

Part 1: Introduction to Computer Graphics

1. What do you mean by computer graphics?

The branch of science and technology concerned with methods and techniques for converting data to or from visual presentation using computers.

- Create an image.
- Store the image in the memory.
- Display the image on display device.
- Make a processing on the images.
- Interact with the image.

2. What are the applications of computer graphics?

1. Computer Aided Design
2. Graphical User Interface
3. Entertainment
4. Simulation and Training
5. Education and Presentation
6. Computer Generated Art
7. Scientific Visualization
8. Image Processing
9. Virtual reality

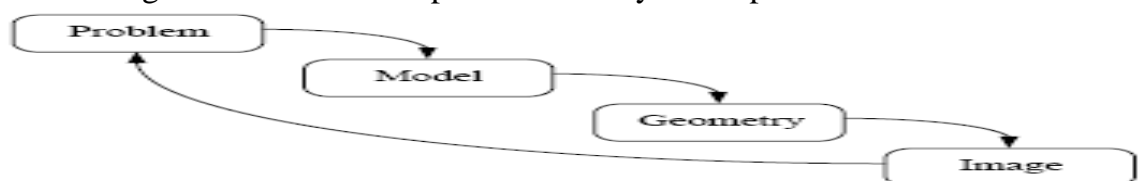
3. What can the programmer do in computer graphics?

- Develop the geometric representation for the geometric objects of the images.
- Assemble these objects into an appropriate geometric space.
- Specify how the scene is to be viewed and how it will be displayed on the graphic device.
- Define some animation for the image.
- Design a ways for the user to interact with the scene as it is presented.

4. How can the computer graphics used in solving problems?

GC can solve a lot of problems:

- Identifying a problem.
- Building the model.
- Represent the problem geometrically and create an image.
- Use the image to understand the problem and try find a possible solution.



5. Computer Graphics API

Graphic API's is a set of tools that allow a programmer to write applications that include the use of interactive computer graphics without dealing with system details for tasks such as window handling and interactions.

6. What do you mean by GUI?

GUI stands for Graphical user interface. A major component of a GUI is a window manager that allows a user to display multiple-window areas. To make a particular window active we simply click in that window using an interactive pointing device. Interfaces also display menus and icons for fast selection of processing options or parameter values.

7. What does it mean by RGB?

The RGB is a color model, in which red, green, and blue light are added together in various ways to reproduce a different array of colors. The name of the model comes from the initials of the three additive primary colors, red, green, and blue.

8. Define refresh buffer/Frame buffer?

Picture definition is stored in a memory area called the refresh buffer or frame buffer. This memory area holds the set of intensity values for all the screen points.

9. Define Pixel.

Each screen point is referred to as a pixel or pel (Picture element).

10. Define bitmap.

On a black and white system with one bit per pixel, the frame buffer is commonly known as a bitmap.

11. What is the role of a video controller?

It is used to control the operation of the display device by accessing the frame buffer to refresh the screen.

12. Define Graphics controller /Display controller/Display processor.

The purpose of the display processor is to free the CPU from graphic chores. A major task of the display processor is digitizing a picture definition given in an application program into a set of pixel intensity values for storage in the frame buffer.

13. What do you mean by scan conversion?

A major task of the display processor is digitizing a picture definition given in an application program into a set of pixel intensity values for storage in the frame buffer. This digitization process is called scan conversion.

The status of each pixel on the screen was stored in a memory location (memory mapped display).

- Each pixel is numbered sequentially.
- By writing values to the correct locations in memory the appearance of the screen can be controlled.
- Found out if it is controlled by a programmer.
- A program can turn a pixel on or off.

FF FF FF	0
FF FF FF	1
FF FF FF	2
FF 00 00	3
↓	
FF FF FF	22
FF 00 00	23
FF FF FF	24

- A computer graphics system is a computer system; that have all the components of a general-purpose computer system.
- There are six major elements in our system:
- Input devices
- Central Processing Unit
- Graphics Processing Unit
- Memory
- Frame buffer
- Output devices

```

graph LR
    subgraph Input
        K[Keyboard] --> Bus
        T[Tablet] --> Bus
        M[Mouse] --> Bus
    end
    subgraph Core
        P[Processor] <--> Mem[Memory]
    end
    Bus --> P
    P --> FB[Frame buffer]
    FB --> Out[Monitor]
  
```

raster scan displays	random scan displays
The electron beam is swept across the screen, one row at a time from top to bottom.	The electron beam is directed to the parts of the screen where a picture is to be drawn.

18. What is the difference between raster storage image and vector storage image?

Raster Image:

The images is considered as rectangular arrays of pixels, each pixel have different colors stored as three numbers, for RGB. In a Monochrome system [black-and-white], each screen point is either on (a bit value of 1) or off (a bit value of 0), so only one bit per pixel is needed to store the intensity of screen positions.

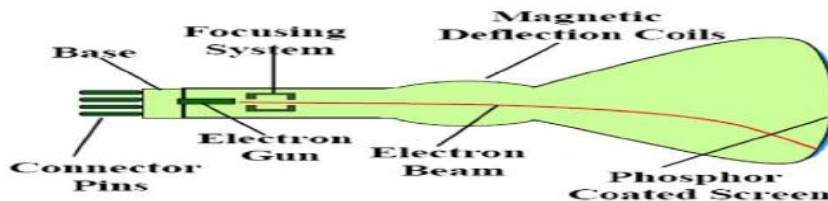
Vector Image:

The image is stored as a set of instructions for displaying the.

- Are often used for text, diagrams, mechanical drawings, and other applications where precision are important and photographic images and complex shading aren't needed.

19. List the operating characteristics for the video display systems based on the CRT technology?

- A beam of electrons emitted by an electron gun, passes through focusing System and deflection systems that direct the beam toward specified positions on the phosphor-coated screen.
- The phosphor-coated screen then emits a small spot of light at each position contacted by the electron beam.
- The light emitted by the phosphor fades very rapidly, the picture is redrawn by quickly directing the electron beam back over the same points. This type of display is called a refresh CRT.

**20. Define persistence in terms of CRT Phosphorous.**

Persistence is the one of the major property of phosphorous used in CRT's. It means how long they continue to emit light after the electron beam is removed.

21. Define resolution.

The maximum number of points that can be displayed without overlap on a CRT monitor.

22. What do you mean by an aspect ratio?

Aspect ratio is the ratio of vertical points to horizontal points necessary to produce equal length lines in both directions on the screen. An aspect ratio of $\frac{3}{4}$ means that a vertical line plotted with three points has same length as a horizontal line plotted with 4 points.

23. What are the different properties of phosphorus?

1. Color
2. Persistence

24. What are the different types of Flat-Screens, and what is the difference between them?

- light-emitting diodes (LEDs) - light-emitting diodes that can be turned on and off
- liquid-crystal displays (LCDs) - polarization of the liquid crystals in the middle panel
- plasma panels - voltages on the grids to energize gases
- Similarities:
 - All use a two-dimensional grid to address individual light-emitting elements.
 - The two outside plates each contain perpendicular parallel grids of wires .
 - Sending electrical signals to a wire in each grid, generates electrical field at the

Intersection of two wires, can control the corresponding element in the middle plate.

25. What do you mean by retracing? Define horizontal as well as vertical retracing.

- Retracing:

At the end of each scan line, the electron beam returns to the left side of the screen to begin displaying the next scan line.

- Horizontal retrace

The return to the left of the screen, after refreshing each scan line.

- Vertical retrace

At the end of each frame, the electron beam returns to the top left corner of the screen to begin the next frame.

26. What do you mean by interlacing?

It is the method of incrementally displaying a visual on a CRT. On some raster scan systems, each frame is displayed in two passes using an interlaced refresh procedure. In the first pass, the beam sweeps across every other scan line from top to bottom. Then after the vertical retrace, the beam sweeps out the remaining scan lines.

27. What is a Beam penetration method?

This technique is used in random scan display systems. Two layers of phosphor (red and green) are coated onto the inside of the CRT screen, the displayed colors depends on how far the electron beam penetrates into the phosphors layers. A slow electron beam excites only the outer red layer. A very fast electron beam penetrates through the red layer and hence excites the green layer. An average electron beam gives the combination of red and green color. That is yellow and orange. This technique only provides four colors.

28. Define shadow masking.

This technique is used in raster scan display devices.

It gives more colors than a beam penetration method.

A shadow Mask CRT has three phosphor color dots at each pixel location (red light, green light and blue light).

This type of CRT also has three electron guns one for each color dot.

A shadow mask grid is installed just behind the phosphor coated screen.

When the three beams pass through a hole in the shadow mask, they activate a dot triangle, which appears as a small color spot on the screen.

More than 17 million different colors can be obtained in a full color system.

29. What are the different types of Flat-Screens, and what is the difference between them?

- light-emitting diodes (LEDs) - light-emitting diodes that can be turned on and off
- liquid-crystal displays (LCDs) - polarization of the liquid crystals in the middle panel
- plasma panels - voltages on the grids to energize gases
- Similarities:
 - All use a two-dimensional grid to address individual light-emitting elements.
 - The two outside plates each contain perpendicular parallel grids of wires.
 - Sending electrical signals to a wire in each grid, generates electrical field at the Intersection of two wires, can control the corresponding element in the middle plate.

30. What are the popular image storage formats?

- **Jpeg format.** This Lossy format compresses image blocks based on thresholds in the human visual system. This format works well for natural images.
- **Tiff format.** This format is most commonly used to hold binary images or lossless compressed 8- or 16-bit RGB although many other options exist.
- **Ppm format.** A lossless, uncompressed format is most often used for 8-bit RGB images although many options exist.
- **Png format.** This is a set of lossless formats with a good set of open source management tools.

31. Consider three different raster systems with resolutions of 640 x 480, 1280 x 1024, and 2560 x 2048.

a. What size is frame buffer (in bytes) for each of these systems to store 12 bits per pixel?

Because eight bits constitute a byte, frame-buffer sizes of the systems are as follows:
 $640 \times 480 \times 12 \text{ bits} / 8 = 450\text{KB};$
 $1280 \times 1024 \times 12 \text{ bits} / 8 = 1920\text{KB};$
 $2560 \times 2048 \times 12 \text{ bits} / 8 = 7680\text{KB};$

b. How much storage (in bytes) is required for each system if 24 bits per pixel are to be stored?

Similarly, each of the above results is just doubled for 24 (12×2) bits of storage per pixel.

32. Q33. Consider two raster systems with the resolutions of 640 x 480 and 1280 x 1024.

a. How many pixels could be accessed per second in each of these systems by a display controller that refreshes the screen at a rate of 60 frames per second?

Since 60 frames are refreshed per second.
Each frame consists of 640 x 480 pixels,
- The access rate of such a system = $(640 \times 480) \times 60 = 1.8432 \times 10^7$ pixels/second.
For the 1280 x 1024 system,
- The access rate is $(1280 \times 1024) \times 60 = 7.86432 \times 10^7$ pixels/second.

b. What is the access time per pixel in each system?

The access time per pixel = $1 / \text{access rate}$.

The access time is around 54 nanoseconds/pixel for the 640 x 480 system,

The access time is around 12.7 nanoseconds/pixel for the 1280x1024 system.

33. Consider a raster system with the resolution of 1024 x 768 pixels and the color palette calls for 65,536 colors. What is the minimum amount of video RAM that the computer must have to support the above-mentioned resolution and number of colors?

No of Colors = 65,536 colors

Number of bits per pixel = $\log_2 (65,536) = 16$ -bit color.

The display's resolution is 1024X768 pixels

Total Number of pixels = 786,432 (1024×768) pixels.

The total number of bits required

$= 786,432 \times 16 = 12,582,912 \text{ Bits} = 1,572,864 \text{ bytes} = 1,536 \text{ KB} = 1.5 \text{ MB}$

34. How Many k bytes does a frame buffer needs in a 600 x 400 pixel?

Given :- Resolution is 600 x 400

Suppose 1 pixel can store n bits

$$\begin{aligned} \text{Then, the size of frame buffer} &= \text{Resolution} \times \text{bits per pixel} \\ &= (600 \times 400) \times n \text{ bits} \\ &= 240000 \text{ } n \text{ bits} \\ &= \frac{240000 \text{ } n}{1024 \times 8} \text{ k bytes} \quad (\text{as } 1\text{kb} = 1024 \text{ bites}) \\ &= 29.30 \text{ } n \text{ k bytes} \end{aligned}$$

35. Q38. How much time is spent scanning across each row of pixels during screen refresh on a raster system with resolution of 1280 X 1024 and a refresh rate of 60 frames per second?

Resolution = 1280 X 1024

That means system contains 1024 scan lines and each scan line contains 128 pixels

Refresh rate = 60 frame/sec.

1 frame takes = $1/60 \text{ sec} = 0.01666 \text{ sec}$.

1 frame buffer consist of 1024 scan lines (It means then 1024 scan lines takes 0.01666 sec) 1

1 scan line takes = $0.01666 / 1024 = 10.6 \mu\text{sec}$

36. Suppose RGB raster system is to be designed using on 8 inch X 10 inch screen with a resolution of 100 pixels per inch in each direction. If we want to store 6 bits per pixel in the frame buffer, how much storage (in bytes) do we need for frame buffer?

Resolution = 8 inch X 10 inch (100 pixels per inch)

Resolution = 8 X 100 by 10 X 100 pixel = 800 X 1000 pixel

1 pixel can be stored in 6 bits

Frame buffer size = $800 \times 100 \times 6 \text{ bits} = 100 \times 100 \times 6 \text{ Byte}$

37. Find out the aspect ratio of the raster system using 8 x 10 inches screen and 100 pixel/inch.

We know that,			
Aspect ratio	=	$\frac{\text{Width}}{\text{Height}}$	
	=	$\frac{8 \times 100}{10 \times 100}$	= 4 / 5
Aspect ratio	=	4 : 5	

38. Consider three different raster systems with resolutions of 640 by 480, 1280 by 1024, and 2560 by 2048. What size frame buffer (in bytes) is needed for each of these systems to store 12 bits per pixel? How much storage is required for each system if 24 bits per pixel are to be stored?

<p>Frame-buffer size for each of the systems is</p> <p>$640 \times 480 \times 12 \text{ bits} \div 8 \text{ bits per byte} = 450 \text{ KB}$</p> <p>$1280 \times 1024 \times 12 \text{ bits} \div 8 \text{ bits per byte} = 1920 \text{ KB}$</p> <p>$2560 \times 2048 \times 12 \text{ bits} \div 8 \text{ bits per byte} = 7680 \text{ KB}$</p> <p>For 24 bits of storage per pixel, each of the above values is doubled.</p> <p>900 KB & 3840 KB & 15360 KB</p>
--

39. How long does it take to load a 640-by-480 frame buffer with 12 bits per pixel, if 10^5 bits can be transferred per second?

<p>Let X the time that will be taken to load a 640-by-480 frame buffer with 12 bits per pixel.</p> <p>Number of bits = $640 * 480 * 12 = 3686400 \text{ bits}$</p> <p>1 sec X 10^5 bits</p> <p>X sec(s) X 3686400 bits</p> <p>Then $X = 3686400 / 10^5 = 36.864 \text{ second}$</p>
--

40. How much time is spent in scanning across each row of pixels during screen refresh on a raster system with a resolution of 1280 by 1024 and refresh rate of 60 frames per second?

<p>The time required for scanning one frame is $1/60 = 0.01666$</p> <p>One frame has 1024</p> <p>The time of scanning on row = $0.01666 / 1024 = 1.627 * 10^{-5} \text{ sec}$</p>

41. Suppose we have a video monitor with a display area with 12 inches width and 9.6 inches high. If the resolution is 1280 X 1024 and the aspect ratio is 1, what are the width and the height of each point on the screen?

Answer:

1280 pixels wide = 12 inches
 1024 pixels high = 9.6 inches
 $1280 / 1024 = 12 / 9.6 = \text{aspect ratio of 1 (so the width should equal height)}$
 1 pixel width = $12 / 1280 = 0.009375 \text{ inches}$
 1 pixel height = $9.6 / 1024 = 0.009375 \text{ inches}$

42. How long would it take to load a 640-by-480 frame buffer with 12 bits per pixel, if 105 bits can be transferred per second? How long would it take to load a 24-bit-per-pixel frame buffer with a resolution of 1280 by 1024 using this same transfer rate?

Total number of bits for the frame = $640 \times 480 \times 12 \text{ bits} = 3686400 \text{ bits}$
 The time needed to load the frame buffer = $3686400 / 10^5 \text{ sec} = 36.864 \text{ sec}$
 Total number of bits for the frame = $1280 \times 1024 \times 24 \text{ bits} = 31457280 \text{ bits}$
 The time needed to load the frame buffer = $31457280 / 10^5 \text{ sec} = 314.5728 \text{ sec}$

43. Suppose an RGB raster system is to be designed using an 8-inch by 10-inch screen with a resolution of 100 pixels per inch in each direction. If we want to store 6 bits per pixel in the frame buffer, how much storage bytes do we need for the frame buffer?

The size of frame buffer is $(8 \times 10 \times 100 \times 100 \times 6) / 8 = 600000 \text{ bytes}$

Part 2: Graphics Primitives

1. How can an application program actually draw something on screen?

In summary, the process of generating a graphics image on the display device can be represented by the following pipeline: Figure 25 - "Programmer's model of a computer graphics system - version 2".



2. Describe what is performed by the following functions:

- Setpixel(x, y, color) - Sets the pixel at position (x, y) to the given color.
- Getpixel(x, y): Gets the color at the pixel at position (x, y).

3. For the brute force line drawing algorithm:

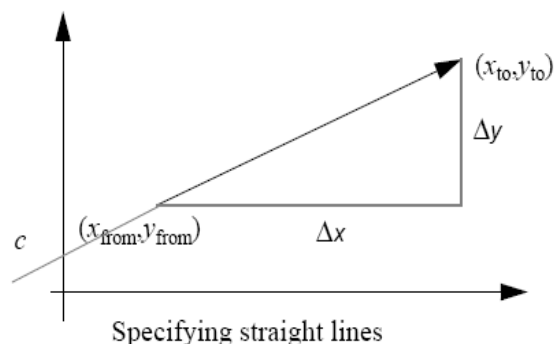
- Analyze the Basic concept of drawing a line using the brute force algorithm?

$y = mx + c$

where m is the gradient of the line:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_{\text{from}} - y_{\text{to}}}{x_{\text{from}} - x_{\text{to}}}$$

and c is its intercept of the y-axis

$$c = y_{\text{from}} - m x_{\text{from}}$$


4. Write pseudo code for applying the algorithm.

```
Method lineBrute (xfrom, yfrom, xto, yto) {
    DeltaY = yto-yfrom;
    DeltaX = xto-xfrom;
    m = DeltaY/DeltaX;
    c=yfrom - (m*xfrom);
    for(int x= xfrom; x<xto; x++) {
        y = (m*x) + c;
        Plot(x, y)
    } }//lineDraw
```

5. Using the Brute force algorithm to digitize a line with end points (20, 10) and (26, 14).

Start Point		End Point		C
20	10	26	14	-3.3333333
X1	Y1	X2	Y2	
				$y=m*x+c$
Delta X	6	Delta Y	4	
Slope (m)	0.666666667			

slope is less than 1 (Loop on X)		
Value Of X	Values of Y	Int Y
20	10	10
21	10.666666667	10
22	11.333333333	11
23	12	12
24	12.666666667	12
25	13.333333333	13
26	14	14

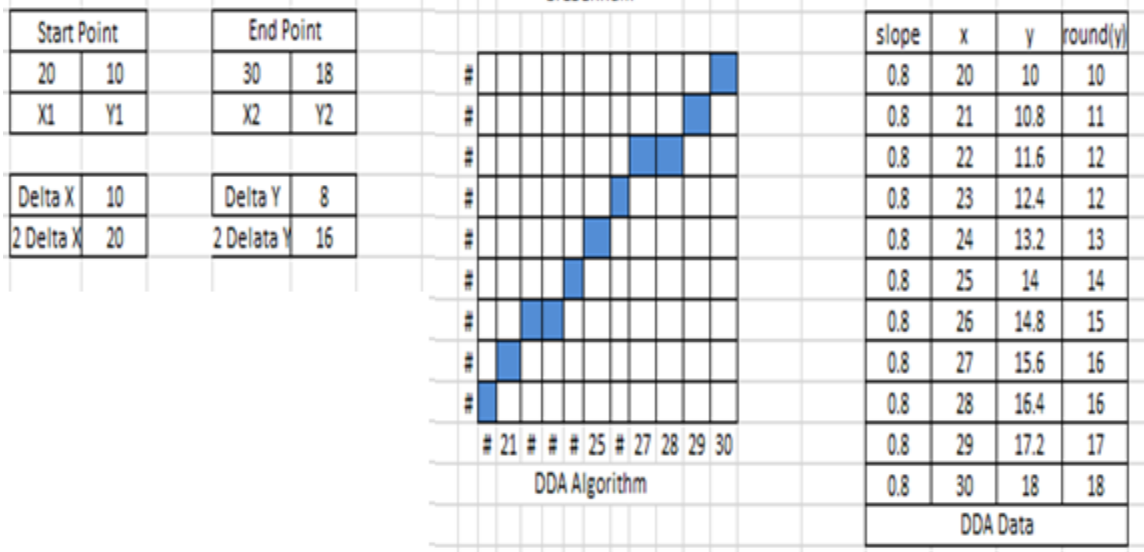
15							
14							
13							
12							
11							
10							
	20	21	22	23	24	25	26

6. Digitize a line with end points (20, 10) and (30, 18).

7. Write the algorithm of the line drawing algorithm using Digital Differential Analyzer (DDA).

1. START
2. Get the values of the starting and ending co-ordinates i.e. , (x_1, y_1) and (x_2, y_2) .
3. Find the value of slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
4. If $|m| \leq 1$ then $\Delta x = 1, \Delta y = m\Delta x$
 $x_k + 1 = x_k + 1, y_k + 1 = y_k + m$
5. If $|m| \geq 1$ then $\Delta y = 1, \Delta x = \Delta y/m$
 $x_k + 1 = x_k + 1/m, y_k + 1 = y_k + 1$
6. STOP

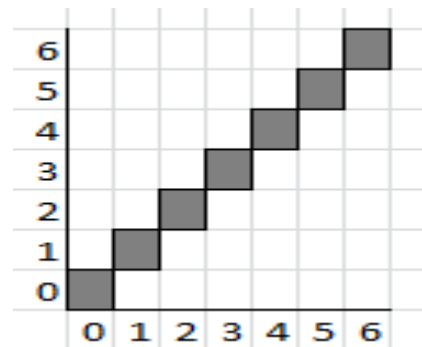
8. Using the DDA algorithm digitize a line with end points (10,15) and (15,30).
9. Digitize a line with end points (20, 10) and (30, 18) using DDA line drawing Algorithm.



10. Implement the DDA algorithm to draw a line from (0,0) to (6,6).

$$M = (6-0)/(6-0) = 6/6 = 1 \quad X_{k+1} = X_k + 1 \quad Y_{k+1} = Y_k + m = Y_k + 1 \quad X_k = 0 \quad Y_k = 0$$

K	X	Y
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6



Eg. Example: Digitize a line with end points (10,15) and (15,30).

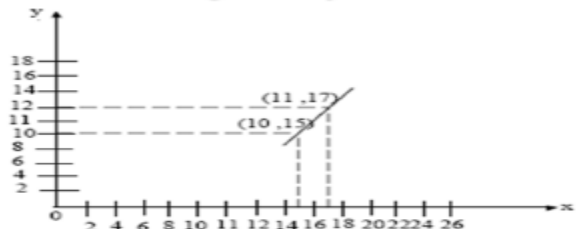
Solution:

$$\text{The slope of line is } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30 - 15}{15 - 10} = \frac{15}{5} = 3$$

$$|m| > 1$$

So we sample at y interval. The formula is given by $x_{k+1} = x_k + 1/m$.

S.N	x	y
1	10	15
2	10	16
3	11	17
4	11	18



11. The process of drawing circle using Brute force method can be enhanced by taking greater advantage of the symmetry in a circle.

Write the complete algorithm used to apply this enhancement.

```

Public void bruteCircleWithCheating (int x0, int y0, int radius)
{
    for(int theta=0; theta<45; theta++)
    {
        int x = round(radius * cos(theta));
        int y = round(radius * sin(theta));
        circlepoints (x0, y0 , x,y)
    }
} //bruteCircle

```

```

/* Center at the origin */
Void circlepoints (int x, int y)
{
    putpixel ( x, y);
    putpixel ( y, x);
    putpixel ( y, -x);
    putpixel ( x, -y);
    putpixel ( -x, -y);
    putpixel ( -y, -x);
    putpixel ( -y, x);
    putpixel ( -x, y);
}

```

Part 3: 2D Transformation

1. Derive the homogenous matrices to represent the following transformation

Translation - Rotation about the origin – Scaling

Homogenous rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogenous scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} xsf & 0 & 0 \\ 0 & ysf & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogenous translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2. Discuss the difference between the rotation about the origin and the rotation about a local point.

Time for some trigonometry:

$$x_2 = r \cos(\theta + \phi)$$

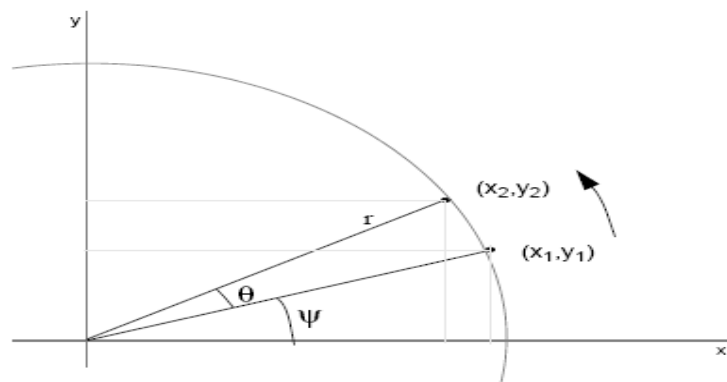
$$y_2 = r \sin(\theta + \phi)$$

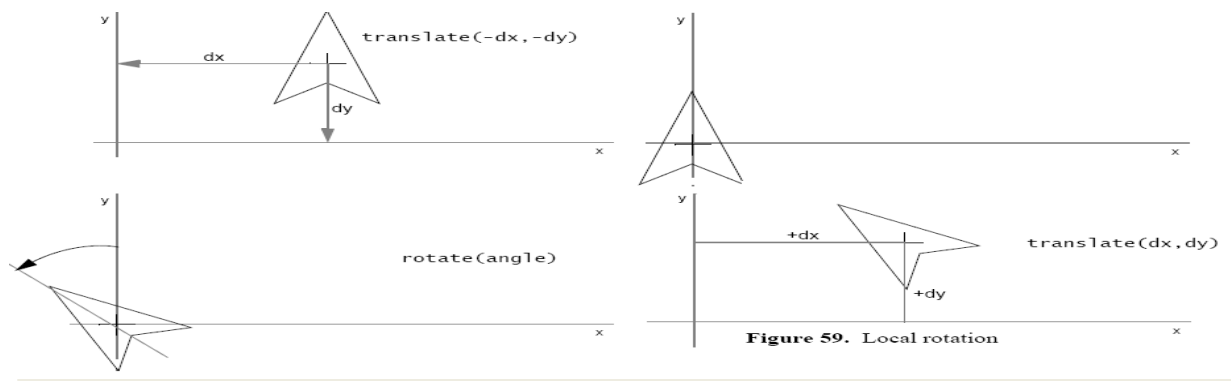
$$x_2 = r (\cos \theta \cos \phi - \sin \theta \sin \phi)$$

$$\text{since } x_1 = r \cos \phi \text{ and } y_1 = r \sin \theta$$

$$x_2 = x_1 \cos \theta - y_1 \sin \theta$$

$$\text{similarly: } y_2 = x_1 \sin \theta + y_1 \cos \theta$$





3. Rotate a triangle A(0,0), B(2,2), C(4,2) about the origin and about P(-2,-2) by an angle of 45 degree.

Sol-

The given triangle ABC can be represented by a matrix, formed from the homogeneous coordinates of the vertices.

$$\begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_{45^\circ} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate about P(-2,-2)

The rotation matrix is given by $R_{45^\circ}.P = T_V.R_{45^\circ}.T_{-V}$

$$R_{45^\circ}.P = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2\sqrt{2}-2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(A'B'C') = R_{45^\circ}.P.(ABC) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2\sqrt{2}-2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -2 & \sqrt{2}-2 \\ 2\sqrt{2}-2 & 4\sqrt{2}-2 & 5\sqrt{2}-2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A' = (-2, 2\sqrt{2}-2), B'(-2, 4\sqrt{2}-2), C'(\sqrt{2}-2, 5\sqrt{2}-2)$$

Draw the triangle ABC and the triangle A', B', C'

4. A unit square is transformed by 2 x 2 transformation matrix. The resulting position vector are:-

$$\begin{pmatrix} 0 & 2 & 8 & 6 \\ 0 & 3 & 4 & 1 \end{pmatrix}, \text{ what is the transformation matrix?}$$

Ans: Suppose the unit square have coordinates

$$(x, y) \quad (x+1, y) \quad (x+1, y+1) \quad (x, y+1)$$

and let the transformation matrix be $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$$\begin{aligned} \text{So, } \begin{pmatrix} 0 & 2 & 8 & 6 \\ 0 & 3 & 4 & 1 \end{pmatrix} &= \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} x & x+1 & x+1 & x \\ y & y & y+1 & y+1 \end{pmatrix} \\ &= \begin{pmatrix} ax+cy & a(x+1)+cy & a(x+1)+c(y+1) & ax+c(y+1) \\ bx+dy & b(x+1)+dy & b(x+1)+d(y+1) & bx+d(y+1) \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{Now, } ax+cy=0 & \quad \text{and} \quad bx+cy=0 \\ a(x+1)+cy=2 & \quad \text{and} \quad b(x+1)+dy=3 \\ a(x+1)+c(y+1)=8 & \quad \text{and} \quad b(x+1)+d(y+1)=4 \\ ax+c(y+1)=6 & \quad \text{and} \quad bx+d(y+1)=1 \end{aligned}$$

from this we get, $a=2, \quad b=3, \quad c=6, \quad d=1$

$$\text{Thus, the transformation matrix is } \begin{pmatrix} 2 & 6 \\ 3 & 1 \end{pmatrix}$$

5. Find the matrix that represents rotation of an object by 45° about the origin. a. What are the new coordinates of the point P (2, -4) after the rotation?

6. A triangle is defined by

$$\begin{pmatrix} 2 & 4 & 4 \\ 2 & 2 & 4 \end{pmatrix}$$

Find the transformed coordinates after the following transformation

- 90° rotation about origin.
- Reflection about line $y = -x$.

7. Translate the square ABCD whose co-ordinate are A(0,0), b(3,0), C(3,3), D(0,3) by 2 units in both direction and then scale it by 1.5 units in x direction and 0.5 units in y direction.

8. Perform a 45o rotation of a triangle A(0,0, B(1,1), C(5,2)

a. About the origin.

b. About the point p(-1,-1)

9. Find the transformation matrix that transforms the square ABCD whose center is at (2, 2) is reduced to half of its size, with center still remaining at (2, 2). The coordinate of square ABCD are A (0, 0), B (0, 4), C (4, 4) and D (4, 0). Find the co-ordinate of new square.

Ans. (HINT: - After scaling the square to half of its size, the new translated square will have center at (1, 1) so, translate again the new square by (1, 1), so that center again reach to (2, 2).)

10. Consider the square A (1, 0), B (0, 0), C (0, 1), D (1, 1). Rotate the square ABCD by 45 degree clockwise about A (1, 0).

HINT:-

- First, translate the square by $T_x = -1$ and $T_y = 0$.

- Then rotate the square by 45 degree.

- Again translate the square by $T_x = 1$ and $T_y = 0$.

11. Magnify the triangle with vertices A (0, 0), B (1, 1) and C (5, 2) to twice its size while keeping C (5, 2) fixed.

Ans. HINT:-

1) First, translate the triangle by $T_x = -5$ and $T_y = -2$

2) Then magnify the triangle by twice its size

3) Again translate the triangle by $T_x = 5$ and $T_y = 2$.