JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY
HYDERABAD

B.Tech III Year II Semester Examinations, February/March 2022

Digital Signal Processing
(Common to ECE, EIE, ETM)

R16

Code: 136BE

1. a, Give the linear shift time invariant specified by following difference equations

$$y(n) + \frac{1}{2}y(n-1) = x(n) - \frac{1}{2}x(n-1), n \geq 0$$

find the frequency response of the system

sol $He^{j\Omega} = \frac{1}{1+\frac{1}{2}e^{j\Omega}} \rightarrow 7m$

b, Determine the Z-transform and the ROC of the following signals.

(i) $x(n) = \left(\frac{1}{2}\right)^n u(n)$ — 4m

(ii) $x(n) = \left(\frac{1}{3}\right)^n u(n-1)$ — 4m

2. a, Find the frequency response of the following causal systems.

$$y(n) = \frac{1}{2}x(n) + x(n-1) + \frac{1}{2}x(n-2)$$

a) Solution — 7m

b, Find the Z-transform and the ROC of the following discrete time signal.

$$x(n) = \left(-\frac{1}{5}\right)^n u(n) + 5\left(\frac{1}{2}\right)^{-n} u(-n-1)$$ - [8m]

3. a, Find the 8 point DFT of the sequence [7m]

$$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$$

Sol 8-point DFT - [4m]

X(k) - 3m

3 b) Compute the FFT of the sequence, [8m]

$x(n) = \{1, 0, 0, 0, 0, 0, 0, 0\}$

A) Equation - 4m

diagram - 4m

4 a) Find the pole locations of a 6th order Butterworth filter with $\Omega_c$ = 1rad/sec. [5m]

A) Design all pole location from 1 to 6 - [5m]

4 b) Using bilinear transformation, design a high pass filter, monotonic in pass band with cuttoff frequency of 1000Hz and down 10dB at 350Hz. The sampling frequency is 500Hz. [10m]

A) find the value N - 5m

H[Z] 5m

5) a) Design a chebyshev filter with a maximum passband attenuation of 2.5dB at $\Omega_p$ = 20rad/sec and stop band attenuation of 30dB at $\Omega_s$ = 50 rad/sec. [9m]

A) values of a, b - 4m

Transfer function H(s) - 5m:

6) a) Using a rectangular window technique design low pass filter with passband gain of unity

cutt-off frequency of 1kHz and working at a sample frequency of 5kHz. The length of the impulse response should be 7. [8m]

A, equation of $h_d(n)$ - 4m.

for $n = 0, 1, 2, 3$ $h_d(n)$ (4m)

6.b, Design an ideal band pass filter with a frequency response $H_d = 1$ $\frac{\pi}{4} < |\omega| \leq \frac{3\pi}{4}$. find the values of $h(n)$ for $N = 11$ and plot the frequency response. [7M]

sd $h(5)$ - [4m]

curve [3m]

7, a, Design a filter with $H_d = e^{-j5\omega}$ $-\frac{\pi}{2} \leq \omega \leq \frac{\pi}{2}$

$= 0$ for $\frac{\pi}{2} < |\omega| < \pi$

using Blackman window with $N = 11$

~~H(n) = 0.65y(n-2) + 0.52~~

sol step 1. : Impulse response sequence - 5m

step 2 : Blackman window sequence - 5m.

7, b, Determine the dead band of the following system $y(n) = 0.65\,y(n-2) + 0.52\,y(n-1) + x(n)$ [5m]

sd. find out dead range. [5m]

8, a, Determine the input output relation as a factor of 2 up sampler in the frequency domain

sol Diagram - 2m

Explaination - 3m.

8 b, Consider a Butterworth low pass filter whose transfer function is [10m]

$$H(z) = \frac{0.05(1+z^{-1})^2}{1-1.2z^{-1}+0.8z^{-2}}$$

Compute the pole position in the z-plane.

sol pole position H[z) ——— 5m.

z — 5m.

poles

R16

B.Tech - III Year - II Semester Exam Feb/March-2022

Digital Signal Processing

Scheme of Evaluation.

1a Given: $y(n) = \frac{1}{2}y(n-1) = x(n) - \frac{1}{2}x(n-1)$ [7m]

w.k.T $H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$ is termed as frequency response.

Taking DFT on B.S

$$Y(e^{j\Omega}) + \frac{1}{2}e^{-j\Omega}Y(e^{j\Omega}) = X(e^{j\Omega}) - \frac{1}{2}e^{-j\Omega}X(e^{j\Omega})$$

$$Y(e^{j\Omega})\left[1+\frac{1}{2}e^{-j\Omega}\right] = X(e^{j\Omega})\left[1-\frac{1}{2}e^{-j\Omega}\right]$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{\left(1-\frac{1}{2}e^{-j\Omega}\right)}{\left(1+\frac{1}{2}e^{-j\Omega}\right)}$$

$$\boxed{H(e^{j\Omega}) = \frac{1}{1+\frac{1}{2}e^{-j\Omega}} - \frac{\frac{1}{2}e^{-j\Omega}}{1+\frac{1}{2}e^{-j\Omega}}.}$$

1b Given:- (i) $x(n) = \left(\frac{1}{2}\right)^n u(n)$ [8m]

w.k.T z transform of signal $x(n)$ is represented as

$$z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Putting the value of $x(n)$ we get

$$X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n u(n) z^{-n} \quad [\because u(n) \text{ exists only +ive}]$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n \cdot 1 \cdot z^{-n} \quad [4M]$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n = 1 + \frac{1}{2} z^{-1} + \left(\frac{1}{2} z^{-1}\right)^2 + \cdots$$

$$= 1 + x + x^2 + \cdots = \frac{1}{1-x} \quad \text{if } |x| < 1$$

$$\boxed{X(z) = \frac{1}{1 - \frac{1}{2} z^{-1}}} \quad \text{if } \left|\frac{1}{2} z^{-1}\right| < 1$$

$$\boxed{ROC = |z| > \frac{1}{2}}$$

(ii) $x(n) = \left(\frac{1}{3}\right)^n u(-n-1)$ [4m]

w.k.T z trasform is repented as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n u(-n-1) z^{-n}$$

$$\Rightarrow -\sum_{n=1}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n$$

$$\Rightarrow 1 + x + x^2 + \cdots = \frac{1}{1-x}$$

$$\boxed{X(z) = \frac{-1}{1 - \frac{1}{3} z^{-1}}} \quad |z| < \frac{1}{3}$$

ROC: $\left|\frac{1}{3} z^{-1}\right| < 1$

$$\boxed{|z| > \frac{1}{3}}$$

(2)



(2a) Given :- $y(n) = \frac{1}{2}x(n) + x(n-1) + \frac{1}{2}x(n-2)$ [7m]

frequency response is given by $H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$

Taking DFT of above expression

$$Y(e^{j\omega}) = \frac{1}{2}X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega}) + \frac{1}{2}e^{-2j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega})\left[\frac{1}{2} + e^{-j\omega} + \frac{1}{2}e^{-2j\omega}\right]$$

$$\therefore H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \left(\frac{1}{2} + e^{-j\omega} + \frac{1}{2}e^{-2j\omega}\right)$$

(2b) Given $\left(\frac{-1}{5}\right)^n u(n) + 5\left(\frac{1}{2}\right)^{-n} u(-n-1)$ [8m]

we know that Z transform is given by $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$$= \sum_{n=0}^{\infty}\left(\frac{-1}{5}\right)^n z^{-n} + 5\sum_{n=-\infty}^{\infty}(2)^n z^{-n}$$

$$= \sum_{n=0}^{\infty}\left(-\frac{1}{5}\right)^n z^{-n} + 5\sum_{n=1}^{\infty}(2^{-1}z)^n$$

The first power series converges for $\left|\frac{-1}{5}z^{-1}\right| < 1$

& $|z| > \frac{1}{5}$

The second power series converges for $\left|\frac{z}{2}\right| < 1$ & $|z| < 2$

$\therefore$ Roc is $\frac{1}{5} < |z| < 2$ &

$$\boxed{X(z) = \frac{1}{1 + \frac{1}{5}z^{-1}} - \frac{5}{1 - 2z^{-1}}}$$ ; ROC : $\frac{1}{5} < |z| < 2$

3b

3a 8 point DFT of $x(n) = \{1,1,1,1,0,0,0,0\}$ $N=8$ [7m]

The twiddle factors $W_8^0 = 1$, $W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $W_8^2 = -j$ $W_8^3 = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$W_8^4 = -1$, $W_8^5 = \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$, $W_8^5 = j$, $W_8^7 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

[4m]

$$= \sum_{n=0}^{7} x(n)\, W_8^{kn}$$

$X(k) = x(0)W_8^0 + x(1)W_8^k + x(2)W_8^{2k} + x(3)W_8^{3k} + x(4)W_8^{4k} + x(5)W_8^{5k} +$

$x(6)W_8^{6k} + x(7)W_8^{7k}$

as we can see $x(4), x(5), x(6), x(7) = 0$

$\therefore$ $X(k) = x(0)W_8^0 + x(1)W_8^k + x(2)W_8^{2k} + x(3)W_8^{3k}$

$X(k) = 1 + W_8^k + W_8^{2k} + W_8^{3k}$

for $k=0$; $X(0) = 1+1+1+1 = 4$

$k=1$; $X(1) = 1 + W_8^1 + W_8^2 + W_8^3 = 1 + \cancel{\frac{1}{\sqrt{2}}} - j\frac{1}{\sqrt{2}} - j - \cancel{\frac{1}{\sqrt{2}}} - j\frac{1}{\sqrt{2}} = 1 - j2.414$

$k=2$, $X(2) = 1 + W_8^2 + W_8^4 + W_8^6 = 1 - j - 1 + j = 0$

$k=3$, $X(3) = 1 + W_8^3 + W_8^6 + W_8^9 = 1 - \cancel{\frac{1}{\sqrt{2}}} - j\frac{1}{\sqrt{2}} - j + \cancel{\frac{1}{\sqrt{2}}} - j\frac{1}{\sqrt{2}} = 1 - j0.414$

$k=4$, $X(4) = 1 + W_8^4 + W_8^8 + W_8^{12} = \cancel{1} \cancel{-1} + \cancel{1} - \cancel{1} = 0$

$k=5$, $X(5) = 1 + W_8^5 + W_8^{10} + W_8^{15} = 1 - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} - j + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} = 1 + j0.414$

$k=6$, $X(6) = 1 + W_8^6 + W_8^{12} + W_8^{18} = 1 + j - 1 - j = 0$

$k=7$, $X(7) = 1 + W_8^7 + W_8^{14} + W_8^{21} = 1 + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} + j - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} = 1 + j2.414$

[3m]

$$\boxed{X(k) = \{4,\ 1-j2.414,\ 0,\ 1-j0.414,\ 0,\ 1+j0.414,\ 0,\ 1+j2.414\}}$$

(3b) Compute FFT of Sequence $x(n) = \{1,0,0,0,0,0,0,0\}$

The given Sequence is $x(n) = \{1,0,0,0,0,0,0,0\}$ [4m]

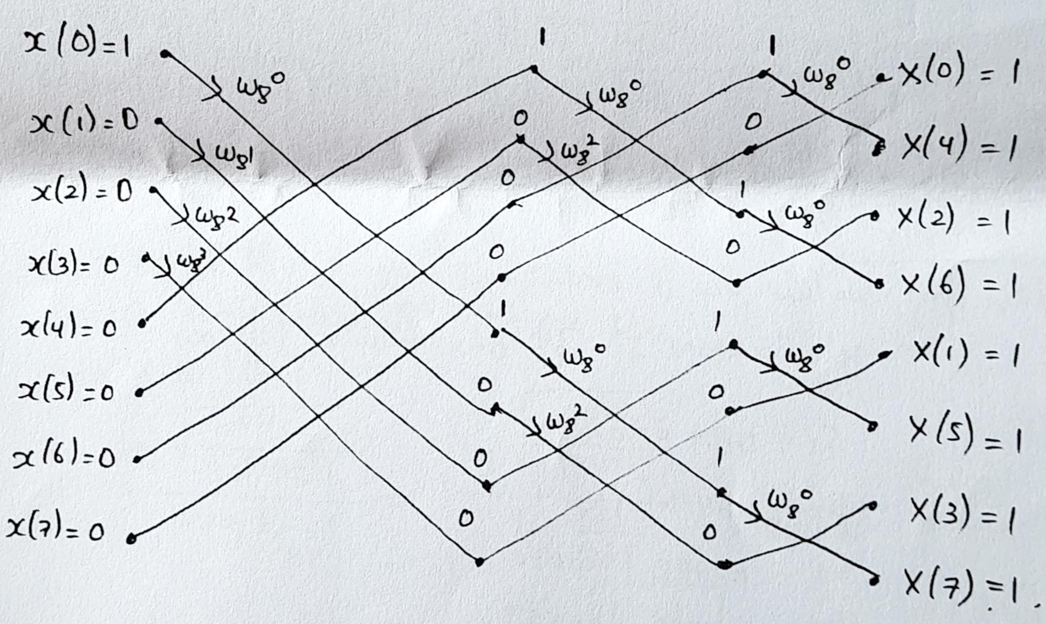
Twiddle factors are:-

$W_8^0 = 1$

$W_8^1 = (e^{-j2\pi/8})^1 = e^{-j\pi/4} \Rightarrow 0.707 - j0.707$

$W_8^2 = (e^{-j2\pi/8})^2 = e^{-j\pi/2} \Rightarrow -j$

$W_8^3 = (e^{-j2\pi/8})^3 = e^{-j3\pi/4} \Rightarrow -0.707 - j0.707$




[4m]

$\therefore X(k) = \{1,1,1,1,1,1,1,1\}$

4a) Pole locations of $6^{th}$ order Butterworth filter with $\Omega_c = 1$ rad/sec. [5m]

Given data $N = 6$

$\Omega_c = 1$ rad/sec

Pole locations :- $S_k = e^{j\left(\pi/2 + \frac{(2k-1)\pi}{2N}\right)}$ ; $k = 1, 2, \ldots N$

$S_k = e^{j\left(\pi/2 + \frac{(2k-1)\pi}{12}\right)}$ ; $k = 1, 2, 3, 4, 5, 6$

$$S_1 = e^{j\left(\frac{\pi}{2} + \frac{(2(1)-1)\pi}{12}\right)} = e^{j\left(\frac{\pi}{2} + \frac{\pi}{12}\right)} = e^{j\frac{7\pi}{12}} = \cos\frac{7\pi}{12} + j\sin\frac{7\pi}{12}$$
$$= 0.99 + j\,0.031$$

$$S_2 = e^{j\left(\frac{\pi}{2} + \frac{3\pi}{12}\right)} = e^{j\frac{3\pi}{4}} = \cos\frac{3\pi}{4} + j\sin\left(\frac{3\pi}{4}\right) = 0.999 + j\,0.041$$

$$S_3 = e^{j\left(\frac{\pi}{2} + \frac{5\pi}{12}\right)} = e^{j\frac{11\pi}{12}} = \cos\frac{11\pi}{12} + j\sin\left(\frac{11\pi}{12}\right) = 0.998 + j\,0.0502.$$

$$S_4 = e^{j\left(\frac{\pi}{2} + \frac{7\pi}{12}\right)} = e^{j\frac{13\pi}{12}} = \cos\frac{13\pi}{12} + j\sin\left(\frac{13\pi}{12}\right) = 0.998 + j\,0.059$$

$$S_5 = e^{j\left(\frac{\pi}{2} + \frac{9\pi}{12}\right)} = e^{j\frac{5\pi}{4}} = \cos\frac{5\pi}{4} + j\sin\left(\frac{5\pi}{4}\right) = 0.997 + j\,0.068$$

$$S_6 = e^{j\left(\frac{\pi}{2} + \frac{11\pi}{12}\right)} = e^{j\frac{17\pi}{12}} = \cos\frac{17\pi}{12} + j\sin\left(\frac{17\pi}{12}\right) = 0.996 + j\,0.077$$

$\therefore$ pole locations for $6^{th}$ order Butterworth filter are

$S_1 = 0.99 + j\,0.031$

$S_2 = 0.999 + j\,0.041$

$S_3 = 0.998 + j\,0.050$

$S_4 = 0.998 + j\,0.059$

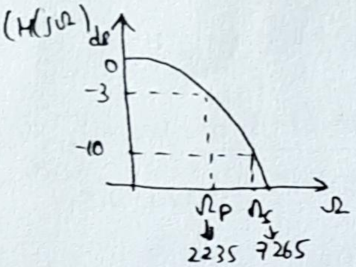
$S_5 = 0.997 + j\,0.068$

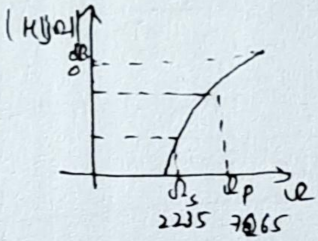
$S_6 = 0.996 + j\,0.077$.

4b Given $\alpha_p = 3dB$, $\omega_c = \omega_p = 2\times\pi\times1000 = 2000\pi$ rad/sec     [10m]

$\alpha_s = 10dB$, $\omega_s = 2\times\pi\times350 = 700\pi$ rad/sec

$$T = \frac{1}{f} = \frac{1}{5000} = 2\times10^{-4}\ sec$$




The characteristic are monotonic in both passband & stopband $\therefore$ the filter is butterworth filter.

Prewraping the digital frequencies we have

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_pT}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{2000\pi\times2\times10^{-4}}{2}\right) = 10^4\tan(0.2\pi)$$

$$\boxed{\Omega_p = 7265\ rad/sec}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_sT}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{700\pi\times2\times10^{-4}}{2}\right) = 10^4\tan(0.07\pi)$$

$$\boxed{\Omega_s = 2235\ rad/sec}$$

→ a low pass filter for given specifications & use suitable transformation to obtain transfer function of High pass filter

Order of the filter :-

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log\sqrt{\frac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log\frac{7265}{2235}} = \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} = 0.932$$

$$\therefore \boxed{N=1}$$     [5 M]

The first order Butterworth filter for $\Omega_c = 1$ rad/sec is $H(s) = \frac{1}{1+s}$

The high pass filter for $\Omega_c = \Omega_p = 7265$ rad/sec can be obtained by using the transformation

$$s \rightarrow \frac{\Omega_c}{s}$$

$$s \rightarrow \frac{(7265)}{s}$$

The transfer function of High pass filter is $H(s) = \left.\frac{1}{s+1}\right|_{s=\frac{7265}{s}}$

$$\Rightarrow \frac{s}{s+7265}$$

using bilinear Transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Big|_{s=\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265} = \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}}$$ [5m]

5a

Given $\Omega_p = 20$ rad/sec ; $\alpha_p = 2.5$ dB [9m]

$\Omega_s = 50$ rad/sec ; $\alpha_s = 30$ dB .

W.K.T $$N = \frac{\cosh^{-1} \lambda/\varepsilon}{\cosh^{-1} 1/k}$$

where $\lambda = \sqrt{10^{0.1\alpha_s}-1} = \sqrt{10^{0.1(30)}-1} = 31.607$

$\varepsilon = \sqrt{10^{0.1\alpha_p}-1} = \sqrt{10^{0.1(2.5)}-1} = 0.882$

$k = \frac{\Omega_p}{\Omega_s} = \frac{20}{50} = 0.4$

Now $$N \geq \frac{\cosh^{-1}\frac{31.607}{0.882}}{\cosh^{-1}\frac{1}{0.4}} = 2.726$$

$\therefore \boxed{N=3}$

$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.65$

$a = \Omega_p\left[\frac{\mu^{1/N}-\mu^{-1/N}}{2}\right] = 20\left[\frac{2.65^{1/3}-2.65^{-1/3}}{2}\right] = 6.6$

$b = \Omega_p\left[\frac{\mu^{1/N}-\mu^{-1/N}}{2}\right] = 20\left[\frac{2.65^{1/3}+2.65^{-1/3}}{2}\right] = 21.06$ [4m]

$s_k = a\cos\phi_k + jb\sin\phi_k$ ; $k = 1,2,3$

$\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi$ ; $k=1,2,3$ .

$\phi_1 = \frac{\pi}{2} + \frac{\pi}{6} = 120^\circ$

$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{6} = 180^\circ$

$\phi_3 = \frac{\pi}{2} + \frac{5\pi}{6} = 240^\circ$

$S_k = a\cos\phi_k + jb\sin\phi_k$ ; $k = 1,2,3$

$S_1 = 6.6\cos(120^\circ) + j(21.06)\sin(120^\circ) = -3.3 + j18.23$

$S_2 = 6.6\cos(180) + j(21.06)\sin(180^\circ) = -6.6$

$S_3 = 6.6\cos(240^\circ) + j(21.06)\sin(240) = -3.3 - j18.23$

Denominator of $H(s) = (s+6.6)(s^2+6.6s+343.2)$

Numerator of $H(s) = (6.6)(343.2) = 2265.27)$ [5m]

$\therefore$ $\boxed{\text{Transfer function } H(s) = \dfrac{2265.27}{(s+6.6)(s^2+6.6s+343.2)}}$

5b) Impulse Variance for $H(z)$ if $T = 1$ sec $H(s) = \dfrac{1}{s^3+3s^2+4s+1}$ [7m]

$$H(s) = \frac{1}{(s+1)^3+s}$$

$$= \frac{1}{(s+0.3176)(s+1.3412+j1.1615)(s+1.3412-j1.1615)}$$

$$= \frac{A}{(s+0.3176)} + \frac{B}{(s+1.341+j1.1615)} + \frac{C}{(s+1.3412-j1.1615)}$$

$$A = (s+0.3176)\frac{1}{(s+0.3176)(s+1.341+j1.1615)} = 0.3176$$

$B = -1.341 + j1.165$ for B $0.889 + j0.792$

$C = -1.341 - j1.1615$ for C $= 0.889 - j0.792$

$$\therefore H(s) = \frac{0.3176}{(s+0.3176)} + \frac{0.889+j0.792}{(s+1.341+j1.1615)} + \frac{0.889-j0.792}{(s+1.3412-j1.1615}$$

$$= \frac{0.3176}{(s-(-0.3176)} + \frac{0.889+j0.792}{(s-(-1.341-j1.1615)} + \frac{0.889-j0.792}{(s-(-1.3412+j1.1615)}$$

an impulse variant

if $H(s) = \sum_{k=1}^{N} \dfrac{C_k}{s-P_k}$ then $H(z) = \sum_{k=1}^{N} \dfrac{C_k}{1-e^{P_kT}z^{-1}}$

$$H(z) = \frac{0.3176}{1-e^{-0.3176}z^{-1}} + \frac{0.889+j0.792}{1-e^{-1.341}e^{-j1.615}z^{-1}} + \frac{0.889-j0.792}{1-e^{-1.341}e^{j1.61}z^{-1}}$$

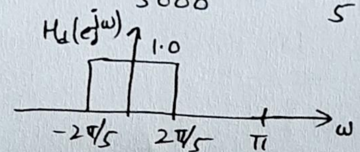
$$H(z) = \frac{0.3176}{1-0.727z^{-1}} + \frac{0.889+j0.792}{1-0.051z^{-1}} + \frac{0.889-j0.792}{1+0.051z^{-1}}$$

6a Given $f_c = 1000\,Hz$, $F = 5000\,Hz$. [8m]

the cutoff freq $\omega_c = 2\pi f T$

$$= \frac{2\pi(1000)}{5000} = \frac{2\pi}{5}$$

freq response




→ The desired freq response of LPF is shown the filter coeffients are given by

$$h_d(n) = \frac{1}{2\pi}\int_{-2\pi/5}^{2\pi/5} e^{j\omega n}\,d\omega$$

$$= \frac{\sin\frac{2\pi}{5}n}{\pi n}; \quad -\infty \le n \le \infty$$ [4M]

the rectangular window for $N = 7$ is given by

$$w_R(n) = \begin{cases} 1 & \text{for } -3 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

for $n = 0$; $h(0) = \lim_{n\to 0} \frac{\sin\frac{2\pi}{5}n}{\pi n} = \frac{2}{5} = 0.4$

$n = 1$; $h(1) = h(-1) = \frac{\sin\left(\frac{2\pi}{5}\right)}{\pi} = 0.3027$

$n = 2$; $h(2) = h(-2) = \frac{\sin(4\pi/5)}{2\pi} = 0.0935$

$n = 3$; $h(3) = h(-3) = \frac{\sin(6\pi/5)}{3\pi} = -0.062$

The filter coeffients of realizable filter are [4M]

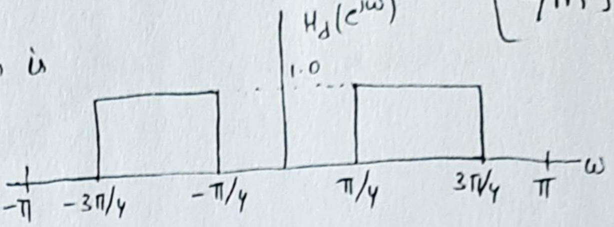
$h(0) = h(6) = -0.06236$; $h(1) = h(5) = 0.0935$; $h(2) = h(4) = 0.3027$

$h(3) = 0.4$.

(4)

[7m]

(6b) The ideal frequency response of filter is shown as




$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}$$

$$= \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{3\pi/4} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi j n}\left[e^{-j\pi n/4} - e^{-j3\pi n/4} + e^{j3\pi n/4} - e^{j\pi n/4}\right]$$

$$= \frac{1}{\pi n}\left[\sin\frac{3\pi}{4}n - \sin\frac{\pi}{4}n\right] \quad -\alpha \le n \le \infty$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = h_d(n) \text{ for } |n| \le 5$$
$$= 0 \text{ otherwise}$$

The filter coeffects are symmetrical about $n=0$ satisfy the condition

$h(n) = h(-n)$

for $n = 0$ $\quad h(0) = \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} 1 \cdot d\omega + \int_{\pi/4}^{3\pi/4} 1 \cdot d\omega\right]$

$$= \frac{1}{2\pi}\left\{-\frac{\pi}{4} + \frac{3\pi}{4} + \frac{3\pi}{4} - \frac{\pi}{4}\right] = \frac{1}{2} = 0.5$$

$$h(1) = h(-1) = \frac{\sin\frac{3\pi}{4} - \sin\frac{\pi}{4}}{\pi} = 0$$

$$h(2) = h(-2) = \frac{\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}}{2\pi} = \frac{-2}{2\pi} = -0.31$$

$$h(3) = h(-3) = \frac{\sin\frac{9\pi}{4} - \sin\frac{3\pi}{4}}{3\pi} = 0$$

[4m]

$$h(4) = h(-4) = \frac{\sin(3\pi) - \sin\pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin\left(\frac{15\pi}{4}\right) - \sin\left(\frac{5\pi}{4}\right)}{5\pi} = 0$$

The transfer function of filter is

$$H(z) = h(0) = \sum_{n=1}^{N-1/2} [h(n)(z^{n} + z^{-n})]$$

$$= 0.5 - 0.3183(z^{2} + z^{-2})$$

The transfer function of realizable filter is

$$H'(z) = z^{-5}[0.5 - 0.3183(z^{2} + z^{-2})] = -0.3183z^{-3} + 0.5z^{-5} - 0.3183z^{-}$$

The filter coefficients of causal filter are

$$h(0) = h(10) = h(1) = h(9) = h(2) = h(8) = h(4) = h(6) = 0$$

$$h(3) = h(7) = -0.3183$$

$$h(5) = 0.5$$

$$\bar{H}(e^{jw}) = \sum_{n=1}^{N-1/2} a(n)\cos wn$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$$a(1) = 2h(5-1) = 2h(4) = 0$$

$$a(2) = 2h(5-2) = 2h(3) = -0.6366$$

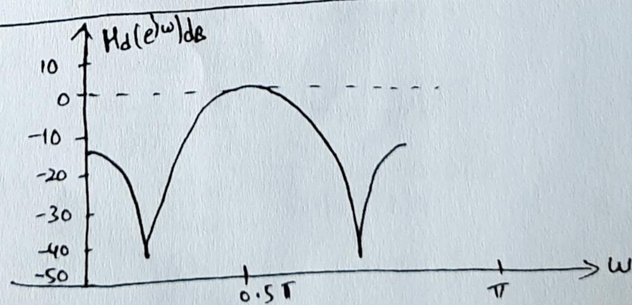
$$a(3) = 2h(5-3) = 2h(2) = 0$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0$$

$$\bar{H}(e^{jw}) = 0.5 - 0.6366\cos 2w$$

| w (in degrees) | 0 | 20 | 30 | 45 | 60 | 75 | 90 | 105 | 135 | 160 | 180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{H}(e^{jw})$ | -0.1366 | 0.012 | 0.181 | 0.5 | 0.81 | 1.05 | 1.136 | 1.05 | 0.5 | 0.012 | -0.136 |
| $\lvert H(e^{jw})\rvert$ dB | -17.3 | -38.1 | -14.8 | -6.02 | -1.74 | 0.43 | 1.11 | 0.43 | -6.02 | -38.17 | -17.3 |




[3m]

7a

(13)

[10m)

(7a) Given $H_d(e^{j\omega}) = e^{-j5\omega}$

$N = 11$

The given filter is Symmetric filter $H_d(e^{j\omega}) = e^{-j5\omega} = e^{-j\frac{(N-1)}{2}\omega}$

Step (i):- The impulse response sequence $h_d(n) =$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n} \cdot d\omega$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{-j5\omega} \cdot e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j(n-5)\omega} \cdot d\omega = \frac{1}{2\pi}\left[\frac{e^{j(n-5)\omega}}{j(n-5)}\right]_{-\pi/2}^{\pi/2}$$

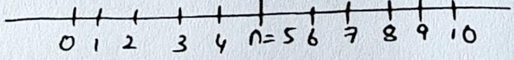
$$= \frac{1}{2j\pi(n-5)}\left[e^{j(n-5)\pi/2} - e^{-j(n-5)\pi/2}\right]$$

$$= \frac{1}{\pi(n-5)}\left[\frac{e^{j(n-5)\pi/2} - e^{-j(n-5)\pi/2}}{2j}\right]$$

$$h_d(n) = \frac{\sin(n-5)\pi/2}{\pi(n-5)} \quad ; \; -\infty \le n \le \infty$$

The impulse response sequence $h_d(n)$ is Symmetric about $n=5$;

$$\alpha = \frac{N-1}{2} = \frac{11-1}{2} = 5$$




$h_d(0) = h_d(10)$ ; $h_d(2) = h_d(8)$

$h_d(1) = h_d(9)$ ; $h_d(3) = h_d(7)$

$h_d(4) = h_d(6)$

[5m]

for $n=0$

$$h_d(0) = \frac{\sin(0-5)\pi/2}{(0-5)\pi} = \frac{\sin(-5\pi/2)}{-5\pi} = \frac{-1}{-5\pi}$$

$$h_d(0) = h_d(10) = 0.0637$$

for $n=1$; $h_d(1) = \frac{\sin(1-5)\pi/2}{(1-5)\pi} = \frac{\sin(-4\pi/2)}{-4\pi} = \frac{\sin(-2\pi)}{-4\pi}$

$$h_d(1) = h_d(9) = 0$$

$$h_d(2) = \frac{\sin(2-5)\pi/2}{(2-5)\pi} = \frac{\sin(-3\pi/2)}{-3\pi} = \frac{1}{-3\pi}$$

$$h_d(2) = -0.106 = h_d(8)$$

for $n=3$

$$h_d(3) = \frac{\sin(3-5)\pi/2}{(3-5)\pi} = \frac{\sin(-3\pi/2)}{-2\pi} = 0$$

$$h_d(3) = h_d(7) = 0$$

for $n=4$

$$h_d(4) = \frac{\sin(4-5)\pi/2}{(4-5)\pi} = \frac{\sin(-\pi/2)}{-\pi} = \frac{-1}{-\pi}$$

$$h_d(4) = h_d(6) = 0.3183$$

for $n=5$, the $h_d(5)$ becomes indeterminate

$$\left[\because \lim_{n\to 0} \frac{\sin an\pi}{n\pi} = a\right]$$

$$h_d(5) = \frac{\sin(n-5)\pi/2}{(n-5)\pi} = \frac{1}{2} = 0.5$$

$$h_d(5) = 0.5$$

Step(2): Blackman window sequence for $N=11$

$$w_B(n) = \begin{cases} 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1} ; & -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

$$w_B(n) = w_B(-n)$$

[5m)

$$w_B(n) = 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1} ; \quad -5 \le n \le 5$$

$$w_B(0) = 0.42 + 0.5\cos 0 + 0.08\cos(0)$$

$$= 0.42 + 0.5 + 0.08 \Rightarrow 1$$

$$w_B(1) = 0.8492 = w_B(-1)$$

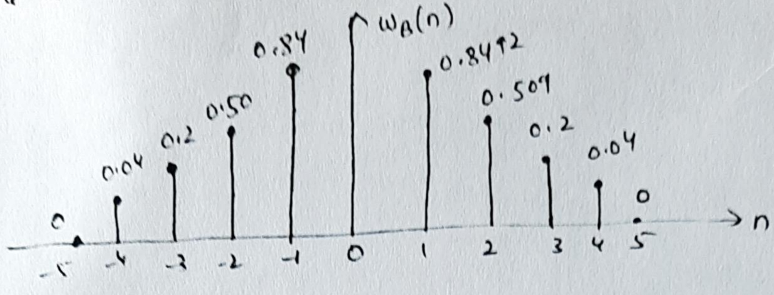
$$w_B(2) = 0.5098 = w_B(-2)$$

$$w_B(3) = 0.2 = w_B(-3)$$

$w_B(4) = 0.04 = w_B(-4)$

$w_B(5) = 0 = w_B(-5)$

The non causal Blackman Window Sequence




$w_B(5) = 1$

$w_B(0) = w_B(10) = 0$

$w_B(1) = w_B(9) = 0.04$

$w_B(2) = w_B(8) = 0.2$

$w_B(3) = w_B(7) = 0.50$

$w_B(4) = w_B(6) = 0.84$

Step (3) : Multiply $h_d(n)$ & $w_B(n)$ we get

$$h(n) = h_d(n) \cdot w_B(n) \quad ; \quad 0 \leq n \leq 10$$

$h(0) = h_d(0) \cdot w_B(0) = 0.0637 \times 0 = 0$

$h(1) = h_d(1)\, w_B(1) = 0 \times 0.04 = 0$

$h(2) = h_d(2)\, w_B(2) = -0.021$

$h(3) = h_d(3)\, w_B(3) = 0$

$h(4) = h_d(4)\, w_B(4) = 0.2703$

$h(5) = h_d(5)\, w_B(5) = 0.5$

Step 4 :- Transfer function of realizable filter

$$H(z) = z^{\frac{-(N-1)}{2}} \left\{ h(0) + \sum_{n=1}^{N-1/2} h(n)\left[z^n + z^{-n}\right] \right\}$$

$$H(z) = z^{-5} \{ h(0) + h(1)[z^1 + z^{-1}] + h(2)[z^2 + z^{-2}] + h(3)[z^3 + z^{-3}) +$$
$$h(4)[z^4 + z^{-4}] + h(5)[z^5 + z^{-5}] \}$$

Sub $h(0), h(1), h(2), h(3), h(4), h(5)$

$$H(z) = z^{-5} \{ -0.0212 z^2 - 0.0212 z^{-2} + 0.2703 z^4 + 0.2703 z^{-4} +$$
$$+ 0.5 z^5 + 0.5 z^{-5}$$

$H(z) = -0.0212z^{-3} - 0.0212z^{-7} + 0.2703z^{-1} + 0.2703z^{-9} + 0.5z^{0} + 0.5z^{-10}$

$H(z) = 0.5 + 0.2703z^{-1} - 0.0212z^{-3} - 0.0212z^{-7} + 0.2703z^{-9} + 0.5z^{-10}$

$\therefore h(n) = \{0.5, 0.2703, 0, -0.021, 0, 0, 0, -0.0212, 0, 0.2703, 0.5\}$

$n=0 \quad n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5 \quad n=6 \quad n=7 \quad n=8 \quad n=9 \quad n=10$

(7b) Dead band for the equation [5m]

$y(n) = 0.65\, y(n-2) + 0.52\, y(n-1) + x(n)$

The Deadband for the system is given by $\pm \dfrac{2^{-b}}{2(1-|\alpha|)}$

Here we used rounding for quantization as a decimal.

$\therefore b = 0$ (no of bits used for quantization)

$\alpha$ is $0.52$

$$\pm \frac{2^{-0}}{2(1-|0.52|)}$$

$$= \pm \frac{1}{0.96}$$

Deadband range $= \pm 1.0$

[5m]

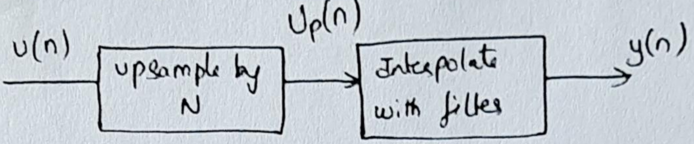
(8a) upsampling :- It is process of converting the sample rate of a digital signal from one rate to another rate is called sampling rate conversion.

→ Increasing the rate of already sampled signal is up-sampling whereas decreasing the rate is called down sampling

(7) (17)

→ Many practical application requires to transmit & receive digital signals with different sampling rates

→ Let us consider, simplest case of up-sampling, we want to double the rate of signal. we need to insert 0's in between two successive samples.



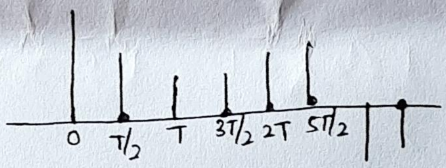

– 2m

→ The up-sample consist of two operations

① Add N-1 zero samples between every sample of input signal by effectively scaling time axis by factor N.

② Filter the resulting sequence in order to create smoothly varying set of sequence sample.

– 3m




upsampled by 2.

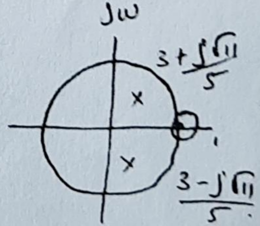
[10m]

(8b) Pole position of $H(z) = \dfrac{0.05(1+z^{-1})^2}{1-1.2z^{-1}+0.8z^{-2}}$.

L 5m.

$$= \frac{0.05(z+1)^2/\cancel{z^2}}{\dfrac{z^2-1.2z+0.8}{\cancel{z^2}}}$$

$$= \frac{0.05(z+1)(z+1)}{\left(z-\frac{3+\sqrt{11}}{5}j\right)\left(z-\frac{3-j\sqrt{11}}{5}\right)}$$




Zeros = $z = -1$

Poles $z = \dfrac{3+j\sqrt{11}}{5}, \dfrac{3-j\sqrt{11}}{5}$

– [5m]