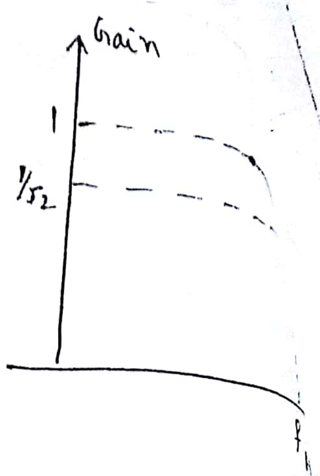
$$|A_v| = \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}}$$



Case - ① , $f = 0$, $|A_v| = 1$

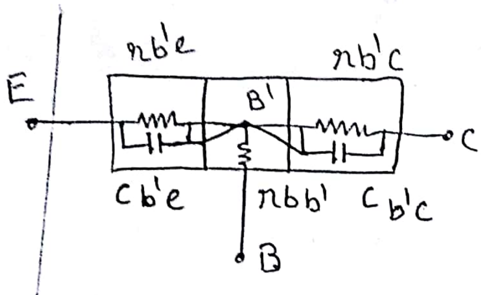
Case - ② , $f = f_H$, $|A_v| = \frac{1}{\sqrt{2}}$

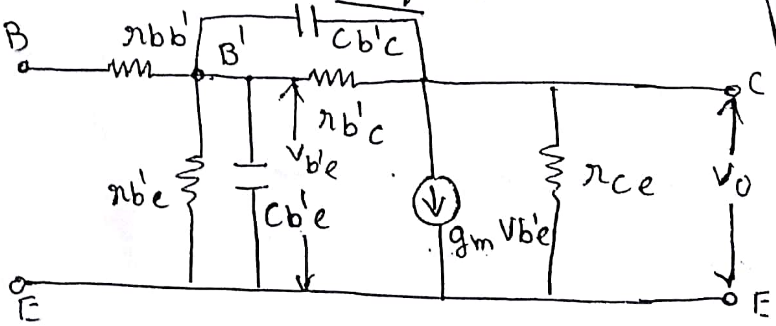
Case - ③ , $f > f_H$, $|A_v|$ = Low

Case - ④ , $f >> f_H$, $|A_v|$ = very low.

→ The freqn at which has $1/\sqrt{2}$ times of maximum value then it is upper cut off frequency.

High frequency response of BJT Amplifier:-





→ Forword biased PN junction exhibits a cappacitive effect called diffusion capacitance. $C_{b'e}$ → diffusion capacitor of Emitter junction
Value = 100 pF

→ Reverse biased PN junction exhibits a capacitive effect called Transition capacitance. $C_{b'c}$ → Transition capacitance of collector junction
Value = 3 pF

→ $r_{bb'}$ – Base Spreading resistance. Its typical value is 100Ω. It is a base series resistance between the external base terminal B and internal base region B'.

$r_{b'e}$ - resistance between the virtual base B' and the emitter terminal E. Typical value is 1 KΩ.

Input resistance from base to emitter with the output shorted is simply $r_{bb'} + r_{b'e}$ and this is the same as h_{ie}. Hence $h_{ie} = r_{bb'} + r_{b'e}$.

$r_{b'c}$ - resistance between the virtual base B' and the collector terminal C. It has a large value = 4 MΩ.

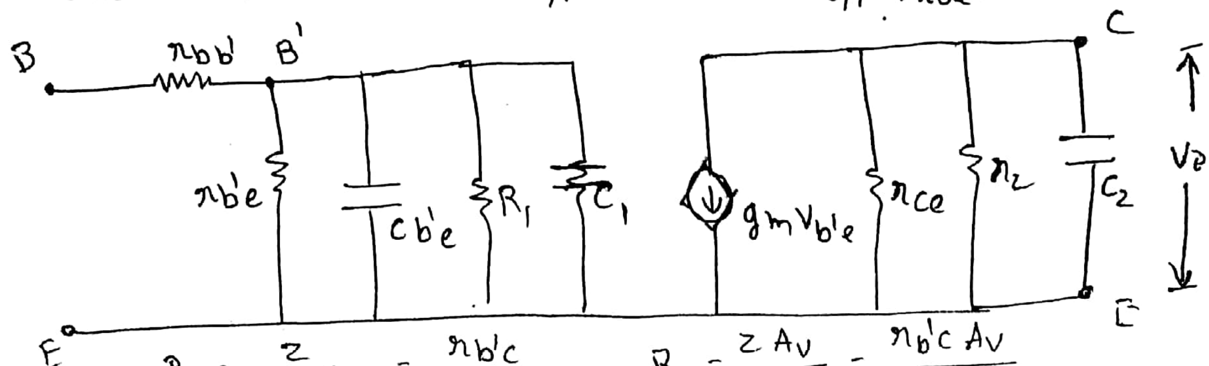
r_{ce} - output resistance with typical value of 80 KΩ.

$r_{ce} >> R_L$, if a load R_L is connected r_{ce} can be neglected.

$g_m V_{b'e}$ - It is a current generator. It is also known as collector current 'I_c'. where g_m is transconductance. which is a ratio of o/p current to input voltage.

$$\therefore g_m = \frac{I_c}{V_{b'e}} \Rightarrow I_c = g_m V_{b'e}$$

→ The above circuit can be modified by Miller's theorem i.e., seperate i/p and o/p capacitance, resistance will exist between i/p side and o/p side.

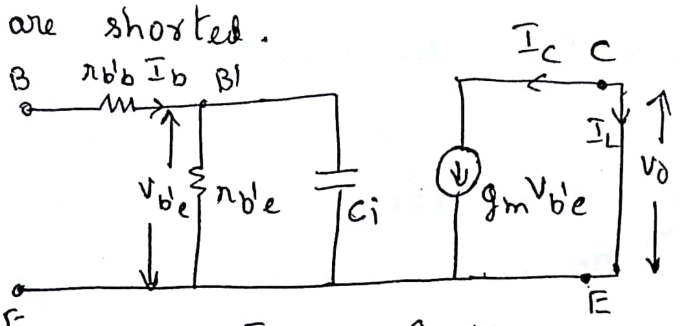


$$R_1 = \frac{Z}{1-A_V} = \frac{r_{b'c}}{1-A_V} \qquad R_2 = \frac{Z A_V}{A_V - 1} = \frac{r_{b'c} A_V}{A_V - 1}$$

$$C_{Mi} = C_f(1-A_V) = C_{b'c}(1-A_V) \qquad C_{Mo} = C_f\left(1-\frac{1}{A_V}\right) = C_{b'c}\left(1-\frac{1}{A_V}\right)$$

CE short circuit current gain at high frequency:-

Short circuited current gain means the O/P terminals are shorted.



→ r_{ce} becomes zero, so neglected.

→ $r_{b'e} \| R_1 \simeq r_{b'e}$

→ $C_i = C_{b'e} + C_1$ $= C_{b'e} + C_{b'c}(1 - A_V)$

O/P = 0 because O/P terminals short circuit

$$A_i = \frac{-I_c}{I_b} = \frac{-g_m V_{b'e}}{I_b} = \frac{I_L}{I_b} \quad (I_L = -I_c)$$

from the circuit $I_b = \dfrac{V_{b'e}}{r_{b'e}} + \dfrac{V_{b'e}}{1/jwC_i} = V_{b'e}\left[\dfrac{1}{r_{b'e}} + jwC_i\right]$

$$\therefore A_i = \frac{-g_m \cancel{V_{b'e}}}{\cancel{V_{b'e}}\left[\frac{1}{r_{b'e}} + jwC_i\right]} = \frac{-g_m r_{b'e}}{1 + jwC_i r_{b'e}}$$

- $g_m r_{b'e} = \dfrac{I_c}{V_{b'e}} \times r_{b'e} = \dfrac{I_c}{I_b} = \beta$

$f_\alpha = \dfrac{f_\beta}{1-\alpha}$

$f_\beta = f_\alpha(1-\alpha)$

$$\therefore A_i = \frac{-\beta}{1 + j2\pi f C_i r_{b'e}} = \frac{-\beta}{1 + j\frac{f}{f_\beta}}$$

where $\boxed{f_\beta = \dfrac{1}{2\pi r_{b'e} C_i}}$

$$|A_i| = \frac{\beta}{\sqrt{1 + (f/f_\beta)^2}}$$

at $f = f_\beta \Rightarrow |A_i| = \dfrac{\beta}{\sqrt{2}}$

$\therefore f_\beta \rightarrow \beta$ cut off frequency.

→ It is the frequency at which the CE short circuit current gain drops by 3dB or $1/\sqrt{2}$ times from its value at low frequency.

→ $\underline{f_\alpha}$:- It is the frequency at which the CB short circuit current gain drops by 3dB or $1/\sqrt{2}$ times from its value at low frequency. $\boxed{f_\alpha = \dfrac{1}{2\pi r_{b'e}(1 + h_{fe}) C_{b'e}}} = \dfrac{1 + h_{fe}}{2\pi r_{b'e} C_{b'e}}$

Unity Gain frequency :- f_T - Transition freq

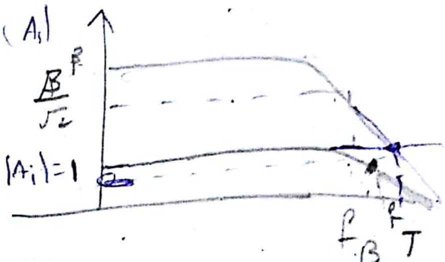
It is the frequency at which CE short circuit current gain becomes unity. Hence it is called unity gain frequency or Band width.

$$|A_i| = \frac{\beta}{\sqrt{1+\left(\frac{f_T}{f_\beta}\right)^2}} = 1 \qquad \because f = f_T \Rightarrow |A_i| = 1$$

The ratio of f_T/f_β is quite large compared to 1, so

$f_T/f_\beta >> 1 \quad \Rightarrow f_T >> f_\beta$

$$1 = \frac{\beta}{f_T/f_\beta} = \frac{\beta f_\beta}{f_T}$$



$$\therefore f_T = \beta f_\beta$$

Where f_T is the product of low frequency current gain h_{fe} or β and β cut off frequency or CE band width.

$$\therefore f_T = h_{fe} \times \frac{1}{2\pi r_{b'e}(C_{b'e}+C_{b'c})}$$

$$= \frac{g_m \cancel{r_{b'e}}}{2\pi \cancel{r_{b'e}}(C_{b'e}+C_{b'c})} = \frac{g_m}{2\pi(C_{b'e}+C_{b'c})}$$

$$\because h_{fe} = \frac{I_C}{I_B} = \frac{I_C}{V_{b'e}/r} = g_m r_{b'e}$$

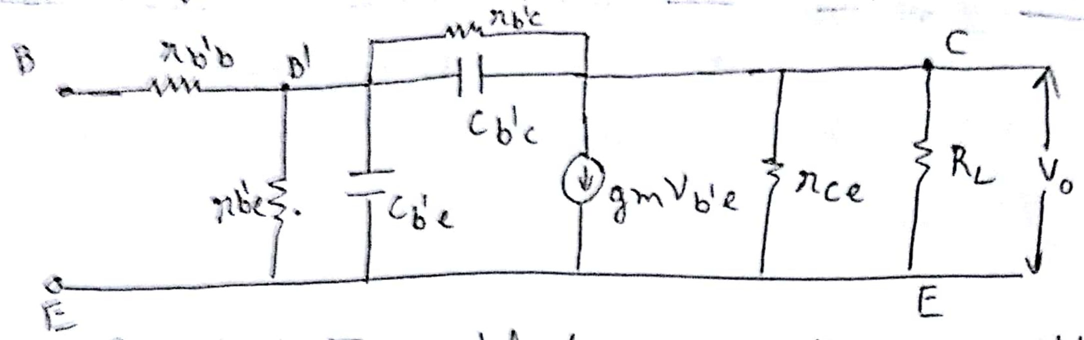
since $C_{b'c} << C_{b'e}$ 3 PF   100 PF

$$\boxed{\therefore f_T = \frac{g_m}{2\pi C_{b'e}}} \qquad \therefore C_{b'e} = \frac{g_m}{2\pi f_T}$$

substituting $f_\beta = \frac{f_T}{\beta}$ in A_i, then $\boxed{A_i = \frac{-\beta}{1+j\frac{\beta f}{f_T}}}$

→ This equation shows the dependence of transistor's short circuit current gain on the transistor's gain at low frequency β and the high frequency characteristics f_T.

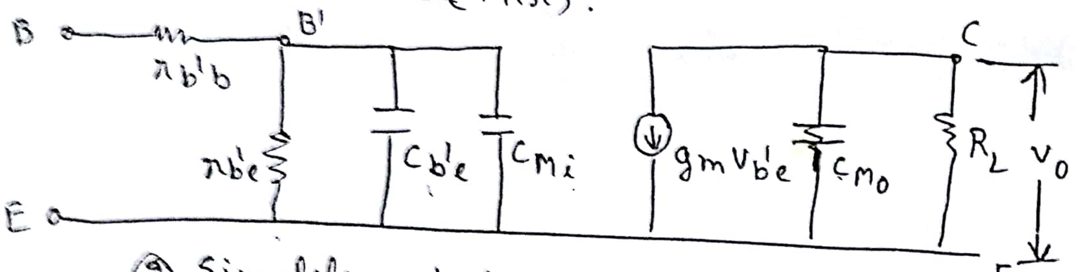
High frequency Current Gain with Resistive Load :-



① Hybrid π - model for a single transistor with Resistive Load

→ In the o/p circuit r_{ce} is parallel with R_L. For high freqn amplifiers R_L is small as compared to r_{ce} (80KΩ) and hence we can neglect r_{ce}.

→ Using Miller's theorem, we can split $r_{b'c}$ and $C_{b'c}$; But $r_{b'c}$ is neglecting here because it is high value (4M comparing to $r_{b'e}$ (1KΩ).



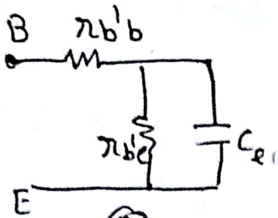
② Simplified hybrid - π model with Resistive Load

At input circuit :

where $C_{Mi} = C_{b'c}(1 - A_V)$

where $A_V = \frac{V_o}{V_i} = \frac{-g_m \cancel{V_{b'e}} R_L}{\cancel{V_{b'e}}} = -g_m R_L$

$\therefore C_{Mi} = C_{b'c}(1 + g_m R_L)$ —— ①



③

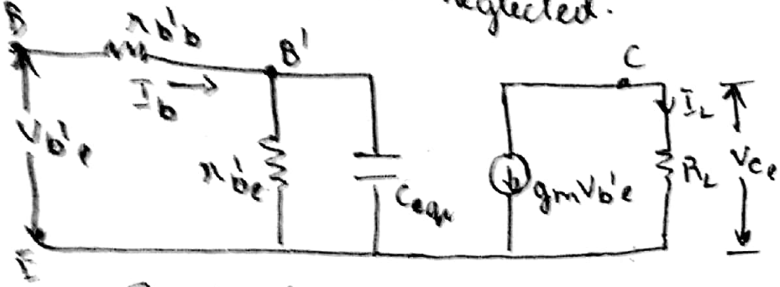
→ As $C_{b'e}$ and C_{Mi} are in parallel, the total capacitance is $C_{eq} = C_{b'e} + C_{b'c}(1 + g_m R_L)$ —— ②

→ Due to this i/p capacitance of the circuit is increased.

At output circuit :-

$$C_{Mo} = C_{b'c}\left(1 - \frac{1}{A_v}\right) \simeq C_{b'c} \quad \text{---(3)} \qquad [\because A_v >> 1]$$

→ As i/p capacitance $C_{b'e} + C_{b'c}(1 + g_m R_L)$ is very high comparing with output capacitance $C_{b'c}$. Hence O/P capacitance is negligible in comparing with the i/p. So, we may be neglected.



(4) Simplified hybrid π - Model

$$z = r_{b'e} \| c_{eq}$$

$$z = \frac{r_{b'e} \times \frac{1}{j\omega C_{eq}}}{r_{b'e} + \frac{1}{j\omega C_{eq}}}$$

$$z = \frac{r_{b'e}}{1 + j\omega r_{b'e} C_{eq}}$$

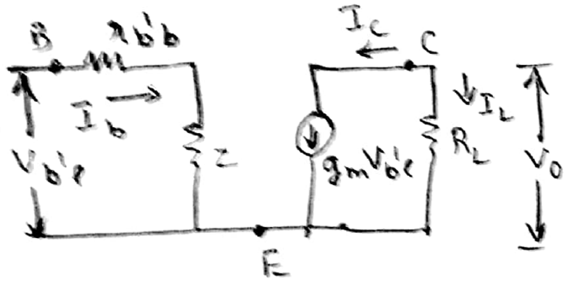


fig-(5)

∴ The current gain for the cir

$$A_i = \frac{-I_c}{I_b} = \frac{-g_m V_{b'e}}{I_b} = -g_m Z$$

$$\therefore A_i = \frac{-g_m r_{b'e}}{1 + j\omega r_{b'e} C_{eq}} = \frac{-\beta}{1 + j2\pi f r_{b'e} C_{eq}} \qquad \left[\because Z = \frac{V_{b'e}}{I_b}\right]$$

Let $f_H = \dfrac{1}{2\pi r_{b'e} C_{eq}}$,

$$\boxed{\therefore A_i = \frac{-\beta}{1 + j\left(\frac{f}{f_H}\right)}} \quad \text{---(5)}$$

$$\boxed{\therefore |A_i| = \frac{\beta}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}} \quad \text{---(6)}$$

if $f = f_H$, then $\boxed{|A_i| = \frac{\beta}{\sqrt{2}}}$ ---(7)

→ The f_H is the frequency at which the transistor's gain drops by 3dB or $1/\sqrt{2}$ times from its value at low frequen

$$\therefore f_H = \frac{1}{2\pi r_{b'e} C_{eq}} = \frac{1}{2\pi r_{b'e}\left[C_{b'e} + C_{b'c}(1+g_m R_L)\right]}$$

At $R_L = 0$, $f_H = \frac{1}{2\pi r_{b'e}(C_{b'e} + C_{b'c})} = f_\beta$ — (9)

→ From eqn-(9) says that maximum Possible value for f_H is f_β. As R_L increases, C_{eq} increases, then decreasing f_H as shown in fig (6).

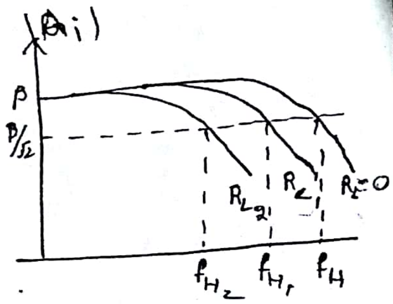


Fig-(6)

Relation between high frequency π and Low freqn h-parameters:-

Let us see how we can obtain all the resistance components in the hybrid-π model from the h-parameters in the CE configuration.

(1) **Transconductance:-**

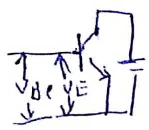
It is defined as the ratio of output current to the input voltage.

$$g_m = \frac{I_C}{V_{b'e}}$$

$g_m = \frac{i_{o/p}}{v_{in}}\Big|_{V_o=0}$

$g_m = \frac{I_C}{V_{be}}\Big|_{V_{CE}=0}$

$r_e = \frac{V_E}{I_E}$ $V_E = V_{b'e}$



We know that $I_C = \alpha I_E$ then $g_m = \frac{\alpha I_E}{V_{b'e}}$ — (1)

but the emitter resistance $r_e = \frac{V_{b'e}}{I_E}$ — (2)

$$\therefore g_m = \frac{\alpha \cancel{V_{b'e}}}{r_e \times \cancel{V_{b'e}}} = \frac{\alpha}{r_e}.$$

$g_m = \frac{I_C}{V_T}$

$g_m = \frac{I_C}{26mV}$

$r_e = \frac{26mV}{I_E}$

$\therefore r_e = \frac{V_T}{I_C}$ ($\because I_E \approx I_C$)

$\boxed{g_m = \frac{1}{r_e}}$

→ The dynamic emitter resistance r_e is given as $r_e = \frac{V_T}{I_E}$

$r_e \approx \frac{V_T}{I}$

where $V_T = \frac{KT}{q}$

Where K = boltzmann Constant = 1.36×10^{-23} J/°K

$r_{b'e} = \beta r_e$

$r_{b'e} = \frac{\beta}{g_m}$

$\boxed{\therefore g_m r_{b'e} = \beta}$

$T = 300°K$, $q = 1.6 \times 10^{-19} C$, $V_T = \frac{T}{11{,}600} = 26\,mV$

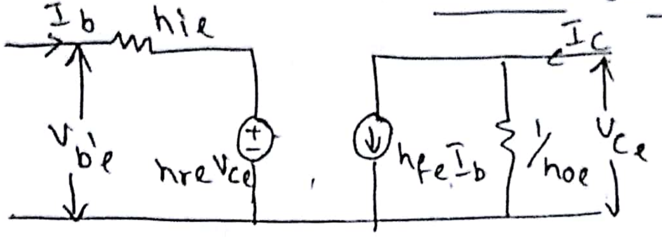
$\therefore g_m = \frac{\alpha}{r_e} = \frac{\alpha}{V_T/I_E} = \frac{\alpha I_E}{V_T} = \frac{I_c}{V_T} = \frac{1.3\times10^{-3}}{26\times10^{-3}} = 50\,mA/V$

Note :-

Transconductance $\propto$ collector current (I_c)

$\propto \frac{1}{Temp}$

(2) Input Resistance of emitter junction ($r_{b'e}$) :-

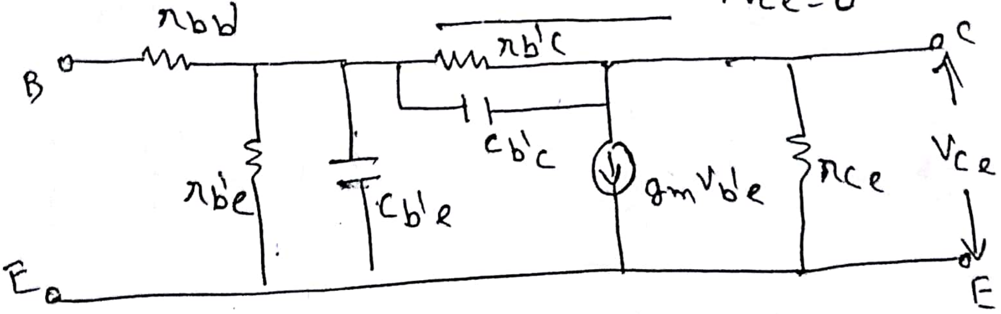


It can be obtained by low frequency h-parameter model.

$\therefore V_{b'e} = h_{ie} I_b + h_{re} V_{ce}$ — (1)

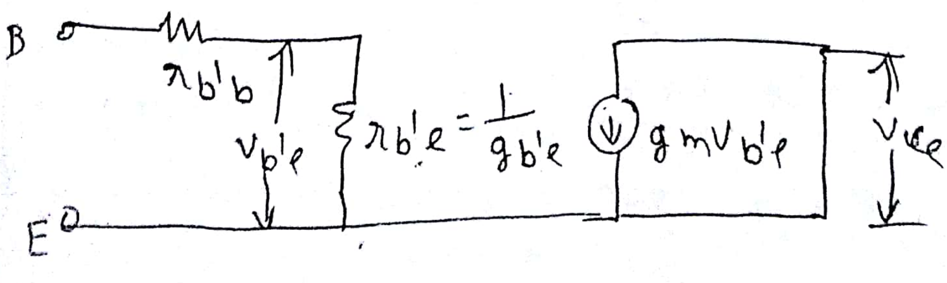
$I_c = h_{fe} I_b + h_{oe} V_{ce}$ — (2)

$\therefore h_{fe} = I_c / I_b \big|_{V_{ce}=0}$ — (3)



At low frequency, the hybrid π model when V_{ce} i.e short circuited at output side, r_{ce} can be neglect

$\rightarrow r_{b'c} >> r_{b'e}$, so $r_{b'c}$ is neglected.



By looking at the o/p circuit, the collector curre

$$I_c = g_m V_{b'e}$$

$$I_c = g_m I_b r_{b'e}$$

$$\frac{I_c}{I_b} = g_m r_{b'e}$$

from eqn (3) $\frac{I_c}{I_b} = h_{fe}$

$$\boxed{\therefore h_{fe} = g_m r_{b'e}}$$

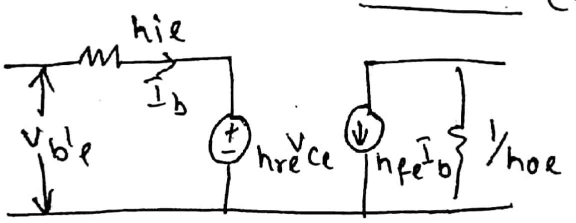
we know that $r_{b'e} = \frac{h_{fe}}{g_m} = \frac{50}{50 \times 10^{-3}} = 1K\Omega$

$$\boxed{\therefore \text{Input conductance } g_{b'e} = \frac{g_m}{h_{fe}}}$$

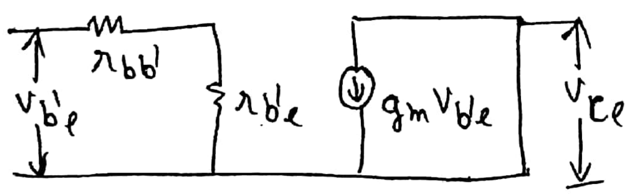
Note:- $g_{b'e} = \frac{g_m}{h_{fe}} = \frac{I_c}{V_T \cdot h_{fe}}$ $\therefore g_{b'e} \propto I_c$

$\propto \frac{1}{Temp.}$

(3) Base Spreading Resistance ($r_{bb'}$):-



$\rightarrow$ $h_{ie} = \left.\frac{V_{b'e}}{I_b}\right|_{V_{ce}=0}$



$\rightarrow$ o/p short circuited

$$h_{ie} = r_{bb'} + r_{b'e}$$

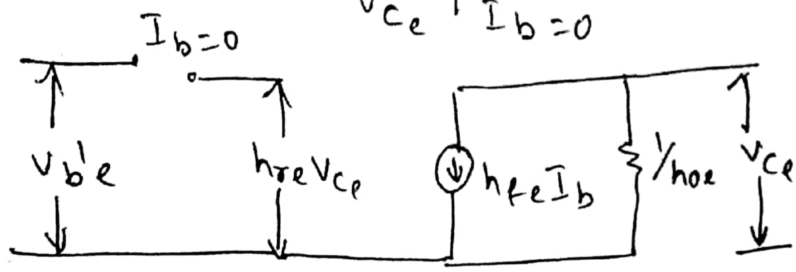
$$\boxed{r_{bb'} = h_{ie} - r_{b'e}}$$

$$= 1100 - 1000 = 100\Omega$$

④ Resistance of collector junction ($r_{b'c}$) or feed back resist

eqn ① $V_{b'e} = h_{ie} I_b + h_{re} V_{ce}$ — ①

$$h_{re} = \left. \frac{V_{b'e}}{V_{ce}} \right|_{I_b = 0}$$



$\therefore V_{b'e} = h_{re} V_{ce}$

$$V_{b'e} = \frac{V_{ce} \times r_{b'e}}{r_{b'e} + r_{b'c}}$$

$$\frac{V_{b'e}}{V_{ce}} = \frac{r_{b'e}}{r_{b'e} + r_{b'c}}$$

$$\therefore h_{re} = \frac{r_{b'e}}{r_{b'e} + r_{b'c}}$$

$$\therefore h_{re}(r_{b'e} + r_{b'c}) = r_{b'e}$$

$$r_{b'c} = \frac{r_{b'e} - h_{re} r_{b'e}}{h_{re}} = 4 M\Omega \simeq 3.999 M\Omega$$

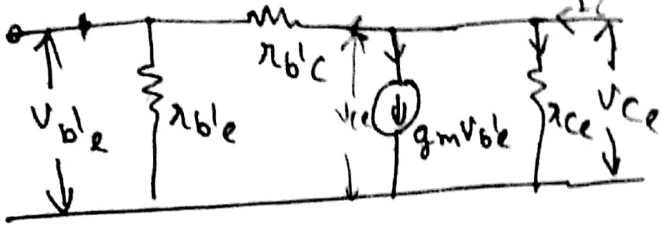
$$r_{b'c} = \frac{r_{b'e}(1 - h_{re})}{h_{re}} = \frac{r_{b'e}}{h_{re}} \quad [\because 1 - h_{re} \simeq 1]$$

$$\boxed{\therefore r_{b'c} = \frac{r_{b'e}}{h_{re}}} \quad \text{(or)} \quad \boxed{g_{b'c} = \frac{h_{re}}{r_{b'e}} = h_{re}\, g_{b'e}}$$

(5) Output Resistance (r_{ce}):-

eqn (2) $I_c = h_{fe} I_b + h_{oe} V_{ce}$

$\therefore h_{oe} = I_c / V_{ce} \Big|_{I_b = 0}$



$$I_c = \frac{V_{ce}}{r_{ce}} + g_m V_{b'e} + \frac{V_{ce}}{r_{b'e} + r_{b'c}}$$

$\cancel{\because V_{b'e} = \frac{V_{ce}\, r_{b'e}}{r_{b'e} + r_{b'c}}}$

$$= V_{ce}\left[\frac{1}{r_{ce}} + \frac{g_m V_{b'e}}{V_{ce}} + \frac{1}{r_{b'e} + r_{b'c}}\right]$$

X ______

$$\therefore h_{oe} = \frac{I_c}{V_{ce}} = g_{ce} + \frac{g_m r_{b'e}}{r_{b'e} + r_{b'c}} + \frac{1}{r_{b'e} + r_{b'c}}$$

$$= g_{ce} + \frac{h_{fe} + 1}{r_{b'e} + r_{b'c}} \qquad [\because g_m r_{b'e} = h_{fe}]$$

$$= g_{ce} + \frac{h_{fe}}{r_{b'c}} \qquad [\because h_{fe} >> 1] \quad [r_{b'c} >> r_{b'e}]$$

$$\boxed{\therefore h_{oe} = g_{ce} + g_{b'c} h_{fe}}$$

$$\boxed{g_{ce} = h_{oe} - g_{b'c} h_{fe} = \frac{1}{r_{ce}}}$$

______ X

$$h_{oe} = \frac{I_c}{V_{ce}} = g_{ce} + h_{fe}\, g_{b'e}\, h_{re} + g_{b'c}$$

$$= g_{ce} + \cancel{g_{b'e}}\, h_{fe} \frac{g_{b'c}}{\cancel{g_{b'e}}} + g_{b'c}$$

$\left[\because g_m = \frac{h_{fe}}{r_{b'e}} = h_{fe}\right.$

$h_{re} = \frac{V_{b'e}}{V_{ce}}$

$\left. r_{b'c} >> r_{b'e}\right]$

$$\boxed{\therefore g_{ce} = h_{oe} - g_{b'c}(1 + h_{fe})}$$

⑥ **Diffusion capacitance ($C_{b'e}$):-**

It is also represented as C_π in hybrid-π model. $C_{b'e}$ is determined from a measurement of f_T, the freq at which the CE short-circuit current gain drops to unity. It is given by $\boxed{C_{b'e} = \frac{g_m}{2\pi f_T}}$ $\left[\because f_T = \frac{g_m}{2\pi C_{b'e}}\right]$

⑦ **Transition capacitance ($C_{b'c}$):-**

It is also represented as C_μ in hybrid-π model. $C_\mu = C_{b'c}$ is measured as a CB o/p capacitance with i/p open ($I_E = 0$), and is usually specified by manufacturer as C_{ob} (open circuit base capacitance.

Relation between Low freq h-parameter and High freq π Parameters.

Hybrid π moden on

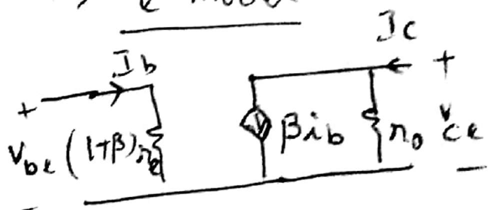
$\rightarrow$ Low freq Small signal

$$g_m = \frac{I_C}{V_T} = 50\, mA/V$$

$$r_{b'e} = \frac{h_{fe}}{g_m} = 1K\Omega = \frac{1}{g_{b'e}}$$

$$r_{bb'} = h_{ie} - r_{b'e} = 100\Omega$$

$\rightarrow r_e$ model



$$r_{b'c} = \frac{1}{g_{b'c}} = \frac{r_{b'e}}{h_{re}}$$

$$g_{ce} = \frac{1}{r_{ce}} = h_{oe} - (1+h_{fe})g_{b'c}$$

$$C_{b'e} = \frac{g_m}{2\pi f_T}$$

VCCS — $g_m V_{b'e}$ 

CCCS — βi_b

3.10 Gain Bandwidth Product

3.10.1 Gain Bandwidth Product for Voltage

The gain bandwidth product for voltage gain is given as

$$|A_{vs\,low}\, f_H| = |A_{vso}\, f_H| = \frac{-h_{fe} R_L}{R_s + h_{ie}} \times \frac{1}{2\pi R_{eq} C_{eq}}$$

$$= \frac{-h_{fe} R_L}{R_s + h_{ie}} \times \frac{1}{2\pi C_{eq} \left[\dfrac{r_{b'e}(r_{bb'} + R_s)}{r_{b'e} + r_{bb'} + R_s}\right]}$$

$$= \frac{-h_{fe} R_L}{R_s + h_{ie}} \times \frac{1}{2\pi C_{eq} \left[\dfrac{r_{b'e}(r_{bb'} + R_s)}{(R_s + h_{ie})}\right]}$$

$$\because \quad h_{ie} = r_{b'e} + r_{bb'}$$

$$= \frac{-h_{fe} R_L}{2\pi C_{eq}\, r_{b'e}(r_{bb'} + R_s)}$$

$$= \frac{-g_m r_{b'e} R_L}{2\pi C_{eq}\, r_{b'e}(r_{bb'} + R_s)} \qquad \because\; h_{fe} = g_m r_{b'e}$$

$$= \frac{-g_m R_L}{2\pi C_{eq}(r_{bb'} + R_s)} \qquad \ldots (1)$$

This equation can be further simplified as follows.

$$|A_{vso} \times f_H| = \frac{g_m}{2\pi[C_e + C_C[1 + g_m R_L]]} \times \frac{R_L}{R_s + r_{bb'}}$$

$$\because \quad C_{eq} = C_e + C_C(1 + g_m R_L)$$

$$= \frac{g_m}{2\pi[C_e + C_C + g_m R_L]} \times \frac{R_L}{R_s + r_{bb'}} \qquad \because\; g_m R_L >> 1$$

$$= \frac{R_L}{R_s + r_{bb'}} \times \frac{2\pi f_T C_e}{2\pi[C_e + C_C(2\pi f_T C_e) R_L]}$$

$$\because \quad g_m = 2\pi f_T C_e$$

$$= \frac{R_L}{R_s + r_{bb'}} \times \frac{2\pi C_e f_T}{2\pi C_e[1 + 2\pi f_T C_C R_L]}$$

$$= \frac{R_L}{R_s + r_{bb'}} \times \frac{f_T}{1 + 2\pi f_T C_C R_L}$$

$$\therefore \quad |A_{vso} \times f_H| = \frac{R_L}{R_s + r_{bb'}} \times \frac{f_T}{1 + 2\pi f_T C_C R_L} \qquad \ldots (2)$$