Two – Port Networks

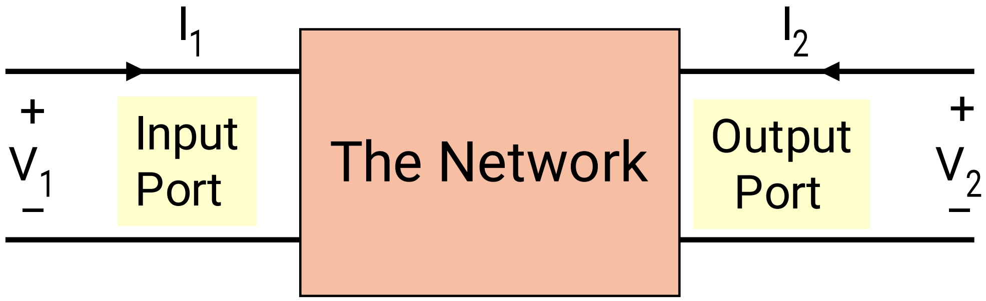
Unit- IV
Analysis and Design of Small Signal Low Frequency BJT Amplifiers:

Transistor Hybrid model, Determination of h-parameters from transistor characteristics, Typical values of h-parameters in CE, CB and CC configurations, Transistor amplifying action, Analysis of CE, CC, CB Amplifiers and CE Amplifier with emitter resistance, low frequency response of BJT Amplifiers, effect of coupling and bypass capacitors on CE Amplifier.

Two Port Networks

Generalitie
s:

The standard configuration of a two port:



The network ?

The voltage and current convention ?

* notes

Two Port Networks

Network Equations:

Impedance
Z parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Admittance
Y parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Hybrid
H parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Transmission
A, B, C, D
parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

- Of these four variables V_1, V_2, i_1 and i_2, two can be selected as independent variables and the remaining two can be expressed in terms of these independent variables. This leads to various two part parameters out of which the following three are more important.

Hybrid Parameters or h-parameters

If the input current i_1 and output Voltage V_2 are takes as independent variables, Then input voltage V_1 and output current i_2 can be written as depandent variable:

$$V_1 = h_{11} i_1 + h_{12} V_2$$

$$i_2 = h_{21} i_1 + h_{22} V_2$$

The four hybrid parameters : h_{11}, h_{12}, h_{21} and h_{22} are defined as follows.

- $h_{11} = [V_1 / i_1]$ with $V_2 = 0$, :I/P Resistance with output port short circuited.
- $h_{12} = [V_1 / V_2]$ with $i_1 = 0$, : reverse voltage transfer ratio with i/p port open circuited.
- $h_{21} = [i_2 / i_1]$ with $V_2 = 0$, : Forward current gain with output part short circuited.
- $h_{22} = [i_2 / V_2]$ with $i_1 = 0$, : Output admittance with input part open circuited.

The dimensions of h – parameters are as follows:

h_{11} - Ω

h_{12} – dimension less.

h_{21} – dimension less.

h_{22} – mhos

as the dimensions are not alike, (i.e) they are hybrid in nature, and these parameters are called as hybrid parameters.

Two Port Networks

Hybrid Parameters:

The equations for the hybrid parameters are:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \frac{V_1}{I_1} \bigg|_{V_2 = 0} \qquad h_{12} = \frac{V_1}{V_2} \bigg|_{I_1 = 0}$$

$$h_{21} = \frac{I_2}{I_1} \bigg|_{V_2 = 0} \qquad h_{22} = \frac{I_2}{V_2} \bigg|_{I_1 = 0}$$

* notes

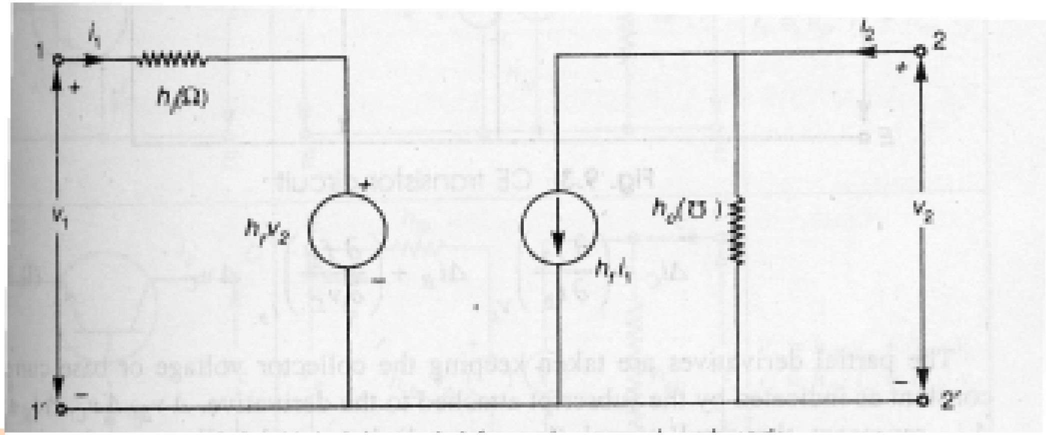
Hybrid Parameters:

The equations for the hybrid parameters are:

$V_1 = h_{11} i_1 + h_{12} V_2$ $\qquad$ $I_2 = h_{21} i_1 + h_{22} V_2$

$\downarrow$ $\qquad$ $\downarrow$

$V_1 = h_i i_1 + h_r V_2$ $\qquad$ $I_2 = h_f i_1 + h_0 V_2$



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_i = \frac{V_1}{I_1} \Bigg|_{V_2 = 0} \qquad h_r = \frac{V_1}{V_2} \Bigg|_{I_1 = 0}$$

$$h_f = \frac{I_2}{I_1} \Bigg|_{V_2 = 0} \qquad h_o = \frac{I_2}{V_2} \Bigg|_{I_1 = 0}$$

Transistor Hybrid Model CE Configuration

In common emitter transistor configuration, the input signal is applied between the base and emitter terminals of the transistor and output appears between the collector and emitter terminals. The input voltage (V_{be}) and the output current (i_c) are given by the following equations:

$$V_{be} = h_{ie}.i_b + h_{re}.V_c$$

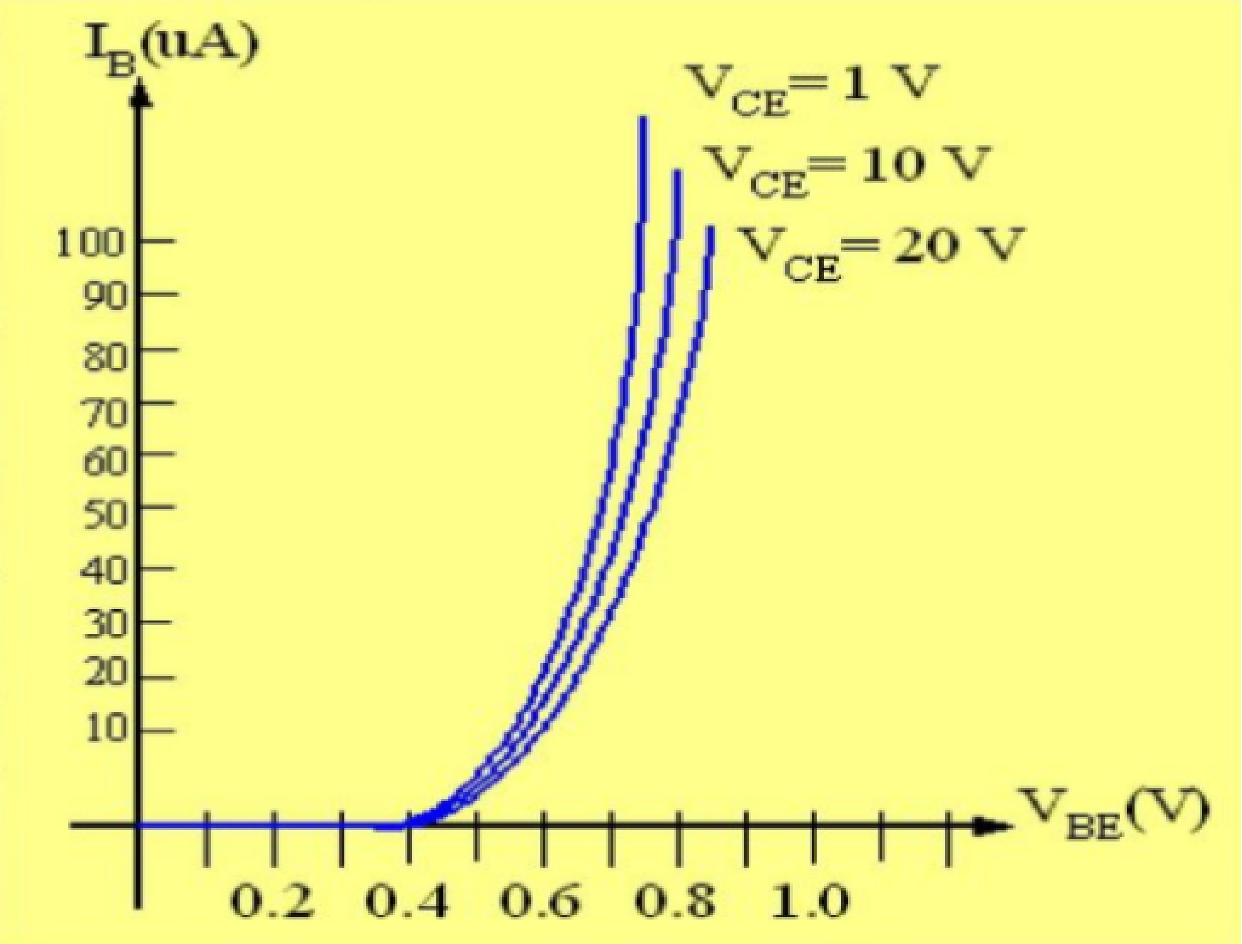
$$i_C = h_{fe}.i_b + h_{oe}.V_c$$

$$h_{ie} = \text{input resistance} = \left.\frac{\Delta v_{be}}{\Delta i_b}\right|_{v_{ce} = constant}$$

$$h_{re} = \text{reverse transfer voltage ratio} = \left.\frac{\Delta v_{be}}{\Delta v_{ce}}\right|_{i_b = constant}$$

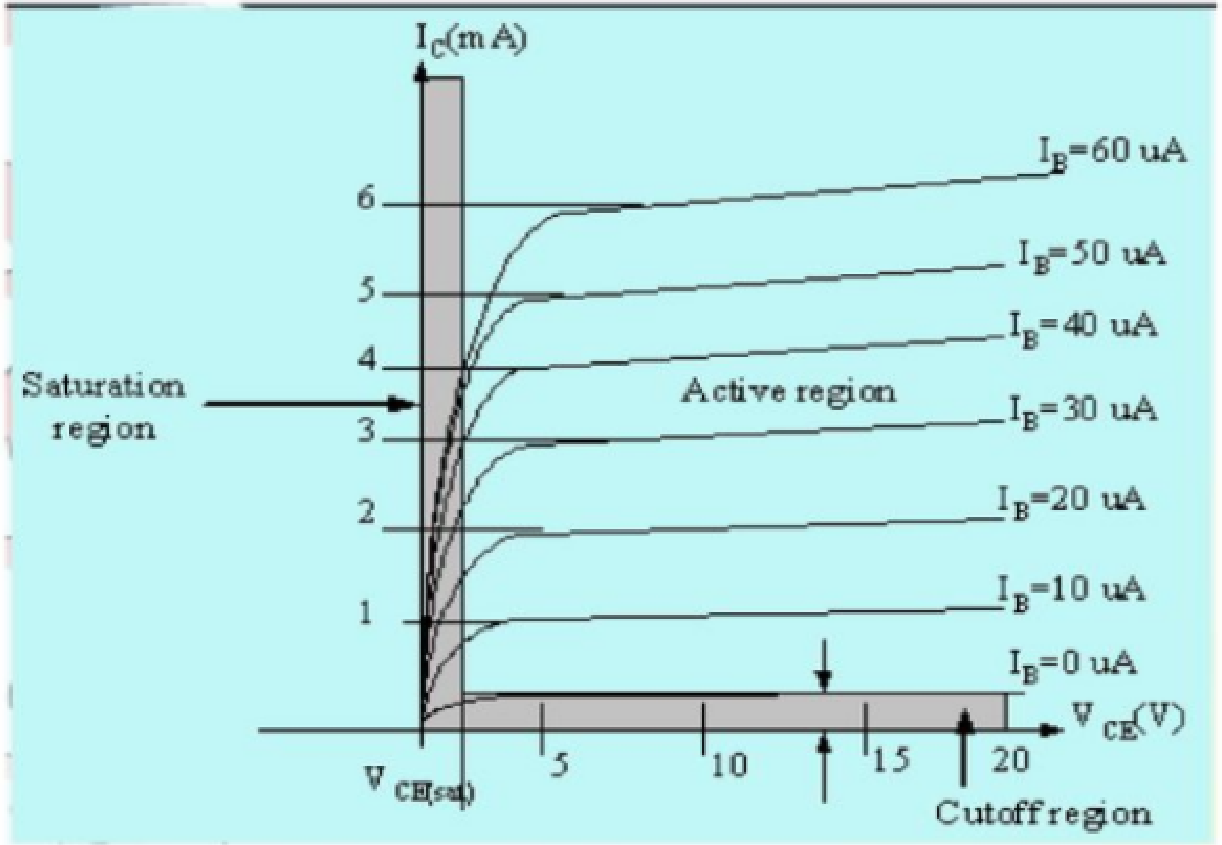
$$h_{fe} = \text{forward transfer current ratio} = \left.\frac{\Delta i_c}{\Delta i_b}\right|_{v_{ce} = constant}$$

$$h_{oe} = \text{output conductance} = \left.\frac{\Delta i_c}{\Delta v_{ce}}\right|_{i_b = constant}$$



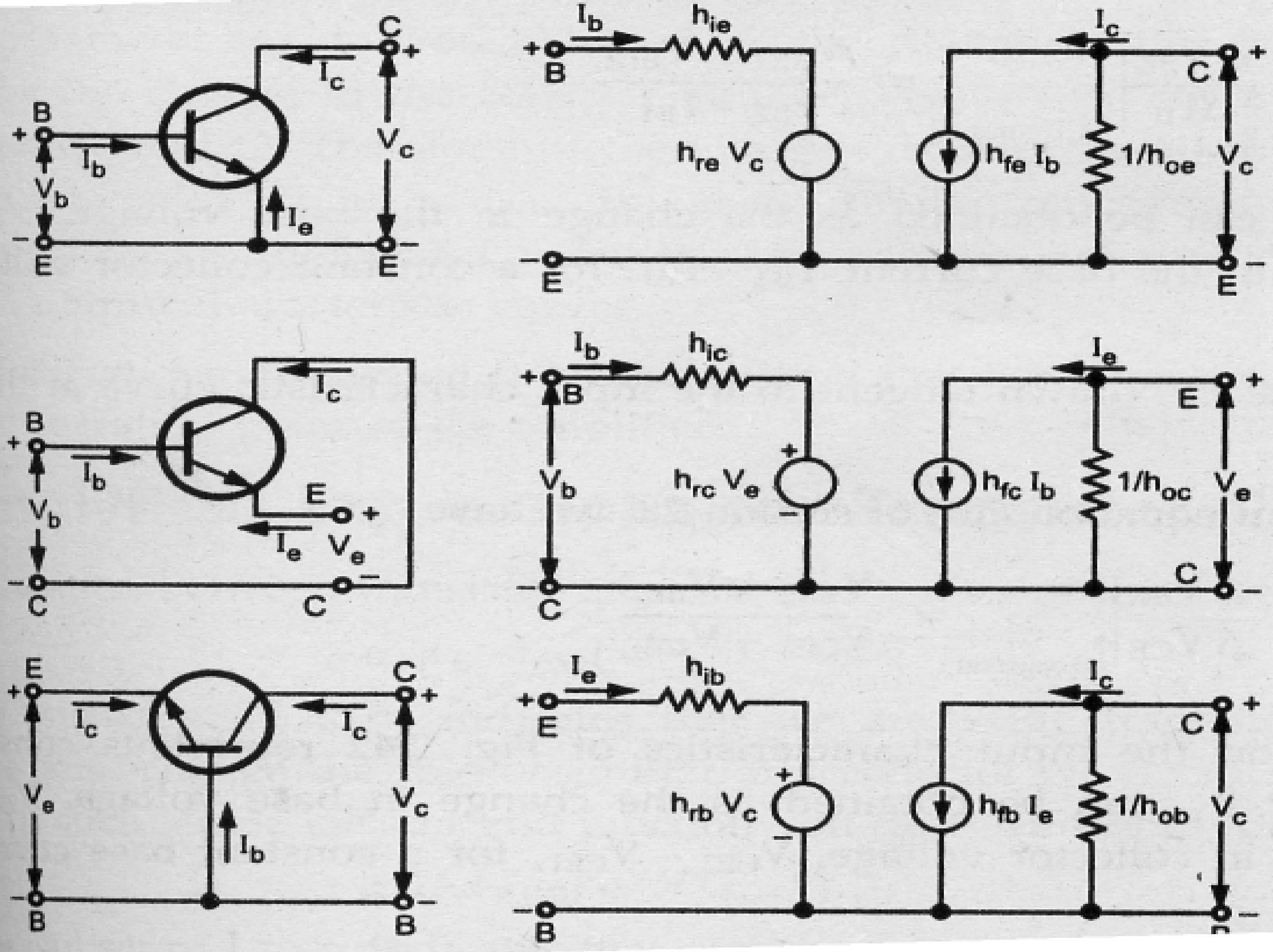
$$h_{ie} = \text{input resistance} = \left.\frac{\Delta v_{be}}{\Delta i_b}\right| v_{ce} = constant$$

$$h_{re} = \text{reverse transfer voltage ratio} = \left.\frac{\Delta v_{be}}{\Delta v_{ce}}\right| i_b = constant$$



$$h_{fe} = \text{forward transfer current ratio} = \left.\frac{\Delta i_c}{\Delta i_b}\right| v_{ce} = constant$$

$$h_{oe} = \text{output conductance} = \left.\frac{\Delta i_c}{\Delta v_{ce}}\right| i_b = constant$$



CE

$$V_b = h_{ie} I_b + h_{re} V_c$$
$$I_c = h_{fe} I_b + h_{oe} V_c$$

CC

$$V_b = h_{ic} I_b + h_{rc} V_e$$
$$I_e = h_{fc} I_b + h_{oc} V_e$$

CB

$$V_e = h_{ib} I_e + h_{rb} V_c$$
$$I_c = h_{fb} I_e + h_{ob} V_c$$

Transistor Hybrid Model

Use of h – parameters to describe a transistor have the following advantages:

- h – parameters are real numbers up to radio frequencies .
- They are easy to measure
- They can be determined from the transistor static characteristics curves.
- They are convenient to use in circuit analysis and design.
- Easily convert able from one configuration to other.
- Readily supplied by manufacturiers.

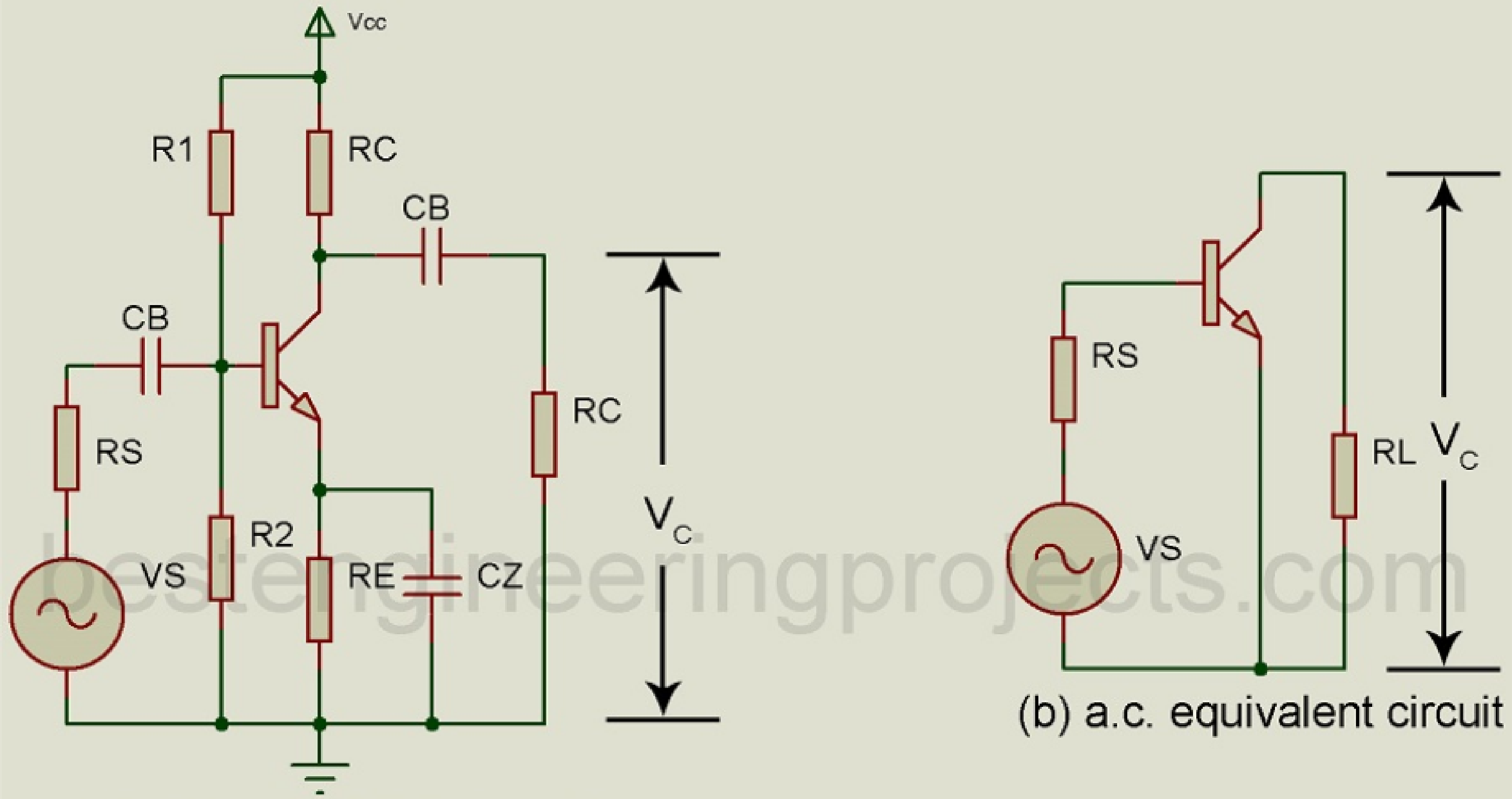
Procedure for DC and AC analysis

The process of dc analysis is as follows :

1. Reduce ac signal source to zero.
2. Open all capacitors.
3. Analyse the resulting *dc equivalent circuit*.

The process of ac analysis is as follows :

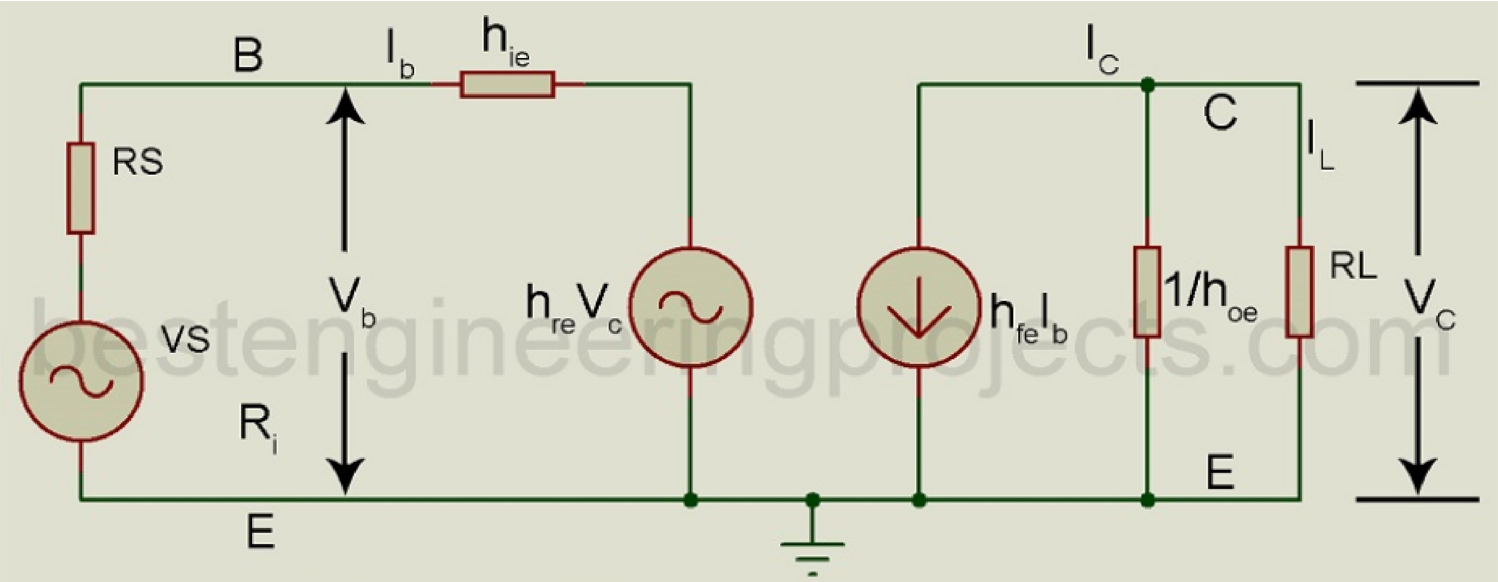
1. Reduce the dc source to zero.
2. Short all capacitors.
3. Analyse the resulting *ac equivalent circuit*.



(a) Circuit of CE Amplifier

(b) a.c. equivalent circuit

CE Amplifier

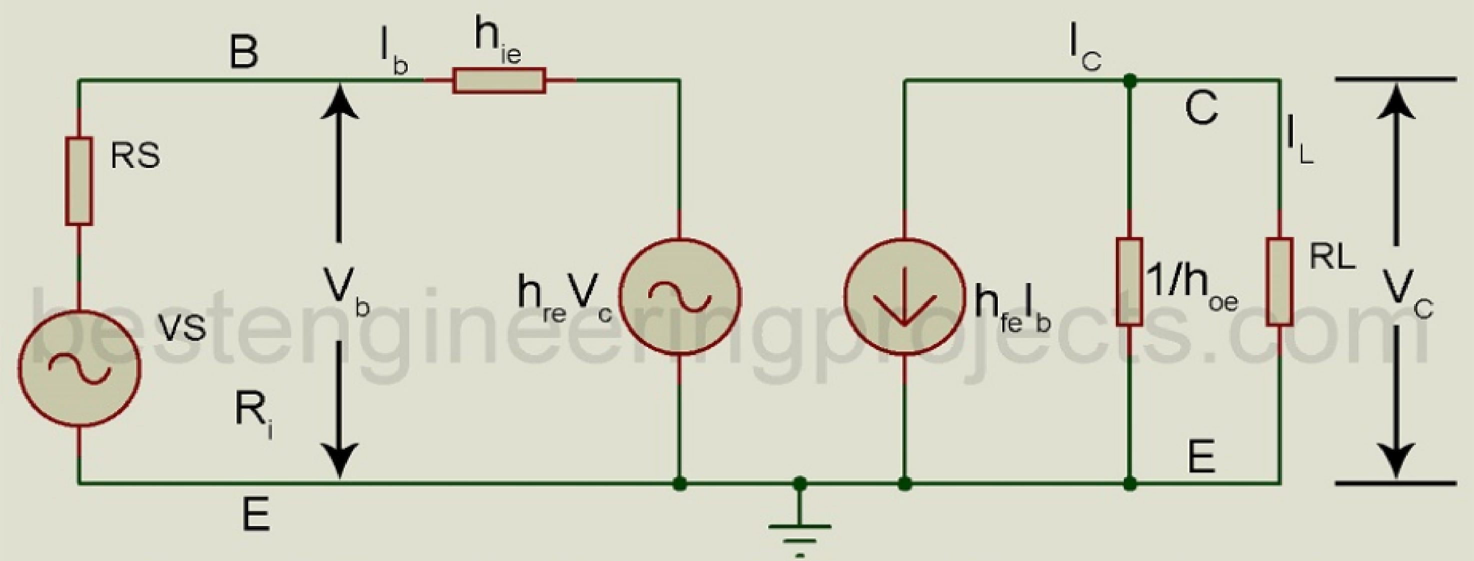


Equivalent Circuit of CE amplifier using h-model

$$A_I = \frac{I_L}{I_b} = -\frac{I_c}{I_b} \qquad I_c = h_{fe} \times I_b + h_{oe} \times V_c \qquad V_c = I_L \times R_L = -I_c \times R_L$$

$$I_c = h_{fe}I_b - h_{oe} \times I_c \times R_L or (1 + h_{oe} \times R_L) I_c = h_{fe} \times I_b$$

Hence Current Gain A_I is $$A_I = -\frac{I_c}{I_b} = -\frac{h_{fe}}{1 + h_{oe} \times R_L}$$

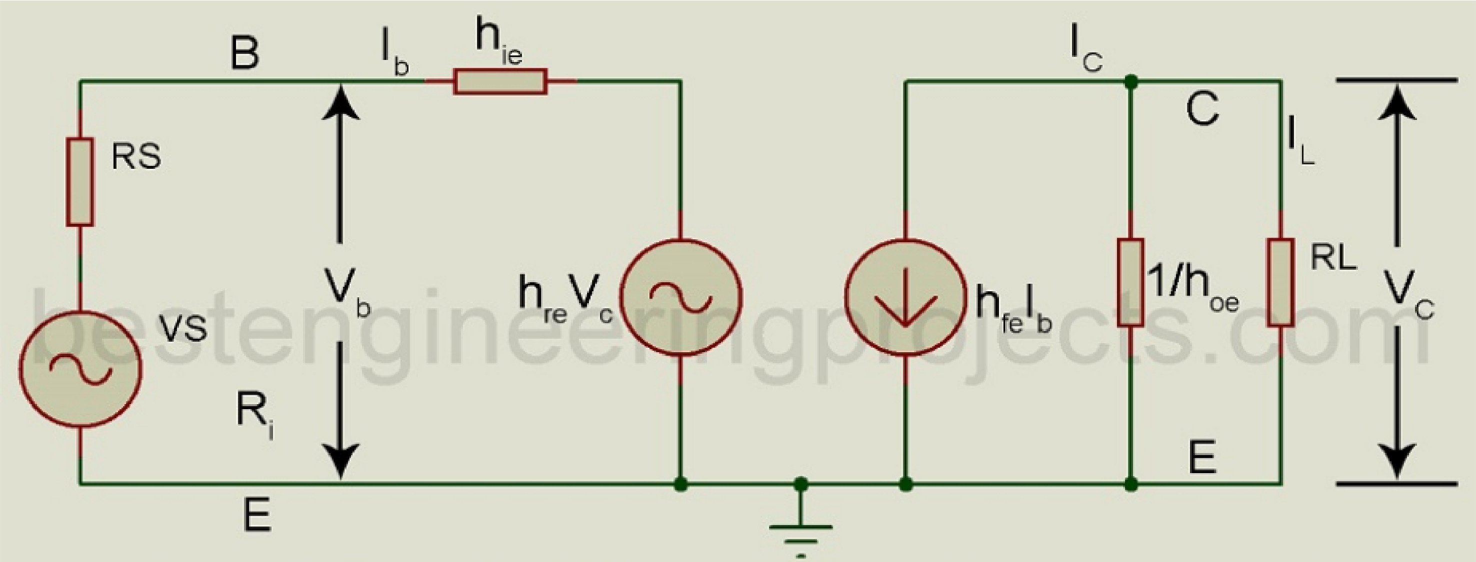


Equivalent Circuit of CE amplifier using h-model

$$R_i = \frac{V_b}{I_b} \qquad V_b = h_{ie} \times I_b + h_{re} \times V_c \qquad V_c = -I_c \times R_L = A_I I_b R_L$$

$$V_b = h_i e \times +h_{re} A_I I_b R_L \qquad R_i = \frac{V_b}{I_b} = h_{ie} + h_{re} A_I R_L \quad = h_{ie} - \frac{h_{fe} h_{re}}{h_{oe} + Y_L}$$

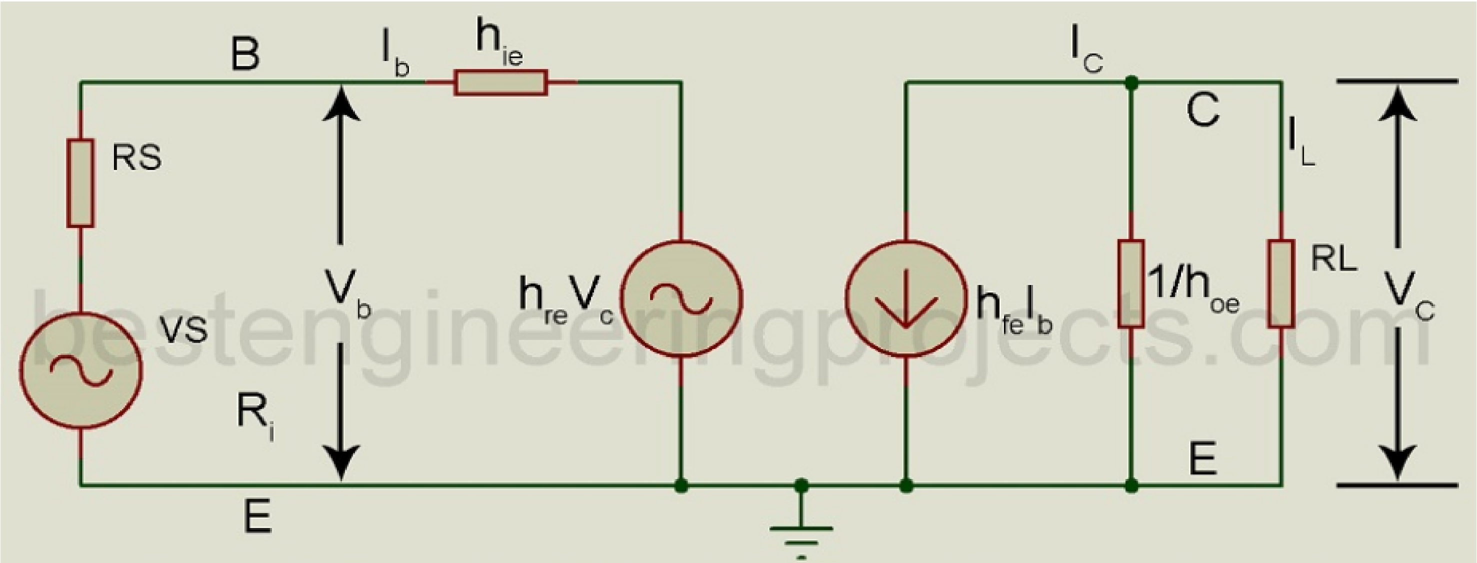
$$Y_L = \frac{1}{R_L}$$



Equivalent Circuit of CE amplifier using h-model

Voltage Gain or Voltage Amplification:

$$A_v = \frac{V_c}{V_b} = -\frac{I_c R_L}{I_b R_i} = \frac{A_I R_L}{R_i}$$



Equivalent Circuit of CE amplifier using h-model

Output Admittance Y0:

$$Y_0 = \frac{I_c}{V_c}$$

With Vs =0

$$Y_0 = h_{fe} \times \frac{I_b}{V_c} + h_{oe}$$

But with Vs = 0, Figure gives (Rs + hie) Ib + hre Vc = 0

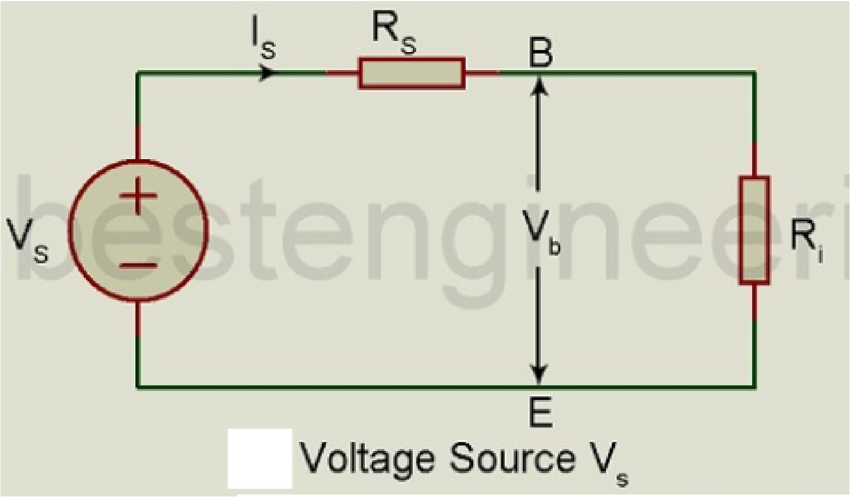
$$\frac{I_b}{V_c} = -\frac{h_{re}}{h_{ie} + R_s}$$

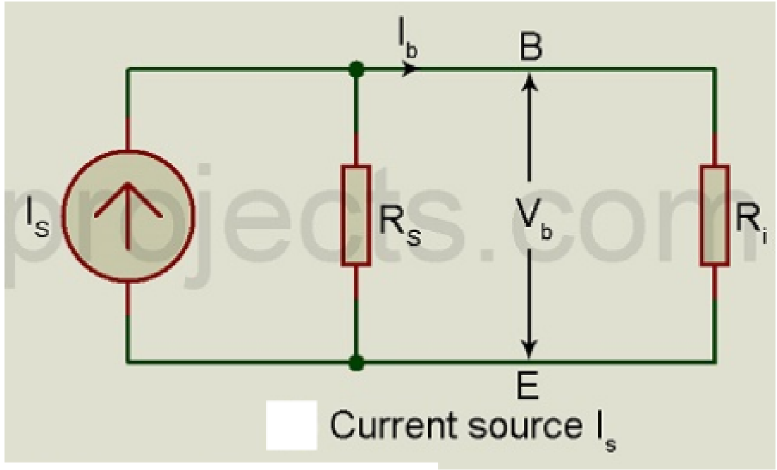
$$Y_0 = h_{oe} - \frac{h_{fe} \times hre}{h_{ie} + R_s}$$

$$R_0 = \frac{1}{Y_0}$$

In the calculation of Y0, RL has been considered external to the amplifier. If we include RL in parallel with R0, we get the output terminal impedance Zt given by

$$Z < sub > t < /sub > = \frac{R_0 R_L}{R_0 + R_L}$$





Overall Voltage Gain Considering Rs:

$$A_{VS} = \frac{V_c}{V_s} = \frac{V_c}{V_b} \times \frac{V_b}{V_s} = A_V = \frac{V_b}{V_s} \qquad V_b = V_s \times \frac{R_i}{R_i + R_s}$$

$$A_{VS} = A_v \times \frac{R_i}{R_i + R_s}$$

If Rs = 0, then AVS = AV. Thus, AV forms a special care of AVS with Rs = 0.

Overall Current Gain Considering Rs:

$$A_{Is} = \frac{I_L}{I_s} = \frac{-I_C}{I_b} \times \frac{I_b}{I_s} = A_I \times \frac{I_b}{I_s}$$

$$I_b = I_s \frac{R_s}{R_s + R_i}$$

$$A_{VS} = A_{IS} \times \frac{R_i}{R_s}$$

$$A_I = -\frac{h_{fe}}{1 + h_{oe} \times R_L}$$

$$R_i = h_{ie} + h_{re} \times A_I \times R_L$$

$$Y_0 = h_{oe} - \frac{h_{re}h_{fe}}{h_{ie} + R_s} = \frac{1}{Z_0}$$

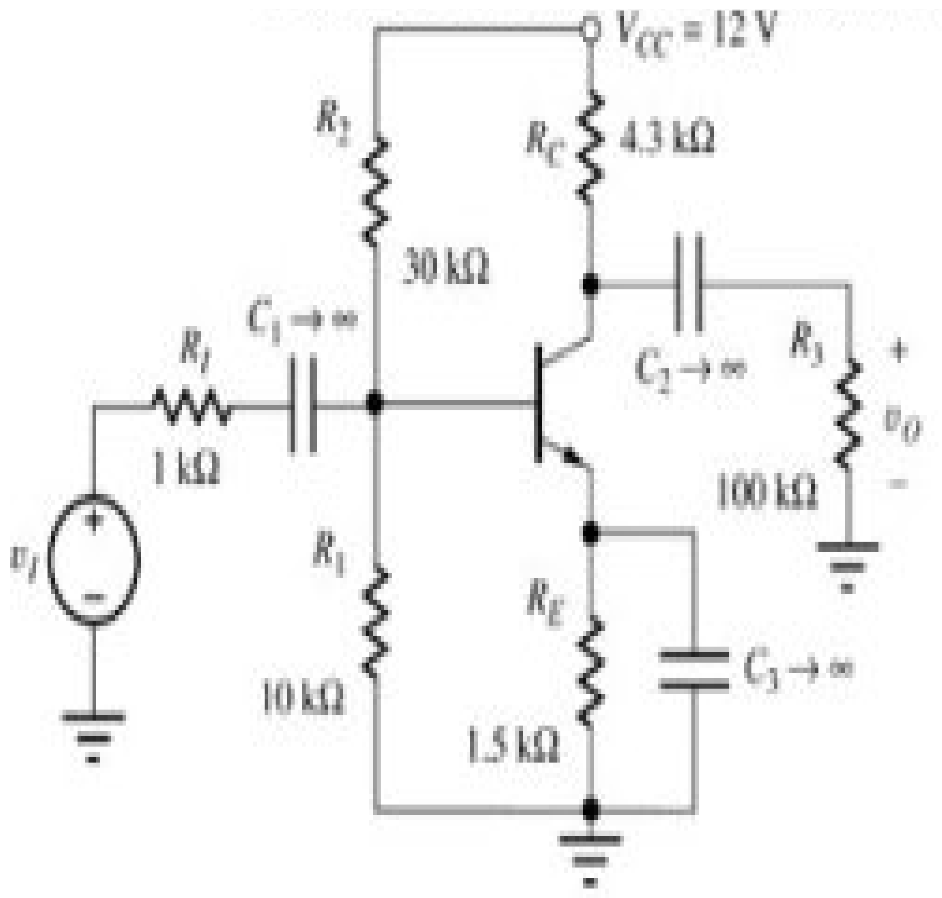
$$A_V = \frac{A_I \times R_L}{R_i}$$

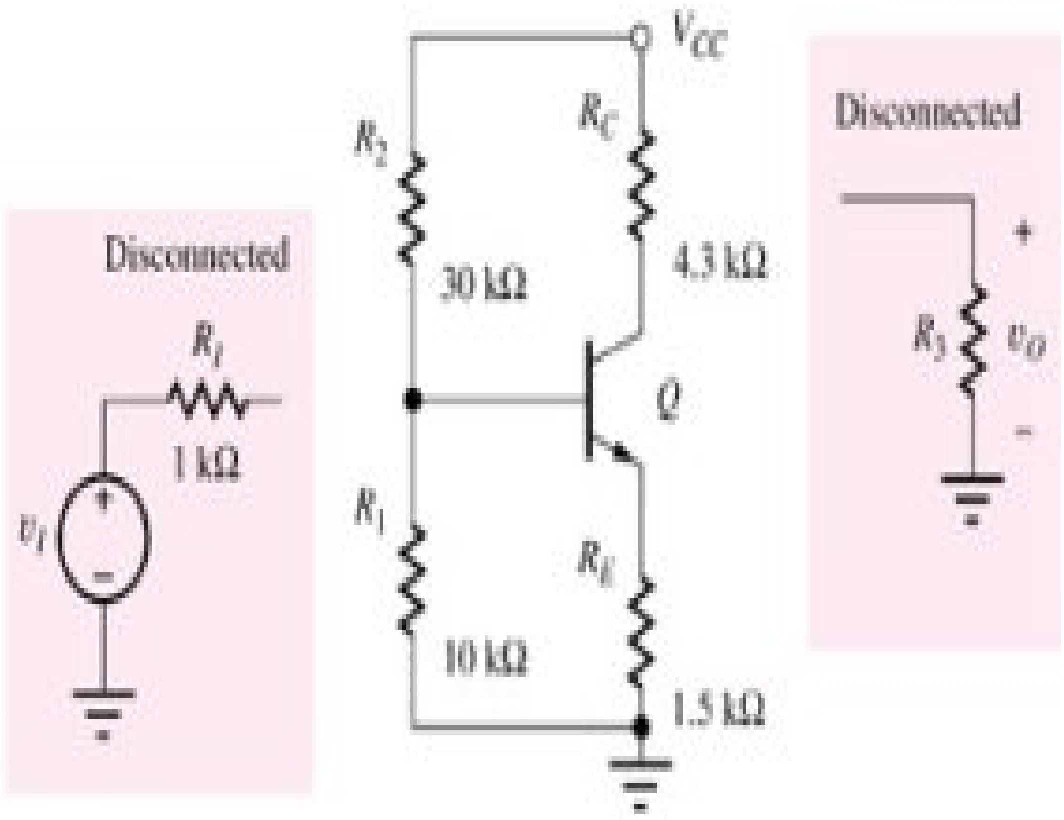
$$A_{VS} \frac{A_V R_i}{R_i + R_s}$$

$$A_{IS} = \frac{A_I R_s}{R_i + R_s}$$

Parameter	*CE*	*CC*	*CB*
h_i	1,100 Ω	1,100 Ω	22 Ω
h_r	2.5×10^{-4}	1	3×10^{-4}
h_f	50	−51	−0.98
h_o	25 $\mu A/V$	25 $\mu A/V$	0.49 $\mu A/V$

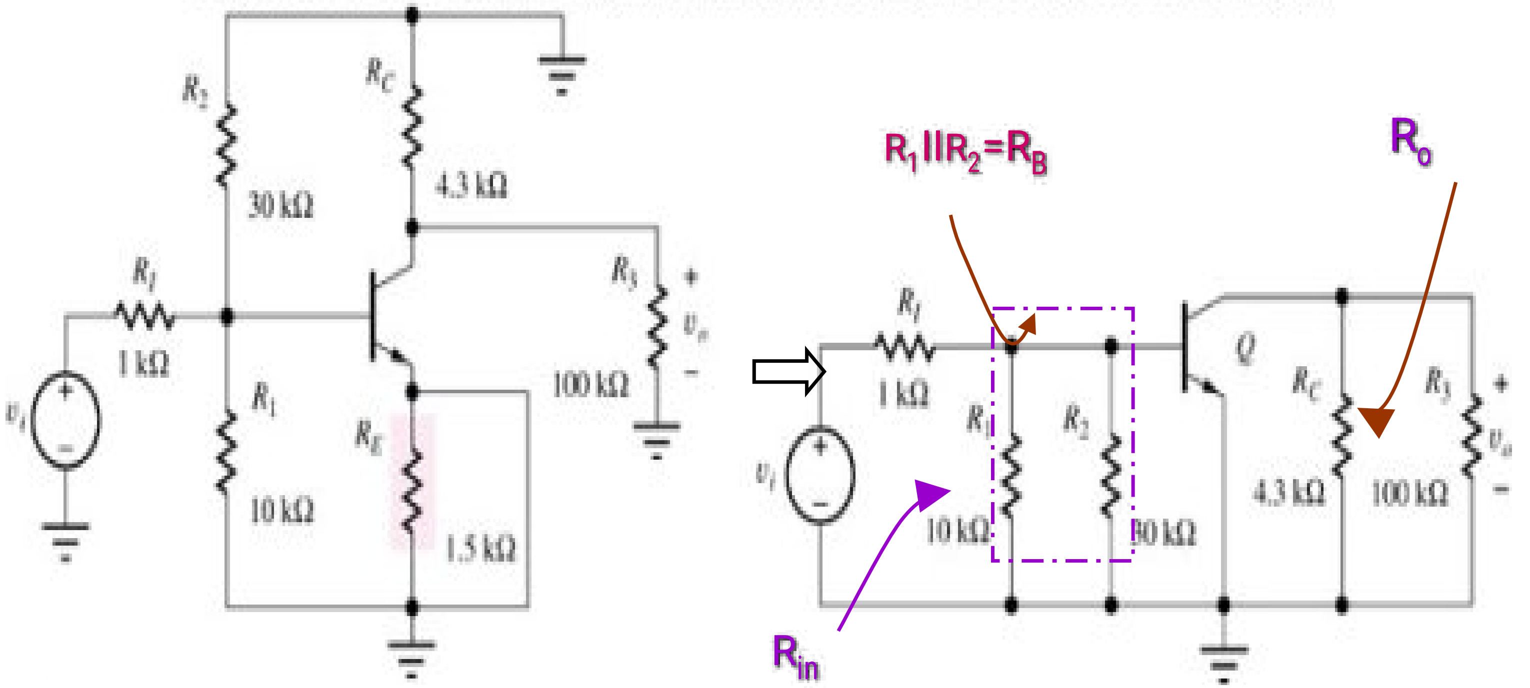
D C Equivalent for the BJT Amplifier (Step1)





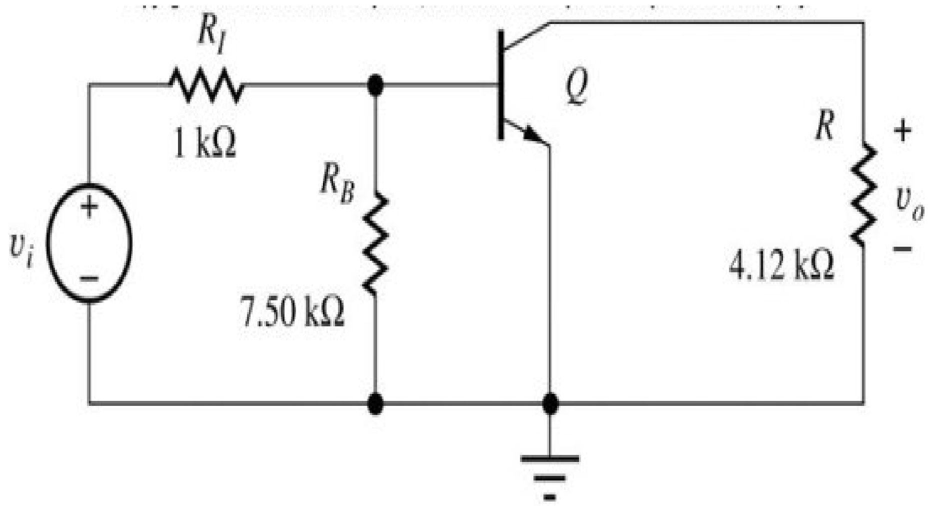
AC Equivalent Circuit

A C Equivalent for the BJT Amplifier (Step 2)



- Coupling capacitor C_C and Emitter bypass capacitor C_E are replaced by short circuits.
- DC voltage supply is replaced with short circuits, which in this case is connected to ground.

A C Equivalent for the BJT Amplifier (continued)



All externally connected capacitors are assumed as short circuited elements for ac signal

$$R_B = R_1 \| R_2 = 10k\ \Omega \| 30k\ \Omega$$

$$R = R_C \| R_3 = 4.3k\ \Omega \| 100k\ \Omega$$

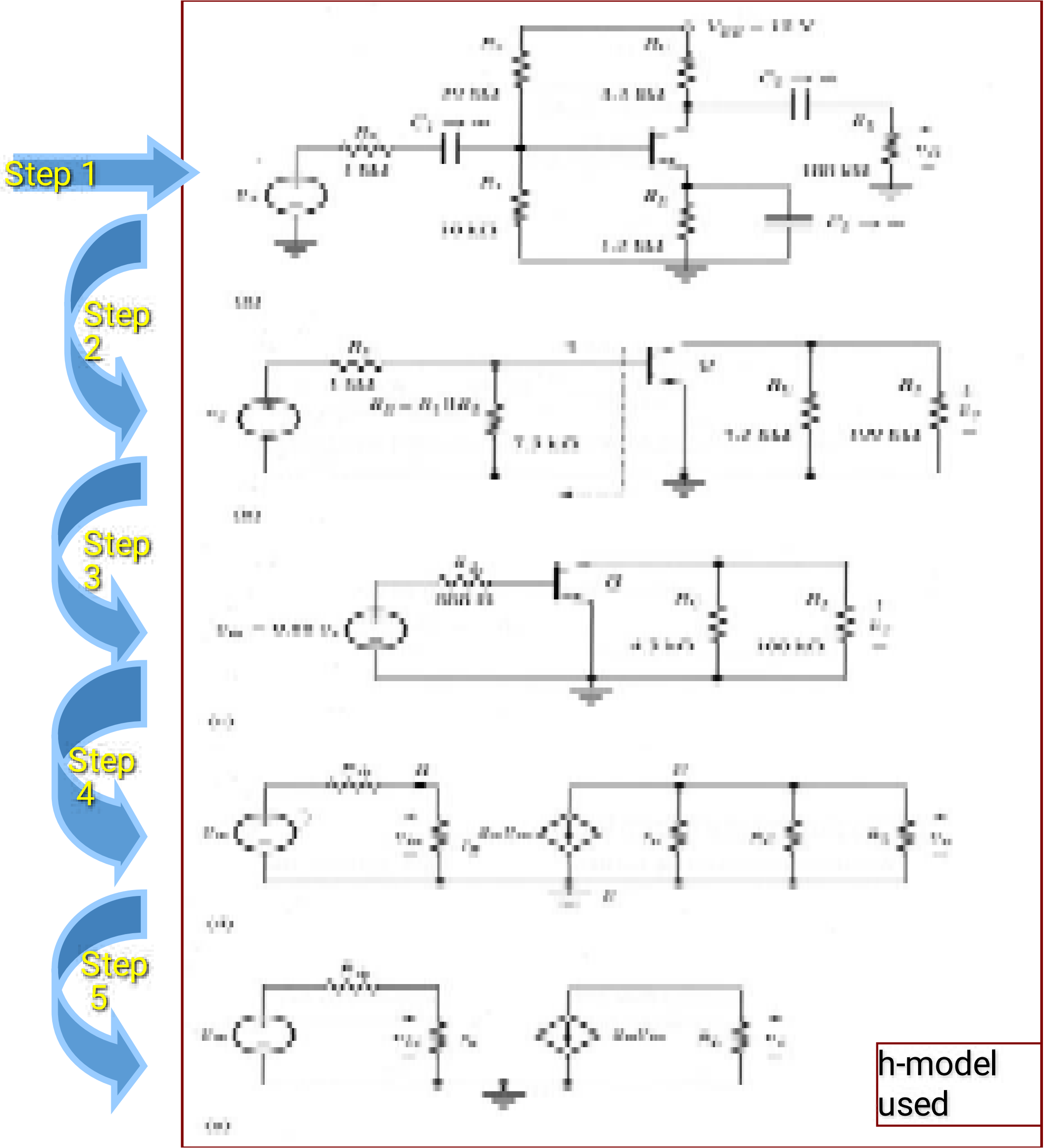
- By combining parallel resistors into equivalent R_B and R, the equivalent AC
  circuit above is constructed. Here, the transistor will be replaced by its
  equivalent small-signal AC model (to be developed).

A C Analysis of CE Amplifier

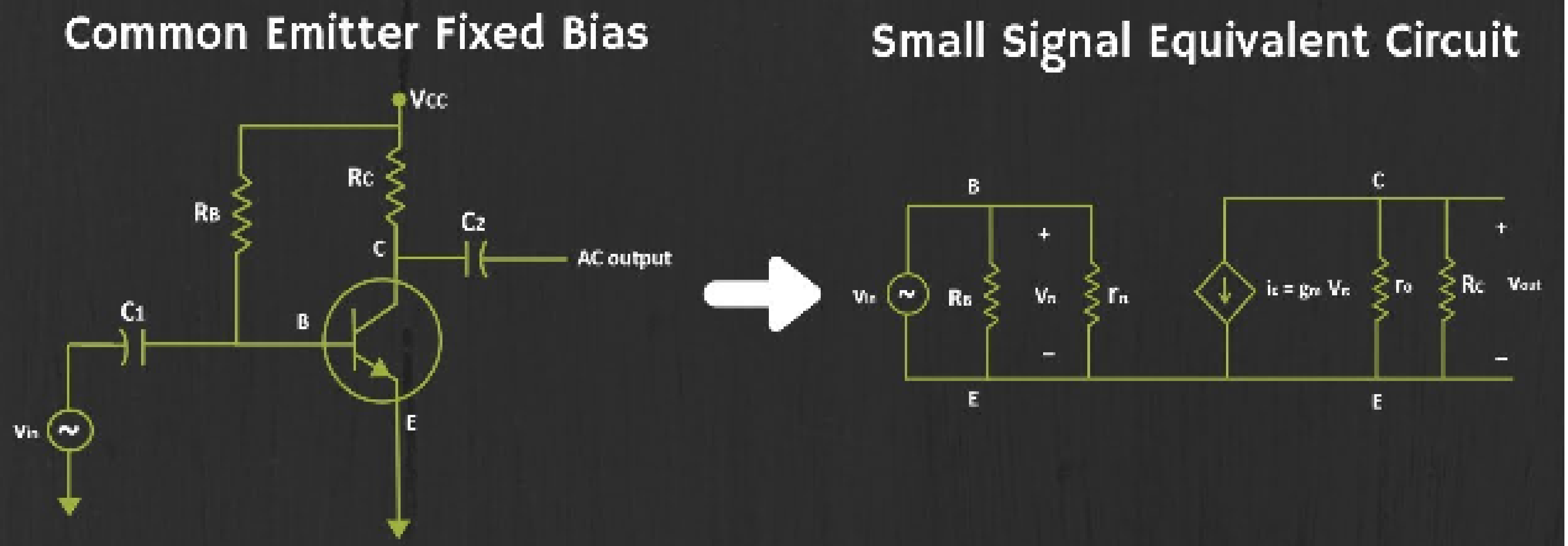
1) Determine DC operating point and calculate small signal parameters
2) Draw the AC equivalent circuit of Amp.
   - DC Voltage sources are shorted to ground
   - DC Current sources are open circuited
   - capacitors are short circuits
   - inductors are open circuits
3) Find the simplified form of network which is a two port network
4) Replace transistor with small signal model
5) Simplify the circuit as much as necessary.

Steps to Analyze a Transistor Amplifier

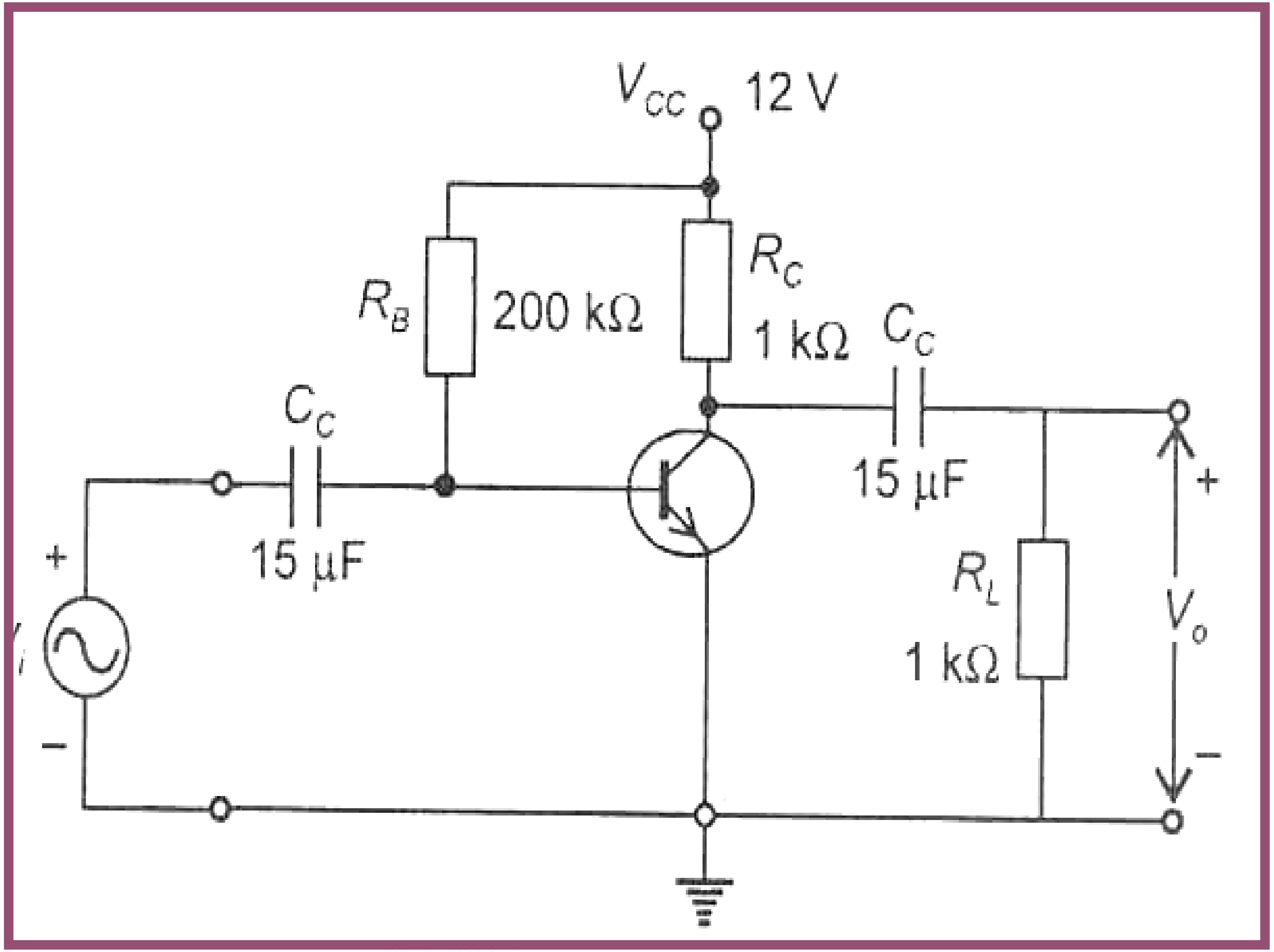
6) Calculate the small signal parameters and gain etc.



BJT Small Signal Analysis



Example : Analyze the CE amplifier circuit using h-parameter model of BJT. hie= 2K hfe = β = 100, i/hoe= 40K hre = .00025



Solution : ***DC Analysis :***

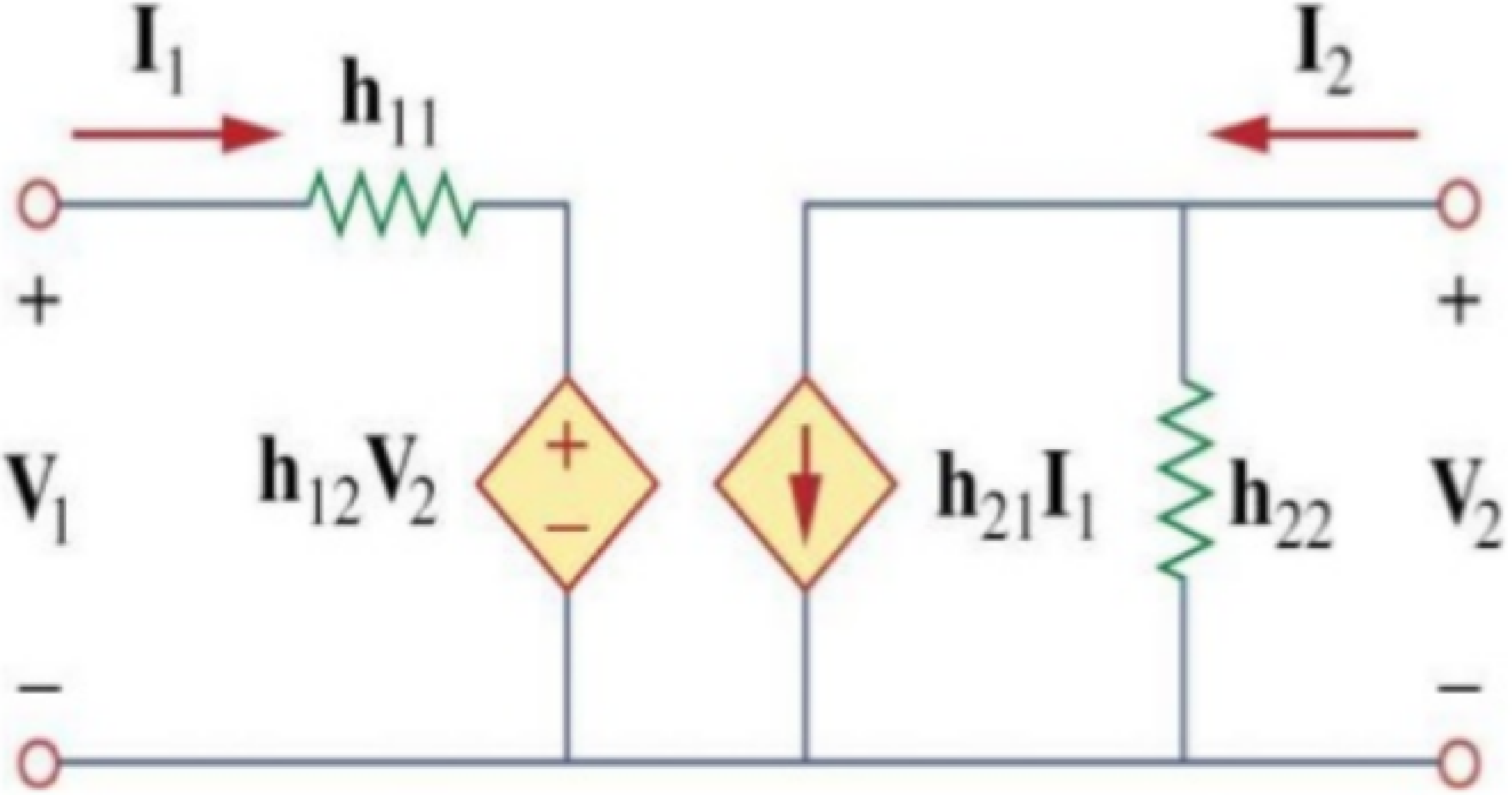
$$I_B = \frac{V_{CC}}{R_B} = \frac{12\text{ V}}{200\text{ k}\Omega} = 60\ \mu\text{A}$$

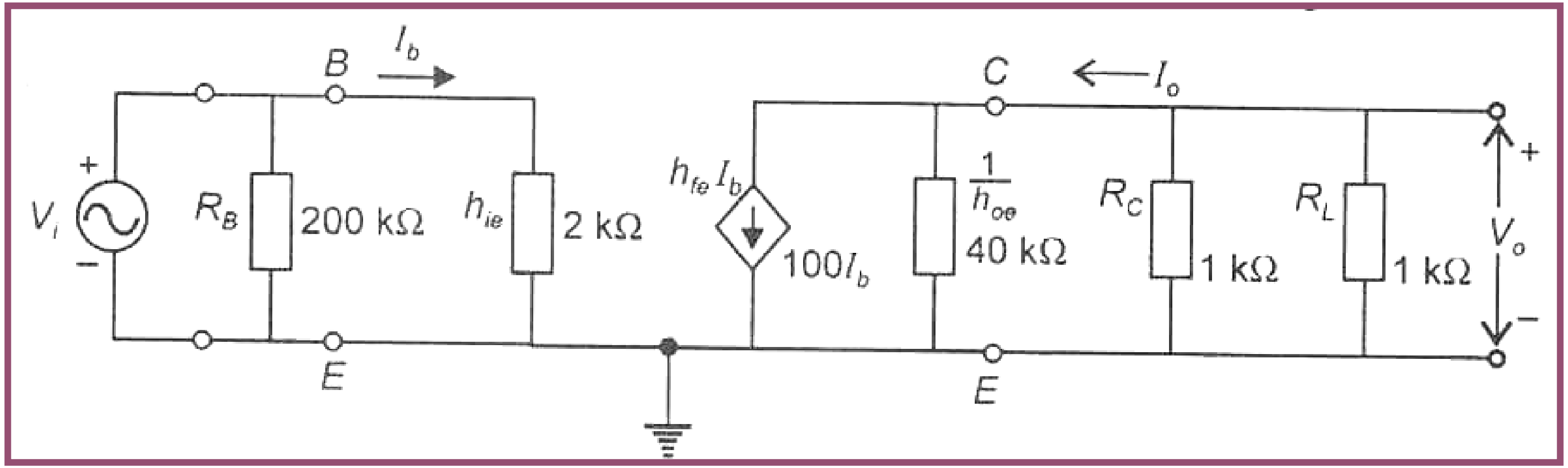
$$\therefore \qquad I_C = \beta_0 I_B = 100 \times 60\ \mu\text{A} = \mathbf{6\ mA};$$

$$I_{C(\text{sat})} = \frac{V_{CC}}{R_C} = \frac{12\text{ V}}{1\text{ k}\Omega} = 12\text{ mA}$$

Thus, the transistor is not in saturation. Then

$$V_{CE} = V_{CC} - I_C R_C = 12\text{ V} - 6\text{ mA} \times 1\text{ k}\Omega = 6\text{ V}$$


I1
h11
I2
+
+
V1
h12V2
+
−
h21I1
h22
V2
−
−



$$R_{ac} = R_C \| R_L = \frac{(1\text{ k}\Omega)(1\text{ k}\Omega)}{(1\text{ k}\Omega + 1\text{ k}\Omega)} = 0.5\text{ k}\Omega$$

$$A_V = -\frac{h_{fe} R_{ac}}{h_{ie}} = -\frac{100 \times 0.5\text{ k}\Omega}{2\text{ k}\Omega} = 25$$

$$A_I = -\frac{h_{fe}}{1 + h_{oe} \times R_L}$$

$$A_V = \frac{A_I \times R_L}{R_i}$$

$$I_o = -100 I_b$$

$$\text{current gain} = \frac{I_o}{I_b} = -100$$

$$R_i = h_{ie} + h_{re} \times A_I \times R_L$$

$$R_i = R_B \,||\, h_{ie} = 200 \text{ k}\Omega \,||\, 2 \text{ k}\Omega = \frac{(200 \text{ k}\Omega)(2 \text{ k}\Omega)}{(200 \text{ k}\Omega + 2 \text{ k}\Omega)} \approx \mathbf{2 \text{ k}\Omega}$$

$$\begin{aligned} R_o &= R_L \,||\, R_C \,||\, (1/h_{oe}) = (1 \text{ k}\Omega) \,||\, (1 \text{ k}\Omega) \,||\, (40 \text{ k}\Omega) \\ &= (0.5 \text{ k}\Omega) \,||\, (40 \text{ k}\Omega) \approx \mathbf{0.5 \text{ k}\Omega} \end{aligned}$$

$$A_I = -\frac{h_{fe}}{1 + h_{oe} \times R_L}$$

$$R_i = h_{ie} + h_{re} \times A_I \times R_L$$

$$A_{VS} \frac{A_V R_i}{R_i + R_s}$$

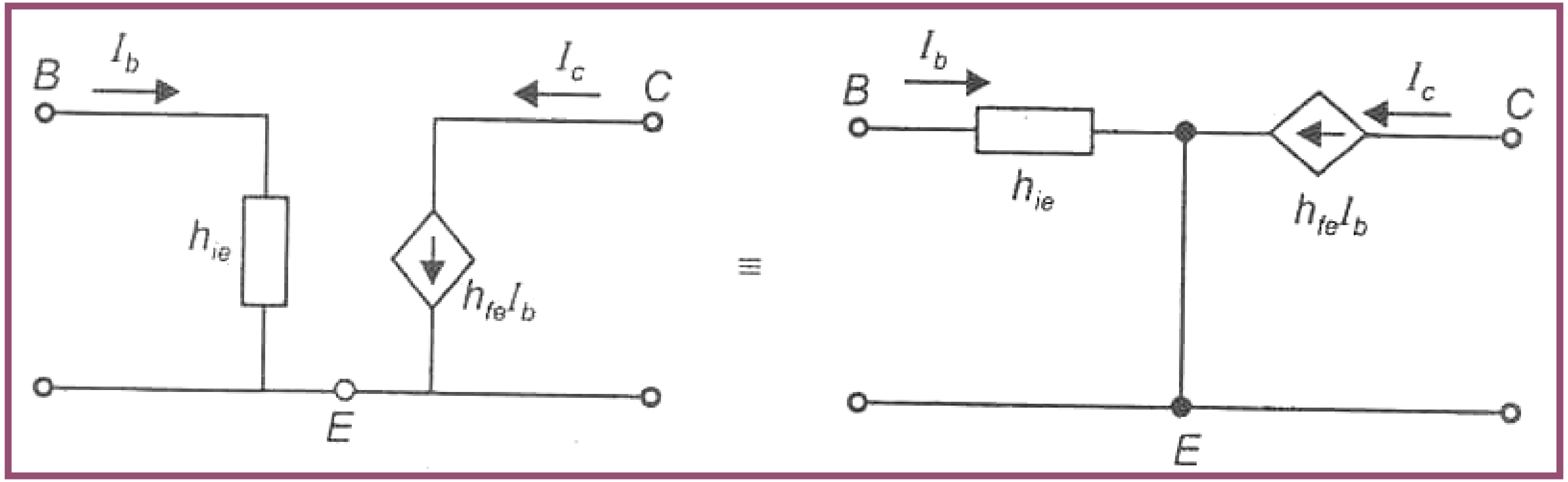
$$A_V = \frac{A_I \times R_L}{R_i}$$

$$Y_0 = h_{oe} - \frac{h_{re} h_{fe}}{h_{ie} + R_s} = \frac{1}{Z_0}$$

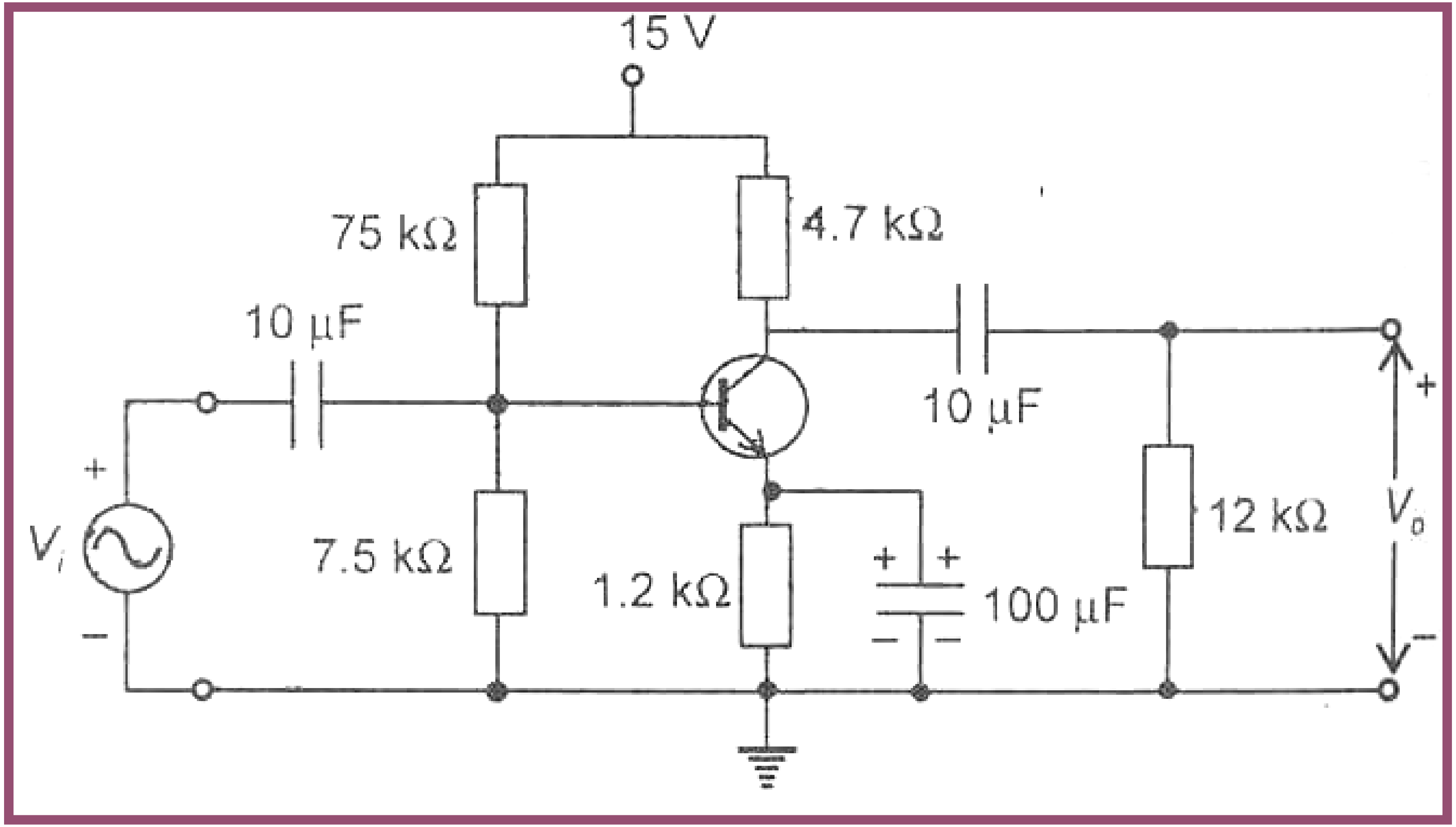
$$A_{IS} = \frac{A_I R_s}{R_i + R_s}$$

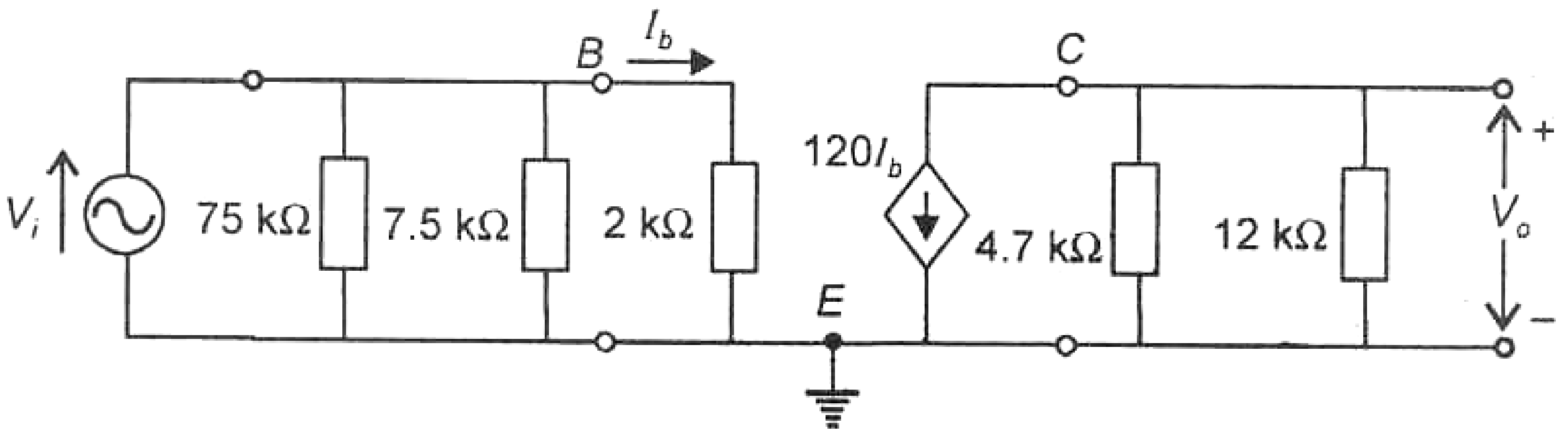
Parameter	CE	CC	CB
h_i	1100 Ω	1100 Ω	22 Ω
h_r	2.5×10^{-4}	1	3×10^{-4}
h_f	50	-51	-0.98
h_o	25 µA/V	25 µA/V	0.49 µA/V

Note : Since $1/h_{oe}$ can be ignored, the h – parameter model is further simplified.



Example 7.7 In the amplifier circuit of Fig. 7.31, the BJT used has $\beta = 150$ and $r_i = 2\ \text{k}\Omega$. Neglecting V_{BE}, determine its Q-point. Draw its ac equivalent circuit and calculate (a) the voltage gain, and (b) the input resistance.


15 V
75 kΩ
4.7 kΩ
10 μF
10 μF
+
V_i
−
7.5 kΩ
1.2 kΩ
+ +
− −
100 μF
12 kΩ
+
V_o
−



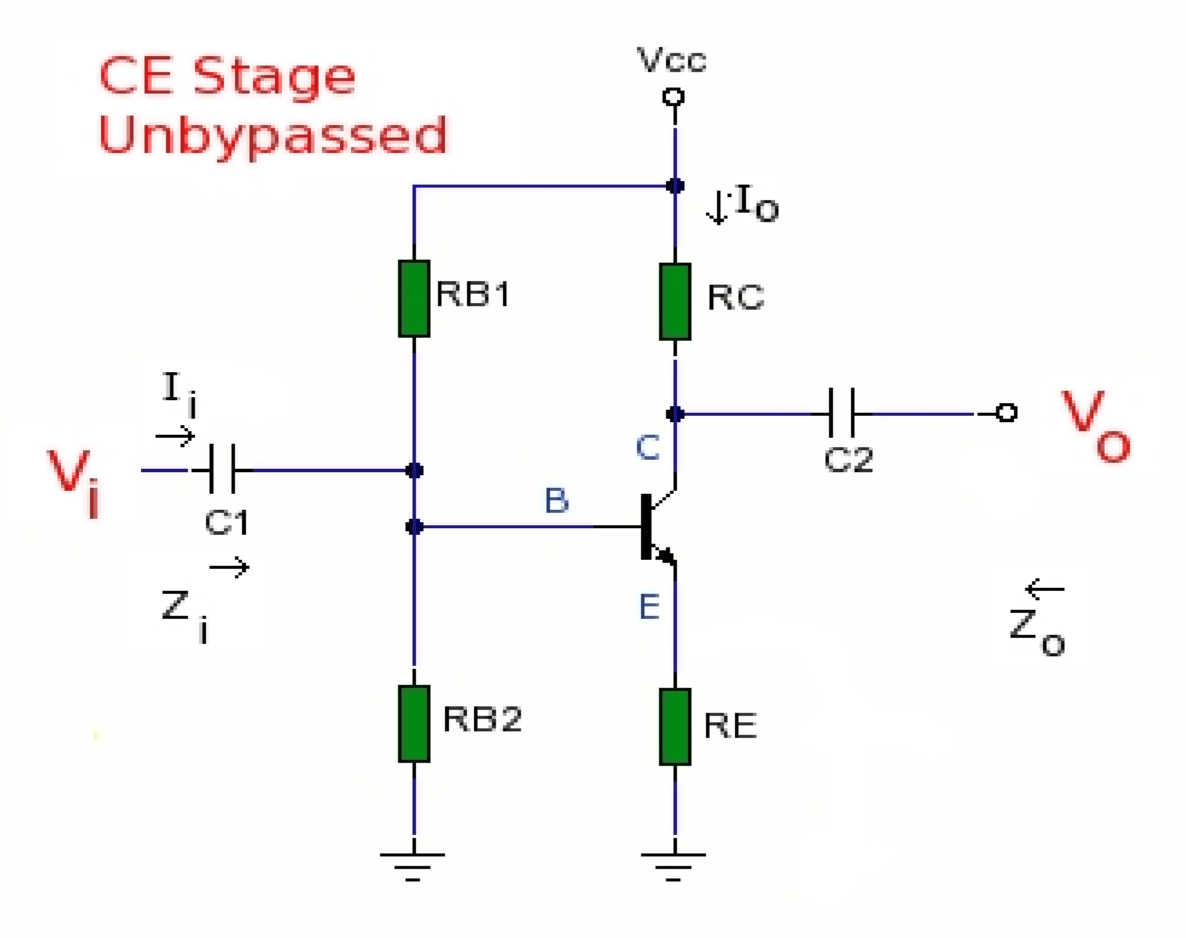
$$R_{ac} = (4.7\ \text{k}\Omega) \,||\, (12\ \text{k}\Omega) = \frac{(4.7\ \text{k}\Omega)\,(12\ \text{k}\Omega)}{(4.7\ \text{k}\Omega + 12\ \text{k}\Omega)} = 3.38\ \text{k}\Omega$$

$$\therefore \text{ The voltage gain, } A_V = -\frac{\beta\, R_{ac}}{r_i} = -\frac{150 \times 3.38\ \text{k}\Omega}{2\ \text{k}\Omega} = -253.5$$

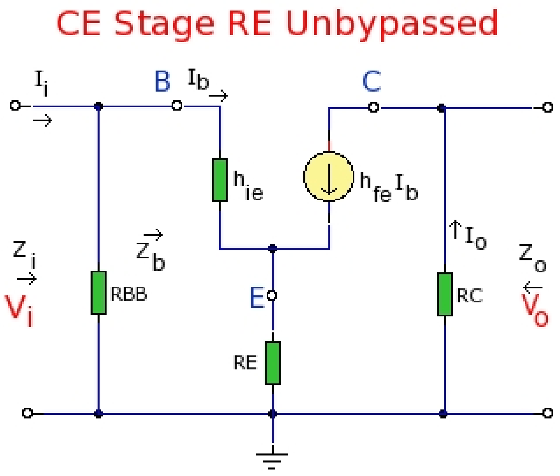
$$\boxed{R'_i = (75\ \text{k}\Omega)\,||\,(7.5\ \text{k}\Omega)\,||\,(2\ \text{k}\Omega) \simeq (7.5\ \text{k}\Omega)\,||\,(2\ \text{k}\Omega)}$$

$$\boxed{= \frac{(7.5\ \text{k}\Omega)\,(2\ \text{k}\Omega)}{(7.5\ \text{k}\Omega + 2\ \text{k}\Omega)} = 1.58\ \text{k}\Omega}$$

The h-parameter model of a common emitter stage with the emitter resistor unbypassed is now shown. The model will be used to build equations for voltage gain, current gain, input and output impedance. The circuit is shown below:

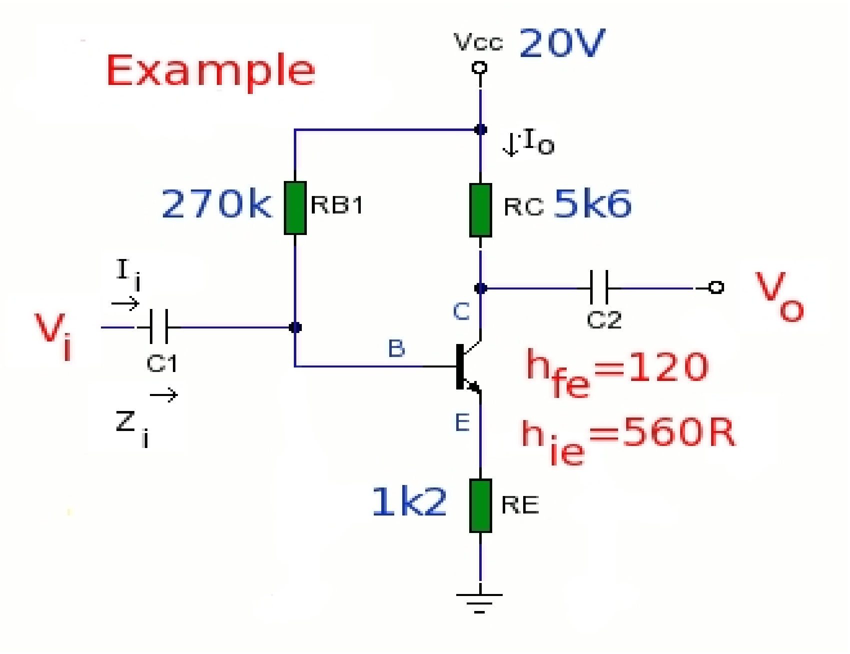


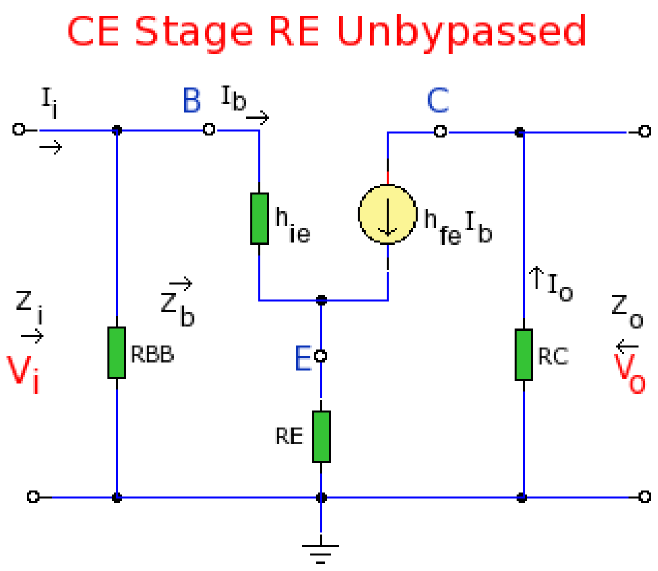
As in the previous example, RB1 and RB2 are in parallel, the bias resistors are replaced by resistance R_{BB}, but as RE is now unbypassed this resistor appears in series with the emitter terminal. The hybrid small signal model is shown below, once again effects of small signal parameters $h_{re}V_{ce}$ and h_{oe} have been omitted.



The input impedance Z_i is the bias resistors R_{BB} in parallel with the impedance of the base, Z_b. $Z_b = h_{ie} + (1 + h_{fe})$ RE Since hfe is normally much larger than 1, the equation can be reduced to: $Z_b = h_{ie} + h_{fe}$ RE $Z_i = R_{BB} \parallel (h_{ie} + h_{fe} RE)$ With Vi set to zero, then Ib = 0 and $h_{fe}I_b$ can be replaced by an open-circuit. The output impedance is: $Z_o = RC$ Note the − sign in the equation, this indicates phase inversion of the output waveform.

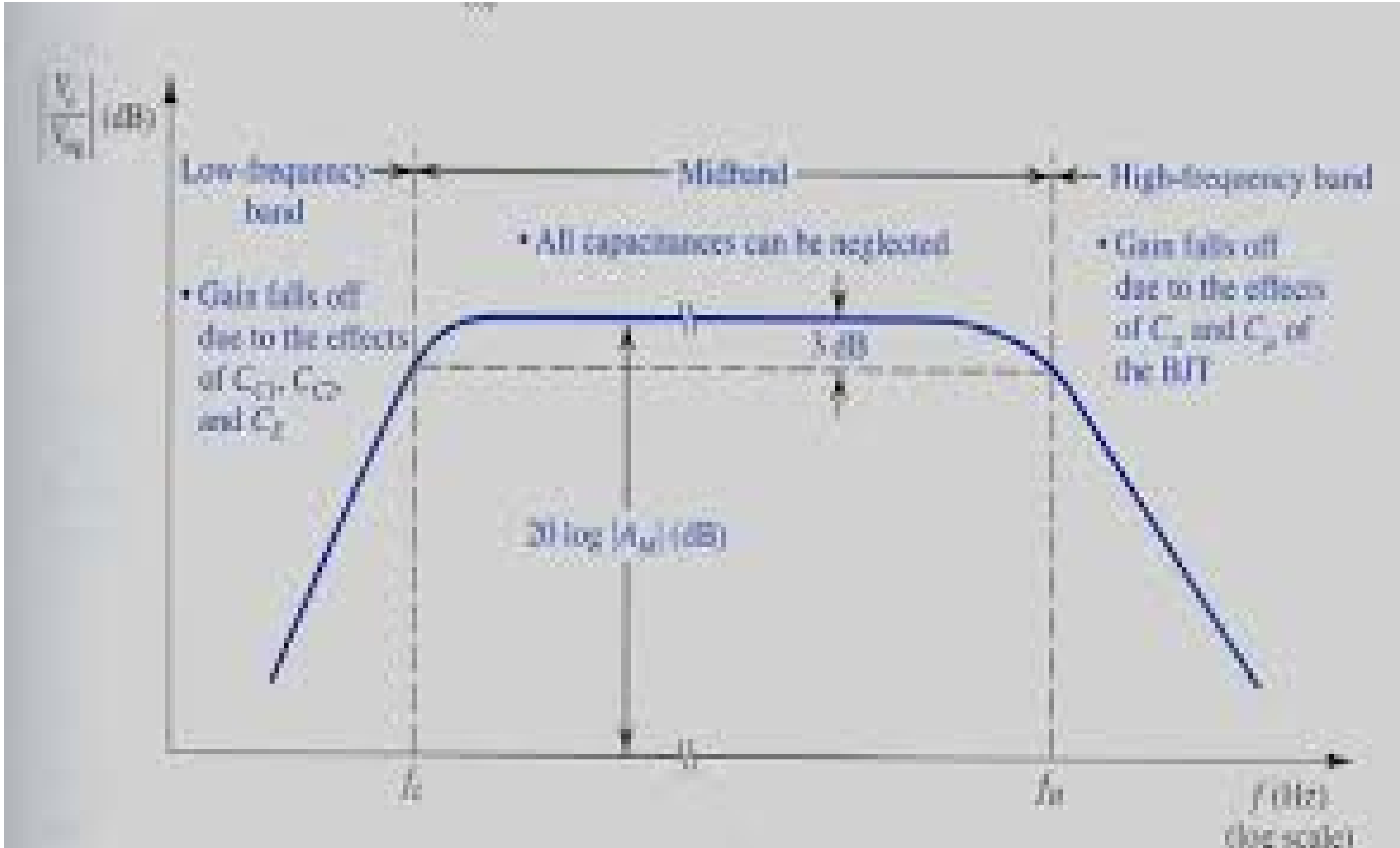
$$V_o = -I_o RC = -h_{fe} I_b RC$$





The hybrid parameters must be known to use the hybrid model, either from the datasheet or measured. In the above circuit, Zi, Zo, Av, and Ai will now be calculated. Note that this CE stage uses a single bias resistor RB1 which is the value RBB.

Parameter	CE	CC	CB
h_i	1100 Ω	1100 Ω	22 Ω
h_r	2.5×10^{-4}	1	3×10^{-4}
h_f	50	-51	-0.98
h_o	25 μA/V	25 μA/V	0.49 μA/V



$\left|\frac{V_o}{V_{sig}}\right|$ (dB)
Low-frequency band
Midband
High-frequency band
• All capacitances can be neglected
• Gain falls off due to the effects of C_{C1}, C_{C2}, and C_E
• Gain falls off due to the effects of C_π and C_μ of the BJT
3 dB
20 log $|A_M|$ (dB)
f_L
f_H
f (Hz) (log scale)

(b)

Conversion from *CE* *h*-parameters to *CB* and *CC* *h*-parameters, and to *r*-parameters.

CE to *CB* *h*-parameters	*CE* to *CC* *h*-parameters	*CE* *h*-parameters to *r*-parameters
$h_{ib} \approx \dfrac{h_{ie}}{1 + h_{fe}}$	$h_{ic} = h_{ie}$	$\alpha \approx \dfrac{h_{fe}}{1 + h_{fe}}$
$h_{rb} \approx \dfrac{h_{ie}\, h_{oe}}{1 + h_{fe}} - h_{re}$	$h_{rc} = 1 - h_{re}$	$r_c = \dfrac{1 + h_{fe}}{h_{oe}}$
$h_{fb} \approx \dfrac{h_{fe}}{1 + h_{fe}}$	$h_{fc} = 1 + h_{fe}$	$r_{\mathrm{e}} = \dfrac{h_{ie}}{1 + h_{fe}} \approx r'_{e}$
$h_{ob} \approx \dfrac{h_{oe}}{1 + h_{fe}}$	$h_{oc} = h_{oe}$	$r_b = h_{ie} - \dfrac{h_{re}(1 + h_{fe})}{h_{oe}}$