International Institute of Information Technology, Bangalore
CS501 Data Structures and Algorithms.
Test 1: September 1, 2017

1. What is the difference between a Monte Carlo algorithm and a Las Vegas algorithm.(**2 marks**)

2. If $T(1) = 1, T(n) = n + T(2n/3) + T(2n/7)$, then prove that $T(n) = \Theta(n)$. (**2 marks**)

3. Let $F(0) = 0, F(1) = 1, F(2) = 2$ and $F(n) = (2 * F(n-1) - F(n-2) + 3 * F(n-3))\%1000$. We would like to have an $O(\log n)$ algorithm to compute $F(n)$ by computing A^n for some matrix. What is the matrix that we should use here. (**2 marks**)

4. Let A and B be two arrays, each having n numbers already in the sorted order. Give an $O(\log n)$ time algorithm to find the median of all $2n$ elements in the arrays A and B.(**4 marks**)

5. Given a array of integers, we would like to compute the smallest subarray which contains all the integers in the range $[1, \ldots k]$, where $k < n$.

   For array $A = \{1, \mathbf{2}, \mathbf{3}, \mathbf{9}, \mathbf{12}, \mathbf{3}, \mathbf{1}, \mathbf{6}, \mathbf{3}, \mathbf{13}, \mathbf{100}, \mathbf{123}, \mathbf{4}, 14, 3, 12, 23\}$ and $k = 4$, the answer is 12 as highlighted.

   Design a linear time ($O(n + k)$) algorithm for this problem.(**5 marks**)

6. Let A be array of lenght n, with each elemets is from the set $\{0, 1, 2\}$ with each of $\{0, 1, 2\}$ appearing atleast once in the given array. We would like to compute i and j such that the number of $0's$ in $A[0], A[1], \ldots A[i]$, the number of $1's$ in $A[i+1], A[i+2], \ldots A[j]$ and the number of $2's$ in $A[j+1], A[j+2], \ldots A[n-1]$ are same.

   For array $A = \{0, 1, 2, 2, 2, 0, 2, 1, \mathbf{2}, \mathbf{1}, \mathbf{0}, \mathbf{2}, \mathbf{1}, \mathbf{0}, \mathbf{2}, \mathbf{2}, \mathbf{0}, 2, 1, 1, 2, 0, 1, 0\}$ and the number of $1's$ in the highlighted subarray is 2, which is same as the number $0's$ before the highlighted subarray and the number $2's$ after the highlighted subarray.

   Give a linear time algorithm to solve this problem.(**5 marks**)