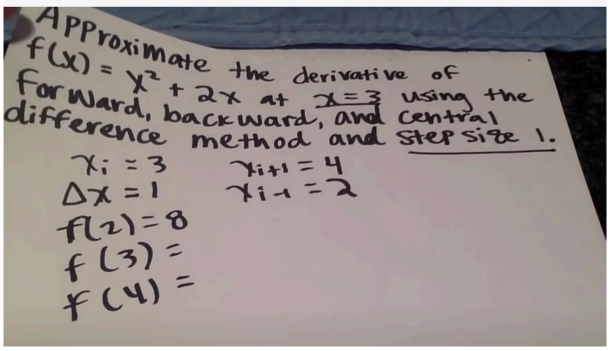
Forward difference Method

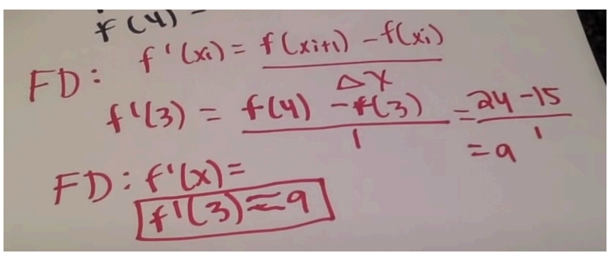
$$f'(x_i) = \frac{f(x_{i+\Delta x}) - f(x_i)}{\Delta X} \longrightarrow \text{Costum}$$

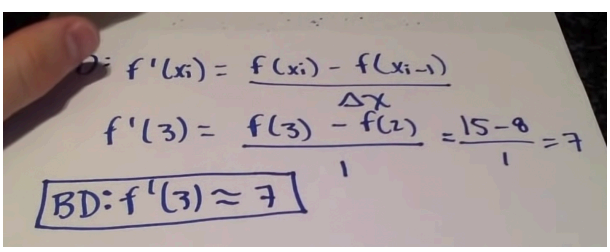
Backward difference Method

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{\Delta X}$$

$$\begin{aligned} \frac{d}{dx}(c) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{c - c}{h} \\ &= \lim_{h\to 0} \frac{0}{h} \\ &= 0. \end{aligned}$$







Central difference Method

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta X}$$

$f(4) = 5.4^2$

$f(3) = 5.3^2$

$$\boxed{f(x) = 5x^2}$$

$\int \to x^n$

$n.x^{n-1}$

$f'(x) = 10x$

$f'(3) = 30$

$$\boxed{\Delta X = 0.1}$$

$$f'(3) = \frac{f(4) - f(3)}{1} = \frac{80 - 45}{1} = 35$$

$$f'(3) = \frac{f(3.1) - f(3)}{0.1} =$$

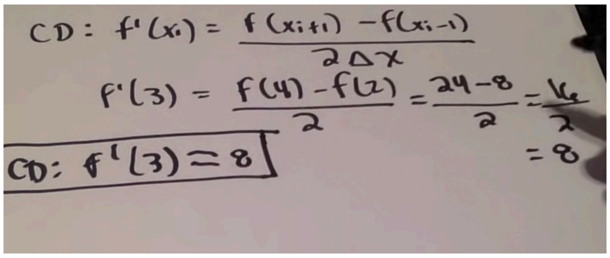
Hasil asli

$f(x) = x^2 + 2x$

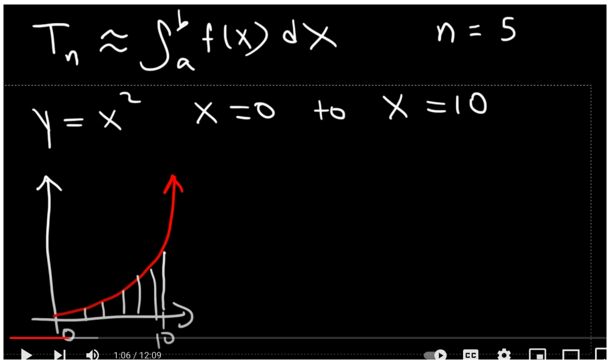
$f'(x) = 2x + 2$

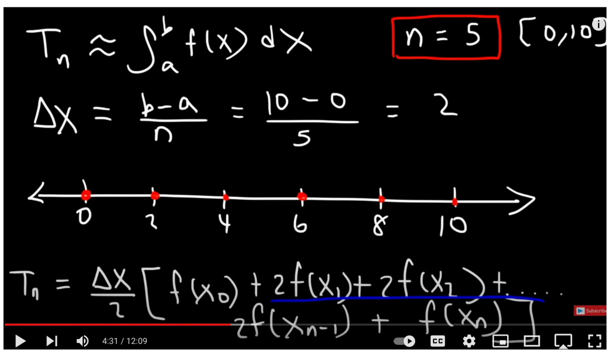
$f(3) = 8$ //

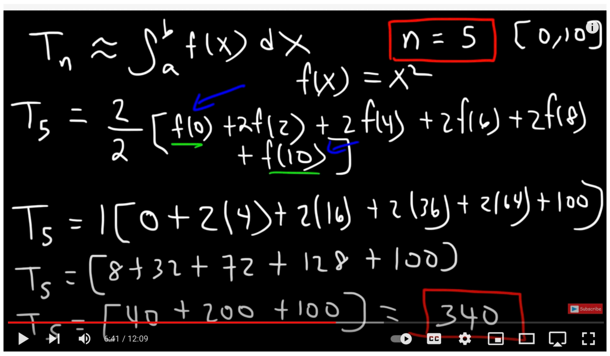
Errornya = 1



Trapezoidal method







$$T_n \approx \int_a^b f(x)\,dx \qquad \boxed{n=5} \qquad [0,10]$$

$$f(x) = x^2$$

$$T_5 = \frac{2}{2}\left[f(0) + 2f(2) + 2f(4) + 2f(6) + 2f(8) + f(10)\right]$$

$$T_5 = 1[0 + 2(4) + 2(16) + 2(36) + 2(64) + 100]$$

$$T_5 = [8 + 32 + 72 + 128 + 100]$$

$$T_5 = [40 + 200 + 100] = \boxed{340}$$

$$\int_2^{10} x^3\,dx$$

$$\int_a^b f(x)\,dx \approx S_n$$

$$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

$$\int_{\boxed{2}}^{\boxed{10}} x^3\,dx \qquad \Delta x = \frac{b-a}{n} \qquad n = 4$$

$$\Delta x = \frac{10-2}{4} = 2$$

2 4 6 8 10

$$S_4 = \frac{2}{3}\left[f(2) + 4f(4) + 2f(6) + 4f(8) + f(10)\right]$$

Runge-kutta method

$$y_{n+1} = y_n + h\,T_4(x_n, y_n, h)$$

$$x_{n+1} = x_n + h$$

$$T_4 = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x, y)$$

$$k_2 = f\left(x + \frac{h}{2}, y + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(x + \frac{h}{2}, y + \frac{h}{2}k_2\right)$$

$$k_4 = f(x + h, y + hk_3)$$

Example $\frac{dy}{dx} = 3x^2 y$ $\quad x_0 = 1,\ y_0 = 2$

Use $h = 0.1$

$$\Rightarrow y_1 = y_0 + h\,T_4(x_0, y_0, h)$$

$$y_1 = 2 + \frac{0.1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_0, y_0) = 3x_0^2 y_0 = 3(1)^2(2) = 6$$

Example $\frac{dy}{dx} = 3x^2 y$ $x_0 = 1,\ y_0 = 2$

Use $h = 0.1$

$\Rightarrow y_1 = y_0 + h T_4(x_0, y_0, h)$

$y_1 = 2 + \frac{0.1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = f(x_0, y_0) = 3x_0^2 y_0 = 3(1)^2(2) = 6$

$k_2 = f(1.05, 2.3) = 3(1.05)^2(2.3) = 7.607$

$k_3 = f(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_2)$

$k_3 = f[1.05, 2 + \frac{0.1}{2}(7.607)] = f(1.05, 2.38)$

$k_3 = 3(1.05)^2(2.38) = \boxed{7.872}$

$k_4 = f(x_0 + h, y_0 + hk_3)$

$k_4 = f[1.10, 2 + 0.1(7.872)]$

$k_4 = f(1.10, 2.787) = 3(1.10)^2(2.787) = 10.117$

$y_1 = y_0 + \frac{h}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 2 + \frac{0.1}{6}[6 + 2(7.607) + 2(7.872) + 10.117]$

$\boxed{y_1 = 2.787 \quad x_1 = 1.10}$

$y_2 = y_1 + h T_4(x_1, y_1)$

$k_1 = f(x_1, y_1)$

euler

$y = x^2$ $\boxed{y(1.5)}$ ←

$\boxed{\frac{dy}{dx} = 2x}$ $f(1,1)$

$y_{n+1} = y_n + h f'(x)_n$

$\boxed{h = 0.1}$

n	x_n	y_n	Actual
0	1	1	1
1	1.1	1.2	1.21
2	1.2	1.42	1.44
3	1.3	1.66	1.69
4	1.4	1.92	1.96
5	1.5	2.2	2.25

$y_3 = 1.42 + 0.1 f'(1.2)$
$= 1.66$

$y_4 = 1.66 + 0.1 f'(1.3)$

$y_5 = 1.92 + 0.1 f'(1.4)$

$\boxed{y(1.5) \approx 2.2}$