

## Contents

```
artikel1
amsmath amssymb amsthm hyperref bookmark % fbb % newpx % liber-
tine % crimson % bm cochineal
% eulervm % math mathpazo % math
tikz-cd enumerate
definition remark
{ colorlinks=true, linkcolor=blue, urlcolor=cyan, citecolor=red }
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## Serge Lang's Algebra Chapter III Solutions

dirichletian

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[(1)]By the second isomorphism theorem, we have

$$\frac{U}{U \cap W} \cong \frac{U + W}{W}.$$

For two vector spaces,  $X \supseteq Y$  over a field  $K$ , we have  $\dim X/Y = \dim X - \dim Y$ . Thus  $\dim U - \dim U \cap W = \dim U + W - \dim W$ . Let  $M$  be a module over a commutative ring  $R$ . Let  $I$  be a maximal ideal of  $R$ . We first assume  $M$  is finite-dimensional. Suppose  $\{v_1, \dots, v_m\}$  and  $\{w_1, \dots, w_n\}$  be two distinct basis sets of  $M$ .