SIGNALS AND SYSTEMS

II B.TECH I SEM

II ECE-2

- Name of the Faculty: Mr.A.BALA RAJU, Assistant.Professor
- Name of the Course: Signals and Systems.
- Class: II B.Tech-ECE 2-I sem.
- Subject Code: EC 304PC.
- Number of Lectures hours/Week: 4
- Number of Tutorial hours/Week: 1
- Number of Credits: 4

COURSE OBJECTIVES

- This gives the basics of Signals and Systems required for all Electrical Engineering related course
- To understand the behaviour of signal in time and frequency domain
- To understand the characteristics of LTI systems
- This gives concepts of Signals and Systems and its analysis using different transform techniques.

COURSE OUTCOMES

Upon completing this course, the student will be able to

- Differentiate various signal functions.
- Represent any arbitrary signal in time and frequency domain.
- Understand the characteristics of linear time invariant systems.
- Analyse the signals with different transform technique
- Understand different sampling techniques and comparison of signals

SYLLABUS

UNIT - I

- **Signal Analysis:** Analogy between Vectors and Signals, Orthogonal Signal Space, Signal approximation using Orthogonal functions, Mean Square Error, Closed or complete set of Orthogonal functions, Orthogonality in Complex functions, Classification of Signals and systems, Exponential and Sinusoidal signals, Concepts of Impulse function, Unit Step function, Signum function

SYLLABUS

UNIT – II

- **Fourier series:** Representation of Fourier series, Continuous time periodic signals, Properties of Fourier Series, Dirichlet's conditions, Trigonometric Fourier Series and Exponential Fourier Series, Complex Fourier spectrum.
- **Fourier Transforms**: Deriving Fourier Transform from Fourier series, Fourier Transform of arbitrary signal, Fourier Transform of standard signals, Fourier Transform of Periodic Signals, Properties of Fourier Transform, Fourier Transforms involving Impulse function and Signum function, Introduction to Hilbert Transform.

SYLLABUS

UNIT – III

- **Signal Transmission through Linear Systems:** Linear System, Impulse response, Response of a Linear System, Linear Time Invariant(LTI) System, Linear Time Variant (LTV) System, Transfer function of a LTI System, Filter characteristic of Linear System, Distortion less transmission through a system, Signal bandwidth, System Bandwidth, Ideal LPF, HPF, and BPF characteristics, Causality and Paley-Wiener criterion for physical realization, Relationship between Bandwidth and rise time, Convolution and Correlation of Signals, Concept of convolution in Time domain and Frequency domain, Graphical representation of Convolution.

SYLLABUS

UNIT – IV

- **Laplace Transforms:** Laplace Transforms (L.T), Inverse Laplace Transform, Concept of Region of Convergence (ROC) for Laplace Transforms, Properties of L.T, Relation between L.T and F.T of a signal, Laplace Transform of certain signals using waveform synthesis.
- **Z–Transforms:** Concept of Z- Transform of a Discrete Sequence, Distinction between Laplace, Fourier and Z Transforms, Region of Convergence in Z-Transform, Constraints on ROC for various classes of signals, Inverse Z-transform, Properties of Z-transforms.

SYLLABUS

UNIT – V

- **Sampling theorem:** Graphical and analytical proof for Band Limited Signals, Impulse Sampling, Natural and Flat top Sampling, Reconstruction of signal from its samples, Effect of under sampling – Aliasing, Introduction to Band Pass Sampling.
- **Correlation:** Cross Correlation and Auto Correlation of Functions, Properties of Correlation Functions, Energy Density Spectrum, Parsevals Theorem, Power Density Spectrum, Relation between Autocorrelation Function and Energy/Power Spectral Density Function, Relation between Convolution and Correlation, Detection of Periodic Signals in the presence of Noise by Correlation, Extraction of Signal from Noise by Filtering.

TEXT BOOKS:

- 1. Signals, Systems & Communications - B.P. Lathi, 2013, BSP.
- 2. Signals and Systems - A.V. Oppenheim, A.S. Willsky and S.H. Nawabi, 2 Ed.

REFERENCE BOOKS:

- 1. Signals and Systems – Simon Haykin and Van Veen, Wiley 2 Ed.,
- 2. Signals and Systems – A. Rama Krishna Rao, 2008, TMH
- 3. Fundamentals of Signals and Systems - Michel J. Robert, 2008, MGH International Edition.
- 4. Signals, Systems and Transforms - C. L. Philips, J.M.Parr and Eve A.Riskin, 3 Ed., 2004, PE.
- 5. Signals and Systems – K. Deergha Rao, Birkhauser, 2018.

CORRELATION

UNIT-V-II

15/2/2021 → college - Labs

- Get one class Mate lab Records
2-side-white

send the pgms -

4-sessions - ~~10~~ | 8-10

5-6. expmts - finish
Record writing

→ Typing - execution - Checking
Results

We need Print outs of Graphs

Left side - Read
↓ Calc Graphs

Internal → 10-Marks exam
15 - Day to Day
Code - allocate Script - execute - show
exam

$\boxed{75} + 25 = 135$

100

Rt side
Code

Internal Lab
Exam
15-20

external
22-27
3 - 3

Signal Comparision → Correlation → Similarity

Comparision — Component of $f_1(t)$ along $f_2(t) \cong C_2 f_2(t)$

$C_2 =$ f_1 & f_2 are

$$\frac{\int_{t_1}^{t_2} f_1(t) f_2(t)\,dt}{\int_{t_1}^{t_2} f_2^2(t)\,dt} = C_2 = 0 \Rightarrow \boxed{I = \int_{t_1}^{t_2} f_1(t) f_2(t)\,dt = 0}$$

basic fn

Radar - Applications — Tx — 1 signal → delay

Rx — signal ←

Time delay — $\frac{\text{Velocity}}{3\times10^8 \, m/s}$ ⇒ distance

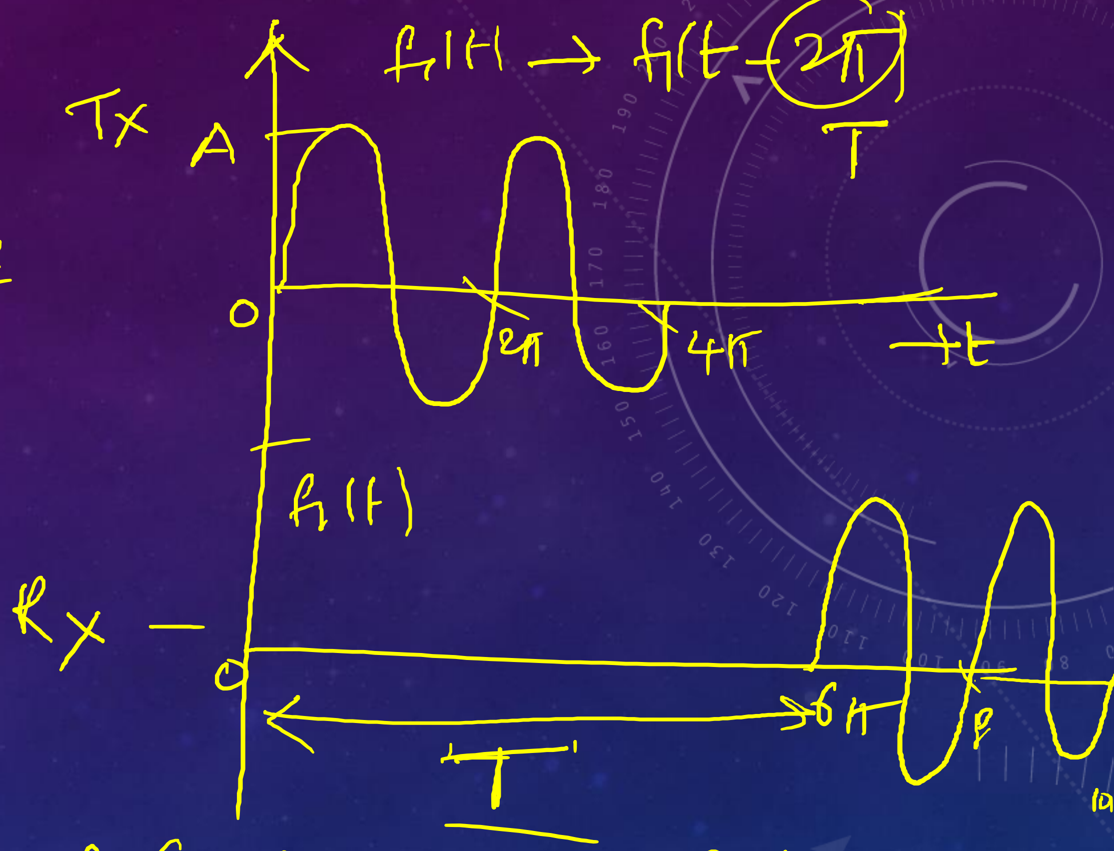
Targets → Warfield

├ Airport

└→ Vehicle / humane / ship

$$I = \int_{t_1 \, -\infty}^{t_2 \, \infty} f_1(t) \cdot f_2(t)\, dt = 0 = \text{orthogonal functions}$$

dissimilar



Time shift — delay

$$\int_{-\infty}^{\infty} f_1(t)\, f_2(t-\tau)\, dt = R_{12}(\tau) = \psi_{12}(\tau) = \text{Cross-correlation function of } f_1(t) \,\&\, f_2(t)$$

function $\tau \rightarrow \tau$ is searching / scanning } Parameter

$$R_{12}(\tau) = \int_{-\infty}^{\infty} f_1(t)\, f_2(t-\tau)\, dt \quad \text{(or)}$$

$$= \int_{-\infty}^{\infty} f_1(t+\tau)\, f_2(t)\, dt$$

$R_{12}(\tau) = 0 \Rightarrow$ two signals are said to be uncorrelated

$$R_{21}(\tau) = \psi_{21}(\tau) = \int_{-\infty}^{\infty} f_2(t) f_1(t-\tau)\,dt$$

$$= \int_{-\infty}^{\infty} f_2(t+\tau) f_1(t)\,dt$$

$$R_{12}(\tau) = \psi_{12}(\tau) = \int_{-\infty}^{\infty} f_1(t) f_2(t-\tau)\,dt$$

$t+\tau = x$

$t = x-\tau$

Replace τ with '$-\tau$'

$$= \int_{-\infty}^{\infty} f_1(t) f_2(t+\tau)\,dt$$

$$R_{12}(\tau) = R_{21}(-\tau)$$

$$= \int_{-\infty}^{\infty} f_1(x-\tau) f_2(x)\,dx$$

$$\boxed{R_{12}(\tau) = R_{21}(-\tau)}$$

Verify

If $f_1(t)$, $f_2(t)$ are complex fns

$$R_{12}(\tau) = \psi_{12}(\tau) = \int_{-\infty}^{\infty} f_1(t)\, f_2^{*}(t-\tau)\, dt$$

$$= \int_{-\infty}^{\infty} f_1(t+\tau)\, f_2^{*}(t)\, dt$$

$R_{21}(\tau) =$

Auto-Correlation →

Correlation — (i) — Cross-correlation → two different fns — $R_{12}(\tau) = \int_{-\infty}^{\infty} f_1(t) f_2(t-\tau) dt$

(ii) Auto-Correlation

↳ Comparing the fn with the same function

$$R_{11}(\tau) = \psi_{11}(\tau) = \int_{-\infty}^{\infty} f_1(t) f_1(t-\tau) dt$$

$$= \int_{-\infty}^{\infty} f_1(t+\tau) f_1(t) dt$$

$R_{11}(\tau) = R_{11}(-\tau) \Rightarrow$ ACR fn is an even fn of 'τ'

Properties of Auto-Correlation Function:

① $R_{11}(0) = \int_{-\infty}^{\infty} f_1^2(t)\,dt = \psi_{11}(0)$ — i.e the value of the ACR fn at the origin is equal to the energy of the signal

$$\int_{-\infty}^{\infty} f^2(t)\,dt = E$$

$$\frac{1}{\pi}\int_{0}^{\infty} |F(\omega)|^2\,d\omega = E$$

ESD

W.K.T $R_{11}(\tau) = \int_{-\infty}^{\infty} f_1(t)\, f_1(t-\tau)\,dt$

Sub. $\tau = 0$ (at origin) $= R_{11}(0) = \int_{-\infty}^{\infty} f_1(t)\, f_1(t)\,dt$

$$R_{11}(0) = \int_{-\infty}^{\infty} f_1^2(t)\,dt = \text{Energy of the Signal}$$

(*) $R_{11}(0) \geq R_{11}(\tau)$

Consider the Integral

$$= \int_{-\alpha}^{\alpha} [f_1(t) \pm f_1(t+\tau)]^2 \, dt = \underline{\text{Positive}}$$

$$= \int_{-\alpha}^{\alpha} f_1^2(t)\,dt \pm \int_{-\alpha}^{\alpha} f_1^2(t+\tau)\,dt \pm 2\int_{-\alpha}^{\alpha} f_1(t) f_1(t+\tau)\,dt \geq 0$$

$$\Rightarrow R_{11}(0) = \int_{-\alpha}^{\alpha} f_1^2(t)\,dt = \text{energy of the signal}$$

$$R_{11}(\tau) = \int_{-\alpha}^{\alpha} f_1^2(t+\tau)\,dt = \text{energy of the shifted signal}$$

$\tau = 0 =$ ACR _ maximum

No shifting

$f_1(t)$ ϕ with $f_1(t)$

$\tau =$ shifting $\uparrow$ = ACR $\downarrow$

$$R_{11}(0) + R_{11}(0) \pm 2 R_{11}(T) \geq 0$$

$$2R_{11}(0) \geq 2R_{11}(T)$$

$$R_{11}(0) \geq R_{11}(T)$$

↳ the max value of ACR occurs at the origin.

the ACR fn becomes smaller at larger values of 'T'

→ For Random & Non-Periodic signals, with zero avg value

$$\boxed{\lim_{T \to \infty} R_1(T) = 0}$$

(3) $R_{11}(\tau) = R_{11}(-\tau) \Rightarrow$ ACR fn is an even function of 'τ'

$$R_{11}(\tau) = \int_{-\alpha}^{\alpha} f_1(t) f_1(t+\tau)\, dt$$

let $t+\tau = t_1$

$t = t_1 - \tau$

$$= \int_{-\alpha}^{\alpha} f_1(t_1-\tau) f_1(t_1)\, dt_1$$

$$= \int_{-\alpha}^{\alpha} f_1(t_1) f_1(t_1+(-\tau))\, dt_1$$

$$\boxed{R_{11}(\tau) = R_{11}(-\tau)}$$

Relationship between Convolution & Correlation :-

$$R_{12}(\tau) = \rho_{12}(\tau) = \int_{-\alpha}^{\alpha} f_1(t)\, f_2(t-\tau)\, dt = \text{Cross correlation}$$

$$R_{21}(\tau) = \rho_{21}(\tau) = \int_{-\alpha}^{\alpha} f_2(t)\, f_1(t-\tau)\, dt =$$

$$R_{11}(\tau) = \rho_{11}(\tau) = \int_{-\alpha}^{\alpha} f_1(t)\, f_1(t-\tau)\, dt = \text{Auto-Correlation}$$

Convolution of two fn $f_1(t)$ & $f_2(t)$ = $f_1(t) * f_2(t)$

$$= \int_{-\alpha}^{\alpha} f_1(\tau)\, f_2(t-\tau)\, d\tau$$

Let us take the Conv. of $f_1(t)$ & it's time reversed function

$$= f_1(t) * f_2(-t)$$

$$f_1(t) * f_2(t) = \int_{-\alpha}^{\alpha} f_1(\tau) f_2(t-\tau)\, d\tau$$

$$f_1(t) * f_2(-t) = \int_{-\alpha}^{\alpha} f_1(\tau) f_2(\tau - t)\, d\tau$$

Replace 'τ' $\Rightarrow$ 'x' $\Rightarrow \int_{-\alpha}^{\alpha} f_1(x) f_2(x-t)\, dx$

Replace 't' with 'τ' $\Rightarrow \int_{-\alpha}^{\alpha} f_1(x) f_2(x-\tau)\, dx$

$$= R_{12}(\tau) = \varphi_{12}(\tau)$$

$R_{12}(\tau) = \psi_{12}(\tau) = \rho_{12}(\tau) = [f_1(t) * f_2(-t)]$

$R_{21}(\tau) = \psi_{21}(\tau) = \rho_{21}(\tau) = [f_2(t) * f_1(-t)]$

$R_{11}(\tau) = \psi_{11}(\tau) = \rho_{11}(\tau) = (f_1(t) * f_1(-t)]$

when $f_1(t)$ is even $\Rightarrow$ $f_1(t) = f_1(-t) \Rightarrow$

$\rho_{11}(\tau) = [f_1(t) * f_1(t)]$ = ACR is equals to the Convovolution

If $f_2(t)$ is an even fn $[f_1(t) * f_2(t)] = \rho_{12}(\tau)$ = Convolution

$\downarrow$

$f_2(-t) = f_2(t)$

Correlation — finding the similarities

→ Two types — (i) <u>cross correlation</u> — comparing two different functions & finding the similarities

$$R_{12}(\tau) = \int_{-\infty}^{\infty} f_1(t)\, f_2(t-\tau)\, dt$$

searching / scanning

as $\tau \to \infty \Rightarrow$ CCR → increases

(ii) <u>Auto-correlation</u> → comparision of the function with itself to measure the similarity

$$R_{11}(\tau) = \int_{-\infty}^{\infty} f_1(t)\, f_1(t-\tau)\, dt$$

ACR ↑ at $\boxed{\tau = 0}$ as $\lim_{\tau \to \infty}$ ACR $= 0$

Correlation → in Radar / sonar Applications

Relation between Correlation & Convolution:

$R_{12}(\tau) = \rho_{12}(\tau) = f_1(t) * f_2(-t) =$ Convolution

If $f_2(t)$ is an even fn $\Rightarrow$ $\boxed{f_2(-t) = f_2(t)}$

$R_{12}(\tau) = \rho_{12}(\tau) = f_1(t) * f_2(t) =$ Normal Convolution

$R_{21}(\tau) = \rho_{21}(\tau) = \left[f_1(-t) * f_2(t)\right]\Big|_{at\ t=\tau}$

If $f_1(t)$ is an even fn $\Rightarrow$ $f_1(-t) = f_1(t)$

$R_{21}(\tau) = f_1(t) * f_2(t) =$ Convolution

for an even fn $\Rightarrow$ The Convolution = Cross Correlation

$\rho_{11}(\tau) = ACR = R_{11}(\tau) = f_1(t) * f_1(-t)$

$= f_1(t) * f_1(t)$ for even

$=$

$f_1(t) \longleftrightarrow F_1(\omega) = \int_{-\infty}^{\infty} f_1(t)\, e^{-j\omega t}\, dt$

$f_1(-t) \longrightarrow \int_{-\infty}^{\infty} f_1(-t)\, e^{-j\omega t}\, dt$

$\int_{-\infty}^{\infty} f_1(x)\, e^{-j(-\omega)x} \cdot dx$

$f_1(-t) \longleftrightarrow F_1(-\omega)$

let $-t = x \Rightarrow t = -x$

$f_1(-t) \longleftrightarrow F_1(-\omega)$

$F_1(-\omega) = \int_{-\infty}^{\infty} f_1(t)\, e^{+j\omega t}\, dt$

$$R_{12}(\tau) = \underbrace{f_1(t) * f_2(-t)}_{\text{Conv. in time Domain}} \xleftrightarrow{FT} \underbrace{F_1(\omega) \cdot F_2(-\omega)}_{\text{multiplication in Freq Domain}}$$

$$R_{21}(\tau) = f_2(t) * f_1(-t) \xleftrightarrow{FT} F_2(\omega) \cdot F_1(-\omega)$$

$$R_{11}(\tau) = f_1(t) * f_1(-t) \xleftrightarrow{FT} F_1(\omega) F_1(-\omega)$$

$$|F_1(\omega)|^2$$

$$\Pi \; S_1(\omega)$$

Energy Spectral Density (ESD)

$$R_{11}(\tau) \xleftrightarrow{FT}$$

ACR is the time domain Counter part of ESD

ACR fn & ESD forms a FT <u>Pair</u>

$$FT[ACR] = FT[R_{11}(\tau)] = \int_{-\infty}^{\infty} R_{11}(\tau)\, e^{-j\omega\tau}\, d\tau.$$

$$R_{11}(\tau) = \psi_{11}(\tau) = \int_{-\infty}^{\infty} f_1(t)\, f_2(t-\tau)\, dt$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f_1(t)\, f_2(t-\tau)\, dt \right) e^{-j\omega\tau}\, . d\tau$$

$$= \int_{-\infty}^{\infty} f_1(t)\, dt \int_{-\infty}^{\infty} f_2(t-\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} f_1(t)\, e^{-j\omega t}\, . dt \, . \int_{-\infty}^{\infty} f_2(t-\tau)\, e^{+j\omega t} \, . \, e^{-j\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} f_1(t) e^{-j\omega t} dt \cdot \int_{-\infty}^{\infty} f_1(t-\tau)\, e^{+j\omega(t-\tau)} . d\tau$$

$$= F_1(\omega) \cdot F_1(-\omega)$$

$$F_1(-\omega) = \int_{-\infty}^{\infty} f_1(t) e^{j\omega t} . dt$$

$$= |F_1(\omega)|^2$$

$$R_{11}(\tau) \longleftrightarrow |F_1(\omega)|^2 = \pi S_1(\omega)$$

$$ACR \xleftrightarrow{FT} ESD \longrightarrow \text{Wiener - khinchines Relationship}$$

Energy Spectral Density (ESD) / (Parseval's Relation)

EDS – Energy Density Spectrum

$$E = \int_{-\alpha}^{\alpha} |f(t)|^2 dt$$ = for finite energy signal

$$= \int_{-\alpha}^{\alpha} f(t) \cdot f(t) dt$$

Spectral Density is defined as the distribution of energy (or) Power per unit Bandwidth as a function of freq.

When Energy Signal / Non Periodic $\longrightarrow$ Energy Spectral Density (ESD)

Power " / Periodic $\longrightarrow$ Power Spectral Density (PDS)

$$E = \int_{-\infty}^{\infty} f^2(t)\,dt = \int_{-\infty}^{\infty} f(t)\cdot f(t)\,dt$$

$$= \int_{-\infty}^{\infty} f(t)\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega\right) dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,d\omega\left(\int_{-\infty}^{\infty} f(t)e^{j\omega t}\,dt\right)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\cdot F(-\omega)\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\,d\omega$$

$$= \int_{-\infty}^{\infty} |F(\omega)|^2\,df$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$$

$$F(-\omega) = \int_{-\infty}^{\infty} f(t)e^{+j\omega t}\,dt$$

$$\omega = 2\pi f$$

$$E = \int_{-\alpha}^{\alpha} f^2(t)\,dt = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} |F(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\cdot 2\int_{0}^{\alpha} |F(\omega)|^2 d\omega = \frac{1}{\pi}\int_{0}^{\alpha} |F(\omega)|^2 d\omega$$

→ energy/unit BW

$$= \frac{1}{\pi}|F(\omega)|^2 \longrightarrow$$ Energy Spectral Density (EDS/EDS)

$$|F(\omega)|^2 = \pi S(\omega)$$

→ Energy from both +ve & -ve freq components

$$ESD = S(\omega) = \frac{1}{\pi}|F(\omega)|^2$$

Power Spectral Density / Power Density Spectrum

ESD → energy signal (E = finite)

ACR $\overset{FT}{\longleftrightarrow}$ ESD

CCR ↔ CPSD

Correlation ↔ Spectral density

Energy & Power are the Parameters to describe the strength of the signal

E = finite = $0 \le E \le \infty$ = Energy signal

E = Infinite → we use Power → to describe signal

Power signal ⇒ E = infinite & Power = finite ($0 \le P \le \infty$)

Periodic

Correlation fns for Non-finite energy Signals

$\int_{-\infty}^{\infty} f^2(t)\,dt = \infty =$ infinite $\therefore$ Power is definite

$$\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)\,dt$$

$$\overline{R_{11}(\tau)} = \overline{\psi_{11}(\tau)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f_1(t)\,f_1(t-\tau)\,dt = \text{ACR of Power Signal} \quad (1)$$

$$\overline{R_{12}(\tau)} = \overline{\psi_{12}(\tau)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f_1(t)\,f_2(t-\tau)\,dt = \text{CCR} \quad \text{"} \quad (2)$$

If $f(t)$ is Periodic $\Rightarrow$ $f(t) = f(t \pm T)$ $\Rightarrow$ then their Corresponding ACR / CCR are also Periodic with Same Period

Auto correlation & Power Spectral Density forms FT Pair

$$\overline{R_{11}(\tau)} = \varphi_{11}(\tau) \longleftrightarrow \pi S(\omega)$$

$$= \lim_{T\to\infty} \frac{1}{T} |F_T(\omega)|^2$$

$$F_T(\omega) = \pi S_T(\omega)$$

$$\boxed{\overline{R_{11}(\tau)} \xleftrightarrow{FT} \lim_{T\to\infty} \frac{\pi}{T} S_T(\omega)}$$

Detection of Periodic Signals in the Presence of Noise By Correlation:

Applications of Correlation

~~(i)~~ Detection of Signals Masked by Noise

(ii) Radar & Sonar Applications

(iii) Detection of Periodic Component in brain Waves

(iv) ocean wave analysis

(v) Geophysics → meteorology

Periodic Signal = s(t) , Noise Signal n(t)

$\rightarrow$ Random & No-Periodicity is Maintained

Detecting s(t) covered / Mixed / Masked by Noise

ACR & CCR – fns Can be used for Detection

Random

$$\overline{R_{sn}(\tau)} = \lim_{T \to \alpha} \frac{1}{T} \int_{-\alpha}^{\alpha} s(t)\, n(t-\tau)\, dt$$

$$\overline{R_{sn}(\tau)} = 0 \quad \text{for all } \tau$$

Noise and Periodic Signals are uncorrelated

Detection by Auto Correlation

$f(t)$ = Received

$f(t) = s(t) + n(t)$

Let $\overline{R_{ff}(\tau)}$, $\overline{R_{ss}(\tau)}$ and $\overline{R_{nn}(\tau)}$ → ACR of noise $n(t)$

↳ ACR fn of Received Signal $f(t)$

ACR of Periodic Signal $s(t)$

$$\overline{R_{ff}(\tau)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f(t) f(t-\tau)\, dt \quad \text{—— } (1)$$

$$f(t) = s(t)+n(t) \Rightarrow f(t-\tau) = s(t-\tau)+n(t-\tau)$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(s(t)+n(t)\right)\left(s(t-\tau)+n(t-\tau)\right) dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\left(\int_{-T/2}^{T/2} \left(s(t)\, s(t-\tau)\right) dt + \int_{-T/2}^{T/2} s(t)\, n(t-\tau)\, dt + \int_{-T/2}^{T/2} n(t)\, s(t-\tau)\, dt + \int_{-T/2}^{T/2} n(t)\, n(t-\tau)\, dt\right)$$

$$= \overline{R_{ss}(\tau)} + \underbrace{\overline{R_{sn}(\tau)}}_{0} + \underbrace{\overline{R_{ns}(\tau)}}_{0} + \overline{R_{nn}(\tau)}$$

as the signal $s(t)$ & Noise $n(t)$ are uncorrelated

i.e $\overline{R_{sn}(\tau)} = \overline{R_{ns}(\tau)} = 0$

$$\overline{R_{ff}(\tau)} = \overline{R_{ss}(\tau)} + \overline{R_{nn}(\tau)}$$

as $s(t)$ - Periodic $\Rightarrow$ ACR fn $\overline{R_{ss}(\tau)}$ also Periodic with the Same Period

as $\tau \to \infty$, $\boxed{\lim_{\tau \to \infty} R_{nn}(\tau) = 0}$

$$\overline{R_{ff}(\tau)} = \overline{R_{ss}(\tau)}$$

Pg: 528
S, S & C
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Detection By Cross-Correlation

$f(t) = s(t) + n(t)$

$n(t) \rightarrow$ noise

$s(t) \rightarrow$ Signal

$f(t) \leftarrow$ Received

→ By finding the CCR of $f(t)$ with the locally generated signal $c(t)$ which is of same freq as that of $f(t)$

$$\bar{R}_{fc}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, c(t-\tau)\, dt$$

$$\overline{R_{fc}(\tau)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} (s(t)+n(t))\, c(t-\tau)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\left(\int_{-T/2}^{T/2} (s(t)\, c(t-\tau))\, dt + \int_{-T/2}^{T/2} n(t)\, c(t-\tau)\, dt\right)$$

$$\overline{R_{fc}(\tau)} = \overline{R_{sc}(\tau)} + \overline{R_{nc}(t-\tau)}$$

$$\boxed{\overline{R_{nc}(t-\tau)} = 0}$$

$$\overline{R_{fc}(\tau)} = R_{sc}(\tau)$$

c(t) Periodic

→ CCR Periodic → Contains Periodic Component dis adv :— the freq of the signal to be detected

c(t) → locally Generated.

but → Radar / Sonar — we will be knowing freq of Tx signal

Extraction of a Signal from Noise By filtering

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Syllabus Completed

Signals, Systems &

Communi

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Red

Extraction of the Signal from Noise By Filtering

531

532

⇒ ACR, CCR ⇒ Apply — Detection of Periodic Signal masked by Noise

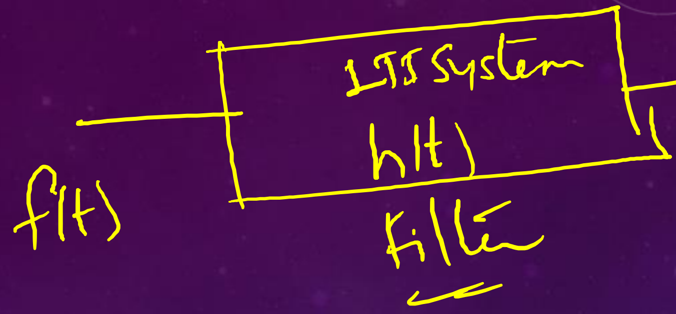
Filtering → Detect – Periodic

CCR + ACR — time domain Representation

Filtering → Freq. Domain operation

Freq

$$\boxed{R_{12}(\tau) = f_1(t) * f_2(-t)} = \psi_{12}(\tau) \xleftrightarrow{FT} F_1(\omega) F_2(-\omega)$$



$y(t) = ?$

$y(t) = f(t) * h(t)$ — Time Domain operation

$Y(\omega) = F(\omega)\, H(\omega)$ — ①

spectrum of system

spectrum of i/p

$y(t) = f_1(t) * f_2(-t) = R_{12}(\tau) = \psi_{12}(\tau)$

$Y(\omega) = F_1(\omega)\, F_2(-\omega)$ — ②

$\boxed{H(\omega) = F_2(-\omega)} \Longrightarrow \boxed{h(t) = f_2(-t)}$

T/F of the system

$Y(\omega) = F(\omega) H(\omega) \rightarrow$ Filtering operation

$Y(\omega) = \underline{\underline{F_1(\omega) F_2(-\omega)}} \rightarrow$ Filter with T/F $\underline{\underline{F_2(-\omega)}}$

Filtering operation - is the FD counterpart of $R_{12}(\tau) = \underline{\underline{\varphi_{12}(\tau)}}$

Detection of Periodic signal using CCR $\rightarrow$ $\underline{\underline{\text{Periodic}}}$

$f_1(t) = s(t) + n(t)$ - input

$f_2(t) = c(t)$ - locally generated carrier with same freq

$f_2(-t) = c(-t)$,

$c(t) \longleftrightarrow C(\omega)$

$c(-t) \longleftrightarrow C(-\omega)$

$$R_{12}(T) = f_1(t) * f_2(-t)$$
$$= f(t) * c(-t)$$
$$= F(\omega)\, C(-\omega)$$

$f(t) = s(t) + r(t)$

$c(t) =$ Periodic Signal with same Periods of $s(t)$

$$C(\omega) = \int_{-\alpha}^{\alpha} c(t)\, e^{-j\omega t}\, dt \Rightarrow C(-\omega) = \int_{-\alpha}^{\alpha} c(t)\, e^{+j\omega t}\, dt$$

$\downarrow$

$$C^*(\omega) = \int_{-\alpha}^{\alpha} c^*(t)\, e^{+j\omega t}\, dt$$

$x = a + jb$

$x^* = a - jb$

for real $c(t) \Rightarrow c(t) = c^*(t)$

$$\boxed{C^*(\omega) = C(-\omega)}$$

$c(t)$ = Periodic

$C(-\omega) = ?$

$$Y(\omega) = F(\omega) . C(-\omega)$$

$$Y(\omega) = F(\omega) . 2\pi \sum_{n=\alpha}^{\alpha} C_n^{*} \delta(\omega - n\omega_0)$$

will eliminate the Noise P

Detects the Periodic signal

FT Repn

FT [Periodic signal $f(t)$]

$$f(t) = \sum_{n=\alpha}^{\alpha} F_n e^{jn\omega_0 t}$$

$$F(\omega) = \sum_{n=\alpha}^{\alpha} F_n \, FT\left(e^{jn\omega_0 t}\right)$$

$$F(\omega) = \sum_{n=\alpha}^{\alpha} F_n 2\pi \delta(\omega - n\omega_0)$$

FS coefficient

$\omega = \pm\omega_0, \pm 2\omega_0 \ldots$ $\pm n\omega_0$

Revision

try to attempt $\boxed{5}$ Full Questions

I. Part - A

1 - 10 (25 M)

Part - B

5 out of 8

$10 \times 5 = 50$

What is a Sampling

(a) Sampling Converts a CTS to DTS

(2)

$f_s = 2f_m$ = Nyquist Rate (3)

Straight forward Answer

(3)

2

2 (a) 5

(b) + '2' (1)

3 (a)

(8) 3 (b)

4 (a)

4 (b)

5 (a)

5 (b)

6 (a)

6 (b)

7 (a) (b) 5

8 (a) (b)

9 (a)

(b) 5

(i) at one place

(ii)

3:00 mins

Script → Cm

Unit-I → Analogy b/n Vector Signals → $V_1 \cdot V_2 = 0$

$$\int_{t_1}^{t_2} f_1(t) \cdot f_2(t)\,dt = 0$$

(t_1, t_2)

$$C_{12} = \frac{\int_{t_1}^{t_2} f_1(t) f_2(t)\,dt}{\int_{t_1}^{t_2} f_2^2(t)\,dt} = 0$$

$$C_r = \frac{\int_{t_1}^{t_2} f(t) g_r(t)\,dt}{\int_{t_1}^{t_2} g_r^2(t)\,dt}$$

$g_r(t)$ = set of mutually orthogonal fn

→ Complete

$$\int_{t_1}^{t_2} x(t) g_r(t)\,dt = 0$$

$$f(t) \simeq C_1 g_1(t) + C_2 g_2(t) + \dots\, C_r g_r(t) + \dots \infty$$

r = fn

$$\mathcal{E} = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} f^2(t)\,dt - \sum C_r^2 K_r\right) = \text{MSE}$$

as $n \uparrow = \mathcal{E} \downarrow$

Complex –

Classification – Basic operations – Problems

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt$$

b_n

unit II → Freq Analysis of the Signal

F.S & F.T – an tool

$$TFS = f(t) = x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

F.S ⇒ Two types –

Sum Sinus & expn

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$EFS = x(t) = \sum_{n=-\alpha}^{\alpha} F_n e^{jn\omega_0 t}$

$F_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} \, dt$

EFS Vs TFS

F_n
F_n
F_0

⇒ Even fns — Cosine — a_n
odd + Sine — b_n
Half-wave sy $\boxed{f(t \pm T/2) = -f(t)}$ → both a_n & b_n but odd harmonics

⇒ Properties → Parseval's theorem

FT → FT from FS ⇒ Basic Signals

existence condition → Dirichlet's Condition

$\int_{-\infty}^{\infty} |f(t)|\,dt$ = finite

absolutely integral

⇒ Sint, Cost, δ(t), u(t), Sgn(t) → limitation

Sufficient but not necessary

Properties →

Signal Tx through Linear System

Linearity =

$a_1 x_1(t) + a_2 x_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$

scaling

additive

Time Invariance

helping in to find the Res

$x(t) \rightarrow y(t)$

$x(t-t_0) \rightarrow y(t-t_0)$

$\boxed{LTI}$ — Conv

$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$

$$\boxed{y(n) = \sum_{k} x(k)\,h(n-k)}$$

= Conv Sum

$$x(n) = \sum_{k} x(k)\,\delta(n-k)$$

Causal

physically Realizable

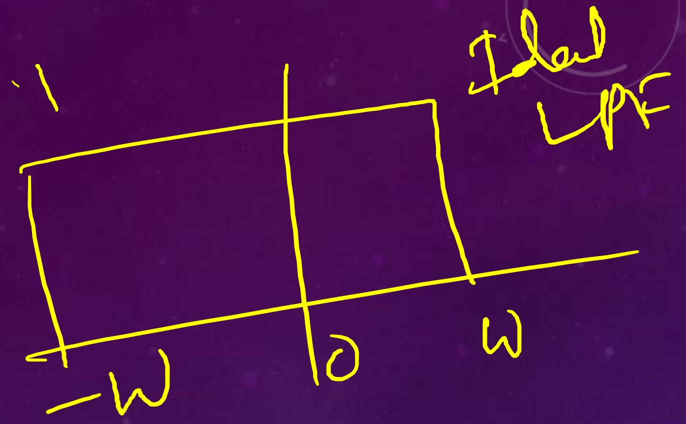
paley-wiener criter

$$I = \int_{-\infty}^{\infty} \frac{|H(\omega)|^2}{1+\omega^2}\, d\omega = \text{finite}$$

⇒ Distortion less Tx —

Ideal - Filter — Non-Causal IR

→ not Practically Realizo



Stability

Causal

Rise Time

V_s BW

Signal BW

System BW

'α'

Convolution

Graphical Convolution

Unit-IV   Laplace T   Z-Transform

$\int_{-\infty}^{\infty} f(t)\,dt$ = Infinite

FT → X

$e^{-at}u(t) = \frac{1}{s+a}$ = ROC = $Re(s) > -a$

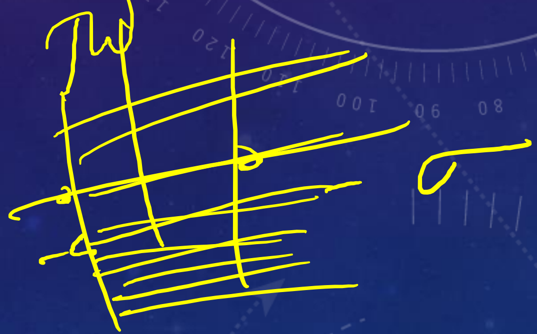
$-e^{-at}u(-t) = \frac{1}{s+a}$ = $Re(s) < -a$

$\int_{-\infty}^{\infty} f(t)\cdot e^{-\sigma t}\,dt$

$\int_{-\infty}^{\infty} \left(f(t)e^{-\sigma t}\right) e^{-j\omega t}\,dt$ → FT is also Converge

$f_1(t) + f_2(t)$

$R_1 \cap R_2 = R$



Prope   unilateral

$ZT \Rightarrow \sum x(t) e^{-j\omega n}$ = DTFT ✗ not summable

$z = re^{j\omega}$ = polar $\Rightarrow X(z) = \sum_n x(n) z^{-n}$

LT & ZT

CTS DTS

solution differential eqn → LT

difference → ZT

Unit - V → Sampling theorem → C.T.S to D.T.S

$\boxed{f_s \geq 2f_m}$ $\boxed{T_s \leq \frac{1}{2f_m}}$

f_m = highest freq com

Bandlimited $f_L = 0$, $f_H \neq 0$

Recovery | Reconstruction

Bandpass $f_L \neq 0$, $f_H \neq 0$

Graphical, Analytical →

$f_s \geq \frac{2f_H}{N}$

Simply - typed —   Instantaneous Samp —   s(t) = Impulse

Natural   ,,   s(t) = pulse train

→ Flat Top —

Correlation

→ ACR — Prope

→ CCR

→ Pr   ESD | PSD

$f_1(t) * f_2(-t) =$ ~~Con~~ ACR

2 — CCR

Applicat

~~ACR~~

CCR → Filtering

$A = x^2 + 2x + 1$

$B = 4x^3 + 3x^2 + 5x + 1$

$A.B = \underline{(x^2+2x+1)}\,(4x^3+3x^2+5x+1)$

$A = [1, 2, 1]\,, \quad B = [4, 3, 5, 1]$

$W = \underline{\underline{conv}}(A, B)$

Interactive → help — Command

Tool Box

Document

Syntax

— description

— examples

plot(x)

stem(x)

Thank you