Angle Modulation – Frequency Modulation

Consider again the general carrier $v_c(t) = V_c \cos(\omega_c t + \varphi_c)$

$(\omega_c t + \varphi_c)$ represents the angle of the carrier.

There are two ways of varying the angle of the carrier.

- By varying the frequency, ω_c – **Frequency Modulation.**
- By varying the phase, ϕ_c – **Phase Modulation**

Frequency Modulation

In FM, the message signal $m(t)$ controls the frequency f_c of the carrier. Consider the carrier

$$v_c(t) = V_c \cos(\omega_c t)$$

then for FM we may write:

FM signal $v_s(t) = V_c \cos(2\pi(f_c + \text{frequency deviation})t)$,where the frequency deviation will depend on $m(t)$.

Given that the carrier frequency will change we may write for an instantaneous carrier signal

$$V_c \cos(\omega_i t) = V_c \cos(2\pi f_i t) = V_c \cos(\varphi_i)$$

where ϕ_i is the instantaneous angle = $\omega_i t = 2\pi f_i t$ and f_i is the instantaneous frequency.

Frequency Modulation

Since $\varphi_i = 2\pi f_i t$ then $\dfrac{d\varphi_i}{dt} = 2\pi f_i$ or $f_i = \dfrac{1}{2\pi}\dfrac{d\varphi_i}{dt}$

i.e. frequency is proportional to the rate of change of angle.

If f_c is the unmodulated carrier and f_m is the modulating frequency, then we may deduce that

$$f_i = f_c + \Delta f_c \cos(\omega_m t) = \frac{1}{2\pi}\frac{d\varphi_i}{dt}$$

Δf_c is the peak deviation of the carrier.

Hence, we have $\dfrac{1}{2\pi}\dfrac{d\varphi_i}{dt} = f_c + \Delta f_c \cos(\omega_m t)$,*i.e.* $\dfrac{d\varphi_i}{dt} = 2\pi f_c + 2\pi \Delta f_c \cos(\omega_m t)$

Frequency Modulation

After integration *i.e.* $\int \left(\omega_c + 2\pi\Delta f_c \cos(\omega_m t)\right)dt$

$$\varphi_i = \omega_c t + \frac{2\pi\Delta f_c \sin(\omega_m t)}{\omega_m}$$

$$\varphi_i = \omega_c t + \frac{\Delta f_c}{f_m}\sin(\omega_m t)$$

Hence for the FM signal, $v_s(t) = V_c \cos(\varphi_i)$

$$v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m}\sin(\omega_m t)\right)$$

Frequency Modulation

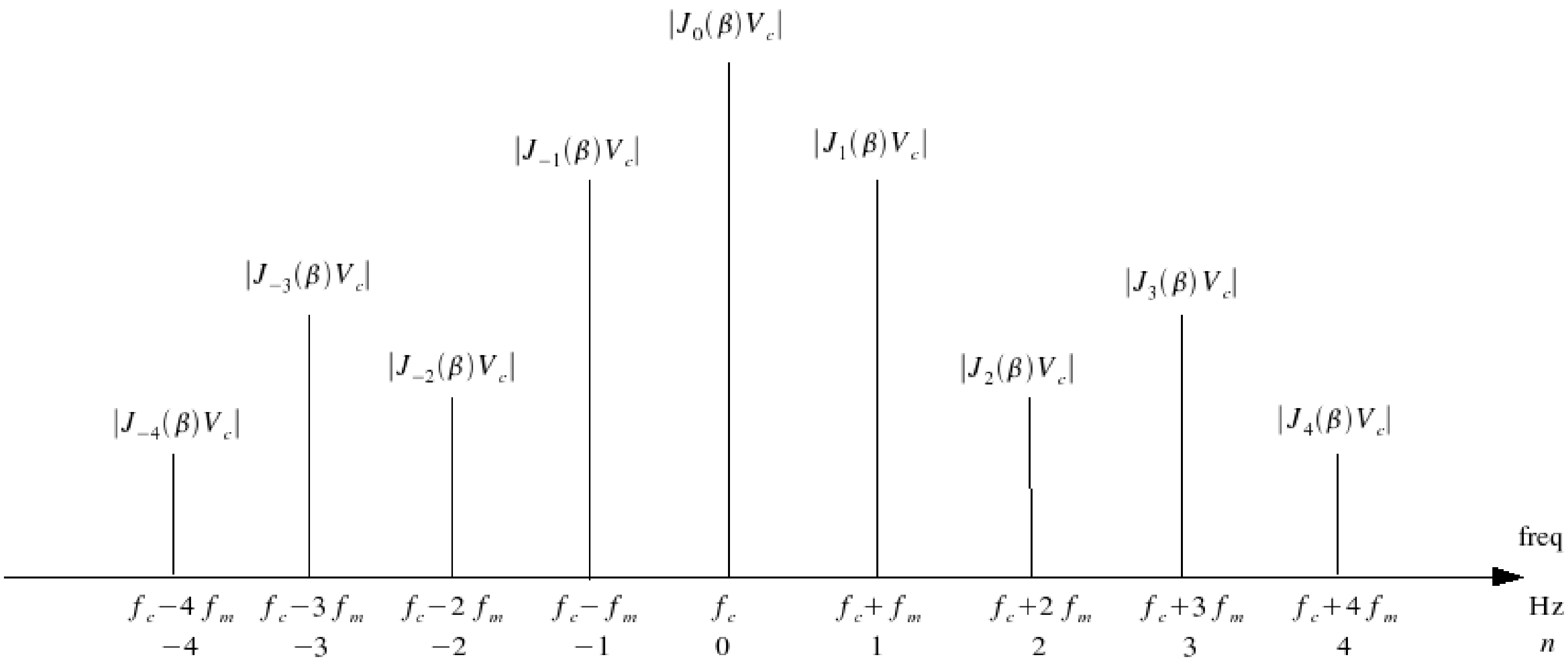
The ratio $\dfrac{\Delta f_c}{f_m}$ is called the **Modulation Index** denoted by β *i.e.*

$$\beta = \frac{\text{Peak frequency deviation}}{\text{modulating frequency}}$$

Note – FM, as implicit in the above equation for $v_s(t)$, is a non-linear process – *i.e.* the principle of superposition does not apply. The FM signal for a message $m(t)$ as a band of signals is very complex. Hence, $m(t)$ is usually considered as a 'single tone modulating signal' of the form

$$m(t) = V_m \cos(\omega_m t)$$

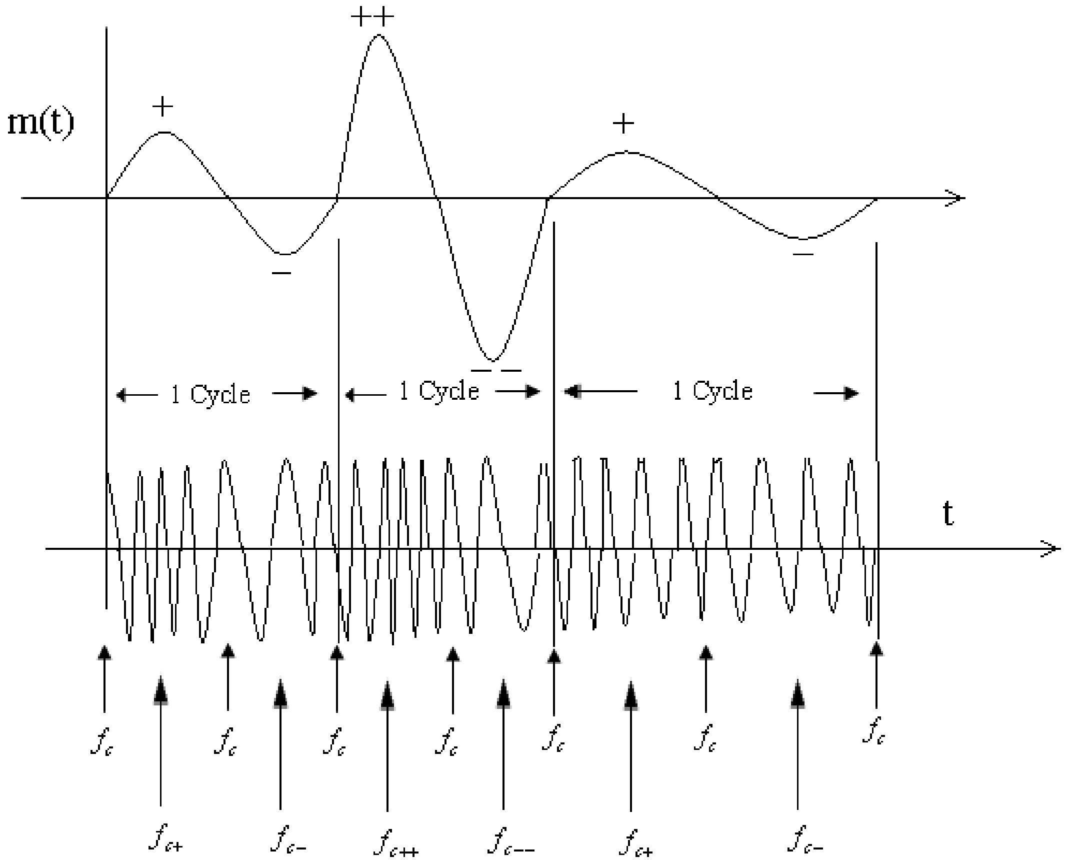
FM Signal Spectrum.

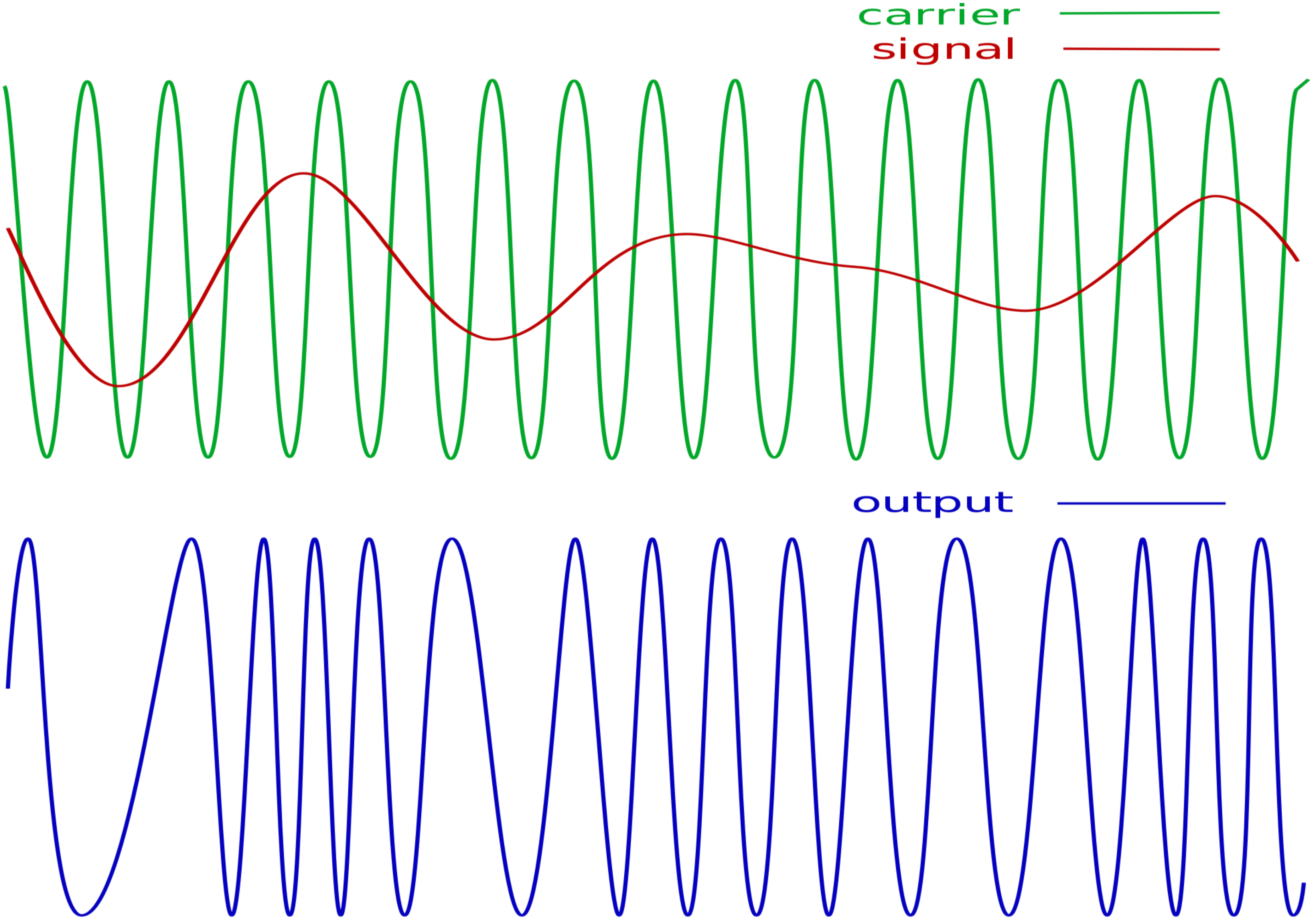


The amplitudes drawn are completely arbitrary, since we have not found any value for $J_n(\beta)$ – this sketch is only to illustrate the spectrum.

FM Signal Waveforms.

Frequency changes at the input are translated to rate of change of frequency at the output. An attempt to illustrate this is shown below:




carrier
signal
output

Carson’s Rule for FM Bandwidth.

An approximation for the bandwidth of an FM signal is given by BW = 2(Maximum frequency deviation + highest modulated frequency)

$$\text{Bandwidth} = 2(\Delta f_c + f_m)$$ **Carson’s Rule**

Advantages of AM Modulation

1.Corverage area of AM Receiver is wider than FM because atmospheric propagation.

2 AM is long distance propagation because λ

3 AM Circuit is cheaper and non complex than FM.

4. AM have bandwidth limited and FM unlimited.

Disdvantages of AM Modulation

1. Signal of AM is not stronger than FM when it propagate through obstacle.

2. Only one sideband of AM transmits Information Signal, So it loss power on other sideband and Carrier.

3. Noise mixes AM Signal in amplitude when it propagates in free space that it make difficulty to recover Original Signal at receiver.

AM	FM
1. In AM modulation, amplitude of the signal is varied, and frequency and phase are kept constant.	1. In FM modulation, frequency of the signal is varied, and amplitude and phase are kept constant.
2. AM has two sidebands	2. FM has infinite number of sidebands
3. The carrier of AM comprises of most of the transmitted power, which contains no information.	3. All transmitted power in FM is useful, and there is no wastage of power unlike AM.
4. Modulation index in AM varies from 0 to 1.	4. Modulation index in FM is always greater than one.
5. AM is more noisy since the AM receivers do not have amplitude limiters.	5. Noise in FM can be reduced by employing amplitude limiters to remove the amplitude variations caused by noise.
6. AM has narrow channel bandwidth which is 2fm.	6. The bandwidth in FM is much higher, up to 10 times as that of AM.
7. In AM if two or more signals received at same frequency, then both will be demodulated, this can lead to interference.	7. In FM if two or more signal received at same frequency, the receiver will capture the stronger signal and eliminate the weaker signal.
8. AM broadcast operates in the medium frequency (MF) and high frequency (HF).	8. FM broadcast operates in the upper VHF and UHF range, where noise effects are minimal.
9. The design of AM transmitter and receiver is not complex for the modulation and demodulation purpose.	9. The design of FM transmitter and receiver is relatively complex for the modulation and demodulation purpose.
10. AM transmission and reception equipments are not that expensive since the circuitry is relatively simple.	10. FM transmission and reception equipment is expensive as the circuitry is complex.

Problem

1) A sinusoidal carrier of 20 V, 2 MHz is frequency modulated by a sinusoidal message signal 0f 10 V, 50 KHz with modulation index 5. Find bandwidth and power ?

$$\text{Bandwidth} = 2\,f_m\,(\beta + 1)$$
$$= 2 \times 50(6)$$
$$= 600\ \text{KHz}$$

$$\text{Power} = \frac{A_c^2}{2R}$$
$$= \frac{400}{2(1)}$$
$$= 200\ \text{W}$$

Problem 2

An FM signal is given by $S(t) = 10\cos(2\pi\times10^6 t + 8\sin 4\pi\times10^3 t)$

Find Δfc , bandwidth and Power ?

$$S(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

$$\Delta f_c = f_m \beta = 16KHz$$

$$\text{Bandwidth} = 2 f_m (\beta + 1)$$

$$= 2\times2(9)$$

$$= 36\,\text{KHz}$$

$$\text{Power} = \frac{A_c^2}{2R}$$

$$= \frac{100}{2(1)}$$

$$= 50\,\text{W}$$