Code No: 137BX

R16

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech IV Year I Semester Examinations, December - 2019
DIGITAL SIGNAL PROCESSING
(Electrical and Electronics Engineering)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Check the stability of the system defined by $y(n) = a\,x(n-7)$. [2]

b) Find the ROC and Z-Transform of the causal sequence $x(n) = \{1, 0, -2, 3, 5, 4\}$ [3]

c) List the difference between Linear and circular convolution. [2]

d) Express the relation between DFT and Z-Transform. [3]

e) Distinguish between IIR and FIR filters. [2]

f) Mention the important features of IIR filters. [3]

g) Write the steps involved in FIR filter design. [2]

h) What is Gibbs phenomenon? [3]

i) Give any applications of multirate DSP system. [2]

j) Show that the decimator is a time variant system. [3]

PART – B

(50 Marks)

2. Determine if the systems described by the following equations are a) Causal or non causal b) Linear or Non-linear.
   i) $y(n) = x(n) + \frac{1}{x(n-1)}$ ✗, ✗
   ii) $y(n) = x(n^2)$ L, ✗
   iii) $y(n) = n\,x(n)$ L, C [10]

OR

3. Realize the system given by difference equation
   $y(n) = -0.1y(n-1)+0.71\,y(n-2)+0.7x(n)-0.252\,x(n-2)$
   in Cascade form and Parallel form. [10]

4. An 8 – point sequence is given by x(n) ={2,1,2,1,3,1,2,1}. Compute 8 –point DFT of x(n) by radix -2 DIT FFT . Also sketch the magnitude and phase spectrum. [10]

OR

5. Perform the linear convolution of the following sequence by
   a) Overlap add method and b) Overlap save method
   x(n) = {1,-1,2,-2,3,-3,4,-4}; h(n) = {-1,1} [10]

6. If the specifications of analog low pass filter are to have a *1dB* attenuation at cut off frequency of *1KHz* and maximum stop band ripple $\delta_s = 0.01$ for $|f| > 5\ KHz$, determine required filter order
a) Butterworth
b) Type-I chebyshev
c) Type-II chebyshev. [10]

OR

7. For the given analog transfer function $H(s) = \frac{2}{(s+2)(S+3)}$ find H(Z) by using the bilinear transformation. [10]

8. Express the different window functions used in FIR filter design and sketch the plots in time domain. [10]

OR

9. Design a low pass filter using a rectangular window technique with pass band gain of unity, cutoff frequency of 1000Hz and working at a sampling frequency of 5Khz .The length of Impulse response should be 7. [10]

10.a) Derive the expression for the spectrum of the output signal of a decimator with an example.
b) Explain process of up-sampling with an example. [5+5]

OR

11. Explain process of Measurement of Coefficient Quantization Effects through Pole-Zero Movement. [10]

---ooOoo---

137BX.

Digital Signal Processing

$y(n) = ax(n-7)$

If $x(n) < M$ then

$y(n) = |a x(n-7)|$

$\le a|(x(n-7)|$

$< a|(M - ax(n-7)|$

Hence the system is stable

b $x(n) = \{1, 0, -2, 3, 5, 4\}$

$X(z) = \sum_{n=-\infty}^{\infty} X(n) z^{-n}$

$= 1 + 2z^{-2} + 3z^{-3} + 5z^{-4} + 4z^{-5}$

$x(z) =$

z-plane, Im, ROC, Re(z)

c The convolution of discrete-time signal is known as linear convolution. The $x(n)$ be the input to an LTI system and $y(n)$ be the output of the system. The output $y(n)$ can be obtained by convolving to Impulse response $h(n)$ and the input signal $\sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$.

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$ (or) $y(n) = \sum_{k=-\infty}^{\infty}$

d DFT and Z-transform

$x(n)$ is $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ with a ROC.

$X(z)$ is sampled at N equally spaced points on the unit circle $z_k = e^{j2\pi k/N}$ we obtain

$z_k = e^{j2\pi k/N}, k = 0, 1, 2 \ldots N-1$

$X(k) = X(z)|_{z=e^{j2\pi k/N}} \quad k = 0, 1 \ldots N-1$

$= \sum_{n=-\infty}^{\infty} (x)(n)\, e^{-j2\pi nk/N}$

If the sequence $x(n)$ has a finite duration of length N, then

$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}$

$= \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi nk/N}\right] z^{-n}$

$= \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} e^{j2\pi k/N} z^{-1}\right]^{n}$

$= \frac{1}{N}\left[X(k) \sum_{k=0}^{N-1}\right.$

$= \frac{1-z^{-N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1-e^{-j2\pi k/N} z^{-1}}$

e

FIR Filter	IIR Filter
1) The impulse response of this filter is restricted to finite number of samples	The impulse response of IIR filter extends over an infinite duration
2) FIR filter can have precisely linear phase	These filters do not have linear phase

closed form design equations do not exist. Greater flexibility to control the shape of their magnitude response	... frequency filter can be designed using closed form design formula. Less flexibility specially for obtaining non standard frequency response

f 1) The Impulse response of this filter extends over an infinite duration
2) These filters do not have linear phase
3) designed using closed-form design formulas
4) Less flexibility for obtaining non-standard frequency responses

g 1) For the desired frequency response $H_d(e^{j\omega})$.

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\,d\omega.$$

2) Truncate $h_d(n)$ at $n = \pm\frac{(N-1)}{2}$ to get the finite duration sequence
3) Find $H(z)$ using the equation

$$H(z) = z^{-(N-1)/2}\left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)(z^{n} + z^{-n})\right].$$

h Gibbs phenomenon:

One possible way of finding an FIR filter that approximates $H(e^{j\omega})$ would be to truncate the finite fourier series at $n = \pm\frac{(N-1)}{2}$. Abrupt function of the series will lead to oscillation both in passband and in stopband. This phenomenon is known as Gibbs phenomenon.

i 1) In transmultiplexers
2) Narrow band filtering for fetal ECG and EEG
3) In video, PAL and NTSC run at different sampling rates.
4) Speech processing to reduce the storage space.

j

$$x(n) \longrightarrow \boxed{h(n)} \xrightarrow{v(n)} \boxed{\downarrow M} \longrightarrow y(n).$$

A decimator.

$$v(n) = x(n) * h(n)$$
$$= \sum_{k=0}^{N-1} h(k)\,x(n-k)$$

The output of $v(n)$ is $y(n) = v(nM)$

$$= \sum_{k=0}^{N-1} h(k)\,x(nM-k)$$

(iii) $y(n) = n\,x(n)$.

$y(n) = T(x(n)) = n(x(n))$

If we delay the output by k units in time

$y(n-k) = (n-k)\,x(n-k)$. $y(n,k) \neq y(n-k)$

The system is time invariant.

The system linear and causal

(iv) $y(n) = x(n^2)$

If $x(n)$ is ... then $|y(n)| = x(n^2)$ is also bounded

Hence the system is stable

The system is linear and causal

(1) $y(n) = x(n) + \frac{1}{x(n-1)}$

The input is delay by k units in time

$y(n,k) = T\left(\frac{1}{x(n-1)}\right) = x\left(\frac{1}{x(n-1)}\right) + n\,x(n+1-k)$.

If the output is delayed by k units in the

$y(n-k) = x(n) + \frac{1}{x(n-1)}$

The system $y(n,k) \neq y(n-k)$ the system is invariant

the system nonlinear and causal

a. $y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252$...

The system function of the difference equation is

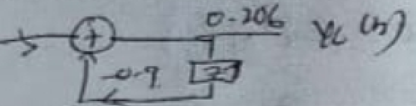
$$H(z) = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

$$= 0.35 + \frac{0.35 - 0.035 z^{-1}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

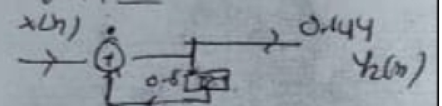
$$= 0.35 + \frac{0.206}{1 + 0.9 z^{-1}} + \frac{0.144}{1 - 0.8 z^{-1}}$$

$$= C + H_1(z) + H_2(z)$$

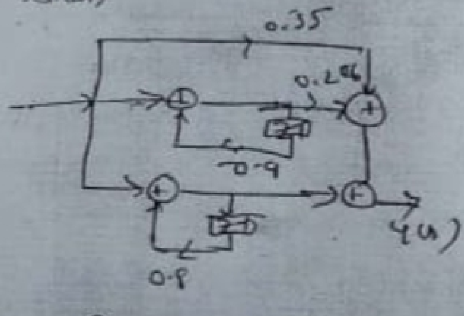
$H_1(z)$ can be realized in direct form II as



$H_2(z)$ can be realized in direct form II



Now the realization of $H(z)$ is shown



Parallel form realization

$x(n) = \{2, 1, 2, 1, 3, 1, 2, 1\}$

DIT algorithm:

$W_8^0 = 1,\ W_8^1 = 0.707 - j0.707,\ W_8^2 = -j,\ W_8^3 = -0.707 - j0.707$

Input

$x(0) = 2$

$x(4) = 3$

$x(2) = 2$

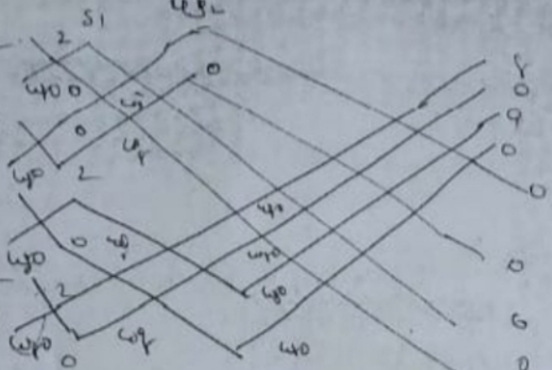
$x(6) = 1$

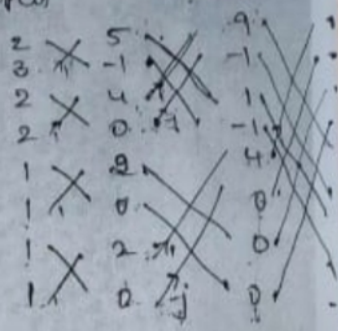
$x(1) = 3$

$x(5) = 1$

$x(3) = 2$

$x(7) = 1$





$x(k) = (8, 0, 0, 0, 0, 0, 0, 0)$

$X(k) = \{13, -1, 1, -1, 5, -1, 1, -1\}$

DIF algorithm

$x(0) = 2$

$x(1) = 1$

$x(2) = 2$

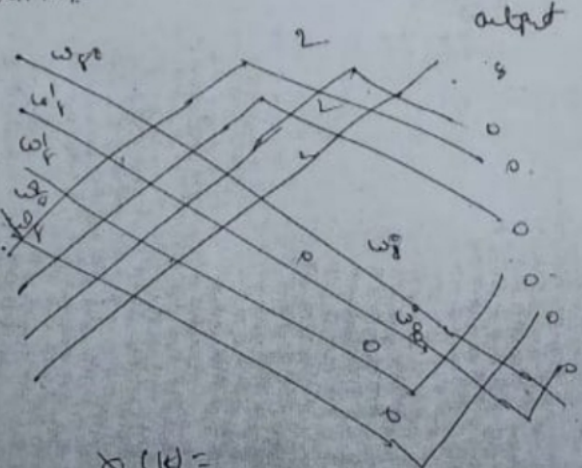
$x(3) = 1$

$x(4) = 3$

$x(5) = 1$

$x(6) = 2$

$x(7) = 1$



$x(k) =$

5) $x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$
$h(n) = \{-1, 1\}$

(a) overlap add method.
b) overlap save method.

(b) overlap save method

The Input sequence can be divided into blocks of data as

$x_1(n) = \{0, \underbrace{1, -1, 2}_{3\,data}\}$ m − 1 zero appended

$x_2(n) = \{\underbrace{2, \underbrace{-2, 3, -3}_{3\,new\,data}}_{}\}$ m − 1 = 1 data from prev block

$x_3(n) = \{-3, 4, -4\}$

Given $h(n) = \{-1, 1\}$ Appending two zero to the sequence we obt

$h(n) = \{-1, 1, 0, 0\}$

$y_1(n) = x_1(n) \,④\, h(n)$

$$= \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \\ -3 \end{bmatrix}$$

$y_2(n) = x_2(n) \,④\, h(n)$

$$= \begin{pmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 3 \\ -3 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ -5 \\ 6 \end{pmatrix}$$

$\boxed{2\ -1\ 2\ -3}$ discard

$\boxed{-5\ 4\ -5\ 6}$ discard

$y(n) = \{-1, 2, -3, 4, -5, 6\}$

(a) Overlap add method.

$x_1(n) = \{1, -1, 2\}$ m − 1 = 1 zero adder

$x_2(n) = \{2, -2, 3, 0\}$

$y_1(n) = x_1(n) \,④\, h(n) = \{1, -1, 2, -2\}$

$y_2(n) = x_2(n) \,④\, h(n) = \{-2, 3, -3, 0\}$.

$\boxed{1, -1, 2, -2}$ swell

m − 1 = 1

$y(n) = \{1, 2, -2, 0, -2, 3, -3\}$ $\boxed{-2, 3, 3, 0}$

1dB attenuation at 1 kHz

$f_{c_1} = 1\,kHz$.

$\delta_s = 0.01$ for $(f) > 5\,kHz$.

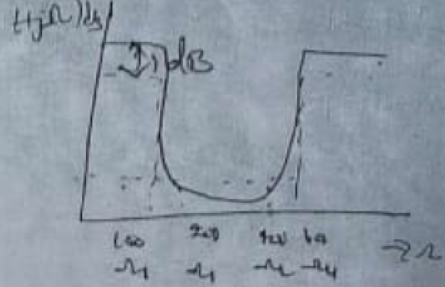
$\Omega_1 = 2\times\pi\times 1000 = 2000\pi$ rad/sec

normalized lowpass filter

$\Omega_r = \min\{|A|, |B|\}$

$$A = \frac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l\Omega_u} =$$

$$B = \frac{\Omega_2(\Omega_u - \Omega_l)}{-\Omega_2^2 + \Omega_l\Omega_u}$$



(a) Butterworth filter

The order of normalized Butterworth filter is

$$N = \frac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\dfrac{\Omega_s}{\Omega_p}}$$

$\alpha_p = 1dB$

$\alpha_s = 20dB$ $\quad \dfrac{\Omega_s}{\Omega_p} = 2 =$

$$= \log\sqrt{\frac{10^2-1}{10^{0.3}-1}} = \frac{0.9975}{0.3010} = 3.32$$

take $N = 4$

$$H(s) = \frac{1}{(s^2+0.765s+1)(s^2+1.848s+1)}$$

$$s \rightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u\Omega_l} \qquad s \rightarrow \frac{s(1000\pi)}{s^2 + 4\times10^4\,\Omega}$$

Chebyshev filter:

$$N = \cosh^{-1}\sqrt{\frac{10^2-1}{10^{0.3}-1}} = 2.75$$

$N = 3$.

$\varepsilon = (10^{0.1\alpha_p} - 1)^{0.5} = (10^{0.1} - 1)^{0.5} = 0.5368$

$\lambda = (10^{0.1\alpha_s} - 1)^{0.5} = (10^2 - 1)^{0.5} = 9.95$

$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 3.47$

$a = \Omega_p\left(\dfrac{\mu^{1/N} - \mu^{-1/N}}{2}\right)$, $b = \Omega_p\left(\dfrac{\mu^{1/N} + \mu^{-1/N}}{2}\right)$

$$= 1\left(\frac{(3.47)^{1/3} - (3.47)^{-1/3}}{2}\right) = 1\left(\frac{(3.47)^{1/3} + (3.47)^{-1/3}}{2}\right),$$

$= 1.10$

$\phi_k = \frac{\pi}{2} + \frac{(k-1)\pi}{2N}$ $\quad k = 1, 2, \ldots$

$\phi_1 = \frac{\pi}{2} + \frac{\pi}{6} = 120°$

$\phi_2 = \frac{\pi}{2} + \frac{\pi}{3} = 150$

$\phi_3 = \frac{\pi}{2} + \frac{5\pi}{6} = 240°$

$s_1 = a\cos\phi_1 + j\,b\sin\phi_1$, $\quad s_3 = a\cos\phi_3 + j\,b\sin\phi_3$

$s_2 = a\cos\phi_2 + j\,b\sin\phi_2$

6 Denominator of the ...

$= (s + 0.476)(s^2 + 0.238s + 0.9559)$

$= (s + 0.476)(s^2 + 0.476s + 0.975)$

Numerator of the transfer function

$= (0.476)(0.9755) = 0.4643$

$$H(s) = \frac{0.4643}{(s+0.476)(s^2+0.476s+0.9755)}$$

$$H(\omega)\Big|_s \rightarrow \frac{s(1600\pi)}{s^2+24\times10^4\pi^2}$$

$$= \frac{0.4643\,(s^2+24\times10^4\pi^2)^3}{(1600\pi s + 0.476(s^2+24\times10^4\pi^2))(s^2(1600\pi)^2 + 0.476((1600\pi)) \ldots}$$

$= \quad s^2 + 24\times10^4\pi^2 + 0.9755\,(s^2+24\times10^4\pi^2)^2$

(5)

$H(s) = \frac{2}{(s+2)(s+3)}$ find $H(z)$

Given $H(s) = \frac{2}{(s+2)(s+3)}$

Substitute $s = 2/T\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$ to get $H(z)$

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{2}{(s+2)(s+3)}\Big|_{s=2/T\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Given $T = 1$ sec

$$H(z) = \frac{2}{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right\}+3}$$

$$= \frac{2(1+z^{-1})^2}{(3-(z^{-1})5)t}$$

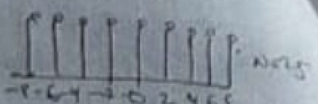
$$= \frac{(1+z^{-1})^2}{6-2z^{-1}} = \frac{0.166(1+z^{-1})^2}{(1-0.33z^{-1})}$$

8. $$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d\, e^{j\theta}\, W(e^{j(\omega-\theta)})$$
$$= H_d(e^{j\omega}) * W(e^{j\omega})$$

① Rectangular window:

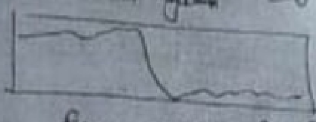
$w_R(n) = 1$ for $-(N-1)/2 \le n \le (N-1)/2$
$= 0$

The spectrum of rectangular window is given by



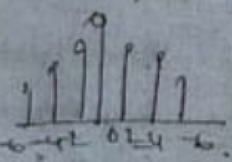
Rectangular window

$$W_R(e^{j\omega}) = \sum_{n=-(N-1)/2}^{(N-1)/2} e^{-j\omega n}$$
$$= \frac{\sin \omega N/2}{\sin \omega/2}$$

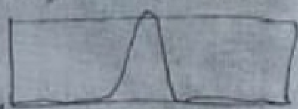

Frequency response of LPF using Rectangular window

② Triangular or Bartlett window:

$$W_T(e^{j\omega}) = \left(\frac{\sin \frac{(N-1)}{4}\omega}{\sin \frac{\omega}{2}}\right)^2$$



③ Raised cosine window.


normalised freq

$$w_\alpha(n) = \alpha + (1-\alpha)\cos\frac{2\pi n}{N-1} \quad \text{for } -(N-1)/2 \le n \le (N-1)/2$$

④ Hanning window:

$$W_H(e^{j\omega}) = 0.5\frac{\sin\frac{\omega N}{2}}{\sin \omega/2} + 0.25\frac{\sin(\omega N/2 - \pi N/(N-1))}{\sin(\omega/2 - \pi/(N-1))}$$
$$+ 0.25\frac{\sin(\omega N/2 + \pi N/(N-1))}{\sin(\omega/2 + \pi/(N-1))}.$$

⑤ Hamming window:

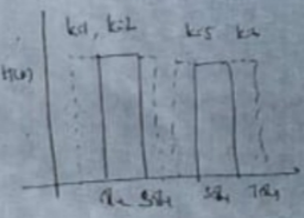
$$W_H(e^{j\omega}) = 0.54\frac{\sin\omega N/2}{\sin\omega/2} + 0.23\frac{\sin(\omega N/2 - \pi N/(N-1))}{\sin(\omega/2 - \pi/(N-1))}$$
$$+ 0.23\frac{\sin(\omega N/2 + \pi N/(N-1))}{\sin(\omega/2 + \pi/(N-1))}.$$

⑥ Blackman window:

$$w_B(n) = 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}$$
for $-(N-1)/2 \le n \le (N-1)/2$.

cut off frequency $f_{c_1} = 1000\,Hz$.

Sampling frequency $F = 5\,kHz$.



$$\omega_{c_1} = 2\pi f_{c_1} T = \frac{2\pi f_1}{F} = \frac{2\pi(1000)}{5000}$$

$$= \frac{2\pi}{5}$$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{7}k} \quad k = 0, 1 \ldots 6$$

$H(k) = 0$ for $k = 0, 3$

$= 1$ for $k = 1, 2$

$\theta = -\left(\frac{N-1}{N}\right)\pi$ for $0 \le k \le \frac{N-1}{2}$

$H(k) = 0 = e^{-j6\pi k/7}$ for $k = 0, 3$

$k = 1, 2$

The filter coefficients are given by

$$h(n) = \frac{1}{N}\left[H(0) + \sum_{k=1}^{N-1} Re\left(H(k)\, e^{j2\pi kn/N}\right)\right]$$

$$= \frac{1}{7}\left[2\sum_{k=1}^{3} Re\left(e^{-j6\pi k/7} e^{j2\pi kn/7}\right)\right)$$

$$= \frac{1}{7}\left(2\sum_{k=1}^{2}\cos\frac{2\pi k}{7}(3-n)\right) = \frac{2}{7}\left(\cos\frac{2\pi}{7}(3-n) + \cos\frac{4\pi}{7}(3-n)\right)$$

$h(0) = h(6) = -0.0792$

$h(1) = h(5) = -0.321$

$h(2) = h(4) = 0.11456$

$h(3) = 0.57$

$N = 7$

Rectangular window $W_R(n) = 1$

$$W_R(e^{j\omega}) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} e^{-j\omega n}.$$

---

a

Input signal $x(n)$ the z-transform of $x(n)$ is signal $x(n)$

$x(n) \rightarrow \boxed{h(n)} \xrightarrow{v(n)} \boxed{M} \rightarrow y(n)$

$Z(x(n))$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$p(n) = \frac{1}{M}\sum_{k=0}^{M-1} e^{j2\pi kn/M}.$$

multiplying the sequence $x(n)$ with $p(n)$

$x'(n) = x(n)\, p(n)$

$$x'(n) = \begin{cases} x(n) & n = 0, \pm M, \pm 2M \\ 0 \end{cases}$$

The z-transform of the output sequency is given by.

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)\, z^{-n}.$$

$$= \sum_{n=-\infty}^{\infty} x'(nM)\, z^{-n} = \sum_{n=-\infty}^{\infty} x'(n)\, z^{-n/M}$$

10 a Input Signal $(x(n))$ the Z transform of the signal $x(n)$ $X(z)$

$x(n) \rightarrow \boxed{h(n)} \xrightarrow{v(n)} \boxed{m} \rightarrow y(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$p(n) = \frac{1}{m} \sum_{k=0}^{m-1} e^{j2\pi kn/m}$$

Multiplying the sequence $x(n)$ with $(p(n))$

$$x'(n) = x(n)\, p(n)$$

$$x'(n) = \begin{cases} x(n) & n=0, \pm m, \pm 2m \\ 0 \end{cases}$$

The z transform of the output sequence is given by.

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}.$$

$$= \sum_{n=-\infty}^{\infty} x'(nm) z^{-n} = \sum_{n=-\infty}^{\infty} x'(n)\, z^{-n(1/m)}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} x(n) \left[\frac{1}{m} \sum_{k=0}^{m-1} e^{j2\pi kn/m}\right] z^{-n/m}.$$

$$= \frac{1}{m} \sum_{k=0}^{m-1} x\left[e^{-j2\pi k/m}\, z^{1/m}\right]$$

$z = e^{j\omega}$

$$Y(e^{j\omega}) = \frac{1}{m} \sum_{k=0}^{m-1} x(e^{-j2\pi k/m} e^{j\omega/m})$$

$$= \frac{1}{m} \sum_{k=0}^{m-1} x(e^{j(\omega-2\pi k)/m})$$

Example: $Y(e^{j\omega}) = \frac{1}{2} \sum_{k=0}^{1} x(e^{j(\omega - \frac{2\pi k}{2})}) = \frac{1}{2}(x(e^{j\omega/2}) + x(-e^{j\omega/2}))$

10 b Up-Sampling: $Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}$

$$= \sum_{n=-\infty}^{\infty} x(n/L) z^{-n} = x(z^{L})$$

Substitute $z = e^{j\omega}$ $\quad Y(e^{j\omega}) = X e^{j\omega L}$

example: $y(n) = x(n/L)$

(7)

... filters coefficients are computed to infinite precision in theory, filter coefficients are represented in binary and are stored in registers. If a bit register is used the filter coefficient must be rounded or truncated to b bits.

Due to quantization of coefficients, the frequency response of the filter may differ appreciably from the desired response and sometimes the filter may actually fail to meet the desired specification.

If the poles of desired filter are close to the unit circle, then those of the filter with quantized coefficients may lie just outside the unit circle leading to instability.

This change in value of filter coefficients modifies the pole zero locations. Sometimes pole location will be changed in such a way that the system may drive into instability.