

# **Teaching Agents with Deep Apprenticeship Learning**

by

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Degree of Master of Science  
in Computer Engineering

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# Dedication

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# **Abstract**

**Teaching Agents with  
Deep Apprenticeship Learning**

**Amar Bhatt**

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As the field of robotic and humanoid systems expand, more research is being done how to best control systems to perform complex, smart tasks. Many supervised learning and classification techniques need large datasets, and only allow the system to copy what it is given. The sequential relationship within datasets used for task learning results in markov decision problems that traditional classification algorithms cannot solve. Reinforcement learning helps to solve these types of problems using a reward/punishment and exploration/exploitation methodology without the need for datasets. While this works for simple systems, complex systems are more difficult to teach using traditional reinforcement learning. Often these systems have complex, non-linear, non-intuitive cost functions which make it near impossible to model. Inverse reinforcement learning learns complex cost functions based on input from an expert system. This is also referred to as apprenticeship learning. Deep learning has also made a large impact in learning complex systems, and has achieved state of the art results in several applications. Using methods from apprenticeship learning and deep learning a variety of systems can be taught complex tasks from an expert. The systems learning may not have the same degrees of freedom as the expert, but will learn how to complete the tasks based on its own functionality and limitations.

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# Chapter 1

## Introduction

When born, animals and humans are thrown into an unknown world forced to use their sensory inputs for survival. As they begin to understand and develop their senses they are able to navigate and interact with their environment. The process in which we learn to do this is called reinforcement learning. This is the idea that learning comes from a series of trial and error where there exists rewards and punishments for every action. The brain naturally logs these events as experiences, and decides new actions based on past experience. An action resulting in a reward will then be higher favored than an action resulting in a punishment. Using this concept, autonomous systems, such as robots, can learn about their environment or how to do tasks in the same way. Reinforcement learning is used in many applications aimed to reflect the way a human/animal's brain learns/reacts [14]. Some of these applications include the inverse pendulum, mountain car problem [22], robotic navigation, and decision based systems. While traditional classification methods can assist in task mimicking, it does not perform well when introduced to new environments or new tasks. Reinforcement learning does not mimic the correct move, but instead develops a policy to choose actions to obtain the highest reward.

Reinforcement learning techniques such as Q-Learning, are useful when the cost system is known, but is not the case for many complex systems. Due to the vast number of variables in tasks such as driving, walking, or medical procedures, cost functions must be learned before they are applied. To do this, inverse reinforcement learning, or apprenticeship learning, is used. Apprenticeship learning extracts an action policy by observing an expert's

transitions over time and determining the cost function from the observations [16]. It is a Markovian decision process, in that the result of the next action is based solely on the present state and not states that happened previously. Therefore, inverse reinforcement learning sets to optimize the set of learned features in a particular state. Traditional inverse reinforcement learning uses this optimization of the linear combination of these features to determine the cost function of the given state. For static environments the traditional inverse reinforcement learning techniques are viable, however this is not the case for dynamic environments [24]. For dynamic networks, state features are constantly changing, which requires that the reward function and features be updated. Unfortunately, this no longer gives the most optimal solution but only an approximate [24].

These traditional methods are not scalable to systems with large number of states. For example, if using images as an input for reinforcement learning or apprenticeship learning, the Q-table would be very large, and often times unsustainable. Therefore, combining these algorithms with deep learning where the rewards for state-action pairs are learned in a neural network, systems can be as complex or as large as needed [11]. Combining deep learning with reinforcement learning opens up a new dimension of learning, where complex correlations of environment and state features can be utilized to learn complex tasks.

# Chapter 2

## Reinforcement Learning

Reinforcement learning is used to teach systems based on trial and error. It uses a reward and punishment based cost function to help determine a policy on which to choose its next action. It is based around how living creatures learn using primitive functions at a young age. This methodology creates a balance between random decisions (exploration) and choosing the best learned policy over time (exploitation). These algorithms work with agent(s), state-action pairs, and policies. An agent is what system is being operated on, such as, a car, robot, humanoid, or game solver. States represent different modes of the system, it can be any combination of environment or system specific features, such as, location of agent, location of obstacles, distance from goal, time, or speed. Actions can be anything that influences a transition from one state to another state, such as moving the agent, throwing at a target, or picking up an object. Actions will lead to some reward/punishment that will help train the system, such as an increase/decrease in score. Choosing an action at a specific state is called the system's policy. This is what will be learned using Reinforcement Learning. Therefore, reinforcement learning algorithms optimize over a known cost function and output a policy.

### 2.0.1 Temporal Difference Learning

Temporal difference learning is branch of reinforcement learning that attempts to model the way an animal learns by predicting a reward in terms of a given stimulus. The heart of this algorithm comes from its prediction

error signal, which constantly adjusts expected rewards based on the current given state [17]. This algorithm estimates the value function of a state, allowing it to estimate the final reward at each state and action taken. If it were not estimated, and instead calculated, the agent would need to wait until the final reward before updating state and action reward values. The non-estimated value function is shown in (2.1) and the temporal difference estimation of the value function is shown in (2.2) [7]. Each of these two functions return a value ( $V$ ) when given a state ( $s$ ) at time step ( $t$ ) which is represented by  $s_t$ . The value  $V(s_t)$  is the anticipated value or reward at specific state. If  $V(s_t)$  is very high, then that state will carry a higher importance upon evaluation. Therefore, when evaluating this type of algorithm, the next states with the largest values will be transitioned to. The learning rate,  $\alpha$ , is used to determine the proportion of update. If  $\alpha$  is very large, then the update at each time step,  $t$ , will also be large.  $R_t$  is the total reward of the system when proceeding from state  $s_t$ . The immediate reward received at  $s_t$  is defined by  $r_t$ , therefore the reward for a state one time step out ( $s_{t+1}$ ) is  $r_{t+1}$ . The discount rate,  $\gamma$ , represents the weight (between 0 and 1) that the next state value will be multiplied with. With a discount rate of 1, the entire value of the next state will be considered in the state value update. A discount rate of less than 1 will decrease the importance of the value of the next state in the state value update.

$$V(s_t) = V(s_t) + \alpha[R_t - V(s_t)] \quad (2.1)$$

$$V(s_t) = V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \quad (2.2)$$

Estimating in this way allows for the continuous update of state values as more information is provided. As shown, the major difference between the original value function and its estimate is the consideration of the reward in the next step as well as the difference in expected reward between the current state and next state. The non-estimating function assumes that the final reward function  $R_t$  exists as shown in (2.3) [11].

$$R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots + \gamma^{n-t} r_n \quad (2.3)$$

The  $r$  value in (2.2) is the estimated reward for the action selected at that state in current time, whereas  $R_t$  in (2.1) is the final reward. The  $\gamma$  value (0 to 1) is the discount factor used for convergence, where a low  $\gamma$  causes the system to over emphasize the current state, and a higher  $\gamma$  will give a higher weight to future states [7]. Temporal difference learning requires several iterations, called episodes, to converge. An episode starts at any given state, but must end at a terminal state. A system can have more than one terminal state (e.g. end of a game, reaching the goal state, etc.). There are two types of Temporal Difference Learning methods; Off-Policy and On-Policy. Both methods choose a different balance between exploration and exploitation policy. Exploration is the idea of choosing random actions to learn the entire state-space. Exploitation is choosing only those actions that proved to be high in reward.

### **Off-Policy: Q-Learning**

Q-learning is an off-policy method of temporal difference learning. It is the one of the most widely used algorithms in this domain. The Bellman Equation shown in (2.4) [11] is used to update its value function denoted as  $Q$ .

$$Q(s_t, a_t) = r + \gamma * \max_{a_{t+1}}(Q(s_{t+1}, a_{t+1})) \quad (2.4)$$

As shown, the *Q-value* at state  $s_t$  and  $a_t$  at time step  $t$  is based on the current value and the next best state-transitions value multiplied by the discount factor. In Q-learning  $Q$  is represented as a table with states as its rows and actions at its columns. Therefore, each Q-value at a particular state-action pair is the total reward at that state assuming that optimal actions are taken from that state onwards. This requires knowledge of future states and rewards. Therefore, this algorithm requires several episodes starting at different states to reach convergence and fill the Q-table [25]. This is because unlike other neural-network based applications, Q-Learning is unsupervised and must learn an environment dynamically. Typically, this causes massive overhead when scaling this algorithm to larger environments, causing the traditional Q-Learning methods to be unpopular [18]. However, recent

advances in this algorithm have generated new ways to define its policy making it capable to scale to larger environments [1]. To iteratively update Q-values over time, the value function in (2.5) is used.

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha[r_{t+1} + \gamma * \max_{a_{t+1}}(Q(s_{t+1}, a_{t+1})) - Q(s_t, a_t)] \quad (2.5)$$

In this equation,  $\alpha$  is the learning rate usually held between 0 and 1. If this value is set to 1 then the Q-value is updated fully without considering the previous value,  $Q(s_t, a_t)$ , which will cause the algorithm to learn very quickly. However, this may result in divergence of the algorithm. If  $\alpha$  is set to 0 then the Q-value is not updated. Therefore, to ensure the system is learning and can converge to a solution  $\alpha$  needs to be set between 0 and 1. The exact value of the learning rate needs to be determined through experimentation.

This algorithm learns soft policies allowing it to have a high factor of exploration. Off-Policy algorithms update value functions using assumed actions which have not been tried, as shown by taking the maximum valued action at any given state. This allows the agent to learn actions it did not actually perform [7]. This shows that this algorithm favors exploration more so than exploitation.

### **On-Policy: Sarsa**

Sarsa is an on-policy method of temporal difference learning [21]. In this case the value function in (2.6) is updated according to a policy designed more around experience. This ties this algorithm into considering control and exploration as one entity, whereas its Off-Policy counterpart can separate the two.

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \quad (2.6)$$

Unlike the Q-Learning value function equation, Sarsa does not consider the maximum value of an action at a given state, but instead takes actions that have been taken in the past [7]. This shows that this algorithm favors

exploitation more so than exploration. This also allows this algorithm to populate Q-values with a closer true approximate than the Q-Learning algorithm [21]. Therefore, it can be said that while Q-Learning updates based on states, Sarsa updates in accordance to state-action pairs. This allows for a better convergence policy, and leads to less unnecessary exploration by the system at hand [22]. However, because this algorithm lacks in exploration, it may converge to a local minima or non-optimal solution.

### Epsilon Selection Policies

Both off-policy and on-policy methods of temporal difference learning rely on the selection of states and actions which is called a policy. Policies are meant to find a balance between exploration and exploitation. Popular policies used in both Q-Learning and Sarsa are  $\epsilon$ -greedy,  $\epsilon$ -soft, and softmax. In both  $\epsilon$ -greedy and  $\epsilon$ -soft selection policies an action is chosen with the highest estimated reward for most of the iterations. They choose a random action with a probability of  $\epsilon$  for  $\epsilon$ -greedy and probability  $1-\epsilon$  for  $\epsilon$ -soft where  $\epsilon$  is a small value between 0 to 1.  $\epsilon$ -greedy is chosen to limit exploration using  $\epsilon$  between 0 to 0.5. This results in 0% to 50% of the trials having actions chosen at random.  $\epsilon$ -soft is chosen to favor exploration using  $\epsilon$  between 0 to 0.5. This results in 50% to 100% of the trials having actions chosen at random [7]. These  $\epsilon$  values can be dynamically adjusted based on number of episodes completed, number of steps within an episode, or based on how well the algorithm is converging. Therefore, common practice is to have high exploration (random action) in the beginning of learning, and high exploitation towards the end of learning [1]. Unlike the  $\epsilon$  selections, softmax chooses random actions with respect to weights assigned to each action. These weights are adjusted over time as the system learns. This makes it so undesirable actions are less likely to be chosen over time, whereas in the  $\epsilon$  selections the worst action has the same probability as the best action of being randomly chosen. Each selection method has its own benefits, but none have proven to be better than the other [7].

## 2.1 Q-Learning and Sarsa Implementation

Using simulated sensory data from ultrasonic sensors, moisture sensors, encoders, shock sensors, pressure sensors, and steepness sensors, a robotic system will be able to make decisions on how to navigate through its environment to reach a goal. The robotic system will not know the source of the data or the terrain it is navigating. Given a map of an open environment simulating an area after a natural disaster, the robot will use model-free temporal difference learning with exploration to find the best path to a goal in terms of distance, safety, and terrain navigation. Two forms of temporal difference learning will be tested; off-policy (Q-Learning) and on-policy (Sarsa). Through experimentation with several world map sizes, it is found that the off-policy algorithm, Q-Learning, is the most reliable and efficient in terms of navigating a known map with unequal states.

### 2.1.1 Application

In many cases robotic path planning is used in controlled environments, whether in a hospital, nursing home, hotel, etc. Reinforcement learning is also used to navigate known city maps and other open environments [18]. However, many of these cases fall into the category of finding the best path of a known map which can also be achieved with less invasive path planning algorithms. Consider the case of a city that underwent a natural disaster. While the city map has stayed the same, the terrain of the city may have vastly altered. Clear roads may be flooded with water, and buildings may have been knocked down creating alternative paths. In this case, it will be pertinent for a land-based robotic system to efficiently navigate the city to reach potential resources and survivors in the safest way. If the system took the known path as shown on a city map before the disaster, the robot may get stuck, hurt, or take too much time. Understanding the landscape and making decisions based on the type of terrain can ensure an efficient and safe path is chosen.

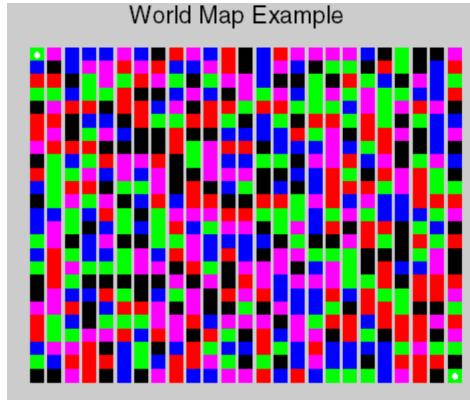


Figure 2.1: Map Example with Terrain (Sand = Magenta, Forest = Green, Pavement = Black, Water = Blue, Misc. Debris = Red, Start/Goal = White dots)

### 2.1.2 Methods

To test the temporal difference learning algorithms, Q-Learning and Sarsa, MATLAB was used. An  $M \times N$  matrix was created consisting of  $M \times N$  states. Each state had between 2-4 neighbors, depending on where on the grid it fell. Each state also was associated to one of five terrain types; sand, forest, pavement, water, mountainous rock/debris. These states represented an area on a map after a natural disaster. This meaning the defined paths from a start to a goal have been severely altered. An example map is shown in Fig. 2.1. Each terrain was associated to certain punishments, as decided by the simulated sensor readings on the robot. The sensors used were, encoders (speed detection), ultrasonic sensors (visibility), shock sensor (smoothness), moisture sensor, pressure sensor (stability), and steepness sensor. Each sensor reading was statically defined and normalized between 0 and 1, where 0 is the safest/most efficient and 1 is the most dangerous/least efficient for that particular sensor. Further testing of the system would call for the sensor readings to be introduced to natural noise.

The world map matrix were at first predefined to validate the two algorithms (defined path from start to goal) and to find appropriate parameter values for the algorithms (punishment weight, discount factor, goal reward, etc). The maps were then randomized for further testing. The simulated robot navigating the map from the start (top left corner) to the goal (bottom right corner) had no prior knowledge of the terrain of the state it was in. It

did know the map, however. As each algorithm was run, the value at each state was updated using a reward for the action chosen as well as a punishment which was determined by the sensor readings for each state. On a normalized average the punishments for each terrain type was as follows: sand 1, forest 2, pavement 0, water 3, mountainous rock/debris 4. This is much like the penalties incurred in James Sutton's puddle world problem, however, in this case the robot does not know about the terrain in its world, only what it reads through its sensors [22]. Varying punishments made one state more favorable than another in terms of proximity to the goal, safeness, and efficiency. A rewards matrix of  $M^*N \times M^*N$  was created, where each row symbolized a state and each column symbolized an action (next state). Each neighbor state in the actions column were given a value of 0, non-neighbors were given a value of  $-\infty$ . The goal state and any state that was a neighbor to the goal received the highest reward value. A Q-matrix was also created the same way as the reward matrix, however, the goal was set to 0 to be updated by the temporal difference learning algorithms. Both the reward and Q-matrix represented the true map of the world without terrain information [2]. Other factors needing tuning were the goal reward, the discount factor, and the punishment weight. These were found for both algorithms through the control testing on a predefined map. Also, each algorithm was run for 10,000 episodes with an epsilon of 0.2 meaning that 20% of the episodes would have next state actions chosen at random under the  $\epsilon$ -greedy selection policy. The updated Q-matrix is used to determine the best path based on the highest rewards for each action at a given state.

### **Q-Learning Algorithm**

To implement the Q-learning algorithm, the high-level steps outlined in Fig. 2.2 were used. As shown by the algorithm, a random state is chosen from the generated Q-matrix used to hold the values to determine the best path. Then an action is taken in the current state based on the  $\epsilon$ -greedy selection policy. Using this action, the next state transition is determined as well as the reward for taking the action. These values then update the Q-matrix at the current state as described by (2.5). This is then repeated with the next

```

Choose a Q(s,a)
For each episode
    Set state s
    while s is not goal
        Choose action a from s using
            epsilon policy from Q
        Take action a, observe reward r
            and next state s'
        Update Q-value as follows
            Q(s,a) <- Q(s,a) + $\alpha [r+ \gamma \max_{a'}{Q(s',a')} - Q(s,a)]
            set s to s'
    Repeat until s is the goal state

```

Figure 2.2: Q-Learning Algorithm [7]

state, until the current state is the goal. As shown in the algorithm, the Q-matrix at each state is updated based on a learning parameter,  $\alpha$  which is set to 1 in this application to assure quick learning. This matrix also depends on a discount factor denoted by  $\gamma$  which is set to 0.8 to give importance to future rewards [7]. This was repeated for 10,000 episodes where at the start of each episode a random starting state was chosen.

### Sarsa Algorithm

To implement the Sarsa algorithm, the high-level steps outlined in Fig. 2.3 were used. As shown by the algorithm a random state is chosen from the generated Q-matrix used to hold the values to determine the best path. Then an action is taken in the current state based on the  $\epsilon$ -greedy selection policy. Then until the state present is not the goal, the action will be taken and the next state transition is determined as well as the reward for taking the action. Another action is chosen using the  $\epsilon$ -greedy selection policy for the next state. The Q-matrix at the current state is then updated as described in (2.6). As shown in the algorithm, the Q-matrix at each state is updated based on a learning parameter,  $\alpha$  which is set to 1 to assure quick learning. This matrix also depends on a discount factor denoted by  $\gamma$  which is set to 0.3 to give importance to future rewards [7]. This was repeated for 10,000 episodes

```

Choose a Q(s,a)
For each episode
    Set state s
    Choose action a from s using epsilon
        policy from Q
    while s is not goal
        Take action a, observe reward r
            and next state s'
        Choose action a' from s' using
            epsilon policy from Q
        Update Q-value as follows
        
$$Q(s,a) \leftarrow Q(s,a) + \alpha [r + \gamma Q(s',a') - Q(s,a)]$$

        set s to s'
        set a to a'
    Repeat until s is the goal state

```

Figure 2.3: Sarsa Algorithm [7]

where at the start of each episode a random starting state was chosen.

### 2.1.3 Results

To validate the effectiveness of the two algorithms, each was run with a 5x5, 10x10, 15x15, and 25x25 size world with randomly generated terrains. Each was run in 10 different terrain maps at each world size. The results from this can be found in Table 2.1. As shown, it is apparent that the Q-Learning algorithm performs best in terms of cost. It chooses the shortest and safest path. Also, as the world size gets bigger, the Q-Learning algorithm takes advantage of the increase in space and terrain to lower its cost per move. Sarsa also does this, but only slightly. The Sarsa algorithm is the fastest, while it may not be the most efficient. It is shown to finish well before the Q-Learning algorithm on larger maps. However, the Q-Learning algorithm has a higher rate of convergence, where the Sarsa algorithm is prone to diverge at larger map sizes. In the 25x25 world size, the Sarsa algorithm failed to converge, increasing the discount factor for the algorithm at this world size to 0.8 allowed for convergence. Examples of paths found by

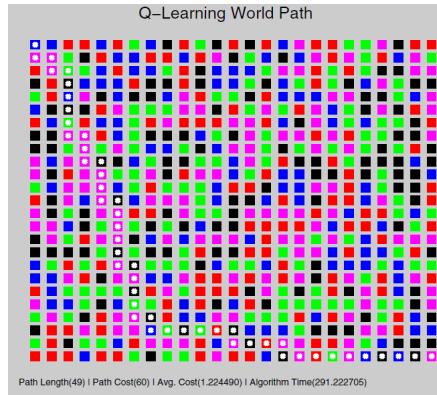


Figure 2.4: Q-Learning Algorithm Path Example with Punishment

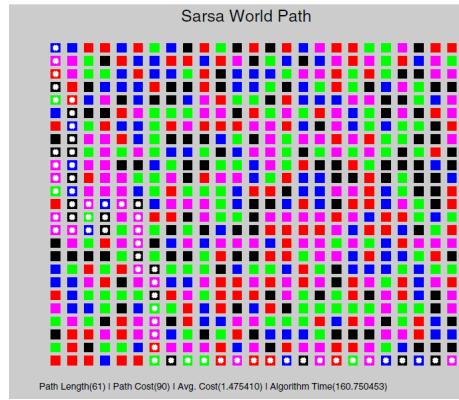


Figure 2.5: Sarsa Algorithm Path Example with Punishment

the Q-Learning algorithm and Sarsa algorithm at a world size of 25x25 are shown in Fig. 2.4 and Fig. 2.5, respectively.

In consideration of natural disaster relief and rebuilding, the Q-Learning algorithm is the best. It takes the longest to converge, but will find the most efficient path, that is time conscience and safe for itself. Sarsa, tends to favor the goal over safety in these situations because of its exploitation over exploration strategy.

#### 2.1.4 Conclusion

Reinforcement learning is a series of methods and algorithms used to pseudo map out the way a living being makes decisions. Just like a living being, a decision cannot be proven right or wrong until it has been made [2]. This

Table 2.1: Q-Learning and Sarsa results across several world map sizes

		<b>Q-Learning</b>	<b>Sarsa</b>
5x5	Avg. Path	9	9
	Avg. Cost	12.3	18
	Avg. Unit Cost	1.3667	2.000
	Avg. Time (s)	2.396	2.2731
	% Converge	100	100
10x10	Avg. Path	19	23.667
	Avg. Cost	23.4	38.8
	Avg. Unit Cost	1.2316	1.6260
	Avg. Time (s)	12.7255	12.2337
	% Converge	100	60
15x15	Avg. Path	29	60.75
	Avg. Cost	32.2	80.0
	Avg. Unit Cost	1.1103	1.3110
	Avg. Time (s)	37.8544	33.4482
	% Converge	100	40
25x25	Avg. Path	49	63.2*
	Avg. Cost	51.3	101*
	Avg. Unit Cost	1.0469	1.6005*
	Avg. Time (s)	337.8118	186.467*
	% Converge	100	50*

\*with a discount factor of 0.8

methodology gives way for a system to learn its environment and discover patterns not easily recognizable. Used in the case of natural disaster response or ruins exploration, reinforcement learning, specifically temporal difference learning, can be used to explore the area, as well as build an efficient navigation map. In terms of the experiments done in this paper Q-Learning does much better than Sarsa in creating an efficient and safe path from a start to a goal. Using multiple synchronized systems across a map, where each system would represent an episode, could reduce the time limitation found in Q-learning. In this type of problem, Q-Learning does the best because of the importance it places on exploration. On known, non-variable maps, Sarsa will do better because it can exploit rewards in future states early.

# Chapter 3

## Apprenticeship Learning (Inverse Reinforcement Learning)

Apprenticeship Learning, or inverse reinforcement learning, is the process of learning a cost/reward function to move from state to state. In these cases, the system observes and attempts to learn from another system that performs some task. Unlike reinforcement learning, the cost-function does not need to be known. Therefore, this type of system takes in an expert's policy (through demonstration) and learns a cost function based on its own state features and features perceived from the expert [9].

In [8] an imitation learning (inverse reinforcement learning) algorithm is used in two ways, one with supervised learning and one with reward shaping. In supervised learning, the imitator observes a demonstrator's motion and attempts to model it. One issue with this is that this is a black box learning technique. The imitator can observe the final state of the demonstrator, but it has no knowledge of hidden states and actions used to create the motion. This can be mitigated if the imitator knows the inverse model of the demonstrator system. Hamahata et al. [8], assumes that the imitator obtains the estimated actions over a discrete time from the demonstrator. This then allows the imitator to find the optimal control to imitate the demonstrator using least squares or ridge regression. In reward shaping a more implicit approach to imitation learning is used. This method attempts to create a reward function from the observations of the demonstrator. In many cases, changing a rewards function also changes the optimal policy of the system. However, the rewards function found is designed to be included as an aggregate term to the underlying rewards function. The additional rewards

function is defined as ( 3.1).

$$r^{\text{sub}}_{t+1} = \gamma\phi(x_{t+1}) - \phi(x_t) [8] \quad (3.1)$$

The value of this is added to the value of the underlying rewards function. In the additional rewards function, the  $\phi$  term is defined when it is equal to the optimal value function. Using this shaping method the imitator has a faster learning rate [8].

One challenge of inverse reinforcement learning is limited demonstrations and not performing the exact task as the demonstrator. In these cases, traditional imitation learning is not a feasible option. In [4] a method is proposed to learn an optimal policy rather than just a reward. This means that the system learns at the task level rather than just matching patterns which is useful for cases where the exact motion of the demonstrator is not learned. This paper explores the inverse pendulum problem, and attempts to solve it using reinforcement learning. This approach proved to be limited by intricately complex movements, however it did demonstrate a sense of learned movement from demonstration [4].

# Chapter 4

## Deep Reinforcement Learning

An advantage to using neural networks with inverse reinforcement learning stems from that fact that not everything can be observed. The demonstrator, or "expert", may not reach every state or move stochastically. Therefore, neural networks allow the system to generalize for unknown states and obstacles. Also, for complex systems with numerous state-action pairs, maintaining a Q-table with that much data becomes infeasible. Neural Networks are great for finding complex features in complex functions, therefore, the traditional *Q-function* is replaced by a network [11]. The Neural Inverse Reinforcement Learning algorithm in [27] is a model-free implementation of deep reinforcement learning. It uses several sensors on the robot as inputs into the neural network. It also uses the current state, and any obstacle detected as inputs. Therefore, this method separates a robot's coordinates with its environment so that it can better generalize its navigation [27].

Atari has been a common platform for Deep Reinforcement Learning. Specifically, Deep Q-networks (DQN) developed by DeepMind (owned by Google) uses pixels and scores from classic Atari games to learn the cost function for the game to maximize the score [13]. Most neural network implementations of Q-learning are unstable or fail to converge due to non-linear functions. By using deep convolutional neural networks and storing an agent's experience in replay memory a more stable loss function can be determined. This method also applies Q-learning updates on mini-batches of experience that are drawn at random from a pool of samples generated by the system's own exploration. Therefore, the system is generating its own dataset to learn from. While this has been a proven method to increase the efficiency of the system, especially when using a GPU, this also allows the

system to not get stuck in a Markovian loop which causes divergence because of high correlations between state spaces. This idea was further built upon by the authors in a paper published one year later [12]. In this paper, the algorithm was modified to use multiple agents to learn rather than batch-processing (replay). This method further uncorrelated state spaces because different agents would be at different states. This method proved to converge much faster, and also learn unknown environments better [12]. However, having multiple agents in a non-simulated environment can prove to be a challenge, therefore batch-processing as introduced in [13] is a more feasible option. Another implementation of this algorithm is Maximum Entropy Deep Inverse Reinforcement Learning from [26]. This algorithm applies wide convolutional layers to learn more relevant spatial features in the data it is trying to learn from. These wider convolutional layers essentially represent fully convolutional neural networks with width one. This essentially means that there is no pooling using this methodology to ensure every bit of raw input is used.

Unfortunately, reinforcement techniques lacked a benchmark. An algorithm could not be compared directly to another algorithm on performance unless the same exact task was being done. However, a benchmark was created in [6] for continuous control reinforcement learning tasks based on control, humanoid locomotion, occluded tasks, sensor limits, delayed action, combinational tasks. To do this all tasks were standardized into a finite-Markovian decision process. Therefore, these benchmarking standards will be used to compare against future advancements in Deep Reinforcement Learning.

# Chapter 5

## Platform

Generally environments that make use of automatic guided vehicles (AGVs) have to plan the path(s) where the robots should go before installing the tracks, like magnetic strips or metal tracks; this is an investment even before using the robots. If any change to the path(s) is required to be made, then more cost is incurred. For this research a four wheeled differential drive robot has been controlled wirelessly to follow paths drawn on a graphical user interface within a workspace of 1.8m by 1.4m. The robot is controlled by correcting its orientation through visual feedback from a camera. Error analysis was performed to investigate how well the robot followed the path drawn. The estimated error of the robot is within a few centimeters of the path and can be reduced by modifying various thresholds.

### 5.1 Background

Robots whose path is controlled by the user are termed as automatic guided vehicles (AGVs). AGVs follow paths using lasers, cameras, or are physically attached to these paths. These mobile robots follow predefined routes and so, are heavily used in industries for materials handling. Apart from transporting loads, these kinds of robots are used in risky environments and explorations, like space, mines and underwater. For guiding some AGVs, lines, magnetic tracts, or wires are installed on warehouse floors. Hence, making use of these robots requires an initial investment. Vision-based AGVs follow a physical line or markers in a room: landmark based navigation. Combining these ideas a robotic system was created for dynamic path following using vision based control.

This research platform uses a wireless automatic guided robot that does not require any kind of intrusive modifications to be made to the environment apart from installing an overhead camera. The camera above the workspace tracks the robot continuously and is used to help correct it to follow a predefined path. The user is allowed to define the path the robot should follow in a GUI created in MATLAB.

Using a live image of the environment from a bird's-eye-view, the robot is tracked in real-time using ArUco fiducial markers. The live images are fed to a software GUI where the user can freehand draw any path they desire directly on top of the image. The system then sends commands to the robot to move it along this path by using the camera and ArUco markers as feedback control. Doing this, the user can draw complicated paths around a room and around obstacles for the robot to navigate.

## 5.2 System Overview

This system consisted of four main sub-components; the Robot, OpenCV, MATLAB, and the Environment workspace itself. Fig. 5.1 shows the entire system architecture and work flow using these four main components.

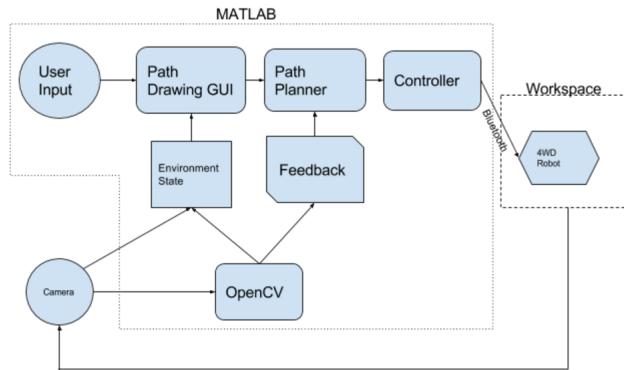


Figure 5.1: System Work Flow

### 5.2.1 Differential Wheel Drive Robot

Four-wheeled differential drive robotics platforms are commonly used in navigation applications. The system used in this setup was the 4WD Rover I from Lynxmotion, as shown in Fig. 5.2.

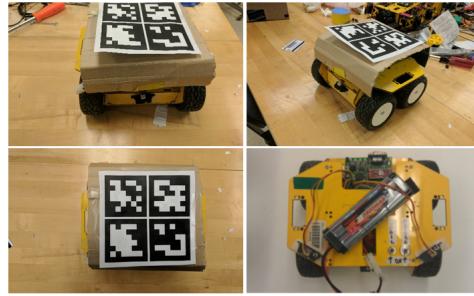


Figure 5.2: 4WD Rover I from Lynxmotion

In this type of system each pair of wheels can be controlled independently from another pair. Therefore, this allows the robot to rotate about its center, which gives it greater control and speed on sharp angled turns. To control these motors, pulse width modulation (PWM) signals are given to each motor. This type of signal is usually at a fixed frequency and is a digital square wave ranging from 0V to  $V_{max}$ .  $V_{max}$  for the specific application performed was 7.2V. The signal determines on and off time for the motor based on its duty cycle. A duty cycle of 50% will result in the motors being powered at half their capacity. A duty cycle of 10% will result in the motors being powered at a tenth of their capacity. A duty cycle of 100% will result in the motors being powered at full capacity which looks like a DC signal at 7.2V. These duty cycles are shown in Fig 5.3. To control the duty cycle, an Arduino Uno was used which is the main controller used for the four-wheeled robot. A value of 0 to 255 was used to set the duty cycle between 0 and 100%. In this case 0 would represent 0% or full stop, 128 would represent 50% or half speed, and 255 would represent 100% or full speed.

This robot was programmed through the Arduino to take in serial commands to determine motion. Single character commands were used as shown in Table 5.1. To take in these serial commands, a Bluetooth module was connected to the Arduino's TX and RX lines. The module used was the

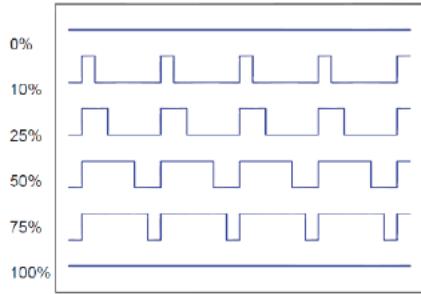


Figure 5.3: Various duty cycles on PWM signals

HC-06, which is a low-power slave Bluetooth module. Once the Arduino receives a movement command it sends the appropriate PWM signals to the Pololu Dual MC3926 motor controller board which was wired directly to the motors, battery, and Arduino Uno.

Table 5.1: Movement Commands

Command	Function	Wheel Movement
F	Move Forward	All wheels turn in same direction, forward
L	Turn Left	Left side wheels reverse, right side wheels forward
R	Turn Right	Right side wheels forward, left side wheels reverse
S	Stop	Duty cycle of all wheels set to 0%, no movement

### 5.2.2 OpenCV

OpenCV is a comprehensive computer vision toolbox used for a variety of real-time applications such as tracking, facial recognition, segmentation, and classification. It is originally written in C++ but wrappers have been made to allow it to be used in Python and MATLAB. For this system, the only component of OpenCV being used is video stream capture and the

ArUco Marker detection. ArUco markers are pattern-based black and white squares that can be easily identified through OpenCV. Since OpenCV knows what patterns to look for, it can identify ArUco markers in an environment and determine the corners of them as well as the orientation. For this application four ArUco markers were used as shown in Fig. 5.4. This marker array was printed to the size 14.5 cm by 14.5 cm and was mounted flat on top of the robot so that the center of the array was at the center of the robot. The top two markers were facing in the direction of the front of the robot and were used as a reference to determine the orientation of the robot.

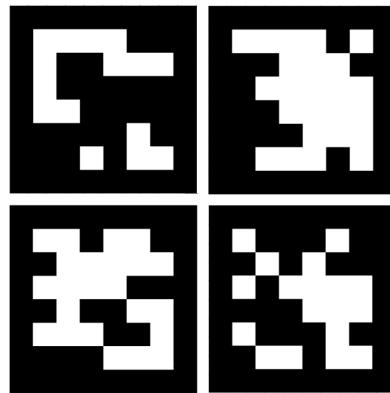


Figure 5.4: ArUco Array of four markers

### 5.2.3 MATLAB

MATLAB is a powerful computing platform. It uses a scripting based language which is ideal for fast prototyping. It has also been built to be computationally efficient especially in terms of matrix math and neural networks. In this system, MATLAB is used to create the main GUI, interface with OpenCV, interface with the camera, path creation and planning, and sending serial commands to the robot via Bluetooth.

### 5.2.4 Environment

The environment that the robot resides in can be variable. However, the surface on which the robot roams needs to be navigable by the robot chassis. Also, the environment needs to be well lit so that the markers can be

detected. A webcam was placed on the ceiling of the room which was about 8 feet from the ground and faced perpendicular to the floor. The webcam used is shown in Fig. 5.5. The webcam itself was used with a resolution of 640 px by 480 px. This created view-able workspace of about 1.8m (6 feet) by 1.4m (4.5 feet). The robot was placed within this static workspace. The webcam was connected to the computer running the MATLAB application through USB.



Figure 5.5: Webcam used for tracking

### 5.3 Procedure

As an overview, the procedure of running this system takes into account all four main components. First, the camera is set-up to view a bird's-eye view of the workspace. The robot is then placed within that workspace with its ArUco markers. The application is then started using MATLAB. OpenCV functions are used to detect the ArUco markers and determine the millimeter to pixel ratio since the ArUco markers are a known size. Once the robot is detected the user is prompted to draw a path over the current image. This path is then fed to the path planner code where the robot iteratively follows the following steps:

- The robot faces desired point
- The robot goes towards point
- Feedback from OpenCV to determine whether or not robot meets success thresholds
- The robot's movement is adjusted to meet threshold if needed

- Once the point is reached, the desired point moves to the next point

Once the robot completes the desired path the errors from the robot movement versus the ground truth path are found. The user is then prompted to draw another path. This basic procedure is shown in the flowchart in Fig 5.6.

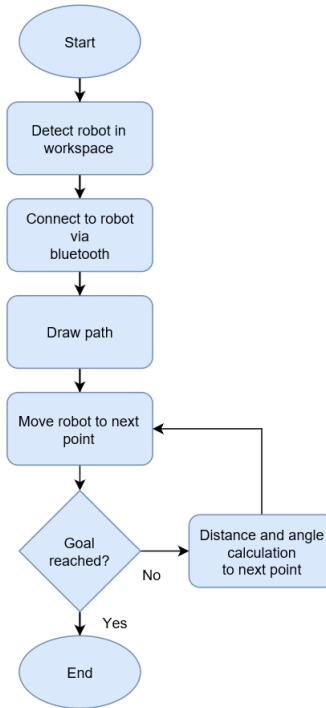


Figure 5.6: Basic Flowchart of Procedure

### 5.3.1 Detecting ArUco Markers

To detect the 4 ArUco markers on the robot, the ArUco marker library within OpenCV was used. This library contains a function *detectMarkers* which takes in an image (that contains the markers), a dictionary of marker definitions, and some optional parameters. It then returns the corner locations and ID numbers for all markers detected. Once the corners and IDs are found, as shown in Fig. 5.7, the center point for the array is also found. This is done by using a centroid function that uses the points of the 4 corners closest to the center of the image. This is shown in Equation 5.1 and 5.2 where n is equal to 4 to represent the four points used. This center point is

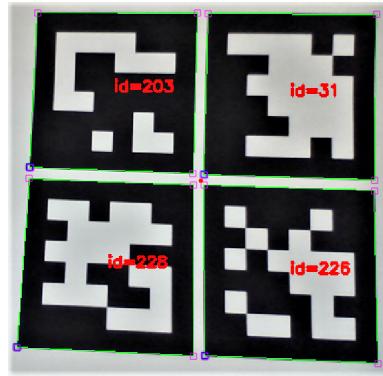


Figure 5.7: Annotated ArUco Array of four markers

used to determine the center of the robot, which is also its turning axis.

$$center_x = \frac{\sum_{k=0}^n x_k}{n} \quad (5.1)$$

$$center_y = \frac{\sum_{k=0}^n y_k}{n} \quad (5.2)$$

### 5.3.2 Drawing a Path

Once the robot and the markers are detected in the frame of the workspace the user is prompted to draw a path. This is done by using the built in *imfreehand* function on the current image of the workspace. This allows the user to freely drawing any path within the workspace as shown in Fig. 5.8. Once the user draws the path the path is represented by a solid red line with a white star at its first point and a yellow star at its end point.

### 5.3.3 Accuracy Thresholds

To ensure the robot follows the path accurately, thresholds were created for the allowable distance it was from the desired point and its orientation with respect to the next point. To identify the distance in pixels between the desired point and the robot, denoted as  $d$ , the euclidean distance between the desired point and the center point of the ArUco marker array was found. This equation is shown in 5.3, where  $x_c$  and  $y_c$  are the pixel coordinates

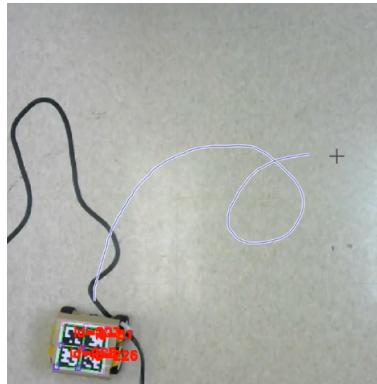


Figure 5.8: Path Drawn by user in GUI

for the center of the robot, and  $x_g$  and  $y_g$  are the pixel coordinates for the desired point (current point on path trying to be reached). If the value of  $d$  was less than the threshold (by default was 20 pixels) then the robot met the distance criteria.

$$d = \sqrt{(x_c - x_g)^2 + (y_c - y_g)^2} \quad (5.3)$$

The second criteria was the orientation of the robot to the next point. This was important to meet so that the robot was always following the trajectory of the path. To do this 3 vectors were found and plotted. The first vector was from the center of the robot to the desired point ( $V_g$ ). The second vector was from the center of the robot to the front of the robot between ArUco markers 31 and 203 ( $V_o$ ). The third vector was from the front of the robot to the desired point ( $V_n$ ). These vectors are shown in Fig. 5.9.

To use these vectors to modify the robot's current heading, they were first normalized by dividing the vector elements ( $x$  and  $y$ ) by the magnitude of the vector. The dot product was then taken between  $V_o$  and  $V_g$  as shown in Equation 5.4. Where  $a$  is a value between 1 and -1. The desired orientation value of the robot is 1 which means that the robot's heading is facing towards (parallel) the desired point. At -1, the robot is facing the opposite way of the desired point. An orientation threshold was defined to be close to 1 to make the robot more accurate in path following.

$$a = V_o \cdot V_g \quad (5.4)$$

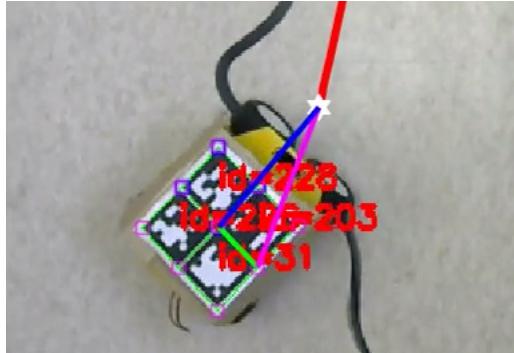


Figure 5.9: Setting Robot Heading Direction: white star indicates desired point; green indicates current heading direction between center and front of robot ( $V_o$ ); blue indicates desired heading direction between center and desired point ( $V_g$ ); magenta indicates difference between the directions which is the vector from the front of the robot to the desired point ( $V_n$ )

The  $a$  value gives us information on heading but does not tell the optimal way for the robot to turn. The system could have been setup to turn left every time the heading was less than the threshold until the orientation threshold was met, but that would not be optimal. To determine the direction the robot should turn to make the most optimal decision to meet the orientation threshold Equation 5.5 is used. Where the two parameters of the  $\text{atan}2$  function are the  $\sin(\theta)$  and  $\cos(\theta)$ . This function returns the four quadrant inverse tangent that will give a  $\theta$  value between  $180^\circ$  and  $-180^\circ$ . This gives us the exact angle between the  $V_g$  and  $V_n$  vectors to determine which direction to turn. Since this angle is in reference to the current heading of the robot, a positive  $\theta$  value will mean that the robot needs to turn left, and a negative  $\theta$  value will mean that the robot needs to turn right.

$$\theta = \text{atan}2(V_{g_x} * V_{n_y} - V_{g_y} * V_{n_x}, V_{g_x} * V_{n_x} + V_{g_y} * V_{n_y}) \quad (5.5)$$

Therefore, using the values  $d$ ,  $a$ , and  $\theta$  the robot's movement is altered using Algorithm 1. To ensure that the robot has enough time to act before another command is sent to it the movement commands were only sent once a certain time threshold was met. Therefore, some movement commands were skipped to allow the robot's movement to be smooth and to ensure that the serial buffer on the robot would not be overflowed.

---

**Algorithm 1** Robot Next Move

---

```

if  $d < distanceThreshold \wedge a > orientationThreshold$  then
     $desiredPoint \leftarrow nextPoint$ 
else if  $a < orientationThreshold$  then
    if  $\theta > 0$  then
        TurnLeft
    else
        TurnRight
else if  $d > distanceThreshold$  then
    MoveForward

```

---

### 5.3.4 Following the Path

Using Algorithm 1, the robot went to each point on the path drawn by the user. As it did this, the center point of the robot was plotted every time a point was reached. Using this data the, the overall error of the physical robot path was determined. Fig. 5.10 shows a snapshot of a robot following a path.

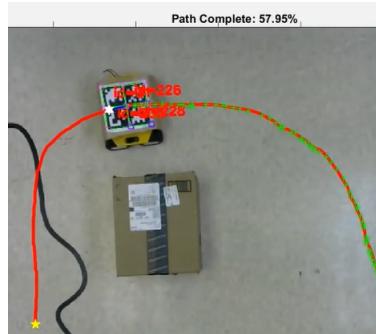


Figure 5.10: Robot following the Path: white star indicates next point to reach; green markings show robot's past positions, yellow star indicates goal

### 5.3.5 Accuracy Analysis

To determine the overall accuracy of the robot, error between the physical path the robot took and the actual path was measured. This accuracy was improved by changing the distance and orientation thresholds for the system. At the end of each path taken, the error magnitude for each point is displayed as well as the standard deviation of error, mean error, maximum

error, minimum error, and estimated error. The estimated error is defined as the error that will be found 95% of the time or two standard deviations ( $\sigma$ ) away from the mean ( $\mu$ ), as shown in Equation 5.6.

$$Est.Error = \mu + 2 * \sigma \quad (5.6)$$

Three trials were tested with varying threshold values. Fig. 5.11, 5.12, and 5.13 show several paths with their corresponding error charts.

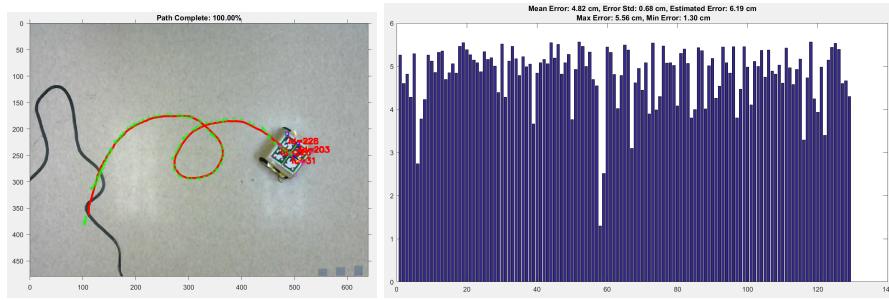


Figure 5.11: Results for robot motion with  $distanceThreshold = 20px$  and  $orientationThreshold = 0.95$

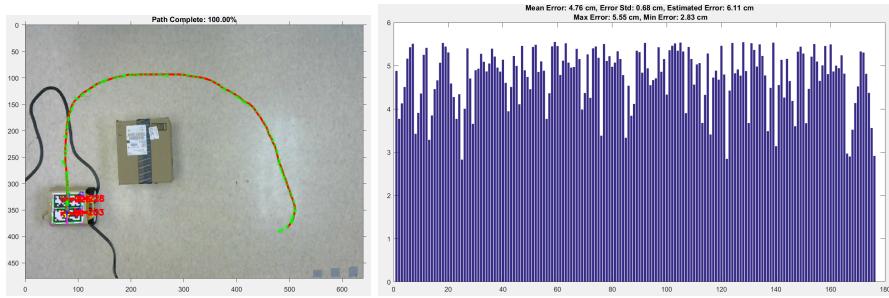


Figure 5.12: Results for robot motion with  $distanceThreshold = 20px$  and  $orientationThreshold = 0.99$

As shown in these figures, the robot's path seems to follow the path drawn by the user. Table 5.2 shows these results in terms of standard deviation of error, mean error, maximum error, minimum error, and estimated error. Each of these values is converted to centimeters from pixels by using the known size of the workspace and the ArUco marker array.

As shown, as the threshold for orientation increased the estimated error decreased. However, the largest contributor to the decrease in estimated

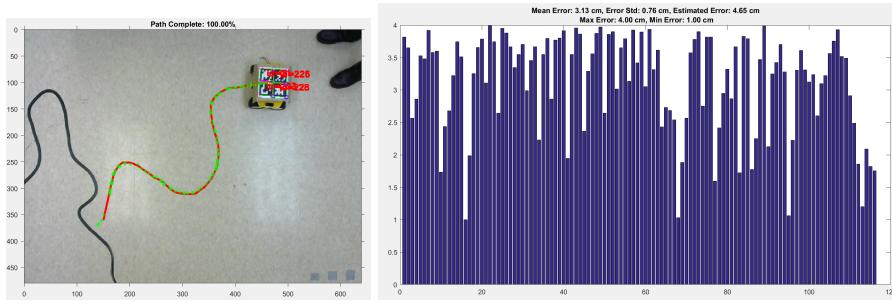


Figure 5.13: Results for robot motion with  $\text{distanceThreshold} = 15\text{px}$  and  $\text{orientationThreshold} = 0.99$

Table 5.2: Error Analysis

$a^*$	$d^*$	STD	Mean Er- ror	Max Er- ror	Min Er- ror	Estimated Error
0.95	20	0.68	4.82	5.56	1.30	6.19
0.99	20	0.68	4.76	5.55	2.83	6.11
0.99	15	0.76	3.13	4.01	1.00	4.65

Error in cm,  $d^*$  in px

$a^*$  = orientation threshold,  $d^*$  = distance threshold

error was the decrease in the distance threshold. Upon performing further tests, it was found that if the distance threshold is decreased then the orientation threshold must be increased, otherwise the robot would reach each point on the path but it will be unable to accurately produce the path due to the over-correction it will need to do. The orientation threshold contributes to lowering the over-correction needed. It was also found that lowering the distance threshold too much would result in the robot not being able to reach a point. This is because while the ArUco marker detection is robust, it is not very stable and does shift by a few pixels causing errors in the system when the distance threshold is too small. Another threshold needing to be modified was the time threshold. This threshold was used to determine how often movement commands would be sent to the robot. If it was too high the robot would overshoot points, if it was too low then the robot would get overloaded with commands. Therefore, a threshold of 50 ms was used after testing various values.

# **Chapter 6**

## **Conclusions**

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## Appendix A

### A Long Proof

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