# Unit - 2

Angle modulation: Basic concepts of phase modulation, FM: Single tone FM, spectrum Analysis of sinusoidal FM using Bessel functions, NBFM, WBFM, constant average power, Transmission B/W of FM wave. Generation of FM signal - Armstrong method. Detection of FM Balanced slope detector, PLL, comparision of FM and AM. Concept of pre-emphasis & De-emphasis.

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**Introduction:** In contrast to linear modulation, Exponential modulation is a non-linear process. The transmission bandwidth of exponential modulation is usually much greater than twice the message Bandwidth. In exponential modulation, the angle of the carrier is varied accordance with the baseband signal. For this reason, exponential modulation is also referred to as angle modulation.

There are 2 forms of angle modulation which may be distinguished

↳ Frequency Modulation (FM)

↳ phase modulation (PM)

**Mathematical representation of frequency and phase modulation**

An angle modulated sinusoidal carrier can be written in the form

$$s(t) = A_c \cos(\theta_i(t)) \rightarrow (1)$$

$$(2\pi f_c t + \theta_0)$$

$\theta_i(t)$ = Angle of the modulated sinusoidal carrier

In the above equation, representation of complete oscillation corresponds to a change of $\theta_i(t)$ by $2\pi$ radians. Assuming $\theta_i(t)$ to increase monotonically with time, the average frequency in Hz over the interval $t$ and $t+\Delta t$ can be written as:

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t} \rightarrow (2)$$

We may therefore define the instantaneous frequency of the angle modulated wave $s(t)$ by

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}$$

$$= \frac{1}{2\pi} \cdot \frac{d\,\theta_i(t)}{dt} \rightarrow (3)$$

Thus angle modulated wave $s(t)$ can be considered as a rotating phasor of length $A_c$ and angle $\theta_i(t)$. The instantaneous angular velocity is given by

$$\omega_i = \frac{d\,\theta_i(t)}{dt} \rightarrow (4)$$

where $\theta_i(t) = 2\pi f_c t + \theta_0$

Now, $e_c(t) = A_c \cos(2\pi f_c t + \theta_0)$

We will

$$P_{S1} = P_{S2} = (0.099)^2 \times 10\,kW$$
$$= 98\,W$$

### 6.5 GENERATION OF FREQUENCY MODULATED WAVES

There are basically two methods of generating frequency modulated signals, namely *indirect method* and *direct method*. In the *indirect method* ...

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We will discuss two commonly used method in which $\theta_i(t)$ may be varied in accordance with the modulating signal.

1. Phase modulation (PM):

Phase modulation is a form of angle modulation in which the angle $\theta_i(t)$ is made to vary linearly with baseband signal $e_m(t)$

Assuming initial phase angle $\theta_0 = 0$, we get

$$\theta_i(t) = \underbrace{2\pi f_c t}_{\text{Angle of unmodulated carrier having frequency } f_c} + \underbrace{K_p}_{\text{phase sensitivity}} e_m(t) \rightarrow (5)$$

Phase modulated wave is represented as.

$$S(t) = \cos\left(2\pi f_c t + K_p e_m(t)\right)$$

2. Frequency modulation (FM):

Frequency modulation is a form of angle modulation in which the instantaneous frequency $f_i(t)$ of carrier is varied linearly with the baseband signal $e_m(t)$. Thus the instantaneous frequency of a FM signal can be represented as:

$$f_i(t) = \underbrace{f_c}_{\text{Unmodulated}} + \underbrace{k_f}_{\text{Frequency sensitivity}} e_m(t) \rightarrow (6)$$

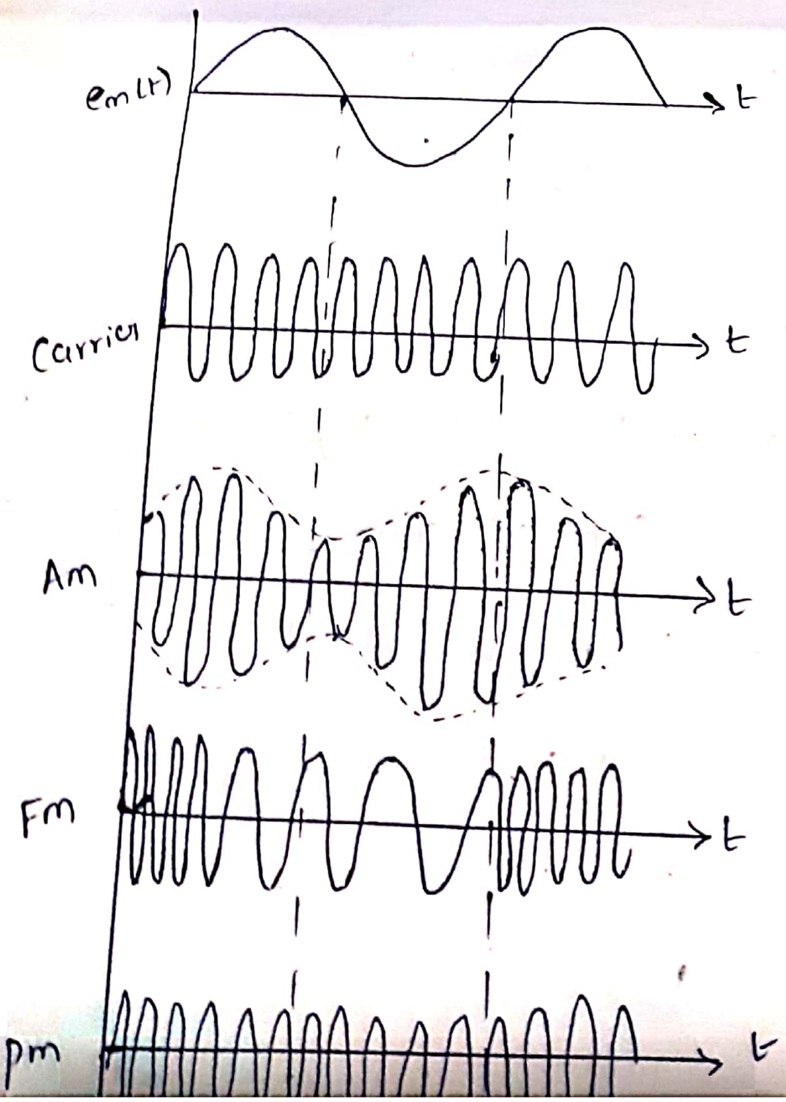
using Equation (4) & (6)

$$\frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + k_f\, e_m(t)$$

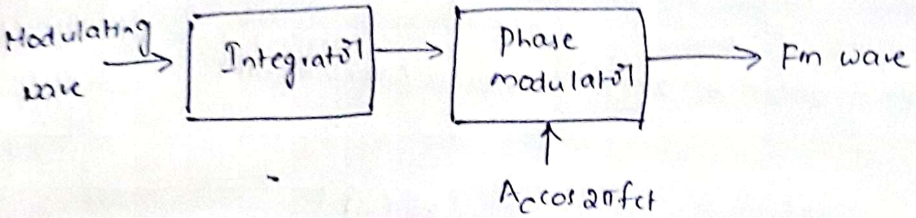
$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t e_m(t)\, dt \rightarrow (7)$$

In the above representation the angle of unmodulated carrier has been assumed to be zero at $t=0$. The frequency modulated wave can thus be represented using the generalised form of angle modulated wave as.

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t e_m(t)\, dt\right) \rightarrow (8)$$



## Generation of FM Using PM:

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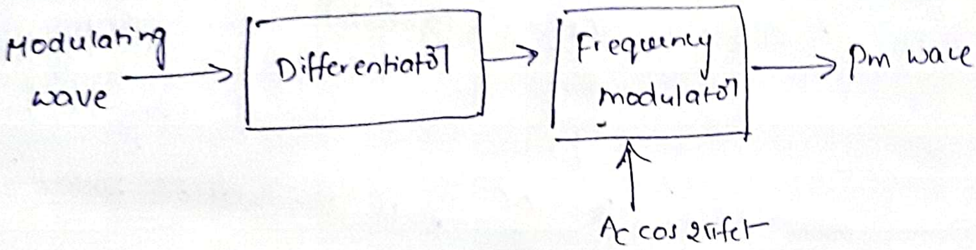



## Single tone FM:

Modulating signal is considered as a single tone (single frequency) sinusoidal signal given by

$$e_m(t) = A_m \cos(2\pi f_m t)$$

Instantaneous frequency of FM can be written as

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$$

$$= f_c + \Delta f \cos(2\pi f_m t)$$

$$\Delta f = k_f A_m$$

Maximum value of instantaneous frequency

$$f_i|_{max} = f_c + \Delta f$$

$$f_i|_{min} = f_c - \Delta f$$

$\Delta f$ = frequency deviation

The instantaneous angle of FM wave

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t)\, dt$$

$$= 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \underbrace{\frac{\Delta f}{f_m}} \sin(2\pi f_m t)$$

$$\downarrow$$

$\beta = \frac{\Delta f}{f_m}$ is modulation index

$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

$$\theta_i\big|_{max} = 2\pi f_c t + \beta$$

$$\theta_i\big|_{min} = 2\pi f_c t - \beta$$

final single tone modulated wave is

$$s(t) = A_c \cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right)$$

Narrow band Fm (NBFm) for which $\beta$ is small compared to 1 radian

Wide band Fm (WBFm) $\beta > 1$ radian

As the names of 2 types Fm indicate, NBFm has got a very narrow bandwidth which is equal to twice the message bw while WBFm has a much larger value which is ideally infinite.

NBFm

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

$$= A_c [\cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t)) - \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))$$

Assuming the modulation index which is basically the phase deviation of Fm, to be compared to 1 radian ($\beta << 1$ radian), we may use the following approximations

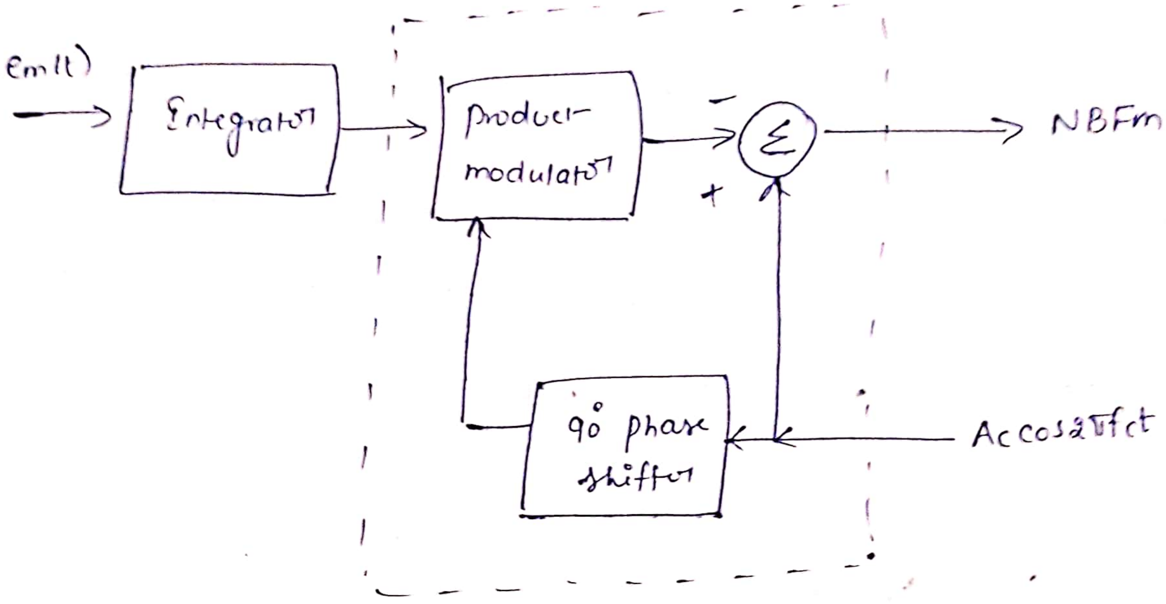
$$\cos(\beta \sin(2\pi f_m t)) \approx 1$$

$$\sin(\beta \sin(2\pi f_m t)) \approx \beta \sin(2\pi f_m t)$$

$$s(t) = A_c \cos 2\pi f_c t - \beta \overset{A_c}{.} \sin(2\pi f_c t) \sin(2\pi f_m t) \rightarrow (1)$$

From the above expression, we can setup the arrangement for generating NBFm.

6.13 Block diagram of indirect method of generating FM wave.




Narrow band phase modulator

The NBFm obtained by using the above method is not perfect and has some distortion. It also differs from ideal fm in a no. of ways

1) The envelope of NBFm contains residual amplitude modulation and therefore varies with time. This can be verified by equation (1) as

$$s(t) = e(t)\cos(2\pi f_c t + \phi) \rightarrow (2)$$

where $$e(t) = \sqrt{A_c^2 + \beta^2 A_c^2 \sin^2(2\pi f_m t)} \rightarrow (3)$$

and $$\phi = \tan^{-1}\left(\frac{\beta A_c \sin(2\pi f_m t)}{A_c}\right)$$

$$= \tan^{-1}(\beta \sin(2\pi f_m t))$$

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## Wide band FM (WBFM):

For larger value of $\beta$ compared to 1 radian, the FM signal becomes wideband. Ideally the bandwidth is infinite in such cases.

For single tone sinusoidal modulation, the FM wave can be expressed as

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

The WBFM signal can be expressed as

$$s(t) = A_c \cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t)) - A_c \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))$$

$$= 9.8\,\text{kW}$$

and the power in each side frequency is

$$P_{S1} = P_{S2} = (0.099)^2 \times 10\,\text{kW}$$
$$= 98\,\text{W}$$

## 6.5 GENERATION OF FREQUENCY MODULATED WAVES

There are basically two methods of generating frequency modulated signals, namely, *indirect method* and *direct method*.In the *indirect method*, a narrow-band FM is first produced by modulating the carrier by the baseband signal.The resulting narrow-band signal is then frequency multiplied to have the desired frequency deviation.In the *direct method*, on the other hand, the carrier frequency is directly varied in accordance with the input baseband signal by means of a voltage controlled oscillator (VCO).In this section we shall discuss the above two methods for generating FM signals.

### 6.5.1 Indirect method of generating FM (Armstrong method)

The indirect method of generating WBFM was first proposed by *E.H.Armstrong* and the method is thus popularly known as *Armstrong method*.A simple block diagram for generating FM signal by this method is shown in Fig.6.13.A NBFM is generated by integrating the baseband signal $e_m(t)$ and using it to phase modulate the carrier generated by a crystal oscillator.In order to minimise the

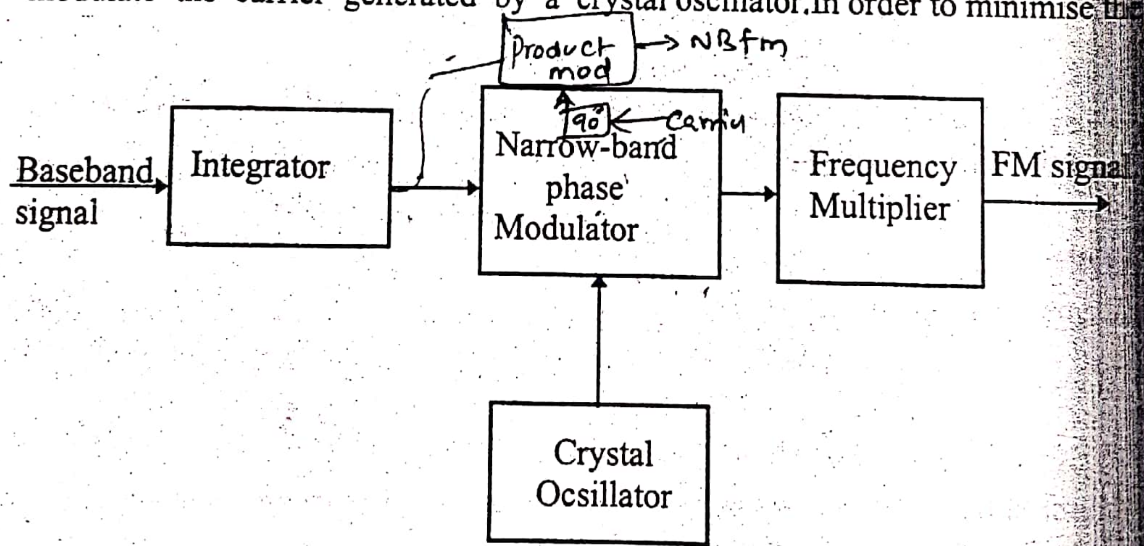


**Fig.6.13** Block diagram of indirect method of generating FM wave.

distortion in the phase modulation, the maximum *phase deviation* or the index of modulation $\beta$ is kept low.This results in NBFM.The NBFM is then converted to WBFM by using frequency multipliers.It should be noted that the carrier frequency of the resulting WBFM changes to a higher value during frequency multiplication.A proper design is necessary for frequency multiplication so that desired values of deviation and the carrier frequency can be achieved simulteneously through frequency multiplication.A typical arrangement is discussed in section 6.5.2.

Let $s_1(t)$ denote the output of the phase modulator which is the NBFM.Thus,

$$s_1(t) = A_1\left(\cos(2\pi f_1 t) + 2\pi k_1 \int_0^t e_m(t)dt\right) \tag{6.52}$$

where $f_1$ is the frequency of the crystal oscillator and $k_1$ is a constant.For sinusoidal modulating signal

$$s_1(t) = A_1\left(\cos(2\pi f_1 t) + \beta_1 \sin(2\pi f_m t)\right) \tag{6.53}$$

where $\beta_1$ is the index of modulation of NBFM, which is kept less than 0.3 radian to keep the distortion minimum.

The phase modulator output is next multiplied *n* times in frequency by using frequency multiplier, producing the desired WBFM, given by

$$s(t) = A_c\left(\cos\left(2\pi n f_1 t + 2\pi n k_1 \int_0^t e_m(t)dt\right)\right) \tag{6.54}$$

Writing $nf_1 = f_c$ and $nk_1 = k_f$ for sinusoidal modulating signal, we get the desired signal as

$$s(t) = A_c\left(\cos\left(2\pi f_c t + 2\pi k_f \int_0^t e_m(t)dt\right)\right)$$

that is,
$$s(t) = A_c\left(\cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right)\right) \tag{6.55}$$

where $\beta = n\beta_1$.

### 6.5.2 Indirect generation of WBFM for practical use

In this section we describe a practical method of generating FM signal using indirect method.A simplified diagram of a commercial FM generation system using Armstrong method is shown in Fig.6.14.

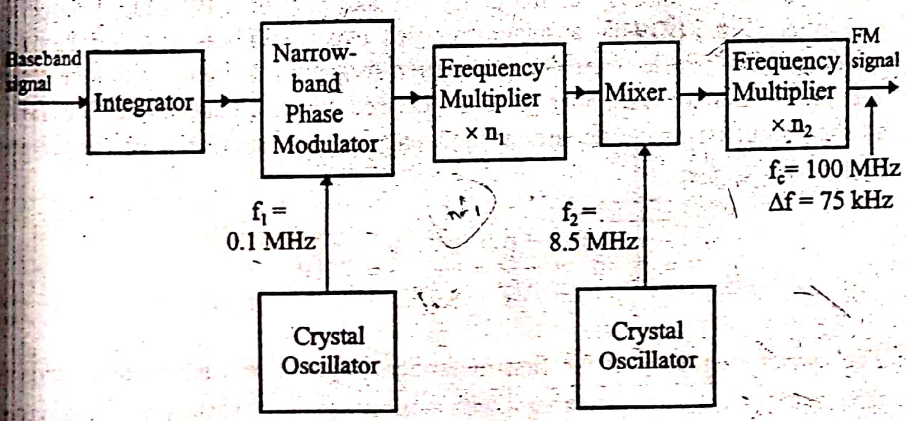


**Fig.6.14 Block diagram of WBFM modulation for practical use (Armstrong method).**

For commercial use it is required to transmit audio signal containing frequencies in the range 50 Hz to 15 kHz and the value of $\Delta f = 75\,\text{kHz}$. Let the final carrier frequency of the FM required is $f_c = 100\,\text{MHz}$. We begin with NBFM with a carrier frequency $f_{c1} = 100\,\text{kHz}$ generated by a crystal oscillator. In order to limit the harmonic distortion produced by the *narrow-band phase modulator*, we restrict the modulation index $\beta$, to a maximum of 0.3 radians.

Let us assume $\beta_1 = 0.2$ radian. The lowest modulation frequency $100\,\text{Hz}$ produces a deviation $\Delta f_1 = 0.2 \times 50\,\text{Hz} = 10\,\text{Hz}$ at the narrow-band phase modulator output while the largest modulation frequency $15\,\text{kHz}$ produces a frequency deviation of

$$\Delta f_2 = 0.2 \times 15\,\text{kHz} = 3\,\text{kHz}$$

The lowest modulation frequency is therefore, of immediate concern. We select the value of $\Delta f_1 = 10\,\text{Hz}$ so that at the highest modulating frequency $\beta$ becomes even less.

In order to produce a frequency deviation of $\Delta f = 75\,\text{kHz}$ at the output, a frequency multiplication is required. For example, $\Delta f_1 = 10\,\text{Hz}$ and the required deviation is $\Delta f = 75\,\text{kHz}$. We therefore, require a total frequency multiplication by a factor

$$n = \frac{75000}{10} = 7500.$$

A straightforward frequency multiplication equal to this value will lead to a very high value of carrier frequency than the desired 100 MHz. In order to achieve the desired deviation and carrier frequency we take help of a two-stage frequency multiplier (Fig.6.14). This arrangement uses two multipliers and a mixer. The mixer enables one to translate the carrier frequency suitably without altering $\Delta f$. The final stage multiplier gives the desired carrier frequency and deviation as shown. Let $n_1$ and $n_2$ are the frequency multiplication factors for the two multipliers, so that

$$n = n_1 \cdot n_2 = \frac{\Delta f}{\Delta f_1} = \frac{75000}{10} = 7500 \tag{6.56}$$

The carrier frequency at the first multiplier output is translated downwards to frequency $(f_2 - nf_1)$ by mixing it with a carrier wave of frequency $f_2$, which is supplied by another oscillator. The carrier frequency at the input of the second frequency of the FM signal generated by direct method.A typical feedback arrangement for frequency stabilisation in direct method is shown in Fig.6.17.The output of the FM generator is applied to a mixer together with the output from a *crystal controlled oscillator* and the difference term is extracted.The mixer output is then applied to *frequency discriminator* and then filtered using a low-pass filter.A frequency discriminator (to be discussed in next section) is a device whose output voltage has an instantaneous amplitude proportional to the instantaneous frequency of the input FM signal.The LPF output is only zero when the generated FM has the correct frequency (same as the reference frequency of the crystal oscillator).However, deviation of the frequency of the generated FM with respect to reference signal will cause the frequency discriminator-LPF combination to develop a d.c. voltage with a polarity determined by the sense of the frequency drift of the modulator.This d.c. volatge after suitable amplification is fed back to the VCO of the frequency modulator in such a way as to modify its frequency in a direction that tends to restore the carrier frequency to its required value.

## 6.6 DEMODULATION OF FM WAVES

*Frequency demodulation* is the process that enables one to extract the original modulating signal (baseband signal) from the frequency modulated wave.This can be achieved by the system which has a transfer chracteristic just inverse of VCO.In other words, a frequency demodulator produces an output voltage whose instantaneous amplitude is directly proportional to the instantaneous frequency of the input FM signal.There are two basic methods for demodulating the FM waves.One method is based on the frequency discrimination and the other is based on *phase locked loops*.The two methods are discussed in the following section.

### 6.6.1 Frequency discriminator

A *frequency discriminator* is an FM demodulator which must produce an output voltage linearly dependent on input frequency.A simple tuned circuit (tuned slightly above or below the resonant frequency) has just the property over a limited range.This type of detection is also known as *slope detection* and is illustrated in Fig.6.18.This can be understood as follows.

Suppose $s(t)$ is a frequency modulated signal with $f_c$ less than the resonant frequency, $f_0$ of the tuned circuit.As the instantaneous frequency $f_i(t)$ of the FM signal swings above or below $f_c$, the amplitude ratio of the tuned circuit converts the frequency variation to an amplitude variation on the top of the FM signal resulting a waveform $s_c(t)$ which is basically a hybrid modulated wave (see Fig.6.18d.).Extracting only the amplitude variation with an envelope detector (including a d.c. block) produces $s_d(t)$, the demodulated signal *i.e.* the baseband signal (Fig.6.18e).

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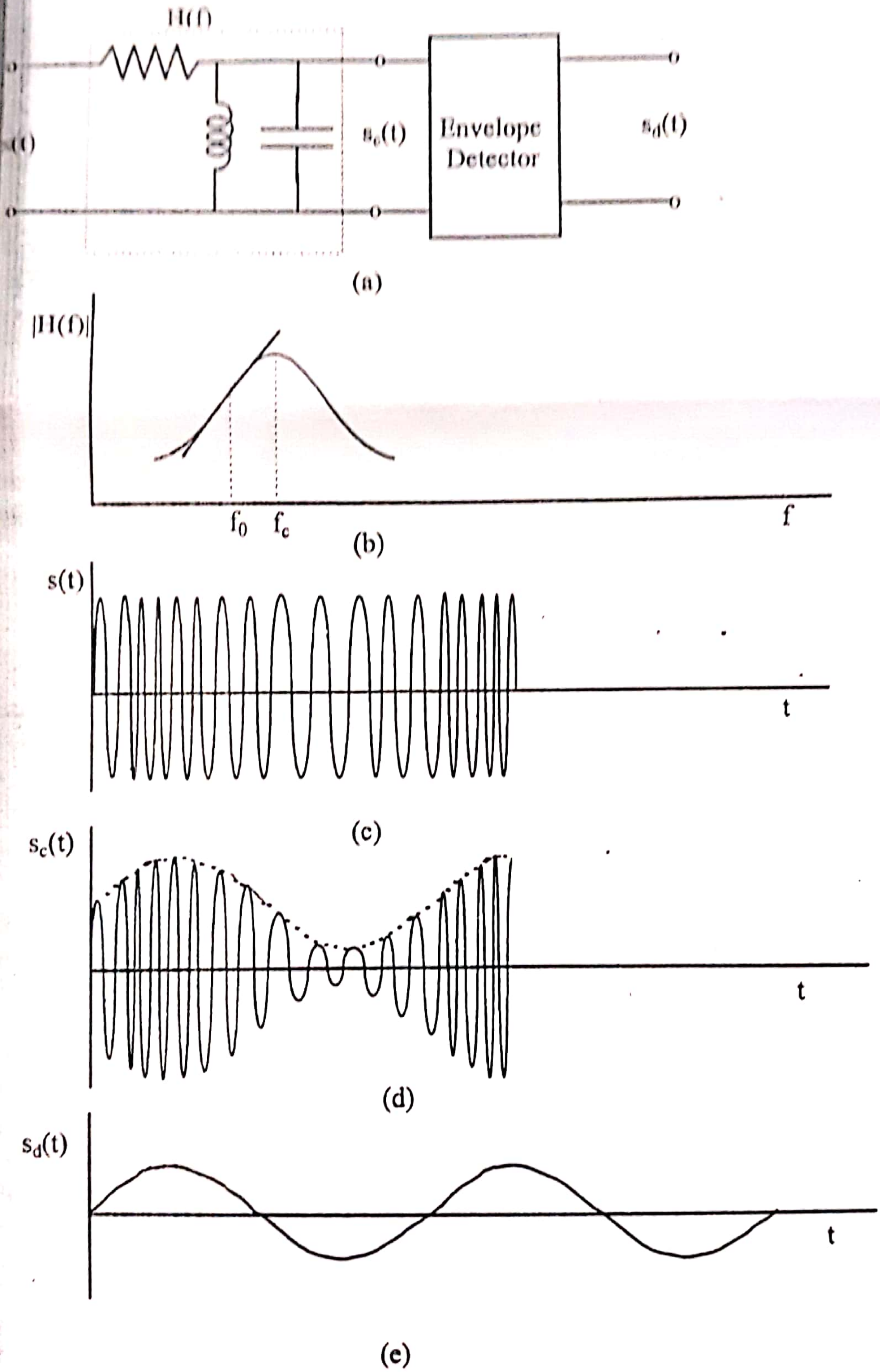


**Fig.6.18 FM slope detector and waveform.**