Transmission lines

Transmission line: The medium which is transfer energy or signal or power from one place to another place is called this arrangement is transmission line.

→ Thus a transmission line is a conductive method of guiding electrical energy from one place to another.

→ In Communications, these lines are used as a link between an antenna and a transmitter & a receiver.

Types of Transmission lines:

There are four types of transmission lines:

1. **Parallel wire type:**

→ It is common form of transmission line also known as open wire line because of its construction.

→ These lines consists of a pair of parallel conducting wires separated by a uniform distance as shown in fig.

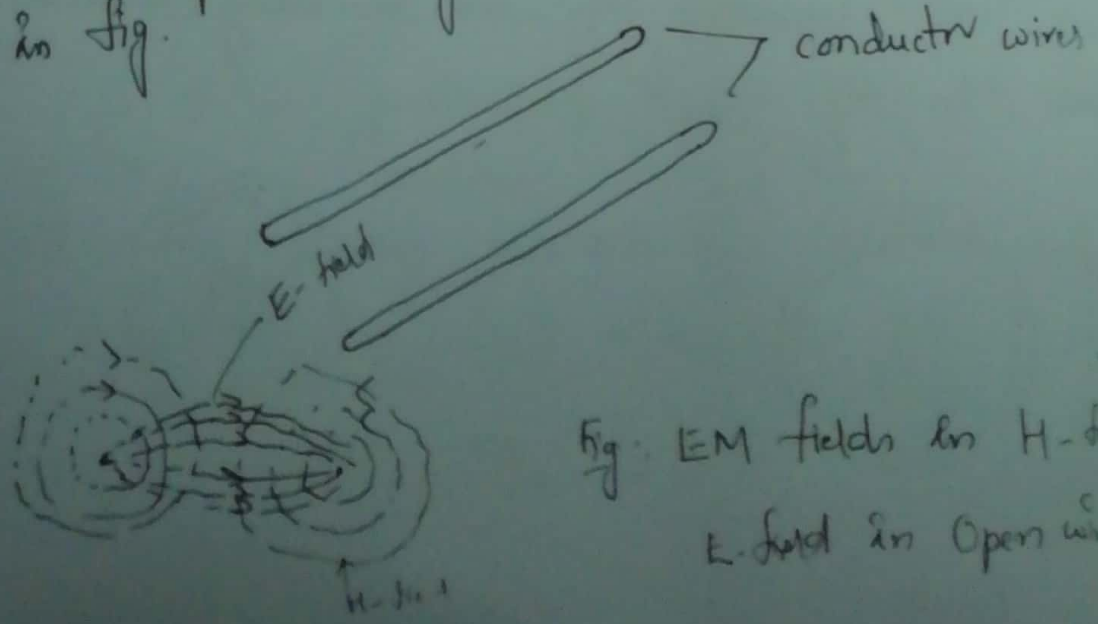


Fig: EM fields in H-field & E-field in Open wire lines.

up electric fields between line conductor.

→ This type of energy transmission is commonly known as TE mode of propagation.

Examples: Telephone lines, telegraphy line and power lines.

Advantages:

↳ Used as antenna feeders and impedance matching purpose.

↳ Easy to construct and very cheaper.

↳ Dielectric loss is very small because the insulation between line conductors is normally air.

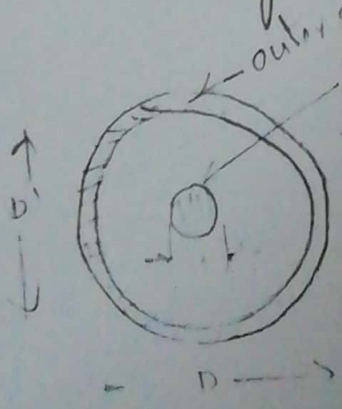
↳ Useful at the range up to 100 MHz.

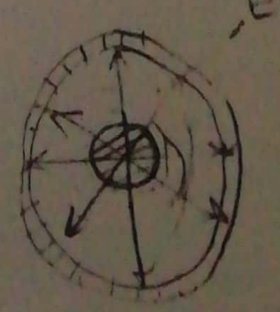
Disadvantage

↳ Significant energy loss due to radiation. Hence, these lines becomes unsuitable for frequencies above 100 MHz.

2. Coaxial Type :

→ Coaxial tube consists of two concentric conductors which are separated by an insulating material as shown in fig





→ In order to avoid the severe radiation le takes place in open wire transmission lines at frequencies beyond 100 MHz, a closed field configuration is employed in coaxial cable by surrounding.

→ Most of the coaxial cables use polyethylene as dielectric between then conductor.

Advantages:

1. No radiation losses, totally eliminated.
2. Suitable for above 100 MHz also.

Disadvantages:

1. Above 1 GHz not suitable.
2. Quite costlier compared to open wire lines.
3. losses in dielectric increases if the signal frequency increases.

Mathematical Analysis:

Let D – diameter of outer conductor.

d – diameter of inner conductor.

μ, ε – permiability & permitivity of insulating medium

Then, the primary constants of Coaxial cable is given

$$L = \frac{\mu}{2\pi} \log_e\left(\frac{D}{d}\right) \ H/m$$

$$C = 2\pi\varepsilon / \log_e\left(\frac{D}{d}\right) \ F/m$$

which may be fitted with a dielectric material and is used to guide electromagnetic waves is called Wave guide.

→ The transmitted wave is reflected back by the internal walls of wave guide and the resulting distribution associated with the wave causes the transmission mode.

→ Then wave guides used in UHF range.

4. Optical fibers :

→ Optical fibers are increasingly replacing wire transmission lines in communication system.

Advantages :

→ Superior transmission quality.

→ higher information carrying capacity.

→ Light weight and smaller size

→ Higher security

→ low attenuation.

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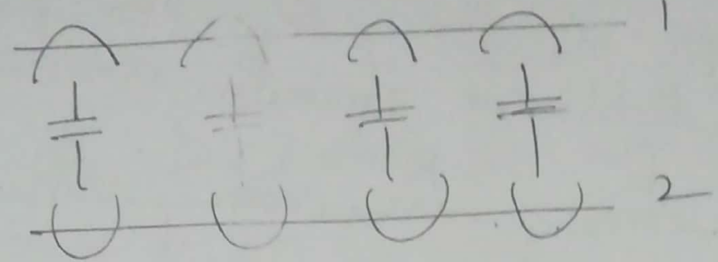
Fundamental Qu... (...



→ Let us consider two parallel uniform conductors is carrying current, there is magnetic field around conductors and voltage drop along them.

→ The magnetic field which is proportional to the current, indicates that the line has series induct L; the voltage drop indicates the presence of series resistance R.

→ Voltage applied across the conductor produces an electric field between the conductor and charges on them. This indicates that the line contains shunt capacitance C & since this capacity is never lossless or perfect, it will have some shunt conductance G, as well.

→ A unit section of line is represented in below fig.

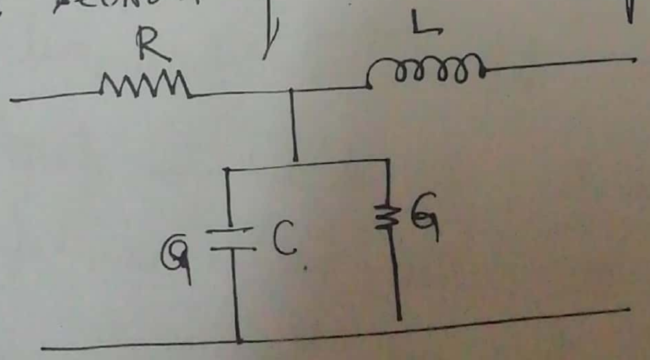
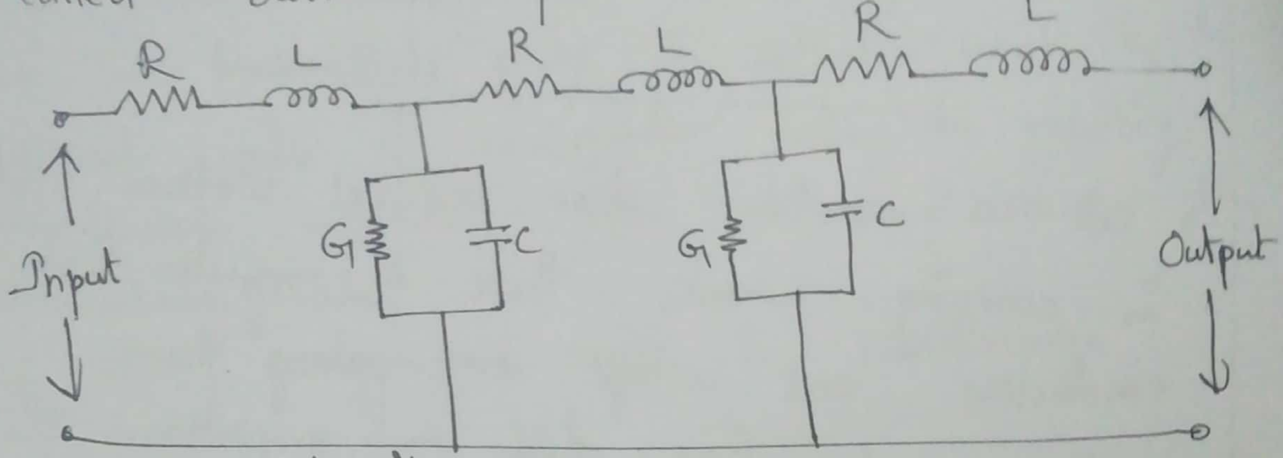


fig. Equivalent ckt of a unit length of Tx line

→ more,
the R, L, C and G are distributed along the whole length of line. So, these parameters are called distributed parameters.



|← One unit section |
(per unit length)

Fig: General Equivalent circuit.

→ So, the above parameters R, L, C and G are so called transmission line parameters.

Primary constants of Transmission Line:

The four line parameters R, L, C and G are termed as primary constants of the transmission line.

They are defined as follows:

1. Resistance R is defined as loop resistance per unit length line. Thus, it is sum of resistance of both wires for unit line length. Its units are Ohms per km.

per unit length line. Its units are ~~ohm~~ Henry per km (H/km)

3. <u>Conductance G</u> is defined as shunt conductance between the two wires per unit length. Its units ~~are~~ is mhos per km (℧/km)

4. <u>Capacitance C</u> is defined as shunt capacitance between two wires per unit length line. Its unit is Farad / km. (F/km).

<u>Note</u>: ① For the purpose of transmission line theory, the above primary constants (R, L, C & G) are ~~will be~~ assumed to independent of frequency.

② The series impedance Z and shunt admittance Y of the line per unit length are defined as

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

Transmission Line Equations

Let the line be for length l and the primary constants of the line R, L, C & G per km.

As already assumed that R, L, C & G are independent of frequency

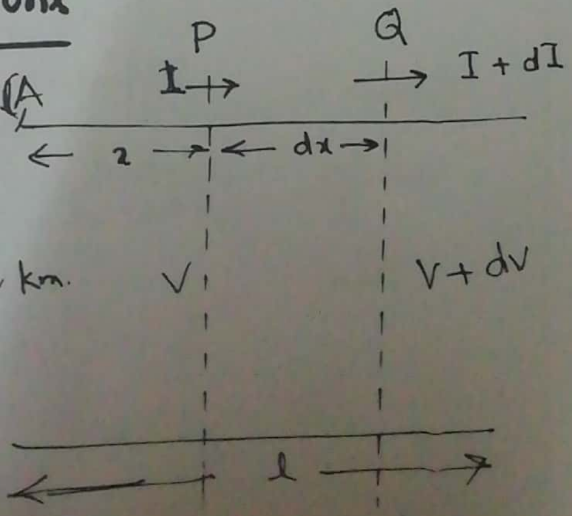


Fig: short section PQ distance x from the sending end of transmission line consider

at a distance from the sending point A as shown in fig.

At P, ~~the~~ let the voltage be V & current I

At Q, the voltage will be V + dv & current I+dI

The series impedance of small section dx will be

$$(R + JωL)\, dx.$$

lly, the shunt admittance of small section dx will be $(G + JωC)\, dx$.

The potential difference between P and Q is due to current flowing through series impedance $(R + JωL)\, dx$

i.e
$$V - (V + dv) = I\,(R + JωL)\, dx$$

$$-dv = (R + JωL)\, I\, dx$$

$$-\frac{dv}{dx} = (R + JωL)\, I \rightarrow (1)$$

Current difference between P & Q is due to voltage applied to shunt admittance $(G + JωC)\, dx$

Then,
$$I - (I + dI) = V\,(G + JωC)\, dx$$

$$-dI = (G + JωC)\, V\, dx$$

$$-\frac{dI}{dx} = (G + JωC)\, V \rightarrow (2)$$

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Eq^n (2) differentiate w.r.t x we get

$$-\frac{d^2V}{dx^2} = (R + J\omega L)\frac{dI}{dx}$$

$$-\frac{d^2V}{dx^2} = -(R + J\omega L)(G + J\omega C)V$$

$$\therefore \boxed{\frac{d^2V}{dx^2} = (R + J\omega L)(G + J\omega C)V} \rightarrow (3)$$

lly
$$-\frac{d^2I}{dx^2} = (G + J\omega C)\frac{dV}{dx}$$

$$-\frac{d^2I}{dx^2} = -(R + J\omega L)(G + J\omega C)I$$

$$\boxed{\frac{d^2I}{dx^2} = (R + J\omega L)(G + J\omega C)I} \rightarrow (4)$$

Let $(R + J\omega L)(G + J\omega C) = \gamma^2$

where γ is propagation constant.

Then Eq^n (3) & (4) becomes

$$\boxed{\begin{aligned} \frac{d^2V}{dx^2} &= \gamma^2 V \\ \frac{d^2I}{dx^2} &= \gamma^2 I \end{aligned}} \rightarrow (5)$$

of the transmission line.

Here, the propagation constant γ gives information about the variation of voltage and current w.r.t to distance along the line.

Then solution of transmission line equation (5) is,

$$V(x) = V^{+}(x) + V^{-}(x)$$

$$\boxed{V(x) = V_0^{+} e^{-\gamma x} + V_0^{-} e^{+\gamma x}} \rightarrow (6)$$

where V_0^+ is the complex amplitude in positive x- direction and V_0^- is the complex amplitude in $-x$ direction.

Illy

$$I(x) = I^{+}(x) + I^{-}(x)$$

$$\boxed{I(x) = I_0^{+} e^{-\gamma x} + I_0^{-} e^{+\gamma x}} \rightarrow (7)$$

The above Eq^n (6) & (7) called as transmission line equations.

The instantaneous expression for voltage can be expressed as

$$V(x,t) = Re\left[V_0(x)\, e^{j\omega t}\right]$$

$$= Re\left[V_0^{+} e^{-\gamma x} e^{j\omega t} + V_0^{-} e^{+\gamma x} e^{j\omega t}\right]$$

here $\gamma = \alpha + j\beta$

$$V(x,t) = Re\left[V_0^+ e \quad e \quad + V_0 \, e \right.$$

$$= V_0^+ e^{-\alpha x} \cos(\omega t - \beta x) + V_0^- e^{\alpha x} \cos(\omega t + \beta x)$$

$$\therefore \boxed{V(x,t) = V_0^+ e^{-\alpha x} \cos(\omega t - \beta x) + V_0^- e^{+\alpha x} \cos(\omega t + \beta x)} \rightarrow (8)$$

Ily,

$$\boxed{I(x,t) = I_0^+ e^{-\alpha x} \cos(\omega t - \beta x) + I_0^- e^{\alpha x} \cos(\omega t + \beta x)} \rightarrow (9)$$

Infinite line

Let us consider the hypothetical line of infinite length.

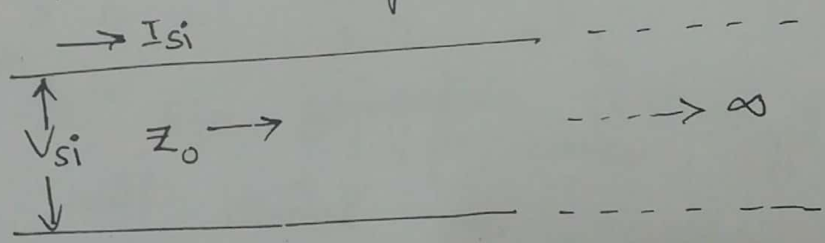


Fig. Infinite line.

A signal fed into a line of infinite length could not reach the far end in a finite time. Consequently, the condition of the far end can have no effect at the input end. For this reason transmission line analysis begins with an infinite line inorder to separate input conditions from output conditions.

When an A.C voltage is applied to the sending end of infinite line, a finite current will flow due

G between two of the

The input impedance of the infinite is ratio of the voltage applied to the current flowing the the line.

This input impedance is known as Characteristic impedance of the line and is denoted by Z_0

$$\therefore \quad Z_0 = \frac{V_{si}}{I_{si}} \longrightarrow (1)$$

Where V_{si} & I_{si} are sending end voltage & current of an infinite line. as shown in fig

Current at any point distance x from the sending end is given by

$$I = c e^{-\gamma x} + d e^{\gamma x} \longrightarrow (2)$$

The values of c and d can be determined by considering an infinite line

At the sending end of the infinite line,

$x = 0$, & $I = I_{si}$

then Eq^n (2) becomes,

$$I_{si} = c + d \longrightarrow (3)$$

However, at the receiving end of the infinite line, $x = \infty$ and $I = 0$ from Eq^n (2) become

$$0 = c \times 0 + d \times \infty$$

Since $e^{\infty} = \infty$ & $e^{-\infty} = 0$

Thus, either $d = 0$ or $\infty = 0$

But ∞ cannot equal to zero.

$$d = 0, \quad I_{s1} = C$$

$\therefore$ Eq^n (2) becomes

$$\boxed{I = I_{si}\, e^{-\gamma x}} \longrightarrow (4)$$

This eq^n (4) gives current at any point of an infinite line.

Ily the voltage at any point at infinite line can be deduced to be

$$\boxed{V = V_{si}\, e^{-\gamma x}} \longrightarrow (5)$$

Then, the def^n of propagation constant per unit length

$$e^{\gamma} = \frac{I_S}{I_1}$$

Where I_1 is the current at a unit distance from sending end.

Then, a distance x from the sending end, we have

$$e^{\gamma x} = \frac{I_S}{I_R} \longrightarrow (6)$$

Where I_R is the current at a distance x

But $\gamma = \alpha + j\beta$

Eq

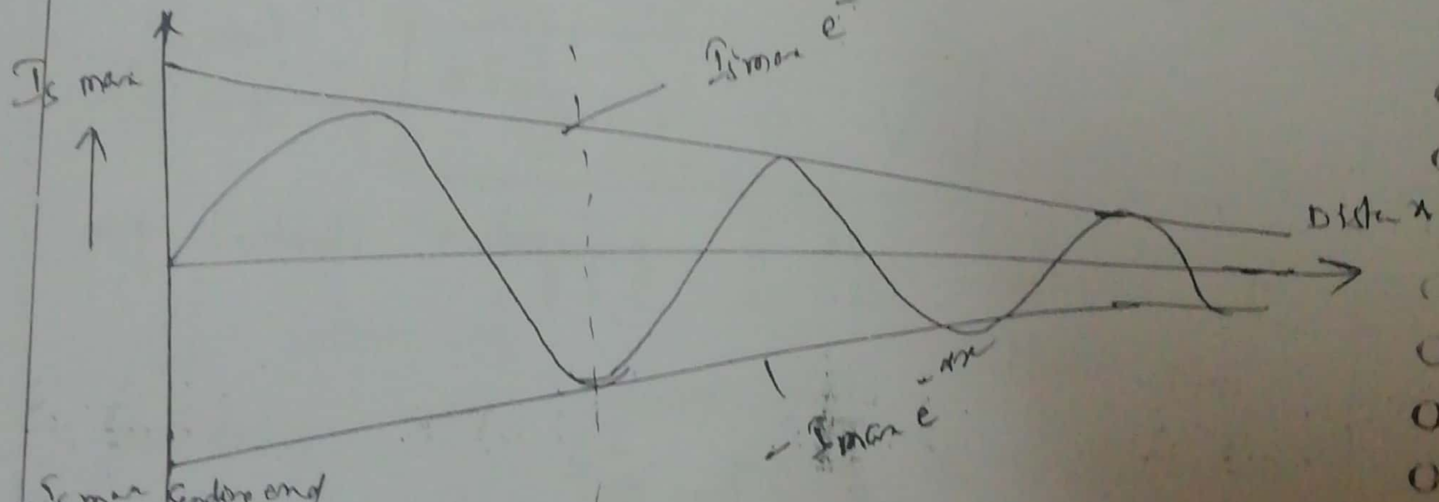
$$\frac{I_s}{I_R} = e^{\alpha x} \angle \beta x$$

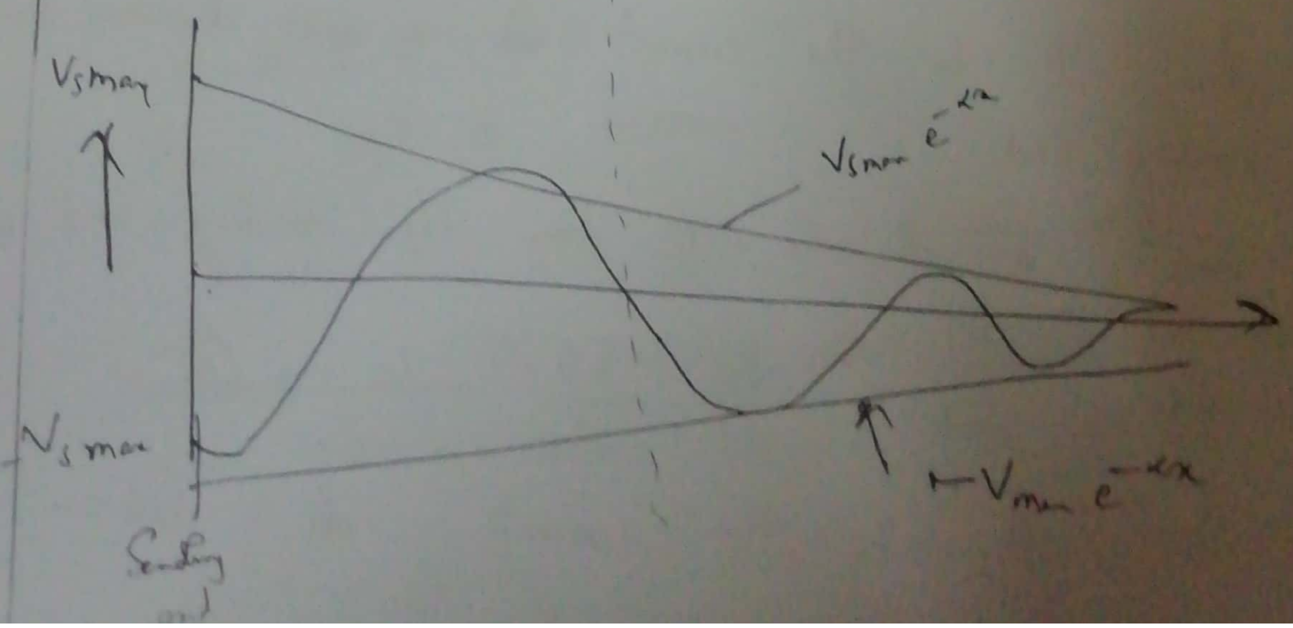
$$I_R = I_s e^{-\alpha x} \angle -\beta x \longrightarrow (7)$$

Eqⁿ (7) represents the current at a distance x down the line.

Ily, the voltage V_R at any point x down the line is given by

$$V_R = V_s e^{-\alpha x} \angle -\beta x \longrightarrow (8)$$

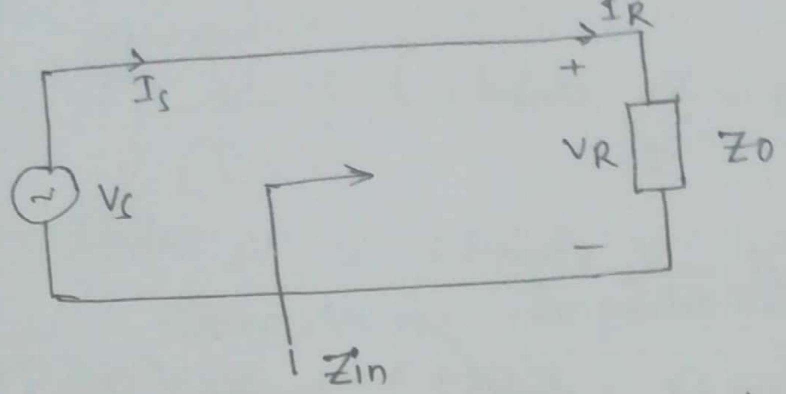




Tran ssion Line

Imp ance Z_0 :

Consider a line of length l which is terminate with characteristic impedance Z_0 as shown in fig.



Let V_R = voltage at termination

I_R = Current at "

V_s = Source voltage

I_s = Source current

According to transmission line equations

$$V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x \longrightarrow (1)$$

$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x \longrightarrow (2)$$

At distance $x = l$,

$$V = V_R, \quad I = I_R$$

$$\therefore V_R = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l \longrightarrow (3)$$

$$I_R = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l \longrightarrow (4)$$

$$Z_0 = \frac{V_R}{I_R} \longrightarrow (5)$$

$$\therefore Z_0 = \frac{V_S \cosh \gamma l - I_S Z_0 \sinh \gamma l}{I_S \cosh \gamma l - \frac{V_S}{Z_0} \sinh \gamma l}$$

$$Z_0 \left(I_S \cosh \gamma l - \frac{V_S}{Z_0} \sinh \gamma l \right) = V_S \cosh \gamma l - I_S Z_0 \sinh \gamma l$$

$$Z_0 I_S \cosh \gamma l - V_S \sinh \gamma l = V_S \cosh \gamma l - I_S Z_0 \sinh \gamma l$$

$$Z_0 I_S (\cosh \gamma l \cancel{+ \sinh \gamma l}) = V_S (\cosh \gamma l \cancel{+ \sinh \gamma l})$$

~~We know that~~

$$Z_0 I_S = V_S$$

$$\therefore \boxed{Z_0 = \frac{V_S}{I_S}} \longrightarrow (6)$$

But $\frac{V_S}{I_S} = Z_{in}$, is the input impedance of transmission line.

$$\boxed{Z_{in} = Z_0}$$

Hence, the input impedance Z_{in} of a finite length line terminated with its characteristic impedance Z_0 is equal to Z_0. Then when the line is matched i.e., the termination impedance is equal to its characteristic impedance Z_0

Two complex quantities γ and Z_0 are called as secondary constants of transmission line.

$\gamma \rightarrow$ propagation constant

$$= \sqrt{(R+J\omega L)(G+J\omega C)} = \sqrt{ZY}$$

$Z_0 \rightarrow$ Characteristic impedance

$$\therefore Z_0 = \sqrt{\frac{R+J\omega L}{G+J\omega C}} = \sqrt{\frac{Z}{Y}}$$

Calculation of Characteristic Impedance :

As mentioned earlier, the characteristic impedance is the input impedance of an infinite line.

We know that

$$* \; -\frac{dV}{dx} = (R+J\omega L) I \longrightarrow (1)$$

for infinite line $V = V_{si} e^{-\gamma x}$

$I = I_{si} e^{-\gamma x}$

then

$$-\frac{d}{dx}\left(V_{si} e^{-\gamma x}\right) = (R+J\omega L) I_{si} e^{-\gamma x}$$

γ V_{si}, e_i

$$\frac{V_{si}}{I_{si}} = \frac{\gamma}{R+J\omega L} \cdot \frac{R+J\omega L}{\gamma}$$

$$\frac{V_{si}}{I_{si}} = \frac{R+J\omega L}{\sqrt{(R+J\omega L)(G+J\omega C)}}$$

$$\frac{V_{si}}{I_{si}} = \sqrt{\frac{R+J\omega L}{G+J\omega C}}$$

But, by the defⁿ input impedance

$$Z_i = \frac{V_{si}}{I_{si}} = Z_0$$

~~Then~~, ∴

$$\boxed{Z_0 = \sqrt{\frac{R+J\omega L}{G+J\omega C}}}$$

Significance :

1. The line is terminated by its Z_0 behaves as an infinite line, means that ~~to~~ there is no reflection.
2. When a line is terminated in its characteristic impedance, it is said to be correctly terminated or properly terminated or non-resonant line.

The propagation constant per unit length of a uniform line may be defined as the natural logarithm of the steady state vector ratio of the current or voltage at any point, to that at a point unit distance further from the source, when the line is infinitely long.

$$\therefore \quad \gamma = \log_e \frac{I_1}{I_2}$$

$$\gamma = \log_e \frac{V_1}{V_2}$$

or

Current I_x at any point, distance x from the sending end is given

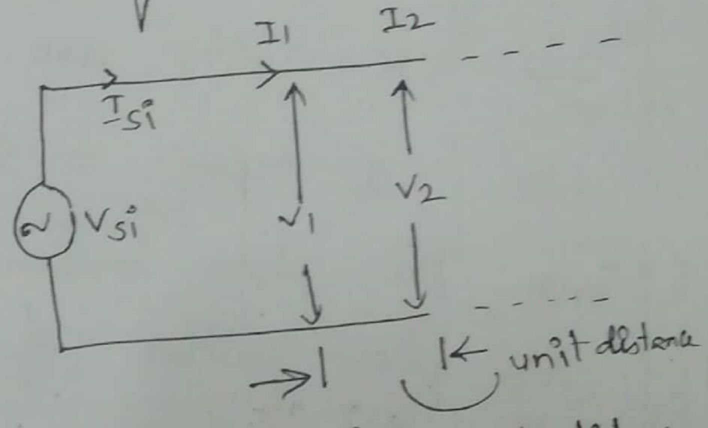


Fig: Voltage & I at unit distance apart.

$$I_x = I_{si}\, e^{-\gamma x}$$

$$e^{\gamma x} = \frac{I_{si}}{I_x}$$

Taking log of both sides $\quad \gamma x = \log_e \frac{I_{si}}{I_x}$

If I_{si} & I_x are unit distance apart, then

$x = 1$

$$\therefore \quad \boxed{\gamma = \log_e \frac{I_{si}}{I_x}} = \text{Propagation constant.}$$

→ In general, the propagation constant is usually a complex quantity and can be expressed as $\gamma = \alpha + j\beta$

α = attenuation constant

β = phase constant.

→ The attenuation constant α determines the reduction or attenuation in voltage and current along the line.

→ α units are neper/km.

→ 1 neper = 8.686 dB.

→ The phase constant β determines the variation in phase position of voltage and current along the line.

→ β units are rad/km.

→ 1 rad = 57.3°.

→ Phase constant when multiplied by the length of transmission line is termed as electrical length of line.

→ Attenuation constant when multiplied by the length of line is termed as total attenuation or simply line attenuation.

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} \rightarrow ①$$

Squaring both sides and equating real parts

$$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C) \rightarrow ②$$

$$\alpha^2 - \beta^2 + 2\alpha\beta j = RG - \omega^2 LC + j(\omega LC + R\omega C$$

then $\alpha^2 - \beta^2 = RG - \omega^2 LC \rightarrow ③$

We know that, the squaring magnitude of γ is

$$|\gamma|^2 = |\alpha + j\beta|^2 = \alpha^2 + \beta^2 = \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)} \rightarrow ④$$

Add Eq^n ③ & ④ we, get

$$2\alpha^2 = (RG - \omega^2 LC) + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}$$

$$\boxed{\alpha = \sqrt{\frac{1}{2}\left[(RG-\omega^2 LC) + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}\right]}} \rightarrow ⑤$$

Subtracting Eq^n ③ from Eq^n ④, we get

$$2\beta^2 = (RG - \omega^2 LC) - \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}$$

$$\beta = \sqrt{\frac{1}{2}\left[(RG-\omega^2 LC) - \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}\right]} \rightarrow ⑥$$

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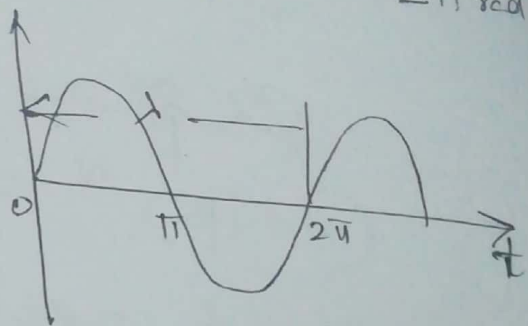
) <u>Wave length</u> (λ): Wave length λ is defined as the distance that a wave travels along the line inorder that the total phase shift is 2π rad.

By definition $\beta\lambda = 2\pi$

Where β is the phaseshift



$$\therefore \boxed{\lambda = \frac{2\pi}{\beta} \text{ meters}}$$

<u>Note</u>: In the case of cable with solid dielectric having dielectric constant K, the wavelength is very closely the free space wavelength, λ_f divided by $\sqrt{K}$, i.e

$$\lambda = \frac{\lambda_f}{\sqrt{K}}.$$

(b) <u>Velocity of Propagation</u> (or) <u>Phase velocity (v_p)</u>:-

<u>Definition</u> Velocity of propagation is defined as the velocity with which a signal of single frequency propagates along the line at a particular frequency f. It is denoted by v_p

→ This is velocity of propagation along the line based on the observations of change in phase along the line. That's why it is often referred to as phase velocity and wave velocity.

one cycle in time t and occurs at a distance of one wavelength λ, then

$$\lambda = v_p \times t$$

$$\lambda = v_p \times \frac{1}{f}$$

$$\therefore \boxed{v_p = \lambda f}$$

But we know that $\lambda = \frac{2\pi}{\beta}$

$$\therefore v_p = \frac{2\pi f}{\beta}$$

$$\boxed{v_p = \frac{\omega}{\beta}}$$

→ In free space, phase velocity is approximately equal to the velocity of light.

$$\therefore v_p = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8 \text{ m/sec}$$

→ In a medium with dielectric constant $\varepsilon = \varepsilon_0 \varepsilon_r$, the phase velocity reduced to

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{3\times 10^8}{\sqrt{\varepsilon_r}} \text{ m/s}$$

R + jωL G + jωC

$$\alpha^2 + \beta^2 = \sqrt{(RG-\omega^2LC) + j\omega(RC+LG)}^{\,2}$$

$$(RG-\omega^2LC)^2 + \omega^2(RC+LG)^2$$

→ In case of a distortionless or lossless line β not a constant multiple of ω. As a result of the components in a complex waveform normally shift in phase relation during propagation.

→ This phenomenon is known as dispersion which results in distortion.

→ When dispersion exists, the significant value of v_p is often difficult to define in complex wave.

→ Thus, in small dispersion, a significant velocity of propagation is group velocity.

→ So, the group velocity is defined as the velocity of the envelope of a complex signal. It is denoted by v_g.

→ The group velocity special importance in transmission of modulated waves, pulses & transmission through waveguides.

→ Mathematically, v_g is the ratio of change in angular frequency to the change in phase constant of the wave.

$$\therefore \quad v_g = \frac{\omega_2 - \omega_1}{\beta_2 - \beta_1} = \frac{d\omega}{d\beta} \text{ m/sec.}$$

here $\frac{d\omega}{d\beta}$ is evaluated at the carrier of centre frequency.

lationship b een group

We know that when a wave travels along a line, the phase velocity is,

$$v_p = \frac{\omega}{\beta}$$

Differentiating w.r.t to ω, we get

$$\frac{dv_p}{d\omega} = \frac{1.\beta - \omega\frac{d\beta}{d\omega}}{\beta^2} = \frac{\cancel{\beta}\left(1 - \frac{\omega}{\beta}\frac{d\beta}{d\omega}\right)}{\beta^{\cancel{2}}}$$

$$\frac{dv_p}{d\omega} = \frac{1 - \frac{\omega}{\beta}\frac{d\beta}{d\omega}}{\beta} \qquad \left(\because v_p = \frac{\omega}{\beta} \quad v_g = \frac{d\omega}{d\beta}\right)$$

$$\beta\frac{dv_p}{d\omega} = 1 - v_p \cdot \frac{1}{v_g}$$

$$\therefore \frac{v_p}{v_g} = 1 - \beta\frac{dv_p}{d\omega}$$

$$\therefore \boxed{v_g = \frac{v_p}{1 - \beta\frac{dv_p}{d\omega}}}$$

If v_p is constant w.r.t ω, then $\frac{dv_p}{d\omega} = 0$

$$\therefore \boxed{v_g = v_p}$$

* Thus if the phase velocity is independent of frequency, it becomes group velocity

Lossless transmission line

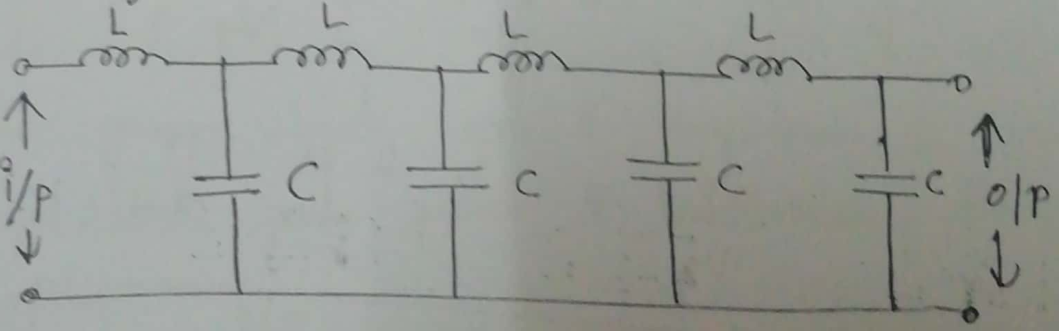
A transmission line is said to be lossless if at radio frequencies the inductive reactance (JωL) is much larger than series resistance R and capacitive reactance (Jωc) is also much larger than shunt conductance G.

Thus, in case of lossless line

$$\boxed{R = 0 \quad \text{and} \quad G = 0}$$

Condition for lossless transmission:

As per our above discussion, the equivalent circuit of a lossless line is shown below



We know that

$$\gamma = \sqrt{(R + J\omega L)(G + J\omega C)}$$

$$Z_0 = \sqrt{\frac{(R + J\omega L)}{G + J\omega C}}$$

By substituting $R = G = 0$

$$\gamma = \alpha + j\beta = \sqrt{(J\omega L)(J\omega C)} = j\omega\sqrt{LC}$$

Thus $\boxed{\alpha = 0 \quad \text{and} \quad \beta = \omega\sqrt{LC}}$

and $$Z_0 = \sqrt{\frac{\cancel{j}\omega L}{\cancel{j}\omega C}} = \sqrt{\frac{L}{C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Phase velocity $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

since $v_p = \frac{1}{\sqrt{\mu\varepsilon}}$

$$\therefore \boxed{LC = \mu\varepsilon}$$

This is the condition for lossless transmission Line.

Properties

1. For lossless line $\alpha = 0$ and $\beta = \omega\sqrt{LC}$.
2. The characteristic impedance is a real quantity. This shows that, lossless line is purely resistive.
3. The Z_0 depend only on L and C values.

If the transmission line (or) transmission medium is such that different frequencies travel with different velocities, then the line (or) the medium is said to be dispersive. In that case, the signals are propagated with a velocity known as group velocity.

Distortion...

"A line is said to be a distortionless line if the attenuation constant α is independent of frequency ω and β is linearly dependent on frequency."

Condition for distortionless transmission

We know that

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{L\left(\frac{R}{L} + j\omega\right) C\left(\frac{G}{C} + j\omega\right)}$$

$$\alpha + j\beta = \sqrt{LC}\sqrt{\left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)}$$

To make the α in the above equation independent of frequency, Let $\frac{R}{L} = \frac{G}{C}$

Then $\alpha + j\beta = \sqrt{LC}\sqrt{\left(\frac{R}{L} + j\omega\right)\left(\frac{R}{L} + j\omega\right)}$

$$\alpha + j\beta = \sqrt{LC}\left(\frac{R}{L} + j\omega\right)$$

Equating real and imaginary terms, we get

$$\boxed{\alpha = \frac{R}{L}\sqrt{LC} = R\sqrt{\frac{C}{L}}}$$

& $\boxed{\beta = \omega\sqrt{LC}}$

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$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L\left(\frac{R}{L} + j\omega\right)}{C\left(\frac{G}{C} + j\omega\right)}}$$

Let $\frac{R}{L} = \frac{G}{C}$

then $$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Therefore, for distortionless transmission

$$\boxed{\alpha = R\sqrt{\frac{C}{L}} = G\sqrt{\frac{L}{C}}}$$

or $$\boxed{\alpha = \frac{R}{Z_0} \text{ or } G Z_0}$$

and $$\boxed{\beta = \omega\sqrt{LC}} \; ; \; \boxed{v_p = \frac{1}{\sqrt{LC}}}$$

∴ The condition for distortionless transmission

is $$\boxed{\frac{R}{L} = \frac{G}{C}} \text{ or } \boxed{RC = LG}$$

Hence, in a distortionless line, the attenuation, characteristic impedance and phase velocity are independent of frequency. But β varies linearly with frequency.

We know that the attenuation constant α is given as

$$\alpha = \sqrt{\frac{1}{2}\left[(RG-\omega^2 LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}$$

The attenuation constant α depends on primary constants R, G, C and L of the line and the frequency of the wave.

To determine the primary constants at which the wave propagates with minimum attenuation on the line.

So, the variation of attenuation should be considered separately w.r.t to L, C, G and R.

(i) **Value of L for Minimum attenuation :-**

While determining the value of L for minimum attenuation, it will be assumed that the other three parameters R, G and C including ω are constant.

Then,

the minimum attenuation occurs at $\frac{d\alpha}{dL} = 0$

$$\frac{d\alpha}{dL} = \frac{1}{2}\frac{1}{\sqrt{\left[(RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]\times\frac{1}{2}}} \times$$

$$\frac{1}{2}\left[-\omega^2C + \frac{\cancel{2}\,\omega^2L(G^2+\omega^2C^2)}{\cancel{2}\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}\right] = 0$$

$$\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\frac{\cancel{\omega^2} L\,(G^2+\omega^2C^2)}{\sqrt{R^2+\omega^2L^2}\,\sqrt{G^2+\omega^2C^2}} = \cancel{\omega^2} C$$

$$L\sqrt{\frac{(G^2+\omega^2C^2)}{R^2+\omega^2L^2}} = C$$

$$L\cdot\sqrt{G^2+\omega^2C^2} = C\sqrt{R^2+\omega^2L^2}$$

Squaring both the sides, we get

$$L^2(G^2+\omega^2C^2) = C^2(R^2+\omega^2L^2)$$

$$L^2G^2 + \cancel{\omega^2L^2C^2} = R^2C^2 + \cancel{\omega^2L^2C^2}$$

$$L^2G^2 = R^2C^2$$

$$\therefore \boxed{LG = RC}$$

or

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

This condition is same as for distortionless line.

$\therefore$ The minimum attenuation occurs at

$$\boxed{L = \frac{RC}{G} \quad H/m}$$

Let the param capacitr C is var
remaining R, L G and ω are constant.

∴ The minimum attenuation occurs at $\frac{d\alpha}{dC} = 0$

Similarly, if the capacitor is varying, the minimum attenuation occurs at

$$\frac{R}{L} = \frac{G}{C}$$

$$\therefore \boxed{C = \frac{LG}{R}} \quad F/m$$

(iii) Values of R and G for Minimum attenuation:-

While keeping all other parameters constant, the minimum attenuation occurs at $\frac{d\alpha}{dR} = 0$ & $\frac{d\alpha}{dG} = 0$.

We know that

$$\alpha = \sqrt{\frac{1}{2}\left[(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}\right]}$$

It is observed that the attenuation $\alpha = 0$ occurs only when $R = 0$ and $G = 0$.

But, in practice, to get minimum attenuation with R and G, these values should be kept as small as possible.

Thus, a lossless line, $\alpha = 0$, is a distortionless line when $R = 0$ and $G = 0$.

"The line characteristics depends on the frequency the signal the presence of many co... nents cause variation in attenuation and phase ...ocity of the wave. This phenomenon is called ... distortion"

Types of line distortion:

In practice, there are two types of line distortions occuring on transmission lines.

1. Frequency distortion.

2. Delay distortion.

1. Frequency distortion: It occurs due to variation of attenuation with frequency. At the receiving end, the wave contains different amplitudes at different frequencies resulting in frequency distortion.

If the attenuation α is made independent of frequency, this distortion will not exist on transmission line.

2. Delay distortion:-

We know that for a line

$$\beta = \sqrt{\frac{1}{2}\left[(\omega^2 LC - RG) + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}\right]}$$

the β also varies with frequency

Now, the velocity v is given by $v = \frac{\omega}{\beta}$

T velocity of propagation, waves are with frequency. Hence, some waves will reach receiving end very fast while some waves will get delayed than the others. Hence, all frequencies will not have same transmission time. Thus, the output wave at the receiving end will not be exact replica of the input wave at the sending end. This type of distortion is called phase distortion or delay distortion.

This type of distortion is very serious in case of video and picture transmission.

The Remedy : Use coaxial cable for TV tran

Note : When v_p is independent of frequency, delay distortion doesnot exist on lines.

line by inserting inductance in series with line is called loading. This line is called load line."

In general, lumped inductors known as loading coils are placed at suitable intervals along the line to increase the effective distributed inductance.

For load line, the attenuation constant very low up to f_c.

It can be calculated as

$$\boxed{f_c = \frac{1}{\pi\sqrt{LCd}} \text{ Hz.}}$$

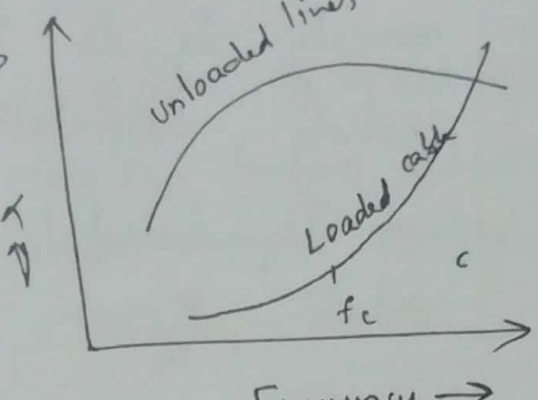


Fig: Effect of loading.

where L = inductance of load line

C = capacitance of the load line

d = length of loaded line in meters,

The cut-off frequency may be raised by decreasing the inductance per coil, or by spacing the coil closer together.

However, decreasing the inductance per coil will increase α in the transmission band, while spacing coil together increases the cost.

Types of Loading :- There are 3 types of loading in practice. i.e

1. Continuous loading.
2. Patch loading.
3. Lumped Loading.

→ It is seen earlier that if the primary of a line, mutually satisfies the relationship $RC = LG$ then the distortionless transmission.

→ But, for a practical line, $\frac{R}{G} >> \frac{L}{C}$ and hence the signal is distorted. Thus, the preventive remedy is to ~~take~~ make the condition $\frac{R}{G} = \frac{L}{C}$ satisfy artificially.

→ To satisfy the condition, it is necessary to reduce $\frac{R}{G}$ or increase $\frac{L}{C}$.

→ Let us consider all the possibilities.

(i) Reduce R : This will decrease attenuation but will require large conductors which in turn causes an increase ~~the~~ in cable size and cost. Reduction of R will also lower $|Z_0|$. Hence, this possibility is uneconomical.

(ii) Increase G : The shunt conductance can be increased by lowering the conductor insulation or by adding shunt conductance along the line. However, this process increases losses and lower $|Z_0|$. This is also worst advice to increase G in the line.

(iii) Decrease C : This will increase the spacing between conductor, resulting increase of cable size and cost. Hence, this possibility is uneconomical.

(iv) Increase L : This decreases α and reduces distortion and hence offers the best approach to achieve distortionless and minimum attenuation condition.

continuous loaded cable ...

nloaded cab

In this way, the cost loading ca reduced.

This sections are separated by 200 to 2 meters.

→ Patch loading is suitable for submarine cables.

3. **Lumped Loading :-**

→ The inductance of a line can be increased by the introduction of loading coils (i.e lumped inductors) of uniform intervals. This is called lumped loading.

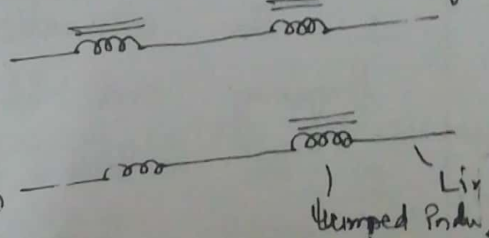
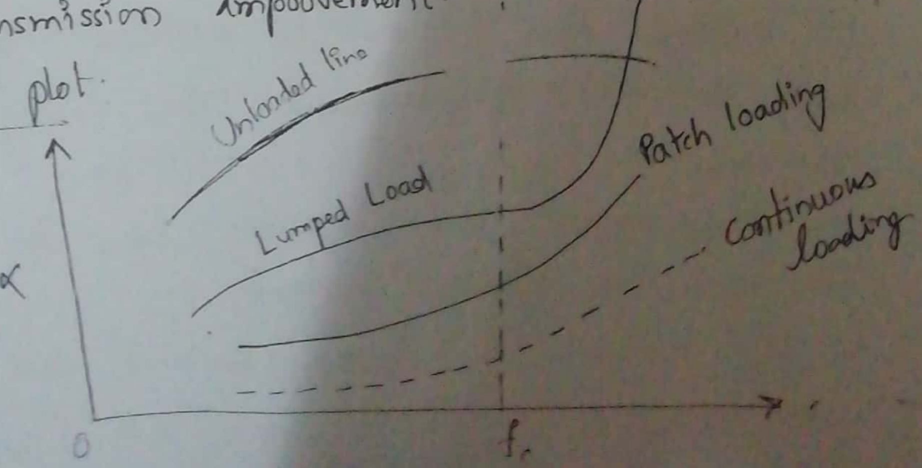


Fig: Lumped loading.

Advantages:

1. More convenient than continuous loading.
2. Produces the high inductance, having small dimensions.
3. Very low eddy current losses.
4. Preferred for open wire lines and cables for the transmission improvement.

parison plot.



→ this ... sheet of ... magnet ... as permalloy ... μ-metal) ... wound round ... cond... in fig

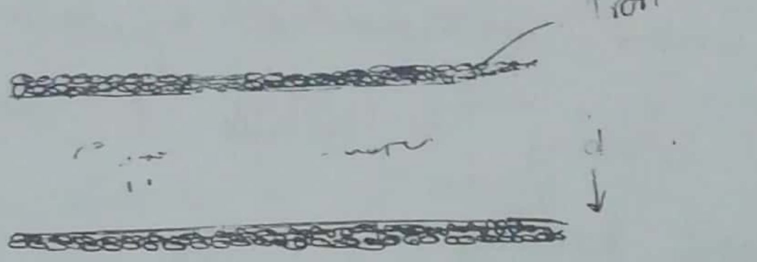


Fig: Continuous Loading

Advantages:

1. The attenuation to the signal is independent of frequency and it is same as to all the frequencies.
2. This method increases the permeability of the surrounding space, there by increasing the conductance up to 100 mH/km.
3. Cut-off frequency is very high compared to lumped loading.

Dis advantages:

This process is laborious and expensive.
2. Existing lines cannot be modified by this method. Hence, the total ~~rep~~ replacement of existing cables with new ones.
3. Eddy current and hysteresis losses in magnetic material increases the primary constant R.
4. This method of loading is not used for the landline but are preferred for Ocean cables (Submarine)

(1) ~~$\alpha = \sqrt{\frac{1}{2}\left[(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)}\right.}$~~

A signal is said to be distorted if the received signal is not the exact replica of the transmitted signal.

The line characteristics depends on the frequency of the signal, the presence of many components cause variation in attenuation & phase velocity of the wave. This phenomenon is called the Transmission line distortion.

__Types:-__

1. Distortion due to Z_0 varying with frequency:-

The characteristic impedance Z_0 of the line varies with frequency while the line is terminated in impedance which does not vary with frequency in similar fashion as that of Z_0. This causes the distortion. The power is absorbed at certain frequencies while it gets reflected for certain frequencies. So there exists the selective power absorption due to this type of distortion.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}}$$

if the condition $LG = CR$ is satisfied, then $L/R = C/G$

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{L/C}$$

For such line, Z_0 does not vary with frequency, ω. Thus line can be easily & correctly terminated in an impedance matching with Z_0 at all the frequencies.

This condition eliminates the selective power absorption & hence the distortion.

2. Frequency distortion :

It occurs due to variation of attenuation with frequency because it's a function frequency. All frequencies of the wave transmitting on line will not be attenuated equally. Hence, the received wave form will not be identical with the input waveform. This variation is known as frequency distortion.

If the attenuation α is made independent of frequency, this distortion will not exist on Tx line.

Condition for distortionless transmission

If a T/x line is to have neither frequency nor delay distortion (phase), then α & the velocity of propagation cannot be functions of frequency

① In view of the fact that $v_p = \omega/\beta$, then β must be a direct function of frequency.

Consideration β eqn

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

shows that if the term under the second radical be reduced to equal $(RG + \omega^2 LC)^2$, then the required condition on β is obtained.

$$(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2 = (RG + \omega^2 LC)^2$$

$$\cancel{R^2 G^2} - 2\omega^2 LCRG + \cancel{\omega^4 L^2 C^2} + \omega^2 L^2 G^2 + \omega^2 R^2 C^2 + 2\omega^2 LGRC = \cancel{R^2 G^2} + \cancel{\omega^4 L^2 C^2} + 2\omega^2 LCRG$$

$$\omega^2 L^2 G^2 + \omega^2 R^2 C^2 - 2\omega^2 LCRG = 0$$

$$L^2 G^2 + R^2 C^2 - 2LCRG = 0$$

$$(LG - RC)^2 = 0$$

$$\boxed{LG = RC}$$

This is the condition that will make β a direct function of frequency.

with this condition

$$\beta = \sqrt{\frac{\omega^2 LC - RG + RG + \omega^2 LC}{2}} = \sqrt{\frac{2\omega^2 LC}{2}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

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The velocity of propagation ... $v_p = \omega/\beta = \frac{\omega}{\omega\sqrt{LC}}$

$= \frac{1}{\sqrt{LC}}$

which is same for all frequencies, thus eliminating delay distortion.

② Consider $\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2(LG+CR)^2}}{2}}$

It will be independent of frequency. If the term under the internal radical is forced to reduce to $(RG + \omega^2 LC)^2$

Therefore the condition that will make 'α' ~~direct~~ independent of frequency is $\underline{\underline{LG = RC}}$

Applying condition

$\alpha = \sqrt{\frac{RG - \cancel{\omega^2 LC} + RG + \cancel{\omega^2 LC}}{2}} = \sqrt{RG}$.

which is independent of frequency, thus eliminating frequency distortion on the line.

To achieve the condition $LG = CR$ (or) $L/C = R/G$ requires a very large value of L, since G is small. If G is intentionally increased, α & the attenuation are increased, resulting in poor line efficiency. To reduce R, raises the size & cost of the conductors above economic limits, so that the hypothetical results cannot be achieved.

1. A Transmission line has the following ...

$R = 10.4\,\Omega$, $L = 3.666\,mH$, $C = 0.00835\,mF$

$G = 0.08\,M\mho$. Calculate Z_0, α, β & V_p at

$w = 5{,}000$ radians per sec.

$Z = R + jwL = 21.05\angle 60.4^\circ$

$Y = G + jwC = 41.77 \times 10^{-6}\angle 89.9^\circ$.

$Z_0 = \sqrt{\frac{Z}{Y}} = 224.5\angle -14.75$

$P = \sqrt{ZY} = 0.2961\angle 75.15)$

$= 0.007561 + j0.02863$

$= 0.007561$ neper/km.

$\alpha = 0.007561$ neper/km.

$\beta = 0.02863$ radian/kms

$V_p = \frac{w}{\beta} = \frac{5{,}000}{0.02863} = 1.746 \times 10^5$ km/sec