 90^\circ = 180^\circ$

$\angle ABD + \angle BAD = 90^\circ$

$\angle BAD = 90^\circ - \angle ABD \rightarrow (1)$

But $\angle A + \angle B + \angle C = 180^\circ$ in $\triangle ABC$

$\angle B + \angle C = 90^\circ$, Because, $\angle A = 90^\circ$

$\angle C = 90^\circ - \angle B \rightarrow (2)$

From equations (1) and (2)

$\angle BAD = \angle C$

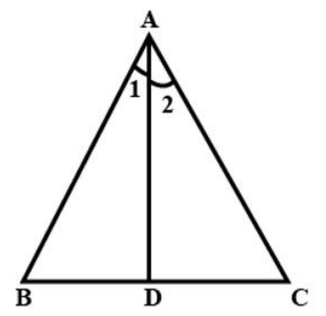
$\angle BAD = \angle ACB$

Hence proved.

14. In $\Delta ABC \angle B=45^{\circ}, \angle C=55^{\circ}$ and bisector $\angle A$ meets BC at a point D. Find $\angle ADB$ and $\angle ADC$

Ans: In ΔABC

$\angle A+\angle B+\angle C=180^{\circ}$ [As we know that the sum of three angle of a Δ is 180°]



$\Rightarrow \angle A+45^{\circ}+55^{\circ}=180^{\circ}$

$\angle A=180^{\circ}-100^{\circ}=80^{\circ}$

AD bisects $\angle A$

$\angle 1=\angle 2=\frac{1}{2}\angle A=\frac{1}{2}\times 80^{\circ}=40^{\circ}$

Now in ΔADB,

We have, $\angle 1+\angle B+\angle ADB=180^{\circ}$

$\Rightarrow 40^{\circ}+45^{\circ}+\angle ADB=180^{\circ}$

$\Rightarrow \angle ADB=180^{\circ}-85^{\circ}=95^{\circ}$

$\angle ADB+\angle ADC=180^{\circ}$

Also $95^{\circ}+\angle ADC=180^{\circ}$

$\angle ADC=180^{\circ}-95^{\circ}=85^{\circ}$

Hence, the value of $\angle ADB=95^{\circ}$ and $\angle ADC=85^{\circ}$

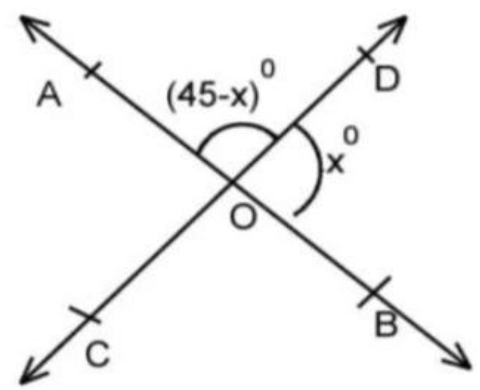
15. In figure two straight lines AB and CD intersect at a point 0 . If $\angle BOD = x^{\circ}$ and $\angle AOD = (45 - x)^{\circ}$. Find the value of x hence find

(a) $\angle BOD$

(b) $\angle AOD$

(c) $\angle AOC$

(d) $\angle BOC$



Ans:

$\angle ADB = \angle AOD + \angle DOB$ By linear pair

$180^{\circ} = 4x - 5 + x$

$180^{\circ} + 5 = 5x$

$5x = 185$

$x = \frac{185}{5} = 37^{\circ}$

So, $\angle AOD = 4x - 5$

$= 4 \times 37 - 5 = 148 - 5$

$= 143^{\circ}$

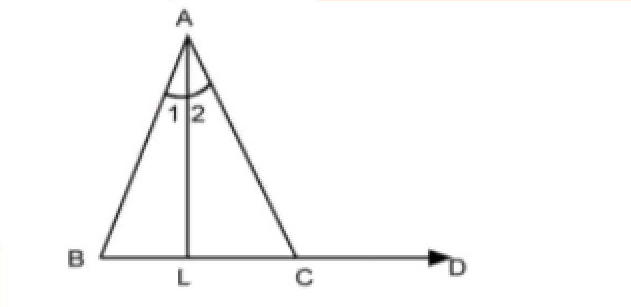
$\angle BOC = 143^{\circ}$

Because, $\angle AOD$ and $\angle BOC$

$\angle BOD = x = 37^\circ$ [Vertically opposite angles]

$\angle BOD = \angle AOC = 37^\circ$

16. The side BC of a ΔABC is produced to D. the bisector of $\angle A$ meets BC at L as shown if fig. prove that $\angle ABC + \angle ACD = 2\angle ALC$



Ans: In ΔABC we have

$\angle ACD = \angle B + \angle A \rightarrow (1)$ [Exterior angle property]

$\Rightarrow \angle ACD = \angle B + 2L1$

[So, $\angle A$ is the bisector of $\angle A = 2L1$]

In ΔABL

$\angle ALC = \angle B + \angle BAL$ [Exterior angle property]

$\angle ALC = \angle B + \angle 1$

$\Rightarrow 2\angle ALC = 2\angle B + 2\angle 1 \ldots (2)$

Subtracting equation (1) from equation (2)

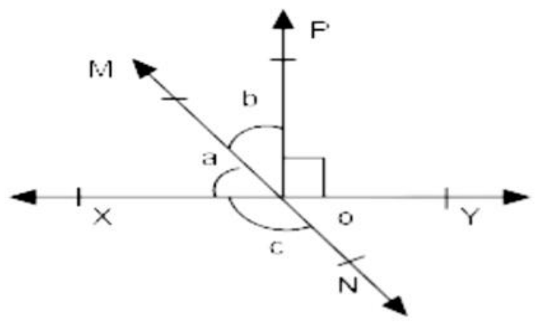
$2\angle ALC - \angle ACD = \angle B$

$2\angle ALC = \angle B + \angle ACD$

$\angle ACD + \angle ABC = 2\angle ALC$

Hence proved.

17. In fig lines XY and MN intersect at O If $\angle POY = 90^\circ$ and a: $b = 2:3$ find $\angle C$



Ans: Lines XY and MN intersect at O.

So, $\angle C = \angle XON = \angle MOY$ [Vertically opposite angle]

$= \angle b + \angle POY$

But,

$\angle POY = 90^\circ$

So, $\angle C = \angle b + 90^\circ \rightarrow (1)$

Also,

$\angle POX = 180^\circ - \angle POY$

Put the value of $\angle POY$ in the above equation.

$= 180^\circ - 90^\circ$

$= 90^\circ$

So, $a + b = 90^\circ$

But,

$a : b = 2:3$ [According to the question]

$a = \frac{2}{5} \times 90^\circ$

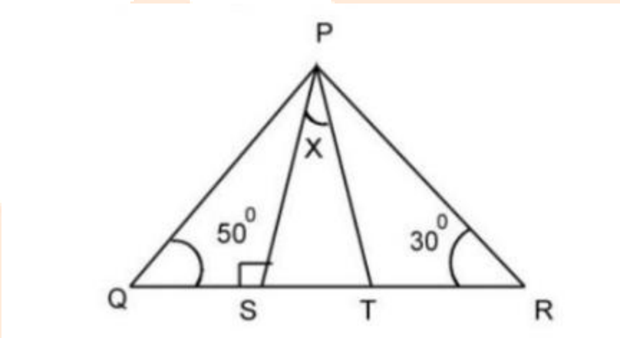
$= 36^\circ \rightarrow (2)$ From equation (1) and equation (2) we get

$b = 90^\circ - 36^\circ = 54^\circ$

$\angle C = 54^\circ + 90^\circ$

Hence, the value of $\angle C = 144^\circ$.

18. In fig PT is the bisector of $\angle QPR$ in ΔPQR and $PS \perp QR$, find the value of x



Ans: $\angle QPR + \angle Q + \angle R = 180^\circ$ [According to the angle sum property of triangle]

$\angle QPR = 180^\circ - 50^\circ - 30^\circ = 100^\circ$

$\angle QPT = \frac{1}{2}\angle QPR$

$= \frac{1}{2} \times 100^\circ = 50^\circ$

$\angle Q + \angle QPS = \angle PST$ [Exterior angle theorem]

$\angle QPS = 90^\circ - \angle Q$

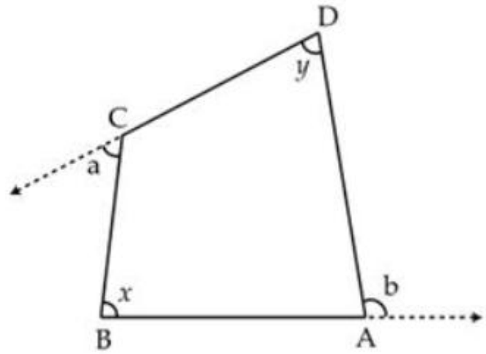
$= 90^\circ - 50^\circ = 40^\circ$

$x = \angle QPT - \angle QPS$

$= 50^\circ - 40^\circ = 10^\circ$

Hence, the value of $x = 10^\circ$.

19. The sides BA and DC of a quadrilateral $ABCD$ are produced as shown in fig show that $\angle X + \angle Y = \angle a + \angle b$



Ans: In given figure join BD

In ΔABD

$\angle b = \angle ABD + \angle BDA$ [Exterior angle theorem]

In ΔCBD

$\angle a = \angle CBD + \angle BDC$

$\angle a + \angle b = \angle CBD + \angle BDC + \angle ABD + \angle BDA$

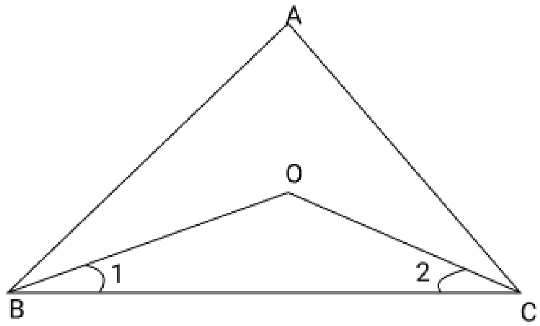
$= (\angle CBD + \angle ABD) + (\angle BDC + \angle BDA)$

$= \angle x + \angle y$

$\angle a + \angle b = \angle x + \angle y$

Hence proved.

20. In the BO and CO are Bisectors of $\angle \mathrm{B}$ and $\angle \mathrm{C}$ of $\Delta \mathrm{ABC}$, show that $\angle \mathrm{BOC} = 90^{\circ} + \frac{1}{2} \angle \mathrm{A}$



Ans: According to the given figure,

$\angle 1 = \frac{1}{2}\angle ABC$

And $\angle 2 = \frac{1}{2}\angle ACB$

So, $\angle 1 + \angle 2 = \frac{1}{2}(\angle ABC + \angle ACB)$ ......(1)

But,

$\angle ABC + ACB + \angle A = 180^{\circ}$

So, $\angle ABC + ACB = 180^{\circ} - \angle A$

But,

$\frac{1}{2}[\angle ABC + ACB] = 90^{\circ} - \frac{1}{2}\angle A$ ....(2)

From equation (1) and equation (2) we get

$\angle 1 + \angle 2 = 90^{\circ} - \frac{1}{2}\angle A$ ....(3)

But,

$\angle BOC + \angle 1 + \angle 2 = 180^{\circ}$ [Angle of a]

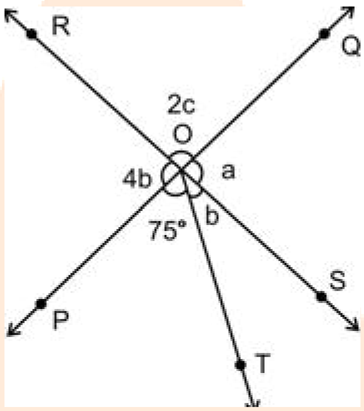
Put the value of $\angle 1 + \angle 2$ in the above equation,

$$=180^\circ-\left(90^\circ-\frac{1}{2}\angle A\right)$$

$$=90^\circ+\frac{1}{2}\angle A$$

Hence proved.

21. In fig two straight lines PQ and RS intersect each other at o, if $\angle POT=75^\circ$ Find the values of a, b and c



Ans: PQ intersect RS at O

So, $\angle QOS=\angle POR$ [vertically opposite angles]

$A=4b$ ……..(1)

Also,

$a+b+75^\circ=180^\circ$ [Because, POQ is a straight lines]

So, $a+b=180^\circ-75^\circ$

$=105^\circ$

Using, equation (1) $4b+b=105^\circ$

$5b=105^\circ$

Or

$b=\frac{105}{5}=21^{\circ}$

So, $a=4b$

$a=4\times 21$

$a=84$

Again, $\angle QOR$ and $\angle QOS$ form a linear pair

So, $a+2c=180^{\circ}$

Using, equation (2)

$84^{\circ}+2c=180^{\circ}$

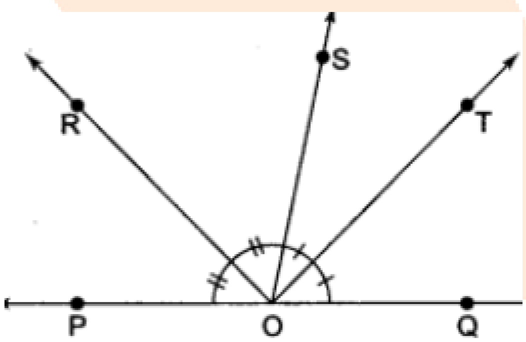
$2c=180^{\circ}-84^{\circ}$

$2c=96^{\circ}$

$c=\frac{96^{\circ}}{2}=48^{\circ}$

Hence, $a=84^{\circ}, b=21^{\circ}$ and $c=48^{\circ}$

22. In figure ray OS stands on a line POQ, ray OR and ray OT are angle bisector of $\angle POS$ and $\angle SOQ$ respectively. If $\angle POS=x$, find $\angle ROT$.



Ans: Ray OS stands on the line POQ

So, $\angle \mathrm{POS}+\angle \mathrm{SOQ}=180^{\circ}$

But $\angle POS = X$

So, $x + \angle SOQ = 180^\circ$

$\angle SOQ = 180^\circ - X$

Now ray OR bisects $\angle POS$,

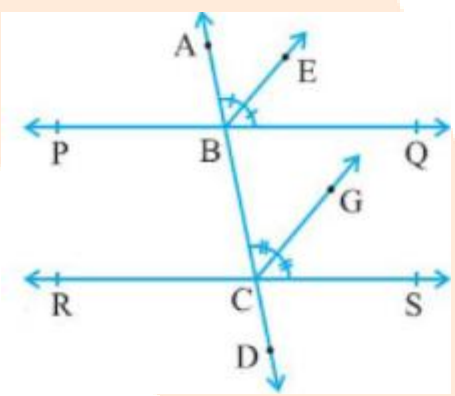
Hence, $\angle ROS = \frac{1}{2} \times \angle POS = \frac{1}{2} \times x = \frac{x}{2}$

Similarly, $\angle SOT = \frac{1}{2} \times \angle SOQ = \frac{1}{2} \times \left(180^\circ - X\right) = 90^\circ - \frac{x}{2}$

$\angle ROT = \angle ROS + \angle SOT = \frac{x}{2} + 90^\circ - \frac{x}{2} = 90^\circ$

Hence, the value of $\angle ROT = 90^\circ$.

23. If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.



Ans: According to the question and figure given that, AD is transversal intersect two lines PQ and RS To prove $PQ \parallel RS$

Proof: BE bisects $\angle ABQ$ $\angle = \frac{1}{2} \angle ABQ \to (1)$

Similarity CG bisects $\angle BCS$

So, $\angle 2 = \frac{1}{2}\angle BCS \rightarrow (2)$

But $\mathrm{BE} \parallel \mathrm{CG}$ and AD is the transversal

So, $\angle 1 = \angle 2$

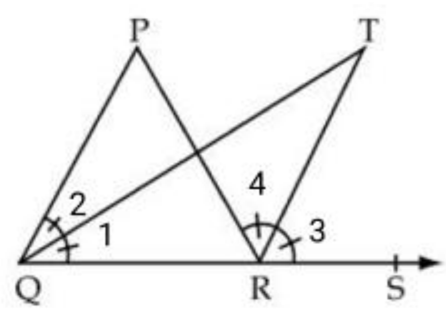
So, $\frac{1}{2}\angle ABQ = \frac{1}{2}\angle BCS$ [By equations (1) and (2)]

$\Rightarrow \angle \mathrm{ABQ} = \angle \mathrm{BCS}$ [Because corresponding angles are equal]

So, $\mathrm{PQ} \parallel \mathrm{RS}$

Hence proved.

24. In figure the sides QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T. Then prove that $\angle QRT = \frac{1}{2}\angle QPR$



Ans: Solution, In $\Delta\mathrm{PQR}$

$\angle \mathrm{PRS} = \angle \mathrm{Q} + \angle \mathrm{P}$ [By exterior angle theorem]

$\angle 4 + \angle 3 = \angle 2 + \angle 1 + \angle \mathrm{P}$

$2\angle 3 = 2\angle 1 + \angle \mathrm{P} \rightarrow (1)$

So, QT and RT are bisectors of $\angle Q$ and $\angle PRS$

In $\triangle QTR$,

$\angle 3 = \angle 1 + \angle T \rightarrow$ (2) [By exterior angle theorem]

By equations (1) and (2) we get

$2[\angle 1 + \angle T] = 2\angle 1 + \angle P$

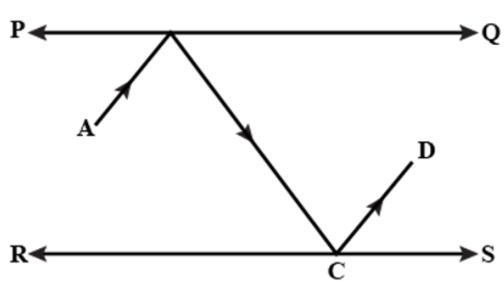
$2\angle 1 + 2\angle T = 2\angle 1 + \angle P$

$\angle T = \frac{1}{2}\angle P$

$\angle QTR = \frac{1}{2}\angle QPR$

Hence proved.

25. In figure PQ and RS are two mirror placed parallel to each other. An incident ray AB striker the mirror PQ at B, the reflected ray moves along the path BC and strike the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.



Ans: Solution, Draw $MB \perp PQ$ and $NC \perp RS$

$\angle 1 = \angle 2 \rightarrow (1)$ [Angle of incident]

And $\angle 3 = \angle 4 \rightarrow (2)$ [is equal to angle of reflection]

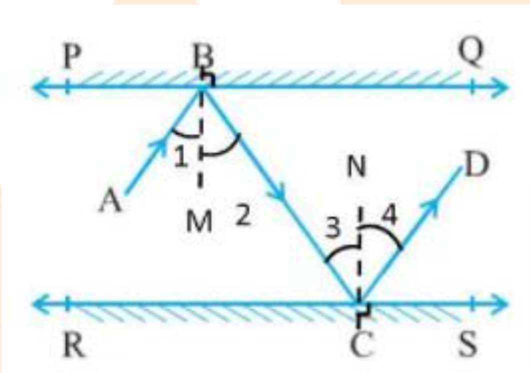
Because, $\angle MBQ = \angle NCS = 90^{\circ}$

So, $MB \parallel NC$ [By corresponding angle property]

Because, $\angle 2 = \angle 3 \rightarrow (3)$ [Alternate interior angle]

By equations $(1), (2)$ and (3)

$\angle 1 = \angle 4$



$\angle 1 + \angle 2 = \angle 4 + \angle 3$

$\Rightarrow \angle ABC = \angle BCD$

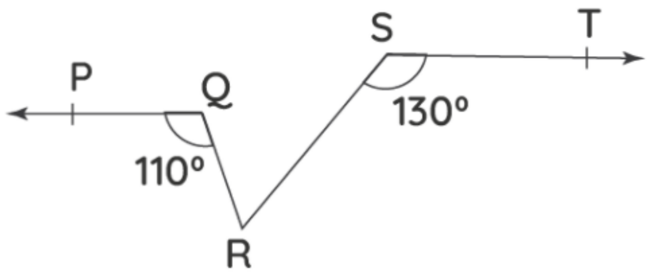
So, $AB \parallel CD$ [By alternate interior angles]

Hence proved.

Long Answer Type Questions **4 Mark**

1. In the given figure, if $PQ \parallel ST, \angle PQR = 110^{\circ}$ and $\angle RST = 130^{\circ}$, find $\angle QRS$.

[Hint: Draw a line parallel to ST through point R.]



Ans: According to the question, given that $PQ \parallel ST, \angle PQR = 110^\circ$ and $\angle RST = 130^\circ$.

As we need to find the value of $\angle QRS$ in the figure.

As we need to draw a line RX that is parallel to the line ST, to get

Therefore, we have $ST \parallel RX$.

As we also know that lines parallel to the same line are also parallel to each other.

As we can determine that $PQ \parallel ST \parallel RX$.

$\angle PQR = \angle QRX$ (Alternate interior angles), or

$\angle QRX = 110^\circ$

As we also know that angles on same side of a transversal are supplementary.

$\angle RST + \angle SRX = 180^\circ$

$\Rightarrow 130^\circ + \angle SRX = 180^\circ$

$\Rightarrow \angle SRX = 180^\circ - 130^\circ = 50^\circ.$

According to the figure, we can determine that

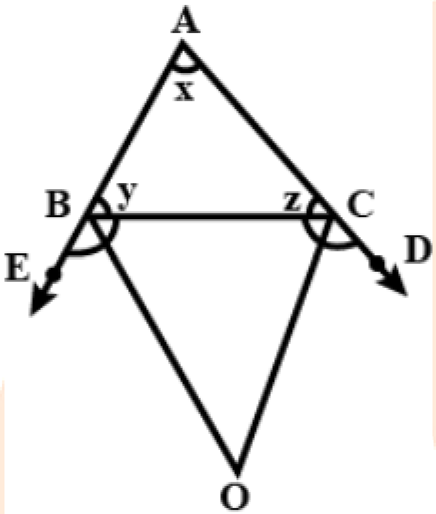
$\angle QRX = \angle SRX + \angle QRS$

$\Rightarrow 110^\circ = 50^\circ + \angle QRS$

$\Rightarrow \angle QRS = 60^\circ$.

Hence, we can determine that $\angle QRS = 60^\circ$.

2. In fig the side AB and AC of $\triangle ABC$ are produced to point E and D respectively. If bisector BO And CO of $\angle \mathrm{CBE}$ and $\angle \mathrm{BCD}$ respectively meet at point O, then prove that $\angle \mathrm{BOC} = 90^{\circ} - \frac{1}{2}\angle \mathrm{BAC}$



Ans: Ray BO bisects $\angle \mathrm{CBE}$

So, $\angle \mathrm{CBO} = \frac{1}{2}\angle \mathrm{CBE}$

$= \frac{1}{2}\left(180^{\circ} - y\right)$ $\left(\text{Because, } \angle \mathrm{CBE} + \mathrm{y} = 180^{\circ}\right)$

$= 90^{\circ} - \frac{y}{2}$........(1)

Similarly, ray CO bisects $\angle BCD$

$\angle \mathrm{BCO} = \frac{1}{2}\angle BCD$

$= \frac{1}{2}\left(180^{\circ} - Z\right)$

$= 90^{\circ} - \frac{Z}{2}$..........(2)

In $\triangle \mathrm{BOC}$

$\angle \mathrm{BOC} + \angle \mathrm{BCO} + \angle \mathrm{CBO} = 180^{\circ}$

$\angle BOC = \frac{1}{2}(y+z)$

But $x+y+z=180^\circ$

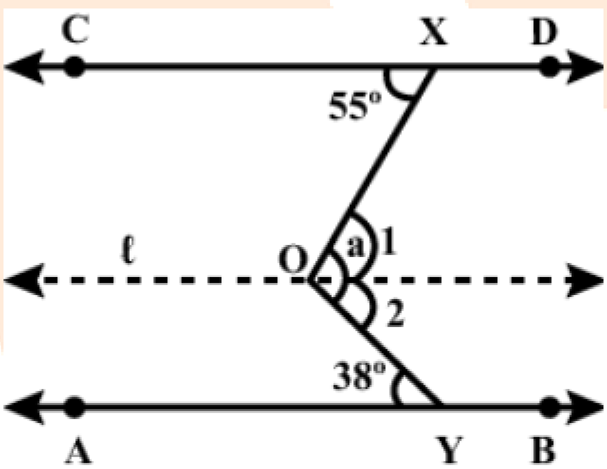
$y+z=180^\circ - x$

$\angle BOC = \frac{1}{2}\left(180^\circ - x\right) = 90^\circ - \frac{x}{2}$

$\angle BOC = 90^\circ - \frac{1}{2}\angle BAC$

Hence proved.

3. In given fig. AB $\parallel$ CD. Determine $\angle a$.



Ans: Through O draw a line l parallel to both AB and CD

Clearly

$\angle a = \angle 1 + \angle 2$

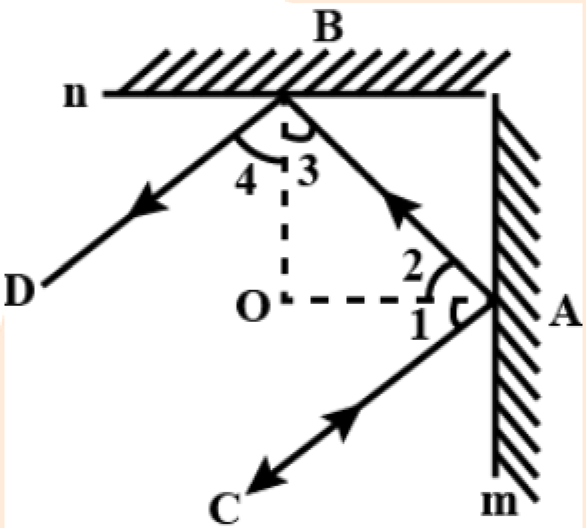
$\angle 1 = 38^\circ$

$\angle 2 = 55^\circ$ [Alternate interior angles]

$\angle a = 55^\circ + 38^\circ$

Hence, the value of $\angle a = 93^{\circ}$.

4. In fig M and N are two plane mirrors perpendicular to each other; prove that the incident ray CA is parallel to reflected ray BD.



Ans: Now we draw $\mathrm{AP} \perp \mathrm{M}$ and $\mathrm{BQ} \perp \mathrm{N}$

So, $BQ \perp N$ and $AP \perp M$ and $\mathrm{M} \perp \mathrm{N}$

So, $\angle \mathrm{BOA} = 90^{\circ}$

$\Rightarrow \mathrm{BQ} \perp \mathrm{AP}$

In $\Delta\mathrm{BOA} \angle 2 + \angle 3 + \angle \mathrm{BOA} = 180^{\circ}$ [By angle sum property]

$\Rightarrow \angle 2 + \angle 3 + 90^{\circ} = 180^{\circ}$

So, $\angle 2 + \angle 3 = 90^{\circ}$

Also $\angle 1 = \angle 2$ and $\angle 4 = \angle 3$

$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3 = 90^{\circ}$

So, $(\angle 1 + \angle 4) + (\angle 2 + \angle 3) = 90^{\circ} + 90^{\circ} = 180^{\circ}$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180^{\circ}$

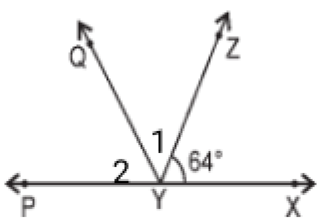
or $\angle \mathrm{CAB} + \angle \mathrm{DBA} = 180^{\circ}$

So, $CA \parallel BD$ [By sum of interior angles of same side of transversal]

Hence proved.

5. It is given that $\angle XYZ = 64^{\circ}$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$. Find $\angle XYQ$ and reflex $\angle QYP$.

Ans:



According to the figure,

YQ bisects $\angle ZYP$

So, $\angle 1 = \angle 2$

$\angle 1 + \angle 2 + \angle 64^{\circ} = 180^{\circ}$[YX is a line]

$\angle 1 + \angle 1 + 64^{\circ} = 180^{\circ}$

$2\angle 1 = 180^{\circ} - 64^{\circ}$

$2\angle 1 = 116^{\circ}$

$\angle 1 = 58^{\circ}$

So, $\angle XYQ = 64^{\circ} + 58^{\circ} = 122^{\circ}$

$\angle 2 + \angle XYQ = 180^{\circ}$

$\angle 1 = \angle 2 = \angle QYP = 58^{\circ}$

$\angle 2 + 122^{\circ} = 180^{\circ}$

$\angle 2 = 180^\circ - 122^\circ$

$\angle QYP = \angle 2 = 58^\circ$

Reflex $\angle QYP = 360^\circ - \angle QYP$

$= 360^\circ - 58^\circ$

$= 302^\circ$

Hence, the value of $\angle XYQ = 122^\circ$ and reflex $\angle QYP = 302^\circ$.