UNIT - 1

ELECTROSTATIC

coloumb's law :- coloumb's law states the force b/w the point charges q_1 and q_2 directly proportional to the product of the charges and inversely proportional to square of the distance between them.

→ point charge is a Hypothetical charge and force acting at the single point in the space is opposite.

→ Mathematically $\boxed{F}$ is directly proportional to $Q_1 Q_2$ and inversely proportional to square of the distance between them

$$F \propto \frac{Q_1 Q_2}{R^2}$$

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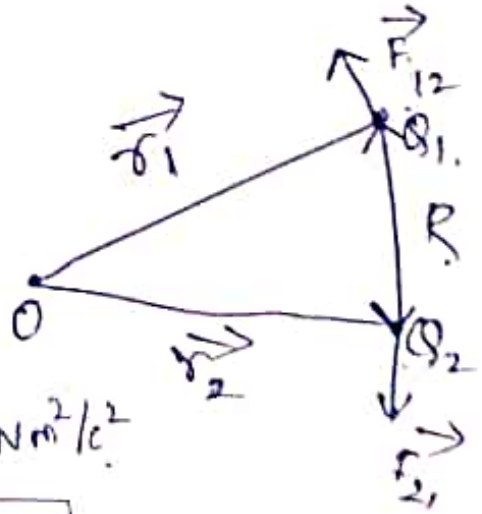
$$F = k \frac{Q_1 Q_2}{R^2}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \ Nm^2/c^2$$

$\boxed{= 9\times10^9 \ Nm^2/c^2}$

$\boxed{\varepsilon_0 = 8.85\times10^{-12} \ F/m}$

where $\boxed{\varepsilon_0 = 8.85\times10^{-12} \ F/m}$



ε_0 → permeability of Free space.

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}} \rightarrow (1)$$

As shown in figure the position vector of the point charge are given by $\vec{r}_1, \vec{r}_2$.

Lets force $\vec{F}_{12}$ represent the force on Q_1 due to the charge Q_2. Another force $\vec{F}_{21}$ represents

Force on Q_2 due the charge Q_1.

These two charges Q_1 and Q_2 are separated by the distance R

Therefore, charges are separated by the distance of

$$R = |\vec{r}_1 - \vec{r}_2| = |\vec{r}_2 - \vec{r}_1|$$

$$|\vec{r}_1 - \vec{r}_2| = 1 = |\vec{r}_2 - \vec{r}_1|$$

$$\frac{R}{|\vec{r}_1 - \vec{r}_2|} = \hat{a}_{12}$$

(3)

$$\boxed{\hat{a}_{012} = \frac{R}{|\vec{r}_1 - \vec{r}_2|}} \longrightarrow (2)$$

→ The force acting on Q_1 charge is $\vec{F}_{12}$ where $\vec{F}_{12}$ is defined as

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2} \cdot \hat{a}_{12}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2} \cdot \frac{R}{|\vec{r}_1 - \vec{r}_2|}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R |\vec{r}_1 - \vec{r}_2|}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R |\vec{r}_1 - \vec{r}_2|}$$

$$\boxed{R = |\vec{r}_1 - \vec{r}_2|}$$

$R = |r_2 - r_2|$

(or)

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{|\vec{r}_1 - \vec{r}_2|^3} |\vec{r}_1 - \vec{r}_2|$$

similarly

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

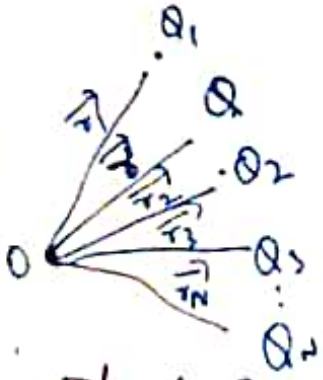
$$|\vec{F}_{21}| = |\vec{F}_{12}|$$

$++ \rightarrow$ Rep
$-- \rightarrow$ Rep
$+- \rightarrow$ Attr.

→ When we have number of point charges to determine the force on a particular charge due to the other charge we apply superposition theorem.

→ If we have N number of charges $Q_1, Q_2, Q_3, Q_4 \ldots Q_N$ located respectively at the point represented by the position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots \vec{r}_N$.

→ The force experience by the charge Q located at the $\vec{r}$. is given by



$$\boxed{\vec{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}} \rightarrow (3)$$

Electric Field Intensity :-

Electric field Intensity (or) electric field strength at a point is defined as force/unit charge. i.e,

$$\boxed{\vec{E} = \frac{\vec{F}}{Q}} \quad \text{—(4)} \cdot \quad \frac{N}{C} \text{ or } \frac{V}{m} \text{ units}$$

The electric field intensity "E" at a point "$\vec{r}$" due to the charge 'Q' located at $\vec{r}'$ is given by

$$\boxed{\vec{E} = \frac{Q(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|^3}} \quad (5) \qquad \vec{E} = \frac{\vec{F}}{Q}$$

for the collection of 'N' point charges $Q_1, Q_2, Q_3 \cdots Q_n$ located at the position vector $\vec{r}_1, \vec{r}_2, \vec{r}_3 \cdots \vec{r}_n$ the electric field intensity at the point $\vec{r}'$.

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_N(\vec{r}-\vec{r}_i')}{|\vec{r}-\vec{r}_i'|^3}} \quad (6)$$

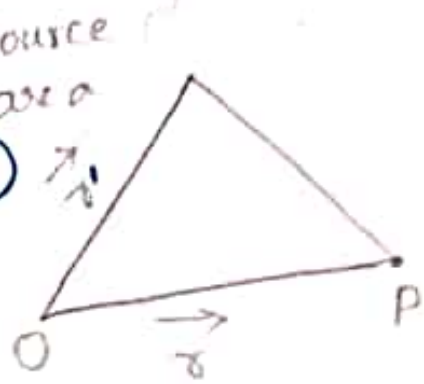


fig:- (2)

Line charge:

$\rho_l = \frac{\text{Total line charge}}{\text{line length}}$, $\frac{Q}{l} = C/m$

$$dQ = \rho(\vec{r}')\, dv'$$

$dQ = \rho_l\, dl$ line

$dQ = \rho_s\, ds$ surface

$dQ = \rho_v\, dv$ volume

$$d(\vec{E}) = \frac{d(Q)(r-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|^3}$$

$$= \frac{\rho(\vec{r}')\, dv\, (r-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_i'|^3}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_N(r-r_i')}{|r-r_i'|^3}$$

Expression-(6) can be modified suitable to compute Electric field due to the continuous distribution of charge.

fig (2) we consider the continuous distribution of the charge in the region is denoted as a source region. $\boxed{\int \vec{r}' \, dv'}$

For an elementary charge. $dQ = \rho(\vec{r}')dv'$ due to the all other charges consider this charge as a point charge, we can write the field expression as

$$\boxed{d(\vec{E}) = \frac{dQ(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|^3}} \quad \text{--- (7)}$$

$$\boxed{d(\vec{E}) = \frac{\int_v (\vec{r}')\, dv' (\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|^3}} \quad \text{--- (8)}$$

with this expression applying integration we get the electric field at point p. due to the distribution of the charges thus the expression of the electric field at point p can be written as

$$\boxed{\vec{E}(r) = \int_v \frac{\rho(\vec{r})(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|^3} dv'} \quad \text{--- (8)}$$

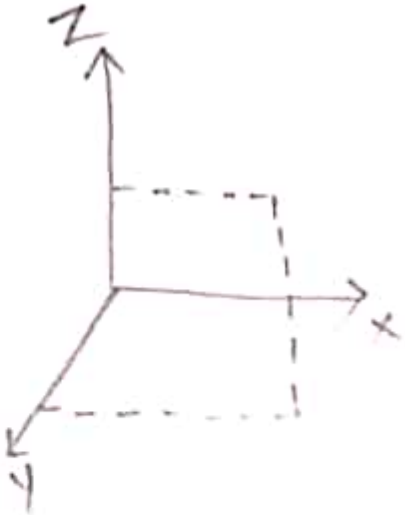
(5) similar techniques can be adopted when the charge distribution is in the form of line charge density (or) surface charge density.

$$\boxed{\vec{E}(r) = \int_L \frac{\rho_l(\vec{r})\,(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_1|^3}\, dL} \quad \text{— (9)}$$

$$\boxed{\vec{E}(r) = \int_S \frac{\rho_s(\vec{r})\,(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_1|^3}\, ds} \quad \text{— (10)}$$

Introduction of EMTL & Basics :-

(A) Co-ordinate system :-



(1) cartesian (x, y, z)
(2) cylinder (ρ, ϕ, z)
(3) spherical (r, θ, ϕ)

→ Transformation :-

(1) (x, y, z) to (ρ, ϕ, z)
(2) (x, y, z) to (r, θ, ϕ)
(3) (ρ, ϕ, z) to (r, θ, ϕ)

vice-versa

Problems

① Two point charges of $10\mu C$ each of the same sign are situated 1.5 cm apart. Calculate the magnitude of repulsive force.

Given data

Two point charges $= 10\mu C$

$Q_1 = Q_2 = 10\mu C = 10 \times 10^{-6} = C$

1M

$R = 1.5cm = 0.015m$

$$\boxed{F = k\frac{Q_1 Q_2}{R^2}}$$

$k = 9.95 \times 10$ 2M

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon R^2} = \frac{10 \times 10^{-6} \times 10 \times 10^{-6}}{4 \times \pi \times 8.854 \times 10^{-12} \times (0.015)^2}$$

$$K = \frac{1}{4\pi\varepsilon}$$

$$= \frac{100 \times 10^{-12}}{4\pi \times 8.854 \times (0.015)^2 \times 10^{-12}}$$

1

$$\boxed{\varepsilon = 8.854 \times 10^{-12}}$$

$$= \frac{100}{4(3.14) \times 8.854 \times (0.015)^2}$$

$\pi = (3.14)$

$$\boxed{F = 4 \times 10^3 N}$$ 1

② Find the force on a charge Q_1 $450\mu C$ located at $(\overset{x}{5}, \overset{y}{0}, \overset{z}{10})$ due to the charge due to $250\mu C$ $(\underset{x}{0}, \underset{y}{4}, \underset{z}{5})$ in free space. ε_0

Sol

Given data

Charge $Q_1 = 450\mu C$ at $(\overset{x}{5},\overset{y}{0},\overset{z}{0})$

$Q_2 = 250\mu C$ at $(\overset{x}{0},\overset{y}{4},\overset{z}{5})$

$Q_1 = 450\times10^{-6}C$

$R_1 = 5\hat{a}_x$

$Q_2 = 250\times10^{-6}C$

$R_2 = 4\hat{a}_y + 5\hat{a}_z$

$R_{21} = \vec{R_1} - \vec{R_2}$

$\vec{R_{21}} = (5-0)\hat{a}_x + (0-4)\hat{a}_y + (0-5\hat{a}_z)$

$|\vec{R_{21}}| = \sqrt{(5)^2 + (-4)^2 + (5)^2}$

$= \sqrt{25+16+25}$

$= \sqrt{66} = 8.21$

$\hat{a}_x \cdot \hat{a}_x =$

$\hat{a}_y \cdot \hat{a}_y =$

$\hat{a}_z \cdot \hat{a}_z =$

$\hat{a}_{21} = \dfrac{\vec{R_{21}}}{|R_{21}|} = \dfrac{5\hat{a}_x - 4\hat{a}_y - 5\hat{a}_z}{8.12}$

$$\boxed{\hat{a}_{21} = 0.615\hat{a}_x - 0.5\hat{a}_y - 0.615\hat{a}_z}$$

$\frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$

$F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 (R_{21})^2} \hat{a}_{21}$

$F \propto \frac{Q_1 Q_2}{R_{21}^2} \hat{a}_{21}$

$F = \frac{450\times10^{-6} \times 250\times10^{-6}}{4\pi\times8.854\times10^{-12}\times(8.12)^2}$

$F = 15.3(\hat{a}_{21})$

$= 15.3(0.615\hat{a}_x - 0.5\hat{a}_y - 0.615\hat{a}_z)$

$$\boxed{F = 9.4\hat{a}_x - 7.65\hat{a}_y - 9.4\hat{a}_z \text{ N}}$$

$$\boxed{\frac{\bar{F}}{\bar{Q}} = \bar{E}}$$

(3) A circular disk 20m with a charge density $\rho_s = 100\sin\phi$ NC/m² is enclosed by surface 'S' with a net flux. Calculate net flux across surface.

Sol Given data

$$\boxed{\rho_s = 100\sin\phi \text{ NC/m}^2}$$

we know that

$$\psi = Q_{enc} = \int \rho_s \, ds$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} (100\sin\phi\times10^{-9})\,\rho\, d\rho\, d\phi$$

$$= 100\times10^{-9}\int_0^{2\pi}\rho\, d\rho \boxed{\int_0^{\pi}\sin\phi\, d\phi} \quad 2$$

$\sin(0^\circ) = 0$

$$= 100\times10^{-9}\times\frac{\pi^2}{2} = \pi$$

$$100\times10^{-9}\left[\frac{\rho^2}{2}\right]_0^{2\pi}\left[2\right] = 100\times10^{-9}\frac{4\pi^2}{\cancel{2}}\cdot\cancel{2}$$

$$= 400\times10^{-9}(3.14)^2$$

$$= \frac{60 \times 10^{-9}}{4\pi \times 129} \cdot \frac{4\hat{a}_x + 7\hat{a}_y - 8\hat{a}_z}{\sqrt{129}}$$

$$= 3.25 \times 10^{-12} (4\hat{a}_x + 7\hat{a}_y - 8\hat{a}_z)$$

4) Determine the Electric field $\vec{E}$ at the Origin due to a point charge of 54.9 nC located at (-4, 5, 3) in air

Sol: Given data $Q = 54.9$ nC at point (-4, 5, 3).

$$\vec{E} = \frac{Q}{4\pi \varepsilon R^2} \cdot \hat{a}_R \qquad \text{in air } \varepsilon = \varepsilon_0$$

$$\vec{E} = \frac{54.9 \times 10^{-9}}{4\pi \varepsilon_0 R^2} \hat{a}_R$$

$R = (x_1 - x_2)\hat{a}_x - (y_1 - y_2)\hat{a}_y - (z_1 - z_2)\hat{a}_z$

Now the distance vector from the point charge to the origin is $R = (0, 0, 0) - (-4, 5, 3) = 4\vec{a}_x - 5\vec{a}_y - 3\vec{a}_z$

$$R = \sqrt{4^2 + 5^2 + 3^2} = \sqrt{50}$$

$$\frac{\vec{R}}{|\vec{R}|} = \hat{a}_R = \frac{4\vec{a}_x - 5\hat{a}_y - 3\hat{a}_z}{\sqrt{50}}$$

$$\therefore E = \frac{54.9 \times 10^{-9}}{4\pi \varepsilon_0 (\sqrt{50})^2} \left(\frac{4\vec{a}_x - 5\vec{a}_y - 3\vec{a}_z}{\sqrt{50}} \right)$$

$$E = 5.6\hat{a}_x - 5.77\hat{a}_y - 4.19\hat{a}_z \quad \text{V/m or N/C}$$

② Evaluate the Electric field intensity at a point $P(-5, 7, -4)$ is free space due to a charge of 0.2 mille coulombs placed at point $R(2, -1, -2)$.

Sol: Given data: $Q = 0.2\ mC$

formula: $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$$\bar{R} = (-5-2)\hat{a}_x + [7-(-1)]\hat{a}_y + [-4-(2)]\hat{a}_z$$

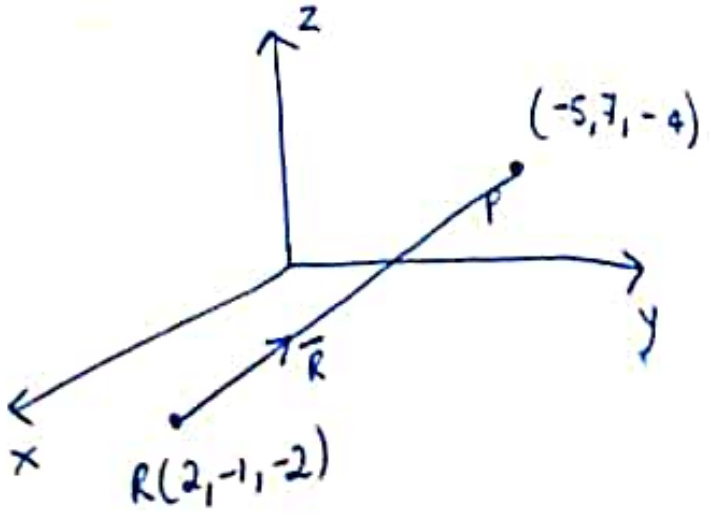
$$\bar{R} = -7\hat{a}_x + 8\hat{a}_y - 2\hat{a}_z$$

$$|\bar{R}| = \sqrt{7^2 + 8^2 + 2^2} = \sqrt{117}$$

$$\boxed{\hat{a}_R = \frac{R}{|\bar{R}|}}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\cdot\hat{a}_R = \frac{0.2\times10^{-3}}{4\pi\times8.854\times10^{-12}\times117}\left[\frac{-7\hat{a}_x + 8\hat{a}_y - 2\hat{a}_z}{\sqrt{117}}\right]$$

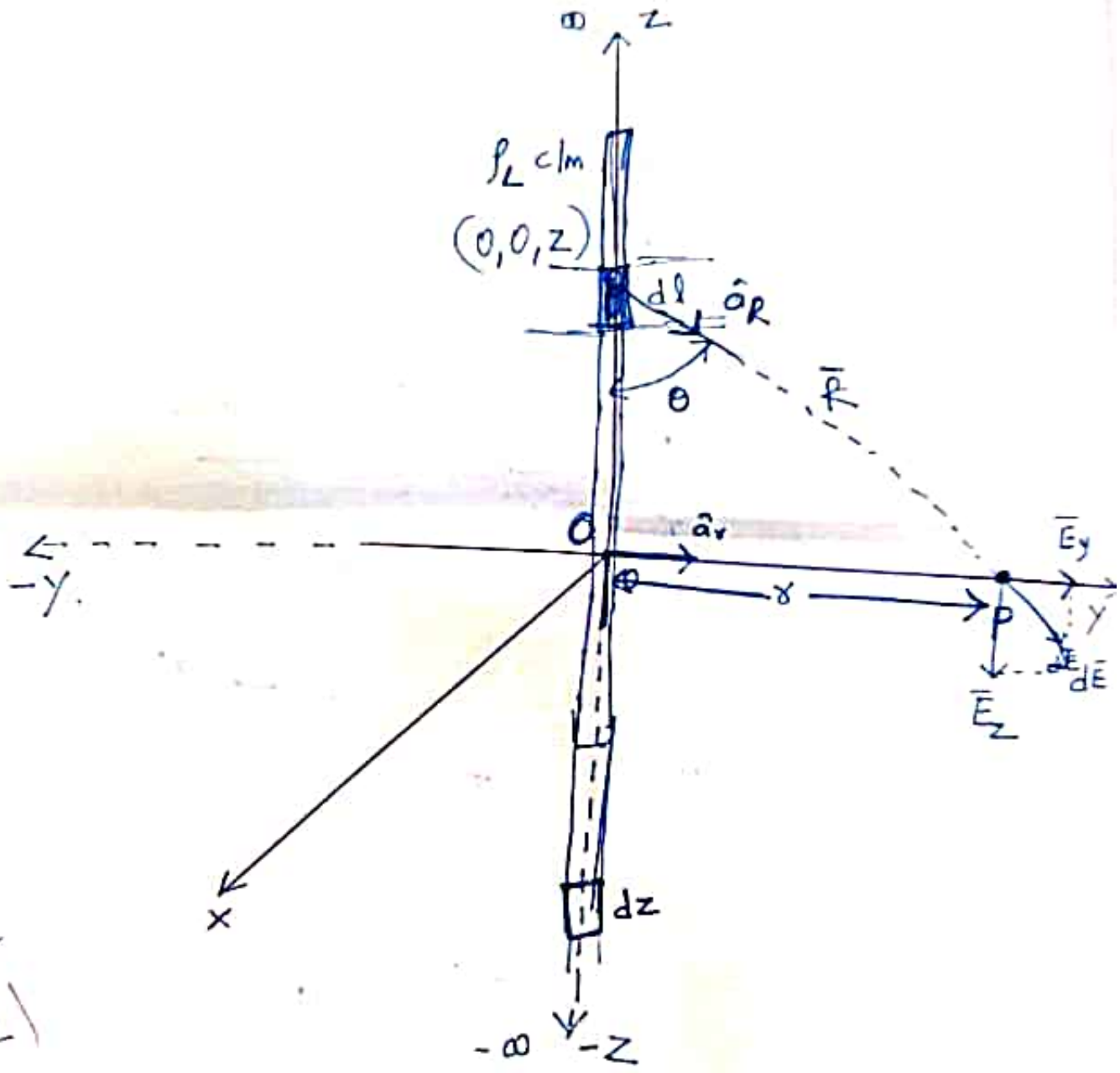
$$\vec{E} = -9.942\hat{a}_x + 11.363\hat{a}_y - 2.84\hat{a}_z\ kV/m$$



* Surface integral

* Electric Field due to line charge, (or)

(*) Electric Field intensity of an infinitely long, straight, line charge of a uniform density ρ_L in air.



$\hat{a}_x \; \hat{a}_y \; \hat{a}_z$

(0,0,z)

(0,r,0)

$\boxed{y = r}$

$dl = dz$

$$\bar{R} = (0-0)\hat{a}_x + (r-0)\hat{a}_y + (0-z)\hat{a}_z = r\hat{a}_y - z\hat{a}_z$$

$$|\bar{R}| = \sqrt{r^2 + z^2}$$

$= \int_S \rho_s \, ds$ Coulombs

$$\hat{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{r\hat{a}_y - z\hat{a}_z}{\sqrt{r^2+z^2}}$$

$$\boxed{d\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2} \cdot \hat{a}_R}$$

$$\boxed{dl = dz}$$

$$Q = \rho_L \, dl$$

$$= \frac{\rho_L \, dl}{4\pi\varepsilon_0 R^2}\left(\frac{r\hat{a}_y - z\hat{a}_z}{\sqrt{r^2+z^2}}\right)$$

$$d\bar{E} = \frac{\rho_L \, dz}{4\pi\varepsilon_0 (r^2+z^2)^2}\left(\frac{r\hat{a}_y - z\hat{a}_z}{\sqrt{r^2+z^2}}\right)$$

$$d\bar{E} = \frac{\rho_L \, dz}{4\pi\varepsilon_0 (\sqrt{r^2+z^2})^2} \frac{r\hat{a}_y}{\sqrt{r^2+z^2}} \left(-\frac{z\hat{a}_z}{\sqrt{r^2+z^2}}\right)$$

for every charge on +ve z-axis there is equal charge present on −ve z-axis

$$d\bar{E} = \frac{\rho_L \, dz}{4\pi\varepsilon_0 (\sqrt{r^2+z^2})^2} \cdot \frac{r\hat{a}_y}{(\sqrt{r^2+z^2})}$$

$$d\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \, r \cdot dz \, \hat{a}_y$$

$$\boxed{\bar{E} = \int_{-\infty}^{\infty} \frac{\rho_L}{4\pi\varepsilon_0 (r^2 + z^2)^{3/2}} \cdot r \cdot dz \, \hat{a}_y}$$

Substitution $\boxed{z = r \tan\theta}$ i.e $r = \frac{z}{\tan\theta}$.

$\boxed{z \text{ varies } -\infty \text{ to } \infty}$ $\boxed{dz = r \sec^2\theta \, d\theta}$

for	for
$z = -\infty$	$z = +\infty$
$\theta = -90^\circ$	$\theta = \tan^{-1}(\infty) = +90^\circ$

$$\therefore \bar{E} = \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_L}{4\pi\varepsilon_0 [r^2 + r^2 \tan^2\theta]^{3/2}} \cdot r \times r \sec^2\theta \, d\theta \cdot \hat{a}_y$$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{r^2 \sec^2\theta \, d\theta}{r^3 [1 + \tan^2\theta]^{3/2}} \cdot \hat{a}_y.$$

$1 + \tan^2\theta = \sec$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{r [1 + \tan^2\theta]^{3/2}} \cdot \hat{a}_y$$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{r \sec^3\theta} \cdot \hat{a}_y$$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0 r}\int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta}\, d\theta\, \hat{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r}\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta\, \hat{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} \cdot \hat{a}_y$$

$\sin \pi/2 = 1$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0 r}\left[1-(-1)\right] \cdot \hat{a}_y$$

$$E = \frac{\rho_L}{\cancel{4}\pi\varepsilon_0 r} \cdot \cancel{2} \cdot \hat{a}_y$$

$$\boxed{E = \frac{\rho_L}{2\pi\varepsilon_0 r} \cdot \hat{a}_y \quad V/m}$$

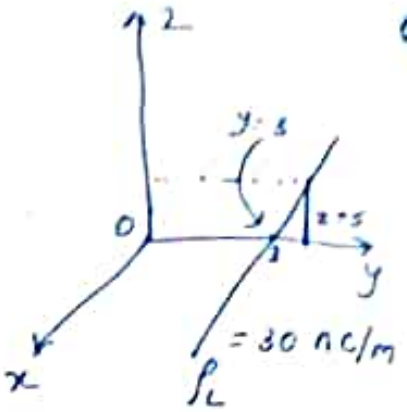
r = $\perp$re distance of Point P from line charge

$\hat{a}_r$ = Unit Vector in the direction of The $\perp$re distance of point P from The line c

(2) An infinitely long uniform line charge is located at $y = 3$, $z = 5$, if $\rho_L = 30\ nC/m$ find field $\bar{E}$ intensity at (i) Origin
(ii) $P(5, 6, 1)$

4) Sol: Given data: $\rho_L = 30\ nC/m = 30 \times 10^{-9}\ C/m$ charge located at $y = 3$; $z = 5$

(i) $\bar{E}$ at Origin $(0,0,0)$ $\bar{r} = (0-3)\hat{a}_y + (0-5)\hat{a}_z$



$\hat{r} = -3\hat{a}_y - 5\hat{a}_z$

$|\hat{r}| = \sqrt{34} = \sqrt{3^2 + 5^2}$

$\therefore \boxed{\bar{E} = \dfrac{\rho_L}{2\pi\varepsilon_0 r} \cdot \hat{a}_r}$ $\boxed{\because \hat{a}_r = \dfrac{\hat{r}}{|\hat{r}|}}$

Note: As charge is parallel to x-axis, there will not be any component of $\bar{E}$ in x-direction hence do not consider x-co-ordinate while calculating $\bar{r}$

$$= \frac{30 \times 10^{-9}}{2\pi \times 8.85 \times 10^{-12} \times \sqrt{34}} \left[\frac{-3\hat{a}_y - 5\hat{a}_z}{\sqrt{34}}\right]$$

$\bar{E} = -47.58\,\hat{a}_y - 79.3\,\hat{a}_z$ V/m

(ii) $\bar{E}$ at $P(5,6,1)$

hence $\bar{r} = (6-3)\hat{a}_y + (1-5)\hat{a}_z$

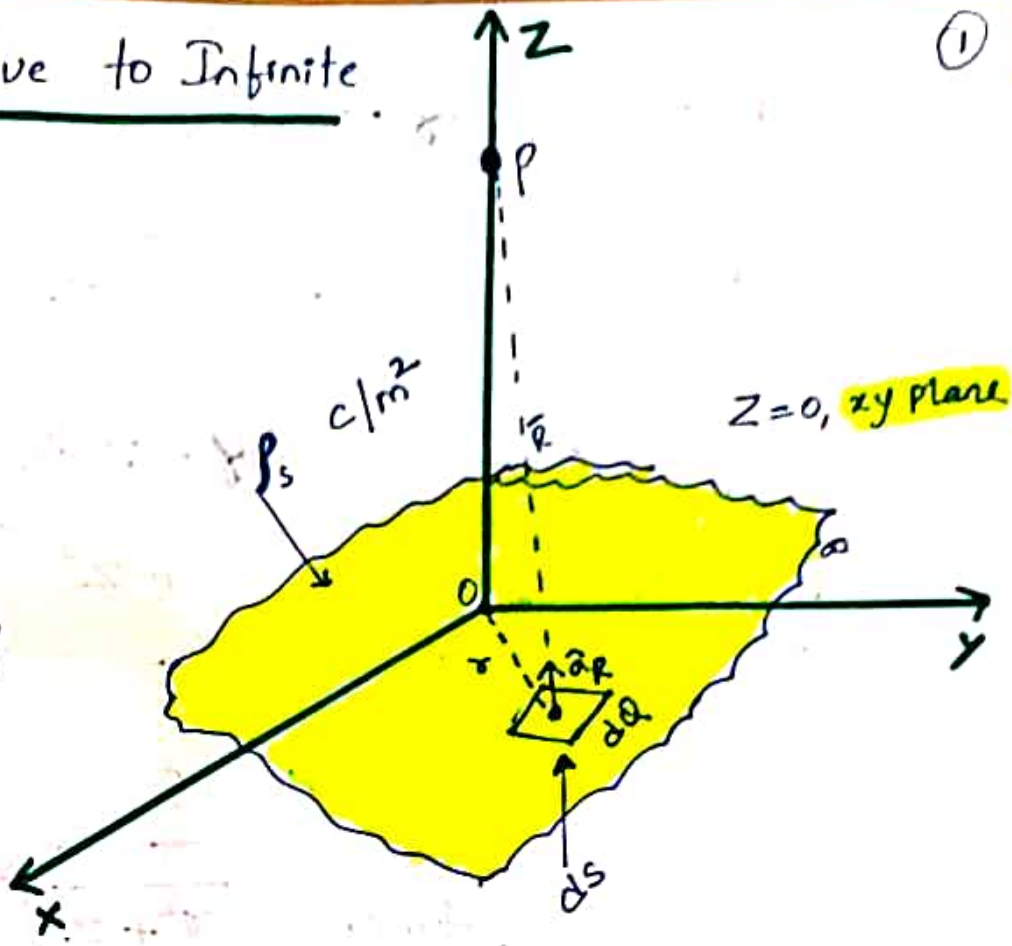
$\bar{r} = 3\hat{a}_y - 4\hat{a}_z$

$|\bar{r}| = \sqrt{25} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$$\bar{E} = \frac{30 \times 10^{-9}}{2\pi \times 8.85 \times 10^{-12} \times 5} \left[\frac{3\hat{a}_y - 4\hat{a}_z}{5}\right]$$

$\boxed{\bar{E} = 64.71\,\hat{a}_y - 86.28\,\hat{a}_z \text{ V/m}}$

Electric Field due to Infinite Sheet of charge.



$$\boxed{ds = r\,dr\,d\phi}$$

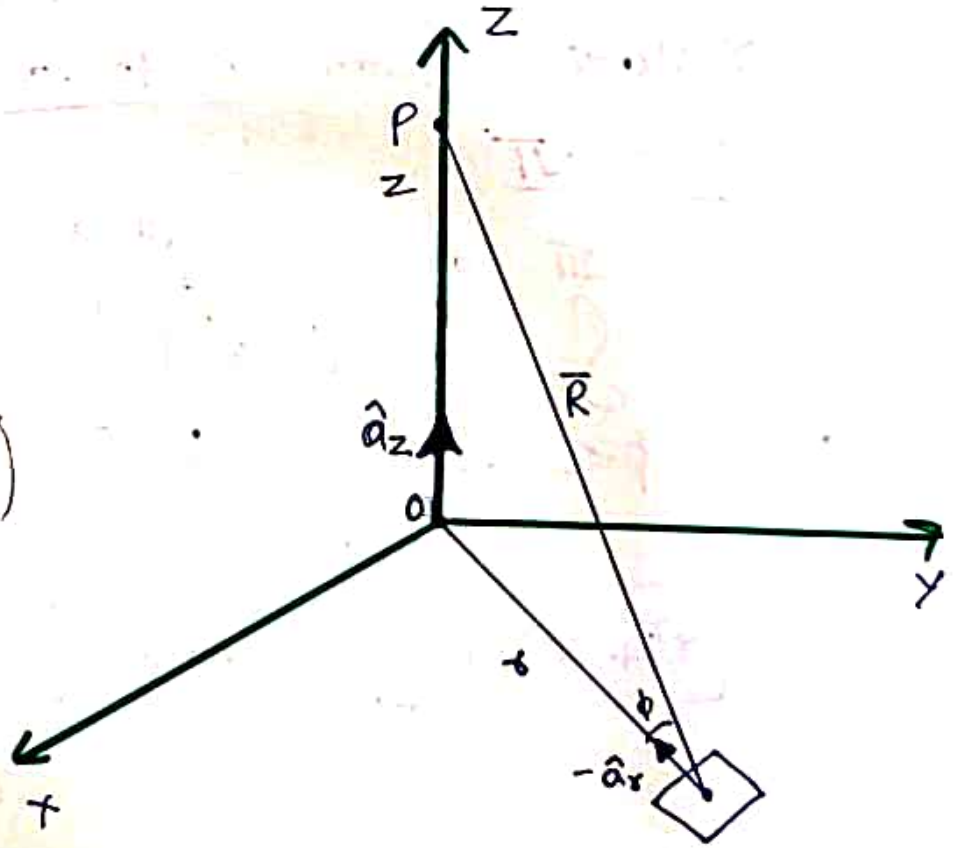
$$\rho_s = \frac{dQ}{ds}$$

$$\boxed{dQ = \rho_s \cdot ds}$$

$$\boxed{dQ = \rho_s \cdot r \cdot dr\,d\phi}$$

$$\boxed{d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2} \cdot \hat{a}_R}$$

$$d\bar{E} = \frac{\rho_s \cdot r \cdot dr \cdot d\phi}{4\pi\varepsilon_0 R^2} \cdot \hat{a}_R$$



$$\boxed{\hat{a}_R = \frac{\bar{R}}{|\bar{R}|}}$$

$$\bar{R} = r\,\hat{a}_r \text{ (or) } -r\,\hat{a}_r + z\,\hat{a}_z$$

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$$\bar{R} = -r\hat{a}_r + z\hat{a}_z$$

$$|\bar{R}| = \sqrt{(-r)^2 + (z)^2} = \sqrt{r^2 + z^2}$$

$$\boxed{\hat{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-r\hat{a}_r + z\hat{a}_z}{\sqrt{r^2 + z^2}}}$$

$$\boxed{d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\hat{a}_R}$$

$$\therefore \quad d\bar{E} = \frac{\rho_s \, r \cdot dr \, d\phi}{4\pi\varepsilon_0 \left(\sqrt{r^2 + z^2}\right)^2} \cdot \left(\frac{-r\hat{a}_r + z\hat{a}_z}{\sqrt{r^2 + z^2}}\right)$$

$$\boxed{d\bar{E} = \frac{\rho_s \, r \cdot dr \, d\phi}{4\pi\varepsilon_0 (\sqrt{r^2 + z^2})^2} \cdot \frac{z\hat{a}_z}{(\sqrt{r^2 + z^2})}}$$

⊙ For infinite sheet in xy plane,

r varies from <u>0 to ∞</u> while ϕ varies from <u>0 to 2π</u>

$$E = \int_{\phi=0}^{2\pi} \int_{r=0}^{\infty} d\bar{E} = \int_0^{2\pi} \int_0^{\infty} \frac{\rho_s \, r \cdot dr \, d\phi}{4\pi\varepsilon_0 (r^2 + z^2)^{3/2}} \cdot (z\hat{a}_z)$$

$\boxed{r^2 + z^2 = u^2}$ hence $\boxed{2r\,dr = 2u\,du}$

$r = 0$; $u = z$ &

$r = \infty$, $u = \infty$

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$$\bar{E} = \int_0^{2\pi}\int_{u=z}^{\infty} \frac{\rho_s}{4\pi\varepsilon_0} \frac{u \cdot du}{(u^2)^{3/2}} \cdot d\phi \cdot (z\,\hat{a}_z)$$

$$= \int_0^{2\pi}\int_{u=z}^{\infty} \frac{\rho_s}{4\pi\varepsilon_0} \frac{du}{u^2}\, d\phi\, (z \cdot \hat{a}_z)$$

$$= \int_0^{2\pi} \frac{\rho_s}{4\pi\varepsilon_0}\, d\phi\, (z \cdot \hat{a}_z) \left[-\frac{1}{u}\right]_z^{\infty}$$

as $\boxed{\int \frac{1}{u^2} = \int u^{-2} = \frac{u^{-1}}{-1} = -\frac{1}{u}}$

$\bar{E} = \cancel{\int}$

$$\bar{E} = \frac{\rho_s}{4\pi\varepsilon_0} [\phi]_0^{2\pi} (z \cdot \hat{a}_z) \left[\underbrace{-\frac{1}{\infty} - \left(-\frac{1}{z}\right)}_{\frac{1}{z}}\right]$$

$$= \frac{\rho_s}{\cancel{4\pi}\varepsilon_0} (\cancel{2\pi})\, \hat{a}_z$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \cdot \hat{a}_z \ \text{V/m}} \quad (\text{or}) \quad \boxed{\frac{\rho_s}{2\varepsilon_0}\, \hat{a}_n \ \text{V/m}}$$

$\hat{a}_n$ = Direction normal to the S. Area

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Prob

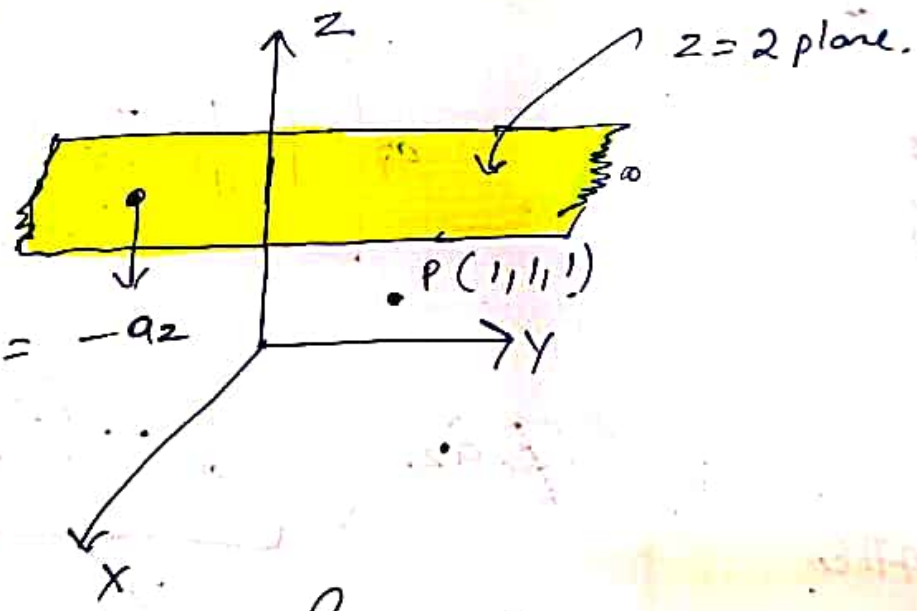
Uniform charge surface $\rho_s = 2\,\mu C/m^2$ is situated at $z=2$ plane. what is the value of $\bar{E}$ (Electric field Intensity) at point $P(1,1,1)$?

Sol:

Given data

$$\rho_s = 2\,\mu C/m^2 = 2\times10^{-6}\ C/m^2$$

$$z = 2 \text{ plane.}$$



$$\hat{a}_n = -a_z$$

$$\bar{E}_p = \frac{\rho_s}{2\varepsilon_0} \cdot \hat{a}_n$$

$$= \frac{2\times10^{-6}}{2\times8.854\times10^{-12}}(-\hat{a}_z)$$

$$\boxed{\bar{E}_p = -112.9433\ \hat{a}_z\ V/m}$$

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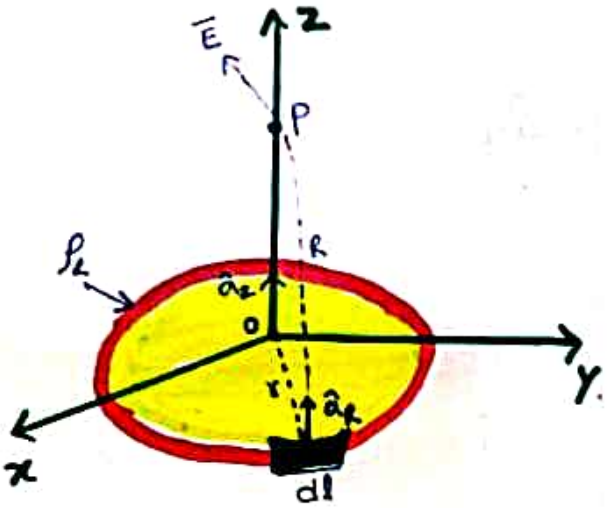
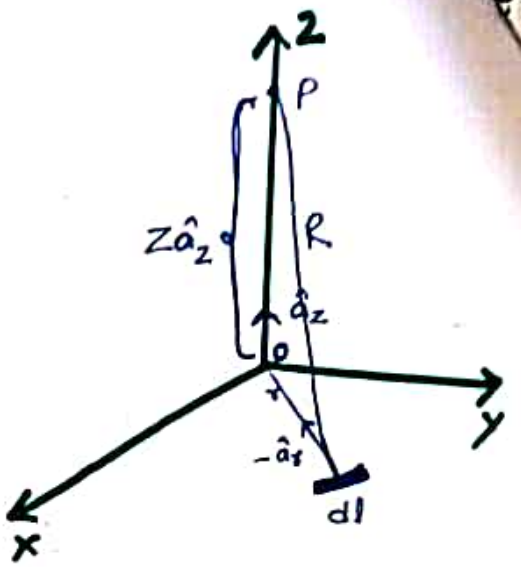
Volume distribution

Derive the Expression for $\bar{E}$ at a point P on the axis of a charged circular ring, carrying a charge uniformly along its circumference with a density ρ_L





$dQ = \rho_L \, dl$ and

$$\boxed{d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2} \cdot \hat{a}_R = \frac{\rho_L \, dl}{4\pi\varepsilon_0 R^2} \cdot \hat{a}_R}$$

- Consider the cylindrical co-ordinate system. For dl we are moving in ϕ direction where $dl = d\phi \cdot r$

  $dl = r \cdot d\phi$

- The two components of R are.
  - (A) Distance r in the direction of $-\hat{a}_r$, radially inward i.e $-r\hat{a}_r$
  - (B) Distance z in the direction of $\hat{a}_z$ i.e $z\hat{a}_z$

$$\bar{R} = -r\hat{a}_r + z\hat{a}_z$$

$$\boxed{|\bar{R}| = \sqrt{r^2 + z^2}}$$

$$\hat{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-r\hat{a}_r + z\hat{a}_r}{\sqrt{r^2+z^2}}$$

$$d\bar{E} = \frac{\rho_L \, dl}{4\pi\varepsilon_0 \left(\sqrt{r^2+z^2}\right)^2} \times \frac{-r\hat{a}_r + z\hat{a}_z}{\sqrt{r^2+z^2}}$$

$$\therefore d\bar{E} = \frac{\rho_L (r \cdot d\phi)}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \left[-r\hat{a}_r + z\hat{a}_z\right]$$

- The radial component of $\bar{E}$ at point P will be symmetrically placed in the plane parallel to xy plane and are going to cancel each other. Hence all $\hat{a}_r$ components cancel each other and only component in the z-direction exist in $\bar{E}$

$$d\bar{E} = \frac{\rho_L (r \cdot d\phi)}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \cdot z\hat{a}_z$$

$$\bar{E} = \int_{\phi=0}^{2\pi} \frac{\rho_L \, r \cdot d\phi}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \cdot z\hat{a}_z$$

$$\bar{E} = \frac{\rho_L \, r}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \cdot z\hat{a}_z \, [\phi]_0^{2\pi}$$

Integration w.r.t ϕ

$$\boxed{\bar{E} = \frac{\rho_L \, r z}{2\varepsilon_0 (r^2+z^2)^{3/2}} \cdot \hat{a}_z}$$

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- Point charge, Q $\quad \bar{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2} \cdot \hat{a}_r$

- Infinite line charge: $\rho_L = \bar{E} = \dfrac{\rho_L}{2\pi\varepsilon_0 r} \cdot \hat{a}_r$

- Infinite sheet of charge $\quad \rho_s = \bar{E} = \dfrac{\rho_s}{2\varepsilon_0} \cdot \hat{a}_n$

- Charged circular ring $\quad \bar{E} = \dfrac{\rho_L r z}{2\varepsilon_0 (r^2+z^2)^{3/2}} \cdot \hat{a}_z$

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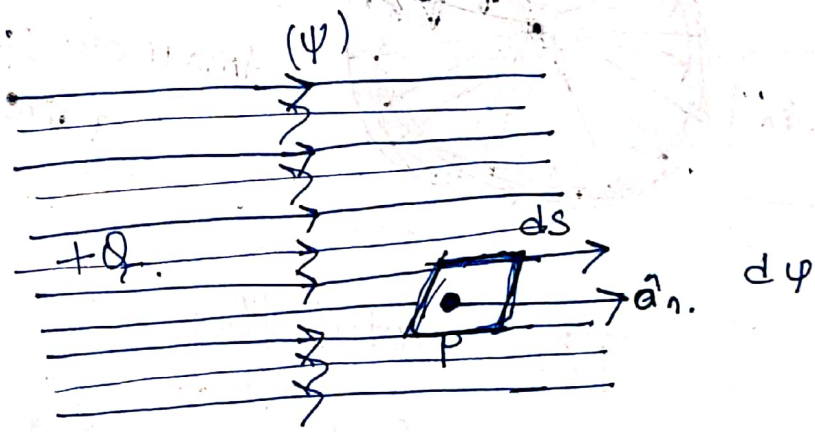
Electric Flux Density ($\vec{D}$)

Def:

The net Electric flux (ψ) passing through the unit Surface area is called as ($\vec{D}$) Electric flux Density.

the direction of flux density is normal to the Surface area ds

$$\boxed{\vec{D} = \frac{d\psi}{ds}\hat{a}_n} \quad C/m^2$$



Electric flux lines.

* the flux Density ($\vec{D}$) at any point depends on the direction of the differential surface area.

if the flux density makes an angle θ with normal to ds then

$$d\psi = D\,ds\cos\theta$$

$$\boxed{d\psi = \vec{D}\cdot d\vec{s}}$$

Note:
$|\vec{A}|\cdot|\vec{B}| = |\vec{A}||\vec{B}|\cos\theta$

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that is, for an Electric flux Density $\vec{D}$, the flux passing through the Surface $\boxed{d\psi = \vec{D} \cdot d\vec{s}}$

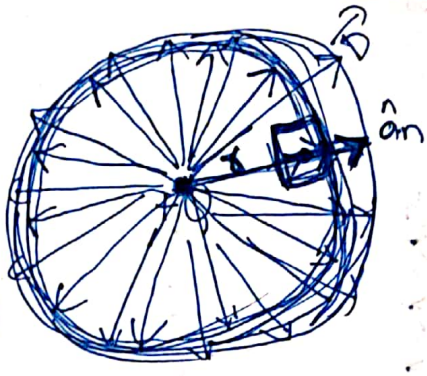
$d(\psi)$

$$\boxed{\psi = \oint_S \vec{D} \cdot d\vec{s}}$$

If A is the total Area,

the total flux is $\boxed{\psi = \vec{D} \cdot \vec{A}}$

$\Rightarrow \vec{D}$ Due to point charge '+Q'



imaginary sphere of radius 'r' carry point charge +Q

$$\boxed{\vec{D} = \frac{d\psi}{ds} \cdot \hat{a}_n}$$

$$\vec{D} = \frac{\text{total flux}}{\text{Surface area of the sphere}} = \frac{\psi}{4\pi r^2} \cdot \hat{a}_n$$

Since $\psi = Q$,

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2} \cdot \hat{a}_n \ C/m^2}$$

The Electric flux density is also called as Displacement flux density

$\boxed{\hat{a}_n = \hat{a}_r}$ The radial direction Normal to the Surface.

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Relationship between $\vec{E}$ and $\vec{D}$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon r^2} \cdot \hat{a}_r}$$

The electric field strength ($\vec{E}$) at a distance r from a +ve point charge Q.

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2} \cdot \hat{a}_r}$$

$$\vec{E} = \frac{1}{\varepsilon} \cdot \vec{D}$$

$$\boxed{\vec{D} = \varepsilon \vec{E}}$$

$\vec{D}$ due charge distribution

Case 1

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon\rho} \hat{a}_\rho$$

then

$$\vec{D} = \varepsilon \vec{E} = \cancel{\varepsilon} \frac{\rho_L}{2\pi\cancel{\varepsilon}\rho} \cdot \hat{a}_\rho = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho.$$

Surface:

$$\vec{E} = \frac{\rho_s}{2\varepsilon} \hat{a}_n$$

$$\vec{D} = \varepsilon \vec{E} = \cancel{\varepsilon} \frac{\rho_s}{2\cancel{\varepsilon}} \hat{a}_n = \frac{\rho_s}{2} \hat{a}_n$$

Volume: $\vec{E} = \frac{Q}{4\pi\varepsilon r^2} \cdot \hat{a}_r$ $\quad \vec{D} = \varepsilon \vec{E} = \frac{Q}{4\pi r^2} \cdot \hat{a}_r = \int_v \frac{\rho_v \cdot dv}{4\pi r^2} \cdot \hat{a}_r$

$\boxed{Q = \int_v \rho_v dv}$

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1.17 Gauss' Law (Integral Form)

Gauss' law states that *the net electric flux passing through any closed surface is equal to the total charge enclosed by that surface.*

Mathematically, it can be expressed as

Flux $$\psi = Q_{encl} = \oint_s \vec{D} \cdot d\vec{S}, \tag{1.49}$$

where Q_{encl} = charge enclosed by the surface, $\vec{D}$ = electric flux density and $d\vec{S}$ = differential surface area.

Proof Consider an arbitrary surface enclosing a charge Q. Let dS be a differential surface area through which the flux passes, as shown in Fig. 1.19. The differential surface area vector is always chosen normal to the surface area.

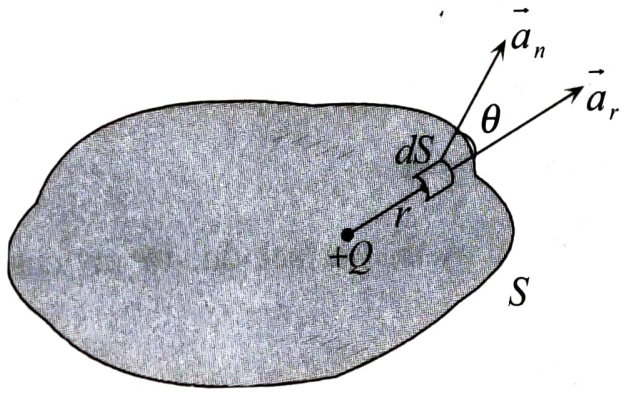


Fig. 1.19 *Charge enclosed by the surface S*

$$d\vec{S} = dS\vec{a}_n.$$

We know that the electric flux density at a distance r from a point charge is

$$\vec{D} = \frac{Q}{4\pi r^2}\vec{a}_r.$$

Taking the dot product with surface area $d\vec{S}$ on both sides,

$$\vec{D} \cdot d\vec{S} = \frac{Q\vec{a}_r}{4\pi r^2} \cdot dS\vec{a}_n.$$

Let θ be the angle between the direction of $\vec{a}_r$ and the normal to the surface, $\vec{a}_n$.

$$\vec{D} \cdot d\vec{S} = \frac{Q}{4\pi r^2} dS\, \vec{a}_r\, \vec{a}_n = \frac{Q}{4\pi r^2} dS \cos\theta.$$

The net flux passing through the closed surface is obtained by taking the closed surface integration on both sides.

$$\oint_S \vec{D}\cdot d\vec{S} = \oint_s \frac{Q}{4\pi r^2} dS \cos\theta$$

$$= \frac{Q}{4\pi}\oint_S \frac{dS\cos\theta}{r^2}.$$

The quantity $dS\cos\theta$ is the component of dS in the radial direction, and $\frac{dS\cos\theta}{r^2}$ is the projection of dS on a spherical surface of unity radius. This represents the solid angle of the surface area dS at the point charge.

Therefore, the solid angle of the sphere is $\oint_S \frac{dS\cos\theta}{r^2} = 4\pi.$

So $\oint_S \vec{D}\cdot dS = \frac{Q}{4\pi}\times 4\pi = Q.$

Therefore, the total displacement through the closed surface is

$$\oint_S \vec{D}\cdot dS = Q = \psi.$$

If Q_{encl} is the total charge enclosed by the surface due to any charge distribution, then the Gauss' law is

$$\oint_S \vec{D}\cdot d\vec{S} = Q_{encl}$$ (Hence proved.)

Note:

1. The surface over which the Gauss' law is applied is called the Gaussian surface.
2. The Gaussian surface is always a closed surface. It may be of any shape, but must have a constant electric field.
3. The surface is chosen such that at each point on the Gaussian surface the flux density $\vec{D}$ is either normal or tangential to the surface.
4. When D is normal to the Gaussian surface, $\vec{D}\cdot d\vec{S} = DdS$ is a constant. If $\vec{D}$ is tangential to the surface, $\vec{D}\cdot d\vec{S} = 0$.
5. If there are more than one charge or charge distribution enclosed by the Gaussian surface, the net charge is the algebraic sum of all the charges.

For a volume charge with charge density ρ_v C/m^3, the total flux is

$$\psi = \int_v \rho_v dv = \oint_S \vec{D}_S \cdot d\vec{S}.$$

For a line charge with charge density ρ_l C/m,

$$\psi = \int \rho_l dl = \oint_S \vec{D}_S \cdot d\vec{S}.$$

For a surface charge with charge density ρ_S C/m^2,

$$\psi = \int_S \rho_S dS = \oint_S \vec{D}_S \cdot d\vec{S}.$$

Gauss' law can be applied to the above described charge distributions when symmetry conditions exist. The use of Gauss' law is easy for the symmetrical distributions because the problem of integration always disappears when a proper closed surface is chosen. Gauss' law is not useful if there is no suitable Gaussian surface.

Example 1.9 A circular disc of radius 20 m with a charge density $\rho_s = 100\sin\phi$ nC/ m^2 is enclosed by surface S. What net flux that crosses the surface?

Solution Given: $\rho_s = 100\sin\phi$ nC/ m^2; radius $a = 20$ m

Flux crossing the surface is $\psi = Q = \int \rho_s dS$

$$= \int_{\phi=0}^{2\pi} \int_{r=0}^{20} (100\sin\phi \times 10^{-9}) \rho \, d\rho \, d\phi$$

$$= 100 \times 10^{-9} \int_0^{20} \rho \, d\rho \int_0^{2\pi} \sin\phi \, d\phi$$

$$= 100 \times 10^{-9} \frac{10^2}{2} \times 0 = 0.$$

Example 1.10 A point charge, $Q = 60$ nC, is located at the origin of a Cartesian coordinate system. Find the electric flux density D at (4, 7, –8) m.

Solution Given: $Q = 60$ nC at origin and a point at (4, 7, –8).

The distance vector is

$$\vec{R} = 4\vec{a}_x + 7a_y - 8\vec{a}_z$$

$$R = \sqrt{4^2 + 7^2 + 8^2} = \sqrt{129}.$$

$$\vec{a}_R = \frac{4\vec{a}_x + 7\vec{a}_y - 8\vec{a}_z}{\sqrt{4^2 + 7^2 + 8^2}} = \frac{4\vec{a}_x + 7\vec{a}_y - 8\vec{a}_z}{\sqrt{129}}.$$

We know that the flux density is

$$\vec{D} = \frac{Q}{4\pi R^2} \vec{a}_R$$

$$= \frac{60 \times 10^{-9}}{4\pi \times 129} \cdot \frac{4\vec{a}_x + 7\vec{a}_y - 8\vec{a}_z}{\sqrt{129}}$$

$$= 3.25 \times 10^{-12} (4\vec{a}_x + 7\vec{a}_y - \vec{a}_z)$$

$$= (13\vec{a}_x + 22.8\vec{a}_y - 26\vec{a}_z) \quad \text{pC/m}^2.$$

1.21 Electric Flux Density Due to Infinite Line Charge Using Gauss' Law

The electric field of an infinity long line charge at any point P can be solved by using Gauss' law.

Consider an infinite line charge of density ρ_l C/m lying along the z-axis extending from $-\infty$ to $+\infty$. Let a point P be at a distance ρ from the line charge.

Consider a cylindrical surface with radius ρ and height z, as shown in Fig. 1.22, as the Gaussian surface, and let dS be a differential surface area on it. Since the line charge is symmetrical about the z-axis, the flux density has only radial component, $\vec{a}_\rho$, normal to the surface.

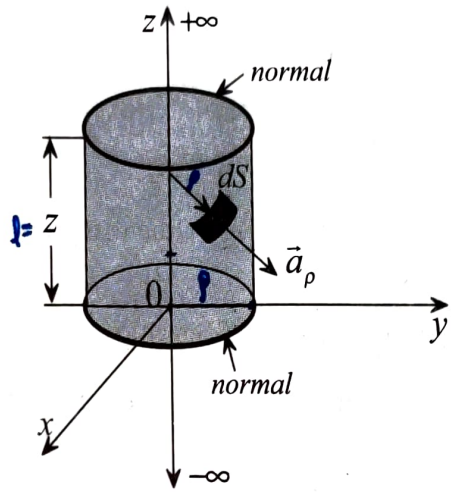


Fig. 1.22 *Infinite line charge*

From Gauss' law, the net charge enclosed by the cylindrical surface is

$$Q = \oint_S \vec{D} \cdot d\vec{S}.$$

The integration is evaluated over the closed surface of the cylinder.

$$Q = \int_{\text{side}} \vec{D} \cdot d\vec{S} + \int_{\text{top}} \vec{D} \cdot d\vec{S} + \int_{\text{bottom}} \vec{D} \cdot d\vec{S}. \tag{1.60}$$

Since $\vec{D}$ has only radial component, no component along the z-axis exist.

Thus, $\int_{\text{top}} \vec{D} \cdot d\vec{S} = \int_{\text{bottom}} \vec{D} \cdot d\vec{S} = 0$

$$\therefore \quad Q = \int_{\text{side}} \vec{D} \cdot d\vec{S}$$

$\hat{a}_\rho = \dfrac{\vec{D}}{|D|}$

$d\vec{S} = \rho \, d\phi \, dz \, \vec{a}_\rho$ and $\vec{D} = D\vec{a}_\rho$.

$$\therefore \quad Q = \int_0^z \int_0^{2\pi} D\vec{a}_\rho \cdot \rho \, d\phi \, dz \, \vec{a}_\rho = \int_0^z \int_0^{2\pi} D \rho \, d\phi \, dz$$

$$Q = D(2\pi \rho z)$$

$\hat{a}_\rho \cdot \hat{a}_\rho = 1$

Q =

$$\text{or,} \quad D = \frac{Q}{2\pi\rho z}.$$

Since $Q = \rho_l z$,

$$D = \frac{\rho_l}{2\pi\rho}.$$

The electric flux density

$$\vec{D} = \frac{\rho_l}{2\pi\rho}\vec{a}_\rho \text{ C/m}^2, \tag{1.61}$$

and the electric field strength

$$\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{\rho_l}{2\pi\varepsilon\rho}\vec{a}_\rho, \tag{1.62}$$

which is same as Eq.(1.32).

Example 1.12 Find the flux density at (2, –2, 1) due to a point charge 10 mC at (1, –4, 0) and a line charge 0.3 mC/m along the *y*-axis.

Solution Given: $Q = 10\times10^{-3}$ C at (1, –4, 0) and $\rho_l = 30\times10^{-3}$ C/m.

To calculate the flux density *D* at (2, –2, 1) due to point charge, we know that

$$\vec{D}_1 = \varepsilon\vec{E} = \frac{Q}{4\pi R^2}\vec{a}_R.$$

The distance vector is

$$\vec{R} = (2,-2,1)-(1,-4,0)$$

$$\vec{R} = (2-1)\vec{a}_x + (-2+4)\vec{a}_y + (1-0)\vec{a}_z = \vec{a}_x + 2\vec{a}_y + \vec{a}_z$$

$$R = \sqrt{1+2^2+1^2} = \sqrt{6}.$$

$$\therefore \quad \vec{D}_1 = \frac{10\times10^{-3}}{4\pi 6}\times\frac{\vec{a}_x + 2\vec{a}_y + \vec{a}_z}{\sqrt{6}}$$

$$= 5.4(\vec{a}_x + 2\vec{a}_y + \vec{a}_z)\times10^{-6} \text{ C/m}^2.$$

Flux density *D* at (2, –2, 1) due to line charge along the *y*-axis:

$$\vec{D}_2 = \varepsilon\vec{E} = \frac{\rho_l}{2\pi R}\vec{a}_R.$$

The point on the line charge along the *y*-axis is (0, – 2, 0)

Distance vector $\vec{R} = (2,-2,1)-(0,-2,0) = 2\vec{a}_x + \vec{a}_z$

$$R = \sqrt{2^2+1} = \sqrt{5}.$$

$$\therefore \quad \vec{D}_2 = \frac{0.3\times10^{-3}}{2\pi\times\sqrt{5}} \times \frac{2\vec{a}_x + \vec{a}_z}{\sqrt{5}}$$

$$= 9.5(2\vec{a}_x + \vec{a}_z)\times10^{-6}\ \text{C/m}^2.$$

al flux density $\vec{D} = \vec{D}_1 + \vec{D}_2 = (24.4\vec{a}_x + 10.8\vec{a}_y + 14.9\vec{a}_z)\ \mu\text{C/m}^2$.

2 Flux Density Due to an Infinite Sheet Charge Using Gauss' Law

ider an infinite sheet of charge with charge density ρ_s C/ m^2 lying along the x–y plane $= 0$ as shown in Fig. 1.23.

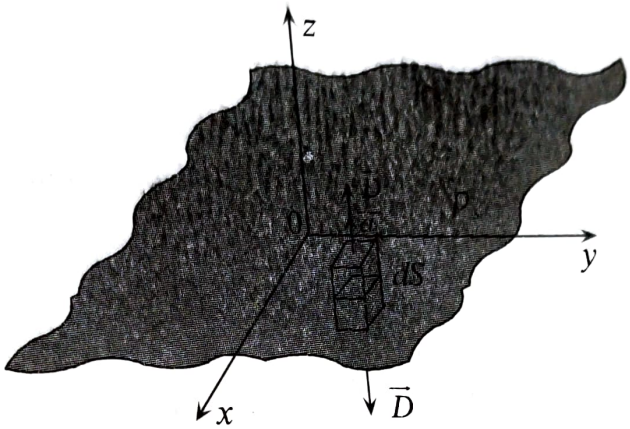


Fig. 1.23 *Field due to infinite sheet of charge*

oose a rectangular box as a Gaussian surface parallel to the z-axis, which cuts the sheet charge with differential surface area dS at an arbitrary point.

The charge enclosed by the surface is

$$Q = \oint_S \vec{D}\cdot d\vec{S} = \int_{\text{sides}} \vec{D}\cdot d\vec{S} + \int_{\text{top}} \vec{D}\cdot d\vec{S} + \int_{\text{bottom}} \vec{D}\cdot d\vec{S}. \tag{1.63}$$

Since $\vec{D}$ has no component in the x- and y- directions,

$$\int_{\text{sides}} \vec{D}\cdot d\vec{S} = 0$$

and $\quad d\vec{S} = dx\,dy\,\vec{a}_z.$

$$\therefore \quad \int_{\text{top}} \vec{D}\cdot d\vec{S} = \int D\vec{a}_z \cdot dx\,dy\,\vec{a}_z = DA;$$

$$\int_{\text{bottom}} \vec{D}\cdot d\vec{S} = \int D(-\vec{a}_z)\,dxdy(-\vec{a}_z) = DA.$$

$$\therefore \quad Q = DA + DA = 2DA.$$

Since $$\rho_s = \frac{Q}{A},$$

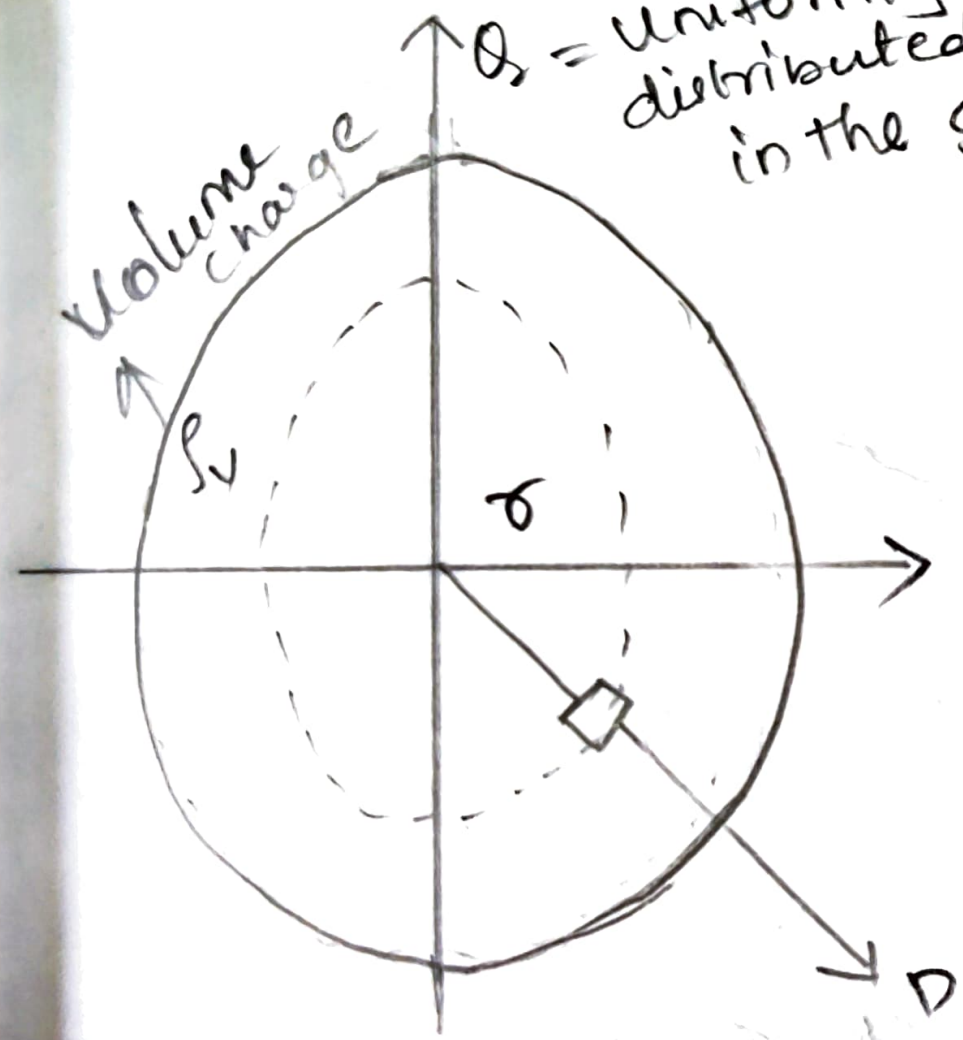
$$D = \frac{Q}{2A} = \frac{\rho_S}{2}.$$

Therefore, $$\vec{D} = \frac{\rho_S}{2}\vec{a}_n$$

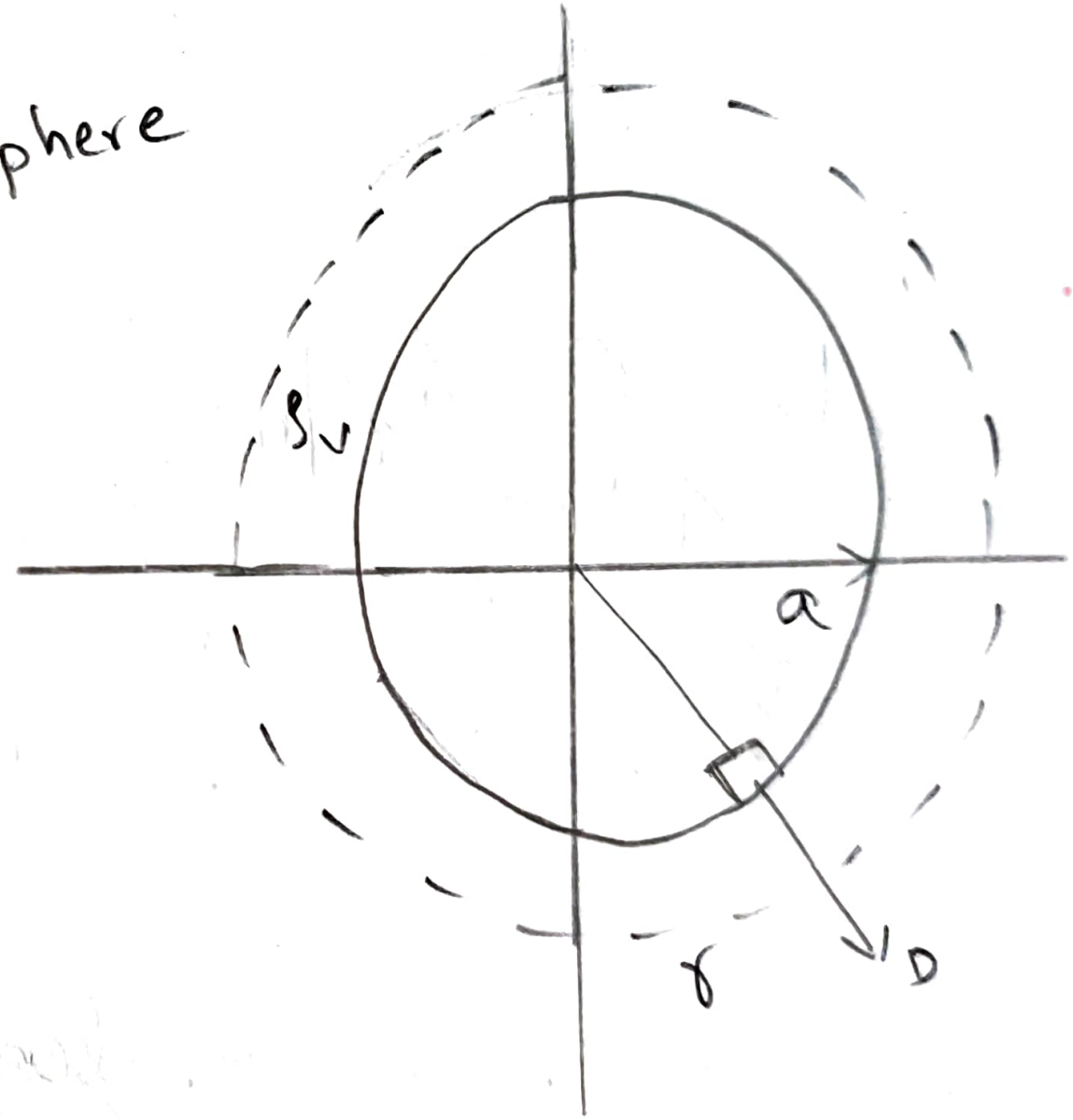
and the field strength $$\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{\rho_S}{2\varepsilon}\vec{a}_n,$$

which is same as Eq. (1.43).

Q = uniformly distributed charge in the sphere



$r < a$



$r > a$

1.25 Flux Density for a Uniformly Charged Sphere

Consider a sphere of radius a as shown in Fig. 1.28. Let charge Q be uniformly distributed in the sphere with volume charge density ρ_v C/ m^3. Also consider an imaginary Gaussian surface of radius r from the centre of the sphere. Let the flux density $\vec{D}$ be radially outward along $\vec{a}_r$ through a differential surface area dS.

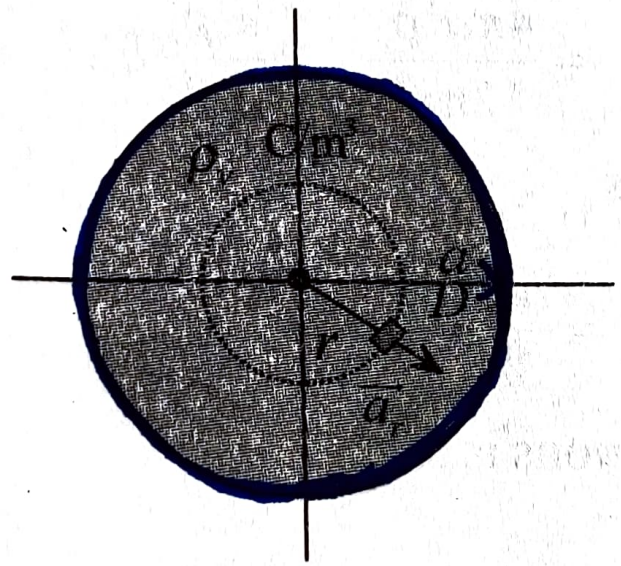


Fig. 1.28 *Sphere of volume charge*

In spherical coordinates,

$$d\vec{S} = r^2 \sin\theta \, d\theta \, d\phi \, \vec{a}_r \qquad (1.$$

and

$$\vec{D} = D \cdot \vec{a}_r.$$

n OnePlus

(1) For $r < a$, the charge enclosed by the surface

$$Q = \oint_S \vec{D} \cdot d\vec{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} D\, r^2 \sin\theta \, d\theta \, d\phi \; \vec{a}_r \cdot \vec{a}_r$$

$$= Dr^2 2\pi[-\cos\theta]_0^{\pi} = 4\pi r^2 D$$

$\cos 0° = 1$

$\cos 180° = -1$

$[-\cos\pi - \cos 0]$

$[-(-1) - 1]$

$1 + 1 = 2$

The flux density $\vec{D} = \dfrac{Q}{4\pi r^2}\vec{a}_r$ C/ m^2.

For volume charge density ρ_v, the charge within the sphere of radius r is

$$Q = \int_v \rho_v dv = \frac{4\pi r^3}{3} \cdot \rho_v$$

$\boxed{V = \frac{4}{3}\pi r^3}$

$\rho_v = \dfrac{Q}{V}$

$d v \rho_v = dQ$

$\boxed{\rho_v \, dv = Q}$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\vec{a}_r}$$

The flux density $\vec{D} = \dfrac{4\pi r^3 \rho_v}{12\pi r^2}\vec{a}_r = \dfrac{r}{3}\rho_v \vec{a}_r$. (1.73)

(2) For $r > a$, the charge enclosed by the sphere of radius a is

$$Q = \frac{4}{3}\pi a^3 \rho_v$$

$\rho_v = \dfrac{Q}{V}$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r}$$

Volume of sphere(V) $= \frac{4}{3}\pi a^3$

$$= \frac{4\pi a^3 \rho_v}{12\pi r^2} = \frac{a^3 \rho_v}{3r^2}\vec{a}_r. \quad (1.74)$$

(3) At $r = a$, the flux density is

$$\vec{D} = \frac{a^3 \rho_v}{3a^2}\vec{a}_r = \frac{a\rho_v}{3}\vec{a}_r. \quad (1.$$

Thus the flux density at all regions is :

$$\vec{D} = \begin{cases} \dfrac{r}{3}\rho_v \vec{a}_r & r < a \\ \dfrac{a\rho_v}{3}\vec{a}_r & r = a \\ \dfrac{a^3 \rho_v}{3r^2}\vec{a}_r & r > a \end{cases} \quad (1$$

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The electric field intensity is $\vec{E} = \vec{D} / \varepsilon$.

$$\vec{E} = \begin{cases} \dfrac{r}{3\varepsilon}\rho_v \vec{a}_r & r < a \\ \dfrac{a\rho_v}{3\varepsilon}\vec{a}_r & r = a \\ \dfrac{a^3\rho_v}{3\varepsilon r^2}\vec{a}_r & r > a \end{cases} \tag{1.77}$$

The Fig. 1.29 shows the variation of electric field intensity for charged sphere.

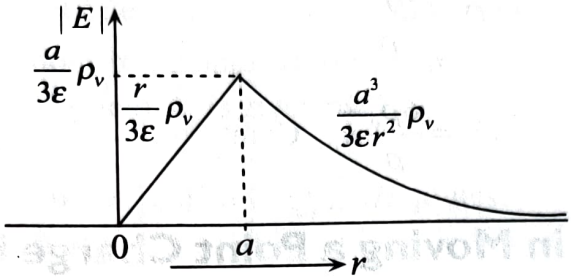

Fig. 1.29 *Field distribution in a charged sphere*

Example 1.13 A volume enclosed between cylindrical coordinates $\rho = 2$ m and $\rho = 4$ m has a uniform charge density of ρ_v C/m^3. Use Gauss' law to find D in all the regions.

Solution Given the volume enclosed by cylindrical coordinates $\rho = 2$ m and $\rho = 4$ m, and charge density ρ_v C/m^3. Assume that the length of the cylinder is L metres. The surface area of the cylinder is $2\pi\rho L$ m^2.

For $0 < \rho < 2$,

$$\int \vec{D} \cdot d\vec{S} = Q.$$

$D \cdot S = D(2\pi\rho L) = 0$ since no charge is enclosed.

$$D = 0.$$

For $2 \le \rho \le 4$,

$$\oint D \cdot dS = Q = \int \rho_v dv$$

$$D \cdot S = \int_0^L \int_{\phi=0}^{2\pi} \int_{\rho=2}^{\rho} \rho_v \rho \, d\rho \, d\phi \, dz$$

$$D\, 2\pi\rho L = 2\pi \rho_v L \left(\frac{\rho^2}{2} \right)_2^{\rho}$$

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By APKK

$$D = \frac{\rho_v}{2\rho}(\rho^2 - 4) \quad \text{C/m}^2$$

$$\vec{D} = \frac{\rho_v}{2\rho}(\rho^2 - 4)\vec{a}_\rho \quad \text{C/m}^2.$$

For $\rho > 4$,

$$D(2\pi\rho L) = \int_0^L \int_0^{2\pi} \int_2^4 \rho_v \rho \, d\rho \, d\phi \, dz = 2\pi\rho_v \frac{(4^2 - 2^2)}{2}$$

$$D = \frac{6\rho_v}{\rho},$$

and

$$\vec{D} = \frac{6\rho_v}{\rho}\vec{a}_\rho \quad \text{C/m}^2.$$

6. If the charge is moving in different fields $\vec{E}_1, \vec{E}_2, \ldots E_N$, and covering distances $d\vec{l}_1, d\vec{l}_2, d\vec{l}_3 \ldots d\vec{l}_N$, respectively, then the total work done is

$$W = -Q\sum_{i=1}^{N} \vec{E}_i \cdot d\vec{l}_i. \tag{1.80}$$

7. The differential length vector in Cartesian coordinate system is

$$d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y + dz\,\vec{a}_z. \tag{1.81}$$

In cylindrical coordinate system it is

$$d\vec{l} = d\rho\,\vec{a}_\rho + \rho d\phi\,\vec{a}_\phi + dz\,\vec{a}_z. \tag{1.82}$$

In spherical coordinate system it is

$$d\vec{l} = dr\,\vec{a}_r + rd\theta\,\vec{a}_\theta + r\sin\theta\, d\phi\,\vec{a}_\phi. \tag{1.83}$$

1.27 Electric Potential

Electric potential at a point P due to a fixed charge Q is defined as *the work done in moving one coulomb of charge from infinity* (∞) *to the point P in an electric field against the force created by the fixed charge.* The work must be done against the field. Electric potential is also defined as the work done per unit charge.

Mathematically, the potential is

$$V_p = \frac{W}{Q} = -\int_{\infty}^{P} \vec{E} \cdot d\vec{l} \quad \text{J/C or volts} \tag{1.84}$$

The potential is a scalar quantity produced at any point due to the charge Q. This potential is also called the scalar potential.

Potential difference

Potential difference is defined as the *work done in moving a unit charge from a point A to another point B against field* $\vec{E}$. *It is denoted by* V_{BA}.

$$V_{BA} = \frac{W}{Q} = -\int_{A}^{B} \vec{E} \cdot d\vec{l}. \tag{1.85}$$

Here, A is the initial point and B the final point. The negative sign indicates loss in potential energy in moving Q from A to B, i.e., the work is being done by the field.

The potential difference is also defined as difference between potentials at given points.

$$V_{BA} = V_B - V_A$$

where $$V_A = -\int_{-\infty}^{A} \vec{E}\cdot d\vec{l} \quad \text{and} \qquad (1.86)$$

$$V_B = -\int_{-\infty}^{B} \vec{E}\cdot d\vec{l} \quad \text{volts.} \qquad (1.87)$$

V_A and V_B are the absolute potentials at A and B, respectively.

The work done is

$$W = Q\,V_{BA}$$

$$W = Q(V_B - V_A).$$

Note:

1. The potential difference depends only on the terminal points. It is independent of the path chosen.
2. The potential at infinity is zero.
3. The potential around a closed path is equal to zero, that is, $V = \oint \vec{E}\cdot d\vec{l} = 0$.
4. One volt potential at a point is equal to one joule of work done in moving a unit charge from infinity to that point in electric field $\vec{E}$.

Example 1.14 Calculate the work done in moving a 40 μC charge from a point $A\,(3, 45^0, 135^0)$ to $B\,(4, 120^0, 75^0)$ in the field having a potential given by $V = 25r\cos^2\theta\sin\phi$ volts.

Solution Given: $V = 25r\cos^2\theta\sin\phi$ volts, $Q = 40$ μC, and the points $A(3, 45^0, 135^0)$ and $B\,(4, 120^0, 75^0)$ in spherical coordinates,

Work done $$W = Q\,V_{BA} = Q(V_B - V_A).$$

But the potential at point B is

$$V_B = 25(4)\cos^2(120)\sin(75)$$

$$= 24.14 \text{ volts}$$

and that at point A is $$V_A = 25\times 3\cos^2(45)\sin(135)$$

$$= 75\times 0.5\times 0.207 = 26.516 \text{ volts.}$$

Work done $$W = Q(V_B - V_A) = 40\times 10^{-6}(24.14 - 26.516) = -95\ \mu\text{J}.$$

The minus sign indicates that the work is being done by the field.

1.35 Relationship between $\vec{E}$ and V: Maxwell's Second Equation

We know that in a given electric field $\vec{E}$, the potential difference between points A and B is independent of the path chosen, i.e.,

$$V_{AB} = -\int_B^A \vec{E}.d\vec{l} \quad \text{and} \quad V_{BA} = -\int_A^B \vec{E}.d\vec{l} = -V_{AB}.$$

$$\therefore \qquad V_{AB} + V_{BA} = \oint \vec{E}.d\vec{l} = 0. \tag{1.110}$$

Thus the potential difference around a closed path must be zero.

Applying Stokes' theorem,

$$\oint \vec{E}.d\vec{l} = \int (\nabla \times \vec{E}).d\vec{S} = 0$$

$$\text{or} \qquad \nabla \times \vec{E} = 0, \tag{1.111}$$

that is, the curl of the electric field must be zero. Any electric field that satisfies the above equation is called conservative field or irrotational.

Now, the potential at any point in the field is

$$V = -\int \vec{E}.d\vec{l}.$$

In Cartesian coordinates, $$dV = -E.dl = -E_x dx - E_y dy - E_z dz. \quad \text{— (1)}$$

Also, $$dV = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz. \quad \text{— (2)}$$

Compare (1) & (2)

Therefore, $$E_x = -\frac{dV}{dx}; \quad E_y = -\frac{\partial V}{\partial y}; \quad \text{and} \quad E_z = -\frac{\partial V}{\partial z}.$$

We know that $$\vec{E} = E_x\vec{a}_z + E_y\vec{a}_y + E_z\vec{a}_z$$

$$= -\frac{\partial V}{\partial x}\vec{a}_x - \frac{\partial V}{\partial y}\vec{a}_y - \frac{\partial V}{\partial z}\vec{a}_z$$

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

or, $$\vec{E} = -\nabla V. \tag{1.112}$$

Thus the electric field intensity is equal to the gradient of the potential. The negative sign indicates that the field direction is opposite to the direction of the potential increment.

1.36 Potential Function (*V*)

An electric potential at any point in the field which depends on the coordinates (or position) is called potential function.

The electrostatic field E can be described completely by means of a potential function $V(x, y, z)$. The potential gradient of V is given by

$$E = -\nabla V,$$

where, V is a scalar potential function. This equation gives a simple method to find the electric field intensity from the potential.

It should be noted that if two points in the electric field have non-zero potential difference, the electric lines of force should be present. These lines of force travel from a point of higher potential to a point of lower potential.

Properties of potential function

The potential function has the following properties.

1. It has a single value at any point in the electrostatic field.
2. It is a continuous function.
3. The difference in potential between any two points does not depend on the path of integration.
4. An electrostatic field is conservative in nature, since $\nabla \times (\nabla V)$, which is the work done in moving a point charge around any closed path in the field, is zero.

1.37 Equipotential Surfaces

An equipotential surface is defined as *a surface in an electric field on which all points on the surface are at the same potential. In other words, the potential difference between any two points on an equipotential surface is always zero*. Hence the work done in moving a charge from one point to another point on an equipotential surface is always zero.

The gradient of the potential and the equipotential surface are always orthogonal to each other.

Energy Density or charge Density

2.42 Energy Stored in an Electrostatic Field

The energy stored in an electric field due to N point charges is expressed as

$$W_E = \frac{1}{2}\sum_{i=1}^{N} V_i Q_i \text{ joules} \tag{2.126}$$

Proof Consider a free space where there is no electric field. Let a point charge Q_1 be moved from infinity to a point 1 in the free space as shown in Fig. 2.41.

Since there is no field, the work done is zero, i.e., $W_1 = 0$. Now due to Q_1 in the free space, an electric field is established.

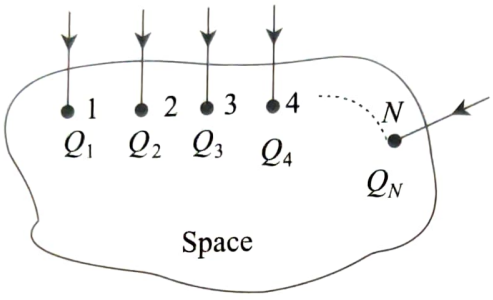


Fig. 2.41 *Point changes in space*

If a point charge Q_2 is moved from infinity to a point 2 in this field, then the work done in keeping Q_2 is

$$W_2 = V_{21}Q_2, \tag{2.127}$$

where V_{21} is the potential at point 2 due to Q_1. Now the field increases due to Q_1 and Q_2.

If another point charge Q_3 is moved from infinity to a point 3 in this field, then the work done in keeping Q_3 is

$$W_3 = V_{31}Q_3 + V_{32}Q_3,$$

where V_{31} is the potential at point 3 due to Q_1, and V_{32} is the potential at point 3 due to Q_2.

Similarly, if the point charges Q_4, $Q_5, \ldots\ldots Q_{N-1}$, etc., are moved into this field, the field strength increases.

Finally, if the point charge Q_N is moved from infinity to a point N in this field, then the work done in keeping Q_N is

$$W_E = V_{N1}Q_N + V_{N2}Q_N + V_{N3}Q_N + \ldots. + V_{NN-1}Q_N.$$

The total work done in positioning all the point charges at their respective points is

$$\begin{aligned} W_E &= W_1 + W_2 + W_3 + \ldots.. + W_N \\ &= 0 + (V_{21}Q_2) + (V_{31}Q_3 + V_{32}Q_3) + (V_{41}Q_4 + V_{42}Q_4 + V_{43}Q_4) \\ &\quad + (V_{51}Q_5 + V_{52}Q_5 + V_{53}Q_5 + V_{54}Q_5) + \ldots.. \\ &\quad + (V_{N1}Q_N + V_{N2}Q_N + V_{N3}Q_N + \ldots\ldots + V_{NN-1}Q_N). \end{aligned} \tag{2.128}$$

The total work done in positioning all the point charges in a space is called the potential energy or the energy stored in an electrostatic field.

To simplify the equation, let us consider that all the point charges are placed in reverse order. That is,

$$Q_i V_{ij} = Q_j V_{ji}.$$

The energy stored in the field can be written as

$$\begin{aligned} W_E = {} & (V_{12}Q_1) + (V_{13}Q_1 + V_{23}Q_2) + (V_{14}Q_1 + V_{24}Q_2 + V_{34}Q_3) \\ & + (V_{15}Q_1 + V_{25}Q_2 + V_{35}Q_3 + V_{45}Q_4) + \ldots\ldots \\ & + V_{1N}Q_1 + V_{2N}Q_2 + V_{3N}Q_3 + \ldots\ldots + V_{N-1N}Q_N \end{aligned} \tag{2.129}$$

Adding Eqs (2.128) and (2.129), we get

$$\begin{aligned} 2W_E = {} & Q(V_{12} + V_{13} + V_{14} + V_{15} + \ldots. + V_{1N}) + Q_2(V_{21} + V_{23} + V_{24} + \ldots. + V_{2N}) \\ & + Q_3(V_{31} + V_{32} + V_{34} + \ldots.. + V_{3N}) \\ & + Q_4(V_{41} + V_{42} + V_{43} + V_{44} + \ldots + V_{4N}) \cdots\cdots \\ & + Q_N(V_{N1} + V_{N2} + N_{N3} + \ldots. + V_{NN}). \end{aligned}$$

Let the potential at Q_1 due to all the other charges be

$$V_1 = V_{12} + V_{13} + V_{14} + \ldots. + V_{1N},$$

and that at Q_2 be $V_2 = V_{21} + V_{23} + V_{24} + \ldots. + V_{2N}$,

$$\vdots$$

and at Q_N $\quad V_N = V_{N1} + V_{N2} + V_{N3} + \ldots. + V_{NN}.$

$$\therefore \quad 2W_E = Q_1V_1 + Q_2V_2 + Q_3V_3 + \ldots. + Q_NV_N.$$

or $\quad W_E = \frac{1}{2}(Q_1V_1 + Q_2V_2 + Q_3V_3 + \ldots. + Q_NV_N).$

Thus the potential energy stored in the field is

(Proved)

$$W_E = \frac{1}{2}\sum_{i=1}^{N} Q_iV_i \quad \text{joules.} \tag{2.130}$$

Similarly, for a line charge of charge density ρ_l,

$$W_E = \frac{1}{2}\int_l V\,\rho_l\,dl \quad \text{joules;}$$

for a surface charge of charge density ρ_s,

$$W_E = \frac{1}{2}\int_s V\,\rho_s\,dS \quad \text{joules;} \tag{2.131}$$

and for a volume charge of charge density ρ_v

$$W_E = \frac{1}{2}\int_v V\,\rho_v\,dv \quad \text{joules.} \tag{2.132}$$

2.43 Energy Stored in Terms of $\vec{E}$ and $\vec{D}$

Consider a volume charge density ρ_v existing in a volume with applied potential, V volts. We know that the energy stored is

$$W_E = \frac{1}{2}\int_v V\,\rho_v\,dv \quad \text{joules.}$$

From Gauss' law, $\rho_v = \nabla\cdot\vec{D} = \nabla\cdot\varepsilon\vec{E}$.

$$\therefore \quad W_E = \frac{\varepsilon}{2}\int_v V(\nabla\cdot E)dv.$$

Using the vector identity

$$(\nabla\cdot\vec{E})V = \nabla\cdot V\vec{E} - \vec{E}\cdot\nabla V.$$

$$W_E = \frac{\varepsilon}{2}\left[\int_v (\nabla\cdot V\vec{E})dv - \int_v \vec{E}\cdot\nabla V dv\right].$$

From the divergence theorem, since no closed surface encloses $\vec{E}$,

$$\int_v (\nabla\cdot V\vec{E})dv = \oint_s V\vec{E}\cdot d\vec{S} = 0.$$

$$\therefore \quad W_E = \frac{-\varepsilon}{2}\int_v \vec{E}\cdot\nabla V dv.$$

We know that $\vec{E} = -\nabla V$.

$$\text{So} \quad W_E = \frac{\varepsilon}{2}\int_v \vec{E}\cdot\vec{E}\cdot dv = \frac{\varepsilon}{2}\int_v E^2 dv$$

$$\text{or} \quad W_E = \int_v \frac{1}{2}\varepsilon E^2 dv = \int_v \frac{1}{2}\vec{D}\cdot\vec{E}dv \quad \text{joules} \tag{2.133}$$

or $$W_E = \int_v \frac{1}{Q\varepsilon} D^2 dv \text{ joules.} \tag{2.134}$$

Therefore the energy density in an electrostatic field is

$$w_E = \frac{W_E}{v} = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2\varepsilon}D^2 \text{ joules/m}^3 \text{ or J/m}^3 \tag{2.135}$$

Additional Solved Problems

Example 2.23 A point charge at $Q_1 = 600\ \mu\text{C}$ located at $(2, -4, -3)$ experiences a force $\vec{F} = 4\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z$ due to a point charge Q_2 at $(4, -6, -4)$. Determine the magnitude of Q_2.

Solution Given: $Q_1 = 600\ \mu\text{C}$ at $(2, -4, -3)$; $\vec{F} = 4\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z$; Q_2 at $(4, -6, -4)$.

We know that $$\vec{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2}\vec{a}_{R_{21}}.$$

The distance vector is

$$\vec{R}_{21} = (2, -4, -3) - (4, -6, -4)$$

$$= (2-4)\vec{a}_x + (-4+6)\vec{a}_y + (-3+4)\vec{a}_z = -2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$$

$$|\vec{R}_{21}| = \sqrt{2^2 + 2^2 + 1} = 3.$$

Since the force is in the direction of the distance vector, magnitude of force is

$$|\vec{F}_{21}| = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2}|\vec{a}_{R_{21}}|.$$

But $|\vec{F}_{21}| = \sqrt{4^2 + 4^2 + 2^2} = 6.$

$$\therefore \quad 6 = \frac{600 \times 10^{-6} \times Q_2}{4\pi \times 8.854 \times 10^{-12} \times 3^2}.$$

$$Q_2 = 10\ \mu\text{C}.$$

CONVENTION AND CONDUCTION CURRENTS

$V = IR$

(or)

$\boxed{I = V/R}$

"Introduction of current w.r.to charge & time"

Introduction: Electric current 'I' is generally caused by motion of electric charge Q.

→ The current 'I' through a given area is a electric charge Q passing through the area per unit time 't'

i.e., Current $\boxed{I = \frac{dQ}{dt}}$ — (1).

C/sec (or) Am

Thus the current of ~~one ampere~~ 1 ampere charge is being transfered at the rate 1c/sec.

let us consider the current density $\vec{J}$ if current ΔI flows through a surface ΔS then the current density $\vec{J}$ is given by

$$J = \frac{\Delta I}{\Delta S} \Rightarrow \Delta I = J_n * \Delta S$$

$$\boxed{\Delta I = J_n \cdot \Delta s} \quad \text{— (2)}$$

The current density $\vec{J}$ is assumed to be perpendicular to the surface

→ The total current (I) ~~the~~ flows through the surface s is given by

$$\boxed{I = \int_s \vec{J} \cdot d\vec{s}} \quad \text{— (4)}$$

→ If the current density is not normal to the surface

$$\boxed{\Delta I = \vec{J} \cdot \Delta s} \quad \text{— (3)}$$

Depending on how I is produced there are different kinds of current densities such as

(i) convention current density.
(ii) conduction current density.
(iii) Displacement current density.

→ we will discuss about conduction and convention current densities when the eq — (4) can be applied to all the current densities

PQV

conduction & conve

CONVENTIONAL current

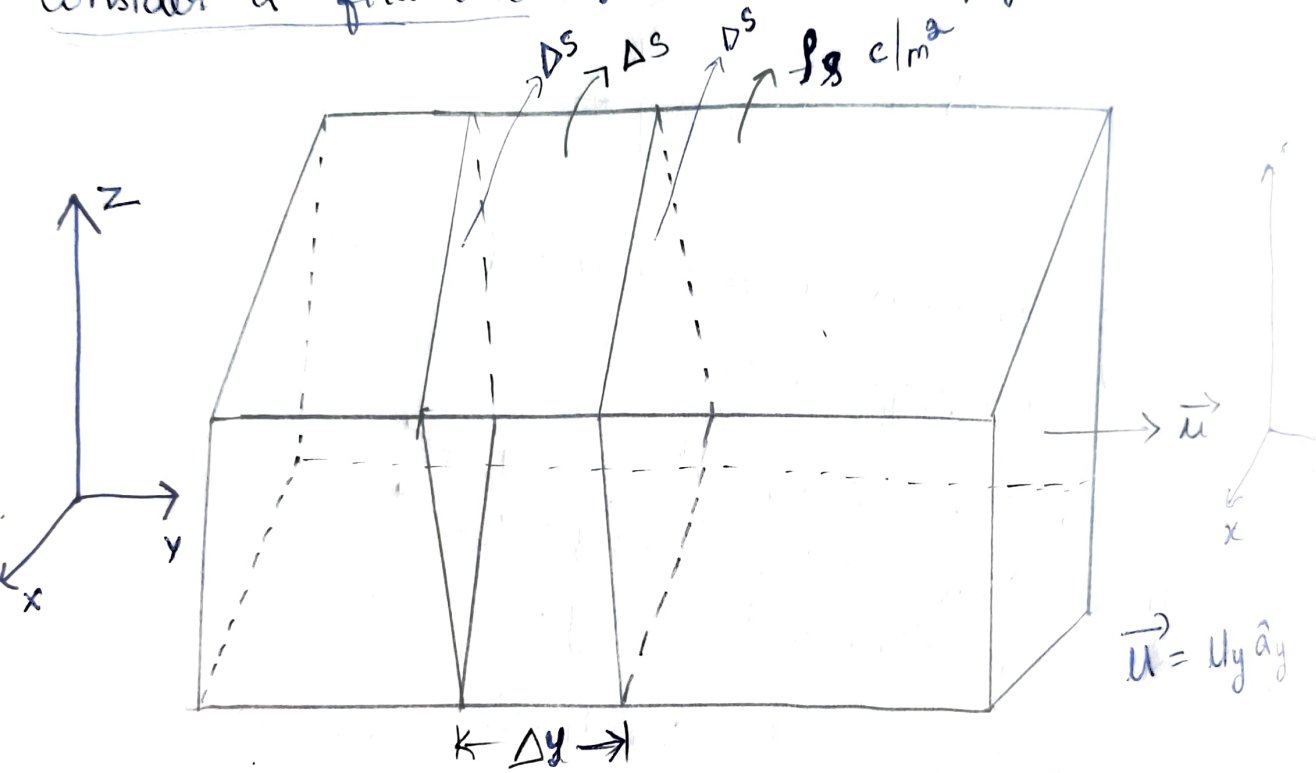
→ this type of current occurs when current flow through an insulating medium such as vaccum, liquid dense rare field gas., etc

Note: ① Does not require any conductor
② Does not obey the ohm's law

Examples:- Conventional current densities of are

① Beam of electrons in a vaccum tube can be considered as convention current.

consider a filament as shown in a figure



If there is a flow of charge, density (ρ_v), the velocity $\bar{u} = \hat{a}_y u_y$

then the current filament is given as

$$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta S \cdot \Delta y}{\Delta t} = \frac{\rho_v \Delta S \cdot \Delta y}{\Delta t}$$

the current thrown the filament ΔI

we know that → $\Delta Q = \rho_v \Delta V$ → multiplication of (3) three parameters so we are writing: $\rho_v \Delta S \frac{\Delta y}{\Delta t}$

where

$$\boxed{\Delta Q = \rho_v \Delta S \, \Delta l_y} \rightarrow \boxed{u_y = \frac{\Delta y}{\Delta t}}$$

$$\boxed{\Delta I = \rho_v \Delta S \, u_y} \quad (5) \qquad \Rightarrow \frac{\Delta I}{\Delta S} = \rho_v u_y = J_y$$

The current density at a given point is the current through the unit normal area. At that point the current density around Y-direction

$$J_y = \frac{\Delta I}{\Delta S}$$

from eq (5) $\boxed{J_y = \rho_v u_y}$

Therefore $J_y = \rho_v u_y$

Here in general $\boxed{\vec{J} = \rho_v \vec{u}}$ (6)
Vector form

The current I is the conventional current and J is conventional current density (A/m^2).

CONDUCTION CURRENT DENSITY:

The conduction current to flow requires a conductor.

The conductor has large amount of free e^- electrons that provide conduction current due to an applied electric field ($\vec{E}$)

when the electric field $\vec{E}$ is applied the force $\vec{F}$ on the electrons with the charge is given by

$$\vec{F} = -e\vec{E} \quad (7)$$

→ since the electrons (e^-) are not in free space it will not be under the influence of Electric field

Rather it suffers constant collision with atomic lattice and drifts from one atom to another. If the electrons with the mass (m) is moving in the electric field $\vec{E}$ with an ~~avro~~ average drift velocity $\bar{u}$. According

~~The drift velocity $\bar{u}$~~

to Newton's law the average change in momentum of free electrons must match the applied force. Thus

$$\boxed{\vec{F} = -e\vec{E}}$$

$$\boxed{\vec{F} = \frac{m\bar{u}}{T}}$$

$$-e\vec{E} = \frac{m\bar{u}}{T} \Rightarrow \boxed{\bar{u} = \frac{-eT\vec{E}}{m}}$$

where T is the average time interval between collision. If they are n electrons present in unit volume. The electron charge density is given by

$$\boxed{\rho_v = -ne}$$

Thus Conduction current density $\boxed{\vec{J} = \rho_v \vec{u}}$

$$\vec{J} = -ne\left(\frac{-eT\vec{E}}{m}\right)$$

$$\vec{J} = \frac{ne^2T\vec{E}}{m} = \sigma\vec{E} = \boxed{\sigma = \frac{ne^2T}{m}}$$

$$\boxed{\vec{J} = \sigma \vec{E}} \quad \text{--- (8)}$$

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$ conductivity,

where <u>conductivity</u> of the conductor (σ) is $\frac{ne^2\tau}{m}$

The above relationship in eq (8) is known as Point form <u>Ohm's law</u>.

Note: σ <u>it obey's Ohm's law</u>)

$\odot$ <u>Conductor required</u>

⊙ Conductor required

Dielectric constant :-

In general all insulators are also called as dielectrics. In perfect dielectric there are no free charge existing.

Polarization in dielectrics:

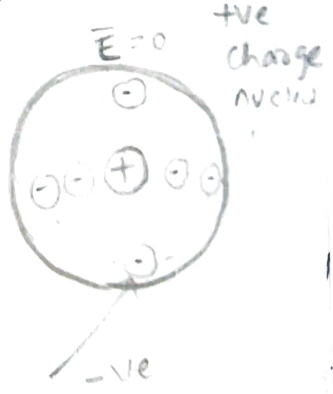
→ consider a atom in dielectric, as consisting of a nucleus with of -ve and +ve charge in the form of revolving electrons in the orbits.

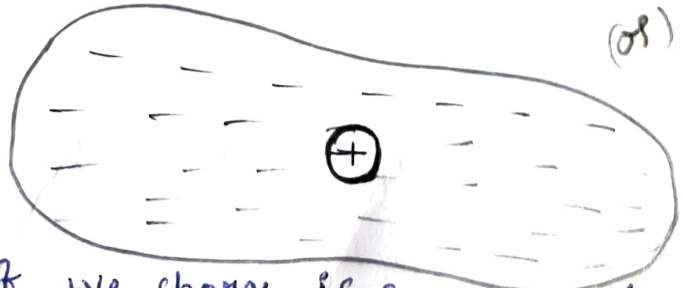
Atom of an Dielectric

$F_+ = Q\vec{E}$

$F_- = -Q\vec{E}$

when electric field is applied to an atom as shown in figure unpolaized atom of dielectric





⊙ if the number of +ve charge is same as -ve charge and hence atom is electrically neutral.

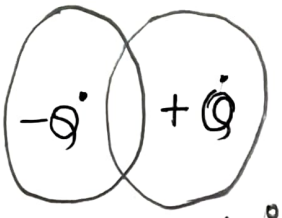
⊙ Due to symmetry, both positive and negative charge can be assumed to be point charge of equal amount, coinciding at the centre. Hence there cannot exist an electric dipole. This is called unpolarized atom.

when the dipole results from the displacement of charges and the dielectric is said to be polarised. In the polarised state the e^- ~~cover~~ cloud is destroyed by the applied electric field $\vec{E}$.

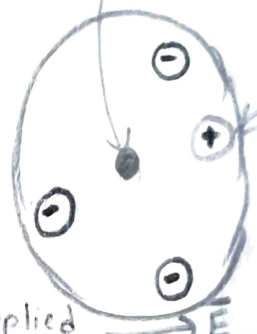
⊙ it can be seen that an electron cloud has a centre separated from the nucleus. center of electron cloud this form an electric dipole

Dipole moment is given as

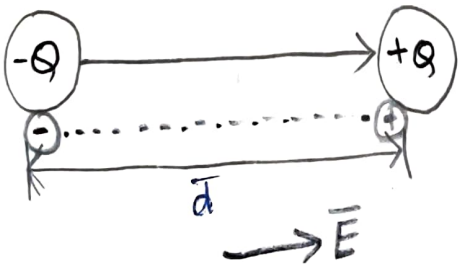
$$\bar{P} = Q\bar{d}$$



electric dipole.



⊙ Now there is a separation b/w the nucleus and the centre of the electron cloud as show in fig such atom is called polarized atom



$\bar{P} = Q\bar{d}$

⊙

where $\bar{d}$ is a distance vector from $-Q$ to $+Q$ of the dipole as shown in a figure.

The sum ~~some~~ of all the dipole moment gives the net electric ~~flux~~ field.

The measure of intensity of the polarisation

$\bar{P}$ (coloumbs/m^2).

polarisation p is a dipole moment per unit volume 'v' of dielectric i.e.,

$$\bar{P} = \frac{\lim_{V \to 0} N \cdot \bar{P}}{\Delta V} \Rightarrow \bar{P} = \frac{\lim_{V \to 0} N \cdot \vec{P}}{\Delta V}$$

where $Q d_i = \bar{P}$ is dipole moment

$$\boxed{\bar{P} = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{\Delta V} N \cdot Q_i d_i}{\Delta V}}$$

$\bar{P}$ is polarization

N is total no of electrons.

units are coulombs per square meter (C/m^2)

if when there is no polarisation, electric flux density is then

$$\boxed{\vec{D} = \varepsilon_0 \vec{E}} \rightarrow (1)$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0}$$

In the presence of polarisation, we have $\rightarrow \vec{E} = \frac{\vec{P}}{\varepsilon_0}$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} - \frac{\vec{P}}{\varepsilon_0}$$

$$\therefore \boxed{\varepsilon_0 \vec{E} = \vec{D} - \vec{P}} \rightarrow (2) \quad \text{or} \quad \boxed{\vec{D} = \varepsilon_0 \vec{E} + P}$$

If the polarisation $\vec{P}$ and electric field intensity are in same direction then

$$\boxed{\vec{P} = \varepsilon_0 \chi_e \vec{E}} \rightarrow (3)$$

where χ_e is known as <u>susceptibility of Material</u>

Sub eq-(3) in eq (2)

$$\varepsilon_0 \vec{E} = \vec{D} - \varepsilon_0 \chi_e \vec{E} \quad \text{(or)} \quad \boxed{\vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E}}$$

$$\varepsilon_0 \vec{E} + \varepsilon_0 \vec{E} x_e = \bar{D}$$

$$\boxed{\bar{D} = \bar{P} + \varepsilon_0 \vec{E}}$$ we know that

$$= \varepsilon_0 \vec{E} [1 + x_e]$$ $\bar{P} = \varepsilon_0 x_e \bar{E}$

$$\vec{D} = \varepsilon_0 (1 + x_e) \vec{E}$$

$$\boxed{1 + x_e = \varepsilon_r}$$

$$\boxed{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}}$$

$$\vec{D} = \varepsilon \vec{E}$$ we know $\boxed{\vec{D} = \varepsilon \vec{E}}$

$$\varepsilon \cancel{\vec{E}} = \varepsilon_0 \varepsilon_r \cancel{\vec{E}}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$ $\boxed{\varepsilon_r = 1 + x_e}$

$$\boxed{1 + x_e = \frac{\varepsilon}{\varepsilon_0}}$$

where ε_0 is the permitivity of free space. $\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m

ε_r is called Relative permitivity or Dielectric Constant.

The dielectric constant (ε_r) is the ratio of permitivity of the dielectric to the permitivity of free space

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$\varepsilon_0 \rightarrow$ free space.

$\varepsilon_r \rightarrow$ relative permitivity

The dielectric constant ε_r and susceptibility of material χ_e are dimensionless quantity also called electric susceptibility χ_e

$\varepsilon_r \geq$ unity (or) 1

$$\boxed{\varepsilon_r = \chi_e + 1}$$

for free space an non-dielectric material such as metals.

→ The minimum value of the electric field at which dielectric breakdown occurs is called dielectric strength of dielectric material.

→ The dielectric strength is maximum electric field that a dielectric can tolerate (or) with stand without breakdown.

~~RELAXATION TIME:~~

Isotropic & Homogeneous.

Isotropic

- If a material has identical values of property in all the direction then the materials is said to be Isotropic in nature.
- In the case of dielectric, it is said to be isotropic if electric flux density (D) and electric field Intensity (E) are in the same direction.

  Eg: Free Space, water, glass., etc.

Homogeneous

- the medium is called homogeneous when the physical Characteristics of the medium do not vary from point to point but remain same every where thoughout the medium.
- In the case of dielectric material, it is called homogeneous if its permittivity is constant in the region and donot vary with co-ordinates of the point in the medium.

  Eg: Free Space water, optical glass.. etc.

Isotropic and Homogeneous dielectric :-

Properties of dielectric.

They are three types.

(i) linear
(ii) Homogeneous
(iii) Isotropic

Linear:

→ A dielectric material is said to be linear if the flux density with in the material $\vec{D}$ where is linearly with $\vec{E}$ i.e.,

The material properties like permitivity should be independant of applied field. Otherwise is in non- linear

Homogeneous :- Eg: Free Space water, optical glass etc

→ A dielectric material is said to be homogeneous if its material properties permitivity ε and Conductivity 'σ' does not vary from point to point in the material i.e, permitivity(ε) and Conductivity σ constant through out the material.

For an homogenous materia (or) non-homogeneous

material i.e, permitivity ('ε' and 'σ' depends on space - co-ordinate or varies conductivity

- Isotropic ; Eg: Free space, water, glass etc.

A dielectric material is said to be isotropic if $\vec{D}$ and $\vec{E}$ are in the same direction i.e, the material properties are independent of the directions. Otherwise the material is non-isotrophic (not independant of direction). Eg:

3.18 Continuity Equation

The continuity equation states that *the current density diverging from a small volume is equal to the time rate of decrease of charge per unit volume at any given point.*

Mathematically,

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t}. \tag{3.59}$$

This equation is called the point form of continuity equation.

The integral form of continuity equation states that *the current flows outward from a closed surface is equal to the rate of decrease of charge inside that surface.*

$$I = \oint_S \vec{J} \cdot d\vec{S} = \frac{-dQ}{dt}. \tag{3.60}$$

Proof Consider a closed surface S. Let a differential surface dS be normal to the direction of current density $\vec{J}$ as shown in Fig. 3.18.

The total current crossing the surface S is

$$I = \oint_S \vec{J} \cdot d\vec{S}.$$

According to the principle of conservation of energy, *charge can neither be created nor be destroyed, that is, the outward rate of flow of positive charge is equal to the rate of decrease of charge inside the closed surface.*

Let Q be the charge within the closed surface. Then the rate of charge decreasing through the surface is $I = \frac{-dQ}{dt}$

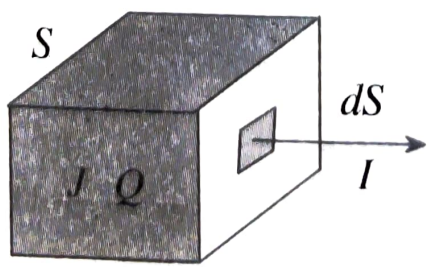


Fig. 3.18 *Current flow through a closed surface*

From Eq.(3.62),

$$I = \oint_S \vec{J} \cdot d\vec{S} = \frac{-dQ}{dt}.$$

From divergence theorem, we know that

$$\oint_S \vec{J} \cdot dS = \int_v \nabla \cdot \vec{J}\, dv$$

Also, $$Q = \int_v \rho_v \cdot dv.$$

$$\therefore \quad I = \int_v \nabla \cdot \vec{J}\, dv = \frac{-d}{dt}\int_v \rho_v \cdot dv$$

or $$\int_v (\nabla \cdot \vec{J}) dv = \int_v \frac{-\partial \rho_v}{\partial t} \cdot dv.$$

Eliminating the volume integration on both sides

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t} \quad \text{— (A)}$$

we can modified Eq, (A)

$$\nabla \cdot \vec{J} = -\dot{\rho}_v = \boxed{-\rho_v = \nabla \cdot \vec{J}}$$ (Proved)

Note: For steady current, the volume charge density is constant with respect to time.

$$\therefore \quad \nabla \cdot \vec{J} = 0 \qquad (3.61)$$

$$-\frac{\partial \rho_v}{\partial t} = 0$$

3.19 Relaxation time (T_r)

The charge density through the closed surface within a conductor decreases exponentially with time. The relaxation time T_r is defined as *the time taken by the charge density to decay to 36.8% of its initial value at a given point.*

It is expressed as

$$T_r = \frac{\varepsilon}{\sigma} \text{ seconds}, \qquad (3.62)$$

where ε = permittivity of the medium and σ = conductivity of the medium.

The relaxation time depends only on medium properties.

Proof Consider a linear and homogeneous medium with constants σ and ε through which current is flowing with current density $\vec{J}$.

We know that $$\vec{J} = \sigma \vec{E} = \sigma \frac{\vec{D}}{\varepsilon}.$$

From continuity equation, we know that

$$\nabla \cdot \vec{J} = -\dot{\rho}_v .$$

$$\therefore \quad \nabla \cdot \left(\frac{\sigma}{\varepsilon}\right)\vec{D} = -\dot{\rho}_v$$

$$\text{or} \quad \frac{\sigma}{\varepsilon}\left(\nabla \cdot \vec{D}\right) = -\dot{\rho}_v$$

from continuity eq

$$-\rho_v = -\frac{\partial \rho_v}{\partial t}$$

From Gauss law, we know that

$$\nabla \cdot \vec{D} = \rho_v$$

$$\therefore \quad \frac{\sigma}{\varepsilon}\rho_v = -\dot{\rho}_v = \frac{-\partial \rho_v}{\partial t}$$

$$\text{or} \quad \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0.$$

This is a first order differential equation, whose solution is

$$\rho_v = A e^{-\sigma t/\varepsilon} .$$

where A is a constant.

Let the initial value at $t = 0, \quad \rho_v = \rho_{v0}$

so that $A = \rho_{v0} .$

$$\therefore \quad \rho_v = \rho_{v0} e^{-\sigma t/\varepsilon} .$$

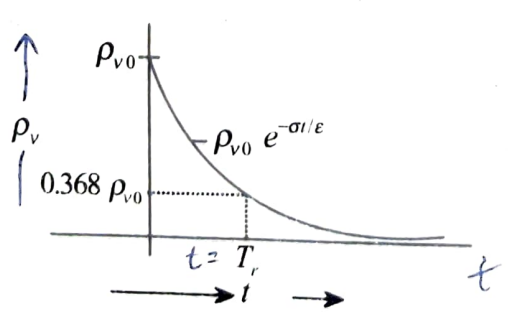
The time constant $\varepsilon / \sigma = T_r$ is the relaxation time in seconds.

$$\rho_v = \rho_{v0} e^{-t/T_r} \tag{3.63}$$

$$\text{At } t = T_r, \quad \rho_v = \frac{\rho_{v0}}{e} = 0.368\, \rho_{v0} . \tag{3.64}$$

$e^{-1} = 0.368$

Therefore the relaxation time $T_r = \dfrac{\varepsilon}{\sigma}$, is the time taken for the charge density to decay to 36.8% or $(1/e)$ times its initial value. This is shown in Fig. 3.19.



$e^{-1} = 36.8\%$

Fig. 3.19 *Charge density and relaxation time*

2.10 Poisson's and Laplace's Equations

A useful approach to finding electric potentials in free space or in a dielectric medium is through the direct solution of Poisson's equation or Laplace's equation. The Poisson's equation states that

In a homogeneous medium, if there exists volume charge density ρ_v C/m^3 *at any point, then the potential at that point is given by*

$$\nabla^2 V = -\rho_v / \varepsilon, \tag{2.18}$$

where ∇ is the del operator and ε is the permittivity of the medium.

If the volume charge density is zero (charge-free region of space), and only either point or charge distribution exist in a medium, then

$$\nabla^2 V = 0. \tag{2.19}$$

This relationship is called Laplace's equation.

Proof: Consider a homogeneous and linear medium where ε is constant throughout the region in which the field is defined. Let ρ_v be the volume charge density. Then, from the point form of Gauss' law, we know that

$$\nabla \cdot \vec{D} = \rho_v$$

Also, $\vec{D} = \varepsilon \vec{E}.$

Then, $\nabla \cdot \varepsilon \vec{E} = \rho_v$

$$\nabla \cdot \vec{E} = \rho_v / \varepsilon.$$

We also know that $\vec{E} = -\nabla V.$

$\therefore$ $\nabla \cdot \nabla V = -\rho_v / \varepsilon$

or $\nabla^2 V = -\rho_v / \varepsilon$ (Proved)

If $\rho_v = 0,$

then $\boxed{\nabla^2 V = 0}$

where ∇^2, called the del square operator, is given in different coordinate systems as:

Cartesian coordinate system

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}. \tag{2.20}$$

Cylindrical coordinate system

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}. \tag{2.21}$$

Spherical coordinate system

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}. \tag{2.22}$$

Expressing the above equation in different coordinate systems to take advantage of the symmetry of a charge distribution helps in solving problems related to electric potential. Laplace's and Poisson's equations are useful for solving problems when the potential and charge are known at some boundary and it is required to find the electric field and potential

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By APKK

Procedure for solving Poisson's or Laplace's equations

1. Solve the given Laplace's equation, $\nabla^2 V = 0$, or the Poisson's equation, $\nabla^2 V = \frac{\rho_V}{\varepsilon}$, to obtain potential by using integration either for a single variable or for multiple variables in a given coordinate system. The solution has two unknown integration constants.
2. Apply boundary conditions to determine the integration constants such that the solution for potential becomes unique.
3. Find the potential by substituting the integration constants.
4. Determine electric field intensity using $\vec{E} = -\nabla V$ and the electric flux density using $\vec{D} = \varepsilon \vec{E}$.
5. The charge induced on a conductor can be obtained from

$$Q = \int_S D \cdot dS = \int_S D_n \, dS$$

   where D_n is the normal component on the conductor.
6. The capacitance between two conductors can be obtained from $C = Q / V$.

Applications of Laplace's and Poisson's equations

(1) The surface charge density $\vec{D}$ and electric field intensity $\vec{E}$ at the given regions can be obtained directly without using Coulumb's or Gauss's law.

(2) By applying boundary conditions on Laplace's and or Poisson's equations, we can calculate the capacitance of different structures.

Capacitance :

It is denoted by 'c'

where $C = \frac{\text{charge } (Q)}{\text{Potential difference } (V)}$

units = Couloumbs/volts (or) Faraday.

$$C = \frac{Q}{V}$$

→ Q is the charges (+Q, −Q), the force acting b/w these charges (+Q, −Q) the electric field flows normally to the conducting surface area 's' and flux flow from +ve charge to −ve charge then the potential difference 'V' is generated.

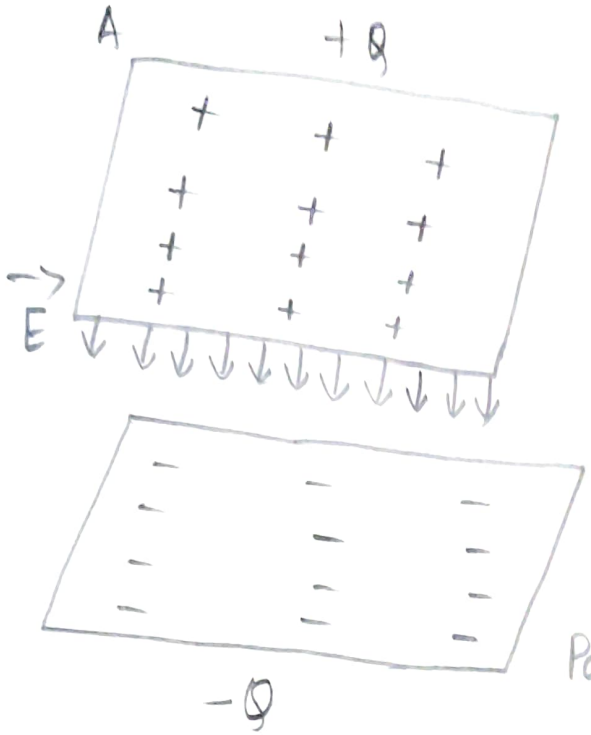
The potential difference across the plate is proportional to the charge on the plate i.e.,

$$Q \propto V \text{ (or) } Q = CV$$

where 'c' is proportionality constant called as capacitance

→ Capacitor consists of two plates A & B separated by the free space (or) dielectric media of thickness 'd' with Permittivity 'ε' of conducted surface area 's' is given by

$$C = \frac{Q}{V} = \frac{\varepsilon s}{d}$$



Parallel plate capacitor

The ratio of the charge (Q) ~~and~~ on the conductor to the Potential difference b/w the conductor is given by

$$C = Q/V = \frac{\varepsilon S}{d} = \text{faradays}$$

S = area of conductor

Conducting plates carries equal and opposite charges.

Derivation of parallel plates :-

Let us consider two parallel plates in the x and y planes Separated by the distance 'd' meter along x-axis. Let ε be the permitivity of the dielectric medium b/w the plates of surface area of the conductor 'S'.

If plate carry equal and opposite charges and electric field developed across the plates (or) b/w the plates. which can be obtained from electric flux density '$\vec{D}$' and electric field intensity '$\vec{E}$'

$$\boxed{\vec{D} = \rho_s \hat{a}_z \Rightarrow \frac{Q}{S}\hat{a}_z} \quad \text{—— (1)}$$

Parallely $$\boxed{\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{Q}{\varepsilon S}\cdot\hat{a}_z} \quad \text{—— (2)}$$

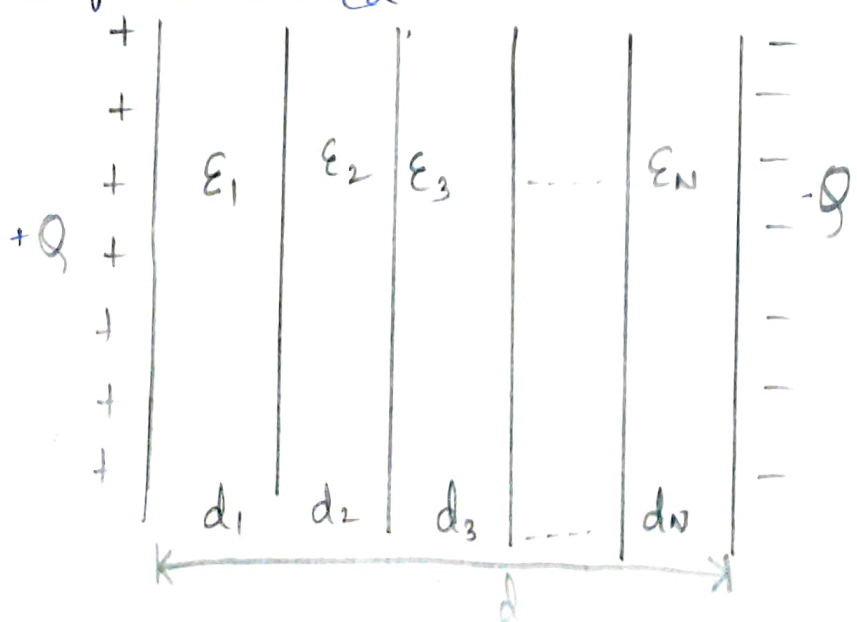
$$\vec{D} = \varepsilon\vec{E}$$

$$V = \int_l \vec{E}\cdot d\vec{l} = \frac{Q}{\varepsilon S}\hat{a}_z\cdot\hat{a}_z d = \frac{Qd}{\varepsilon S} = \frac{Q}{V} = C$$

Capacitors with different dielectric media:-

Capo Consider two parallel plates medium Capacitor separated by a distance 'd' of dielectric medium(d) as shown in figure

let $\varepsilon_1, \varepsilon_2, \varepsilon_3 \ldots \varepsilon_N$ be the permitivity of dielectric media/material of material thickness $d_1, d_2, d_3 \ldots d_n$ of same area



we know that potential difference 'V' =

$$V = \int_{l} \vec{v} \cdot dl$$

i.e,

$$\varepsilon_1 d_1 + \varepsilon_2 d_2 + \varepsilon_3 d_3 + \dots + \varepsilon_n d_n .$$

We also know that flux density $\vec{D} = \varepsilon \vec{E}$ $\quad \vec{E} = \frac{\vec{D}}{\varepsilon}$

$$D = \frac{Q}{S}$$

we know that $\vec{D} = \frac{Q}{S}$

$$\vec{E} = \frac{Q}{S\varepsilon}$$

Potential $V = \frac{D_1 d_1}{\varepsilon_1} + \frac{D_2 d_2}{\varepsilon_2} + \dots \frac{D_N d_n}{\varepsilon_n}$ — (2)

since $D = \frac{Q}{S}$ $\quad V = \frac{Q d_1}{S\varepsilon_1} + \frac{Q d_2}{S\varepsilon_2} + \frac{Q d_3}{S\varepsilon_3} + \dots \frac{Q d_N}{S\varepsilon_N}$

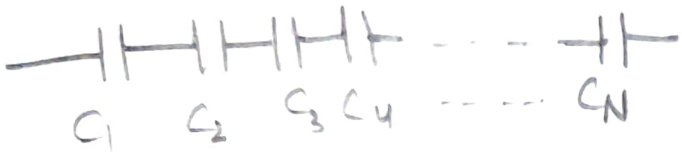
$$V = \frac{Q}{S}\left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} + \frac{d_3}{\varepsilon_3} + \dots + \frac{d_N}{\varepsilon_N}\right]$$

$$\frac{Q}{V} = \frac{S}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} + \frac{d_3}{\varepsilon_3} + \dots + \frac{d_N}{\varepsilon_N}} \quad \text{— (3)}$$

units → farad

Capacitors connected in series:-

If 'N' number of capacitors of different ~~directi~~ dimensions are connected in series as shown in figure.



The potential difference (V)

$$V = V_1 + V_2 + V_3 ---- V_N$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + -- \frac{Q}{C_N}$$

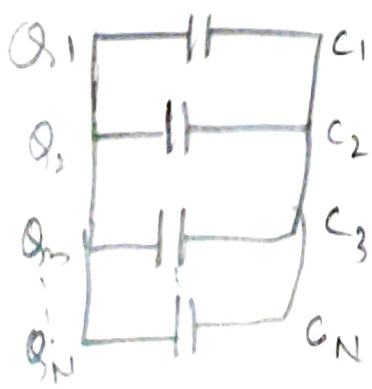
$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + -- + \frac{1}{C_N}\right]$$

$$\frac{1}{C} = \frac{V}{Q^2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ---- + \frac{1}{C_N}\right]$$

$$C = C_1 + C_2 + C_3 + -- C_N)$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + --- + \frac{1}{C_N}$$

Capacitors connected in parallel:- (with different dimensions)



$$Q = Q_1 + Q_2 + Q_3 + \dots + Q_N$$

$$Q = C_1 V + C_2 V + C_3 V + \dots C_N V$$

$$C = \frac{Q}{V} \Rightarrow C = C_1 + C_2 + C_3 + C_4 \dots C_N$$

if N capacitors are connected parallel as shown in fig. potential a/c is V. Then total charge on all the capacitor is

$$Q = Q_1 + Q_2 + Q_3 + \dots + Q_N$$

$$Q = C_1 V + C_2 V + \dots + C_N V$$

$$C = Q/V = C_1 + C_2 + \dots + C_N$$

where $C = C_1 + C_2 + \dots + C_N$

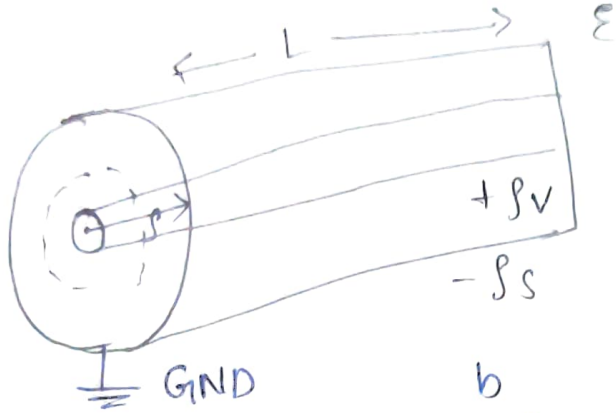
Note :- Total capacitance = sum of individual capacitors where they are connected parallely.

Capacitors of co-axial :-

Consider two co-axial conducting cylinder separated by a dielectric material of permitivity ε. The radius of inner conductor is "a" and other conductors is 'b' and height is 'L' as shown in fig.

→ Let ρ_s be the charge density on the inner conductor and $-\rho_s$ charge density on other conductor. Electric field intensity b/w conductor

$$E = \frac{\rho_s}{\varepsilon} \hat{a}_\rho \quad Q = \rho_s 2\pi r L$$



Potential difference $V = \int_a^b \vec{E} \cdot d\vec{l}$

$$= \int_a^b \frac{\rho_s}{\epsilon} \hat{a}_\rho \, d\rho \, \hat{a}_\rho$$

$$= \int_a^b \frac{Q}{2\pi\rho\epsilon L} d\rho = \frac{Q}{2\pi\epsilon L} \int_a^b 1/\rho \, d\rho$$

$$V = \frac{Q}{2\pi\epsilon L} \ln(b/a)$$

the capacitance is

$$C = Q/V = \frac{2\pi\epsilon L}{\ln(b/a)} \text{ F}$$

* Spherical capacitors:-

Capacitances b/w two concentric spheres consider two connecentric sphere of conductor with radius 'a' & 'b' where $b > a$. The sphere are filled with dielectric material of permitivity ϵ. The inner conductor carry a +ve charge and outer conductor carry a -ve charge -Q consider a gaussians spherical surface of radius 'r' denoted in fig are dotted line.

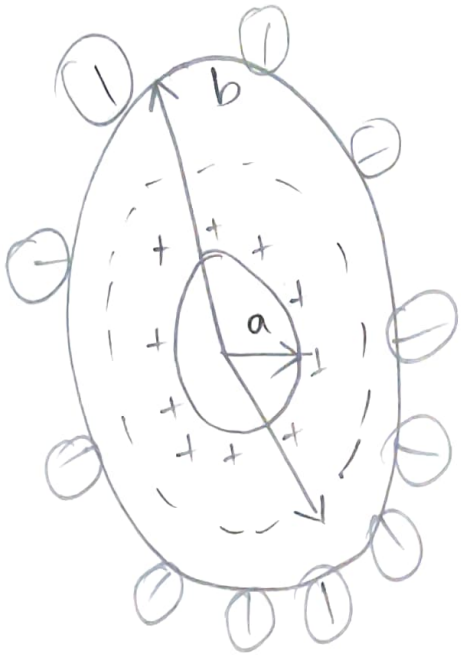


fig :- Capacitor 'b/w 2 concentric sphere

The field intensity of radial direction at radius 'r'

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r \ V/m \quad -(1)$$

the potential difference is

$$V = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r \cdot dr \cdot \hat{a}_r$$

$$V = \frac{Q}{4\pi\varepsilon}\int_a^b 1/r^2 \, dr.$$

$$V = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right) \Rightarrow \frac{Q}{V} = \frac{4\pi\varepsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)} = C$$

Note :- If there is only one sphere of radius 'a' carrying +ve charge Q its the outer sphere is at ∞ ie, b = ∞ substituting in eqn (2) then

$$= \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{\infty}\right)}$$

$$= \frac{4\pi\varepsilon}{\left(\frac{1}{a} - 0\right)}$$

$$= \frac{4\pi\varepsilon}{1/a}$$

$$C = 4\pi\varepsilon a \quad \text{--- (2)}$$

Note (2) :- capacitance of single isolated sphere radius 'a'

$C = 4\pi\varepsilon a$ (for single sphere)

$C = 4\pi\varepsilon\left(\frac{1}{a} - \frac{1}{b}\right)$ (for two solid sphere)