R18

Code No: 156AR

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech III Year II Semester Examinations, February/March - 2022

DIGITAL SIGNAL PROCESSING

(Common to ECE, EIE)

Time: 3 Hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

- - -

1.a) What are the conditions for stability and causality of an LTI system? Explain.
b) Explain in detail the classification of discrete-time systems.
c) What is the need for multi-stage implementation of sampling rate converters? Explain with an example. [5+5+5]

2.a) Find 8-point DFT X(K) of the real sequence.
$x(n) = \{0.707, 1, 0.707, 0, -0.707, -1, -0.707, 0\}$ by using DIF radix-2 FFT
b) Find the N-point DFT of $x(n) = b^n \cos an$ using the linearity property. [8+7]

3.a) State and prove any two properties of Discrete Fourier series.
b) Given $x(n) = 2^n$ and N=8, find X(k) using DIT-FFT algorithm. [6+9]

4.a) Design a digital low pass filter using Chebyshev filter that meets the following specifications: Passband magnitude characteristics that is constant to within 1dB for recurrences below $\omega = 0.2\pi$ and stopband attenuation of atleast 15dB for frequencies between $\omega = 0.3\pi$ and π. Use bilinear transformation.
b) Derive the relation between digital and analog frequencies in bilinear transformation. [10+5]

5.a) Design a Butterworth analog high pass filter that will meet the following specifications
i) Maximum pass band attenuation = 2dB
ii) Passband edge frequency = 200rad/sec
iii) Minimum stopband attenuation=20dB
iv) Stop band edge frequency = 100 rad/sec.
b) Prove that for a linear phase FIR filter the impulse response is symmetric. [8+7]

6.a) Explain the type II frequency sampling method of designing an FIR digital filter.
b) Design a band pass filter which approximates the ideal filter with cutoff-frequencies at 0.2rad/sec and 0.3rad/sec. The filter order is M=7. Use the Hanning window function. [5+10]

7.a) Explain coefficient quantization of IIR filters.
b) What is Round-off Noise in IIR Digital Filters? Discuss its effects in IIR filters. [7+8]

8.a) Describe various Structures of IIR filters with suitable diagrams.
b) Explain the limit cycle oscillations due to product round-off and overflow errors.[10+5]

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Digital Signal Processing

(Common to ECE, EIE)

A

1.a This property says that the output of a causal system depends only on the present and past values of the input to the system

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \quad - 2.5M$$

For a causal system the impulse response $h(n)=0$ for $n<0$

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k) \quad - 2$$

An LTI system is stable if it produces a bounded o/p sequence for every bounded i/p sequence. - 2.5M

$$|y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \right.$$

the necessary and sufficient condition for stability is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty.$$

1.b. 1. static & Dynamic system.

2. causal & non causal system

3. Linear & non linear system

me va

FIR &

6 stable

me variant and Time invarint System.

FIR & IIR Systems

6 stable & unstable systems.

1. static & Dynamic Systems: output at any instant n depends on the i/p samples at the same time, but not on past or future samples of the i/p. —(1)

$$y(n) = ax(n)$$

2. causal & non causal systems: A causal system if the o/p of the system at any time n depends only at present and past i/p. but does not depend on future i/p. —(1)

$$y(n) = x(n) + x(n-1)$$

3. Linear & non linear: A system follows super position principle weighted sum of signals should be equal to the corresponding weighted sum of the o/p of the system.

$$T\{a_1x_1(n) + a_2x_2(n)\} = a_1T[x_1(n)] + a_2T[x_2(n)]$$ —(1)

4. Time variant and invariant system: A system is Time invariant if the characteristics of the system do not change with time.

$$y(n,k) = T[x(n-k)]$$ —(1)

the o/p sequence by k samples $y(n-k)$

$$y(n,k) = y(n-k)$$

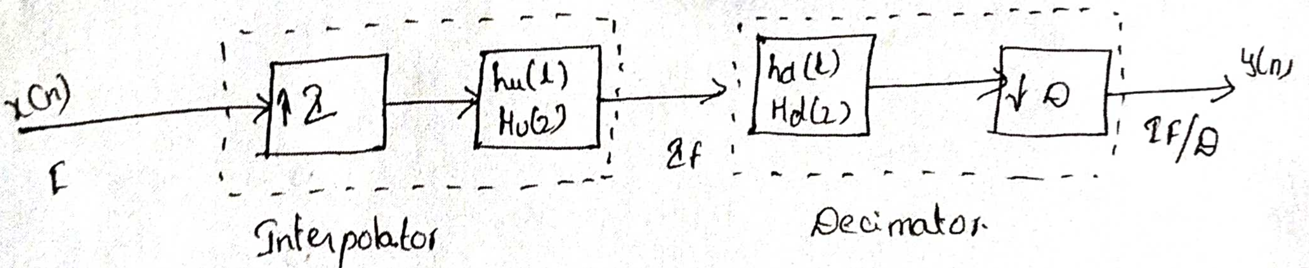
5. FIR & IIR systems: The impulse response of the system is of finite duration, then the system is called FIR

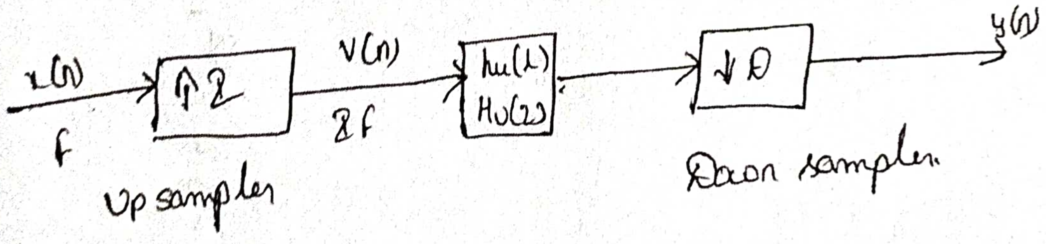
IIR :- An infinite impulse response system. impulse response for duration

$$h(n) = a^n u(n)$$ — (1)

1.C Some applications sampling rate conversion by a non linear factor may be required for example transfering data from a compact disc at a rate of 44.1 kHz to a digital audio tape at 48kHz. Here the sampling rate conversion factor is 48/44.1, which is a non integer.

A sampling rate conversion by a factor I/D can be achieved by first performing interpolation by factor I and then performing decimation by factor D. The cascade configuration of interpolator and decimator. The anti imaging filter $H_u(z)$ and the anti aliasing filter $H_d(z)$ are operated at the sampling rate, hence can be replaced by a simple low pass filter with transfer function $H(z)$, where the low-pass filter has a cut off frequency of $\omega_c = \min\left[\frac{\pi}{I}, \frac{\pi}{D}\right]$ — (3M)





— (2M)

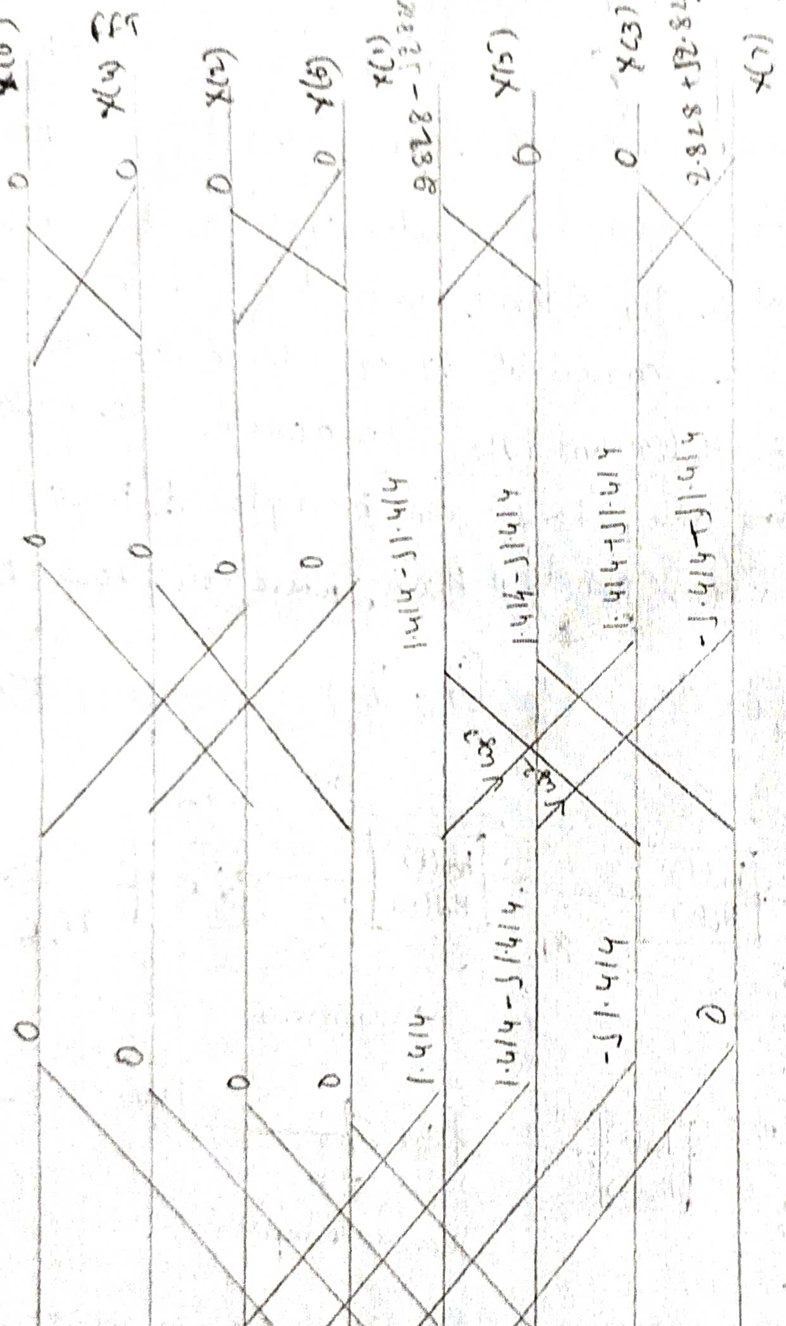
time domain and frequency domain relations of sampling rate converters

$$H_u(\omega) = \begin{cases} I & -\pi/I \leq \omega \leq \pi/I \\ 0 & \end{cases}$$

$H_d(\omega) = \begin{cases} 1 & -\pi/D \le \omega \le \pi/D \\ 0 & \text{else where} \end{cases}$

2.a $x(n) : \{0.707, 1, 0, 0.707, 0, -0.707, -1, -0.707, 0\}$. DIF radix 2 FFT

$W_8^0 = 1$ $W_8^1 = 0.707 - j0.707$ $W_8^2 = -j$ $W_8^3 = -0.707 - j0.707$



$(0.707 + 0.707)1$

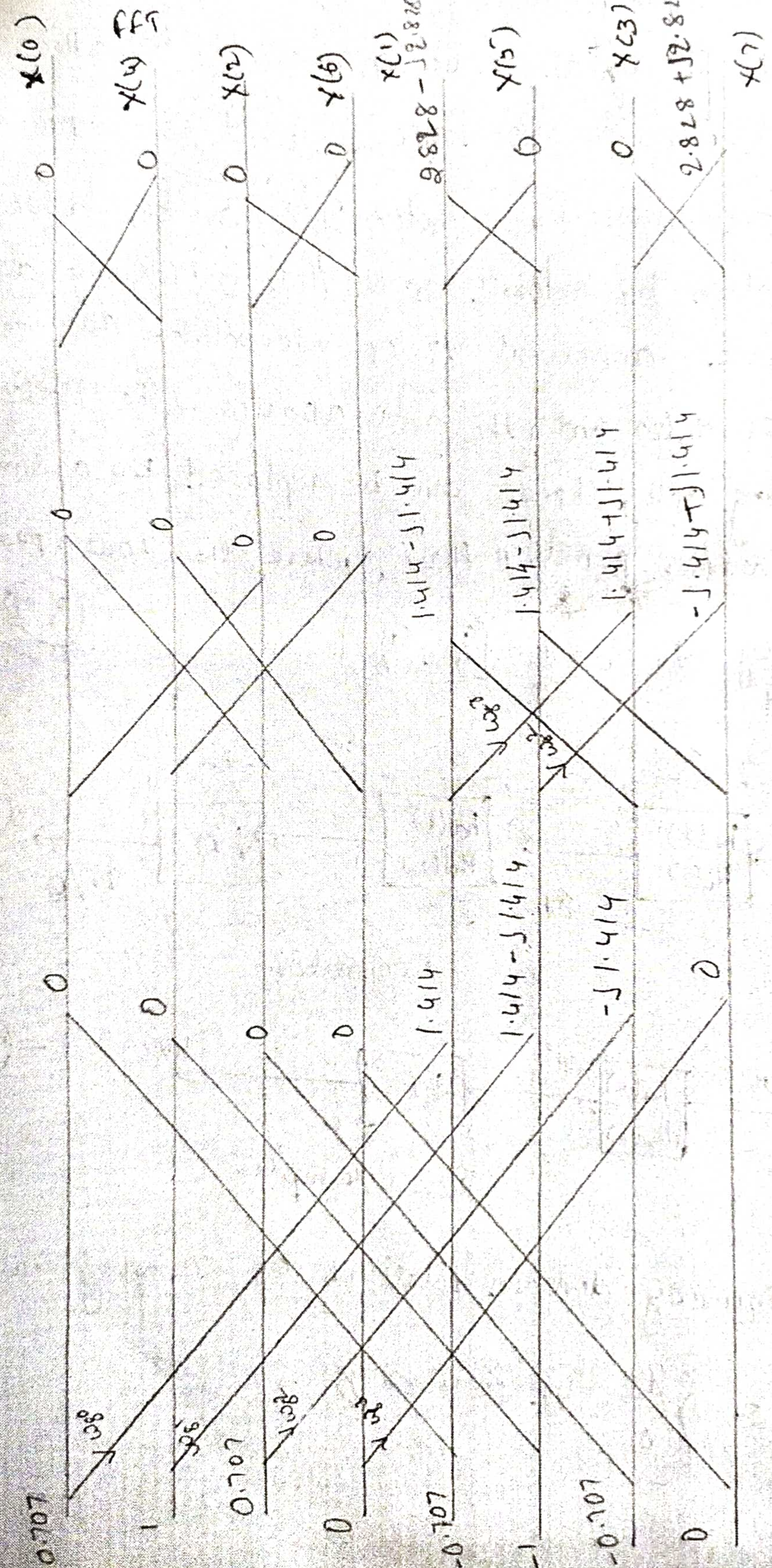
$\{\cancel{20}, 2.828 - j2.828, 0, 0, 0, 0, 0, 2.828 + j2.828\}$

8M

$$H_d(\omega) = \begin{cases} 1 & -\pi/2 \le \omega \le \pi/2 \\ 0 & \text{else where} \end{cases}$$

2.a $x(n) = \{0.707, 1, 0, 0.707, 0, -0.707, -1, -0.707, 0\}$. DIF radix 2 FFT

$W_8^0 = 1 \quad W_8^1 = 0.707 - j0.707 \quad W_8^2 = -j \quad W_8^3 = -0.707 - j0.707$



$(0.707 + 0.707)$

$\{0, 2.828 - j2.828, 0, 0, 0, 0, 0, 2.828 + j2.828\}$

8M

2.b $x(n) = b^n \cos an$

$$b^n \cos an = b^n \left[\frac{e^{jan} + e^{-jan}}{2} \right] \quad -(2M) \qquad -(7M)$$

$$DFT = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi kn}{N}} \quad -(1M)$$

$$= \frac{1}{2} \left[\sum_{n=0}^{N-1} b^n e^{jan} \cdot e^{-\frac{j2\pi kn}{N}} + \sum_{n=0}^{N-1} b^n e^{-jan} e^{-\frac{j2\pi kn}{N}} \right]$$

$$= \frac{1}{2} \left\{ \left[\sum_{n=0}^{N-1} \left(b e^{ja} e^{-\frac{j2\pi k}{N}} \right)^n + \sum_{n=0}^{N-1} \left(b e^{-ja} e^{-\frac{j2\pi k}{N}} \right)^n \right] \right\}$$

$$= \frac{1}{2} \cdot \sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a} \quad -(1M)$$

$$= \frac{1}{2} \left[\frac{1-\left(be^{ja}e^{-\frac{j2\pi k}{N}}\right)^N}{1-be^{ja}e^{-\frac{j2\pi k}{N}}} + \frac{1-\left(be^{-ja}e^{-\frac{j2\pi k}{N}}\right)^N}{1-be^{-ja}e^{-\frac{j2\pi k}{N}}} \right]$$

$$= \frac{1}{2} \left\{ \frac{1-\left[be^{j\left[a-\frac{2\pi k}{N}\right]}\right]^N}{1-be^{j\left(a-\frac{2\pi k}{N}\right)}} + \frac{1-\left(be^{-j\left[a+\frac{2\pi k}{N}\right)}\right)^N}{1-be^{-j\left(a+\frac{2\pi k}{N}\right)}} \right\}$$

$-(3M)$

Any two properties of discrete fourier Series

Linearity: consider two periodic sequences $x_{1p}(n)$ & $x_{2p}(n)$ both with period N, such that

$$DFS[x_{1p}(n)] = X_{1p}(k) \text{ and}$$

$$DFS[x_{2p}(n)] = X_{2p}(k)$$

then $DFS[a_1 x_{1p}(n) + a_2 x_{2p}(n)] = a_1 X_{1p}(k) + a_2 X_{2p}(k)$

- Periodic convolution: Let $x_{1p}(n)$ & $x_2p(n)$ be two periodic sequences with period N with

$$DFS[x_{1p}(n)] = X_{1p}(k)$$

$$DFS[x_{2p}(n)] = X_{2p}(k)$$

3M

If $X_{3p}(k) = X_{1p}(k)\, X_{2p}(k)$ then the periodic sequence $x_{3p}(n)$ with fourier series coefficients $X_{3p}(k)$ is

$$x_{3p}(n) = \sum_{m=0}^{N-1} x_{1p}(m)\, x_{2p}(n-m)$$

- Symmetry Property: If $DFS[x_p(n)] = X_p(k)$ then

3M

$$DFS[x_p^*(n)] = X_p^*(-k)$$

$$[x_p^*(-n)] = X_p^*(k)$$

$$DFS\{Re[x_p(n)]\} = DFS\left[\frac{x_p(n) + x_p^*(n)}{2}\right] = \frac{1}{2}\left[X_p(k) + X_p^*(-k)\right]$$

$$= X_{pe}(k)$$

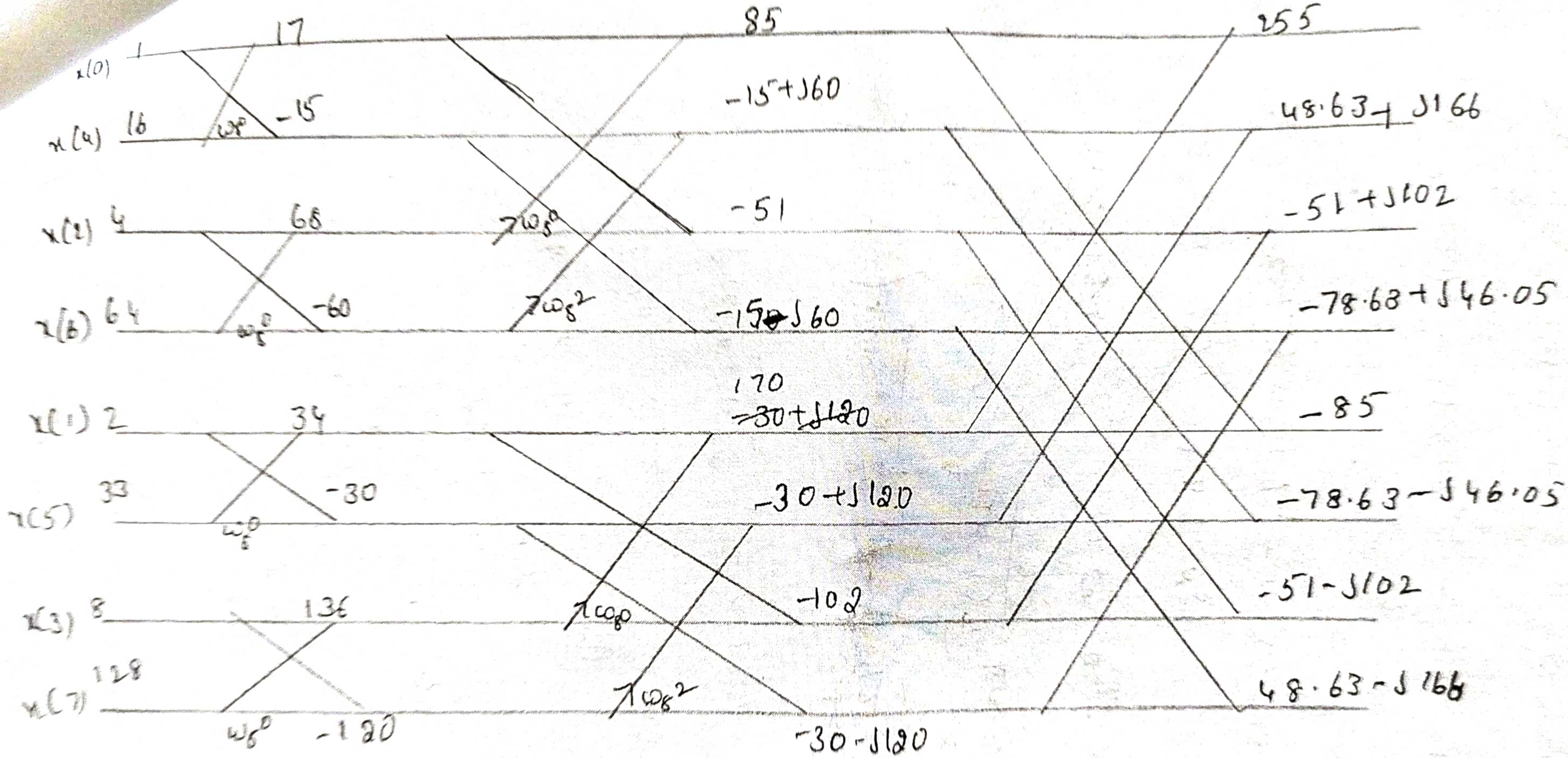
$$DFS\{j\, im[x_p(n)]\} = DFS\left[\frac{x_p(n) - x_p^*(n)}{2}\right]$$

$$= \frac{1}{2}\left[X_p(k) - X_p^*(-k)\right)$$

$$= X_{po}(k).$$

3) $x(n) = 2^n$, $N = 8$ find DIT-FFT

$x(n) = 2^n = \{1, 2, 4, 8, 16, 32, 64, 128\}$



$X(k) = \{255, 48.63+j166, -51+j102, -78.63+j46.05, -85, -78.63-j46.05, -51-j102, 48.63-j166\}$

9M

$\alpha_p = 1dB \quad \omega_p = 0.2\pi \quad \alpha_s = 15dB \quad \omega_s = 0.3\pi \quad T = 1\,sec$

$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 2\tan\frac{0.2\pi}{2} = 0.65$

$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = 1.02$

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\frac{\Omega_s}{\Omega_p}} \geq 3.01 \quad -(2M)$$

Let us take $N = 4$

$\varepsilon = \sqrt{10^{0.1\alpha_p}-1} = 0.508$ -(1M) $\quad \mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 4.17$ -(1M)

$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.237$

$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.692$

$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, 3, 4 \quad - 4M$

$\phi_1 = 112.5^\circ \quad \phi_2 = 157.5^\circ \quad \phi_3 = 202.5^\circ \quad \phi_4 = 247.5^\circ$.

$S_k = a\cos\phi_k + jb\sin\phi_k$.

$S_1 = a\cos\phi_1 + jb\sin\phi_1 = 0.237\cos 112.5^\circ + 0.6918\sin 112.5^\circ$

$= -0.0907 + j0.639$

$S_2 = a\cos\phi_2 + jb\sin\phi_2 = -0.2189 + j0.2647$

$S_3 = a\cos\phi_3 + jb\sin\phi_3 = -0.2189 - j0.2647$

$S_4 = a\cos\phi_4 + jb\sin\phi_4 = -0.0907 - j0.639.$ -(1M)

The denominator polynomial of

$$H(s) = [(s+0.0907)^2 + (0.639)^2][(s+0.2189)^2 + (0.2647)^2]$$

$$= (s^2 + 0.1814s + 0.4165)(s^2 + 0.4378s + 0.118 \quad - (2M)$$

As N is even the numerator of $H(s) = \frac{(0.4165)(0.118)}{\sqrt{1+\varepsilon^2}} = 0.04381$

the transfer function $H(s) = \frac{0.04381}{(s^2+0.1814s+0.4165)(s^2+0.4378s+0.118)}$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]} \quad - (2M)$$

$$H(z) = \frac{0.04381}{(s^2+0.1814s+0.4165)(s^2+0.4378s+0.118)}\Bigg|_{s=2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$T = 1sec$

$$= \frac{0.001836(1+z^{-1})^4}{(1-1.4992z^{-1}+0.8482z^{-2})(1-1.5548z^{-1}+0.6493z^{-2})}$$

4.b The bilinear transformation is a conformal mapping that transforms the $j\Omega$ axis into the unit circle in the z-plane only once, thus avoiding aliasing of freq components. All points in the LHP of 's' are mapped inside the unit circle in the z-plane and all points in the RHP of 's' are mapped into outside the unit circle in the z-plane

$$H(s) = \frac{b}{s+a}$$

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

$$sY(s) + aY(s) = bX(s) \quad - (2M)$$

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

trapezoidal formula

$$y(t) = \int_{t_0}^{t} y'(\tau)\,d\tau + y(t_0)$$

$$y(nT) = \frac{T}{2}\left[y'(nT) + y'(nT-T)\right] + y(nT-T)$$

$$y'(nT) = -a\,y(nT) + b\,x(nT)$$

the z-transform of this difference eq is

$$\left[1+\frac{aT}{2}\right]Y(z) - \left[1-\frac{aT}{2}\right]z^{-1}Y(z) = \frac{bT}{2}\left[1+z^{-1}\right]X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{bT}{2}\left[1+z^{-1}\right]}{1+\frac{aT}{2} - \left[1-\frac{aT}{2}\right]z^{-1}}$$

dividing nu & de by $\frac{T}{2}\left[1+z^{-1}\right]$

$$H(z) = \frac{b}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+a}$$

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] \quad - (2M)$$

$z = re^{j\omega}$ $\quad s = \sigma + j\Omega$

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2} \quad - (1M)$$

5a

$\alpha_p = 2\,dB$ $\quad \alpha_s = 20\,dB$ $\quad \Omega_p = 200\,rad/sec$ $\quad \Omega_s = 100\,rad/sec$

$\Omega_s = 200\,rad/sec$ $\quad \Omega_p = 100\,rad/sec$

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \Omega_s/\Omega_p} = \frac{\log\sqrt{\frac{10^{2}-1}{10^{0.2}-1}}}{0.301} = \frac{\log\sqrt{99/0.58}}{0.301}$$

$$= \frac{1.12}{0.301} = 3.7 \quad - (2M)$$

$N = 4$

$\Omega_c = 1\ rad/sec \quad N = 4$ is

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.8477s+1)} \quad - (2M)$$

to get high pass filter having cut off freq

$\Omega_c = \Omega_p = 200\ rad/se$

$$s \to \frac{200}{s} \qquad H_a(s) = H(s)\Big|_{s \to \frac{200}{s}} \quad - (2M)$$

$$= \frac{1}{(s^2+0.76537s+1)(s^2+1.8477s+1)}\Big|_{s\to\frac{200}{s}}$$

$$= \frac{s^4}{((200)^2 + 153.1s + s^2)((200)^2 + 369.54s + s^2)}$$

$$= \frac{s^4}{(s^2+153.1s+(200)^2)(s^2+369.54s+(200)^2)} \quad - (2M)$$

5.3 Symmetrical impulse response N is odd

$$H(e^{j\omega}) = \sum_{n=0}^{6} h(n)\, e^{-j\omega n}$$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega(N-1)/2} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n)\, e^{-j\omega n}$$

let $n = N-1-m$ we have

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega(N-1)/2} + \sum_{m=0}^{\frac{N-3}{2}} h(N-1-m)\, e^{-j\omega(N-1-m)} \quad (3M)$$

Symmetrical impulse response $h(n) = h(N-1-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega(N-1)/2} + \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega(N-1)/2}\left[\sum_{n=0}^{\frac{N-3}{2}} 2h(n)\cos\omega\left(\frac{N-1}{2}-n\right) + h\left(\frac{N-1}{2}\right)\right]$$

let $\frac{N-1}{2} - n = P$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[\sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-n\right)\cos\omega n + h\left(\frac{N-1}{2}\right)\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n.$$

$$a(0) = h\left(\frac{N-1}{2}\right) \qquad a(n) = 2h\left(\frac{N-1}{2}-n\right) \quad - \text{4M}$$

(or)

Symmetric impulse response for N even.

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega n} + \sum_{n=N/2}^{N-1} h(n)e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{N-2}{2}} h(N-1-n)e^{-j\omega(N-1-n)}$$

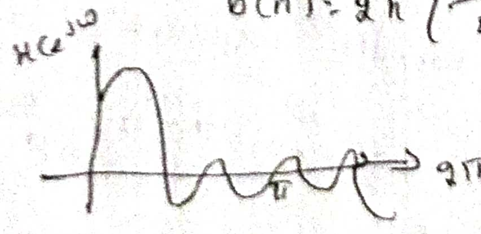
$$h(n) = h(N-1-n)$$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega(N-1)/2}\left[\sum_{n=0}^{\frac{N-2}{2}} 2h(n)\cos\omega\left(\frac{N-1}{2}-n\right)\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\sum_{n=1}^{N/2} b(n)\cos(n-1/2)\omega$$

$$b(n) = 2h\left(\frac{N}{2}-n\right)$$



6.a. Frequency sampling method of designing FIR filter

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k)\, e^{j2\pi kn/N} \qquad n = 0, 1, \dots N-1$$

&

$$H(k) = \sum_{k=0}^{N-1} h(n)\, e^{-j2\pi kn/N}$$

The DFT samples $H(k)$ for an FIR sequence can be regarded as samples of the filter z-transform

$$H(k) = H(z)\Big|_{z = e^{j2\pi k/N}}$$

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

$$H(z) = \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} H(k)\, e^{j2\pi kn/N}\right] z^{-n}$$

$$= \sum_{k=0}^{N-1} \frac{H(k)}{N} \sum_{n=0}^{N-1} H(k) \left(e^{j2\pi k/N} z^{-1}\right)^{n}$$

$$= \sum_{k=0}^{N-1} \frac{H(k)}{N} \frac{1 - \left(e^{j2\pi k/N} z^{-1}\right)^{N}}{1 - e^{j2\pi k/N} z^{-1}}$$

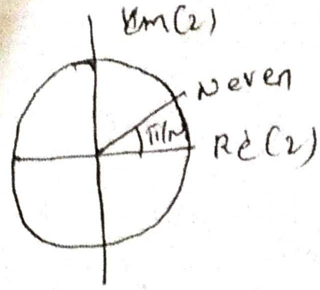
$$= \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{j2\pi k/N} z^{-1}}$$

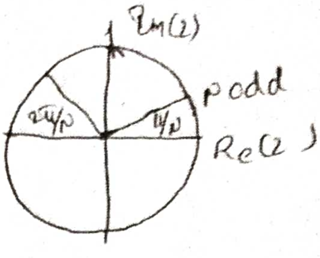
$$H(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}} = H\left(e^{j2\pi kn/N}\right) = H(k) \quad \text{— 3M}$$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}(k + 1/2)}$$

$$= H_d\left(e^{j\pi(2k+1)/N}\right) \qquad k = 0, 1, \dots N-1$$

$$h(n) = \sum_{k=0}^{N-1} H(k)\, e^{j2\pi kn/N}.$$





— (1M)

$$H(N-k-1) = H^*(k)$$

$$H\left(\frac{N-1}{2}\right) = 0$$

$$h(n) = \frac{2}{N} \sum_{K=0}^{\frac{N-3}{2}} Re\left[H(K)\, e^{j2\pi n(2k+1)/N}\right] \quad - (4M)$$

6.b

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-0.3}^{-0.2} e^{j\omega n} d\omega + \int_{0.2}^{0.3} e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi j n} \left[e^{-j0.2n} - e^{-j0.3n} + e^{j0.3n} - e^{j0.2n} \right]$$

$$= \frac{1}{\pi n} \left[\sin 0.3n - \sin 0.2n \right] \quad - (3M)$$

$h_d(n)$ to 7 samples.

$$h(n) = h_d(n)\, \omega(n) \quad \text{for } |n| \leq 3 \quad - (2M)$$

$$h(n) = h(-n)$$

$$h(0) = \frac{1}{2\pi} \left[\int_{-0.3}^{-0.2} d\omega + \int_{0.2}^{0.3} d\omega \right]$$

$$= \frac{1}{2\pi} \left[-0.2 + 0.3 + 0.3 - 0.2 \right] = \frac{0.2}{2\pi} = \frac{2}{20\pi} = 0.1\pi \quad - (1M)$$

$$h(1) = h(-1) = \frac{\sin 0.3 - \sin 0.2}{\pi} = 0$$

$$h(2) = h(-2) = \frac{\sin 0.6 - \sin 0.4}{2\pi} = 0$$

$$h(3) = h(-3) = \frac{\sin 0.9 - \sin 0.6}{3\pi} = 0$$

$N = 7$

$\omega_{Hn}(n) = 0.5 + 0.5 \cos \frac{\pi n}{5}$ for $-\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right)$ — 1M

$\omega_{Hn}(n) = 0.5 + 0.5 \cos \frac{\pi n}{5}$

$\omega_{Hn}(0) = 0.5 + 0.5 = 1$

$h(n) = h_d(n) \cdot \omega_{Hn}(n)$

$h(0) = 0.1\pi \times 1 = 0.1\pi$ $\quad h(1) = 0 ; h(2) = 0 ; h(3) = 0$

— 2M

transfer function

$H(z) = h(0) = 0.1\pi$

$H'(z) = z^{-3}\, 0.1\pi$

$= 0.1\pi z^{-3}$

$h(0) = 0, \; h(1) = 0, \; h(2) = 0, \; h(3) = 0.1\pi \; h = 4 = 0$

$h(5) = 0 \quad h(6) = 0$

2M

$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$

$a(0) = h\left(\frac{N-1}{2}\right) = h(3) = 0.1\pi$

$a(n) = 0$

7a The coefficient Quantization of IIR filters.

In the design of a digital filter, the co-efficients are evaluated with infinite precision. So the coefficients are to be represented with words of infinite length. In the realization IIR & FIR filters in hardware or software, the accuracy with which filter coefficients can be specified is limited by the word length of the register used to store the coefficients. Usually the filter coefficients are quantized to the word size of the register to store

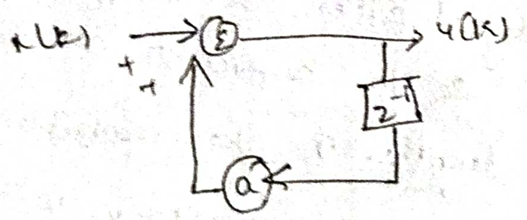
Other ... have finite ... digital filter ... coeffi...

(1M) either by truncation or rounding. But when they are quantized have finite word length, the transfer function H(z) of the digital filter implemented in hardware or software form with quantized coefficients is different from the desired transfer function H(z). The main effect of the coefficient quantization is, therefore, on the poles and zeros that move to different locations from the original desired locations. The freq response of the actual filter deviates from that which would have been obtained with an infinite word length representation, and the filter may actually fails to meet the desired specifications. The sensitivity of the filter freq response characteristics to quantization of filter co-eff is minimized by realizing the filter having large no. of poles and zeros as an interconnection of second order sections. - (1M)

7. b

A simple one - pole digital filter can be represented by the following difference equation

$$y(k) - a\,y(k-1) = x(k)$$

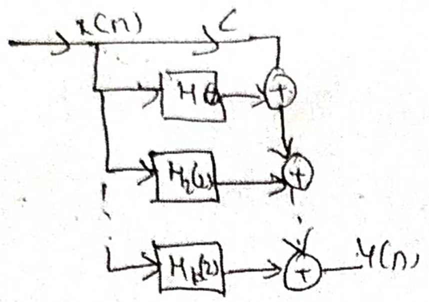


- (2M)

Quantizer is placed in the f/b path after multiplication by the coefficient a. The quantized f/b s/l is summed with the i/p s/l if the i/p s/l is quantized with the same granularity as that of the f/b s/l, addition could take place with no further parallel form

$$H(z) = C + \sum_{k=1}^{N} \frac{C_k}{1 - P_k z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = C + H_1(z) + H_2(z) - - -$$



— (2M)

8.b Statistical analysis is simple but quantization is truly a non linear effect, and should be analyzed as a deterministic process

Ex $y(k) = -0.625\, y(k-1) + u(k)$ (5M)

4-bit rounding arithmetic

$u(k) = 0 \quad y(0) = 3/8$

$y(k) = 3/8, -1/4, 1/8 --$

Oscillations in the absence of i/p ($u(k) = 0$) are called zero i/p limit cycle oscillations

Ex $y(n) = 0.95\, y(n-1) + x(n)$

$y(n) = x(n) + q\,[0.95\, y(n-1)]$

$n=0 \quad x(0) = 0.875 \quad y(-1) = 0$

$y(0) = 0.875 + 0 = 0.875$

$n=1 \quad y(1) = x(1) + q\{0.95\, y(0)$

$y(1) = q\,[0.83125]$

$(0.83125)_{10} = (\,?\,)_2$

$(0.1101)_2 = (0.8125)_{10}$

$y(2) = q[0.771875]$

Round off $(0.7125)_{10} = (0.1100)_2 = (0.75)_{10}$

$y(4) = (0.653125)_{10} = (0.101001\ldots)_2$

Round off $(0.653125)_{10} = (0.1010)_2 = 0.625$

$y(5) = (0.59375)_{10} = (0.10011)_2 = 0.625$

$y(6) = 0.625$

Dead band $= \dfrac{\frac{1}{2} 2^{-b}}{1-|\alpha|}$

$= \dfrac{\frac{1}{2} \cdot 2^{-4}}{1-0.95} = \underline{0.625}$