A Sorted RSSI Quantization Based Algorithm for Sensor Network Localization

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Abstract

Range estimation is essential in many sensor network localization algorithms. Although wireless sensor systems usually have available received signal strength indication (RSSI) readings, this information has not been effectively used in the existing localization algorithms. In this paper, we present a sensor network localization algorithm based on a sorted RSSI quantization scheme that can improve the range estimation accuracy when distance information is not available. The range level used in the quantization process can be decided by each node using an adaptive quantization scheme. The new algorithm can be implemented in a distributed way and has significant improvement over existing range-free algorithms. The performance advantage for various sensor networks is shown with experimental results from our extensive simulation with a more realistic radio model.

1. Introduction

Localization plays an important role in wireless sensor network applications when the positions of nodes cannot be decided beforehand or, if nodes are mobile. Future sensor networks will involve a large number of sensor nodes densely deployed over physical space [5]. Many applications require knowing the positions of the nodes. Nodes could be equipped with a global positioning system (GPS) to provide them with absolute positions, but this is currently an infeasible solution. With a network of thousands of nodes, it is unlikely that the position of each node can be pre-determined.

The existing work on sensor network localization can be categorized into range-free/aware and anchor-free/aware approaches [1, 3, 4, 14, 17, 19, 22]. In the range-free (also known as connectivity-based, connectivity-only) approach, the algorithms do not need range hardware support and are immune to range measurement errors (assuming that range error does not affect connectivity) while providing less accurate localization results. In the range-aware approach, the algorithms require more sophisticated range hardware support while providing more accurate localization results than the range-free algorithms when the range measurement is accurate. The range-aware algorithms include the convex constraint satisfaction method [4], the DV-based approach [14, 19], collaborative multilateration [1], and MDS-MAP [22]. Among them, the DV-based approach [14, 19] and MDS-MAP also deal with the range-free case.

Much less work has been done on utilizing other range-related information for sensor network localization. Received signal strength indication (RSSI) is a measure of the RF energy received and is closely related to range. RSSI is supported by sensor node hardware such as the Berkeley motes [23]. For the localization purpose, the information provided by RSSI or similar types of measurements is less than range but more than a connectivity-only hop count, and it can be used to improve the accuracy of connectivity-only localization algorithms.

There are other types of measurements related to range, such as sound strength and light luminosity and brightness. We refer to this range-related information as partial range information (PRI). PRI is defined as any type of measurement being monotonically increasing or decreasing and bijective to the range measurement. A similar requirement of RSSI can be found in RightSpot [9].It is called partial range information because these types of measurement can not be easily converted to accurate distance measurement due to the unknown exact mapping between PRI and range, yet the "comparing" of PRI values can be arranged so that it corresponds to comparing of distance values based on their monotonic bijective relationship.

The usage of PRI enlarges the scope of the "range" parameter in the localization spectrum, which can significantly improve the performance of localization algorithms. The process that uses PRI values is called range quantization. Range quantization (RangeQ) [12] associates each 1-hop connection of unknown distance to a value referred to

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as range level. RangeQ uses range information (pseudo-PRI since range is bijective to itself) instead of RSSI to test the effectiveness of range quantization.

In this paper, we present our sorted RSSI quantization scheme (SRangeQ) that uses sorted RSSI readings, and we compare SRangeQ with the quantized RSS method (RSSQ) [16] using simulation with a more realistic radio model for setting up the connectivity of the network.

2. Related work

Yemini [24] presents some theoretical algorithmic aspects of the position-location problem. A detailed survey of the area is provided by Hightower and Borriello [7]. Many systems use some kind of range or distance information and rely on powerful anchor nodes with unusual capabilities, such as directional signals and radio or laser ranging devices. Distance estimates can also be obtained from time-of-arrival (ToA) measurements. ToA methods have better accuracy, but may require additional hardware at the sensor nodes to receive a signal that has a smaller propagation speed than radio, such as ultrasound [20, 13]. Localization techniques for mobile robots use additional odometric measurements for estimating the initial positions [8, 18], which are not available on regular nodes in wireless sensor networks.

Doherty's [4] convex constraint satisfaction approach describes a method for localizing sensor nodes based only on connectivity. The method formulates the localization problem with isotropic communication as a feasibility problem with convex radial constraints. The method requires centralized computation. For the method to work well, it needs anchor nodes to be placed on the outer boundary, preferably at the corners, to make the constraints sufficiently tight. When all anchors are located in the interior of the network, the position estimation of outer nodes can easily collapse toward the center, which leads to large estimation errors.

The "DV-based" approach by Niculescu and Nath [14] is simple and efficient. It is among the best of triangulation-based methods. Bischoff and Wattenhofer [2] have obtained theoretical lower bounds for connectivity-based algorithms in their recent work. Similar to APS, their unit-disk model of the connectivity keeps the connectivity unchanged in an erroneous environment. A more realistic model is assumed for our SRangeQ.

MDS-MAP[22, 21] uses connectivity or local distance measures between unknown nodes as well as those between unknown and anchors, thus leading to better localization results. It uses an optimization method referred to as multi-dimensional scaling (MDS) to solve the localization problem. In the connectivity-only approach, MDS-MAP obtains each pairwise distances by multiplying the shortest-hop-distances with the radio range, which can be improved with available PRI measurements.

Another so-called range-free algorithm [6] uses special long-radio-range anchors to form multiple triangles to cover the whole network. To check the status of a node being inside of a triangle, this algorithm requires RSSI readings so that each node can determine if it is farther away or closer to a fixed node than some other nodes. Our method also uses RSSI for comparison, but the purpose of comparing is to find the range information, not inside or outside of a triangle.

Although we have developed RSSI quantization without being aware of the recent work of Patwari and Hero [16] on the idea of using proximity and quantized RSS (received signal strength) to improve the localization accuracy, our approach and their approach share the same objective, but use the RSS values in different ways. In their approach, an RSS-to-range mapping function is needed to convert the quantized RSS to range. This can be a difficult task due to the multiple existence of radio propagation models. Finding the right model imposes a considerable amount of field-test overhead. Our method does not need a mapping function. It obtains the range estimation via quantization, not by direct conversion between RSSI and distance. RightSpot [9] also uses RSSI for the localization purpose, but the algorithm itself is not a triangulation-based method.

The existing RSSI based localization algorithms use RSSI in two different ways: direct mapping between RSSI and distance (location) and indirect mapping. Ours is in the category of indirect mapping. The most significant difference of our approach with other indirect mapping algorithms is that we uses RSSI to obtain estimated distances and we use a special quantization model to approximate the mapping, which is shown to be more accurate and robust than pure range-free algorithms according to our simulation results.

3. Range estimation

For the range estimation problem, the network is modelled as a connected, undirected graph $G = (V, E)$, where V is the set of sensor nodes, and E is the set of edges connecting neighboring nodes. Each edge $e(v, w)$ is associated with a value $z \in Z$. Let (x, y) be the unknown coordinates of $v \in V$. Let $A \subset V$ be the set of anchors with known coordinates. The range estimation problem is to find the Euclidean distance $d \in D$ for each $e(v, w)$.

In a general localization model, the localization problem is solved if the 1-hop z value is the measured distance $d \in D$. If the range information is not available, d can be assigned the same value (radius in MDS [22], corrected hop distance in APS [14]) for each of the 1-hop connections. In our RSSI approach, z is the value of RSSI.

The advantages of utilizing RSSI are as follows:

1. RSSI is supported by standard 802.11 NICs [15]. It is also provided by some sensor nodes, e.g., the Berkeley motes [23]. Therefore, it does not need extra hardware while providing this information.
2. RSSI is not used by the existing connectivity-based algorithms. The localization accuracy can be improved with the help of RSSI.
3. The RSSI-aided range estimation module can be plugged into any shortest-path-distance based connectivity-only algorithms. The cost of implementation is low.
4. SRangeQ is distributed, and there is no flooding in communication. The mapping computation from RSSI to range is $O(n\log(n))$ for each node, where n is the number of nodes in a 1-hop neighborhood. Therefore, both the communication and computation costs are relatively low, making the additional costs considerably small.

4. Sorted RSSI quantization

The concept of sorted RSSI quantization is similar to that of image quantization in image processing except that the quantization is not from continuous RSSI to discrete RSSI. The process of sorted RSSI quantization starts with sorting RSSI readings to obtain a sorted range list. It then applies a quantizer on the list to generate range estimation. In the quantization process, range level represents the level of range in a hop, which is similar to the gray level representing the intensity of a pixel in a gray-level image. The number of range levels in range quantization is referred to as range-level resolution. For example, the range-level resolution in range-free localization algorithms is $s = 1$, since each 1-hop connection has one range level with the same range-level value (radius as in MDS-MAP [22], distance correction as in APS [14]). For the sorted RSSI quantization algorithm, the range-level resolution is an integer s, where $s = 2, 3, \cdots, n$.

Given a range-level resolution, a quantization scheme assigns a quantized range-level value to a 1-hop connection based on its RSSI value. The quantization scheme with a fixed range-level resolution for all the sensor nodes is referred to as fixed quantization. By using this method, one has to find the best range-level resolution on a trial-and-error basis for the whole network. The range value assignment can be defined as a mapping between an RSSI set and a range level set with known RSSI values and a fixed range-level resolution s. Since we assume there is no known mapping function between RSSI and its corresponding range value, new mapping models need to be found on a trial-and-error basis. Based on extensive simulations, an area-proportional model is found to be effective and accurate in mapping RSSI to quantized range.

To start the quantization process, a predefined number of range-level resolution and the RSSI values from each node to its neighboring nodes are needed. Each node needs one round of communication to obtain the RSSI readings from its neighboring nodes. Therefore, the local RSSI set does not have to be calibrated for the whole network, which is a difficult task due to shadow fading, Rayleigh fading, and multipath in radio propagation [11]. After obtaining the RSSI values, the sorted RSSI quantization algorithm follows 2 steps to assign a range value for each 1-hop connection.

4.1. Step 1: Sorting RSSI values

Step 1 is a sorting process, the first step toward the mapping of RSSI to range. At the end of step 1 each node i knows the order of closeness of all its neighboring nodes. As the only known information about the mapping between RSSI and range is that it is monotonic decreasing, the RSSI value set needs to be sorted in the descending order in order to obtain the sorted range list in the ascending order.

For a node i in a randomly-deployed network, it has a set of connected 1-hop neighbors Nb. Let $p(i, g)$ be the $RSSI$ between node i and $g \in Nb$. All $rssi(i, g)$'s for $g \in Nb$ can be sorted, and the nodes in Nb can be rearranged in the descending order of their RSSI values. The result of this step is an ordered node list $g \in Nb$ in the increasing range order.

4.2. Step 2: Quantization

The goal of this process is to estimate $\hat{d}$, the distance of each 1-hop connection. The existing range-free algorithms, APS [14] and MDS [22], sets all the 1-hop distances to the same value.In the sorted RSSI quantization, a hop is subdivided into s sub-unit hops of size u, where $u = R/s$, R is the radius and s is the range-level resolution. Each 1-hop connection is assigned a range-level value of $u, 2u, \cdots$ or su. If the assignment is correct, it obtains a range-level value closer to the true distance for each 1-hop connection than that of R as in MDS [22] and a distance correction as in APS [14]. The problem left unsolved is, to find an effective distance distribution model so that the number of nodes falls into each range level j can be estimated with the help of the range order obtained from Step 1.

For each node i, let the size of its neighboring node list Nb be n and Nb be arranged in the increasing range order which is obtained from Step 1. The quantization process involves dividing the maximum 1-hop range (R) into s smaller quantities of size u and assign an appropriate quantized range-level value, ju, to each neighbor in Nb.

With range-level resolution s, Nb is divided into s clusters with m_j nodes in the j-th cluster C_j, where $\sum m_j = n$, $j = 1, 2, \cdots, s$. The range-level value for each node $k \in C_j$ is set to $\hat{d}_{ik} = ju$.

We have considered the following two models: simple linear distribution model and area-proportional distribution model. In the simple linear model, the distribution adopts a uniform model, where $f_{(j)} = j/s, j = 1, 2, \cdots, s$, which is proved to be not accurate after extensive simulation. The second one improves the localization accuracy significantly, which is explained in details as follows.

The area-proportional model does not use any mapping function, which is similar to the simple linear model. Unlike the simple linear model, this model does not divide the RSSI set evenly. Instead, it estimates the distribution of nodes in the neighborhood. Assuming that the nodes are randomly distributed, we know that the nodes are equally likely to fall into any spot in a circle with $radius = R$. We cut the whole circle into s annuli. The area of the j-th annulus formed by two circles of radii $(j-1)u$ and ju is:

$$A_j = \pi((ju)^2 - ((j-1)u)^2) = \pi(2j-1)u^2 \qquad (1)$$

The expected number of nodes falling into the s annuli are:

$$\lambda\pi(2j-1)u^2 \qquad (2)$$

where λ is the node density.

The expected number of nodes falling into the circle of radius R is:

$$\lambda\pi R^2 = n \qquad (3)$$

From (3), we obtain

$$\lambda = \frac{n}{\pi R^2} = \frac{n}{\pi(su)^2} \qquad (4)$$

Based on (2) and (4), the expected number of nodes falling into the j-th annulus is:

$$\frac{\pi(n(2j-1)u^2}{\pi(su)^2} = \frac{n(2j-1)}{s^2} \qquad (5)$$

Then, the expected numbers of nodes falling into the s annuli are:

$$\frac{n}{s^2}, \frac{3n}{s^2}, \ldots, \frac{n(2j-1)}{s^2}, \ldots, \frac{n(2s-1)}{s^2} \qquad (6)$$

Therefore, m_j, the number of nodes for the j-th range-level is obtained. The corresponding quantized range-level values for the s clusters are:

$$u, 2u, \cdots, ju, \cdots, su \qquad (7)$$

The final step involves rounding the number of nodes in (6) to an integer, m_j, and make sure that $\sum m_j = n$. Since Nb is sorted in the increasing range order, C_j is obtained by picking each group of nodes of size m_j from Nb starting from $j = 1$ to $j = s$.

5. Simulation results

The simulation was carried out using MATLAB. One hundred networks were randomly created for each simulation parameter set. The communication between neighboring nodes is assumed bidirectional. A snapshot of the network deployment and the localization results are illustrated in Fig. 1(a), Fig. 1(b) and Fig. 1(c).

5.1. Simulation parameters

The simulated $RSSI$ values are converted from distances by using the power loss function:

$$\text{Path Loss} = \text{Unit Loss} + 10n_p \log(d)$$

where

Unit loss = Power Loss (dB) at $1m$ distance ($30dB$)

$n - p$ = power-delay index (3.5)

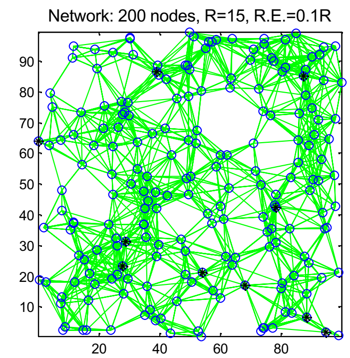
d = distance between transmitter and receiver

The first set of simulations was performed to compare the range estimation accuracy between RSSQ [16] and the SRangeQ algorithm. The range level chosen was 10 making sure that using RSSI would not make a significant difference for SRangeQ since the levels allowed for RSSI is much larger than 10. The second set of simulations was performed to observe the localization accuracy performance of APS based on the range estimation of SRangeQ. Networks of dimension 10×10 were deployed randomly, with radio range from 10.00 to 20.00. The connectivity was not guaranteed for a pair of nodes within the radio range because of environment uncertainties and vise versa for nodes outside of each other's radio range. There were 200 nodes in each network.

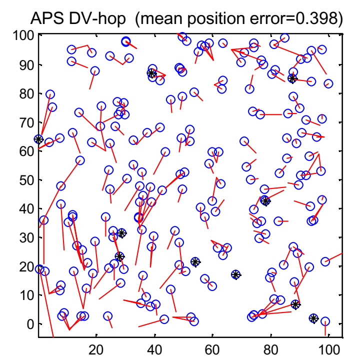
- Network size: 100×100
- Notes deployed: 200
- Radio range: $10.00 - 20.00$
- Localization algorithm: APS [14]
- Anchor rate: 5%
- Fixed range-level resolutions: $s = 2, 3, \cdots$ ($s = 1$: range-free)
- Adaptive range-level resolutions: 0.25, 0.5, 0.75, 1, 1.25, 1.5$\times$ neighborhood size.

5.2. Range estimation accuracy: SRangeQ vs. RSSQ

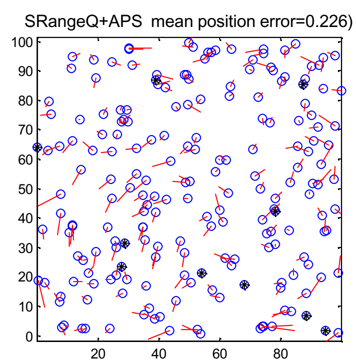
RSSQ [16] proposes a way to utilize RSSI by quantizing RSSI and then converting the quantized RSSI into a Euclidean distance using a path loss formula. In the simulation, the values of n_p were created using a Gaussian distribution with $\mu = 3.5$ and $\sigma = 0.00 - 0.50$. A threshold



(a) Network topology



(b) APS range-free localization result



(c) SRangeQ localization result with range-level resolution $s = 10$

Figure 1. Network layout and localization results of SRangeQ-APS and APS range-free with the position error presented by each short line.

$minrssi$ was set to test if a pair of nodes were connected or not. The RSSI values created in the simulation are the same for RSSQ and SRangeQ. The difference lies in their different ways to convert RSSI to range. RSSQ uses 2 steps in this operation: (1) quantize RSSI, (2) convert the quantized RSSI to distance by using the path loss function, where $n_p = \mu$.

To evaluate the performance of RangeQ and RSSQ, we designed three scenarios of the error pattern for the power-delay index n_p.

- Case 1: grid. The network is divided into 4 grids. Each $RSSI$ in the same grid shares the same n_p. This roughly mimics a network partitioned into different areas each sharing the same indoor/ourdoor environment.
- Case 2: cell. The neighbors of a node shares the same n_p. This is similar to Case 1 except that the partition is smaller.
- Case 3: random. This case assumes the corresponding n_p for each node is random. This is an extreme case of wireless communication scenario.

Fig. 2 shows the range estimation simulation result of SRangeQ and RSSQ. We observe from the result that:

- RSSQ performs better when the error of n_p is less than 0.1. This suggests that RSSQ works well when n_p is stable and known beforehand.
- The range estimation error of SRangeQ does not change much when n_p increases, which means SRangeQ is not sensitive to n_p errors. When the error of n_p is more than 0.4, SRangeQ is about 50% more accurate than RSSQ for the average 1-hop range estimation.
- In Case 3 when none of the $RSSI$ shares the same n_p, RSSQ obtains better results when $sigma$ is less than 0.2. For the other two cases, SRangeQ is more accurate when $sigma$ is more than 0.1.

From the above observation, we conclude that the advantage of SRangeQ comes from the fact that it does not need the value of n_p. Therefore, it has no overhead for the calculation and calibration of n_p, and its accuracy is relatively insensitive to n_p fluctuations. Therefore, SRangeQ is a better choice when the operating environment changes or is unknown.

5.3. Localization based on SRangeQ

Connectivity-based localization algorithms usually use hops multiplied by the radio range or a corrected hop-distance to estimate the shortest-path distance between a node and an anchor. This is not accurate because (1) the

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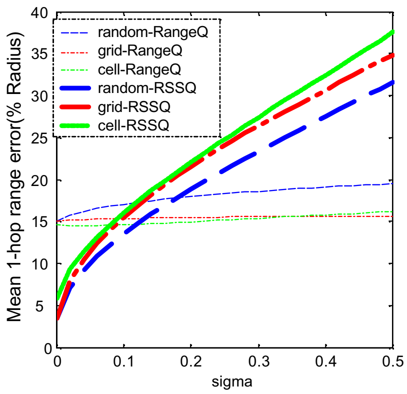

Figure 2. Range estimation error: 100×100 **network,** 200 **nodes, radius** $= 15$**, range error** $= 100\times$ **average 1-hop distance error** $/$ **radius**

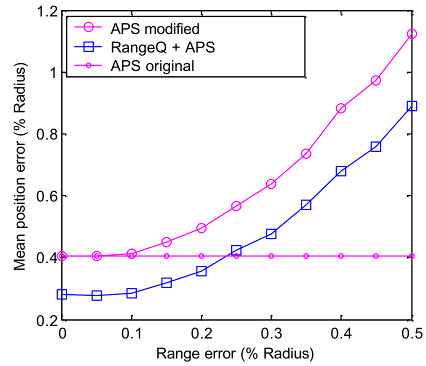

Figure 3. APS and RangeQ: 100×100 **network ,** 200 **nodes, radius** $= 15$ **(The values of range errors were converted from** $RSSI$ **errors.)**

variance of 1-hop distances can not be neglected in random networks and (2) the radio signal coverage is not a perfect disk, making the radio range variable among different nodes.

This inaccuracy can be improved by using the SRangeQ procedure first for range measurement and using a localization algorithm for estimating the positions based on the range measurement generated by SRangeQ. The localization algorithm can be any connectivity-based algorithm. APS [14] is chosen in the simulation for its simplicity and accuracy as a connectivity-based localization algorithm.

Fig. 3 shows the simulation results of SRangeQ+APS, APS-modified, and APS-original. The result of APS-original is obtained by assuming the connectivity is not affected by any communication errors, which is suggested in the original work [14]. In reality, nodes can be reached beyond the assumed radius as well as being rejected within radio range because of the uncertainty of wireless communication. With this assumption, we have the modifed APS algorithm, APS-modified. The performance of APS-modified deteriorates while the $RSSI$ error increases. SRangeQ+APS is 12% more accurate than APS-modified, despite the increasing range error.

Fig. 4 was obtained by increasing the range level by one each time, starting from the connectivity-only case (range-level resolution $= 1$). The result shows that the accuracy does not always increase when the range-level resolution increases. There is a saturation point which sets the limit of the area-proportional quantization model. In the adaptive case (Fig. 5), the accuracy is similar to the fixed one. The advantage of using adaptive range-level resolution is that no configuration is needed for the range-level resolution parameter before deploying the network. This is useful when there are thousands of nodes involved.

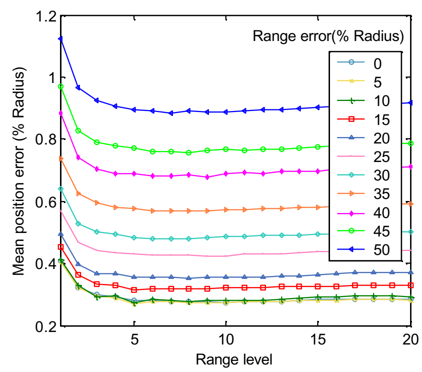

Figure 4. Fixed range-level resolution: 100×100 **network,** 200 **nodes, radius** $= 15$

6. Conclusions

The simulation results show the effectiveness of SRangeQ on range estimation and localization. A summary of the performance of SRangeQ is listed below:

1. SRangeQ is insensitive to $RSSI$ error on range estimation, and it is $10 - 50\%$ more accurate than a direct $RSSI$ quantization method when n_p error is larger than 0.1.

2. SRangeQ is distributed and can be plugged into other connectivity-based localization algorithms.

3. SRangeQ+APS, with the improved range estimation plugged into APS localization algorithm, is over 10% more accurate than APS without SRangeQ.

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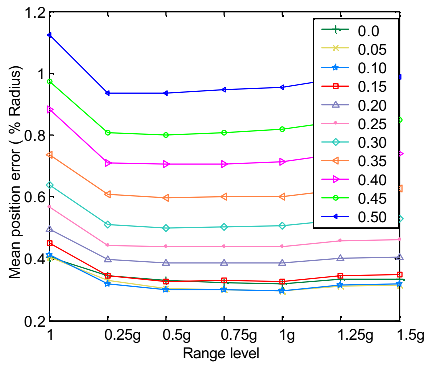

Figure 5. Adaptive range-level resolution: 100×100 **network,** 200 **nodes, radius** $= 15$

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