International Institute of Information Technology, Bangalore.
CS 511 Algorithms. Practice Problems

1. Design an efficient algorithm to decide if the mimimum spanning tree of a given undirected graph is unique. What is the complexity of your algorithm ?

2. There are n chairs arranged around a circular table numbering from $1, 2, 3 \ldots n$. Each chair has a number written on it, say $a_1, a_2, \ldots a_n$ are the numbers written on each chair (a_i is the number on the ith chiar). A person sitting on a chair i can jump a_i number of chairs towards left or right. For example, if you are sitting on chair 4 and it has number 2 written on it, then you can either jump to chair number 2 or chair number 6.

   Given X and Y, design an efficient algorithm to find the minimum number of jumps required go from chair X to chair Y. If its impossible to go from chair X to chair Y, then print -1. What is the complexity of your algorithm ?

3. You are given a directed graph with weights on each node of the graph. Design an efficient algorithm to find a pair of nodes, say i, j such that j is reachable from i and $w(j) - w(i)$ is maximum. What is the complexity of your algorithm ?

4. You are given a tree having n nodes, rooted at node 1. Every node has a weight associated with the node. A pair of nodes say i and j, are called beautiful pair of nodes if node i is ancestor of node j and $w(i) > w(j)$. Recall that a node i is a ancestor of node j, if i is a node on the unique path from 1 to j.

   Design an efficient algorithm to count the total number of beautiful pair of nodes.What is the complexity of your algorithm ?

5. Let us assume that you are working for a company,which specialize in moving large number luggage from one place to another. There are n movers and m packages. The packages come in a single line, one behind the other, and then the movers come one by one to pick the packages. A mover can come only once, **can only pick the first few packages in the line**, and then take them with him. He should choose at least one packet. Another mover comes, and repeats the process, till all

the packages are taken. The movers need to carry of all the packages. Design an efficient algorithm to minimize the maximum load that any mover gets ?

For example, if there are two movers and the packets have weights $5, 10, 21, 20$, then the maximum load is 36, the first movers picks $5, 10, 21$ and the second mover picks 20.

6. You are given a binary string of length n. $Flip(l, u)$ is a function, which flips all the bits between l and r, you are allowed to call this function ONLY once. Design a **linear (time and space)** algorithm to find the values of l and u, such that after one call to the function $Flip(l, u)$, the binary string will have maximum number of 1's.

7. There are a group of n companies, over the past few days the companies had made several purchases among themselves, and thus owe each other certain amounts of money (say imbalance.) They dont care from which companies their money is coming from, as long as that they get / pay their share of imbalance. E.g : If three companies A, B, and C have imbalances $+20M, -15M$ and $-5M$ respectively, A could pay off B and C directly via a $15M$ and a $5M$ transaction, or A could pay B and $C, 10M$ each and then C would pay $5M$ to B via a third transaction, resulting in all transactions being $\leq 10M$. Given $a_1, a_2, \ldots a_n$ which are the imbalance of the n companies, such that $\sum a_i = 0$, Design an efficient algorithm to find the minimum amount x, such that all imbalances can be satisfied by transactions involving $\leq x$ amount of money (thereby evading taxes as much as possible.) Note that some companies can have 0 imbalance, but will still help other companies for future favors. Also note that the government has imposed restrictions such that there can be only one transaction between any two companies, and in only one direction (i.e A gives some amount to B or B gives some amount to A, but not both). Also A cannot pay B twice. Furthermore, all transactions must in integral multiples of the unit currency.

8. A cell phone company is trying out its new model of cell phone. Here's how its structure is:

   The keypad has 11 buttons corresponding to digits from 0 to 9 and one additional button called *Add*. After pressing any button from 0 to 9, the corresponding digit appears on the screen. The *Add* button replaces

the last two digits appearing on the screen with their sum taken *mod* 10. If there are less than two digits currently on the screen, pressing *Add* does nothing. Each button has a non-negative cost of pressing associated with it. The cost of pressing *Add* button is always 0. Given the cost of pressing each button and the target phone number, design an efficient algorithm to find the minimum cost of feeding that number into the phone screen using a sequence of button presses.

9. In this problem you are asked to convert a string into a palindrome with minimum cost of operations. The operations allowed are

   (a) Add any character at any position

   (b) Remove any character from any position

   (c) Replace any character at any position with another character

   Every operation you do on the string would count for a different cost, say i, d, r respectively. Youd have to keep the total cost as low as possible. Design an efficient algorithm for this problem. What is the complexity of your algorithm ?

10. Suppose you own two stores, A and B . On each day you can be either at A or B. If you are currently at store A (or B) then moving to store B the next day (or A) will cost C amount of money. For each day $i, i = 1, 2, \ldots n$, you are also given the profits $P^A(i)$ and $P^B(i)$ that you will make, if you are at store A or B on day i respectively. Design an efficient algorithm, to give a schedule which tells, where you should be on each day so that the overall money earned (profit minus the cost of moving between the stores) is maximized. What is the complexity of your algorithm ?