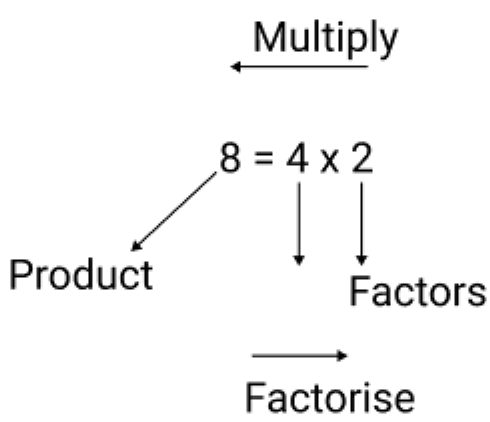
Revision Notes

Class – 9 Mathematics

Chapter 2 – Polynomials

Introduction

An algebraic expression of the form $a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ where $a_0, a_1, a_2, \ldots a_n$ are real numbers, n is a positive integer is called a polynomial in x. In the case of numbers, you are familiar with factors and products. For instance, 8 is the result of multiplying 4 and 2. The factors of eight are 4 and 2.



In the same way, the algebraic expression $a.b.c = abc$ will be written as $1.a.b.c$ or $1.ab.c$ or $1.bc.a$ or $1.ac$ or $1, a, b, c, ab, bc, ac, abc$ are all factors of $a.b.c$ and $a.b.c$ is a product. The term "factorization" refers to the process of expressing a given expression or number as the product of its components.

(1) Polynomials in One Variable

The formulas with only one variable are known as polynomials in one variable. A polynomial is a mathematical statement made up of variables and coefficients that involves the operations of addition, subtraction, multiplication, and exponentiation.

Below are some instances of polynomials in one variable:

- x^2+3x-2

- $3y^3+2y^2-y+1$

- m^4-5m^2+8m-3

(2) Coefficient of polynomials.

A coefficient is a number or quantity that is associated with a variable. It's generally an integer multiplied by the variable immediately adjacent to it.

For example, in the expression $3 x, 3$ is the coefficient but in the expression $x^2+3,1$ is the coefficient of x^2.

(3) Terms of polynomial.

Polynomial terms are the portions of the equation that are usually separated by "+" or "-" marks. As a result, each term in a polynomial equation is a component of the polynomial. The number of terms in a polynomial like 2^2+5+4 is 3.

(4) Types of polynomials.

Types of polynomials.	Meaning	Example
Zero or constant polynomial	Zero polynomials are polynomials having 0 degrees.	3 or $3x^0$
Linear polynomial	Linear polynomials are polynomials having a degree of 1 as the degree of the polynomial. The greatest exponent of the variable(s) in linear polynomials is 1.	$x+y-4$ $5m+7n$ $2p$

Quadratic polynomial	Quadratic polynomials are polynomials having a degree of 2 as the degree of the polynomial.	$8x^2+7y-9$ m^2+mn-6
Cubic polynomial	Cubic polynomials are polynomials having a degree of 3 as the degree of the polynomial.	$3x^3$ p^3+pq+7

(5) Degree of polynomial.

The largest exponential power in a polynomial equation is called its degree. Only variables are taken into account when determining the degree of any polynomial; coefficients are ignored.

$4x^5+2x^3-20$ In the above polynomial degree will be 5.

(6) Zeros of polynomials.

The polynomial zeros are the x values that fulfil the equation y = f(x). The zeros of the polynomial are the values of x for which the y value is equal to zero, and f(x) is a function of x. The degree of the equation y = f(x), determines the number of zeros in a polynomial.

Factorization of Polynomials

You know that any polynomial of the form $\mathrm{p(a)}$ can also be written as $p(a)=g(a)\times h(a)+R(a)$

Dividend = Quotient $\times$ Divisor + Remainder

If the remainder is zero, then $\mathrm{p(a)=g(a)xh(a)}$. That is, the polynomial $\mathrm{p(a)}$ is a product of two other polynomials g (a) and $\mathrm{h(a)}$. For example, $3a+6a^2=3a\times(1+2a)$.

A polynomial may be expressed in more than one way as the product of two or more polynomials.

Study the polynomial $3a+6a^2=3ax(1+2a)$.

This can also be factorised as $3a+6a^2=6a\times\left(\frac{1}{2}+a\right)$.

Methods of Factorizing Polynomials

A polynomial can be factorised in a number of ways.

1. Factorization, which is done by dividing the expression by the HCF of the words in the provided expression.
2. Factorization by grouping the terms of the expression.
3. Factorization using identities.

Factorization is achieved by dividing the expression by the HCF of the given expression's terms.

The biggest monomial in a polynomial is the HCF, which is a factor of each term in the polynomial. We can factorise a polynomial by determining the expression's Highest Common Factor (HCF) and then dividing each term by its HCF. The factors of the above equation are HCF and the quotient achieved.

Steps for Factorization

Determine the HCF of the supplied expression's terms.

Find the quotient by dividing each term of the provided equation by the HCF.

As a product of HCF and quotient, write the given expression.

Factorization by grouping the expression's terms

We come encounter polynomials in a variety of circumstances, and they may or may not contain common factors among their components. In such instances, we arrange the expression's terms so that common factors exist among the terms of the resulting groups.

Steps for Factorization by Grouping

If required, rearrange the terms.

Assemble the provided phrase into groups, each with its own common component.

Determine each group's HCF.

Find out what the other component is.

Convert the phrase to a product of the common and additional factors.

Factorization using Identities

To locate the products, recall the following identities:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Observe that the LHS in the identities are all factors and the RHS are their products. Thus, we can write the factors as follows:

Factors of $a^2 - 2ab + b^2$ are $(a-b)$ and $(a-b)$ Factors of $a^2 + 2ab + b^2$ are $(a+b)$ and $(a+b)$ Factors of $a^2 - b^2$ are $(a+b)$ and $(a-b)$ Factors of $a^3 + 3a^2b + 3ab^2 + b^3$ are $(a+b),(a+b)$ and $(a+b)$

Factors of $a^3-3a^2b+3ab^2-b^3$ are $(a-b),(a-b)$ and $(a-b)$ Factors of $a^2+b^2+c^2+2ab+2bc+2ca$ are$(a+b+c)$ and $(a+b+c)$

We may deduce from the preceding identities that a given statement in the form of an identity can be expressed in terms of its components.

Steps for factorization using Identities

Recognize the correct persona.

In the form of the identity, rewrite the provided statement.

Using the identity, write the factors of the given equation.

$$a^3 \pm b^3 \pm 3ab(a \pm b) = (a \pm b)^3$$

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$x^3 \pm y^3 = (x \pm y)\left(x^2 \pm xy + y^2\right)$$

Factorization of Trinomials of the form x^2+bx+c

Trinomials are expressions with three terms. For example, $x^2+14x+49$ is a trinomial. All trinomials cannot be factorised using a single approach. We must investigate the pattern in trinomials and select the best approach for factorising the given trinomial.

Factorizing a Trinomial by Splitting the Middle Term

The product of two binomials of the type $(x+a)$ and $(x+b)$ is $(x+a)\times(x+b) = x^2+x(a+b)+ab[a$ trinomial]

Examine the relationship between the middle and last words in these examples.

Middle Term	Observe that the numerical	**Factors of last term**

a) 12x = 9x + 3x	Coefficient of the middle term is the sum of factors of last	27 = 9 x 3
b) 12s = 8s + 4s	Therefore to factorise a trinomial expression of the	32 = 8 x 4
c) 12m = 7m + 5m	type x^2+cx+d look for this relation.	35 = 7 x 5

Therefore, to factorize expressions of the type $\left(x^2+cx+d\right)$, we need to discover two elements that meet the aforementioned criteria. That is, the middle word must be divided so that the product of the components equals the last term.

Steps to Factorize a Trinomial of the form x^2+bx+c where b and c are Integers:

Find all pairs of components whose product is the trinomial's final term.

Choose a pair of factors whose total equals the coefficient of the trinomial's middle word from the pairs of factors from step 1.

Using the pair of components from step 2, split the middle term and rebuild the trinomial.

Factorize the words from step 3 by grouping them together.

Double-check the answer.

Factoring a trinomial of the type ax $+bx^2+c(a\neq 1)$ by splitting the middle term

To factorize expressions of the type x^2+bx+c, you will find two numbers a and b such that their sum is equal to the coefficient of the middle term and their product is equal to the last term(constant).

Steps for factoring $ax^2+bx+c(a\neq 1)$ by grouping

- Find the product (ac), of the coefficient of x^2 and the last term.

Make a list of ac's factor pairs.

Select a factor pair whose total equals the middle term's coefficient.

Split the middle term to rewrite the polynomial.

Reorganize and factorise the data.

Remainder Theorem

If $f(x)$ is a polynomial in x and is divided by x-a; the remainder is the value of $f(x)$ at $x=a$ i.e., Remainder $=f(a)$ Proof:

Let $p(x)$ be a polynomial divided by $(x-a)$. Let $q(x)$ be the quotient and R be the remainder. By division algorithm, Dividend $=($ Divisor x quotient $)+$ Remainder $p(x)=q(x)\cdot(x-a)+R$

Substitute $x=a$,

So, we get

$p(a)=q(a)(a-a)+R$

$p(a)=R(a-a=0,0-q(a)=0)$

Hence Remainder $=p(a)$.

Steps for Factorization using Remainder Theorem

Using the trial-and-error approach, determine the constant factor for which the given expression equals zero.

Subtract the factor calculated in step 1 from the expression.

The quotient should be factored. Factorize the quotient further if it is a trinomial.

If the expression is of the fourth degree, the first step will be to reduce it to a trinomial, which will then be factorised further.

Factor Theorem

Statement:

If $\mathrm{p(x)}$, a polynomial in x is divided by $\mathrm{x-a}$ and the remainder $=\mathrm{p}$ (a) is zero, then $(\mathrm{x-a})$ is a factor of $p(x)$

Proof:

When $\mathrm{p(x)}$ is divided by $\mathrm{x-a}$, $\mathrm{R=p(a)}$ (by remainder theorem) $\mathrm{p(x)=(x-a)\cdot q(x)+p(a)}$

(Dividend = Divisor x quotient + Remainder Division Algorithm) But $\mathrm{p(a)=0}$ is given Hence $\mathrm{p(x)=(x-a)\cdot q(x)}$

$\Rightarrow (x-a)$ is a factor of $p(x)$ Conversely if $\mathrm{x-a}$ is a factor of $\mathrm{p(x)}$ then $\mathrm{p(a)=0}$. $p(x)=(x-a)\cdot q(x)+R$

If $(\mathrm{x-a})$ is a factor, then the remainder should be zero $(\mathrm{x}$ - a divides $\mathrm{p(x)}$ exactly) $\mathrm{R=0}$

By remainder theorem, $\mathrm{R=p(a)}$ $\Rightarrow \mathrm{p(a)=0}$