PERCEPTRON: PROOF OF CONVERGENCE

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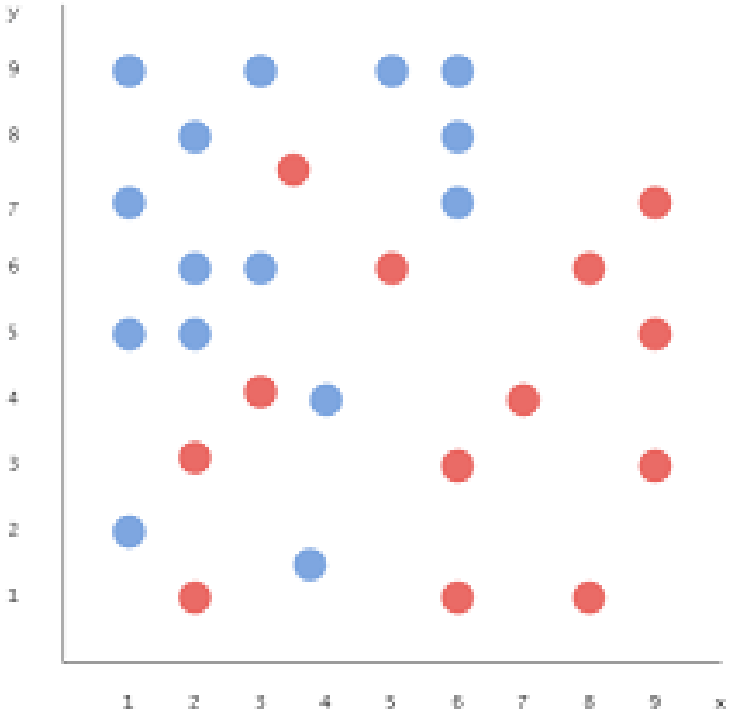
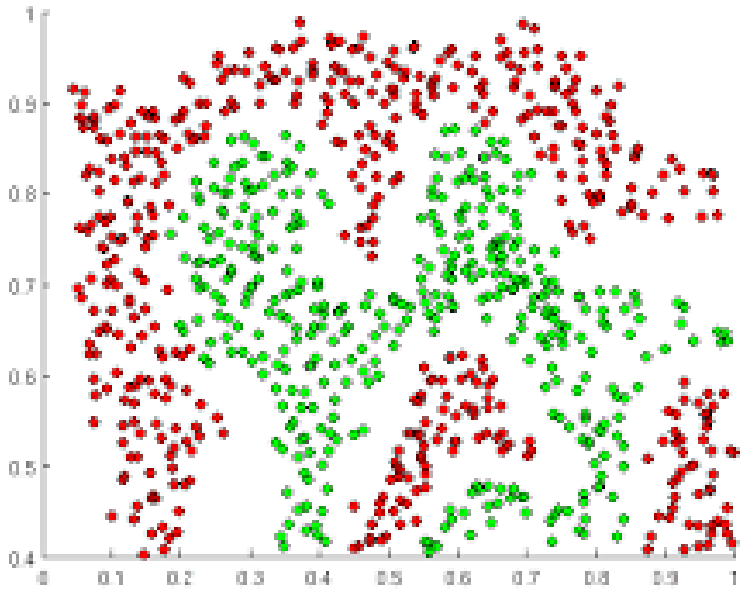
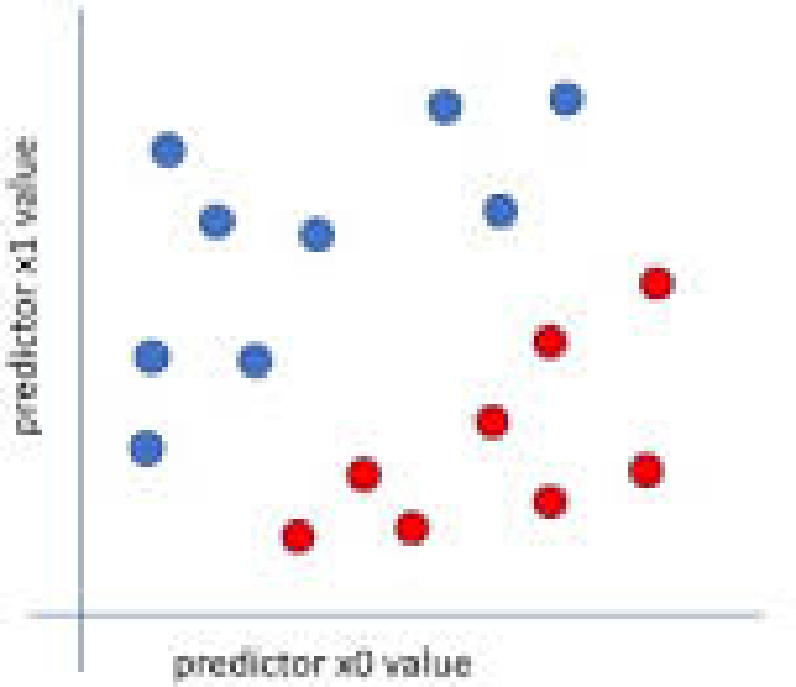
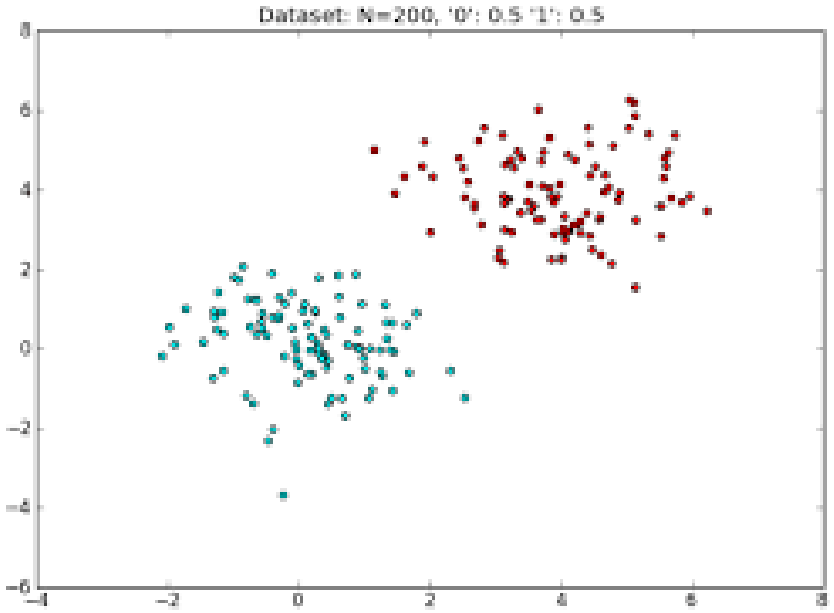
What classifier do you use?





What classifier do you use?

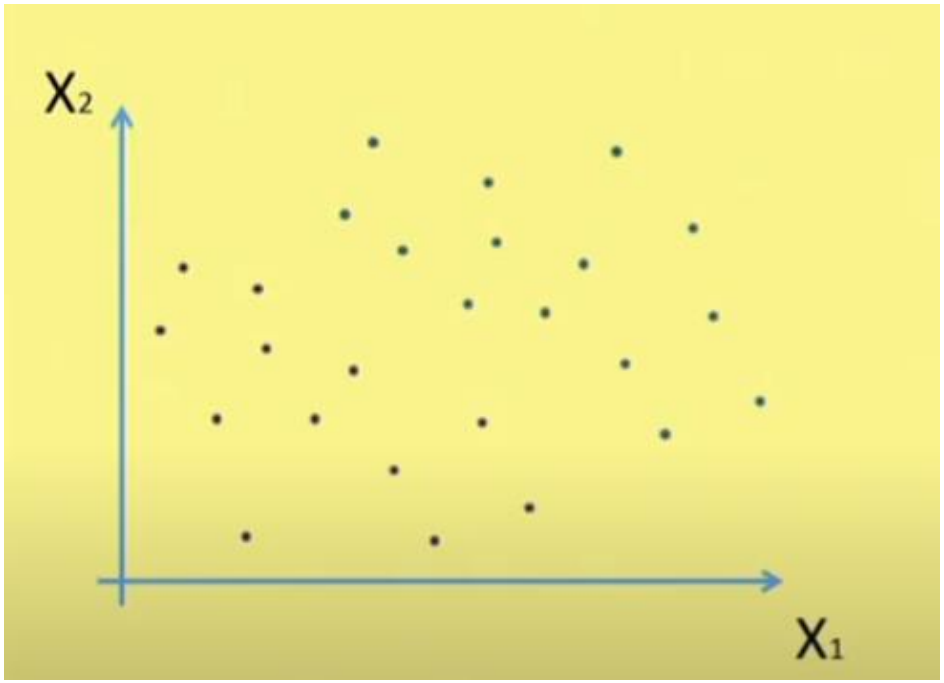




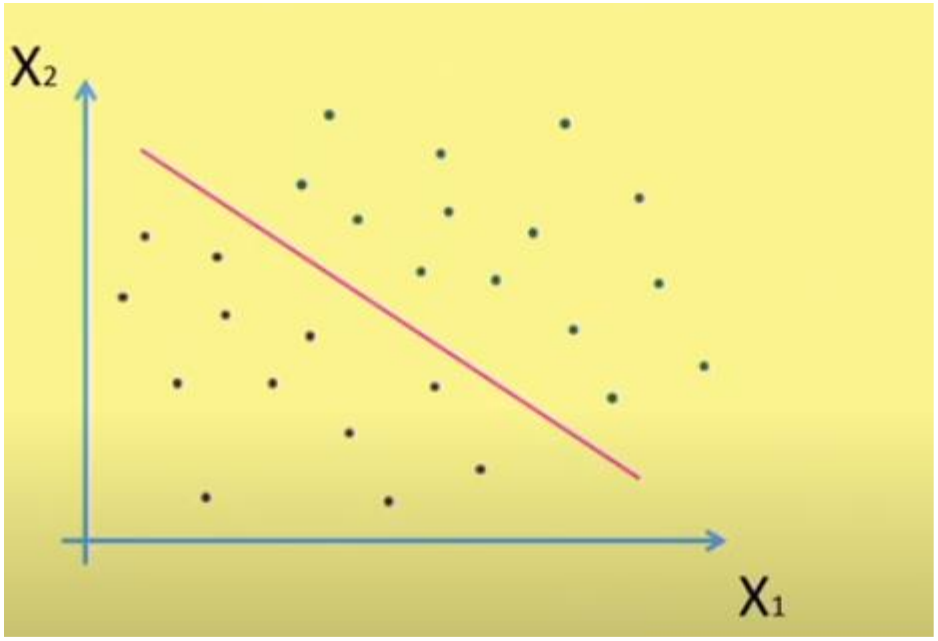
Possible solution for previous problems

- K-NN search
  - Testing: It is time consuming as it should compare the distance with all the sample
  - This methods works well if the number of samples are less
- Linear discriminant function (LDF)
  - It uses sample to estimate the parameter such as **w** and $\mathbf{w_0}$
  - Testing: It just substitute the value of x in g(x) and take the decision
  - If g(x) > 0 then class 1, otherwise class 2

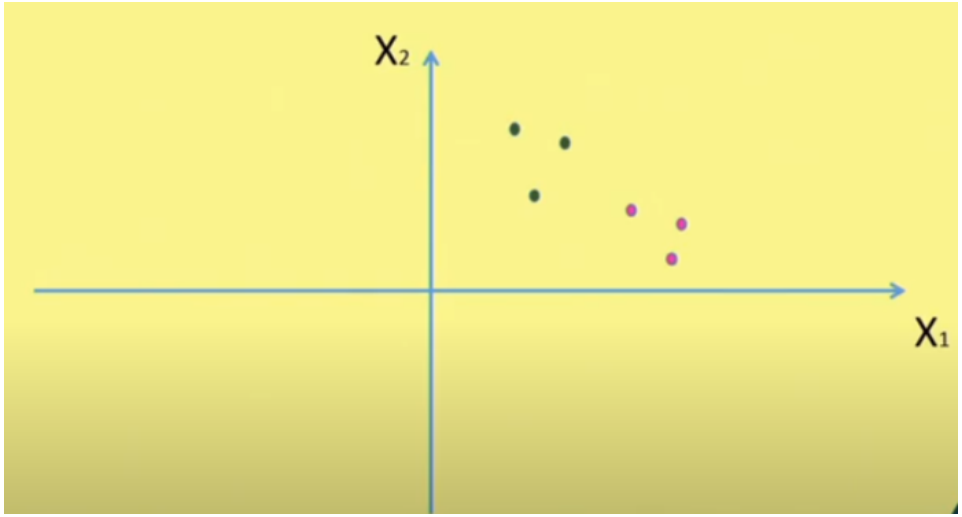
Perceptron: two class problem



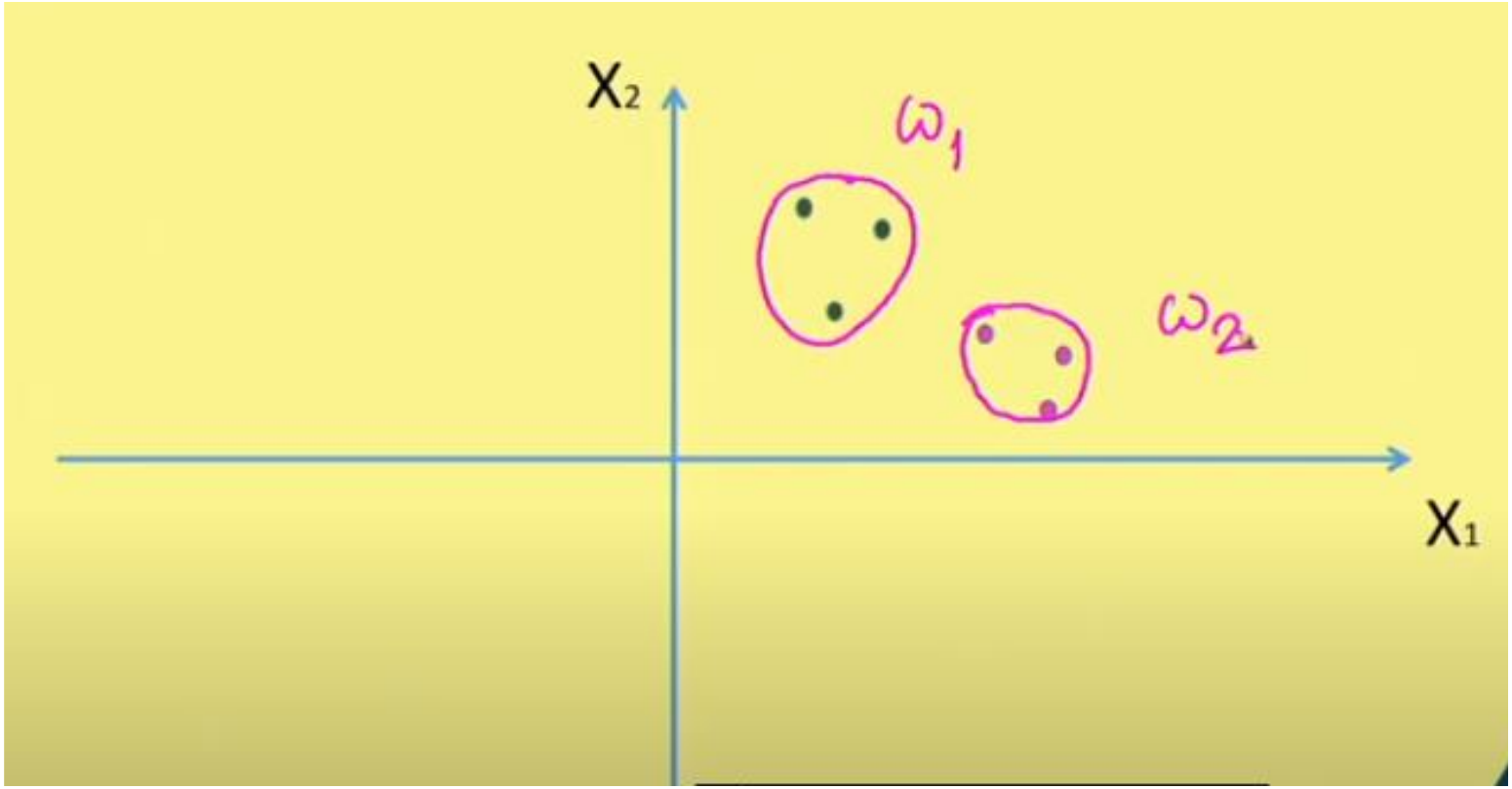
Perceptron - two class problem



Perceptron -two class problem

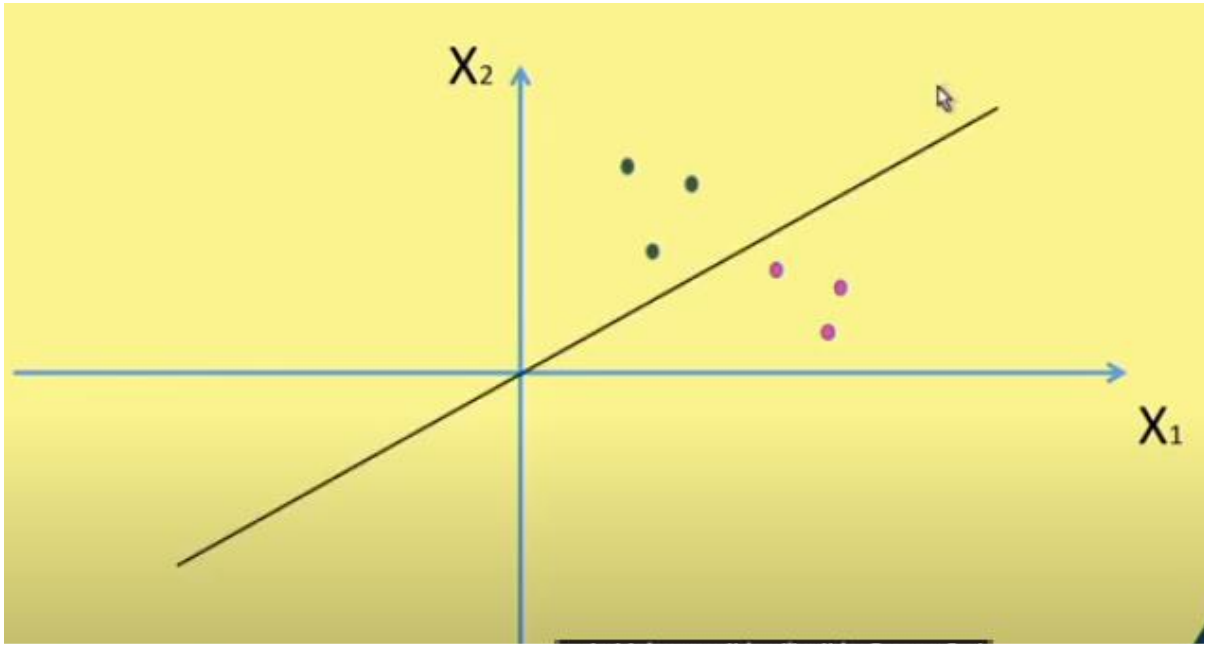


Linear Classifier: Perceptron



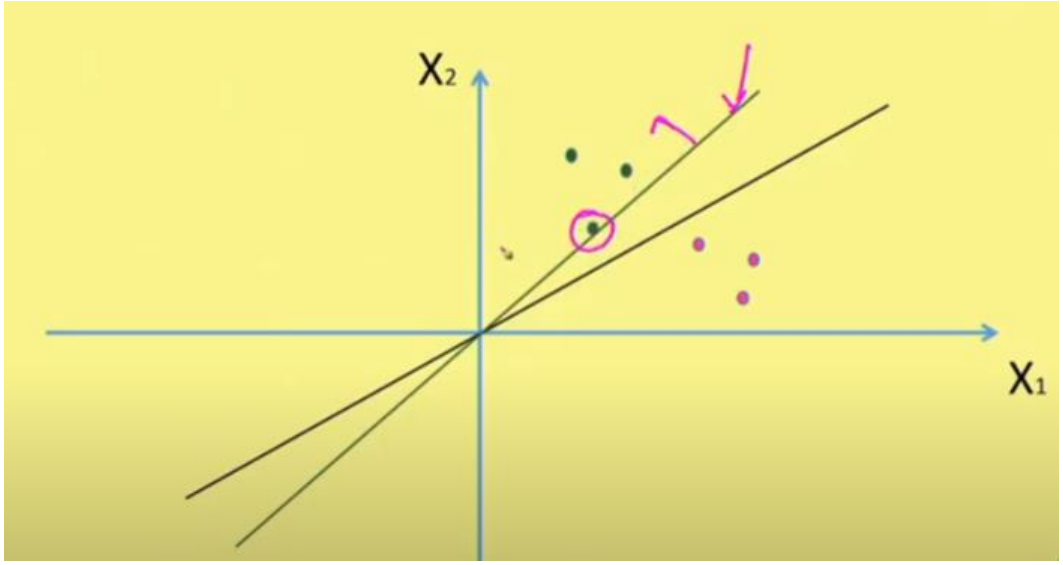
Linear Classifier: Perceptron

- This linear line clearly separates the two classes
- Also, we can see there is a limit of orientation of this particular line (decision boundary)



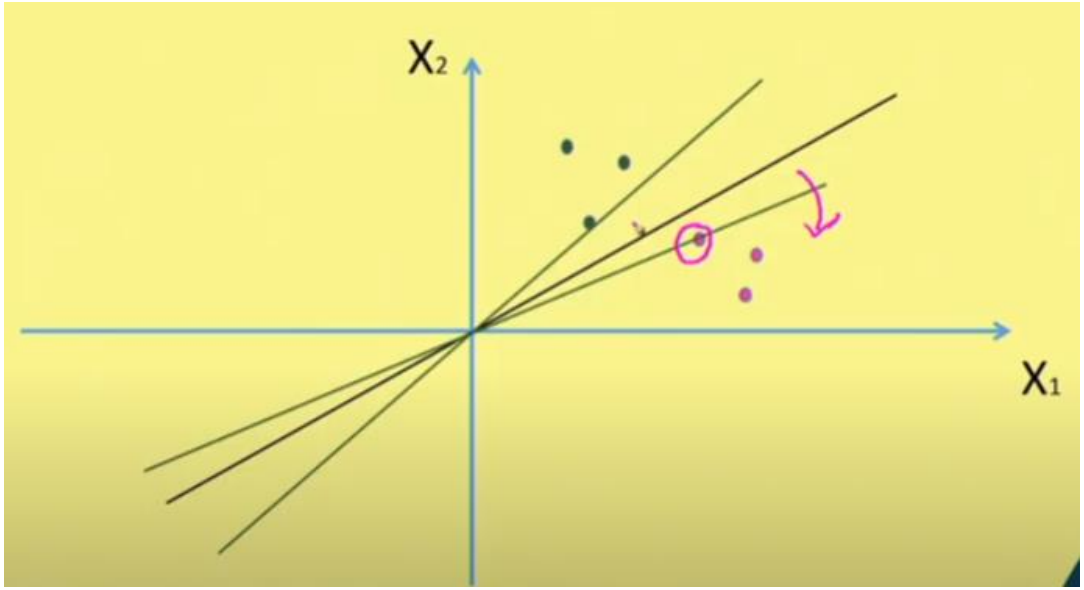
Linear Classifier: Perceptron

- If we rotate further in anti-clock wise direction, the misclassified sample is the one marked as circle.



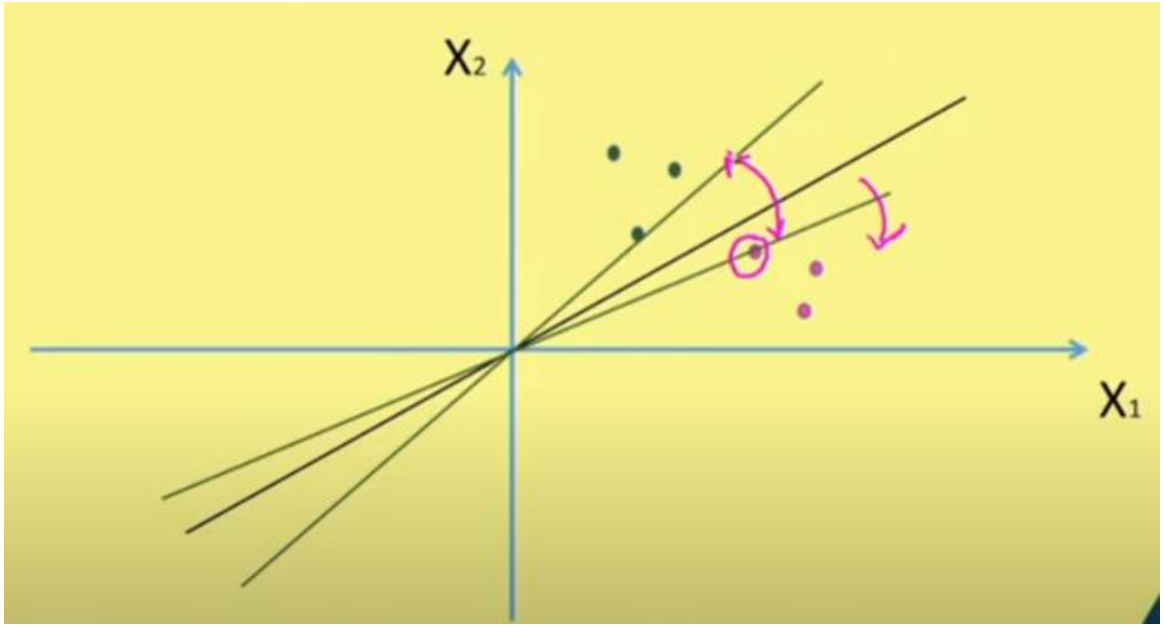
Linear Classifier: Perceptron

- Similarly, if we come to the other side, this is the limit
- If we rotate it further in clock wise direction; the misclassified sample is shown in circle



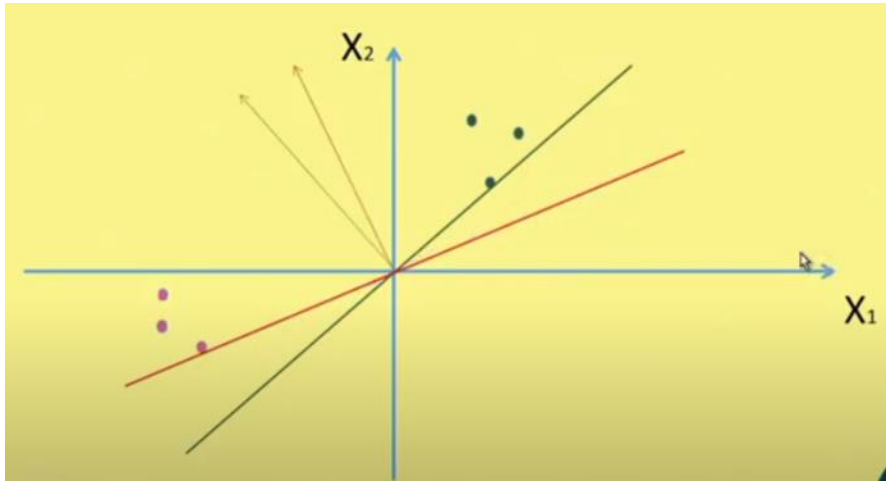
Linear Classifier: Perceptron

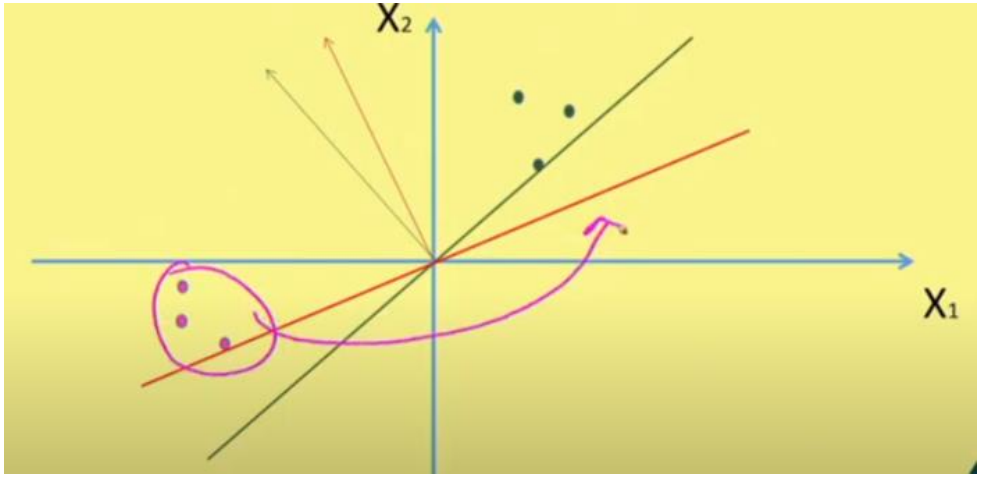
- So the range, that we could have is shown here.



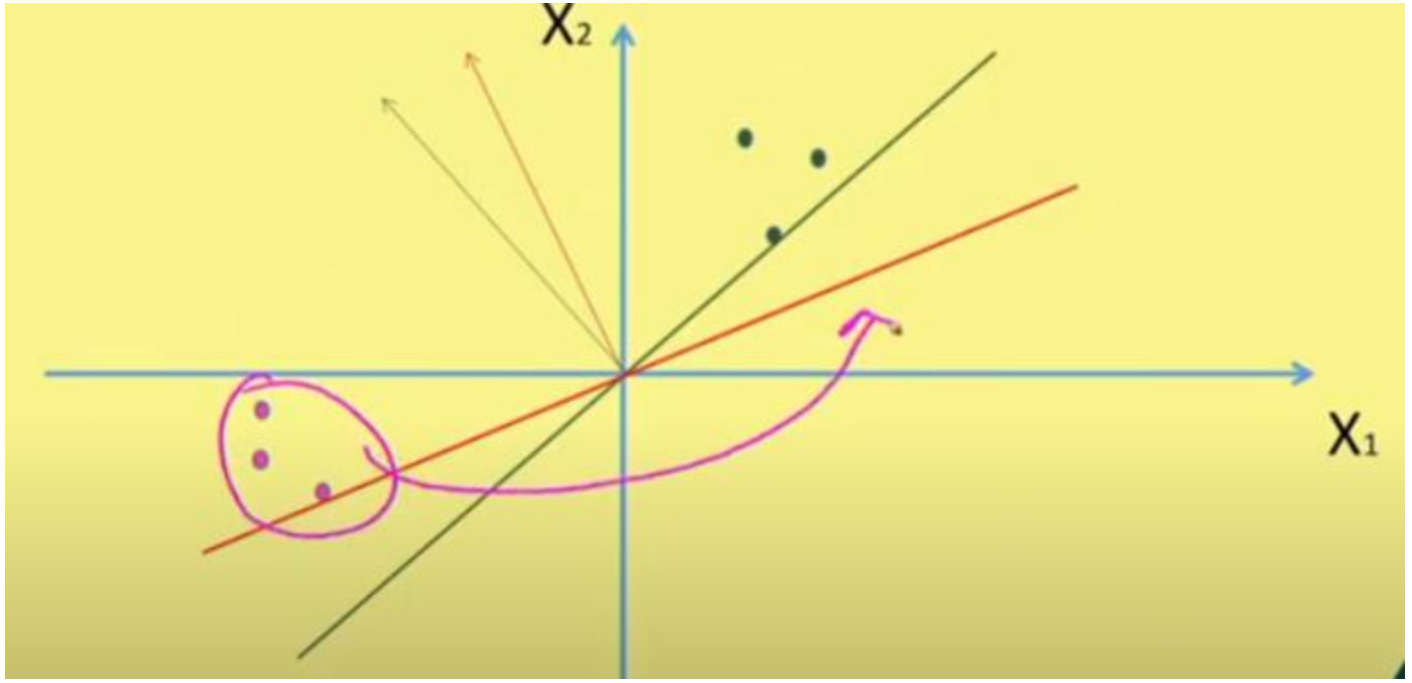
Linear Classifier: Perceptron

- After taking negation of all sample for class 2



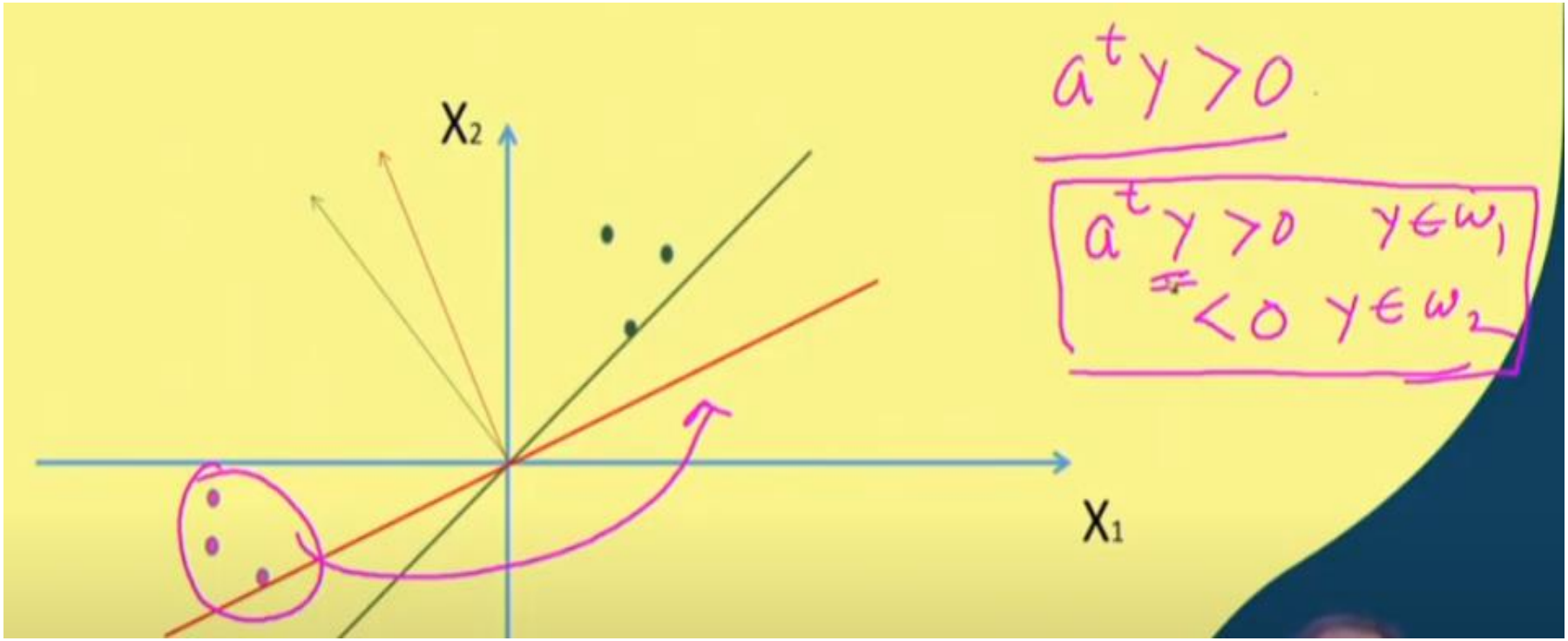


Linear Classifier: Perceptron

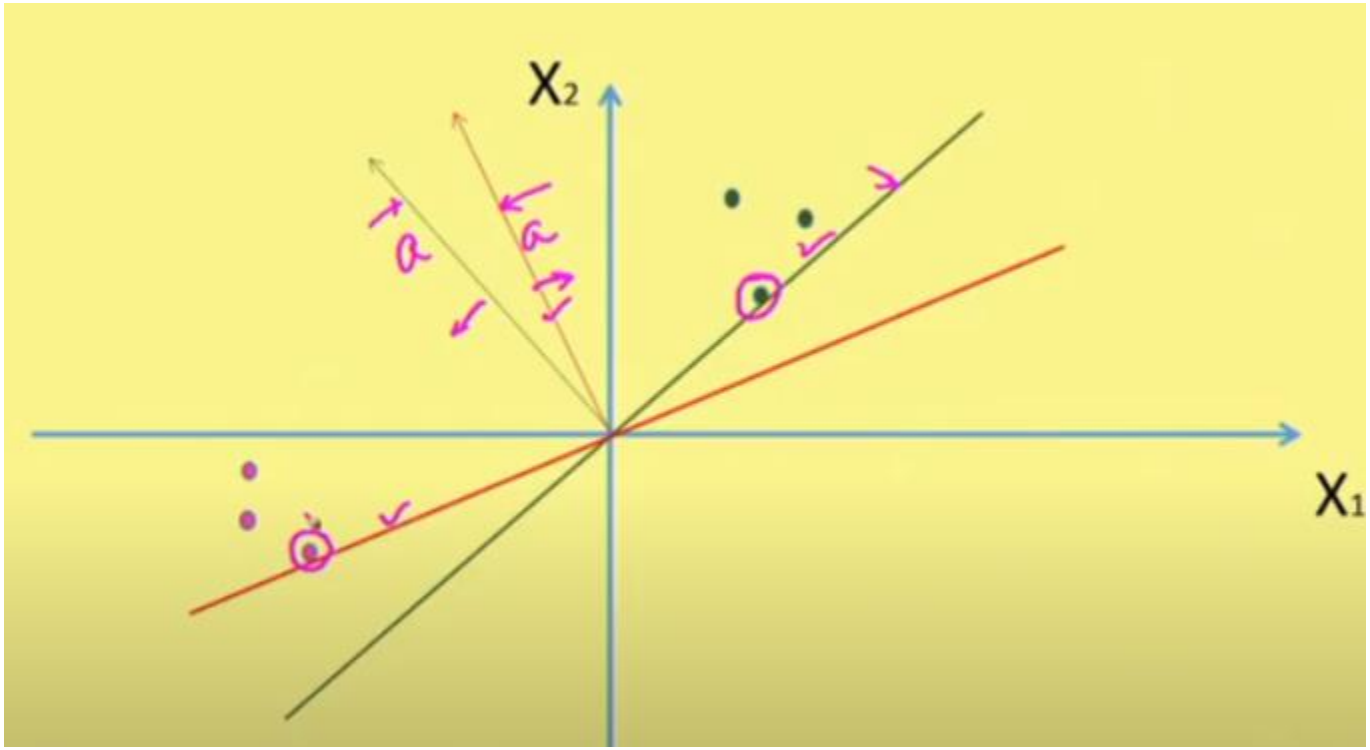


Linear Classifier: Perceptron

- For design purpose alone we use $a^t y > 0$
- For testing we use if $a^t y > 0$, then y is class 1
- If if $a^t y < 0$, then y is class 2

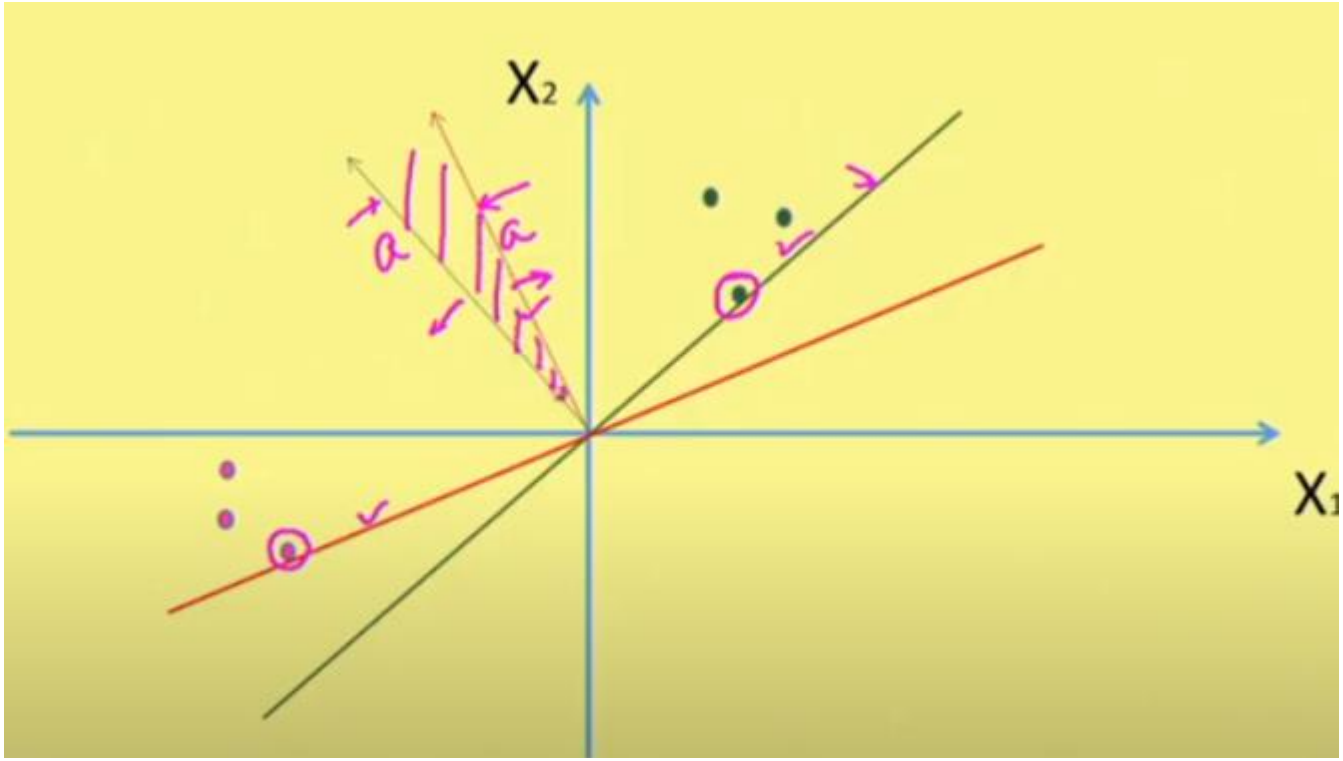


Linear Classifier: Perceptron



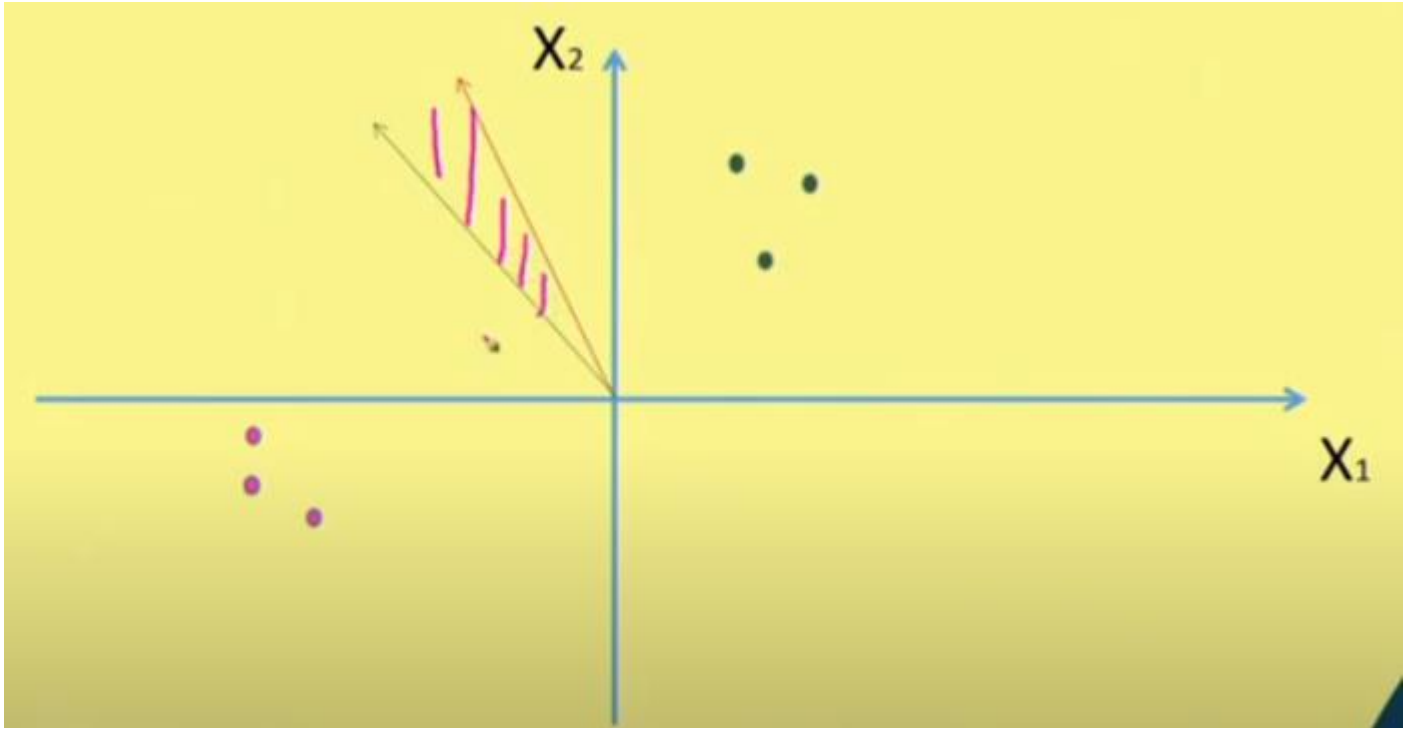
Linear Classifier: Perceptron

- So the solution vector ‘a’ must lie on this solution region.



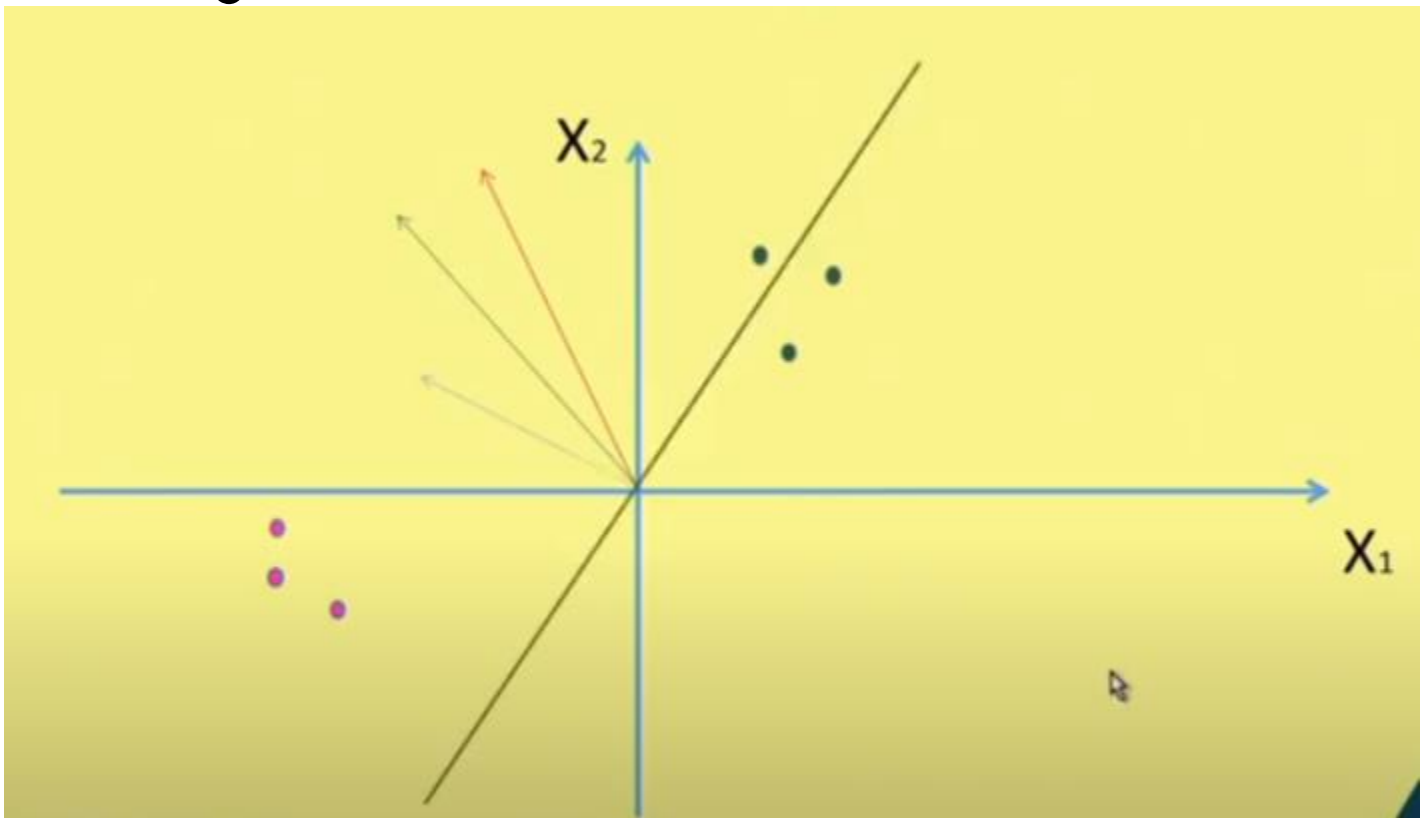
Linear Classifier: Perceptron

- How do we make the solution vector ‘a’ should land in solution region?

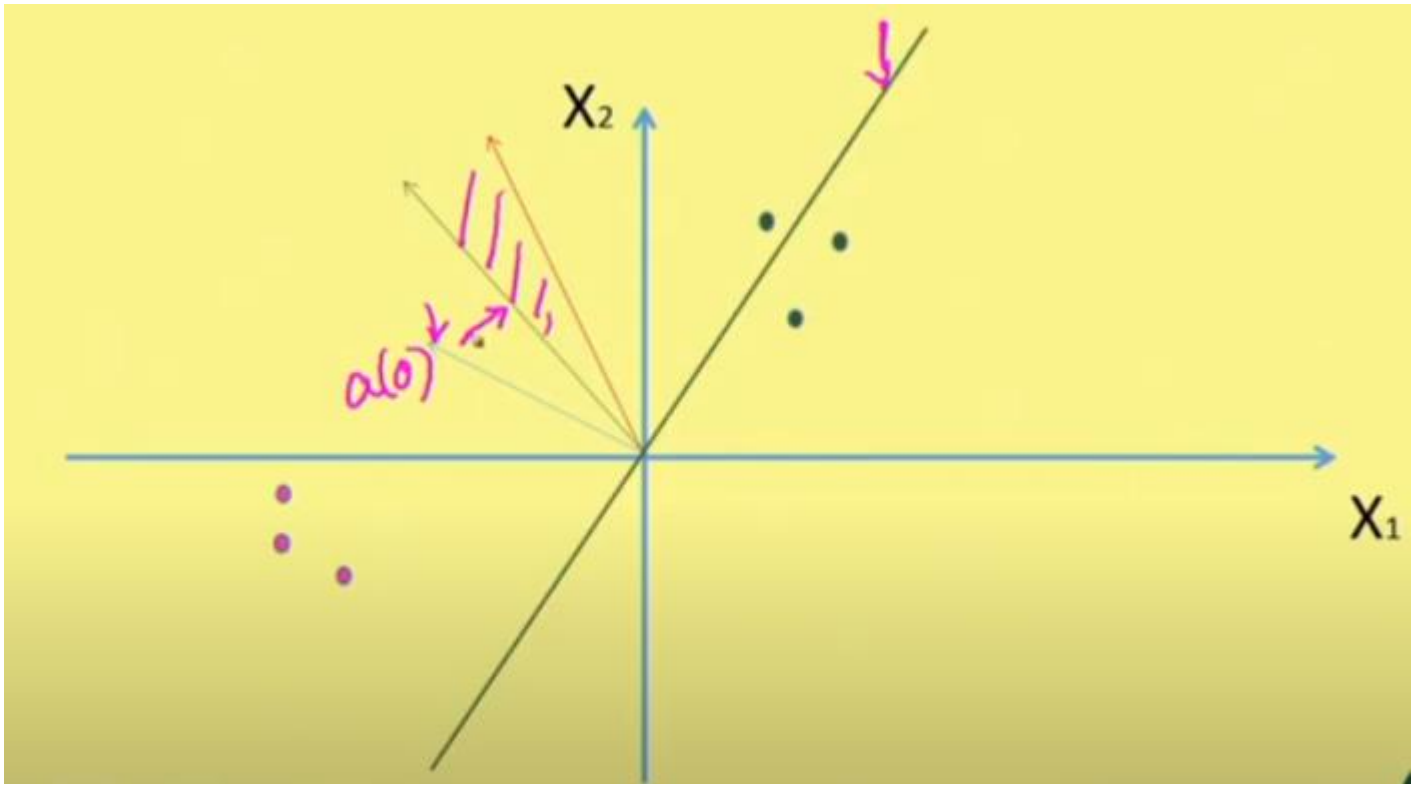


Linear Classifier: Perceptron

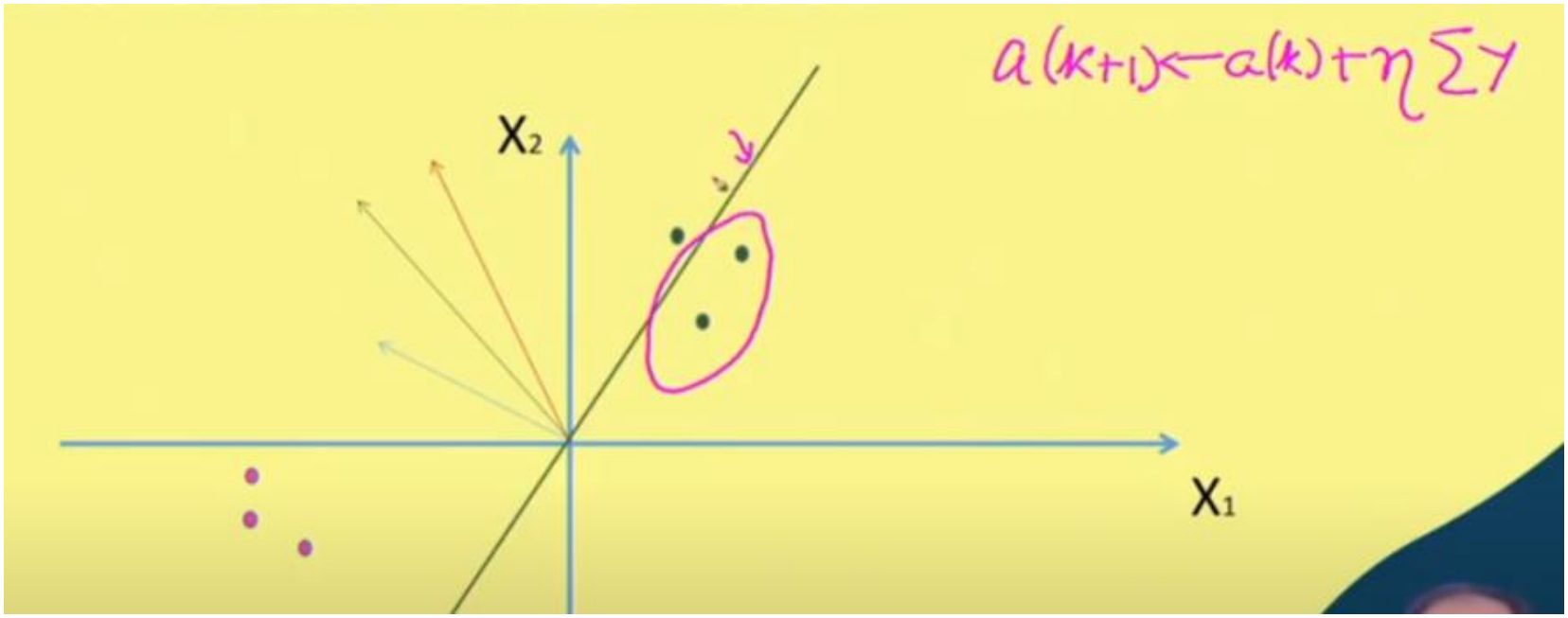
- Suppose, initial weight vector a(0) is chosen something like this



Linear Classifier: Perceptron

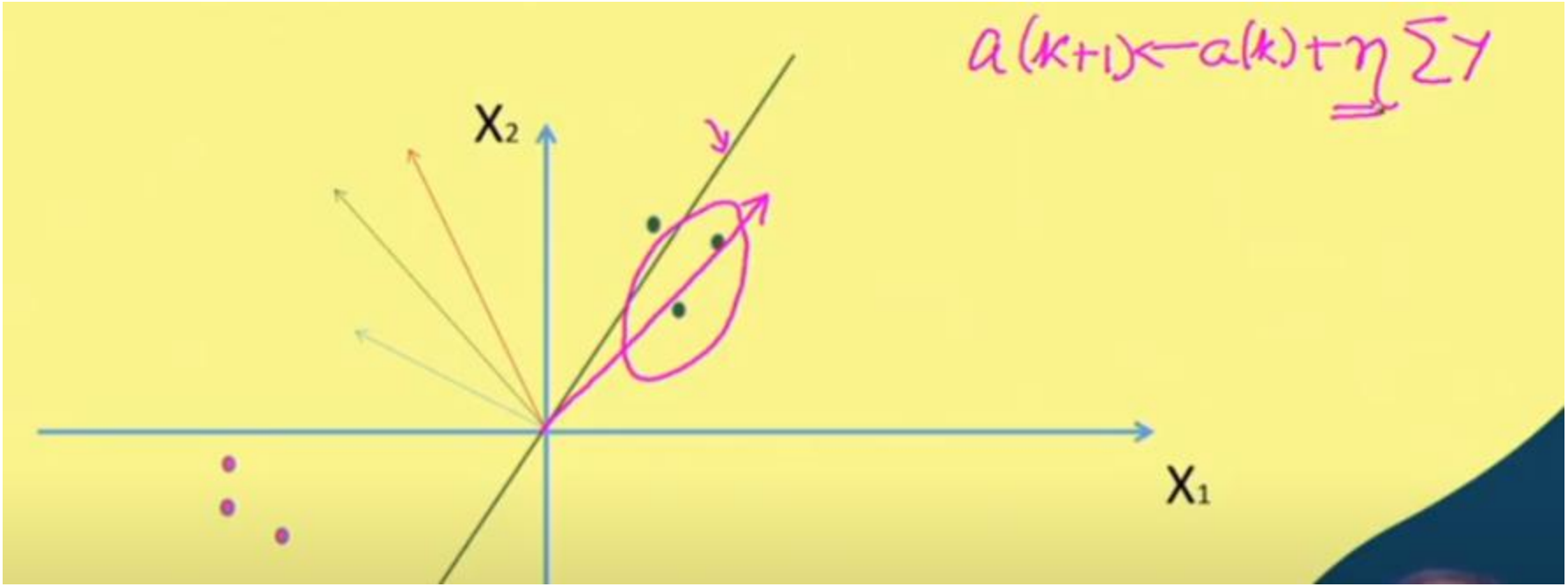


Linear Classifier: Perceptron



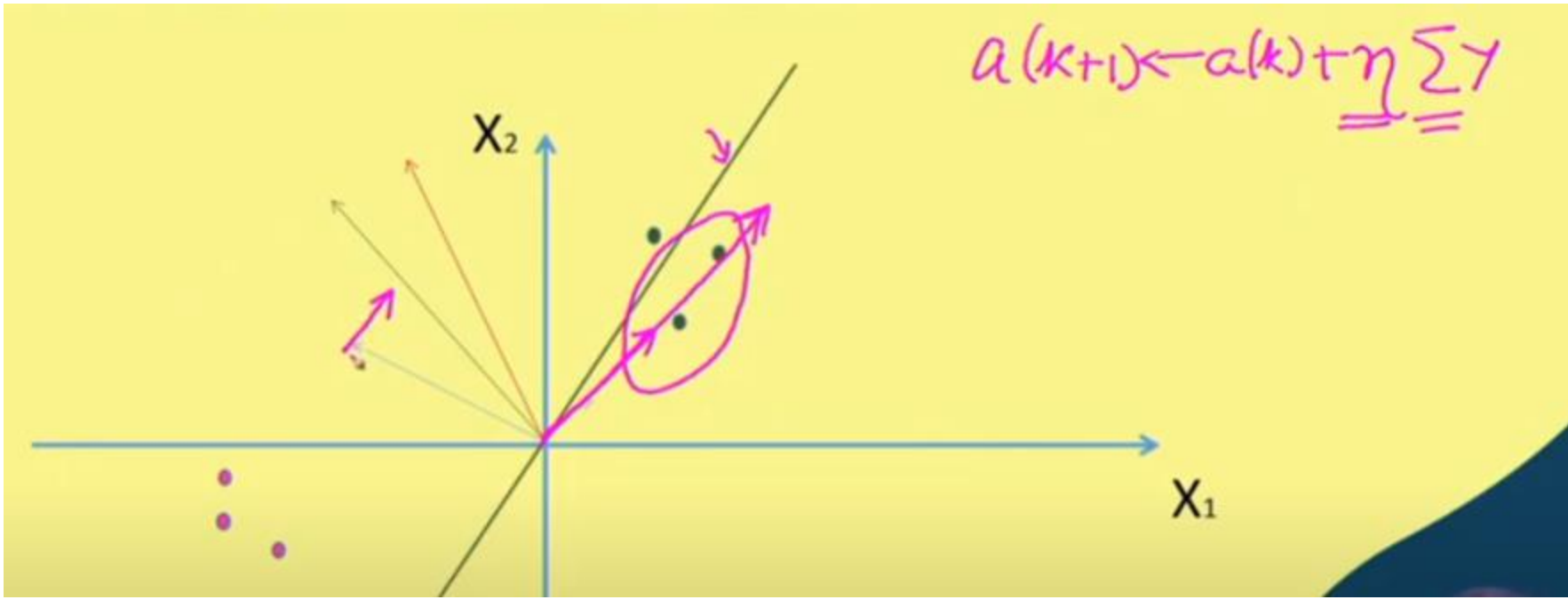
Linear Classifier: Perceptron

- All mis-classified samples are in the direction shown



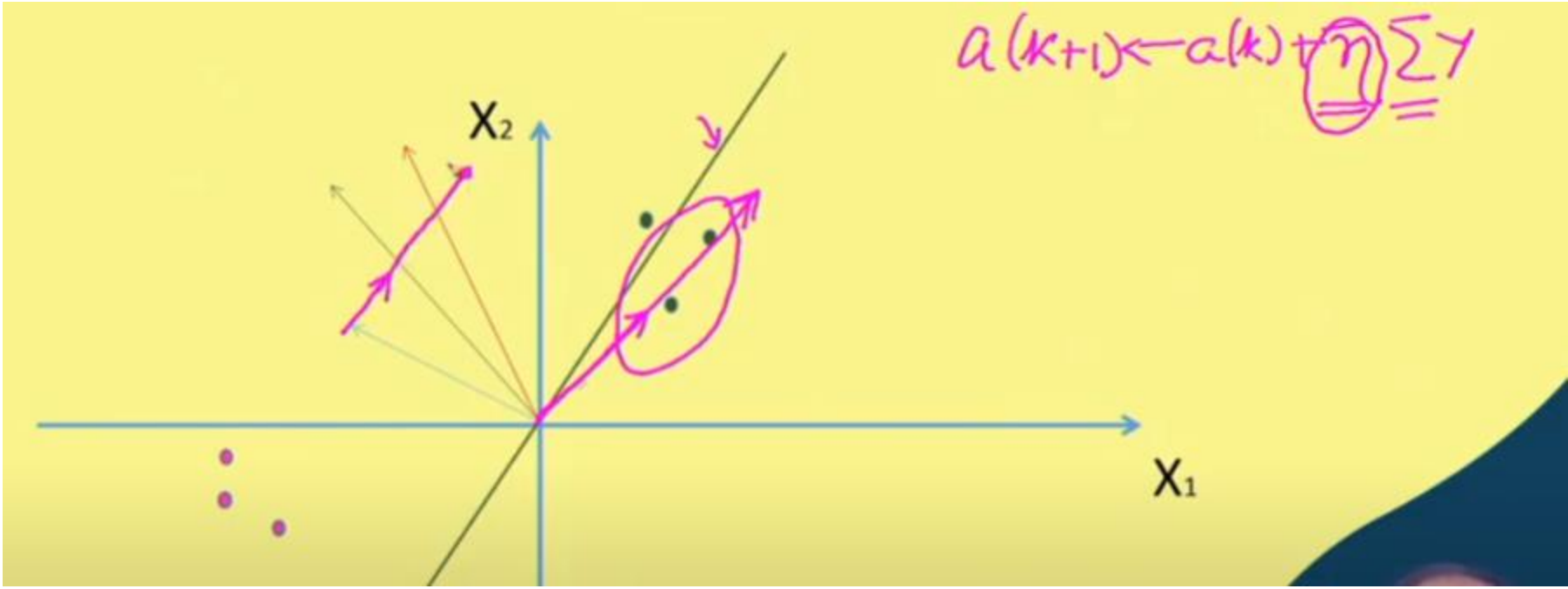
Linear Classifier: Perceptron

- So, a(0) is moved in the direction as shown and slowly it land in the solution region.



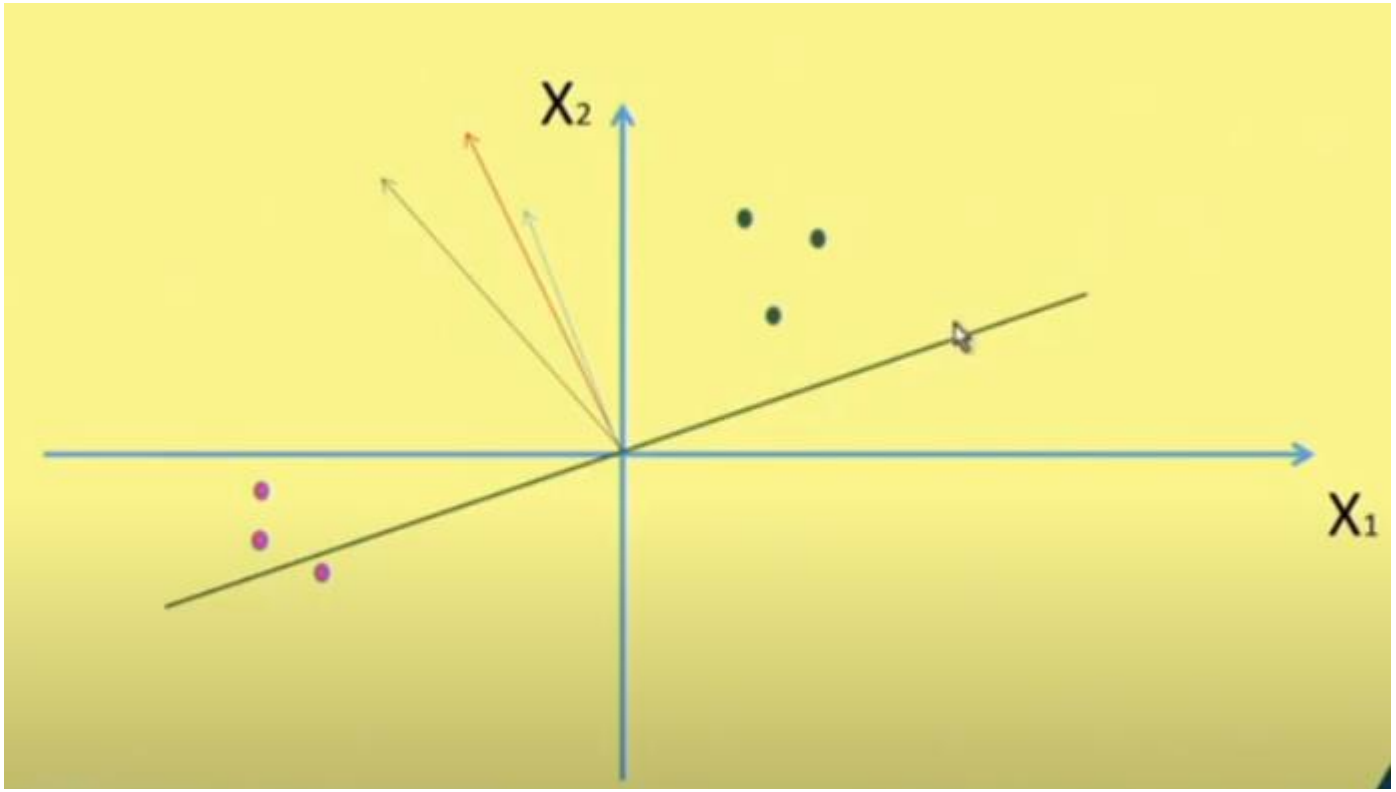
Linear Classifier: Perceptron

- If the learning rate η is very large then solution vector is over shooted.

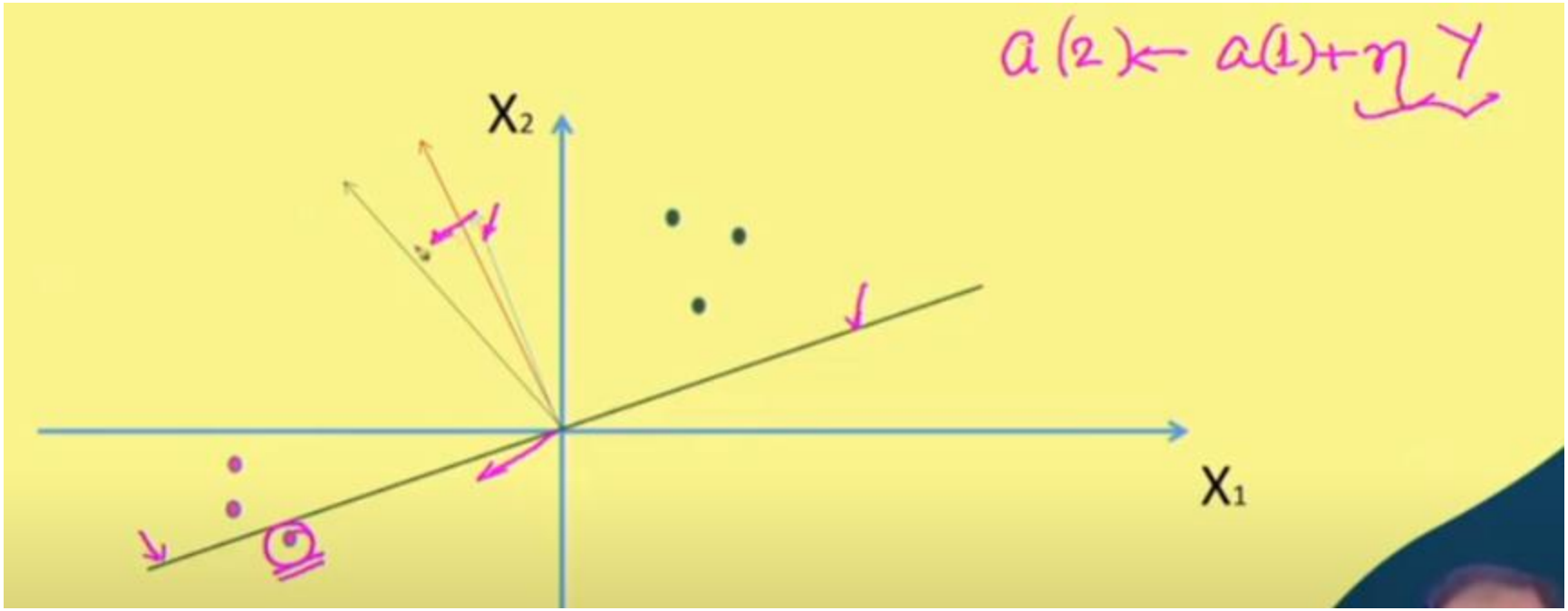


Linear Classifier: Perceptron

- If the learning rate η is very large then solution vector is crossed the solution region.

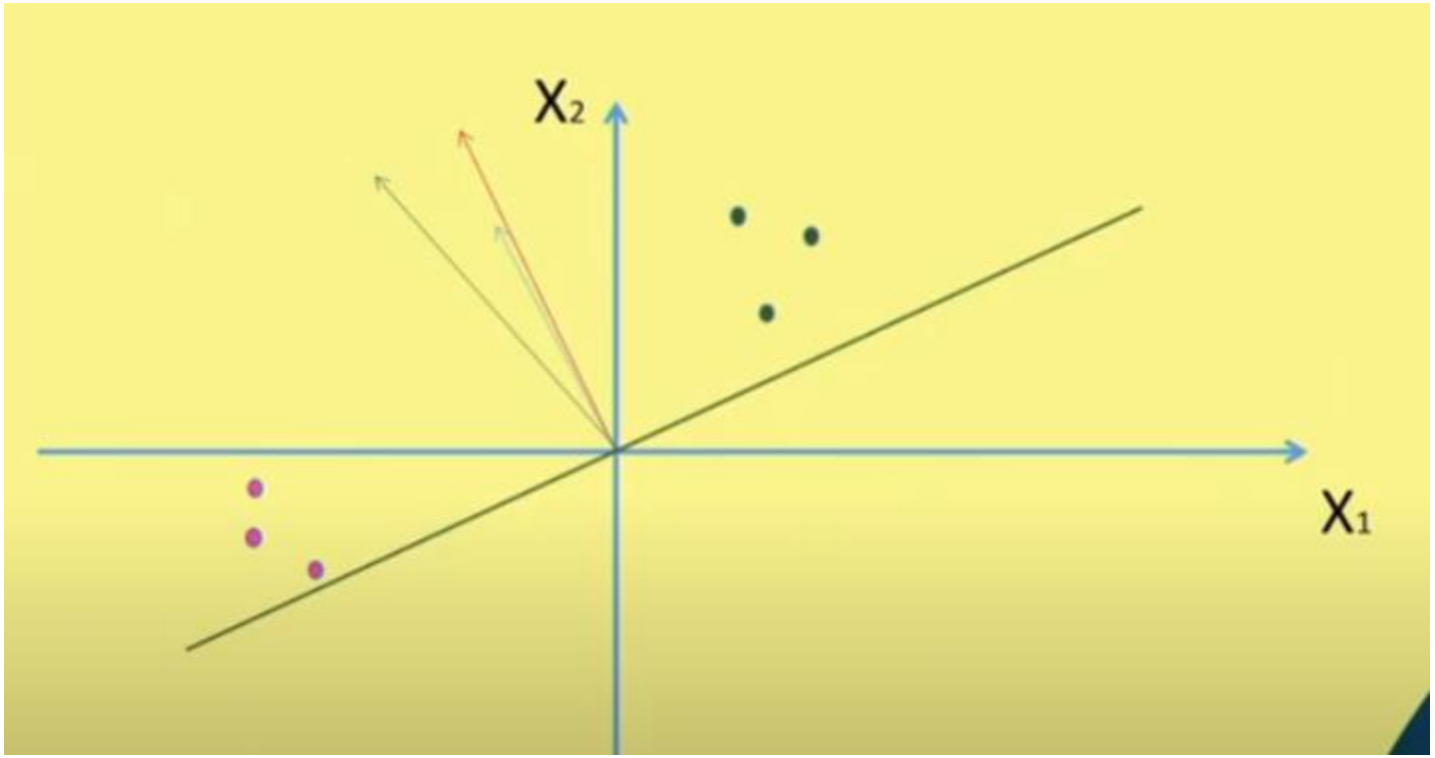


Linear Classifier: Perceptron



Linear Classifier: Perceptron

- This approach clearly says that by taking gradient descent approach it is possible the weight vector converge in the solution region.



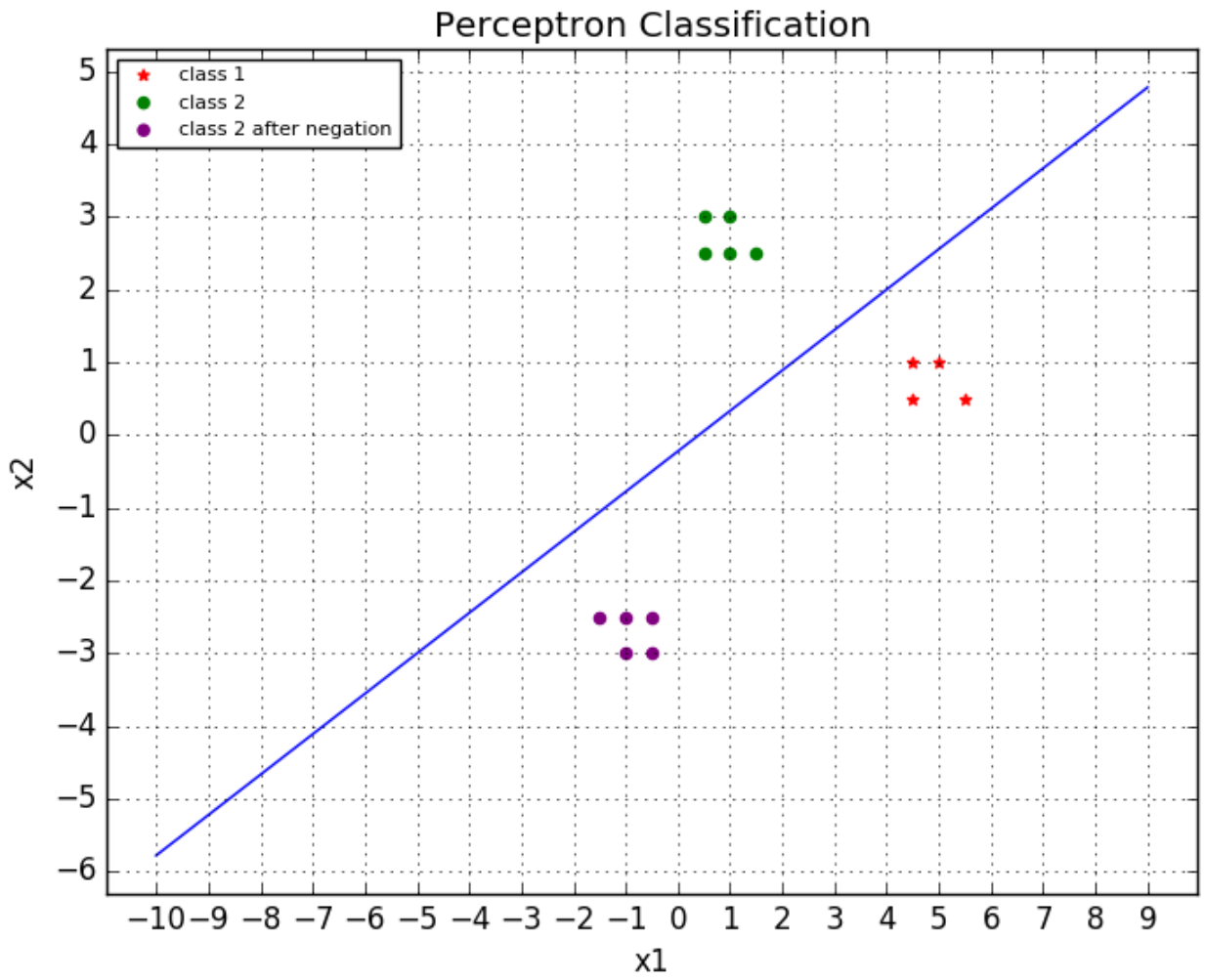
Perceptron criteria

- The approach can be $J(a) = \sum - (a^T y)$
- Using **gradient decent algorithm**
- It works well if the **classes are linearly separable**

MSE criteria

- If the classes are linearly separable, then it is possible to have linear decision boundary like this
- If the classes are not linearly separable then the approach is based on minimum error criteria (Mean Square Error criteria)

Linear Classifier: Perceptron



Summary

- Convergence Procedure of Perceptron Algorithm

Thank you