Analog communications

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Introduction to communication system:

"Communication" is the process of conveying message at a distance. If the distance involved is beyond the direct communication, then communication engineering comes into the picture. The branch of Engineering which deals with communication systems is known as "Telecommunication Engineering".

The communication system consists of mainly three blocks

1. Transmitter
2. Channel (or) Transmission media
3. Reciever

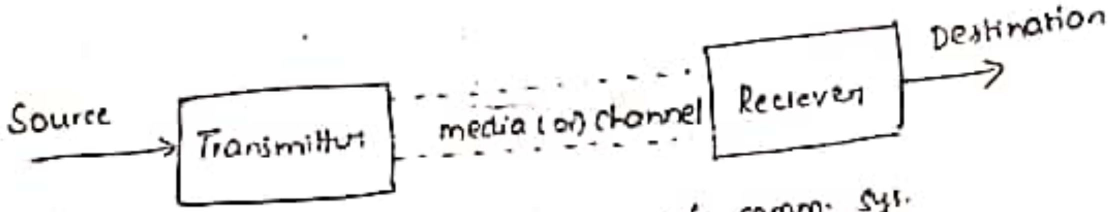

fig. Basic block diagram of comm. sys.

From the above diagram, transmitter takes the information from source and sends the information to the channel (or) media. Reciever recieves the information from channel and finally sends to destination.

The information from the source can be of many kinds such as sounds, words, pictures etc... we cannot transmit these kind of physical messages through communication channel. So, all these physical messages are converted into electrical signals and then transmitted through the channel. This conversion process is done by the device called as "Transducer"

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Transducer: Transducer is a device which converts one energy system to another energy system. Eg: Generator (which converts mechanical energy to electrical energy)

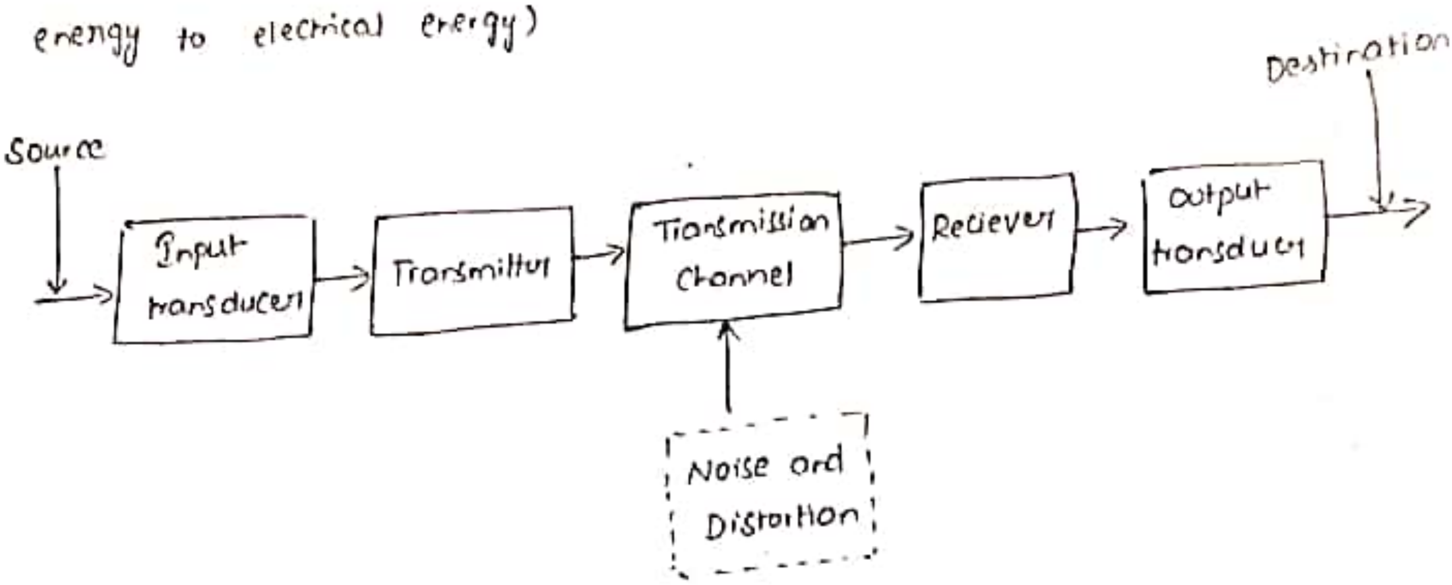

fig: communication system

Source: Sends the information or message to the system

Input transducer: This is a device which converts physical message (like words, sounds, pictures etc...) into appropriate electrical signals and gives to the transmitter.

Transmitter: The purpose of a transmitter is to modify the message signal (input signal or information signal) in a suitable form for transmission over the communication channel. This can be achieved through the process known as "Modulation".

Transmission channel: This is basically a medium which electrically connects the transmitter to the Reciever. It may be a pair of wires, a coaxial cable, free space, optical fiber or even a laser beam. The channel introduces 2 major problems such as "Noise and Distortion" for any communication system.

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Receiver: The main function of this unit is to reproduce the original message from the distorted signal available at the input of it. The reproduction of the signal is accomplished by the process known as "demodulation or detection" which is basically the reverse process of the modulation used in the transmitter.

Output transducer: It converts electrical signals into appropriate physical message which is exact replica of input signal.

Destination: It receives the information or message finally.

Modulation:

Simply to say, Modulation is "Modification of signal". The purpose of modulation is to convert the signal to a suitable form to match the transmission media. This is necessary because the message signal being a low frequency signal cannot be transmitted efficiently over the channel directly. This transmission channel is best suited for high freq signal transmission. The high frequency signals are called "carriers". Modulation is a scheme which alters some characteristics of the high frequency carrier in accordance with the low frequency message signal (i/p signal) called the modulating signal.

Need for modulation:

The process of modulation serves the following purposes.

(1) **Efficient Radiation:** In radio communication the information is transmitted in the form of electromagnetic waves from the transmitting antenna. However, for efficient radiation from the radiating element it is necessary that

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$\lambda = \frac{c}{f}$

the size of the element should be of the order of $\lambda/10$, 'λ' being the wavelength of the signal to be radiated. Unfortunately, many signals like audio signals have frequency components down to 100Hz or even less. Efficient radiation at 100Hz requires an antenna of length 300km. This is quite impractical. With the help of modulation this low frequency signal can be translated to higher frequency range and subsequently radiated efficiently from reduced size antenna.

Frequency translation: Modulation enables one to translate the signals occupying similar frequency ranges to different regions in the frequency spectrum. This allows a user to tune his radio or television set to a particular broadcasting station. In the absence of such a scheme the signals from various stations would have resulted in a jumble of interfering signals.

Multiplexing: Sometimes it is necessary to send a number of signals simultaneously between 2 points. Modulation scheme enables one to multiplex a number of signals at the same time in the single channel without any interference among themselves. This multiplexing is utilized in long distance telephony, data telemetry etc.

Reduction of Noise: Noise and Interference are 2 major limitations of communication system. These effects cannot be eliminated totally. However certain modulation schemes can suppress the noise and interference to some extent.

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- Radio frequency spectrum:

	Frequency	Designation
1	30 – 300 Hz	Extremely low frequency (ELF)
2.	300 – 3000 Hz	Voice frequency (VF)
3.	3 – 30 kHz	Very low frequency (VLF)
4.	30 – 300 kHz	Low frequency (LF)
5.	0.3 – 3 MHz	Medium frequency (MF)
6.	3 – 30 MHz	High frequency (HF)
7.	30 – 300 MHz	Very high frequency (VHF)
8.	0.3 – 3 GHz	Ultra high frequency (UHF)
9	3 – 30 GHz	Super high frequency (SHF)
10.	30 – 300 GHz	Extra high frequency (EHF)
11.	300 GHz – 300 THz	Infrared light

12. 300 THz – 3 PHz – visible light
13. 3 PHz – 30 PHz – UV light
14. 30 PHz – 300 PHz – X-rays
15. 300 PHz – 3 EHz – γ-rays
16. 3 EHz – 30 EHz – cosmic Rays.

Tera – 10^{12}
Peta – 10^{15}
Exa – 10^{18}

1 GHz = 10^9 Hz
1 MHz = 10^6 Hz
1 KHz = 10^3 Hz

Frequency division multiplexing (FDM)

It is often desirable to transmit several messages over a single channel. Multiplexing is a technique which enables one to achieve this. There are basically 2 techniques for Multiplexing namely,

→ Time division multiplexing (TDM)

→ frequency division multiplexing (FDM)

In TDM the number of signals are multiplexed in a single channel on time sharing basis. On the other hand in FDM, the multiplexed signals are kept seperated by translating them in various ranges of frequency spectrum. As an example of FDM, consider voice signals (300 Hz to 3.4 kHz) transmitted over telephone systems.

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To transmit a number of such signals over the same channel, the signals must be kept apart (by proper frequency translation) so that they do not interfere with each other and can be seperated at the reciever end.

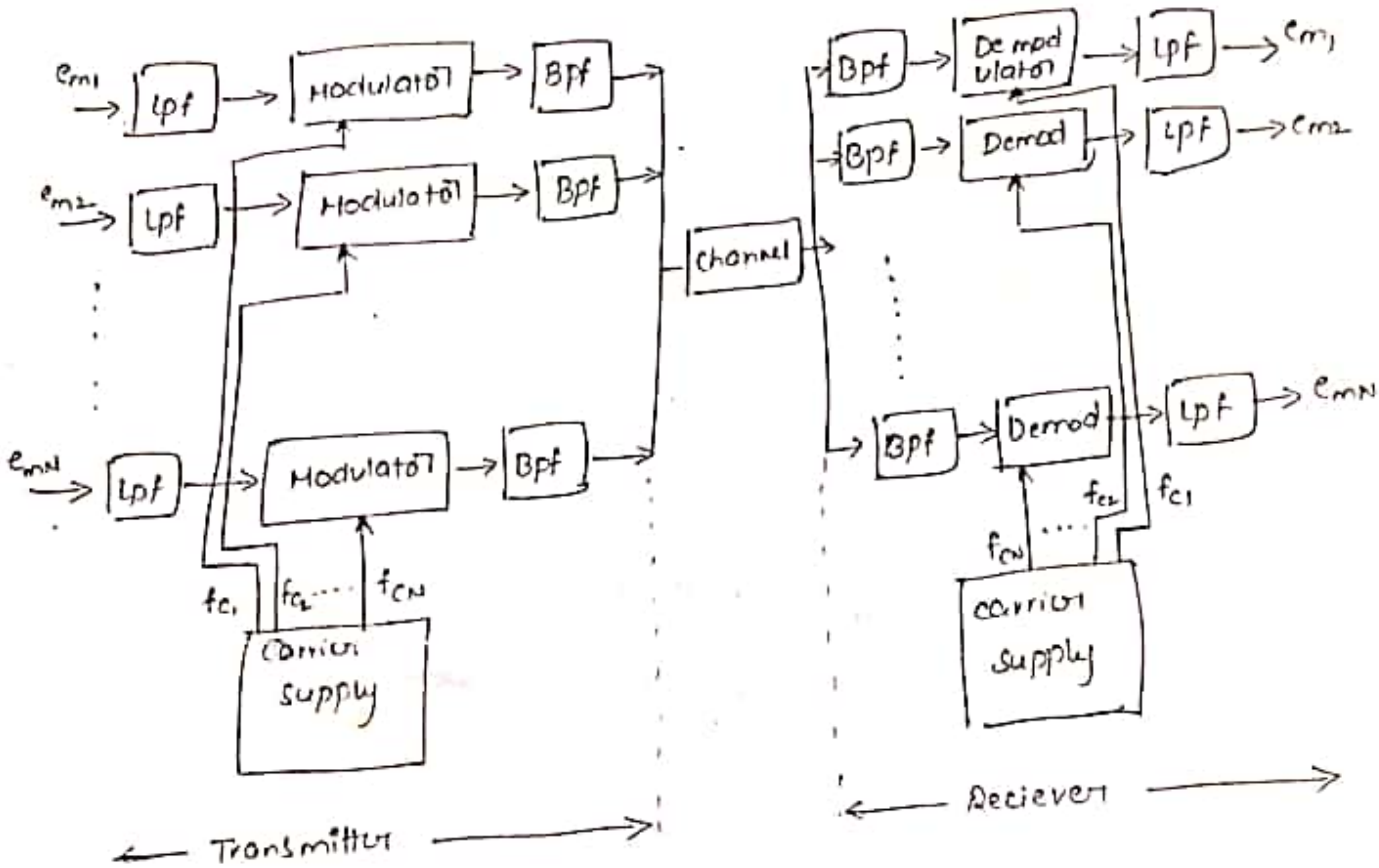

fig: Block diagram of FDM system.

In the above diagram, several inputs of a message signals individually modulate the subcarriers f_{c1}, f_{c2} .. after passing through Lpf to limit the message bandwidth. Most commonly used modulation scheme in FDM system is SSB-SC scheme in which the bandwidth of the modulated signal is equal to the BW of individual signals. The BPF's after the modulator are used to restrict the band of each modulated wave to the pescribed range. The o/p's of the BPF's are combained in parallel to form the input to the common channel.

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In the recieving end, a bank of BPF's, with their inputs connected in parallel is used to seperate the translated message signals on a frequency occupancy basis. Finally the original message signals are recovered by individual demodulators. The figure permits one-way transmission only. To provide a two way transmission (as needed in telephony) we have to duplicate the multiplexing facilities with components connected in the reverse order and with signal proceeding from right to left.

Amplitude Modulation:

Modulation → Continuous wave (CW) modulation (If carrier is sinusoidal)
Modulation → Pulse modulation (If carrier is pulse)

Definition: Amplitude of the carrier is varied in accordance with the message signal keeping frequency and phase constant.

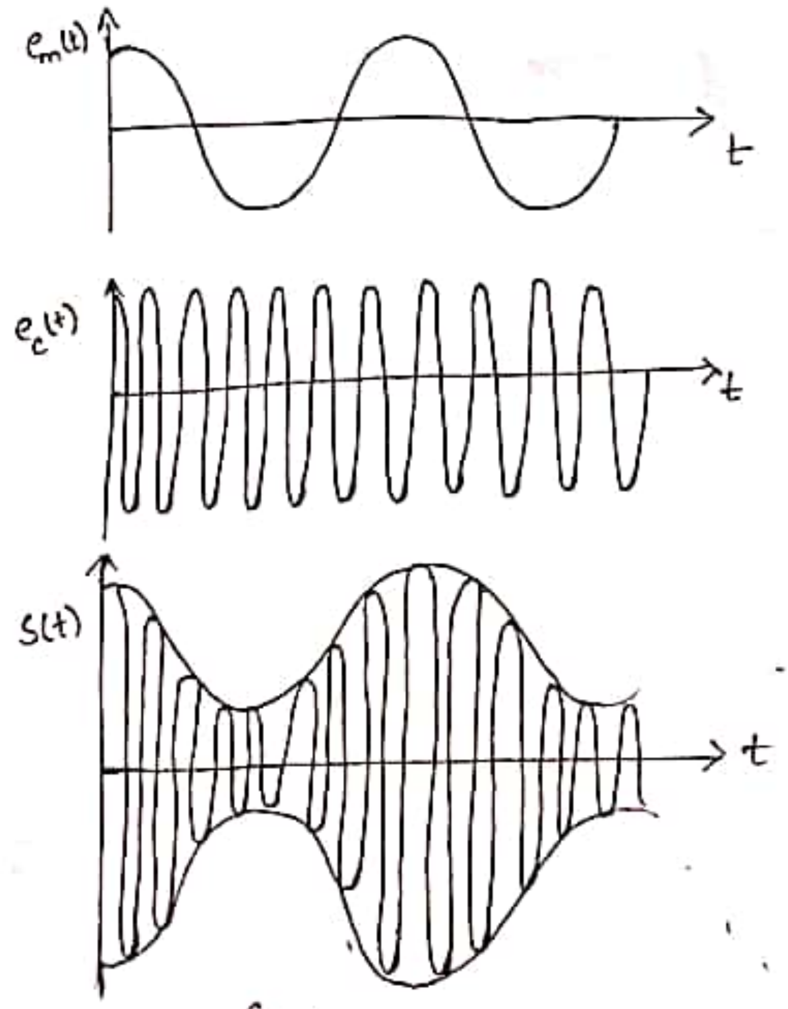

fig: A

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Time domain representation:

Consider a carrier signal $e_c(t)$ defined by

$$e_c(t) = A_c \cos(2\pi f_c t) + \phi$$

where A_c = Amplitude
f_c = Frequency
t = time (as if it is time domain)
ϕ : phase angle (constant) we can neglect ϕ

phase angle of carrier is assumed to be zero. Let the baseband or message signal be denoted by $e_m(t) = A_m \cos(2\pi f_m t)$

By definition, Amplitude modulated wave (o/p wave) is represented by the equation

$$\boxed{S(t) = A_c [1 + k_a e_m(t)] \cos 2\pi f_c t}$$

k_a = Amplitude sensitivity which is constant

The frequency of carrier assumed to be much larger than the highest frequency present in baseband signal. A closer examination of figure reveals that the envelope of the resultant modulated wave corresponds to the baseband signal (or message signal) when following requirements are satisfied.

(i) The amplitude of $k_a e_m(t)$ is always less than unity that is

$$\boxed{|k_a e_m(t)| < 1 \quad \forall t}$$

If $|k_a e_m(t)| > 1$ then o/p wave changes and results in "over modulation"

This causes phase reversal of carrier every time the factor $[1 + k_a e_m(t)]$ crosses zero.

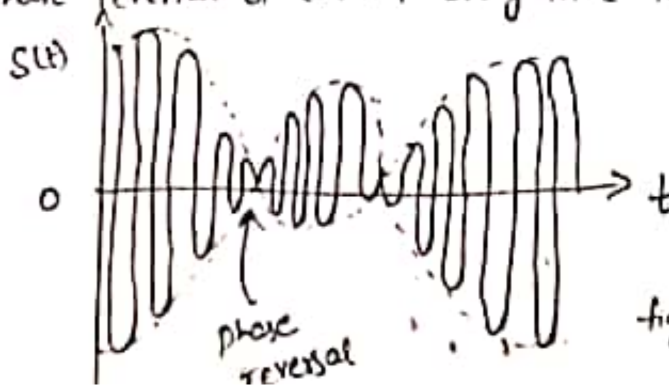

fig: over modulation when $[k_a e_m(t)] > 1$

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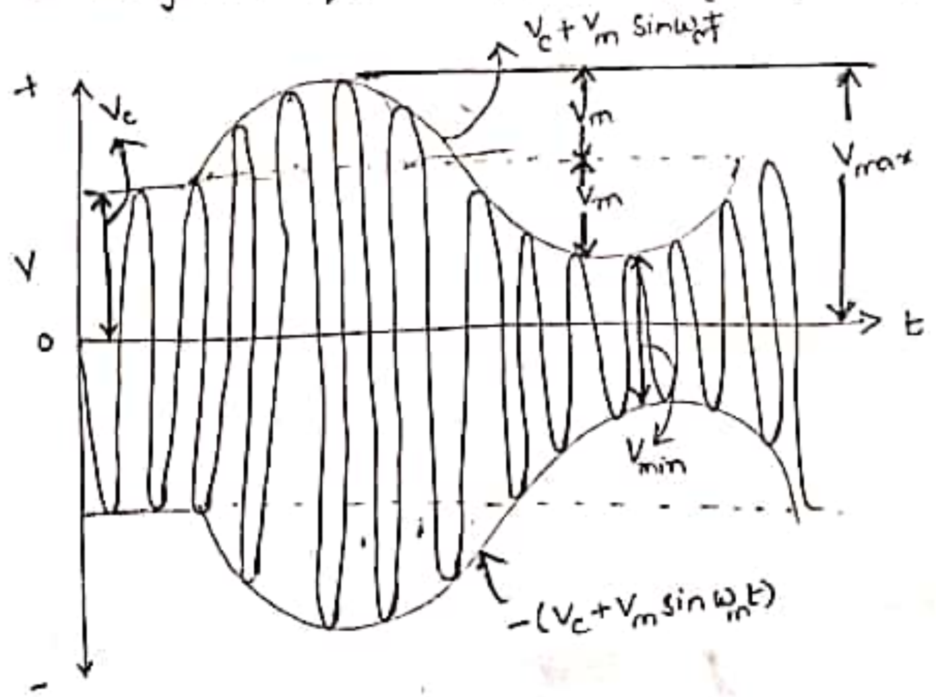
The carrier frequency f_c is much greater than the highest frequency component W in the baseband signal that is

$$f_c >> W$$

where W is message bandwidth

→ How do we get the equation $s(t) = A_c[1 + k_a\, m(t)] \cos 2\pi f_c t$?



The top envelope of AM wave given by the relation

$$A = V_c + V_m \sin\omega_m t$$

$$-A = -(V_c + V_m \sin\omega_m t)$$

Distortion will occur if V_m greater than V_c. This, and the fact that the ratio V_m/V_c often occurs, leads to definition of "modulation index"

$$m = \frac{V_m}{V_c}$$

The modulation index is a number lying b/w 0 and 1, and it is very often expressed as "percentage modulation"

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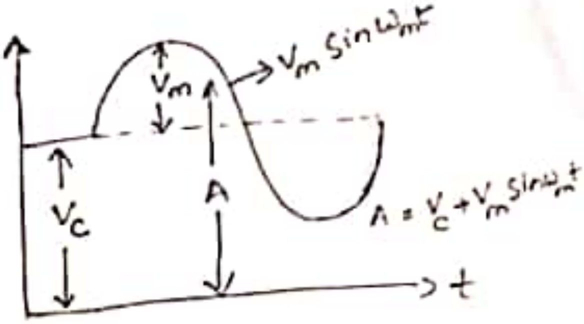
The equation for amplitude of Amplitude modulated voltage

$$A = V_c + V_m$$

$$A = V_c + V_m \sin \omega_m t$$

$$A = V_c + m V_c \sin \omega_m t$$

$$A = V_c (1 + m \sin \omega_m t)$$



The instantaneous voltage of resulting amplitude modulated wave is

$$V = A \sin \theta$$

$$\boxed{V = V_c (1 + m \sin \omega_m t) \sin \omega_c t}$$

Note: we can use A_c in place of V_c

U can use any variables, suffixes of ur own interest. No need to follow the same.

$$\sin x \sin y = \frac{1}{2} (\cos(x-y) - \cos(x+y))$$

$$\boxed{V = V_c \sin \omega_c t + \frac{m V_c}{2} \cos(\omega_c - \omega_m)t - \frac{m V_c}{2} \cos(\omega_c + \omega_m)t} \quad \text{--- (1)}$$

From previous figure

$$V_m = \frac{V_{max} - V_{min}}{2} \rightarrow (1) \quad \text{and} \quad V_c = V_{max} - V_m$$

$$V_c = V_{max} - \frac{V_{max} - V_{min}}{2}$$

$$V_c = \frac{V_{max} + V_{min}}{2} \rightarrow (2)$$

Dividing Eq. (1) & (2)

$$\boxed{m = \frac{V_m}{V_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}}$$

In eq^n (1), 1st term → Unmod. carrier
$f_c - f_m$ → LSB $f_c + f_m$ → USB.
→ BW reqd for AM is twice the freq. of modulating signal.

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Frequency domain description:

Representation of Amplitude wave in time domain is

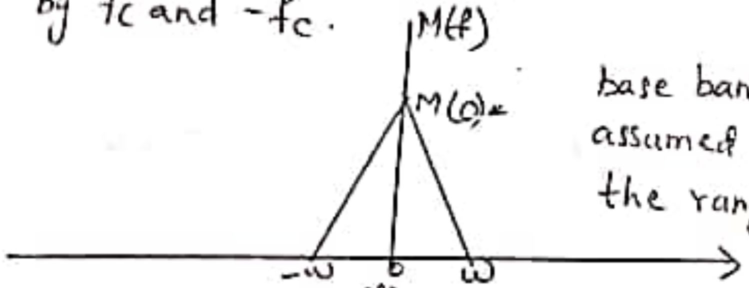
$$s(t) = A_c\left[1 + k_a e_m(t)\right]\cos 2\pi f_c t$$

To convert time domain into frequency domain, apply fourier transform on both sides.

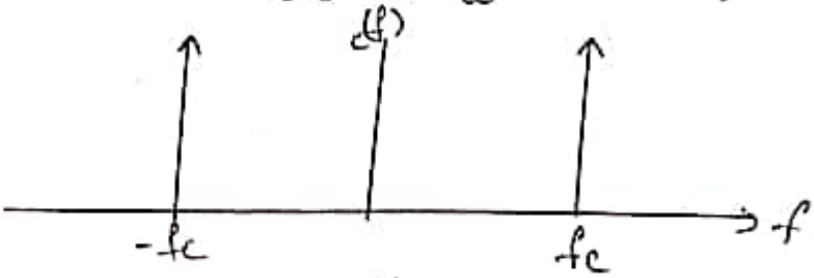
let $C(f)$, $M(f)$ and $S(f)_c$ be fourier transforms of $e_c(t)$, $e_m(t)$ and $s(t)$, we get

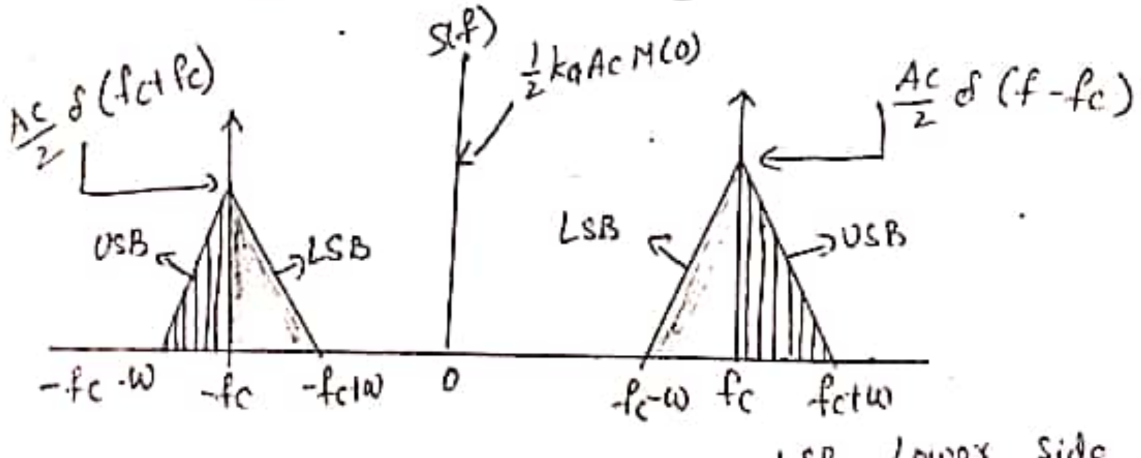
$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{k_a A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

The spectrum of amplitude modulated wave consist impulse functions occuring at f_c and $-f_c$, and original spectrum $M(f)$ shifted in frequency domain by f_c and $-f_c$.



base band signal has been assumed to be bandlimited in the range $-\omega \le f \le \omega$.





LSB - Lower Side band

USB - Upper Side band

fig: B

(i) → For positive frequencies, a portion of the spectrum of AM wave lying above the carrier frequency f_c.

The band of frequency which is lying above f_c is "upper sideband" (USB) whereas symmetrical portion below f_c is called "lower sideband" (LSB)

> For negative frequency, the upper side band is represented by the portion of the spectrum below $-f_c$ and lower side band by symmetrical portion above $-f_c$. The condition $f_c > w$ ensures that the 2 sidebands do not overlap.

(ii) For positive frequencies, the highest frequency component represent in spectrum of AM wave is f_c+W and lowest f_c-W. The difference between these 2 extreme frequencies defines the bandwidth, B_T of AM wave

$$B_T = (f_c+W) - (f_c-W)$$
$$= 2W$$

Thus, the bandwidth of the amplitude modulated wave is twice the maximum frequency present in baseband signal.

Single tone modulation:

A single tone modulation corresponds to a scheme that uses a modulating wave $e_m(t)$ consisting of single tone or frequency component.

For Eg: $e_m(t) = \underbrace{A_m}_{\text{Amplitude}} \cos(2\pi \underbrace{f_m}_{\text{frequency}} t)$

For carrier $e_c(t) = A_c \cos(2\pi f_c t)$

Am wave can be written as,

$$s(t) = A_c [1 + K_a A_m \cos 2\pi f_m t] \cos 2\pi f_c t$$
$$= A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t \text{, where } \boxed{\mu = K_a A_m}$$

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$\mu = k_a A_m$ which is the "maximum deviation" from the unmodulated carrier amplitude. (7)

'μ' is dimensionless and is called "modulation factor or modulation index"

when expressed in percentage, μ is called the "percentage modulation index"

In order to avoid overmodulation, $\mu < 1$.

The single tone amplitude modulation in time-domain is illustrated in fig: A and fig B (in before pages)

$$S(t) = A_c \cos(2\pi f_c t) + \mu A_c \cos(2\pi f_c t) \cos(2\pi f_m t)$$

Expressing product of cosine terms as sum of 2 sinusoidal waves we get.

$$S(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} [\cos(2\pi(f_c+f_m)t) + \cos(2\pi(f_c-f_m)t)]$$

The spectrum of resulting signal can be obtained by taking fourier transform on both sides

$$S(f) = \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)] + \frac{\mu A_c}{4}[\delta(f-f_c-f_m) + \delta(f+f_c+f_m)]$$
$$+ \frac{\mu A_c}{4}[\delta(f-f_c+f_m) + \delta(f+f_c-f_m)]$$

Note! Modulation index can be represented by μ or m

For detailed description for modulation index see page (5) (and include modulation index equations in this topic also)

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Power relations in Am waves:

It has been shown that the carrier component of the modulated wave has the same amplitude as the unmodulated carrier. That is, the amplitude of the carrier is unchanged: energy is either added or substracted. The modulated wave contains extra energy in sideband components. Therefore, the modulated wave contains more power than the carrier had before modulation took place. Since the amplitude of sidebands depends on the modulation index, V_m/V_c, it is anticipated that the total power in modulated wave will depend on modulation index also.

The total power in modulated wave will be

$$P_t = \frac{V_{carr}^2}{R} - \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R} \text{ (rms)} \rightarrow ①$$

Where all three voltages are (rms) values ($\sqrt{2}$ converted 2 peak) and 'R' is resistance (Eg. Antenna resistance) in which Grato power is dissipated. The first term of equation is the unmodulated carrier power and is given by

$$P_c = \frac{V_{carr}^2}{R} = \frac{(V_c/\sqrt{2})^2}{R} \qquad m = \frac{V_m}{V_c}$$

$$P_c = \frac{V_c^2}{2R} \rightarrow ②$$

$$\text{|||}^{ly} \quad P_{LSB} = P_{USB} = \frac{V_{SB}^2}{R} = \left(\frac{mV_c/2}{\sqrt{2}}\right)^2 \div R = \frac{m^2V_c^2}{8R}$$

$$= \frac{m^2}{4} \cdot \frac{V_c^2}{2R} \rightarrow ③$$

Substituting ② & ③ in ①

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$$P_t = \frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R}$$

$$P_t = P_c + \frac{m^2}{4}P_c + \frac{m^2}{4}P_c$$

$$\boxed{\frac{P_t}{P_c} = 1 + \frac{m^2}{2}}$$

Note: u can take 'm' or 'μ'

The final equation is total power in amplitude-modulated wave to the unmodulated carrier power.

Note: * $\boxed{P_t = 1.5\,P_c \quad \text{when } m = 1}$ → It is the max. power that relavant amplifiers must be capable of handlin w/o distortion.

Current calculations:

$$\frac{P_t}{P_c} = \frac{I_t^2 \cancel{R}}{I_c^2 \cancel{R}}$$

I_c – Unmodulated current

I_t – Modulated or total current

$$= \frac{I_t^2}{I_c^2} = \left(\frac{I_t}{I_c}\right)^2$$

$$\left(\frac{I_t}{I_c}\right)^2 = 1 + \frac{m^2}{2}$$

$$\frac{I_t}{I_c} = \sqrt{1 + \frac{m^2}{2}}$$

(or)

$$I_t = I_c\sqrt{1 + \frac{m^2}{2}}$$

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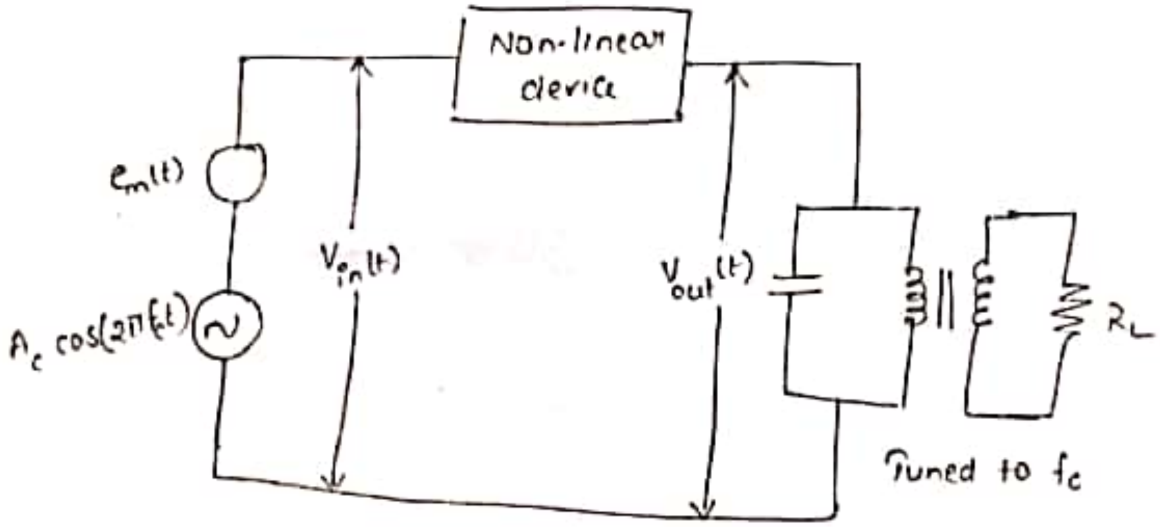
Generation of Am waves:

The device that generates amplitude modulated (Am) wave is called an "Amplitude Modulator". In this section we describe 2 methods for generating Am waves, namely (1) Square law modulator (or power law)
(2) Switching modulator

Both the methods require the use of non-linear element for implimentation. For the case of switching modulator, the nonlinear charecteristics of the element is considered to be piece-wise linear.

Square-law modulator:



The square law modulator requires a means to add up the carrier and modulating waves, a non-linear element and a Band pass filter for extracting the desired modulated wave.

The non-linear element used in square-law modulator may be diode or a transistor. commonly used filter is usually singly or doubly tuned. when a non-linear element such as a diode is suitably biased and operated in a restricted portion of its charectoristic curve, we find the transfer charecteristics of diode load resistor combination can be represented closely as "square law" i.e

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$$V_{out}(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t) \rightarrow ①$$

where a_1 and a_2 are constants

$V_{in}(t)$ and $V_{out}(t)$ are the input and output voltages respectively.

The input voltage $V_{in}(t)$ consists of carrier wave + Modulating wave

$$V_{in}(t) = A_c \cos(2\pi f_c t) + e_m(t) \rightarrow ②$$

Substitute Eq ② into Eq ①, we get the resulting voltage developed across the primary winding of the output transformer is

$$V_{out}(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} e_m(t) \right] \cos(2\pi f_c t) + a_1 e_m(t)$$

$$+ a_2 e_m^2(t) + a_2 A_c^2 \cos^2(2\pi f_c t) \rightarrow ③$$

The first term in Eq ③ is amplitude modulated (Am) wave with amplitude sensitivity $K_a = \frac{2a_2}{a_1}$

The remaining 3 terms are unwanted and can be removed by filtering.

$$V_{out}(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} e_m(t) \right] \cos(2\pi f_c t) + a_1 e_m(t)$$

$$+ a_2 e_m^2(t) + \frac{a_2 A_c^2}{2} \left(1 + \cos(2\pi \cdot 2 f_c t) \right) \Big]$$

$\because \cos^2\theta = 1 + \cos 2\theta$

Taking F.T

$$V_{out}(f) = \frac{a_1 A_c}{2} \left(\delta(f-f_c) + \delta(f+f_c) \right) + a_2 A_c \left(M(f-f_c) + M(f+f_c) \right) +$$

$$a_1 M(f) + a_2 M(f) * M(f) + \frac{a_2 A_c^2}{2} \delta(f) + \frac{a_2 A_c^2}{4} \left(\delta(f-2f_c) + \delta(f+2f_c) \right)$$

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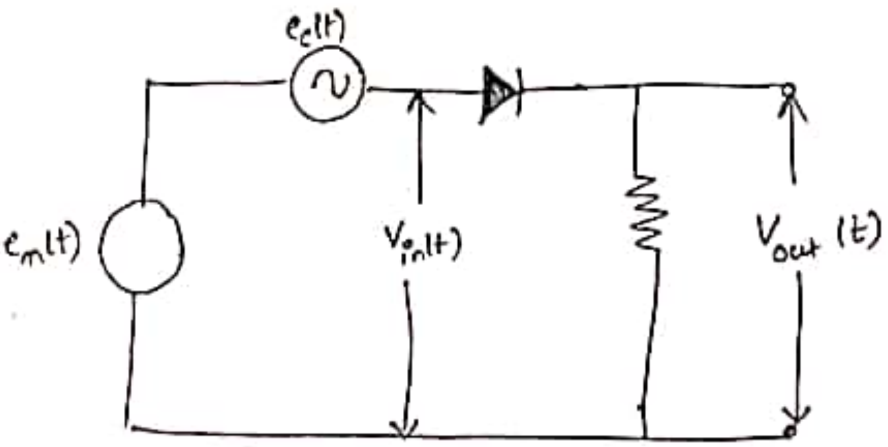
. It may not be out of point to mention here that Amplitude modulation can be classified under 2 heads namely low-level and high-level modulation. The former refers to a scheme which uses modulation at power levels lower than the final power to be transmitted. On the other hand the highlevel modulation is a scheme in which the modulation is done at higher power level which means that small amount of amplification is needed in this case after modulation to attain the final o/p power to be transmitted.

Thus substantial amount of linear amplification is necessary in case of lowlevel - modulation to bring the power upto the final value of power to be transmitted. Because of heavy filtering required, square law modulators are used primarily for low-level modulation. But RF power amplifiers of required linearity are very difficult to design and it is better to have high-level modulation if the transmitted power is to be large.

Switching Modulator:

Efficient high-level modulators are arranged so that undesired modulation products never fully develop and neednot be filtered out. This can be accomplished with the help of switching device.



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The carrier wave $e_c(t)$ applied to the diode has been assumed to be of large amplitude so as to swing right across the charecteristics curve of the diode. The diode has been assumed to be ideal in the sense that it offers zero resistance in the forward direction ($e_c(t) > 0$) and infinite resistance in reverse direction ($e_c(t) < 0$).

Thus, we approximate the transfer charecteristics of the diode load resistor combination by a piece-wise linear charecteristics.

The input voltage $V_{in}(t)$ can be written as

$$V_{in}(t) = e_c(t) + e_m(t)$$

$$= A_c \cos(2\pi f_c t) + e_m(t) \quad \text{where } |e_m(t)| << A_c$$

The resulting load voltage $V_{out}(t)$ is

$$V_{out}(t) \cong V_{in}(t) \quad ; e_c(t) > 0$$

$$\cong 0 \quad ; e_c(t) < 0$$

This means that load voltage $V_{out}(t)$ varies periodically b/w the values $V_{in}(t)$ and zero at the rate equal to carrier freq f_c.

Demodulation of Am waves:

Demodulation or Detection is the process by which the message is recovered from the modulating wave at the reciever. Demodulation is reverse of modulation.

There are 2 methods for demodulation

1) Square law demodulator
2) Envelope detector.

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Square law demodulator:

The demodulation can be achieved by using a square-law device. consider a fim non-linear device having a transfer charecteristic described by

$$V_{out}(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t) \rightarrow ①$$

where V_{in} and V_{out} are input and output voltages respectively and a_1 and a_2 are constants. In case of demodulation, input signal is AM wave (i.e o/p of modulator is i/p for demodulator). Thus

$$V_{in}(t) = A_c [1 + k_a e_m(t)] \cos 2\pi f_c t \rightarrow ②$$

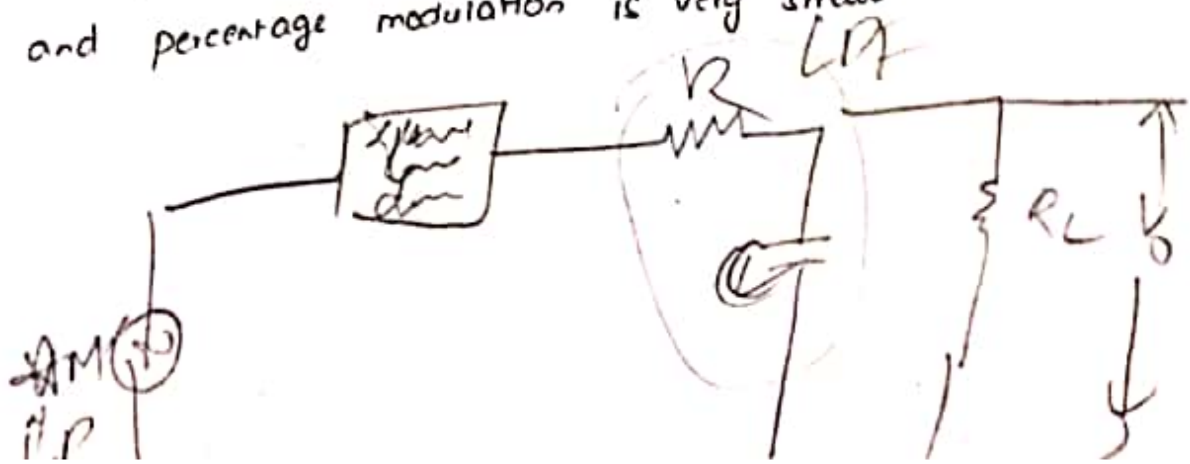
Substituting eq ② in ① we get

$$V_{out}(t) = a_1 A_c [1 + k_a e_m(t)] \cos 2\pi f_c t +$$

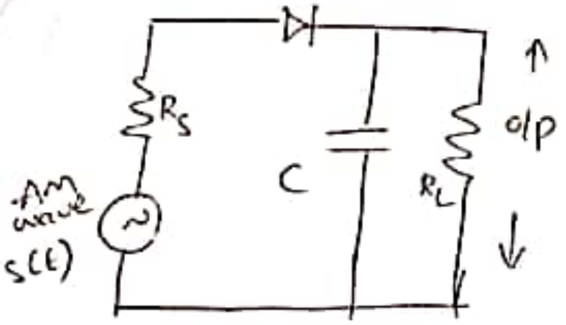
$$\frac{1}{2} a_2 A_c^2 [1 + 2 k_a e_m(t) + k_a^2 e_m^2(t)] [1 + \cos 2\pi \cdot 2 f_c t]$$

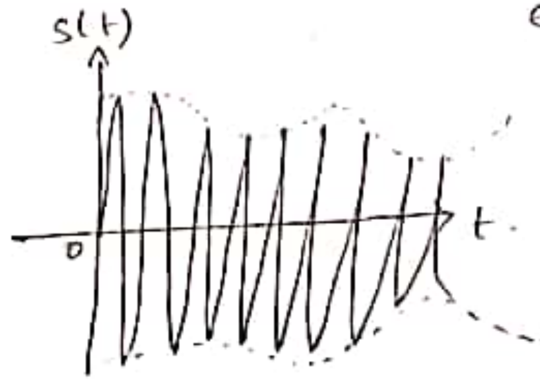
The desired signal $a_2 A_c^2 k_a e_m(t)$ is due to the square term $a_2 V_{in}^2(t)$ and hence the name square law demodulator. This component can be extracted by means of low pass filter.

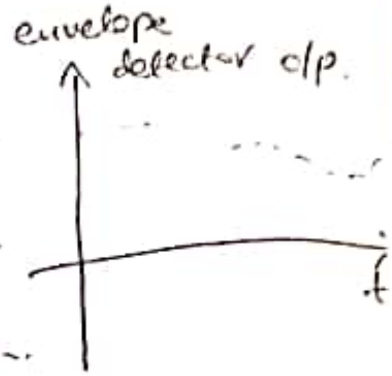
Thus we conclude that distortionless recovery of the baseband signal is possible by square law demodulator when the applied AM wave is weak and percentage modulation is very small



Envelope Detector (ED)







It is highly suited for the demodulation of a narrow band AM wave ($f_c > W$) for which $m \leq 100\%$. Ideally an envelope detector produces an o/p signal that follows the envelope of the i/p signal waveform exactly, hence the name.

ED consists of a diode & a Resistor - capacitor filter

$$R_S C << 1/f_c$$ (∵ Time const c carrier period)

Hence, 'C' charges rapidly & thereby follows the applied vol. upto +ve peak when diode is conducting.

Discharging current time const $R_L C$ is long to ensure that cap discharges slowly through R_L bet. +ve peaks of carrier wave, but not so long the V_C will not discharge @ max. rate of change of modulating wave,

$$\frac{1}{f_c} << R_L C << 1/W$$ W - message BW.

DSB Modulation

Suppressed Carrier Modulation:

The carrier of amplitude modulated wave does not convey any information. It is obvious from the fact that the carrier component remains constant in amplitude and frequency, no matter what the modulating signal does. It is thus seen that no information is carried through carrier.
For 100% modulation about 67% of total power is required for transmitting the carrier which does not contain any information.

Thus if carrier is suppressed, only the sidebands remain and a saving of two-thirds power can be achieved at 100% modulation. Such suppression of carrier does not effect the message signal in any way. The idea has resulted in evolution of "suppressed carrier modulation." Thus the short coming of conventional AM in regard of power wastage is overcome by suppressing the carrier from modulating wave, resulting in "Double sideband suppressed carrier (DSB-SC) modulation"

It is further seen that the 2 sidebands in AM are "images" of each other and both of them are effected by changes in modulating voltage amplitude through the factor $\frac{\mu A_c}{2}$. It is thus evident that all information can be conveyed by any one of the sidebands. It is therefore sufficient to transmit one sideband by suppressing the carrier and other sideband from the modulated wave.

Suppressing one sideband will also enable one to save an amount of power equal to $\frac{P_c \mu^2}{4}$ from total sideband power $\frac{P_c \mu^2}{2}$, 'P_c' being carrier power. This is new scheme called "single-sideband suppressed carrier modulation" (SSB-SC)

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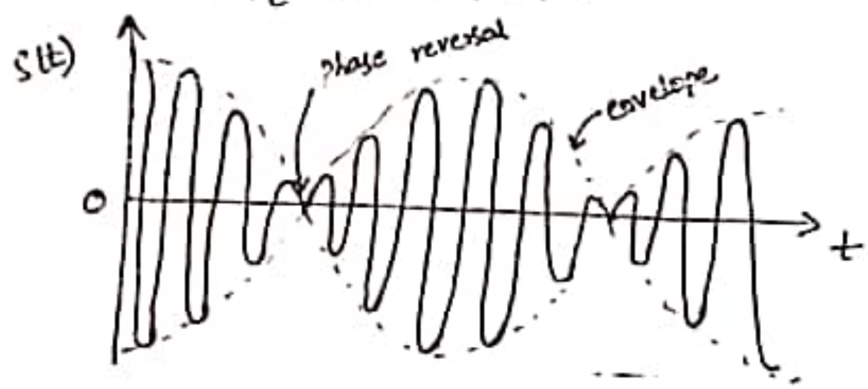
Double sideband suppressed carrier modulation (DSB-SC)

The carrier is completely independent of baseband signal $e_m(t)$ which means that the transmission of carrier wave represents a waste of power. The efficiency of transmission can be improved by suppressing the carrier from the modulating wave, resulting in DSB-SC modulation. It can be easily seen from equation (below) that by suppressing the carrier, we obtain a modulated wave that is proportional to the product of carrier wave and the baseband signal.

A DSB-SC wave as a function of time can be written as

$$\boxed{s(t) = e_c(t)\, e_m(t)} \rightarrow \text{Time domain}$$

$$= A_c\, e_m(t) \cos(2\pi f_c t)$$



1) This modulating wave undergoes a phase reversal whenever the baseband signal $e_m(t)$ acrosses zero.

2) A comparision of this signal with that of conventional Am wave reveals that the envelope of DSB-SC wave is different from baseband signal.

Taking fourier transform of s(t) in equation we get

$$\boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]} \rightarrow \text{Freq. domain.}$$

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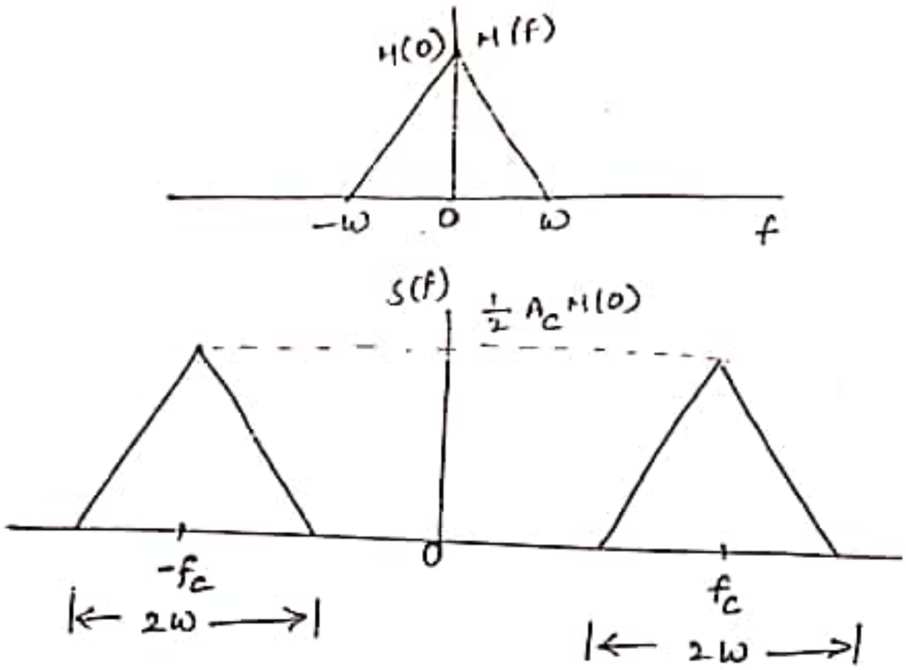

The spectrum of DSB-SC signal is shown above for baseband signal $e_m(t)$ limited to interval $-W \le f \le W$. This modulation process simply translates the baseband spectrum by $\pm f_c$. The bandwidth of the DSB-SC signal is same as that of conventional AM that is 2W (twice the bandwidth of baseband signal)

Generation of DSB-SC Signals:

"A DSB-SC signal is basically the product of the baseband signal and the carrier wave". Unfortunately, a single electronic device or component cannot generate this signal. The system for achieving this is called "product modulator" (PM).

Two types of product modulators are namely

1) Balanced Modulator
2) Ring Modulator

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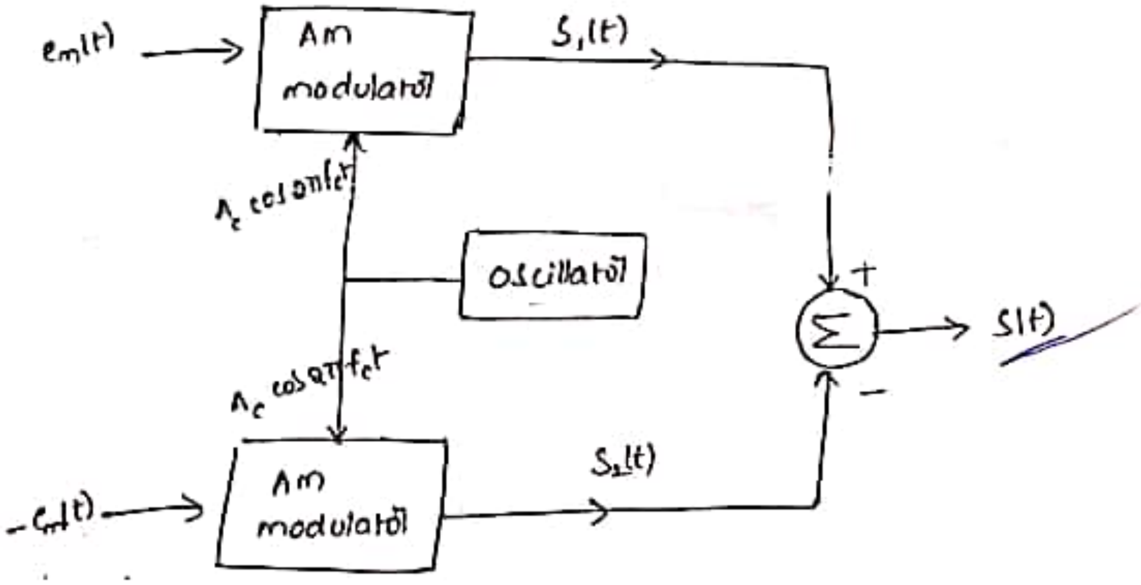
Balanced Modulator:

This is one of the possible method for generating a DSB-SC signal. It consists of two Am modulators arranged in balanced configuration to suppress the carrier. we assume that two modulators are identical. One input to each modulator is from an oscillator which generates sinusoidal carrier. other input is modulating wave.

Note: The baseband signal applied to one of the modulators has a sign reversal.



The output of two Am modulators can be expressed as

$s(t) = e_c(t) e_m(t)$

$$S_1(t) = A_c \left(1 + k_a e_m(t)\right) \cos 2\pi f_c t$$

$$S_2(t) = A_c \left(1 - k_a e_m(t)\right) \cos 2\pi f_c t$$

Substracting $S_2(t)$ from $S_1(t)$, we get

$$S(t) = S_1(t) - S_2(t)$$

$$= \cancel{A_c \cos 2\pi f_c t} + k_a e_m(t) A_c \cos 2\pi f_c t - \cancel{A_c \cos 2\pi f_c t} + A_c \cos 2\pi f_c t \, k_a e_m(t)$$

$$\boxed{S(t) = 2 A_c k_a e_m(t) \cos 2\pi f_c t}$$

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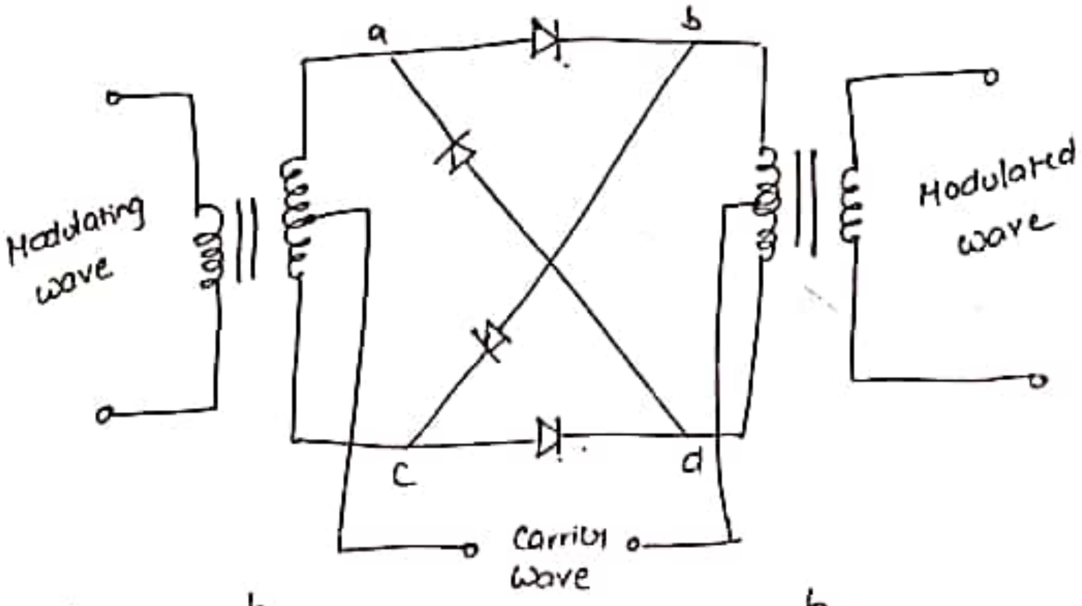
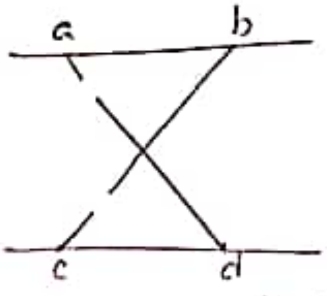
Thus, expect for a scaling factor 2ka, the balance modulator output is equal to the product of modulating wave and carrier, which is nothing but a DSB-SC wave.

Ring Modulator

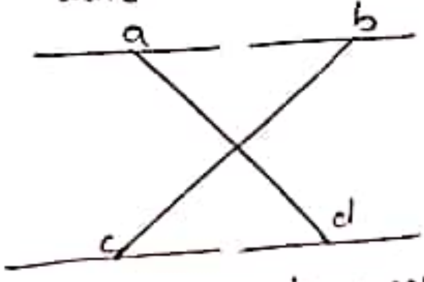
In Ring modulator 4 diodes are connected in the form of a ring in which they all point in the same way. The diodes are controlled by square wave carrier $c(t)$ of frequency f_c applied through the center tapped transformer. For ideal diodes and perfectly balanced transformers, the outer diodes are switched on when the carrier supply is positive while the inner diodes are switched off presenting high impedence.

When carrier supply is positive, the modulator multiplies baseband signal with +1. When carrier supply is negitive, the modulator multiplies baseband signal with −1 ($e_m(t)$ by −1)





When carrier is positive



When carrier is negitive.

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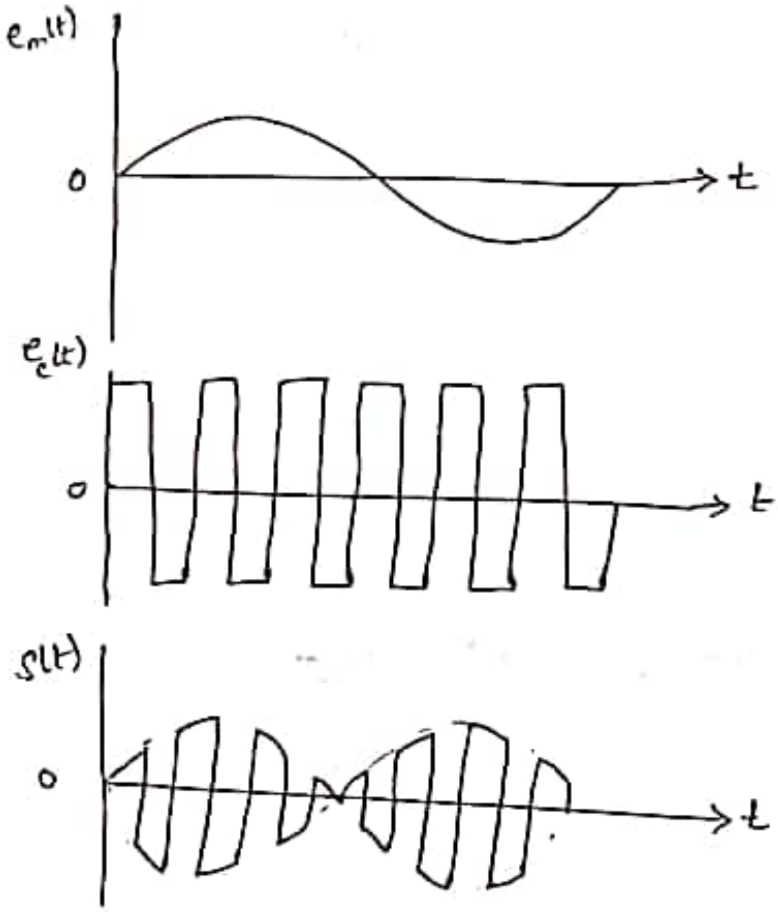
Thus, the ring modulator is a product modulator multiply the baseban for a square wave carrier and the baseband signal. The modulating wave has been assumed to be sinusoidal.



A ring modulator is often called a double-balanced modulator, (or) lattice because it is balanced with respect to the baseband signal as well as the square wave carrier.

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Demodulation or Detection of DSB-SC signals:

The DSB-SC signal can be detected by syncronous detector as well as by envelope detector after suitable carrier re-insertion.

Syncronous detection:

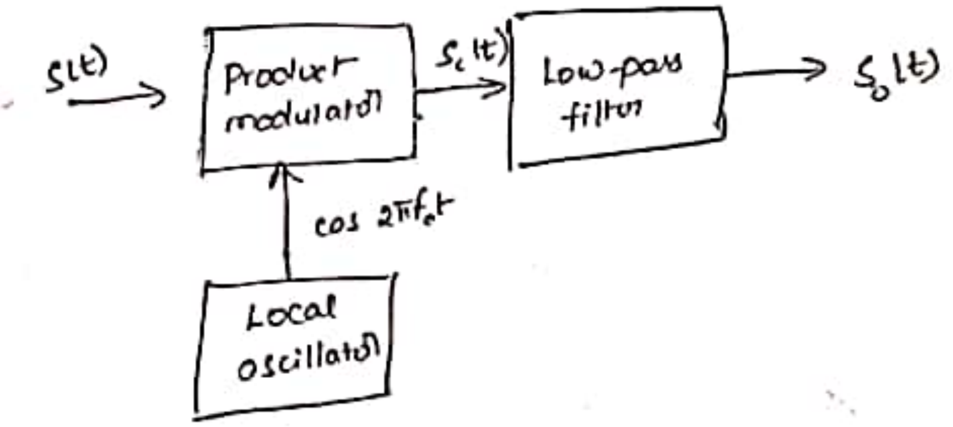
The baseband signal can be uniquely recovered from a DSB-SC wave $s(t)$ by first multiplying $s(t)$ with a locally generated carrier wave and then lowpass filtering the product. we may simply note that the output of product modulator is

$$S_c(t) = A_c\, e_m(t) \cos(2\pi f_c t) \cos(2\pi f_c t)$$

$$= A_c e_m(t) \cos^2(2\pi f_c t)$$

$$= A_c e_m(t)\ \frac{1}{2}\left(1+\cos(2\pi\, 2f_c t)\right)$$

$$= A_c \frac{1}{2}\, e_m(t) + \frac{A_c}{2} e_m(t) \cos(2\pi\, 2f_c t)$$



Thus, the baseband signal reappears after filtering out the highpass signal corresponding to second term. The second term respresents a DSB-SC wave with carrier frequency $2f_c$.

In the above case the local oscillator signal has been assumed to have exactly the same phase and frequency as the carrier used in the generation of DSB-SC signal.

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This means the local oscillator must be coherent or synchronous in respect of frequency and phase of original carrier used in the modulation. This method is therefore known as coherent or synchronous detection.

In order to demodulate DSB-SC signal by synchronous detection technique, one must generate a local carrier of same frequency and phase angle at receiver side. Any discrepancy in frequency and phase of local carrier gives rise to a distortion in detector output.

We consider the following two situations.

1) The local oscillator has an identical frequency but arbitrary phase difference ϕ measured with respect to carrier $e_c(t)$: phase error

2) The local oscillator has identical phase but a difference in frequency with respect to carrier $e_c(t)$: frequency error

Phase error:

Denoting the multiplying carrier by $A_c' \cos(2\pi f_c t + \phi)$, '$\phi$' being phase difference between local oscillator signal and carrier $e_c(t)$, we get

$$s_c(t) = A_c A_c' \cos(2\pi f_c t + \phi) \cos(2\pi f_c t)\ e_m(t)$$

$$= \frac{A_c A_c'}{2} e_m(t) \left[\cos(2\pi 2f_c t + \phi) + \cos\phi \right]$$

$$= \frac{A_c A_c'}{2} e_m(t) \cos(2\pi 2f_c t + \phi) + \frac{A_c A_c'}{2} e_m(t) \cos\phi$$

The first term represents a DSB-SC wave with a carrier $2f_c$ whereas the second term is proportional to baseband signal $e_m(t)$.

First term is easily removed by LPF

The cutoff frequency of this filter is greater than W but less than $2f_c - W$. Thus at the output of filter we obtain,

$$S_o(t) = \frac{1}{2} A_c A_c' \, e_m(t) \cos\phi$$

The demodulator output is thus proportional to $e_m(t)$ when phase error ϕ is constant. The amplitude of demodulated signal is maximum when $\phi = 0$ and minimum (zero) when $\phi = \frac{\pi}{2}$. The zero demodulated signal, which occurs for $\phi = \pm\frac{\pi}{2}$ represents "Quadrature null effect" of coherent detector. So long as the phase error ϕ is constant, it provides an undistorted version of the original baseband signal $e_m(t)$. Unfortunately, due to random variation in the comm. channel, the multiplication factor $\cos\phi$ varies randomly with time. This results in distortion of the signal. Therefore necessary arrangements should be made at the receiver end to maintain the local oscillator in perfect syncronism, in both frequency and phase with the carrier wave used to generate DSB-SC wave in the transmitter. This increases "complexity of Reciever" and increases the cost.

Frequency error: Suppose that the local oscillator signal is not precisely at frequency f_c but is instead $f_c + \Delta f$ while the phase difference between the two local oscillator signal and the $e_c(t)$ is zero. The o/p of product modulator will be

$$S_c(t) = A_c A_c' \, e_m(t) \cos(2\pi f_c t) \cos(2\pi (f_c + \Delta f)t)$$

$$= \frac{A_c A_c'}{2} e_m(t) \left[\cos\left[2\pi (2f_c + \Delta f)t\right] + \cos(2\pi \Delta f t) \right]$$

when passed through LPF we get $S_o(t) = \frac{A_c A_c'}{2} e_m(t) \cos(2\pi \Delta f t)$

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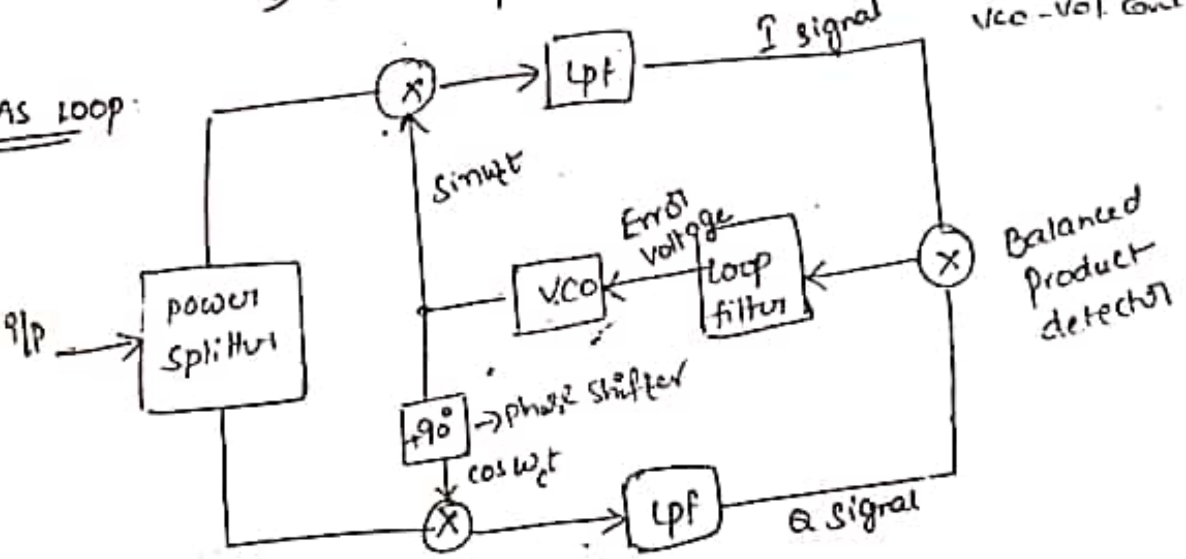
The resulting signal will wax and wane or may even be entirely acceptable if Δf is comparable to, or larger than, the frequency present in the baseband signal (message)

The frequency and phase of carrier is recovered by using two different techniques

1) Squaring Loop
2) COSTAS LOOP.

vco - vol. controlled osc.

COSTAS LOOP:



This method of carrier recovery is called COSTAS Loop or quadrature loop. This recovery scheme uses a parallel tracking Loops (I and Q) simultaneously to derive the product of I and Q components of the signal that derives the vco. The Inphase (I) loop uses vco as in pll and the quadrature (Q) loop uses a $90°$ shifted vco signal. Once the frequency of the vco is equal to the suppressed - carrier frequency, the product of the I and Q signals will produce an error voltage proportional to any phase error in the vco. The error voltage controls the phase and thus the frequency of the vco.

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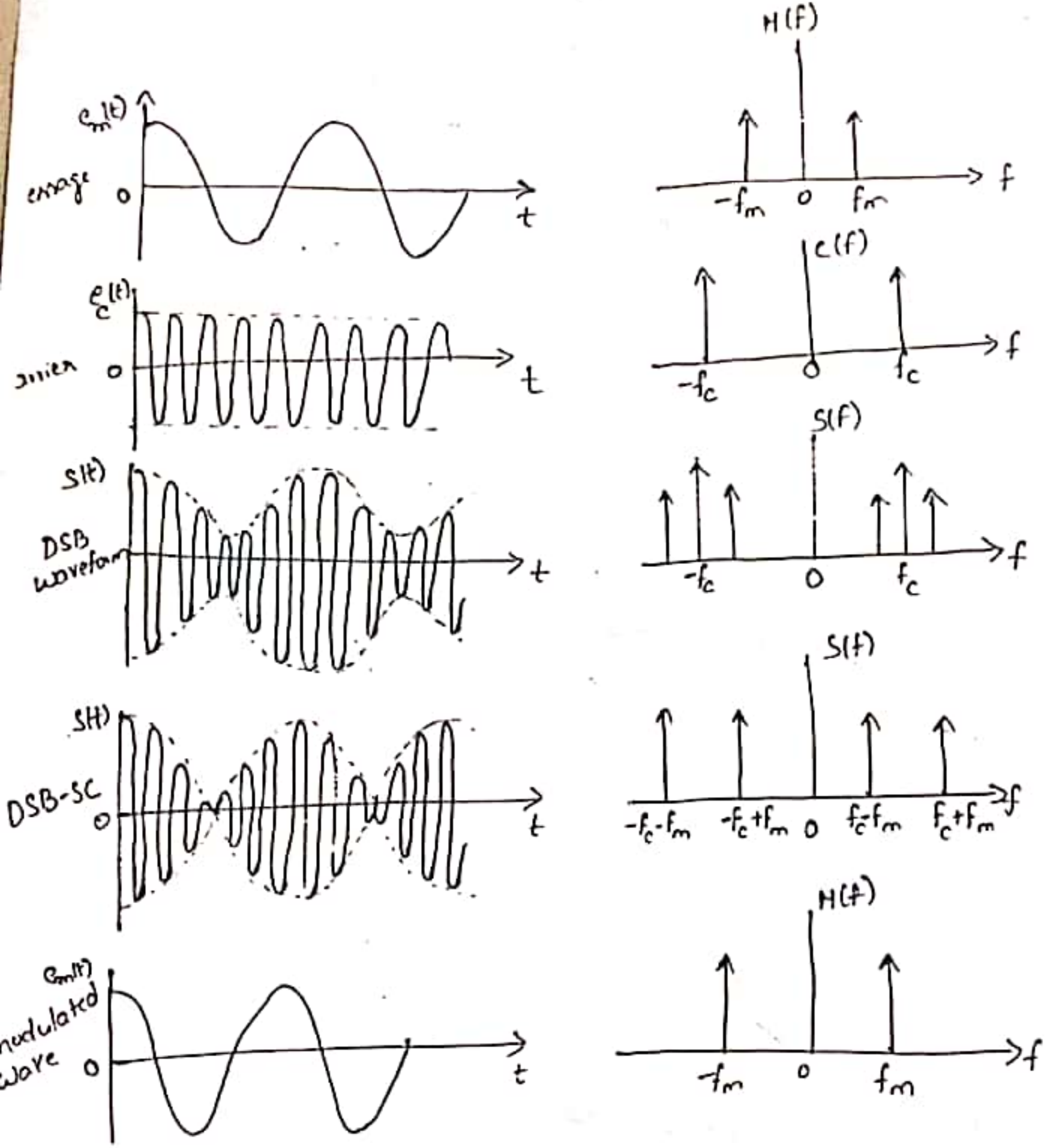
∴ Waveforms in time domain & frequency domain.



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Single tone modulated DSB-SC

Assuming the baseband signal to be a single tone sinusoidal signal of frequency f_m, we write

$$e_m(t) = A_m \cos(2\pi f_m t)$$

$$e_c(t) = A_c \cos(2\pi f_c t)$$

$$s(t) = e_m(t) \cdot e_c(t)$$

It is in form of $\cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$

$$= A_c A_m \underbrace{\cos(2\pi f_c t)(\cos 2\pi f_m t)}$$

$$= \frac{A_c A_m}{2}\left[\cos(2\pi(f_c+f_m)t) + \cos(2\pi(f_c-f_m)t)\right]$$ → Time domain

Frequency spectrum is

$$S(f) = \frac{A_c A_m}{4}\left[\delta(f-f_c-f_m) + \delta(f+f_c+f_m) + \delta(f-f_c+f_m) + \delta(f+f_c-f_m)\right]$$

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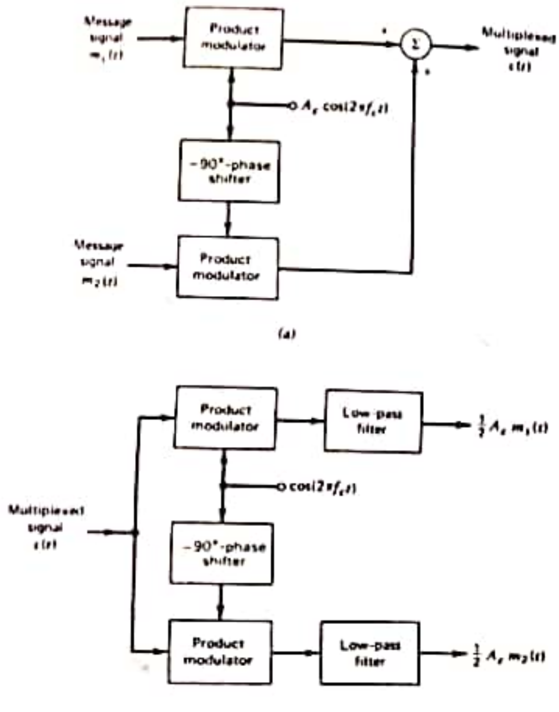


Figure 7.16
Quadrature-carrier multiplexing system. (a) Transmitter. (b) Receiver.

the system. This requirement may be satisfied, for example, by using a Costas loop; see Section 7.2.

7.4 SINGLE-SIDEBAND MODULATION

Standard amplitude modulation and double-sideband suppressed-carrier modulation are wasteful of bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth. In either

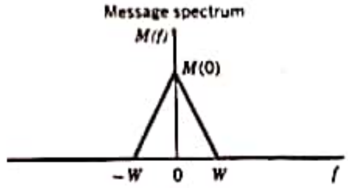
related to each other by virtue of their symmetry about the carrier frequency; that is, given the amplitude and phase spectra of either sideband, we can uniquely determine the other. This means that insofar as the transmission of information is concerned, only one sideband is necessary, and if both the carrier and the other sideband are suppressed at the transmitter, no information is lost. Thus the channel needs to provide only the same bandwidth as the message signal, a conclusion that is intuitively satisfying. When only one sideband is transmitted, the modulation is referred to as *single-sideband modulation*.

In the study of standard amplitude modulation and double sideband-suppressed carrier modulation, pursued in Sections 7.1 and 7.2, we first formulated a time-domain description of the modulated wave and then moved on to its frequency-domain description. In the study of single-sideband modulation, we find it easier in conceptual terms to reverse the order in which these two descriptions are presented.

FREQUENCY-DOMAIN DESCRIPTION

The precise frequency-domain description of a *single-sideband (SSB) modulated* wave depends on which sideband is transmitted. Consider a message signal $m(t)$ with a spectrum $M(f)$ limited to the band $-W \le f \le W$, as in Fig. 7.17*a*. The spectrum of the DSBSC modulated wave, obtained by multiplying $m(t)$ by the carrier wave $A_c \cos(2\pi f_c t)$, is as shown in Fig. 7.17*b*. The upper sideband is represented in duplicate by the frequencies above f_c and those below $-f_c$; and when only the upper sideband is transmitted, the resulting SSB modulated wave has the spectrum shown in Fig. 7.17*c*. Likewise, the lower sideband is represented in duplicate by the frequencies below f_c (for positive frequencies) and those above $-f_c$ (for negative frequencies); and when only the lower sideband is transmitted, the spectrum of the corresponding SSB modulated wave is as shown in Fig. 7.17*d*. Thus the essential function of SSB modulation is to translate the spectrum of the modulating wave, either with or without inversion, to

Message spectrum



(*a*)

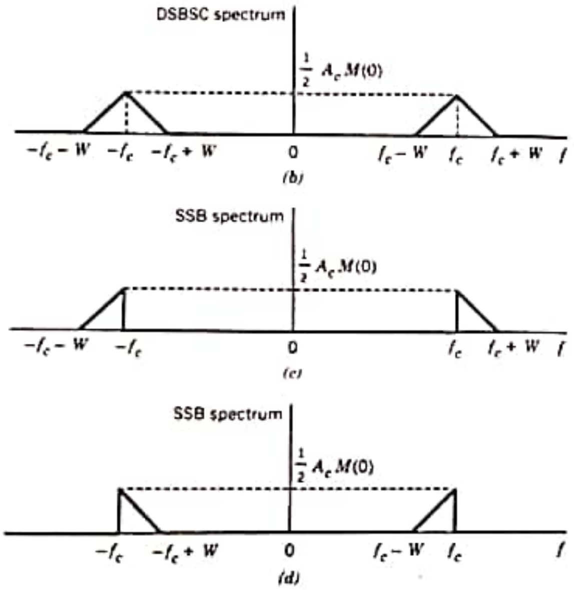


Figure 7.17 *(continued)*

a new location in the frequency domain. Moreover, the transmission bandwidth requirement of an SSB modulation system is one half that of a standard AM or DSBSC modulation system. The benefit of using SSB modulation is therefore derived principally from the reduced bandwidth requirement and the elimination of the high-power carrier wave. The principal disadvantage of SSB modulation, however, is the cost and complexity of its implementation.

FREQUENCY DISCRIMINATION METHOD FOR GENERATING AN SSB MODULATED WAVE

The frequency-domain description presented for SSB modulation leads us naturally to the *frequency discrimination method* for generating an SSB modulated wave. Application of the method, however, requires that the message signal satisfy two conditions:

(speech or music). In telephony, for example, the useful frequency content of a speech signal is restricted to the band 0.3–3.4 kHz, thereby creating an *energy gap* from zero to 300 Hz.

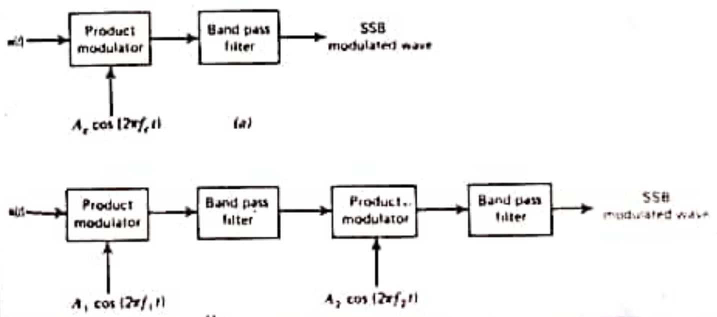
2. The highest frequency component W of the message signal $m(t)$ is much less than the carrier frequency f_c.

Then, under these conditions, the desired sideband will appear in a nonoverlapping interval in the spectrum in such a way that it may be selected by an appropriate filter. Thus an SSB modulator based on frequency discrimination consists basically of a product modulator and a filter designed to pass the desired sideband of the DSBSC modulated wave at the product modulator output and reject the other sideband. A block diagram of this modulator is shown in Fig. 7.18*a*. The most severe requirement of this method of SSB generation usually arises from the unwanted sideband, the nearest frequency component of which is separated from the desired sideband by twice the lowest frequency component of the message signal.

In designing the band-pass filter in the SSB modulation scheme of Fig. 7.18*a*, we must therefore satisfy two basic requirements:

1. The passband of the filter occupies the same frequency range as the spectrum of the desired SSB modulated wave.
2. The width of the guardband of the filter, separating the passband from the stopband where the unwanted sideband of the filter input lies, is twice the lowest frequency component of the message signal.

We usually find that this kind of frequency discrimination can be satisfied only by using highly selective filters, which can be realized using crystal resonators with a Q factor per resonator in the range of 1000 to 2000.



When it is necessary to generate an SSB modulated wave occupying a frequency band that is much higher than that of the message signal (e.g., translating a voice signal to the high-frequency region of the radio spectrum), it becomes very difficult to design an appropriate filter that will pass the desired sideband and reject the other using the simple arrangement of Fig. 7.18*a*. In such a situation it is necessary to resort to a multiple-modulation process so as to ease the filtering requirement. This approach is illustrated in Fig. 7.18*b* involving two stages of modulation. The SSB modulated wave at the first filter output is used as the modulating wave for the second product modulator, which produces a DSBSC modulated wave with a spectrum that is symmetrically spaced about the second carrier frequency f_2. The frequency separation between the sidebands of this DSBSC modulated wave is effectively twice the first carrier frequency f_1, thereby permitting the second filter to remove the unwanted sideband.

TIME-DOMAIN DESCRIPTION

The spectra shown in Fig. 7.17 clearly display the frequency-domain description of SSB modulated waves; also, they highlight the relation between this frequency-domain description and that of the message signal. It is interesting to observe that we were able to relate the spectral content of SSB modulated waves to that of the message signal without having to resort to the use of mathematics. But how do we define an SSB modulated wave in the time domain? The answer to this question is desired not only because it completes the description of SSB modulated waves but also it provides the mathematical basis of another method for their generation. Unfortunately, the task of developing the time-domain description of SSB modulated waves is mathematically more difficult than that of standard AM or DSBSC modulated waves. To solve the problem, we use the idea of a complex envelope, which was discussed in Section 3.5.

Consider first the mathematical representation of an SSB modulated wave $s_u(t)$, in which only the upper sideband is retained. The spectrum of this modulated wave is depicted in Fig. 7.17*c*. We recognize that $s_u(t)$ may be generated by passing a DSBSC modulated wave through a band-pass filter of transfer function $H_u(f)$. The DSBSC spectrum is illustrated in Fig. 7.17*b*, which corresponds to the message spectrum $M(f)$ of Fig. 7.17*a*. As for the transfer function $H_u(f)$, ideally, it has the frequency dependence shown in Fig. 7.19*a*.

The DSBSC modulated wave is defined by

$$s_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t) \tag{7.39}$$

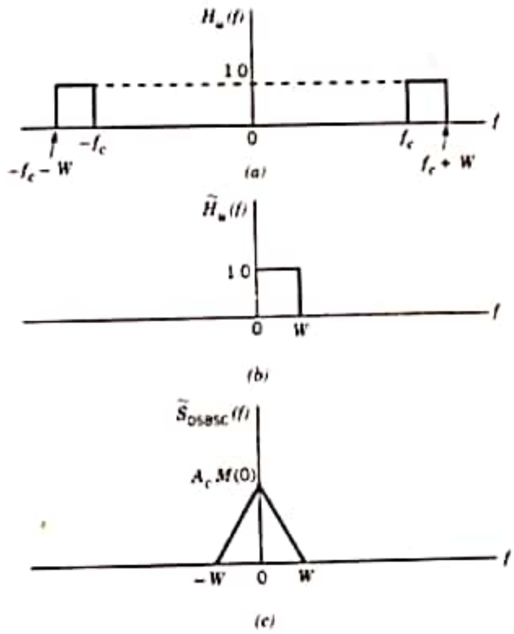


Figure 7.19
(a) Frequency response of ideal band-pass filter for selecting the upper sideband of a DSBC modulated wave. (b) Frequency response of equivalent low-pass filter. (c) Spectrum of complex envelope of DSBSC modulated wave.

low-pass *complex envelope* of the DSBSC modulated wave is given by

$$\tilde{s}_{DSBSC}(t) = A_c m(t) \tag{7.40}$$

The SSB modulated wave $s_u(t)$ is also a band-pass signal. However, unlike the DSBSC modulated wave, it has a quadrature as well as an in-phase component. Let the low-pass signal $\tilde{s}_u(t)$ denote the complex envelope of $s_u(t)$. We may then write

$$s_u(t) = \mathrm{Re}[\tilde{s}_u(t) \exp(j2\pi f_c t)] \tag{7.41}$$

To determine $\tilde{s}_u(t)$, we proceed as follows (see Section 3.5):

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7.19b. From this figure, we see that $\tilde{H}_u(f)$ may be expressed as

$$\tilde{H}_u(f) = \begin{cases} \tfrac{1}{2}[1 + \operatorname{sgn}(f)], & 0 < f < W \\ 0, & \text{otherwise} \end{cases} \tag{7.42}$$

where $\operatorname{sgn}(f)$ is the signum function.

2. The DSBSC modulated wave is replaced by its complex envelope. The spectrum of this envelope is as shown in Fig. 7.19c, which follows from Eq. 7.40. That is to say,

$$\tilde{S}_{DSBSC}(f) = A_c M(f) \tag{7.43}$$

3. The desired complex envelope $\tilde{s}_u(t)$ is determined by evaluating the inverse Fourier transform of the product $\tilde{H}_u(f)\tilde{S}_{DSBSC}(f)$. Since, by definition, the message spectrum $M(f)$ is zero outside the frequency interval $-W < f < W$, we find from Eqs. 7.42 and 7.43 that

$$\tilde{H}_u(f)\tilde{S}_{DSBSC}(f) = \frac{A_c}{2}[1 + \operatorname{sgn}(f)]M(f) \tag{7.44}$$

Given that $m(t) \rightleftharpoons M(f)$, we find (from Example 3 of Chapter 3) that the corresponding Fourier transform pair for $\hat{m}(t)$, the Hilbert transform of $m(t)$, is

$$\hat{m}(t) \rightleftharpoons -j \operatorname{sgn}(f) M(f) \tag{7.45}$$

Accordingly, the inverse Fourier transformation of Eq. 7.44 yields

$$\tilde{s}_u(t) = \frac{A_c}{2}[m(t) + j\hat{m}(t)] \tag{7.46}$$

which is the desired result.

Having determined $\tilde{s}_u(t)$, we are now ready to formulate the mathematical description of the SSB modulated wave $s_u(t)$. Specifically, placing Eq. 7.46 in Eq. 7.41, we get

$$s_u(t) = \frac{A_c}{2}[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)] \tag{7.47}$$

This equation reveals that, except for a scaling factor, a modulated wave containing only an upper sideband has an in-phase component equal to

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EXERCISE 8 Let $s_l(t)$ denote an SSB modulated wave in which only the lower sideband is retained. To determine $s_l(t)$, proceed as follows:

1. Identify the transfer function $H_l(f)$ of a band-pass filter the output of which equals $s_l(t)$ in response to a DSBSC modulated wave.
2. Determine the transfer function $\tilde{H}_l(f)$ of the equivalent low-pass filter corresponding to $H_l(f)$.
3. Hence, using the results in parts (1) and (2), show that $s_l(t)$ is given by

$$s_l(t) = \frac{A_c}{2}[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)] \tag{7.48}$$

What are the in-phase and quadrature components of $s_l(t)$?

DISCUSSION

Equations 7.47 and 7.48 are *canonical* representations of upper and lower sidebands modulated on a carrier of frequency f_c. These two equations clearly demonstrate how the upper and lower sidebands can be isolated from each other by subtracting or adding the outputs of two product modulators. The modulators differ from each other by the insertion of $-90°$ phase shifts between the modulating waves as well as between the carrier waves at their inputs; we will have more to say on this issue when we revisit the generation of SSB modulated waves. The mathematical complexity of Eqs. 7.47 and 7.48, involving not only the message signal $m(t)$ but also its Hilbert transform $\hat{m}(t)$, makes it difficult for us to sketch the waveforms of SSB modulated waves, in general. We therefore have to resort to the use of single-tone modulation in order to infer time-domain properties of SSB modulation.

EXAMPLE 3 SINGLE-TONE MODULATION (CONTINUED)

Consider again the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t) \tag{7.49}$$

The Hilbert transform of this signal is obtained by passing it through a $-90°$ phase shifter, which yields

$$\hat{m}(t) = A_m \sin(2\pi f_m t) \tag{7.50}$$

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by

$$\begin{aligned} s(t) &= \tfrac{1}{2}A_cA_m[\cos(2\pi f_m t)\cos(2\pi f_c t) - \sin(2\pi f_m t)\sin(2\pi f_c t)] \\ &= \tfrac{1}{2}A_cA_m\cos[2\pi(f_c + f_m)t] \end{aligned} \tag{7.51}$$

This is exactly the same as the result obtained by suppressing the lower side-frequency $f_c - f_m$ of the corresponding DSBSC wave of Eq. 7.35. The SSB wave of Eq. 7.51 and its spectrum are illustrated in Fig. 7.3*e*.

Next, using Eq. 7.48, we find that the SSB wave, obtained by transmitting only the lower side-frequency, is defined by

$$\begin{aligned} s(t) &= \tfrac{1}{2}A_cA_m[\cos(2\pi f_m t)\cos(2\pi f_c t) + \sin(2\pi f_c t)\sin(2\pi f_m t)] \\ &= \tfrac{1}{2}A_cA_m\cos[2\pi(f_c - f_m)t] \end{aligned} \tag{7.52}$$

which is exactly the same as the result obtained by suppressing the upper side-frequency $f_c + f_m$ of the DSBSC wave of Eq. 7.35. The SSB wave of Eq. 7.52 and its spectrum are illustrated in Fig. 7.3*f*.

PHASE DISCRIMINATION METHOD FOR GENERATING AN SSB MODULATED WAVE

The *phase discrimination method* of generating an SSB modulated wave involves two separate simultaneous modulation processes and subsequent combination of the resulting modulation products, as shown in Fig. 7.20. The derivation of this system follows directly from Eq. 7.47 or 7.48, which defines the canonical representation of SSB modulated waves in the time-domain. The system uses two product modulators, I and Q, supplied with carrier waves in phase quadrature to each other. The incoming baseband signal $m(t)$ is applied to product modulator I, producing a modulated DSBSC wave that contains *reference phase* sidebands symmetrically spaced about carrier frequency f_c. The Hilbert transform $\hat{m}(t)$ of $m(t)$ is applied to product modulator Q, producing a DSBSC modulated wave that contains sidebands having identical amplitude spectra to those of modulator I, but with phase spectra such that vector addition or subtraction of the two modulator outputs results in cancellation of one set of sidebands and reinforcement of the other set. The use of a plus sign at the summing junction yields an SSB wave with only the lower sideband, whereas the use of a minus sign yields an SSB wave with only the upper sideband. In this way

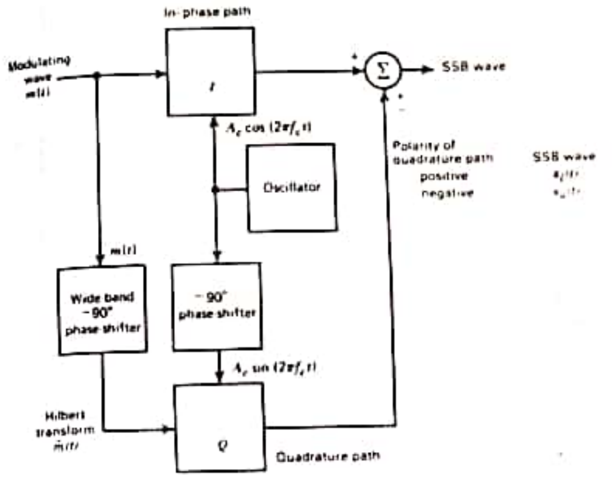

Figure 7.20
Block diagram of the phase discrimination method for generating SSB modulated waves.

DEMODULATION OF SSB WAVES

To recover the baseband signal $m(t)$ from the SSB wave $s(t)$, equal to $s_u(t)$ or $s_l(t)$, we have to shift the spectrum in Fig. 7.17*c* or *d* by the amounts $\pm f_c$ so as to convert the transmitted sideband back into the baseband signal. This can be accomplished using coherent detection, which involves applying the SSB wave $s(t)$, together with a locally generated carrier $\cos(2\pi f_c t)$, assumed to be of unit amplitude for convenience, to a product modulator and then low-pass filtering the modulator output, as in Fig. 7.21. Thus, using Eq. 7.47 or 7.48, we find that the product modulator output is given by

$$\begin{aligned} v(t) &= \cos(2\pi f_c t)s(t) \\ &= \tfrac{1}{2}A_c\cos(2\pi f_c t)[m(t)\cos(2\pi f_c t) \mp \hat{m}(t)\sin(2\pi f_c t)] \\ &= \underbrace{\tfrac{1}{4}A_c m(t)}_{\text{Scaled}} + \underbrace{\tfrac{1}{4}A_c[m(t)\cos(4\pi f_c t) \mp \hat{m}(t)\sin(4\pi f_c t)]}_{\text{Unwanted component}} \end{aligned}$$

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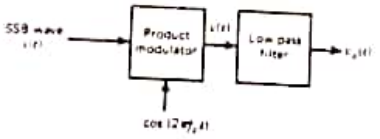

Figure 7.21
Coherent detection of an SSB modulated wave

The first term in Eq. 7.53 is the desired message signal. The combination of the remaining terms represents an SSB modulated wave with a carrier frequency of $2f_c$; as such, it represents an unwanted component in the product modulator output that is removed by low-pass filtering.

The detection of SSB modulated waves, just presented, assumes ideal conditions, namely, perfect synchronization between the local carrier and that in the transmitter both in frequency and phase. The effect of a phase error ϕ in the locally generated carrier wave is to modify the detector output as follows:

$$v_o(t) = \frac{1}{4}A_c m(t)\cos\phi \mp \frac{1}{4}A_c \hat{m}(t)\sin\phi \tag{7.54}$$

where the plus sign applies to an incoming SSB modulated wave containing only the upper sideband (i.e., the modulated wave of Eq. 7.47), and the minus sign applies to one containing only the lower sideband (i.e., the modulated wave of Eq. 7.48). Owing to the phase error ϕ, the detector output $v_o(t)$ contains not only the message signal $m(t)$ but also its Hilbert transform $\hat{m}(t)$. Consequently, the detector output suffers from *phase distortion*. This phase distortion is usually not serious with voice communications because the human ear is relatively insensitive to phase distortion. The presence of phase distortion gives rise to what is called the Donald Duck voice effect. In the transmission of music and video signals, on the other hand, phase distortion in the form of a constant phase difference in all components can be intolerable.

EXERCISE 9 **Show that the low-pass filter in the coherent detector of Fig. 7.21 only passes the message signal component of the product modulator output, provided it satisfies the following conditions:**

(a) Bandwidth = W
(b) Width of guardband $\leq 2f_c - aW$, where $a = 1$ for an SSB modulated wave containing only the upper sideband, and $a = 2$ for an SSB modulated wave containing only the lower sideband.

EXERCISE 10 Let $\cos(2\pi f_c t + \phi)$ denote the local carrier applied to the product modulator in Fig. 7.21. Show that the effect of the phase error ϕ is to modify the detector output $v_o(t)$ as in Eq. 7.54.

7.5 VESTIGIAL SIDEBAND MODULATION

Single-sideband modulation is well-suited for the transmission of voice because of the energy gap that exists in the spectrum of voice signals between zero and a few hundred hertz. When the message signal contains significant components at extremely low frequencies (as in the case of television signals and wideband data), the upper and lower sidebands meet at the carrier frequency. This means that the use of SSB modulation is inappropriate for the transmission of such message signals owing to the difficulty of isolating one sideband. This difficulty suggests another scheme known as *vestigial sideband* modulation (VSB), which is a compromise between SSB and DSBSC modulation. In this modulation scheme, one sideband is passed almost completely whereas just a trace, or *vestige*, of the other sideband is retained.

FREQUENCY-DOMAIN DESCRIPTION

Figure 7.22 illustrates the spectrum of a *vestigial sideband (VSB) modulated wave* $s(t)$ in relation to that of the message signal $m(t)$, assuming that the lower sideband is modified into the vestigial sideband. Specifically, the transmitted vestige of the lower sideband compensates for the amount removed from the upper sideband. The *transmission* bandwidth required by the VSB modulated wave is therefore given by

$$B = W + f_v \tag{7.55}$$

where W is the message bandwidth and f_v is the width of the vestigial sideband.

Vestigial sideband modulation has the virtue of conserving bandwidth almost as efficiently as single-sideband modulation, while retaining the excellent low-frequency baseband characteristics of double-sideband modulation. Thus VSB modulation has become standard for the transmission

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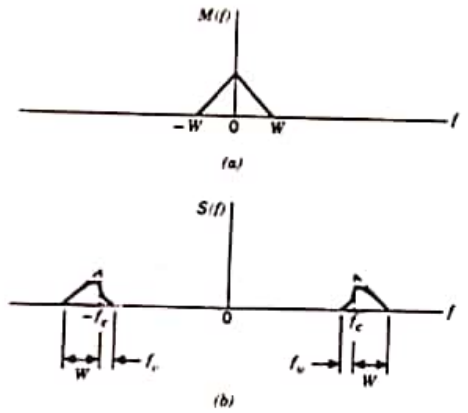


Figure 7.22
(a) Spectrum of message signal. (b) Spectrum of VSB modulated wave containing a vestige of the lower sideband.

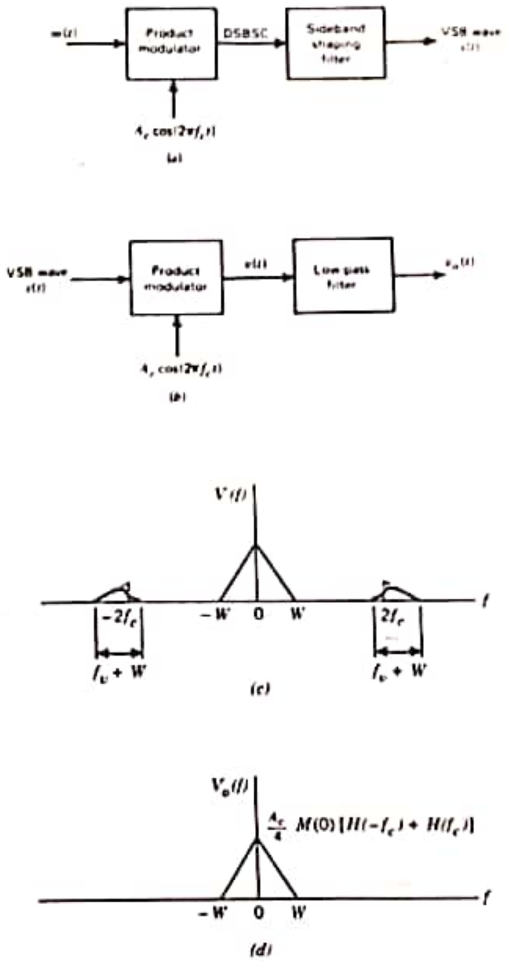
GENERATION OF VSB MODULATED WAVE

To generate a VSB modulated wave, we pass a DSBSC modulated wave through a *sideband shaping filter*, as in Fig. 7.23*a*. The exact design of this filter depends on the desired spectrum of the VSB modulated wave. The relation between the transfer function $H(f)$ of the filter and the spectrum $S(f)$ of the VSB modulated wave $s(t)$ is defined by

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]H(f) \qquad (7.56)$$

where $M(f)$ is the message spectrum. We wish to determine the specification of the filter transfer function $H(f)$, so that $S(f)$ defines the spectrum of the desired VSB wave $s(t)$. This can be established by passing $s(t)$ through a coherent detector and then determining the necessary condition for the detector output to provide an undistorted version of the original message signal $m(t)$. Thus, multiplying $s(t)$ by a locally generated sine-wave $\cos(2\pi f_c t)$, which is synchronous with the carrier wave $A_c \cos(2\pi f_c t)$ in both frequency and phase, as in Fig. 7.23*b*, we get

$$v(t) = \cos(2\pi f_c t)s(t) \qquad (7.57)$$



(d) Spectrum of the demodulated signal v.(t)

Figure 7.23
Scheme for the generation and demodulation of a VSB modulated wave. (a) Block diagram of VSB modulator. (b) Block diagram of VSB demodulator. (c) Spectrum of

the PM o/p v(t) in demod. scheme

transform of $v(t)$ as

$$V(f) = \frac{1}{2}[S(f - f_c) + S(f + f_c)] \tag{7.58}$$

Therefore, substitution of Eq. 7.56 in 7.58 yields

$$\begin{aligned} V(f) = & \frac{A_c}{4} M(f)[H(f - f_c) + H(f + f_c)] \\ & + \frac{A_c}{4}[M(f - 2f_c)H(f - f_c) + M(f + 2f_c)H(f + f_c)] \end{aligned} \tag{7.59}$$

The spectrum $V(f)$ is illustrated in Fig. 7.23c. The second term in Eq. 7.59 represents a VSB wave corresponding to carrier frequency $2f_c$. This term is removed by the low-pass filter in Fig. 7.23b to produce an output $v_o(t)$, the spectrum of which is given by

$$V_o(f) = \frac{A_c}{4} M(f)[H(f - f_c) + H(f + f_c)] \tag{7.60}$$

The spectrum $V_o(f)$ is illustrated in Fig. 7.23d. For a distortionless reproduction of the original baseband signal $m(t)$ at the coherent detector output, we require $V_o(f)$ to be a scaled version of $M(f)$. This means, therefore, that the transfer function $H(f)$ must satisfy the condition

$$H(f - f_c) + H(f + f_c) = 2H(f_c) \tag{7.61}$$

where $H(f_c)$ is a constant. With the message spectrum $M(f)$ assumed to be essentially zero outside the interval $-W \le f \le W$, we need to satisfy Eq. 7.61 only for values of f in this interval.

The requirement of Eq. 7.61 is satisfied by using a filter with a frequency response $H(f)$ such as that shown in Fig. 7.24 for positive frequencies. This response is normalized so that $H(f)$ falls to one half at the carrier frequency f_c. The cutoff portion of this response around f_c exhibits odd symmetry in the sense that inside the transition interval defined by $f_c - f_v \le f \le f_c + f_v$, the sum of the values of $H(f)$ at any two frequencies equally displaced above and below f_c is unity. Such a filter is much less elaborate than that required if one sideband is to be completely suppressed.

In general, to preserve the baseband spectrum, the phase response of the sideband shaping filter in Fig. 7.23a must exhibit odd symmetry about the carrier frequency f_c. Specifically, it must be linear over the frequency intervals $f_c - f_v \le |f| \le f_c + W$, and its value at the frequency f_c has to

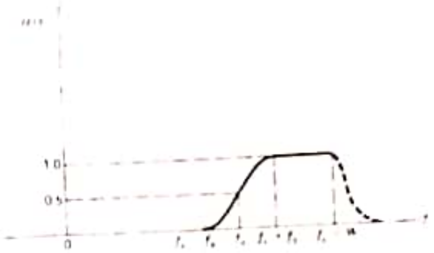


Figure 7.24
Frequency response of sideband shaping filter for a VSB modulated wave containing a vestige of lower sideband; only the positive frequency portion is shown.

The frequency response of Fig. 7.24 pertains to a VSB modulated wave containing a vestige of the lower sideband. In the situation depicted here control over the frequency response of the sideband shaping filter need only be exercised over the band $f_c - f_v \le |f| \le f_c + W$. This is the reason for showing the frequency response of the sideband shaping filter in Fig. 7.24 for $f > f_c + W$ as a dashed line.

EXERCISE 11 **Construct the positive-frequency portion of the frequency response of a sideband shaping filter for a VSB modulated wave that contains a vestige of the upper sideband.**

TIME-DOMAIN DESCRIPTION

Our next task is to determine the time-domain description of a VSB modulated wave. To do this, we follow a procedure similar to that used for SSB modulated waves in Section 7.4.

Let $s(t)$ denote a VSB modulated wave containing a vestige of the lower sideband. This modulated wave may be viewed as the output of a sideband shaping filter produced in response to a DSBSC modulated wave defined in Eq. 7.39. The filter has a transfer function $H(f)$ as illustrated in Fig. 7.24. Using the band-pass to low-pass transformation technique of Section 3.5, we may replace the sideband shaping filter by an equivalent complex-

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$\frac{A_c}{2}$

$s(t) = \frac{A_c}{2} m_I(t)\cos\omega_c t - \frac{A_c}{2} m_Q(t)\sin\omega_c t$.

$m_Q(t)$ is the response produced by passing the message signal $m(t)$ through a LPF of impulse response $h_Q(t)$.

h_Q is the desired resp. for a VSB modulated wave containing a vestige of the LSB.

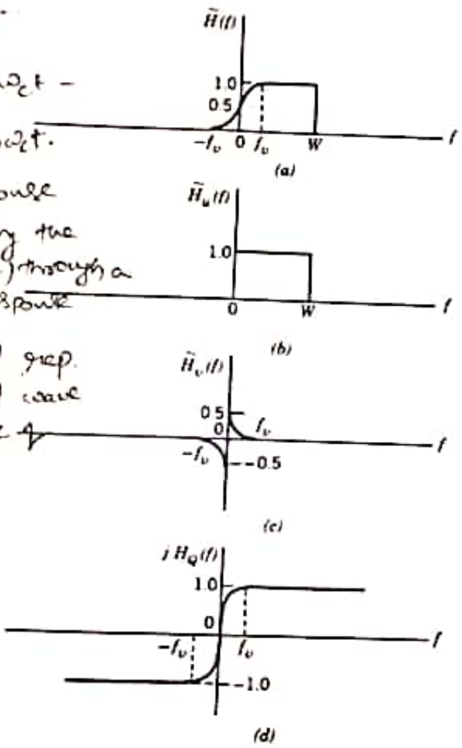

Figure 7.25
(a) Idealized frequency response $\tilde{H}(f)$ of a low-pass filter equivalent to the sideband shaping filter that passes a vestige of the lower sideband. (b) First component of $\tilde{H}(f)$. (c) Second component of $\tilde{H}(f)$. (d) Frequency response of a filter with transfer function $jH_Q(f)$.

we may express $\tilde{H}(f)$ as the difference between two components $\tilde{H}_s(f)$ and $\tilde{H}_v(f)$ as shown by

$$\tilde{H}(f) = \tilde{H}_s(f) - \tilde{H}_v(f) \qquad (7.62)$$

These two components are described individually as follows:

2. The transfer function $\tilde{H}_v(f)$, shown in Fig. 7.25c, accounts for both the generation of a vestige of the lower sideband and the removal of a corresponding portion from the upper sideband.

Thus, substituting Eq. 7.42 in 7.62, we may redefine the transfer function $\tilde{H}(f)$ as

$$\tilde{H}(f) = \begin{cases} \frac{1}{2}[1 + \mathrm{sgn}(f) - 2\tilde{H}_v(f)], & -f_v < f < W \\ 0, & \text{otherwise} \end{cases} \qquad (7.63)$$

The signum function $\mathrm{sgn}(f)$ and the transfer function $\tilde{H}_v(f)$ are both *odd* functions of the frequency f. Hence, they both have *purely imaginary* inverse Fourier transforms. Accordingly, we may introduce a new transfer function

$$H_Q(f) = \frac{1}{j}[\mathrm{sgn}(f) - 2\tilde{H}_v(f)] \qquad (7.64)$$

that has a *purely real* inverse Fourier transform. Let $h_Q(t)$ denote the inverse Fourier transform of $H_Q(f)$; that is,

$$h_Q(t) \rightleftharpoons H_Q(f) \qquad (7.65)$$

Figure 7.25*d* shows a plot of $jH_Q(f)$ as a function of frequency in accordance with both Eq. 7.64 and Fig. 7.25c. To go on with our task, we rewrite Eq. 7.63 in terms of $H_Q(f)$ as

$$\tilde{H}(f) = \begin{cases} \frac{1}{2}[1 + jH_Q(f)], & -f_v < f < W \\ 0, & \text{elsewhere} \end{cases} \qquad (7.66)$$

We are now ready to determine the VSB modulated wave $s(t)$. First, we write

$$s(t) = \mathrm{Re}[\tilde{s}(t)\exp(j2\pi f_c t)] \qquad (7.67)$$

where $\tilde{s}(t)$ is the complex envelope of $s(t)$. Since $\tilde{s}(t)$ is the output of the complex low-pass filter of transfer function $\tilde{H}(f)$, which is produced in response to the complex envelope of the DSBSC modulated wave, we may express the spectrum of $\tilde{s}(t)$ as

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The component $\frac{A_c}{2} m(t)$ constitutes the inphase comp of this VSB modulated wave & $\frac{A_c}{2} m_Q(t)$ constitutes the quadrature comp.

DSBSC & SSB waves may be regarded as special cases of VSB mod. wave. If VSB is ↑ to the width of full side band, resulting wave becomes a DSBSC wave with the result that $m_Q(t)$ vanishes. If the width of VSB is f_0, resulting wave becomes an SSB wave containing the USB, with the result that $m_Q(t) = \hat{m}(t)$; $\hat{m}(t)$ Hilbert transform of $m(t)$.

$$s(t) = A_c\left(1 + \tfrac{1}{2}K_a m(t)\right)\cos 2\pi f_c t - \frac{1}{2} k_a A_c m_Q(t) \sin(2\pi f_c t)$$

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