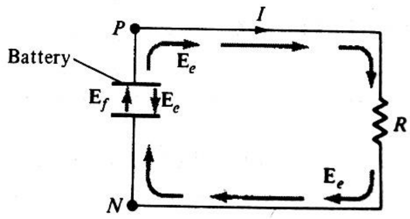
CHAPTER-3

Faraday's Law: States that the induced emf, V_{emf} (volts), in any closed circuit is equal to the time rate of change of magnetic flux linkage by the circuit.

- $V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\psi}{dt}$ $\lambda = N\psi$ is the flux linkage
- The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it. This is known as *Lenz's* law
- $\mathbf{E} = \mathbf{E}_e + \mathbf{E}_f$
- $\mathbf{E}_f$ is zero outside the battery and $\mathbf{E}_f$ and $\mathbf{E}_e$ have opposite directions inside the battery



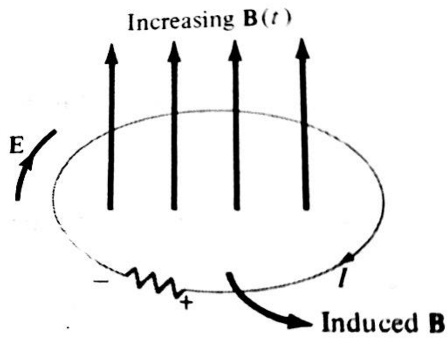
$$\oint_L \mathbf{E}.d\mathbf{l} = \oint_L \mathbf{E}_f.d\mathbf{l} + \oint_L \mathbf{E}_e.d\mathbf{l} = \int_N^P \mathbf{E}_f.d\mathbf{l}$$

$$V_{emf} = \int_N^P \mathbf{E}_f.d\mathbf{l} = -\int_N^P \mathbf{E}_e.d\mathbf{l} = IR$$

- An electrostatic field cannot maintain a steady current in a closed circuit since $\oint_L \mathbf{E}_e.d\mathbf{l} = 0 = IR$
- An emf-produced field $\mathbf{E}_f$ is nonconservative.
- Except in electrostatics, voltage and potential difference are usually not equivalent.

Transformer and Motional EMFs

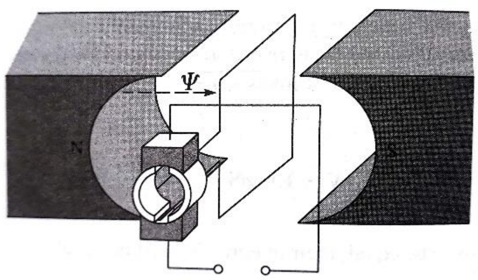
- $V_{emf} = -\frac{d\psi}{dt} = \oint_L \mathbf{E}.d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}.d\mathbf{S}$
- *Stationary loop in time-varying B Field (transformer EMF)*
- $V_{emf} = \oint_L \mathbf{E}.d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t}.d\mathbf{S}$
- this emf induced by the time-varying current (producing the time-varying B filed) in a stationary loop is referred to as transformer emf.
- Applying Stoke's theorem



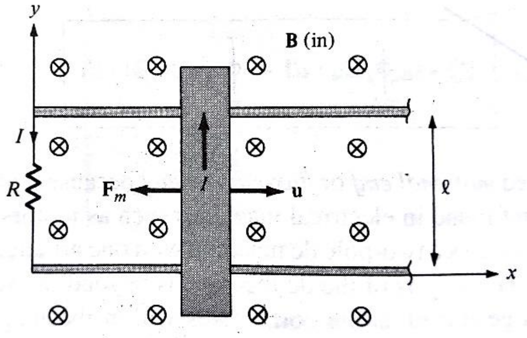
$$\oint_L \mathbf{E}.d\mathbf{l} = -\int_S (\nabla \times \mathbf{E}).\ d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t}.d\mathbf{S}$$

$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ ------------→ Maxwell's eqn. for time-varying fields

- The time-varying E field is not conservative.
- *Moving loop in Static B Field (Motional EMF)*
- $\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$
- Motional electric filed $\mathbf{E}_m$ is $\mathbf{E}_m = \frac{\mathbf{F}_m}{Q} = \mathbf{u} \times \mathbf{B}$
- $V_{emf} = \oint_L \mathbf{E}.d\mathbf{l} = \oint_L (\mathbf{u} \times \mathbf{B}).\ d\mathbf{l}$



- This emf is said to be motional emf or self-cutting emf. This kind of emf found in electrical machines such as motors, generators, and alternators.
- $\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$
- $I = \frac{Q}{t}$
- $Q = It$
- $\mathbf{F}_m = It(\mathbf{u} \times \mathbf{B}) = I\mathbf{l} \times \mathbf{B}$
- $F_m = IlB$
- $V_{emf} = uBl$



- $\int_S (\nabla \times \mathbf{E}_m) .\ d\mathbf{S} = \int_S\ \nabla \times (\mathbf{u} \times \mathbf{B}) .\ d\mathbf{S}$
- $\nabla \times \mathbf{E}_m = \nabla \times (\mathbf{u} \times \mathbf{B})$
- *Moving loop in time-varying B Field*
- In this case, both transformer and motional emf are present.

$$V_{emf} = \oint_L \mathbf{E}.d\mathbf{l} = = -\int_S \frac{\partial \mathbf{B}}{\partial t}.d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) .\ d\mathbf{l}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

Displacement Current

- $\nabla \times \mathbf{H} = \mathbf{J}$
- $\nabla.(\nabla \times \mathbf{H}) = \nabla.\mathbf{J} = \mathbf{0}$
- From continuity eqn. $\nabla.\mathbf{J} = -\frac{\partial \rho_v}{\partial t} \neq 0$
- $\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d$
- $\nabla.\mathbf{J}_d = -\nabla.\mathbf{J} = \frac{\partial \rho_v}{\partial t} = \nabla.\frac{\partial \mathbf{D}}{\partial t}$
- $\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$------------→ displacement current density
- $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ ------------------→ Maxwell's eqn. for time-varying fields

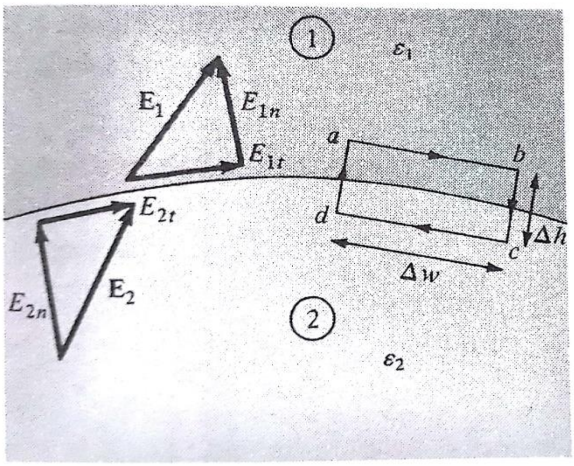
Maxwell's Equations in Final Forms

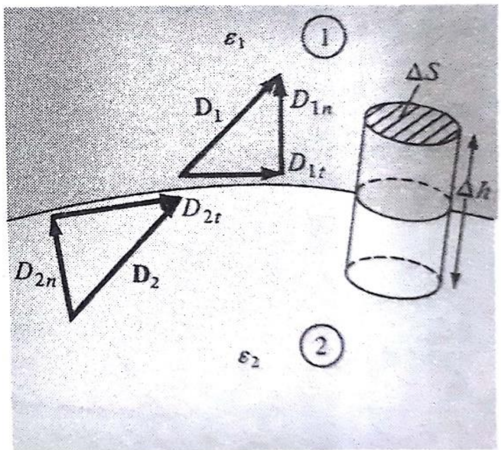
Differential (Point Form)	Integral Form	Remarks
$\nabla.\mathbf{D} = \rho_v$	$\oint_S \mathbf{D}.d\mathbf{S} = \int_v \rho_v dv$	Gauss's Law
$\nabla.\mathbf{B} = 0$	$\oint_S \mathbf{B}.d\mathbf{S} = 0$	Nonexistence of magnetic monopole
$\nabla \times \mathbf{E} = 0$	$\oint_L \mathbf{E}.d\mathbf{l} = -\frac{\partial}{\partial t}\int_S \mathbf{B}.d\mathbf{S}$	Faraday's Law
$\nabla \times \mathbf{H} = \mathbf{J}$	$\oint_L \mathbf{H}.d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right).d\mathbf{S}$	Ampere's circuit law

Electric Boundary Conditions:

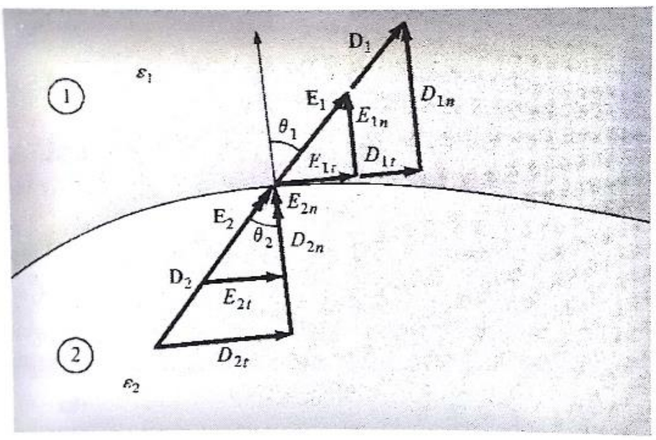
- Used to determine the fields in a different medium

  Dielectric (ϵ_{r_1}) – Dielectric (ϵ_{r_2}):





- $\mathbf{E} = \mathbf{E}_t + \mathbf{E}_n$
- $\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n}$ and $\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n}$
- $\oint_L \mathbf{E}.d\mathbf{l} = 0$
- $E_{1t} = E_{2t}$
- The tangential components of **E** are the same on the two sides of the boundary.
- $\mathbf{D} = \epsilon\mathbf{E} = \mathbf{D}_t + \mathbf{D}_n$
- $\frac{D_{1t}}{\epsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\epsilon_2}$
- $D_{1n} - D_{2n} = \rho_S$
- If no free charge exists, $\rho_S = 0$, at the interface,
- $D_{1n} = D_{2n}$
- The normal components of **D** are continuous across the boundary.
- $\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$
- Refractive index:
- Consider $\mathbf{D}_1$ or $\mathbf{E}_1$ and $\mathbf{D}_2$ or $\mathbf{E}_2$ making angles θ_1 and θ_2 with the normal to the interface

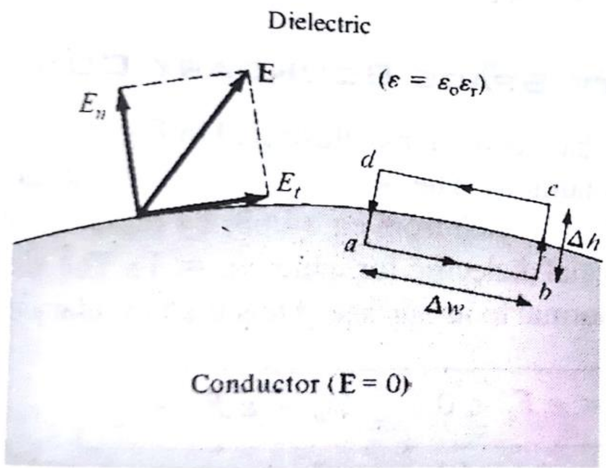


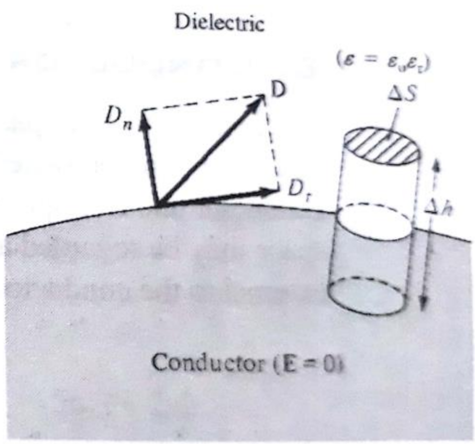
$$E_1 \sin\theta_1 = E_{1t} = E_{2t} = E_2 \sin\theta_2$$

$$\epsilon_1 E_1 \cos\theta_1 = D_{1n} = D_{2n} = \epsilon_2 E_2 \cos\theta_2$$

- $(\tan\theta_1)/\epsilon_1 = (\tan\theta_2)/\epsilon_2$
- $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$
- This is the law of refraction of the electric field at a boundary free of charge ($\rho_S = 0$)

  <u>**Conductor – Dielectric:**</u>
- The conductor is assumed to be perfect for which ($\sigma \to \infty, \rho_C \to 0$)
- $E_t = 0$ and $D_n = \rho_S$



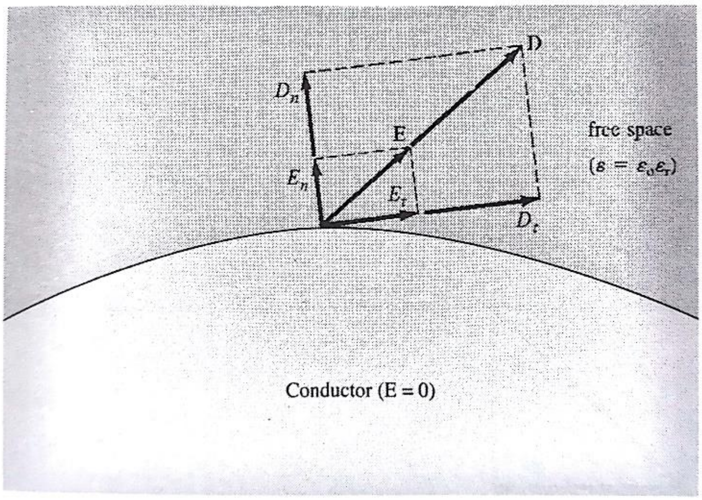


- Under static conditions, below are the points about a perfect conductor
  - No electric field may exist within a conductor ($\rho_v = 0, \mathbf{E} = 0$)
  - Then can be no potential difference between any two points in the conductor, i.e., a conductor is an equipotential body
  - An electric field is external to the conductor and is normal to its surface

$$D_t = \epsilon_o \epsilon_r E_t = 0, \qquad D_n = \epsilon_o \epsilon_r E_n = \rho_S$$

- This concept of $\mathbf{E} = 0$ inside a conductor is applied in electrostatic screening and shielding.

Conductor – Free-space:

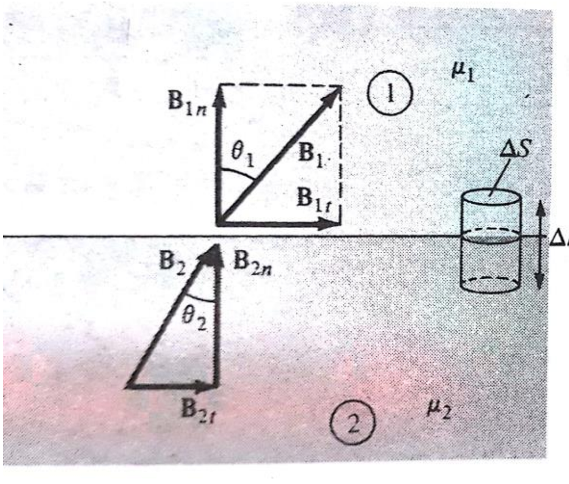


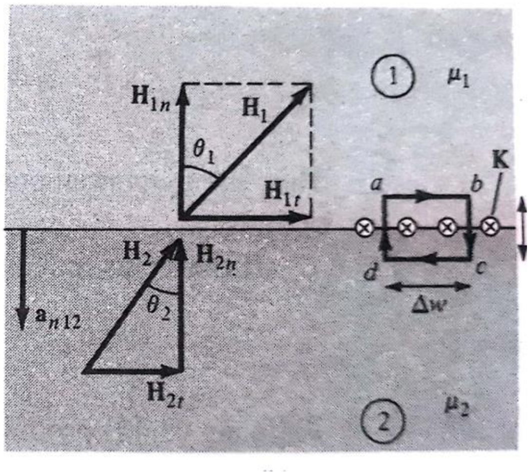
- Boundary conditions are the same except that $\epsilon_r = 1$

$$D_t = \epsilon_o E_t = 0, \qquad D_n = \epsilon_o E_n = \rho_S$$

Magnetic Boundary Conditions:

- $\oint_S \mathbf{B}.d\mathbf{S} = 0$
- $\oint_L \mathbf{H}.d\mathbf{l} = I$





- $B_{1n}\Delta S - B_{2n}\Delta S = 0$
- $\mathbf{B}_{1n} = \mathbf{B}_{2n}$ or $\mu_1\mathbf{H}_{1n} = \mu_2\mathbf{H}_{2n}$
- The normal components of **D** are continuous whereas the normal components of **H** are discontinuous.
- $H_{1t} - H_{2t} = K \quad \frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$
- $\mathbf{H}_{1t} = \mathbf{H}_{2t}$ and $\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$ when the boundary current free (K=0)
- <u>Refractive index:</u>

- Consider $\mathbf{B}_1$ or $\mathbf{H}_1$ and $\mathbf{B}_2$ or $\mathbf{H}_2$ making angles θ_1 and θ_2 with the normal to the interface

$$B_1 \cos\theta_1 = B_{1n} = B_{2n} = B_2 \cos\theta_2$$

$$\frac{B_1}{\mu_1}\sin\theta_1 = H_{1t} = H_{2t} = \frac{B_2}{\mu_2}\sin\theta_2$$

- $(\tan\theta_1)/\mu_1 \quad =(\tan\theta_2)/\mu_2$
- $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$
- This is the law of refraction of the electric field at a boundary free of charge ($K = 0$)