Important Questions for Class 9

Maths

Chapter 8 - Quadrilaterals

Very Short Answer Type Questions **1 Mark**

1. A quadrilateral ABCD **is a parallelogram if**

(a) $AB = CD$

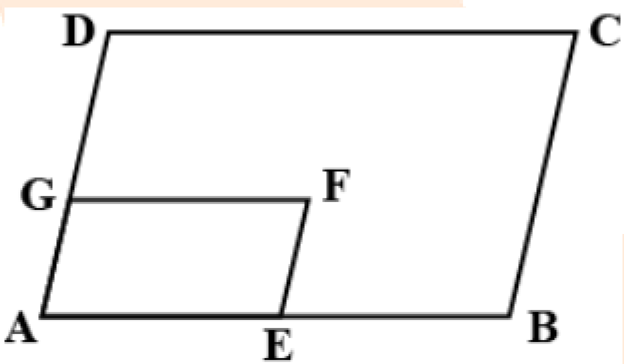
(b) $AB \parallel BC$

(c) $\angle A = 60^\circ, \angle C = 60^\circ, \angle B = 120^\circ$

(d) $AB = AD$

Ans: (c) $\angle A = 60^\circ, \angle C = 60^\circ, \angle B = 120^\circ$

2. In figure, ABCD and AEFG are both parallelogram if $\angle C = 80^\circ$**, then** $\angle DGF$ **is**



(a) 100°

(b) 60°

(c) 80°

(d) 120°

Ans: (c) 80°

3. In a square ABCD, the diagonals AC and BD bisects at O . Then $\triangle AOB$ is

(a) Acute angled

(b) Obtuse angled

(c) Equilateral

(d) Right angled

Ans:(d) Right angled

4. ABCD **is a rhombus. If $\angle ACB = 30^{\circ}$, then $\angle ADB$ is**

(a) 30°

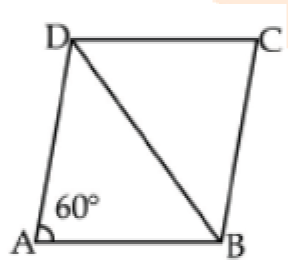
(b) 120°

(c) 60°

(d) 45°

Ans: (c) 60°

5. In fig ABCD **is a parallelogram. If $\angle DAB = 60^{\circ}$ and $\angle DBC = 80^{\circ}$ then $\angle CDB$ is**



(A) 80°

(B) 60°

(C) 20°

(D) 40°

Ans: (D) 40°

6. If the diagonals of a quadrilateral bisect each other, then the quadrilateral must be.

(a) Square

(b) Parallelogram

(c) Rhombus

(d) Rectangle

Ans: (b) Parallelogram

7. The diagonal AC and BD of quadrilateral ABCD are equal and are perpendicular bisector of each other then quadrilateral ABCD is a

(a) Kite

(b) Square

(c) Trapezium

(d) Rectangle

Ans: (b) Square

8. The quadrilateral formed by joining the mid points of the sides of a quadrilateral ABCD taken in order, is a rectangle if

(a) ABCD **is a parallelogram**

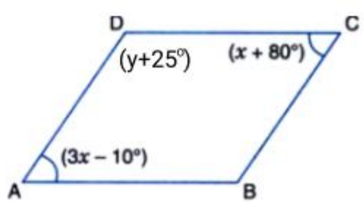
(b) $ABCD$ **is a rut angle**

(c) Diagonals AC **and** BD **are perpendicular**

(d) $AC = BD$

Ans: (a) $ABCD$ is a parallelogram

9. In the fig $ABCD$ **is a Parallelogram. The values of** x **and** y **are**



(a) 30,35

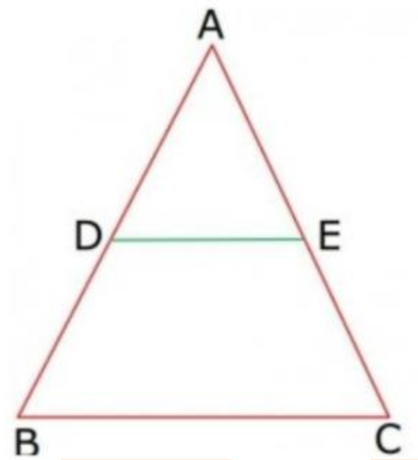
(b) 45,30

(c) 45,45

(d) 55,35

Ans: (b) 45,30

10. In fig if $DE = 8\text{cm}$ **and** D **is the mid-Point of** AB**, then the true statement is**



(a) $AB = AC$

(b) $DE||BC$

(c) E **is not mid-Point of AC**

(d) $DE \neq BC$

Ans: (c) E is not mid-Point of AC

11. The sides of a quadrilateral extended in order to form exterior angler. The sum ofthese exterior angle is

(a) 180°

(b) 270°

(c) 90°

(d) 360°

Ans: (d) 360°

12. $ABCD$ **is rhombus with** $\angle ABC = 40^{\circ}$**. The measure of** $\angle ACD$ **is**

(a) 90°

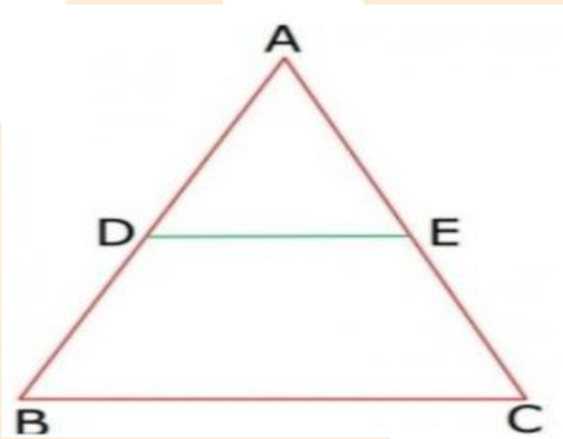
(b) 20°

(c) 40°

(d) 70°

Ans: (b) 20°

13: In fig D is mid-point of AB and $DE \parallel$ BC then AE is equal to



(a) AD

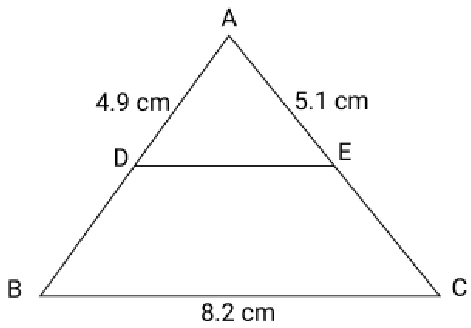
(b) EC

(c) DB

(d) BC

Ans: (b) EC

14. In fig D and E are mid-points of AB and AC respectively. The length of DE is



(a) 8.2cm

(b) 5.1cm

(c) 4.9cm

(d) 4.1cm

Ans: (d) 4.1cm

15. A diagonal of a parallelogram divides it into

(a) Two congruent triangles

(b) Two similes triangles

(c) Two equilateral triangles

(d) None of these

Ans: (a) Two congruent triangles

16. A quadrilateral is a, if it's opposite sides are equal:

(a) Kite

(b) Trapezium

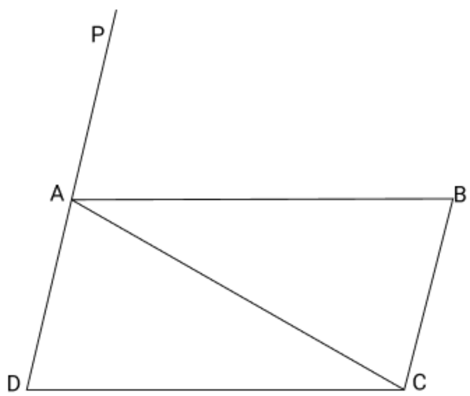
(c) Cyclic quadrilateral

(d) Parallelogram

Ans: (d) Parallelogram

17. In the adjoining Fig. $\mathrm{AB} = \mathrm{AC}$. $\mathrm{CD} \| \mathrm{BA}$ **and** AD **is the bisector of** $\angle PAC$ **prove that**

(a) $\angle DAC = \angle BCA$ **and**



Ans:

In $\Delta ABC \quad AB = AC$

$\Rightarrow \angle BCA = \angle BAC$ [Opposite angle of equal sides are equal]

$\angle CAD = \angle BCA + \angle ABC$ [Exterior angle]

$\Rightarrow \angle PAC = \angle BCA$

Now, $\angle PAC = \angle BCA$

$\Rightarrow AP \parallel BC$

Also $CD \| BA$ (According to the question)

So, $ABCD$ is a parallelogram

(ii) ABCD is a parallelogram

18. Which of the following is not a parallelogram?

(a) Rhombus

(b) Square

(c) Trapezium

(d) Rectangle

Ans: (c) Trapezium

19. The sum of all the four angles of a quadrilateral is

(a) 180^0

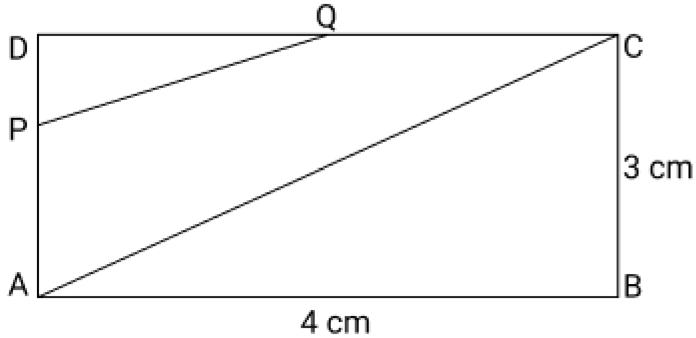
(b) 360^0

(c) 270^0

(d) 90^0

Ans: (b) 360°

20. In Fig ABCD is a rectangle P and Q are mid-points of AD and DC respectively. Then length of PQ is



(a) 5 cm

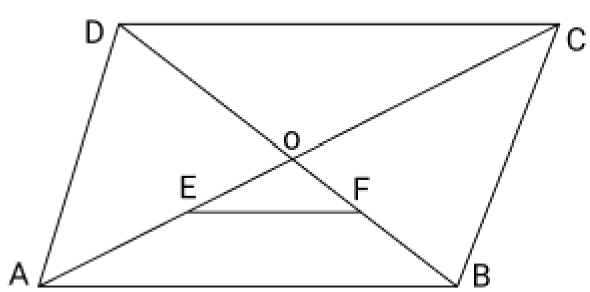
(b) 4cm

(c) 2.5cm

(d) 2cm

Ans: (c) 2.5cm

21: In Fig ABCD is a rhombus. Diagonals AC and BD intersect at $0.\text{E}$ **and** F **are mid points of AO and BO respectively. If** $AC = 16\text{cm}$ **and** $BD = 12\text{cm}$ **then EF is**



(a) 10cm

(b) 5cm

(c) 8cm

(d) 6cm

Ans:(b) 5cm

Short Answer Type Questions **2 Marks**

1. The angles of a quadrilateral are in the ratio 3: 5: 9: 13. Find all angles of the quadrilateral

Ans: Assume that in quadrilateral $\text{ABCD}, \angle\text{A} = 3x, \angle\text{B} = 5x, \angle\text{C} = 9x$ and $\angle\text{D} = 13x$.

We know that the sum of all the angles of a quadrilateral $= 360^{\circ}$

So, $\angle A+\angle B+\angle C+\angle D=360^{\circ} \Rightarrow 3x+5x+9x+13x=360^{\circ}$

$\Rightarrow 30x=360^{\circ} \Rightarrow x=12^{\circ}$

Now, $\angle A=3x=3\times 12=36^{\circ}$

$\angle B=5x=5\times 12=60^{\circ}$

$\angle C=9x=9\times 12=108^{\circ}$

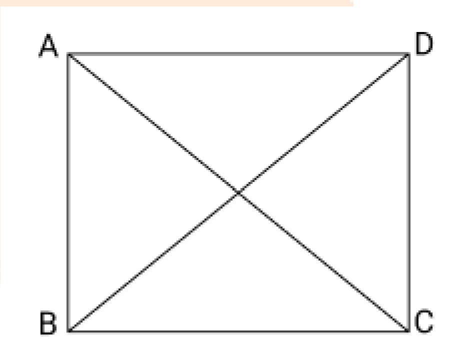
And $\angle D=13x=13\times 12=156^{\circ}$

Therefore, angles of given quadrilateral are $36^{\circ},60^{\circ},108^{\circ}$ and 156°.

2. If the diagonals of a parallelogram are equal, show that it is a rectangle.

Ans: According to the question: $ABCD$ is a parallelogram with diagonal $AC=$ diagonal BD

To prove: $ABCD$ is a rectangle.



Proof: In triangles ABC and ABD,

$AB=AB$ [Common]

$AC=BD$ [According to the question]

$AD=BC$ [Opposite Sides of a $\parallel$ gm]

So, $\Delta ABC\cong \Delta BAD$ [By SSS congruency]

$\Rightarrow \angle DAB=\angle CBA$ [By C.P.C.T.]………..(i)

But $\angle DAB+\angle CBA=180^{\circ}$……… (ii)

[Because, $AD \parallel BC$ and AB cuts them, the sum of the interior angles of the same side of transversal is 180°]

According to the equations (i) and (ii),

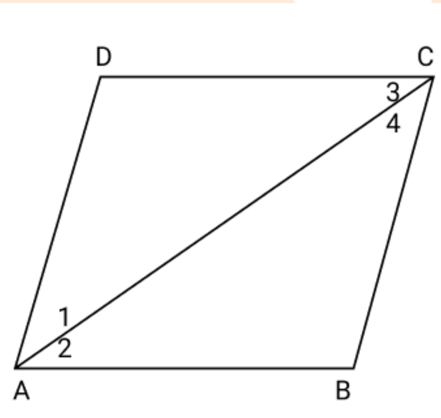
$\angle DAB = \angle CBA = 90^\circ$

Therefore, $ABCD$ is a rectangle.

3. Diagonal AC of a parallelogram ABCD bisects $\angle$ A (See figure). Show that:

(i) It bisects $\angle C$ also.

(ii) $ABCD$ is a rhombus.



Ans:

Diagonal AC bisects $\angle A$ of the parallelogram $ABCD$.

(i) Since $AB \parallel DC$ and AC intersects them.

So, $\angle 1 = \angle 3$ [Alternate angles] ....(i)

Similarly $\angle 2 = \angle 4$………… (ii)

But $\angle 1 = \angle 2$ [Given] ....(iii)

So, $\angle 3 = \angle 4$ [Using eq. (i), (ii) and (iii)]

Hence AC bisects $\angle C$.

(ii) $\angle 2 = \angle 3 = \angle 4 = \angle 1$

$\Rightarrow AD = CD$ [Sides opposite to equal angles]

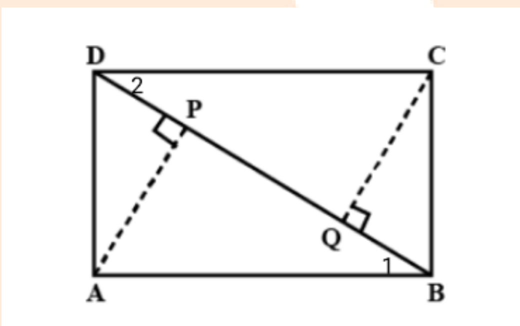
So, $AB = CD = AD = BC$

Therefore ABCD is a rhombus.

4. ABCD is a parallelogram and AP and Care the perpendiculars from vertices A and C on its diagonal BD (See figure). Show that:

(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$



Ans:

According to the question,

ABCD is a parallelogram.

$AP \perp BD$ and $CQ \perp$ BD

To prove:

(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$

Proof:

(i) In ΔAPB and ΔCQD,

$\angle 1 = \angle 2$ [Alternate interior angles]

$AB = CD$ [Opposite sides of a parallelogram are equal]

$\angle \mathrm{APB} = \angle \mathrm{CQD} = 90^{\circ}$

So, $\Delta \mathrm{APB} \cong \Delta \mathrm{CQD}$ [By ASA Congruency]

(ii) Since $\Delta \mathrm{APB} \cong \Delta \mathrm{CQD}$

$\mathrm{AP} = \mathrm{CQ}$ [By C.P.C.T.]

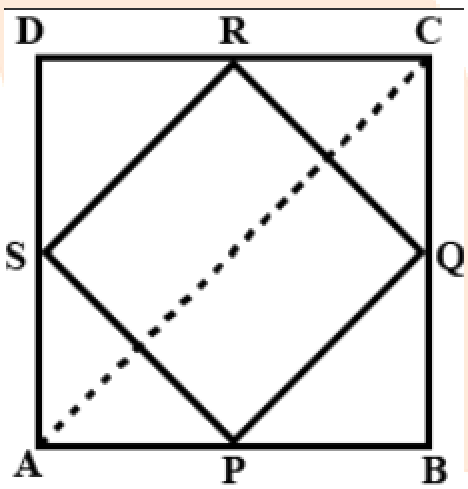
Hence proved.

5. ABCD is a quadrilateral in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively (See figure). AC is a diagonal. Show that:

(i) $\mathrm{SR} \parallel \mathrm{AC}$ **and** $\mathrm{SR} = \frac{1}{2}\mathrm{AC}$

(ii) $\mathrm{PQ} = \mathrm{SR}$

(iii) PQRS is a parallelogram.



Ans:

In $\Delta \mathrm{ABC}, \mathrm{P}$ is the mid-point of AB and Q is the mid-point of BC.

Then $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

(i) In $\Delta ACD, R$ is the mid-point of CD and S is the mid-point of AD.

Then $SR \parallel AC$ and $SR = \frac{1}{2}AC$

(ii) Since $PQ = \frac{1}{2}AC$ and $SR = \frac{1}{2}AC$

Hence, $PQ = SR$

(iii) Since $PQ \parallel AC$ and $SR \parallel AC$

Hence, P|| SR [two lines parallel to given line are parallel to each other]

Now $PQ = SR$ and $PQ \parallel SR$

Hence, PQRS is a parallelogram.

6. The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.

Ans:

Assume that angles of quadrilateral $ABCD$ are $3x, 5x, 9x$, and $13x$

$\angle A + \angle B + \angle C + \angle D = 360^{\circ}$ [We know that the sum of angles of a quadrilateral is 360°]

$30x = 360^{\circ}$

$x = 12^{\circ}$

So, $\angle A = 3x = 3 \times 12 = 36^{\circ}$

$\angle B = 5x = 5 \times 12 = 60^{\circ}$

$\angle C = 9x = 9 \times 12 = 108^{\circ}$

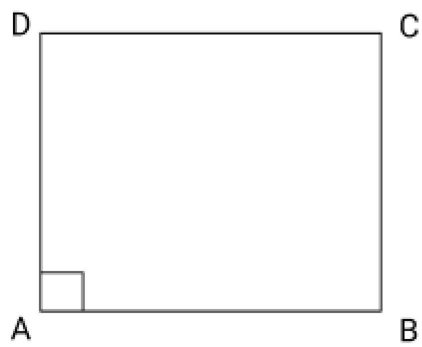
$\angle D = 13x = 13 \times 12 = 156^{\circ}$

Hence, the value of all angles are $36^{\circ}, 60^{\circ}, 108^{\circ}, 156^{\circ}$.

7: Show that each angle of a rectangle is a right angle.

Ans: As we also know that rectangle is a parallelogram whose one angle is right angle.

Assume that ABCD be a rectangle.



$\angle A = 90^{\circ}$

To prove $\angle B = \angle C = \angle D = 90^{\circ}$

Proof:

$\because AD \parallel BC$ and AB is transversal

So, $\angle A + \angle B = 180^{\circ}$

$90^{\circ} + \angle B = 180^{\circ}$

$\angle B = 180^{\circ} - 90^{\circ} = 90^{\circ}$

$\angle C = \angle A$

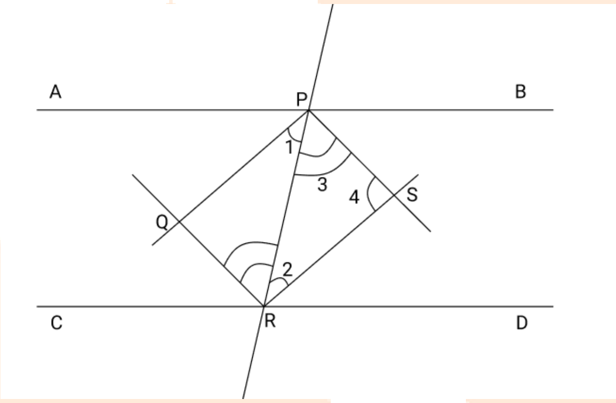
So, $\angle C = 90^{\circ}$

$\angle D = \angle B$

So, $\angle D = 90^\circ$

Hence proved.

8. A transversal cuts two parallel lines prove that the bisectors of the interior angles enclose a rectangle.



Ans: According to the question,

$\because AB \parallel CD$ and EF cuts them at P and R.

$\angle APR = \angle PRD$ [Alternate interior angles]

So, $\frac{1}{2}\angle APR = \frac{1}{2}\angle PRD$

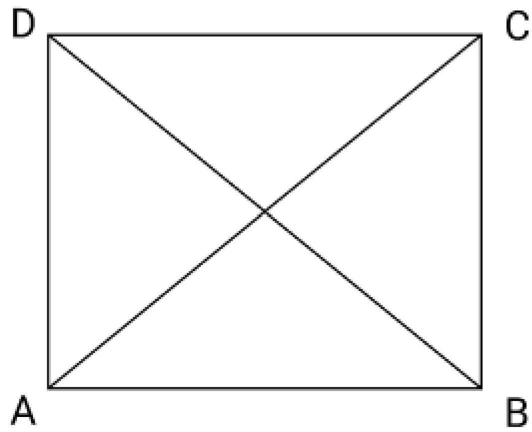
i.e. $\angle 1 = \angle 2$

$\therefore PQ \parallel RS$ [Alternate]

Hence, proved.

9. Prove that diagonals of a rectangle are equal in length.

Ans: ABCD is a rectangle and AC and BD are diagonals.



To prove $AC = BD$

Proof:

In ΔDAB and CBA

$\mathrm{AD} = \mathrm{BC}$ [In a rectangle opposite sides are equal]

$\angle A = \angle B \quad \left[90^{\circ} \text{each}\right]$

$\mathrm{AB} = \mathrm{AB}$ [common]

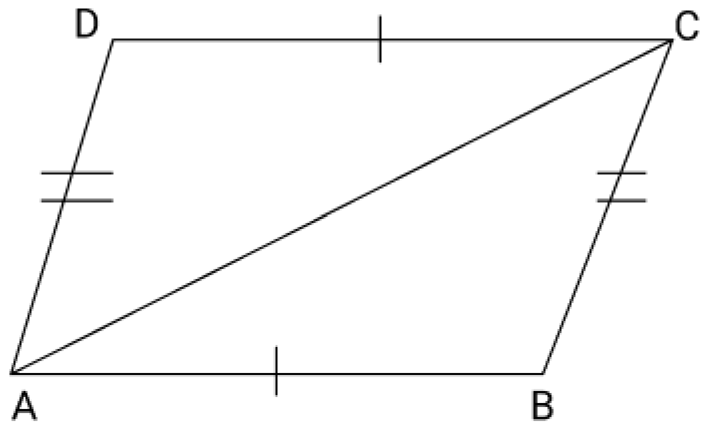
So, $\Delta DAB \cong \Delta CAB$ [By SAS]

So, $AC = BD$ [By C.P.C.T]

10. If each pair of opposite sides of a quadrilateral is equal, then prove that it is a parallelogram.

Ans: According to the question,

A quadrilateral ABCD in which $\mathrm{AB} = \mathrm{DC}$ and $\mathrm{AD} = \mathrm{BC}$



To prove:

ABCD is a parallelogram

We construct a line AC to with join point A with point C.

Proof: In $\triangle ABC$ and $\triangle ADC$

$\mathrm{AD} = \mathrm{BC}$ (According to the question)

$\mathrm{AB} = \mathrm{DC}$

$\mathrm{AC} = \mathrm{AC}$ [common]

So, $\Delta ABC \cong \Delta ADC$ [By SSS]

So, $\angle BAC = \angle DAC$ [By CPCT]

So, $ABCD$ is a parallelogram.

12. Show that the line segments joining the mid points of opposite sides of a quadrilateral bisect each other.

Ans: According to the question,

ABCD is quadrilateral E, F, G, H are mid points of the side AB, BC, CD and DA respectively

To prove:

EG and HF bisect each other.

In $\triangle ABC$, E is mid-point of AB and F is mid-point of BC

So, $EF \parallel AC$ and $EF = \frac{1}{2}AC \ldots\ldots(i)$

Similarly, $HG \parallel AC$ and $HG = \frac{1}{2}AC \ldots\ldots(ii)$

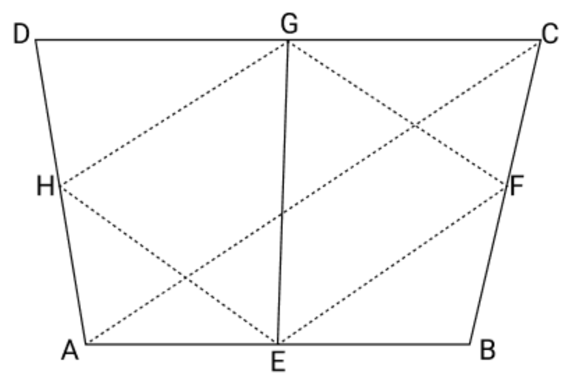
According to equations (i) and (ii),

$EF \parallel HG$ and $EF = GH$

So, $EFGH$ is a parallelogram and EG and HF are its diagonals

The diagonals of a parallelogram bisect each other

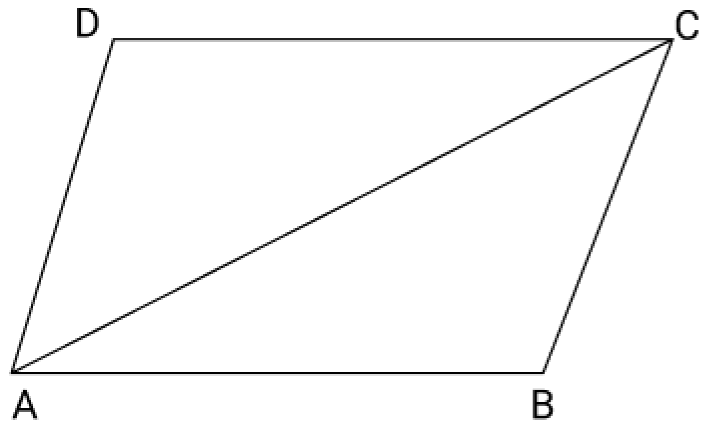
Hence, EG and HF bisect each other.



13. ABCD is a rhombus show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$

Ans: According to the question,

ABCD is a rhombus



In ΔABC and ΔADC

$AB = AD$ [Sides of a rhombus]

$BC = DC$ [Sides of a rhombus]

$AC = AC$ [Common]

So, $\Delta ABC \cong \Delta ADC$ [By SSS Congruency]

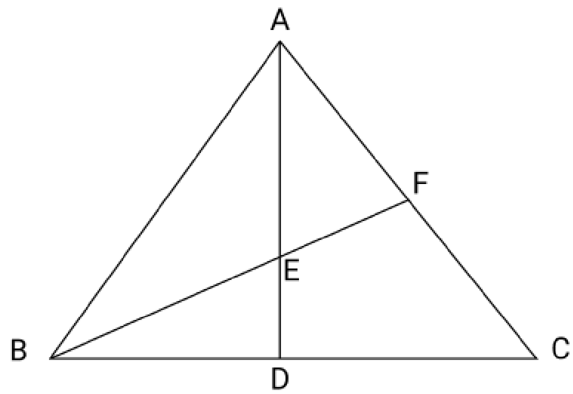
So, $\angle CAB = \angle CAD$ and $\angle ACB = \angle ACD$

Therefore, AC bisects $\angle A$ as well as $\angle C$

Similarly, by joining B to D, we can prove that $\triangle ABD \cong \Delta CBD$

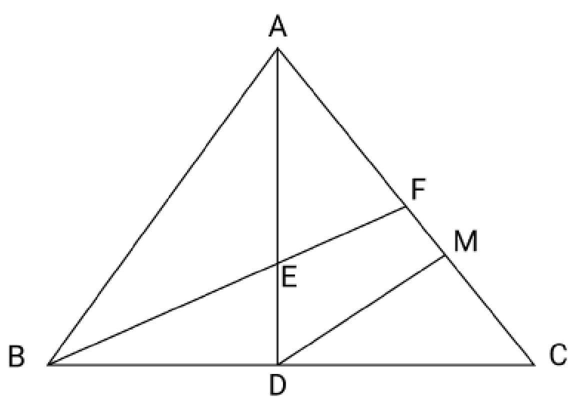
Therefore, BD bisects $\angle B$ as well as $\angle D$

14. In fig AD is a median of $\triangle ABC, E$ is mid-Point of AD. BE produced meet AC at F. Show that $AF = \frac{1}{3}AC$



Ans: Suppose M is mid-Point of CF Join DM

So, $DM \parallel BF$



In $\triangle ADM, E$ is mid- Point of AD and

$DM \parallel EF \Rightarrow F$ is mid-point of AM

So, $AF = FM$

$\text{FM} = \text{MC}$

So, $AF = FM = MC$

So, $AC = AF + FM + MC$

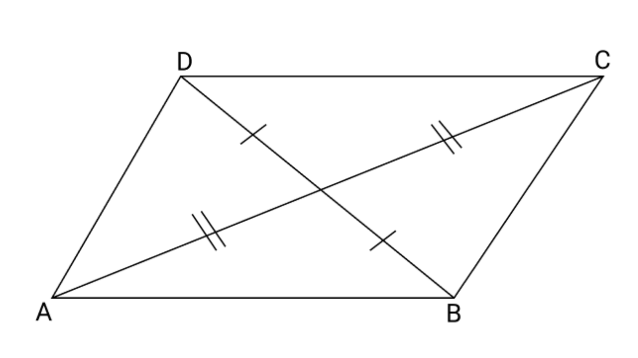
$= AF + AF + AF$

$\Rightarrow AC = 3AF$

$\Rightarrow AF = \frac{1}{3} AC$

Hence Proved.

15. Prove that a quadrilateral is a parallelogram if the diagonals bisect each other.



Ans: According to the given figure,

ABCD is a quadrilateral in which diagonals AC and BD intersect each other at O

In $\triangle AOB$ and $\triangle DOC$

$OA = OC$ [Given]

$OB = OD$ [Given]

And $\angle AOB = \angle COD$ [Vertically apposite angle]

So, $\triangle AOB \cong \Delta COD$ [By SAS]

So, $\angle OAB = \angle OCD$ [By C.P.C.T]

But this is Pair of alternate interior angles

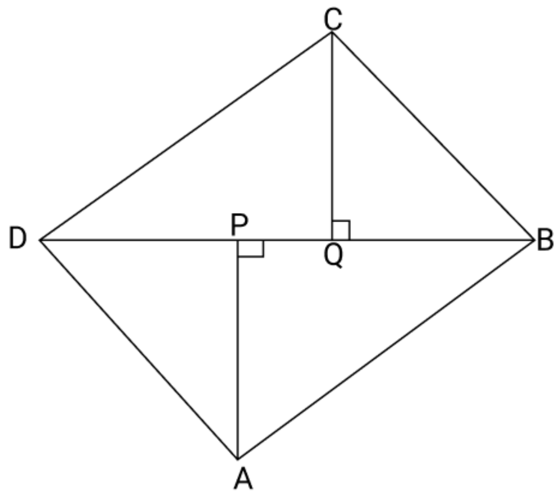
So, $AB \parallel CD$

So, $AB \parallel CD$

Similarly $AD || BC$

So, Quadrilateral ABCD is a Parallelogram.

16. In fig ABCD is a Parallelogram. AP and Care Perpendiculars from the Vertices A and C on diagonal BD.



Show that

(i) $\triangle APB \cong \Delta CQD$

(ii) $AP = CQ$

Ans:

(I) in ΔAPB and ΔCQD

$\mathrm{AB} = \mathrm{DC}$ [Opposite sides of a Parallelogram]

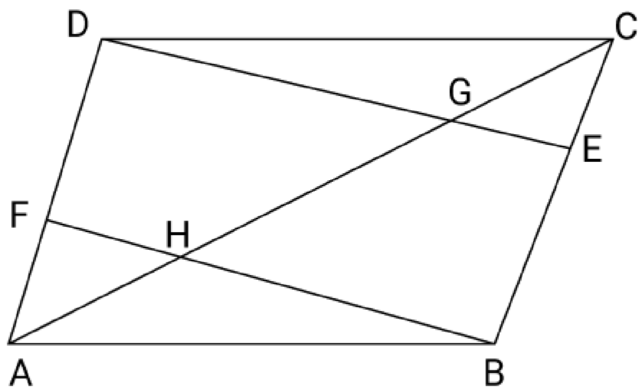
$\angle P = \angle Q$ [each 90°]

And $\angle ABP = \angle CDQ$

So, $\Delta APB \cong \Delta CQD$ [By ASA]

(II) So, $AP = CQ$ (By C.P.C.T)

17. ABCD is a Parallelogram E and F are the mid-Points of BC and AD respectively. Show that the segments BF and DE trisect the diagonal AC.



Ans: $FD \parallel BE$ and $FD = BE$

So, BEDF Is a Parallelogram

$EG \parallel BH$ and E is the mid-Point of BC

So, G is the mid-point of HC

Or $HG = GC$………….. (i)

Similarly $AH = HG$………….. (ii)

According to equations (i) and (ii) we get

$AH = HG = GC$

Hence, the segments BF and DE bisects the diagonal AC.

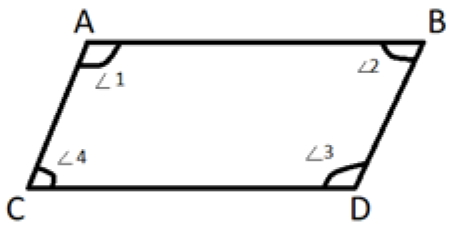
18. Prove that if each pair of apposite angles of a quadrilateral is equal, then it is a parallelogram.

Ans: According to the figure and question,

$ABCD$ is a quadrilateral in which $\angle A = \angle C$ and $\angle B = \angle D$

To Prove:

ABCD is a parallelogram



Proof:

$\angle A = \angle C$ [According to the question]

$\angle B = \angle D$ [According to the question]

$\angle A + \angle B = \angle C + \angle D \ldots\ldots(i)$

In quadrilateral. ABCD

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$(\angle A + \angle B) + (\angle A + \angle B) = 360^\circ$ [By….(i)]

$\angle A + \angle B = 180^\circ$

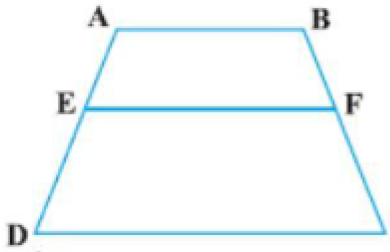
$\angle A + \angle B = \angle C + \angle D = 180^\circ$

These are sum of interior angles on the same side of transversal

So, $AD \parallel BC$ and $AB \parallel DC$

So, ABCD is a parallelogram.

19. In Fig. ABCD is a trapezium in which $\mathrm{AB} \parallel \mathrm{DCE}$ is the mid-point of AD. A line through E is parallel to AB show that l bisects the side BC



Ans: We draw a line AC with join point A to Point C.

In ΔADC

E is mid-point of AD and $\text{EO} \parallel \text{DC}$

So, O is mid point of AC [A line segment joining the midpoint of one side of a triangle parallel to second side and bisect the third side]

In ΔACB

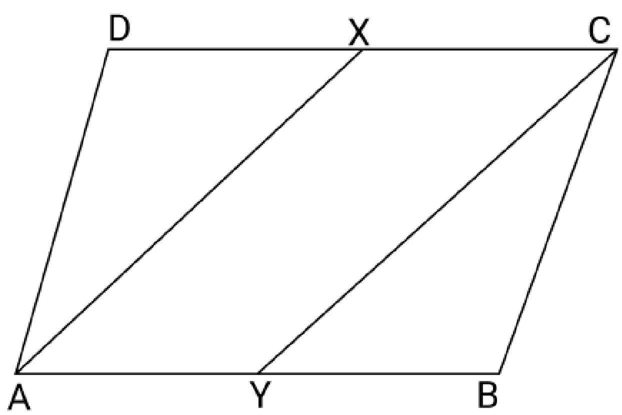
O is mid point of AC

$\text{OF} \parallel AB$

So, F is mid point of BC

So, l Bisect BC.

20. In Fig. ABCD is a parallelogram in which X and Y are the mid-points of the sides DC and AB respectively. Prove that AXCY is a parallelogram



Ans: According to the given figure,

ABCD is a parallelogram

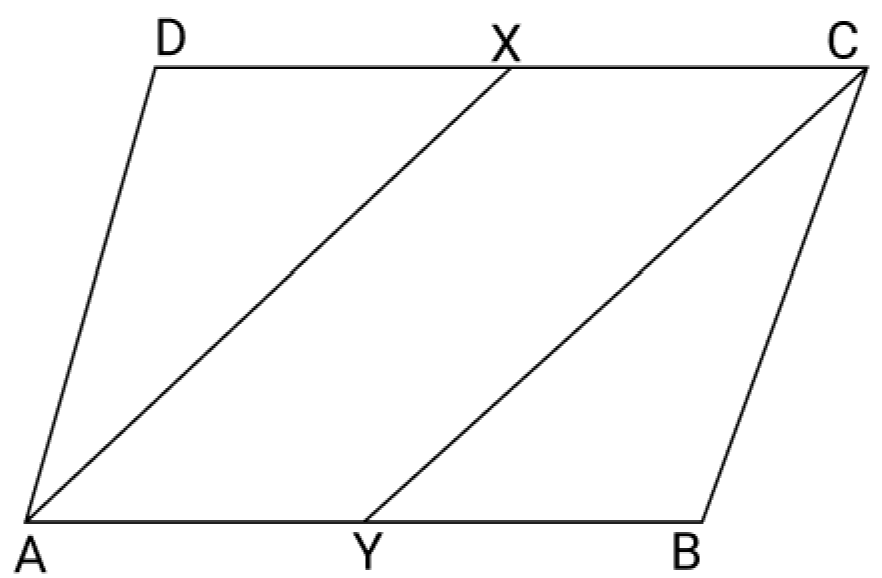
So, $\mathrm{AB} \parallel \mathrm{CD}$ and $\mathrm{AB} = \mathrm{CD}$

$\Rightarrow \frac{1}{2} AB \parallel \frac{1}{2} CD$ And $\frac{1}{2} AB = \frac{1}{2} CD$

$\Rightarrow XC \parallel AY$ And $XC = AY$

[X and Y are mid-point of DC and AB respectively]

$\Rightarrow AXCY$ is a parallelogram



21. The angles of quadrilateral are in the ratio 3:5:10:12. Find all the angles of the quadrilateral.

Ans: Assume that angles of quadrilaterals are

3x, 5x, 10x, and 12x

$\angle A = 3x, \angle B = 5x, \angle C = 10x, \angle D = 12x$

In a quadrilateral

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$3x + 5x + 10x + 12x = 360^\circ$

$30x = 360$

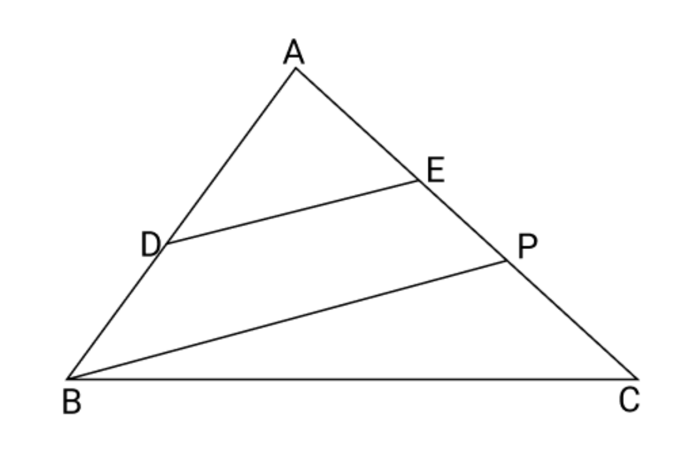
$x = \dfrac{360}{30} = 12$

$\angle A = 3 \times 12 = 36^\circ, \angle B = 5 \times 12 = 60^\circ$

$\angle C = 10 \times 12 = 120^\circ, \angle D = 12 \times 12 = 144^\circ$

Hence, the values of all angles are $36^\circ, 60^\circ, 120^\circ, 144^\circ$.

22. In fig D is mid-points of AB. P is on AC such that $PC = \frac{1}{2}AP$ and $DE \parallel BP$ show that $AE = \frac{1}{3}AC$



Ans: According to Δ ABP

D is mid points of AB and DE|| BP

E is midpoint of AP

So, $AE = EP$ also $PC = \frac{1}{2}AP$

$2PC = AP$

$2PC = 2AE$

$\Rightarrow PC = AE$

So, $AE = PE = PC$

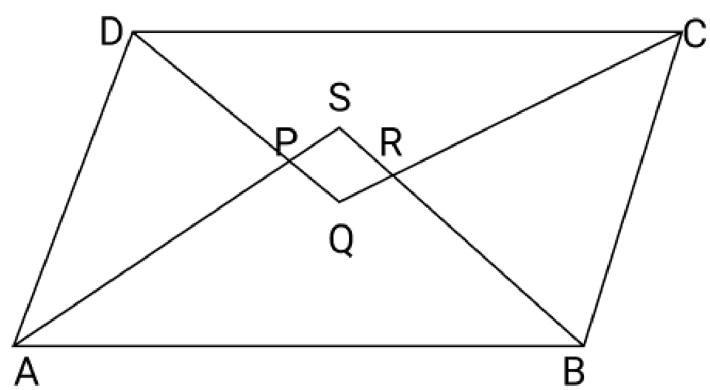
So, $AC = AE + EP + PC$

$AC = AE + AE + AE$

$\Rightarrow AE = \frac{1}{3}AC$

Hence Proved.

23: Prove that the bisectors of the angles of a Parallelogram enclose a rectangle. It is given that adjacent sides of the parallelogram are unequal.



Ans: According to the given figure,

$ABCD$ is a parallelogram

So, $\angle A + \angle D = 180^{\circ}$

or $\frac{1}{2}(\angle A + \angle D) = 90^{\circ}$

Or $\angle APD = 90^{\circ}$ [We know that the sum of all angles of a triangle is 180°]

So, $\angle SPQ = \angle APD = 90^{\circ}$

Similarly, $\angle QRS = 90^{\circ}$ and $\angle PQR = 90^{\circ}$

$\angle P + \angle Q + \angle R + \angle S = 360^{\circ}$

So, $\angle PSR = 90^{\circ}$.

Hence, each angle of quadrilateral PQRS is 90° .

Therefore, PQRS is a rectangle.

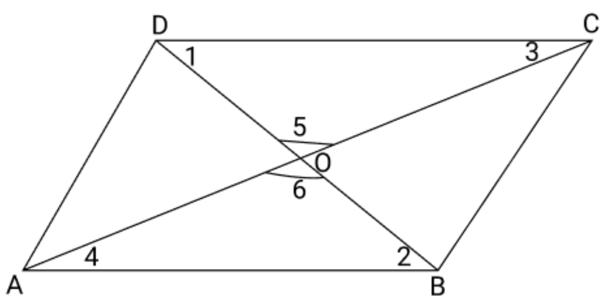
24. Prove that a quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal

Ans: According to the question and figure,

ABCD is a quadrilateral in which $\mathrm{AB} \parallel \mathrm{DC}$ and $\mathrm{BC} \parallel \mathrm{AD}$.

To Prove:

ABCD is a parallelogram



Construction: Join AC and BD intersect each other at O.

Proof:

$\Delta AOB \cong \Delta DOC$ [By AAA]

Because $\angle 1 = \angle 2$

$\angle 3 = \angle 4$ and $\angle 5 = \angle 6$

So, $AO = OC$

And $BO = OD$

So, $ABCD$ is a parallelogram

Because, diagonals of a parallelogram bisect each other.

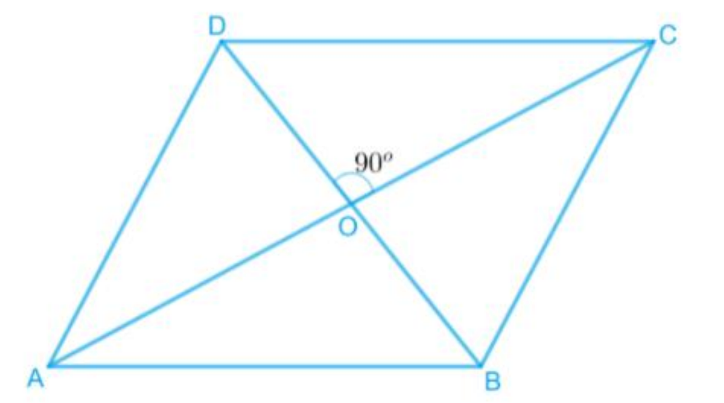
Short Answer Type Questions **3 Marks**

1. Show that is diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Ans: According to the question and figure,

Suppose $ABCD$ is a quadrilateral.

Suppose its diagonal AC and BD bisect each other at right angle at point O.



So, $OA = OC, OB = OD$

And $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^{\circ}$

To prove:

$ABCD$ is a rhombus.

Proof:

In ΔAOD and ΔBOC,

$OA = OC$ [According to the figure]

$\angle AOD = \angle BOC$ [According to the figure]

$OB = OD$ [According to the figure]

So, $\Delta AOD \cong \Delta COB$ [By SAS congruency]

$\Rightarrow AD = CB$ [By C.P.C.T.]..........(i)

Again, In ΔAOB and ΔCOD,

$OA = OC$ [According to the figure]

$\angle AOB = \angle COD$ [According to the figure]

$OB = OD$ [According to the figure]

So, $\Delta AOB \cong \Delta COD$ [By SAS congruency]

$\Rightarrow AD = CB$ [By C.P.C.T.] ....(ii)

Now In ΔAOD and ΔBOC,

$OA = OC$ [According to the figure]

$\angle AOB = \angle BOC$ [According to the figure]

$OB = OB$ [Common]

So, $\Delta AOB \cong \Delta COB$ [By SAS congruency]

$\Rightarrow AB = BC$ [By C.P.C.T.]..........(iii)

According to the equations (i), (ii) and (iii),

$AD = BC = CD = AB$

And the diagonals of quadrilateral ABCD bisect each other at right angle.

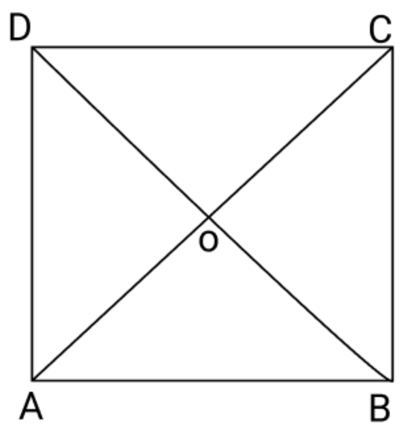
Hence, $ABCD$ is a rhombus.

2. Show that the diagonals of a square are equal and bisect each other at right angles.

Ans: According to the question and figure,

ABCD is a square.

AC and BD are its diagonals bisect each other at point O.



To prove:

$AC = BD$ and $AC \perp BD$ at point O.

Proof:

In triangles ABC and BAD,

$AB = AB$ [Common]

$\angle ABC = \angle BAD = 90^\circ$

$BC = AD$ [Sides of a square]

So, $\Delta ABC \cong \Delta BAD$ [By SAS congruency]

$\Rightarrow AC = BD$ [By C.P.C.T.]

Hence proved.

Now in triangles AOB and AOD,

$AO = AO$ [Common]

$AB = AD$ [Sides of a square]

OB = OD [Diagonals of a square bisect each other]

So, $\Delta AOB \cong \Delta AOD$ [By SSS congruency]

$\angle AOB = \angle AOD$ [By C.P.C.T.]

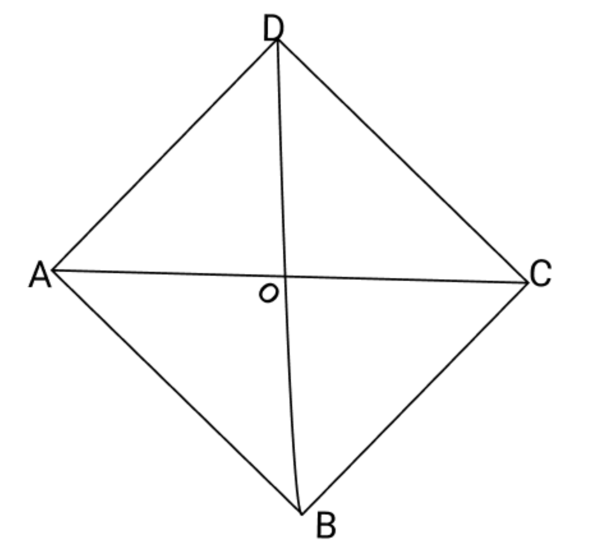
But $\angle AOB + \angle AOD = 180^\circ$ [Linear pair]

So, $\angle AOB = \angle AOD = 90^\circ$

$\Rightarrow OA \perp BD$ or $AC \perp BD$

Hence proved.

3. ABCD is a rhombus. Show that the diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.



Ans: ABCD is a rhombus.

Hence, $AB = BC = CD = AD$

Suppose O be the point of bisection of diagonals.

So, $OA = OC$ and $OB = OD$

In ΔAOB and ΔAOD,

$OA = OA$ [Common]

$AB = AD$ [Equal sides of rhombus]

$OB = OD$ [Diagonals of rhombus bisect each other]

So, $\Delta AOB \cong \Delta AOD$ [By SSS congruency]

$\Rightarrow \angle OAD = \angle OAB$ [By C.P.C.T.]

$\Rightarrow$ OA bisects $\angle A$ ....(i)

Similarly $\Delta BOC \cong \Delta DOC$ [By SSS congruency]

$\Rightarrow \angle OCB = \angle OCD$ [By C.P.C.T.]

$\Rightarrow OC$ bisects $\angle C$..........(ii)

According to the equations (i) and (ii), we can say that diagonal AC bisects $\angle A$ and $\angle C$.

Now in ΔAOB and ΔBOC,

$OB = OB$ [Common]

$AB = BC$ [Equal sides of rhombus]

$OA =$ OC [Diagonals of rhombus bisect each other]

So, $\Delta AOB \cong \Delta COB$ [By SSS congruency]

$\Rightarrow \angle OBA = \angle OBC$ [By C.P.C.T.]

$\Rightarrow OB$ bisects $\angle B$……… (iii)

Similarly $\Delta AOD \cong \Delta COD$ [By SSS congruency]

$\Rightarrow \angle ODA = \angle ODC$ [By C.P.C.T.]

$\Rightarrow$ BD bisects $\angle D$………..(iv)

According to the equations (iii) and (iv), we can say that diagonal BD bisects $\angle B$ and $\angle D$.

4. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = B(See figure). Show that:

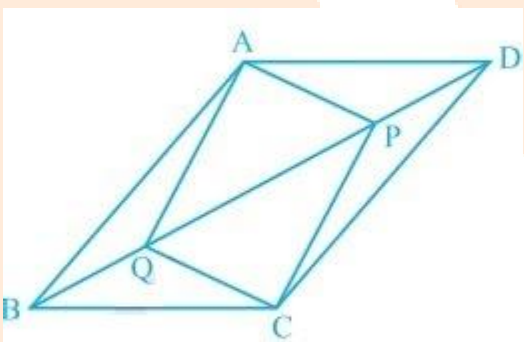
(i) $\Delta APD \cong \Delta CQB$

(ii) $AP = CQ$

(iii) $\Delta AQB \cong \Delta CPD$

(iv) $AQ = CP$

(v) APCis a parallelogram.



Ans:

(i) In Δ APD and Δ CQB,

$DP = BQ$ [According to the figure]

$\angle ADP = \angle QBC$ [Alternate angles $(AD \parallel BC$ and BD is transversal)]

$AD = CB$ [Opposite sides of parallelogram]

So, $\Delta APD \cong \Delta CQB$ [By SAS congruency]

(ii) Since $\Delta APD \cong \Delta CQB$

$\Rightarrow AP = CQ$ [By C.P.C.T.]

(iii) In ΔAQB and ΔCPD,

$BQ = DP$ [According to the figure]

$\angle ABQ = \angle PDC$ [Alternate angles $(AB \parallel CD$ and BD is transversal)]

$AB = CD$ [Opposite sides of parallelogram]

$\Delta AQB \cong \Delta CPD$ [By SAS congruency]

(iv) Since $\Delta AQB \cong \Delta CPD$

$\Rightarrow AQ = CP$ [By C.P.C.T.]

(v) In quadrilateral APCQ,

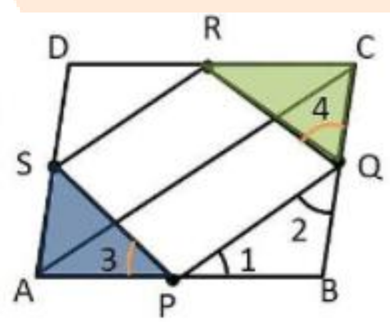
$AP =$ C[Proved in part (i)]

$AQ = CP$ [Provedin part (iv)]

Since opposite sides of quadrilateral APCare equal.

Therefore, APCis a parallelogram.

5. ABCD is a rhombus and P, Q, R, S are mid-points of AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rectangle.



Ans: According to the question and figure,

P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

To prove:

PQRS is a rectangle.

We draw a line AC with join point A to Point C.

Proof:

In $\Delta ABC, P$ is the mid-point of AB and Q is the mid-point of BC.

So, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$……… (i)

In $\Delta ADC, R$ is the mid-point of CD and S is the mid-point of AD.

So, $SR \parallel AC$ and $SR = \frac{1}{2}$ AC..........(ii)

According to the equations (i) and (ii),

$PQ \parallel SR$ and $PQ = SR$

So, PQRS is a parallelogram.

Now ABCD is a rhombus. [According to the question]

So, $AB = BC$

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC \Rightarrow PB = BQ$

So, $\angle 1 = \angle 2$ [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

$AP = CQ$ [P and Q are the mid-points of AB and BC and $AB = BC$]

Similarly AS = CR and PS = QR [Opposite sides of a parallelogram]

$\Delta APS \cong \Delta CQR$ [By SSS congreuancy]

$\Rightarrow \angle 3 = \angle 4$ [By C.P.C.T.]

Now we have $\angle 1 + \angle SPQ + \angle 3 = 180^\circ$

And $\angle 2 + \angle PQR + \angle 4 = 180^\circ$ [Linear pairs]

So, $\angle 1+\angle SPQ+\angle 3=\angle 2+\angle PQR+\angle 4$

Since $\angle 1=\angle 2$ and $\angle 3=\angle 4$ [Proved above]

So, $\angle SPQ=\angle PQR$………. (iii)

Now PQRS is a parallelogram [Proved above]

So, $\angle SPQ+\angle PQR=180^{\circ}$……….(iv) [Interior angles]

Using equations (iii) and (iv),

$\angle SPQ+\angle SPQ=180^{\circ} \Rightarrow 2\angle SPQ=180^{\circ}$

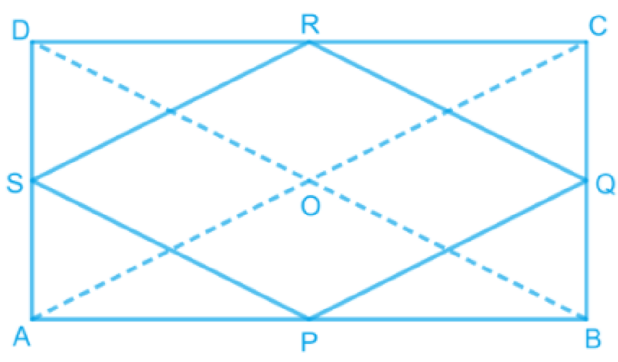
$\Rightarrow \angle SPQ=90^{\circ}$

Hence, PQRS is a rectangle.

6. $ABCD$ **is a rectangle and P, Q, R and S are the mid-points of the sides** AB, BC, CD **and DA respectively. Show that the quadrilateral PQRS is a rhombus.**

Ans: According to the question,

A rectangle $ABCD$ in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To prove:

PQRS is a rhombus.

We draw a line AC with join point A to Point C.

Proof:

In $\Delta ABC, P$ and Q are the mid-points of sides AB, BC respectively.

So, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$………. (i)

In $\Delta ADC, R$ and S are the mid-points of sides CD, AD respectively.

So, $SR \parallel AC$ and $SR = \frac{1}{2}AC$……… (ii)

According to the equations (i) and (ii),

$PQ \| SR$ and $PQ = SR$ ....(iii)

So, PQRS is a parallelogram.

Now $ABCD$ is a rectangle. [According to the question]

So, $AD = BC$

$\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC \Rightarrow AS = BQ$………. (iv)

In triangles APS and BPQ,

$AP = BP$ [P is the mid-point of AB]

$\angle PAS = \angle PBQ \left[\text{Each } 90^\circ\right]$

And $AS = BQ$ [From equation (iv)]

So, $\Delta APS \cong \Delta BPQ$ [By SAS congruency]

$\Rightarrow PS = PQ$ [By C.P.C.T.].........(v)

According to the equation (iii) and (v),

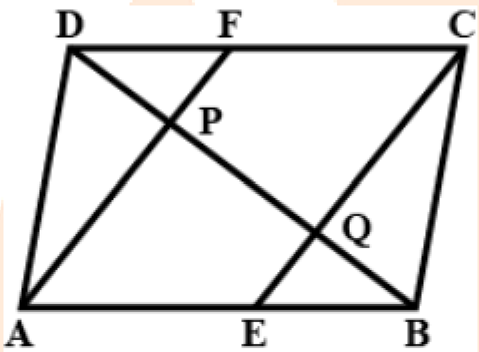
We get that PQRS is a parallelogram.

$\Rightarrow PS = PQ$

$\Rightarrow$ Two adjacent sides are equal.

Therefore, PQRS is a rhombus.

7. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (See figure). Show that the line segments AF and EC trisect the diagonal BD.



Ans: According to the question,

E and F are the mid-points of AB and CD respectively.

So, $\mathrm{AE}=\frac{1}{2}\mathrm{AB}$ and $\mathrm{CF}=\frac{1}{2}\mathrm{CD}$……… (i)

But ABCD is a parallelogram.

So, $\mathrm{AB}=\mathrm{CD}$ and $\mathrm{AB}\parallel\mathrm{DC}$

$\Rightarrow\frac{1}{2}\mathrm{AB}=\frac{1}{2}\mathrm{CD}$ and $\mathrm{AB}\parallel\mathrm{DC}$

$\Rightarrow\mathrm{AE}=\mathrm{FC}$ and $\mathrm{AE}\parallel\mathrm{FC}$ [From equation (i)]

AECF is a parallelogram.

$\Rightarrow\mathrm{FA}\parallel\mathrm{CE}\Rightarrow\mathrm{FP}\parallel\mathrm{CQ}$ [FP is a part of FA and CQ is a part of CE] ....(ii)

Because a segment traced at the midpoint of one of a triangle's sides and parallel to the other side bisects the third.

In $\Delta DCQ, F$ is the mid-point of CD and $\Rightarrow FP \parallel CQ$

So, P is the mid-point of DQ.

$\Rightarrow DP = PQ$……….(iii)

Similarly, In $\Delta ABP, E$ is the mid-point of AB and $\Rightarrow EQ \parallel$ AP

So, Q is the mid-point of BP.

$\Rightarrow BQ = PQ$……….(iv)

Acccording to the equations (iii) and (iv),

$DP = PQ = BQ$……….(v)

Now $BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ$

$\Rightarrow BQ = \frac{1}{3} BD$……… (vi)

According to the equations (v) and (vi),

$DP = PQ = BQ = \frac{1}{3} BD$

$\Rightarrow$ Points P and trisects BD.

So AF and CE trisects BD.

8. ABC **is a triangle right angled at** C**. A line through the mid-point M of hypotenuse** AB **and parallel to BC intersects AC at D.**

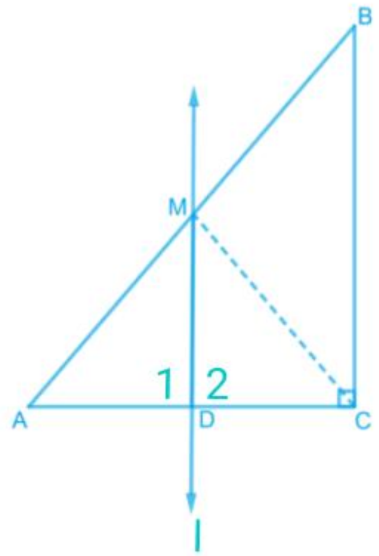
Ans:

(i) In $\Delta ABC, M$ is the mid-point of AB [According to the question]

$MD \parallel BC$

So, $AD = DC$ [Converse of mid-point theorem]

Hence D is the mid-point of AC.



(ii) $l \parallel BC$ (given) consider AC as a transversal.

So, $\angle 1 = \angle C$ [Corresponding angles]

$\Rightarrow \angle 1 = 90^{\circ} \left[\angle C = 90^{\circ}\right]$

Hence, $MD \perp AC$.

(iii) In ΔAMD and ΔCMD,

$AD = DC$ [Proved above]

$\angle 1 = \angle 2 = 90^{\circ}$ [Proved above]

$MD = MD$ [Common]

So, $\Delta AMD \cong \Delta CMD$ [By SAS congruency]

$\Rightarrow AM = CM$ [By C.P.C.T.]..........(i)

According to the question M is the mid-point of AB.

So, $AM = \frac{1}{2} AB$……….(ii)

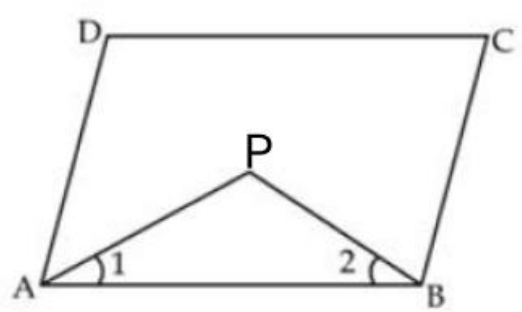
According to the equations (i) and (ii),

$$CM = AM = \frac{1}{2}AB$$

9. In a parallelogram ABCD, bisectors of adjacent angles A and B intersect each other at P. prove that $\angle APB = 90^{\circ}$

Ans: According to the question,

$ABCD$ is a parallelogram is and bisectors of $\angle A$ and $\angle B$ intersect each other at P.



To prove:

$\angle APB = 90^{\circ}$

Proof:

$$\angle 1 + \angle 2 = \frac{1}{2}\angle A + \frac{1}{2}\angle B$$

$$= \frac{1}{2}(\angle A + \angle B) \quad \rightarrow (i)$$

But $ABCD$ is a parallelogram and $AD \parallel BC$

So, $\angle A + \angle B = 180^{\circ}$

So, $\angle 1 + \angle 2 = \frac{1}{2} \times 180^{\circ} = 90^{\circ}$

In $\triangle APB$

$\angle 1 + \angle 2 + \angle APB = 180^{\circ}$

$90^{\circ} + \angle APB = 180^{\circ}$

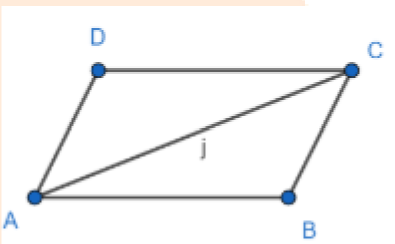
$\angle APB = 90^{\circ}$

Hence Proved.

10. In figure diagonal AC of parallelogram ABCD bisects $\angle A$ show that

(i) if bisects $\angle C$

ABCD is a rhombus



Ans:

(i) $AB \parallel DC$ and AC is transversal

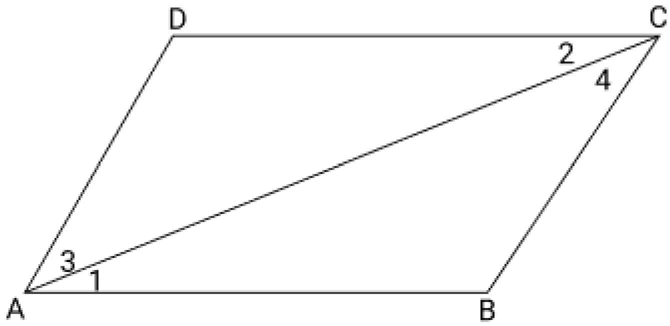
So, $\angle 1 = \angle 2$ (Alternate angles)

And $\angle 3 = \angle 4$ (Alternate angles)

But, $\angle 1 = \angle 3$

So, $\angle 2 = \angle 4$

So, AC bisecsts $\angle C$



(ii) In $\triangle ABC$ and $\triangle ADC$

$\mathrm{AC} = \mathrm{AC}$ [Common]

$\angle 1 = \angle 3$ [Given]

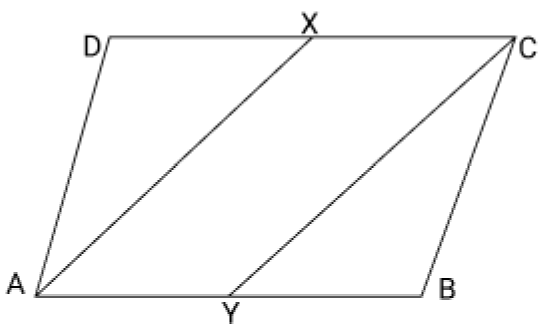
$\angle 2 = \angle 4$ [Proved]

So, $\triangle ABC \cong \triangle ADC$

So, $AB = AD$ [By CPCT]

So, $ABCD$ is a rhombus.

11. In figure ABCD is a parallelogram. AX and CY bisects angles A and C. prove that AYCX is a parallelogram.



Ans: According to the question,

In a parallelogram AX and CY bisects $\angle A$ and $\angle C$ respectively and we have to show that AYCX in a parallelogram.

In ΔADX and ΔCBY

$\angle D = \angle B$…..(i) [Opposite angles of parallelogram]

$\angle DAX = \frac{1}{2}\angle A$ [Given] ...(ii)

And $\angle \mathrm{BCY} = \frac{1}{2}\angle \mathrm{C}$ [Given] .....(iii)

But $\angle \mathrm{A} = \angle \mathrm{C}$

By equations (2) and (3), we get

$\angle DAX = \angle BCY \rightarrow (iv)$

Also, $AD = BC$ [opposite sides of parallelogram] ....(v)

According to the equations (i), (iv) and (v), we get

$\Delta ADX \cong \Delta CBY$ [By ASA]

So, $DX = BY$ [By CPCT]

But, $AB = CD$ [opposite sides of parallelogram]

$\mathrm{AB} - \mathrm{BY} = \mathrm{CD} - \mathrm{DX}$

Or

$\mathrm{AY} = \mathrm{CX}$

But $AY \parallel XC$ [Because, $ABCD$ is $a \parallel gm$]

So, $AYCX$ is a parallelogram.

12. Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side.

Ans: According to the question,

ΔABC in which E and F are mid points of side AB and AC respectively.

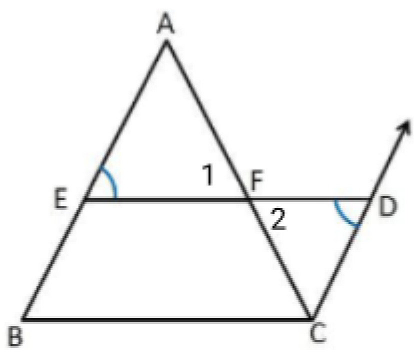
To prove:

$EF \parallel BC$

Construction:

Produce EF to D such that $EF =$ FD. Join CD

Proof: In ΔAEF and ΔCDF



$AF = FC$ [Because, F is mid-point of AC]

$\angle 1 = \angle 2$ [Vertically opposite angles]

EF=FD [By construction]

So, $\Delta AEF \cong \Delta CDF$ [By SAS]

And So, $AE = CD$ [By CPCT]

$AE = BE$ [Because, E is the mid-point]

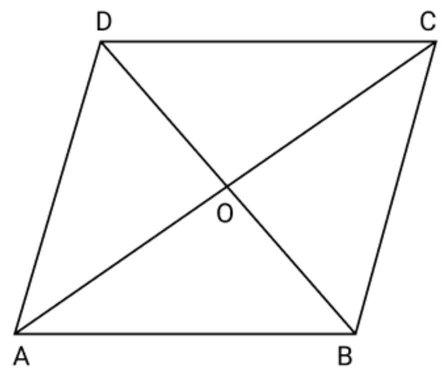
And So, $BE = CD$

$AB \parallel CD$ [So, $\angle BAC = \angle ACD$]

So, $BCDE$ is a parallelogrom $EF \parallel BC$.

Hence proved.

13. Prove that a quadrilateral is a rhombus if its diagonals bisect each other at right angles.



Ans: According to the question and figure,

ABCD is a quadrilateral diagonals AC and BD bisect each other at O at right angles

To Prove:

ABCD is a rhombus

Proof:

Because, diagonals AC and BD bisect each other at O.

So, $OA = OC, OB = OD$ And $\angle 1 = \angle 2 = \angle 3 = 90^{\circ}$

Now in ΔBOA And ΔBOC

$OA = OC$ [Given]

$OB = OB$ [Common]

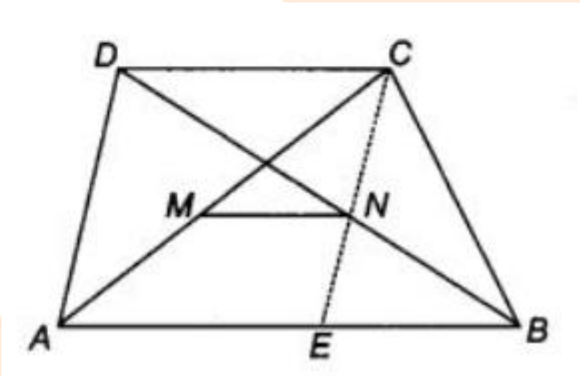
And $\angle 1 = \angle 2 = 90^{\circ}$ (Given)

So, $\Delta BOA = \Delta BOC$ [By SAS]

So, $BA = BC$ (By C.P.C.T.)

Similarly, $BC = CD, CD = DA$ and $DA = AB$, Hence, ABCD is a rhombus.

14. Prove that the straight line joining the mid points of the diagonals of a trapezium is parallel to the parallel sides.



Ans: According to the question and figure,

Given a trapezium ABCD in which $AB \parallel DC$ and M, N are the mid Points of the diagonals AC and BD.

As we need to prove that $MN \parallel AB \parallel DC$

Join CN and let it meet AB at E

Now in ΔCDN and ΔEBN

$\angle DCN = \angle BEN$ [Alternate angles]

$\angle CDN = \angle BEN$ [Alternate angles]

And $DN = BN$ [Given]

$\Delta CDN \cong \Delta EBN$ [By ASA]

So, $CN = EN$ [By C.P.C.T]

Now in $\Delta ACE, M$ and N are the mid points of the sides AC and CE respectively.

So, $MN \parallel AE$ Or $MN \parallel AB$

Also $AB \parallel DC$

So, $MN \parallel AB \parallel DC$

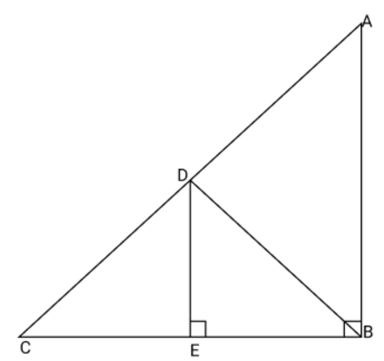
15. In fig $\angle B$ is a right angle in $\triangle ABCD$ is the mid-point of AC. $DE \parallel AB$ intersects BC

at E**. show that**

(i) E **is the mid-point of** BC

(ii) $\mathrm{DE} \perp \mathrm{BC}$

(ii) $\mathrm{BD} = \mathrm{AD}$



Ans: Proof:

So, $DE \parallel AB$ and D is mid points of AC

In ΔDCE and ΔDBE

$\mathrm{CE} = \mathrm{BE}$

$\mathrm{DE} = \mathrm{DE}$

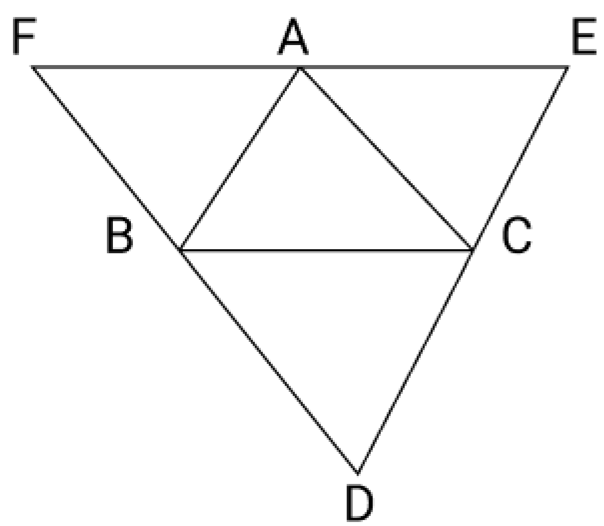
And $\angle DEC = \angle DEB = 90^{\circ}$

So, $\Delta DCE = \Delta DBE$

So, $\Delta DCE \cong \Delta DBE$

So, $CD = BD$

16: ABC is a triangle and through vertices A, B and C lines are drawn parallel to BC, AC and AB respectively intersecting at D, E and F. Prove that perimeter of ΔDEF is double the perimeter of $\triangle ABC$.



Ans: According to the figure,

$BCAF$ is a parallelogram

So, $BC = AF$

According to the figure,

$ABCE$ is a parallelogram

So, $BC = AE$

$AF + AE = 2BC$

Or $EF = 2BC$

Similarly,

$ED = 2AB$ and $FD = 2AC$

Because, Perimeter of $\Delta ABC = AB + BC + AC$

Perimeter of $\Delta DEF = DE + EF + DF$

$= 2AB + 2BC + 2AC$

$= 2[AB + BC + AC]$

$= 2$ Perimeter of ΔABC

Hence Proved.

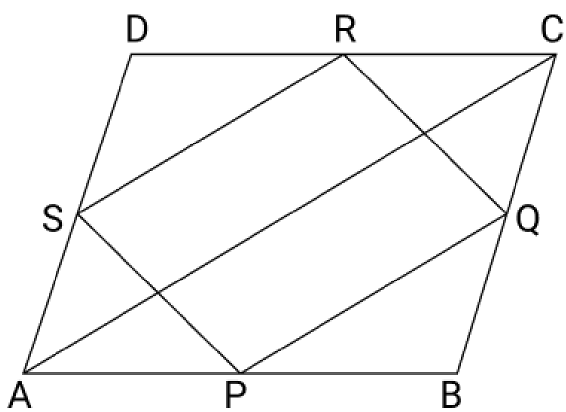
17. In fig ABCD is a quadrilateral P, Q, R and S are the mid Points of the sides AB, BC, CD and DA, AC is diagonal. Show that

(i) $SR \parallel AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram

(iv) PR and Sbisect each other



Ans: In $\Delta ABC, P$ and Q are the mid-points of the sides AB and BC respectively

(i) So, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$

(ii) Similarly $\mathrm{SR} \parallel \mathrm{AC}$ and $\mathrm{SR} = \frac{1}{2}\mathrm{AC}$

So, $PQ \parallel SR$ and $PQ = SR$

(iii) Therefore, PQRS is a Parallelogram.

Because, $PQ \parallel SR$.

(iv) PR and Sbisect each other.

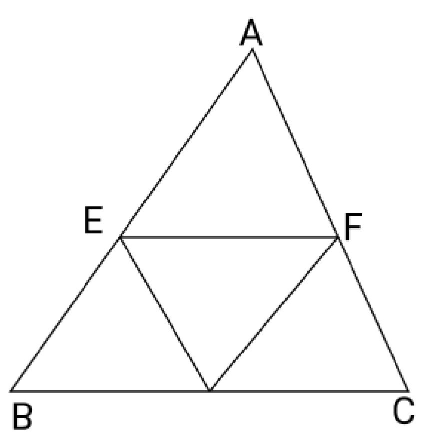
Because, $PQ \parallel SR$.

If we meet point P to R and point S to they bisect each other.

18. In $\Delta ABC, D, E, F$ are respectively the mid-Points of sides AB, DC and CA. show that $\triangle ABC$ is divided into four congruent triangles by Joining D,E,F.

Ans: According to the question,

D and E are mid-Points of sides AB and BC of $\Delta\mathrm{ABC}$



So, $\mathrm{DE} \parallel \mathrm{AC}$ $\{$Because a line segment joining the mid-Point of any two sides of a triangle parallel to third side $\}$

Similarly, $\mathrm{DF} \parallel \mathrm{BC}$ and $\mathrm{EF} \parallel \mathrm{AB}$

ADEF, BDEF and DFCE are all Parallelograms.

DE is diagonal of Parallelogram BDFE

$\Delta BDE \cong \Delta FED$

Similarly, $\Delta \text{DAF} \cong \Delta \text{FED}$

And $\Delta \text{EFC} \cong \Delta \text{FED}$

So all triangles are congruent

19. ABCD is a Parallelogram is which P and are mid-points of opposite sides AB and CD. If Aintersect DP at S, Bintersects CP at R, show that

(i) APCis a Parallelogram

(ii) DPBis a parallelogram

(iii) PSQR is a parallelogram

Ans:

(i) In quadrilateral APCQ

$\text{AP} \parallel \text{QC}$ [Because, $\text{AB} \parallel \text{CD}$]……… (i)

$\text{AP} = \frac{1}{2}\text{AB}, \text{CQ} = \frac{1}{2}\text{CD}$ (Given)

Also $\text{AB} = \text{CD}$

So $\text{AP} = \text{QC}$…………..(ii)

Therefore, APCis a parallelogram

[If any two sides of a quadrilateral equal and parallel then quad is a parallelogram]

(ii) Similarly, quadrilateral DPBis a Parallelogram because $\text{DQ} \parallel \text{PB}$ and $\text{DQ} = \text{PB}$

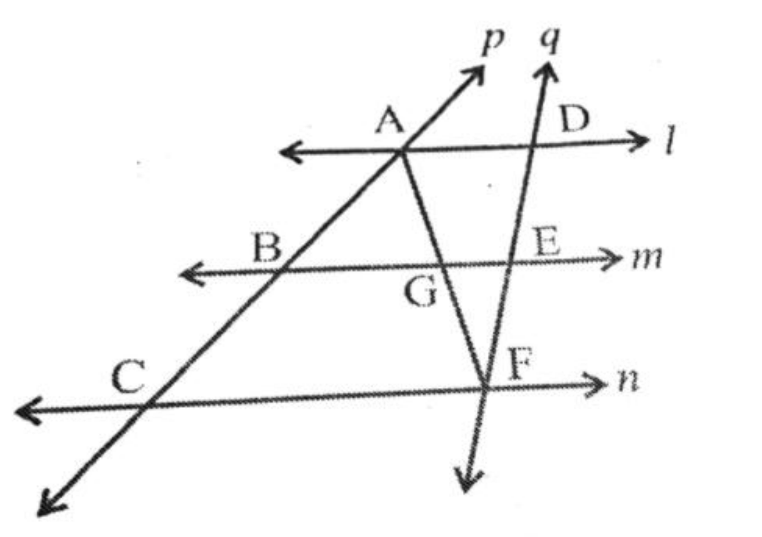
(iii) In quadrilateral PSQR,

$SP \parallel QR$ [SP is a part of DP and QR is a Part of QB]

Similarly, $SQ \parallel PR$

So, PSQR is also parallelogram.

20. l, m, n are three parallel lines intersected by transversals P and such that l, m and n cut off equal intercepts AB and BC on P .In fig Show that l, m, n cut off equal intercepts DE and EF on also.



Ans: In fig l, m, n are 3 parallel lines intersected by two transversal P and Q.

To Prove:

$DE = EF$

Proof:

In ΔACF

B is mid-point of AC

And $BG \parallel CF$

So, G is mid-point of AF [By mid-point theorem]

Now In ΔAFD

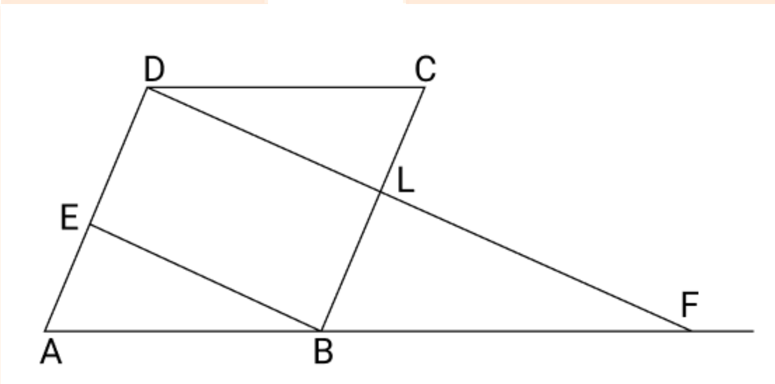
G is mid-point of AF and $\mathrm{GE} \| \mathrm{AD}$.

So, E is mid-point of FD [By mid-point theorem]

So, $\mathrm{DE} = \mathrm{EF}$

Hence Proved.

21. ABCD is a parallelogram in which E is mid-point of AD. $\mathrm{DF} \| \mathrm{EB}$ meeting AB produced at F and BC at L prove that $\mathrm{DF} = 2\mathrm{DL}$



Ans: In $\triangle AFD$

Because, E is mid-point of AD (According to the question)

$\mathrm{BE} \| \mathrm{DF}$ (According to the question)

So, by converse of mid-point theorem B is mid-point of AF

So, $AB = BF \ldots.(i)$

ABCD is parallelogram

So, $AB = CD \ldots\ldots(ii)$

According to the equation (i) and (ii)

$\mathrm{CD} = \mathrm{BF}$

Consider ΔDLC and ΔFLB

$\mathrm{DC} = \mathrm{FB}$ [Proved above]

$\angle DCL = \angle FBL$ [Alternate angles]

$\angle DLC = \angle FLB$ [Vertically opposite angles]

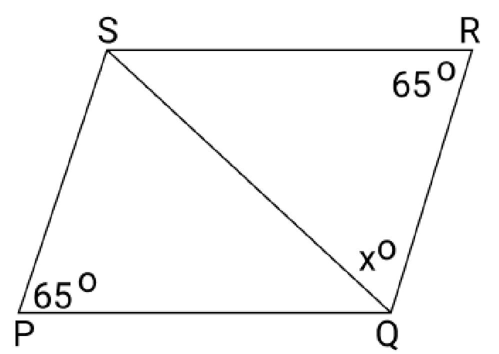
$\Delta DLC = \Delta FLB$ [By ASA]

So, $\mathrm{DL} = \mathrm{LF}$

So, $\mathrm{DF} = 2\mathrm{DL}$

Hence proved.

22. PQRS is a rhombus if $\angle P = 65^{\circ}$ find $\angle RSQ$.



Ans: According to the question and figure,

$\angle R = \angle P = 65^{\circ}$ [Opposite angles of a parallelogram are equal]

Suppose, $\angle RSQ = x^{\circ}$

In ΔRSQ

We have $\mathrm{RS} = \mathrm{RQ}$

$\angle RQS = \angle RSQ = x^\circ$ [Opposite Sides of equal angles are equal]

In $\triangle RSQ$

$\angle S + \angle Q + \angle R = 180^\circ$ [By angle sum property]

$x^\circ + x^\circ + 65^\circ = 180^\circ$

$2x^\circ = 180^\circ - 65^\circ$

$2x^\circ = 115^\circ$

$x = \dfrac{115}{2} = 57.5^\circ$

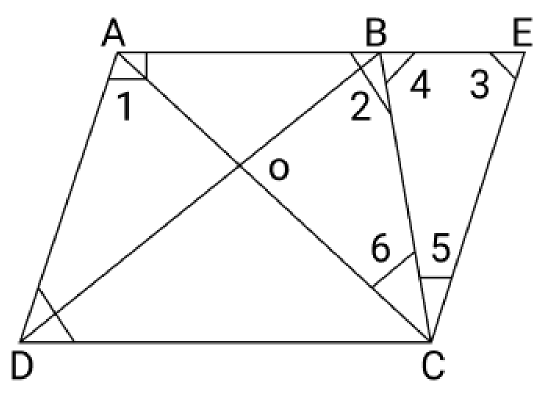
Hence, the value of $\angle RSQ = 57.5^\circ$.

23. ABCD **is a trapezium in which** $\mathrm{AB} \parallel \mathrm{CD}$ **and** $\mathrm{AD} = \mathrm{BC}$ **show that**

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$



Ans:

(i) Produce AB and Draw a line Parallel to DA meeting at E

Because, $AD \parallel EC$

$\angle 1 + \angle 3 = 180^{\circ}$ …. (1) [We know that the sum of interior angles on the some side of transversal is 180°]

In ΔBEC

BC=CE (given)

So, $\angle 3 = \angle 4$.....(2) [In a triangle equal side to opposite angles are equal]

$\angle 2 + \angle 4 = 180^{\circ}$ …….(3)

By equation (1) and (3)

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle 3 = \angle 4$

So, $\angle 1 = \angle 2$

So, $\angle A = \angle B$

(ii) Because, $AD \parallel EC$

$\angle D + \angle 6 + \angle 5 = 180^{\circ}$ ……(i)

$AE \parallel DC$

$\angle 6 + \angle 5 + \angle 3 = 180^{\circ}$ …….(ii)

$\angle D + \angle 6 + \angle 5 = \angle 6 + \angle 5 + \angle 3$

$\angle D = \angle 3 = \angle 4$

So, $\angle C = \angle D$.

(iii) In $\Delta \mathrm{ABC}$ and $\Delta \mathrm{BAD}$

$\mathrm{AB} = \mathrm{AB}$ [common]

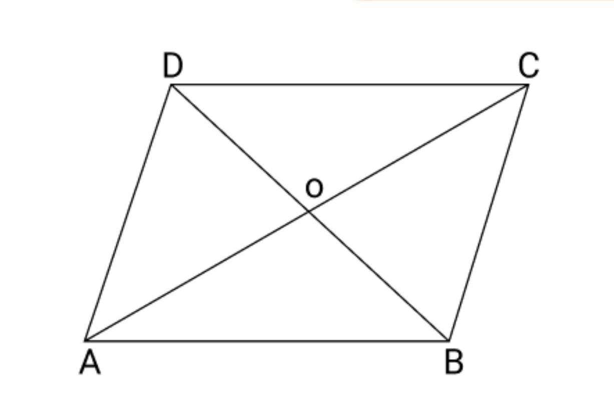
$\angle 1 = \angle 2$ [Proved above]

$\mathrm{AD} = \mathrm{BC}$ [According to the question]

So, $\Delta ABC \cong \Delta BAD$ [By SAS]

24. Show that diagonals of a rhombus are perpendicular to each other.

Ans:



According to the figure,

A rhombus ABCD whose diagonals AC and BD intersect at a Point O.

To Prove:

$\angle BOC = \angle DOC = \angle AOD = \angle AOB = 90^{\circ}$

Proof:

Clearly ABCD is a Parallelogram in which

$\mathrm{AB} = \mathrm{BC} = \mathrm{CD} = \mathrm{DA}$

As we know that diagonals of a Parallelogram bisect each other

So, $\mathrm{OA} = \mathrm{OC}$ and $\mathrm{OB} = \mathrm{OD}$

Now in $\Delta\mathrm{BOC}$ and $\Delta\mathrm{DOC}$, we have

$\mathrm{OB} = \mathrm{OD}$

$\mathrm{BC} = \mathrm{DC}$

$\mathrm{OC} = \mathrm{OC}$

So, $\Delta\mathrm{BOC} \cong \Delta\mathrm{DOC}$ [By SSS]

So, $\angle BOC = \angle DOC$ [By C.P.C.T]

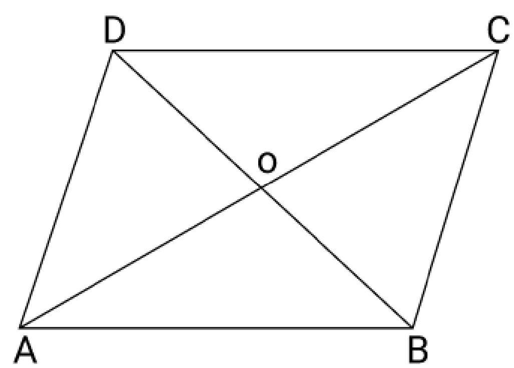
But $\angle BOC + \angle DOC = 180^{\circ}$

So, $\angle BOC = \angle DOC = 90^{\circ}$

Similarly, $\angle AOB = \angle AOD = 90^{\circ}$

Therefore, diagonals of a rhombus bisect each other at 90°.

25. Prove that the diagonals of a rhombus bisect each other at right angles.



Ans: According to the question and figure given a rhombus ABCD whose diagonals AC and BD intersect each other at O.

As we need to prove that $\mathrm{OA} = \mathrm{OC}, \mathrm{OB} = \mathrm{OD}$ and $\angle AOB = 90^{\circ}$

In $\triangle AOB$ and ΔCOD

$\mathrm{AB} = \mathrm{CD}$ [Sides of rhombus]

$\angle AOB = \angle COD$ [Vertically opposite angles]

And $\angle ABO = \angle CDO$ [Alternate angles]

So, $\Delta \mathrm{AOB} \cong \Delta \mathrm{COD}$ [By ASA]

So, $\mathrm{OA} = \mathrm{OC}$

And $\mathrm{OB} = \mathrm{OD}$ [By C.P.C.T]

Also in ΔAOB and ΔCOB

OA=OC [Proved]

$AB = CB$ [Sides of rhombus]

And $OB = OB$ [Common]

$\Delta AOB \cong \Delta COB$ [By SSS]

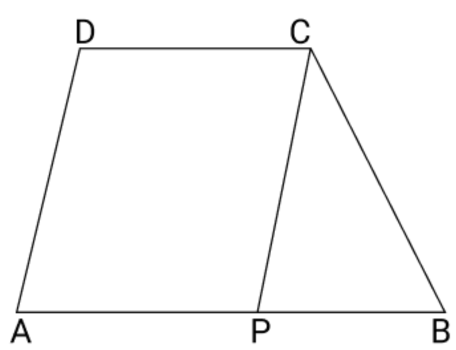
So, $\angle AOB = \angle COB$ [By C.P.C.T]

But $\angle AOB + \angle COB = 180^{\circ}$ [linear pair]

So, $\angle AOB = \angle COB = 90^{\circ}$

Hence proved.

26. In fig $ABCD$ is a trapezium in which $AB \parallel DC$ and $AD = BC$. Show that $\angle A = \angle B$



Ans: To show that $\angle A = \angle B$,

Draw $CP \parallel DA$ meeting AB at P

Because, $AP \parallel DC$ and $CP \parallel DA$

So, $APCD$ is a parallelogram

Again in ΔCPB

$CP = CB$ [Because, $BC = AD$] (Given)

$\angle CPB = \angle CBP \ldots(i)$ [Angles opposite to equal sides]

But $\angle CPA + \angle CPB = 180^{\circ}$[By linear pair]

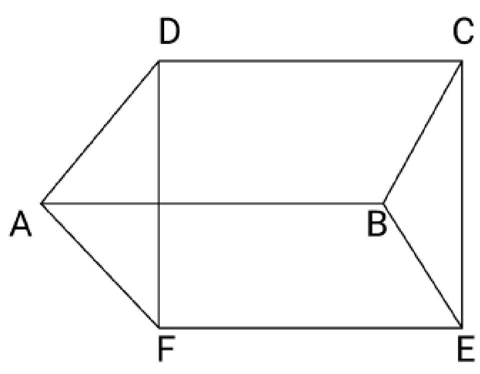
Also $\angle A + \angle CPA = 180^{\circ}$ [Because, $APCD$ is a parallelogram]

So, $\angle A + \angle CPA = \angle CPA + \angle CPB$ Or $\angle A = \angle CPB$

$= \angle CB$

Hence proved.

27. In fig ABCD and ABEF are Parallelogram, prove that CDFE is also a parallelogram.



Ans: According to the question,

$ABCD$ is a parallelogram

So, $AB = DC$ also $AB \| DC$………….. (i)

Also ABEF is a parallelogram

Because, $AB = FE$ and $AB \| FE$………..(ii)

By equations (i) and (ii)

$AB = DC = FE$

So, $AB = FE$

And $AB \parallel DC \parallel FE$

So, $AB \parallel FE$

So, CDEF is a parallelogram.

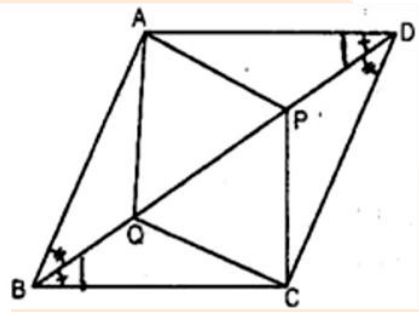
Hence Proved.

Long Answer Type Questions **4 Marks**

1. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

(i) ABCD is a square.

(ii) Diagonal BD bisects both $\angle B$ as well as $\angle D$.



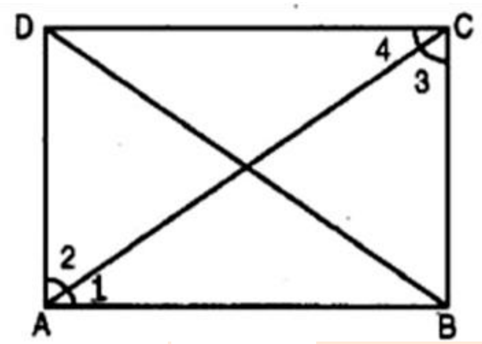
Ans: According to the question,

$ABCD$ is a rectangle.

Hence $AB = DC$ ....(i)

And $BC = AD$

Also $\angle A = \angle B = \angle C = \angle D = 90^{\circ}$



(i) In ΔABC and ΔADC

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

[AC bisects $\angle A$ and $\angle C$ (According to the question)]

$AC = AC$ [Common]

So, $\Delta ABC \cong \Delta ADC$ [By ASA congruency]

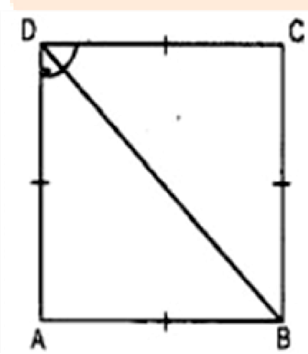
$\Rightarrow AB = AD$……….. (ii)

According to the equations (i) and (ii),

$AB = BC = CD = AD$

Therefore, $ABCD$ is a square.

(ii) In ΔABC and ΔADC



$AB = BA$ [Since $ABCD$ is a square]

$AD = DC$ [Since $ABCD$ is a square]

$BD = BD$ [Common]

So, $\Delta ABD \cong \Delta CBD$ [By SSS congruency]

$\Rightarrow \angle ABD = \angle CBD$ [By C.P.C.T.] ....(iii)

And $\angle ADB = \angle CDB$ [By C.P.C.T.] ....(iv)

According to the equations (iii) and (iv),

It is clear that diagonal BD bisects both $\angle B$ and $\angle D$.

2. An ΔABC and $\Delta DEF, AB = DE, AB \parallel DE, BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (See figure). Show that:

(i) Quadrilateral ABED is a parallelogram.

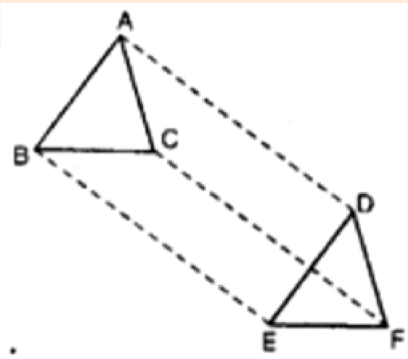
(ii) Quadrilateral BEFC is a parallelogram.

(iii) $AD \parallel CF$ **and** $AD = CF$

(iv) Quadrilateral ACFD is a parallelogram.

(v) $AC = DF$

(vi) $\Delta ABC \cong \Delta DEF$



Ans:

(i) In ΔABC and ΔDEF

$AB = DE$ [According to the question]

And $AB \parallel DE$ [According to the question]

So, ABED is a parallelogram.

(ii) In ΔABC and ΔDEF

$BC =$ EF [According to the question]

And $BC \| EF$ [According to the question]

So, BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

So, $AD \| BE$ and $AD = BE$ ....(i)

Also BEFC is a parallelogram.

So, $CF \| BE$ and $CF = BE$………. (ii)

According to the equations (i) and (ii), we get

So, $AD \| CF$ and $AD = CF$

(iv) As $AD \| CF$ and $AD = CF$

$\Rightarrow$ ACFD is a parallelogram.

(v) As ACFD is a parallelogram.

So, $AC = DF$

(vi) In ΔABC and ΔDEF,

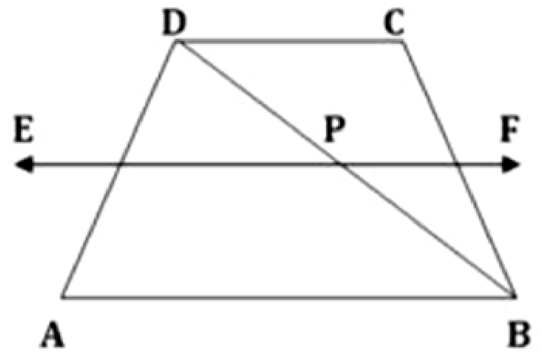
$AB = DE$ [According to the question]

$BC = EF$ [According to the question]

$AC = DF$ [Proved]

So, $\Delta ABC \cong \Delta DEF$ [By SSS congruency]

3. ABCD is a trapezium, in which $AB \parallel DC, BD$ is a diagonal and E is the mid-point of AD. A line is drawn through E, parallel to AB intersecting BC at F (See figure). Show that F is the mid-point of BC.



Ans: Suppose diagonal BD intersect line EF at point P.

In ΔDAB,

E is the mid-point of AD and $EP \parallel AB$ [Because, $EF \parallel AB$ (According to the question) P is the part of EF]

So, P is the mid-point of other side, BD of ΔDAB.

[A line drawn parallel to one side of a triangle and through the mid-point of the other intersects the third side at the mid-point.]

Now in ΔBCD,

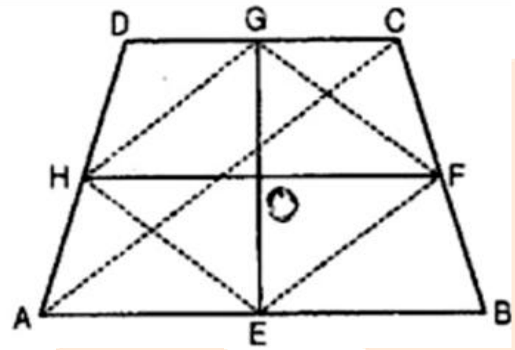
P is the mid-point of BD and $PF \parallel DC$ [Because, $EF \parallel AB$ (according to the question) and $AB \parallel DC$ (according to the question)]

So, $EF \parallel DC$ and PF is a part of EF.

So, F is the mid-point of other side, BC of ΔBCD. [Converse of mid-point of theorem]

4. Show that the line segments joining the mid-points of opposite sides of a quadrilateral bisect each other.

Ans: According to the question and figure, A quadrilateral ABCD in which EG and FH are the line-segments joining the midpoints of opposite sides of a quadrilateral.



To prove:

EG and FH bisect each other.

Construction:

Join AC, EF, FG, GH and HE.

Proof:

In $\Delta ABC, E$ and F are the mid-points of respective sides AB and BC.

So, $EF \parallel AC$ and $EF = \frac{1}{2}AC$………. (i)

Similarly, in Δ ADC,

G and H are the mid-points of respective sides CD and AD.

So, $HG \parallel AC$ and $HG = \frac{1}{2}AC$……… (ii)

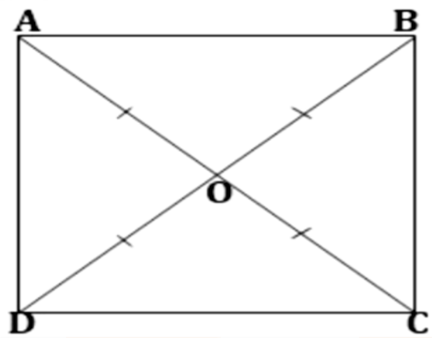
According to the equations (i) and (ii),

$EF \parallel HG$ and $EF = HG$

So, EFGH is a parallelogram.

Line segments (i.e. diagonals) EG and FH (of parallelogram EFGH) bisect each other because the diagonals of a parallelogram bisect each other.

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



Ans: Suppose $ABCD$ be a quadrilateral in which equal diagonals AC and BD bisect each other at right angle at point O.

We have $AC = BD$ and $OA = OC$

And $OB = OD$……….(ii)

Now $OA + OC = OB + OD$

$\Rightarrow OC + OC = OB + OB$ [Using equation (i) and (ii)]

$\Rightarrow 2OC = 2OB$

$\Rightarrow OC = OB$……….. (iii)

According to the equations (i), (ii) and (iii),

we get,

$OA = OB = OC = OD$..........(iv)

Now in ΔAOB and ΔCOD,

$OA =$ OD [Proved]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$OB = OC$ [Proved]

So, $\Delta AOB \cong \Delta DOC$ [By SAS congruency]

$\Rightarrow AB = DC$ [By C.P.C.T.] ....(v)

Similarly,

$\Delta BOC \cong \Delta AOD$ [By SAS congruency]

$\Rightarrow BC = AD$ [By C.P.C.T.] ....(vi)

According to the equation (v) and (vi), it is determined that $ABCD$ is a parallelogram because opposite sides of a quadrilateral are equal.

Now in ΔABC and ΔBAD,

$AB = BA$ [Common]

$BC = AD$ [Proved above]

$AC = BD$ [According to the figure]

So, $\Delta ABC \cong \Delta BAD$ [By SSS congruency]

$\Rightarrow \angle ABC = \angle BAD$ [By C.P.C.T.] ………… (vii)

But $\angle ABC + \angle BAD = 180^\circ$ [$ABCD$ is a parallelogram] ....(viii)

So, $AD \parallel BC$ and AB is a transversal.

$\Rightarrow \angle ABC + \angle ABC = 180^\circ$ [Using equations (vii) and (viii)]

$\Rightarrow 2\angle ABC = 180^\circ \Rightarrow \angle ABC = 90^\circ$

So, $\angle ABC = \angle BAD = 90^\circ$ ……… (ix)

Opposite angles of a parallelogram are equal.

But $\angle ABC = \angle BAD =$

So, $\angle ABC = \angle ADC = 90^\circ$ ……….(x)

So, $\angle BAD = \angle BDC = 90^\circ$ ……….(xi)

According to the equations (x) and (xi), we get

$\angle ABC = \angle ADC = \angle BAD = \angle BDC = 90^\circ$ …………(xii)

Now in ΔAOB and ΔBOC,

$OA = OC$ [Given]

$\angle AOB = \angle BOC = 90^\circ$ [Given]

$OB = OB$ [Common]

So, $\Delta AOB \cong \Delta COB$ [By SAS congruency]

$\Rightarrow AB = BC$.........(xiii)

According to the equations (v), (vi) and (xiii), we get,

$AB = BC = CD = AD$..........(xiv)

Now, according to the equations (xii) and (xiv), we have a quadrilateral whose equal diagonals bisect each other at right angle.

Also sides are equal make an angle of 90° with each other.

So, ABCD is a square.

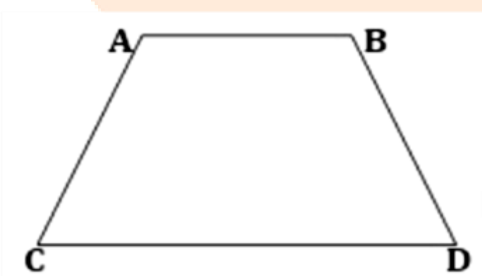
6. $ABCD$ **is a trapezium in which** $AB \parallel CD$ **and** $AD = BC$ **(See figure). Show that:**

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) Diagonal AC = Diagonal BD



Ans: According to the question,

$ABCD$ is a trapezium.

$AB \parallel CD$ and $AD = BC$

To prove:

(i) $\angle A = \angle B$

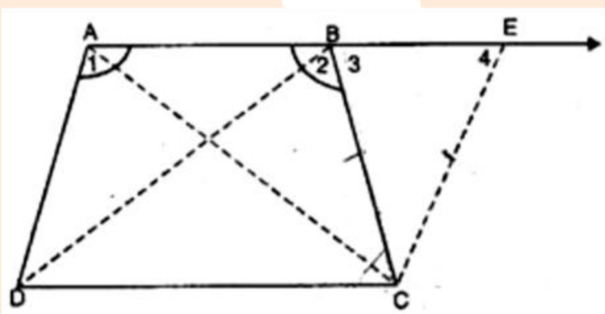
(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) Diagonal AC = Diagonal BD

Construction:

Draw CE $\parallel$ AD and extend AB to intersect CE at E.



Proof:

(i) As AECD is a parallelogram. [By construction]

So, $AD = EC$

But $AD = BC$ [According to the question]

So, $BC = EC$

$\Rightarrow \angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

Now $\angle 1 + \angle 4 = 180^\circ$ [Interior angles]

And $\angle 2 + \angle 3 = 180^\circ$ [Linear pair]

$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 2$ [Because, $\angle 3 = \angle 4$]

$\Rightarrow \angle A = \angle B$

(ii) $\angle 3 = \angle C$ [Alternate interior angles]

And $\angle D = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [ΔBCE is an isosceles triangle]

So, $\angle C = \angle D$

(iii) In ΔABC and ΔBAD,

$AB = AB$ [Common]

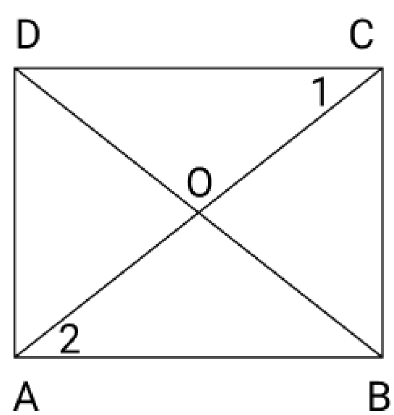
$\angle 1 = \angle 2$ [Proved]

$AD = BC$ [According to the question]

So, $\Delta ABC \cong \Delta BAD$ [By SAS congruency]

(iv) $AC = BD$ [By C.P.C.T.]

7. Prove that if the diagonals of a quadrilateral are equal and bisect each other at right angles then it is a square.



Ans: According to the question and figure,

In a quadrilateral $ABCD, AC = BD, AO = OC$ and $BO = OD$ and $\angle AOB = 90^{\circ}$

To prove:

ABCD is a square.

Proof:

In $\triangle AOB$ and ΔCOD

$\mathrm{OA}=\mathrm{OC}$

$\mathrm{OB}=\mathrm{OD}$ [According to the question and figure]

And

$\angle AOB=\angle COD$ [Vertically opposite angles]

So, $\Delta AOB\cong\Delta COD$ [By SAS]

So, $AB=CD$ [By C.P.C.T.]

$\angle 1=\angle 2$ [By C.P.C.T] But these are alternate angles So, $AB\parallel CD$

ABCD is a parallelogram whose diagonals bisects each other at right angles.

So, $ABCD$ is a rhombus

Again in $\triangle ABD$ and ΔBCA

$\mathrm{AB}=\mathrm{BC}$ [Sides of a rhombus]

$\mathrm{AD}=\mathrm{AB}$ [Sides of a rhombus]

And $\mathrm{BD}=\mathrm{CA}$ [Given]

So, $\triangle ABD\cong\Delta BCA$

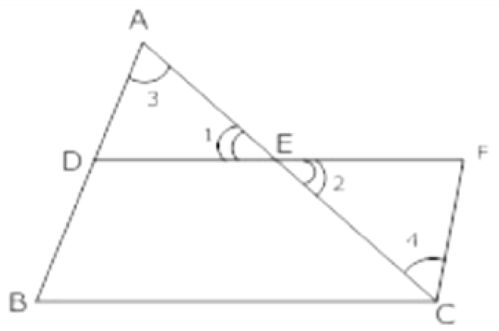
So, $\angle BAD=\angle CBA$ [By CPCT]

These are alternate angles of these same side of transversal

So, $\angle BAD+\angle CBA=180^{\circ}$ or $\angle BAD=\angle CBA=90^{\circ}$

Therefore, ABCD is a square.

8. Prove that in a triangle, the line segment joining the mid points of any two sides is parallel to the third side.



Ans: According to the question and figure,

A ΔABC in which D and E are mid-points of the side AB and AC respectively

To Prove: $DE \parallel BC$ and $DE = \frac{1}{2}BC$

Construction: Draw $CF \parallel BA$

Proof: In ΔADE and ΔCFE

$\angle 1 = \angle 2$ [Vertically opposite angles]

$\mathrm{AE} = \mathrm{CE}$ [Given]

And $\angle 3 = \angle 4$ [Alternate interior angles]

So, $\Delta ADE \cong \Delta CFE$ [By ASA]

So, $\mathrm{DE} = \mathrm{FE}$ [By C.P.C.T]

But DA = DB

So, $\mathrm{DB} = \mathrm{FC}$

Now $\mathrm{DB} \parallel \mathrm{FC}$

So, DBCF is a parallelogram

So, $\mathrm{DE} \parallel \mathrm{BC}$

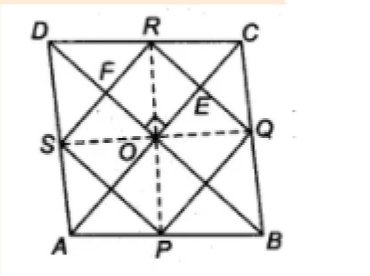
Also $\mathrm{DE}=\mathrm{EF}=\frac{1}{2}\mathrm{BC}$

Hence, $DE \parallel BC$ and $DE=\frac{1}{2}BC$.

9. ABCD **is a rhombus and P, Q, R, and** S **are the mid-Points of the sides** $\mathrm{AB}, \mathrm{BC}, \mathrm{CD}$ **and DA respectively. Show that quadrilateral PQRS is a rectangle.**

Ans: Join AC and BD which intersect at O let BD intersect RS at E and AC intersect Rat F

In $\Delta\mathrm{ABD}$, P and S are mid-points of sides AB and AD.



So, $\mathrm{PS} \| \mathrm{BD}$ and $\mathrm{PS}=\frac{1}{2}BD$

Similarly, $\mathrm{RQ} \| \mathrm{DB}$ and $\mathrm{RQ}=\frac{1}{2}\mathrm{BD}$

So, $\mathrm{RS} \| \mathrm{BD} \| \mathrm{RQ}$ and $\mathrm{PS}=\frac{1}{2}BD=RQ$

$\mathrm{PS}=\mathrm{RQ}$ and $\mathrm{PS} \| \mathrm{RQ}$

So, PQRS is a parallelogram

Now, $\mathrm{RF} \| \mathrm{EO}$ and $\mathrm{RE} \| \mathrm{FO}$

So, OFRE is also a parallelogram.

Again, we also know that diagonals of a rhombus bisect each other at right angles.

Because, $\angle EOF=90^{\circ}$

Because, $\angle EOF = \angle ERF$ [Opposite angles of a parallelogram]

So, $\angle ERF = 90^{\circ}$

So, each angle of the parallelogram PQRS is 90°

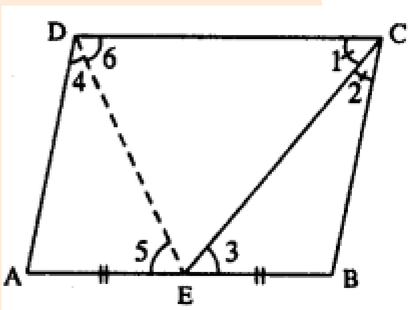
Therefore, PQRS is a rectangle.

10. In the given Fig. ABCD is a parallelogram E is mid-point of AB and CE bisects $\angle BCD$ Prove that:

(i) $AE = AD$

(ii) DE **bisects** $\angle ADC$

(iii) $\angle DEC = 90^{\circ}$



Ans: According to the question,

ABCD is a parallelogram

So, $AB \parallel CD$ and EC cuts them

$\Rightarrow \angle BEC = \angle ECD$ [Alternate interior angle]

$\Rightarrow \angle BEC = \angle ECB \quad [\angle ECD = \angle ECB]$

$\Rightarrow EB = BC$

$\Rightarrow AE = AD$

(i) Now $AE = AD$

$\Rightarrow \angle ADE = \angle AED$

$\Rightarrow \angle ADE = \angle EAC$ [So, $\angle AED = \angle EDC$ Alternate interior angles]

(ii) So, DE bisects $\angle ADC$

(iii) Now $\angle ADC + \angle BCD = 180^{\circ}$

$\Rightarrow \frac{1}{2}\angle ADC + \frac{1}{2}\angle BCD = 90^{\circ}$

$\Rightarrow \angle EDC + \angle DCE = 90^{\circ}$

But, the sum of all the angles of the triangle is 180°

$\Rightarrow 90^{\circ} + \angle DEC = 180^{\circ}$

$\Rightarrow \angle DEC = 90^{\circ}$

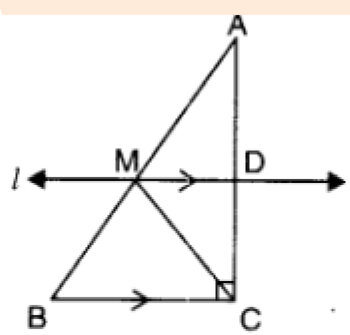
Hence proved.

11. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. show that

(i) D **is mid-point of** AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2}AB$



Ans: According to the question,

ABC is a triangle right angle at C

(i) M is mid-point of AB

And $\mathrm{MD} \parallel \mathrm{BC}$

So, D is mid-Point of AC [a line through midpoint of one side of a triangle parallel to another side bisect the third side]

(ii) Because, $\mathrm{MD} \parallel \mathrm{BC}$

$\angle ADM = \angle DCB$ [Corresponding angles]

$\angle ADM = 90^\circ$

(iii) In ΔADM and ΔCDM

$\mathrm{AD} = \mathrm{DC}$ [Because D is mid-point of AC]

$\mathrm{DM} = \mathrm{DM}$ [Common]

So, $\Delta \mathrm{ADM} \cong \Delta \mathrm{CDM}$ [By SAS]

$\mathrm{AM} = \mathrm{CM}$ [By C.P.C.T]

$\mathrm{AM} = \mathrm{CM} = \mathrm{MB}$ [Because M is mid-point of AB]

So, $\mathrm{CM} = \mathrm{MA} = \dfrac{1}{2}\mathrm{AB}$

Hence proved.