and $Y(t)$ are statistically independent.

8.6 NOISE

8.6.1 Introduction to Noise

In the context of signals, noise may be defined as an undesired or unwanted electrical signal, which is accompanied by the desired signal. Signals, in transmission always pick up some undesired signals. Noise is an unintentional fluctuations that tend to disturb transmission and reproduction of transmitted signals. Many disturbances of an electrical nature produce noise in communication systems. Noise signals may or may not be predictable in nature.

The presence of noise increases system complexity. Noise present at all frequencies. Pridictable noise is a known noise and can be eleminated easly. Unpredictable noise is a random noise. We can eliminate or minimize the effect of random noise by knowing its statistical behavior.

Some examples are

1. Power supply fluctuations produce hum noise.
2. In radio receivers noise may produce hiss in the loud speaker output.
3. In TV receivers noise appear as " SNOW" or confetti (colored snow) i.e. white/color spots on the screen along with the picture.
4. Radiation pickup generated by electrical/electronics devices.

The effects of noise are

1. Noise may produce unwanted pulses or may cancel wanted pulses in pulse communication systems.
2. Noise may cause serious mathematical errors in computation.
3. Noise can limit the range of systems for a given transmitted power.
4. It effects the sensitivity of receivers and may reduce the bandwidth of the system.

8.7 CLASSIFICATION OF NOISE

Noise is classified into two broad groups depending on the source

1. External noise
2. Internal noise

Different types of noise can be categorized on the basis of their statistical properties in time and frequency domain. Noise power spectral density $S_N(\omega)$ is the important statistical property for analysis. Here we will discuss some important noise sources.

8.7.1 External Noise

Noise whose sources are external to the receiver/system is called external noise such as atmospheric noise, extraterrestrial noise etc. Most of the external noise is added into the desined signal in communication channels.

(i) Atmospheric Noise

It is also called static noise. It is in the form of impulses caused by lightning discharges in thunderstorms and other natural electric disturbances occurring in the atmosphere. It consists of spurious radio signals distributed in the broad range of frequency spectrum. It is propagated over earth in the same as ordinary radio waves of the same frequencies. Since the higher frequencies are limited to line-of-sight propagation, atmosphere noise becomes less severe at frequencies above 30 MHz. The field strength is inversely proportional to the frequency. The noise will interfere more with the reception of radio than that of TV.

(ii) Extra Terrestrial Noise

There are many types of noise sources called space noise available in the space. Space noise is observable at frequencies in the range from about 8 MHz to somewhat above 1.43 GHz . This noise has strongest components over the range of about 20 to 120 MHz. For convenience, all space noise sources are grouped into two subgroups.

(a) Solar noise

The sun always radiates heat. Under normal "quiet" conditions, there is a constant noise radiation from the sun, because it has very high temperature (over 6000^{oC} on the surface). It therefore radiates noise with very wide band frequency spectrum which includes all the frequencies used for communication. However, the sun is constantly changing star which undergoes cycles of peak activity from which electrical disturbances erupt, such as corona flares and sunspots. The noise produced in these periods is much greater than that received during periods of quiet sun. The solar cycle disturbances repeat themselves approximately every 11 years. In addition, every 100 years a super cycle exists with very high intensity. Evidence shows that the year 1957 was not only a peak but the highest such peak on record. When sun radiates more energy during the peaks, entire system is distorted and signals are effected.

(b) Cosmic Noise

Since distant stars are also suns and have high temperatures, they radiate noise in the same manner as our sun. Some times the noise may become significant, since it is originated from a large number of stars. This noise is distributed fairly uniformly over the entire sky. We also receive noise from our own galaxy (the Milky Way), from other galaxies, and from other virtual point sources such as "quasars" and "pulsars". This noise is called 'Galactic noise' and it is very intense in the peak periods.

(iii) Industrial Noise

The noise sources such as electrical disturbances in industries, automobile and aircraft ignition, sparks from electric motors and switching equipment, leakage from high voltage lines, cross talk between channels etc., generates noise known as 'industrial noise'. It is also called man made noise. This noise is more intense in industrially densely populated areas. Fluorescent lights are another powerful source of such noise and therefore should not be used where sensitive receiver reception or testing is being conducted. The nature of industrial noise is so variable that it is very difficult to analyze theoretically. But it obeys the general principle that received noise increases as the receiver bandwidth increases. This noise exists in the frequency range from 1 MHz to 600MHz

It is under human control and can be eliminated by removing the noise source.

8.7.2 Internal Noise

The noise created within a device or a system is called internal noise. The internal noise is generated by any of the active or passive devices found in the systems. This noise is also called function noise. Since the noise is randomly distributed over the entire radio spectrum, it can be easily described by statistical methods. The main property is, the random noise power is proportional to the bandwidth over which it is measured.

(a) Shot Noise

In 1918, while conducting some experiments on vacuum tubes Schotlky had observed that the fluctations are exist in the anode current at the output terminal of a vacuum diode. These fluctuations are named as shot noise. Shot noise fluctuations spread evenly over all frequencies. It appears as a randomly varying noise current superimposed on the output current. Shot noise follows Poission's distribution, since there is a randomnees in the number of particles arrived at the output. Some examples of shot noise are emission of electrons from the cathode of a vacuum tube, flow and recombination of electrons and holes in semiconductors. In bipolar transistors shot noise is mainly due to the random drift of current carriers across the junctions and also due to random generation and recombination of electron hole pair.

Due to shot noise, the external current at the output composed of a large number of random independent current pulses, as shown in Fig. 8.9(a). The shot noise current can be represented interms of its mean-square variable about the mean value.

$$\overline{i_n^2} = A\left[i - I_D\right]^2 = \underset{T\to\infty}{lt} \frac{1}{T}\int_0^T (i - I_D)^2\, dt$$

$$\overline{i_n^2} = 2qeI_D\, B$$

The shot noise rms current

$$i_{sh} = \sqrt{\overline{i_n^2}} = \sqrt{2qeI_D B}$$

Where q_e = electron charge $= 1.6\times10^{-19} C$

I_D = DC current

B = Band width of the system.

The total current can be expressed as

$$i(t) = i_{sh} + I_D$$

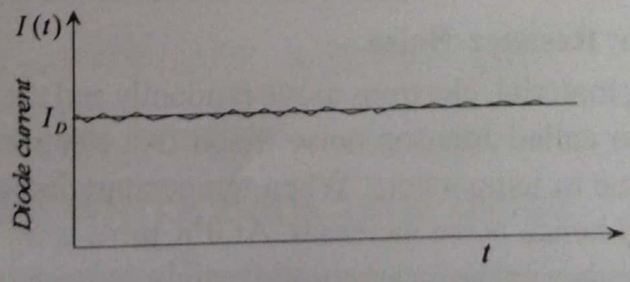


Fig. 8.9(a): Shot noise current

Power density spectrum:

In shot noise the number of fluctuations during the observed time is very large. The camplete random process may be considered as statistically independent and WSS. Therefore the shot noise distribution may be approximated as Gaussion distribution with zero mean value.

Shot noise is more effective at very high frequencies than at low frequencies. The time taken to reach the electron from cathode to anode is called transition time τ sec. The power density spectrum is independent of frequency up to $\frac{1}{\tau}Hz$. Beyond this frequency the power density varies with frequency. The most convenient method of dealing with shot noise is to find the value or formula for an equivalent input-noise resistor R_{eq}.

The power density spectrum of shot noise is

$$S_N(\omega) = q_e I_D \quad \text{or} \qquad \text{... (8.55)}$$

$$S_N(\omega) = 4KTR_{eq} \qquad \text{... (8.56)}$$

Shot noise voltage is

$$v_n(t) = \frac{i_n(t)}{g_m} \qquad \text{... (8.57)}$$

Where g_m the transconductance of the device.

(b) Transit-time Noise

At frequencies of VHF and EHF, the time taken by an of electron to travel from the emitter to the collector of a transistor becomes significant to the period the signal being amplified. The electron reaches the collector at different time periods called transit time effect, produces random noise.

(c) Flicker noise

At low audio frequencies a noise called Flicker noise is formed in transistors. It is proportional to emitter current and junction temperature and inversely proportional to frequency. It may be completely ignored above 500 Hz. It is seen in CRO screens and TV screens. This is also called 'modulation noise'. The picture is not clear and the picture is obtained before other picture is gone, i.e, the picture blinks on the screen because of flickering effect.

(d) Thermal Noise or Resister Noise

In any conducting material, electrons move randomly and the noise produced is called thermal noise. It is also called Johnson noise. Each free electron inside of a conducting medium is in motion due to temperature. When temperature increases, random motion of electrons increases, and hence noise increases. At 0^0K there is no motion of electrons and hence noise is zero. The thermal noise amplitude mainly depends on resistance. So it is also called resister noise. More noise is generated for high resistive path.

Thermal noise power is proportional to the temperature in degree kelvin and the bandwidth of the system.

$$P_n \alpha TB \qquad \text{...(8.58)}$$

Where $T =$ temperature, in ^0K, $B =$ bandwidth in Hz.

Noise power is $P_n = kTB\ watts$

$k =$ Boltzmans constant given by

$$k = 1.38 \times 10^{-23}\ J/^0K$$

The power density spectrum of noise voltage contributing the thermal noise is given by

$$S_v(\omega) = \frac{2kTR}{1+\left(\frac{\omega}{\alpha}\right)^2}$$

where R is the resistance and α is the average of collisions per second per electron

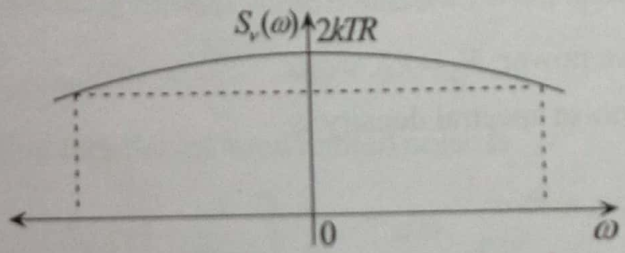


Fig. 8.9(b): Power density spectrum of thermal noise

The power density spectrum is shown in Fig.8.9(b). Since number of free electrons in a conductor are very large, the value of α is in the order of 10^{14}, the spectrum may be considered to be flat up to the frequency of $10^{13}\ Hz$. Hence for all practical communication systems, the power spectral density, $S(\omega)$ is considered to be independent of frequency. The thermal noise distribution may be approximated as Gaussian distribution with zero mean value.

i.e., $\quad S_v(\omega) = 2kTR$

Also, the power density spectrum for noise current is

$$S_I(\omega) = S_v(\omega)/R^2 = 2kT/R = 2kTG$$

where G is a conductance of the resistor

8.8 WHITE NOISE OR WHITE GAUSSIAN NOISE

White noise is a thermal noise whose frequency components are extend from zero to infinity. Consider a WSS thermal noise process $N(t)$. If it is called white noise, the power spectral density of $N(t)$ is a constant at all frequencies. That is the power density of white noise is independent of frequency. It can be expressed as

$$S_{NN}(\omega) = \frac{N_0}{2}$$

Where N_0 is the noise power.

According to the central limit theorem, the probability density function of white noise can be assumed as Gaussian distribution with zero mean. Hence it is known as white Gaussian noise. For all practical purposes, shot noise and thermal noise may be considered as white Gaussian noise. The bandwidth of the white noise is infinity. Also it has infinite average power so it is not physically realizable. If the white noise exists within a frequency range i.e. band limited then it is called *colored noise.* Band limited noise can be realizable.

8.8.1 Power Spectrum of White Noise

The power density spectrum of white noise is given by

$$S_{NN}(\omega) = \frac{\text{Noise power}}{2 \times \text{Bandwidth}} = \frac{kTB}{2B} = \frac{kT}{2} \; watts / Hz \qquad \text{... (8.59)}$$

Since noise power $N_0 = kT$ *watts*

Then the two sided noise spectral density is

$$S_{NN}(\omega) = \frac{N_0}{2} \quad -\infty < \omega < \infty \qquad \text{... (8.60)}$$

The autocorrelation function is

$$R_{NN}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{NN}(\omega) e^{j\omega\tau} d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{N_0}{2} e^{j\omega\tau} d\tau = \frac{N_0}{2}\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega\tau} d\tau$$

$$R_{NN}(\tau) = \frac{N_0}{2}\delta(\tau)$$

The power spectral density and autocorrelation of white noise shown in Fig.8.10.

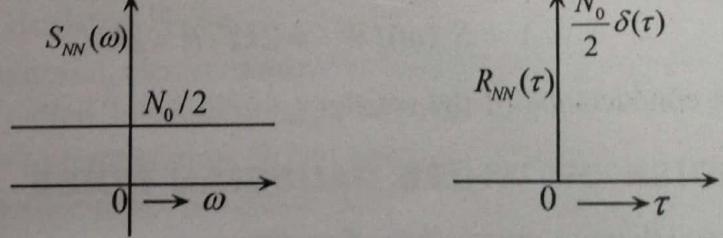


Fig. 8.10: (a) Power density spectrum of white noise *(b) Autocorrelation of white noise*

8.8.2 Band limited White Noise

Fig.8.11(a): Power spectrum of lowpass ... limited noise

8.10 RESISTOR NOISE VOLTAGE

Thermal noise generated by a resistor is proportional to its absolute temperature and bandwidth.

The noise power is $P_n = kTB$ Watts ... (8.65)

we know that the maximum power across the load resistance R is

$$P_n = \frac{v_n^2}{4R} \quad \text{... (8.66)}$$

where v_n is noise source voltage

For thermal noise, $P_n = kTB = \frac{v_n^2}{4R}$

or $v_n^2 = 4kTRB$

$\therefore$ The rms noise voltage is $v_n = \sqrt{4kTRB}$ volts ... (8.67)

The rms noise current is $i_n = \frac{v_n}{R} = \frac{\sqrt{4kTRB}}{R} = \sqrt{4kTGB}$ amps ... (8.68)

where $G = \frac{1}{R}$ □

Therefore the equivalent rms noise voltage is proportional to the absolute temperature resistance and bandwidth over which the noise is measured.

8.11 EQUIVALENT NOISE RESISTOR

The rms noise voltage generated in a resistor may be considered as noise voltage source. Therefore the noisy resistor can be modeled as Thevenin's equivalent circuit as shown in Fig. 8.11(a)

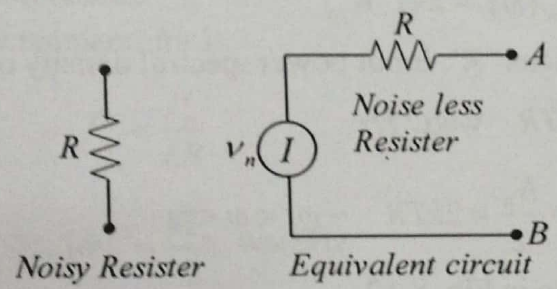


Fig. 8.11: (a) Noisy resister and equivalent circuit

Similarly consider a network with terminals A, B. Let the thevenin's impedance across A, B is Z_{eq} as shown in Fig.8.11(a)

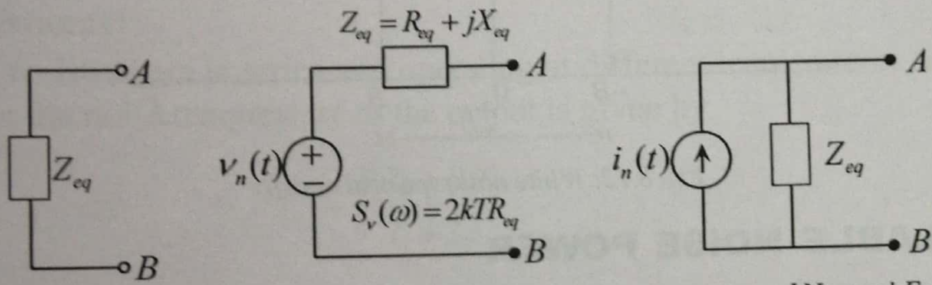


Fig. 8.11: (b) Noisy impedance (c) Thevenin's Equivalent noise source and Norton'sEquivalent noise source

Fig.8.11(c) shows the equivalent noise source of the impedance Z_{eq}. The mean square value of the noise voltage is

$$v_n^2 = 4kTBR_{eq}$$

$$\therefore \quad v_n = \sqrt{4kTB\,R_{eq}} \text{ volts} \qquad \text{... (8.69)}$$

and

$$i_n = \sqrt{4kTBG_{eq}}$$

Where R_{eq} is the real value of the thevenin's impediance and $G_{eq} = \frac{1}{R_{eq}}$

8.12 RESISTOR NOISE SPECTRAL DENSITY

Consider the noise spectral density for band limited noise is

$$S_{NN}(\omega) = \frac{N_0}{2} \quad -\omega_0 < \omega < \omega_0$$

where ω_0 is the bandwidth $\omega_0 = 2\pi B$

we know that

$$S_{NN}(\omega) = \frac{v_n^2}{2B} = \frac{4kTB\ \mathrm{R}_{eq}}{2B}$$

$$S_{NN}(\omega) = 2kT\, R_{eq} \qquad \text{...(8.70)}$$

For thermal noise resistor '*R*', input power spectral density of noise is,

$$S_{NN}(\omega) = 2kTR \quad \text{watt}/Hz$$

$$\therefore \quad S_{NN}(\omega) = \frac{N_0}{2} = 2kTR \quad -\omega_0 < \omega < \omega \qquad \text{...(8.71)}$$

The spectrum is shown in Fig. 8.12.

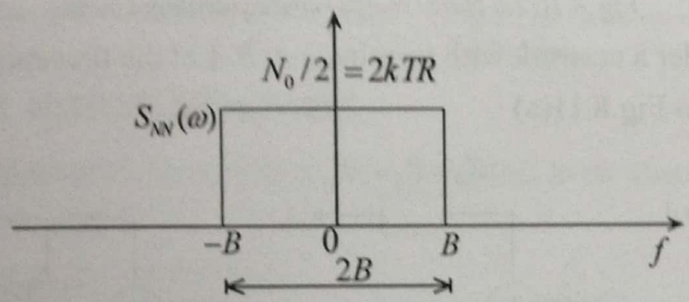


Fig. 8.12: White noise spectral density

8.13 AVAILABLE NOISE POWER

Available noise power of a noise source is the maximum power that can be drawn from the noise source.

$$P_a = \frac{v_n^2}{4\,R_{eq}}$$

$$R_{eq} = R_s = \text{Thevenin's resistance}$$

$$\therefore \quad P_a = \frac{4kTB\ R_{eq}}{4\,R_{eq}}$$

$$P_a = kTB \ \text{watts} \qquad \text{...(8.72)}$$

8.14 EQUIVALENT NOISE TEMPERATURE

It is defined as the noise temperature with respect to the available noise power or power spectral density. For a passive RLC network the noise temperature is given by

$$T = \frac{P_a}{kB} \ \text{Kelvin.} \qquad \text{...(8.73)}$$

This is same as physical temperature.

Suppose for active devices, where the power or power spectral density is amplified then the temperature increases as the density increases. This temperature is called equivalent or effective noise temperature.

The effective noise temperature is

$$T_n = \frac{P_a}{kB}$$

Also $$S_{NN}(\omega) = \frac{kT_n}{2} \text{ watt/Hz}$$

or $$T_n = \frac{2S_{NN}(\omega)}{k} \qquad \text{... (8.74)}$$

The equivalent noise temperature depends on power spectral density of noise.

Series resistance:

If two resistors are in series and operating at different temperatures as shown in Fig. 8.13(a), then the noise temperature at the output is given by

$$T_n = \frac{R_1T_1 + R_2T_2}{T_1 + T_2} \qquad \text{... (8.75)}$$

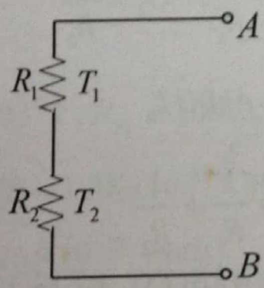


Fig. 8.13(a): Two series resistors at T_1 and T_2

Proof

we know that for resistor R_1, the mean square noise voltage is

$$v_{n1}^2 = 4KT_1R_1B$$

for R_2, $$v_{n2}^2 = 4KT_2R_2B$$

and for R_1 and R_2 $$v_n^2 = 4KB(R_1T_1 + R_2T_2) \qquad \text{... (8.76)}$$

If T_n is the noise temperature of R_1 and R_2 then

$$4KT_nBR = 4KB(R_1T_1 + R_2T_2)$$

$$\therefore \quad T_n = (R_1T_1 + R_2T_2)/R$$

or $$T_n = \frac{(R_1T_1 + R_2T_2)}{R_1 + R_2} \qquad \text{... (8.77)}$$

Parallel resistance

If two resistors R_1 and R_2 at temperature T_1 and T_2 are connected in parallel, the noise temperature of the combination is given by

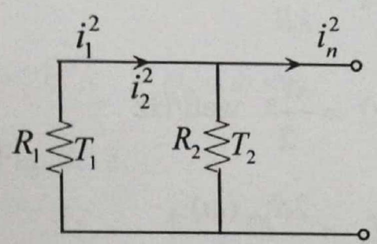


Fig. 8.13(b): Two parallel resistors at T_1 and T_2

From Fig. 8.13(b) the mean square noise current is

$$i_n^2 = i_1^2 + i_2^2$$

$$i_n^2 = 4kT_1BG_1 + 4kT_2BG_2$$

where $$G_1 = \frac{1}{R_1}, \quad G_2 = \frac{1}{R_2}$$

But $$i_n^2 = 4kBGT_n$$

where $$G = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \quad 4kGBT_n = 4kB(T_1G_1 + T_2G_2)$$

$$\therefore \quad T_n = \frac{T_1G_1 + T_2G_2}{G} = \frac{T_1R_2 + R_2R_1}{R_1 + R_2}$$

8.15 NOISE THROUGH TWO PORT NETWORKS

Consider a two port network with input spectral density $S_{ni}(\omega)$ and output spectral density $S_{no}(\omega)$ with network transfer function $H(\omega)$ as shown in Fig.8.14.

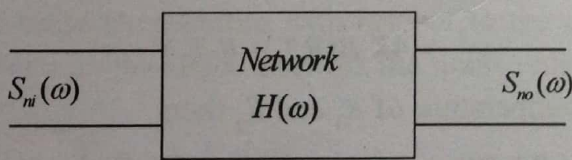


Fig. 8.14: Two port network

When the message signal is passed through the two port network, the noise available at the output is contributed by

(i) input noise and (ii) noise generated internally by the system.

Since the resistors and active devices in a two port network acts as independent noise sources, the signal quality degrades at the output. The signal which is corrupted by the noise at the output can be measured interms of signal to noise ratio, noise figure, equivalent noise temperature etc.

The output noise spectral density is

$$S_{no}(\omega) = S_{ni}(\omega)|H(\omega)|^2$$

Output noise power is

$$P_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{no}(\omega)d\omega$$

and the rms noise voltage is $$V_n = \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{no}(\omega)d\omega\right] \quad ...(8.78)$$

Example 8.3

The input impedance of an amplifier is 200Ω. Amplifier is operating at 100 kHz to 1 MHz, temperature is 30^0C. Find out rms noise voltage and noise power at the input of the amplifier.

Solution

Given $T = 30^0 C = 273 + 30 = 303^0 k$, $R_{eq} = 200\Omega$, $B = 10^6 - 10^5 = 900\times10^3\ Hz$

Noise power, $$P_n = kTB = 1.38\times10^{-23}\times303\times900\times10^3$$

$$= 3.76\times10^{-15}\ watts$$

rms noise voltage

$$v_n = \sqrt{4k\,TBR_{eq}} = 1.734\ volts$$

8.16 SINGLE TO NOISE RATIO

The ratio of the signal power to the accompanying noise power is called signal to noise ratio. It is given by

$$\frac{S}{N} = \frac{\text{Signal power}}{\text{noise power}} = \frac{v_s^2}{v_n^2}$$

or $$\frac{S}{N} = \frac{S_S(\omega)}{S_n(\omega)} = \frac{\text{Signal power spectral density}}{\text{Noise power spectral density}}$$

Input signal to noise ratio is

$$\left(\frac{S}{N}\right)_i = \frac{S_{si}(\omega)}{S'_{ni}(\omega)}$$

output signal to noise ratio

$$\left(\frac{S}{N}\right)_0 = \frac{S_{so}(\omega)}{S'_{no}(\omega)}$$

Where $S_{si}(\omega)$ = input signal power spectral density

$S_{so}(\omega)$ = output signal power spectral density

$S'_{ni}(\omega)$ = input noise power spectral density

$S'_{no}(\omega)$ = output noise power spectral density

The signal to noise ratio at the output of the noise-free two port network is same as at the input. But for noisy networks, since the noise power increases, the signal to noise ratio at the output deteriorates.

8.17 AVAILABLE POWER GAIN

The available power gain of the system, is defined as

$$G_a(\omega) = \frac{\text{Maximum psd of the signal at the output}}{\text{Maximum psd of the signal at the input}}$$

$$\therefore \quad G_a(\omega) = S_{so}(\omega)/S_{si}(\omega) \qquad \ldots(8.79)$$

The available output power is

$$P_{ao} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{so}(\omega)d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} G_a(\omega)S_{si}(\omega)d\omega$$

8.18 EQUIVALENT NOISE BANDWIDTH

The equivalent noise bandwidth of a system is defined as the bandwidth of the ideal system, which is having the same output power as the actual system with infinite bandwidth.

It is expressed as
$$W_N = \frac{\int_0^{\infty} |H(\omega)|^2\, d\omega}{|H(0)|^2} \; rad/\text{sec} \qquad \ldots(8.80)$$

and
$$B_N = \frac{W_N}{2\pi} \; Hz$$

Proof

Consider a low pass filter as shown in Fig. 8.15 and assume that a white noise is applied at the input of the filter.

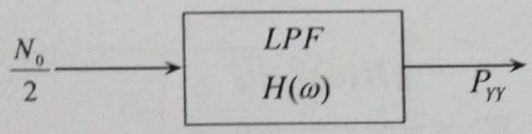


Fig. 8.15: Low pass filter

The output power is $$P_{YY} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{no}(\omega)d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{N_0}{2}|H(\omega)|^2\, d\omega$$

$$P_{YY} = \frac{N_0}{2\pi}\int_{0}^{\infty} |H(\omega)|^2\, d\omega \qquad \text{... (8.81)}$$

Now, consider an ideal system, with bandwidth '$2W_N$'.

$$H(\omega) = \begin{cases} H(0) & |\omega| < W_N \\ 0 & \text{otherwise} \end{cases}$$

$$P_{ideal} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{N_0}{2}|H(\omega)|^2\, d\omega$$

$$= \frac{No}{4\pi}\int_{-W_N}^{W_N} |H(0)|^2\, d\omega$$

$$P_{ideal} = \frac{N_0}{4\pi}|H(0)|^2 \int_{-W_N}^{W_N} d\omega = \frac{N_0}{4\pi}|H(0)|^2\, 2W_N \qquad \text{... (8.82)}$$

Equating eq.(8.82) and (8.83)

$$\frac{N_0}{2\pi}\int_0^{\infty}|H(\omega)|^2\, d\omega = \frac{N_0}{2\pi}|H(\omega)|^2 W_N$$

$$\therefore \quad W_N = \frac{\int_0^{\infty}|H(\omega)|^2\, d\omega}{|H(0)|^2}$$

Noise bandwidth is always greater than 3-dB bandwidth

Similarly for band pass filter, the equivalent nose bandwidth is the bandwidth of the ideal filter which produces the same output power as the actual system with infinite bandwidth.

$$\therefore \quad W_N = \frac{\int_0^\infty |H(\omega)|^2\, d\omega}{|H(\omega_0)|^2} \qquad \text{...(8.83)}$$

8.19 EQUIVALENT (EFFECTIVE) INPUT NOISE TEMPERATURE

The input noise temperature of the network, which is due to the noise internally generated by the network is known as *effective input noise temperature* or *equivalent input noise temperature*. It is denoted by T_e where $T_e = \dfrac{2S''_{ni}(\omega)}{k}$

The total input noise spectral density is equal to the spectral density of the input noise plus spectral density of noise generated internally by the system.

i.e $$S_{ni}(\omega) = S'_{ni}(\omega) + S''_{ni}(\omega)$$

where $S'_{ni}(\omega) = \dfrac{kT}{2}$ is the power spectral density of the input noise and $S_{ni}''(\omega) = \frac{kT_e}{2}$ is the power spectral density of the noise generated internally by the system.

The output power spectral density is

$$S_{no}(\omega) = G_a(\omega)\, S_{ni}(\omega)$$

$$S_{no}(\omega) = G_a(\omega)\left(\frac{kT}{2} + \frac{kT_e}{2}\right)$$

$$S_{no}(\omega) = \frac{kG_a(\omega)(T + T_e)}{2}$$

For noiseless system, $T_e = 0$

Then $$S_{no}(\omega) = \frac{kTG_a(\omega)}{2}$$

8.20 NOISE FIGURE

Noise Figure gives the amount of noise internally generated by the system. It is the ratio of power density of the total noise available at the output of network to the power density at the output only due to the input noise source. It gives the measure of system performance of the noise.

$$F = \frac{S_{no}(\omega)}{S_{no}'(\omega)} = \frac{S_{no}'(\omega) + S_{no}''(\omega)}{S_{no}'(\omega)} = 1 + \frac{S_{no}''(\omega)}{S_{no}'(\omega)} \qquad \text{...(8.84)}$$

Where $S_{no}(\omega) =$ the total noise power spectral density at the output.

$S_{no}'(\omega)$ noise power spectral density at the output due input noise and

$S_{no}''(\omega) =$ noise power spectral density at the output due to the noise generated internally by the system.

If for noiseless system, $S_{no}''(\omega) = 0$

and $S_{no}(\omega) = S_{no}'(\omega)$

Then $F = 1$

If $F > 1$, the system is said to be noisy system.

The range of F is $1 < F < \infty$, as fincreases, the system becomes noisy.

8.20.1 Noise Figure in Terms of Available Power Gain

$$F = \frac{S_{no}(\omega)}{S_{no}'(\omega)}$$

$$S_{no}(\omega) = G_a(\omega)S_{ni}'(\omega)$$
$$= G_a(\omega)kT/2$$

$$\therefore \quad F = \frac{S_{no}(\omega)}{G_a(\omega)kT/2} \quad \text{... (8.85)}$$

or $$S_{no}(\omega) = G_a(\omega)FkT/2 \quad \text{... (8.86)}$$

8.20.2 Noise Figure in Terms of Input Noise Temperature

$$S_{no}(\omega) = G_a(\omega)S_{ni}(\omega) \quad \text{... (8.87)}$$
$$= G_a(\omega)(S_{ni}'(\omega) + S_{ni}''(\omega))$$
$$= G_a(\omega)(kT/2 + kT_e/2)$$
$$= G_a(\omega)k(T + T_e)/2 \quad \text{... (8.88)}$$

Compare eq.(8.91) and eq.(8.93)

$$F = (T + T_e)/T$$

or $$T_e = T(F-1)$$

$$\text{F in } dB = 10\log_{10}(F)$$

For noiseless system Noise Figure is $F = 0dB$

8.20.3 Noise Figure in terms of Signal to Noise Ratio

We know that signal power spectral density at the output

$$S_{so}(\omega) = G_a(\omega)S_{si}(\omega) \quad \text{... (8.89)}$$

$$S_{no}'(\omega) = G_a(\omega)S_{ni}(\omega) \quad \text{... (8.90)}$$

and $$S_{no}(\omega) = F\,S_{no}'(\omega)$$
$$= G_a(\omega)F\,S_{ni}(\omega)$$
$$= \frac{S_{so}(\omega)}{S_{si}(\omega)}F\,S_{ni}(\omega)$$

$$\therefore \quad F = \frac{S_{si}(\omega)/S_{ni}(\omega)}{S_{so}(\omega)/S_{no}(\omega)} \quad \text{... (8.91)}$$

$$F = \frac{(S/N)_i}{(S/N)_0} \quad \text{... (8.92)}$$

where $\quad (S/N)_0 = \dfrac{S_{si}(\omega)}{S_{ni}(\omega)}$ = Input signal power to noise ratio

and $\quad (S/N)_0 = \dfrac{S_{so}(\omega)}{S_{no}(\omega)}$ = Output signal power to noise ratio

Therefore Noise Figure is also defined as signal power to noise ratio of the input to signal power to noise ratio of the output.

Note :

If $\quad (S/N)_i = (S/N)_o$, then $F = 1$ represent ideal system ...(8.93)

For real systems, $(S/N)_i$ is always greater than $(S/N)_o$

$\therefore$ F is always greater than 1.

8.20.4 Noise Figure Interms of Network Transfer Function

Consider a two port network has the transfer function $H(\omega)$

The output signal power spectral density for a system is

$$S_{so}(\omega) = S_{si}(\omega)|H(\omega)|^2 \quad ...(8.94)$$

we know that the noise figure of the system is

$$F = \frac{S_{si}(\omega)/S_{ni}(\omega)}{S_{so}(\omega)/S_{no}(\omega)}$$

$$F = \frac{S_{si}(\omega)}{S_{so}(\omega)} \times \frac{S_{no}(\omega)}{S_{ni}(\omega)}$$

$$\therefore \quad F = \frac{S_{no}(\omega)}{S_{ni}(\omega)|H(\omega)|^2} \quad ...(8.95)$$

or $\quad S_{no}(\omega) = F\left|H(\omega)\right|^2 S_{ni}(\omega)$

8.20.5 Average Operating Noise Figure

In the previous sections we have defined the Noise Figure which is constant with respect to frequency. It is also called spot noise figure. But in practical cases the Noise Figure depends on frequency. So the average Noise Figure of the system should be considered.

The average Noise Figure $\overline{F}$ is defined as the ratio of the total output available noise power N_0 to the total output available noise power N due to the source alone. It can be expressed as

$$\overline{F} = \frac{N_0}{N_{SO}} \quad \text{... (8.96)}$$

where $\quad N_0 = N_{SO} + N_{sys}$

$$N_{SO} = \frac{1}{\pi}\int_0^\infty G_a S_{ni}(\omega)d\omega$$

But $\quad S_{ni}(\omega) = \frac{KT_e}{2}$

From eq.(8.96) $\quad = \frac{k}{2\pi}\int_0^\infty T_e G_a d\omega$

and $\quad N_0 = \frac{k}{2\pi}\int_0^\infty F T_e G_a d\omega$

$$\therefore \quad \overline{F} = \frac{\frac{k}{2\pi}\int_0^\infty F T_e G_a d\omega}{\frac{k}{2\pi} T_e G_a d\omega}$$

$$\therefore \quad \overline{F} = \frac{\int_0^\infty F T_e G_a d\omega}{\int_0^\infty T_e G_a d\omega}$$

If T_e is constant then

Average noise figure is $\overline{F} = \frac{\int_0^\infty F G_a d\omega}{\int_0^\infty G_a d\omega}$ ... (8.97)

8.21 OUTPUT NOISE POWER AND SYSTEM NOISE POWER

The output noise power is equal to the sum of the noise power available at the output due to external noise and the noise power of the system.

$$N_0 = G_a(\omega)N_i + N_{sys} \quad \text{... (8.98)}$$

$$F = \frac{S_i / N_i}{S_0 / N_0} = \frac{N_0}{G_a(\omega)N_i}$$

$$N_0 = G_a(\omega)N_i F$$

$$\therefore \; G_a(\omega)N_i F = G_a(\omega)N_i + N_{sys}$$

Noise power of the system is $N_{sys} = G_a(\omega)N_i(F-1)$ ...(8.99)

Example 8.5

An amplifier has input resistance 200Ω and equivalent input noise temperature $400^0 K$. Find the Noise Figure a system at room temperature $27^0 C$.

Solution

Given $T = 27^0 C = 300^0 K,\; T_e = 400^0 K$

we know that $T_e = T(F-1)$

Noise figure $F = 1 + \frac{T_e}{T}$

$$\therefore \quad F = 1 + \frac{400}{300} = 2.33$$

8.22 NOISE IN CASCADE AMPLIFIERS

(1) Noise Figure:

Consider two amplifiers are cascaded

G_{a_1} - Power gain of amplifier 1.

G_{a_2} - Power gain of amplifier 2.

T_{e_1} - Equivalent input noise temperature of amplifier 1.

T_{e_2} - Equivalent input noise temperature of amplifier 2.

F_1 - Noise Figure of amplifier 1.

F_2 - Noise Figure of amplifier 2.

The overall gain is

$$G_a = G_{a_1} \times G_{a_2} \quad .(8.100)$$

The total output noise power available is

$$N_0 = N_{0_1} + N_{0_2} + N_{0_3}$$

where N_{01} = output noise power due to input noise power N_i

$$\therefore \quad N_{0_1} = G_{a_1} G_{a_2} N_i = G_a N_i \quad .(8.101)$$

N_{0_2} = output noise power due to noise power generated internally by first amplifier

From eq.8.99

$$N_{sys1} = G_{a_1} N_i (F_1 - 1)\, G_{a_2}$$

$$\therefore \quad N_{0_2} = G_a N_i (F_1 - 1) \qquad ..(8.102)$$

N_{0_3} = output noise power due to noise power generated by second amplifier

$$N_{0_3} = G_{a_2} N_i (F_2 - 1) \qquad ..(8.103)$$

we know that overall Noise Figure is

$$F = \frac{N_0}{G_a N_i}$$

or $$N_0 = G_a N_i F$$

Now $$N_0 = N_{0_1} + N_{0_2} + N_{0_3}$$

$$G_a N_i F = G_a N_i + G_a N_i (F_1 - 1) + G_{a_2} N_i (F_2 - 1) \qquad ..(8.104)$$

$$F = 1 + F_1 - 1 + \frac{(F_2 - 1) G_{a_2}}{G_{a_1} G_{a_2}}$$

or $$F = F_1 + \frac{F_2 - 1}{G_{a_1}}$$

Similarly for 3 amplifier cascaded

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1} G_{a2}}$$

For N amplifiers cascaded as shown in Fig.8.17,

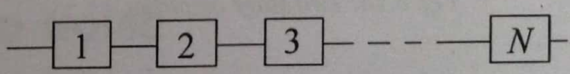


Fig. 8.17: Cascaded networks

The overall Noise Figure is

$$F = F_1 + \frac{F_2 - 1}{G_{a_1}} + \frac{F_3 - 1}{G_{a_1} G_{a_2}} + \ldots\ldots + \frac{F_n - 1}{G_{a_1} G_{a_2} \ldots G_{a_{n-1}}} \qquad .(8.105)$$

This equation is called Fris's Formula. It shows that the contribution to overall Noise Figure is mainly by the first stage.

(2) Equivalent Noise Temperature

we know that, $F = 1 + T_e / T$

$$F_1 = 1 + T_{e_1}/T$$

$$F_2 = 1 + T_{e_2}/T$$

The overall Noise Figure for two amplifiers coscaded is $F = F_1 + \dfrac{F_2 - 1}{G_{a_1}}$

$$\therefore \qquad 1 + \frac{T_e}{T} = 1 + \frac{T_{e_1}}{T} + \frac{T_{e_2}}{G_{a_1} T}$$

$\therefore$ The equivalent Noise Temperature is

$$T_e = T_{e_1} + \frac{T_{e_2}}{G_{a_1}} \qquad ..(8.106)$$

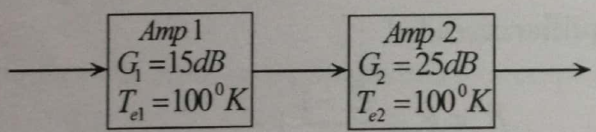
For N amplifiers, cascaded

The equivalent Noise Temperature is

$$T_e = T_{e_1} + \frac{T_{e_2}}{G_{a_1}} + \frac{T_{e_3}}{G_{a_1} G_{a_2}} + ... + \frac{T_{e_n}}{G_{a_1} G_{a_2} .....G_{a_{n-1}}} \qquad ..(8.107)$$

Example 8.6

Find the overall Noise Figure and equivalent input noise temperature of the system shown in Fig. 8.18



Room temperature $= 27^0 C$

Fig. 8.18: Two stage amplifier

Solution

Given
$$G_1 = 15dB = 10^{1.5} = 31.62$$

$$G_2 = 25dB = 10^{2.5} = 316.23$$

$$T_{e1} = 100^0 K$$

$$T_{e2} = 150^0 K$$

$$T = 27^0 C = 300^0 K$$

Noise Figure $\quad F_1 = 1 + \dfrac{T_{e_1}}{T} = 1 + \dfrac{100}{300} = 1.33$

$$F_2 = 1 + \frac{T_{e_2}}{T} = 1 + \frac{150}{300} = 1.5$$

Overall Noise Figure $\quad F = F_1 + \frac{F_2 - 1}{G_1}$

$$= 1.33 + \frac{1.5 - 1}{31.62}$$

$$F = 1.33 + 0.02 = 1.35$$

Equivalent noise temperature is

$$T_e = T_{e_1} + \frac{T_{e_2}}{G_h} = 100 + \frac{150}{31.62}$$

$$T_e = 104.94^0 K$$

also $\quad F = 1 + \frac{T_e}{T} = 1.35$

8.23 ANTENNA NOISE TEMPERATURE

Antenna noise temperature is the equivalent temperature of the noise with respect to the available spectral density of the noise at the antenna.

It is denoted by T_{ant}.

When the antenna is connected to receiver, the input spectral density of noise is given by

$$S_i(\omega) = \frac{k(T_{ant} + T_e)}{2} \quad ..(8.108)$$

and input noise power, $\quad N_i = k(T_{ant} + T_e)B_N \quad ..(8.109)$

8.24 NARROW-BAND NOISE

If white noise is passed through bandpass filter, whose $W_N << \omega_0$ the noise process appearing at the output is called narrow-band noise. The spectral components of narrow-band noise are concentrated about some mid-band frequency $\pm\omega_0$.

We can represent the narrow band noise in canonical form as

$$N(t) = N_I(t)\cos(\omega_0 t) - N_Q(t)\sin(\omega_0 t) \quad ..(8.110)$$

Where $n_I(t)$ = in-phase component

and $n_Q(t)$ = quadrature phase component.

given by $$N_I(t) = N(t)\cos\omega_0 t + \hat{N}(t)\sin\omega_0 t$$

and $$N_Q(t) = \hat{N}(t)\cos\omega_0 t - N(t)\sin\omega_0 t$$

where $\hat{N}(t)$ = Hilbert transform of $N(t)$

In polar form

$$r(t) = \sqrt{N_I^2(t) + N_Q^2(t)}$$

$$\psi(t) = \tan^{-1}\left[\frac{N_Q(t)}{N_I(t)}\right]$$

$$N(t) = r(t)\cos[\omega_0 t + \psi(t)]$$

where $r(t)$ = envolope process of $N(t)$

$\psi(t)$ = phase process of $N(t)$

Therefore narrow band noise process have both amplitude and phase processes.

8.24.1 In-Phase and Quadrature Components of a Narrow Band Noise

The in-phase and quadrature components of narrow band noise can be extracted from the noise signal using product modulator and low pass filter as shown in Fig.8.20(a).

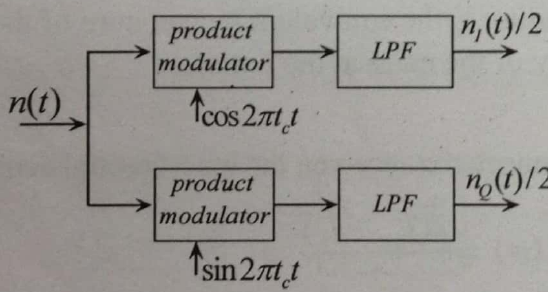


Fig. 8.20(a): Extraction of $n_I(t)$ and $n_Q(t)$ from $n(t)$

The generation of narrow band process from its in phase and quadrature components using product modulator and adder is shown in Fig.8.20(b)

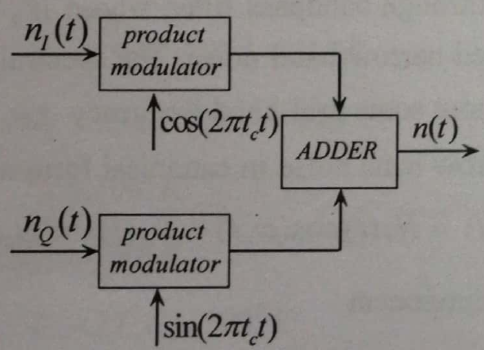


Fig. 8.20:(b) Generation of $n(t)$ from $n_I(t)$ and $n_Q(t)$

8.25 PROPERTIES OF NARROW-BAND NOISE

Some of important properties of narrow-band noise are

(1) The in-phase component $N_1(t)$ and quadrature phase component $N_Q(t)$ of narrow-band noise $N(t)$ have zero mean, since $N(t)$ has zero mean.

(2) If the narrow band noise $N(t)$ is Gaussian, then its in phase component $N_1(t)$ and quadrature component, $N_Q(t)$ are jointly Gaussian.

(3) If the narrow band noise $N(t)$ is wide-sense stationary, then its in phase component $N_1(t)$ and quadrature component $N_Q(t)$ are jointly wide-sense stationary.

(4) Both in phase component $N_1(t)$ and quadrature component $N_Q(t)$ have the same power spectral density, which is related to the power spectral density $S_N(\omega)$ of the narrow-band noise $N(t)$ as follows,

$$S_{N_1}(\omega) = S_{NQ}(\omega) = S_N(\omega-\omega_c) + S_N(\omega+\omega_c) \quad -\omega_0 \le \omega \le \omega_0 \qquad ..(8.111)$$

where it assumed that $S_N(\omega)$ occupies the frequency interval

$\omega_c - \omega_0 \le |\omega| \le \omega_c + \omega_0$ and $\omega_c > \omega_0$

(5) Quadrature components $N_Q(t)$ and $N_1(t)$ has the same variance as the narrow-band noise $N(t)$

(6) The cross-spectral densities of the quadrature components of a narrow-band noise are purely imaginary.

$$S_{N_I N_Q}(\omega) = -S_{N_Q N_I}(\omega) \text{ i.e} \qquad ..(8.112)$$

$$S_{N_I N_Q}(\omega) = \begin{cases} \frac{-j}{2}[S_N(\omega+\omega_c) - S_N(\omega-\omega_c)] & -\omega_0 \le \omega \le \omega_0 \\ 0 & \text{elesewhere} \end{cases}$$

(7) If narrow band noise $N(t)$ is Gaussian with zero mean and a power spectral density $S_N(\omega)$ that is locally symmetric about the mid-band frequency $\pm\omega_0$, then the in-phase noise $N_1(t)$ and the quadrature noise $N_Q(t)$ are statistically independent.

i.e., $R_{N_I N_Q}(0) = 0$