IIIT-Bangalore,
Mathematics for ML (GEN 512)
Assignment Set 3

(Eigenvalues and Eigenvectors)

August 22, 2018

1. Prove the following properties.

   (i) If A be a singular matrix, then 0 is an eigenvalue of A and conversely.

   (ii) If A be a non-singular matrix and λ be an eigenvalue of A, then λ^{-1} is an eigenvalue of A^{-1}.

   (iii) If A and B be two matrices of same order, then AB and BA have the same eigenvalues.

   (iv) If λ is an eigenvalue of A, then

      (a) $\lambda + k$ is an eigenvalue of $A + kI$ (k is a scalar),

      (b) λk is an eigenvalue of kA

      (c) λ^m is an eigenvalue of A^m.

   (v) If λ is an eigenvalue of a nonsingular matrix A, then $\frac{\det A}{\lambda}$ is an eigenvalue of adj A.

   (vi) The eigenvalues of a diagonal matrix are its diagonal elements.

   (vii) (a) The product of n eigenvalues of A equals to the determinant, (b) The sum of n eigenvalues equals the sum of n diagonal entries.

   (viii) Let A and P be both $n \times n$ matrices and P be non-singular, then A and $P^{-1}AP$ have the same eigenvalues.

2. Find a matrix S such that $S^{-1}AS$ is a diagonal matrix, where

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

3. Find a matrix S such that $S^{-1}AS$ is a diagonal matrix, where

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}.$$

4. Diagonalize the (symmetric) matrix

$$A = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}.$$