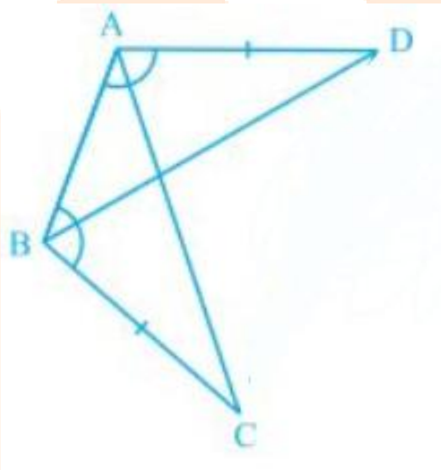
Important Questions for Class 9

Maths

Chapter 7 – Triangles

Very Short Answer Type Questions **1 Mark**

1. In fig, if AD =BC and $\angle$ BAD = $\angle$ ABC, then $\angle$ ACB is equal to



a. $\angle$**ABD**
b. $\angle$ **BAD**
c. $\angle$**BAC**
d. $\angle$**BDA**

Ans. in ΔABC *and* ΔABD

AD =BC (given)

$\angle$ BAD = $\angle$ ABC (Given)

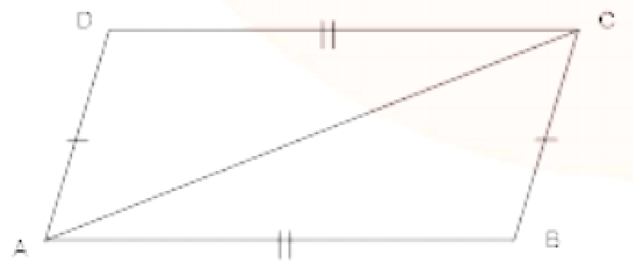
AB = AB (Common side)

$\therefore \Delta ABC \cong \Delta ABD$

By CPCT theorem $\angle ACB = \angle BDA$ **(By SAS Congruency)**

Correct option is (D) $\angle$BDA

2. In fig, if ABCD is a quadrilateral in which AD= CB, AB=CD, and ∠ D= ∠ B, then ∠CAB is equal to



a. ∠ ACD
b. ∠ CAD
c. ∠ ACD
d. ∠ BAD

Ans: in $\Delta ABC\ and\ \Delta CDA$

CB = AD (Given)

AB=CD (Given)

∠B = ∠D (Given)

$\therefore \Delta ABC \cong \Delta CDA$ **(By SAS Congruency)**

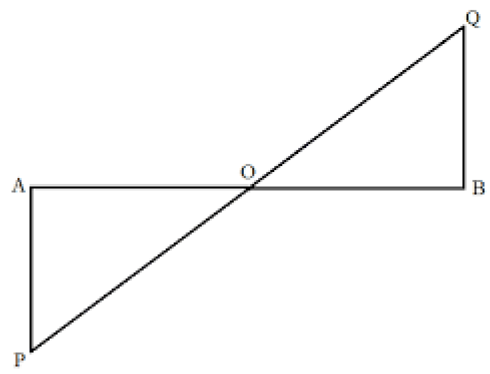
By CPCT theorem $\angle CAB = \angle ACD$

Option (C) ∠ ACD is correct.

3. If O is the mid – point of AB and ∠ BQO = ∠ APO, then ∠ OAP is equal to

a. ∠ QPA
b. ∠ OQB
c. ∠ QBO
d. ∠ BOQ

Ans:



In ΔAOP and ΔBOQ

$AO = BO$ ..... (O is midpoint of AB)

$\angle APO = \angle BQO$ (Given)

$\angle AOP = \angle BOQ$ (Vertically opposite Angles)

$\therefore \Delta AOP \cong \Delta BOQ$ **(By AAS Congruency)**

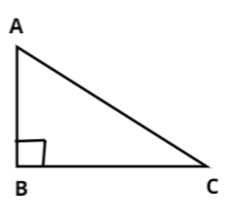
By CPCT $\angle OAP = \angle QBO$

Correct option is (C) $\angle$ QBO

4. IF AB$\perp$BC and $\angle$ A = $\angle$ C, then the true statement is

a. **AB$\neq$AC**
b. **AB=BC**
c. **AB=AD**
d. **AB=AC**

Ans:



ΔABC

$\angle A = \angle C$

Sides opposite to equal angles are also equal

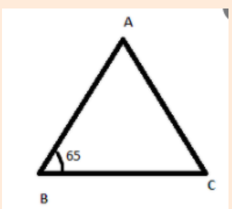
AB = BC

Correct option is (B) AB=BC

5. If � ABC is an isosceles triangle and $\angle$ B = 65^0 **, find $\angle$ A.**

a. 60^0
b. 70^0
c. 50^0
d. none of these

Ans:



Since $\triangle ABC$ is on isosceles triangle

$\therefore \angle B = \angle C$

$\therefore \angle B = 65^\circ$

$\therefore \angle C = 65^\circ$

$\therefore \angle A + \angle B + \angle C = 180^\circ$

$\therefore \angle A + 130^\circ = 180^\circ$

$\therefore \angle A = 180^\circ - 130^\circ$

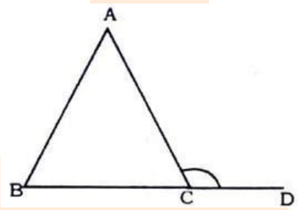
$\therefore \angle A = 50^\circ$

Correct option is (c) 50^0

6. If AB=AC and $\angle$ ACD=120^0 , find $\angle$A

- **a.** 50^0
- **b.** 60^0
- **c.** 70^0
- **d.** **none of these**

Ans:



$\because AB = AC$

$\Rightarrow \angle ABC = \angle ACB = x(say)$

Let $\angle BAC = y$

We know,

Exterior angles = sum of interior opposite angles

$120^\circ = \angle ABC + \angle BAC$

$120^\circ = x + y - - -(1)$

Again, $\angle ACB + \angle ACD = 180^\circ$

$x + 120^\circ = 180^\circ$

$\therefore x = 60^\circ$

From(1),

$60^\circ + y = 120^\circ$

$\Rightarrow y = 60^\circ$

$\Rightarrow \angle A = 60^\circ$

Correct option is (b) 60^0

7. What is the sum of the angles of a quadrilateral?

- **a.** 260^0
- **b.** 360^0
- **c.** 180^0
- **d.** 90^0

Ans. (b) 360^0

8. The sum of the angles of a triangle will be:

- **a.** 360^0
- **b.** 270^0
- **c.** 180^0
- **d.** 90^0

Ans. (c) 180^0

9. An angle is 14^0 more than its complement. Find its measure.

- **a. 42**
- **b. 32**
- **c. 52**
- **d. 62**

Ans: Two angles having sum equals to 90 degrees are called complementary angles.

let first angle = x

it's Complement = $90^\circ - x$

A.T.Q

$x = 14^\circ + 90^\circ - x$

$x = 104 - x$

$\Rightarrow 2x = 104^{\circ}$

$\Rightarrow x = \frac{104}{2}$

$\therefore x = 52^{\circ}$

Correct option is (C) 52

10. An angle is 4 time its complement. Find measure.

a. **62**
b. **72**
c. **52**
d. **42**

Ans: Two angles having sum equals to 90 degrees are called complementary angles.

let angle = x

Therefore, it's complement = $90^{\circ} - x$

A.T.Q

$x = 4(90^{\circ} - x)$

$x = 360^{\circ} - 4x$

$x + 4x = 360^{\circ}$

$\Rightarrow 5x = 360^{\circ}$

$\Rightarrow x = \frac{360^{\circ}}{5}$

$\therefore x = 72^{\circ}$

Correct option (B) 72

11. Find the measure of angles which is equal to its supplementary.

- **a.** 120^0
- **b.** 60^0
- **c.** 45^0
- **d.** 90^0

Ans: Two angles having sum equals to 180 degrees are called supplementary angles.

$$x = 180^\circ - x$$

$$2x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{2}$$

$$\therefore x = 90^\circ$$

Correct option is (D) 90^0

12. Which of the following pairs of angle are supplementary?

- **a.** $30^0, 120^0$
- **b.** $45^0, 135^0$
- **c.** $120^0, 30^0$
- **d. None of these.**

Ans: (B) $45^0, 135^0$

Because $45^\circ + 135^\circ = 180^\circ$

13. Find the measure of each exterior angle of an equilateral triangle.

- **a.** 110^0
- **b.** 100^0
- **c.** 120^0
- **d.** 150^0

Ans. (C)120^0

Because $180^\circ - 60^\circ = 120^\circ$

14. In an isosceles Δ ABC, if AB=AC and $\angle A = 90^0$, Find $\angle$ B.

- **a.** 45^0
- **b.** 80^0
- **c.** 95^0
- **d.** 60^0

Ans: AB = AC

$\Rightarrow \angle C = \angle B$ **(angles opposite equal sides are also equal)**

In ΔABC

$\angle A + \angle B + \angle C = 180^\circ$

$90^\circ + 2\angle B = 180^\circ$

$\Rightarrow 2\angle B = 180^\circ - 90^\circ$

$\Rightarrow \angle B = \frac{90^\circ}{2}$

$\Rightarrow \angle B = 45^\circ$

(A)45^0

15. In a � ABC, if $\angle$B= $\angle$C=45^0 , Which is the longest side.

- **a. BC**
- **b. AC**
- **c. CA**
- **d. None of these.**

Ans: �ABC

$\angle A+\angle B+\angle C=180^{\circ}$

$\angle A+45^{\circ}+45^{\circ}=180^{\circ}$

$\Rightarrow \angle A=180^{\circ}-90^{\circ}=90^{\circ}$

This is a right angles triangle in which right angle is at $\angle A$

Therefore, Side opposite to $\angle A$ is the longest side (hypotenuse)

Correct option is (A) BC

16. In a $\triangle$ABC, if AB=AC and $\angle$ B= 70^{0} , Find

- **a.** 40^{0}
- **b.** 50^{0}
- **c.** 45^{0}
- **d.** 60^{0}

Ans: In ΔABC

$AB=AC$

$\angle C=\angle B=70^{\circ}$

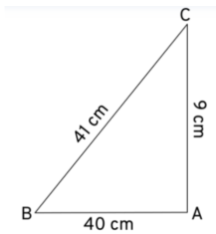
$\angle A+\angle B+\angle C=180^{\circ}$

$\angle A+70^{\circ}+70^{\circ}=180^{\circ}$

$\Rightarrow \angle A=180^{\circ}-140^{\circ}=40^{\circ}$

Correct option is $\text{(A)}\,40^{0}$

17. Determine the shortest sides of the triangles.



a. AC
b. BC
c. CA
d. none of these

Ans. (B) BC

18. In an ΔABC, if $\angle A = 45^0$ and $\angle B = 70^0$, determine the longest sides of the triangle.

a. AC
b. AB
c. BC
d. none of these

Ans: Angle opposite to longest side is largest

Side opposite to $\angle B$ is AC

Correct option is (a) AC

19. The sum of two angles of a triangle is equal to its third angle. Find the third angles.

a. 90^0
b. 45^0
c. 60^0
d. 70^0

Ans. (a) 90^0

20. Two angles of triangles are $60^\circ, 70^\circ$ respectively. Find third angles.

a. 90^0
b. 45^0
c. 60^0
d. 50^0

Ans. (d) 50^0

21. Δ**ABC is an isosceles triangle with AB=AC and** $\angle B = 45^0$ **, find** $\angle A$ **.**

Ans: In ΔABC

$AB = AC$

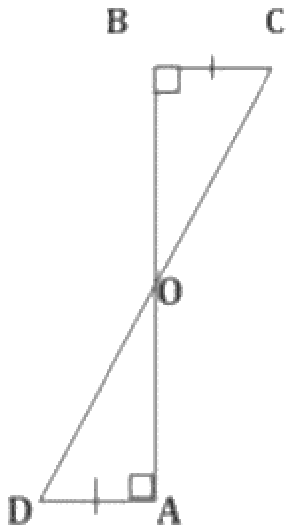
$\angle C = \angle B = 45^{\circ}$

$\angle A + \angle B + \angle C = 180^{\circ}$

$\angle A + 45^{\circ} + 45^{\circ} = 180^{\circ}$

$\Rightarrow \angle A = 180^{\circ} - 90^{\circ} = 90^{\circ}$

22. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB (See figure)



Ans: In ΔBOC and ΔAOD

$OBC = OAD =$ [Given]

$BOC = AOD$ [Vertically Opposite angles]

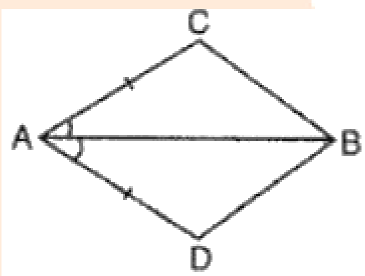
$BC = AD$ [Given]

$BOC = AOD$ [By ASA congruency]

$OB = OA$ and $OC = OD$ [By C.P.C.T.]

Short Answer Type Questions **2 Marks**

1. In quadrilateral ABCD (See figure). AC = AD and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD?



Ans: Given: In quadrilateral $ABCD$

$AC = AD$ and AB bisects $\angle A$.

To prove: $\Delta ABC \cong \Delta ABD$

Proof: In ΔABC and ΔABD,

$AC = AD$ [Given]

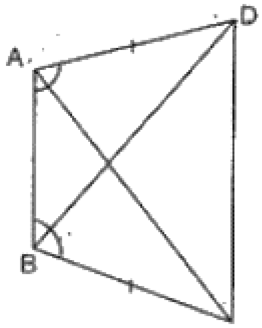
$\angle BAC = \angle BAD$ [AB bisects $\angle A$]

$AB = AB$ [Common]

$\Delta ABC \cong \Delta ABD$ [By SAS congruency]

Thus $BC = BD$ [By C.P.C.T.]

2. ABCD is a quadrilateral in which AD = BC and $\angle$ DAB = $\angle$CBA. (See figure). Prove that:



(i) ΔABD $\cong$ ΔBAC

Ans: In ΔABC and ΔABD,

$BC = AD$ [Given]

$DAB = CBA$ [Given]

$AB = AB$ [Common]

$\Delta ABC \cong \Delta ABD$ [By SAS congruency]

Thus $AC = BD$ [By C.P.C.T.]

(ii) BD = AC

Ans: Since $\Delta ABC \cong \Delta ABD$

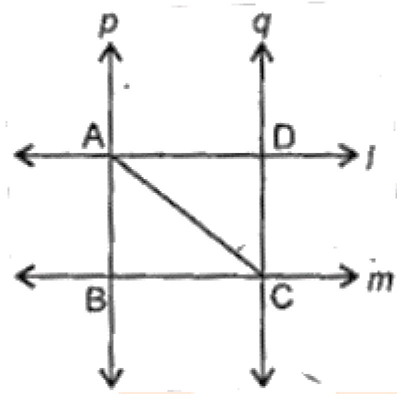
$AC = BD$ [By C.P.C.T.]

(iii) $\angle$ ABD = $\angle$ BAC

Ans: Since $\Delta ABC \cong \Delta ABD$

$\angle ABD = \angle BAC$ [By C.P.C.T.]

3. *l* and *m* are two parallel lines intersected by another pair of parallel lines *p* and *q* (See figure). Show that ΔABC $\cong$ ΔCDA.



Ans: AC being a transversal. [Given]

Therefore $\angle DAC = \angle ACB$ [Alternate angles]

Now $p \parallel q$ [Given]

And AC being a transversal. [Given]

Therefore $\angle BAC = \angle ACD$ [Alternate angles]

Now In ΔABC and ΔADC,

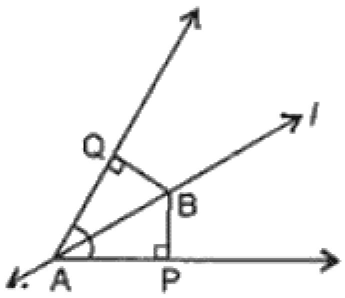
$\angle ACB = \angle DAC$ [Proved above]

$\angle BAC = \angle ACD$ [Proved above]

$AC = AC$ [Common]

$\Delta ABC \cong \Delta CDA$ [By ASA congruency]

4. Line is the bisector of the angle A and B is any point on BP and BQ are perpendiculars from B to the arms of A. Show that:



(i). $\Delta APB \cong \Delta AQB$

Ans: Given: Linel bisects $\angle A$

$\angle BAP = \angle BAQ$

In ΔABP and ΔABQ

$\angle BAP = \angle BAQ$ [Given]

$\angle BPA = \angle BQA = $ [Given]

$AB = AB$ [Common]

$\Delta APB \cong \Delta AQB$ [By ASA congruency]

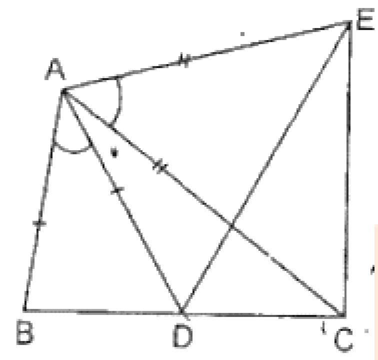
(ii). $BP = BQ$ **or** P **is equidistant from the arms of** $\angle A$ **(see figure)**

Ans: Since $\Delta APB \cong \Delta AQB$

$BP = BQ$ [By C.P.C.T.]

B is equidistant from the arms of $\angle A$.

5. In figure, AC = AB, AB = AD and $\angle$ BAD = $\angle$ EAC. Show that BC = DE



Ans: Given that $\angle BAD = \angle EAC$

Adding $\angle DAC$ on both sides,

we get, $\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$BAC = EAD$ ....(i)

Now in ΔABC *and* ΔADE

$AB = AD$ [Given]

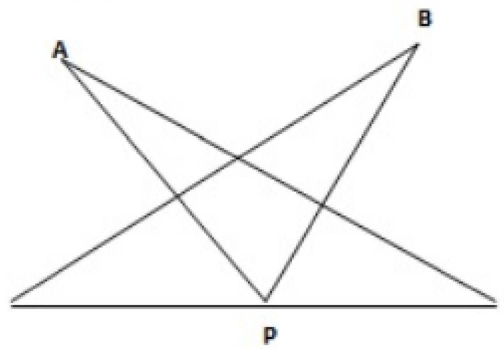
$AC = AE$ [Given]

$\angle BAC = \angle DAE$ [From eq. (i)]

$\Delta ABC \cong \Delta ADE$ [By SAS congruency]

$BC = DE$ [By C.P.C.T.]

6. AB is a line segment and P is the mid-point. D and E are points on the same side of AB such that BAD = ABE and EPA = DPB. Show that: (i) DAF FBPE D (ii) AD = BE (See figure)



Ans: Given that $\angle EPA = \angle DPB$

Adding $\angle EPD$ on both sides, we get,

$\angle EPA + \angle EPD = \angle DPB + \angle EPD$

$\angle APD = \angle BPE$ ....(i)

Now in ΔAPD and ΔBPE,

$\angle PAD = \angle PBE[\angle BAD = \angle ABE \text{ (given)}]$

$\angle PAD = \angle PBE$

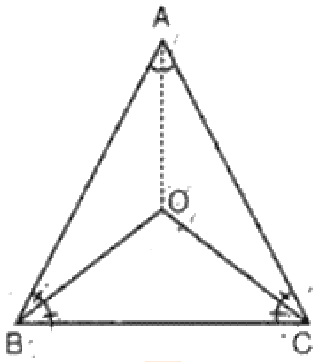
$AP = PB$ [P is the mid - point of AB]

$\angle APD = \angle BPE$ [From eq. (i)]

$\angle DPA = \angle EBP$ [By ASA congruency]

$AD = BE$ [By C.P.C.T.]

7. In an isosceles triangle ABC, with AB = AC, the bisectors of B and C intersect each other at O. Join A to O. Show that:



(i) OB = OC

Ans: ABC is an isosceles triangle in which

$AB = AC$

$\Rightarrow C = B$ [Angles opposite to equal sides]

$\Delta OCA + \Delta OCB = \Delta OBA + \Delta OBC$

OB bisects $\angle B$ and OC bisects $\angle C$

$\angle OBA = \angle OBC$ and $\angle OCA = \angle OCB$

$\angle OCB + \angle OCB = \angle OBC + \angle OBC$

$2\angle OCB = 2\angle OBC$

$\angle OCB = \angle OBC$

Now in ΔOBC,

$\angle OCB = \angle OBC$ [Proved above]

$OB = OC$ [Sides opposite to equal sides]

(ii) AO bisects A

Ans: In ΔAOB and ΔAOC,

$AB = AC$ [Given]

$\angle OBA = \angle OCA$ [Given]

And $\angle B = \angle C$

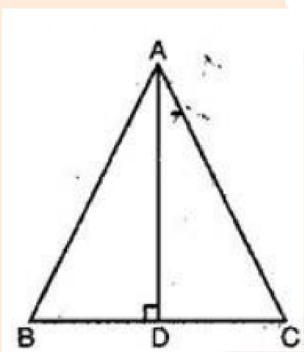
$\frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\angle OBA = \angle OCA$

$OB = OC$ [Proved above]

$\Delta AOB \cong \Delta AOC$ [By SAS congruency]

8. In ABC, AD is the perpendicular bisector of BC (See figure). Show that ABC is an isosceles triangle in which AB = AC.



Ans: In ΔAOB and ΔAOC, ,

$BD = CD$ [AD bisects BC]

$\angle ADB = \angle ADC$ [[Since $AD = BC$]

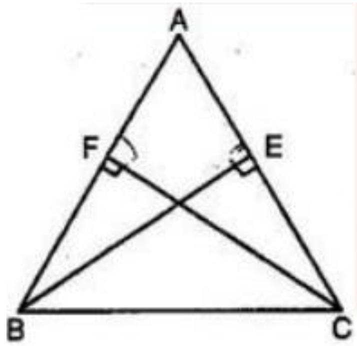
$AD = AD$ [Common]

$\therefore \Delta ABD \cong \Delta ACD$ [By SAS congruency]

therefore, $AB = AC$ [By C.P.C.T.]

Therefore, ABC is an isosceles triangle.

9. ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively (See figure). Show that these altitudes are equal.



Ans: In ΔABE and ΔACF,

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC$ [Since $AD \perp BC$] [Given]

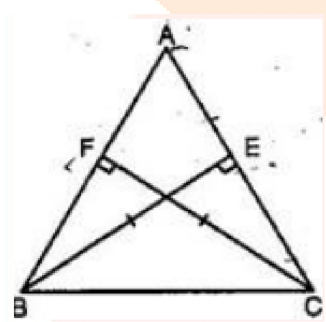
$AB = AC$ [Given]

$\Delta ABE \cong \Delta ACF$ [By ASA congruency]

$BE = CF$ [By C.P.C.T.]

Altitudes are equal.

10. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that:



(i). In ΔABE and ΔACF,

Ans: In ΔABE and ΔACF,

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC = 90^0$ [Given]

$BE = CF$ [Given]

$\Delta ABE \cong \Delta ACF$ [By ASA congruency]

(ii) AB = AC or ΔABC is an isosceles triangle.

Ans: Since, $\Delta ABE \cong \Delta ACF$

$BE = CF$ [By C.P.C.T.]

ΔABC is an isosceles triangle.

11. ABC and DBC are two isosceles triangles on the same base BC (See figure). Show that $\angle$ABD = $\angle$ACD.

Ans: In isosceles triangle ABC

$AB = AC$ [Given]

$\angle ACB = \angle ABC$ ....(i) [Angles opposite to equal sides]

Also in Isosceles triangle BCD.

$BD = DC$

$\angle BCD = \angle CBD$ ....(ii) [Angles opposite to equal sides]

Adding eq. (i) and (ii)

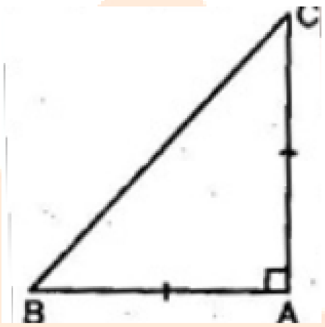
$\angle ACB + \angle BCD = \angle ABC + \angle CBD$

$\angle \text{ACD} = \angle \text{ABD}$ **or**

$\angle \text{ABD} = \angle \text{ACD}$

12. ABC is a right angled triangle in which $\angle A = 90^0$ and AB = AC. Find $\angle B$ and $\angle C$.

Ans: Δ ABC is a right triangle in which,



$\angle A = 90^0$ And $AB = AC$

In ΔABC,

$AB = AC \Rightarrow \angle C = \angle B$ ....(i)

We know that, in ΔABC, $\angle A + \angle B + \angle C = 180^0$ [Angle sum property]

$90^0 + B + B = 180^0$ [$\angle A = 90^0$ (given) and $\angle B = \angle C$ (from eq. (i))]

$2\angle B = 90^0$

$\angle B = 90^0$ Also $\angle C = 45^0$ [$\angle B = \angle C$]

13. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that:

(i) AD bisects BC.

Ans: In ΔABD and ΔACD,

$AB = AC$ [Given]

$\Delta ADB = \Delta ADC = 90^0$ [$AD \perp BC$]

$AD = AD$ [Common]

$\Delta ABD \cong \Delta ACD$ [RHS rule of congruency]

$BD = DC$ [By C.P.C.T.]

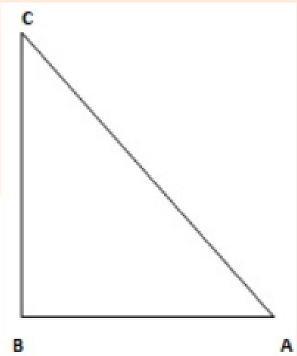
AD bisects BC

(ii) AD bisects A.

Ans: Since, $\angle BAD = \angle CAD$ [By C.P.C.T.]

AD bisects $\angle A$.

14. Show that in a right angles triangle, the hypotenuse is the longest side.



Ans: Given: Let ABC be a right angled triangle, right angled at B.

To prove: Hypotenuse AC is the longest side.

$\angle A + \angle B + \angle C = \angle A + 90^0 + \angle C = [\angle B = 90^0]$

$\angle A + \angle C = 180^0 - 90^0$

And $\angle B = 90^0$

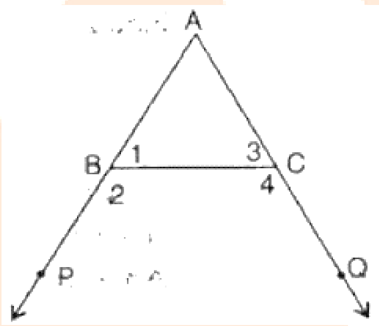
$\angle B > \angle C$ and $\angle B > \angle A$

Since the greater angle has a longer side opposite to it.

$AC > AB \text{ and } AC > AB$

Therefore $\angle B$ being the greatest angle has the longest opposite side AC, i.e. hypotenuse.

15. In figure, sides AB and AC of ABC are extended to points P and Q respectively. Also PBC< QCB. Show that AC > AB.



Ans: Given: In ΔABC, $\angle PBC < \angle QCB$

To prove: $AC > AB$

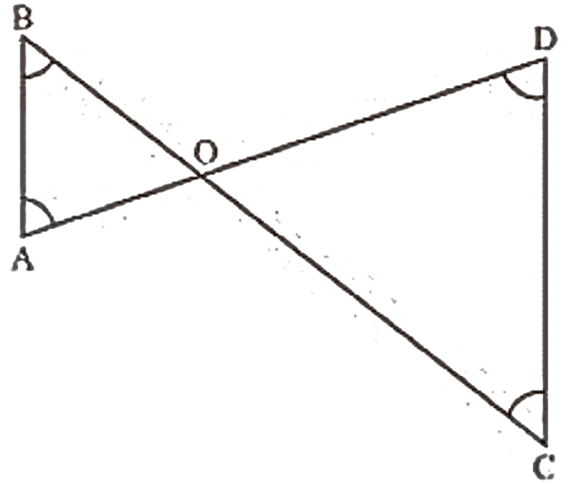
Proof : In ΔABC, $\angle 4 > \angle 2$ [Given]

Now $\angle 1 + \angle 2 = \angle 3 + \angle 4 = 180^0$ [Linear pair]

$\angle 1 > \angle 3 [\because \angle 4 > \angle 2]$

$AC > AB$ [Side opposite to greater angle is longer]

16. In figure, B < A and C < D. Show that AD < BC.



Ans: In ΔAOB

$\angle B < \angle A$ [Given]

$OA < OB \ldots\ldots\ldots(i)$ [Side opposite to greater angle is longer]

In $\Delta COD, \angle C < \angle D$ [Given]

$OD < OC \ldots\ldots\ldots(ii)$ [Side opposite to greater angle is longer]

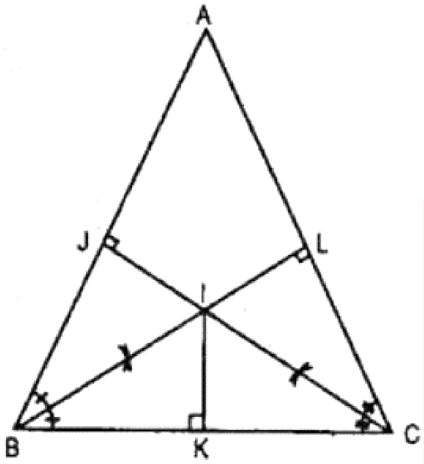
Adding eq. (i) and (ii),

$OA + OD < OB + OC$

$AD < BC$

17. In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Ans: Let ΔABC be a triangle.



Drawbisectorsof $\angle$Band $\angle$C.

Let these angle bisectors intersect each other at point I.

Draw$IK \perp BC$

Also draw $IJ \perp AB$ and $IL \perp AC$.

Join AI.

In ΔBIK and ΔBIJ,

$\angle IKB = \angle IJB = 90^0$ [By construction]

$\angle IBK = \angle IBJ$

BIisthebisectorof $\angle B$ (By construction)]

$BI = BI$ [Common]

$\Delta BIK \cong \Delta BIJ$ [ASA criteria of congruency]

$IK = IJ$ [By C.P.C.T.] ………..(i)

Similarly, $\Delta CIK \cong \Delta CIL$

$\therefore IK = IL$ [By C.P.C.T.] ………..(ii)

From eq (i) and (ii),

$IK = IJ = IL$

Hence, I is the point of intersection of angle bisectors of any two angles of ΔABC equidistant from its sides.

18. In quadrilateral ACBD, AB=AD and AC bisects A. show ABC ACD?

Ans: In ΔABC and ΔACD,

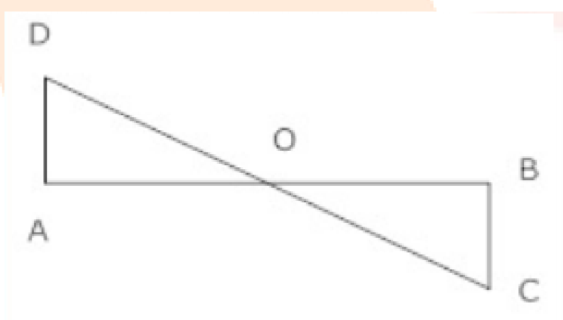
$AD = AB \ldots\ldots\ldots (\text{Given})$

$\angle BAC = \angle CAD \ldots\ldots. (\text{AC bisects A})$

$\text{And} AC = AC \ldots\ldots\ldots\ldots.. (\text{Common})$

$\Delta ABC \cong \Delta ACD \ldots\ldots\ldots.. (\text{SAS axiom})$

19. If DA and CB are equal perpendiculars to a line segment AB. Show that CD bisects AB.



Ans: In ΔAOD and ΔBOC,

$AD = BC \ldots\ldots\ldots.. (\text{Given})$

$\angle A = \angle B$ and

$\angle AOD = \angle BOC (\text{vert opp. Angles})$

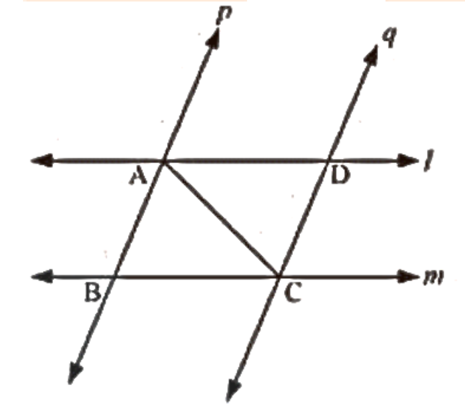
$\therefore \angle AOD = \angle BOC(\text{AAS rule})$

$\therefore OA = OB(\text{CPCT})$

Hence, CD bisects AB.

20. l and m, two parallel lines, are intersected by Another pair of parallel lines p and C. show that $\Delta ABC \cong \Delta CDA$.

Ans: L || M and AC cuts them (given)



$\therefore \angle ACB = \angle CAD(\text{alternateangles})$

P || Q and AC cuts them (Given)

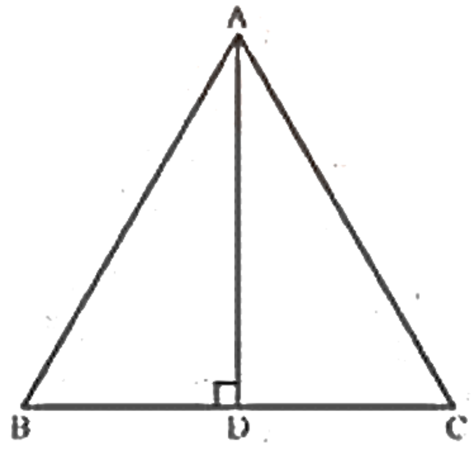
$\therefore \angle CAB = \angle ACD(\text{Alternateangles})$

$AC = CA(\text{common})$

$\therefore \Delta ABC \cong \Delta CDA(\text{ASA rule})$

21. In fig, the bisector AD of ΔABC is $\perp$ to the opposite side BC at D. show that ΔABC is isosceles?

Ans: In ΔABD and ΔACD



Let $\angle BAD = \angle 1$ and $\angle DAC = \angle 2$

$\angle 1 = \angle 2$….(AD is the bisector of $\angle A$)

And $\angle ADB = \angle ADC = 90^{\circ}$………(AD $\perp$ BC)

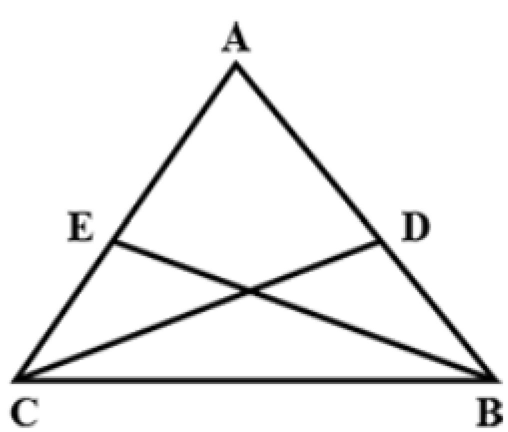
$\therefore$ AD = AD…….(common)

$\Delta ABD \cong \Delta ACD$…… (ASA rule)

$\therefore AB = AC$ (C.P.C.T)

Hence ΔABC is isosceles.

22. If AE=AD and BD=CE. Prove that $\Delta AEB \cong \Delta ADC$



Ans: We have,

$AE = AD and CE = BD$

$\Rightarrow AE + CE = AD + BD$

$\Rightarrow AC = AB (i)$

Now, in ΔAEB and ΔADC,

$AE = AD[given]$

$\angle EAB = \angle DAC[common]$

$AB = AC[from (i)]$

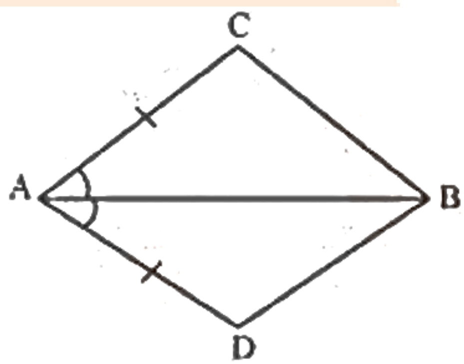
$AE = AD[given]$

$\angle EAB = \angle DAC[common]$

$AB = AC[from (i)]$

$\Delta AEB \cong \Delta ADC[by SAS]$

23. In quadrilateral ACBD, AC=AD and AB bisects $\angle$A. show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?



Ans: In $\Delta ABC \cong \Delta ABD$,

$AC = AD\ [given]$

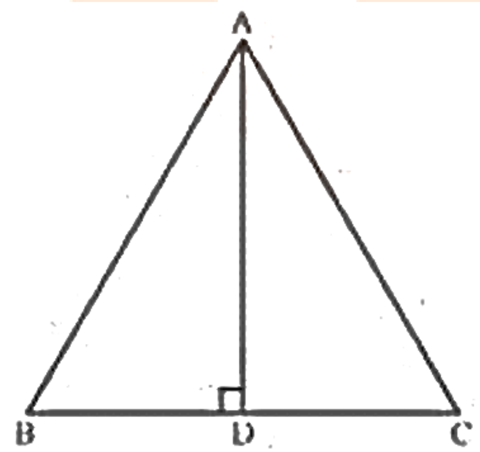
$\angle CAB = \angle DAB [AB\ bisects\ \angle A]$

$AB = AB$ [common]

$\Delta ABC \cong \Delta ABD$ [SAS criterion]

$\therefore BC = BD$ [CPCT]

24. In ΔABC, the median AD is $\perp$ to BC. Prove that ΔABC is an isosceles triangle



Ans: In ΔABD and ΔACD

$BD = CD$ [D is mid - point of BC]

$AD = AD$ [Common]

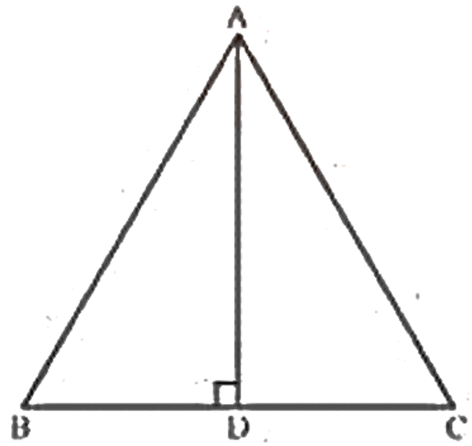
$\angle ADB = \angle ADC$ [each 90° $\because AD \perp BC$]

$\Delta ABD \cong \Delta ACD$ [By SAS]

$\therefore AB = AC$ [CPCT]

Hence, triangle ABC is an isosceles triangle.

25. Prove that ABC is isosceles if altitude AD bisects BAC



Ans: In ΔABD and ΔACD

$\angle ADB = \angle ADC$ [Each $90°$ [$AD \perp BC$]

$\angle BAD = \angle CAD$[AD bisects $\angle BAC$]

$AD = AD$[common]

$\Delta ABD \cong \Delta ACD$[By AAS]

$\Rightarrow AB = AC$[CPCT]

Thus, ΔABC is an isosceles triangle.

26. ABC is An isosceles triangle in which altitudes BE and CF are drawn to side AC and AB respectively. Show that these altitudes are equals.

Ans: In ΔABE and ΔACF,

$\angle A = \angle A\left[\text{Common}\right]$

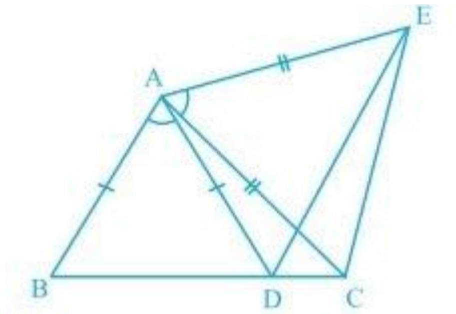
$\angle AEB = \angle AFC = 90^0$

$AB = AC$[given]

$\Delta ABE \cong \Delta ACF$[By AAS]

$\Rightarrow BE = CF$[CPCT]

27. If AC= AE, AB=AD and $\angle BAD = \angle EAC$ show that BC =DE.



Ans: In ΔBAC and ΔDAE,

$AB = AD$[given]

$AC = AE$[given]

Also, $\angle BAD = \angle EAC$[given]

$\therefore \angle BAC + \angle DAC = \angle EAC + \angle CAD$

$\Rightarrow \angle BAC = \angle EAD$

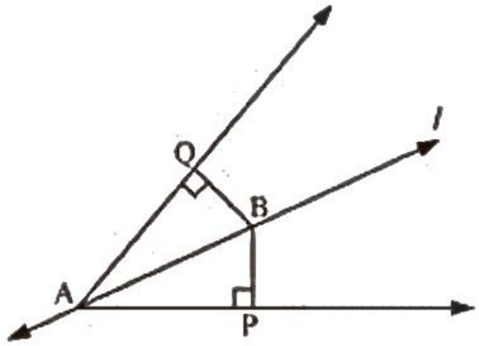
$\therefore \Delta BAC \cong \Delta DAE$[SAScriterior]

$\Rightarrow$ BC=DE[CPCT]

28. Line $\angle$ is the bisector of an angle $\angle$A and $\angle$B is any point on line l. BP and BQ are from $\angle$B to the arms of $\angle$A show that :

(i). $\Delta APB \cong \Delta AQB$

Ans: In $\Delta APB \cong \Delta AQB$,



$\angle BAP = \angle BAQ$[given]

$\angle APB = \angle AQB = 90^0$[common]

AB=AB[common]

$\therefore \Delta APB \cong \Delta AQB$[AAS rule]

(ii). BP = BQ or B is A equidistant from the arms of $\angle A$

Ans. BP=BQ[CPCT]

29. In the given figure, ΔABC is an isosceles triangle and $\angle B = 75^0$, find x.

Ans: In ΔABC,

$AB = AC$

$\Rightarrow \angle B = \angle C$[Angles opposite to equal sides are equal]

$\therefore \angle B = 75^\circ$

$\therefore \quad \angle B = \angle C = 75^\circ$

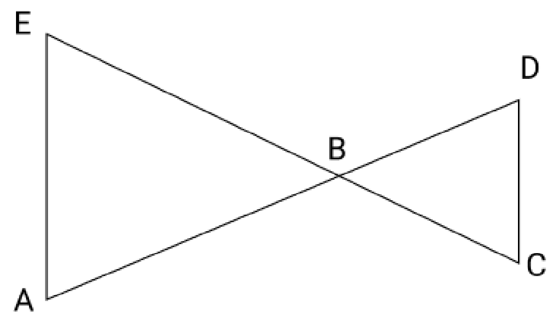
$\angle A + \angle B + \angle C = 180^{\circ}$

$x + 150 = 180^{\circ}$

$x = 30^{\circ}$

30. If E> A and C> D. prove that AD>EC.

Ans: In ΔABE,



$\angle E > \angle A$ [given]

$\Rightarrow AB > EB$[Side opposite to greater angle is larger].....(i)

Similarly,in ΔBCD,

$\angle C > \angle D$ [Given]

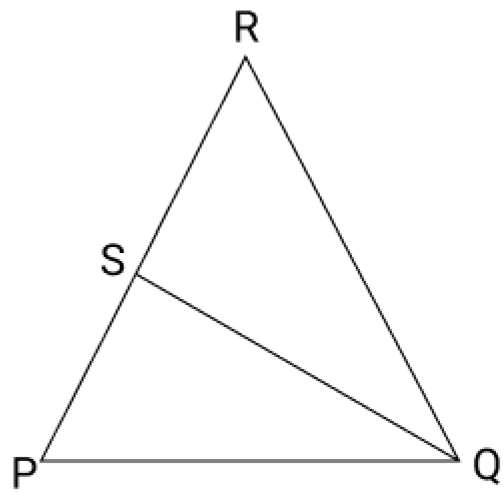
$\Rightarrow BD > BC \quad \rightarrow (ii)$

Adding (i) and (ii)

$AB + BD > EB + BC$

or $AD > EC$

31. If PQ= PR and S is any point on side PR. Prove that RS<QS.



Ans: In ΔPQR

PQ-PR[given]

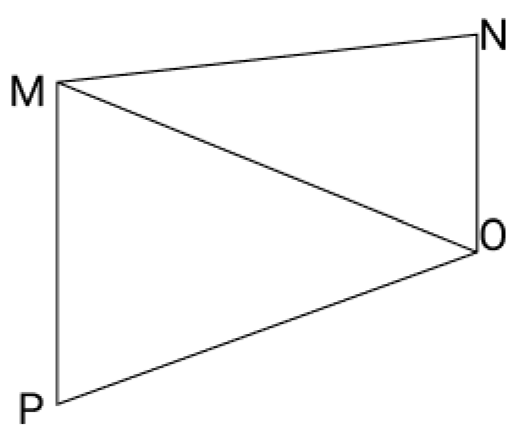
$\angle PRQ = \angle PQR$[angleoppositetoequalsideareequal]

Now,$\angle SQR < \angle PQR$[$\angle SQR$ is a part of $\angle PQR$]

$\angle SQR < \angle PRQ$ or $\angle SRQ$

$\Rightarrow RS < QS$[sideoppositetosmallerangleinΔSRQ]

32. Prove that MN+NO +OP+PM>2MO.



Ans: In ΔMON

$MN + NO > MO$ [Sum of any two side of Δ is greater than third sides]...(i)

Similarly in ΔMPQ

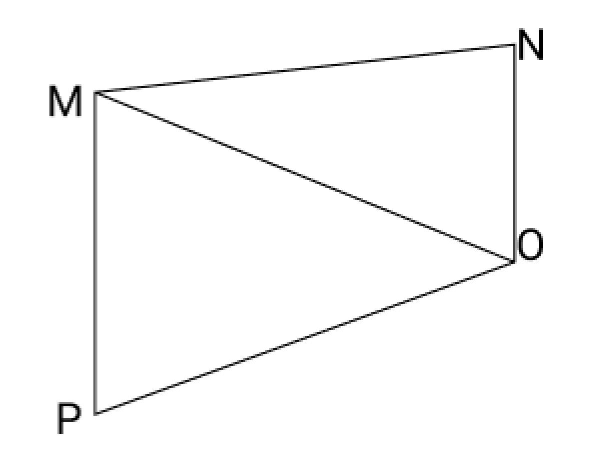
$OP + PM > MO$

Hence from (i) and (ii)

or $MN + NO + OP + PM > 2MO$

33. Prove that MN+NO+OP>PM.

Ans: In ΔMON



$MN + NO > MO$ [Sum of any two side of Δ is greater than third sides]...(i)

Similarly in ΔMOQ,

$MO + OP > PM$......(ii)

Hence from (i) and (ii)

or $MN + NO + OP + MO > MO + PM$

or $MN + NO + OP > PM$

34. ΔABC **is an equilateral triangle and** $\angle B = 60^0$**, find** $\angle C$ **.**

Ans: In ΔABC,

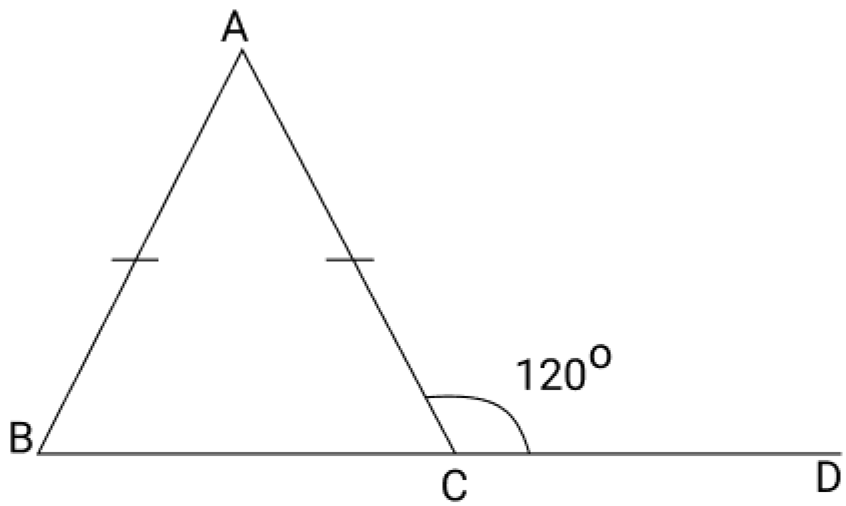
$AB = AC$

$\angle B = \angle C$[angleoppositetoequalsidesareequal]

but $\angle B = 60^0$

$so, \angle C = 60^0$

35. In the figure, AB = AC and $\angle ACD = 120^0$**, find** $\angle B$ **.**



Ans: Since, in ΔABC, AB=AC

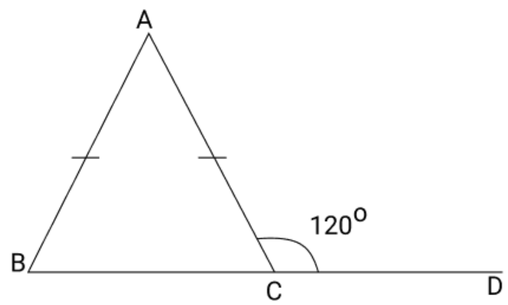
$\angle B = \angle C$[angleoppositetoequalsidesareequal]

but $\angle ACB + \angle ACD = 180^0$[Linear pair]

so, $\Delta ACB = 180^0 - 120^0$

and, $\angle C = \angle B = 60^0$

36. In the given figure, find $\angle A$



Ans: In ΔABC

$\angle A + \angle B + \angle C = 180^0$[sum of angles of a triangle]

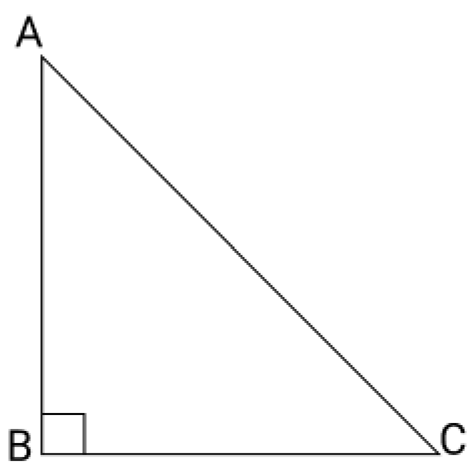
$\angle A + 60^0 + 60^0 = 180^0$

$\Rightarrow \angle A = 180^0 - 120^0$

$\angle A = 60^0$

Short Answer Type Questions **3 Marks**

1. Prove that in a right triangle, hypotenuse is the longest (or largest) side.



Ans: Given a right angled triangle ABC in which $\angle B = 90^0$

Therefore, AC is hypotenuse.

Now, since

$\angle B=90^{\circ}$

$\therefore A+\angle B+\angle C=180^{\circ}$

$\angle A+\angle C=180^{\circ}-90^{\circ}=90^{\circ}$

ie. $\angle B=\angle A+\angle C$

$\Rightarrow \angle B>\angle A$ and $\angle B>\angle C$

$\therefore AB=BC=AC\Rightarrow AB=BC$

$\Rightarrow \angle C=\angle A.\dots\dots..(i)$

Similarly, $AB=AC$

$\Rightarrow \angle C=\angle B\dots\dots\dots$ (ii)

From eq.(i)and(ii),

$\angle \mathrm{A}=\angle \mathrm{B}=\angle \mathrm{C}\ \Delta ABC$

$\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}=180^{\circ}\dots\dots\dots$(iv)

$\Rightarrow \angle \mathrm{A}+\angle \mathrm{A}+\angle \mathrm{A}=180^{\circ}\Rightarrow 3\angle \mathrm{A}=180^{\circ}$

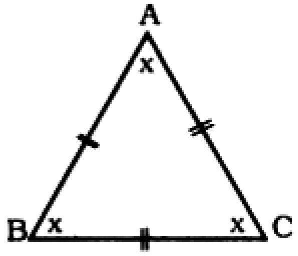
$\Rightarrow \angle \mathrm{A}=60^{\circ}$

Since $\angle \mathrm{A}=\angle \mathrm{B}=\angle \mathrm{c}$[Fromeq.(iii)]

$\therefore \angle \mathrm{A}=\angle \mathrm{B}=\angle c=60^{\circ}$

2. Show that the angles of an equilateral triangle are 60 degree each.

Ans: Let ABC be an equilateral triangle.



$\therefore AB = BC = AC \Rightarrow AB = BC$

$\Rightarrow \angle C = \angle A \ldots \ldots ....(i)$

Similarly, $AB = AC$

$\Rightarrow \angle C = \angle B \ldots \ldots ..6(ii)$ From eq.(i) and (ii).

$\angle A = \angle B = \angle C$

Now _ in _ ΔABC

$\angle A + \angle B + \angle C = 180^\circ \ldots \ldots ...$ (iv)

$\Rightarrow \angle A + \angle A + \angle A = 180^\circ$

$\Rightarrow 3\angle A = 180$

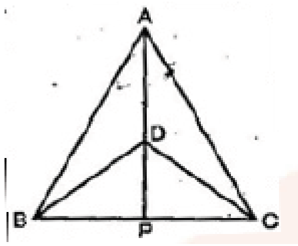
$\Rightarrow \angle A = 60^\circ$

Since $\angle A = \angle B = \angle C$[*F*rom eq. (iii)]

$\therefore \angle A = \angle B = \angle C = 60^\circ$

Hence, each angle of equilateral triangle is 60^0

3. ΔABC and ΔDBC **are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:**



(i). ABD $\cong$ ACD

DBC is an isosceles triangle.

$\mathrm{BD} = \mathrm{CD}$

Now in ABD and ACD,

AB = AC[Given]

$\mathrm{BD} = \mathrm{CD}[Given]$

$\mathrm{AD} = \mathrm{AD}[\mathrm{Common}]$

$\therefore \Delta \mathrm{ABD} \cong \Delta ACD[By\ SSS\ \text{congruency}]$

$\Rightarrow \angle \mathrm{BAD} = \angle \mathrm{CAD}[\text{By } C.P.C.T.] \ldots \ldots ...(\mathrm{i})$

(ii). $\Delta \mathrm{ABP} \cong \Delta \mathrm{ACP}$

Now in ΔABP and ΔACP,

$AB = AC[\text{Given}]$

$\angle BAD = \angle CAD[\text{From eq.}(i)]$

$\mathrm{AP} = \mathrm{AP}$

$\therefore \Delta \mathrm{ABP} = \Delta \mathrm{ACP}[\text{By SAS congruency}]$

(iii). AP bisects $\angle A$ as well as $\angle D$

Ans: since $\Delta \mathrm{ABP} \cong \Delta \mathrm{ACP}$

$\Rightarrow \angle \mathrm{BAP} = \angle \mathrm{CAP}[\mathrm{By C.P.C.T.}]$

$\Rightarrow$ AP bisects $\angle$A.

Since $\Delta ABD \cong \Delta ACD$ [From part (i)]

$\Rightarrow \angle ADB = \angle ADC$ [By C.P.C.T.] ……… (ii)

Now $\angle ADB + \angle BDP = 180^\circ$ [Linear pair]……….(iii)

And $\angle ADC + \angle CDP = 180^\circ$ [Linear pair]……….(iv)

From eq.(iii) and (iv),

$\angle ADB + \angle BDP = \angle ADC + \angle CDP$

$\Rightarrow \angle ADB + \angle BDP = \angle ADB + \angle CDP$ [Using (ii)]

$\Rightarrow \angle BDP = \angle CDP$

$\Rightarrow$ DP bisects $\angle D$ or AP bisects $\angle D$.

(iv). AP is the perpendicular bisector of BC.

Ans: $\therefore BP = PC$ [By C.P.C.T.]

And $\angle APB = \angle APC$ [By C.P.C.T.]………(vi)

Now $\angle APB + \angle APC = 180^\circ$ [Linear Pair]

$\Rightarrow \angle APB + \angle APC = 180^\circ$ [Using eq.(vi)]

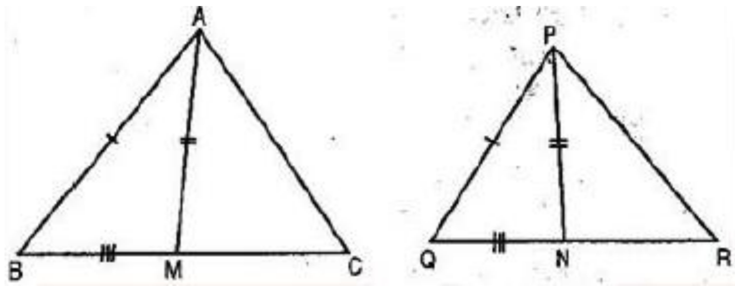
$\Rightarrow 2\angle APB = 180^\circ$

$\Rightarrow \angle APB = 90^\circ$

$\Rightarrow AP \perp BC$…………(vii)

From eq. (v), we have BP $\perp$ PC and from (vii), we have proved $AP \perp BC$. So, collectively AP is perpendicular bisector of BC.

4. Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of PQR (See figure). Show that: ABM PQN



Ans: AM is the median of VABC

$\therefore \text{BM} = \text{MC} = \frac{1}{2}\text{BC}\ldots\ldots\ldots\text{(i)}$

PN is the median of VPQR

$\therefore QN = NR = \frac{1}{2}QR\ldots\ldots\ldots(ii)$

Now $\text{BC} = \text{QR[Given]}$

$\Rightarrow \frac{1}{2}\text{BC} = \frac{1}{2}\text{QR}$

$\therefore \text{BM} = \text{QN}$

(i). $\Delta ABM \cong \Delta PQN$

Now in $\Delta ABM \cong \Delta PQN$,

AB=PQ[given]

AM=PN[given]

BM=QN[from eq.(iii)]

$\therefore \Delta ABM \cong \Delta PQN$[by SSS congruency]

$\Rightarrow \angle B = \angle Q[by\ \text{CPCT}]........(iv)$

(ii). $\vartriangle ABC \cong \vartriangle PQR$

In **$ABC{\text{ }}and{\text{ }}\vartriangle PQR$**

${\text{AB = PQ[given]}} $

$\angle B{\text{ = }}\angle {\text{Q[proven above]}}$

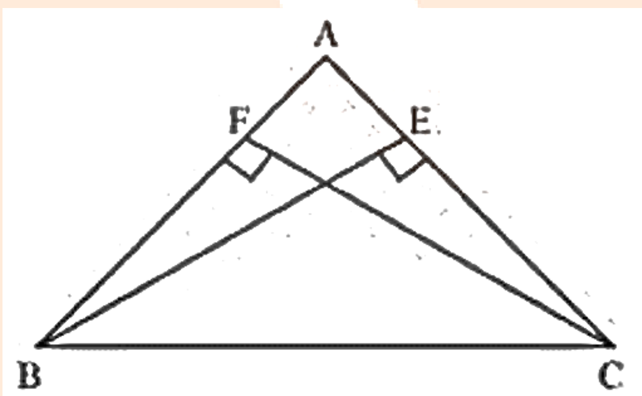
${\text{BC = QR[given]}}$

$\therefore \vartriangle ABC \cong \vartriangle PQR[{\text{by SAS congruency}}]$

5. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans. In $\vartriangle BEC{\text{ and }}\vartriangle CFB,$

$\angle BEC{\text{ = }}\angle CFB[each\,{\text{ }}{90^0}]$



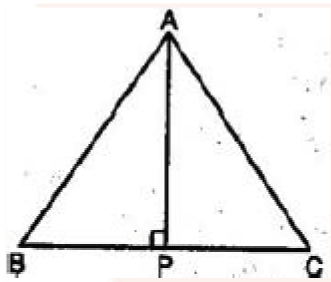
${\text{BC = BC[Common]}}$

${\text{BE = CF[Given]}}$

${\text{\Delta BEC}} \cong {\text{\Delta CFB[RHScongruency]}}$

$\Rightarrow {\text{EC = FB[By C}}{\text{.P}}{\text{.C}}{\text{.T}}{\text{.]}}.....{\text{(i)}}$

6. ABC is an isosceles triangles with AB = AC. Draw AP$\bot$ BC and show that $\angle$B = $\angle$C.



Ans. Given: ABC is an isosceles triangle in which AB = AC

${\text{Toprove:}}\angle {\text{B = }}\angle {\text{C}} $

${\text{Construction:Draw}}\;{\text{AP = BC}} $

${\text{Proof:In }}\vartriangle {\text{ABP and}}\;\vartriangle {\text{ACP}} $

$\angle {\text{APB}} = \angle {\text{APC}} = {90^\circ }[{\text{By}}\;{\text{construction}}) $

${\text{AB}} = {\text{AC[Given}}] $

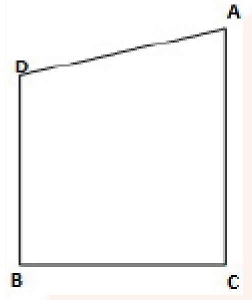
${\text{AP}} = {\text{APICommon}}] $

$

\therefore \vartriangle {\text{ABP}} \cong \vartriangle {\text{ACP[RHS congruency]}}$

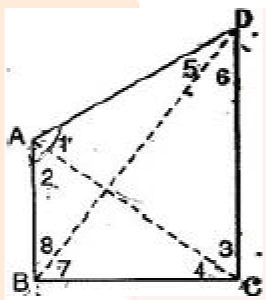
$\Rightarrow \angle B = \angle C[{\text{by CPCT}}] $

7. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (See figure). Show that $\angle $A >$\angle $ C and$\angle $ B > $\angle $D.



Ans. Given: ABCD is a quadrilateral with AB as smallest and CD as longest side.

i. **$\angle A < \angle C$**



${\text{Construction: Join AC and BD}} $

${\text{Proof:(i)In }}\vartriangle {\text{ABC,AB is the smallest side}}{\text{.}} $

$\angle {\text{4 < }}\angle {\text{2}}...........{\text{(i)}} $

${\text{[Angle opposite to smaller side is smaller]}} $

${\text{In }}\vartriangle {\text{ADC, DC is the longest side}}{\text{.}} $

$\angle {\text{3 < }}\angle {\text{1}}...........{\text{(ii)}} $

${\text{[Angle opposite to longer side is longer]}} $

${\text{Adding eq}}{\text{.(i) and (ii),}} $

$\angle {\text{4 + }}\angle {\text{3 < }}\angle {\text{1 + }}\angle {\text{2}} $

ii. $\angle B > \angle D$

${\text{In }}\vartriangle {\text{ABD, AB is the smallest side}}{\text{.}}$

$\therefore \angle {\text{5 < }}\angle {\text{8}}........{\text{(iii)}}$

${\text{[Angle opposite to smaller side is smaller]}}$

${\text{In }}\vartriangle {\text{BDC, DC is the longest side}}{\text{.}}$

$\angle {\text{6 < }}\angle {\text{7}}..........{\text{(iv)}}$

${\text{[Angle opposite to longer side is longer]}}$

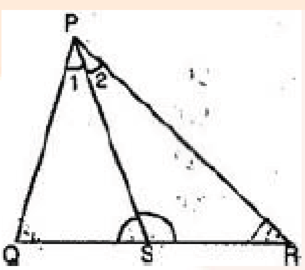
${\text{Adding eq}}{\text{.(iii) and (iv),}}$

$\angle {\text{5 + }}\angle {\text{6 < }}\angle {\text{7 + }}\angle {\text{8}}$

$\Rightarrow \angle {\text{D < }}\angle {\text{B}}$

8. In figure, PR > PQ and PS bisects QPR. Prove that PSR> PSQ.

Ans:



Ans.

${\text{In }}\vartriangle {\text{POR}},$

${\text{PR}} > {\text{PQ}}[G{\text{iven}}]$

$\therefore \angle {\text{PQR}} > \angle {\text{PR}}{{\text{Q}}_{ - \ldots }}({\text{i}})[{\text{Angle opposite to longer side is greater}}]$

${\text{Again }}\angle 1 = \angle 2 \ldots ({\text{ii}})$

$\because PS{\text{ is the bisector of }}\angle P]$

$\therefore \angle {\text{PQR}} + \angle 1 > \angle {\text{PRQ}} + \angle 2 \ldots \ldots \ldots ({\text{iii}})$

But $\angle PQS + \angle 1 + \angle PSQ = \angle PRS + \angle 2 + \angle PSR = {180^\circ }$

${\text{[Angle sum property]}}$

$\Rightarrow \angle {\text{PQR}} + \angle 1 + \angle {\text{PSQ}} = \angle {\text{PRQ}} + \angle 2 + \angle {\text{PSR}} \ldots \ldots ({\text{iv}})$

$[\angle {\text{PRS}} = \angle {\text{PRQ and }}\angle {\text{PQS}} = \angle {\text{PQR}}]$

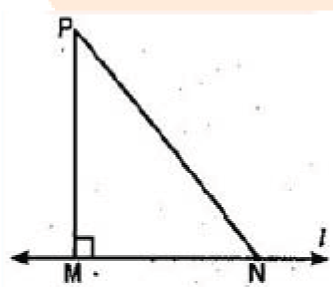
From eq.(iii) and (iv),

$\angle {\text{PSQ}} < \angle {\text{PSR}}$

${\text{ Or }}\angle {\text{PSR}} > \angle {\text{PSQ}}$

9. **Show that all the line segments drawn from a given point not on it, the perpendicular line segment is the shortest.**

Ans.



Given: l is a line and P is point not lying on I.

${\text{PM}} \bot {\text{I}}$

${\text{N is any point on other than M}}{\text{.}}$

${\text{Proof:In \Delta PMN,}} $

$\angle {\text{M is the right angle}}{\text{.}} $

$\therefore {\text{N is an acute angle}}{\text{.(Angle sum property of \Delta)}} $

$\therefore \angle {\text{M > }}\angle {\text{N}} $

$\therefore {\text{PN > PM[Side opposite greater angle]}} $

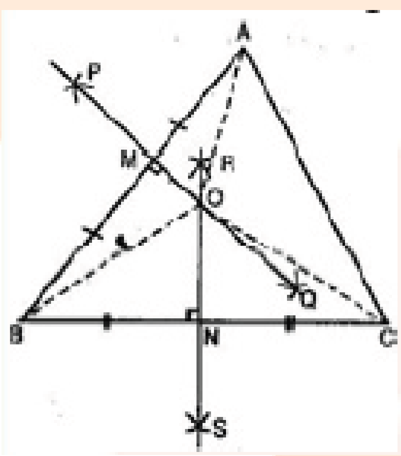
$\Rightarrow {\text{PM < PN}} $

Hence of all line segments drawn from a given point not on it, the perpendicular is the

Shortest.

10. ABC is a triangle. Locate a point in the interior of $\vartriangle ABC$ which is equidistant from all the vertices of$\vartriangle ABC$.

Ans. Let ABC be a triangle.



Draw perpendicular bisectors PQ and RS of sides AB and BC respectively of triangle ABC. Let PQ bisects AB at M and RS bisects BC at point N.

Let PQ and RS intersect at point O.

Join OA, OB and OC.

Now in AOM and BOM,

AM = MB [By construction]

$\angle AMO = \angle BMO = 90°$ [Byconstruction]

OM=OM[Common]

$\therefore \Delta AOM \cong \Delta BOM$[BySAScongruency]

$\Rightarrow$ OA=OB[ByC.P.C.T.]…...(i)

Similarly$\Delta BON \cong \Delta CON$

$\Rightarrow$ OB=OC[ByC.P.C.T.].....(ii)

Fromeq.(i)and(ii),

OA = OB = OC

Hence O, the point of intersection of perpendicular bisectors of any two sides of ΔABC equidistant from its vertices.

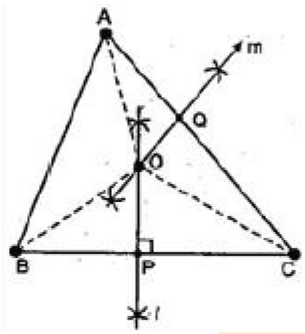
11. In a huge park, people are concentrated at three points (See figure).

A: where there are different slides and swings for children.

B: near which a man-made lake is situated.

C: which is near to a large parking and exit.

Where should an ice cream parlour be set up so that maximum number of persons can approach it?



Ans: The parlour should be equidistant from A, B and C.

For this let we draw perpendicular bisector say l of line joining points B and C also draw perpendicular bisector say m of line joining points A and C.

Let l and m intersect each other at point O.

Now point O is equidistant from points A, B and C.

Join OA, OB and OC.

Proof: in $\Delta BOP and \Delta COP$,

$OP = OP [Common]$

$\angle OPB = \angle OPC = 90° [By construction]$

$BP = PC [midpoint of BC]$

$\therefore \Delta BOP \cong \Delta COP [By SAS congruency]$

$\Rightarrow OB = OC [By C.P.C.T.] \ldots..(i)$

$Similarly \Delta AOQ \cong \Delta COQ$

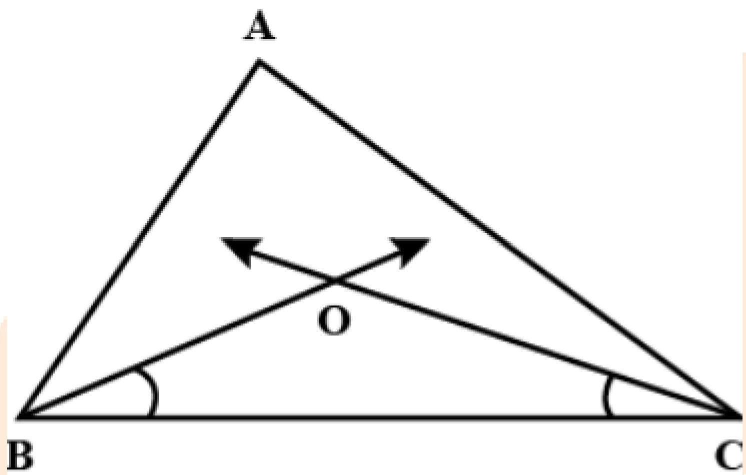
$\Rightarrow OA = OC [By C.P.C.T.].....(ii)$

$From eq. (i) and (ii),$

$OA = OB = OC$

Therefore, ice cream parlour should be set up at point O, the point of intersection of perpendicular bisectors of any two sides out of three formed by joining these points.

12. If ABC, the bisector of ABC and BCA intersect each other at the point O prove that $\angle BOC = 90° + \frac{\angle A}{2}$



Ans: In ΔBOC, we know,

$\angle 1 + \angle 2 + \angle BOC = 180°$.....(l)

In �ABC,we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 2(\angle 1) + 2(\angle 2) = 180°$

$\Rightarrow \frac{\angle A}{2} + \angle 1 + \angle 2 = 90^{\circ}$

$\Rightarrow \angle 1 + \angle 2 = 90^0 - \frac{\angle A}{2}$

Substituting this value of $\angle 1 + \angle 2$ in (1)

$90° - \frac{\angle A}{2} + \angle BOC = 18°$

$\angle BOC = 90° + \frac{\angle A}{2}$

So, $\angle BOC = 90^{\circ} + \frac{\angle A}{2}$

13. Prove that if one angle of a triangle is equal to the sum of the other two angles, the triangle is right angled

Ans: $\angle A + \angle B + \angle C = 180^{\circ}$[Sum of three angles of triangle is180°......(1)

Given that:$\angle A + \angle C = \angle B$....(2)

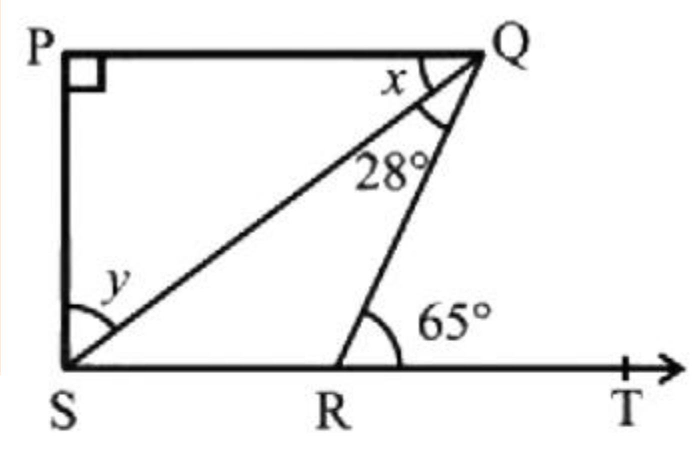
From(1)and(2),

$\angle B + \angle B = 180°$

$\Rightarrow \angle B = \dfrac{180^{\circ}}{2} = 90^{\circ}$

HenceΔABCisrightangled.

14. If fig, if PQ$\perp$ PS, PQ $\perp$SR, $\angle$SQR = $28^{0}$and $\angle$QRT =65^{0} , then find the values of X and Y.



Ans: $PQ \parallel SR$ and QR is the transversal,

$\therefore \angle PQR = \angle QRT$[pairof alternateangles]

or$\angle PQS + \angle SQR = \angle QRT$

or x + 28°= 65°

x = 65°- 28°= 37°

Also in ΔPQS,

$\angle SPQ + \angle PSQ + \angle PQS = 180°$

$\Rightarrow 90° + y + x = 180°$

or $90° + y + 37° = 180°$

$y = 53°$

15. If AD= AE and D and E are point on BC such that BD=EC prove that AB=AC.

Ans: In �ADE,

$AD = AE[Given]$

$\therefore \angle ADE = \angle AED$[angles opposite to equal side are equal]

Now, $\angle ADE + \angle ADB = 180^{\circ}[linearpair]$

Also, $\angle AED + \angle AEC = 180^{\circ}[linearpair]$

$\Rightarrow \angle ADE + \angle ADB = \angle AED + \angle AEC$

But, $\angle ADE = \angle AED$

Now in, ΔABD *and* ΔACE,

$BD = CE$

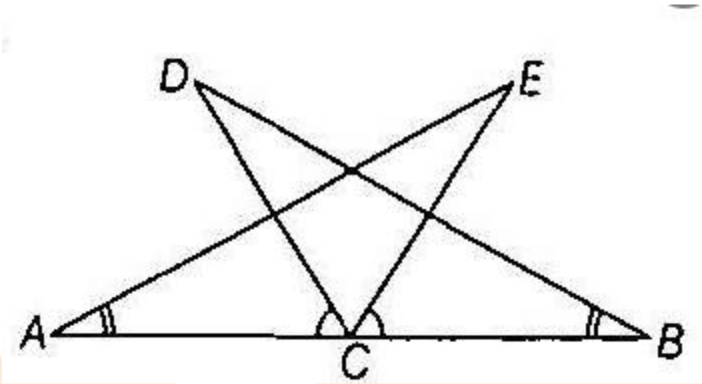
$AD = AE$

$\angle ADB = \angle AEC$

$\therefore \Delta ABC \cong \Delta ACE\ [By SAS]$

$\Rightarrow AB = AC[CPCT]$

16. In the given figure, AC=BC, $\angle$ DCA= $\angle$ECB and $\angle$DBC=$\angle$ EAC. Prove that ΔDBC and ΔEAC are congruent and hence DC=EC.



Ans: We know,

$\angle DCA = \angle ECB [Given]$

$\Rightarrow \angle DCA + \angle ECD = \angle ECB + \angle ECD [adding \angle ECD on both sides]$

$\Rightarrow \angle ECA = \angle DCB ...(i)$

$\angle DCB = \angle ECA [From (i)]$

Now in, ΔDBC *and* ΔEAC

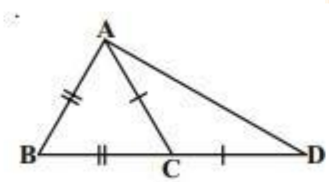
$BC = AC [given]$

$\angle DBC = \angle EAC [given]$

$\Delta DBC \cong \Delta EAC [By SAS]$

$\Rightarrow DC = EC [CPCT]$

17. From the following figure, prove that $\angle BAD = 3\angle ADB$.



Ans: Let $\angle ADC = Q$

$\Rightarrow \angle CAD = Q[\because, CA = CD]$

Exterior $\angle ACB = \angle CAD = Q + Q = 2Q$

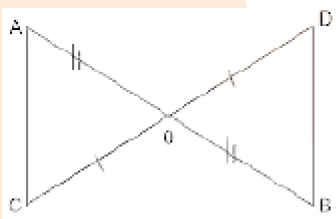
$\Rightarrow \angle BAC = 2Q[\because BA = BC]$

$\angle BAD = \angle BAC + \angle CAD$

Hence $= 2Q + Q = 3Q$

$\Rightarrow 3\angle ADC = 3\angle ADB$

18. Is the mid-point of AB and CD. Prove that AC=BD and $AC\parallel BD$.



Ans: In ΔAOC *and* ΔBOD

$AO = OB$[O is the mid - point of AB]

$\angle AOC = \angle BOD$[vertically opposite angles]

$CO = OD$[O is the mid - point of CD]

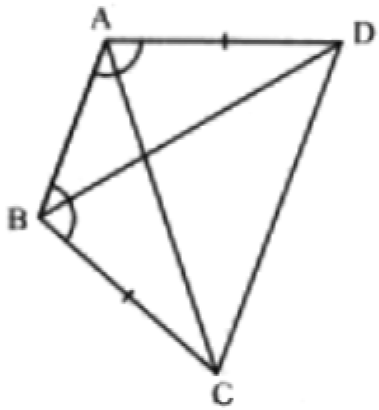
$\Delta AOC \cong \Delta BOD$[By SAS]

$AC = BD$[CPCT]

$\Rightarrow \angle CAO = \angle DBO$[CPCT]

Now, AC and BD are two lines inter sected by a transversal AB such that i.e. alternate angle are equal

19. ABCD is a quadrilateral in which AD=BC and $\angle$DAB= $\angle$CBA. Prove that.



(i). $\Delta ABD \cong \Delta BAC$

Ans: In ΔABD and ΔBAC,

AD=BC[given]

$\angle DAB = \angle CBA$[given]

AB=AB[Common]

because $\Delta ABD \cong \Delta BAC$[SAS criterion]

(ii). $BD = AC$

Ans: $BD = AC$ [by CPCT]

(iii). $\angle ABD = \angle BAC$

Ans: In reference with the first part

$\angle ABD = \angle BAC$

20. AB is a line segment. AX and BY are equal two equal line segments drawn on opposite side of line AB such that $AX \parallel BY$. If AB and XY intersect each other at P. prove that

(i). $\Delta APX \cong \Delta BPY$

Ans: In ΔAPX and ΔBPY ,

$\angle 1 = \angle 2$[alternate angle]

$\angle 3 = \angle 4$[verticallyoppositeangle]

AX = BY [given]

$\therefore \Delta APX \cong \Delta BPY\ [By AAS]$

(ii). AB and XY bisect each other at P.

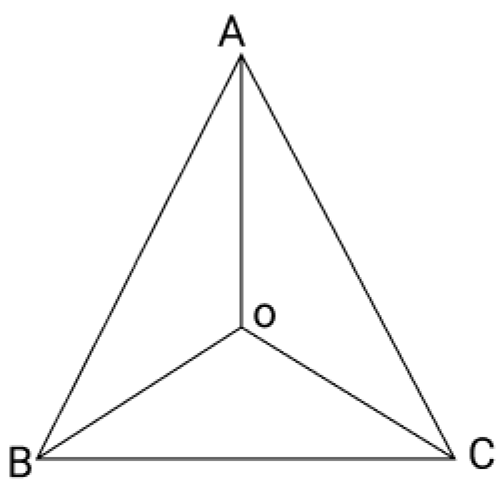
Ans: $AX = BY[given]$

$\therefore \Delta APX \cong \Delta BPY\ [By AAS]$

$\Rightarrow AP = BP\ and\ PX = PY$[CPCT]

$\Rightarrow$ AB and XY bisects each other at P

21. In an isosceles ΔABC**, with AB =AC, the bisector of** $\angle$**B and** $\angle$**C intersect each other at O, join A to O. show that:**



(i). OB=OC

Ans: In ΔABC

$AB = AC[\text{given}]$

$\angle ACB = \angle ABC[\text{angles opposite to equal side}]$

$\therefore \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

or $\angle OCB = \angle OBC$

$\Rightarrow OB = OC[\text{side opposite to equal angle}]$

(ii). AO bisects $\angle A$.

Ans: In ΔAOB and ΔAOC

$AB = AC[\text{given}]$

$\angle ABO = \angle ACO[\text{Halves of equals}]$

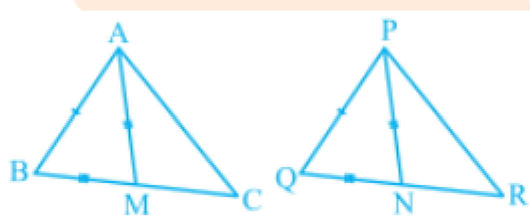
$OB = OC[\text{proved}]$

$\therefore \Delta AOB \cong \Delta AOC[\text{SAS rule}]$

$\Rightarrow \angle BAO = \angle CAO[\text{CPCT}]$

i.e. AO bisects $\angle A$

22. Two side AB and BC and median AM of a triangle ABC are respectively equal to side PQ and QR and median PN of ΔPQR, show that



(i). $\Delta ABM \cong \Delta PQN$

Ans: *In* ΔABM *and* ΔPQN

$AB = PQ[Given]$

$BM = QN[\mathit{Halves\ of\ equal}]$

AP = PN[Given]

$\therefore \Delta ABM \cong \Delta PQN[\mathit{SSS\ rules}]$

(ii). $\Delta ABC \cong \Delta PQR$

Ans: $\Rightarrow \angle B = \angle Q$

Now, in ΔABC *and* PQR,

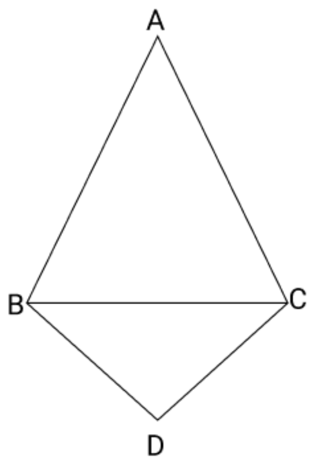
$AB = PQ[\mathit{Given}]$

BC = QR[Given]

$\angle B = \angle Q[\mathit{Proved}]$

$\therefore \Delta ABC \cong \Delta PQR.[\mathit{SASrule}]$

23. In the given figure, ABC and DBC are two triangles on the same base BC such that AB=AC and DB=DC. Prove that $\angle ABD = \angle ACD.$,



Ans: In ΔABC,

$AB = AC[\mathit{Given}]$

$\therefore \angle ABC = \angle ACB$[angles opposite to equal side are equals]

Similarly in, ΔDBC,

$DB = DC[Given].....(1)$

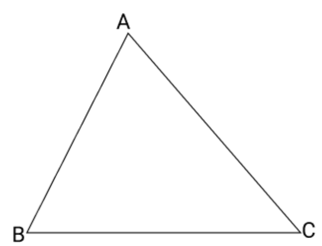
$\therefore \angle DBC = \angle DCB$

Adding(1)and(2)

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

or; $\angle ABD = \angle ACD$

24. Prove that the Angle opposite to the greatest side of a triangle is greater than two third of a right angle i.e. greater than 60^0



Ans: In ΔABC,

$AB > BC[given]$

$\angle C > \angle A[angle$ opposite to large side is greater$].....(i)$

similarly,

$AB > AC$

$\therefore \angle C > \angle B............(ii)$

adding (i) and (ii),

$2\angle C > (\angle A + \angle B)$

adding $\angle$C to both sides,

$3\angle C > (\angle A + \angle B + \angle C)$

$3\angle C > 180^0$[sum of three angles of a triangle is 180^0]

or $\angle C > 60^0$

25. AD is the bisector of $\angle$A of ABC, where D lies on BC. Prove that AB>BD and AC>CD.

Ans: In ΔADC

$\angle 3 > \angle 2$[Exterior angles of Δ is greater than each of the interior opposite angles.

but $\angle 2 = \angle 1$[AD bisects $\angle A$]

$\therefore \angle 3 = \angle 1$[Sideoppositetogreaterangleislarger]

$\Rightarrow AB > BD$

In ΔABD

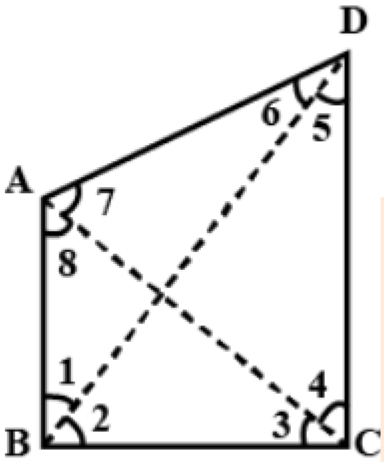
$\angle 4 > \angle 1$[ExterioranglesofΔ isgreaterthaneachoftheinterioroppositeangles]

But, $\angle 1 = \angle 2$

$\therefore \angle 4 > \angle 2$

$\Rightarrow AC > CD$[Sideoppositetogreaterangleislarger].

26. In the given figure, AB and CD are respectively the smallest and the largest side of a quadrilateral ABCD. Prove that $\angle$A> $\angle$C and $\angle$B>$\angle$ D.



Ans: JoinAC.

In ΔABC

$BC > AB$ [ABisthesmallestsidesofquadrilateralABCD]

$\Rightarrow \angle 1 > \angle 3$[Angle opposite to larger side is greater]...(i)

In ΔADC

$CD > AD$[CD is the largest side of quadrilateral ABCD]

$\angle 2 > \angle 4$[angle opposite to larger side is greater]....(ii)

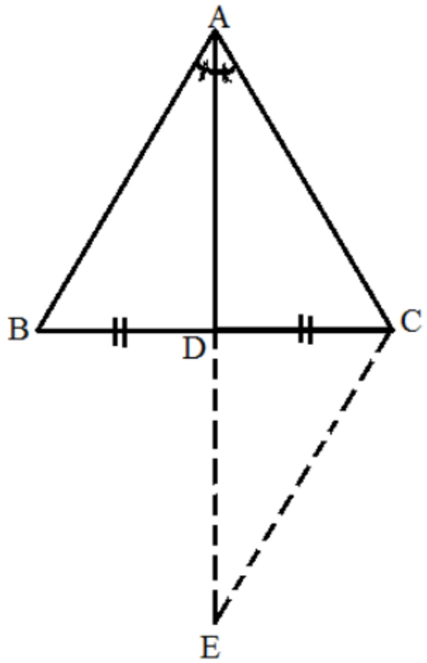
Adding (i) and (ii)

$\angle 1 + \angle 2 > \angle 3 + \angle 4$

Or $\angle A > \angle C$

Similarly,by joining BD,we can show that$\angle B > \angle D$

27. If the bisector of a vertical angle of a triangle also bisects the opposite side; prove that the triangle is an isosceles triangle.



Ans: In ΔADC and ΔEDB,

$DC = DB$[Given]

$AD = ED$[By construction]

$\angle ADC = \angle EDB$ [vertically opposite angle]

$\therefore \Delta ADC \cong \angle EDB$[By SAS]

$\Rightarrow AC = EB$ and $\angle DAC = \angle DEB$[CPCT]

But, $\angle DAC = \angle BAD$[$\because$ AD bisects $\angle A$]

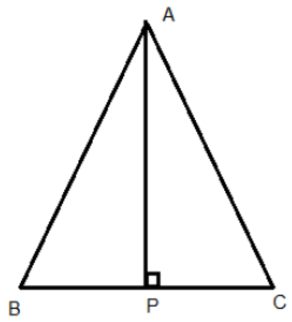
$\therefore \angle BAD = \angle DEB$

$\Rightarrow AB = BE$

But BE=AC[Proved above]

$\therefore AB = AC$

28. ABC is an isosceles triangle with AB = AC. Draw AP $\perp$BC to show that $\angle B = \angle C$



Ans: In a right angled triangle APB and APC,

AP=AP[common]

HypotenuseAB = HypotenuseAC[Given]

$\therefore \Delta APB \cong \Delta APC$[RHSrule]

$\Rightarrow \angle$B = $\angle$C[CPCT]

29. AD is an altitude of an isosceles triangle ABC in which AB = AC. Prove that:

(i). AD bisects BC

Ans: In right triangle ABD and ACD,

Side AD = Side AD[common]

Hypotenuse AB = Hypotenuse AC [Given]

$\therefore \Delta ABD \cong \Delta ACD$[by RSH]

$\Rightarrow BD$ = CD[CPCT]

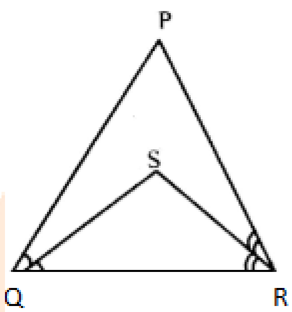
Also, AD bisects BC

(ii). AD bisects $\angle A$

Ans: $\angle BAD$ = $\angle$CAD[CPCT]

$\therefore$ AD bisects $\angle A$

30. In the given figure, PQ>PR, QS and RS are the bisectors of the $\angle$Q and $\angle$R respectively. Prove that SQ>SR.



Ans: Since PQ>PR

$\therefore \angle R > \angle Q$[angle opposite to larger side is larger]

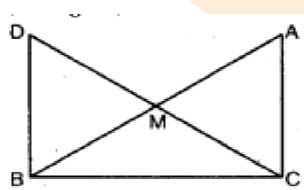
$\Rightarrow \frac{1}{2}\angle R > \frac{1}{2}\angle Q$

$\Rightarrow \angle SRQ > \angle SQR$

$\Rightarrow SQ > SR$[Sideoppositetogreaterangleislarger]

Long Answer Type Questions **4 Marks**

1. In right triangle ABC, right angled at C, M is the m`id-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. (See figure)



Show that:

(i) $\Delta AMC \cong \Delta BMD$

Ans: In ΔAMC and ΔBMD

AM=BM[ABisthemid-pointofAB]

$\angle AMC = \angle BMD$[Verticallyoppositeangles]

CM=DM[Given]

$\therefore \Delta AMC \cong \Delta BMD$[BySAScongruency]

$\therefore \angle ACM = \angle BDM$......(i)

$\angle CAM = \angle DBM$andAC=BD[byC.P.C.T.]

(ii) $\Delta DBC \cong \Delta ABC$

Ans: Now in ΔDBC and ΔABC

DB=AC[Proved-inpart(i)]

$\angle DBC = \angle ACB = 90°$[Provedinpart(ii)]

BC=BC[Common]

$\therefore \Delta DBC \cong \Delta ACB$[BySAScongruency]

(iii) CM = AB

Ans: Since $\Delta DBC \cong \Delta ACB$ [Proved above]

$\therefore DC = AB$

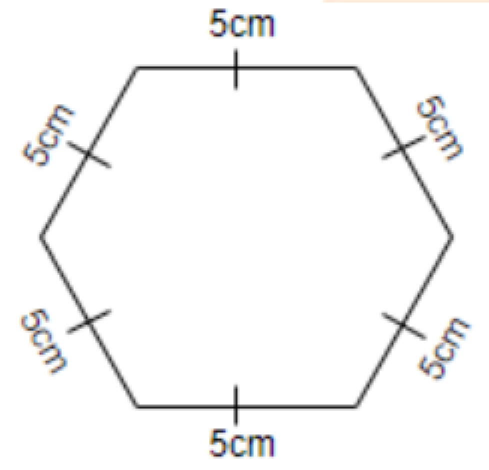
$\Rightarrow AM + CM = AB$

$\Rightarrow CM + CM = AB \ [\because DM = CM]$

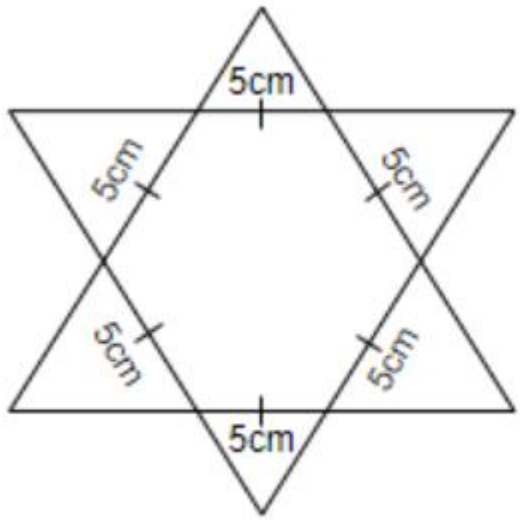
$\Rightarrow 2CM = AB$

$\Rightarrow CM = \frac{1}{2} AB$

2. Complete the hexagonal rangoli and the star rangolies (See figure) but filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



(i)



(ii)

Ans: In hexagonal rangoli, Number of equilateral triangles each of side 5 cm are 6.

Area of equilateral triangle $=\frac{\sqrt{3}}{4}(\text{ side })^2=\frac{\sqrt{3}}{4}(5)^2=\frac{\sqrt{3}}{4}\times 25$ sq. cm

Area of hexagonal rangoli = 6 x Area of an equilateral triangle

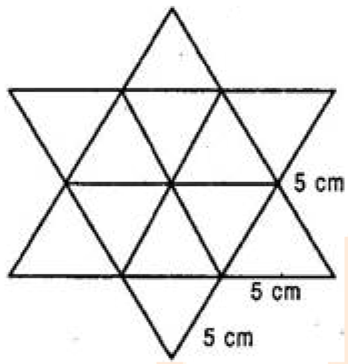
$=6\times\frac{\sqrt{3}}{4}\times 25=150\times\frac{\sqrt{3}}{4}\text{sq}\cdot\text{cm}\ldots\ldots$ (1)

Now area of equilateral triangle of side 1 cm= $\frac{\sqrt{3}}{4}(\text{ side })^2=\frac{\sqrt{3}}{4}(1)^2=\frac{\sqrt{3}}{4}$ sq. cm. ...(2)

Number of equilateral triangles each of side 1 cm in hexagonal rangoli

$=150\times\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}=150\times\frac{\sqrt{3}}{4}\times\frac{4}{\sqrt{3}}-150\ldots.(3)$

Now in Star rangoli,



Number of equilateral triangles each of side 5cm = 12

Therefore, total area of star rangoli = 12

Area of an equilateral triangle of side 5cm

$$= 12 \times \left(\frac{\sqrt{3}}{4} (5)^2 \right)$$

$$= 12 \times \frac{\sqrt{3}}{4} \times 25$$

$$= 300 \frac{\sqrt{3}}{4} sq, cm \ldots \ldots \ldots (iv)$$

Number of equilateral triangles each of side 1 cm in star rangoli

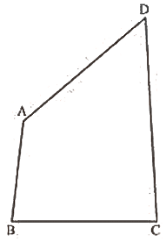
$$= 300 \frac{\sqrt{3}}{4} \div \frac{\sqrt{3}}{4}$$

$$= 300 \frac{\sqrt{3}}{4} \times \frac{4}{\sqrt{3}}$$

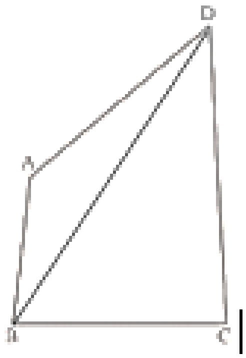
$$= 300 \ldots \ldots (v)$$

From eq. (iii) and (v), we observe that star rangoli has more equilateral triangles each of side=1 cm

3. Prove that the sum of the quadrilateral is 360^0 ?



Ans:



Join B and D

to obtain two triangles

ABD $\perp BCD$

$\angle BAD + \angle ABD + \angle BDA = 180^\circ$ [sum of three angles of Δ is $180°$].....(1)

$\angle CBD + \angle BCD + \angle CDB = 180^\circ$ [sum of three angles of Δ is $180°$]...(2)

Adding, (1) and (2)

$\angle BAD + \angle ABD + \angle BDA + \angle CBD + \angle BGCD + \angle BCD + \angle CDB = 360^\circ$

or $\angle BAD + (\angle ABD + \angle CBD) + \angle BCD + (\angle CDB + \angle BDA) = 360^\circ$

or $\angle BAD + \angle ABC + \angle BCD + \angle CDA = 360^\circ$

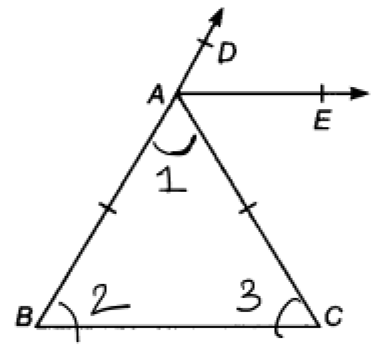
$\angle A + \angle B + \angle C + \angle D = 360^\circ$

So, Sum of quadrilateral is 360^0

Hence, proved.

4. ABC is an isosceles triangle with AB=AC. AD bisects the exterior A. Prove that $AD\parallel BC$.

Ans: Since AD bisects the exterior A



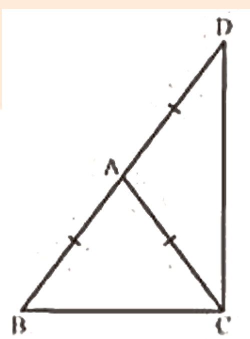
,

$\angle \text{EAD} = \frac{1}{2}\angle EAC$

$= \frac{1}{2}\left[180^{\circ} - \angle 1\right] = 90^{\circ} - \frac{1}{2}\angle 1 \ldots (i)$

$\left[\therefore \angle 1 + \angle EAC = 180^{\circ} (\text{ Linear pair })\right]$

5. ABC is an isosceles triangle in which AB=AC and side BA is produced to D such that AD=AB. Show that BCD is a right angle.



Ans: $\angle ABC = \angle ACB$[anglesoppositetoequalside]

Also, $\angle \text{ACD} = \angle \text{ADC}$[angles opposite to equal side]

Now

$\angle BAC+\angle CAD=180^{\circ}$[linear pair]

Also, $\angle CAD=\angle ABC+\angle ACB$[exterior angle of ΔABC]

$=2\angle \mathrm{ACB}$[exteriorangleof $\Delta \mathrm{ABC}$]

Also, $\angle BAC=\angle ACD+\angle ADE$

$=2\angle \mathrm{ACD}$

$\therefore \angle \mathrm{BAC}+\angle \mathrm{CAD}$

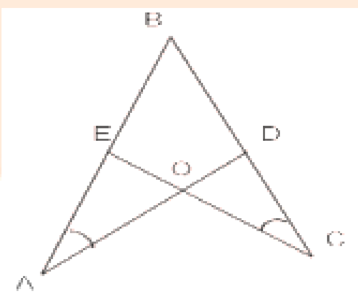
$=2(\angle \mathrm{ACD}+\angle \mathrm{ACB})$

$=2\angle \mathrm{BCD}$

i.e. $2\angle \mathrm{BCD}=180^{\circ}$

or $\angle \mathrm{BCD}=90^{\circ}$

6. In the given figure, $\angle$ A= $\angle$ C and AB =BC. Prove that $\Delta \mathrm{ABD}\cong \Delta CBE$



Ans: In ΔAOE and ΔCOD

$\angle \mathrm{A}=\angle \mathrm{C}$[Given]

$\angle AOE=\angle \mathrm{COD}$[verticallyoppositeangle]

$\therefore \angle \mathrm{A}+\angle AOE=\angle C+\angle COD$

$\Rightarrow 180^{\circ}-\angle AEO=180^{\circ}-\angle CDO$

$\angle \mathrm{A}+\angle AOE+\angle AEO=180^{\circ}$ and

$\Rightarrow \angle AEO = \angle CDO \rightarrow$ (i)

Now

$\angle AEO + \angle OEB = 180^{\circ}$[linear pair]

And$\angle CDO + \angle ODB = 180^{\circ}$[linear pair]

$\Rightarrow \angle AEO + \angle OEB = \angle CDO + \angle ODB$

$\Rightarrow \angle OEB = \angle ODB$(Using(i)]

$\Rightarrow \angle CEB = \angle ADB \rightarrow (ii)$

Now,in ΔABD and ΔCBE,

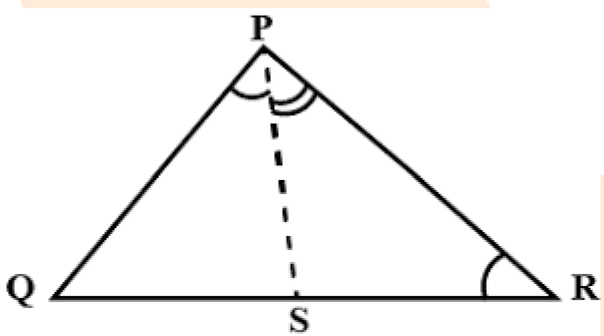
$\angle A = \angle C$[Given]

$\angle ADB = \angle CEB$[From(ii)]

$AB = CB$

$\Delta ABD \cong \Delta CBE$[ByAAS]

7. In the given figure, PR>PQ and PS is the bisector of Prove that $\angle PSR > \angle PSQ$



Ans: In ΔPOR

$PR > PQ$

$\Rightarrow \angle 3 > \angle 4$[angle opposite to larger side....(i)

Also, $\angle 6 = \angle 1 + \angle 3$[Exterior angle theorem]....(ii)

Similarly, $\angle 5 = \angle 2 + \angle 4$

But, $\angle 2 = \angle 1$[PS bisects $\angle$QPR]

$\therefore \angle 5 = \angle 1 + \angle 4 \ldots$.(iii)

Subtracting (iii) from (ii)

$\angle 6 - \angle 5 = (\angle 1 + \angle 3) - (\angle 1 + \angle 4)$

Or $\angle 6 - \angle 5 = \angle 3 - \angle 4 \ldots$.(iv)

Now

$\angle 3 > \angle 4$

$\Rightarrow \angle 3 - \angle 4 = 0 \quad \rightarrow (v)$

From (iv) and (iii)

$\angle 6 - \angle 5 > 0$

$\angle 6 > \angle 5$

Or $\angle$PSR > $\angle$PSQ