Assignment

Q. With out using Routh criterion, determine if the following systems are asymptotically stable, marginally stable or, unstable

(a) $M(s) = \frac{10(s+2)}{s^3+3s^2+5s}$ (b) $\frac{s-1}{(s+5)(s^2+2)}$ (c) $\frac{K}{s^2+5s+5}$ (d) $\frac{100(s-1)}{(s+5)(s^2+2s+2)}$

(e) $\frac{100}{s^3-2s^2-3s+10}$ (f) $\frac{10(s+12.5)}{s^6+4s^4+5s^2+2}$

(a) ch eqn

$s^3+3s^2+5s=0$

$s(s^2+3s+5)=0$

$s\left(s+1.5\pm\frac{j\sqrt{11}}{4}\right)=0$

$s=0, -1.5\pm\frac{j\sqrt{11}}{4}$

Two Poles lies at Left Half S-Plane complex conjugate and one Pole lies at origin so System is Marginally Stable

(b) ch eqn

$(s+5)(s^2+2)=0$

$s=-5, \pm j\sqrt{2}$

roots are at

$s=-5, -j2, j2$

There is one pair of conjugate poles of the system on the imaginary axis of the S-Plane System is marginally stable

(c) ch eqn

$s^2+5s+5=0$

$(s+3.168)(s+1.382)=0$

$s=-3.168, -1.382$

All Poles lies at left Half of S-Plane then ~~so~~ System is Stable (asymptotically)

(d) CH. eqn

$(s+5)(s^2+2s+2)=0$

$(s+5)(s+1-j1)(s+1+j1)$

$s=-5, -1+j1, -1-j1$

All the root at LH of S-Plane system is asymptotically stable)

(e) $s^3-2s^2-3s+10=0$

$(s+2)(s^2-4s+5)=0$

$(s+2)(s-2-j1)(s-2+j1)=0$

$s=-2, s=2+j1, s=2+j1$

Complex Conjugate roots at RH of S-Plane

unstable

(f) $s^6+4s^4+5s^2+2=0$

$(s^2+1)(s^4+3s^2+2)=0$

$(s^2+1)(s^2+1)(s^2+2)$

$s=\pm j1, \pm j1, s=\pm j\sqrt{2}$

repeated root at $j\omega$ axis

unstable

Q Find the stability of system whose ch eqn is

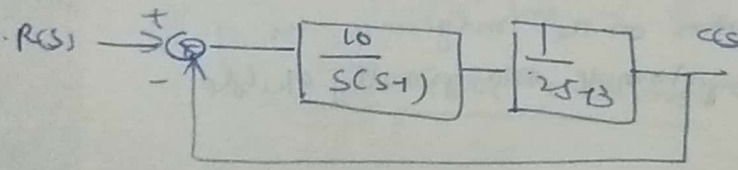
$$s^5+s^4+6s^3+12s^2+18s+6=0$$

s^5	1	6	18
s^4	1	12	6
s^3	$\frac{1\times6-1\times12}{1}=-6$	$\frac{1\times18-1\times6}{1}=12$	0
s^2	$\frac{-6\times12-1\times12}{-6}=14$	$\frac{-6\times6-1\times0}{-6}=6$	0
s^1	$\frac{14\times12-(-6\times6)}{14}=14.57$	0	
s^0	$\frac{14.57\times6-14\times0}{14.57}=6$		

change (between s^4 and s^3)
change (between s^3 and s^2)

There are two sign changes in the element of first column of the routh array. Hence there are two roots of ch eqn in the Right half (RH) of s-plane. Hence system is unstable.

Q



Find system stable or not

Here $G(s)=\frac{10}{s(s+1)(2s+3)}$, $H(s)=1$

CE $1+G(s)H(s)=0$

$$s(s-1)(2s+3)+10=0$$

$$2s^3+s^2-3s+10=0$$ Not satisfy necessary condition

Here coefficients of s is negative so system is unstable as to find location of roots use Routh criter

s^3	2	-3	
s^2	1	10	
s^1	$\frac{-3-20}{1}=-23$	0	
s^0	$\frac{-23\times10-1\times0}{10}=10$		

Sign Change (between s^2 and s^1)
Sign Change (between s^1 and s^0)

Two sign change in first column means two poles lies in Right half of s-Plane

System is unstable.

Q Test the stability of system whose ch is given below

$s^4 + 2s^3 + 10s^2 + 20s + 5 = 0$

s^4	1	10	5
s^3	2	20	
s^2	$\frac{2\times10-20\times1}{2}=0$	$\frac{2\times5-1\times0}{2}=5$	
s^2	ϵ	5	
s^1	$\frac{20\epsilon-10}{\epsilon}$	0	
s^0	5		

The 1st element of s^2 row is zero so Routh column test fails so system is unstable. To find no of roots in RH of s-plane replace zero with small positive no ϵ

If $\epsilon \to 0$ then coefficient of s^1 is negative there are two sign change between $s^2 \to s^1$ and $s^1 \to s^0$ row. So two roots lie in RH of s-plane

<u>Alternate method</u> replace $s \to \frac{1}{z}$ in ch eqn

$(\frac{1}{z})^4 + 2(\frac{1}{z})^3 + 10(\frac{1}{z})^2 + 20(\frac{1}{z}) + 5 = 0$

or, $5z^4 + 20z^3 + 10z^2 + 2z + 1 = 0$

z^4	5	10	1
z^3	20	2	0
z^2	$\frac{20\times10-5\times2}{20}=9.5$,	$\frac{20\times1-5\times0}{20}=1$	
z	$\frac{9.5\times2-20\times1}{9.5}=-0.105$	0	
z^0	$\frac{-0.105\times1-9.5\times0}{-0.105}=1$		

Two sign changes
Two roots at RH of s-plane unstable System

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Q. $q(s) = s^6 + 3s^5 + 5s^4 + 9s^3 + 8s^2 + 6s + 4 = 0$

s^6	1	5	8	4
s^5	3	9	6	0
s^4	2	6	4	
s^3	$\boxed{0}$	$\boxed{0}$		

Row of s^3 is zero (Type II) difficulty - 2. The system is unstable. To complete routh array, form auxiliary eqn, using the coefficient of the s^4 row (the row just above the row of zeros)

$A(s) = 2s^4 + 6s^2 + 4 = 0$

take 1st derivative of the eqn

$$\frac{dA(s)}{d(s)} = 8s^3 + 12s = 0$$

$$\text{or, } 2s^3 + 3s = 0$$

Now replace the row of zeros by the elements of the first derivative of the auxiliary eqn and proceed with the formulation of the routh table as below

s^6	1	5	8	4
s^5	3	9	6	0
s^4	2	6	4	
s^3	2	3	0	
s^2	3	4		
s^1	0.333			
s^0	4			

coefficient are taken from $\frac{dA(s)}{ds}$

There are no sign change in 1st column, which indicates no root of the ch eqn in RH of s-plane. Still system is unstable due to existance of the row of zeros, which means that there must be roots on the imaginary axis of s-plane to determine this solve the auxiliary eqn $A(s) = 2s^4 + 6s^2 + 4 = 0$

$2s^4 + 6s^2 + 4 = 0$

$s^4 + 3s^2 + 2 = 0$

$(s^2+1)(s^2+2) = 0$

$\boxed{s = \pm j1 \text{ or } s = \pm j\sqrt{2}}$

After solving this eqn we find

$\boxed{s = -1, \quad s = -2, \quad s = \pm j1, \quad s = \pm j\sqrt{2}}$

There are two pairs of non repeated roots on the imaginary axis. So the system oscillates and it is marginally stable. (unstable)

Determine the range of values of K for the system to stable

Q ① $s^4 + 4s^3 + 13s^2 + 36s + K = 0 \rightarrow$ $K >$ 0.526 and -2.52

② $s^3 + 3Ks^2 + (K+2)s + 4 = 0 \rightarrow$ $0 < K < 36$

③ $s^4 + 20Ks^3 + 5s^2 + 10s + 15 = 0 \rightarrow$ $K > 0$, $K > 0.1$

K must be complex but must be real the third condition not satisfied so system always stable.

$$s^4 + 4s^3 + 13s^2 + 36s + K = 0$$

s^4	1	13	K
s^3	4	36	0
s^2	4	K	
s^1	$36-K$		
s^0	K		

for stability, all the elements in the 1st column of the routh array must be positive. Therefore, from the s^0 row

$$K > 0$$

and from s^1 row

$$36 - K > 0 \Rightarrow K < \underline{36}$$

Since the open loop gain K must be positive, the range of value of K for stability is $0 < K < 36$

② $s^3 + 3Ks^2 + (K+2)s + 4 = 0$

s^3	1	$K+2$
s^2	$3K$	4
s^1	$\frac{3K(K+2)-4}{3K}$	0
s^0	4	

For stability, all the elements in the 1st column of the routh array must be positive therefore, from the s^2 row

$$3K > 0 \text{ ie } K > 0$$

and from the s^1 row

$$3K^2 + 6K - 4 > 0$$

$$K > \frac{-6 \pm \sqrt{6^2 - 4 \times 3 \times -4}}{2 \times 3} \Rightarrow K > 0.526 \text{ and } -2.52$$

Assignment

determine the Stability of the System whose ch eq are given below. if system is unstable then find out No of roots on RH of s-plane

ⓐ $s^5+s^4+24s^3+48s^2-25s-5=0$ — unstable three root of RHS

ⓑ $s^4+2s^3+10s^2+8s+3=0$ — Stable

ⓒ $s^6+2s^5+8s^4+15s^3+20s^2+16s+16=0$ — unstable 4 roots at RHS

ⓐ

s^5	1	24	-25
s^4	1	48	-5
s^3	-24	-20	
s^2	47.16	-5	
s^1	-22.54	0	
s^0	-5		

Sign changes 3 times

There are three sign changes in the element of 1st column of the Routh array. Hence three roots of ch eqn in the RH of s-plane.

→ coefficient of s constant term is negative Hence necessary condition not satisfied system is unstable

ⓑ $s^4+2s^3+10s^2+8s+3=0$

s^4	1	10	3
s^3	2	8	0
s^2	6	3	
s^1	7	0	
s^0	3		

All element of routh array are Positive. So the system is Stable.

ⓒ $s^6+2s^5+8s^4+15s^3+20s^2+16s+16=0$

s^6	1	8	20	16
s^5	2	15	16	0
s^4	0.5	12	16	
s^3	-33	-48		
s^2	11.27	16		
s^1	-1.16			
s^0	16			

4 times Sign change

There are Four sign changes in first column of the Routh array. Hence there are four roots of ch eqn in the RH of the s-Plane So The system is unstable

Q $s^6+2s^5+s^4+2s^3+3s^2+4s+5=0$

s^6	1	1	3	5
s^5	2	2	4	0
s^4	~~0~~ ϵ	1	5	
s^3	∞			

The first element of s^4 row is zero, whereas there are some non-zero elements in the same row (i.e difficulty 1 arises). So the system is unstable. To find the location of the roots. replace the first zero element by a small positive number ϵ and proceed with the formulation of the Routh table

s^6	1	1	3	5
s^5	2	2	4	0
s^4	ϵ	1	5	
s^3	$\frac{2\epsilon-2}{\epsilon}$	$\frac{4\epsilon-10}{\epsilon}$		

s^2 $\dfrac{\frac{2\epsilon-2}{\epsilon}\times 1-\epsilon\left(\frac{4\epsilon-10}{\epsilon}\right)}{\frac{2\epsilon-2}{\epsilon}} = \dfrac{4\epsilon^2+12\epsilon-2}{\epsilon}$ 5

s^1 $\dfrac{\left[-\frac{4\epsilon^2+12\epsilon-2}{\epsilon}\right]\left[\frac{4\epsilon-10}{\epsilon}\right]-5\left[\frac{2\epsilon-2}{\epsilon}\right]}{\frac{-4\epsilon^2+12-2}{\epsilon}}$ 0

s^0 5

As $\epsilon \to 0$, there are two sign changes in the 1st column ~~of~~ elements. means two root of ch eqn lie in the RHS of s-plane system is unstable.

Alternate method: replace $s \to 1/z$ s-plane to z-plane eqn will be

$$\left(\tfrac{1}{z}\right)^6+2\left(\tfrac{1}{z}\right)^5+\left(\tfrac{1}{z}\right)^4+2\left(\tfrac{1}{z}\right)^3+3\left(\tfrac{1}{z}\right)^2+4\left(\tfrac{1}{z}\right)+5=0$$

or $q(z)=5z^6+4z^5+3z^4+2z^3+z^2+2z+1=0$

z^6	5	3	1	1
z^5	4	2	2	0
z^4	0.5	-1.5	1	
z^3	14	-6		
z^2	-1.285	1		
z	4.88			
z^0	1			

Two sign changes

There are two sign changes means two root of ch eqn in the RHS of s-plane.

Q

$$q(s) = s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 2s + 1 = 0$$

s^6	1	3	3	1
s^5	1	3	2	0
s^4	ϵ	1	1	
s^3	$\frac{3\epsilon - 1}{\epsilon}$	$\frac{2\epsilon - 1}{\epsilon}$		
s^2	$\frac{-2\epsilon^2 + 4\epsilon - 1}{3\epsilon - 1}$	1		
s^1	$\frac{4\epsilon^2 - \epsilon}{2\epsilon^2 - 4\epsilon + 1}$			
s^0	1			

As $\epsilon \to 0$ the element of s^1 row tend to zero. This indicates that there are symetrically located roots in the s-plane. We therefore need to examine the auxiliary polynomial to find possibility of imaginary axis roots. If no such roots exist, the usual procedure of replacing the all-zero row by coefficents of the derivative of the auxiliary polynomial is adopted. If the imaginery-axis roots are found to exist, the original polynomial is divided out by auxiliary eqn and test is performed on the remainder polynomial. The example under consideration, the auxiliary eqn is (lift $\epsilon \to 0$ in s^2-row)

$$s^2 + 1 = 0$$

yielding two roots on imaginery axis. Dividing the original polynomial $q(s)$ by s^2+1, we get

$$q'(s) = s^4 + s^3 + 2s^2 + 2s + 1$$

$s^6 + 2s^5 + s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$

5

Routh array for this polynomial is

s^4	1	2 1
s^3	1	2
s^2	ϵ	1
s^1	$\frac{2\epsilon - 1}{\epsilon}$	
s^0	1	

As $\epsilon \to 0$, there are two sign changes in the first - column elements. This indicates that there are two roots in the RHS.

Difficulty 2 When all the elements in any row of Routh array are zero. This condition indicates that:

- There are symmetrically located roots in the S-Plane (Pair of real roots with opposite signs and...
  or,
- Pair of conjugate roots on the imaginary axis
  or,
- Complex conjugate roots forming quadrates in the S-Plane. The Polynomial whose coefficients are the elements of the row just above the row of zeros in the Routh array is called an auxilary (eq^n) Polynomial. This Polynomial gives the number and location of root pairs of the characteristics eq^n which are symmetrically located in the S-Plane.

The order of Auxiliary equation is always even.

Q Determine the stability of sixth order eq^n.

$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

s^6	1	8	20	16
s^5	2	12	16	0
s^5	1	6	8	0
s^4	2	12	16	
s^4	1	6	8	
s^3	0	0		
s^3	4	12		
s^2	3	8		
s^1	1/3			
s^0	8			

$\leftarrow$ All element of row are zero.

$\leftarrow$ new row is coefficient of $\frac{dA(s)}{ds}$

$A(s) = s^4 + 6s^2 + 8 = 0$

$\frac{dA(s)}{ds} = 4s^3 + 12s$

$s^6 + 2s^5 + s^4 + 2s + 3$ ... 5

There is no change in sign in first column of new array

By solving auxiliary eqn we find ~~roots of~~ eqn.

$$s^4 + 6s^2 + 8 = 0$$

$$s^4 + 4s^2 + 2s^2 + 8 = 0$$

$$s^2(s^2+4) + 2(s^2+4) = 0$$

$$(s^2+4)(s^2+2) = 0$$

$$s = \pm j2, \quad s = \pm j\sqrt{2}$$

These two pairs of roots are also roots of original CH eqn.

There are no ~~real part~~ root of ch eqn has positive real part. There system under consideration is limitedly stable.

Assignment

Stability or instability is a property of the system i.e closed loop poles of the system and does not depend on input or driving function. The poles of i/p do not affect stability of system, they affect steady state o/p.

$$q(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n = 0 \quad -① $$

may be written in factor form

$$q(s) = a_0 \prod (s + \sigma_K) \prod \left[(s + \sigma)^2 + \omega_e^2 \right] \quad -②$$

For the system to be stable, the root should have negative real part, which is satisfied if ~~eq^n②~~ in eq^n② all σ_K and σ_e are the real. It means that all the factor of eq^n② have positive terms only.

It may be noted that none of the coefficients can be zero or negative unless one (or more than one) of the following occurs

i) one or more roots have positive real parts;

ii) a root (or roots) at origin i.e, $\sigma_K = 0$, and hence $a_n = 0$;

3) $\sigma_l = 0$ for some l, which implies the presence of roots on $j\omega$ axis

We therefore conclude that the absence or negativeness of any of the coefficients of $q(s)$ (with $a_0 > 0$) indicates that the system is either unstable or at most limitedly stable

HCO Find Range of K for Stable System

Q $s^4 + Ks^3 + 2s^2 + s + 3 = 0$

s^4	1	2	3
s^3	K	1	
s^2	$\frac{2K-1}{K}$	3	
s^1	$\frac{-3K^2+2K-1}{2K-1}$	0	
s^0	3		

① $K > 0$

② $2K-1 > 0 \Rightarrow K > 0.5$

③ $-3K^2+2K-1 > 0$

↳ Complex value of K

unstable system

No value of K for which above system is stable

Q $s^4 + 5s^3 + 3s^2 + (3+K)s + K = 0$

s^4	1	3	K
s^3	5	$3+K$	
s^2	$\frac{12-K}{5}$		
s^1	$\frac{36-16K-K^2}{12-K}$		
s^0	K		

① $12-K > 0 \Rightarrow K < 12$

② $36-16K-K^2 > 0$

$\Rightarrow K < 2$

③ $K > 0$

Range

$0 < K < 2$

Q $s^4 + 20Ks^3 + 5s^2 + 10s + 15 = 0$

s^4	1	5	15
s^3	$20K$	10	
s^2	$\frac{100K-10}{20K}$	15	
s^1	$\frac{100K-10-600K^2}{100K-1}$		
s^0	15		

① $20K > 0$ $K > 0$

② $100K - 10 > 0$ $K > 0.1$

③ $100K - 10 - 600K^2 > 0$

$10K - 1 - 60K^2 > 0$

→ Complex value

K can't be complex

System is always unstable

Q $s^6 + 2s^5 + s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$

Q $G(s)H(s) = \dfrac{k e^{-s}}{s(s^2+5s+9)}$ find range of k for stable system

(Hint) $e^{-s} = (1-s)$

$1 + G(s)H(s) = 0$

$$1 + \frac{k(1-s)}{s(s^2+5s+9)} = 0$$

$$s^3 + 5s^2 + (9-k)s + k = 0$$

Routh array is

s^3	1	$9-k$
s^2	5	k
s^1	$\frac{45-6k}{5}$	0
s^0	k	

① s^1 row $45 - 6k > 0$

$k < 7.5$

② s^0 row $k > 0$

range $0 < k < 7.5$ for stable system.

Q $G(s) = \dfrac{e^{-sT}}{s(s+1)}$ find stability using RH

The characteristic eq $1+G(s)=0$

$$1+\frac{e^{-sT}}{s(s+1)} = 0$$

$$e^{-sT} = \left(1 - sT + \frac{s^2T^2}{\lfloor 2} + \cdots\right)$$
$$= (1-sT)$$

$$s^2+s+e^{-sT} = 0$$

$$s^2+s+1-sT = 0$$

or $s^2+s(1-T)+1 = 0$

RH array

s^2	1	1
s^1	$1-T$	0
s^0	1	

for stability $1-T > 0$

$\boxed{\text{or, } T < 1 \text{ for stability}}$

Q $G(s) = \dfrac{K e^{-s}}{s(s^2+5s+9)}$ find stability

Let $e^{-s} = (1-s)$

Q $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$

s^5	1	2	3
s^4	1	2	15
s^3	0 ϵ	-12	0
s^2	$\frac{2\epsilon + 12}{\epsilon}$	15	
s^1	$\frac{-24\epsilon - 144 - 15\epsilon^2}{2\epsilon + 12}$		
s^0	15		

$$\lim_{\epsilon \to 0} \frac{2\epsilon + 12}{\epsilon} = 2 + \lim_{\epsilon \to 0} \frac{12}{\epsilon}$$
$$= 2 + \infty = \infty$$

$$\lim_{\epsilon \to 0} \frac{-24\epsilon - 144 - 15\epsilon^2}{2\epsilon + 12}$$
$$= -\frac{144}{12} = -12$$

	s^5	1	2	3
	s^4	1	2	15
	s^3	ϵ	-12	0
Sign change →	s^2	∞	15	0
Sign change →	s^1	-12	0	
	s^0	15		

Two sign change System is unstable

Assignment

Q1 Test the stability of

$s^5+2s^4+3s^3+6s^2+2s+1=0$

s^5	1	3	2
s^4	2	6	1
s^3	$\frac{2\times3-6\times1}{2}=0$	$\frac{2\times2-1}{2}=\frac{3}{2}$	
s^3	ϵ	1.5	0
s^2	$\frac{6\epsilon-3}{\epsilon}=-\infty$	1	0
s^1	$\frac{1.5\frac{(6\epsilon-3)}{\epsilon}-\epsilon}{6\epsilon-3/\epsilon}=1.5$	0	
s^0	1		

① To examine sign change

$\lim_{\epsilon\to0}\left(\frac{6\epsilon-3}{\epsilon}\right)=6-\lim_{\epsilon\to0}\frac{3}{\epsilon}$

$=6-\infty \quad =-\infty$

② $\lim \frac{1.5\frac{(6\epsilon-3)}{\epsilon}-\epsilon^2}{6\epsilon-3}$

$=+1.5$

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s^5	1	3	2
s^4	2	6	1
s^3	ϵ	1.5	0
Sign change $\rightarrow s^2$	$-\infty$	1	0
Sign change $\rightarrow s^1$	1.5	0	0
s^0	1	0	0

Two Sign change System is unstable.

Second method Replace s — by 1/2 in original eq^n take LCM rearrange characteristics eq^n.

$s^5+2s^4+3s^3+6s^2+2s+1$

$(\frac{1}{z})^5+2(\frac{1}{z})^4+3(\frac{1}{z})^3+6(\frac{1}{z})^2+2(\frac{1}{z})+1=0 \Rightarrow \frac{1}{z^5}+\frac{2}{z^4}+\frac{3}{z^3}+\frac{6}{z^2}+\frac{2}{z}+1=0$

or $z^5+2z^4+6z^3+3z^2+2z+1=0$

z^5	1	6	2
z^4	2	3	1
z^3	4.5	1.5	0
z^2	2.33	1	0
$\rightarrow z^1$	-0.429	0	
$\rightarrow z^0$	1		

Two Sign changes, System is unstable

Q Using RH investigate the stability of a unity FB Sys. whose open loop TF is

$$G(s) = \frac{e^{-sT}}{s(s+1)}$$

The characteristic fn is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{e^{-sT}}{s(s+1)} = 0$$

$$s^2 + s + e^{-sT} = 0$$

Now e^{-sT} can be expressed in series from as

$$e^{-sT} = \left[1 - sT + \frac{s^2T^2}{2!} + \cdots\right]$$

Truncating the series and considering only first two terms we get

$$e^{-sT} \simeq 1 - sT$$

$$s^2 + s + 1 - sT = 0$$

$$s^2 + s(1-T) + 1 = 0$$

So Routh's array is

s^2	1	1
s	$1-T$	0
s^0	1	

For stability $1 - T > 0$ for stability

$$T < 1$$

This is the required condition for stability of the system.