ELECTROMAGNETIC WAVES:

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \rightarrow ①$$

$$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} \rightarrow ②$$

$$\nabla \cdot \bar{D} = \rho_v$$

$$\nabla \cdot \bar{B} = 0$$

$$\bar{D} = \epsilon \bar{E}$$

$$\bar{B} = \mu \bar{H}$$

$$\bar{J} = \sigma \bar{E}$$

Homogeneous Medium: is one for which the quantities ϵ, μ, and σ are constant throughout the medium.

Isotropic Medium: The medium is isotropic if ϵ is a scalar constant, so that $\bar{D}$ and $\bar{E}$ have everywhere the same direction.

* The Maxwell's equations ① & ② are for source-free regions, that is, regions in which there are no impressed voltages or currents (no generators).

* Substituting $\bar{D}$, $\bar{B}$, and $\bar{J}$ in Maxwell's eqn.

$$\left.\begin{aligned} \nabla \times \bar{H} &= \sigma \bar{E} + \epsilon \frac{\partial E}{\partial t} \\ \nabla \times \bar{E} &= -\mu \frac{\partial \bar{B}}{\partial t} \end{aligned}\right\} \text{differential eqns}$$

Differential form	Integral Form	Law
$\nabla \cdot \bar{D} = \rho_v$	$\oint_S \bar{D} \cdot d\bar{s} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B} \cdot d\bar{s} = 0$	Nonexistence of isolated magnetic charge
$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$	$\oint_L \bar{E} \cdot d\bar{l} = \frac{-\partial}{\partial t} \int_S \bar{B} \cdot d\bar{s}$	Faraday's law
$\nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t}$	$\oint_L \bar{H} \cdot d\bar{l} = \int_S \left(J + \frac{\partial \bar{D}}{\partial t} \right) d\bar{s}$	Ampere's circuit law.

Time-harmonic Fields:

- a time-harmonic field is one that varies periodically or sinusoidally with time.

- $$z = x + iy = r\angle\phi = r e^{j\phi}$$

$$r = \sqrt{x^2+y^2} \qquad \phi = \tan^{-1}(y/x)$$

- $$\phi = \omega t + \theta$$

$$z = r e^{j(\omega t+\theta)} = r e^{j\omega t} \cdot e^{j\theta}$$

- $$\bar{A} = Re\{A_s e^{j\omega t}\} \qquad \bar{A}_s = r e^{j\theta}$$

- Maxwell's equations

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \qquad \nabla \cdot \bar{D} = \rho_v$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \nabla \cdot \bar{B} = 0$$

Let $\bar{H} = \bar{H}_s e^{j\omega t}$, $\bar{E} = \bar{E}_s e^{j\omega t}$

$$\nabla \times \bar{H}_s e^{j\omega t} = \sigma \bar{E} + \epsilon \frac{\partial E}{\partial t}$$

$$= \sigma \bar{E}_s e^{j\omega t} + \epsilon \frac{\partial}{\partial t}\left(\bar{E}_s e^{j\omega t}\right)$$

$$= \sigma \bar{E}_s e^{j\omega t} + j\omega\epsilon \bar{E}_s e^{j\omega t}$$

$$\boxed{\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon)\bar{E}_s} \longrightarrow ③$$

Ily for $\nabla \times E = -\frac{\partial \bar{B}}{\partial t}$

and $\nabla \times \bar{E}_s e^{j\omega t} = -\mu \frac{\partial}{\partial t}\left[\bar{H}_s e^{j\omega t}\right]$

$$\boxed{\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s} \rightarrow (4)$$

now, from vector identities

$$\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$$

$$\Rightarrow \nabla \times (\nabla \times \bar{H}_s) = \nabla(\underbrace{\cancel{\nabla \cdot \bar{H}_s}}_{0}) - \nabla^2 \bar{H}_s = \nabla \times \left[(\sigma + j\omega\epsilon)\bar{E}_s\right]$$

$$\Rightarrow -\nabla^2 \bar{H}_s = [\sigma + j\omega\epsilon](\nabla \times \bar{E}_s)$$

$$\Rightarrow -\nabla^2 \bar{H}_s = [\sigma + j\omega\epsilon](-j\omega\mu \bar{H}_s) \quad \{\because \text{ eqn } (4)$$

$$\Rightarrow -\nabla^2 \bar{H}_s = [\omega^2\mu\epsilon - j\sigma\omega\mu]\bar{H}_s$$

$$\Rightarrow \nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s = 0$$

where $\gamma^2 = -[\omega^2\mu\epsilon - j\sigma\omega\mu] = +j\omega\mu[\sigma + j\omega\epsilon]$

Ily for $\nabla \times (\nabla \times \bar{E}_s) = \nabla(\underbrace{\cancel{\nabla \cdot \bar{E}_s}}_{0}) - \nabla^2 \bar{E}_s = \nabla \times \left[-j\omega\mu \bar{H}_s\right]$

$$\Rightarrow -\nabla^2 \bar{E}_s = -j\omega\mu[\nabla \times \bar{H}_s] = -j\omega\mu[\sigma + j\omega\epsilon]\bar{E}_s$$

$$\Rightarrow \nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0 \quad \text{where } \boxed{\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)}$$

$\rightarrow$ (5)

where γ is the propagation constant of the medium.

and $\nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s = 0$ and $\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$

are the homogeneous vector Helmholtz's equations(or) simply vector wave equations.

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta \quad \text{and} \quad |\gamma^2| = \alpha^2 + \beta^2 \rightarrow (6)$$

$$Re\{\gamma^2\} = \alpha^2 - \beta^2 \rightarrow (7)$$

from (5)
we know

$$\left.\begin{aligned}\gamma^2 &= j\omega\mu(\sigma + j\omega\epsilon) \\ &= j\sigma\omega\mu - \omega^2\mu\epsilon\end{aligned}\right\} \quad |\gamma^2| = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2} \rightarrow (9)$$

$$Re\{\gamma^2\} = -\omega^2\mu\epsilon \rightarrow (8)$$

from (7) & (7) $\quad \alpha^2 - \beta^2 = -\omega^2\mu\epsilon \rightarrow (10)$

equating (6) & (9) $\quad \alpha^2 + \beta^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2} \rightarrow (11)$

$$2\alpha^2 = -\omega^2\mu\epsilon + \omega\mu\sqrt{\sigma^2 + \omega^2\epsilon^2}$$

$$\alpha^2 = \frac{-\omega^2\mu\epsilon + \omega^2\mu\epsilon\sqrt{1 + \frac{\sigma^2}{\epsilon^2\omega^2}}}{2}$$

$$\alpha^2 = \frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\epsilon^2\omega^2}} - 1\right]$$

$$\therefore \boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} - 1\right]}} \rightarrow (12)$$

and $\quad -2\beta^2 = -\omega^2\mu\epsilon - \omega\mu\sqrt{\sigma^2+\omega^2\epsilon^2}$

$$\beta^2 = \frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}+1\right]$$

$$\therefore \quad \boxed{\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}+1\right]}} \rightarrow (13)$$

- lossy dielectric: is a medium in which an EM wave, as it propagates, losing power owing to imperfect dielectric

In this case, α and β are same as given in (12) & (13).

- lossless dielectric: in lossless dielectrics, $\sigma << \omega\epsilon$

i.e., $\sigma \simeq 0$, $\epsilon = \epsilon_0\epsilon_r$, $\mu = \mu_0\mu_r$

Substituting in (12) & (13).

$$\alpha = 0, \qquad \beta = \omega\sqrt{\frac{\mu\epsilon}{\cancel{2}}\left[\sqrt{1+0}+1\right]} = \omega\sqrt{\mu\epsilon}$$

- free space: $\sigma = 0$, $\epsilon = \epsilon_0$, and $\mu = \mu_0$

$$\alpha = 0, \quad \beta = \omega\sqrt{\mu_0\epsilon_0}$$

– Good conductors:

A perfect ~~or~~ a good conductor is one, in which

$\sigma >> \omega \epsilon$ $\quad \frac{\sigma}{\omega \epsilon} >> 1$

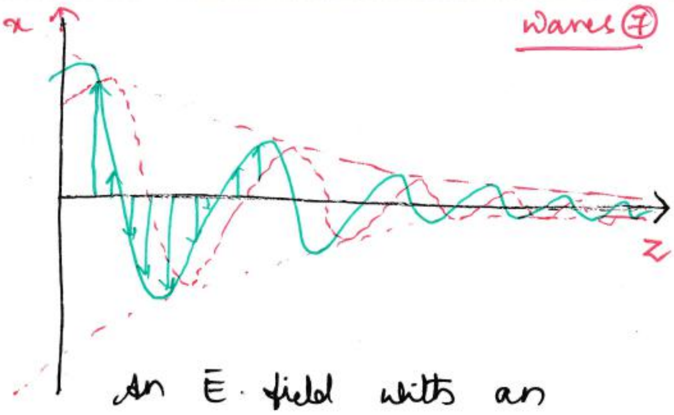
$\sigma \simeq \infty, \quad \epsilon = \epsilon_0, \quad \mu = \mu_0 \mu_r$

now $\alpha = \omega \sqrt{\frac{\mu \epsilon}{2}\left[\frac{\sigma}{\epsilon \omega} - 1\right]} = \omega \sqrt{\mu_0 \mu_r \epsilon_0 (\sigma / \epsilon \omega)}$

$= \sqrt{\frac{\omega^2 \mu_0 \mu_r \cancel{\epsilon_0} \sigma}{2 \cancel{\epsilon_0} \omega}} = \sqrt{\frac{\omega \mu \sigma}{2}}$

||ly $\beta = \omega \sqrt{\frac{\mu \epsilon}{2}\left[\frac{\sigma}{\epsilon \omega} + 1\right]} = \sqrt{\frac{\omega^2 \mu \cancel{\epsilon} \sigma}{2 \cancel{\epsilon} \omega}} = \sqrt{\frac{\omega \mu \sigma}{2}}$

$\therefore \quad \alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}}$ in a good conductor.



An $\bar{E}$ field with an x-component traveling in the + z-direction.

let $\bar{E}_s = E_{xs}(z)\,\bar{a}_x$

from $\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$

$\Rightarrow \nabla^2 E_{xs}(z) - \gamma^2 E_{xs}(z) = 0$

$\Rightarrow (\nabla^2 - \gamma^2)\, E_{xs}(z) = 0$

we know,

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

here $V = E_{xs}(z)$

then $\underbrace{\cancel{\frac{\partial^2}{\partial x^2} E_{xs}(z)}}_{0} + \underbrace{\cancel{\frac{\partial^2}{\partial y^2} E_{xs}(z)}}_{0} + \frac{\partial^2}{\partial z^2} E_{xs}(z) - \gamma^2 E_{xs}(z) = 0$

$\therefore \frac{\partial^2}{\partial z^2} E_{xs}(z) - \gamma^2 E_{xs}(z) = 0$

$\left[\frac{d^2}{dz^2} - \gamma^2\right] E_{xs}(z) = 0.$ $\longrightarrow$ This is a scalar wave equation, also, a linear homogeneous DE.

the Soln. $E_{xs}(z) = E_0 e^{-\gamma z} + E_0' e^{\gamma z}$ E_0, E_0' are constants

the term $E_0' e^{-\gamma(-z)}$ denotes a wave travelling along $-\bar{a}_z$, whereas we assume wave propagation along $\bar{a}_z$, hence, E_0' becomes zero.

$\Rightarrow$ $$E_{xs}(z) = E_0 e^{-\gamma z}$$

$$\bar{E}(z,t) = Re\{E_{xs}(z) e^{j\omega t} \bar{a}_x\}$$

Substitute $E_{xs}(z)$ in the above

$$\bar{E}(z,t) = Re\{E_0 e^{-\gamma z} e^{j\omega t} \bar{a}_x\} \quad \text{and } \gamma = \alpha + j\beta$$

$$= Re\{E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \bar{a}_x\}$$

$$\boxed{\bar{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_x} \rightarrow (14)$$

Ily we can define $\bar{H}(z,t)$ as below,

$$\bar{H}(z,t) = Re\{H_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \bar{a}_y\}$$

where $H_0 = \dfrac{E_0}{\eta}$

here η: characteristic or intrinsic impedance of the medium.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta| e^{j\theta_\eta}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}, \quad \boxed{\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}} \rightarrow (16)$$

here $0 \le \theta_\eta \le 45^\circ$

$$\bar{H}(z,t) = Re\left\{\frac{E_0}{|\eta| e^{j\theta_\eta}} e^{-\alpha z} e^{j(\omega t - \beta z)} \bar{a}_y\right\}$$

$$\boxed{\bar{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \bar{a}_y} \rightarrow (15)$$

If we observe eqns (14) & (15), the magnitude 'E_0' decreases with $e^{-\alpha z}$ ($z>1$), as the wave propagates along the z-axis ($\bar{a}_z$).

α ⇒ Hence, 'α' is known as the attenuation constant & attenuation coefficient of the medium

α ⇒ It is a measure of the spatial rate of decay of the wave in the medium, measured in Np/m and can be expressed in decibels/m (dB/m).

α ⇒ An attenuation of 1 Np denotes a reduction to e^{-1} of the original value, whereas, an increase of 1 Np indicates an increase by a factor of 'e'.

$$1\,Np = 20 \log_{10} e = 8.686\,dB.$$

α ⇒ if $\sigma=0$, which is the case of a lossless medium or free space, $\alpha=0$ and the wave doesn't get attenuated as it propagates

β ⇒ the quantity 'β' is a measure of the phase shift per unit length in rad/m. Hence, 'β' is called a phase constant and ~~phase~~ wave number.

$$u = \frac{\omega}{\beta}, \quad \lambda = \frac{2\pi}{\beta}$$

$\Rightarrow$ Eqns (14) & (15) show that, $\bar{E}$ and $\bar{H}$ are out of phase by θ_η at any instant of time due to the complex intrinsic impedance of the medium.

$\Rightarrow$ at any time E leads $\bar{H}$ (or $\bar{H}$ lags $\bar{E}$) by θ_η.

$\Rightarrow$ The conduction current density

$$\bar{J} = \sigma \bar{E}$$

and the displacement current density $\bar{J}_d = \dfrac{\partial \bar{D}}{\partial t}$

$$= \epsilon \frac{\partial}{\partial t}\left[E_s e^{j\omega t}\right]$$

$$= j\omega\epsilon \bar{E}_s e^{j\omega t}$$

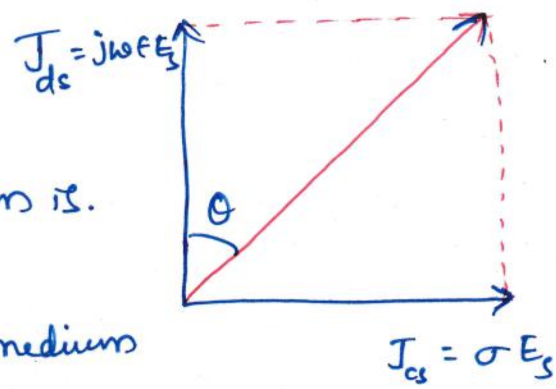
$$= j\omega\epsilon \bar{E}$$

the ratio of $\bar{J}/\bar{J}_d = \dfrac{\sigma \cancel{\bar{E}}}{j\omega\epsilon \cancel{\bar{E}}} = \dfrac{\sigma}{j\omega\epsilon}$

$$\left|\bar{J}/\bar{J}_d\right| = \boxed{\frac{\sigma}{\omega\epsilon} = \tan\theta.} \rightarrow (17)$$

$\tan\theta$: is the lossy tangent, θ: loss angle of the medium.

$\Rightarrow$ $\tan\theta$ & θ can be used to determine how lossy a medium is.



$\Rightarrow$ $\sigma \ll \omega\epsilon \rightarrow \tan\theta \rightarrow 0$, the medium is said to be lossless or perfect dielectric.

$\Rightarrow$ $\sigma \gg \omega\epsilon \rightarrow \tan\theta \rightarrow \infty$, the medium is a good conductor.

⇒ The characteristic behavior of a medium depends not only on the constitutive parameters σ, ϵ, and μ but also on the frequency of operation.

⇒ A medium that is regarded as a good conductor at low frequencies, may be a good dielectric at higher frequencies.

⇒ from eqns (15) & (17)

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon} \qquad \tan\theta = \frac{\sigma}{\omega\epsilon}$$

$$\Rightarrow \quad \theta = 2\theta_\eta$$

and we know $\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon)\, E_s$

$$= j\omega\epsilon\left[1 - \frac{j\sigma}{\omega\epsilon}\right] E_s$$

$$= j\omega\epsilon_c E_s$$

where $\boxed{\epsilon_c = \epsilon\left[1 - \frac{j\sigma}{\omega\epsilon}\right]} \rightarrow (18)$

$$= \epsilon' - j\epsilon'' \quad , \text{ where } \epsilon' = \epsilon \text{ and } \epsilon'' = \sigma/\omega$$

ϵ_c : is the complex permittivity of the medium

the ratio of ϵ' and ϵ'' ⇒ $\epsilon'/\epsilon'' = \frac{\omega\epsilon}{\sigma} \Rightarrow \epsilon''/\epsilon' = \frac{\sigma}{\epsilon\omega}$

which is the loss tangent of the medium.

Uniform plane waves:

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} \qquad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

- curl operation is a differentiation wrt space

$$\nabla \times \bar{H} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\bar{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\bar{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\bar{a}_z$$

$$\nabla \times \bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\frac{\partial}{\partial t}\left[\nabla \times \bar{H}\right] = \left[\frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial t}\right) - \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial t}\right)\right]\bar{a}_x + \left[\frac{\partial}{\partial z}\left(\frac{\partial A_x}{\partial t}\right) - \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial t}\right)\right]\bar{a}_y + \left[\frac{\partial}{\partial x}\left(\frac{\partial A_y}{\partial t}\right) - \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial t}\right)\right]\bar{a}_z$$

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\nabla \times (\nabla \times E) = -\mu \, \nabla \times \left(\frac{\partial \bar{H}}{\partial t}\right)$$

$$\epsilon \frac{\partial E}{\partial t}$$

$$\nabla(\nabla \cdot E) - \nabla^2 \bar{E} = -\mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla \cdot E = 0 \Rightarrow \quad \nabla^2 \bar{E} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\text{IIIy} \quad \nabla^2 \bar{H} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}$$

These are said to be wave equations. (in non-conducting ~~free space~~)

— A uniform plane wave has the same magnitude of electric and magnetic fields in the direction of the wave propagation.

— In other words, E & $\bar{H}$ are considered to be independent of their respective dimensions.

Let

x - direction of wave propagation

y - " $\bar{E}$

z - " $\bar{H}$

Then $\nabla^2 \bar{E}$ has only x-component.

$$\nabla^2 \bar{E} = \frac{\partial^2 \bar{E}}{\partial x^2} \rightarrow (19)$$

and the eqn $\nabla^2 \bar{E} = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} \rightarrow (20)$

$$\frac{\partial^2 \bar{E}}{\partial x^2} = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla \times \nabla \times \bar{A} = \nabla^2 A_x \bar{a}_x + \nabla^2 A_y \bar{a}_y + \nabla^2 A_z \bar{a}_z$$

Since the direction of wave propagation is 'x' and 'E' has y (or) z components.

Let us say 'E' has y component E_y.

then $$\frac{\partial^2 E_y}{\partial x^2} = \mu \epsilon \frac{\partial^2 E_y}{\partial t^2} \longrightarrow (21)$$

This is a second-order PDE. This is the PDE for voltage (or) current along a lossless transmission medium. The general form of solution is

$$\bar{E} = f_1(x - v_0 t) + f_2(x + v_0 t)$$

where $v_0 = 1/\sqrt{\mu \epsilon}$

func f_2 becomes zero, since the direction of wave propagation is +ve 'x'.

$$\therefore \bar{E} = f_1(x - v_0 t)$$

$$\frac{\partial^2 \bar{E}}{\partial x^2} = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial x^2} = \mu \epsilon \frac{\partial^2 E_x}{\partial t^2}, \quad \frac{\partial^2 E_y}{\partial x^2} = \mu \epsilon \frac{\partial^2 E_y}{\partial t^2}, \quad \frac{\partial^2 E_z}{\partial x^2} = \mu \epsilon \frac{\partial^2 E_z}{\partial t^2}$$

in a charge-free region $\rho_v = 0 \Rightarrow \nabla \cdot \bar{E} = 0$.

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \longrightarrow (22)$$

Since the direction of wave propagation is 'x' and according to the definition of uniform planewave, $\bar{E}$ is independent of y and z.

$$\therefore \frac{\partial E_y}{\partial y} = \frac{\partial E_z}{\partial z} = 0$$

now eqn (22) becomes $\frac{\partial E_x}{\partial x} = 0$

$\Rightarrow$ from $\frac{\partial^2 E_x}{\partial x^2} = \mu \epsilon \frac{\partial^2 E_x}{\partial t^2}$ and $\frac{\partial E_x}{\partial x} = 0$

$$\Rightarrow \frac{\partial^2 E_x}{\partial t^2} = 0$$

'E_x' : is either zero, constant in time, or increasing uniformly with time.

$\Rightarrow$ if a field is either constant in time or increasing uniformly with time is not part of the wave motion.

$\Rightarrow$ therefore $E_x = 0$ is the only possible definition that says a uniform planewave progressing in the $+x$-direction has no component of '$\bar{E}$'.

⇒ A similar analysis would show that there is no x-component of $\bar{H}$.

⇒ Finally, we can conclude that uniform plane electromagnetic waves are transverse and have components of $\bar{E}$ and $\bar{H}$ only in directions perpendicular to the direction of propagation.

$$\therefore \quad \nabla \times \bar{E} = \frac{-\partial E_z}{\partial x}\bar{a}_y + \frac{\partial E_y}{\partial x}\bar{a}_z$$

$$\nabla \times \bar{H} = \frac{-\partial H_z}{\partial x}\bar{a}_y + \frac{\partial H_y}{\partial x}\bar{a}_z$$

we know in free space $\nabla \times \bar{H} = \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial E}{\partial t}$

$$\therefore \quad \frac{-\partial H_z}{\partial x}\bar{a}_y + \frac{\partial H_y}{\partial x}\bar{a}_z = \cancel{\epsilon_0\left[\frac{-\partial E_z}{\partial x}\bar{a}_y + \frac{\partial E_y}{\partial x}\bar{a}_z\right]}$$

$$= \epsilon_0\left[\frac{\partial E_y}{\partial t}\bar{a}_y + \frac{\partial E_z}{\partial t}\bar{a}_z\right]$$

$$\text{Ily} \quad \frac{-\partial E_z}{\partial x}\bar{a}_y + \frac{\partial E_y}{\partial x}\bar{a}_z = -\mu_0\left[\frac{\partial H_y}{\partial t}\bar{a}_y + \frac{\partial H_z}{\partial t}\bar{a}_z\right]$$

equating $\bar{a}_y$ & $\bar{a}_z$ terms

$$-\frac{\partial H_z}{\partial x} = \epsilon_0 \frac{\partial E_y}{\partial t} \rightarrow 23(a)$$

$$\frac{\partial H_y}{\partial x} = \epsilon_0 \frac{\partial E_z}{\partial t} \rightarrow 23(b)$$

$$+\frac{\partial E_z}{\partial x} = \mu_0 \frac{\partial H_y}{\partial t} \rightarrow 23(c)$$

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \rightarrow 23(d)$$

now. $\bar{E} = f_1(x - v_0 t)$

and if $E_y = f_1(x - v_0 t)$, then

$$\cancel{\frac{\partial E_y}{\partial t} = \frac{\partial f_1(}{\partial(x - v_0 t)} \cdot \frac{\partial(x - v_0 t)}{\partial t}}$$

$$\frac{\partial E_y}{\partial t} = f_1'(x - v_0 t)(-v_0) \longrightarrow \text{substitute this in } (23(a))$$

now $\cancel{+}\frac{\partial H_z}{\partial x} = \cancel{+} v_0 \epsilon_0 f_1'$

$v_0 = \sqrt{\mu_0 \epsilon_0}$ or $\sqrt{\mu \epsilon}$

$$H_z = \int v_0 \epsilon_0 f_1' \, dx + C$$

$$= \sqrt{\frac{\epsilon_0}{\mu_0}} \int f_1' \, dx + C$$

if $f_1(x - v_0 t)$

$\frac{\partial f_1}{\partial x} = f_1'$

$$= \sqrt{\frac{\epsilon_0}{\mu_0}} \int \frac{\partial f_1}{\partial x} \, dx + C.$$

$$= \sqrt{\frac{\epsilon_0}{\mu_0}} f_1 + C.$$

$$H_z = \sqrt{\frac{\epsilon_0}{\mu_0}} E_y + C.$$

→ 'c' indicates the presence of field independent of 'x'.

→ since this is not part of the wave motion, it will be neglected and ~~that~~ hence

$$H_z = \sqrt{\frac{\epsilon_0}{\mu_0}} E_y \Rightarrow \frac{E_y}{H_z} = \sqrt{\frac{\mu_0}{\epsilon_0}} \text{ (or) } \sqrt{\frac{\mu}{\epsilon}}$$

$|||^{ly}$ $\frac{E_z}{H_y} = -\sqrt{\mu/\epsilon}$.

$$\cancel{\frac{E_y}{H_z} = \sqrt{\frac{\epsilon_0}{\mu_0}}} \text{ (or) } \boxed{\cancel{\frac{E_y}{H_z} = \sqrt{\frac{\epsilon}{\mu}}}}$$

$E = \sqrt{E_y^2 + E_z^2}$ and $H = \sqrt{H_y^2 + H_z^2}$

$$\boxed{E/H = \sqrt{\mu/\epsilon}}$$ ohms

$\therefore \sqrt{\frac{\mu}{\epsilon}}$ is said to be characteristic impedance (or) intrinsic impedance. of a non-conducting medium.

for free - space.

$\epsilon = \epsilon_0$, $\mu = \mu_0$

$$\sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \simeq 120\pi \simeq 377\,\Omega$$

$$\boxed{\eta = \sqrt{\mu/\epsilon}}$$

Skin depth:

- In good conductors $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \beta$

$$= \sqrt{\pi f \mu \sigma}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} \Rightarrow \boxed{\lambda = \frac{u}{f} = \frac{2\pi}{\beta}}$$

- $\eta = \left|\frac{\bar{E}}{\bar{H}}\right| = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$$= \sqrt{\frac{\cancel{j\omega}\mu}{\cancel{j\omega}\epsilon\left(1+\frac{\sigma}{j\omega\epsilon}\right)}} = \sqrt{\frac{\mu}{\left(\frac{\sigma}{j\omega\epsilon}\right)}} = \sqrt{\frac{j\omega\mu\epsilon}{\sigma}}$$

for good conductors $\sigma >> \omega\epsilon$

$$\therefore \eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$|\eta| = \sqrt{\frac{\omega\mu}{\sigma}}$$

$$2\theta_\eta = \tan^{-1}\left(\frac{\sqrt{\frac{j\omega\mu}{\sigma}}}{0}\right) = 90^\circ$$

$$\theta_\eta = 45^\circ$$

$$\therefore \eta = \sqrt{\frac{\mu\omega}{\sigma}} \angle 45^\circ$$

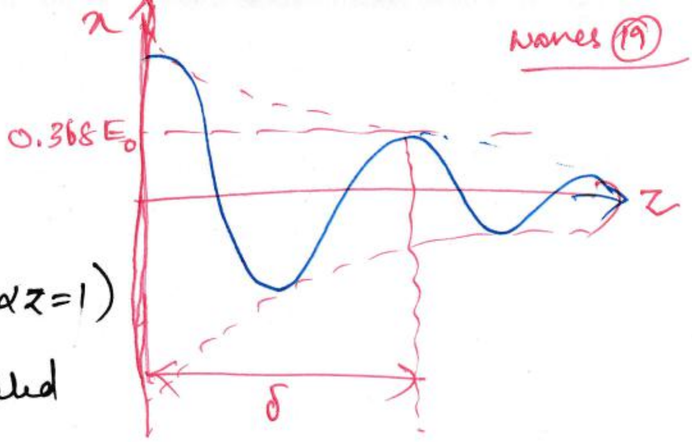
$\Rightarrow$ Thus $\bar{E}$ leads $\bar{H}$ by 45°.

$$\bar{E} = E_0 e^{-\alpha z} \cos(\beta z - \omega t)\, \bar{a}_x$$

$$\bar{H} = H_0 e^{-\alpha z} \cos(\beta z - \omega t - 45^\circ)\, \bar{a}_y$$

$$\boxed{\bar{H} = \frac{E_0}{\sqrt{\frac{\omega\mu}{\sigma}}} e^{-\alpha z} \cos(\beta z - \omega t - 45^\circ)\, \bar{a}_y}$$

* The distance 'δ', through which the wave amplitude is attenuated by the factor $e^{-\alpha z}$ $(\alpha z = 1)$ $= e^{-1}$ ($\simeq 37\%$ of max value) is called skin depth.



⇒ This is also known as penetration depth.

$$\therefore \quad \alpha\delta = 1 \quad \Rightarrow \quad \delta = 1/\alpha$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \Rightarrow \quad \delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

⇒ The skin depth (δ) is a measure of the depth to which an EM wave can penetrate the medium.

⇒ Skin depth (δ) is useful in calculating the ac resistance due to skin effect.

We know the dc resistance

$$R_{dc} = \frac{l}{\sigma S}$$

for a good conductor, skin resistance (R_s) surface resistance is $\mathrm{Re}\{\eta\}$,

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}, \quad \eta = \sqrt{\frac{j\omega\mu}{\sigma}}, \quad \alpha = \sqrt{\pi f\mu\sigma}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ = \sqrt{\frac{2\pi f\mu\sigma}{\sigma^2}}\, e^{j\pi/4} = \sqrt{\frac{2}{\sigma^2}}\cdot\frac{1}{\delta}\, e^{j\pi/4}.$$

$$\eta = \frac{\sqrt{2}}{\sigma\delta}\left(\frac{1+j}{\sqrt{2}}\right) = \frac{1+j}{\sigma\delta}$$

since the ac resistance / skin resistance is real part of 'η'.

$$\mathrm{Re}\{\eta\} = \mathrm{Re}\left\{\frac{1+j}{\sigma\delta}\right\} = \frac{1}{\sigma\delta}$$

$$\therefore \quad \boxed{R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}}$$

⇒ The above eqn is the resistance of a unit width and unit length of the conductor. It is equivalent to the dc resistance for a unit length of the conductor having cross-sectional area $1 \times \delta$.

⇒ Thus, for a given width w, and length l, the ac resistance is

$$\boxed{R_{ac} = \frac{l}{\sigma\delta w} = \frac{R_s l}{w}}$$

⇒ Let $S = \delta w$ and $w = 2\pi a$

then $R_{dc} = \dfrac{l}{\sigma(\pi a^2)}$ and $R_{ac} = \dfrac{l}{\sigma\delta(2\pi a)}$

$$\frac{R_{ac}}{R_{dc}} = \frac{\cancel{l}}{\cancel{\sigma}\delta(2\cancel{\pi}\cancel{a})} \times \frac{\cancel{\sigma}(\cancel{\pi} a^{\cancel{2}})}{\cancel{l}} = \frac{a}{2\delta} = \frac{a}{2}\sqrt{\pi f \mu \sigma}$$

$$\boxed{R_{ac}/R_{dc} = \frac{a}{2}\sqrt{\pi f \mu \sigma}}$$

Power and the Poynting Vector:

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{\partial E}{\partial t}$$

$$\bar{E} \cdot (\nabla \times \bar{H}) = \sigma E^2 + \bar{E} \cdot \epsilon \frac{\partial E}{\partial t} \longrightarrow (1)$$

let $\bar{A} = \bar{H}$ & $\bar{B} = \bar{E}$

$$\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A} \cdot (\nabla \times \bar{B})$$

$$\nabla \cdot (\bar{H} \times \bar{E}) = \bar{E} \cdot (\nabla \times \bar{H}) - \bar{H} \cdot (\nabla \times \bar{E})$$

$$\Rightarrow \bar{E} \cdot (\nabla \times \bar{H}) = \nabla \cdot (\bar{H} \times \bar{E}) + \bar{H} \cdot (\nabla \times \bar{E}) \longrightarrow (2)$$

from (1) & (2)

$$\nabla \cdot (\bar{H} \times \bar{E}) + \bar{H} \cdot (\nabla \times \bar{E}) = \sigma E^2 + \bar{E} \cdot \epsilon \frac{\partial E}{\partial t} \longrightarrow (3)$$

$$= \cancel{\sigma E^2 + \tfrac{1}{2} \epsilon \frac{\partial}{\partial t} E^2}$$

let

$$\epsilon \bar{E} \cdot \frac{\partial \bar{E}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2} \bar{D} \cdot E\right)$$

and

$$\mu \bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2} \bar{B} \cdot \bar{H}\right)$$

Illy

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\bar{H} \cdot (\nabla \times \bar{E}) = \bar{H} \cdot \left(-\mu \frac{\partial \bar{H}}{\partial t}\right) \longrightarrow (4)$$

Substituting (4) in (3)

$$\nabla \cdot (\bar{H} \times \bar{E}) + \bar{H} \cdot \left(-\mu \frac{\partial \bar{H}}{\partial t}\right) = \sigma E^2 + \bar{E} \cdot \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\Rightarrow \nabla \cdot (\bar{H} \times \bar{E}) = \sigma E^2 + \frac{\partial}{\partial t}\left(\frac{1}{2} \bar{D} \cdot E\right) + \frac{\partial}{\partial t}\left(\frac{1}{2} \bar{B} \cdot \bar{H}\right)$$

$$\Rightarrow \quad \nabla \cdot (\bar{E} \times \bar{H}) = \frac{-\partial}{\partial t}\left[\frac{1}{2}\bar{D}\cdot\bar{E} + \frac{1}{2}\bar{B}\cdot\bar{H}\right] - \sigma E^2$$

$$\Rightarrow \quad \int_v \nabla \cdot (\bar{E} \times \bar{H})\, dv = -\frac{\partial}{\partial t}\left[\int_v \left(\frac{1}{2}\bar{D}\cdot\bar{E} + \frac{1}{2}\bar{B}\cdot\bar{H}\right) dv\right] - \sigma \int_v E^2 dv$$

Applying Divergence Theorem to the left-hand side

$$\Rightarrow \quad \underbrace{\oint_s (\bar{E} \times \bar{H}) \cdot d\bar{s}}_{\text{total power leaving the volume}} = \underbrace{\frac{-\partial}{\partial t}\left[\int_v \frac{1}{2}(\bar{D}\cdot\bar{E}) + \frac{1}{2}(\bar{B}\cdot\bar{H})\right]}_{\text{rate of decrease in energy stored in } \bar{E} \text{ and } \bar{H} \text{ fields.}} - \underbrace{\int_v \sigma E^2 dv.}_{\text{Ohmic power dissipated.}}$$

$\Rightarrow \quad \bar{E} \times \bar{H} = \bar{\mathcal{P}}$ is known as the Poynting vector. (W/m^2)

Poynting's theorem: states that the net power flowing out of a given volume 'v' is equal to the time rate of decrease in the energy stored within 'v' minus the ohmic losses.

$$\bar{E}(z,t) = E_0 e^{-\alpha t} \cos(\omega t - \beta z)\, \bar{a}_x$$

$$\bar{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \bar{a}_y$$

$$\bar{\mathcal{P}}(z,t) = \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta)\, \bar{a}_z$$

$$= \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \left[\cos\theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)\right] \bar{a}_z$$

The average Poynting vector

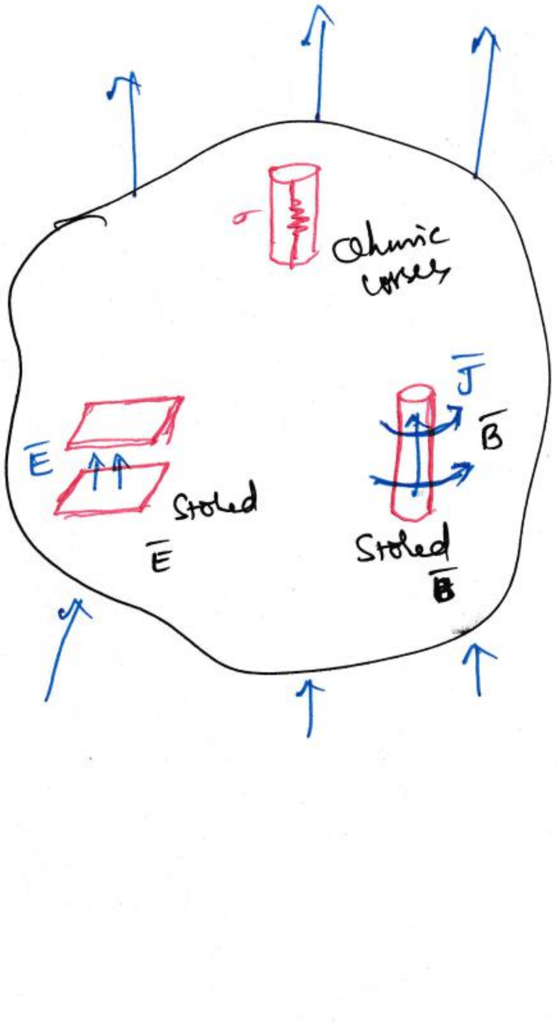
$$\bar{P}_{avg}(z) = \frac{1}{T}\int_0^T P(z,t)\,dt$$

$$\bar{P}_{avg}(z) = \frac{1}{2}\,\mathrm{Re}\left(E_s \times H_s^*\right)$$

$$\boxed{P_{avg}(z) = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta\, \bar{a}_z}$$

The total avg power crossing a given surface 'S'.

$$P_{avg} = \int_S P_{ave} \cdot d\bar{S}$$



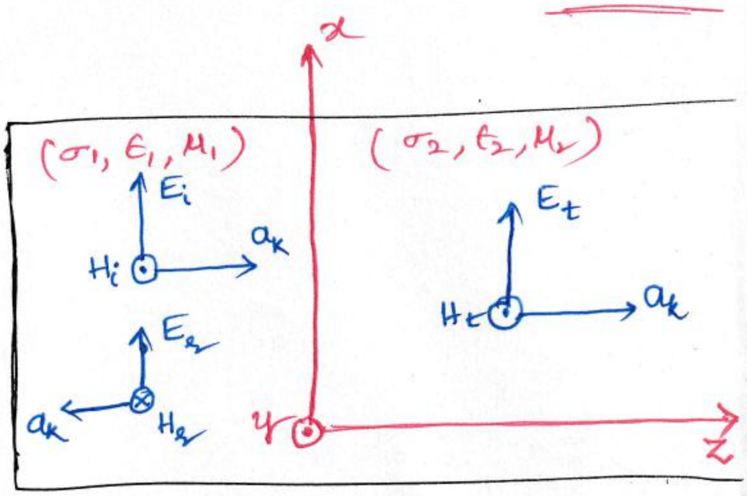
Reflection of a plane wave:

At normal incidence:

i – incident

r – reflected

t – transmitted / (refracted)



$$\bar{E}_{is}(z) = E_{io}\, e^{-\gamma_1 z}\, \bar{a}_x$$

$$\bar{H}_{is}(z) = H_{io}\, e^{-\gamma_1 z}\, \bar{a}_y = \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, \bar{a}_y$$

$$\bar{E}_{rs}(z) = E_{ro}\, e^{-\gamma_1(-z)}\, \bar{a}_x = E_{ro}\, e^{+\gamma_1 z}\, \bar{a}_x$$

$$\bar{H}_{rs}(z) = H_{ro}\, e^{-\gamma_1(-z)}\, (-\bar{a}_y) = \frac{-E_{ro}}{\eta_1}\, e^{\gamma_1 z}\, \bar{a}_y$$

$$\bar{E}_{ts}(z) = E_{to}\, e^{-\gamma_2 z}\, \bar{a}_x$$

$$\bar{H}_{ts}(z) = H_{to}\, e^{-\gamma_2 z}\, \bar{a}_y = \frac{E_{to}}{\eta_2}\, e^{-\gamma_2 z}\, \bar{a}_y$$

⇒ The total field in medium 1 is

$$\bar{E}_1 = \bar{E}_i + \bar{E}_r \qquad\qquad \bar{E}_2 = \bar{E}_t$$

$$\bar{H}_1 = \bar{H}_i + \bar{H}_r \qquad\qquad \bar{H}_2 = \bar{H}_t$$

From the boundary conditions defined, the tangential components of $\bar{E}$ & $\bar{H}$ are continuous at the boundary

$\therefore \quad E_i(0) + E_r(0) = E_t(0)$

$$E_{i0} + E_{r0} = E_{t0} \quad \rightarrow (1)$$

Ily $\quad H_i(0) + H_r(0) = H_t(0) \quad \Rightarrow H_{i0} - H_{r0} = H_{t0}$

$$\Rightarrow \frac{1}{\eta_1}\left[E_{i0} - E_{r0}\right] = \frac{1}{\eta_2} E_{t0} \quad \rightarrow (2)$$

$$E_{i0} + E_{r0} = E_{t0}$$

$$E_{i0} - E_{r0} = \frac{\eta_1}{\eta_2} E_{t0}$$

$$\Rightarrow 2E_{i0} = \left(1 + \frac{\eta_1}{\eta_2}\right) E_{t0}$$

$$\Rightarrow E_{t0} = \left(\frac{2\eta_2}{\eta_1 + \eta_2}\right) E_{i0}$$

Ily

$$2E_{r0} = \left(1 - \frac{\eta_1}{\eta_2}\right) E_{t0}$$

$$E_{r0} = \frac{\cancel{2}}{\cancel{2}}\left(\frac{\eta_2 - \eta_1}{\cancel{\eta_2}}\right)\left(\frac{\cancel{2}\cancel{\eta_2}}{\eta_1 + \eta_2}\right) E_{i0}$$

$$= \left(\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\right) E_{i0}$$

The reflection coefficient

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \Rightarrow \quad E_{r0} = \Gamma E_{i0}$$

The transmission coefficient

$$\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_1 + \eta_2} \quad \Rightarrow \quad E_{t0} = \tau E_{i0}$$

$\Rightarrow$ a) $1 + \Gamma = \tau$

b) Γ, τ are dimensionless and may be complex.

c) $0 \leq |\Gamma| \leq 1$

Case (i) : Medium 1 is a perfect dielectric ($\sigma_1 = 0, \mu_1, \epsilon_1$)

and Medium 2 is a perfect conductor ($\sigma_2 \simeq \infty, \mu_2, \epsilon_2$).

for a perfect conductor $\eta_2 = \sqrt{\frac{\omega\mu_2}{\sigma_2}} \angle 45^\circ$

Substituting $\sigma_2 = 0 \Rightarrow \eta_2 \to 0$.

then $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{-\eta_1}{\eta_1} = -1$

and $\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = 0$

$\Rightarrow$ This shows the wave is totally reflected. This is due to the fields in a perfect conductor vanished so there can be no transmitted wave. ($E_2 = 0$)

$\Rightarrow$ The totally reflected wave combines with the standing wave. A standing wave doesn't move/travel, it consists of two waves (E_i and E_r) of equal amplitude but opposite directions.

$$\bar{E}_{1s} = \bar{E}_{is} + \bar{E}_{rs} = \left(E_{i0} e^{-\gamma_1 z} + E_{r0} e^{\gamma_1 z} \right) \bar{a}_x$$

lly $\bar{H}_{1s} = \bar{H}_{is} - \bar{H}_{rs} = \left(H_{i0} e^{-\gamma_1 z} - H_{r0} e^{\gamma_1 z} \right) \bar{a}_y$

and $\Gamma = -1, \quad \sigma_1 = 0, \quad \alpha_1 = 0, \quad \beta_1, \quad \Rightarrow \gamma = j\beta_1$

$$\bar{E}_{1s} = -E_{i0}\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\bar{a}_x$$

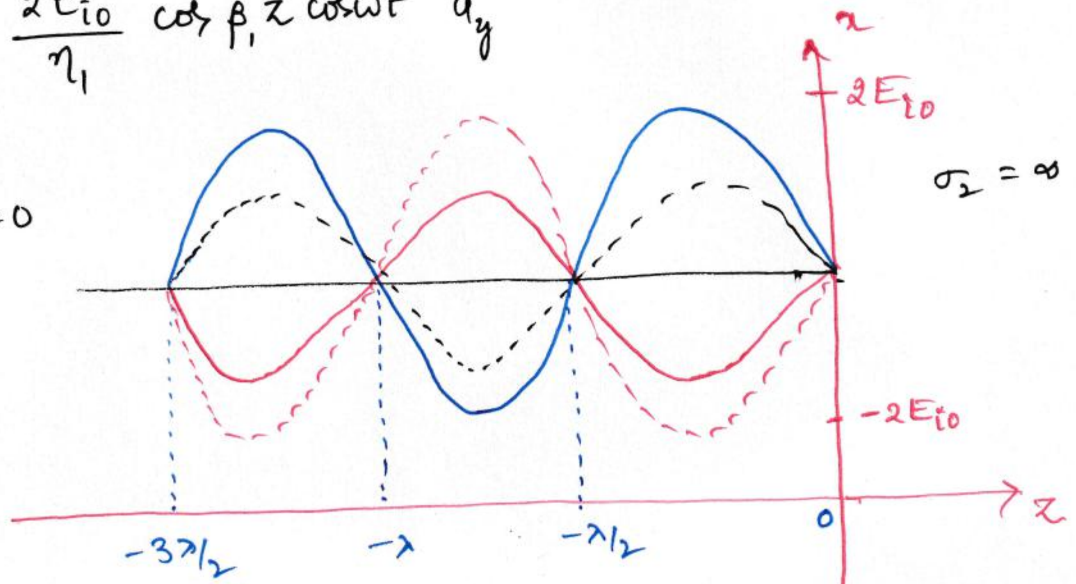
$$= -2jE_{i0}\sin\beta_1 z\,\bar{a}_x$$

$$\bar{E}_1 = \mathrm{Re}\left\{\bar{E}_{1s}e^{j\omega t}\right\} = \mathrm{Re}\left\{-2jE_{i0}\sin\beta_1 z\,(\cos\omega t + j\sin\omega t)\right\}\bar{a}_x$$

$$= \mathrm{Re}\left\{-2jE_{i0}\sin\beta_1 z\cos\omega t + 2E_{i0}\sin\beta_1 z\sin\omega t\right\}\bar{a}_x$$

$$\bar{E}_1 = 2E_{i0}\sin\beta_1 z\sin\omega t\,\bar{a}_x$$

lly for $\bar{H}_1 = \dfrac{2E_{i0}}{\eta_1}\cos\beta_1 z\cos\omega t\,\bar{a}_y$



$\lambda = \dfrac{2\pi}{\beta}$

$$\bar{E}_1 = 2E_{i0}\sin\beta_1 z\,\sin\omega t\,\bar{a}_x$$

$t = 0,\ T/8,\ T/4,\ 3T/8,\ T/2,\ \ldots \qquad T = \dfrac{2\pi}{\omega}$

$t = 0 \Rightarrow \bar{E}_1 = 0$

$t = T/8 \Rightarrow \dfrac{2\pi}{8\omega} \Rightarrow \bar{E}_1 = 2E_{i0}\sin\beta_1 z\sin\left[\omega\cdot\dfrac{2\pi}{8\omega}\right]^{\Rightarrow\pi/4} = \sqrt{2}E_{i0}\sin\beta_1 z$

$t = T/4 \Rightarrow \dfrac{2\pi}{4\omega} \Rightarrow \bar{E}_1 = 2E_{i0}\sin\beta_1 z\sin\left[\omega\cdot\dfrac{2\pi}{4\omega}\right] = 2E_{i0}\sin\beta_1 z$

$t = 3T/8 \Rightarrow \dfrac{6\pi}{8\omega} \Rightarrow \bar{E}_1 = 2E_{i0}\sin\beta_1 z\sin\left[\omega\cdot\dfrac{6\pi}{8\omega}\right] = -\sqrt{2}E_{i0}\sin\beta_1 z$

$t = T/2 \Rightarrow \dfrac{2\pi}{2\omega} \Rightarrow \bar{E}_1 = 2E_{i0}\sin\beta_1 z\sin\left[\omega\cdot\dfrac{2\pi}{2\omega}\right] = 0$

Case (2): When both medium 1 & 2 are lossless.

(a) if $\eta_2 > \eta_1$, $\Gamma > 0$ $\sigma_1 = 0 = \sigma_2$.

⇒ now, there will be a standing wave in medium 1 but also a transmitted wave in medium 2.

⇒ However, the incident and reflected waves have amplitudes that are not equal in magnitude.

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \Rightarrow E_{ro} = \Gamma E_{io}$$

$$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} \Rightarrow E_{to} = \tau E_{io}$$

$$\bar{E}_{1s} = \bar{E}_{is} + \bar{E}_{rs} \leftarrow \text{total field in medium 1.}$$

$$\bar{E}_{2s} = \bar{E}_{ts} \leftarrow \text{field in medium 2.}$$

here $\sigma_1 = 0 = \sigma_2$, $\alpha_1 = 0 = \alpha_2$

$\gamma_1 = j\beta_1$ and $\gamma_2 = j\beta_2$

now $$\bar{E}_{1s} = \left(E_{io} e^{-\gamma_1 z} + E_{ro} e^{\gamma_1 z}\right) \bar{a}_x$$

$\because E_{ro} = \Gamma E_{io}$ $\Rightarrow$ $$\bar{E}_{1s} = E_{io}\left(e^{-\gamma_1 z} + \Gamma e^{+\gamma_1 z}\right) \bar{a}_x$$

$$= \left(E_{io} e^{-j\beta_1 z} + \Gamma E_{io} e^{j\beta_1 z}\right) \bar{a}_x$$

$$\bar{E}_1 = \mathrm{Re}\left\{\bar{E}_{1s} e^{j\omega t}\right\}$$

$$= \mathrm{Re}\left\{E_{io} e^{-j\beta_1 z} e^{j\omega t} + \Gamma E_{io} e^{j\beta_1 z} e^{j\omega t}\right\} \bar{a}_x$$

$$= \mathrm{Re}\left\{E_{io}\left[\cos(\omega t - \beta_1 z) - j\sin(\omega t - \beta_1 z)\right] + \Gamma E_{io}\left[\cos(\omega t + \beta_1 z) + j\sin(\omega t + \beta_1 z)\right]\right\} \bar{a}_x$$

$$\bar{E}_1 = \left[E_{i0} \cos(\omega t - \beta_1 z) + \Gamma E_{i0} \cos(\omega t + \beta_1 z) \right] \bar{a}_x$$

$$\bar{E}_i = E_{i0} \cos(\omega t - \beta_1 z) \; \bar{a}_x$$

$$\bar{E}_r = E_{r0} \cos(\omega t + \beta_1 z) = \Gamma E_{i0} \cos(\omega t + \beta_1 z) \; \bar{a}_x$$

Maximum of $|E_1|$ occurs at

$$-\beta_1 z_{max} = n\pi$$

$$z_{max} = \frac{-n\pi}{\beta_1} = \frac{-n\lambda_1}{2} \quad , \; n = 0, 1, 2, \ldots$$

and the minimum values of $|E_1|$ occurs at

$$-\beta_1 z_{min} = (2n+1)\frac{\pi}{2}$$

$$z_{min} = \frac{-(2n+1)\pi}{2\beta_1} = \frac{-(2n+1)\lambda_1}{4} \quad , \; n = 0, 1, 2, \ldots$$

Case (2b) : if $\eta_2 < \eta_1$, $\Gamma < 0$

locations of maximum value of $|E_1|$

$$z_{max} = \frac{-(2n+1)\pi}{2\beta_1} = \frac{-(2n+1)\lambda_1}{4} \quad , \; n = 0, 1, 2, \ldots$$

locations of minimum value of $|E_1|$

$$z_{min} = \frac{-n\pi}{\beta_1} = \frac{-n\lambda_1}{2} \quad , \; n = 0, 1, 2, \ldots$$

Note :

1. $|H_1|$ minimum occurs whenever there is $|E_1|$ maximum and vice-versa

2. The transmitted wave in medium 2 is purely a travelling wave, and consequently there are no maxima & minima in this region.

$\Rightarrow$ The ratio of $|E_1|_{max}$ to $|E_1|_{min}$ or $\left(|H_1|_{max} \text{ to } |H_1|_{min}\right)$ is called the standing wave ratio.

$$S = \frac{|E_1|_{max}}{|E_1|_{min}} = \frac{|H_1|_{max}}{|H_1|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

(or) $$|\Gamma| = \frac{S-1}{S+1}$$

Since $0 \leq |\Gamma| \leq 1$, $1 \leq S \leq \infty$.

— SWR is dimensionless and expressed in decibels (dB).

$$S \text{ dB} = 20\log_{10} S$$