DM systems are subjected to 2 types of quantizing error -

1) slope - overload distortion.

2) Granular noise.

let $q(nT_S)$ denote the quantizing error.

from DPCM WKT i/p to prediction filter is

$$u(nT_S) = x(nT_S) + q(nT_S) \quad \text{—} (1)$$

from DM WKT ⇒

$$e(nT_S) = x(nT_S) - u(nT_S - T_S) \quad \text{—} (2)$$

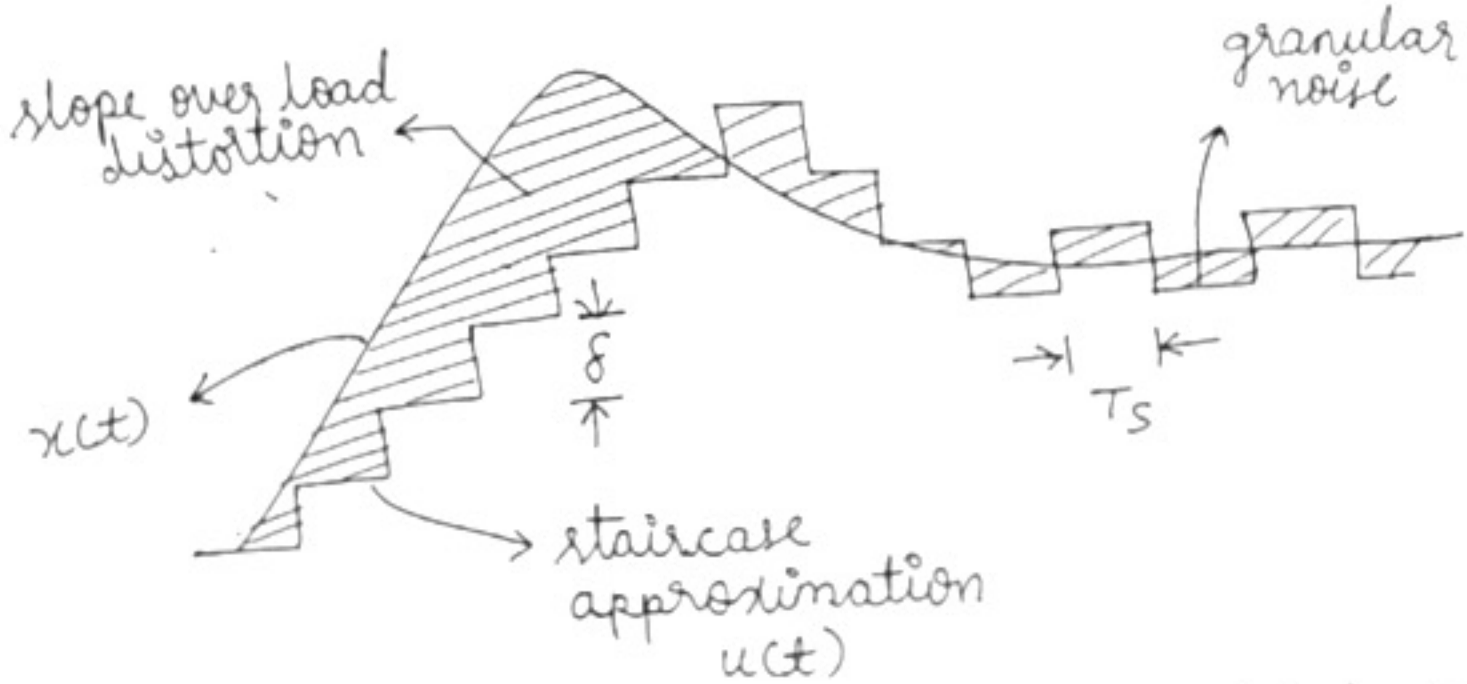
Put (1) in (2) ⇒

$$e(nT_S) = x(nT_S) - x(nT_S - T_S) - q(nT_S - T_S) \quad \text{—} (3)$$

Thus except for quantization error $q(nT_S - T_S)$, the quantizer i/p is a first backward difference of i/p signal, which may be viewed as a digital approximation to the derivative of the i/p signal.

if we consider the maximum slope of original i/p waveform $x(t)$, it is clear that in order for sequence of samples $u(nT_S)$ to increase as fast as the input sequence of samples $x(nT_S)$ in a region of maximum slope of $x(t)$, we require the condition below to be satisfied ⇒

$$\frac{\delta}{T_S} \geq \max \left| \frac{dx(t)}{dt} \right| \quad \text{—} (4)$$



The staircase approximation $u(t)$ falls behind $x(t)$ because the step size δ is too small to trace the i/p signal $x(t)$. This condition is called slope-overload distortion – quantisation error.

"If the step size δ is too large relative to the local slope characteristics of i/p waveform $x(t)$, it causes the staircase approximation $u(t)$ to hunt around a relatively flat segment of the i/p waveform & this phenomenon is called as granular noise."

Thus we require a large step size to accomodate a wide dynamic range, where as a small step size is required for the accurate representation of relatively low-level signals.

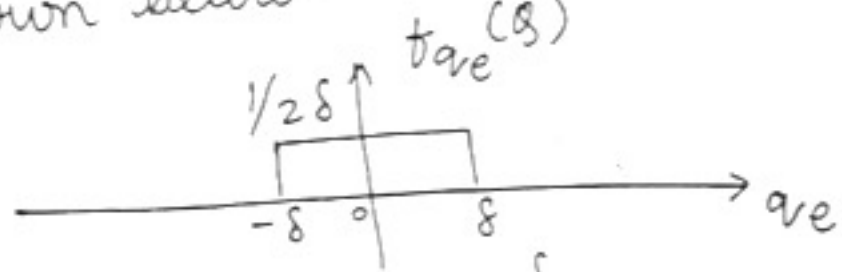
∴ choice of optimum step size is very important to minimize the quantizing error in a linear delta modulator which is compramise b/w slope overload distortion & granular noise.

If slope overload distortion is not there, then max^m quantisation error is $\pm\delta$.

$\therefore$ let this be uniformly distributed with variance $\frac{\delta^2}{3}$. At the receiver, a LPF is designed with a bandwidth 'w' Hz, ($f_m \leq w$)

$$\therefore \text{(Avg o/p noise power)} = \frac{w}{f_s} \times \frac{\delta^2}{3}$$

In delta modulation (DM) the quantisation error varies between $[-\delta$ & $\delta]$ & its pdf is as shown below-



$$\text{variance } \sigma_{q_e}^2 = \int_{-\delta}^{\delta} q_e^2 \, f_{q_e}(q) \, dq_e$$

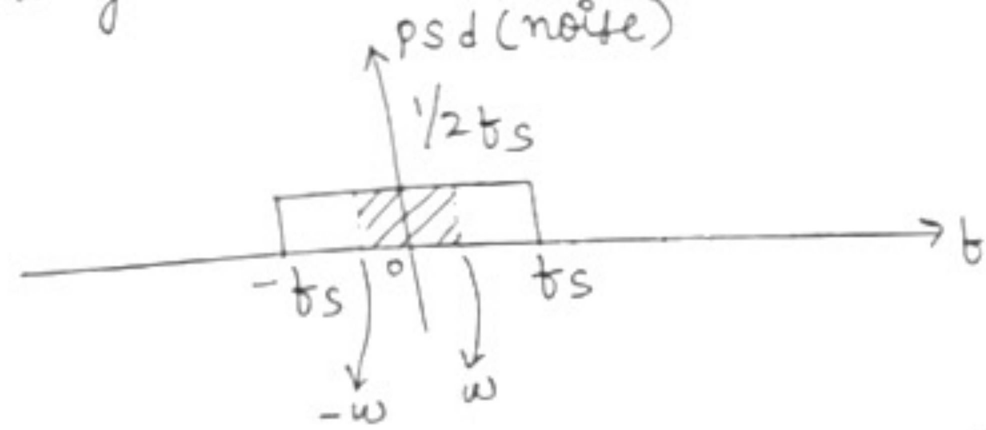
$$\sigma_{q_e}^2 = \frac{1}{2\delta} \left. \frac{q_e^3}{3} \right|_{-\delta}^{\delta}$$

$$\sigma_{q_e}^2 = \frac{1}{2\delta}\left[\frac{\delta^3}{3} - \left(\frac{-\delta^3}{3}\right)\right]$$

$$\sigma_{q_e}^2 = \frac{\cancel{2}\delta^3}{\cancel{2}\delta 3}$$

$$\sigma_{q_e}^2 = \frac{\delta^2}{3}$$

This noise power is uniformly distributed over $-f_s$ to f_s range. This is shown in below figure -



ie average o/p noise power $= \left(\frac{w}{f_s}\right) \times \frac{\delta^2}{3}$

$$N = \frac{w\delta^2}{3f_s}$$

$$\therefore \quad (SNR)_{o\,max} = \frac{\text{signal power}}{\text{noise power}}$$

$$(SNR)_{o\,max} = \frac{\frac{\delta^2}{8\pi^2 f_m^2 T_s^2}}{\frac{w\delta^2}{3f_s}}$$

$$(SNR)_{o\,max} = \frac{\cancel{\delta^2}}{8\pi^2 f_m^2 T_s^2} \times \frac{3f_s}{w\cancel{\delta^2}}$$

$$(SNR)_{o\,max} = \frac{3}{8wT_s^3\pi^2 f_m^2}$$

(post-filtered o/p SNR) $\Rightarrow$ $$\boxed{(SNR)_{o\,max} = \frac{3}{8\pi^2 w f_m^2 T_s^3}}$$

At no slope overload distortion, the o/p SNR is inversely proportional to cube of sampling interval T_s.