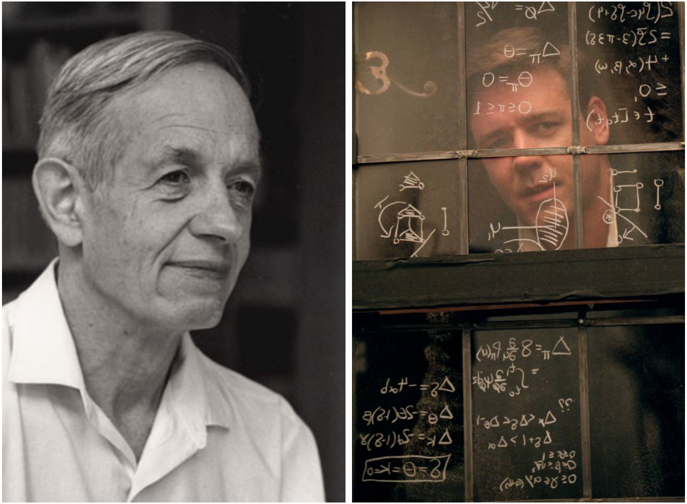
Nash Equilibrium

Frederik Mallmann-Trenn
6CCS3AIN

Nash Equilibrium



John Forbes Nash.

(princeton.edu, Universal Pictures/DreamWorks)

Nash Equilibrium

- In general, we will say that two strategies s_1 and s_2 are in Nash equilibrium (NE) if:
  1. under the assumption that agent i plays s_1, agent j can do no better than play s_2; and
  2. under the assumption that agent j plays s_2, agent i can do no better than play s_1.
- Neither agent has any incentive to deviate from a NE.
- Eh?

Nash Equilibrium

- Let's consider the payoff matrix for the grade game:

		j	
		Y	X
	Y	2	1
i		2	4
	X	4	3
		1	3

- Here the Nash equilibrium is (Y, Y).
- If i assumes that j is playing Y, then i's best response is to play Y.
- Similarly for j.

Nash Equilibrium

- If two strategies are best responses to each other, then they are in Nash equilibrium.

Nash Equilibrium

- In a game like this you can find the NE by cycling through the outcomes, asking if either agent can improve its payoff by switching its strategy.

		j	
		Y	X
	Y	2	1
i		2	4
	X	4	3
		1	3

- Thus, for example, (X, Y) is not an NE because i can switch its payoff from 1 to 2 by switching from X to Y.

Nash Equilibrium

- More formally:

  A pair of strategies (i^*, j^*) is a Nash equilibrium solution to the game (A, B) if:

$$\begin{aligned} \forall i, a_{i*,j*} &\geqslant a_{i,j*} \\ \forall j, b_{i*,j*} &\geqslant b_{i*,j} \end{aligned}$$

- That is, (i^*, j^*) is a Nash equilibrium if:
  - If j plays j^*, then i^* gives the best outcome for i.
  - If i plays i^*, then j^* gives the best outcome for i.

Nash Equilibrium

- Unfortunately:
  1. Not every interaction scenario has a pure strategy NE.
  2. Some interaction scenarios have more than one NE.

Nash Equilibrium

- This game has two pure strategy NEs, (C, C) and (D, D):

		j	
		D	C
	D	5	1
i		3	2
	C	0	3
		2	3

- In both cases, a single agent can't unilaterally improve its payoff.

Nash Equilibrium

- This game has no pure strategy NE:

		j	
		D	C
i	D	2 1	1 2
	C	0 2	1 1

- For every outcome, one of the agents will improve its utility by switching its strategy.
- We can find a form of NE in such games, but we need to go beyond pure strategies.

Nash equilibria?

- Consider this scenario (again):

		j	
		C	D
i	A	1 2	4 3
	B	2 3	3 2

- Are there any Nash equilibria?