ATTENUATOR:

An attenuator is a two port resistive network. It is used to reduce the signal level when used between a generator and load. Attenuators may be symmetrical (or) assymmetrical. They may provide fixed (or) variable attenuation. A fixed attenuator is also called pad. The attenuation is expressed in decibels.

$$\text{Attenuation in dB} = 10 \log_{10} \frac{P_1}{P_2}$$

where P_1 is the input power, P_2 is o/p power.

for a properly matched N/w

$$P_1 = I_1^2 R_0 = \frac{V_1^2}{R_0}$$

$$P_2 = I_2^2 R_0 = \frac{V_2^2}{R_0}$$

where R_0 is the characteristic resistance of the N/w

$$\text{attenuation in dB} = 20 \log_{10} \frac{V_1}{V_2} = 20 \log_{10} \frac{I_1}{I_2}$$

Attenuation is also expressed in nepers as the natural logarithm of the voltage (or) current ratio.

$$\text{Attenuation in nepers} = \ln \frac{V_1}{V_2} = \ln \frac{I_1}{I_2} = \frac{1}{2} \ln \frac{P_1}{P_2}$$

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = N \text{ then } \frac{P_1}{P_2} = N^2$$

$$\text{Attenuation in dB} = 20 \log_{10} N$$

$$N = \text{antilog}\left(\frac{dB}{20}\right)$$

where N is the attenuation nepers.

Relation btwn Decible & Neper

$$A \cdot N = \ln \frac{V_1}{V_2} \quad A \cdot D = 20 \log_{10} \frac{V_1}{V_2}$$

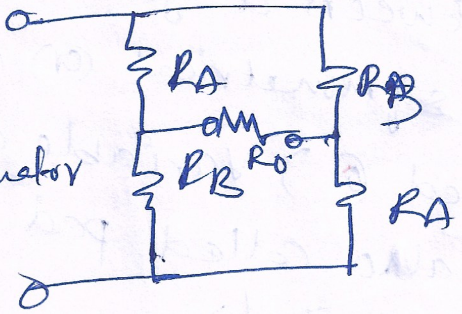
Changing the base of logari

$$A \cdot D = \frac{20 \ln \frac{V_1}{V_2}}{\ln 10} = \frac{20}{2.303} \ln \frac{V_1}{V_2} = 8.686 \times \text{atten in nepers}$$

$A \cdot N =$ ... $= 0.115 \times$ attenuation dB.

LATTICE ATTENUATOR

A lattice is a general type of symmetrical balanced and balanced N/W. Any symmetrical balanced (or) unbalanced N/W can be transformed into an equivalent lattice N/W.

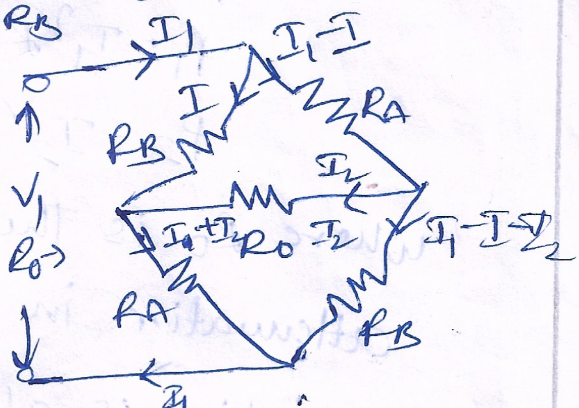


The elements of a lattice attenuator can be specified in terms of characteristic impedance & propagation constant

$$Z_0 = \sqrt{Z_{OC} Z_{SC}}$$

$$Z_{OC} = \frac{R_A + R_B}{2} \qquad Z_{SC} = \frac{2R_A R_B}{R_A + R_B}$$

$$Z_0 = R_0 = \sqrt{\frac{R_A + R_B}{2} \times \frac{2R_A R_B}{R_A + R_B}} = \sqrt{R_A R_B}$$



Apply KVL to the loop.

$$V_1 = R_0 I_1 = R_A (I_1 - I) + R_0 I_2 + R_A (I_1 + I_2)$$

$$R_0 I_1 = R_A (I_1 + I_2) + R_0 I_2$$

$$(R_0 - R_A) I_1 = (R_A + R_0) I_2$$

$$\frac{I_1}{I_2} = \frac{R_A + R_0}{R_0 - R_A} = \frac{1 + \frac{R_A}{R_0}}{1 - \frac{R_A}{R_0}}$$

$$N = e^{\alpha} = \frac{I_1}{I_2} = \frac{1 + R_A/R_0}{1 - R_A/R_0}$$

$$N\left(1 - \frac{R_A}{R_0}\right) = 1 + R_A/R_0$$

$$R_A = R_0 \left(\frac{N-1}{N+1}\right)$$

$$R_B = \frac{R_0^2}{R_A} = \frac{R_0^2}{R_0 \left(\frac{N-1}{N+1}\right)}$$

$$= R_0 \left(\frac{N+1}{N-1}\right)$$

$R_A = R_0 \tanh \frac{\alpha}{2}$ (through arms)

$$= R_0 \left(\frac{e^{\alpha/2} - e^{-\alpha/2}}{e^{\alpha/2} + e^{-\alpha/2}}\right)$$

$$= R_0 \left(\frac{e^{\alpha} - 1}{e^{\alpha} + 1}\right)$$

$$= R_0 \left(\frac{N-1}{N+1}\right)$$

$R_B = R_0 \coth \frac{\alpha}{2}$ (cross arms)

$$= R_0 \left(\frac{e^{\alpha/2} + e^{-\alpha/2}}{e^{\alpha/2} - e^{-\alpha/2}}\right)$$

$$= R_0 \left(\frac{e^{\alpha} + 1}{e^{\alpha} - 1}\right)$$

$$= R_0 \left(\frac{N+1}{N-1}\right)$$

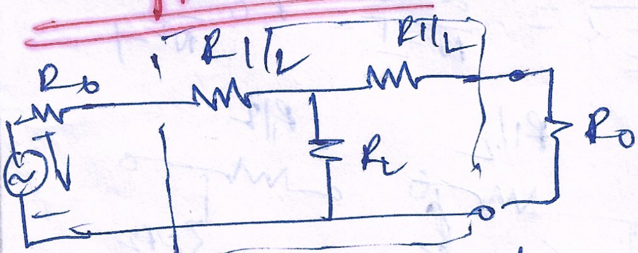
Bisection theorem:-

A N/w said to have been bisected when the open-circuited & short-circuited i/p impedances of the two bisected networks are equivalent. Also, the square root of the product of these impedances is the characteristic impedance of the original whole N/w.

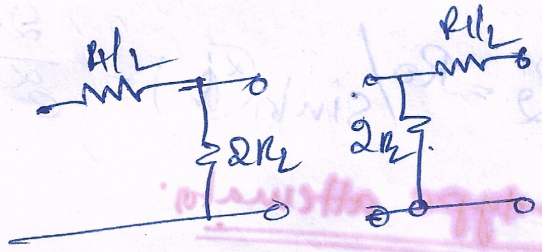
A bisection theorem states that 'any symmetrical balanced (or) unbalanced N/w can be transformed into an equivalent lattice network. The series arm of the lattice is equal to the short-circuited impedance of the bisected N/w and the diagonal arm is equal to the open-circuited impedance of the bisected N/w.

The lattice N/w ~~R_A~~ & R_B equations are used to find ~~latti~~ ~~T&π N/ws~~ equivalent lattice network for T&π networks using the bisection theorem.

T-type attenuators:-



2.(a) T-type attenuator



2(b) bisection of T-type attenuator

From the bisection theorem, the series arm of the lattice N/w is equal to the short-circuited i/p resistance of the bisected network and the diagonal arm is equal to the open-circuited input resistance of the bisected network.

$R_{SC} = R1/2$ $\quad R_{OC} = R1/2 + 2R_2$

$R_A = R_{SC}$, $R_B = R_{OC}$

$R_A = R_0\left(\frac{N-1}{\ }\right)$ $R_B = R_0\left(\frac{N+1}{N-1}\right)$

$R_1 = 2R_A$

$= 2R_0\left(\frac{N-1}{N+1}\right)$

$$2R_2 = \frac{R_B - \frac{R_1}{2}}{\cancel{2}}$$

$$= R_0\left(\frac{N+1}{N-1}\right) - \frac{2R_0\left(\frac{N-1}{N+1}\right)}{2}$$

$$2R_2 = \frac{R_0\left[(N+1)^2 - (N-1)^2\right]}{(N-1)(N+1)}$$

$$= R_0\frac{\left[\cancel{N^2} + 2N + \cancel{1} - \cancel{N^2} - \cancel{1} + 2N\right]}{(N-1)(N+1)}$$

$$R_2 = \frac{R_0}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}N}{N^2-1} = \underline{\underline{\frac{2NR_0}{N^2-1}}} = \left(\frac{2N}{N^2-1}\right)R_0$$

(or) series & Shunt arms of the symmetrical T-Network can be given as

$$\frac{R_1}{2} = R_0 \tanh\frac{\alpha}{2} = R_0\left(\frac{e^{\alpha/2} - e^{-\alpha/2}}{e^{\alpha/2} + e^{\alpha/2}}\right) = R_0\left(\frac{e^{\alpha}-1}{e^{\alpha}+1}\right)$$

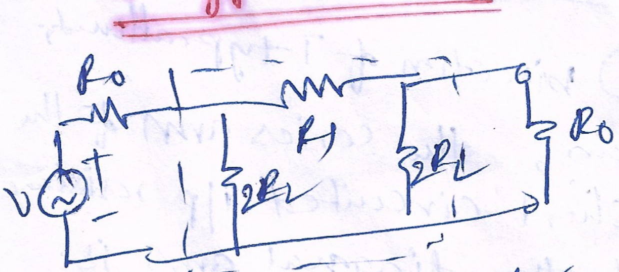
$$= R_0\left(\frac{N-1}{N+1}\right)$$

$$R_1 = 2R_0\left(\frac{N-1}{N+1}\right)$$

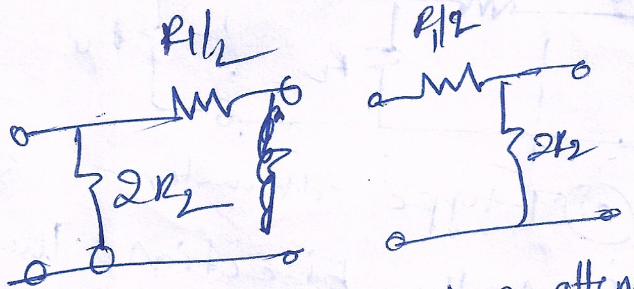
~~T-type attenuator~~

$$R_2 = R_0/\sinh(\alpha) = \frac{2R_0}{e^{\alpha} - e^{-\alpha}} = \frac{2R_0}{N - \frac{1}{N}} = R_0\frac{2N}{N^2-1}$$

π-type attenuator



3(a) π-attenuator



3(b) Bisection of π-type attenuator

from bisection Theorem, the series arm of lattice N/W is equal to the ~~R~~ short-circuited i/p resistance of the bisected N/W & the diagonal arm is equal to the open-circuit i/p resistance of the bisected network.

$$R_{SC} = \frac{2R_2 \times \frac{R_1}{2}}{\frac{R_1}{2} + 2R_2} = R_A$$

$$R_{SC} = \frac{R_1 R_2}{\frac{R_1}{2} + 2R_2} = R_A \Rightarrow ①$$

$$R_{OC} = 2R_2 = R_B .$$

$$R_A = R_0\left(\frac{N-1}{N+1}\right)$$

$$R_B = R_0\left(\frac{N+1}{N-1}\right)$$

$$2R_2 = R_0\left(\frac{N+1}{N-1}\right)$$

$$\boxed{R_2 = \frac{R_0}{2}\left(\frac{N+1}{N-1}\right)} \text{ A.}$$

$$\frac{R_1 \times R_B/2}{\frac{R_1}{2} + \cancel{2}\frac{R_B}{\cancel{2}}} = R_A$$

$$\frac{R_1 R_B/\cancel{2}}{R_1 + 2R_B/\cancel{2}} = R_A$$

$$R_1 R_B = R_1 R_A + 2R_A R_B$$

$$R_1(R_B - R_A) = 2R_A R_B$$

$$R_1 = \frac{2R_A R_B}{R_B - R_A}$$

$$\frac{R_1 \times \frac{R_0}{2}\left(\frac{N+1}{N-1}\right)}{\frac{R_1}{2} + \cancel{2}\frac{R_0}{\cancel{2}}\left(\frac{N+1}{N-1}\right)} = \frac{R_0(N-1)}{N+1}$$

$$\boxed{R_1 = R_0\left(\frac{N^2-1}{2N}\right)}$$

$$R_1 = \frac{2 \times R_0\left(\frac{N+1}{N-1}\right)\left(\frac{N-1}{N+1}\right)}{R_0\left(\frac{N+1}{N-1}\right) - R_0\left(\frac{N-1}{N+1}\right)}$$

$$= \frac{2R_0}{\frac{(N+1)^2 - (N-1)^2}{N^2-1}}$$

$$= \cancel{2}R_0\left(\frac{N^2-1}{\cancel{4}_2 N}\right)$$

$$\boxed{R_1 = R_0\left(\frac{N^2-1}{2N}\right)}$$

Series & shunt arms of π-type attenuators are given

$$R_1 = R_0 \sinh\alpha = R_0\left(\frac{e^{\alpha} - e^{-\alpha}}{2}\right) = R_0 \frac{N - 1/N}{2} = \boxed{R_0\left(\frac{N^2-1}{2N}\right)} = R_1$$

$$2R_2 = R_0 \coth\alpha/2 = R_0\left(\frac{e^{\alpha/2} + e^{-\alpha/2}}{e^{\alpha/2} - e^{-\alpha/2}}\right) = R_0\left(\frac{e^{\alpha}+1}{e^{\alpha}-1}\right) = R_0\left(\frac{N+1}{N-1}\right)$$

$$\boxed{R_2 = \frac{R_0}{2}\left(\frac{N+1}{N-1}\right)} \text{ A}$$

1. Design T-pad attenuator to give the attenuation of 60 dB and to work in a line of 500 Ω impedance.

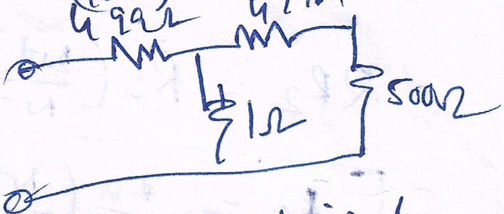
$(N)\,dB = 60\,dB \quad R_0 = 500\,\Omega$

$60 = 20\log_{10}(N)$

$N = 1000$

$R_1 = 2R_0\left(\frac{N-1}{N+1}\right) = 2\times500\left(\frac{1000-1}{1000+1}\right) = 998\,\Omega$

$R_2 = R_0\left(\frac{2N}{N^2-1}\right) = 500\left(\frac{2\times1000}{(1000)^2-1}\right) = 1\,\Omega$



2. An attenuator is composed of symmetrical T-section having a series arm of 175 Ω & shunt arm of 350 Ω. Find the characteristic impedance & attenuation in dB.

$\frac{R_1}{2} = 175\,\Omega, \quad R_1 = 350\,\Omega, \quad R_2 = 350\,\Omega$

$R_1 = 2R_0\left(\frac{N-1}{N+1}\right) = 350 \rightarrow (1)$

$R_2 = R_0\left(\frac{2N}{N^2-1}\right) = 350 \rightarrow (2)$

divide eqn (2) by (1).

$$\frac{2N/N^2-1}{2(N-1/N+1)} = 1$$

$$\frac{N}{(N+1)(N-1)} = \frac{N-1}{(N+1)} \Rightarrow N = (N-1)^2.$$

$N^2 - 3N + 1 = 0 \qquad N = 2.611$ or $N = 0.3819$.

N in dB $= 20\log_{10} 2.611 = 8.36\,dB$.

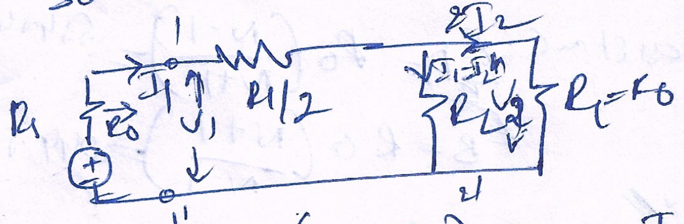
(or) N in dB $= 20\log_{10} 0.3819 = -8.36\,dB$. Since attenuation cannot be negative

$N = 2.618$ Neper, $= 8.36\,dB$

$350 = 2R_0\left(\frac{2.618-1}{2.618+1}\right) \Rightarrow R_0 = 391.32\,\Omega$

L-TYPE ATTENUATOR:-

The attenuator is connected between a source with source resistance $R_s = R_0$ and load resistance $R_L = R_0$.



$$V_2 = (I_1 - I_2) R_2 = I_2 R_L = I_2 R_0$$

$$I_1 R_2 = I_2 (R_2 + R_L)$$

$$\frac{I_1}{I_2} = \frac{R_2 + R_L}{R_2} = N$$

$$1 + \frac{R_L}{R_2} = N$$

$$R_2 = \frac{R_L}{N-1} = \frac{R_0}{N-1} \rightarrow (1)$$

The resistance of the network as viewed from 1-1' into the network is

$$R_0 = \frac{R_1}{2} + \frac{R_2 R_0}{R_2 + R_0}$$

$$\frac{R_1}{2} = \frac{R_0^2}{R_2 + R_0}$$

Substituting the value of R_2 from eqn (1).

$$\frac{R_1}{2} = R_0^2 \Big/ \frac{R_0}{N-1} + R_0 = \frac{R_0^2 (N-1)}{R_0 + R_0(N-1)}$$

$$\frac{R_1}{2} = \frac{R_0^2 (N-1)}{\cancel{R_0}(1 + N - \cancel{1})} = R_0 \frac{(N-1)}{N} \rightarrow (2).$$

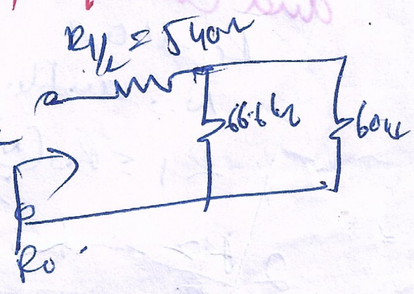
Attenuation N can ~~be~~ of the N/w can be varied by varying the values of R_1 & R_2.

1. Design L-type attenuator to operate into a load resistance of 600 Ω with attenuation of 20 dB.

$$N = \text{antilog}\frac{dB}{20} = \text{antilog}\frac{20}{20} = 10$$

$$\frac{R_1}{2} = R_0\left(\frac{N-1}{N}\right) = 600\left(\frac{10-1}{10}\right) = 540\Omega$$

$$R_2 = R_0/N-1 = \frac{600}{10-1} = 66.66\Omega$$

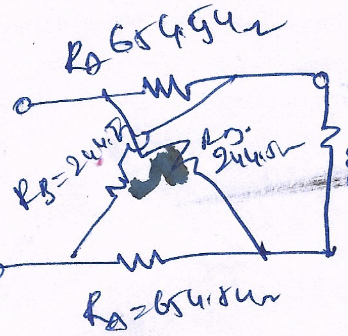


2. Design a lattice attenuator if the characteristic impedance of 800Ω and attenuation of 20 dB

$R_o = 800\Omega$ $D = 20\,dB$ $N = \text{antilog}\frac{D}{20} = \text{antilog}\frac{20}{20} = 10$

series-arm resistance $R_A = R_o\left(\frac{N-1}{N+1}\right) = 654.54\Omega$

$R_B = R_o\left(\frac{N+1}{N-1}\right) = 244.5\Omega$



3. A attenuator composed of symmetrical Π-section having series arm of 275Ω and each shunt arm of 450Ω. Find the characteristic impedance & attenuation in dB.

$R_1 = 275\Omega$ $2R_2 = 450\Omega \Rightarrow R_2 = 225\Omega$

$R_1 = R_o\left(\frac{N^2-1}{2N}\right) = 275 \rightarrow (1)$

$R_2 = \frac{R_o}{2}\left(\frac{N+1}{N-1}\right) = 225 \rightarrow (2)$

divide eqn (2) by (1).

$\frac{N^2-1/2N}{1/2(N+1)(N-1)} = \frac{\cancel{(N+1)(N-1)}}{\frac{275}{225}}$

(0.8181) $N = 2.872$ (or) $N = 0.348$

$(N-1)^2 = 2N$

$N^2 - 3.222N + 1 = 0$ $N = 2.872 = 9.11\,dB$

$N\text{ in }dB = 20\log_{10} 2.872$

$N\text{ in }dB = 20\log_{10} 0.348 = -9.16\,dB$ (it cannot be)

$275 = R_o\left(\frac{2.872^2-1}{2\times 2.872}\right) \Rightarrow 217.92\Omega$

3. Design a Π-type attenuator to give attenuation of 20 dB and characteristic resistance of 100Ω

$R_o = 100\Omega$ $D = 20\,dB$

$N = \text{antilog} = D/20 = 10$

$R_1 = R_o\left(\frac{N^2-1}{2N}\right) = 495\Omega$

$2R_2 = R_o\left(\frac{N+1}{N-1}\right) = 122.22\Omega$