Exercise 2.1

Question 1:
Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$
(ii) $y^2 + \sqrt{2}$
(iii) $3\sqrt{t} + t\sqrt{2}$
(iv) $y + \frac{2}{y}$
(v) $y + 2y^{-1}$

Solution 1:
i) $4x^2 - 3x + 7$
One variable is involved in given polynomial which is 'x'
Therefore, it is a polynomial in one variable 'x'.

(ii) $y^2 + \sqrt{2}$
One variable is involved in given polynomial which is 'y'
Therefore, it is a polynomial in one variable 'y'.

(iii) $3\sqrt{t} + t\sqrt{2}$

No. It can be observed that the exponent of variable t in term $3\sqrt{t}$ is $\frac{1}{2}$, which is nota whole number. Therefore, this expression is not a polynomial.

(iv) $y + \frac{2}{y}$
$= y + 2y^{-1}$

The power of variable 'y' is -1 which is not a whole number.
Therefore, it is not a polynomial in one variable

No. It can be observed that the exponent of variable y in term $\frac{2}{y}$ is -1, which is not a whole number. Therefore, this expression is not a polynomial.
(v) $x^{10} + y^3 + t^{50}$

In the given expression there are 3 variables which are 'x, y, t' involved.

Therefore, it is not a polynomial in one variable.

Question 2:

Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

(ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$

(iv) $\sqrt{2}x-1$

Solution 2:

(i) $2+x^2+x^3$

$=2+1(x^2)+x$

The coefficient of x^2 is 1.

(ii) $2-x^2+x^3$

$=2-1(x^2)+x$

The coefficient of x^2 is -1.

(iii) $\frac{\pi}{2}x^2+x$

The coefficient x^2 of is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1=0x^2+\sqrt{2}x-1$

The coefficient of x^2 is 0.

Question 3:

Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution 3 :

Binomial of degree 35 means a polynomial is having

1. Two terms
2. Highest degree is 35

Example: $x^{35} + x^{34}$

Monomial of degree 100 means a polynomial is having

1. One term
2. Highest degree is 100

Example : x^{100}.

Question 4:

Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$

(ii) $4 - y^2$

(iii) $5t - \sqrt{7}$

(iv) 3

Solution 4:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $5x^3 + 4x^2 + 7x$

Highest power of variable ‘x’ is 3. Therefore, the degree of this polynomial is 3

(ii) $4 - y^2$

Highest power of variable ‘y’ is 2. Therefore, the degree of this polynomial is 2.

(iii) $5t - \sqrt{7}$

Highest power of variable ‘t’ is 1. Therefore, the degree of this polynomial is 1.

(iv) 3

This is a constant polynomial. Degree of a constant polynomial is always 0.

Question 5: Classify the following as linear, quadratic and cubic polynomial:

(i) $x^2 + x$

(ii) $x - x^3$

(iii) $y + y^2 + 4$

(iv) $1+x$
(v) 3t
(vi) r^2
(vii) $7x^2$ $7x^3$

Solution 5:
Linear polynomial – whose variable power is '1'

Quadratic polynomial - whose variable highest power is '2'
Cubic polynomial- whose variable highest power is '3'

(i) x^2+x is a quadratic polynomial as its highest degree is 2.
(ii) $x-x^3$ is a cubic polynomial as its highest degree is 3.
(iii) $y+y^2+4$ is a quadratic polynomial as its highest degree is 2.
(iv) 1 + x is a linear polynomial as its degree is 1.
(v) 3t is a linear polynomial as its degree is 1.
(vi) r^2 is a quadratic polynomial as its degree is 2.
(vii) $7x^2$ $7x^3$ is a cubic polynomial as highest its degree is 3.

Exercise 2.2

Question 1:
Find the value of the polynomial at $5x-4x^2+3$ at
(i) x = 0
(ii) x = −1
(iii) x = 2

Solution 1:
(i) $p(x)=5x-4x^2+3$
$p(0)=5(0)-4(0)^2+3 = 3$

(ii) $p(x)=5x-4x^2+3$

$p(-1)=5(-1)-4(-1)^2+3$
$= -5-4(1)+3=-6$

(iii) $p(x)=5x-4x^2+3$
$p(2)=5(2)-4(2)^2+3 = 10-16+3=-3$

Question 2:

Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$
(ii) $p(t) = 2 + t + 2t^2 - t3$
(iii) $p(x) = x^3$
(iv) $p(x) = (x - 1)(x + 1)$

Solution 2:

(i) $p(y) = y^2 - y + 1$

- $p(0) = (0)^2 - (0) + 1 = 1$
- $p(1) = (1)^2 - (1) + 1 = 1 - 1 + 1 = 1$
- $p(2) = (2)^2 - (2) + 1 = 4 - 2 + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

- $p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$
- $p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$
- $p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

- $p(0) = (0)^3 = 0$
- $p(1) = (1)^3 = 1$
- $p(2) = (2)^3 = 8$

(v) $p(x) = (x - 1)(x + 1)$

- $p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$
- $p(1) = (1 - 1)(1 + 1) = 0(2) = 0$
- $p(2) = (2 - 1)(2 + 1) = 1(3) = 3$

Question 3:

Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1, x=-\frac{1}{3}$

(ii) $p(x)=5x-\pi, x=\frac{4}{5}$

(iii) $p(x)=x^2-1, x=1,-1$

(iv) $p(x)=(x+1)(x-2), x=-1, 2$

(v) $p(x)=x^2, x=0$

(vi) $p(x)=lm+m, x=-\frac{m}{l}$

(vii) $p(x)=3x^2-1, x=-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x)=2x+1, x=\frac{1}{2}$

Solution 3:

(i) If $x=-\frac{1}{3}$ is a zero of given polynomial p(x) = 3x + 1, then $p\left(-\frac{1}{3}\right)$ should be 0.

Here, $p\left(-\frac{1}{3}\right)=3\left(-\frac{1}{3}\right)+1 = -1+1=0$

Therefore, is a zero of the given polynomial.

(ii) If $x=\frac{4}{5}$ is a zero of polynomial p(x) = 5x − π, then $p\left(\frac{4}{5}\right)$ should be 0.

Here, $p\left(\frac{4}{5}\right)=5\left(\frac{4}{5}\right)-\pi = 4-\pi$

As $p\left(\frac{4}{5}\right) \neq 0$

Therefore, $x=\frac{4}{5}$ is not a zero of the given polynomial.

(iii) If x = 1 and x = −1 are zeroes of polynomial $p(x) = x^2 - 1$, then p(1) and p(−1)should be 0.

Here, $p(1) = (1)^2 - 1 = 0$, and

$p(-1) = (-1)^2 - 1 = 0$

Hence, x = 1 and −1 are zeroes of the given polynomial.

(iv) If x = −1 and x = 2 are zeroes of polynomial p(x) = (x +1) (x − 2), then p(−1) and p(2)should be 0.

Here, p(−1) = (−1 + 1) (−1 − 2) = 0(−3) = 0, and

p(2) = (2 + 1) (2 − 2) = 3(0) = 0

Therefore, x = −1 and x = 2 are zeroes of the given polynomial.

(v) If x = 0 is a zero of polynomial $p(x) = x^2$, then p(0) should be zero.

Here, $p(0) = (0)^2 = 0$

Hence, x = 0 is a zero of the given polynomial.

(vi) If $p\left(\frac{-m}{l}\right)$ is a zero of polynomial p(x) = lx + m, then $p\left(\frac{-m}{l}\right)$ should be 0.

Here, $p\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = 0$

Therefore, $x = \frac{-m}{l}$ is a zero of the given polynomial.

(vii) If $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 - 1$, then

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

Here, $p\left(\frac{-1}{\sqrt{3}}\right)=3\left(\frac{-1}{\sqrt{3}}\right)^2-1=3\left(\frac{1}{3}\right)-1=1-1=0$, and

$p\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-1=3\left(\frac{4}{3}\right)-1=4-1=3$

Hence, $x=\frac{-1}{\sqrt{3}}$ is a zero of the given polynomial.

However, $x=\frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

(viii) If $x=\frac{1}{2}$ is a zero of polynomial p(x) = 2x + 1, then $p\left(\frac{1}{2}\right)$ should be 0.

Here, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

As $p\left(\frac{1}{2}\right)\neq 0$,

Therefore, $x=\frac{1}{2}$ is not a zero of the given polynomial.

Question 4:

Find the zero of the polynomial in each of the following cases:

(i) p(x) = x + 5

(ii) p(x) = x – 5

(iii)p(x) = 2x + 5

(iv)p(x) = 3x – 2

(v) p(x) = 3x

(vi)p(x) = ax, a $\neq$ 0

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution 4:

Zero of a polynomial is that value of the variable at which the value of thepolynomial is obtained as 0.

(i) p(x) = x + 5
Let p(x) = 0
x + 5 = 0
x = − 5
Therefore, for x = −5, the value of the polynomial is 0 and hence, x = −5 is a zero of the given polynomial.

(ii) p(x) = x − 5
Let p(x) = 0
x − 5 = 0
x = 5
Therefore, for x = 5, the value of the polynomial is 0 and hence, x = 5 is a zero of the given polynomial.

(iii) p(x) = 2x + 5
Let p(x) = 0
2x + 5 = 0
2x = − 5

$$x = -\frac{5}{2}$$

Therefore, for $x = -\frac{5}{2}$, the value of the polynomial is 0 and hence, $x = -\frac{5}{2}$ is a zero of the given polynomial.

(iv) p(x) = 3x – 2

p(x) = 0
3x − 2 = 0
Therefore, for $x = \frac{2}{3}$, the value of the polynomial is 0 and hence, $x = \frac{2}{3}$ is a zero of the given polynomial.

(v) $p(x) = 3x$
Let $p(x) = 0$
$3x = 0$
$x = 0$
Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vi) $p(x) = ax$
Let $p(x) = 0$
$ax = 0$
$x = 0$
Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vii) $p(x) = cx + d$
Let $p(x) = 0$
$cx + d = 0$

$$x = \frac{-d}{c}$$

Therefore, for $x = \frac{-d}{c}$, the value of the polynomial is 0 and hence, $x = \frac{-d}{c}$ is a zero of the given polynomial.

Exercise 2.3

Question 1:
Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$
(ii) $x - \frac{1}{2}$
(iii) x
(iv) $x + \pi$
(v) $5 + 2x$

Solution 1:
(i) $x^3 + 3x^2 + 3x + 1 \div x + 1$
By long division, we get

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ \hline x+1 & \cancel{x^3} + 3x^2 + 3x + 1 \\ & \cancel{x^3} + x^2 \\ & - \quad - \\ \hline & \quad 2\cancel{x^2} + 3x + 1 \\ & \quad 2\cancel{x^2} + 2x \\ & \quad - \quad - \\ \hline & \qquad \cancel{x} + 1 \\ & \qquad \cancel{x} + 1 \\ & \qquad - \; - \\ \hline & \qquad 0 \end{array}$$

Therefore, the remainder is 0.

(ii) $x^3 + 3x^2 + 3x + 1 \div x - \frac{1}{2}$
By long division,

$$
\begin{array}{r|l}
 & x^2+\frac{7}{2}x+\frac{19}{4} \\
\hline
x-\frac{1}{2} & \cancel{x^3}+3x^2+3x+1 \\
 & \cancel{x^3}-\frac{x^2}{2} \\
 & -\quad + \\
\hline
 & \cancel{\frac{7}{2}x^2}+3x+1 \\
 & \cancel{\frac{7}{2}x^2}-\frac{7}{4}x \\
 & -\quad + \\
\hline
 & \cancel{\frac{19}{4}x}+1 \\
 & \cancel{\frac{19}{4}x}-\frac{19}{8} \\
 & -\quad + \\
\hline
 & \frac{27}{8} \\
\hline
\end{array}
$$

Therefore, the remainder is $\frac{27}{8}$.

(iii) $x^3 + 3x^2 + 3x + 1 \div x$

By long division,

$$
\begin{array}{r|l}
 & x^2+3x+3 \\
\hline
x & \cancel{x^3}+3x^2+3x+1 \\
 & \cancel{x^3} \\
 & - \\
\hline
 & \cancel{3x^2}+3x+1 \\
 & \cancel{3x^2} \\
 & - \\
\hline
 & \cancel{3x}+1 \\
 & \cancel{3x} \\
 & - \\
\hline
 & 1 \\
\hline
\end{array}
$$

Therefore, the remainder is 1.

(iv) $x^3 + 3x^2 + 3x + 1 \div x + \pi$
By long division, we get

$$
\begin{array}{r l}
 & x^2 + (3-\pi)x + (3 - 3\pi + \pi^2) \\
\cline{2-2}
x+\pi \big) & x^3 + 3x^2 + 3x + 1 \\
 & x^3 + \pi x^2 \\
 & - \quad - \\
\cline{2-2}
 & (3-\pi)x^2 + 3x + 1 \\
 & (3-\pi)x^2 + (3-\pi)\pi x \\
 & - \qquad\qquad - \\
\cline{2-2}
 & \left[3 - 3\pi + \pi^2\right]x + 1 \\
 & \left[3 - 3\pi + \pi^2\right]x + (3 - 3\pi + \pi^2)\pi \\
 & - \qquad\qquad\qquad - \\
\cline{2-2}
 & \left[1 - 3\pi + 3\pi^2 - \pi^3\right] \\
\cline{2-2}
\end{array}
$$

Therefore, the remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1$.

(v) 5 + 2x
By long division, we get

$$\frac{x^2}{2}+\frac{x}{4}+\frac{7}{8}$$

$$2x+5\,\overline{)\,x^3+3x^2+3x+1}$$

$$\cancel{x^3}+\frac{5}{2}x^2$$

$$-\quad -$$

$$\frac{x^2}{\cancel{2}}+3x+1$$

$$\frac{\cancel{x^2}}{2}+\frac{5x}{4}$$

$$-\quad -$$

$$\frac{7x}{\cancel{4}}+1$$

$$\frac{\cancel{7}}{4}x+\frac{35}{8}$$

$$-\quad -$$

$$-\frac{27}{8}$$

Therefore, the remainder is $-\dfrac{27}{8}$.

Question 2:

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Solution 2:

$x^3 - ax^2 + 6x - a \div x - a$

By long division,

$$x^2+6$$

$$x-a\,\overline{)\,x^3-ax^2+6x-a}$$

$$x^3-ax^2$$

$$-\quad +$$

$$6x-a$$

$$6x-6a$$

$$-\quad +$$

$$5a$$

Therefore, when $x^3 - ax^2 + 6x - a$ is divided by $x - a$, the remainder obtained is 5a.

Question 3:

Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Solution 3:

Let us divide $(3x^3 + 7x)$ by $(7 + 3x)$.

By long division, we get

$$
\begin{array}{r|l}
 & x^2 - \frac{7}{3}x + \frac{70}{9} \\ \hline
3x+7 & 3x^3 + 0x^2 + 7x \\
 & 3x^3 + 7x^2 \\
 & - \quad\; - \\ \hline
 & -7x^2 + 7x \\
 & -7x^2 - \frac{49x}{3} \\
 & + \quad\;\; + \\ \hline
 & \frac{70x}{3} \\
 & \frac{70x}{3} + \frac{490}{9} \\
 & - \quad\;\; - \\ \hline
 & -\frac{490}{9} \\ \hline
\end{array}
$$

The remainder is not zero,

Therefore, $7 + 3x$ is not a factor of $3x^3 + 7x$.