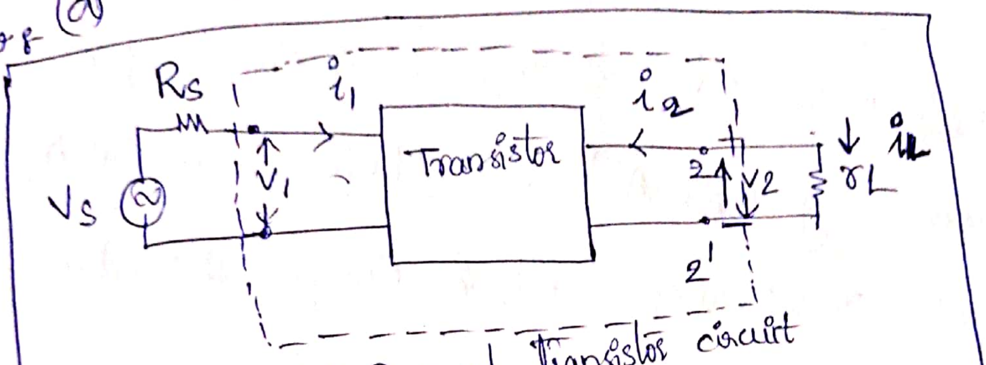
Analysis of a transistor amplifier circuit using h-parameters:- (Amplifier Expressions)

Consider the general transistor amplifier circuit as shown below:- (a)



(a) General transistor circuit

(b) Transistor amplifier circuit

The amplifier can be formed by connecting a signal source at it's input terminals and load resistance at it's output terminals and 'R_s' is the internal resistance of the voltage source. The figure shows the hybrid equivalent circuit of a general transistor amplifier circuit ~~(b)~~ (c)

The performance of the transistor can be analysed by calculating the four parameters, they are (i) current gain ('A_i') (iv) Output resistance ('R_o')
(ii) Voltage gain ('A_v')
(iii) Input resistance ('R_i')

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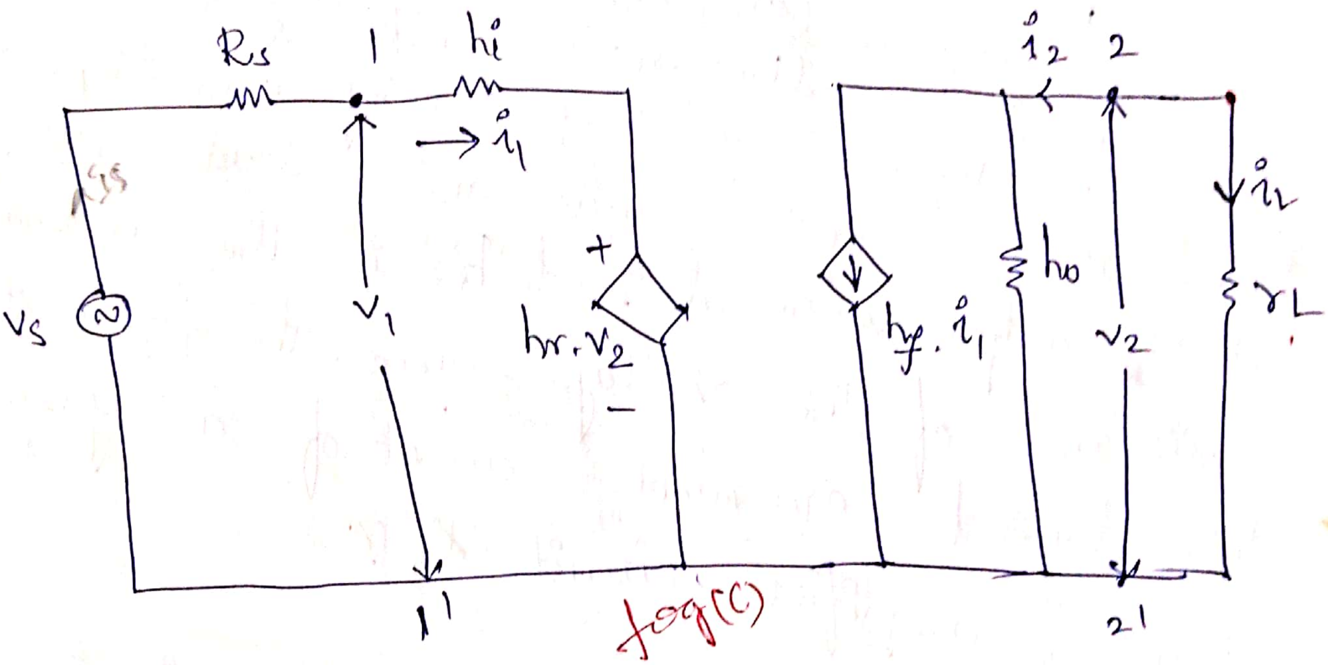
The following figure shows the small signal hybrid equivalent circuit of general transistor amplifier.

we shall derive the expressions for current gain (Ai), voltage gain (Av), input resistance (Ri) & Output resistance (Ro)



Hybrid Equivalent circuit for general amplifier circuit.

The expressions can be obtained by using following relations-

$$V_1 = h_i \cdot i_1 + h_r \cdot V_2 \quad \text{—— (1)}$$

$$i_2 = h_f \cdot i_1 + h_o \cdot V_2 \quad \text{—— (2)}$$

The voltage drop across 'r_L' is equal to voltage across output terminals 2-2'.

$$\therefore V_2 = i_L \cdot r_L \quad (\because i_L = -i_2)$$

$$\boxed{V_2 = -i_2 \cdot r_L}$$

(i) Current gain ($-A_i$)

The ratio of output to input current

$$A_i = \frac{i_L}{i_1} = \frac{-i_2}{i_1}$$

Now substituting, $V_2 = -i_2 \cdot r_L$ in equation (2)

$$i_2 = h_f \cdot i_1 + h_o \cdot V_2$$

$$i_2 = h_f \cdot i_1 + h_o \cdot (-i_2 \cdot r_L)$$

$$i_2 = h_f \cdot i_1 - h_o i_2 \cdot r_L$$

$$i_2 + h_o i_2 r_L = h_f \cdot i_1$$

$$i_2 (1 + h_o r_L) = h_f i_1$$

$$\boxed{\therefore \frac{i_2}{i_1} = \frac{h_f}{1 + h_o r_L}}$$

$$\therefore A_i = -\frac{i_2}{i_1}$$

$$\therefore \boxed{A_i = \frac{-h_f}{1+h_o \cdot r_L}} \quad \text{--- (3)}$$

(ii) <u>Input Resistance (R_i)</u>:-

Input resistance which we see looking into input terminals 1-1' is called amplifier input resistance.

$$R_i = \frac{V_1}{i_1}$$

Substituting $V_2 = -i_2 r_L$ in equation (1)

$$V_1 = h_i \cdot i_1 + h_r \cdot V_2$$

$$V_1 = h_i \cdot i_1 + h_r \cdot (-i_2 r_L)$$

$$V_1 = h_i \cdot i_1 - h_r \cdot i_2 r_L$$

Divide this equation both sides by 'i_1'.

$$\frac{V_1}{i_1} = h_i - \frac{h_r \, i_2 \cdot r_L}{i_1}$$

Replacing '$\frac{i_2}{i_1}$' with "$-A_i$"

$$\boxed{R_i = h_i + h_r \cdot A_i \cdot r_L} \quad (4)$$

Substituting $A_i = \frac{-h_f}{1+h_o r_L}$ in the above equation,

Now, Substitute $R_i = h_i + h_r\left(\frac{-h_f}{1+h_o r_L}\right) r_L$

$$\boxed{\therefore R_i = h_i - \frac{h_r h_f}{h_o + \frac{1}{r_L}}} \quad (5)$$

(iii) Voltage Gain (A_v):-

It is defined as the ratio of output voltage 'V_2' to input voltage 'V_1'.

$$\therefore A_v = \frac{V_2}{V_1} = \frac{-i_2 r_L}{V_1}$$

We know $A_i = -\frac{i_2}{i_1}$

$$\Rightarrow i_2 = -A_i \cdot i_1$$

Sub. in 'A_v' we get,

$$\therefore A_v = A_i \cdot r_L \left(\frac{i_1}{v_1}\right)$$

$$\boxed{A_v = \frac{A_i \cdot r_L}{R_i}} \quad \text{—} (6)$$

Substituting $A_i = -\dfrac{h_f}{1+h_o r_L}$ in equation (6)

& $R_i = h_i - \dfrac{h_r h_f}{h_o + 1/r_L}$ in above equation

$$A_v = \frac{\left(\dfrac{-h_f r_L}{1+h_o r_L}\right)}{\left(h_i - \dfrac{h_r h_f}{h_o + 1/r_L}\right)}$$

$\rightarrow \dfrac{h_r h_f r_L}{h_o r_L + 1}$

$$A_v = \frac{-h_f r_L / \cancel{1+h_o r_L}}{h_i(h_o r_L + 1) - h_r h_f r_L / \cancel{1+h_o r_L}}$$

$$\boxed{A_v = \frac{-h_f r_L}{h_i + (h_i h_o - h_r h_f) r_L}}$$

Replace $h_i h_o - h_r h_f$ with Δh in above equation

$$\therefore \boxed{A_V = -\frac{h_f \cdot r_L}{h_i + \Delta h \cdot r_L}} \quad -(7)$$

(iv) Output Resistance (R_o):-

It is obtained by setting the voltage source V_s to '0' & Load resistance 'R_L' to '∞' and by driving the output terminals 2-2' from a generator V_2 as shown in the figure below:

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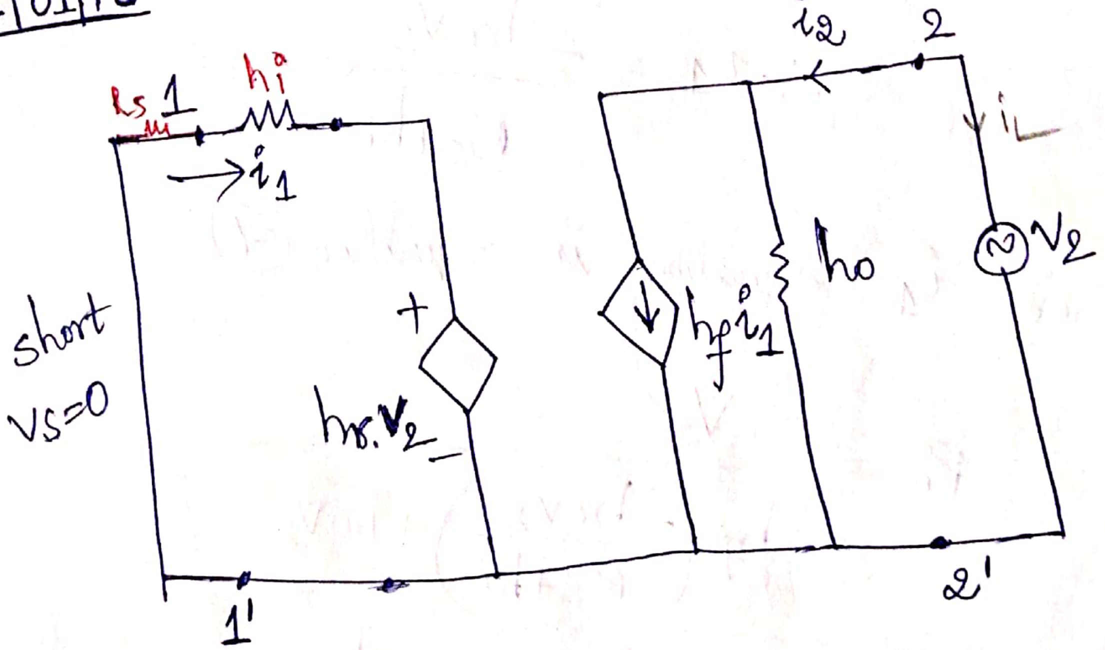

The output resistance 'R_o' is defined as output voltage 'V_2' to output current 'i_2' to the current drawn from V_2.

$$R_o = \frac{V_2}{i_2}$$

$$R_o = \frac{V_2}{h_f i_1 + h_o V_2} \quad \text{--- (7)}$$

The current 'i_1' is obtained applying KVL to the input circuit.

$$R_s i_1 + h_i i_1 + h_r V_2 = 0$$

$$\therefore i_1 = -\frac{h_r V_2}{R_s + h_i}$$

Sub. i_1 equation in equation (7)

$$\therefore R_o = \frac{V_2}{h_f \left(-\frac{h_r V_2}{R_s + h_i} \right) + h_o V_2}$$

$$\boxed{R_o = \frac{R_s + h_i}{(R_s + h_i) h_o - h_r h_f}}$$

$$R_o = \frac{R_s + h_i}{R_s h_o + (h_i h_o - h_r h_f)}$$

$$\boxed{R_o = \frac{R_s + h_i}{R_s h_o + \Delta h}} \quad — (8)$$

If $R_s = 0$; $\boxed{R_o = \frac{h_i}{\Delta h}}$

Output admittance:

$$Y_o = \frac{i_2}{V_2}$$

But $i_2 = h_f i_1 + h_o . V_2$

divide by V_2; we get

$$\frac{i_2}{V_2} = h_f \frac{i_1}{V_2} + h_o . \cancel{\frac{V_2}{V_2}}$$

Apply KVL to input,

$$R_s i_1 + h_i i_1 + h_r V_2 = 0$$

$$i_1 (R_s + h_i) + h_r V_2 = 0$$

From; $\frac{i_1}{V_2} = \frac{-h_r}{R_s + h_i}$

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$$Y_0 = \frac{I_2}{V_2} = h_f\left(\frac{-h_r}{R_s+h_i}\right)+h_o$$

$$\boxed{Y_0 = h_o - \frac{h_r h_f}{R_s+h_i}} \quad \text{--- (9)}$$

Note: Output admittance is a function of source resistance R_s.

Power Gain: (A_p)

It is defined as output power delivered to the load to the input power. $= \frac{P_2}{P_1}$

$$P_2 = V_2 \times i_L = -V_2 \times i_2 \qquad A_I = \frac{-i_2}{i_1}$$

$$P_1 = V_1 \times i_1$$

$$\therefore A_p = \frac{-V_2 \times i_2}{V_1 \times i_1} = + A_V \times A_I$$

$$A_p = A_i \times \frac{r_L}{R_i} \times A_i$$

$$A_V = \frac{A_i \times r_L}{R_i}$$

$$\boxed{\therefore A_p = (A_i)^2 \times \frac{r_L}{R_i}} \quad \text{--- (10)}$$

Overall voltage Gain: (A_{vs}) / Voltage Amplification taking into account the resistance of Source R_s / Effect of Source resistance on voltage gain:

It is important to take into account the source resistance R_s while calculating the voltage & current gains, results a overall voltage gain (A_{vs}) and overall current gain (A_{Is}).

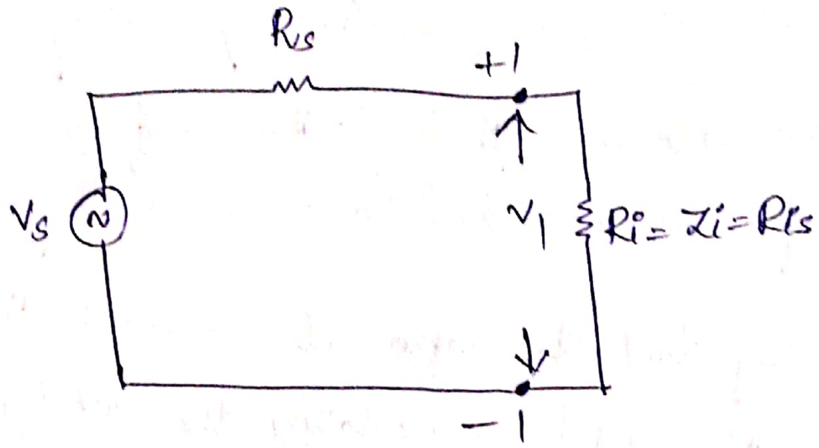
Overall voltage Gain: (A_{vs})

It is defined as the ratio of output voltage 'V_2' to Source voltage 'V_s'.

$$A_{vs} = \frac{V_2}{V_s} \times \frac{V_1}{V_1}$$

$$A_{vs} = A_v \times \frac{V_1}{V_s}$$

Consider the ~~nortons~~ Thevenin's equivalent circuit for the Source as shown below:



voltage drop across input resistance (Ri) is

$$V_1 = V_s \times \frac{R_i}{R_i+R_s}$$

$$\Rightarrow \frac{V_1}{V_s} = \frac{R_i}{R_i+R_s}$$

$$A_{vs} = A_v \times \frac{V_1}{V_s}$$

$$A_V = \frac{A_i \times r_L}{R_i}$$

$$= A_v \times \frac{R_i}{R_i+R_s}$$

$$A_{vs} = A_I \times \frac{r_L}{\cancel{R_i}} \times \frac{\cancel{R_i}}{R_i+R_s}$$

$$\boxed{A_{vs} = \frac{A_I \times r_L}{R_i+R_s}} \quad \text{— (ii)}$$

NOTE: If voltage source is a ideal voltage source, $R_s = 0$ then $\boxed{A_{vs} = \frac{A_I r_L}{R_i} = A_v}$

Overall Current Gain (A_{IS}) / Effect of Source resistance on Current gain / Current amplification taking into account the source resistance R_S:-

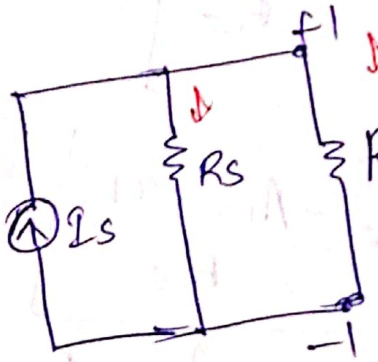
$$A_{is} = \frac{i_L}{i_s} = \frac{-i_2}{i_s}$$

$A_i = \frac{-i_2}{i_1}$

$$A_{is} = \frac{-i_2}{i_s} \times \frac{i_1}{i_1}$$

$$A_{is} = A_I \times \frac{i_1}{i_s}$$

Considering the Norton's equivalent circuit for source as shown below:



Current flowing through the amplifier input resistance may be written as

$$i_1 = i_s \times \frac{R_s}{R_s + R_i}$$

$$\Rightarrow \boxed{\frac{i_1}{i_s} = \frac{R_s}{R_s + R_i}}$$

$$\boxed{A_{is} = A_i \times \frac{R_s}{R_s + R_i}} \quad \text{--- (12)}$$

Relationship between A_{vs} & A_{is}:

we know that $A_{vs} = \dfrac{A_i \times r_L}{R_s + R_i}$

& $A_{is} = \dfrac{A_i \times R_s}{R_s + R_i}$

Taking the ratio of above two equations:

$$\boxed{\frac{A_{vs}}{A_{is}} = \frac{r_L}{R_s}} \quad \text{--- (13)}$$

$$\Rightarrow \boxed{A_{vs} = \frac{A_{is} \times r_L}{R_s}}$$

$$\Rightarrow \boxed{A_{is} = \frac{A_{vs} \times R_s}{r_L}}$$

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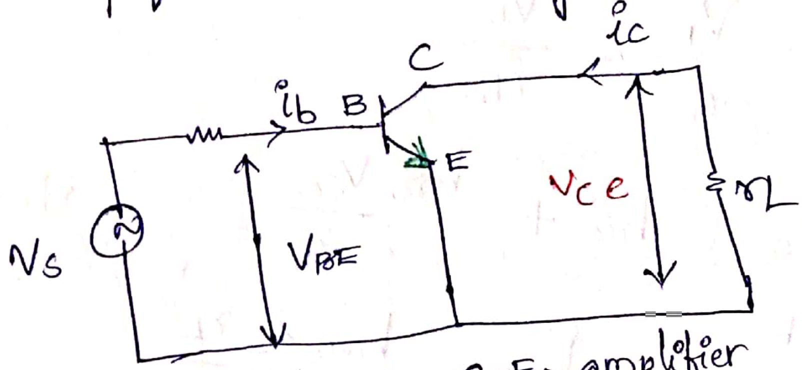
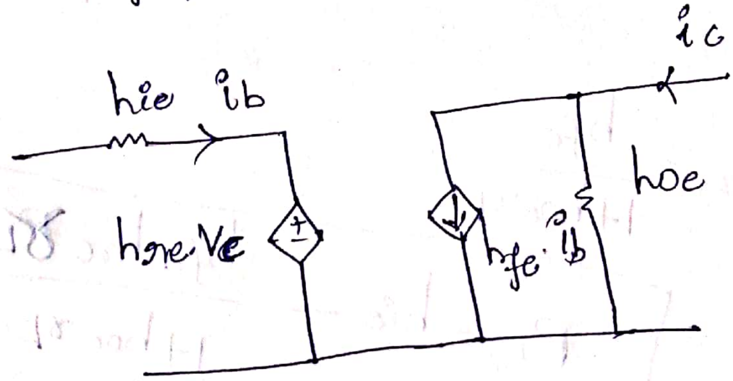
C.E amplifier – Exact analysis:-



freq (a) C.E. amplifier



$$R_i = h_{ie} - \frac{h_{fe} h_{re} r_L}{1 + h_{oe} r_L}$$

Hybrid parameters equations

$$\boxed{\begin{aligned} V_{be} &= h_{ie} i_b + h_{re} V_{ce} \quad (1) \\ i_c &= h_{fe} i_b + h_{oe} V_{ce} \quad (2) \end{aligned}}$$

1) Current gain (A_i):- $-\frac{i_2}{i_1} = -\frac{i_c}{i_b}$

$$-A_i = \frac{-h_{fe}}{1 + h_{oe} r_L} = \left(-\frac{i_c}{i_b}\right)$$

obtained by dividing (2) by i_b

(2)

2) Input resistance (R_{ie}):-

$$R_i = \frac{V_1}{i_1} = \frac{V_{be}}{i_b}$$

$$\frac{V_{be}}{i_b} = h_{ie} + h_{re}\frac{V_{ce}}{i_b}$$

$V_{ce} = -i_c r_L$

$$= h_{ie} - h_{re}\frac{i_c r_L}{i_b}$$

$$-\frac{i_c}{i_b} = -\frac{h_{fe}}{1+h_{oe}r_L}$$

$$\boxed{R_i = h_{ie} - \frac{h_{fe} h_{re} r_L}{1+h_{oe} r_L}}$$

3) Voltage gain (A_v):-

$V_2 = -i_c r_L$

$V_1 = i_1 r_L$

$$A_v = \frac{V_2}{V_1} = \frac{V_{ce}}{V_{be}}$$

$$A_v = -\frac{i_c r_L}{V_{be}}$$

$$A_v = -\frac{i_c}{i_b} \times r_L \cdot \frac{i_b}{V_{be}}$$

$$\boxed{A_v = \frac{A_{ie} r_L}{R_{ie}}}$$

4) <u>Output admittance (Y_o):-</u>

$$Y_o = \frac{i_2}{V_2} = \frac{i_{ce}}{V_{ce}}$$

$$i_{ce} = h_{fe} i_b + h_{oe} V_{ce}$$

$$\frac{i_{ce}}{V_{ce}} = h_{fe} \frac{i_b}{V_{ce}} + h_{oe}$$

$$Y_o = \frac{i_{ce}}{V_{ce}} = h_{oe} + h_{fe} \frac{i_b}{V_{ce}}$$

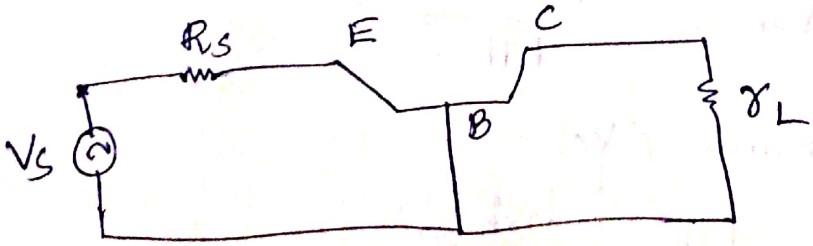
$$Y_o = h_{oe} + h_{fe} \cdot \frac{i_b}{V_{ce}}$$

Applying KVL $\quad R_s i_b + h_{ie} i_b + h_{ie} V_{ce} = 0$

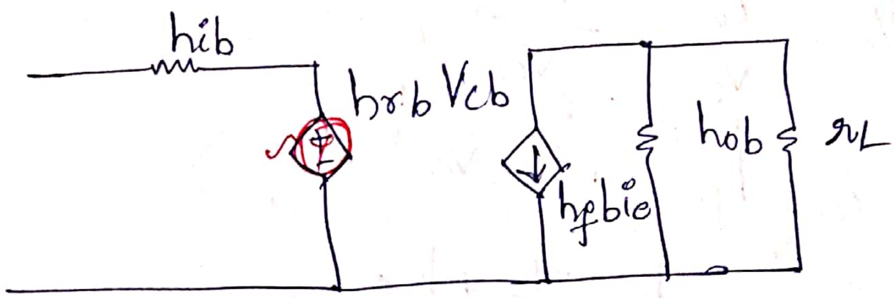
$$\frac{i_b}{V_{ce}} = -\frac{h_{re}}{R_s + h_{ie}}$$

$$\boxed{Y_o = h_{oe} - \frac{h_{re}}{R_s + h_{ie}}} \quad h_{fe}$$

2) C.B. amplifier exact analysis



Hybrid model



Hybrid equations :-

$$V_{eb} = h_{ib}\, i_e + h_{rb}\, V_{cb}$$

$$i_{cb} = h_{fb} i_e + h_{ob} V_{cb}$$

1) current gain (A_{ib})

$$A_{ib} = \frac{i_2}{i_1} = \frac{-i_{c}}{i_e} = \frac{-h_{fb}\, r_L}{1+h_{ob} r_L}$$

2) Input resistance (R_i)

$$R_i = h_{ib} - \frac{h_{fb}\, h_{rb}\, r_L}{1+h_{ob} r_L}$$

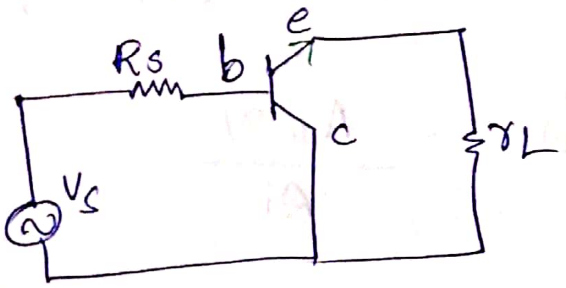
3) voltage gain (A_{Vb})

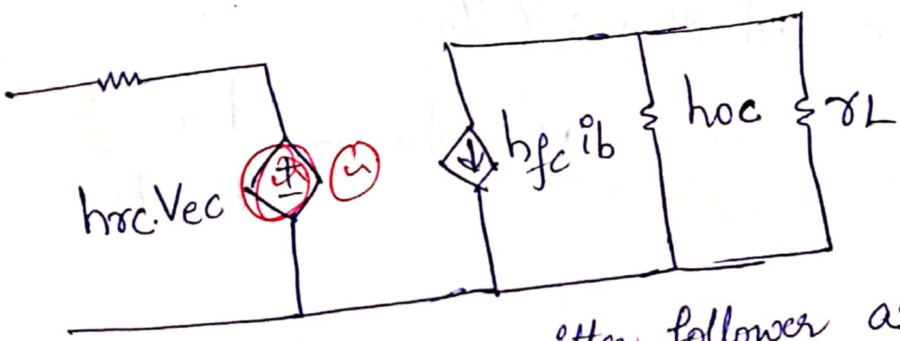
$$A_{Vb} = \frac{A_{ib} \cdot r_L}{R_{ib}}$$

4) Output admittance (Y_o)

$$Y_o = h_{ob} - \frac{h_{rb}\, h_{fb}}{R_s + h_{ib}}$$

3) C.C exact analysis (emitter follower):-





C.C is also Known as emitter follower as it's voltage gain is unity and hence any change in the base voltage appears as an equal change across the load at the emitter ie the o/p signal at the emitter follows the input signal at the base.

Hybrid equations of CC amplifier model

$$V_{bc} = h_{ic} i_b + h_{rc} V_{ec}$$

$$i_{ec} = h_{fc} i_b + h_{oc} V_{ec}$$

1) Current gain $(A_i) = \dfrac{-h_{fc}}{1+h_{oc}R_L}$

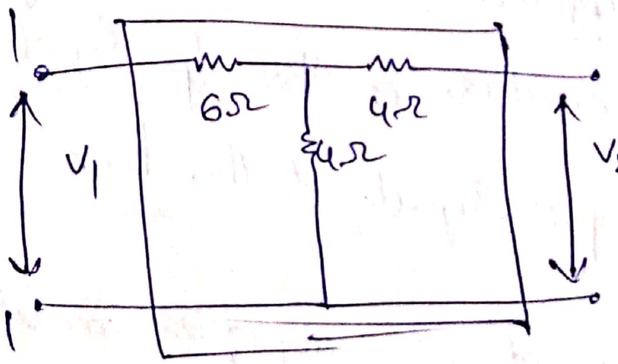
2) Input resistance (R_i) $= h_{ic} - \dfrac{h_{fc} h_{rc} r_L}{1+h_{oc} r_L}$

3) Voltage gain (A_v) $= \dfrac{-A_{ic} r_L}{R_{ic}}$

4) output admittance (Y_o)

$$Y_o = h_{oc} - \frac{h_{rc} \times h_{fc}}{R_s + h_{ic}}$$

Determine the value of h-parameters for the ckt shown below:



$V_1 = 6I_1 + 4(I_1+I_2) \Rightarrow V_1 = 10I_1 + 4I_2$

$V_2 = 4I_2 + 4(I_1+I_2) \Rightarrow V_2 = 4I_1 + 8I_2$

$V_1 = 10I_1$

$V_2 = 4I_1$

$\Rightarrow \dfrac{V_1}{V_2} = 2.5$

$h_{11} = \dfrac{V_1}{I_1} = 8\Omega$

$h_{21} = \dfrac{i_2}{i_1}$

$h_{12} = \frac{1}{2}$, $h_{21} = \frac{I_2}{I_1} = \frac{1}{2}$

$h_{22} = 1/8\ \Omega$

2) A transistor is used in a CB amplifier has the following values of h-parameters $h_{ib} = 28\Omega$, $h_{fb} = -0.98A$, $h_{rb} = 5\times10^{-4}$, $h_{ob} = 0.34\times10^{-6} S$

Calculate the values of I/p resistance, O/p resistance, current & voltage gain if $R_L = 1.2K\Omega$. assumes $R_s = 0$

Sol

$$A_i = \frac{-h_{fb}}{1+h_{or}L} = 0.98$$

$$R_i = h_{ib} - \frac{h_{fb}h_{rb}R_L}{1+h_{or}L} = 28.6$$

$$A_v = \frac{A_i r_L}{Z_i} = 41.11$$

$$R_o = h_{ob} - \frac{h_i h_f}{R_s + h_i} = 56K\Omega = 55.8\ k\Omega$$

The h-parameter of a transistor used in a CE are $h_{ie} = 1K\Omega$, $h_{re} = 1\times10^{-4}$, $h_{fe} = 50$, $h_{oe} = 100\ \mu S$

Supplying $r_L = 1K\Omega$, the transistor is $R_s = 100\Omega$

Sol

$$A_i = \frac{-h_{fe}}{1+h_{oe}r_L} = -45.45$$

$$R_i = h_{ie} - \frac{h_{fe} h_{re} r_L}{1 + h_{oe} r_L} = 995.45$$

$A_v = -45.65$

$R_o = 10.2 K\Omega$

$R_{os} = R_o \| r_L = 911\Omega$

$$\frac{A_i\, r_L}{R_i + R_s}$$

$$A_{vs} = \frac{A_i \cdot r_L}{R_i + R_s} = -22.63$$

$\boxed{R_{os} = R_o \| r_L}$

$$A_{is} = \frac{A_i \times R_s}{R_s + R_i}$$

$R_{is} = R_i + R_s$

3) A transistor used in a c.c circuit as shown below has the following set of h-parameters.

$h_{ic} = 2K\Omega$, $h_{fc} = -51$, $h_{rc} = 1$, $h_{oc} = 25 \times 10^{-6}$ mhos.

Find the values of I/p & o/p resistances current & voltage gain of amplifier stage. use the exact hybrid formula's.