SIGNALS AND SYSTEMS

II B.TECH I SEM

II ECE-2

- Name of the Faculty: Mr.A.BALA RAJU, Assistant. Professor
- Name of the Course: Signals and Systems.
- Class: II B.Tech-ECE 2-I sem.
- Subject Code: EC 304PC.
- Number of Lectures hours/Week: 4
- Number of Tutorial hours/Week: 1
- Number of Credits: 4

COURSE OBJECTIVES

- This gives the basics of Signals and Systems required for all Electrical Engineering related course
- To understand the behaviour of signal in time and frequency domain
- To understand the characteristics of LTI systems
- This gives concepts of Signals and Systems and its analysis using different transform techniques.

COURSE OUTCOMES

Upon completing this course, the student will be able to

- Differentiate various signal functions.
- Represent any arbitrary signal in time and frequency domain.
- Understand the characteristics of linear time invariant systems.
- Analyse the signals with different transform technique
- Understand different sampling techniques and comparison of signals

SYLLABUS

UNIT - I

- **Signal Analysis:** Analogy between Vectors and Signals, Orthogonal Signal Space, Signal approximation using Orthogonal functions, Mean Square Error, Closed or complete set of Orthogonal functions, Orthogonality in Complex functions, Classification of Signals and systems, Exponential and Sinusoidal signals, Concepts of Impulse function, Unit Step function, Signum function

SYLLABUS

UNIT – II

- **Fourier series:** Representation of Fourier series, Continuous time periodic signals, Properties of Fourier Series, Dirichlet's conditions, Trigonometric Fourier Series and Exponential Fourier Series, Complex Fourier spectrum.
- **Fourier Transforms:** Deriving Fourier Transform from Fourier series, Fourier Transform of arbitrary signal, Fourier Transform of standard signals, Fourier Transform of Periodic Signals, Properties of Fourier Transform, Fourier Transforms involving Impulse function and Signum function, Introduction to Hilbert Transform.

SYLLABUS

UNIT – III

- **Signal Transmission through Linear Systems:** Linear System, Impulse response, Response of a Linear System, Linear Time Invariant(LTI) System, Linear Time Variant (LTV) System, Transfer function of a LTI System, Filter characteristic of Linear System, Distortion less transmission through a system, Signal bandwidth, System Bandwidth, Ideal LPF, HPF, and BPF characteristics, Causality and Paley-Wiener criterion for physical realization, Relationship between Bandwidth and rise time, Convolution and Correlation of Signals, Concept of convolution in Time domain and Frequency domain, Graphical representation of Convolution.

SYLLABUS

UNIT – IV

- **Laplace Transforms:** Laplace Transforms (L.T), Inverse Laplace Transform, Concept of Region of Convergence (ROC) for Laplace Transforms, Properties of L.T, Relation between L.T and F.T of a signal, Laplace Transform of certain signals using waveform synthesis.
- **Z–Transforms:** Concept of Z- Transform of a Discrete Sequence, Distinction between Laplace, Fourier and Z Transforms, Region of Convergence in Z-Transform, Constraints on ROC for various classes of signals, Inverse Z-transform, Properties of Z-transforms.

SYLLABUS

UNIT – V

- **Sampling theorem:** Graphical and analytical proof for Band Limited Signals, Impulse Sampling, Natural and Flat top Sampling, Reconstruction of signal from its samples, Effect of under sampling – Aliasing, Introduction to Band Pass Sampling.

- **Correlation:** Cross Correlation and Auto Correlation of Functions, Properties of Correlation Functions, Energy Density Spectrum, Parsevals Theorem, Power Density Spectrum, Relation between Autocorrelation Function and Energy/Power Spectral Density Function, Relation between Convolution and Correlation, Detection of Periodic Signals in the presence of Noise by Correlation, Extraction of Signal from Noise by Filtering.

TEXT BOOKS:

- 1. Signals, Systems & Communications - B.P. Lathi, 2013, BSP.
- 2. Signals and Systems - A.V. Oppenheim, A.S. Willsky and S.H. Nawabi, 2 Ed.

REFERENCE BOOKS:

- 1. Signals and Systems – Simon Haykin and Van Veen, Wiley 2 Ed.,
- 2. Signals and Systems – A. Rama Krishna Rao, 2008, TMH
- 3. Fundamentals of Signals and Systems - Michel J. Robert, 2008, MGH International Edition.
- 4. Signals, Systems and Transforms - C. L. Philips, J.M.Parr and Eve A. Riskin, 3 Ed., 2004, PE.
- 5. Signals and Systems – K. Deergha Rao, Birkhauser, 2018.

Z-TRANSFORMS

UNIT-IV-II

Classification of Signals — Continuous time

- Continuous time
  - Periodic
    - CT Fourier Series
  - Non-Periodic
    - CT Fourier Transform (FT)
- Discrete time
  - DT Periodic
    - DTFS
  - DT Non-Periodic
    - Discrete Time Fourier Transform (DTFT)

CTFT → existence Conditions – $\int_{-\infty}^{\infty} |f(t)|\,dt < \infty$ = absolutely Integrable

$\int_{-\infty}^{\infty} |f(t)|\,dt = \infty$ = Not absolutely Integrable

Laplace Transforms → continuous time signal

DTFT → $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$ = existence Condition = $\sum_{n=-\infty}^{\infty} |x(n)| < \infty$

DT

absolutely summable

if Not absolutely summable → $\boxed{\text{Z-Transforms}}$

Discrete Time Counter part of LT

Z-Transform : Discrete Time Signals $\longrightarrow$ Sequence $\rightarrow x[n]$

$x(n)$

Ilpt Variable time

$x(n) =$

$= \quad x(n) = [2, 1, 3, -1]$

seq ✓

if $x(n)$ DT seq $\rightarrow$ $\boxed{ZT[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}}$

$Z =$ Complex Variable $=$ polar form $= re^{jw}$

$S =$ " $=$ Rectangler $= \sigma + jw$

$Z = re^{j\omega} \Rightarrow$ r = magnitude of the Complex Variable Z

ω = angle " " Z

$$DTFT(x(n)) = X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} \quad \text{——— (1)}$$

$$ZT(x(n)) = X(z) = \sum x(n)z^{-n}$$

Let us Replace $Z = re^{j\omega} \Rightarrow X(z) = \sum_{n=-\infty}^{\infty} x(n)(re^{j\omega})^{-n}$

$$X(z) = \sum_{n=-\infty}^{\infty} \underbrace{(x(n)r^{-n})}_{f(n)} e^{-j\omega n} \quad \text{——— (2)}$$

$$= \sum f(n)e^{-j\omega n} \longrightarrow DTFT(f(n))$$

$$X(re^{j\omega}) = \sum x(n) r^{-n} e^{-j\omega n}$$

$x(n)$ — absolutely NOT summable

if $\sum_{n=-\infty}^{\infty} |x(n) r^{-n}| < \infty$ = absolutely summable

r^{-n} = real exponential decaying & increasing

$\downarrow$ FT is also converging ✓

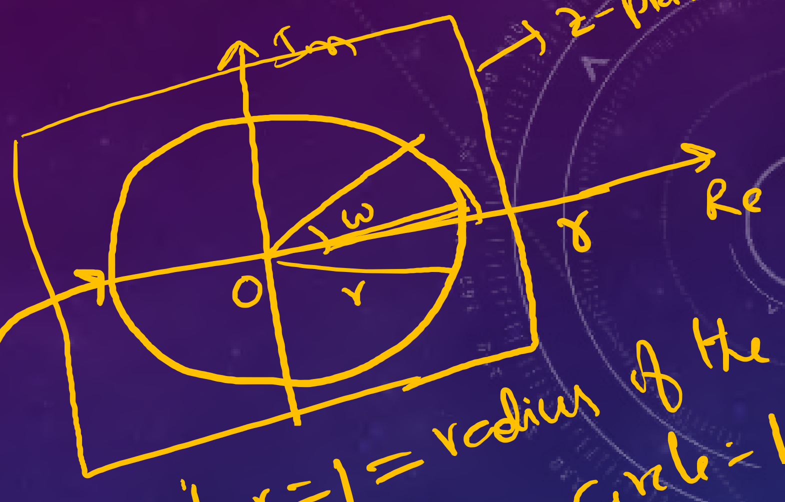
$x(n) r^{-n}$ — absolutely summable $\rightarrow$ ZT $\rightarrow$ defined (DTFT)

$|z| = r$ = radius $= 1$ = unit radius

$$X(e^{j\omega}) = \sum x(n) e^{-j\omega n}$$

$$X(z)\Big|_{\text{at } r=1=|z|} = X(e^{j\omega}) = X(\omega) = \underline{\text{DTFT}}$$

→ Unit-circle is playing the similar role as $j\omega$ axis in L-Transforms



if $r = 1$ = radius of the circle $= 1$

Unit-circle

find the ZT of $x(n) = a^n u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} =$$ Bilateral / two-sided Z-Transform.

$u(n) = 1 \quad n \geq 0$

$\quad\;\; = 0 \quad n < 0$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n$$

$$X(z) = \frac{1}{1-az^{-1}} = \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}$$

absolutely summable $= \sum_n |az^{-1}| < 1$

$$\left|\frac{a}{z}\right| < 1 \Rightarrow |a| < |z|$$

$$\Rightarrow |z| > |a|$$

$\sum_{n=0}^{\infty} (2)^n \quad a > 1$

$1+2+4+8+16 \ldots$

Infin

$x(n) = a^n u(n) \Rightarrow X(z) = \frac{1}{1-az^{-1}} = \frac{z}{z-a}$; $|z|>|a|$

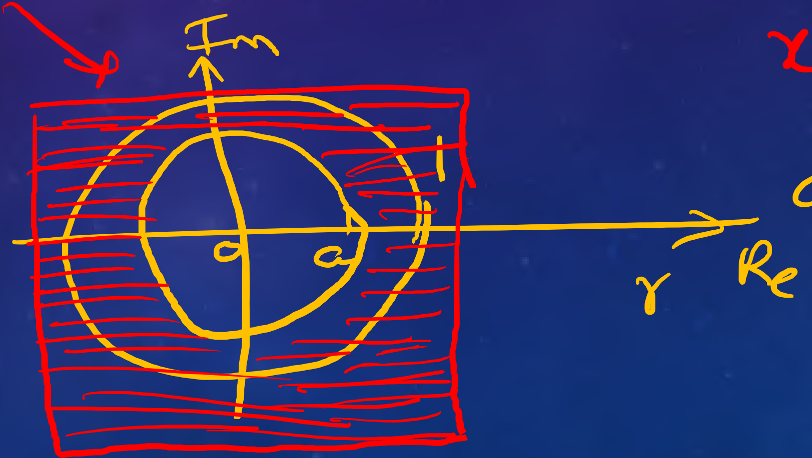
$|z|>|a| \rightarrow$ Range of Values for which $X(z)$ is Valid
Converging

Region of Convergence — ROC

$X(z) = \frac{z}{z-a}$; $|z|>|a| \rightarrow$ Circle with radius 'a'

$x(n) \xleftrightarrow{ZT} X(z)$

$a^n u(n) \leftrightarrow \frac{z}{z-a}$



roots of the numerator are called — Zeros : $z=0$ — zero

$\hookrightarrow$ roots " denominator " — poles $z=a$

pole-zero plot

$u(n)=1$ $n\geq 0$

$u(-n)=1$ $n\leq 0$

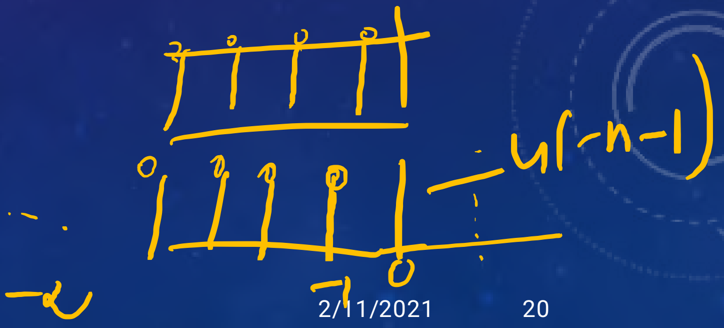
$x(n) = -a^n u(-n-1)$

= left sided seq $(-\infty$ to $-1)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} -a^n u(-n-1) z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} a^n z^{-n}$$



$$X(z) = -\sum_{n=1}^{\infty} a^{-n} z^{n} = -\left[\sum_{n=0}^{\infty} (a^{-1}z^{1})^{n} - 1\right]$$

$a^0 = 1 - 1$

$$= 1 - \sum_{n=0}^{\infty} (a^{-1}z^{1})^{n}$$

$$= 1 - \frac{1}{1 - a^{-1}z^{1}} = \frac{\cancel{1} - a^{-1}z - \cancel{1}}{1 - a^{-1}z}$$

$$= \frac{-z/a}{1 - z/a} = \frac{-z}{\cancel{a}} \times \frac{\cancel{a}}{a - z}$$

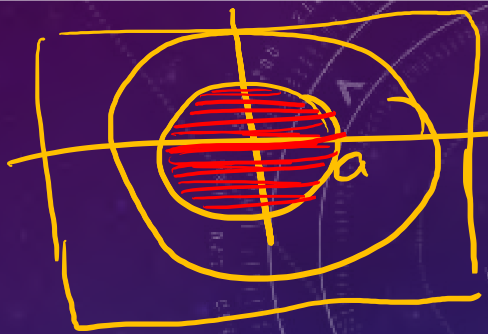
$$X(z) = \frac{z}{z-a}$$

for $x(n) = -a^{n}u(-n-1) \Rightarrow X(z) = \frac{z}{z-a}$ ✓

$\Sigma\, |a^{-1}z| < 1 \Rightarrow \frac{|z|}{a} < 1 \Rightarrow |z| < |a|$

$X(z) = \frac{z}{z-a}$, $|z| < |a|$ — ROC

$x(n) = -a^n u(-n-1)$



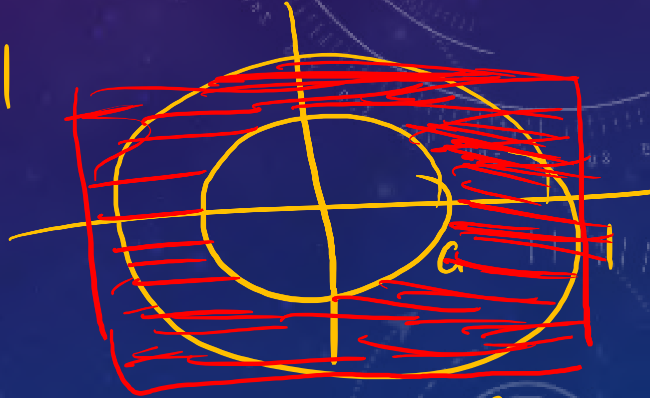
Left sided

$0 < a < 1$

$X(z) = \frac{z}{z-a}$, $|z| > |a|$

$x(n) = a^n u(n)$

$|z| > |a|$



Rt sided

* $x(n) = 7\left(\frac{1}{3}\right)^n u(n) - 6\left(\frac{1}{2}\right)^n u(n)$, $X(z) = ?$ ROC $= ?$

$$X(z) = \frac{7}{1-\frac{1}{3}z^{-1}} - \frac{6}{1-\frac{1}{2}z^{-1}}$$

$$= \frac{7 \cdot z}{z-1/3} - \frac{6 \cdot z}{z-1/2} = \frac{z(z-3/2)}{(z-1/3)(z-1/2)} = X(z)$$

$\left|\frac{1}{3}z^{-1}\right| < 1$

$\left|\frac{1}{3z}\right| < 1$

$\left|\frac{1}{3}\right| < |z|$

$|z| > 1/3$

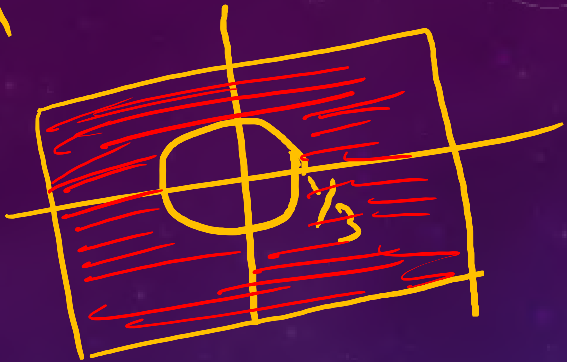
$\left|\frac{1}{2}z^{-1}\right| < 1$

$\left|\frac{1}{2z}\right| < 1$

$\left|\frac{1}{2}\right| < |z|$

$|z| > 1/2$

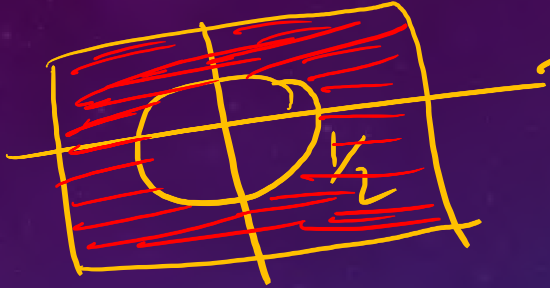
$\frac{7}{1-\frac{1}{3}z^{-1}}$



ROC $|z|>1/3$

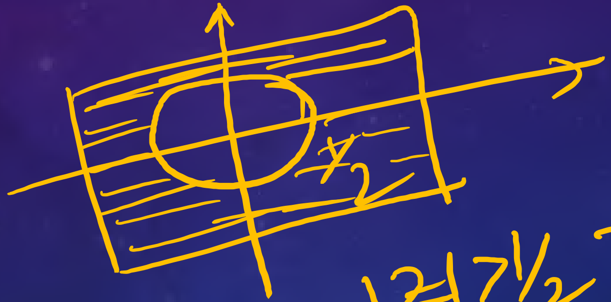
$\cap$

$\frac{6}{1-\frac{1}{2}z^{-1}}$



$|z|>1/2$

$\cap$



$|z|>1/2$ - Common ROC

$$x(n) = \{\underset{\uparrow}{1}, 2, 5, 7, 0, 1\} \qquad X(z) = ?$$

$$x(n) = \begin{cases} 1 & n=0 \\ 2 & n=1 \\ 5 & n=2 \\ 7 & n=3 \\ 0 & n=4 \\ 1 & n=5 \end{cases}$$

$$x(n) = \sum_{k} x(k)\,\delta(n-k)$$

$$x(n) = \delta(n) + 2\delta(n-1) + 5\delta(n-2) + 7\delta(n-3) + 0\,\delta(n-4) + 1\cdot\delta(n-5)$$

$$X(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + 0 + z^{-5}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + \cdots$$

$$\boxed{X(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}}$$

$\cancel{x(n)} = \delta(n) \Rightarrow X(z) = 1$ → ZT Impulse

$a^n u(n) = x(n) \Rightarrow a = 1 \Rightarrow x(n) = u(n)$ → ZT of step fn

$$\boxed{X(z) = \frac{z}{z-1}}$$

$$X(z) = \frac{z}{z-a}$$

$\cos w_0 n = \left(e^{jw_0 n} + e^{-jw_0 n}\right)$

$x(n) = \cos w_0 n\, u(n)$ —

$\cancel{x(n)} = \sin w_0 n\, u(n)$

$x(n) = a^n \cos w_0 n\, u(n)$

$$x(n) = \sin\omega_0 n \, u(n) \Rightarrow X(z) = ?$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\sin\omega_0 n = \frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}$$

$$X(z) = \frac{1}{2j} \sum_{n=0}^{\infty} \left(e^{j\omega_0 n} - e^{-j\omega_0 n}\right) z^{-n}$$

$$= \left(\sum_{n=0}^{\infty} e^{j\omega n} z^{-n}\right) - \sum_{n=0}^{\infty} e^{-j\omega_0 n} z^{-n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \left(e^{j\omega} z^{-1}\right)^n - \frac{1}{2j}\sum_{n=0}^{\infty} \left(e^{-j\omega} z^{-1}\right)^n$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

$$X(z) = \frac{1}{2j}\left(\frac{1}{1-e^{j\omega} z^{-1}} - \frac{1}{1-e^{-j\omega} z^{-1}}\right)$$

$$= \frac{1}{2j}\left(\frac{\cancel{1} - e^{-j\omega} z^{-1} - \cancel{1} + e^{j\omega} z^{-1}}{(1-e^{j\omega} z^{-1})(1-e^{-j\omega} z^{-1})}\right)$$

$$= \frac{1}{\cancel{2j}} \frac{\cancel{2j} \sin\omega \, z^{-1}}{1 - e^{-j\omega} z^{-1} - e^{j\omega} z^{-1} + z^{-2}}$$

$$X(z) = \frac{\sin\omega \, z^{-1}}{1 - z^{-1} \cos\omega + z^{-2}}$$

* $a^n \cos wn\, u(n) = x(n) \Rightarrow X(z) = ?$

$$X(z) = \frac{1}{2}\left(\frac{1}{1 - ae^{jw}z^{-1}} + \frac{1}{1 - ae^{-jw}z^{-1}}\right)$$

$$X(z) = \frac{z^2 - az\cos w}{z^2 - 2az\cos w + a^2}$$

$x(n) = \left(\frac{1}{3}\right)^n \sin\left(\frac{\pi}{4}n\right) u(n)$

$$= \frac{\frac{1}{3} z \sin \pi/4}{z^2 - 2\cdot\frac{1}{3} z \cos\pi/4 + (1/3)^2}$$

$$X(z) = \frac{3z}{9\sqrt{2}z^2 - 6z + \sqrt{2}}$$

* $\sin wn\, u(n) = x(n) \Rightarrow X(z) = \dfrac{z \sin w}{z^2 - 2z\cos w + 1}$

* $a^n \sin wn\, u(n) \Rightarrow X(z) = \dfrac{az \sin w}{z^2 - 2az\cos w + a^2}$

$a \to 1/3$, $w \to \pi/4$

Properties of Z-Transforms :-

$$x(n) \leftrightarrow X(z)$$

① Linearity Property :-

$$x(n) \longleftrightarrow X(z)$$

$x(n) \to x_1(n), \; x_2(n), \; x_k(n)$

$X(z) \to X_1(z), \; X_2(z), \; \ldots \; X_K(z)$

$\Rightarrow$ Superposition Principle

additivity   Scaling

If $x_1(n) \leftrightarrow X_1(z)$ & $x_2(n) \leftrightarrow X_2(z)$

then

$$a x_1(n) + b x_2(n) \leftrightarrow a X_1(z) + b X_2(z)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \qquad a x_1(n) + b x_2(n)$$

$$= \sum_n \left(a x_1(n) + b x_2(n)\right) z^{-n}$$

$$= \sum_n a x_1(n) z^{-n} + \sum_n b x_2(n) z^{-n}$$

$$= a X_1(z) + b X_2(z)$$

* Time Shifting Property

if $x(n) \longleftrightarrow X(z)$

$$x(n-n_0) \longleftrightarrow z^{-n_0} X(z)$$

time shifting (const)

$$X(z) = \sum_n x(n) z^{-n}$$

$$X(z) = \sum x(n-n_0) z^{-n}$$

Sol.

let $n-n_0 = l \Rightarrow n = l+n_0$

$$X(z) = \sum_l x(l)\, z^{-(l+n_0)}$$

$$\sum_l x(l)\, z^{-l} \cdot z^{-n_0}$$

$$= z^{-n_0} \cdot X(z)$$

$$\boxed{x(n-n_0) \longleftrightarrow z^{-n_0} X(z)}$$

$\delta(n) \longleftrightarrow 1$

$\delta(n-k) \longleftrightarrow z^{-k} \cdot 1$

* Scaling Property:

If $x(n) \longleftrightarrow X(z)$

$$a^n x(n) \longleftrightarrow X(z/a)$$

Sol:-

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= \sum_{n} a^n x(n) z^{-n}$$

$$= \sum_{n} x(n) \left(\frac{1}{z} a^{-1}\right)^{-n}$$

$$= \sum_{n} x(n) (z/a)^{-n}$$

$$= X(z/a)$$

$$a^n x(n) \longleftrightarrow X(z/a)$$

* Time Reversal Prop:

If $x(n) \longleftrightarrow X(z)$

$$x(-n) \longleftrightarrow X(z^{-1}) \Rightarrow X(1/z)$$

$$X(z) = \sum x(n) z^{-n} \qquad \sum x(-n) z^{-n}$$

let $-n = m \Rightarrow n = -m$

$$= \sum x(m) z^{m} = \sum x(m) (z^{-1})^{-m}$$

$$= X(z^{-1})$$

$$= X(1/z)$$

$$x(-n) \longleftrightarrow X\left(\frac{1}{z}\right)$$

$$x(n) = x_1(n) * x_2(n)$$

$$\text{Conv} = x_1(n) * x_2(n) = \sum_{k} x_1(k) x_2(n-k)$$

$\Leftarrow$ Conv. Sum

DT. signals

$$X(z) = \sum x(n) z^{-n}$$

$$= \sum (x_1(n) * x_2(n)) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\sum_{k=0}^{\infty} x_1(k) x_2(n-k) \right) z^{-n}$$

Conv

⊛ Convolution Property

If $x_1(n) \Leftrightarrow X_1(z)$ &

$x_2(n) \Leftrightarrow X_2(z)$ then

$x($

$$= \sum_{k=-\infty}^{\infty} x_1(k) \sum_{n=-\infty}^{\infty} x_2(n-k) z^{-n}$$

$$ZT[x_2(n-k)]$$

$$= \sum_{k} x_1(k) z^{-k} \cdot X_2(z)$$

$$= X_1(z)\, X_2(z)$$

$$x_1(n) * x_2(n) \longleftrightarrow X_1(z) \cdot X_2(z)$$

Convolution in time domain is the multiplication in Z-domain

$$x_2(n-k) \longleftrightarrow z^{-k} X_2(z)$$

Time shifting Prop

* Differentiation in z-domain

if $x(n) \leftrightarrow X(z)$ then

$$n x(n) \longleftrightarrow -z \frac{d}{dz} X(z)$$

Sol. $X(z) = \sum_n x(n) z^{-n}$

$$\frac{d}{dz} X(z) = \sum_n x(n) \frac{d}{dz}(z^{-n})$$

$$= \sum_n x(n) \, -n z^{-n-1}$$

$$-z^{-1} \sum (n x(n)) z^{-n} \cdot \cancel{z}$$

$$\frac{d}{dz} X(z) = -\frac{1}{z} \sum (n x(n)) z^{-n}$$

$$n x(n) \longleftrightarrow -z \frac{d}{dz}(X(z))$$

Initial & Final value theorem

Initial Value theorem

$x(n) = 0 \; n < 0$

for a causal seq $x(n)$

$$\lim_{n \to 0} x(n) = x(0) = \lim_{z \to \infty} X(z)$$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= x(0) + x(1) z^{-1} + x(2) z^{-2} + \cdots + x(n) z^{-n}$$

$$\lim_{z \to \infty} X(z) = \lim_{z \to \infty} \left(x(0) + \frac{x(1)}{z} + \frac{x(2)}{z^2} + \frac{x(3)}{z^3} + \cdots \right)$$

$$\boxed{x(0) = \lim_{z \to \infty} X(z)}$$

Final Value theorem

for a Causal seq. $x(n) = 0 \quad n<0$

$$\boxed{\lim_{n\to\infty} x(n) = x(\infty) = \lim_{z\to 1} (z-1)X(z)}$$

Sol:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \Rightarrow \text{Consider } Z[x(n) - x(n-1)]$$

$$X(z) - z^{-1}X(z) = \sum_{n=0}^{\infty} (x(n) - x(n-1)) z^{-n}$$

$$= \left(\cancel{x(0)} - \cancel{x(-1)} + \cancel{x(1)} - \cancel{x(0)} + \cancel{x(2)} - \cancel{x(1)} + x(3) - \cancel{x(2)} \dots + x(\infty) \right)$$

$$X(z)(1 - z^{-1}) = X(z)\left(1 - \frac{1}{z}\right) = X(z)\left(\frac{z-1}{z}\right)$$

$\lim_{z \to 1} X(z)(z-1) = \lim_{z \to 1} z.(\cdots \cdots x(\infty))$

$$\boxed{x(\infty) = \lim_{z \to 1} (z-1) X(z)}$$

(i) find the Z.T of $\delta(n-k)$

W.K.T $\Rightarrow$ $\delta(n) \xleftrightarrow{ZT} 1$

$\delta(n-k)$ = time shifting Prop. of ZT

shifting

$x(n-n_0) \leftrightarrow z^{-n_0} X(z)$

$x(n) = \delta(n) \Rightarrow Z\{\delta(n)\} = 1 = X(z)$

$\delta(n-k) = z^{-k} \cdot 1 = z^{-k}$

(ii) $n\left(\frac{1}{4}\right)^n u(n)$

$a^n u(n) \longleftrightarrow \frac{z}{z-a}$

$n a^n u(n) \longleftrightarrow -z \cdot \frac{d}{dz}(X(z))$

let $\left(\frac{1}{4}\right)^n u(n) = x(n) \Rightarrow X(z) = \frac{z}{z - 1/4}$

$n \cdot \left(\frac{1}{4}\right)^n u(n) = -z \cdot \frac{d}{dz}\left(\frac{z}{z-1/4}\right)$

$= -z\left(\frac{(z-1/4) \cdot 1 - z(1)}{(z-1/4)^2}\right) = \frac{1/4 \cdot z}{(z-1/4)^2} = \frac{a \cdot z}{(z-a)^2}$

Inverse Z. Transforms:

When $X(z)$ = Rational function = $\frac{A(z)}{B(z)}$

(i) Partial fractional Expansion Method

(ii) Long Division Method (Power Series Method)

(iii) Residue Method

(i) Partial fraction Expansion Method

Let $X(z) = \frac{A(z)}{B(z)}$

$$X(z) = \frac{A(z)}{B(z)} = \frac{A_1}{z-z_1} + \frac{A_2}{z-z_2} + \cdots \frac{A_k}{z-z_k}$$

Simple poles $z_1, z_2 \cdots z_k$

$IZT[X(z)] = Z^{-1}(X(z)) = Z^{-1}\left(\frac{A_1}{z-z_1}\right) + Z^{-1}\left(\frac{A_2}{z-z_2}\right) + \dots Z^{-1}\left(\frac{A_k}{z-z_k}\right)$

We know that $a^n u(n) \xleftrightarrow{ZT} \frac{z}{z-a}$ for $|z|>|a|$

ROC

(i) for $|z|>|a|$ i.e Right hand Sequence

$$Z^{-1}\left(\frac{z}{z-a}\right) = a^n u(n)$$

(ii) for $|z|<|a| \Rightarrow$ Left hand Sequence

$$Z^{-1}\left(\frac{z}{z-a}\right) = -a^n u(-n-1)$$

$z \cdot \frac{10}{z-z_1}$

$X(z)$ = given

$$\frac{X(z)}{z} = \frac{A_1}{z-a} = \frac{10}{z-a}$$

$$X(z) = 10\left(\frac{z}{z-a}\right) = 10\, a^n u(n)$$

find the IZT of $X(z) = \dfrac{z}{3z^2-4z+1} =$

$$\frac{X(z)}{z} = \frac{1}{3(z^2-4/3\,z+1/3)} \Rightarrow \qquad z^2-\frac{4}{3}z+\frac{1}{3} = (z-1)(z-1/3)$$

$$= \frac{1}{3(z-1)(z-1/3)} = \frac{A_1}{z-1} + \frac{A_2}{z-1/3}$$

$A_1 = 1/2, \quad A_2 = -1/2$

$$\frac{X(z)}{z} = \frac{1/2}{z-1} - \frac{1/2}{z-1/3} \Rightarrow X(z) = \frac{1}{2}\frac{z}{z-1} - \frac{1}{2}\frac{z}{z-1/3}$$

$$X(z) = \frac{z^2}{z} \times \frac{1}{3z^2-4z+1} = \frac{A}{z-1} + \frac{B}{z-1/3} = \frac{1}{2}\,\frac{1}{z-1} + \frac{1}{2}\,\frac{1}{z-1/3}$$

$$\frac{z}{z-1}$$

$$\frac{X(z)}{z} = \frac{1}{z(3z^2-4z+1)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-1/3}$$

$$X(z) = \frac{1}{2}\,\frac{z}{z-1} - \frac{1}{2}\,\frac{z}{z-1/3}$$

$\boxed{(i)}$

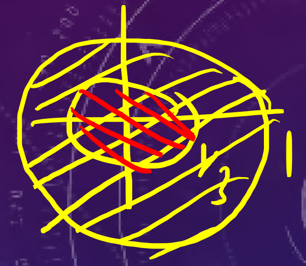
$$|z| > 1 \ \cap \ |z| > 1/3$$

$$|z| > 1$$

$$x(n) = \frac{1}{2}(1)^n u(n) - \frac{1}{2}\left(\frac{1}{3}\right)^n u(n)$$

$$X(z) = \frac{1}{2}\frac{z}{z-1} - \frac{1}{2}\frac{z}{z-1/3}$$

$|z| < 1 \quad \cap \quad |z| < 1/3$

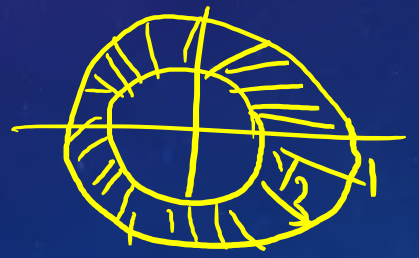


$|z| < 1/3$

(i)

$$x(n) = \frac{1}{2}\left(-(1)^n u(-n-1)\right) + \frac{1}{2}\left((1/3)^n u(-n-1)\right)$$

(ii) $\frac{1}{3} < |z| < 1$



$$x(n) = -\frac{1}{2}(1)^n u(-n-1) - \frac{1}{2}(1/3)^n u(n)$$

$$X(z) = \frac{z(z^2-4z+5)}{(z-1)(z-2)(z-3)} \Rightarrow \frac{X(z)}{z} = \frac{z^2-4z+5}{(z-1)(z-2)(z-3)}$$

$$\frac{X(z)}{z} = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{z-3}$$

$$A = \left.\frac{z^2-4z+5}{\cancel{(z-1)}(z-2)(z-3)} \times \cancel{(z-1)}\right|_{z=1} = \frac{2}{2} = 1$$

$$B = \left.\frac{z^2-4z+5}{(z-1)\cancel{(z-2)}(z-3)} \times \cancel{(z-2)}\right|_{z=2} = \frac{4-8+5}{1\times -1} = -1$$

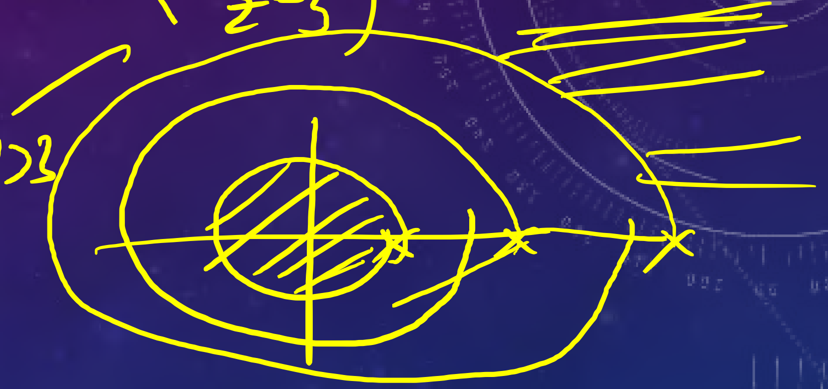
$$C = \left.\frac{z^2-4z+5}{(z-1)(z-2)\cancel{(z-3)}} \times \cancel{(z-3)}\right|_{z=3} = \frac{9-12+5}{2\times 1} = \frac{2}{2} = 1$$

$$X(z) = \frac{z}{z-1} - \frac{z}{z-2} + \frac{z}{z-3}$$

$$Z^{-1}(X(z)) = Z^{-1}\left(\frac{z}{z-1}\right) - Z^{-1}\left(\frac{z}{z-2}\right) + Z^{-1}\left(\frac{z}{z-3}\right)$$

(i) $|z|>3$ $\quad |z|>1 \cap |z|>2 \cap |z|>3$

$$\boxed{|z|>3}$$



$$(1)^n u(n) - (2)^n u(n) + (3)^n u(n)$$

(ii) $|z|<1$ $\quad x(n) = -(1)^n u(-n-1) + 2^n u(-n-1) - 3^n u(-n-1)$

$$= \frac{z}{z-1} - \frac{z}{z-2} + \frac{z}{z-3}$$

(iii) $1 \le |z| < 3$

$$x(n) = (1)^n u(n) + (2)^n u(-n-1) - (3)^n u(-n-1)$$

(*) $X(z) = \dfrac{(z+1)}{3z^2-4z+1} \Rightarrow \dfrac{z(z+1)}{z(3z^2-4z+1)}$

$$\frac{X(z)}{z} = \frac{z+1}{z(3z^2-4z+1)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-1/3}$$

$A = 1,\ B = 1,\ C = -2$

$$X(z) = \frac{z}{2} + \frac{z}{z-1} - \frac{z}{z-\frac{1}{3}}$$

$$x(n) = \delta(n) + (1)^n u(n) - \left(\frac{1}{3}\right)^n u(n) \quad - \quad \underline{\underline{|z| > 1}}$$

⊛ Long Division Method (Power Series Expansion Method)

$$X(z) = \frac{A(z)}{B(z)} \Rightarrow B(z)\,\big)\,A(z)\,\big($$

$2)\,7(3$
$\quad 6$

$$X(z) = \frac{1}{1-az^{-1}} \qquad \boxed{|z|>a}$$

$$1-az^{-1}\,\big)\,1\,\big(\,1+az^{-1}+a^2z^{-2}$$

$\cancel{1}-az^{-1}$
$+$

az^{-1}
$\cancel{az^{-1}}-a^2z^{-2}$
$+$

a^2z^{-2}
$a^2z^{-2}-a^3z^{-3}$

$$1+az^{-1}+a^2z^{-2}+a^3z^{-3}+\ldots$$

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n}$$

$$x(0)+x(1)z^{-1}+x(2)z^{-2}+\ldots$$

$$(1, a, a^2, a^3 \ldots a^n)\,x(n)z^{-n} \Rightarrow x(n) = a^n u(n)$$

$x(n) = a^n u(n)$

$\boxed{|z| > a}$

$$\frac{1}{1-az^{-1}} = \frac{z}{z-a} = z-a \,\big|\, \cancel{z} \,\big|\, 1 + az^{-1} + a^2 z^{-2}$$

$\cancel{z} - a$

$- \quad +$

$\cancel{a}$

$\cancel{a} - a^2 z^{-1}$

$- \quad +$

$\cancel{a^2 z^{-1}}$

$\cancel{a^2 z^{-1}} - a^3 z^{-2}$

$- \quad +$

$= 1 + az^{-1} + a^2 z^{-2} + \ldots$

$= \quad a^0 = 1$

$a^1 = a$

$a^2 = a^2$

$a^n = a^n$

$$\boxed{x(n) = a^n u(n)}$$

for Right side seq ($|z|>|a|$)

Arrange $X(z)$ in decreasing Powers of z (or) increasing Powers of z^{-1}

for left sided seq $|z|<a$

Arrange $X(z)$ in increasing Powers of z (or) decreasing Powers of z^{-1}

$$X(z) = \frac{1}{1-az^{-1}} \qquad |z| < |a| \quad \text{— left hand seq}$$

$$-az^{-1}+1 \,\big)\, 1 \,\big(\, -a^{-1}z - a^{-2}z^{2} - a^{-3}z^{3} - a^{-4}z^{4} - a^{-5}z^{5} - \cdots$$

$$\cancel{1} - a^{-1}z$$

$$- \quad +$$

$$a^{-1}\cancel{z}$$

$$\cancel{a^{-1}z} - a^{-2}z^{2}$$

$$- \quad +$$

$$a^{-2}z^{2}$$

$$-a^{-1} - a^{-2} - a^{-3} \ldots a^{-n}$$

$$\boxed{x(n) = -a^{n}u(-n-1)}$$

$$X(z) = \frac{z}{z-a} \Rightarrow -a+z \,|\, z \,|\, -a^{-1}z$$

$$X(z) = \frac{2}{2-3z^{-1}+z^{-2}} = \frac{2}{2-\frac{3}{z}+\frac{1}{z^2}} = \frac{2z^2}{2z^2-z+1}$$

Inverse ZT using Contour Integral (or) Residue theorem

$$IZT(X(z)) = x(n) = \frac{1}{2\pi j}\oint X(z)\, z^{n-1}\, dz$$

$$= \text{Sum of Residues of } X(z)\, z^{n-1} \text{ at Pole } z = z_i$$

$$x(n) = R_1 + R_2 + R_3$$

Thank you