Electricity + Magnetism (current causes magnetic field) current → flow of charge
↓ **Electromagnetic fields and Waves.**
(flow of e)

1. Electrostatics.

Introduction:

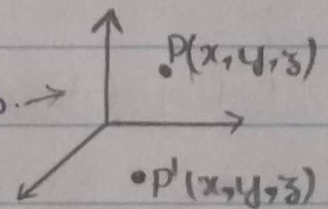
charges (4)	currents (3)	(8) → used to derive EM wave eqⁿ. maxwell's equations gives
- Q → electron	I - current (no point current)	the relations between
Q dl - line charge	line current - I dl.	charges and fields:
dl × dw = Q ds - surface charge; Q → surface charge density	$\bar{K} \cdot d\bar{s}$; K → surface current density	electric flux → charges - coulombs (C)
→ $\rho_v\,dv$; ρ_v → volume charge density	$\bar{J}\,dv$; J → volume current density	permittivity - Farad/meter
		Electric field - volts/meter
		magnetic field - Henry/meter

$e = 1.602 \times 10^{-19}\,C$ $\quad \varepsilon_0 = 8.85 \times 10^{-12}\,F/m$ $\quad c = 3 \times 10^{8}\,m/s$

$m = 9.1 \times 10^{-31}\,kg$ $\quad \mu_0 = 4\pi \times 10^{-7}\,H/m$

Electrostatics → charges are at rest.

Coordinate system - ① Rectangular/cartesian →
② Cylindrical → (ρ, ϕ, z)
③ Spherical → (r, θ, ϕ)



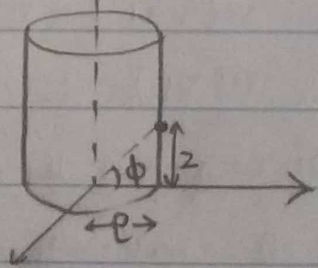
Two points are connected in form of curve then it is cylindrical and spherical.

Spherical co-ordinate system is an extension of cylindrical.

$0 \le \rho < \infty$ $\quad x = \rho\cos\phi$

$0 \le \phi \le 2\pi$ $\quad y = \rho\sin\phi$

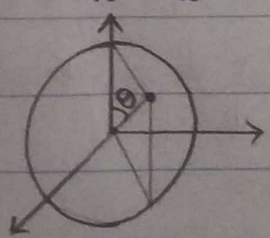
$-\infty < z < \infty$ $\quad z = z$



Illy, $\rho = \sqrt{x^2+y^2}$, $\phi = \tan^{-1}(y/x)$, $z = z$

$\bar{a}_x \cdot \bar{a}_x = 1$, $\bar{a}_x \cdot \bar{a}_y = 0$.

$\bar{a}_x \times \bar{a}_x = 0$, $\bar{a}_x \times \bar{a}_y = \bar{a}_z$, $\bar{a}_y \times \bar{a}_z = \bar{a}_x$, $\bar{a}_z \times \bar{a}_x = \bar{a}_y$



$\bar{A} = A_r\bar{a}_r + A_\theta\bar{a}_\theta + A_\phi\bar{a}_\phi$

$x = r\sin\theta\cos\phi$ $\quad$ Illy, $r = \sqrt{x^2+y^2+z^2}$

$y = r\sin\theta\sin\phi$ $\quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$z = r\cos\phi$ $\quad \phi = \tan^{-1}(y/x)$

$0 \le r < \infty$

$0 \le \phi \le \pi$

$0 \le \theta \le 2\pi$

Vector calculus: differential length, area, volume

1) Cartesian co-ordinate system:

$$d\bar{l} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

$$d\bar{s} = dx\,dy\,\bar{a}_z = dy\,dz\,\bar{a}_x = dx\,\cancel{dy}\,dz\,\bar{a}_y$$

$$d\bar{v} = dx\,dy\,dz.$$

2) cylindrical: $d\bar{l} = d\rho\,\bar{a}_\rho + \rho\cdot d\phi\,\bar{a}_\phi + dz\,\bar{a}_z$

$$d\bar{s} = \rho\,d\rho\,d\phi\,\bar{a}_z = \rho\cdot d\phi\,dz\,\bar{a}_\rho + d\rho\cdot dz\,\bar{a}_\phi$$

$$d\bar{v} = \rho\,d\rho\,d\phi\,dz$$

3) spherical: $d\bar{l} = dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi$

$$d\bar{s} = r^2\sin\theta\,d\theta\,d\phi\,\bar{a}_r = r\sin\theta\,dr\cdot d\phi\,\bar{a}_\theta$$

$$= r\,dr\,d\theta\,\bar{a}_\phi$$

$$d\bar{v} = r^2\sin\theta\,dr\,d\theta\,d\phi.$$

Del operator or differential operator:

$$\nabla = \frac{\partial}{\partial x}\cdot\bar{a}_x + \frac{\partial}{\partial y}\cdot\bar{a}_y + \frac{\partial}{\partial z}\cdot\bar{a}_z$$

gradient of V $\rightarrow \nabla V$. divergence of $\bar{A} \rightarrow \nabla\cdot\bar{A}$

curl of $\bar{A} \rightarrow \nabla\times\bar{A}$ Laplacian of V $\rightarrow \nabla^2 V$

$\nabla(V+U) = \nabla V + \nabla U$ $\nabla\left(\frac{V}{U}\right) = \frac{U\nabla V - V\nabla U}{U^2}$

$\nabla(VU) = U\nabla V + V\nabla U.$ $\nabla V^n = n\cdot V^{n-1}\nabla V.$

Divergence theorem: Gauss-Ostrogradsky theorem.

It states that the total outward flux of a vector field $\bar{A}$ through the closed surface 'S' is same as the volume integral of the divergence of $\bar{A}$

$$\oint_S \bar{A}\cdot d\bar{s} = \int_V \nabla\cdot\bar{A}\,dv$$ ~~i.e./closed~~ ~~surface integral~~

i) $\nabla\cdot\bar{A} \rightarrow$ scalar

ii) $\nabla\cdot(\bar{A}+\bar{B}) \rightarrow \nabla\cdot\bar{A} + \nabla\cdot\bar{B}$

iii) $\nabla\cdot(U\bar{A}) \rightarrow U(\nabla\cdot\bar{A}) + \bar{A}\cdot\nabla U$, $U \rightarrow$ scalar

Stokes theorem: States that the circulation of a vector field $\bar{A}$ around a closed path 'L' is equal to the surface integral of the curl of $\bar{A}$ over the open surface 'S' bounded by 'L'.

$$\oint_L \underset{\text{vector}}{\bar{A}\cdot d\bar{l}} = \int_S (\nabla\times\bar{A})\cdot d\bar{s} \quad = \text{scalar}$$

$$\oint \bar{A}\cdot d\bar{s} = \int (\nabla\cdot\bar{A})\cdot dv \quad = \text{scalar}$$

i) Curl of a vector is a vector

ii) $\nabla\times(\bar{A}+\bar{B}) = \nabla\times\bar{A} + \nabla\times\bar{B}$

iii) $\nabla\times(\bar{A}\times\bar{B}) = \bar{A}(\nabla\cdot\bar{B}) - \bar{B}(\nabla\cdot\bar{A}) + \bar{B}\cdot\nabla\bar{A} - (\bar{A}\cdot\nabla)\bar{B}$

iv) $\nabla\times(V\bar{A}) = V(\nabla\times\bar{A}) + \nabla V\times\bar{A}$.

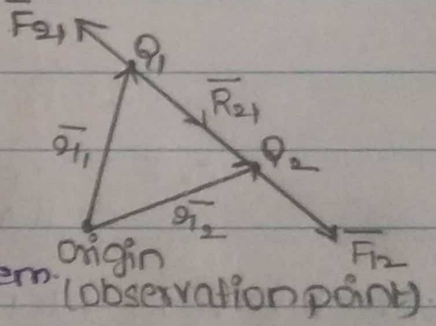
v) $\nabla\cdot(\nabla\times\bar{A}) = 0$. , $\nabla\times(\nabla V) = 0$.

* **Coulomb's law:** States that the force F between two point charges Q_1 and Q_2 is,

i) Exists along the line joining them.

ii) Directly proportional to the product of Q_1 and Q_2

iii) Inversely proportional to the square of distance $\bar{R}$ between them.



$$F \propto \frac{Q_1 Q_2}{R^2}$$

$$\bar{F} = \frac{K\, Q_1 Q_2}{R^2}\ N, \text{ where } K = \frac{1}{4\pi\epsilon_0} \rightarrow \text{proportionality constant}$$

ϵ_0 = permittivity of free space $= 8.85\times10^{-12}\, F/m \cong \frac{10^{-9}}{36\pi}\, F/m$

$$\therefore K = \frac{1}{4\pi\epsilon_0} = 9\times10^{9}\, m/F$$

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 \cdot R^2}\ N$$

If point charges Q_1 and Q_2 are located at points having position vectors r_1 and r_2 then the force on

Q_2 due to Q_1 is $\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 \cdot R^2}\, \overline{a}_{R_{12}}$

where, $\overline{a}_{R_{12}}$ is the vector $= \frac{\overline{R}_{12}}{|\overline{R}_{12}|}$, $\overline{R}_{12} = \overline{r}_2 - \overline{r}_1$

$$= \frac{\overline{R}_{12}}{R}$$

$$\therefore \overline{F}_{12} = \frac{Q_1 \cdot Q_2}{4\pi\epsilon_0 \cdot R^3} \cdot \overline{R}_{12}$$

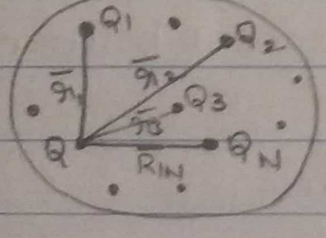
$$\therefore \overline{F}_{12} = \frac{Q_1 Q_2 (\overline{r}_2 - \overline{r}_1)}{4\pi\epsilon_0 |\overline{r}_2 - \overline{r}_1|^3} \ N.$$

$\overline{a}_{\overline{R}_{21}} = -\overline{a}_{\overline{R}_{12}}$

$F_{12} \rightarrow$ force on Q_2 due to Q_1 ('$-$' is due to direction)

$F_{21} \rightarrow$ force of Q_1 due to $Q_2 \Rightarrow \overline{F}_{21} = -\overline{F}_{12}$

If we have more than 2 point charges, 'N' i.e, $Q_1, Q_2 ---- Q_N$ we can use the principle of superposition to determine the force on a particular charge



$$\overline{F} = \frac{Q Q_1 (r - \overline{r}_1)}{4\pi\epsilon_0 |r - \overline{r}_1|^3} + \frac{Q Q_2 (r - \overline{r}_2)}{4\pi\epsilon_0 |r - \overline{r}_2|^3} + ----$$

$$+ \frac{Q \cdot Q_N (r - \overline{r}_N)}{4\pi\epsilon_0 |r - \overline{r}_N|^3}$$

$$\therefore \overline{F} = \overline{F}_1 + \overline{F}_2 + ---- \overline{F}_N$$

$$\therefore \overline{F} = \frac{Q}{4\pi\epsilon_0} \sum_{K=1}^{N} \frac{Q_K (\overline{r} - \overline{r}_K)}{|\overline{r} - \overline{r}_K|^3}$$

Electric field intensity: $\overline{E} \rightarrow$ The force per unit charge when placed in an electric field

$$\overline{E} = \frac{\overline{F}}{Q} \ N/c \ \& \ V/m.$$

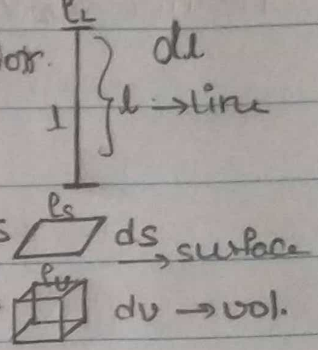
$\rightarrow$ Electric field intensity at point $\overline{r}$, due to a point charge located at r' can be ~~*~~ written as,

$$\because \ E = \frac{\overline{F}}{Q} = \frac{\frac{Q}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_K (r - \overline{r}_K)}{|r - \overline{r}_K|^3}}{Q}$$

$$\overline{E} = \frac{\cancel{Q}\,Q'}{4\pi\epsilon_0} \cdot \frac{(\overline{r} - r')}{|r - r'|^3} \times \frac{1}{\cancel{Q}}$$

$$\therefore \bar{E} = \frac{Q(\bar{r}-\bar{r}')}{4\pi\epsilon_0|\bar{r}-\bar{r}'|^3} \text{ V/m.}$$

Electric fields due to continuous charge distributions:



$\rho_L \to$ amount of charge in line conductor.
$\to$ line charge density (C/m)

|||ly, ρ_s (C/m²) $\to$ surface charge density

ρ_v (C/m³) $\to$ vol. charge density

$$dQ = \rho_L d\bar{l} = \rho_s d\bar{s} = \rho_v dv \cdot C.$$

$$Q = \int_L \rho_L d\bar{l} = \int_S \rho_s d\bar{s} = \int_V \rho_v dv.$$

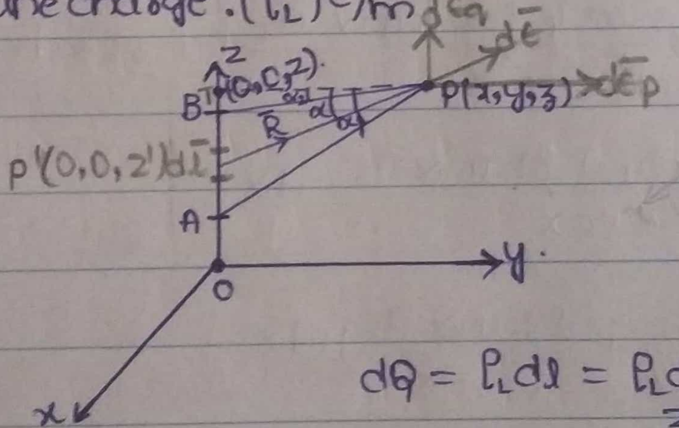
substituting "Q" in $\bar{E} = \frac{Q(\bar{r}-\bar{r}')}{4\pi\epsilon_0|\bar{r}-\bar{r}'|^3} = \frac{Q}{4\pi\epsilon_0 R^2} \cdot \bar{a}_R$

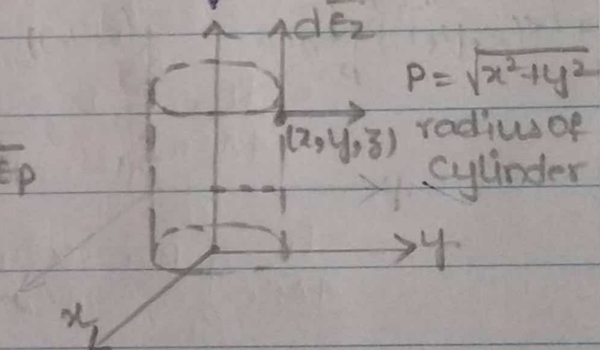
where, $\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{\bar{r}-\bar{r}'}{|\bar{r}-\bar{r}'|}$.

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 R^2} \cdot \bar{a}_R \text{ V/m.}$$

$$\bar{E} = \int_L \frac{\rho_L d\bar{l}}{4\pi\epsilon_0 R^2}\bar{a}_R = \int_S \frac{\rho_s d\bar{s}}{4\pi\epsilon_0 R^2}\bar{a}_R = \int_V \frac{\rho_v dv}{4\pi\epsilon_0 R^2}\bar{a}_R \text{ V/m.}$$

i) line charge : (ρ_L) C/m





$$dQ = \rho_L dl = \rho_L dz$$

$$Q = \int_L \rho_L dz = \int_{z_A}^{z_B} \rho_L dz$$

$(\bar{R} = P(x,y,z) - P'(0,0,z'))$

$(\bar{R} = x\bar{a}_x + y\bar{a}_y + (z-z')\bar{a}_z)$

$(|R| = \sqrt{x^2+y^2+(z-z')^2})$

$(\bar{R} = \rho\bar{a}_\rho + (z-z')\bar{a}_z)$

$(|R| = \sqrt{\rho^2+(z-z')^2})$

$$\bar{E} = \int_L \frac{\rho_L d\bar{l}}{4\pi\epsilon_0 R^2}\bar{a}_R$$

$d\bar{l} = d\bar{z}'$ at $(0,0,z')$

$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|}$

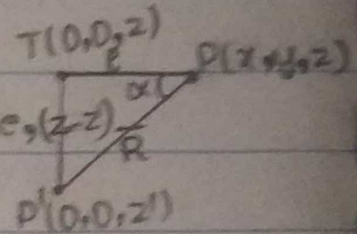
$$\boxed{\therefore \bar{E} = \frac{\rho_L}{4\pi\epsilon_0}\int_L \frac{\rho\bar{a}_\rho + (z-z')\bar{a}_z}{(\rho^2+(z-z')^2)^{3/2}}}$$

$$\bar{a}_R = \frac{\rho\bar{a}_\rho + (z-z')\bar{a}_z}{[\rho^2+(z-z')^2]^{1/2}}$$

$$\therefore \bar{E} = \frac{\rho_L}{4\pi\epsilon_0}\int_{z_A}^{z_B} \frac{(\rho\bar{a}_\rho + (z-z')\bar{a}_z)}{(\rho^2+(z-z')^2)^{3/2}} . dz'$$

Derive $\alpha, \alpha_1, \alpha_2$: from the diagram,

assuming $TP'P$ as right angle triangle,



$$R = \sqrt{\rho^2 + (z-z')^2}$$

$$\cos\alpha = \frac{\rho}{R} \Rightarrow R = \frac{\rho}{\cos\alpha} = \rho\sec\alpha.$$

$$\sin\alpha = \frac{z-z'}{R} \Rightarrow R = \frac{(z-z')}{\sin\alpha} = (z-z')\operatorname{cosec}\alpha$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{z-z'}{\rho} \Rightarrow (\underset{\text{const}}{z}-z') = \rho\tan\alpha$$

$$-dz' = \rho\cdot\sec^2\alpha\, d\alpha.$$

$$dz' = -\rho\sec^2\alpha . d\alpha.$$

$$\because R = \rho\sec\alpha \text{ , } dz' = -\rho\sec^2\alpha\, d\alpha$$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{(\rho\bar{a}_\rho + (z-z')\bar{a}_z).dz'}{R^3}$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{R\cos\alpha\cdot\bar{a}_\rho + \rho\cdot\tan\alpha\cdot\bar{a}_z}{\rho^{\cancel{3}2}\cdot\sec^{\cancel{3}}\alpha} . (-\cancel{\rho}\cancel{\sec^2}\alpha\, d\alpha)$$

$$= \frac{-\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{R\cos\alpha\bar{a}_\rho + \rho\tan\alpha\bar{a}_z}{\rho^2\cdot\sec\alpha} . d\alpha$$

$$= \frac{-\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \left(\frac{\rho\bar{a}_\rho}{\rho^2\sec\alpha} + \frac{\rho\tan\alpha\cdot\bar{a}_z}{\rho^2 . \sec\alpha}\right) . d\alpha.$$

$$= \frac{-\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \left(\frac{\bar{a}_\rho}{\rho\sec\alpha} + \frac{\tan\alpha\,\bar{a}_z}{\rho\cdot\sec\alpha}\right) d\alpha$$

$$= \frac{-\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \left(\frac{\cos\alpha}{\rho} . \bar{a}_\rho + \frac{\sin\alpha}{\rho}\bar{a}_z\right) d\alpha$$

$$= \frac{-\rho_L}{4\pi\epsilon_0\rho}\int_{\alpha_1}^{\alpha_2} (\cos\alpha\bar{a}_\rho + \sin\alpha\bar{a}_z) d\alpha$$

$$= \frac{-\rho_L}{4\pi\epsilon_0\rho}\left[(\sin\alpha_2 - \sin\alpha_1)\bar{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\bar{a}_z\right]$$

$$\boxed{\bar{E} = \frac{\rho_L}{4\pi\epsilon_0\rho}\left[(\cos\alpha_2 - \cos\alpha_1)\cdot\bar{a}_z - (\sin\alpha_2 - \sin\alpha_1)\cdot\bar{a}_\rho\right]}$$

Case1: An infinite line charge extending between ~~A(0,0,0) and~~ A(0,0,−∞) and B(0,0,∞).

i.e, $\alpha_1 = \pi/2$ and $\alpha_2 = -\pi/2$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0\rho}\left[-(\sin(-\pi/2) - \sin\pi/2) + 0\right] = \frac{2\rho_L}{4\pi\epsilon_0\rho}\bar{a}_\rho.$$

$$\boxed{\therefore \bar{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\cdot\bar{a}_\rho \ V/m}$$

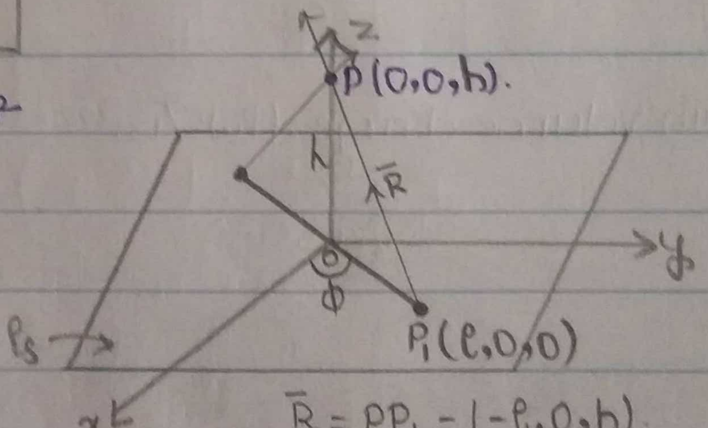
case2: A semi-infinite line charge extending between A(0,0,0) and B(0,0,∞).

i.e, $\alpha_1 = 0$ and $\alpha_2 = +\pi/2$.

$$\epsilon = \frac{\rho_L}{4\pi\epsilon_0\rho}\left((-\sin(\pi/2) - 0)\bar{a}_\rho + (0 - \cos(0))\bar{a}_z\right)$$

$$\boxed{\therefore \bar{E} = \frac{\rho_L}{4\pi\epsilon_0\rho}(-\bar{a}_\rho - \bar{a}_z).}$$

ii) surface charge: (ρ_s) C/m^2



$dQ = \rho_s ds$

$$Q = \int_S \rho_s ds$$

$$\bar{E} = \int_S \frac{\rho_s d\bar{s}}{4\pi\epsilon_0 R^2}\bar{a}_R$$

$\bar{R} = PP_1 = (-\rho, 0, h)$

$$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{\bar{R}}{R}, \quad d\bar{s} = \rho\cdot d\rho d\phi.$$

$$= \frac{-\rho\cdot\bar{a}_\rho + h\bar{a}_z}{\sqrt{\rho^2+h^2}}$$

$$\bar{E} = \int_S \frac{\rho_s \rho\cdot d\rho d\phi}{4\pi\epsilon_0}\cdot\frac{(-\rho\cdot\bar{a}_\rho + h\bar{a}_z)}{(\rho^2+h^2)^{3/2}}$$

$$\therefore \bar{E} = \frac{\rho_s}{4\pi\epsilon_0}\int_s \frac{\rho\cdot d\rho\, d\phi(-\rho\bar{a}_\rho + h\bar{a}_z)}{(\rho^2+h^2)^{3/2}}$$

$E = E\bar{a}_\rho + E\bar{a}_z$

charge is distributed equally on the surface

$$\bar{E} = \frac{\rho_s}{4\pi\epsilon_0}\int_s \frac{\rho\cdot d\rho\cdot d\phi\, h}{(\rho^2+h^2)^{3/2}}\bar{a}_z$$

$$= \frac{\rho_s}{4\pi\epsilon_0}\int_{\rho=0}^{\infty}\int_{\phi=0}^{2\pi} \frac{\rho\cdot h\cdot d\rho\cdot d\phi}{(\rho^2+h^2)^{3/2}}\bar{a}_z \qquad \left(\because \int_0^{2\pi} d\phi = 2\pi\right)$$

$\rho^2+h^2 = t \Rightarrow 2\rho\cdot d\rho = dt$

$$\cancel{= \frac{\rho_s\, 2\pi}{4\pi\epsilon_0}\int_{t=0}^{\infty}\frac{dt\cdot h}{t^{3/2}}} \qquad = \frac{\rho_s\, 2\pi h}{8\pi\epsilon_0}\int_0^{\infty}(t+h^2)^{-3/2}\, dt$$

$$= \frac{\rho_s h}{4\epsilon_0}\left[(\infty+h^2)^{-1/2} - (0+h^2)^{-1/2}\right]\bar{a}_z$$

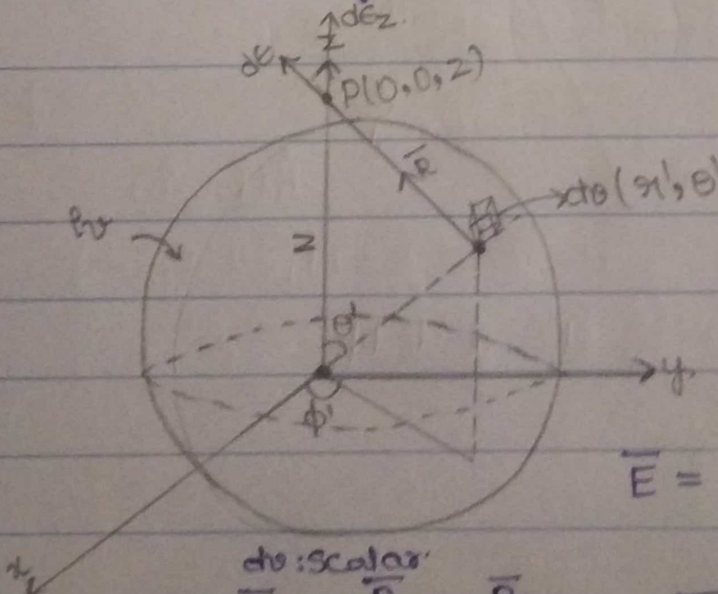
$$\boxed{\therefore \bar{E} = \frac{\rho_s}{2\epsilon_0}\cdot\bar{a}_z}$$

$\bar{a}_z$ is normal to the surface of the charge considered

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\bar{a}_n}$$

$\bar{a}_n \rightarrow$ unit normal vector.

iii) volume charge : $(\rho_v)\, C/m^3$.



$dQ = \rho_v dv$.

$$Q = \int_v \rho_v dv = \rho_v \int_v dv$$

$$Q = \rho_v\left(\frac{4\pi a^3}{3}\right)$$

$$\bar{E} = \int_v \frac{\rho_v\cdot dv}{4\pi\epsilon_0 R^2}\bar{a}_R$$

dv : scalar

$$\bar{a}_R = \frac{\bar{R}}{|R|} = \frac{\bar{R}}{R}$$

$$\bar{a}_R = \cos\alpha\,\bar{a}_z + \sin\alpha\,\bar{a}_\rho$$

$$\bar{E} = \int_v \frac{\rho_v dv}{4\pi\epsilon_0 R^2}(\cos\alpha\,\bar{a}_z + \sin\alpha$$

$$\bar{E} = \int_v \frac{\rho_v dv}{4\pi\epsilon_0 R^2} \cos\alpha \bar{a}_z + \int_v \frac{\rho_v dv}{4\pi\epsilon_0 R^2} \sin\alpha \bar{a}_\rho$$

*on the surface of the conductor, the field is zero.

$\therefore a_\rho = 0$

$$\boxed{\therefore \bar{E} = \int_v \frac{\rho_v \cdot dv}{4\pi\epsilon_0 R^2} \cos\alpha \bar{a}_z}$$

$$dv = r'^2 \cdot \sin\theta' dr' \cdot d\theta' \cdot d\phi'$$

$$\cos\alpha = \frac{z^2 + R^2 - r'^2}{2zR} \quad (\because \text{cosine rule})$$

$$R^2 = z^2 + r'^2 - 2zr'\cos\theta'$$

Let, $\cos\theta' = \frac{z^2 + r'^2 - R^2}{2zr'}$ and $z, r' \rightarrow$ const

$$+\sin\theta' \cdot d\theta' = \frac{+\cancel{2}RdR}{\cancel{2}zr'} \Rightarrow \sin\theta' \cdot d\theta' = \frac{RdR}{zr'}$$

Substituting in dv,

$$dv = r'^2 \cdot \sin\theta' \cdot d\theta' \cdot dr' \cdot d\phi' = r'^{\cancel{2}} \cdot \frac{RdR}{z\cancel{r'}} dr' \cdot d\phi'$$

$$= r' \cdot R\,dR \cdot dr' d\phi'$$

$$E_z = \bar{E} \cdot \bar{a}_z = \int_v \frac{\rho_v \cdot \cancel{R} dR \cdot dr' \cdot d\phi' \cdot \cos\alpha}{4\pi\epsilon_0 \cdot z \cdot R^{\cancel{2}} \cdot r'}$$

$$= \frac{\rho_v}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} \int_{r'=0}^{a} \int_{R=z-r'}^{z+r'} \frac{\cancel{r'} \cos\alpha}{zR\cancel{r'}} \cdot dR \cdot dr' \cdot d\phi'$$

$$= \frac{\rho_v}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^a \int_{z-r'}^{z+r'} \frac{dR \cdot dr'}{zR \cdot r'} \left(\frac{z^2 + R^2 - r'^2}{2zR} \right)$$

$$= \frac{\rho_v (2\cancel{\pi})}{2\cancel{4\pi}\epsilon_0} \int_0^a \int_{z-r'}^{z+r'} \frac{r'^{\cancel{2}} \cdot RdR}{z\cancel{r'}} \cdot dr' \cdot \left(\frac{z^2 + R^2 - r'^2}{2zR} \right)$$

$$= \frac{\rho_v}{2\epsilon_0} \int_0^a \int_{z-r'}^{z+r'} r' \left[1 + \frac{z^2 - r'^2}{R^2} \right] \cdot dR\, dr'$$

$$E_z = \frac{\rho_v}{2\epsilon_0(2z^2)} \int_{r'=0}^{a}\int_{R=z-r'}^{z+r'} r'\left(1+\frac{z^2-r'^2}{R^2}\right) . dR \cdot dr'$$

$$= \frac{\rho_v}{2\epsilon_0(2z^2)}\int_{r'=0}^{a}\left(r'\left(R\right]_{z-r'}^{z+r'} + r'(z^2-r'^2)\left(\frac{-1}{R}\right]_{z-r'}^{z+r'}\right) dr'$$

$$= \frac{\rho_v}{2\epsilon_0(2z^2)}\int_{r'=0}^{a}\left(r'(2r') + r'(z^2-r'^2)\left(\frac{-1}{z+r'}+\frac{1}{z-r'}\right)\right) dr'$$

$$= \frac{\rho_v}{2\epsilon_0(2z^2)}\int_{r'=0}^{a}\left(2r'^2 + r'(\cancel{z^2-r'^2})\left(\frac{-z+r'+z+r'}{\cancel{z^2-r'^2}}\right)\right) dr'$$

$$= \frac{\rho_v}{2\epsilon_0(2z^2)}\int_{r'=0}^{a}(2r'^2+2r'^2) . dr'$$

$$= \frac{\rho_v}{4\epsilon_0 z^2}\left(2\left(\frac{r'^3}{3}+\frac{r'^3}{3}\right)_0^a\right)$$

$$\therefore E_z = \frac{\rho_v}{4\epsilon_0 z^2}\left(\frac{\overset{4}{\cancel{2}}}{3}a^3 \cancel{+a^2}\right)$$

$$\boxed{\therefore E_z = \frac{\rho_v}{3\epsilon_0 z^2}a^3}$$

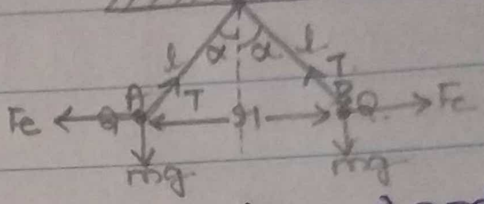
$$E_z = \frac{1}{\epsilon_0 z^2}\left(\frac{a^3}{3}\rho_v\right)$$

$$= \frac{4\pi a^3\rho_v}{12\pi\epsilon_0 z^2} = \frac{1}{4\pi\epsilon_0 z^2}\left(\frac{4\pi a^3}{3}\rho_v\right) = \frac{Q}{4\pi\epsilon_0 z^2}$$

$$\boxed{\therefore \bar{E} = \frac{Q}{4\pi\epsilon_0 z^2}\bar{a}_z} \text{ V/m}$$

Q) Point charges 5nC, -2nC are located at (2,0,4) & (-3,0,5) respectively. Determine the force on 1nC point charge located at (1,-3,7). Also calculate the electric field at that point.

Q) Two point charges of equal mass 'm' & charge 'Q' are suspended at a common point with two threads of negligible mass and length 'l'. Show that at equilibrium the inclination angle 'α' of each thread to the vertical is given by $Q^2 = 16\pi\epsilon_0 mgl^2 \sin^2\alpha \tan\alpha$. If α is too small, show that $\alpha = \sqrt[3]{\frac{Q^2}{16\pi\epsilon_0 mgl^2}}$



Q) Point charges 1nC and -2nC are located at (3,2,-1) and (-1,-1,4). Determine the force on a 10nC point charge located at (0,3,1). Also calculate the electric field at that point

Q) The circular ring of radius 'a' carries uniform charge ρ_L (C/m) and is placed on the xy plane with axis the same as z-axis. Show that $\bar{E}$ at (0,0,h) is

i) $\bar{E} = \frac{\rho_L \cdot ah}{2\epsilon_0(h^2+a^2)^{3/2}} \cdot \bar{a}_z$

ii) What values of h gives the max. value of '$\bar{E}$'.

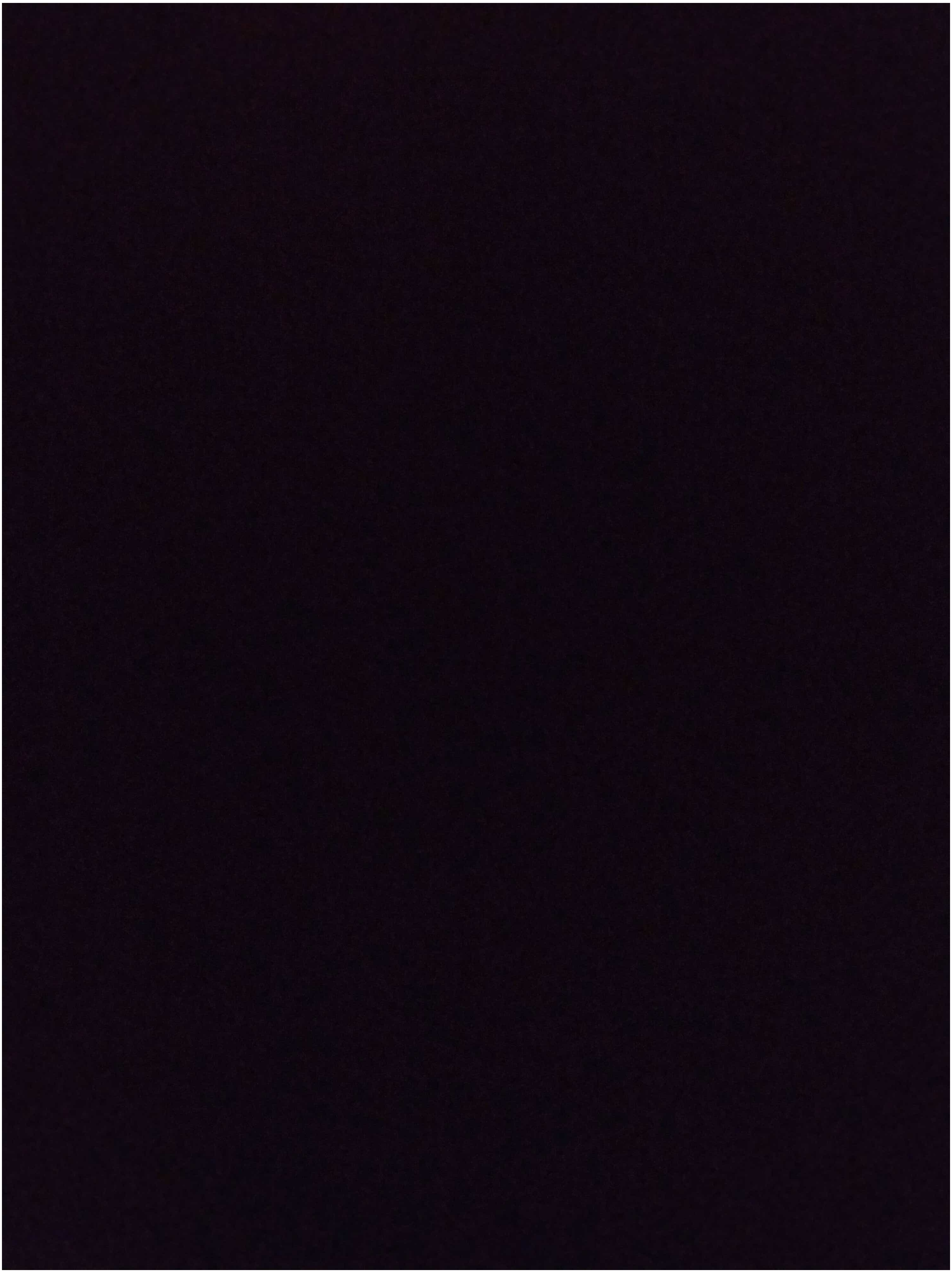
iii) If the total charge on ring is Q. Find $\bar{E}$ as $a \to 0$

Q) The circular disc of radius 'a' carries uniform charge ρ_s (C/m²) and is placed on the xy plane with its axis along z-axis.

a) at (0,0,h) that $\bar{E} = \frac{\rho_s}{2\epsilon_0}\left\{1 - \frac{h}{(h^2+a^2)^{1/2}}\right\}\bar{a}_z$

b) Derive the $\bar{E}$ due to an infinite sheet of charge on z=0 place

c) If a<<h show that $\bar{E}$ is similar to the field due to a point charge



Flux: {flow of charge from one place to another place}

relation between fluxdensity & electricfield $\Rightarrow \bar{D} = \epsilon_0 \bar{E}$ V/m

$\epsilon_0 \Rightarrow$ permittivity $= 8.85 \times 10^{-12}$ F/m $= \frac{10^{-9}}{36\pi}$ F/m

$\psi = \int_S \bar{D} \cdot d\bar{s}$ coulombs (c).

electric flux/electric displacement (scalar quantity).

$d\bar{s} = m^2$
$\bar{D} \cdot d\bar{s} =$ Scalar
$\psi = C$

$\bar{D} = \delta$ C/m² $\rightarrow$ charge density.

$\rho_s \rightarrow$ surface chargedensity

Electric flux density: Flux is line showing the displacement of charge (+ve or -ve) measured in colournbs. ψ = Colournbs $\rightarrow$ scalar.

Flux density among the charge measured in unitarea

$\bar{D} = \delta$ C/m² $\rightarrow$ vector

Gauss's law: The total electric flux 'ψ' through any closed surface is equal to the total charge enclosed by that surface.

$$\psi = Q_{enc} = \oint_S d\psi.$$

total charge enclosed by surface

$$Q_{enc} = \int_V \rho_v dv$$

$$\psi = \int_S \bar{D} \cdot d\bar{s} \Rightarrow \oint_S d\psi = \oint_S \bar{D} \cdot dv$$

$$\oint_S d\psi = \int_V \rho_v dv \Rightarrow \oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_v dv$$

using divergence theorem that states closed surface integral by S = volume integral of divergence of field $\bar{A}$

$$\oint_S \bar{A} d\bar{s} = \int_V (\nabla \cdot \bar{A}) \cdot dv.$$

$$\Rightarrow \oint_v (\nabla \cdot \bar{D})\, dv = \oint_v \rho_v \cdot dv$$

$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$

divergence of $\bar{D}$ scalar

The volume charge density 'ρ_v' is same as that of the divergence of the electric flux density $\bar{D}$. This is called Maxwell's equation for electrostatic fields.

$$\boxed{\rho_v = \nabla \cdot \bar{D}}$$

Applications of Gauss law:

① Relationship between ψ and electric field.

② Maxwell's equation.

③ Calculate of variation charge distribution

When charge distribution's uniform → assume boundary that has uniform charge density.

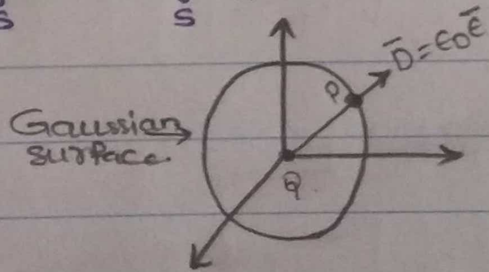
The distribution is uniform → Boundary that has uniform charge distribution is called Gaussian Surface.

$$\boxed{Q_{enc} = \psi = \oint_S \bar{D} \cdot d\bar{s}}$$

point charge:

$$Q = \oint_S \bar{D} \cdot d\bar{s} = \oint_S D_r\, ds.$$

field is existing in all the omnidirectional.



$$= D_r \oint_S d\bar{s}$$

$d\bar{s} = r\,dr\,d\theta\, \bar{a}_\phi$

$= r^2 \sin\theta\, d\theta\, d\phi\, \bar{a}_r$

$$= D_r \oint_S r^2 \sin\theta\, d\theta\, d\phi$$

$$= D_r \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta\, d\theta\, d\phi$$

$$= D_r \cdot r^2 (2\pi) \int_{\theta=0}^{\pi} \sin\theta d\theta.$$

$$= D_r \cdot 2\pi r^2 \cdot \left[-\cos\theta\right]_0^{\pi}$$

$$= D_r 2\pi r^2 \left(-(-1-1)\right)$$

$$= 4 D_r \pi r^2$$

$$\therefore Q = 4\pi r^2 D_r$$

$$D_r = \frac{Q}{4\pi r^2}$$

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2} \cdot \bar{a}_r}$$

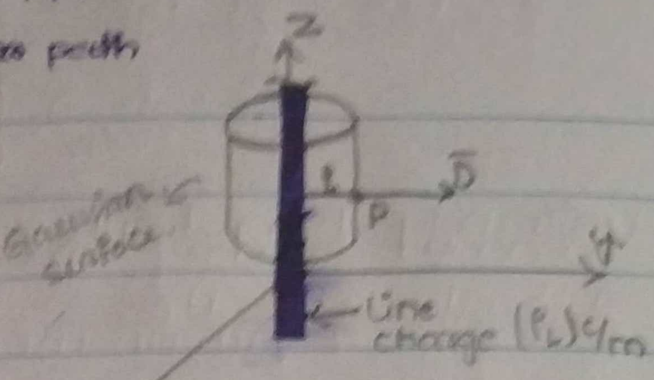
where $\bar{D} = \epsilon_0 \bar{E} \Rightarrow \bar{E} = \frac{\bar{D}}{\epsilon_0}$

$$\boxed{\bar{E} = \frac{\bar{D}}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2} \bar{a}_r} \ \text{V/m.}$$

Infinite line charge using Gauss laws:

The normal & perpendicular path is in 'ρ' direction. ($\bar{a}_\rho$)

$$\bar{D} = D_\rho \bar{a}_\rho$$



WKT, $Q_{enc} = \oint_S \bar{D}\cdot d\bar{s}$

$$\oint_S \bar{D}\cdot d\bar{s} = \oint_S (D_\rho \bar{a}_\rho)\cdot(\rho\, d\phi\, dz\, \bar{a}_\rho)$$

$$= D_\rho \int_{\phi=0}^{2\pi}\int_{z=0}^{l} \rho\, d\phi\, dz$$

$$= \rho\, D_\rho\, (2\pi)\, l$$

$$= 2\pi\rho l D_\rho \rightarrow \text{RHS}$$

$dQ = \rho_L\, dl$

$Q = \int_L \rho_L\, dl$

$= \rho_L\, l$

$d\bar{s} = \rho\, d\phi\, dz\, \bar{a}_\rho$

ρ → radius of the cylinder.

from Gauss's law, $\rho_L \cancel{l} = 2\pi\rho \cancel{l} D_\rho$

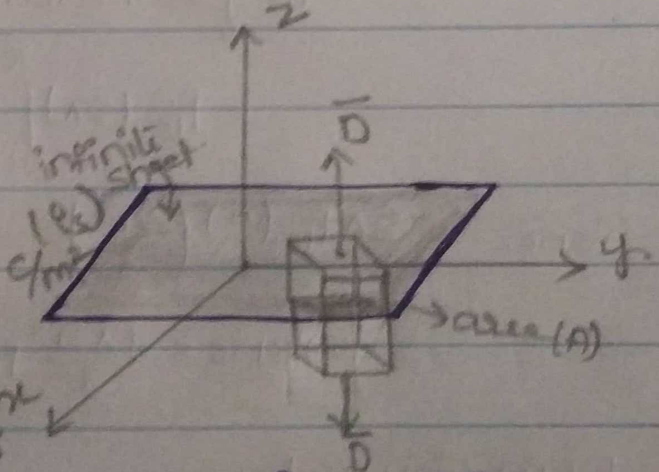
$$D_\rho = \frac{\rho_L}{2\pi\rho} \Rightarrow \boxed{\bar{D} = \frac{\rho_L}{2\pi\rho}\bar{a}_\rho}$$

Electric field $\bar{D} = \epsilon_0 \bar{E} \Rightarrow \bar{E} = \frac{\bar{D}}{\epsilon_0}$

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 \cdot \rho}\bar{a}_\rho \text{ V/m}}$$

Surface charge or infinite sheet charge:

$$Q_{enc} = \oint_S \bar{D}\cdot d\bar{s}$$



$$\oint_S \bar{D}\cdot d\bar{s} = \int_{top} \bar{D}\cdot d\bar{s} + \int_{bottom} \bar{D}\cdot d\bar{s}$$

$$= D_z\left[\int_{top} ds + \int_{bottom} ds\right]$$

$$= D_z[A + A] = 2AD_z \rightarrow \text{RHS}$$

$$Q_{enc} = \int_S \rho_s\, ds = \rho_s \int_S ds = \rho_s A$$

from gauss's law,

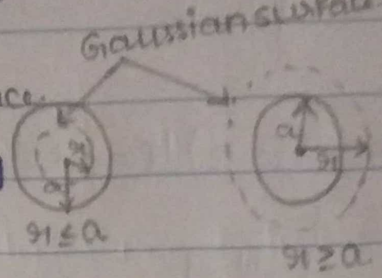
$$\rho_s \cancel{A} = 2\cancel{A} D_z \Rightarrow D_z = \frac{\rho_s}{2} \Rightarrow \boxed{\bar{D} = \frac{\rho_s}{2}\bar{a}_z}$$

$$\bar{D} = \epsilon_0 \bar{E} \Rightarrow \bar{E} = \frac{\bar{D}}{\epsilon_0} = \frac{\rho_s}{2\epsilon_0}\bar{a}_z$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\bar{a}_z}$$

Uniformly charged sphere:

$a \rightarrow$ radius of the sphere
$r \rightarrow$ radius of gaussian surface
$\rho_v \rightarrow$ volume charge density (C/m^3).



$\Psi = Q_{enc}$

$dQ_{enc} = \rho_v \, dv$.

$\Psi = \oint_S \bar{D} \cdot d\bar{s}$

$Q_{enc} = \int_v \rho_v \, dv$.

$\bar{D} = D_r \bar{a}_r$

$d\bar{s} = r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r$

$\bar{D} \cdot d\bar{s} = D_r r^2 \sin\theta \, d\theta \, d\phi$.

$$\Psi = \oint_S D_r r^2 \sin\theta \, d\theta \, d\phi = D_r r^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \, d\phi$$

$$\Psi = D_r r^2 (2\pi) \cdot (-\cos\theta)_0^{\pi} = 2\pi r^2 D_r (-1-1)$$

$$\boxed{\Psi = -4\pi r^2 D_r} \rightarrow \text{LHS.}$$

$dv = r^2 \sin\theta \, d\theta \, d\phi \, dr$

$$Q_{enc} = \int_v \rho_v \, dv = \rho_v \int_v dv$$

$$= \rho_v \int_v r^2 \sin\theta \, d\theta \, d\phi \, dr = \rho_v \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{r} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \rho_v (2\pi) \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{r=0}^{r} r^2 \, dr$$

$$Q_{enc} = 2\pi \rho_v (-2) \left(\frac{r^3}{3}\right) = \frac{-4\pi r^3}{3} \rho_v \text{ C.}$$

$$\boxed{Q_{enc} = \frac{-4\pi r^3}{3} \rho_v} \rightarrow \text{RHS}$$

From Gauss law,

$$\Psi = Q_{enc} \Rightarrow \cancel{-4\pi r^2} D_r = \frac{\cancel{-4\pi} r^3}{3} \rho_v$$

$$\Rightarrow \boxed{D_r = \frac{r}{3} \rho_v} \text{ for } r \leq a$$

for $r \geq a$, $\psi = Q_{enc}$.

$\psi \rightarrow$ common for both $r \leq a$ and $r \geq a$.

$\psi = D_r 4\pi r^2$

$$Q_{enc} = \int_v \rho_v dv = \rho_v \int r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \rho_v \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} r^2 \sin\theta\, d\theta\, dr\, d\phi.$$

$$= \rho_v (2\pi)(-2)\left(\frac{a^3}{3}\right) = -\frac{4\pi a^3}{3}\rho_v.$$

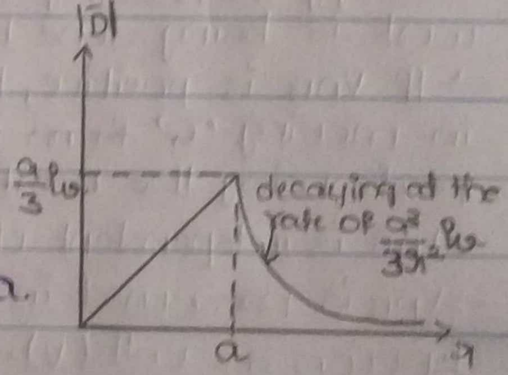
From Gauss law, $\psi = Q_{enc}$. $\Rightarrow \cancel{-} D_r 4\pi r^2 = \cancel{-}\frac{4\pi a^3}{3}\rho_v$.

$$\Rightarrow \boxed{D_r = \frac{a^3}{3r^2}\rho_v} \text{ for } r \geq a$$

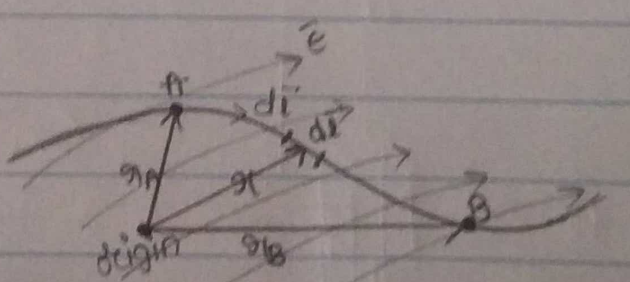
$$\therefore \bar{D} = \begin{cases} \frac{r}{3}\rho_v \cdot \bar{a}_r & \text{for } r \leq a \\ \frac{a^3}{3r^2}\rho_v \cdot \bar{a}_r & \text{for } r \geq a \end{cases}$$

$\bar{D} = \epsilon_0 \bar{E} \Rightarrow \bar{E} = \frac{\bar{D}}{\epsilon_0}$.

$$\bar{E} = \begin{cases} \frac{\rho_v r}{3\epsilon_0}\bar{a}_r & \text{for } r \leq a \\ \frac{\rho_v a^3}{3\epsilon_0 r^2} \cdot \bar{a}_r & \text{for } r \geq a. \end{cases}$$



Electric potential:



$dW = -\bar{F} \cdot d\bar{l}$

$dW = -Q\bar{E} \cdot d\bar{l}$

$W = \int -Q\bar{E} \cdot d\bar{l}$

$W = -Q\int_A^B \bar{E} \cdot d\bar{l} \rightarrow$ work done to move point charge from A to B in the presence of Electric field.

$$\therefore W = -Q\int_A^B \bar{E}\cdot d\bar{l} \text{ J}$$

work per unit charge $\Rightarrow \frac{W}{Q} \rightarrow \frac{J}{C}$ = 'V'

$$V = \frac{W}{Q} = -\int_A^B \bar{E}\cdot d\bar{l}$$

$$\therefore \text{Electric potential} = V = -\int_A^B \bar{E}\cdot d\bar{l}$$

This is called potential difference between points A and B denoted as V_{AB}.

where, $V_{AB} = V_B - V_A = -\int_A^B \bar{E}\cdot d\bar{l}$.

→ In determining V_{AB}, A is the initial point and B is the final point

→ If V_{AB} is negative, there is a loss in potential energy in moving 'Q' from A to B. This implies that work is being done by the field.

→ If V_{AB} is positive, there is a gain in potential energy in moving 'Q' from A to B i.e, an external agent is performing the work

→ V_{AB} is independent of the path taken.

→ V_{AB} is measured in Joules per coloumb (J/C) commonly known as volts (V).