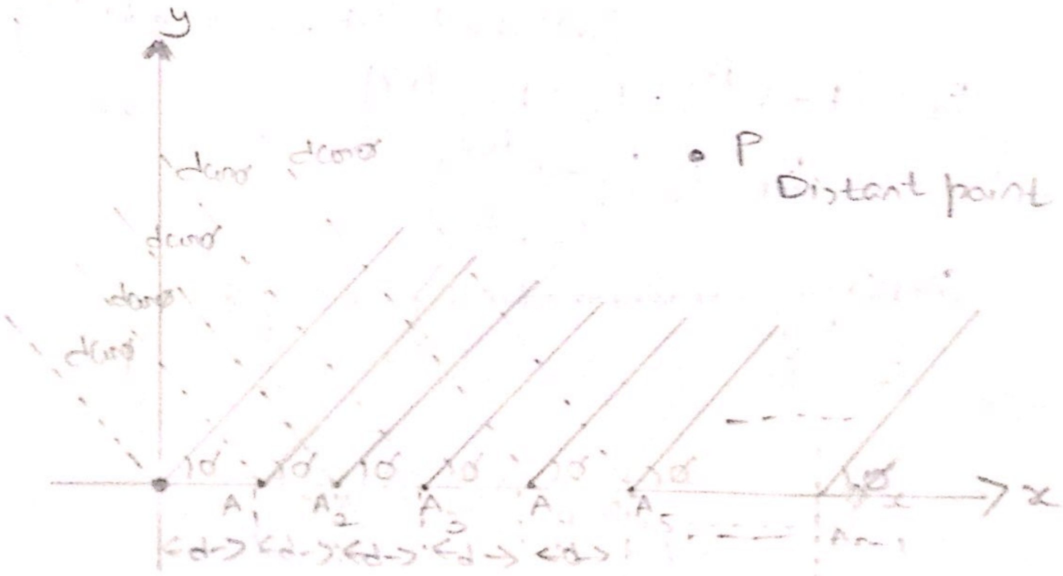
1Q) What is a uniform linear array? obtain the expressions for resultant field for n element case and get the normalized expression ①

Element Uniform Linear Arrays :-

At higher frequencies, for Point communications it is necessary to have a Pattern with single beam radiation. such highly directive single beam Pattern can be obtained by increasing the Point sources.

An array of n elements is said to be a linear array if all the individual elements are spaced equally along a line. An array is said to be uniform array if the elements in the array are fed with currents with equal magnitudes and with uniform Progressive Phase shift along the line.

Consider a general n element linear and uniform array with all the individual elements spaced equally at a distance d from each other and all elements are fed with current equal in magnitude and uniform Progressive Phase shift along line as shown in the fig.



The total resultant field at the distant point P is obtained by adding the fields due to n individual sources, vectorically. Hence we can write

$$E_r = E_0 e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \dots E_0 e^{j(n-1)\psi}$$

$$\boxed{E_T = E_0 (1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi})} \quad (1)$$

Note that $\psi = (\beta d \cos\phi + \alpha)$ indicates the total phase difference of the fields from adjacent sources calculated at point P. Similary α is the progressive phase shift between the two adjacent point sources. The value of α may lie between $0°$ and $180°$. If $\alpha = 0°$ we get n element uniform linear broad side array. If $\alpha = 180°$ we get n element uniform linear end fine array.

Multiplying equation (1) we get,

$$E_T e^{j\psi} = E_0 (e^{j\psi} + e^{j2\psi} + e^{j3\psi} + e^{jn\psi} \quad (2)$$

Subtraction equation (2) from (1) we get

$$E_T - E_T e^{j\psi} = E_0 \{(1 + e^{j2\psi} + \dots + e^{j(n-1)\psi}) - (e^{j\psi} + e^{j2\psi} + \dots + e^{jn\psi})\}$$

$$E_T = (1 - e^{j\psi}) = E_0 (1 - e^{jn\psi})$$

$$E_T = E_0 \left(\frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \right)$$

Simplifying mathematically we get

$$E_T = E_0 \left[\frac{\left(-j 2 \sin\frac{n\psi}{2}\right) e^{j\frac{n\psi}{2}}}{\left(-j 2 \sin\frac{\psi}{2}\right) e^{j\frac{\psi}{2}}} \right]$$

$$E_T = E_0 \left(\frac{\sin \frac{\psi}{2}}{\sin \frac{\psi}{2}} \right) e^{i\left(\frac{n-1}{2}\right)v}$$

This equation (4) indicates the resultant field due to n element array at distant Point P

$$E_T = E_0 \left[\frac{\sin n\frac{v}{2}}{\sin \frac{v}{2}} \right]$$

The Phase angle θ of the resultant field at Point P is given by

$$\theta = \frac{n-1}{2} v = \frac{n-1}{2} (\beta d \cos \phi + \alpha)$$

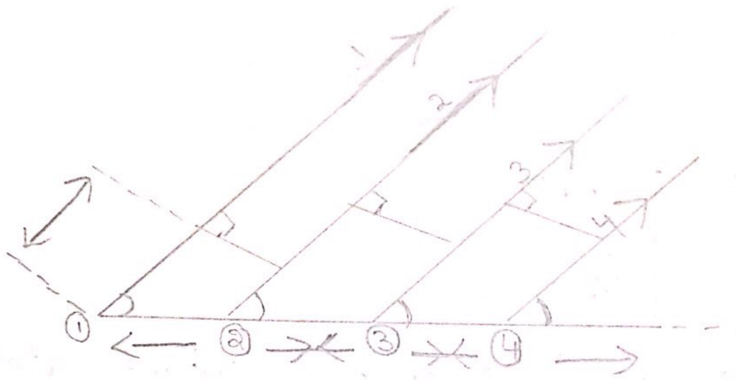
2Q) What is principle of pattern multiplication? Explain with the help of example patterns.

Ans) Pattern Multiplication Method:-

In the Previous sections, we have discussed antena arrays and the methods of obtaining the radiation patterns of the same. other than this straight forward mathematical method, there is another simple method of obtaining the same patterns of the arrays. This method is known as Pattern Multiplication Method. The method is a very useful in the design of arrays because it makes possible to draw the patterns of complicated rays rapidly almost by inspection.

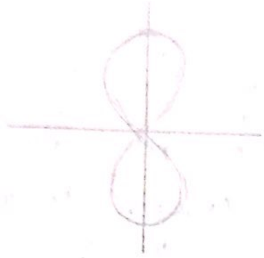
To illustrate the method consider 4 elements array of equispaced identical antennas. Let the spacing between two units be $d = \frac{\lambda}{2}$ Also assume that all the elements are supplied with equal

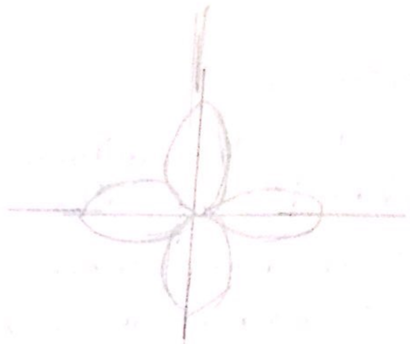
Magnitude currents which are in Phase.



As the Point P at which the resultant field has to be obtained is far away we can assume the radiation from the antenna in the form of Parllel lines.

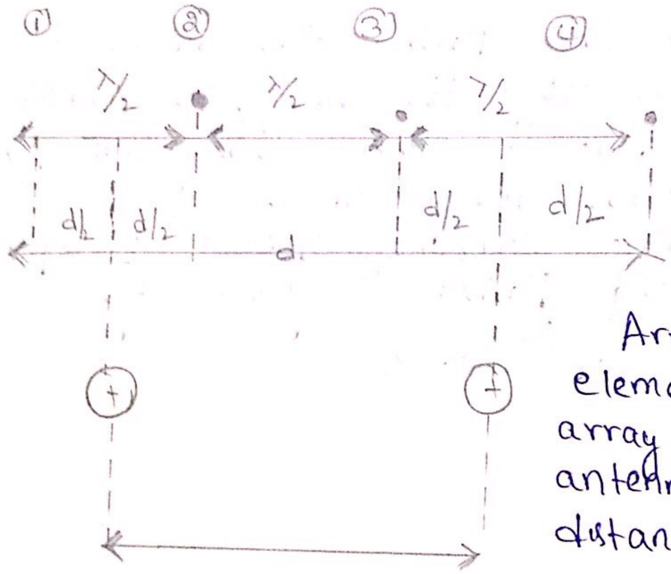
The radiation Pattern of the antennas (1) & (2) treated to be operating as a single unit is similar to the radiation Pattern of the antennas (3) and (4) spaced $\frac{\lambda}{2}$ distance apart and fed with equal current in phase treated to be operated as single unit. Now instead of considering two two seperate elements (1) and (2) we are replace it by a single antenna located at a Point mid way between them $(\frac{d}{2})$ Similarly replacing antennas (3) and (4) by single antenna having same Pattern Now both the antennas have bidirection pattern. i.e figure eight Pattern spaced distance λ apart from each other fed with equal current in Phase is also (b) Now the resultant radiation Pattern of four element array can be obtained as the multiplication of Pattern (d) Note that this multiplication for different value of d





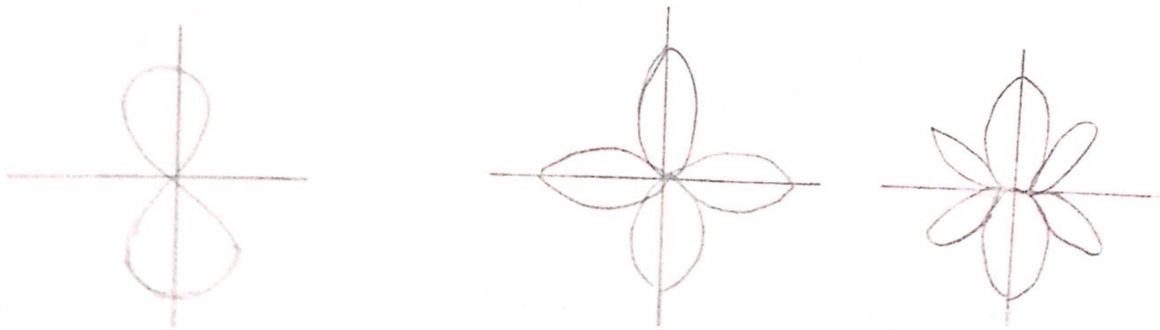
(a) Radiation Pattern of two antennas spaced at distance $\frac{\lambda}{2}$ and fed with equal current in Phase.

(b) Radiation Pattern of two antennas spaced at distance λ and fed with equal currents in Phase.



Array of 4 identical element Replacement of array by two single antennas placed at distance λ apart.

(C) antenna (1) and (2) and (3) and (4) replaced by single antenna seperately.



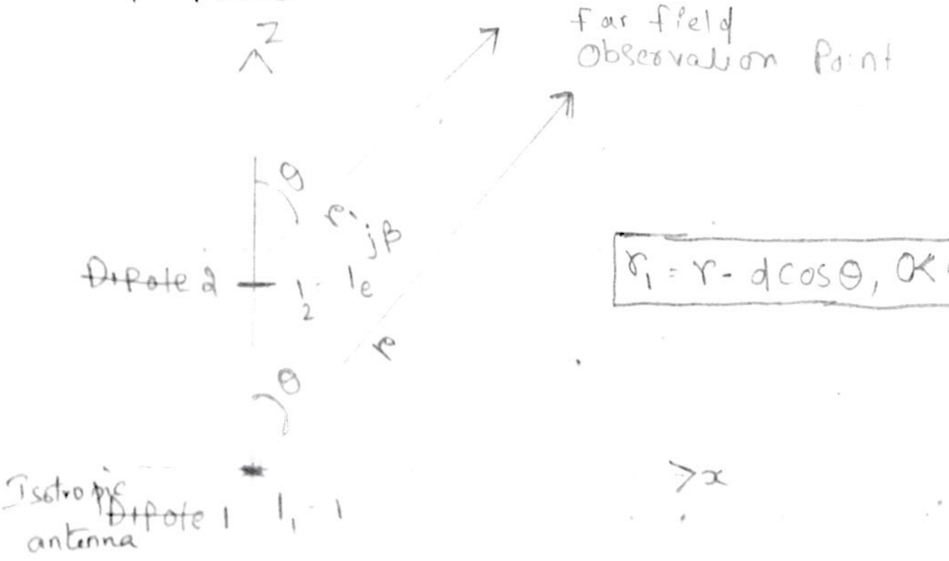
Multiplication of Pattern

Q3)

Array of 2 Point Sources: Various cases

Array is an assembly of antennas in an electrical and geometrical of such nature that the radiation from each element add up to give a maximum field intensity in a particular direction & cancels in other directions. An important characteristics of an Array is the change of its radiation pattern in response to different excitations of its antenna elements.

CASE 1: Istropic Point Sources of Same Amplitude and Phase:



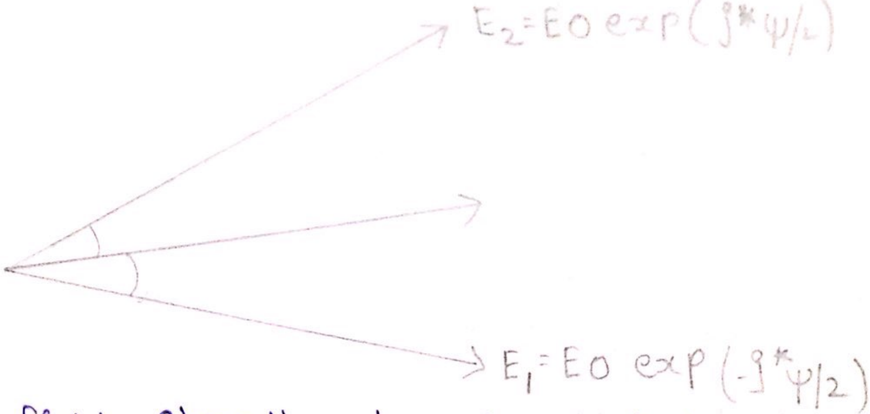
$$\boxed{r_1 = r - d\cos\theta,\ 0 < \theta < \pi}$$

Phase difference $= \beta d/2 * \cos\theta = 2\pi/\lambda * d/2 * \cos\theta$

Point Sources and Arrays:

β: Propogation Constant i.e : $1/2 * d_r * \cos\theta$

Where $d_r = \beta d = 2\pi/\lambda * d$ = Path difference



The total field strength at a large distance r in the direction θ is $E = E_1 + E_2 = E_0 \,(\exp(j\psi/2)$

Therefore $E = 2E_0 \cos\psi/2$ .... (*) (by trigonometry)

ψ = Phase difference b/w E_1 & E_2 & $\psi/2 = d_r/2 * \cos\theta$

E_0 = amplitude of the field at a distance by single isotropic antenna

Substituting for ψ in (*) & normalizing

$E = 2E_0 \cos(2\pi/\lambda * d/2 * \cos\theta)$

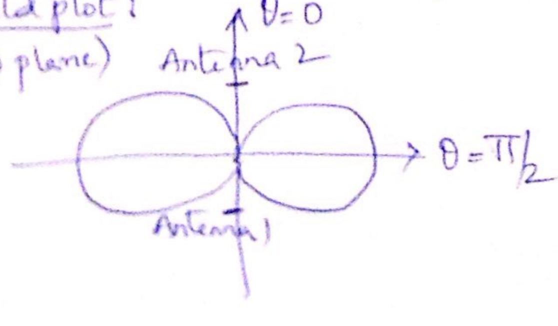
for $d = \lambda/2$ $\quad E = \cos(\pi/2 * \cos\theta$

At $\theta = \pi/2$ $\quad E = 1$ Point of maxima $= \pi/2$ (or) $3\pi/2$

At $\theta = 0$. $E = 0$ Point of minima = 0

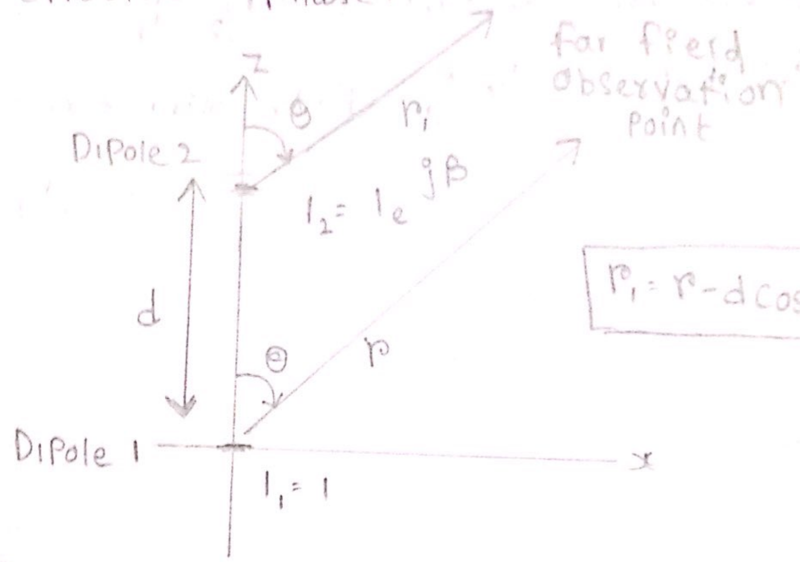
At $\theta = \pm\pi/3$ $\quad E = 1/\sqrt{2}$ 3db bandwidth Point $= \pm\pi/3$

E field plot :
(θ plane)



Case 2:

Isotropic Point Sources of Same Amplitude, but opposite Phase:



$\boxed{r_1 = r - d\cos\theta,\ 0<\theta<\pi}$

The total field strength at a large distance r in the direction θ is: $E = E1 + E2 = E0(\exp(j*\psi/2 - \exp(-j\psi/2))$

Therefore $E = 2jE0\sin(\psi/2)$ ..... (*)

ψ = Phase difference B/w E1 & E2

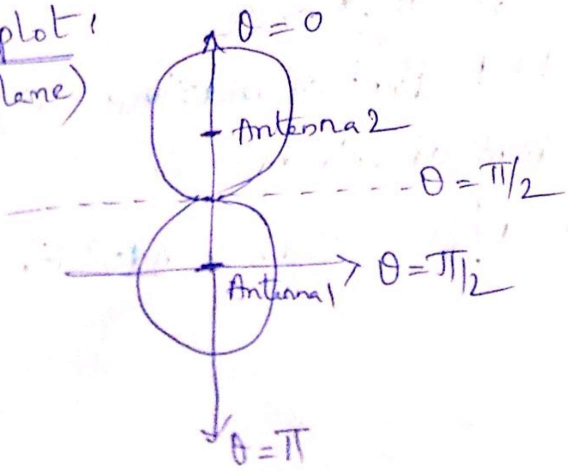
$\psi/2 = dr/2 * \cos\theta$

E0 = amplitude of the field at a distance by single isotropic antenna

At K = 0 E = 1 Point of maxima = 0 (or) π

At K = 0 θ = π/2 E = 0 Point of minima = π/2 (or) -π/2

E-field plot: (θ plane)



4. What is array factor? What is a Broad Side Array and End Fire Array? What is a scanning array or phased array?

Array Factor :- The array factor is the ratio of the magnitude of the resultant field to the magnitude of the maximum field.

$$\text{Array factor} : A.F = \frac{|E_T|}{|E_{max}|}$$

But maximum field is $E_{max} = 2E_0$

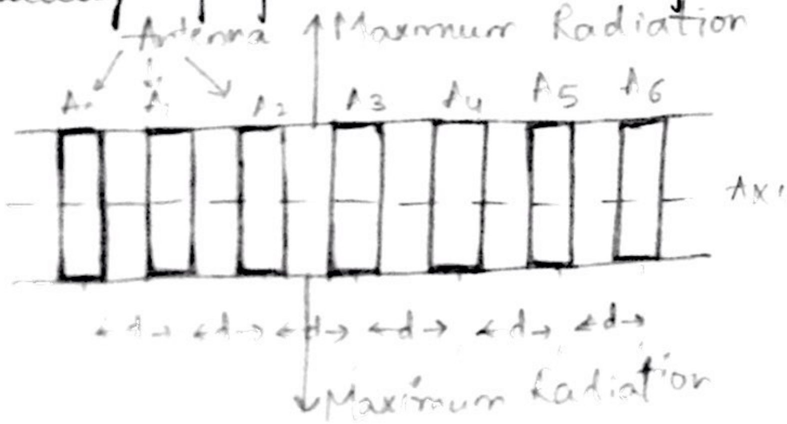
$$A.T = \frac{|E_T|}{|E_0 \times 2|} = \frac{|E_T|}{|2E_0|} = \cos\left(\pi \frac{d}{\lambda} \cos\phi\right) \text{ — for 2 element array}$$

$$= \frac{1}{n} \sin \frac{n\psi}{2} \text{ — for n element array}$$

The array-factor represents the relative value of the field as a function of ϕ_i or θ. It defines the radiation pattern in a plane containing the line of the array.

$$(\psi = \beta d \cos\phi)$$

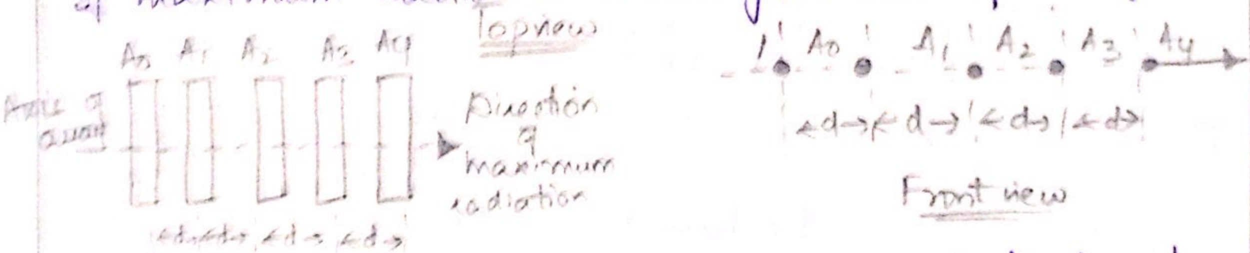
Broad Side Array :- The broad side array is the array of antennas in which all the elements are placed parallel to each other and the direction of maximum radiation is always perpendicular to the plane consisting elements.



Typical Pattern:

A broad side array consists of number of identical antennas placed parallel to each other along a straight line. The straight line is perpendicular to the axis of individual antenna. It is known as axis of antenna array. Thus, each element is perpendicular to the axis of antenna array. All the individual antennas are spaced equally along the axis of antenna array and the spacing between the two elements is denoted by "d".

<u>End Fire Array</u>:- The End Fire Array is very much similar to the broadside array from the point of view of arrangement. But, the difference is in the direction of maximum radiation. In the end fire array, the direction of maximum radiation is along the axis of array.

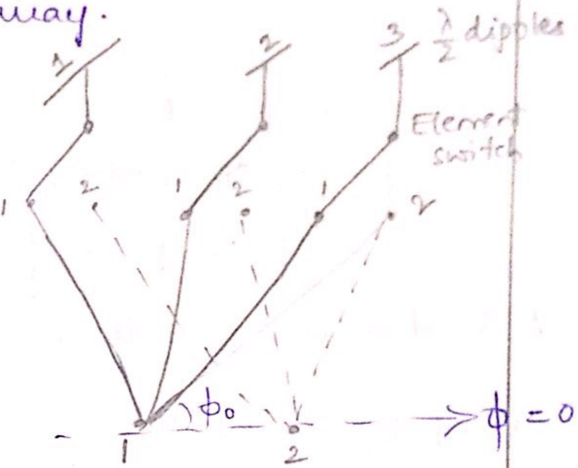


Thus in the end fire array number of identical antennas are spaced equally along a line. All the antennas are fed individually with currents of equal magnitude but their phases vary progressively along the line to get entire arrangement unidirectional finally.
It can also be defined as an array with direction of maximum radiation coincides with the direction of the axis of array to get a unidirectional radiation.

Typical pattern:

Scanning Array (or) Phased Array:- The array in which the phase ~~and the amplitude of most~~ of the elements is variable provided that the direction of maximum radiation[(beam)-(direction)] and pattern shape along with the side lobes is controlled, is called as phased array.

Suppose the array gives maximum radiation in direction $\phi = \phi_0$, where $0 \le \phi_0 \le 180°$, then the phase shift that must be controlled can be obtained as follows:-



Phased array with mechanical switches of elements and feed point

$$\psi = kd\cos\phi + \alpha \Big|_{\phi=\phi_0} = 0$$

$$kd\cos\phi_0 + \alpha = 0$$

$$\boxed{\therefore \alpha = -kd\cos\phi_0}$$

Thus, maximum radiation can be achieved in any direction if the progressive phase direction between the elements is controlled.

5. Get the expressions for Directivity and HPBW of BSA.

Directivity of Broad Side Array (BSA):- The directivity in case of Broad side array is defined as:

$$G_{D_{max}} = D = \frac{U_{max}}{U_0} = \left[\frac{\text{Max}}{\text{Avg}}\right] \text{Radiation Intensity}.$$

where $U_0 = \frac{P_{rad}}{4\pi} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} |E(\theta,\phi)|^2 \sin\theta \, d\theta \, d\phi. \rightarrow ①$

Expression of ratio of magnitudes, $\left|\frac{E_T}{E_0}\right| = n$

$|E_T| = n|E_0|$

$\therefore E_{normalised} = \left|\frac{E_T}{E_{Tmax}}\right| = \frac{|E_0|}{n|E_0|} = \frac{1}{n}$

Hence the field is given by, $E_{Normalised} = \frac{\sin \frac{n\psi}{2}}{n\left(\sin\frac{\psi}{2}\right)}$

where $\psi = \beta d \cos\phi$

$$\therefore E = \frac{1}{n}\left[\frac{\sin \frac{n\beta d\cos\phi}{2}}{\sin \frac{\beta d\cos\phi}{2}}\right] \rightarrow ②$$

But, d is very small i.e, $\sin\frac{\beta d\cos\phi}{2} \simeq \frac{\beta d\cos\phi}{2}$

Sub eqn ② in eqn ①

$$U_0 = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\left[\frac{\sin\frac{n\beta d\cos\phi}{2}}{\sin\frac{\beta d\cos\phi}{2}}\right]^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{4\pi}\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi}\left[\frac{\sin\frac{n\beta d\cos\phi}{2}}{\sin\frac{\beta d\cos\phi}{2}}\right]^2 \sin\theta\, d\theta$$

$$= \frac{1}{4\pi}(2\pi)\int_{\theta=0}^{\pi}\left(\frac{\sin z}{z}\right)^2 \sin\theta\, d\theta$$

Limits:-

$\theta = \pi \rightarrow z = -\frac{n}{2}\beta d$

$\theta = 0 \rightarrow z = \frac{n}{2}\beta d$

let $z = \frac{n}{2}\beta d\cos\theta$

$dz = -\frac{n}{2}\beta d\sin\theta\, d\theta$

$\therefore \sin\theta\, d\theta = \frac{-dz}{\frac{n}{2}\beta d}$

$$\therefore U_0 = \frac{1}{2}\int_{\frac{n}{2}\beta d}^{-\frac{n}{2}\beta d}\left(\frac{\sin z}{z}\right)^2 \cdot \frac{-dz}{\frac{n}{2}\beta d}$$

$$U_0 = \frac{-1}{n\beta d}\int_{\frac{n}{2}\beta d}^{-n/2\beta d}\left(\frac{\sin z}{z}\right)^2 dz$$

$\therefore U_0 = \frac{1}{n\beta d}\int_{-\infty}^{\infty}\left(\frac{\sin z}{z}\right)^2 dz$; As n is large; hence $n\beta d$ is also very large for large array

We know: $\int_{-\infty}^{\infty}\left(\frac{\sin z}{z}\right)^2 dz = \pi$

$$\therefore U_0 = \frac{1}{n\beta d}(\pi) = \frac{\pi}{n\beta d}$$

$$\therefore G_{Dmax} = D = \frac{U_{max}}{U_0} = \frac{1}{\pi/n\beta d} = \frac{n\beta d}{\pi}$$

$$= \frac{n}{\pi}\left(\frac{2\pi}{\lambda}\right)d = \frac{2nd}{\lambda} = 2n\left(\frac{d}{\lambda}\right)$$

$$\boxed{\therefore G_{Dmax} = 2n\left(\frac{d}{\lambda}\right)}$$

The total length of the array is given by; $L = (n-1)d \simeq nd$ where n is large

$$\boxed{\therefore G_{Dmax} = \frac{2L}{\lambda}}$$ For Broad Side Array.

HPBW (Half Power Beam Width):- ($\to$ BSA)

The beamwidth between first nulls is given by,

$$BWFN = 2\times\gamma \quad \text{where } \gamma = 90-\phi$$

$$\therefore \phi_{min} = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right) \text{; where } m = 1,2,3\ldots$$

Also $90-\phi_{min} = \gamma$; i.e $90-\gamma = \phi_{min}$

$$90-\gamma = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right)$$

$$\cos(90-\gamma) = \cos\left[\cos^{-1}\left(\pm\frac{m\lambda}{nd}\right)\right]$$

$$\sin\gamma = \pm\frac{m\lambda}{nd}$$

As γ is small; $\sin\gamma \simeq \gamma$ $\quad \therefore \gamma = \pm\frac{m\lambda}{nd}$

For 1^{st} null: $m=1 \Rightarrow \gamma = \frac{1\lambda}{nd}$

$$\boxed{BWFN = 2\gamma = \frac{2\lambda}{nd} = \frac{2\lambda}{L} = \frac{2}{L_\lambda}} = \frac{114.6}{L/\lambda} \text{ degrees}$$

$$HPBW = \gamma = \frac{\lambda}{nd}.$$

But $nd = (n-1)d$: n is very large ; it is denoted by "L" ie, total length of the array.

$$\therefore HPBW = \frac{BWFN}{2} = \frac{1}{(L/\lambda)} \text{ rad}$$

Expressing, HPBW in degrees,

$$\boxed{\therefore HPBW = \frac{57.3}{(L/\lambda)} \text{ degrees}}$$

---

6. Get the expressions for Directivity and HPBW of End Fire Array (EFA)

Directivity of End Fire Array:- The directivity in case of End Fire Array is defined as

$$G_{Dmax} = D = \frac{U_{max}}{U_0} = \left[\frac{Max}{Avg}\right] \text{ Radiation Intensity.}$$

where $U_0 = \frac{P_{rad}}{4\pi} = \frac{1}{4\pi} \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} |E(\theta,\phi)|^2 \sin\theta \, d\theta \, d\phi \longrightarrow (1)$

Expression of ratio of magnitude, $\frac{|E_T|}{|E_0|} = n \Rightarrow |E_T| = n|E_0|$

$$\therefore E_{normalised} = \left|\frac{E_T}{E_{max}}\right| = \frac{|E_0|}{n|E_0|} = \frac{1}{n}$$

Hence, the field is given by, $E_{normalised} = \frac{\sin \frac{n\psi}{2}}{n(\sin \psi/2)}$

where $\psi = \beta d \cos\phi$

$$\therefore F = \frac{1}{n}\left[\frac{\sin \frac{n\beta d\cos\phi}{2}}{\sin \frac{\beta d\cos\phi}{2}}\right] \rightarrow ②$$

But; d is very small i.e, $\sin\frac{\beta d\cos\phi}{2} \simeq \frac{\beta d\cos\phi}{2}$

Sub eqn ② in eqn ①

$$U_0 = \frac{1}{4\pi}\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi}\left[\frac{\sin \frac{n\beta d\cos\phi}{2}}{\sin \frac{\beta d\cos\phi}{2}}\right]^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{4\pi}\int_{\phi=0}^{\pi} d\phi \int_{\theta=0}^{\pi}\left[\frac{\sin \frac{n\beta d\cos\phi}{2}}{\sin \frac{\beta d\cos\phi}{2}}\right]^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{4\pi}(\pi)\int_{\theta=0}^{\pi}\left(\frac{\sin z}{z}\right)^2 \sin\theta \, d\theta$$

Let $z = \frac{n}{2}\beta d\cos\theta$

$dz = -\frac{n\beta d}{2}\sin\theta \, d\theta$

$\sin\theta \, d\theta = \frac{-dz}{\frac{n}{2}\beta d}$

Limits

$\theta = \pi \rightarrow z = \frac{-n}{2}\beta d$

$\theta = 0 \rightarrow z = \frac{n}{2}\beta d$

$$\therefore U_0 = \frac{1}{4}\int_{n/2\beta d}^{-n/2\beta d}\left(\frac{\sin z}{z}\right)^2 \cdot \frac{-dz}{\frac{n\beta d}{2}} = \frac{-1}{2n\beta d}\int_{\frac{n}{2}\beta d}^{-n/2\beta d}\left(\frac{\sin z}{z}\right)^2 dz$$

$$\therefore U_0 = \frac{1}{2n\beta d}\int_{-\infty}^{\infty}\left(\frac{\sin z}{z}\right)^2 dz$$; As n is large; hence $n\beta d$ is very large, for large array

We know; $\int_{-\infty}^{\infty}\left(\frac{\sin z}{z}\right)^2 dz = \pi$

$$\therefore U_0 = \frac{1}{2n\beta d}(\pi) = \frac{\pi}{2n\beta d}$$

$$\therefore G_{Dmax} = \frac{U_{max}}{U_0} = \frac{1}{\pi/2n\beta d} = \frac{2n\beta d}{\pi}$$

$\therefore G_{Dmax} = \frac{2n}{\pi}\left(\frac{2\pi}{\lambda}\right)(d) = \frac{4nd}{\lambda} = \frac{4L}{\lambda}$ as $L = nd$

i.e, the total length of the array is given by: $L = (n-1)d \approx nd$
$\because n$ is large

$$\boxed{\therefore G_{Dmax} = \frac{4L}{\lambda}}$$ For End Fire Array

HPBW (Half Power Band Width): ($\rightarrow$ EFA)

The beamwidth of the end fire array is greater than that of broad side array.

Beamwidth = 2 x Angle between first nulls and maximum of the major lobe i.e θ_{min}

we have,

$$\phi_{min} = 2\sin^{-1}\left(\pm\sqrt{\frac{m\lambda}{2nd}}\right)$$

$$\therefore \sin\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

If ϕ_{min} is very low, then we can write $\sin\frac{\phi_{min}}{2} \approx \frac{\phi_{min}}{2}$.

$$\therefore \frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

$$\therefore \phi_{min} = \pm\sqrt{\frac{4m\lambda}{2nd}} = \pm\sqrt{\frac{2m\lambda}{nd}}$$

But $nd = L \rightarrow \phi_{min} = \pm\sqrt{\frac{2m\lambda}{L}} = \pm\sqrt{\frac{2m}{(L/\lambda)}}$ ($m = 1$ for first null)

$$\therefore BWFN = \pm 114.6\sqrt{\frac{2m}{(L/\lambda)}} \text{ in degrees}$$

$$\boxed{\therefore HPBW = \pm 57.3\sqrt{\frac{2m}{(L/\lambda)}} \text{ in degrees.}}$$

7) Get the expressions for Directivity and HPBW of modified EFA.

Ans: End Fire Antenna with Increased Directivity: (Hansen-Woodyard Array)

Directivity of EFA can be increased without affecting other characteristics. This array is referred to Hansen-Woodyard Array. or end fire antenna with increased directivity.

The phase shift between closely spaced elements of very long array is given by.

$$\alpha = -\left[\beta d + \frac{\pi}{n}\right]$$

$$= -\left[\beta d + \frac{2.94}{n}\right] \quad \text{Max in } \theta = 0^\circ \rightarrow (1)$$

$$\alpha = +\left[\beta d + \frac{\pi}{n}\right]$$

$$= +\left[\beta d + \frac{2.94}{n}\right] \quad \text{Max in } \theta = \pi^\circ \rightarrow (2)$$

Then Directivity of array factor is calculated as

$$D = \frac{4\pi \phi_{max}}{P_{rad}}$$

$$D = \frac{\phi_{max}}{\frac{P_{rad}}{4\pi}} = \frac{\phi_{max}}{\phi_0}$$

where $\phi_0 \rightarrow$ Average radiation intensity.

$$\phi_0 = \frac{P_{rad}}{4\pi}$$

$$= \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \phi(\theta) \sin\theta \, d\theta \, d\phi$$

$$= \frac{2\pi}{4\pi} \int_0^{\pi} \left| \frac{z'}{\sin z'} \cdot \frac{\sin z}{z} \right|^2 \sin\theta \, d\theta \qquad \because \int_0^{2\pi} \phi(\theta) \, d\phi$$

Since

$$\phi(\theta) = \left[\frac{z'}{\sin z'} \cdot \frac{\sin z}{z}\right]^2$$

where

$$z = q[\beta d \cos\theta - p], \quad q = \frac{nd}{2}, \quad z' = q[\beta - p]$$

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$$\phi_0 = \frac{1}{2}\left[\left|\frac{z'}{\sin z'}\right|^2\right] \int_0^{\pi} \left|\frac{\sin z}{z}\right|^2 \sin\theta \, d\theta$$

$$\phi_0 = \frac{1}{2}\left[\frac{q(\beta-p)}{\sin q(\beta-p)}\right]^2 \times \int_0^{\pi} \left[\frac{\sin q(p\cos\theta-p)}{q(p\cos\theta-p)}\right]^2 \sin\theta \, d\theta \rightarrow (4)$$

Now, max directivity can be obtained by minimizing above equation

$$\Rightarrow \phi_0 = \frac{1}{4pq}\left[\frac{v}{\sin v}\right]^2 \times \left[\frac{\pi}{2} + \frac{\cos(2v)-1}{2v} + \sin 2v\right] \rightarrow (5)$$

where

$v = q(\beta - p)$

But, in case of increased or improved EFA can be reduced to

$$\phi_0 = \frac{1}{n\beta d}\left(\frac{\pi}{2}\right)^2 \left[\frac{\pi}{2} + \frac{2}{\pi} - 1.8515\right]$$

$\phi_0 = \frac{0.878}{n\beta d}$ By substituting all values

$$\Rightarrow \phi_0 = \frac{0.878}{n\beta d} \times \frac{2\pi}{2\pi}$$

$$\phi_0 = \frac{0.878 \times 2}{\pi} \times \frac{\pi}{2n\beta d} = 0.5589\left[\frac{\pi}{2n\beta d}\right]$$

$$\phi_0 = 0.5589\left[\frac{\pi}{2n\beta d}\right]$$

$D = \frac{\phi_{max}}{\phi_0}$ $\phi_{max} = 1$

$$D = \frac{2n\beta d}{\pi} \times \frac{1}{0.5589} = \frac{2n \times 2\pi \times d}{\pi\lambda} \times \frac{1}{0.5589}$$

$$D = 1.789\left[4n\left(\frac{d}{\lambda}\right)\right]$$

$L = (n-1) \approx nd.$

$$\Rightarrow \boxed{D = 1.789\left[4\left(\frac{L}{\lambda}\right)\right]}$$

Directivity of an increased EFA is improved by 1.789 times the ordinary EFA.

8) Write notes on Binomial Array?

Ans) An Array which is formed by arranging the amplitudes of antenna elements in the array as the coefficients of binomial expansion, then this type of array is referred as binomial array. It is constructed by using $N-1$ complex plane nulls, which all are in same z-plane location $z = -1$.

If the element spacing in array is $\lambda/2$ or less, then there are no side lobes. Generally side lobes are produced for element spacing greater than '$\lambda/2$'.

Special characteristics :

Binomial array exhibits special characteristics under wavelength is less than '$\lambda/2$'. First one is that the amplitude pattern (or) radiation pattern of a binomial array has only single null, which means that directivity of antenna increases very slowly as function of antenna number (element number).
It is clear that element number weakly affects the radiation pattern of array. On the contrary phase pattern is strongly affected by element number.
Second one is excitation of binomial array is always symmetrical & real, if phase shift is not considered. Therefore, EFA pattern due to Binomial array can be implemented without any tunable phase shifters.

1	n = 1
1 1	n = 2
1 2 1	n = 3
1 3 3 1	n = 4
1 4 6 4 1	n = 5
1 5 10 10 5 1	n = 6
1 6 15 20 15 6 1	n = 7

Pascal's triangle displaying coefficients of the binomial series.

In general, the pattern for the binomial array is given by

$$\boxed{E = \cos^{n-1}\left[\frac{\pi}{2}\cos\theta\right]}$$

Use of Binomial Array:

1) To reduce secondary lobes
2) In derivation of pattern through principle of pattern multiplication

Disadvantage of Binomial array:

1) if Directivity decreases then HPBW increases
2) To design a large array, larger amplitude ratios are needed (complex)
3) Main lobe beams width is large.

---

3) Design a 7 element Broad side Array required to achieve gain of 14dB.

sol:

WKT

$$D = 2\left[\frac{nd}{\lambda}\right] \qquad 2\,\frac{(n-1)d}{\lambda}$$

Here $n = 7$, $G_D = 14dB \Rightarrow G = 25.12$

Amplitudes of lobes of field pattern relative amplitudes is given by pascal triangle with reference as number of sources

n.	Relative Amplitudes
3	1 2 1
4	1 3 3 1
5	1 4 6 4 1
6	1 5 10 10 5 1
→ 7	1 6 15 20 15 6 1

Relative amp ratio is 1:1:1:1

$$G_D = 2\left(\frac{L}{\lambda}\right) \Rightarrow L = \frac{G_D \cdot \lambda}{2} = \frac{14\lambda}{2} = 7\lambda$$

10. Design a 8 element EFA required to achieve a gain of 16dB.

A. Given,

$n =$ number of elements $= 8$.

$G_{D\,max} = 16\,dB$.

We know that

$G = \frac{4(n-1)d}{\lambda}$ $\quad G_{D\,max}$ of EFA $= 4\left(\frac{nd}{\lambda}\right)$

$$4 \times \frac{8 \times d}{\lambda} = G_{D\,max}.$$

$$16 = 10 \log_{10} x.$$

$$x = 10^{1.6}$$

$$\boxed{D = 39.8107}$$

$$39.8107 = \frac{32 \times d}{\lambda}$$

$$\frac{d}{\lambda} = 1.244$$

$$\boxed{d = 1.244\lambda}$$

$I_0 \cos\omega t$

x x x x • x x x x

$i_1(t)$ $i_2(t)$ $i_3(t)$ $i_4(t)$ $\quad$ $i_5(t)$ $i_6(t)$ $i_7(t)$ $i_8(t)$.

$$\delta = \pm \beta d.$$

$$= \pm \frac{2\pi}{\lambda} \times 1.244\lambda = 7.816\pi \text{ rad}$$

$$= 1.816\pi \text{ rad}$$

$I_0 \cos\omega t \angle \delta/2 = i_5(t) = I_0 \cos\left(\omega t + \frac{1.816\pi}{2}\right) = I_0 \cos(\omega t + 0.908\pi)$

$I_0 \cos\omega t \angle 3\delta/2 = i_6(t) = I_0 \cos\left(\omega t + \frac{3\times 1.816\pi}{2}\right) = I_0 \cos(\omega t + 2.724\pi)$

$i_7(t) = I_0 \cos\left(\omega t + \frac{5\times 1.816\pi}{2}\right) = I_0 \cos(\omega t + 4.54\pi)$

$i_8(t) = I_0 \cos\left(\omega t + \frac{7\times 1.816\pi}{2}\right) = I_0 \cos(\omega t + 6.356\pi)$

$i_4(t) = I_0 \cos(\omega t - 0.908\pi)$

$i_3(t) = I_0 \cos(\omega t - 2.724\pi)$

$i_2(t) = I_0 \cos(\omega t - 4.54\pi)$

$i_1(t) = I_0 \cos(\omega t - 6.356\pi)$

11Q. What is an optimal array?

A. users specify the array requirements in terms of directivity and side lobe levels.

A uniform BSA does not always satisfy user specifications.

A BSA with non uniform amplitude distribution helps here consider the following cases:

Case (i):

D= 22dB

First side lobe level = -13.26dB.

× × × × ×

1 : 1 : 1 : 1

Case (ii) :

D=-20dB

I side lobe level = -11dB.

× × × × × ×

1 : 1.6 : 1.9 : 1.6 : 1

Case (iii) :

There are no side lobes.

D=16dB

I side lobe level = $-\infty$ dB

× × × × ×

1 4 6 4 1

in case (i) it is a uniform linear array it has high side lobe level and high directivity. It is not preferrable as it is having high side lobe level.

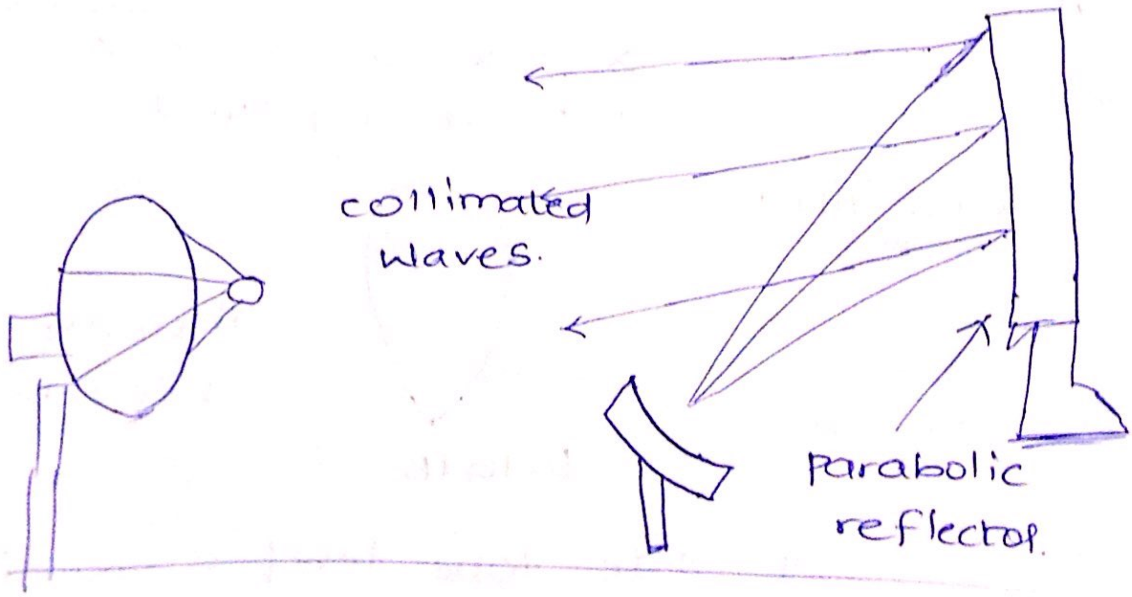
The case (ii) shows that side lobe levels has been reduced and the pattern is sharp. This is type of array design is known as optimal antenna.

Dolph Tchebyscheff's polynomials are used in the design to meet the user's specifications.

In case (iii) there are no side lobes which is not preferred.

12. What is the significance of Fraunhofer distance in antenna Measurements?

A

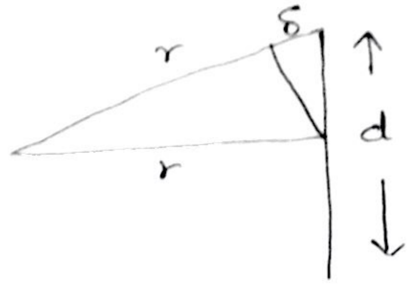


Johnson compact range

A threshold limit on the maximum phase error i.e seen in wireless communication.

Phase error $\leq 22.5^\circ$

i.e, path difference, $\delta \leq \frac{\lambda}{16}$



$$(r+\delta)^2 = r^2 + \left(\frac{d}{2}\right)^2$$

$$2r\delta + \delta^2 = \frac{d^2}{4}$$

$$\boxed{r^2 = \frac{d^2}{8\delta}}$$

$$\delta \leq \frac{\lambda}{16} \Rightarrow \boxed{r \geq \frac{2d^2}{\lambda}}$$

↳ Fraunhoffer distance.

$r > \frac{2d^2}{\lambda}$ $\cancel{\geq \frac{\lambda}{2\pi}}$ ~~all the~~ distances ~~are~~ known as Fraunhofer zone.

(Plane wavefront is seen here)

* Also write the advantages of far field measurement

(1)

18 Write notes on the following as applicable to antenna measurements

1) Sources of error.

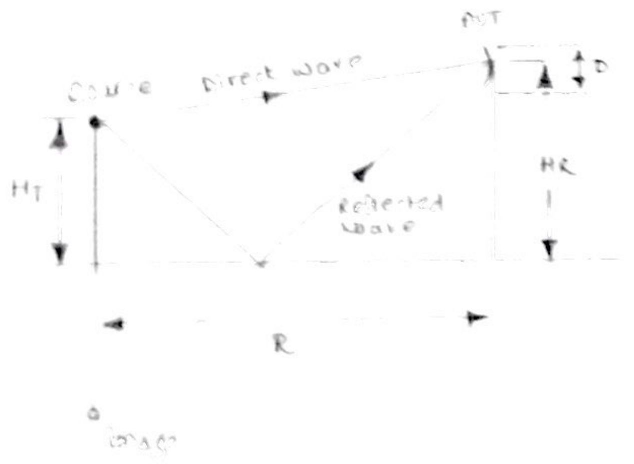
Ans The Source of errors in antenna measurement are

1) Finite measurement distance between antennas.
2) Reflection from surroundings.
3) Coupling in the reactive near field.
4) Misalignment of antenna.
5) Manmade errors.
6) Atmospheric effects
7) Cables
8) Impedance mismatch.
9) Imperfections of instruments.

ii) Antenna reflection ranges.

a) Reflecting range:

It is also known as Ground-Reflection range. At VHF & lower frequencies, ground reflections are difficult to avoid because a directional source antenna is very large and a ground-reflection range can be used.

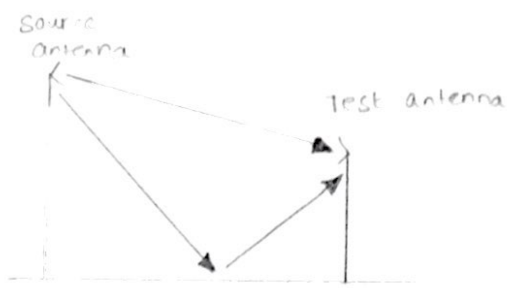


b) Slant range :

It is the line-of-sight distance between two points, not at the same level

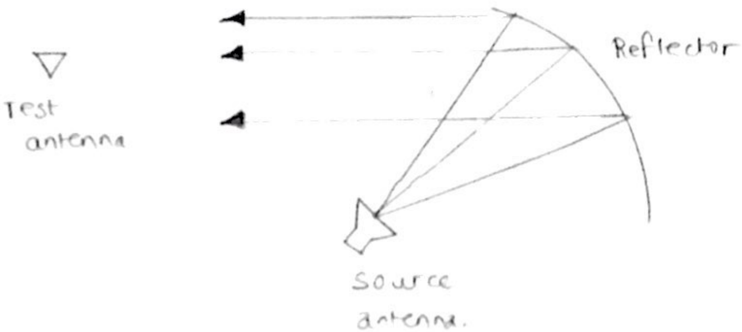
c) Elevated range :-

Elevated ranges are outdoor ranges. These antennas can be on mountains, towers, buildings etc. This is often done for very large antennas or at low frequencies (VHF and below, < 100MHz).



d) Compact range :-

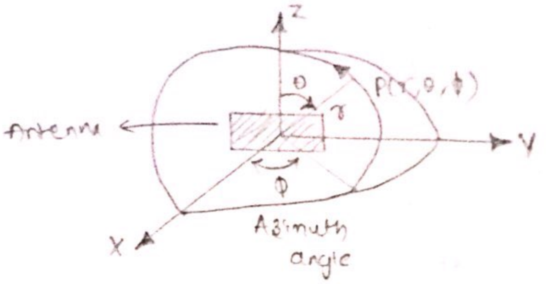
The source antenna must be placed in the far field of the test antenna. Since, the wave received by the test antenna should be a plane wave.



14 Describe the method of measuring radiation pattern of an antenna.

Ans The radiation pattern is nothing but the plot of the intensity of radiation taken at different points that are at equal distance from the antenna.

Let us assume a 3-coordinate cartesian system in which the antenna, whose pattern is to be measured, is placed at the origin as shown in figure.



Block diagram? and explanation?

The plane xy is horizontal plane. For horizontal plane antenna, the two patterns exhibited are,

1) ϕ component of E-field (horizontal) is measured as a function of ϕ in xy plane ($\theta=90°$). This is indicated as $E_\phi(\theta=90°, \phi)$ and called as E-plane pattern.

2) The ϕ component of E-field is measured as a function of θ in xz plane ($\phi=0°$). This is represented as $E_\phi = (\theta, \phi=0°)$ and called as H-plane pattern.

These E-plane & H-plane patterns are mutually ⊥r to major lobe.

The plane xz is called vertical plane and the two patterns are to be measured in this plane are,

i) The 'θ' component of E-field is measured as a function of ϕ in xy plane ($\theta=90°$). This is represented by $E_\theta(\theta=90°, \phi)$ and called as H-plane pattern.

ii) The θ component of E field is measured as a function of θ in x-z plane ($\phi = 0$) This is represented as $E_\theta(\theta, \phi = 0°)$ and called as E plane pattern.

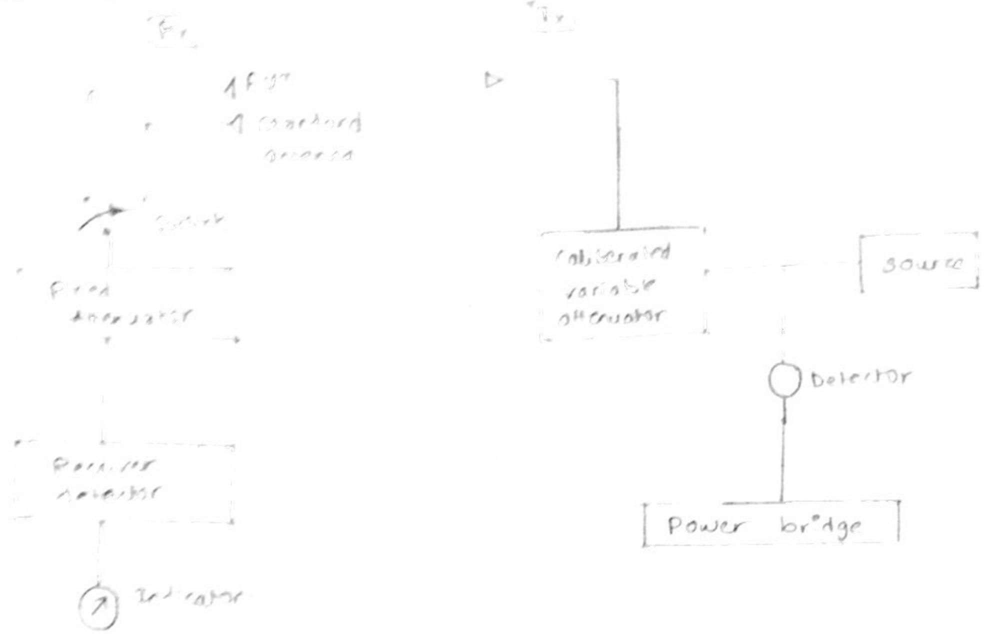
5 Describe the method of measuring gain of antenna (Discuss both the methods).

Ans The 2 methods of measuring gain of antenna are

i) Direct comparison method set up

2) Absolute gain method.

i) Direct comparison method setup:



When power bridge setting is not required,

$$\boxed{G_{AUT} = G_{std} \times \frac{P_2}{P_1} \times \frac{A_2}{A_1}}$$

By taking $\log_{10}$ on both sides

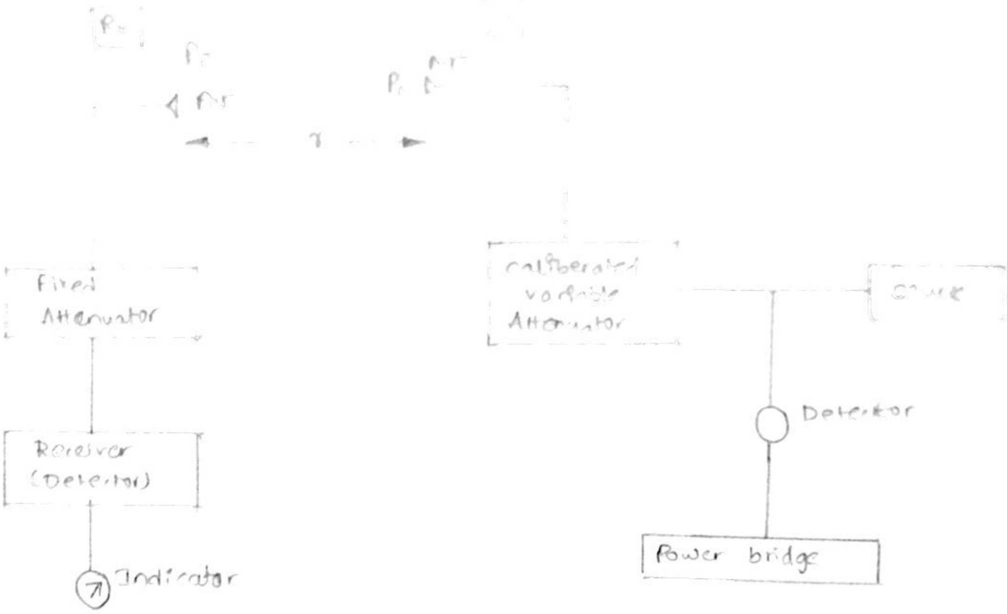
$$G_{AUT}(dB) = G_{std}(dB) + P_2(dB) - P_1(dB) + A_2(dB) - A_1(dB)$$

When power bridge factor setting is required

$$\boxed{G_{AUT} = G_{std} \times \frac{P_2}{P_1} \times \frac{A_2}{A_1} \times \frac{PB_1}{PB_2}}$$

where AF - Attenuation factor , G_{std} - Gain of standard antenna
PB - power bridge factor , G_{AUT} - Gain of unknown antenna
P - power received

2) Absolute gain method :-



For a maximum signal, antenna direction are adjusted. then the input to the transmitting antenna is adjusted to a specified level & the corresponding receiver reading is recorded i.e the attenuator dial setting & power bridge readings are recorded as W_t and P_{t_1} respectively.

Then by using through pads the transmitter is converted to the receiver. the attenuator dial is adjusted until the receiver shows the same previous level. Now the attenuator dial setting and the power bridge readings are recorded as W_r and P_{t_2}.

Since the two antennas are identical $P_{t_1} = P_{t_2}$ and the gain 'G_0' is calculated as

$$\boxed{G_0 = G_t = G_r = \frac{4\pi r}{\lambda}\sqrt{\frac{W_r}{W_t}}}$$