Matrix Multiplication

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Matrix

$$\begin{array}{c|cccc}
 & 1 & 2 & \dots & n \\
\hline
1 & a_{11} & a_{12} & \dots & a_{1n} \\
2 & a_{21} & a_{22} & \dots & a_{2n} \\
3 & a_{31} & a_{32} & \dots & a_{3n} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
m & a_{m1} & a_{m2} & \dots & a_{mn}
\end{array}$$

- Here we have a $m \times n$ (m by n) matrix.
- (image source: wikipedia)

Matrix Multiplication

$$\begin{pmatrix} \boxed{c_{1,1}} & c_{1,2} & c_{1,3} \\ c_{2,1} & c_{2,2} & c_{2,3} \\ c_{3,1} & c_{3,2} & c_{3,3} \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \cdot \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,1} & b_{2,2} & b_{2,3} \\ b_{3,1} & b_{3,2} & b_{3,3} \end{pmatrix} \tag{1}$$

- From left matrix: select matching row
- From right matrix: select matching column
- Multiply them component-wise

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- Formula $c_{i,j} = \sum_k a_{i,k} \cdot b_{k,j}$
- $c_{1,1} = a_{1,1} \cdot b_{1,1} + a_{1,2} \cdot b_{2,1} + a_{1,3} \cdot b_{3,1}$

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$$\begin{pmatrix} c_{1,1} & c_{1,2} & c_{1,3} \\ c_{2,1} & c_{2,2} & \boxed{c_{2,3}} \\ c_{3,1} & c_{3,2} & c_{3,3} \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ \boxed{a_{2,1}} & \boxed{a_{2,2}} & \boxed{a_{2,3}} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \cdot \begin{pmatrix} b_{1,1} & b_{1,2} & \boxed{b_{1,3}} \\ b_{2,1} & b_{2,2} & \boxed{b_{2,3}} \\ b_{3,1} & b_{3,2} & \boxed{b_{3,3}} \end{pmatrix} \tag{2}$$

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Matrix Multiplication: example

- What happens if you multiply

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix} \tag{3}$$

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- The outcome is

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \tag{4}$$

Transpose

- The transpose of a matrix A is an operation which flips the matrix along its diagonal (switches the row and column indices of the matrix)

$$\mathbf{A} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \end{pmatrix} \tag{5}$$

- becomes

$$\mathbf{A}^T = \begin{pmatrix} a & e \\ b & f \\ c & g \\ d & h \end{pmatrix} \tag{6}$$

- Note that $\left(\mathbf{A}^T\right)^T = \mathbf{A}$