## Systems Thinking for Design

Session 8

https://sites.google.com/a/iiitdm.ac.in/sudhirvs/courses/systems-thinking-for-design



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# Session outline: Diagnosis using Properties of Complex Networks

Introduction to complex networks

Properties of complex networks

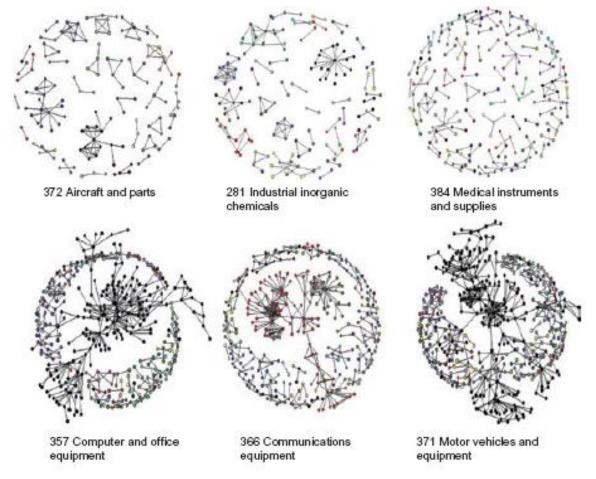
#### Introduction

- We have already seen how one could use the in-, out- and totaldegree to understand the nature of the network (random, power law) and classify the key elements
- In this session we will see how one could use other properties of complex networks to understand the problem situation
- We will analyze the discovery matrix (of the problem situation) to extract
  - Cores
  - Clusters
  - Centrality of nodes

### What is a network perspective?

- A network perspective is about looking at the
  - Real world as elements (nodes) and relationships (edges/arcs)
  - With no clear 'boundary' (or the difference between inside and outside)

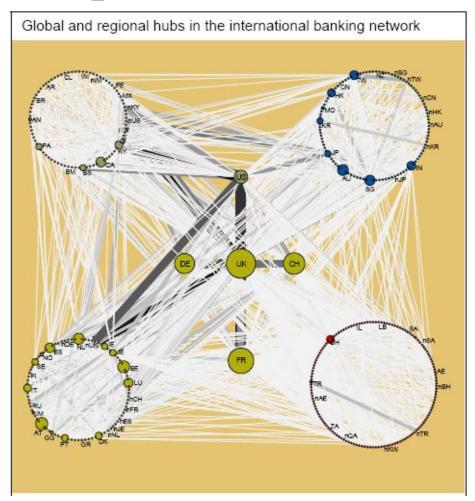
### Complex networks in the real-world (1/5)



Strategic Alliance Networks in different industries

Figure 2. Nine industry alliance networks-graphical visualizations

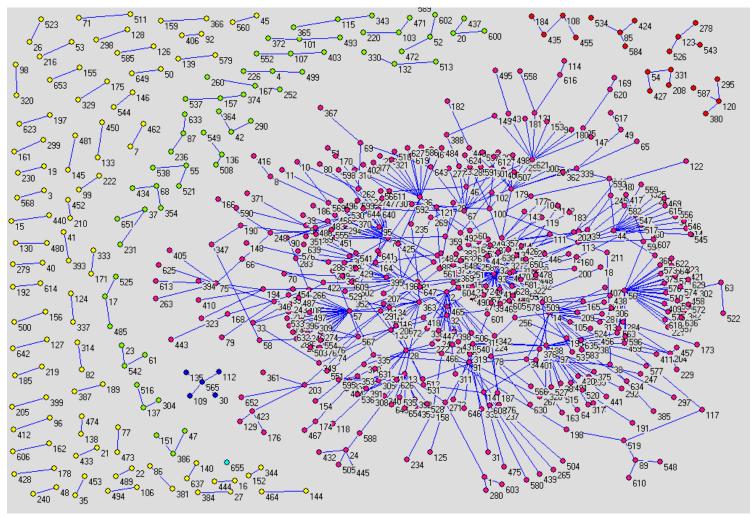
### Complex networks in real-world (2/5)



The graph shows the linkages between 212 banking centres and their linkages with 212 non-banks.

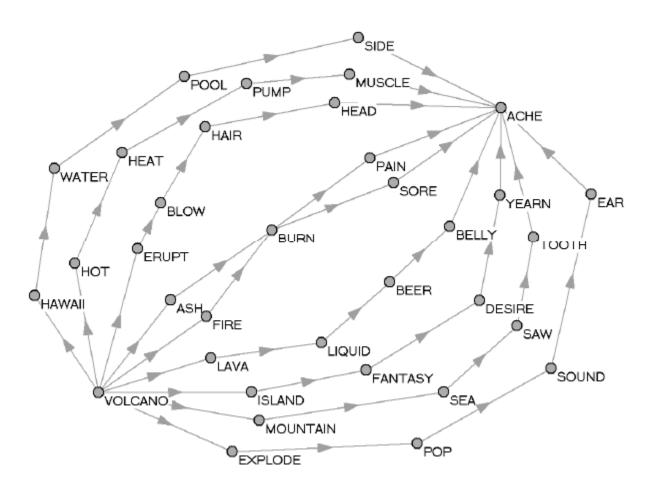
Each location is represented by a node. The color of the nodes represents the continent (red for Africa and the Middle East, green for the Americas, blue for Asia-Pacific and mustard for Europe).

### Complex networks in the real-world (3/5)



IT outsourcing contracts in Global Banking

### Complex networks in the real-world (4/5)

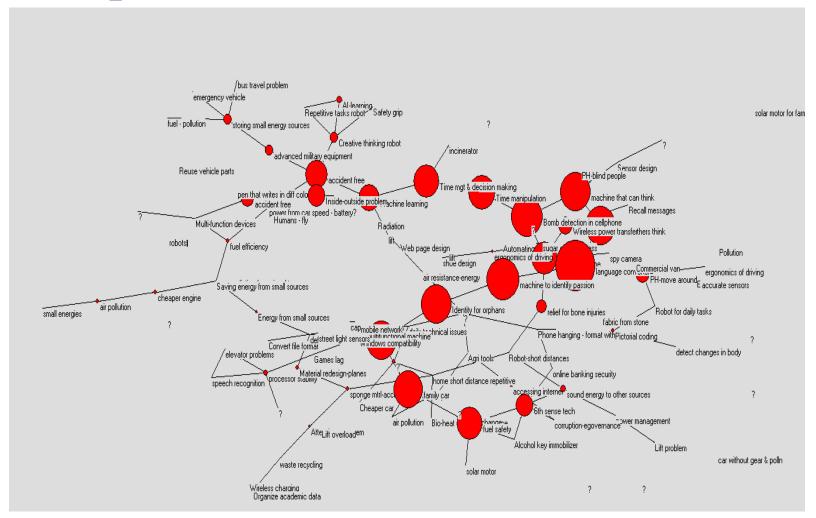


Part of the semantic network formed by free association.

Each directed edge illustrates an association between a cue and a response.

A sample of associative directed paths from VOLCANO to ACHE is shown (the shortest path length is four)

### Complex networks in the real-world (5/5)



Potential areas of collaboration among ideas of students

# Session outline: Diagnosis using Properties of Complex Networks

Introduction to complex networks

Properties of complex networks

### Concepts of complex networks

Question	Local metrics	Global metrics
How many different influences does an entity receive?	degree, in-degree, out-degree	degree distribution
How many other entities does it influence?		
Do some nodes in a network play an important role in connecting the whole network?	Centrality	degree distribution
How tightly are the entities (nodes) grouped together (by edges)?	clique, n-clique, k-core	clustering coefficient
How long does communication between nodes take?	shortest-path length	diameter, radius

### Different types of networks

Random ... Erdos & Renyi

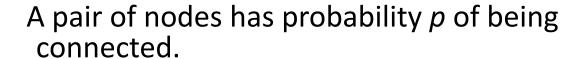
Scale free (power law) ... Albert, Barabasi

• Small world (six degrees, searchable) ... Watts, Strogatz, Kleinberg

### Random graphs (1/5)

Erdős and Renyi (1959)

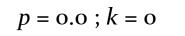
**N** nodes



Average degree,  $k \approx pN$ 

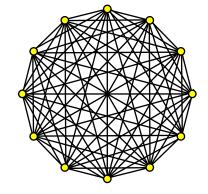
What interesting things can be said for different values of p or k?

(that are true as  $N \rightarrow \infty$ )

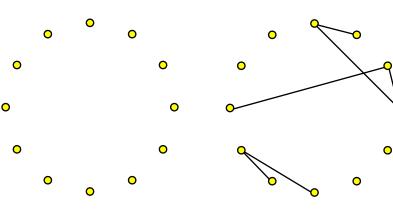


$$p = 0.09$$
;  $k = 1$ 

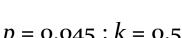
$$p = 1.0 ; k \approx \frac{1}{2}N^2$$

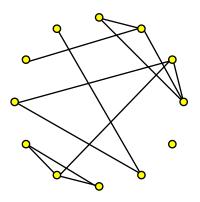


### Random Graphs (2/5)

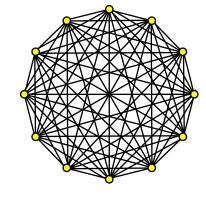


$$p = 0 \circ k = 0$$





$$p = 0.0$$
;  $k = 0$   $p = 0.045$ ;  $k = 0.5$   $p = 0.09$ ;  $k = 1$   $p = 1.0$ ;  $k \approx \frac{1}{2}N^2$ 



$$p = 1.0 \; ; k \approx \frac{1}{2}N^2$$

Size of largest component

11

12

Diameter of largest component

Average path length between nodes

0.0

2.0

1.0

### Random graphs (3/5)

#### If *k* < 1:

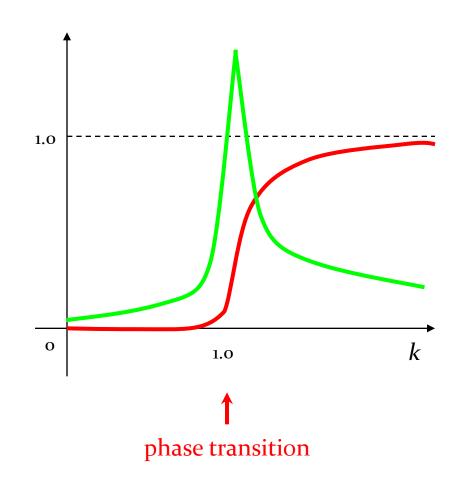
- small, isolated clusters
- small diameters
- short path lengths

#### At k = 1:

- a giant component appears
- diameter peaks
- path lengths are high

#### For k > 1:

- almost all nodes connected
- diameter shrinks
- path lengths shorten



### Random graphs (4/5)

What does this mean?

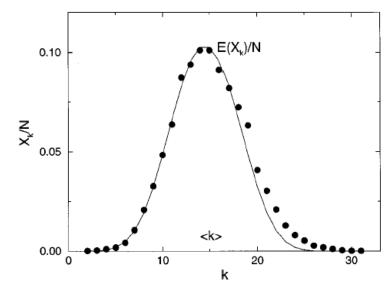
If connections between people can be modeled as a random graph, then...

- Because the average person easily knows more than one person (k >> 1),
- We live in a "small world" where within a few links, we are connected to anyone in the world
- Erdős and Renyi showed that average path length between connected nodes is

$$\frac{\ln N}{\ln k}$$

### Random graphs (5/5)

Albert and Barabasi (1999)



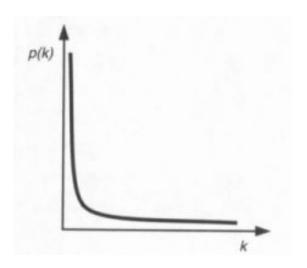
Degree distribution of a random graph, N = 10,000 p = 0.0015 k = 15. (Curve is a Poisson curve, for comparison.)

What's the degree (number of edges) distribution over a graph, for realworld graphs?

Random-graph model results in Poisson distribution.

But, many real-world networks exhibit a *power-law* distribution.

### Scale-free networks (1/3)



Typical shape of a power-law distribution.

What's the degree (number of edges) distribution over a graph, for real-world graphs?

Random-graph model results in Poisson distribution.

But, many real-world networks exhibit a power-law distribution.

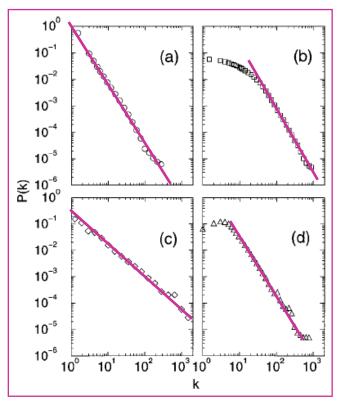
### Scale-free networks (2/3)

Power-law distributions are straight lines in log-log space.

How should random graphs be generated to create a power-law distribution of node degrees?

#### Hint:

Pareto's\* Law: Wealth distribution follows a power law.



Power laws in real networks:

- (a) WWW hyperlinks
- (b) co-starring in movies
- (c) co-authorship of physicists
- (d) co-authorship of neuroscientists

### Scale-free networks (3/3)

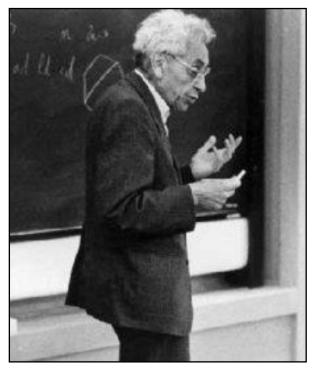
"The rich get richer!"

Power-law distribution of node distribution arises if

- Number of nodes grow;
- Edges are added in proportion to the number of edges a node already has.

Additional variable fitness coefficient allows for some nodes to grow faster than others.

#### Erdős number



Paul Erdős (1913-1996)

Number of links required to connect scholars to Erdős, via co-authorship of papers

Erdős wrote 1500+ papers with 507 co-authors.

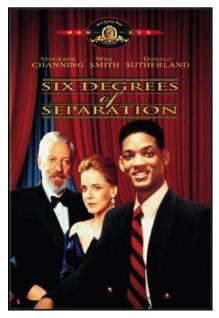
Jerry Grossman's (Oakland Univ.) website allows mathematicians to compute their Erdos numbers:

http://www.oakland.edu/enp/

Connecting path lengths, among mathematicians only:

- average is *4.65*
- maximum is 13

### Six degrees of separation (1/2)



John Guare wrote a play called *Six Degrees* of Separation, based on this concept.

"Everybody on this planet is separated by only six other people. Six degrees of separation. Between us and everybody else on this planet. The president of the United States. A gondolier in Venice... It's not just the big names. It's anyone. A native in a rain forest. A Tierra del Fuegan. An Eskimo. I am bound to everyone on this planet by a trail of six people..."

### Six degrees of separation (2/2)

Milgram (1967)

#### The experiment:

Random people from Nebraska were to send a letter (via intermediaries) to a stock broker in Boston.

Could only send to someone with whom they were related on a first-name basis.

Among the letters that found the target, the average number of links was six.



Stanley Milgram (1933-1984)

### Small-world networks (1/5)

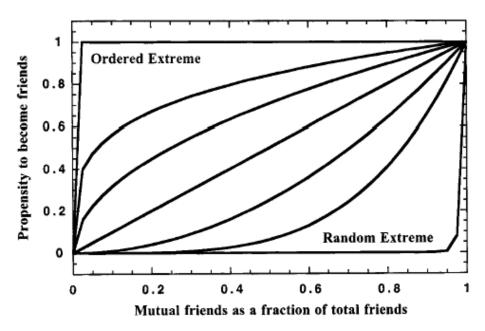
Watts (1999)

The people you know aren't randomly chosen.

People tend to get to know those who are two links away (Rapoport, 1957).

The real world exhibits a lot of *clustering*.

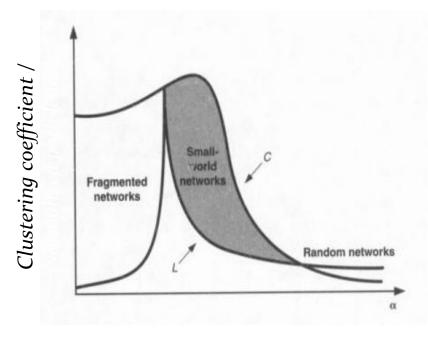
### Small-world networks, alpha model (2/5)



 $\alpha$  model: Add edges to nodes, as in random graphs, but makes links more likely when two nodes have a common friend.

Probability of linkage as a function of number of mutual friends ( $\alpha$  is o in upper left, 1 in diagonal, and  $\infty$  in bottom right curves.)

### Small-world networks, alpha model (3/5)



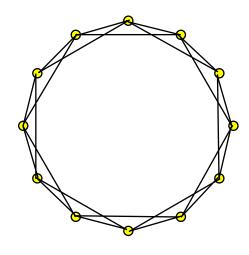
Clustering coefficient (C) and average path length (L) plotted against  $\alpha$ 

 $\alpha$  model: Add edges to nodes, as in random graphs, but makes links more likely when two nodes have a common friend.

For a range of  $\alpha$  values:

- The world is small (average path length is short), and
- Groups tend to form (high clustering coefficient).

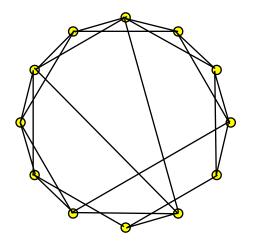
### Small-world networks, beta model (4/5)



$$\beta = 0$$

People know their neighbors.

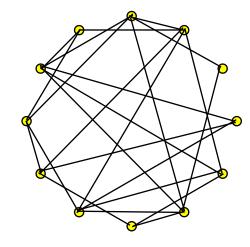
Clustered, but not a "small world"



$$\beta$$
 = 0.125

People know their neighbors, and a few distant people.

Clustered and "small world"



$$\beta = 1$$

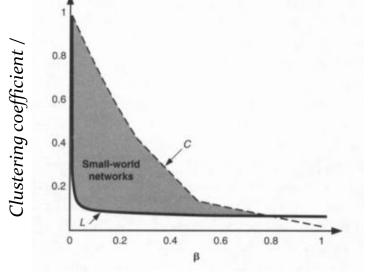
People know others at random.

Not clustered, but "small world"

### Small-world networks, beta model (5/5)

First five random links reduce the average path length of the network by half, regardless of N!

Both  $\alpha$  and  $\beta$  models reproduce shortpath results of random graphs, but also allow for clustering.

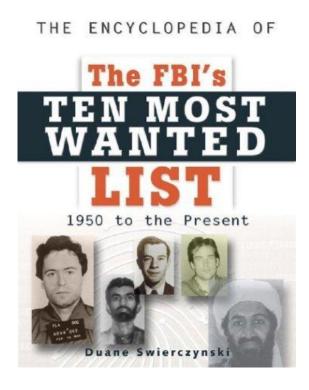


Clustering coefficient (C) and average path length (L) plotted against  $\beta$ 

Small-world phenomena occur at threshold between order and chaos.

### Searchable networks (1/3)

Kleinberg (2000)



Just because a short path exists, doesn't mean you can easily find it.

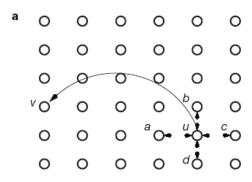
You don't know all of the people whom your friends know.

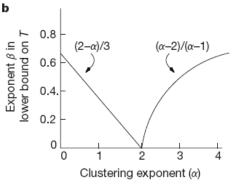
Under what conditions is a network *searchable*?

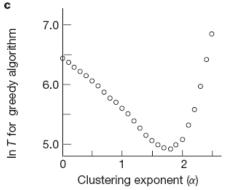
### Searchable networks (2/3)

#### Kleinberg (2000)

- a) Variation of Watts's  $\beta$  model:
  - Lattice is d-dimensional (d=2).
  - One random link per node.
  - Parameter  $\alpha$  controls probability of random link greater for closer nodes.
- b) For d=2, dip in time-to-search at  $\alpha=2$ 
  - For low  $\alpha$ , random graph; no "geographic" correlation in links
  - For high  $\alpha$ , not a small world; no short paths to be found.
- c) Searchability dips at  $\alpha$ =2, in simulation



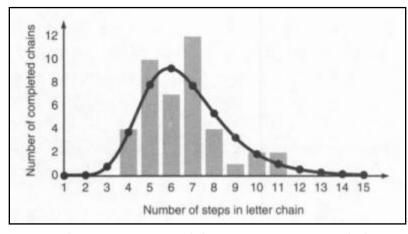




### Searchable networks (3/3)

Watts, Dodds, Newman (2002) show that for d = 2 or 3, real networks are quite searchable.

Killworth and Bernard (1978) found that people tended to search their networks by d = 2: geography and profession.



The Watts-Dodds-Newman model closely fitting a real-world experiment

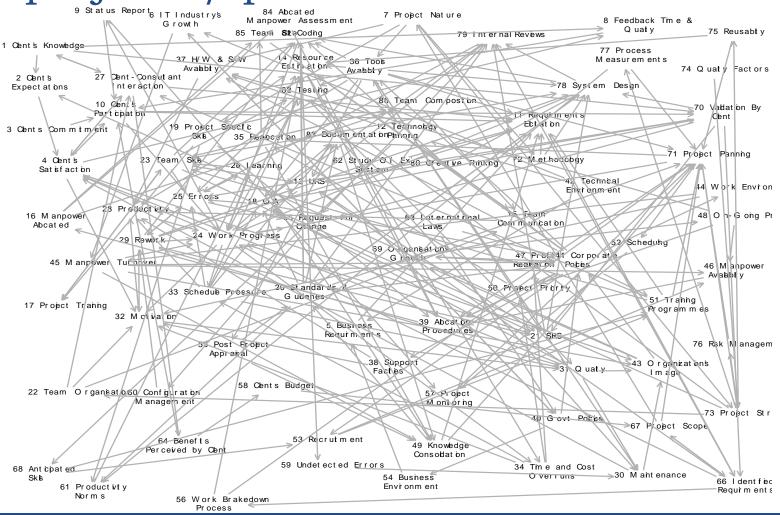
### Network analysis tools

• For scholarly research tools like *UCINet*, *Pajek*, *ORA*, the *statnet* suite of packages in *R*, and *GUESS* are popular.

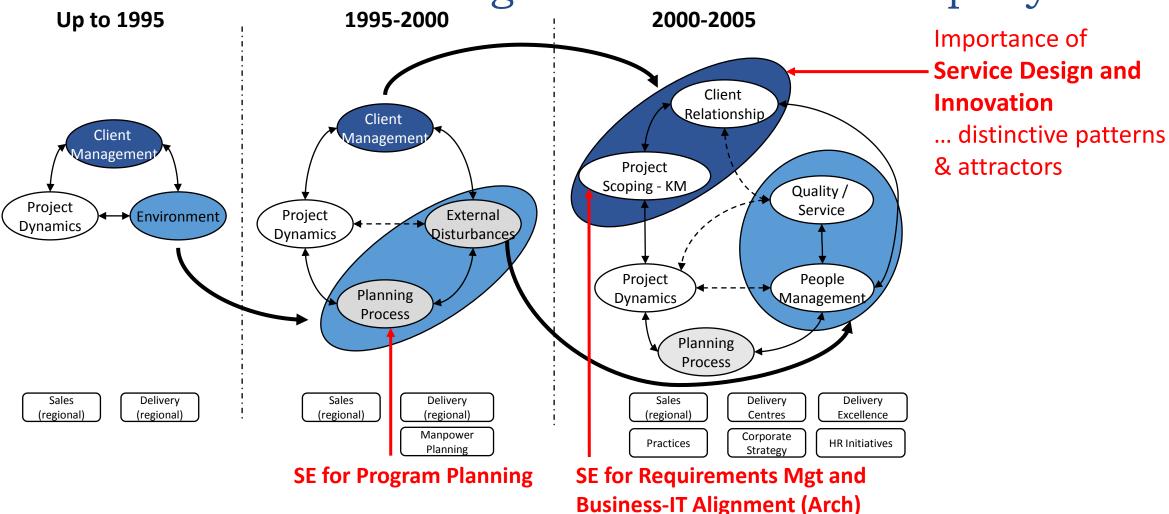
• Examples of business oriented social network tools include InFlow, Keyhubs, NetMiner, Sentinel Visualizer.

http://www.insna.org/

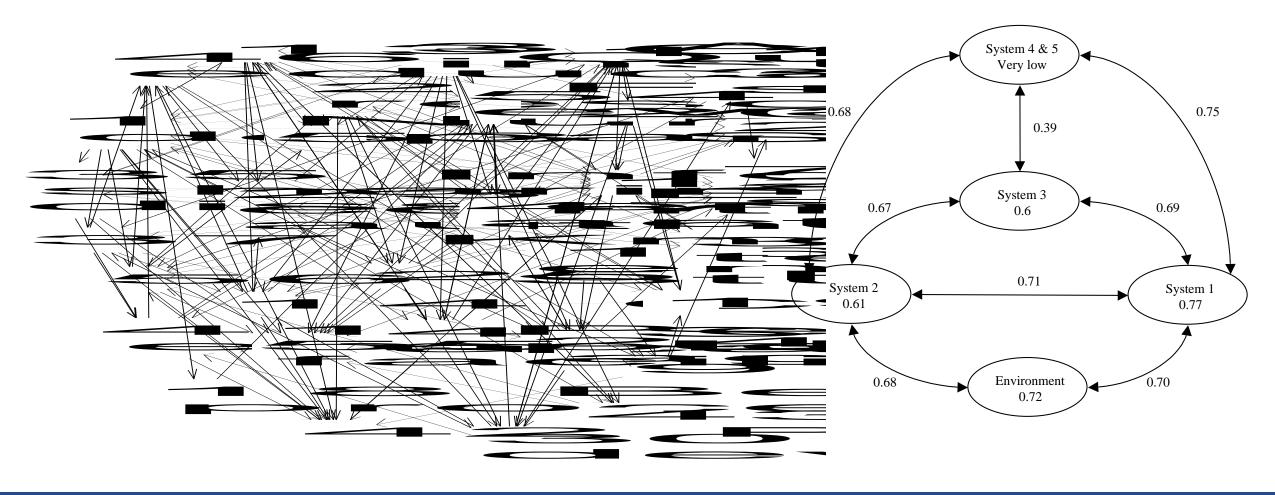
# Example: A large number of factors are involved in complex projects / products



Example: Patterns can be extracted from systems models – evolution of a global IT services company



### View of an Engineering College



## What is your network?

Reflect on today's session and post your comments.

