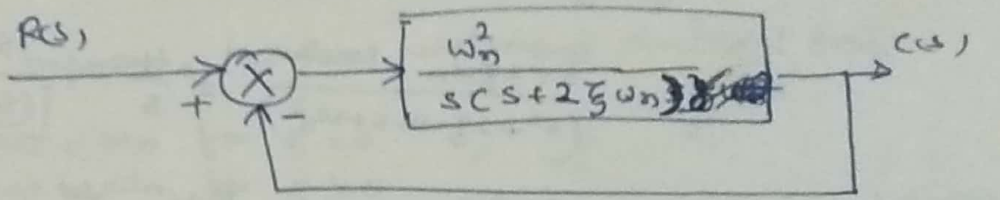
③ ①

Second order System

Response of second order system to the unit step i/p

Consider the second order system shown in fig 1 The closed loop transfer function $C(s)/R(s)$ of the system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \text{—} ①$$



where ξ = damping ratio

$\omega_n \rightarrow$ undamped natural freq

The characteristics eqn of TF is (denominator Polynomial $q(s)$)

$$q(s) = 0$$

or, $$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad \text{—} ②$$

the roots of characteristics eqn are given by

$$s^2 + 2\xi\omega_n s + \omega_n^2 = (s - s_1)(s - s_2)$$

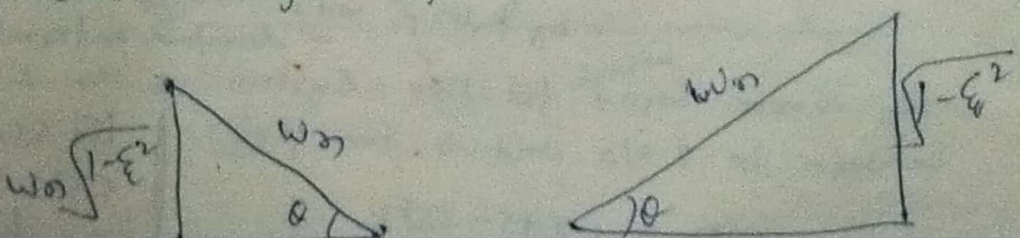
for $\xi < 1$ $\quad s_1, s_2 = -2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4\times\omega_n^2}/2$ $\qquad j = \sqrt{-1}$

$$s_1, s_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \quad \text{—} ③$$

$$= -\xi\omega_n \pm j\omega_d$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$ is called damped natural freq

Most of Control System with exception of robotic control system are design with damping factor $\xi < 1$. to have a high response speed.



Response of under damped System :- $(0 < \xi < 1)$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\xi\omega_n + j\omega_d)(s+\xi\omega_n - j\omega_d)}$$

For a unit step i/p $R(s) = 1/s$, Therefore above eqn

$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$

$$= \cancel{\frac{\omega_n^2}{(s+\xi\omega_n+j\omega_d)(s+\xi\omega_n-j\omega_d)}}$$

$$= \frac{1}{s} - \frac{s+2\xi\omega_n}{(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{1}{s} - \frac{s+2\xi\omega_n}{\left[(s+\xi\omega_n)^2+\omega_n^2-\omega_n^2\xi^2\right]}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

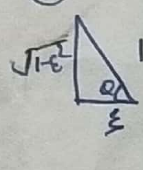
$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi}{\sqrt{1-\xi^2}}\cdot\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2} \qquad \therefore \omega_d = \omega_n\sqrt{1-\xi^2}$$

taking Laplace TF

$$C(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\omega_d t$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right) \quad -(4)$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\theta\cos\omega_d t + \cos\theta\sin\omega_d t\right] \quad -(5)$$



$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t + \theta)\right] \quad -(6)$$

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_n t\sqrt{1-\xi^2} + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] \text{ for } t \geq 0 \quad -(7)$$

$\therefore$ by putting $\omega_d = \omega_n\sqrt{1-\xi^2}$ and $\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

– damped natural freq

The error signal for this System is the difference between i/p & o/p and is from eqn (4)

$$e(t) = r(t) - C(t)$$

The Steady State value of $C(t)$ is given

$$C_{ss} = \lim_{t\to\infty} C(t) = 1$$

$$= e^{-\xi \omega_n t}\left[\cos \omega_d t + \frac{\xi}{\sqrt{1-\xi^2}} \sin \omega_d t\right], \text{ for } t \geq 0$$

The error signal exhibits a damped sinusoidal osc, At steady state or, $t = \infty$, no error exists between i/p & o/p.

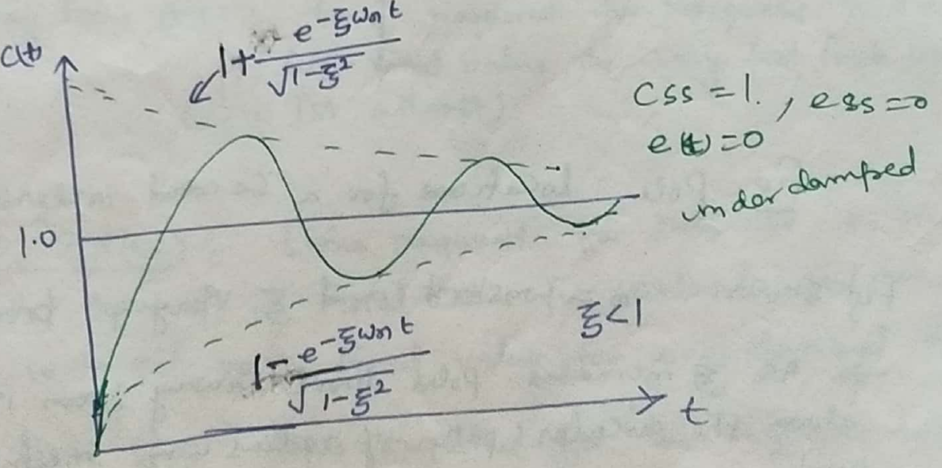
if $\xi = 0$, the response became undamped and oscillation continue indefinitely, the response $c(t)$ for the zero damping case is

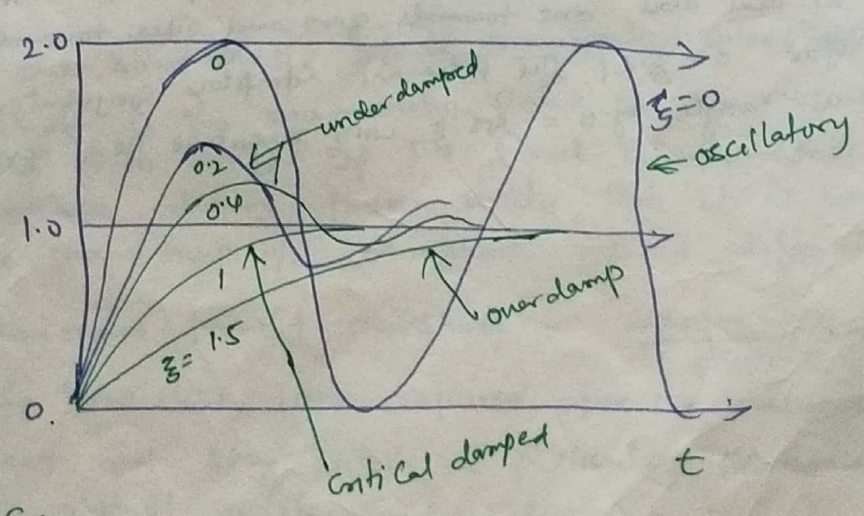
$$c(t) = 1 - \cos \omega_n t \quad \text{for } t \geq 0 \qquad \therefore \omega_n = \omega_d \text{ if } \xi = 0.$$

Thus ω_n represents the ~~undamp~~ undamped natural freq

$$\omega_d = \omega_n \text{ for } \xi = 0$$
$$\omega_d < \omega_n \text{ for } \xi > 0$$

The time response of an under damped ($\xi < 1$) second order system





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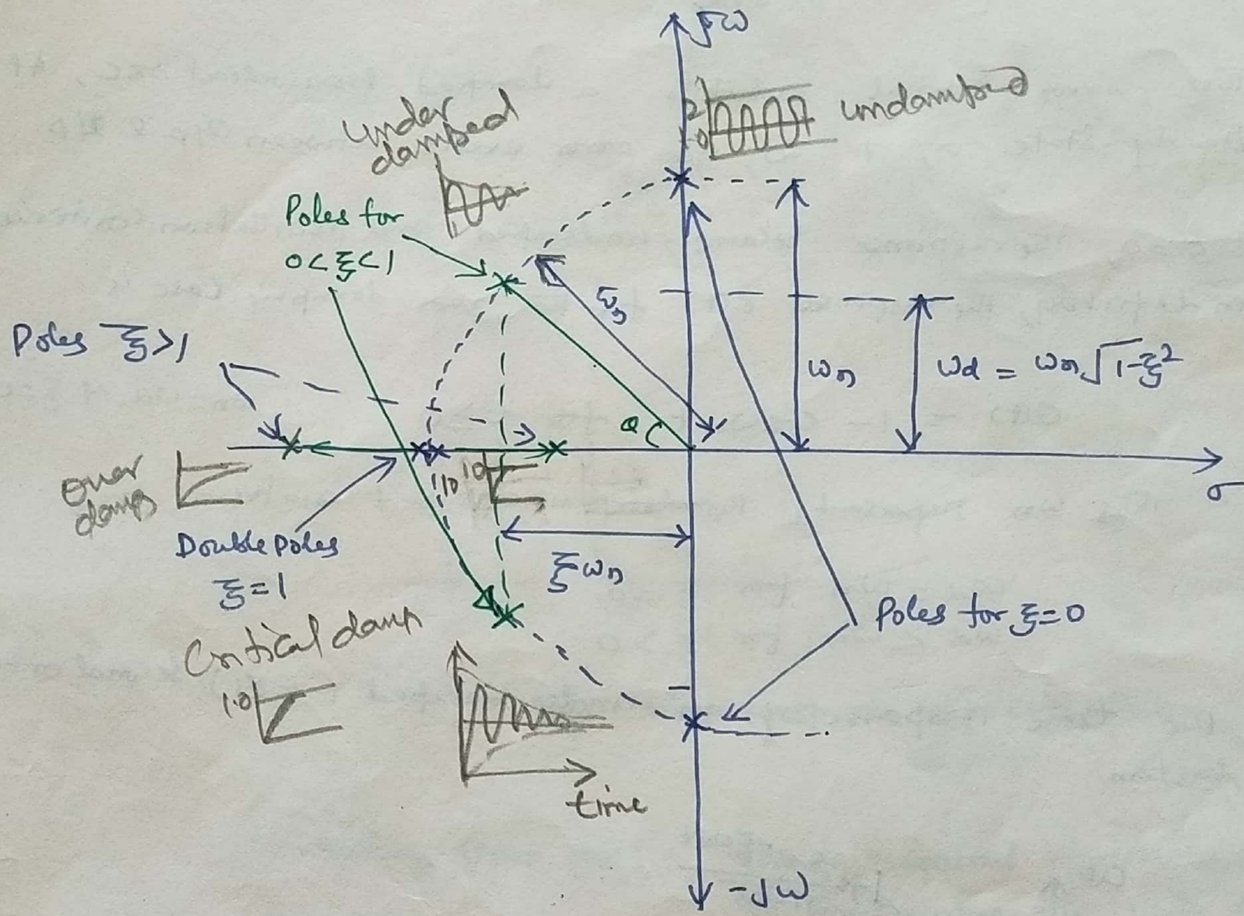


Fig Pole locations for a second order system

Fig shows ω_n = constant and ξ varying from 0 to ∞

→ As ξ increases poles move away from imaginary axis along the circular path of radius ω_n meeting at the point $\sigma = -\omega_n$ and then separating and travelling along the real axis, one towards zero and other towards infinity. For $0<\xi<1$, the poles are complex conjugate pair making an angle of $\theta = \cos^{-1}\xi$ with negative real axis

Time response Specification.

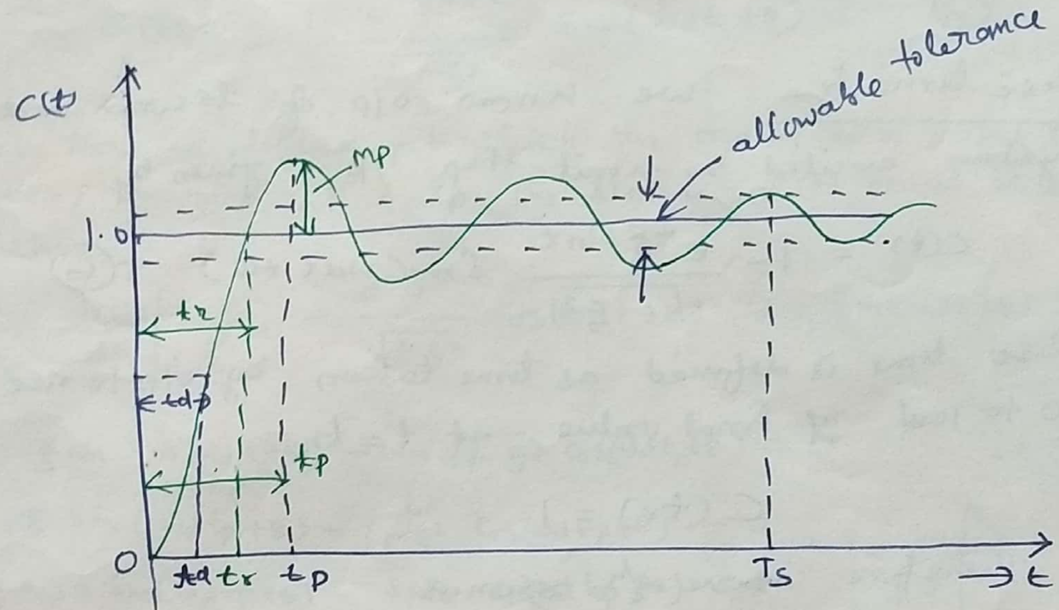


fig: time response specification

① Delay time (td): time required for response to reach 50% of final value the very ~~fast~~ first time (i.e. Ist attempt)

② Rise time (tr) :- time required for response to rise from 0 to 100% of final value for underdamped and ~~10-90%~~ from 10 to 90% of the final value for overdamped systems

③ Peak time (tp) time required for the response to reach the first peak of the overshoot.

④ Peak over shoot Mp: ~~it is~~ maximum overshoot or peak overshoot is the maximum value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum % overshoot. It is defined ~~as~~ by

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

⑤ Settling time (ts) Time required for the response curve to reach and stay within a particular tolerance band usually 2 to 5% of its final value

⑥ Steady State error (ess) It indicate the error between the actual o/p and the desired o/p as t tends to infinity

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$$

Rise time t_r — We know o/p of second order system excited by unit-step i/p is given by

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad \text{— (6)}$$

Rise time is defined as time taken by o/p to rise 0 to 100% of final value, at $t = t_r$

$$c(t_r) = 1$$

Therefore above eqn becomes

$$1 = 1 - \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta)$$

or,

$$\frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

$$\frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \neq 0$$ Then $\sin(\omega_d t_r + \theta)$ must be equal to zero. Therefore,

$$\sin(\omega_d t_r + \theta) = 0 = \sin \pi$$

$$\omega_d t_r + \theta = \pi$$

$$\omega_d t_r = \pi - \theta$$

$$t_r = \frac{\pi - \theta}{\omega_d}$$

Therefore, rise time

$$t_r = \frac{\pi - \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n \sqrt{1-\xi^2}}$$

for small value of t_r, ω_d must be large

Peak time (t_p) we know from eqn (6)

$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad -(6)$$

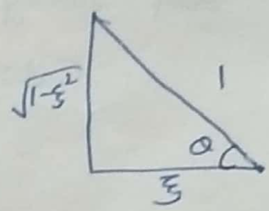
Peak time is defined at which the maximum value magnitude occurs, therefore, at $t = t_p$ the slope of $c(t)$ must be zero, Therefore,

$$\left.\frac{dc(t)}{dt}\right|_{t=t_p} = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t + \theta)\cdot \omega_d - \sin(\omega_d t + \theta)\cdot \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot(-\xi\omega_n)\Big|_{t_p} = 0$$

or, $\xi \omega_n \sin(\omega_d t_p + \theta) - \omega_n \sqrt{1-\xi^2} \cos(\omega_d t_p + \theta) = 0$

or, $\xi \sin(\omega_d t_p + \theta) - \sqrt{1-\xi^2}\cos(\omega_d t_p + \theta) = 0$

or. $\cos\theta \sin(\omega_d t_p + \theta) - \sin\theta \cos(\omega_d t_p + \theta) = 0$

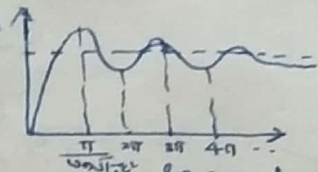


or, $\sin(\omega_d t_p + \theta - \theta) = 0 = \sin \pi$

$\sin(\omega_d t_p) = \sin \pi$ or $\sin n\pi$, $n = 1, 2, 3 \ldots$

or, $\omega_d t_p = n\pi$ $\quad n = 1$ for T_p

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$



The first under shoot occurs at $t = \frac{2\pi}{\omega_d}$, and second ~~under~~ over shoot occurs at $t = \frac{3\pi}{\omega_d}$, and so on.

Peak over shoot m_p The peak over shoot is the difference between the peak value and the reference input, Therefore

$$C(t_p) = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

$$M_P = c(t_p) - 1 = -\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

Substituting the value of $t_p = \pi/\omega_n\sqrt{1-\xi^2}$ in above eqn

$t_p = \pi/\omega_d$

$$m_p = -\frac{e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \frac{\pi}{\omega_d} + \theta\right)$$

$$= -\frac{e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}}(-\sin\theta)$$

$$= +\frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\cancel{\sqrt{1-\xi^2}}}\,\cancel{\sqrt{1-\xi^2}}) = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

therefore, Peak % over shoot $= \underline{100\, e^{-\pi\xi/\sqrt{1-\xi^2}}\ \%}$

Settling time t_s :- Settling time is given by time

~~$t_s = 4T$~~ =

required to settle down within $\pm 2\%$ of final value or,

t_s is time taken by o/p to reach 98% of final value and remains with $\pm 2\%$ as $t \to \infty$.

$$C(t)\Big|_{t=t_s} = 98$$

and oscillatory term completely vanishes the term which control the amplitude of the output within $\pm 2\%$ is $e^{-\xi \omega_n t}$

Hence t_s is obtained by considering only exponential ~~term~~ ~~only~~ decay envelope, neglecting all other terms.

$$\therefore \; C(t)\Big|_{t=t_s} = 1 - \frac{e^{-\xi\omega_n t_s}}{\sqrt{1-\xi^2}}$$

$$0.98 = 1 - \frac{e^{-\xi\omega_n t_s}}{\sqrt{1-\xi^2}}$$

or, $$e^{-\xi\omega_n t_s} = 0.02 \quad (\text{For low value of } \xi)$$

$$-\xi\omega_n t_s = \log_e(0.02)$$

$$t_s = \frac{3.912}{\xi\omega_n}$$

In practice settling time is assumed to be

$$\boxed{t_s = \frac{4}{\xi\omega_n}} \quad \text{for } 2\% \text{ tolerance}$$

$$T = \frac{1}{\xi\omega_n} = \text{time constant of the system}$$

t_s for 5% tolerance $= \dfrac{3}{\xi\omega_n}$

Summary

$$t_d = \frac{1+0.7\xi}{\omega_n} \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} \text{ sec}$$

$$t_r = \frac{\pi-\theta}{\omega_d} \text{ sec}$$

$$t_s = \frac{4}{\xi\omega_n} \text{ sec for } \pm 2\%$$

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

Steady state error (e_{ss}) we know that

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

for unit step $r(t) = 1$,

$$e_{ss} = \lim_{t\to\infty}(r(t) - c(t)) \quad \text{or,}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty}[r(t) - c(t)]$$

$$\lim_{t\to\infty}\left[\cancel{1} - \cancel{1} + \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)\right]$$

$$\lim_{t\to\infty}\left[\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)\right] = 0$$

Thus second order system has zero steady state error

$$\boxed{e_{ss} = 0}$$

for ramp i/p $r(t) = t$, $R(s) = 1/s^2$,

$$c(t) = L^{-1}\left[\frac{\omega_n^2}{s^2(s^2 + 2\xi\omega_n s + \omega_n^2)}\right]$$

$$= t - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t}}{\omega_n\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\,t + \theta\right]$$

$$e_{ss} = \lim_{t\to\infty}[r(t) - c(t)] = \frac{2\xi}{\omega_n} = \frac{1}{K_v}$$

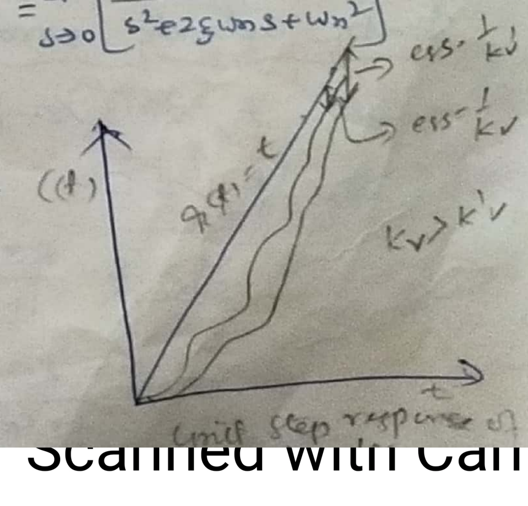
using final value theorm

$$e_{ss} = \lim_{s\to 0}[s\,E(s)] = \lim_{s\to 0} s[R(s) - C(s)]$$

$$\lim_{s\to 0} s\left[\frac{1}{s^2} - \frac{\omega_n^2}{s^2(s^2 + 2\xi\omega_n s + \omega_n^2)}\right]$$

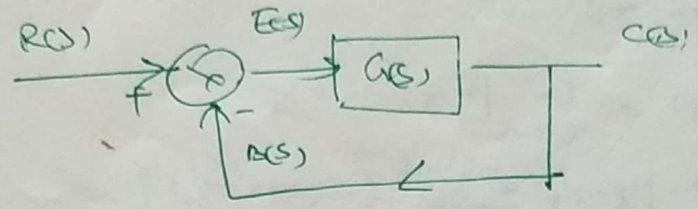
$$\lim_{s\to 0} s\left[\frac{s^2 + 2\xi\omega_n s + \cancel{\omega_n^2} - \cancel{\omega_n^2}}{s^2(s^2 + 2\xi\omega_n s + \omega_n^2)}\right] = \lim_{s\to 0}\left[\frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}\right]$$

$$= \frac{2\xi\omega_n}{\omega_n^2} = \frac{2\xi}{\omega_n} = \frac{1}{K_v}$$



Steady State Errors and Errors Constant :→

Let unity feedback system.



Here $\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)}$

$\therefore G(S)\,E(S) = C(S)$

$E(S) = \frac{C(S)}{G(S)}$

$C(S) = \frac{G(S)\,R(S)}{1+G(S)}$

$E(S) = \frac{R(S)}{1+G(S)}$

or $\frac{E(S)}{R(S)} = \frac{1}{1+G(S)}$

The steady state error ess may be found by using final value theorem.

$$e_{ss} = \underset{t\to\infty}{Lt}\ e(t) = \underset{s\to 0}{Lt}\ \frac{s\,R(S)}{1+G(S)}$$

ess depends upon R(S) & G(S).

Static Position Constant (K_p) the steady state error of the system for a unit step ($r(t)=1$, $R(S)=1/s$] is

$$e_{ss} = \underset{s\to 0}{Lt}\ s\,E(S) = \underset{s\to 0}{Lt}\ s\cdot\frac{R(S)}{1+G(S)}$$

$$= \underset{s\to 0}{Lt}\ \frac{s\cdot\frac{1}{s}}{1+G(S)} = \frac{1}{1+G(0)} = \frac{1}{1+K_p}$$

Where $K_p = \underset{s\to 0}{Lt}\ G(S) = G(0)$ is defined Position error constant

Static velocity error constant ess for unit ramp i/p $r(t) = t$, $R(S) = 1/s^2$] is

$$e_{ss} = \underset{s\to 0}{Lt}\ s\,E(S)$$

$$= \underset{s\to 0}{Lt}\ s\cdot\frac{R(S)}{1+G(S)}$$

$$= \underset{s\to 0}{Lt}\ s\cdot 1/s^2 \quad Lt\ \frac{1}{\quad} = \underset{s\to 0}{Lt}\ \frac{1}{s\,G(S)} = \frac{1}{K_v}$$

K_v :

$\omega_n = \sqrt{\frac{K_v}{\tau}} = \sqrt{\frac{K}{J}}$

$\xi = \frac{1}{2\sqrt{K_v \tau}} = \frac{f}{2\sqrt{KJ}}$

Static Acceleration error Constant :-

The steady state error with unit Parabolic input $\left[r(t) = \frac{t^2}{2},\ R(s) = 1/s^3\right]$ is

$$e_{ss} = \lim_{s\to 0} s\,E(s)$$

$$= \lim_{s\to 0} s \cdot \frac{R(s)}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{s \cdot \frac{1}{s^3}}{1+G(s)}$$

$$= \frac{1}{\lim_{s\to 0} s^2 G(s)} = \frac{1}{K_a}$$

where $K_a = \lim_{s\to 0} s^2 G(s)$ is defined as the acceleration error constant.

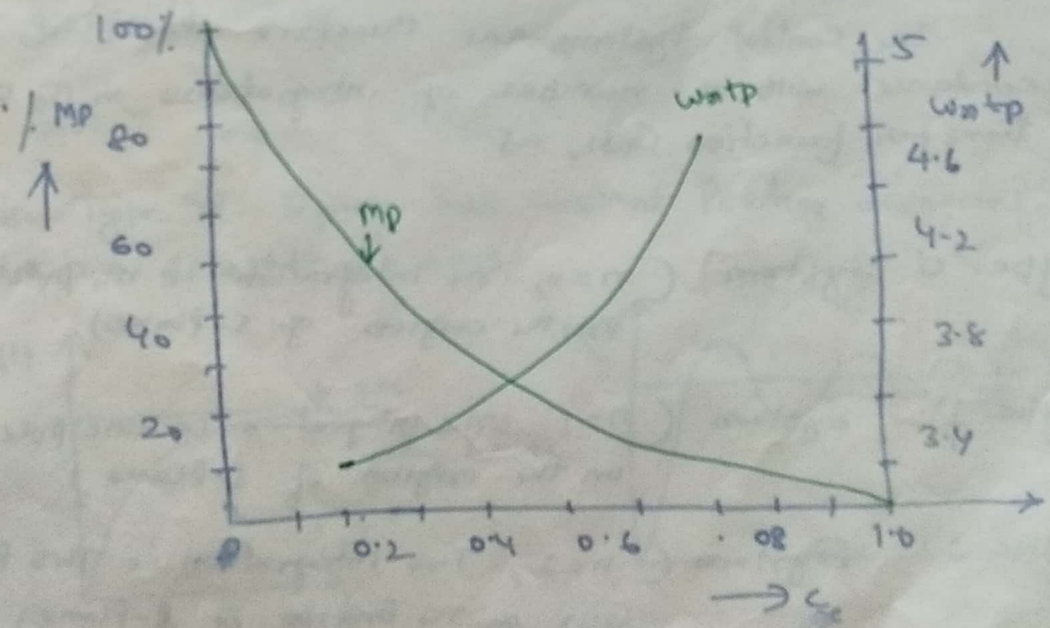
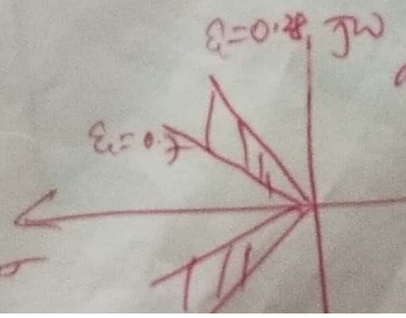


fig mp and $\omega_n T_p$ Versus ξ for a Second order Systems

$$M_p = 100\,e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\omega_n T_p = \frac{\pi}{\sqrt{1-\xi^2}}$$



desirable region for location of poles

of second order

Types of Control System :

The open loop Transfer function of a unity feedback system can be written in two standard forms: The Time constant form and Pole-Zero form. In these two forms, G(s) is given as follows:

$$G(s) = \frac{K(1+T_{z_1}s)(1+T_{z_2}s)\cdots}{s^n(1+T_{p_1}s)(1+T_{p_2}s)\cdots} \quad \text{(Time constant form)}$$

$$= \frac{K'(s+z_1)(s+z_2)\cdots}{s^n(s+P_1)(s+P_2)\cdots} \quad \text{Pole zero form}$$

The gain in the two forms are related by

$$K = K' \frac{\Pi_i z_i}{\Pi_j P_j}, \quad i = 1, 2, \cdots \quad j = 1, 2 \cdots$$

The term s^n in the denominator corresponds to the number of integrations in the system. As $s \to 0$ this term dominate in determing the steady state error.

The control systems are therefore classified in the accordance with the number of integrations in the open loop transfer function G(s), as

Type '0' System ($n=0$, no integration i.e no pole of G(s) on the origin of s-plane)

Type '1' System ($n=1$ one integration i.e one pole of G(s) on the origin of s-plane)

Type '2' System ($n=2$, Two integration i.e Two poles of G(s) on the origin of s-plane)

Steady State error : Type-0-System

For Type '0'

$$G(S) = \frac{K(1+T_{z_1}S)(1+T_{z_2}S)\cdots}{(1+T_{P_1}S)(1+T_{P_2}S)\cdots}$$

$$K_P = \lim_{S\to 0} \frac{K(1+T_{z_1}S)(1+T_{z_2}S)\cdots}{(1+T_{P_1}S)(1+T_{P_2}S)\cdots}$$

$$K_P = K$$

$$\therefore\ e_{ss} = \frac{1}{1+K_P} = \frac{1}{1+K} = \text{finite value}$$

error can be minimize by increase Gain K (forward path) but stability may effected.

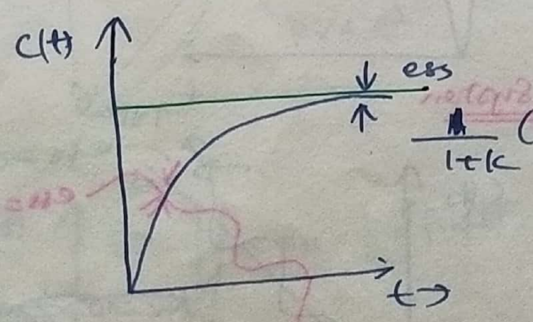
$$K_V = \lim_{S\to 0} S\,G(S) = \lim_{S\to 0} S \cdot \frac{K(1+T_{z_1}S)(1+T_{z_2}S)}{(1+T_{P_1}S)(1+T_{P_2}S)} = 0$$

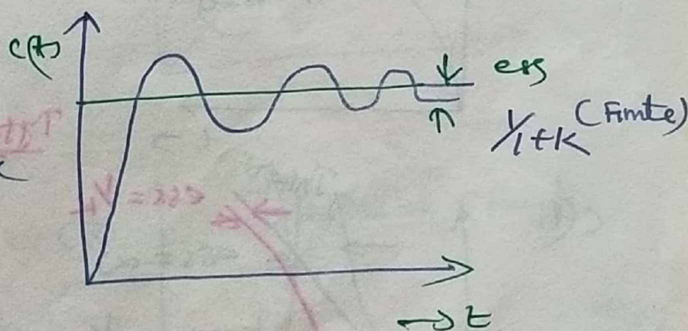
$$\therefore\ e_{ss} = \frac{1}{K_V} = \frac{1}{0} = \infty$$

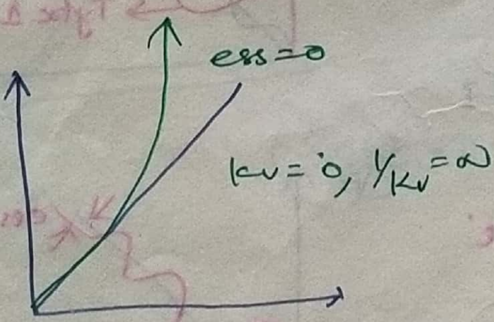
$$K_a = \lim_{S\to 0} S^2 G(S) = 0$$

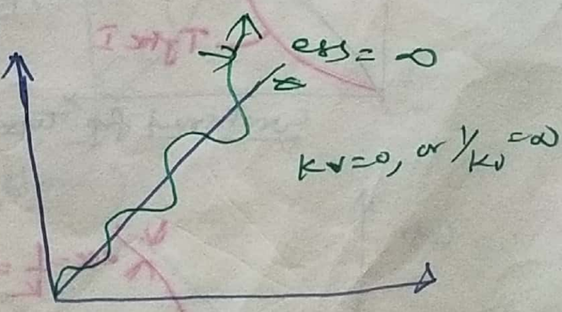
$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Thus Type '0' System has constant Position error, infinite velocity & accerlation errors.









Steady State error: Type 1 System

For type 1 - System

$$G(s) = \frac{K(1+T_{z_1}s)(1+T_{z_2}s)\ldots}{s(1+T_{p_1}s)(1+T_{p_2}s)\ldots}$$

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{K(1+T_{z_1}s)(1+T_{z_2}s)\ldots}{s(1+T_{p_1}s)(1+T_{p_2}s)\ldots} = \infty$$

$$\boxed{K_p = \infty}$$

$$\therefore \underset{(\text{Position})}{e_{ss}} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

$$K_v = \lim_{s\to 0} s\,G(s) = \lim_{s\to 0} \frac{K(1+T_{z_1}s)(1+T_{z_2}s)\ldots}{(1+T_{p_1}s)(1+T_{p_2}s)\ldots} = K$$

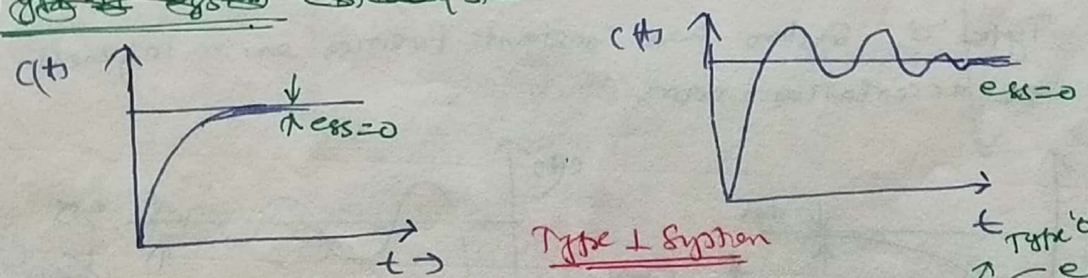
$$\therefore e_{ss}(\text{velocity}) = \frac{1}{K_v} = \frac{1}{K} = \text{finite value}$$

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} \frac{s(1+T_{z_1}s)(1+T_{z_2}s)K\ldots}{(1+T_{p_1}s)(1+T_{p_2}s)\ldots} = 0$$

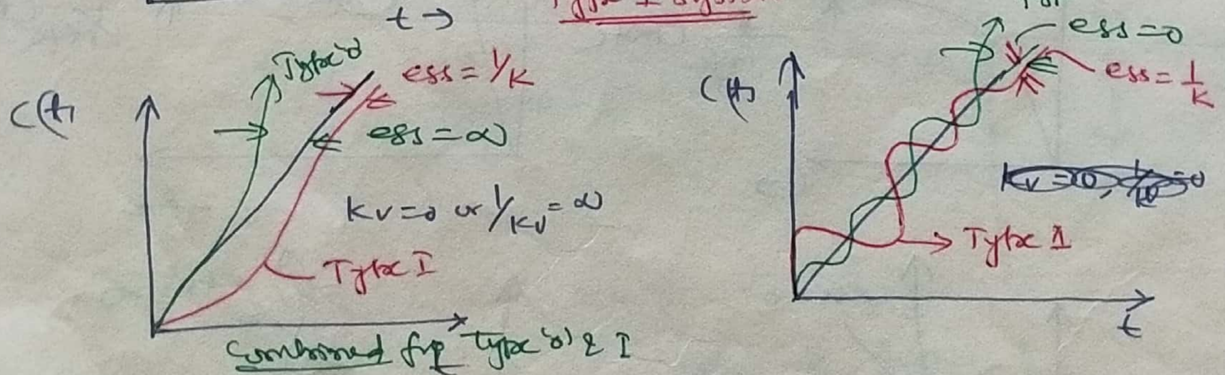
$$\therefore e_{ss}(\text{acceleration}) = \frac{1}{K_a} = \frac{1}{0} = \infty$$

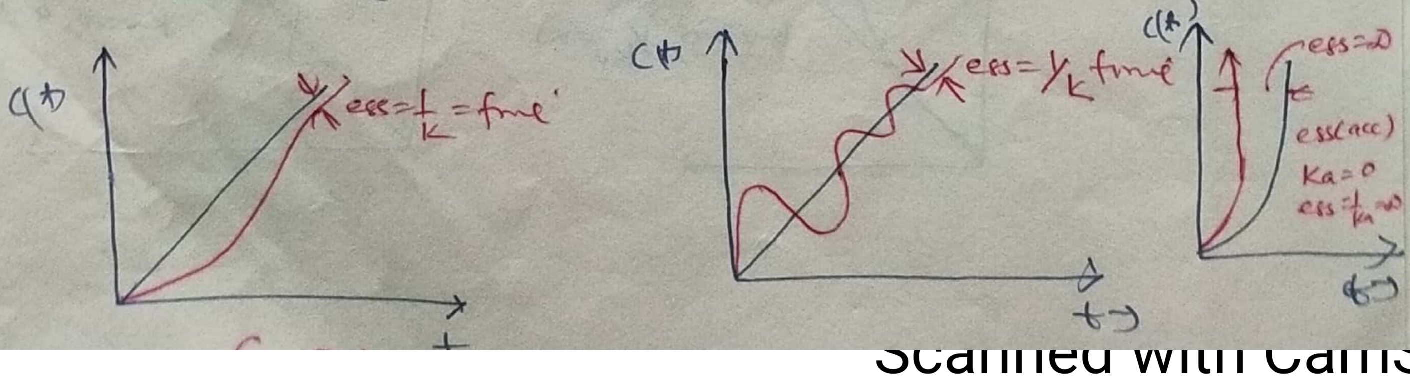
Thus the system with $n=1$, (Type-1) has zero position error, a constant velocity error and an infinite acceleration error at the steady state.

~~Type II system Esteady state~~



Type 1 System





Steady state error (Type-II-system) :-

For Type '2' system

$$G(s) = \frac{K(1+T_{z_1}s)(1+T_{z_2}s)\,----}{s^2(1+T_{p_1}s)(1+T_{p_2}s)\,----}$$

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{K(1+T_{z_1}s)(1+T_{z_2}s)\,---}{s^2(1+T_{p_1}s)(1+T_{p_2}s)\,----} = \infty$$

$$\therefore\ e_{ss}(\text{Position}) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

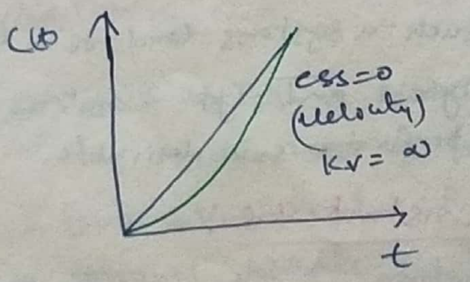
$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{K(1+T_{z_1}s)\,---}{s(1+T_{p_1}s)\,---} = \infty$$

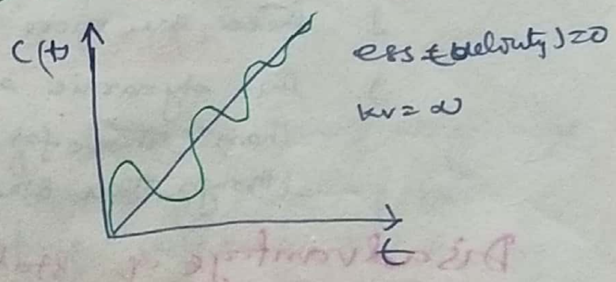
$$\therefore\ e_{ss}(\text{velocity}) = \frac{1}{K_v} = \frac{1}{\infty} = 0$$

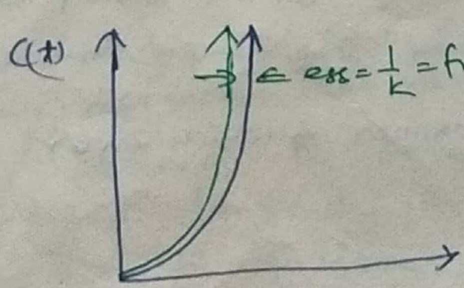
$$K_a = \lim_{s\to 0} s^2G(s) = \lim_{s\to 0} \frac{K(1+T_{z_1}s)(1+T_{z_2}s)\,---}{(1+T_{p_1}s)(1+T_{p_2}s)\,---} = K$$

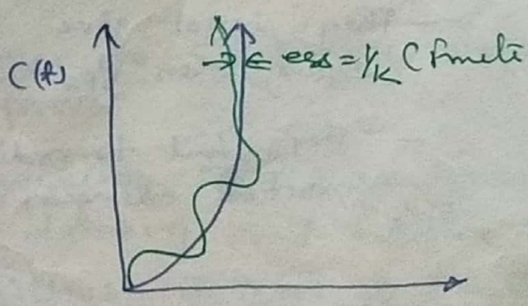
$$e_{ss}(\text{acceleration}) = \frac{1}{K_a} = \frac{1}{K} = (\text{finite})$$

thus, for $n=2$ (Type II) has zero Position error, zero velocity error and, and a constant acceleration error and it is given by K i.e. open loop gain of (forward Path) TF.









(Type-II System)

Steady State errors for various inputs and Systems :-

Types of input	Steady State error		
	Types-0-	Type-1	Type-2
unit Step	$\frac{1}{1+k_p}$ $k_p = \lim_{s\to 0} G(s) = K$	0	0
unit ramp	∞	$1/K_v$ $K_v = \lim_{s\to 0} sG(s) = K$	0
unit Parabolic	∞	∞	$1/K_a$ $K_a = \lim_{s\to 0} s^2 G(s) = K$

→ The error k_p, k_v and k_a describe the ability of a System to reduce or eliminate Steady State errors. Therefore They are indicative of Steady State performance.

→ As the type of System becomes higher, Progressively more Steady State errors are eliminated.

→ System higher than Type-II are not employed in practice because of two Reasons.

1. these are more difficult to Stabilize
2. The dynamic errors for Such a Systems tend to be larger than Those for Type-0, Type-I and Type-2 Systems. although their Steady State performance is desirable.

Disadvantage of Static error Constant are :

- They donot give any information on the Steady State errors, when i/ps are other than Step, Ramp & Parabola.
- they fail to indicate the manner in which error function changes with time.

Effect of Adding Poles and Zeros to Transfer function :-

Addition of Pole to the Forward path of Transfer function

- Addition of Poles increases the order of the system
- increases overshoot of closed loop system
- reduces stability
- increases rise time of step response
- reduces bandwidth

Addition of Pole to the closed loop Transfer function

- increases the rise time
- decreases overshoot
- as far as overshoot is concerned Adding a pole to closed loop TF has just opposite effect to that of adding a pole to the forward path TF

Addition of zero to the closed loop TF

- Decreases rise time
- increase maximum overshoot of step I/p.

Addition of a zero to the Forward path TF

- when zero is very far away from the imaginary axis, the overshoot is large and damping is very poor.
- Overshoot reduced and damping improves when the zero moves ~~.....~~ to the right.
- when the zero moves closer to origin, the overshoot increases but damping improves.

although the characteristics equation roots are generally used to study the relative damping and relative stability of linear control systems. the zero of TF should not be overlooked in their effects on the transient performance of the system.

Dominant Poles of Transfer function

The Poles which have dominant effect on the transient response is called dominant poles

or,

The poles that are close to the imaginary axis in the left s-plane give rise to transient responses that will decay relatively slowly and are called dominant poles whereas the poles that are far away from the imaginary axis (relative dominant poles) corresponds to fast decaying time responses and are called insignificant poles.

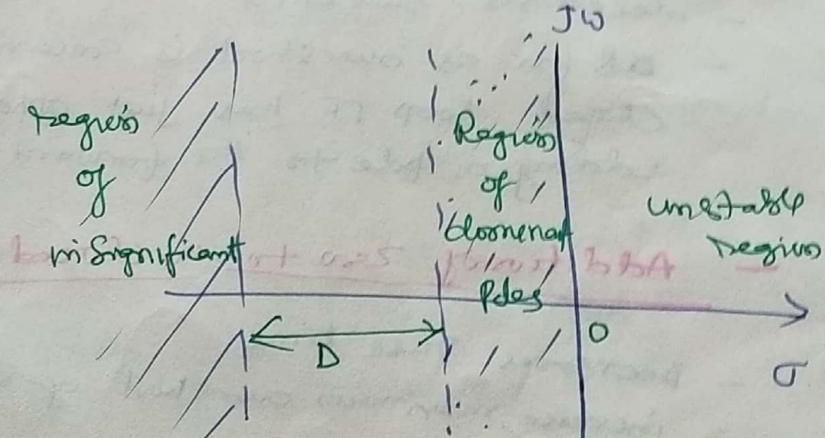


Fig regions of dominant & insignificant poles in s-plane

It has been recognized in practice and in the literature that if the magnetude of real part of a pole is at least 5 to 10 times that of the dominant poles or pair of complex dominant poles, the pole may be regarded as insignificant in sofar as the transient response is concerned.

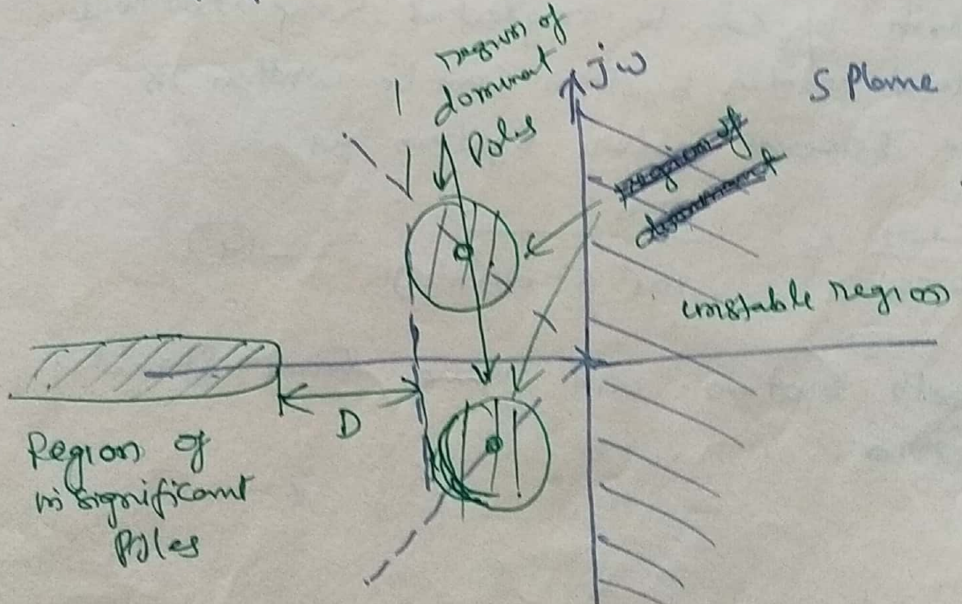
Relative damping ratio :-

When a system is higher than second order, we can no longer strictly use the damping ratio ξ and the undamped natural frequency ω_n, which are defined for prototype second order systems. However, if the system dynamics can be accurately represented by a pair of complex conjugate dominant poles, then we can still use ξ and ω_n to indicate the dynamics of the transient response and damping ratio in this case is referred to as the relative damping ratio of the system.

For example, consider a closed loop TF

$$M(s) = \frac{C(s)}{R(s)} = \frac{20}{(s+10)(s^2+2s+2)} \quad \text{—— ①}$$

The poles at $s=-10$, is ten times the real part of complex conjugate poles, at $-1 \pm j1$, we can refer relative damping ratio of the system as 0.707



Here in eqn ①

$\therefore 2\xi\omega_n = 2$

$\therefore \omega_n = \sqrt{2}$

$\xi = 0.707$

$= \frac{1}{\sqrt{2}}$

fig: regions of dominant and insignificant poles in the S-Plane.

Proper way of neglecting insignificant Poles with consideration of steady state response

If the insignificant Poles are neglected as they are the the transient response is unchanged but the steady state response gets affected.

Consider the following closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{20}{(s+10)(s^2+2s+2)} \quad \text{— ①}$$

neglecting the pole far away from the origin, the 3rd order system can be approximated by the second order system. with TF

$$\frac{C(s)}{R(s)} = \frac{20}{s^2+2s+2} \quad \text{— ②}$$

The transient response of above two system will be same but their steady state responses will be different. To maintain the same steady state response, the TF can be written as

$$\frac{C(s)}{R(s)} = \frac{20}{10(s/10+1)(s^2+2s+2)} \quad \text{— ③}$$

The term $s/10$ can be neglected compared to 1.

So the third order system can be written in term of second order system as

$$\frac{C(s)}{R(s)} = \frac{20}{10(s^2+2s+2)} \quad \text{— ④}$$

Now both system will have same steady state error.