	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India	**COM-205T** `Discrete Structures for Computing` Quiz 1 26-Aug-2015 Duration: 1 hr Marks: 15

Name: **Roll no:**

`0. (0 marks) Prove that` $3^0 = 1$.

1. (1 mark) Show that $P \to (Q \to R) \leftrightarrow (P \to Q) \to (P \to R)$

2. (2 marks) Express the following using logic. P and Q are propositions. You are allowed to use only the following logical connectives: $\neg, \vee, \wedge, \to$ and any other operator is not allowed.

   - P is necessary for Q
   - P unless Q
   - P is necessary for Q whereas P is not sufficient for Q.
   - Either P or Q.

3. (1 mark) Negate the following: $\forall x \ \ \exists \epsilon((x > 0 \wedge \epsilon > 0) \wedge \forall y(y > 0 \to x - y \geq \epsilon))$

4. (1.5 marks) Consider the assertion: Discrete Mathematics grading is transparent only if I study Discrete Mathematics. For the given assertion, write the

   - Converse:

   - Inverse:

   - Contrapositive:

5. (0.5 marks) Write the following logical expression using only $\wedge$ and $\neg$. $P \rightarrow (Q \rightarrow P)$.

6. (2 marks) Consider the following two implications. Of the two, one is true and the other one is false. Justify your answer with a proof/counter example.
   (i) $\exists x(P(x) \wedge Q(x)) \rightarrow \exists x P(x) \wedge \exists x Q(x)$
   (ii) $\exists x P(x) \wedge \exists x Q(x) \rightarrow \exists x(P(x) \wedge Q(x))$

7. (1 mark) Check the validity of the argument.
   Some trigonometric functions are periodic. Some periodic functions are continuous. Therefore, some trigonometric functions are continuous.

   - Write the above argument using predicate logic.

   - Prove or Disprove.

8. (2 marks) Prove or Disprove.

$$[\exists x P(x) \rightarrow \forall x Q(x)] \rightarrow \forall x[P(x) \rightarrow Q(x)]$$
$$\forall x[P(x) \rightarrow Q(x)] \rightarrow [\exists x P(x) \rightarrow \forall x Q(x)]$$

9. (1 mark) Express $\exists ! x P(x)$ using $\forall x P(x)$ and $\exists x P(x)$. Your expression must involve both $\forall$ and $\exists$ and logically equivalent to $\exists ! x P(x)$. Any other assumption must be stated clearly.

10. (1.5 marks) Express the following using First Order Logic. Clearly, mention UOD and the set of predicates used.
Some Republicans like all Democrats.
No Republican likes any Socialist.
Therefore, no Democrat is a Socialist.

11. (1.5 marks)`Scenario:` Five persons A, B, C, D, E are in a compartment in a train. A, C, E are men and B, D are women. The train passes through a tunnel and when it emerges, it is found that E is murdered. An inquiry is held, A, B, C, D make the following statements. Each makes two statements.

A says: I am innocent. B was talking to E when the train was passing through the tunnel.
B says: I am innocent. I was not talking to E when the train was passing through the tunnel.
C says: I am innocent. D committed the murder.
D says: I am innocent. One of the men committed the murder.

Out of 8 statements given above, 4 are true and 4 are false. Who is the murderer. Support your answer with a precise and concise justification.

Extra Credit: Prove or Disprove. All scientists are human beings. Therefore, all children of scientists are children of human beings.

ROUGH WORK

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India	**COM-205T** `Discrete Structures for Computing` Quiz 1 05-Oct-2015 Duration: 1 hr Marks: 15

Name: **Roll no:**

0. `(0 marks) What is your source (class notes, text books, internet) of preparation for COM 205T`

1. (1.5 mark) How many transitive relations are there on a set of size two. List all of them.

2. (1.5 marks) Consider the set $A = \{1, 2, \phi, \{1, 2\}\}$. Say true or false for the following.
   - $1, 2 \in A$
   - $\{1, 2\} \subset A$
   - $\{1, 2\} \in A$
   - $\phi \in A$
   - $\phi \subset A$
   - $\{\phi\} \subset A$

3. (1.5 marks) Consider the set of integers and the binary relation $R = \{(a, b) \mid \text{ a divides b}\}$. Is R an equivalence relation. Justify your answer with a proof/counter example.

4. (1 mark) How many binary relations are there on a finite set of size n that are symmetric and asymmetric. Justify.

5. (1 mark) `Claim:` If a binary relation R is symmetric and transitive, then R is an equivalence relation. `Proof:` Since R is symmetric, both (a, b) and (b, a) are in R and given that R is transitive, it follows that $(a, a) \in R$. Therefore, R is reflexive. From the above arguments, it follows that R is an equivalence relation. Is the proof correct. Justify your answer.

6. (1 mark) Prove or Disprove: $R_1R_2 \cap R_1R_3 \subset R_1(R_2 \cap R_3)$, where $R_1 \subseteq A \times B, R_2, R_3 \subseteq B \times C$.

7. (1.5 marks) Is it true that in a group of 5 people there exist 3 mutual friends or a pair of enemies (2 mutual enemies).

8. (1.5 marks) Prove or Disprove: in any set of 8 distinct integers there exist two whose sum or difference is divisible by 7.

9. (1.5 marks) Present a Direct Proof: $\forall n, 2^n \leq n! \leq n^n$

10. (1.5 marks) Present a proof using mathematical induction: $\forall n, 2^n \leq n! \leq n^n$

11. (1.5 marks) What is wrong with this `'proof'`. **Theorem:** For every positive integer n, if x and y are positive integers with $max(x, y) = n$, then $x = y$.

    Basis Step: Suppose that $n = 1$. If $max(x, y) = 1$ and x and y are positive integers, we have $x = 1$ and $y = 1$.
    Inductive Step: Let k be a positive integer. Assume that whenever $max(x, y) = k$ and x and y are positive integers, then $x = y$. Now let $max(x, y) = k + 1$, where x and y are positive integers. Then $max(x - 1, y - 1) = k$, so by the inductive hypothesis, $x - 1 = y - 1$. It follows that $x = y$, completing the inductive step.

Extra Credit: How many equivalence (binary) relations are there on a set of size n. Justify.

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India	**COM-205T** `Discrete Structures for Computing` End Semester 02-Dec-2015 Duration: 3 hrs Marks: 50

Name: **Roll no:**

`0. The name of the Movie that narrates the discoveries of Prof.Nash Williams`

1 Light Dose

`Credits: 1 mark each`

1. Write the power set of $\{\emptyset, \{\emptyset\}, \{1,2\}\}$

2. `Statement:` A graph G is 2-colorable is a necessary condition for G to be bipartite. Write the converse and contrapositive.

3. Write the definition of Weak induction and Strong induction using the first order logic.

4. Let R_1, R_2 be relations defined on a finite set A and $t(R_1)$ is the transitive closure of R_1. Prove or Disprove. $t(R_1 \cup R_2) = t(R_1) \cup t(R_2)$

5. Given a function $f : A \to B$, what is the necessary and sufficient condition for f^{-1} to exist (inverse of f).

6. Let $A = \{1, \ldots, n\}$. Given a function $f : A \to A$ is onto, does it follow that f is 1-1. Prove or Disprove.

7. How many onto functions are there from a set of size 3 to a set of size 2.

8. How many binary strings are there of length 20 with exact 4 zeros.

9. Show that the greatest lower bound is unique.

10. A bag contains 3 red, 4 black, 5 blue balls. The minimum number of balls to be taken in any draw so that we get to see 3 balls of the same color.

11. Show that the set of composite numbers is infinite.

12. A= set of C-programs. B= set of C++ programs. Which set is bigger. Justify.

13. Draw a graph on 5 vertices such that G and $\bar{G}$ (complement of G) are same.

14. The maximum number of edges in a simple graph with 8 vertices and 4 components. Draw one such graph.

15. Is the number of graphs on n vertices with chromatic number 3 finite or infinite. Justify. Note: n is a fixed integer.

2 Medium Dose

`Credits: 1.5 marks each`

1. Express the following using the first order logic by clearly mentioning UOD, predicates used: Everyone who gets admitted into an IIT gets a job. Therefore, if there are no jobs, then nobody gets admitted into any IIT.

2. Suppose S and T are two sets and $f : S \to T$ is a function. Let R_1 be an equivalence relation on T. Let R_2 be a binary relation on S such that $(x, y) \in R_2$ iff $(f(x), f(y)) \in R_1$. Is R_2 an equivalence relation. Prove or Disprove.

3. Suppose R_1 and R_2 are equivalence relations (defined on a finite set A) inducing partitions $P1$ and $P2$. Let $R = R_1 \cap R_2$. How do you obtain the partition P induced by R using $P1$ and $P2$.

4. `Claim:` All students in DM course get 'S' grade. We now present a proof using mathematical induction on the number of students. `Base:` $n = 1$. 'Renjith' gets 'S' grade. `Hypothesis:` Assume $n = k$ students get 'S' grade. `Induction Step:` Consider a set of $k + 1$ students. The set $\{s_1, \ldots, s_{k+1}\}$ of students contain $\{s_1, \ldots, s_k\}$ and $\{s_2, \ldots, s_{k+1}\}$. Clearly both the sets are of size k and by the hypothesis all students in $\{s_1, \ldots, s_k\}$ get 'S' grade and all students in $\{s_2, \ldots, s_{k+1}\}$ get 'S' grade. Therefore, all students in $\{s_1, \ldots, s_{k+1}\}$ get 'S' grade. This completes the induction. Is the proof correct. If not, identify the flaw in this argument.

5. A= set of prime numbers and the binary relation R is 'divides' ; Is R a partial order. Is R a well-order. What are the minimal elements of the set $\{2, 3, 5, 7\}$. What are the minimal elements of the set A.

6. Let A be a finite set and R be a binary relation on A. Count the following sets.
   - The number of irreflexive and symmetric binary relations
   - The number of irreflexive and anti-symmetric binary relations
   - The number of irreflexive and asymmetric binary relations

7. Show that one of any n consecutive integers is divisible by n.

8. Show that the number of derangements on n items is $\displaystyle\sum_{i=2}^{n}(-1)^{i-1}\frac{n!}{i!}$.

9. Show that the set $[3, 4]$ is uncountable.

10. Prove that G is bipartite if and only if G is 2-colorable. Be precise and formal.

3 Strong Dose

`Credits: 2 marks each`

1. All horses are animals. Therefore, heads of horses are heads of animals. Prove or Disprove.

2. How many partial orders are there on a set of size 3. List all of them.

- How of them are total order.

- How many of them are well-order.

3. Given \$4 and \$5 currency, is it possible give change for \$$n$ using these denominations. If yes, prove using Mathematical Induction.

4. Two disks, one smaller than the other , are each divided into 200 congruent sectors. In the larger disk 100 of the sectors are chosen arbitrarily and painted red; the other 100 sectors are painted blue. In the smaller disk each sector is painted either red or blue with no stipulation on the number of red and blue sectors. The small disk is then placed on the larger disk so that their centers coincide. Show that it is possible to align the two disks so that the number of sectors of small disk whose color matches the corresponding sector of the large disk is atleast 100. (Hint: PHP)

5. An infinite integer array is passed as an input to a sorting program. How many different inputs are possible, i.e., is it finite or countably infinite or uncountable. Justify.

6. Draw two non-isomorphic graphs with degree sequence $(3, 3, 3, 3, 3, 3, 3, 3, 3, 3)$ if it exists. Intuitively, argue that the two graphs drawn are non-isomorphic. Justify if no such graphs exist for this degree sequence.

7. With suitable justifications, find the cardinality of the following sets (finite, countably infinite, uncountable)

   - The number of acyclic graphs on n-vertices, n is a fixed integer.

   - The number of bipartite graphs on n-vertices, $n \in \mathbf{N}$. Note: n is a variable.

8. Mention a set and a relation satisfying the following conditions
   - a subset with no maximal element and no minimal element.
   - a subset with no lub and no glb
9. Seven students go on holidays. They decide that each will send a post card to three of the others. Is it possible that every student receives post cards from precisely the three to whom he sent postcards.
10. How many binary equivalence relations are there on a set of size n. Prove your answer. Be precise.

SPACE FOR ROUGH WORK

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM 205T - Discrete Mathematics	Quiz 1 30-Aug-2016 Duration: 1hr Marks: 15

Roll No: Name:

0. (0 marks) Name the scientist with whom mathematician Ramanujam had a good academic career

1. (1 mark) I prepare well for exams is sufficient for me to get good grades. And, I secure good grades only if I maintain a good CGPA.

2. (1 mark) $\exists x(P(x) \wedge Q(x)) \rightarrow \exists x P(x) \wedge \exists x Q(x)$. Let us attempt a proof.
By definition; $(P(0) \wedge Q(0)) \vee (P(1) \wedge Q(1)) \vee (P(2) \wedge Q(2)) \vee \ldots$
What is the next step ? Complete the proof. Do not attempt any other proof technique.

3. (1 mark) Negate the following and simplify.
   $\forall n \exists z \forall k(|z| = k \rightarrow \exists u \exists v \exists w((z = uvw \wedge |uv| \geq k \wedge |v| \geq 1) \wedge \forall i(i \geq 0 \rightarrow uv^i w \in S)))$

4. (1 mark) Prove or Disprove: $\forall x(P(x) \leftrightarrow Q(x)) \leftrightarrow \exists x(P(x) \leftrightarrow Q(x))$

5. (1 mark) What is the underlying meaning of the following logical expression; P is some predicate.
   $\exists x(P(x) \wedge \forall y(P(y) \leftrightarrow y = x))$

6. (3 marks) Write logical notation for each of the following; for each, write an expression using only existential quantifier and an another expression using only universal quantifier. UOD: Set of students. PREDICATES: $Boy(x)$ x is a boy, $SMART(x)$ x is smart. Do NOT use any other predicates.

   (a) Some boys are smart.

   Using only $\exists$

   Using only $\forall$

   (b) Not all boys are smart.

   Using only $\exists$

   Using only $\forall$

   (c) All boys are not smart.

   Using only $\exists$

   Using only $\forall$

7. (2 marks) Some students of DM are well motivated by a faculty. All students of DM likes all faculty. Therefore, some students of DM likes a faculty who motivates them. UOD: Set of students and faculty, PREDICATES: $STUD(x)$: x is a student. $FACULTY(x)$: x is a faculty. $LIKES(x, y)$: x likes y. $MOTIVATES(x, y)$: x motivates y.

   - Write the above argument in FOL.

- Is the above argument true ?

8. (2 marks) There exists a IIIT where many students are studying. There is a IIIT with no students. Therefore, there are two IIITs such that a student is part of one IIIT whereas he is not part of the other. UOD: Set of students and IIITs. PREDICATES: $STUD(x)$: x is a student. $IIIT(x)$: x is a IIIT. $STUDY(x, y)$: x is studying in y. Do NOT use any other predicates.

   - Write the above argument in FOL.

   - Is the above argument true ?

9. (3 marks) Consider the academic timetable at IIITDM. UOD: Set of students,courses and time slots. PREDICATES: $STUD(x)$: x is a student. $ELECOURSE(x)$: x is an elective course. $COURSE(x)$: x is a course. $TIMESLOT(x)$: x is a time slot. $TAKEN(x,y)$: x has taken course y. $DAY(x,y)$: (course) x is offered on (day) y. $COURSE\text{-}OFFERED\text{-}SLOT(x,y)$: x is offered in time slot y. Write the FOL for the following.

- Each student has taken at least two elective courses.

- There exists a student who has courses in all time slots. (there exists a student who has taken at least one course in each time slot)

- There is a student who has not taken a course on any of the time slots on Wednesday.

Indian Institute of Information Technology Design and Manufacturing, Kancheepuram
Chennai 600 127, India

COM-205T
Discrete Structures for Computing
End Semester
02-Dec-2015
Duration: 3 hrs
Marks: 50

Name: **Roll no:**

0. The name of the Movie that narrates the discoveries of Prof.Nash Williams

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9. Seven students go on holidays. They decide that each will send a post card to three of the others. Is it possible that every student receives post cards from precisely the three to whom he sent postcards.

10. How many binary equivalence relations are there on a set of size n. Prove your answer. Be precise.

SPACE FOR ROUGH WORK

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India	**COM-205T** `Discrete Structures for Computing` Quiz 1 26-Aug-2015 Duration: 1 hr Marks: 15

Name: **Roll no:**

0. `(0 marks) Prove that` $3^0 = 1$.

1. (1 mark) Show that $P \to (Q \to R) \leftrightarrow (P \to Q) \to (P \to R)$

2. (2 marks) Express the following using logic. P and Q are propositions. You are allowed to use only the following logical connectives: $\neg, \vee, \wedge, \to$ and any other operator is not allowed.

   - P is necessary for Q
   - P unless Q
   - P is necessary for Q whereas P is not sufficient for Q.
   - Either P or Q.

3. (1 mark) Negate the following: $\forall x \ \ \exists \epsilon((x > 0 \wedge \epsilon > 0) \wedge \forall y(y > 0 \to x - y \geq \epsilon))$

4. (1.5 marks) Consider the assertion: Discrete Mathematics grading is transparent only if I study Discrete Mathematics. For the given assertion, write the

   - Converse:

   - Inverse:

   - Contrapositive:

5. (0.5 marks) Write the following logical expression using only $\wedge$ and $\neg$. $P \to (Q \to P)$.

6. (2 marks) Consider the following two implications. Of the two, one is true and the other one is false. Justify your answer with a proof/counter example.
   (i) $\exists x(P(x) \wedge Q(x)) \to \exists x P(x) \wedge \exists x Q(x)$
   (ii) $\exists x P(x) \wedge \exists x Q(x) \to \exists x(P(x) \wedge Q(x))$

7. (1 mark) Check the validity of the argument.
Some trigonometric functions are periodic. Some periodic functions are continuous. Therefore, some trigonometric functions are continuous.

   - Write the above argument using predicate logic.

   - Prove or Disprove.

8. (2 marks) Prove or Disprove.

$$[\exists x P(x) \to \forall x Q(x)] \to \forall x[P(x) \to Q(x)]$$
$$\forall x[P(x) \to Q(x)] \to [\exists x P(x) \to \forall x Q(x)]$$

9. (1 mark) Express $\exists! x P(x)$ using $\forall x P(x)$ and $\exists x P(x)$. Your expression must involve both $\forall$ and $\exists$ and logically equivalent to $\exists! x P(x)$. Any other assumption must be stated clearly.

10. (1.5 marks) Express the following using First Order Logic. Clearly, mention UOD and the set of predicates used.
Some Republicans like all Democrats.
No Republican likes any Socialist.
Therefore, no Democrat is a Socialist.

11. (1.5 marks)`Scenario:` Five persons A, B, C, D, E are in a compartment in a train. A, C, E are men and B, D are women. The train passes through a tunnel and when it emerges, it is found that E is murdered. An inquiry is held, A, B, C, D make the following statements. Each makes two statements.

    A says: I am innocent. B was talking to E when the train was passing through the tunnel.
    B says: I am innocent. I was not talking to E when the train was passing through the tunnel.
    C says: I am innocent. D committed the murder.
    D says: I am innocent. One of the men committed the murder.

    Out of 8 statements given above, 4 are true and 4 are false. Who is the murderer. Support your answer with a precise and concise justification.

Extra Credit: Prove or Disprove. All scientists are human beings. Therefore, all children of scientists are children of human beings.

ROUGH WORK

	Indian Institute of Information Technology Design and Manufacturing, Kancheepuram Chennai 600 127, India An Autonomous Institute under MHRD, Govt of India An Institute of National Importance COM - Discrete Structures for Computing	Quiz 2 05-Oct-2017 Duration: 1hr Marks: 15

`Roll No:` `Name:`

0. (0 marks) How many books did the mathematician Ramanujan read during his research career.

1. (0.5 marks) What is the power set of $\{1, 2, \{1, 2\}\}$.

2. (1 mark) Say true or false with exactly one line justification.
   (i) $\{1, 2\} \in \{1, 2, \{1, 2\}\}$

   (ii) $\{1, 2\} \subset \{1, 2, \{1, 2\}\}$

3. (1.5 marks) Let $A = \{1, 2, 3\}$ and R be a binary relation defined on A. Present an example relation R such that
   (i) R is symmetric and anti symmetric

   (ii) R is anti symmetric but not reflexive

   (iii) R is neither symmetric nor transitive

4. (1.5 mark) Five distinct non-negative numbers are chosen randomly from the set of integers. Prove or disprove: there exists two in the chosen set such that their sum or difference is divisible by 6.

5. (1 mark) 21 numbers are chosen randomly from the set of integers. What is the tight lower bound on the set of integers that are divisible by 3 in the chosen set. Justify.

6. (2 marks) Prove or disprove: For every integer k, there are more than $k+3$ prime numbers.

7. (2 marks) Show that $\sqrt{5}$ is irrational.

8. (2 marks) Claim: in any group of 13 people, there exists 4 mutual friends or 3 mutual enemies. Present a proof or a counter example.

9. (1.5 marks) Write the base cases and the inductive hypothesis for the following claim. Do NOT prove this claim. A monkey is asked to climb a ladder of size n (n steps). Each time, it takes either 1 step or 2 steps or 3 steps. Claim: The number of ways of climbing up the ladder is at most 4^n.

10. (2 marks) Let A be a set. Like binary, ternary relations, are there unary relations defined on A. What are they and how many are there. Prove your answer.

Extra credit: (2.5 marks) What is the minimum number of people in a group so that we either find 4 mutual enemies or 4 mutual friends. Prove your answer.

Indian Institute of Information Technology
Design and Manufacturing, Kancheepuram
Chennai 600 127, India
An Autonomous Institute under MHRD, Govt of India
An Institute of National Importance
COM 205T - Discrete Mathematics

End Semester
17-Nov-2017
Duration: 3hrs
Marks: 40

Roll No: Name:

0. Name the scientist who discovered the theory of infinite sets

1 Light Dose

1 mark each

1. Write converse and inverse for the statement 'I drink coffee whenever I get headache'.
   Converse:

   Inverse:

2. Out of the following the four logical expressions, identify the two that are equivalent. NO justification is needed.
   (a) $\forall x(P(x) \rightarrow Q(x))$
   (b) $\neg\exists x(P(x) \wedge \neg Q(x))$
   (c) $\neg\neg\exists x(P(x) \vee \neg Q(x))$
   (d) $\forall x(P(x) \vee \neg Q(x))$

3. Is there a binary relation which is both reflexive and irreflexive. Mention one, if exists.

4. How many 1-1 functions are there from a domain of size 4 to co-domain of size 3.

5. Prove or Disprove: In a group of 5 people there exists 3 mutual friends or 3 mutual enemies.

6. Draw a graph for the degree sequence $(3, 3, 3, 1, 1, 1)$.

7. Consider the above graph drawn as a binary relation, and find the transitive closure.

8. How many onto functions are there from a domain of size 4 to a co-domain of size 2.

9. What is the chromatic number of a complete graph on $n \geq 2$ vertices.

10. Draw a planar graph for the degree sequence $(4, 4, 4, 4, 1, 1, 1, 1)$, if it exists.

2 Medium Dose

`2 marks each`

1. Is the Peterson graph an Eulerian graph. How about the Line graph of the Peterson graph. Justify.

2. Show that for any planar graph, $V - E + F = 2$.

3. Show that $[5, 9]$ is uncountable.

4. On a set of size n, how many binary equivalence relations are there ? Prove your answer.

5. Present an example set and a subset for each of the following
   - Minimum and Maximum elements exist, however neither least nor greatest elements exist
   - Upper and lower bounds exist, however neither greatest LB nor least UB exist
6. How many binary relations are there that are neither reflexive nor antisymmetric.

7. The set A consists of composite numbers and the set B consists of prime numbers. Which set is larger. Justify.

8. In how many different ways can k pigeons be distributed into n pigeonholes such that each pigeon has at least two pigeons.

9. $\forall x(P(x) \to Q(x)) \to \forall x P(x) \vee \forall x Q(x)$. Is this true ? How about the converse ?

3 Strong Dose

3 marks each

1. Draw example graphs satisfying (i) Both G and $\bar{G}$ (complement of G) are planar (ii) G is planar whereas $\bar{G}$ is non-planar (iii) Both G and $\bar{G}$ are non-planar

2. Present a **good** lower and upper bound for non-transitive binary relations. Justify.

3. Express the following in FOL: Some logicians are good at proof techniques. Not all logicians are good at graph theory, however all logicians are good at some topics in infinite sets. Therefore, there are logicians who are neither good at functions nor relations.

4. Consider the series 1 2 2 4 8 32 256 .... What is the n^{th} number in this series. Present a good upper bound and a proof of correctness if deriving exact number is challenging.

Extra Credit: (3 marks) Express in FOL. All horses are animals. Therefore, heads of horses are heads of animals. Prove or disprove.