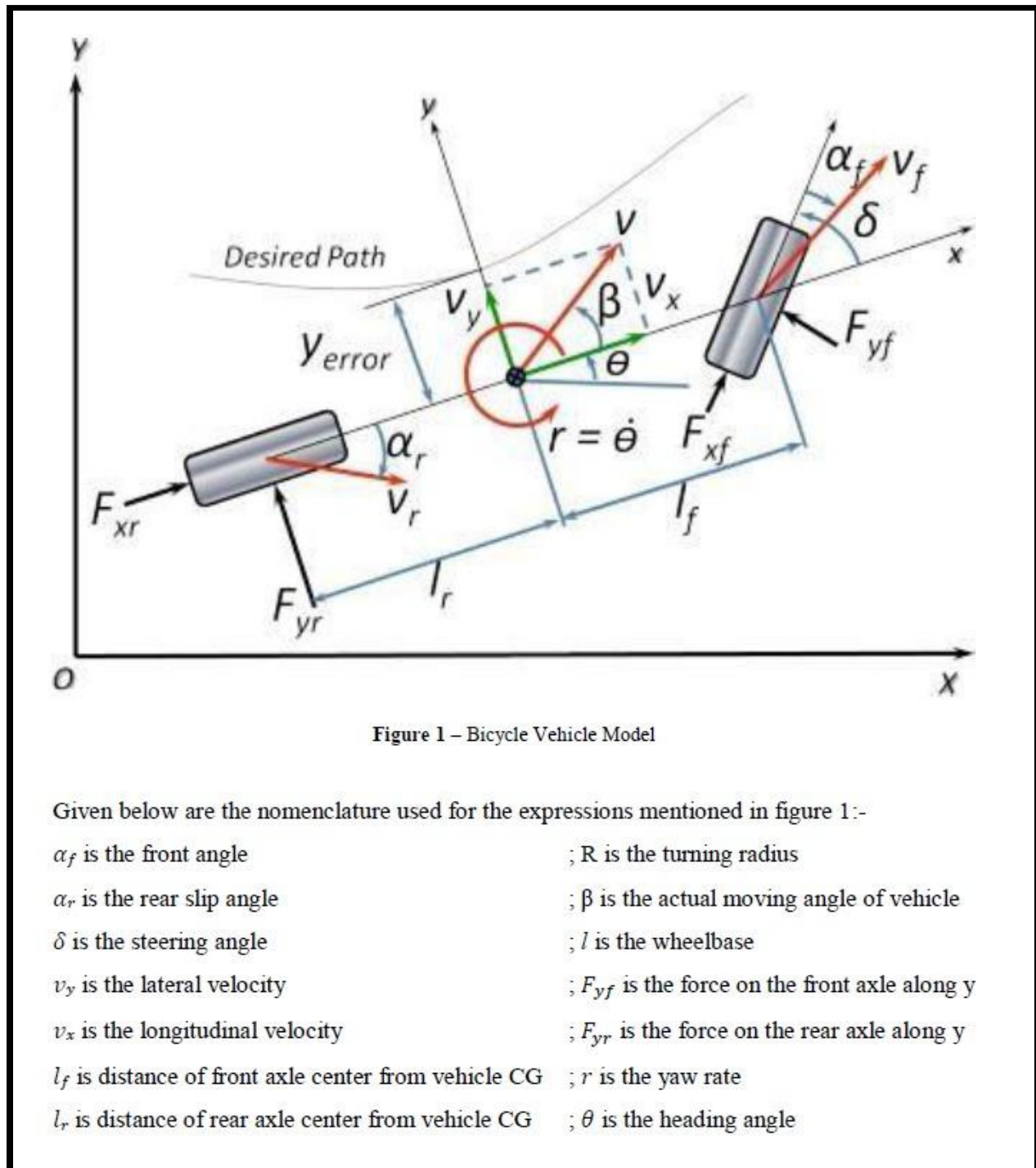


VEHICLE DYNAMICS-ASSIGNMENT 4

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ED21S005



From figure,

$$m(\dot{v}_y + r v_x) = F_{yf} + F_{yr}$$

$$L_z \dot{\theta} = l_f \cdot F_{yf} - l_r \cdot F_{yr}$$

Also, we can see from figure,

$$\tan(\alpha_r) = \frac{-(l_r \cdot r - v_y)}{v_x}$$

$$\beta = \frac{v_y}{v_x}$$

let α_r be very small,

$$\tan(\alpha_r) \approx \alpha_r = -\frac{l_r \cdot r}{v_x} + \frac{v_y}{v_x}$$

$$\boxed{\alpha_r = \beta - \frac{l_r}{R}}$$

Similarly,

$$\tan(\alpha_f) \approx \alpha_f = -\left(\delta - \frac{l_f r + v_y}{v_x}\right)$$

α_f becomes negative because as it is in the negative z-direction.

$$\alpha_f = -\left(\delta - \frac{l_f}{R} - \beta\right)$$

$$\delta_f = \frac{l_f}{R} + \alpha_r + \frac{l_r}{R} - \alpha_f$$

$$\boxed{\delta = \frac{l_f + l_r}{R} + \alpha_r - \alpha_f}$$

Taking a linear tire model,

$$F_y = -C \cdot \alpha.$$

(negative α produces a +ve F_y .)

$$F_{yf} = -C_f \cdot \alpha_f \quad \& \quad F_{yr} = -C_r \cdot \alpha_r.$$

$$\therefore m(\dot{v}_y + r v_x) = F_{yf} + F_{yr}.$$

$$m(\dot{v}_y + r v_x) = -C_f \alpha_f - C_r \alpha_r.$$

$$m(\dot{v}_y + r v_x) = -C_f \left(\frac{l_f + l_r}{R} - \delta + \beta - \frac{l_r}{R} \right) - C_r \left(\beta - \frac{l_r}{R} \right).$$

$$m(\dot{v}_y + r v_x) = \left(-\frac{C_f l_f}{v_x} + \frac{C_r l_r}{v_x} \right) r - (C_f + C_r) \beta + C_f \delta$$

$$m \dot{v}_y + m r v_x + \left(\frac{C_f l_f}{v_x} - \frac{C_r l_r}{v_x} \right) r + (C_f + C_r) \beta = C_f \delta$$

$$\therefore \boxed{m \dot{v}_y + \left(\frac{C_f + C_r}{v_x} \right) v_y + \left(m v_x + \frac{C_f l_f - C_r l_r}{v_x} \right) r = C_f \delta.}$$

\Rightarrow lateral Motion of the Vehicle.

$$\therefore \boxed{\dot{v}_y = -\left(\frac{C_f + C_r}{m v_x} \right) v_y + \left(\frac{C_f l_f - C_r l_r}{m v_x} - v_x \right) r + \left(\frac{C_f}{m} \right) \delta.}$$

From the torque equation,

$$I_z \dot{\gamma} = l_f \cdot F_{yf} - l_r \cdot F_{yr}.$$

$$I_z \cdot \dot{\gamma} = -l_f C_f \alpha_f + l_r C_r \alpha_r.$$

$$I_z \cdot \dot{\gamma} = -l_f C_f \left(\frac{l_f + l_H}{R} - \delta + \beta - \frac{l_r}{R} \right) + l_r C_r \left(\beta - \frac{l_r}{R} \right).$$

$$I_z \dot{\gamma} = \frac{-l_f^2 C_f}{v_x} \gamma + l_f C_f \delta - \frac{l_f v_y C_f}{v_x} + \frac{l_r v_y \cdot C_r}{v_x} - \frac{l_r^2 C_r \cdot \gamma}{v_x}.$$

\therefore

$$I_z \dot{\gamma} + \left(\frac{C_f l_f - C_r l_r}{v_x} \right) v_y + \left(\frac{C_f l_f^2 + C_r l_r^2}{v_x} \right) \gamma = C_f l_f \delta.$$

That is,

$$\dot{\gamma} = - \left(\frac{C_f l_f - C_r l_r}{I_z \cdot v_x} \right) v_y - \left(\frac{C_f l_f^2 + C_r l_r^2}{I_z \cdot v_x} \right) \gamma + \left(\frac{C_f l_f}{I_z} \right) \delta.$$

Writing equations for \dot{v}_y and \dot{r} in state-space form,

$$\begin{aligned} (A) \quad \begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} &= \begin{bmatrix} -\frac{(c_f + cr)}{m v_x} & -\frac{l_f c_f + l_r c_r - v_x}{m v_x} \\ -\frac{(l_f c_f - l_r cr)}{I_z v_x} & -\frac{(l_f^2 c_f + l_r^2 cr)}{I_z v_x} \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} \\ &+ \begin{bmatrix} c_f/m \\ \frac{l_f l_f}{I_z} \end{bmatrix} \delta. \end{aligned}$$

If output required is v_y , $C = [1 \ 0]$.

If output required is r , $C = [0 \ 1]$.

$$\underline{\underline{[D] = 0.}}$$

Parameters Given,

$$C_f \Rightarrow C_{af} = 60000 \text{ N/rad.}$$

$$C_r \Rightarrow C_{ar} = 60000 \text{ N/rad.}$$

$$m \Rightarrow m = 1000 \text{ kg.}$$

$$I_z = 1650 \text{ kg m}^2.$$

$$l_f = a = 1. \text{ m.}$$

$$l_r = b = 1.5 \text{ m}$$

$$V_x = 16 \text{ m/s.}$$

Calculating Matrices A, B, C and D. using the above parameters,

~~A =~~

$$A = \begin{bmatrix} -7.5 & -14.125 \\ 1.13636 & -7.38636 \end{bmatrix}$$

$$B = \begin{bmatrix} 60 \\ 36.3636 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$D = 0.$$

Since $C = [0 \ 1]$, output is r .

\therefore Converting it into transfer function form,

$$\frac{r(s)}{\delta(s)} = C.(sI - A)^{-1}B + D.$$

For getting both responses (lateral velocity
and Yaw rate) , $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Mathematica was ~~also~~ used to solve the
matrix equations and get Transfer Functions
for both the outputs.

Mathematica

Steering angle input (step) = 8°

$$8^\circ \rightarrow 8 \times \frac{\pi}{180} = \underline{\underline{\frac{2\pi}{45} \text{ rad.}}}$$

$$\underline{\underline{\delta(s) = \frac{2\pi}{45} s.}}$$

Now,,

$$\begin{aligned} \frac{r(s)}{\delta(s)} &= \frac{36.3636s + 340.909}{s^2 + 14.8864s + 71.4488} \\ \frac{v_y(s)}{\delta(s)} &= \frac{60s + -70.4543}{s^2 + 14.8864s + 71.4488} \end{aligned}$$

Considering $\delta(s) = \frac{2\pi}{45s}$,

$$r(s) = \frac{5.0774s + 47.6}{s(s^2 + 14.8864s + 71.4487)}.$$

$$v_y(s) = \frac{8.37758s + -9.83727}{s(s^2 + 14.8864s + 71.4487)}.$$

Taking partial Fractions,

$$r(s) = \frac{0.666209}{s} + \frac{-4.84011 - 0.666209s}{s^2 + 14.8864s + 71.4488}$$

$$v_y(s) = \frac{-0.13768}{s} + \frac{10.4272 + 0.137683s}{s^2 + 14.8864s + 71.4488}.$$

Taking Inverse Laplace Transform,

$$r(t) = 0.666209 + \left[0.02965 \sin(4.006t) - 0.666209 \cos(4.006t) \right] \cdot e^{-7.443t}$$

$$v_y(t) = \left[2.3471 \sin(4.006t) + 0.137683 \cos(4.006t) \right] \cdot e^{-7.443t} - 0.137683.$$

\Rightarrow The above two are the ss-model Analytical solutions.

(2). (c).

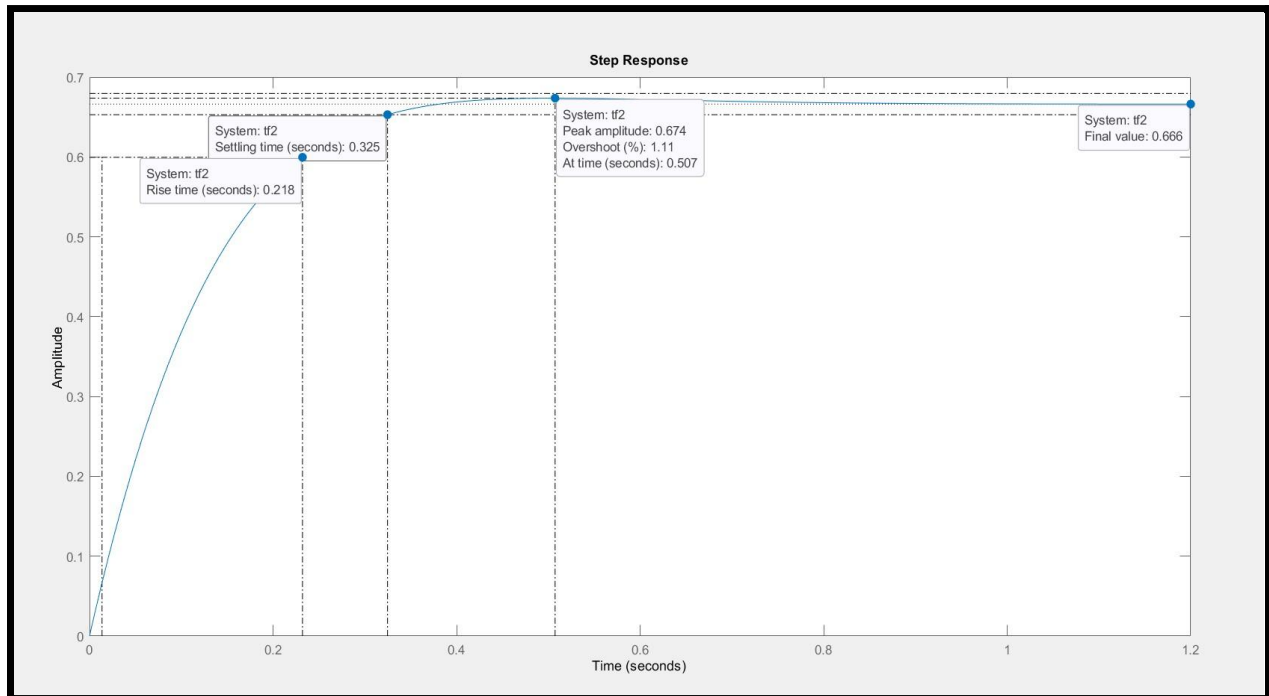
(i) Peak value = 0.674 rad/s .

\Rightarrow It overshoots by 1.1% .

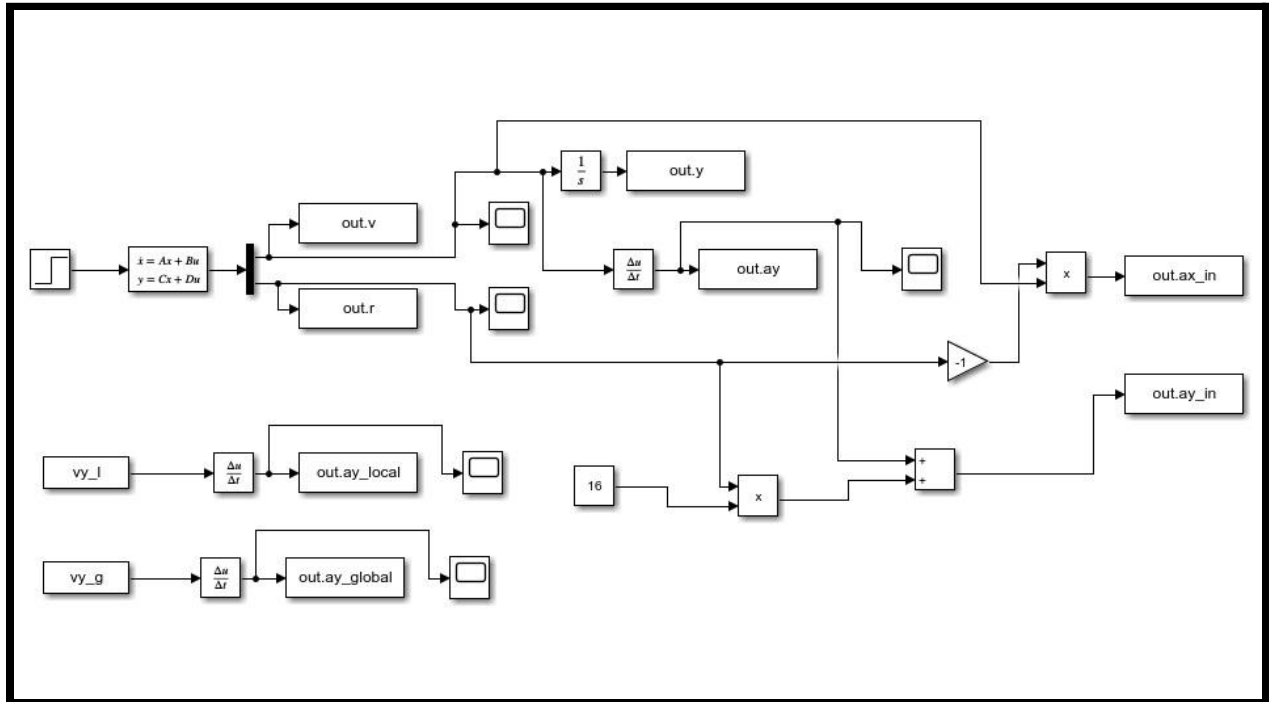
(ii) Steady state value = 0.666 rad/s .

(iii) Settling time = 0.325 seconds

(iv) Peak response time = 0.501 seconds .



Simulink Model



```

In[1]:= A = {{-7.5, -14.125}, {1.13636, -7.38636}};
        B = {{60}, {36.3636}};
        C2 = {0, 1};
        C1 = {1, 0};
        i = IdentityMatrix[2];

```

```

In[6]:= Mat1 = C1.Inverse[S.i - A].B;
        Mat2 = C2.Inverse[S.i - A].B;

```

```

In[8]:= Mat = {{s + 7.5, 14.125}, {-1.13636, s + 7.38636}};

```

```

In[9]:= Inmat = Inverse[Mat];

```

```

In[10]:= Tf2 = C2.Inmat.B // Simplify

```

$$\text{Out[10]} = \left\{ 0. + \frac{340.909}{71.4488 + 14.8864 s + s^2} + \frac{36.3636 s}{71.4488 + 14.8864 s + s^2} \right\}$$

```

In[11]:= Tf1 = C1.Inmat.B // Simplify

```

$$\text{Out[11]} = \left\{ 0. - \frac{70.4543}{71.4488 + 14.8864 s + s^2} + \frac{60. s}{71.4488 + 14.8864 s + s^2} \right\}$$

(b) Analytical Solution From the State Space Model and Transfer Function

In simulink, step input was given at 3 seconds and simulation was run for 12 seconds.

For better interpretability, xlim has been used in the plots.

```
%For Lateral Speed (analytical)
t=0:0.001:12;
v=(2.3471*sin(4.006*t)+0.137683*cos(4.006*t)).*exp(-7.443*t)-0.137683;
figure;
plot(t,v,LineWidth=2) ;hold on;
```

```
%For Lateral Speed (Transfer Function)
tf1=tf([60*(2*pi)/45,-70.4543*(2*pi)/45],[1,14.8864,71.4488])
```

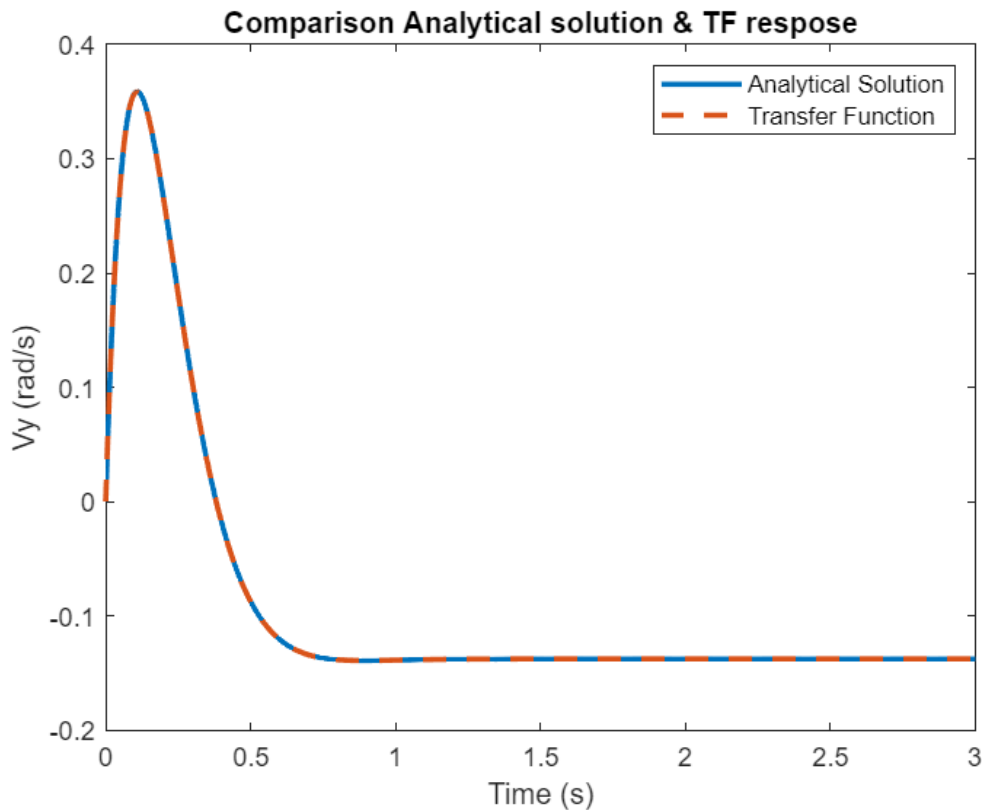
```
tf1 =

      8.378 s - 9.837
-----
      s^2 + 14.89 s + 71.45
```

Continuous-time transfer function.

```
stepres=step(tf1,t);
plot(t,stepres,'LineStyle','--',LineWidth=2)

title('Comparison Analytical solution & TF respose')
xlabel('Time (s)')
ylabel('Vy (rad/s)')
xlim([0,3])
legend('Analytical Solution','Transfer Function')
```



```
%For Yaw Rate (analytical)
```

```
r=(0.02965*sin(4.006*t)-0.666212*cos(4.006*t)).*exp(-7.443*t)+0.666212;
figure;
plot(t,r,LineWidth=2); hold on;
```

```
%For yaw rate (Transfer Functon)
```

```
tf2=tf([36.3636*(2*pi)/45,340.909*(2*pi)/45],[1,14.8864,71.4488])
```

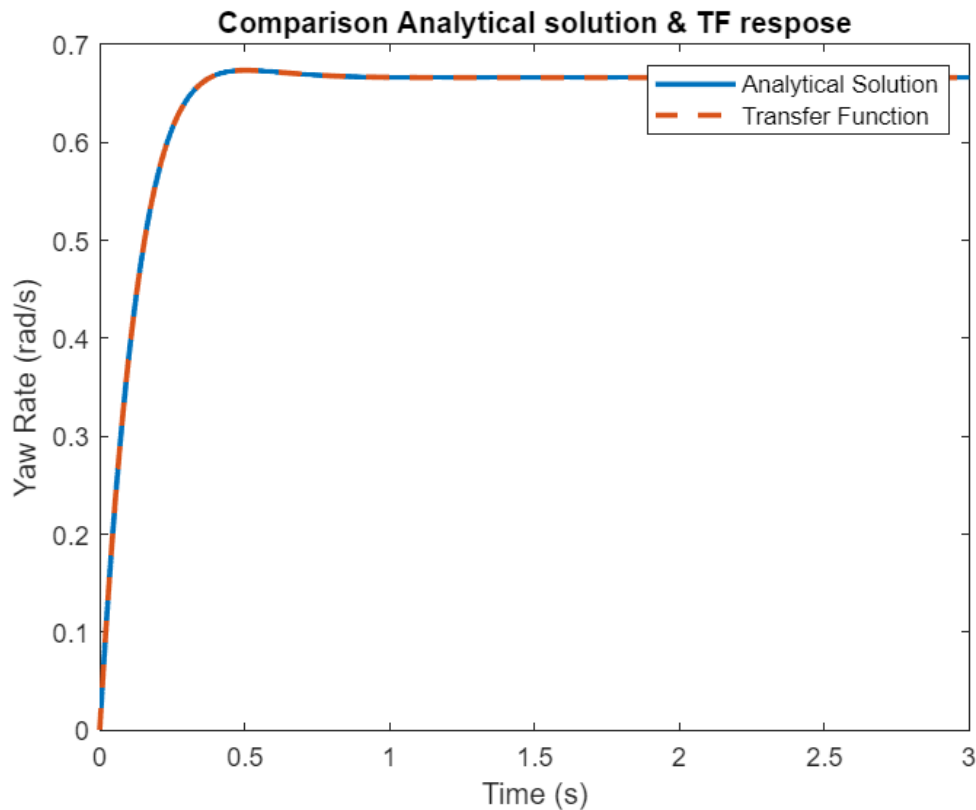
```
tf2 =
```

```
      5.077 s + 47.6
-----
s^2 + 14.89 s + 71.45
```

```
Continuous-time transfer function.
```

```
stepres2=step(tf2,t);
plot(t,stepres2,'LineStyle','--',LineWidth=2)

title('Comparison Analytical solution & TF respose')
xlabel('Time (s)')
ylabel('Yaw Rate (rad/s)')
xlim([0,3])
legend('Analytical Solution','Transfer Function')
```



Frame of Reference changing

```

vx=16;
yaw_rate =out.r.Data;          %y(:,1)';
vy_local = out.v.Data ;      %y(:,2)';
vx_global = zeros(1,length(t));
vy_global = zeros(1,length(t));
yaw = cumtrapz(t,yaw_rate);
for k = 1:length(t)
    R = [cos(yaw(k)), -sin(yaw(k)); ...
         sin(yaw(k)), cos(yaw(k))];
    temp = R\[vy_local(k); vx];
    vy_global(k) = temp(1,:);
    vx_global(k) = temp(2,:);
end
vy_g=[t;vy_global]';
vy_l=[t;vy_local]';
x_g=cumtrapz(t,vx_global);
y_g=cumtrapz(t,vy_global);

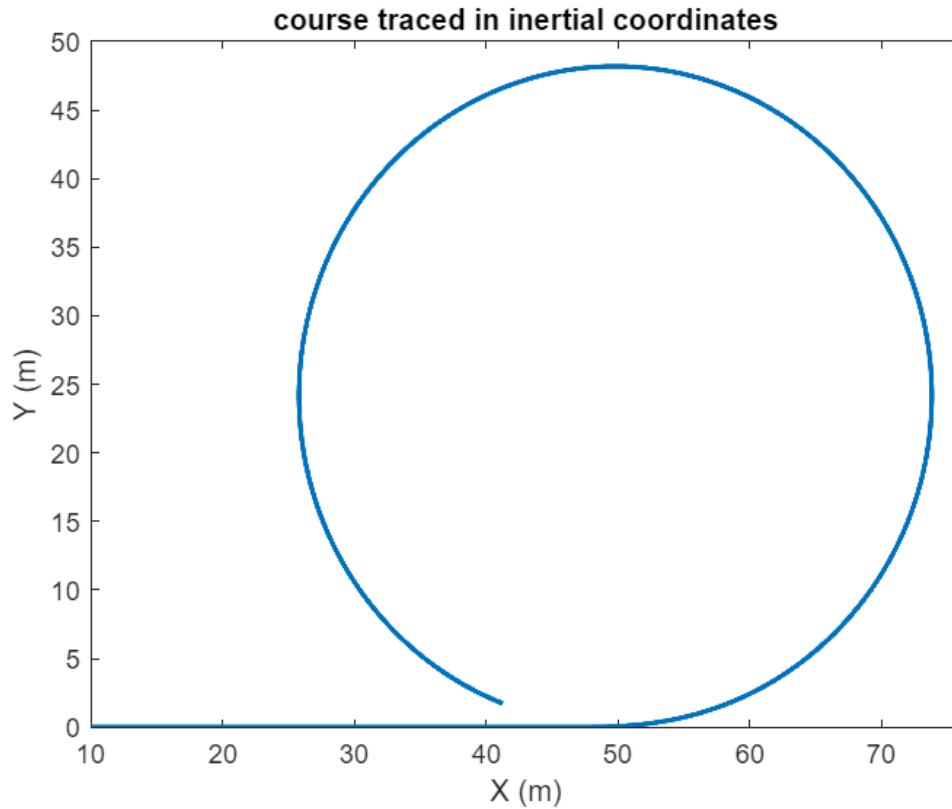
```

(d)Course traced

The Longitudinal velocity of the vehicle is 16 m/s.

Before giving the step input, with 3 seconds, it runs for 48 meters in the x direction and this is visible in the plot.


```
% for plotting the course of the vehicle
figure;
plot(x_g,y_g,LineWidth=2)
xlim([10,76])
title('Course traced in Inertial Coordinates')
xlabel('X (m)')
ylabel('Y (m)')
```



(e) Accelerations

```
figure;
plot(t,out.ay_global.Data(:,1),LineWidth=2);hold on;
plot(t,out.ay_local.Data,'linestyle','--',LineWidth=2)
title('Lateral Acceleration')

xlim([2,12])
xlabel('Time')
ylabel('ay (m/s^2)')
legend('ay_global','ay_local')
```

