An implementation of the Filtered Back Projection algorithm using Matlab

Tim Schaeps, Maggie Goossens, Joost Batenburg, Jan Sijbers, and Tim Dams.

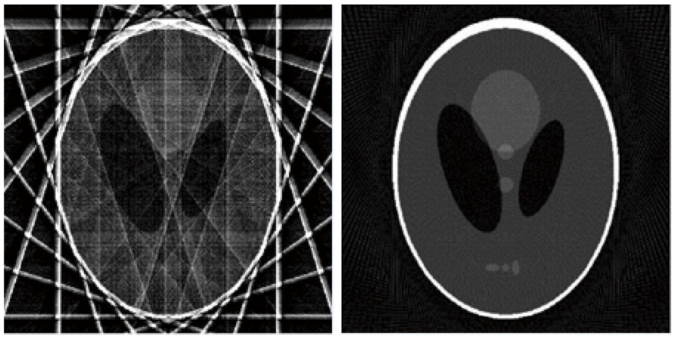
Abstract—Tomography is a technique developed with the purpose of getting structural information out of an object as realistic as possible without opening the object. Evidently one cannot take just a picture. With the use of several types of scanners, data needed to reconstruct this information is delivered. There are several ways to achieve this, but one of them, the filtered back projection algorithm, will be discussed in this paper. The goal is to give a representation of the original internal structure. Different possibilities for implementation will be taken into account with the purpose of giving a detailed overview of this technique.

Index Terms—filter back projection, tomography, medical imaging, matlab.

1 INTRODUCTION

TOMOGRAPHY is a technique used to get an overview of structures inside for example a human body or non biological object without the need to open the object. One can also say that this is a nondestructive way of obtaining visual data from the inside of an object. When an X-ray scanner makes projections at different angles around the object, the data obtained using this process have to be reconstructed in one way or another. The data without reconstruction are meaningless. [?] describes all the essential factors that have to be taken into account. One technique to achieve this is the filtered back projection (FBP) method. It is based on the Back Projection (BP) algorithm [?]. BP is not exact and the reconstructed image is blurry. This is a reason why this technique isn't qualified for clinical applications. That is where FBP comes in: Projected data are filtered and "smeared" out at the same angles at which the projections were taken. FBP is a very popular technique that is still preferred over relatively young and promising iterative techniques such as ART [1]. We also need to consider the intended quality of the reconstructed image. The size of angles at which the projections were made during a CT-scan determines the quality of the reconstructed image. When a big angle (θ) between projections is used, for example (e.g. $\pi/18$ rad) there are fewer projections than when a smaller angle is used (e.g. $\pi/90$ rad). This can also be seen in figure 1.

It is evident that the latter method, using smaller angles, gives a far better result than the first one, because there are more projections and as a result more detail in the final image. The counter side however is the fact that a smaller angle also results in more data and thus more computing time needed. Considering this, we need to decide whether quality is more important, or fast visualization. This decision can be made case by case. There are still other possibilities available, which will be discussed together with the technique in detail further on.



(a) 10 Projections (b) 100 Projections

Fig. 1. Difference between different amounts of projections

1.1 Main goal

The main goal of this research was to create an all-in application, which gives an overview of the complete process of image-data acquisition of an object. There are several techniques available. In this paper the FBP method will be discussed. Using Matlab, this method will be implemented in a toolbox which has an educational purpose. To achieve a maximal overview, the different steps in the project will be developed and discussed in modules. These modules will be discussed in detail in the next section.

1.1.1 Theory

A solid mathematical foundation is obtained through the following explanation [?]. First we consider a coordinate system (t,s) to be a rotated version of the well known original (x,y) system.

$$\begin{bmatrix} x2 \\ y2 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} * \begin{bmatrix} x1 \\ y1 \end{bmatrix} \tag{1}$$

A projection along lines of t is written as

$$P_\theta(t) = \int_{-\infty}^{\infty} f(t,s)ds \tag{2}$$

and it's fourier transform can be written as

$$S_\theta(w) = \int_{-\infty}^{\infty} P_\theta(t)e^{-j2\pi wt}dt \tag{3}$$

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Substituting the last two formula's (defenition of a projection into its Fourier transform) gives:

$$S_\theta(w) = \int_{-\infty}^{\infty} [f(t,s)ds]\, e^{-j2\pi wt} dt \tag{4}$$

Converting the resulting formula to the (x,y) coordinate system using (1) has the following result:

$$S_\theta(w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi w(x*\cos(\theta)+y*\sin(\theta))}dxdy \tag{5}$$

This can also be written as

$$S_\theta(w) = F(w,\theta) = F(w\cos\theta, w\sin\theta) \tag{6}$$

This represents the 2D fourier transform of a spatial frequency. When this is applied on the several projections, incrementing θ, taken of an object, the values of F(u,v) can be determined. The more projections, the more points can be found in the (u,v) system and as a result of this, the quality of the reconstructed image will improve if an inverse Fourier transformation is applied.

2 Conclusion

The main goal for this project was to develop an educational toolbox which shows the entire process of performing a CT-scan end reconstructing the obtained data in order to visualize the internal structure of an object. Mostly, when the FBP is applied, only the result is shown. In educational environments it is difficult to visualize the different components of the algorithm, especially when the user doesn't possess enough knowledge on the subject. This is the reason why each fase in the reconstruction process has been shown, considering each detail.

Knowing that the implementation of the Filtered Back Projection isn't a real innovation, implementing a self-developed algorithm for this technique was quite a challenge.

References

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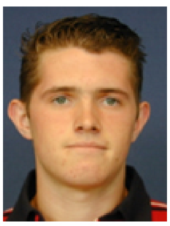

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