Object Representation, Description and Classification

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Overview

1. Object Representation
   - Boundary Based Object Representation
   - Region Based Object Representation
2. Object Description
   - Boundary Based Object Description
   - Region Based Object Description
3. Object Classification
   - Fishers Linear Discriminant Analysis
     - Minimum Distance Classifier
     - Bayes Classifier

Boundary Based Object Representation

- Set of boundary pixels
- Set of lines approximating boundary
- Vertices of minimal polygon containing the boundary
- chain code
- Signature

Region Based Object Representation

- Set of all pixels in the region of object
- Run length code of region

Boundary Based Object Description

- Length of boundary
- Diameter: max dist of two points on the boundary
- Eccentricity: Ratio of minor and major axis of minimum bounding rectangle
- Curvature: Rate of change of slope
- Fourier Descriptor: FT of $(x_i + jy_i)$, $1 \leq i \leq n$, where (x_i, y_i) s are points on the boundary
- Shape Number: Smallest cyclic difference chain code
  - it is translation invariant as chain code does not change during translation
  - it is scaling invariant due to normalization of grid size w.r.t the grid size of template object
  - it is rotation invariant as cyclic difference is the same for all rotated version
- moments : The ith moment is defined as $m_i = \sum_{r=0}^{L-1}(r-\mu)^i p(r)$ -Translation, rotation and spatial scaling invariant, using normalization

Region Based Object Description

- area: Number pixels
- compactness: $perimeter^2/area$
- moment: The ith moment is defined as $m_i = \sum_{r=0}^{L-1}(r-\mu)^i p(r)$
  -Translation, rotation and spatial scaling invariant, using normalization, scaling in amplitude can also be maid

Object Classification

- Object recognition is required in solving many of the real world problems using images.
- Object recognition can be done using object classification.
- Using the object descriptors, we define a vector called pattern vector
- The pattern vector is a vector whose components are object descriptors.

Object Classification

- For instance, if we want to classify the given fruit into any one of the two categories: Lemon and Orange, the pattern vector can be described as $X = (x_1, x_2)$, where x_1 is weight and x_2 is radius
- Here radius can be computed using the image of the fruit, but the weight can't be computed from image.
- We can use weighing sensor on the conveyer belt where the physical object is kept.
- As the lemon will have less wight and less radius compared to that of orange, all pattern vectors corresponding to lemon and orange will make different clusters in the XY plane.

Fishers Linear Discriminant Analysis

- Let $C_1, C_2, ..., C_k$ be k classes.
- Define decision function d_i for class C_i for all $1 \leq i \leq k$
- if($d_i(X) > d_j(X)$) for all $j \neq i$ then assign patters X to class C_i

Classifiers following Fishers Linear Discriminant Analysis

- Minimum distance classifier
- Bayes classifier (An Optimum Statistical Classifier)

Minimum distance classifier

- Training:
  - i/p: Set of patterns with known label: (X, C_i) for $1 \leq i \leq k$
  - o/p: Mean of each class C_i
  - Find mean vector for each class C_i: $M_i = (1/|C_i|) \sum_{x \in C_i} x$
- Testing
  - i/p: Pattern X; o/p: class label to which X needs to be assigned
  - $if(||M_i - X|| \leq ||M_j - X||)$ for all $j \neq i$, then assign pattern X to the class C_i
- Here the parameter learnt is the mean value of each of the class using the samples with labels.

Bayes classifier

Training

- i/p: (x_i, w_i), where x_i is pattern of object i, and w_i is its class label for all $1 \leq i \leq n$
- o/p: Probability distribution $P(x/w_i)$ and $P(w_i)$ for each i

Testing

If $P(x/w_i).P(w_i) \geq P(x/w_j).P(w_j) \quad \forall i \neq j$, then assign x to class w_i

Proof of optimality of Bayes classification

Bayes Classifier

If $P(x/w_i).P(w_i) \geq P(x/w_j).P(w_j) \quad \forall i \neq j$, then $x \in C_i$

To prove: Bayes classifier is the statistically optimum classifier.

Proof.

We define the risk of assigning x to w_j as

$$r_j(x) = \sum_{k=1}^{w} L_{k_j} P(w_k/x) \tag{1}$$

L_{k_j} is the loss incurred when x is assigned to w_j but x belong to w_k with probability $P(w_k/x)$.
An optimum classifier needs to minimize the risk
Hence the optimal classifier is:

$$\text{If } r_i(x) \leq r_j(x) \;\; \forall j \neq i \; then\ assign\ x\ to\ w_i \tag{2}$$

Proof. contd.

$$(2) \iff \text{If } \sum_{k=1}^{w} L_{k_i} P(w_k/x) \leq \sum_{k=1}^{w} L_{k_j} P(w_k/x) \quad \forall i \neq j \tag{3}$$

then assign x to w_i

$$\text{Now } L_{k_i} = 1 - \delta_{k_i} \tag{4}$$

where $\delta_{k_i} = 1$ when $k = i$
0 otherwise.

From 3 and 4

$$\iff \text{If } \sum_{k=1}^{w} (1 - \delta_{k_i}) P(w_k/x) \leq \sum_{k=1}^{w} (1 - \delta_{k_j}) P(w_k/x) \quad \forall i \neq j \tag{5}$$

then assign x to w_i

$\sum_{k=1}^{w} (1 - \delta_{k_i})\ P(w_k/x) = \sum_{k=1}^{w}\ P(w_k/x) - \sum_{k=1}^{w} \delta_{k_i}\ P(w_k/x)$

$[\because P(A/B) = P(B/A).\frac{P(A)}{P(B)}]$

Proof. contd.

$= \sum_{k=1}^{w} P(x/w_k).\frac{P(w_k)}{P(x)} - \sum_{k=1}^{w} \delta_{k_i} P(x/w_k).\frac{P(w_k)}{P(x)}$
$= \frac{1}{P(x)}\left[\sum_{k=1}^{w} P(x/w_k).P(w_k) - \sum_{k=1}^{w} \delta_{k_i} P(x/w_k).P(w_k)\right]$
Now by definition of δ_{k_i}

$$\delta_{k_i} = \begin{cases} 1, & \text{when } k = i \\ 0, & \text{otherwise} \end{cases}$$

$= \frac{1}{P(x)}\left[\sum_{k=1}^{w} P(x/w_k).P(w_k) - P(x/w_i).P(w_i)\right]$
$= \frac{1}{P(x)}\left[P(x) - P(x/w_i).P(w_i)\right]$
From 5,
If $\frac{1}{P(x)}\left[P(x) - P(x/w_i).P(w_i)\right] \leq \frac{1}{P(x)}\left[P(x) - P(x/w_j).P(w_j)\right] \ \forall i \neq j$
then assign x to w_i.
$P(x) > 0$, hence can be canceled out.
$\iff$ if $P(x) - P(x/w_i).P(w_i) \leq P(x) - P(x/w_j).P(w_j) \ \forall i \neq j$
then assign x to w_i

Proof. contd.

$\Longleftrightarrow$ if $P(x/w_i).P(w_i) \geq P(x/w_j).P(w_j) \;\; \forall i \neq j$

then assign x to w_i,

which is the Bayes classifier.

Hence Bayes classifier is statistically optimum classifier. $\square$