

Assignment 2

Due: Sep 10, No extension will be given as I will hand out A3 on Sep 10.

1. Refer a standard text on calculus and write the definition of limit in FOL. Also, find the negation for the expression.
2. Write the definition of linear search in FOL.
3. Prove or Disprove:
 - (a) $\exists x(P(x) \wedge Q(x)) \rightarrow \exists xP(x) \wedge \exists xQ(x)$
 - (b) $\exists xP(x) \wedge \exists xQ(x) \rightarrow \exists x(P(x) \wedge Q(x))$
4. Write FOL: No one who has no complete knowledge of himself will ever have a true understanding of another.
5. Write FOL: Thought or thinking is what set human beings apart from other living things.
6. Check the validity of the following implications
 - $[(P \rightarrow Q) \vee (R \rightarrow S)] \rightarrow [(P \vee R) \rightarrow (Q \vee S)]$
7. For a matrix A , the 'row-major' order refers to

- (i) sorting within the row
- (ii) the last element of the first row is smaller than the first element of the second row. In general, the last element of the i^{th} row is smaller than the first element of the $(i + 1)^{th}$ row.

Which of the following represents the definition or row major precisely.

Solution: 1

$\{\forall i \forall j [1 \leq i \leq 20, 1 \leq j \leq 29 \rightarrow A[i][j] \leq A[i][j + 1]] \wedge [\forall i 1 \leq i \leq 19 \rightarrow A[i][30] \leq A[i + 1][1]]\}$

Solution:2

$\forall i \forall j \forall k \forall h (1 \leq i, k \leq 30 \wedge 1 \leq j, h \leq 20 \rightarrow (i < k \rightarrow A[j][i] < A[j][k]) \wedge (j < h \rightarrow A[j][i] < A[h][i]))$

Solution: 3

Fine tune the above expression so that it works for any two cols instead of a fixed one.

$\forall i \forall j \forall k \forall h (1 \leq i, k \leq 30 \wedge 1 \leq j, h \leq 20 \rightarrow (i < k \leftrightarrow A[j][i] < A[j][k]) \wedge (j < h \leftrightarrow A[j][i] < A[h][k]))$

8. Show the validity of the argument. All lions are animals. Therefore, all heads of lions are heads of animals.
9. Rewrite each proposition symbolically, when universe of discourse is a set of real numbers.
 - (a) For each integer x , there exists an integer y such that $x + y = 0$
 - (b) There exists an integer x such that $x + y = y$ for every integer y
 - (c) For all integers x and y , $x.y = y.x$
 - (d) There are integers x and y such that $x + y = 5$
10. Express the following using First Order Logic. Clearly, mention UOD and the set of predicates used.

Prove or disprove

Some Republicans like all Democrats.

No Republican likes any Socialist.

Therefore, no Democrat is a Socialist.