Prior Sampling

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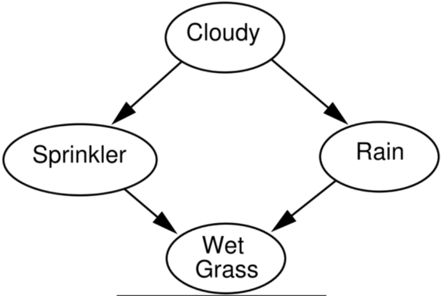
6CCS3AIN

Prior sampling

- Let's use stochastic sampling instead!

Prior sampling

P(C)
.50



C	P(S|C)
T	.10
F	.50

C	P(R|C)
T	.80
F	.20

S	R	P(W|S,R)
T	T	.99
T	F	.90
F	T	.90
F	F	.01

- How would you estimate $P(c, \neg s, r, w)$?

Take a step back

- How would you estimate $P(\text{die shows } 7)$?

Stochastic simulation

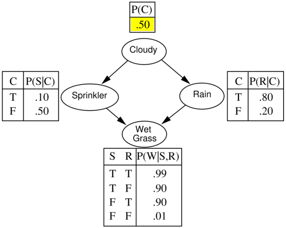
- How would you estimate $P(\text{die shows } 7)$?
- Simple: you take n random samples from the network
- Let X_i be the binary r.v. that is 1 if the event sampled in the ith run is 7
- Simply output

$$\hat{P}(7) = \frac{\sum_{i=1}^{n} X_i}{n}$$

- Law of large numbers says that $\lim_{n\to\infty} \hat{P} = P$.

Stochastic simulation

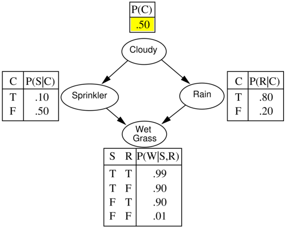
- Back to our Bayesian network with cloudy, sprinkler, rain, and wet grass.



- How would you estimate $P(c, \neg s, r, w)$?

Stochastic simulation

- Back to our Bayesian network with cloudy, sprinkler, rain, and wet grass.



- How would you estimate $P(c, \neg s, r, w)$?
- Simple you just take say n random samples from the network
- Let X_i be the binary r.v. that is 1 if the event sampled in the ith run is $c, \neg s, r, w$
- Simply output

$$\hat{P}(c, \neg s, r, w) = \frac{\sum_{i=1}^{n} X_i}{n}$$

- Law of large numbers says that $\lim_{n\to\infty} \hat{P} = P$.

Prior sampling. Let's generate a random sample!

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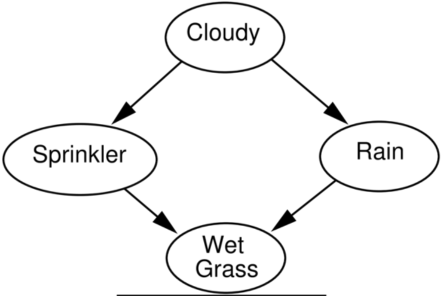

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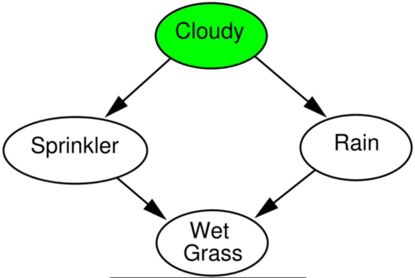
Prior sampling.

- List of randomly generated numbers: 0.4, 0.2, 0.71, 0.2 for this run.
- We have 0.4 small than $P(C)$, so $Cloudy = true$

P(C)
.50



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Prior sampling

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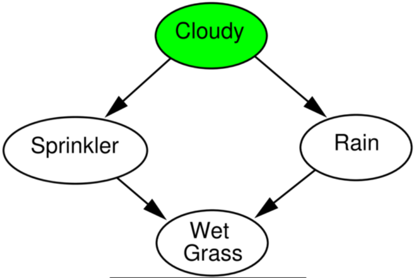

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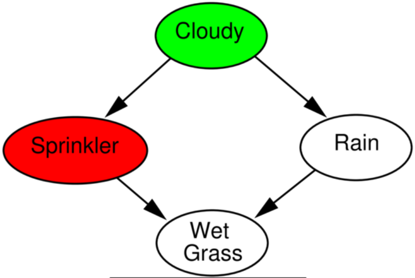
Prior sampling

- List of randomly generated numbers: 0.4, 0.2, 0.71, 0.2 for this run
- We have $0.2 \geqslant P(S \mid C)$ and hence, $Sprinkler = false$.

P(C)
.50



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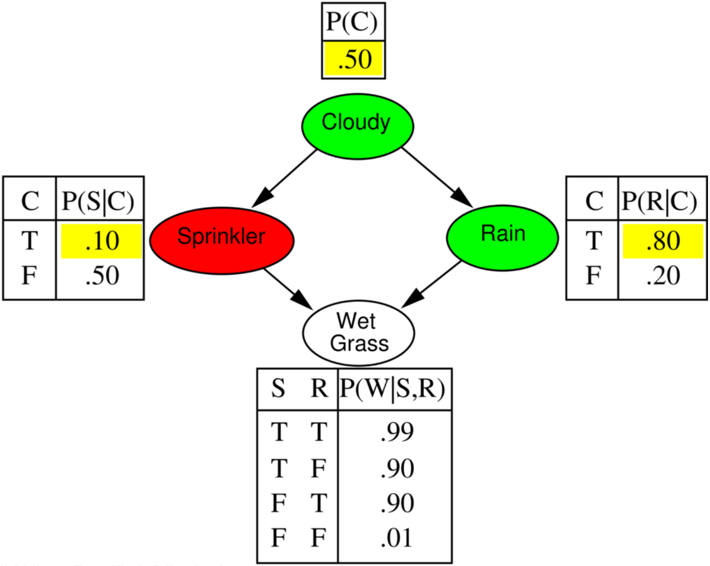
Prior sampling

- List of randomly generated numbers: 0.4, 0.2, 0.71, 0.2 for this run.
- We have $0.71 < P(R \mid C)$ and hence, $Rain = true$



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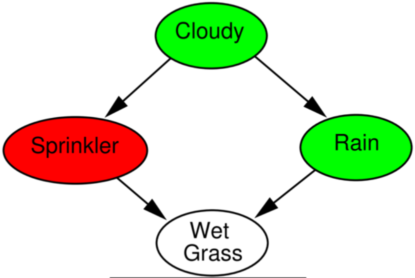
Prior sampling



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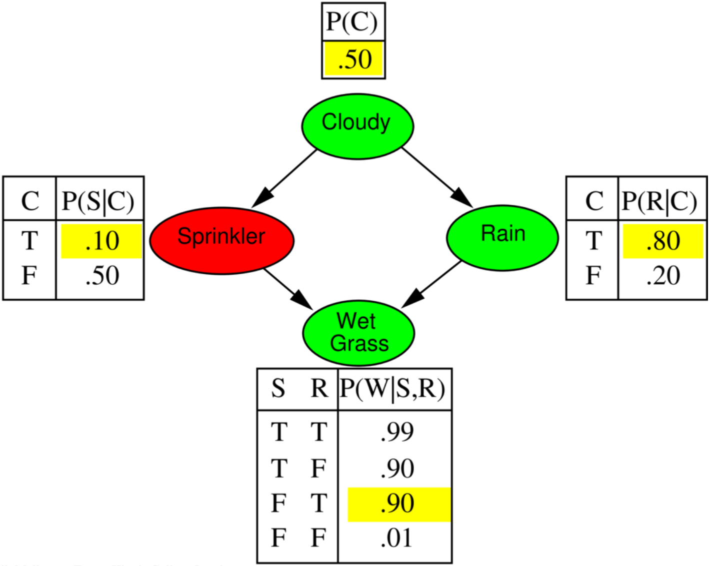
Prior sampling

- List of randomly generated numbers: 0.4, 0.2, 0.71, 0.2 for this run.
- We have $0.2 < P(W \mid S, R)$ and hence, $WetGrass = true$



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Prior sampling

- So, this time we get the event

$$[Cloudy = true, Sprinkler = false, Rain = true, WetGrass = true]$$

- Will write this:

$$[true, false, true, true]$$

Prior sampling

- If we repeat the process many times, we can count the number of times $[true, false, true, true]$ is the result.
- The proportion of this to the total number of runs is:

$$P(c, \neg s, r, w)$$

- The more runs, the more accurate the probability.
- Similarly for other joint probabilities.

Prior sampling

function PRIOR-SAMPLE(*bn*) **returns** an event sampled from *bn*
inputs: *bn*, a belief network specifying joint distribution $\mathbf{P}(X_1, \ldots, X_n)$

$\mathbf{x} \leftarrow$ an event with n elements
for $i = 1$ **to** n **do**
$x_i \leftarrow$ a random sample from $\mathbf{P}(X_i \mid parents(X_i))$
given the values of $Parents(X_i)$ in $\mathbf{x}$
return $\mathbf{x}$

Prior sampling limitation

- How would you get the following marginal distribution?

$$\mathbf{P}(X|e)$$