Proposition. *Given a triangle with sides a, b and c, the area of the triange can be found by*

$$A = \sqrt{S(S-a)(S-b)(S-c)},$$

where $2S = a + b + c$.

Proof. WLOG, we will show that

$$16A^2 = h^2c^2 = (a+b+c)(b+c-a)(a+c-b)(a+b-c),$$

where h is (one of) the height(s) perpendicular to the side with length c, i.e., the base. Let h divide the base in to two segments with length $c-r$ and r, where $r \in \mathbb{R}$. Again, WLOG we can assume that

$$h^2 = a^2 - (c-r)^2,$$
$$h^2 = b^2 - r^2.$$

Solving the above two equations simeltaneously, we get

$$r = \frac{b^2+c^2-a^2}{2c}, \qquad h^2 = b^2 - \left(\frac{b^2+c^2-a^2}{2c}\right)^2.$$

Thus,

$$\begin{aligned}
4A^2 &= h^2c^2 \\
&= b^2c^2 - \left(\frac{b^2+c^2-a^2}{2}\right)^2 \\
&= \left(bc - \frac{b^2+c^2-a^2}{2}\right)\left(bc + \frac{b^2+c^2-a^2}{2}\right) \\
&= \left(\frac{-(b^2-2bc+c^2)+a^2}{2}\right)\left(\frac{b^2+2bc+c^2-a^2}{2}\right) \\
&= \left(\frac{a^2-(b-c)^2}{2}\right)\left(\frac{(b+c)^2-a^2}{2}\right) \\
&= \left(\frac{[a-b+c)][a+b-c]}{2}\right)\left(\frac{[b+c-a][b+c+a]}{2}\right) \\
&= \frac{(a-b+c)(a+b-c)(b+c-a)(a+b+c)}{4}.
\end{aligned}$$

$\square$