RSS-Based Location Estimation with Unknown Pathloss Model

Xinrong Li, *Member, IEEE*

Abstract— **Recently, received signal strength (RSS)-based location estimation technique has been proposed as a low-cost, low-complexity solution for many novel location-aware applications. In the existing studies, radio propagation pathloss model is assumed known a priori, which is an oversimplification in many application scenarios. In this paper we present a detailed study on the RSS-based joint estimation of unknown location coordinates and distance-power gradient, a parameter of pathloss model. A nonlinear least-square estimator is presented and the performance of the algorithm is studied based on CRB and various simulation results. From simulation results it is shown that the proposed joint estimator is especially useful for location estimation in unknown or changing environments.**

Index Terms— **Cramer-Rao bound (CRB), least square estimation, location estimation, received signal strength (RSS).**

I. Introduction

In recent years, location estimation in wireless networks has attracted great interests from researchers in the field of signal processing and wireless communications [1]–[14]. Location estimation is an important task in many innovative applications of wireless systems. For example, location estimation in cellular networks is mandated by FCC to support E-911 services for mobile subscribers [1], [2]. Location estimation is also desired in local environments to provide positioning, tracking, and navigation services to people with special needs, such as firefighters and soldiers in their mission, elderly and patients under special care, and etc. [3]. In addition, location estimation is an essential building block of wireless sensor networks [5]. Widely envisioned applications of sensor networks include environmental and structural monitoring, and many military and public safety applications [15]–[17]. Sensing data without knowing the sensor's location is meaningless [4].

A variety of location estimation techniques have been proposed for wireless networks, including time-of-arrival (TOA), angle-of-arrival (AOA), received signal strength (RSS), pattern recognition, and Bayesian filters [5]–[14]. It is well known that TOA-based systems require high-precision timing and synchronization components; signal processing techniques to combat multipath effects further increase system complexity [6], [10]. AOA-based systems require antenna array, which significantly increases system complexity. Pattern recognition and Bayesian filtering methods require very high computational capacity. In contrast, RSS-based techniques have the lowest complexity and cost among the alternatives since RSS measurements are readily available in most of wireless systems [5]. A widely known disadvantage of RSS-based techniques is their relatively low accuracy as compared to TOA-based techniques. An implication of such an observation is that careful study is needed to determine whether the achievable performance meets application requirements.

Existing studies of RSS-based techniques, e.g. [5], [11]–[14], are based on the classic radio propagation pathloss model [19]. In all of these studies the pathloss model is considered known a priori by assuming either the channel is perfect free-space or extensive channel measurement and modeling is performed prior to system deployment. Such an assumption is an oversimplification in many application scenarios such as monitoring and surveillance applications in hostile or inaccessible environments, where no extensive channel measurement is possible. In some other scenarios, the channel characteristics of application environments tend to change considerably over a long period of time due to various seasonal and accidental reasons [20]. Thus, a reliable location estimation algorithm needs to optimally accommodate and adapt to changing or unknown channel conditions. In this paper we propose to jointly estimate unknown location coordinates and distance-power gradient, a parameter of pathloss model, based on RSS, which eliminates the need for extensive channel measurement. A nonlinear least square (LS) solution of the joint estimation problem is presented in this paper; the performance of the proposed method is studied based on Cramer-Rao Bound (CRB) and various simulation results.

The rest of the paper is organized as follows. In Section II, we first define the joint estimation problem and present a nonlinear LS solution of the problem. Then, performance lower bounds and accuracy measures are derived in Section III. Performance of the proposed joint estimator is analyzed based on CRB with numerical examples in Section IV. Various simulation results are presented in Section V to study the performance of the proposed algorithm. Finally, conclusions are summarized in Section VI.

II. RSS-Based Location Estimation Algorithms

A. Problem Formulation

Suppose a target node is located at some unknown location (x, y) and n reference nodes are located at the known locations (x_i, y_i), $1 \le i \le n$. The problem of RSS-based location estimation is to estimate the unknown location coordinates of target node from RSS measurements of the radio signals transmitted from reference nodes to target node. The unknown

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X. Li is with the Department of Electrical Engineering, University of North Texas, 3940 N. Elm St., Denton, TX 76207 USA (e-mail: xinrong@unt.edu).

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location coordinates can be estimated based on distance measurements by solving a system of nonlinear equations [1]

$$\sqrt{(x-x_i)^2+(y-y_i)^2}=d_i,\ 1\le i\le n, \tag{1}$$

where d_i is the measured distance between target node and the ith reference node.

The nonlinear equations for distance measurement-based location estimation can be related to RSS through the classic narrowband radio propagation pathloss model [19]

$$L_p = L_0 + 10\alpha\log_{10} d + v, \tag{2}$$

where L_0 is the signal power loss in dB unit at 1 m distance, L_p is signal power loss at a distance d ($d \ge 1$ m), the parameter α is distance-power gradient (also known as pathloss exponent), and v is a Gaussian random variable representing log-normal shadow fading effects in multipath environments. In radio propagation channel studies, the random variable v is considered zero-mean, *i.e.*, $v \sim N(0, \sigma_v^2)$, while its standard deviation σ_v depends on the characteristics of a specific multipath environment [19]. For a given wireless system, L_0 can be calculated or measured during system calibration period and $L_p = P_t - P_r$ (dB) can be determined real-time at receiver node by measuring the received signal power P_r if the transmit signal power P_t is known.

Here we consider α as unknown parameter that we shall estimate simultaneously with the unknown location coordinates of target node. Usually, α can be assumed constant in an environment for a certain period of time. Define $p = L_p - L_0$, which is the observed pathloss in dB from 1 m distance to d, or equivalently, $p = P_0 - P_r$ (dB), where P_0 and P_r are received powers in dBm unit at a distance of 1 m and d, respectively. By substituting (1) into (2), the system of nonlinear equations for location estimation can be rewritten as

$$p_i = g_i(\theta) + v_i,\ 1\le i\le n, \tag{3}$$

where $g_i(\theta) = 10\alpha\log_{10} d_i$ and the unknown vector parameter $\theta = [\theta_1, \theta_2, \theta_3]^T = [x, y, \alpha]^T$. Equivalently, we can formulate (3) into the following vector form

$$\mathbf{p} = \mathbf{g}(\theta) + \mathbf{v}, \tag{4}$$

where $\mathbf{p} = [p_1, \ldots, p_n]^T$, $\mathbf{g}(\theta) = [g_1(\theta), \ldots, g_n(\theta)]^T$, and $\mathbf{v} = [v_1, \ldots, v_n]^T$. Now the problem of RSS-based location estimation is formulated into a system of nonlinear equations with unknown vector parameter θ. In the next section, a nonlinear LS solution of this problem is presented.

B. LS Location Estimation Algorithms

The LS solution of the system of nonlinear equations in (4) can be formed as the θ that minimizes the loss function

$$\xi(\theta) = (\mathbf{p}-\mathbf{g}(\theta))^T(\mathbf{p}-\mathbf{g}(\theta)). \tag{5}$$

To apply linear LS algorithm, the nonlinear functions vector $\mathbf{g}(\theta)$ is linearized first using the first-order Taylor series expansion [21], [22]

$$\mathbf{g}(\theta) = \mathbf{g}(\theta^{(0)}) + \mathbf{J}^{(0)}\cdot(\theta-\theta^{(0)}),$$

where $\theta^{(0)}$ is an initial estimate of the unknown parameter and $\mathbf{J}^{(0)}$ is the Jacobian matrix of $\mathbf{g}(\theta)$ at $\theta^{(0)}$. For n nonlinear functions and m unknown parameters, the Jacobian matrix

$$[\,\mathbf{J}^{(0)}\,]_{ij} = \left.\frac{\partial g_i(\theta)}{\partial\theta_j}\right|_{\theta=\theta^{(0)}}, \tag{6}$$

where $1\le i\le n$ and $1\le j\le m$. The LS solution of the linearized problem is obtained as [21]

$$\hat{\theta} = \theta^{(0)} + ((\mathbf{J}^{(0)})^T\mathbf{J}^{(0)})^{-1}(\mathbf{J}^{(0)})^T(\mathbf{p}-\mathbf{g}(\theta^{(0)})). \tag{7}$$

Given $\mathbf{g}(\theta)$ in (4), the Jacobian matrix can be derived as

$$\begin{aligned}\frac{\partial g_i(\theta)}{\partial\theta_1} &= \frac{\partial g_i(\theta)}{\partial x} = \frac{-10\alpha}{\ln 10}\cdot\frac{u_{ix}}{d_i},\\ \frac{\partial g_i(\theta)}{\partial\theta_2} &= \frac{\partial g_i(\theta)}{\partial y} = \frac{-10\alpha}{\ln 10}\cdot\frac{u_{iy}}{d_i},\\ \frac{\partial g_i(\theta)}{\partial\theta_3} &= \frac{\partial g_i(\theta)}{\partial \alpha} = \frac{10}{\ln 10}\ln d_i,\end{aligned} \tag{8}$$

where $u_{ix} = (x_i - x)/d_i$, $u_{iy} = (y_i - y)/d_i$, d_i is the distance between target node and the ith reference node, and $\mathbf{u}_i = u_{ix}\mathbf{u}_x + u_{iy}\mathbf{u}_y$ is a unit geometric vector from target node to the ith reference node while $\mathbf{u}_x$ and $\mathbf{u}_y$ are unit vectors in the directions of x and y axes, respectively.

This approach introduces error when the linearized function does not accurately approximate the nonlinear function, especially when the initial estimate is far away from the optimum. An iterative algorithm, known as Gauss-Newton algorithm, can be used to alleviate such a problem [21]; that is,

$$\hat{\theta}^{(k+1)} = \hat{\theta}^{(k)} + ((\mathbf{J}^{(k)})^T\mathbf{J}^{(k)})^{-1}(\mathbf{J}^{(k)})^T(\mathbf{p}-\mathbf{g}(\hat{\theta}^{(k)})), \tag{9}$$

where $\hat{\theta}^{(k)}$ and $\mathbf{J}^{(k)}$ are the estimated parameter and Jacobian matrix at the kth iteration, respectively. The iteration will be stopped when some stopping criteria is met. For example, for a given small tolerance δ, the algorithm is stopped if $|\xi(\hat{\theta}^{(k+1)})-\xi(\hat{\theta}^{(k)})| < \delta$ or if a maximum number of iteration has been performed. The same as with any iterative algorithms, there is no guarantee for the iterative Gauss-Newton algorithm in (9) to converge to the global optimum; with a random initial estimate, the algorithm may converge to a local optimum [22]. The readers are cautioned that in the implementation of the Gauss-Newton algorithm, many safeguard measures need to be incorporated to cope with various challenging issues such as ill-conditioning, divergence, and lack of identifiability of parameters, which might be inherent in nonlinear models. Detailed discussion on these issues and a range of techniques for modifying the basic algorithm in (9) to make it reliable can be found in [22]. In the simulations, we employ the Levenberg-Marquardt method, which is widely regarded as the most robust modified Gauss-Newton algorithm.

For the ease of discussion, here we refer to the estimator in (9) as RSS LS location estimator with unknown distance-power gradient, or simply, RSS-UDPG LSE. The RSS LS location estimator with known distance-power gradient, referred to as RSS LSE, is essentially the same as (9) except that the unknown vector parameter only contains location coordinates of target node, *i.e.*, $\theta = [x, y]^T$ for 2-D location estimation, which reduces the $n\times 3$ Jacobian matrix to an $n\times 2$ matrix. In the following two sections, we present CRB analysis of unbiased RSS-based location estimators. Then, the performance of the LS algorithms presented in this section is studied through various simulation results in Section V.

III. Cramer-Rao Bound Analysis

A. Derivation of CRB

In radio propagation channel studies, the random variable v in the pathloss model (2), which represents shadow fading effects in multipath environments, is considered zero-mean Gaussian random variable, *i.e.*, $v \sim N(0, \sigma_v^2)$, while its standard deviation σ_v depends on the characteristics of a specific multipath environment [19]. Then, the pathloss observation p_i, defined in (3), has probability density function

$$f(p_i;\,\theta) = \frac{1}{\sqrt{2\pi}\sigma_v}\exp\{-\frac{(p_i - g_i(\theta))^2}{2\sigma_v^2}\}, \tag{10}$$

which is parameterized by unknown vector parameter θ. If we assume p_i, $1 \le i \le n$, are statistically independent observations, joint distribution of observation vector $\mathbf{p}$ becomes

$$f(\mathbf{p};\,\theta) = \prod_{i=1}^{n} f(p_i;\,\theta). \tag{11}$$

The CRB on the covariance matrix of any unbiased estimator $\hat{\theta}$ is defined as (in the positive semidefinite sense)

$$\mathrm{cov}(\hat{\theta}) - \mathbf{F}^{-1} \ge 0, \tag{12}$$

where $\mathbf{F} = -E[\nabla_\theta(\nabla_\theta \ln f(\mathbf{p};\,\theta))^T]$ is the Fisher information matrix [21]. If the shadow fading random variables v_i, $1 \le i \le n$, are IID zero-mean Gaussian random variables, *i.e.*, the shadow fading effects follow log-normal distribution, the LS estimator that minimizes (5) is the maximum likelihood estimator (MLE) and it asymptotically achieves CRB with equality. However, it should be noted that as pointed out in [14] the observed pathloss p_i could not be assumed ergodic random variable and CRB gives a lower bound on the ensemble variance over different random shadowing environments.

Given the joint distribution of observation vector $\mathbf{p}$ in (11), equivalent log-likelihood function can be defined as

$$l(\theta) = -\frac{1}{2\sigma_v^2}\sum_{i=1}^{n}(p_i - g_i(\theta))^2, \tag{13}$$

and the Fisher information matrix can be derived as

$$\begin{aligned} F_{kj} = [\,\mathbf{F}\,]_{kj} &= -E[\frac{\partial^2 l(\theta)}{\partial\theta_k \partial\theta_j}] \\ &= \begin{cases} \frac{1}{\sigma_v^2}\sum_{i=1}^{n}(\frac{\partial g_i(\theta)}{\partial\theta_j})^2, & \text{if } k = j, \\ \frac{1}{\sigma_v^2}\sum_{i=1}^{n}\frac{\partial g_i(\theta)}{\partial\theta_k}\frac{\partial g_i(\theta)}{\partial\theta_j}, & \text{if } k \ne j, \end{cases} \end{aligned} \tag{14}$$

where the gradients of $g_i(\theta)$ are derived in (8). Then, the CRB of any unbiased estimator can be derived from the Fisher information matrix as shown in (12). In the next section, we further define quantitative performance measures for location estimators based on CRB.

B. Location Estimation Accuracy Measures

Let $(\hat{x},\,\hat{y})$ be any unbiased 2-D location estimator. Then, the CRB in (12) gives a lower bound on the variance of the unbiased estimator $(\hat{x},\,\hat{y})$; that is,

$$E[(\hat{x} - x)^2] \ge [\,\mathbf{F}^{-1}\,]_{11},\; E[(\hat{y} - y)^2] \ge [\,\mathbf{F}^{-1}\,]_{22}. \tag{15}$$

Instead of the variance of location coordinate estimators along individual coordinate axes, in location estimation applications a more meaningful performance measure of location estimators is based on the geometric location estimation error $\varepsilon = \sqrt{(\hat{x} - x)^2 + (\hat{y} - y)^2}$. From (15), we can readily see that the mean-squared error (MSE) of any unbiased location estimator is lower bounded as

$$\varepsilon_{rms}^2 = E[\varepsilon^2] \ge [\,\mathbf{F}^{-1}\,]_{11} + [\,\mathbf{F}^{-1}\,]_{22}, \tag{16}$$

where ε_{rms} is defined as the root-MSE (RMSE) of location estimators. It is worth to note that in general the MSE of any estimator $\hat{\theta}$ can be determined as [21]

$$\mathrm{mse}(\hat{\theta}) = \mathrm{var}(\hat{\theta}) + b^2(\hat{\theta}), \tag{17}$$

where $b(\hat{\theta}) = E[\hat{\theta}] - \theta$ is the bias of the estimator. Therefore, for any unbiased estimator, the MSE equals to the variance.

From (16), we can determine the lower bound on the MSE of any unbiased location estimator

$$\varepsilon_{rms}^2 \ge (F_{22}F_{33} - F_{23}^2 + F_{11}F_{33} - F_{13}^2)/|\,\mathbf{F}\,|, \tag{18}$$

where $|\,\mathbf{F}\,|$ is the determinant of the Fisher information matrix. Similarly, we can determine the lower bound on the variance of any unbiased estimator of distance-power gradient $\hat{\alpha}$ as

$$\sigma_{\hat{\alpha}}^2 \ge (F_{11}F_{22} - F_{12}^2)/|\,\mathbf{F}\,|. \tag{19}$$

Similar to the discussions in [23], we can simplify the lower bounds in (18) and (19) and derive an interesting interpretation of these accuracy measures by utilizing the definitions of cross product and dot product of geometric vectors. Given two geometric vectors $\mathbf{a} = a_x\mathbf{u}_x + a_y\mathbf{u}_y$ and $\mathbf{b} = b_x\mathbf{u}_x + b_y\mathbf{u}_y$, the cross product of two vectors can be written as $\mathbf{a} \times \mathbf{b} = (a_x b_y - a_y b_x)\mathbf{u}_z$, where $\mathbf{u}_z = \mathbf{u}_x \times \mathbf{u}_y$ is the unit vector in the direction of z axis. If we further define the angle of a vector, denoted by ϕ, as the angle measured from x axis in counter-clockwise direction as in polar coordinate system, the cross product of two vectors becomes $\mathbf{a} \times \mathbf{b} = |\,\mathbf{a}\,||\,\mathbf{b}\,|\sin\phi_{ab}\mathbf{u}_z$, where $\phi_{ab} = \phi_b - \phi_a$ with ϕ_a and ϕ_b being the angles of vectors $\mathbf{a}$ and $\mathbf{b}$. On the other hand, the dot product of two vectors can be written as $\mathbf{a} \bullet \mathbf{b} = a_x b_x + a_y b_y = |\,\mathbf{a}\,||\,\mathbf{b}\,|\cos\phi_{ab}$. Here we apply these definitions to two vectors $\mathbf{u}_i = u_{ix}\mathbf{u}_x + u_{iy}\mathbf{u}_y$ and $\mathbf{u}_j = u_{jx}\mathbf{u}_x + u_{jy}\mathbf{u}_y$, which are unit vectors from target node to the ith and jth reference nodes, respectively. By defining $\phi_{ij} = \phi_j - \phi_i$, we can derive that

$$\begin{aligned} \mathbf{u}_i \times \mathbf{u}_j &= (u_{ix}u_{jy} - u_{iy}u_{jx})\mathbf{u}_z = \sin\phi_{ij}\mathbf{u}_z, \\ \mathbf{u}_i \bullet \mathbf{u}_j &= u_{ix}u_{jx} + u_{iy}u_{jy} = \cos\phi_{ij}. \end{aligned}$$

After some tedious derivation, the lower bound on the accuracy measures defined in (18) and (19) can be written as

$$\varepsilon_{rms}^2 \ge (\sigma_v/\alpha)^2 \cdot GDOP_{xy,u}^2, \tag{20}$$

$$\sigma_{\hat{\alpha}}^2 \ge \sigma_v^2 \cdot GDOP_{\alpha}^2, \tag{21}$$

$$GDOP_{xy,u} = \frac{\ln 10}{10}\sqrt{\frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\{(\frac{\ln d_j}{d_i} - \frac{\ln d_i}{d_j})^2 + 2\frac{\ln d_i \ln d_j}{d_i d_j}(1-\cos\phi_{ij})\}}{\sum_{k=1}^{n}\{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\ln^2 d_k}{d_i^2 d_j^2}\sin^2\phi_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\ln d_j \ln d_k}{d_i^2 d_j d_k}\sin\phi_{ij}\sin\phi_{ki}\}}}$$

$$GDOP_{\alpha} = \frac{\ln 10}{10}\sqrt{\frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{1}{d_i^2 d_j^2}\sin^2\phi_{ij}}{\sum_{k=1}^{n}\{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\ln^2 d_k}{d_i^2 d_j^2}\sin^2\phi_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\ln d_j \ln d_k}{d_i^2 d_j d_k}\sin\phi_{ij}\sin\phi_{ki}\}}}$$

where $GDOP_{xy,u}$ and $GDOP_{\alpha}$ are geometric dilution of precision (GDOP) defined following the convention in the literature [14], [23] as shown at the top of the page.

The subscript u in $GDOP_{xy,u}$ is used to distinguish the GDOP of the location estimators with unknown distance-power gradient α from that of location estimators with known α. By defining GDOP, we have separated the effects of environmental parameters, α and σ_v, and geometric parameters, d_i and ϕ_i. It is worth to note that the geometric conditioning A_{ij} defined in [14], [23] only depends on ϕ_{ij}, *i.e.*, $A_{ij} = |\sin\phi_{ij}|$, since the parallelogram used in defining A_{ij} is determined by two unit-length vectors. From these results we can observe that the lower bounds in (20) and (21) depend not only on angular distribution of reference nodes, but also on the distances from target node to reference nodes.

The CRB for any unbiased RSS location estimators with known distance power gradient has been derived in [14]. Similar to (20), the lower bound on the MSE of any unbiased RSS location estimators (with known α) can be derived as

$$\varepsilon_{rms}^2 \geq (\sigma_v/\alpha)^2 \cdot GDOP_{xy}^2, \tag{22}$$

where

$$GDOP_{xy} = \frac{\ln 10}{10}\sqrt{(\sum_{i=1}^{n} d_i^{-2})/(\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\sin^2\phi_{ij}}{d_i^2 d_j^2})}.$$

By comparing (20) and (22), we notice that the existence of the unknown parameter α complicates the CRB of location estimators. In the next section, the GDOP of unbiased location estimators are analyzed with numerical examples.

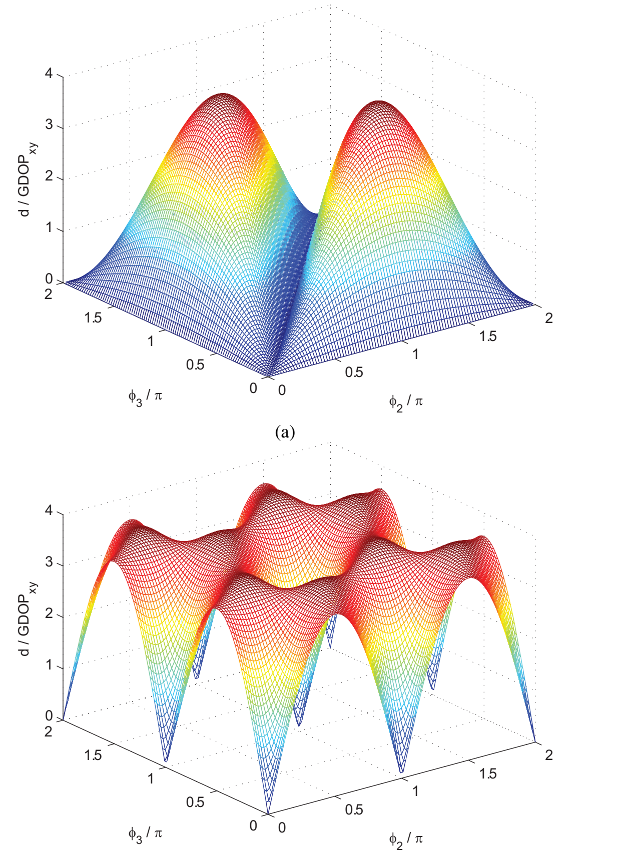


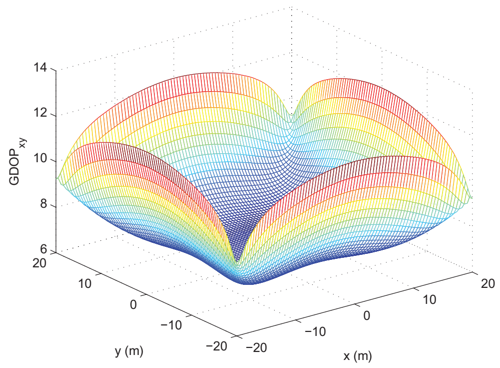
Fig. 1. (a) Plot of $d/GDOP_{xy}$ versus ϕ_2 and ϕ_3 when $n = 3$, $\phi_1 = 0$, and $d_i = d$, $1 \leq i \leq n$, with unknown α, and (b) with known α.

IV. Numerical Analysis of GDOP

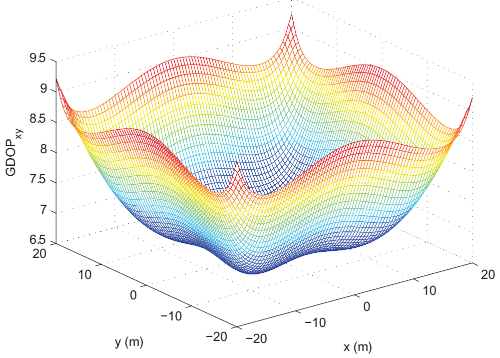
From (20)-(22), it is noted that the performance lower bounds of any unbiased RSS-UDPG and RSS location estimators depend on environmental parameters, α and σ_v, which vary significantly in different application environments. For example, in [14] it is determined through channel measurements that $\sigma_v/\alpha = 1.7$. More measurement results of multipath channels can be found in [19]. In this section, the effects of geometric parameters on the performance bounds are analyzed based on GDOP, which may facilitate the design of optimal strategy for reference node deployment.

A. Effects of Angular Distribution of Reference Nodes

A close look at the performance bounds derived in (20)-(22) reveals that the effects of distances d_i and angles ϕ_i can only be separated when d_i is a constant. In order to study the effects of angular distribution of reference nodes on the performance bounds, here we assume $d_i = d$, $1 \leq i \leq n$. Then, the ratio $d/GDOP_{xy}$ only depends on ϕ_i for both RSS-UDPG and RSS location estimators. Fig. 1 shows three-dimensional plots of this ratio versus ϕ_2 and ϕ_3 when $n = 3$ and $\phi_1 = 0$. It is interesting to note that under such conditions the shapes of the $d/GDOP_{xy}$ of RSS-UDPG and RSS location estimators are almost exactly the same as that of $1/GDOP_{xy}$ of hyperbolic trilateration and circular trilateration, respectively, shown in [23]. Thus, the same analysis in [23] applies here; that is, for unbiased RSS-UDPG location estimators, GDOP tends to infinity when at least $n-1$ reference nodes lie along a half straight line originated from target node, while for unbiased RSS location estimators, GDOP tends to infinity when all reference nodes lie along a straight line passing through target node. Such an observation indicates that RSS-UDPG estimators are more sensitive to geometric parameters than RSS estimators and RSS-UDPG



(a)



(b)

Fig. 2. (a) Plot of $GDOP_{xy}$ with $L = 40$ m, $\Delta = 1$ m, and unknown α, and (b) with known α.

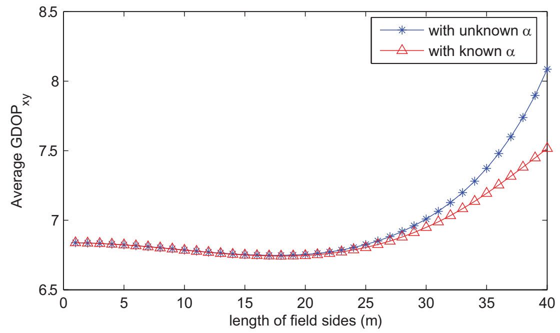


Fig. 3. Average $GDOP_{xy}$ versus the length of field sides when reference nodes are fixed at four corners of a square with the sides of length 42 m and the deployment field is in the center of the square.

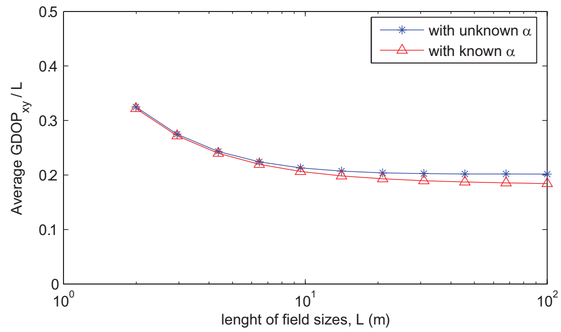


Fig. 4. Average $GDOP_{xy}$ divided by the length of square field sides L while reference nodes are at four corners of the field with $\Delta = 1$ m.

estimators require more careful planning of reference node deployment structure to minimize the occurrence of the worst-case scenario, *i.e.*, alignment of $n-1$ reference nodes on a half straight line originated from target node.

The preceding analysis is based on the assumption that $d_i = d$, $1 \leq i \leq n$, which is only a special scenario in practice. In the next section, the combined effects of geometric parameters, ϕ_i and d_i, $1 \leq i \leq n$, on GDOP are analyzed with a specific deployment structure of reference nodes.

B. Combined Effects of Geometric Parameters

In this section we analyze the combined effects of geometric parameters, ϕ_i and d_i, on GDOP using a simple square deployment of four reference nodes. Such a deployment scheme is chosen for the ease of graphical illustration as shown in Fig. 2. The analysis presented in this section provides useful insights to more complex deployment structures.

For a square field with the sides of length L, four reference nodes are placed at the corners of the field with some margin Δ. The use of a margin is justified since in the problem definition in (3) it is assumed that $d_i \geq 1$ m. Fig. 2 shows three dimensional plot of $GDOP_{xy}$ with $L = 40$ m and $\Delta = 1$ m for both unbiased RSS-UDPG and RSS location estimators. From the results we can observe that for both types of estimators $GDOP_{xy}$ is small in the center of the field and becomes large as target node moves towards boundary areas. In order to observe this phenomenon more clearly, we fix the reference nodes at the same locations, but vary the field size L while keeping the field in the center of the square defined by four reference nodes. Average GDOP in the field is calculated for each value of L and the results are plotted in Fig. 3. From the results, we can clearly observe that on average GDOP remains almost constant and same for both types of estimators when L is less than 30 m, but increases steadily beyond this point. In addition, RSS-UDPG location estimators have larger GDOP than RSS location estimators at boundary areas.

As another example, we vary the field size L and the location of reference nodes simultaneously while keeping the margin constant, $\Delta = 1$ m. The average GDOP in the field is calculated for each value of L for both types of estimators. The ratio of average GDOP and L is plotted in Fig. 4, which indicates that the performance bounds of the two estimators are very close to each other. In addition, when L is greater than 10 m, the average GDOP remains almost constant at about 20% of L while it becomes a little larger as L decreases below 10 m. This result is important in that it provides an easy way to quickly predict performance lower bounds of any unbiased location estimators in a specific application environment with the same configuration of reference nodes.

In this section we studied the effects of geometric parameters on the performance bounds of unbiased RSS-based location estimators based on GDOP. In the next section, we present various simulation results of the LS location estimation algorithms given in Section II to study the behavior of the proposed algorithms in complex application scenarios. Simulation results are also compared with the performance lower bounds that we derived in this paper.

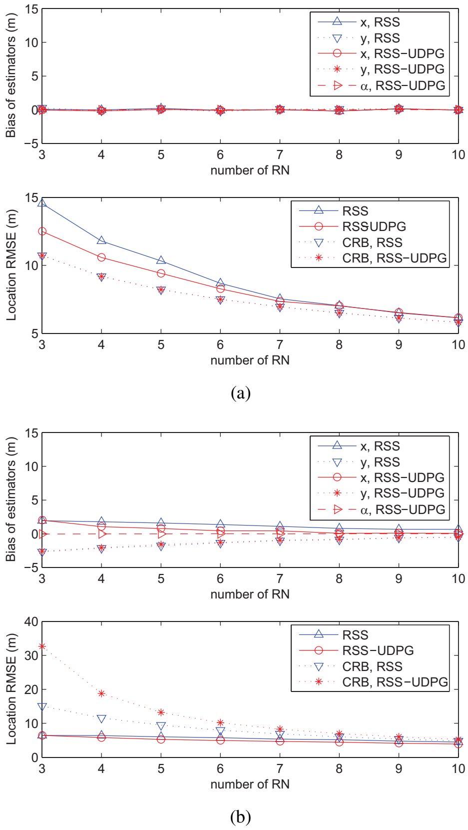


Fig. 5. (a) Bias and location RMSE of RSS LSE and RSS-UDPG LSE versus n at location $(0, 1)$, and (b) at location $(17, 3)$ when $\alpha = 3$ and $\sigma_v = 6$.

V. Simulation Results

Without loss of generality, in the simulations we consider circular deployment of reference nodes to conveniently study the effects of the number of reference nodes, which has significant effects on the performance of location estimators. In the circular deployment scheme, reference nodes are evenly spaced on a circle; the inside of the circle is considered as deployment field for target node. The center of the field is fixed at $(0, 0)$ and the radius of the field is set to 20 m. Random target node locations are uniformly sampled in the field. At each random location, we determine CRB and perform 200 simulations to determine the bias and RMSE of location estimators. In the following, we first exam the bias and efficiency of RSS-based estimators. Then, the effects of inaccurate prior estimate of distance-power gradient are studied to demonstrate the importance of the proposed joint estimator. Some initial simulation results can also be found in [24].

A. Bias and Efficiency of RSS-Based Estimators

The LSE presented in Section II.B is a MLE if the shadow fading variables v_i in (3) are indeed IID Gaussian random variables. It is well known that MLE is asymptotically unbiased and asymptotically efficient. However, with nonlinear models and finite data records, *i.e.*, finite number of reference nodes in location estimation problems, usually we can say nothing about the optimality of MLE and CRB may not closely bound estimators' performance [21]. The required data record length

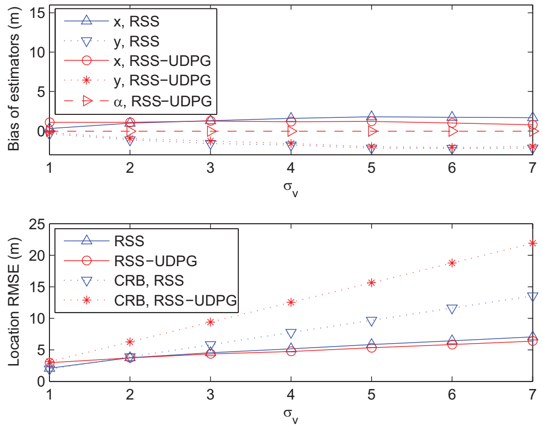


Fig. 6. Bias and location RMSE of RSS LSE and RSS-UDPG LSE versus σ_v at location $(17, 3)$ when $\alpha = 3$ and $n = 4$.

for the asymptotic results to apply depends on signal-to-noise ratio. From simulation results, we notice that the required number of reference nodes for the asymptotic results to apply in location estimation problems also significantly depends on the relative deployment geometry of the nodes.

Fig. 5 shows simulation results at two randomly selected target node locations $(0, 1)$ and $(17, 3)$, one close to the center of field and the other close to the edge. It is clear that at location $(0, 1)$ all estimators have no significant bias and CRB clearly lower-bounds estimators' performance. However, at location $(17, 3)$ estimators are noticeably biased when n is small. For example, when $n = 3$ estimates of coordinate y with both RSS and RSS-UDPG estimators have about 2.5 m bias. CRB is a lower bound for unbiased estimators and it does not closely bounds the performance of biased estimators. In addition, as can be seen from Fig. 5 the variance of biased estimators can even go below CRB [21], which indicates that the biased LSEs have much better performance in terms of location RMSE than any unbiased estimators, performance of which is lower-bounded by the CRB. As the number of reference nodes increases, all estimators tend to be unbiased and efficient at both locations. In order to exam the effects of noise level on the required data record length for the asymptotic results of MLE to apply, in the second simulation we vary the standard deviation of log-normal fading variable σ_v while the target node location is at $(17, 3)$. The results presented in Fig. 6 show that the value of σ_v has significant effect on the bias and efficiency of location estimators.

In order to evaluate the performance of location estimators at random target node locations, in the next simulation we uniformly sample 100 random target node locations in the field, determine location RMSE and CRB of both RSS-UDPG LSE and RSS LSE at each random location, and then average the location RMSE and the CRB over the 100 random locations to compare the average performance of the estimators. From the results shown in Fig. 7(a), we can observe that when the number of reference node is small, RSS-UDPG LSE is slightly better than RSS LSE and significantly better than any unbiased estimators, performance of which is lower-bounded by CRB. We also vary the value of σ_v while keeping

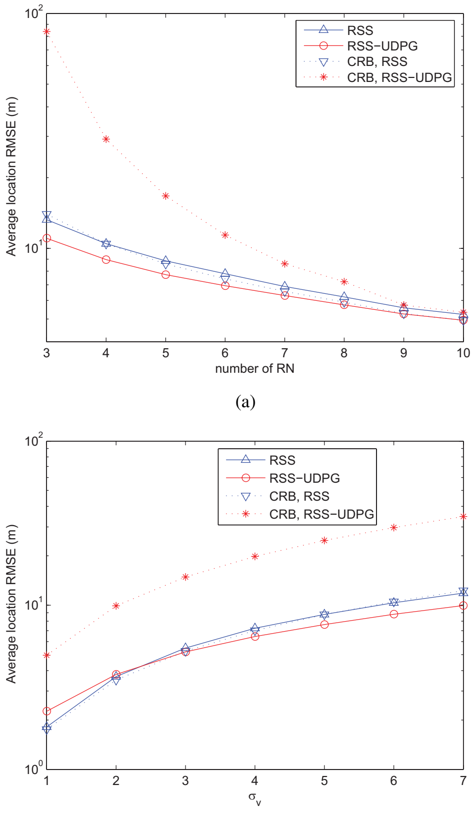


Fig. 7. (a) Average location RMSE of RSS LSE and RSS-UDPG LSE versus n when $\alpha = 3$ and $\sigma_v = 6$, and (b) versus σ_v when $\alpha = 3$ and $n = 4$.

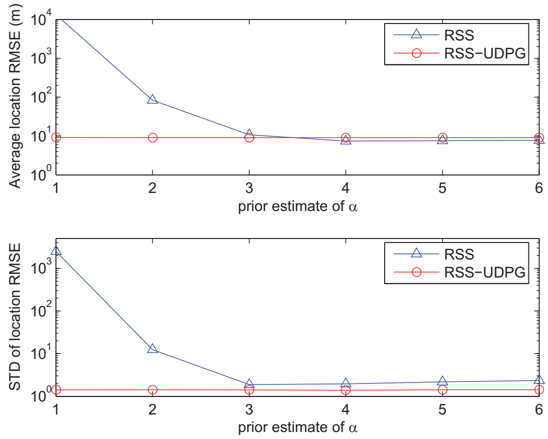


Fig. 8. Average and standard deviation of location RMSE of RSS LSE and RSS-UDPG LSE versus prior estimate of α when its actual value is 3, $n = 4$ and $\sigma_v = 6$.

$n = 4$. The results presented in Fig. 7(b) show that when σ_v is relatively large, on average RSS-UDPG LSE performs slightly better than RSS LSE, and it performs significantly better than CRB.

In [14], the authors show that RSS MLE is inherently biased due to biased range estimation with RSS-based methods; then, based on biased gradient norm of the MLE, it is shown that no unbiased estimator is possible in the close vicinity of reference nodes. Interested readers may refer to [14] for more discussions. However, the results shown in this section is very encouraging in that the RSS-based LS location estimators (or MLE) have much better performance than that predicted by the CRB. In "good" scenarios, for example, at the center of field and/or when large number of reference nodes are available and/or when standard deviation of log-normal shadow fading variable is very low, both RSS LSE and RSS-UDPG LSE are almost unbiased and their performances are closely bounded by CRB. However, in "bad" scenarios, which are bad in the sense that the CRB in these scenarios is significantly (or sometimes dramatically) larger than that of "good" scenarios, for example, at the edge of field and/or when very small number of reference nodes are available and/or when standard deviation of fading variable is large, performance of both estimators only slightly degrades in terms of location RMSE as compared to that of "good" scenarios at the cost of bias. Such an observation is clearly more promising than what is predicted by CRB for unbiased estimators.

It is very important to note that in all of the simulations presented in this section, the actual value of the parameter distance-power gradient α is assumed known *a priori* to the estimator RSS LSE, which is a very ideal condition. In the next section, we continue to compare the performance of RSS-UDPG LSE and RSS LSE based on simulations for more complex (and realistic) scenarios where no accurate prior estimate of the parameter α is available.

B. Effects of Inaccurate Prior Estimate of α

In the literature of RSS-based location estimation techniques, it is usually assumed that the parameter distance-power gradient α is accurately known a priori either through extensive channel measurement or by assuming a perfect free-space channel condition. Such an assumption is certainly an oversimplification for many practical application scenarios, where no extensive channel measurement is possible or the channel characteristics tend to vary significantly during a long period of operation of location estimation systems. Therefore, in these application scenarios it is strongly desirable to jointly estimate distance-power gradient, a parameter of radio propagation pathloss model, and the unknown location coordinates.

Fig. 8 shows simulation results of RSS-UDPG LSE and RSS LSE versus prior estimate of distance-power gradient when its actual value is 3. From these results, we can observe that inaccurate prior estimate of α has almost no effect on the performance of RSS-UDPG LSE since it is able to estimate the value of distance-power gradient as an unknown parameter. However, the prior estimate of α has tremendous effect on the performance of the location-only estimator RSS LSE; inaccurate prior estimate of α results in dramatically large location errors for RSS LSE due to the divergence of the algorithm, especially when the prior estimate of α is less than its actual value. From these results we can see that it is extremely harmful to design RSS LSE with perfect free space channel model, *i.e.*, with $\alpha = 2$, but apply the algorithm in complex multipath environments where the actual value of α is much greater than 2. Such an observation clearly demonstrates the importance of the proposed joint estimator.

Due to space limitation, analysis and simulation results on the estimation performance of the parameter α, which can be considered as a nuisance parameter in location estimation problems, are not shown in this paper. But from extensive simulation results we have observed that in general the parameter α can be estimated very accurately with the proposed joint estimator. For example, from Fig. 8 we can clearly observe that inaccurate prior estimate of the parameter α has almost no effect on the estimation results, which indicates that starting with any random initial estimate the iterative estimator of distance-power gradient can almost always converge to the same optimal value.

VI. Conclusion

In this paper, we present a detailed study on the joint estimation of distance-power gradient, a parameter of radio propagation pathloss model, and unknown location coordinates in RSS-based location estimation systems. Jointly estimating distance-power gradient eliminates the need for extensive channel measurement and modeling, and enables the algorithm to adaptively optimize its performance in dynamically changing environmental conditions.

The joint estimation problem is formulated into a system of nonlinear equations and a LS solution is presented, which is referred to as RSS-UDPG LSE in contrast to the conventional location-only estimator RSS LSE. Performance of the joint estimator is studied based on CRB analysis and simulation results. Quantitative expressions of accuracy measures have been derived; performance bounds of joint estimators and location-only estimators are compared with numerical examples. Based on CRB analysis, it is shown that unbiased RSS-UDPG estimators are more sensitive to geometric parameters than unbiased RSS estimators, which indicates that jointly estimating distance-power gradient requires more careful planning of reference node deployment than the conventional location-only estimator. However, based on various simulation results it is demonstrated that on average RSS-UDPG LSE has slightly better performance than RSS LSE when accurate estimate of distance-power gradient is assumed known a priori, and significantly better than that predicted by CRB for any unbiased estimators. The mismatch between CRB and simulation results is analyzed based on bias and asymptotic property of MLE. In addition, when prior estimate of distance-power gradient is inaccurate, the proposed joint estimator has significantly better performance as compared to the location-only estimator, which clearly demonstrates the value of the proposed method in many complex practical application scenarios.

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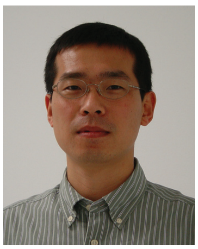

Xinrong Li (S’00-M’04) received B.E. degree from the University of Science and Technology of China, Hefei, China, in 1995, M.E. degree from the National University of Singapore in 1999, and Ph.D. degree from Worcester Polytechnic Institute (WPI), Worcester, MA, in 2003, all in electrical engineering. From 2003 to 2004, he was a Post-doc Research Fellow at the Center for Wireless Information Network Studies, WPI. Since 2004, he has been with the Department of Electrical Engineering, University of North Texas, Denton, TX, as an Assistant Professor. His recent research has been focused on statistical signal processing for geolocation, wireless communications, and sensor networks.