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Mid Term - 1 Solved Paper
AI - 2072 : Probabilistic Graphical Methods

Question: 1 What are Probabilistic Graphical Models. Differentiate between directed graphical models and undirected graphical models.

Answer- Probabilistic Graphical Models are a type of statistical model that probabilistic relationships among a set of variables. They use a graph structure to visually depict these relationships. These are two main types: Directed Graphical models (DAG's) and undirected graphical models (UGM'S).

* Directed Graphical Models :-

- Structure : DAG'S use directed edges, meaning, there's a clear arrow indicating the direction of influence between variables.
- Interpretations : The edges represent direct dependencies and can be interpreted casually. If A influences B, there's an arrow from A to B.
- Example : Bayesian Network are a common types of DAG. Think of it like a family tree which each node is a person and edges represent parent-child relationships.

* Undirected Graphical Models :-

- Structure :- UGM'S are undirected edges, implying a symmetric relationship between variables.
- Interpretations :- Edges represent's associations, but without a clear direction of influence. It's more about captoning correlations than causation.
- Example :- Markov Random Field are a classic UGM. Imagine a social network where nodes are people and edges represent friendship. It's not about who influences whom just that there's a connection.

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Question 2 Explain Bayesian Belief Network and Conditional Independence with example.

Answer - Bayesian Belief Networks (BBN's), also known as Bayes Nets, are graphical models that represent probabilistic relationships among a set of variables. They are named after the Bayesian probability theory, which deals with updating belief's based on evidence.

* Example of Bayesian Belif Network & Conditional Independence with an example.

Bayesian Belief Network (BBN) :
Imagine a diagnostic system for a rare disease. We have three variables :

1) Test Result (T) : Positive or negative result from a diagnostic test.
2) Disease Presence (D) : Whether the patient actually has the disease or not.
3) Symptoms (S) : Presence or absence of symptoms related to the disease.

Now, let's construct a Bayesian Belief Network :-

• Structure :-

$T \rightarrow D \rightarrow S$

The arrow for the test result to disease presence indication that the tests result's depends on whether the disease is present. The arrow from disease presence to symptoms shows that symptoms depends on whether the disease is present.

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- Probabilities :-
  - P(D) : Prior probability of the disease.
  - P(T|D) : Probability of the test result given the disease.
  - P(S|D) : Probability of syptoms given the disease.

This network allows us to model and update our beliefs about the likelihood of disease presence based on test result's & symptoms.

★ Conditional Independence :- In conditional independence winthin this network. In this context, it means that certain variables become independent when you know the value of other variables.

- Example :-
  - Conditional Independence Claims : symptomps (S) are conditionally independent of the test result (T) given the disease presence (D)
  - Mathematically : $P(S|T,D) = P(S|D)$
  - Interpretation : If you know whether the disease is present (D), the information about the test result (T) doesn't provide any additional information about the symptoms (S).

Question → 3 Define

i) Prior Probability

- Definition :- Prior probability refers to the probability assigned to an event before any new data or evidence is taken into account. It represents the initial belief or probability distribution based on existing information or knowledge.
- Symbolically : P(A) represents the prior probability of event A.

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2. Conditional Probability :-
Conditional Probability is the probability given that the another event has already occured. It quantifies the likelihood of an event under a specific condition
• Symbolically - P(A|B) represents the conditional probability of event A given that event B has occured.

3. Posterior Probability :-
Posterior Probability is the updated probability of an event incorporating new evidence or data. It is derived by applying Bayes' Theorem, which combines the prior probability and the likelihood of the observed data.
• Symbolically - P(A|B), where A is the event of interest, B is the observed evidence or data.

Question → 4 Explain Markov Random Field's with an example?
Answer → Markov Random Fields → MRF's are a type of probabilistic graphical model that represents dependencies between random variables in a structure way. They're often used in image processing, computer vision and the other fields where spatial or temporal relationship matter's.

Graph Structure → Unlike Bayesian Belief Network, MRF's use an undirected graph. Nodes represent random variables and edges indicate dependencies or interaction between variables.

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* Example : Image Segmentation with MRF's :-
Imagine you have an image and you want to segment it into regions based on pixels intensities. Each pixel can be considered as a random variables, MRF's can model the relationship between neighbouring pixels.

- Nodes : Each pixel in the image is a node in the MRF.
- Edge : An edge connects neighbouring pixels, indicating a dependence b/w them.
- Potential Functions : MRF uses potential functions to quantify the compatability between neighbouring pixels. These functions encode the preference for certain configurations of neighbouring pixels.
- Objective : The goal is to find the configuration (assignment values to pixels) that maximises the joint probability of the entire configuration in MRF is often proportional to the product of potential functions.

$$P\,(\text{configuration}) \propto \prod_{\text{edges}} (\text{Potential nodes on the edges})$$

Question → 5 Explain the Bayesian networks over three variables, encoding different types of dependencies ; cascade, common parent, V-structure.

Answer → 1) Cascade Dependency :-
- Scenario :
  - Imagine three variables : A, B and C
  - A cause B, and B cause C
- Graph structure

$$A \rightarrow B \rightarrow C$$

An arrow from A to B indicates that A influences B, similarly and Arrow from B to C indicates that B influences C.

* Changes in A lead to changes in B, which in turn lead to a change in C. It's like a cascade effect.

2) Common Parent Dependency :-

A variable A & B are both influenced by a common parent, C.

- Graph Structure :
  - C ← A
  - C ← B

- C is a common cause for Both A & B. Changes in C can both influence A & B.

3) V-Structure (collider) :-

Variable A & B both influence C.

- Graph Structure :-
  - A → C ← B

A and B are parents of C, and there's no direct edge between A & B.

- When A & B both happen, it increases the likelihood of C. However, if you observe C, it provides evidence against the independence of A & B.

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* Example →

Let's say A represents "Rain", B represents "Traffic" and C represents "Late to work".

1) Cascade →
If it rains (A), it might cause traffic (B), you might be late to work (C).

2) Common Parents →
The common parents, C (Late to work), could be "Bad weather" influencing both "traffic" (B) and "Car troubles" (A).

3) V- structure →
Both "Bad weather" (A) and "car troubles" (B) could contribute to being "Late to work" (C).