IIIT-Bangalore,
Mathematics for ML (GEN 512)
Assignment Set 5

(Pseudoinverse and Linear Transformation)

1. Try the following problems from the Book (by Gilbert Strang, 4th Edition) **Problem Set 7.3, Page 406-407**–Problem nos.-1, 2, 4, 5, 6, 7, 8, 9, 15.

2. Let $T : V \to W$ be a linear transformation between the vector spaces V and W (over $\mathbb{R}$). Then prove prove the following properties:

   (a) T is injective if and only if $\mathrm{Ker}T = \{\mathbf{0}\}$.

   (b) If $\mathrm{Ker}T = \{\mathbf{0}\}$, then the images of a linearly independent set of vectors in V are linearly independent in W.

3. Prove that the mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x+z, y+z)$, $(x, y, z) \in \mathbb{R}^3$ is linear. Describe $\mathrm{Ker}T$ and $\mathrm{Im}T$ and find their dimensions.

4. Prove that the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x+y, y+z, z+x)$, $(x, y, z) \in \mathbb{R}^3$ is one-to-one and onto.

5. Determine the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$ which maps the basis vectors $(1, 0, 0)$, $(0, 1, 0)$ $(0, 0, 1)$ of $\mathbb{R}^3$ to the vectors $(1, 1)$, $(2, 3)$, $(-1, 2)$ respectively. Find $T(1, 2, 0)$.

6. A linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by $T(x_1, x_2, x_2) = (3x_1 - 2x_2 + x_3, x_1 - 3x_2 - 2x_3)$, $(x_1, x_2, x_3) \in \mathbb{R}^3$. Find the matrix of T relative to

   (a) the ordered based $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ of $\mathbb{R}^3$ and $(1, 0)$, $(0, 1)$ of $\mathbb{R}^2$.

   (b) the ordered based $(0, 1, 1)$, $(1, 0, 1)$, $(1, 1, 0)$ of $\mathbb{R}^3$ and $(1, 0)$, $(0, 1)$ of $\mathbb{R}^2$.