PATTERN RECOGNITION
LECTURE 4
PROXIMITY MEASURES

LECTURE NOTES

10-01-2019

Proximity Measures

- In order to classify patterns they need to be compared against each other and against a standard.
- To determine this similarity/dissimilarity, proximity measures are used.
- The distance between two patterns is used as a proximity measure.
- It is necessary to classify a new pattern.

Distance Measures

- A distance measure is used to find the dissimilarity between pattern representations.
- Patterns which are more similar should be closer.
- The distance function could be a metric or a non-metric.

Properties

- A metric is a measure for which the following properties hold:
  - Positive Reflexivity : $D(X, X) = 0 \; \forall \; X$
  - Symmetry : $D(X, Y) = D(Y, X) \; \forall X, Y$ and
  - Triangular Inequality:

$$D(X, Z) + D(Z, Y) \geq D(X, Y) \; \forall X, Y, Z$$

where $D(X, Y)$ gives the distance between X and Y.

Metric Similarity Function

- Minkowski Metric/ L_p Norm
- Quadratic Form Distance
- Edit Distance

1. Minkowski Metric/ L_p Norm

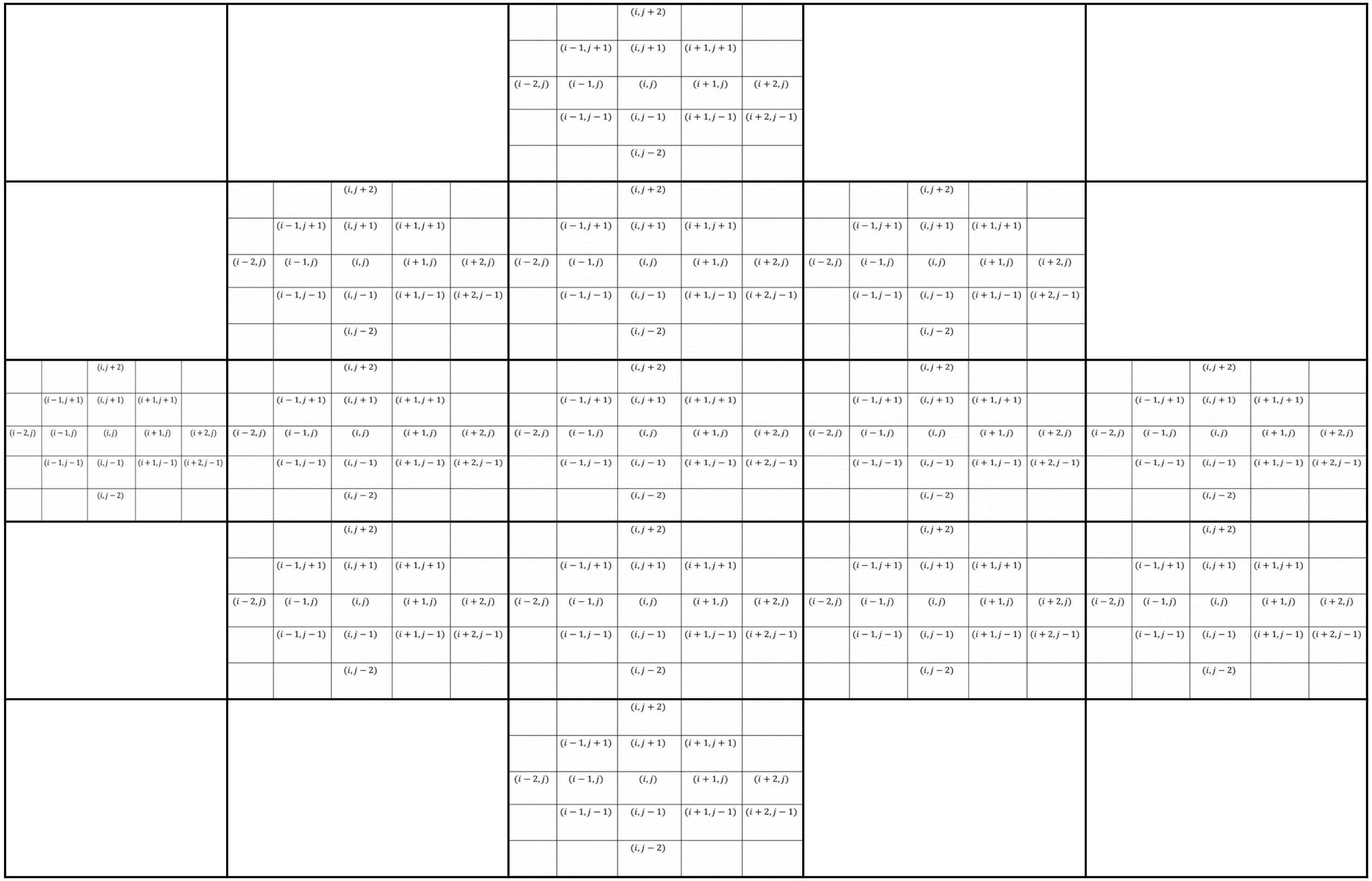
- The popularly used distance metric, called the *Minkowski Metric* is of the form :

$$L_p(X,Y) = \left(\sum_{i=1}^{d} | x_i - y_i |^p\right)^{\frac{1}{p}} \text{------}eqn(1)$$

$$where\ p = 1,2,\ldots,\infty\ and$$
$$d\ is\ the\ dimension$$

- This is also called the L_p norm.
- Depending on the value of p, we get different distance measures.

Lp norm (contd)

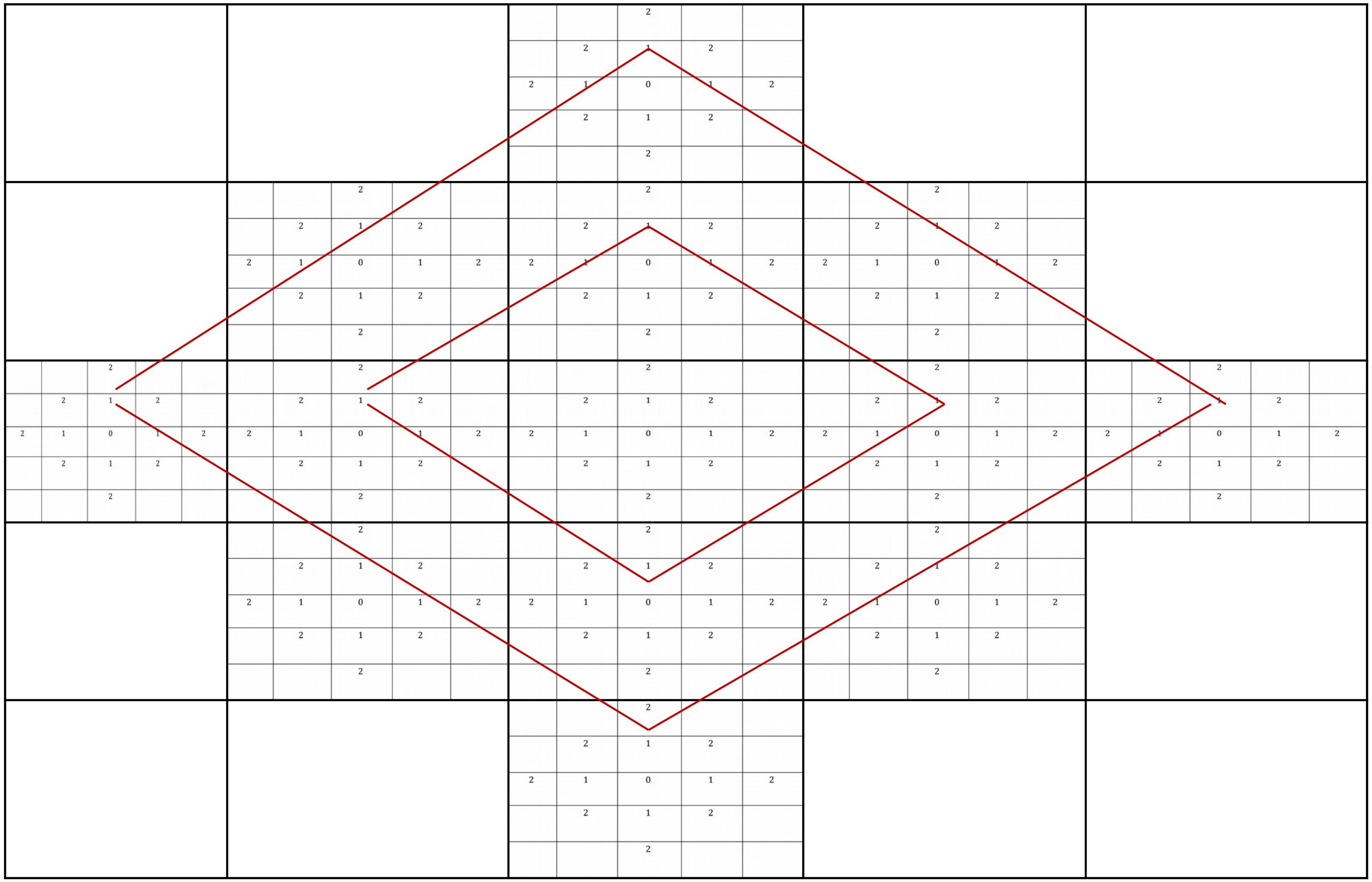


Manhattan Distance/ L_1 Norm

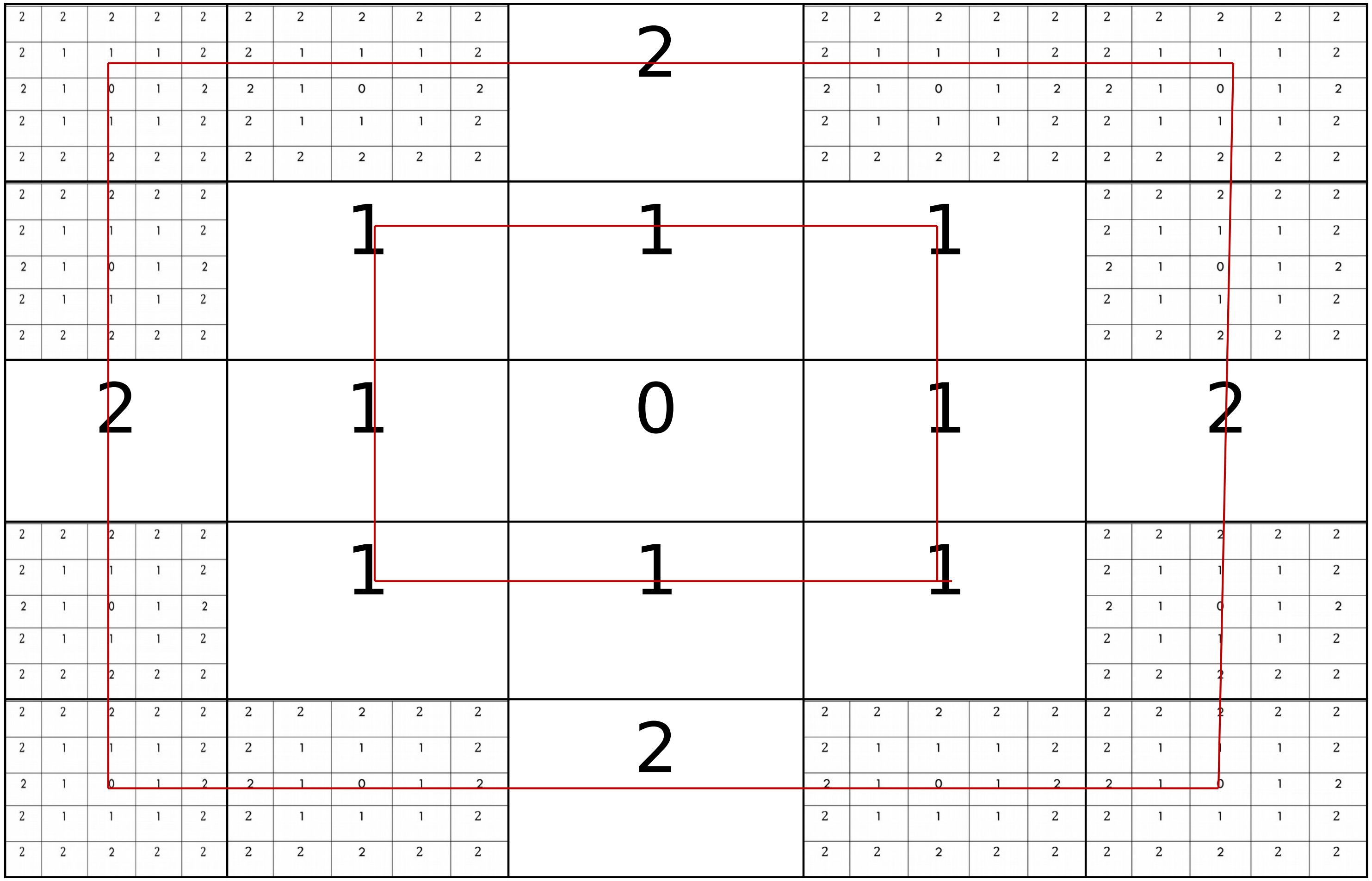
- When $p = 1$ is substituted in *eqn(1)* we get the *Manhattan* or the *city block distance.*
- This is also called the L_1 norm.
- This can be written as:

$$D(X,Y) = \left(\sum_{i=1}^{d} |x_i - y_i|\right)$$

L1 norm called as City Block Distance/ taxi-cab distance: Based on 4-connectivity



L1 norm called as Chessboard Distance:
Based on 8-connectivity

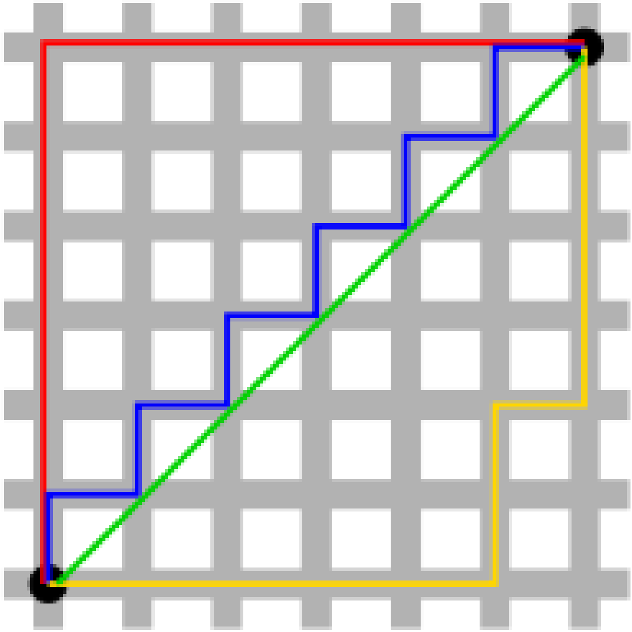


Euclidean Distance/ L_2 Norm

- When $p = 2$ is substituted in Minkowski metric, we get the *Euclidean distance.*
- This is also called the L_2 norm.

$$D(X,Y) = \left(\sum_{i=1}^{d} | x_i - y_i |^2 \right)^{\frac{1}{2}}$$

Taxicab or city block distance vs Euclidean Distance



An illustration comparing the taxicab metric to the Euclidean metric on the plane: According to the taxicab metric the red, yellow, and blue paths have the same length which is 12. According to the Euclidean metric, the green path has length $6\sqrt{2} \approx 8.49$, and is the unique shortest path.

L_∞ Norm and $L_{-\infty}$ Norm

- The L_∞ norm is :

$$D(X,Y) = max|x_i - y_i|$$
$$where\ i = 1,2,\ldots,d.$$

- The $L_{-\infty}$ norm is :

$$D(X,Y) = min|x_i - y_i|$$
$$where\ i = 1,2,\ldots,d.$$

Lp norm (contd)

- Is L_p norm a metric?

  Yes, $\forall\ 1 \leq p \leq \infty$

Histogram distance

- https://stats.stackexchange.com/questions/7400/how-to-assess-the-similarity-of-two-histograms

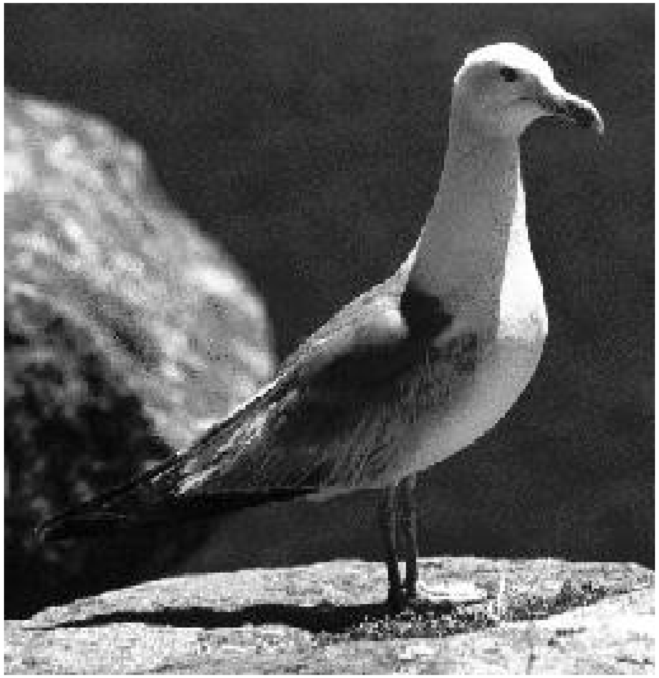
- https://mpatacchiola.github.io/blog/2016/11/12/the-simplest-classifier-histogram-intersection.html

2. Quadratic Form Distance

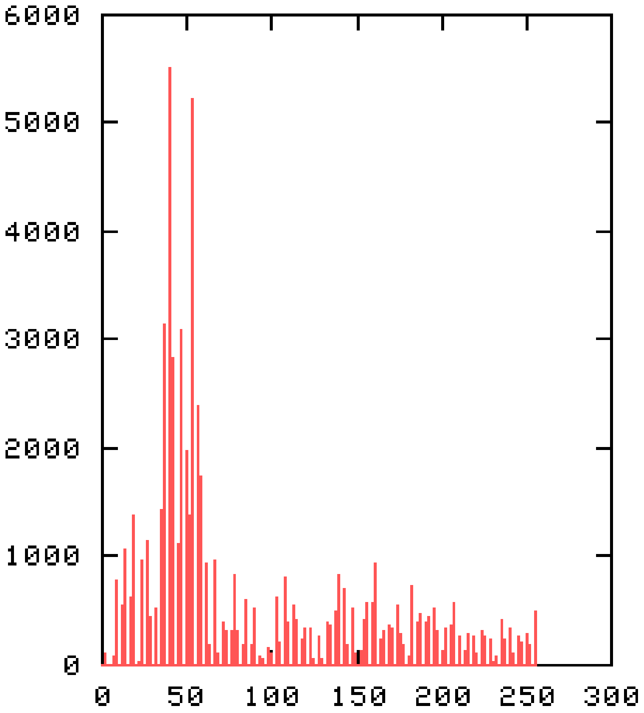
- It allows comparison across different bin locations.
- Quadratic form metrics consider the cross relation of the bins.
- In a naïve implementation, the color histogram quadratic distance is computationally expensive.
- The quadratic form distance between color histograms h_q and h_t is given by:

$$D(h_q, h_t) = (h_q - h_t)^T A (h_q - h_t)$$

where A is the transform matrix.

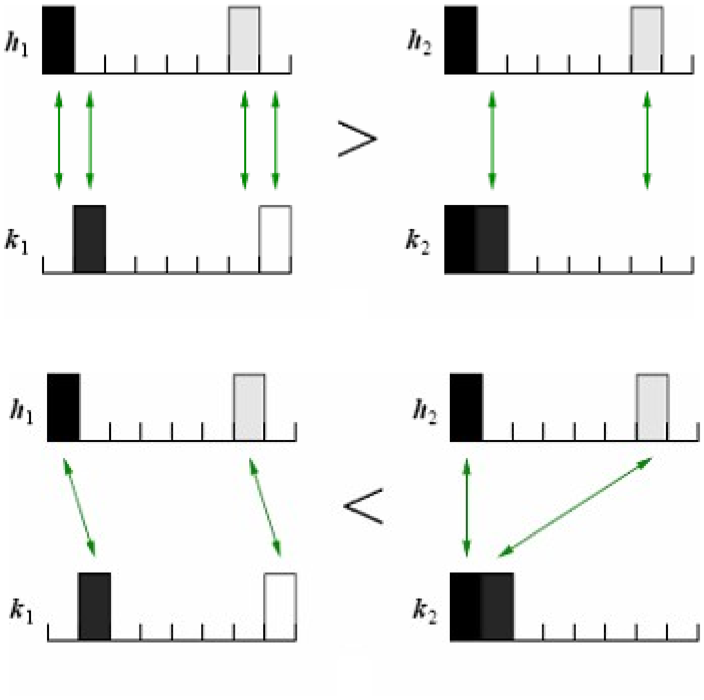


Image

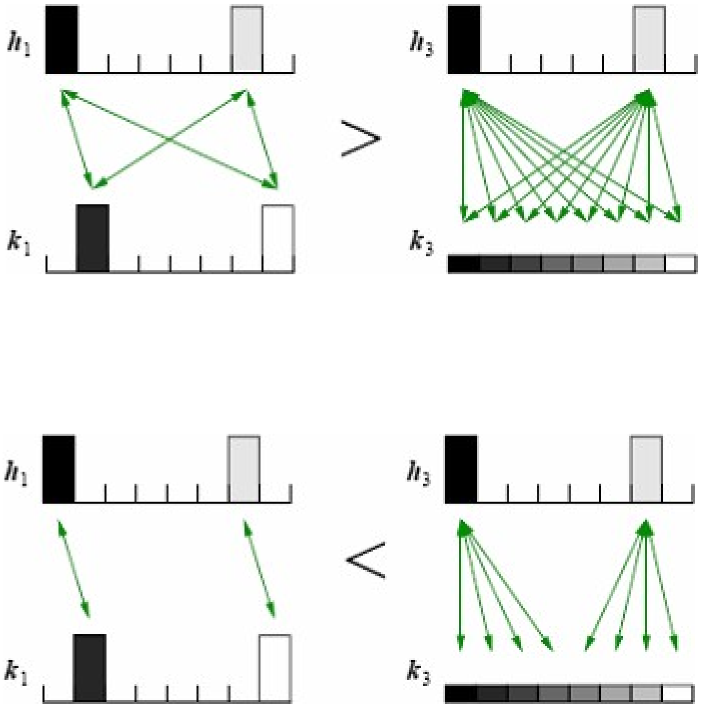


Histogram of the given image

Bin by bin relation



Cross bin relation



Histogram L_1 Distance

- If histograms are normalized,

  $$D\left(h_q, h_t\right) = \sum_{i=1}^{d} [h_q(i) - h_t(i)]$$

  where $d\ is\ the\ dimension$

Cases of Quadratic Form Distance

- Squared Euclidean Distance

$$D(h_q, h_t) = (h_q - h_t)^T A (h_q - h_t)$$

$$where\ A = I, Identity\ matrix$$

$$D_{L_2}(h_q, h_t) = (h_q - h_t)^T (h_q - h_t)$$

Here, cross bin relations are not considered.

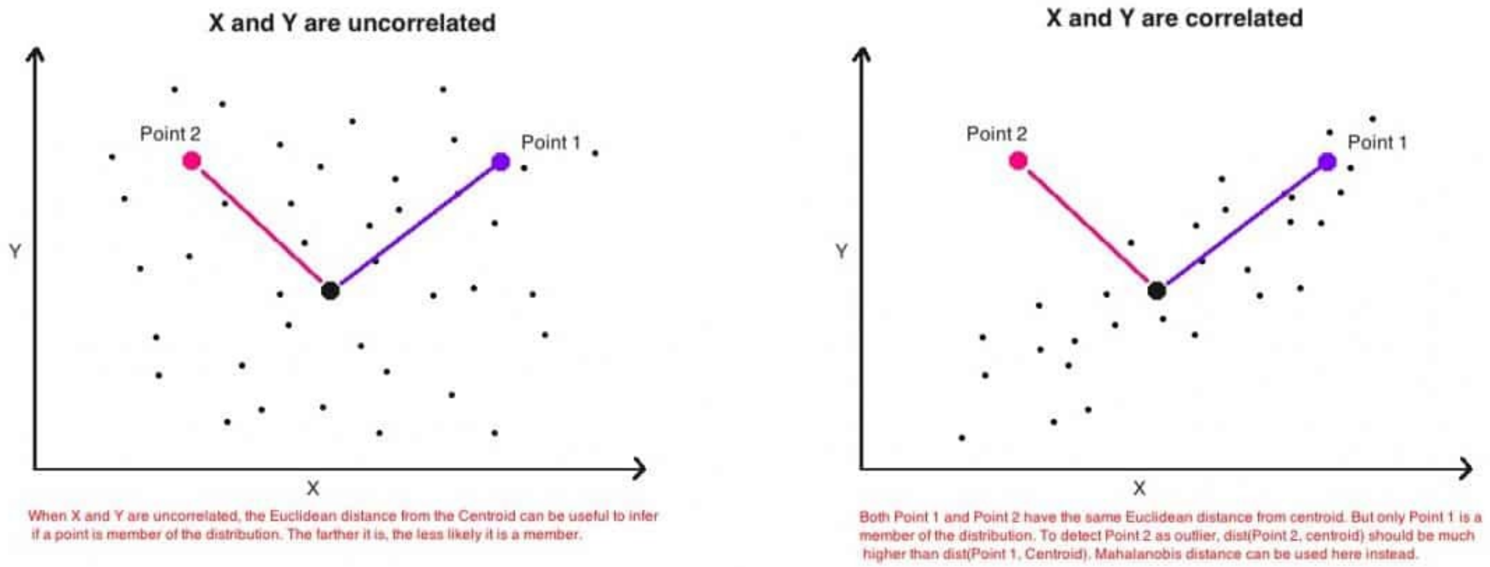
$$\therefore\ D_{L_2}(h_q, h_t) = (h_q - h_t)^2$$

$$= (\sum_{i=1}^{d} [h_q(i) - h_t(i)])^2$$

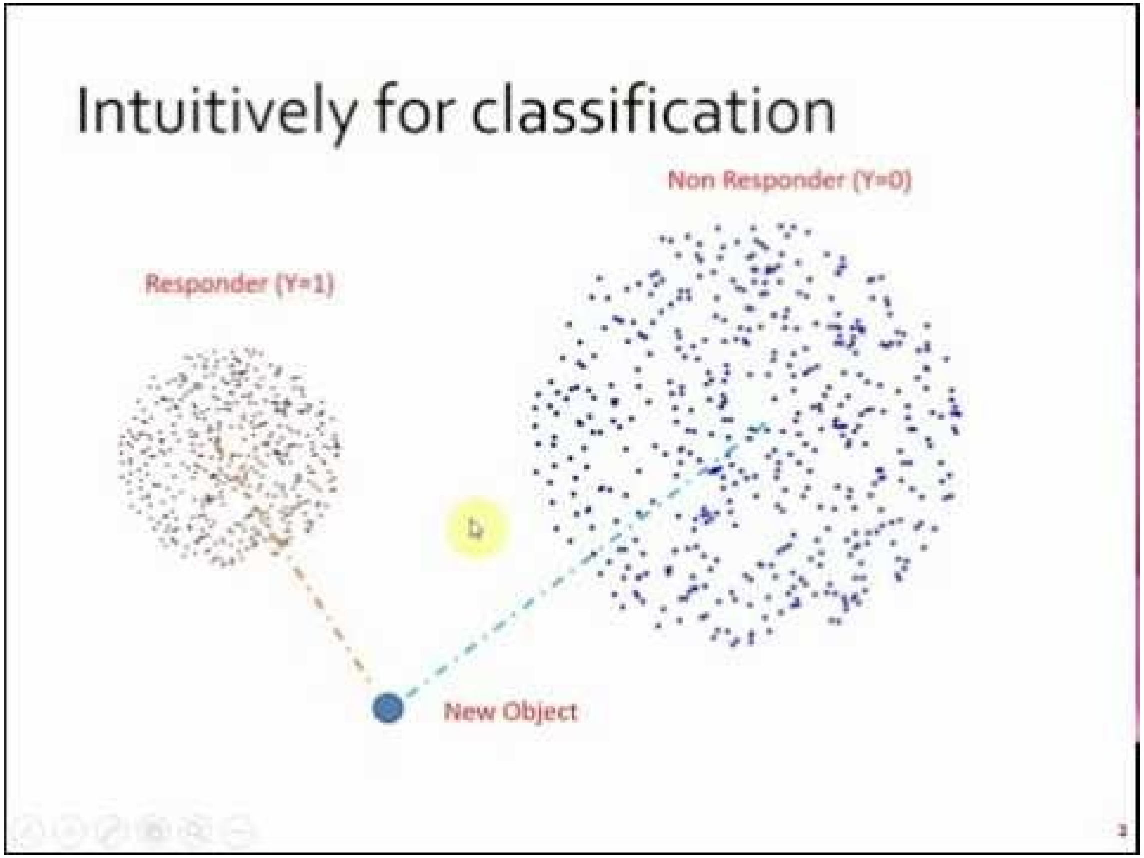
Cases of Quadratic Form Distance

- Squared Mahalanobis Distance
  - Another special case of the quadratic form metric in which the transform matrix A is given by the covariance matrix
  - i.e. $A = \Sigma^{-1}$
  - $D(h_q, h_t) = (h_q - h_t)^T \Sigma^{-1} (h_q - h_t)$

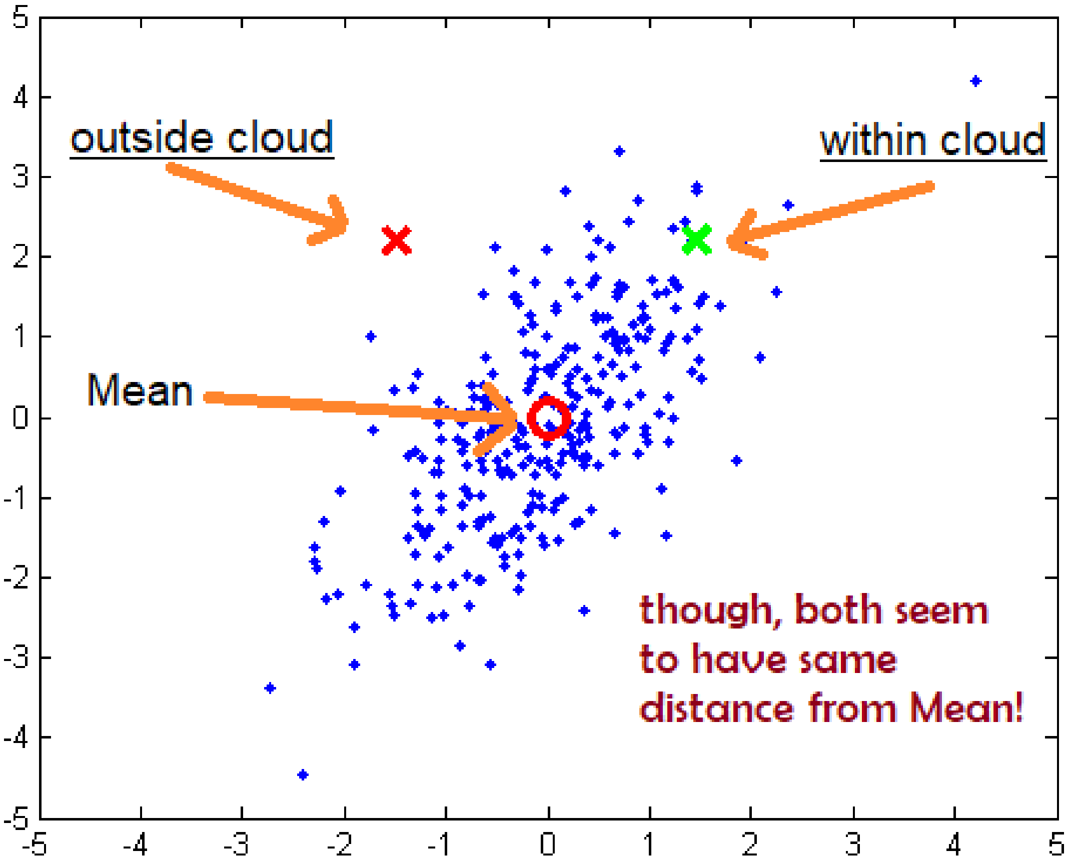
Intuition of Mahalanobis Distance (MD)



Intuition of MD: Classification



Intuition of MD: Finding Outliers



Mahalanobis Distance : Example 1

- Suppose you have data for five people, and each person vector has a height, score on some test, and an age.

X	Y	Z
Height	Score	Age
64	580	29
66	570	33
68	590	37
69	660	46
73	600	55
Mean 68	600	40

- The mean of the data is $(68.0, 600.0, 40.0)$.
- Now suppose you want to know how far another person, $v = (66, 640, 44)$, is from this data.
- It turns out the Mahalanobis Distance is 5.33 (no units).

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- $Z = \begin{bmatrix} -4 & -20 & -11 \\ -2 & -30 & -7 \\ 0 & -10 & -3 \\ 1 & 60 & 6 \\ 5 & 0 & 15 \end{bmatrix}$
- Find the covariance matrix
- $\Sigma = \frac{1}{N-1} Z^T Z$

$$\Sigma = \frac{1}{4}\begin{bmatrix} -4 & -2 & 0 & 1 & 5 \\ -20 & -30 & -10 & 60 & 0 \\ -11 & -7 & -3 & 6 & 15 \end{bmatrix}\begin{bmatrix} -4 & -20 & -11 \\ -2 & -30 & -7 \\ 0 & -10 & -3 \\ 1 & 60 & 6 \\ 5 & 0 & 15 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 46 & 200 & 139 \\ 200 & 5000 & 820 \\ 139 & 820 & 440 \end{bmatrix}$$

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- $\Sigma = \begin{bmatrix} 11.50 & 50 & 34.75 \\ 50 & 1250 & 205 \\ 34.75 & 205 & 110 \end{bmatrix}$
- $\Sigma^{-1} = \begin{bmatrix} 3.6885 & 0.0627 & -1.2821 \\ 0.0627 & 0.0022 & -0.0240 \\ -1.2821 & -0.0240 & 0.4588 \end{bmatrix}$
- $D(h_q, h_t) = (h_q - h_t)^T A (h_q - h_t) - - -Eqn(1)$
- Here $A = \Sigma^{-1}$

- $Eqn(1)$ becomes

$$= [-2 \quad 40 \quad 4]\begin{bmatrix} 3.6885 & 0.0627 & -1.2821 \\ 0.0627 & 0.0022 & -0.0240 \\ -1.2821 & -0.0240 & 0.4588 \end{bmatrix}\begin{bmatrix} -2 \\ 40 \\ 4 \end{bmatrix}$$

$$= [-9.9964 \quad -0.1325 \quad 3.4413]\begin{bmatrix} -2 \\ 40 \\ 4 \end{bmatrix}$$

$$= \sqrt{28.4573}$$

$$= 5.33$$

Example 2

- Given the histogram of a pure red image

  $h_q = [1\,,0\,,0]^{\,T}$

  and a pure orange image:

  $h_t = [0\,,1\,,0]^{\,T}$

- The transform matrix $A = \begin{bmatrix} 1 & 0.9 & 0 \\ 0.9 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- Find the quadratic form distance and the Euclidean distance between them.

Quadratic form distance-Example 2

- $D(h_q, h_t) = (h_q - h_t)^T A (h_q - h_t) --- Eqn(1)$
- $h_q = [1\ 0\ \ 0]^T$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

- $h_t = [0\ \ 1\ \ \ 0]^T$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Quadratic form distance-Example 2

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- $h_q - h_t = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$
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- $(h_q - h_t)^T = [1 \quad -1 \quad 0] - - - Eqn(2)$
- Substituting Eqn (2) and value of A in Eqn (1),
- $D(h_q, h_t) = [1 \ -1 \ 0] \begin{bmatrix} 1 & 0.9 & 0 \\ 0.9 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Quadratic form distance-Example 2

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- $$D\left(h_q, h_t\right) = [1\ \ -1\ \ 0]\begin{bmatrix} 1 & 0.9 & 0 \\ 0.9 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$
$$= [0.1\ \ -0.1\ \ 0]\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$
$$= (0.1) + (0.1) + 0$$
$$= 0.2$$

Euclidean Distance-Example 2

- $D^2\left(h_q, h_t\right) = (h_q - h_t)^2$

$$= \underline{=}[1 \quad -1 \quad 0] \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$=[1 \; + 1 + \; 0]$$

$$= 2$$

- $D\left(h_q, h_t\right) \; = \sqrt{2}$

3. Edit Distance

- The popularly used distance metric, called the *Minkowski Metric* is of the form :

$$L_p(X,Y) = \left(\sum_{i=1}^{d} | x_i - y_i |^p\right)^{\frac{1}{p}} \text{------} eqn(1)$$

$$where\ p = 1,2,\ldots,\infty\ and$$

$$d\ is\ the\ dimension$$

- This is also called the L_p norm.
- Depending on the value of p, we get different distance measures.

Example

		$(i, j+2)$		
	$(i-1, j+1)$	$(i, j+1)$	$(i+1, j+1)$	
$(i-2, j)$	$(i-1, j)$	(i, j)	$(i+1, j)$	$(i+2, j)$
	$(i-1, j-1)$	$(i, j-1)$	$(i+1, j-1)$	$(i+2, j-1)$
		$(i, j-2)$		

So, we get the edit distance to be 3.

THANK YOU