Consider a transmission line along the x axis as shown in and terminated with impedance $Z_R$ as shown in fig.

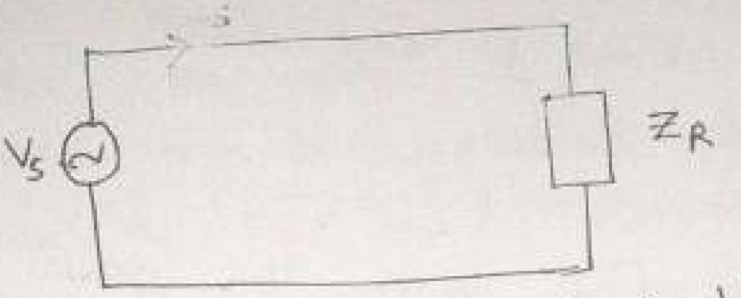



Fig: Transmission line terminated with $Z_R$.

Incident wave: When the source is applied on the line, the voltage and current components of the travelling wave decrease exponentially along the line with a decaying factor $e^{-\gamma x}$. This wave is called the incident wave.

Reflected wave: At the far end termination, due to impedance mismatch, the wave reflects back and travels in the opposite direction. This wave is called reflected wave. The voltage and current components of the reflected wave again decreases exponentially along the line in the negative x-direction with a decaying factor $e^{\gamma x}$.

Standing wave: The incident and reflected waves having the same frequency combined on the line results in standing waves.

The voltage and current distribution of standing wave at a given frequency is given by

$V = \quad e$

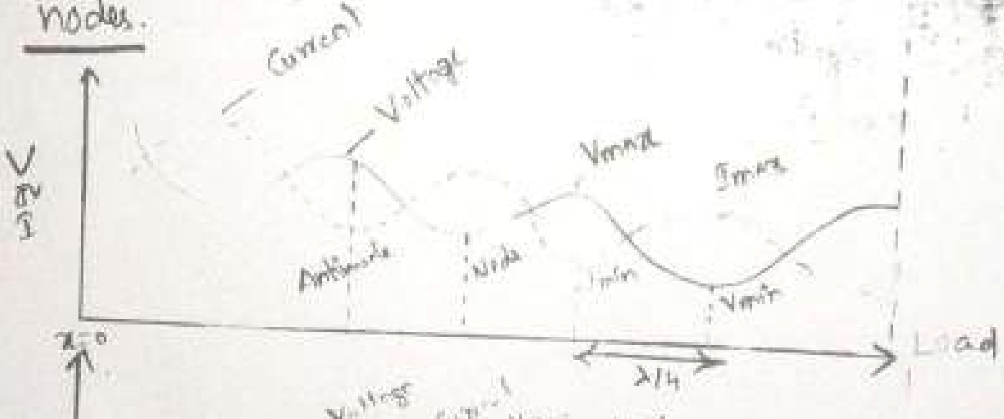
$I = \frac{1}{Z_0}\left(be^{-\gamma x} \quad ae^{\gamma x}\right)$

where $Z_c$ is characteristic impedan.

a and b are constants

Properties of standing waves:

1. In standing waves, two waves will travel in opposite directions between source and load.
2. At some points on the line, then two waves are in-phase, while at other points, they are out of phase.
3. The in-phase components added give maximum voltage at points called anti-nodes.
4. While the out-phase components subtracted give minimum voltage at other points called nodes.



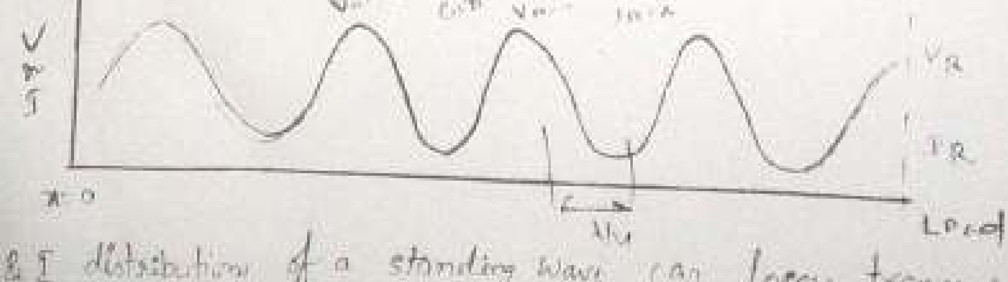




Fig. V & I distribution of a standing wave on lossy transmission line
(b) lossless transmission line

→ In any transmission li[illegible] the load [illegible] the [illegible] establishes the current [illegible] will [illegible] determines the [illegible] the input impedance.

→ Therefore, the ~~various~~ follow ways in which the voltage and current may be distributed along the line

(i) When the load end i.e terminating end is open

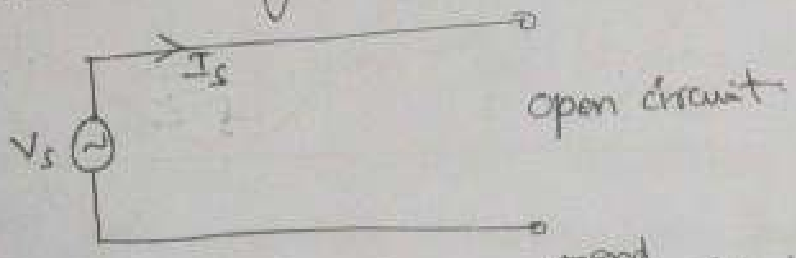
(ii) When the load end is shorted.

(iii) When the load is equal to its characteristic impedance.

## (i) Open circuited Lines :

→ The transmission line whose far end i.e terminating end is open is called open-circuited line

→ Consider a transmission line open at termination as shown in fig




→ The impedance at open ~~line~~ end will be infinite and no current will flow.

→ Thus, at the open end termination, the voltage becomes maximum and current becomes zero.

→ But at quarter wavelength ($\lambda/4$) distance from the termination, the incident wave is at 90° and the reflected wave is at −90°. Thus, both waves are 180° out of phase and voltage becomes minimum

Summary

→ The standi wave patte is repeated very half-wave lengths i.e maxima are spaced half-wavelength apart on the transmission line and minima are also spaced half-wavelength apart.

→ The distance between a maximum and minimum is a quarter wavelength.

→ The current maximum occurs at a point of minimum voltage and vice-versa.

→ The current and voltage distribution along the open-circuited line is shown in below fig.

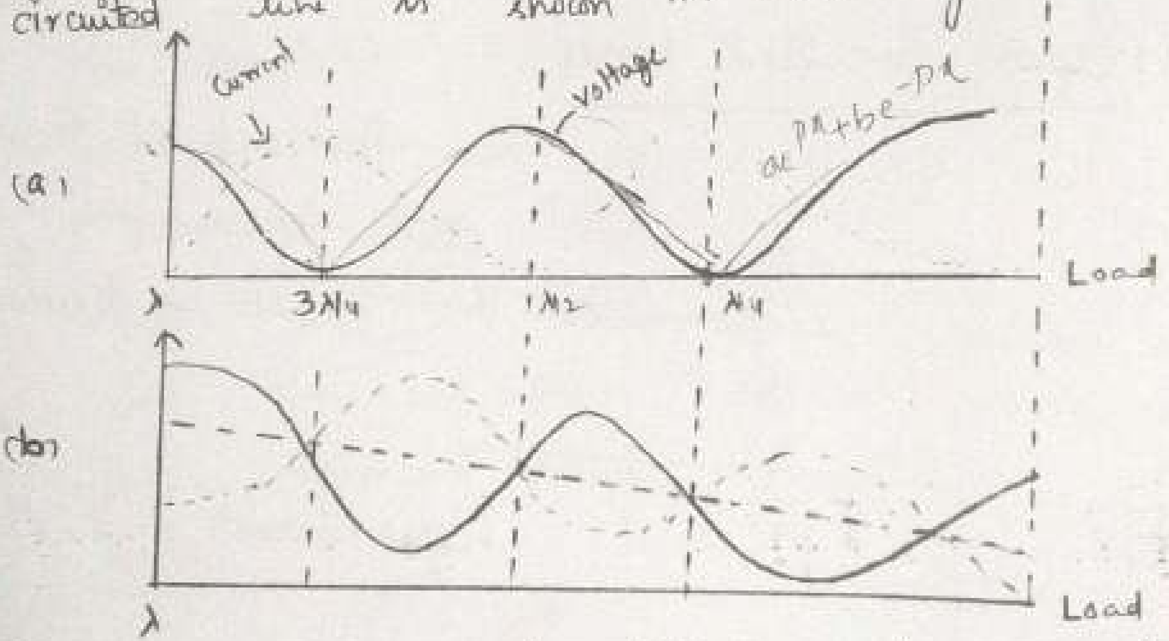



Fig: Voltage and Current distribution along open circuited line for (a) Lossless Line (b) Lossy line.

→ In a high-frequency lossless lines the values of the different maximum are equal as shown in fig (a).

→ However, in lossy line, they go on decreasing due to attenuation of the line as shown in fig(b).

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→ Short-circuited line is defined as transmission line whose far end is shorted.

→ Consider a transmission line short-circuited at termination as shown in below fig.

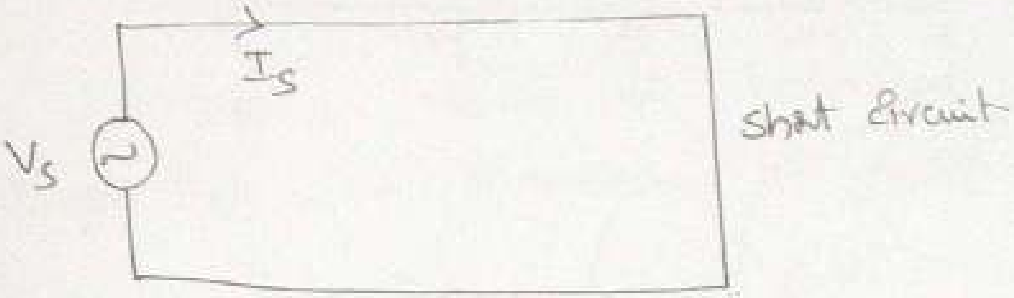



Fig: short-circuit line

## Standing Waves in Short-Circuited Lines

→ At short-circuit termination, since the impedance is zero, the current becomes maximum and voltage becomes zero.

→ The standing wave has a node or minimum at the short-circuited end and at every half-wavelength from the end.

→ At $\lambda/4$ distance from the termination, both incident and reflected waves are in-phase, so the voltage becomes maximum and the current becomes minimum.

→ Similarly, at every $\lambda/4$ distance, the voltage becomes minimum and maximum alternatively.

→ Voltage and current distributions of short-circuited lines differ from the open-circuited lines can only in the voltage and current are interchanged.

circuited lines are ... in fig.

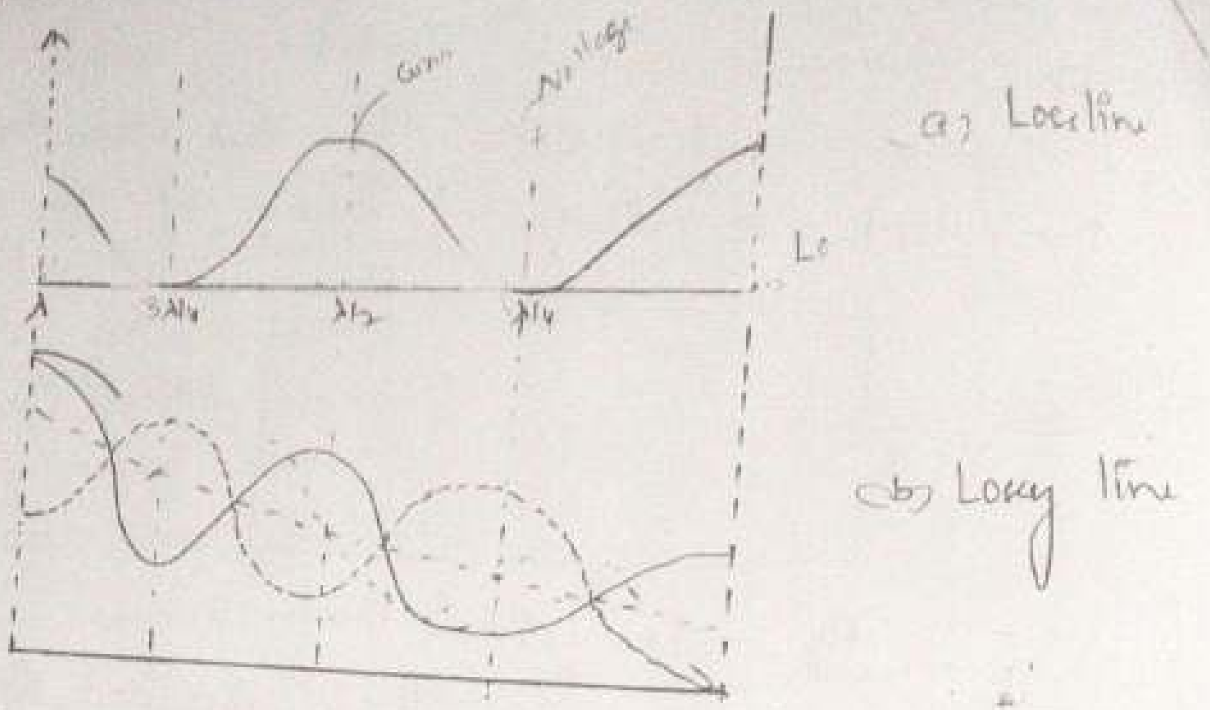



fig: Short - circuited Line

<u>Note</u> :-

1. It is observed that in an open - circuited line, the voltage becomes maximum and current becomes minimum at <u>even</u> multiples of $\lambda/4$ distance from the load towards the source.
2. Similarly, in short - circuited line, the voltage becomes maximum and current becomes minimum at <u>odd</u> multiples of $\lambda/4$ distance from the load toward the source.

Consider a transmission line of length $l$.

Let $V_S$ - voltage at the source

$I_S$ - Current at the source.

$V_R$ - voltage at termination

$I_R$ - Current at termination

We know that, the transmission line equations

$$V = V_S \cosh\gamma x - I_S Z_0 \sinh\gamma x \longrightarrow (1)$$

$$I = I_S \cosh\gamma x - \frac{V_S}{Z_0} \sinh\gamma x \longrightarrow (2)$$

At $x = l$, $V = V_R$ & $I = I_R$, then

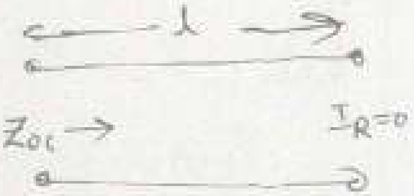
Eq$^n$ (1) & (2) becomes,

$$V_R = V_S \cosh\gamma l - I_S Z_0 \sinh\gamma l \longrightarrow (3)$$

$$I_R = I_S \cosh\gamma l - \frac{V_S}{Z_0} \sinh\gamma l \longrightarrow (4)$$

## Input impedance of open-circuited line :-

When the load is open, termination current becomes zero.




i.e $I_R = 0$

Now, Eq$^n$ (4) becomes

$$I_S \cosh\gamma l - \frac{V_S}{Z_0} \sinh\gamma l = 0$$

$$I_S \cosh\gamma l = \frac{V_S}{Z_0} \sinh\gamma l$$

$$Z_0 \frac{\cosh\gamma l}{\sinh\gamma l} = \frac{V_S}{I_S}$$

$I_s$

Let be the input impedance of the open

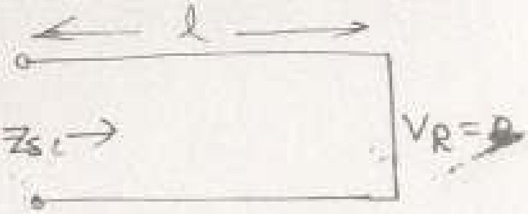
Then, $Z_{OC} = \frac{V_s}{I_s} = Z_0 \coth \gamma l$

$$\boxed{Z_{OC} = Z_0 \coth \gamma l} \longrightarrow (5)$$

## Input impedance of short-circuited line :-

When the load is shorted, the termination voltage becomes zero.

i.e $V_R = 0$




Now, Eq$^n$ (3) becomes,

$$V_S \cosh \gamma l - I_S Z_0 \sinh \gamma l = 0$$

$$V_S \cosh \gamma l = I_S Z_0 \sinh \gamma l$$

$$\frac{V_S}{I_S} = Z_0 \frac{\sinh \gamma l}{\cosh \gamma l}$$

$$\frac{V_S}{I_S} = Z_0 \tanh \gamma l$$

Let $Z_{SC}$ be the input impedance of short-circuit line

Then, $Z_{SC} = \frac{V_S}{I_S} = Z_0 \tanh \gamma l$

$$\therefore \boxed{Z_{SC} = Z_0 \tanh \gamma l}$$

When multiplying $Z_{oc}$ and $Z_{sc}$, we get

$$Z_{oc} \times Z_{sc} = Z_0 \coth \ldots$$

$$Z_{oc} Z_{sc} = Z_0^2$$

$$\boxed{Z_0 = \sqrt{Z_{oc} Z_{sc}}}$$

Thus, for any uniform and symmetrical line, the characteristic impedance is the geometric mean of the open and short circuited impedances.

When dividing $Z_{sc}$ by $Z_{oc}$, we get

$$\frac{Z_{sc}}{Z_{oc}} = \frac{\cancel{Z_0} \tanh \gamma l}{\cancel{Z_0} \coth \gamma l} = \tanh^2 \gamma l$$

$$\therefore \tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\therefore \boxed{\gamma = \frac{1}{l} \tanh^{-1}\left(\sqrt{\frac{Z_{sc}}{Z_{oc}}}\right)}$$

Transmission line

Consider a wave is applied on a transmission line. 2) the line of infinite length & if line is terminated with characteristic (Zo), then is no reflected wave.

Until that we have discussed three mode of termination viz

(1) line terminated its characteristic impedance.

(2) short - circuited termination.

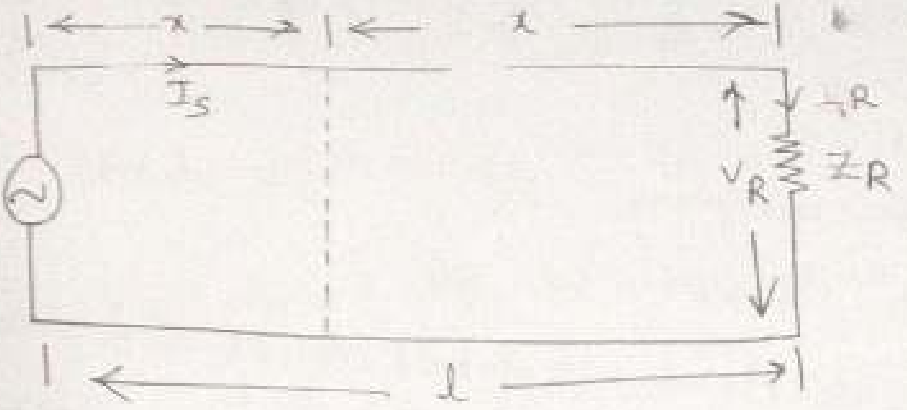
(3) open - circuited termination.

There will be another mode of termination i.e the termination $Z_R$, other than characteristic impedance of the line.

So, if the line is terminated with load impedance $Z_R$, some part of the energy will transfer to the load and the other part of it will be reflected back. Thus, standing waves appears on the line, resulting in loss of power.


For maximum power transfer to the load, the load impedance $Z_R$ should be complex conjugate of the source impedance.

Consider transmission of length $l$ terminating by with an impedance $Z$ as shown in fig.




Let $V_R$ be the voltage at $Z_R$ and $I_R$ be the current through $Z_R$.

The voltage and current at any point on the transmission line are given by

$$V = A\cosh\gamma x + B\sinh\gamma x \rightarrow (1)$$

$$I = -\frac{1}{Z_0}(B\cosh\gamma x + A\sinh\gamma x) \rightarrow (2)$$

where A and B are constants

At $x = l$, $V = V_R$ & $I = I_R$

then Eq$^n$ (1) & (2) becomes,

$$V_R = A\cosh\gamma l + B\sinh\gamma l \longrightarrow (3)$$

$$I_R = -\frac{A}{Z_0}\sinh\gamma l - \frac{B}{Z_0}\cosh\gamma l \longrightarrow (4)$$

Multiply Eq$^n$ (3) with $\cosh\gamma l$ & Eq$^n$ (4) with $\sinh\gamma l$

$$\cancel{V_R} \; V_R\cosh\gamma l = A\cosh^2\gamma l + B\sinh\gamma l\cosh\gamma l$$

$$Z_0 I_R\sinh\gamma l = -A\sinh^2\gamma l - B\cosh\gamma l\sinh\gamma l$$

$V_R \cosh\gamma l \quad Z_0 I_R \sinh = A(\cos^2 l - \sinh^2\gamma l$

$$A = V_R \cosh\gamma l + Z_0 I_R \sinh\gamma l \longrightarrow (5)$$

$(\because \cosh^2\gamma l - \sinh^2\gamma l = 1)$

Again multiplying Eq$^n$ (3) with $\sinh\gamma l$ and Eq$^n$ (4) with $\cosh\gamma l$, we get

$$V_R \sinh\gamma l = A \sinh\gamma l \cosh\gamma l + B \sinh^2\gamma l \longrightarrow (6)$$

$$Z_0 I_R \cosh\gamma l = -A \sinh\gamma l \cosh\gamma l - B \cosh^2\gamma l \longrightarrow (7)$$

Adding Eq$^n$ (6) & (7), we get

$$V_R \sinh\gamma l + Z_0 I_R \cosh\gamma l = -B(\cosh^2\gamma l - \sinh^2\gamma l)$$

$$\therefore B = -V_R \sinh\gamma l - Z_0 I_R \cosh\gamma l \longrightarrow (8)$$

Substituting the values of A and B in Eq$^n$ (1) & (2), we get

$$V = (V_R \cosh\gamma l + Z_0 I_R \sinh\gamma l)\cosh\gamma x - (V_R \sinh\gamma l + I_R Z_0 \cosh\gamma l)\sinh\gamma x$$

$$= V_R(\cosh\gamma l \cosh\gamma x - \sinh\gamma l \sinh\gamma x) + Z_0 I_R(\sinh\gamma l \cosh\gamma x - \cosh\gamma l \sinh\gamma x)$$

$$V = V_R \cosh\gamma(l-x) + Z_0 I_R \sinh\gamma(l-x) \longrightarrow (9)$$

$$I = -\frac{1}{Z_0}(B \cosh\gamma x + A \sinh\gamma x)$$

$$Z_0 I = -B \cosh\gamma x - A \sinh\gamma x$$

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$Z_0 -$

$- ( V_R \cosh\gamma \quad Z_0 I_R \sinh\gamma l \quad \sinh\gamma x$

$Z_0 I = V \sinh\gamma l \cosh\gamma x + Z_0 I_R \cosh \quad \cosh\gamma x$

$V_R \cosh\gamma l \sinh\gamma x - Z_0 I_R \sinh\gamma l \sinh\gamma x$

$Z_0 I = V_R (\sinh\gamma l \cosh\gamma x - \cosh\gamma l \sinh\gamma x)$

$+ Z_0 I_R (\cosh\gamma l \cosh\gamma x - \sinh\gamma l \sinh\gamma x)$

$Z_0 I = V_R \sinh\gamma(l-x) + Z_0 I_R \cosh\gamma(l-x)$

$$\boxed{I = I_R \cosh\gamma(l-x) + \frac{V_R}{Z_0} \sinh\gamma(l-x)} \longrightarrow (10)$$

Now rewrite Eq$^n$ (9)

$$\boxed{V = V_R \cosh\gamma(l-x) + Z_0 I_R \sinh\gamma(l-x)}$$

If $y = l-x$ is the distance from the load point, the voltage and current distribution at point y is

$$\boxed{\begin{aligned} V &= V_R \cosh\gamma y + Z_0 I_R \sinh\gamma y \quad \longrightarrow (11) \\ I &= I_R \cosh\gamma y + \frac{V_R}{Z_0} \sinh\gamma y \quad \longrightarrow (12) \end{aligned}}$$

Eq$^n$ (11) & (12) represents the general line equations of voltage and current at a point of distance y from the receiving end in terms of terminal voltage and current.

Calculat...

Input impedance of transmission line is defined the impedance measured across the input terminals of the transmission line.

It is normally denoted by $Z_{in}$

$$\therefore Z_{in} = \frac{V_S}{I_S}$$

At $x = l$, i.e. terminating end, the voltage and current distributions are

$$V = V_R \cosh \gamma l + Z_0 I_R \sinh \gamma l$$

$$I = I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l$$

At $x = 0$ or $y = l$, $V = V_S$ & $I = I_S$

$\therefore$ The input impedance is given by

$$Z_S = Z_{in} = \frac{V_S}{I_S} = \frac{V_R \cosh \gamma l + Z_0 I_R \sinh \gamma l}{I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l}$$

$$\boxed{Z_{in} = Z_0 \cdot \frac{V_R \cosh \gamma l + Z_0 I_R \sinh \gamma l}{V_R \sinh \gamma l + I_R Z_0 \cosh \gamma l}}$$

$Z_{in}$ in terms of $Z_R$:

$$Z_{in} = Z_0 \left[ \frac{\frac{V_R}{I_R} \cosh \gamma l + Z_0 \sinh \gamma l}{\frac{V_R}{I_R} \sinh \gamma l + Z_0 \cosh \gamma l} \right]$$

$$Z_{in} = Z_0 \left[ \frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_R \sinh \gamma l + Z_0 \cosh \gamma l} \right]$$

or
$$\boxed{Z_{in} = Z_0 \left[ \frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]}$$

If $Z_R = Z_0$ then $Z_{in} = Z_0$

Line Impedance :- Line impedance is the impedance at any point on the line.

We know that, the voltage and current distribution of line at any point distance $y$ from the load

$$V = V_R \cosh \gamma y + Z_0 I_R \sinh \gamma y$$

$$I = I_R \cosh \gamma y + \frac{V_R}{Z_0} \sinh \gamma y$$

$$\therefore Z = \frac{V}{I} = \frac{V_R \cosh \gamma y + Z_0 I_R \sinh \gamma y}{I_R \cosh \gamma y + \frac{V_R}{Z_0} \sinh \gamma y}$$

$$= Z_0 \left[ \frac{V_R \cosh \gamma y + Z_0 I_R \sinh \gamma y}{Z_0 I_R \cosh \gamma y + V_R \sinh \gamma y} \right]$$

$$= Z_0 \left[ \frac{\frac{V_R}{I_R} \cosh \gamma y + Z_0 \sinh \gamma y}{\frac{V_R}{I_R} \sinh \gamma y + Z_0 \cosh \gamma y} \right]$$

$$Z = \frac{\quad}{Z_0 \cosh \gamma y + Z_R \sinh \gamma y}$$

$$\boxed{Z = \frac{Z_R + Z_0 \tanh \gamma y}{Z_0 + Z_R \tanh \gamma y}}$$

Note:- The condition for a perfectly matched transmission line is

$$Z_{in} = Z_R = Z_R.$$

When the transmission line is perfectly matched, no standing wave exit.

**Calculation of input impedance of lossless line**

We know that

$$V = V_R \cosh \gamma l + Z_0 I_R \sinh \gamma l$$

$$I = I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l$$

For lossless line $\alpha = 0 \Rightarrow \gamma = j\beta$

$$\therefore V = V_R \cosh j\beta l + Z_0 I_R \sinh j\beta l$$

$$I = I_R \cosh j\beta l + \frac{V_R}{Z_0} \sinh j\beta l$$

$$\frac{V}{I} = \frac{V_R \cos \beta l + j Z_0 I_R \sin \beta l}{I_R \cos \beta l + j \frac{V_R}{Z_0} \sin \beta l}$$

$$Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}$$

Since $\beta = \frac{2\pi}{\lambda}$

$$Z_{in} = Z_0 \frac{Z_R + j Z_0 \tan \frac{2\pi l}{\lambda}}{Z_0 + j Z_R \tan \frac{2\pi l}{\lambda}}$$

we know that $\beta l = \frac{2\pi}{\lambda} l = \omega \sqrt{LC}\, l$

$$\therefore Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan(\omega l \sqrt{LC})}{Z_0 + jZ_R \tan(\omega l \sqrt{LC})}$$

where L and C are primary constant of the line

Input impedance of the line in terms of exponentials

$\cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$

$\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$

$$Z_{in} = Z_0 \left[ \frac{Z_R\left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_0\left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)}{Z_R\left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right) + Z_0\left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right)} \right]$$

$$= Z_0 \frac{(Z_0 + Z_R)e^{\gamma l} - (Z_0 - Z_R)e^{-\gamma l}}{(Z_0 + Z_R)e^{\gamma l} + (Z_0 - Z_R)e^{-\gamma l}}$$

$$= Z_0 \left( \frac{1 - \frac{Z_0 - Z_R}{Z_0 + Z_R} e^{-2\gamma l}}{1 + \frac{Z_0 - Z_R}{Z_0 + Z_R} e^{-2\gamma l}} \right)$$

$$Z_{in} = Z_0 \frac{1 + K e^{-\gamma l}}{1 - K_r e^{-2\gamma l}}$$

→ When the parameters of the line are not uniform along the line or the termination impedance is different from the characteristic impedance ($Z_R \neq Z_0$), the incident wave is reflected back at the load.

**Reflection:** The phenomenon of a wave being reflected at the load due to improper termination is called reflection.

→ The major problem with the reflection, so there is a significant power loss occurs along the line.

## Reflection coefficient (K):

Reflection coefficient is defined as the ratio of reflected voltage to the incident voltage or reflected current to the incident current.

* It is a vector quantity
* Generally, it is denoted by K.

Mathematically,

Reflection coefficient $\boxed{K = \frac{V_r}{V_i}}$

or $\boxed{K = \frac{-I_r}{I_i}}$

Here, negative sign indicates that, $I_r$ is in opposite direction to $I_i$.

According to voltage and current distribution at any point in transmission line is given by

$$V = a e^{\gamma x} + b e^{-\gamma x} \longrightarrow ①$$

$$I = \frac{1}{Z_0}\left( b e^{-\gamma x} - a e^{\gamma x} \right) \longrightarrow ②$$

If $y$ is the distance measured from the termination $Z_R$, then by substituting $\boxed{-y}$ in place of $x$ we get,

$$V = b e^{+\gamma y} + a e^{-\gamma y} \longrightarrow ③$$

$$I = \frac{1}{Z_0}\left[ b e^{\gamma y} - a e^{-\gamma y} \right] \longrightarrow ④$$

At load point, the first term is incident voltage / current

$$\therefore V_i = b e^{\gamma y} \quad \& \quad I_i = \frac{b}{Z_0} e^{\gamma y} \longrightarrow ⑤$$

and the second term is reflected voltage / current

$$V_r = a e^{-\gamma y} \quad \& \quad I_r = -\frac{a}{Z_0} e^{-\gamma y} \longrightarrow ⑥$$

To find constants :

The conditions at load are at $y = 0$, $V = V_R$ and $I = I_R$. Substituting these in Eq$^n$ ③ & ④ we get,

$V_R = b + a \rightarrow (8)$

$I_R = \frac{1}{Z_0}(b-a) \rightarrow$

Solving the above Equations

$2a = \frac{1}{2} V_R - I_R Z_0$

$a = \frac{1}{2}(V_R - I_R Z_0)$

$b = \frac{1}{2}(V_R + I_R Z_0)$

∴ The voltage reflection coefficient is

$$K = \frac{V_r}{V_i} = \frac{a e^{-\gamma y}}{b e^{\gamma y}} = \frac{a}{b} e^{-2\gamma y}$$

At the termination, $y=0$, then $K = \frac{a}{b}$

$$K = \frac{\frac{1}{2}(V_R - I_R Z_0)}{\frac{1}{2}(V_R + I_R Z_0)} = \frac{\frac{V_R}{I_R} - Z_0}{\frac{V_R}{I_R} + Z_0}$$

But, we know that $Z_R = \frac{V_R}{I_R}$,

∴ The Reflection coefficient $\boxed{K = \frac{Z_R - Z_0}{Z_R + Z_0}}$

So, the voltage reflection coefficient $K$ is completely dependent only on load impedance and characteristic impedance.

$K$ is a complex quantity

$\boxed{-1 \leq K \leq 1}$

For matche termination i.e $Z_R = Z$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_R - Z_R}{Z_R + Z_R}$$

$$\boxed{K = 0}$$

It shows that the reflected wave is zero.

Also, $V_R = V_S$ or $V_S = Z_0 I_S$ & $V_R = Z_0 V_R$

(b) For short circuited termination i.e $Z_R = 0$:

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{-Z_0}{Z_0} = -1$$

$$\therefore \boxed{K = -1}$$

It shows that the entire incident wave reflected back with $180^\circ$ phase shift.

Also $V_R = 0$ & $I_R = \frac{2V_S}{Z_0}$

(c) For Open circuited termination i.e $Z_R = \infty$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{1 - Z_0/Z_R}{1 + Z_0/Z_R} = \frac{1-0}{1+0}$$

$$\boxed{K = 1}$$

It shows that the entire incident wave reflected back with the same phase.

So, $V_R = 2V_S$ & $I_R = 0$

Reflection -

Reflection ... is the p... loss due refl...

So, Reflection loss is defined as the ratio of ... power to the incident power. It is expressed in decibles

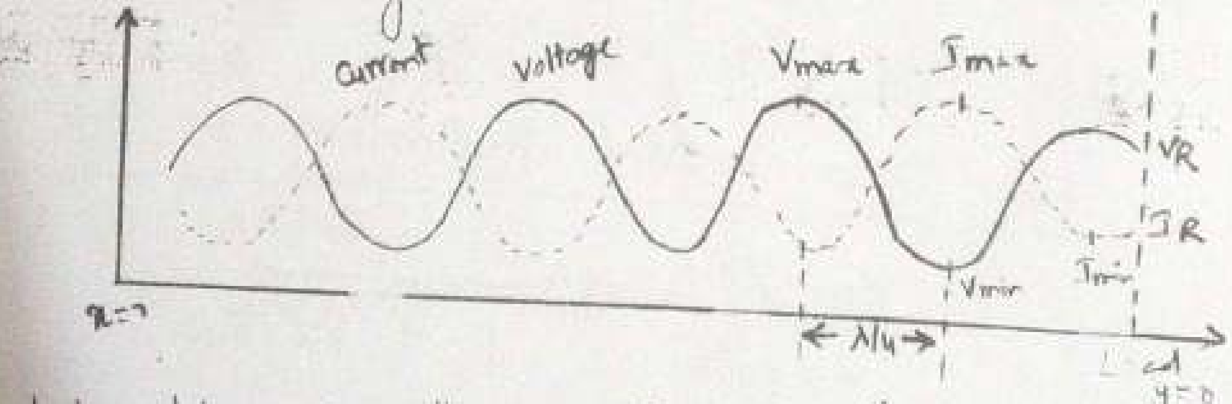
$$K_l = 10 \log\left[\frac{\text{Load Power}}{\text{Incident Power}}\right] = 20 \log\left|\frac{1}{K_f}\right|$$

Where $K_f$ is the reflection factor given by

$$\boxed{K_f = \sqrt{1 - K^2} = \frac{2\sqrt{Z_0 Z_R}}{Z_R + Z_0}}$$

## Standing Wave Ratio

When the lossless line is not terminated with characteristic impedance, the combination of incident wave and reflected wave gives rise to standing waves as shown in fig.




Let $V_{max}$ be the maximum voltage

$V_{min}$ be the minimum voltage

$I_{max}$ be the maximum current

$I_{min}$ be the minimum current

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points $\lambda/2$.

The maximum values occur wh... and reflected ... .

i.e $|V_{max}| = |V_r| + |V_i| \longrightarrow (1)$

& $|I_{max}| = |I_r| + |I_i| \longrightarrow (2)$

The minimum values occur when the incident and reflected waves are subtracted.

i.e $|V_{min}| = |V_i| - |V_r| \longrightarrow (3)$

$|I_{min}| = |I_i| - |I_r| \longrightarrow (4)$

* **Voltage standing Wave Ratio (VSWR):**

The ratio of the maximum magnitude of the voltage to the minimum magnitude of the voltage is called voltage standing wave ratio. It is abbreviated as VSWR and denoted by S.

$$\boxed{VSWR = S = \frac{|V_{max}|}{|V_{min}|}}$$

**Current standing Wave Ra... (ISWR):**

The ratio of maximum current to the minimum current value is called current standing wave ratio. It is abbreviated as ISWR | CSWR

$$\boxed{ISWR = \frac{|I_{max}|}{|I_{min}|}}$$

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## VSWR ......

We know that

$$S = \frac{|V_{max}|}{|V_{min}|}$$

$$S = \frac{|V_r| + |V_i|}{|V_i| - |V_r|} = \frac{1 + \left|\frac{V_r}{V_i}\right|}{1 - \left|\frac{V_r}{V_i}\right|}$$

— But we know that

$$K = \frac{V_r}{V_i} \quad \text{or} \quad |K| = \left|\frac{V_r}{V_i}\right|$$

$$\boxed{S = \frac{1+|K|}{1-|K|}}$$

or $$\boxed{|K| = \frac{S-1}{S+1}}$$

∴ VSWR is a real quantity.

It is always greater than 1.

$$\boxed{1 \leq S < \infty}$$

**Note:**

1. If the line is perfectly matched, $Z_R = Z_0$,

then $K = 0$

$$S = \frac{1+K}{1-K} = \frac{1+0}{1-0} = 1$$

Hence, there is no reflection.

then $S = \frac{1+|K|}{1-|K|} = \frac{1+1}{1-1} = \infty$

i.e when the ... is completely reflected back, VSWR becomes infinity.

3. The range of reflection coefficient & VSWR is

$$\boxed{-1 \leq K \leq 1}$$

$$\boxed{0 < S \leq \infty}$$

4. VSWR is more preferable compared to the current standing wave ratio (ISWR) because vivn at different points on the line can be easily measured by using voltmeter. It is very difficult to measure current on the line.

5. The maximum and minimum positions of the standing wave are separated by a distance of $\lambda/4$.

6. The consecutive two maxima or two minima of standing wave are separated by a distance of $\lambda/2$.

We know that the line impedance at a point $y$ on the lossless line is

$$Z = Z_0 \frac{1 + K e^{-j2\beta y}}{1 - K e^{-j2\beta y}}$$

The line impedance is complex, but for lossless line the impedances at $V_{max}$ and $V_{min}$ positions are always real;

<u>Impedance Maxima ($Z_{max}$):</u> At voltage maximum (anti-node position), the line impedance is called impedance maxima, $Z_{max}$.

It can be expressed as

$$Z_{max} = \frac{V_{max}}{I_{min}} = \frac{V_{max}}{V_{min}/Z_0} = Z_0 \left|\frac{V_{max}}{V_{min}}\right|$$

$$\boxed{Z_{max} = Z_0 S} \quad \left( S = \left|\frac{V_{max}}{V_{min}}\right| \right)$$

Hence, the impedance maxima $Z_{max}$ at any point on the line is the product of $Z_0$ & $S$.

<u>Impedance Minima, ($Z_{min}$):</u> At voltage minimum (node position), the line impedance is called impedance minima, $Z_{min}$.

It can be expressed as $Z_{min} = \frac{V_{min}}{I}$

$$Z_{min} = \frac{Z_0}{S}$$

Thus, the impedance on the line varies from maximum value $Z_{max} = Z_0 S$ at the anti-node to minimum value $Z_{min} = \frac{Z_0}{S}$ at the node point.

## * Input impedance of Lossless SC and OC lines:

We know that input impedance of short-circuited (SC) and open-circuited (OC) lines are

$$Z_{SC} = Z_0 \tanh \gamma l$$

$$Z_{OC} = Z_0 \coth \gamma l$$

For lossless/HF lines transmission line, $\alpha = 0$

then $\gamma = \alpha + j\beta = j\beta$

$$\boxed{Z_{SC} = Z_0 \tanh j\beta l = j Z_0 \tan\beta l}$$

$$\boxed{Z_{OC} = Z_0 \coth j\beta l = -j Z_0 \cot\beta l}$$

Hence, $Z_0$ is resistive, the input impedance for both open & short circuited lossless lines is pure reactive. Depending on the length, the transmission line can provide either a capacitive or an inductive effect.

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Let $l = \lambda/8$, then $\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$

$\therefore Z_{sc} = j Z_0 \tan(\pi/4) = j Z_0$ (inductive)

& $Z_{oc} = -j Z_0 \cot(\pi/4) = -j Z_0$ (capacitive)

$\therefore$ the quarter wavelength $(0 < l < \lambda/4)$

SC lines acts as an inductive

OC lines acts as capacitive.

Case (b): $\lambda/4 < l < \lambda/2$

Let $l = \lambda/3$, then $\beta l = \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{3} = \frac{2\pi}{3}$

$\therefore Z_{sc} = j Z_0 \tan\left(\frac{2\pi}{3}\right) = -j\sqrt{3} Z_0$ (capacitive)

& $Z_{oc} = -j Z_0 \cot\left(\frac{2\pi}{3}\right) = j\sqrt{3} Z_0$ (inductive)

Therefore, for $(\lambda/4 < l < \lambda/2)$,

the SC lines acts as capacitive.

OC lines acts as an inductive.

Case (c): $l = \lambda/4$

$\beta l = \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{4} = \frac{\pi}{2}$

$Z_{sc} = j Z_0 \tan(\pi/2) = \pm\infty$

& $Z_{oc} = -j Z_0 \cot(\pi/2) = 0$

$\therefore$ for $l = \lambda/4$, SC line acts as open circuit

OC line acts as short circuit.

$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$

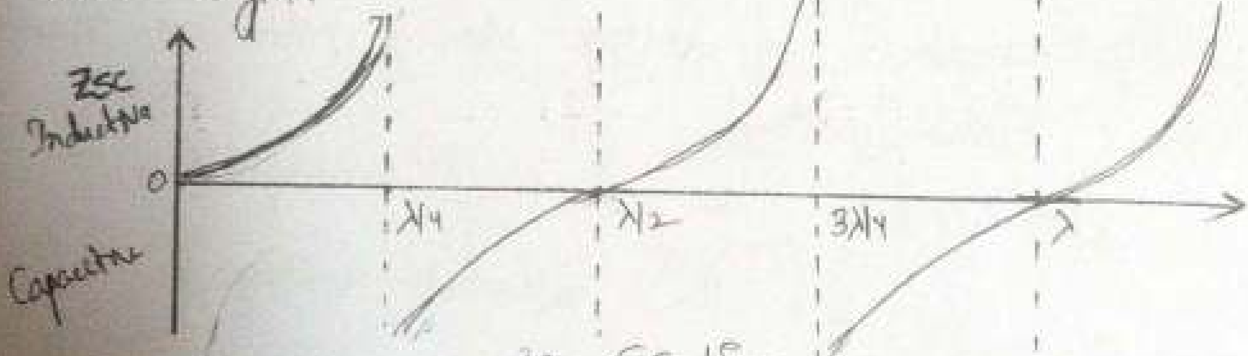
$Z_{sc}$ ... $\tan(\pi)$ ...

$\therefore l = \lambda/2$, the SC line acts as short ckt,

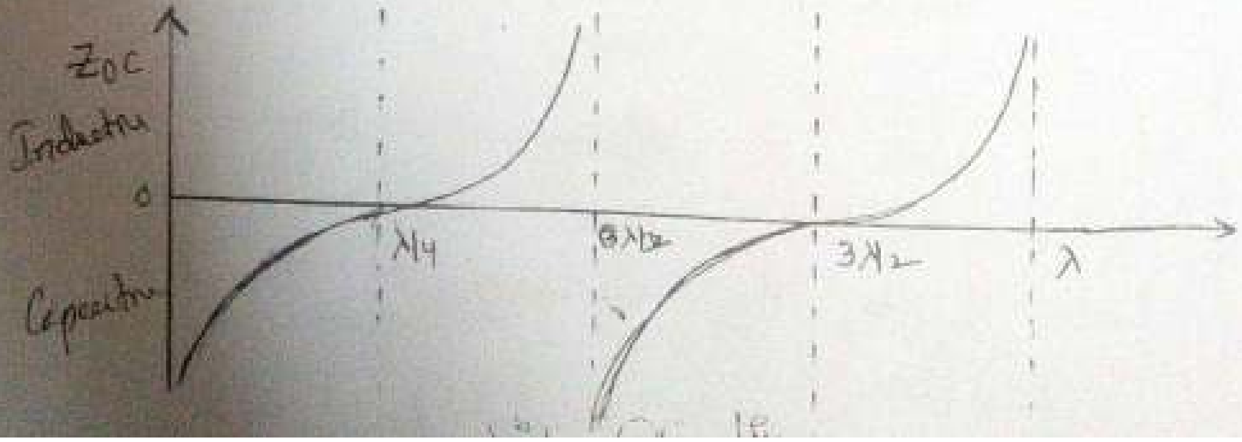
OC line acts as open ckt.

Conclusion is that, after each quarter wavelength of the line, the nature of reactance reverses. The same reactances values repeat every half wave length distance.

The below fig shows the variation of input impedance for lossless SC & OC lines with length.




i) SC Line



Consider a lossless transmission line + nat... with load impedance $Z_R \neq Z_0$. When the ... is incident, standing waves exist on the line with definite maxima and minima of voltage along the line as shown in fig. When the line is lossless at high frequency, the attenuation of the wave is zero.

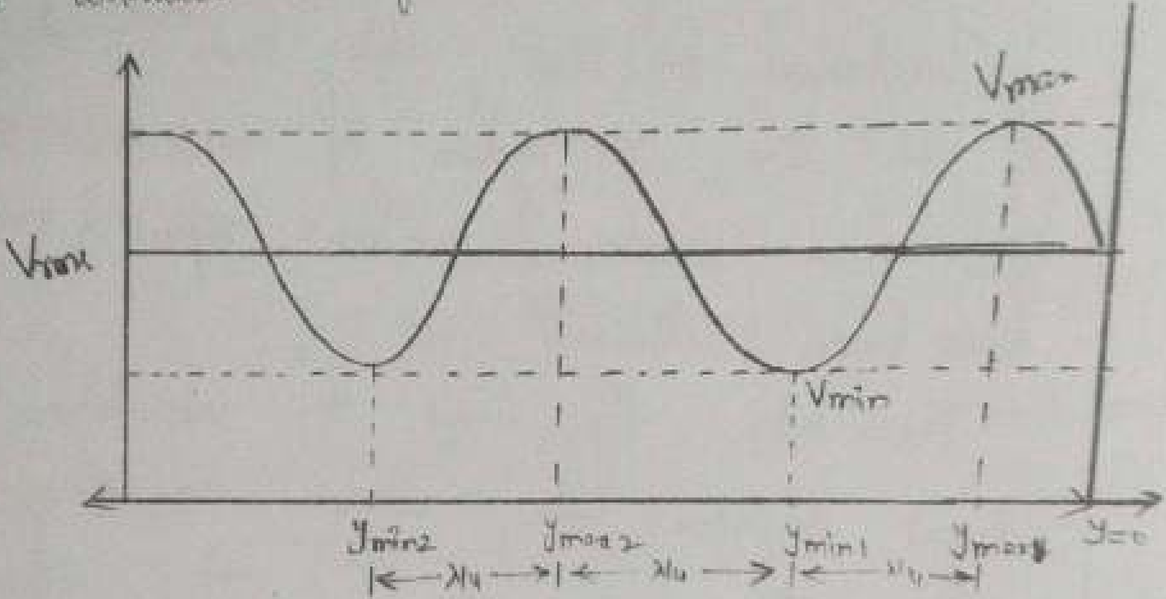



Fig: Location of voltage Max & Min on standing wave pattern.

Let $V_i$ be the incident voltage

$V_r$ be the reflected voltage

$V_{max}$ = Maximum voltage of the standing wave.

$V_{min}$ = Minimum voltage of the standing wave.

Let $y$ be the distance of the line from termination line from termination is,

$$V = b e^{\gamma} + a e^{-\gamma y}$$

Since, the line is lossless, $\gamma = j\beta$

$$V = b e^{j\beta y} + a e^{-j\beta y} \longrightarrow (1)$$

The reflected and incident voltages are

$$V_r = a e^{-j\beta y} \quad \& \quad V_i = b e^{j\beta y} \longrightarrow (2)$$

Then, the reflection coefficient K is

$$K = \frac{V_r}{V_i} \quad (or) \quad K = |K| e^{j\phi} = \frac{a e^{-j\beta y}}{b e^{j\beta y}}$$

where $\phi$ is the phase angle of the reflection coefficient.

At the termination, $y = 0$

$$\therefore |K| e^{j\phi} = \frac{a}{b}$$

$$a = b |K| e^{j\phi} \longrightarrow (3)$$

Substitute this a value in eqn (1), we get,

$$V = b e^{j\beta y} + b |K| e^{j\phi} e^{-j\beta y}$$

$$= b e^{j\beta y} \left(1 + |K| e^{-j(2\beta y - \phi)}\right)$$

$$V = b \left(1 + |K| e^{-j(2\beta y - \phi)}\right) e^{j\beta y} \longrightarrow (4)$$

Taking the magnitude of the voltage

$$|V| = |b| \left|1 + |K| e^{j(2\beta y - \phi)}\right| \rightarrow 4(a)$$

... voltages are in
incident and reflected
even
2 x integer multiple of π

$$\text{At } y = y_{max},\quad 2\beta y_{max} - \phi = 2n\pi$$

where n is an integer n = 0, 1, 2, 3 ...

$$\therefore \boxed{y_{max} = \frac{1}{2\beta}(2n\pi + \phi)} \longrightarrow (6)$$

Substituting in Eq$^n$ (4)(a), the maximum amplitude of the voltage is

$$|V_{max}| = |b|\left(1 + |K| e^{-j2n\pi}\right)$$

$$\therefore \boxed{|V_{max}| = |b|\,[1 + |K|]} \quad \left(\because e^{-j2n\pi} = 1\right) \longrightarrow (7)$$

Similarly, the minimum voltage occurs when both the incident and reflected voltages are out-of-phase or odd integer multiple of π.

$$\text{At } y = y_{min}$$

$$2\beta y_{min} - \phi = (2n+1)\pi$$

$$\boxed{y_{min} = \frac{1}{2\beta}\left[(2n+1)\pi + \phi\right]} \longrightarrow (8)$$

Substituting in Eq$^n$ (4)(a), the minimum amplitude of the voltage is

$$\boxed{|V_{min}| = |b|\,[1 - |K|]} \longrightarrow (9)$$

$$\frac{V_{min}}{}\qquad \frac{}{i\sqrt{[1-|K|]}}$$

or

$$S = \frac{1+|K|}{1-|K|} \longrightarrow (10)$$

Finding of the position of first maxima and minima from the termination :-

For finding position of the first maxima i.e $y_{max1}$, put $n=0$ in Eq$^n$ (6), we get

$$y_{max1} = \frac{\phi}{2\beta} = \frac{\phi\lambda}{2\times 2\pi} = \frac{\phi\lambda}{4\pi}$$

$$\boxed{y_{max1} = \frac{\phi\lambda}{4\pi}}$$

first minima, $y_{min1}$, put $n=0$ in Eq$^n$ (8)

$$y_{min1} = \frac{1}{2\beta}(\phi+\pi)$$

$$y_{min1} = \frac{\lambda}{2\times 2\pi}(\phi+\pi) = \frac{\phi}{\pi}\frac{\lambda}{4}+\frac{\lambda}{4}$$

$$\boxed{y_{min1} = y_{max1}+\frac{\lambda}{4}}$$

Note :- For resistive load, if $Z_L > Z_0$, the first voltage maximum occurs near the load, & if $Z_L < Z_0$, the first voltage minimum occurs near the load.

a "If the load impedance not equal the complex conjugate of the line impedance the transmission line is added a short length of transmission line to the line to achieve maximum power transfer. This is called impedance transformation or impedance matching device."

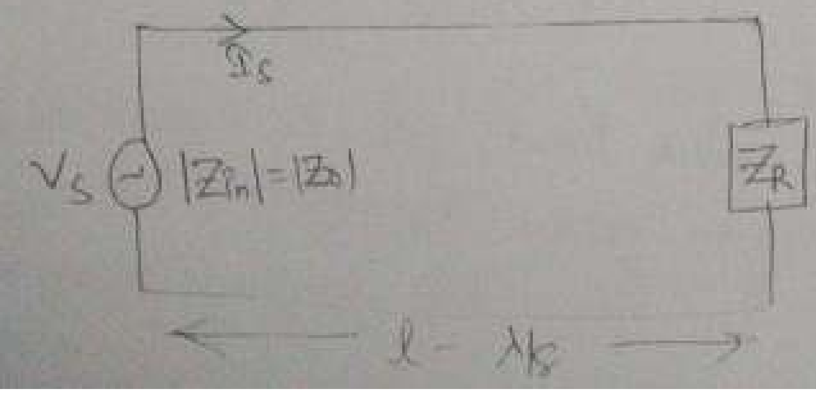
The following short length transmission lines used in impedance matching device

1. The eighth wave ($\lambda/8$ length) transmission line
2. The quarter wave ($\lambda/4$ length) " "
3. The Half wave ($\lambda/2$ length) " "

1. Eighth wave ($\lambda/8$ length) transmission line:

The length of the eighth wave transmission line is $\lambda/8$, where $\lambda$ is the wave length.

Let us consider a $\lambda/8$ length transmission line terminated with impedance $Z_R$ and characteristic impedance $Z_0$ as shown in fig



transmission line

$$Z_{in} = Z_0 \left[ \frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$$

For lossless line, $\alpha = 0$, $\gamma = j\beta$

$$Z_{in} = Z_0 \left[ \frac{Z_R + Z_0 \tanh j\beta l}{Z_0 + Z_R \tanh j\beta l} \right]$$

$$Z_{in} = Z_0 \left[ \frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l} \right]$$

For length $l = \lambda/8$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan(\pi/4)}{Z_0 + jZ_R \tan(\pi/4)}$$

$$Z_{in} = Z_0 \left[ \frac{Z_R + jZ_0}{Z_0 + jZ_R} \right]$$

$$|Z_{in}| = |Z_0| \left| \frac{Z_R + jZ_0}{Z_0 + jZ_R} \right|$$

$$\therefore |Z_{in}| = |Z_0| \cdot 1$$

$$\boxed{|Z_{in}| = |Z_0|}$$

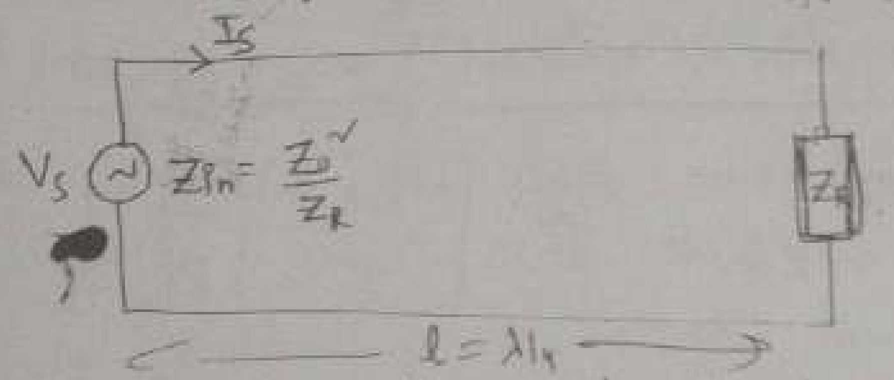
equal the magnitude of the characteristic impedance and is independent of load impedance ($Z_R$).

* Thus a $\lambda/2$ length transmission line is used to transform any impedance ($Z_R$) to a magnitude of characteristic impedance.

## 2. Quarter wave ($\lambda/4$ length) transmission Line:

Consider a $\lambda/4$ length transmission line with load impedance $Z_R$ and characteristic impedance $Z_0$ as shown in fig.

This $\lambda/4$ length transmission line is also called a quarter wave transformer.




We know that, the input impedance of a distortionless / lossless line is

$$Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}$$

For length $l = \lambda/4$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\frac{Z_0}{\tan\beta l} - jZ_R$$

Substitute $\beta l = \frac{\pi}{2}$ in

$$Z_{in} = Z_0 \frac{jZ_0}{jZ_R}$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_R}}$$

(or) $$\boxed{Z_0 = \sqrt{Z_{in} \cdot Z_R}}$$

Thus, a $\lambda/4$ length line is considered as a transformer that matches a load of $Z_R$ to a source impedance of $Z_{in}$.

The transformer characteristic impedance is equal to the geometric mean of the source and load impedance.

* **Applications of a $\lambda/4$ line transformer:**

1. To match the impedance between a transmission line and an antenna.

For a example, if a line with characteristic impedance $Z_0$ is connected to a main transmission line, ~~then~~ if there is mismatch occurs, then a quarter wave transformer is inserted in between the line and load.

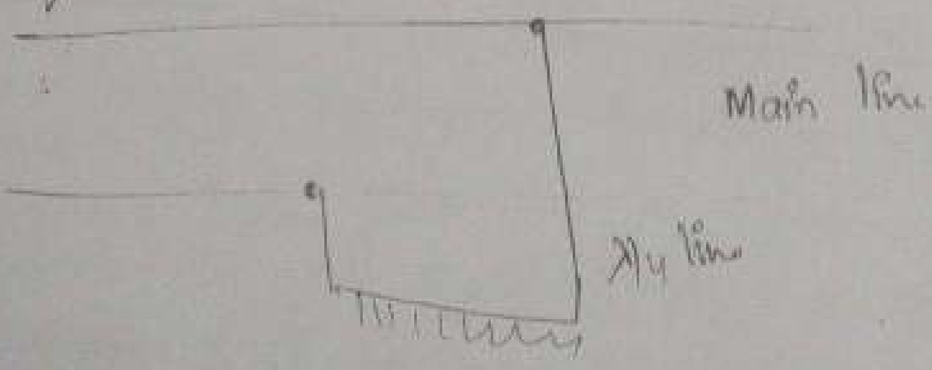
The quarter ... having a characteristic impedance

$$Z_0 = \sqrt{Z_L \times R_{IN}}$$


Fig: A quarter wave transformer for matching.

2. To step up or step-down the characteristic impedance $Z_0$ of transmission line. Hence a line of quarter-wave length acts as an impedance converter.

3. It can provide a mechanical support to the transmission line in addition to the impedance. For example, the line connected between the transmission line and the ground act as an insulator at the point of contact as shown in fig




Disadvantage: It is sensitive to change in frequency, for a new wavelength the section will no longer be $\lambda/4$ in length.

The length of the half wave transmission line is $\lambda/2$.

We know that, the input impedance of a lossless transmission line is,

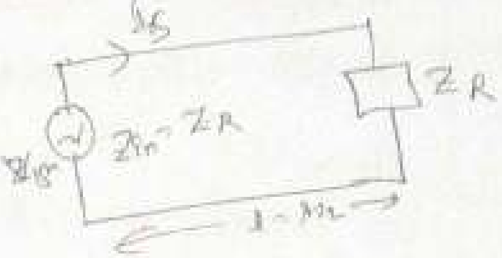
$$Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}$$

For length $l = \lambda/2$,

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$\therefore Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan\pi}{Z_0 + jZ_R \tan\pi}$$

$$Z_{in} = \cancel{Z_0} \frac{Z_R}{\cancel{Z_0}}$$




$$\boxed{Z_{in} = Z_R}$$

Thus, the input impedance of a half wave line is equal to its termination impedance.

Application: If the load and source cannot be made adjacent, a half wavelength line may be connected at the load point for accurate measurements.

* ... UHF line is terminated ...

→ When a UHF line is [illegible] impedance which is not equal to the characteristic impedance of the line mismatch occurs.

→ Mismatch reduces efficiency and increases power loss.

→ To avoid mismatching, it is necessary to add impedance matching device between the load and the line.

→ A quarter wave transformer ($\lambda/4$ length line) can be used to achieve impedance matching.

→ So we have to cut the line to insert a transformer in between the line and the load.

→ The other method is to use open or short circuited line as a matching device, which can be connected in parallel to the line at a certain distance from the load. This matching device is called stub matching.

<u>Stub</u> :- The short lengths of OC or SC line is called stub.

<u>Advantages of stub matching</u>:

1. The length & characteristic impedance of the line remains the same.
2. Since, the stub is added in shunt, there is no need to cut the line.

for perfect matching.

**ethods of stub Match:**

1. Single stub matching.

2. Double stub matching.

## 1. Single stub matching :

→ In this method, to achieve impedance matching an open and short circuited short length transmission line connected in parallel to the main line at a certain distance from the load.

→ Since, the stub is connected in parallel, it is easy to use admittance instead of impedance for analysis.

→ We know that, when the load admittance $Y_R (= \frac{1}{Z_R})$ is not equal to the characteristic admittance $Y_0$ (i.e $Y_R \neq Y_0$), a mismatch occurs. Hence, standing wave exist on the line.

→ When we move from the load towards the source, the admittance on the line varies from the max/min value to the min/max value depending on the line length. This admittance variation repeats after every $\lambda/2$ length.

→ At some point on the line, the real part of the admittance is equal to the characteristic admittance

i.e $Re[Y] = Re[Y_0]$.

characteristic admittance ($Y_0$) is added to the admittance at this point is matched to the load and stub combination.

→ To avoid losses, the stub should be connected as near the load as possible

Design Procedure :-

1. Consider a transmission line terminated with load admittance $Y_R$. Let a short circuited stub of length $l_t$ at a distance $l_s$ from the load be connected to the line as shown in below fig

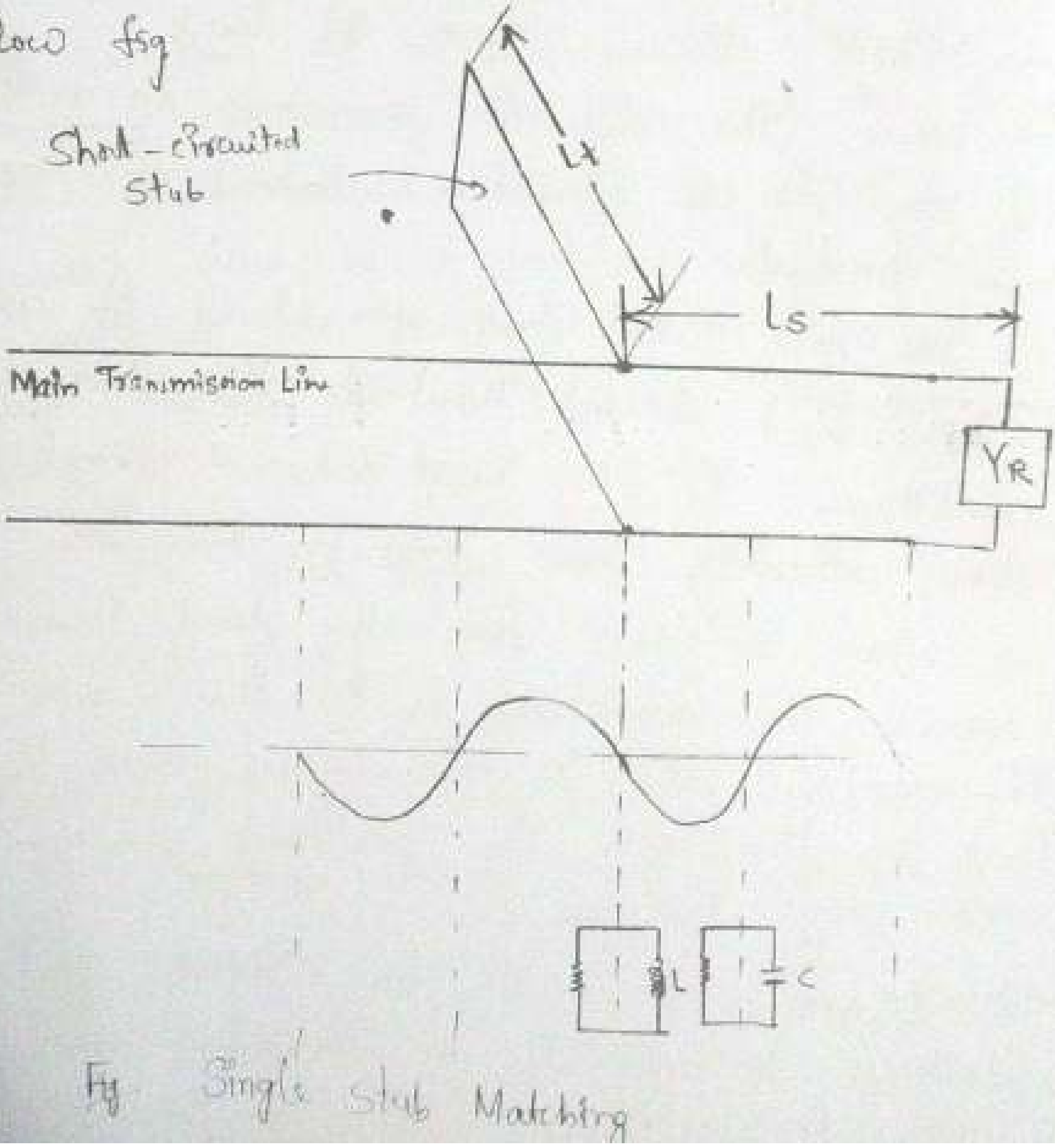



Fig Single stub Matching

We know that the input impedance at any point on the line is

$$Z_{in} = Z_0 \frac{Z_R \cosh\gamma y + Z_0 \sinh\gamma y}{Z_0 \cosh\gamma y + Z_R \sinh\gamma y}$$

$$= Z_0 \frac{Z_R + Z_0 \tanh\gamma y}{Z_0 + Z_R \tanh\gamma y}$$

where y is the distance from the load.

At UHF for lossless line, $\alpha = 0$, $\gamma = j\beta$

$$\therefore Z_{in} = Z_0 \frac{Z_R + Z_0 \tanh j\beta y}{Z_0 + Z_R \tanh j\beta y}$$

$$Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan\beta y}{Z_0 + jZ_R \tan\beta y} \Rightarrow (1)$$

Converting the impedance into admittance,

$$Y_{in} = \frac{1}{Z_{in}} = Y_0 \frac{Y_R + jY_0 \tan\beta y}{Y_0 + jY_R \tan\beta y} \Rightarrow (2)$$

Since, the load is variable, it has to be normalized.

Normalized load is $y_r = \frac{Y_R}{Y_0}$

Normalised

$$y_{in} = \frac{Y_{in}}{Y_0} = \frac{\frac{Y_R}{Y_0} + j\tan\beta y}{1 + j\frac{Y_R}{Y_0}\tan\beta y}$$

$$y_{in} = \frac{y_R + j\tan\beta y}{1 + j y_R \tan\beta y} \rightarrow (3)$$

Let $Y_R$ and $Y_0$ be real, Separating real & imaginary terms in $y_{in}$

$$\therefore y_{in} = \left(\frac{y_R + j\tan\beta y}{1 + j y_R \tan\beta y}\right)\left(\frac{1 - j y_R \tan\beta y}{1 - j y_R \tan\beta y}\right)$$

$$= \frac{y_R + y_R\tan^2\beta y - j y_R^2 \tan\beta y + j\tan\beta y}{1 + y_R^2\tan^2\beta y}$$

$$\therefore y_{in} = \frac{y_R(1+\tan^2\beta y)}{1+y_R^2\tan^2\beta y} + j\frac{(1-y_R^2)\tan\beta y}{1+y_R^2\tan^2\beta y} \rightarrow (4)$$

For no reflection, at a distance $y = l_s$, the real part of the normalised admittance should be equal to unity.

i.e $\quad Re[Y_{in}] = Y_0 \quad \& \quad Re[y_{in}] = 1.$

$$\therefore \text{ at } y = l_s, \quad \frac{y_R(1+\tan^2\beta l_s)}{1+y_R^2\tan^2\beta l_s} = 1$$

& $\quad (y_R + y_R\tan^2\beta l_s) = 1 + y_R^2\tan^2\beta l_s$

$$\tan^2 \beta l_s = \frac{1 - \cancel{y_R}}{y_R(1 - \cancel{y_R})} = \frac{1}{y_R}$$

$$\tan \beta l_s = \frac{1}{\sqrt{y_R}} = \sqrt{\frac{Y_0}{Y_R}} \quad (\because y_R = \frac{Y_R}{Y_0})$$

$$\boxed{l_s = \frac{1}{\beta} \tan^{-1} \sqrt{\frac{Y_0}{Y_R}}} \longrightarrow (5)$$

Therefore, the location of the short circuited stub is,

$$\boxed{l_s = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_R}{Z_0}}} \longrightarrow (6) \quad (\because \beta = \frac{2\pi}{\lambda})$$

At this location, the imaginary part of $y_{in}$ (susceptance) from Eq$^n$ (4)

Let $\quad I_m [y_{in}] = b_s$

$$\therefore b_s = \frac{(1 - y_R^2) \tan \beta l_s}{1 + y_R^2 \tan^2 \beta l_s} \longrightarrow (7)$$

Substitute the $\tan \beta l_s$ value from Eq$^n$ (5), we get

$$b_s = \frac{(1 - y_R^2) \frac{1}{\sqrt{y_R}}}{1 + \cancel{y_R^2} \cdot \frac{1}{\cancel{y_R}}} = \frac{1 - y_R^2}{\sqrt{y_R} (1 + y_R)}$$

$$= \frac{\cancel{(1 + y_R)} (1 - y_R)}{\sqrt{y_R} \cancel{(1 + y_R)}} = \frac{1 - y_R}{\sqrt{y_R}}$$

$$b_s = \frac{1}{\sqrt{\frac{Y_R}{Y_0}}} - \frac{1}{\sqrt{Y_0 Y_R}}$$

$$\boxed{b_s = \frac{Y_0 - Y_R}{\sqrt{Y_0 Y_R}}} \longrightarrow (8)$$

Therefore, at length $l_s$,

$$y_{in} = 1 + j b_s \longrightarrow (9)$$

If a short-circuited stub is added in parallel at this point with susceptance equal to $-j b_s$, then admittance is

$$\boxed{\begin{aligned} y_{in} &= 1 + j b_s - j b_s \\ y_{in} &= 1 \quad \& \quad Y_{in} = Y_0 \end{aligned}} \longrightarrow (10)$$

Thus, matching is achieved at a distance $l_s$ from the load.

<u>Step 2</u>: To find the length of the short-circuited stub.

We know that, the impedance of the short-circuited stub line with length $l_t$ is

$$Z_{sc} = j Z_0 \tan \beta l_t \longrightarrow (11)$$

The susceptance is

$$-j B_s = \frac{1}{Z_{sc}} = \frac{1}{j Z_0 \tan \beta l_t} \quad \& \quad B_s = Y_0 \cot \beta l_t$$

Compare Eq$^n$ ⑧ with Eq$^n$ ⑫ we get

$$\cot\beta l_t = \frac{Y_0 - Y_R}{\sqrt{Y_0 Y_R}}$$

$$\therefore \tan\beta l_t = \frac{\sqrt{Y_0 Y_R}}{Y_0 - Y_R}$$

$$\boxed{l_t = \frac{1}{\beta}\tan^{-1}\left(\frac{\sqrt{Y_0 Y_R}}{Y_0 - Y_R}\right)} \rightarrow (14)$$

In terms of impedances,

$$\boxed{l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{Z_0 Z_R}}{Z_0 - Z_R}\right)} \rightarrow (15)$$

The length of the short-circuited stub is

$$\boxed{l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{Z_0 Z_R}}{Z_0 - Z_R}\right)} \quad \text{for } (Z_R > Z_0)$$

If $Z_R > Z_0$, to obtain a positive length

$$l_t = \frac{\lambda}{2\pi}\left[\pi - \tan^{-1}\left(\frac{\sqrt{Z_R Z_0}}{Z_0 - Z_R}\right)\right] \quad \text{for } Z_R > Z_0$$

$(\because \tan^{-1}(-x) = -\tan^{-1}x)$

Hence, the stub susceptance cancels the susceptance of the line at stub point, so that the line is terminated with the characteristic impedance $Z_0$ at the place of stub connection.

Disadvantages

1. The location and length of the single stub match depends on the frequency. If the frequency of the wave changes, the location and length of the stub should be changed. However, it is very difficult to change the stub once it is fixed.

2. In practical cases, the location of the stub has to be moved along the line for final adjustment. This tuning is possible only on open-wire lines. However, it is very difficult to place a stub on coaxial cables.

Q* Why short circuited stub is generally preferred over open-circuited stub?

Ans A short-circuited stub is generally preferred over open-circuited stub because of the following reasons.

1. It provides strong construction and supports to the main line.

2. The short-circuited stub can be easily established with a large metal plate.

3. Radiation loss is very less compared to the open-circuited stub.

→ To overcome the disadvantages of single stub matching, two stubs can be used. This is called double stub matching.

→ Consider a double stub matching system consisting of two short-circuited stubs connected in parallel to the line near the load as shown in fig.

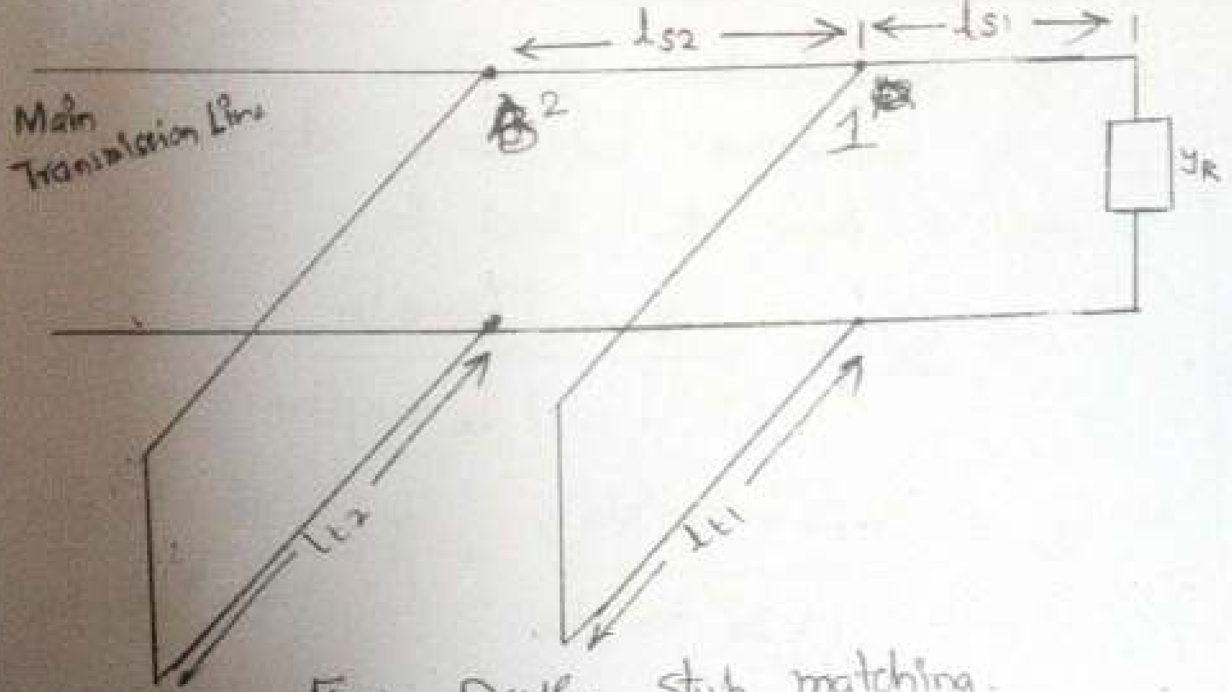



Fig: Double stub matching.

Let
$l_{s1}$ = location of stub A from load.
$l_{s2}$ = location of stub B from load.
$l_{t1}$ = length of stub A. $l_{t2}$ = length of stub B
$l_s$ = separation between stubs, $Y_R$ = load admittance.

→ The characteristic admittance of the stubs should be equal to the line characteristic admittance $Y_0$.

→ Since the admittance repeats at every $\lambda/2$, the total distance $l_{s1}+l_{s2}$ can never be more than or equal to $\lambda/2$.

i.e. $\lambda/4$ & $3\lambda/8$ distance

→ The total distance of the double stub matching should be kept as small as possible.

→ Therefore, in the design of a double stub matching it is better to keep the location of the stubs fixed.

→ When the signal frequency changes, the stub lengths can be adjusted to achieve impedance matching.

## Design of double stub matching:

1. Fix the location of stubs.
2. The normalized admittance at the location of stub 1 from the load is

$$y_A = \frac{y_R + j\tan\beta l_{s1}}{1 + j y_R \tan\beta l_{s1}}$$

or

$$y_A = \frac{y_R(1+\tan^2\beta l_{s1})}{1+y_R^2\tan^2\beta l_{s1}} + j\frac{(1-y_R^2)\tan\beta l_{s1}}{1+y_R^2\tan^2\beta l_{s1}}$$

$$= g_A + j b_A$$

3. Connect a short-circuited stub 1 having susceptance $\pm j b_1$ to the line at 1.

The admittance at 1 after stub connection is

$$y_A' = g_A + j(b_A \pm b_1).$$

4. For impedance matching, the admittance of the line at the location of stub 2 should be equal to the characteristic admittance, i.e.

$$Y_B' = Y_0.$$

equal to

$$y_B = 1 \pm j b_2.$$

5. Connect a short-circuited stub 2 having suscepta
$\mp j b_2$ to the line at z.

6. The stub lengths can be obtained from

$$b_1 = \cot \beta l_{t1}, \quad b_2 = \cot \beta l_{t2}.$$

P.H Smith of [illegible] transmission line a polar chart for calculating transmission line characteristics. This chart is called "Smith chart".

This chart consists of two sets of orthogonal circles which represents the values of normalized impedance.

One set of circles represents the resistive component R, called R circles, and the other set of circles represents the ~~relative~~ Reactive component X, called X circle.

## Derivation of R-circles and X-circles :

Consider a transmission line having characteristic impedance $Z_0$ terminated with $Z_R$

The reflection coefficient is

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{\frac{Z_R}{Z_0} - 1}{\frac{Z_R}{Z_0} + 1} = \frac{\bar{Z}_R - 1}{\bar{Z}_R + 1}$$

where $\bar{Z}_R = \frac{Z_R}{Z_0}$ is the normalized load impedance

$$\bar{Z}_R = \frac{1+K}{1-K}$$

Since $Z_R$ & $K$ are complex quantities, we have

$$R + jX = \frac{1 + K_r + jK_x}{1 - K_r - jK_x} \quad (\because K = K_r + jK_x)$$

where r and x are the real and imaginary parts of $Z_R$, $K_r$ & $K_x$ are the real and imaginary

Separating the real and imaginary parts,

$$R+jx = \frac{(1+K_r+jK_x)(1-K_r+jK_x)}{(1-K_r-jK_x)(1-K_r+jK_x)}$$

$$R+jx = \frac{1-K_r+jK_x+K_r+jK_rK_x+jK_x-jK_rK_x}{(1-K_r)^2+K_x^2}$$

$$\therefore \quad R+jx = \frac{1-K_r^2-K_x^2+j2K_x}{(1-K_r)^2+K_x^2} \longrightarrow (1)$$

$$R = \frac{1-K_r^2-K_x^2}{(1-K_r)^2+K_x^2} \longrightarrow (2)$$

$$\& \quad x = \frac{2K_x}{(1-K_r)^2+K_x^2} \longrightarrow (3)$$

Arranging the Eq$^n$ (2) in standard form, then

$$R = \frac{1-K_r^2-K_x^2}{(1-K_r)^2+K_x^2}$$

$$R(1-K_r)^2+RK_x^2 = 1-K_r^2-K_x^2$$

$$R(1+K_r^2-2K_r)+RK_x^2 = 1-K_r^2-K_x^2$$

$$(1+R)K_x^2+(1+R)K_r^2+R-2RK_r = 1$$

$$K_r^2+K_x^2-\frac{2R}{(1+R)}K_r = \frac{1-R}{1+R}$$

$$\left(K_r-\frac{R}{1+R}\right)^2-\frac{R^2}{(1+R)^2}+K_x^2 = \frac{1-R}{1+R}$$

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$\left(K_r - \frac{\cdot\cdot}{1+R}\right) \quad \frac{}{1+R} \quad (1+R)$

$$\boxed{\left(K_r - \frac{R}{1+R}\right)^2 + R_x^2 = \frac{1}{(1+R)^2}}$$

This equation represents a family of R circles on the K-plane as shown in fig.

→ These circles are called <u>constant R circles</u>, having centres at $\left(\frac{R}{1+R}, 0\right)$ and radii of $\left[\frac{1}{(1+R)}\right]$.

→ A set of circles generated at different values of R.

→ At $R=0$, the centre of the circle is at $(0,0)$ and the radius is 1. This is an outer circle.

→ As R increases, the circle radius decreases.

→ $R=\infty$, represents at $(1,0)$, All circles touches the point $(1,0)$.

Now, the imaginary part is Eq. (3),

$$X = \frac{2K_x}{(1-K_r)^2 + K_x^2}$$

$$(1-K_r)^2 + K_x^2 = \frac{2K_x}{X}$$

$$(1-K_r)^2 + K_x^2 - \frac{2K_x}{X} = 0$$

$$(K_r-1)^2 + \left(K_x - \frac{1}{X}\right)^2 - \frac{1}{X^2} = 0$$

$$\boxed{(K_r-1)^2 + \left(K_x - \frac{1}{X}\right)^2 - \frac{1}{X^2} = 0}$$

This equation represents another family of circles on the K-plane as shown in fig.

→ These circles are called constant X-circles, having centres at $(1, 1/X)$ and radii of $1/X$.

→ $X=0$ represents a straight line along the $K_r$-axis

→ If $X$ is positive, then $K_x$ is ~~negative~~ positive and the circles are generated above the $K_R = 0$ axis.

→ If $X$ is negative, and the circles are generated below the $K_r=0$ axis, the $K_r = 0$ axis.

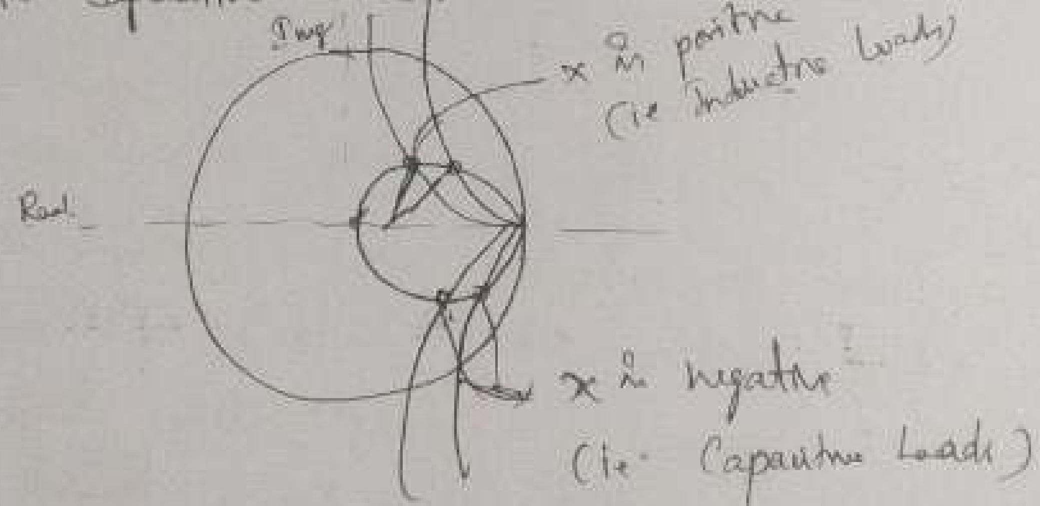
→ $X = \infty$ represents a point at $(1,0)$.

→ All circles touch the point $(1,0)$.

Properti...

1. **Normalizing Impedance**: The Smith chart represents the normalized values of R and X circles. In fact, they are $R/Z_0$ circles and $jX/Z_0$ circles. Thus, to obtain actual values, the values from the chart should be multiplied by $Z_0$.

2. **Plotting of Load Impedance**: The intersection points of R and X circles give normalised load impedance ($Z_R$) values. If x is positive, the point is above the $K_R = 0$ axis i.e inductive load, and if x is negative, the point below $K_R = 0$ axis i.e capacitive load.




3. **Determination of VSWR**: The VSWR values can be obtained by drawing S-circles on the chart as shown in fig. The circles with a centre at the origin (0,0), with radius $|Z_R|$ are called S-circles.

The radius of the S-circle $|Z_R|$ gives VSWR value $S = \left|\frac{Z_R}{Z_0}\right|$.

at the point at the right side of the centre of voltage maxima ($V_{max}$). In the given chart, the points M and L' respectively gives the position voltage maximum and voltage minimum.

The location of the first $V_{max}$ can be obtained from the wavelength scale on the outer circle. The arc AP' gives the distance of $V_{min}$ from the load. Similarly, the Arc P'AB gives the distance of the first $V_{max}$ from the load.

6. Open and short-circuited Line: At point B on the right side end of the horizontal axis, both R and X are infinite which represents an open-circuit termination of the line.

Similarly, at point A on the left side end of the horizontal axis, both R and X are zero which represents short-circuit termination of the line.

7 Movement along the periphery of the chart:

On the outer circle, or periphery of the chart, moving in the clock-wise direction corresponds to travelling from the load towards the generator.

Similarly, moving in the anti-clockwise direction corresponds to travelling from the generator towards the load. The full rotation around the chart gives a distance of $\lambda/2$. If the line length is greater than $\lambda/2$, rotate around the circle n times to reach the line length.

the horizontal axis is, the normalised resistance at M is equal to the value of VSWR i.e.
S = OM.

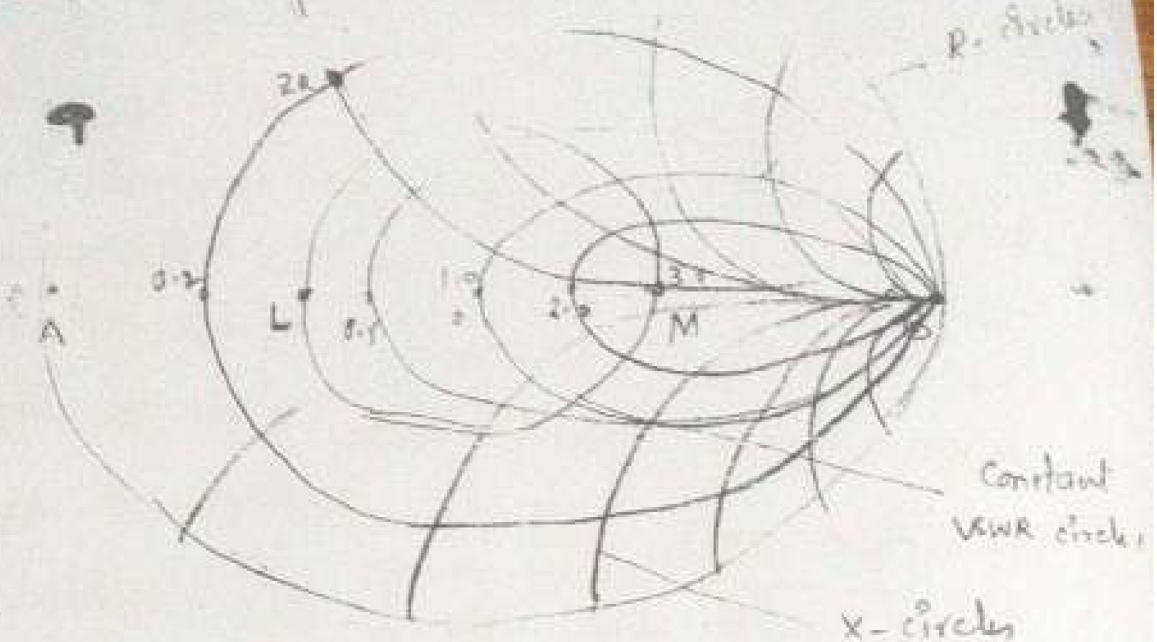



Fig: S-circles on smith chart.

4. Determination of K in magnitude & direction

Draw the line OP and extend it to the outer circle, it cuts the outer circle at P'. The angles are indicates on the outer circle. The angle (θ) of the line OP gives the angle of reflection coefficient. The value (K_R, K_X) at the point P gives the magnitude of K. |K| can be obtained from the K scale provided in the chart. The length of OP on the K scale gives the magnitude of K. Also |K| can be calculated directly as $|K| = \frac{OP}{OP'}$

5 Location of Voltage Maximum and Minimum:

There are two intersection points of the S-circles with the horizontal axis AB. The point at the

...atched L... The ... R=1 repres...

$\bar{Z}_R = \frac{R_e[Z_R]}{Z_0} = 1$ ...21 pass... ...gh the point o...

The reactive part of the load impedance is equal to $Z_0$ of the line. This circle represents impedance only when the reactive component varies on the line. A stub can be used at this location to nullify the reactive component.

Therefore, the centre point of the chart is known as the matched load point.

## Applications of Smith Chart :

1. Used as admittance diagram.
2. Conversion of impedance to admittance
3. Determination of input impedance.
4. Determination of load impedance
5. Input impedance and admittance of an SC line.
6. Input impedance & admittance of an OC line.
7. Determination of location & lengths of stubs by Smith chart

Problems using Smith Chart

1. Solve the problems with the given data

Characteristic impedance

Load impedance

wave length / frequency of the signal

Calculate the followings using Smith Chart

(a) VSWR

(b) Load admittance

(c) Impedance of transmission line at voltage maxima & minima (ie Zmax & Zmin)

(d) Distance between load and first voltage maximum for transmission line.

(e) Voltage Reflection Coefficient K

(f) Input impedance of the line

(g) Location of voltage maximum.

Reflection loss & Reflection factor

Reflection is either due to impedance irregularity (or) when the line is not correctly terminated.

Reflection results in power loss which is termed as reflection loss which is defined as the ratio of power to load to the incident power & is normally denoted by the letter $F_l$

In decibel, $F_l = 10\log_{10}\frac{\text{power to load}}{\text{incident power}} = 10\log_{10}\frac{4(\sqrt{Z_R Z_0})^2}{(Z_R+Z_0)^2}$

$$= 20\log_{10}\left(\frac{2\sqrt{Z_R Z_0}}{Z_R+Z_0}\right) \text{dB}.$$

The reflection loss can also be computed from reflection factor by the relationship

$F_l = 20\log F_r$, where $F_r$ is the reflection factor

Reflection factor, $F_r$ is the geometric mean of the two impedances divided by the arithmetic mean:

$$F_r = \frac{\sqrt{Z_R Z_0}}{\frac{Z_R+Z_0}{2}} = \frac{2\sqrt{Z_R Z_0}}{Z_R+Z_0}$$

The reflection factor can also be calculated from the reflection co-efficient by the relation

$$F_r = \sqrt{1-K^2} = \sqrt{1-\left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)^2} = \sqrt{\frac{(Z_R+Z_0)^2-(Z_R-Z_0)^2}{(Z_R+Z_0)^2}}$$

$$= \frac{2\sqrt{Z_0 Z_R}}{Z_R+Z_0}$$

⇒ If the impedances have different phase angle the reflection factor may be either larger (or) smaller than unity. But if they have same phase angle but different magnitude, the reflection factor is always less than unity.