ELECTRO MAGNETIC WAVES 'N' TRANSMISSION LINES

UNIT-1

ELECTRO STATICS:-

Colb coulomb's law, Electric field intensity, Fields due to different charge distibutions, Electric Flux density, Guass law and its applications, Electric Pottential, Relation b/w E and V, Maxwell's two equations for electrostatic fields, Energy density, Maxwells 2 eqns for electrostatic fields, energy density. R^n Energy density. (olve e.p w=1/2 QV)

Illustrative Problems:

convection & conduction currents, Dielectric constant, Isotropic and Homogenous Dielectrics, continuity equation and Rexalation time, Poisson's and Laplace's equations, capacitance - parallel plate, coaxial and spherical capacitors, Illustrative Problems.

UNIT-2:

MAGNETOSTATICS:

Biot - savart law, Ampere's circuital law and applicatns, Magnetic flux density, Maxwell's two equations for Magneto static fields, Magnetic scalar and vector pottentials, forces due to Magnetic Fields, Ampere's force law, [Forces due to] Inductances and magnetic energy, Illustrative Problems: Lorentz eq^n Maxwells for diff fields.

Maxwell's equations (Time varying fields): Faradays law and transformer emf, Inconsistence of Ampere's Law and displacement current density, Maxwell's equations in different final forms and word statements, conditions at boundary surface: Dielectric-Dielectric force dielectric: conductor Interfaces.

UNIT-III (south)

EM CHARACTERISTICS-I, for free space $\mu=\mu_0, \varepsilon=\varepsilon_0$

Wave eqns for conducting and perfect dielectric Media, uniform plane waves- definition, All Relations between E and H, sinusoidal variations, wave propagation in Lossless and conducting media, conductors and dielectrics - characterization, wave propagation in good conductors and good Dielectrics, Polarization, Illustrative problems:

EM WAVE CHARACTERISTICS-II.

Reflection and Refraction of plane waves- Normal and oblique indices for perfect dielectric, Brewster angle, critical Angle, Total Internal Reflection, surface impedance, Poynting vector Poynting theorem - Applications, Power loss in plane conductor, Illustrative problems

UNIT-IV.

Transmission Lines-I

Types, Parameters, Transmissions line equations, Primary and Secondary constants, expressions for characteristics Impedance Propagation constant, phase and group velocities, Infinite Line concepts, Losslessness/Low Loss characterization, Distortion condition for distortionlessness and Minimum Attenuation, Loading - types of loading, Illustrative problems.

UNIT - V

TRANSMISSION LINES-II:

Input Impedances Relations, SC and OC Lines, Reflection coefficient, VSWR, UHF Lines as circuit Elements, $\lambda/4$, $\lambda/2$ and $\lambda/8$ Lines - Impedances transformations, significance of Z_{min} and Z_{max}, Smith chart - configuration and Applications, Single and double stub Matching, Illustrative Problems.

TEXT BOOKS:

1. Matthew N.O. sadiku (2008), Principles Elements of electromagnetics, 4rd edition, oxford university press, New Delhi.

2. Umesh sinha, Satya prakashan (2001), Transmission Lines & Networks, Tech India Publications, India.

REFERENCE BOOKS:

1. William H. Hayt Jr., John A. Buck (2006), Engineering electromagnetics, 7th edition, TATA MC Graw Hill, India.

2. C. Jordon, K.G. Balmain (2000), Electromagnetic waves and Radiating systems, 2nd edition, Prentice Hall of India, New Delhi

3. John . D. Kraus (2007), Electromagnetics, 6th edition, Mc Graw Hill, New Delhi.

Nanapeneni Narayana Roa (2006), Elements of engineering Electromagnetics, 6th edition.

[VECTORS:-]*

FIELD:-

It is a region or space which is surrounded by some electric or magnetic charge and which is a function which specifies some quantity.

ELECTRIC FIELD:

It is an area region or space which is surrounded by any electric field component and which is effected by that component.

MAGNETIC FIELD:

It is an area region or space which is surrounded by any magnetic field component and which is effected by that component.

SCALAR:

It is a single valued real no. (either +ve or -ve) which specifies the magnitude of any physical quantity.

Ex: Speed, mass, distance, area etc

VECTOR:

It is a function of any physical quantity which not only specifies the magnitude but also direction of that physical quantity.

Ex: Force, velocity, flux etc

UNIT VECTOR:

It is a vector whose magnitude is one and specifies the direction of the considered vector.

suppose if $\bar{a}$ is a unit vector then it is given by

$$\boxed{\bar{a} = \bar{a}_{OA} = \frac{\overline{OA}}{|\overline{OA}|}}$$

$\bar{A} = \overline{OA}$

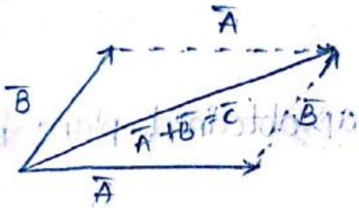
$\because |\overline{OA}| = 1$

$\bar{a} = \overline{OA}$

OPERATIONS ON ~~M~~ VECTORS:-

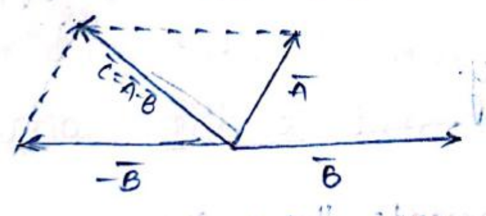
i) ADDITION OF VECTORS:

suppose $\bar{A}$ and $\bar{B}$ are 2 vectors then it's sum can be calculated by parallelogram law (rule) and it is given by $\bar{C} = \bar{A} + \bar{B}$ (or) $\overline{OC} = \overline{OA} + \overline{OB}$



ii) substraction OF VECTORS:

suppose $\bar{A}$ and $\bar{B}$ are 2 vectors then it's difference can also be calculated by parallelogram law (rule) and it is given by $\bar{C} = \bar{A} - \bar{B}$



iii) SCALING OF VECTORS:

suppose a vector $\overline{OA}$ is considered and let 'k' is a constant then $\xrightarrow[\overline{OA}]{}$

if $k > 1$ $\xrightarrow[K \cdot \overline{OA}]{}$

if $K < 1$ $\xrightarrow[K \cdot \overline{OA}]{}$

if k is -ve (-1) $\xrightarrow[\overline{OA}]{}$ $\xleftarrow[K \cdot \overline{OA}]{}$

IDENTICAL VECTORS:

If two vectors are identical in terms of magnitude and direction then those vectors are called identical vectors.

COPLANAR VECTORS:

The vectors which lie in same plane are called coplanar vectors.

CO ORDINATE SYSTEM :-

1) CARTESIAN COORDINATE SYSTEM / RECTANGULAR COORDINATE SYS

- It has 3 planes
  - * XY Plane $\perp^{r}$ to Z axis
  - * YZ plane $\perp^{r}$ to X axis
  - * XZ plane $\perp^{r}$ to Y axis

If x is a constant then the ~~ap~~ obtained plane is $\|^{el}$ to YZ plane.

similarly Y → constant $\|^{el}$ to XZ plane.

Similarly Z → constant $\|^{el}$ to XY plane.

* BASE VECTOR:

It is a unit vector along the coordinate axis i.e for cartesian coordinate system the base vectors are $\bar{a}_x, \bar{a}_y, \bar{a}_z$ along x, y, z axis respectively.

* POSITION VECTOR:

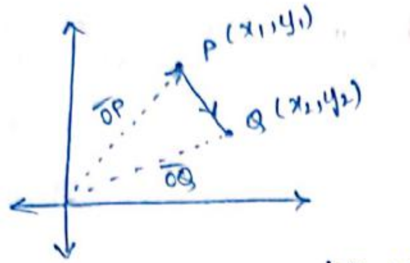
It is a vector which represents the position of a point which is located in coordinate system.

Let $P(x,y,z)$ be a point in cartesian coordinate system then $\overline{OP} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$ is called position vector.

$$\bar{P} = \overline{OP} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

* The magnitude of position vector is given by $|\overline{OP}| = \sqrt{x^2+y^2+z^2}$

* DISTANCE VECTOR / SEPERATION VECTOR:-



Let P and Q be any points on a cartesian coordinate system such that $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are seperated by some distance then distance vector $\overline{PQ}$ is given by.

$\overline{PQ} = \overline{OQ} + -\overline{OP}$

$\overline{OP} = x_1\bar{a}_x + y_1\bar{a}_y + z_1(\bar{a}_z)$

$\overline{OQ} = x_2\bar{a}_x + y_2\bar{a}_y + z_2\bar{a}_z$

$\overline{OP}$ and $\overline{OQ}$ are position vectors of P and Q respectively.

$\therefore \overline{PQ} = (x_2-x_1)\bar{a}_x + (y_2-y_1)\bar{a}_y + (z_2-z_1)\bar{a}_z$

$|\overline{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ is magnitude of distance vector.

$\boxed{\overline{PQ} \neq \overline{QP}}$ (direction differs but magnitudes are same)

$\boxed{|\overline{PQ}| = |\overline{QP}|}$

* DIFFERENTIAL LENGTH :

Suppose 'P' be any point in a cartesian coordinate system with coordinates (x, y, z) Let 'P'' be another differential point with a displacement of 'dx' units in x-direction 'dy' units in y direction and 'dz' units in z-direction then differential length $\overline{dL}$ from P to P' is given by $[\overline{dL} = d\bar{x} + d\bar{y} + d\bar{z}]$ x

$\overline{dL} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$

* DIFFERENTIAL SURFACE AREA:

For same assumptions as above:

It Differential surface area which is normal to x-direction is given by $\overline{ds}_x = dy\,dz \cdot \bar{a}_x$ $\overline{ds}_y$ =

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$\overline{ds_y} = dx\,dz \cdot \overline{a_y}$ differential surface area normal to y direction

$\overline{ds_z} = d_x\,dy \cdot \overline{a_z}$ differential " " " " z direction

$\overline{dv}$

DIFFERENTIAL VOLUME:($\overline{dv}$)

$$\overline{dv} = dx \cdot dy \cdot dz$$

* CYLINDRICAL COORDINATE SYSTEM:-

cylindrical coordinates are r, θ, z with [unit vectors] base vectors $\overline{a_r}, \overline{a_\theta}, \overline{a_z}$ respectively.

Limits of $r \rightarrow (0, \infty)$

$\theta \rightarrow (0, 2\pi)$

$z \rightarrow (-\infty, \infty)$

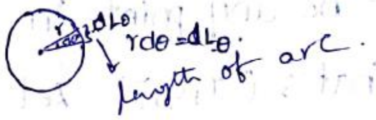
DIFFERENTIAL LENGTH:

Differential length in r direction:-

$$\overline{dL_r} = dr \cdot \overline{a_r}$$

in θ direction:-

$$\overline{dL_\theta} = r\,d\theta \cdot \overline{a_\theta}$$



in z direction:-

$$\overline{dL_z} = dz \cdot \overline{a_z}$$

DIFFERENTIAL SURFACE AREAS:

i) normal to r direction:

$$\overline{ds_r} = r\,d\theta \cdot dz\,\overline{a_r}$$

ii) normal to θ direction

$$\overline{ds_\theta} = dr\,dz\,\overline{a_\theta}$$

iii) $\overline{ds_z} = r\,d\theta\,dr\,\overline{a_z}$

DIFFERENTIAL VOLUME:-

$\overline{dV} = r\,d\theta\,dr\,dz$

TRANFORMATION: (CARTESIAN-to CYLINDRICAL)

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}(y/x)$

$z = z$

(CYLINDRICAL to CARTESIAN)

$x = r\cos\theta$.

$y = r\sin\theta$.

$z = z$.

* SPHERICAL COORDINATE SYSTEM!

coordinates are r, θ, ϕ with base vectors $\overline{a_r}, \overline{a_\theta}, \overline{a_\phi}$

Limits

$r \Rightarrow (0 \text{ to } \infty)$

$\theta = (0 \text{ to } 2\pi)$

$\phi = (0, \pi)$

ORIENTATION →

To Cart

$x = r\sin\phi\cos\theta;$

$y = r\sin\phi\sin\theta;$

$z = r\cos\phi.$

to sph

$r = \sqrt{x^2+y^2+z^2}$

$\theta = \tan^{-1}(y/x)$

$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

DIFFERENTIAL LENGTH

in r direction:

$\overline{dL_r} = dr\,\overline{a_r}$

$\overline{dL_\theta} = r\,d\theta \cdot \overline{a_\theta}$

$\overline{dL_\phi} = [r^2 d\phi\,\overline{a_\phi}]^{x}\ r\sin\phi\,d\phi\,\overline{a_\phi}$

DIFFERENTIAL AREAS:

$\overline{ds_r} = r^2 d\theta \sin\phi \overset{d\phi}{\cdot} \overline{a_r}$

$\overline{ds_\theta} = r\sin\phi\,dr\,d\phi\,\overline{a_\theta}$

$\overline{ds_\phi} = r\,d\theta\,dr\,\overline{a_\phi}$

DIFFERENTIAL VOLUME:

$\overline{dV} = r^2\,dr\,d\theta\,\sin\phi\,d\phi$

COLOUMB'S LAW:

It states that force b/w two charges [is directio] acts along line joining 2 charges and it is

* directly proportional to product of two charges &
* inversely proportional to square of the distance b/w them.

$F \propto \frac{q_1 q_2}{r^2}$

k → decided by medium.

$F \propto Q_1 Q_2$

$F \propto 1/r^2$

$k = 1/4\pi\epsilon_0$

ϵ → permittivity of medium

$F \propto \frac{Q_1 Q_2}{r^2}$

$\epsilon = \epsilon_r \epsilon_0$

ϵ_0 → absolute permittivity.

$F = k\frac{Q_1 Q_2}{r^2}$

$\epsilon_r = 1$

$\epsilon = \epsilon_0$

ϵ_r → relative permittivity.

$k \simeq 9 \times 10^9 \left[\frac{1}{4\pi\epsilon_0}\right] m/F$

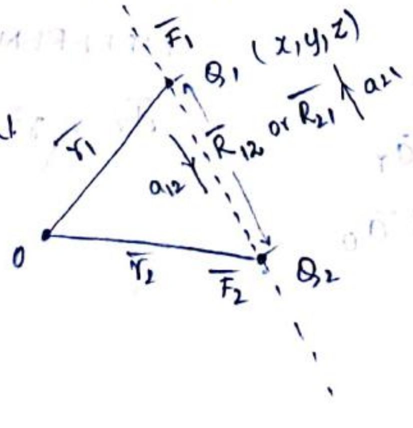
$\epsilon_0 = 8.85 \times 10^{-12} F/m$

$\boxed{F = 9\times10^9 \frac{Q_1 Q_2}{r^2}}$ N (if air medium) (or) free space.

Note:

* coloumb's law is an experimental work which can't be mathematically derived.
* It is evident that like charges repel each other unlike charge attract each other.

• VECTOR FORM OF COLOUMB'S LAW



Let Q_1 and Q_2 are two point charges whose position vectors are $\bar{r}_1$ and $\bar{r}_2$ which are seperated by a distance $\bar{R}$ as shown in fig:

Let $\bar{F}_1$ be force experienced by charge Q_1 (i.e force exerted by Q_2) and $\bar{F}_2$ be force experienced by charge Q_2. Also the distance vector along the force is $\bar{R}$

consider charge Q_1 is exerting force of Q_2 therefore force experienced by charge Q_2 is given by

$$\bar{F}_2 = K \frac{Q_1 Q_2}{|\bar{R}_{12}|^2} \bar{a}_{12}$$

$$= \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2}{|\bar{r}_2 - \bar{r}_1|^2} \frac{(\bar{R}_{12})}{|\bar{R}_{12}|}$$

where $\underbrace{\bar{R}_{12}}_{\text{Distance vectors}} = \underbrace{\bar{r}_2 - \bar{r}_1}_{\text{Position vectors}}$ and $\bar{a}_{12}$ is unit vector along force direction

$$= \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2 (\bar{r}_2 - \bar{r}_1)}{|\bar{r}_2 - \bar{r}_1|^3}$$

$$\boxed{\bar{F}_2 = \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2}{|r_2 - r_1|^3} (\bar{r}_2 - \bar{r}_1)}$$

$_{\text{1ll}}$arly.

force exerted by Q_2 on Q_1 (or) force experienced by charge Q_1 is given by:

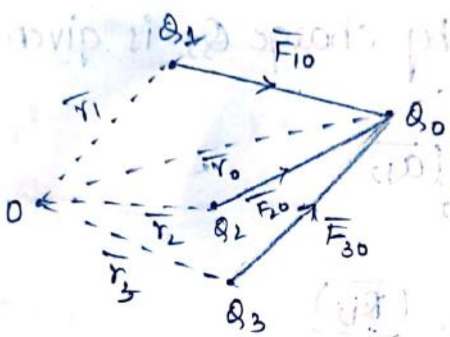
$$\bar{F}_1 = K \frac{Q_1 Q_2}{|\bar{R}_{21}|} \bar{a}_{21}$$

$$\boxed{F_1 = K \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2}{|r_1 - r_2|^3} (r_1 - r_2)}$$

• COLOUMB'S LAW FOR 'N' POINT CHARGES (OR)

FORCE DUE TO 'N' CHARGES

Let Q_1, Q_2, Q_3 be 3 charges which are located in vector space with position vectors $\bar{r}_1, \bar{r}_2, \bar{r}_3$. Let there is another charge Q_0 whose position vector is $\bar{r}_0$ as shown in fig.



Now force experienced by charges Q_0 due to charges Q_1 Q_2 & Q_3 can be calculated using superposition principle.

Let $\bar{F}_{10}$ be force experienced by $\vec{Q}_0$ due to charge Q_1

so from coloumb's law

$$\bar{F}_{10} = \frac{1}{4\pi\epsilon} \frac{Q_1 Q_0}{|\bar{R}_{10}|^2} \bar{a}_{10}$$

Where $\bar{R}_{10}$ is distance vector along the force $= \bar{r}_0 - \bar{r}_1$

$\bar{a}_{10}$ is unit vector along force direction

$$\bar{a}_{10} = \frac{\bar{R}_{10}}{|\bar{R}_{10}|} = \frac{\bar{r}_0 - \bar{r}_1}{|(\bar{r}_0 - \bar{r}_1)|}$$

$$\bar{F}_{10} = \frac{1}{4\pi\epsilon} \frac{Q_1 Q_0}{|r_0 - r_1|^3} (\bar{r}_0 - \bar{r}_1) \rightarrow (1)$$

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$$\text{Ilarly}\quad \overline{F_{20}} = \frac{1}{4\pi\epsilon}\frac{Q_2 Q_0}{|\overline{r_0}-\overline{r_2}|^3}(\overline{r_0}-\overline{r_2}) \rightarrow (2)$$

$$\overline{F_{30}} = \frac{1}{4\pi\epsilon}\frac{Q_3 Q_0}{|\overline{r_0}-\overline{r_3}|^3}(\overline{r_0}-\overline{r_3}) \rightarrow (3)$$

$\overline{F_{20}} \rightarrow$ force experienced by $\vec{Q}_0$ due to charge Q_2

$\overline{F_{30}} \rightarrow$ force experienced by Q_0 due to charge Q_3.

By superposition theorem:

force, acting on charge Q_0 is given by
overall

$$\overline{F_0} = \overline{F_{10}} + \overline{F_{20}} + \overline{F_{30}}$$

$$\overline{F_0} = \frac{Q_0}{4\pi\epsilon}\left(\frac{Q_1(\overline{r_0}-\overline{r_1})}{|\overline{r_0}-\overline{r_1}|^3} + \frac{Q_2(\overline{r_0}-\overline{r_2})}{|\overline{r_0}-\overline{r_2}|^3} + \frac{Q_3(\overline{r_0}-\overline{r_3})}{|\overline{r_0}-\overline{r_3}|^3}\right)$$

The same concept is extended for 'n' charges and net force acting on Q_0:

$$\overline{F_0} = \frac{Q_0}{4\pi\epsilon}\left[\frac{Q_1}{|\overline{R_{10}}|^2}\overline{a_{10}} + \frac{Q_2}{|\overline{R_{20}}|^2}\overline{a_{20}} + \cdots + \frac{Q_n}{|\overline{R_{n0}}|^2}\overline{a_{n0}}\right]$$

$$\overline{F_0} = \frac{Q_0}{4\pi\epsilon}\left[\sum_{N=1}^{n}\frac{Q_n}{|\overline{R_{N0}}|^2}\overline{a_{N0}}\right]$$

$$\boxed{\overline{F_0} = \frac{Q_0}{4\pi\epsilon}\left[\sum_{i=1}^{n}\frac{Q_i}{|\overline{R_{i0}}|^2}\overline{a_{i0}}\right]}$$

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Problems :

1) A charge $Q_1 = -20\mu C$ is located at $(-6,4,6)$ and another charge $Q_2 = 50\mu C$ is located at $(5,8,2)$ find the force exerted by

i) Q_2 on Q_1 " $= \bar{F_1}$

ii) Q_1 on Q_2 " $= \bar{F_2}$

Given that the point charges are located at points A & B resp. $Q_1 = -20\mu C$ is located at point A. $Q_2 = 50\mu C$ is located at point B.

$\therefore$ positions vector for two point charges will be

$\overline{OA} = -6\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z$

$\overline{OB} = 5\bar{a}_x + 8\bar{a}_y - 2\bar{a}_z$

$\overline{R_{AB}} = 11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z = \overline{OB} - \overline{OA}$

$\overline{R_{BA}} = -11\bar{a}_x - 4\bar{a}_y + 8\bar{a}_z = \overline{OA} - \overline{OB}$

$|\overline{R_{AB}}| = |\overline{R_{BA}}| = \sqrt{(11)^2 + (4)^2 + (8)^2}$

$= 14.17$

from coloumb's law:

$$\bar{F_1} = k \frac{Q_1 Q_2}{R_{21}} \bar{a}_{BA}$$

$$= 9\times10^9 \frac{-20\times10^{-6} \times 50\times10^{-6}}{(14.17)^2}$$

$$\bar{a}_{BA} = \frac{\overline{R_{BA}}}{|\overline{R_{BA}}|} = \frac{-11\bar{a}_x - 4\bar{a}_y + 8\bar{a}_z}{(14.17)}$$

$$\bar{F_1} = \frac{-9\times10^{11} \times 10^{-12} (10)}{(14.17)^3} (-11\bar{a}_x - 4\bar{a}_y + 8\bar{a}_z)$$

$$\bar{F_1} = -3.16\times10^{-3}\ (-11\bar{a}_x - 4\bar{a}_y + 8\bar{a}_z)$$

$$= 3.16\times10^{-3}\ (11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z) \Rightarrow 0.03\bar{a}_x + 0.012\bar{a}_y - 0.024\bar{a}_z$$

$$\bar{F_2} = 9\times10^{9}\ \frac{-20\times10^{-6}\times50\times10^{-6}}{(14.17)^3}\ (11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z)$$

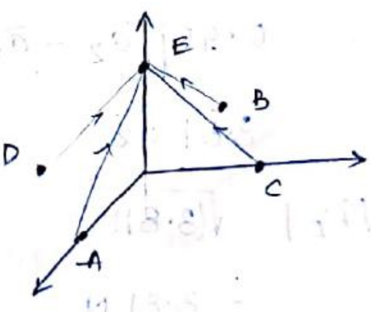
$$= \frac{-9}{(14.17)^3}\ (11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z)$$

$$= -0.03\bar{a}_x - 0.012\bar{a}_y + 0.024\bar{a}_z$$

$$\bar{F_1} = -\bar{F_2}$$

$$|\bar{F_1}| = |\bar{F_2}| = 0.040\ N$$

2) 4 point charges are located at a vector space at points $(1,0,0)$, $(-1,0,0)$, $(0,1,0)$, $(0,-1,0)$ respectively each point charge is of $10\,\mu C$. Determine force experienced by a point charge of $30\,\mu C$ which is located at $(0,0,1)$



$$\bar{OA} = 1\bar{a}_x + 0\bar{a}_y + 0\bar{a}_z = \bar{a}_x$$
$$\bar{OB} = -1\bar{a}_x + 0\bar{a}_y + 0\bar{a}_z = -\bar{a}_x$$
$$\bar{OC} = 0\bar{a}_x + 1\bar{a}_y + 0\bar{a}_z = \bar{a}_y$$
$$\bar{OD} = 0\bar{a}_x - 1\bar{a}_y + 0\bar{a}_z = -\bar{a}_y$$
$$\bar{OE} = \bar{a}_z$$

$$\bar{R}_{AE} = \bar{OE} - \bar{OA} = \bar{a}_z - \bar{a}_x$$
$$\bar{R}_{BE} = \bar{OE} - \bar{OB} = \bar{a}_z + \bar{a}_x$$
$$\bar{R}_{CE} = \bar{OE} - \bar{OC} = \bar{a}_z - \bar{a}_y$$
$$\bar{R}_{DE} = \bar{OE} - \bar{OD} = \bar{a}_z + \bar{a}_y$$

$$|\bar{R}_{AE}| = |\bar{R}_{EA}| = \sqrt{1+1} = \sqrt{2} \qquad |\bar{R}_{DE}| = \sqrt{1+1} = \sqrt{2}.$$

$$|\bar{R}_{BE}| = |\bar{R}_{EB}| = \sqrt{1+1} = \sqrt{2}.$$

$$|\bar{R}_{CE}| = |\bar{R}_{EC}| = \sqrt{1+1} = \sqrt{2}$$

$$\bar{F}_E = \bar{F}_{AE} + \bar{F}_{BE} + \bar{F}_{CE} + \bar{F}_{DE}$$

$$\bar{F}_{AE} = \frac{K Q_A Q_E}{|\bar{R}_{AE}|^2} \bar{a}_{AE} \qquad \bar{a}_{AE} = \frac{\bar{R}_{AE}}{|\bar{R}_{AE}|} = \frac{\bar{a}_z - \bar{a}_x}{\sqrt{2}}.$$

$$= \frac{9\times10^9 \times 10\times10^{-6} \times 30\times10^{-6}}{2\sqrt{2}} (\bar{a}_z - \bar{a}_x)$$

$$= 0.95(\bar{a}_z - \bar{a}_x) \quad \left[|\bar{F}_{AE}| = \sqrt{0.95^2 + 0.95^2} = 1.34\right] \times$$

$$\bar{F}_{BE} = \frac{9\times10^9 \times 10\times10^{-6} \times 30\times10^{-6}}{2\sqrt{2}} (\bar{a}_z + \bar{a}_x)$$

$$= 0.95(\bar{a}_z + \bar{a}_x) \quad \left[|\bar{F}_{BE}| = \sqrt{0.95^2 + 0.95^2} = 1.34\right] \times$$

$$\bar{F}_{CE} = \frac{9\times10^9 \times 10\times10^{-6} \times 30\times10^{-6}}{2\sqrt{2}} (\bar{a}_z - \bar{a}_y)$$

$$= 0.95(\bar{a}_z - \bar{a}_y)$$

$$\bar{F}_{DE} = \frac{9\times10^9 \times 10\times10^{-6} \times 30\times10^{-6}}{2\sqrt{2}} (\bar{a}_z + \bar{a}_y)$$

$$= 0.95(\bar{a}_z + \bar{a}_y)$$

$$\bar{F}_E = \bar{F}_{AE} + \bar{F}_{BE} + \bar{F}_{CE} + \bar{F}_{DE}$$

$$= 0.95\left[\bar{a}_z - \cancel{\bar{a}_x} + \bar{a}_z + \cancel{\bar{a}_x} + \bar{a}_z - \cancel{\bar{a}_y} + \bar{a}_z + \cancel{\bar{a}_y}\right]$$

$$= 3.81\,\bar{a}_z$$

$$|\bar{F}_E| = \sqrt{(3.81)^2}$$

$$= 3.81 \text{ N}$$

Electric field (vector) (or) Electric field intensity:
Strength

The force experienced by a unit +ve charge when it is brought near to another charge is called El-field intensity.

denoted by $\overline{E}$

units: N/C

vector form of electric field:

Let Q_1 be any charge and Q_2 be unit positive charge then from defination of el-field strength and coloumbs law we can write

$$\overline{F_2} = K \frac{Q_1 Q_2}{|\overline{R_{12}}|^2} \overline{a_{12}}$$

where $\overline{R_{12}}$ is distance vector b/w the point charges and $\overline{a_{12}}$ is unit vector along the force.

$\overline{R_{12}} = \overline{r_2} - \overline{r_1}$ where $\overline{r_1}, \overline{r_2}$ are position vectors

and $\overline{a_{12}} = \frac{\overline{R_{12}}}{|\overline{R_{12}}|}$

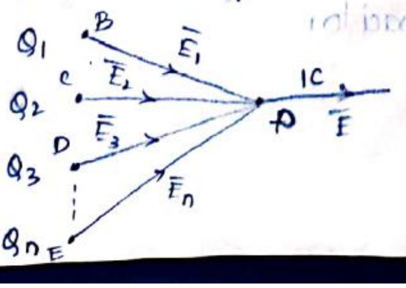
Let $Q_2 = 1C$ then

$$\frac{\overline{F_2}}{Q_2} = \overline{E} = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{|\overline{R_{12}}|^2} \overline{a_{12}}$$

There fore $\overline{E}$ or electric field strength is generalised

as $$\overline{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\overline{R_{12}}|^2} \overline{a_{12}}$$

Electric field intensity for 'n' point charges :-



consider 'n' point charges $Q_1, Q_2, \ldots Q_n$ which are placed in vectors space and where positions vectors are $\bar{r}_1, \bar{r}_2, \ldots \bar{r}_n$

Let there be any other point 'p' whose position vector is $\bar{r}_p$

Now to calculate $\bar{E}$ at point 'p' due to 'n' point charges we apply the principle of superposition.

Now $\bar{E}$ at point 'p' due to Q_1

$$\bar{E}_{1P} = \frac{1}{4\pi\epsilon} \frac{Q_1}{|R_{1P}|^2} \bar{a}_{1P}$$

11ly.

$\bar{E}$ at point 'p' due to Q_n

$$\bar{E}_{nP} = \frac{1}{4\pi\epsilon} \frac{Q_n}{|\bar{R}_{nP}|^2} \bar{a}_{nP}$$

By superposition principle: Net el- field due to 'n' point charges is $\bar{E} = \bar{E}_{1P} + \bar{E}_{2P} + \cdots + \bar{E}_{nP}$

$$\bar{E} = \frac{1}{4\pi\epsilon}\left[\frac{Q_1}{|\bar{R}_{1P}|^2} \bar{a}_{1P} + \frac{Q_2}{|\bar{R}_{2P}|^2} \bar{a}_{2P} + \cdots + \frac{Q_n}{|\bar{R}_{nP}|^2} \bar{a}_{nP} \right]$$

$$= \frac{1}{4\pi\epsilon}\left[\sum_{i=1}^{n} \frac{Q_i}{|R_{iP}|^2} \bar{a}_{iP} \right]$$

CHARGE DISTRIBUTION :-

- TYPES
  - Point charge.
  - Line charge. current carrying conductor
  - Surface charge. capacitor
  - volume charge.

• Point charge distribution:

→ point whose dimensions are very small compared to surface carrying charge.

• Line charge distribution:

If charges are spread along a line either finite or infinite then it is called uniform line charge.

charge density for a line charge is given by:

$$\rho_L = \frac{Q}{L} \quad (\text{units} = C/m) \quad \text{where } \rho_L \rightarrow \text{charge per unit length.}$$

• Surface charge distribution:

If charges are distributed uniformly along a surface then charge distribution is called uniform surface charge distribution.

uniform surface charge density (ρ_s) and is given by

$$\rho_s = \frac{Q}{V} \quad \text{unit} = \frac{\text{charge}}{\text{unit surface area (S.A)}} = \frac{dQ}{dS} \quad \text{units } (C/m^2)$$

• Volume charge distribution:

If charges are distributed uniformly along a unit volume then distribution is said to be volume charge distribution.

volume charge density (ρ_v) and is given by

$$\rho_v = \frac{\text{charge}}{\text{unit volume}} = \frac{dQ}{dv}$$

units → C/m^3.

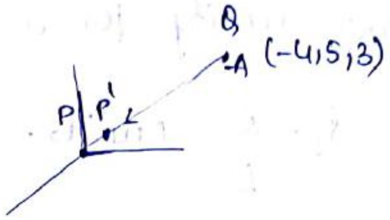
Find the electric field intensity at origin due to a point charge 54.9nC located at (-4,5,3) i)-for air medium ii) find at the point the electric field at point (1,3,5)

Given that a charge of 54.9nC placed at (-4,5,3)

$A = (-4,5,3)$

$Q = 54.9\,nC$.

Let P be a point where electric field is to be calculated.



$P = (0,0,0)$

$\overline{OA} = -4\bar{a}_x + 5\bar{a}_y + 3\bar{a}_z$

$\overline{R}_{AP} = \overline{OP} - \overline{OA}$

$= 4\bar{a}_x - 5\bar{a}_y - 3\bar{a}_z$

$|\overline{R}_{AP}| = \sqrt{16+25+9}$

$= 7.071$

$$\bar{a}_{AP} = \frac{4\bar{a}_x - 5\bar{a}_y - 3\bar{a}_z}{7.071}$$

$= 0.56\bar{a}_x - 0.70\bar{a}_y - 0.42\bar{a}_z$

$$\overline{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{|\overline{R}_{AP}|^2}\bar{a}_{AP}$$

$$= \frac{9\times10^{9}\times 54.9\times10^{-9}}{(7.07)^2}\times(0.56\bar{a}_x - 0.70\bar{a}_y - 0.42\bar{a}_z$$

$\overline{E} = \cancel{9.88}\ 5.53\bar{a}_x - 6.9\,\bar{a}_y - 4.14\bar{a}_z$

$|\overline{E}| = 9.97\,N/C$.

ii) $P = (1,3,5)$

$R_{AP} = \overline{OP} - \overline{OA} = 5\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z$

$|\overline{R}_{AP}| = 5.74$

$$\bar{a}_{AP'} = \frac{4\bar{a}_x - 5\bar{a}_y - 3\bar{a}_z}{(5.74)}$$

$$= 0.121\bar{a}_x - 0.151\bar{a}_y - 0.091\bar{a}_z$$

$$\bar{E} = \frac{9\times10^9 \times 549\times10^{-9}}{(5.74)^2} \cdot (0.121\bar{a}_x - 0.151\bar{a}_y - 0.091\bar{a}_z)$$

$\rightarrow (13.03\bar{a}_x - 5.212\bar{a}_y + 5.212\bar{a}_z)$

$$= 14.99$$

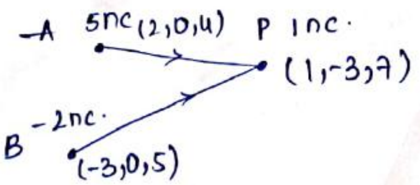
$$\bar{E} = 1.81\bar{a}_x - 2.2\bar{a}_y - 1.36\bar{a}_y$$

$$|\bar{E}| = 3.15 \text{ N/C}.$$

$$= \cancel{0.69\bar{a}_x} - 0.\cancel{67}\bar{a}_y - \cancel{0.52\bar{a}_z}$$

$$\bar{a}_{AP'} = \frac{5\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z}{5.74}$$

Two point charges of 5 nC and -2nC are located at (2,0,4) and (-3,0,5) i) Determine force on a point charge of 1nC located at (1,-3,7)
ii) find electric field $\bar{E}$ at (1,-3,7)



i) $\overline{OA} = 2\bar{a}_x + 4\bar{a}_z$

ii) $\overline{OB} = -3\bar{a}_x + 5\bar{a}_z$ $\qquad R_{AB} =$

$\overline{OP} = \bar{a}_x - 3\bar{a}_y + 7\bar{a}_z$

$\bar{R}_{AP} = -\bar{a}_x - 3\bar{a}_y + 3\bar{a}_z$ $\qquad |\bar{R}_{AP}| = 4.35$

$\bar{R}_{BP} = 4\bar{a}_x - 3\bar{a}_y + 2\bar{a}_z$ $\qquad |\bar{R}_{BP}| = 5.38$

$$\bar{a}_{AP} = \frac{-\bar{a}_x - 3\bar{a}_y + 3\bar{a}_z}{4.35} = -0.229\,\bar{a}_x - 0.689\bar{a}_y + 0.689\bar{a}_z$$

$$\bar{a}_{BP} = \frac{4\bar{a}_x - 3\bar{a}_y + 2\bar{a}_z}{5.38} = 0.743\bar{a}_x - 0.557\bar{a}_y + 0.371\bar{a}_z$$

$$P \quad \bar{E}_{AP} = \frac{9\times10^{9} \times 5\times10^{-9} \times 1\times10^{-9}}{(4.358)^2}(-0.229\bar{a}_x - 0.68\bar{a}_y + 0.68\bar{a}_z)$$

$$\bar{E}_{AP} = 1.04\times10^{-9}\,\bar{a}_x + 0.708\times10^{-9}\,\bar{a}_y - 0.708\bar{a}_z\times10^{-9}$$

$$|\bar{E}_{AP}| = 1.44\times10^{-9}\ N/C.$$

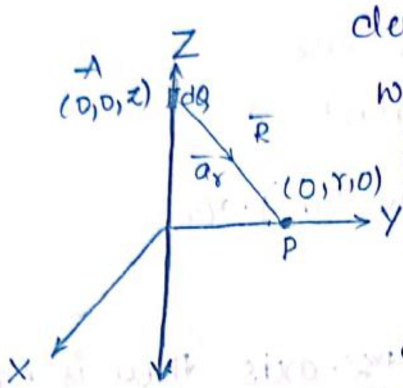
$$\bar{E}_{BP} = (-2.29\ \bar{a}_x + 1.70\bar{a}_y - 1.14\bar{a}_z)\times10^{-9}$$

$$|\bar{E}_{BP}| = \sqrt{(2.29)^2 + (1.70)^2 + (1.14)^2}$$

$$= 3.071\times10^{-9}\ N/C.$$

ELECTRIC FIELD INTENSITY DUE TO AN INFINITE LINE CHARGE:

consider an point infinite line charge which is placed around the z axis. consider a point charge of line charge density $\bar{\rho}_L$ on the infinite line charge



where the differential charge is dQ. Let 'p' be any point on Y axis which is at a radial distance 'r' from the origin. consider a small differential length 'dL' carrying differential charge 'dQ'

Let the coordinates of differential charge element are (0,0,z).

$$\rho_L = \frac{dQ}{dL}$$

$$dQ = \rho_L dL$$

Now the position vectors for differential charge and point 'p' is given by

$$\overline{OA} = z \cdot \bar{a}_z$$

$$\overline{OP} = r \cdot \bar{a}_y$$

The distance vector $\bar{R}$ along the direction of electric cha field is given by:

$$\bar{R} = \bar{R}_{AP} = \overline{OP} - \overline{OA}$$

$$= r\bar{a}_y - z\bar{a}_z$$

$$|\bar{R}| = \sqrt{r^2+z^2}$$

$$\bar{a}_r = \frac{r \cdot \bar{a}_y - z\bar{a}_z}{\sqrt{r^2+z^2}}$$

from coloumbs law $\bar{E}$ due to a differential charge element dQ is given by

$$d\bar{E} = \frac{1}{4\pi\epsilon} \frac{dQ}{|\bar{R}|^2} \bar{a}_r \rightarrow ①$$

From line charge density $\rho_L = \frac{dQ}{dL}$

$$\Rightarrow dQ = \rho_L \, d_L \rightarrow (2)$$

From (1) & (2)

$$\bar{dE} = \frac{1}{4\pi\epsilon(\sqrt{(r^2+z^2)})^2} \frac{\rho_L dL}{} \frac{r\bar{a}_y - z\bar{a}_z}{\sqrt{r^2+z^2}}$$

$$= \frac{1}{4\pi\epsilon} \frac{\rho_L dL}{(r^2+z^2)^{3/2}} (r\bar{a}_y - z\bar{a}_z) \rightarrow (3)$$

** For every [positive] charge on +ve z-axis there is equal and opposite charge on -ve z-axis so el- field components at point 'P' will not have z-components.

$\bar{a}_z$ ||el z-axis so $\bar{a}_z = 0$

$$\bar{dE} = \frac{1}{4\pi\epsilon} \frac{\rho_L dL}{(r^2+z^2)^{3/2}} r\bar{a}_y \rightarrow (4)$$

By Integrating above eqn along it's length (along z-axis) we get overall electric field due to infinite line charge is obtained.

$$\bar{E} = \int_{z=-\infty}^{\infty} \bar{dE} \, d\bar{z}$$

$$= \frac{1}{4\pi\epsilon} \int_{z=-\infty}^{\infty} \frac{\rho_L \bar{dL}}{(r^2+z^2)^{3/2}} r \bar{a}_y \, d\bar{z}$$

$$= \frac{\rho_L}{4\pi\epsilon} \int_{-\infty}^{\infty} \frac{2r}{(r^2+z^2)^{3/2}} \bar{a}_y \, d\bar{z}$$

= consider $z = r\tan\theta$

$$\theta = \tan^{-1}(z/r)$$

$$dz = r\sec^2\theta \, d\theta$$

$z \Rightarrow -\infty, \infty$

$\theta \Rightarrow -\pi/2, \pi/2.$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon} \int_{-\pi/2}^{\pi/2} \frac{r}{(r^2+r^2\tan^2\theta)^{3/2}} r\sec^2\theta \, d\theta \, \bar{a_y}$$

$$= \frac{\rho_L}{4\pi\epsilon} \int_{-\pi/2}^{\pi/2} \frac{r^2\sec^2\theta}{r^3(1+\tan^2\theta)^{3/2}} d\theta \, \bar{a_y}$$

$$= \frac{\rho_L}{4\pi\epsilon} \int_{-\pi/2}^{\pi/2} \frac{r^2\sec^2\theta}{r^3(\sec^3\theta)} d\theta \, \bar{a_y}$$

$$= \frac{\rho_L}{4\pi\epsilon} \int_{-\pi/2}^{\pi/2} \frac{1}{r} \cos\theta \, d\theta \, \bar{a_y}$$

$$= \frac{\rho_L}{4\pi r\epsilon} (-\sin\theta)_{-\pi/2}^{\pi/2} \bar{a_y}$$

$$= \frac{2\rho_L}{4\pi\epsilon r} \bar{a_y}$$

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi r\epsilon} \bar{a_y} \ N/C \ (\text{or}) \ V/m} \rightarrow 5$$

eqn 5 can be generalised by assuming a unit vector along the distance vector $\bar{R}$ which is $\perp$'r to the line charge. The $\perp$'r distance is r then

the $\bar{E} = \frac{\rho_L}{2\pi\epsilon r} \bar{a_r}$

where $\bar{a_r}$ is the unit vector along the $\perp$'r distance 'r' (in radial direction).

Note: Field intensity $\bar{E}$ at any point has no components in the direction ||el to the line along which line charge is located.

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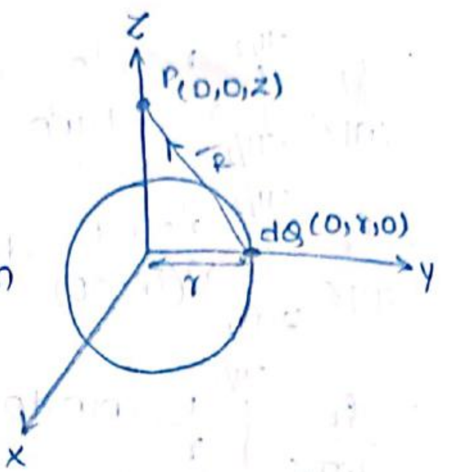
ELECTRIC FIELD INTENSITY DUE TO CHARGED CIRCULAR RING:

consider a charged circular ring of radius 'r' which is placed on XY plane with its center at the origin. It is carrying a uniform charge along the circumference of the ring. Let the charge



density be ρ_L. consider any point 'p' which is at a ⊥r distance 'z' from the base. Therefore the coordinates are point 'p' and differential charge element will become (0,0,z) and (0,r,0) respectively. Now the differential charge dQ is given by

$$\rho_L = \frac{dQ}{dL}$$

$$\bar{dQ} = \rho_L \bar{dL}$$

Now electric field at point 'p' due to differential charge element dQ is given by $\bar{dE} = \frac{1}{4\pi\epsilon}\frac{dQ}{|R|^2}\bar{a}_r$

distance vector $\overline{OP} - \overline{OA} = z\bar{a}_z - r\bar{a}_r$

$$|\bar{R}| = \sqrt{r^2+z^2}$$

$$\bar{a}_r = \frac{z\bar{a}_z - r\bar{a}_r}{\sqrt{r^2+z^2}}$$

$$\bar{dE} = \frac{1}{4\pi\epsilon}\frac{\rho_L\, dL}{(r^2+z^2)^{3/2}}(z\bar{a}_z - r\bar{a}_r)$$

$$\bar{dE} = \frac{\rho_L\, dL}{4\pi\epsilon(r^2+z^2)^{3/2}}(z\bar{a}_z - r\bar{a}_r) \rightarrow ①$$

from above equation radial components of $\bar{E}$ at point 'P' are symmetrically placed in a plane parallel to xy plane. Hence the radial components of el- field will cancel each other i.e $\bar{a}_r = 0$

$$\bar{dE} = \frac{\rho_L}{4\pi\epsilon} dL(z\bar{a}_z) \rightarrow (2)$$

To find over all el- field intensity due to a charged circular ring we integrate the above equation (2) along it's perimeter ~~in~~ θ i.e θ = '0 to 2π'

$$\bar{E} = \int_0^{2\pi} \frac{\rho_L\, dL}{4\pi\epsilon} \frac{z\bar{a}_z\, d\theta}{(r^2+z^2)^{3/2}}$$

$\therefore d\theta = dL$
so neglect dL.

$$= \frac{\rho_L}{4\pi\epsilon(r^2+z^2)^{3/2}} z\bar{a}_z \int_0^{2\pi} d\theta$$

$$= \frac{2\pi \rho_L z\bar{a}_z}{4\pi\epsilon(r^2+z^2)^{3/2}}$$

$$\boxed{E = \frac{\rho_L z\bar{a}_z}{2\epsilon(r^2+z^2)^{3/2}} \; N/C \text{ (or) } V/m}$$

Note: for air medium or freespace $\epsilon = \epsilon_0 \epsilon_r$ & $\epsilon_r = 1$

$$\boxed{\epsilon = \epsilon_0}$$

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Note: The electric field intensity for an infinite sheet of charge density is given by $\bar{E} = \frac{\rho_s}{2\epsilon_0} \bar{a}_z$

Electric flux: It Lines of force along the charge. It is also known as 'displacement flux'.

Properties:

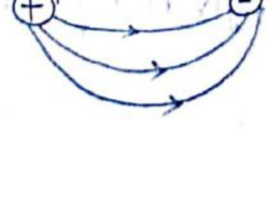
- They always originate at +ve charge and terminate at -ve charge.
- If -ve charge is not present they terminate at ∞
- More the no. of flux lines stronger the [strength of]* electric field.
- Flux lines are always $\parallel^{el}$ to each other and never cross each other.
- Flux lines are always normal to surface in which charged body is present.

DEFINATION OF ELECTRIC FLUX:

If the charge on the body is 'Q' then the total no. of lines originated from point charge is also 'Q' i.e total no. of flux lines is equal to charge on the surface so electric flux $(\psi) = Q$

$$\psi = Q$$

units for electric flux $\rightarrow$ coloumbs.



ELECTRIC FLUX DENSITY : ($\bar{D}$)

It is defined as the net flux lines passing through a unit surface area which is normal to charged body.

It is denoted by $\bar{D}$

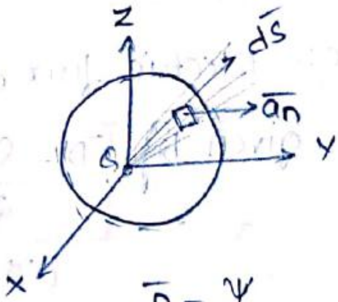
It is given by electric flux density = $\frac{\text{electric flux}}{\text{unit surface area}}$

i.e $\bar{D} = \frac{\Psi}{S}$ or $\bar{D} = \frac{d\Psi}{ds}$

units: It is measured in C/m^2

***Note: $\bar{D}$ has a specific direction which is normal to surface under consideration.

FLUX DENSITY DUE TO VARIOUS CHARGE DISTRIBUTIONS:



consider a sphere in which a point charge 'Q' is placed at center. consider a differential surface area $\bar{ds}$ which is normal to the flux lines originating from the charge as shown in figure.

$\bar{D} = \frac{\Psi}{S}$

for small area

$\bar{D} = \frac{d\Psi}{ds}$

Let $\bar{a}_n$ be a unit vector along flux lines which is normal to surface. so el- flux density due to this point charge is given by.

$$\bar{D} = \frac{d\Psi}{ds} \text{ (or) } \bar{D} = \frac{\Psi}{s}.$$

From defination of flux we have $\Psi = Q$.

and surface area of sphere is $S = 4\pi r^2$.

$$\boxed{\therefore \bar{D} = \frac{Q}{4\pi r^2}\bar{a}_n \ c/m^2}$$

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RELATION B/W ELECTRIC FLUX DENSITY ($\bar{D}$) & ELECTRIC FIELD INTENSITY ($\bar{E}$)

$$\bar{D} = \frac{Q}{4\pi r^2}\,\bar{a}_r$$

We know that,

$$\bar{E} = \frac{1}{4\pi\epsilon}\,\frac{Q}{R^2}\,\frac{Q}{|R|^2}\,\bar{a}_r$$

$$\epsilon\bar{E} = \frac{Q}{4\pi|R|^2}\,\bar{a}_r = \bar{D}$$

$$\bar{D} = \epsilon\cdot\bar{E}$$

for free space

$$\boxed{\bar{D} = \epsilon_0\bar{E}}$$

Note: Electric flux density due to an infinite line charge is given by $\boxed{\bar{D} = \frac{\rho_L}{2\pi r}\,\bar{a}_r}$

$$\left(\because \bar{E} = \frac{\rho_L}{2\pi r\epsilon}\,\bar{a}_r\right)$$

The el- flux density due to charged circular ring will be

$$\boxed{\bar{D} = \frac{\rho_L\cdot r}{2(r^2+z^2)^{3/2}}\cdot\bar{a}_r}$$

$$\left(\because \bar{E} = \frac{\rho_L r}{2(r^2+z^2)^{3/2}\epsilon}\,\bar{a}_r\right)$$

El- flux density due to an infinite ~~line~~ sheet of charge.

$$\boxed{\bar{D} = \frac{\rho_s}{2}\,\bar{a}_r} \qquad \bar{E} = \frac{\rho_s}{2\epsilon}\,\bar{a}_r$$

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* GUASS LAW:

It states that total flux through any closed surface is equal to the charge enclosed by that surface (i.e)

$\Psi =$ charge enclosed by the surface.

From defination of electric flux. we have

$$\left[\bar{D} = \frac{Q}{S}\right] \times \qquad \bar{D} = \frac{d\Psi}{ds}$$

$$d\Psi = \bar{D}\cdot\vec{ds}$$

$$\Psi = \int_S \bar{D}\cdot ds \rightarrow (1)$$

If a volume charge is considered with a charge density of ρ_v then the total charge enclosed is given by

$$\rho_v = \frac{dQ}{dv} \Rightarrow dQ = \rho_v \, dv$$

$$dQ = \int_V \rho_v dv \rightarrow (2)$$

Applying guass law from eq (1) & (2)

$$\int_S \bar{D}\cdot ds = \int_V \rho_v dv \rightarrow (3)$$

[it states that outward flux for closed surface is same as volume integral of divergence]

Applying divergence theorem: $\left[\because \int_S \bar{A}\cdot\bar{ds} = \int_V \nabla \bar{A}\cdot dv\right]$

$$(3) \longleftrightarrow \int_V \nabla \bar{D}\cdot dv = \int_V \rho_v dv \rightarrow (4)$$ [(Maxwell's 1st equation. integral form)] ×

Also

$$\nabla\cdot\bar{D} = \rho_v \rightarrow (5)$$

equations (3) and (5) are called maxwell's 1st equations in integral form and differential form respectively.

-APPLICATIONS OF GAUSS LAW:

- To calculate el flux density using guass law we follow 3 steps.

i) we need to check whether the charged surface is symmetrical about coordinate system or not.

ii) construct a mathematical closed surface called gaussian surface around the charge.

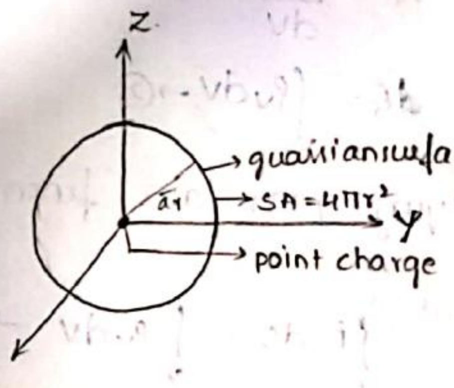
iii) surface is choosen such that flux density $\bar{D}$ is normal to the guassian surface and then we apply guass law

- Finding electric flux density for a point charge:

consider a point charge which is placed at origin

$\psi = Q$.

$\psi = \oint_S \bar{D}$.



consider a point which is place at origin.

step-1: choose the spherical coordinate system as point charge emits flux lines in all 3 directions.

step-2: construct a guassian surface around the spherical body as shown in figure.

step-3: Now flux lines from origin will be normal to the assumed guassian surface. Now we apply guass law i.e total flux falling on a surface is same as the charge enclosed by the body.

$\therefore$ total flux $\psi = \int_S \bar{D} \cdot ds$

$= D \cdot S$ ($\because S \rightarrow$ total surface area of sphere)

$= D \cdot 4\pi r^2 \rightarrow ①$

and charge enclosed is equal to $= Q \rightarrow ②$

$\therefore \psi = Q$

$Q = \bar{D} \cdot 4\pi r^2$

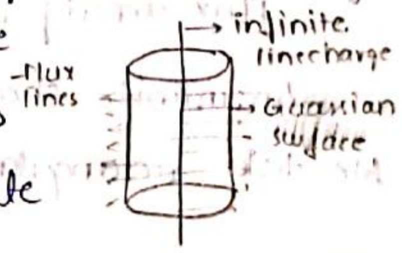
$$\boxed{\bar{D} = \frac{Q}{4\pi r^2} \bar{a}_r}$$

$\bar{a}_r$ is unit vector along flux lines (normal to guassian surface).

$$\boxed{\bar{E} = \frac{Q}{4\pi \epsilon r^2} \bar{a}_r}$$

- Flux density due to infinite line charge

consider an infinite line charge whose line charge density is ρ_L. To apply guass law we assume cylindrical coordinate system.



We construct gaussian surface around the ch line charge.

Now flux lines will meet gaussian surface in radial direction which is normal to the assumed gaussian surface.

Applying gauss law we k T

$\rho = \frac{dQ}{dL}$ $dQ = \rho_L dL$ (L is Length of Assumed portion of infinite line)

$Q = \rho_L l \rightarrow ①$

Total flux due to line charge is given by

$\psi = \int \bar{D} \cdot ds$

$= \bar{D} \cdot S$

$= 2\pi r l \cdot \bar{D} \rightarrow ②$

∴ surface area of gaussian surface is curved surface area of cylinder.

-Applying gauss law we have

-total charge = Ψ

$$\bar{D} \cdot 2\pi r l = \rho_L l.$$

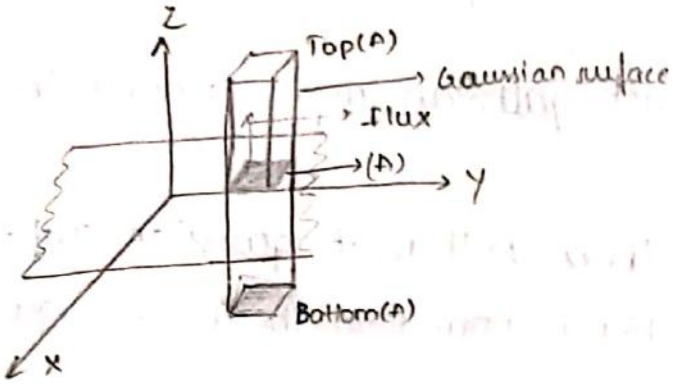
$$\boxed{\bar{D} = \frac{\rho_L}{2\pi r}\bar{a}_r}$$

$\bar{a}_r$ is unit vector along directn of flux Normal to gaussian surface

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon r}\bar{a}_r}$$

- Flux density due to an infinite sheet of charge:-

-An infinite sheet of charge placed on xy plane.
We take rectangular coordinate system to calculate flux density



Let ρ_s be charge density of infinite sheet;
we construct gaussian surface above & below the infinite sheet as shown in fig.
Here flux lines are normal to assumed gaussian surface which are in the direction of Z axis.
-According to gauss law total flux passing through gaussian surface is given by.

$$\psi = \int \bar{D} \cdot ds$$
$$= \bar{D} \left[\int_{top} ds + \int_{bottom} ds \right]$$
$$= \bar{D} \left[A + A \right]$$
$$= 2A\bar{D} \rightarrow (1)$$

Now total charge enclosed by sheet of charge is given by.

$$\rho_s = \frac{dQ}{ds}$$

$$dQ = \rho_s ds$$

$$Q = \int \rho_s ds.$$

$$= \rho_s \int_{\text{charged body}} ds$$

$$Q = \rho_s A \rightarrow (2)$$

Q. from gauss law

we have.

$$2A\bar{D} = \rho_s A$$

$$\boxed{\bar{D} = \frac{\rho_s}{2} \bar{a}_z}$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon} \bar{a}_z}$$

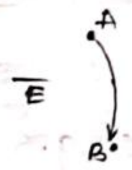
- Flux density due to uniformly charged sphere:-

consider a uniformly charged sphere of radius 'a' which is placed at origin.

ELECTRIC POTTENTIAL :

consider a point charge 'Q' which is moved from 'A' to 'B' in an electric field as shown in figure.

$\bar{E}$ canbe calculated from coloumb's law and Gauss law.



The other way of calculating $\bar{E}$ is from "scalar" electric potential.

From coloumb's law $\bar{E} = \frac{\bar{F}}{Q}$

$$\bar{F} = \bar{E} Q$$

Work done indisplacing a point charge with a displacement of $d\bar{L}$ is given by

$$dw = -\bar{F} \cdot d\bar{L}$$ (As work done is due to external field $\bar{E}$ so negative)

(or)

$$dw = -\bar{F} \cdot dL$$

$$\therefore \; dw = -\bar{E} Q \cdot dL$$

-ve sign indicates work is done by an external agent i.e $\bar{E}$. so work done is moving a point charge 'Q' from point 'A' to point 'B' is otherwise called Potential energy needed to move point charge from 'A' to 'B' and is given by work done (W)

$$W = -\int_A^B \bar{E} Q \cdot dL$$

$$\frac{W}{Q} = -\int_A^B \bar{E} \cdot dL.$$

$$V_{AB} = \frac{W}{Q} = -\int_A^B \bar{E} \cdot dL$$

V_{AB} = P.E for displacing point charge from A to B.

if V_{AB} is -ve then there is a loss in P.E in moving a charge from A to B. and vice versa.

This P.E is measured in 'J/c'.

It is generally known as 'volt'

* Mathematical representation of Electric potential:

consider an electric field which is due to a point charge 'Q' consider 2 points A and B which are in el- field. whose position vectors are $\bar{r}_A$ and $\bar{r}_B$

From coloumb's law

$\bar{E}$ due to a charge 'Q' is given by.

$\bar{E}$ A → B

$$\bar{E} = \frac{Q}{4\pi\epsilon} \frac{1}{|\bar{R}|^2} \bar{a}_R$$

From def of $\bar{E}$ el- field potential required to move a charge from A to B is given by.

$\bar{A} \cdot \bar{A} = 1$

$\bar{A} \cdot \bar{B} = 0$

$$V_{BA} = -\int_A^B \bar{E} \cdot \bar{dL}$$

$R = r$

$$= -\int_A^B \frac{1}{4\pi\epsilon} \frac{Q}{|\bar{R}|^2} \bar{a}_R \cdot \bar{dL}$$

differential length in spherical coordinates is given by.

$$\bar{dL} = dr\bar{a}_r + rd\theta\bar{a}_\theta + r\sin\theta \cdot \bar{a}_\phi d\phi$$

$$V_{AB} = -\int_A^B \frac{Q}{4\pi\epsilon |\bar{R}|^2} \bar{a}_r \left[dr\bar{a}_r + rd\theta\bar{a}_\theta + r\sin\theta\, \bar{a}_\phi \right]$$

r^{-2}

$\bar{a}_r \cdot \bar{a}_r = 1$

$\bar{a}_r \cdot \bar{a}_\theta = 0$

$\bar{a}_r \cdot \bar{a}_\phi = 0$

$$V_{AB} = -\int_A^B \frac{Q}{4\pi\epsilon|\bar{R}|^2} dr \cdot 1$$

$$= \frac{-Q}{4\pi\epsilon} \int_A^B \frac{1}{r^2} dr.$$

r^{-2} $\frac{r^{-2+1}}{-2+1}$ → $-\frac{1}{r}$

$$= \frac{-Q}{4\pi\epsilon}\left[\frac{-1}{r}\right]_A^B$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_B}-\frac{1}{r_A}\right]$$

$$\boxed{V_{AB} = V_{BA} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_B}-\frac{1}{r_A}\right]}$$

$$V_{AB} = V_{BA} = \frac{Q}{4\pi\epsilon}\frac{1}{r_B} - \frac{Q}{4\pi\epsilon}\frac{1}{r_A}.$$

$$V_{AB}\ \cancel{V_{BA}} = V_B - V_A.$$

V_B and V_A are called absolute pottentials at points A and B

// So V_{AB} is regarded as the pottential of B with refeence to the potential of A.

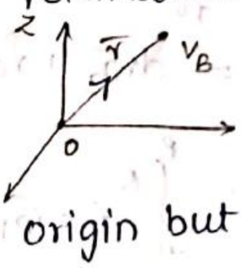
* POTENTIAL ON VARIOUS CHARGE DISTRIBUTIONS:

If the charge is moved from '∞' to point 'B' the potential at point 'B' is given by

ie $V_{AB} = V_B = \frac{Q}{4\pi\epsilon r_B}$ ($\because$ pottential at '∞' is '0' $1/r_A = 1/\infty = 0$)

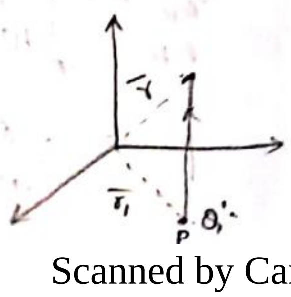
In general potential at a radius 'r' from a point charge at origin is called absolute pot'ential and is given by

$$V = \frac{Q}{4\pi\epsilon r} \Rightarrow$$



If point charge is not located at origin but located at a point whose position vector is $\bar{r}_1$ then the pottential at the point is given by

$$V_p = \frac{Q}{4\pi\epsilon|\bar{r}-\bar{r}_1|}$$



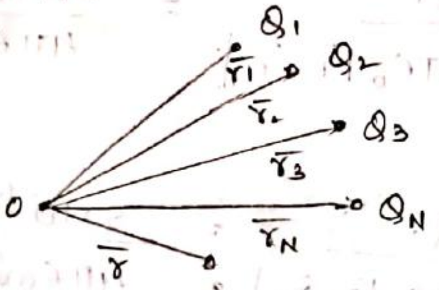
It is evident that electric potential

Problems:

1) Two point charges $-4\mu C$ and $5\mu C$ are located at $(2,-1,3)$ and $(0,4,-2)$. find the potential at $(1,0,1)$ assuming '0' potential at '∞'.

NOTE:

If N point charges $Q_1, Q_2, Q_3 \cdots Q_n$ are located at points whose position vectors are $\bar{r}_1, \bar{r}_2, \cdots \bar{r}_N$ then by applying principle of superposition we calculate overall Potential difference at a point whose position vector is $\bar{r}$



$$V_1 = \frac{Q_1}{4\pi\epsilon|\bar{r}-\bar{r}_1|}$$

$$V_2 = \frac{Q_2}{4\pi\epsilon||\bar{r}-\bar{r}_2||}$$

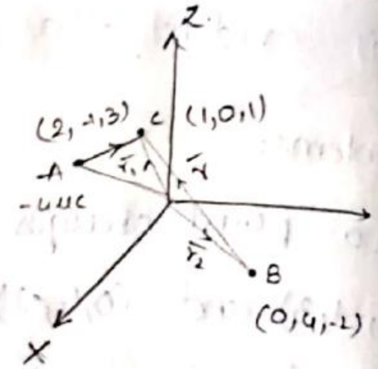
$$V = V_1 + V_2 + V_3 + \cdots + V_N$$

$$= \frac{Q_1}{4\pi\epsilon|\bar{r}-\bar{r}_1|} + \frac{Q_2}{4\pi\epsilon|\bar{r}-\bar{r}_2|} + \cdots + \frac{Q_N}{4\pi\epsilon||\bar{r}-\bar{r}_8|}$$

$$\boxed{V(\bar{r}) = \frac{1}{4\pi\epsilon}\sum_{i=1}^{N}\frac{Q_i}{|\bar{r}-\bar{r}_i|}}$$

$$V_{AC} = \frac{-4\times10^{-6}}{4\pi\epsilon|\bar{r}-\bar{r}_1|}$$

$$= \frac{9\times10^{9}\times(-4)\times10^{-6}}{}$$



$$\bar{r} = \bar{a}_x + \bar{a}_z$$

$$\bar{r}_1 = 2\bar{a}_x - \bar{a}_y + \bar{a}_z$$

$$\bar{r}_2 = 4\bar{a}_y - 2\bar{a}_z$$

$$\bar{r}-\bar{r}_1 = -\bar{a}_x + \bar{a}_y - 2\bar{a}_z \quad |\bar{r}-\bar{r}_1| = \sqrt{1+1+4} = \sqrt{6}$$

$$\bar{r}-\bar{r}_2 = \bar{a}_x - 4\bar{a}_y + 3\bar{a}_z \quad |\bar{r}-\bar{r}_2| = \sqrt{1+16+9} = \sqrt{26}$$

$$V_A = \frac{Q_1}{4\pi\epsilon_0|\bar{r}-\bar{r}_1|} = \frac{-4\times10^{-6}}{4\pi\epsilon_0\sqrt{6}} = -14.69\times10^{3}$$

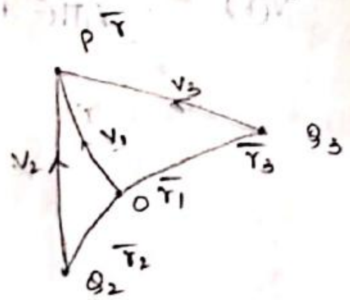
$$V_B = \frac{Q_2}{4\pi\epsilon_0|\bar{r}-\bar{r}_2|} = \frac{5\times10^{-6}}{4\pi\epsilon_0\sqrt{26}} = 8.825\times10^{3}$$

V_{AB}

$$V(r) = V_A + V_B$$

$$V(r) = -5.865\times10^{3}\,J/c.$$

2) Point charge of 3μC is located at origin and 2 other charges of -4μC and 5μC are located at (2,-1,3) and (0,4,-2) respectively. find pottential at (-1,5,2) assuming zero pottential at '∞'.



$Q_1 = 3μC$ (0,0,0) A

$Q_2 = -4μC$ (2,-1,3) B

$Q_3 = 5μC.$ (0,4,-2) C

Given pottential must be calculated at (-1,5,2) P

Position vector of $\bar{P} = \bar{r} = -\bar{a}_x + 5\bar{a}_y + 2\bar{a}_z$;

position vector of $A = \bar{r}_1 = 0$

Position vector of $C = \bar{r}_3 = 4\bar{a}_y - 2\bar{a}_z$;

$|\bar{r}-\bar{r}_1| = \sqrt{30}$

$|\bar{r}-\bar{r}_2| = \sqrt{46}$

$|\bar{r}-\bar{r}_3| = \sqrt{18}$

$$V_r = \frac{Q_1}{4\pi\epsilon_0|\bar{r}-\bar{r}_1|} + \frac{Q_2}{4\pi\epsilon_0|\bar{r}-\bar{r}_2|} + \frac{Q_3}{4\pi\epsilon_0|\bar{r}-\bar{r}_3|}$$

$$= 9\times10^9\left[\frac{3\times10^{-6}}{\sqrt{30}} + -\frac{4\times10^{-6}}{\sqrt{46}} + \frac{5\times10^{-6}}{\sqrt{18}}\right]$$

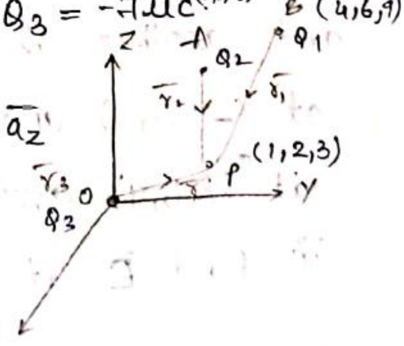
$$= 10228.194 \text{ J/c.}$$

$$= 10.22\times10^{3} \text{ J/c.}$$

3) calculate work done in displacing a point charge from infinity to point (1,2,3).

Let

i) Q_1, Q_2, Q_3 be charges located at (1,5,8), (4,6,9), (0,0,0). $Q_1 = 3\mu C$, $Q_2 = -5\mu C$, $Q_3 = -7\mu C$



position vector of $P = \bar{r} = \bar{a}_x + 2\bar{a}_y + 3\bar{a}_z$

Position vector of $\bar{A} = \bar{a}_x + 5\bar{a}_y + 8\bar{a}_z = \bar{r}_2$

Position vector of $\bar{B}$ $(\bar{r}_1)$ $= 4\bar{a}_x + 6\bar{a}_y + 9\bar{a}_z$

Position vector of $\bar{O} = \bar{r}_3 = 0$

$|\bar{r}-\bar{r}_1| = |-3\bar{a}_x - 4\bar{a}_y - 6\bar{a}_z| = \sqrt{9+16+36} = \sqrt{61}$

$|\bar{r}-\bar{r}_2| = |-3\bar{a}_y - 5\bar{a}_z| = \sqrt{34}$

$|\bar{r}-\bar{r}_3| = |\bar{a}_x+2\bar{a}_y+3\bar{a}_z|$

$= \sqrt{14}$

$V = V_1+V_2+V_3+ \cdots$

$= \sum \frac{Q_1}{4\pi\epsilon_0|\bar{r}-\bar{r}_1|} = 9\times10^9\left[\frac{-5\times10^{-6}}{\sqrt{61}} + \frac{3\times10^{-6}}{\sqrt{34}} - \frac{7\times10^{-6}}{\sqrt{14}}\right]$

$= 9\times10^3\left[\quad\right]$

$= -17968.65\ J/c$

$V = -17.968\times10^3\ J/c.$

4) If a point charge of 5 μc is located at (1,2,3) and another 2 charges at of same charge located at (−1,2,3) and (1,−2,3) respectively i) find potential at (3,6,9) assuming zero potential at '∞'
ii) find $\bar{E}$ at (3,6,9) due to 3 point charges

i) $Q_1 = 5\mu c$ — A (1,2,3) $P = (3,6,9)$

$Q_2 = 5\mu c$ B (−1,2,3)

$Q_3 = 5\mu c$. C. (1,−2,3)

Position vectors of

- 'P' $= \bar{r} = 3\bar{a}_x + 6\bar{a}_y + 9\bar{a}_z$

'A' $= \bar{r}_1 = \bar{a}_x + 2\bar{a}_y + 3\bar{a}_z$

$\bar{B} = \bar{r}_2 = -\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z$

$C = \bar{r}_3 = +\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z$

$|\bar{r} - \bar{r}_1| = |2\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z| = \sqrt{4+16+36} = \sqrt{56}$

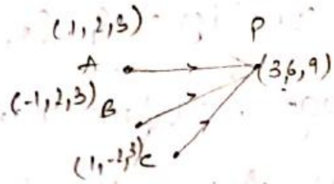
$|\bar{r} - \bar{r}_2| = |4\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z| = \sqrt{16+16+36} = \sqrt{68}$

$|\bar{r} - \bar{r}_3| = |2\bar{a}_x + 8\bar{a}_y + 6\bar{a}_z| = \sqrt{4+64+36} = \sqrt{104}$

$V = V_1 + V_2 + V_3$

$= 5\times10^{-6} \times 9\times10^{9}\left[\frac{1}{\sqrt{56}} + \frac{1}{\sqrt{68}} + \frac{1}{\sqrt{104}}\right]$

$= 15.8\times10^{3}\ J/C.$



ii) $|R_{AP}| = |\bar{r}_p - \bar{r}_a|$

$= |\bar{r} - \bar{r}_1| = \sqrt{56}$

$|R_{BP}| = |\bar{r} - \bar{r}_2| = \sqrt{68}$ $\qquad a_{xy} = \frac{R_{xy}}{|R_{xy}|}$

$|R_{CP}| = |\bar{r} - \bar{r}_3| = \sqrt{104}$

$\bar{a}_{AP} = \frac{2\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z}{\sqrt{56}} = 0.267\bar{a}_x + 0.534\bar{a}_y + 0.801\bar{a}_z$

$\bar{a}_{BP} = \frac{4\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z}{\sqrt{68}} = 0.485\bar{a}_x + 0.485\bar{a}_y + 0.727\bar{a}_z$

$\bar{a}_{CP} = \frac{2\bar{a}_x + 8\bar{a}_y + 6\bar{a}_z}{\sqrt{104}} = 0.196\bar{a}_x + 0.784\bar{a}_y + 0.588\bar{a}_z$

$F_{AP} = \frac{Q}{4\pi\epsilon_0 |R_{AP}|^2}[0.267\bar{a}_x + 0.534\bar{a}_y + 0.801\bar{a}_z]$

$= \cancel{-8.03\times10^{-4}}\ 803.57[0.267\bar{a}_x + 0.534\bar{a}_y + 0.801\bar{a}_z]$

$F_{BP} = \frac{Q}{4\pi\epsilon_0 |R_{BP}|^2}[0.485\bar{a}_x + 0.485\bar{a}_y + 0.727\bar{a}_z]$

$= 661.76[0.485\bar{a}_x + 0.485\bar{a}_y + 0.727\bar{a}_z]$

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$$\bar{E}_{CP} = \frac{5\times10^{-6}\times9\times10^{9}}{104}\left[0.196\,\bar{a}_x + 0.784\,\bar{a}_y + 0.588\,\bar{a}_z\right]$$

$$= 432.6\left[0.196\,\bar{a}_x + 0.784\,\bar{a}_y + 0.588\,\bar{a}_z\right]$$

$$\bar{E} = \bar{E}_{AP} + \bar{E}_{BP} + \bar{E}_{CP}$$

$$= 820.99\,\bar{a}_x + 1089.21\,\bar{a}_y + 1379.127\,\bar{a}_z$$

$$|\bar{E}| = 1863.633 \text{ N/c}$$

$$= 1.863 \text{ N/c or V/m}$$

* POTENTIAL DIFFERENCE DUE TO VARIOUS CHARGE DISTRIBUTION

consider ρ_L, ρ_s, ρ_v be charge densities due to line charge, surface charge and volume charge respectively. then the potential difference due to various charge distribution is given by.

-for line charge distribution.

$$V = \frac{\int_L \rho_L \cdot d_L}{4\pi\epsilon r} \quad (\text{or}) \quad \frac{\int \rho_L \cdot d_L}{4\pi\epsilon|\bar{r}-\bar{r}_1|}$$

If charge is not located at origin.

surface charge distribution:

$$V = \frac{\int_S \rho_s \cdot ds}{4\pi\epsilon r} \quad (\text{or}) \quad V = \frac{\int_S \rho_s \cdot d_s}{4\pi\epsilon|\bar{r}-\bar{r}_1|}$$

volume charge distribution:

$$V = \frac{\int_V \rho_v\, dv}{4\pi\epsilon r} \quad \text{or} \quad V = \frac{\int_V \rho_v\, dv}{4\pi\epsilon|\bar{r}-\bar{r}_1|}$$

* MAXWELL 2^{nd} EQUATION: (OR) RELATION B/W $\bar{E}$ and $\bar{V}$

potential difference b/w 2 points A and B is independent of the path taken i-e $V_{AB} = -V_{BA}$.

$$\Rightarrow -V_{AB} + V_{BA} = 0$$

$$\oint_L \bar{E} \cdot dL = 0 \rightarrow (1)$$

It shows that Line integral along a closed path is '0'.

It shows that no net work is done in displacing a point charge from A to B and B to A along a closed path.

WKT from stoke's theorem we get

$$\oint_L \bar{E} \cdot d\bar{L} = \int_S (\nabla \times \bar{E}) \, d\bar{s}$$

$$\Rightarrow \int_S (\nabla \times \bar{E}) \, ds = 0 \rightarrow (2)$$

also

$$\boxed{\nabla \times \bar{E} = 0} \rightarrow (3)$$

NOTE

Eqn's 2 and 3 are same which are called Maxwell's equations.

eqn (2) $\rightarrow$ 2nd maxwell's eqn in integral form.

eqn (3) $\rightarrow$ " " " " point / differential form

* E AND V RELATION:

* $\bar{E}$ and V RELATION:

From defination of potential we have

$$V = \int \bar{E} \cdot \bar{dL}$$

$\because \Delta \cdot \bar{A} = \text{Div}(\bar{A})$
$\Delta \times \bar{A} = \text{curl}(\bar{A})$
$\Delta \cdot A = \text{gradient}($

differentiating on b.s

$$dV = -\bar{E} \cdot \bar{dL}$$

The electric field is decomposed into E_x, E_y and E_z components and differential length has a displacement of dx, dy, dz in x, y, z directions respectively then we replace

dv with ∇V and gradient of v is given by

$$\nabla V = \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} \rightarrow (1)$$

$$-\bar{E} \cdot \bar{dL} = -[\bar{E}_x \cdot dx\, \bar{a}_x + \bar{E}_y \cdot dy\, \bar{a}_y + \bar{E}_z\, dz\, \bar{a}_z] \rightarrow (2)$$

from eqn's (1) and (2) we can ~~say~~ write that the corresponding field components in x, y, z direction is

$$\frac{dv}{dx} = -\bar{E}_x, \quad \frac{dv}{dy} = -\bar{E}_y, \quad \frac{dv}{dz} = -\bar{E}_z$$

$$\bar{E} = \bar{E}_x + \bar{E}_y + \bar{E}_z$$

$\therefore$ above eqn's can be generalised as $\bar{E} = -\nabla V$

i.e $\bar{E}$ is same as gradient of electric potential

5) Find potential at (-2,4,3) due to a point charge of -2.5nC which is at (0,-5,1). Assume zero potential at '∞'

P(-2,4,3)

A(0,-5,1)

$OP = -2\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z$

$OA = 0 - 5\bar{a}_y + \bar{a}_z$

$\overline{AP} = -2\bar{a}_x + 9\bar{a}_y + 2\bar{a}_z$

$$V = \frac{Q}{4\pi\epsilon_0 |\bar{r}-\bar{r}_1|}$$

$$= \frac{-2.5\times10^{-9}\times9\times10^{9}}{\sqrt{89}} = -2.38\,J/C$$

6) A point charge of 10nC is located at (-1,2,0) while a line charge at y=1 and z=2 carries a uniform line charge density $\rho_L = 4nC/m$. Find V at point 'A' (1,0,2)

6 I. Sol

Given that 10 nC $= Q_1$ is located at (-1,2,0) = 'P'

Also " " line charge of $\rho_L = 4nC/m$ is at y=1 and z=2

Now net potential at point A(1,0,2) will be due point charge as well as the line charge i.e $V = V_1 + V_2$. where

$V_1 \rightarrow$ potential due to point charge

$V_2 \rightarrow$ " " " line "

$$V_1 = \frac{Q}{4\pi\epsilon_0 |\bar{r}-\bar{r}_1|} = \frac{10\times10^{-9}\times9\times10^{9}}{\sqrt{12}}$$

$OP = -\bar{a}_x + 2\bar{a}_y + 0 = \bar{r}_1$

$OA = \bar{a}_x + 2\bar{a}_z = \bar{r}$

$$= \frac{90}{\sqrt{12}} = 25.98\,J/C$$

$PA = 2\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z$

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*NOTE: for an infinite line charge of charge density ρ_L which is along z axis the potential difference $V_{BA} = V_B - V_A$.

$$V_{BA} = \frac{\rho_L}{2\epsilon_0}\left[\ln(r_A - r_B)\right]$$

$$V_{BA} = -\int \bar{E}\cdot d\bar{L}$$

$$= -\int \frac{\rho_L}{2\pi\varepsilon r}\,\bar{a}_r\,(dr\cdot\bar{a}_r)$$

$$= -\int \frac{\rho_L}{2\pi\varepsilon r}\cdot dr$$

$$= -\frac{\rho_L}{2\pi\varepsilon}\int \frac{1}{r}\,dr$$

$$= \frac{-\rho_L}{2\pi\varepsilon}\left[\log(r)\right]_{r_B}^{r_A} = \frac{\rho_L}{2\pi\varepsilon}\log(r_A - r_B)$$

* If point A is the on line charge then $r_A = 0 \Rightarrow$

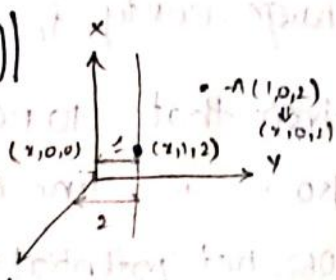
$$V_B = \frac{-\rho_L}{2\pi\varepsilon}\ln(r_B)$$

6 II. <u>sol</u>

$$|\bar{r}_B| = \left|(x\bar{a}_x + 2\bar{a}_z) - (x\bar{a}_x + \bar{a}_y + 2\bar{a}_z)\right|$$

$$= |-\bar{a}_y|$$

$$= \sqrt{1} = 1.$$



$$\left[\ |\bar{r}_A| = \left|(x\bar{a}_x) - (x\bar{a}_x + 2\bar{a}_z)\right|\right.$$

$$= |2\bar{a}_z|$$

$$\left.= \sqrt{4} = 2.\ \right]^{\times}$$

$$|\bar{r}_A| = \left|(x\bar{a}_x) - (x\bar{a}_x + \bar{a}_y + 2\bar{a}_z)\right|$$

$$= |\bar{a}_y + 2\bar{a}_z|$$

$$= \sqrt{1+4} = \sqrt{5}$$

$$V_2 = \frac{\rho_L}{2\pi\varepsilon} \ln\left(\frac{\sqrt{5}}{1}\right)$$

$$= \frac{\rho_L}{2\pi\varepsilon_0} \ln(\sqrt{3})$$

$$= \frac{4\times10^{-9}\times9\times10^{9}}{2\times2} \ln(\sqrt{3})^{1/2}$$

$= \cancel{14.4 J/c}$ $57.88 J/c$.

$= 58 J/c$.

$V = V_1 + V_2$

$= 83.81 J/c$.

* ENERGY DENSITIES

Consider a planar surface 'S' onto which 3 point charges Q_1, Q_2, Q_3 are to be moved. Let the point charges are at '∞' and they are moved to points P_1, P_2, P_3 respectively. From def of electric pottential the work done can be calculated as

work done (W) = charge (Q) x pottential (V)

It can be observed that work done to move 1st point charge to the surface is 'zero' since no charges are present on the surface and no force will act on 'Q_1' i.e work done to move a point charge 'Q_1' to point 'P_1' is $W_1 = 0$

work done to move a point charges 'Q_2' 'Q_3' to points 'P_2' and 'P_3' are given as

$Q_2 \rightarrow P_2$ $\qquad$ $Q_3 \rightarrow P_3$

$W_2 = Q_2 \cdot V_{21}$ $\qquad$ $W_3 = Q_3[V_{31} + V_{32}]$

$\therefore$ Total work done in moving 3 point charges on to the surface is $W_E = W_1 + W_2 + W_3$

$$= Q_2 V_{21} + Q_3 V_{31} + Q_3 V_{32} \rightarrow (1)$$

suppose if charges are moved in reverse order then work done to move point charge Q_3 to P_3 is

$$W_3 = 0$$

work done to move Q_2 to P_2

$$W_2 = Q_2 \cdot V_{23}$$

work done to move Q_1 to P_1

$$W_1 = Q_1 [V_{12} + V_{13}]$$

$$W_1 = Q_1 V_{12} + Q_1 V_{13}$$

$$W_E = W_1 + W_2 + W_3$$

$$= Q_2 V_{23} + Q_1 V_{12} + Q_1 V_{13} \rightarrow (2)$$

Adding equations (1) and (2)

$$2W_E = Q_1 [V_{12} + V_{13}] + Q_2 [V_{23} + V_{21}] + Q_3 [V_{31} + V_{32}]$$

Here potential at 'P_1' is due to potential of all charges at 'P_1'

$$V_1 = V_{12} + V_{13} + V_{14} + \cdots + V_{1n}$$

$$2W_E = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

Generalising above equation we get

$$\boxed{W_E = \frac{1}{2} \left[\sum_{i=1}^{n} Q_i V_i \right]}$$

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* In case of continous charge distribution 'Σ' is replaced with integral and we express charge in terms of charge densities

$$W_E = \frac{1}{2} \int QV$$

$$W_E = \frac{1}{2} \int_L \rho_L V \cdot dL \quad \text{(Line charge distribution)}$$

$$W_E = \frac{1}{2} \int_S \rho_S V \cdot ds \quad \text{(surface " ")}$$

$$W_E = \frac{1}{2} \int_V \rho_V \cdot V dv \quad \text{(volume " ")}$$

* RELATION B/W ENERGY STORED, FIELD INTENSITY AND FLUX DENSITY :-

Consider a volume of charge whose charge density is ρ_V then the energy stored in electrostatic field is given by

$$W_E = \frac{1}{2} \int_V \rho_V \cdot V \cdot dv$$

From Maxwell's 1st equation

$$\nabla \cdot \bar{D} = \rho_V$$

$$\therefore W_E = \frac{1}{2} \int_V (\nabla \cdot \bar{D}) V \cdot dv \rightarrow ①$$

From vector identities we have

$$(\nabla \cdot \bar{A}) V = \nabla \cdot V\bar{A} - \bar{A}(\nabla V)$$

$$\therefore W_E = \frac{1}{2} \int_V ((\nabla \cdot V\bar{D} - \bar{D}(\nabla V)) dv$$

$$V = \frac{Q}{4\pi\varepsilon r} \qquad V\bar{D} \propto 1/r^3$$

$$\bar{D} = \frac{Q}{4\pi\varepsilon r^2}$$

As surface ↑ $V\bar{D}$ ↓

for large surface $V \cdot \bar{D}$ is '0'

$$W_E = \frac{1}{2}\left[\int_V \nabla \cdot V\bar{D}\, dv - \int_V \bar{D} \cdot \nabla V\, dv\right]$$

$$= \frac{1}{2}\left[\oint_S V \bar{D} ds - \int_V \bar{D} \cdot \nabla V dv\right]$$

as $\nabla \cdot \bar{D}$ is neglected

$$W_E = -\frac{1}{2} \int_V \bar{D} \cdot \nabla V\, dv$$

$$= \frac{1}{2} \int_V \bar{D} \cdot (-\nabla V)\, dv$$ (from Maxwell's 2nd eqn $\boxed{\bar{E} = -\nabla V}$)

$$= \frac{1}{2} \int_V \bar{D} \cdot \bar{E}\, dv \rightarrow (a)$$

$$\bar{D} = \epsilon \bar{E}$$

$$W_E = \frac{1}{2} \int_V \epsilon \bar{E}^2 dv \rightarrow (b)$$

$$W_E = \frac{1}{2} \int_V \frac{D^2}{\epsilon} \cdot dv \rightarrow (c)$$

$$a = b = c$$

* CURRENT AND CURRENT DENSITY :-

current: Rate of flow of charges at a specified point or across a surface is called electric current.

It is measured in coloumbs/sec which is generally known as Amperes.

It is denoted with 'I' and is given by

$$I = \frac{dq}{dt} \text{ or } \frac{Q}{t}$$

$$C/s = Amp$$

It is considered as motion of +ve charges. So direction of current is observed to be opp to the direction of els

TYPES

i) conduction current:

current which is existing in conductors due to displacement of elections under influence of applied pottentialis called conduction current / Drift current.

Ex: current passing through copper wire

ii) convection current

In case of dielectrics flow of charges is under the influence of el field. In such cases the current produced is called diffusion/displacement/ convection current

Ex: current in $\|^{el}$ plate capacitor.

Note:

ohm's law can't be applied in finding convection currents In such cases we define current density to calculate currents.

CURRENT DENSITY:-

current passing through unit surface area when it is held normal to the direction of current is called current density.

Denoted by $\overline{J}$

consider a unit surface area

Let $\overline{a}_n$ be unit vector which is normal to the considered surface. Then current density $\overline{J}$ is given by

$$\overline{J} = \frac{\Delta I}{\Delta S}$$

$$\Delta I = \overline{J}\,\Delta S$$

$$= \overline{J} \cdot \overline{dS}$$

$\overline{J} \cdot \overline{dS}$ → dot product

It is measured in Amp/m^2

Total current can be calculated from current density as

$$\boxed{I = \int_s \overline{J} \cdot \overline{ds}}$$

3M

* RELATION B/W CURRENT DENSITY AND VOLUME CHARGE DENSITY:

consider a volume of charge whose charge density is ρ_v. Let ΔQ be the differential charge in given volume. then from def of vol. charge density.

$$\rho_v = \frac{\Delta Q}{\Delta v} \text{ (or) } \frac{dQ}{dv} \text{ (or) } \frac{\Delta Q}{dv}$$

$$\Rightarrow \Delta Q = \rho_v dv$$

we can write differential volume 'dv' as

$$dv = ds \cdot dl \quad \text{(or)} \quad dv = \Delta s \cdot \Delta l$$

$$\therefore \Delta Q = \rho_v \, ds \, dl.$$

(or)

$$\Delta Q = \rho_v \Delta s \Delta l \rightarrow ①$$

from def of current differential current is given by

$$\Delta I = \frac{\Delta Q}{\Delta t}$$

$$\Delta I = \frac{\rho_v \Delta s \Delta l}{\Delta t}$$

WKT the rate of change of displacement is called velocity.

$$\therefore \Delta I = \rho_v \Delta s \, v_l \rightarrow ②$$

$$v_l = \frac{\Delta l}{\Delta t}$$

$v_l \rightarrow$ velocity in length direction.

from def of current density we have.

$$\bar{J} = \frac{\Delta I}{\Delta S}$$

from eq ②

$$\frac{\Delta I}{\Delta S} = \rho_v v_d$$

$$\bar{J} = \rho_v v_d$$

above equation can be generalised as

$$\bar{J} = \rho_v v_x$$

where v_x is the velocity of charged particle in 'x' direction

*** CONTINUITY EQUATION:-**

consider a closed surface with current density $\bar{J}$. Then from the definition of current density the total current passing through the surface (s) is given by

$$I = \int_S \bar{J} \cdot \bar{ds} \longrightarrow ①$$

we know that current is the flow of +ve charges if Q_i is charge enclosed on the surface then from definition of current the rate of change of charge

$$\frac{dQ_i}{dt} = I \longrightarrow ②$$

from the concept of law of conservation of energy, the total charge leaving from one surface, is equal to the total charge entering into another surface.

i.e rate of ↓ of charge from one surface is same as (rate of ↑)^x the outward flow of current in another surface.

$\therefore$ we can write:

$$I = -\frac{dQ_i}{dt} = \int_S \bar{J}\cdot\bar{ds} \rightarrow (3)$$

-ve sign indicates ↓ of charge on one surface.

equation (3) is called "CONTINUITY EQN | CURRENT CONTINUITY EQN IN INTEGRAL FORM:"

from def. of volume charge density.

$$Q_i = \int_V \rho_v \, dv$$

from eqns (3) & (4) we can write

$$I = -\frac{dQ_i}{dt} = \int_S \bar{J}.\bar{ds}$$

$$\Rightarrow -\frac{d}{dt}\left[\int_V \rho_v dv\right] = \int_S \bar{J}\cdot\bar{ds}$$

-Applying guass divergence theorem:

$$\int_V \left(-\frac{d}{dt}\rho_v\right)\cdot dv = \int_V (\nabla\cdot\bar{J})\cdot dv$$

$$\Rightarrow \boxed{\frac{-\partial \rho_v}{\partial t} = \nabla\bar{J}} \rightarrow (5)$$

eqn (5) is called current continuity eqn in differential form or point form.

*** HOMOGENEOUS AND ISOTROPHIC MEDIUM:**

- Linear and non Linear medium:

Mediums are said to be linear if they obey coloumbs law. i.e materials in which Electric flux density is $\propto$ Electric field intensity $(\bar{D} \propto \bar{E})$

(or) $\bar{D} = \epsilon\bar{E}$

such mediums are called linear mediums. If material medium is not const then they are called non-linear mediums.

- ISOTROPIC AND NON ISOTROPIC MEDIUM:

Isotropic mediums are those whose properties are independent of direction. i.e The materials whose molecules are randomly oriented [are called iso tropic]*. (Resistor)
If they don't possess independence of direction then those materials are called non/AN isotropic mediums.
(Diode)

- HOMOGENEOUS AND NON HOMOGENEOUS MEDIUM:

- A Medium is said to be homogeneous if its physical properties like mass, molecular weight etc. doesn't vary from one point to another point.
If they vary. then they are called non homogenous Medium.

2M
* POISSON'S EQUATION:-

We know that from Maxwell's 1st equation

$$\nabla \cdot \bar{D} = \rho_v$$

we also know that

$$\bar{D} = \epsilon_0 E$$

$$\rho_v = \nabla \cdot \epsilon_0 \cdot \bar{E} \rightarrow (1)$$

WKT Electric field is expressed interms of electric potential as $\bar{E} = -\nabla \bar{V} \rightarrow (2)$

from (1) & (2)

$$\rho_v = \epsilon_0 \nabla(-\nabla V)$$

$$-\frac{\rho_v}{\epsilon_0} = \nabla^2 V \Rightarrow \nabla^2 V = \frac{-\rho_v}{\epsilon} \rightarrow (5)$$

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The above eqn is poison eqn which relates potential and volume charge density.

LAPLACE EQN:

Consider charge density in medium whose volume charge distribution density is absent i.e the charge density.

Let the combination of line and surface charge densities but volume charge density is zero.

Then from above eqn ③ $\nabla^2 V = 0 \rightarrow ④$

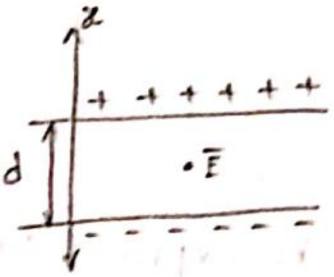
eqn④ is called Laplace equation which is derived from poisons eqn.

CAPACITANCE:

PARALLEL PLATE CAPACITOR:

CAPACITOR: Device which stores energy in the form of electric field

— Assume a parallel plate capacitor in which two ||ᵉˡ plates are seperated by distance of 'd'. Let surface area of each plate be 's'



$E = \frac{Q}{4\pi\epsilon_0 r^2} \bar{a}_r$

WKT $\bar{E}$ due to an infinite sheet of charge is given by

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}$$

But a parallel plate capacitor total field is given by

$$\bar{E} = \underbrace{\rho_s/2\epsilon_0}_{\text{upper plate}} + \underbrace{\rho_s/2\epsilon_0}_{\text{lower plate}} = \frac{\rho_s}{\epsilon_0}$$

$$\bar{E} = \frac{\rho_s}{\epsilon_0}$$

WKT E and V are related as

$$V = -\int_0^d \bar{E}\cdot\bar{dL} \quad (\because \bar{E} = -\nabla V)$$

$$V = -\int_0^d \frac{\rho_s}{\epsilon_0}\,\bar{dL}$$

WKT surface charge density is given by

$$\rho_s = Q/S.$$

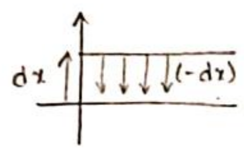
$$V = -\int \frac{Q/S}{\epsilon_0}\,\bar{dx}$$

$$= -\int \frac{Q}{\epsilon_0 S}(-\bar{dx})$$

$$= \frac{+Q}{\epsilon_0 S}\int_0^d dx.$$

$$= +\frac{Q}{S\epsilon_0}d.$$

$-dx$ as charges in capacitor move ↓ and dx ↑



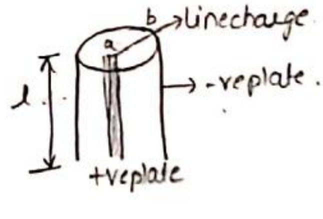
$$V = \frac{Qd}{S\epsilon_0}$$

$$\boxed{C = Q/V = \frac{\epsilon_0 S}{d}}$$

* COAXIAL CAPACITOR:

WKT the electric field intensity $\bar{E}$ is equal to due to infinite line charge

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 r}\bar{a_r}$$



$$\text{WKT} \Rightarrow \rho_L = \frac{Q}{l}.$$

$$\bar{E} = \frac{Q}{2\pi\epsilon_0 l r}\cdot\bar{a_r}$$

$$V = -\int_a^b \bar{E}\cdot\bar{dL}.$$

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$$= -\int_a^b \frac{Q}{2\pi\epsilon_0 l r} \bar{a}_r (-dr)\bar{a}_r$$

$-dr$ as we are calculating V from b to a. a to b $\rightarrow dr$

$$= \frac{Q}{2\pi\epsilon_0 l} \cdot \int_a^b \frac{1}{r} \cdot dr = \frac{Q}{2\pi\epsilon_0 l} \log(b/a)$$

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 l}{\log(b/a)}$$

* **SPHERICAL CAPACITOR:**

We know that electric field intensity due to spherical charge is $\bar{E} = \frac{Q}{4\pi\epsilon r^2} \cdot \bar{a}_r$.

WKT potential $V = -\int \bar{E} \cdot \bar{dL}$

$$= -\int_a^b \frac{Q}{4\pi\epsilon r^2} \bar{a}_r \cdot \bar{dL}$$

$$\because \bar{dL} = (-dr)\bar{a}_r$$

$$V = \int_a^b \frac{Q}{4\pi\epsilon r^2} dr \quad [\bar{a}_r \cdot \bar{a}_r = 1]$$

$$= \frac{Q}{4\pi\epsilon} \int_a^b \frac{1}{r^2} dr$$

$$= \frac{Q}{4\pi\epsilon} \left[\frac{-1}{r}\right]_a^b$$

$$= \frac{Q}{4\pi\epsilon} \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\Rightarrow \boxed{C = \frac{Q}{V} = \frac{4\pi\epsilon}{1/a - 1/b}}$$

27/1/18

UNIT-2

MAGNETOSTATICS:

A current carrying conductor behaves as a magnet.

* Magnetic field strength / Magnetic field intensity: ($\bar{H}$)
  - It is the qual tative measure of strength or weakness of magnetic field.
  - It is defined as the force experienced by a unit north pole of 1wb strength which is placed at that point
  - Magnetic field Intensity is measured in N/wb (or) Henry/H

* Magnetic flux density: ($\bar{B}$)
  - The magnetic lines of force that the magnetic flux is crossing at a unit area which is held at a right angle to the direction of magnetic flux is called Magnetic flux density.
  - Measured in wb/m^2 or tesla (T)

* Relation b/w $\bar{B}$ and $\bar{H}$:

  - $\bar{B}$ is given by

$$\bar{B} = \mu\bar{H}$$

where 'μ' is permeability of the medium

It is given by $\mu = \mu_0 \mu_r$

$\mu_0 \rightarrow$ permeability of free space.

$\mu_r \rightarrow$ relative permeability.

$\mu_r = 1$ (for free space)

$\mu_0 = 4\pi \times 10^{-7}$ H/m

* BIOT - SAVART'S LAW:

It states that differential magnetic field strength $d\bar{H}$ produced at a point 'P' because of a current carrying conductor of differential current $\bar{I}\cdot d\bar{L}$ is directly

$$\left[d\bar{H} \propto \bar{I}d\bar{L}\sin\theta\right]$$

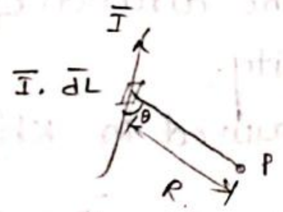
proportional to product of I and $d\bar{L}$ and sin angle b/w the element and line joining 'P'

and it is inversely proportional to the square of the distance b/w point 'p' and differential current element

$$d\bar{H} \propto \frac{\bar{I}\cdot d\bar{L}\sin\theta}{R^2}.$$

we introduce proportionality constant 'K'

where $K = 1/4\pi$



So $$d\bar{H} = \frac{K\,\bar{I}d\bar{L}\sin\theta}{R^2}$$

$$\boxed{d\bar{H} = \frac{\bar{I}\cdot d\bar{L}\sin\theta}{4\pi R^2}}$$

$\bar{a}\times\bar{b} = ab\sin\theta.$

from defination of cross product we have.

$\bar{a}\times\bar{b} = ab\sin\theta.$

$$\therefore\ d\bar{H} = \frac{\bar{I}\cdot d\bar{L}\cdot 1\,\sin\theta}{4\pi R^2}$$

$$= \frac{\bar{I}\cdot d\bar{L}\times\bar{a}_r}{4\pi R^2}$$

$\bar{a}_r \rightarrow$ unit vector (as 1 is magnitude (scalar) $\bar{a}_r$ is written)

$\bar{a}_r \rightarrow$ unit vector along distance vector

$$\bar{a}_r = \frac{\bar{R}}{|\bar{R}|}$$

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$$d\bar{H} = \frac{\bar{I}\cdot \bar{dL} \times \bar{R}}{4\pi R^2 |\bar{R}|}$$

$$\boxed{d\bar{H} = \frac{\bar{I}\cdot \bar{dL} \times \bar{R}}{4\pi R^3}} \quad (\because |\bar{R}| = R.)$$

- Differential magnetic field strength for various ~~charge~~ ~~dt~~ / current distribution:-

Biot savart's law can be extended for various current distributions in a similar manner.

The differential magnetic field strength can be calculated for the surface current density $\bar{K}\cdot\bar{ds}$ (measured in Amp/m) and for a volume current density $\bar{J}\cdot\bar{dv}$ (measured in amp/m²)

Line current density:

$$\bar{H} = \int_L \frac{\bar{I}\cdot \bar{dL} \times \bar{R}}{4\pi R^3} \text{ (or) } = \int_L \frac{\bar{I}\,\bar{dL} \times \bar{a}_{12}}{4\pi R^2}$$

surface current density:

$$\bar{H} = \int_S \frac{\bar{K}\cdot\bar{ds} \times \bar{R}}{4\pi R^3} \text{ (or) } \int_S \frac{\bar{K}\cdot\bar{ds} \times \bar{a}_{12}}{4\pi R^2}$$

volume current density.

$$\bar{H} = \int_V \frac{\bar{J}\cdot\bar{dv} \times \bar{R}_{12}}{4\pi R^3} \text{ (or) } \int_V \frac{\bar{J}\cdot\bar{dv} \times \bar{a}_{12}}{4\pi R^2}$$

* AMPERES CIRCUIT LAW:

STATEMENT : It states that line integral of $\bar{H}$ around a closed path is same as net current enclosed by the path.

It states that circulation of $\bar{H}$ equals the current enclosed. i.e $I_{enclosed} = \oint_L \bar{H}\,\bar{dL}$

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* MAXWELL'S 3rd EQUATION :-

Applying stokes theorem to amperes circuital law.

$$I_{enc} = \oint_L \overline{H}\cdot\overline{dL}$$

$$I_{enc} = \iint_S (\nabla \times \overline{H})\,\overline{ds} \rightarrow ①$$

from def of current density we have.

$$I = \int_S \overline{J}\cdot\overline{ds} \rightarrow ②$$

combining eqn's ① & ②

$$\int_S \overline{J}\cdot\overline{ds} = \int_S (\nabla\times\overline{H})\cdot\overline{ds}$$

$$\boxed{\int_S \overline{J}\cdot\overline{ds} = \oint_L \overline{H}\cdot\overline{dL}} \rightarrow ③$$

$$\boxed{\nabla \times \overline{H} = \overline{J} \quad \rightarrow ④}$$

eqn's ③ and ④ are same which are called maxwell's 3rd equations. eq ③ is in integral form

eq ④ is in point/differential form.

* MAXWELL'S 4th EQUATION:

"Magnetic unit poles doesn't exist".

It states that isolated magnetic charge doesn't exist

i.e $\oint_S \overline{B}\cdot\overline{ds} = 0$

Applying gauss divergence theorem

$$\oint_S \overline{B}\cdot\overline{ds} = 0 \Rightarrow \oint_V \nabla\cdot\overline{B}\,dv = 0$$

$$\boxed{\nabla\cdot\overline{B} = 0} \rightarrow ②$$

eqns ① & ② are Maxwell's 4^{th} equations.

eqn① is integral form & ② is differential form

** MAXWELL EQUATIONS

• for static el- magnetic fields.

Equation	point form	Integral form	Principles
Maxwell's 1^{st} eqn	$\nabla \cdot \bar{D} = \rho_v$	$\int_S \bar{D} \cdot \bar{ds} = \int_V \rho_v dv$	Gauss law
Maxwell's 2^{nd} eqn	$\nabla \times \bar{E} = 0$	$\oint \bar{E} \cdot \bar{dL} = 0$ (or) $\int_S \nabla \times \bar{E} = 0$	conservative nature of el- field
Maxwell's 3^{rd} eqn	$\nabla \times \bar{H} = \bar{J}$	$\int \bar{H} \cdot \bar{dL} = \int \bar{J} \cdot \bar{ds}$	Ampere's circuit law.
Maxwell's 4^{th} eqn	$\nabla \cdot \bar{B} = 0$	$\oint \bar{B} \cdot \bar{ds} = 0$	conservative nature of magnetic fields / Non existance of magnetic monopoles

*APPLICATIONS OF AMPERE'S CIRCUIT LAW:

We follow the following steps to calculate the $\bar{H}$ from ampere's circuit law:

- consider a closed path preferably symmetrical such that it encloses direct current 'I' once. This path is called 'AMPERIAN PATH'.
- consider differential length $\bar{dL}$ depending on the coordinate system used.
- Identify the symmetry and find in which direction $\bar{H}$ exists.
- find $\bar{H} \cdot \bar{dL}$ which is the dot product (make sure that $\bar{H}$ are $\bar{dL}$ are in same direction

- Find the integral of $\bar{H}\cdot\bar{dL}$ around a closed path which is assumed and equaled with total current enclosed by that path.

* MAGNETIC FIELD STRENGTH DUE TO AN INFINITE LINE CURRENT :

Consider an infinite line of current which is along z axi and carrying a current 'I'. consider a point 'P', which is on closed path constructed the around the conductor. This path is called Amperial path. Point 'P' is at a ⊥r distance of 'r' from the line carrying current. Now $\bar{H}$ is to be calculated at point 'P'. It is clear that if ⊥r distance r ↑ $\bar{H}$ (field strength) ↓ and vice versa.

Also direction of $\bar{H}$ is along $\bar{a}_\theta$ since the componen of $\bar{a}_r$ and $\bar{a}_z$ will not be available at point 'P'. This is because the radial components will cancel each other and there is no field in the direction of z axis as current is flowing in z-direction.

$$H_r = H_z = 0 \quad H_\theta \neq 0$$

and $\bar{H} = \bar{H}_\theta \cdot \bar{a}_\theta$

The differential component in cylindrical components is given by :

$$\bar{dL} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z \text{ (in cartesian)}$$

$$\bar{dL} = dr\,\bar{a}_r + r\,d\theta\cdot\bar{a}_\theta + dz\,\bar{a}_z$$

Here $\bar{a}_r$ and $\bar{a}_z = 0$

$$\Rightarrow \bar{dL} = r\,d\theta\,\bar{a}_\theta$$

Now applying amperes circuit law:

$$I_{enc} = \oint \bar{H}\cdot\bar{dL}$$

$$I = \oint H_\theta\,\bar{a}_\theta\,r\,d\theta\,\bar{a}_\theta$$

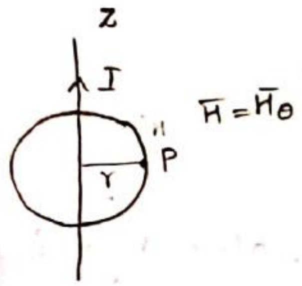
$$I = \oint H_\theta \, r \cdot d\theta$$

$$= \int_0^{2\pi} H_\theta \, r \, d\theta$$

$$= H_\theta \, r \, (\theta)_0^{2\pi}$$

$$= H_\theta \, r \, (2\pi)$$

$$= 2\pi H_\theta r$$



$$\boxed{H_\theta = \frac{I}{2\pi r}}$$

∴ The magnetic field strength at a point 'p' can be generalised as

$$\boxed{\bar{H} = \frac{I}{2\pi r} \bar{a}_\theta \quad A/m \text{ (or) } N/wb \text{ or } H}$$

$\bar{H}$ DUE TO AN INFINITE SHEET OF CURRENT

consider an infinite sheet of conductor in x-y plane at z=0. Let the conductor has the surface current density of $\bar{K}$ in +ve y direction. i-e $\bar{K} = k_y \bar{a}_y$

consider a closed path of width 'b' and length 'a' ⊥r to xy plane. then current flowing across the width 'b' is given by $I_{enc} = \int k_y \cdot b \rightarrow (1)$

$$\bar{K} = \frac{I}{L}$$

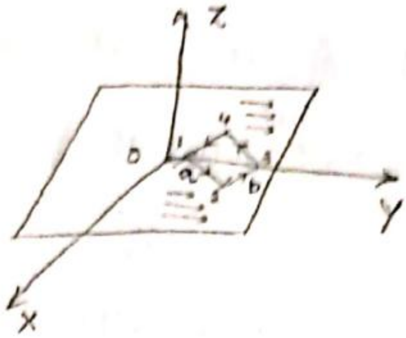
$$k_y = \frac{I}{b} \quad (\because I = k_y b)$$

For above consideration

if we apply amperes circuit law $\bar{H}$ can be calculated.

$\bar{H}_y = 0$ (field in y direction is 0 [current is in y direction so current field on +ve z axis and -ve z axis are compensated and get cancelled. $\bar{H}_z = 0$]

I_{enc}



Calculated by using formula

$$I_{enc} = \oint \bar{H} \cdot \bar{dl}$$

To evaluate the above integral we consider amperial path 1-2-3-4-1 as shown in figure. since the sheet extends in x-y plane due to symmetry field along z directio is zero. i.e $H_z = 0$. As the current is flowing in +ve y direction the component $\bar{H}_y$ is also '0'

$$\bar{H} = H_x \bar{a}_x$$

$\bar{H} \cdot \bar{dl}$ for amperian path can be calculated as

$$\oint \bar{H} \cdot \bar{dl} = 0(-a) + (-H_x)(-b) + 0(a) + H_x(b)$$

$$= 2H_x b \rightarrow (2)$$

$$\oint \bar{H} \cdot \bar{dl} = \int_1^2 \bar{H} \cdot \bar{dL} + \int_2^3 \bar{H} \cdot \bar{dl} + \int_3^4 \bar{H} \cdot \bar{dl} + \int_4^1 \bar{H} \cdot \bar{dl}$$

from (1) & (2)

$$k_y b = 2bH_x$$

$$k_y = 2H_x$$

$$\boxed{H_x = \frac{k_y \bar{a}_x}{2}} \quad \text{for } \forall z \geq 0$$

$$\text{if } z < 0 \quad H_x = -\frac{k_y}{2}$$

$$\therefore \bar{H} = \frac{k_y}{2} \bar{a}_x \ \forall z \geq 0$$

* MAGNETIC POTTENTIALS:

Magnetic pottential is defined as work done in displacing a unit north pole from 1point to other point in presence of magnetic field.

There are two types of pottentials for magnetic field.

→ i) Scalar pottential. (V_m)

ii) vector pottential ($\bar{A}$)

NOTE! from vector identities we have.

i) curl(grad(scalar)) = 0

$$\nabla \times (\nabla A) = 0$$

ii) Divergence of curl of a vector is 0

$$\cancel{\nabla \cdot (\nabla \times \bar{A})} \quad (\nabla \cdot (\nabla \times \bar{A})) = 0$$

$\nabla \times \bar{A}$ = curl

$\nabla \cdot \bar{A}$ = div

$\nabla \cdot A$ = grad.

~~Scal~~

1) SCALAR POTTENTIAL AND LAPLACE EQUATION:

Scalar pottential: (V_m)

from vector identity we have

$$\nabla \times (\nabla V_m) = 0 \rightarrow (1)$$

Also from pottentials we can write $\bar{H} = -\nabla V_m \rightarrow (2)$

$$\Rightarrow \quad D \times \bar{H} = 0 \rightarrow (3)$$

but from Maxwell's 3rd eqn we have

$$\nabla \times \bar{H} = \bar{J} \rightarrow (4)$$

comparing (3) and (4) if $\bar{J} = 0$ then we can define

$\bar{H} = -\nabla V_m$ where V_m is scalar ~~po~~ magnetic pottential.

For a specified magnetic pottential V_{mab} is givenby

$$V_{m\,ab} = -\int_b^a \overline{H} \cdot \overline{dl}$$

Laplace equation:

from maxwell's 4^{th} equation:

$$\nabla \cdot \overline{B} = 0$$

$$\therefore \nabla \cdot (\mu_0 \overline{H}) = 0$$

$$\Rightarrow \nabla \cdot \overline{H} = 0 \quad \mu_0 \rightarrow \text{constant}$$

from potentials $\overline{H} = -\nabla V_m$

$$\nabla \cdot (-\nabla V_m) = 0$$

(or) $\boxed{\nabla^2 V_m = 0}$

Above eq^n is called laplace eq^n

* VECTOR MAGNETIC POTTENTIAL & POISSON'S EQUATION:

We know that divergence of curl of a vector is '0'

$$\nabla \cdot (\nabla \times \overline{A}) = 0 \rightarrow (1)$$

from maxwell's 4^{th} equation

$$\nabla \cdot \overline{B} = 0 \rightarrow (2)$$

from (1) & (2)

$$\overline{B} = \nabla \times \overline{A}$$

this eq^n relates vector magnetic pottential and magnetic flux density.

→ Poissons equation:

from vector identities we have $\nabla \times (\nabla \times A)$

$$= \nabla (\nabla \cdot \overline{A}) - \nabla^2 A$$

from maxwell's 3rd eq^n

$$\nabla \times \bar{H} = \bar{J}$$

$$\bar{B} = \mu_0 \bar{H}$$

$$\bar{H} = \frac{\bar{B}}{\mu_0}$$

$$\nabla \times \frac{\bar{B}}{\mu_0} = \bar{J}$$

$$\nabla \times \bar{B} = \mu_0 \bar{J}$$

$$\nabla \times (\nabla \times \bar{A}) = \mu_0 \bar{J}$$

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$$

$$\bar{J} = \frac{\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}}{\mu_0}$$

$$\boxed{\bar{J} = -\frac{\nabla^2 \bar{A}}{\mu_0}} \quad \text{if } \nabla \cdot \bar{A} = 0$$

* FORCE ON A MOVING CHARGE / LORENTZ EQN :-

from coloumb's law force exerted on a charge is given by

+ve -ve.

+ve charge →

-ve charge ←

$$\bar{F}_e = Q\bar{E} \rightarrow (A)$$

$\parallel^{rly}$ Magnetic force experienced by charge Q if it is moving with a velocity $\bar{v}$ in the presence of magnetic field.

$\bar{F}_m$ is given by

$$\bar{F}_m = Q(\bar{v} \times \bar{B})$$

∴ net force experienced by a charge due to both stationary and el and mag fields is given by.

$$\bar{F} = \bar{F}_e + \bar{F}_m$$

$$F = Q[\bar{E} + (\bar{v} \times \bar{B})] \rightarrow (1)$$

but from Newton force eq^n mechanical force is given

by $\bar{F} = ma$

$= m\frac{dv}{dt} \rightarrow$ ② from ① & ②

$$\bar{F} = m\frac{dv}{dt} = Q\,[E + (\bar{v} \times \bar{B})] \rightarrow ③$$

above eq^n ③ is called Lorentz eq^n which relates Electric & mechanical forces.

* FORCE DUE TO DIFFERENTIAL CURRENT ELEMENT:

A magnetic force experienced by charge Q if it moving with a velocity 'v' in presence of magnetic field $\bar{B}$

$\bar{F}_m = Q(\bar{v} \times \bar{B})$. force experienced by diff current element $d\bar{F}_m$ of diffi charge moving in steady magnetic field is given by $d\bar{F}_m = dQ(\bar{v} \times \bar{B})$

But we know that

$$\rho_v = \frac{dQ}{dv} \Rightarrow \rho_v\, dv = dQ$$

$$d\bar{F}_m = \rho_v\, dv\,(\bar{v} \times \bar{B})$$

$$= (\rho_v\, \bar{v} \times \bar{B})\, dv$$

relatn b/w vol charge density and velocity is given

by $\bar{J} = \rho_v v_x$ $\quad v_x \rightarrow$ velocity in x-directn

$$\therefore d\bar{F}_m = \rho_v(\bar{J} \times \bar{B})\, dv$$

WKT various current distributions are related as

$\bar{J} \cdot \bar{dv} \equiv \bar{K}\, d\bar{S} \equiv \bar{I} \cdot \bar{dL}$
$\hookrightarrow$ 11larly to

for a surface current distribution differential force is

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given by $d\bar{F}_m = (\bar{K} \times \bar{B})\, ds$

for line current

$$d\bar{F}_m = (\bar{I} \times \bar{B})\, dl$$

total force can be calculated by Integrating the differential force w.r.t. volume, surface or line current distribution.

for volume current.

$$\bar{F}_m = \int_v (\bar{J} \times \bar{B})\, dv$$

surface current

$$\bar{F}_m = \int_s (\bar{K} \times \bar{B})\, ds.$$

line current

$$\bar{F}_m = \int_l (\bar{I} \times \bar{B})\, dl.$$

Note: for line current distribution.

$$\int d\bar{F}_m = \int (\bar{I} \times \bar{B})\, d\bar{l}$$

$$\bar{F}_m = (\bar{I} \times \bar{B}) \int d\bar{l}$$

$$= (\bar{I} \times \bar{B}) \cdot L$$

$$= IB \sin\theta \cdot L$$

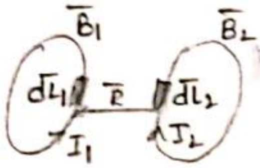
$$F_m = BIL \cdot \sin\theta$$

AMPERE'S FORCE LAW (or)

FORCE B/W TWO current carrying elements :-

If current carrying conductors are brought near to each other they experience a force of attraction or repulsion b/w. them. Due to the magnetic field around

the coil. If both currents are in same directn then they experience a force of attraction and viceversa.



consider two current carrying conductors carrying currents I_1 and I_2 as shown in fig. Let they are seperated by a distance $\bar{R}$ consider two differential elements on the coil with differential length $\bar{dL}_1$ and $\bar{dL}_2$. The force exerted by 2nd coil and the diff current element in 1st coil due to diff flux density $d\bar{B}_2$ is given by.

$$d\bar{F}_1 = I_1 \bar{dL}_1 \times \bar{B}_2$$

&

$$d(d\bar{F}_1) = I_1 \bar{dL}_1 \times d\bar{B}_2 \rightarrow ①$$

from biot savart's law

$$\bar{dH} = \frac{I \cdot \bar{dL} \times \bar{a}_R}{4\pi |\bar{R}|^2}$$

The magnetic field strength due to second coil is given by $\bar{dH}_2 = \frac{I_2 \bar{dL}_2 \times \bar{a}_R}{4\pi |\bar{R}|^2}$

$$d\bar{B}_2 = \mu \, \bar{dH}_2$$

$$= \mu \frac{I_2 \bar{dL}_2 \times \bar{a}_R}{4\pi |\bar{R}|^2} \rightarrow ②$$

subistituting ② in ①

$$d(d\bar{F}_1) = I_1 \bar{dL}_1 \times \frac{\mu I_2 \bar{dL}_2 \times \bar{a}_R}{4\pi |\bar{R}|^2} \rightarrow ③$$

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$$= \frac{\mu}{4\pi|\bar{R}|^2} \bar{I_1}\bar{I_2}(\bar{dL_1} \times \bar{dL_2} \times \bar{a_R}) \rightarrow (4)$$

To calc entire force acting on 1st conductor we get

integrate along length of 2 conductors

$$\bar{F_1} = \frac{\mu}{4\pi} \int_{L_1}\int_{L_2} \bar{I_1}\cdot\bar{I_2} \frac{\bar{dL_1} \times \bar{dL_2} \times \bar{a_R}}{|\bar{R}|^2}$$

$L_1 \rightarrow$ length of 1st conductor.

The force experienced by 1st conductor is given by.

$$\bar{F_{21}} = \mu I_1 I_2 \int_{L_1}\int_{L_2} \frac{\bar{dL_1} \times (\bar{dL_2} \times \bar{a_{R21}})}{4\pi|\bar{R_{21}}|^2}$$

similarly force acting on 2nd conductor is given by

$$\bar{F_2} = \mu I_1 I_2 \int_{L_1}\int_{L_2} \frac{\bar{dL_2} \times (\bar{dL_1} \times \bar{a_{R12}})}{4\pi|\bar{R_{12}}|^2}$$

INDUCTANCE:

If there are 'N' turns in a coil with a flux 'ϕ' then product $N\phi$ is called linkage flux. It is measured in weber turns.

Total linkage flux of the coil α current through the coil.

$N\phi \alpha I$

i.e $L = \frac{N\phi}{I}$

$N\phi = LI$.

Inductance is measured in Henry.

Energy stored in inductor $W_m = \frac{1}{2}Li^2$

Time varying fields

* FARADAY'S LAWS:

* Electro statics: study of ch stationary charges

* FARADAY'S LAW: *** Long.

According to faraday's exp a static magnetic field can't produce any current but with time varying fields an electro motive force (emf) is induced which may drive the current in closed path or circuit.

faraday discussed that the induced emf is equal to the rate of change of magnetic flux linked to the closed ckt i.e It can be written as

$$e = -N\frac{d\phi}{dt} \rightarrow ①$$

where $N \rightarrow$ no. of turns in the coil

$\phi \rightarrow$ magnetic flux

$e \rightarrow$ induced emf.

If assume a coil with single turn then emf is

$$e = \frac{-d\phi}{dt} \rightarrow ②$$

here -ve sign indicates the directn of induced emf such that it produces a current which will produce the magnetic field in opp direction to the original.

"The induced emf acts to produce an opposing flux," this is the concept postulated by HFE Lenz

The induced emf is a scalar quantity measured in volts and induced emf is given by $e = \int \bar{E} \cdot \bar{dL} \rightarrow ③$

Induced emf indicates a voltage about a closed path such that if any part of the path changes then induced emf also changes. WKT magnetic flux through a specified area is given as

$$\phi = \int_S \bar{B}\cdot\bar{ds} \qquad \left(\because \bar{B} = \frac{d\phi}{ds}\right) \rightarrow (4)$$

from equations (2) and (4)

$$e = \int \bar{E}\cdot\bar{dl} = -d/dt\,(\phi)$$

$$\boxed{\int \bar{E}\cdot\bar{dl} = -d/dt\left[\int_S \bar{B}\cdot\bar{ds}\right] \rightarrow (5)}$$

from eqn (5) there are 2 conditions for induced emf

case i) transformer emf

The closed path in which emf is induced, is stationary and magnetic flux is sinusoidally varying with time. so from eqn (5)

only $\bar{B}$ is a time varying quantity.

so eqn (5) can be rearranged as

$$\int_L \bar{E}\cdot\bar{dL} = \int_S -\frac{d\bar{B}}{dt}\cdot\bar{ds}$$

Applying stokes theorem to above eqn we get

$$\int_L \bar{E}\cdot\bar{dl} = \int_S (\nabla\times\bar{E})\bar{ds} = \int_S -\frac{d\bar{B}}{dt}\cdot\bar{ds} \rightarrow (6)$$

$$\therefore \nabla\times\bar{E} = \frac{d\bar{B}}{dt} = \frac{-\partial\bar{B}}{\partial t} \rightarrow (7)$$

eqn 6 is similar to transformer action as applied el-field produces magnetic flux. eqn 7 is same as Maxwell's second eqn

It is clear that if $\bar{B}$ is not varying with time then the above eqn can be written as

$$\boxed{\nabla \times \bar{E} = 0} \left[\bar{B} \text{ is const } \& -\frac{\partial \bar{B}}{\partial t} = 0 \right]$$

case 2: Generator emf:

From faraday's law we have (eqn ⑤)

$$\int \bar{E} \cdot \bar{dL} = -d/dt \left[\int \bar{B} \cdot \bar{ds} \right] \rightarrow ⑤$$

$$\int \bar{E} \cdot \bar{dL} = -d - \int \left[d/dt \ \bar{B} \cdot \bar{ds} \right] \rightarrow 5$$

from def of mag field intensity force due to mag field is $\bar{F}m = Q(\bar{v} \times \bar{B})$

$$\therefore \ \bar{E} = \frac{\bar{F}m}{Q} = \bar{v} \times \bar{B} \quad \bar{E} \rightarrow \text{field strength} \rightarrow ⑧$$

$\therefore$ ⑧ in ⑤

$$\int_L \bar{E} \cdot \bar{dL} = \int_S -\frac{\partial \bar{B}}{\partial t} \cdot \bar{ds}$$

$$\int_L \bar{v} \times \bar{B} = \int_S -\frac{\partial \bar{B}}{\partial t} \cdot \bar{ds}$$

applying stokes theorem surface integral of

$$\int_S (\nabla \times \bar{v} \times \bar{B}) \, \bar{ds} = \int_S -\frac{\partial \bar{B}}{\partial t} \, \bar{ds} \rightarrow ⑨$$

$$\boxed{\nabla \times \bar{v} \times \bar{B} = -\frac{\partial \bar{B}}{\partial t} \rightarrow ⑩}$$

$$\nabla \times \bar{v} \times \bar{B} = -\frac{\partial \bar{B}}{\partial t} = \nabla \times \bar{E}$$

eq^n 10 represents total emf induced when a conductor is moved in uniform magnetic flux. Here field is stationary while a closed path is moved freely. Hence this represents generator action so it represents generator emf.

Note: If a coil is moved in EM field it experiences both transformer and generator emf's so induced emf is given by

Induced emf = transformer emf + generator emf

$$e = \int \bar{E} \cdot \bar{dl} = \int_s -\frac{\partial \bar{B}}{\partial t} \cdot \bar{ds} + \int_s (\nabla \times \bar{v} \times \bar{B}) \bar{ds}$$

* INCONSISTANCY OF AMPERE'S CIRCUIT LAW :- (or)

-AMPERE'S CIRCUIT LAW FOR TIME VARYING FIELD :-

From Maxwell's 3rd eq^n we have $\nabla \times \bar{H} = \bar{J}$

-Applying divergence on b.s we have.

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J}$$

from vector identities $\nabla \cdot (\nabla \times \bar{H}) = 0$

$$\therefore \quad \nabla \cdot \bar{J} = \nabla \cdot (\nabla \times \bar{H}) = 0 \quad \rightarrow (1)$$

But from continuity eq^n

$$\nabla \cdot \bar{J} = \frac{-\partial \rho_v}{\partial t} \rightarrow (2)$$

from (1) & (2)

We write $\nabla \cdot \bar{J} = \frac{-\partial \rho_v}{\partial t} = 0 \rightarrow (3)$

but for time varying fields $\frac{d\rho_v}{\partial t} \neq 0 \rightarrow (4)$

from (3) & (4)

We can say that Ampere's circuit law is inconsistent for time varying fields. This is because eqns 1,3,4 are incompatible.

So in order to satisfy eqn ① we can write eqn ① as

$$\nabla\cdot(\nabla\times\bar{H}) = \nabla\cdot\bar{J} + \nabla\cdot\bar{N} \rightarrow ⑤$$

$$= -\frac{\partial \rho_v}{\partial t} + \nabla\cdot\bar{N}$$

$$\nabla\cdot\bar{N} = \frac{\partial \rho_v}{\partial t}$$

from eqn 5 if we apply guass law i.e

$$\nabla\cdot\bar{D} = \rho_v$$

$$\therefore \nabla\cdot\bar{N} = -\nabla\cdot\bar{J} = \frac{\partial \rho_v}{\partial t}$$

$$= -\frac{-\partial \rho_v}{\partial t}$$

$$= \frac{\partial \rho_v}{\partial t}$$

$$= \frac{\partial}{\partial t}(\nabla\cdot\bar{D})$$

$$\boxed{\nabla\cdot\bar{N} = \nabla\left(\frac{\partial \bar{D}}{\partial t}\right)}$$

$$\Rightarrow \nabla\times\bar{H} = \bar{J} + \bar{J}_d$$

$$\left[\nabla\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}\right]$$

$$\nabla\times\bar{H} = \bar{J} + \bar{N}$$

$$\nabla\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \rightarrow ⑥$$

where $\bar{J}$ or $\bar{J}_c$ is conduction current density which is due to the moment of charges & $\bar{J}_d$ is displacement current density which is due to time varying electric flux.

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eqn (6) is called modified ampere ckt law or ampere ckt law for time varying fields or 3rd maxwells eqn for time varying fields.

MAXWELL'S EQUATIONS AND FINAL FORM:

1st MAXWELL'S EQUATIONS:

- According to guass law total flux out of a closed surface is equal to the total charges enclosed by the surface.

$$\int_S \bar{D}\cdot\bar{ds} = \int_V \rho_v dV$$

- By applying stoke's theorem

$$\nabla\cdot\bar{D} = \rho_v$$

2nd MAX

from faradays law/ transformer emf equation we have $\int_L \bar{E}\cdot\bar{dL} = -\int_S \frac{\partial \bar{B}}{\partial t}\,\bar{ds}$

In other words $\nabla\times\bar{E} = -\frac{\partial B}{\partial t}$

3rd MAXWELLS EQUATION:

from Inconsistancy of Ampere ckt law/ Modified ampere ckt law we have

$$\nabla\times\bar{H} = \bar{J}+\frac{\partial \bar{B}}{\partial t} \text{ (Pointform)}$$

$$\oint\bar{H}\cdot\bar{dL} = \int_S \left[\bar{J}\cdot\bar{ds} + \frac{\partial \bar{D}}{\partial t}\right]\bar{ds}$$ if Stokes theorem applied.

$$\int_S(\nabla\times\bar{H})\cdot\bar{ds} = \int_S\left[\bar{J}+\frac{\partial \bar{D}}{\partial t}\right]ds.$$

MAXWELL'S 4^{th} EQN:

In any magnetic field the conservation of magnetic flux the total flux enclosed by surface is always 0. It gives the non existence of magnetic monopoles.

$$\nabla \cdot \bar{B} = 0$$

$$\int_S B \cdot \bar{ds} = 0$$

* Maxwells equation for the stationary Electro mag waves:

eqn	Point form	Integral form	Principle.
1.	$\nabla \cdot \bar{D} = \rho_v$	$\int_S \bar{D} \cdot \bar{ds} = \int_V \rho_v dv$	Guass law
2.	$\nabla \times \bar{E} = 0$	$\oint \bar{E} \cdot \bar{dL} = 0 / \int_S \nabla \times \bar{E} = 0$	conservative nature of field
3.	$\nabla \times \bar{H} = \bar{J}$	$\int_S \bar{H} \cdot \bar{dL} = \int_S \bar{J} \cdot \bar{ds}$	Ampere circuit law
4.	$\nabla \cdot \bar{B} = 0$	$\int_S \bar{B} \cdot ds = 0$	conservative nature of magnetic fields / Non existance of magnetic monopoles.

* Maxwell's equations for time varying fields:

eqn	Point	Integral	Principle.
1	$\nabla \cdot \bar{D} = \rho_v$	$\int_S \bar{D} \cdot \bar{ds} = \int_V \rho_v dv$	Guass law
2.	$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$	$\int_L \bar{E} \cdot \bar{dl} = \int_S -\frac{\partial \bar{B}}{\partial t} ds$	Faraday's law
3.	$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	$\int_L \bar{H} \bar{dL} = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) \bar{ds}$	modified ampere's ckt law

4. $\nabla \cdot \bar{B} = 0$ $\quad \int_S B \cdot \bar{ds} = 0$ conservation of magnetic flux

* Maxwells equations for a perfect conducting medium:

for a perfect conducting medium we have $\rho_v = 0$ and σ is $\sigma \gg \omega\varepsilon$ (or) $J \gg \frac{\partial D}{\partial t}$

∴ the maxwell's equations for perfect conducting medium becomes.

eqn

1. $\nabla \cdot \bar{D} = 0$ $\quad \int \bar{D} \cdot \bar{ds} = 0$

2. $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$ $\quad \int_L \bar{E} \cdot \bar{dL} = \int_S -\frac{\partial \bar{B}}{\partial t} \bar{ds}$

3. $\nabla \times \bar{H} = \bar{J}$ $\quad \int_L \bar{H} \cdot \bar{dL} = \int_S \bar{J} \cdot \bar{ds}$

4. $\nabla \cdot \bar{B} = 0$ $\quad \oint B \cdot \bar{ds} = 0$

MAXWELL's equations for free space:-

for free space $\rho_v = 0$, $\bar{J} = 0$ ($\because \sigma = 0$ conductivity so $\bar{J} = 0$)

1. $\nabla \cdot \bar{D} = 0$ $\quad \int_S \bar{D} \cdot \bar{ds} = 0$

2. $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$ $\quad \int_L \bar{E} \cdot \bar{dL} = \int_S -\frac{\partial \bar{B}}{\partial t} \bar{ds}$

3. $\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$ $\quad \int_L \bar{H} \cdot \bar{dL} = \int_S \frac{\partial \bar{D}}{\partial t} \cdot \bar{dS}$

4. $\nabla \cdot \bar{B} = 0$ $\quad \oint B \cdot \bar{ds} = 0$

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* Maxwell's equations for harmonically / sinusoidally varying field:-

for harmonically varying fields the flux densities are given by $\bar{D} = \bar{D}_0 e^{j\omega t}$.

$$\bar{B} = \bar{B}_0 e^{j\omega t}$$

consider $\frac{\partial \bar{D}}{\partial t} = \frac{\partial}{\partial t}\left[\bar{D}_0 e^{j\omega t}\right] = \bar{D}_0 e^{j\omega t}(j\omega)$

$$\frac{\partial \bar{D}}{\partial t} = j\omega \bar{D}$$

llarly $\frac{\partial \bar{B}}{\partial t} = j\omega \bar{B}$

1. $\nabla \cdot \bar{D} = \rho_v$ $\quad \int_S \bar{D}\cdot\bar{ds} = \int_V \rho_v \, dv$

2. $\nabla \times \bar{E} = -j\omega \bar{B}$ $\quad \int_L \bar{E}\cdot\bar{dl} = -j\omega \int_S \bar{B}\cdot\bar{ds}$

3. $\nabla \times \bar{H} = \bar{J} + \underbrace{\frac{\partial \bar{D}}{\partial t}}_{j\omega \bar{D}}$ $\quad \int_L \bar{H}\cdot\bar{dl} = \int_S (J + j\omega\bar{D})\bar{ds}$

4. $\nabla \cdot \bar{B} = 0$ $\quad \int_S \bar{B}\cdot\bar{ds} = 0$

* BOUNDARY CONDITIONS:-

1. DIELECTRIC – DIELECTRIC BOUNDARY:

To determine boundary conditions we use maxwells equn's for static electric fields.

i.e $\int_S \bar{D}\cdot\bar{ds} = \int_S \rho_v \, dv$

$$\oint \bar{E}\cdot\bar{dl} = 0$$

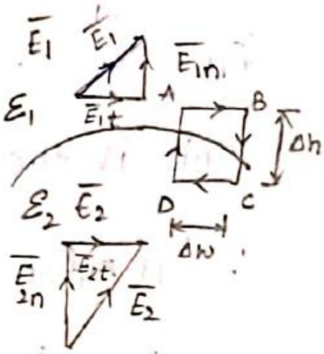
we decompose el- field and el- flux into their tangential and normal components.

i.e $\bar{D} = \bar{D}_t + \bar{D}_n$ and $\bar{E} = \bar{E}_t + \bar{E}_n$

consider 2 diel⁻ mediums with permitivities ε_1 and ε_2 such that

$$\varepsilon_1 = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_2 = \varepsilon_0 \varepsilon_r$$



Let $\bar{E}_1$ and $\bar{E}_2$ be the field strengths of medium 1 and medium 2 respectively. Let $\bar{E}_{1t}$ and $\bar{E}_{2t}$ be the tangential components of el⁻ fields and $\bar{E}_{1n}$ and $\bar{E}_{2n}$ are the normal components of el⁻ fields in mediums 1 and 2 respectively.

$\therefore$ we can write that $\bar{E}_1 = \bar{E}_{1t} + \bar{E}_{2n}$

$$\bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2n}$$

consider a closed path A-B-C-D-A with Δw and Δh as length and height of considered path w.r.t boundary.

Now if we apply maxwell 2nd eqn to the closed path A-B-C-D-A we get

$$\oint \bar{E} \cdot \bar{dl} = 0$$

$$\int_A^B \bar{E} \cdot \bar{dl} + \int_B^C \bar{E} \cdot \bar{dl} + \int_C^D \bar{E} \cdot \bar{dl} + \int_D^A \bar{E} \cdot \bar{dl} = 0$$

$$\bar{E}_{1t} \Delta w - \bar{E}_{1n}\left(\frac{\Delta h}{2}\right) - \bar{E}_{2n}\left(\frac{\Delta h}{2}\right) - \bar{E}_{2t} \Delta w + \bar{E}_{1n}\left(\frac{\Delta h}{2}\right)$$

$$+ \bar{E}_{2n}\left(\frac{\Delta h}{2}\right) = 0$$

$$|\bar{E}_{1t}| = E_1 \quad \& \quad |\bar{E}_{2t}| = \bar{E}_2$$

That is we neglect direction of tangential comp and to attain boundary condition we take $\Delta h \to 0$

$\therefore$ The above eqn becomes $\bar{E}_{1t} \Delta w - \bar{E}_{2t} \Delta w = 0$

$(\bar{E}_{1t} - \bar{E}_{2t})\,\Delta w = 0$ As $\Delta w \neq 0$ as Δh is already 0

$$\boxed{\bar{E}_{1t} = \bar{E}_{2t}} \rightarrow (1)$$

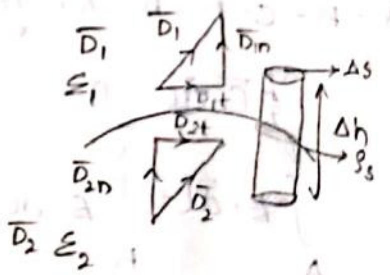
This is the 1st boundary condition.

$\therefore\ \bar{E}_{1t} = \bar{E}_{2t}$

$$\Rightarrow \boxed{\frac{\bar{D}_{1t}}{\varepsilon_1} = \frac{\bar{D}_{2t}}{\varepsilon_2}} \rightarrow (2)$$

from (1) & (2) it is understood that tagential components of $\bar{E}$ are continous along the boundary while tagential components of el- flux density are discontinous along the boundary (as $\bar{D}_{1t} \neq D_{2t}$ depends on ε_1 and ε_2).

To calculate the normal components of el-field and el-flux we apply maxwell's 1st eqn & we consider a cylindrical surface (Pill box) to which gauss law is applied.



from defination of charge density

$$\rho_s = \frac{\Delta Q}{\Delta s}$$

$$\Rightarrow \Delta Q = \rho_s \Delta s \rightarrow (1)$$

from gauss law & def of $\bar{D}$, flux density we have $\bar{D} = \frac{\Delta \psi}{\Delta s}$

$$\Delta \psi = \bar{D} \cdot \Delta s \rightarrow (2)$$

Also $\Delta Q = \Delta \psi$

$$\rho_s \Delta s = \bar{D} \cdot \Delta s$$

$$\rho_s \Delta s = (\bar{D}_{1n} - \bar{D}_{2n})\,\Delta s$$

To attain boundary conditions we take $\Delta h \to 0$ and as surfaces are similar we write $(\bar{D}_{1n} - \bar{D}_{2n}) = \rho_s$

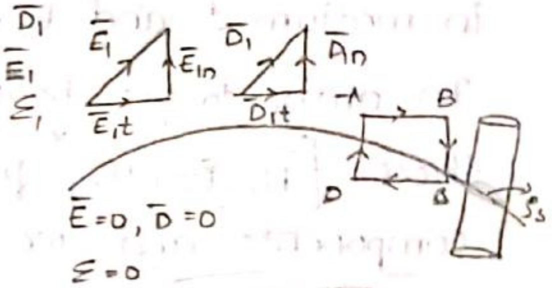
if we take a charge free surface then $\rho_s = 0 \Rightarrow \boxed{\bar{D}_{1n} = \bar{D}_{2n}} \rightarrow (3)$

$\boxed{\varepsilon_1 \bar{E}_{1n} = \varepsilon_2 \bar{E}_{2n}} \rightarrow (4)$

from (3) & (4) normal comp of $\bar{E}$ is not continous and normal comp of $\bar{D}$ is continous along boundary.

* DIELECTRIC CONDUCTOR BOUNDARY:

In this case we assume a perfect conducting medium i.e $\rho_v = 0$ and conductivity tend to ∞



$\sigma \rightarrow \infty, \rho_v = 0$

and WKT along the conductor the field components are zeroes i.e $\bar{E}$ and $\bar{D} = 0$ ($\bar{E} = 0$ & $\bar{D} = 0$) at conductor boundary.

By to following the same procedure that we have applied for Diel - Diel boundary we get the boundaries for Diel conductor boundary as

i) $\bar{E}_{1t} = 0 \quad (\because \bar{E}_{1t} = \bar{E}_{2t}$ and $\bar{E}_{2t} = 0)$

ii) $\bar{D}_{1n} = \rho_s \quad (\bar{D}_{1n} = \bar{D}_{2n}$ & $\bar{D}_{2n} = 0)$

Generalised as $\left[(\bar{D}_{1n} - \bar{D}_{2n}) = \rho_s \right]$

$\bar{D}_n = \rho_s \qquad \varepsilon \bar{E}_1 = \rho_s \qquad \bar{E}_1 = \rho_s / N$ (3)rd conclusion

The following conclusions can be made out of these boundary conditions.

1) No el field exist in conductor.

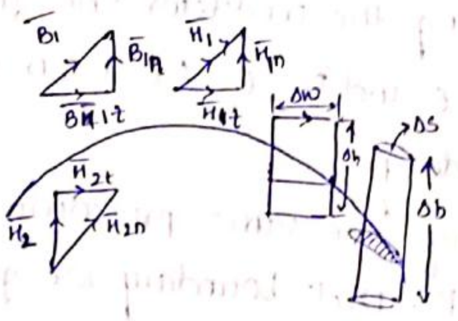
2) As $\bar{E}$ is 0 then potential diff b/w any two point in conductor is 0 so conductor is an equi potential body.

3. e^- field is external to conductor & it must be normal to the surface.

* MAGNETIC CONDUCTOR BOUNDARY CONDITIONS:-

Consider two magnetic mediums having permeability μ_1 and μ_2. Let their field components be $\bar{B}_1, \bar{H}_1$ in medium 1 and $\bar{B}_2, \bar{H}_2$ in medium 2.

In order to get boundary condition we decompose these $[\bar{H}_2, \bar{B}_2, \bar{\mu}_2]^x$ field components into tangential components and we apply maxwell's 3rd equation and 4th equation.



$\Delta \times \bar{H} = \bar{J}$ or $\int \bar{H} \cdot d\bar{L} = I$ (or) $\int \bar{H} \cdot d\bar{L} = \int \bar{J} \cdot ds$ (3rd eqn)

$\nabla \cdot \bar{B} = 0$ (or) $\int \bar{B} \cdot \bar{ds} = 0$ (or) $\int (\nabla \cdot \bar{B}) dV = 0$

By applying maxwell's equation we assumed a closed path, a,b,c,d and cylindrical surfaces ΔS as shown in figure.

Applying maxwells 4th equation to the cylindrical surface (pill box) and assuming the negligible height of the cylinder. $\Delta b \to 0$ then

$$\int \bar{B} \cdot \bar{ds} = 0$$

$$\bar{B} \cdot \Delta S = 0$$

$$(\bar{B}_{1n} - \bar{B}_{2n}) \Delta S = 0$$

$\overline{B_{1n}} - \overline{B_{2n}} = 0$

$\overline{B_{1n}} = \overline{B_{2n}}$

we know that $\overline{B} = \mu \overline{H}$

$\mu_1 \overline{H_{1n}} = \mu_2 \overline{H_{2n}}$

The normal components of magnetic flux density are continous along the boundary where as magnetic fields components are discontinous along the boundary

Applying maxwells 3rd equation to the closed path abcda then

i.e $\int \overline{H} \cdot \overline{dl} = I$.

$\overline{H_1} \Delta w - \overline{H_{1n}} \frac{\Delta h}{2} - \overline{H_{2n}} \frac{\Delta h}{2} - \overline{H_{2t}} \Delta w + \overline{H_{2n}} \frac{\Delta h}{2} + \overline{H_{1n}} \frac{\Delta h}{2} = I$.

$\overline{H_{1t}} \Delta w - \overline{H_{2t}} \Delta w = I$.

Assuming boundary condtions i.e $\Delta h \to 0$

from the def of surface current density, we have

$I = \int \overline{K} \cdot ds$

$I = \overline{K} \cdot \Delta s$

$I = \overline{K} \cdot \Delta w \cdot \Delta h$

$(\overline{H_{1t}} - \overline{H_{2t}}) \Delta w = \overline{K} \cdot \Delta s$

$(\overline{H_{1t}} - \overline{H_{2t}}) \Delta w = \overline{K} \cdot \Delta w$

$\overline{H_{1t}} - \overline{H_{2t}} = \overline{K}$

If we assume surface current density to be zero

$\overline{H_{1t}} - \overline{H_{2t}} = 0$

$\overline{H_{1t}} = \overline{H_{2t}}$

where $\overline{H} = \overline{B}/\mu$

$\frac{\overline{B_{1t}}}{\mu_1} = \frac{\overline{B_{2t}}}{\mu_2}$

The tangential components of magnetic field are continous along the boundary and magnetic flux density are discontinous along the boundary.

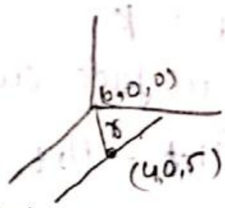
Note:

Same as electro boundary conditions.

* - A current filament $I = 100A$ which is along Y-axis is passing through a point $(4, 0, -5)$. find the magnetic field strength at origin.

Note: The magnetic field intensity due to an infinitely long conductor with a magnetic path of radius 'R' is given by $\bar{H} = \frac{I}{4\pi l} \bar{a}_\phi$

here length l can be calculated as $l = 2\pi r$



Given $I = 100 Amp$ and a point on current element is $(4, 0, -5)$

$\therefore$ current at Magnetic field strength at origin is given by

where $r \rightarrow$ distance b/w conductor and origin.

$$\bar{H} = \frac{I}{2\pi r}\bar{a}_\phi$$

$$|\bar{r}| = \sqrt{4^2+5^2} = \sqrt{41}$$

$$\bar{H} = \frac{I}{2\pi\sqrt{41}}\bar{a}_\phi$$

$$= 9.48\,\bar{a}_\phi \; A/m$$

* Given magnetic flux density $\bar{B} = r\sin\theta\,\bar{a}_\theta$. find the total flux ~~passin~~ crossing through a surface defined by $\theta = \pi/4$, $(1 \le r \le 2)$ and $(0 \le z \le 5)$

$$\bar{B} = \frac{\phi}{S} \quad (\text{or}) \quad \bar{B} = \frac{d\phi}{ds}$$

$$\phi = \int_S \bar{B}\cdot\bar{ds}$$

$$\phi = \int r\sin\theta\,\bar{a}_\theta\,\bar{ds}$$

given $\theta = \pi/4$

$$\bar{B} = r\sin\theta\,\bar{a}_\theta$$

$$\bar{ds}_\theta = \cancel{r\,dr\,d\phi\,\bar{a}_\phi}\; dr\,dz\,\bar{a}_\theta$$

$$\phi = \int_S \bar{B}\cdot\bar{ds}$$

$$= \int_S r\sin\theta\,\bar{a}_\theta\cdot dr\,dz\,\bar{a}_\theta$$

$$= \int_S r\sin\theta\,\cancel{a_\theta}\,dr\,dz$$

$$= \int_{r=1}^{2}\int_{z=0}^{5} r\sin\theta\,dr\cdot dz = \sin\theta\left[\frac{r^2}{2}\right]_0^2 \left[z\right]_0^5$$

$$= 15/2\,\sin\theta$$

$$\phi = \frac{15}{2}\frac{1}{\sqrt{2}} = 5.303\,wb.$$

14/2/18 UNIT-3 EM WAVE CHARACTERISTICS:-

EM wave is an electro magnetic wave which transports Energy in various mediums.

EM wave propagates energy in transverse direction (i.e $\perp^{lr}$ to both electric and magnetic fields).

[wave → used]ˣ

Important wave parameters (or) characteristics:

i) speed /velocity
ii) wave mode.
iii) Energy
iv) Attenuation
v) distance travelled

EXAMPLES :-

Light wave.
Radio waves
Micro waves etc.

* wave characteristics depend on medium of propagation

sound wave velocity under water 1500 m/sec.
" " velocity in free space 340 m/sec.

[For exam]ˣ

* PARAMETERS OF VARIOUS MEDIUMS:

→ perfect dielectric (free space) :-

for this medium $\rho_v = 0$, $\bar{J} = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$

$\sigma <<< \omega\varepsilon_0$ so $\sigma = 0$

→ Loss less dielectric:

$\bar{J} = 0$, $\sigma = 0$, $\varepsilon = \varepsilon_0\varepsilon_r$, $\mu = \mu_0\mu_r$, $\sigma << \omega\varepsilon$

→ Lossy dielectric:-

$\sigma \neq 0$, $\varepsilon = \varepsilon_0\varepsilon_{r}$, $\mu = \mu_0\mu_r$

* perfect conductor: $\varepsilon = \varepsilon_0,\ \mu = \mu_0 \times \mu_r$

$\rho_v = 0,\ \sigma = \infty,\ \bar{J} \neq 0$ and $\bar{J} >> \frac{\partial D}{\partial t},\ \sigma >> \omega\varepsilon$

* EM WAVE EQN FOR PERFECT DIELECTRIC MEDIUM:-

for free space medium parameters are

$\rho_v = 0,\ \bar{J} = 0,\ \varepsilon = \varepsilon_0,\ \mu = \mu_0,\ \sigma = 0$

The maxwell's equations for free space are given by

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow \nabla \times \bar{E} = -\partial/_{\partial t}(\mu_0 \bar{H}) = -\mu_0 \frac{\partial \bar{H}}{\partial t} \rightarrow ①$$

($\because$ $\bar{B} = \mu\bar{H}$ for free space
$\bar{B} = \mu_0 \bar{H}$)

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} \Rightarrow \nabla \times \bar{H} = \varepsilon_0 \frac{\partial E}{\partial t} \rightarrow ②$$

($\because$ $\bar{D} = \varepsilon \bar{E} \Rightarrow \bar{D} = \varepsilon_0 \bar{E}$)

$$\nabla \cdot \bar{D} = 0 \Rightarrow \nabla \cdot \bar{E} = 0 \rightarrow ③$$

$$\nabla \cdot \bar{B} = 0 \Rightarrow \nabla \cdot \bar{H} = 0 \rightarrow ④$$

- Applying Partial derivatives on b.s for eqn ① & ②

$$\partial/_{\partial t}[\nabla \times \bar{E}] = \partial/_{\partial t}\left[-\mu_0 \frac{\partial \bar{H}}{\partial t}\right]$$

$$\nabla \times \frac{\partial \bar{E}}{\partial t} = -\mu_0 \frac{\partial^2 \bar{H}}{\partial t^2} \rightarrow ⑤$$

||'y

$$\nabla \times \frac{\partial \bar{H}}{\partial t} = \varepsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} \rightarrow ⑥$$

Taking curl on b.s for eqn's ① & ②

$$\nabla \times \nabla \times \bar{E} = \nabla \times \left(-\mu_0 \frac{\partial \bar{H}}{\partial t}\right)$$

$$\nabla \times \nabla \times \bar{E} = -\mu_0 \left(\nabla \times \frac{\partial \bar{H}}{\partial t}\right) \rightarrow ⑦$$

$$\nabla \times \nabla \times \bar{H} = \nabla \times \left(\varepsilon_0 \frac{\partial \bar{E}}{\partial t}\right)$$

$$\nabla \times \nabla \times \bar{H} = \varepsilon_0 \left[\nabla \times \frac{\partial \bar{E}}{\partial t}\right] \rightarrow ⑧$$

14/2/1.

using vector identities we have

$$\nabla\times\nabla\times\bar{E} = \nabla(\nabla\cdot\bar{E}) - \nabla^2\bar{E} \rightarrow (9)$$

||^ly

$$\nabla\times\nabla\times\bar{H} = \nabla(\nabla\cdot\bar{H}) - \nabla^2\bar{H} \rightarrow (10)$$

from eqns (3) & (4) we can write

$$(11) \leftarrow \nabla\times\nabla\times\bar{E} = -\nabla^2\bar{E} \quad (\because \nabla\cdot\bar{D}=0 \Rightarrow \nabla\cdot\bar{E}=0)$$

$$(12) \leftarrow \nabla\times\nabla\times\bar{H} = -\nabla^2\bar{H} \quad (\because \nabla\cdot\bar{B}=0 \Rightarrow \nabla\cdot\bar{H}=0)$$

from (7) & (11)

$$-\nabla^2\bar{E} = -\mu_0\left[\nabla\times\frac{\partial\bar{H}}{\partial t}\right]$$

from (6)

$$-\nabla^2\bar{E} = -\mu_0\left[\varepsilon_0\frac{\partial^2\bar{E}}{\partial t^2}\right]$$

$$\boxed{\nabla^2\bar{E} = \mu_0\varepsilon_0\frac{\partial^2\bar{E}}{\partial t^2}} \rightarrow (13)$$

(8) in (12)

$$\nabla\times\nabla\times\bar{H} \quad -\nabla^2\bar{H} = \varepsilon_0\left[\nabla\times\frac{\partial\bar{E}}{\partial t}\right] \text{ from (5)}$$

$$-\nabla^2\bar{H} = -\mu_0\varepsilon_0\frac{\partial^2\bar{H}}{\partial t}$$

$$\boxed{\nabla^2\bar{H} = \mu_0\varepsilon_0\frac{\partial^2\bar{H}}{\partial t^2}} \rightarrow (14)$$

Eqns (13) & (14) are EM wave equations for free space (or) perfect dielectric.

The velocity of EM wave in free space is

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8 \text{ m/s}$$

* WAVE EQUATION IN A CONDUCTING MEDIUM :-

medium parameters for conducting medium are

$\sigma = \infty, \ \bar{J} \neq 0, \ \rho_v = 0, \ J >> \frac{\partial \bar{D}}{\partial t}$

$\varepsilon = \varepsilon_0 \cdot \ \mu = \mu_0 \mu_r$

Maxwell's eqns for conducting medium are

$[\nabla \cdot \bar{D} = \rho_v \ 0$

$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \approx \bar{J} \left[as \ \frac{\partial \bar{D}}{\partial t} <<< \bar{J} \right] \Big]$

$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow \nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \rightarrow (1)$

$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \Rightarrow \nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t} \rightarrow (2)$

$(\because \bar{J} = \sigma \bar{E} \ , \bar{D} = \varepsilon \bar{E})$

$\nabla \cdot \bar{D} = \rho_v = 0 \Rightarrow \nabla \cdot \bar{E} = 0 \rightarrow (3)$

$\nabla \cdot \bar{B} = 0 \Rightarrow \nabla \cdot \bar{H} = 0 \rightarrow (4)$

Taking partial differentiation on b.s for (1) & (2)

$\nabla \times \left(\frac{\partial \bar{E}}{\partial t} \right) = -\mu \frac{\partial^2 \bar{H}}{\partial t^2} \rightarrow (5)$

$\nabla \times \frac{\partial \bar{H}}{\partial t} = \partial/\partial t \left(\sigma \bar{E} + \varepsilon \frac{\partial E}{\partial t} \right) \cdot$

$= \sigma \partial \bar{E}/\partial t + \varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} \rightarrow (6)$

Taking curl on b.s

$\nabla \times \nabla \times \bar{H} = \sigma (\nabla \times \bar{E}) + \epsilon \left(\nabla \times \frac{\partial \bar{E}}{\partial t} \right) \rightarrow (8)$

$\nabla \times \nabla \times \bar{E} = \mu \left(\nabla \times \frac{\partial \bar{H}}{\partial t} \right) \rightarrow (7)$

from vector identities

$\nabla \times \nabla \times \bar{E} = \nabla \cdot (\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\nabla^2 \bar{E} \rightarrow (9)$

$$\nabla \times \nabla \times \bar{H} = -\nabla^2 \bar{H} \rightarrow (10)$$

from eqns (6), (7), (9)

$$-\nabla^2 \bar{E} = -\mu\left[\sigma \frac{\partial \bar{E}}{\partial t} + \varepsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2}\right]$$

$$\boxed{\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}} \rightarrow (11)$$

Sub (8), (1), (5)

$$-\nabla^2 H = \sigma\left(-\mu \frac{\partial \bar{H}}{\partial t}\right) + \varepsilon_0\left(-\mu \frac{\partial^2 \bar{H}}{\partial t^2}\right)$$

$$\boxed{\nabla^2 \bar{H} = \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t}} \rightarrow (12)$$

Equations (11) & (12) are the wave eqns for EM wave in conducting medium.

* WAVE PROPAGATION IN HARMONIC MEDIUMS: (OR) TIME HARMONIC WAVE EQN (OR) PHASOR [EQN OF] NOTATION OF EM WAVE:

To obtain wave equation in phasor or time harmonic medium we define $\bar{E}$ and $\bar{H}$ in phasor form i.e $\bar{E} = |\bar{E}| e^{j\omega t}$ & $\bar{H} = |\bar{H}| e^{j\omega t}$

consider $\frac{\partial \bar{E}}{\partial t} = |\bar{E}| e^{j\omega t} (j\omega)$

$$\Rightarrow \frac{\partial \bar{E}}{\partial t} = \bar{E}(j\omega)$$

consider 2nd order differential of $\bar{E}$.

$$\frac{\partial^2 E}{\partial t^2} = j\omega \cdot \frac{\partial}{\partial t}\left[|\bar{E}| e^{j\omega t}\right]$$

$$= (j\omega)^2 \bar{E} = -\omega^2 \bar{E}$$

llarly

$$\frac{\partial \bar{H}}{\partial t} = (j\omega)\bar{H}$$

$$\frac{\partial^2 \bar{H}}{\partial t^2} = (j\omega)^2 \bar{H} = -\omega^2 \bar{H}$$

Here ω is angular frequency which is measured in rad/sec.

fields are consideed in medium which is homogenous linear isotrophic and source free.

we know that wave equations for el-field in ~~toma~~ time domain for a conducting medium is

$$\nabla^2 \bar{E} - \mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

$$\Rightarrow \nabla^2 \bar{E} - \mu\sigma (j\omega \bar{E}) - \mu\varepsilon(-\omega^2 \bar{E}) = 0$$

$$\Rightarrow \nabla^2 \bar{E} - j\omega\mu\sigma \bar{E} + \mu\varepsilon\omega^2 \bar{E} = 0$$

$$\Rightarrow \nabla^2 \bar{E} - \bar{E}\underbrace{[j\omega\mu\sigma - \omega^2\mu\varepsilon]}_{\text{Propagation constant}} = 0 \rightarrow (1)$$

$\bar{E}$ depends on it

llarly

$$\nabla^2 \bar{H} - [j\omega\mu\sigma - \omega^2\mu\varepsilon] \cdot \bar{H} = 0 \rightarrow (2)$$

from eqn's (1) & (2)

they are time harmonic wave eqn's for conducting medium

for loss less medium : $\sigma = 0$ $\therefore$ eq's (1) & (2) become

$$\left.\begin{aligned} \nabla^2 \bar{E} + \omega^2\mu\varepsilon \bar{E} = 0 \\ \nabla^2 \bar{H} + \omega^2\mu\varepsilon \bar{H} = 0 \end{aligned}\right\} \text{Helmholtz's wave eqn.}$$

* UNIFORM PLANE WAVE :

It is EM WAVE in which el⁻ and magnetic fields are directed in fixed directions in space and constant (on) infinite planes ⊥lr to direction of propagation.

1. The plane wave has no el⁻ and magnetic components along direction of propagation.

2. E and H fields have const amp and phase on infinite planes ⊥lr to direction of propagation

3. We cannot generate uniform plane wave in practise since it is not possible to keep E and H field as const on infinite planes.

* UNIFORM PLANE WAVE EQUATION :

consider uniform plane wave which travelling in z direction in a free space medium. Assume that

→ The magnetic field is in Y direction

el⁻ X

considering cartesian equation coordinates the plane wave eqn is given by

for conducting medium

$$\nabla^2 \bar{E} - (j\omega\mu\sigma - \omega^2\mu\epsilon)\bar{E} = 0 \rightarrow (1)$$

$$\nabla^2 \bar{H} - (j\omega\mu\sigma - \omega^2\mu\epsilon)\bar{H} = 0 \rightarrow (2)$$

But

$$\nabla^2 \bar{E} = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2}$$

since wave is travelling in z direction.

$$\frac{\partial^2 E_x}{\partial x^2} = \frac{\partial^2 E_y}{\partial y^2} = 0$$

eqn① becomes

$$\nabla^2 \bar{E} - (j\omega\mu\sigma - \omega^2 \mu\varepsilon)\bar{E} = 0 \rightarrow (3)$$

from ③

The wave propagates in perfect dielectric the $\sigma = 0$, &

$\varepsilon = \varepsilon_0$ & $\mu = \mu_0$

$$\frac{\partial^2 \bar{E}}{\partial z^2} + \omega^2 \mu_0 \varepsilon_0 \bar{E} = 0$$

we take $\beta^2 = \omega^2 \mu_0 \varepsilon_0$

$$\beta = \omega\sqrt{\mu_0 \varepsilon_0}$$

Here β is called phase constant which is measured in rad/m

$$\boxed{\frac{\partial^2 \bar{E}}{\partial z^2} + \beta^2 \bar{E} = 0} \rightarrow (4)$$

eqn④ is called equation of plane wave in general form

$^{||}$ly. $\boxed{\frac{\partial^2 \bar{H}}{\partial z^2} + \beta^2 \bar{H} = 0} \rightarrow (5)$

When uniform plane wave propagating in z-direction with el- field component in x-direction and magnetic field along y direction then clearly we can write $E_z = H_z = 0$

$\therefore$ eqn④ and ⑤ can be altered as.

$$\boxed{\begin{aligned} \frac{\partial^2 E_x}{\partial z^2} + \beta^2 E_x = 0 \\ \frac{\partial^2 H_y}{\partial z^2} + \beta^2 H_y = 0 \end{aligned}}$$

* solution for uniform plane wave equation:

General solution to uniform plane wave equation is given by $E_x(z) = E^+ e^{-j\beta z} + E^- e^{j\beta z}$

where E^+ and E^- are complex arbitrary constants which will be determined using boundary conditions.

E^+ represents el field component when wave is propagating in +ve z axis and E^- represents el field component when wave is propagating in negative z-axis.

-Assume that the initial phase angle of the wave is 'zero' in time domain the wave can be expressed as $E_x(z,t) = Re[E_x(z) e^{j\omega t}]$

$$= Re[(E^+ e^{-j\beta z} + E^- e^{j\beta z}) e^{j\omega t}]$$

$$= Re[E^+ e^{j\omega t - j\beta z} + E^- e^{j\omega t + j\beta z}]$$

$$= Re[E^+ e^{j(\omega t - \beta z)} + E^- e^{j(\omega t + \beta z)}]$$

$[\because Re(e^{j\theta}) = \cos\theta]$

$$= E^+ \cos(\omega t - \beta z) + E^- \cos(\omega t + \beta z)$$

||arly

$$H_y(z,t) = H^+ \cos(\omega t - \beta z) + H^- \cos(\omega t + \beta z)$$

Note: If the wave travels only in +ve z axis directn then

$$E_{x}(z,t) = E^+ \cos(\omega t - \beta z)$$

$$H_{y}(z,t) = H^+ \cos(\omega t - \beta z)$$

If wave travels only -ve z Y direction:

$E_x(z,t) = E^- \cos(\omega t + \beta z)$

$H_y(z,t) = H^- \cos(\omega t + \beta z)$

PHASE VELOCITY:

It is defined as the velocity at which wave travels with constant phase.

phase velocity is denoted with v_p

from solution of uniform plane wave eqn

phase of the wave is $(\omega t - \beta z)$

$\omega t - \beta z$ = constant to get phase velocity.

$\beta z = \omega t - \text{const}$

$z = \frac{\omega t}{\beta} - \frac{\text{const}}{\beta}$

consider differential of time for above equation.

$(\because z \to \text{distance} \quad \frac{\partial z}{\partial t} = \text{velocity})$

$\frac{dz}{dt} = d/dt\left[\frac{\omega t}{\beta} - \text{constant}\right]$

$= \omega/\beta - 0$

$\frac{dz}{dt} = \omega/\beta = v_p$

As phase is constant we call it phase velocity.

$v_p = \omega/\beta$ $\qquad \boxed{\beta = \omega\sqrt{\mu\varepsilon}}$ rad/m

$= \frac{\omega}{\omega\sqrt{\mu\varepsilon}}$

$= \frac{1}{\sqrt{\mu\varepsilon}}$ $\qquad$ for free space $v_p = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$= 3\times10^8$ m/sec.

NOTE : i) for mediums other than free space phase velocity is as:

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

ii) phase velocity can be represented interms of wavelength as

$$v_p = \frac{\omega}{\beta}$$

$$= \frac{2\pi f}{\frac{2\pi}{\lambda}}$$

$$v_p = \lambda f$$

* CHARACTERISTIC IMPEDANCE OF EM WAVE :- (η)

When EM wave is travelling in a medium the ratio b/w el field and mag field is constant and that ratio is called wave impedance / intrinsic / characteristic impedance. Denoted by (η)

and Defined as $\eta = \frac{\bar{E}}{\bar{H}}$

units : ohms (Ω)

* EXPRESSION FOR CHARACTERISTIC IMPEDANCE :-

consider a uniform plane wave travelling in loss less medium so $\bar{J} = 0$

$E_z = H_z = 0$

WKT from Maxwell's equations

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t} \rightarrow ②$$

$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \varepsilon \frac{\partial \bar{E}}{\partial t} \rightarrow (1) \quad (\because \bar{J} = 0 \quad \bar{D} = \varepsilon \bar{E}$

from (1) & (2)

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = \varepsilon \left[\frac{\partial \bar{E}_x}{dt} \bar{a}_x + \frac{\partial \bar{E}_y}{dt} \bar{a}_y + \frac{\partial \bar{E}_z}{\partial t} \cdot \bar{a}_z \right]$$

$H_z = 0$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & 0 \end{vmatrix} = \varepsilon \left[\frac{\partial \bar{E}_x}{\partial t} \bar{a}_x + \frac{\partial \bar{E}_y}{\partial t} \bar{a}_y + \frac{\partial \bar{E}_z}{\partial t} \bar{a}_z \right]$$

$$\bar{a}_x \left[0 - \frac{\partial H_y}{\partial z} \right] - \bar{a}_y \left[0 - \frac{\partial H_x}{\partial z} \right] + \bar{a}_z \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right] =$$

$$\varepsilon \left[\frac{\partial \bar{E}_x}{\partial t} \bar{a}_x + \frac{\partial \bar{E}_y}{\partial t} \bar{a}_y + \frac{\partial \bar{E}_z}{\partial t} \bar{a}_z \right]$$

comparing the coefficients of on b.s

$$-\frac{\partial H_y}{\partial z} = \varepsilon \frac{\partial E_x}{\partial t} \rightarrow (3)$$

$$\frac{\partial H_z}{\partial z} = \varepsilon \frac{\partial E_y}{\partial t} \rightarrow (4)$$

$$\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \quad (\because E_z = 0)$$

from harmonic equations we can write

$$\frac{\partial}{\partial t} H_y = \varepsilon j\omega E_x - j\omega \varepsilon E_x \rightarrow (5)$$

$$\frac{\partial}{\partial t} H_x = j\omega \varepsilon \bar{E}_y \rightarrow (6)$$

$E_x = E_x(z,t) = E^+ e^{-j\beta z}$

$\frac{\partial E_x}{\partial z} = E^+ \partial/\partial t [e^{-j\beta z}]$

$= -j\beta E^+ e^{-j\beta z}$.

$\partial/\partial z \; E_x = -j\beta E_x \rightarrow (7)$

Illarly.

$\partial/\partial z \; H_y = -j\beta H_y \rightarrow (8)$

$+j\beta H_y = j\omega \varepsilon E_x$

$$\frac{E_x}{H_y} = \frac{+j\beta}{j\omega\varepsilon}$$

$$\frac{\beta}{\omega\varepsilon} = \frac{+E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}.$$

Illarly if we consider.

$$\frac{E_y}{H_x} = \sqrt{\frac{\mu}{\varepsilon}}$$

for free space.

Characteristic impedance $\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\Omega$.

28/2/18:

* WAVE PROPAGATION IN CONDUCTION MEDIUM:-

considering uniform plane wave which is propagating in conducting medium then the time harmonic wave equation is given by

$$\left.\begin{aligned} \nabla^2 \bar{E} &= (j\omega\mu\sigma - \omega^2\mu\varepsilon)\,\bar{E} = 0 \\ \nabla^2 \bar{H} &= (j\omega\mu\sigma - \omega^2\mu\varepsilon)\,\bar{H} = 0 \end{aligned}\right\} \rightarrow (1)$$

we can also write.

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0 \qquad (\because \gamma^2 = (j\omega\mu\sigma - \omega^2\mu\varepsilon))$$

$$\nabla^2 \bar{H} - \gamma^2 \bar{H} = 0$$

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$$

$\gamma \rightarrow$ propagation const.

$$\gamma = \alpha + j\omega$$

where α is attenuation constant

β is phase constant

when a wave is propagating in conduction medium its power reduces due to losses in medium. the amount of attenuation in wave energy is represented by attenuation constant 'α'. It is measured in neper/decibel.

$$\boxed{1\,\text{neper} = 8.686\,\text{dB}}$$

A change in phase shift of the wave canbe represented by phase constant 'β' and is measured in rad/m

$$\gamma = \alpha + j\cancel{\beta}\,\beta$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

for loss less medium attenuation constant is '0'. i.e $\boxed{\alpha = 0}$

no loss so no attenuation the medium doesn't conduct so no loss.

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2/3/18

The plane wave eqn for el- field component is given by $\frac{\partial^2}{\partial z^2} E_x - \gamma^2 E_x = 0 \rightarrow$ ①

The general solution for differential equation is given by

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z} \qquad (\gamma = \alpha + j\beta)$$

$$[E_x(z,t) = Re[E_x(z) \cdot e^{j\omega t}]^{\times}$$

$E^+ \rightarrow$ El- field direct represent el- field components when wave propogates in positive z direction.

$E^- \rightarrow$ -ve z direction..

in time domain the el- field component can be represented as

$$E_x(z,t) = Re[E_x(z) e^{j\omega t}]$$

$$= Re[(E^+ e^{-\gamma z} + E^- e^{\gamma z}) e^{j\omega t}]$$

$$= Re[(E^+ e^{-(\alpha+j\beta)z} + E^- e^{+(\alpha+j\beta)z}) e^{j\omega t}]$$

$$E_x(z,t) = E^+ e^{-\alpha z} \cos(\omega t \mp \beta z) + E^- e^{\alpha z} \cdot \cos(\omega t \pm \beta z) \quad ②$$

||arly.

$$H_y(z,t) = H^+ e^{-\alpha z} \cos(\omega t - \beta z) + H^- e^{\alpha z} \cos(\omega t + \beta z)$$

$H^+ \rightarrow$ directn of mag field comp when wave (propagates) travels in +ve z direction.

Note:

If uniform plane wave Propagates in free space or lossless dielectric medium then the wave components can be derived by substituting $\alpha=0$ in eqn ② & ③

i.e $E_x(z,t) = E^+ \cos(\omega t-\beta t) + E^- \cos(\omega t+\beta t)$

* INTRINSIC IMPEDANCE FOR LOSSY DIELECTRIC OR CONDUCTING MEDIUM:

LOSSY Dielectric $\longrightarrow$ medium conducts
so some energy is lost.

Perfect diel$^-$ = freespace.

consider a uniform plane wave in a conducting medium.

we know that for uniform plane wave.

$E_z = H_z = 0$ ($\because$ no field comp exists in direction of propagation.

As medium is conducting medium $\bar{J} = \sigma\bar{E}$, $\sigma \neq 0$

$\bar{J} \neq 0$

for a uniform plane wave $\frac{\partial E_x}{\partial y} = 0$, $\frac{\partial E_y}{\partial x} = 0$,

$\frac{\partial H_x}{\partial y} = 0$, $\frac{\partial H_y}{\partial x} = 0$

For Maxwell's eqns for conducting medium are.

$$\nabla \times \bar{H} = \sigma\bar{E} + \varepsilon\frac{\partial \bar{E}}{\partial t} \rightarrow ① \quad (\because \bar{J} = \sigma\bar{E}, \ \bar{D} = \varepsilon\bar{E})$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{E} = -\mu\frac{\partial \bar{H}}{\partial t} \rightarrow ②$$

consider $\frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t} \rightarrow ③$

from intrinsic impedance

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial \cancel{t}} \rightarrow (3)$$

We have from intrinsic impedance (η) from eqn (3)

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial \cancel{t}} = -\mu j\omega H_y = -j\omega\mu H_y \rightarrow (4)$$

consider a uniform plane wave which is propagating in z direction then its electric field is given by

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z}$$

consider the wave is propagating in +ve z-direction only. i.e $E_x(z) = E^+ e^{-\gamma z}$

consider

$$\partial/_{\partial z}[E_x(z)] = \frac{\partial E_x}{\partial z} = \partial/_{\partial z}[E^+ e^{-\gamma z}]$$

$$= E^+(-\gamma) e^{-\gamma z}$$

$$\partial/_{\partial z} E_x = -\gamma E_x \rightarrow (5)$$

from eqn (4) and (5)

$$\frac{\partial E_x}{\partial z} \Rightarrow -j\omega\mu H_y = -\gamma E_x$$

$$\frac{E_x}{H_y} = \frac{+j\omega \cancel{E_x} \mu}{\gamma} \rightarrow (6)$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\frac{E_x}{H_y} = \eta = \frac{+j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega)}}$$

$$\frac{E_x}{E_y} = \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \rightarrow (7)$$

eqn's (6) and (7) are nothing but the intrinsic impedance in terms of ~~pro~~propagation constant & conductivity.

NOTE: To derive ~~con~~ value of characteristic impedance for free space / perfect dielectric we substitute $\sigma = 0$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi = 377\Omega.$$

WAVE PROPAGATION IN GOOD DIELECTRIC:

not needed

WKT, wave eqn's for perfect dielectric is given by

$$\nabla^2 \bar{E} + \omega^2 \mu\varepsilon \bar{E} = 0$$

wave eqn in terms of propagation constant is

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0$$

we ~~know~~ can write $\gamma^2 = -\omega^2\mu\varepsilon$

$$\gamma = \sqrt{-\omega^2\mu\varepsilon}$$

$$= j\omega\sqrt{\mu\varepsilon}$$

$$\gamma = \alpha + j\beta$$

$$\boxed{\beta = \omega\sqrt{\mu\varepsilon}}$$

Also $\alpha = 0$; since the medium is perfect dielectric so attenuation is zero

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$V_p = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \rightarrow \text{free space.}$$

$v_p \cong c = 3\times10^8\ m/s$

$$\eta = \frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

Not needed

* RELATION BETWEEN ATTENUATION and PHASE CONSTANTS

WKT, $\gamma = \alpha + j\beta$

$$\gamma^2 = (\alpha + j\beta)^2$$

$$j\omega\mu(\sigma + j\omega\varepsilon) = \alpha^2 - \beta^2 + 2j\alpha\beta$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\varepsilon$$

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon \rightarrow (1)$$

$$2\alpha\beta = \omega\mu\sigma \rightarrow (2)$$

$$(a+b)^2 = (a-b)^2 + 4ab$$

$$(\alpha^2+\beta^2)^2 = (\alpha^2-\beta^2)^2 + 4\alpha^2\beta^2$$

$$(\alpha^2+\beta^2)^2 = \omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2$$

$$\alpha^2+\beta^2 = \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2} \rightarrow (3)$$

α^2: adding (1) + (3)

$$2\alpha^2 = -\omega^2\mu\varepsilon + \omega\mu\sqrt{\omega^2\varepsilon^2+\sigma^2}$$

$$\alpha^2 = \frac{\omega\mu}{2}\left(-\omega\varepsilon + \sqrt{\omega^2\varepsilon^2+\sigma^2}\right)$$

$$\alpha = \sqrt{\frac{\omega\mu}{2}\left(-\omega\varepsilon + \sqrt{\omega^2\varepsilon^2+\sigma^2}\right)}$$

sub ④ & ③

$$\alpha = \sqrt{\frac{\omega^2 \mu \varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$$

substract ① & ②

$$\alpha^2+\beta^2-\alpha^2+\beta^2 = \sqrt{\omega^2\mu\varepsilon+\omega^2\mu^2\sigma^2}+\omega^2\mu\varepsilon$$

$$2\beta^2 = \sqrt{\omega^2\mu\varepsilon+\omega^2\mu^2\sigma^2}+\omega^2\mu\varepsilon$$

llarly.

$$\beta = \sqrt{\frac{\omega\mu}{2}\left(\omega\varepsilon+\sqrt{\omega^2\varepsilon^2+\sigma^2}\right)}$$

PROPAGATION FOR GOOD CONDUCTORS & GOOD DIELECTRIC:-

consider, a good conducting medium whose conductivity is $\sigma >> \omega\varepsilon$ i.e for good conductors conductivity is very high w.r.to frequency.

From the def. of propagation constant, we have

$$\gamma = \sqrt{j\omega\mu(\sigma+j\omega\varepsilon)}$$

As $\sigma >> \omega\varepsilon$

$$\gamma = \sqrt{j\omega\mu\sigma}$$

$\sqrt{j} = \angle 45^\circ$

$a\angle 45^\circ$

$= a\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right]$

$$\gamma = \alpha + j\beta$$

$$\gamma = \sqrt{\omega\mu\sigma}\cdot\sqrt{j}$$

$$= \sqrt{\omega\mu\sigma}\ \angle 45^\circ$$

$$= \sqrt{\omega\mu\sigma}\left[\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right]$$

$$= \frac{\sqrt{\omega\mu\sigma}}{\sqrt{2}} + j\frac{\sqrt{\omega\mu\sigma}}{\sqrt{2}}$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

for good conductors the value of attenuation constant is same as value of phase constant.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}}$$

Phase velocity $V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}}$ $\quad V_p = \sqrt{\frac{2\omega}{\mu\sigma}}$

WAVE PROPAGATION IN GOOD DIELECTRICS / PROPAGATION CONSTANT FOR GOOD DIELECTRIC MEDIUMS:

consider, a wave propagating in a good dielectric medium

for good dielectrics σ is very low when compared with $\omega\varepsilon$ i.e $\sigma \lll \omega\varepsilon$

WKT propagation constant γ is $\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$$\gamma = \sqrt{j\omega\mu\left(j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)\right)} \quad \left(\because \frac{\sigma}{\omega\varepsilon} \ll 1\right)$$

$$= j\omega\sqrt{\mu\varepsilon}\left[\sqrt{1+\frac{\sigma}{j\omega\varepsilon}}\right]$$

$$= j\omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{1/2}$$

WKT from binomial expression we have.

$$(1+x)^{1/2} = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \cdots$$

$$\gamma = j\omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma}{2j\omega\varepsilon} - \frac{\sigma^2}{8j^2\omega^2\varepsilon^2} + \cdots\right]$$

$$= j\omega\sqrt{\mu\varepsilon} + j\omega\sqrt{\mu\varepsilon}\,\frac{\sigma}{2j\omega\varepsilon} + j\omega\sqrt{\mu\varepsilon}\,\frac{\sigma^2}{8\omega^2\varepsilon^2}$$

$$= \frac{\sqrt{\mu}\,\sigma}{\sqrt{\varepsilon}\cdot 2} + j\omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\alpha = \frac{\sigma}{2}\sqrt{\mu/\varepsilon} \qquad \beta = \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]$$

$$(1+x)^n = {}^nC_0\, 1^n x^0 + {}^nC_1\, 1^{n-1} x^1 + {}^nC_2\, 1^{n-2} x^2 + \dots$$

$$= 1 + \frac{n!}{(n-1)!\,1!} x^1 + \frac{n!}{(n-2)!\,2!} x^2 + \dots$$

$$= 1 + \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}\,1!} x + \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!} x^2 + \dots$$

$$= 1 + nx + \frac{n(n-1)}{2} x^2 + \dots$$

$$(1+x)^{1/2} = 1 + x/2 + \frac{1/2\,(1/2 - 1)}{2} x^2 + \dots$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$$

* SKIN DEPTH AND POLARISATION :-

SKIN DEPTH / DEPTH OF PENETRATION :

When an EM wave is propagating in conducting medium It gets attenuated. As frequency ↑ the rate of attenuation will also ↑ ($\because \alpha = \omega\sqrt{\mu\varepsilon}$). The EM wave penetrates only a very small depth before it gets attenuated to a negligible value.

- Depth of Penetration is defined as the distance at which wave attenuates to $1/e$ times it's maximum Value. Approximately it is 37% of its max Amp. It is also defined as the reciprocal of attenuation const skindepth (δ) = $1/\alpha$

consider an EM wave whose el-field is given by

$\bar{E} = E_0 e^{-\alpha z}$ at skindepth el- field is 37%

of its max value (i.e $1/e$ times)

$\therefore$ at $z = \delta$

$$\bar{E} = E_0 e^{-\alpha \delta}$$

$$= E_0 \cdot e^{-\alpha \delta}$$

$$\frac{\bar{E}}{E_0} = e^{-\alpha \delta}$$

$$= 1/e = e^{-1} = e^{-\alpha \delta}$$

$$\alpha \delta = 1$$

$$\alpha = 1/\delta \Rightarrow \delta = 1/\alpha$$

for lossy medium skindepth is given by.

$$\delta = \frac{1}{\alpha} = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)}}$$

-for [good conductors] dielectric.

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\delta = 1/\alpha$$

$$= \frac{1}{\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}$$

-for goodconductors:

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

POLARISATION OF EM WAVE:

It
electric field strength $\bar{E}$ at a given point in space.

Depending on direction / orientation of el field there are 3 types of polarisations.

i) Linear polarisation
ii) elliptical
iii) circular

i) Linear polarisation:

- A wave is said to be linearly polarised if el-field strength remains along straight line at a given point in space.

linear polarisation
- → horizontal polarisation
- → vertical polarisation

The el-field vector in time domain is given by

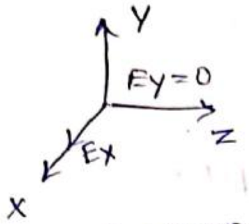
$$\bar{E}(z,t) = \bar{E}_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_x + \bar{E}_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_y$$

$$\bar{E}(z,t) = E_x \bar{a}_x + E_y \bar{a}_y$$

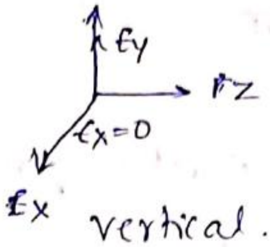
∴ electric field has 2 components E_x and E_y

if E_x is present and E_y is 0 the wave is said to be polarised in X direction, this phenomenon is called horizontal polarisation

llorly E_y is Present and $E_x = 0$ → vertical polarisation
wave is polarised in Y direction

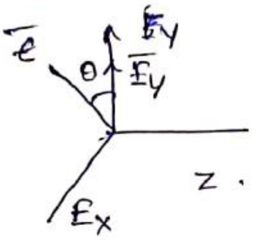


horizontal



vertical.

If Ex, Ey both exists



Polarised wave in X & Y

energy densities $w_E = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i$

charge densities for line charge distibution

$$w_E = \frac{1}{2}\int \rho_v \cdot V \cdot \overline{dL}$$

for surface charge $w_E = \frac{1}{2}\int \rho_s \cdot V ds$

volume " $w_E = \frac{1}{2}\int \rho_v V dv$

current density $\overline{J} = \frac{\Delta I}{\Delta S}$

total current $I = \int_s \overline{J} \cdot \overline{ds}$

Relation b/w $\overline{J}$ & ρ_v

$$\overline{J} = \rho_v \cdot v_x$$

continuity eq^n

$$-\frac{d}{dt}\left(\int_v \rho_v dv\right) = \int_s \overline{J} \cdot \overline{ds}$$

continuity eq^n of differential form

$$\frac{-d\rho_v}{dt} = \nabla \overline{J}$$

Poissons eqn $\nabla^2 V = \frac{-\rho_s}{2\epsilon_0}$

if $\nabla^2 V = 0$ then it is Laplaced eq^n

capacitance of 11el plate capacitor

$$C = \frac{Q}{V} = \frac{S\epsilon_0}{d}$$

capacitance of coaxial capacitor

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 l}{\log(b/a)}$$

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surface charge distribution:

$$V = \frac{\int_S \rho_s \, ds}{4\pi\epsilon r}$$

volume charge distribution

$$V = \frac{\int_v \rho_v \, dv}{4\pi\epsilon r}$$

Maxwell's 2nd equation: / Relation b/w $\bar{E}$ and V.

$$V_{AB} = -V_{BA}.$$

$$V_{AB} + V_{BA} = 0$$

$$\oint \bar{E} \cdot dL = 0$$

stoke's theorem

$$\oint_L \bar{E} \cdot dL = \int_S (\nabla \times \bar{E}) \bar{ds}$$

$$\int_S (\nabla \times \bar{E}) \bar{ds} = 0 \rightarrow \text{integral form}$$

$$\nabla \times \bar{E} = 0 \rightarrow \text{differential form.}$$

$\bar{E}$ and $\bar{V}$ relation:

$$V = \int \bar{E} \cdot \bar{dL}$$

$$dv = -\bar{E} \cdot \bar{dL}$$

$$\nabla V = \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} \rightarrow ①$$

$$-\bar{E} \cdot \bar{dL} = -[\bar{E}_x \, dx \, \bar{a}_x + \bar{E}_y \, dy \, \bar{a}_y + \bar{E}_z \, dz \, \bar{a}_z] \rightarrow ②$$

① & ②

$$\frac{\partial V}{\partial x} = -\cancel{\bar{E}_x dx \bar{a}_x} \qquad \frac{\partial V}{\partial x} = -\bar{E}_x$$

$$\frac{dv}{dy} = -\bar{E}_y \qquad \frac{dv}{dz} = -\bar{E}_z$$

$$\bar{E} = \bar{E}_x + \bar{E}_y + \bar{E}_z$$

above eqns are generalised as $\bar{E} = -\nabla V$

V potential:

$\to$ scalar

$V = \omega/Q$

(J/C)

$$\overline{E} = \frac{\overline{F}}{Q}$$

$$d\omega = -\overline{F}\cdot\overline{dL}$$

$$= -\overline{E}Q\cdot\overline{dL}$$

$$\omega = -\int_A^B \overline{E}\, Q\, dL$$

$$\frac{\omega}{Q} = -\int_A^B \overline{E}\cdot dL$$

$$V_{AB} = -\int_A^B \overline{E}\cdot dL$$

$$\overline{a_r}\cdot\overline{a_r} = 1$$
$$\overline{a_r}\cdot\overline{a_\theta} = 0$$

$$\nabla\cdot\overline{A} = \mathrm{Div}(\overline{A})$$
$$\nabla\times\overline{A} = \mathrm{curl}(\overline{A})$$
$$\nabla\cdot A = \text{gradient}(A)$$

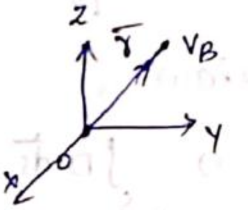
V_{AB} = Potential for displacing point charge from A to B

$$V_{AB} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \Rightarrow V_{AB} = V_B - V_A$$

charge moved from '∞' to 'B' the $V_B = V_{AB} = \dfrac{Q}{4\pi\epsilon r_B}$.

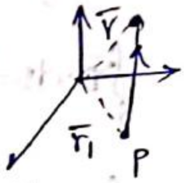
absolute potential

$$V_r = \frac{Q}{4\pi\epsilon r}$$



(Point at origin)

$$V_p = \frac{Q}{4\pi\epsilon|\overline{r}-\overline{r_1}|}$$



(Point not at origin

for n charges

$$V = \frac{1}{4\pi\epsilon}\sum_{i=1}^{n}\frac{Q_i}{|\overline{r}-\overline{r_i}|}$$

V for line charge distribution:

$$V = \int\frac{\rho_L\cdot dL}{4\pi\epsilon r} \cdot \text{(or)} \int\frac{\rho_L\, dL}{4\pi\epsilon|\overline{r}-\overline{r_1}|}$$

if point is not at origin

$\bar{D} = \epsilon \bar{E}$

for infinite line charge $\bar{D} = \frac{\rho_L}{2\pi r} \bar{a_r}$

$\left(\because \bar{E} = \frac{\rho_L}{2\pi r \epsilon} \bar{a}_r\right)$

charged circular ring:

$\bar{D} = \frac{\rho_L r \cdot \bar{a_r}}{2(r^2+z^2)^{3/2}}$

infinite sheet : $\bar{D} = \frac{\rho_s}{2} \bar{a_r}$

Guass law:

ψ = total charge enclosed by surface.

$\bar{D} = \frac{\partial \psi}{\partial s}$

$\psi = \int_S \bar{D} \cdot \bar{ds} \rightarrow (1)$

$Q = \int_V \rho_v \, dv \rightarrow (2)$

Divergence theorem $\rightarrow \left[\int_S \bar{A} \cdot \bar{ds} = \int_V \nabla \cdot A \, \bar{dv}\right]$

from guass law

$\psi = Q \Rightarrow \int_S \bar{D} \cdot \bar{ds} = \int_V \rho_v dv \rightarrow (3)$

$\int_S \bar{D} \cdot ds = \int_V \nabla \cdot \bar{D} \, dv = \int_V \rho_v dv$

$\rho_v = \nabla \cdot \bar{D}$

Maxwell's 1st eqn : differential form $\rho_v = \nabla \cdot \bar{D}$

integral " $\int_S \bar{D} \cdot \bar{ds} = \int_V \rho_v \, dv$

$\bar{D}$ for uniformly charged sphere:

if $r > a$ → sphere. $\bar{D} = \frac{\rho_v a^3}{3r^2} \bar{a}_r$

quassian surface.

if $r < a$ $\bar{D} = \frac{\rho_v r}{3} \bar{a}_r$

Electric field strength / Electric field intensity.

$$\bar{E} = \frac{1}{4\pi\epsilon}\frac{Q_1}{|\bar{R}_{12}|^2}\bar{a}_{12} \quad \text{(units N/c)}$$

$$\bar{E} = \frac{\bar{F}}{Q_2}$$

'n' charges

$$\bar{E} = \frac{1}{4\pi\epsilon}\left[\sum_{i=1}^{n}\frac{Q_i}{|\bar{R}_{ip}|^2}\bar{a}_{ip}\right]$$

$$\rho_L = \frac{Q}{L} \qquad \rho_s = \frac{Q}{S} \qquad \rho_V = \frac{Q}{V}$$

Electric field intensity due to an infinite line charge

$$\bar{E} = \frac{\rho_L}{2\pi r\epsilon}\bar{a}_y \quad \text{N/c or V/m}$$

charged circular ring

$$\bar{E} = \frac{\rho_L z\bar{a}_z}{2\epsilon(r^2+z^2)^{3/2}} \quad \text{N/c (or) V/m.}$$

infinite sheet $\rightarrow \frac{\rho_s}{2\epsilon_0}\bar{a}_z = \bar{E}$

Flux (ϕ, ψ)

$$\psi = Q.$$

Electric flux density ($\bar{D}$)

$$\bar{D} = \frac{d\psi}{ds} \quad (\text{c/m}^2)$$

$$\bar{D} = \psi/s$$

$$\psi = Q$$

so $\bar{D} = \frac{Q}{s}$

for sphere $\bar{D} = \frac{Q}{4\pi r^2}\bar{a}_n \ \text{c/m}^2$

differential length

$$d\bar{L} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

differential surface area normal to x direction

$$\bar{ds}_x = dy\,dz \cdot \bar{a}_x$$

perfect dielectric → free space.

Lossy diel → conducting medium

differential volume

$$\bar{dv} = dx\,dy\,dz$$

cylindrical coordinate system

differential length in 'r' direction $\bar{dL}_r = dr\,\bar{a}_r$

'θ' $\bar{dL}_\theta = r\,d\theta \cdot \bar{a}_\theta$

'z' $\bar{dL}_z = dz\,\bar{a}_z$

surface area $\bar{ds}_r = r\,d\theta \cdot dz\,\bar{a}_r$

$\bar{ds}_\theta = dr\,dz\,\bar{a}_\theta$

$\bar{ds}_z = r\,d\theta\,dr\,\bar{a}_z$

volume $\bar{dv} = r\,d\theta\,dr\,dz$

$$\boxed{\begin{aligned} &\nabla \times (\nabla \times A) \\ &= \nabla(\nabla \cdot \bar{A}) - \nabla^2 A \\ &\nabla \cdot (\nabla \times \bar{A}) = 0 \end{aligned}}$$

Sph spherical

$$\bar{dL}_r = dr\,\bar{a}_r$$
$$\bar{dL}_\theta = r\,d\theta \cdot \bar{a}_\theta$$
$$\bar{dL}_\phi = r\sin\phi\,d\phi\,\bar{a}_\phi$$

coloumbs law:

$$\oint_S D \cdot ds = \int_V \nabla \cdot D\,dv$$

$$\oint_L \bar{E} \cdot dL = \int_S \nabla \times \bar{E}\,ds$$

$$F \propto \frac{q_1 q_2}{r^2} \qquad \epsilon_0 = 8.85 \times 10^{-12}\ F/m$$

$$F = 9 \times 10^9 \frac{Q_1 Q_2}{r^2}$$

vector form :-

$$\bar{F}_2 = \frac{k Q_1 Q_2}{|\bar{R}_{12}|}\,\bar{a}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{|\bar{r}_2 - \bar{r}_1|^3} (\bar{r}_2 - \bar{r}_1)$$

for 'n' charges

$$F_0 = \frac{Q_0}{4\pi\epsilon} \left[\sum_{i=1}^{n} \frac{Q_i}{|R_{i0}|^2}\,\bar{a}_{i0} \right]$$

1) Define Mag fl strength and derive an exp for diff mag fl strength due to a current carrying conductor. (BIOT SAVART'S LAW)

2) using amp ckt law derive exp for mag fl strngth due to an inf line of current due to an inf sheet of current

3) Four Maxwell eqns for EMagnetics with their word statements.

Conversion Formulae

4) derive an exp for generator & transformer emf from Faraday's law.

5) define current density / disp^n density. inconsistency of amp ckt law for time varying fields explain.

Original Unit	Converted Unit	Multiplication Factor	Original Unit	Converted Unit	Multiplication Factor
LENGTH			**VOLUME**		
Inches	Centimeters	2.541	Cubic Centimeters	Cubic Inches	16.387
Yards	Meters	0.9144	Cubic Meters	Cubic Feet	35.3144
Miles	Kilometers	1.6093 cm	Cubic Meters	Cubic Yards	1.3081
			Liters	Gallons	0.22
AREA			**MASS**		
Sq. Inches	Sq. Centimeters	6.4516	Pounds	Kilograms	0.4536
Sq. Meters	Sq. Feet	10.7639	Pounds	Tonnes	.0004536