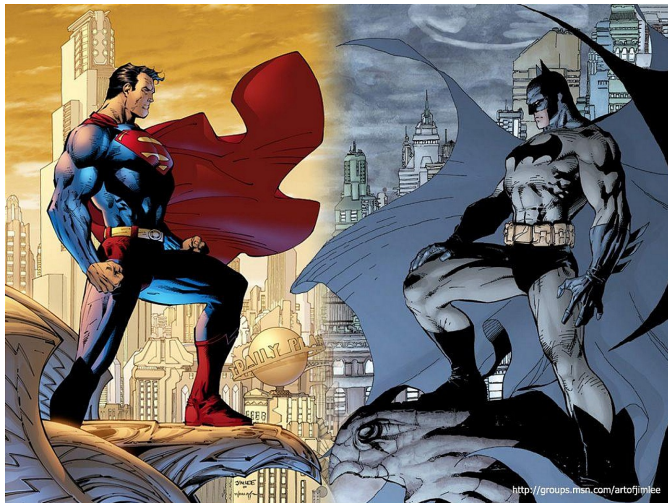# Intro to Game Theory/ Strategic Reasoning
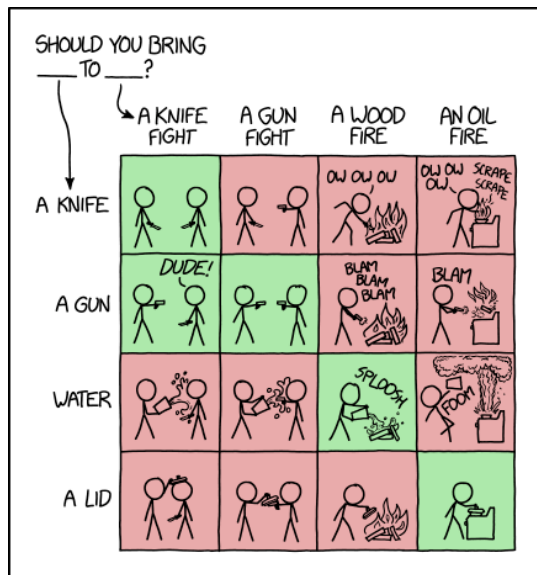

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Competition between agents

(groups.msn.com/artofjimlee)

Multi-party decision making

(xkcd.com/1890/)

In the real world

(U.S. Army Military History Institute)

- Strategic analysis.

In the real world

(Adi Talwar/citylimits.org)

- School admissions

In the real world

(froedtert.com)

- Kidney exchange
- Longest chain involved 28 donors and recipients.

The grade game

(Lyndon Johnson's Report Card, US National Archives)

The grade game

- Imagine in this module we assign grades as follows
- You are randomly paired with a partner (you do not know who!)
- You have to write X or Y on the piece of paper
- You will get a grade based on the following rules:
- If both you and your partner write X, then you both get a B
- If you write X and your partner writes Y then you get D and your partner gets A
- If you write Y and your partner writes X then you get A and your partner gets D
- If both you and your partner write Y then you both get C

  (The Grade Game, Ben Polak)

The grade game

- What would you do?
- What you get depends also on the choice of your partner.
- This is the blueprint of strategic interaction.
- There is a poll on KEATS to find out ...

Choose the side

- Which side of the road to drive on?

(haulagetoday.com)

Choose the side

- Which side of the road to drive on?

(businessinsider.com)

Choose the side

- Which side of the road to drive on?

Any fule kno that.

(Geoffrey Williams/Ronald Searle)

Choose the side

- **Which side of the road to drive on?**

- **Same side as everyone else**

  (berkshireeagle.com)

Choose the side

- How do you choose when you don't know what "everyone else" is doing.

Game theory

- Game theory is a framework for analysing interactions between a set of agents.
- Abstract specification of interactions.
- Describes each agent's preferences in terms of their utility.
    - Assume agents want to maximise utility.
- Give us a range of solution strategies with which we can make some predictions about how agents will/should interact.

Payoff Matrices

- We can characterise the "choose side" scenario in a payoff matrix

$$j$$

	left	right
left	1	0
	1	0
right	0	1
	0	1

i

- we have two agents, each player picking a (pure) strategy
- Agent i is the row player
  gets the lower reward in a cell.
- Agent j is the column player
  gets the upper reward in a cell.

Payoff Matrices

- We can characterise the grade game scenario in a payoff matrix

		j	
		Y	X
i	Y	2 2	1 4
	X	4 1	3 3

- Payoffs are the US grade points that correspond to the problem statement.
- From the game earlier: Grade A is 4, Grade B is 3 etc.

Outcomes

- An outcome is what we get when we combine the actions of all the players.
- An outcome corresponds to an element of the payoff matrix

<div align="center">

j

	left	right
up	1	0
	1	0
down	0	1
	0	1

</div>

i

- We identify outcomes by the moves the players make:

$$(what\ i\ plays, what\ j\ plays)$$

- Thus $(up, right)$ identifies the outcome in which
  i plays up and j plays $right$

Payoff Matrices

- Actually there are two matrices here, one (call it A) that specifies the payoff to i and another B that specifies the payoff to j.

- Sometimes we'll write the payoff matrix as (A, B) in recognition of this.

- $A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ is the payoff matrix for i from the following table

		j	
		left	right
i	up	2	0
		1	0
	down	0	1
		3	1

- Note that $a_{i',j'}$ is the payoff if i picks action i' and j picks action j'