16/1/2019 UNIT-I - NETWORK TOPOLOGY Tuesday

→ In a network topology, only geometrical structure of a network is considered.

→ Basically, a network is an interconnection of various elements which are either active or passive

→ In the network topology, only geometrical pattern is considered but not diff. network elements.

* Elements in network topology:

1. Node: It is defined as the pt at which two or more elements are having common connection.
2. Branch: It is a line connecting a pair of nodes, the line representing a single element and series of connected elements.
3. loop: When there is more than 1 path b/w 2 nodes there it is a circuit or loop.
4. Mesh: It is a loop which does not contain any other loops in it.
5. Linear graph: It is defined as collection of nodes and branches The nodes are joined together by branches.

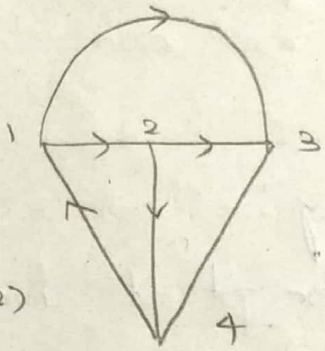
* Graphs:

1. Oriented graph: If every branch of a graph has a direction or each element in the connected

graph is given direction, it is called oriented or directed graph.

→ Otherwise

2. <u>Planar graph</u>: A graph is said to be planar if it can be drawn on a plane surface such that no 2 branches cross each other



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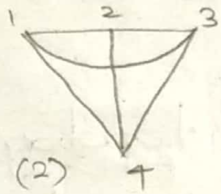

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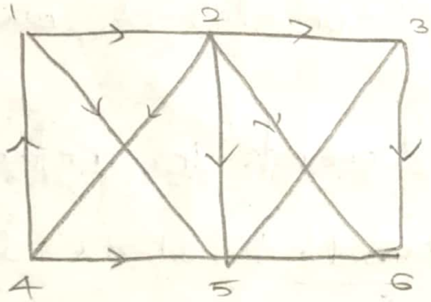


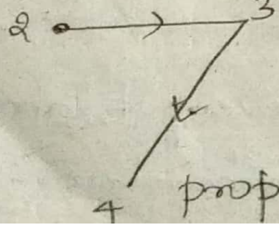
(3)

3. <u>Non-planar graph</u>: A non-planar graph is a graph drawn on 2D plane such that 2 or more branches intersect each other at a pt other than a node.

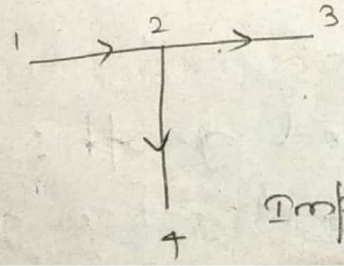
4. <u>Sub graph</u>: It is a subset of branches & nodes of a graph. There are 2 types of subgraphs.

→ If the subgraph contains branches & nodes less in no. then it is called <u>proper subgraph</u>.

→ If the subgraph contains branches & nodes in less no. & all nodes respec. is called <u>improper subgraph</u>.



proper



Improper

5. <u>Path</u> : A path is an improper subgraph.

→ At terminating node, only 1 branch is incident, at the remaining nodes 2 branches are incident

6. <u>Connected graph</u> : A graph is said to be connected if there exists a path b/w any pair of nodes.

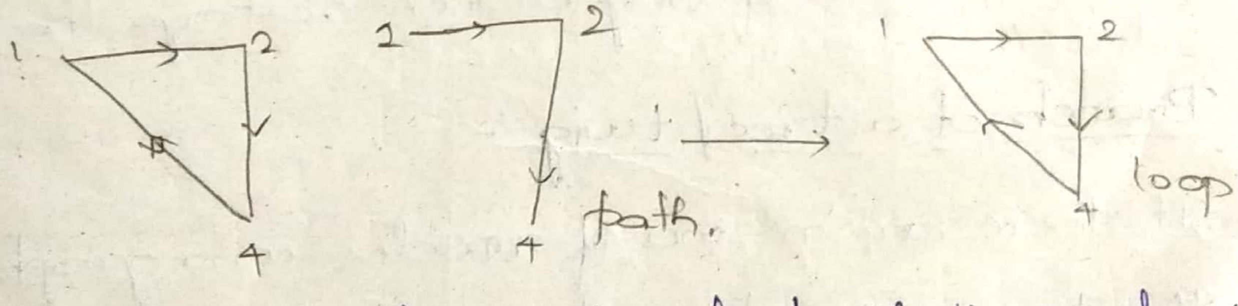
7. <u>loop or circuit</u> : It is a connected subgraph of connected graph at each node of which are incident exact at 2 branches.

→ If 2 terminals of a path are made to coincide it result a loop of circuit.

→ There are atleast 2 branches in a loop.

→ There are exactly 2 path b/w any pair of nodes in a loop.

→ The max. no. of possible branches is = to the no. of nodes.



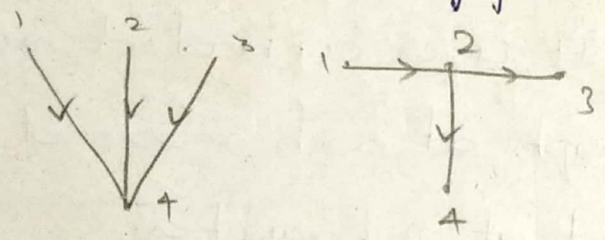
→ A node & branch are incident if the node is a terminal of branch.

→ Nodes can be incident to 1 or more elements.

→ The no. of branches incident to the node indicate the degree of node.

8. <u>Tree</u>: A tree is a connected subgraph of network which consists of all the nodes of original graph but no closed path.

→ Tree contains all the nodes of graph, there exists only 1 path b/w any pair of nodes.



→ If 'n' is the no. of nodes of graph there are n-1 branches in the tree.

→ The tree does not contain any loop.

→ Every connected graph has atleast 1 tree.

→ The min. terminal nodes in a tree are two.

→ A tree is a set of branches with every node connected to every other node in such a way that removal of any branch destroys the property.

9. <u>Branch of a tree</u> / <u>twig</u> :-

→ If there are 'n' no. of nodes in a graph then the tree of graph contains n-1 twigs.

10. <u>Cotree</u>:

→ The left over branches after forming a tree constitute a cotree.

→ Branch of cotree is called chord / link.

→ A set of branches forming a compliment of tree is called cotree.

→ The no. of branches of a cotree is $b-(n-1)$ where b is no. of branches of graph.

→ If there exists 'n' no of nodes/vertices then rank of a graph $r = n-1$

* <u>Incidence matrix</u> :-

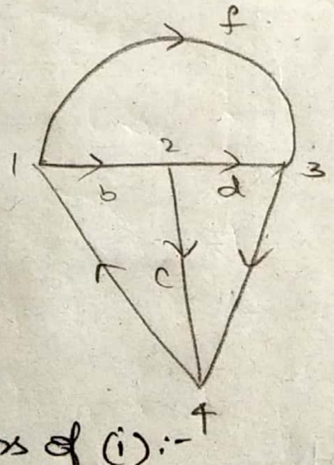
→ An oriented graph can be completely explained or represented with the help of a matrix known as incidence matrix.

→ The incidence matrix gives information abt the branches incident on which nodes, orientation at the node (whether towards/away from node).

→ <u>Types</u> :- (i) complete incidence (A_a)

(ii) Reduced incidence (A)

→



$a_{nk} = +1, 0, -1$

$+1 \Rightarrow$ leaving

$-1 \Rightarrow$ entering

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & f \end{array} \\
\begin{bmatrix}
-1 & +1 & 0 & 0 & 0 & +1 \\
0 & -1 & +1 & +1 & 0 & 0 \\
0 & 0 & 0 & -1 & +1 & -1 \\
+1 & 0 & -1 & 0 & -1 & 0
\end{bmatrix}
\end{array}
$$

Props of (i) :-

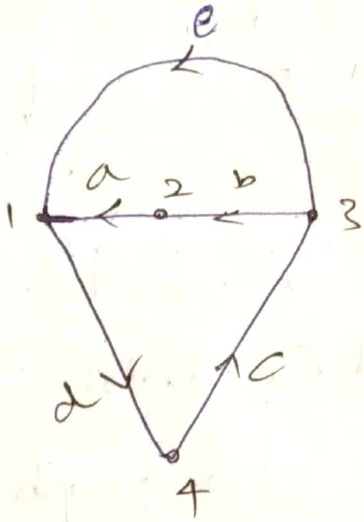
→ The sum of entries in any column is zero

→ The rank of complete incidence matrix of connected graph is n-1.

→ The determinant of a loop of complete incidence matrix is always zero.

→

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \end{array}$$



(ii) Reduced incidence matrix :

→ when any 1 row from the complete incidence matrix is eliminated by using mathematical manipulation then it is called reduced incident matrix.

22/7/19 monday

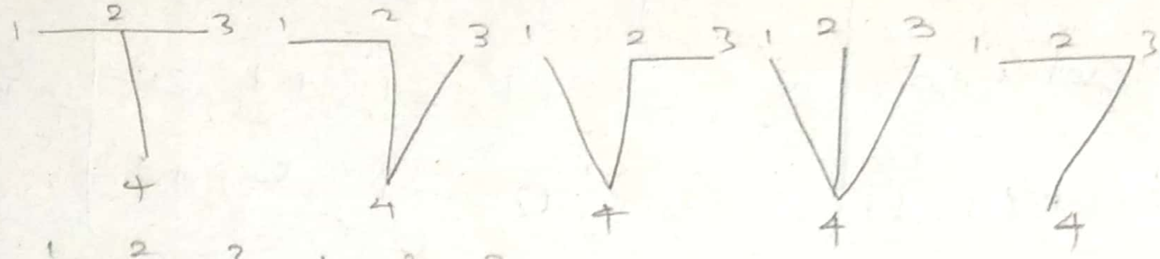
$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} -1 & +1 & 0 & 0 & 0 \\ 0 & -1 & +1 & +1 & 0 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

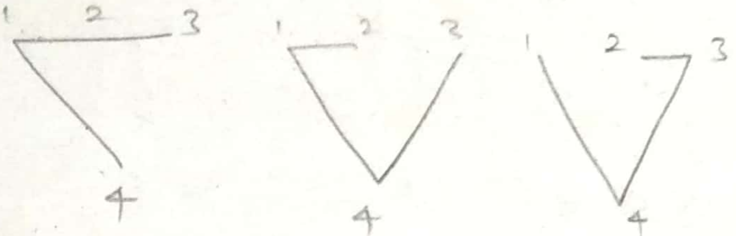
$$A^T = \begin{bmatrix} -1 & 0 & 0 \\ +1 & -1 & 0 \\ 0 & +1 & 0 \\ 0 & +1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\det(AA^T) = 8$$

→ The no. of possible trees is det $|AA^T|$

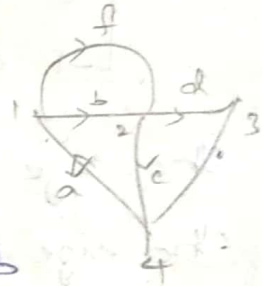
→





From this,
$AI_b = 0$
$V_b = A^T V_n$.

* loop / circuit matrix :- (B_a) $l \times b$



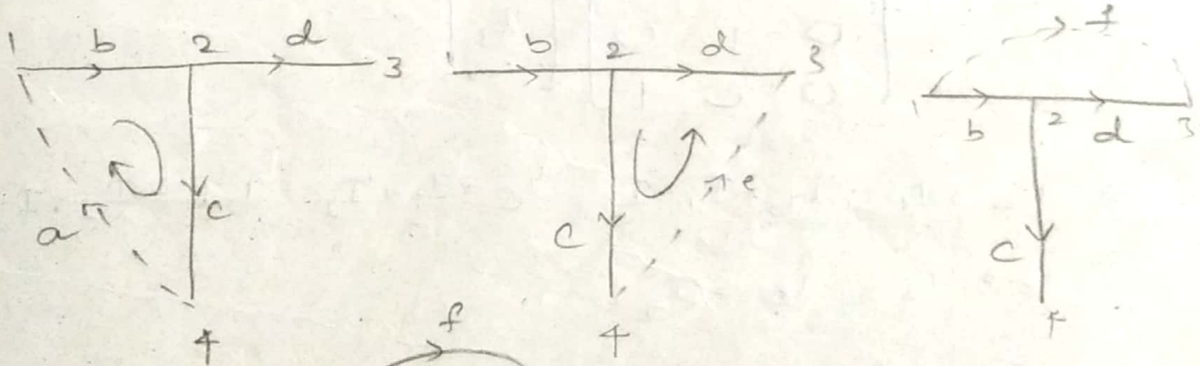
→ $b_{hk} = +1; -1; 0$

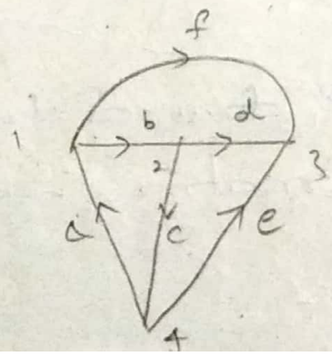
→ $b_{hk} = +1$ → if branch k is in loop & its orientation coincides with loop

-1 → if branch k is in loop & its orientation does not coincide with loop orientation

0 → if the branch k is not in the loop.

* Fundamental circuit matrix / Fundamental tieset :- [B]





$$\begin{array}{l} \\ \overset{I_1}{\text{tieset }(a,b,c)} \\ \overset{I_2}{\text{tieset }(e,c,d)} \\ \overset{I_3}{\text{tieset }(f,b,d)} \end{array} \begin{bmatrix} a & b & c & d & e & f \\ +1 & +1 & +1 & 0 & 0 & 0 \\ 0 & 0 & +1 & -1 & +1 & 0 \\ 0 & -1 & 0 & -1 & 0 & +1 \end{bmatrix} = B$$

→ $B = B_t : B_l$

$$\begin{array}{ccc} b\,c\,d & : & a\,e\,f \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & : & 1 & 0 & 0 \\ 0 & 1 & -1 & : & 0 & 1 & 0 \\ -1 & 0 & -1 & : & 0 & 0 & 1 \end{bmatrix}$$

→ The size f-tieset matrix is $b-(n-1) \times b$ [B]

→ The size of twig matrix $B_T = b-(n-1) \times n-1$

→ The size of link matrix $B_l = b-(n-1) \times b-(n-1)$

→ The branch current $i_b = B^T I_l$ (mesh analysis)

$$i_b = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$\Rightarrow i_a = I_1$; $i_b = I_1 - I_3$ $\quad i_c = I_1 + I_2 \quad i_d = -I_2 - I_3$

$i_e = I_2$; $i_f = I_3$

→ KVL equations can be formed from tieset matrix using rows of tieset matrix, we get

$V_a + V_b + V_c = 0$

$V_c - V_d + V_e = 0$

$-V_b - V_d + V_f = 0$ $\boxed{BV_b = 0}$

→ $\sum_{k=1}^{b} v_{ik} = 0$; $\sum_{k=1}^{b} b_{ik} V_{ik} = 0$ $i = 1, \ldots l$.

$b_{ik} = +1 ; -1$

23/7/2019 Tuesday

* Orthogonal relationship :-

→ $A_a B_a^T = A_a^T B_a = 0$

→ $AB^T = A^T B = 0$

* Cut-set matrix

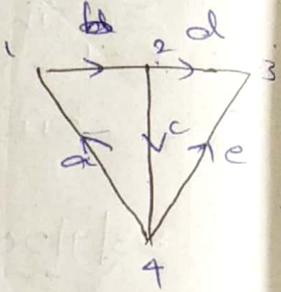
→ A connected graph can be separated into 2 parts by removing certain branches of graph. This is =ent to cutting a graph into 2 parts. This is refered as cut-set.

→ A cut-set is a minimal set of branches of a connected graph such that after removal of these branches, graph gets separated into 2 distinct parts each of which a connected graph, if the condition that replacing any 1 branch from the cut-set makes the graph connected.

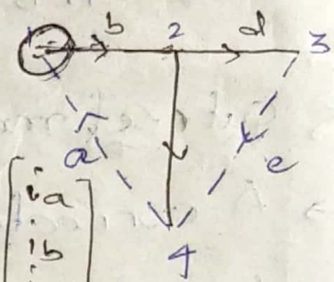
→ cutset matrix is represented by Q_a.

→ The size of cutset matrix is $c \times b$

$$
\begin{array}{l}
 \\ \text{f-cutset }(a,b) \\ \text{fcutset }(b,c,d) \\ \text{fcutset }(d,e) \\ \text{fcutset }(a,c,e) \\ \text{fcutset }(b,c,e) \\ \text{fcutset }(a,c,d)
\end{array}
\begin{array}{c}
\begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{bmatrix}
+1 & -1 & 0 & 0 & 0 \\
0 & +1 & -1 & -1 & 0 \\
0 & 0 & 0 & +1 & +1 \\
-1 & 0 & +1 & 0 & -1 \\
0 & -1 & +1 & 0 & -1 \\
+1 & 0 & -1 & -1 & 0
\end{bmatrix}
\end{array}
$$



→



$$
\begin{array}{l}
 \\ V_1 \ \text{fcutset}_1(b,a) \\ V_3 \ \text{fcutset}_2(d,e) \\ V_4 \ \text{fcutset}_3(c,a,e)
\end{array}
\begin{array}{c}
\begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{bmatrix}
-1 & +1 & 0 & 0 & 0 \\
0 & 0 & 0 & +1 & -1 \\
-1 & 0 & +1 & 0 & +1
\end{bmatrix}
\end{array}
\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \end{bmatrix}
$$

→ The above matrix is denoted by Q.

→ The size of Q is $n-1 \times b$ (nodal analysis)

→ $Q = Q_T : Q_L = [U : Q_L]$

$$
= \begin{array}{ccccc}
b & d & c & : & a \quad e \\
1 & 0 & 0 & & -1 \quad 0 \\
0 & 1 & 0 & & 0 \quad -1 \\
0 & 0 & 1 & & -1 \quad +1
\end{array}
$$

$\boxed{Q I_b = 0}$

→ The size of Q_T is $(n-1) \times (n-1)$

→ The size of Q_L is $(n-1) \times b-(n-1)$

→ $V_b = Q^T V_t$

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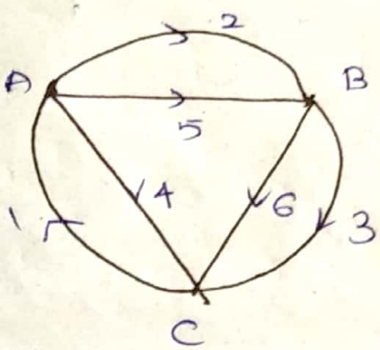
→ we get KCL equations from this, i.e.,

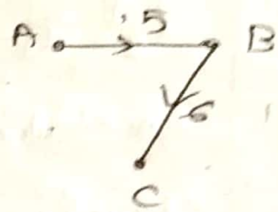
$$\left.\begin{aligned} -i_a + i_b &= 0 \\ i_d - i_e &= 0 \\ -i_a + i_c + i_c &= 0 \end{aligned}\right\} \text{KCL}$$

→ $V_a = -V_1 - V_4$; $V_b = V_1$; $V_c = V_4$; $V_d = V_3$; $V_e = V_4 - V_3$

Q.



find CIM, RIM, no. of possible trees, funda tieset, funda-cut-set. Consider



26/7/2019

Friday

Sol: CIM: $n \times b$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} -1 & +1 & 0 & +1 & +1 & 0 \\ 0 & -1 & +1 & 0 & -1 & +1 \\ +1 & 0 & -1 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

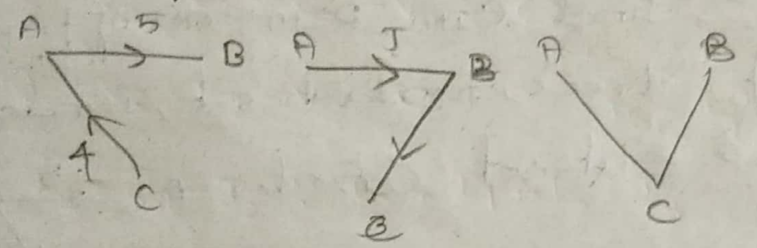
$$\left(\begin{bmatrix} -1 & +1 & 0 & +1 & +1 & 0 \\ 0 & -1 & +1 & 0 & -1 & +1 \\ +1 & 0 & -1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & +1 \\ +1 & -1 & 0 \\ 0 & +1 & -1 \\ +1 & 0 & -1 \\ +1 & -1 & 0 \\ 0 & +1 & -1 \end{bmatrix} \right.$$

$= 1+1+1+1 \quad -1-1 \quad -1-1 \;) \times$

-1

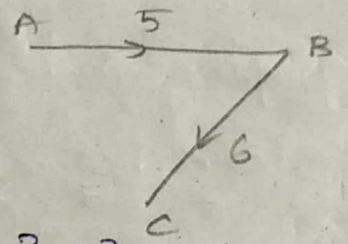
$$\left|\begin{bmatrix} -1 & +1 & 0 & +1 & +1 & 0 \\ 0 & -1 & +1 & 0 & -1 & +1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ +1 & -1 \\ 0 & +1 \\ +1 & 0 \\ +1 & -1 \\ 0 & +1 \end{bmatrix}\right| = 12.$$

No. of possible trees = 12



Fundamental tieset :-

Consider a tree



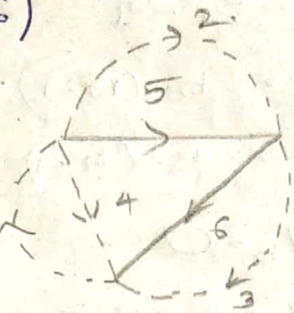
$$\begin{array}{l} \\ \text{tieset}(1,5,6) \\ \text{tieset}(2,5,6) \\ \text{tieset}(3,5,6) \\ \text{tieset}(4,5,6) \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & 0 & +1 & +1 \\ 0 & +1 & 0 & 0 & -1 & 0 \\ 0 & 0 & +1 & 0 & 0 & -1 \\ 0 & 0 & 0 & +1 & -1 & -1 \end{bmatrix} \end{array}$$

$B_T : B_l$

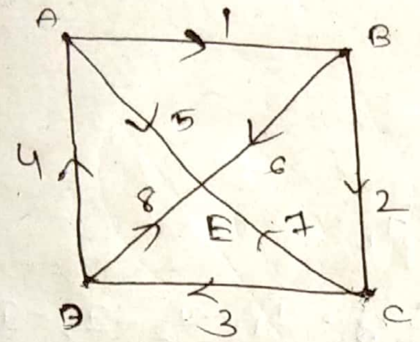
5	6	:	1	2	3	4
1	1		1	0	0	0
-1	0		0	1	0	0
0	-1		0	0	1	0
-1	-1		0	0	0	1

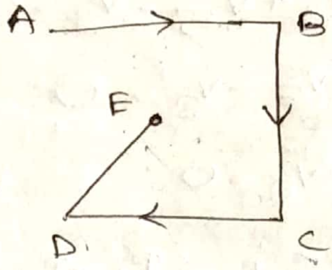
Fundamental cutset :- Q (2 × 6)

$$
\begin{array}{c} \\ CS(5,1,2,4) \\ CS(6,1,3,4) \end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix} -1 & +1 & 0 & +1 & +1 & 0 \\ -1 & 0 & +1 & +1 & 0 & -1 \end{bmatrix}
\end{array}
$$



(ii)





30/7/2019
Tuesday

→ Consider this tree

CIM: n×b = 5×8 (A_a)

$$
\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
A & +1 & 0 & 0 & -1 & +1 & 0 & 0 & 0 \\
B & -1 & +1 & 0 & 0 & 0 & +1 & 0 & 0 \\
C & 0 & -1 & +1 & 0 & 0 & 0 & +1 & 0 \\
D & 0 & 0 & -1 & +1 & 0 & 0 & 0 & +1 \\
E & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1
\end{array}
$$

RIM: n-1×b = 4 × 8 (A)

$\det|AA^T| = 45$

Fundamental tieset : b-(n-1)×b
(B)

$$
\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
I_1 \text{ tieset1}(4,1,2,3,8) & +1 & +1 & +1 & +1 & 0 & 0 & 0 & 0 \\
I_2 \text{ tieset 2}(5,1,2,3,8) & -1 & -1 & -1 & 0 & +1 & 0 & 0 & -1 \\
I_3 \text{ tieset3}(6,1,2,3,8) & 0 & -1 & -1 & 0 & 0 & +1 & 0 & -1 \\
I_4 \text{ tieset4}(7,1,2,3,8) & 0 & 0 & -1 & 0 & 0 & 0 & +1 & -1
\end{array}
$$

$B = B_T \vdots B_l.$

$B_T = b-(n-1) \times n-1 \longrightarrow 4 \times 4$

$B_l = b-(n-1) \times b-(n-1) \longrightarrow 4 \times 4$

$BV_b = 0$

$\Rightarrow v_1 + v_2 + v_3 + v_4 = 0$

$\Rightarrow -v_1 - v_2 - v_3 + v_5 - v_8 = 0$

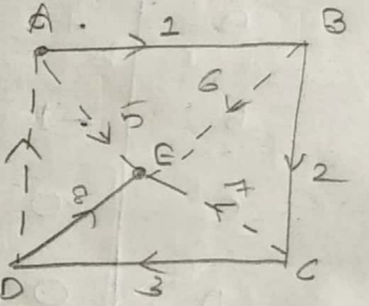
$\Rightarrow -v_2 - v_3 + v_6 - v_8 = 0$

$\Rightarrow -v_3 + v_7 - v_8 = 0$ $\qquad i_b = B^T I_L$

$i_1 = I_1 - I_2$; $i_2 = I_1 - I_2 - I_3$; $i_3 = I_1 - I_2 - I_3 - I_4$

$i_4 = I_1$; $i_5 = I_2$; $i_6 = I_3$; $i_7 = I_4$; $i_8 = -I_2 - I_3 - I_4$

Fundamental cutset: $Q\ (n-1) \times b = 4 \times 8$



$$
\begin{array}{l}
 \\
v_1 \text{ cutset } 1(1,4,5) \\
v_8 \text{ cutset } 2(8,5,6,7) \\
v_2 \text{ cutset } 3(2,4,5,6) \\
v_2 \text{ cutset } 4(3,4,5,6,7)
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\left[\begin{array}{cccccccc}
+1 & 0 & 0 & -1 & +1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & +1 & +1 & +1 & +1 \\
0 & +1 & 0 & -1 & +1 & +1 & 0 & 0 \\
0 & 0 & +1 & -1 & +1 & +1 & +1 & 0
\end{array}\right]
\end{array}
$$

$Q_T : Q_L \Rightarrow I : Q_L$

$Q_T \Rightarrow (n-1) \times (n-1) \Rightarrow 4 \times 4$

$Q_L \rightarrow (n-1) \times b-(n-1) \Rightarrow 4 \times 4$

Q $I_b = 0$

$i_1 - i_4 + i_5 = 0$; $i_5 + i_6 + i_7 + i_8 = 0$

$i_2 - i_4 + i_5 + i_6 = 0$; $i_3 - i_4 + i_5 + i_6 + i_7 = 0$

~~$v_1 = V_1$; $v_2 = V_3$; $v_3 = V_4$;~~

$v_1 = V_1$; $v_2 = V_2$; $v_3 = V_3$; $v_4 = -V_1 - V_2 - V_3$

$v_5 = V_1 + V_2 + V_3 + V_4$; $v_6 = V_2 + V_3 + V_8$; $v_7 = V_3 + V_8$

$v_8 = V_8$.

2/8/2019 Part-2 Magnetic Circuits Friday

* Coupled circuits :

→ When energy transfer takes place from one to another without any electrical connection.

→ Coupled circuits are used in network analysis and synthesis. Eg/. Transformer and gyrator.

→ The requirements for induction –

(i) Coil or conductor.

(ii) permanent magnet or electromagnet.

(iii) Relative motion b/w coil & magnet (acheived by moving coil w.r.t flux, moving magnet w.r.t coil).

→ The phenomena of cutting flux lines by the conductor to get the induced emf is called electromagnetic induction.

→ Faraday 1st law :- Whenever the no. of magnetic lines of force (flux) linking with a coil changes, an emf gets induced in that coil or conductor.

→ Farady's 2nd law: The magnitude of induced emf is directly α to rate of change of flux linkages.

Flux linkage = $N \times \phi$

$t = t_1, \phi_1 \Rightarrow N\phi_1$

$t = t_2, \phi_2 \Rightarrow N\phi_2$

$$e \propto \frac{N\phi_2 - N\phi_1}{t_2 - t_1}$$

$$e \propto \frac{Nd\phi}{dt} \Rightarrow e = KN\frac{d\phi}{dt} \Rightarrow e = -N\frac{d\phi}{dt} \quad [\because K = 1]$$

→ $\phi \propto I$

$\phi = kI$

$$e = -N\frac{d\phi}{dt}$$

$$e = -Nk\frac{dI}{dt} \Rightarrow e = -\left(\frac{N\phi}{I}\right)\frac{dI}{dt} \rightarrow e = -L\frac{dI}{dt}$$

L = coefficient of self inductance.

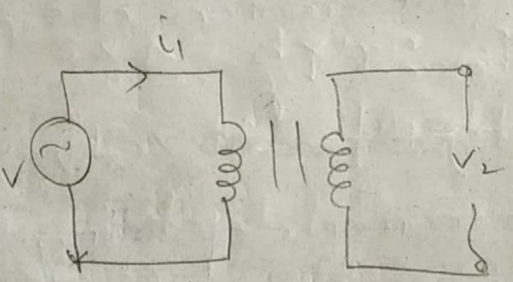
→ $\phi_{12} \propto I_1$

$$\phi_{12} = kI_1 \Rightarrow k = \frac{\phi_{12}}{I_1}$$

$$e = -N\frac{d\phi}{dt}$$

$$V_2 = -N_2\frac{d\phi_{12}}{dt}$$

$$= -N_2 k\frac{dI_1}{dt}$$



$$V_2 = -\frac{N_2\phi_{12}}{I_1}\frac{dI_1}{dt}$$

$$V_2 = -M_{12}\frac{dI_1}{dt} \quad \text{|||ly}$$

$$V_1 = -M_{21}\frac{dI_2}{dt}$$ [M_{12}, M_{21} are mutual inductances]

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$\rightarrow$ $M_{12} = M_{21} = M$

$$M \times M = \frac{N_2 \phi_{12}}{I_1} \times \frac{N_1 \phi_{21}}{I_2}$$

$$= \frac{N_2 K_1 \phi_1}{I_1} \times \frac{N_1 K_2 \phi_2}{I_2} \quad [\because \phi_{12} = K_1 \phi_1 ; \phi_{21} = K_2 \phi_2]$$

$$= K_1 K_2 \frac{N_1 \phi_1}{I_1} \frac{N_2 \phi_2}{I_2}$$

$$M^2 = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2} \sqrt{L_1 L_2}$$

$$M = K \sqrt{L_1 L_2}$$

$$\Rightarrow K = \frac{M}{\sqrt{L_1 L_2}} = \text{coefficient of capling.}$$

Q. When one of magnetically coupled coils has a current of 3A. The resultant fluxes ϕ_{11} & ϕ_{12} are 0.3mWb & 0.5mWb. If $N_1 = 600$, $N_2 = 1200$ find L_1, L_2, M, K.

Sol! $I_1 = 3A$

$\phi_{11} = 0.3 mWb$

$\phi_{12} = 0.5 mWb$

$N_1 = 600$

$N_2 = 1200$

$\phi_1 = \phi_{11} + \phi_{12}$

$= 0.3 + 0.5 = 0.8 mWb$

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{600 \times 0.8 \times 10^{-3}}{3} = \frac{0.6 \times 0.8}{3}$$

$= 0.2 \times 0.8$

$= 0.16$ Henry

$$K = \frac{\phi_{12}}{\phi_1} = \frac{0.5}{0.8} = 0.625$$

$$M = \cancel{1200}\ \frac{N_2 \phi_{12}}{I_1}$$

$$= 200mH$$

$$M = K\sqrt{L_1 L_2}$$

$$L_2 = \frac{M^2}{K^2 L_1} \Rightarrow L_2 = \frac{(200\times10^{-3})^2}{(0.625)^2\times0.16}$$

$$= 640mH$$

Q. If a coil of $L_1 = 100mH$ is magnetically coupled to another coil of $L_2 = 400mH$ ~~in~~ the coefficient of coupling is 0.6. The second coil has 1500 turns. $I_1 = 4\sin 200t$ Amp. Determine the voltage at the coil and flux ϕ_1.

Sol: $L_1 = 100mH$, $L_2 = 400mH$

$K = 0.6$, $I_1 = 4\sin 200t$ Amp.

$N_2 = 1500$

$$M = K\sqrt{L_1 L_2}$$

$$= 0.6\sqrt{100\times10^{-3}\times400\times10^{-3}}$$

$$= 0.6\sqrt{0.1\times0.4}$$

$$= 0.6\sqrt{0.04}$$

$$= 0.6\sqrt{4\times10^{-2}} = 0.6\times10^{-1}\times2$$

$$= 1.2\times10^{-1}$$

$$= 0.12 = 120mH.$$

$$V_2 = M\frac{dI_1}{dt} = 120\times10^{-3}\times\frac{d}{dt}(4\sin 200t)$$

$$= 12\cancel{0}\times1\cancel{0^{-3}}\times4\times2\cancel{00}\times\cos 200t$$

$$= 96\cos 200t$$

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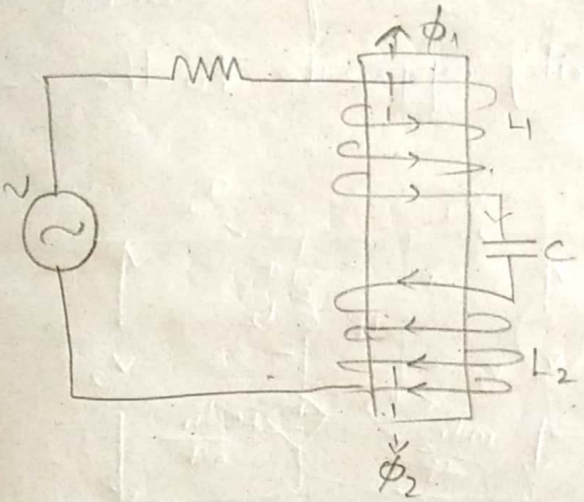
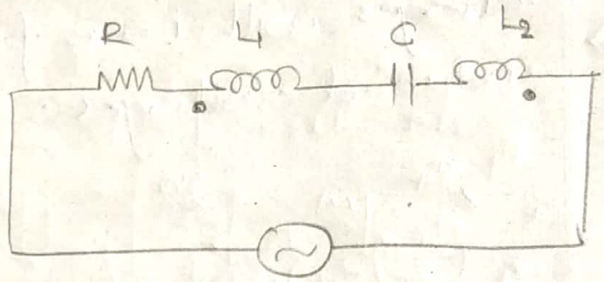
$$M = \frac{N_2 \phi_{12}}{I_1}$$

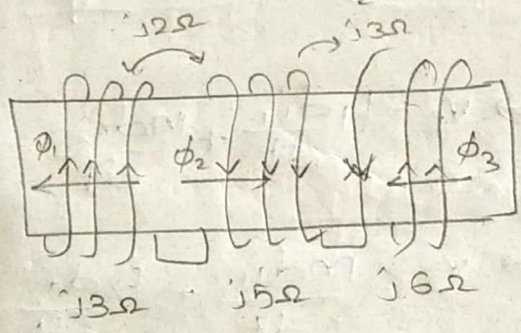
$$M = \frac{N_2 \, K\phi_1}{I_1} \Rightarrow \phi_1 = \frac{M I_1}{N_2 K}$$

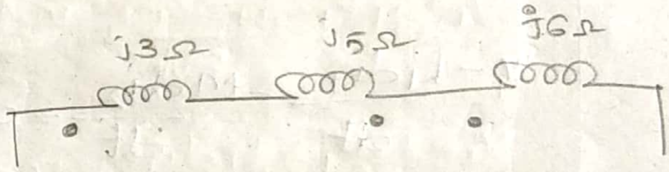
$$\phi_1 = 0.5333 \sin 200t \text{ mWb.}$$

* **Dot convention:-**









→ The sign of mutually induced voltage depends on the winding of coils.

→ But it is very inconvienient to supply the information about winding of coils. Hence, dot conventions are used for indicating direction of coil windings.

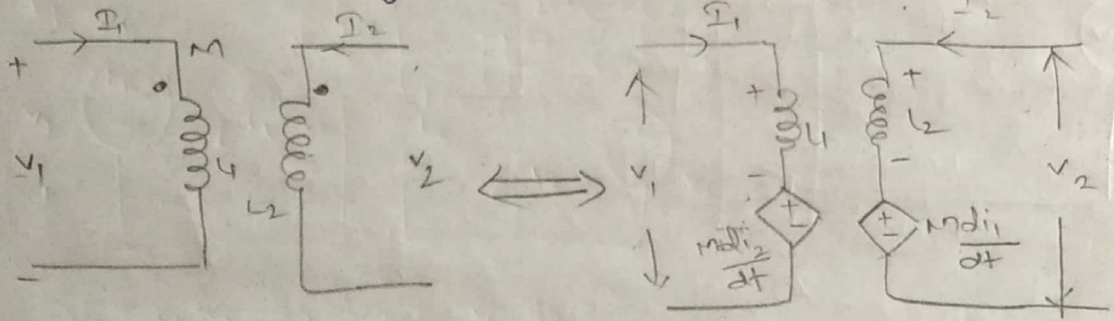
→ If a positive current enters into the dots of both the coils or out of both coils then mutually induced emf of both the coils, add to the self induced voltages. Hence mutually induced voltages will have same polarity as that of self induced voltage.

6/8/2019

Tuesday

→ If the current enters into or out of the dot in 1 coil & in the other coil current flows out of or into the dot then the mutually induced voltages will have polarity opposite to the self induced voltage.

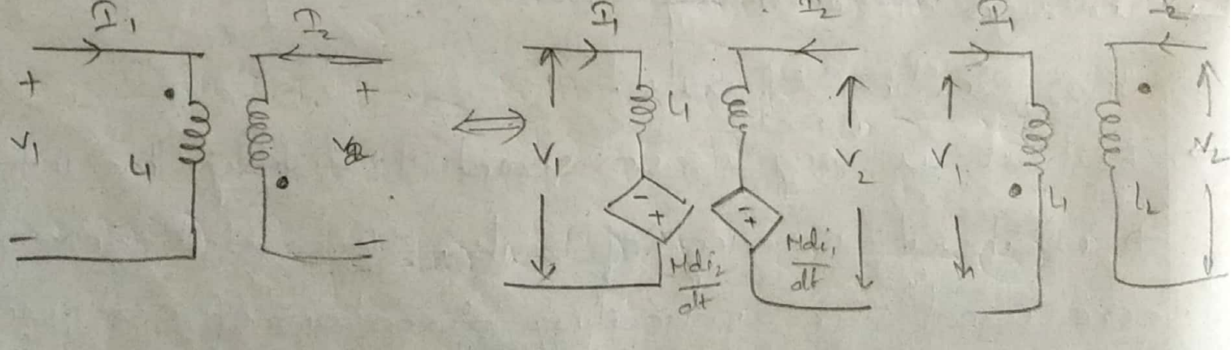
→



$$v_1 - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0 \Rightarrow v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 - L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0 \Rightarrow v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

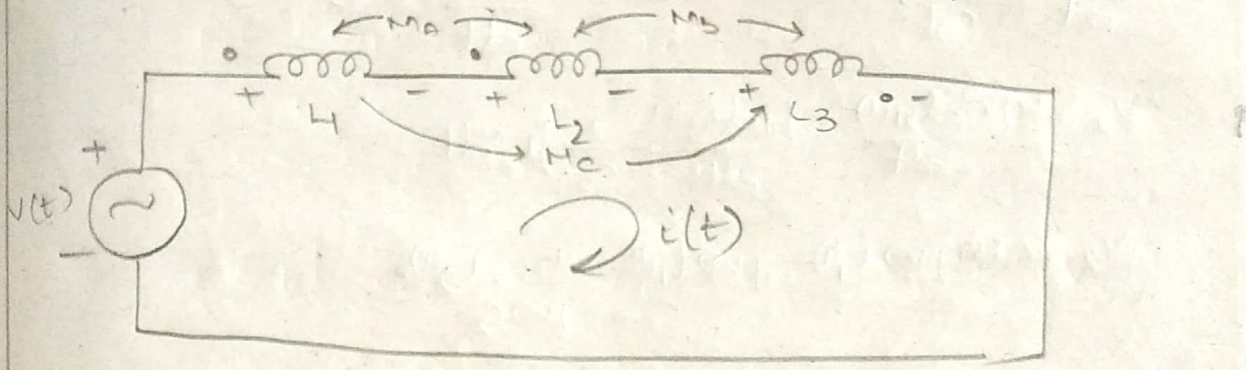
→



$$v_1 - L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = 0 \Rightarrow v_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$v_2 - L_2\frac{di_2}{dt} + M\frac{di_1}{dt} = 0 \Rightarrow v_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$

Q. Write the eq. for the figure showing network.

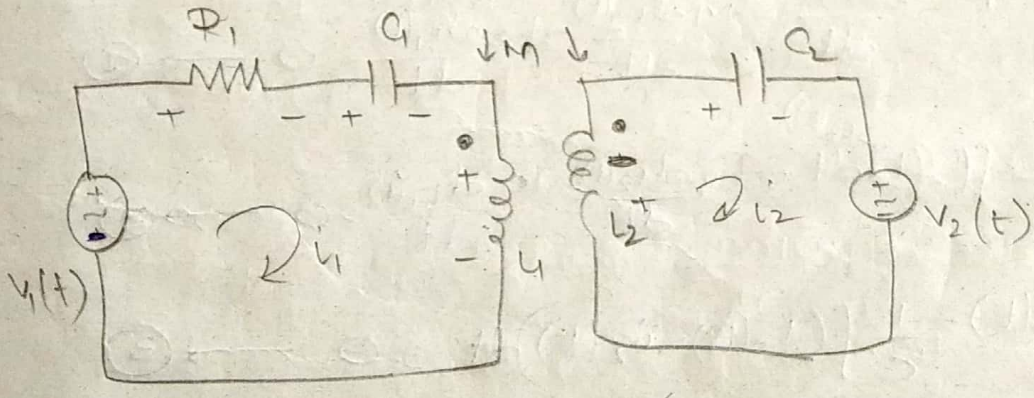


$$v(t) - L_1\frac{di(t)}{dt} - M_A\frac{di(t)}{dt} - L_2\frac{di(t)}{dt} + M_B\frac{di(t)}{dt} - L_3\frac{di(t)}{dt}$$
$$+ M_C\frac{di(t)}{dt} - M_A\frac{di(t)}{dt} + M_B\frac{di(t)}{dt} + M_C\frac{di(t)}{dt} +$$
$$M_B\frac{di(t)}{dt} = 0.$$

$$v(t) - (L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C)\frac{di(t)}{dt}$$

Effective inductance $= L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C$.

Q. Write the loop equations for the network.



$$\cancel{V_1 + i_1R_1}\ V_1 - i_1R_1 - \frac{1}{C_1}\int i_1\,dt - L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = 0$$

$$V_1 - i_1R_1 - \frac{1}{j\omega C_1} - j\omega L_1 i_1 + j\omega M I_2 = 0$$

$$V_1 = \left(R_1 + \frac{1}{j\omega C_1} + j\omega L_1\right)I_1 - j\omega I_2$$

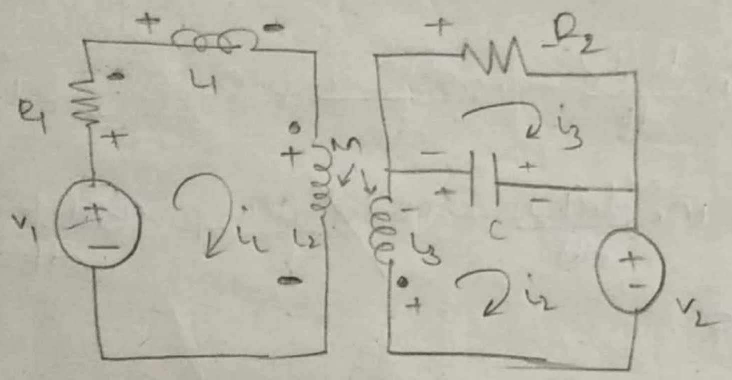
$Q = CV$ $\quad \frac{Q}{V} = C$ $\quad \frac{\int i\,dt}{V}$

$$-L_1\frac{di_2(t)}{dt}-\frac{1}{C_2}\int i_2(t)dt - v_2 + M\frac{di_1(t)}{dt} = 0$$

$$v_2 = M\frac{di_1(t)}{dt} - L_1\frac{di_2(t)}{dt} - \frac{1}{C_2}\int i_2(t)dt.$$

$$v_2 = Mj\omega i_1(t) - j\omega L_1 i_2(t) - \frac{1}{j\omega C_2} i_2(t).$$

Q



$$\cancel{v_1 + i_1R_1 - L_1\frac{di_1}{dt} + M\frac{di_2}{dt}}.$$

$$\cancel{v_1 - i_1R_1 - L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt}}$$

$$v_1 - i_1(t)R_1 - L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} - L_2\frac{di_1(t)}{dt} = 0 \longrightarrow (1)$$

$$-v_2 - \frac{1}{C}\int(i_2(t) - i_3(t))dt - L_3\frac{di_2(t)}{dt} - M\frac{di_1(t)}{dt} = 0 \longrightarrow (2)$$

$$-R_2 i_3(t) - \frac{1}{C}\int(i_3(t) - i_2(t))dt = 0 \longrightarrow (3)$$

9/8/2019

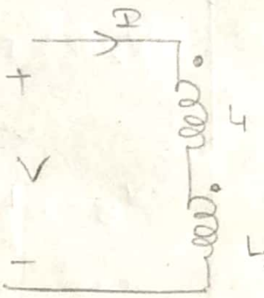
Friday

→ series aiding → series (cumulative capling)

series opposing → series (differential capling)

* Series aiding :-

$I_1 = I_2 = I$ since it is in series.



$$V_1 = -L_1 \frac{dI_1}{dt} = -L_1 \frac{dI}{dt}$$

$$V_2 = -L_2 \frac{dI_2}{dt} = -L_2 \frac{dI}{dt}$$

$$V_3 = -M \frac{dI_2}{dt} = -M \frac{dI}{dt}$$

$$V_4 = -M \frac{dI_2}{dt} = -M \frac{dI}{dt}$$

$$V = V_1 + V_2 + V_3 + V_4$$

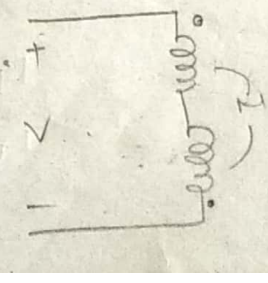
$$= -(L_1 + L_2 + 2M)\frac{dI}{dt} \Rightarrow L_{eff} = L_1 + L_2 + 2M$$

→ Two coils are connected in series and the current through the 2 coils is in same direction inorder to produce flux in the same direction. Such coupling is termed as series aiding.

* Series opposing :-

→ Two coils are connected in series and the current through the 2 coils are in opposite inorder to produce fluxes in opposite direction. Such coupling is termed as series opposing.

$I_1 = I_2 = I$



$$V_1 = -L_1 \frac{dI_1}{dt} = -L_1 \frac{dI}{dt}$$

$$v_2 = -L_2 \frac{dI_2}{dt} = L_2 \frac{dI}{dt}$$

$$v_3 = M\frac{dI_2}{dt} = M\frac{dI}{dt}$$

$$V_4 = M\frac{dI_1}{dt} = M\frac{dI}{dt}.$$

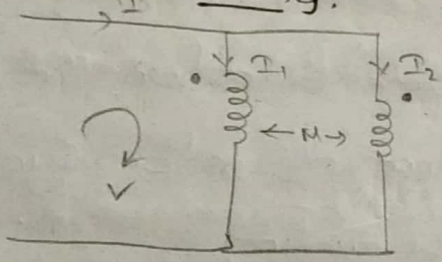
$$v_1 + V_2 + V_3 + V_4 = V$$

$$v = -(L_1 + L_2 - 2M)\frac{dI}{dt}$$

$$L_{eff} = L_1 + L_2 - 2M$$

→ parallel aiding → parallel (cumulative coupling)
parallel opposing → parallel (differential coupling)

* **parallel aiding:**



$$I = I_1 + I_2$$

$$V - L_1 \frac{dI_1}{dt} = 0$$

$$v = j\omega L_1 I_1 + j\omega M I_2 \rightarrow (1)$$

$$v = j\omega L_2 I_2 + j\omega M I_1 \rightarrow (2)$$

(1) = (2)

$$\Rightarrow j\omega L_1 I_1 + j\omega M I_2 = j\omega L_2 I_2 + j\omega M I_1 \longrightarrow (3)$$

$$I = I_1 + I_2 \Rightarrow I_2 = I - I_1 \longrightarrow (4)$$

Sub (4) in (3)

$$j\omega L_1 I_1 + j\omega M(I - I_1) = j\omega L_2 (I - I_1) + j\omega M I_1$$

$$L_1 I_1 + MI - MI_1 = L_2 I - L_2 I_1 + M I_1$$

$$L_1 I_1 + MI - MI_1 + L_2 I_1 - MI_1 = L_2 I$$

$$I_1 = \frac{(L_2 - M) I}{L_1 + L_2 - 2M} \quad \text{—— (5)}$$

from eq (4), we get $I_2 = I - I_1$

$$\Rightarrow I_2 = I - \frac{(L_2 - M) I}{L_1 + L_2 - 2M}$$

$$\Rightarrow I_2 = \frac{I(L_1 + L_2 - 2M) - (L_2 - M) I}{L_1 + L_2 - 2M}$$

$$= \frac{IL_1 + \cancel{IL_2} - 2MI - \cancel{L_2 I} + MI}{L_1 + L_2 - 2M}$$

$$\Rightarrow I_2 = \frac{(L_1 - M) I}{L_1 + L_2 - 2M} \longrightarrow (6)$$

Sub (5) & (6) in eq (1)

$$V = j\omega \left[L_1 \left(\frac{(L_2 - M) I}{L_1 + L_2 - 2M} \right) \right] + j\omega M \left[\frac{(L_1 - M) I}{L_1 + L_2 - 2M} \right]$$

$$= \frac{j\omega I}{L_1 + L_2 - 2M} \left[\frac{L_1 L_2 - L_1 M + ML_1 - M^2}{I} \right]$$

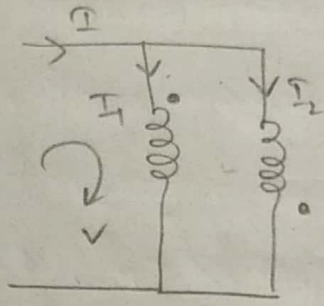
$$V = j\omega \left[\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \right] I$$

$$L_{eff} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

→ Two coils connected in $\parallel^{lel}$ such that fluxes produced by the coils act in same direction such connection is called parallel aiding

* Parallel opposing:

Two coils connected in $||^{lel}$ such that fluxes produced by the coils act in opposite direction. Such connection is called parallel opposing.



$I = I_1 + I_2$

$V - L_1 \frac{dI}{dt} = 0$

$V = j\omega L_1 I_1 - j\omega M I_2 \longrightarrow (1)$

$V = j\omega L_2 I_2 - j\omega M I_1 \longrightarrow (2)$

(1) = (2)

$j\omega[L_1 I_1 - M I_2] = j\omega[L_2 I_2 - M I_1] \longrightarrow (3)$

$I_2 \Rightarrow I - I_1 \longrightarrow (4)$

Sub (4) in (3),

$L_1 I_1 - MI + MI_1 = L_2 I - L_2 I_1 - M I_1$

$\Rightarrow I_1 [L_1 + M + M + L_2] = (L_2 + M) I$

$\Rightarrow I_1 = \frac{(L_2 + M) I}{L_1 + L_2 + 2M}$

$|||^{ly}$ $I_2 = \frac{(L_1 + M) I}{L_1 + L_2 + 2M}$

$|||^{ly}$ $V =$

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Q. Two coils with coeff of coupling is 0.6 b/w them that are connected in series so as to magnetize in (i) same direction (ii) opposite direction. The total inductance in same direction is 1.5H, and in opposite direction is 0.5H.

Sol: $L_{cumulative} = 1.5H$

$L_{differential} = 0.5H$

$\Rightarrow L_1 + L_2 + 2M = 1.5$

$L_1 + L_2 - 2M = 0.5$

(+)

$L_1 + L_2 = 1$

$L_1 + L_2 + 2M = 1.5$

$L_1 + L_2 - 2M = 0.5$

(−)

$M = 0.25$

$K = \dfrac{M}{\sqrt{L_1 L_2}} \Rightarrow L_1 L_2 = \dfrac{M^2}{K^2} = \dfrac{\left(\frac{1}{4}\right)^2}{(0.6)^2} = 0.17361$

$L_1 L_2 = 0.1736$

$L_1 = \dfrac{0.1736}{L_2}$

$\dfrac{0.1736}{L_2} + L_2 + 2(0.25) = 1.5$

$L_2 = 0.78H$

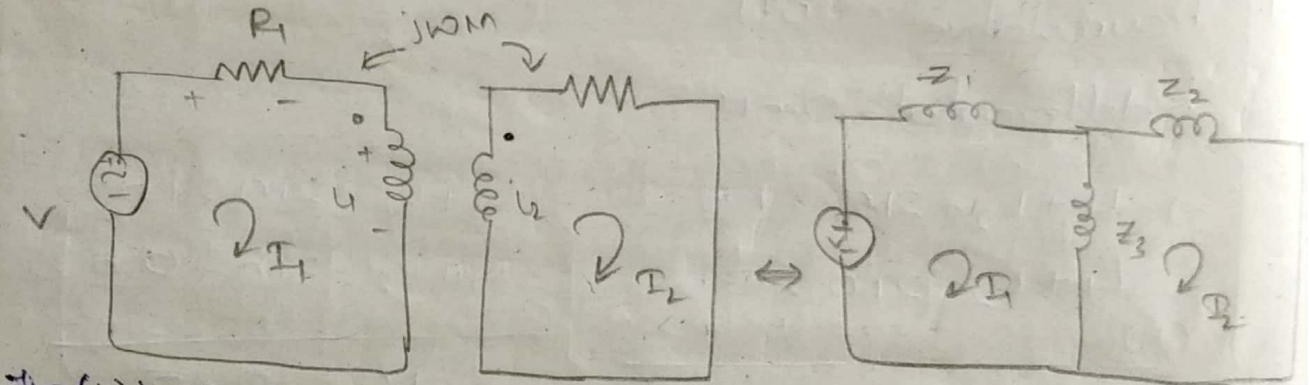
$|||^{ly}$ $L_1 = \dfrac{0.1736}{0.78} = 0.22H$

* Conductively coupled equivalent T circuit :-

→ For easy circuit analysis, it is desirable to replace magnetically coupled circuit with an equivalent network called conductively coupled circuit.

→ In this circuit, no magnetic couplings are involved, the dot connection is not needed.

→ It can be analysed by general network simplification techniques such as mesh analysis, nodal analysis.



Fig(1):-

$$V = (R_1 + j\omega L_1)I_1 - j\omega M I_2 \longrightarrow (1)$$

$$(R_2 + j\omega L_2)I_2 - j\omega m I_1 = 0 \longrightarrow (2)$$

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix}$$

Fig(2)

$$V = (Z_1 + Z_2)I_1 - Z_2 I_2 \longrightarrow (1)$$

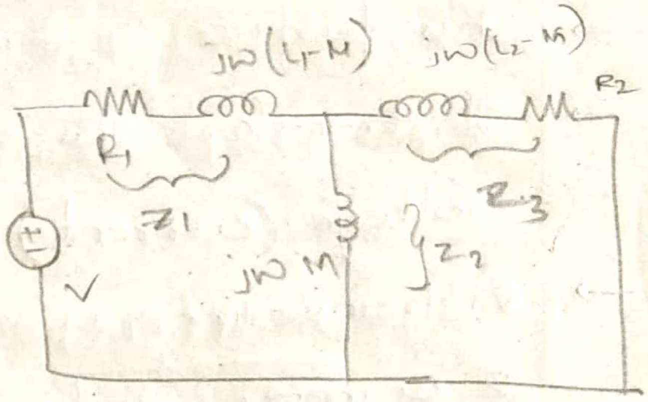
$$-Z_2 I + (Z_2 + Z_3) I_2 \longrightarrow (2)$$

$$\begin{bmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \sim \\ 0 \end{bmatrix}$$

$Z_1 + Z_2 = R_1 + j\omega L_1$

$Z_2 = j\omega M$

$Z_2 + Z_3 = R_2 + j\omega L_2$



$\Rightarrow Z_1 = R_1 + j\omega L_1 - Z_2$

$= R_1 + j\omega L_1 - j\omega M$

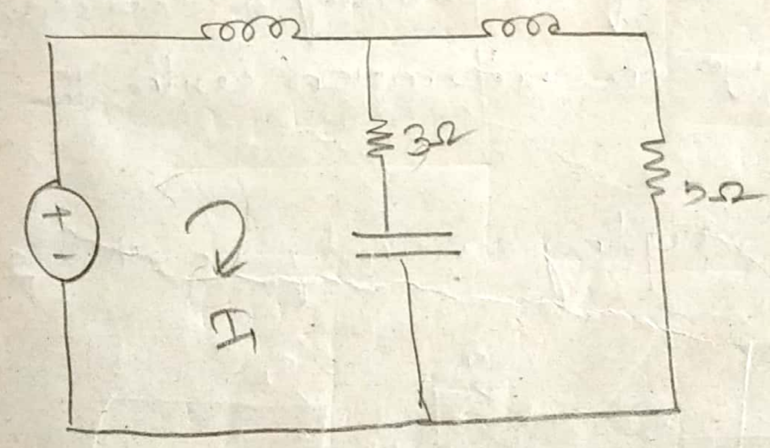
$= R_1 + j\omega (L_1 - M)$

$\Rightarrow Z_3 = R_2 + j\omega L_2 - Z_2$

$= R_2 + j\omega L_2 - j\omega M$

$= R_2 + j\omega (L_2 - M)$

Q.



Dot connection for multiple inductors:

* Ideal transformer :-

→ Transfer of energy from 1 circuit to another circuit through mutual induction is widely used in power systems.

→ This purpose is served by transformer.

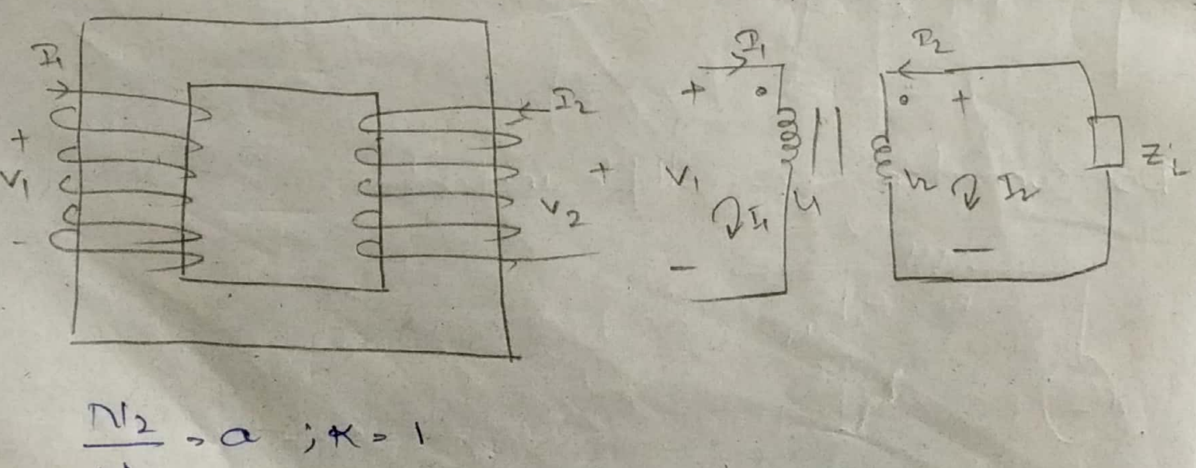
→ It transfers the energy at one voltage to another voltage (current) without change in transformer.

→ A transformer is a static piece of operators have 2 or more windings or coils arranged on a common magnetic pole.

→ Ideal transformer is characterized by assuming zero resistance (zero power dissipation in the primary & secondary windings).

→ The self inductances of the primary & secondary are extremely large in comparision with the load impedance.

→ The coefficient of coupling is unity i.e., $K = 1$.



$\frac{N_2}{N_1} = a$; $K = 1$

$$v = L\frac{di}{dt} \longrightarrow (1)$$

$$v = N\frac{d\phi}{dt} \longrightarrow (2)$$

$$(1) = (2)$$

$$\Rightarrow L\frac{di}{dt} = N\frac{d\phi}{dt} \Rightarrow L = N\frac{d\phi}{di} \rightarrow L = \frac{N^2}{R}$$

$$\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = a^2$$

$$I = \frac{v}{R} = \frac{emf}{Resistance}$$

$$\phi = \frac{mmf}{Reluctance} = \frac{NI}{R}$$

w.k.that, $v_1 = N_1\frac{d\phi}{dt}$

$$v_2 = N_2\frac{d\phi}{dt}$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = a.$$

from the figure,

$$v_1 = j\omega L_1 I_1 - j\omega M I_2 \longrightarrow (1)$$

$$0 = -j\omega M I_1 + (j\omega L_2 + Z_L) I_2 \longrightarrow (2)$$

$$\Rightarrow I_2 = \frac{j\omega M I_1}{j\omega L_2 + Z_L} \longrightarrow (3)$$

Sub (3) in (1)

$$v_1 = j\omega L_1 I_1 - j\omega M \left[\frac{j\omega M I_1}{j\omega L_2 + Z_L}\right]$$

$$= I_1 j\omega L_1 + \frac{I_1 \omega^2 m^2}{Z_L + j\omega L_2}$$

$$Z_{in} = \frac{V_1}{I_1}$$

$$\Rightarrow Z_{in} = j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2}$$

we know that, $k = 1 \rightarrow M = \sqrt{L_1 L_2}$

$$Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{Z_L + j\omega L_2}$$

$$= \frac{j\omega L_1 (Z_L + j\omega L_2) + \omega^2 L_1 L_2}{Z_L + j\omega L_2}$$

$$= \frac{j\omega L_1 Z_L - \omega^2 L_1 L_2 + \omega^2 L_1 L_2}{Z_L + j\omega L_2}$$

$$Z_{in} = \frac{Z_L L_1 j\omega}{Z_L + j\omega L_2}$$

$$j\omega L_2 >>> Z_L$$

$$Z_{in} = \frac{Z_L L_1 j\omega}{j\omega L_2}$$

$$Z_{in} = \frac{Z_L L_1}{L_2}$$

$$Z_{in} = \frac{Z_L}{a^2}.$$

Consider eq (2),

$$j\omega M I_1 = (j\omega L_2 + Z_L) I_2$$

$$\frac{I_1}{I_2} = \frac{Z_L + j\omega L_2}{j\omega M}$$

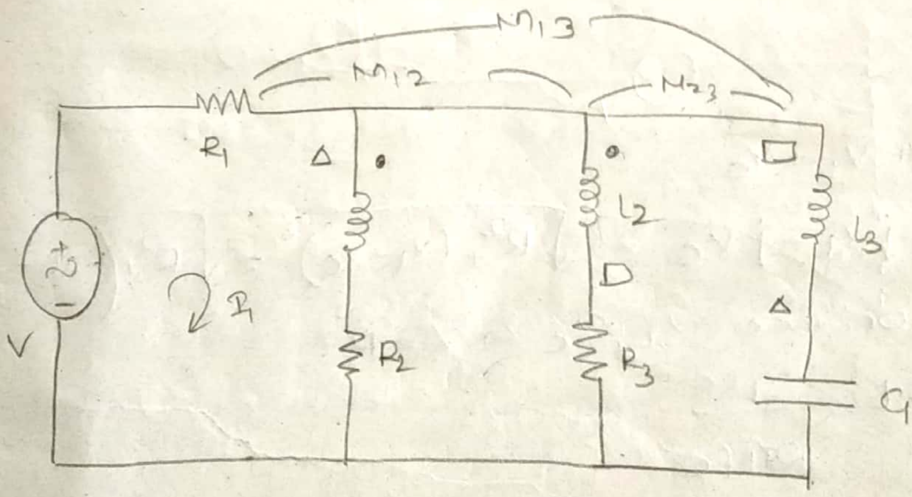
$$\frac{I_1}{I_2} = \frac{j\omega L_2}{j\omega M}$$

$$= \frac{L_2}{M} = \frac{L_2}{\sqrt{L_1 L_2}} = \sqrt{\frac{L_2}{L_1}} = a$$

$$\therefore a = \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}} = \frac{I_1}{I_2}$$

Q. An amplifier of output resistance 1936Ω is to feed a loud speaker with an impedance of 4Ω. (i) Calculate the desired turns ratio for an ideal transformer to connect the 2 systems.
(ii) An RMS current of 20mA at 500Hz is flowing in the primary. Calculate the RMS Value of current in secondary winding. (iii) What is the power delivered to the load?

→



$$V - R_1 I_1 - j\omega L_1 (I_1 - I_2) - R_2 (I_1 - I_2) - j\omega M_{12} (I_2 - I_3) + j\omega M_{13} I_3$$

or

$$+ j\omega M (I_3 - I_2) = 0$$

* Resonance :

→ It is defined as a phenomena in which applied voltage and resulting current are in phase.

→ An AC circuit is said to be in resonance if it exhibits unity power factor condition.

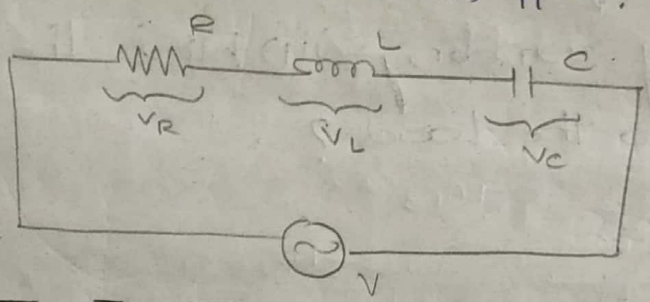
→ There are 2 types of resonance.

(i) Series resonance or resonance

(ii) parallel resonance or anti resonance.

→ The reactances are cancelled if the inductive and capacitive reactances are in series.

→ The susceptance get cancelled if the inductive & capacitive reactances are in ||'el.



$$\bar{V} = \bar{V_R} + \bar{V_L} + \bar{V_C}$$

$$Z = R + jX_L - jX_C = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

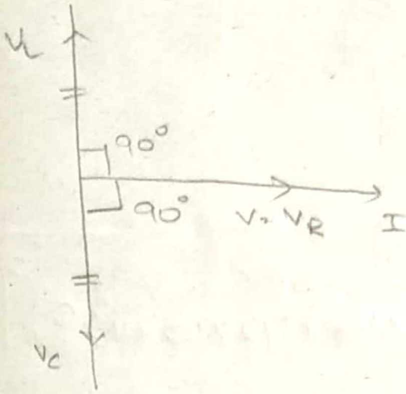
At resonance $X_L = X_C$ & $Z = R$

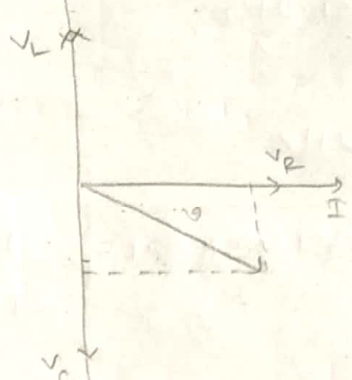
$$\Rightarrow 2\pi f L = \frac{1}{2\pi f C}$$

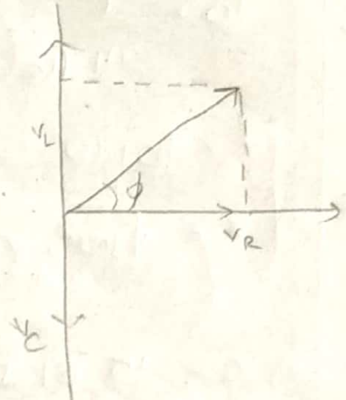
$$\Rightarrow f_0^2 = \frac{1}{(2\pi)^2 LC}$$

$$\rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$\rightarrow X_L = X_C$ $\quad\quad \rightarrow X_L < X_C$ $\quad\quad \rightarrow X_L > X_C$







$\cos\phi = \cos 0 = 1$
$\bar{V} = \bar{V_R}$
RC circuit

$\cos(-\phi)$, leading
$\bar{V} = \bar{V_R} + (\overline{V_C - V_L})$
resistive circuit

$\cos\phi$, lagging
$V = V_R + (V_L - V_C)$
RL circuit.

Q. Determine the value of capacitive reactance and impedance for the circuit. $R = 50\Omega$, $L = +j25$, $C = -jX_C$

Sol! $X_C = 25$.

$Z = R = 50\Omega$.

Q. Determine f_0 where $R = 10\Omega$, $L = 0.5mH$, $C = 10\mu F$

Sol! $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi \times \sqrt{0.5 \times 10^{-3} \times 10 \times 10^{-6}}}$

$= \dfrac{1}{2\pi \times \sqrt{10^{-8} \times 0.5}} = \dfrac{1}{2\pi \times 10^{-4} \times \sqrt{0.5}} = 2.25 KHz$

Q. $f_0 = 5KHz$, $L = 1mH$ $R = 2\Omega$ find $C = ?$

Sol: $f = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow 5 \times 10^3 = \dfrac{1}{2\pi\sqrt{C \times 10^{-3}}}$ $\quad X_L = \omega L$

$\Rightarrow C =$

Q. Determine the impedance, f_0, 10Hz $\uparrow f_0$, 10Hz $\downarrow f_0$

$R = 10\Omega$, $0.1H = L$, $C = 10\mu F$.

Sol! At f_0, $Z = R = 10\Omega$

At $f = f_0 + 10$,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 159.2\,Hz$$

$$f = f_0 + 10 = 169.2\,Hz$$

$$\Rightarrow Z = \sqrt{10^2 + \left(2\pi \times 169.92 \times 0.1 - \frac{1}{2\pi \times 169.92 \times 10 \times 10^{-6}}\right)^2}$$

$$\Rightarrow Z = 15.81\,\Omega$$

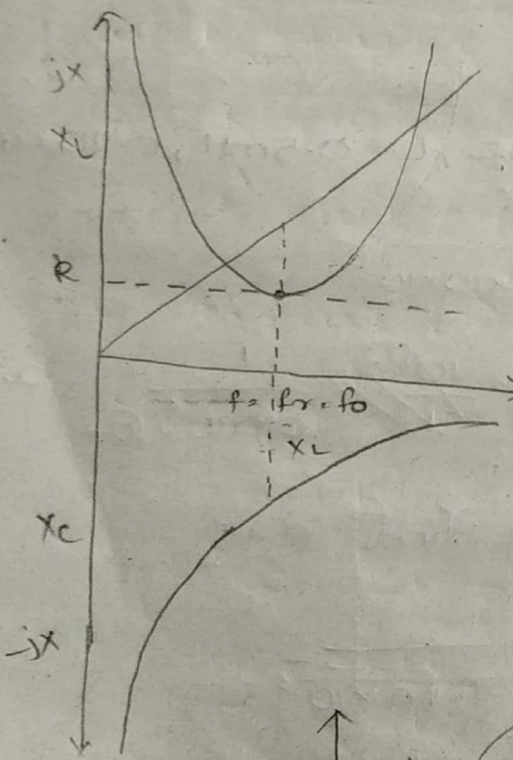
At $f = f_0 - 10$

$$f = 169.62 - 10 = 149.62\,Hz.$$

$$\Rightarrow Z = \sqrt{10^2 + \left(2\pi \times 149.62 \times 0.1 - \frac{1}{2\pi \times 149.62 \times 10^{-6} \times 10}\right)^2}$$

$$\Rightarrow Z = 16.28\,\Omega$$

→



$X_L = 2\pi f L$

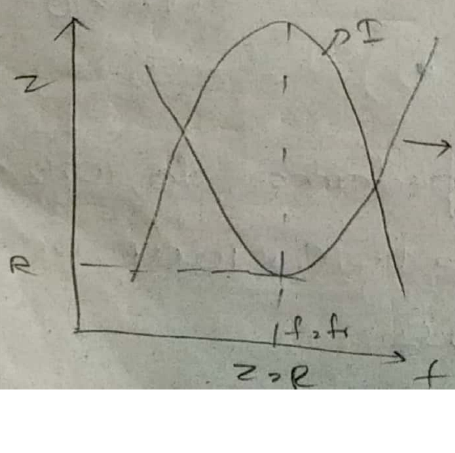
$f = 0,\ X_L = 0$

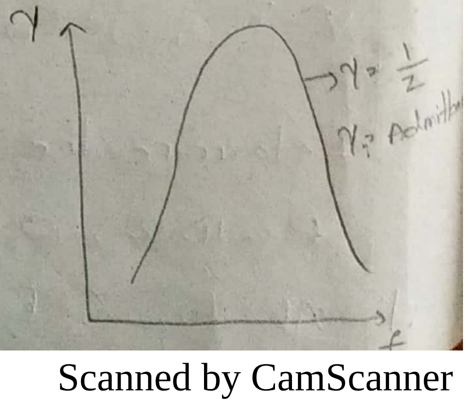
$f = \infty,\ X_L = \infty$

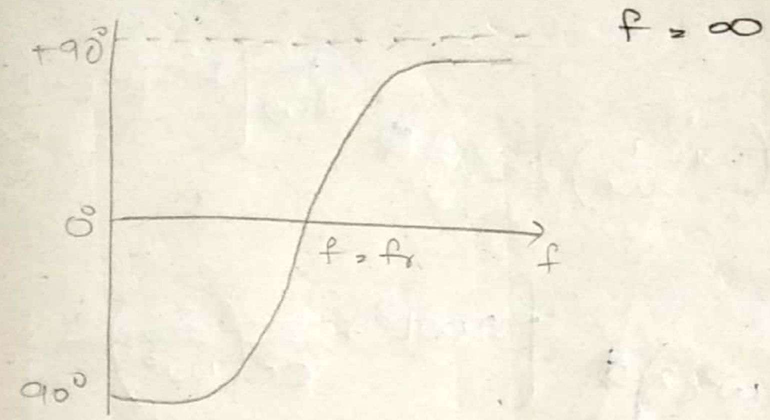
$X_C = \frac{1}{2\pi f C}$

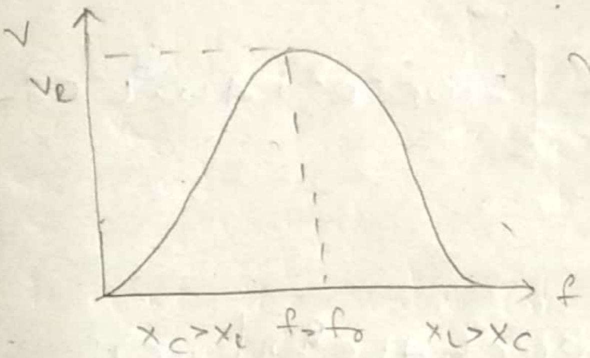
$f = 0,\ X_C = \infty$

$f = \infty,\ X_C = 0$









16/8/2019

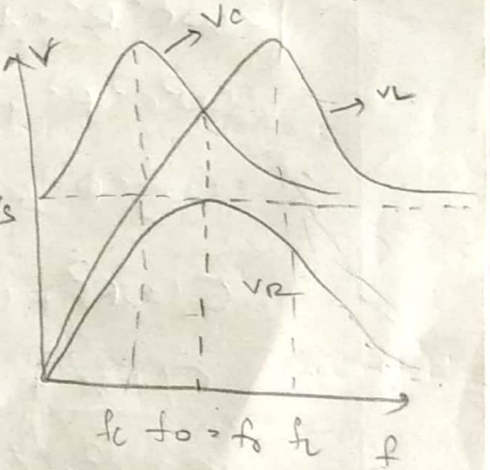
Friday

**→ For a resistor, C, L –

$V_R = I \times R$

$V_C = I \times \frac{1}{\omega C} = I \times \frac{1}{2\pi f C}$

$V_L = I \times \omega L = I \times 2\pi f L$



**→ $V_C = I \times \frac{1}{\omega C}$

$$= \frac{V_S}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\ \omega C}$$

$$V_C^2 = \frac{V_S^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \omega^2 C^2}$$

$\frac{dV_C^2}{d\omega} = 0$ To get max. value

$$0 = V_s^2 \frac{-1}{\left(\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)\omega^2 C^2\right)^2} \cdot \frac{d}{d\omega}\left[\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)\omega^2 C^2\right]$$

$$0 = \frac{-V_s^2}{\left(\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)\omega^2 C^2\right)^2} \cdot \left[\cancel{2\omega L^2 + \frac{1}{C^2}\left(\frac{-2}{\omega^3}\right)}\right] \times$$

$$\frac{-V_s^2}{\left(\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)\omega^2 C^2\right)^2} \cdot \left[2\omega C^2 R^2 + 4\omega^3 L^2 C^2 - 4LC\omega\right] = 0$$

$$\Rightarrow -V_s^2\left[2\omega C^2 R^2 + 4\omega^3 L^2 C^2 - 4LC\omega\right] = 0$$

$$V_s \neq 0$$

$$2\omega C^2 R^2 + 4\omega^3 L^2 C^2 - 4LC\omega = 0$$

$$\omega C^2 R^2 + \omega^3 L^2 C^2 (2) - 2LC\omega = 0$$

$$\omega C^2 R^2 + \omega^2 L^2 C^2 (2) - 2LC = 0$$

$$C\omega R^2 + \omega^2 L^2 (2C) - 2L = 0$$

$$C\omega R^2 + 2\omega^2 L^2 C - 2L = 0$$

$$C2L^2\omega^2 + R^2\omega C - 2L = 0$$

$$2L^2\omega^2 = 2LC - R^2\omega$$

$$\omega^2 = \frac{2LC}{2L^2} - \frac{R^2\omega}{2L^2}$$

$$\omega^2 = \frac{C}{L} - \frac{R^2\omega}{2L^2}$$

$$\omega = \sqrt{\frac{C}{L} - \frac{\omega R^2}{2L^2}} \times$$

$$\cancel{2L^2C\omega^2 + R^2\omega C - 2L = 0}$$

$$2\omega R^2C^2 + 4\omega^3L^2C^2 - 4LC\omega = 0$$

$$2R^2C^2 + 4\omega^2L^2C^2 - 4LC = 0$$

$$R^2C + 2C\omega^2L^2 - 2L = 0$$

$$\omega^2 = \frac{2L - R^2C}{2CL^2}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$2\pi f_C = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$f_C = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R^2C}{2L}}$$

$$f_C = f_r\sqrt{1 - \frac{R^2C}{2L}}$$

→ $V_L = I \times \omega L$

$$= \frac{V_S\,\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_L^2 = \frac{V_S^2\,\omega^2 L^2}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$\frac{1}{\omega_2}$

$$\frac{dV_L^2}{d\omega} = 0$$

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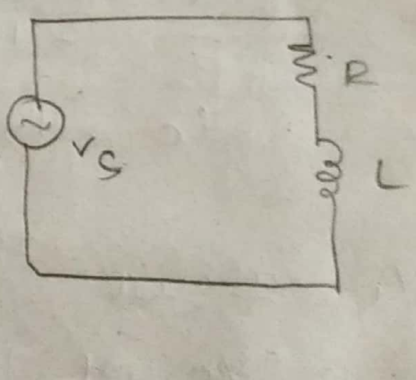
* Quality factor :-

→ Figure of merit is (given by) called quality factor.

→ $Q = \frac{\text{max. energy stored per cy.}}{\text{max. energy dissipated per cycle}} \times 2\pi$

→ $E_L = \frac{1}{2} L I_m^2$



$P_D = I^2 R$

$= \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{I_m^2 R}{2}$

P_D per cycle $= \frac{I_m^2 R}{2f}$

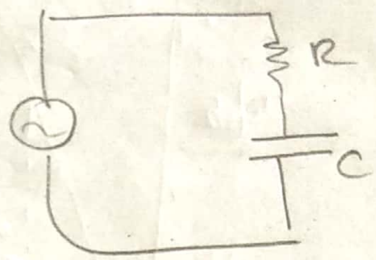
$Q = 2\pi \times \frac{\frac{1}{2} L I_m^2}{I_m^2 R / 2f}$

$= \frac{\pi \times 2f \times L}{R} = \frac{\omega L}{R}$ = Quality factor of Inductor

→ $E_C = \frac{1}{2} C V_m^2 = \frac{1}{2} C \left[\frac{I_m}{\omega C}\right]^2$

$P_D = I^2 R$

$= \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{I_m^2 R}{2}$



P_D per cycle $= \frac{I_m^2 R}{2f}$

$Q = 2\pi \times \frac{\frac{1}{2} C V_m^2}{I_m^2/2f} = \frac{1}{\omega C R}$ = quality factor of capacitor

→ $Q = \frac{1}{\omega C R}$; $Q = \frac{\omega L}{R}$.

$Q \times Q = \frac{\omega L}{R} \times \frac{1}{\omega C R}$

$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ = Quality factor of LCR circuit.

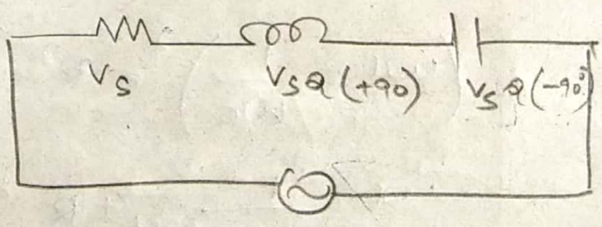
→ The series resonance circuit acts like a voltage amplifier.

$V_{in} = V_s$

$V_L = I \times j \times_L$ $\left[\because \text{At resonance}, I = \frac{V_s}{R}\right]$

$= \frac{V_s}{R} \omega L (+90)$

$V_L = V_s Q_0 \angle 90°$



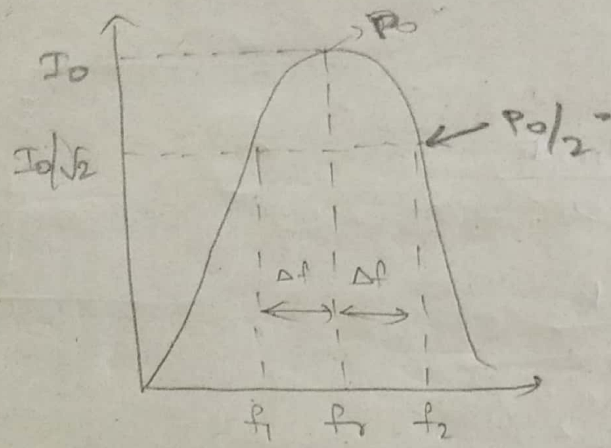
|||ly $V_C = I \times -j X_C$

$= \frac{V_s}{R} \times \frac{1}{\omega C} \angle -90°$

$= V_s Q_0 \angle -90°$

9/8/2019 Monday

* Bandwidth of series resonant circuit :



Bandwidth $= f_2 - f_1 = 2\Delta f$

$$P' = \frac{P_0}{2}$$

$$= \frac{I_0^2}{2} R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

$$I_0 = \frac{V}{R}$$

$$\frac{I_0}{\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{Z}$$

$$\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \longrightarrow (1)$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \longrightarrow (2)$$

(1) + (2)

$$(\omega_1 + \omega_2) L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} \right) \frac{1}{C} = 0$$

$$(\omega_1 + \omega_2) L - \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \right) \frac{1}{C} = 0$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$f_1 f_2 = \frac{1}{4\pi^2 LC} = f_r^2 \quad \left[\because f_r = \frac{1}{2\pi\sqrt{LC}} \right]$$

$$\boxed{\Rightarrow f_r = \sqrt{f_1 f_2}}$$

(1) − (2)

$$(\omega_2 - \omega_1) L + \left(\frac{1}{\omega_1} - \frac{1}{\omega_2} \right) \frac{1}{C} = 2R$$

$$\omega_2 - \omega_1 + \left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right) \frac{1}{LC} = \frac{2R}{L}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_r^2 = \frac{1}{LC}$$

$$\Rightarrow \omega_2 - \omega_1 = \frac{R}{L}$$

$$f_2 - f_1 = \frac{R}{2\pi L}$$

$$f_1 = f_r - \Delta f$$

$$= f_r - \frac{R}{4\pi L}$$

$$f_2 = f_r + \Delta f$$

$$= f_r + \frac{R}{4\pi L}$$

Selectivity :-

$$\text{Selectivity} = \frac{f_r}{BW}$$

$$= \frac{f_r}{f_2 - f_1} = \frac{f_r}{R/2\pi L} = \frac{2\pi f_r L}{R} = \frac{\omega L}{R}$$

$$\text{Quality factor} = \frac{f_r}{BW}$$

Q. A series RLC circuit consists of resistance 1 KΩ, L = 100mH, C = 10pF. Find Resonant f_r, I_0, Q-factor, f_1, f_2

Sol:

$R = 1K\Omega = 10^3 \Omega$ $\quad V = 100V$

$L = 100 \times 10^{-3} = 0.1 H$

$C = 10 \times 10^{-12} = 10^{-11} F$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^{-1} \times 10^{-11}}} = \frac{1}{2\pi \times 10^{-6}} = \frac{10^6}{2\pi}$$

$= 159235.6 Hz$

$$I_0 = \frac{V}{R} = \frac{100}{10^3} = 0.1A$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10^3}\sqrt{\frac{10^{-1}}{10^{-11}}}$$

$$= \frac{10^5}{10^3} = 100$$

$$f_1 = f_r - \frac{R}{4\pi L} = 159235.6 - \frac{10^3}{4\pi \times 10^{-1}}$$

$$= 159235.6 - \frac{10^4}{4\pi}$$

$$= 159235.6 - 795.77 = 158439.83$$

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$f_2 = f_r + \frac{R}{4\pi L} = 159235.6 + 795.77$

$= 160031.37$

Q. A series RLC circuit with $R = 8\Omega$, B.W = 50Hz, Determine the value of L and C so that the circuit resonates at 250Hz

Sol: $BW = \frac{R}{2\pi L}$

$50 = \frac{R}{2\pi L} \Rightarrow 2\pi L = \frac{8}{50}$

$\Rightarrow L = \frac{8}{2\pi \times 50} = 0.025H$

$f_r = \frac{1}{2\pi\sqrt{LC}}$

$250 = \frac{1}{2\pi\sqrt{0.025 \times C}}$

$\sqrt{0.025 \times C} = \frac{1}{2\pi \times 250}$

$0.025 \times C = \frac{1}{4\pi^2 \times 250 \times 250}$

$C = \frac{1}{4\pi^2 \times 250 \times 250 \times 0.025} = 1.62113 \times 10^{-5} F$

Q. A max. current of 0.1A flows through the circuit when a capacitor value is 5μF with a fixed frequency & voltage of 5V. Determine the frequency at which the circuit resonates, the BW, Q, the value of R at resonance.

$L = 0.1H$

$$I_0 = \frac{V}{R}$$

$$R = \frac{V}{I_0} = \frac{5}{0.1} = 50\Omega$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{50}\sqrt{\frac{0.1}{5\times10^{-6}}}$$

$$= \frac{1}{50}\sqrt{\frac{10^{6}\times10^{-1}}{5}}$$

$$= \frac{1}{50}\sqrt{\frac{10^5}{5}} = 2.828.$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1\times5\times10^{-6}}}$$

$$= \frac{1}{2\pi\sqrt{5\times10^{-7}}} = 225.07\ Hz$$

$$BW = \frac{R}{2\pi L} = \frac{50}{2\pi\times10^{-1}} = \frac{500}{2\pi} = 79.57\ Hz.$$

Q. $R = 10\Omega$, $L = 60mH$, $f = 25Hz$ are present in series RLC Circuit. The P.F of the circuit is 45° leading. At wt frequency the circuit resonates.

Sol: $R = 10\Omega$

$L = 60mH$

$f = 25Hz$

$$\tan\phi = \frac{X_C - X_L}{R}$$

$$\tan 45^\circ = \frac{X_C - X_L}{10}$$

$$X_C - X_L = 10$$

$$\cos\phi = R/Z$$

$\frac{1}{\sqrt{2}} = \frac{10}{z} \Rightarrow z = 10\sqrt{2}$

Q. A series RLC circuit has $R = 4\Omega$, $L = 25mH$.

(a) Calculate 'C' if $Q = 50$.

(b) Calculate, ω_1, ω_2 & BW.

(c) Calculate Avg power at $\omega_0 = \omega_1, \omega_2$ take $V_m = 100V$

Sol: $R = 4\Omega$, $L = 25mH$

(i) $Q = 50$

$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow 50 = \frac{1}{4}\sqrt{\frac{25\times10^{-3}}{C}}$

$\Rightarrow C =$

(ii) $\omega_1 = \omega_r - \frac{R}{2L}$; $\omega_2 = \omega_r + \frac{R}{2L}$

$Q = \frac{\omega_r L}{R}$

$50 \times 4 = \omega_r L \Rightarrow \omega_r = \frac{50\times4}{25\times10^{-3}} = \frac{200\times10^{+3}}{25}$

$\Rightarrow \omega_r = \frac{\cancel{0.2}}{\cancel{25}} \frac{200000}{25} =$

$\Rightarrow \omega_r = 8\times10^{-3}$ Hz

$\omega_1 = 8\times10^{-3} - \frac{4\times10^{+3}}{2\times25} = 10^{-3}\left[8 - \frac{2}{25}\right]$

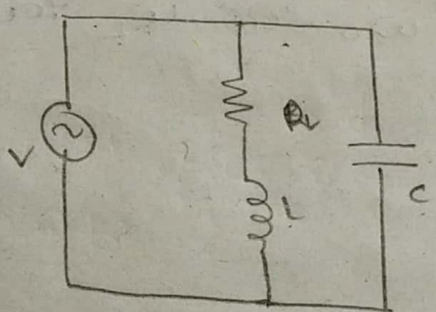
$\omega_1 = 8\times10^{-3} - 8$

$= 8(10^{-3} - 1)$

20/8/2019

Tuesday

* Parallel Resonance :-



$Z = R \pm jX \ \Omega$

$Y = G \pm jB \ ℧$

$Z_1 = R + jX_L$

$Y_1 = \frac{1}{Z_1} = \frac{1}{R+jX_L} = \frac{R-jX_L}{R^2+X_L^2}$

$Z_2 = \cancel{\frac{1}{jX_C}} \ -jX_C$

$Y_2 = \frac{1}{Z_2} = \cancel{jX_C} \ \frac{j}{X_C}$

$Y = Y_1 + Y_2$

$= \frac{R-jX_L}{R^2+X_L^2} + j/X_C$

$Y_T = \frac{R}{R^2+X_L^2} - \frac{jX_L}{R^2+X_L^2} + \frac{j}{X_C}$

$$Y_T = \frac{R}{R^2+X_L^2} - j\left[\frac{X_L}{R^2+X_L^2} - \frac{1}{X_C}\right]$$

$$G = \frac{R}{R^2+X_L^2} \; ; \; B = \frac{X_L}{R^2+X_L^2} - \frac{1}{X_C} = 0$$

$$\Rightarrow \frac{X_L}{R^2+X_L^2} = \frac{1}{X_C}$$

$$\Rightarrow X_L X_C = R^2 + X_L^2$$

$$\Rightarrow R^2 + X_L^2 = \omega_{ar} L \frac{1}{\omega_{ar} C}$$

$$\Rightarrow R^2 + X_L^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + \omega_{ar}^2 L^2 = \frac{L}{C}$$

$$\Rightarrow \omega_{ar}^2 L^2 = \frac{L}{C} - R^2 \Rightarrow \omega_{ar}^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\Rightarrow \omega_{ar} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_{ar} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

f_{ar} = frequency at resonance in ||le'

ar = anti resonance.

$$\frac{1}{LC} >> \frac{R^2}{L^2}$$

$$Y = \frac{R}{R^2+X_L^2}$$

$$Z = \frac{R^2+X_L^2}{R} = \frac{L}{RC}.$$

→ $R^2 + X_L^2 = \frac{L}{C}$

$$R^2\left[1 + \frac{\omega_{ar}^2 C^2}{R^2}\right] = \frac{L}{C}$$

$$R^2(1+Q^2) = \frac{L}{C}$$

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$\Rightarrow R(1+Q^2) = \frac{L}{CR}$

$\Rightarrow R(1+Q^2) = Z_{ar}$

→ Z_{ar} is also called dynamic resistance.

→ $f_{ar} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-\frac{R^2C}{L}}$

$Q \times Q = \frac{\omega L}{R} \times \frac{1}{\omega RC}$

$Q^2 = \frac{L}{R^2C}$

$f_{ar} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-\frac{1}{Q^2}}$

→ If $Q\uparrow$ (very large) then $f_{ar} = f_r$.

→ $\parallel^{lel}$ resonance circuit acts like a current amplifier.

→ Series resonance circuit acts like a voltage amplifier.

→ $V = IZ_{ar} = I \times \frac{L}{CR}$

$I_C = \frac{V}{X_C} = V\omega_{ar}C$

$= \frac{IL}{RC} \times \omega_{ar}C$

$= I \times \left(\frac{\omega_a L}{R}\right)$

$I_C = IQ$

Hence, current is amplified by Q times, across the capacitor.

$P = I_L^2 R$

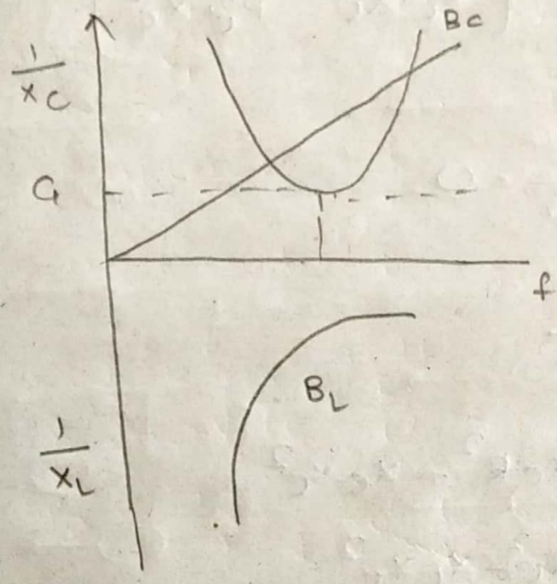
$P = I^2 Z_{ar}$

$I^2 Z_{ar} = I_L^2 R$

$$I_L^2 = \frac{I^2 \cdot \frac{1}{RC}}{R} \Rightarrow I_L^2 = I^2 \times \frac{L}{R^2 C}$$

$$\Rightarrow I_L = \sqrt{I^2 \frac{L}{R^2 C}}$$

$$\Rightarrow I_L = I \times \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow I_L = I \times Q.$$

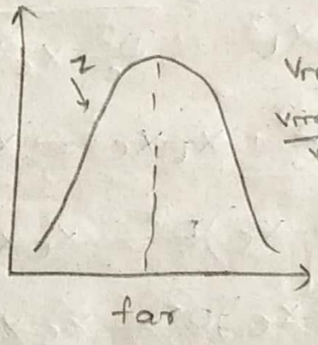


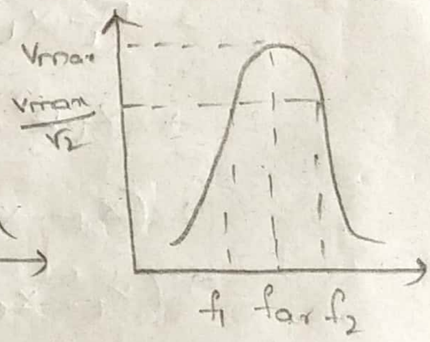
X_C

$B_C = \frac{1}{X_C} = \omega C$

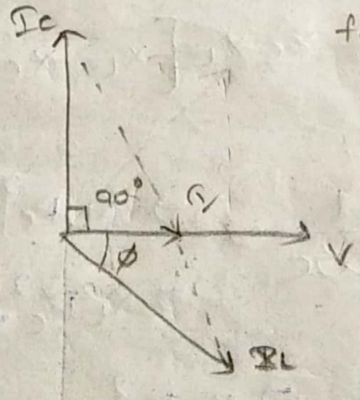
$B_L = \frac{1}{X_L} = \frac{1}{\omega L}$

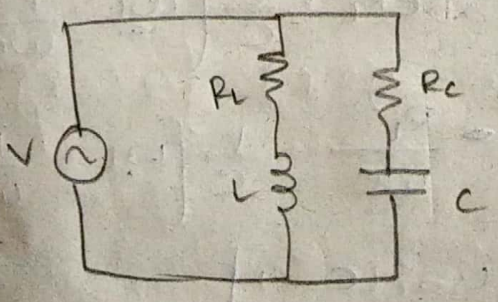
$Y = G \pm jB$

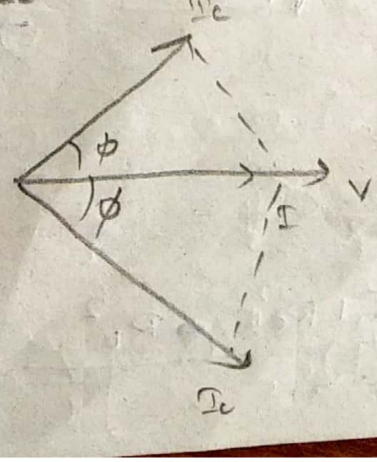




$$Q = \frac{f_{ar}}{f_2 - f_1}$$







$Z_1 = R_L + jX_L$

$$Y_1 = \frac{1}{Z_1} = \frac{R_L - jX_L}{R_L^2 + X_L^2}$$

$Z_2 = R - jX_L$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R - jX_L} = \frac{R_C + jX_L}{R_C^2 + X_C^2}$$

$Y = Y_1 + Y_2$

$$= \frac{R_L}{R_L^2+X_L^2} + \frac{R_C}{R_C^2+X_C^2} - j\left[\frac{X_L}{R_L^2+X_L^2} - \frac{X_C}{R_C^2+X_C^2}\right]$$

$B = 0$

$$\Rightarrow \frac{X_L}{R_L^2+X_L^2} - \frac{X_C}{R_C^2+X_C^2} = 0$$

$$\Rightarrow \frac{X_L}{R_L^2+X_L^2} = \frac{X_C}{R_C^2+X_C^2}$$

$\Rightarrow \cancel{X_L R^2 + X_L X_C^2 = X_C R^2 + X_C X_L^2}$

$\Rightarrow \cancel{(X_L - X_C)R^2 = X_C X_L^2 - X_L X_C^2}$

$\Rightarrow \cancel{(X_L - X_C)R^2 = X_C X_L [X_L - X_C]}$

$\Rightarrow \cancel{R^2 = X_C X_L}$

$\Rightarrow \cancel{R^2 = \omega L \times \frac{1}{\omega C}}$

$\Rightarrow \cancel{R = \sqrt{\frac{L}{C}}}$

for

$\Rightarrow X_L R_C^2 + X_L X_C^2 = X_C R_L^2 + X_C X_L^2$

$\Rightarrow X_L R_C^2 - X_C R_L^2 = X_C X_L [X_L - X_C]$

$$\Rightarrow X_L R_C^2 - X_C R_L^2 = \frac{L}{C}\left[\omega_{ar} L - \frac{1}{\omega_{ar} C}\right]$$

$$\Rightarrow \omega_{ar} L R_C^2 - \frac{R_L^2}{\omega_{ar} C} = \frac{L}{C}\left[\omega_{ar} L - \frac{1}{\omega_{ar} C}\right]$$

$$\Rightarrow \frac{\omega_{ar}^2 L R_C^2 C - R_L^2}{\omega_{ar} C} = \frac{L[\omega_{ar}^2 LC - 1]}{\omega_{ar} C^2}$$

$$\Rightarrow \omega_{ar}^2 R_C^2 LC - R_L^2 = \frac{1}{C}\left[\omega_{ar}^2 LC - 1\right]$$

$$\Rightarrow \omega_{ar}^2 R_C^2 LC^2 - R_L^2 C = \omega_{ar}^2 L^2 C - L$$

$$\Rightarrow \omega_{ar}^2 R_C^2 LC^2 - \omega_{ar}^2 L^2 C = R_L^2 C - L$$

$$\Rightarrow \omega_{ar}^2 LC\left[CR_C^2 - L\right] = R_L^2 C - L$$

$$\Rightarrow \omega_{ar}^2 = \frac{1}{LC}\left[\frac{R_L^2 C - L}{R_C^2 C - L}\right]$$

$$\Rightarrow \omega_{ar}^2 = \frac{1}{LC}\left[\frac{R_L^2 - L/C}{R_C^2 - L/C}\right]$$

$$\Rightarrow \omega_{ar} = \sqrt{\frac{1}{LC}\left[\frac{R_L^2 - L/C}{R_C^2 - L/C}\right]}$$

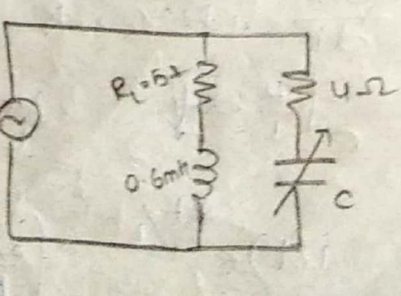
$$\Rightarrow 2\pi f_{ar} = \sqrt{\frac{1}{LC}\left[\frac{R_L^2 - L/C}{R_C^2 - L/C}\right]}$$

$$\Rightarrow f_{ar} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\left[\frac{R_L^2 - L/C}{R_C^2 - L/C}\right]}$$

$$\Rightarrow f_{ar} = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(\frac{R_L^2 - L/C}{R_C^2 - L/C}\right)}$$

Q. Find the value of c such that the circuit resonates at $\omega_{ar} = 5000$ rad/sec

Sol: $\omega_{ar} = \frac{1}{\sqrt{LC}}\sqrt{\left(\frac{R_L^2 - L/C}{R_C^2 - L/C}\right)}$



$$5000 = \frac{1}{\sqrt{0.6\times10^{-3}\times C}}\sqrt{\frac{25 - 0.6\times10^{-3}/C}{16 - 0.6\times10^{-3}/C}}$$

Solving this, we get a quadratic equation in terms of 'C':

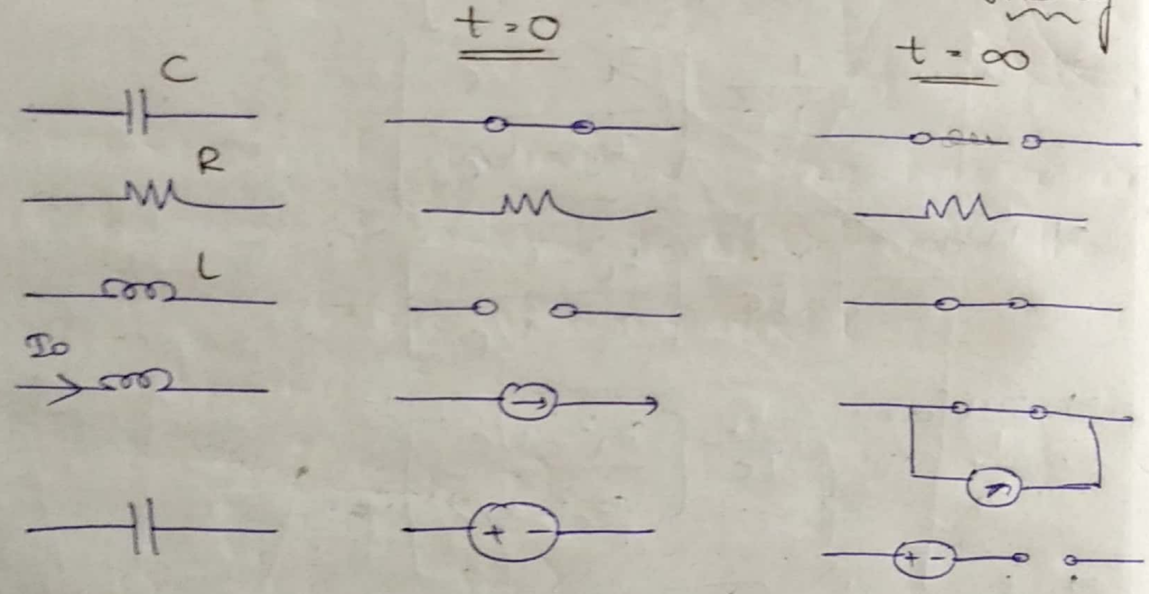
$C_1 = 20.66\,\mu F$

$C_2 = 121\,\mu F$

Q. In $||^{lel}$ resonant circuit find the resonant frequency $L = 1mH, R_L = 2\Omega$, $C = 10H$.

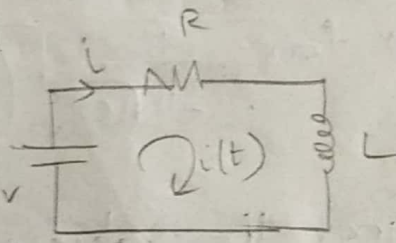
29/8/2019

Friday



* Transcient analysis:

DC Step input



$$V - Ri(t) - L\frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} + \frac{Ri(t)}{L} = \frac{V}{L}$$

$$\frac{dx}{dt} + Px = Q \text{ then soln } x(I.F) = \int Q\,(I.F)\,dt$$

$$e^{\int \frac{R}{L}dt} = e^{\frac{Rt}{L}}$$

$$i(t)e^{Rt/L} = \int \frac{V}{L} e^{Rt/L}dt + C$$

$$i(t)e^{Rt/L} = \frac{V}{L}\int e^{Rt/L}\,dt + C$$

$$i(t)e^{Rt/L} = \frac{V}{L}\frac{\left(e^{Rt/L}\right)}{R/L} + C$$

$$\boxed{i(t) = \frac{V}{R} + Ce^{-Rt/L}}$$

t = 0⁻ just b4 switching
t = 0⁺ just aft switching
t = ∞ steady state.

$$i(t) = \frac{V}{R} + Ce^{-Rt/L} \longrightarrow (1)$$

$$t = 0^- \rightarrow i(0^-) = 0$$

$$t = 0^+ \rightarrow i(0^+) = 0$$

$$i(0^-) = \frac{V}{R} + Ce^{-R(0)/L}$$

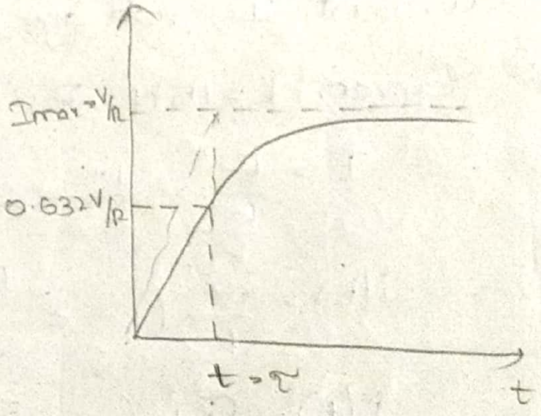
$$0 = \frac{V}{R} + C \Rightarrow C = -\frac{V}{R} \longrightarrow (2)$$

Sub (2) in (1),

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-Rt/L}$$

$$i(t) = \frac{V}{R}\left[1 - e^{-R/L\,t}\right)$$

$$\boxed{\tau = \frac{L}{R}}$$



If $t = \tau = \frac{L}{R}$

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-R/L \times L/R}$$

$$= 0.632\frac{V}{R}$$

$$i(t) = 63.2\%\, V/R$$

$$V_R = Ri(t) = R\times\frac{V}{R}\left(1 - e^{-R/L\,t}\right)$$

$$= V\left(1 - e^{-R/L\,t}\right)$$

$$V_L = L\frac{di(t)}{dt} = L\frac{V}{R}\left(-e^{-R/L\,t}\right)\times\left(\frac{-R}{L}\right)$$

$v_L = v e^{-R/L\, t}$

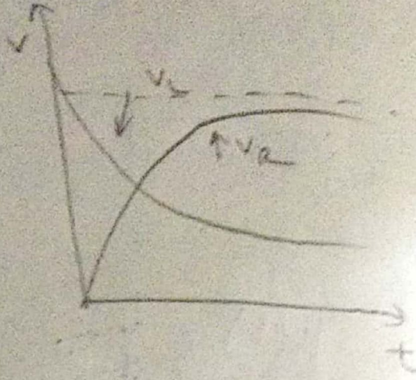
$P_R = v_R\, i(t)$

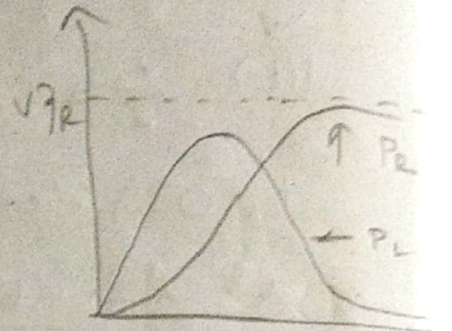
$= v(1-e^{-t/\tau})\frac{v}{R}(1-e^{-t/\tau})$

$= \frac{v^2}{R}(1-e^{-t/\tau})^2$

$P_L = v_L \times i(t) = v e^{-t/\tau} \frac{v}{R}(1-e^{-t/\tau})$

$= \frac{v^2}{R}\left(e^{-t/\tau} - e^{-2t/\tau}\right)$





Q. Series RL circuit, L = 15H, R = 30Ω has a const voltage 60V applied at t = 0. Determine the current I, voltage across R and L

Sol: Given L = 15H, R = 30Ω

$\tau = \frac{L}{R} = \frac{15}{30} = \frac{1}{2}$

$i(t) = \frac{v}{R}(1-e^{-t/\tau})$

$i(t) = \frac{60}{30}(1-e^{-0}) = 0$ at $t=0$

$i(t) = 2(1-e^{-2t})$

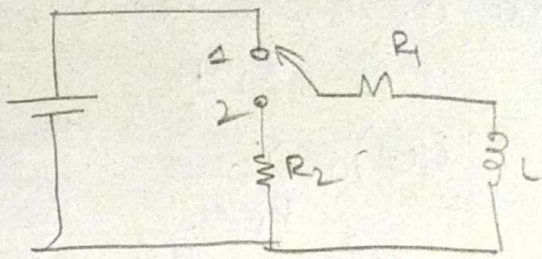
$v_R = 60(1-e^{-2t})$

$v_L = 60(e^{-2t})$

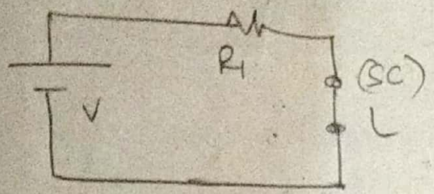
Q. In a network of figure, the switch is initially at position 1. On the steady state having reached the switch is changed to position 2. Find i(t).

At position 1,

$$i(0^-) = \frac{V}{R_1}$$



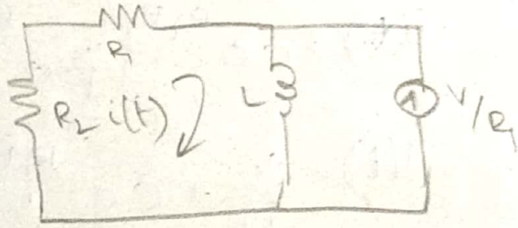
then the eq. ckt is



Since, the inductor does not accept sudden changes in current,

$$i(0^+) = \frac{V}{R_1} \quad \text{(At position 2)}$$

Writing KVL for adj. fig.,



$$-R_2 i(t) - R_1 i(t) - L\frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} + \left(\frac{R_1+R_2}{L}\right) i(t) = 0$$

$$i(t) = Ce^{-\left(\frac{R_1+R_2}{L}\right)t} \longrightarrow (1)$$

$$t=0 \Rightarrow i(0) = Ce^{-\left(\frac{R_1+R_2}{L}\right)(0)}$$

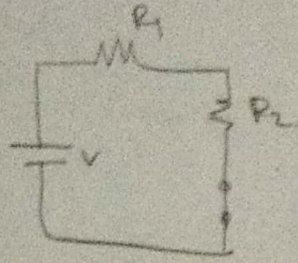
$$\Rightarrow \frac{V}{R_1} = C \longrightarrow (2)$$

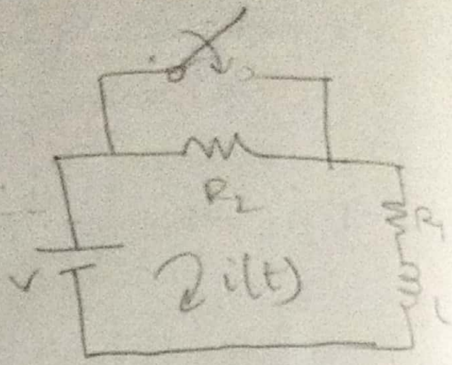
(2) in (1)

$$i(t) = \frac{V}{R_1} e^{-\left(\frac{R_1+R_2}{L}\right)t}$$

Q. A switch is closed at t=0. A steady state having previously attained. Find the current i(t).

Sol: $i(0^-) = \frac{V}{R_1+R_2}$

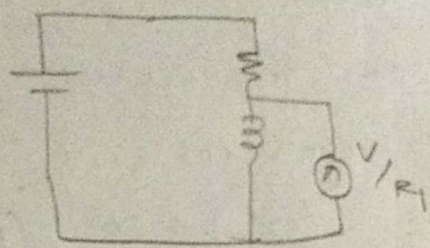




$i(0^+) = \frac{V}{R_1+R_2}$

$$V - R_1 i(t) - L\frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} + \frac{R_1}{L}i(t) = \frac{V}{L}$$



$$i(t) = \cancel{\frac{V}{R_1}\left(1-e^{-\frac{R_1}{L}t}\right)}$$

$$i(t) = ce^{-R/_L t}\int \frac{V}{L} e^{\frac{Rt}{L}} dt + c e^{-\frac{Rt}{L}}$$

$$i(t) = \frac{V}{R_1} + ce^{-\frac{R_1 t}{L}}$$

At $t=0$,

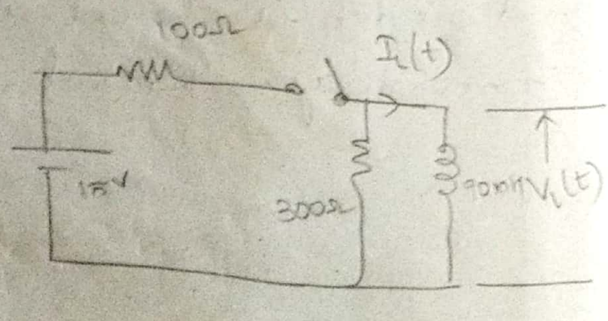
$$i(0) = \frac{V}{R_1} + ce^{-R/_L(0)}$$

$$\frac{V}{R_1+R_2} = \frac{V}{R_1} + C \Rightarrow C = \frac{V}{R_1+R_2} - \frac{V}{R_1}$$

$$i(t) = \frac{V}{R_1} + \left(\frac{V}{R_1+R_2} - \frac{V}{R_1}\right)e^{-R_1/_L t}$$

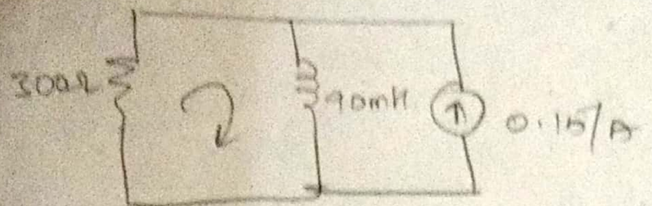
Q. Steady state is reached with switch closed, the switch is open at $t=0$. Find $v_L(t)$, $I_L(t)$

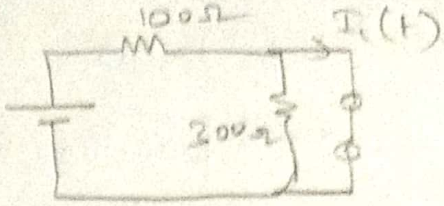
Sol: At $t=0^-$



$$i(0^-) = \frac{15}{100} = 0.15A$$

$V_L(0^-) = 0V$





$$300\,i(t) + 90\times10^{-3}\frac{d}{dt}i(t) = 0$$

$$\frac{d}{dt}i(t) + \frac{300}{90\times10^{-3}}i(t) = 0$$

$$i(t) = c\,e^{-3.33\times10^{3}t}$$

At $t = 0$

$$i(0) = c\,e^{-3.33\times10^{3}(0)}$$

$$c = 0.15A$$

$$i(t) = 0.15\,e^{-3.33\times10^{3}(t)}$$

$$V_L(t) = L\frac{di(t)}{dt}$$

$$= 90\times10^{-3}\frac{d}{dt}\,0.15\,e^{-3.33\times10^{3}t}$$

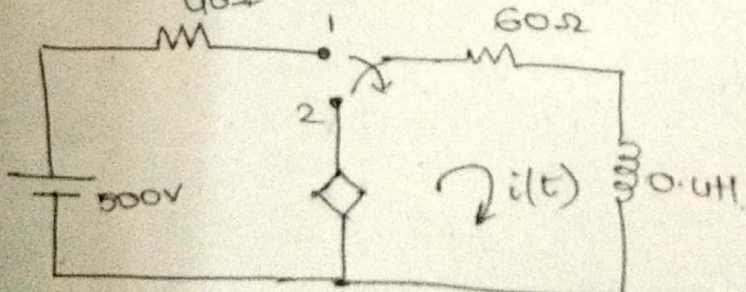
$$= -450\,e^{-33.33\times10^{3}t}$$

At $t = 0$, $V_L(t) = -450V$

26/8/2019

Monday

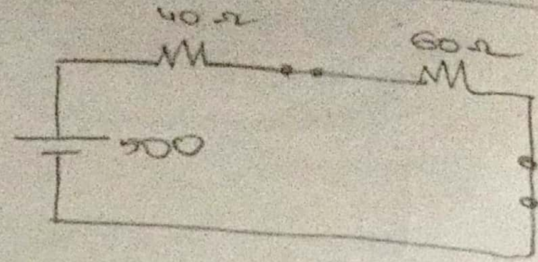
Q.



find $i(t)$ when switch is changed from ① to ② at $t = 0$.

$t = 0^-$

$$i(0^-) = \frac{500}{100} = 5A$$

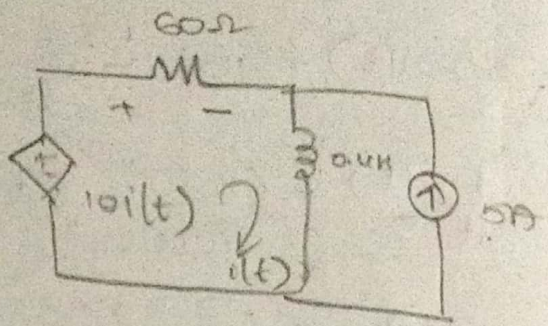


$i(0^+) = 5A$

when position of switch is changed from 1 to 2,

Applying KVL,

$10i(t) - 60i(t) - 0.4\frac{d}{dt}i(t) = 0$



$\frac{di(t)}{dt} + \frac{50}{0.4}i(t) = 0$

$i(t) = ce^{-pt}$

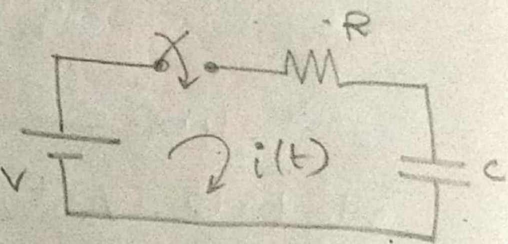
$= ce^{-125t}$

At $t=0$, $c=5$

$i(t) = 5e^{-125t} \quad (t>0)$

RC circuit :

$V - Ri(t) - \frac{1}{C}\int i(t)dt = 0$



$0 - R\frac{d}{dt}i(t) - \frac{1}{C}i(t) = 0$

$\frac{di(t)}{dt} + \frac{1}{RC}i(t) = 0 \longrightarrow (1)$

$i(t) = ce^{-pt} = ce^{-t/RC}$

$\boxed{RC = \tau}$

$t = 0^+ \Rightarrow i(0^+) = \frac{V}{R}$.

$\frac{V}{R} = ce^{-0/RC}$

$\Rightarrow c = \frac{V}{R}$.

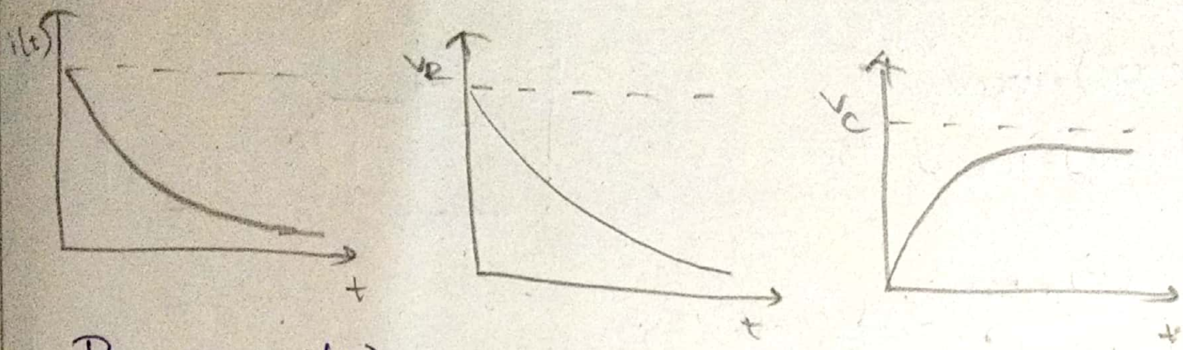
$\Rightarrow i(t) = \frac{V}{R}e^{-t/RC}$

$V_R = Ri(t) = Ve^{-t/RC}$

$V_C = \frac{1}{C}\int_0^t i(t)\,dt = \frac{1}{C}\int_0^t \frac{V}{R}e^{-t/RC}dt$

$$v_C = \frac{V}{RC}\left[e^{-t/RC}\right]_0^t \times (-RC)$$

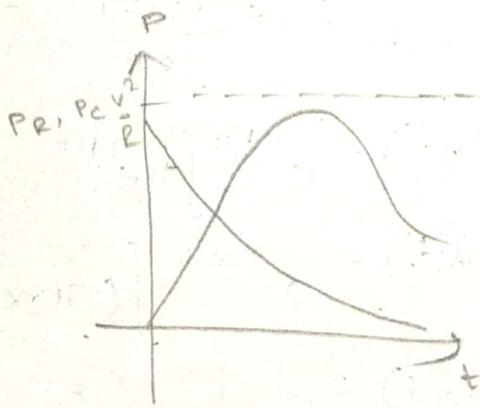
$$= V\left[e^{-t/RC} - e^{-0}\right] = V\left[1 - e^{-t/RC}\right]$$



$P_R = V_R \times i(t)$

$$= Ve^{-t/RC} \times \frac{V}{R} e^{-t/RC}$$

$$= \frac{V^2}{R} e^{-2t/RC}$$



$P_C = V_C \times i(t)$

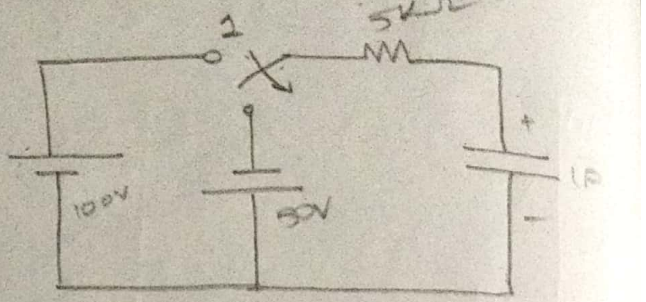
$$= V\left(1 - e^{-t/RC}\right)\frac{V}{R} e^{-t/RC}$$

$$= \frac{V^2}{R}\left[e^{-t/RC} - e^{-2t/RC}\right]$$

Q. A series RC circuit consists of 10Ω Resistor, 0.1F capacitor. A constant voltage of 20V is applied at $t=0$. Apply obtain the current eq, V_R, V_C.

Sol: $i(t) = 2$

Q. The switch in the circuit of figure moved from position 1 to 2 at $t=0$. find $v_c(t)$.

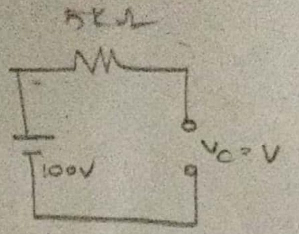


Sol:

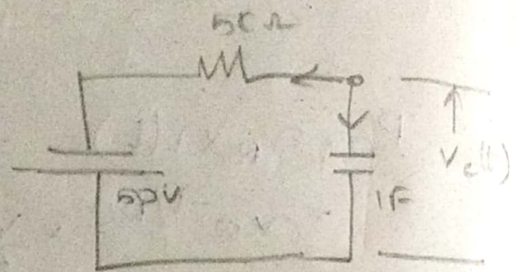
$t=0^-$

$v_c(0^-) = 100V$

$v_c(0^+) = 100V$



$$\frac{v_c(t)+50}{5k} + \frac{d}{dt}v_c(t) = 0$$



$$\frac{d\,v_c(t)}{dt} + \frac{v_c(t)}{5k} = -10\times10^{-3}$$

$$v_c(t) = e^{-pt}\int Ke^{+pt} + ce^{-pt}$$

$$v_c(t) = e^{-\frac{1}{5k}t}\int(-10\times10^{-3})e^{+\frac{t}{5k}} + ce^{-\frac{t}{5k}}$$

$$\left[v_c(t) = -0.25\times10^{-5}e^{-t/5k} + ce^{-t/5k}\right]\times$$

$$v_c(t) = 50 + ce^{-t/5k}$$

$t=0 \Rightarrow 100 = 50 + C$

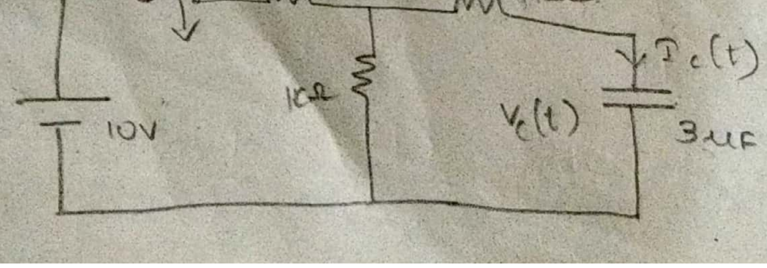
$\Rightarrow C = 50$

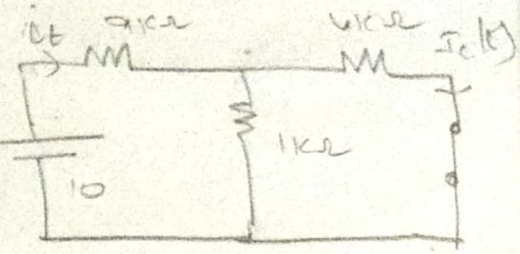
$$v_c = 50(1+e^{-t/5k})$$

27/8/2019

Tuesday

Q. In a network, the switch is closed at $t=0$, the capacitor is initially uncharged. Find $v_c(t)$ & $i_c(t)$





At $= 0^+$

$v_c(0^+) = 0$

$I_c(0^+) = ?$

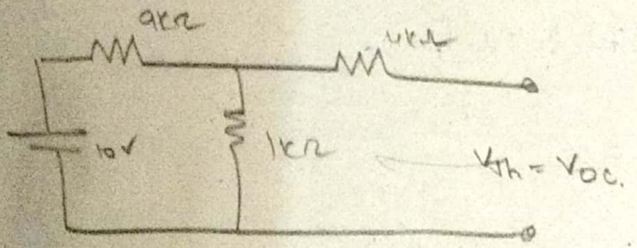
$$I_T(0^+) = \frac{10V}{\left(\frac{4\times1\times10^{6}}{5\times10^{3}}\right)+9k}$$

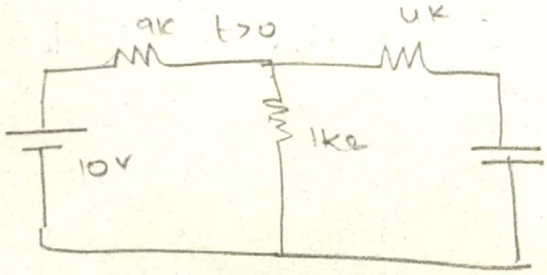
$$= \frac{10}{\frac{4}{5}\times10^{3}+9\times10^{+3}}$$

$$= \frac{10}{(0.8+9)\times10^{+3}}$$

$$= \frac{10}{9.8\times10^{3}} = \frac{1}{9.8\times10^{2}} = \frac{10^{-2}}{9.8} = 1.02\,mA.$$

$$I_c(0^+) = I_T(0^+) \times \frac{1k}{5k} = \frac{1.02\,mA}{5} = 0.204\,mA$$



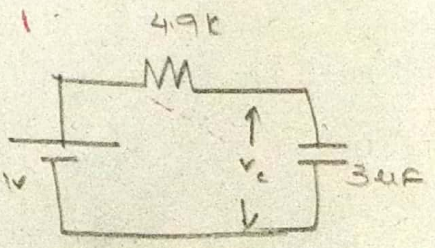


$10 - 10ki = 0$

$$i = \frac{10}{10k} = 1mA$$

$V_{th} = V_{oc} = 1k \times 1m = 1V$

$$R_{th} = \frac{9\times1k}{(9+1)k} + 4k = 4.9k\Omega$$



~~$v_c(t) = e^{-pt}$~~

$$\frac{v_c(t)-1}{4.9k} + 3\times10^{-6}\frac{d}{dt}v_c(t) = 0$$

$$\frac{dv_c(t)}{dt} + \frac{v_c(t)}{4.9\times10^{3}\times3\times10^{-6}} = \frac{1}{4.9k\times3\times10^{-6}}$$

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$$v_c(t) = e^{-Pt}\int \alpha e^{Pt}\,dt + ce^{-Pt}$$

$$= e^{-68.02t}\int 68.02\, e^{68.02t}\,dt + ce^{-68.02t}$$

$$= e^{-68.02t}\, e^{68.02t} \times \frac{68.02}{68.02} + Ce^{-68.02t}$$

$$= 1 + Ce^{-68.02t}$$

At $t = 0^{-}$ $\rightarrow c = -1$

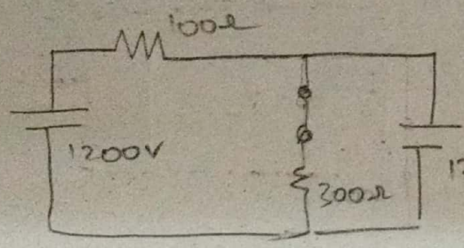
$\Rightarrow v_c(t) = 1 - e^{-68.02t}$

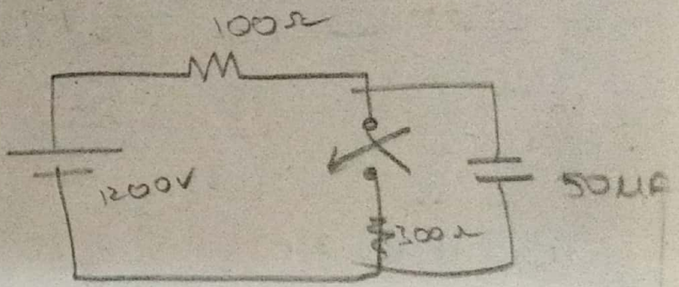
$$i(t) = \frac{C\,d v_c(t)}{dt} = 3\times10^{-6}\frac{d}{dt}\left[1 - e^{-68.02t}\right]$$

$$i(t) = 2.04\times10^{-4}\, e^{-68.02t}.$$

Q. For the network, the switch is opened for long time and closes at $t=0$. Find $v_c(t)$.

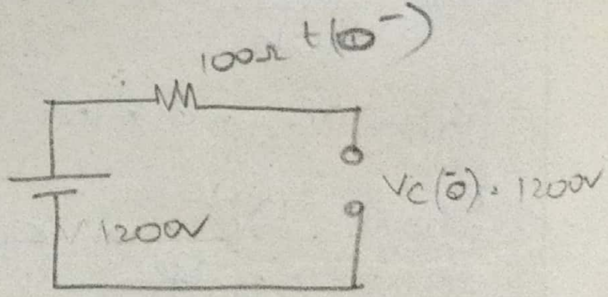
Sol:





$v_c(0^+) = 1200V$

$$I_c(0^+) = \frac{1200}{100} = 12A$$



$$\frac{v_c(t) - 1200}{100} + \frac{v_c(t)}{300} + 50\times10^{-6}\frac{d\,v_c(t)}{dt} = 0$$

$$\Rightarrow \frac{v_c(t) - 1200}{100} + \frac{v_c(t)}{300} = -50\times10^{-6}\frac{d}{dt}v_c(t)$$

$$\rightarrow \cancel{2v_c(t)}\ \frac{v_c(t)}{100} + \frac{v_c(t)}{300} - 12 = -50\times10^{-6}\frac{d}{dt}v_c(t)$$

$$\left(-50\times10^{-6}\frac{d}{dt}v_c(t) = 4\times10^{-3}v_c(t) - 12\right.$$

$$\Rightarrow \frac{d}{dt}v_c(t) = \frac{4\times10^{-3}}{-50\times10^{-6}}v_c(t) - \frac{12}{-50\times10^{-6}}$$

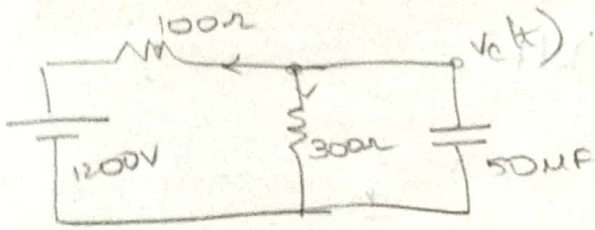
$$\Rightarrow \frac{d}{dt}v_c(t) = -\frac{2\times10^{3}}{25}v_c(t) + \frac{12\times10^{6}}{50}$$

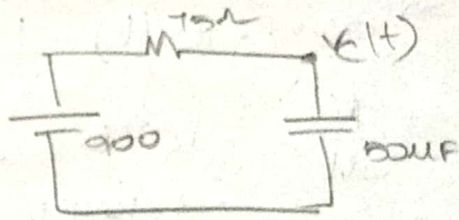
$$\Rightarrow \frac{d}{dt}v_c(t) + \frac{2\times10^{3}}{25}v_c(t) = \frac{12\times10^{6}}{50}$$

$$\Rightarrow \frac{d}{dt}v_c(t) + 80v_c(t) = 24\times10^{4}$$

$$\Rightarrow v_c(t) = e^{-80t}\int(\;)x$$

$v_c(t) = 900 + 300e^{-266.6t}$ for $t > 0$

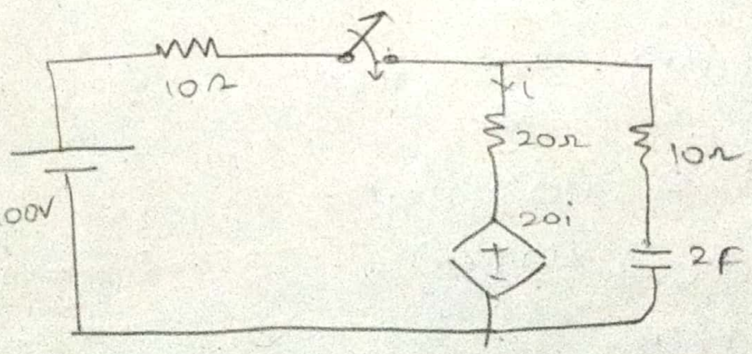


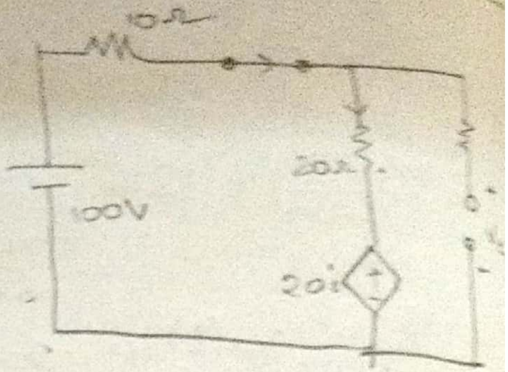


$$V_{oc} = V_{Th} = \frac{1200}{400}\times300 = 900V.$$

$$R_{th} = \frac{100\times300}{400} = \frac{300}{4} = 75\Omega$$

Q. Find the current $i(t)$ when the switch is opened at $t=0$





$100 - 10i(0^-) - 20i(0^-)$
$-20i(0^-) = 0$

$100 = 50i(0^-)$

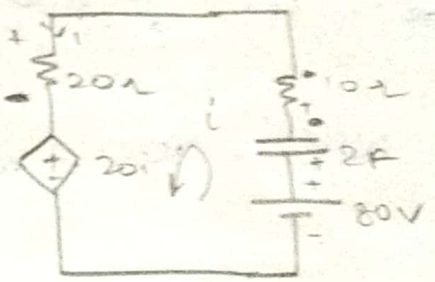
$i(0^-) = \frac{100}{50} = 2A$

$100 - 10i(0^-) = V_c(0^-)$

$100 - 10\times2 = V_c(0^-)$

$V_c(0^-) = 80V$

$i(0^-) = 2A$, $V_c(0^-) = 80V$



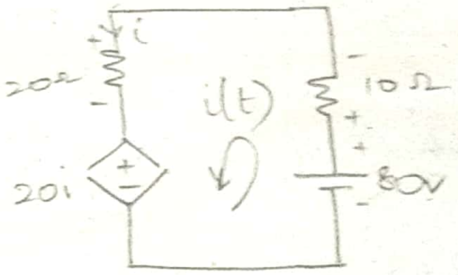
$Y(20i(t) - 20i(t) - 10i(t)$
$-\frac{1}{2}\int i(t)dt - 80 = 0$

$\Rightarrow 80 = -10i(t) - \frac{1}{2}\int i(t)dt)$

$\rightarrow \frac{di}{dt} + 10^4 i = 0$

$\Rightarrow i = Ce^{-1\times10^4 t}$

At $t = 0^+$,



30/8/2019

Friday

Q! $80 - 10i - 20i - 20i = 0$

$80 = 50i$

$i(0^+) = \frac{80}{50} = 1.6A$

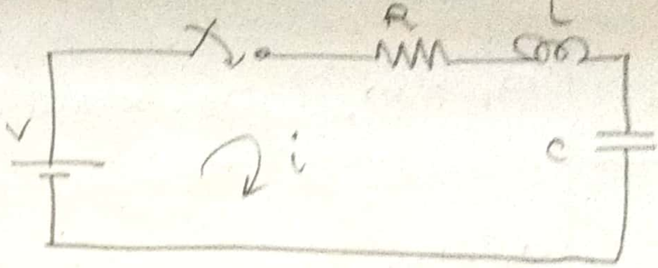
put $t = 0$, $i(0) = 1.6A$

$1.6 = Ce^{-1\times10^4(0)}$

$C = 1.6A$

$i = 1.6e^{-1\times10^4 t}$.

* RLC circuit:



$$V - Ri(t) - \frac{1}{C}\int i(t)dt - L\frac{di(t)}{dt} = 0$$

$$0 - R\frac{d}{dt}i(t) - \frac{1}{C}i(t) - L\frac{d^2}{dt^2}i(t) = 0$$

$$R\frac{d}{dt}i(t) + \frac{1}{C}i(t) + L\frac{d^2}{dt^2}i(t) = 0$$

$$L\frac{d^2}{dt^2}i(t) + \frac{1}{C}i(t) + R\frac{d}{dt}i(t) = 0$$

$$\frac{d^2}{dt^2}i(t) + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = 0.$$

$$S_1, S_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\left[\frac{d^2i}{dt^2} + a_1\frac{di}{dt} + a_2 i = 0\right.$$

$$\left. S_1, S_2 = -\frac{a_1}{2} \pm \sqrt{\left(\frac{a_1}{2}\right)^2 - a_2}\right]$$

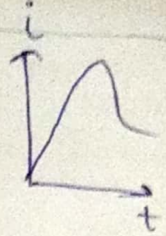
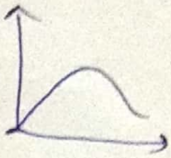
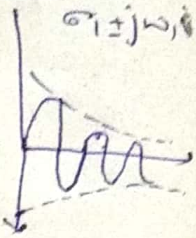
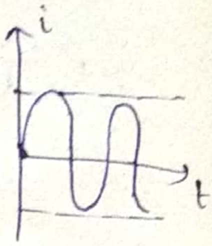
→ Critical resistance R_{cr} is given by,

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

$$\frac{R_{cr}^2}{4L^2} = \frac{1}{LC}$$

$$\Rightarrow R_{cr} = 2\sqrt{\frac{L}{C}}$$

Note:

Coeff of condition	Nature of roots	Descriptive name	Soln	Graph
$a_1^2 > 4a_2$	-ve, real & $\neq$	overdamped	$i = k_1 e^{s_1 t} + k_2 e^{s_2 t}$	
$a_1^2 = 4a_2$	-ve, real & $=$	critically damped	$i = k_1 e^{s_1 t} + k_2 t e^{s_2 t}$	
$a_1^2 < 4a_2$	Complex conj. real part -ve	under damped	$i = e^{\sigma_1 t}(k_1 \cos\omega_1 t + k_2 \sin\omega_1 t)$	$S_1, S_2 = \sigma_1 \pm j\omega_1$ 
$a = 0$, $a_2 \neq b$	conjugate imaginary	oscillatory	$i = k_1 \cos\omega_1 t + k_2 \sin\omega_1 t$, $S_1, S_2 = \pm j\omega_1$	

→ Damping ratio is the ratio of actual resistance to the critical value of resistance.

$$\text{Damping ratio} = \frac{R}{R_{cr}} = \frac{R}{2}\sqrt{\frac{C}{L}} = \zeta$$

→ $$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$2\zeta\omega_n = 2 \times \frac{R}{2}\sqrt{\frac{C}{L}} \times \frac{1}{\sqrt{LC}} = \frac{R}{L}$$

$$\frac{d^2i}{dt^2} + 2\zeta\omega_n\frac{di}{dt} + \omega_n^2 i = 0$$

$$s_1, s_2 = -\zeta\omega_n \pm \left(\sqrt{\omega_n^2}\right)\sqrt{(\zeta^2-1)} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$

$$Q = \omega_n\left(\frac{1}{R}\right) \Rightarrow \omega_n \times \frac{1}{2\zeta\omega_n} = \frac{1}{2\zeta}$$

$$\Rightarrow 2\zeta = \frac{1}{Q}$$

$$\frac{d^2i}{dt^2} + \frac{\omega_n}{Q}\frac{di}{dt} + \omega_n^2 i = 0.$$

$$i = K_1e^{s_1t} + K_2e^{s_2t}$$

$$= K_1e^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right)t} + K_2e^{\left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right)t}$$

→ Damping ratio (ζ) varies from 0 to ∞ ($\because R = 0$ to ∞)

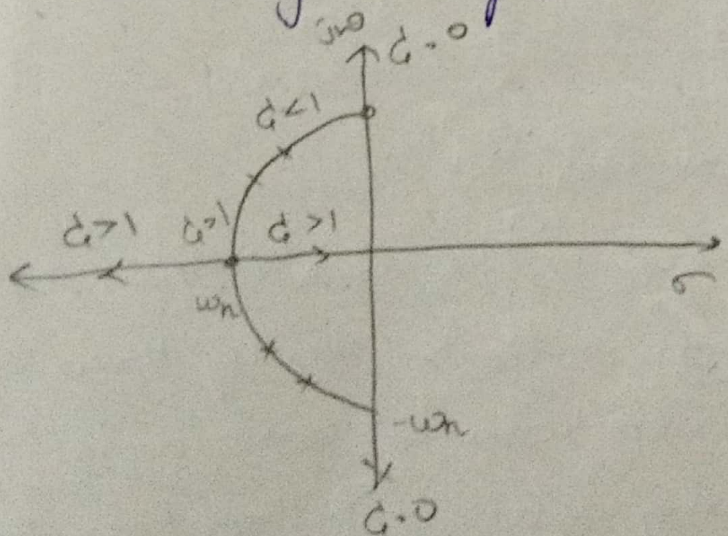
→ $\zeta > 1$ – overdamped case

$\zeta = 1$ – critically damped case.

$\zeta < 1$ – under damped case.

$\zeta = 0$ – oscillatory ($R = 0$).

→ Root locus is given by,



3/9/19 UNIT-III - Two port networks Tuesday

Two port network parameters

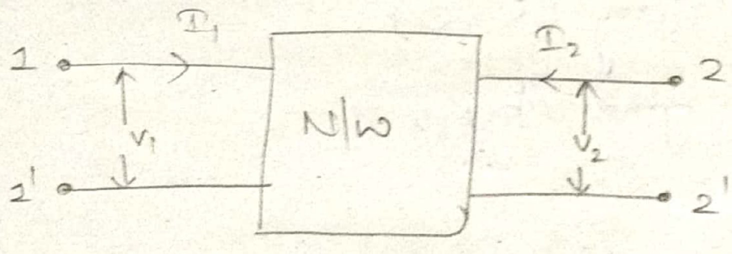
→ Generally any network may be represented by a pair of terminals at which a signal may enter or leave the port.

→ A port is defined as any pair of terminals into which energy is supplied or from which energy is withdrawn where network variables may be measured.

→ A 2-port network is simply ~~a~~ inside a black box and network has only 2 pairs of accessible network terminals.

→ Usually, 1 pair represents i/p and 2nd pair represents o/p.

→ Such a network is common in electronics, communication, transmission and distribution



2port network.

→ 2 ports containing no sources in their branches are called passive ports. Among them are power transmission lines & transformers.

→ 2 ports containing sources in their branches are called Active ports. A voltage or current is

Two port network parameters

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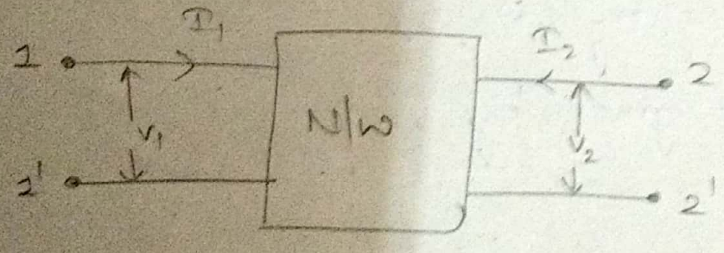


2 port network.

→ 2 ports containing no sources in their branches are called passive ports. Among them are power transmission lines & transformers.

→ 2 ports containing sources in their branches are called Active ports. A voltage or current is assigned to each of 2 ports.

→ Possible combination with V_1, V_2, I_1, I_2.

Parameters.	Expression	In terms of
1. Z parameters	$V_1 V_2$	$I_1 I_2$
2. Y parameters	$I_1 I_2$	$V_1 V_2$
3. Transmission	$V_1 I_1$	$V_2 I_2$
4. Inverse transmission	$V_2 I_2$	$V_1 I_1$
5. Hybrid	$V_1 I_2$	$I_1 V_2$
6. Inverse hybrid	$I_1 V_2$	$V_1 I_2$

* Z-parameters :

→ These are also called open circuit parameters.

→ Z_{11} - Input impedance when o/p port is open circuited and it is also called as driving point i/p impedance.

→ Z_{21} - forward transfer impedance when o/p port is open circuited. It is also called as O.C forward transfer impedance.

→ Z_{12} - Reverse transfer impedance when i/p port is OC and is also called as O.C reverse transfer impedance.

→ Z_{22} - o/p impedance when o/p port is O.C and is also called open circuit o/p impedance

→ $V_1, V_2 = f(I_1, I_2)$

$V_1 = aI_1 + bI_2$

$V_2 = cI_1 + dI_2$

$V_1 = Z_{11}I_1 + Z_{12}I_2$

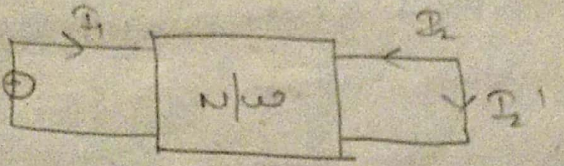
$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(•) **Conditions for reciprocity**:

→ A network is said to be reciprocal, if the ratio of excitation of 1 port to response at other port is same if excitation & response are interchanged.

Case-1:



$V_1 = V_S$ $\quad$ $V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = 0$ $\quad$ $V_2 = Z_{21}I_1 + Z_{22}I_2$

$V_S = Z_{11}I_1 - Z_{12}I_2'$

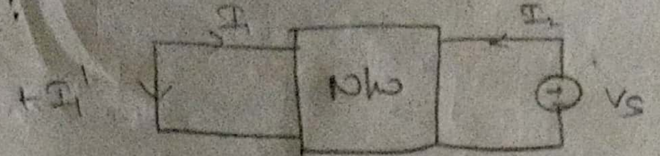
$0 = Z_{21}I_1 - Z_{22}I_2'$

$\Rightarrow I_1 = \dfrac{Z_{22}}{Z_{21}} I_2'$

$\Rightarrow V_S = Z_{11}\left[\dfrac{Z_{22}}{Z_{21}} I_2'\right] - Z_{12}I_2'$

$\Rightarrow \dfrac{V_S}{I_2'} = \dfrac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$

Case-2:



$V_2 = V_S \quad V_1 = 0 \quad I_1 = -I_1'$

$$\frac{V_S}{I_1'} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{12}}$$

$$\frac{V_S}{I_2'} = \frac{V_S}{I_1'} \text{ if } Z_{12} = Z_{21}$$

(c) Conditions for symmetry:

→ For a n/w to be symmetrical, the voltage to current ratio at one port should be same as the voltage to current ratio at another port with one of the port open circuited.

Case-1: If o/p is O.C

$I_2 = 0$

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$\rightarrow Z_{11} = V_1 / I_1$

Case-2: If i/p is O.C

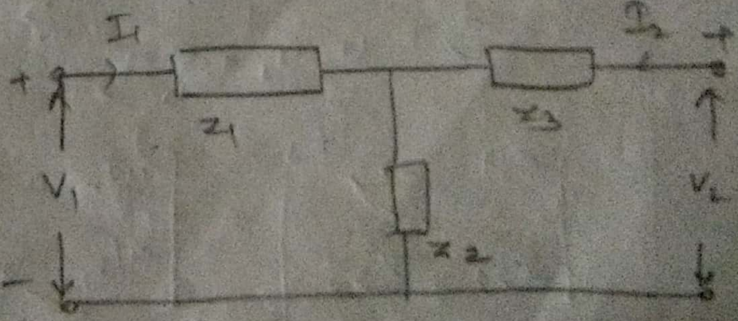
$I_1 = 0$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$Z_{22} = V_2 / I_2$

It is symmetrical, if $\frac{V_1}{I_1} = \frac{V_2}{I_2} \Rightarrow Z_{11} = Z_{22}$

Q. Find the Z parameters.



put $I_2 = 0$

$V_1 = Z_1 I_1 + Z_2 (I_1 - I_2)$

$V_1 = (Z_1 + Z_2) I_1$

$Z_{11} = \frac{V_1}{I_1} = Z_1 + Z_2$

$\Rightarrow V_2 = I_1 Z_2$

$Z_{21} = \frac{V_2}{I_1} = Z_2$

|||^{ly} put $I_1 = 0 \Rightarrow Z_{22} = \frac{V_2}{I_2} = Z_2 + Z_3$

$Z_{12} = \frac{V_1}{I_2} = Z_2$

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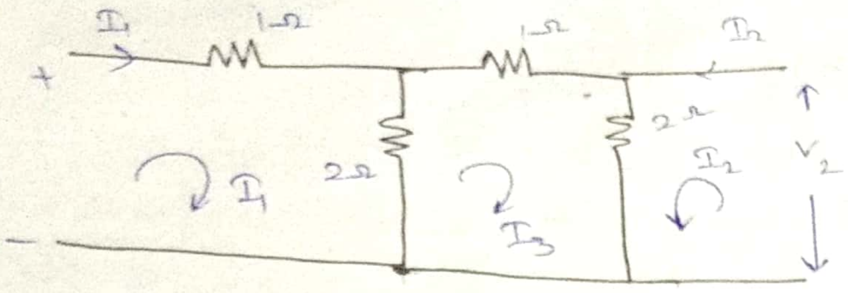
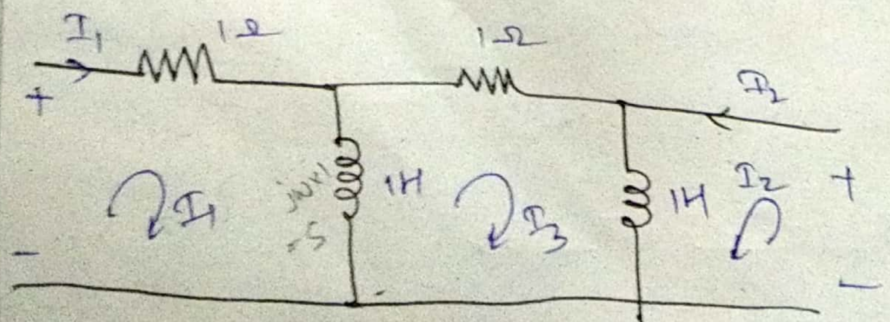
Q. 

Find z parameters

Sol:
$$V_1 = 3I_1 - 2I_3 \longrightarrow (1)$$
$$V_2 = 2(I_2 + I_3) = 2I_2 + 2I_3 \longrightarrow (2)$$
$$-2I_1 + 2I_2 + 5I_3 = 0 \longrightarrow (3)$$
$$I_3 = \frac{-2I_2 + 2I_1}{5}$$
$$\Rightarrow I_3 = 0.4I_1 - 0.4I_2 \longrightarrow (4)$$

Sub (4) in (1) & (2)

$$\Rightarrow V_1 = 3I_1 - 2\left[\frac{2}{5}I_1 - \frac{2}{5}I_2\right]$$
$$= 3I_1 - \frac{4}{5}I_1 + \frac{4}{5}I_2$$
$$\Rightarrow V_1 = \frac{11}{5}I_1 + \frac{4}{5}I_2$$
$$V_2 = 2I_2 + 2\left[\frac{2}{5}I_1 - \frac{2}{5}I_2\right]$$
$$= 2I_2 + \frac{4}{5}I_1 - \frac{4}{5}I_2$$
$$V_2 = \frac{4}{5}I_1 + \frac{6}{5}I_2$$
$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 11/5 & 4/5 \\ 4/5 & 6/5 \end{bmatrix}$$

Q. 

Find z parameters

$V_1 = I_1 + SI_1 - SI_3$ ——— (1)

$V_2 = S(I_2 + I_3) = SI_2 + SI_3$ ——— (2)

$2SI_3 + I_3 - SI_1 + SI_2 = 0$

$2SI_3 + I_3 + S(I_2 - I_1) = 0$

$(2S+1) I_3 + S(I_2 - I_1) = 0$

$$I_3 = \frac{-S(I_2 - I_1)}{2S+1}$$

$$I_3 = \frac{S(I_1 - I_2)}{2S+1} \text{ ——— (3)}$$

Sub (3) in (1) & (2),

$$V_1 = I_1 + SI_1 - S\left[\frac{S(I_1 - I_2)}{2S+1}\right]$$

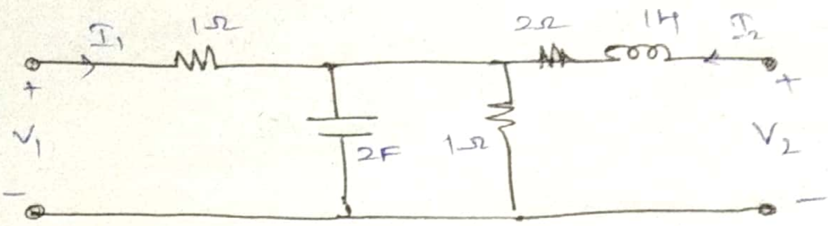
$$= I_1 + SI_1 - \frac{S^2 I_1}{2S+1} + \frac{S^2 I_2}{2S+1}$$

$$= I_1\left[1 + S - \frac{S^2}{2S+1}\right] + \frac{S^2 I_2}{2S+1}$$

$$= I_1\left[\frac{2S+1+2S^2+S-S^2}{2S+1}\right] + \frac{S^2 I_2}{2S+1}$$

$$\text{IIIly } V_2 = \frac{S^2}{2S+1} I_1 + \frac{S^2+S}{2S+1} I_2$$

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{S^2+3S+1}{2S+1} & \frac{S^2}{2S+1} \\ \frac{S^2}{2S+1} & \frac{S^2+S}{2S+1} \end{bmatrix}$$

Q. 

Find Z parameters.

For T circuit,

$Z_{11} = Z_1 + Z_2$

$Z_{22} = Z_2 + Z_3$

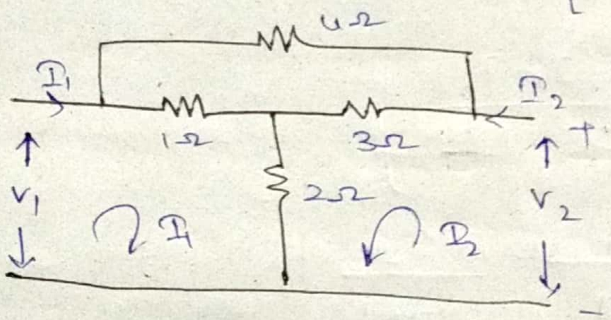
$Z_{12} = Z_{21} = Z_2$

$Z_1 = 1$

$Z_2 = 2F \| 1\Omega$

$Z_3 = 2+S$

we get, $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{2S+2}{2S+1} & \frac{1}{2S+1} \\ \frac{1}{2S+1} & \frac{2S^2+5S+3}{2S+1} \end{bmatrix}$

Q. 

Find Z parameters.

* Y-parameters :-

→ These are also called ~~impedance~~ Admittance parameters, short circuit parameters.

→ $(I_1, I_2) = f(V_1, V_2)$

$I_1 = Y_{11} V_1 + Y_{12} V_2$ —— (1)

$I_2 = Y_{21} V_1 + Y_{22} V_2$ —— (2)

Case-1:

shortcircuit at o/p port, $V_2 = 0$

$I_1 = Y_{11} V_1$

$Y_{11} = I_1 / V_1 \Big/_{V_2 = 0}$

$I_2 = Y_{21} V_1$

$Y_{21} = \frac{I_2}{V_1} \Big/ V_2 = 0$

Case-2 :

Shortcircuit at i/p port, $V_1 = 0$

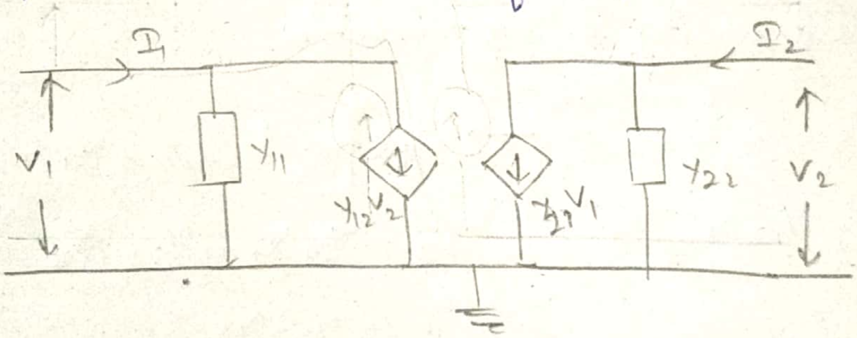
$I_1 = Y_{12} V_2$

$Y_1 = \frac{I_1}{V_2} \Big/ V_1 = 0$

$I_2 = Y_{22} V_2$

$Y_{22} = \frac{I_2}{V_2} \Big/ V_1 = 0.$

→ Equivalent circuit of Y parameters is



$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

→ Y_{11} is the driving pt admittance with o/p port SC.

→ Y_{11} is also called short circuit i/p admittance.

→ Y_{21} is the transfer admittance with o/p port SC.

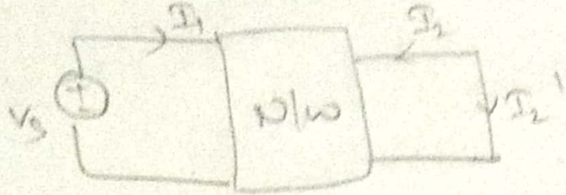
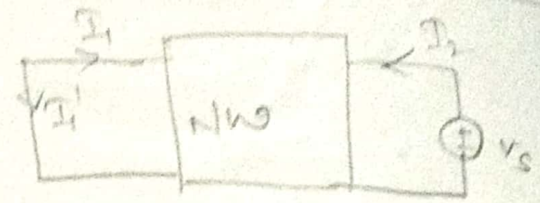
→ Y_{21} is also called short circuit transfer (forward) admittance

→ Y_{12} is the transfer admittance with i/p port SC

→ Y_{12} is also called SC reverse transfer admittance.

→ Y_{22} is the SC driving pt admitance with i/p port SC

→ Y_{22} is also called SC o/p admittance.

→  → 

$V_1 = V_S$ $\qquad\qquad$ $V_2 = V_S$

$V_2 = 0$ $\qquad\qquad$ $V_1 = 0$

$I_2 = -I_2'$ $\qquad\qquad$ $I_1 = -I_1'$

$I_1 = Y_{11} V_S$ $\qquad\qquad$ $-I_1' = Y_{12} V_2 = Y_{12} V_S$

$-I_2' = Y_{21} V_S$ $\qquad\qquad$ $\frac{V_S}{I_1'} = -\frac{1}{Y_{12}}$

$\frac{V_S}{I_2'} = -\frac{1}{Y_{21}}$

$\Rightarrow \frac{V_S}{I_2'} = \frac{V_S}{I_1'} \Rightarrow -\frac{1}{Y_{21}} = -\frac{1}{Y_{12}} \Rightarrow Y_{21} = Y_{12}$

Hence, reciprocity is applicable.

→ o/p port S.C, $V_2 = 0$ $\qquad$ → i/p port S.C, $V_1 = 0$

$I_1 = Y_{11} V_S + Y_{12} V_2$ $\qquad\qquad$ $I_2 = Y_{21} V_1 + Y_{22} V_2$

$I_1 = Y_{11} V_S$ $\qquad\qquad$ $I_2 = Y_{22} V_S$

$\frac{V_S}{I_1} = \frac{1}{Y_{11}}$ $\qquad\qquad$ $\frac{V_S}{I_2} = \frac{1}{Y_{22}}$

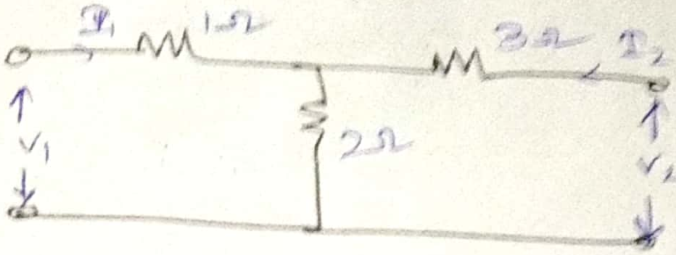
$\Rightarrow \frac{V_S}{I_1} = \frac{V_S}{I_2} \Rightarrow \frac{1}{Y_{11}} = \frac{1}{Y_{22}} \Rightarrow Y_{11} = Y_{22}$

Hence, symmetry is applicable.

Q. Test results of a 2 port networks are (a) port 2 is SC, V_1 $\qquad$, $I_1 = 2.1\angle 30^\circ A$, $I_2 = -1.1\angle -20^\circ A$

(b) port 1 is SC $V_2 = 50\angle 0$, $I_2 = +3\angle -50$, $I_1 = -1.1\angle -20$ A

Q. Find the Y parameters for the network



Case-1: S.C o/p port, $V_2 = 0$

$Y_{11} = \frac{I_1}{V_1}$; $Y_{21} = \frac{I_2}{V_1}$

$V_1 = ((3 \| 2) + 1) I_1$

$Y_{11} = \frac{I_1}{V_1} = \frac{5}{11}$ ℧

$I_2 = -I_1 \times \frac{2}{2+3}$

$I_2 = -\frac{5}{11} V_1 \times \frac{2}{5}$

$I_2 = -\frac{2V_1}{11}$

$\frac{I_2}{V_1} = \frac{-2}{11}$ ℧ $= Y_{21}$