Problems -

1. A signal with peak-to-peak 4 Volts is to be quantized with quantizer levels 3, 4, 5. Sketch its characteristics & mention the type of quantizer.

Sol.

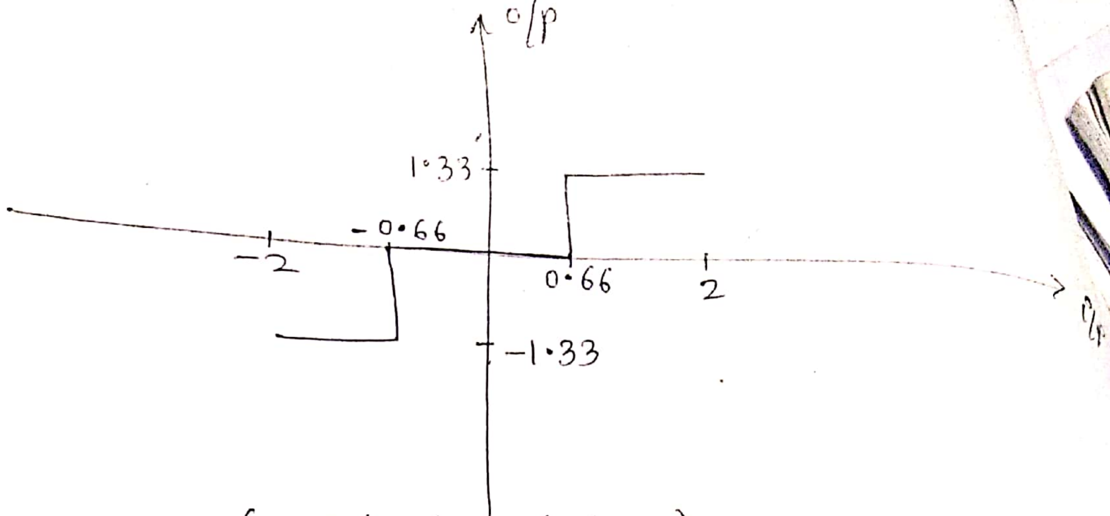
range $\Rightarrow$ $-2V$ to $+2V$

(i) $L = 3$ $\therefore$ $\Delta = \frac{4V}{3}$

$\Delta = 1.34V$

$\Rightarrow [-2, \ -0.66, \ 0.66, \ +2] \Rightarrow$ decision levels.

$\Rightarrow [\ -1.34, \ 0, \ 1.33] \Rightarrow$ representation levels



(mid-tread type)

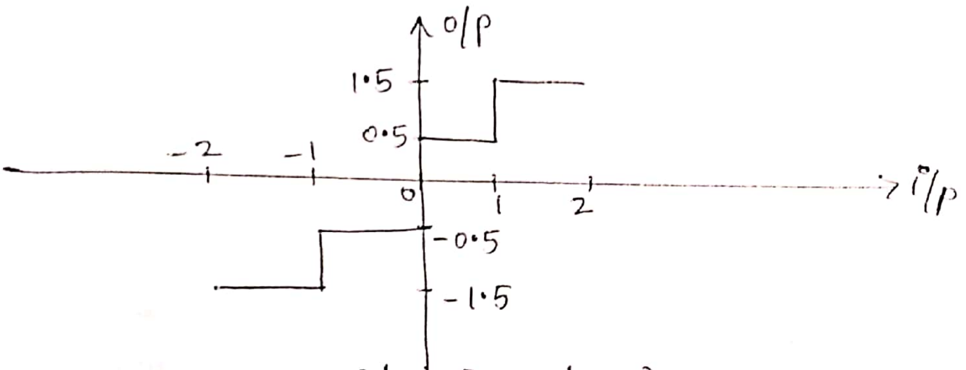
(ii) $L = 4$

$$\therefore \quad \Delta = \frac{4V}{4}$$

$$\Delta = 1V$$

$\Rightarrow$ $[-2, -1, 0, 1, 2]$ $\Rightarrow$ decision levels

$\Rightarrow$ $[-1.5, -0.5, 0.5, 1.5]$ $\Rightarrow$ representation levels



(mid-riser type)

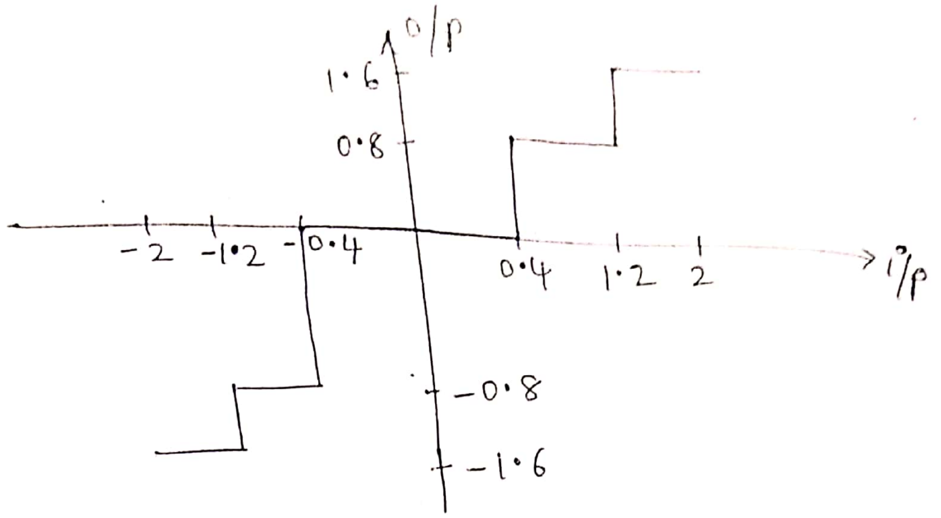
(iii) $L = 5$

$$\therefore \quad \Delta = \frac{4V}{5} = 0.8$$

$\Rightarrow$ $[-2, -1.2, -0.4, 0.4, 1.2, 2]$ $\Rightarrow$ decision levels

$\Rightarrow$ $[-1.6, -0.8, 0, 0.8, 1.6]$ $\Rightarrow$ representation levels.

PTO -->



(mid-tread type)

2. The signals $g_1(t) = 10\cos(100\pi t)$ & $g_2(t) = 10\cos 50\pi t$ are both sampled with $f_s = 75\,Hz$. Show that the two sequences so obtained are identical.

(2013 05M)

Sol.

$$g_1(t) = 10\cos(100\pi t)$$

this signal is sampled in time domain with $T_s = \frac{1}{f_s} = \frac{1}{75}$ secs.

$$g_{\delta_1}(t) = \sum_{n=-\infty}^{+\infty} g_1(nT_s)\,\delta(t - nT_s)$$

$$g_{\delta_1}(t) = \sum_{n=-\infty}^{+\infty} g_1\left(\frac{n}{75}\right)\delta\left(t - \frac{n}{75}\right)$$

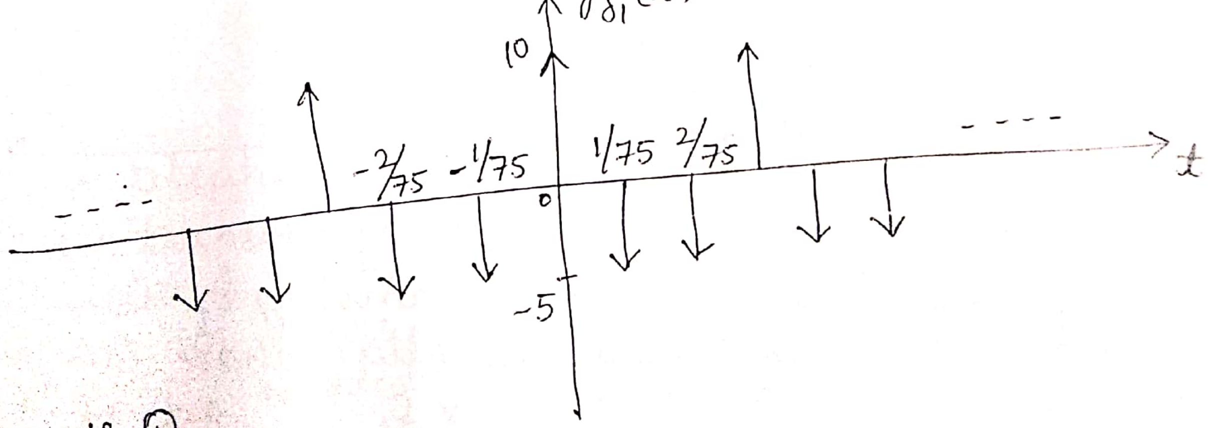


fig ①.

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Now $g_2(t) = 10\cos(50\pi t)$

$$g_{\delta_2}(t) = \sum_{n=-\infty}^{+\infty} g_2(nT_s)\,\delta(t-nT_s)$$

$$g_{\delta_2}(t) = \sum_{n=-\infty}^{+\infty} g_2\left(\frac{n}{75}\right)\,\delta\left(t-\frac{n}{75}\right)$$

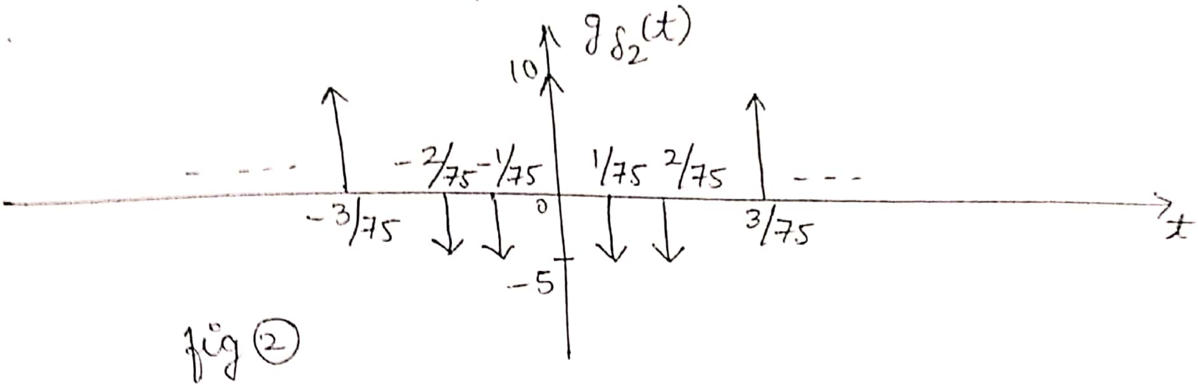
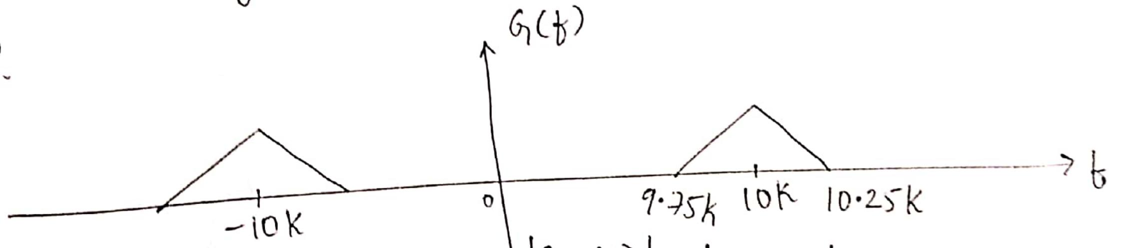


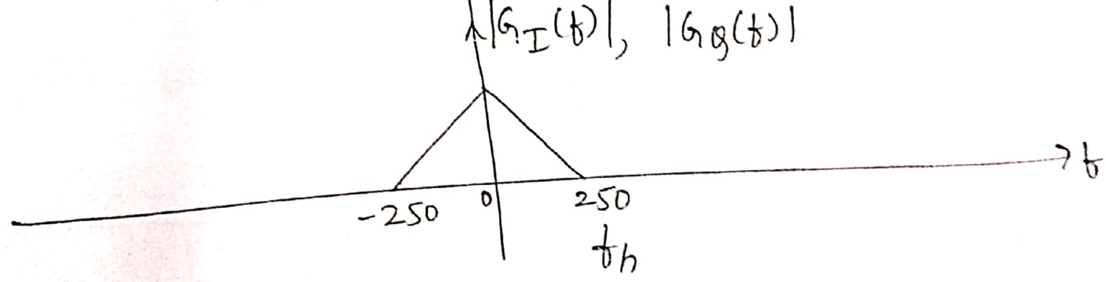
fig ②

from fig ① & ②, the two sequences so obtained are identical.

3. The spectrum of a bandpass signal occupies a bandwidth 0.5 KHz, centered around ±10 KHz. Find the nyquist rate for quadrature sampling of the inphase & quadrature component of the signal (2014 4m)

Soln.





∴ nyquist rate for sampling of inphase & quadrature phase is $f_{nyq} = 2 \times f_b$

$f_{nyq} = 2 \times 250$

$f_{nyq} = 500\,Hz$ //

A PCM system uses uniform quantizer & a 7-bit encoder with bit rate of 50MBPs. Find maximum msg bandwidth. Find the o/p SNR for f_m of 1MHz.

(6m)
(2007)

Sl^n.

$n = 7$ bits

$r = 50$ MBPS

$f_m = 1$ MHz

bit rate or signalling rate $r = n f_s$ bits/sec

max^m msg bandwidth $B_T = \frac{r}{2}$

$$B_T = \frac{50 MBPS}{2}$$

$$B_T = 25 MHz$$

$$(SNR)_0 = 1.8 + 6n$$

$$= 1.8 + 6 \times 7$$

$$(SNR)_0 = 43.8 dB$$

$$10 \log_{10} (SNR)_0 = 43.8 dB$$

$$\log (SNR)_0 = \frac{43.8}{10}$$

$$(SNR)_0 = 10^{4.38}$$

$$(SNR)_0 = 23.98 K$$

5. A telephone signal bandlimited to 4KHz is to be transmitted by PCM. The signal to quantization noise is to be atleast 40dB. find the nos of levels into which the signal has to be encoded also find the bandwidth of transmission.

Note – $(SQNR)_{dB} = 1.8 + 6n$ → for sinusoidal i/p signal.

$(SQNR)_{dB} = 4.8 + 6n$ → for any other i/p signal.

$\underline{S^n}$

$f_m = 4 KHz$
$(SNR)_o = 40 dB$
$L = ?$
$n = ?$
$B_T = ?$

given is a telephone signal, which is not sinusoida

$\therefore (SNR)_o = 4.8 + 6n \ dB$

$40 = 4.8 + 6n$

$6n = 35.2$

$n = 5.8667$

$n \approx 6$ bits //

$\therefore L = 2^n$

$L = 64$ levels //

$$\text{NOW } B_T = \frac{r}{2}$$

$$= \frac{nf_s}{2}$$

$$= \frac{6 \times (2 \times f_m)}{2}$$

$$= \frac{6 \times 8K}{2}$$

$B_T = 24 \ KHz$ //

6. A telephone signal with BW 4KHz is digitized into an 8-bit PCM, sampled at nyquist rate. calculate PCM transⁿ BW & signal to quantis- - ation noise ratio. (2008 4m)

$\underline{S^n}$.

$n = 8$ bits
$f_m = 4 KHz$
$f_s = 2f_m = 8 KHz$
$B_T = ?$
$(SNR)_o = ?$

$$B_T = \frac{r}{2}$$

$$= \frac{nf_s}{2}$$

$$= \frac{8 \times 8K}{2}$$

$$B_T = 32\,KHz$$

$\Rightarrow$ not a sinusoidal i/p signal //

Now $(SNR)_0 = 4.8 + 6n$

$$= 4.8 + 6 \times 8$$

$$(SNR)_0 = 52.8\,dB \;//$$

7. A signal $x(t) = 3\sin(500t)$ is sampled & quantized using 10bit PCM.
(i) find (SNR) in dB, step size & nos of levels.
(ii) if above signal is quantized using N-bit PCM wave, find N to achieve a (SNR) of atleast 50dB. Also find the new nos of levels & new value of step size. (7m)

$\underline{S}^n$.

$$x(t) = 3\sin(500t)$$

$$A_m = 3V$$

$$f_m = \frac{500}{2\pi}$$

$$f_m = 79.57\,Hz$$

$$n = 10\,bits$$

$$(SNR)_0 = 1.8 + 6n$$

$$= 1.8 + 6 \times 10$$

$$(SNR)_0 = 61.8\,dB$$

WKT $\quad L\Delta = 2A_m$

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$$\Delta = \frac{2A_m}{L} = \frac{2A_m}{2^n}$$

$$\Delta = \frac{2 \times 3}{2^{10}}$$

$$\Delta = 5.85\ m\ volts/level$$

$$L = 2^n = 2^{10} = 1024\ levels.$$

(ii) $$(SNR)_o = 1.8 + 6n$$

$$50 = 1.8 + 6n$$

$$n = 8.03$$

$$N = n \approx 9\ bits$$

$$L = 2^N = 512\ levels.$$

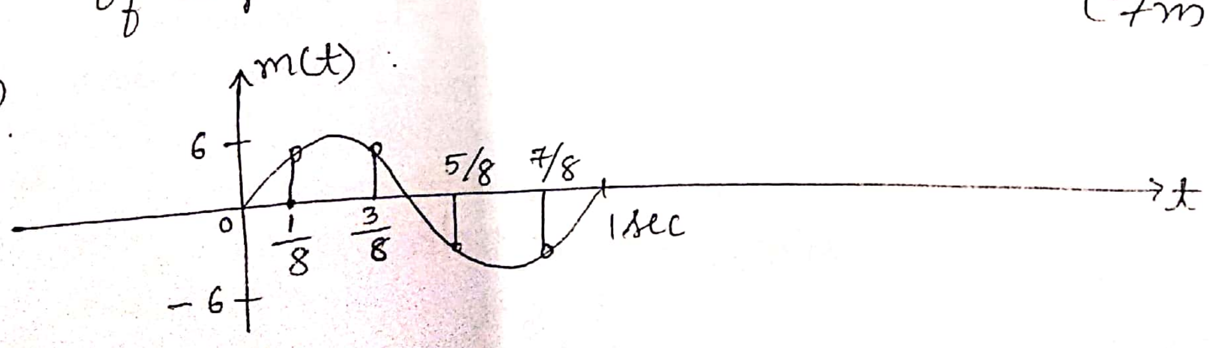
step size, $$\Delta = \frac{2A_m}{L}$$

$$\Delta = \frac{2 \times 3}{512}$$

$$\Delta = 11.72\ mV/level$$

8. The signal $m(t) = 6 \sin(2\pi t)$ V is transmitted using n bit binary PCM system. The quantizer is of midriser type with a step size of 1 volt. The sampling freq is 4 Hz with samples taken at $t = \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}$ .... sec. Sketch PCM wave for one complete cycle of input. (2013 7m)

$S1^n$.



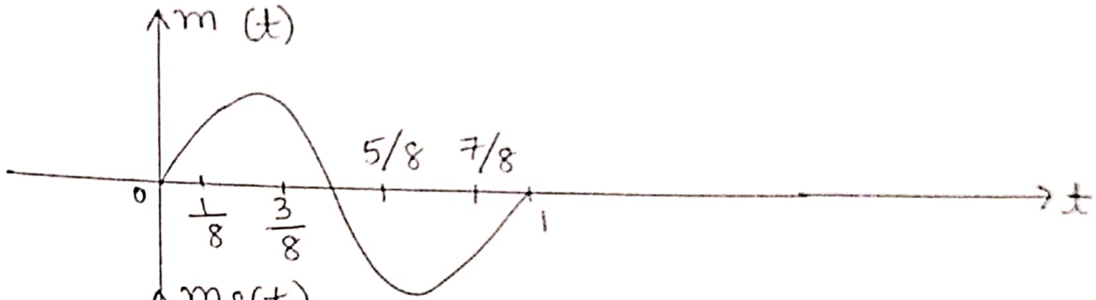
$m\left(\frac{1}{8}\right) = 6 \sin\left(2\pi \frac{1}{8}\right)$

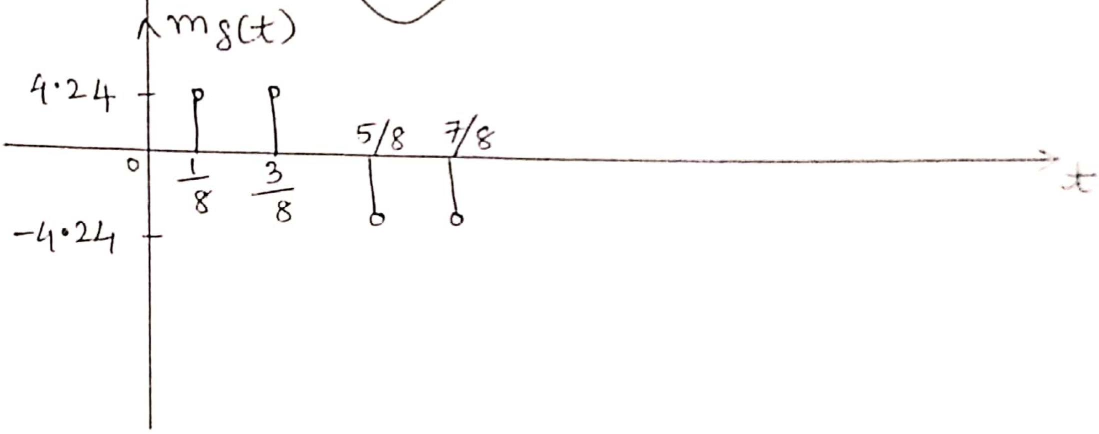
$m\left(\frac{1}{8}\right) = 4.24 \text{ V}$

$m\left(\frac{3}{8}\right) = 6 \sin\left(2\pi \frac{3}{8}\right) = 4.24 \text{ V}$

$m\left(\frac{5}{8}\right) = 6 \sin\left(2\pi \frac{5}{8}\right) = -4.24 \text{V}$

$m\left(\frac{7}{8}\right) = 6 \sin\left(2\pi \frac{7}{8}\right) = -4.24 \text{V}$





given $\Delta = 1 \text{volt}$

$\Delta = \frac{2A_m}{L}$

$L = \frac{2A_m}{\Delta} = \frac{2 \times 6}{1}$

$L = 12$ levels

ie $2^n = 12$

$n = 4$ bits

$(-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6) \Rightarrow$ DL

$(-5.5, -4.5, -3.5, -2.5, -1.5, -0.5, 0.5, 1.5,$
$2.5, 3.5, 4.5, 5.5) \Rightarrow$ RL

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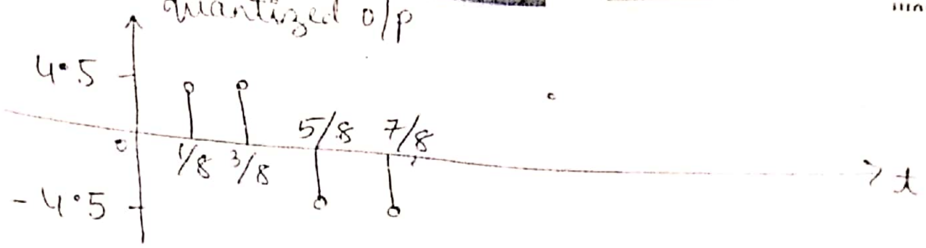


quantisation levels	codeword (n = 4 bits)
-5.5	0000
-4.5	0001
-3.5	0010
-2.5	0011
-1.5	0100
-0.5	0101
0.5	0110
1.5	0111
2.5	1000
3.5	1001
4.5	1010
5.5	1011

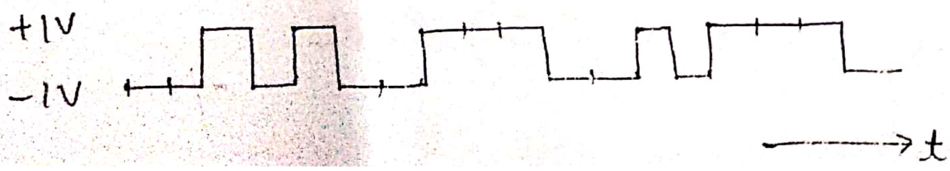
encoded o/p -

$\Rightarrow$ 1010 1010 0001 0001

PCM wave for one complete cycle -

$\Rightarrow$ 1 0 1 0 1 0 1 0 0 0 0 1 0 0 0 1

9. Fig below shows a PCM wave in which the amplitude levels of +1V & -1V are used to represent binary symbols 1 & 0 respectively. The codeword used consists of three bits. Find the sampled version of an analog signal from which this PCM wave is derived.



Sol^n

encoded o/p => 001010011100101110

$n = 3$ bits

$L = 2^n = 8$ levels

let us take $A_m = 3$ volts

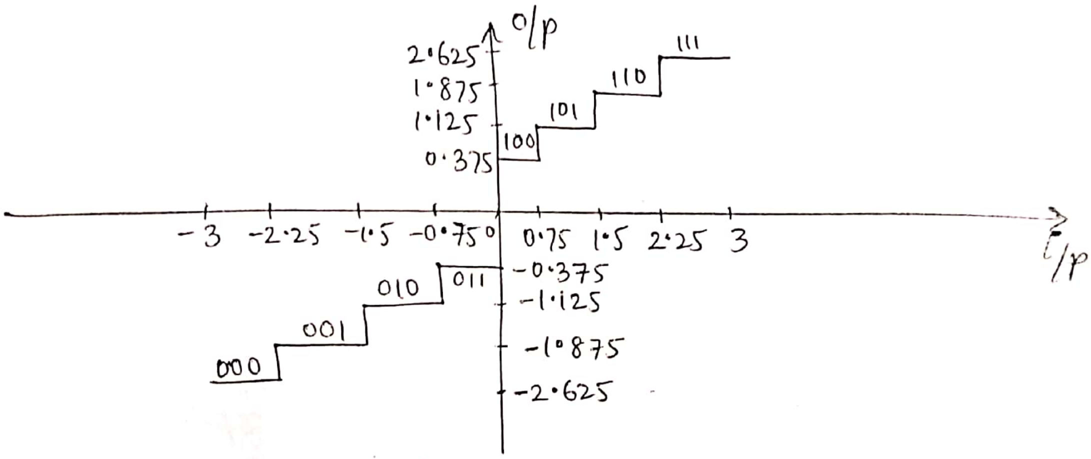
$\therefore$ $\Delta = \dfrac{2A_m}{L}$

$\Delta = \dfrac{2 \times 3}{8}$

$\Delta = 0.75$

(−3, −2.25, −1.5, −0.75, 0, 0.75, 1.5, 2.25, 3) => DL

(−2.625, −1.875, −1.125, −0.375, 0.375, 1.125, 1.875, 2.625) => RL



encoded o/p => 001 010 011 100 101 110

decoded o/p => (−1.875, −1.125, −0.375, 0.375, 1.125, 1.875)

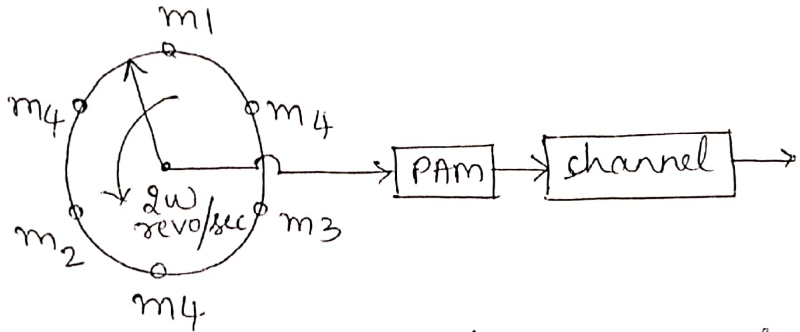
10. Four messages bandlimited to w, w, w & 3w are to be time division multiplexed, with w being 2000 Hz. Set up a scheme for accomplishing this multiplexing requirement

0X0

with each msg signal sampled at its nyquist rate. Also find the speed of the commutator in samples per second.

Sol.

msgs	BW	$f_s = 2BW$	samples per revolution	angle of separation
m_1	W	$2W$	1	360°
m_2	W	$2W$	1	360°
m_3	W	$2W$	1	360°
m_4	$3W$	$6W$	3	120°
		$12W$ samples per sec	6 samples per revolution	



Speed of commutator $= 2W$ rps

$= 2 \times 2000$ rps

$= 4000$ rps

But each revolution $= 6$ samples.

$\therefore$ speed of commutator $= 6 \times 4000 \dfrac{\text{samples}}{\text{sec}}$

$= 24000$ samples/sec

11. The signals $g_1(t) = 10\cos(100\pi t)$ & $g_2(t) = 10\cos 50\pi t$ are both sampled with $f_s = 75Hz$. Show that the two sequences so obtained are identical in both time & frequency domain.

$f_s = 75 Hz$

Time domain -

$T_s = \frac{1}{f_s} = \frac{1}{75}$ sec.

$g_1(t) = 10 \cos(100\pi t)$

put $t = nT_s$

$t = \frac{n}{75}$

$g_1\left(\frac{n}{75}\right) = 10 \cos\left(100\pi\left(\frac{n}{75}\right)\right)$

$g_1\left(\frac{n}{75}\right) = 10 \cos\left(\frac{4\pi n}{3}\right)$ —(1)

Now $g_2(t) = 10 \cos(50\pi t)$

put $t = nT_s$

$t = \frac{n}{75}$

$g_2(t) = 10 \cos\left(\frac{50\pi n}{75}\right)$

$g_2\left(\frac{n}{75}\right) = 10 \cos\left(\frac{2\pi n}{3}\right)$ —(2)

eqn (1) => $g_1\left(\frac{n}{75}\right) = 10 \cos\left[2\pi - \frac{2\pi}{3}\right]n$

$g_1\left(\frac{n}{75}\right) = 10 \cos\left(\frac{2\pi n}{3}\right)$ —(3)

From (2) & (3) => after sampling both $g_1(t)$ & $g_2(t)$ are identical in time domain.

Frequency domain -

$g_1(t) = 10 \cos 100\pi t$

FT obs =>

$G_1(f) = \frac{10}{2}\left[\delta(f-50) + \delta(f+50)\right]$ —(4)

Now $\Rightarrow$ $g_2(t) = 10\cos 50\pi t$

FT obs $\Rightarrow$

$$G_2(f) = \frac{10}{2}\left[\delta(f-25) + \delta(f+25)\right] \quad (5)$$

Now $G_{1\delta}(f) = f_s \sum_{n=-\infty}^{+\infty} G(f - nf_s)$

$$G_{1\delta}(f) = 5 \times 75 \sum_{n=-\infty}^{+\infty}\left[\delta(f-50-75n) + \delta(f+50-75n)\right] \quad (6)$$

also $G_{2\delta}(f) = 5 \times 75 \sum_{n=-\infty}^{+\infty}\left[\delta(f-25-75n) + \delta(f+25 - 75n)\right]$

put $n = l-1$ put $n = k+1$

$$G_{2\delta}(f) = 375 \sum_{l=-\infty}^{+\infty} \delta(f-25-75(l-1)) + 375 \sum_{k=-\infty}^{+\infty} \delta(f+25 - 75(k+1))$$

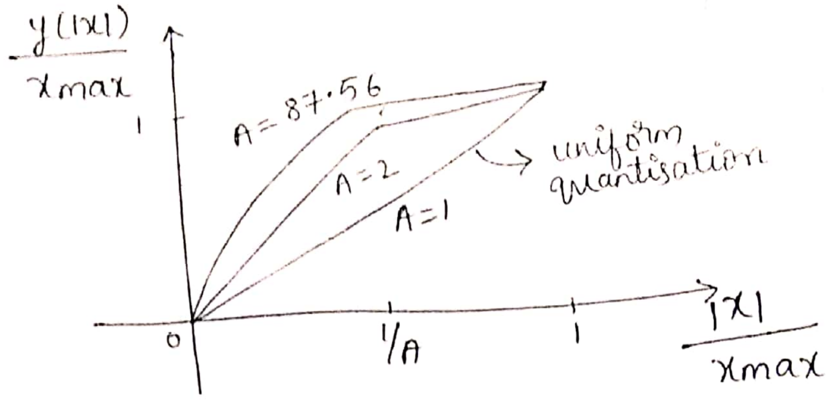
$$G_{2\delta}(f) = 375 \sum_{l=-\infty}^{+\infty} \delta(f-50-75l) + 375 \sum_{k=-\infty}^{+\infty} \delta(f+50 - 75k)$$

$$G_{2\delta}(f) = 375\left[\sum_{n=-\infty}^{+\infty} \delta(f-50-75n) + \delta(f+50-75n)\right] \quad (7)$$

By eq^n (6) & (7) $\Rightarrow$

after sampling, the spectrums are identical in frequency domain.

This is because of aliasing as $g_1(t)$ is sampled at less than its nyquist rate $\Rightarrow$ 100 Hz.
But there is no aliasing in $g_2(t)$.



<u>Note</u>- SQNR for μ-law companding is given by =>

$$(SNR)_0 = \frac{3L^2}{[\ln(1+\mu)]^2}$$

<u>Problems</u> -

1. A PCM system which employs uniform quantisation & produces a binary o/p, given an i/p signal whose amplitude varies from 4V to -4V & having avg power of 40mW calculate the nos of bits/sample of required signal to noise ratio is 20dB.

<u>Sl^n</u>. range => -4V to 4V

P = 40mW

$$(SNR)_0 = \frac{\sigma_x^2}{\sigma_{Qe}^2} = \frac{P}{(\Delta^2/12)}$$

$$(SNR)_0 = \frac{12P}{\Delta^2}$$

But $\Delta = \frac{V_{pp}}{L}$

$$\therefore (SNR)_0 = \frac{12P}{V_{pp}^2/L^2} = \frac{12PL^2}{V_{pp}^2}$$

$$(SNR)_0 = \frac{12 \times 40m \times L^2}{(8)^2} \quad -(1)$$

$$SNR = 10 \log_{10} SNR \text{ dB}$$

$$\frac{20}{10} = \log_{10} SNR$$

ie $SNR = 100$

$\therefore$ eqⁿ ① $\Rightarrow$

$$L^2 = \frac{100 \times 8^2}{12 \times 40m \times 1}$$

$$L^2 = 13.334K$$

$$L = 115.47$$

$$L \approx 128$$

$$2^n \approx 128$$

$$\therefore \boxed{n = 7 \text{ bits}}$$

2. The signal $g(t) = 2\cos(2000\pi t) - 4\sin(4000\pi t)$ is quantized by rounding off, using a 12-bit quantizer. what is rms quantisation error & SQNR.

Solⁿ.

$Am_1 = 2V$, $Am_2 = 4V$

$\therefore$ $V_{max} = 6V$

$$\Delta = \frac{2V_{max}}{L}$$

$$= \frac{2 \times 6}{2^n}$$

$$= \frac{12}{2^{12}}$$

$$\Delta = 2.93 \times 10^{-3} \text{ V/level}$$

rms value of quantizer error $= \sqrt{\sigma_{qe}^2}$

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rms value of error = $\sqrt{\Delta^2/12}$

$= 845.82\,\mu V$

$SQNR = 1.8 + 6n$

$SQNR = 1.8 + 6 \times 12$

$SQNR = 73.8\,dB$

3. For a binary PCM signal, determine L, if the compression parameter μ is 100 & the maximum $(SNR)_{dB}$ is 45dB. Determine $(SNR)_{dB}$ with this value of L.

Sol^n.

$(SNR)_{dB} = 45\,dB$

$(SNR) = 31622.7766$

WKT $(SNR) = \dfrac{3L^2}{\ln(1+\mu)^2}$

$31622.7 = \dfrac{3L^2}{\ln(1+100)}$

$L^2 = 224514.68$

$L = 473.829$

$L \approx 512 = 2^9$ levels.

$(SNR) = \dfrac{3L^2}{\ln(1+\mu)^2}$

$SNR = \dfrac{3 \times 512^2}{\ln(1+100)^2}$

$SNR = 36.923\,k$

$(SNR)_{dB} = 10 \log_{10}(36.923k) = 45.67\,dB$