Normal Form Games

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Normal form games

(Pendleton Ward/Cartoon Network)

Normal form games

- An n-person, finite, normal form game is a tuple (N, A, u), where
  - N is a finite set of players.
  - $A = A_1 \times \ldots \times A_n$ where A_i is a finite set of actions available to i. Each $a = (a_1, \ldots, a_n) \in A$ is an action profile.
  - $u = (u_1, \ldots, u_n)$ where $u_i : A \mapsto \Re$ is a real-valued utility function for i.
- Naturally represented by an n-dimensional matrix

Strategies

- We analyze games in terms of strategies, that is what agents decide to do.
  - Combined with what the other agent(s) do(es) this jointly determines the payoff.
- An agent's strategy set is its set of available choices.
- Can just be the set of actions — pure strategies.
- We need more than just pure strategies in many cases.
  - Will discuss this later

Payoff matrix

- Here is the payoff matrix from the "choose which side" (of the road) game:

i \ j	left	right
left	1 1	0 0
right	0 0	1 1

- We can classify games by the form of the payoff matrix.

Common payoff games

- "Choose which side" game

	left	right
left	1 1	0 0
right	0 0	1 1

  Also called the *coordination game*

- Any game with $u_i(a) = u_j(a)$ for all $a \in A_i \times A_j$ is a common payoff game.

Common payoff games

- The misanthropes' (un)coordination game:

	left	right
left	0 0	1 1
right	1 1	0 0

Here we try to avoid each other.

Misanthrope

http://www.thebadchemicals.com

Constant sum games

- Matching pennies

	heads	tails
heads	-1 1	1 -1
tails	1 -1	-1 1

- Any game with $u_i(a) + u_j(a) = c$ for all $a \in A_i \times A_j$ is a constant sum game.

Zero-sum games

- A particular category of constant sum games are zero-sum games.
- Where utilities sum to zero:

$$u_1(a) + u_j(a) = 0 \quad \text{for all } a \in A_i \times A_j$$

Zero-sum games

- Where preferences of agents are diametrically opposed we have strictly competitive scenarios.

(Library of Congress)

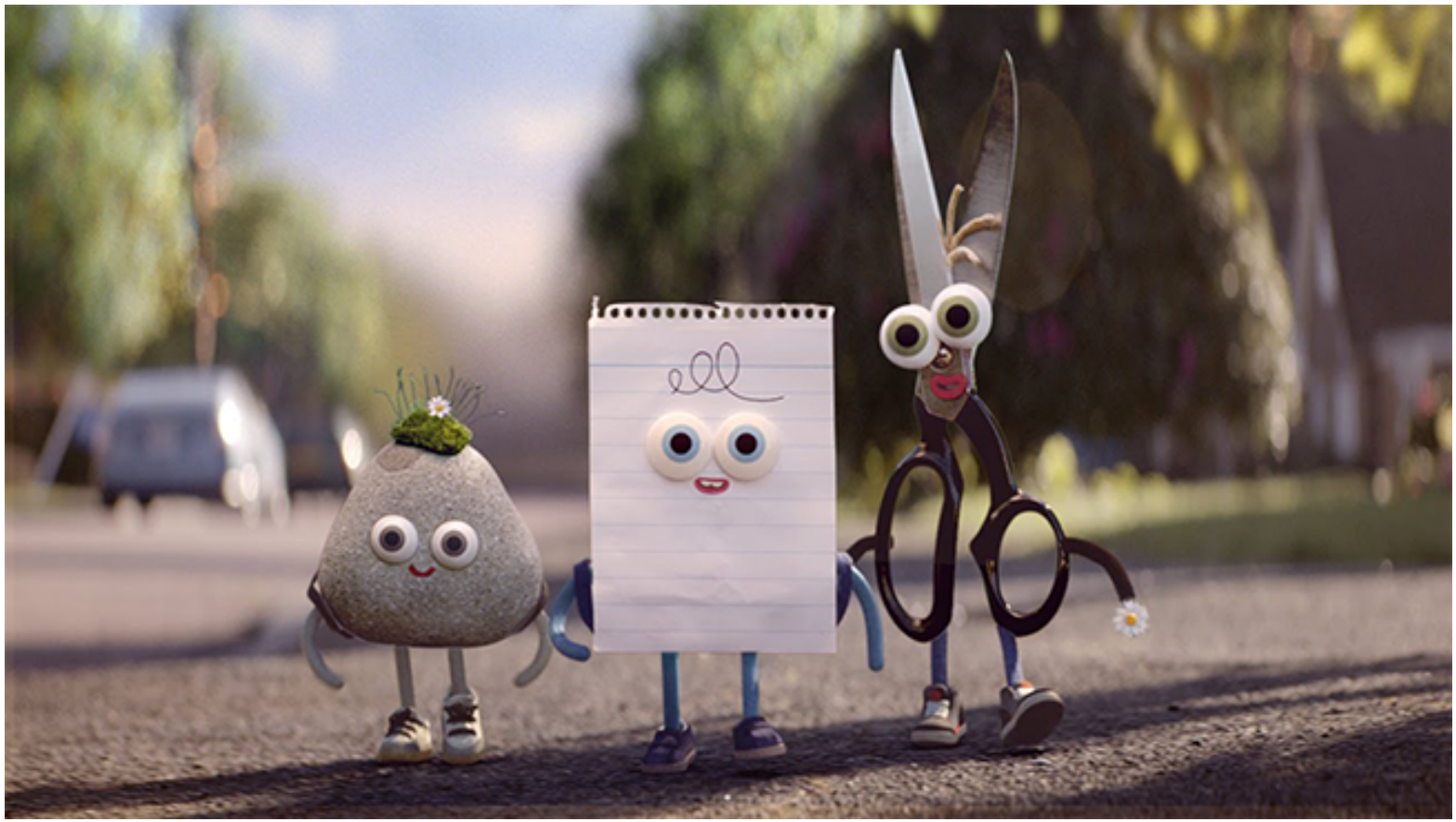
- Zero sum implies strictly competitive.

Zero-sum games

- Zero-sum encounters in real life are very rare . . . but people tend to act in many scenarios as if they were zero-sum.
- Most encounters have some room in the set of outcomes for agents to find (somewhat) mutually beneficial outcomes.

Zero-sum games

- Rock, paper, scissors:



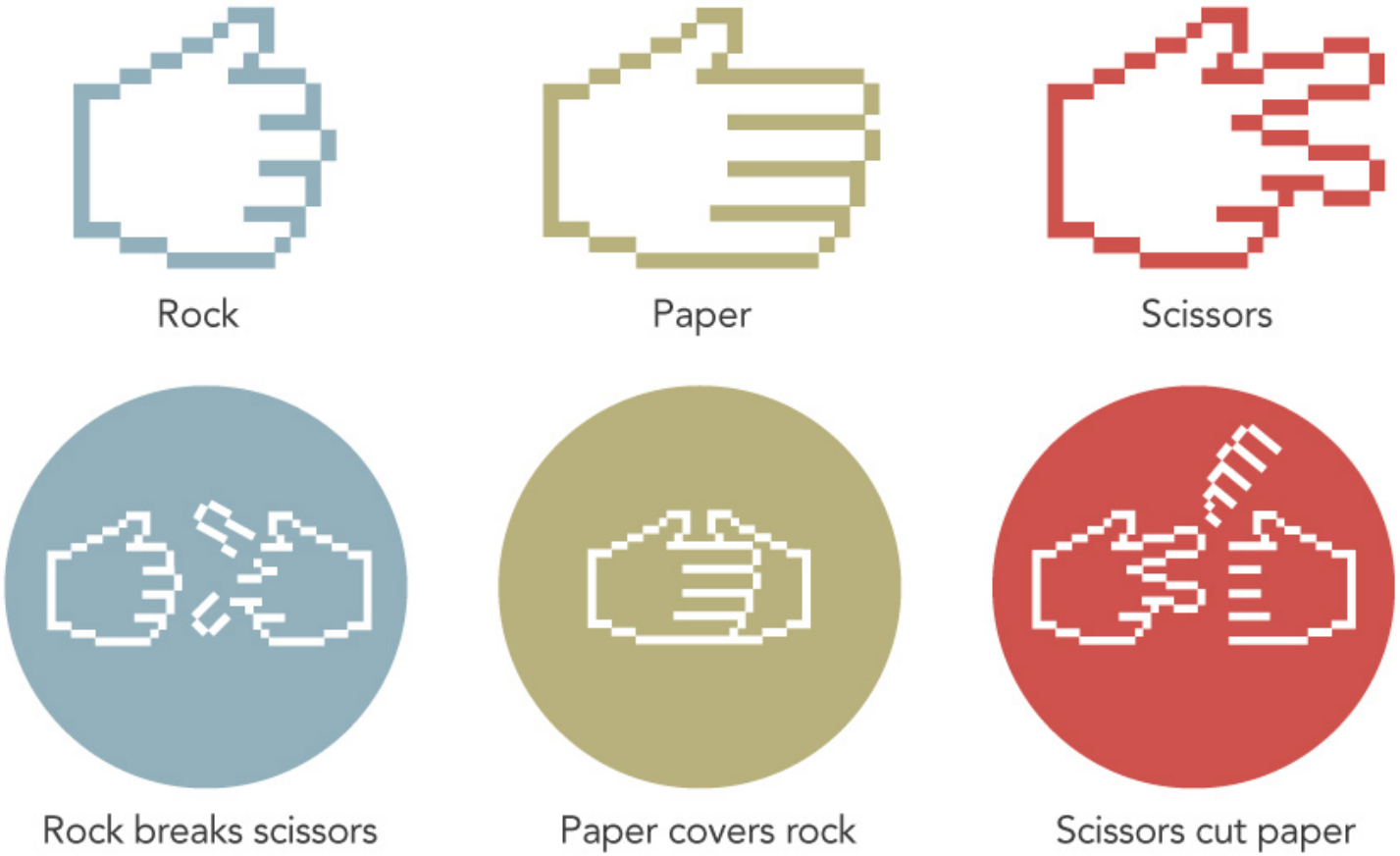
(Google/Droga5)

is another constant/zero sum game.

- Game in two senses.

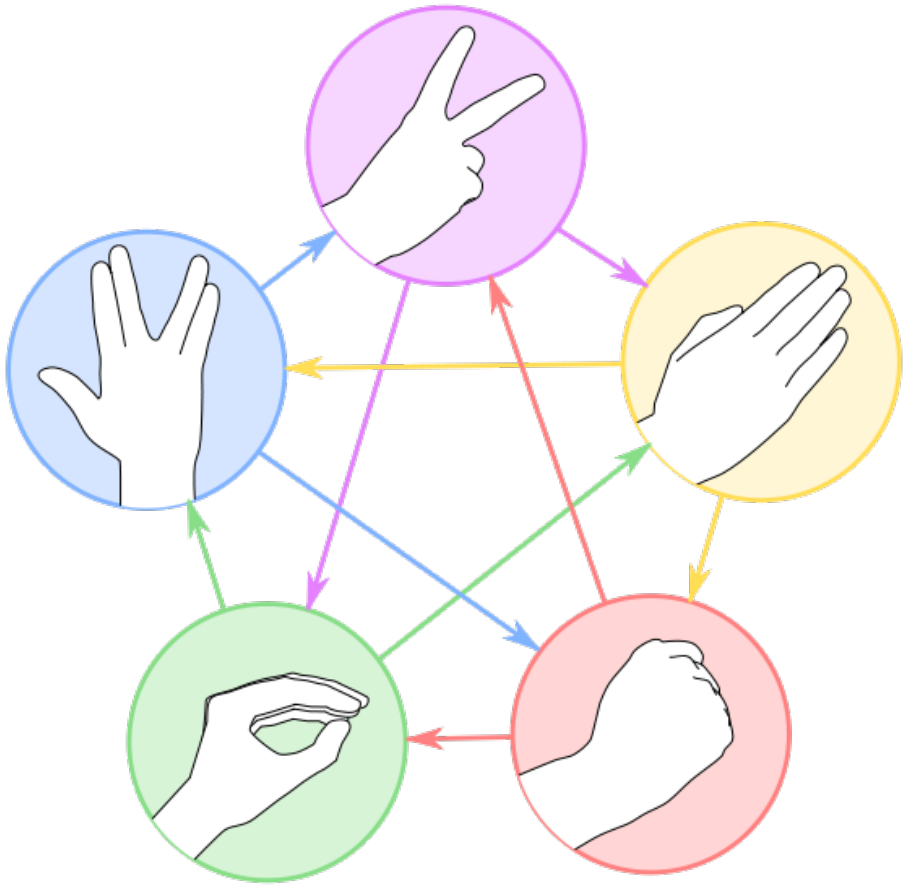
The rules

- Rules for “rock, paper, scissors”.



(eyemotive.com)

Rock, paper scissors, lizard, Spock



(DMacks/Nojhan/Wikipedia)

`http://www.youtube.com/watch?v=x5Q6-wMx-K8`

- Scissors cuts paper, paper covers rock, rock crushes lizard, lizard poisons Spock, Spock smashes scissors, scissors decapitates lizard, lizard eats paper, paper disproves Spock, Spock vaporizes rock, and as it always has, rock crushes scissors.

Rock, paper, scissors

i \ j	rock	paper	scissors
rock	0 0	1 -1	-1 1
paper	-1 1	0 0	1 -1
scissors	1 -1	-1 1	0 0