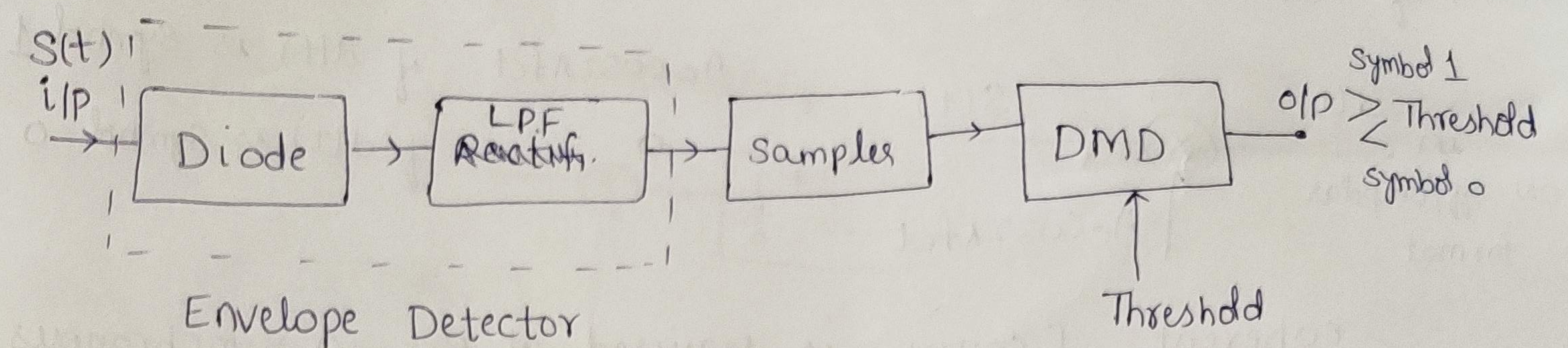
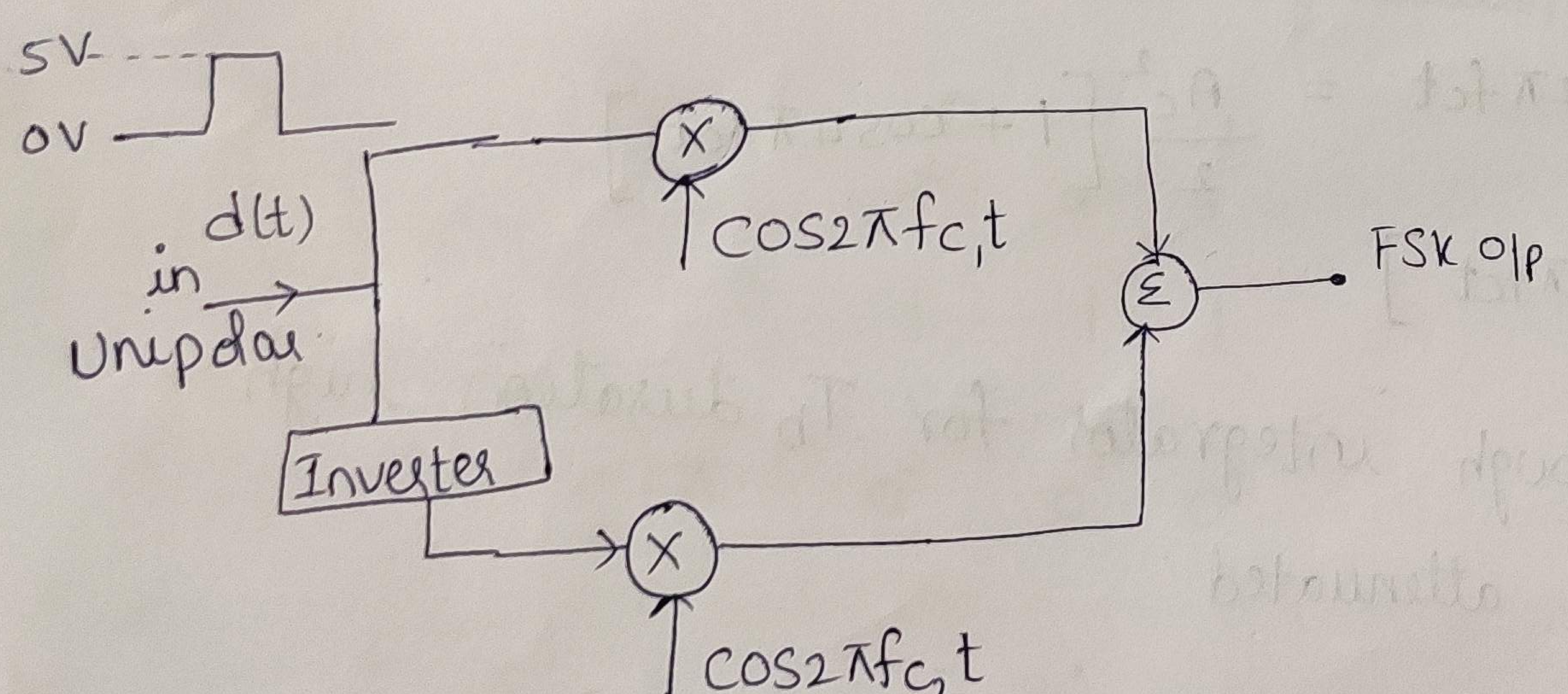
Non Coherent Detection :



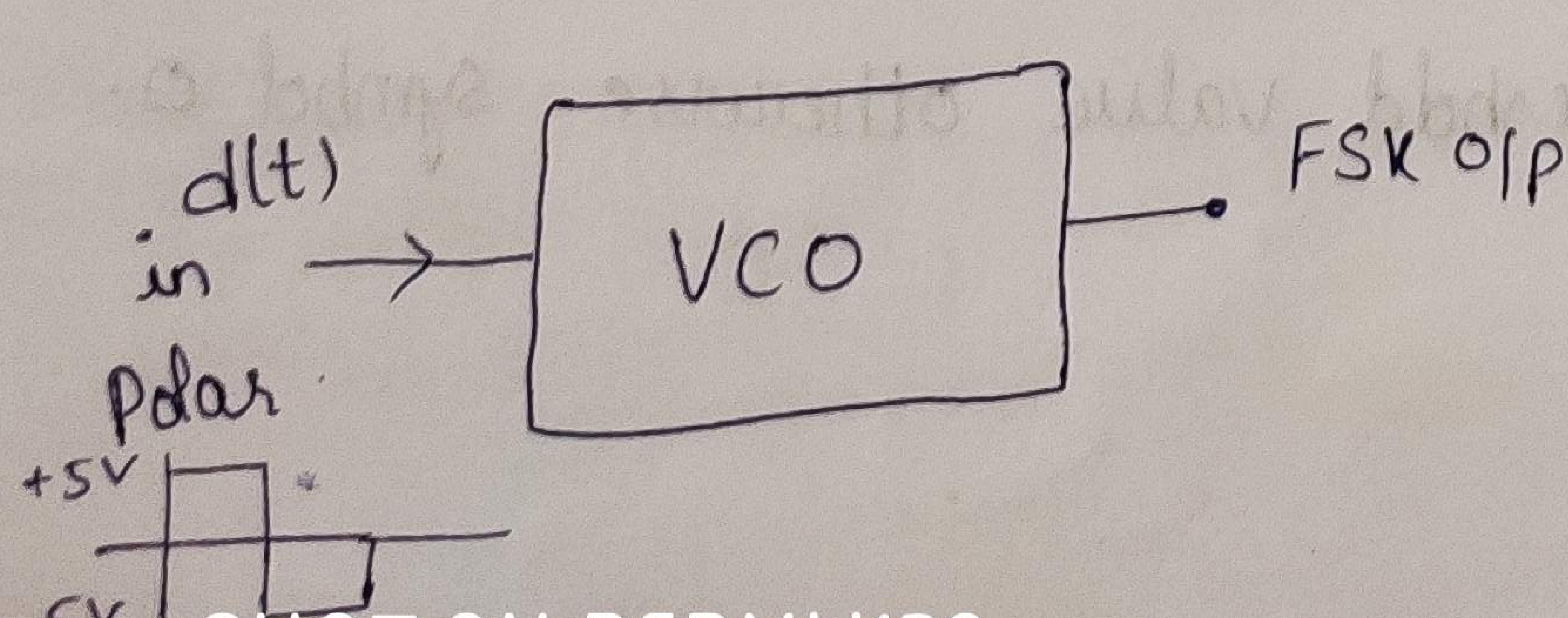

- Modulated signal is passed through the envelope detector in which envelope of ASK is detected.
- Detected envelope is sampled, and each sample is compared with preset threshold value and decision is taken in favour of symbol 1 or symbol 0.
- Threshold value depends on the type of line coding technique.

FSK Generation : -

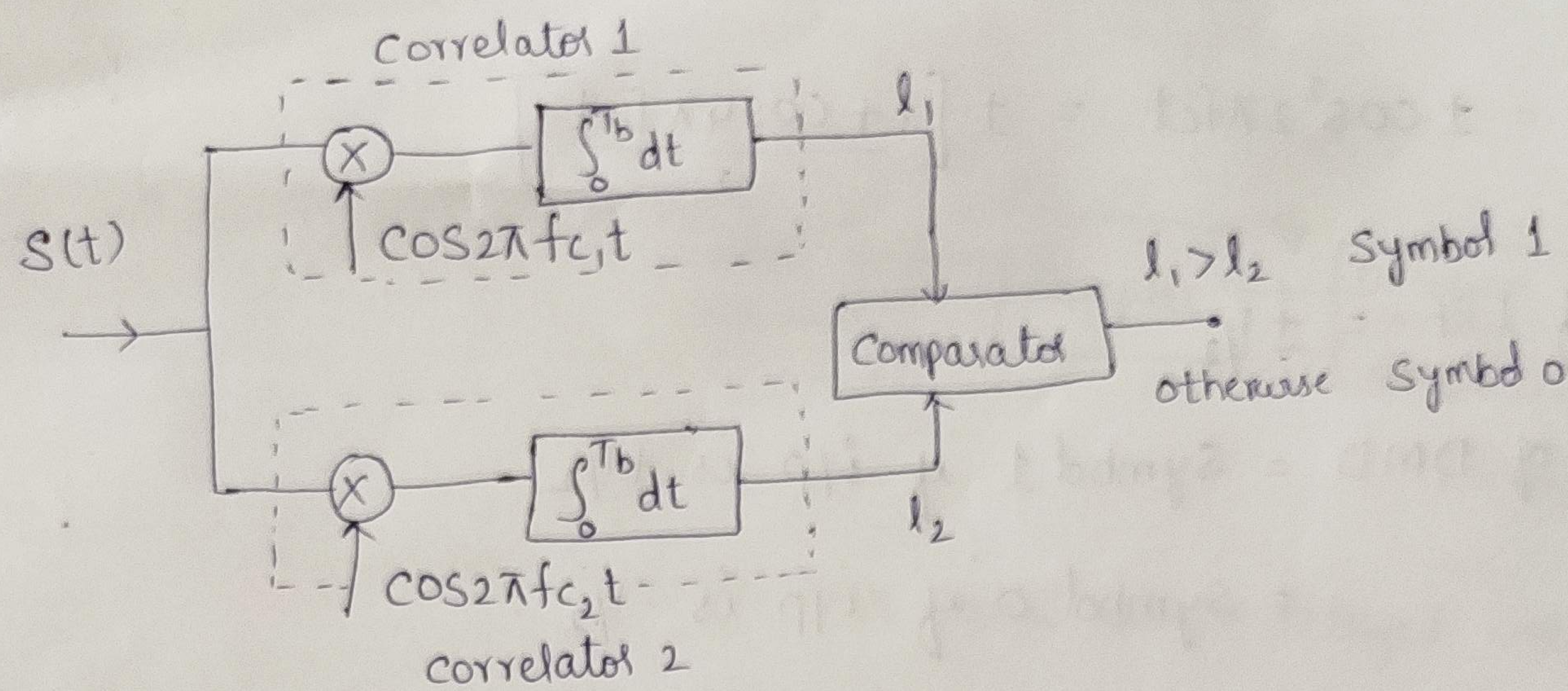



$S(t) = \cos 2\pi f_{c_1} t$ Symbol 1

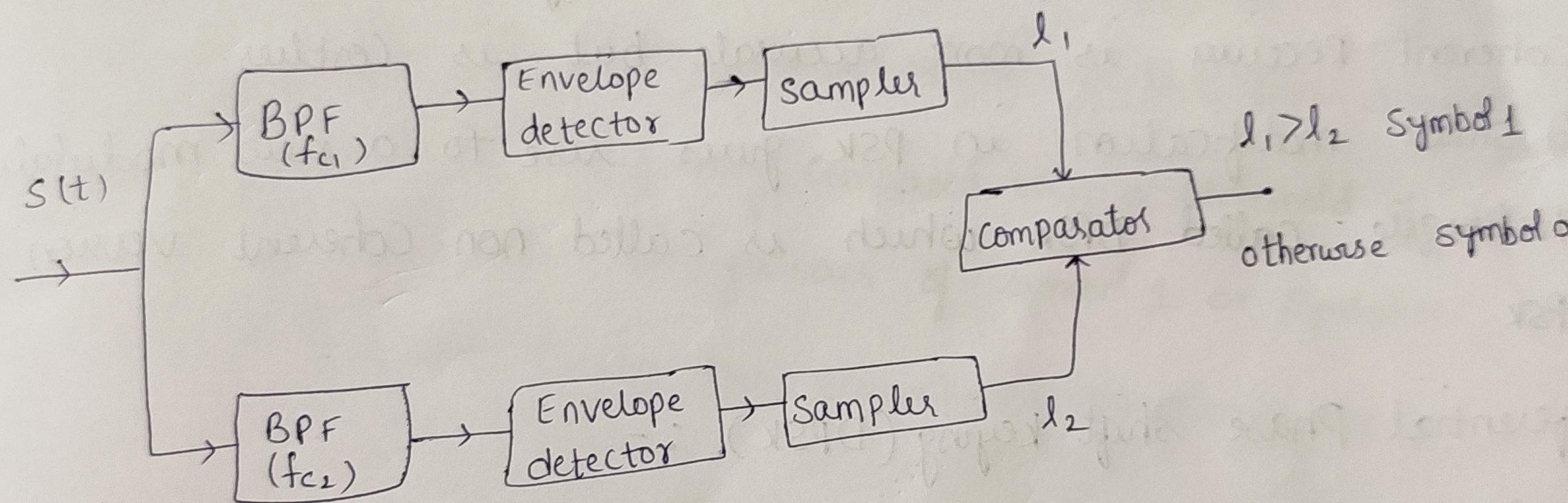
$\cos 2\pi f_{c_2} t$ Symbol 0



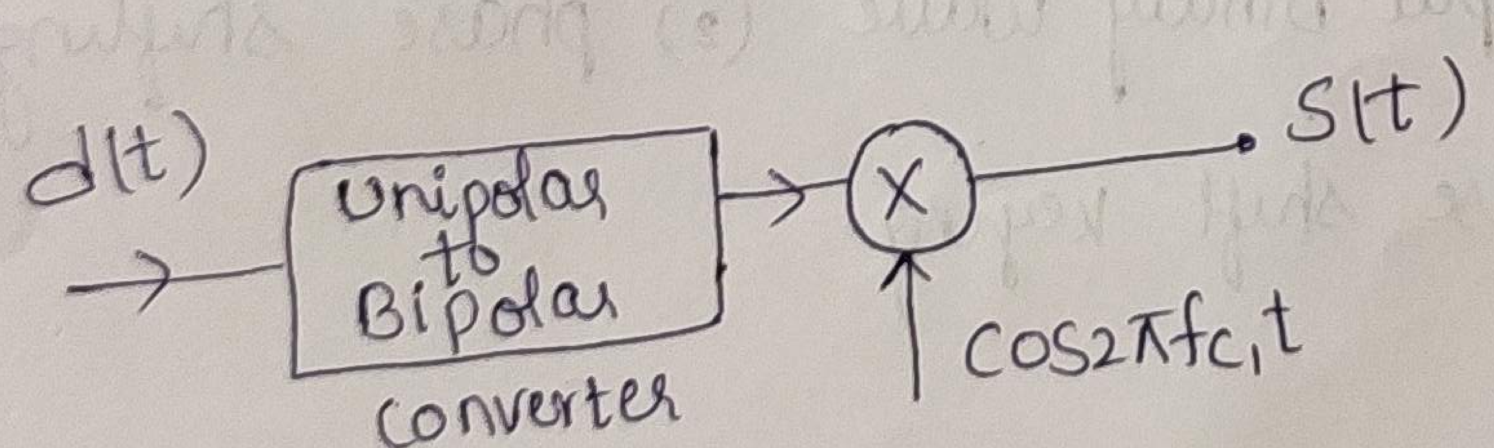
Coherent detection :




Non- Coherent Detection :




PSK Generation :

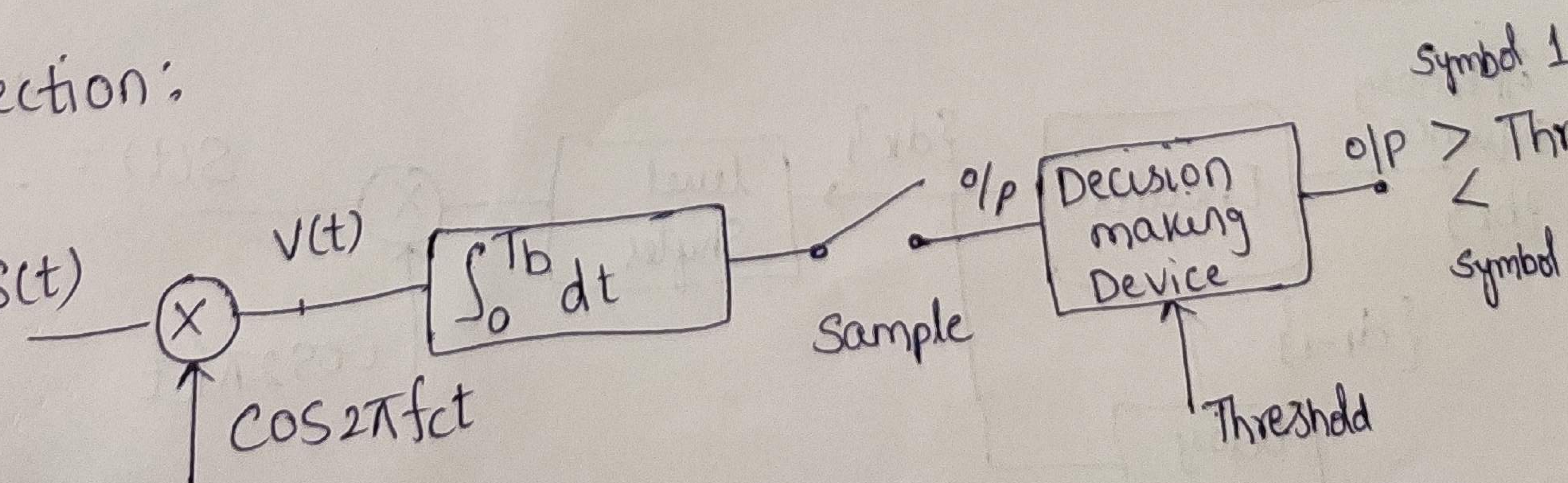



$S(t) = A_c \cos 2\pi f_c t$ symbol 1

$- A_c \cos 2\pi f_c t$ symbol 0

$\cos(2\pi f_c t + \pi) = - \cos 2\pi f_c t$

Coherent Detection:



$V(t) = S(t) \times c(t)$

$V(t) = \pm \cos^2 2\pi f_c t = \pm \left[\frac{1+\cos 4\pi f_c t}{2}\right]$

o/p of LPF $= \pm 1/2$

o/p of DMD = Symbol 1 if i/p is $+1/2$

= Symbol 0 if i/p is $-1/2$.

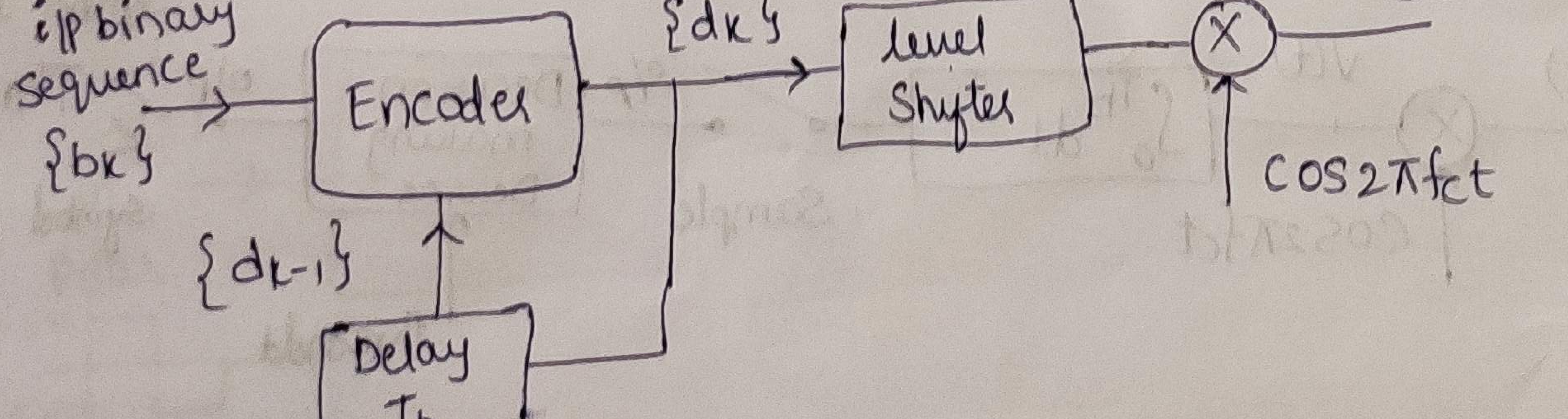
- Non Coherent Detection is not possible in PSK because it has constant envelope, and constant frequency.
- Coherent Receiver is more accurate but is Costlier.
- A small modification in PSK gives rise to a new modulation Technique called DPSK which is called non-Coherent version of PSK.

## Differential Phase shift keying (DPSK) :-

It eliminates the need for a coherent reference signal at the receiver by combining two basic operations at the transmitter. (1) differential encoding of the input binary wave (2) phase shifting. hence the name differential phase shift keying.

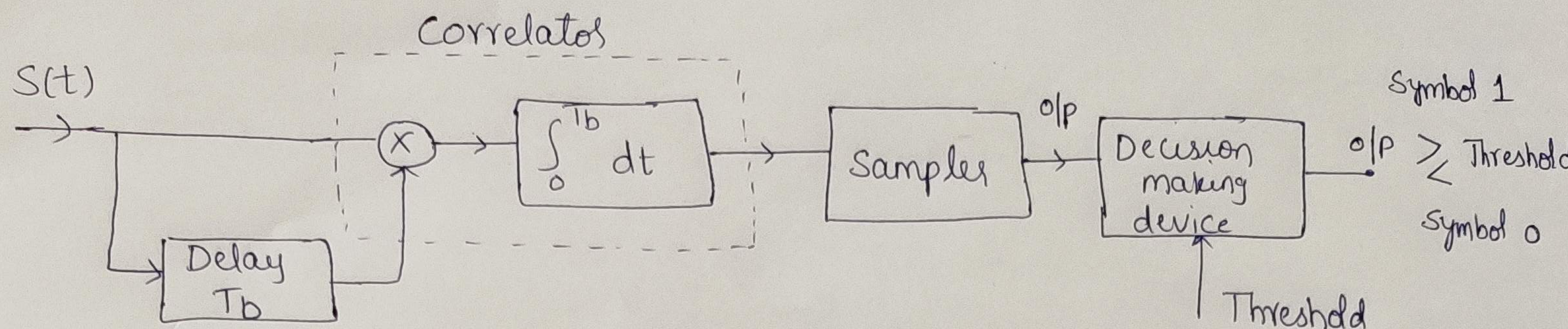
### DPSK Transmitter:

$$S(t) = \begin{cases} +\cos 2\pi f_c t & S-1 \\ -\cos 2\pi f_c t & S-0 \end{cases}$$




The Encoder used is EX-NOR gate $[d_k = b_k d_{k-1} + b_k' d_{k-1}']$

## DPSK Receiver :




At the receiver input, the received DPSK signal and a delayed version of it are applied to a correlator.

The resulting correlator output is proportional to the cosine of the difference between the carrier phase angles in the two correlator inputs.

The correlator output is finally compared with a threshold of zero volts and a decision is taken in favour of symbol 1 or symbol 0.

The below table illustrates the generation & detection of DPSK.

| | | | | | | |
|---|---|---|---|---|---|---|
| $\{b_k\}$ | | 0 | 1 | 0 | 1 | 1 |
| $\{d_k\}$ | $1^*$ | 0 | 0 | 1 | 1 | 1 |
| DPSK | $0°$ | $\pi$ | $\pi$ | $0°$ | $0°$ | $0°$ |
| Rxed DPSK | 0 | $\pi$ | $\pi$ | $0°$ | $0°$ | $0°$ |
| o/p of Correlator | | -1 | +1 | -1 | +1 | +1 |
| DMD o/p | | 0 | 1 | 0 | 1 | 1 |

* indicates an arbitrary value. It can be symbol 1 (or) symbol 0.

# M - ary Modulation Techinques:

M-ary data - more than 2 Symbols

$M = 2^n \rightarrow$ no. of bits in each symbol

↓

no. of symbols

Binary modulation is a special case of M-ary with $M = 2$

$M = 2^1$

Symbols in binary - $\{1, 0\}$

If $n = 2$, $M = 4$, Symbols are $\{\overset{\text{dibit}}{00}, 01, 10, 11\}$

If $n = 3$, $M = 8$, Symbols are $\{\underset{\text{Tribit}}{000}, 001, 010, ---- 111\}$

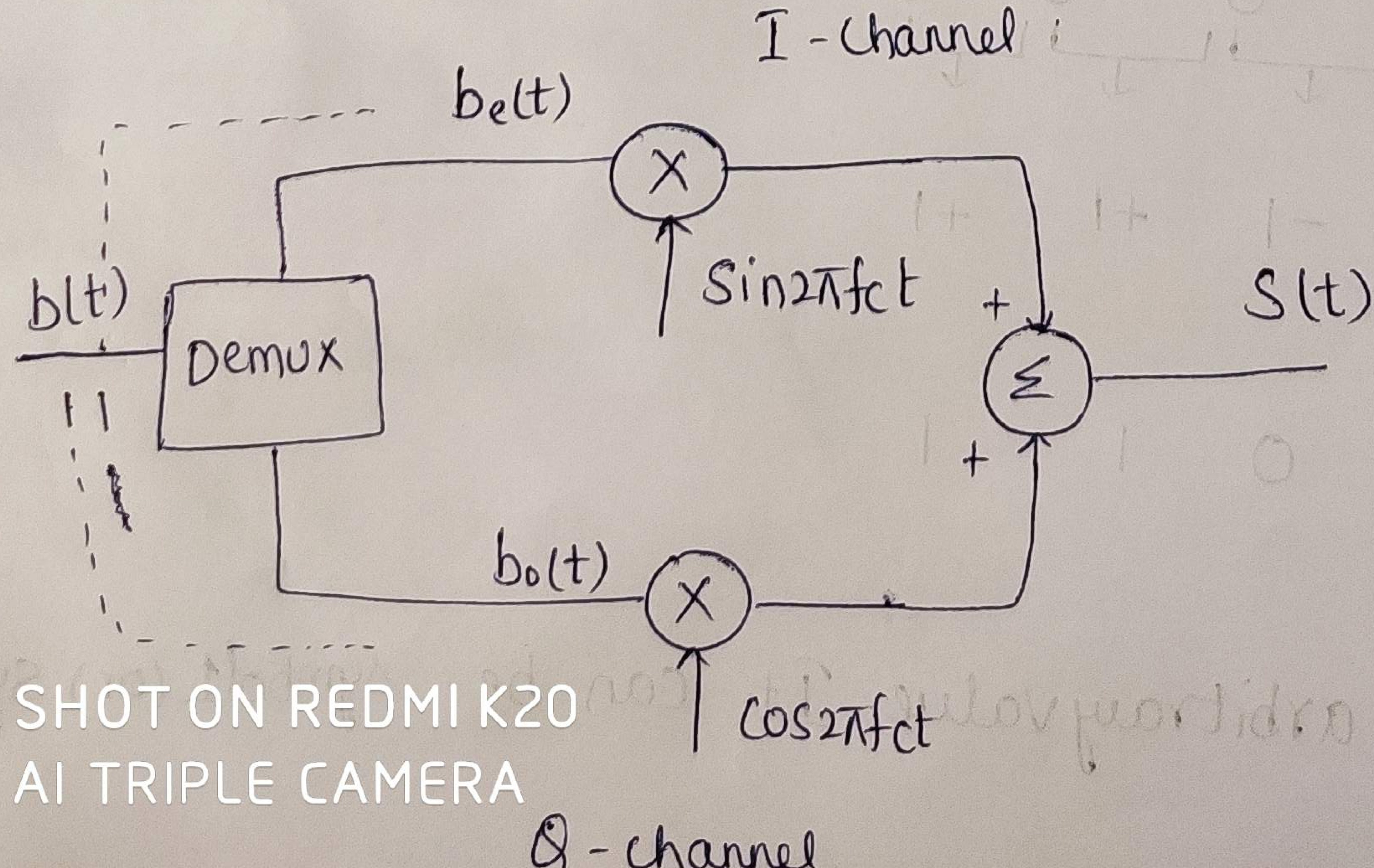
M-ary Signalling Schemes are preferred over binary to conserve bandwidth at the expense of increasing power.

$$T_s = nT_b$$

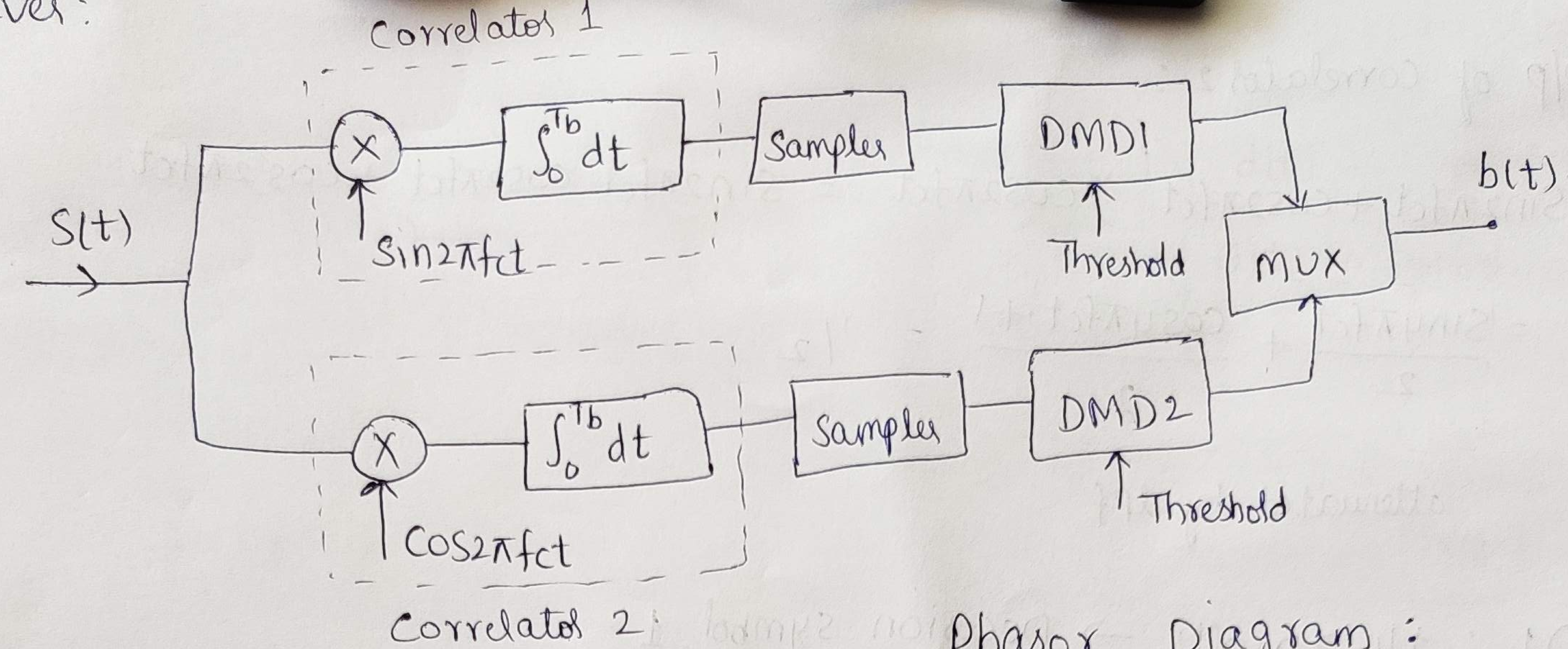
$T_s$ = symbol duration, $T_b$ = bit duration.

With $M=4$ and Preferring PSK we get Quadrature phase shift keying.

QPSK:



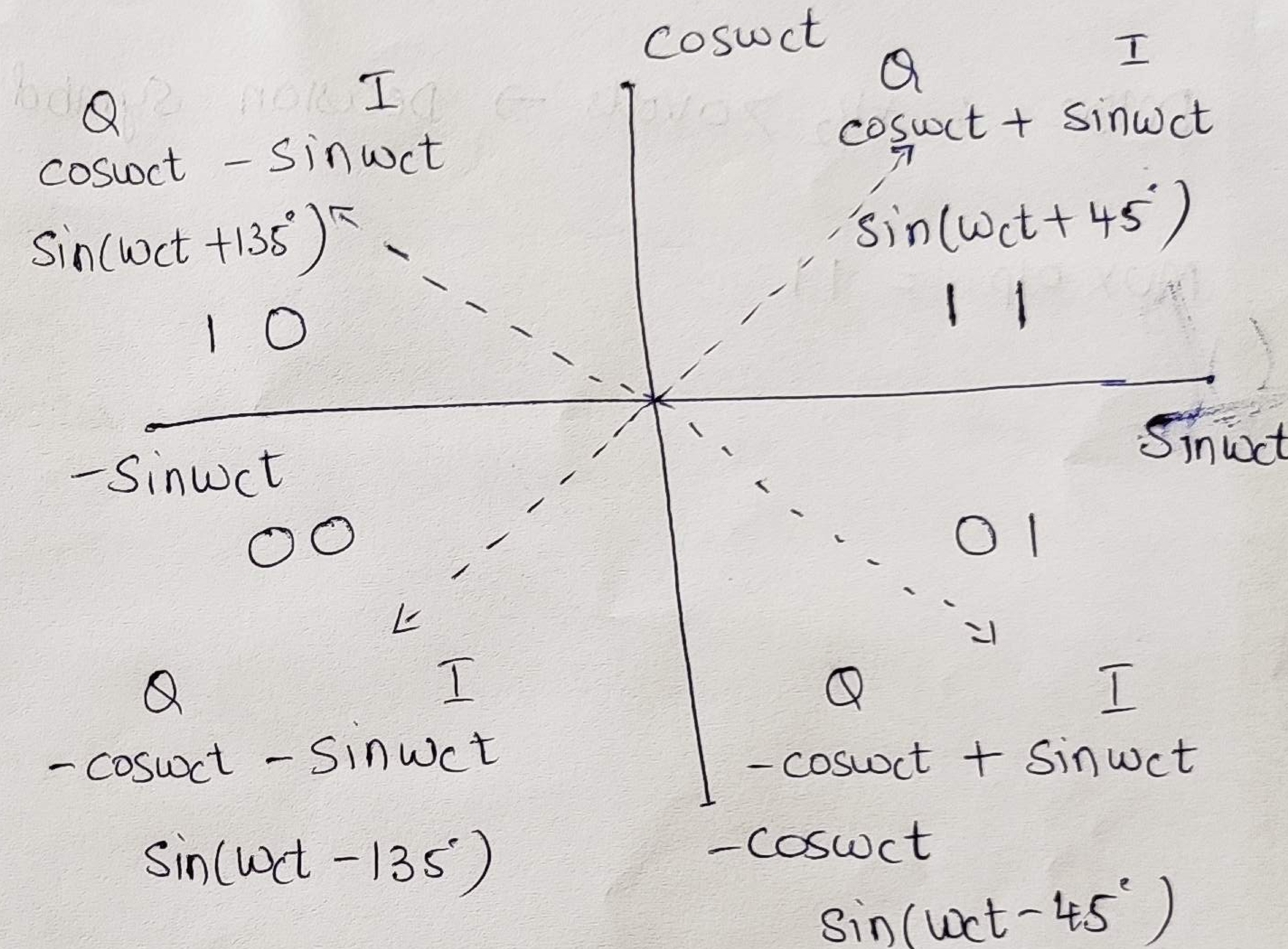
Receiver:




Truth table :

| Q | I | Phase angle |
|---|---|---|
| 1 | 1 | $+45^\circ$ |
| 1 | 0 | $+135^\circ$ |
| 0 | 0 | $-135^\circ / 225^\circ$ |
| 0 | 1 | $-45^\circ / 315^\circ$ |

Phasor Diagram :

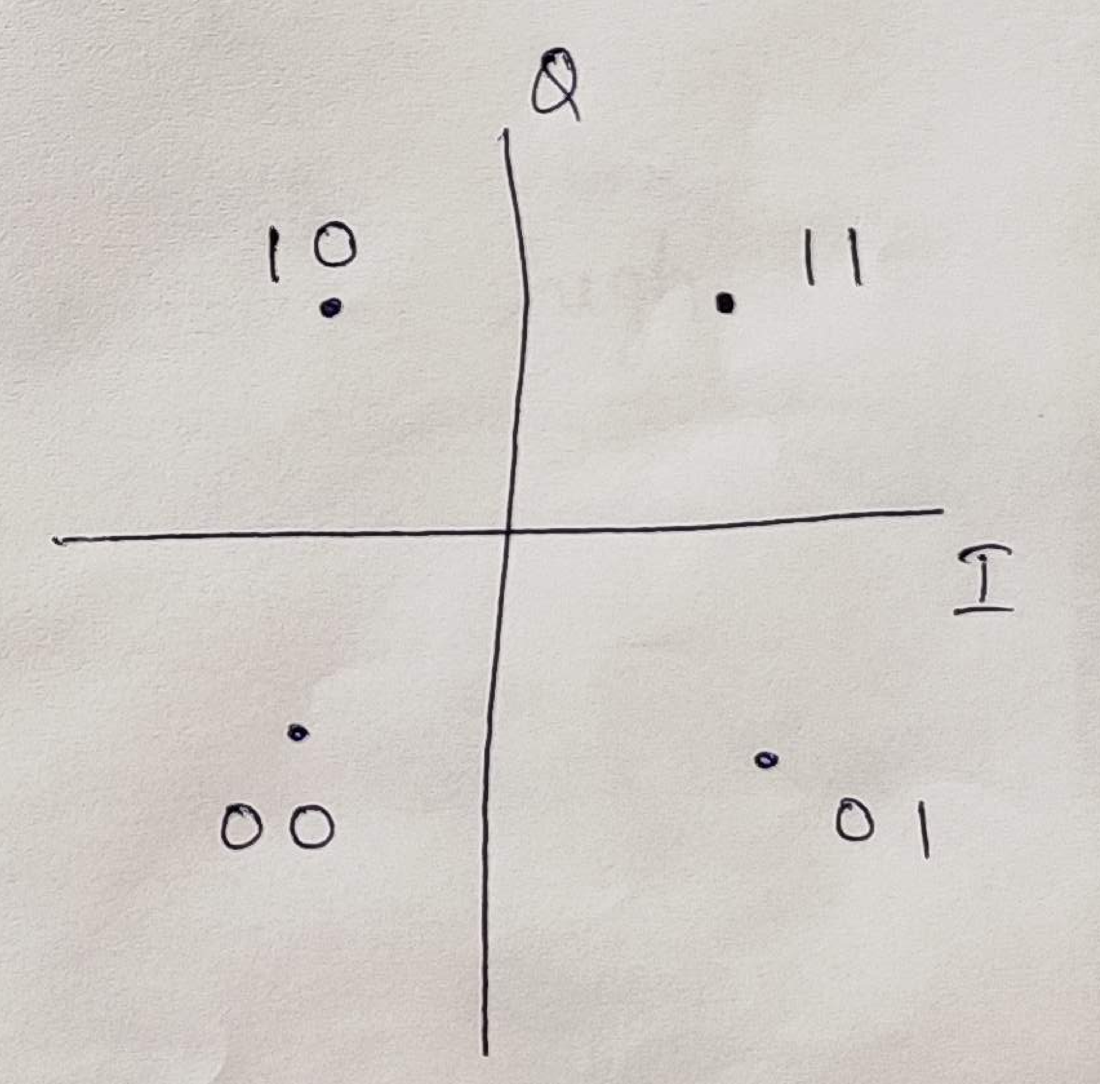



If input is 11 then

o/p of I-channel - $\sin\omega_c t$

o/p of Q-channel - $\cos\omega_c t$

o/p of summer - $\sin\omega_c t + \cos\omega_c t$




Constellation diagram

At the receiver if this o/p is received then

o/p of correlator 1 :

$$(+\sin\omega_c t + \cos\omega_c t)\sin 2\pi f_c t = \sin^2 2\pi f_c t + \sin 2\pi f_c t \cos 2\pi f_c t$$

$$= \frac{1 - \cos 4\pi f_c t}{2} + \boxed{\frac{\sin 4\pi f_c t}{2}}$$ Attenuated by lpf.

SHOT ON REDMI K20
AI TRIPLE CAMERA

o/p of Correlator 2 :

$(\sin 2\pi f_c t + \cos 2\pi f_c t)\cos 2\pi f_c t = \sin 2\pi f_c t \cdot \cos 2\pi f_c t + \cos^2 2\pi f_c t$

$= \underbrace{\frac{\sin 4\pi f_c t}{2} + \frac{\cos 4\pi f_c t + 1}{2}}_{\text{attenuated by lpf}} = 1/2$

DMD1 : $+1/2 > 0$ volts $\rightarrow$ Decision symbol 1

DMD2 : $+1/2 > 0$ volts $\rightarrow$ Decision symbol 1

MUX o/p = 11

I balanced modulator $=(-1)(\sin \omega_c t) = -1 \sin \omega_c t$

Q balanced modulator $=(-1)(\cos \omega_c t) = -1 \cos \omega_c t$ and the output of the linear summer is
$-1 \cos \omega_c t - 1 \sin \omega_c t = 1.414 \sin(\omega_c t - 135°)$

For the remaining dibit codes (01, 10, and 11), the procedure is the same. The results are shown in Figure 2-18a.

| Binary input Q | Binary input I | QPSK output phase |
|---|---|---|
| 0 | 0 | -135° |
| 0 | 1 | -45° |
| 1 | 0 | +135° |
| 1 | 1 | +45° |

(a)

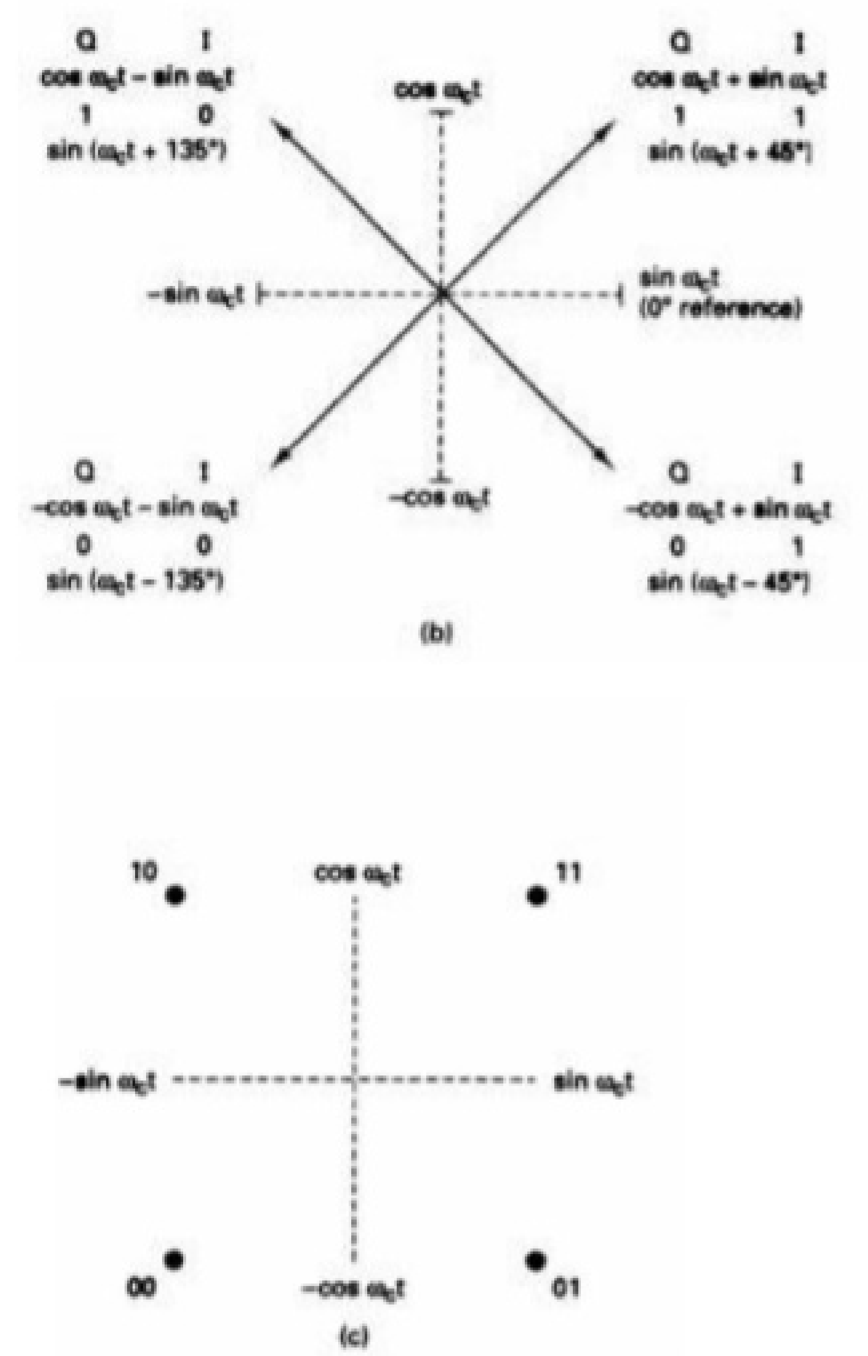



**FIGURE 2-18 QPSK modulator: (a) truth table; (b) phasor diagram; (c) constellation diagram**

In Figures 2-18b and c, it can be seen that with QPSK each of the four possible output phasors has exactly the same amplitude. Therefore, the binary information must be encoded entirely in the phase of the output signal



Figure 2-18b, it can be seen that the angular separation between any two adjacent phasors in QPSK is 90°.Therefore, a QPSK signal can undergo almost a+45° or -45° shift in phase during transmission and still retain the correct encoded information when demodulated at the receiver.

Figure 2-19 shows the output phase-versus-time relationship for a QPSK modulator.

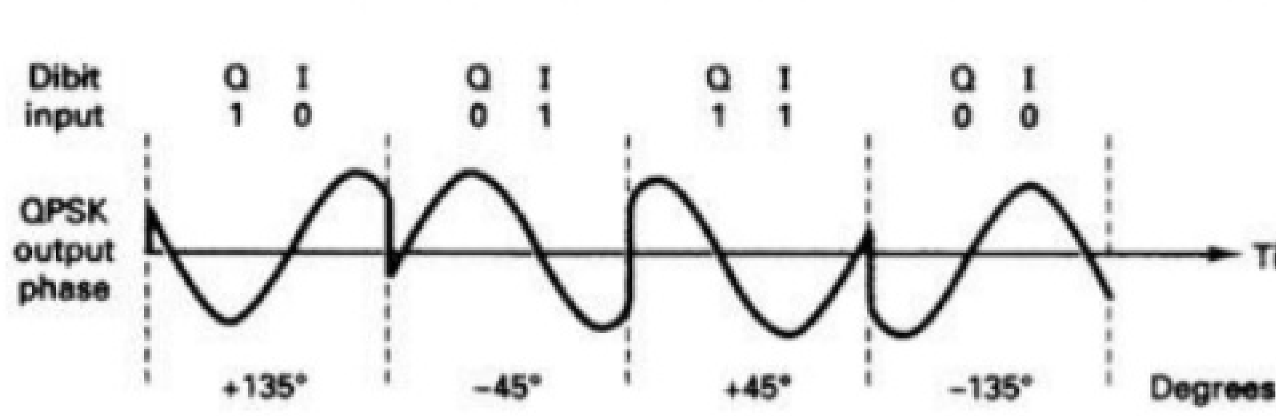



**FIGURE 2-19 Output phase-versus-time relationship for a PSK modulator**

PART-2

DATATRANSMISSION

**BASE BAND SIGNAL RECEIVER:**

**Consider that a binary encoded signal consists of a time sequence of voltage levels +V or –V. if there is a guard interval between the bits, the signal forms a sequence of positive and negative pulses. in either case there is no particular interest in preserving the waveform of the signal after reception .we are interested only in knowing within each bit interval whether the transmitted voltage was +V or –V. With noise present, the receives signal and noise together will yield sample values generally different from ±V. In this case, what deduction shall we make from the sample value concerning the transmitted bit?**

Suppose that the noise is gaussian and therefore the noise voltage has a probability density which is entirely symmetrical with respect to zero volts. Then the probability that the noise has increased the sample value is the same as the probability that the noise has decreased the sample value. It then seems entirely reasonable that we can do no better than to assume that if the sample value is positive the transmitted level was $+V$, and if the sample value is negative the transmitted level was $-V$. It is, of course, possible that at the sampling time the noise voltage may be of magnitude larger than $V$ and of a polarity opposite to the polarity assigned to the transmitted bit. In this case an error will be made as indicated in Fig. 11.1-1. Here the transmitted bit is represented by the voltage $+V$ which is sustained over an interval $T$ from $t_1$ to $t_2$. Noise has been superimposed on the level $+V$ so that the voltage $v$ represents the received signal and noise. If now the sampling should happen to take place at a time $t = t_1 + \Delta t$, an error will have been made.

We can reduce the probability of error by processing the received signal plus noise in such a manner that we are then able to find a sample time where the sample voltage due to the signal is emphasized relative to the sample voltage due to the noise. Such a processer (receiver) is shown in Fig. 11.1-2. The signal input during a bit interval is indicated. As a matter of convenience we have set $t = 0$ at the beginning of the interval. The waveform of the signal $s(t)$ before $t = 0$ and after $t = T$ has not been indicated since, as will appear, the operation of the receiver during each bit interval is independent of the waveform during past and future bit intervals.

The signal $s(t)$ with added white gaussian noise $n(t)$ of power spectral density $\eta/2$ is presented to an integrator. At time $t = 0+$ we require that capacitor $C$ be uncharged. Such a discharged condition may be ensured by a brief closing of switch $SW_1$ at time $t = 0-$, thus relieving $C$ of any charge it may have acquired during the previous interval. The sample is taken at the output of the integrator by closing this sampling switch $SW_2$. This sample is taken at the end of the bit interval, at $t = T$. The signal processing indicated in Fig. 11.1-2 is described by the phrase *integrate and dump*, the term *dump* referring to the abrupt discharge of the capacitor after each sampling.

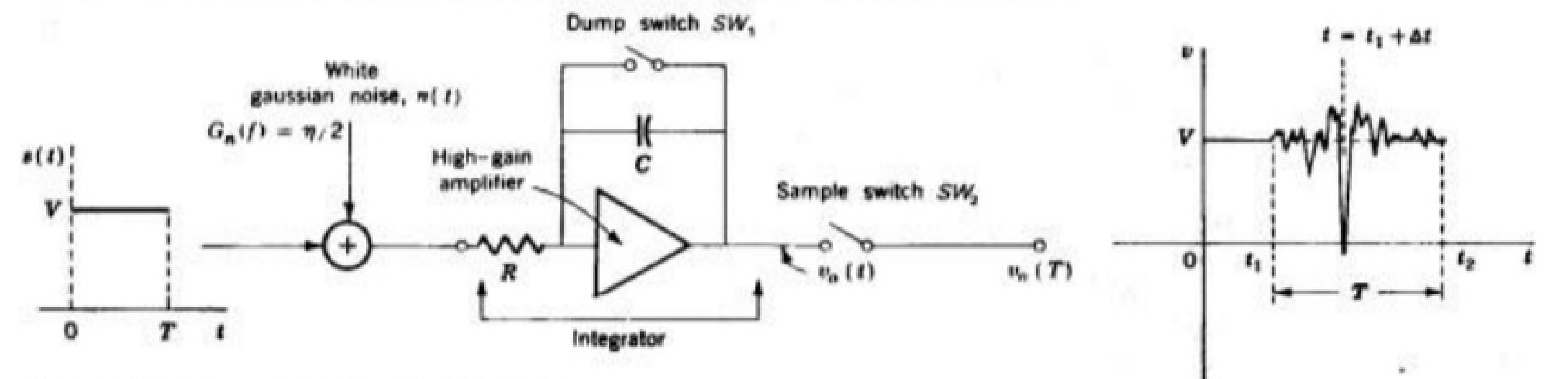



Figure 11.1-2 A receiver for a binary coded signal.

## Peak Signal to RMS Noise Output Voltage Ratio

The integrator yields an output which is the integral of its input multiplied by $1/RC$. Using $\tau = RC$, we have

$$v_o(T) = \frac{1}{\tau}\int_0^T [s(t) + n(t)]\, dt = \frac{1}{\tau}\int_0^T s(t)\, dt + \frac{1}{\tau}\int_0^T n(t)\, dt \tag{11.1-1}$$

The sample voltage due to the signal is

$$s_o(T) = \frac{1}{\tau}\int_0^T V\, dt = \frac{VT}{\tau} \tag{11.1-2}$$

The sample voltage due to the noise is

$$n_o(T) = \frac{1}{\tau}\int_0^T n(t)\, dt \tag{11.1-3}$$

This noise-sampling voltage $n_o(T)$ is a gaussian random variable in contrast with $n(t)$, which is a gaussian random process.

The variance of $n_o(T)$ was found in Sec. 7.9 [see Eq. (7.9-17)] to be

$$\sigma_o^2 = \overline{n_o^2(T)} = \frac{\eta T}{2\tau^2} \tag{11.1-4}$$

and, as noted in Sec. 7.3, $n_o(T)$ has a gaussian probability density.

The output of the integrator, before the sampling switch, is $v_o(t) = s_o(t) + n_o(t)$. As shown in Fig. 11.1-3$a$, the signal output $s_o(t)$ is a ramp, in each bit interval, of duration $T$. At the end of the interval the ramp attains the voltage $s_o(T)$ which is $+VT/\tau$ or $-VT/\tau$, depending on whether the bit is a 1 or a 0. At the end of each interval the switch $SW_1$ in Fig. 11.1-2 closes momentarily to discharge the capacitor so that $s_o(t)$ drops to zero. The noise $n_o(t)$, shown in Fig. 11.1-3$b$, also starts each interval with $n_o(0) = 0$ and has the random value $n_o(T)$ at the end of each interval. The sampling switch $SW_2$ closes briefly just before the closing of $SW_1$ and hence reads the voltage

$$v_o(T) = s_o(T) + n_o(T) \tag{11.1-5}$$

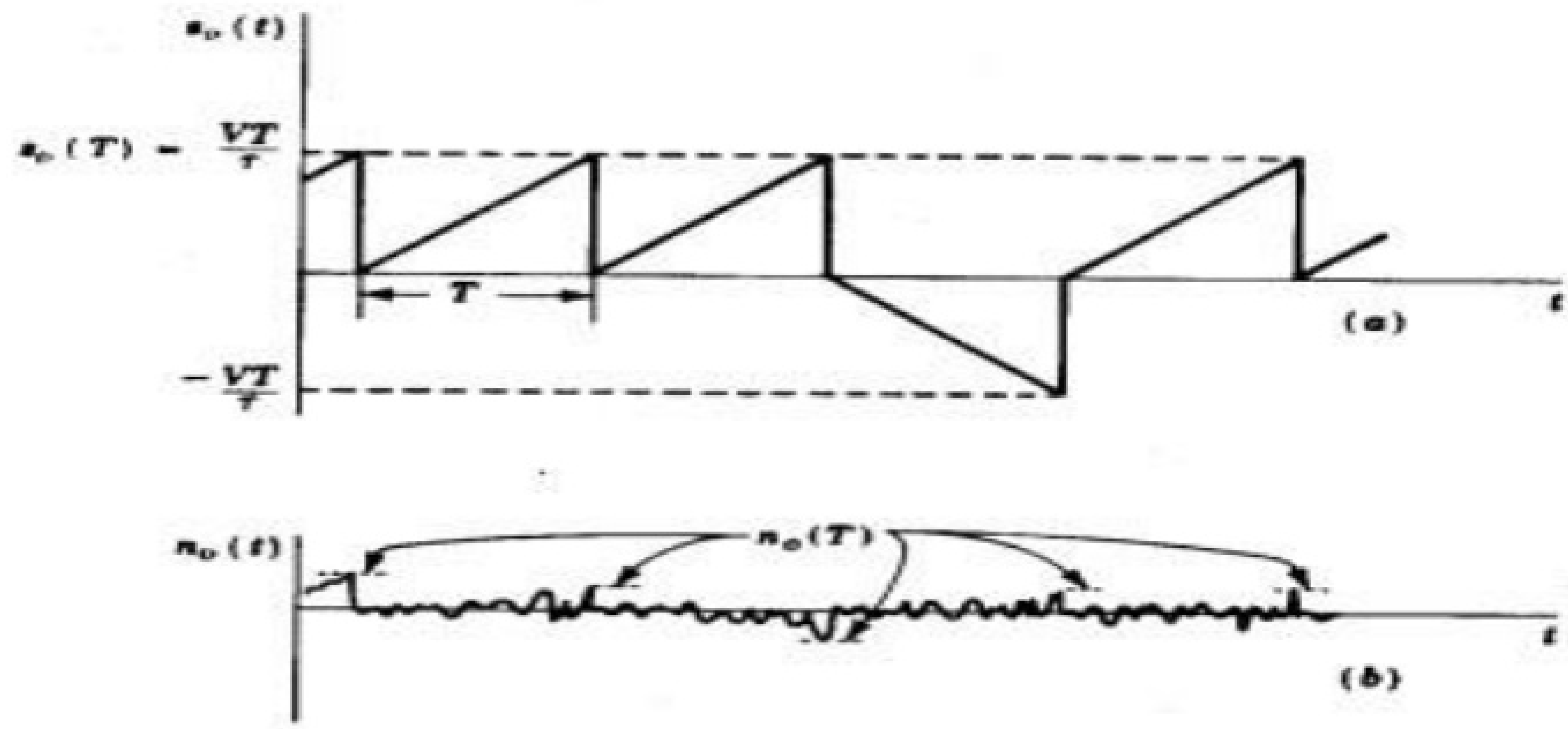



Figure 11.1-3 (a) The signal output and (b) the noise output of the integrator of Fig. 11.1-2.

We would naturally like the output signal voltage to be as large as possible in comparison with the noise voltage. Hence a figure of merit of interest is the signal-to-noise ratio

$$\frac{[s_o(T)]^2}{\overline{[n_o(T)]^2}} = \frac{2}{\eta} V^2 T \qquad (11.1\text{-}6)$$

This result is calculated from Eqs. (11.1-2) and (11.1-4). Note that the signal-to-noise ratio increases with increasing bit duration $T$ and that it depends on $V^2T$ which is the normalized energy of the bit signal. Therefore, a bit represented by a narrow, high amplitude signal and one by a wide, low amplitude signal are equally effective, provided $V^2T$ is kept constant.

It is instructive to note that the integrator filters the signal and the noise such that the signal voltage increases linearly with time, while the standard deviation (rms value) of the noise increases more slowly, as $\sqrt{T}$. Thus, the integrator enhances the signal relative to the noise, and this enhancement increases with time as shown in Eq. (11.1-6).

**PROBABILITY OF ERROR**

**Since the function of a receiver of a data transmission is to ditinguish the bit 1 from the bit 0 in the presence of noise, a most important charcteristic is the probability that an error will be made in such a determination.we now calculate this error probabilty $P_e$ for the integrate and dump receiver of fig 11.1-2**

We have seen that the probability density of the noise sample $n_o(T)$ is gaussian and hence appears as in Fig. 11.2-1. The density is therefore given by

$$f[n_o(T)] = \frac{e^{-n_o^2(T)/2\sigma_o^2}}{\sqrt{2\pi\sigma_o^2}} \tag{11.2-1}$$

where $\sigma_o^2$, the variance, is $\sigma_o^2 \equiv \overline{n_o^2(T)}$ given by Eq. (11.1-4). Suppose, then, that during some bit interval the input-signal voltage is held at, say, $-V$. Then, at the sample time, the signal sample voltage is $s_o(T) = -VT/\tau$, while the noise sample is $n_o(T)$. If $n_o(T)$ is positive and larger in magnitude than $VT/\tau$, the total sample voltage $v_o(T) = s_o(T) + n_o(T)$ will be positive. Such a positive sample voltage will result in an error, since as noted earlier, we have instructed the receiver to interpret such a positive sample voltage to mean that the signal voltage was $+V$ during the bit interval. The probability of such a misinterpretation, that is, the probability that $n_o(T) > VT/\tau$, is given by the area of the shaded region in Fig. 11.2-1. The probability of error is, using Eq. (11.2-1),

$$P_e = \int_{VT/\tau}^{\infty} f[n_o(T)]\, dn_o(T) = \int_{VT/\tau}^{\infty} \frac{e^{-n_o^2(T)/2\sigma_o^2}}{\sqrt{2\pi\sigma_o^2}}\, dn_o(T) \tag{11.2-2}$$

Defining $x \equiv n_o(T)/\sqrt{2}\sigma_o$, and using Eq. (11.1-4), Eq. (11.2-2) may be rewritten as

$$\begin{aligned} P_e &= \frac{1}{2}\frac{2}{\sqrt{\pi}}\int_{x=V\sqrt{T/\eta}}^{\infty} e^{-x^2}\, dx \\ &= \frac{1}{2}\operatorname{erfc}\left(V\sqrt{\frac{T}{\eta}}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{V^2T}{\eta}\right)^{1/2} = \frac{1}{2}\operatorname{erfc}\left(\frac{E_s}{\eta}\right)^{1/2} \end{aligned} \tag{11.2-3}$$

in which $E_s = V^2T$ is the signal energy of a bit.

If the signal voltage were held instead at $+V$ during some bit interval, then it is clear from the symmetry of the situation that the probability of error would again be given by $P_e$ in Eq. (11.2-3). Hence Eq. (11.2-3) gives $P_e$ quite generally.

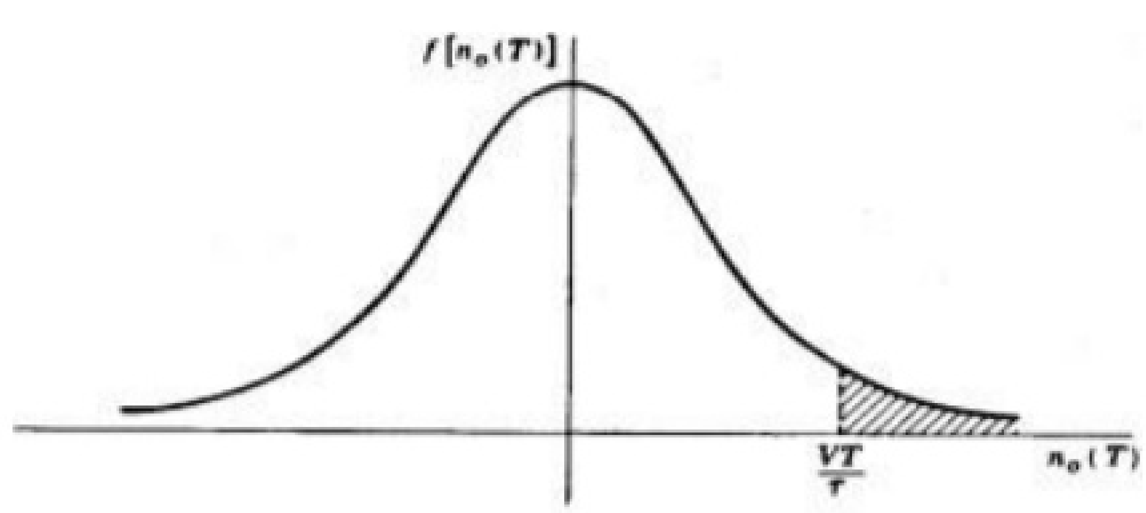



**Figure 11.2-1** The gaussian probability density of the noise sample $n_o(T)$.

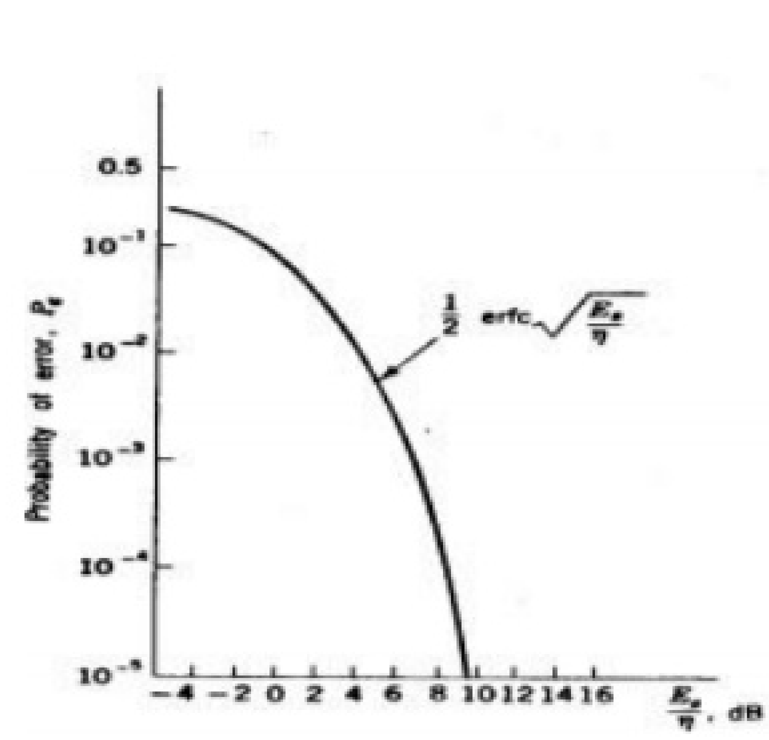



**Figure 11.2-2** Variation of $P_e$ versus $E_s/\eta$.



The probability of error $p_e$, as given in eq.(11.2-3),is plotted in fig.11.2-2.note that $p_e$ decreases rapidly as $E_s/\eta$ increases. The maximum value of $p_e$ is ½.thus ,even if the signal is entirely lost in the noise so that any determination of the receiver is a sheer guess, the receiver cannot bi wrong more than half the time on the average.

The probability of error $p_e$, as given in eq.(11.2-3),is plotted in fig.11.2-2.note that $p_e$ decreases rapidly as $E_s/\eta$ increases. The maximum value of $p_e$ is ½.thus ,even if the signal is entirely lost in the noise so that any determination of the receiver is a sheer guess, the receiver cannot bi wrong more than half the time on the average.

**THE OPTIMUM FILTER:**

In the receiver system of Fig 11.1-2, the signal was passed through a filter(integrator),so that at the sampling time the signal voltage might be emphasized in comparison with the noise voltage. We are naturally led to risk whether the integrator is the optimum filter for the purpose of minimizing the probability of error. We shall find that the received signal contemplated in system of fig 11.1-2 the integrator is indeed the optimum filter. However, before returning specifically to the integrator receiver.

We assume that the received signal is a binary waveform. One binary digit is represented by a signal waveform$S_1$ (t) which persists for time T, while the4 other bit is represented by the waveform $S_2$(t) which also lasts for an interval T. For example, in the transmission at baseband, as shown in fig 11.1-2 $S_1$(t)=+V; for other modulation systems, different waveforms are transmitted. for example for PSK signaling , $S_1$(t)=Acos$w_0$t and $S_2$(t)=-Acos$w_0$t;while for FSK, $S_1$(t)=Acos($w_{0+\Omega}$)t.

As shown in Fig. 11.3-1 the input, which is $s_1(t)$ or $s_2(t)$, is corrupted by the addition of noise $n(t)$. The noise is gaussian and has a spectral density $G(f)$. [In most cases of interest the noise is white, so that $G(f) = \eta/2$. However, we shall assume the more general possibility, since it introduces no complication to do so.] The signal and noise are filtered and then sampled at the end of each bit interval. The output sample is either $v_o(T) = s_{o1}(T) + n_o(T)$ or $v_o(T) = s_{o2}(T) + n_o(T)$. We assume that immediately after each sample, every energy-storing element in the filter has been discharged.

We have already considered in Sec. 2.22, the matter of signal determination in the presence of noise. Thus, we note that in the absence of noise the output sample would be $v_o(T) = s_{o1}(T)$ or $s_{o2}(T)$. When noise is present we have shown that to minimize the probability of error one should assume that $s_1(t)$ has been transmitted if $v_o(T)$ is *closer* to $s_{o1}(T)$ than to $s_{o2}(T)$. Similarly, we assume $s_2(t)$ has been transmitted if $v_o(T)$ is *closer* to $s_{o2}(T)$. The decision boundary is therefore midway between $s_{o1}(T)$ and $s_{o2}(T)$. For example, in the baseband system of Fig. 11.1-2, where $s_{o1}(T) = VT/\tau$ and $s_{o2}(T) = -VT/\tau$, the decision boundary is $v_o(T) = 0$. In general, we shall take the decision boundary to be

$$v_o(T) = \frac{s_{o1}(T) + s_{o2}(T)}{2} \qquad (11.3\text{-}1)$$

The probability of error for this general case may be deduced as an extension of the considerations used in the baseband case. Suppose that $s_{o1}(T) > s_{o2}(T)$ and that $s_2(t)$ was transmitted. If, at the sampling time, the noise $n_o(T)$ is positive and larger in magnitude than the voltage difference $\frac{1}{2}[s_{o1}(T) + s_{o2}(T)] - s_{o2}(T)$, an error will have been made. That is, an error [we decide that $s_1(t)$ is transmitted rather than $s_2(t)$] will result if

$$n_o(T) \geq \frac{s_{o1}(T) - s_{o2}(T)}{2} \qquad (11.3\text{-}2)$$

Hence probability of error is

$$P_e = \int_{[s_{o1}(T)-s_{o2}(T)]/2}^{\infty} \frac{e^{-n_o^2(T)/2\sigma_o^2}}{\sqrt{2\pi\sigma_o^2}}\, dn_o(T) \tag{11.3-3}$$

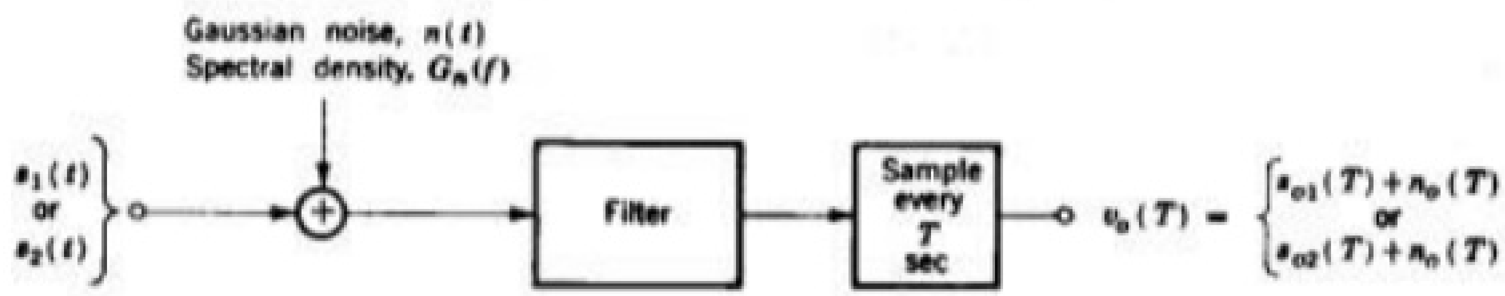



**Figure 11.3-1** A receiver for binary coded signalling.

If we make the substitution $x \equiv n_o(T)/\sqrt{2}\sigma_o$, Eq. (11.3-3) becomes

$$P_e = \frac{1}{2}\frac{2}{\sqrt{\pi}} \int_{[s_{o1}(T)-s_{o2}(T)]/2\sqrt{2}\sigma_o}^{\infty} e^{-x^2}\, dx \tag{11.3-4a}$$

$$P_e = \frac{1}{2}\,\text{erfc}\left[\frac{s_{o1}(T)-s_{o2}(T)}{2\sqrt{2}\sigma_o}\right] \tag{11.3-4b}$$

Note that for the case $s_{o1}(T) = VT/\tau$ and $s_{o2}(T) = -VT/\tau$, and, using Eq. (11.1-4), Eq. (11.3-4b) reduces to Eq. (11.2-3) as expected.

The complementary error function is a monotonically decreasing function of its argument. (See Fig. 11.2-2.) Hence, as is to be anticipated, $P_e$ decreases as the difference $s_{o1}(T) - s_{o2}(T)$ becomes larger and as the rms noise voltage $\sigma_o$ becomes smaller. The optimum filter, then, is the filter which maximizes the ratio

$$\gamma = \frac{s_{o1}(T)-s_{o2}(T)}{\sigma_o} \tag{11.3-5}$$

We now calculate the transfer function $H(f)$ of this optimum filter. As a matter of mathematical convenience we shall actually maximize $\gamma^2$ rather than $\gamma$.

mathematical convenience we shall actually maximize $\gamma^2$ rather than $\gamma$.

**Calculation of the Optimum-Filter Transfer Function $H(f)$**

The fundamental requirement we make of a binary encoded data receiver is that it distinguishes the voltages $s_1(t) + n(t)$ and $s_2(t) + n(t)$. We have seen that the ability of the receiver to do so depends on how large a particular receiver can make $\gamma$. It is important to note that $\gamma$ is proportional not to $s_1(t)$ nor to $s_2(t)$, but rather to the *difference* between them. For example, in the baseband system we represented the signals by voltage levels $+V$ and $-V$. But clearly, if our only interest was in distinguishing levels, we would do just as well to use $+2$ volts and 0 volt, or $+8$ volts and $+6$ volts, etc. (The $+V$ and $-V$ levels, however, have the advantage of requiring the least average power to be transmitted.) Hence, while $s_1(t)$ or $s_2(t)$ is the received signal, the signal which is to be compared with the noise, i.e., the signal which is relevant in all our error-probability calculations, is the difference signal

$$p(t) \equiv s_1(t) - s_2(t) \tag{11.3-6}$$

Thus, for the purpose of calculating the minimum error probability, we shall assume that the input signal to the optimum filter is $p(t)$. The corresponding *output signal* of the filter is then

$$p_o(t) \equiv s_{o1}(t) - s_{o2}(t) \tag{11.3-7}$$



We shall let $P(f)$ and $P_o(f)$ be the Fourier transforms, respectively, of $p(t)$ and $p_o(t)$.

If $H(f)$ is the transfer function of the filter,

$$P_o(f) = H(f)P(f) \tag{11.3-8}$$

and

$$p_o(T) = \int_{-\infty}^{\infty} P_o(f)e^{j2\pi fT}\, df = \int_{-\infty}^{\infty} H(f)P(f)e^{j2\pi fT}\, df \tag{11.3-9}$$

The input noise to the optimum filter is $n(t)$. The output noise is $n_o(t)$ which has a power spectral density $G_{n_o}(f)$ and is related to the power spectral density of the input noise $G_n(f)$ by

$$G_{n_o}(f) = |H(f)|^2 G_n(f) \tag{11.3-10}$$

Using Parseval's theorem (Eq. 1.13-5), we find that the normalized output noise power, i.e., the noise variance $\sigma_o^2$, is

$$\sigma_o^2 = \int_{-\infty}^{\infty} G_{n_o}(f)\, df = \int_{-\infty}^{\infty} |H(f)|^2 G_n(f)\, df \tag{11.3-11}$$

From Eqs. (11.3-9) and (11.3-11) we now find that

$$\gamma^2 = \frac{p_o^2(T)}{\sigma_o^2} = \frac{|\int_{-\infty}^{\infty} H(f)P(f)e^{j2\pi Tf}\, df|^2}{\int_{-\infty}^{\infty} |H(f)|^2 G_n(f)\, df} \tag{11.3-12}$$

Equation (11.3-12) is unaltered by the inclusion or deletion of the absolute value sign in the numerator since the quantity within the magnitude sign $p_o(T)$ is a positive real number. The sign has been included, however, in order to allow further development of the equation through the use of the *Schwarz inequality*.

The *Schwarz inequality* states that given arbitrary complex functions $X(f)$ and $Y(f)$ of a common variable $f$, then

$$\left|\int_{-\infty}^{\infty} X(f)Y(f)\, df\right|^2 \le \int_{-\infty}^{\infty} |X(f)|^2\, df \int_{-\infty}^{\infty} |Y(f)|^2\, df \tag{11.3-13}$$

The equal sign applies when

$$X(f) = KY^*(f) \tag{11.3-14}$$

where $K$ is an arbitrary constant and $Y^*(f)$ is the complex conjugate of $Y(f)$.

We now apply the Schwarz inequality to Eq. (11.3-12) by making the identification

$$X(f) \equiv \sqrt{G_n(f)}\, H(f) \tag{11.3-15}$$

and

$$Y(f) \equiv \frac{1}{\sqrt{G_n(f)}} P(f)e^{j2\pi Tf} \tag{11.3-16}$$

Using Eqs. (11.3-15) and (11.3-16) and using the Schwarz inequality, Eq. (11.3-13), we may rewrite Eq. (11.3-12) as

$$\frac{p_o^2(T)}{\sigma_o^2} = \frac{|\int_{-\infty}^{\infty} X(f)Y(f)\, df|^2}{\int_{-\infty}^{\infty} |X(f)|^2\, df} \le \int_{-\infty}^{\infty} |Y(f)|^2\, df \tag{11.3-17}$$

or, using Eq. (11.3-16),

$$\frac{p_o^2(T)}{\sigma_o^2} \le \int_{-\infty}^{\infty} |Y(f)|^2\, df = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{G_n(f)}\, df \tag{11.3-18}$$

The ratio $p_o^2(T)/\sigma_o^2$ will attain its maximum value when the equal sign in Eq. (11.3-18) may be employed as is the case when $X(f) = KY^*(f)$. We then find from Eqs. (11.3-15) and (11.3-16) that the optimum filter which yields such a maximum ratio $p_o^2(T)/\sigma_o^2$ has a transfer function

$$H(f) = K\frac{P^*(f)}{G_n(f)} e^{-j2\pi fT} \tag{11.3-19}$$

Correspondingly, the maximum ratio is, from Eq. (11.3-18),

$$\left[\frac{p_o^2(T)}{\sigma_o^2}\right]_{\max} = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{G_n(f)}\, df \tag{11.3-20}$$

In succeeding sections we shall have occasion to apply Eqs. (11.13-19) and (11.13-20) to a number of cases of interest.

## 11.4 WHITE NOISE: THE MATCHED FILTER

An optimum filter which yields a maximum ratio $p_o^2(T)/\sigma_o^2$ is called a *matched filter* when the input noise is *white*. In this case $G_n(f) = \eta/2$, and Eq. (11.3-19) becomes

$$H(f) = K\frac{P^*(f)}{\eta/2} e^{-j2\pi fT} \tag{11.4-1}$$

The impulsive response of this filter, i.e., the response of the filter to a unit strength impulse applied at $t = 0$, is

$$h(t) = \mathscr{F}^{-1}[H(f)] = \frac{2K}{\eta}\int_{-\infty}^{\infty} P^*(f)e^{-j2\pi fT}e^{j2\pi ft}\, df \tag{11.4-2a}$$

$$= \frac{2K}{\eta}\int_{-\infty}^{\infty} P^*(f)e^{j2\pi f(t-T)}\, df \tag{11.4-2b}$$

A physically realizable filter will have an impulse response which is real, i.e., not complex. Therefore $h(t) = h^*(t)$. Replacing the right-hand member of Eq. (11.4-2b) by its complex conjugate, an operation which leaves the equation unaltered, we have

$$h(t) = \frac{2K}{\eta}\int_{-\infty}^{\infty} P(f)e^{j2\pi f(T-t)}\, df \tag{11.4-3a}$$

$$= \frac{2K}{\eta} p(T-t) \tag{11.4-3b}$$

Finally, since $p(t) \equiv s_1(t) - s_2(t)$ [see Eq. (11.3-6)], we have

$$h(t) = \frac{2K}{\eta}[s_1(T-t) - s_2(T-t)] \tag{11.4-4}$$

## COHERENT RECEPTION: CORRELATION:

We discuss now an alternative type of receiving system which, as we shall see, is identical in performance with the matched filter receiver. Again, as shown in Fig. 11.6-1, the input is a binary data waveform $s_1(t)$ or $s_2(t)$ corrupted by noise $n(t)$. The bit length is $T$. The received signal plus noise $v_i(t)$ is multiplied by a locally generated waveform $s_1(t) - s_2(t)$. The output of the multiplier is passed through an integrator whose output is sampled at $t = T$. As before, immediately after each sampling, at the beginning of each new bit interval, all energy-storing elements in the integrator are discharged. This type of receiver is called a *correlator*, since we are *correlating* the received signal and noise with the waveform $s_1(t) - s_2(t)$.

The output signal and noise of the correlator shown in Fig. 11.6-1 are

$$s_o(T) = \frac{1}{\tau}\int_0^T s_i(t)[s_1(t) - s_2(t)]\,dt \qquad (11.6\text{-}1)$$



$$n_o(T) = \frac{1}{\tau}\int_0^T n(t)[s_1(t) - s_2(t)]\,dt \qquad (11.6\text{-}2)$$

Where $s_i(t)$ is either $s_1(t)$ or $s_2(t)$,and wthere π is the constant of the integrator(i.e.,the integrator output is 1/π times the integral of its input).we now compare these outputs with the matched filter outputs.

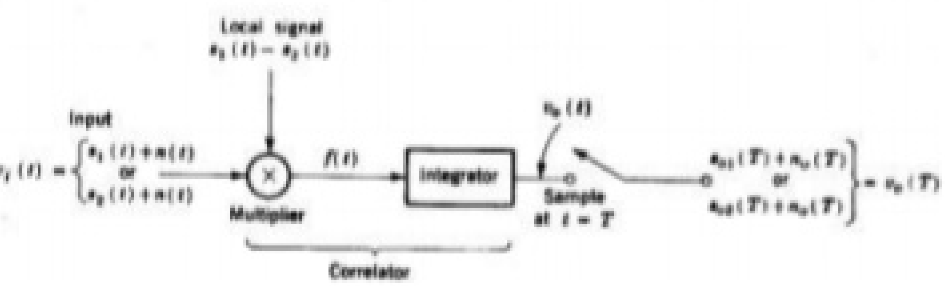



**Fig:11.6-1 Coherent system of signal reception**

If h(t) is the impulsive response of the matched filter ,then the output of the matched filter $v_o(t)$ can be found using the convolution integral. we have

$$v_o(t) = \int_{-\infty}^{\infty} v_i(\lambda)h(t-\lambda)\,d\lambda = \int_0^T v_i(\lambda)h(t-\lambda)\,d\lambda \qquad (11.6\text{-}3)$$

The limits on the integral have been charged to 0 and T since we are interested in the filter response to a bit which extends only over that interval. Using Eq.(11.4-4) which gives h(t) for the matched filter, we have

$$h(t) = \frac{2K}{\eta}[s_1(T-t) - s_2(T-t)] \qquad (11.6\text{-}4)$$

so that

$$h(t-\lambda) = \frac{2K}{\eta}[s_1(T-t+\lambda) - s_2(T-t+\lambda)] \qquad (11.6\text{-}5)$$

sub 11.6-5 in 11.6-3

$$v_o(t) = \frac{2K}{\eta}\int_0^T v_i(\lambda)[s_1(T-t+\lambda) - s_2(T-t+\lambda)]\,d\lambda \qquad (11.6\text{-}6)$$

Since $v_i(\lambda) = s_i(\lambda) + n(\lambda)$, and $v_o(t) = s_o(t) + n_o(t)$, setting $t = T$ yields

$$s_o(T) = \frac{2K}{\eta}\int_0^T s_i(\lambda)[s_1(\lambda) - s_2(\lambda)]\,d\lambda \qquad (11.6\text{-}7)$$

where $s_i(\lambda)$ is equal to $s_1(\lambda)$ or $s_2(\lambda)$. Similarly we find that

$$n_o(T) = \frac{2K}{\eta}\int_0^T n(\lambda)[s_1(\lambda) - s_2(\lambda)]\,d\lambda \qquad (11.6\text{-}8)$$

Thus $s_o(T)$ and $n_o(T)$, as calculated from eqs.(11.6-1) and (11.6-2) for the correlation receiver, and as calculated from eqs.(11.6-7) and (11.6-8) for the matched filter receiver, are identical .hence the performances of the two systems are identical. The matched filter and the correlator are not simply the right by amount $T$.

## 11.5 PROBABILITY OF ERROR OF THE MATCHED FILTER

The probability of error which results when employing a matched filter, may be found by evaluating the maximum signal-to-noise ratio $[p_o^2(T)/\sigma_o^2]_{max}$ given by Eq. (11.3-20). With $G_n(f) = \eta/2$, Eq. (11.3-20) becomes

$$\left[\frac{p_o^2(T)}{\sigma_o^2}\right]_{max} = \frac{2}{\eta}\int_{-\infty}^{\infty} |P(f)|^2\, df \qquad (11.5\text{-}1)$$



From parseval's theorem we have

$$\int_{-\infty}^{\infty} |P(f)|^2\, df = \int_{-\infty}^{\infty} p^2(t)\, dt = \int_0^T p^2(t)\, dt \qquad (11.5\text{-}2)$$

In the last integral in Eq. (11.5-2), the limits take account of the fact that $p(t)$ persists for only a time $T$. With $p(t) = s_1(t) - s_2(t)$, and using Eq. (11.5-2), we may write Eq. (11.5-1) as

$$\left[\frac{p_o^2(T)}{\sigma_o^2}\right]_{max} = \frac{2}{\eta}\int_0^T [s_1(t) - s_2(t)]^2\, dt \qquad (11.5\text{-}3a)$$

$$= \frac{2}{\eta}\left[\int_0^T s_1^2(t)\, dt + \int_0^T s_2^2(t)\, dt - 2\int_0^T s_1(t)s_2(t)\, dt\right] \qquad (11.5\text{-}3b)$$

$$= \frac{2}{\eta}(E_{s1} + E_{s2} - 2E_{s12}) \qquad (11.5\text{-}3c)$$

Here $E_{s1}$ and $E_{s2}$ are the energies, respectively, in $s_1(t)$ and $s_2(t)$, while $E_{s12}$ is the energy due to the correlation between $s_1(t)$ and $s_2(t)$.

Suppose that we have selected $s_1(t)$, and let $s_1(t)$ have an energy $E_{s1}$. Then it can be shown that if $s_2(t)$ is to have the *same energy*, the optimum choice of $s_2(t)$ is

$$s_2(t) = -s_1(t) \qquad (11.5\text{-}4)$$

The choice is optimum in that it yields a maximum output signal $p_o^2(T)$ for a given signal energy. Letting $s_2(t) = -s_1(t)$, we find

$$E_{s1} = E_{s2} = -E_{s12} \equiv E_s$$

and Eq. (11.5-3c) becomes

$$\left[\frac{p_o^2(T)}{\sigma_o^2}\right]_{max} = \frac{8E_s}{\eta} \qquad (11.5\text{-}5)$$

Rewriting Eq. (11.3-4b) using $p_o(T) = s_{o1}(T) - s_{o2}(T)$, we have

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{p_o(T)}{2\sqrt{2}\,\sigma_o}\right] = \frac{1}{2}\,\mathrm{erfc}\left[\frac{p_o^2(T)}{8\sigma_o^2}\right]^{1/2} \qquad (11.5\text{-}6)$$

Combining Eq. (11.5-6) with (11.5-5), we find that the minimum error probability $(P_e)_{min}$ corresponding to a maximum value of $p_o^2(T)/\sigma_o^2$ is

$$(P_e)_{min} = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{1}{8}\left[\frac{p_o^2(T)}{\sigma_o^2}\right]_{max}\right\}^{1/2} \qquad (11.5\text{-}7)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{E_s}{\eta}\right)^{1/2} \qquad (11.5\text{-}8)$$

We note that Eq. (11.5-8) establishes more generally the idea that the error probability depends only on the signal energy and not on the signal waveshape. Previously we had established this point only for signals which had constant voltage levels.

# Intersymbol Interference (ISI)

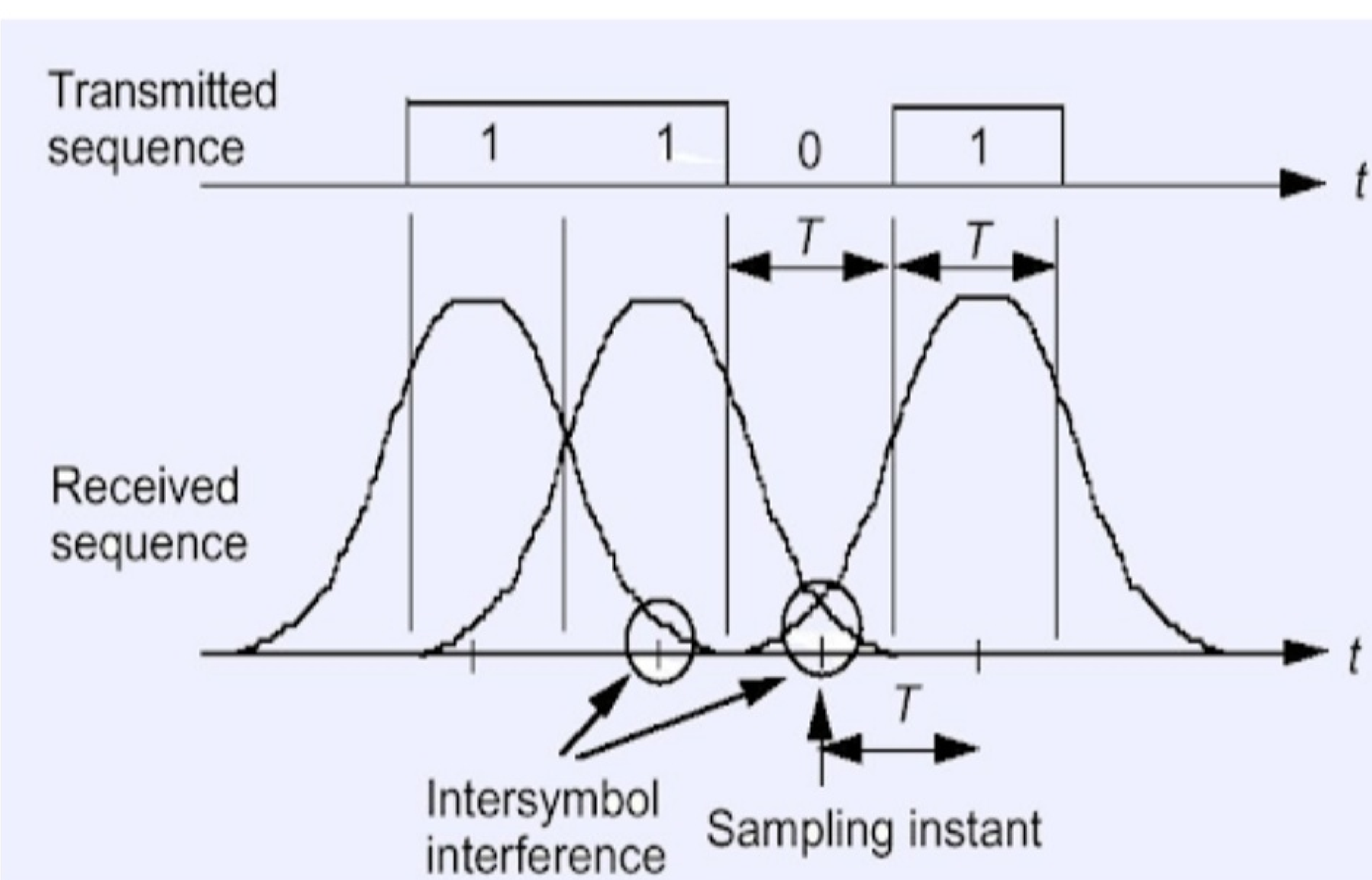
- If the transmission channel is bandlimited, then high frequency components will be cut off
  - Hence, the pulses will spread out
  - If the pulse spread out into the ***adjacent symbol* periods**, then it is said that intersymbol interference (ISI) has occurred

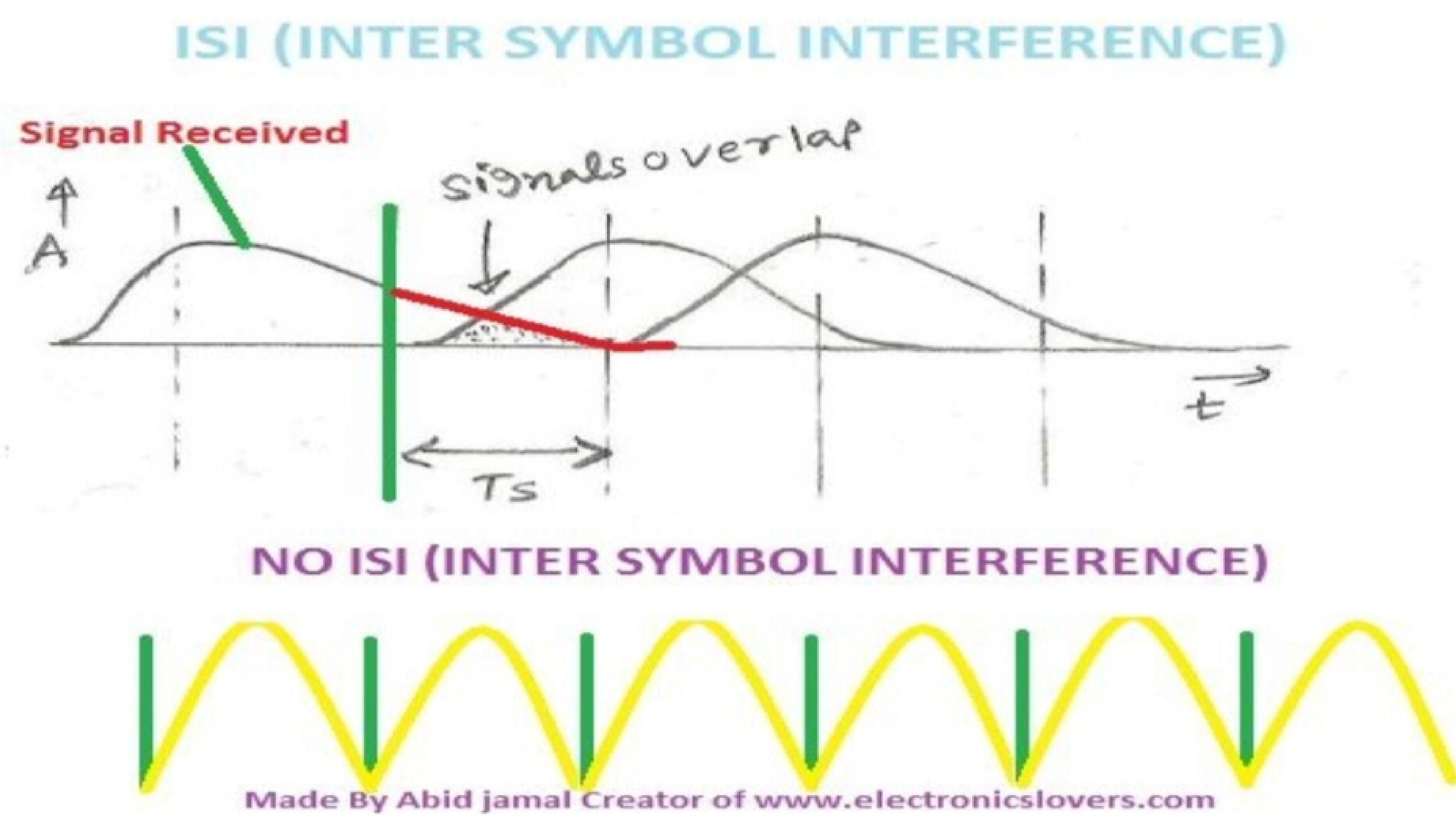
**Intersymbol Interference (ISI)**

- Intersymbol interference (ISI) occurs when a pulse spreads out in such a way that it interferes with adjacent pulses at the ***sample instant***
- **Causes**
  - Channel induced distortion which spreads or disperses the pulses
  - Multipath effec



# Steps in designing the receiver






New Note

# Inter sym ference(ISI)

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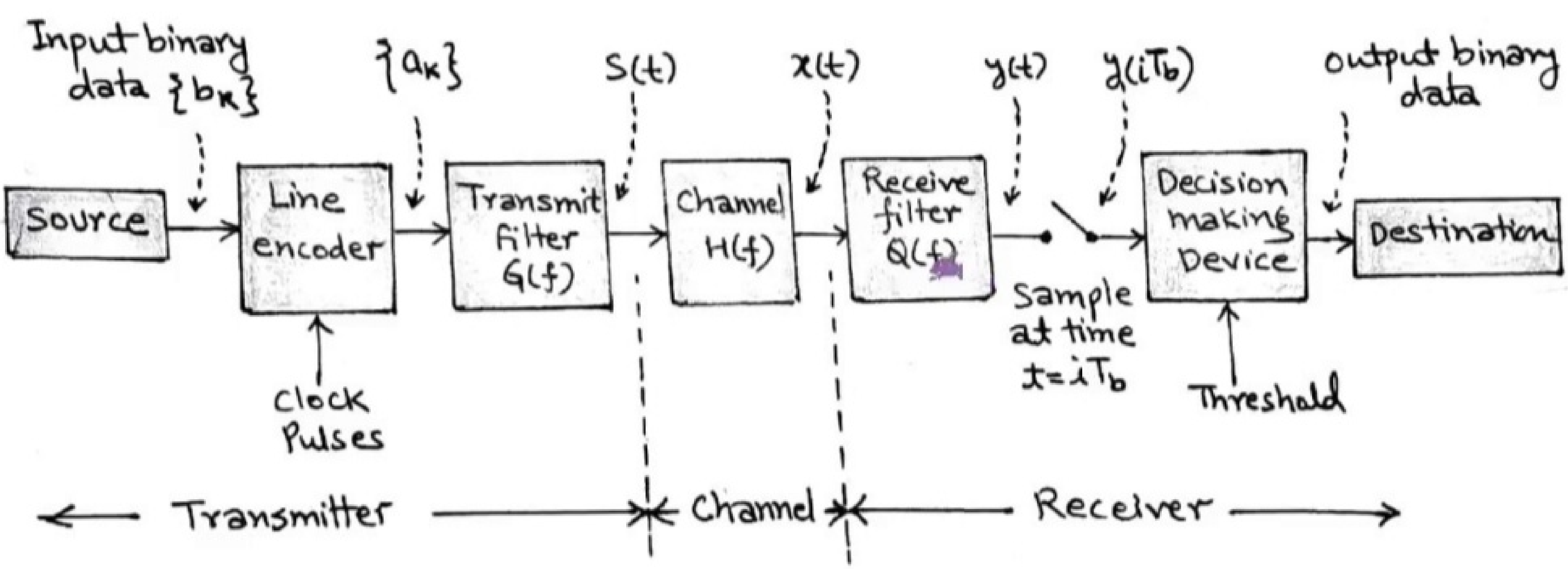


Element bk ~ represent binary symbol

# Eye diagram

## Eye diagram