IIIT-Bangalore,
Mathematics for ML (GEN 512)
Assignment Set 1

(Vector Spaces and Subspaces)

03-07-2018

1. We are given three different vectors $\mathbf{b_1}$, $\mathbf{b_2}$, $\mathbf{b_3}$. Construct a matrix so that the equations $Ax = b_1$ and $Ax = b_2$ are solvable, but $Ax = b_3$ is not solvable. How can you decide if this is possible? How could you construct A?

2. Describe a subspace $\mathbf{S}$ of each vector space V, and then a subspace $\mathbf{SS}$ of $\mathbf{S}$.

$$\begin{aligned}
V_1 &= \text{all combinations of } (1,1,0,0) \text{ and } (1,1,1,0) \text{ and } (1,1,1,1) \\
V_2 &= \text{all vectors } vperpendicular tou = (1,2,1), \text{ so } \mathbf{u}\cdot\mathbf{v} = 0 \\
V_3 &= \text{all symmetric } 2\times 2 \text{ matrices (a subspace of } \mathbf{M}) \\
V_4 &= \text{all solutions to equations } d^4y/dx^4 = 0 (\text{ a subspace of } \mathbf{F})
\end{aligned}$$

3. Create a 3×4 matrix whose special solutions to $Ax = \mathbf{0}$ are $\mathbf{s_1}$ and $\mathbf{s_2}$:

$$\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ 0 \\ -6 \\ 1 \end{bmatrix} \quad \text{pivot columns 1 and 3; free variables } x_2 \text{ and } x_4$$

   You could create the matrix A in row reduced form R. Then describe all possible matrices A with the required nullspace $N(A) =$ all combinations of s_1 and s_2.

4. Find the complete solution $\mathbf{x} = \mathbf{x_p} + \mathbf{x_n}$ by forward elimination on $[A\ \mathbf{b}]$:

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 4 & 8 \\ 4 & 8 & 6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}$$

   Find numbers y_1, y_2, y_3 so that $y_1(\text{row } 1) + y_2(\text{row } 2) + y_3(\text{row } 3) =$ zero row. Check that $\mathbf{b} =$ (4, 2, 10) satisfies the condition $y_1b_1 + y_2b_2 + y_3b_3 = 0$. Why is this the condition for the equations to be solvable and $\mathbf{b}$ to be in the column space.

5. Start with the vectors $\mathbf{v_1} = (1,2,0)$ and $\mathbf{v_2} = (2,3,0)$. (a) Are they linearly independent? (b) Are they a basis for any space? (c) What space $\mathbf{V}$ do they span? (d) What is the dimension of $\mathbf{V}$? (e) Which matrices A have $\mathbf{V}$ as their column space? (f) Which matrices have $\mathbf{V}$ as their nullspace? (g) Describe all vectors $\mathbf{v_3}$ that complete a basis $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ for $\mathbf{R}^3$.

6. Suppose $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}$ is a basis for $\mathbf{R}^n$ and the $n \times n$ matrix A is invertible. Show that $A\mathbf{v_1}, A\mathbf{v_2}, \ldots, A\mathbf{v_n}$ is also a basis for $\mathbf{R}^n$.