International Institute of Information Technology, Bangalore.
CS 501 Data Structures and Algorithms.
Mid Term Exam, 18 September 2017.

1. You are given a sequence which is increasing till say k and then decreasing. That is first k, $0 \leq k \leq n$ elements are increasing. For example $2, 7, 19, 23, 56, 8, 3, 2, 1$ is an increasing decreasing sequence with $k = 5$. Given an increasing decreasing sequence of n numbers, design an $O(\log n)$ algorithm to find the value of k. (**4 marks**)

2. A sequence of numbers is said to be **differbyone**, if the sequnce of these numbers when sorted in increasing order, each element is greater than the previous by exactly one.

   For example: the array $3, 6, 5, 4$ is **differbyone** and $1, 2, 4, 5$ is not.

   Given an array a of length n, congaing all integers (exactly once) from 1 to n, Design an efficient algorithm to count the number of contiguous sub-arrays which are **differbyone** whose length is at least l. What is the complexity of your algorithm ?(**4 marks**)

3. Let A be array of lenght n, with each elemets is from the set $\{0, 1, 2\}$ with each of $\{0, 1, 2\}$ appearing atleast once in the given array. We would like to compute i and j such that the number of $0's$ in $A[0], A[1], \ldots A[i]$, the number of $1's$ in $A[i+1], A[i+2], \ldots A[j]$ and the number of $2's$ in $A[j+1], A[j+2], \ldots A[n-1]$ are same.

   For array $A = \{0, 1, 2, 2, 2, 0, 2, 1, \mathbf{2}, \mathbf{1}, \mathbf{0}, \mathbf{2}, \mathbf{1}, \mathbf{0}, \mathbf{2}, \mathbf{2}, \mathbf{0}, 2, 1, 1, 2, 0, 1, 0\}$ and the number of $1's$ in the highlighted subarray is 2, which is same as the number $0's$ before the highlighted subarray and the number $2's$ after the highlighted subarray.

   Give a linear time algorithm to solve this problem.(**4 marks**)

4. Design a data structure *MinStack* that supports all the stack operations like $Push(), Pop(), Top()$ and an additional operation $getMin()$ which will return the minimum element from the *MinStack*. All these operations on *MinStack* must take only constant time. (**4 marks**)

5. Design a data structure which allows the following operations efficiently.(**4 marks**)

   - Add(X) - Adds a positive integer X.
   - DeleteMedian() -Prints the median of the intergers in the data structure and remove it from the data structure. Take the smaller element as the median in case of even number of elements. For Example, Median of $4, 3, 1, 2$ is 2 and median of $3, 1, 2$ is 2.

6. A full binary tree is a binary tree in which every node other than the leaf nodes have two children.

   Given the Pre-order Traversal of a full binary tree, give a linear time algorithm to compute the height of the full binary tree.(**4 marks**)

   Input: Binary sequence, idicating if the node is an internal node or not (1 if the node is an internal node, 0 if it is leaf node).

   Example: Height of 1110000 is 3

   Height of 1011000 is 3

7. Let P be a priority queue that performs insert, deletemin, and merge in $O(logn)$ time, and performs makeheap in $O(n)$ time where n is the size of resulting priority queue.

   Show that P can be modified to perform insert in $O(1)$ amortized time, without affecting the cost of deletemin or merge ($i.e.O(logn)$ amortized time). (**4 marks**)

8. You are given two strings X and Y of length n. If $Y = y_0y_1 \dots y_{n-1}$, then after a circular shift you get $Y_1 = y_1y_2 \dots y_{n-1}y_0$. After k circular shift you get $Y_k = y_ky_{k+1} \dots y_{n-1}y_0y_1 \dots y_{k-1}$.

   Give a linear time algorithm to find the minimim possible circular shift on Y, say k circular shifts, so that Y_k has the longest common prefix with X. (**4 marks**)