

Forward difference Method

$$f'(x_i) = \frac{f(x_{i+\Delta x}) - f(x_i)}{\Delta x} \rightarrow \text{Costum}$$

Backward difference Method

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{\Delta x}$$

$$\begin{aligned} \frac{d}{dx}(c) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{c - c}{h} \\ &= \lim_{h \rightarrow 0} \frac{0}{h} \\ &= 0. \end{aligned}$$

Central difference Method

$$f'(x_i) = \frac{f(x_{i+\Delta x}) - f(x_{i-\Delta x})}{2\Delta x}$$

$$\begin{aligned} f(4) &= 5 \cdot 4^2 \\ f(3) &= 5 \cdot 3^2 \end{aligned} \quad \left[f(x) = 5x^2 \right] \quad \int x^n \rightarrow \frac{x^{n+1}}{n+1}$$

$$f'(x) = 10x$$

$$f'(3) = 30$$

$$\Delta x = 0.1$$

$$f'(3) = \frac{f(4) - f(3)}{1}$$

$$= \frac{80 - 45}{1} = 35$$

$$f'(3) = \frac{f(3.1) - f(3)}{0.1} =$$

Approximate the derivative of $f(x) = x^2 + 2x$ at $x=3$ using the forward, backward, and central difference method and step size 1.

$x_i = 3$ $x_{i+1} = 4$
 $\Delta x = 1$ $x_{i-1} = 2$
 $f(2) = 8$
 $f(3) =$
 $f(4) =$

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$$f(x) = x^2 + 2x$$

$$f'(x) = 2x + 2$$

$$f(3) = 8$$

Errornya = 1

$$\begin{aligned} f'(x_i) &= \frac{f(x_{i+\Delta x}) - f(x_i)}{\Delta x} \\ \text{FD: } f'(3) &= \frac{f(4) - f(3)}{1} = \frac{24 - 15}{1} = 9 \end{aligned}$$

$$\text{FD: } f'(x) = \boxed{f'(3) \approx 9}$$

$$\begin{aligned} f'(x_i) &= \frac{f(x_i) - f(x_{i-1})}{\Delta x} \\ f'(3) &= \frac{f(3) - f(2)}{1} = \frac{15 - 8}{1} = 7 \end{aligned}$$

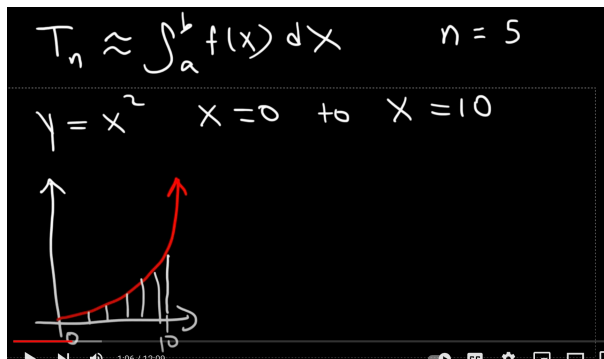
$$\text{BD: } f'(3) \approx 7$$

$$CD: f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}))}{2\Delta x}$$

$$f'(3) = \frac{f(4) - f(2)}{2} = \frac{24 - 8}{2} = \frac{16}{2} = 8$$

CD: $f'(3) \approx 8$

Trapezoidal method



$$T_n \approx \int_a^b f(x) dx \quad n = 5 \quad [0, 10]$$

$$\Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2$$

$$T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$T_n \approx \int_a^b f(x) dx \quad n = 5 \quad [0, 10]$$

$$f(x) = x^2$$

$$T_5 = \frac{2}{2} [f(0) + 2f(2) + 2f(4) + 2f(6) + 2f(8) + f(10)]$$

$$T_5 = 1 [0 + 2(4) + 2(16) + 2(36) + 2(64) + 100]$$

$$T_5 = [8 + 32 + 72 + 128 + 100]$$

$$T_5 = [40 + 200 + 100] = 340$$

$$T_n \approx \int_a^b f(x) dx \quad n=5 \quad [0, 100]$$

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$$T_5 = [40 + 200 + 100] = 340$$

$$\int_2^{10} x^3 dx$$

$$\int_a^b f(x) dx \approx S_n$$

$$S_n = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_{n-1}) + 4f(x_n) + f(x_n)]$$

$$\int_2^{10} x^3 dx \quad \Delta x = \frac{b-a}{n} \quad n=4$$

$$\Delta x = \frac{10-2}{4} = 2$$

$$\leftarrow \begin{array}{c} | & | & | & | & | \\ 2 & 4 & 6 & 8 & 10 \end{array} \rightarrow$$

$$S_4 = \frac{2}{3} [f(2) + 4f(4) + 2f(6) + 4f(8) + f(10)]$$

Runge-kutta method

$$y_{n+1} = y_n + h T_4(x_n, y_n, h)$$

$$x_{n+1} = x_n + h$$

$$T_4 = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x, y)$$

$$k_2 = f(x + \frac{h}{2}, y + \frac{h}{2} k_1)$$

$$k_3 = f(x + \frac{h}{2}, y + \frac{h}{2} k_2)$$

$$k_4 = f(x + h, y + h k_3)$$

Example $\frac{dy}{dx} = 3x^2 y$ $x_0 = 1, y_0 = 2$
Use $h = 0.1$

$$\Rightarrow y_1 = y_0 + h T_4(x_0, y_0, h)$$

$$y_1 = 2 + \frac{0.1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_0, y_0) = 3x_0^2 y_0 = 3(1)^2(2) = 6$$

Example $\frac{dy}{dx} = 3x^2y$ $x_0=1, y_0=2$

Use $h=0.1$

$\Rightarrow y_1 = y_0 + hT_4(x_0, y_0, h)$

$y_1 = 2 + \frac{0.1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = f(x_0, y_0) = 3x_0^2y_0 = 3(1)^2(2) = 6$

$k_2 = f(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_1) = 3(1.05)^2(2.3) = 2.607$

$k_3 = f(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_1)$

$k_3 = f(1.05, 2 + \frac{0.1}{2}(2.607)) = f(1.05, 2.38)$

$k_3 = 3(1.05)^2(2.38) = 2.872$

$k_4 = f(x_0 + h, y_0 + hk_1)$

$k_4 = f(1.10, 2 + 0.1(2.872))$

$k_4 = f(1.10, 2.2872) = 3(1.10)^2(2.2872) = 10.117$

$y_1 = y_0 + \frac{h}{6} [k_1 + 2k_2 + 2k_3 + k_4]$

$= 2 + \frac{0.1}{6} [6 + 2(2.607) + 2(2.872) + 10.117]$

$y_1 = 2.287 \quad x_1 = 1.10$

$y_2 = y_1 + hT_4(x_1, y_1)$

$k_1 = f(x_1, y_1)$

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$y = x^2$ $y(1.5)$ $y_{n+1} = y_n + h f'(x_n)$

$\frac{dy}{dx} = 2x$ $P(1,1)$ $h=0.1$

n	x_n	y_n	Actual
0	1	1	1
1	1.1	1.2	1.21
2	1.2	1.42	1.44
3	1.3	1.66	1.69
4	1.4	1.92	1.96
5	1.5	2.2	2.25

$y_3 = 1.42 + 0.1 f'(1.2)$
 $= 1.66$
 $y_4 = 1.66 + 0.1 f'(1.3)$
 $y_5 = 1.92 + 0.1 f'(1.4)$
 $y(1.5) \approx 2.2$