

# Pareto Optimality and Social Welfare



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# Pareto Optimality

- An outcome is said to be **Pareto optimal** (or **Pareto efficient**) if there is no other outcome that makes one agent **better off** without making another agent **worse off**.
- If an outcome is Pareto optimal, then at least one agent will be reluctant to move away from it (because this agent will be worse off).
- If an outcome  $\omega$  is **not** Pareto optimal, then there is another outcome  $\omega'$  that makes **everyone** as happy, if not happier, than  $\omega$ .
- “Reasonable” agents would agree to move to  $\omega'$  from  $\omega$  if  $\omega$  is **not** Pareto optimal and  $\omega'$  is.

# Pareto Optimality

- This game has one Pareto efficient outcome,  $(D, D)$ .

		$j$	
		D	C
$i$	D	5 3	1 2
	C	0 2	0 1

- There is no solution in which either agent does better.

# Pareto Optimality

- This next game has two Pareto efficient outcomes,  $(C, D)$  and  $(D, C)$ .

		$j$	
		D	C
$i$	D	1 3	4 1
	C	1 4	1 1

- Note that Pareto efficiency doesn't necessarily mean *fair*.
- Just that you can't move away and make one agent better off without making the other worse off.
- Different way of thinking about this: Ignore players and imagine you are buying socks online. All options cost the same
- $i$  is the number of stars (rating) and  $j$  is the number of pairs you get.
- Trade-off: all reasonable options are Pareto efficient

# Pareto optimal?

- Consider this scenario (again):

		$j$	
		C	D
$i$	A	1 2	4 3
	B	2 3	3 2

- Are there any Pareto optimal outcomes?

# Pareto optimal?

- Consider this scenario (again):

		$j$	
		C	D
$i$	A	1 2	4 3
	B	2 3	3 2

- Are there any Pareto optimal outcomes?
- (A,D)

# Pareto Optimality

- Pareto optimality is a rather weak concept.



*(coolfunpedia.blogspot.co.uk)*

- What is the Pareto optimal way to divide a pile of money between player  $i$  and player  $j$ ?

# Social Welfare

- The social welfare of an outcome  $\omega$  is the sum of the utilities that each agent gets from  $\omega$ :

$$\sum_{i \in Ag} u_i(\omega)$$

- Think of it as the “total amount of money in the system”.
- As a solution concept, may be appropriate when the whole system (all agents) has a single owner (then overall benefit of the system is important, not individuals).



# Social Welfare

- As a solution concept it doesn't consider the benefits to individuals.



*(telegraph.co.uk)*

- A very skewed outcome can maximise social welfare.

# Social Welfare

- In both these games,  $(C, C)$  maximises social welfare.

		$j$	
		D	C
$i$	D	2	1
	C	3	4

		$j$	
		D	C
$i$	D	2	1
	C	3	9