NCERT Solutions for Class 9

Maths

Chapter 1 – Number System

Exercise 1.3

1. Convert the following numbers in decimal form and state what kind of decimal expansion each has:

(i) $\frac{36}{100}$

Ans: Divide 36 by 100.

$$\begin{array}{r|l} & 0.36 \\ \hline 100 & 36 \\ & -0 \\ \hline & 360 \\ & -300 \\ \hline & 600 \\ & -600 \\ \hline & 0 \end{array}$$

So, $\frac{36}{100} = 0.36$ and it is a terminating decimal number.

(ii) $\frac{1}{11}$

Ans: Divide 1 by 11.

$$
\begin{array}{r}
0.0909....\\
11\overline{)\quad 1}\\
\underline{-0}\\
10\\
\underline{-0}\\
100\\
\underline{-99}\\
10\\
\underline{-0}\\
100\\
\underline{-99}\\
1
\end{array}
$$

It is noticed that while dividing 1 by 11, in the quotient 09 is repeated.

So, $\frac{1}{11}=0.0909.....$ or

$\frac{1}{11}=0.\overline{09}$

and it is a non-terminating and recurring decimal number.

(iii) $\mathbf{4\frac{1}{8}}$

Ans: $4\frac{1}{8}=4+\frac{1}{8}=\frac{32+1}{8}=\frac{33}{8}$

Divide 33 by 8.

$$
\begin{array}{r}
4.125\\
8\overline{)\ 33}\\
\underline{-32}\\
10\\
\underline{-8}\\
20\\
\underline{-16}\\
40\\
\underline{-40}\\
\underline{0}
\end{array}
$$

Notice that, after dividing 33 by 8, the remainder is found as 0.

So, $4\frac{1}{8}=4.125$ and it is a terminating decimal number.

(iv) $\mathbf{\frac{3}{13}}$

Ans: Divide 3 by 13.

$$
\begin{array}{r}
0.230769 \\
13\overline{)\quad\quad 3} \\
\underline{-0} \\
30 \\
\underline{-26} \\
40 \\
\underline{-39} \\
10 \\
\underline{-0} \\
100 \\
\underline{-91} \\
90 \\
\underline{-78} \\
120 \\
\underline{-117} \\
\underline{3}
\end{array}
$$

It is observed that while dividing 3 by 13, the remainder is found as 3 and that is repeated after each 6 continuous divisions.

So, $\frac{3}{13}=0.230769.......$ or

$\frac{3}{13}=0.\overline{230769}$

and it is a non-terminating and recurring decimal number.

(v) $\mathbf{\frac{2}{11}}$

Ans: Divide 2 by 11.

$$\begin{array}{r}
0.1818.....\\
11\overline{)\quad 2}\\
\underline{-0}\\
20\\
\underline{-11}\\
90\\
\underline{-88}\\
20\\
\underline{-11}\\
90\\
\underline{-88}\\
\underline{2}
\end{array}$$

It can be noticed that while dividing 2 by 11, the remainder is obtained as 2 and then 9, and these two numbers are repeated infinitely as remainders.

So, $\frac{2}{11} = 0.1818.....$ or

$$\frac{2}{11} = 0.\overline{18}$$

and it is a non-terminating and recurring decimal number.

(vi) $\mathbf{\frac{329}{400}}$

Ans: Divide 329 by 400.

$$\begin{array}{r|l} & 0.8225 \\ 400 & 329 \\ & -0 \\ & 3290 \\ & -3200 \\ & 900 \\ & -800 \\ & 1000 \\ & -800 \\ & 2000 \\ & -2000 \\ & 0 \end{array}$$

It can be seen that while dividing 329 by 400, the remainder is obtained as 0.

So, $\frac{329}{400} = 0.8225$ and is a terminating decimal number.

2. If $\frac{1}{7} = 0.142857...$, then predict the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ without calculating the long division?

Ans: Note that, $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as $2 \times \frac{1}{7}$, $3 \times \frac{1}{7}$, $4 \times \frac{1}{7}$, $5 \times \frac{1}{7}$, and $6 \times \frac{1}{7}$

Substituting the value of $\frac{1}{7} = 0.142857$, gives

$2 \times \frac{1}{7} = 2 \times 0.142857... = 0.285714...$

$3 \times \frac{1}{7} = 3 \times 0.428571... = 0.428571...$

$4 \times \frac{1}{7} = 4 \times 0.142857... = 0.571428...$

$5 \times \frac{1}{7} = 5 \times 0.71425... = 0.714285...$

$6\times\frac{1}{7}=6\times0.142857...=0.857142...$

So, the values of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$ obtained without performing long division are

$\frac{2}{7}=0.\overline{285714}$

$\frac{3}{7}=0.\overline{428571}$

$\frac{4}{7}=0.\overline{571428}$

$\frac{5}{7}=0.\overline{714285}$

$\frac{6}{7}=0.\overline{857142}$

3. Convert the following decimal numbers into the form of $\frac{p}{q}$, where p and q are integers and $q\neq0$.

(i) $0.\overline{6}$

Ans: Let $x=0.\overline{6}$

$\Rightarrow x=0.6666$ .... (1)

Multiplying both sides of the equation (1) by 10, gives

$10x=0.6666\times10$

$10x=6.6666.....$ ...... (2)

Subtracting the equation (1) from (2), gives

$10x=6.6666.....$

$\underline{-x=0.6666.....}$

$9x=6$

$9x=6$

$x=\frac{6}{9}=\frac{2}{3}$

So, the decimal number becomes

$0.\overline{6}=\frac{2}{3}$ and it is in the required $\frac{p}{q}$ form.

(ii) $0.4\overline{7}$

Ans: Let $x = 0.4\overline{7}$

$\Rightarrow x = 0.47777.....$ $\quad\ldots\ldots(a)$

Multiplying both sides of the equation (a) by 10, gives

$10x = 4.7777.....$ $\quad\ldots\ldots(b)$

Subtracting the equation (a) from (b), gives

$$\begin{aligned} 10x &= 4.7777..... \\ \underline{-x} &\underline{= 0.4777.....} \\ 9x &= 4.3 \end{aligned}$$

Therefore,

$$x = \frac{4.3}{9} \times \frac{10}{10}$$

$$\Rightarrow x = \frac{43}{90}$$

So, the decimal number becomes

$0.4\overline{7} = \frac{43}{90}$ and it is in the required $\frac{p}{q}$ form.

(iii) $0.\overline{001}$

Ans: Let $x = 0.\overline{001} \Rightarrow$ $\quad\ldots\ldots (1)$

Since the number of recurring decimal number is 3, so multiplying both sides of the equation (1) by 1000, gives

$1000 \times x = 1000 \times 0.001001.....$ $\quad\ldots\ldots (2)$

Subtracting the equation (1) from (2) gives

$$\begin{aligned} 1000x &= 1.001001..... \\ \underline{-x} &\underline{= 0.001001.....} \\ 999x &= 1 \end{aligned}$$

$$\Rightarrow x = \frac{1}{999}$$

Hence, the decimal number becomes

$0.\overline{001} = \frac{1}{999}$ and it is in the $\frac{p}{q}$ form.

4. Represent the nonterminating decimal number $0.99999.....$ into the form of $\frac{p}{q}$. Did you expect this type of answer? Explain why the answer is appropriate.

Ans: Let $x = 0.99999.....$ .... (a)

Multiplying by 10 both sides of the equation (a), gives

$10x = 9.9999.....$ ...... (b)

Now, subtracting the equation (a) from (b), gives

$$\begin{array}{l} 10x = 9.99999..... \\ \underline{-x = 0.99999.....} \\ 9x = 9 \end{array}$$

$$\Rightarrow x = \frac{9}{9}$$

$$\Rightarrow x = 1.$$

So, the decimal number becomes

$0.99999... = \frac{1}{1}$ which is in the $\frac{p}{q}$ form.

Yes, for a moment we are amazed by our answer, but when we observe that $0.9999.........$ is extending infinitely, then the answer makes sense.

Therefore, there is no difference between 1 and $0.9999.........$ and hence these two numbers are equal.

5. Find the maximum number of digits in the recurring block of digits in the decimal expansion of $\frac{1}{17}$ by performing the long division.

Ans: Here the number of digits in the recurring block of $\frac{1}{17}$ is to be determined.

So, let us calculate the long division to obtain the recurring block of $\frac{1}{17}$.

Dividing 1 by 17 gives

$$
\begin{array}{rr}
 & 0.0588235294117647....\\
17 & \overline{)\qquad\qquad\qquad 1}\\
 & \underline{-0}\\
 & 10\\
 & \underline{-0}\\
 & 100\\
 & \underline{-85}\\
 & 150\\
 & \underline{-136}\\
 & 140\\
 & \underline{-136}\\
 & 40\\
 & \underline{-34}\\
 & 60\\
 & \underline{-51}\\
 & 90\\
 & \underline{-85}\\
 & 50\\
 & \underline{-34}\\
 & 160\\
 & \underline{-153}\\
 & 70\\
 & \underline{-68}\\
 & 20\\
 & \underline{-17}\\
 & 30\\
 & \underline{-17}\\
 & 130\\
 & \underline{-119}\\
 & 110\\
 & \underline{-102}
\end{array}
$$

$$\begin{array}{r} 80 \\ \underline{-68} \\ 120 \\ \underline{-119} \\ 1 \end{array}$$

Thus, it is noticed that while dividing 1 by 17, we found 16 number of digits in the repeating block of decimal expansion that will continue to be 1 after going through 16 continuous divisions.

Hence, it is concluded that $\frac{1}{17} = 0.0588235294117647.....$ or $\frac{1}{17} = 0.\overline{0588235294117647}$ and it is a recurring and non-terminating decimal number.

6. Observe at several examples of rational numbers in the form $\frac{p}{q}(q \neq 0)$, where p and q are integers with H.C.F between them is 1 and having terminating decimal representations. Guess the property that q must satisfy?

Ans: Let us consider the examples of such rational numbers $\frac{5}{2}, \frac{5}{4}, \frac{2}{5}, \frac{2}{10}, \frac{5}{16}$ of the form $\frac{p}{q}$ which have terminating decimal representations.

$\frac{5}{2} = 2.5$

$\frac{5}{4} = 1.25$

$\frac{2}{5} = 0.4$

$\frac{2}{10} = 0.2$

$\frac{5}{16} = 0.3125$

In each of the above examples, it can be noticed that the denominators of the rational numbers have powers of $2, 5$ or both.

So, q must satisfy the form either 2^m, or 5^n, or both $2^m \times 5^n$ (where $m = 0,1,2,3.....$ and $n = 0,1,2,3.....$) in the form of $\frac{p}{q}$.

7. Give examples of three numbers whose decimal representations are non-terminating and non-recurring.

Ans: All the irrational numbers are non-terminating and non-recurring, because irrational numbers do not have any representations of the form of $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers. For example:

$\sqrt{2} = 1.41421.....$,

$\sqrt{3} = 1.73205...$

$\sqrt{7} = 2.645751....$

are the numbers whose decimal representations are non-terminating and non-recurring.

8. Write any three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Ans: Converting $\frac{5}{7}$ and $\frac{9}{11}$ into the decimal form gives

$\frac{5}{7} = 0.714285.....$ and

$\frac{9}{11} = 0.818181.....$

Therefore, 3 irrational numbers that are contained between 0.714285...... and 0.818181.....

are:

0.73073007300073......

0.74074007400074......

0.76076007600076......

Hence, three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ are

0.73073007300073......

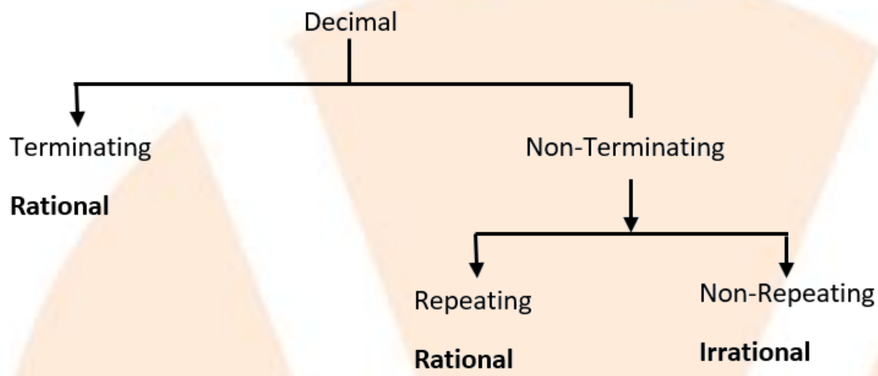
0.74074007400074......

0.76076007600076......

9. Classify the following numbers and state whether it is rational or irrational:

(i) $\sqrt{23}$

Ans: The following diagram reminds us of the distinctions among the types of rational and irrational numbers.



After evaluating the square root gives

$\sqrt{23} = 4.795831.....$, which is an irrational number.

(ii) $\sqrt{225}$

Ans: After evaluating the square root gives

$\sqrt{225} = 15$, which is a rational number.

That is, $\sqrt{225}$ is a rational number.

(iii) 0.3796

Ans: The given number is 0.3796. It is terminating decimal.

So, 0.3796 is a rational number.

(iv) 7.478478

Ans: The given number is $7.478478\ldots$.

It is a non-terminating and recurring decimal that can be written in the $\frac{p}{q}$ form.

Let $\quad x = 7.478478\ldots. \qquad \ldots\ldots(a)$

Multiplying the equation (a) both sides by 100 gives

$\Rightarrow 1000x = 7478.478478..... \qquad \ldots\ldots(b)$

Subtracting the equation (a) from (b), gives

$$1000x = 7478.478478....$$
$$\underline{\quad -x = \quad 7.478478....}$$
$$999x = 7471$$
$$999x = 7471$$
$$x = \frac{7471}{999}$$

Therefore, $7.478478..... = \frac{7471}{999}$, which is in the form of $\frac{p}{q}$

So, $7.478478...$ is a rational number.

(v) 1.101001000100001.....

Ans: The given number is $1.101001000100001....$

It can be clearly seen that the number $1.101001000100001....$ is a non-terminating and non recurring decimal and it is known that non-terminating non-recurring decimals cannot be written in the form of $\frac{p}{q}$.

Hence, the number $1.101001000100001....$ is an irrational number.