UNIT - II

ANGLE MODULATION.

Angle modulation

frequency modulation　　　　phase modulation

frequency modulation :- The carrier frequency is varied in accordance to the amplitude of message signal.

Phase modulation :- The carrier phase is varied in accordance with the amplitude of the message signal.

* Both FM and PM are non-linear modulations. (varied exponentially).

PHASE MODULATION :-

$$c(t) = A_c \cos 2\pi f_c t$$

$$c(t) = A_c \cos \theta(t)$$ [un modulated carrier signal].

$$\boxed{\theta(t) = 2\pi f_c t + \phi.}$$

If carrier is phase modulated, then the instantaneous phase angle is given as,

$$\boxed{\theta_i(t) = 2\pi f_c t + k_p m(t).}$$

where k_p - phase sensitivity.

The phase modulated signal is

$$\boxed{S_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]} \quad \text{—①}$$

FREQUENCY MODULATION:-

Un modulated carrier is given by

$$c(t) = A_c \cos \theta(t).$$

where $\boxed{\theta(t) = 2\pi f_c t + \phi.}$

if carrier is frequency modulated the instantaneous frequency of the carrier signal is given as.

$$\boxed{f_i(t) = f_c + K_f m(t)}$$

K_f = frequency sensitivity.

$$S_{FM}(t) = A_c \cos \theta_i(t)$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \qquad f_c + K_f m(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$2\pi [f_c + K_f m(t)] = \frac{d}{dt}\theta(t)$$

$$\theta(t) = \int 2\pi [f_c + K_f m(t)]\, dt$$

$$\theta(t) = 2\pi f_c t + 2\pi K_f \int m(t)\,dt$$

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int m(t)\,dt.$$

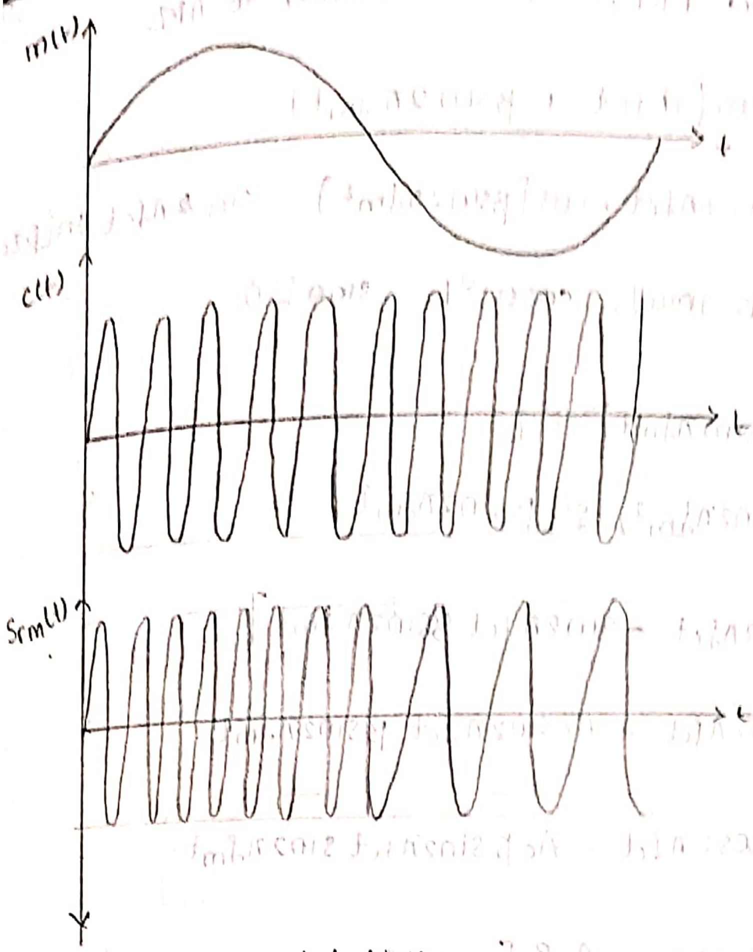
$$\boxed{S_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int m(t)\,dt\right]}$$

* FM can be generated using a phase modulator by providing the integrated input to the modulator.

$$m(t) \longrightarrow \boxed{\text{Integrator}} \longrightarrow \boxed{PM} \longrightarrow S_{FM}(t)$$

* PM can be generated using a frequency modulator by providing the differentiator i/p to the modulator.

$$m(t) \longrightarrow \boxed{\text{differentiator}} \longrightarrow \boxed{FM} \longrightarrow S_{PM}(t)$$



Single tone modulation:-

$m(t) = A_m \cos 2\pi f_m t$

$s(t) = A_c \cos(2\pi f_c t + K_f 2\pi \int m(t)\, dt]$

$f_i(t) = f_c + K_f\, m(t).$ $\qquad f_i(t) = f_c + K_f A_m \cos 2\pi f_m t.$

$$\boxed{\Delta f = K_f A_m}$$

$f_i(t) = f_c + \Delta f \cos 2\pi f_m t$

$f_i(t)_{max} = f_c + \Delta f$

$f_i(t)_{min} = f_c - \Delta f$

$s(t) = A_c \cos[2\pi f_c t + 2\pi K_f \int A_m \cos 2\pi f_m t\, dt].$

$= A_c \cos\left[2\pi f_c t + \frac{2\pi}{2\pi} \frac{\Delta f}{f_m} \sin 2\pi f_m t\right]$

$= A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$

β = modulation index $\qquad \beta = \frac{\Delta f}{f_m}$

β: <1 NBFM, >1 WBFM.

Narrow Band FM :- $(\beta < 1)$ similar to AM.

$$S_{fm}(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

$$s(t) = A_c \{\cos 2\pi f_c t \cdot \cos[\beta \sin 2\pi f_m t] - \sin 2\pi f_c t \sin(\beta \sin 2\pi f_m t)$$

if θ is small, $\cos\theta \cong 1$ $\quad \sin\theta \cong 0$

$\beta < 1$ $\quad \cos(\beta \sin 2\pi f_m t) \cong 1$

$$\sin(\beta \sin 2\pi f_m t) \cong \beta \sin 2\pi f_m t$$

$$s(t) = A_c[\cos 2\pi f_c t - \sin 2\pi f_c t \,\beta \sin 2\pi f_m t]$$

$$s(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t \,\beta \sin 2\pi f_m t$$

$$\boxed{s_{NBFM}(t) = A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \sin 2\pi f_m t}$$

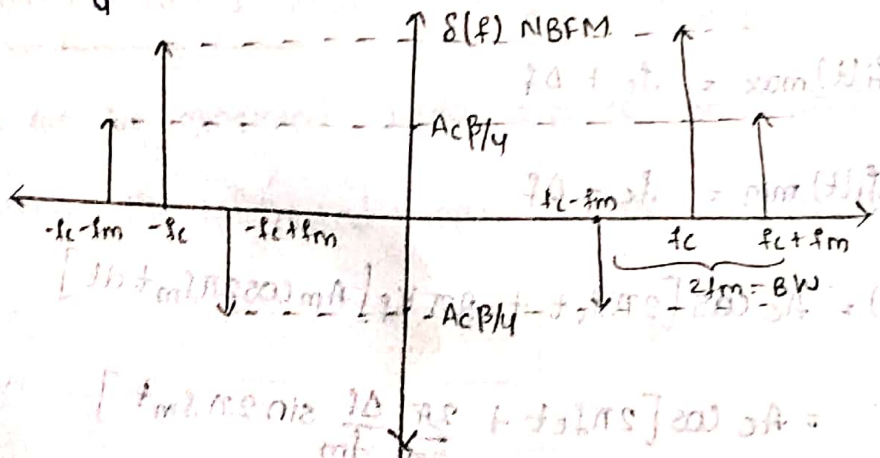
$$s_{NBFM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \beta}{2}[\cos 2\pi(f_m + f_c)t - \cos 2\pi(f_c - f_m)t]$$

$\Rightarrow$ Similar to AM concept.

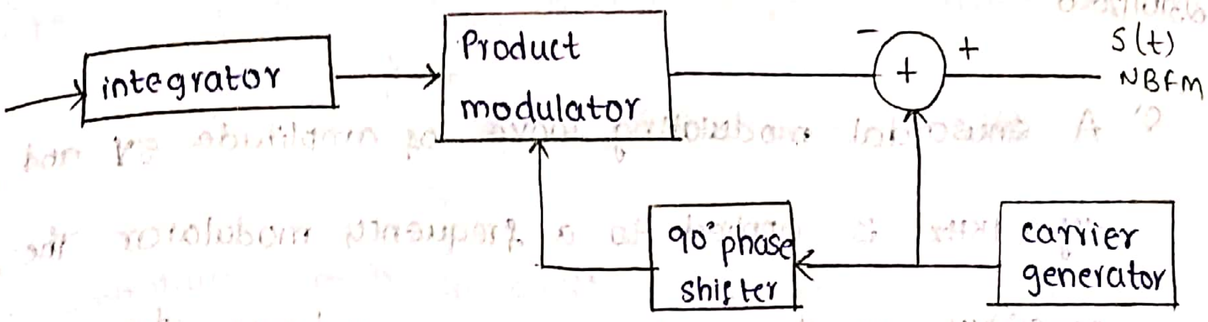
BW of NBFM is same as AM (i.e $2f_m$)

$$S(f) = \frac{A_c}{2}[\delta(f+f_c) + \delta(f-f_c)] + \frac{A_c\beta}{4}[\delta(f+f_c+f_m) + \delta(f \mp f_c \mp f_m)]$$

$$- \frac{A_c\beta}{4}[\delta(f+f_c-f_m) + \delta(f-(f_c-f_m))].$$



If we add NBFM & AM we get SSBFC with upper side band.



$A\cos\theta + B\sin\theta$

$$max = \sqrt{A^2+B^2} \qquad \theta = \tan^{-1}\left(\frac{-B}{A}\right)$$

S(t) NBFM magnitude $= \sqrt{A_c^2 + A_c^2\beta^2\sin^2 2\pi f_m t}$

$$\theta = \tan^{-1}\left[\frac{-(-A_c\beta\sin 2\pi f_m t)}{A_c}\right]$$

$$\theta = \tan^{-1}(\beta\sin 2\pi f_m t)$$

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{1}{5}x^5 + \cdots$$

Limitations:- Amplitude is varying with time higher order harmonic distortions.

* If we consider $\beta < 0.3$ amplitude variations and higher harmonic components will be minimized.

23/01/2020

Q) A sinusoidal modulating wave of amplitude 5V and frequency 1KHz is applied to a frequency modulator. The frequency sensitivity of the modulator is 40Hz/volts. The carrier frequency is 100KHz. Calculate frequency deviation and modulation index.

Δf = frequency deviation.

K_f = frequency sensitivity

$\Delta f = K_f \cdot A_m \Rightarrow 40 \times 5$

$= 200Hz$

$$\beta = \frac{\Delta f}{f_m} = \frac{200}{1000} \Rightarrow 0.2$$

Wide Band (f_m) frequency Modulation ($\beta > 1$) :-

$$S_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t) \quad - (1)$$

$$S(t) = Re\left[A_c \exp^{j[2\pi f_c t + \beta \sin 2\pi f_m t]}\right] - (2)$$

$$s(t) = Re\left[\tilde{s}(t) \cdot e^{j2\pi f_c t}\right] - (3)$$

where $\tilde{s}(t)$ = complex envolope of FM signal which equal

$$\tilde{s}(t) = A_c \exp^{j\beta \sin 2\pi f_m t} \quad - (4)$$

$\tilde{s}(t)$ is a period function of time with a fundamental frequency 'f_m'. we may therefore expand $\tilde{s}(t)$ in the form of complex fourier series as follows:-

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n \exp(j2\pi n f_m t) - (5)$$

where C_n is complex fourier coefficient equals to

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp(-j2\pi n f_m t)\,dt \quad — ⑥$$

substitute eq ④ in eq ⑥

we get ! $C_n = A_c f_m \int_{-1/2f_m}^{1/2f_m} \exp j[\beta \sin 2\pi f_m t - 2\pi n f_m t]\,dt$

let $x = 2\pi f_m t$ $\quad dx = 2\pi f_m dt$ $\quad dt = \dfrac{dx}{2\pi f_m}$

$$C_n = \frac{f_m A_c}{2\pi f_m} \int_{-\pi}^{\pi} e^{j[\beta \sin x - xn]}\,dx$$

$x = 2\pi f_m \times -\frac{1}{2f_m}$

$= -\pi$

$$C_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j[\beta \sin x - nx]}\,dx \quad — ⑦$$

The integral on RHS of eq ⑦ is n^{th} order bessel function of the first kind and argument β. This function is commonly denoted by the symbol $J_n(\beta)$ which is given as

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp j[\sin x\beta - nx]\,dx \quad — ⑧$$

$$C_n = A_c J_n(\beta) \quad — ⑨$$

substitute ⑨ in ⑤

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t) \quad — ⑩$$

???

$$s(t) = \mathrm{Re}\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t) \exp(j2\pi f_c t)\right].$$

$$s(t) = \mathrm{Re}\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp j[2\pi f_c t + 2\pi n f_m t]\right] \quad — ⑪$$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, \mathrm{Re}[\exp(j2\pi f_c t + j2\pi n f_m t]$$

∞ SB components

$$\boxed{s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]} \quad — ⑫$$

$n = 0$ carrier

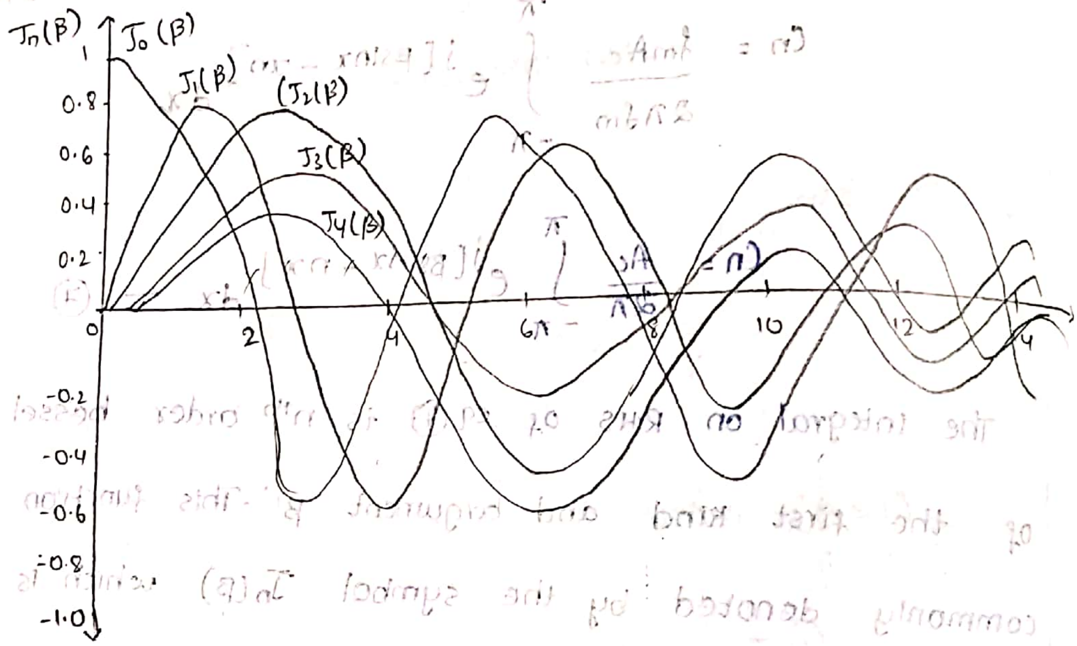
27/01

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f+f_c+nf_m) + \delta(f-f_c-nf_m) \right] \quad (13)$$

The below plots shows $J_n(\beta)$ Vs β for $n = 0,1,2,3,4$.

These plots shows that for fixed 'n', $J_n(\beta)$ alternates b/w +ve and -ve values for Increase in β and $|J_n(\beta)|$ approaches zero as β approaches '∞'.

Bessel fn $\ln$ Vs β.



As β value increases mag decreases.

for small values of β

$J_0(\beta) \simeq 1$

$J_1(\beta) \simeq \beta/2$

$J_n(\beta) = 0$

$n > 1$

$$\boxed{\begin{array}{ll} J_{-n}(\beta) = J_n(\beta) & \text{for } n \text{ even} \\ J_{-n}(\beta) = -J_n(\beta) & \text{for } n \text{ odd.} \end{array}}$$

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$s(t) = A_c J_0(\beta) \cos 2\pi f_c t + A_c J_1(\beta) \cos 2\pi (f_c + f_m)t + A_c J_{-1}(\beta)$

$\cos (2\pi)(f_c - f_m)t.$ → for smaller values of β.

for smaller values of β.

$$= A_c \cos 2\pi f_c t + \frac{A_c \beta}{2} \cos 2\pi (f_c + f_m)t - A_c J_1(\beta) \cos 2\pi (f_c - f_m)t$$

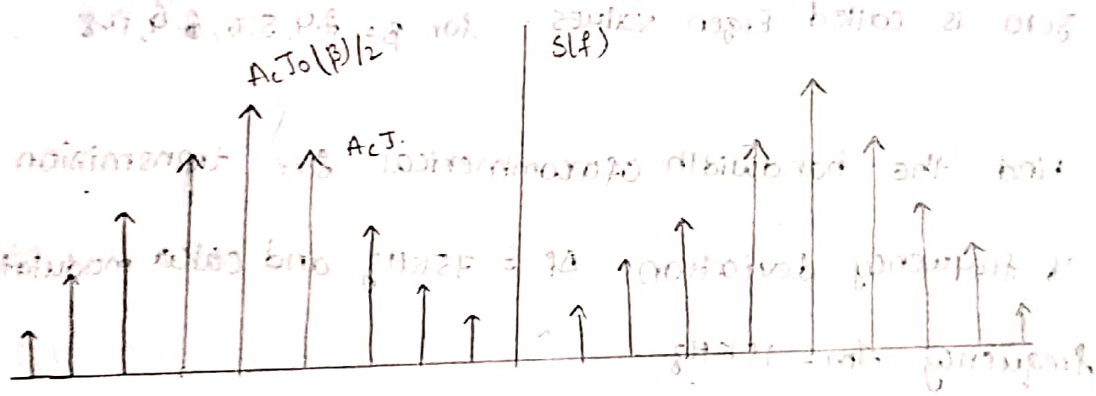
$$s(t) = A_c \cos 2\pi f_c t + \frac{A_c \beta}{2} \cos 2\pi (f_c + f_m)t - \frac{A_c \beta}{2} \cos 2\pi (f_c - f_m)t.$$

for larger values of β :-

$s(t) = A_c J_0(\beta) \cos 2\pi f_c t + A_c J_1(\beta) \cos 2\pi (f_c + f_m)t - A_c J_1(\beta) \cos 2\pi (f_c - f_m)t$

$A_c J_2(\beta) \cos 2\pi (f_c + 2f_m)t - A_c J_2(\beta) \cos 2\pi (f_c - 2f_m)t + A_c J_3(\beta)$

$\cos (2\pi)(f_c + 3f_m)t - A_c J_3(\beta) \cos 2\pi (f_c - 3f_m)t + \dots\dots$



for smaller values $BW = 2f_m$

for larger values $BW = 2\Delta f$.

Practical Bandwidth $BW \cong 2f_m + 2\Delta f$
for FM wave

$$\cong 2f_m \left(1 + \frac{\Delta f}{f_m}\right)$$

$BW \cong 2f_m(1+\beta)$ → carson's rule

ideally = ∞

30/01

Generation of FM :-

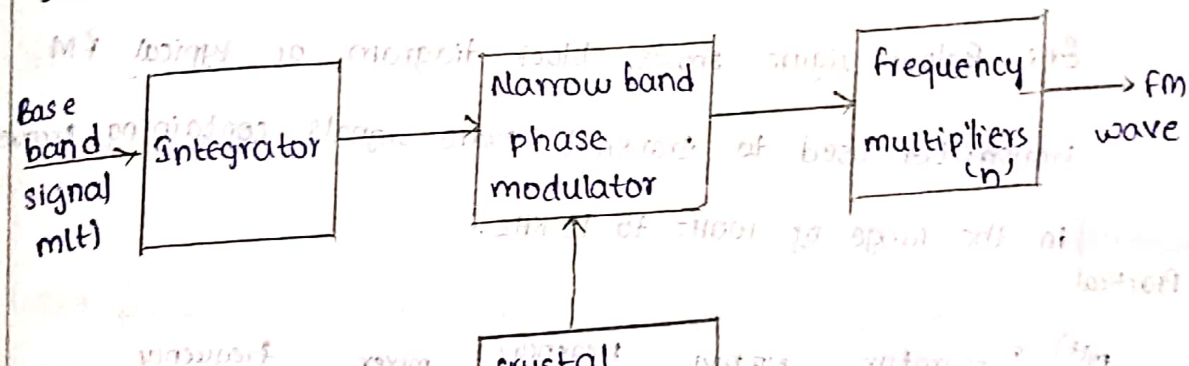
i) Indirect method (or) Armstrong method :-

ii) Direct method:-

* Phase modulator (provides) produces the narrow band FM which is then converted into wide band FM using phase multipliers so, this generation of FM is an indirect process. So this is called as indirect method.

* In direct method we use Vco's to generate FM.

Block diagram of Indiret method:-



β should be less than 0.3 to minimise envelope and phase distortions in the WBFM.

31/01/20. $f_i(t) = f_c + K_f\, m(t)$.

$= f_c + K_f A_m \cos 2\pi f_m t$.

$n f_i(t) = n f_c + \underbrace{n K_f A_m}_{\Delta f} \cos 2\pi f_m t$.

31/01/20

let $S_1(t)$ denote the o/p of phase modulator

$$S_1(t) = A_1 \cos\left[2\pi f_1 t + 2\pi K_f \int_0^t m(t)\,dt\right] \quad (1)$$

for a sinusoidal signal the o/p $S_1(t)$ can be written as

single tone modulation

$$S_1(t) = A_1 \cos\left[2\pi f_1 t + \beta_1 \sin 2\pi f_m t\right] \quad (2)$$

$$\beta_1 = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m} \quad (\beta_1 < 0.3)$$

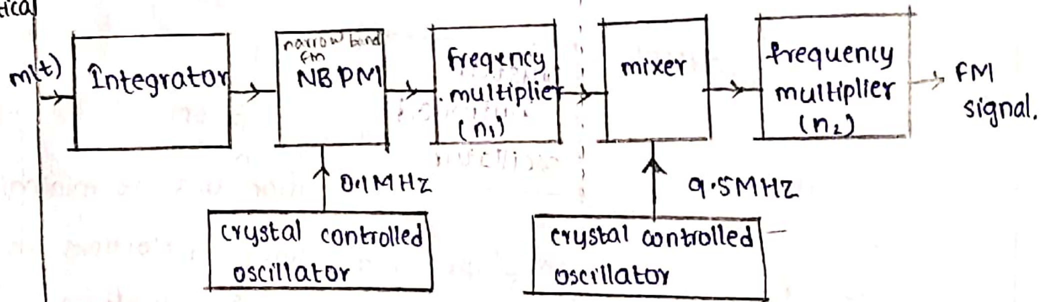
The phase modulator o/p is next multiplied 'n' times in frequency by the frequency multiplier producing the desired wide band fm.

$$S(t) = A_1 \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right] \quad (3)$$

where $f_c = n f_1$; $\beta = n\beta_1$

Ex: Below figure shows block diagram of typical FM transmitter used to transmit audio signals containing frequencies in the range of 100Hz to 15 KHz.

Practical



* The narrowband phase modulator is supplied with carrier frequency 0.1MHz by a crystal controlled oscillator. The desired FM wave at the transmitter o/p has carrier freq 100MHz and the frequency deviation 75KHz

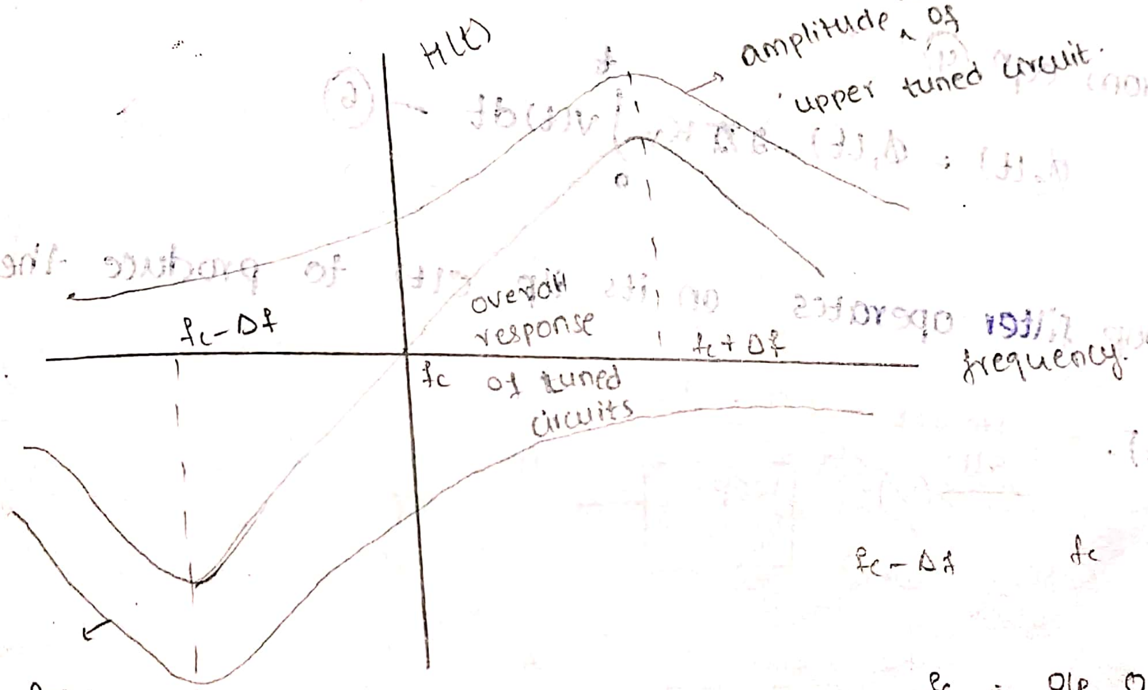
Ex: $f_c = 100$ MHz ; $\Delta f = 75$ KHz fixed.

$f_m = 100$Hz to 15KHz $\quad f_1 = 0.1$MHz $\quad f_2 = 9.5$MHz variable. $\beta = 0.2$

$\beta = \frac{\Delta f}{f_m} \Rightarrow \Delta f_1 = 20$Hz to 3KHz. $\quad n = \frac{\Delta f}{\Delta f_1} = 3750$

Direct FM :- in LICA clw. voltage controlled oscillator.

04/02 Balanced slope detector :-



Amplitude response of lower tuned circuit

$f_c - \Delta f$ $\quad$ f_c $\quad$ $f + \Delta f$

f_c = o/p 0

$f_c + \Delta f$ + max volt / − min volt $\quad$ + voltage

$f_c - \Delta f$ + min volt / − max volt $\quad$ − voltage

-0.2

I/P to PLL is FM

$$s(t) = A_c \sin\left[2\pi f_c t + \phi_1(t)\right] \quad -① $$

with modulating signal $m(t)$

$$\phi_1(t) = 2\pi k_f \int_0^t m(t)\,dt \quad - ②$$

k_f = frequency sensitivity of modulator

o/p of VCO

$$r(t) = A_v \cos\left[2\pi f_c t + \phi_2(t)\right] \quad - ③$$

$$\phi_2(t) = 2\pi K_v \int_0^t v(t)\,dt \quad - ④$$

K_v is frequency sensitivity of VCO

The o/p multiplier $v(t) \propto m(t)$

$$s(t)\, r(t) = K_m A_c A_v \left\{ \sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right] + \sin\left[\phi_1(t) + \phi_2(t)\right] \right.$$

$$e(t) = K_m A_c A_v \sin\left[\phi_1(t) - \phi_2(t)\right]$$

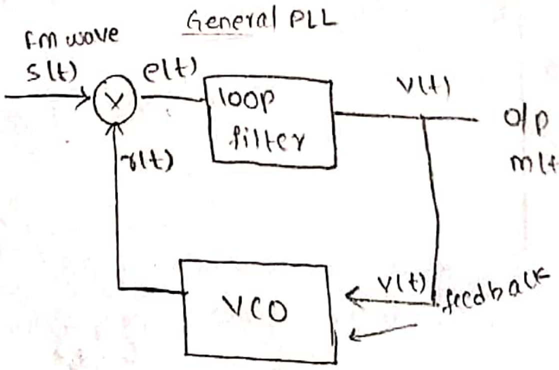
$$e(t) = K_m A_c A_v \sin\left[\phi_e(t)\right] \quad - ⑤$$

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

from eq^n ④

$$\phi_e(t) = \phi_1(t) - 2\pi K_v \int_0^t v(t)\,dt \quad - ⑥$$

The loop filter operates on its i/p $e(t)$ to produce the o/p $v(t)$.



$$v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau \quad - ⑦$$

$h(t)$ is impulse response of filter.

substitute ⑦ in ⑥.

Synchronous detector

$$\phi_e(t) = \phi_1(t) - 2\pi K_v \int_0^t \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau\, dt \quad - ⑧$$

By differentiating ⑧ w.r.t t we get

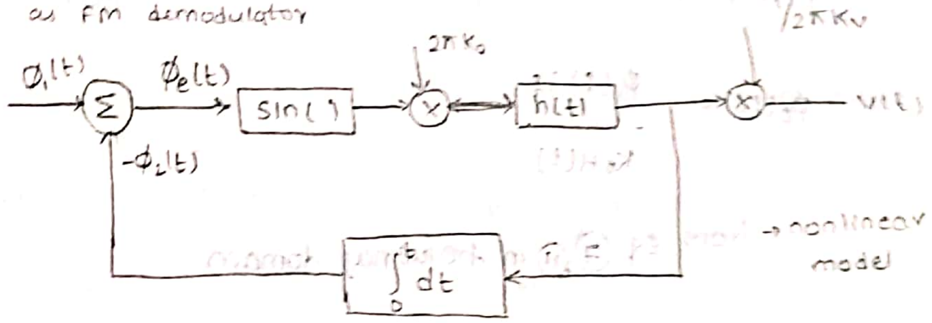
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_v \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau \quad - ⑨$$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_v K_m A_c A_v \int_{-\infty}^{\infty} \sin\phi_e(\tau)\, h(t-\tau)\, d\tau$$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin\phi_e(\tau)\, h(t-\tau)\, d\tau \quad - ⑩$$

where $K_0 = A_c A_v K_m K_v$ K_0 = loop parameter.

06/02 PLL as FM demodulator



VCO replaced by integrator

aim: to make $\phi_e(t) \cong 0$. incoming & VCO are locked range.

(or) both are in same phase.

linear model:- removing sin( ) from non-linear model.

when $\phi_e(t) = 0$, PLL is said to be in lock range [phase lock].

$$\sin[\phi_e(t)] \cong \phi_e(t) \quad - ⑪$$

from eq ⑩ we can write

$$\frac{d\phi_e(t)}{dt} + 2\pi K_0 \int_{-\infty}^{\infty} \phi_e(\tau)\, h(t-\tau)\, d\tau = \frac{d\phi_1(t)}{dt} \quad - ⑫$$

apply F.T

$$j2\pi f\, \phi_e(t) + 2\pi K_0\, \phi_e(f)\, H(f) = j2\pi f\, \phi_1(f)$$

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$$\phi_e(f)\left[j2\pi f + 2\pi K_o H(f)\right] = j2\pi f \phi_1(f)$$

$$\phi_e(f) = \frac{j2\pi f \phi_1(f)}{j2\pi f + 2\pi K_o H(f)}$$

$$\phi_e(f) = \frac{j\cancel{2\pi} f \phi_1(f)}{j2\cancel{\pi} f\left[1 + \frac{K_o H(f)}{jf}\right]}$$

$$\phi_e(f) = \frac{\phi_1(f)}{\left[1 + \frac{K_o H(f)}{jf}\right]}$$

$L(f) = \frac{K_o H(f)}{jf}$ → openloop transfer function.

$|L(f)| > 1$

$$\phi_e(f) = \frac{\phi_1(f)\, jf}{K_o H(f)}$$

from eq 7 11 in frequency domain.

$$V(f) \cong \phi_e(f) H(f)$$

$$V(f) \cong \frac{\phi_1(f)\, jf}{K_o \cancel{H(f)}} \cancel{H(f)}$$

$$V(f) \cong \frac{jf\, \phi_1(f)}{K_o}$$

$$V(t) \cong \frac{1}{2\pi K_o} \frac{d\phi_1(t)}{dt}$$

from eq 2

$$V(t) \cong \frac{\cancel{2\pi} K_f\, m(t)}{\cancel{2\pi} K_o}$$

$$V_o(t) \cong \frac{K_f}{K_o} m(t) \quad \text{— (21)}$$

$$\boxed{V(t) \propto m(t)}$$

Amplitude Modulation	Frequency Modulation.
* Amplitude of carrier changes in accordance with the amplitude of the message signal.	* frequency of carrier changes in accordance with the amplitude of the message signal.
* Noise is more in AM	* Noise is less in FM.
* $BW = 2f_m$	* $BW = 2(\beta+1)f_m = 2\Delta f + f_m 2$.
* BW required is less	* BW required is more.
* Power required for AM $P_t = P_c\left[1+\frac{\mu^2}{2}\right]$	* Power required for FM. $P_t = P_c = \frac{A_c^2}{2}$.
* Power required is more	* Power required is less.
* 2 side bands in AM..	* side bands depends on β.
* Transmittion efficiency is more in AM.	* Transmittion efficiency is in FM.
* AM is simple.	* FM is complex.
* Standard broad cast range 540Kz - 1640KHz.	* Standard broadcast range (88 - 108) MHz
* Commericial bandwidth of Am = 10KHz	* Commerical bandwidth = 200KHz Δf = 75KHz.
* IF (intermediate frequency) = 455K	* IF = 10.7MHz
* applications	* applications.
* linear modulation	* non-linear modulation.