6CCS3AIN 2020
Lecture 8B: Argumentation

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(with thanks to Elizabeth Black)

This Week (Lecture 8): Argumentation Theory

Part A

- Introduction
- Abstract argumentation
- Extension-based semantics

Part B

- Complete semantics
- Grounded semantics
- Preferred semantics
- Stable semantics
- Argument acceptance

Lecture for Week 8, Part B

This is a continuation from Part A of the lecture on Argumentation.

These slides will only make sense after viewing Part A.

Complete semantics

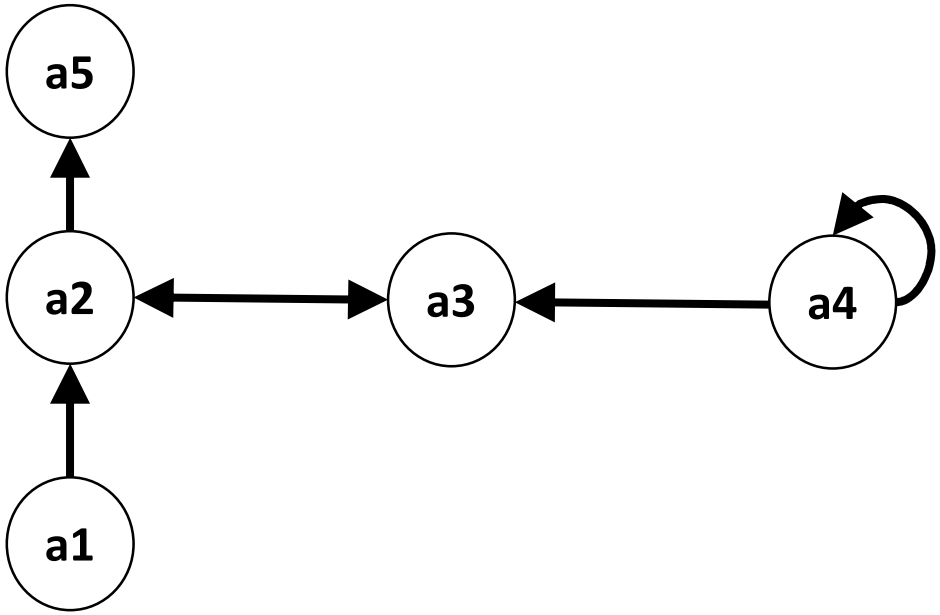
The complete semantics aims to include in a subset of arguments all those arguments that the arguments in the subset can defend.

The intuitive notion is that a collection of arguments work together to support a claim, both directly supporting the claim and by attacking other arguments which attack the claim.

So the Complete Semantics aims to include in the set of acceptable arguments all the arguments which jointly support one another (provided they don't conflict with one another).

Complete extensions

A **complete extension** is an admissible subset that includes **all** arguments it defends.



Note that we sometimes use a double-headed arrow when there are attacks in both directions.

Example 2

$E = \{a1, a5\}$ is a complete extension, since it is admissible and it defends both $a1$ and $a5$ (and doesn't defend any other arguments).

Note that $a3$ is not defended by E, since there is no argument in E that attacks the attacker $a4$, and $a4$ cannot be part of a conflict-free set.

The subset $\{a1\}$ is not a complete extension, since it does not include $a5$, which it defends.

Exercise 4 (Example 3)

Consider the set of arguments
$S = \{a1, a2, a3, a4, a5\}$
and the attack relation $R =$
$\{(a1, a2), (a3, a2), (a3, a4), (a4, a3), (a4, a5), (a5, a5)\}$.

- Draw the argumentation framework
- Identify the conflict-free subsets
- Identify all admissible subsets of S
- Identify all complete extensions of $\langle S, R \rangle$.

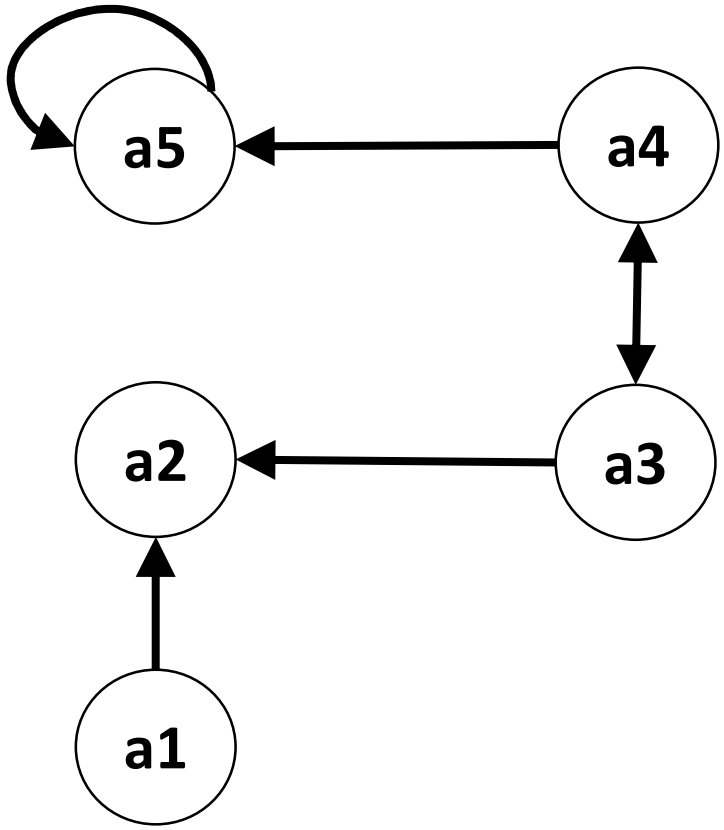
Remember:

A set $A \subseteq S$ is **admissible** if and only if A is conflict-free and A defends each argument that is a member of A.

A **complete extension** is an admissible set that includes all arguments it defends.

Pause to do Exercise

Exercise 4 Solutions (Example 3)



Example 3

Conflict-free subsets:

$\{\}, \{a1\}, \{a2\}, \{a3\}, \{a4\},$
$\{a1, a3\}, \{a1, a4\}, \{a2, a4\}$

Admissible subsets:

$\{\}, \{a1\}, \{a3\}, \{a4\}, \{a1, a3\},$
$\{a1, a4\}$

Complete extensions:

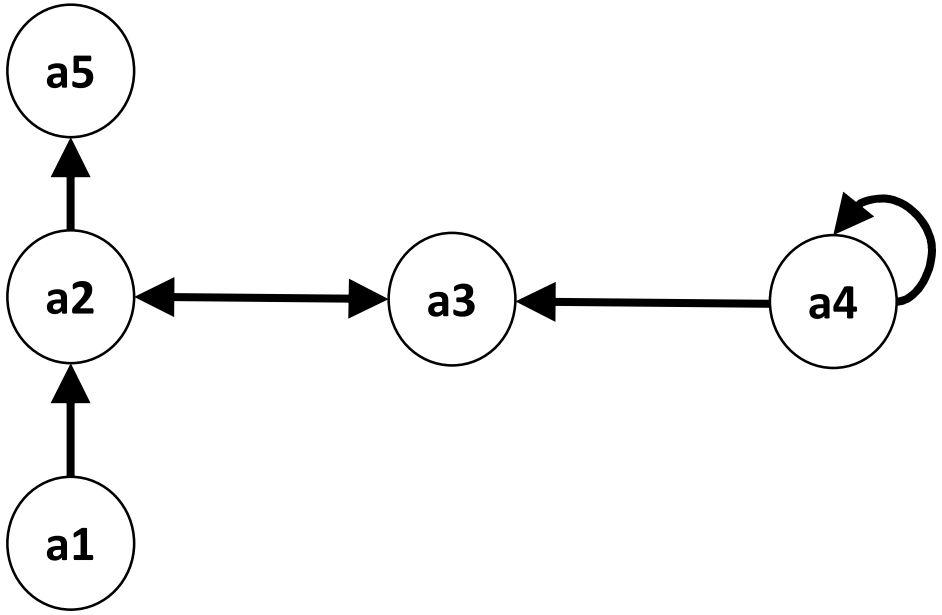
$\{a1\}, \{a1, a3\}, \{a1, a4\}$

Note: The empty set defends a1, but does not contain it.

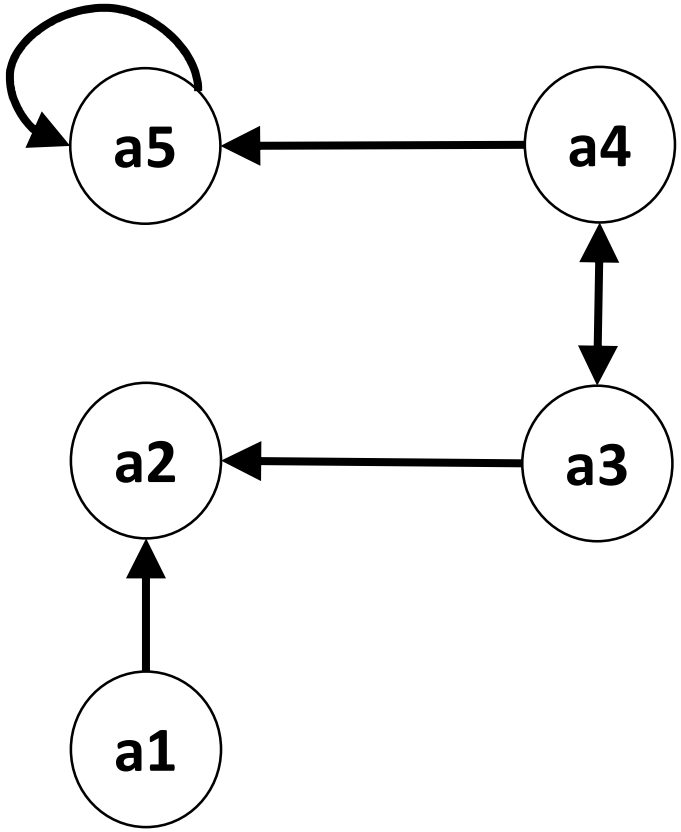
A set $A \subseteq S$ is **admissible** if and only if A is conflict-free and A defends each argument that is a member of A.

A **complete extension** is an admissible set that includes all arguments it defends.

Note: Example 2 ≠ Example 3



Example 2



Example 3

Maximal and minimal subsets

Take $C \subseteq 2^S$, a **maximal subset** of S in C is a set $T \in C$ such that no other set in C strictly includes T.

A maximal subset is as large as it can be.

Formally, T is a **maximal subset** of S in C if and only if $\nexists T' \in C$ such that $T \subsetneq T'$.

Example.

Take $S = \{a1, a2, a3\}$ and let $C = \{\{a1\}, \{a2\}, \{a1, a2\}, \{a2, a3\}\}$.

The maximal subsets of C in S are $\{a1, a2\}$ and $\{a2, a3\}$.

Maximal and minimal subsets

Take $C \subseteq 2^S$, a **minimal subset** of S in C is a set $T \in C$ such that no other set in C is strictly included in T.

Formally, T is a **minimal subset** of S in C if and only if $\nexists T' \in C$ such that $T' \subsetneq T$.

A minimal subset is as small as it can be.

Example.

Take $S = \{a1, a2, a3\}$ and let $C = \big\{\{a1\}, \{a2\}, \{a1, a2\}, \{a2, a3\}\big\}$.

The minimal subsets of C in S are $\{a1\}$ and $\{a2\}$.

Take S = {a1, a2, a3, a4} and let C = {{}, {a1}, {a2, a3}, {a2, a3, a4}}. Which of the following statements are true?

{a2, a3, a4} is a maximal subset of C in S.

{a1} is a maximal subset of C in S.

{a1} is a minimal subset of C in S.

{a2, a3} is a minimal subset of C in S.

Take S = {a1, a2, a3, a4} and let C = {{}, {a1}, {a2, a3}, {a2, a3, a4}}. Which of the following statements are true?

{a2, a3, a4} is a maximal subset of C in S.

{a1} is a maximal subset of C in S.

{a1} is a minimal subset of C in S.

{a2, a3} is a minimal subset of C in S.

Correct answers:
{a2, a3, a4} is a maximal subset of C in S.
{a1} is a maximal subset of C in S.

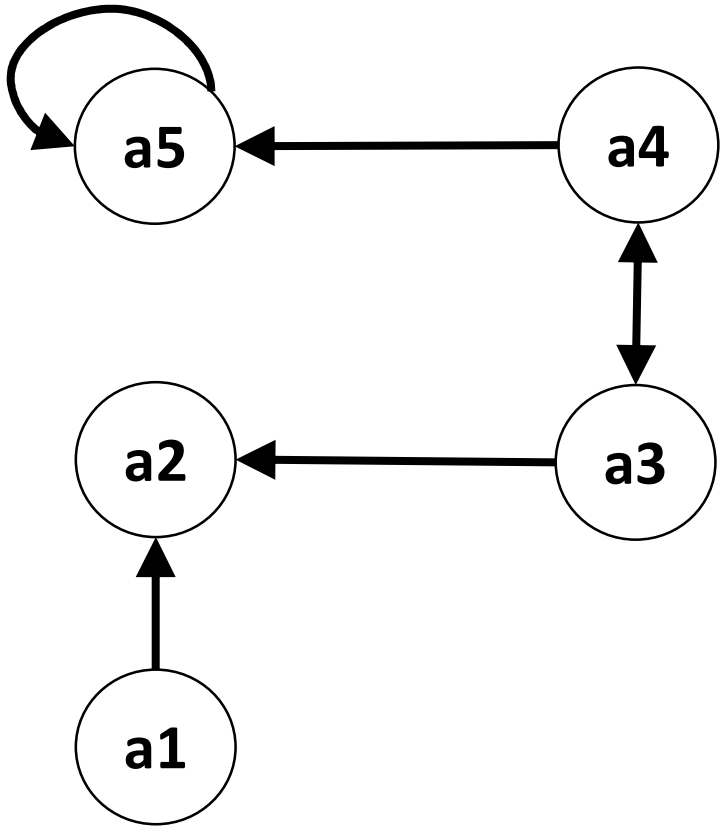
Grounded semantics

The grounded semantics aims to be cautious in the acceptance of arguments.

You can think of it as: *“Accept only what is not controversial”.*

Grounded extension

The **grounded extension** is the minimal complete extension with respect to set inclusion (ie, the minimal subset of the set of complete extensions).



Example 3

Conflict-free subsets:

$\{\}, \{a1\}, \{a2\}, \{a3\}, \{a4\},$

$\{a1, a3\}, \{a1, a4\}, \{a2,a4\}$

Admissible subsets:

$\{\}, \{a1\}, \{a3\}, \{a4\}, \{a1, a3\},$
$\{a1, a4\}$

Complete extensions:

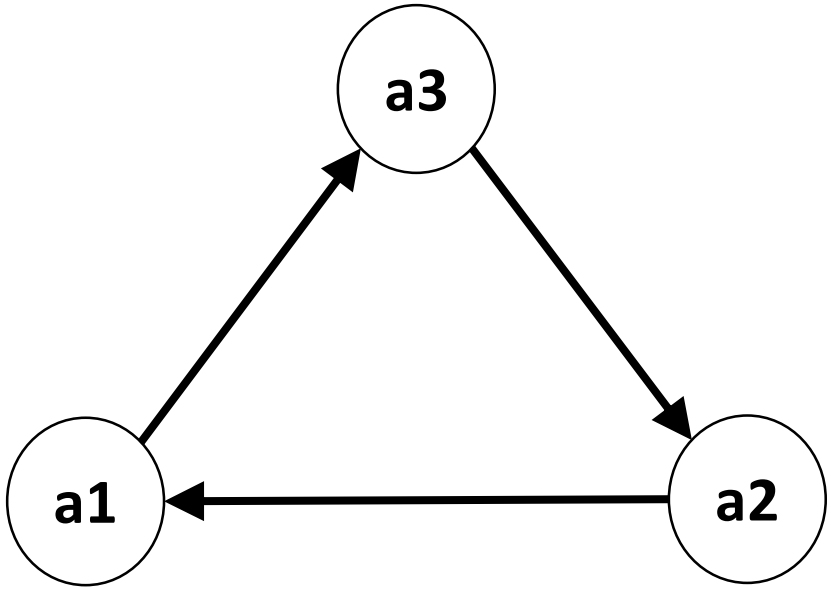
$\{a1\}, \{a1, a3\}, \{a1, a4\}.$

Grounded extension: $\{a1\}$.

Grounded extension

The grounded extension always exists and it is unique.

The grounded extension could be empty.



Example 4

Conflict-free subsets: { }, $\{a1\}$, $\{a2\}$ and $\{a3\}$.

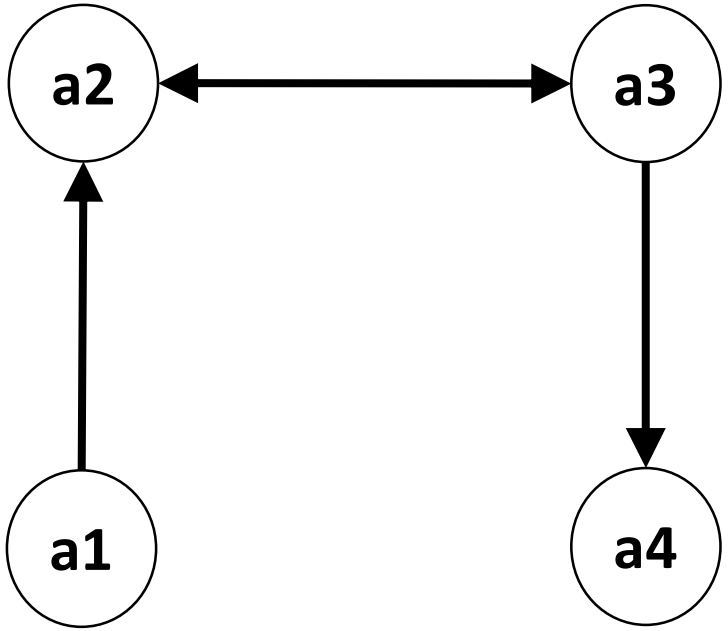
Admissible subsets: { }

Complete extensions: { }

Grounded extension: { }

Exercise 5 (Example 5)

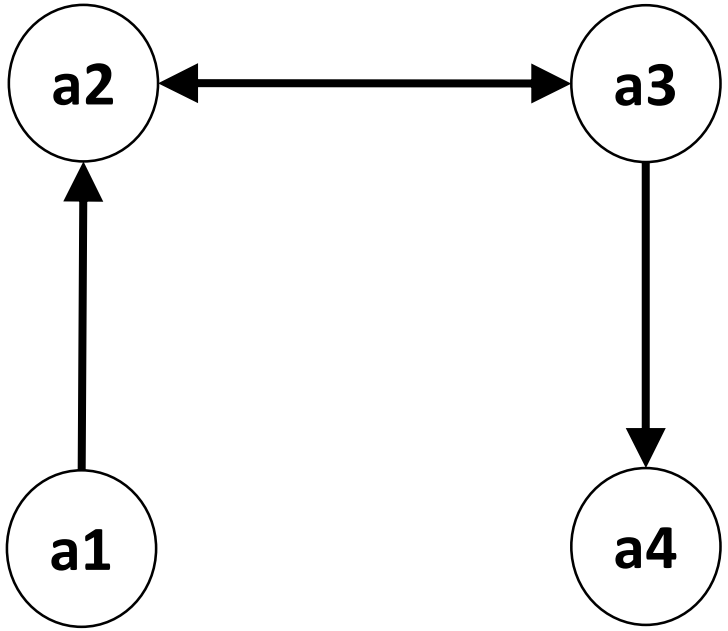
What is the grounded extension of the argumentation framework below?



Pause to do Exercise

Exercise 5 Solution

What is the grounded extension of the argumentation framework below?



Example 5

Conflict-free subsets: { }, $\{a1\}$, $\{a2\}$, $\{a3\}$, $\{a4\}$, $\{a1, a3\}$, $\{a1, a4\}$, $\{a2, a4\}$

Admissible subsets: { }, $\{a1\}$, {a3}, $\{a1, a3\}$

Complete extensions: $\{a1, a3\}$

Grounded extension: $\{a1, a3\}$

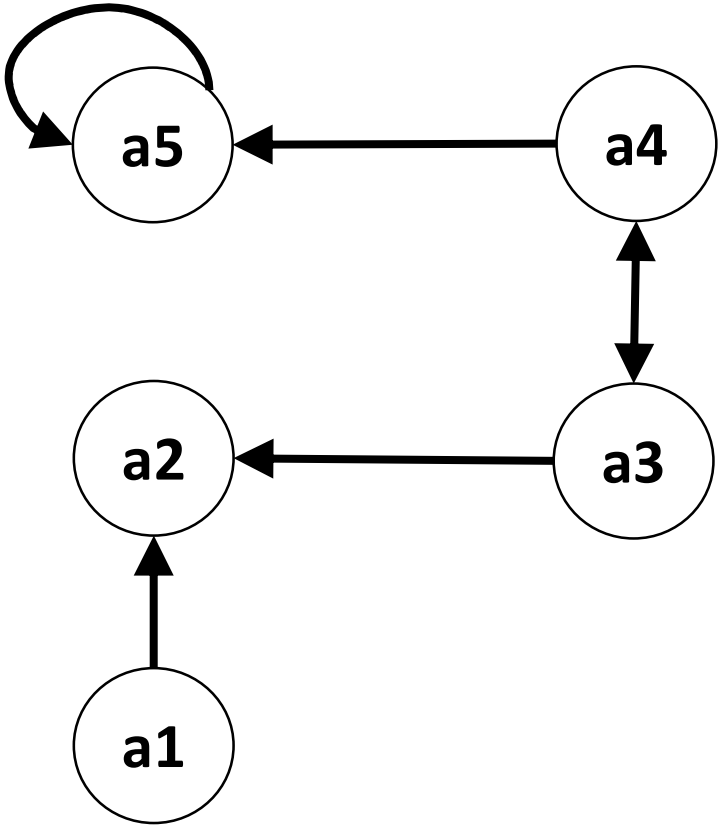
Preferred semantics

The preferred semantics tries to maximise the acceptance of arguments.

You can think of it as: *“Accept as much as you can defend”.*

Preferred extension

A **preferred extension** is a complete extension that is maximal with respect to set inclusion (ie, a maximal subset of the set of all complete extensions).



Example 3

Conflict-free subsets:

$\{\}$, $\{a1\}$, {a2}, $\{a3\}$, $\{a4\}$,

$\{a1, a3\}$, $\{a1, a4\}$, {a2,a4}

Admissible subsets:

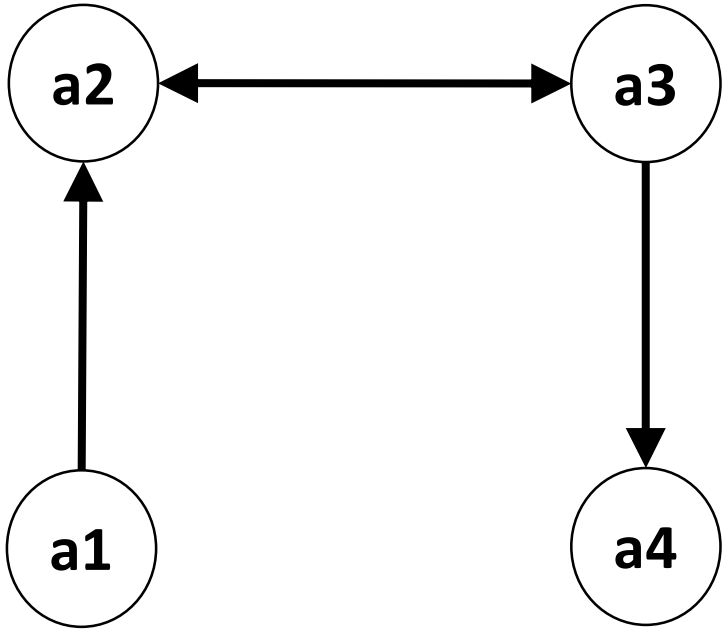
$\{\}$, $\{a1\}$, $\{a3\}$, $\{a4\}$, $\{a1, a3\}$, $\{a1, a4\}$

Complete extensions: $\{a1\}$, $\{a1, a3\}$, $\{a1, a4\}$.

Preferred extensions: $\{a1, a3\}$, $\{a1, a4\}$.

Exercise 6 (Example 5)

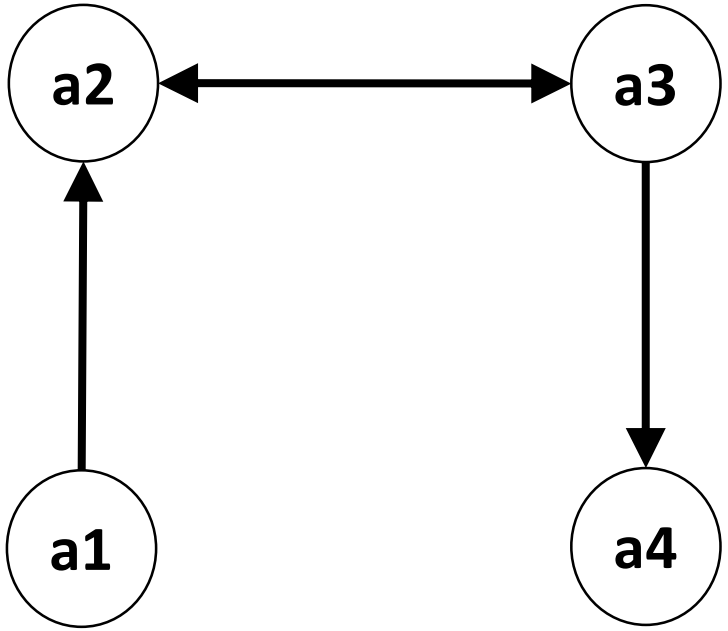
What are the preferred extensions of the argumentation framework below (Example 5)?



Pause to do Exercise

Exercise 6 Solution

What are the preferred extensions of the argumentation framework below?



Example 5

Conflict-free subsets: $\{\}$, $\{a1\}$, $\{a2\}$, $\{a3\}$, $\{a4\}$, $\{a1, a3\}$, $\{a1, a4\}$, $\{a2, a4\}$

Admissible subsets: $\{\}$, $\{a1\}$, $\{a3\}$, $\{a1, a3\}$

Complete extensions: $\{a1, a3\}$

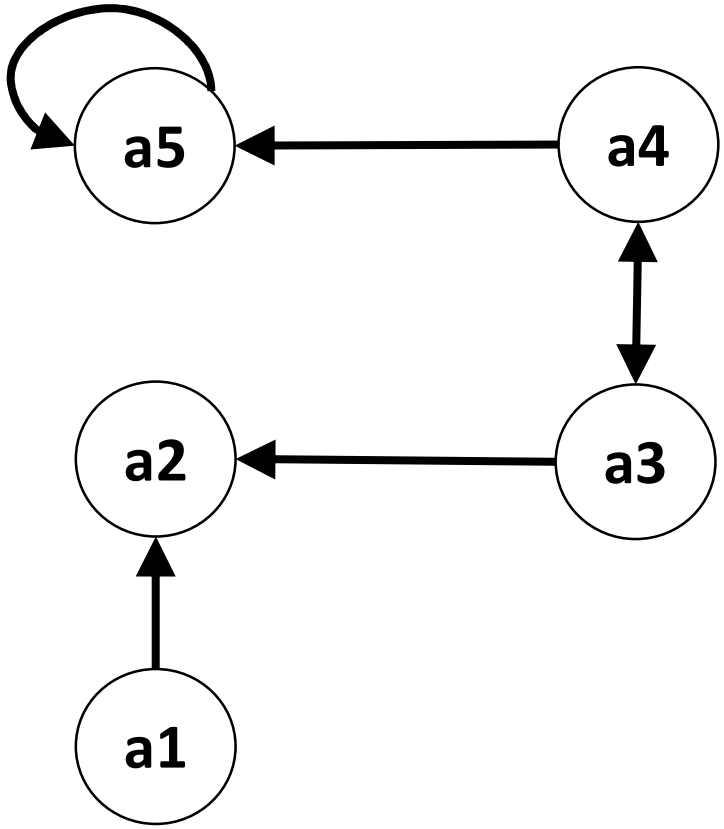
Preferred extensions: $\{a1, a3\}$

Grounded, complete & preferred extensions for Example 3

Complete extensions: $\{a1\}$, $\{a1, a3\}$, $\{a1, a4\}$.

Preferred extensions: $\{a1, a3\}$, $\{a1, a4\}$.

Grounded extension: $\{a1\}$.



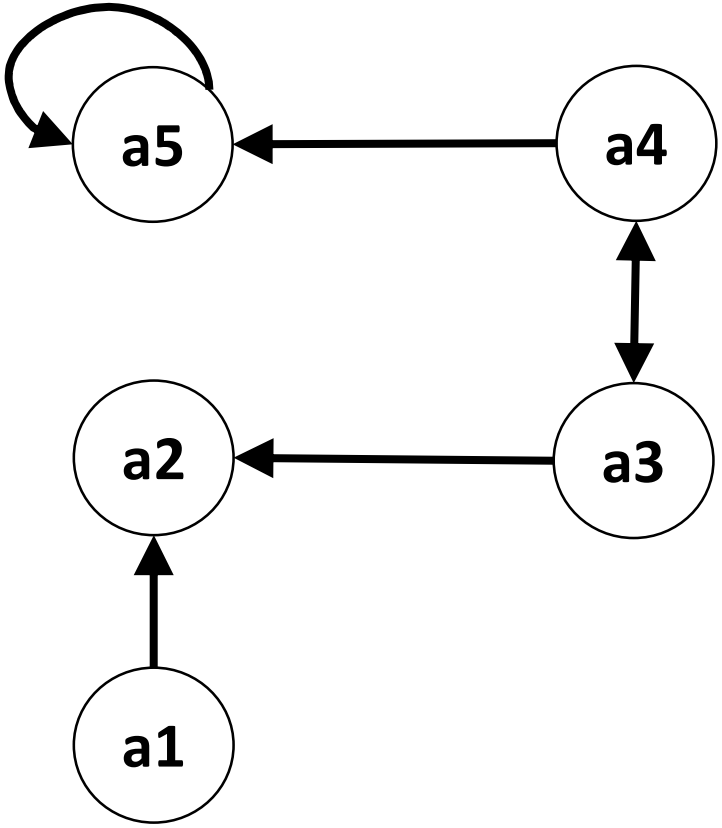
Example 3

The grounded extension is included in all preferred extensions.

The grounded extension coincides with the intersection of all complete extensions.

Stable extensions

A **stable extension** of an argumentation framework $\langle S, R \rangle$ is a preferred extension E such that for all $y \in S \setminus E$ there exists $x \in E$ such that $(x, y) \in R$ (in other words, for every argument y that isn't part of E, there is an argument in E that attacks y).



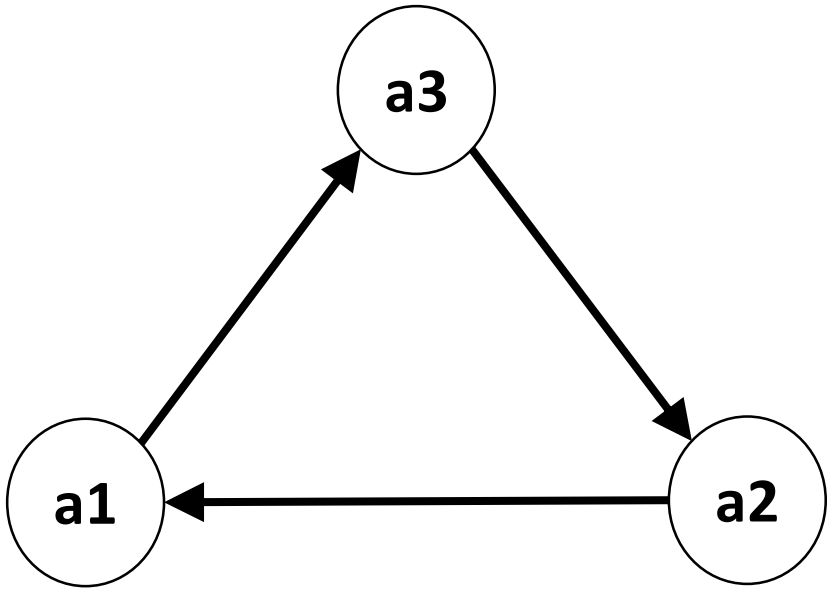
Example 3

Preferred extensions: $\{a1, a3\}$, $\{a1, a4\}$.

Stable extensions: $\{a1, a4\}$.

Stable extensions

Stable extensions do not always exist. In the framework of Example 4, there is no stable extension.



Example 4

Conflict-free subsets: { }

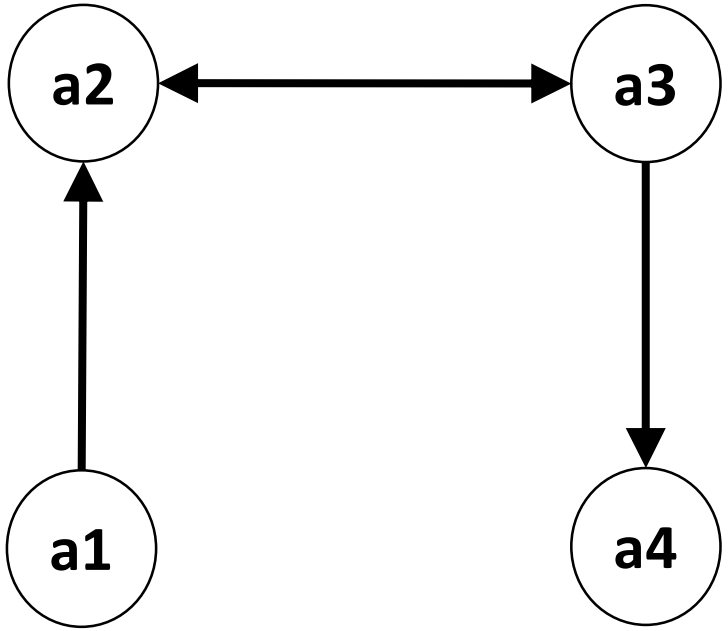
Admissible subsets: { }

Complete extensions: { }

No stable extension.

Exercise 7

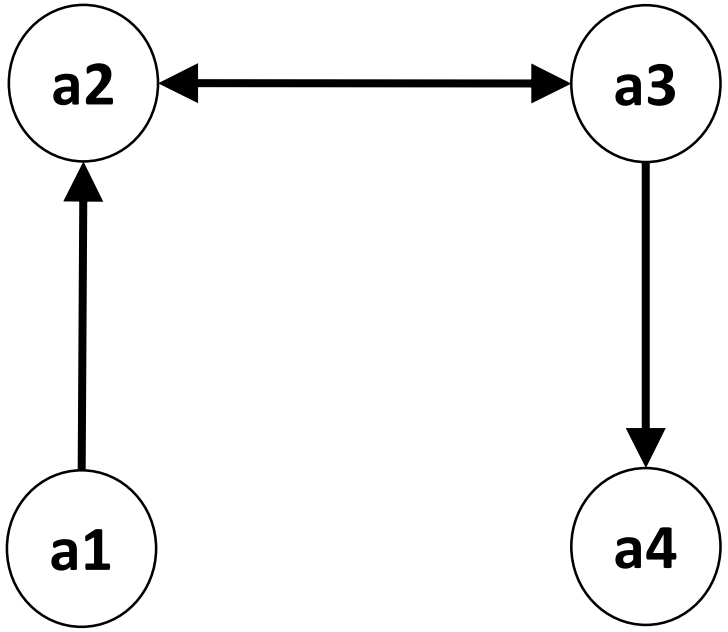
What are the stable extensions of the argumentation framework for Example 5?



Pause to do Exercise

Exercise 7 Solution

What are the stable extensions of the argumentation framework for Example 5?



Example 5

Conflict-free subsets: $\{\ \}$, $\{a1\}$, $\{a2\}$, $\{a3\}$, $\{a4\}$, $\{a1, a3\}$, $\{a1, a4\}$, $\{a2, a4\}$

Admissible subsets: $\{\ \}$, $\{a1\}$, $\{a3\}$, $\{a1, a3\}$

Complete extensions: $\{a1, a3\}$

Preferred extensions: $\{a1, a3\}$

Stable extensions: $\{a1, a3\}$

Credulous acceptance

An argument is **credulously accepted** by an argumentation framework under a particular semantics if and only if it is part of *at least one* of the extensions generated by those semantics.

For example, an argument is credulously accepted under the complete semantics if and only if it is part of at least one complete extension.

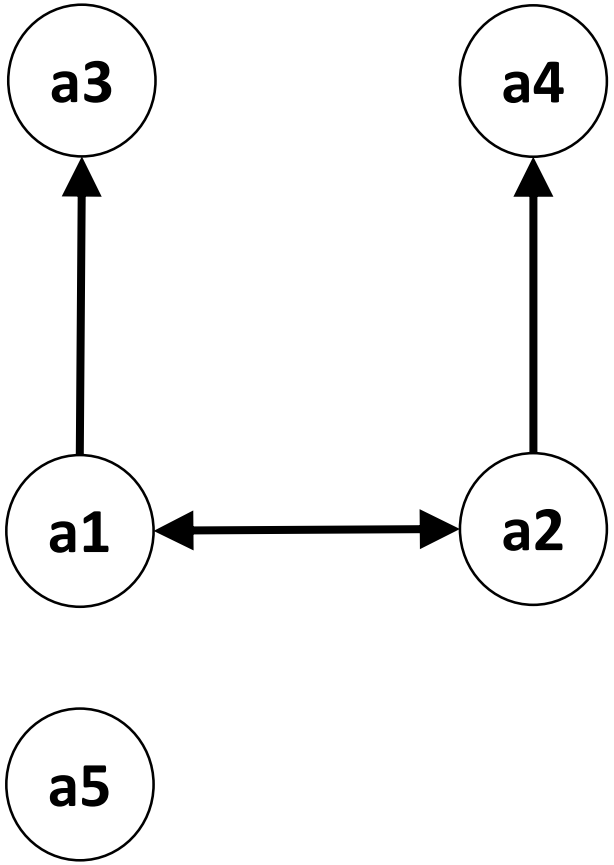
Skeptical acceptance

An argument is **skeptically accepted** by an argumentation framework under a particular semantics if and only if it is part of *all* of the extensions generated by those semantics.

For example, an argument is skeptically accepted under the complete semantics if and only if it is part of all of the complete extensions.

Exercise 8 – Example 6

Task: For each of the following semantics, determine which arguments are credulously accepted and which arguments are skeptically accepted: complete, grounded, preferred, stable.



Example 6

There are 5 arguments, a1, a2, . . . , a5.

There are 4 attacks:

(a1, a2)

(a1, a3)

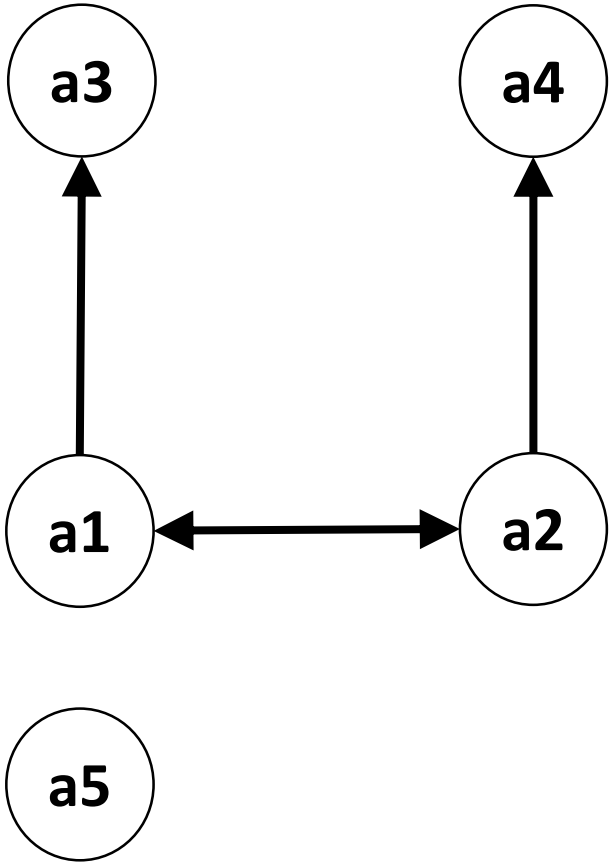
(a2, a1)

(a2, a4).

Pause to do Exercise

Exercise 8 Solutions

Task: For each of the following semantics, determine which arguments are credulously accepted and which arguments are skeptically accepted: complete, grounded, preferred, stable.



The different semantics

Conflict-free subsets: $\{\}$, $\{a1\}$, $\{a2\}$, $\{a3\}$, $\{a4\}$, $\{a5\}$, $\{a1, a4\}$, $\{a1, a5\}$, $\{a2, a3\}$, $\{a2, a5\}$, $\{a3, a4\}$, $\{a3, a5\}$, $\{a4, a5\}$, $\{a1, a4, a5\}$, $\{a2, a3, a5\}$, $\{a3, a4, a5\}$

Admissible subsets: $\{\}$, $\{a1\}$, $\{a2\}$, $\{a5\}$, $\{a1, a4\}$, $\{a1, a5\}$, $\{a2, a3\}$, $\{a2, a5\}$, $\{a1, a4, a5\}$, $\{a2, a3, a5\}$

Complete extensions: $\{a5\}$, $\{a1, a4, a5\}$, $\{a2, a3, a5\}$

Grounded extension: $\{a5\}$

Preferred extensions: $\{a1, a4, a5\}$, $\{a2, a3, a5\}$

Stable extensions: $\{a1, a4, a5\}$, $\{a2, a3, a5\}$

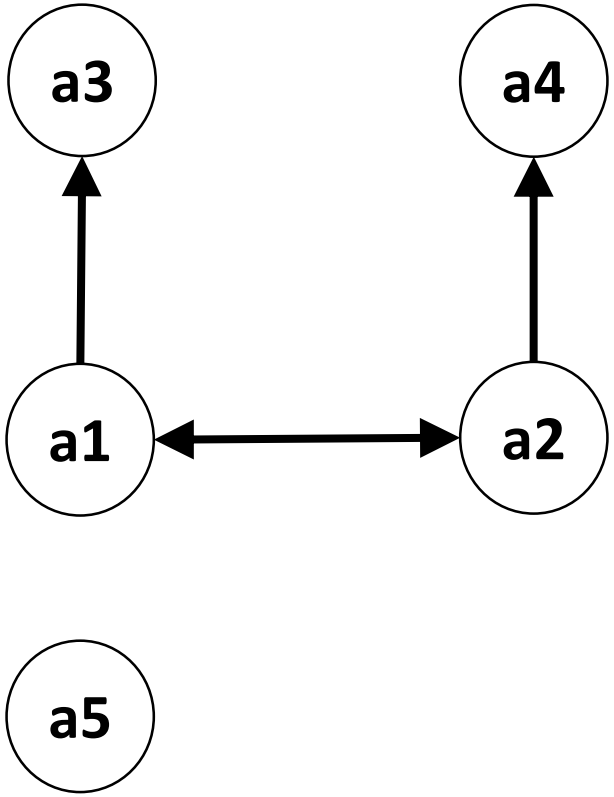
Exercise 8 Solutions

Complete extensions: $\{a5\}$, $\{a1, a4, a5\}$, $\{a2, a3, a5\}$

Grounded extension: $\{a5\}$

Preferred extensions: $\{a1, a4, a5\}$, $\{a2, a3, a5\}$

Stable extensions: $\{a1, a4, a5\}$, $\{a2, a3, a5\}$



Credulously accepted complete: all arguments.

Skeptically accepted complete: $a5$.

Credulously/skeptically accepted grounded: $a5$.

Credulously accepted preferred/stable: all arguments.

Skeptically accepted preferred/stable: $a5$.

Summary

- This week we have looked at different extension-based semantics that can be used to determine subsets of arguments one can reasonably accept.

- There are several different ways of determining (or calculating) the semantics for argumentation frameworks and different algorithms for these. We don't consider them in this module.