Mixed strategy

Frederik Mallmann-Trenn
6CCS3AIN

Mixed strategy

- Chances are you would play a mixed strategy.
- You would:
  - sometimes play rock,
  - sometimes play paper; and
  - sometimes play scissors.
- A fixed/pure strategy is easy for an adaptive player to beat.

Mixed strategy

- A mixed strategy is just a probability distribution across a set of pure strategies.
- So, for a game where agent i has two actions a_1 and a_2, a mixed strategy for i is a probability distirbution:
$$MS_i = \{P(a_1), P(a_2)\}$$
- Given this mixed strategy, when i comes to play, they pick action a_1 with probability $P(a_1)$ and a_2 with probability $P(a_2)$.

Mixed strategy

- To determine the mixed strategy, i can compute the **best values** of $P(a_1)$ and $P(a_2)$.
- These will be the values which **give i the highest expected payoff** given the options that j can choose and the joint payoffs that result.
- We could write down the expected payoffs of different mixed strategies and pick the one that optimises expected payoff.
- There is also a simple graphical method which works for very simple cases.
- Will look at this method next.

Mixed strategy

- Let's consider the payoff matrix:

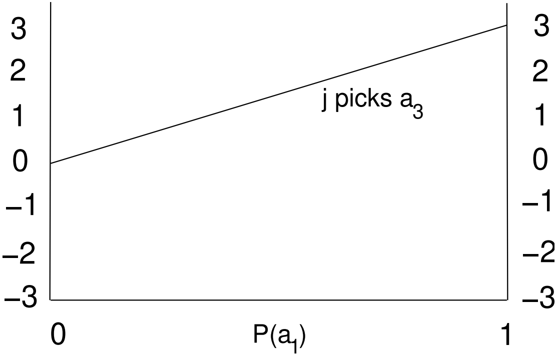
		j	
		a_3	a_4
i	a_1	-3 3	1 -1
	a_2	0 0	-1 1

- We want to compute mixed strategies to be used by the players.
- That means decide $P(a_1)$ and $P(a_2)$ etc.

Mixed strategy

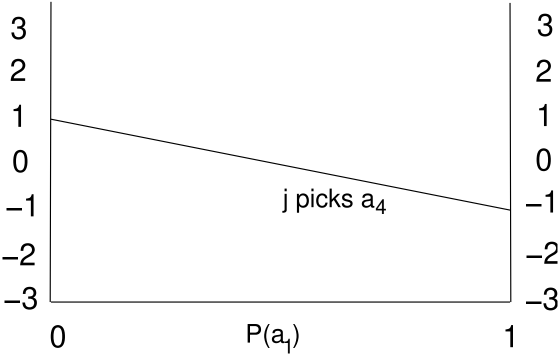
- i's analysis of this game would be something like this.

Mixed strategy



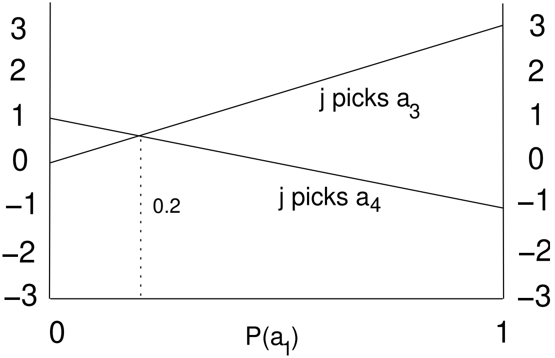
- Consider it from i's perspective. Let's say you know that j plays a_3.
- i's payoff will be 3 or 0 depending on whether i picks a_1 or a_2.
- The expected payoff therefore varies along the line,
  as $P(a_1)$ varies from 0 to 1.

Mixed strategy



- Consider it from i's perspective. Let's say you know that j plays a_4.
- i's payoff will be -1 or 1 depending on whether i picks a_1 or a_2.
- The expected payoff therefore varies along the line, as $P(a_1)$ varies from 0 to 1.

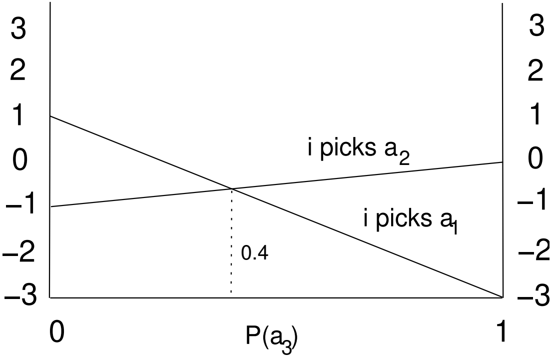
Mixed strategy



- Where the lines intersect, i has the same expected payoff whatever j does.
- This is a rational choice of mixed strategy.

Mixed strategy

- j can do the same kind of analysis:



Mixed strategy

- This analysis will help i and j choose a mixed strategy in zero-sum games.

(Archives of the Institute of Advanced Study, Princeton)

- This approach is due to von Neumann.

General sum games

- Battle of the Outmoded Gender Stereotypes
  - aka Battle of the Sexes

	this	that
this	1	0
	2	0
that	0	2
	0	1

(Time-Life/Getty)

- Game contains elements of cooperation and competition.
- The interplay between these is what makes general sum games interesting.

Negotiation

- Interplay between cooperation and competition leads to negotiation
- See, for example, the work of Sarit Kraus.

(law-train.eu)

Nash equilibrium

- Earlier we introduced the notion of Nash equilibrium as a solution concept for general sum games.
- (We didn't describe it in exactly those terms.)
- Looked at pure strategy Nash equilibrium.
- Issue was that not every game has a pure strategy Nash equilibrium.

Nash equilibrium

- For example:

		j			
		D		C	
i	D		2		1
		1		2	
	C		0		1
		2		1	

- Has no pure strategy NE.

Nash equilibrium

- The notion of Nash equilibrium extends to mixed strategies.
- And every game has at least one mixed strategy Nash equilibrium.

Nash equilibrium

- For a game with payoff matrices A (to i) and B (to j), a mixed strategy (x^*, y^*) is a Nash equilibrium solution if:

$$\begin{aligned} \forall x, x^* A y^{*T} &\geqslant x A y^{*T} \\ \forall y, x^* B y^{*T} &\geqslant x^* B y^T \end{aligned}$$

- In other words, x^* gives a higher expected value to i than any other strategy when j plays y^*.
- Similarly, y^* gives a higher expected value to j than any other strategy when i plays x^*.

Nash equilibrium

- Unfortunately, this doesn't solve the problem of which Nash equilibrium you should play.