NCERT Solutions for Class 9 Maths

Chapter 9 - Circles

Exercise 9.1

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Ans: As we know that a circle is a collection of points therefore, they are equidistant from a fixed point. Now, this fixed point will be the centre of the circle and the equal distance between these points will be the radius of the circle. Hence, the shape of a circle will depend on its radius. Therefore, when we superimpose two circles of equal radius, then both the circles will cover each other. Thus, these two circles will be congruent when they have equal radius. Now, let us assume that two congruent circles have a common centre: O and O', AB and CD are the two chords of same length.

In ΔAOB and $\Delta CO'D$, we can observe that

$AB = CD$ as they are chords of the same length.

$OA = O'C$ as they are radii of congruent circles,

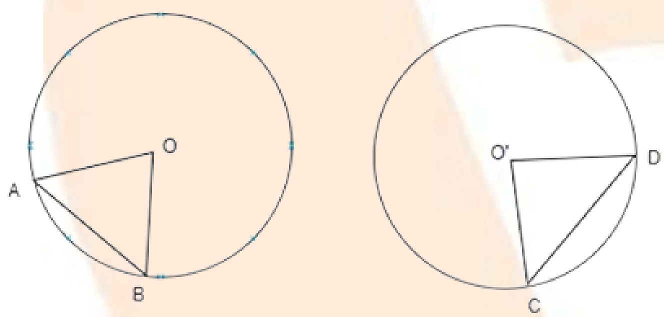
$OB = O'D$

as they are radii of congruent circles.

Therefore, $\Delta AOB \cong \Delta CO'D$ by the SSS congruence rule. This implies $\angle AOB \cong \angle CO'D$ By CPCT. Hence, equal chords of congruent circles subtend equal angles at their centres.

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Ans: Let us assume that there are two congruent circles with the same radii that have centres as O and O'.



In ΔAOB and $\Delta CO'D$,

$\angle AOB = \angle CO'D$ (Given)

$OA = O'C$ as they are radii of congruent circles

$OB = O'D$ as they are radii of congruent circles

Therefore,

$\Delta AOB \cong \Delta CO'D$ by the SSS congruence rule.

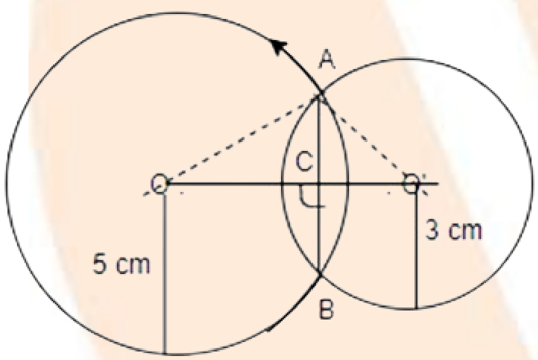
$\Rightarrow AB = CD$ (By CPCT)

Hence, if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Exercise 9.2

1. Two circles of radii 5cm and 3cm intersect at two points and the distance between their centres is 4cm. Find the length of the common chord.

Ans: Let us assume that the radius of the circle which is centred at O and O' be 5cm and 3cm.



Therefore, OA=OB

$\Rightarrow 5cm$

Similarly,

$O'A = O'B$

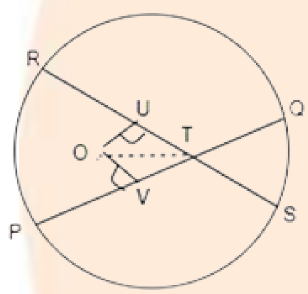
$\Rightarrow 3cm$

Now, the line segment OO' will be the perpendicular bisector of the chord AB. True.

We know that the points on the circle are always on equal distances from the centre of the circle and hence, this equal distance is defined as the radius of the circle. This is why a line segment joining the centre to any point on the circle is a radius of the circle.

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Ans: Let us assume that PQ and RS are the two chords of equal length of a circle and they are intersecting at a common point T.



So, let us draw two perpendicular bisectors OV and OU on these chords.

In ΔOVT and ΔOUT,

We have $OV = OU$ as they are equal chords of a circle and are equidistant from the centre.

Also, $\angle OVT = \angle OUT$.

Therefore,

$\Delta OVT \cong \Delta OUT$, by the RHS congruent rule.

$\Rightarrow VT = UT$ by CPCT.

Now, we have given that –

$PQ = RS$

$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}RS$

$\Rightarrow PV = RU$.

Now, let us add both the conditions as –

$PV + VT = RU + UT$

$\Rightarrow PT = RT$.

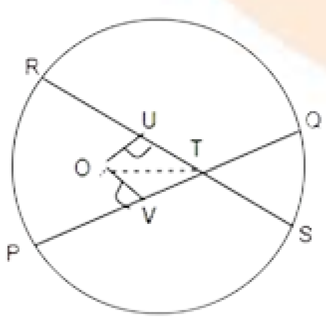
On subtracting we get –

$PQ - PT = RS - RT$

This equation indicates that a corresponding segment of the chords are congruent to each other. Hence, proved.

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Ans: Let us assume that PQ and RS are the two chords of the same length of a circle which are intersecting at a common point T .



So, let us draw two perpendicular bisectors OV and OU on these chords.

In ΔOVT and ΔOUT ,

We have $OV = OU$ as they are equal chords of a circle and are equidistant from the centre.

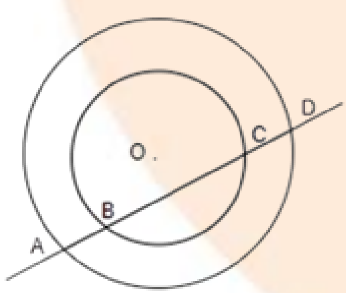
Also, $\angle OVT = \angle OUT$.

Therefore,

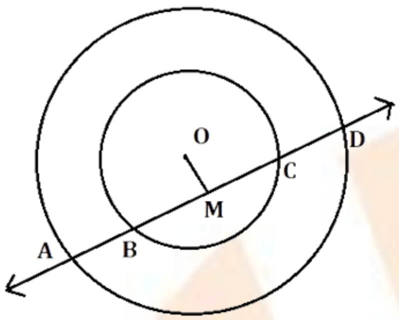
$\Delta OVT \cong \Delta OUT$, by the RHS congruence rule.

Therefore, we can conclude that $\angle OVT = \angle OUT$ by CPCT. Hence, if two equal chords of a circle intersect within the circle, then the line joining the point of intersection to the centre makes equal angles with the chords. Hence, proved.

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$.



Ans: In the figure, let us draw a perpendicular OM bisecting the chord BC and AD.



We can observe from the figure that $BC < AD$.

Hence, we have –

$BM = MC$ and

$AM = MD$.

On subtracting both equations, we get – $AM - BM = MD - MC$

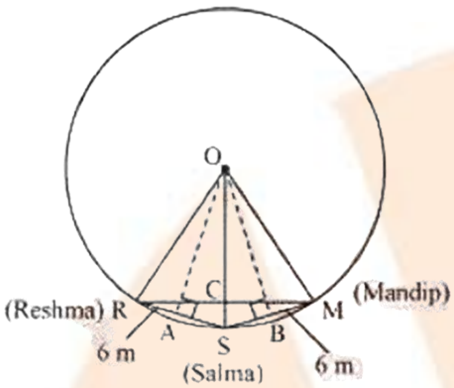
$\Rightarrow AB = CD$.

Hence, proved.

5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma and Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Ans: Let us assume that OA and OB are the two perpendiculars of RS and SM

As shown in the figure below.



Hence, we have

AR=AS

$\Rightarrow 3m$.

Also, $OR = OS = OM = 5m$.

In ΔOAR,

$OA^2 + AR^2 = OR^2$

$\Rightarrow OA^2 = 25 - 9$

$\Rightarrow OA = 4m$.

As, from the figure we can observe that ORSM is a kite. Now, we know that the diagonals of a kite are perpendicular.

Therefore,

$\angle RCS = 90^\circ$ and $RC = CM$.

Area of the $\Delta ORS = \frac{1}{2} \times OA \times RS$

$\Rightarrow \frac{1}{2} \times RC \times OS = \frac{1}{2} \times 4 \times 6$

$\Rightarrow RC = 4.8$

Hence,

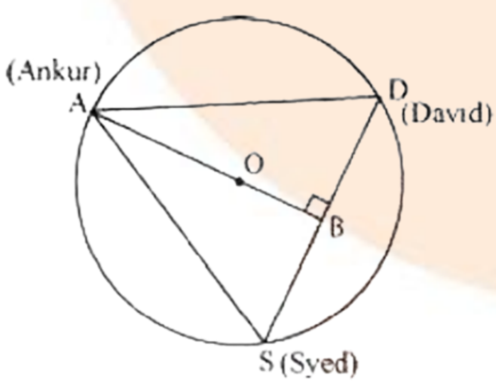
$RM = 2RC$

$\Rightarrow RM = 9.6m$.

Therefore, the distance between Reshma and Mandip will be 9.6m.

6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Ans: Let us draw a figure as –



From the figure, we can observe that AS=SD=DA.

Hence, ΔASD will be an equilateral triangle and $OA = 20m$.

Now, we know that the medians of an equilateral triangle will pass through the centre. Also, the medians will intersect each other at the ratio 2:1.

Therefore, the median AB is –

$$\frac{OA}{OB} = \frac{2}{1}$$

$$\Rightarrow \frac{20}{OB} = \frac{2}{1}$$

$$\Rightarrow OB = 10m$$

Hence, $AB = OA + OB$

$$\Rightarrow AB = 30m.$$

In ΔABD, we have –

$$AD^2 = AB^2 + BD^2$$

$$\Rightarrow AD^2 = 900 + \left(\frac{AD}{2}\right)^2$$

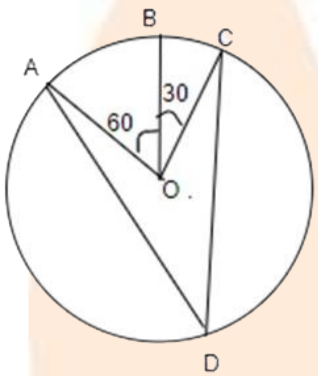
$$\Rightarrow 3AD^2 = 3600$$

$$\Rightarrow AD = 20\sqrt{3}$$

Hence, the length of the string of each phone will be $20\sqrt{3}m$.

Exercise 9.3

1. In the given figure, A, B, and C are three points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$



Ans: From the figure, we can observe that –

$\angle AOC = \angle AOB + \angle BOC$

$\Rightarrow \angle AOC = 60^\circ + 30^\circ$

$\Rightarrow \angle AOC = 90^\circ$.

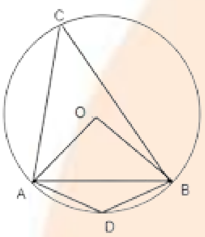
As, the angle subtended by the arc at the centre will be twice the angle on the remaining part. Therefore,

$\Rightarrow \angle ADC = \frac{1}{2}(90^\circ)$

$\Rightarrow \angle ADC = 45^\circ$.

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Ans: In ΔOAB,



We have –

$AB = OA = OB$ as radius.

Hence, ΔOAB will be an equilateral triangle.

This implies that each interior angle of the equilateral triangle will be 60°.

$\Rightarrow \angle AOB = 60^{\circ}$

$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB$

$\Rightarrow \frac{1}{2}(60^{\circ}) = 30^{\circ}$.

In quadrilateral ACBD,

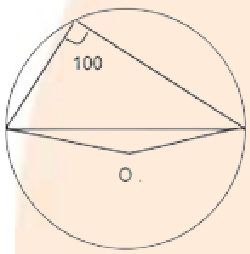
We have –

$\angle ACB + \angle ADB = 180^{\circ}$

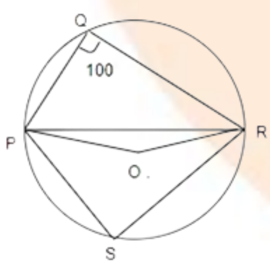
$\Rightarrow \angle ADB = 150^{\circ}$.

Therefore, the angle subtended by the chord on the major and minor arc will be 30° and 150°

3. In the given figure, $\angle PQR = 100^{\circ}$, where $P,Q,$ and R are points on a circle with centre O. Find $\angle OPR$.



Ans: Let us assume that PR is a chord of the circle and S is any point on the major arc.



PQRS is a cyclic quadrilateral.

Hence, we have –

$\angle PQR + \angle PSR = 180^\circ$

$\Rightarrow \angle PSR = 80^\circ$

Now, we know that the angle subtended by the arc at centre will be double the angle subtended by it.

Therefore,

$\angle PQR = 2\angle PSR$

$\Rightarrow \angle POR = 160^\circ$

In ΔPOR,

We can observe that –

$OP = PR$.

$\Rightarrow \angle OPR = \angle ORP$ as they are opposite angles of equal sides of a triangle.

$\Rightarrow \angle OPR + \angle ORP + \angle POR = 180^\circ$ which is the angle sum property of a triangle.

$\Rightarrow 2\angle OPR + 160^\circ = 180^\circ$

$\Rightarrow \angle OPR = 10^\circ$

Therefore, $\angle OPR = 10^\circ$.

4. In figure, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$?

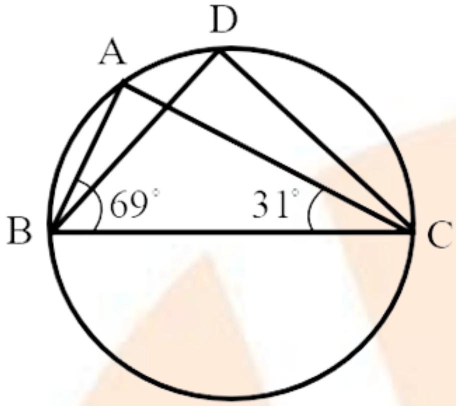


Fig. 10.38

Ans: From the given figure, we have –

$\angle BAC = \angle BDC$.

In ΔABC,

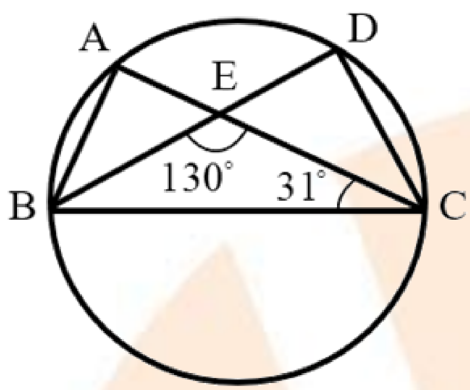
$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 69^\circ - 31^\circ$

$\Rightarrow \angle BAC = 80^\circ$.

Therefore, we have $\angle BDC = 80^\circ$.

5. In the given figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.



Ans: From the given figure, we have –

In ΔCDE,

$\angle CDE + \angle DCE = \angle CEB$

$\Rightarrow \angle CDE = 130^\circ - 20^\circ$

$\Rightarrow \angle CDE = 110^\circ$.

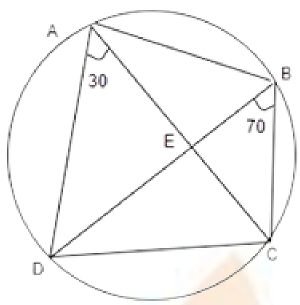
But we know that $\angle CDE = \angle BAC$

Therefore,

$\angle BAC = 110^\circ$.

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC = 30^\circ$, find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Ans: The figure will be as –



From figure, we can observe that –

$\angle CBD = \angle CAD$

$\Rightarrow \angle CAD = 70^\circ$.

$\Rightarrow \angle BAD = \angle BAC + \angle CAD$

$\Rightarrow \angle BAD = 100^\circ$

Therefore, we have –

$\angle BCD + \angle BAD = 180^\circ$

$\Rightarrow \angle BCD = 80^\circ$.

Now, in $\Delta ABC,$ we have –

$AB = BC$

$\Rightarrow \angle BCA = \angle CAB$

$\Rightarrow \angle BCA = 30^\circ$.

Also, we have –

$\angle BCD = 80^\circ$

$\Rightarrow \angle BCA + \angle ACD = 80^{\circ}$

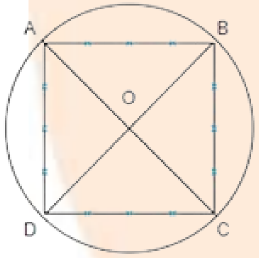
$\Rightarrow \angle ACD = 80^{\circ} - 30^{\circ}$

$\Rightarrow \angle ACD = 50^{\circ}$

$\Rightarrow \angle ECD = 50^{\circ}$.

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Ans: Let us assume a cyclic quadrilateral ABCD having diagonals BD and AC, intersecting at a common point O.



$\angle BAD = \frac{1}{2} \angle BOD$

$\Rightarrow \angle BAD = 90^{\circ}$

Now, $\angle BCD + \angle BAD = 180^{\circ}$

$\Rightarrow \angle BCD = 90^{\circ}$.

$\angle ADC = \frac{1}{2} \angle AOC$

$\Rightarrow \angle ADC = 90^\circ$

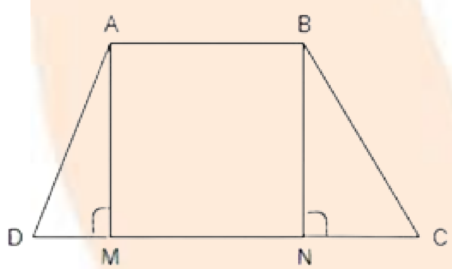
$\Rightarrow \angle ADC + \angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 90^\circ$.

Therefore, each interior angle of the quadrilateral is 90° which implies that ABCD is a rectangle.

8. If the non – parallel sides of a trapezium are equal, prove that it is cyclic.

Ans: Let us assume a trapezium ABCD with $AB \parallel CD$ and $BC = AD$ as shown in the figure below.



From the figure, we can observe that $AM \perp CD$ and $BN \perp CD$.

Therefore, in ΔAMD and ΔBNC,

$AD = BC$.

$\Rightarrow \angle AMD = \angle BNC$

$AM = BN$

$\Rightarrow \Delta AMD \cong \Delta BNC$ by the RHS congruence rule.

Therefore, $\angle ADC = \angle BCD$.

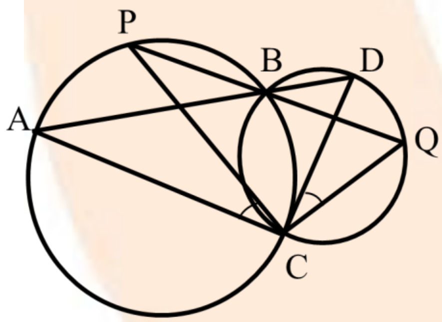
$\angle BAD$ and $\angle ADC$ are on the same side.

Therefore, $\angle BAD + \angle ADC = 180^{\circ}$

$\angle BAD + \angle BCD = 180^{\circ}$

Hence, the angles are supplementary. Therefore, ABCD is a cyclic quadrilateral.

9. Two circles intersect at two points B and C. Through B, two-line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$.



Ans: Let us join the chords AP and DQ.

Therefore,

$\angle PBA = \angle ACP$,

Also, $\angle DBQ = \angle QCD$.

Now, we know that ABD and PBQ are the line segments intersecting at common point B.

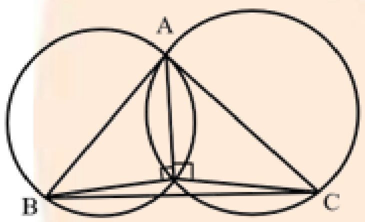
Therefore,

$\angle PBA = \angle DBQ$

Hence, we have $\angle ACP = \angle QCD$.

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Ans: Let us consider a triangle ΔABC in the figure given below –



We can observe that two circles are drawn by taking the diameters AB and AC. We will let the points B and C intersect each other at a common point D which does not lie on the line segment BC.

Therefore, after joining AD we have –

$\angle ADB = 90^\circ$

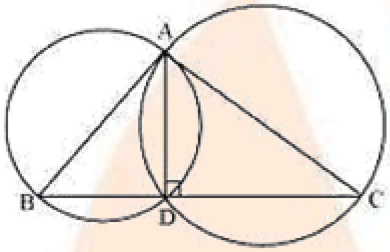
$\Rightarrow \angle ADC = 90^\circ$

$\Rightarrow \angle BDC = \angle ADB + \angle ADC$

$\Rightarrow \angle BDC = 180^\circ$.

Hence, we have a straight line as BD. This implies that the assumption that we considered was wrong.

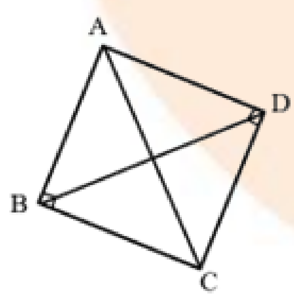
Therefore, the point of intersection D will lie on the line segment BC.



11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Ans:

From the figure we know that in ΔABC,



$\angle ABC + \angle BCA + \angle CAB = 180^{\circ}$

$\Rightarrow \angle BCA + \angle CAB = 90^{\circ}$.

In ΔADC,

$\angle CDA + \angle ACD + \angle DAC = 180^{\circ}$

$\Rightarrow \angle ACD + \angle DAC = 90^{\circ}$

After adding both the conditions, we get –

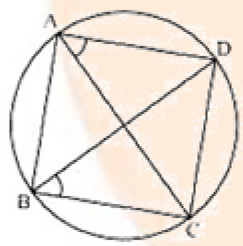
$\angle BCA + \angle CAB + \angle ACD + \angle DAC = 180^{\circ}$

$\Rightarrow (\angle BCA + \angle ACD) + (\angle CAB + \angle DAC) = 180^{\circ}$

$\Rightarrow \angle BCD + \angle DAB = 180^{\circ}$.

Now, we know that $\angle B + \angle D = 180^{\circ}$.

Therefore, we can observe from the sum of each interior angle that it is a cyclic quadrilateral.

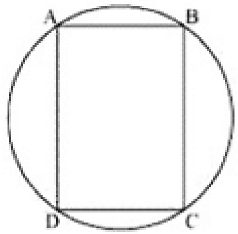


Hence,

$\angle CAD = \angle CBD$.

12. Prove that a cyclic parallelogram is a rectangle.

Ans: Let us assume a cyclic parallelogram ABCD as shown in the figure below –



We have –

$\angle A + \angle C = 180^{\circ}$.

Now, we know that in a parallelogram opposite angles are always equal.

Therefore,

$\angle A = \angle C$ and

$\angle B = \angle D$.

$\Rightarrow \angle A + \angle C = 180^{\circ}$

$\Rightarrow \angle A = 90^{\circ}$.

Similarly,

$\Rightarrow \angle B = 90^{\circ}$.

Therefore, all the interior angles of the parallelogram are 90° which implies it is a rectangle. Hence, proved.