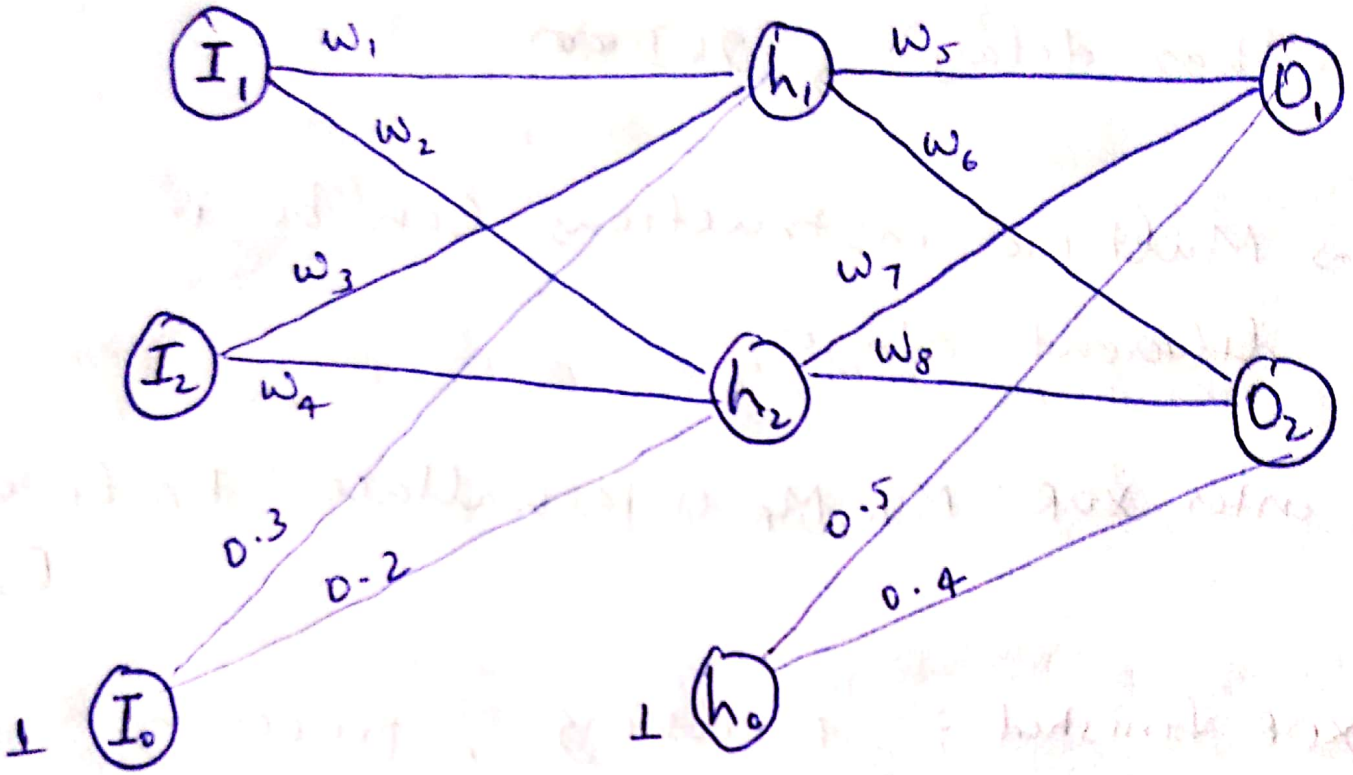

$I_1 = 0.1$ ~~(target)~~

$I_2 = 0.2$ ~~(target)~~

~~target~~ $O_1 = 0.4$

target $O_2 = 0.3$

$\eta = 0.1$

$w_1 = 0.1$

$w_2 = 0.1$

$w_3 = 0.2$

$w_4 = 0.2$

$w_5 = 0.1$

$w_6 = 0.1$

$w_7 = 0.2$

$w_8 = 0.2$

let sigmoid function be $\frac{e^x}{e^x+1}$

$h_i = f(\sum w_{ij} * x_i)$ where f is sigmoid function

$net_{h_1} = 0.3 * I_0 + w_1 * I_1 + w_3 * I_2$

$= 0.3 * 1 + 0.1 * 0.1 + 0.2 * 0.2$

$= 0.35$

$Out_{h_1} = \frac{e^{net_{h_1}}}{1+e^{net_{h_1}}} = \frac{1.41906755}{2.41906755} = 0.58661758$

$net_{h_2} = 0.2 * I_0 + w_2 * I_1 + w_4 * I_2$

$= 0.2 * 1 + 0.1 * 0.1 + 0.2 * 0.2$

$= 0.25$

$out_{h_2} = \frac{e^{net_{h_2}}}{1+e^{net_{h_2}}} = \frac{1.28402542}{2.28402542} = 0.5621765$

$net_{o_1} = 0.5 * h_0 + w_5 * h_1 + w_7 * h_2$ ($h_1 \rightarrow out_{h_1}$, $h_2 \rightarrow out_{h_2}$)

$= 0.5 * 1 + 0.1 * 0.58661758 + 0.2 * 0.5621765$

$= 0.67109706$

$Out_{o_1} = \frac{e^{neto_1}}{1+e^{neto_1}} = \frac{1.95638241}{2.95638241} = 0.66174877$

$net_{o_2} = 0.4 * h_0 + w_6 * h_1 + w_8 * h_2$ ($h_1 \rightarrow out_{h_1}$, $h_2 \rightarrow out_{h_2}$)

$= 0.4 * 1 + 0.1 * 0.5866175 8 + 0.2 * 0.5621765$

$= 0.57109706$

$Out_{o_2} = \frac{e^{neto_2}}{1+e^{neto_2}} = \frac{1.77020801}{2.77020801} = 0.63901628$

$target_{o_1} = 0.4 \quad target_{o_2} = 0.3$

$$E_{o_1} = \frac{1}{2}(target_{o_1} - out_{o_1})^2 \qquad E_{o_2} = \frac{1}{2}(target_{o_2} - out_{o_2})^2$$

$$E_{total} = E_{o_1} + E_{o_2}$$

$$E_{o_1} = \frac{1}{2} * (0.4 - 0.66174877)^2 = 0.03425621$$

$$E_{o_2} = \frac{1}{2} * (0.3 - 0.63901628)^2 = 0.05746602$$

$$E_{total} = 0.03425621 + 0.05746602$$

$$= 0.09172223$$

we get total error as 0.09172223

Now, we need to calculate new w_i to reduce error using back propagation

For minimizing error, we need to modify weights

consider w_5

To modify w_5, we need to calculate $\frac{\partial E_{total}}{\partial w_5}$

because according to weight updation rule

$$w(k+1) = w(k) - \eta * \frac{\partial E_{total}}{\partial w_i}$$

$$E_{total} = E_{o_1} + E_2 \quad ; \quad E_{o_1} = \frac{1}{2}(target_{o_1} - out_{o_1})^2$$

$$out_{o_1} = \frac{e^{net_{o_1}}}{1 + e^{net_{o_1}}}$$

$$net_{o_1} = 0.5 * 1 + w_5 * out_{h_1} + w_7 * out_{h_2}$$

Hence,

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial Out_{o_1}} * \frac{\partial Out_{o_1}}{\partial net_{o_1}} * \frac{\partial net_{o_1}}{\partial w_5}$$

Similarly,

$$\frac{\partial E_{total}}{\partial w_6} = \frac{\partial E_{total}}{\partial Out_{o_2}} * \frac{\partial Out_{o_2}}{\partial net_{o_2}} * \frac{\partial net_{o_2}}{\partial w_6}$$

$$\frac{\partial E_{total}}{\partial w_7} = \frac{\partial E_{total}}{\partial Out_{o_1}} * \frac{\partial Out_{o_1}}{\partial net_{o_1}} * \frac{\partial net_{o_1}}{\partial w_7}$$

$$\frac{\partial E_{total}}{\partial w_8} = \frac{\partial E_{total}}{\partial Out_{o_2}} * \frac{\partial Out_{o_2}}{\partial net_{o_2}} * \frac{\partial net_{o_2}}{\partial w_8}$$

$$\frac{\partial E_{total}}{\partial Out_{o_1}} = 2 * \frac{1}{2} (target_{o_1} - out_{o_1}) * (-1) + 0$$

$$= -(target_{o_1} - out_{o_1})$$

$$= -(0.4 - 0.66174877)$$

$$= 0.2617488$$

Similarly

$$\frac{\partial E_{total}}{\partial Out_{o_2}} = -(target_{o_2} - out_{o_2})$$

$$= -(0.3 - 0.63901628)$$

$$= 0.33901628$$

we know that

$$\frac{d}{dx} f(x) = f(x)(1 - f(x)) \quad \text{where} \quad f(x) = \frac{e^x}{1+e^x}$$

$$\therefore \frac{\partial Out_{o_1}}{\partial net_{o_1}} = Out_{o_1}(1 - Out_{o_1}) = 0.22383734$$

and $$\frac{\partial Out_{o_2}}{\partial net_{o_2}} = Out_{o_2}(1 - Out_{o_2}) = 0.23067447$$

$$\frac{\partial net_{o_1}}{\partial w_5} = out_{h_1} = 0.58661758$$

$$\frac{\partial net_{o_2}}{\partial w_6} = out_{h_1} = 0.58661758$$

$$\frac{\partial net_{o_1}}{\partial w_7} = out_{h_2} = 0.5621765$$

$$\frac{\partial net_{o_2}}{\partial w_8} = out_{h_2} = 0.5621765$$

$$\therefore \frac{\partial E_{total}}{\partial w_5} = 0.2617488 * 0.22383734 * 0.58661758$$

$$= 0.03436943$$

$$\frac{\partial E_{total}}{\partial w_6} = 0.33901628 * 0.23067447 * 0.58661758$$

$$= 0.0458749$$

$$\frac{\partial E_{total}}{\partial w_7} = 0.2617488 * 0.22383734 * 0.5621765$$

$$= 0.03293745$$

$$\frac{\partial E_{total}}{\partial w_8} = 0.33901628 * 0.23067447 * 0.5621765$$

$$= 0.04396355$$

$$\text{new } w_i = \text{previous } w_i - \eta * \frac{\partial E_{total}}{\partial w_i}$$

$$\therefore \text{new } w_5 = 0.1 - 0.1 * 0.03436943$$

$$= 0.09656306$$

$$\text{new } w_6 = 0.1 - 0.1 * 0.0458749$$

$$= 0.09541251$$

new $w_7 = 0.2 - 0.1 * 0.03293745$

$= 0.19670625$

new $w_8 = 0.2 - 0.1 * 0.04396355$

$= 0.19560365$

Now Consider w_1

For updating w_1, we need to calculate $\frac{\partial E_{total}}{\partial w_1}$

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h_1}} * \frac{\partial out_{h_1}}{\partial net_{h_1}} * \frac{\partial net_{h_1}}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial w_2} = \frac{\partial E_{total}}{\partial out_{h_2}} * \frac{\partial out_{h_2}}{\partial net_{h_2}} * \frac{\partial net_{h_2}}{\partial w_2}$$

$$\frac{\partial E_{total}}{\partial w_3} = \frac{\partial E_{total}}{\partial out_{h_1}} * \frac{\partial out_{h_1}}{\partial net_{h_1}} * \frac{\partial net_{h_1}}{\partial w_3}$$

$$\frac{\partial E_{total}}{\partial w_4} = \frac{\partial E_{total}}{\partial out_{h_2}} * \frac{\partial out_{h_2}}{\partial net_{h_2}} * \frac{\partial net_{h_2}}{\partial w_4}$$

$$\frac{\partial E_{total}}{\partial out_{h_1}} = \frac{\partial E_{o_1}}{\partial out_{h_1}} + \frac{\partial E_{o_2}}{\partial out_{h_1}}$$

$$\frac{\partial E_{total}}{\partial out_{h_2}} = \frac{\partial E_{o_1}}{\partial out_{h_2}} + \frac{\partial E_{o_2}}{\partial out_{h_2}}$$

$$\frac{\partial E_{o_1}}{\partial out_{h_1}} = \frac{\partial E_{o_1}}{\partial out_{o_1}} * \frac{\partial out_{o_1}}{\partial net_{o_1}} * \frac{\partial net_{o_1}}{\partial out_{h_1}}$$

$$\frac{\partial E_{o_2}}{\partial out_{h_2}} = \frac{\partial E_{o_2}}{\partial out_{o_2}} * \frac{\partial out_{o_2}}{\partial net_{o_2}} * \frac{\partial net_{o_2}}{\partial out_{h_2}}$$

Here $\frac{\partial E_{o_1}}{\partial out_{o_1}}$ is same as $\frac{\partial E_{total}}{\partial out_{o_1}}$ and

$\frac{\partial E_{o_2}}{\partial out_{o_2}}$ is same as $\frac{\partial E_{total}}{\partial out_{o_2}}$

$$\frac{\partial net_{o_1}}{\partial out_{h_1}} = w_5 \qquad \frac{\partial net_{o_2}}{\partial out_{h_2}} = w_8$$

$$\frac{\partial net_{o_1}}{\partial out_{h_1}} = w_6 \qquad \frac{\partial net_{o_1}}{\partial out_{h_2}} = w_7$$

$$\therefore \frac{\partial E_{o_1}}{\partial out_{h_1}} = 0.2617488 * 0.22383734 * 0.1$$

$$= 0.00585891 6$$

$$\frac{\partial E_{o_2}}{\partial out_{h_2}} = 0.33901628 * 0.23067447 * 0.2$$

$$= 0.01564048$$

$$\frac{\partial E_{total}}{\partial out_{h_1}} = 0.00585891 6$$

$$\frac{\partial E_{total}}{\partial out_{h_2}} = 0.01564048$$

$$\frac{\partial E_{o_2}}{\partial out_{h_1}} = \frac{\partial E_{o_2}}{\partial out_{o_2}} * \frac{\partial out_{o_2}}{\partial net_{o_2}} * \frac{\partial net_{o_2}}{\partial out_{h_1}}$$

$$\frac{\partial E_{o_1}}{\partial out_{h_2}} = \frac{\partial E_{o_1}}{\partial out_{o_1}} * \frac{\partial out_{o_1}}{\partial net_{o_1}} * \frac{\partial net_{o_1}}{\partial out_{h_2}}$$

$$\frac{\partial E_{o_2}}{\partial out_{h_1}} = 0.33901628 * 0.23067447 * 0.1$$

$$= 0.00782024$$

$$\frac{\partial E_{o_1}}{\partial out_{h_2}} = 0.2617488 * 0.2238373 4 * 0.2$$

$$= 0.01171783$$

$$\frac{\partial E_{total}}{\partial out_{h_1}} = 0.00585891 6 + 0.00782024$$

$$= 0.01367916$$

$$\frac{\partial E_{total}}{\partial out_{h_2}} = 0.01564048 + 0.01171783$$

$$= 0.02735831$$

Similar to $\frac{\partial out_{o_1}}{\partial net_{o_1}}$

we can calculate $\frac{\partial out_{h_1}}{\partial net_{h_1}} = out_{h_1}(1 - out_{h_1})$

$$= 0.24249739$$

$$\frac{\partial out_{h_2}}{\partial net_{h_2}} = out_{h_2}(1 - out_{h_2})$$

$$= 0.24613408$$

Now

$$\frac{\partial net_{h_1}}{\partial w_1} = I_1 \qquad \frac{\partial net_{h_2}}{\partial w_2} = I_1$$

$$\frac{\partial net_{h_1}}{\partial w_3} = I_2 \qquad \frac{\partial net_{h_2}}{\partial w_4} = I_2$$

$$\therefore \frac{\partial E_{total}}{\partial w_1} = 0.01367916 * 0.24249739 * 0.1$$

$$= 0.00033171 61$$

$$\frac{\partial E_{total}}{\partial w_2} = 0.02735831 * 0.24613408 * 0.1$$

$$= 0.00067338 12$$

$$\frac{\partial E_{total}}{\partial w_3} = 0.01367916 * 0.24249739 * 0.2$$

$$= 0.00066343 22$$

$$\frac{\partial E_{total}}{\partial w_4} = 0.02735831 * 0.24613408 * 0.2$$

$$= 0.00134676 2$$

$$\text{new } w_i = \text{previous } w_i - \eta * \frac{\partial E_{total}}{\partial w_i}$$

new w_1 = 0.1 − 0.1 * 0.000331761

= 0.09996682 8

new w_2 = 0.1 − 0.1 * 0.000673312

= 0.09993266 2

new w_3 = 0.2 − 0.1 * 0.000663 4322

= 0.19993365 7

new w_4 = 0.2 − 0.1 * 0.001346762

= 0.19986532 4

Now we have got new w_i $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$

So we will calculate new error

Now net h_1 = 0.3 * I_0 + new w_1 * I_1 + new w_3 * I_2

= 0.34998341

out h_1 = 0.58661356

net h_2 = 0.2 * I_0 + new w_2 * I_1 + new w_4 * I_2

= 0.24996633

out h_2 = 0.56216821

net o_1 = 0.5 * h_0 + new w_5 * out h_1 + new w_7 * out h_2

= 0.6672272

out o_1 = 0.66088201

net o_2 = 0.4 * h_0 + new w_6 * out h_1 + new w_8 * out h_2

= 0.51555918

= 0.62610877

target $O_1 = 0.4$ target $O_2 = 0.3$

$$E_{O_1} = \frac{1}{2}(0.4 - 0.66088201)^2 = 0.03402971$$

$$E_{O_2} = \frac{1}{2}(0.3 - 0.62610877)^2 = 0.05317347$$

$$E_{total} = E_{O_1} + E_{O_2} = 0.08720318$$

Now error has reduced on modifying w_i

error reduced from 0.09172223 to 0.08720318