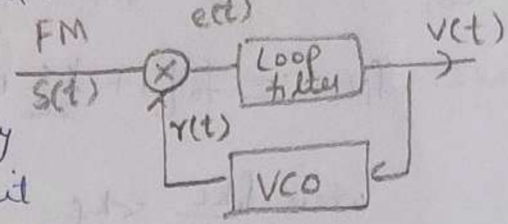
Phase locked loop:

PLL is a negative feedback system that consists of 3 major components. multiplier, VCO, loop filter connected together in the form of a feedback loop. The VCO is a sine wave generator whose frequency is determined by a voltage applied to it from an external source. In effect, any frequency modulator may serve as VCO



we assume initially, we have adjusted VCO so that when the control voltage is zero two conditions are satisfied (1) frequency of VCO is precisely set to unmodulated f_c, (2) the VCO o/p has a 90° phase shift w.r. to unmodulated carrier wave.

I/P applied to PLL is an FM wave defined by

$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \quad \text{—} \ ①$$

with a modulating wave $m(t)$ we have

$$\phi_1(t) = 2\pi k_f \int_0^t m(t)\, dt \quad \text{—} \ ②$$

where k_f frequency sensitivity of the frequency modulator.

let VCO o/p be defined by.

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)] \quad \text{—} \ ③$$

with a control voltage $v(t)$ applied to the VCO i/p we have.

$$\phi_2(t) = 2\pi k_v \int_0^t v(t)\, dt \quad \text{—} \ ④$$

k_v is the frequency sensitivity of VCO. (Hz/V)

The incoming FM wave $s(t)$ & the VCO o/p $r(t)$ are applied to the multiplier producing two components.

1) A high frequency component represented by

$$K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$$

2) A low frequency component represented by

$$K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$

where K_m is the multiplier gain $[V^{-1}]$

High frequency component is eliminated by the LPF.

I/P to the loop filter is given by

$$e(t) = K_m A_c A_v \sin[\phi_e(t)] \quad \text{--- (5)}$$

where $\phi_e(t)$ is the phase error defined by

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$= \phi_1(t) - 2\pi k_v \int v(t)\,dt \quad \text{--- (6)}$$

The loop filter operates on its input $e(t)$ to produce the output

$$v(t) = \int_{-\infty}^{\infty} e(\tau)h(t-\tau)\,d\tau \quad \text{--- (7)}$$

where $h(t)$ is the impulse response of the filter

using equations from eqn (5) to (7) to relate $\phi_e(t)$ & $\phi_1(t)$ & differentiating with respect to time we obtain

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{d(t)} - 2\pi k_0 \int_0^t \int_{-\infty}^{\infty} e(\tau)\,h(t-\tau)\,d\tau\,dt$$

$$- 2\pi k_0 \int_{-\infty}^{\infty} \sin[\phi_e(\tau)]\,h(t-\tau)\,d\tau \quad \text{--- (8)}$$

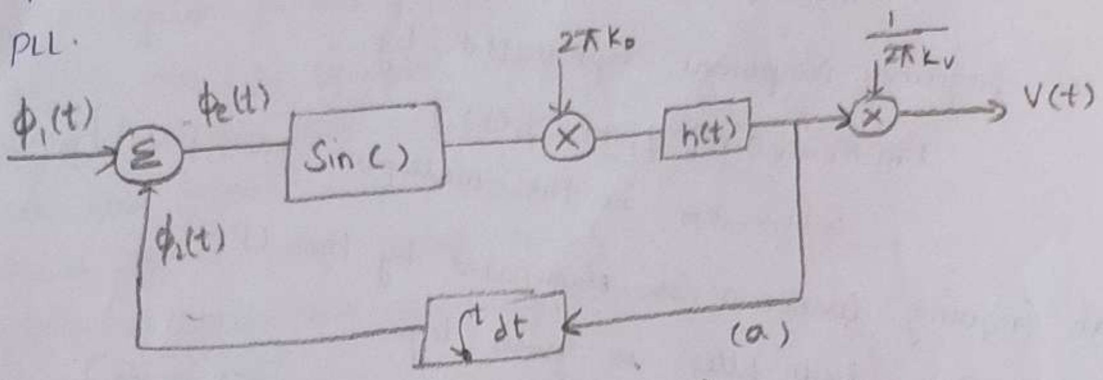
Loop parameter $K_0 = K_m k_v A_c A_v$ --- (9)

Eqn (8) suggests the representation or model of below figure.

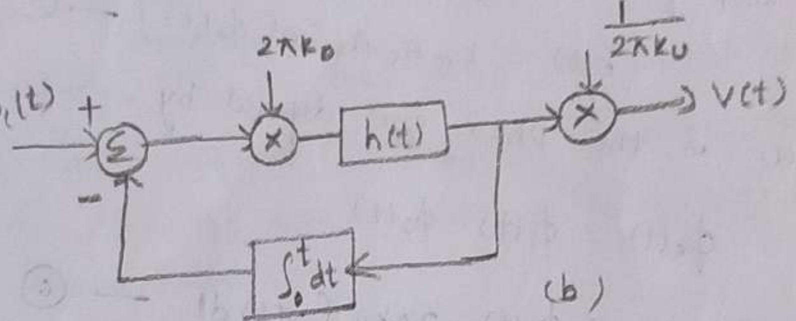
In this model we have also included the relationship btn $v(t)$ & $e(t)$ as represented by eqn (5) & (7)

we see that model [below fig] resembles block diagram. The multiplier is replaced by a subtractor & a sinusoidal non-linearity, & the VCO by an integrator.

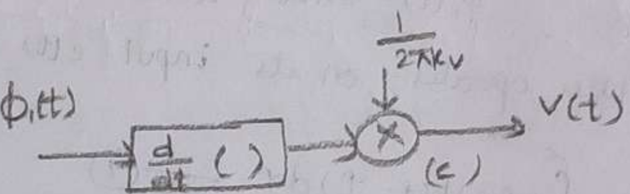
The loop parameter k_0 plays an important role in the operation of a PLL.



(a)



(b)



(c)

$$K_0 = A_c A_v K_m k_v$$
(Hz) (V V V^{-1} Hz/V).

Linearized model:

When the phase error $\phi_e(t)$ is zero, the PLL is said to be in phase lock. When $\phi_e(t)$ is small,

$$\sin[\phi_e(t)] \simeq \phi_e(t) \quad — (10)$$

From eqn (8) we get

$$\frac{d\phi_e(t)}{dt} + 2\pi k_0 \int_{-\infty}^{\infty} \phi_e(\tau)\, h(t-\tau)\, d\tau = \frac{d\phi_1(t)}{dt} \quad — (11)$$

$$j2\pi f\, \phi_e(f) + 2\pi k_0\, \phi_e(f) H(f) = j2\pi f\, \phi_1(f) \quad — (12)$$

$$\phi_e(f) \times [jf + k_0 H(f)] = jf\, \phi_1(f)$$

$$\phi_e(f) = \frac{jf}{jf + K_0 H(f)} \phi_1(f)$$

From eqn 7

$$V(f) = \phi_e(f) H(f)$$

$$V(f) = \frac{1}{1 + \frac{K_0 H(f)}{jf}} \cdot \phi_1(f) \cdot H(f)$$

$$V(f) \simeq \frac{jf}{K_0 \cancel{H(f)}} \phi_1(f) \cdot \cancel{H(f)}$$

$$V(f) \simeq \frac{jf \phi_1(f)}{K_0}$$

$$V(t) \simeq \frac{1}{2\pi K_0} \frac{d\phi_1(t)}{dt}$$

$$\left[\because \phi_1(t) = 2\pi K_f \int_0^t m(t) dt\right]$$

$$V(t) \simeq \frac{\cancel{2\pi} K_f}{\cancel{2\pi} K_0} m(t)$$

$$V(t) \simeq \frac{K_f}{K_0} m(t)$$

$$V(t) \propto m(t)$$