NCERT Solutions for Class 9

Maths

Chapter 1 – Number System

Exercise 1.1

1. Do you think zero is a rational number? If it is, then can it be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$? Describe it.

Ans: Remember that, according to the definition of rational number, a rational number is a number that can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Now, notice that zero can be represented as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5}.....$

Also, it can be expressed as $\frac{0}{-1}, \frac{0}{-2}, \frac{0}{-3}, \frac{0}{-4}.....$

Therefore, it is concluded from here that 0 can be expressed in the form of $\frac{p}{q}$, where p and q are integers.

Hence, zero must be a rational number.

2. Write any 6 rational numbers between 3 and 4.

Ans: It is known that there are infinitely many rational numbers between any two numbers. Since we need to find 6 rational numbers between 3 and 4, so multiply and divide the numbers by 7 (or by any number greater than 6)

Then it gives,

$$3 = 3 \times \frac{7}{7} = \frac{21}{7}$$

$$4 = 4 \times \frac{7}{7} = \frac{28}{7}$$

Hence, 6 rational numbers found between 3 and 4 are $\frac{22}{7},\frac{23}{7},\frac{24}{7},\frac{25}{7},\frac{26}{7},\frac{27}{7}$.

3. Write any five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Ans: It is known that there are infinitely many rational numbers between any two numbers.

Since here we need to find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, so multiply and divide by 6 (or by any number greater than 5).

Then it gives,

$$\frac{3}{5}=\frac{3}{5}\times\frac{6}{6}=\frac{18}{30},$$

$$\frac{4}{5}=\frac{4}{5}\times\frac{6}{6}=\frac{24}{30}.$$

Hence, 5 rational numbers found between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{19}{30},\frac{20}{30},\frac{21}{30},\frac{22}{30},\frac{23}{30}$.

4. Verify all the statements given below and state whether they are true or false. Show proper reasons for your answers.

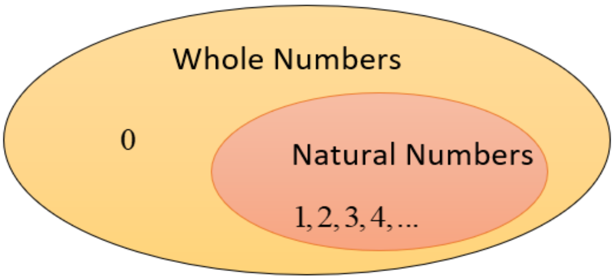
i. Statement: Every natural number is a whole number.

Ans: Write the whole numbers and natural numbers in a separate manner.

It is known that the whole number series is $0,1,2,3,4,5\ldots\ldots$ and

the natural number series is $1,2,3,4,5\ldots\ldots$

Therefore, it is concluded that all the natural numbers lie in the whole number series as represented in the diagram given below.



Thus, it is concluded that every natural number is a whole number.

Hence, the given statement is true.

ii. Statement: Every integer is a whole number.

Ans: Write the integers and whole numbers in a separate manner.

It is known that integers are those rational numbers that can be expressed in the form of $\frac{p}{q}$, where $q=1$.

Now, the series of integers is like $0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots$.

But the whole numbers are $0, 1, 2, 3, 4, \ldots$.

Therefore, it is seen that all the whole numbers lie within the integer numbers, but the negative integers are not included in the whole number series.

Thus, it can be concluded from here that every integer is not a whole number.

Hence, the given statement is false.

iii. Statement: Every rational number is a whole number.

Ans: Write the rational numbers and whole numbers in a separate manner.

It is known that rational numbers are the numbers that can be expressed in the form $\frac{p}{q}$, where $q \neq 0$ and the whole numbers are represented as $0, 1, 2, 3, 4, 5, \ldots$

Now, notice that every whole number can be expressed in the form of $\frac{p}{q}$ as $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \ldots$

Thus, every whole number is a rational number, but all the rational numbers are not whole numbers. For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ are not whole numbers.

Therefore, it is concluded from here that every rational number is not a whole number.

Hence, the given statement is false.

Exercise 1.2

1. Verify all the statements given below and state whether they are true or false. Give proper reasons for your answers.

i. Every irrational number is a real number.

Ans: Write the irrational numbers and the real numbers in a separate manner.

- The irrational numbers are the numbers that cannot be represented in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  For example, $\sqrt{2}, 3\pi,\ .011011011...$ are all irrational numbers.

- The real number is the collection of both the rational numbers and irrational numbers.

  For example, $0, \pm\frac{1}{2}, \pm\sqrt{2}, \pm\pi, ...$ are all real numbers.

Thus, it is concluded that every irrational number is a real number.

Hence, the given statement is true.

ii. Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

Ans: Consider points on a number line to represent negative as well as positive numbers.

Observe that, positive numbers on the number line can be expressed as $\sqrt{1}, \sqrt{1.1}, \sqrt{1.2}, \sqrt{1.3}, ...$, but any negative number on the number line cannot be expressed as $\sqrt{-1}, \sqrt{-1.1}, \sqrt{-1.2}, \sqrt{-1.3}, ...$, because these are not real numbers.

Therefore, it is concluded from here that every number point on the number line is not of the form $= \sqrt{m}$, where m is a natural number.

Hence, the given statement is false.

iii. Every real number is an irrational number.

Ans: Write the irrational numbers and the real numbers in a separate manner.

- The irrational numbers are the numbers that cannot be represented in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  For example, $\sqrt{2}, 3\pi,\ .011011011...$ are all irrational numbers.

- Real numbers are the collection of rational numbers (Ex: $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{7}, \ldots\ldots$) and the irrational numbers (Ex: $\sqrt{2}, 3\pi,\ .011011011...$).

Therefore, it can be concluded that every irrational number is a real number, but every real number cannot be an irrational number.

Hence, the given statement is false.

2. Are the square roots of all positive integer numbers irrational? If not, provide an example of the square root of a number that is not an irrational number.

Ans: Square root of every positive integer does not give an integer.

For example: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, ...$ are not integers, and hence these are irrational numbers. But $\sqrt{4}$ gives ± 2, these are integers and so, $\sqrt{4}$ is not an irrational number.

Therefore, it is concluded that the square root of every positive integer is not an irrational number.

3. Represent $\sqrt{5}$ on the number line.

Ans: Follow the procedures to get $\sqrt{5}$ on the number line.

- Firstly, Draw a line segment AB of 2 unit on the number line.
- Secondly, draw a perpendicular line segment BC at B of 1 units.
- Thirdly, join the points C and A, to form a line segment AC.
- Fourthly, apply the Pythagoras Theorem as

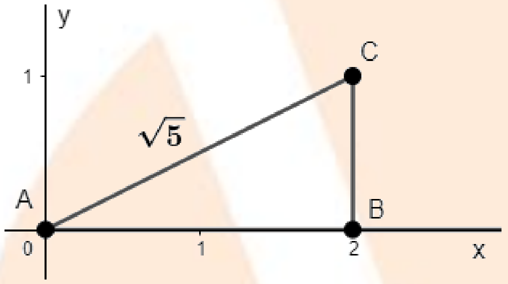
$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 2^2 + 1^2$$

$$AC^2 = 4 + 1 = 5$$

$$AC = \sqrt{5}$$

- Finally, draw the arc ACD, to find the number $\sqrt{5}$ on the number line as given in the diagram below.



Exercise 1.3

1. Convert the following numbers in decimal form and state what kind of decimal expansion each has:

i. $\mathbf{\dfrac{36}{100}}$

Ans: Divide 36 by 100.

$$\begin{array}{r|l} & 0.36 \\ 100 & 36 \\ & \underline{-0} \\ & 360 \\ & \underline{-300} \\ & 600 \\ & \underline{-600} \\ & \underline{0} \end{array}$$

So, $\dfrac{36}{100} = 0.36$ and it is a terminating decimal number.

ii. $\mathbf{\frac{1}{11}}$

Ans: Divide 1 by 11.

$$
\begin{array}{r|l}
 & 0.0909.... \\
11 & \quad 1 \\
 & -0 \\
 & \quad 10 \\
 & -0 \\
 & \quad 100 \\
 & -99 \\
 & \quad 10 \\
 & -0 \\
 & \quad 100 \\
 & -99 \\
 & \quad 1
\end{array}
$$

It is noticed that while dividing 1 by 11, in the quotient 09 is repeated.

So, $\frac{1}{11} = 0.0909.....$ or

$\frac{1}{11} = 0.\overline{09}$ and it is a non-terminating and recurring decimal number.

iii. $\mathbf{4\frac{1}{8}}$

Ans: $4\frac{1}{8} = 4 + \frac{1}{8} = \frac{32+1}{8} = \frac{33}{8}$

Divide 33 by 8.

$$
\begin{array}{r|l}
 & 4.125 \\
8 & \quad 33 \\
 & -32 \\
 & \quad 10
\end{array}
$$

$$\begin{array}{r}
\underline{-8} \\
20 \\
\underline{-16} \\
40 \\
\underline{-40} \\
\underline{0}
\end{array}$$

Notice that, after dividing 33 by 8, the remainder is found as 0.

So, $4\frac{1}{8} = 4.125$ and it is a terminating decimal number.

iv. $\mathbf{\frac{3}{13}}$

Ans: Divide 3 by 13.

$$\begin{array}{r}
0.230769 \\
13\overline{)\quad\quad\quad 3} \\
\underline{-0} \\
30 \\
\underline{-26} \\
40 \\
\underline{-39} \\
10 \\
\underline{-0} \\
100 \\
\underline{-91} \\
90 \\
\underline{-78} \\
120 \\
\underline{-117} \\
\underline{3}
\end{array}$$

It is observed that while dividing 3 by 13, the remainder is found as 3 and that is repeated after each 6 continuous divisions.

So, $\frac{3}{13} = 0.230769.......$ or

$\frac{3}{13} = 0.\overline{230769}$ and it is a non-terminating and recurring decimal number.

v. $\mathbf{\frac{2}{11}}$

Ans: Divide 2 by 11.

$$
\begin{array}{r|l}
 & 0.1818..... \\
11 & \quad 2 \\
 & \underline{-0} \\
 & \quad 20 \\
 & \underline{-11} \\
 & \quad 90 \\
 & \underline{-88} \\
 & \quad 20 \\
 & \underline{-11} \\
 & \quad 90 \\
 & \underline{-88} \\
 & \quad \underline{2}
\end{array}
$$

It can be noticed that while dividing 2 by 11, the remainder is obtained as 2 and then 9, and these two numbers are repeated infinitely as remainders.

So, $\frac{2}{11} = 0.1818.....$ or

$\frac{2}{11} = 0.\overline{18}$ and it is a non-terminating and recurring decimal number.

vi. $\mathbf{\frac{329}{400}}$

Ans: Divide 329 by 400.

$$
\begin{array}{r}
0.8225 \\
400\overline{)329} \\
\underline{-0} \\
3290 \\
\underline{-3200} \\
900 \\
\underline{-800} \\
1000 \\
\underline{-800} \\
2000 \\
\underline{-2000} \\
\underline{0}
\end{array}
$$

It can be seen that while dividing 329 by 400, the remainder is obtained as 0.

So, $\frac{329}{400} = 0.8225$ and is a terminating decimal number.

2. If $\frac{1}{7} = 0.142857...$, then predict the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ without calculating the long division?

Ans: Note that, $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as $2\times\frac{1}{7}$, $3\times\frac{1}{7}$, $4\times\frac{1}{7}$, $5\times\frac{1}{7}$, and $6\times\frac{1}{7}$

Substituting the value of $\frac{1}{7} = 0.142857$, gives

$$2\times\frac{1}{7} = 2\times 0.142857... = 0.285714...$$

$$3\times\frac{1}{7} = 3\times 0.428571... = 0.428571...$$

$$4\times\frac{1}{7} = 4\times 0.142857... = 0.571428...$$

$5\times\frac{1}{7}=5\times0.71425\ldots\quad=0.714285\ldots$

$6\times\frac{1}{7}=6\times0.142857\ldots=0.857142\ldots$

So, the values of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ obtained without performing long division are $\frac{2}{7}=0.\overline{285714}$

$\frac{3}{7}=0.\overline{428571}$

$\frac{4}{7}=0.\overline{571428}$

$\frac{5}{7}=0.\overline{714285}$

$\frac{6}{7}=0.\overline{857142}$

3. Convert the following decimal numbers into the form of $\mathbf{\frac{p}{q}}$, where $\mathbf{p}$ and $\mathbf{q}$ are integers and $\mathbf{q\neq0}$.

i. $\mathbf{0.\overline{6}}$

Ans: Let $x=0.\overline{6}$

$\Rightarrow x=0.6666$ .... (1)

Multiplying both sides of the equation (1) by 10, gives

$10x=0.6666\times10$

o$10x=6.6666$ ...... (2)

Subtracting the equation (1) from (2), gives

$10x=6.6666.....$

$\underline{-x=0.6666.....}$

$9x=6$

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

So, the decimal number becomes

$0.\bar{6} = \frac{2}{3}$ and it is in the required $\frac{p}{q}$ form.

ii. $0.4\overline{7}$

Ans: Let $x = 0.4\overline{7}$

$\Rightarrow x = 0.47777.....$ ……(a)

Multiplying both sides of the equation (a) by 10, gives

$10x = 4.7777.....$ ……(b)

Subtracting the equation (a) from (b), gives

$$\begin{array}{l} 10x = 4.7777..... \\ \underline{-x = 0.4777.....} \\ 9x = 4.3 \end{array}$$

Therefore,

$x = \frac{4.3}{9} \times \frac{10}{10}$

$\Rightarrow x = \frac{43}{90}$

So, the decimal number becomes

$0.4\overline{7} = \frac{43}{90}$ and it is in the required $\frac{p}{q}$ form.

iii. $0.\overline{001}$

Ans: Let $x = 0.\overline{001} \Rightarrow$ …… (1)

Since the number of recurring decimal number is 3, so multiplying both sides of the equation (1) by 1000, gives

$1000 \times x = 1000 \times 0.001001.....$ ...... (2)

Subtracting the equation (1) from (2) gives

$$\begin{array}{r} 1000x = 1.001001..... \\ -x = 0.001001..... \\ \hline 999x = 1 \end{array}$$

$$\Rightarrow x = \frac{1}{999}$$

Hence, the decimal number becomes

$0.\overline{001} = \frac{1}{999}$ and it is in the $\frac{p}{q}$ form.

4. Represent the nonterminating decimal number $0.99999.....$ into the form of $\frac{p}{q}$. Did you expect this type of answer? Explain why the answer is appropriate.

Ans: Let $x = 0.99999.....$ .... (a)

Multiplying by 10 both sides of the equation (a), gives

$10x = 9.9999.....$ ...... (b)

Now, subtracting the equation (a) from (b), gives

$$\begin{array}{r} 10x = 9.99999..... \\ -x = 0.99999..... \\ \hline 9x = 9 \end{array}$$

$$\Rightarrow x = \frac{9}{9}$$

$$\Rightarrow x = 1.$$

So, the decimal number becomes

$0.99999... = \frac{1}{1}$ which is in the $\frac{p}{q}$ form.

Yes, for a moment we are amazed by our answer, but when we observe that $0.9999.........$ is extending infinitely, then the answer makes sense.

Therefore, there is no difference between 1 and $0.9999.........$ and hence these two numbers are equal.

5. Find the maximum number of digits in the recurring block of digits in the decimal expansion of $\mathbf{\frac{1}{17}}$ by performing the long division.

Ans: Here the number of digits in the recurring block of $\frac{1}{17}$ is to be determined.

So, let us calculate the long division to obtain the recurring block of $\frac{1}{17}$.

Dividing 1 by 17 gives

$$
\begin{array}{r|l}
 & \overline{0.0588235294117647....} \\
17 & 1 \\
 & \underline{-0} \\
 & 10 \\
 & \underline{-0} \\
 & 100 \\
 & \underline{-85} \\
 & 150 \\
 & \underline{-136} \\
 & 140 \\
 & \underline{-136} \\
 & 40 \\
 & \underline{-34} \\
 & 60 \\
 & \underline{-85} \\
 & 150 \\
 & \underline{-136} \\
 & 140
\end{array}
$$

$$
\begin{array}{r}
\underline{-136} \\
40 \\
\underline{-34} \\
60 \\
\underline{-51} \\
90 \\
\underline{-85} \\
50 \\
\underline{-34} \\
160 \\
\underline{-153} \\
70 \\
\underline{-68} \\
20 \\
\underline{-17} \\
30 \\
\underline{-17} \\
130 \\
\underline{-119} \\
110 \\
\underline{-102} \\
80 \\
\underline{-68} \\
120 \\
\underline{-119} \\
\underline{1}
\end{array}
$$

Thus, it is noticed that while dividing 1 by 17, we found 16 number of digits in the repeating block of decimal expansion that will continue to be 1 after going through 16 continuous divisions.

Hence, it is concluded that $\frac{1}{17} = 0.0588235294117647.....$ or

$\frac{1}{17} = 0.\overline{0588235294117647}$ and it is a recurring and non-terminating decimal number.

6. Observe at several examples of rational numbers in the form $\frac{p}{q}(q \neq 0)$, where p and q are integers with H.C.F between them is 1 and having terminating decimal representations. Guess the property that q must satisfy?

Ans: Let us consider the examples of such rational numbers $\frac{5}{2}, \frac{5}{4}, \frac{2}{5}, \frac{2}{10}, \frac{5}{16}$ of the form $\frac{p}{q}$ which have terminating decimal representations.

$\frac{5}{2} = 2.5$

$\frac{5}{4} = 1.25$

$\frac{2}{5} = 0.4$

$\frac{2}{10} = 0.2$

$\frac{5}{16} = 0.3125$

In each of the above examples, it can be noticed that the denominators of the rational numbers have powers of $2, 5$ or both.

So, q must satisfy the form either 2^m, or 5^n, or both $2^m \times 5^n$ (where $m = 0,1,2,3.....$ and $n = 0,1,2,3.....$) in the form of $\frac{p}{q}$.

7. Give examples of three numbers whose decimal representations are non-terminating and non-recurring.

Ans: All the irrational numbers are non-terminating and non-recurring, because irrational numbers do not have any representations of the form of $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers. For example:

$\sqrt{2} = 1.41421.....$,

$\sqrt{3} = 1.73205...$

$\sqrt{7} = 2.645751....$

are the numbers whose decimal representations are non-terminating and non-recurring.

8. Write any three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Ans: Converting $\frac{5}{7}$ and $\frac{9}{11}$ into the decimal form gives

$\frac{5}{7} = 0.714285.....$ and

$\frac{9}{11} = 0.818181.....$

Therefore, 3 irrational numbers that are contained between 0.714285...... and 0.818181.....

are:

0.73073007300073......

0.74074007400074......

0.76076007600076......

Hence, three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ are

0.73073007300073......

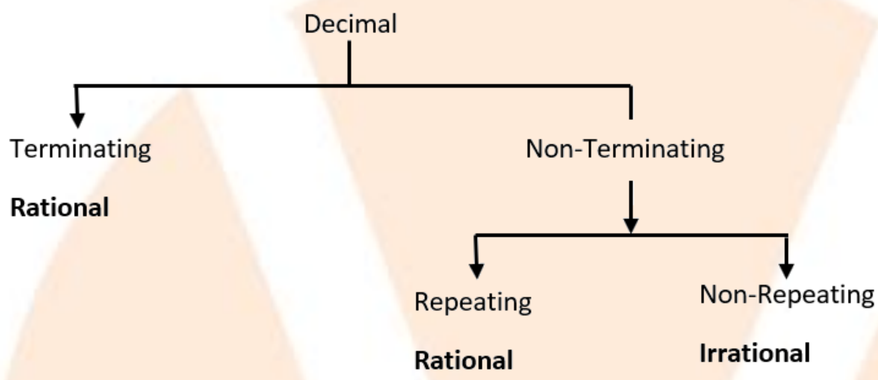
0.74074007400074......

0.76076007600076......

9. Classify the following numbers and state whether it is rational or irrational:

(i) $\sqrt{23}$

Ans: The following diagram reminds us of the distinctions among the types of rational and irrational numbers.



After evaluating the square root gives

$\sqrt{23} = 4.795831.....$, which is an irrational number.

(ii) $\sqrt{225}$

Ans: After evaluating the square root gives

$\sqrt{225} = 15$, which is a rational number.

That is, $\sqrt{225}$ is a rational number.

(iii) 0.3796

Ans: The given number is 0.3796. It is terminating decimal.

So, 0.3796 is a rational number.

(iv) 7.478478

Ans: The given number is $7.478478\ldots$.

It is a non-terminating and recurring decimal that can be written in the $\frac{p}{q}$ form.

Let $\quad x = 7.478478\ldots$ $\quad\ldots\ldots(a)$

Multiplying the equation (a) both sides by 100 gives

$\Rightarrow 1000x = 7478.478478.....$ $\quad\ldots\ldots(b)$

Subtracting the equation (a) from (b), gives

$$\begin{aligned} 1000x &= 7478.478478.... \\ -x &= \quad\ \ 7.478478\ldots \\ \hline 999x &= 7471 \end{aligned}$$

$999x = 7471$

$$x = \frac{7471}{999}$$

Therefore, $7.478478..... = \frac{7471}{999}$, which is in the form of $\frac{p}{q}$

So, $7.478478...$ is a rational number.

(v) 1.101001000100001.....

Ans: The given number is $1.101001000100001....$

It can be clearly seen that the number $1.101001000100001....$ is a non-terminating and non recurring decimal and it is known that non-terminating non-recurring decimals cannot be written in the form of $\frac{p}{q}$.

Hence, the number $1.101001000100001....$ is an irrational number.

Exercise 1.4

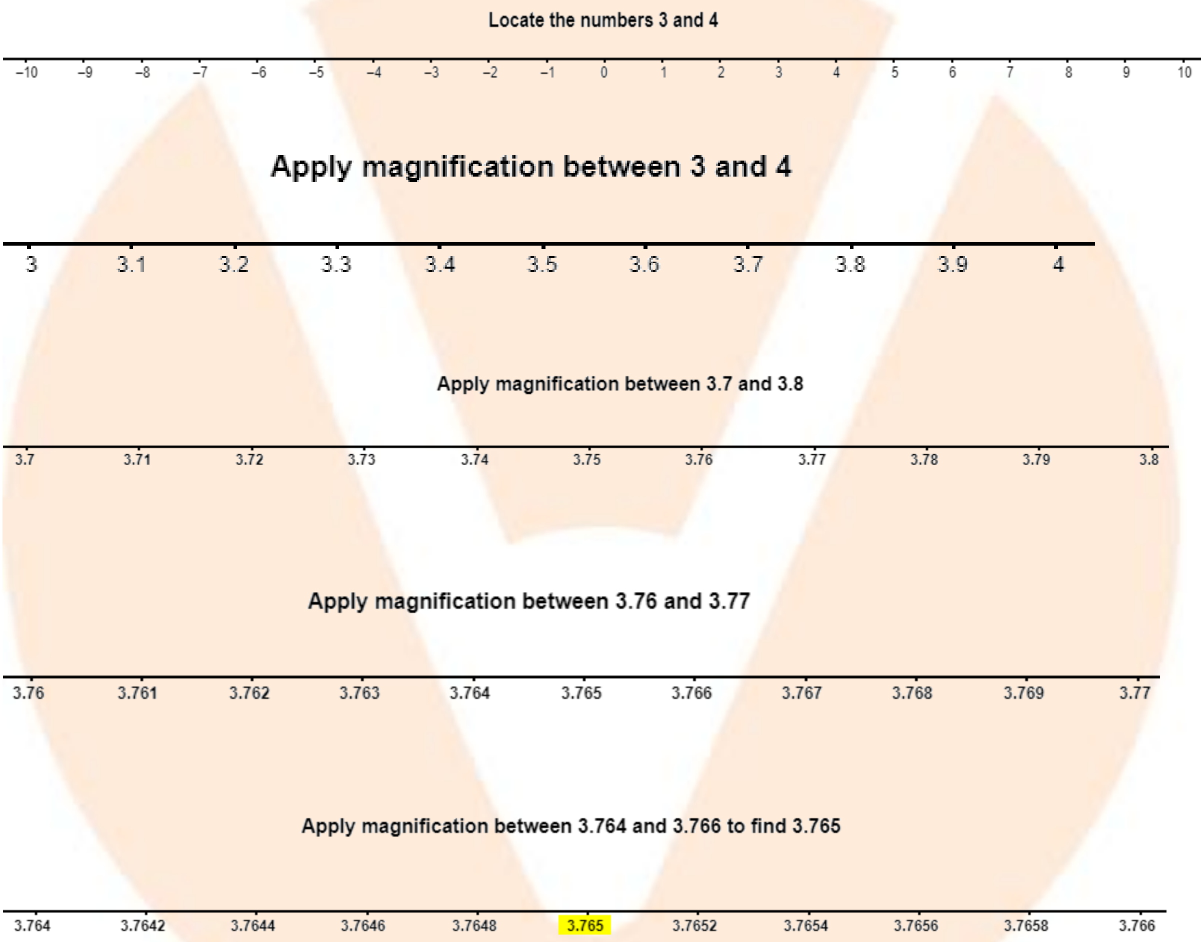
1. Apply some successive magnification to visualize 3.765 on the number line.

Ans:

- It is clear that the value 3.765 lies between the numbers 3 and 4.
- Also, the number 3.7 and 3.8 lie between the numbers 3 and 4.
- The number 3.76 and 3.77 lie between the numbers 3 and 4.

- Again, the numbers 3.764 and 3.766 lie between the numbers 3.76 and 3.77
- Thus, the number 3.765 lies between the numbers 3.764 and 3.766.

So, first locate the numbers 3 and 4 on the number line, then use the successive magnification as shown in the diagrams below.

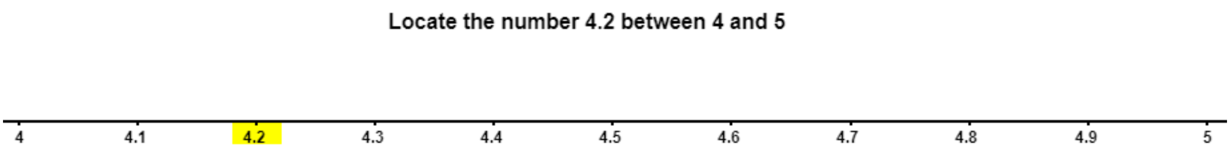


2. Visualize $4.\overline{26}$ on the number line, up to 4 decimal places.

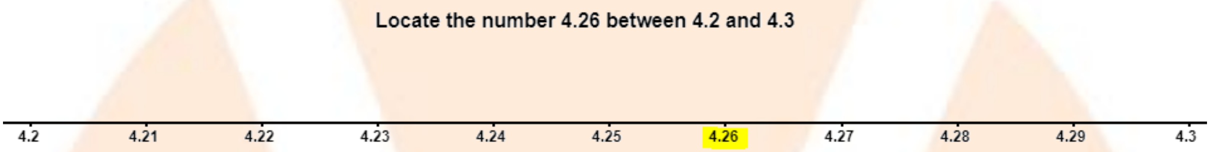
Ans: The number $4.\overline{26}$ can be represented as $4.262.....$

Apply successive magnification, after locating the numbers 4 and 5 on the number line and visualize the number up to 4 decimal places as given in the following diagrams.

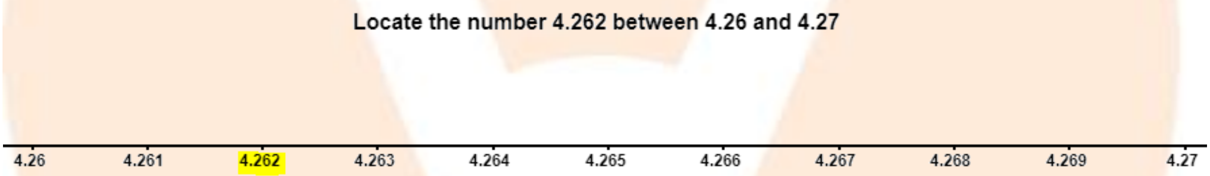
The number 4.2 is located between 4 and 5 .



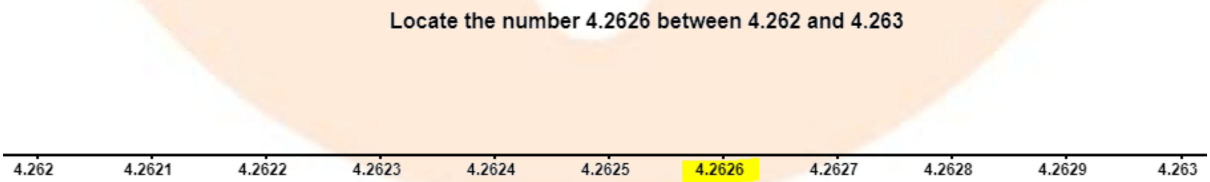
The number 4.26 is located between 4.2 and 4.3.



The number 4.262 is located between 4.26 and 4.27.



The number 4.2626 is located between 4.262 and 4.263.



Exercise 1.5

1. Determine whether the following numbers are rational or irrational:

(i) $2 - \sqrt{5}$

Ans: The given number is $2-\sqrt{5}$.

Here, $\sqrt{5}=2.236.....$ and it is a non-repeating and non-terminating irrational number.

Therefore, substituting the value of $\sqrt{5}$ gives

$2-\sqrt{5}=2-2.236.....$

$=-0.236.....$, which is an irrational number.

So, $2-\sqrt{5}$ is an irrational number.

(ii) $\left(3+\sqrt{23}\right)-\left(\sqrt{23}\right)$

Ans: The given number is $\left(3+\sqrt{23}\right)-\left(\sqrt{23}\right)$.

The number can be written as

$\left(3+\sqrt{23}\right)-\sqrt{23}=3+\sqrt{23}-\sqrt{23}$

$=3$

$=\frac{3}{1}$, which is in the $\frac{p}{q}$ form and so, it is a rational number.

Hence, the number $\left(3+\sqrt{23}\right)-\sqrt{23}$ is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

Ans: The given number is $\frac{2\sqrt{7}}{7\sqrt{7}}$.

The number can be written as

$\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$, which is in the $\frac{p}{q}$ form and so, it is a rational number.

Hence, the number $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

(iv) $\frac{1}{\sqrt{2}}$

Ans: The given number is $\frac{1}{\sqrt{2}}$.

It is known that, $\sqrt{2}=1.414.....$ and it is a non-repeating and non-terminating irrational number.

Hence, the number $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

Ans: The given number is 2π.

It is known that, $\pi=3.1415$ and it is an irrational number.

Now remember that, Rational $\times$ Irrational = Irrational.

Hence, 2π is also an irrational number.

2. Simplify each of the numbers given below:

(i) $\left(3+\sqrt{3}\right)+\left(2+\sqrt{2}\right)$

Ans: The given number is $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$.

By calculating the multiplication, it can be written as

$\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$.

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$.

(ii) $\left(3+\sqrt{3}\right)+\left(3-\sqrt{3}\right)$

Ans: The given number is $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$.

By applying the formula $\left(a+b\right)\left(a-b\right)=a^2-b^2$, the number can be written as

$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=3^2-\left(\sqrt{3}\right)^2=9-3=6$.

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

Ans: The given number is $\left(\sqrt{5}+\sqrt{2}\right)^2$.

Applying the formula $\left(a+b\right)^2=a^{2+}2ab+b^2$, the number can be written as

$$\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+2\sqrt{5}\sqrt{2}+\left(\sqrt{2}\right)^2$$

$$=5+2\sqrt{10}+2$$

$$=7+2\sqrt{10}.$$

(iv) $\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Ans: The given number is $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$.

Applying the formula $\left(a+b\right)\left(a-b\right)=a^2-b^2$, the number can be expressed as

$$\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$$

$$=5-2$$

$$=3.$$

3. Remember that, π is the ratio of the circumference c of a circle to its diameter d, that is, $\pi=\frac{c}{d}$. It seems that the definition of π contradicts the fact that it is an irrational number. Explain why it is not actually a contradiction?

Ans: It is known that, $\pi=\frac{22}{7}$, which is a rational number. But, note that this value of π is an approximation.

On dividing 22 by 7, the quotient $3.14...$ is a non-recurring and non-terminating number. Therefore, it is an irrational number.

In order of increasing accuracy, approximate fractions are

$$\frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{52163}{16604}, \frac{103993}{33102}, \text{ and } \frac{245850922}{78256779}.$$

Each of the above quotient has the value $3.14...$, which is a non-recurring and non-terminating number.

Thus, π is irrational.

So, either circumference (c) or diameter (d) or both should be irrational numbers.

Hence, it is concluded that there is no contradiction regarding the value of π and it is made out that the value of π is irrational.

4. Visualize $\sqrt{9.3}$ on the number line.

Ans: Follow the procedure given below to represent the number $\sqrt{9.3}$.

• First, mark the distance 9.3 units from a fixed-point A on the number line to get a point B. Then $AB = 9.3$ units.

• Secondly, from the point B mark a distance of 1 unit and denote the ending point as C.

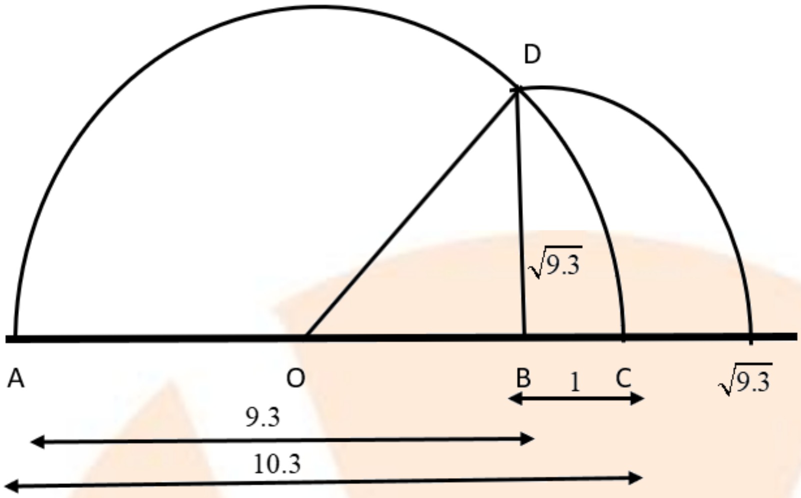
• Thirdly, locate the midpoint of AC and denote as O.

• Fourthly, draw a semi-circle to the centre O with the radius $OC = 5.15$ units. Then

$$\begin{aligned} AC &= AB + BC \\ &= 9.3 + 1 \\ &= 10.3 \end{aligned}$$

So, $OC = \frac{AC}{2} = \frac{10.3}{2} = 5.15$.

• Finally, draw a perpendicular line at B and draw an arc to the centre B and then let it meet at the semicircle AC at D as given in the diagram below.



5. Rationalize the denominators for each of the numbers given below:

(i) $\dfrac{1}{\sqrt{7}}$

Ans: The given number is $\dfrac{1}{\sqrt{7}}$.

Multiplying and dividing by $\sqrt{7}$ to the number gives

$$\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{7}}{7}.$$

(ii) $\dfrac{1}{\sqrt{7}-\sqrt{6}}$

Ans: The given number is $\dfrac{1}{\sqrt{7}-\sqrt{6}}$.

Multiplying and dividing by $\sqrt{7}+\sqrt{6}$ to the number gives

$$\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}$$

Now, applying the formula $(a-b)(a+b)=a^2-b^2$ to the denominator gives

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{1}.$$

(iii) $\mathbf{\frac{1}{\sqrt{5}+\sqrt{2}}}$

Ans: The given number is $\frac{1}{\sqrt{5}+\sqrt{2}}$.

Multiplying and dividing by $\sqrt{5}-\sqrt{2}$ to the number gives $\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$

Now, applying the formula $(a+b)(a-b)=a^2-b^2$ to the denominator gives

$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}.$$

(iv) $\mathbf{\frac{1}{\sqrt{7}-2}}$

Ans: The given number is $\frac{1}{\sqrt{7}-2}$.

Multiplying and dividing by $\sqrt{7}+2$ to the number gives

$$\frac{1}{\sqrt{7}-2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}.$$

Now, applying the formula $\left(a+b\right)\left(a-b\right)=a^2-b^2$ to the denominator gives

$$\frac{1}{\sqrt{7}-2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-\left(2\right)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}$$

$$=\frac{\sqrt{7}+2}{3}.$$

Exercise 1.6

1. Compute the value of each of the following expressions:

(i) $64^{\frac{1}{2}}$

Ans: The given number is $64^{\frac{1}{2}}$.

By the laws of indices,

$a^{\frac{m}{n}}=\sqrt[n]{a^m}$, where $a>0$.

Therefore,

$$64^{\frac{1}{2}}=\sqrt[2]{64}$$
$$=\sqrt[2]{8\times 8}$$
$$=8.$$

Hence, the value of $64^{\frac{1}{2}}$ is 8.

(ii) $32^{\frac{1}{5}}$

Ans: The given number is $32^{\frac{1}{5}}$.

By the laws of indices,

$a^{\frac{m}{n}} = \sqrt[m]{a^m}$, where $a > 0$

$32^{\frac{1}{5}} = \sqrt[5]{32}$

$= \sqrt[5]{2\times2\times2\times2\times2}$

$= \sqrt[5]{2^5}$

$= 2.$

Alternative method:

By the law of indices $\left(a^m\right)^n = a^{mn}$, then it gives

$32^{\frac{1}{5}} = (2\times2\times2\times2\times2)^{\frac{1}{5}}$

$= \left(2^5\right)^{\frac{1}{5}}$

$= 2^{\frac{5}{5}}$

$= 2.$

Hence, the value of the expression $32^{\frac{1}{5}}$ is 2.

(iii) $125^{\frac{1}{5}}$

Ans.

The given number is $125^{\frac{1}{3}}$.

By the laws of indices

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ where $a > 0$.

Therefore,

$125^{\frac{1}{3}} = \sqrt[3]{125}$

$= \sqrt[3]{5 \times 5 \times 5}$

$= 5.$

Hence, the value of the expression $125^{\frac{1}{3}}$ is 5.

2. Compute the value of each of the following expressions:

(i) $\mathbf{9^{\frac{3}{2}}}$

Ans: The given number is $9^{\frac{3}{2}}$.

By the laws of indices,

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ where $a > 0$.

Therefore,

$9^{\frac{3}{2}} = \sqrt[2]{(9)^3}$

$= \sqrt[2]{9 \times 9 \times 9}$

$= \sqrt[2]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= 3 \times 3 \times 3$

$= 27.$

Alternative Method:

By the laws of indices, $\left(a^m\right)^n = a^{mn}$, then it gives

$9^{\frac{3}{2}} = (3 \times 3)^{\frac{3}{2}}$

$= \left(3^2\right)^{\frac{3}{2}}$

$= 3^{2 \times \frac{3}{2}}$

$= 3^3$

That is,

$9^{\frac{3}{2}} = 27.$

Hence, the value of the expression $9^{\frac{3}{2}}$ is 27.

(ii) $\mathbf{32^{\frac{2}{5}}}$

Ans: We know that $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ where $a > 0$.

We conclude that $32^{\frac{2}{5}}$ can also be written as

$\sqrt[5]{(32)^2} = \sqrt[5]{(2\times2\times2\times2\times2)\times(2\times2\times2\times2\times2)}$

$= 2\times2$

$= 4$

Therefore, the value of $32^{\frac{2}{5}}$ is 4.

(iii) $\mathbf{16^{\frac{3}{4}}}$

Ans: The given number is $16^{\frac{3}{4}}$.

By the laws of indices,

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$, where $a > 0$.

Therefore,

$16^{\frac{3}{4}} = \sqrt[4]{(16)^3}$

$= \sqrt[4]{(2\times2\times2\times2)\times(2\times2\times2\times2)\times(2\times2\times2\times2)}$

$= 2\times2\times2$

$= 8.$

Hence, the value of the expression $16^{\frac{3}{4}}$ is 8.

Alternative Method:

By the laws of indices,

$(a^m)^n = a^{mn}$, where $a > 0$.

Therefore,

$16^{\frac{3}{4}} = (4\times4)^{\frac{3}{4}}$

$= (4^2)^{\frac{3}{4}}$

$= (4)^{2\times\frac{3}{4}}$

$= (2^2)^{2\times\frac{3}{4}}$

$= 2^{2\times2\times\frac{3}{4}}$

$= 2^3$

$= 8.$

Hence, the value of the expression is $16^{\frac{3}{4}} = 8$.

(iv) $125^{-\frac{1}{3}}$

Ans: The given number is $125^{-\frac{1}{3}}$.

By the laws of indices, it is known that

$a^{-n} = \frac{1}{a^n}$, where $a > 0$.

Therefore,

$125^{-\frac{1}{3}} = \frac{1}{125^{\frac{1}{3}}}$

$= \left(\frac{1}{125}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\left(\frac{1}{125}\right)}$

$= \sqrt[3]{\left(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\right)}$

$= \frac{1}{5}.$

Hence, the value of the expression $125^{-\frac{1}{3}}$ is $\frac{1}{5}$.

3. Simplify and evaluate each of the expression:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$

Ans: The given expression is $2^{\frac{2}{3}}.2^{\frac{1}{5}}$.

By the laws of indices, it is known that

$a^m \cdot a^n = a^{m+n}$, where $a > 0$.

Therefore,

$2^{\frac{2}{3}}.2^{\frac{1}{5}} = (2)^{\frac{2}{3}+\frac{1}{5}}$

$= (2)^{\frac{10+3}{15}}$

$= 2^{\frac{13}{15}}.$

Hence, the value of the expression $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ is $2^{\frac{13}{15}}$.

(ii) $\left(3^{\frac{1}{3}}\right)^7$

Ans: The given expression is $\left(3^{\frac{1}{3}}\right)^7$.

It is known by the laws of indices that,

$(a^m)^n = a^{mn}$, where $a > 0$.

Therefore,

$$\left(3^{\frac{1}{3}}\right)^7 = 3^{\frac{7}{3}}.$$

Hence, the value of the expression $\left(3^{\frac{1}{3}}\right)^7$ is $3^{\frac{7}{3}}$.

(iii) $\mathbf{\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}}$

Ans: The given number is $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$.

It is known by the Laws of Indices that

$\dfrac{a^m}{a^n} = a^{m-n}$, where $a > 0$.

Therefore,

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$$

$$= 11^{\frac{2-1}{4}}$$

$$= 11^{\frac{1}{4}}.$$

Hence, the value of the expression $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ is $11^{\frac{1}{4}}$.

(iv) $\mathbf{7^{\frac{1}{2}}.8^{\frac{1}{2}}}$

Ans: The given expression is $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$.

It is known by the Laws of Indices that

$a^m \cdot b^m = (a \cdot b)^m$, where $a > 0$.

Therefore,

$$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$$

$$= (56)^{\frac{1}{2}}.$$

Hence, the value of the expression $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$ is $(56)^{\frac{1}{2}}$.