Code No: 136BE

R16

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech III Year II Semester Examinations, July/August - 2021

DIGITAL SIGNAL PROCESSING

(Common to ECE, EIE, ETM)

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

\- - -

1.a) Obtain the cascade realization for the given system function

$$H(Z)=\frac{\left(1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}\right)\left(1-\frac{3}{2}z^{-1}+z^{-2}\right)}{\left(1+z^{-1}+\frac{1}{4}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)}$$

b) Represent discrete time signal and systems in frequency domain. [10+5]

2.a) Determine and sketch the magnitude plot of $y(n)=\frac{1}{2}[x(n)+x(n-2)]$

b) Determine whether the given systems are stable or not. [7+8]

i) y(n)=x(2n) ii) y(n)=x(-n)

3.a) Compute the 8-point DFT of the sequence of x(n)={1,1,2,2,3,3,4,4}.

b) State and prove the circular shift of a sequence. [9+6]

4.a) Obtain the output response y(n), if h(n)={1,2,2,1}; x(n)={1,-1,1,-1} without using DFT.

b) How can we calculate IDFT using FFT algorithm? [9+6]

5.a) For the given analog transfer function $H(s)=\frac{10}{s^2-3s-28}$, determine H(z) using impulse invariant and bilinear transform method for T=1 sec.

b) Compare IIR and FIR filters. [9+6]

6.a) Design a Butterworth filter using the impulse invariance method for the following specifications.

$$0.9\le|H(j\Omega)|\le 1 \text{ for } 0\le\Omega\le 0.1\pi,\ |H(j\Omega)|\le 0.2 \text{ for } 0.3\pi\le\Omega\le\pi$$

b) Explain the procedure to design an FIR filter. [7+8]

7. Design an ideal low pass filter with frequency response

$$H_d(e^{j\omega})=1 \text{ for } -\frac{\pi}{2}\le\omega\le\frac{\pi}{2}$$

$$=0 \text{ for } \frac{\pi}{2}\le|\omega|\le\pi$$

Find the values of h(n) for N=11. Find the H(z). [15]

8.a) Brief out the applications of multi rate signal processing.

b) Explain the method of scaling to prevent overflow limit cycle oscillations. [8+7]

---ooOoo---

T. Jyothsna

Assoc. Prof

Code : 136BE

Reg: R16

B.Tech III Yr II sem Examinations, July/Aug - 2021

Digital signal processing

1 (a) $$H(z) = \frac{(1+3/2\,z^{-1}+\frac{1}{2}z^{-2})(1-3/2\,z^{-1}+z^{-2})}{(1+z^{-1}+1/4\,z^{-2})(1+1/4\,z^{-1}+\frac{1}{2}z^{-2})}$$ (10 M)

sol:- Cascade Realization :-

let $H_1(z) = \dfrac{1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}}{1+z^{-1}+\frac{1}{4}z^{-2}}$; $H_2(z) = \dfrac{1-3/2\,z^{-1}+z^{-2}}{1+1/4\,z^{-1}+1/2\,z^{-2}}$

(4 Marks)

Representing $H_1(z)$ in Direct form - II

$$H_1(z) = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X_1(z)}$$

$\therefore \dfrac{Y_1(z)}{W_1(z)} = 1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}$; $\dfrac{W_1(z)}{X_1(z)} = \dfrac{1}{1+z^{-1}+\frac{1}{4}z^{-2}}$

$Y_1(z) = W_1(z)+\frac{3}{2}z^{-1}W_1(z)+\frac{1}{2}z^{-2}W_1(z)$; $W_1(z)+z^{-1}W_1(z)+\frac{1}{4}z^{-2}W_1(z)=X_1(z)$

Apply I.Z.T

$$\boxed{y_1(n) = w_1(n)+\frac{3}{2}w_1(n-1)+\frac{1}{2}w_1(n-2)}$$

$$\boxed{w_1(n) = x_1(n) - w_1(n-1) - \frac{1}{4}w_1(n-2)}$$

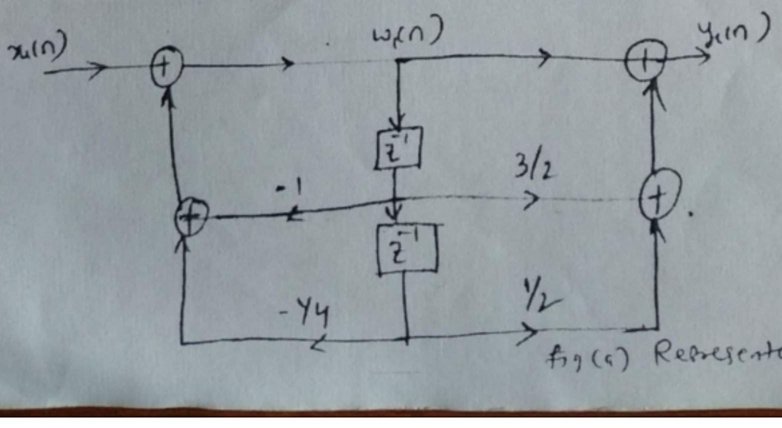


fig (a) Representation of $H_1(z)$

$H_2(z)$ in D.F. II ; $H_2(z) = \dfrac{1-\frac{3}{2}z^{-1}+z^{-2}}{1+\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}}$; $H_2(z) = \dfrac{Y_2(z)}{W_2(z)} \cdot \dfrac{W_2(z)}{X_2(z)}$

$$y_2(n) = w_2(n) - \frac{3}{2} w_2(n-1) + w_2(n-2)$$

(4 Marks)

&

$$w_2(n) = x_2(n) - \frac{1}{4} w_2(n-1) - \frac{1}{2} w_2(n-2)$$

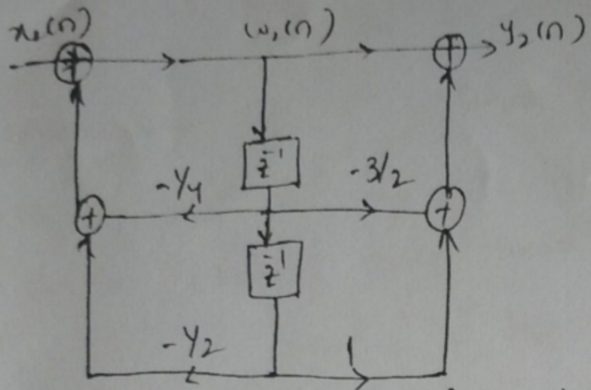
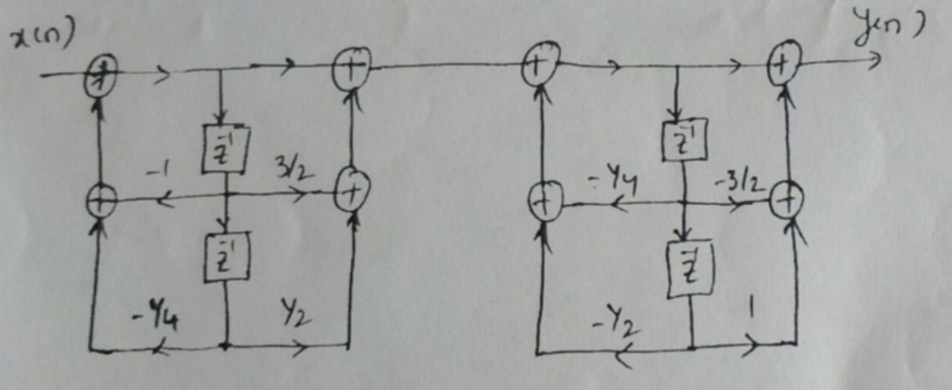


fig (b) Representation of $H_2(z)$

(2 Marks)

Now cascade both the figs



1(b)

(1 Mark)

Discrete time fourier series (Representation of DT signal in freq domain)

Periodic:-

$$x(n) = \sum_{k=0}^{N-1} C_k e^{\frac{j2\pi nk}{N}} \quad ; \; n = 0,1 \cdots N-1$$

periodic signal

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}} \quad ; \; k = 0,1 \cdots N-1$$

(2)

(1 Mark)

Aperiodic signals

If $x(n)$ Aperiodic signal

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

Representation of DT systems in frequency domain (3 Marks)

The o/p $y(n)$ of any LTI system to any i/p signal $x(n)$ can be obtained using Convol^n sum

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

if $x(n) = e^{j\omega n}$ → complex exponential signal

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) e^{j\omega(n-k)}$$

$$= e^{j\omega n} \sum_{k=-\infty}^{\infty} h(k) e^{-j\omega k}$$

$$y(n) = e^{j\omega n} \left[H(e^{j\omega})\right]$$

↓ input ↓ freq Response

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h(k) e^{-j\omega k}$$

$$H(e^{j\omega}) = |H(e^{j\omega})| \angle H(e^{j\omega})$$

↓ magnitude Response → even fun

↓ phase Response → odd fun

$$|H(e^{j\omega})| = |H(e^{-j\omega})|$$

$$\theta(\omega) = -\theta(-\omega)$$

ations, July/Augu...
...eING ...arks: 75

(ii) $y(n) = \frac{1}{2}[x(n) + x(n-2)]$ (3 Marks)

Sol.-
$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n) e^{-j\omega n}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty}[x(n) + x(n-2)]e^{-j\omega n} = \frac{1}{2}\left[X(e^{j\omega}) + e^{-2j\omega}X(e^{j\omega})\right]$$

$$Y(e^{j\omega}) = \frac{X(e^{j\omega})}{2}\left[1 + e^{-2j\omega}\right] \; ; \; H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + e^{-j2\omega}}{2}$$

$$H(e^{j\omega}) = \frac{1 + \cos 2\omega - j\sin 2\omega}{2}$$

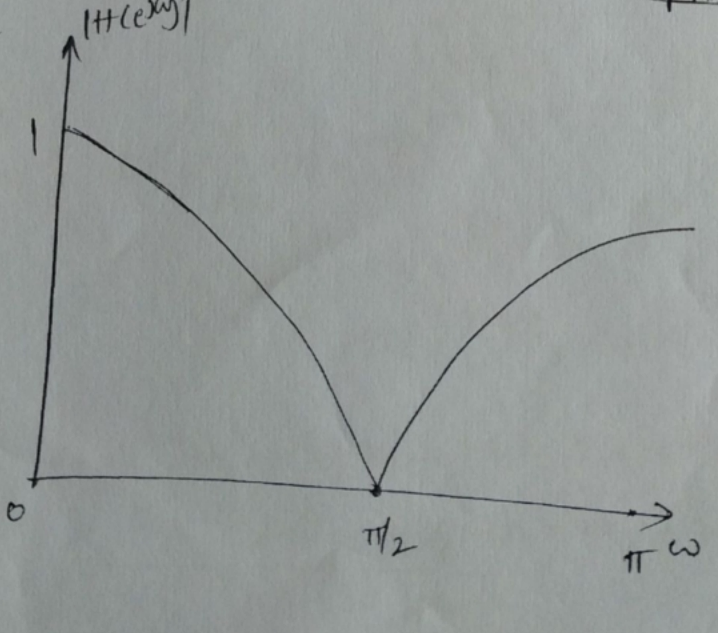
Mag Response $|H(e^{j\omega})| = \frac{1}{2}\left[\sqrt{(1+\cos 2\omega)^2 + \sin^2 2\omega}\right] = \cos\omega$

$$\boxed{|H(e^{j\omega})| = \cos\omega}$$

(2 Marks)

ω	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$H(e^{j\omega})$	1	0.812	0.707	0.5	0	-0.5	-0.707	-1
$\lvert H(e^{j\omega})\rvert$	1	0.812	0.707	0.5	0	0.5	0.707	1

Magnitude plot



(2 Marks)

③

(b) Determine whether the given systems are stable or not (4 Marks)

Sol:- (i) $y(n) = x(2n)$ (4 Marks)

stability condition

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

if $x(n) = \delta(n)$; $y(n) = h(n)$.

$\therefore h(n) = \delta(2n)$ $\left[\because \begin{array}{l}\delta(n)=1;\ n=0 \\ \delta(2n)=1;\ n=0\end{array}\right.$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |\delta(2n)| = 1 < \infty$$

$\therefore$ stable system.

(ii) $y(n) = x(-n)$

if $x(n) = \delta(n)$; $y(n) = h(n)$

$h(n) = \delta(-n)$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |\delta(-n)|$$

$= 1 < \infty$

$\therefore$ stable system

(9 Marks)

3(a) 8-point DFT of a seq $x(n) = \{\underbrace{1, 1, 2, 2, 3, 3, 4, 4}_{N=8}\}$

Sol:- WKT $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$; $k = 0, 1 \dots N-1$, $n = 0, 1 \dots N-1$

$k=0$:-

$$X(0) = \sum_{n=0}^{7} x(n) e^{-j2\pi n(0)/8} = \sum_{n=0}^{7} x(n) = x(0)+x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)$$

$= 1+1+2+2+3+3+4+4$

$$\boxed{X(0) = 20}$$

$k=1$; $X(1) = \sum_{n=0}^{7} x(n) e^{-j2\pi n(1)/8}$

$= x(0)e^{0} + x(1)e^{-j2\pi/8} + x(2)e^{-j4\pi/8} + x(3)e^{-j6\pi/8} + x(4)e^{-j8\pi/8}$

$+ x(5)e^{-j10\pi/8} + x(6)e^{-j12\pi/8} + x(7)e^{-j14\pi/8}$

$= 1 + (0.707 - j0.707) + 2(-j) + 2(-0.707 - j0.707)$

$+ 3(-1) + 3(0.707 + j0.707) + 4(j) + 4(0.707 + j0.707)$

$$\boxed{X(1) = -2 + j\,j4.848}$$

$$X(2) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi n(2)}{8}}$$

$$\boxed{X(2) = -2+2j}$$

$k=3$

$$X(3) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi n(3)}{8}}$$

$$\boxed{X(3) = -2 + j0.828}$$

$k=4$

$$X(4) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi n(4)}{8}}$$

$$\boxed{X(4) = 0}$$

$k=5$

$$X(5) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi n(5)}{8}}$$

$$\boxed{X(5) = -2 - j0.828}$$

$k=6$

$$X(6) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi n(6)}{8}}$$

$$\boxed{X(6) = -2-2j}$$

$k=7$

$$X(7) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi n(7)}{8}}$$

$$\boxed{X(7) = -2 - j4.828}$$

$$\boxed{X(k) = \{20, -2+j4.828, -2+2j, -2+j0.828, 0, -2-j0.828, -2-2j, -2-j4.828\}}$$

5(b) Circular shift of a sequence :- (6 Marks)

Statement: If $DFT[x(n)] = X(k)$ (1 Mark)

Then $DFT[x((n-m))_N] = e^{\frac{-j2\pi km}{N}} X(k)$

Proof :- (5 Marks)

$$DFT[x((n-m))_N] = \sum_{n=0}^{N-1} x((n-m))_N e^{-j2\pi nk/N}$$

$$= \sum_{n=0}^{m-1} x((n-m))_N e^{\frac{-j2\pi nk}{N}} + \sum_{n=m}^{N-1} x((n-m))_N e^{\frac{-j2\pi nk}{N}} \longrightarrow (1)$$

$x((n-m))_N = x(N-m+n)$ (4)

$\therefore \sum_{n=0}^{m-1} x((n-m))_N e^{\frac{-j2\pi nk}{N}} = \sum_{n=0}^{m-1} x(N-m+n) e^{-j2\pi nk/N}$

Let $N-m+n = l$

$\therefore \sum_{n=0}^{m-1} x((n-m))_N e^{\frac{-j2\pi nk}{N}} = \sum_{l=N-m}^{N-1} x(l) e^{-j2\pi k\frac{(-N+m+l)}{N}}$

$= \sum_{l=N-m}^{N-1} x(l) e^{-j2\pi k\frac{(l+m)}{N}} \longrightarrow (2)$ $\quad \because e^{j2\pi k} = 1$ for $k=0,1,2,\ldots$

lly $\sum_{n=m}^{N-1} x((n-m))_N e^{\frac{-j2\pi nk}{N}} = \sum_{l=0}^{N-1-m} x(l) e^{-j2\pi k(m+l)/N} \longrightarrow (3)$

Sub (2) & (3) in (1)

$DFT\left[x((n-m))_N\right] = \sum_{l=N-m}^{N-1} x(l) e^{-j2\pi k\frac{(m+l)}{N}} + \sum_{l=0}^{N-m-1} x(l) e^{-j2\pi k\frac{(m+l)}{N}}$

$= e^{\frac{-j2\pi mk}{N}} \sum_{l=0}^{N-1} x(l) e^{-j2\pi lk/N}$

$$\boxed{DFT\left[x((n-m))_N\right] = e^{\frac{-j2\pi mk}{N}} X(k)}$$

(9 Marks)

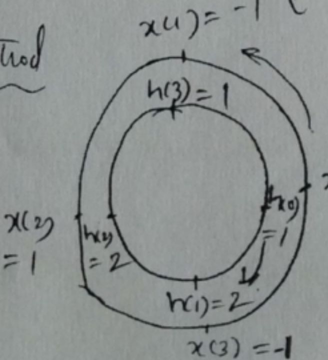
4(a) obtain the o/p $y(n)$; if $h(n) = \{1, 2, 2, 1\}$
$x(n) = \{1, -1, 1, -1\}$.

Sol:- circular convolⁿ

Concentric circle method

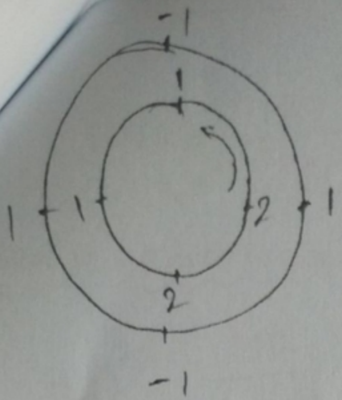
(2 Marks)

$x(n) \to$ Anti clockwise
$h(n) \to$ clockwise
rotation $\to$ Anticlockwise

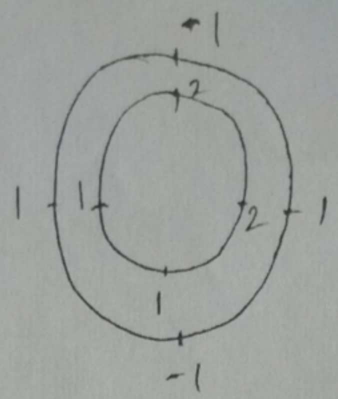


$y(0) = 1 - 2 + 2 - 1$

$\boxed{y(0) = 0}$



(2 Marks)



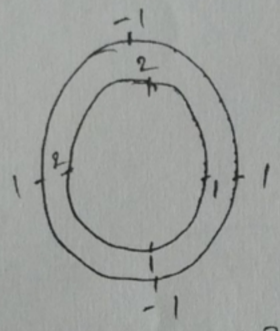
(2 Marks)

$y(1) = 2 - 2 + 1 - 1$

$\boxed{y(1) = 0}$

$y(2) = 2 - 1 + 1 - 2$

$\boxed{y(2) = 0}$



(2 Marks)

$y(3) = 1 - 1 + 2 - 2 = 0$

$\boxed{y(3) = 0}$

$\boxed{y(n) = \{0, 0, 0, 0\}}$ → (1 Mark)

[if student done linear conv^n also give marks]

(4(b)) IDFT using FFT algorithm :- (6 Marks)

$$\text{IDFT}[X(k)] = x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W^{-nk}$$

$$W = e^{-j2\pi/N}$$

Take complex conjugate, multiply by N

$$N x^*(n) = \sum_{k=0}^{N-1} X^*(k) W^{nk}$$

$X^*(k)$ can be computed by using DIT FFT (or) DIF FFT.

$$\rightarrow x(n) = \frac{1}{N}\left[\sum_{k=0}^{N-1} x^*(n) W^{nk}\right]^*$$

for example :-

(5)

$$H(S) = \frac{10}{S^2 - 3S - 28} \quad ; \; T = 1 \text{ sec} \quad (9 \text{ Marks})$$

Sol. Impulse Invariance Method (5 Marks)

$$H(s) = \frac{10}{(S-7)(S+4)} = \frac{A}{S-7} + \frac{B}{S+4}$$

$$A = \cancel{(S-7)} \frac{10}{\cancel{(S-7)}(S+4)} \Bigg|_{S=7} = 10/11 \quad ; \quad B = \cancel{(S+4)} \frac{10}{(S-7)\cancel{(S+4)}} \Bigg|_{S=-4}$$

$$\boxed{B = -10/11}$$

$$\boxed{A = \frac{10}{11}}$$

$$H(S) = \frac{10/11}{S-7} + \frac{-10/11}{S+4}$$

$$\text{if } H(S) = \sum_{k=1}^{N} \frac{C_k}{S - P_k} \quad ; \quad H(z) = \sum_{k=1}^{N} \frac{C_k}{1 - e^{P_k T} z^{-1}}$$

$$\therefore \quad H(z) = \frac{10/11}{1 - e^{7} z^{-1}} - \frac{10/11}{1 - e^{-4} z^{-1}}$$

$$H(z) = \frac{10}{11}\left[\frac{1}{1 - e^{7} z^{-1}} - \frac{1}{1 - e^{-4} z^{-1}}\right] = \boxed{\frac{996.36}{1 - 1096 z^{-1} + 20 z^{-2}}}$$

Bilinear Transformation: (4 Marks)

$$H(z) = H(S) \Big|_{S = \frac{2}{T}\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]}$$

$$H(z) = \frac{10}{S^2 - 3S - 28} \Bigg|_{S = 2\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]}$$

$$H(z) = \frac{10}{\left[2\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]\right]^2 - 3\left(2\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]\right) - 28}$$

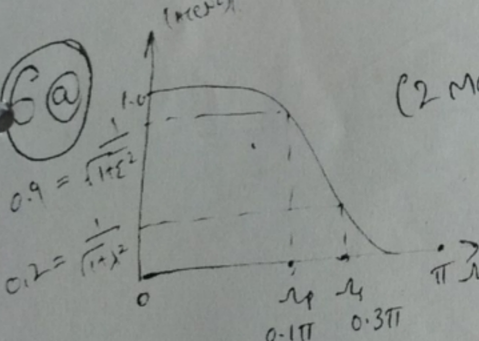
$$= \frac{10(1 + z^{-1})^2}{4[1 - z^{-1}]^2 - 6(1 - z^{-1})(1 + z^{-1}) - 28(1 + z^{-1})^2}$$

$$\boxed{H(z) = \frac{-[10 + 20 z^{-1} + 10 z^{-2}]}{30 + 64 z^{-1} + 18 z^{-2}}}$$

5(b) Compare IIR and FIR filter (6 M) (1 Mark each point)

FIR filters	IIR filters
① can be easily designed to have perfectly linear phase	① Do not have linear phase.
② Can be realized recursively and non-recursively.	② Easily realized recursively.
③ Greater flexibility to control the shape of their magnitude response.	③ Less flexibility, usually limited to specific filters.
④ Errors due to roundoff noise are less severe in FIR filters	④ Roundoff noise is more.
⑤ Always stable	⑤ Not always stable.
⑥ poles are fixed at origin	⑥ poles can be placed anywhere inside the circle.

6 (a)



(2 Marks)

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.9 \qquad \frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\boxed{\varepsilon = 0.4843} \qquad \boxed{\lambda = 4.8989}$$

$$N \geq \frac{\log(\lambda/\varepsilon)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\log\left(\frac{4.8989}{0.4843}\right)}{\log\left(\frac{0.3\pi}{0.1\pi}\right)}$$

$$N \geq 2.106 \quad \Rightarrow \quad \boxed{N = 3}$$

for $N = 3$ (2 Marks)

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$\Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}} = \frac{0.1\pi}{(0.4843)^{1/3}} = 0.127\pi$$

Sub $s \rightarrow \frac{s}{\Omega_c}$ in $H(s)$

$$H(s) = \frac{1}{\left(\frac{s}{0.127\pi}+1\right)\left(\left(\frac{s}{0.127\pi}\right)^2+\left(\frac{s}{0.127\pi}\right)+1\right)}$$

$$H(s) = \frac{(0.127\pi)^3}{(s+0.127\pi)\left(s^2+0.127\pi s+(0.127\pi)^2\right)}$$

Partial invariance Method (3 Marks) ⑥

$$H(s) = \frac{0.0635}{(s+0.398)(s^2+0.398s+0.159)}$$

$$H(s) = \frac{0.0635}{(s+0.4)(s^2+0.4s+0.16)}$$

$$H(s) = \frac{0.0635}{(s+0.4)(s-(-0.2+j0.346))(s-(-0.2-j0.346))}$$

$s_1 = -0.2 + j0.346$

$s_2 = -0.2 - j0.346$

$$H(s) = \frac{A}{s+0.4} + \frac{B}{s-(-0.2+j0.346)} + \frac{C}{s-(-0.2-j0.346)}$$

$$A = \cancel{(s+0.4)}\frac{0.0635}{\cancel{(s+0.4)}(s-(-0.2+j0.346))(s-(-0.2-j0.346))}\Bigg|_{s=-0.4}$$

$$\boxed{A = 3.97}$$

$$B = \frac{\cancel{(s-(-0.2+j0.346))}(0.0635)}{(s+0.4)\cancel{(s-(-0.2+j0.346))}(s-(-0.2-j0.346))}\Bigg|_{s=-0.2+j0.346}$$

$$\boxed{B = -0.24 + j0.138}$$

$$C = \frac{\cancel{(s-(-0.2-j0.346))}(0.0635)}{(s+0.4)(s-(-0.2+j0.346))\cancel{(s-(-0.2-j0.346))}}\Bigg|_{s=-0.2-j0.346}$$

$$\boxed{C = B^* = -0.24 - j0.138}$$

$$H(S) = \frac{3.97}{S+0.4} + \frac{-0.24+j0.138}{S-(-0.2+j0.346)} + \frac{-0.24-j0.138}{S-(-0.2-j0.346)}$$

In Impulse Invariance method

$$\text{if } H(S) = \sum_{K=1}^{N} \frac{C_K}{S-P_K} \quad \text{Then } H(Z) = \sum_{K=1}^{N} \frac{C_K}{1-e^{P_K T} Z^{-1}}$$

Assume $T = 1$ sec.

$$\therefore H(Z) = \frac{3.97}{1-e^{-0.4(1)} Z^{-1}} + \frac{-0.24+j0.138}{1-e^{(-0.24+j0.346)} Z^{-1}} + \frac{-0.24-j0.138}{1-e^{(-0.2-j0.346)} Z^{-1}}$$

(b) Fourier Series Method of Designing FIR filters (8 Marks) (4)

Sol:- ① The desired freq Response of an FIR filter canbe represented by the fourier series

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n}$$

Fourier coefficients $h_d(n)$ are the desired impulse response seq of the filter

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

② Truncate $h_d(n)$ at $n = \pm\left(\frac{N-1}{2}\right)$ to get the finite duration seq $h(n)$.

③ Find $H(z)$ using the eqn $H(z) = z^{-\left(\frac{N-1}{2}\right)}\left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)(z^{n} + z^{-n})\right]$

Design Using windows :-

① Same as Fourier series method.

② $h(n) = h_d(n)\, w(n)$ for $|n| \leq \frac{N-1}{2}$

↳ window sequence.

$= 0$ otherwise.

③ Same as F.S. method.

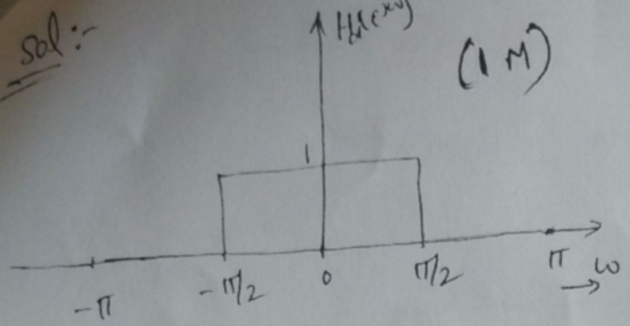
(15 marks)

$H_d(e^{j\omega}) = 1$ for $-\pi/2 \le \omega \le \pi/2$; $N = 11$

$= 0$ for $\pi/2 \le |\omega| \le \pi$

(3 Marks)

Sol :-



(1 M)

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} (1) e^{j\omega n}\, d\omega = \frac{1}{2\pi j n}\left(e^{j\omega n}\right)_{-\pi/2}^{\pi/2}$$

$$\boxed{h_d(n) = \frac{\sin n\pi/2}{n\pi}} \quad -\infty \le n \le \infty$$

symmetry is at $\alpha = \frac{N-1}{2} = \frac{11-1}{2} = 5$

(5 marks)

Truncating $h_d(n)$ ~~at~~ $|n| \le \frac{N-1}{2}$ to get $h(n)$

$h(n) = h_d(n)$ for $|n| \le 5$

$h(n) = \frac{\sin n\pi/2}{n\pi}$ for $|n| \le 5$

$h(0) = \lim_{n\to 0} \frac{1}{2} \frac{\sin n\pi/2}{n\pi/2} = \frac{1}{2} = 0.5$

$h(1) = h(-1) = \frac{\sin \pi/2}{\pi} = 0.3183$

$h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0$

$h(3) = h(-3) = \frac{\sin 3\pi/2}{3\pi} = -0.106$

$h(4) = h(-4) = \frac{\sin 2\pi}{4\pi} = 0$

$h(5) = h(-5) = \frac{\sin 5\pi/2}{5\pi} = \frac{1}{5\pi}$

$= 0.06366$

T.F of the filter is

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)(z^n + z^{-n})$$ (1 mark)

(5 marks)

$$H(z) = 0.5 + \sum_{n=1}^{5} h(n)(z^n + z^{-n})$$

$$H(z) = 0.5 + 0.3183(z^1 + z^{-1}) - 0.106(z^3 + z^{-3}) + 0.06366(z^5 + z^{-5})$$

T.F of Realizable filter

$$H'(z) = z^{-5} H(z)$$

$$H'(z) = 0.06366 - 0.106 z^{-2} + 0.318 z^{-4} + 0.5 z^{-5} + 0.3183 z^{-6} - 0.106 z^{-8} + 0.06366 z^{-10}$$

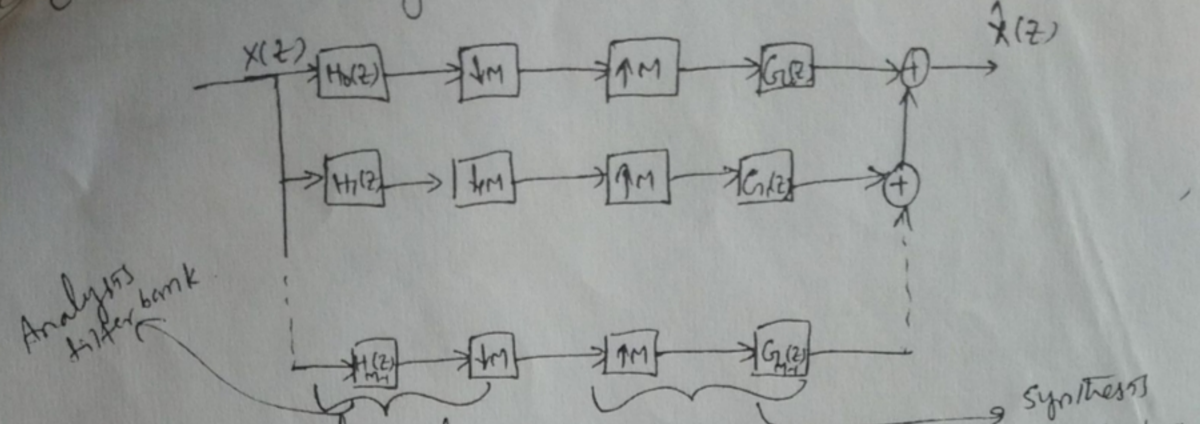
$h(0) = h(10) = 0.06366$
$h(1) = h(9) = 0$
$h(2) = h(8) = -0.106$
$h(3) = h(7) = 0$
$h(4) = h(6) = 0.3183$
$h(5) = 0.5$

filter Coefficients of causal filter

8

Appl^ns of MultiRate Signal Processing (8 Marks)

1. Subband coding of speech signals



2. In digital audio systems. the sampling rates of broadcasted signal, CD, DVD etc. are different. Hence to access signals from these devices, sampling rate converters are needed in digital audio systems.

3. In video broadcasting, The American standard NTSC and European standard PAL employ different sampling rates and sampling rate conversion is required. Hence to R_x both signals, sampling rate conversion is required.

4. Oversampling D/A and A/D converters for high quality digital audio systems and data loggers.

5. Digital transmultiplexers.

6. Narrowband FIR & IIR filters for various applications

7. Quadrature Mirror filter banks.

8(b) (7 Marks).

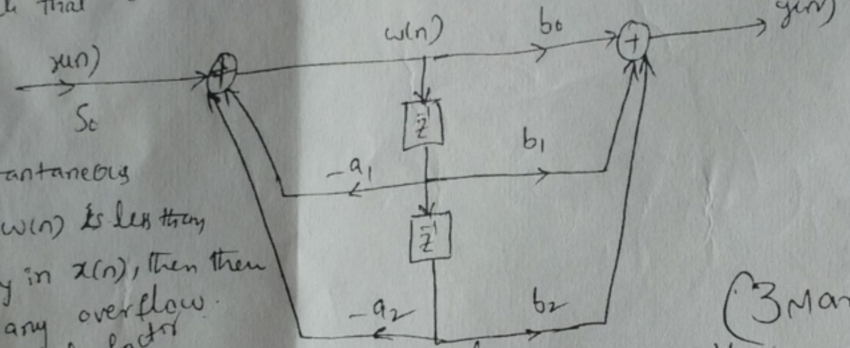
Scaling :-

(2 M)

Saturation arithmetic eliminates limit cycles due to overflow, but it causes undesirable signal distortion due to the non linearity of the clipper.

Inorder to limit the amount of non-linear distortion, it is important to scale the i/p signal & the unit sample response b/w the input any internal summing node in the system such that overflow becomes a rare event.

(2M)



If the instantaneous energy in w(n) is less than finite energy in x(n), then there will not any overflow.

(3 Marks)

Realization of 2nd order IIR filter

scale factor

$$H(z) = S_0 \frac{b_0+b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

$$= S_0 \frac{N(z)}{D(z)}$$

$$H'(z) = \frac{W(z)}{X(z)} = \frac{S_0}{1+a_1 z^{-1}+a_2 z^{-2}} = \frac{S_0}{D(z)}$$

$$W(z) = \frac{S_0 X(z)}{D(z)} = S_0 S(z) X(z)$$

$$\left(\because S(z) = \frac{1}{D(z)}\right)$$

$$w(n) = \frac{S_0}{2\pi}\int S(e^{j\theta}) X(e^{j\theta}) e^{jn\theta} d\theta$$

$$w^2(n) \le S_0^2 \left[\frac{1}{2\pi}\int_{2\pi} (S(e^{j\theta}))^2 d\theta\right]\left[\frac{1}{2\pi}\int_{2\pi} (X(e^{j\theta}))^2 d\theta\right]$$

$$z = e^{j\theta} \quad ; \quad dz = je^{j\theta} d\theta$$

$$d\theta = \frac{dz}{jz}$$

$$w^2(n) \le S_0^2 \sum_{n=0}^{\infty} x^2(n) \frac{1}{2\pi j}\oint_c S(z) S(z^{-1}) z^{-1} dz$$

we can obtain

$$w^2(n) \le \sum_{n=0}^{\infty} x^2(n).$$

when

$$S_0^2 \frac{1}{2\pi j}\oint_c S(z) S(z^{-1}) dz = 1$$

$$S_0^2 = \frac{1}{\frac{1}{2\pi j}\oint_c S(z) S(z^{-1}) z^{-1} dz}$$

$$\boxed{S_0^2 = \frac{1}{I}} \quad \left(\because I = \frac{1}{2\pi j}\oint_c S(z) S(z^{-1}) dz\right)$$