NCERT Solutions for Class 9

Maths

Chapter 2 – Polynomials

Exercise 2.4

1. Use suitable identities to find the following products:

(i) $\mathbf{(x+4)(x+10)}$

Ans: Using the identity, $(x+a)(x+b)=x^2+(a+b)x+ab$

Here we have, $a=4, b=10$

We get,

$(x+4)(x+10)=x^2+(4+10)x+(4)(10)$

$=x^2+14x+40$

(ii) $\mathbf{(x+8)(x-10)}$

Ans: Using the identity, $(x+a)(x+b)=x^2+(a+b)x+ab$

Here we have, $a=8, b=-10$

We get,

$(x+8)(x+(-10))=x^2+(8+(-10))x+(8)(-10)$

$(x+8)(x-10)=x^2+(8-10)x-80$

$=x^2-2x-80$

(iii) $\mathbf{(3x+4)(3x-5)}$

Ans: Using the identity, $(x+a)(x+b)=x^2+(a+b)x+ab$

Here we have, $a=4, b=-5$

We get,

$(3x+4)(3x+(-5))=(3x)^2+(4+(-5))3x+(4)(-5)$

$(3x+4)(3x-5)=9x^2+(4-5)3x-20$

$=9x^2-3x-20$

(iv) $\mathbf{\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)}$

Ans: Using the identity, $(x+y)(x-y)=x^2-y^2$

Here we have, $x=y^2, y=\frac{3}{2}$

We get,

$$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=\left(y^2\right)^2-\left(\frac{3}{2}\right)^2$$

$$=y^4-\frac{9}{4}$$

(v) $(3-2x)(3+2x)$

Ans: Using the identity, $(x+y)(x-y)=x^2-y^2$

Here we have, $x=3, y=2x$

We get,

$$(3+2x)(3-2x)=(3)^2-(2x)^2$$

$$=9-4x^2$$

2. Evaluate the following products without multiplying directly:

(i) 103×107

Ans: $103\times107=(100+3)\times(100+7)$

By using the identity, $(x+a)(x+b)=x^2+(a+b)x+ab$

Here we have, $x=100,\ a=3,\ b=7$

We get,

$$(100+3)(100+7)=(100)^2+(3+7)100+(3)(7)$$

$$(103)\times(107)=10000+1000+21$$

$$=11021$$

(ii) 95×96

Ans: $95\times96=(100-5)\times(100-4)$

By using the identity, $(x-a)(x-b)=x^2-(a+b)x+ab$

Here we have, $x=100,\ a=5,\ b=4$

We get,

$$(100-5)(100-4)=(100)^2-(5+4)100+(5)(4)$$

$$(95)\times(96)=10000-900+20$$

$$=9120$$

(iii) 104×96

Ans: $104\times 96=(100+4)\times(100-4)$

By using the identity, $(x+y)(x-y)=x^2-y^2$

Here we have, $x=100,\ y=4$

We get,

$$(100+4)(100-4)=(100)^2-(4)^2$$

$$(104)\times(96)=10000-16$$

$$=9984$$

3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Ans: $9x^2+6xy+y^2=(3x)^2+2(3x)(y)+(y)^2$

By using the identity, $x^2+2xy+y^2=(x+y)^2$

Here, $x=3x,\ y=y$

$$9x^2+6xy+y^2=(3x)^2+2(3x)(y)+(y)^2$$

$$=(3x+y)^2$$

$$=(3x+y)(3x+y)$$

(ii) $4y^2-4y+1$

Ans: $4y^2-4y+1=(2y)^2-2(2y)(1)+(1)^2$

By using the identity, $x^2-2xy+y^2=(x-y)^2$

Here, $x=2y,\ y=1$

$$4y^2-4y+1=(2y)^2-2(2y)(1)+(1)^2$$

$$=(2y-1)^2$$

$= (2y-1)(2y-1)$

(iii) $\mathbf{x^2 - \frac{y^2}{100}}$

Ans: $x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2$

By using the identity, $x^2 - y^2 = (x+y)(x-y)$

Here, $x = x,\ y = \frac{y}{10}$

$x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2$

$= \left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

4. Expand each of the following, using suitable identities:

(i) $\mathbf{(x + 2y + 4z)^2}$

Ans: By using the identity, $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = x,\ y = 2y,\ z = 4z$

$(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $\mathbf{(2x - y + z)^2}$

Ans: By using the identity, $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = 2x,\ y = -y,\ z = z$

$(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $\mathbf{(-2x + 3y + 2z)^2}$

Ans: By using the identity, $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = -2x,\ y = 3y,\ z = 2z$

$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) $(3a - 7b - c)^2$

Ans: By using the identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = 3a,\ y = -7b,\ z = -c$

$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(v) $(-2x + 5y - 3z)^2$

Ans: By using the identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = -2x,\ y = 5y,\ z = -3z$

$(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

Ans: By using the identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = \frac{1}{4}a,\ y = -\frac{1}{2}b,\ z = 1$

$\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

5. Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Ans: By using the identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can see that, $x^2+y^2+z^2+2xy+2yz+2zx=(x+y+z)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz=(2x)^2+(3y)^2+(-4z)^2$

$+2(2x)(3y)+2(3y)(-4z)+2(-4z)(2x)$

$=(2x+3y-4z)^2$

$=(2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Ans: By using the identity, $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

We can see that, $x^2+y^2+z^2+2xy+2yz+2zx=(x+y+z)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz=\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\left(-\sqrt{2}x\right)(y)$

$+2(y)\left(2\sqrt{2}z\right)+2\left(2\sqrt{2}z\right)\left(-\sqrt{2}x\right)$

$=\left(-\sqrt{2}x+y-2\sqrt{2}z\right)^2$

$=\left(-\sqrt{2}x+y-2\sqrt{2}z\right)\left(-\sqrt{2}x+y-2\sqrt{2}z\right)$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

Ans: By using the identity, $(x+y)^3=x^3+y^3+3xy(x+y)$

$(2x+1)^3=(2x)^3+(1)^3+3(2x)(1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+1+12x^2+6x$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Ans: By using the identity, $(x-y)^3=x^3-y^3-3xy(x-y)$

$$\left(2a-3b\right)^3=\left(2a\right)^3-\left(3b\right)^3-3\left(2a\right)\left(3b\right)\left(2a-3b\right)$$

$$=8x^3-27b^3-18ab\left(2a-3b\right)$$

$$=8x^3-27b^3-36a^2b+54ab^2$$

(iii) $\left(\frac{3}{2}x+1\right)^3$

Ans: By using the identity, $\left(x+y\right)^3=x^3+y^3+3xy\left(x+y\right)$

$$\left(\frac{3}{2}x+1\right)^3=\left(\frac{3}{2}x\right)^3+(1)^3+3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$$

$$=\frac{27}{8}x^3+1+\frac{9}{2}x\left(\frac{3}{2}x+1\right)$$

$$=\frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$$

$$=\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$$

(iv) $\left(x-\frac{2}{3}y\right)^3$

Ans: By using the identity, $\left(x-y\right)^3=x^3-y^3-3xy\left(x-y\right)$

$$\left(x-\frac{2}{3}y\right)^3=(x)^3-\left(\frac{2}{3}y\right)^3-3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$$

$$=x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$$

$$=x^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$$

7. Evaluate the following using suitable identities:

(i) $(99)^3$

Ans: Here we can write $\left(99\right)^3$ as $\left(100-1\right)^3$

By using the identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(100-1)^3 = (100)^3 - (1)^3 - 3(100)(1)(100-1)$

$= 1000000 - 1 - 300(100-1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3$

Ans: Here we can write $(102)^3$ as $(100+2)^3$

By using the identity, $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(100+2)^3 = (100)^3 + (2)^3 + 3(100)(2)(100+2)$

$= 1000000 + 8 + 600(100+2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Ans: Here we can write $(998)^3$ as $(1000-2)^3$

By using the identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(1000-2)^3 = (1000)^3 - (2)^3 - 3(1000)(2)(1000-2)$

$= 1000000000 - 8 - 6000(1000-2)$

$= 1000000000 - 8 - 6000000 + 12000$

$= 994011992$

8. Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

Ans: Here we can write $8a^3 + b^3 + 12a^2b + 6ab^2$ as

$(2a)^3 + (b)^3 + 3(2a)^2(b) + 3(2a)(b)^2$

By using the identity, $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

Here, $x = 2a,\ y = b$

$$8a^3+b^3+12a^2b+6ab^2=(2a)^3+(b)^3+3(2a)^2(b)+3(2a)(b)^2$$
$$=(2a+b)^3$$
$$=(2a+b)(2a+b)(2a+b)$$

(ii) $8a^3-b^3-12a^2b+6ab^2$

Ans: Here we can write $8a^3-b^3-12a^2b+6ab^2$ as

$(2a)^3-(b)^3-3(2a)^2(b)+3(2a)(b)^2$

By using the identity, $(x-y)^3=x^3-y^3-3xy(x-y)$

Here, $x=2a,\ y=b$

$$8a^3-b^3-12a^2b+6ab^2=(2a)^3-(b)^3-3(2a)^2(b)+3(2a)(b)^2$$
$$=(2a-b)^3$$
$$=(2a-b)(2a-b)(2a-b)$$

(iii) $27-125a^3-135a+225a^2$

Ans: Here we can write $27-125a^3-135a+225a^2$ as

$(3)^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

By using the identity, $(x-y)^3=x^3-y^3-3xy(x-y)$

Here, $x=3,\ y=5a$

$$27-125a^3-135a+225a^2=(3)^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$$
$$=(3-5a)^3$$
$$=(3-5a)(3-5a)(3-5a)$$

(iv) $64a^3-27b^3-144a^2b+108ab^2$

Ans: Here we can write $64a^3-27b^3-144a^2b+108ab^2$ as

$(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

By using the identity, $(x-y)^3=x^3-y^3-3xy(x-y)$

Here, $x=4a,\ y=3b$

$$64a^3-27b^3-144a^2b+108ab^2=(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$$

$=(4a-3b)^3$
$=(4a-3b)(4a-3b)(4a-3b)$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

Ans: Here we can write $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ as

$(3p)^3-\left(\frac{1}{6}\right)^3-3(3p)^2\left(\frac{1}{6}\right)+3(3p)\left(\frac{1}{6}\right)^2$

By using the identity, $(x-y)^3=x^3-y^3-3xy(x-y)$

Here, $x=3p,\ y=\frac{1}{6}$

$27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p=(3p)^3-\left(\frac{1}{6}\right)^3-3(3p)^2\left(\frac{1}{6}\right)+3(3p)\left(\frac{1}{6}\right)^2$

$=\left(3p-\frac{1}{6}\right)^3$

$=\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

9. Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

Ans: By using the identity, $(x+y)^3=x^3+y^3+3xy(x+y)$

$x^3+y^3=(x+y)^3-3xy(x+y)$

$x^3+y^3=(x+y)\left[(x+y)^2-3xy\right]$ Taking $(x+y)$ common

$x^3+y^3=(x+y)\left[(x^2+y^2+2xy)-3xy\right]$

$\Rightarrow x^3+y^3=(x+y)(x^2+y^2-xy)$

Hence, verified.

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Ans: By using the identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$$x^3 - y^3 = (x-y)^3 + 3xy(x+y)$$

$$x^3 - y^3 = (x-y)\left[(x-y)^2 + 3xy\right] \qquad \text{Taking } (x-y) \text{ common}$$

$$x^3 - y^3 = (x-y)\left[\left(x^2 + y^2 - 2xy\right) + 3xy\right]$$

$$\Rightarrow x^3 - y^3 = (x-y)\left(x^2 + y^2 + xy\right)$$

Hence, verified.

10. Factorise each of the following:

(i) $27y^3 + 125z^3$

Ans: Here $27y^3 + 125z^3$ can be written as $(3y)^3 + (5z)^3$

$$27y^3 + 125z^3 = (3y)^3 + (5z)^3$$

As we know that, $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$

$$27y^3 + 125z^3 = (3y)^3 + (5z)^3$$

$$27y^3 + 125z^3 = (3y+5z)\left[(3y)^2 - (3y)(5z) + (5z)^2\right]$$

$$= (3y+5z)\left(9y^2 - 15yz + 25z^2\right)$$

(ii) $64m^3 - 343n^3$

Ans: Here $64m^3 - 343n^3$ can be written as $(4y)^3 - (7z)^3$

$$64m^3 - 343n^3 = (4y)^3 - (7z)^3$$

As we know that, $x^3 - y^3 = (x-y)\left(x^2 + xy + y^2\right)$

$$64m^3 - 343n^3 = (4y)^3 - (7z)^3$$

$$64m^3 - 343n^3 = (4m-7n)\left[(4m)^2 + (4m)(7n) + (7n)^2\right]$$

$$= (4m-7n)\left(16m^2 + 28mn + 49n^2\right)$$

11. Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Ans: Here $27x^3 + y^3 + z^3 - 9xyz$ can be written as $(3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$$27x^3+y^3+z^3-9xyz=(3x)^3+(y)^3+(z)^3-3(3x)(y)(z)$$

We know that, $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$$27x^3+y^3+z^3-9xyz=(3x)^3+(y)^3+(z)^3-3(3x)(y)(z)$$

$$=(3x+y+z)\left[(3x)^2+y^2+z^2-3xy-yz-3xz\right]$$

$$=(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$$

12. Verify that $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x^2)\right]$

Ans: As we know that, $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Dividing the equation by $\frac{1}{2}$ and multiply by 2

$$\Rightarrow x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[2(x^2+y^2+z^2-xy-yz-zx)\right]$$

$$=\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$$

$$=\frac{1}{2}(x+y+z)\left[(x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx)\right]$$

$$=\frac{1}{2}(x+y+z)\left[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2zx)\right]$$

$$=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

13. If $x+y+z=0$, show that $x^3+y^3+z^3=3xyz$

Ans: As we know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Given, $x+y+z=0$,then

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

$$x^3+y^3+z^3-3xyz=(0)(x^2+y^2+z^2-xy-yz-zx)$$

$$x^3+y^3+z^3-3xyz=0$$

$$x^3+y^3+z^3=3xyz$$

Hence, proved.

14. Without actually calculating the cubes, find the value of each of the following:

(i) $\mathbf{(-12)^3+(7)^3+(5)^3}$

Ans: Let

$(-12)^3+(7)^3+(5)^3$

$a=-12,\ \ b=7,\ \ c=5$

We know that if, $x+y+z=0$ then $x^3+y^3+z^3=3xyz$

Here, $-12+7+5=0$

$(-12)^3+(7)^3+(5)^3=3xyz$

$=3(-12)(7)(5)$

$=-1260$

(ii) $\mathbf{(28)^3+(-15)^3+(-13)^3}$

Ans: Let

$(28)^3+(-15)^3+(-13)^3$

$a=28,\ \ b=-15,\ \ c=-13$

We know that if, $x+y+z=0$ then $x^3+y^3+z^3=3xyz$

Here, $28-15-13=0$

$(28)^3+(-15)^3+(-13)^3=3xyz$

$=3(28)(-15)(-13)$

$=16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) $\boxed{\textbf{Area: } 25a^2-35a+12}$

Ans: Area: $25a^2-35a+12$

Using the splitting the middle term method,

We've to find a number whose sum $=-35$ and product $25\times12=300$

We'll get -15 and -20 as the numbers $[-15-20=-35]$ and $[-15\times(-20)=300]$

$25a^2-35a+12$

$25a^2-15a-20a+12$

$5a(5a-3)-4(5a-3)$

$(5a-3)(5a-4)$

Possible expression for length $=(5a-4)$

Possible expression for breadth $=(5a-3)$

(ii) $\boxed{\textbf{Area: } 35y^2+13y-12}$

Ans: Area: $35y^2+13y-12$

Using the splitting the middle term method,

We've to find a number whose sum $=13$and product $35\times 12=420$

We'll get -15 and 28 as the numbers $[-15+28=13]$ and $[15\times 28=420]$

$35y^2+13y-12$

$35y^2-15a+28y-12$

$5y(7y-3)+4(7y-3)$

$(7y-3)(5y+4)$

Possible expression for length $=(5y+4)$

Possible expression for breadth $=(7y-3)$

16. What are the possible expressions for the dimensions of the cuboids whose volume are given below?

(i) Volume: $3x^2-12x$

Ans: $3x^2-12x$ can be written as $3x(x-4)$ by taking $3x$ common from both the terms.

Possible expression for length $=3$

Possible expression for length $=x$

Possible expression for length $=(x-4)$

(ii) Volume: $12ky^2+8ky-20k$

Ans: $12ky^2 + 8ky - 20k$ can be written as $4k\left(3y^2 + 2y - 5\right)$ by taking $4k$ common from both the terms.

$12ky^2 + 8ky - 20k = 4k\left(3y^2 + 2y - 5\right)$

Here, we can write $4k\left(3y^2 + 2y - 5\right)$ as $4k\left(3y^2 + 5y - 3y - 5\right)$ by using the splitting the middle term method

$4k\left(3y^2 + 5y - 3y - 5\right)$

$4k\left[y(3y+5) - 1(3y+5)\right]$

$4k(3y+5)(y-1)$

Possible expression for length $= 4k$

Possible expression for length $= (3y+5)$

Possible expression for length $= (y-1)$