Patch Antenna Design by using Achievement Scalarization Function with Non-Linear Programming

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Abstract

Achievement scalarization function (ASF) is used to convert a multi-objective problem into a single objective problem. The difference of this method from other scalarization approaches are its perfomance and utilization of the reference point set. To solve this single objective problem, Sequential Quadratic Programming (SQP) is used. This ASF+SQP couple is first implemented to solve two standard benchmark problems ZDT1 and ZDT2 before applying it to get the ideal dimensions of the patch antenna. This method allows researchers to select the optimal substrate dimensions of the patch antenna in accordance with the desired frequency.

1 Introduction

1.1 Multi-Objective Problem

Multi-objective optimization or Pareto optimization is an area of multiple-criteria decision making that is concerned with more than one objective function to be optimized simultaneously. For a multi-objective function, it is not guaranteed that a single solution simultaneously optimizes each objective. The objective functions are said to be conflicting. A solution is called Pareto optimal if none of the objective functions can be improved in value without degrading some of the other objective values.

For a set of objective functions $f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x})$, the multi-objective problem is defined as:

$$\min \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})\}$$

An Achievement Scalarization Function is used to convert this multi-objective function into a single objective function. There are many types of Achievement Scalarizing functions.

1. Weighted Sum Scalarization $F(\mathbf{x}) = \sum_{i=1}^{M} w_i \cdot f_i(\mathbf{x})$
2. Weighted Product Scalarization $F(\mathbf{x}) = \Pi_{i=1}^{M} w_i \cdot f_i(\mathbf{x})$
3. Chebyshev Scalarization $F(\mathbf{x}) = \max_{i=1}^{M}\{\frac{f_i(\mathbf{x})-z_i^*}{w_i}\}, ||w|| = 1$

In all the above cases, $F(\mathbf{x})$ is the single required objective function and $w_i, i = 0, 1, \ldots, M$ with $w_i \geq 0$ are the weights and $z_i, i = 0, 1, \ldots, M$ are the ideal values for the i-th objective function. In this paper, **Chebyshev Scalarization** is used to solve the Multi-objective problem as it accounts for the ideal values. But, using this method introduces a constraint which is added to the set of decision variables.[1]

$$\frac{f_i(x)-z_i}{w_i} \leq \alpha, i = 1, 2, \ldots, M$$

α is added to the set of decision variables, $\mathbf{x}$ so as to minimize its value.

1.2 Single-Objective Problem

To solve this single objective problem, **Sequential Quadratic Programming** (SQP) is used. SQP is an iterative method in constrained non-linear optimization. It is used when the constraints of the problem are twice continuously differentiable but not necessarily convex. The minimum problem is given below:

$$\begin{gathered}\min_{\mathbf{x}} f(\mathbf{x}) \text{ (Optimization problem)} \\ g_i(\mathbf{x}) \leq 0, i = 1, 2, \ldots, n_e \text{ (Inequality constraints)} \\ h_i(\mathbf{x}) = 0, i = 1, 2, \ldots, n_e \text{ (Equality constraints)} \\ x_L \leq x_i \leq x_U, i = 1, 2, \ldots, N \text{ (Bounds for } \mathbf{x}\text{)}\end{gathered}$$

All the linear and non-linear parts of the problem are assumed to be differentiable. The difference of this algorithm is based on the linearization of the non-linear constraints and approximation of the Lagrangian. The idea of SQP is dependent on formulating sub-quadratic problems (QPs) by using approximated constraints and Lagrangian. The quadratic problem is given below:

$$\begin{gathered}\min\{\frac{d^T B_k d}{2} + \nabla f(\mathbf{x}_k)^T d\} \\ \nabla g_i(\mathbf{x}_k)^T d + g_i(\mathbf{x}_k) = 0 \\ \nabla h_i(\mathbf{x}_k)^T d + h_i(\mathbf{x}_k) \geq 0\end{gathered}$$

where B is the approximation of the Hessian of the Lagrangian, d is the direction vector. The algorithm is given as:

1. Formulate the QP problem by using the Hessian Matrix
2. Choose step length α by using one dimensional line search (Ex: Newton's method: $\alpha = -\frac{\nabla f}{\nabla^2 f}$)
3. Update solution $\mathbf{x}$ and Lagrangian multiplier vectors by $\mathbf{x}_{k+1} = \mathbf{x} + \alpha d$
4. Repeat the process by $k = k + 1$

Finally, SQP is used to solve this objective problem.

2 Application

2.1 Benchmarks

The above mentioned algorithm is applied to two standard benchmark problems: Zitzler-Deb-Thiele (ZDT) 1, ZDT2. ZDT is a benchmark problem set used in the field of multi-objective optimization problems to evaluate the perfomance of multi-objective optimization algorithms[2]. The ZDT set contains six problems out of which two problems are selected to evaluate the perfomance of the above discussed algorithm. The characteristics of the two problems are given below:

1. ZDT1: non-linear problem with two objective functions, has a convex Pareto front
2. ZDT2: non-linear problem with two objective functions, has a concave Pareto front

The problems ZDT1, ZDT2 are described as follows:

ZDT1

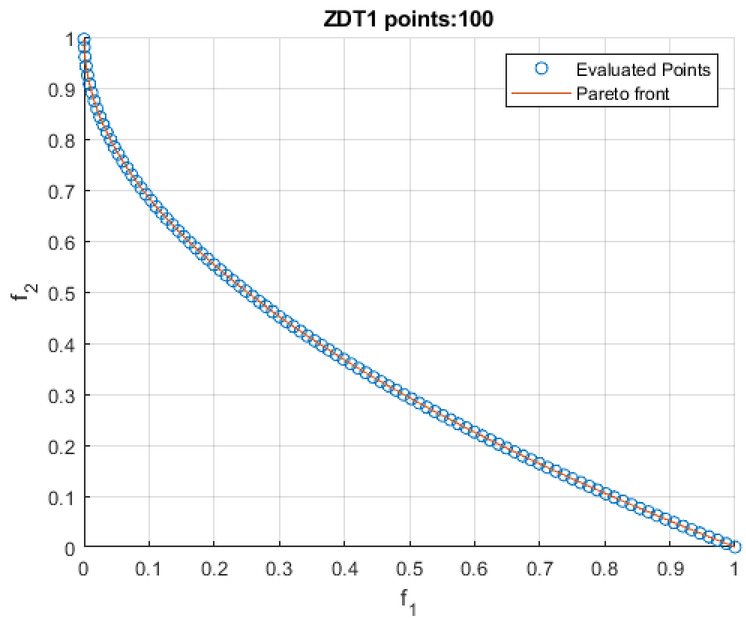
$$\textbf{Minimize } f_1 = x_1$$
$$\textbf{Minimize } f_2 = g(1 - \sqrt{\tfrac{f_1}{g}})$$
$$g = (1 + \textstyle\sum_{i=2}^{N} x_i)$$

ZDT2

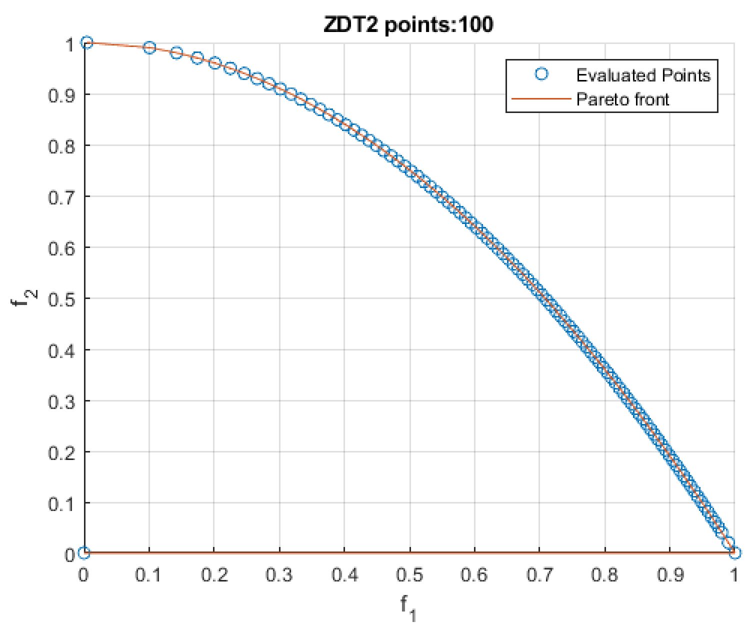
$$\textbf{Minimize } f_1 = x_1$$
$$\textbf{Minimize } f_2 = g(1 - (\tfrac{f_1}{g})^2)$$
$$g = (1 + \textstyle\sum_{i=2}^{N} x_i)$$

where N is the number of variables in the vector $\mathbf{x}$ and f_1, f_2 are the objective functions to. While ZDT1/ZDT2 leave $\mathbf{x}$ unconstrained, we choose the bounds of $\mathbf{x}$ to be $[0, 1]$

Upon implementing the algorithm and testing it on ZDT1, a convex Pareto front is obtained as shown below.



Upon implementing the algorithm and testing it on ZDT2, a concave Pareto front is obtained as shown below.



3 Patch Antenna

A Patch antenna is a type of antenna with a low profile which can be mounted on a surface. It consists of a planar rectangular, circular, triangular, or any geometrical sheet or ”patch” of metal mounted over a larger sheet of metal called ground plane. They are the original type of microstrip antenna described in 1972. The two metal sheets together form a resonant piece of microstrip transmission line with a length of approximately one-half wavelength of the radio waves. It has a number of advantages over other antennas: lightweight, inexpensive and integrabilibity.[3]

One important parameter of an antenna is the bandwidth it covers. For a microstrip antenna, the bandwidth depends on the dimensions of the patch — mainly the height. To make the antenna work at a specific resonance frequency, the dimensions of the antenna should be adjusted. The desired frequency is related to its dimensions by the following relations:

$$f_r = \frac{c_0}{2(L+\Delta W)\sqrt{\epsilon_e(W)}}$$
$$\epsilon_e(W) = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2(1+\frac{10h}{W})^{0.5}}$$
$$\Delta W = 0.412h\frac{(\epsilon_e(W)+0.3)(W/h+0.264)}{(\epsilon_e(W)-0.258)(W/h+0.813)}$$

where f_r is the calculated frequency, l is the length of the patch, W is the width of the patch, h is the thickness of the patch, ϵ is the dielectric constant of the medium, c_0 is the speed of light in vacuum, $\epsilon_e(W)$ is the effective dielectric constant.

3.1 Problem definition

Since we are aiming to optimize the dimensions of the rectangular patch antennas for a desired resonance frequency, the first objective function is defined as follows:

$$\textbf{Minimize } f_1 = |f_{r_{desired}} - f_{r_{calculated}}|$$

The second objective is the height of the antenna which is desired to be minimized in order to reduce substrate costs.

$$\textbf{Minimize } f_2 = h$$

3.2 Results

The following antenna configurations were used to determine the dimensions of the antenna.

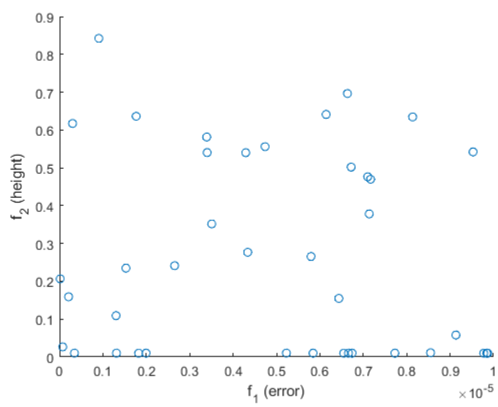
Table 1: Different antenna configurations

Ant.	$f_v(GHz)$	ϵ_r
1	6.2	2.55
2	8.45	2.22
3	7.74	2.22
4	3.97	2.22
5	5.06	2.33

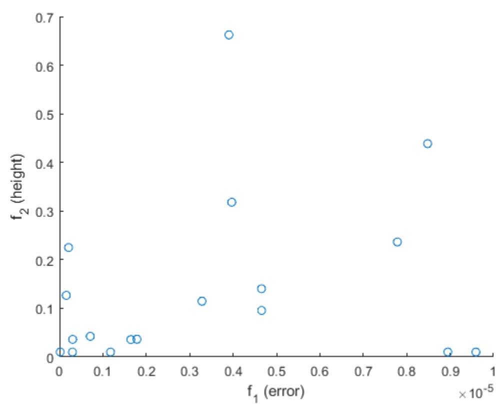
Finally, after applying the implemented algorithm on 100 reference points, the optimal results obtained are shown in table 3. The error values and the dimensions given by this implementation match with those of the original paper.

Table 2: Obtained dimensions for antenna (1) using a reference point set

length (mm)	width (mm)	height (mm)
15.2173	4.0069	0.0299
15.8135	1.0277	0.1006
15.7278	2.2833	0.2085
15.3896	2.5741	0.0724
15.4305	5.8117	0.2701
16.0000	6.5000	0.3438
15.1765	3.6514	0.0100
15.1756	3.7474	0.0100
16.0000	1.8500	0.0100



(a) The values of objective functions for antenna (1)



(b) The values of objective functions for antenna (2)

Table 3: Obtained optimal dimensions for all the antenna configurations in Table 1

Ant. No.	length (mm)	width (mm)	height (mm)	Error
1	15.3660	23.2010	0.0100	5.1e-8
2	11.9422	17.8914	0.0100	5.33e-7
3	13.0150	6.6325	0.0100	1.71e-6
4	25.0000	25.000	0.0573	5.8e-2
5	19.4751	2.2548	0.0100	2.22e-7

4 Conclusion

In this study, the application of Achievement Scalarization Function (ASF) with Sequential Quadratic Programming (SQP) to address the challenge of optimizing patch antenna design is studied. ASF has proven to be a valuable tool for transforming a multi-objective problem into a single objective function which allows efficient solution strategy. The implementation of ASF+SQP was validated by applying it on two standard benchmark problems, ZDT1 and ZDT2. One notable advantage of ASF is its incorporation of a reference point set, which allows a better way of exploring the solution space. The focus on determining the ideal substrate dimensions for the patch antenna is an example of the practical utility of this technique, particularly in customizing antenna characteristics to meet desired frequency specifications. In conclusion, the combination of ASF and SQP is an effective approach for solving optimization problems in antenna design.

References

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[3] Indrasen Singh, VS Tripathi, et al. "Micro strip patch antenna and its applications: a survey". In: *Int. J. Comp. Tech. Appl* 2.5 (2011), pp. 1595–1599.