UNIT - 3

Active Filters & Oscillators

Filter is basically a "Frequency selective circuits".

Filters
- → Analog or Digital Filters
- → Active or passive Filters
- → Audio or Radio Filters.

passive Filters:- uses only passive components. [R, C, L]

Eg:- RC, RL, RLC

Active Filters:- uses Active Components.

Eg:- Transistors, op-amp e.t.c

Advantages of Active filters:-

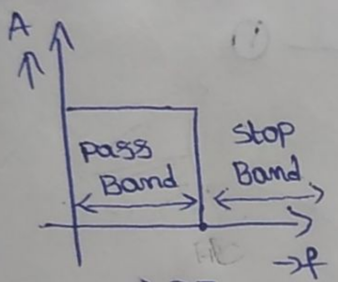
1. Flexibility in gain and Frequency Adjustment
2. No loading problem.
3. Low cost
4. pass band gain
5. Small component size.
6. Use of Inductors can be avoided.
7. control of impedence.

* The gain will be decrease by using passive components.

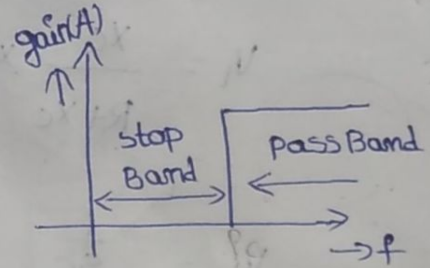
Types of Active filters:-

1. Low pass filter (LPF)
2. High pass filter (HPF)
3. Band pass filter (BPF) — Wide Band pass filter; Narrow Band pass filter
4. Band Reject filter (BRF) — Wide Band Reject filter; Narrow Band Reject filter
5. All pass filter

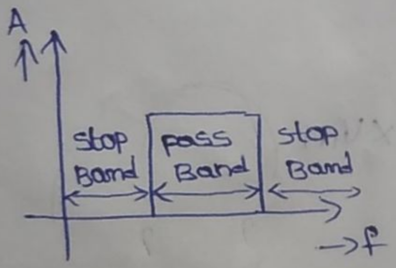
Ideal characteristics:-



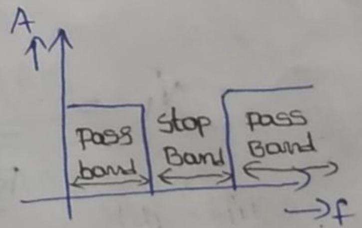
LPF



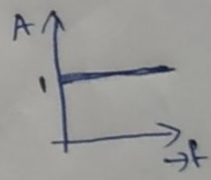
HPF



BPF



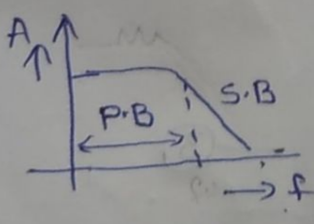
BRF



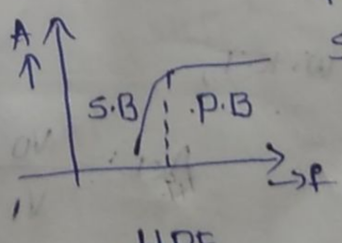
A.P.F

practical characteristics:-

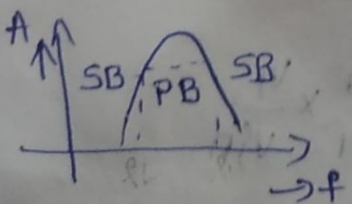
PB → pass Band
SB → stop Band



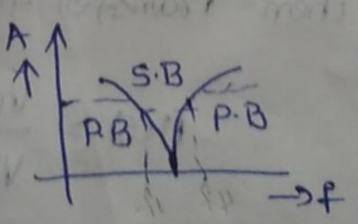
LPF



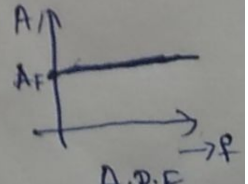
HPF



BPF

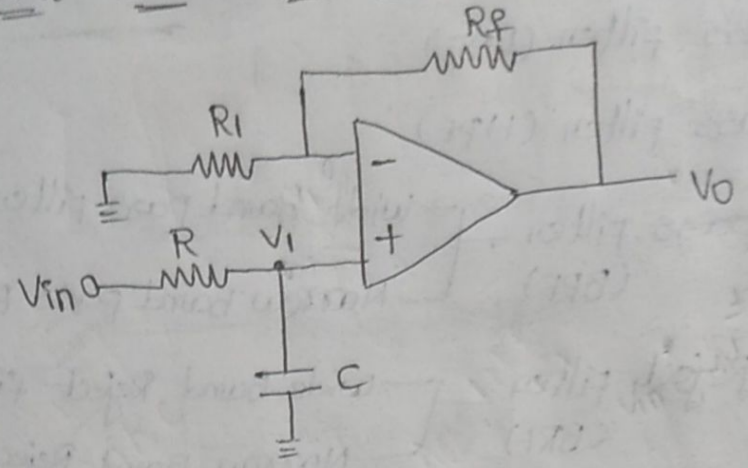


B.R.F



A.P.F

* First order Low pass Filter:-



According to the voltage-divider rule, the voltage at non-inverting terminal is.

$$V_1 = \frac{X_C}{R+X_C} \times V_{in} \quad \text{--- (1)}$$

$$V_1 = \frac{\frac{1}{SC}}{R+\frac{1}{SC}} \times V_{in}$$

$$V_1 = \frac{\frac{1}{SC}}{\frac{RSC+1}{SC}} \times V_{in}$$

$$\frac{V_1}{V_{in}} = \frac{1}{1+S(RC)} \quad \text{--- (2)}$$

W.K.T,

$$A_{CL} = \frac{V_0}{V_1} = 1+\frac{R_f}{R_1} \quad \text{--- (3)}$$

then Transfer function

$$H(S) = \frac{V_0}{V_{in}} = \frac{V_0}{V_1} \times \frac{V_1}{V_{in}}$$

From eq (2) & (3)

$$\frac{V_O}{V_{in}} = \left(1+\frac{R_f}{R_1}\right) \times \frac{1}{1+S(RC)}$$

Let $\boxed{1+\frac{R_f}{R_1} = A_F}$

$$\boxed{H(S) = \frac{A_F}{1+S(RC)}} \quad \text{— (4)}$$

→ To Find frequency response take $S = j\omega$

$$H(j\omega) = \frac{A_F}{1+j\omega RC}$$

$$|H(j\omega)| = \frac{|A_F|}{\sqrt{1+(\omega RC)^2}}$$

Here, $\omega_c = \frac{1}{RC}$

$$\left|\frac{V_O}{V_{in}}\right| = \frac{|A_F|}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$$

Here, $\omega = 2\pi f$

$$\left|\frac{V_O}{V_{in}}\right| = \frac{|A_F|}{\sqrt{1+\left(\frac{2\pi f}{2\pi f_c}\right)^2}}$$

$$|H(f)| = \boxed{\left|\frac{V_O}{V_{in}}\right| = \frac{|A_F|}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}}$$ Here, $f_c = \frac{1}{2\pi RC}$

Here,

$|A_F| = 1+\frac{R_f}{R_1}$ = pass band gain of the filter.

f = frequency of input signal

$f_H = f_c = \frac{1}{2\pi RC}$ = high cut-off frequency of filter.

* if $f << f_c$, [At low frequencies]

$$H(f) = \left| \frac{V_o}{V_{in}} \right| \cong |A_F|$$

* if $f = f_c$,

$$H(f) = \left| \frac{V_o}{V_{in}} \right| = \frac{A_F}{\sqrt{1+\left(\frac{1}{1}\right)^2}} = \frac{A_F}{\sqrt{2}} = 0.707|A_F|$$

* if $f >> f_c$,

$$H(f) = \left| \frac{V_o}{V_{in}} \right| \cong 0$$

$$\therefore \phi = -\tan^{-1}\left(\frac{f}{f_c}\right)$$

where ϕ is the phase angle in degrees.

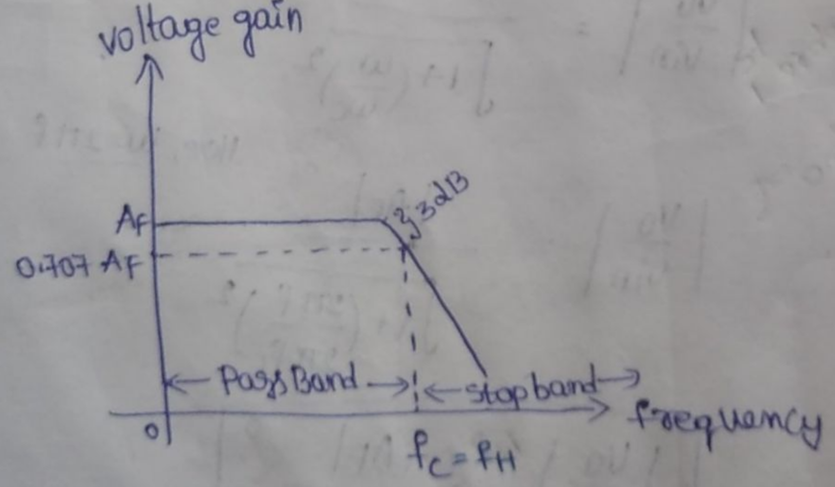
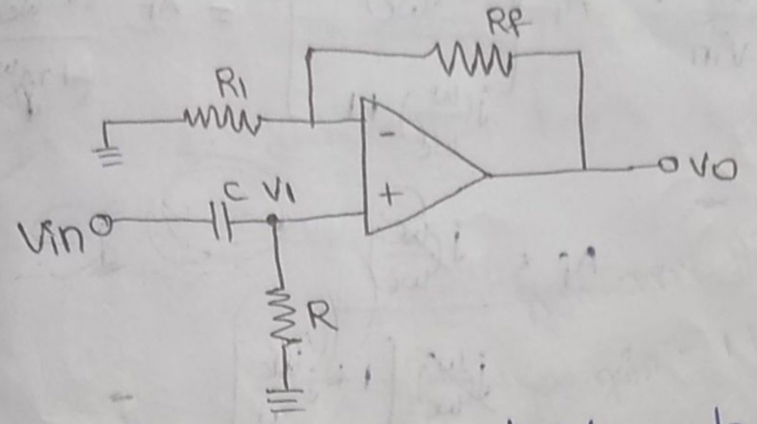


fig:- Frequency response of LPF

* High pass Filter for first order:-



According to the voltage-divider rule, the voltage at non-inverting terminal is,

$$V_1 = \frac{R}{R+X_C} \times V_{in}$$

$$V_1 = \frac{R}{R+\frac{1}{SC}} \times V_{in} = \frac{R(SC)}{R(SC)+1}$$

$$\frac{V_1}{V_{in}} = \frac{R(SC)}{R(SC)+1} \quad \text{—— ①}$$

W.K.T

$$A_{CL} = \boxed{1+\frac{R_f}{R_1} = \frac{V_O}{V_1}} \quad \text{—— ②}$$

From ① & ②

$$\frac{V_O}{V_{in}} = \frac{V_O}{V_1} \times \frac{V_1}{V_{in}}$$

$$= \left(1+\frac{R_f}{R_1}\right) \times \frac{R(SC)}{R(SC)+1}$$

here $S = j\omega$,

$$1+\frac{R_f}{R_1} = A_F$$

$$= A_F \times \frac{Rj\omega C}{Rj\omega C+1}$$

Here $\omega_c = \frac{1}{RC}$

$$\frac{V_o}{V_{in}} = A_F \times \left[\frac{j\left(\frac{\omega}{\omega_c}\right)}{j\left(\frac{\omega}{\omega_c}\right)+1} \right] = A_F \frac{\left(j\frac{\omega}{\omega_c}\right)}{\left(1+j\left(\omega/\omega_c\right)\right)}$$

$$= A_f \times \frac{\cancel{j\left(\frac{\omega}{\omega_c}\right)}}{\cancel{j\left(\frac{\omega}{\omega_c}\right)}\left[1+j\frac{\omega_c}{\omega}\right]}$$

$$\frac{V_o}{V_{in}} = \frac{A_F}{1+j\left(\frac{\omega_c}{\omega}\right)} \quad (\text{or}) \quad \frac{A_F\left[j\frac{\omega}{\omega_c}\right]}{1+j\left(\frac{\omega}{\omega_c}\right)}$$

$$\left|\frac{V_o}{V_{in}}\right| = \frac{|A_F|}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}} \quad (\text{or}) \quad \frac{|A_F|\left(\frac{\omega}{\omega_c}\right)}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}} \qquad \because \omega = 2\pi f, \quad \because \omega_c = 2\pi f_c$$

$$H(f) = \left|\frac{V_o}{V_{in}}\right| = \frac{|A_F|}{\sqrt{1+\left(\frac{f_c}{f}\right)^2}} \quad (\text{or}) \quad \frac{|A_F|\left(\frac{f}{f_c}\right)}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}$$

Here $f_c = f_l$ = low cut-off frequency.

if $f \ll f_c$

$$H(f) = \left|\frac{V_o}{V_{in}}\right| \cong 0$$

if $f = f_c$

$$H(f) = \frac{|A_F|}{\sqrt{2}} = 0.707|A_F|$$

if $f \gg f_c$

$$H(f) = \frac{|A_F|}{\sqrt{1}} = |A_F|$$

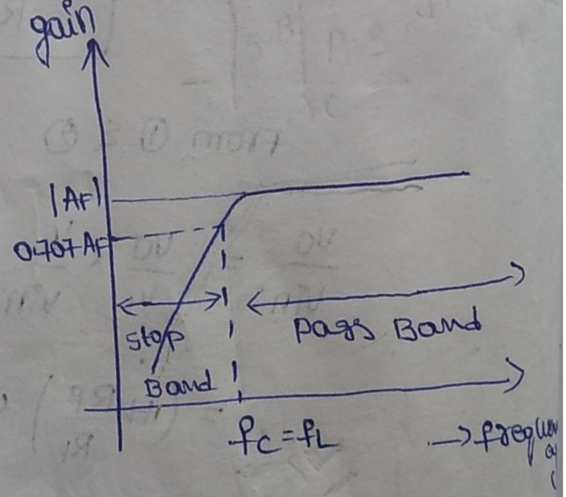


fig:- Frequency response of HPF

* Band pass Filter:-

A Bandpass Filter is basically a frequency selector. It allows one particular band of frequencies to pass.

* Based upon the Quality factor, the Bandpass filter is classified into 2 types

1. Narrow Band pass Filter. $[Q>10]$
2. Wide Band pass Filter $[Q<10]$

wide Band pass Filter:-

The wide band pass Filter is realized by cascading a high pass filter and low pass filter.

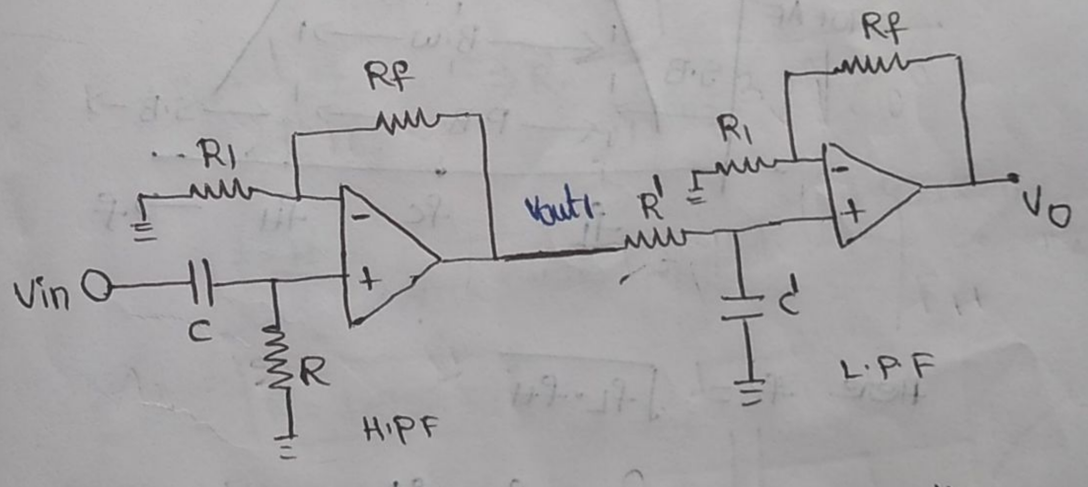


fig:- wide Band pass filter.

Transfer Function of wide Bandpass filter is

$$H(S) = \frac{V_O}{V_{in}} = \frac{V_O}{V_{out1}} \cdot \frac{V_{out1}}{V_{in}}$$

$$\left|\frac{V_O}{V_{in}}\right| = \frac{|A_{O1}|}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}} \cdot \frac{|A_{O2}|}{\sqrt{1+\left(\frac{f_L}{f}\right)^2}}$$

$$H(s) = \left|\frac{V_o}{V_{in}}\right| = \frac{A_{FT}}{\sqrt{\left[1+\frac{f}{f_H}\right]^2 \left[1+\frac{\cancel{f_L}}{f}\right]^2}} \quad (or)$$

$$H(s) = \left|\frac{V_o}{V_{in}}\right| = \frac{A_{FT}\left(f/f_L\right)}{\sqrt{\left(1+\frac{f}{f_H}\right)^2 \left(1+\frac{f}{f_L}\right)^2}}$$

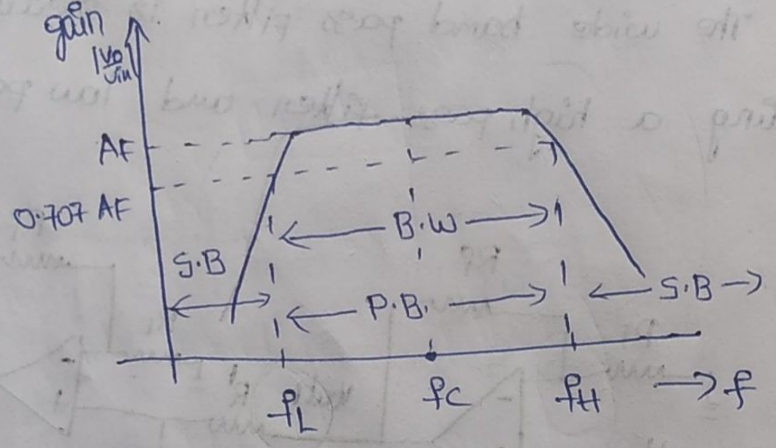
Here $A_{FT} = A_{01} \cdot A_{02}$

f = Input frequency.

f_L = Lower cut-off frequency.

f_H = Higher-cut-off frequency.

The frequency response,



Here $f_C = \sqrt{f_L \cdot f_H}$

Bandwidth $= f_H - f_L$

then $Q = \frac{f_C}{B.W} = \frac{f_C}{f_H - f_L}$

In the wide Band pass filter, the quality factor is less than 10. i.e, Band width is ~~high~~. large.

→ Design a wide Band pass filter with $f_L = 400\,Hz$ and $f_H = 2\,KHz$ and pass band gain is 4. and capacitor $C = 0.05\,\mu F$, $C_1 = 0.01\,\mu F$. and find quality factor.

Sol:- given that,

$f_L = 400\,Hz$

$f_H = 2\,KHz$

$A_F = 4$

$C = 0.05\,\mu F$

$C_1' = 0.01\,\mu F$

$R_F = ?,\ R_1 = ?,\ R = ?\ R' = ?,\ Q = ?$

For L.P.F,

$f_H = 2\,KHz$

$c' = 0.01\,\mu F$

$f_H = \frac{1}{2\pi R' c'} \Rightarrow R' = \frac{1}{2\pi f_H C}$

$\Rightarrow R' = \frac{1}{2 \times \pi \times 2 \times 10^3 \times 0.01 \times 10^{-6}} = 7.957\,K\Omega$

$\boxed{R' = 7.957\,K\Omega}$

For H.P.F,

$f_L = 400\,Hz$

$C = 0.05\,\mu F$

$f_L = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi f_L C}$

$R = \frac{1}{2 \times \pi \times 400 \times 0.05 \times 10^{-6}} = 7.95\,K\Omega$

$\boxed{R = 7.95\,K\Omega}$

$A_{FT} = 4.$

$A_{FT} = A_{O1} \cdot A_{O2} = 4$

$A_{O1} = A_{O2} = 2$

$$\boxed{A_{O1} = A_{O2} = 1 + \frac{R_F}{R_1}}$$

$$1 + \frac{R_F}{R_1} = 2$$

$$\frac{R_F}{R_1} = 2 - 1$$

$$\frac{R_F}{R_1} = 1$$

$$\Rightarrow \boxed{R_F = R_1}$$

$\therefore R_F = R_1 = 10k\Omega$

The wide Band pass filter,

Quality factor $= Q = \frac{f_c}{B.W}$

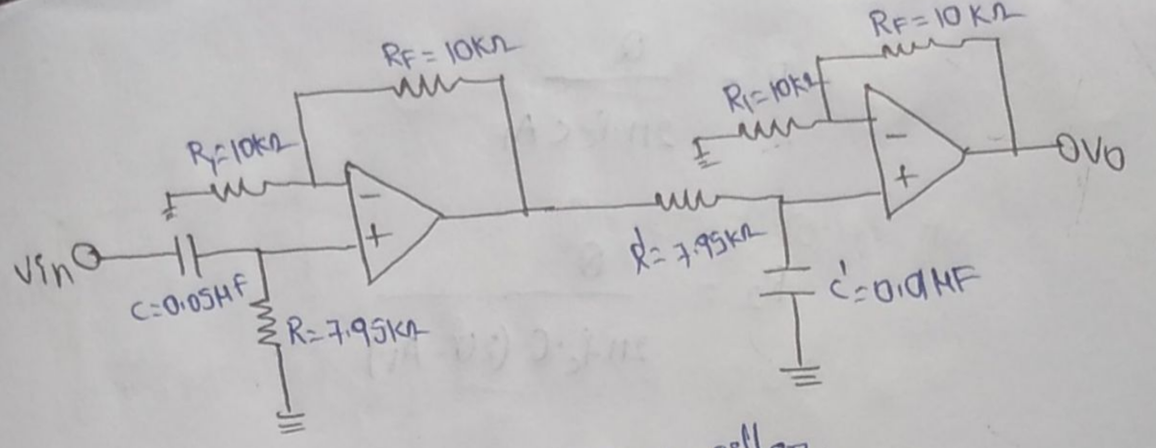
$B.W = f_H - f_l$

$= 2000 - 400 = 1600Hz$

$f_c = \sqrt{f_l \cdot f_H} = \sqrt{2 \times 10^3 \times 400} = 894 Hz.$

$Q = \frac{f_c}{B.W} = \frac{894}{1600} = 0.56 \Rightarrow \boxed{Q = 0.56}$

$\Rightarrow \boxed{Q < 10}$



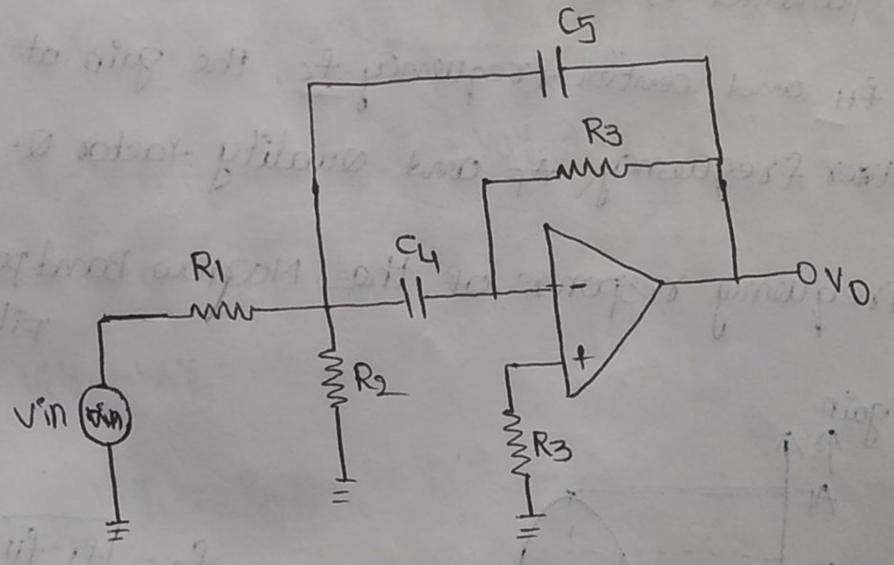
Wide Band pass filter.

Narrow Band pass filter:-

The Narrow Band pass filter uses only one op-amp. It has following features.

1. It has two feed back paths, hence the name multiple-feedback filter.

2. The op-amp is used in the inverting mode.



The circuit components are determined from the following relations.

choose $C_4 = C_5 = C$

* The resistance R_3 connect to non-inverting i/p terminal is offset compensating resistance.

$$R_1 = \frac{Q}{2\pi f_c C A_F}$$

$$R_2 = \frac{Q}{2\pi f_c \cdot C\,(2Q^2 - A_F)}$$

$$R_3 = \frac{Q}{\pi f_c \cdot C}$$

where A_F is the gain at f_c,

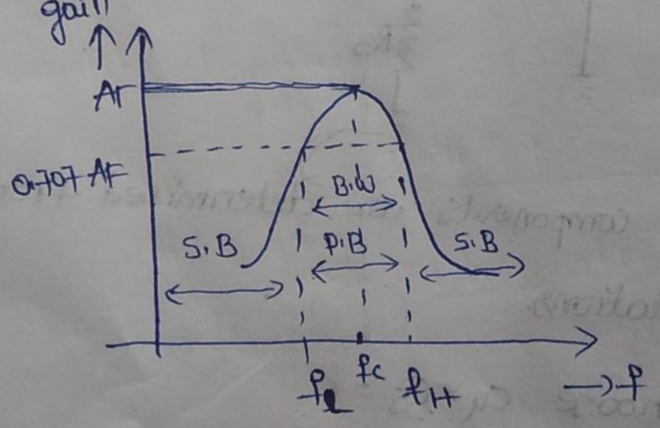
$$A_F = \frac{R_3}{2R_1}$$

The gain A_F must satisfy the equation,

$A_F < 2Q^2$

* The parameters of Narrow band pass filter are f_L, f_H and center frequency f_c, the gain at center frequency A_F and quality factor Q.

The frequency response of the Narrow Band pass filter,

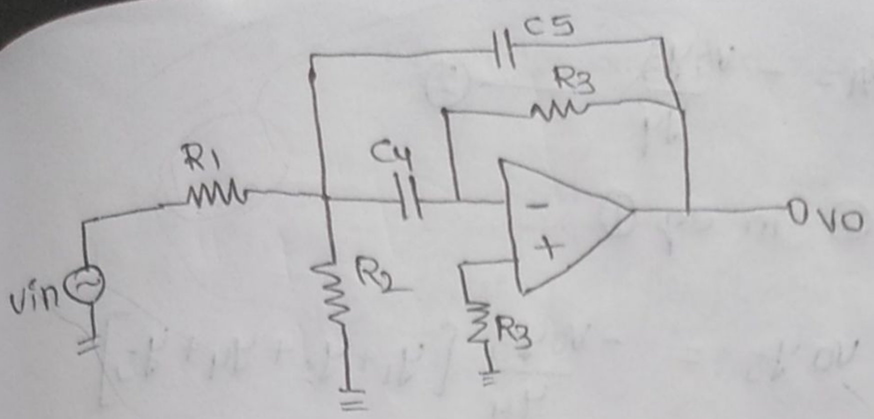


$f_c = \sqrt{f_L \cdot f_H}$

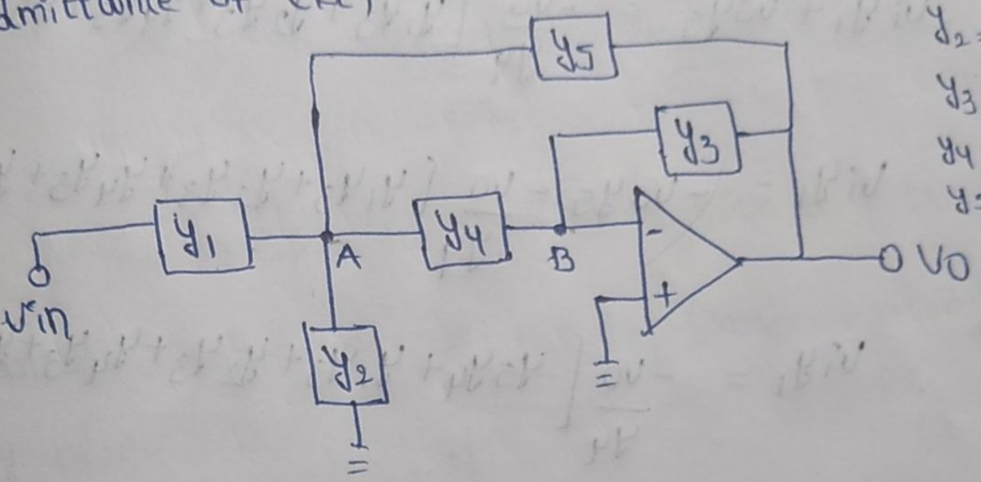
$B.W = f_H - f_L$

$Q = \frac{f_c}{B.W}$

Here $\boxed{Q > 10}$



Admittance of ckt,



Here $y_1 = \frac{1}{R_1}$

$y_2 = 1/R_2$

$y_3 = 1/R_3$

$y_4 = sC_4$

$y_5 = sC_5$

At node A,

$$(V_i - V_A) y_1 = (V_A - 0) y_2 + (V_A - V_0) y_5 + (V_A - V_B) y_4 .$$

$$V_i y_1 - V_A y_1 = V_A y_2 + V_A y_5 - V_0 y_5 + V_A y_4 - V_B y_4$$

Here $V_B = 0$

$$V_i y_1 - V_A y_1 = V_A y_2 + V_A y_5 - V_0 y_5 + V_A y_4$$

$$V_i y_1 = V_A y_1 + V_A y_2 + V_A y_5 + V_A y_4 - V_0 y_5$$

$$V_i y_1 + V_0 y_5 = V_A \left[y_1 + y_2 + y_4 + y_5 \right] \quad \text{—— ①}$$

At node B,

$$(V_A - V_B) y_4 = (V_B - V_0) y_3$$

Here $V_B = 0$

$$V_A y_4 = -V_0 y_3$$

$$V_A = -\frac{V_0 Y_3}{Y_4} \quad \text{—— (2)}$$

sub eq (2) in eq (1)

$$V_i Y_1 + V_0 Y_5 = -\frac{V_0 Y_3}{Y_4}\left[Y_1 + Y_2 + Y_4 + Y_5\right]$$

$$V_i Y_1 + V_0 Y_5 = -\frac{V_0}{Y_4}\left[Y_1Y_3 + Y_2Y_3 + Y_4Y_3 + Y_5Y_3\right]$$

$$V_i Y_1 = -V_0 Y_5 - \frac{V_0}{Y_4}\left[Y_1Y_3 + Y_2Y_3 + Y_4Y_3 + Y_5Y_3\right]$$

$$V_i Y_1 = -\frac{V_0}{Y_4}\left[Y_5Y_4 + Y_1Y_3 + Y_2Y_3 + Y_4Y_3 + Y_5Y_3\right]$$

$$\frac{V_0}{V_i} = \frac{-Y_4 Y_1}{Y_5Y_4 + Y_1Y_3 + Y_2Y_3 + Y_4Y_3 + Y_5Y_3} \quad \text{—— (3)}$$

Here,

$$Y_1 = \frac{1}{R_1} = G_1 \; ; \; Y_2 = \frac{1}{R_2} = G_2 \; ; \; Y_3 = \frac{1}{R_3} = G_3$$

$$Y_4 = SC_4 \; ; \; Y_5 = SC_5$$

substitue the values.

$$\frac{V_0}{V_i} = \frac{-G_1 SC_4}{SC_5 \cdot SC_4 + G_1G_3 + G_2G_3 + SC_4 \cdot G_3 + SC_5 \cdot G_3}$$

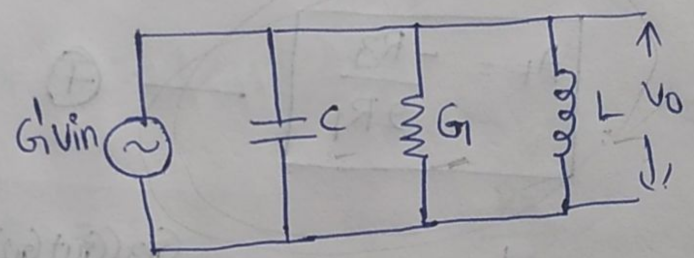
$$\frac{V_0}{V_i} = \frac{-G_1 SC_4}{G_3(G_1+G_2) + SG_3\left[C_4+C_5\right] + S^2(C_4 \cdot C_5)} \quad \text{—— (4)}$$

Divide numerator and denominator by SC_4

$$\frac{V_0}{V_i} = \frac{-G_1}{\frac{G_3(G_1+G_2)}{SC_4} + \frac{\cancel{S}G_3(C_4+C_5)}{\cancel{S}C_4} + \frac{S^{\cancel{2}}\cancel{C_4}C_5}{\cancel{SC_4}}}$$

$$\frac{V_0}{V_i} = \frac{-G_1}{\frac{G_3(G_1+G_2)}{SC_4} + \frac{G_3(C_4+C_5)}{C_4} + SC_5} \quad \text{—} (5)$$

This Transfer function is equivalent to parrall combination of RLC ckt [Resonance ckt] driven by $G'V_i$



$$\frac{V_0}{V_{in}} = H(S) = \frac{-G'}{SC + \frac{1}{SL} + G} \quad \text{—} (6)$$

Compare the equations (5) & (6)

$$G' = G_1$$

$$\frac{1}{SL} = \frac{G_3(G_1+G_2)}{SCL} \Rightarrow \boxed{L = \frac{C_4}{G_3(G_1+G_2)}}$$

$$SC = SC_5 \Rightarrow \boxed{C = C_5}$$

$$\boxed{G = \frac{G_3(C_4+C_5)}{C_4}}$$

To find gain,

$$A_F = \frac{V_0}{V_{in}} = \frac{-G'}{G} = \frac{-G_1}{\frac{G_3(C_4+C_5)}{C_4}}$$

Here $C_4 = C_5 = C$,

$$A_F = \frac{-G_1 C}{G_3(C+C)} = \frac{-G_1 \cdot \cancel{C}}{G_3 \cdot 2\cancel{C}}$$

$$A_F = \frac{-G_1}{2G_3}$$

Here $G_1 = 1/R_1$

$G_3 = 1/R_3$

$$A_F = \frac{-1/R_1}{2\, 1/R_3}$$

$$\boxed{A_F = \frac{-R_3}{2R_1}} \quad \text{—— } (7)$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{\frac{C_4}{G_3(G_1+G_2)} \cdot C_5} = \frac{G_3(G_1+G_2)}{C_4 C_5}$$

$$Q_0 = \omega_0 RC = \frac{\omega_0 C}{G}$$

$\omega = 2\pi f$

$f = \omega/2\pi$

To find B.W,

$$B.W = \frac{f_0}{Q_0} = \frac{\frac{\omega_0}{2\pi}}{\frac{\omega_0 C}{G}} = \frac{\omega_0}{2\pi} \times \frac{G}{\omega_0 C}$$

$$\boxed{B.W = \frac{G}{2\pi C}} \Rightarrow \frac{\frac{G_3(C_4+C_5)}{C_4}}{2\pi C} = \frac{G_3 \cdot 2\cancel{C}}{2\pi C \cdot \cancel{C}}$$

$$\therefore B.W = \frac{f_0}{Q_0} = \frac{G_3}{\pi C}$$

Here, $G_3 = 1/R_3$

$$\frac{f_0}{Q_0} = \frac{1/R_3}{\pi C}$$

$$\boxed{B.W = \frac{f_0}{Q_0} = \frac{1}{R_3 \pi C}} \quad \text{—— (8)}$$

$$R_3 = \frac{1}{\pi C} \times \frac{Q_0}{f_0}$$

$$\boxed{R_3 = \frac{Q}{f_c \pi C}} \quad \text{—— (9)}$$

To Find R_1,

$$A_F = -\frac{R_3}{2R_1} \text{ from (7)}$$

$$A_F = -\frac{R_3}{2R_1} \Rightarrow R_1 = -\frac{R_3}{2A_F}$$

$$R_1 = \frac{\frac{Q}{f_c \pi C}}{2A_F}$$

$$\boxed{R_1 = \frac{Q}{2A_F f_c \pi C}} \quad \text{—— (10)}$$

To Find R_2,

$$\boxed{w_0^2 = \frac{1}{LC} = \frac{G_3(G_1+G_2)}{C_4 C_5}}$$

Here $C_4 = C_5 = C$

$$w^2 = \frac{G_3(G_1+G_2)}{C \cdot C}$$

$$w^2 C^2 = G_3(G_1+G_2)$$

$$G_1+G_2 = \frac{w_0^2 C^2}{G_3}$$

$$G_2 = \frac{w_0^2 C^2}{G_3} - G_1 \qquad G_3 = \frac{1}{R_3}, \; G_1 = \frac{1}{R_1}$$

$$\frac{1}{R_2} = (2\pi f_c)^2 \cdot C^2 \cdot R_3 - \frac{1}{R_1}$$

$$\frac{1}{R_2} = \frac{(2\pi f_c)^2 \cdot C^2 \cdot R_3 \cdot R_1 - 1}{R_1}$$

$$\frac{1}{R_2} = \frac{\cancel{2\pi f_c} \times 2\cancel{\pi f_c} \times \cancel{C} \cdot \cancel{C} \times \frac{Q}{\cancel{f_c \pi C}} \cdot \frac{Q}{\cancel{f_c \pi C} \cdot \cancel{2} A_F} - 1}{\frac{Q}{f_c \pi C \, 2A_F}}$$

$$\frac{1}{R_2} = \frac{2 \cdot Q \cdot \frac{Q}{A_F} - 1}{\frac{Q}{f_c \pi C \cdot 2A_F}} = \frac{2Q^2 - A_F}{\left(\frac{Q \cdot \cancel{A_F}}{f_c \pi C \, 2\cancel{A_F}}\right)}$$

$$\frac{1}{R_2} = \frac{(2Q^2 - A_F)\, f_c \pi C \cdot 2}{Q}$$

$$\boxed{R_2 = \frac{Q}{2\pi f_c C\left[2Q^2 - A_F\right]}} \quad \text{— (11)}$$

substitute the values in eq (4)

$$H(s) = \frac{V_o}{V_{in}} = \frac{-A_o(\omega_o/Q)s}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

$$\boxed{H(s) = \frac{V_o}{V_{in}} = \frac{-A_o \alpha \omega_o s}{s^2 + \alpha \omega_o s + \omega_o^2}}$$

Here $\alpha = 1/Q$.

* Band Reject filter:-

The Band-Reject filter is also called as band-stop or Band-elimination filter.

Based on the quality factor, the Band Reject filter is classified into 2 types.

1. wide Band Reject filter. $[Q < 10]$
2. Narrow Band Reject filter. $[Q > 10]$

Wide Band Reject filter:-

* wide Band Reject filter using a Lowpass filter, highpass filter and summing amplifier.
* To realize a band-reject response, the low-cut-off frequency f_L of high-pass filter must be larger than the high cut-off frequency f_H of the low-pass filter.

* The pass band gain of both the high-pass and low-pass filters must be equal.

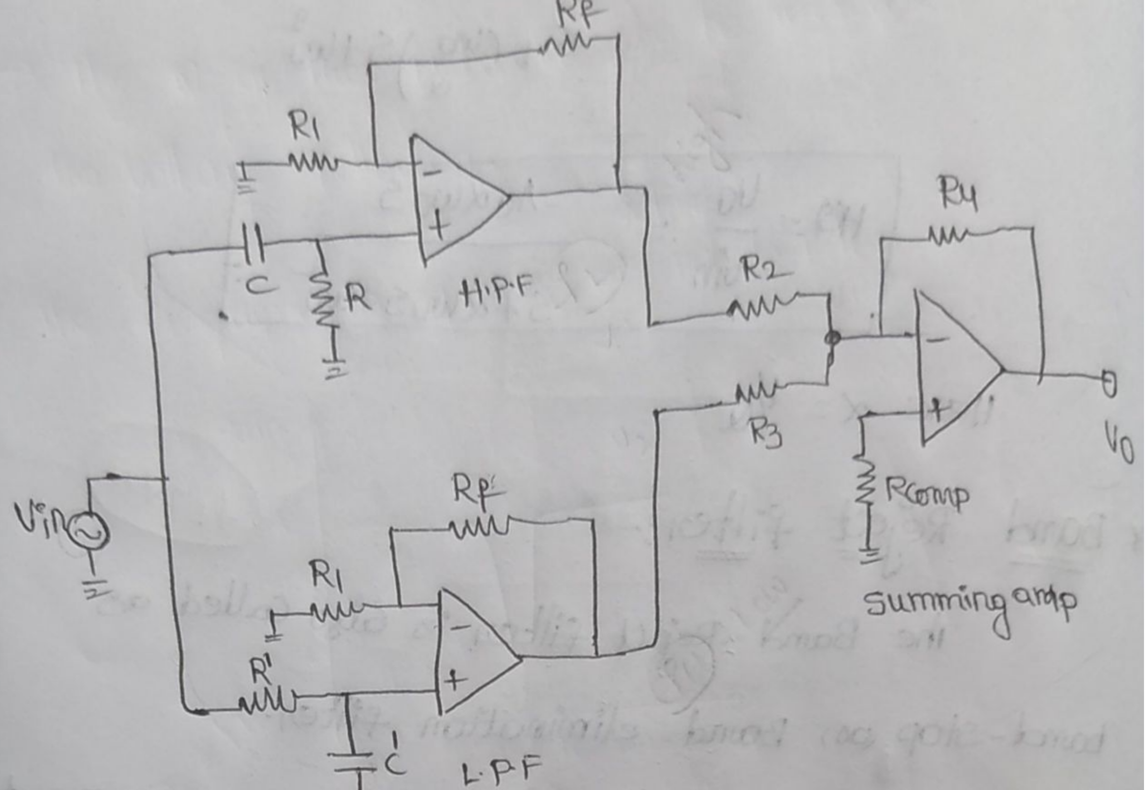
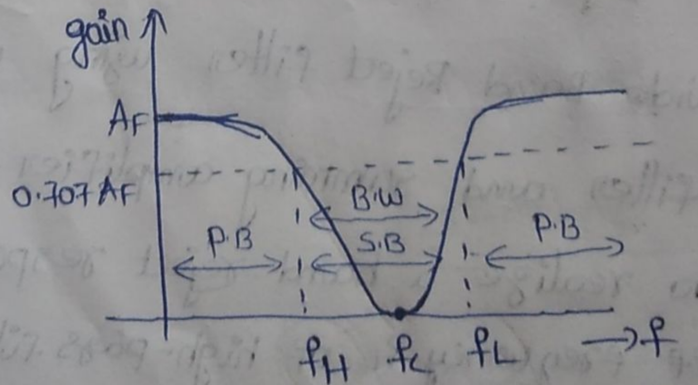


fig:- wide band reject filter.

* Both high pass & Low pass filters provide attenuation in the stop band between f_H and f_L

The frequency response.



Here $f_c = \sqrt{f_L \cdot f_H}$, $B.W = f_L - f_H$

$$Q = \frac{f_c}{B.W}$$

Narrow Band Reject filter:-

The narrow band reject filter, often called as the notch filter. The most commonly used notch filter is the twin-T Network. [i.e two T Networks].

* one T Network is made up of two resistors and a capacitor, while the other uses two capacitors and a resistor.

* The notch-out frequency is the frequency at which maximum attenuation occurs, it is

$$f_N = \frac{1}{2\pi RC}$$

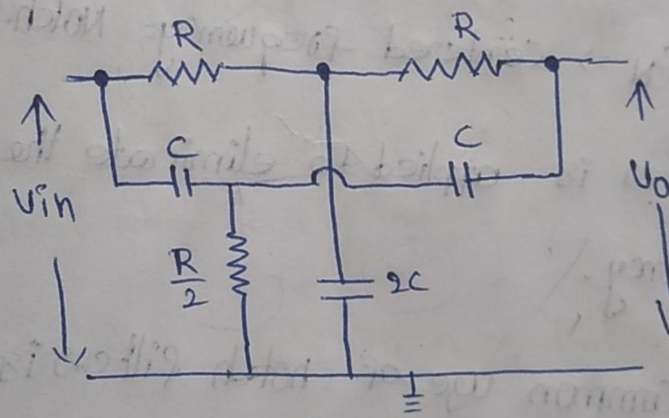


fig:- passive twin-T Network

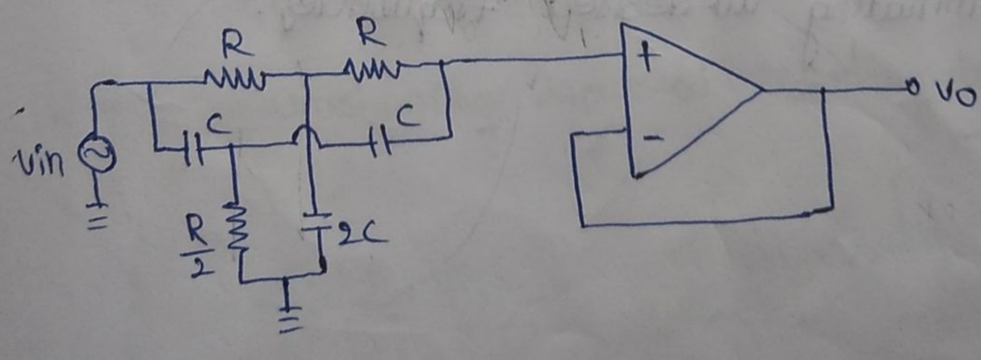
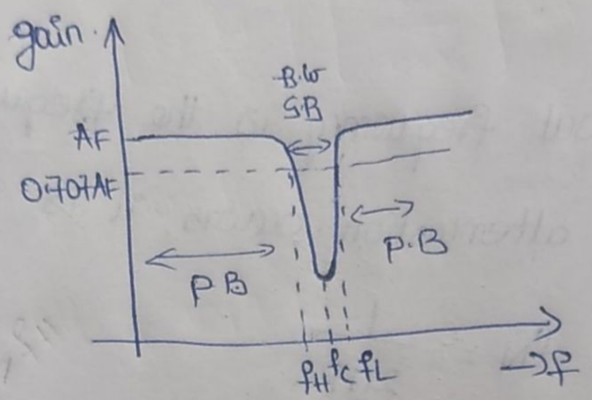


fig:- Active notch filter.

→ unfortunately, the passive twin - T Network has low figure of merit Q.

→ The Q of the Network can be increased with the voltage follower.

The frequency response of Active notch filter,



Here, $f_c = f_N$ => Centered frequency = Notch frequency

* This Active notch filter is applied to eliminate the single frequency.

* The most common use of notch filters is in Communications and biomedical instruments for eliminating un-desired frequencies.

All pass filter:-

- It allows all frequency signals without attenuation.
- It introduce predictable phase shifts.
- when signals are transmitted over transmission line, they undergo change in phase. To compensate for these phase changes; All pass filters are required.
- All pass filters are also called as

  Delay equalizers (or)

  phase correctors.

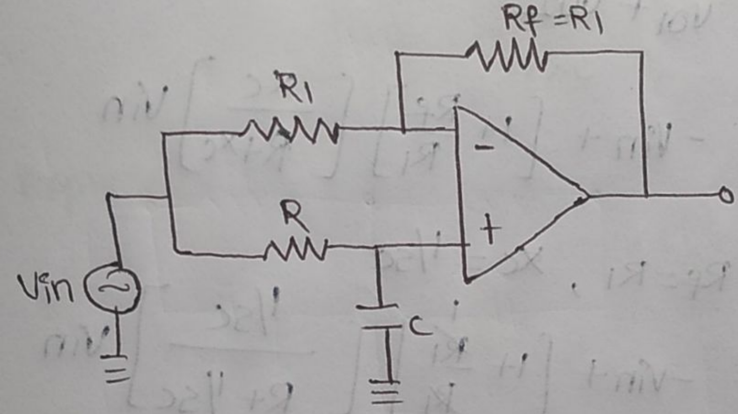
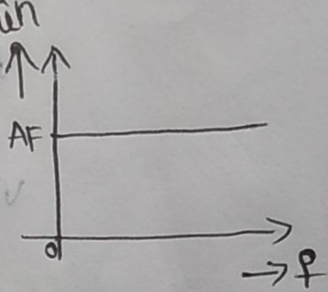


fig:- All pass filter.

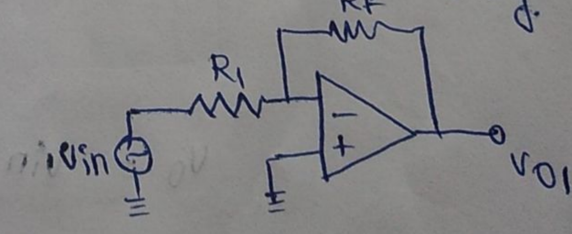


frequency response of All pass filter.

Expression for output or gain:-

From superposition theorem,

Case(i):- Considering input at inverting terminal only.

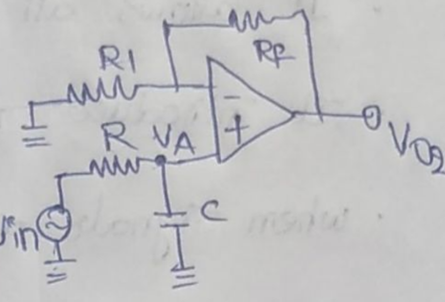


$$V_{o1} = -\frac{R_f}{R_1} \cdot V_{in}$$

if $R_f = R_1$

$$V_{o1} = -\frac{\cancel{R_1}}{\cancel{R_1}} \cdot V_{in} \Rightarrow \boxed{V_{o1} = -V_{in}}$$

Case 2:- Considering input at non-inverting terminal only.



$$V_{o2} = \left[1+\frac{R_f}{R_1}\right] V_A$$

Here, $V_A = \left(\frac{X_C}{R+X_C}\right) V_{in}$

$$\therefore V_{o2} = \left[1+\frac{R_f}{R_1}\right]\left[\frac{X_C}{R+X_C}\right] V_{in} \quad \text{——— (2)}$$

By superposition theorem,

$$V_O = V_{O1} + V_{O2}$$

$$V_O = -V_{in} + \left[1+\frac{R_f}{R_1}\right]\left[\frac{X_C}{R+X_C}\right] V_{in}$$

Here, $R_f = R_1$, $X_C = 1/sC$

$$V_O = -V_{in} + \left[1+\frac{\cancel{R_1}}{\cancel{R_1}}\right]\left[\frac{1/sC}{R+1/sC}\right] V_{in}$$

$\because [s = j\omega]$

$$V_O = -V_{in} + 2\cdot\left[\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}\right]\cdot V_{in}$$

$$V_O = V_{in}\left[-1 + 2\cdot\left[\frac{\frac{1}{\cancel{j\omega C}}}{\frac{Rj\omega C+1}{\cancel{j\omega C}}}\right]\right]$$

$$V_O = V_{in}\left[-1+\frac{2}{1+j\omega RC}\right]$$

$$V_O = V_{in}\left[\frac{-1-j\omega RC+2}{1+j\omega RC}\right]$$

$$V_O = V_{in}\left[\frac{1-j\omega RC}{1+j\omega RC}\right]$$

$\because \omega = 2\pi f$

$$V_O = V_{in}\left[\frac{1-j2\pi fRC}{1+j2\pi fRC}\right] \quad \text{——— (3)}$$

gain,

$$A_F = \frac{V_O}{V_{in}} = \left[\frac{1-j2\pi fRC}{1+j2\pi fRC}\right]$$

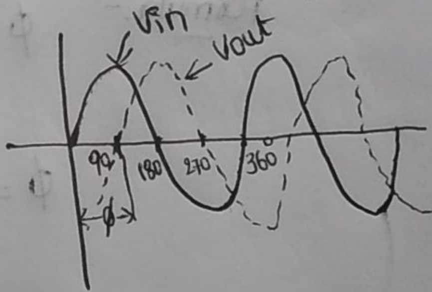
$$\left|\frac{V_O}{V_{in}}\right| = \frac{\sqrt{\cancel{1+(2\pi fRC)^2}}}{\sqrt{\cancel{1+(2\pi fRC)^2}}}$$

$$\boxed{\left|\frac{V_O}{V_{in}}\right| = 1}$$

Expression for phase angle (ϕ) :-

$$\left[\begin{aligned} z &= x+jy \\ \phi &= \tan^{-1}\left[\frac{y}{x}\right] \end{aligned}\right]$$

$$\frac{V_O}{V_{in}} = \frac{1-j2\pi fRC}{1+j2\pi fRC}$$



$$\phi = \frac{\tan^{-1}\left(\frac{-2\pi fRC}{1}\right)}{\tan^{-1}\left(\frac{2\pi fRC}{1}\right)}$$

$$\phi = \tan^{-1}(-2\pi fRC) - \tan^{-1}(2\pi fRC)$$

$$\because \tan^{-1}(-\theta) = -\tan^{-1}(\theta)$$

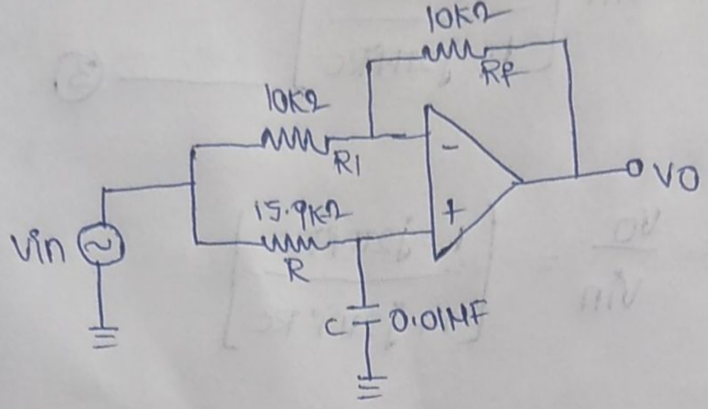
$$\phi = -\tan^{-1}(2\pi fRC) - \tan^{-1}(2\pi fRC)$$

$$\boxed{\phi = -2\tan^{-1}(2\pi fRC)}$$

* If the positions of R and C are interchanged, we get the positive phase shift.

Example:-

Find the phase angle ϕ if the frequency of Vin is 1KHz for the all-pass filter of following circuit.



Sol:- given that,

$R_1 = R_f = 10K\Omega$

$R = 15.9K\Omega$

$C = 0.01\mu F$

$f = 1KHz$

phase angle ϕ = ?

Formula:- $\phi = -2\tan^{-1}\left(\frac{2\pi f RC}{1}\right)$

$$\phi = -2\tan^{-1}\left[2\times\pi\times 1\times10^{3}\times 15.9\times10^{3}\times 0.01\times10^{-6}\right]$$

$$= -2\tan^{-1}\left(2\pi\times10^{3}\times 15.9\times10^{3}\times10^{-8}\right)$$

$$\boxed{\phi = -90^{\circ}}$$

$\therefore$ phase angle $\phi = -90^{\circ}$

This means that the output voltage vo has the same frequency and amplitude but lags Vin by 90°.