Department of Astronomy, Astrophysics and Space Engineering (DAASE)

AA608: Astrostatistics

Importance Sampling

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Contents

1 Markov Chain Monte Carlo :-

Markov chain Monte Carlo (MCMC) methods comprise a class of algorithms for sampling from a probability distribution. By constructing a Markov chain that has the desired distribution as its equilibrium distribution, one can obtain a sample of the desired distribution by recording states from the chain. The more steps that are included, the more closely the distribution of the sample matches the actual desired distribution. Various algorithms exist for constructing chains, including the Metropolis–Hastings algorithm.

2 Metropolis–Hastings algorithm :-

the Metropolis–Hastings algorithm is a Markov chain Monte Carlo (MCMC) method for obtaining a sequence of random samples from a probability distribution from which direct sampling is difficult. This sequence can be used to approximate the distribution (e.g. to generate a histogram) or to compute an integral (e.g. an expected value). Metropolis–Hastings and other MCMC algorithms are generally used for sampling from multi-dimensional distributions, especially when the number of dimensions is high.

The most commonly used proposal distribution is

$$Pr(x_{trail}|x_{s-1} = normal(x_t rail; x_{s-1}, \Sigma)$$

$$Pr(x_{trail}|x_{s-1} = \frac{1}{(2\pi)^N |\Sigma|)^{1/2}} \exp\left[\frac{1}{2}(x_{trail} - x_{s-1})^T \Sigma^{-1} (x_{trail} - x_{s-1})\right]$$

3 Problem Discussion:-

After code the problem which is given in the python file with necessary comment .It clear that for high and low value of proposal distribution we obtain the diffrent values of acceptance ratio ,omega matter value and scaling factor h values.

3.1 distribution we have

- Value of omega matter without prior : 0.2970424377314992
- Value of h without prior : 0.7025176380617021
- Value of acceptance ratio : 8.34
- Value of omega matter with prior : 0.29003948729625917
- Value of h with prior : 0.7059657690785828
- Diffrence Between with and without prior in Omega : 0.0070029504352400185
- Diffrence Between with and without prior in h : 0.0034481310168806845

4 MH-MCMC With Prior(Importance Sampling)

In first case we take constant prior this porblem we are defining the prior which is a normal distribution with mean 0.738 and standard deviation 0.024 and taking the ln of the prior

$$Prior = -\frac{1}{2}\left(\frac{h-\mu}{\sigma}\right)^2 - \log(\sigma \times \sqrt{2\pi})$$

5 Gelman-Rubin Test

It is used to check the convergence of Markov Chain Monte Carlo.If we were to start multiple parallel chains in many different starting values, the theory claims that they should all eventually converge to the stationary distribution.So after some amount of time, it should be impossible to distinguish between the multiple chains.

Let $x_1^j, x_2^j, x_3^j, ...$ be sample from the Jth Markov chain and J chains runs in parallel with diffrent starting value then For each chain discard values as burn-in and keep the ramining L values

$$\text{Chain Mean } \bar{x_j} = \tfrac{1}{L}\sum_{t=1}^{L} x_t^j$$

$$\text{Grand Mean } \bar{x} = \tfrac{1}{J}\sum_{j=1}^{J} \bar{x_j}$$

$$\text{Between chain variance } B = \tfrac{L}{J-1}\sum_{j=1}^{J} (\bar{x_j} - \bar{x})^2$$

$$\text{With in chain variance } s_j^2 = \tfrac{1}{L-1}\sum_{t=1}^{L} (x_t^j = \bar{x_j})^2$$

$$W = \tfrac{1}{J}\sum_{j=1}^{J} s_j^2$$

$$\boxed{R = \frac{\frac{L-1}{L}W + \frac{1}{L}B}{W}}$$

If $L \to \infty$ and $B \downarrow 0$ R approaches the value of 1.In our probelm we get R value as

- Gelman rubin convergence value for Omega matter - 0.9999900002215799
- Gelman rubin convergence value for Parameter h - 0.9999900002284954