

International Institute of Information Technology, Bangalore
CS101 Algorithms Mid Term Exam , 4 October 2011

Answer any FIVE of the following questions.

1. Given a sorted array of distinct integers $A[0] < A[1] < \dots A[n-1]$, design an $O(\log n)$ algorithm for the following
 - (a) Decide whether there is an index i such that $A[i] = i$.
 - (b) Given x and y , find the number of integers in the given array which are strictly greater than x , but strictly smaller than y .
2. Let A , B and C be three sequences of n integers each. Design an $O(n^2)$ algorithm to determine if there are three integers $a \in A, b \in B, c \in C$ such that $c = a + b$.
3.
 - (a) The longest path problem is to find a simple path between two vertices in a graph such that the weight of the path is maximized. Suppose we want to solve the single-source longest path problem in a directed acyclic graph. Can we modify Dijkstra's algorithm to solve this problem by changing minimum to maximum? If so, then prove your algorithm correct. If not, then provide a counterexample.
 - (b) In the art gallery guarding problem, we are given a line that represents a long hallway in an art gallery. We are also given a set $X = \{x_0, x_1, \dots, x_{n-1}\}$ of real numbers that specify the positions of paintings in this hallway. Suppose that a single guard can protect all the paintings within distance at most 1 of his or her position (both sides). Design an efficient algorithm for finding the placement of guards that uses the minimum number of guards to guard all the paintings with positions in X . Argue that your algorithm is correct.
4. Design an efficient algorithm to find the number of shortest paths between every pair of vertices in a weighted directed graph. What is the complexity of your algorithm?
5. Let G be a directed graph represented using an adjacency list and each of its vertices are assigned weights $w : V \rightarrow R$. Give an efficient algorithm that computes, for every node, the highest valued node reachable from that node. What is the complexity of your algorithm?
6. An independent set in a graph G is a subset I of the vertices such that no two vertices in I are adjacent in G . The maximum independent set problem is to find the largest independent set in G . Design an efficient algorithm to find a maximum independent set in a tree. What is the complexity of your algorithm?
7.
 - (a) Find $3^{21636764539378748743} \bmod 25$
 - (b) Given a positive integer n , design an efficient algorithm to decide whether there exist integers $a, b > 1$ such that $n = a^b$. What is the complexity of your algorithm?