BASIC ELECTRICAL ENGINEERING

UNIT 1 : D.C. (Currents) Circuits

UNIT 2 : A.C. (Currents) Circuits

UNIT 3 : Transformers

UNIT 4 : Electrical Machines

UNIT 5 : Electrical Installations

UNIT ① : D.C. (CURRENTS) CIRCUITS

→ Electrical circuit elements

→ Voltage & Current Sources.

→ KVL & KCL

→ Analysis with DC excitation

→ Theorems : a) Super position theorem

b) Thevenin's theorem

c) Norton's theorem

→ Time domain analysis of RL & RC circuits.

Charge : It is the nature (or) behaviour of sub-atomic particles like protons and electrons

→ charge is indicated with 'q'

Units : Coloumb [C]

Current : The rate of flow of charge is called current.

→ It is indicated with 'i'

$i = dq/dt$

Units : coloumb/second [C/s] (or) Ampere

Voltage : The work done in moving the unit charge in a closed path is called voltage.

$$V = \frac{dw}{dq}$$

Units : Joule/coloumb $= J/C$ (or) Volts (V)

Power : The rate of change of work done is called power.

$$P = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt}$$

$$\boxed{P = V \times I}$$

Units : Watt (W), Kilowatt (KW)

Energy (E (or) W) :

$$P = \frac{dw}{dt}$$

$$dw = P \cdot dt$$

$$\int dw = \int P \cdot dt$$

$$w = P \times t$$

$$\Rightarrow \boxed{E = P \times t}$$

Units : Watt.sec

$$\boxed{1 \text{ unit} = 1 \text{ KWH}}$$

Classification of Elements:

8/8/2018

1) Active and Passive elements:

The elements which deliver the energy are called active elements. And also said to be energy sources.

Eg: Voltage source, Current source.

The elements which absorb the energy are called passive elements. And also called as energy sinks.

Eg: Resistor, Inductor, capacitor.

2) Linear and Non-linear elements:

The input-output characteristics (V-I) for an element is a straight line and passes through the origin is called linear element $\left\{\frac{V}{I} = \text{const}\right\}$

Eg: Resistor (R), inductor (L), capacitor (c)

The input-output characteristics (V-I) for an element is other than straight line is called non-linear element.

Eg: Transistor, diode.

Note: The linear elements satisfies the principle of super position.

3) Unilateral and Bilateral elements:

If the element offers different resistances (or) different opposition for either direction of current flow it is called unilateral.

Eg: diode.

If the element offers same resistance for either direction of current flow then it is called bilateral.

Eg: Resistor.

4) Lumped and Distributed elements :

The elements which are small and physically separable are called lumped elements.

Eg: Small circuits.

The elements which are not separable for analytical purpose are called distributed elements.

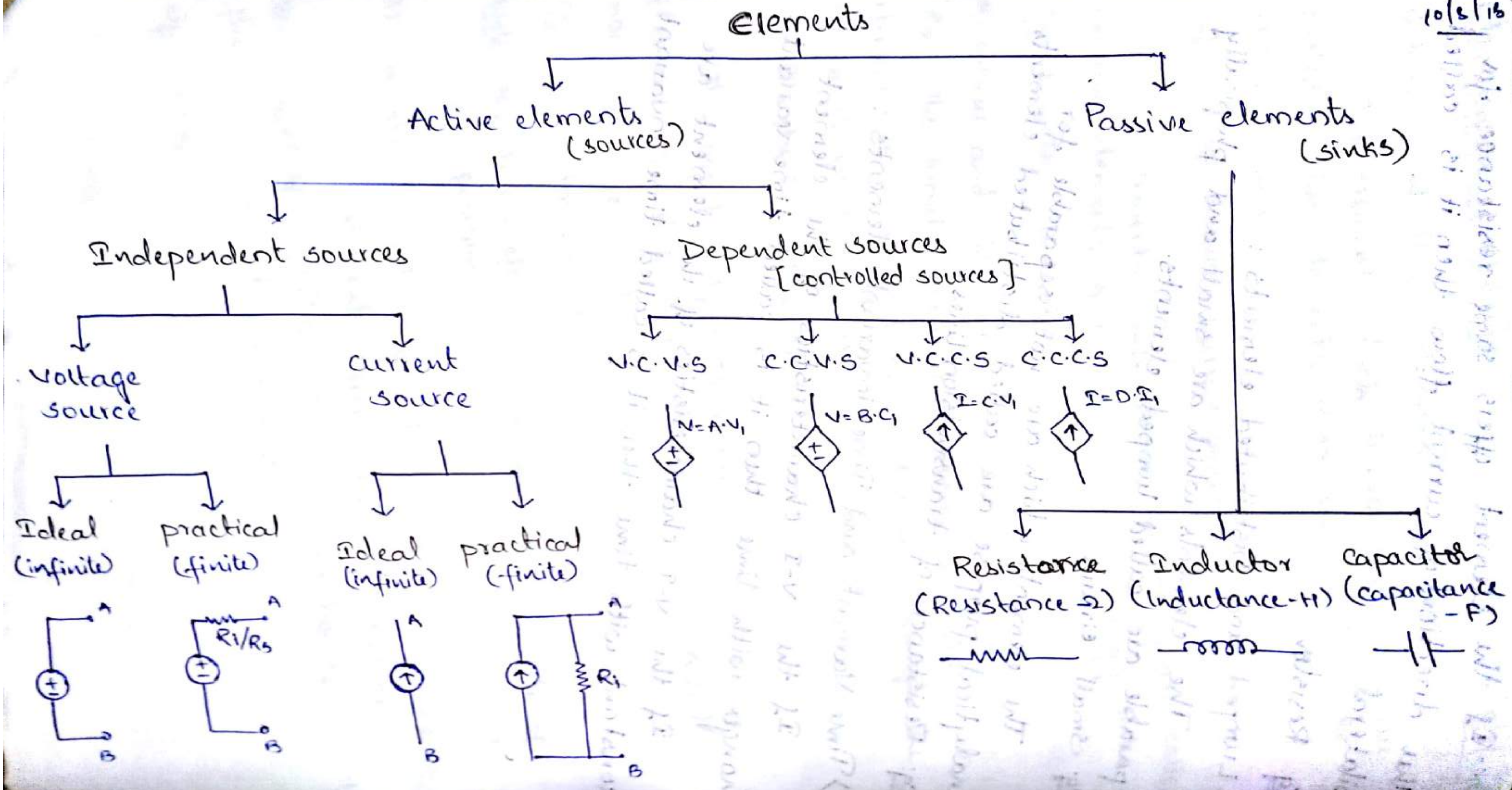
Eg: Resistance of transmission lines.

5) Time Variant and Time invariant elements :

If the V-I characteristics of the element changes with time then it is called time variant.

If the V-I characteristics of the element are constant with time then it is called time invariant.

10/8/18


Elements
Active elements (sources)
Passive elements (sinks)
Independent sources
Dependent sources [controlled sources]
voltage source
current source
V.C.V.S
C.C.V.S
V.C.C.S
C.C.C.S
N=A·V1
V=B·C1
I=C·V1
I=D·I1
Ideal (infinite)
practical (finite)
Ideal (infinite)
practical (-finite)
A
B
R1/Rs
R1
Resistance (Resistance Ω)
Inductor (Inductance-H)
capacitor (capacitance -F)

Resistance (R):

It is the property of the material to oppose the flow of current and it is represented by the letter 'R'.

Units: Ohm (Ω)

Symbol: —wwv—

NOTE: Resistor dissipates the energy in the form of heat and this process is irreversible.

13/8/2018

$\Rightarrow R = \frac{\rho l}{A}$

where, ρ = resistivity / specific resistance of material ($\Omega \cdot m$)

A = Area of cross-section (m^2)

l = length of the material (m)

Conductance (G):

It is the measure of ease with which a conductor will pass the current through it.

$G = \frac{1}{R}$

Units: mho ($\mho$) or Siemen (S)

* Resistance measured for 1 m along the diagonal of a cube is called resistivity.

$\Omega \quad R \longrightarrow G \quad (R = \frac{1}{G}) \quad \mho$ (or) S

$(\Omega \cdot m) \quad \rho \longrightarrow \sigma \quad (\sigma = \frac{1}{\rho}) \quad \sigma = \frac{1}{\Omega \cdot m} = Sm^{-1}$

Resistivity conductivity

OHM'S LAW:

Ohm's law gives relation between voltage, current resistance in a element. At constant temperature, it states that the current flowing through a element is proportional to the applied voltage. provided th

$i \propto V$

$$i = \frac{V}{R}$$

$$\boxed{V = iR} \quad (or) \quad \boxed{i = VG}$$

Power , $P = VI$

$$P = (IR)I$$

$$P = I^2 R$$

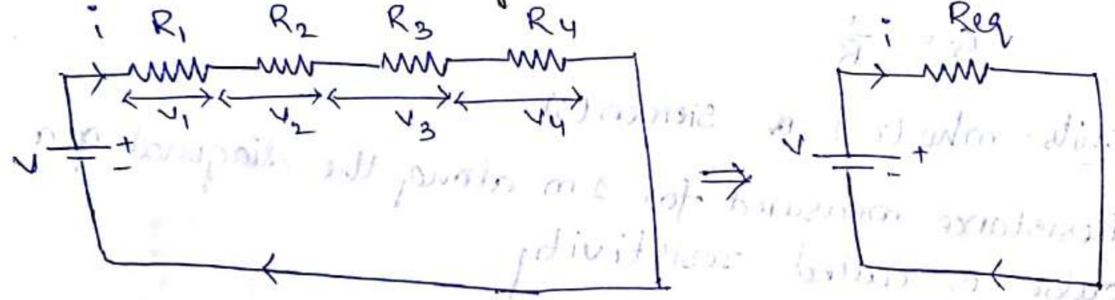
$$P = V\left(\frac{V}{R}\right)$$

$$P = \frac{V^2}{R}$$

$$\boxed{\therefore \; P = VI = I^2 R = \frac{V^2}{R}}$$

Note: When temperature increases for most of the material the resistance increases.

Resistance in Series Combination :
(voltage Divider circuit)



$$V = V_1 + V_2 + V_3 + V_4$$

$$V = iR_1 + iR_2 + iR_3 + iR_4$$

$$V = i(R_1 + R_2 + R_3 + R_4) \rightarrow (1)$$

$$V = iReq \rightarrow (2)$$

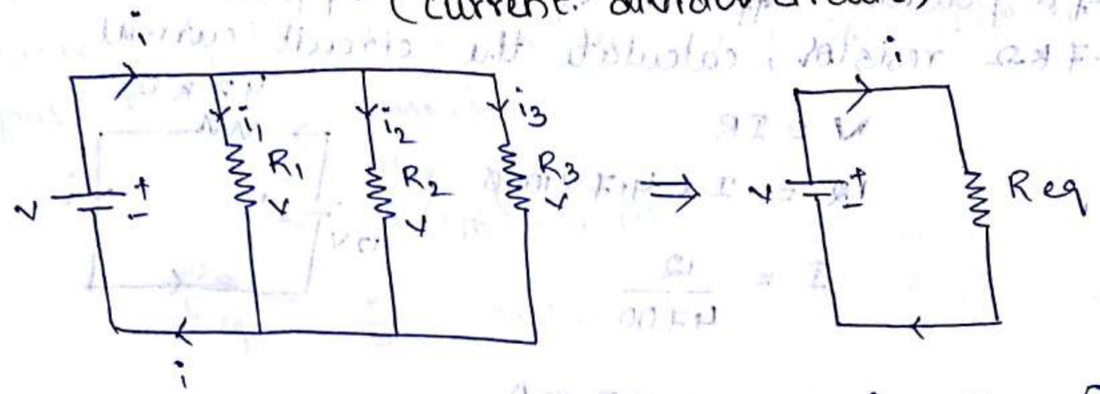
(1) & (2)

$$\cancel{i}Req = \cancel{i}(R_1 + R_2 + R_3 + R_4)$$

$$\boxed{Req = R_1 + R_2 + R_3 + R_4}$$

Resistance in Parallel Combination :

(current divider circuit)



$$i = i_1 + i_2 + i_3 \qquad i = \frac{V}{Req} \rightarrow ②$$

$$i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

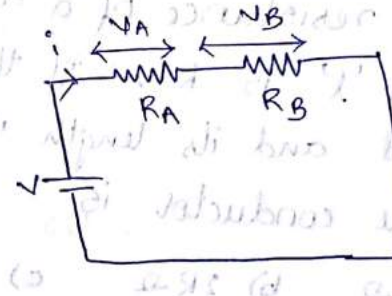
$$i = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \rightarrow ①$$

① & ②

$$\frac{\cancel{V}}{Req} = \cancel{V}\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\boxed{\frac{1}{Req} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Voltage division formula for two elements :

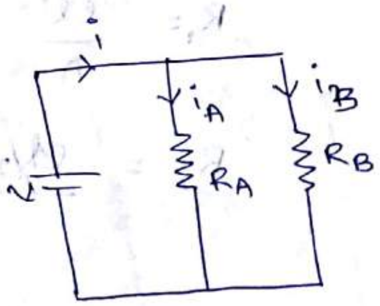


$$V = V_A + V_B$$

$$V_A = V \cdot \frac{R_A}{R_A + R_B}$$

$$V_B = V \cdot \frac{R_B}{R_A + R_B}.$$

Current division formula for two elements :



$$i = i_A + i_B$$

$$i_A = i \cdot \frac{R_B}{R_A + R_B}$$

$$i_B = i \cdot \frac{R_A}{R_A + R_B}$$

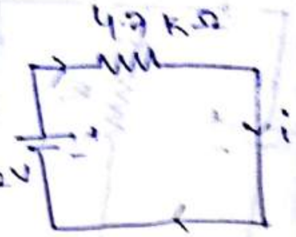
Short Answer Questions :

1) If a potential difference of 12V is applied to a 4.7 kΩ resistor, calculate the circuit current.

$V = IR$

$12 = I \times 4.7 \times 1000$

$I = \frac{12}{4700}$



$I = 2.55\ mA$

2) If 1A of current flows in a circuit the number of electrons flowing in the circuit is

a) 0.625×10^{19} b) 1.6×10^{19} c) 1.6×10^{-19} D) 0.625×10^{-19}

$i = \frac{q}{t}$

$1 = \frac{ne}{1}$

$1 = n \times 1.6 \times 10^{-19}$

$\frac{10}{16} \times 10^{19} = n$

$0.625 \times 10^{19} = n$

3) If resistance of a conductor of diameter 'd' and length 'l' is R Ω. If the diameter of the conductor is halved and its length is doubled, the resistance of the conductor is

a) R Ω b) 2R Ω c) 4R Ω d) 8R Ω

$R_1 = \frac{\rho l}{A} = \frac{\rho l}{\pi \left(\frac{d}{2}\right)^2} = R \Rightarrow \frac{4\rho l}{\pi d^2} = R$

$R_2 = \frac{\rho l/2}{} \quad \frac{\rho (2l)}{\pi \left(\frac{d}{4}\right)^2}$

$= \frac{2 \rho l (16)}{\pi d^2} = 8 \frac{(4 \rho l)}{\pi d^2}$

$= 8R\ \Omega$

4) The current flowing in a series circuit having four equal resistances is I A. What is the magnitude of current if the four resistances are connected in parallel?

case i

$$V = i R_{eq}$$

$$V = i(4R) \rightarrow ①$$

$$R_{eq} = \frac{R}{4} \quad \text{case ii}$$

$$\Rightarrow \quad V = i_2\left(\frac{R}{4}\right)$$

$$4iR = i_2 \frac{\cancel{R}}{4}$$

$$i_2 = 16i$$

5) The value of resistance R is 10 Ω. If the length is doubled and area of cross section becomes half, then resistivity of the element is

a) increases b) decreases c) remains constant d) None

Remains constant.

6) How many coloumbs of charge flows through a circuit carrying a current of 10 A in 1 min?

a) 10 b) 60 c) 600 d) 1200

$$i = \frac{q}{t}$$

$$10 = \frac{q}{60} \Rightarrow q = 600\ C$$

Long Answer Questions:

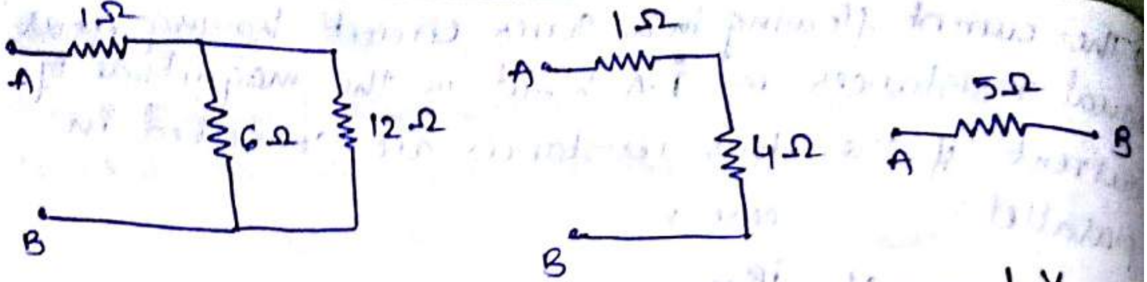
1) Find the equivalent resistance between the terminals A and B.

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{6}$$

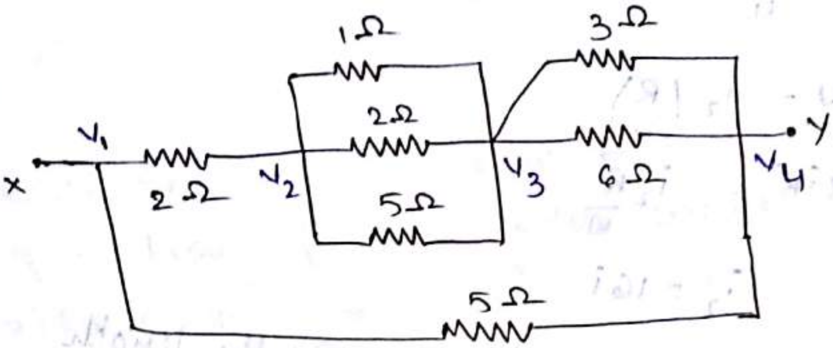
$$R' = \frac{\cancel{12}^{4} \times \cancel{6}}{\cancel{18}_{3}}$$

$$R' = 4$$

$$R_{eq} = 4 + 1 = 5\ \Omega$$



2) Find the equivalent resistance between x and Y terminals of the given circuit.

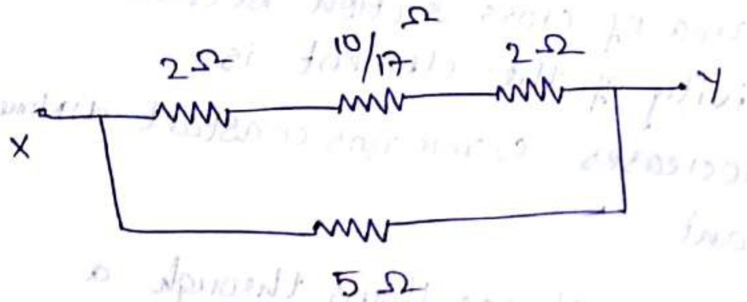


$$\frac{1}{R_1} = \frac{1}{1} + \frac{1}{2} + \frac{1}{5}$$

$$\frac{1}{R_1} = \frac{10+5+2}{10}$$

$$\frac{1}{R_1} = \frac{17}{10}$$

$$R_1 = \frac{10}{17}\ \Omega$$



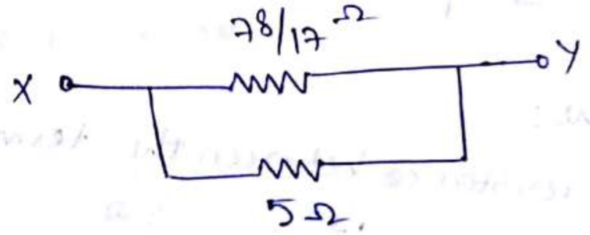
$$\frac{1}{R_2} = \frac{1}{3} + \frac{1}{6}$$

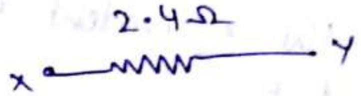
$$\frac{1}{R_2} = \frac{\cancel{3}}{\cancel{6}_2}$$

$$R_2 = 2\ \Omega$$

$$R_3 = 4 + \frac{10}{17}$$

$$R_3 = \frac{68+10}{17} = \frac{78}{17}\ \Omega$$





$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{17}{78}$$

$$\frac{1}{R_{eq}} = \frac{78+85}{390}$$

$$R_{eq} = \frac{390}{163} = 2.39\ \Omega$$
$$\simeq 2.4\ \Omega$$

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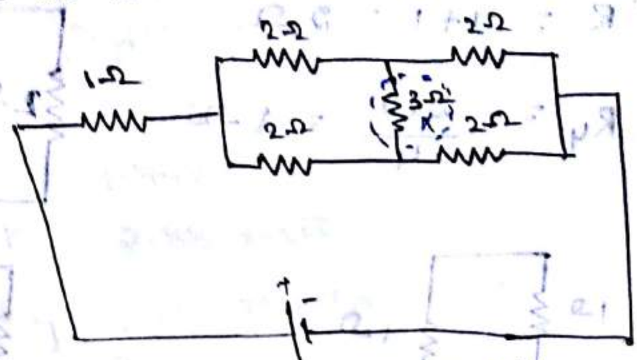
3) Find the power loss in 1Ω resistor shown in the figure.



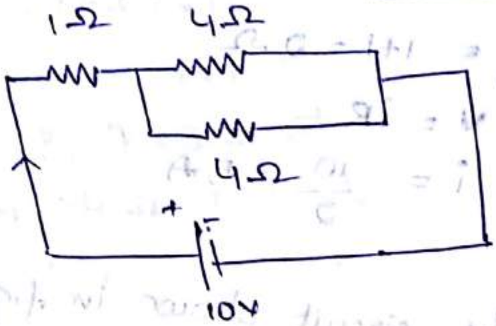
What stone's Bridge

$R_{eq} = 6\Omega$

$P = \frac{V^2}{R_{eq}}$

$P = \frac{10\times 10}{6}$

$P = \frac{100}{6}$



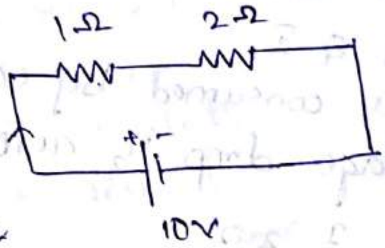
$P = i^2 R$

$\frac{100}{6} = i^2 \times 1$

~~$R_{eq} = 1+2 = 3\Omega$~~

$P = \frac{V^2}{R_{eq}}$

$P = \frac{10\times 10}{3} = \frac{100}{3}$



~~Power loss in 1Ω resistor~~

$i =$

$R_{eq} = 1+2 = 3\Omega$

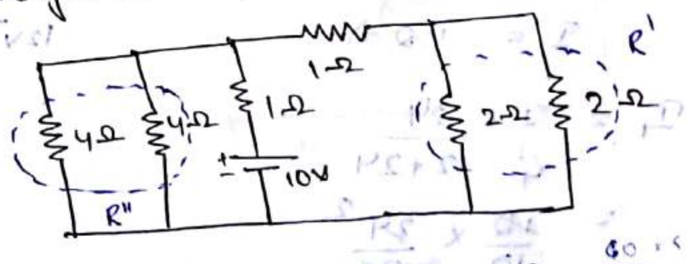
$i = \frac{V}{R_{eq}} = \frac{10}{3}$

Power loss in 1Ω resistor $P = i^2 R$

$= \frac{100}{9} \times 1$

$= 11.11$ watt

4) What is the magnitude of current drained from 10V source.



$R' = \frac{2\times 2}{2+2}$

$= 1\Omega$

$R'' = \frac{4\times 4}{4+4}$

$= 2\Omega$

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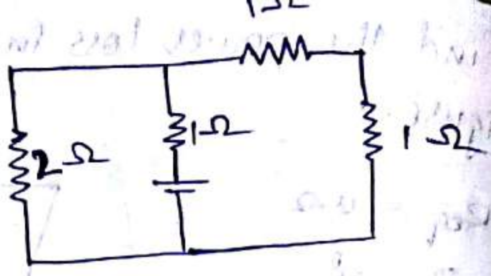
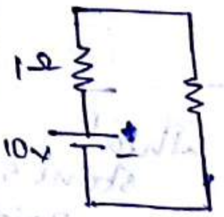
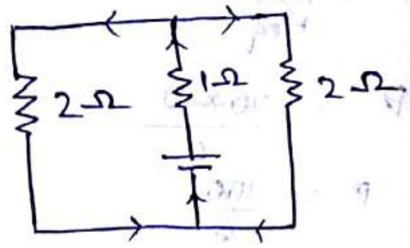
$R''' = 1+1 = 2\Omega$



$R_4 = \frac{2\times2}{4} = 1\Omega$

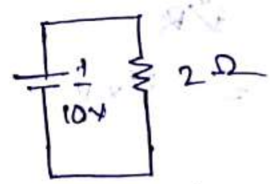




$R_{eq} = 1+1 = 2\Omega$

$V = iR$

$i = \frac{10}{2} = 5A$

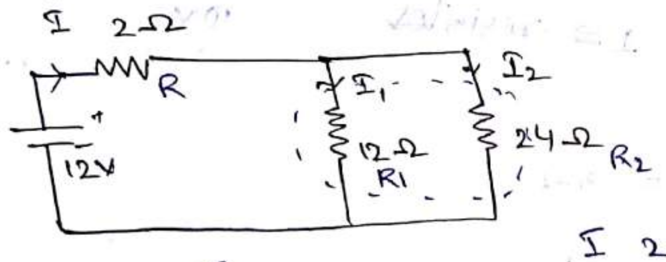


5) In the circuit shown in figure, calculate

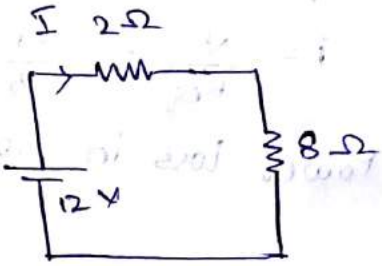
a) I, I_1 & I_2

b) power consumed by each resistor

c) voltage drop V_2 across 2Ω resistor



$R' = \frac{\cancel{12}\times\cancel{24}^{8}}{\cancel{36}_{3}} = 8\Omega$

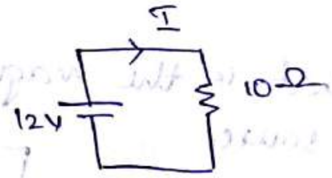


$R_{eq} = 2+8 = 10\Omega$

a) $V = IR$

$12 = I\times10$

$I = 1.2A$



$I_1 = I\times\frac{24}{12+24}$

$= \frac{\cancel{12}^{4}}{10}\times\frac{\cancel{24}^{2}}{\cancel{36}_{3}}$

$= \frac{8}{10}$

$= 0.8A$

$\therefore I_2 = (1.2 - 0.8)A$

$= 0.4A$

b) Power consumed by 2Ω resistor

$$P = i^2 R$$
$$P = (1.2)^2\, 2$$
$$P = 1.44 \times 2$$
$$P = 2.88 \text{ Watt}$$

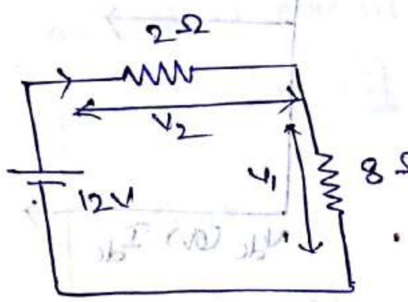
Power consumed by 12Ω resistor

$$P_1 = i_1^2 R_1$$
$$P_1 = (0.8)^2 \times 12$$
$$= 0.64 \times 12$$
$$= 7.68 \text{ Watt}$$

Power consumed by 24Ω resistor

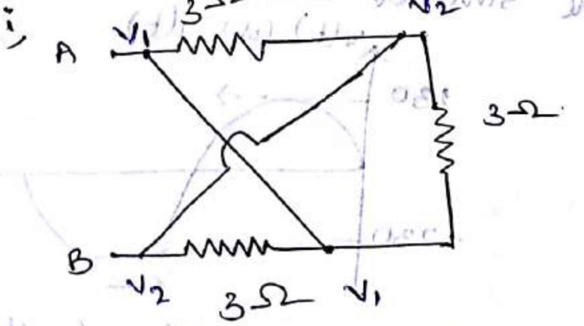
$$P_2 = i_2^2 R_2$$
$$= (0.4)^2 \times 24$$
$$= 0.16 \times 24$$
$$= 3.84 \text{ Watt}$$

c) voltage drop across 2Ω resistor

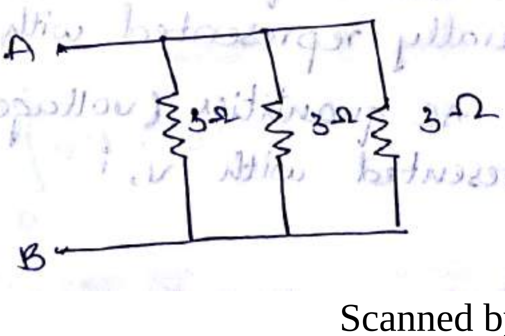


$$V_2 = 12 \times \frac{2}{2+8}$$
$$= 12 \times \frac{2}{10}$$
$$= \frac{24}{10}$$
$$= 2.4V$$

6) Find the resistance between A and B terminals



$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$
$$\frac{1}{R_{eq}} = 1$$
$$R_{eq} = 1\Omega$$



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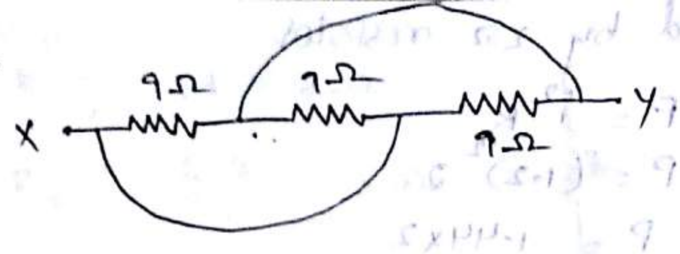
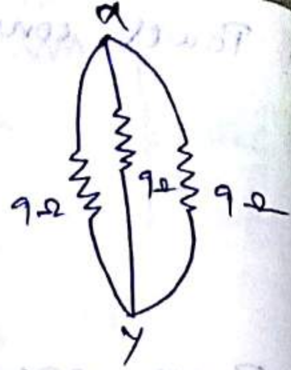
ii,





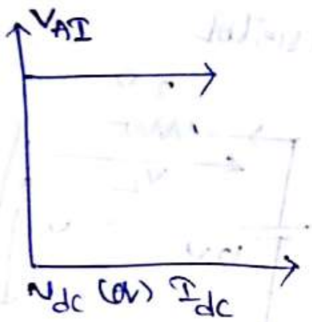
$$\frac{1}{R_{eq}} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$$

$$\frac{1}{R_{eq}} = \frac{3}{9}$$

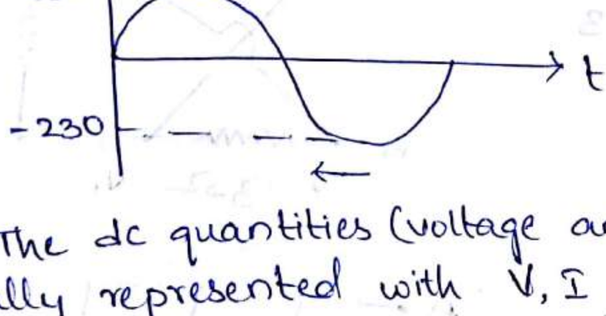
$$R_{eq} = 3\Omega$$

DC Supply and AC Supply:

DC Current: Current that flows continously in one direction with constant magnitude is called direct current. This is the kind of current supplied from the battery.



AC current: current that flows first in one direction for a time and reverse the direction of flow for the similar time.

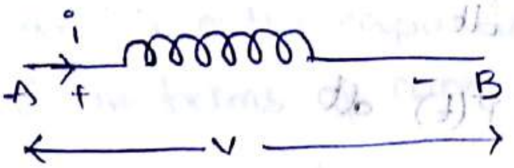


NOTE: The dc quantities (voltage and current) are generally represented with V, I

AC quantities (voltage and current) are represented with v, i

INDUCTOR :

A conductor of certain length, when twisted like a coil (spring) becomes a basic inductor. Inductance is represented by the letter 'L' and the units of inductance is Hendry (H)



spring/coil

The total flux is directly proportional to current.

flux linkage (ψ) α i

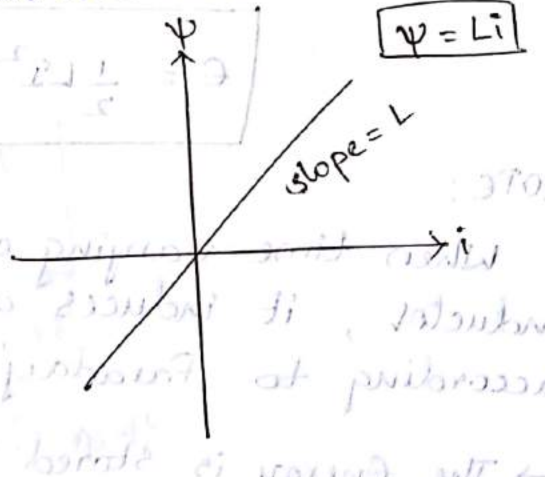
$$\boxed{\psi = Li}$$

which is input-output reaction for inductor

NOTE: For DC currents $\frac{di}{dt}$ is zero that means voltage across the inductor is zero.

→ For DC currents inductor acts as a short circuit.

$$\boxed{\psi = Li}$$



$$\boxed{v = L\frac{di}{dt}}$$

$$\Rightarrow di = \frac{1}{L} v \cdot dt$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v\, dt$$

$$i = \frac{1}{L}\int_{0}^{t} v\, dt + i(0)$$

$$\boxed{i = \frac{1}{L}\int_{0}^{t} v\, dt + i(0)}$$

$$v(t) = L \cdot \frac{di(t)}{dt}$$

$$\int d\, i(t) = \frac{1}{L}\int_{0}^{t} v(t)\, dt$$

$$i(t) = \frac{1}{L}\int_0^t v(t)\,dt$$

$$\text{Power} = v(t) \cdot i(t)$$

$$P(t) = L \cdot \frac{di(t)}{dt}\, i(t)$$

$$E \text{ (or) } W = \int P(t)\,dt$$

$$E = \int L \frac{di(t)}{dt}\, i(t) \cdot dt$$

$$E = L \int i(t)\, d\,i(t)$$

$$E = L \frac{d[i(t)]^2}{2}$$

$$\boxed{E = \frac{1}{2} L\, i(t)^2}$$

$$\boxed{E = \frac{1}{2} L I^2 \text{ in joules}}$$

NOTE:

When time varying current flows in an inductor, it induces a voltage in the inductor according to Faraday's law.

→ The Energy is stored in the inductor in the form of magnetic field.

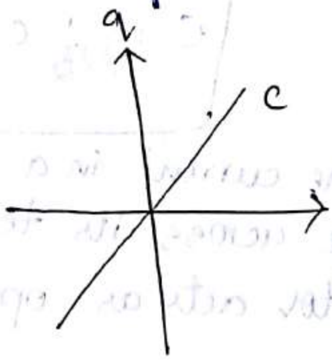
→ The stored energy E is $\boxed{E = \frac{1}{2} L I^2}$

Capacitor:

A capacitor consists of 2 parallel conducting plates (electrodes) separated by any insulating material called dielectric.

The ability of the capacitor to store charge is measured in terms of capacitance.

Capacitance is defined as charge stored per volt applied.



$$\boxed{C = \frac{q}{v}} \Rightarrow q = cv$$

$$i = \frac{dq}{dt}$$

$$i = \frac{d}{dt}(cv)$$

$$\boxed{i = c \cdot \frac{dv}{dt}}$$

$$\boxed{i(t) = c \cdot \frac{d\,v(t)}{dt}}$$

$$\Rightarrow dv = \frac{1}{c} \cdot i \cdot dt$$

$$v = \frac{1}{c} \int_{-\infty}^{t} i(t)\,dt$$

$$v = \frac{1}{c} \int_{0}^{t} i(t)\,dt + v(0)$$

$$\boxed{v = \frac{1}{c} \int_{0}^{t} i(t)\,dt + v(0)}$$

$$P(t) = v(t) \times i(t)$$

$$= v(t) \times c \cdot \frac{dv(t)}{dt}$$

$$E = \int P(t)\, dt$$

$$E = \int v(t) \times c \cdot \frac{dv(t)}{\cancel{dt}} \cdot \cancel{dt}$$

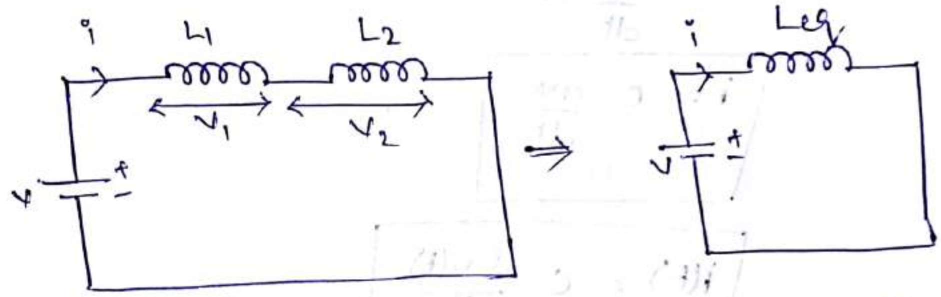
$$E = c \int v(t)\, d\,v(t)$$

$$E = \frac{c\,[v(t)]^2}{2}$$

$$\boxed{E = \frac{1}{2} C v^2}$$

Note: The current in a capacitor is zero if the voltage across its terminals is constant ie., capacitor acts as open circuit to DC.

Inductors in series :



By KVL,

$$v = L_{eq} \frac{di}{dt} \rightarrow ②$$

$$v = V_1 + V_2$$

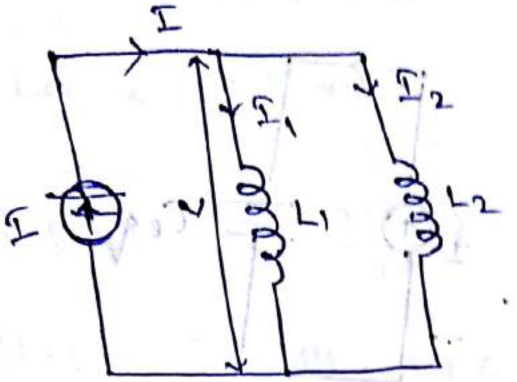
$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \rightarrow ①$$

$$\Rightarrow \frac{di}{dt} L_{eq} = L_1 \frac{di}{dt} + L_2 \cdot \frac{di}{dt}$$

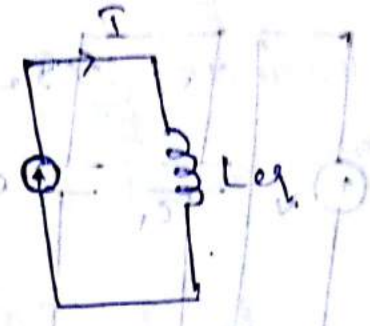
$$\Rightarrow L_{eq} \frac{di}{dt} = \frac{di}{dt}(L_1 + L_2)$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2}$$

Inductors in parallel:



⇒



$$I = \frac{1}{L_{eq}} \int v\,dt \to (4)$$

By KCL,

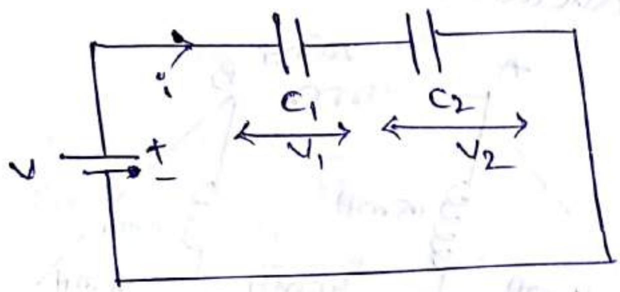
$$I = I_1 + I_2$$

$$I = \frac{1}{L_1} \int v\,dt + \frac{1}{L_2} \int v\,dt \to (3)$$

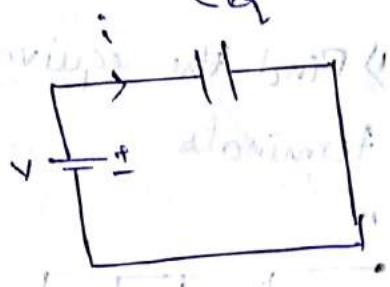
$$\Rightarrow \frac{1}{L_{eq}} \int v\,dt = \frac{1}{L_1} \int v\,dt + \frac{1}{L_2} \int v\,dt$$

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

Capacitors in series:



⇒



$$V = \frac{q}{C_{eq}} \to (ii)$$

By KVL,

$$V = V_1 + V_2$$

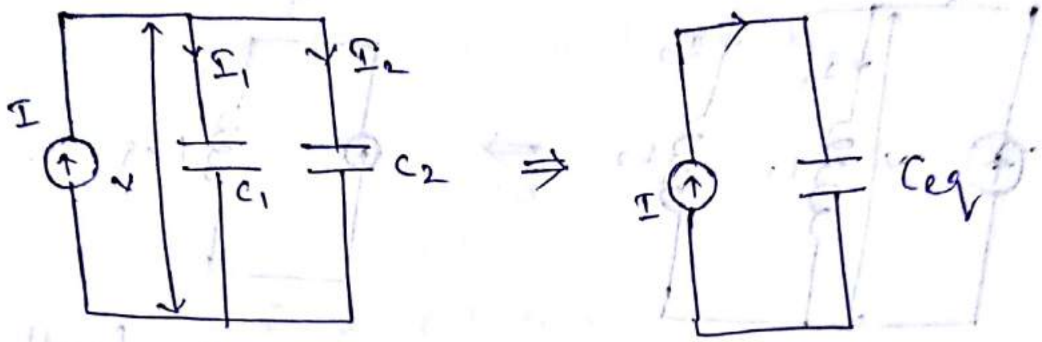
$$V = \frac{1}{C_{eq}} \int i\,dt \to (2)$$

$$V = \left(\frac{q}{C_1} + \frac{q}{C_2}\right) \qquad \int i\,dt \to (1)$$

$$\Rightarrow \frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Capacitors in parallel:



$$I = I_1 + I_2$$

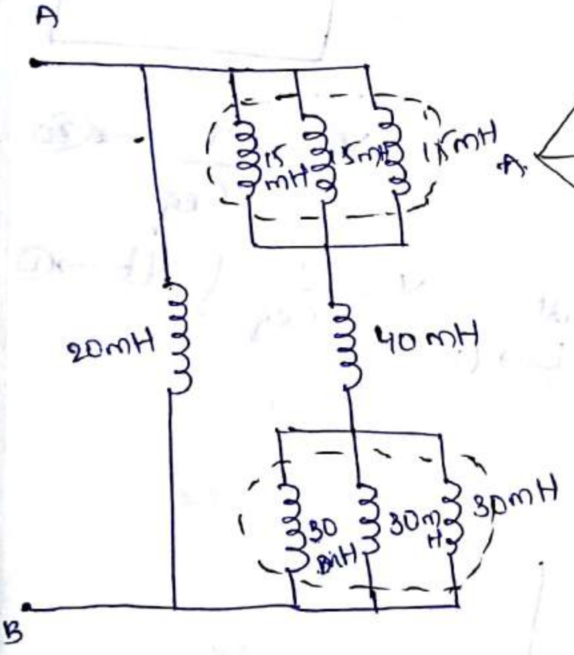
$$I = C_{eq} \frac{dv}{dt} \rightarrow (2)$$

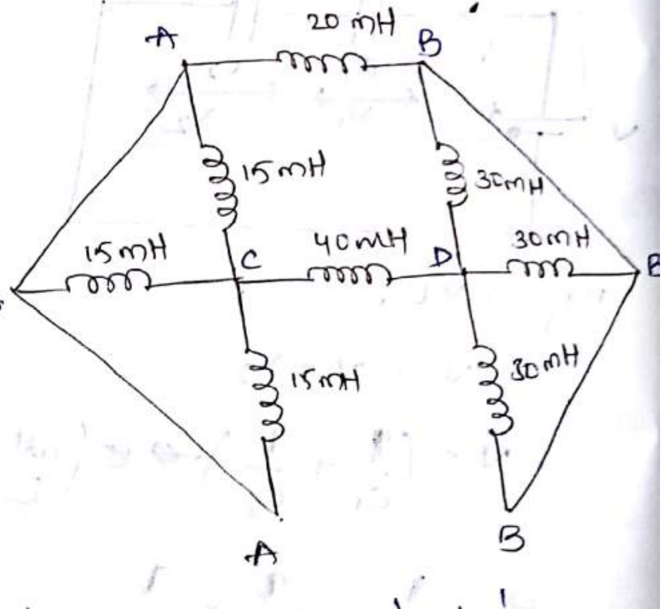
$$I_t = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} \rightarrow (1)$$

$$\Rightarrow \quad C_{eq} \frac{dv}{dt} = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$

$$\boxed{C_{eq} = C_1 + C_2}$$

1) Find the equivalent inductance between A and B terminals.





$$\frac{1}{(L_{eq})_1} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$$

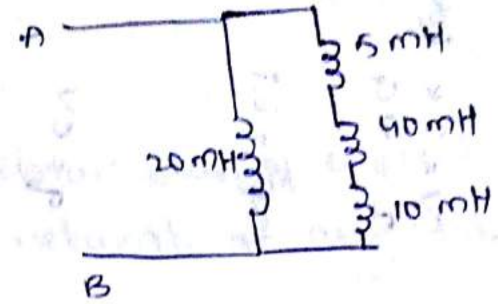
$$(L_{eq})_1 = 5 \text{ mH}$$

$$\frac{1}{(L_{eq})_2} = \frac{1}{30} + \frac{1}{30} + \frac{1}{30}$$

$$(L_{eq})_2 = 10 \text{ mH}$$

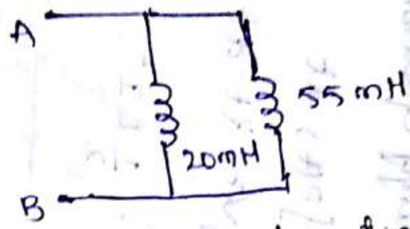
$(L_{eq})_3 = 5 + 40 + 10$

$(L_{eq})_3 = 55 \text{ mH}$

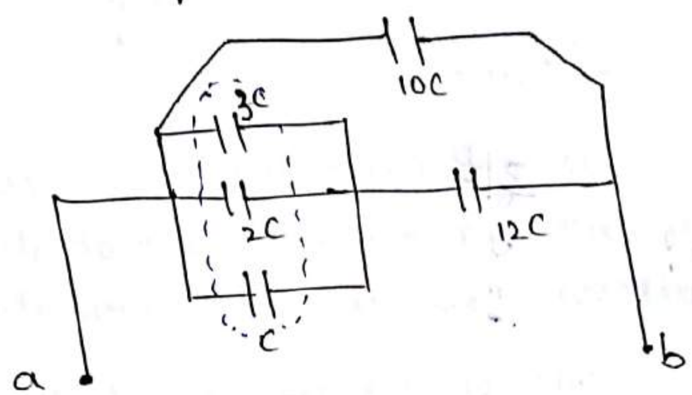


$(L_{eq})_4 = \frac{\cancel{4}}{\cancel{3}} \frac{20 \times 55}{75}$

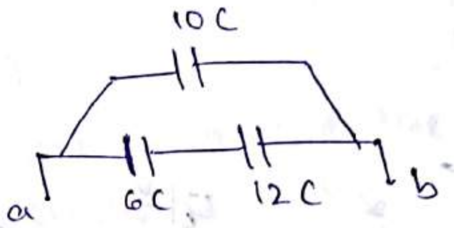
$(L_{eq})_4 = \frac{44}{3} = 14.6 \text{ mH}$



2) Find the equivalent capacitance of the combination

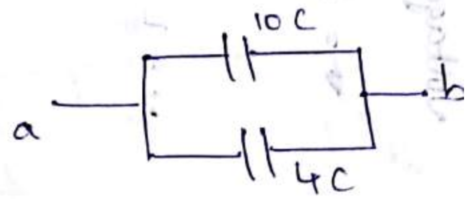


$C_{eq} = C + 2C + 3C = 6C$

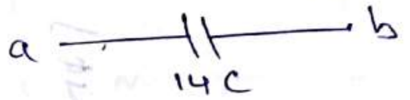


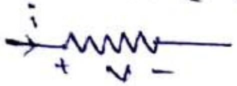
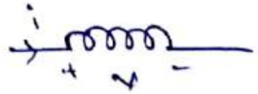
$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12}$

$C_{eq} = \frac{\cancel{12}^{4} \times \cancel{6}}{\cancel{18}_{3}} = 4C$



$C_{eq} = 10 + 4 = 14C$

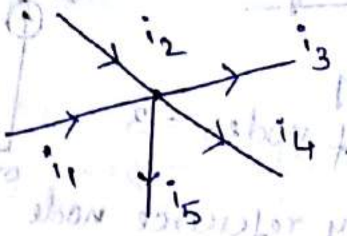


Element	Basic Relation	Voltage equation	Current Equation	Energy equation	i/p - o/p relation
Resistor (Ω) 	$R = \frac{V}{I}$	$v = IR$	$I = \frac{V}{R}$	$W = \int P(t)\,dt = \int v(t)\,i(t)\,dt$	$V = IR$ (or) $I = VG$
Inductor (H) 	$L = \frac{N\phi}{i}$ $[\psi = Li = N\phi]$	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt$	$W = \frac{1}{2}LI^2$	$\psi = Li$
capacitor (F) 	$C = \frac{q}{v}$	$v = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$	$i = C\frac{dv}{dt}$	$W = \frac{1}{2}CV^2$	$q = CV$

Kirchoff's Laws:

1) Kirchoff's current law (KCL) :

It states that the algebraic sum of currents at a node (junction) in a network at any instant of time is zero.



$$\boxed{\sum_{j=1}^{n} i_j = 0}$$

$$\boxed{+i_1 + i_2 - i_3 - i_4 - i_5 = 0}$$

$$i_1 + i_2 = i_3 + i_4 + i_5$$

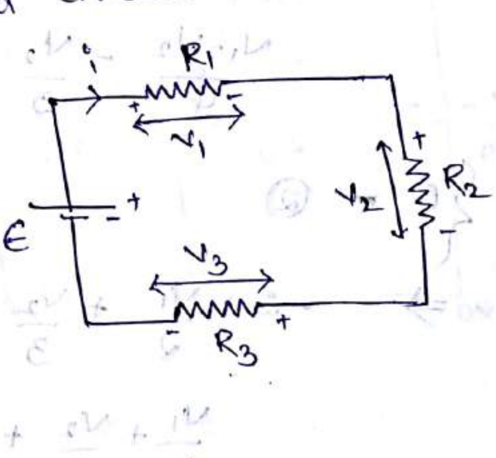
Note: It is general convention to consider entering currents to the node as positive and leaving currents from the node are considered as negative (-)

→ KCL is the consequence of conservation of charge.

2) Kirchaff's voltage Law (KVL) :

It states that at any instant of time the sum of voltages in a closed circuit is zero.

$$\boxed{\sum_{j=1}^{n} V_j = 0}$$



$$+\epsilon - V_1 - V_2 - V_3 = 0$$

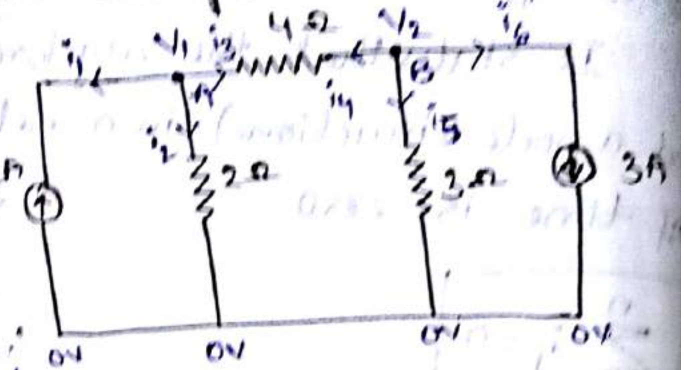
$$\boxed{\epsilon = V_1 + V_2 + V_3}$$

Note :

1) If the voltage drops, +ve to -ve is assigned a '-ve' sign and if the voltage rise from -ve to +ve, it is assumed as '+ve'.

2) KVL is the consequence of the fact that no energy is lost / created in a electric circuit which is based on L.C.E

Q→ Using the Nodal analysis for the circuit diagram given below, find the voltages and currents in the resistors.



Nodal-Analysis:

i, Identify

no. of nodes = 3

ii, Identify reference node

iii, Find V_1, V_2

KCL at point A:

$$i_1 + i_2 + i_3 = 0$$

$$-\cancel{2} + \frac{\cancel{V_1}-0}{\cancel{2}} + \frac{V_{\cancel{1}}}{\cancel{4}} = \cancel{0} \qquad -2 + \frac{V_1-0}{2} + \frac{V_1-V_2}{4} = 0 \rightarrow ①$$

KCL at point B:

$$\frac{V_2-V_1}{4} + \frac{V_2-0}{3} + 3 = 0$$

$$\frac{V_1-V_2}{4} - \frac{V_2}{3} = 3 \longrightarrow ②$$

{① + ②

$$\Rightarrow \quad -2 + \frac{V_1}{2} + \frac{V_2}{3} + 3 = 0$$

$$\frac{V_1}{2} + \frac{V_2}{3} + 1 = 0 \Rightarrow \boxed{3V_1 + 2V_2 = -6} \longrightarrow ③$$

① + ②:

$$-2 + \frac{V_1}{2} + \frac{V_1-V_2}{4} + \frac{V_1-V_2}{4} - V_2 - 3 = 0$$

$$\frac{V_1-V_2}{2} + \frac{V_1-2V_2}{2} = 5 \}$$

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① ⇒ $\frac{V_1}{2} + \frac{V_1 - V_2}{4} = 2$

$2V_1 + V_1 - V_2 = 8$

$3V_1 - V_2 = 8 \longrightarrow$ ③

② ⇒ $\frac{3V_1 - 3V_2 - 4V_2}{12} = 3$

$3V_1 - 7V_2 = 36$

$-3V_1 + 7V_2 = -36 \longrightarrow$ ④

③ + ④ ⇒ $V_2 = \frac{-28}{6} = -4.6\,V$

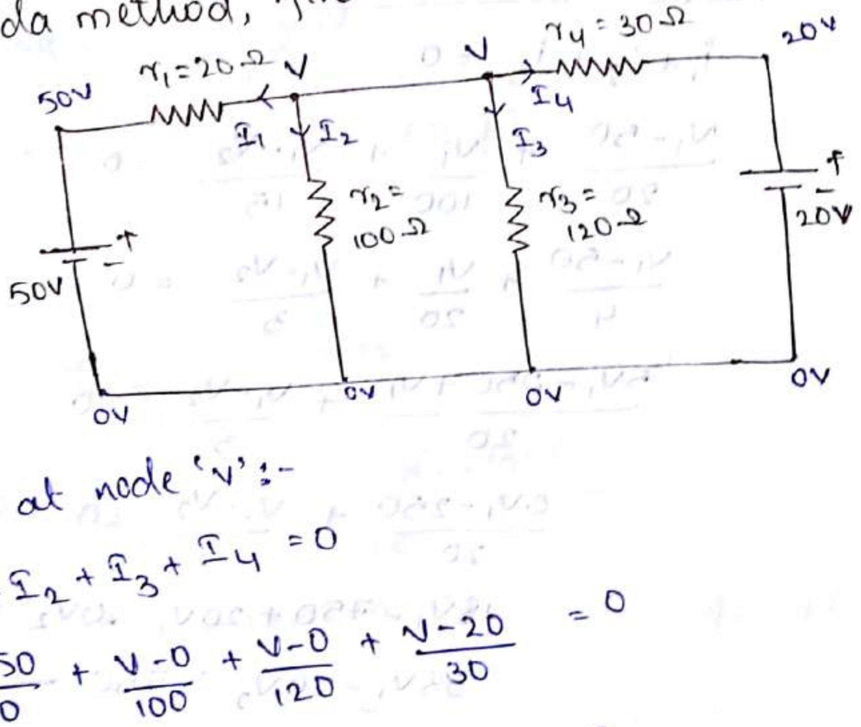
③ ⇒ $V_1 = \frac{10}{9} = 1.11\,V$

$I_{2\Omega} = \frac{V_1}{2} = \frac{1.11}{2} = 0.55\,A$

$I_{4\Omega} = \frac{V_1 - V_2}{4} = \frac{1.11 + 4.66}{4} = \frac{5.77}{4} = 1.44\,A$

$I_{3\Omega} = \frac{V_2}{3} = \frac{4.66}{3} = 1.55\,A$

Q→ Using noda method, find the current through r_2.



Apply KCL at node 'V':-

$I_1 + I_2 + I_3 + I_4 = 0$

$\frac{V-50}{20} + \frac{V-0}{100} + \frac{V-0}{120} + \frac{V-20}{30} = 0$

$\frac{V-50}{2} + \frac{V}{10} + \frac{V}{12} + \frac{V-20}{3} = 0$

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$$\frac{3V-150+2V-40}{6} + \frac{12V+10V}{120} = 0$$

$$\frac{5V-190}{6} + \frac{22V}{120} = 0$$

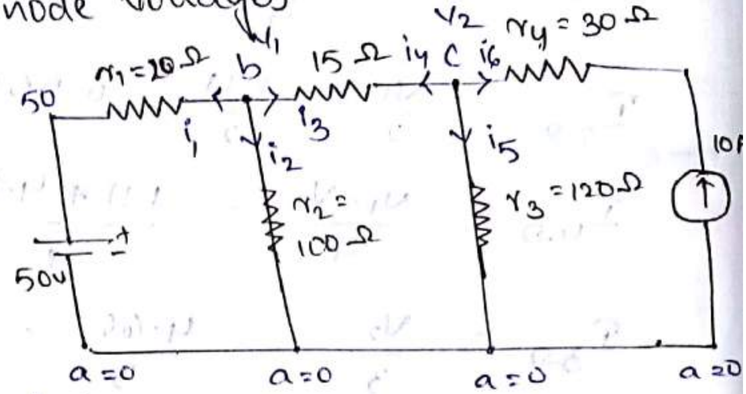
$$100V - 3800 + 22V = 0$$

$$122V = 3800$$

$$V = \frac{3800}{122} = 31.14\,V$$

$$I_{r_2} = \frac{31.14}{100} = 0.3114\,A = 311.4\,mA$$

Q→ ~~(Identify)~~ Find the node voltages in the below given circuit.



Apply KCL at node b :

$$i_1 + i_2 + i_3 = 0.$$

$$\frac{V_1-50}{20} + \frac{V_1}{100} + \frac{V_1-V_2}{15} = 0$$

$$\frac{V_1-50}{4} + \frac{V_1}{20} + \frac{V_1-V_2}{3} = 0$$

$$\frac{5V_1-250+V_1}{20} + \frac{V_1-V_2}{3} = 0$$

$$\frac{6V_1-250}{20} + \frac{V_1-V_2}{3} = 0$$

$$18V_1 - 750 + 20V_1 - 20V_2 = 0$$

$$38V_1 - 20V_2 = 750 \longrightarrow (1)$$

Apply KCL at node c:

$$i_4 + i_5 + i_6 = 0$$

$$\frac{V_2 - V_1}{15} + \frac{V_2}{120} + 10 = 0.$$

$$\frac{4V_2 - 4V_1 + V_2}{120} = -10$$

$$-4V_1 + 5V_2 = -1200$$

$$\Rightarrow 4V_1 - 5V_2 = 1200 \longrightarrow (2).$$

solving equations (1) & (2)

$$(1) \Rightarrow \quad 38V_1 - \cancel{20V_2} = 750$$

$$(2)\times 4 \Rightarrow \quad -\;16V_1 \underset{+}{-} \cancel{20V_2} = \underset{-}{4800}$$

$$22V_1 = \cancel{8880}\ -4050$$

$$V_1 = \cancel{\frac{5550}{22}}\ -\frac{4050}{22}$$

$$V_1 = \cancel{252.27\text{ volts}}\ \ -184 \text{ volts.}$$

substitute $V_1 = \cancel{252.27}$ -184 volts in eqn (2)

$$\Rightarrow \cancel{V_1} = \cancel{\frac{1200 + 5V_2}{4}}$$

$$\cancel{V_1} = \cancel{\frac{1200 + 5(252.27)}{4}}$$

$$\cancel{V_1}.$$

$$\frac{4V_1 - 1200}{5} = V_2$$

$$\Rightarrow V_2 = \frac{4(\overset{-184}{\cancel{252.27}}) - 1200}{5}$$

$$\cancel{V_2 = -38.18\text{ volts}}.$$

$$V_2 = \frac{-1936}{5}$$

$$V_2 = -387.2 \text{ volts.}$$

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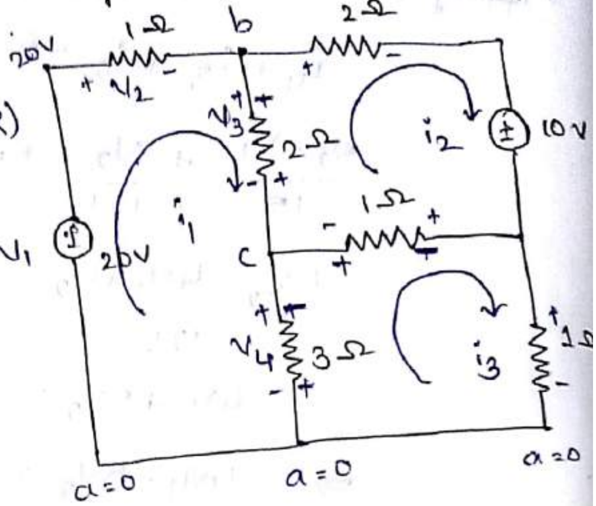
Q→ Using Mesh Current analysis, find the voltage across 3Ω resistor.



Mesh analysis (KVL + V = IR)

i, Meshes = 3

ii, ~~Assume~~ current in clockwise direction.

KVL for loop ① :

$$V_1 + V_2 + V_3 + V_4 = 0$$

$$20 - \cancel{i_1(1)} - \cancel{i_1(2)} - \cancel{i_1(3)}$$

$$20 - i_1(1) - (i_1 - i_2)2 - (i_1 - i_3)3 = 0$$

$$20 - i_1 - 2i_1 + 2i_2 - 3i_1 + 3i_3 = 0$$

$$20 - 6i_1 + 2i_2 + 3i_3 = 0$$

$$-6i_1 + 2i_2 + 3i_3 = -20 \longrightarrow ①$$

KVL for loop ② :

$$-(i_2 - i_1)2 - 2i_2 - 10 - (i_2 - i_3)1 = 0$$

$$-2i_2 + 2i_1 - 2i_2 - 10 - i_2 + i_3 = 0$$

$$2i_1 - 5i_2 + i_3 = 10 \longrightarrow ②$$

KVL for loop ③ :

$$-(i_3 - i_1)3 - (i_3 - i_2)1 - i_3(1) = 0$$

$$-3i_3 + 3i_1 - i_3 + i_2 - i_3 = 0$$

$$3i_1 + i_2 - 5i_3 = 0 \longrightarrow ③$$

For verification:

$$R_{3\times3} I_{3\times1} = V_{3\times1}$$

$$\begin{bmatrix} 6 & -2 & -3 \\ -2 & 5 & -1 \\ -3 & -1 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ -10 \\ 0 \end{bmatrix}$$

$$① \Rightarrow \quad -6i_1 + 2i_2 + 3i_3 = -20$$

$$② \times 3 \Rightarrow \quad 6i_1 - 15i_2 + 3i_3 = 30$$

$$-13i_2 + 6i_3 = 10 \longrightarrow ④$$

$$① \Rightarrow \quad -6i_1 + 2i_2 + 3i_3 = -20$$

$$③ \times 2 \Rightarrow \quad 6i_1 + 2i_2 - 10i_3 = 0$$

$$4i_2 - 7i_3 = -20 \longrightarrow ⑤$$

By solving eqn ④ and eqn ⑤

$$i_2 = 0.746 \text{ Amp}$$

$$i_3 = 3.283 \text{ Amp}$$

By substituting $i_2 = 0.746$ and $i_3 = 3.283A$ in eqn ③, we get value of i_1

$$3i_1 + 0.746 - 5(3.283) = 0$$

$$i_1 = 5.223 \text{ Amp.}$$

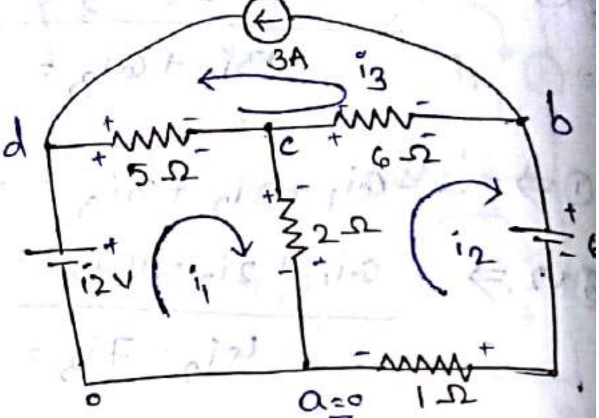
voltage in 3Ω resistor

$$V_{3\Omega} = (i_1 - i_3)3$$

$$= (5.223 - 3.283)3$$

$$= 5.82 \text{ V}$$

Q → Determine the node voltages and current through the resistors using mesh method for the network shown in figure.



Apply KVL for loop ①:

$12 - (i_1+i_3)5 - (i_1-i_2)2 = 0$

$12 - 5i_1 - 5i_3 - 2i_1 + 2i_2 = 0$

$12 - 7i_1 + 2i_2 - 5i_3 = 0$

$7i_1 - 2i_2 + 5i_3 = 12 \longrightarrow ①$

Apply KVL for loop ②:

$-6 - i_2(1) - (i_2-i_1)2 - (i_2+i_3)6 = 0$

$-6 - i_2 - 2i_2 + 2i_1 - 6i_2 - 6i_3 = 0$

$2i_1 - 9i_2 - 6i_3 = 6 \longrightarrow ②$

Apply KVL for loop ③:

From loop ③, $i_3 = 3A$.

substitute in eq^n ①

$① \Rightarrow \quad 7i_1 - 2i_2 + 15 = 12$

$7i_1 - 2i_2 = -3 \longrightarrow ③$

$② \Rightarrow \quad 2i_1 - 9i_2 - 18 = 6$

$2i_1 - 9i_2 = 24 \longrightarrow ④$

$③ \times 2 \Rightarrow \quad \cancel{14i_1} - 4i_2 = -6$

$④ \times 7 \Rightarrow \quad \underline{-\cancel{14i_1} \underset{+}{-} 63i_2 = \underset{-}{}168}$

$59i_2 = -174$

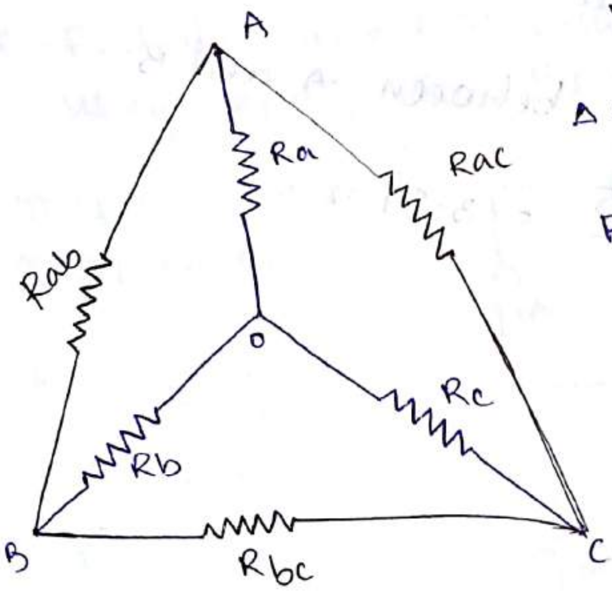
$i_2 = \frac{-174}{59} = -2.95 A$

$$③ \Rightarrow 7i_1 - 2(-2.94) = -3$$
$$7i_1 + 5.88 = -3$$
$$7i_1 = -8.88$$
$$i_1 = -1.27\,A.$$

$$I_{5\Omega} = i_1 + i_3 = -1.27 + 3 = \cancel{4.27\,A} + 1.73\,A$$
$$I_{2\Omega} = i_1 + i_2 = -1.27 + 2.94 = 1.68\,A$$
$$I_{6\Omega} = i_2 + i_3 = -2.95 + 3 = 0.05\,A$$
$$I_{1\Omega} = i_2 = -2.95\,A.$$

Voltage at node (a) = 0V
$$V_b = i_2(1) + 6$$
$$V_b = -2.95 + 6 = 3.05\,V$$
$$V_c = 1.68 \times 2 = 3.36\,V$$
$$V_d = 12\,V$$

Q→ STAR ⟷ DELTA Transformation :



$Y - R_a, R_b, R_c$

$$\Delta \rightarrow R_{ab} = \frac{R_a \cdot R_b + R_b \cdot R_c + R_c \cdot R_a}{R_c}$$

$$R_{bc} = \frac{R_a \cdot R_b + R_b \cdot R_c + R_c \cdot R_a}{R_a}$$

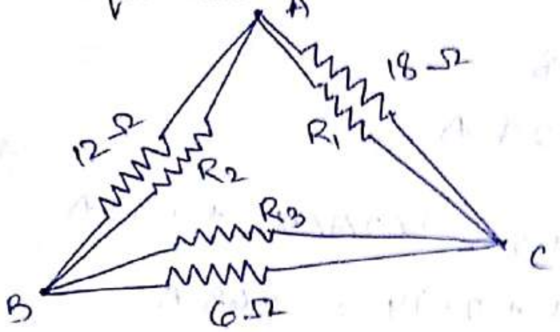
$$R_{ac} = \frac{R_a \cdot R_b + R_b \cdot R_c + R_c \cdot R_a}{R_b}$$

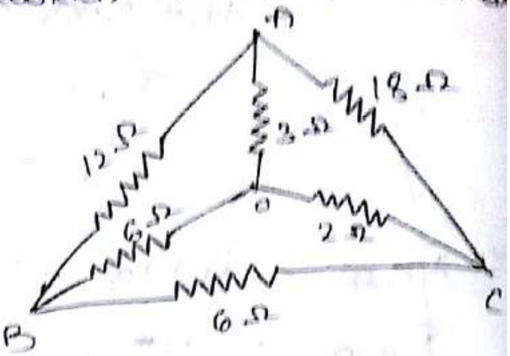
DELTA ⟷ STAR :

$$R_a = \frac{R_{ab} \cdot R_{ac}}{R_{ab} + R_{ac} + R_{bc}} \; ; \; R_b = \frac{R_{ab} \cdot R_{bc}}{R_{ab} + R_{ac} + R_{bc}}$$

$$R_c = \frac{R_{bc} \cdot R_{ac}}{R_{ab} + R_{bc} + R_{ca}}$$

Q→ For the network shown in figure, calculate the equivalent resistance between A and B terminal

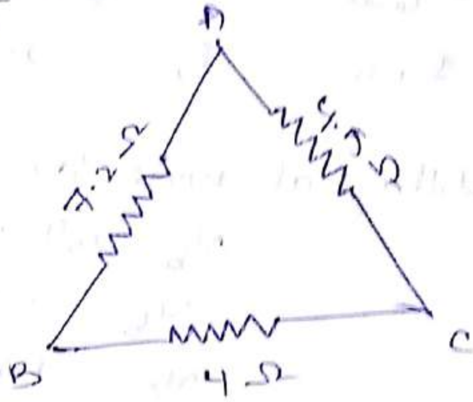




$$R_1 = \frac{6+12+18}{6} = \frac{36}{6} = 6\,\Omega$$

$$R_2 = \frac{36}{2} = 18\,\Omega$$

$$R_3 = \frac{36}{3} = 12\,\Omega$$



Equivalent resistance between A and B

$$R_{eq} = \frac{12 \times \cancel{18}3}{\frac{\cancel{30}}{5}} = \frac{36}{5} = 7.2\,\Omega$$

Equivalent resistance between A and B

$$R_{eq} = \frac{7.2 \times 8.5}{15.7} = 3.89\,\Omega$$

Network Theorems:

*1) Super position theorem

*2) Thevenin's theorem

*3) Norton's theorem

4) Maximum power transfer theorem

5) Millman's theorem

6) Tellegen's theorem

7) Substitution theorem

8) Compensating theorem

Super position theorem:

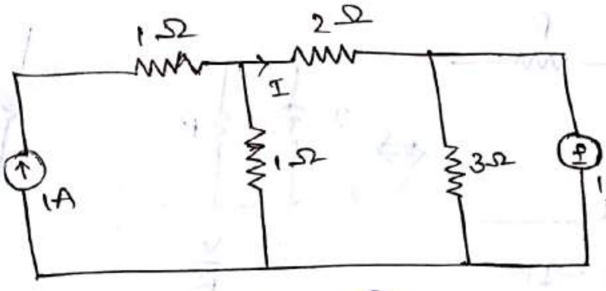
Stmt: If number of voltage (or) current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the currents that would be produced in it when each source acts alone. By replacing all the other sources by their internal resistances.

Total response = Sum of individual responses.

Note:

1. This theorem is applicable for DC and AC circuits
2. voltage source is replaced with short circuit and current source is replaced with open circuit,
3. The practical sources are replaced with their internal resistances.

Q→ Find the current 'I' in the circuit shown below using super position theorem.

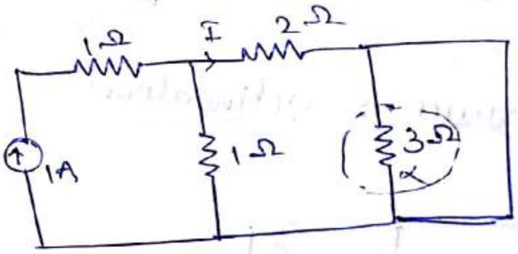


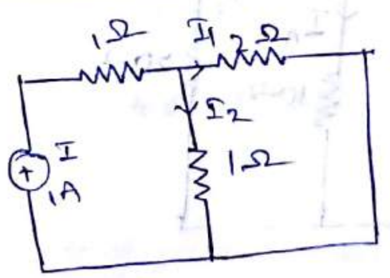
1A current source is acting alone:

$I_1 = I \cdot \frac{1}{1+2}$

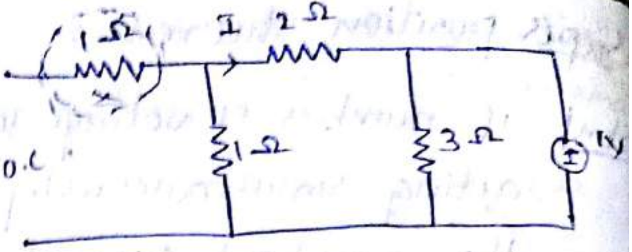
$I_1 = 1 \times \frac{1}{3}$

$I_1 = \frac{1}{3}$ A.





1V voltage source is acting alone:



$$I_1 = I \cdot \frac{3}{3+3}$$

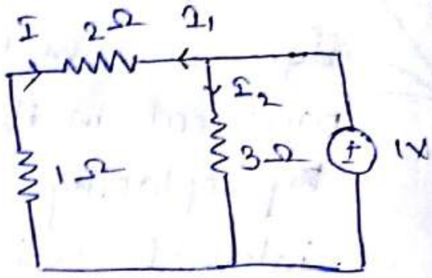
$$I_1 = \frac{I}{2}$$

$$I_1 = \frac{1}{2} \times \frac{2}{3}$$

$$I_1 = \frac{1}{3}\ A$$

$$I = \frac{V}{R}$$

$$I = \frac{1(2)}{3}$$

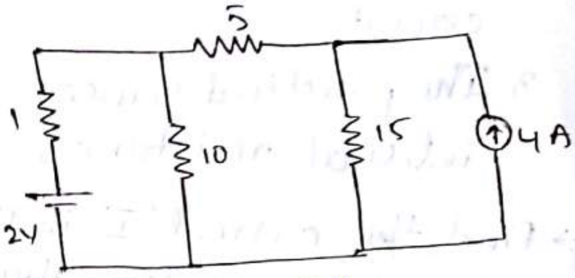


From the two cases

$$I = I_1(\text{case i}) + I_1(\text{case ii})$$

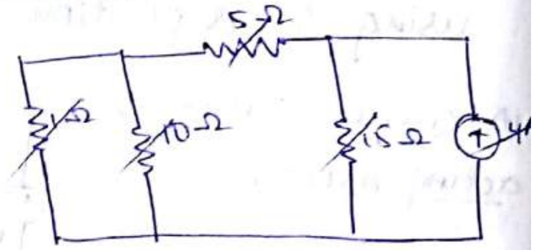
$$I_1 = \frac{1}{3} + \left(-\frac{1}{3}\right) = 0\ \text{Amps}$$

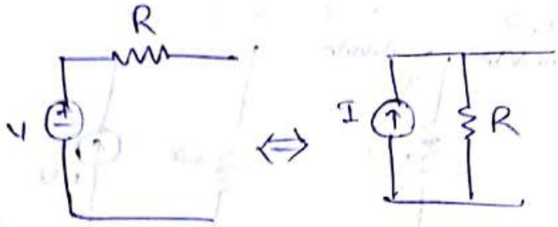
Q→ Solve the network for power delivered to 10 Ω resistor in the circuit shown in figure. All the resistances are in Ohm.



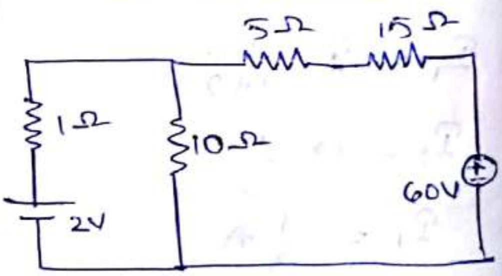
When only 4A current source is acting alone:

Source Transformations

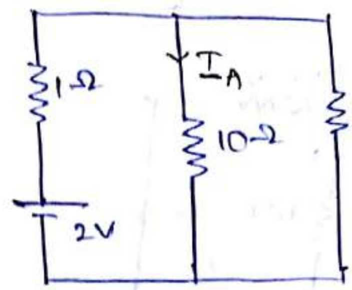


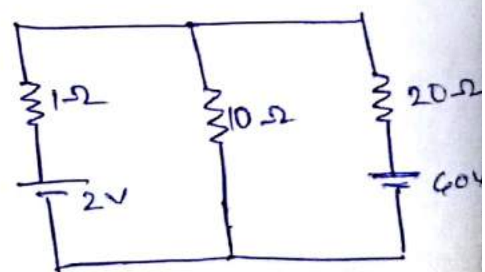


$V = IR$ $\qquad$ $I = \frac{V}{R}$



2V source is acting alone:





$$I = \frac{V}{R_{eq}}$$

$$R' = \frac{10 \times 20}{30} = \frac{20}{3}$$

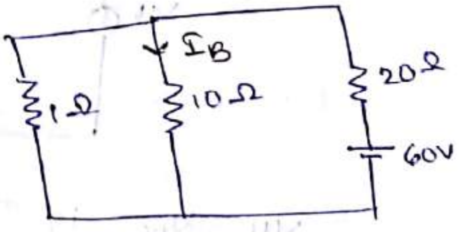
$$R_{eq} = \frac{20}{3} + 1 = \frac{23}{3} = 7.66\ \Omega$$

$$I = \frac{2}{7.66} = 0.2610\ A$$

$$I_A = I \cdot \frac{20}{30} = 0.2610 \times \frac{2}{3}$$

$$I_A = 0.174\ A$$

<u>60V source is acting alone:</u>



$$I = \frac{V}{R_{eq}}$$

$$R' = \frac{1 \times 10}{11} = \frac{10}{11}$$

$$R_{eq} = \frac{10}{11} + 20 = 0.9 + 20 = 20.9\ \Omega$$

$$I = \frac{60}{10} \times 11 \simeq 66\ A$$

$$I_B = I \cdot \frac{1}{11} = 66 \times \frac{1}{11} = 6\ A$$

$$I = \frac{60}{20.9} = 2.87\ A$$

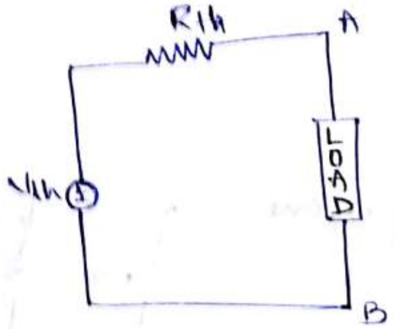
$$I_B = I \cdot \frac{1}{11} = \frac{2.87}{11} = 0.26\ A$$

Total current through 10 Ω resistor $= 0.174 + 0.260 = 0.434\ A$

Power in 10 Ω resistor $= I^2 R = (0.434)^2 \times 10 = 1.88$ Watt

Thevenin's Theorem :

Stmt :- Any linear and bilateral two terminal network consisting of independent and dependent sources and passive elements can be replaced by an equivalent circuit consisting of voltage source with a series resistance.

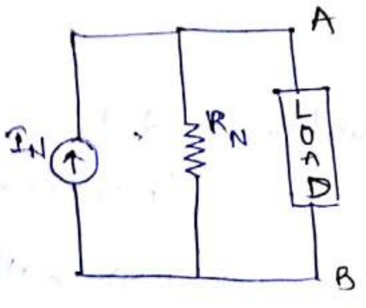


$V_{th} = V_{oc}$ is Thevenin's equivalent voltage (or) open circuit voltage across the load terminals (A-B)

R_{th} is Thevenin's equivalent resistance when looking through load terminals.

Norton's Theorem :

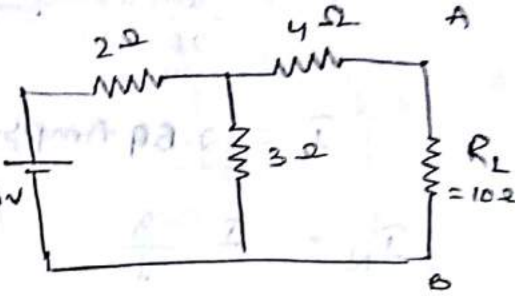
Stmt :- Any linear and bilateral two terminal network consisting of independent and dependent sources and passive elements can be replaced by an equivalent circuit consisting of one current source with parallel resistor.



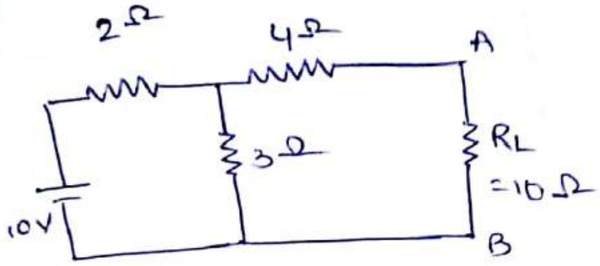
I_{sc} / I_N = Norton's current (or) short circuit current across load.

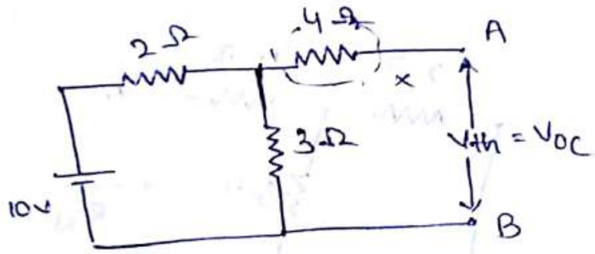
R_N = Norton's equivalent resistance when looking through load terminals (A-B)

Q→ Find the Thevenin's and Norton's equivalent of the network shown below, when the load resistance is 10 Ω. And find the load current.



Thevenin's theorem :





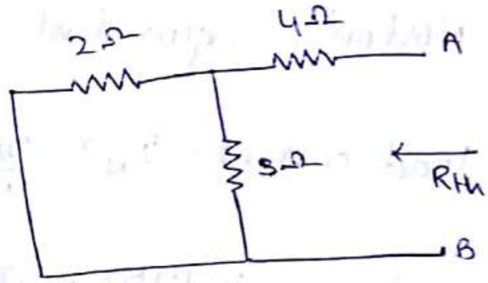
$R_{eq} = 3 + 2 = 5\ \Omega$

$I = \frac{10}{5} = 2\ \text{Amp}.$

$I_{3\Omega} = 2\ \text{Amp}$

$V_{3\Omega} = I_{3\Omega} \times 3$
$= 2 \times 3 = 6\ \text{V}.$

$\therefore V_{th} = V_{OC} = 6\ \text{V}$

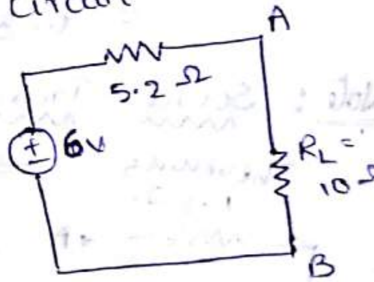


$R_{th} = 4 \,||\, \frac{6}{5}$

$= 4 + \frac{6}{5} = \frac{26}{5}$

$= 5.2\ \Omega$

Thevenin's equivalent circuit.

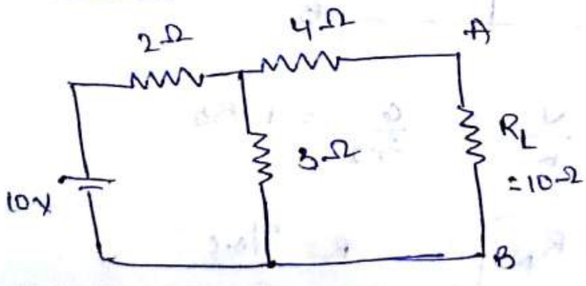


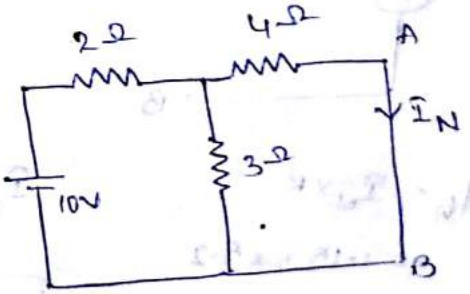
load current $= \frac{V_{th}}{R_{th} + R_L}$

$= \frac{6}{15.2}$

$= 0.394\ \text{Amps}$

Norton's theorem :





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$$R_{eq} = 2 + \frac{12}{7} = \frac{26}{7}$$

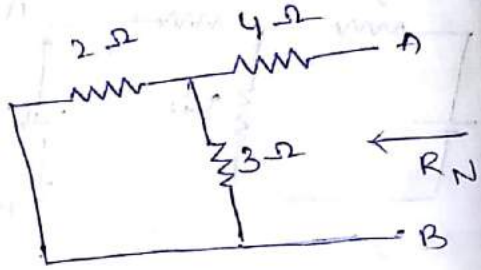
$$I = \frac{10(7)}{26} = \frac{35}{13}$$

$$I = 2.69 \text{ Amps.}$$

$$I_N = I \cdot \frac{3}{7}$$

$$= \frac{2.69 \times 3}{7}$$

$$= 1.153 \text{ Amps.}$$

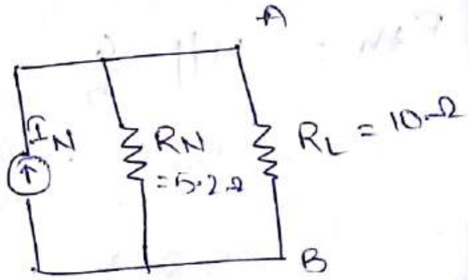


$$R_N = \frac{6}{5} + 4$$

$$= \frac{26}{5}$$

$$= 5.2\ \Omega$$

Norton's Equivalent circuit



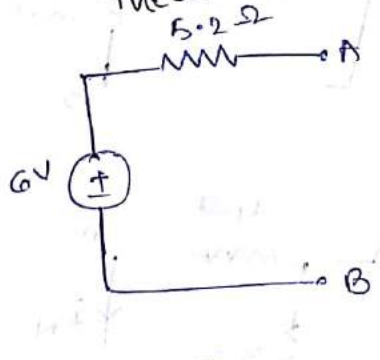
$$\text{load current} = I_N \times \frac{5.2}{15.2}$$

$$= \frac{1.153 \times 52}{152}$$

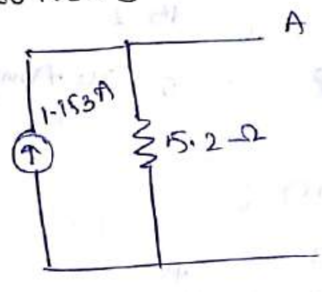
$$= 0.3944 \text{ Amp.}$$

__Note:__ Source Transformation

Thevenin's $\Rightarrow$ Norton's.



$\Rightarrow$



$$I_N = \frac{V}{R} = \frac{6}{5.2} = 1.153$$

$$V_{th} = I_N \times R$$

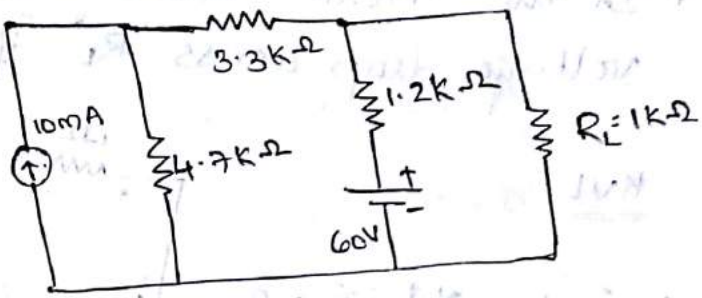
$$= 1.153 \times 5.2$$

$$= 6V$$

$$\boxed{R_{th} = R_N}$$

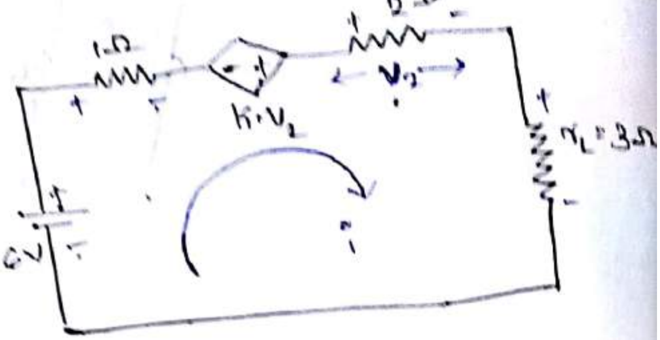
$$R = \frac{V_{o.c}}{I_{sc}}$$

Q→ Find the Thevenin's equivalent circuit for the given circuit and calculate the output voltage for $R_L = 1k\Omega$



Problems on dependent sources:

1) In the circuit shown in the figure, find the voltage drop across 'R_L' if $k = 2$.



KVL

$$6 - i + kV_2 - 2i - 3i = 0$$

$$6 - 4i + 2V_2 - V_2 = 0$$

$$6 - 4i + V_2 = 0$$

$$\cancel{6 - 4i - 2i = 0}$$

$$\cancel{6 = 6i} \Rightarrow \cancel{i = 1A}$$

$$\cancel{V_2 = 2i = 2(1) = 2V}$$

$$6 - i + 2V_2 - 2i - 3i = 0$$

$$6 - 6i + 2V_2 = 0$$

$$6 - 6i + 2(2i) = 0$$

$$6 - 6i + 4i = 0$$

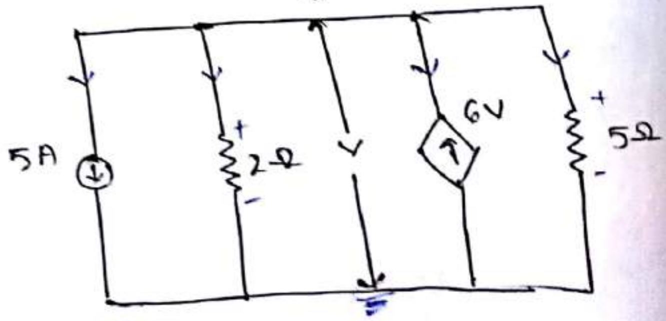
$$6 - 2i = 0$$

$$6 = 2i \Rightarrow \boxed{i = 3A}$$

$$V_{r_L} = i \times r_L$$

$$= 3 \times 3 = 9V.$$

2) Find the 'v' in the circuit shown in figure and also determine the numerical value of dependent source.



Apply nodal analysis at node 'x':

$$5 + \frac{V-0}{2} - 6V + \frac{V-0}{5} = 0$$

$$\frac{5V+2V}{10} = 1 \qquad 5 + \frac{V}{2} - 6V + \frac{V}{5} = 0$$

$$7V = 10 \qquad 5 + \frac{7V}{10} - 6V = 0$$

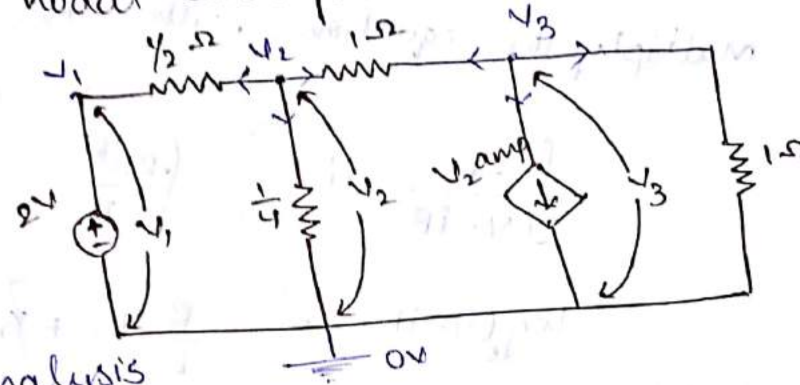
$$5 = \frac{53V}{10}$$

$$V = \frac{50}{53} = 0.943$$

V.C.C.S value

$\therefore 6V = 6(0.943) = 5.658\ A$

3) In the network shown in figure find V_1, V_2 and V_3 by nodal analysis.



$V_1 = 2V$

Apply nodal analysis at V_2:

$$2(V_2 - V_1) + 4V_2 + V_2 - V_3 = 0$$

$$2V_2 - 2V_1 + 5V_2 - V_3 = 0$$

$$7V_2 - 2V_1 - V_3 = 0 \longrightarrow (1)$$

Apply nodal analysis at V_3:

$$V_3 - V_2 + V_2 + V_3 = 0$$

$$2V_3 = 0$$

$$V_3 = 0$$

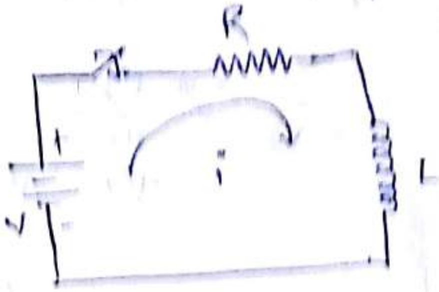
$$(1) \Rightarrow \quad 7V_2 - 4 - 0 = 0$$

$$V_2 = \frac{4}{7} = 0.571\ V$$

Time Domain Analysis of RL series circuit :-

— also called as Transient analysis.

By KVL,



$$V = V_R + V_L$$

$$V = iR + L\cdot\frac{di}{dt}$$

$$V - iR = L\frac{di}{dt}$$

$$\frac{V-iR}{di} = \frac{L}{dt}$$

$$\frac{di}{V-iR} = \frac{dt}{L}$$

Multiply the equation with '$-R$' & integrate

$$\int \frac{-R}{V-iR}\,di = \int \frac{-R\,dt}{L}$$

$$\log_e(V-iR) = -\frac{R}{L}(t) + K$$

K = integration constant

By using initial values / conditions, K is known

At $t=0$, $i=0$

$$\Rightarrow \log_e(V) = -\frac{R}{L}(0) + K$$

$$K = \log_e(V)$$

$$\log_e(V-iR) = -\frac{R}{L}t + \log_e V$$

$$\log_e\left(\frac{V-iR}{V}\right) = -\frac{R}{L}t$$

$$\frac{V-iR}{V} = e^{-\frac{R}{L}t}$$

$$V - iR = V e^{-R/L\, t}$$

$$V - Ve^{-R/L\, t} = iR$$

$$i = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$$

$$\Rightarrow \boxed{i(t) = \frac{V}{R}(1 - e^{-\frac{R}{L}t})}$$

$\tau = \lambda = \frac{L}{R}$ = time constant of series circuit

$$\Rightarrow \boxed{i(t) = \frac{V}{R}(1 - e^{-t/\lambda})}$$

Note :

* Any circuit reaches its final value within 5 time constants.

* Time constant $t = 1\lambda = \frac{L}{R}$

$$i = \frac{V}{R}(1 - e^{-1})$$

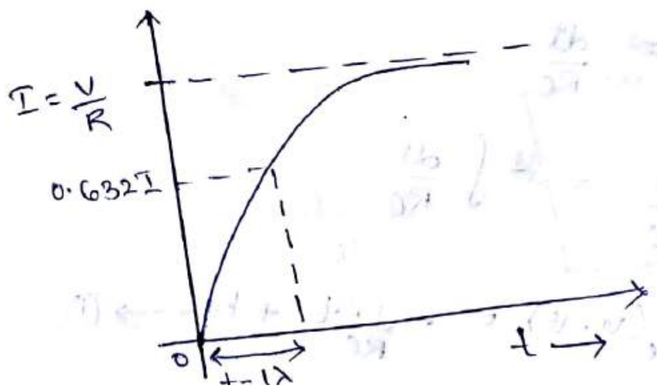
$$\boxed{i = 0.632 \times \frac{V}{R}}$$

* $i_{max} = \frac{V}{R} = I$, when inductor acts as wire / short circuit

$$\boxed{i = 0.632 \times I}$$

* Time constant : The time required for the current in an RL circuit to reach 0.632 times of its steady value while raising is called one time constant of the circuit.

Graph :



Current Decay:

$$iR + L\frac{di}{dt} = 0$$

$$iR = -L\frac{di}{dt}$$

$$\int \frac{di}{i} = \int -\frac{R}{L}\,dt$$

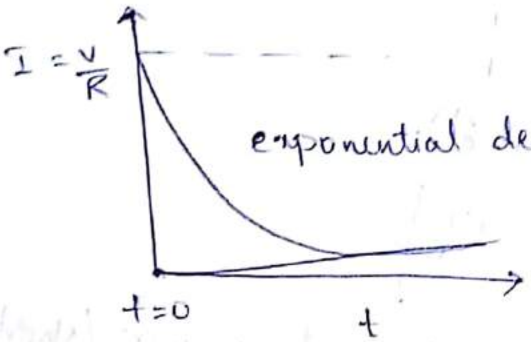
$$\log_e(i) = -\frac{R}{L}t + K$$

At $t = 0$, $i = \frac{V}{R} = I$

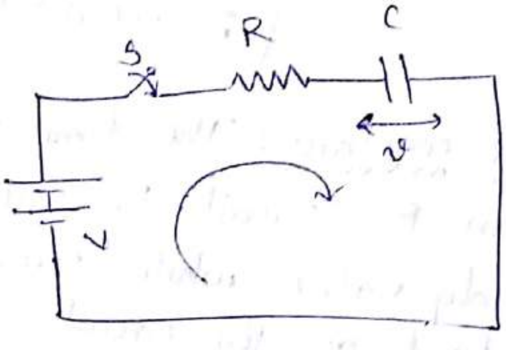
$$K = \log_e I$$

$$\boxed{i(t) = Ie^{-R/L\,t}}$$

Graph:



Time Domain analysis of R-C series circuit:



By KVL, $V = V_R + V_C$

$$V = i\cdot R + v$$

$$V = C\cdot\frac{dv}{dt}R + v$$

$$V - v = RC\frac{dv}{dt}$$

$$\frac{dv}{V - v} = \frac{dt}{RC}$$

$$\int -\frac{dv}{V - v} = -\int \frac{dt}{RC}$$

$$\log_e(V - v) = -\frac{1}{RC}t + K \longrightarrow (1)$$

K = integration constant.

Using initial conditions, value of K is known.

$t = 0, t = 0^+, v_c = 0$

$$\log_e (v - 0) = \frac{-1}{RC}(0) + K$$

$$\Rightarrow K = \log_e v$$

$$① \Rightarrow \log_e (v - v) = \frac{-1}{RC} t + \log_e v$$

$$\log_e\left(\frac{v - v}{v}\right) = \frac{-t}{RC}$$

$$\frac{v - v}{v} = e^{-1/RC\, t}$$

$$v - v = v e^{-t/RC}$$

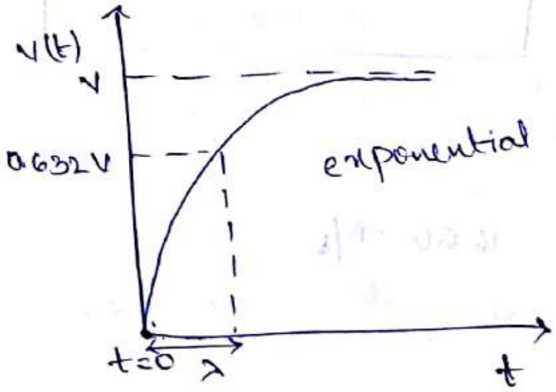
$$\boxed{v_c = v(1 - e^{-t/RC})}$$

$$v_c(t) = v(1 - e^{-t/RC})$$

$\lambda = RC$ = time constant

$$\boxed{v_c(t) = v(1 - e^{-t/\lambda})}$$

Graph:



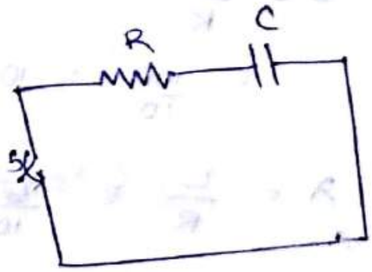
Decay:

By KVL, $0 = v_R + v_C$

$$0 = v + iR$$

$$0 = v + C\frac{dv}{dt}R$$

$$-v = RC\frac{dv}{dt}$$

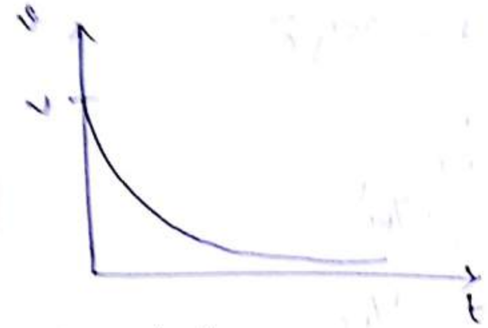


$$-\frac{dv}{v} = \frac{dt}{RC}$$

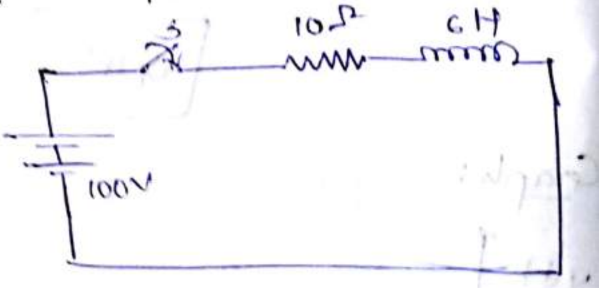
$$-\log_e(v) = \frac{1}{RC}(t) + c$$

$$\Rightarrow \boxed{v(t) = V \cdot e^{-\frac{1}{RC}t}}$$

Graph:



Q→ A coil having resistance of 10 Ω and inductance of 6H is connected to a constant supply voltage of 100 V. i, Find the rate of change of current at the instant of closing the switch. ii, Final steady state value. iii, time constant (λ) of the circuit. iv, time taken by the circuit to reach to a value of 4 Amps.



i, By KVL,

$$V = iR + L \cdot \frac{di}{dt}$$

$t = 0^+$, $i = 0$, $V = 100$

$$100 = 0(10) + 6\frac{di}{dt}$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{100}{6} = 16.66 \text{ A/s}.$$

ii,

$$I = \frac{V}{R}$$

$$= \frac{100}{10} = 10 \text{ A}$$

iii,

$$\lambda = \frac{L}{R} = \frac{6}{10} = 0.6 \text{ sec}.$$

iv, $i = \frac{v}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

$$4 = \frac{10\cancel{0}}{1\cancel{0}}\left(1 - e^{-\frac{10}{6}t}\right)$$

$$0.4 = 1 - e^{-\frac{10t}{6}}$$

$$e^{-10t/6} = 0.6$$

$$-\frac{10t}{6} = \log_e 0.6$$

$$\cancel{-\frac{5t}{3}} = \cancel{-0.221}$$

$$\cancel{t = 0.1326\ sec}$$

$$\cancel{-}\frac{5t}{3} = \cancel{-}0.510$$

$$t = 0.306\ sec.$$

Q→ A capacitor is charged by a dc source through a resistance of 1.2 MΩ. If in one second the potential difference across capacitor reaches 75% of the final value, calculate the capacitance of the capacitor.

$$R = 1.2 M\Omega$$
$$= 1.2 \times 10^6\ \Omega$$

1 sec → v_c = 75% of final value

$$v_c = v\left(1 - e^{-t/RC}\right)$$

$t = 1$ sec, $v_c = \frac{3}{4}v$

$$\Rightarrow \frac{3}{4}\cancel{v} = \cancel{v}\left(1 - e^{-\frac{1}{1.2\times10^6 \cdot C}}\right)$$

$$e^{-\frac{1}{RC}} = 0.25$$

$$\frac{-1}{12\times10^5 \times C} = \ln(0.25)$$

$$\frac{1}{12\times10^{5}\times C} = 1.386$$

$$C = \frac{1}{12\times10^{5}\times 1.386}$$

$$C = \frac{10^{-5}}{16.632}$$

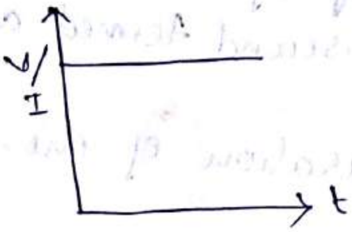
$$C = 0.060\times10^{-5}$$

$$C = 0.6\times10^{-6}$$

$$C = 0.6\ \mu F$$

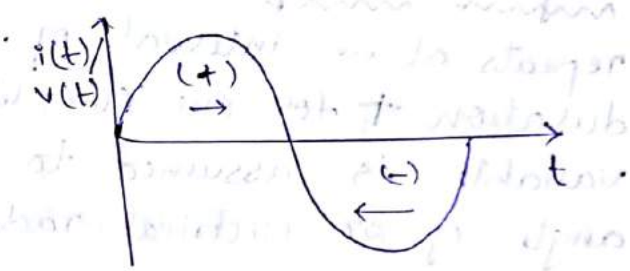
2/7/18 **AC Circuits**

DC: Direct current (or) direct voltage means a quantity of fixed magnitude independent of time and current flows in one direction only.



AC: Alternating current (or) alternating voltage means a sinusoidal varying current (or) voltage.

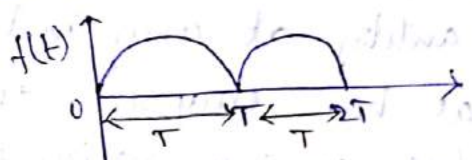
→ For AC current flows in one direction for positive half cycle and flows in opposite direction for negative half cycle.



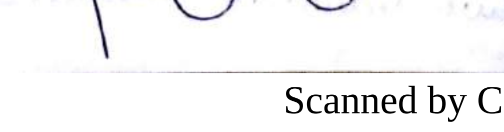
Terminology in AC wave form:

1) Periodic function: The function f(t) is said to be periodic function in time 't' with a periodic of 'T' seconds. If the same pattern of the function repeats after every 'T' seconds.

$$\boxed{f(t) = f(t+T)}$$



2) Wave form: The shape of the curve obtained when the instantaneous values of a periodic variable are plotted along the Y-axis with time as the X-axis is known as the wave form of the variable.



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Cycle: Each repetition of the set of values of a periodic function in equal intervals is termed as cycle.

Frequency: The number of cycles described by a periodic function per second termed as frequency.

Period: (T) The time duration of one cycle is termed as its period.

$$\boxed{T = \frac{1}{f}}$$

Angular velocity (ω): A sinusoidal quantity repeats at an interval of 2π radians. The duration ~~T~~ for one complete cycle of a periodic variable is assumed to correspond to a time angle of 2π electrical radians.

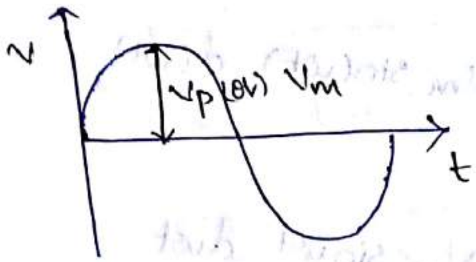
$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f} \quad \text{ele. rad/sec}$$

Instantaneous value: It is the magnitude of the quantity at the given instant of the time.

phase: The phase of any alternating (variable) quantity at any instant is the time interval that has ellapsed since the instantaneous value of the variable last passed through zero from negative to positive direction.

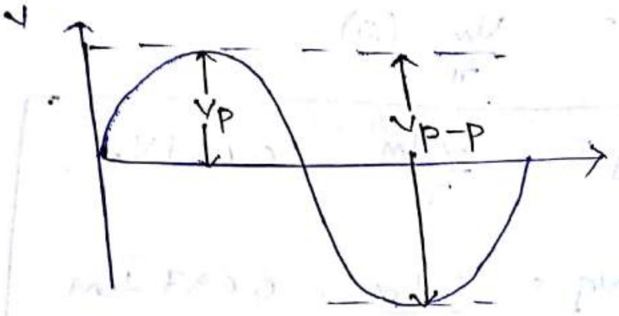
phase difference (ϕ): The phase difference between the periodic variables of the same frequency is defined as the difference in phase between the two variables measured at any instant of time.

Peak value: The maximum magnitude of the instantaneous value is defined as peak value.



Peak-to-peak value: The maximum variation between instantaneous values of positive and negative value of an alternating quantity.

$$\boxed{V_{p-p} = 2V_p}$$



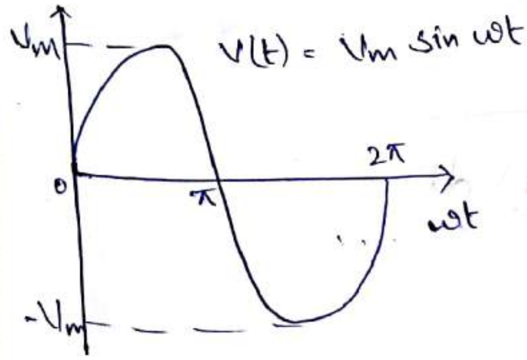
Average Value: The average value of a periodic function $x(t)$ can be defined as

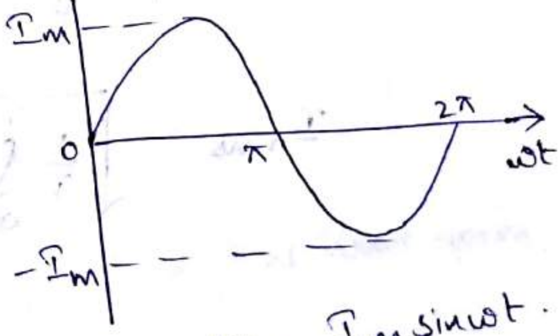
$$x_{avg} = \frac{1}{T}\int_0^T x(t)\,dt$$

where, T = time period of function $x(t)$

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$I_{avg} = \frac{1}{T}\int_0^T i(t)\,dt$$





$I(t) = I_m \sin \omega t$.

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{T}\int_0^T V_m \sin(\omega t)\,d(\omega t)$$

$$= \frac{1}{\pi}\int_0^\pi V_m \sin\omega t\,d\omega t$$

$$= \frac{1}{\pi} V_m (-\cos\omega t)_0^\pi$$

$$= \frac{V_m}{\pi}(2)$$

$$\boxed{V_{avg} = \frac{2V_m}{\pi} = 0.637 V_m \qquad I_{avg} = \frac{2I_m}{\pi} = 0.637 I_m}$$

RMS value / Effective value: It can be defined as the square root of average of the squares of the instantaneous values the alternating quantity.

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

T = Time period of the function x(t)

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2(t)\,dt}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}[V_m \sin(\omega t)]^2\,d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\sin^2\omega t\,d\omega t}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\omega t)\,d\omega t}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}(2\pi - 0)}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\boxed{\begin{aligned} V_{rms} &= 0.707\,V_m \\ I_{rms} &= 0.707\,I_m \end{aligned}}$$

Peak factor / crest factor : It is the ratio of peak value to RMS value.

$$\text{peak factor} = \frac{\text{peak value}}{\text{RMS value}}$$

$$= \frac{\cancel{V_m}(\sqrt{2})}{\cancel{V_m}}$$

$= 1.414$ (for sine wave form)

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Form factor: It is the ratio of RMS value to average value.

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

$$= \frac{V_m/\sqrt{2}}{2V_m/\pi}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\pi}{2} = 1.11 \text{ (for sine wave)}$$

- The voltage wave is represented by $v = 200 \sin 314t$. Find a) maximum value b) RMS value c) Average value d) frequency e) time period f) Instantaneous value after 0.05 sec.

$$v = V_m \sin \omega t$$

Given $v = 200 \sin 314t$

a) $V_m = 200\ V$

b) $V_{rms} = 0.707 V_m = 200 \times 0.707$

$= 141.4\ V$

c) $V_{avg} = 0.637 V_m$

$= 0.637 \times 200$

$= 127.4\ V$

d) $\omega = 314$

$2\pi f = 314$

$f = \frac{314}{2 \times 3.14} = 50\ Hz$

e) $T = \frac{1}{f} = \frac{1}{50} = 0.02$ sec

f) $v(0.05) = 200 \sin 314(0.05)$

$= 200\ \cancel{\sin(15.70)}\ \sin(n\pi)$

$= \cancel{54.12\ V}\ 0$

→ A voltage of 'v' volts is applied in a circuit contains only resistance of 10Ω. If the voltage wave is represented by $v = 20\sin(314t)$. Find peak current, RMS current, Average current and power loss in resistor.

$$v = 20\sin(314t)$$

$$V_m = 20\,V$$

a) $I_m = \dfrac{V_m}{R} = \dfrac{20}{10} = 2$ Amp

b) $V_{rms} = 0.707\,V_m = 0.707 \times 20 = 14.14\,V$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{14.14}{10} \simeq 1.414 \text{ Amps}$$

c) $V_{avg} = 0.637\,V_m = 0.637 \times 20 = 12.74\,V$

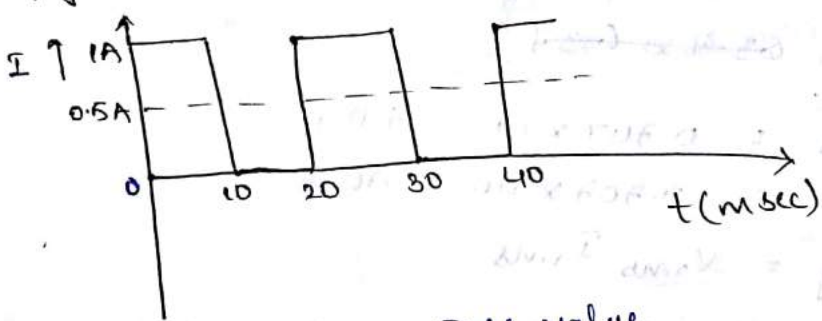
$$I_{avg} = \frac{V_{avg}}{R} = \frac{12.74}{10} = 1.274 \text{ Amps}$$

d) power loss in 10Ω resistor $= V_{rms} I_{rms}$

$$P = (14.14) \times (1.414)$$

$$P = 19.99 \text{ Watts} \simeq 20 \text{ Watts.}$$

→ A non-alternating periodic wave form is shown in the figure. Find its form factor.



$$\text{Form factor} = \frac{\text{RMS value}}{\text{Avg value}}$$

$I(t) = 1\,A \qquad 0 < t < 10$ msec

$I(t) = 0\,A \qquad 10 < t < 20$ msec.

$$I_{rms} = \sqrt{\frac{1}{20 \times 10^{-3}}\left\{\int_0^{10} I^2(t)\,dt + \int_{10}^{20} I^2(t)\,dt\right\}}$$

$I_{rms} = \sqrt{\frac{1}{20}(10-0)} = \frac{1}{\sqrt{2}}$ Amps $= 0.707$ amps

$I_{avg} = \frac{2 I_m}{\pi} = \frac{2 \times 1}{\pi} = \frac{2}{\pi}$ $\left\{ I_{avg} = \frac{1}{T}\int_0^T i(t)\,dt \right.$

Form factor $= \frac{1}{\sqrt{2}} \times \frac{\pi}{2} = 1.11$

$I_{avg} = \frac{1}{20}\left\{ \int_0^{10} 1\,dt + \int_{10}^{20} 0\,dt \right\}$

$= \frac{1}{20}(10-0) = 0.5$ Amp

Form factor $= \frac{0.707}{0.5} = 1.414$

→ The voltage and current in a AC circuit are given by $V = 100 \sin 314t$, $I = 10 \sin 314t$. Determine the average power.

$V = 100 \sin 314t$; $V_m = 100$ volts

$I = 10 \sin 314t$; $I_m = 10$ Amps

$\{ P_{avg} = V_{avg} I_{avg}$

$V_{avg} = 0.637 \times V_m = 0.637 \times 100 = 63.7$ V

$I_{avg} = 0.637 \times I_m = 0.637 \times 10 = 6.37$ Amps

$P_{avg} = \cancel{63.7 \times 6.37} \}$

$I_{rms} = 0.707 \times 10 = 7.07$

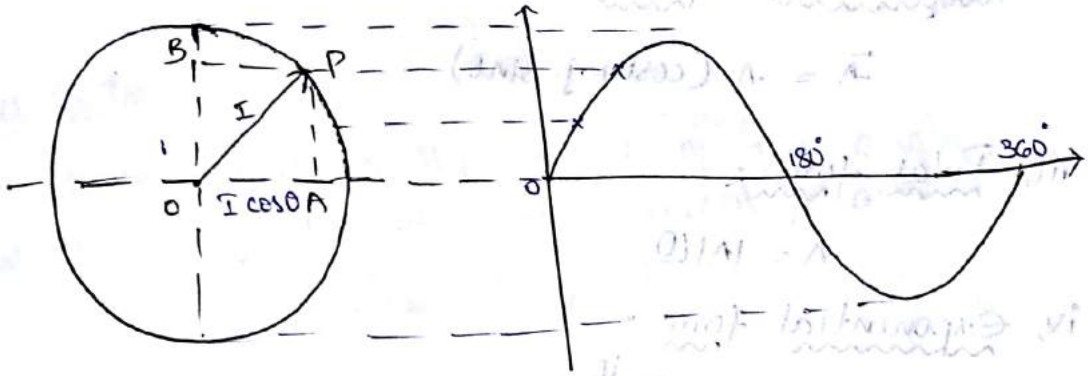
$V_{rms} = 0.707 \times 100 = 70.7$

$P_{avg} = V_{rms} I_{rms}$

$= 7.07 \times 70.7 = 500$ Watts.

Phasor representation of alternating quantities:

A phasor is a complex number that contains the magnitude and phase angle information of a sinusoidal function. The concept of phasor can be developed by using Euler's identity which relates the exponential function to trignometric function.



$$I = \text{real} + \text{quadrature}$$

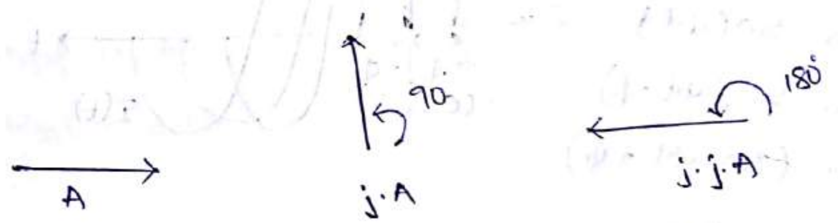
$$\overline{OP} = \overline{OA} + \overline{OB}$$

$$I = I\cos\theta + jI\sin\theta$$

$$= I(\cos\theta + j\sin\theta)$$

$$= I e^{j\theta}$$

j-operator: The multiplication of any phasor by the operator 'j' can rotate the phasor through 90° in clock-wise direction without affecting the magnitude.



$$j^2 A = -A$$

$$j = \sqrt{-1}$$

$$j^2 = -1$$

$$j^3 = -j$$

Representation of phasors in a complex plane:

The alternating quantity can be represented in the following four forms

i, Rectangular (or) Cartesian form:

$$A = a \pm j.b \quad , \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

ii, Trignometric form:

$$\bar{A} = A(\cos\theta + j\sin\theta)$$

iii, Polar form:

$$A = |A| \angle\theta$$

iv, Exponential form:

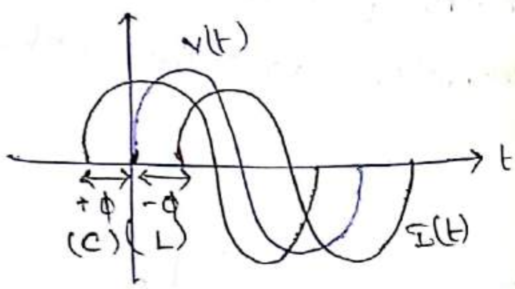
$$A = |A| e^{j\theta}$$

* $V = v(t) = V_m \sin\omega t = V_m \sin\theta = V_m \sin(2\pi f)t = V_m \angle 0^\circ$
$= V_m \cdot (\cos 0^\circ + j\sin 0^\circ)$

* $i = i(t) = I_M \sin(\omega t - \phi) = I_m \angle -\phi = I_m \cos\phi - j I_m \sin\phi$

Relation between Real power, and apparent power, Reactive power and power factor

$$P_{dc} = V_{dc} \times I_{dc}$$



$v(t) = V_m \sin\omega t$

$I_R(t) = I_m \sin(\omega t)$

$I_L(t) = I_m \sin(\omega t - \phi)$

$I_C(t) = I_m(\sin\omega t + \phi)$

$P_{ac} = v(t) \times i(t)$

$= V_m \sin\omega t \times I_M \sin(\omega t - \phi)$

$P_{ac} = V_{rms} \cdot I_{rms} \cos\phi$
$\begin{cases} \phi = 0^\circ \text{ for R - Load} \\ \phi = -90^\circ \text{ for L - load} \\ \phi = +90^\circ \text{ for C - load} \end{cases}$

(complex power)

-Apparent power = Real power + Reactive power
(active/true/average) (imag/wattless/quadrative)

$$V \cdot I = V \cdot I \cdot \cos\phi + j \cdot V \cdot I \cdot \sin\phi$$

$$S = P + j \cdot Q$$

(Volt-Ampere) (Watt) (Volt-ampere-Reactive)

$$S_{(VA)} = P_{(W)} \pm j \cdot Q_{(VAR)}$$

Power factor:

power factor is the cosine of the angle between voltage phasor and current phasor.

$$\boxed{\text{power factor} = \cos\phi}$$

where, ϕ = phase ~~ang~~ difference ($V \hat{} I$)

$$\cos\phi = \frac{\text{Active power}}{\text{Apparent power}} = \frac{P}{S} = \frac{VI\cos\phi}{VI} = \cos\phi$$

Q→ Find the apparent power, true power and reactive power if voltage and currents are measured as $E = 120V$, $I = 100A$ with the current lagging the voltage by an angle of $33.5°$. Also find the new value of current at unity power factor.

(power factor value varies from 0 to 1)

Apparent power = $V \cdot I$ = 120×100 = 12000 V.A = 12 KV.A}

Real power (const) = $V \cdot I \cos\phi$
$= 12000 \cos(33.5)$
$= 10006.629$ Watt $\simeq$ 10 KW

Reactive power = $V \cdot I \cdot \sin\phi$
$= 12000 \sin(33.5)$
$= 6.6$ KVAR

Given $\cos\phi = \cos(33.5°) = 0.83$

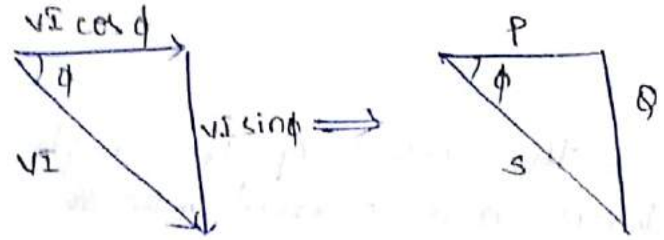
If $\cos\phi = 1$, $P = V \cdot I \cos\phi$

$$10\,KW = 120\, I \times 1$$

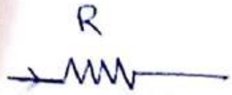
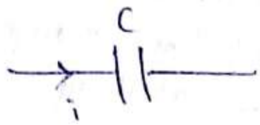
$$I = \frac{10{,}00\cancel{0}}{12\cancel{0}} = 83.3 \text{ Amps}$$

Power Triangle:

It is geomitrical representation of apparent power, real power and reactive power.



Single phase analysis of R, L & C elements:

	DC circuit	AC circuit
	Opposition = R	Impedence = Z = R
	Open circuit (initial) Short circuit (final)	Z = Reactance = X_L $= L\omega = 2\pi f \cdot L$
	S.C (initial) O.C (final)	Z = Reactance = X_C = $\frac{1}{C\omega} = \frac{1}{2\pi f \cdot C}$

$$\boxed{R = \frac{1}{G}}$$

$$\boxed{\text{Reactance } (X) = \frac{1}{\text{Susceptance } (B)}}$$

$$\boxed{\text{Impedence } (Z) = \frac{1}{\text{Admittance } (Y)}}$$

* Impedence (Ω) = Resistance (Ω) + Reactance (Ω)

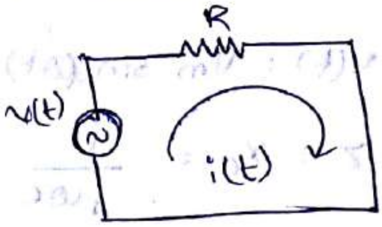
$$Z = R \pm jX$$

* Admittance $(\mho)$ = Conductance $(\mho)$ + Susceptance $(\mho)$

$$Y = G \pm jB$$

Single phase analysis with 'R' Load:

$v(t) = V_m \sin \omega t$ volts



Impedance $(Z) = R$

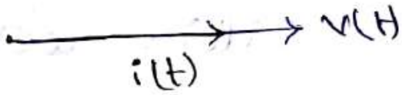
$$i(t) = \frac{v(t)}{Z}$$

$$i(t) = \frac{V_m \sin \omega t}{Z}$$

$$i(t) = \frac{V_m}{R} \sin \omega t$$

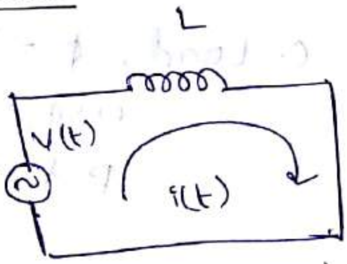
$$i(t) = I_m \sin \omega t \qquad \left(\because I_m = \frac{V_m}{R}\right)$$

for R-Load $\Rightarrow \phi = 0^\circ$ (in phase)



$\cos\phi = \cos 0 = 1$

Single phase analysis with 'L' load:



$v(t) = V_m \sin \omega t$

Impedance $(Z) = X_L = j\omega L$

$$i(t) = \frac{v(t)}{Z}$$

$$i(t) = \frac{V_m \sin \omega t}{Z} = \frac{V_m}{X_L} \sin \omega t$$

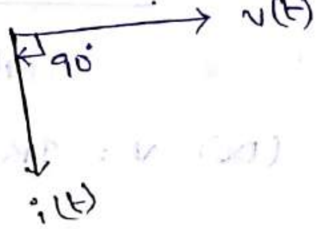
$$i(t) = \frac{V_m}{j\omega L} \sin \omega t$$

$$i(t) = \frac{V_m}{\omega L} \sin(\omega t - 90^\circ)$$

$$i(t) = \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$$

$$i(t) = I_m \sin(\omega t - \pi/2) \qquad \left\{\because I_m = \frac{V_m}{\omega L}\right\}$$

For L-Load $\Rightarrow \phi = 90^\circ$



$\cos\phi = \cos 90^\circ$

$p.f = 0.$

Note:

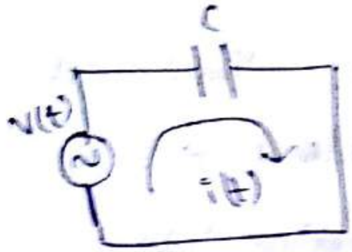
In pure inductor, $\phi = 90^\circ$

power factor $= 0$

Single phase analysis with C-Load:

$v(t) = V_m \sin(\omega t)$

$Z = X_C = \frac{1}{j\omega C}$



$i(t) = \frac{v(t)}{Z}$

$i(t) = \frac{V_m \sin(\omega t)}{(1/j\omega C)}$

$i(t) = (j\omega C) V_m \sin \omega t$

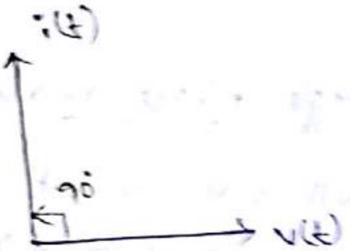
$i(t) = \omega C V_m \sin(\omega t + \pi/2) \quad \{\because I_m = V_m \omega C\}$

$i(t) = I_m \sin(\omega t + \pi/2)$

For C-Load, $\phi = \pi/2$

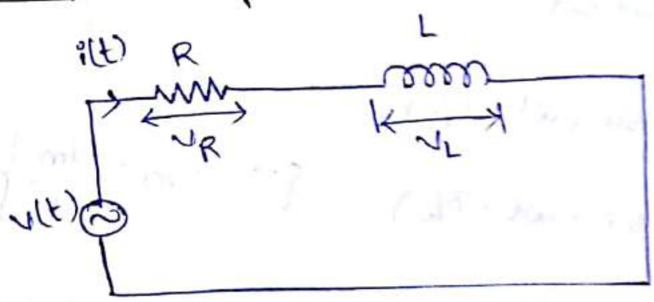
$\cos\phi = \cos \pi/2$

$p.f. = 0.$



* The p.f for pure inductor and pure capacitor is zero. Which indicates they only store the energy. Whereas p.f for resistance is unity. It will only dissipates the energy.

Sinusoidal Analysis of Series R-L Circuit:



$v(t) = V_m \sin \omega t$

(DC) $V = IR \Rightarrow I = V \cdot \frac{1}{R} \Rightarrow I = V.G$

(AC) $V = IZ \quad I = V \cdot \frac{1}{Z} \Rightarrow I = V.Y$

By KVL,

$$v(t) = v_R + v_L$$

$$v_R = I \cdot R$$

$$v_L = j\,(I \cdot X_L)$$

$$v(t) = I \cdot R + j \cdot I X_L$$

$$v(t) = I\,(R + jX_L)$$

$$\frac{v(t)}{R + jX_L} = I(t)$$

$$I(t) = \frac{v(t)}{Z} \quad \{\because Z = R + jX_L\}$$

$x + iy$; magnitude $= \sqrt{x^2 + y^2}$

$$Z = R + jX_L$$

$$= R + jL\omega$$

$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$

$$i(t) = \frac{v(t)}{\sqrt{R^2 + (\omega L)^2}}$$

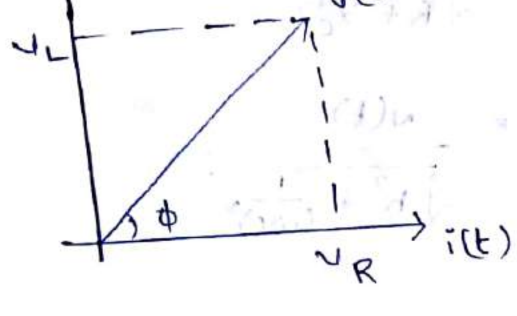
$$\{\because Z = R + j\omega L\}$$

$$\cos\phi = \frac{\text{Resistance}}{\text{Impedance}} = \frac{R}{Z}$$

$$\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

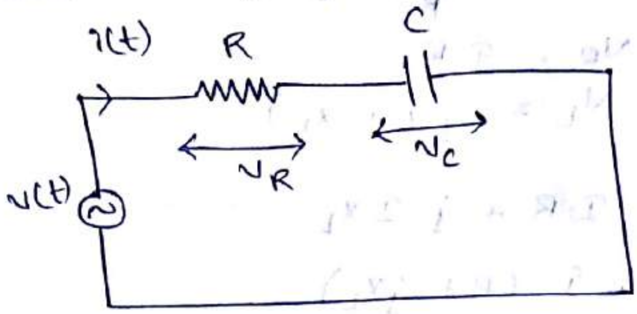
* Full current lags behind voltage by $90°$ (or)
  v leads i by $90°$

phase diagram:



$$v(t) = v_R + v_L$$

Sinusoidal analysis of series R-C circuit:



$$v(t) = V_m \sin \omega t$$

$$Z = R + X_C$$

$$= R + \frac{1}{j\omega C}$$

$$= R - j \cdot \frac{1}{\omega C}$$

$$Z = R - jX_C$$

By KVL,

$$v(t) = v_R + v_C$$

$$v_R = I \cdot R$$

$$v_C = -j \cdot I \cdot X_C$$

$$v(t) = I \cdot R - j\, I X_C$$

$$v(t) = I(R - jX_C)$$

$$v(t) = I(t)\,(R - jX_C)$$

$$i(t) = \frac{v(t)}{R - jX_C}$$

$$i(t) = \frac{v(t)}{Z}$$

$x + iy$: magnitude $= \sqrt{x^2 + y^2}$

$$Z = R - jX_C$$

$$= R - \frac{j}{\omega C}$$

$$|Z| = \sqrt{R^2 + X_C^2}.$$

$$i(t) = \frac{v(t)}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$$

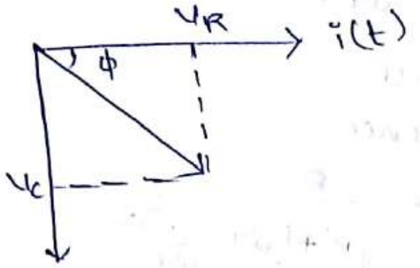
$$Z = R - \frac{j}{\omega C}$$

$$\cos\phi = \frac{\text{Resistance}}{\text{Impedance}}$$

$$= \frac{R}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$$

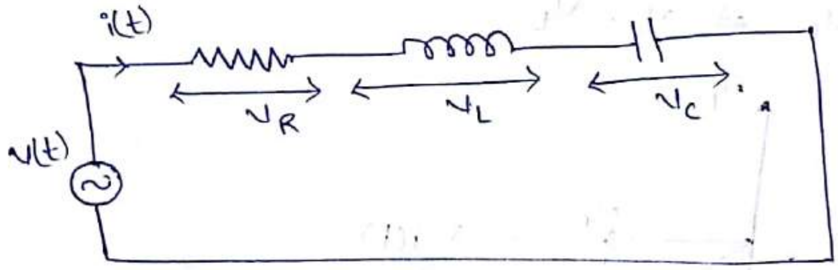
* Current leads voltage by 90° (or) v lags behind i by 90°

<u>phasor diagram</u>:



$$V = V_R + V_C$$

<u>Sinusoidal Analysis of series R-L-C circuit</u>:



$$v(t) = V_m \sin\omega t$$

$$Z = \overleftarrow{R} + \overleftarrow{L} + \overleftarrow{C}$$

$$Z = R + (jX_L) + (-jX_C)$$

$$= R + j(X_L - X_C)$$

$$= R + j\left(L\omega - \frac{1}{\omega C}\right)$$

By KVL,

$$v(t) = V_R + V_C + V_L$$

$$V_R = I \cdot R$$

$$V_C = -jIX_C$$

$$V_L = j \cdot IX_L$$

$$v(t) = IR + jIX_L - jIX_C$$
$$= I\,[R + j\,(X_L - X_C)]$$

$$i(t) = \frac{v(t)}{R + j(X_L - X_C)}$$

$$i(t) = \frac{v(t)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$Z = R + j(X_L - X_C)$$

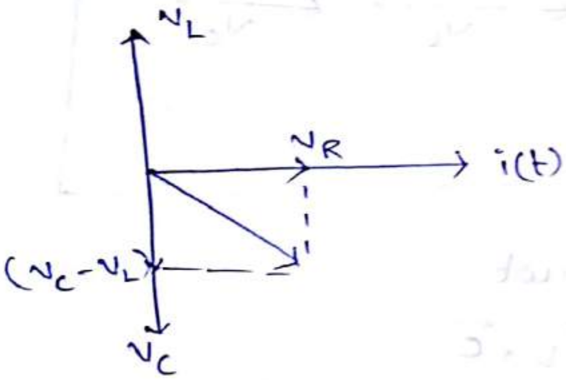
$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\cos\phi = \frac{\text{Resistance}}{\text{Impedence}}$$

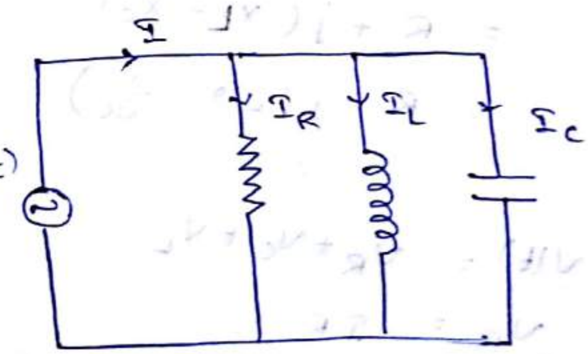
$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

phaser diagram:

$V_C > V_L \Rightarrow V_C - V_L$



Sinusoidal Analysis of parallel R-L-C circuit:



$$v(t) = V_m \sin \omega t$$

$$i_R = \frac{v(t)}{R}$$

$$i_L = \frac{V(t)}{X_L} = \frac{V(t)}{jX_L}$$

$$i_C = \frac{V(t)}{X_C} = \frac{V(t)}{-jX_C}$$

By KVL,

$$i(t) = i_R + i_L + i_C$$

$$i(t) = \frac{V(t)}{R} + \frac{V(t)}{jX_L} - \frac{V(t)}{jX_C}$$

$$= V(t)\left[\frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}\right]$$

$$= V(t)\left[\frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)\right]$$

$$i(t) = V(t)\left[\frac{1}{R} + j\left(\omega C - \frac{1}{L\omega}\right)\right]$$

$$I = V \cdot Y$$

In series		in parallel
$\frac{1}{R}$	=	G
$\frac{1}{X}$	=	B
$\frac{1}{Z}$	=	Y

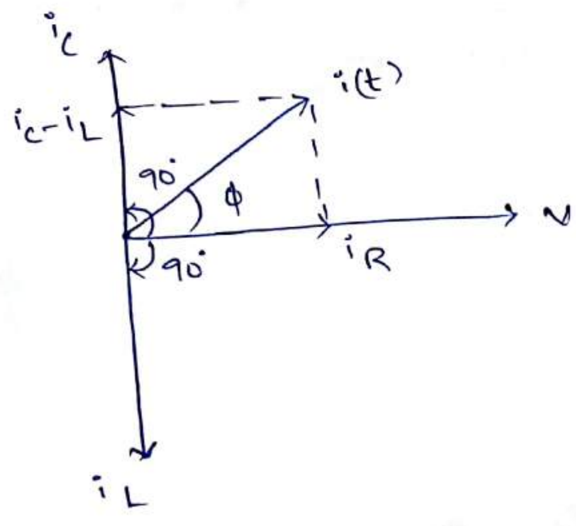
$$i = V\left[G + j(B_C - B_L)\right]$$

$$|Y| = \sqrt{G^2 + (B_C - B_L)^2}$$

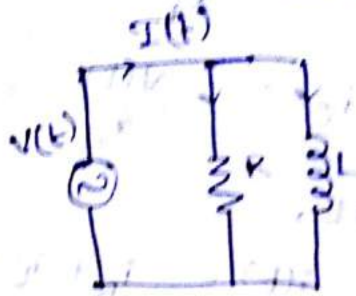
$$|Z| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\cos\phi = \frac{R}{Z}$$

$i_C > i_L$



Sinusoidal Analysis of parallel R-L circuit:



$$v(t) = V_m \sin \omega t$$

$$i_R = \frac{v(t)}{R}$$

$$i_L = \frac{v(t)}{X_L}, \frac{v(t)}{jX_L}$$

By KVL,

$$i(t) = i_R + i_L$$

$$i(t) = \frac{v(t)}{R} + \frac{v(t)}{jX_L}$$

$$i(t) = v(t)\left[\frac{1}{R} + \frac{1}{jL\omega}\right]$$

$$i(t) = v(t)\left[\frac{1}{R} - \frac{j}{L\omega}\right]$$

$$|Z| = \sqrt{(1/R)^2 + \left(\frac{-1}{L\omega}\right)^2}$$

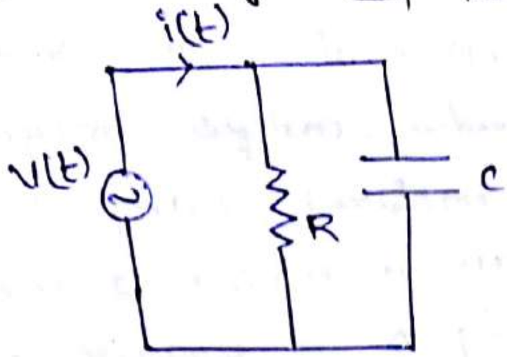
$$|Z| = \sqrt{\frac{1}{R^2} + \frac{1}{L^2\omega^2}}$$

$$\cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{R}{\sqrt{\frac{1}{R^2} + \frac{1}{L^2\omega^2}}}$$

phasor diagram

Sinusoidal Analysis of parallel R-C circuit:



$$v(t) = V_m \sin \omega t$$

$$i_R = \frac{v(t)}{R}$$

$$i_C = \frac{v(t)}{X_C} = \frac{v(t)}{-jX_C}$$

By KVL,

$$i(t) = i_R + i_C$$

$$i(t) = \frac{v(t)}{R} + \frac{v(t)}{-jX_C}$$

$$i(t) = \frac{v(t)}{R} + \frac{j\,v(t)}{1/C\omega}$$

$$i(t) = v(t)\left[\frac{1}{R} + jC\omega\right]$$

$$|Z| = \sqrt{\left(\frac{1}{R}\right)^2 + C^2\omega^2}$$

$$\cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{R}{\sqrt{1/R^2 + C^2\omega^2}}$$

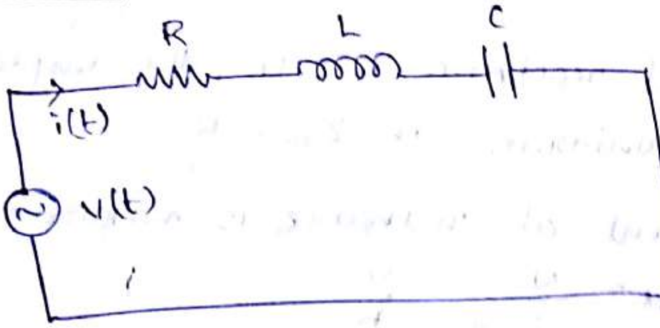
Resonance : 15/9/18

Resonance refers to a condition existing in any physical system, when a fixed amplitude sinusoidal forcing function (i/p voltage) produces a response of maximum amplitude. And the resultant response is in phase with the source function.

→ For resonance, two types of independent energy storage elements capable of interchanging energy between one other must be present.
Eg: Inductance and capacitance for electrical circuit.

→ Resonance is two types based on connection of elements.
i, series resonance
ii, parallel resonance.

Series Resonance :



$$Z = R + jX_L - jX_C$$

$$Z = R + j(X_L - X_C) \quad \{\because X_L = X_C\}$$

$$\boxed{Z = R}$$

Resonance condition

Inductive reactance = Capacitive reactance

$$X_L = X_C$$

$$L\omega = \frac{1}{C\omega}$$

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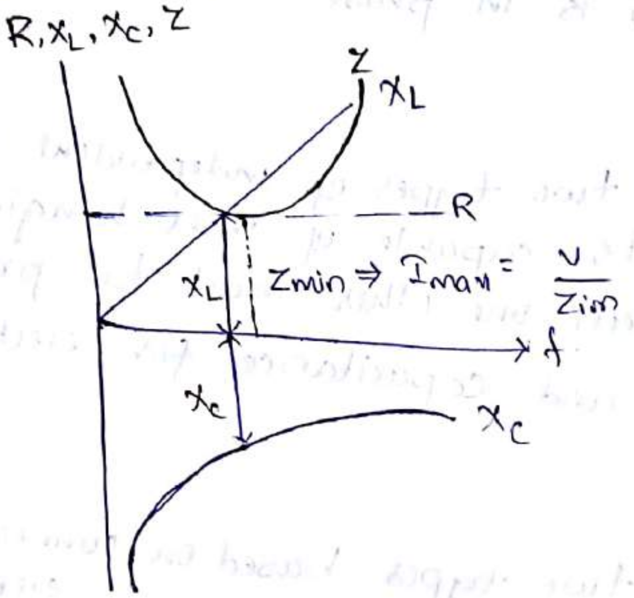
$$\omega^2 = \frac{1}{LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

Graph:



Note :

i, At resonance, inductive reactance = capacitive reactan

i.e., $X_L = X_C$

ii, As the net reactance is zero, the impedence becomes minimum. i.e, $Z_{min} = R$

iii, The current at resonance is maximum

i.e., $I_{max} = \frac{V}{Z_{min}} = \frac{V}{R}$

iv, At resonance, circuit behaves like pure resistive circuit.

v, The phase angle between voltage and current is zero ie., the output current is inphase with applied voltage.

vi, At resonance, the power factor of the circuit is unity $\{\cos\phi = \cos 0 = 1\}$

Quality factor:

It is the ratio of reactive voltage to the resistive voltage.

$$Q = \frac{V_L}{V_R} \text{ (or) } \frac{V_C}{V_R}$$

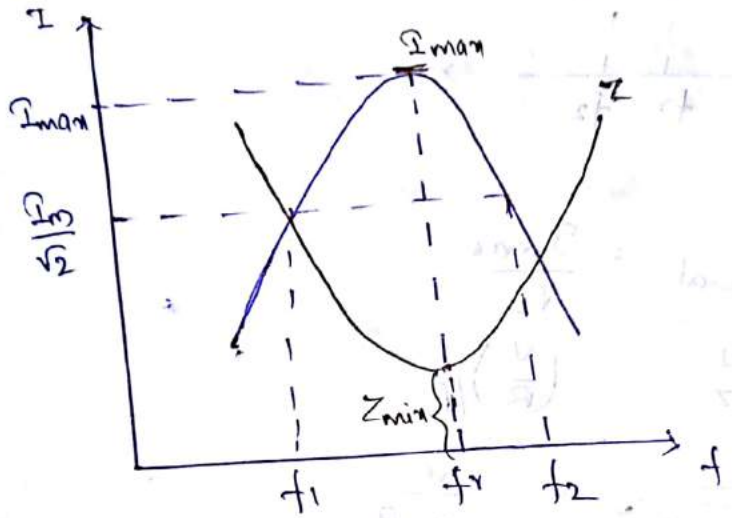
$$= \frac{I \cdot X_L}{I \cdot R} \text{ (or) } \frac{I \cdot X_C}{I \cdot R}$$

$$\frac{X_L}{R} \text{ (or) } \frac{X_C}{R}$$

$$\boxed{Q = \frac{\omega L}{R}} \text{ (or) } \boxed{Q = \frac{1}{\omega C R}} \text{ (or) } \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

At resonance frequency (f_0), $\boxed{Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}}$

Graph



f_r = resonant frequency
f_1 : Lower half-power frequency
f_2 : Higher half-power frequency

* Band width = $(f_2 - f_1)$ Hz.

→ Voltage across inductor (V_L) & voltage across the capacitor (V_C) at resonance.

$V_L = I X_L$ $\qquad$ $V_C = I X_C$

At resonance $\quad V_L = I_0 X_L \qquad V_C = I_0 X_C$

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$$V_L = \frac{V}{R} X_L$$

$$V_L = \left(\frac{V}{R}\right) L\omega$$

$$V_L = \left(\frac{\omega L}{R}\right) V$$

$$\boxed{V_L = Q_0 V}$$

$$V_C = \frac{V}{R} X_C$$

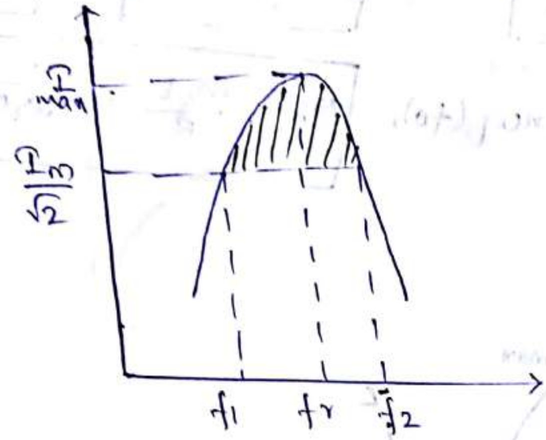
$$V_C = \frac{V}{R} \cdot \frac{1}{C\omega}$$

$$V_C = \left(\frac{1}{C\omega R}\right) \cdot V$$

$$\boxed{V_C = Q_0 V}$$

* Q is also called voltage magnification factor.

Relation between band width, Quality factor and Resonance frequency:



$$I_{general} = \frac{I_{max}}{\sqrt{2}}$$

$$\frac{V}{Z} = \left(\frac{V}{R}\right) / \sqrt{2}$$

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{C\omega}\right)^2}} = \frac{V}{\sqrt{2} \cdot R}$$

$$2R^2 = R^2 + \left(\omega L - \frac{1}{C\omega}\right)^2$$

$$\left(\omega L - \frac{1}{C\omega}\right)^2 = R^2$$

$$\Rightarrow \quad \omega_1 L - \frac{1}{\omega_1 C} = -R \longrightarrow (1)$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \longrightarrow (2)$$

(1) + (2) $\quad L(\omega_1 + \omega_2) - \frac{1}{C}\left[\frac{1}{\omega_1} + \frac{1}{\omega_2}\right] = 0$

Divide by L

$$\Rightarrow (\omega_1+\omega_2) - \frac{1}{LC}\left[\frac{\omega_1+\omega_2}{\omega_1\omega_2}\right] = 0$$

$$\cancel{\omega_1+\omega_2} = \frac{1}{LC}\left[\frac{\cancel{\omega_1+\omega_2}}{\omega_1\omega_2}\right]$$

$$\boxed{LC = \frac{1}{\omega_1\omega_2}}$$

② − ①

$$\Rightarrow (\omega_2-\omega_1)L - \frac{1}{C}\left[\frac{1}{\omega_2} - \frac{1}{\omega_1}\right] = R-(-R)$$

$$L(\omega_2-\omega_1) + \frac{1}{C}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] = 2R$$

$$L(\omega_2-\omega_1) + \frac{1}{C}\left[\frac{\omega_2-\omega_1}{\omega_2\omega_1}\right] = 2R$$

$$(\omega_2-\omega_1) + \frac{1}{LC}\left[\frac{\omega_2-\omega_1}{\omega_2\omega_1}\right] = \frac{2R}{L}$$

$$(\omega_2-\omega_1) + \cancel{\omega_1\omega_2}\left[\frac{\omega_2-\omega_1}{\cancel{\omega_1\omega_2}}\right] = \frac{2R}{L}$$

$$\cancel{2}(\omega_2-\omega_1) = \frac{\cancel{2}R}{L}$$

$$\omega_2-\omega_1 = \frac{R}{L}$$

$$\omega_2-\omega_1 = \frac{\omega_0}{Q} \qquad \left\{\because Q = \frac{\omega_0 L}{R}\right\}$$

$$\boxed{Q = \frac{\omega_0}{\omega_2-\omega_1}}$$

$$\boxed{Q = \frac{f_0}{f_2-f_1}}$$

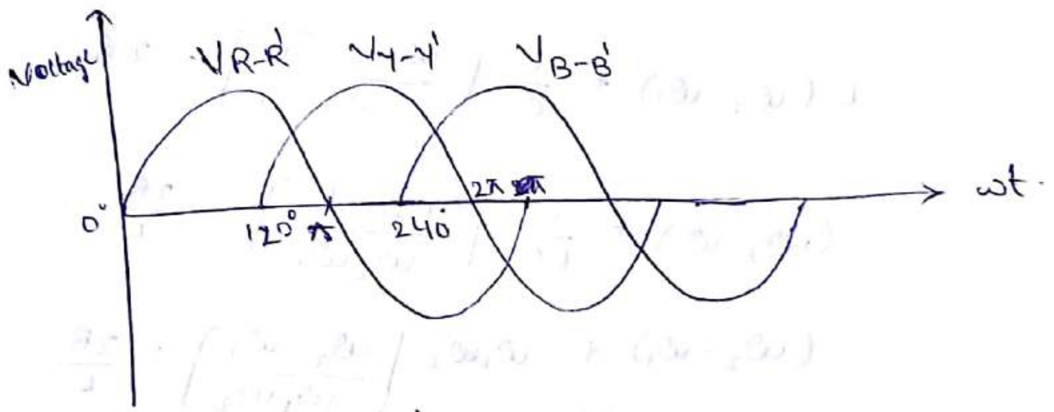
$$\boxed{\text{Quality factor} = \frac{\text{Resonance frequency}}{\text{Band width}}}$$

Note :

* Quality factor is also termed as selectivity.

Three phase circuits :

A Three-phase system of voltages (currents) is simply a combination of three single phase system of voltages (currents) of which the three voltages (currents) are same in magnitude and frequency but differ in phase by 120° electrical from each other in a particular phase sequence.



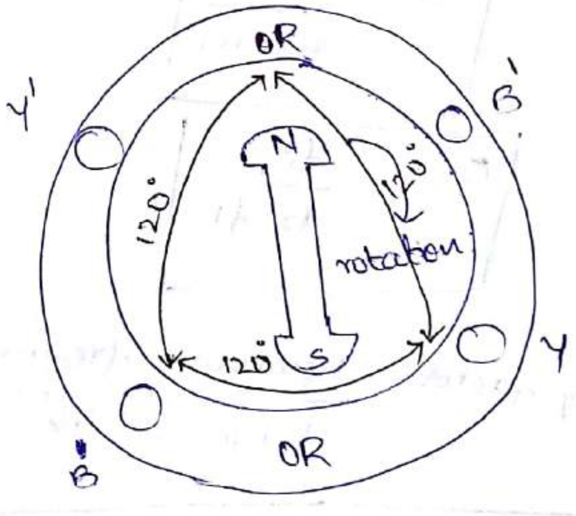
$v(t) = V_m \sin \omega t$

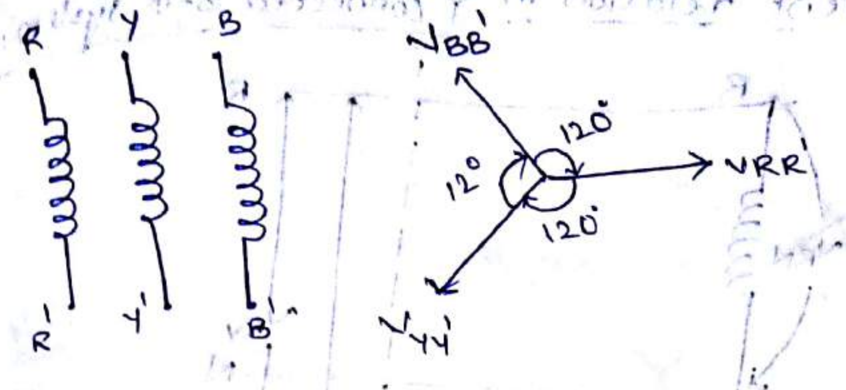
$V_{RR'} = V_m \sin \omega t$

$V_{Y-Y'} = V_m \sin(\omega t - 120°)$

$V_{B-B'} = V_m \sin(\omega t - 240°)$

At any instant of time, $V_{RR'} + V_{YY'} + V_{BB'} = 0$

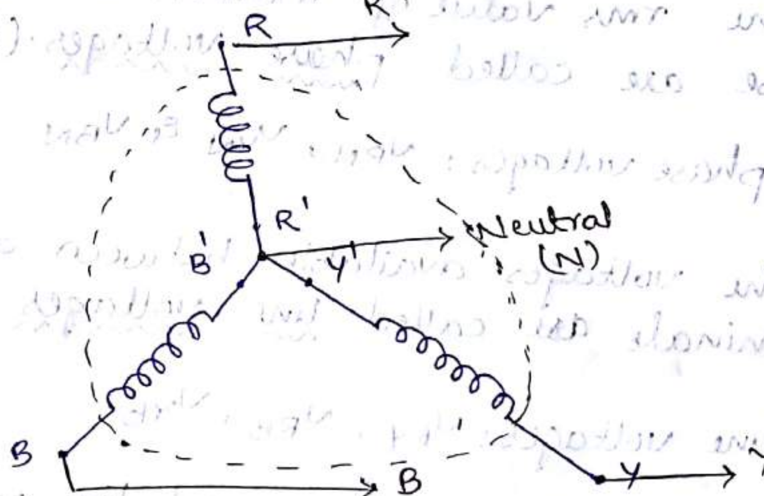




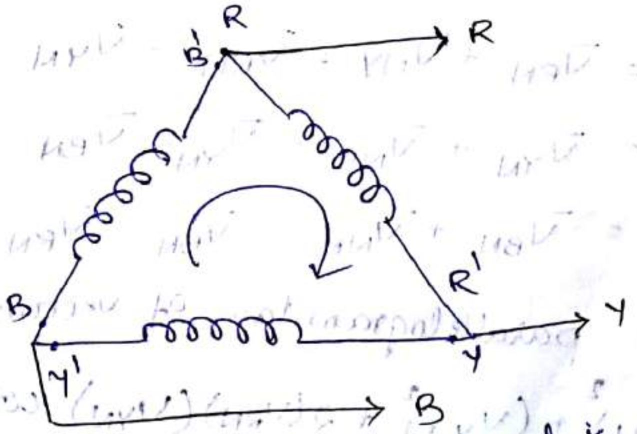
A three phase generator have three different / distinct windings which are displaced by 120° electrical to each other. so that generated voltages in the ~~so~~ consecutive R, Y and B phases differ by 120° in phase.

Inter-connection of 3-φ windings:

1) Y-star

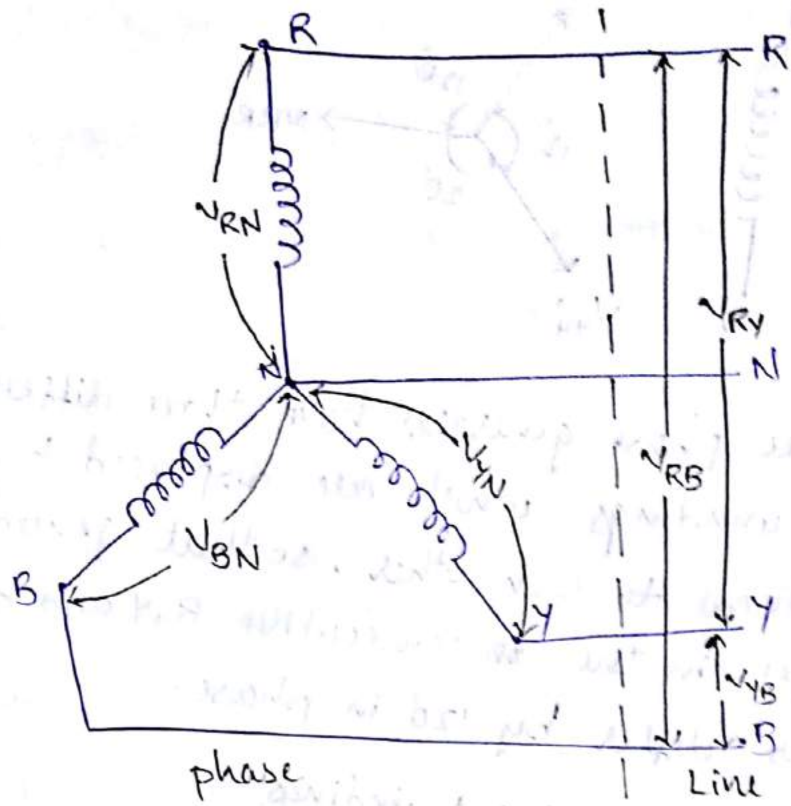


2) Δ - Delta



* If it is 3-φ generator, current is drawn. If it is 3-φ motor, current is supplied.

Voltage & current relation in Y connected 3-ϕ system:



* The rms value of induced voltages in each phase are called phase voltages (V_{ph})

phase voltages: V_{RN}, V_{YN} & V_{BN}

* The voltages available between any pair of terminals are called line voltages (V_L)

line voltages: V_{RY}, V_{RB}, V_{YB}

Relation between phase and line voltages: In 3-ϕ

V_{ph} : V_{RN}, V_{YN}, V_{BN}

$\underline{V_L}$: $V_{RY} = \bar{V}_{RN} + \bar{V}_{NY} = \bar{V}_{RN} - \bar{V}_{YN}$

$V_{YB} = \bar{V}_{YN} + \bar{V}_{NB} = \bar{V}_{YN} - \bar{V}_{BN}$

$V_{BR} = \bar{V}_{BN} + \bar{V}_{NR} = \bar{V}_{BN} - \bar{V}_{RN}$

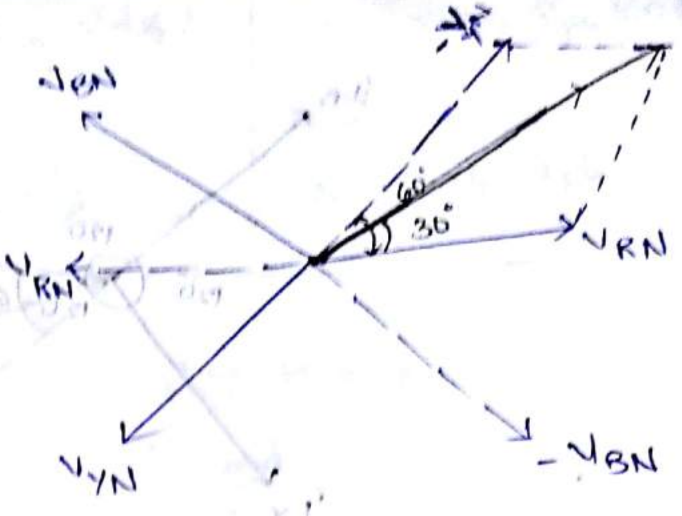
According to parallelogram law of vectors.

$(V_{RY})^2 = (V_{RN})^2 + (V_{YN})^2 + 2(V_{RN})(V_{YN})\cos 60^\circ$

$V_L^2 = (V_{ph})^2 + (V_{ph})^2 + 2(V_{ph})(V_{ph}) \cdot \frac{1}{2}$

$V_L^2 = 3(V_{ph})^2$

$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$



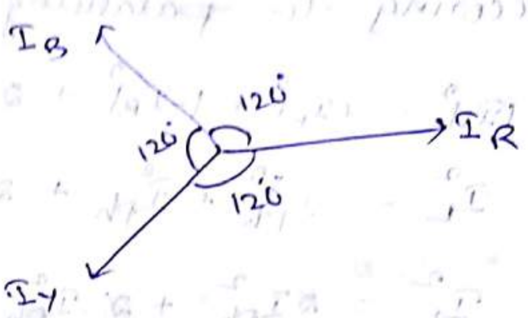
Note:

1) The V_L is $\sqrt{3}$ times the V_{ph} and

2) The V_L are 30° ahead of V_{ph}

* Line currents = phase currents in star connection

$I_L = I_{ph}$



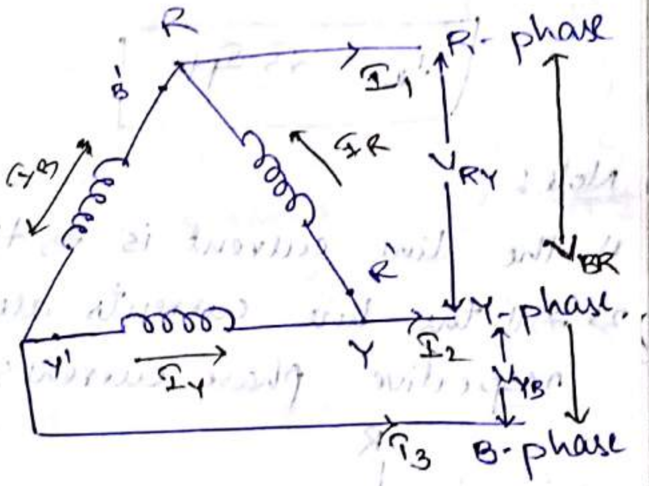
Three phase Delta connection:

phase voltages are equal to Line voltages in 3-ϕ delta connection.

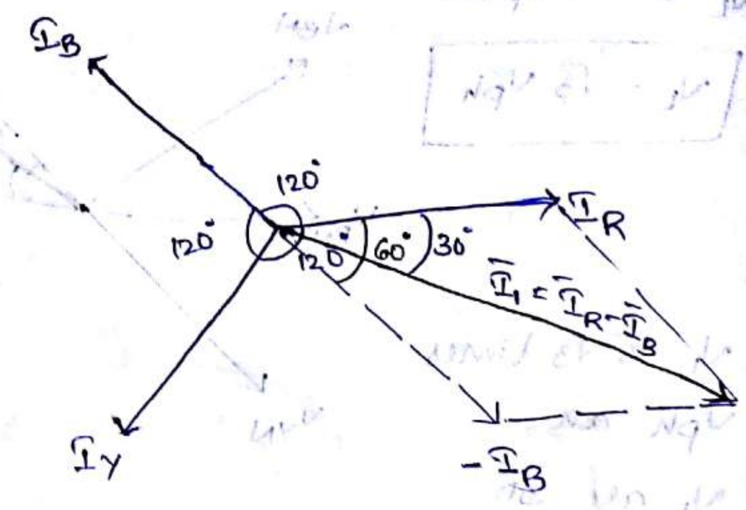
$I_R = I_m \sin \omega t$

$I_Y = I_m \sin(\omega t - 120°)$

$I_B = I_m \sin(\omega t - 240°)$



phase current	Line current;
I_R	$\bar{I}_1 = \bar{I}_R - \bar{I}_B$
I_Y	$\bar{I}_2 = \bar{I}_Y - \bar{I}_R$
I_B	$\bar{I}_3 = \bar{I}_B - \bar{I}_Y$



Accoring to parallelogram law of vectors.

$$|\bar I_1|^2 = |\bar I_R|^2 + |-\bar I_B|^2 + 2|\bar I_R||-\bar I_B| \cos(\bar I_R \hat{\ } \bar I_B)$$

$$I_L^2 = I_{ph}^2 + I_{ph}^2 + 2 I_{ph}^2 \cos 60°$$

$$I_L^2 = 2 I_{ph}^2 + \cancel{2} I_{ph}^2 \cdot \frac{1}{\cancel{2}}$$

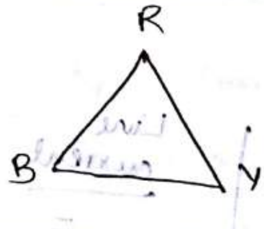
$$I_L^2 = 3 I_{ph}^2$$

$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

Note:

1) The line current is $\sqrt{3}$ times the phase current.

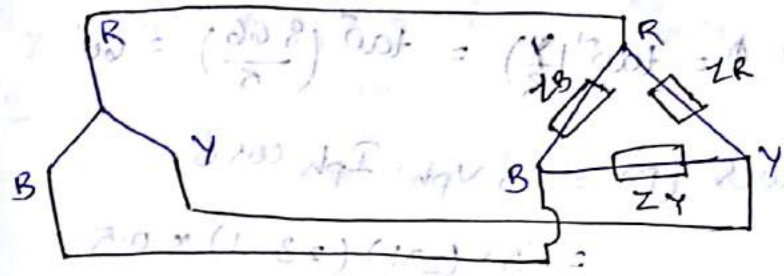
2) All the line currents are 30° behind the respective phase currents.

Connection / Parameter		
Voltages:	$V_L = \sqrt{3}\, V_{ph}$	$V_L = V_{ph}$
Currents:	$I_L = I_{ph}$	$I_L = \sqrt{3}\, I_{ph}$.
Power Real	$\sqrt{3}\, V_L I_L \cos\phi$ (or) $3 V_{ph} I_{ph} \cos\phi$	$\sqrt{3}\, V_L I_L \cos\phi$ (or) $3 V_{ph} I_{ph} \cos\phi$

Reactive power :	$Q = 3 V_{ph} I_{ph} \sin\phi$	$Q = 3 V_{ph} I_{ph} \sin\phi$
Apparent power :	$Q = 3 V_{ph} I_{ph}$	$Q = 3 V_{ph} I_{ph}$

Balanced circuit:

Source Load.



$Z_R = Z_Y = Z_B$

$\Rightarrow I_R = I_Y = I_B$

$\Rightarrow V_R = V_Y = V_B$

For Balance circuits: $V_{RN} + V_{YN} + V_{BN} = 0$ (ph V = 0)

$V_{RY} + V_{YB} + V_{BR} = 0$ (L.V = 0).

$I_R + I_Y + I_B = 0$ (P.C = 0)

$I_1 + I_2 + I_3 = 0$ (L.C = 0).

Q→ Three equal impedences each of $(5 + j8.68)\Omega$ are connected in star to a 400V 3-phase 50Hz supply. Calculate i, line current, {Here mentioned is Line voltage / Line currents}

ii, power factor

iii, Active power consumed.

$Z = 5 + 8.68j$ $\left\{ 10 = \sqrt{5^2 + (8.68)^2} \right.$

$= 10\angle 60°$ $\left. 60° = \tan^{-1}\left(\frac{8.68}{5}\right) \right\}$

i, In star, $I_L = I_{ph}$

$I_L = \frac{V}{Z}$

$V_L = 400V$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\angle 0°$

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$$I_L = I_{ph} = \frac{V_{ph}}{Z}$$

$$= \frac{231\angle 0^\circ}{10\angle 60^\circ} = 23.1 \angle -60^\circ$$

ii, power factor

$$\phi = \cos^{-1}(R/Z)$$

$$= \cos^{-1}\left(\frac{5}{10}\right) = 60^\circ$$

phase angle $\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{8.66}{5}\right) = 60^\circ$

iii, Active power $(P) = 3 V_{ph} \cdot I_{ph} \cos\phi$

$$= 3 \times (231)(23.1) \times 0.5$$

$$= 8000 \text{ Watt}$$

$$= 8 \text{ kW}.$$

Note: (Problem on resonance)

1) A coil has a resistance of 20 Ω and inductance of 80mH and is connected in series with a 100pF capacitor. Determine the resonance frequency and impedence at resonance. If supplied by a 50V source. Find the circuit current and voltage across capacitor.

i, $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{80\times10^{-3}\times100\times10^{-12}}} = 56.29 \text{ Hz}$

ii, $Z_{rc} = \cancel{Z_R}\, R = 20\,\Omega$

iii, $I = \frac{V}{Z} = \frac{50}{20} = 2.5 \text{ Amps}$

iv, $V_C = I X_C$

$$= 2.5 \cdot \frac{1}{\omega C}$$

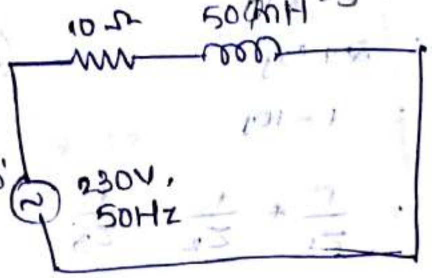
$$= \frac{2.5}{2\pi\times56.29\times100\times10^{-12}}$$

$$= 70.68 \text{ kV}$$

Problems on sinusoidal analysis for R-L-C circuits:

1) A resistance of $10\,\Omega$ is connected in series with 50 mH inductance across a 230 V, 50 Hz supply. calculate (a) current flowing in the circuit.
b) phase angle of the circuit.

$I = 12.35\,A$
-57.52°



$X_L = 2\pi f L$
$= 2\pi \times 50 \times 50 \times 10^{-3}$
$= 15.7$

$Z = 10 + 15.7j = 18.61 \angle 57.50^\circ$

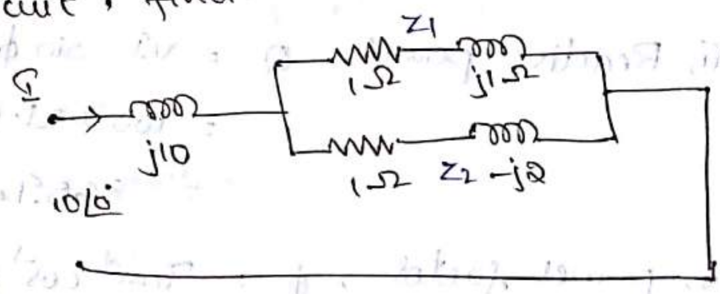
$I = \dfrac{V}{Z} = \dfrac{230 \angle 0^\circ}{18.61 \angle 57.50^\circ}$

$I = 12.35 \angle -57.50^\circ$

phase angle $= -57.50^\circ$
(or) current is lagging by 57.50°

$\left\{ \begin{array}{l} \text{power factor} = \cos\phi \\ = \cos(-57.50^\circ) \\ = 0.53 \end{array} \right\}$

2) In the following circuit, find current I.



$Z = j10$
$z_1 = (1 + j\,\Omega)$
$z_2 = (1 - 2j)\,\Omega$

$Z' = \dfrac{z_1 z_2}{z_1 + z_2} = \dfrac{(1+j)(1-2j)}{2-j} = \dfrac{3-j}{2-j} \times \dfrac{2+j}{2+j}$
$= \dfrac{(3-j)(2+j)}{5}$

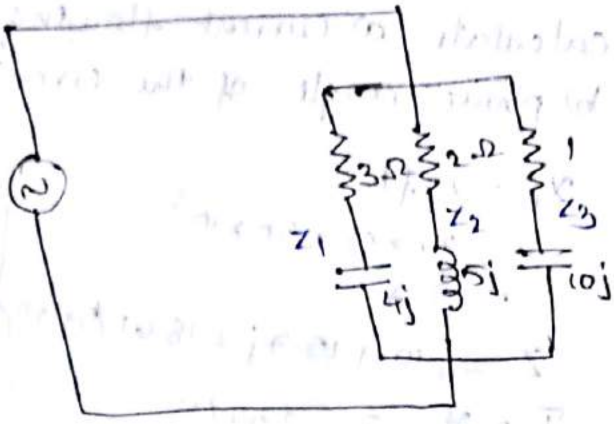
$Z' = \dfrac{7+j}{5}$

$Z = (j10) + \dfrac{7+j}{5} = 14 + 102j$

$Z = 10.29 \angle 82^\circ$

$I = \dfrac{V}{Z} = \dfrac{10 \angle 0^\circ}{10.29 \angle 82^\circ} = 0.97 \angle -82^\circ$

3) The circuit shown in figure, find the current I, active power, reactive power and power factor of the circuit. Assume voltage is 100V and frequency is 60 HZ.



$Z_1 = 3-4j$

$Z_2 = 2+5j$

$Z_3 = 1-10j$

$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\frac{1}{Z_t} = \frac{1}{3-4j} + \frac{1}{2+5j} + \frac{1}{1-10j}$$

$Z_t = 4.22 - 1.84j$

$= 4.61 \angle -23.53^\circ$

i, $I = \frac{V}{Z} = \frac{100 \angle 0^\circ}{4.61 \angle -23.53^\circ} = 21.69 \angle 23.53^\circ$ Amps

ii, Active power, $P = VI \cos\phi$

$= 100 \times 21.69 \cos(23.53^\circ)$

$= 1988.60$ Watt

iii, Reactive power, $Q = VI \sin\phi$

$= 100 \times 21.69 \sin(23.53^\circ)$

$= 865.9$ VAR

iv, power factor, $\phi = \cos^{-1}\left(\frac{R}{Z}\right)$

$= \cos^{-1}\left(\frac{4.22}{4.61}\right)$

$= \cos(23.53)$

$= 0.916$ lead.

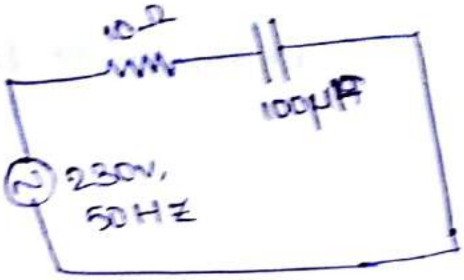
If the current is having positive sign in angle, it is leading.

4) A resistance of $10\,\Omega$ is connected in series with $100\,\mu F$ capacitor through a 230V, 50 Hz supply. Find a) impedence b) current c) power factor d) phaseangle e) voltage across the resistor and capacitor.

$\cancel{Z = 10 - j100\times10^{-6}}$

$\cancel{Z = 10 - j10^{-6}}$

$\cancel{I = \frac{V}{-Z} = \frac{230\angle 0^\circ}{}}$



a) impedence $Z = R - jX_C$

$$X_C = \frac{1}{C\omega} = \frac{1}{10^{-4} \times 2\pi \times 50}$$

$$X_C = \frac{0.318}{10^{-2}} = 31.8$$

$$Z = 10 - j(31.8)$$

b) current $I = \frac{V}{Z}$

$$I = \frac{230\angle 0^\circ}{33.33\angle -72.54^\circ}$$

$$I = 6.900\angle 72.54^\circ$$

c) power factor, $\cos\phi$

$$= \cos(72.54^\circ)$$

$$= 0.300$$

d) phase angle $= 72.54^\circ$

e) Voltage across resistor $V_R = 6.900 \times 10$
$= 69.00\ V$

voltage across capacitor $V_C = I X_C$
$= 6.900 \times 31.8$

$V_C = 219.42\ V.$

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UNIT ② : TRANSFORMERS

→ Ideal and practical transformers.

→ Equivalent circuit.

→ Losses

→ Regulation and Efficiency.

→ Auto - transformer.

→ 3-ϕ transformer connection

20/9/18

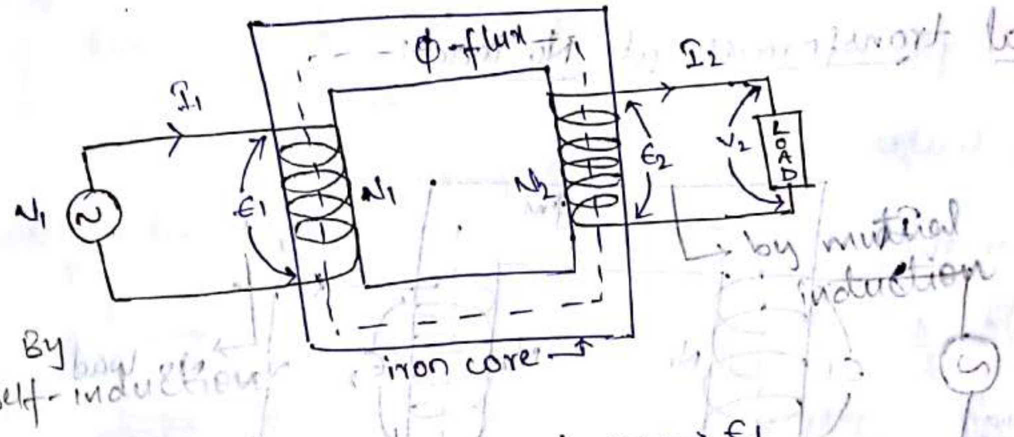
Transformer

- It is a static device which transfers the energy from one voltage level to another voltage level with corresponding change in current without changing the frequency.

Working principle:

(Faraday - mutal induction)

$$\epsilon = -N\frac{d\phi}{dt}$$



$$V_1 \to I_1 \to \phi \to \begin{cases} N_1 \text{ turns} \to \epsilon_1 \\ N_2 \text{ turns} \to \epsilon_2 \to I_2 \to V_2 \end{cases}$$

input power (i/p) = output power (o/p)

$$P_{in} = P_{out}$$

$$V_1 I_1 = V_2 I_2 \Rightarrow \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$\epsilon_1 I_1 = \epsilon_2 I_2$$

Transformation ratio :-

$$\boxed{K = \frac{V_2}{V_1} = \frac{\epsilon_2}{\epsilon_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}}$$

* Laminations made of silicon steel are used in transformer, to minimize the loses.

Assumption for ideal transformer:

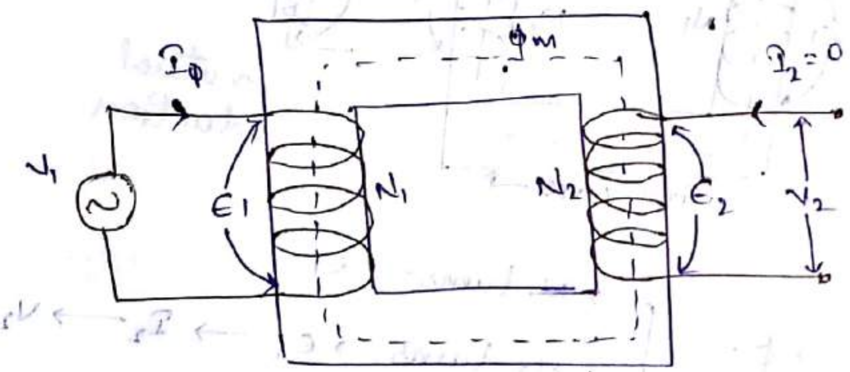
1) Primary and secondary windings have no resistance

2) All the flux produced by the primary winding links with the secondary winding i.e., there is no leakage of flux.

3) Permeability of the core is infinite.

4) Eddy current and hysteresis losses are neglected.

Ideal transformer at No load:



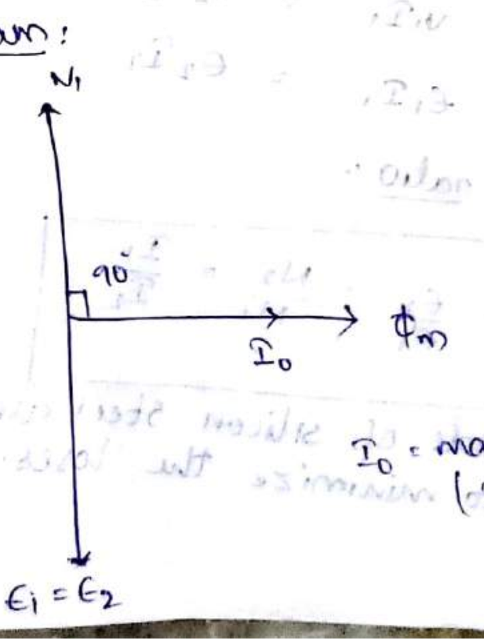
Since, Primary resistance = secondary resistance = 0

it is purely inductive.

$\therefore$ $V_{ea} = V_m \sin \omega t$

$i = I_m \sin(\omega t - \pi/2)$

Phasor diagram:



I_0 = magnetising current (I_μ / I_m)

Transformation - EMF equation:

V_1 = primary winding voltage

V_2 = secondary terminal voltage

I_1 = primary current

I_2 = secondary current

E_1 = primary induced Emf

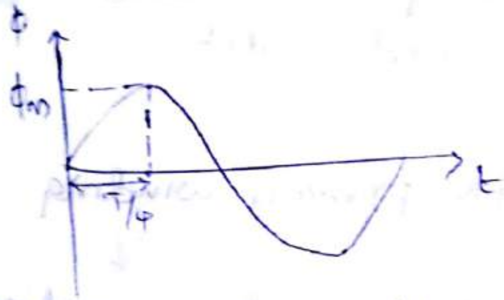
E_2 = secondary induced EMF

N_1 = primary winding turns

N_2 = secondary winding turns

ϕ_m = maximum flux

f = frequency of supply voltage



1 cycle = $T = \frac{1}{f}$

$\frac{1}{4}$ cycle = $\frac{T}{4}$

voltage induced = $\frac{d\phi}{dt}$

Average EMF induced pattern = $\boxed{\frac{\phi_m - 0}{\frac{1}{4f} - 0}}$

$E_{avg} = 4f\phi_m$

form factor = $\frac{\text{RMS value}}{\text{Average value}}$ = 1.11 (sine wave)

RMS value = 1.11 × Avg value

= $1.11 \times 4f\phi_m$

= $4.44 f\phi_m$

EMF across primary winding having N_1 turns (E_1)

= $4.44 N_1 f \phi_m$

EMF across secondary winding having N_2 turns (E_2)

= $4.44 N_2 f \phi_m$

28/7/18

VA Rating of Transformer :

While transfering energy to the transformer, there are few power losses mean while. And these losses appear in the form of heat. Generally the rating of any machine is limited by thermal limit.

→ As the temperature is due to the two types of losses.

i. Iron losses / core losses proportional to Voltage

ii. Copper losses proportional to the current

i.e. losses depends upon voltage & current

→ So the transformer rating is expressed in terms of product of voltage and current.

Rating: $\frac{V_1 I_1}{1000}$ = KVA for primary winding

$\frac{V_2 I_2}{1000}$ = KVA for secondary winding

Note: $\boxed{V_1 I_1 = V_2 I_2}$

* Transformer works with AC only. If DC supply is given, mutual induction won't take place as it requires uniformly changing current. And the winding will draw high current as the inductive reactance is zero.

In transformer (AC) $Z = R + jX_L$

for DC, $X_L = 0$ $\left\{ \because X_L = L\omega = 2\pi f L \quad \boxed{\because f = 0} \right\}$

$\therefore \boxed{Z = R}$

Since impedence is minimum, current will be high $\left\{ I = \frac{V}{R_{min}} \right\}$

With this DC supply, the impedence is very low, which results in burning of the winding.

Losses in Transformer:

Losses

- (Iron losses) $\propto V$
  (core loss / magnetic loss)
  ↓
  [fixed / constant loss]
- (copper losses) $\propto I$
  [variable loss]
  (I depends upon load)

Total loss in T/F = fixed losses + variable losses
= iron losses + copper losses

Fixed losses

- Eddy current losses
- Hysteresis losses
  (B-H curve)

where B = Magnetic field intensity
H = magnetic strength

* These losses depends upon
  i, voltage
  ii, flux
  iii, materials with what they are prepared

Ideal Transformer:

It is a transformer with all losses neglected.

i, It's windings havee zero resistance

ii, Leakage flux is zero.

iii, Permeability of core is high.
(easily allows flux)

iv, $V_1 = E_1$ & $V_2 = E_2$.

Ideal transformer (No Load) :,

I_1 = primary current just sufficient to produce the flux in the core.

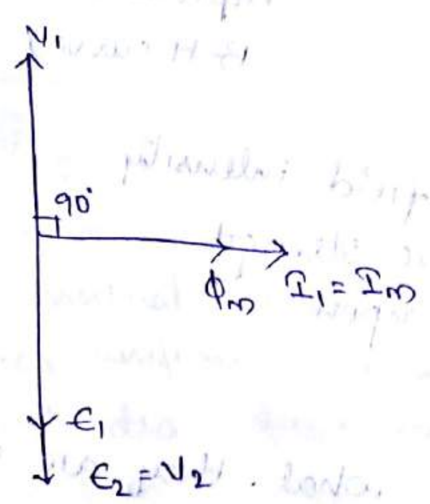
* I_1 is producing the flux(ϕ). so $I_1 = I_m$ where I_m is magnetising current.

→ The magnetising current (I_m) lags the supply voltage by an angle of 90°.

→ And I_1 produces the flux which is in phase with it.

→ flux (ϕ) induces E_1 and E_2 which are anti-phase with V_1.

phasor diagram:



* E_1 & E_2 are anti-phase with V_1 according to Lenz's law [The induced EMF always opposes the very first cause of it]

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→ The power input to the ideal transformer is zero.

$$P_1 = V_1 I_1 \cos\phi_1$$
$$= V_1 I_1 \cos 90^\circ$$
$$= 0.$$

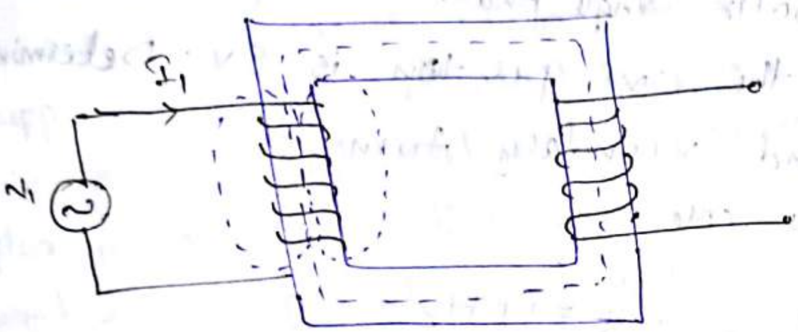
Note:

The ideal transformer will not take any power to maintain the magnetic flux in the transformer.

Practical Transformer: (No load)

Losses :- i, Resistance (I^2R)

ii, Leakage reactance (IX_L)



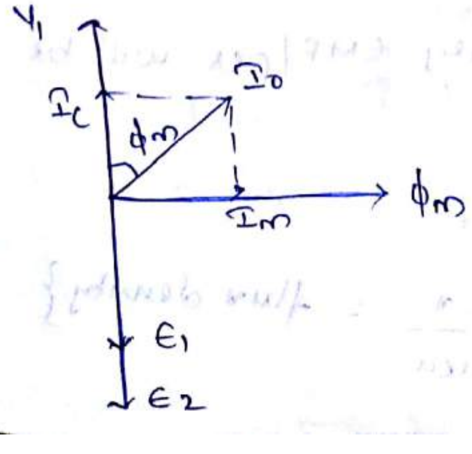
Total losses = Iron loss + copper loss

Iron loss ↑ Magnetizing current I_m

copper loss ↑ working current I_c (or) I_w

$$I_1 = I_m + I_w$$
$$I_0 = I_m + I_w \quad \{\text{At no. load},\ I_1 = I_0\}$$

Phasor diagram:



$$0^\circ < \phi_m < 90^\circ$$

$$\bar{I}_0 = \bar{I}_m + \bar{I}_c$$
$$\bar{I}_0 = I_0 \sin\phi_0 + I_0 \cos\phi_c$$

Note:

→ Total power input $P_1 = V_1 I_1 \cos\phi_1$

But for no-load case $P_0 / W_0 = V_1 I_0 \cos\phi$

$$\boxed{W_0 = V_1 I_c}$$

Note:

→ The no-load current I_0 is 3-5% of full load current. Hence primary copper losses are neglected. So I_c is called core loss (or) iron loss component. Hence power input represents no-load loss (or) iron loss.

* The maximum flux density in the core of a 250/3000V, 50Hz single phase transformer is 1.2 Wb/m². If the EMF per turn is 8V. Determine
a) primary and secondary turns
b) area of the core.

$\frac{E_1}{E_2} = \frac{250}{3000V}$, $f = 50Hz$

$B_m = 1.2\ Wb/m^2$

EMF / turn = 8V

a) 1 turn ⟶ 8V

? ⟵ 250V

$N_1 = \frac{250}{8}$ turns $= 31.25 = 32$ turns

Similarly, $N_2 = \frac{3000}{8} = 375$ turns

{∵ For primary & secondary EMF/per will be constant}

b)

$E_1 = 4.44\,\phi_m f N_1$

$B_m = \frac{\phi_m}{A}$ $\left\{\frac{\text{flux}}{\text{area}} = \text{flux density}\right\}$

$$\Rightarrow A = \frac{\phi_m}{B_m}$$

$$250 = 4.44 \times \phi_m \times 50 \times 32$$

$$\phi_m = \frac{250}{7104}$$

$$\therefore A = \frac{250/7104}{1.2} = 0.029$$

$$A = 0.03\ m^2$$

* The no load current of a transformer is 5 Amps at 0.3 power factor, when supplied at 230V, 50Hz supply. The number of the turns on the primary winding is 100. Calculate i, maximum value of flux in the core. ii, magnetising current and working component of current. iii, core losses in the transformer.

Given $I_0 = 5\ A$.

$\cos\phi_0 = 0.3$

i, $E_1 = 4.44\, \phi_m f N_1$

$I_m \rightarrow$ used for generating ϕ

$I_w \rightarrow$ losses

$$230 = 4.44\, \phi_m \times 50 \times 100$$

$$\phi_m = \frac{230}{444 \times 50}$$

$$\phi_m = 0.0103\ Wb.$$

ii, For No. load current $\bar{I}_0 = \bar{I}_m + \bar{I}_w$ (I_w / I_c)

$$I_0 = I_0 \sin\phi_0 + I_0 \cos\phi_0$$

$$\therefore I_m = I_0 \sin\phi_0 = 5 \times \sin\phi\ 0.953 = 4.765\ A$$

$$I_w \text{ (or) } I_c = I_0 \cos\phi_0$$

$$= 5 \times 0.3$$

$$= 1.5\ Amps.$$

iii, core losses in transformer

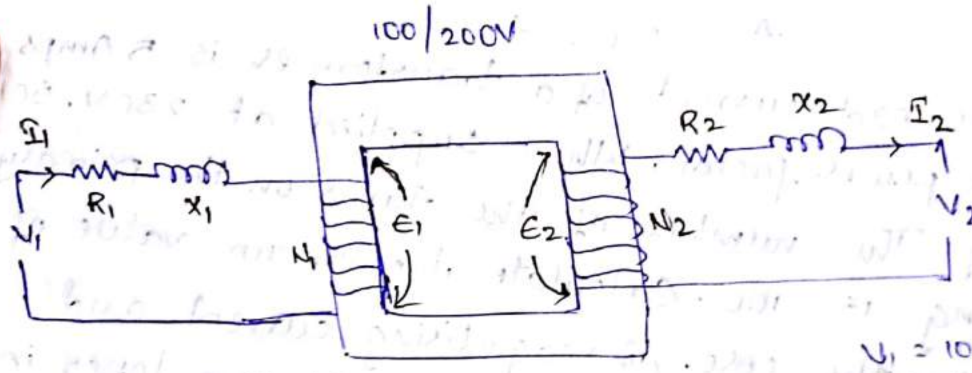
$$P_o = 230 \times 4.7668 \; I_o \cos\phi \quad \{ P_o = V_1 I_o \cos\phi$$

$$P = 230 \times 5 \times 0.3$$

$$P = 1150 \text{ Watt} \times 0.3 = 345 \text{ Watt}$$

{∵ No-load input = No-load losses}

Transformer Equivalent Resistance, Equivalent Reactance and equivalent impedence:



$V_1 = 100$ V
⇒ $I_1 = 10$
$V_2 = 200$V.
$I_2 = 5$ Amp
T/F = 1 KVA

$$V_1 = \text{v. drop} + E_1$$

$$V_1 = (I_1R_1 + jX_1I_1) + E_1$$

$$V_1 = I_1(R_1 + jX_1) + E_1$$

$$\boxed{V_1 = I_1Z_1 + E_1}$$

$$E_2 = \text{v. drop} + V_2$$

$$E_2 = (I_2R_2 + jX_2I_2) + V_2$$

$$E_2 = I_2(R_2 + jX_2) + V_2$$

$$\boxed{E_2 = I_2Z_2 + V_2}$$

For Example: $V_1 = I_1Z_1 + E_1$

$100 = 3 + 97.$

$\cancel{V_2 = I_2Z_2 + E_2}$ $\quad E_2 = I_2Z_2 + V_2$

$\cancel{200 = 5 + 195}$ $\quad 205 = 5 + 200.$

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Secondary parameters referred to Primary ckt parameters :

$R_2 \longrightarrow R_2'$

$X_2 \longrightarrow X_2'$

$Z_2 \longrightarrow Z_2'$

$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

sec. Resistance referred to i ckt $\boxed{R_2' = \frac{R_2}{K^2}}$

Similarly $\boxed{X_2' = \frac{X_2}{K^2}}$

$$\boxed{Z_2' = \frac{Z_2}{K^2}}$$

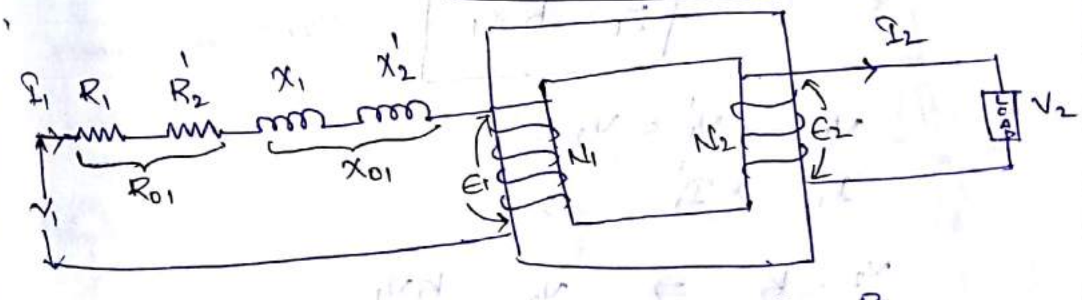
$V_2 \longrightarrow V_2'$

$I_2 \longrightarrow I_2'$

$\boxed{V_2' = V_1 = \frac{V_2}{K}}$ $\left\{\because \frac{V_2}{V_1} = K\right\}$

$$\frac{I_1}{I_2} = K \Rightarrow I_1 = K \cdot I_2$$

$$\boxed{I_2' = I_1 = K \cdot I_2}$$



Total resistance refferred to primary $R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$

Total reactance referred to primary $X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$

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Total impedence referred to primary $Z_{01} = R_{01} + jX_{01}$

Total power loss in resistance $= I_1^2 R_{01}$

Total power loss in reactance $= I_1^2 X_{01}$

Total power loss in T/F $= I_1^2 Z_{01}$

Voltage drop (V_{drop}) $= I_1 Z_{01}$ {in total T/F ckt}

- Primary parameters referred to secondary ckt parameters:

$$R_1 \longrightarrow R_1'$$
$$X_1 \longrightarrow X_1'$$
$$Z_1 \longrightarrow Z_1'$$

$$I_1^2 R_1 = I_2^2 R_1'$$

$$R_1' = \frac{I_1^2}{I_2^2} R_1$$

$$\boxed{R_1' = K^2 R_1}$$

Similarly $\boxed{X_1' = K^2 \cancel{R}_1}$

$$\boxed{Z_1' = K^2 Z_1}$$

$$V_1 \longrightarrow V_1' = V_2$$
$$I_1 \longrightarrow I_1' = I_2$$

$$\frac{V_2}{V_1} = K \Rightarrow V_2 = KV_1$$

$$\boxed{V_1' = V_2 = KV_1}$$

$$\frac{I_1}{I_2} = K \Rightarrow I_2 = \frac{I_1}{K}$$

$$\boxed{I_1' = I_2 = \frac{I_1}{K}}$$

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Total resistance in T/F referred to secondary $R_{02} = R_2 + R_1'$

Total reactance referred to secondary $X_{02} = X_2 + X_1'$

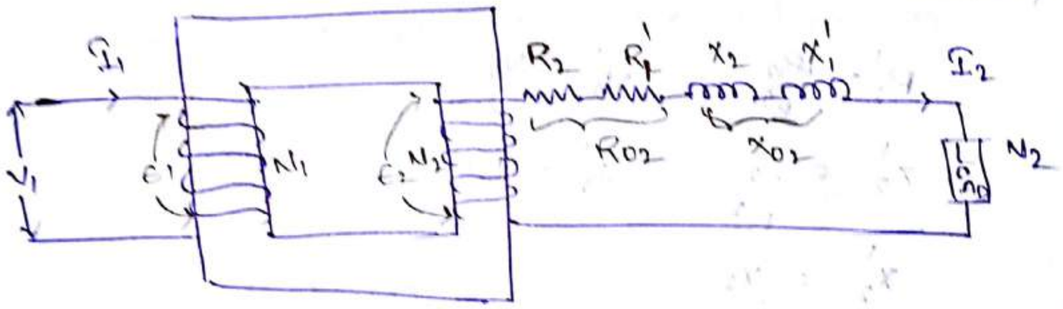
Total impedence referred to secondary $Z_{02} = R_{02} + jX_{02}$

Total power loss in resistance $= I_2^2 R_{02}$

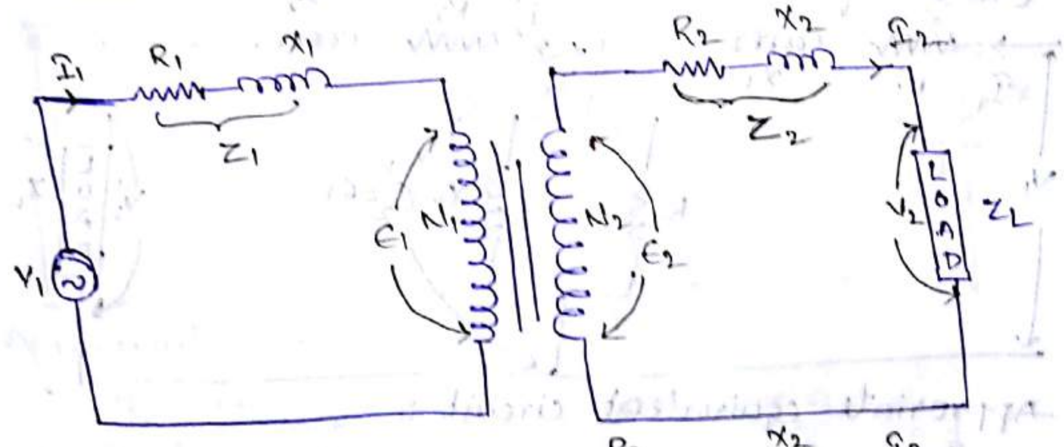
Total power loss in reactance $= I_2^2 X_{02}$

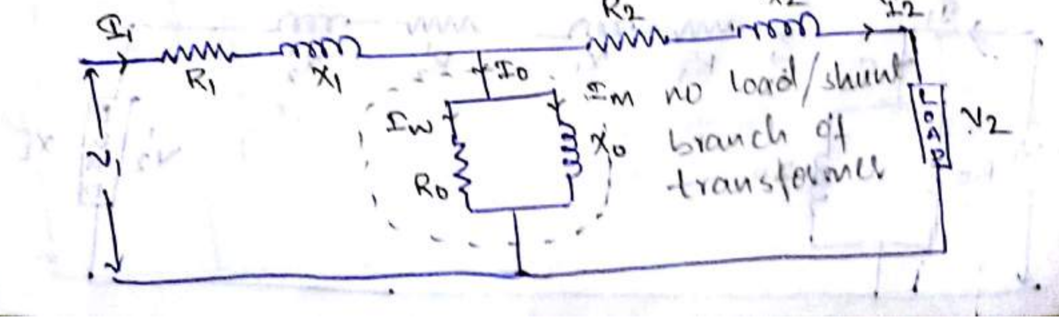
Total power loss in T/F $= I_2^2 Z_{02}$

Voltage drop (V_{drop}) $= I_2 Z_{02}$ {in total T/F ckt}



Equivalent Circuit of Transformer:





Primary current = No load current + primary component of sec. circuit

$$I_1 = I_0 + I_2'$$

No load current $I_0 = I_m + I_w$

I_0 = magnetising current + working component of current

$$\boxed{I_0 = I_0 \sin\phi_0 + I_0 \cos\phi_0}$$

$$R_0 = \frac{V_1}{I_w}, \quad X_0 = \frac{V_1}{I_m}$$

$$\left\{\begin{array}{l} I_w = I_c \text{ (core loss current)} \\ I_m = I_\mu \end{array}\right\}$$

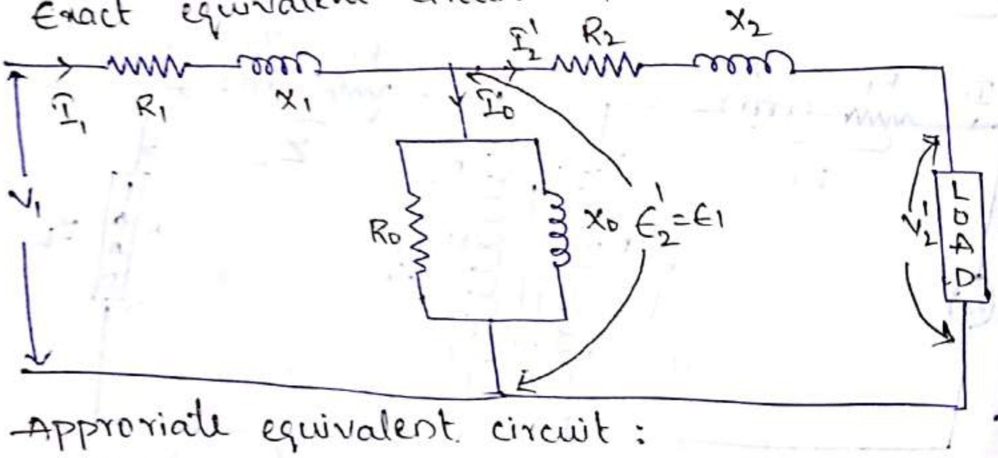
Equivalent circuit of Transformer (secondary parameters referred to primary)

$$R_2' = \frac{R_2}{K^2} \qquad V_2' = \frac{V_2}{K}$$

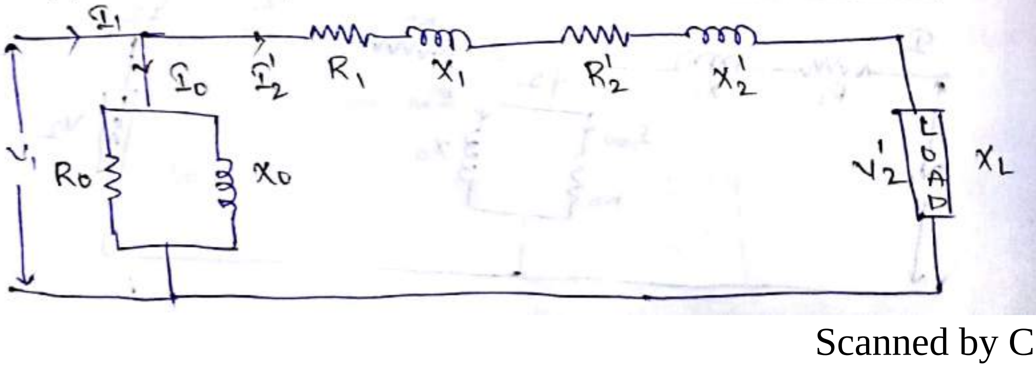
$$Z_2' = \frac{Z_2}{K^2} \qquad I_2' = I_2 \cdot K$$

$$X_2' = \frac{X_2}{K^2}$$

Exact equivalent circuit:



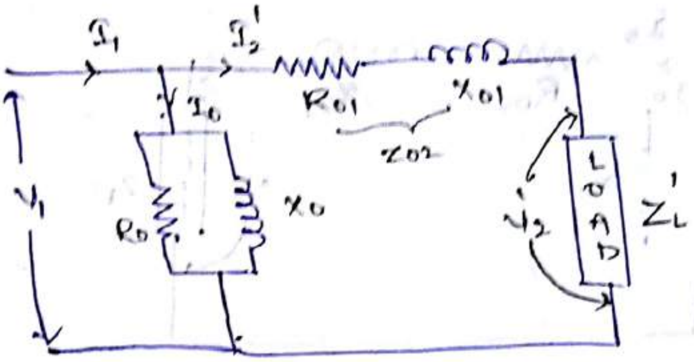
Approriate equivalent circuit:



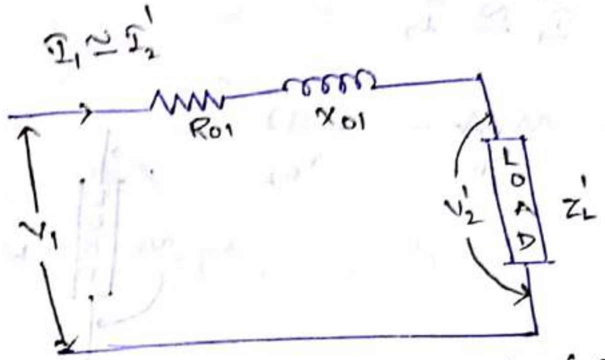
$R_{01} = R_1 + R_2'$

$X_{01} = X_1 + X_2'$

$$\boxed{Z_{01} = R_{01} + jX_{01}}$$



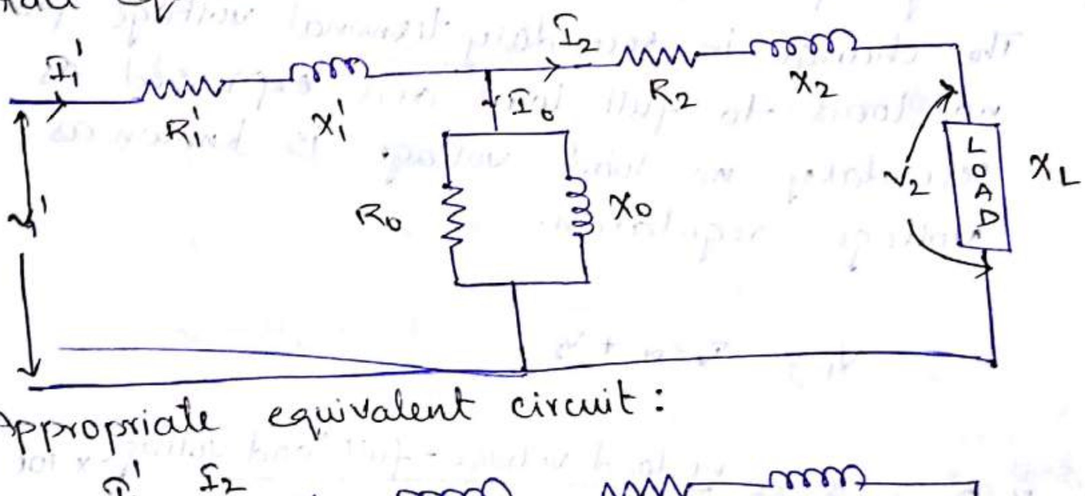
$\Rightarrow$



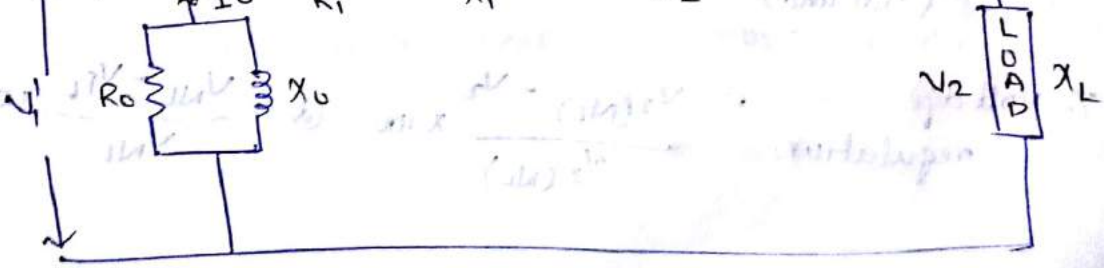
Equivalent circuit of Transformer (Primary parameters referred to secondary ckt)

$R_1' = K^2 R_1$, $V_1' = V_1 \cdot K$, $I_1' = \frac{I_1}{K}$

Exact equivalent circuit :



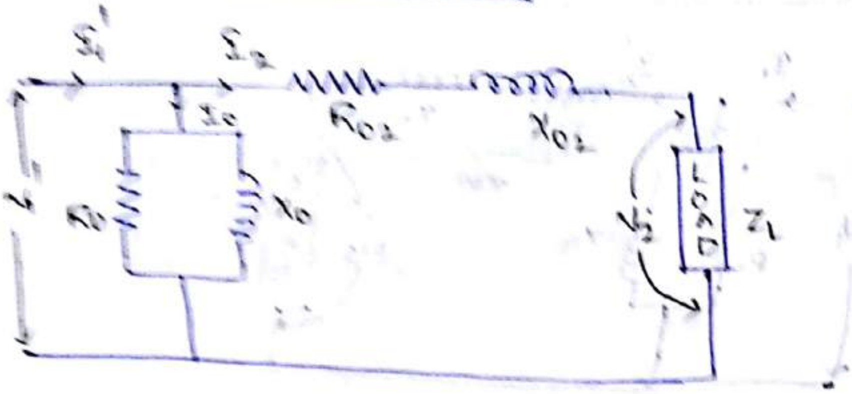
Appropriate equivalent circuit :



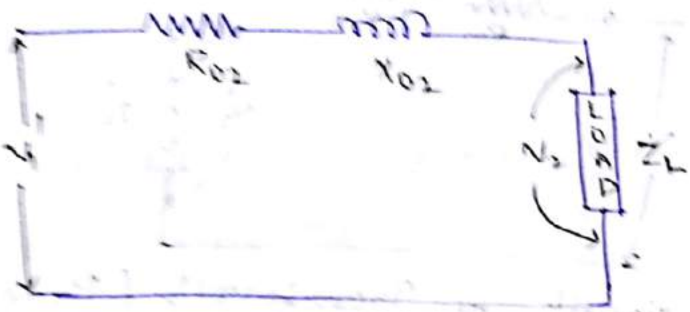
$R_{02} = R_1' + R_2$

$X_{02} = X_1' + X_2$

$$\boxed{Z_{02} = R_{02} + j X_{02}}$$



$I_1' \simeq I_2$



Voltage Regulation of Transformer :

— When the transformer is loaded it's terminal voltage falls from no load to full load. The change in secondary terminal voltage from no load to full load and expressed as secondary no load voltage is known as voltage regulation.

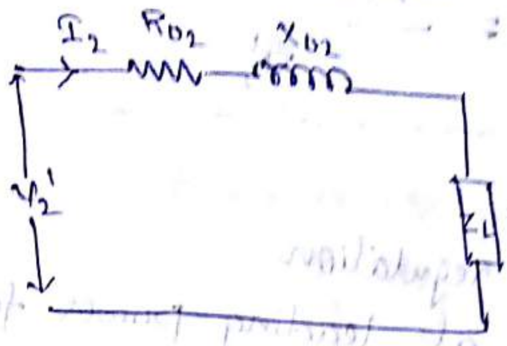
$$V_1 = I_2 Z_{01} + V_2$$

$$\%\ \text{Regulation (Full load)} = \frac{\text{no load voltage} - \text{full load voltage}}{\text{no load voltage}} \times 100$$

$$\%\ \text{voltage regulation} = \frac{V_{2(NL)} - V_2}{V_{2(NL)}} \times 100 \quad (or) \quad \frac{V_{NL} - V_{FL}}{V_{NL}} \times 100$$

% Regulation (Load) = $\frac{V_{NL} - V_L}{V_{NL}} \times 100$

equivalent circuit (referred to secondary)

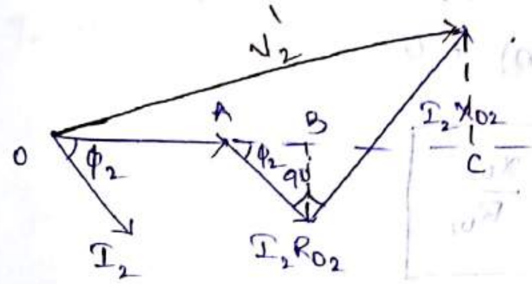


$R_{02} = R_1' + R_2$

$X_{02} = X_1' + X_2$

Phasor Diagram :

→ Assume I_2 is lagging by ϕ_2



$\therefore V_2' = V_2 + I_2 R_{02} + j I_2 X_{02}$

$V_2' = V_2 + I_2 R_{02} \cos\phi_2 + j I_2 X_{02} \sin\phi_2$

$OC = OA + AB + BC$

$V_2' - V_2 = I_2 (R_{02} \cos\phi_2 + j X_{02} \sin\phi_2)$

$V_2' - V_2 = I_2 (R_{02} \cos\phi_2 \pm j X_{02} \sin\phi_2)$

* If current lags, then $I_2 (R_{02} \cos\phi_2 + j X_{02} \sin\phi_2)$

If current leads, then $I_2 (R_{02} \cos\phi_2 - j X_{02} \sin\phi_2)$

% Regulation = $\frac{V_2' - V_2}{V_2'} \times 100$

$$\boxed{\%\ \text{Voltage Regulation} = \frac{I_2(R_{02}\cos\phi_2 \pm X_{02}\sin\phi_2)}{V_2'} \times 100}$$

Note :

1) Condition for zero regulation
- it is possible at leading power factor

$$R_{02}\cos\phi_2 - X_{02}\sin\phi_2 = 0$$

$$\boxed{\tan\phi_2 = \frac{R_{02}}{X_{02}}}$$

2) Condition for maximum regulation
- it is possible at lagging power factor.

$$\frac{d}{d\phi}(\%\ \text{regulation}) = 0$$

$$\boxed{\tan\phi_2 = \frac{X_{02}}{R_{02}}}$$

Q→ A single phase 40 KVA, 6600/250V transformer has primary and secondary resistances; $R_1 = 10\Omega$ and $R_2 = 0.02\Omega$ respectively. The equivalent leakage reactance as referred to primary is 35Ω; Find the

i, full load regulation at unity power factor

ii, half load regulation at 0.8 p.f lagging.

iii, 3/4th load regulation at 0.8 p.f leading.

$R_1 = 10\Omega$; $X_{01} = 35\Omega$

$R_2 = 0.02\Omega$

$V_1 = 6600V$

$V_2 = 250V$

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$R_{02} = R_1' + R_2 = K^2 \cdot R_1 + R_2 = 10(0.0014) + 0.02$
$= 0.0343\ \Omega$

$K = \dfrac{\cancel{250}^{5}}{\cancel{6600}_{132}} = \dfrac{5}{132} = 0.0379$

$X_{02} = X_1' + X_2$

$= K^2 \cdot X_{01}$

$= (0.0379)^2 35$

$= 0.0014 \times 35$

$= 0.0502\ \Omega$

$V_2 I_2 = 40 \times 10^3$

$I_2 = \dfrac{40 \times \cancel{1000}^{4}}{\cancel{250}} = 160$ Amps

i, $I_2 = 160A$

$\%$ regulation $= \dfrac{I_2 (R_{02} \cos\phi_2 + j X_{02} \sin\phi_2)}{V_2'} \times 100$

$\cos\phi_2 = 1$

$\sin\phi_2 = 0$

$\Rightarrow \dfrac{I_2 (0.0343 \times 1 + 0)}{250} \times 100$

$\Rightarrow \dfrac{16\cancel{0} (0.0343)}{\cancel{250}} \times \cancel{100}^{4}$

$= 2.19\%$

ii, $\%$ reg $= \dfrac{8\cancel{0} (0.0343 \times 0.8 + 0.0502 \times 0.6)}{\cancel{250}} \times \cancel{100}^{4}$

$= 32 (0.02744 + 0.03012)$

$= 32 (0.05756)$

$= 1.84\%$

iii) % reg $= \frac{120\,(0.0343\times0.8 - 0.0502\times0.6)}{250}\times 100$

$= 48\,(0.02744 - 0.03012)$

$= 48\,(-0.00268)$

$= -0.128\ \%$

Losses and Efficiency in Transformer :

η = Efficiency $= \frac{\text{output power}}{\text{input power}} \times 100$

$$\eta = \frac{\text{output power}}{\text{output power} + \text{losses}} \times 100$$

$$\eta = \frac{\text{input power} - \text{losses}}{\text{input power}} \times 100$$

Total losses
- iron/core losses
  - hysteresis losses
  - Eddy current losses
- copper losses (copper winding pri + secondary)

Hysteresis losses-

The alternating flux setter setup in magnetic core of transformer undergoes a cycle of magnetisation and demagnetisation resulting in loss of energy which is termed as hysteresis loss.

$$\boxed{W_h = \eta\, B_{max}^{1.4} f \cdot V}$$

η = hysteresis coefficient of the core material

B_{max} = Maximum flux density.

f = frequency of supply.

v = volume of core.

eddy Current losses -

When alternating flux linking the core which will induce an EMF in the core called Eddy EMF due to which a current called Eddy current is being circulated in the core. Due to resistance of the core and with Eddy current circulation, Eddy current losses occur in the form of heat.

$$\boxed{W_e = K_e \, B_{max}^2 \, f^2 \, t^2}$$

Where; K = Eddy current constant

B_{max} = Maximum flux density

f = frequency of supply

t = thickness of lamination.

Note:

Both the hysteresis and Eddy losses depends upon flux density and frequency which are constant based on the A.C supply. Hence these losses are practically constant for (all cores) different codings on transformer.

$W_h \propto B_{max}^x f$ $\quad$ $W_e \propto B_{max}^2 f$

* Hence iron losses are called constant losses (or) fixed losses. (i.e, supply is constant)

Copper losses:

Due to the resistances of primary and secondary windings, power is wasted.

$$\text{copper loss} = I_1^2 R_1 + I_2^2 R_2$$
$$= I_1^2 R_{01}$$
$$= I_2^2 R_{02}$$

* copper losses are called variable losses as these losses depends upon the current which changes based on Load values.

Total losses = iron losses + copper losses
= constant losses + variable losses.
= w_i (or) w_o + w_{cu}

Note:

1. Iron losses are accounted for 10% and copper losses are accounted for 90% of total losses.
2. Hysteresis losses are minimised by using steel of high silicon content.
3. Eddy current losses are minimised by using low resistant and good quality copper material.

~~Efficiency~~:
Efficiency:

$$\eta = \frac{\text{output power}}{\text{output power + total losses}} \times 100$$

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + (w_i + w_{cu})} \times 100$$

$$\eta \text{ at 'x' load} = \frac{x(V_2 I_2 \cos\phi_2)}{x(V_2 I_2 \cos\phi_2) + (w_i + x^2 w_{FL \cdot cu})} \times 100$$

$$\eta_x = \frac{x \cdot KVA \cos\phi_2}{x(KVA \cos\phi_2) + (w_i + x^2 w_{FL \cdot cu})} \times 100$$

where, x = fraction of load

* Condition for maximum efficiency

$$\frac{d(\eta)}{dI_L} = 0$$

$$w_i = w_{cu \cdot Loss}$$

$$w_i = x^2 \, w_{cu \cdot loss}$$

$$\boxed{x = \sqrt{\frac{w_i}{w_{cu \cdot loss}}}}$$

i.e., $\boxed{\text{iron losses} = \text{copper losses}}$

Q→ For a single phase, 50 HZ, 150 KVA transformer the required no load voltage ratio is 5000/250 V. Find
a) The no. of turns in each winding for a maximum core flux of 0.06 Wb.
b) The efficiency at half rated KVA and unity power factor
c) Efficiency at full load with 0.8 p.f lagging
d) The KVA load for maximum efficiency. if full load copper loss are 1800 Watt and core loss are 1500 Watt.

$1-\phi$, 50 Hz, $W_{Cu\,FL} = 1800\,W$

150 KVA, $W_i = 1500\,W$

5000/250 V. $\phi = 0.06$ Wb

a) $E_1 = 4.44\,\phi f N_1$

$E_2 = 4.44\,\phi f N_2$

$5000 = 4.44 \times 0.06 \times 50 \times N_1$

$$N_1 = \frac{5000}{4.44 \times 0.06 \times 50}$$

$$N_1 = \frac{5000}{13.32}$$

$$= 375.37 = 375 \text{ turns}$$

$E_2 = 4.44\,\phi f N_2$

$$N_2 = \frac{250}{4.44 \times 0.06 \times 50}$$

$$= \frac{250}{13.32}$$

$$= 18.76 = 19 \text{ turns}$$

(b) $$\eta_x = \frac{x \cdot KVA \cos\phi_2}{x \cdot KVA \cos\phi_2 + W_i + x^2 W_{F.L\,Cu}} \times 100$$

$$\eta_{1/2} = \frac{0.5 \times 150 \times 1 \times 1}{75000 + 1500 + \frac{\cancel{1800}^{450}}{4}} \times 100$$

$$= \frac{75000}{75000 + 1500 + 450} \times 100$$

$$= \frac{\cancel{750} \cancel{\times 100}}{2025}$$

$$\eta_{1/2} = \cancel{3.703\%}$$

$$\eta_{1/2} = \frac{75000}{76950} \times 100$$

$$\eta_{1/2} = 97.46\%$$

c) $$\eta = \frac{1 \times 75000 \times 0.8 \times 2}{1 \times 75000 \times 0.8 \times 2 + 1500 + 1800} \times 100$$

$$\eta = \frac{\cancel{60000 \times 2}}{\cancel{63300}} \cancel{\times 100}$$

$$\eta = 94.78\% \qquad \eta = \frac{12000}{15300} \times 100$$

$$\eta =$$

c) $$\eta = \frac{1 \times 150000 \times 0.8}{150000 \times 0.8 + 1500 + 1800} \times 100$$

$$= \frac{120000}{120000 + 1500 + 1800} \times 100$$

$$= \frac{120000}{123300} \times 100$$

$$= 97.3\%$$

d) for maximum efficiency, copper loss = iron loss.

$$x = \sqrt{\frac{w_i}{w_{F.L.cu}}}$$

$$x = \sqrt{\frac{1500}{1800}} = \sqrt{\frac{5}{6}}$$

$$x = 0.913$$

$$\eta_{max} = 0.913 \times 150$$

$$= 137 \text{ KVA} \quad \{ \text{KVA for } \eta_{max} = x \cdot \text{KVA} \}$$

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Q→ A 50 KVA transformer, on full load has a copper loss of 600 Watt and iron loss of 500 Watt. Calculate maximum efficiency and load at which it occurs. Assume p.f as unity.

For maximum efficiency, $x = \sqrt{\frac{500}{600}} = 0.913$

$\eta_{max} = x \cdot KVA$

$= 0.913 \times 50$

$= 45.65$ KVA

$\cancel{45.65} = \dfrac{x \cdot \cancel{50000 \times 1}}{x \cdot \cancel{50000 + 1100}}$

$\cancel{228250x} + 50215 = \cancel{50000x}$

$x = \cancel{\dfrac{50215}{27175}} = \cancel{1.84}$

$\eta_{max} = \dfrac{45.65K}{45.65K + 0.5 + 0.6x^2} \times 100$

$= \dfrac{45650}{45650 + 500 + 500.14} \times 100$

$= \dfrac{45650}{46650.14} \times 100$

$= 97.85\%$

Auto Transformer :

A transformer in which a part of winding is common to both primary and secondary circuits is called a auto transformer. With this auto transformer, we get variable secondary voltage where primary and secondary circuits are windings are connected electrically and magnetically.

Because of its only one winding, it uses less copper and hence it is cheaper.

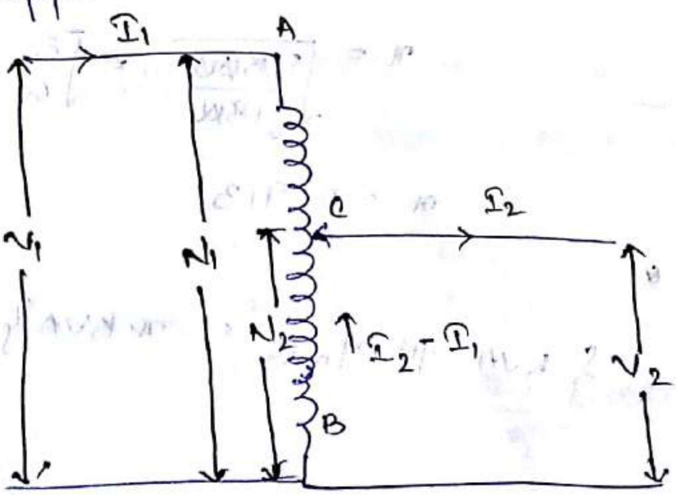


Fig : a

step down - Auto - T/F

$(I_2 > I_1)$

$(V_1 < V_2)$

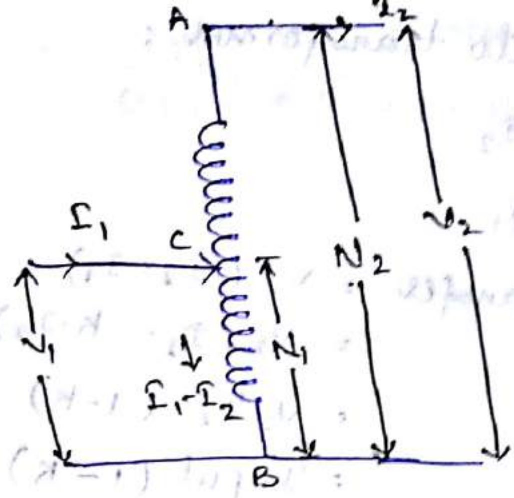


Fig:b
step-up auto transformer
$(I_1 > I_2)$

Fig:a

Primary winding	Secondary winding
AB	CB
V_1	V_2
I_1	I_2
N_1	N_2

Fig:b

Primary winding	Secondary/common
CB	AB
V_1	V_2
N_1	N_2
I_1	I_2

* Transformation ratio (k) :- $k = \frac{V_2}{V_1} = \frac{I_1}{I_2}$

AC CB

$$(V_1 - V_2) I_1 = V_2 (I_2 - I_1)$$

$$V_1 I_1 - \cancel{V_2 I_1} = V_2 I_2 - \cancel{V_2 I_1}$$

$$V_1 I_1 = V_2 I_2$$

input KVA = output KVA

$$\boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2}}$$

voltage per turn is constant.

$$\frac{V_2}{V_1 - V_2} = \frac{N_2}{N_1 - N_2}$$

$$V_2 N_1 - \cancel{V_2 N_2} = V_1 N_2 - \cancel{V_2 N_2}$$

$$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1}}$$

Power transfer in auto transformer:

Output power $= V_2 I_2$

Input power $= V_1 I_1$

Inductive power transfer $= V_2 (I_2 - I_1)$

$= V_2 (I_2 - K \cdot I_2)$

$= V_2 I_2 (1-K)$

$=$ Input $(1-K)$

Conductive power transfer $=$ input $-$ inductive power transfer

$=$ input $-$ input $(1-K)$

$=$ input $[1-1+K]$

$= K \cdot$ input.

Input $=$ inductive $+$ conductive $=$ output

For Two winding transformer

$100 = 100 + 0 = 100$

For auto transformer (if $K = 0.2$)

$100 = 80 + 20 = 100$

Copper Saving in auto transformer compared to two winding transformer :-

Rating (or) copper weight $=$ turns

$= N \cdot I$

(length) (cross section)

For two-winding T/F

copper weight $= N_1 I_1 + N_2 I_2 \longrightarrow (1)$

for auto T/F

copper weight $= I_1 (N_1 - N_2) + N_2 (I_2 - I_1)$

$= I_1 N_1 + I_2 N_2 - 2 I_1 N_2 \longrightarrow (2)$

$$\frac{②}{①} = \frac{\text{copper in auto T/F}}{\text{copper in 2-winding T/F}} = \frac{I_1N_1 + I_2N_2 - 2I_1N_2}{I_1N_1 + I_2N_2}$$

$$= 1 - \frac{2I_1N_2}{I_1N_1 + I_2N_2}$$

$$= 1 - \frac{\cancel{2}\cancel{I_1}N_2}{\cancel{2}\cancel{I_1}N_1}$$

$$= 1 - \frac{N_2}{N_1}$$

$$= (1-K)$$

Copper in auto T/F = (1-K) copper in two winding T/F.

* It is applicable, if they are of same length.

<u>Note</u>: If K = 0.4, then copper saving in auto transformer is 40% of the two winding transformer.

→ Saving in copper increases, as the K value reaches near to unity.

* The primary and secondary windings voltages of auto T/F are 500V and 400V respectively. show with the help of diagram, the current distribution in winding and secondary current is 100A, calculate the economy in copper saving.

$V_1 = 500V$, $V_2 = 400V$

$$K = \frac{V_2}{V_1} = \frac{400}{500} = 0.8$$

$$K = \frac{I_1}{I_2} \Rightarrow I_1 = K \cdot I_2$$

$I_1 = 0.8 \times 100$

$I_1 = 80A.$

Copper saving in Auto T/F = K W'
$= 0.8 \times 100$
$= 80\%$

copper in auto T/F = (1-K) copper in 2 winding T/F

Advantages of Auto transformer:

- It requires less conducting material than 2-winding transformer.
- Efficiency is higher.
- Size and cost is less.
- Better voltage regulation
- A smooth and continuous variation of voltage is possible.

Disadvantages of Auto transformer:

- The two windings are not electrically separate and incase of failure of insulation a server shock will damage the low voltage side.
- The use of auto transformer is economical only when the transformation ratio 'K' is nearer to unity.

Applications of Auto transformer:

- Used for starting of induction motor
- Used for continuous variable supply.
- Used for inter connection of transformer in substations.
- Used in control equipment in locomotives.

Three phase Transformer Connections:

→ For 3-ϕ ~~s~~ system of T/F, we can use a bank of three 1-ϕ transformers (or) one 3-ϕ transformer.

→ The 3-ϕ winding can be prepared with Y (or) Δ connection of windings. Therefore the 3-ϕ transformer can be of following four categories/ combinations:

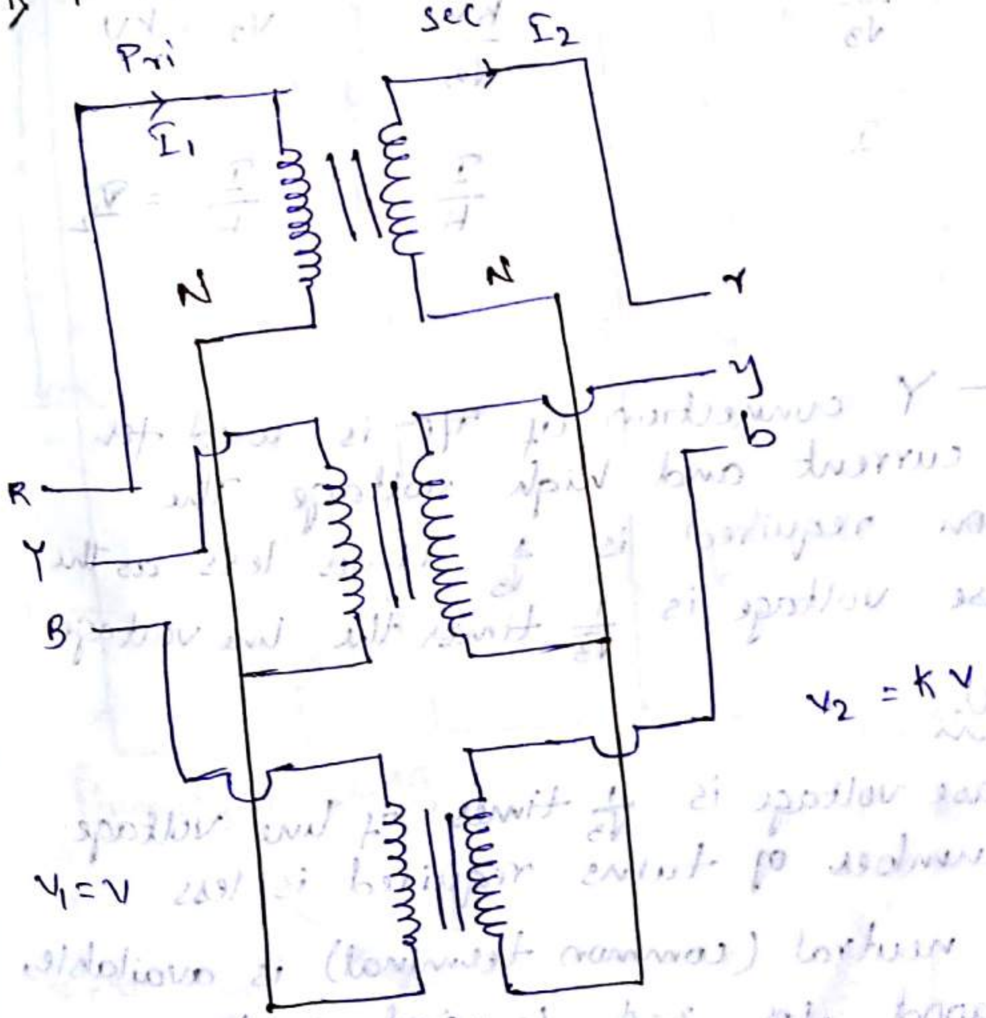
i, Y — Y

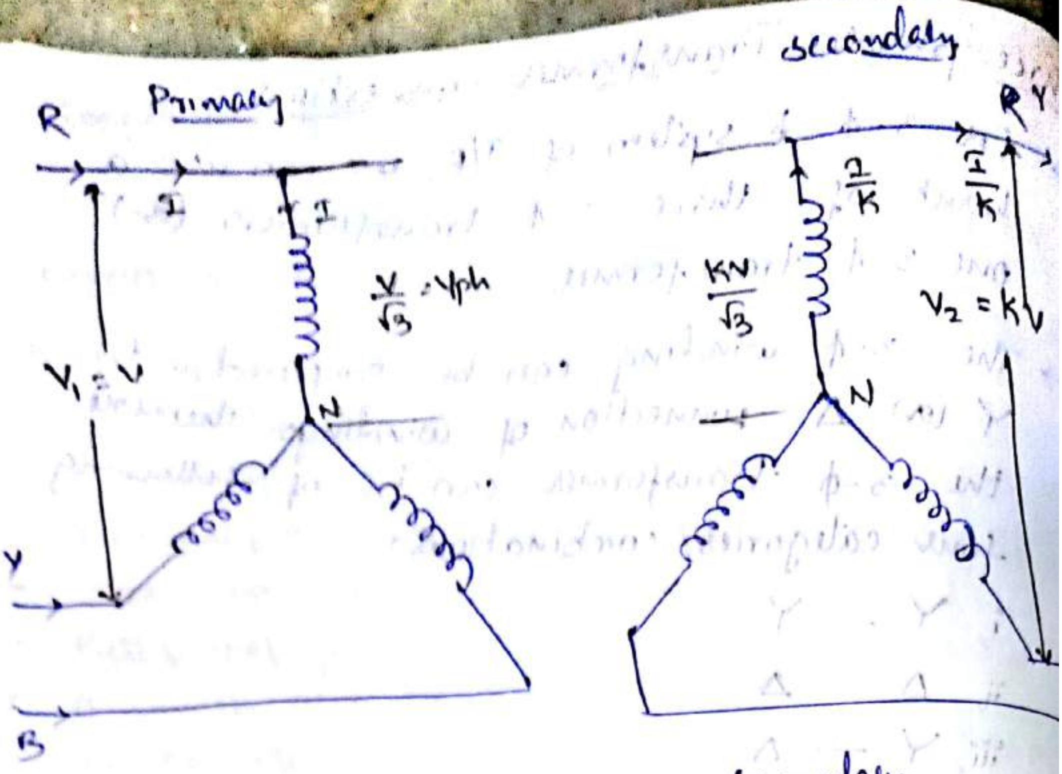
ii, Δ — Δ

iii, Y — Δ

iv, Δ — Y

1) Y to Y 3-ϕ T/F connections:-





Primary

Secondary

Line	phase	phase	line
$V_1 = V$	$\frac{V}{\sqrt{3}}$	$\frac{KV}{\sqrt{3}}$	$V_2 = KV$
$I_1 = I$	I	$\frac{I}{K}$	$\frac{I}{K} = I_2$

Note:

→ ~~In~~ Y – Y connection of T/F is used for small current and high voltage. The insulation required is $\frac{1}{\sqrt{3}}$ times less as the phase voltage is $\frac{1}{\sqrt{3}}$ times the line voltage.

Advantages:

1) The phase voltage is $\frac{1}{\sqrt{3}}$ times of line voltage. So the number of turns required is less.

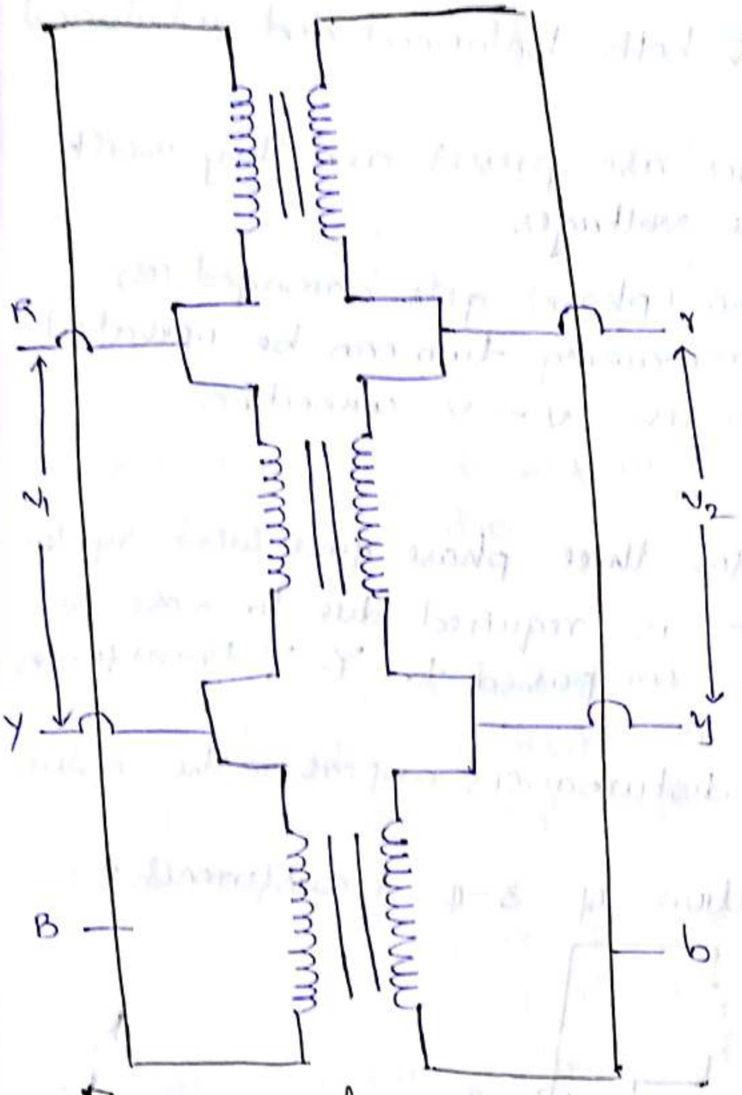
2) As the neutral (common terminal) is available it is good for 3-ϕ 4 ~~wired~~ wire system

Disadvantages:

1) Without neutral, the phase voltages become unbalanced, then the load is unbalanced.

2) The third harmonics will appear in secondary voltage will cause distortion in the secondary phase voltage.

2) Δ to Δ connection of 3-ϕ transformer:



* $K = \frac{V_2}{V_1} = \frac{V_2 \text{ line}}{V_1 \text{ line}}$

star – $V_L = \sqrt{3} V_{ph}$

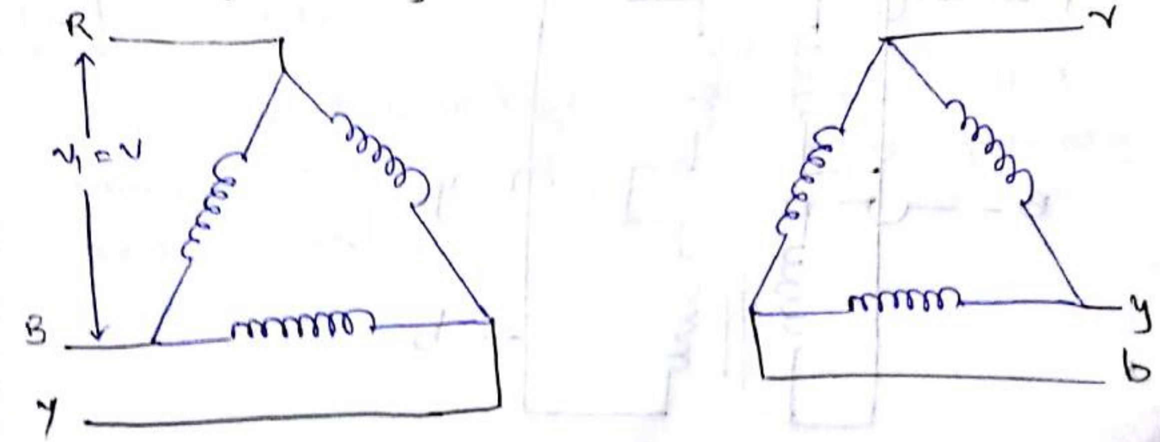
$I_L = I_{ph}$

Delta – $V_L = V_{ph}$

$I_L = \sqrt{3} I_{ph}$

Primary winding

Secondary winding



line	phase	phase	line
$V_1 = V$	V	KV	KV
$I_1 = I$	$\frac{I}{\sqrt{3}}$	$\frac{I}{\sqrt{3}K}$	$\frac{I}{K}$

Note:

→ It is used for large currents and low voltage transformer.

Advantages:

→ Satisfactory for both balanced and unbalanced load.

→ Third harmonics are present and they won't appear in line voltages.

→ If one winding (phase) gets damaged (or) removed, the remaining two can be operated in open delta (or) V – V connection.

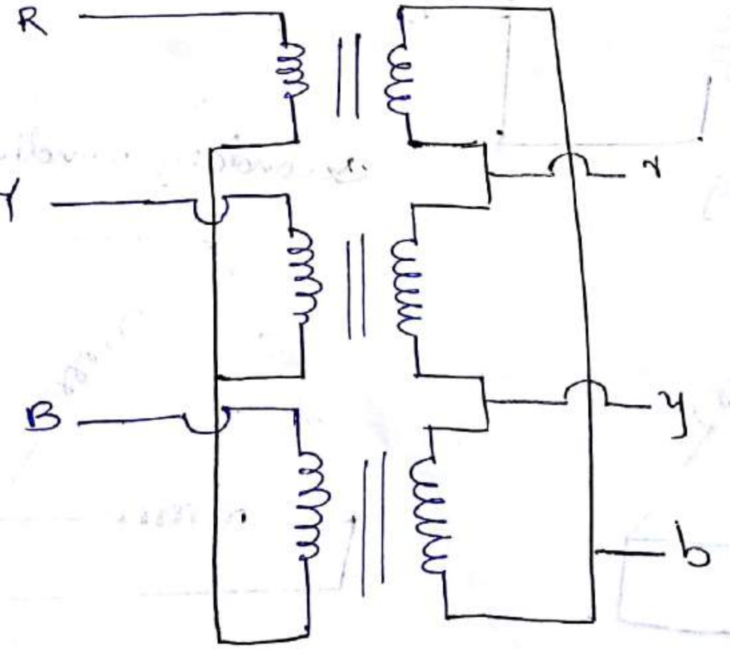
Disadvantages:

→ Not suitable for three phase four wire system

→ More insulation is required due to more no. of turns when compared to Y – Y transformer

* Third harmonics: disturbances appear in the system.

3) Y to Δ connection of 3-φ transformer:



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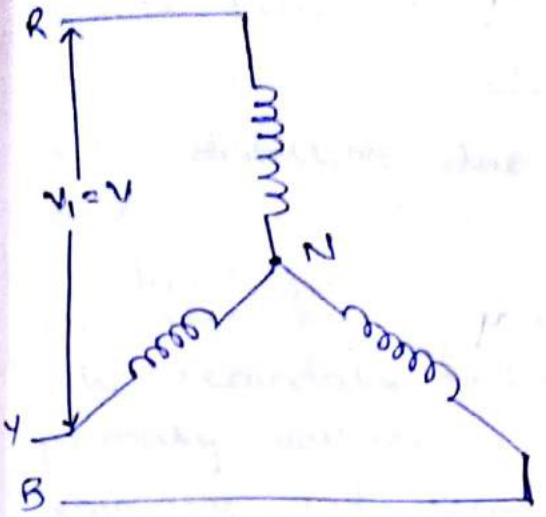
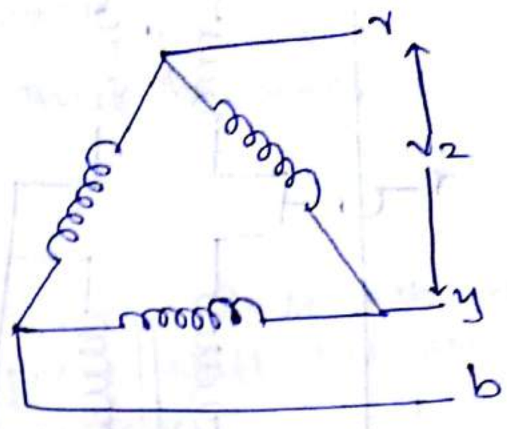
Primary winding   Secondary windings





line	phase	phase	line
$V_1 = V$	$\frac{V}{\sqrt{3}}$	$\frac{KV}{\sqrt{3}}$	$\frac{KV}{\sqrt{3}}$
$I_1 = I$	I	$\frac{I}{K}$	$\frac{\sqrt{3}I}{K}$

→ star to delta transformation is used for step-down operation.

Advantages:

→ Primary star connected. Hence less no. of turns are required. This makes the connection a economical for large high voltage step-down transformers.

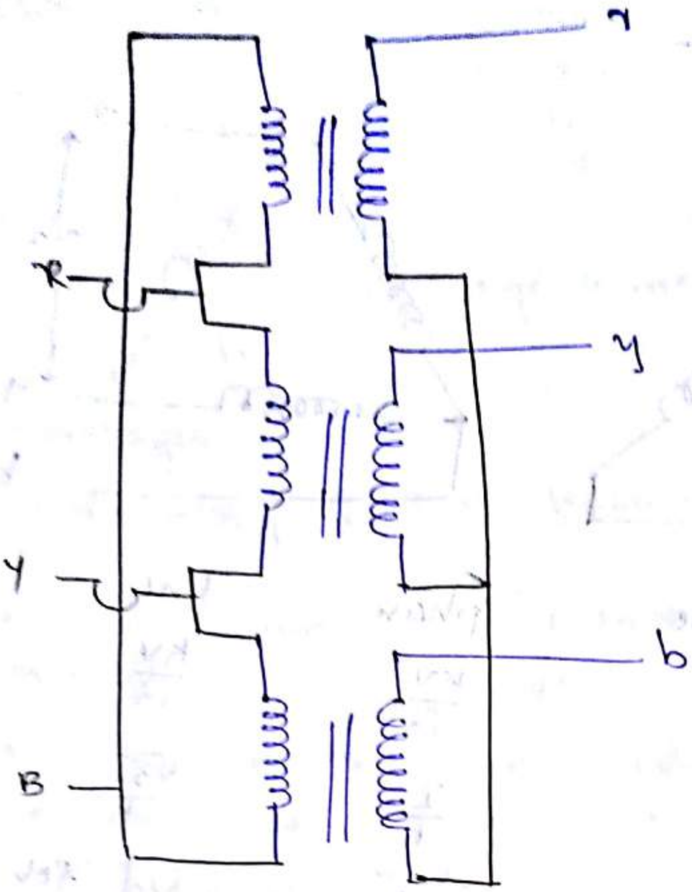
→ Neutral available on the primary can be earthened to avoid disturbances.

→ large unbalanced loads can be handle easily.

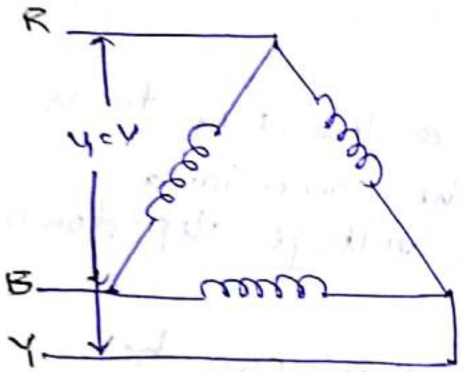
Disadvantages:

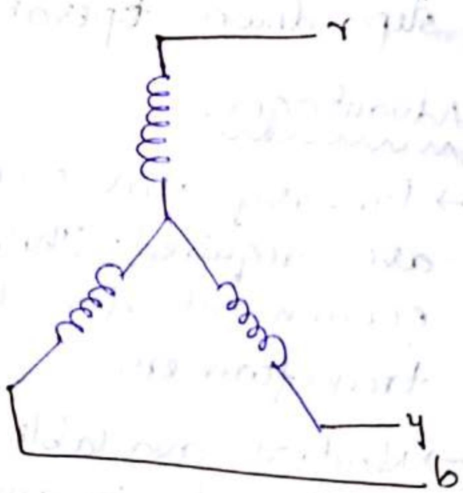
→ The secondary voltage is not in face with the primary i.e., 30° phase shift between primary and secondary voltages.

4) Δ to Y connection of 3-φ transformer :



Primary winding   Secondary winding





line	phase	line	phase
$V_1 = V$	V	KV	$\sqrt{3}\,KV$
$I_1 = I$	$\frac{I}{\sqrt{3}}$	$\frac{I}{\sqrt{3}K}$	$\frac{I}{\sqrt{3}K}$

* This connection is used in step-up transformer

Advantages:

→ With three phase four wire connection of star can eliminate distortion

→ No distortion due to third harmonics.

→ 2

Disadvantages:

→ The secondary voltage is not in phase with the primary voltage i.e. 30° phase shift between primary and secondary voltages.

* A 3-ϕ transformer has delta connected primary winding and star connected secondary winding of 15 Hz 3-ϕ supply the line voltages of Primary and secondary windings are 6600/600 V respectively. The line current in primary is 15A and secondary has a balanced load of 0.8 p.f lagging. Calculate secondary phase voltage, line current and output power.

I_1 Line = 15 Amps.

V_1 Line = 6600 V

V_2 Line = 600 V

Primary

line $V_1 = 6600$

phase $\frac{6600}{\sqrt{3}}$

secondary

phase $KV = \frac{600}{\sqrt{3}}$

line $\sqrt{3}KV = V_2$

$600 = V_2$

$$K = \frac{V_2 ph}{V_1 ph}$$

$$K = \frac{600/\sqrt{3}}{6600} = \frac{1}{11\sqrt{3}}$$

$$K = 0.0525$$

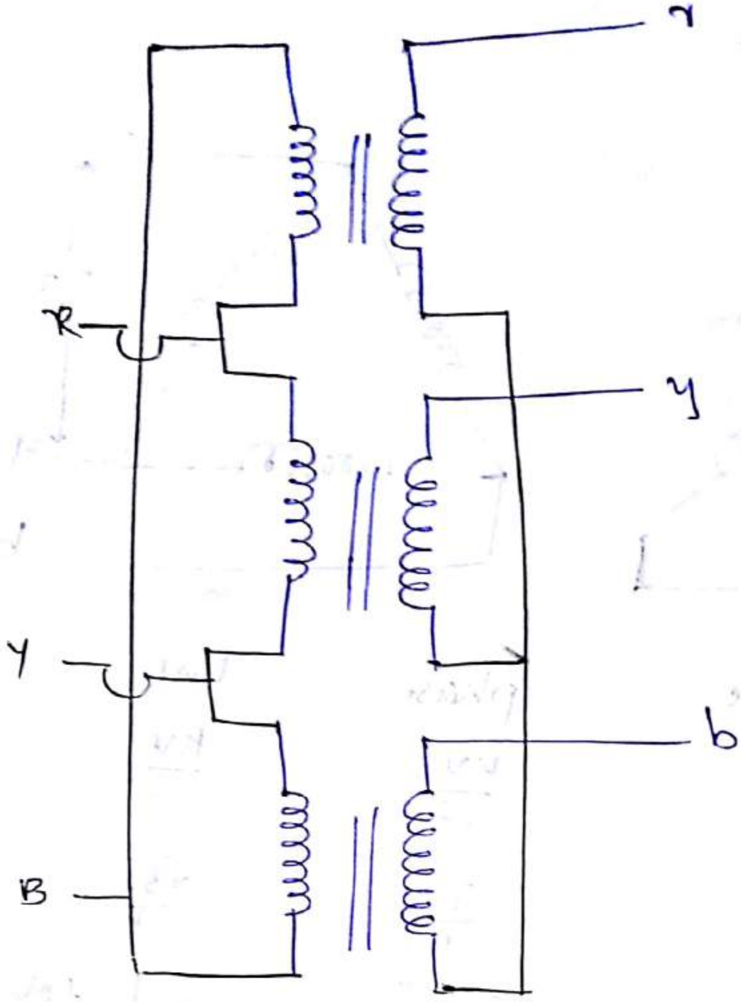
$$K = \frac{I_1 ph}{I_2 ph}$$

$$I_2 ph = \frac{15/\sqrt{3}}{0.0525} = 165$$

$$P = \sqrt{3} V_L I_L \cos\phi$$

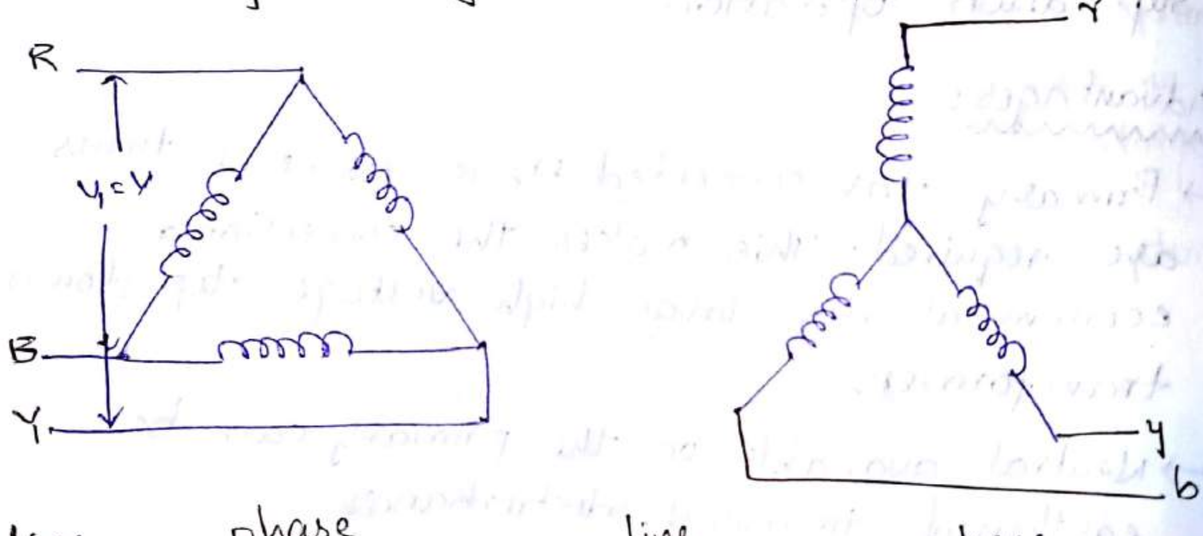
$$= \sqrt{3}\ 600 \times 165 \times 0.8 = 137 \text{ KW}$$

4) Δ to Y connection of 3-ϕ transformer :



Primary winding

Secondary winding



line	phase	line	phase
$V_1 = V$	V	KV	$\sqrt{3}\,KV$
$I_1 = I$	$\frac{I}{\sqrt{3}}$	$\frac{I}{\sqrt{3}K}$	$\frac{I}{\sqrt{3}K}$

* This connection is used in step-up transformer

Advantages:

→ With three phase four wire connection of star can eliminate distortion

→ No distortion due to third harmonics

→ 2

Disadvantages:

→ The secondary voltage is not in phase with the primary voltage i.e. 30° phase shift between primary and secondary voltages.

* A 3-φ transformer has delta connected primary winding and star connected secondary winding of 15 Hz 3-φ supply the line voltages of Primary and secondary windings are 6600/600 V respectively. The line current in primary is 15A and secondary has a balanced load of 0.8 p.f lagging. Calculate secondary phase voltage, line current and output power.

V_1 Line = 6600 V I_1 Line = 15 Amps.

V_2 Line = 600 V

Primary   secondary

line   phase   phase   line

$V_1 = 6600$ $\frac{6600}{\sqrt{3}}$ $KV = \frac{600}{\sqrt{3}}$ $\sqrt{3}KV = V_2$

$600 = V_2$

$K = \frac{V_2 ph}{V_1 ph}$

$K = \frac{600/\sqrt{3}}{6600} = \frac{1}{11\sqrt{3}}$

$K = 0.0525$

$K = \frac{I_1 ph}{I_2 ph}$

$I_2 ph = \frac{15/\sqrt{3}}{0.0525} = 165$

$P.\phi = \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3}\ 600 \times 165 \times 0.8 = 137\ KW$

Problems on EMF equation :

1) A single phase 50 HZ T/F has 30 primary turns and 350 secondary turns. The net cross-sectional area of the core is 250 cm^2. If the primary winding is connected to 230 volt, 50 HZ supply calculate a) peak value of flux density in the core

b) Voltage induced in the secondary winding

c) The primary current when the secondary current is 100 Amp.

$N_1 = 30$; $N_2 = 350$

$A = 250 \, cm^2$, $f = 50 \, HZ$

$V_1 = 230 \, V$

a)

$$E_1 = 4.44 \, \phi_m \, f \, N_1$$

$$E_1 = 4.44 \, (B_{max} \, A) \, f \cdot N_1 \qquad \{\because \phi = BA\}$$

$$230 = 4.44 \times B_{max} \times 250 \times 10^{-4} \times 30 \times 50$$

$$B_{max} = \frac{23}{4.44 \times 25 \times 10^{-4} \times 1500}$$

$$= \frac{23}{444 \times 375 \times 10^{-4}}$$

$$= 1.38 \, wb/m^2 \text{ (or) } T$$

b)

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$E_2 = \frac{350}{30} \times 230$$

$$= 2683.3 \, V$$

c)

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$I_1 = \frac{350}{30} \times 100$$

$$I_1 = 1166.6 \, A.$$

Problem on no-load current

3) A 1-ϕ, 230/110V, 50HZ transformer takes an input of 350 VA at no-load while working at rated voltage. The core loss is 110W, find the loss components of no-load current, the magnetising component of no-load current and no-load power factor.

$$I_0 = I_C + I_M$$

$$I_0 = I_0 \cos\phi_0 + I_0 \sin\phi_0$$

$V_1 = 230V$, $V_2 = 110V$, $f = 50HZ$

No-load input power $= V_1 \cdot I_0 = 350\ VA$

$$230 \times I_0 = 350$$

$$I_0 = \frac{35}{23} = 1.52A$$

No-load loss (or) core loss $= V_1 I_0 \cos\phi_0$

$$110 = 230 \times 1.52 \cos\phi_0$$

$$\cos\phi_0 = \frac{11}{23 \times 1.52}$$

$$\cos\phi_0 = 0.3146$$

$$\Rightarrow \sin\phi_0 = 0.9492$$

$$I_m = I_0 \sin\phi_0 = 1.52 \times 0.9492 = 1.44\ A$$

$$I_c = I_0 \cos\phi_0 = 1.52 \times 0.3146 = 0.478\ A$$

* I_c is called as loss component.

Problem on Equivalent circuit:

3) A 1-ϕ, 50KVA, 4400/220V, 50HZ transformer has primary and secondary resistances $R_1 = 3.45\,\Omega$ and $R_2 = 0.009\,\Omega$ respectively. The value of leakage reactances are $X_1 = 5.2\,\Omega$ and $X_2 = 0.015\,\Omega$ respectively calculate for this transformer.

a) the equivalent resistance referred to primary and secondary windings.

b) the equivalent reactances referred to primary and secondary windings.

c) the equivalent impedences referred to primary and secondary windings

d) Total copper losses in the transformer.

$R_1 = 3.45\,\Omega$; $R_2 = 0.009\,\Omega$

$X_1 = 5.2\,\Omega$; $X_2 = 0.015\,\Omega$

$$K = \frac{220}{4400} = 0.05$$

a) $$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

$$= 3.45 + \frac{0.009}{0.0025}$$

$$= 3.45 + 3.6 = 7.05\,\Omega$$

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1$$

$$= 0.009 + (0.0025)\,3.45$$

$$= 0.009 + 0.0086$$

$$= 0.0176\,\Omega$$

b) $$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

$$= 5.2 + \frac{0.015}{0.0025}$$

$$= 11.2\,\Omega$$

$$X_{02} = X_2 + X_1' = 0.015 + K^2 X_1$$

$$= 0.015 + (0.0025)5.2$$

$$= 0.015 + 0.013$$

$$= 0.028\,\Omega$$

c) $Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2}$

$= \sqrt{49.7 + 125.44}$

$= \sqrt{175.14} = 13.23\,\Omega$

$Z_{02} = \sqrt{(R_{02})^2 + (X_{02})^2}$

$= \sqrt{0.00030 + 0.00078}$

$= \sqrt{0.00108} = 0.0331\,\Omega$

d) copper loss $= I_1^2 R_1 + I_2^2 R_2$

$P = 50\text{ KVA}$

$V_1 I_1 = 50 \times 10^3$

$I_1 = \dfrac{50 \times 10^3}{4400} = \dfrac{500}{44} = 11.36\text{ A}$

$V_2 I_2 = 50 \times 10^3$

$I_2 = \dfrac{50 \times 10^3}{220} = 227.27\text{ A}$

copper loss $= (11.36)^2\, 3.45 + (227.27)^2\, 0.009$

$= 129.0 \times 3.45 + 51619.84 \times 0.009$

$= 445.22 + 464.57$

$= 909.79$

$\simeq 910\text{ W}$

Problem on voltage regulation

4) A 1-ϕ, 80 KVA, 6600/250V transformer has primary and secondary resistances $R_1 = 10\,\Omega$ and $R_2 = 0.02\,\Omega$ respectively. The equivalent leakage reactance as referred to primary is $35\,\Omega$. Find the full load regulation for load power factor of

a) unity p.f

b) 0.8 lagging

c) 0.6 leading

$$\% \text{ voltage regulation} = \frac{V_{NL} - V_{FL}}{V_{NL}} \times 100$$

$$= \frac{V_2' - V_2}{V_2} \times 100.$$

$$= \frac{I_2 (R_{02} \cos\phi_2 \pm X_{02} \sin\phi_2)}{V_2} \times$$

$$V_1 I_1 = 80 \text{ KVA}$$

$$I_1 = \frac{80 \times 10^3}{6600} = 12.12 \text{ A}$$

$$K = \frac{25\cancel{0}}{660\cancel{0}} = 0.0378$$

$$I_2 = \frac{12.12}{0.0378} = 320 \text{ A}.$$

$$R_{02} = R_2 + R_1' = 0.02 + K^2 \cdot R_1$$

$$= 0.02 + (0.0378)^2 \times 10$$

$$= 0.02 + 0.014$$

$$= 0.034 \,\Omega.$$

$$X_{02} = X_2 + X_1' = X_{01} \cdot K^2 = 0.0502 \,\Omega$$

a) $\cos\phi = 1$

$$\eta\% = \frac{32\cancel{0} (0.0343 \times 1 + 0.0502 \times 0)}{25\cancel{0}} \times \cancel{100}^{4}$$

$$= 128 (0.0343)$$

$$= 4.3904\%$$

b) 0.8 lagging

$$\eta\% = \frac{32\cancel{0} (0.0343 \times 0.8 + 0.0502 \times 0.6)}{25\cancel{0}} \times \cancel{100}^{4}$$

$$= 128 (0.02744 + 0.03012)$$

$$= 128 (0.05756)$$

$$= 7.36768\%$$

c) 0.6 leading

$$\eta\% = \frac{320\,(0.0343 \times 0.6 - 0.0502 \times 0.8)}{250} \times 100$$

$$= 128\,(0.02058 - 0.04016)$$

$$= 128\,(-0.01958)$$

$$= -2.506\%$$

Problem on Efficiency

5) A 50 KVA transformer on full load has copper loss of 600 W and iron loss of 500 W. Calculate
a) maximum efficiency and the load at which it occurs. Assume p.f as unity.
b) the efficiency at 75% of full-load current at 0.7 p.f lagging.

50 KVA , $W_{cu \cdot FL} = 600\,W$, $W_i = 500\,W$

a) Efficiency is maximum,

iron loss = copper loss

$$x = \sqrt{\frac{W_i}{W_{cu}}} = \sqrt{\frac{500}{600}} = 0.9129$$

$$\eta_{max} = \frac{x\ KVA\ \cos\phi}{x\ KVA\ \cos\phi + W_i + x^2\ W_{cu \cdot FL}} \times 100$$

$$= \frac{0.9129 \times 50 \times 10^3 \times 1}{0.9129 \times 50 \times 10^3 + 500 + (0.9129)^2\, 600} \times 100$$

$$= \frac{45645}{45645 + 500 + 500} \times 100$$

$$= \frac{45645}{46645} \times 100$$

$$= 97.86\%$$

b) $$\eta_{75\%} = \frac{0.75 \times 50 \times 10^3 \times 0.7}{0.75 \times 35 \times 10^3 + 500 + (0.75)^2 600} \times 10$$

$$= \frac{26250}{26250 + 500 + 337.5} \times 100$$

$$= \frac{26250}{27087.5} \times 100$$

$$= 96.9\%$$

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UNIT ④ : ELECTRICAL MACHINES

* 3-ϕ Induction motor
  - construction & working
  - Production of R.M.F
  - Significance of T.N Charateristics
  - Loss components & efficiency
  - Starting
  - speed control.

* 1-ϕ Induction motor
  - construction & working.

* Separately excited DC motor
  - speed control
  - T.N characteristics.

* synchronous generator
  - construction & working.

Motor

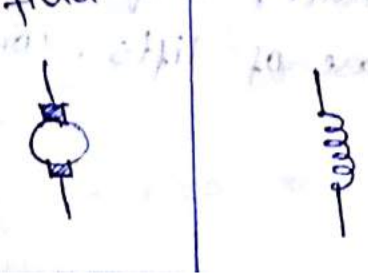
- DC motor
  - self-excited
    - series
    - shunt
    - compound
  - Separately excited
  - Permanent magnet
- AC motor
  - Induction motor
    - squirrel cage
    - slipring
  - synchronous motor

electrical i/p → Motor → Mechanical o/p {Both AC & DC}

Mechanical i/p → Generator → electrical o/p (Both AC & DC)

electrical i/p → Transformer → electrical o/p (only AC)

stator	Rotor
armature	field
field	armature



3-ϕ induction motor :

→ 80-90% of motors are induction motors and

→ It is singly excited machine.

→ Asynchronous motor

It is an AC electric motor in which the rotor needed to produce torque is obtained by electromagnetic induction from the magnetic field of the stator winding.

→ Induction motor and Transformer works on mutual induction.

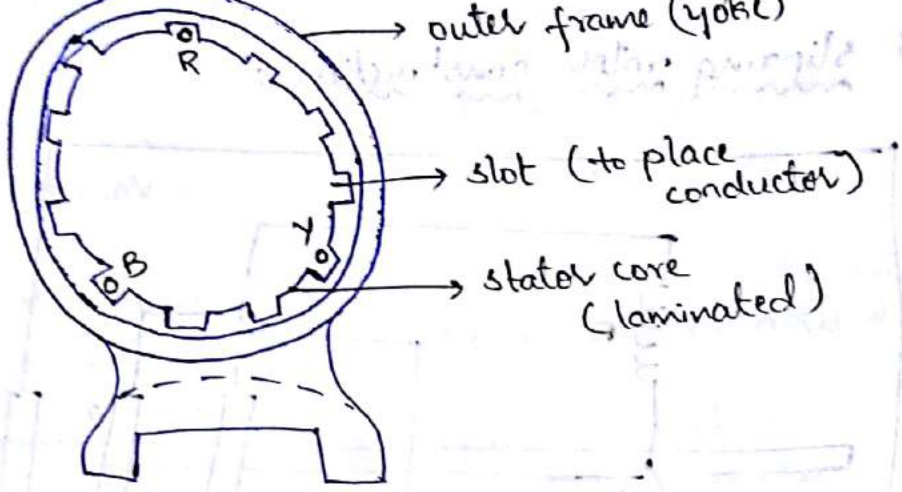
→ Induction motor can be analysed as transformer with secondary rotating

Construction of 3-ϕ induction motor :

Induction motor
- Stator
- Rotor
  - squirrel cage rotor (Resistance cannot be added)
  - slipring rotor (Resistance can be added)

→ By adding resistance, torque can be increased. It is used in case of lifts, conveyor belts etc.

Stator construction :

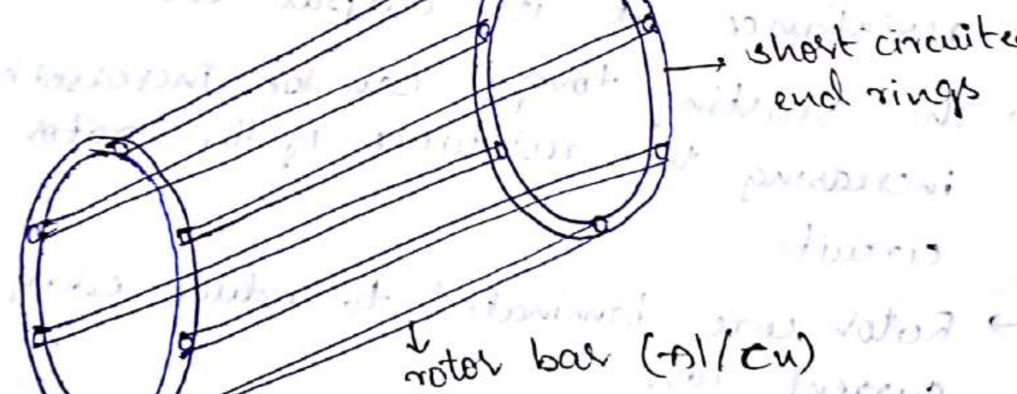


When a 3-ϕ supply is given to the stator winding, ~~the~~ a rotating magnetic field (RMF) of constant magnitude is produced. This RMF induces an EMF in the rotor winding by mutual induction principle. Hence this machine is called induction motor.

Rotor construction :

stator pole = Rotor pole

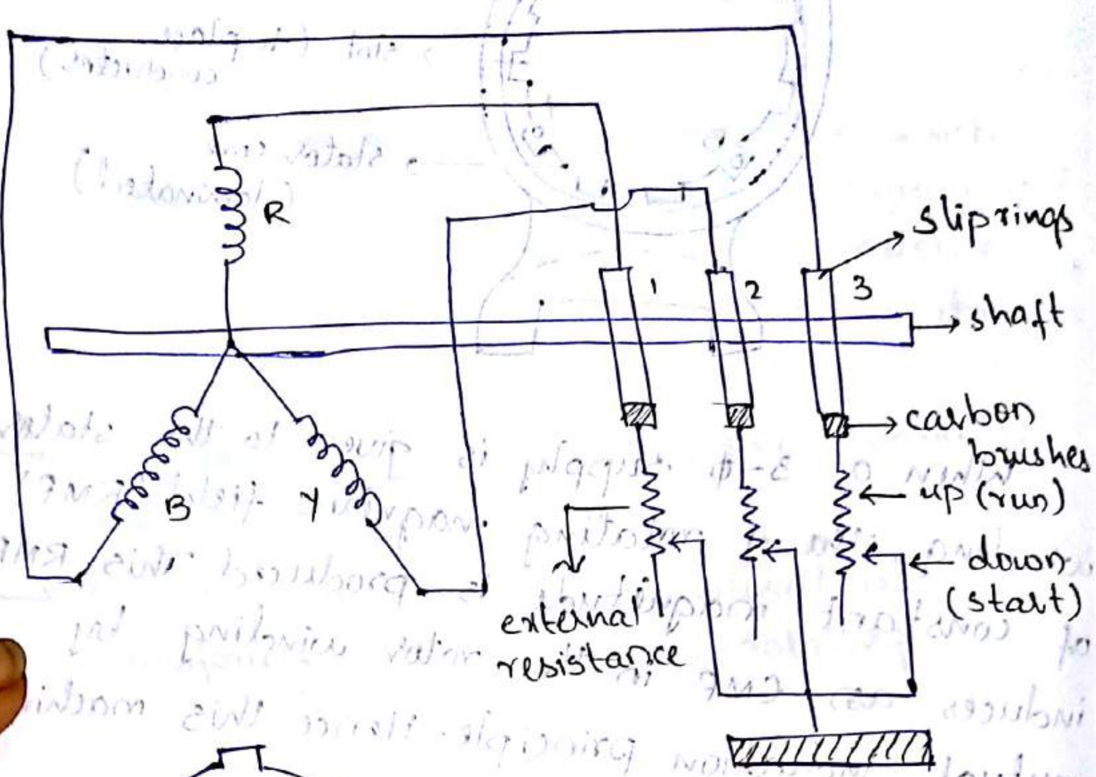
i, squirrel cage construction:

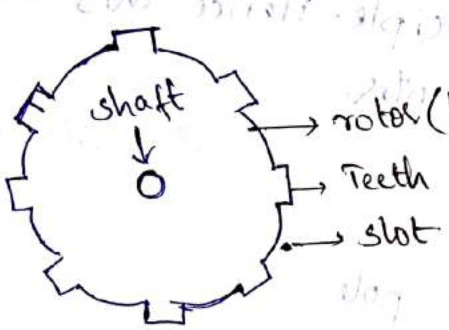


→ If area of crossection of bar is more, resistance will be less.

→ It is useful as there are no damaging parts like wires.

ii, sliprings rotor construction :





→ It is also called wound rotor. Slipring type rotor has the facility to insert external resistance at its output terminals.

→ The starting torque can be increased by increasing the resistance of the rotor circuit.

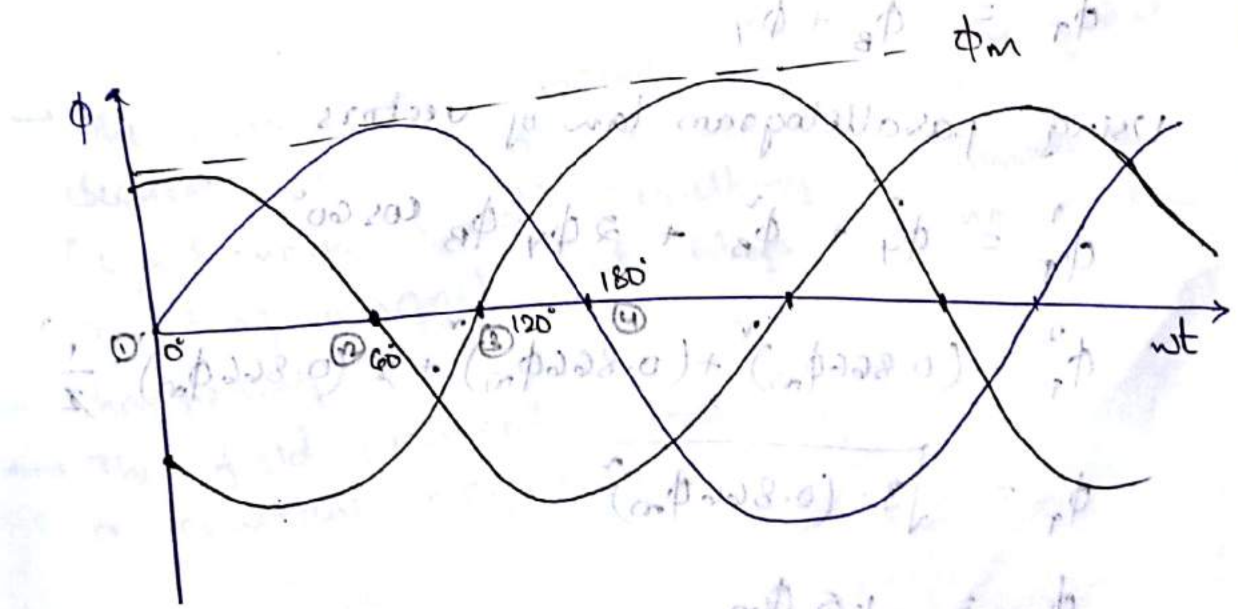
→ Rotor core laminated to reduce eddy current loss.

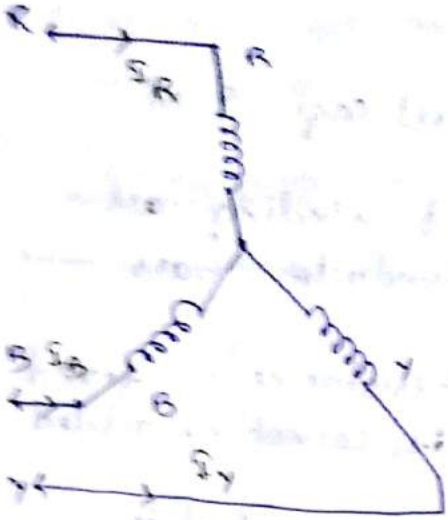
→ sliprings will be equal to the no. of phases.

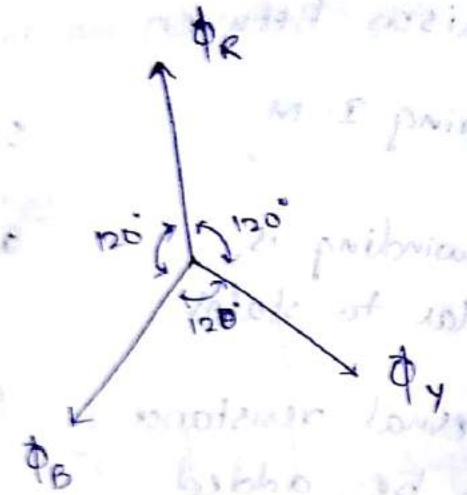
Comparison between

slipring I.M	squirrel cage I.M.
① 3-ϕ winding is similar to stator	① 3-ϕ winding are conductor bars.
② External resistance can be added	② Resistance is fixed i.e, cannot be added.
③ speed control is possible for both stator and delta rotor	③ speed control is possible for stator.
④ sliprings and brushes are present	④ sliprings and brushes are absent
⑤ construction is complex	⑤ construction is simple
⑥ Maintenance is more	⑥ Maintenance is less.
⑦ starting torque is high	⑦ starting torque is low.
⑧ It is used in lift, conveyor	⑧ It is used in fan, blower, grinder

* Production of rotating magnetic field [R.M.F]:







$\phi_R = \phi_m \sin \omega t$

$\phi_Y = \phi_m \sin(\omega t - 120°)$

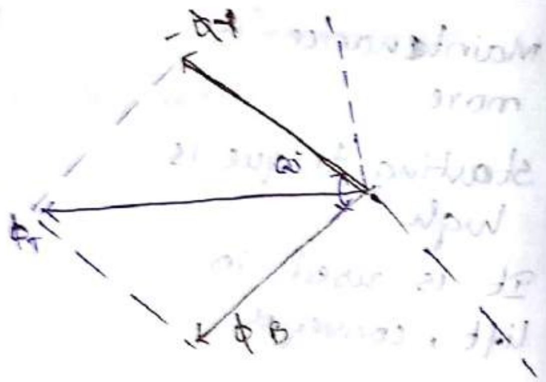
$\phi_B = \phi_m \sin(\omega t - 240°)$

When $\omega t = 0°$ (1st case)

$\phi_R = 0$

$\phi_Y = -0.866\,\phi_m$

$\phi_B = 0.866\,\phi_m$



Resultant :

$\bar{\phi}_T = \bar{\phi}_R + \bar{\phi}_B + \bar{\phi}_Y$

$\bar{\phi}_T = \bar{\phi}_B + \bar{\phi}_Y$

Using parallelogram law of vectors

$\phi_T^2 = \phi_Y^2 + \phi_B^2 + 2\phi_Y \cdot \phi_B \cos 60°$

$\phi_T^2 = (0.866\phi_m)^2 + (0.866\phi_m)^2 + \cancel{2} \cdot (0.866\phi_m)^2 \cdot \frac{1}{\cancel{2}}$

$\phi_T = \sqrt{3 \cdot (0.866\phi_m)^2}$

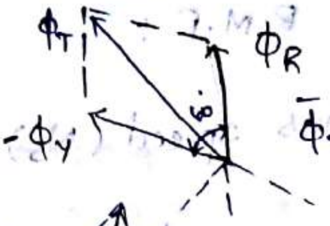
$\phi_T = 1.5\,\phi_m$

When $\omega t = 60°$

$\phi_R = 0.866\,\phi_m$

$\phi_Y = -0.866\,\phi_m$

$\phi_B = 0$



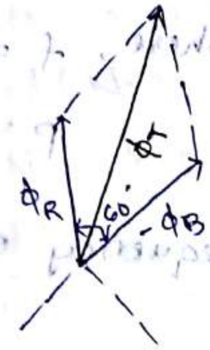
$\bar{\phi}_T = \bar{\phi}_R + \bar{\phi}_Y$

When $\omega t = 120°$

$\phi_R = 0.866\,\phi_m$

$\phi_Y = 0$

$\phi_B = -0.866\,\phi_m$



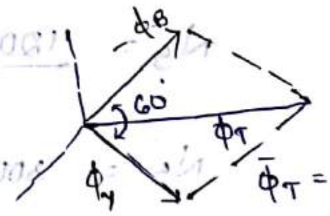
$\bar{\phi}_T = \bar{\phi}_R + \bar{\phi}_B$

When $\omega t = 180°$

$\phi_R = 0°$

$\phi_Y = 0.866\,\phi_m$

$\phi_B = -0.866\,\phi_m$



$\bar{\phi}_T = \bar{\phi}_Y + \bar{\phi}_B$

similarly in the remaining cases also, the resultant $\phi_T = 1.5\,\phi_m$

Note:

From the above four cases the resulting flux at any instant having a constant magnitude of $1.5\,\phi_m$ and it is rotating in clock-wise sequence (R-Y-B)

$$\boxed{\phi_T = 1.5\,\phi_m}$$

→ The fluxes are rotating, based on the phase sequence RYB with a constant speed (synchronous speed) resulting in illusion of rotating magnet but physically no rotation happening.

→ The field produced is rotating in space having a constant magnitude.

Speed of R.M.F :

Synchronous speed (N_S) = $\frac{120f}{P}$

where, f = frequency of supply

P = ~~power~~ No. of poles

→ In India, frequency of power supply is 50 HZ.

$$N_S = \frac{120 \times 50}{P}$$

N_S = 3000 rpm (P = 2)

= 1500 (rpm) (P = 4)

= 1000 rpm. (P = 6)

* Phase sequence may be clockwise (R-Y-B), (or) anti-clockwise (R-B-Y)

Working principle of 3-ϕ I.M :-

Faraday law of Mutual Induction

Due to relative speed (N_S - N), we get

$$e.M.F = \frac{d\phi}{dt}$$

↓

current in rotor

↓

flux in rotor

Lenz law: Effect oppose the cause

∴ Rotor ~~co~~ windings will rotate in such a way that reduce relative speed of RMF

synchronous speed (speed of stator RMF)

$$N_s = \frac{120f}{P}$$

f = frequency of supply

P = no. of poles = 2, 4, 6, 8 ...

N = rotor speed (N_r)

Relative speed = $N_s - N$ = slip speed

$$\%\ \text{slip} = \frac{N_s - N}{N_s} \times 100$$

$$s = 1 - \frac{N}{N_s}$$

$$\frac{N}{N_s} = 1 - s$$

$$N = (1-s) N_s$$

* slip = 1 ⟶ when motor is at rest (N = 0)

slip = 0 ⟶ when N = N_s (practically not possible)

Eg: For a 3-ϕ 50 HZ, 4 pole induction machine is running at 1430 rpm. calculate slip speed and percentage slip

solⁿ synchronous speed, $N_s = \frac{120 \times 50}{4} = 1500$ rpm

slip speed = $N_s - N$

= 1500 − 1430 = 70 rpm

$$\%\ \text{slip} = \frac{N_s - N}{N_s} \times 100 = \frac{70}{1500} \times 100 = 4.6$$

* If slip = 3% calculate speed of machine, which is at 60 HZ supply with 2 poles

$$\%\ \text{slip} = \frac{N_s - N}{N_s} \times 100 = 3\%$$

$$N_s = \frac{120 \times 60}{2} = 3600$$

$$\%\ \text{slip} = \frac{3600 - N}{3600} \times 100 = 3$$

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$$3600 - N = 108$$

$$N = 3600 - 108 = 3492 \text{ rpm.}$$

* For a 6 pole, 50 Hz induction motor, rotating at 570 rpm at full load. Determine the slip.

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Relative speed $= N_s - N$

$$= 1000 - 570 = 430 \text{ rpm}$$

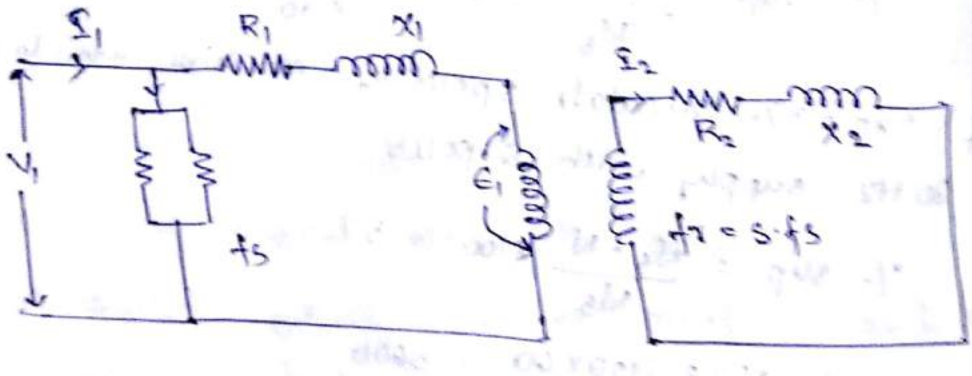
$$\%\text{ slip} = \frac{N_s - N}{N_s} \times 100 = \frac{430}{1000} \times 100 = 43\%$$

Effect of slip on rotor parameters:

Stator	Rotor
frequency: $N_s = \frac{120f}{P}$	$f_r = N_s - N$
$f = \frac{N_s P}{120}$	$\frac{f_r}{f_s} = \frac{N_s - N}{N}$
$f_s \propto N_s$	$\frac{f_r}{f_s} = s$
	$f_r = s \cdot f_s$

Induced emf:

At rest	At running.
$E_2 \propto N_s$	$E_{2r} \propto N_s - N$
	$E_{2r} = s \cdot E_2$



Rotor resistance & reactance

At rest

R_2 , X_2

At running

$R_{2r} = R_2$

$X_2 = s \cdot X_2$

current $I_2 = \frac{E_2}{Z_2}$

$= \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$

$I_{2r} = \frac{sE_2}{Z_{2r}}$

$= \frac{s \cdot E_2}{\sqrt{R_2^2 + (sX_2)^2}}$

power-factor $\cos\phi = \frac{R_2}{Z_2}$

$\cos\phi = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$

$\cos\phi_{2r} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$

* A 8 pole, 3-ϕ induction motor operates from supply of 50 Hz frequency calculate the following
a) speed at which magnetic field is rotating.
b) speed of the rotor at the slip of 0.02
c) frequency of rotor current at slip of 4%.

a) speed of R.M.F , $N_s = \frac{120f}{P}$

$N_s = \frac{120 \times 50}{8} = 750$ rpm

b) Given slip = 0.02

$N = N_s(1-s)$

$= 750(1-0.02)$

$= 735$ rpm

c) $f_r = sf_s$

$f_r = \frac{4}{100} \times 50 = 2Hz$.

* A 50 Hz, 4 pole induction motor connected in Δ manner have a full load slip of 5%. if the supply frequency is 50 Hz, find
1. full load speed
2. Synchronous speed
3. Rotor frequency.

Synchronous speed, $N_s = \frac{120f}{P}$

$N_s = \frac{120 \times 50}{4} = 1500$

% slip = 5%

$N = N_s(1-s)$

$= 1500(1-0.05)$

$= 1500 \times 0.95$

$= 1425$

Rotor frequency, $f_r = s \cdot f_s$

$f_r = 0.05 \times 50$

$= 2.5$ Hz

Torque equation in 3-ϕ Induction Motor : 5/11/18

$$\boxed{T = K \cdot E_2 \cdot I_2 \cos\phi_2}$$

where, $K = \left[\frac{3}{(2\pi N_s/60)}\right]$

→ Torque depends upon the induced emf E_2, power factor of rotor $\cos\phi_2$, I_2

starting Torque :

$$T_{st} = K \cdot E_2 \cdot I_2 \cdot \cos\phi_2$$

$$= K \cdot E_2 \cdot \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$= K \cdot \frac{E_2^2 \cdot R_2}{R_2^2 + X_2^2}$$

$$\boxed{T_{st} = K \cdot \frac{E_2^2 \cdot R_2}{R_2^2 + X_2^2}}$$

K = constant , E_2 = constant

$$\boxed{T_{st} \propto \frac{R_2}{R_2^2 + X_2^2}}$$

Condition for maximum starting torque :

$$\frac{d}{dR_2}(T_{st}) = 0$$

$$\Rightarrow \frac{d}{dR_2}\left(\frac{R_2}{R_2^2 + X_2^2}\right) = 0$$

$$(R_2^2 + X_2^2)1 - R_2(2R_2) = 0$$

$$X_2^2 - R_2^2 = 0$$

$$\boxed{X_2 = R_2}$$

Note:

1) The condition for maximum starting torque is rotor reactance should be equal to the rotor resistance.

Full load Torque : T_{FL}

$$T_{FL} = K \cdot E_{2r} \cdot I_{2r} \cos\phi_{2r}$$

$$T_{FL} = K \cdot (sE_2) \cdot \frac{sE_2}{\sqrt{R_2^2+(sx_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2+(sx_2)^2}}$$

$$T_{FL} = K \cdot \frac{s^2 E_2^2 \, R_2}{R_2^2 + s^2 x_2^2}$$

$$\boxed{T_{FL} = K \cdot \frac{R_2 (sE_2)^2}{R_2^2 + (sx_2)^2}}$$

K = constant , E_2 = constant

$$\boxed{T_{FL} \propto \frac{s^2 \cdot R_2}{R_2^2 + (sx_2)^2}}$$

Note: The full load torque is proportional to slip (s) and (voltage)^2

Condition for maximum full load torque:

Maximum torque under running condition (max. T_{FL}) is $\boxed{R_2 = s x_2}$

Note :

- The ratio of full load torque to maximum torque

$$\frac{T_{FL}}{T_{max}} = \frac{2 \cdot a \cdot S_f}{a^2 + S_f^2}$$

where, S_f = slip at full load

$a = R_2/X_2$

- The ratio of starting torque to maximum torque

$$\frac{T_{st}}{T_{max}} = \frac{2 \cdot a \cdot S}{a^2 + S^2}$$

* ~~A 3 point 3KV, 24 pole~~ A 3.3 KV, 24 poles 50 HZ, 3-ϕ, star connected induction motor has slipring rotor with resistance of 0.016 Ω and standstill reactance of 0.265 Ω/ph. calculate

i, speed at maximum torque.

ii, ratio of full load to maximum torque., if the full load torque is obtained at 247 rpm.

i, RMS synchronous speed $= \frac{120 f}{P}$

$$= \frac{\cancel{120}^{10} \times 50}{\cancel{24}_{2}}$$

$N_s = 250$ rpm.

Speed at $T_{max} = (1-S) N_s$

$R_2 = S X_2$

$$S = \frac{R_2}{X_2} = \frac{0.016}{0.265} = 0.06$$

Speed at $T_{max} = (1-0.06) 250$

$= 0.94 \times 250$

$= 235$ rpm

11. $$\frac{T_{FL}}{T_{max}} = \frac{2a\, s_f}{a^2 + s_f^2}$$

$$S_{full} = \frac{N_s - N}{N_s} = \frac{250 - 247}{250}$$

$$s_f = 0.012$$

$$a = \frac{R_2}{X_2} = 0.06$$

$$\frac{T_{FL}}{T_{max}} = \frac{2 \times 0.06 \times 0.012}{36 \times 10^{-4} + 144 \times 10^{-6}}$$

$$= \frac{0.12 \times 0.012}{37.44 \times 10^{-4}}$$

$$= \frac{14.4}{37.44}$$

$$= 0.3846$$

* The full load slip of a 500HP, 3-ϕ induction motor is 0.02. The rotor winding has a resistance of 0.25 Ω/ph. Calculate slip and power output if the external resistance of 2 Ω each are inserted in each rotor phase. Assume the torque remains unaltered.

$s_f = 0.02$; $R_2 = 0.25\ \Omega/ph$

extra resistance $= 2\ \Omega/ph$.

$$P = \frac{2\pi N T}{60}$$

$R_2 = 0.25\ \Omega/ph \rightarrow s_f = 0.02 \rightarrow P = 500hp$

$R_2' = (2 + 0.25)\ \Omega/ph \rightarrow s = ? \rightarrow P = ?$

$$s' = \left(\frac{s}{R_2}\right) R_2'$$

$$= \frac{0.02}{0.25} \times 2.25 = 0.18$$

$P \propto N$

where $N = (1-s) N_s$

Here, $N_1 = (1-0.02) N_s = 0.98 N_s$

$N_2 = (1-0.18) N_s = 0.82 N_s$

$P_1 \propto 0.98 N_s$

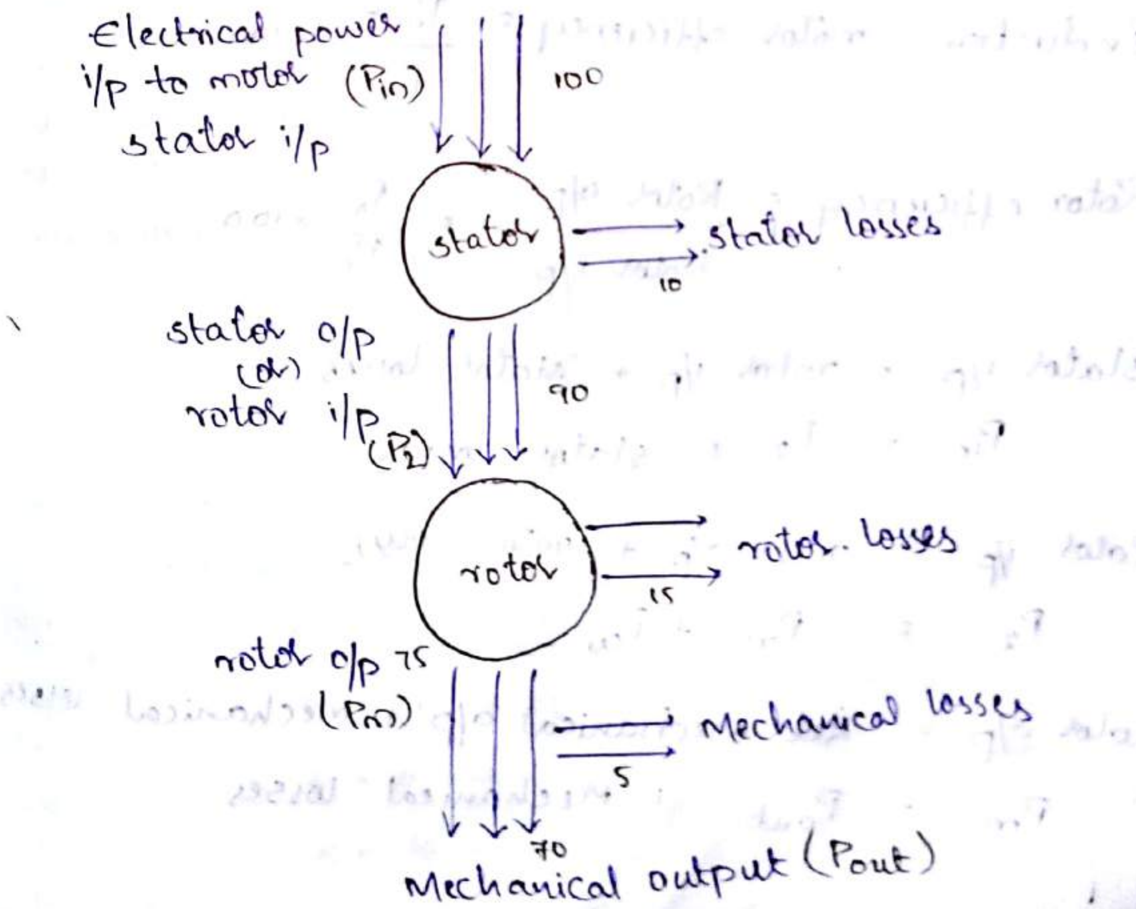
$P_2 \propto 0.82 N_s$

$$\frac{P_1}{P_2} = \frac{0.98 \cancel{N_s}}{0.82 \cancel{N_s}}$$

$$\frac{500}{P_2} = \frac{98}{82}$$

$$\Rightarrow P_2 = \frac{500 \times 82}{98} = 418.36 \text{ h.p}$$

Power transfer chain stages in Induction Motor :



- Motor i/p
- Stator i/p (P_{in})
  - stator losses
    - iron
    - copper
  - stator output (or) (P_2) rotor i/p
    - rotor losses
      - iron
      - copper
    - rotor o/p (P_m) (Gross mechanical power developed)
      - Mechanical losses
        - friction
        - windage.
      - Net mechanical power developed (or) motor o/p (or) shaft power. (P_{out})

Induction motor efficiency $= \frac{P_{out}}{P_{in}} \times 100$

Rotor efficiency $= \frac{\text{Rotor o/p}}{\text{Rotor i/p}} = \frac{P_m}{P_2} \times 100.$

stator i/p = rotor i/p + stator losses

$P_{in} = P_2 +$ stator losses.

Rotor i/p = rotor o/p + rotor losses

$P_2 = P_m + P_{cu}$

Rotor o/p = Net mechanical o/p + mechanical losses

$P_m = P_{out}$ + Mechanical losses

$P_2 = P_m + P_{cu}$

$$\boxed{P_2 = (1-s)P_2 + sP_2}$$

P_2 = rotor i/p

$P_{cu} = s \cdot P_2$ = rotor losses

$P_m = (1-s)P_2$ = rotor o/p.

$$\frac{P_2}{P_{cu}} = \frac{1}{s}$$

$$\frac{P_2}{P_m} = \frac{1}{1-s}$$

$$\boxed{P_2 : P_m : P_{cu} = 1 : 1-s : s}$$

* A 4 pole, 400 V 3-ϕ 50Hz induction motor runs at 1440 rpm at 0.8 p.f lagging. And develops an output of 10.8 KW. The stator losses are 1060 W, frictional & windages losses are 390 W. Calculate

i, slip

ii, rotor copper loss

iii, rotor frequency

iv, line current

v, rotor efficiency

vi, motor efficiency.

i, $N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500$

$N = (1-s)N_s$

$1440 = (1-s)1500$

$s = 1 - \frac{144}{150}$

$s = 1 - 0.96$

$s = 0.04$

$s = 4\%$.

ii, Mechanical losses = 0.39 kW.

Mechanical output = 10.8 kW.

rotor o/p = (10.8 + 0.39) kW

P_m = 11.19 kW.

$(1-s)P_2$ = 11.19 kW

$(1-0.04)P_2$ = 11.19 kW

$P_2 = \frac{11.19}{0.96}$ kW

P_2 = 11.65 kW. ∴ $P_{cu} = P_2 - P_m$
= 0.466 kW

iii, $f_r = s \cdot f_s$
$= 0.04 \times 50$
$= 2$ HZ

iv, Line current

P_2 = 11.65 kW

stator losses = 1.06 kW

P_{in} = 11.65 + 1.06 = 12.71 kW.

$P_{in} = \sqrt{3} V_L I_L \cos\phi$

$12.71 \times 1000 = 400 \cdot I \cdot \sqrt{3} \times 0.8$

$I = \frac{1271}{40} = \frac{31.775 \text{ A}}{\sqrt{3} \times 0.8}$

I = 22.932 A.

v, rotor efficiency = $\frac{\text{rotor o/p}}{\text{rotor i/p}} \times 100$ (or) $\frac{N_r}{N_s} \times 100$

$= \frac{11.19}{11.65} \times 100$ $\qquad \frac{1440}{1500} \times 100$

96%.

= 96.05 %

vi, motor efficiency $= \frac{P_{out}}{P_{in}} \times 100$

$$= \frac{10.8}{12.71} \times 100$$

$$= 84.97\% \approx 85\%$$

Note :

Rotor efficiency $= \frac{P_m}{P_2} \times 100$ (or) $\frac{N_r}{N_s} \times 100$

$\because P_2 : P_m : P_{cu} = 1 : 1-S : S$

* The power i/p to the rotor of a 440V, 50HZ, 3-ϕ, 6 pole I.M is 50 KW. It is observed that rotor E.M.F makes 120 complete cyles per minute. calculate

i, slip

ii, rotor speed

iii, rotor copper loss per phase.

iv, rotor out put

v, rotor resistance per phase if I = 50A.

$P_2 = 50$ KW.

i, slip

$$N_s = \frac{\overset{20}{\cancel{120}} \times 50}{\cancel{6}} = 1000 \text{ rpm.}$$

$f_2 = 120$ cycle/min $= 2$ cycle/sec

$f_r = 2$ Hz , $f = 50$ HZ.

$$\frac{\cancel{2}}{\underset{25}{\cancel{50}}} = S \Rightarrow S = \frac{1}{25} = 0.04$$

ii,

$$N = (1-S) N_s$$

$$= (1-0.04) 1000$$

$$= 960 \text{ rpm.}$$

iii, $P_2 = 50 \, kW$

$P_2 : P_m : P_{cu} = 1 : 1-s : s$

$$\frac{P_2}{P_{cu}} = \frac{1}{s}$$

$$\frac{50}{P_{cu}} = \frac{1}{0.04}$$

$$P_{cu} = \frac{50 \times 4}{100} = 2 \, kW$$

$$P_{cu}/ph = 2/3 \times 1000 \, W = 667 \, Watt/ph.$$

iv, $$\frac{P_2}{P_m} = \frac{1}{1-s}$$

$$\frac{50}{P_m} = \frac{1}{0.96}$$

rotor o/p = $P_m = 50 \times 0.96 = 48 \, kW$

v, Rotor resistance = R_2 = ?

Rotor copper loss = ~~2000 W~~ 667 Watt/ph.

$$\frac{V_2^2}{R_2} = \cancel{2000} \qquad \cancel{I_2^2 \cdot R_2 = 2000}$$

$$\cancel{440 \times 440} \qquad \cancel{50 \times 50 \cdot R_2 = 2000}$$

$$\cancel{R_2 = \frac{4}{5}}$$

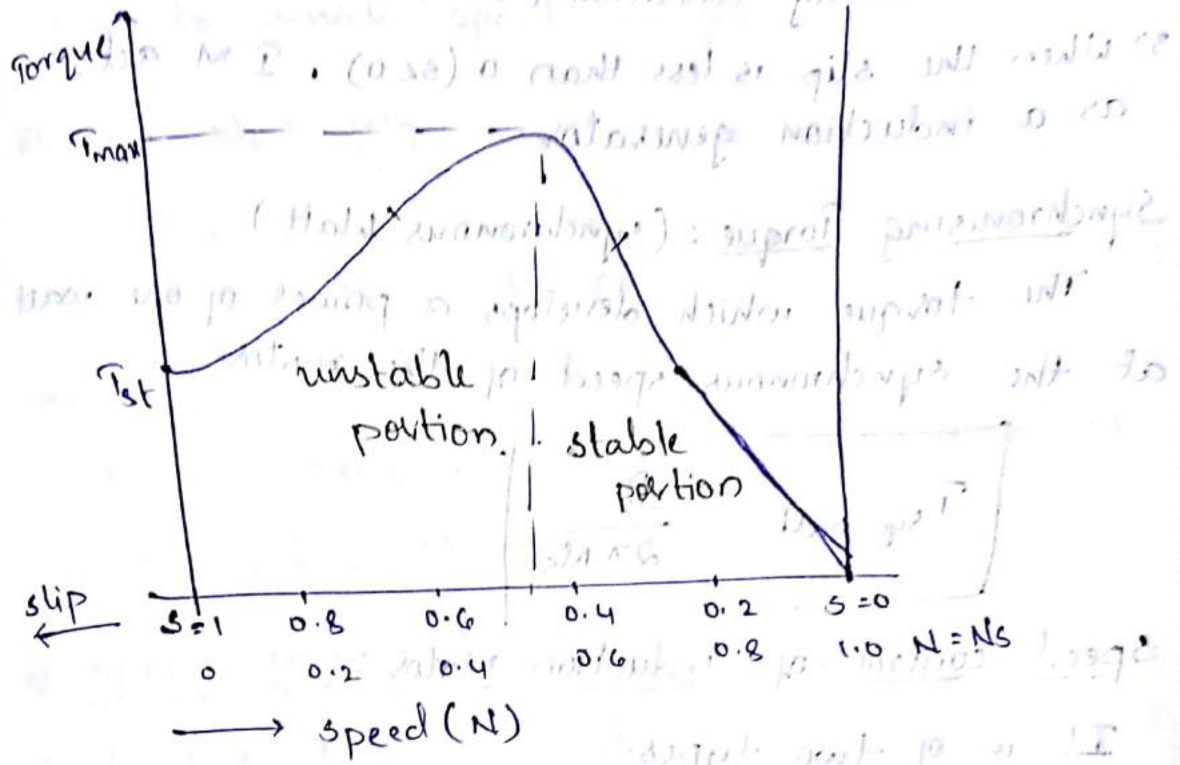
$$\cancel{R_2 = 0.8 \, \Omega/ph}$$

$$I_2^2 \cdot R_2 = 667$$

$$R_2 = \frac{667}{2500} = 0.26 \, \Omega/ph.$$

Torque - slip characteristics of Induction Motor :-

T - S (or) T - N



i, <u>slip is low</u>

$$T = \frac{K \cdot S \cdot E_2^2 \cdot R_2}{(R_2)^2 + (S X_2)^2}$$

$$T \propto S \cdot R_2$$

ii, <u>slip is medium</u>

$$T = \frac{K \cdot S \cdot E_2^2 \cdot R_2}{(R_2)^2 + (S X_2)^2}$$

$$T \propto \frac{S}{S^2 \cdot X_2^2}$$

$$T \propto \frac{1}{S}$$

iii, <u>Slip is high</u>

$$T \propto \frac{1}{S}$$

Note: 1) For Induction motor, slip is varying from 0 to 1.

2) If slip is greater than 1, this I.M is working under breaking condition ($s>1$)

3) When the slip is less than 0 ($s<0$), I.M acts as a induction generator.

Synchronising Torque: (synchronous Watt)

The torque which develops a power of one watt at the synchronous speed of the motor.

$$\boxed{T_{syn\ watt} = \frac{P_2}{2\pi N_s}}$$

Speed control of Induction Motor:

It is of two types.

a) Speed control from stator side:

i, frequency control ($\frac{V}{f}$ control)

ii, voltage control

iii, stator poles changing

iv, stator - Rheostat method

i, frequency control

$$N_s = \frac{120f}{P}$$

$$V = E = K \cdot 4.44\, \phi f \cdot N$$

$$\phi = \frac{V}{4.44\, \phi f\, N}$$

flux must be constant.

so, along with frequency, voltage also must be altered.

ii, voltage control.

$T \propto s E_2^2 \cdot R_2$.

when voltage is altered, slip is also varied. Due to which speed changes.

iii, $N_s = \frac{120f}{P}$

$P = 4$, $N_s = 1500$ rpm

$P = 6$, $N_s = 1000$ rpm

iv, $T \propto s \cdot E_2^2 R_2$

Due to resistance, voltage drop is observed.

As $E_2 \downarrow$, $s \uparrow \rightarrow N \downarrow$

b) speed control from rotor side :

i, Adding external resistance.

ii, cascaded connection of motors

iii, Injecting EMF with rotor frequency.

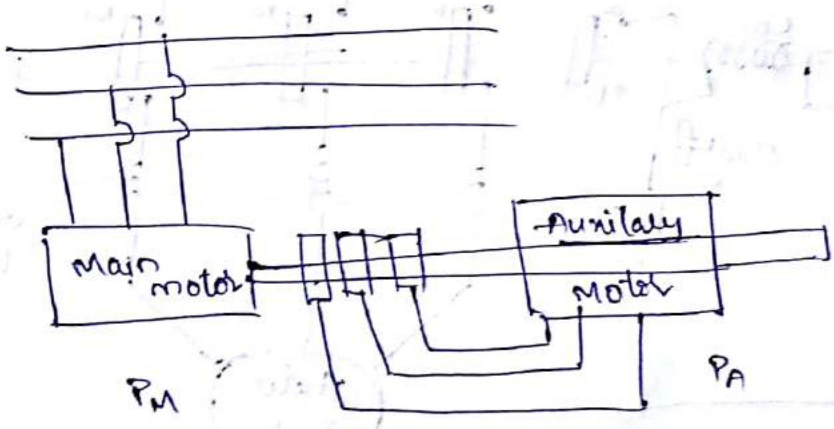
i, $T = K \cdot \frac{s E_2^2 \cdot R_2}{(R_2)^2 + (sX_2)^2}$

For low slip, $T \propto \frac{s \cdot E_2^2 \cdot R_2}{R_2^2}$

$T \propto \frac{s}{R_2}$.

If $R_2 \uparrow$, $s \uparrow$ as a result $N \downarrow$

ii,



i. $N_s = \frac{120f}{P_M}$

ii. $N_s = \frac{120f}{P_A}$

iii. cummulative mode $\cdot$ $N = \frac{120f}{P_M + P_A}$

iv. Differential mode $\cdot$ $N = \frac{120f}{P_M - P_A}$

iii. $E_2 - V_2$ substractive mode

$$T \propto \frac{S \uparrow}{R_2 \uparrow} \quad \therefore N \downarrow$$

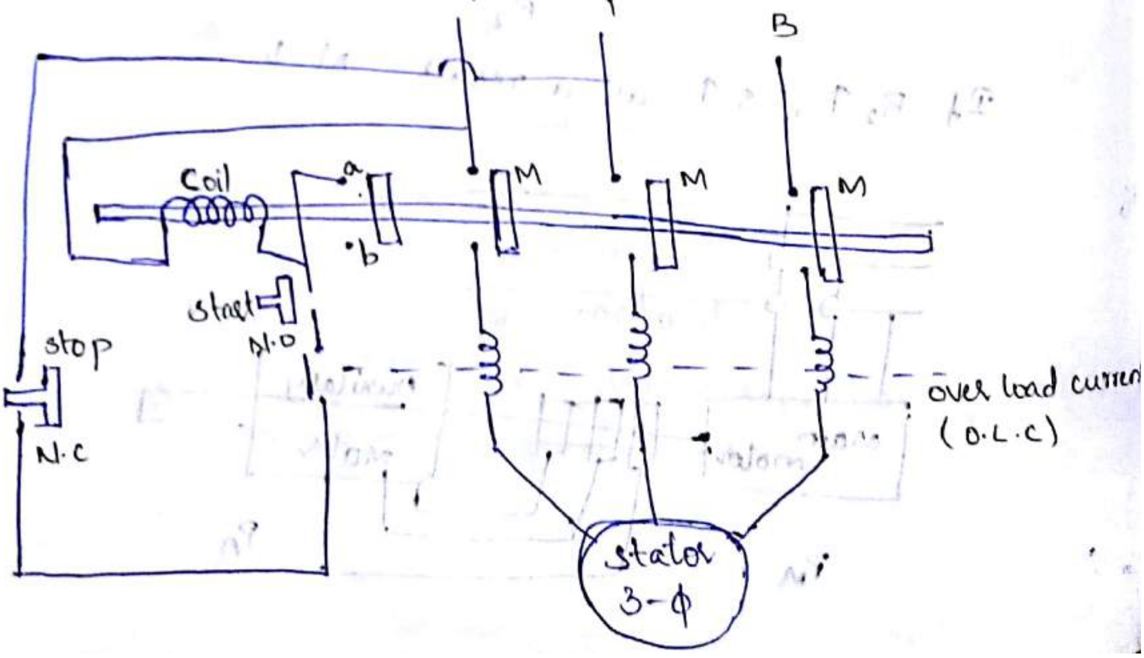
$E_2 + V_2$ addition mode

$$T \propto \frac{S \downarrow}{R_2 \downarrow} \quad \therefore N \uparrow$$

Starting Methods of 3-ϕ Induction motor :: 12/11/1[illegible]

A 3-ϕ induction motor is self-starting. But we prefer starter for limiting high starting current thereby over heating (I^2R). And the starter is also used for over load protection and low voltage protection.

1) Direct Online starter (D.O.L starter)



→ Generally D.O.L starter is used for 5 HP. Motor contactors are used for safe & the safety purpose of motors.

$$P_{cu} = s \cdot P_2$$

$$I_{2r}^2 \cdot R_2 = s \frac{2\pi N_s T}{60}$$

$$T \propto \frac{I_{2r}^2}{s}$$

$$T_{FL} \propto \frac{I_{FL}^2}{S_f}$$

$$T_{st} \propto I_{st}^2 \propto I_{sc}^2 \quad \{\because \text{slip} = 1\}$$

$$\boxed{\frac{T_{st}}{T_{FL}} = \left(\frac{I_{sc}}{I_{FL}}\right)^2 \times S_f}$$

→ starting current is called short circuit current as the rotor is blocked and acts as short circuit at stand still position $\{I_{st} = I_{sc}\}$

Y – Δ starter:

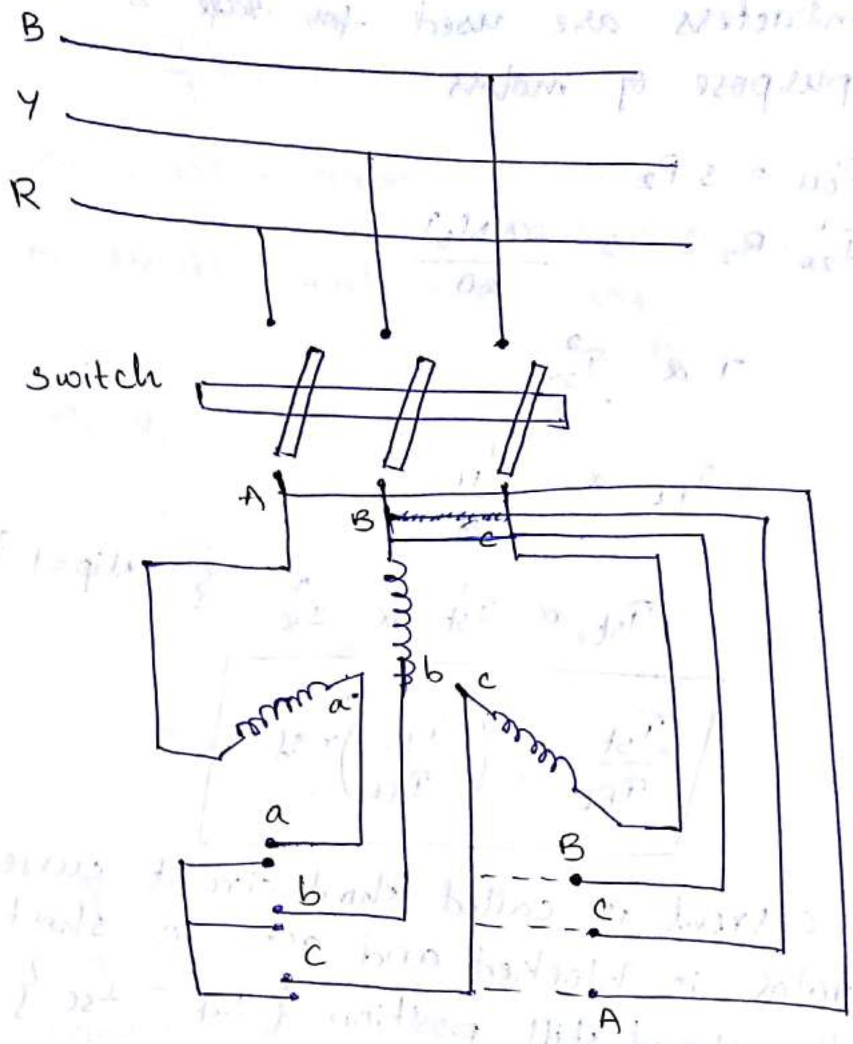
Advantages:

→ It's construction is simple.
→ So, it is not expensive.

Disadvantages:

→ Voltage dip is observed when the motor is started.
→ High current is required during to start the motor.

2) Y – Δ starter:



Advantages:

- price is low
- no limits for operation (durability - long life)
- starting torque is reduced

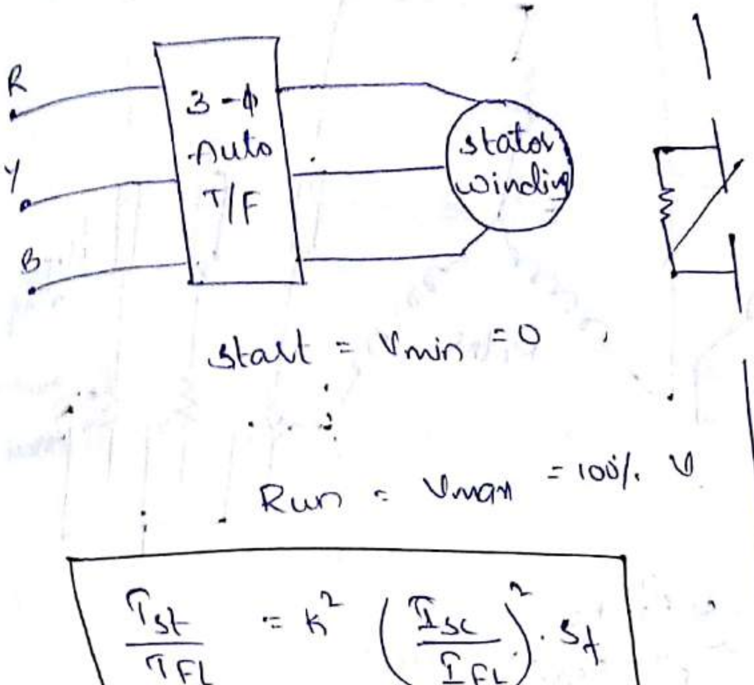
{ In Y connection $V_L = \sqrt{3} V_{ph} \Rightarrow V_{ph} = \frac{V_L}{\sqrt{3}}$

$T \propto V^2$

$T \propto \frac{V_L}{3}$ }

$$\boxed{\frac{T_{st}}{T_{FL}} = \frac{1}{3}\left(\frac{I_{sc}}{I_{FL}}\right)^2 s_f}$$

3) Auto transformer starter :



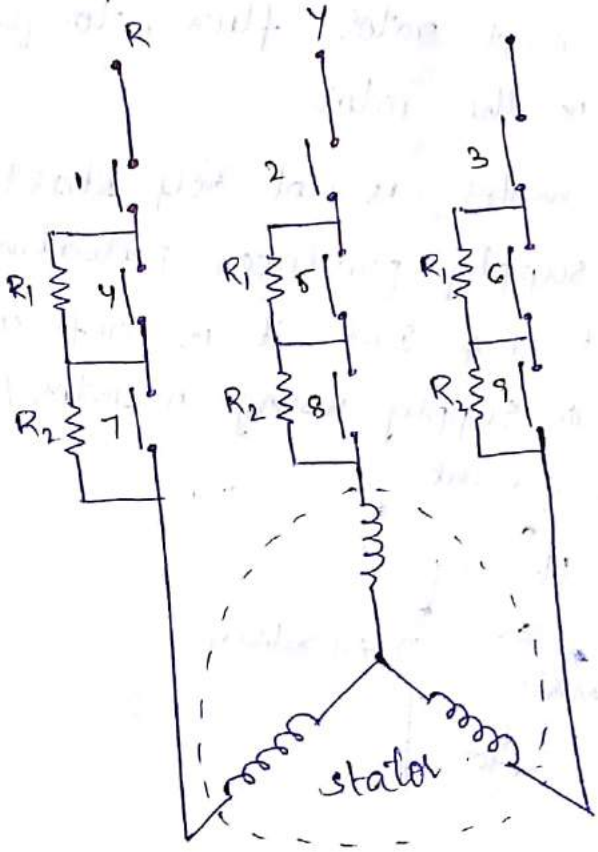
start = $V_{min} = 0$

Run = $V_{max} = 100\%\ V$

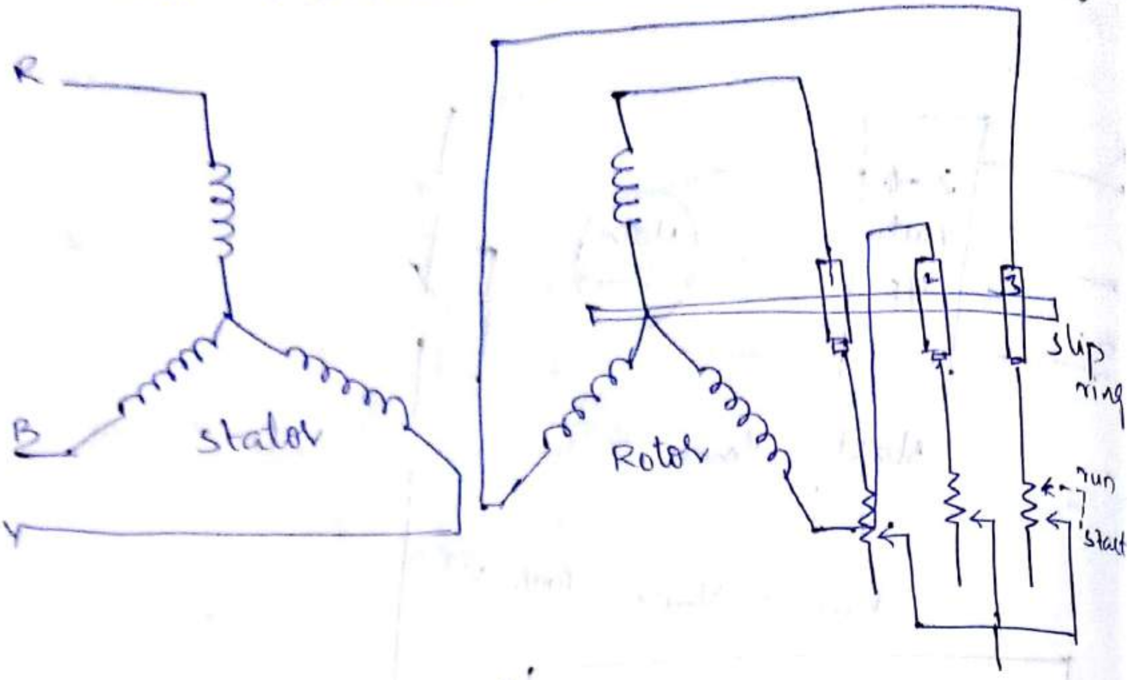
$$\boxed{\frac{T_{st}}{T_{FL}} = K^2 \left(\frac{I_{sc}}{I_{FL}}\right)^2 \cdot S_f}$$

- It is useful, as the voltage can be changed according to our wish but the maintenance is hard, it is expensive. durability will be less.

iv, stator Resistance starter :



5) Rotor resistance starter :

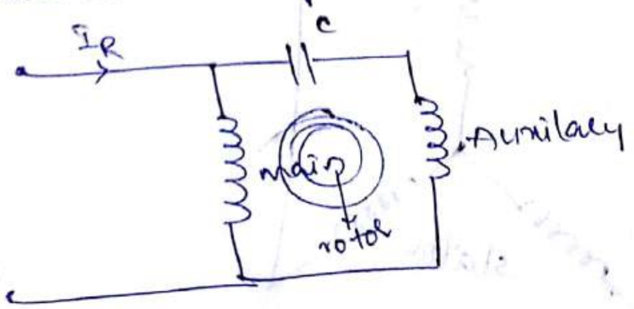


$$T = \frac{s \cdot E_2^2 \cdot R_2}{R_2^2 + (sX_2)^2}$$

Note :

→ 3-ϕ I.M is self starting.
3-ϕ supply produces rotating magnetic field. This R.M.F links with the rotor circuit, induces rotor current. This rotor current produces rotor flux. Due to the interaction of stator RMF and rotor flux, torque will be developed in the rotor.

→ 1-ϕ Induction motor is not self starting because 1-ϕ supply produces pulsating torque. To start 1-ϕ I.M it is preferable to split the 1-ϕ supply using resistor / capacitor arrangement.



Types of 1-ϕ IM based on starting mechanism:

1) slip split phase (R/C) I.M
2) capacitor start I.M
3) capacitor run I.M
4) capacitor start and capacitor run I.M
5) shaded pole I.M

→ 1-ϕ I.M is simple in construction, reliable and easy to repair and comparatively cheaper.

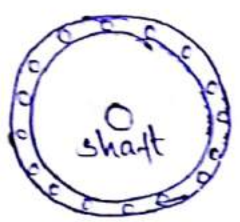
→ I.M is used in fans, refrigerators, washing machines, vacuum cleaner, blowers, water pumps and some kitchen appliances

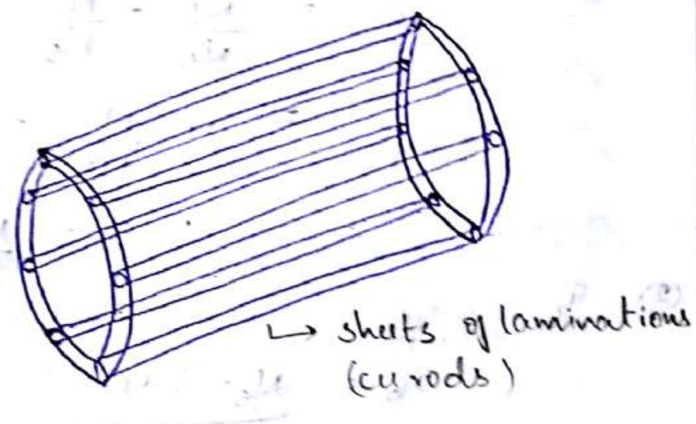
Single phase (1-ϕ) Induction Motor:

Construction:-

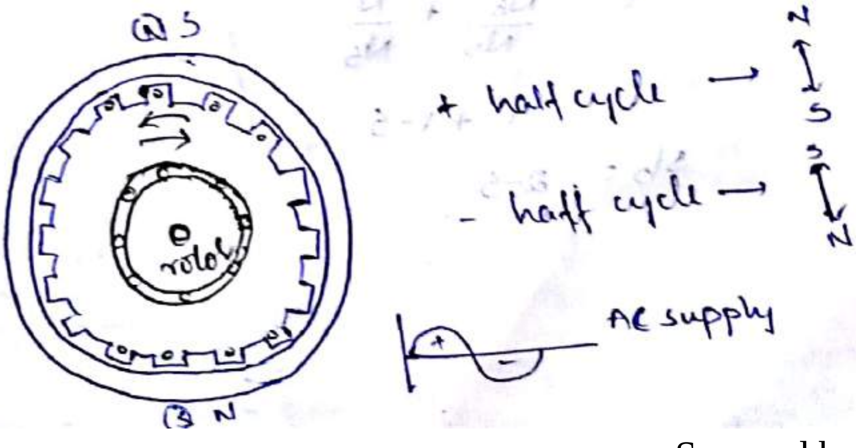
① Rotor:

Squirrel cage rotor





② stator:



→ With pulsating torque, there will not be any rotation i.e. torque will be zero.

→ Two theories are used to explain working of 1-ϕ induction motor
1) Double field revolving theory
2) Cross field revolving theory.

1) Double field revolving theory:-

Torque $\phi_T = \phi_m \cos \omega t$

$$= \phi_m \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2} \right)$$

$$= \frac{\phi_m e^{j\omega t}}{2} + \frac{\phi_m e^{-j\omega t}}{2}$$

$$= \underset{(N_s)}{\phi_f} + \underset{(-N_s)}{\phi_b} \quad (T_{total} = T_f + T_b)$$

① T_f

$$S_f = \frac{N_s - N}{N_s} = S$$

$$= \frac{N_s}{N_s} - \frac{N}{N_s}$$

$$\Rightarrow 1 - \frac{N}{N_s} = S$$

$$1 - S = \frac{N}{N_s}$$

② T_b

$$S_b = \frac{N_s - (-N)}{N_s}$$

$$= \frac{N_s}{N_s} + \frac{N}{N_s}$$

$$= 1 + 1 - S$$

$$S_b = 2 - S$$

$$P_2 : P_m : P_{cu} = 1 : (1-s) : s$$

$$\frac{P_2}{P_{cu}} = \frac{1}{s}$$

$$P_{cu} = s \cdot P_2$$

$$I_2^2 R_2 = s \cdot P_2 \Rightarrow P_2 = \frac{I_2^2 R_2}{s}$$

$$P_2 \propto N \cdot T$$

$$P_2 = \frac{2\pi N T}{60}$$

$$P_2 = \omega T \Rightarrow T = \frac{P_2}{\omega}$$

$$T \propto P_2 \propto \frac{I_2^2 R_2}{s}$$

forward torque $(T_f) = K \cdot \frac{I_2^2 \cdot R_2}{s_f}$

backward torque $(T_b) = -K \cdot \frac{I_2^2 R_2}{s_b}$ { T_f & T_b are equal and opposite }

Total resultant torque $(T) = T_f + T_b$

$$= K \frac{I_2^2 R_2}{s_f} - K \cdot \frac{I_2^2 R_2}{s_b}$$

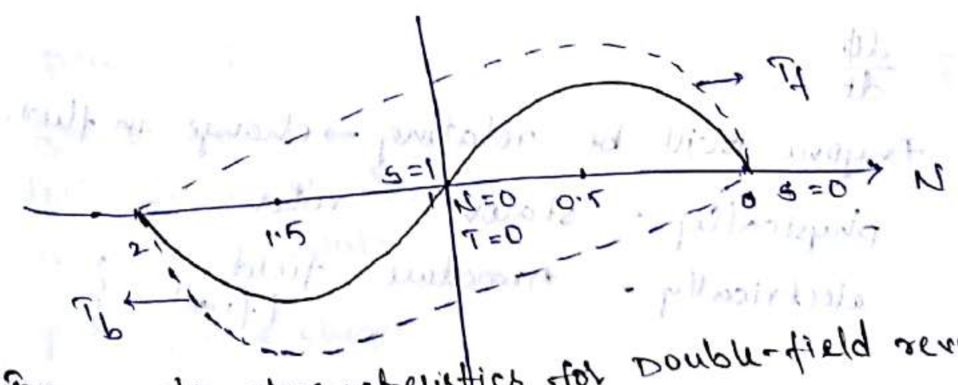
$$= \frac{K \cdot I_2^2 R_2}{s} - \frac{K \cdot I_2^2 R_2}{2-s}$$

In stand still position, $s = 1$

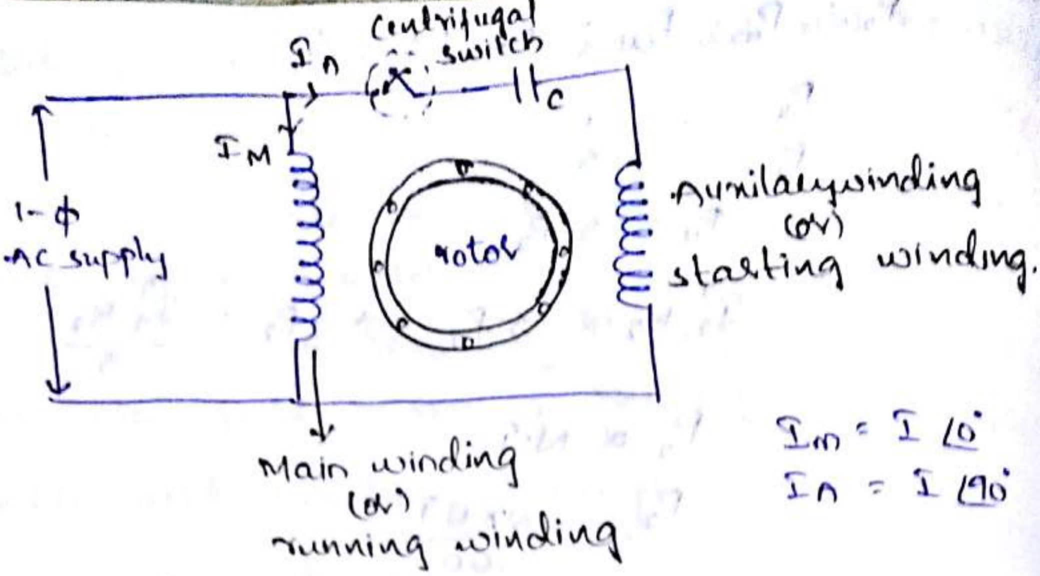
$$T = K I_2^2 R_2 - K I_2^2 R_2$$

$$\boxed{T = 0}$$

*



Torque-slip characteristics for Double-field revolving theory



$I_m = I\angle 0^\circ$
$I_A = I\angle 90^\circ$

Synchronous Generator / Alternator :

→ A.C generator is called alternator.

Synchronous Generator: Rotating machines that rotate at a particular speed fixed by the supplied frequency and no. of poles are called Sychronous machines.

$$N_S = \frac{120f}{P}$$

Construction :

1. Rotating armature and stationary field system is employed in D.C machines
2. stationary armature and rotating field system is employed in A.C machines/generators.

$$\left\{ e = \frac{d\phi}{dt} \right.$$

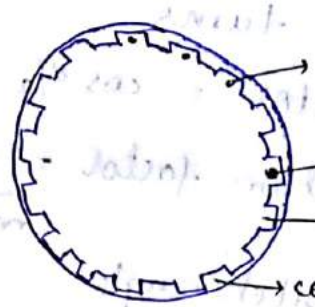
∵ Anyone will be rotating → change in flux.

physically - stator rotor

electrically - Armature field (fixed) }

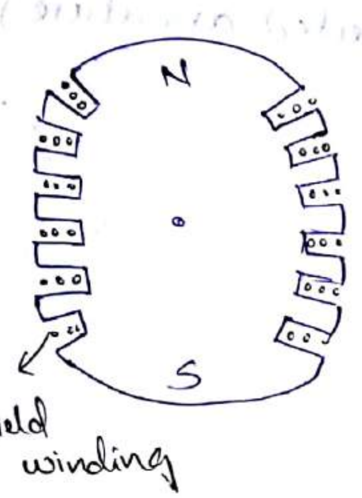
- field winding produces the flux, in armature winding, emf / current is induced.

stator :



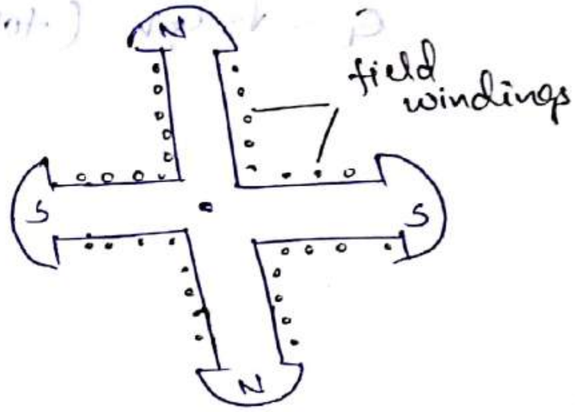
Rotor :

i, cylindrical rotor



- used in thermal stations
- have mechanical strength
- More axial, less diameter.
- speed : 3000 rpm
- 1/3rd of body is left for poles, 2/3rd is left for slots
- cylindrical in shape.
- Air gap is uniform.

ii, saliant pole rotor



- used in hydral stations.
- less mechanical strength
- less axial, more diameter
- speed : 5 - 100 rpm
- Air gap is not uniform, ϕ will also not be uniform when placed in stator
- It is projected.

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EMF Equation of synchronous alternator :

$$\boxed{E_{ph} = 4.44 \phi K_c K_d f T \text{ volts.}}$$

where, T = number of turns

K_c = pitch factor = $\cos \alpha/2$

K_d = distribution factor = $\dfrac{\sin \frac{m\beta}{2}}{m \sin \beta/2}$

α = short pitch angle

β = slot angle

m = phase breadth angle.

$E_L = \sqrt{3} E_{ph}$ (for Y- connected armature)

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Separately Excited DC motor:

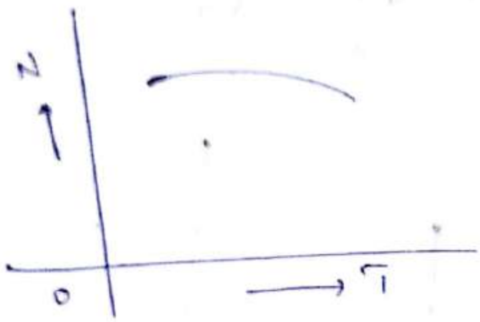
construction:

1) Outer frame / yoke
2) Armature core
3) Armature winding.
4) Field poles
5) Field windings (coils)
6) commutator : converts AC signal to DC signal (Mechanical Rectifier)
7) Brushes & brush holders
8) Inter poles : to avoid armature reactions
9) shaft & bearings

Principle :

Speed - Torque characteristics :-

speed V/s Torque



When load increases, armature current (I_a) increases but speed decreases slightly. Thus with ~~i~~ increase in load (or) torque, the speed ↓ slightly.

Speed control :

$$E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$$

where, ϕ = flux

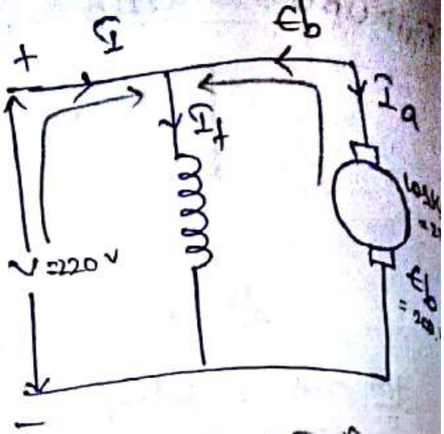
Z = no. of conductors

N = speed

P = no. of poles.

$$V = E_b + I_a R_a$$

$$I_a = \frac{V - E_b}{R_a}$$



$$P = V \cdot I = E_b I_a$$

$$V = E_b + I_a R_a$$

$$220 = 200 + 20$$

$$P = \frac{2\pi N T}{60}$$

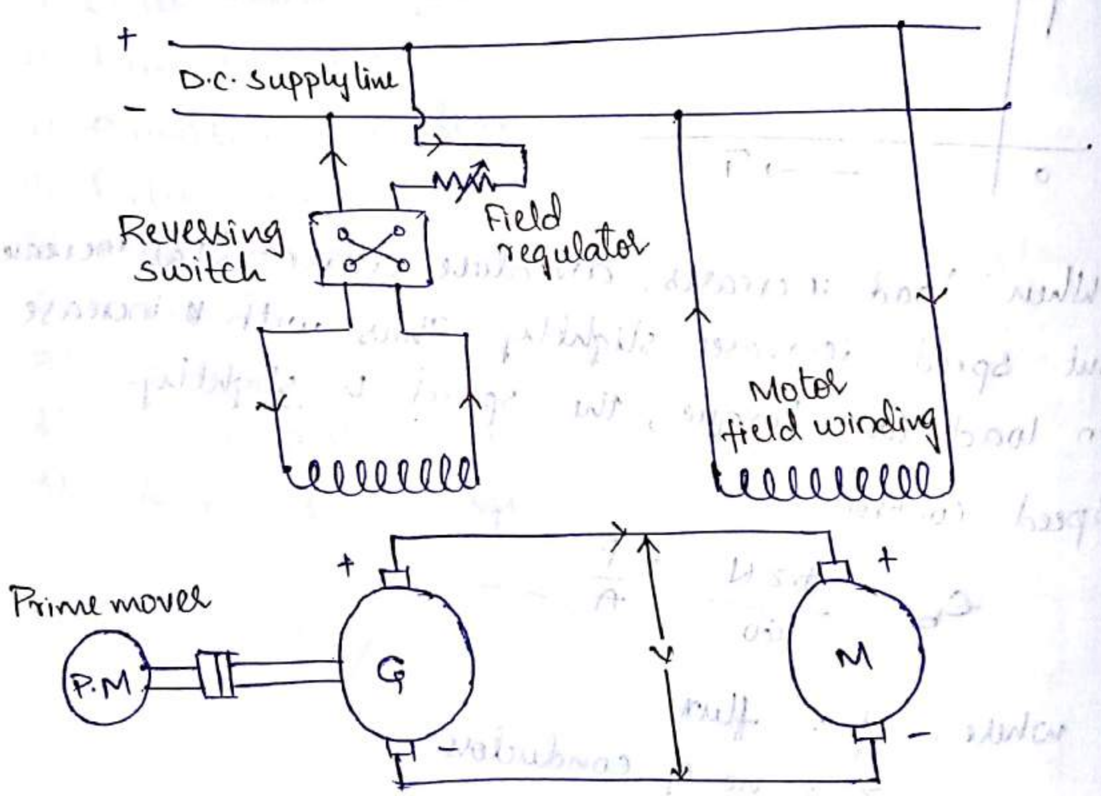
$$P = \omega T$$

$$T = \frac{P}{\omega}$$

$$T = \frac{1}{\omega} \cdot E_b \cdot I_a$$

$$T = \frac{\cancel{60}}{2\pi \cancel{N}} \cdot \frac{\phi Z \cancel{N}}{\cancel{60}} \times \frac{P}{A} \cdot I_a$$

$$T = 0.159 \times \frac{\phi Z P}{A} \cdot I_a$$



$$E \propto I_f \propto \phi$$

UNIT : 5 ELECTRICAL INSTALLATIONS

→ components of LT switchgear.

→ Types of wires & cables

→ Earthing and types of earthing

→ Types of batteries, characteristics & battery backup

→ Elementary calculation of energy consumption.

→ power factor improvement.

Switchgear: In electrical power system, switchgear refers to the devices used for switching, controlling and protecting the electrical circuits and components. The switchgear compriss of wide range of components whose primary functions are switching and interrupting currents during both normal and fault conditions.

→ The different protective equipment used in switch gear

1) switch
2) fuse
3) circuit breaker.
4) protective Relay

1) Switch: The device which is used to open (or) close an electrical circuit in a most conventional way is called switch.

→ It cannot interrupt the current in faulty condition.

2) Fuse: A simple protective device used to protect the cables and equipment under over load and short circuit conditions, is called fuse.

→ It is a small piece of thin strip of wire which melts when fault current flows through it for a sufficient time.

3) circuit breaker :

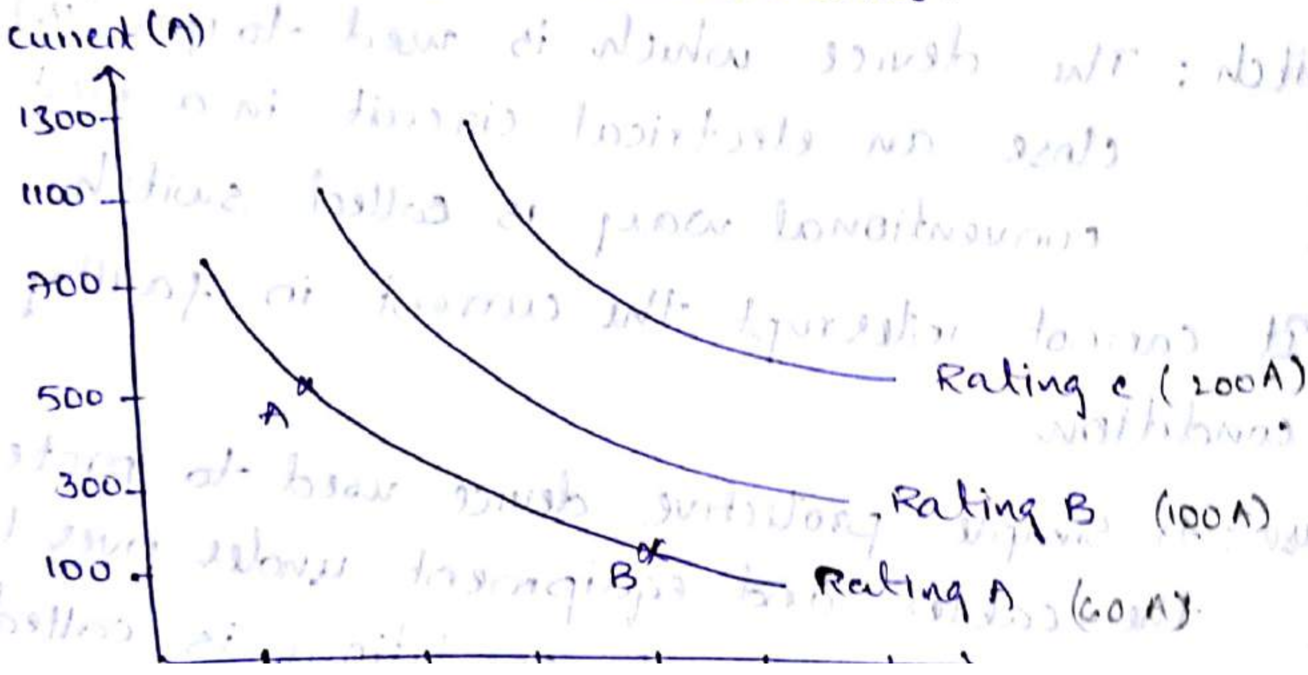
-A switching device which can be used to make (or) break a circuit manually (or) automatically (or) with the help of remote control under different conditions i.e., under normal & fault conditions.

4) Protective Relay :

It will monitor the system for abnormalities in current and voltages and give operation command to circuit breaker under fault conditions.

Fuse :

* Inverse - current characteristics.



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Fuse :

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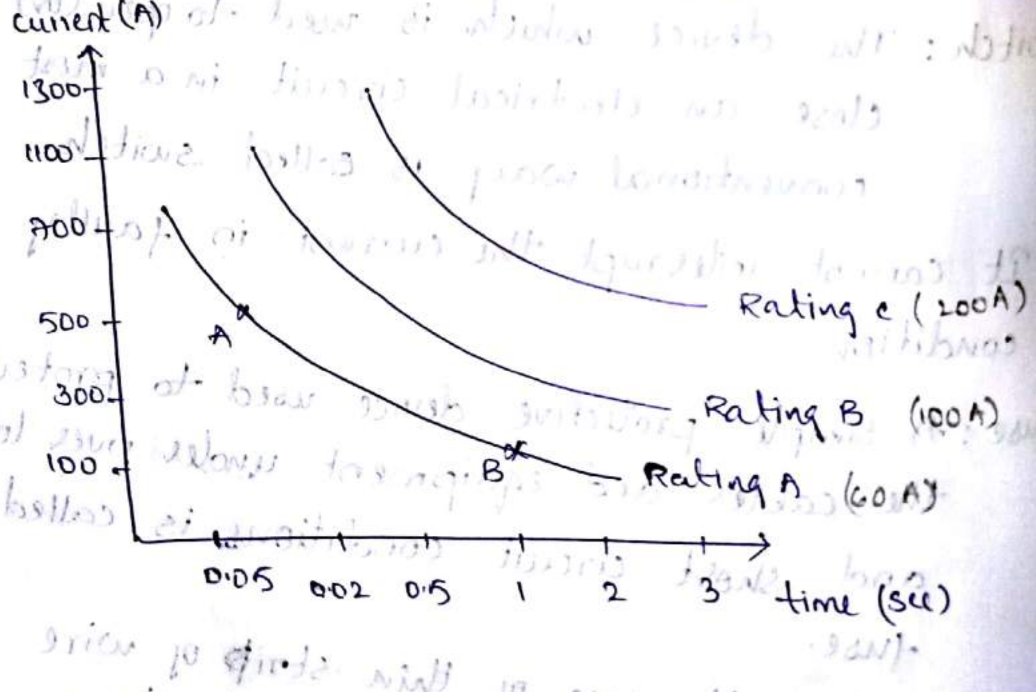


1) fuse must be affordable
2) have low melting point
3) high conductivity
4) less effective on oxidation are the good characteristics of fuse.

Advantages of fuse :

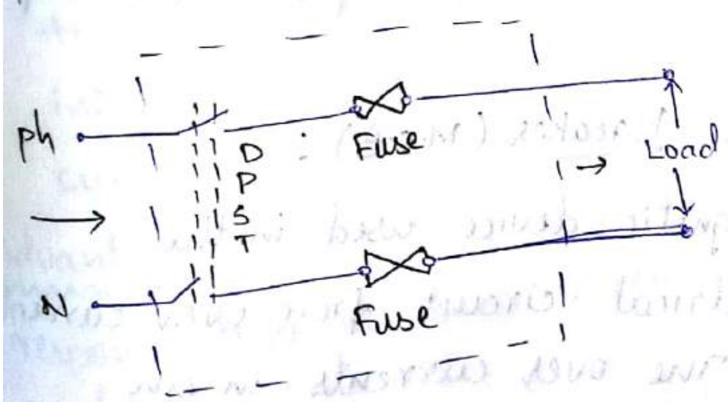
1) It is pollution free.
2) cheapest
3) can be operated without any skilled operator
4) Maintenance is not required.
5) It is automatic / easily breaks.
6) Requires less time
7) Used for over current protection.

Disadvantages of fuse.

1) We should replace manually.
2) To operate fuse in co-ordination with other circuit breakers is difficult. As the characteristics of fuse will not match with other C.B

Switch Fuse Unit (S.F.U) :-

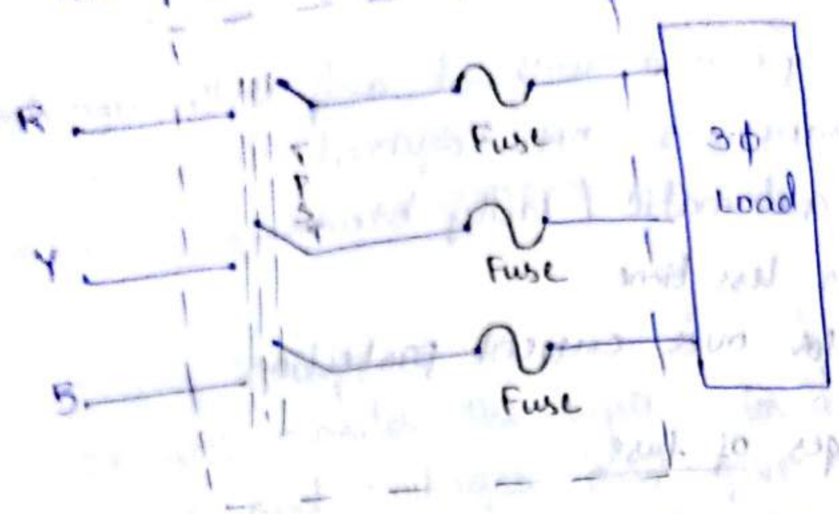
A unit which consists of combination of fuse and switch together
fuse should be always to the right of the switch.



1-ϕ S.F.U

→ Through service mains, energy (current) is transfered from electric pole to the energy meter at house.

→ Difficulty with fuse is to select the correct rating fuse.



3-ϕ S.F.U

→ For 3-ϕ we use triple pole single through (TPST) switch. For 1-ϕ we use double pole single through (DPST) switch.

Note:

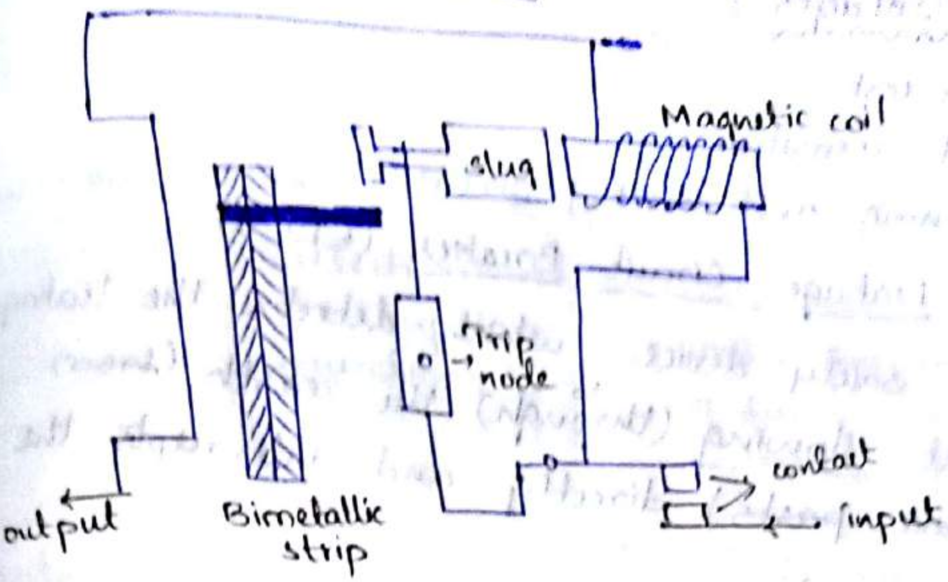
→ The fuse plugs are made up of porcelein rewirable fuse, fitted with their conducting paths.

→ The fuse material is silver plated electrolytic copper.

Miniature Circuit Breaker (M.C.B):

An electromagnetic device used in the protection of electrical circuit from over currents is called MCB. The over currents in an electrical circuit occurs due to short circuit (or) over load (or) any other fault conditions

Construction & Working:



→ MCB is made with flame retardant plastic moulding and materials with high melting point, high dielectric strength.

C.B —→ Magnetic strip (short circuit fault)
C.B —→ Thermal trip (overloading)

→ The current at which the magnetic coil is going to attract the plunge/slug there by interrupting the circuit is called pickup current.

Advantages:

- response time is fast under short circuit condition.
- Faster in overloading and under voltage conditions.
- High reliability
- Less maintenance and replacement cost.
- Easier and safer to operate MCB compared to fuse.

Disadvantages :

- High cost
- Heat sensitive
- Agening and wear problem.

Earth Leakage Circuit Breaker (E.L.C.B)

A safety device which detects the leakage current flowing (~~through~~) to the earth (from the ~~live parts~~) directly and interrupts the power supply.

Earth leakage current :

Current flowing to earth from the live parts of installation without the insulation failure is called earth leakage current.

Earth fault current :

current flowing to earth when there exists an insulation failure in equipment (or) installation.

→ The tolerable limits for human are 65 V for 10 sec and 250 V for 100ms.

Moulded Case Circuit Breaker (M.C.C.B)

A device which uses a moulded case to accomodate and support different current carrying components in addition to insulation system is called MCCB. The practical range of MCCB is from 60 to 300 Amps.

**Comparison between a fuse and circuit breaker.

Particular	Fuse	Circuit Breaker.
function	It performs both detection and interruption functions.	It performs interruption function only. The detection of fault is made by relay system.
operation	Inherently compeletely automatic	Requires elaborate equipment (i.e, relays) for automatic action
Breaking capacity	small	very large.
Operating time	very small (0.002 sec or so)	comparatively large (0.1 to 0.2 sec)
Replacement	Requires replacement after every operation.	No replacement after operation.

Wires and cables

→ Conductors without any covering are known as bare conductors.

Eg: Transmission line (aluminium)

Wire: Bare conductor with insulation is known as wire. Insulation separates the conductor electrically from other conductors.

Cable: It consists of one or more conductors covered with suitable insulation and surrounded by a protective cover.

Classification of wires / cables:

1) According to the conductor material used

a. Copper conductor cables

b. Aluminium conductor cables.

2) According to the number of cores

a) Single core cables

b) Double core cables

c) Three core cables

d) Four core cables

e) Two core with earth continuity conductor cables

3) According to the type of insulation

a) VIR - Vulcanized Indian Rubber

b) Poly vinyl chloride cables

c) Lead sheathed cables

d) Weather proof cables.

4) ~~According to the operating voltage~~

Need for Earthing:

1) Protect the human life and electrical equipment from fault currents.

2) Maintain the voltage at constant level. even when a fault occurs in any part of the system.

3) Protect the electrical equipment and buildings from over voltages occuring due to lightening.

4) Provides a return path for the fault currents occuring in the system.

5) Prevent fire in the electrical systems.

Battery:

A device that converts the stored chemical energy into electrical energy using chemical action is called battery.

- A cell is a device that consists of two electrodes and an electrolyte.
- A battery is a single unit which comprises of two (or) more cells which are connected together electrically.

Selection of Earthing

S.No	Type of Earthing	Application
1.	Plate Earthing	Transmission towers, substations.
2.	Pipe Earthing	Induction motors, coolers, heaters, transformers.
3.	Rod Earthing	Loose (or) Sandy soil.
4.	Strip (or) Wire Earthing	Rocky areas.

Batteries:

- Battery output voltage and current ratings depends upon the elements used for electrodes, size of electrodes and type of electrolyte.
- Battery provides a steady DC voltage at the output terminals.

Primary cell / Batteries :

The batteries that are not rechargable are called primary batteries.

Eg: Disposable batteries (AA, AAA type) use in wall clocks, remotes etc.

Secondary cell / Batteries :

The batteries that are rechargable are called secondary batteries.

Eg: Batteries used in mobile phones, MP3 players.

Types of Primary Batteries/cells :

1) carbon-zinc dry cell
2) Alkaline cell
3) Zinc-chloride cell
4) Mercury cell.
5) silver oxide cell

Types of secondary Batteries/cells :

1) Nickel Iron (Edison) Battery.
2) Fuel cell
3) solar cell

Characteristics of Batteries :

1) chemistry
2) Rated voltage
3) Battery capacity.

- The energy stored in a battery is called battery capacity. It is measured in Ampere hours or Watt hour (or) KWH.

4) cold cranking-Amperes (CCA) [-18°C]

5) Battery life cycle.

6) Usage

Comparison of Primary and secondary cells

S.No	Primary cell	secondary cell
1.	cannot be recharged	can be recharged
2.	light weight	heavy weight
3.	short life and low cost	Long life and high cost.
4.	Low power output and low efficiency	High power output and high efficiency.
5.	Not used continously	Used continuously
6.	Less maintenance	High maintenance

Elementary calculations for energy consumption :-

Tariff : The reasonable price (or) rate at which the produced electrical energy is supplied to the consumer is defined as tariff.

→ The tariff at which the electrical energy is charged is not uniform for all the ~~customers~~ customers

Factors effecting the tariff :-

1) Nature of load.
2) Maximum demand
3) Load requirement time
4) Load powerfactor.

Characteristics of Tariff:

1) proper return
2) fairness
3) Simplicity
4) Reasonable profit
5) Attractive

Types of Tariff:

1) Simple tariff
2) flat tariff
3) Block rate tariff
4) Two part tariff
5) Maximum demand tariff
6) Power factor tariff
7) Three part tariff.

* For domestic usage (houses), block rate tariff is used.

* The monthly electrical consumption of a residential house is 123 units. Block rate tariff is used to determine the monthly bill and the rate for different blocks are 2.74₹ for first 15 units, 2.70₹ for next 25 units and 2.32 for remaining units. Also a constant charge of 12 ₹ for month is charged. Determine the monthly bill.

$$\text{Bill} = [15 \times 2.74 + 25 \times 2.70 + 83 \times 2.32] + 12$$

$$= 313.16 \text{ ₹}.$$

Power factor Improvement :

Cosine of phase angle between voltage and current signals is called power factor.

$$\text{Power factor} = \frac{\text{real power (P)}}{\text{Apparent power (S)}} = \frac{VI\cos\phi}{VI} = \cos\phi$$

$$= \cos(V'I)$$

$\cos\phi = \frac{R}{Z}$ [It's range is 0 to 1]

① static capacitor
② synchronous condensor (over excited synchronous motor)
③ Phase advancers (Induction motor)