Dominant Strategies

Frederik Mallmann-Trenn
6CCS3AIN

Solution Concepts

- How will a rational agent behave in any given scenario?
- Play…
  - Dominant strategy;
  - Nash equilibrium strategy;
  - Pareto optimal strategies;
  - Strategies that maximise social welfare.

Dominant Strategies

- Given any particular strategy s (either C or D) agent i, there will be a number of possible outcomes.
- We say s_1 dominates s_2 if every outcome possible by i playing s_1 is preferred over every outcome possible by i playing s_2.
- Thus in this game:

j

		D		C	
	D		1		4
i		2		2	
	C		1		4
		5		5	

C dominates D for both players.

Dominant Strategies

- Two senses of "preferred"
- s_1 strongly dominates s_2 if the utility of every outcome possible by i playing s_1 is strictly greater than every outcome possible by i playing s_2.
- In other words, $u(s_1) > u(s_2)$, for all outcomes.
- s_1 weakly dominates s_2 if the utility of every outcome possible by i playing s_1 is no less than every outcome possible by i playing s_2.
- In other words, $u(s_1) \geqslant u(s_2)$, for all outcomes.

Dominant Strategies

- A rational agent will never play a dominated strategy.
- So in deciding what to do, we can delete dominated strategies.
- Unfortunately, there isn't always a unique undominated strategy (see later).

Dominant Strategies

- Game with dominated strategies

	L	C	R
U	1	1	0
	3	0	0
M	1	1	0
	1	1	5
D	1	1	0
	0	4	0

- Can eliminate the dominated strategies and simplify the game
- Which strategy is dominated?

Dominant Strategies

- Let's look at the pay-off matrices A and B
- For the column player j we get $B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$
- We can think of this as three vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Dominant Strategies

- Let's look at the pay-off matrices A and B
- For the column player j we get $B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$
- We can think of this as three vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
- We can see that every **component** of R is dominated by L (and actually also C)
- So we can remove R

Dominant Strategies

- Game with dominated strategies

	L	C
U	3, 1	0, 1
M	1, 1	1, 1
D	0, 1	4, 1

- Can eliminate the dominated strategies and simplify the game
- Remove R (dominated by L).

Dominant Strategies

- Let's look at the pay-off matrices A
- For the **row** player i we get $A = \begin{pmatrix} 3 & 0 \\ 1 & 1 \\ 0 & 4 \end{pmatrix}$
- We can think of this as three (row) vectors $\begin{pmatrix} 3 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 4 \end{pmatrix}$

Dominant Strategies

- Let's look at the pay-off matrices A
- For the **row** player i we get $A = \begin{pmatrix} 3 & 0 \\ 1 & 1 \\ 0 & 4 \end{pmatrix}$
- We can think of this as three (row) vectors $\begin{pmatrix} 3 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 4 \end{pmatrix}$
- No strategy here is dominated by any other ...
- So we cannot remove anything else

Dominant Strategies

- If we are lucky, we can eliminate enough strategies so that the choice of action is obvious.
- In general we aren't that lucky.

Dominant Strategies

- Consider this scenario:

j

		C	D
	A	1	4
i		2	3
	B	2	3
		3	2

- Are there any dominated strategies?

Are there any dominated strategies?

- D is dominating!