Bayes' Rule

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Bayes' Rule

- Probability that some hypothesis h is true, given that some event e has occured.

$$P(h|e) = \frac{P(e|h)P(h)}{P(e)}$$

- Now, since:

$$P(e) = P(e|h)P(h) + P(e|\neg h)P(\neg h)$$

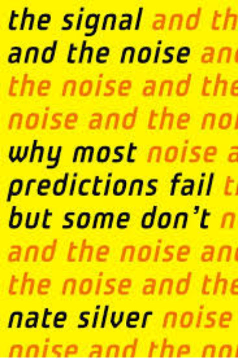
  we can rewrite this in a form in which it is commonly applied:

$$P(h|e) = \frac{P(e|h)P(h)}{P(e|h)P(h) + P(e|\neg h)(1 - P(h))}$$

  where variables are binary valued.

Some examples

- From:



Nate Silver

The wrong underwear

You live with your partner.
You come home from a business trip to discover a strange pair of underwear in your drawer.
What should you think?

In popular culture

(Metro-Goldwyn-Mayer)

- Major plot device in the 2003 romantic comedy.

The wrong underwear

- What do you think?

What do you think?

What would you guess is the probability? Just a rough estimate...

The wrong underwear

- Probability that some hypothesis is true, given that some event has occured.

$$P(h|e) = \frac{P(e|h)P(h)}{P(e|h)P(h) + P(e|\neg h)(1 - P(h))}$$

- event = underwear
- hypothesis = partner cheating
- To apply Bayes rule we need:
  - $P(e|h)$
  - $P(e|\neg h)$
  - $P(h)$

The wrong underwear

- $P(e|h)$
- If they cheated, how likely is the underwear?
- Well, if they cheated, then maybe it is likely the underwear would appear.
- But also, wouldn't they be more careful?
- Say 50% chance.

The wrong underwear

- $P(e|\neg h)$
- Is there an innocent explanation?
- A friend stayed over? Something was left in the dryer?
- Say 5% chance

The wrong underwear

- $P(h)$?
- Prior probability they cheated — before there was any evidence.
- Hard to quantify.
- Studies suggest about 4% of married partners cheat in a given year.
- Say 4%.

The wrong underwear

- The verdict?

$$
\begin{aligned}
P(h|e) &= \frac{P(e|h)P(h)}{P(e|h)P(h) + P(e|\neg h)(1 - P(h))} \\
&= \frac{0.5 \cdot 0.04}{0.5 \cdot 0.04 + 0.05 \cdot (1 - 0.04)} \\
&= 0.294
\end{aligned}
$$

- Low because of the low prior.

The wrong underwear

- Now it happens again.

The wrong underwear

- What do you think?

The wrong underwear

- Well now the prior is 0.294

$$
\begin{aligned}
P(h|e) &= \frac{P(e|h)P(h)}{P(e|h)P(h) + P(e|\neg h)(1 - P(h))} \\
&= \frac{0.5 \cdot 0.294}{0.5 \cdot 0.294 + 0.05 \cdot (1 - 0.294)} \\
&= 0.806
\end{aligned}
$$

- and the verdict is pretty clear.

A medical example

- Let M be meningitis, S be stiff neck:

$$\begin{aligned} P(m|s) &= \frac{P(s|m)P(m)}{P(s)} \\ &= \frac{0.8 \cdot 0.0001}{0.1} \\ &= 0.0008 \end{aligned}$$

- Posterior probability of meningitis still very small, again because of low prior.
- Common pattern in medical test results.
- Note that here we used a slightly different formulation of Bayes Rule.

Bayes rule & knowledge representation

- Useful for assessing diagnostic probability from causal probability:

$$P(Cause|Effect) = \frac{P(Effect|Cause)P(Cause)}{P(Effect)}$$

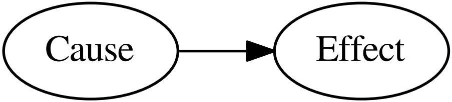
- We saw this in both examples.

Bayes rule & knowledge representation

- Often easier to assess causal probabilities.

$$P(Cause|Effect) = \frac{P(Effect|Cause)P(Cause)}{P(Effect)}$$

- Can visualise this as:

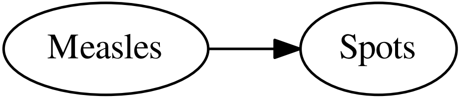


Bayes rule & knowledge representation

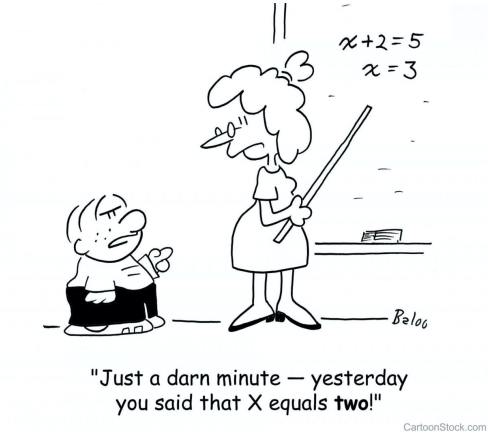
- Often easier to assess causal probabilities.

$$P(Measles|Spots) = \frac{P(Spots|Measles)P(Measles)}{P(Spots)}$$

- Can visualise this as:



Mathematical!



What we learned so far

- Probability is a rigorous formalism for uncertain knowledge
- Joint probability distribution specifies probability of every atomic event
- Queries can be answered by summing over atomic events
- For nontrivial domains, we must find a way to reduce the joint size
- Independence and conditional independence provide the tools for efficient computation along with Bayes' rule.
- Next week we'll look at how they are used.