NCERT Solutions for Class 9 Maths

Chapter 2 – Polynomials

Exercise 2.3

1. Determine which of the following polynomials has $(x+1)$ a factor:

i. x^3+x^2+x+1

Ans: We know that Zero of $x+1$ is -1

Given that, $p(x)=x^3+x^2+x+1$

Now, for $x=-1$

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$p(-1)=-1+1-1+1$

$p(-1)=0$

Therefore, by the Factor Theorem, $x+1$ is a factor of x^3+x^2+x+1.

ii. $x^4+x^3+x^2+x+1$

Ans: We know that Zero of $x+1$ is -1

Given that, $p(x)=x^4+x^3+x^2+x+1$

Now, for $x = -1$

$$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$

$$p(-1) = 1 - 1 + 1 - 1 + 1$$

$$p(-1) = 1$$

Therefore, by the Factor Theorem, $x+1$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

iii. $x^4 + 3x^3 + 3x^2 + x + 1$

Ans: We know that Zero of $x+1$ is -1

Given that, $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Now, for $x = -1$

$$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$

$$p(-1) = 1 - 3 + 3 - 1 + 1$$

$$p(-1) = 1$$

Therefore, by the Factor Theorem, $x+1$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

iv. $x^3 + x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$

Ans: We know that, Zero of $x+1$ is -1

Given that, $p(x) = x^3 + x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$

Now, for $x = -1$

$$p(-1) = (-1)^3 + (-1)^2 - \left(2+\sqrt{2}\right)(-1) + \sqrt{2}$$

$$p(-1) = -1+1+2-\sqrt{2}+\sqrt{2}$$

$$p(-1) = 2+2\sqrt{2}$$

Therefore, by the Factor Theorem, $x+1$ is not a factor of $x^3 + x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$.

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

i. $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x+1$

Ans: Given that, $p(x) = 2x^3 + x^2 - 2x - 1$ $g(x) = x+1$

We know that Zero of $g(x)$ is -1

Now, $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$$p(-1) = -2+1+2-1$$

$$p(-1) = 0$$

Therefore, $g(x) = x+1$ is a factor of $p(x) = 2x^3 + x^2 - 2x - 1$.

ii. $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x+2$

Ans: Given that, $p(x)=x^3+3x^2+3x+1 \quad g(x)=x+2$

We know that Zero of $g(x)$ is -2

Now, $p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$p(-2)=-8+12-6+1$

$p(-2)=-1$

Therefore, $g(x)=x+2$ is not a factor of $p(x)=x^3+3x^2+3x+1$.

iii. $p(x)=x^3-4x^2+x+6$, $g(x)=x-3$

Ans: Given that, $p(x)=x^3-4x^2+x+6 \quad g(x)=x-3$

We know that Zero of $g(x)$ is 3

Now, $p(3)=(3)^3-4(3)^2+(3)+6$

$p(3)=27-36+3+6$

$p(3)=0$

Therefore, $g(x)=x-3$ is a factor of $p(x)=x^3-4x^2+x+6$.

3. Find the value of k, if $x-1$ is a factor of $p(x)$ in each of the following cases:

i. $p(x)=x^2+x+k$

Ans: Given that $x-1$ is a factor of $p(x)=x^2+x+k$

Thus, 1 is the zero of the given $p(x)$

$\Rightarrow p(1)=0$

$\Rightarrow p(1)=(1)^2+(1)+k=0$

$\Rightarrow 1+1+k=0$

$\Rightarrow k=-2$

Therefore, the value of k, if $x-1$ is a factor of $p(x)=x^2+x+k$ is -2.

ii. $p(x)=2x^2+kx+\sqrt{2}$

Ans: Given that $x-1$ is a factor of $p(x)=2x^2+kx+\sqrt{2}$

Thus, 1 is the zero of the given $p(x)$

$\Rightarrow p(1)=0$

$\Rightarrow p(1)=2(1)^2+k(1)+\sqrt{2}=0$

$\Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k=-\left(2+\sqrt{2}\right)$

Therefore, the value of k, if $x-1$ is a factor of $p(x)=2x^2+kx+\sqrt{2}$ is $-\left(2+\sqrt{2}\right)$.

iii. $p(x)=kx^2-\sqrt{2}x+1$

Ans: Given that $x-1$ is a factor of $p(x)=kx^2-\sqrt{2}x+1$

Thus, 1 is the zero of the given $p(x)$

$\Rightarrow p(1)=0$

$\Rightarrow p(1)=k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k-\sqrt{2}+1=0$

$\Rightarrow k=\sqrt{2}-1$

Therefore, the value of k, if $x-1$ is a factor of $p(x)=kx^2-\sqrt{2}x+1$ is $(\sqrt{2}-1)$.

iv. $p(x)=kx^2+3x+k$

Ans: Given that $x-1$ is a factor of $p(x)=kx^2+3x+k$

Thus, 1 is the zero of the given $p(x)$

$\Rightarrow p(1)=0$

$\Rightarrow p(1)=k(1)^2+3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow k=\dfrac{3}{2}$

Therefore, the value of k, if $x-1$ is a factor of $p(x)=kx^2+3x+k$ is $\frac{3}{2}$.

4. Factorise:

i. $12x^2-7x+1$

Ans: Given that, $p(x)=12x^2-7x+1$

Splitting the middle term

$\Rightarrow 12x^2-4x+3x+1$

$\Rightarrow 4x(3x-1)-1(3x-1)$

$\Rightarrow (3x-1)(4x-1)$

Therefore, $12x^2-4x+3x+1=(3x-1)(4x-1)$.

ii. $2x^2+7x+3$

Ans: Given that, $p(x)=2x^2+7x+3$

Splitting the middle term

$\Rightarrow 2x^2+x+6x+3$

$\Rightarrow x(2x+1)+3(2x-1)$

$\Rightarrow (2x+1)(x+3)$

Therefore, $2x^2+x+6x+3=(2x+1)(x+3)$.

iii. $6x^2+5x-6$

Ans: Given that, $p(x)=6x^2+5x-6$

Splitting the middle term

$\Rightarrow 6x^2+9x-4x-6$

$\Rightarrow 3x(2x+3)-2(2x+3)$

$\Rightarrow (2x+3)(3x-2)$

Therefore, $6x^2+9x-4x-6=(2x+3)(3x-2)$.

iv. $3x^2-x-4$

Ans: Given that, $p(x)=3x^2-x-4$

Splitting the middle term

$\Rightarrow 3x^2-4x+3x-4$

$\Rightarrow x(3x-4)+1(3x-4)$

$\Rightarrow (3x-4)(x+1)$

Therefore, $3x^2-4x+3x-4=(3x-4)(x+1)$.

5. Factorise:

i. x^3-2x^2-x+2

Ans: Given that, $p(x)=x^3-2x^2-x+2$

Rearranging the above,

$\Rightarrow x^3-x-2x^2+2$

$\Rightarrow x(x^2-1)-2(x^2-1)$

$\Rightarrow (x^2-1)(x-2)$

$\Rightarrow (x+1)(x-1)(x-2)$

Therefore, $x^3-2x^2-x+2=(x+1)(x-1)(x-2)$.

ii. x^3-3x^2-9x-5

Ans: Given that, $p(x)=x^3-3x^2-9x-5$

$\Rightarrow x^3+x^2-4x^2-4x-5x-5$

$\Rightarrow x^2(x+1)-4x(x+1)-5(x+1)$

$\Rightarrow (x+1)(x^2-4x-5)$

$\Rightarrow (x+1)(x^2-5x+x-5)$

$\Rightarrow (x+1)[x(x-5)+1(x-5)]$

$\Rightarrow (x+1)(x+1)(x-5)$

Therefore, $x^3-3x^2-9x-5=(x+1)(x+1)(x-5)$.

iii. $x^3+13x^2+32x+20$

Ans: Given that, $p(x)=x^3+13x^2+32x+20$

$\Rightarrow x^3+x^2+12x^2+12x+20x+20$

$\Rightarrow x^2(x+1)+12x(x+1)+20(x+1)$

$\Rightarrow (x+1)(x^2+12x+20)$

$\Rightarrow (x+1)(x^2+2x+10x+20)$

$\Rightarrow (x+1)[x(x+2)+10(x+2)]$

$\Rightarrow (x+1)(x+10)(x+2)$

Therefore, $x^3+13x^2+32x+20=(x+1)(x+10)(x+2)$.

iv. $2y^3+y^2-2y-1$

Ans: Given that, $p(y)=2y^3+y^2-2y-1$

$\Rightarrow 2y^3-2y^2+3y^2-3y+y-1$

$\Rightarrow 2y^2(y-1)+3y(y-1)+1(y-1)$

$\Rightarrow (y-1)(2y^2+3y+1)$

$\Rightarrow (y-1)(2y^2+2y+y+1)$

$\Rightarrow (y-1)[2y(y+1)+1(y+1)]$

$\Rightarrow (y-1)(2y+1)(y+1)$

Therefore, $2y^3+y^2-2y-1=(y-1)(2y+1)(y+1)$.