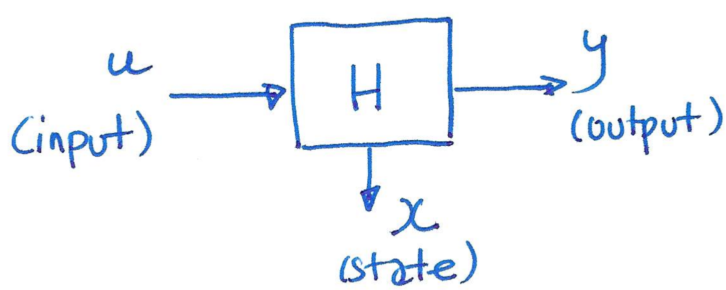
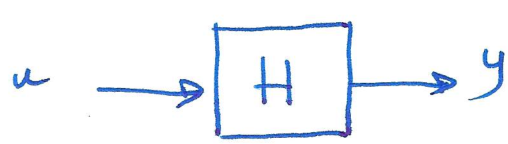
Systems (time-domain)

- systems are objects which accept inputs, generate outputs and have states
- often, we ignore the state variables and consider the input-output model of a system.
- we should distinguish between real systems that exist in the "real world" with physical interactions, and system models. Always keep in mind that our models are only representations, and can describe real systems only approximately.
- we use block diagrams to represent systems as



note that u, x or y may be scalars or vectors

or, without state as



- systems can work with discrete- or continuous-time signals.

- if a system is in continuous time, we often use <u>differential equations</u> to describe it; for example:

state model:

$$\left.\begin{aligned} \dot{x}_1 &= \theta_1 x_2 + x_1 x_2 + u \\ \dot{x}_2 &= \theta_2 x_1 \\ y &= \theta_3 x_1 + x_2 \end{aligned}\right\}$$

1-input (u)
1-output (y)
2 states (x_1, x_2)
3 parameters ($\theta_1, \theta_2, \theta_3$)

input-output model:

$$\ddot{y} + \alpha_1 \dot{y} + \alpha_0 y + \beta_0 \dot{y}^2 = \gamma_1 u + \gamma_2 \dot{u}$$

(2nd-order nonlinear input-output model with one input, u, and one output, y)

- if a system is in discrete-time, we often use <u>difference equations</u> to describe it; for example:

state model:

$$\begin{aligned} x_1(n+1) &= \theta_1 x_2(n) + x_1(n) x_2(n) + u(n) \\ x_2(n+1) &= \theta_2 x_1(n) \\ y(n) &= \theta_3 x_1(n) + x_2(n) \end{aligned}$$

input-output model:

$$y(n-2) + \alpha_1 y(n-1) + \alpha_0 y(n) + \beta_0 y^2(n-1) = \gamma_1 u(n) + \gamma_2 u(n-1)$$

- we also can use transformations such as those which convert system from time- to frequency-domain descriptions, but we will consider these later.

- an important class of systems are those which are <u>linear</u>; this is especially true in signal processing, where we often wish to manipulate signals without introducing <u>nonlinear effects</u> such as distortion.

<u>Principle of Superposition</u>

Consider a transformation $T(\,)$

$$u \longrightarrow \boxed{T} \longrightarrow y$$

we say that T obeys the principle of superposition if for any u_1 and u_2,

$$T(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 T(u_1) + \alpha_2 T(u_2)$$

is true for any $\alpha_1, \alpha_2 \in \mathbb{R}$.

If a system H exhibits the property of superposition (usually we set the initial conditions of H to zero before checking!), then H is said to be <u>linear</u>.

Furthermore, if for any Δ,

CT: $u(t-\Delta) \Rightarrow y = H\,u(t-\Delta) = y(t-\Delta)$, where $y(t) = Hu(t)$

DT: $u(n-\Delta) \Rightarrow y = Hu(n-\Delta) = y(n-\Delta)$, where $y(n) = Hu(n)$

then we say that H is time-invariant (CT) or shift-invariant (DT).

Convolution

Systems which are linear and time/shift-invariant have an input-output description (states are assumed to be initially $\emptyset$ or "at rest") given by the convolution integral or sum:

CT: $$y(t) = \int_{-\infty}^{\infty} h(\tau)\,u(t-\tau)\,d\tau = \int_{-\infty}^{\infty} u(\tau)\,h(t-\tau)\,d\tau$$

DT: $$y(n) = \sum_{k=-\infty}^{\infty} h(k)\,u(n-k) = \sum_{k=-\infty}^{\infty} u(k)\,h(n-k)$$

NOTE:

- when a system is causal, that is, not dependent on future inputs, the upper limits of the integral/sum can be changed to t and n, respectively.
- $h(t)$ (or $h(n)$) are known as the impulse response of the system.