International Institute of Information Technology, Bangalore
CS511 Algorithms. Problems, August 2018.

1. Using Masters Theorem, give asymptotic upper and lower bounds for the following recurrences, assuming that $T(n)$ is a constant for all $n < 5$.

   (a) $T(n) = 3T(n/3) + 1$.
   (b) $T(n) = 3T(n/3) + n$.
   (c) $T(n) = T(n/3) + 1$.
   (d) $T(n) = 2T(n/3) + n$.
   (e) $T(n) = T(n/3) + n$.
   (f) $T(n) = T(2n/3) + 1$.
   (g) $T(n) = 2T(n/3) + n \log n$.
   (h) $T(n) = 3T(n/3) + n^2$.
   (i) $T(n) = T(n/3) + n^3$.
   (j) $T(n) = T(n-1) + n$.
   (k) $T(n) = 3T(n/3) + \log n$.
   (l) $T(n) = T(n/3) + \log n$.

2. The *addBlock*() operation is used to add m integers to to an existing sorted n integers, where $m << n$ (m is very small compared to n). One of your senior student gave me the following algorithm for this problem: the algorithm simply creates an array of length $n+m$, copies over the old n values into the new array, copies over the m values to the end of the array, and finally insertion sort is used (from the nth location onwards) to bring everything into order.

   (a) What is the complexity of the above algorithm.
   (b) Design an efficient algorithm for this problem.

3. Given a sorted array of distinct integers $A[0] < A[1] < \ldots A[n-1]$, design an $O(\log n)$ algorithm for the following

   (a) Decide whether there is an index i such the $A[i] = i$.

(b) Given x and y, find the number of integers in the given array which are strictly greater than x, but strictly smaller than y.

4. You are given a sequence which is increasing till say k and then decreasing. That is first k, $0 \leq k \leq n$ elements are increasing. For example $2, 7, 19, 23, 56, 8, 3, 2, 1$ is an increasing decreasing sequence with $k = 5$. Given an increasing decreasing sequence of n numbers, design an $O(\log n)$ algorithm to find the value of k.

5. Give a $O(n \log k)$ time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists.

6. Given two arrays A and B, containing m and n integers with $n > m$, design an $O(n \log n)$ algorithm to determine how many points are in common in the given two arrays.

7. You are given a sequence of $10 < n < 10^9$ numbers and a number $10 < k < 10^5$ such that, $1 \leq k \leq n$. Write an efficient algorithm to list the k smallest numbers among the given sequence of numbers. What is the complexity of the algorithm ?

8. Suppose, you are a manager in a company and your job is to organize, say n meetings in a day. Each meeting has a start time and an end time. Each meeting will be held between the start time and the end time. You may assume, that the these are in minutes, will have integer values between 0 and 1440. Given the, start time and the end time of n meetings, you would to know the time intervals when no meeting is held. Write an algorithm to solve this problem. What is the complexity of the algorithm ?

   For Example, if the input is $[234, 765], [874, 1232], [654, 1440], [1120, 1345]$ then the output is $[0, 234], [765, 654]$.

9. Let A, B and C be three sequence of n integers each. Design an $O(n^2)$ algorithm to determine if there are three integers $a \in A, b \in B, c \in C$ such that $c = a + b$.

10. Given an array of integers, $a_0, a_1, a_2, \ldots a_{n-1}$ and $l, 1 < l < n$. design a linear time algorithm to compute maximum sum the subsequcen, whose length is at most l.

11. Suppose you are given two sorted arrays of integers $A[1..m]$ and $B[1..n]$ and an integer k. Describe an efficient algorithm to find the kth smallest element in the union of A and B.

    For example, given the input $A[1..10] = [0, 1, 3, 6, 11, 13, 15, 22, 32, 45]$, $B[1..5] = [2, 5, 8, 17, 29]$, $k = 9$, your algorithm should return 13. You can assume that the arrays contain no duplicates.

12. An element in a given sequence $a_1, a_2, \ldots a_n$ is said to be a majority element, if it repeats at least $n/3 + 1$ times. Design a linear time algorithm to decided if a given sequence has a majority element.