Quine MC cluskey Method (or) Tabulation Method

we discussed K-map method, which is a convenient method for minimizing Boolean functions up to 5 variables. But, it is difficult to simplify the Boolean functions having more than 5 variables by using this method.

Quine-McClukey tabular method is a tabular method based on the concept of prime implicants.

Procedure of Quine-McCluskey Tabular Method

f(W,X,Y,Z)=∑m(2,6,8,9,10,11,14,15)

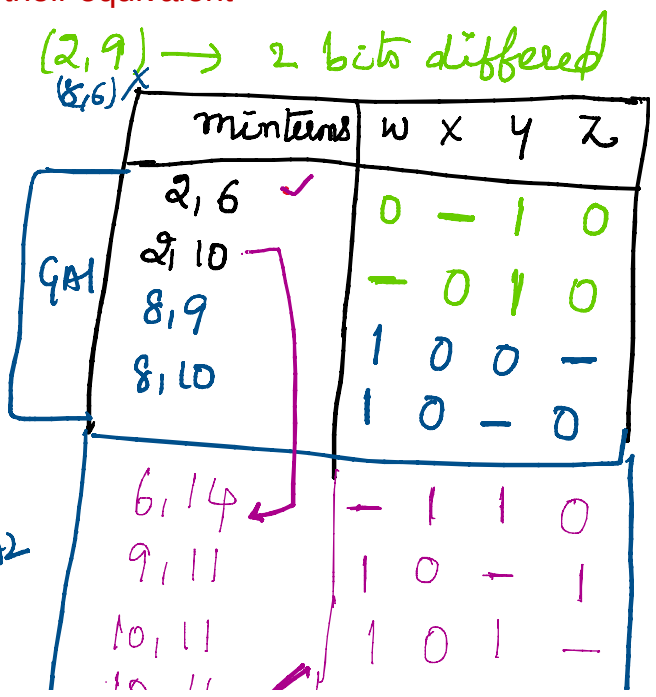
Step 1 − Arrange the given min terms in an ascending order and make the groups based on the number of ones present in their binary representations.

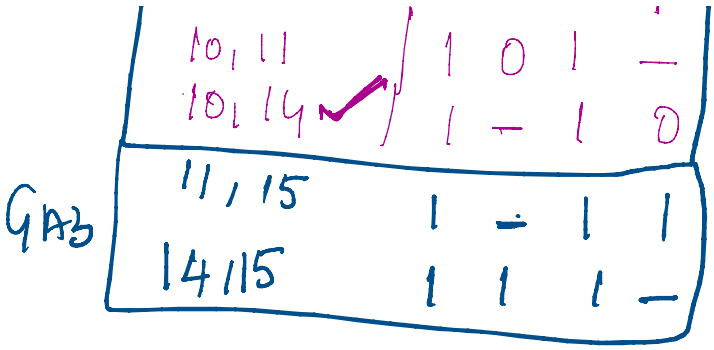
The given Boolean function is in sum of min terms form. It is having 4 variables W, X, Y & Z.
The given min terms are 2, 6, 8, 9, 10, 11, 14 and 15.

The ascending order of these min terms based on the number of ones present in their binary equivalent is 2, 8, 6, 9, 10, 11, 14 and 15.

The following table shows these min terms and their equivalent binary representations.

Group Name	Min terms	W	X	Y	Z
GA1	2	0	0	1	0
	8	1	0	0	0
GA2	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
GA3	11	1	0	1	1
	14	1	1	1	0
GA4	15	1	1	1	1



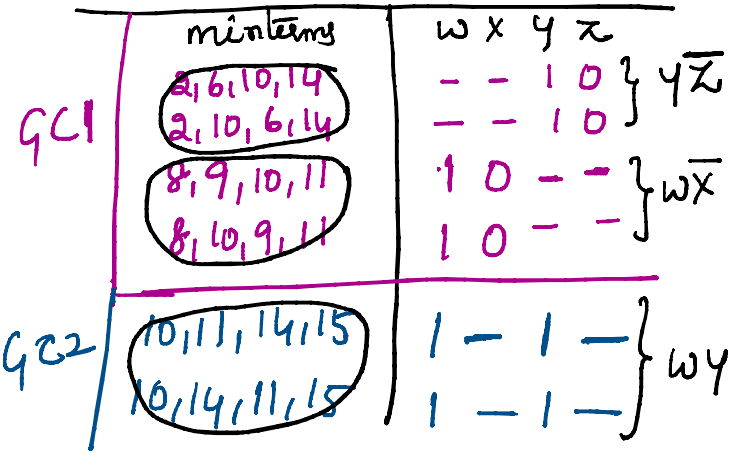


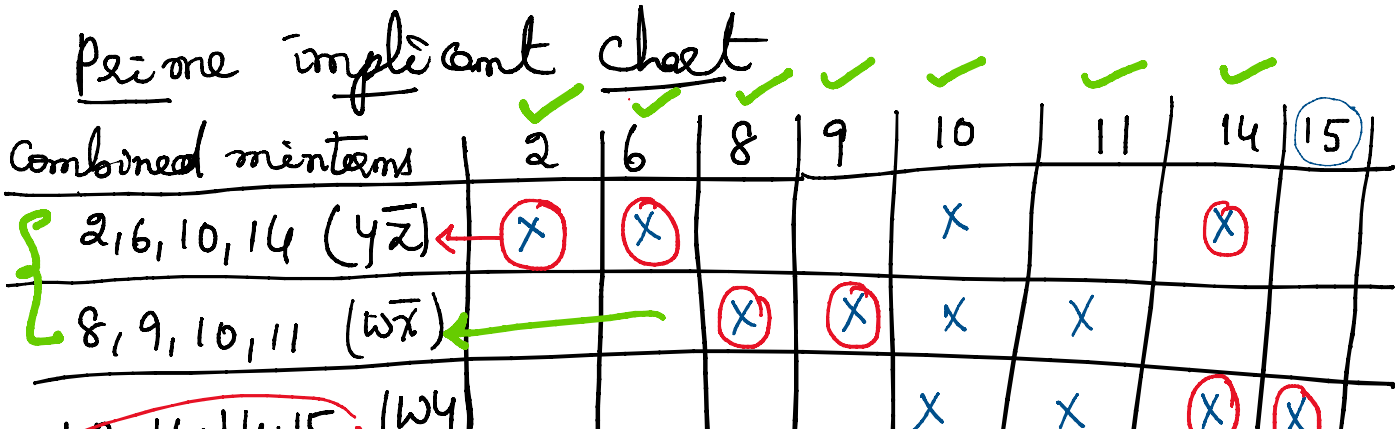
Step 2 − Compare the min terms present in successive groups. If there is a change in only one-bit position, then take the pair of those two min terms. Place this symbol '_' in the differed bit position and keep the remaining bits as it is.

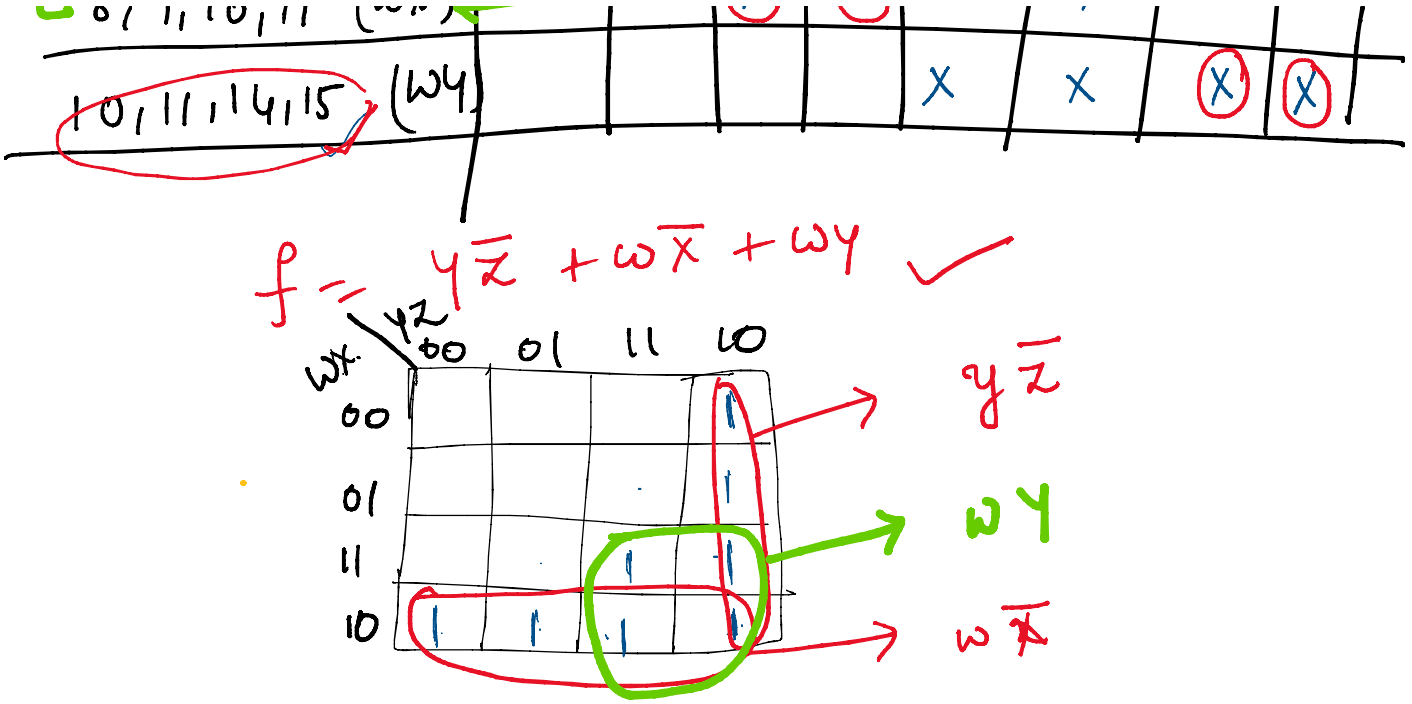
The min terms, which are differed in only one-bit position from adjacent groups are merged.
That differed bit is represented with this symbol, '-'.
In this case, there are three groups and each group contains combinations of two min terms.

The following table shows the possible merging of min term pairs from adjacent groups

Group Name	Min terms	W	X	Y	Z
GB1	2,6	0	-	1	0
	2,10	-	0	1	0
	8,9	1	0	0	-
	8,10	1	0	-	0
GB2	6,14	-	1	1	0
	9,11	1	0	-	1
	10,11	1	0	1	-
	10,14	1	-	1	0
GB3	11,15	1	-	1	1
	14,15	1	1	1	-







Step 3 − Repeat step2 with newly formed terms till we get all prime implicants.

Group Name	Min terms	W	X	Y	Z
GB1	2,6,10,14	-	-	1	0
	2,10,6,14	-	-	1	0
	8,9,10,11	1	0	-	-
	8,10,9,11	1	0	-	-
GB2	10,11,14,15	1	-	1	-
	10,14,11,15	1	-	1	-

Step 4 − Formulate the prime implicant table. It consists of set of rows and columns. Prime implicants can be placed in row wise and min terms can be placed in column wise. Place ‘1’ in the cells corresponding to the min terms that are covered in each prime implicant.

Group Name	Min terms	W	X	Y	Z
GC1	2,6,10,14	-	-	1	0
	8,9,10,11	1	0	-	-
GC2	10,11,14,15	1	-	1	-

Further merging of the combinations of min terms from adjacent groups is not possible, since they are differed in more than one-bit position. There are three rows in the above table. So, each row will give one prime implicant. Therefore, the prime implicants are YZ', WX' & WY.

The prime implicant table is shown below.

Min terms / Prime Implicants	2	6	8	9	10	11	14	15
YZ'	1	1			1		1	
WX'			1	1	1	1		
WY					1	1	1	1

The prime implicants are placed in row wise and min terms are placed in column wise.
1s are placed in the common cells of prime implicant rows and the corresponding min term columns.
The min terms 2 and 6 are covered only by one prime implicant YZ'. So, it is an essential prime implicant.
This will be part of simplified Boolean function.
Now, remove this prime implicant row and the corresponding min term columns. The reduced prime implicant table is shown below.

Step 5 − Find the essential prime implicants by observing each column. If the min term is covered only by one prime implicant, then it is essential prime implicant. Those essential prime implicants will be part of the simplified Boolean function

Min terms / Prime Implicants	8	9	11	15
WX'	1	1	1	
WY			1	1

The min terms 8 and 9 are covered only by one prime implicant WX'. So, it is an essential prime implicant. This will be part of simplified Boolean function. Now, remove this prime implicant row and the corresponding min term columns. The reduced prime implicant table is shown below.

Step 6 − Reduce the prime implicant table by removing the row of each essential prime implicant and the columns corresponding to the min terms that are covered in that essential prime implicant. Repeat step 5 for Reduced prime implicant table. Stop this process when all min terms of given Boolean function are over.

The min terms 8 and 9 are covered only by one prime implicant WX'. So, it is an essential prime implicant. This will be part of simplified Boolean function. Now, remove this prime implicant row and the corresponding min term columns. The reduced prime implicant table is shown below.

Min terms / Prime Implicants	15
WY	1

The min term 15 is covered only by one prime implicant WY. So, it is an essential prime implicant. This will be part of simplified Boolean function.
In this example problem, we got three prime implicants and all the three are essential. Therefore, the simplified Boolean function is

F = YZ' + WX' + WY.