Unit I Introduction

Signal: A signal is defined as any physical quantity that varies with time, space or any other variable.

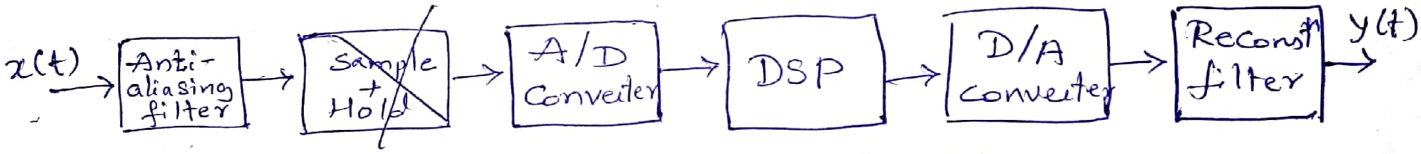
System: A system is defined as a physical device that performs an op^n on a signal.

Signal processing is an op^n that changes the charac's of a signal like amp, freq, phase and shape.

Analog signal processing system

Eg: Amplifiers, filters, freq analyzers.

Digital signal processing system.



Advantages:

1) Greater accuracy.
2) cheaper
3) Easy for data storage
4) Flexibility in conf^n.
5) Time sharing

Limitations:

1) System complexity
2) B.W. limited by sampling rate
3) Power consumption.

App^{n}s of DSP:

1) Telecomm^{n}s:
   - Modems
   - Line repeaters
   - Data encryption
   - Video conferencing

2) Consumer electronics:
   - Digital TV
   - FM stereo app^{n}s
   - Sound recording app^{n}s

3) Instrumentation & control
   - Robort control
   - Servo control
   - Spectrum analysis.

4) Image processing
   - Compression
   - Enhancement
   - Analysis & recognition

5) Medical field
   - CT scan
   - X-ray
   - MRI

6)

6) Speech processing

- speech recog^n
- Speaker verification
- speaker identification } speech analysis
- speech synthesis.
- ~~Speech~~

7) Seismology:

- Exploration of oil & gas
- Detection of undeground nuclear explosion
- Earthquake monitoring.

8) Military:

- Radar signal processing
- Sonal " "
- Navigation
- Secure comm^{n}s.

Classification of Signals: Time-domain Analysis of signals

1) Continuous -time signals 2) Discrete-Time signals.

classification of Discrete-Time signals:

ia) Energy and power signals:

Energy E is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

(A signal is an energy signal, if and only if the total energy is finite). $P=0$ for energy signal

Power is defined as

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

(For power signals, power is finite and energy is infinite).

Problems:

1. Find power and energy for the following.

a) $x(n) = \left(\frac{1}{6}\right)^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{6}\right)^n u(n)\right|^2$$

$$= \sum_{n=0}^{\infty} \left|\left(\frac{1}{6}\right)^{2n}\right|$$

$$= 1 + \frac{1}{36} + \left(\frac{1}{36}\right)^2 + \cdots$$

$$= \frac{1}{1-\frac{1}{36}} = \frac{36}{35} J$$

$\therefore$ It is energy signal

$$1 + a + a^2 + \cdots = \frac{1}{1-a} \text{ for } a<1$$

$$P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=0}^{N} \left(\frac{1}{36}\right)^n$$

for finite N $\leftarrow$ $\boxed{\frac{1-a^{N+1}}{1-a} \text{ for } a<1}$

$$= \lim_{N\to\infty} \frac{1}{(2N+1)} \left[\frac{1-\left(\frac{1}{36}\right)^{N+1}}{1-\frac{1}{36}}\right]$$

$$= 0$$

$\boxed{\frac{a^{N+1}-1}{a-1} \text{ for } a>1}$

b) $x(n) = \sin\left(\frac{\pi}{4}n\right)$

$$E = \sum_{n=-\infty}^{\infty} \left|\sin^2\left(\frac{\pi}{4}n\right)\right|$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1-\cos\left(\frac{\pi}{2}n\right)}{2}\right]$$

$$= \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\sin^2\left(\frac{\pi}{4}n\right)\right|$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos\frac{\pi}{2}n}{2}$$

$$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (1)$$

$$= \frac{1}{2} \quad \left(\because \sum_{n=-N}^{N} 1 = 2N+1\right)$$

$\therefore$ It is power signal

c) $x(n) = e^{2n}u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} e^{4n}u(n)$$

$$= 1 + e^4 + e^8 + \cdots \infty$$

$$= \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} e^{4n}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\frac{e^{4(N+1)}-1}{e^4-1}\right]$$

$$= \infty$$

$\therefore$ It is neither energy nor power signal.

ii) Periodic and aperiodic signals:

A signal is periodic with period N if and only if $x(n+N) = x(n)$ for all n.

Problems:

2) Check the periodicity of the following signals:

a) $x(n) = \cos\frac{3\pi}{4}n$

$\omega_0 = \frac{3\pi}{4}$; $2\pi f_0 = \frac{3\pi}{4}$

$\Rightarrow f_0 = \frac{3}{8} = \frac{K}{N}$

$\therefore$ periodicity $= 8$.

b) $x(n) = e^{j\frac{3}{5}n}$

$\omega_0 = \frac{3}{5}$

$2\pi f_0 = \frac{3}{5}$

$\Rightarrow f_0 = \frac{3}{10\pi}$ → integer, not rational, → Not an integer

$\therefore$ Non-periodic.

	Stability	B.W.	Signal components	ROC
Freq. domain	L.T – Analog; Z.T – Discrete	F.T.	F.T. for noisy signals	L.T – Continuous signals; Z.T – Discrete sequences
Time domain	Impulse response	step response	Not possible for noisy signals	

iii) Symmetric and antisymmetric signals:

If $x(n) = x(-n)$ then it is symmetric/even signal.

If $x(n) = -x(-n)$ then it is antisymmetric/odd signal.

iv) Causal and Non Causal signals:

$x(n)$ is said to be causal if its value is zero for $n<0$.

otherwise it is noncausal signal.

$x(n) = [\underset{\uparrow}{1} \ 2 \ -2 \ -1]$ Causal

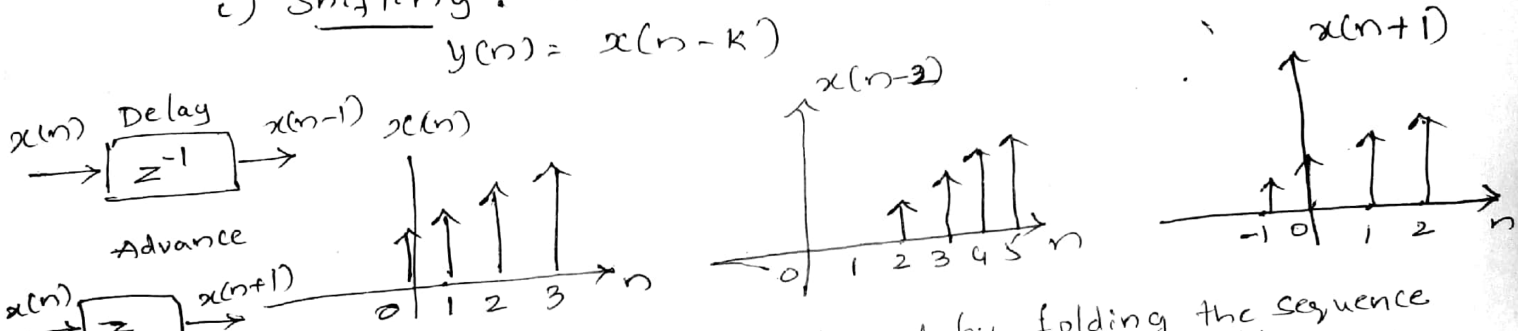
$x(n) = [1 \ \underset{\uparrow}{2} \ -2 \ -1]$ Non causal.

V) Deterministic and random Signals:

Operations on signals:

i) Shifting: It is obtained by shifting $x(n)$.

$y(n) = x(n-k)$



ii) Time reversal: It is obtained by folding the sequence abt $n=0$.

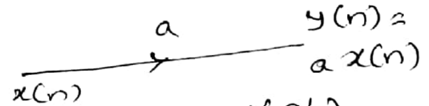
$y(n) = x(-n)$

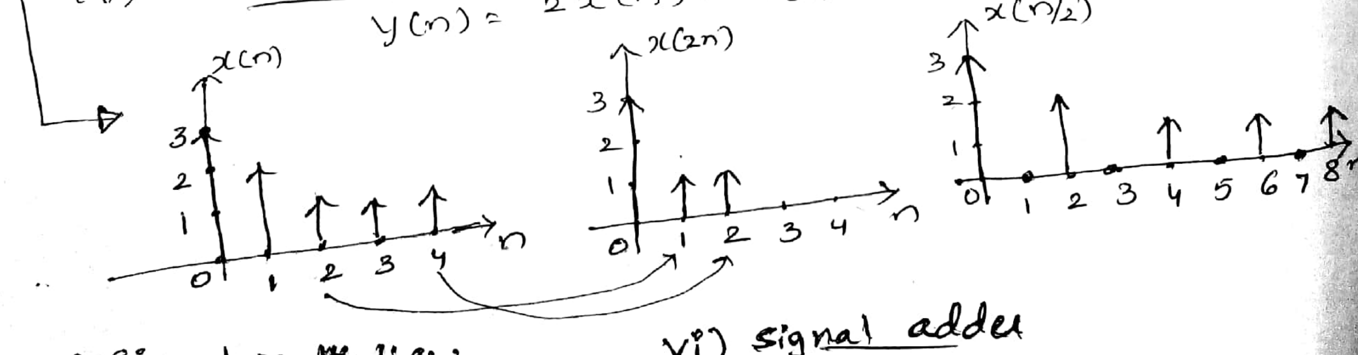
iii) Time scaling:

$y(n) = x(2n)$

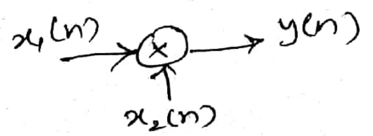
iv) Scalar Multiplication:

$y(n) = 2x(n)$.





v) Signal Multiplier:



vi) signal adder

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Time-domain Analysis of systems

Discrete-Time system: It is a device that operates on discrete-Time i/p signal $x(n)$ to produce discrete-time signal $y(n)$ (o/p signal).

Classification of discrete-time systems:

a) Static & dynamic systems.
b) Time-variant and invariant systems
c) Causal and Non-causal systems
d) Linear and Non-linear systems
e) FIR and IIR systems
f) Stable & unstable systems.

a) **Static & dynamic systems:**

If the o/p depends on only present values of i/p, then it is called a static system. (Memory is not used)

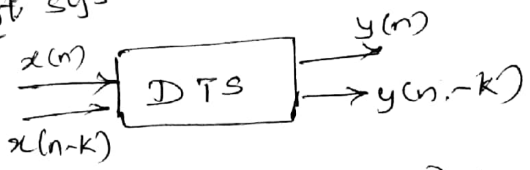
Eg: $y(n) = ax(n)$
$y(n) = ax^2(n)$

If the o/p depends on past or future values of i/p, then it is called a dynamic system. (Memory ~~loss~~).

Eg: $y(n) = x(n-1) + x(n+1)$; $y(n) = x(2n)$.

b) **Time-Variant and Invariant systems:**

If I/p-O/p characteristics of a system do not change with time then it is a time-invariant system.

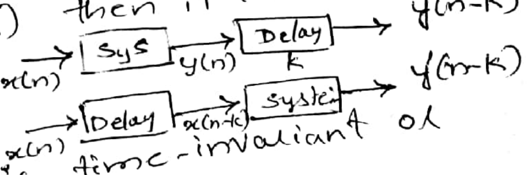


① Delay i/p by K samples

$y(n,K) = T[x(n-K)]$

② Delay o/p by K samples & represent as $y(n-K)$

If $y(n,k) = y(n-K)$ then it is time-invariant else it is time-variant.



Problems:

3. Determine whether the system is time-invariant or time-variant.

a) $y(n) = x(n) + x(n-1)$

b) $y(n) = n\,x(n)$

c) $y(n) = e^{x(n)}$

Time variant: ① Rocket.

As ~~the~~ it ~~fuel~~ burns tremendous amt of fuel, the mass reduce in short periods of time.

② Temp ③ Video

Time-invariant: ① Computer
② Bicycle, car

sol:- a) $y(n,k) = x(n-k) + x(n-1-k)$

$y(n-k) = x(n-k) + x(n-k-1)$

$\therefore y(n,k) = y(n-k) \Rightarrow$ Time-invariant system.

b) $y(n) = n\,x(n)$

$y(n,k) = n\,x(n-k)$

$y(n-k) = (n-k)\,x(n-k)$

$\therefore y(n,k) \neq y(n-k) \Rightarrow$ Time-variant system.

c) Causal & Non-Causal systems:

If the o/p depends on present & past values of i/p, then the system is causal and if the o/p depends on present & future values of i/p then the system is non-causal.

Problem

4. check for causality:

a) $y(n) = x(n) + \frac{1}{x(n-1)}$

sol:-

For $n=-1$

$y(-1) = x(-1) + \frac{1}{x(-2)}$

for $n=0$

$y(0) = x(0) + \frac{1}{x(-1)}$

$\therefore$ It is causal system.

b) $y(n) = x(n^2)$

For $n=-1$

$y(-1) = x(1)$

for $n=0$

$y(0) = x(0)$

$\therefore$ It is non-causal system.

d) Linear & Non-linear systems:

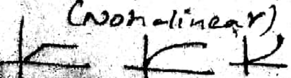
A system that satisfies the superposition principle is said to be a linear system.

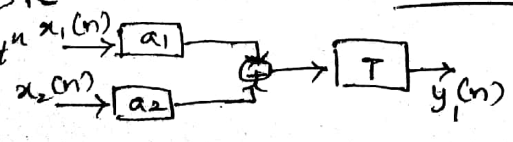
Eg: ampn/attn (Linear)

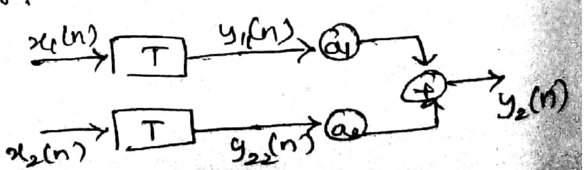
$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

A relaxed system that does not satisfy the superposition principle is called non-linear.

Eg: Systems with threshold, Satn (Non-linear)







$\boxed{\text{Square law}}$ 1) $y(n) = e^{x(n)}$
2) $y(n) = 3x(n+2)$

Problem:

5. check for linearity. $y(n) = \cancel{a}x^2(n)$ Non-linear

a) $y(n) = x(n) + \frac{1}{x(n-1)}$ ✗

$y_1(n) = T[x_1(n)] = x_1(n) + \frac{1}{x_1(n-1)}$

$y_2(n) = x_2(n) + \frac{1}{x_2(n-1)}$

$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$

$= a_1 x_1(n) + a_2 x_2(n) + \frac{1}{a_1 x_1(n-1) + a_2 x_2(n-1)}$

∴ It is a non-linear system.

ii) $y(n) = nx(n)$

$y_1(n) = n x_1(n)$

$y_2(n) = n x_2(n)$

$y_3(n) = a_1 n x_1(n) + a_2 n x_2(n)$

$y_4(n) = a_1 n x_1(n) + a_2 n x_2(n)$

∴ It is linear.

iii) $y(n) = x^2(n)$.

Try. (Non-linear)

iv) $y(n) = \cos[x(n)]$

e) FIR & IIR systems:

If the impulse response is of finite duration then the system is FIR.

If the impulse response is of infinite duration then the system is IIR.

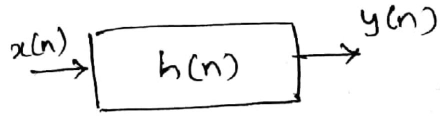
FIR (Eg): $h(n) = \begin{cases} -1 & n = 0, 1, 2 \\ 1 & n = 4, 6 \\ 0 & \text{otherwise} \end{cases}$

IIR (Eg): $h(n) = a^n u(n)$

f) Stable and Unstable systems:

A LTI system is stable if it produces a bounded o/p sequence for every bounded i/p sequence.

If, for bounded i/p sequence the o/p is unbounded, then the system is unstable.



$$y(n) \leq \sum_{K=-\infty}^{\infty} |h(K)| \, |x(n-K)|$$

$$y(n) \leq M \sum_{K=-\infty}^{\infty} |h(K)|$$

The necessary & sufficient condn for ~~To make~~ the system stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Impulse response should be absolutely summable.

Problem:

6. Check for stability $h(n) = \left(\frac{1}{2}\right)^n u(n)$ and find the response if the i/p is $2^n u(n)$

sol: $$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \left|\left(\tfrac{1}{2}\right)^n u(n)\right|$$

$$= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n = 1 + \tfrac{1}{2} + \left(\tfrac{1}{2}\right)^2 + \dots \infty$$

$1 + a + \dots \infty = \frac{1}{1-a}$

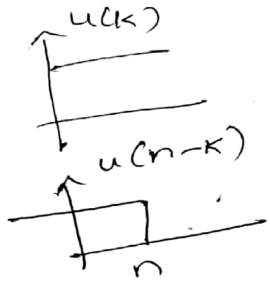
$$= \frac{1}{1 - \frac{1}{2}} = \tfrac{1}{2} < \infty$$

$\therefore$ It is stable.

Response: $$y(n) = x(n) * h(n)$$

$$= \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

$$= \sum_{K=-\infty}^{\infty} 2^K \left(\tfrac{1}{2}\right)^{n-K} u(K)\, u(n-K)$$



if $a<0$ (-ve) $\left[\frac{1-(a)^{n+1}}{1-a}\right]$

$$= \sum_{K=0}^{n} 2^{2K} \left(\tfrac{1}{2}\right)^n.$$

$$= \left(\tfrac{1}{2}\right)^n \sum_{K=0}^{n} 4^K$$

$$= \left(\tfrac{1}{2}\right)^n \left[1 + 4 + 4^2 + \dots 4^n\right]$$

$1 + a + a^2 + \dots a^n$

$\left(\frac{a^{n+1}-1}{a-1}\right)$ for $a>1$

$\left(\frac{1}{1-a}\right)$ $0 \leq a < 1$

$$\therefore \boxed{y(n) = \left(\tfrac{1}{2}\right)^n \left[\frac{4^{n+1}-1}{4-1}\right] \text{ for } n \geq 0 \\ = 0 \quad n < 0}$$

7. Check for stability & causality.

i) $h(n) = 2^n u(-n)$ ii) $h(n) = \delta(n) + \sin(\pi n)$

iii) $h(n) = e^{2n} u(n-1)$

Sol: i) Causality: $h(n) = 0$ for $n<0$ ($\because u(n) = 1$ for $n \geq 0$, $= 0$ for $n<0$)

$h(-1) = 2^{-1} u(1) = \frac{1}{2} \neq 0$

$\therefore$ Non-Causal system.

stability: $\sum_{K=-\infty}^{\infty} |h(K)| < \infty$

$$\sum_{K=-\infty}^{0} 2^K (1) = 1 + \tfrac{1}{2} + \left(\tfrac{1}{2}\right)^2 + \dots$$

$$= \frac{1}{1-\frac{1}{2}} = 2 < \infty$$

$\therefore$ stable system.

ii) Causality: $h(n) = 0$ for $n<0$

$$h(-1) = \delta(-1) + \sin(-\pi) = 0$$
$$h(-2) = \delta(-2) + \sin(-2\pi) = 0$$
$$\vdots$$

$\therefore$ It is a causal system.

Stability: $\sum_{k=0} |\delta(k) + \sin \pi k| = \delta(0) + \sin 0$
$= 1 + 0 = 1$ finite.

$\therefore$ It is a stable system.

iii) $h(n) = e^{2n} u(n-1)$

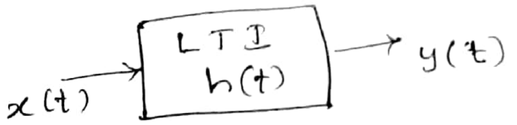
Causality: $h(-1) = e^{-2} u(-2) = 0$
$h(-2) = e^{-4} u(-3) = 0$
$\vdots$

$\therefore$ Causal system.

stability: $\sum_{k=0}^{\infty} |e^{2k} u(k-1)| = \sum_{k=1}^{\infty} |e^{2k}| = \infty$

$\therefore$ Unstable system.

LTI system:



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$
$$y(t) = x(t) * h(t)$$

similarly for discrete systems

$$y(n) = x(n) * h(n)$$
$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$
$$= \sum_{K=-\infty}^{\infty} x(n-k)\, h(k).$$

LTI systems are memory systems, invertible, casual, real and stable.
Eg: seismology.

Problems:

8. Find the response of LTI system if $x(n) = u(n)$ and $h(n) = 5\left(-\frac{1}{2}\right)^n u(n)$

sol:-
$$y(n) = \sum_{K=-\infty}^{\infty} u(k) \cdot 5\left(-\frac{1}{2}\right)^{n-k} u(n-k)$$
$$= \sum_{K=0}^{n} 5\left(-\frac{1}{2}\right)^{n-k} = 5\left(-\frac{1}{2}\right)^n \left[1 - 2 + 2^2 - 2^3 + \dots\right]$$
$$= 5\left(-\frac{1}{2}\right)^n \left[\frac{1-(-2)^{n+1}}{(1-(-2))}\right] = 5\left(\frac{-1}{2}\right)^n \left[\frac{1-(-2)^{n}}{3}\right]$$

$$\boxed{\sum_{K=0}^{n} \left(-\frac{1}{2}\right)^{-k} = 1 - 2 + 2^2 - 2^3 + \dots (-2)^{-n}}$$

9) Determine the range of values of a and b for which LTI system with impulse response $h(n) = a^n$ for $n \geq 0$ is stable.
$= b^n$ for $n < 0$

Sol:-

$$\left(\sum_{k=-\infty}^{-1} |b|^k + \sum_{k=0}^{\infty} |a^k| \right) < \infty$$

$$[1 + |a| + |a|^2 + \cdots \infty] + [|b^{-1}| + |b^{-2}| + \cdots]$$

$$\underbrace{\frac{1}{1-|a|}}_{|a|<1} + |b^{-1}| \left| \frac{1}{1-|b^{-1}|} \right| < \infty$$

$$|b^{-1}| < 1$$

$$\therefore |a| < 1 < |b|$$

Frequency domain Analysis of discrete-time signals

Freq analysis is used to know the freq. components in a given signal to know the signal bandwidth.

Any periodic signal $x(t)$ can be represented as weighted sum of exponentials.

$$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$$

$$\Rightarrow c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

This is called Discrete Time Fourier series

Fourier Transform is

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt.$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} dt.$$

def Discrete-time Fourier Transform is

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

where $x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega.$

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Parsevals relation for periodic & aperiodic discrete time signals.

a) Power spectrum density of periodic signals :

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, x^*(n)$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x(n)\left[\sum_{n=0}^{N-1} c_k^* \, e^{-j2\pi kn/N}\right]$$

$$= \sum_{n=0}^{N-1} c_k^* \left[\frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}\right].$$

$$\therefore P = \sum_{n=0}^{N-1} |c_k|^2 = P_{xx}(k)$$

b) Energy density spectrum of aperiodic signals:-

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, x^*(n)$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\omega}) e^{-j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\omega}) \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\omega})\, X(e^{j\omega})\, d\omega$$

$$\therefore E = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = S_{xx}(\omega)$$

Fourier transform exists only for sequences that are absolutely summable.

i.e. $\sum_{n=-\infty}^{\infty} |x(n)| < \infty$

Properties of DTFT :

1) Linearity : $a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow{DTFT} a_1 X_1(\omega) + a_2 X_2(\omega)$

2) Periodicity : $X(\omega) = X(\omega + 2\pi)$

3) Time shifting : $x(n-k) \xleftrightarrow{DTFT} e^{-j\omega k} X(\omega)$.

4) Freq. shifting : $e^{j\omega_0 n} x(n) \xleftrightarrow{DTFT} X(\omega - \omega_0)$.

5) Time reversal : $x(-n) \xleftrightarrow{DTFT} X(-\omega)$

6) Freq. differentiation : $n x(n) \xleftrightarrow{DTFT} j \frac{d}{d\omega} X(\omega)$

7) Time Convolution : $x(n) * h(n) \xleftrightarrow{DTFT} X(\omega) \cdot H(\omega)$

8) Time diff^n : $\frac{d}{dn}(x(n)) \leftrightarrow j\omega X(\omega)$

9) Scaling : $x(an) \leftrightarrow X\left(\frac{\omega}{a}\right)$.

10) Symmetry : $X(n) \leftrightarrow 2\pi x(-\omega)$.

Proofs :

2) Periodicity :

$$X(\omega + 2\pi) = \sum_{n=-\infty}^{\infty} x(n) \, e^{-j(\omega+2\pi)n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \, e^{-j\omega n} \, e^{-j2\pi n}$$

$$= X(\omega). \quad (\because e^{-j2\pi n} = 1).$$

3) Time-shifting :

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \, e^{-j\omega n}.$$

$$F[x(n-n_0)] = \sum_{n=-\infty}^{\infty} x(n-n_0) \, e^{-j\omega n}$$

- let $n - n_0 = p$

$$= \sum_{p=-\infty}^{\infty} x(p) \, e^{-j\omega(p+n_0)}$$

$$= \sum_{p=-\infty}^{\infty} x(p) \, e^{-j\omega p} \, e^{-j\omega n_0}$$

$$\therefore F[x(n-n_0)] = e^{-j\omega n_0} X(\omega).$$

4) Freq - shifting:

$$DTFT\left[x(n)\, e^{j\omega_0 n}\right] = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n} \cdot e^{j\omega_0 n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-jn(\omega-\omega_0)}$$

$$= X(\omega-\omega_0)$$

5) Time - reversal:

$$DTFT\left[x(-n)\right] = \sum_{n=-\infty}^{\infty} x(-n)\, e^{-j\omega n}.$$

let $-n = p$

$$= \sum_{p=-\infty}^{\infty} x(p)\, e^{j\omega p}$$

$$= \sum_{p=-\infty}^{\infty} x(p)\, e^{-j(-\omega)p}$$

$$= X(-\omega).$$

9) $\boxed{\text{Scaling:}}$

$$DTFT[x(an)] = \sum_{n=-\infty}^{\infty} x(an)\, e^{-j\omega n}$$

$an = p$

$$\sum_{p=-\infty}^{\infty} x(p)\, e^{-\frac{j\omega p}{a}}$$

$$= X\left(\frac{\omega}{a}\right)$$

6) Freq - differentiation:

$$X(\omega) = DTFT[x(n)] = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}.$$

$$\frac{d}{d\omega} X(\omega) = \frac{d}{d\omega} \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{d}{d\omega}\left(e^{-j\omega n}\right)$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} (-jn)$$

$$\frac{d}{d\omega} X(\omega) = -j \sum_{n=-\infty}^{\infty} n\, x(n)\, e^{-j\omega n}$$

$$\therefore\ j\frac{d}{d\omega} X(\omega) = DTFT[n\, x(n)].$$

10) $\boxed{\begin{array}{l}\text{Symm prop:} \\ x(n) \rightleftharpoons X(\omega) \\ X(n) \rightleftharpoons 2\pi\, x(-\omega)\end{array}}$

$$DTFT\left[\frac{d}{dn}(x(n))\right] = j\omega\, X(\omega).$$

8) Time diff^n:-

$$DTFT\left[\frac{d}{dn} x(n)\right] =$$

$$f^{-1}[X(\omega)] = x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) \frac{d}{dn}\left(e^{j\omega n}\right) d\omega$$

$$= j\omega \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega n}\, d\omega$$

7) Time-Convolution:

$$DTFT[x(n) * h(n)] = \sum_{n=-\infty}^{\infty} x(n) * h(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{K=-\infty}^{\infty} x(K) h(n-K) e^{-j\omega(n-K)} e^{-j\omega K}$$

$$= \sum_{K=-\infty}^{\infty} x(K) e^{-j\omega K} \sum_{n=-\infty}^{\infty} h(n-K) e^{-j\omega(n-K)}$$

$$= X(\omega) H(\omega).$$

Problems:

10) Find DTFT of $x(n) = (\frac{1}{2})^{n-1} u(n-1)$ using time-shifting property.

sol:-

$$X(\omega) = \sum_{n=-\infty}^{\infty} (\tfrac{1}{2})^{n-1} u(n-1) e^{-j\omega n}$$

$$= \sum_{n=1}^{\infty} (\tfrac{1}{2})^{n-1} \times 1 \times e^{-j\omega n}$$

$$= 2 \sum_{n=1}^{\infty} (\tfrac{1}{2})^{n} e^{-j\omega n}$$

$$= 2\left[\tfrac{1}{2} e^{-j\omega} + (\tfrac{1}{2})^2 e^{-j2\omega} + \cdots\right]$$

$$= \cancel{2} \times \tfrac{1}{\cancel{2}} e^{-j\omega}\left[1 + \tfrac{1}{2} e^{-j\omega} + \cdots\right]$$

$$= e^{-j\omega}\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] = \frac{2e^{-j\omega}}{2-e^{-j\omega}}$$

Time-shifting

$$= e^{-j\omega n_0}[X(\omega)]$$

$$= e^{-j\omega}\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right]$$

11) Find DTFT of $[e^{j3n} u(n)]$ using freq. shifting.

$$DTFT[e^{j3n} u(n)] = \frac{1}{1-e^{-j(\omega-3)}}$$

12) Find $x(n)$ if $X(\omega) = e^{-j\omega}[\frac{1}{2} + \frac{1}{2}\cos\omega]$

$$X(\omega) = e^{-j\omega}\left[\tfrac{1}{2} + \tfrac{1}{4}(e^{j\omega} + e^{-j\omega})\right]$$

$$= \tfrac{1}{4} + \tfrac{1}{2} e^{-j\omega} + \tfrac{1}{4} e^{-2j\omega}$$

$x(0) = x(2) = \frac{1}{4}$; $x(1) = \frac{1}{2}$ and $x(n) = 0$ otherwise.

$$x(n) = [\tfrac{1}{4} \ \ \tfrac{1}{2} \ \ \tfrac{1}{4}]$$

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Freq domain analysis on Systems (8)

Freq. Response analysis of Discrete-time systems.

Response of first-order IIR low pass system:

$y(n) - a\,y(n-1) = x(n)$. (Difference eq^n for a first-order system)

Taking fourier transform,

$Y(\omega) - a e^{-j\omega} Y(\omega) = X(\omega)$

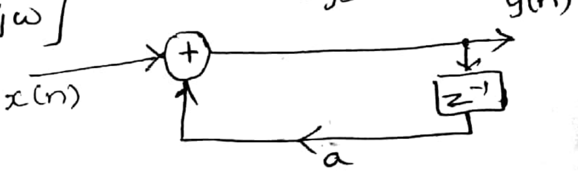
Transfer fun^n:

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{1 - a e^{-j\omega}}$$

$$h(n) = \mathcal{F}^{-1}\left[\frac{1}{1 - a e^{-j\omega}}\right]$$

$$h(n) = a^n u(n)$$

Realization of the system.



$$|H(\omega)| = \frac{1}{\sqrt{(1 - a\cos\omega)^2 + a^2 \sin^2\omega}}$$

$$= \frac{1}{\sqrt{1 + a^2 - 2a\cos\omega}}$$

at

$\omega = 0 \quad |H(\omega)| = \frac{1}{\sqrt{(1-a)^2}} = \frac{1}{1-a}$

$\omega = \pi \quad |H(\omega)| = \frac{1}{\sqrt{(1+a)^2}} = \frac{1}{1+a}$

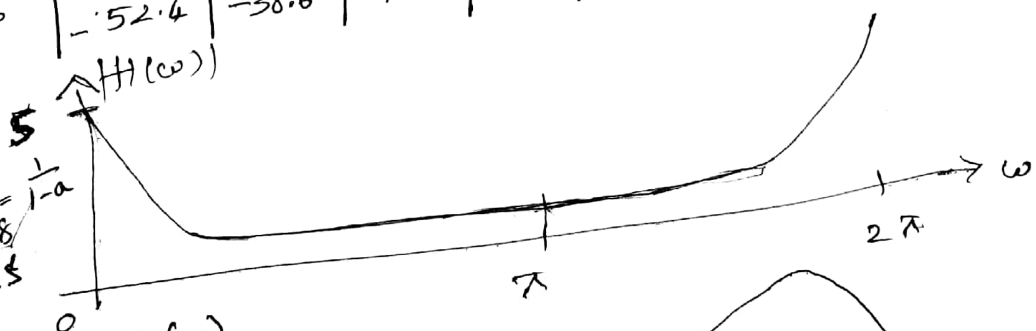
$\omega = 2\pi \quad |H(\omega)| = \frac{1}{\sqrt{(1-a)^2}} = \frac{1}{1-a}$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{a\sin\omega}{1 - a\cos\omega}\right)$$

for $a = 0.8$

ω	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\lvert H(\omega)\rvert$	5	1.402	0.78	0.6	0.55	0.6	0.78	1.4	5
$\angle H(\omega)$	0°	-52.4°	-38.6°	-19.8°	0	19.8°	38.6°	52.4°	0°

$\frac{5}{\sqrt{2}}$



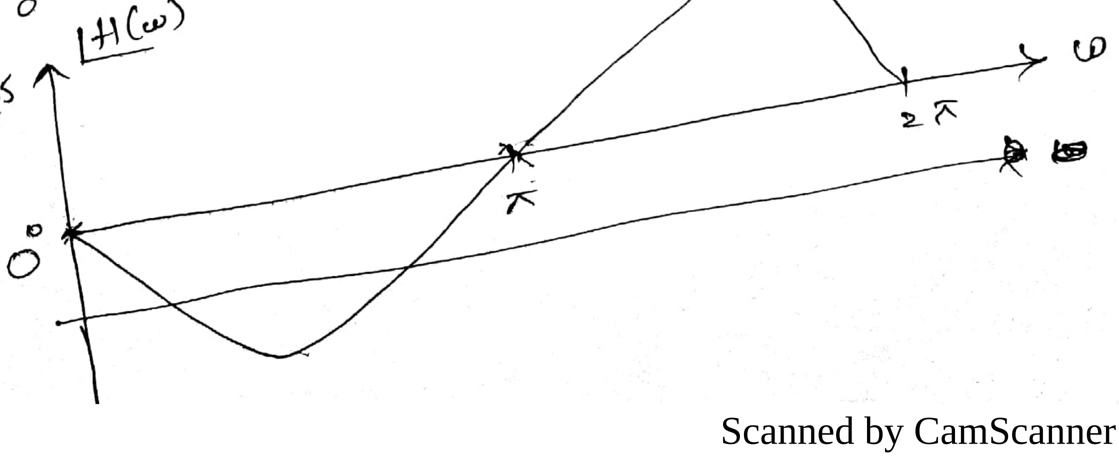
$|H(0)| = \frac{1}{1-0.8} = \frac{1}{1-a}$

$= \frac{1}{0.2} = 5$

$|H(\pi)| = \frac{1}{1+a}$

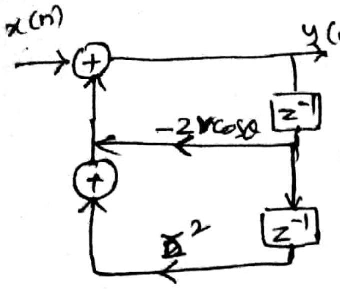
$|H(\pi)| = \frac{1}{1.8} = 0.55$

$|H(2\pi)| = \frac{1}{1-a}$



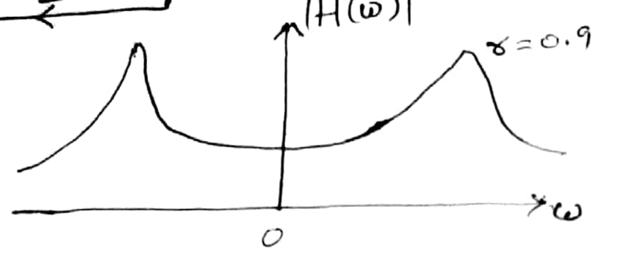
Response of secondorder system:

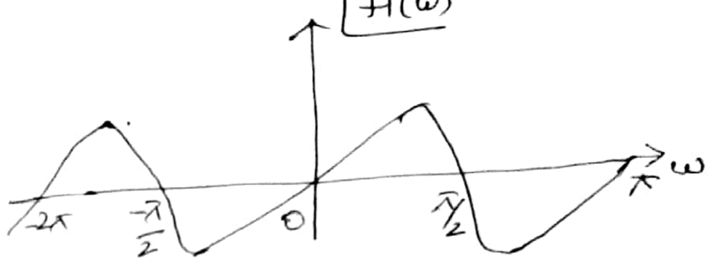
The diff. eqⁿ for a second order system



$$a_0 y(n) - a_1 y(n-1) - a_2 y(n-2) = b_0 x(n)$$

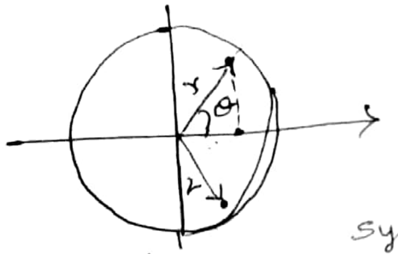
$$\Rightarrow H(\omega) = \frac{1}{1 - 2r e^{-j\omega}\cos\theta + r^2 e^{-2j\omega}}$$





Mag. spectrum Phase spectrum.

$$\boxed{a_1 = -2r\cos\theta \qquad a_2 = +r^2}$$



z-plane

A resonator is a recursive linear (IIR) system having complex conjugate pair of poles located inside the unit circle of the z-plane.

Poles at $r(\cos\theta + i\sin\theta)$ and $r(\cos\theta - i\sin\theta)$

$$\therefore H(z) = \frac{1}{[1 - r(\cos\theta + i\sin\theta)z^{-1}][1 - r(\cos\theta - i\sin\theta)z^{-1}]}$$

$$= \frac{1}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}}$$

Poles at $x + jy$ & $x - jy$
hence $P_1 = r\cos\theta + jr\sin\theta$
$P_2 = r\cos\theta - jr\sin\theta$.

$\therefore$ Numerator coefficient is $b_0 = 1$

Denominator coeffs are $a_0 = 1$, $a_1 = -2r\cos\theta$
$a_2 = r^2$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots}$$

For digital resonator:

$$H(z) = \frac{b_0}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

When r is close to 1
B.W. is narrow.

When r is small
B.W. is wide

B.W $\propto (1-r)$

Linear Convolution :

No. of points in convolved sequence is N_1+N_2-1.

steps :
a) Folding
b) shifting
c) Multiplying
d) Integrating or summing up.

Problems

13. Convolve $x(n) = \{\underset{\uparrow}{3}, 2, 1, 2\}$ & $h(n) = \{1, \underset{\uparrow}{2}, 1, 2\}$ using graphical & tabulation procedures.

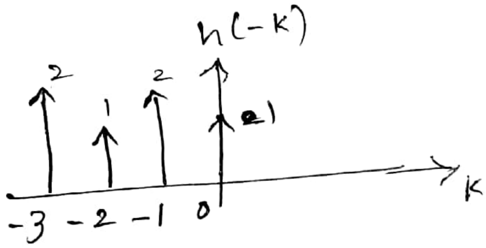
sol:- Graphical method:

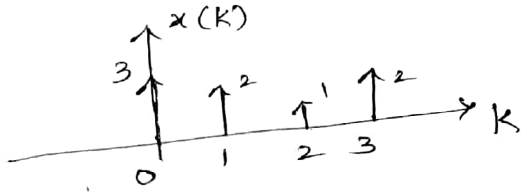
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(0) = \sum x(k)\, h(-k)$$

$x(k) = [\overset{\downarrow}{3} \quad 2 \quad 1 \quad 2]$

$h(-k) = [2 \quad 1 \quad 2 \quad \underset{\uparrow}{1}]$





$\Rightarrow y(0) = 3 \times 1 = 3$

$y(1) = \sum x(k)\, h(1-k)$

$= 3\times2 + 1\times2 = 8$

$y(2) = \sum x(k)\, h(2-k)$

$= 3\times1 + 2\times2 + 1\times1$

$= 8$

$y(3) = \sum x(k)\, h(3-k)$

$= 3\times2 + 2\times1 + 1\times2 + 2\times1$

$= 12$

$y(4) = 9$

$y(5) = 4$

$y(6) = 4$.

$\therefore y(n) = [3 \quad 8 \quad 8 \quad 12 \quad 9 \quad 4 \quad 4]$

Tabulation method:

k	-3	-2	-1	0	1	2	3	4	5	6	
$x(k)$				3	2	1	2				
step¹ $h(-k)$	2	1	2	1							$\Rightarrow 3\times1=3$
step² $h(1-k)$		2	1	2	1						$\Rightarrow 3\times2+2\times1=8$
$h(2-k)$			2	1	2	1					$\Rightarrow 8$
$h(3-k)$				2	1	2	1				$\Rightarrow 12$
$h(4-k)$					2	1	2	1			$\Rightarrow 9$
$h(5-k)$						2	1	2	1		$\Rightarrow 4$
$h(6-k)$							2	1	2	1	$\Rightarrow 4$

$\Rightarrow y(n) = [3 \quad 8 \quad 8 \quad 12 \quad 9 \quad 4 \quad 4]$

Verification:

	3	2	1	2
1	3	2	1	2
2	6	4	2	4
1	3	2	1	2
2	6	4	2	4

$[3 \quad 8 \quad 8 \quad 12 \quad 9 \quad 4 \quad 4]$

Circular Convolution:-

Three methods to perform circular convolution:

a) Concentric circle method

b) Matrix method

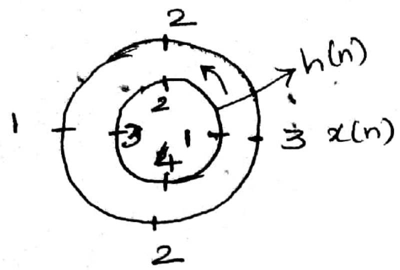
c) DFT & IDFT method.

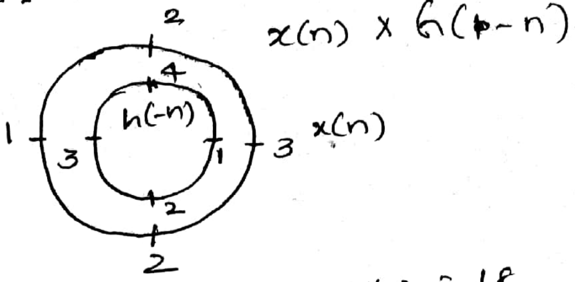
Problem:

14. Perform circular convolution for the given sequences $x(n) = [3 \quad 2 \quad 1 \quad 2]$ $h(n) = [1 \quad 2 \quad 4]$ using concentric circle & matrix methods.

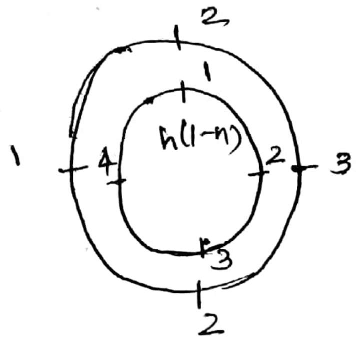
(10)

sol:- Concentric circle method:

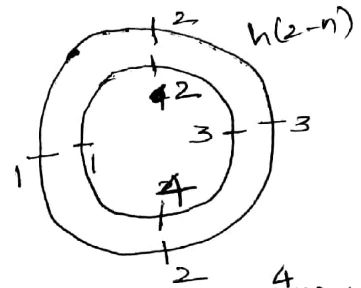




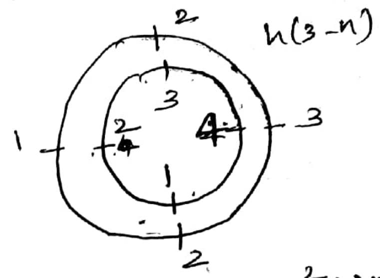
$1\times3 + 2\times4 + 1\times3 + 2\times2 = 18$



$2\times3+2\times1+1\times4+3\times2$
$= 18$



$3\times3 + 2\times2 + 1\times1 + 4\times2$
$= 22$



$3\times4 + 3\times2 + 1\times2 + 1\times2$
$= 22$

$\therefore y(n) = [18 \quad 18 \quad 22 \quad 22]$

Matrix method:

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 18 \\ 18 \\ 22 \\ 22 \end{bmatrix}$$

15. Perform circular convolution of $x(n) = [1 \ -1 \ -2 \ 3 \ -1]$ & $h(n) = [1 \quad 2 \quad 3]$

sol:-

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{bmatrix}$$

To obtain linear convolution result from circular convolution both the sequences should be made to the length of N_1+N_2-1 by appending zeros at the end.

Eg: Linear Convolution from Circular convolution

$x(n) = [3 \ 2 \ 1 \ 2] \qquad h(n) = [1 \ 2 \ 1 \ 2]$

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 2 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 2 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 & 2 \\ 2 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 2 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 2 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 8 \\ 12 \\ 9 \\ 4 \\ 4 \end{bmatrix}$$

Problem on freq response :-

16. In a discrete time system with i/p $x(n)$ & $y(n)$ is described in freq domain i.e. $Y(\omega) = e^{-j2\omega} X(\omega) + \frac{d}{d\omega} X(\omega)$. Compute the response for the i/p $x(n) = \delta(n)$.

sol:- $Y(\omega) = e^{-j2\omega} X(\omega) + \frac{d}{d\omega} X(\omega) = e^{-2j\omega} \times 1 + \frac{d}{d\omega}(1)$

$= e^{-2j\omega} \Rightarrow y(n) = F^{-1}\left[e^{-j2\omega}\right] = \delta(n-2)$.

17. A Causal & Stable LTI system has a property that $\left(\frac{4}{5}\right)^n u(n)$ produces an o/p of $n\left(\frac{4}{5}\right)^n u(n)$.

a) Det. freq. response $H(\omega)$ for the system

b) Det. the difference eqn.

sol:- $y(n) = n\, x(n)$.

$$\left(\tfrac{4}{5}\right)^n u(n) \longrightarrow \boxed{h(n)} \longrightarrow n\left(\tfrac{4}{5}\right)^n u(n)$$

$Y(\omega) = DTFT[y(n)] = DTFT[n\,x(n)]$

$= j\frac{d}{d\omega}(X(\omega)) = j\frac{d}{d\omega}\left(\frac{5}{5-4e^{-j\omega}}\right)$

$$\boxed{F.T.\{a^n u(n)\} = \frac{1}{1-ae^{-j\omega}}}$$

$X(\omega) = DTFT\left[\left(\frac{4}{5}\right)^n u(n)\right] = \sum_{n=-\infty}^{\infty} \left(\frac{4}{5}\right)^n u(n) e^{-j\omega n}$

$= \sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{4}{5} e^{-j\omega}\right)^n$

$= 1 + \frac{4}{5} e^{-j\omega} + \left(\frac{4}{5} e^{-j\omega}\right)^2 + \cdots$

$= \frac{1}{1-\left(\frac{4}{5} e^{-j\omega}\right)} = \frac{5}{5-4e^{-j\omega}}$

$Y(\omega) = \frac{j}{5}\left(\frac{5}{5-4e^{-j\omega}}\right)^2 (-4j\, e^{-j\omega}) = \frac{1}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2}\left(\frac{4}{5} e^{-j\omega}\right)$

$\Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\frac{4}{5} e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^{\cancel{2}}} \times \cancel{\left(1-\frac{4}{5}e^{-j\omega}\right)}$

$= \frac{\frac{4}{5} e^{-j\omega}}{1-\frac{4}{5} e^{-j\omega}}$

$\Rightarrow Y(\omega) - \frac{4}{5} e^{-j\omega} Y(\omega) = X(\omega) \frac{4}{5} e^{-j\omega}$

$\Rightarrow y(n) - \frac{4}{5} y(n-1) = \frac{4}{5} x(n-1)$

Z - Transform

The Z-Transform is a mathematical tool for the analysis of linear time-invariant discrete-time systems in the freq. domain.

The freq response of a discrete-time system can be determined by evaluating the transfer function on the unit circle of z-plane.

The z-transform is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}.$$

where z is a complex variable. In polar form z can be expressed as $z = re^{j\omega}$.

r is the radius of the circle.

$$\Rightarrow X(re^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, r^{-n} e^{-j\omega n}$$

For $r=1$ it leads to unit circle.

For causal sequence $x(n)=0$ for $n<0$

$$\Rightarrow X(z) = \sum_{n=0}^{\infty} x(n)\, z^{-n}$$

$$\therefore X(z) = Z[x(n)]$$

Since z-transform is an infinite power series, it exists only for those values of z for which this series converges. The region of Convergence (ROC) of $X(z)$ is set for all values of z for which $X(z)$ attains a finite value.

ROC of finite Sequences:

Eg: 1 Find z-transform and ROC of the causal sequence (right-handed sequence) $x(n) = [\underset{\uparrow}{1}\ 2\ 0\ -4]$.

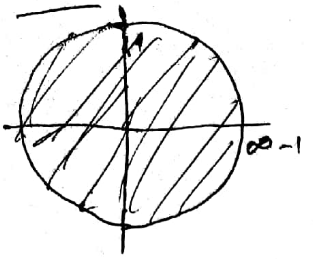
Sol:- $X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$

$= 1 \times z^{0} + 2 \times z^{-1} - 4z^{-3} = 1 + 2z^{-1} - 4z^{-3}$

$X(z)$ converges for all values of z except at $z=0$.

Eg 2: Find Z-transform & ROC of a Anti causal sequence (Left-sided sequence) $x(n) = [-3 \quad 2 \quad -1 \quad \underset{\uparrow}{0}]$.

Sol:-



$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= -3z^{+3} + 2z^{+2} - z + 1$$

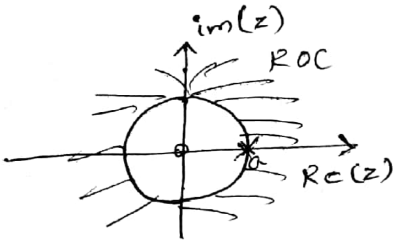
$$= 1 - z + 2z^2 - 3z^3$$

$\therefore$ $X(z)$ Converges for all values of z except at $z = \infty$.

<u>Problems on causality:-</u> Infinite sequences

18. a) Det. Z-transform and ROC of the signal $x(n) = a^n u(n)$

Sol: It is a right sided sequence.



$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1-az^{-1}}$$

$$\Rightarrow |az^{-1}| < 1 \Rightarrow |z| > a.$$

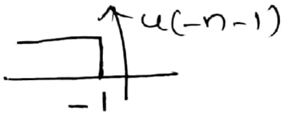
$X(z) = \frac{z}{z-a}$ ROC is $|z| > a$.

$\therefore$ ROC is exterior of a circle having radius $|a|$.

$\therefore$ It is a causal signal

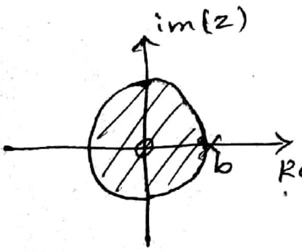
b) Det. z-transform and ROC of the signal $x(n) = -b^n u(-n-1)$.

It is a left-sided sequence.



$$\Rightarrow X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} b^n z^{-n} = -\sum_{n=1}^{\infty} (b^{-1}z)^n = -\left[\sum_{n=0}^{\infty} (b^{-1}z)^n - 1\right]$$



$\therefore$ The series Converges for $|b^{-1}z| < 1$ i.e. $|z| < b$.

$$X(z) = -\left[\frac{1}{1-b^{-1}z} - 1\right] = \frac{z}{z-b}; \text{ROC}: |z| < b.$$

$\therefore$ ROC is interior of a circle having radius b.

$\therefore$ It is a anti-causal signal.

$\therefore$ For diff signals z-transform may be same but ROCs are diff. Hence ROC has to be found out.

ROC of two-sided sequence.

c) Find z-transform of $x(n) = a^n u(n) - b^n u(-n-1)$

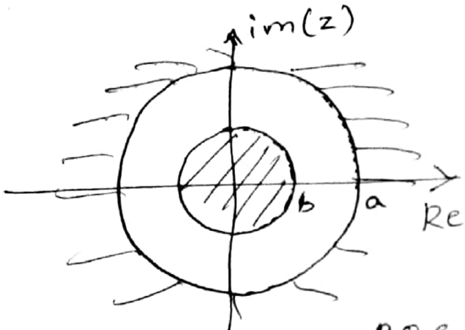
provided i) $|a| > |b|$ ii) $|a| < |b|$.

sol:- $$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} - \sum_{n=-\infty}^{-1} b^n z^{-n}$$

$$= \frac{z}{z-a} + \frac{z}{z-b}$$

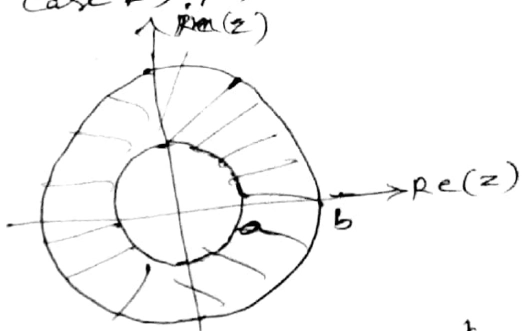
$\therefore$ ROC is $|z| > a$ & $|z| < b$

Case i) $|a| > |b|$



No common ROC
X(z) doesn't exist

Case ii) $|a| < |b|$



Common ROC exists.

Properties of z-transform:-

1. Time-shifting:

$$x(n-m) \overset{Z}{\longleftrightarrow} z^{-m} X(z)$$

$$Z[x(n-m)] = \sum_{n=-\infty}^{\infty} x(n-m) z^{-n}$$

let $n-m=p \Rightarrow \sum_{p=-\infty}^{\infty} x(p) z^{-(p+m)}$

$$= z^{-m} X(p).$$

$$\therefore Z[x(n-m)] = z^{-m} X(z).$$

2. Time reversal: $x(-n) \overset{Z}{\longleftrightarrow} X(z^{-1})$

$$Z[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$$

let $-n=p$ $$= \sum_{p=\infty}^{-\infty} x(p) z^{p}$$

$$= \sum_{p=-\infty}^{\infty} x(p) (z^{-1})^{-p}$$

$$\Rightarrow Z[x(-n)] = X(z^{-1}).$$

3. Freq. differentiation.

$$n x(n) \overset{Z}{\longleftrightarrow} -z \frac{d}{dz} X(z)$$

$$\frac{d}{dz}(X(z)) = \frac{d}{dz}\left[\sum_{n=-\infty}^{\infty} x(n) z^{-n}\right]$$

$$= \sum_{n=-\infty}^{\infty} x(n)(-n) z^{-n-1}.$$

$$-\frac{d}{dz}(X(z)) = \sum_{n=-\infty}^{\infty} n\, x(n) z^{-n} z^{-1}.$$

$$\Rightarrow -z\frac{d}{dz}(X(z)) = z[n x(n)].$$

4. Linearity:

$$Z[a x_1(n) + b x_2(n)] = a X_1(z) + b X_2(z)$$

$$Z\{a x_1(n) + b x_2(n)\} = \sum_{n=-\infty}^{\infty} [a x_1(n) + b x_2(n)] z^{-n}$$

$$= a \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} + b \sum_{n=-\infty}^{\infty} x_2(n) z^{-n}$$

$$= a X_1(z) + b X_2(z).$$

5. Convolution: $x(n) * h(n) \overset{Z}{\longleftrightarrow} X(z) H(z)$

$$Z[x(n) * h(n)] = \sum_{n=-\infty}^{\infty} [x(n) * h(n)] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(k) h(n-k) z^{-n}.$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(k) z^{-k} h(n-k) z^{-(n-k)}$$

$$\therefore z[x(n) * h(n)] = X(z) \cdot H(z).$$

Initial & final value thms.

$x(0) = \lim_{z\to\infty} X(z)$

$x(\infty) = \lim_{z\to 1} (z-1) X(z)$

Appns of Z-Transform:

1. Analyzes the ~~whole~~ discrete time signals & systems
1. Analyzes the discrete time signals & systems. (second reading)

Problem:

19. Find z-transform of a) $x(n) = \frac{1}{2}\delta(n) + \delta(n-1) - \frac{1}{3}\delta(n-2)$

b) $x(n) = u(n-2)$

sol:

a)
$$X(z) = \frac{1}{2} + z^{-1} - \frac{1}{3}z^{-2}$$

ROC: All values of z except at $z=0$

b) $$Z[u(n)] = \frac{z}{z-1}$$

Using time-shifting property

$$Z[x(n-m)] = z^{-m}X(z)$$

$$\Rightarrow Z[u(n-2)] = z^{-2}\left(\frac{z}{z-1}\right) = \frac{z^{-1}}{z-1} \quad \text{ROC: } |z|>1$$

20. Find Inverse z-transform of $X(z) = \log(1-0.5z^{-1})$; $|z| > 0.5$ using differentiation property.

sol:- $$X(z) = \log(1-0.5z^{-1})$$

Differentiating on both sides. ~~& result~~

$$\frac{d}{dz}(X(z)) = \frac{+0.5z^{-2}}{1-0.5z^{-1}}$$

Multiplying $-z$ on both sides

$$-z\frac{d}{dz}(X(z)) = \frac{-0.5z^{-1}}{1-0.5z^{-1}} = \frac{-0.5}{z-0.5}$$

$$Z[n\,x(n)] = -z\frac{d}{dz}X(z) = \frac{-0.5z^{-1}}{1-0.5z^{-1}}$$

$$= -0.5\, Z[(0.5)^{n-1}u(n-1)]$$

$$\therefore n\,x(n) = -0.5\,(0.5)^{n-1}u(n-1)$$

$$\Rightarrow x(n) = -\frac{(0.5)^n}{n}u(n-1)$$

$\because Z[(a)^n u(n)] = \frac{1}{1-az^{-1}}$

$Z[(a)^{n-1}u(n-1)] = \frac{z^{-1}}{1-0.5z^{-1}}$

from time-shifting prop.

System function: Any system is described by a difference eqn.

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

Taking z-transform on both sides, applying time shifting property, we get.

$$Y(z) = -\sum_{k=1}^{N} a_k\, Y(z) z^{-k} + \sum_{k=0}^{M} b_k\, X(z)\, z^{-k}$$

$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$\Rightarrow H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$H(z)$ is known as transfer funn or system funn.

$h(n)$ is impulse response.

Poles & zeros of a system funn:

The zeros of ~~the~~ $H(z)$ are the values of z for which $H(z) = 0$.

The poles of $H(z)$ are the values of z for which $H(z) = \infty$.

$H(z)$ has M zeros and N poles.

if $b_k = 0$ for $1 \le k \le M$ then $H(z)$ reduces to

$$H(z) = \frac{b_0}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$ (All-pole system). IIR system.

stability criterion:

$$|H(z)| = \left|\sum_{n=0}^{\infty} h(n) z^{-n}\right| \le \sum_{n=0}^{\infty} |h(n)|\, |z^{-n}|.$$

when evaluated on unit circle $|z| = 1$

$$\Rightarrow H(z) \le \sum_{n=0}^{\infty} |h(n)| < \infty$$

∴ For a stable system, ROC of the system funn include the unit circle.

Relationship b/w Fourier transform & Z-transform.

Problems at the end.

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

where $z = re^{j\omega}$

$$H(re^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) (re^{j\omega})^{-n}$$

Fourier transform is

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}, \quad \text{for } r=1$$

Fourier transform & z-transform are identical

for $r=1. \Rightarrow |z| = 1.$

$$\therefore H(e^{j\omega}) = H(z) \Big|_{\boxed{z=e^{j\omega}}}.$$

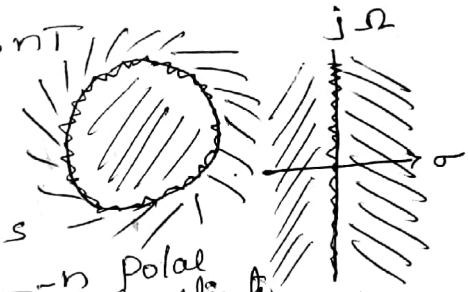
Relationship b/w s-plane and z-plane.

The Laplace transform of a continuous causal signal $x(t)$ is defined as

$$X(s) = \int_0^{\infty} x(t) e^{-st} dt$$

for $t = nT$ (Discrete signal)

$$X(s) = \sum_{n=0}^{\infty} x(nT) e^{-snT}$$



Let $z = e^{sT}$ we get z-transform of $x(nT)$ is

$$X(z) = \sum_{n=0}^{\infty} x(nT) z^{-n}$$

Polal Coordinate

Cartesian Coordinate

$\therefore$ Relation b/w s-plane and z-plane is given by

$$\boxed{z = e^{sT}}$$

Let $z = re^{j\omega}$ & $s = \sigma + j\Omega$

$$\Rightarrow z = re^{j\omega} = e^{(\sigma + j\Omega)T} = e^{\sigma T} \times e^{j\Omega T}$$

$$\Rightarrow |z| = e^{\sigma T} = r \quad \angle z = \omega = \Omega T.$$

$\sigma = 0 \Rightarrow |z| = r = 1$ (~~poles on~~ Imaginary axis of s-plane mapped on to the unit circle of z-plane).

$\sigma < 0 \Rightarrow |z| < 1$ (Left half of s-plane maps into the inside of the unit circle).

$\sigma > 0 \Rightarrow |z| > 1$ (Right half of s-plane maps into the outside of the unit circle).

Realization of digital filters

A digital filter can be realised in two ways.

1. Recursive filter 2. Non-recursive filter.

1. Recursive: If $y(n)$ is a funⁿ of past o/ps, past and present i/ps then it corresponds to recursive in other words Infinite Impulse Response (IIR) filter

$$y(n) = -\sum_{K=1}^{N} a_K\, y(n-K) + \sum_{k=0}^{M} b_K\, x(n-K).$$

2. Non-recursive: If the response of a system is a funⁿ of only past and present i/ps then it corresponds to non-recursive in other words Finite Impulse Response (FIR) filter.

$$y(n) = \sum_{K=0}^{M} b_K\, x(n-K).$$

Recursive / IIR filter realization:

1. Direct form - I
2. Direct form - II / canonical form
3. Transposed direct form
4. Cascade form
5. Parallel form.

1. Direct - form - I :

$$y(n) = +a_1 y(n-1) + a_2 y(n-2) \dots + a_{N-1} y(n-(N-1)) + a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \dots b_M x(n-M)$$

Let

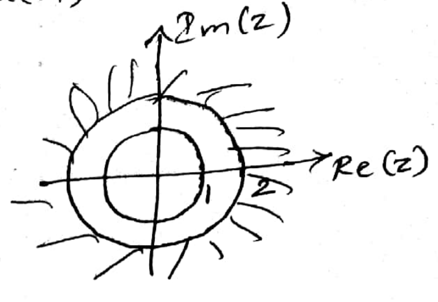
$$b_0 x(n) + b_1 x(n-1) + \dots b_M x(n-M) = w(n)$$

Then

$$y(n) = + a_1 y(n-1) + a_2 y(n-2) + \dots a_N y(n-N) + w(n).$$

Problems on stability :

21. check the ~~stability~~ Convergence of the signal $x(n) = 2^n u(n)$



Sol:- $X(z) = \sum_{n=-\infty}^{\infty} 2^n u(n) z^{-n}$

$= \sum_{n=0}^{\infty} (2z^{-1})^n$

$\Rightarrow (2z^{-1}) < 1 \Rightarrow \frac{z}{z-2}$

$\therefore$ for $|z| > 2$; ~~An unstable system signal~~ the signal Converges.

22. check the stability & draw the pole-zero plot for the system described by the difference equation

$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) - x(n-1)$

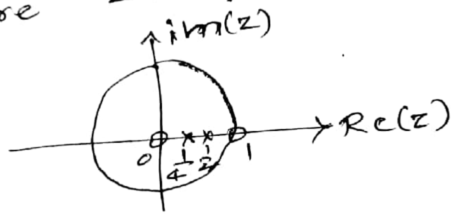
Sol:- $Y(z) \cdot \left[1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right] = X(z)\left[1 - z^{-1}\right]$

$\Rightarrow H(z) = \frac{z(z-1)}{z^2 - \frac{3}{4}z + \frac{1}{8}} = \frac{z(z-1)}{(z-\frac{1}{4})(z-\frac{1}{2})}$

$\therefore$ poles are $z = \frac{1}{4}$, $z = \frac{1}{2}$ &

zeros are $z = 0$, $z = 1$.

Pole-zero plot :-



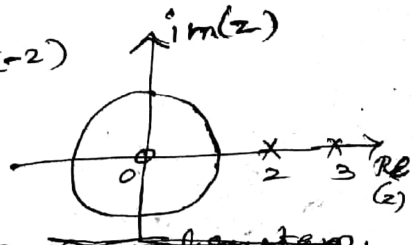
All poles lie inside the unit circle.

$\therefore$ Stable system ~~but not causal~~.

23. Find the stability of $y(n) - 5y(n-1) + 6y(n-2) = x(n)$

Sol:- $Y(z) - 5z^{-1}Y(z) + 6z^{-2}Y(z) = X(z)$

$\Rightarrow H(z) = \frac{1}{1 - 5z^{-1} + 6z^{-2}} = \frac{z^2}{(z-3)(z-2)}$



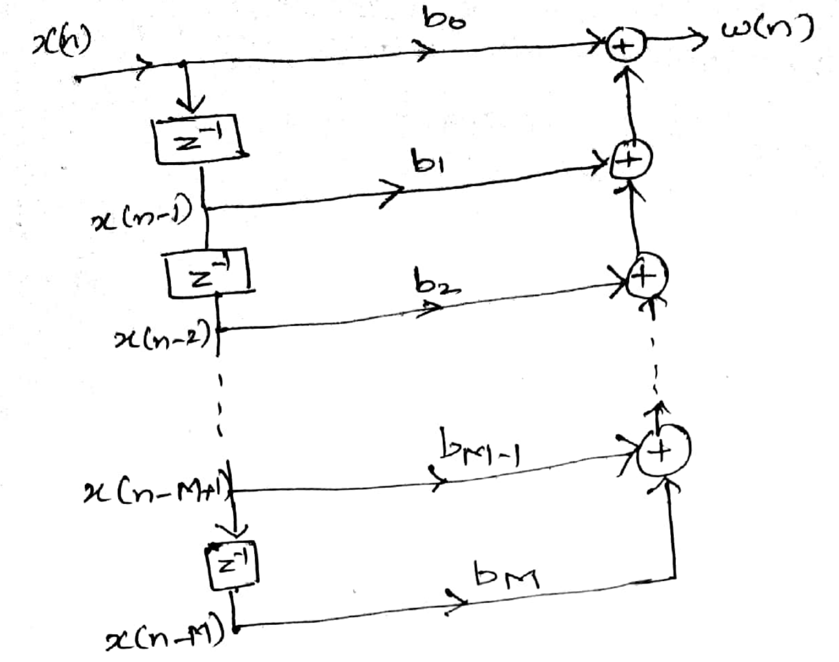
Poles at $z = 3, 2$

zeros at $z = 0$

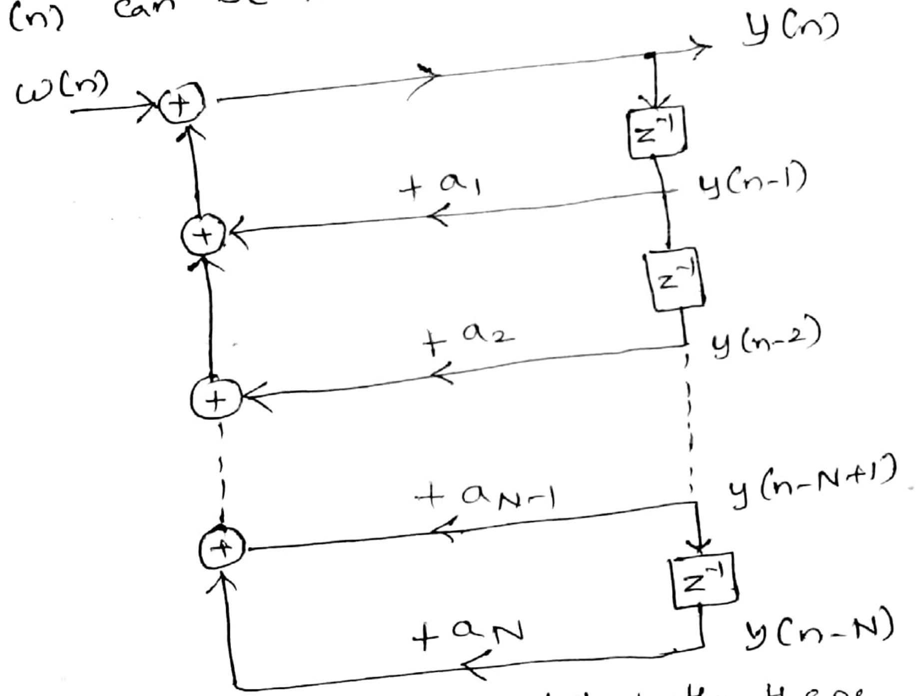
$\therefore$ Unstable system ~~& non causal system~~.

As the poles lie outside the circle.

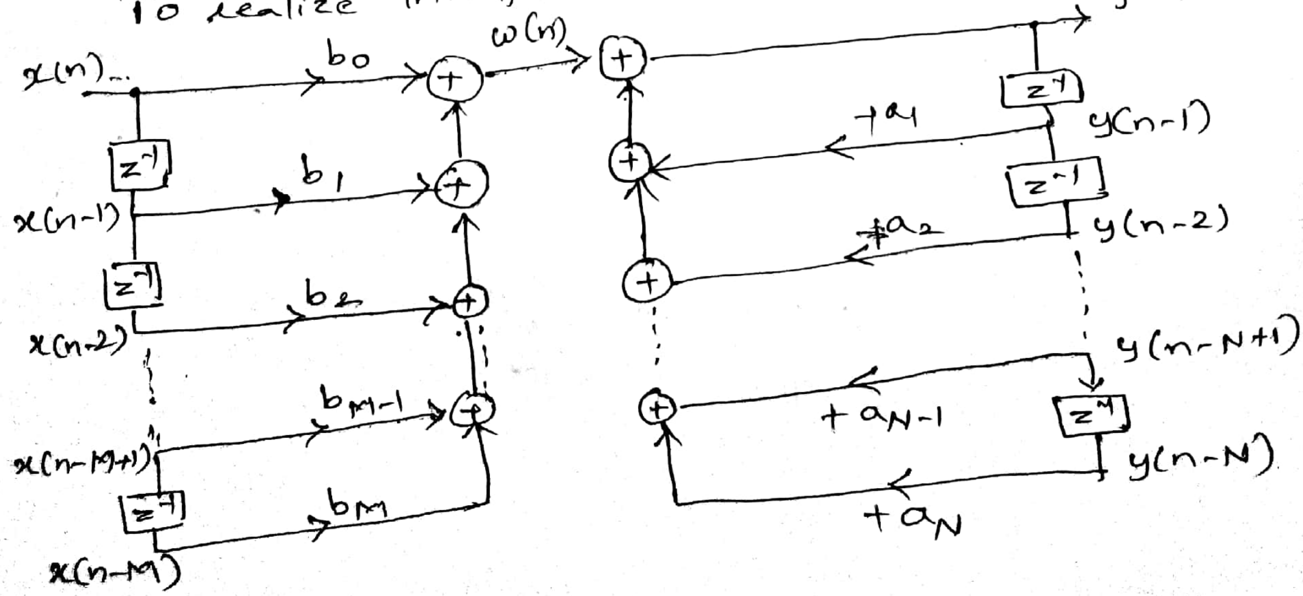
$w(n)$ can be realized as



$y(n)$ can be realized as



To realize the filter club both these.



2. Direct form-II realization: (Canonical)

$$y(n) = +\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k \cancel{x(n)} z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}.$$

Let $\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)}$

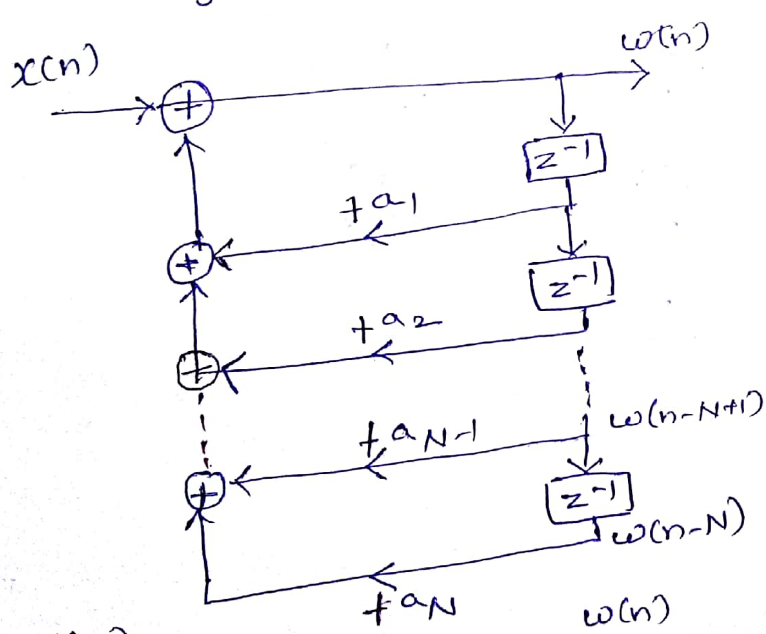
$$\frac{W(z)}{X(z)} = \frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}} \quad \& \quad \frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k}.$$

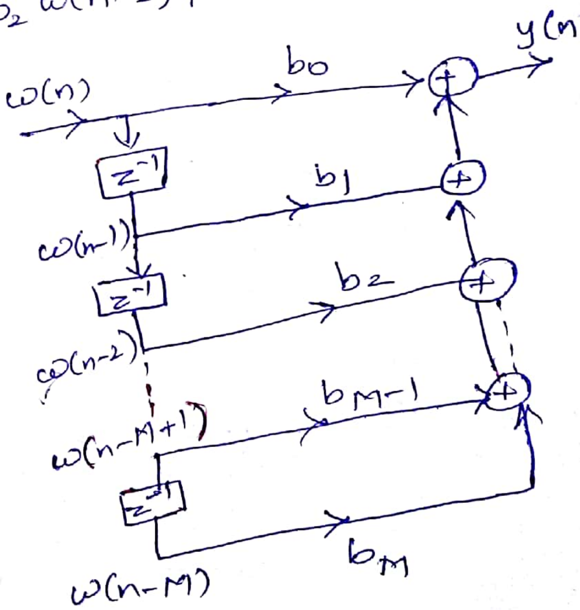
$$W(z) = X(z) + a_1 z^{-1} W(z) + a_2 z^{-2} W(z) + \cdots a_N z^{-N} W(z)$$

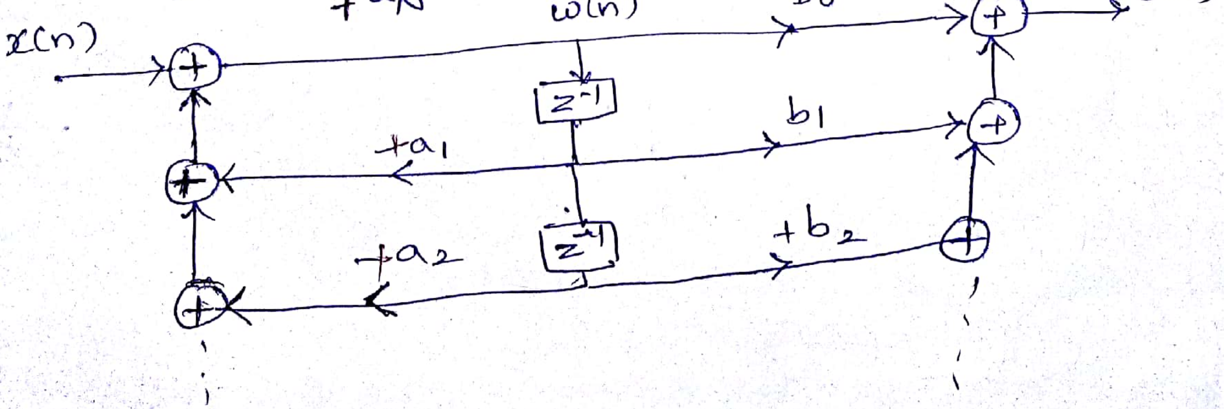
$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + b_2 z^{-2} W(z) + \cdots b_M z^{-M} W(z)$$

$$w(n) = x(n) + a_1 w(n-1) + a_2 w(n-2) + \cdots + a_N w(n-N)$$

$$\& \quad y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \cdots + b_M w(n-M)$$







3. Transposed structure :

steps:

a) First get the direct form realization:

b) Get the signal flow graph from it

c) Reverse the direction of all branches in the signal flow graph

d) Interchange the i/ps and o/ps

e) Change summing points to branching points and viceversa.

Problem :

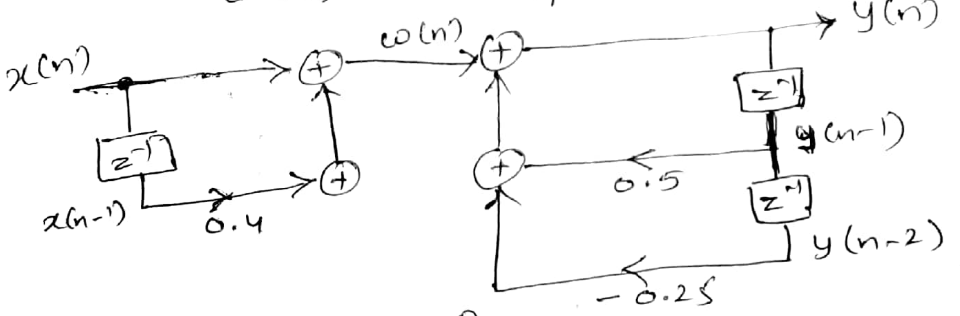
24) Obtain direct form - I, II and transpose direct form - II for the given system.

$$y(n) = \frac{1}{2} y(n-1) - \frac{1}{4} y(n-2) + x(n) + \frac{1}{4} x(n-1).$$

Sol:- Direct form - I :

$$y(n) = \frac{1}{2} y(n-1) - \frac{1}{4} y(n-2) + w(n)$$

$$w(n) = x(n) + \frac{1}{4} x(n-1)$$



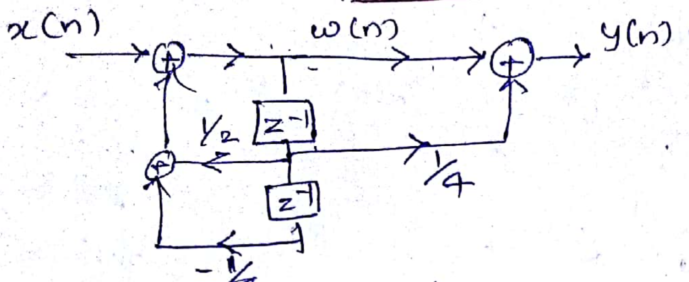
Direct-form - II :

$$\frac{Y(z)}{X(z)} = \frac{1+\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)}$$

$$\frac{Y(z)}{W(z)} = 1+\frac{1}{4}z^{-1} \quad \& \quad \frac{W(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

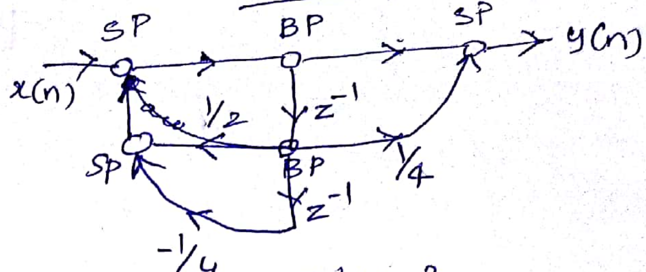
$$\Rightarrow y(n) = w(n) + \frac{1}{4} w(n-1) \quad \& \quad w(n) = x(n) + \frac{1}{2} w(n-1) - \frac{1}{4} w(n-2)$$

3. Transpose direct form:



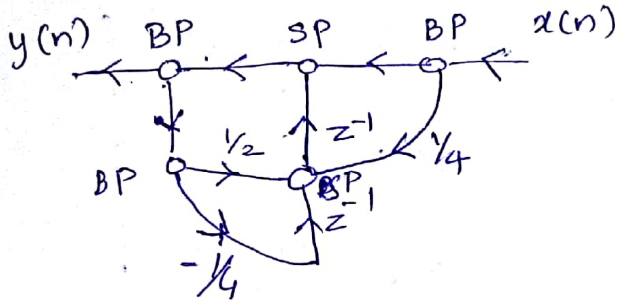
step 1

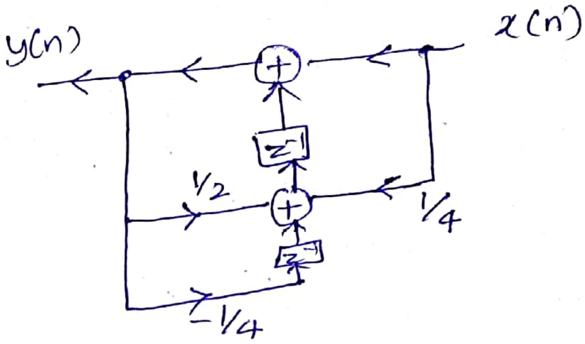
signal flow graph.

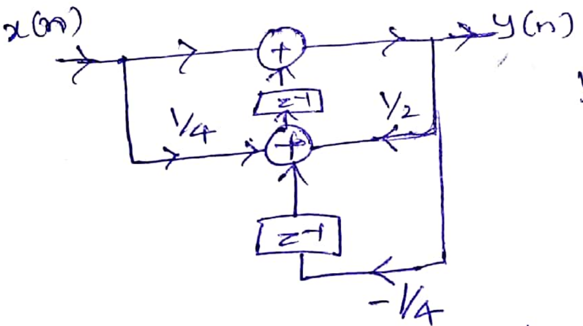


step 2

step 3, 4, 5







$$y(n) = x(n) + \frac{1}{4}x(n-1) + \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2).$$

25) Obtain transpose direct form-II realization
$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6\,x(n-1) + 0.6\,x(n-2)$.

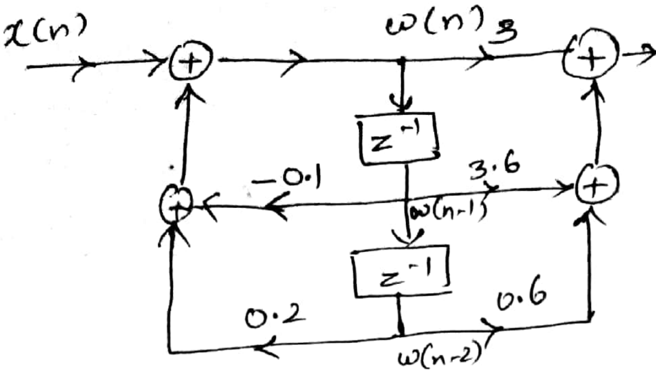
Sol:-
$$\frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}} = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)}$$

$$\Rightarrow \frac{Y(z)}{W(z)} = 3 + 3.6z^{-1} + 0.6z^{-2} \quad \& \quad \frac{W(z)}{X(z)} = \frac{1}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

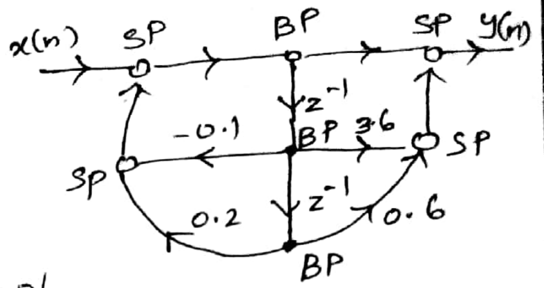
$$\Rightarrow y(n) = 3w(n) + 3.6w(n-1) + 0.6w(n-2)$$

&

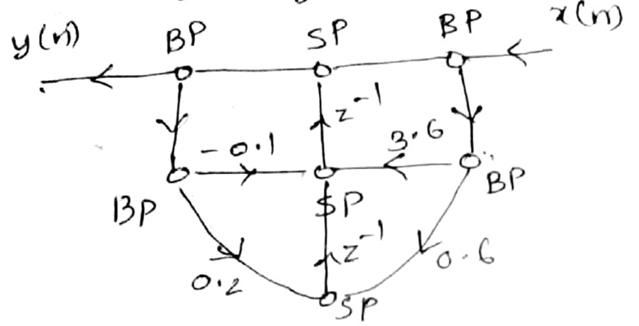
$$w(n) = x(n) - 0.1\,w(n-1) + 0.2\,w(n-2)$$

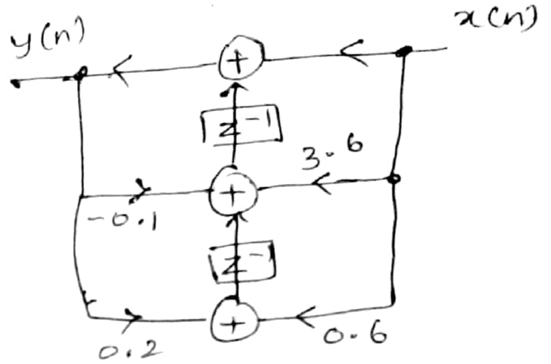


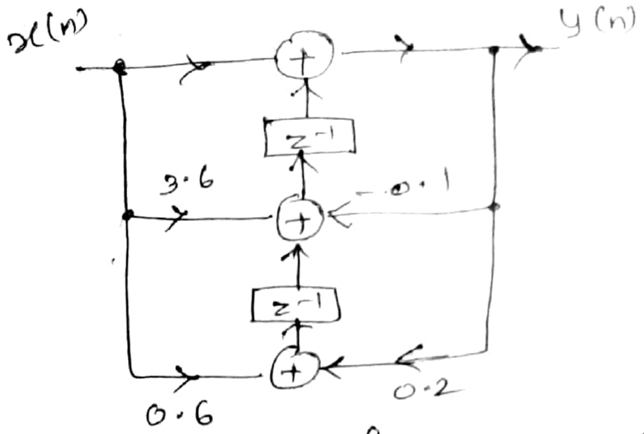
signal flowgraph.



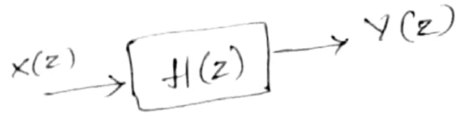
Change the signal flowgraph

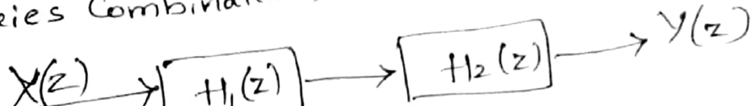






4. <u>Cascade form:</u>
(Series Combination)





26. Realize the system using cascade form whose D.E. is $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$.

<u>Sol:-</u> $Y(z) = \frac{3}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) + X(z) + \frac{1}{3}z^{-1}X(z)$

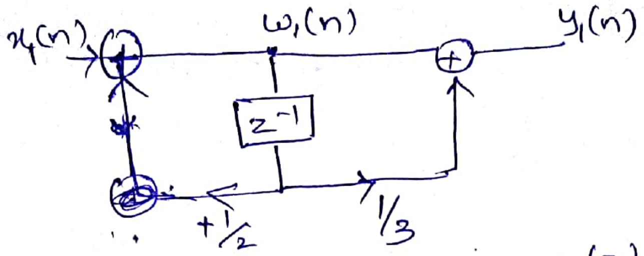
$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{1+\frac{1}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

Let $H_1(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$ & $H_2(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$

$$\frac{Y_1(z)}{W_1(z)} = 1+\frac{1}{3}z^{-1} \quad \& \quad \frac{W_1(z)}{X_1(z)} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

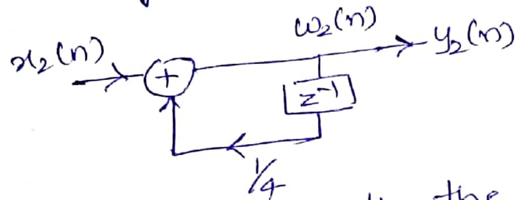
$$\Rightarrow y_1(n) = \omega_1(n) + \frac{1}{3}\omega_1(n-1) \quad \& \quad \omega_1(n) = x_1(n) + \frac{1}{2}\omega_1(n-1)$$

$H_1(z) \Rightarrow$

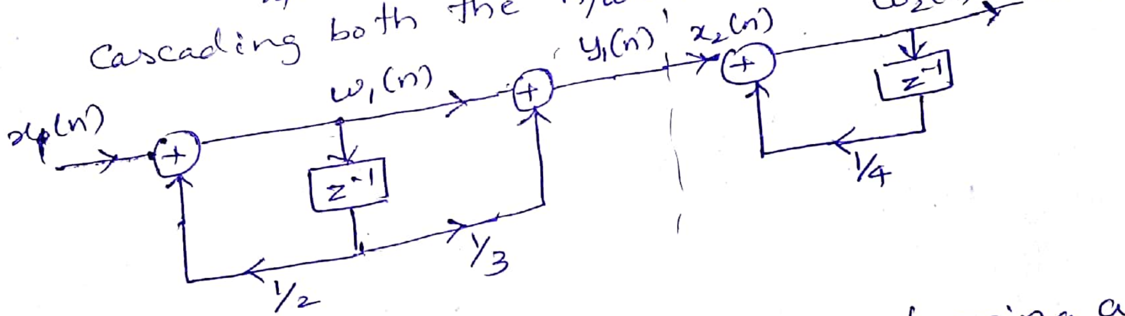


$$H_2(z) = \frac{1}{1-\frac{1}{4}z^{-1}} \; ; \; \frac{Y_2(z)}{W_2(z)} = 1 \quad \& \quad \frac{W_2(z)}{X_2(z)} = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$\Rightarrow y_2(n) = \omega_2(n) \quad \& \quad \omega_2(n) = x_2(n) + \frac{1}{4}\omega_2(n-1)$$



Cascading both the n/ws.



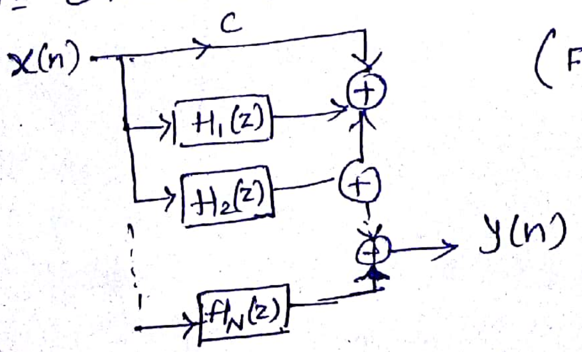
5. Parallel form: It is obtained by performing a partial expansion of $H(z) = c + \sum_{k=1}^{N} \frac{C_k}{1-P_k z^{-1}}$

P_k are poles.

$$\Rightarrow H(z) = c + \frac{C_1}{1-P_1 z^{-1}} + \frac{C_2}{1-P_2 z^{-1}} + \cdots \frac{C_N}{1-P_N z^{-1}}$$

$$H(z) = c + H_1(z) + H_2(z) + \cdots H_N(z).$$

$$Y(z) = cX(z) + H_1(z)X(z) + H_2(z)X(z) + \cdots H_N(z)X(z)$$



(Feedback Comb^n)

27. Realize the system $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ in parallel form.

Sol:- $H(z) = \dfrac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$.

$$= 0.35 + \frac{0.35 - 0.035z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$\begin{array}{r} 0.35 \\ 1 + 0.1z^{-1} - 0.72z^{-2} \,\big)\; 0.7 - 0.252z^{-2} \\ 0.35 + 0.035z^{-1} - 0.252z^{-2} \\ \hline 0.35 - 0.035z^{-1} \end{array}$$

$$= 0.35 + \frac{A}{1 + 0.9z^{-1}} + \frac{B}{1 - 0.8z^{-1}}$$

$A(1 - 0.8z^{-1}) + B(1 + 0.9z^{-1}) = 0.35 - 0.035z^{-1}$

$A + B = 0.35$

$0.9B - 0.8A = 0.035$

$\Rightarrow A = 0.206$

$B = 0.144$

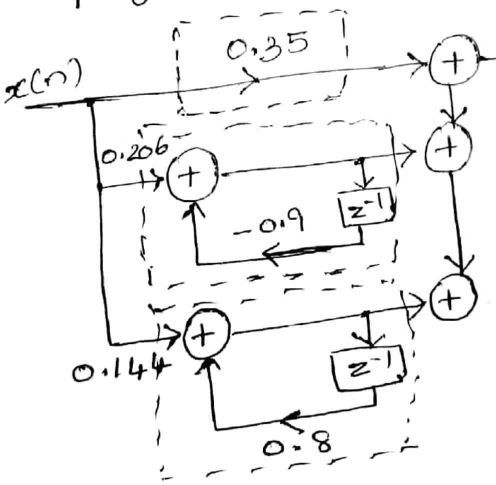
$$H(z) = 0.35 + \frac{0.206}{1 + 0.9z^{-1}} + \frac{0.144}{1 - 0.8z^{-1}}$$

$$H_1(z) = \frac{0.206}{1 + 0.9z^{-1}} = \frac{Y_1(z)}{X_1(z)}$$

$$y_1(n) + 0.9y_1(n-1) = x_1(n)(0.206)$$

$$\Rightarrow y_1(n) = 0.206x_1(n) - 0.9y_1(n-1)$$

$$H_2(z) = \frac{0.144}{1 - 0.8z^{-1}} = \frac{Y_2(z)}{X_2(z)} \;;\; y_2(n) = 0.144x_2(n) + 0.8y_2(n-1)$$



→ Z-transform
→ Relationship with other transforms
→ Z-T & ROC for signals
→ Infinite causal (Exterior)
→ Infinite Non-causal (Interior)
→ Finite causal (Except z=0)
→ Finite non-causal (Except z=∞)
→ Z-T of systems: plot poles & zeros

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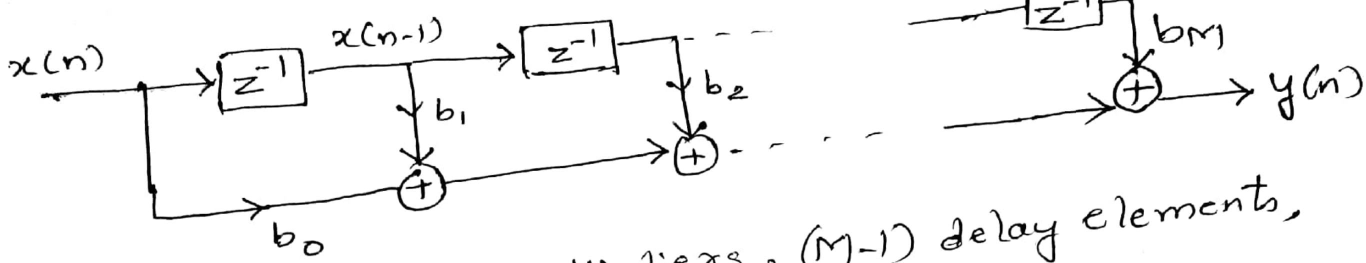
FIR realization:

Direct-form:

$$H(z) = \sum_{K=0}^{M} b_K z^{-K}.$$

$$Y(z) = (b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_M z^{-M}) X(z)$$

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots b_M x(n-M).$$



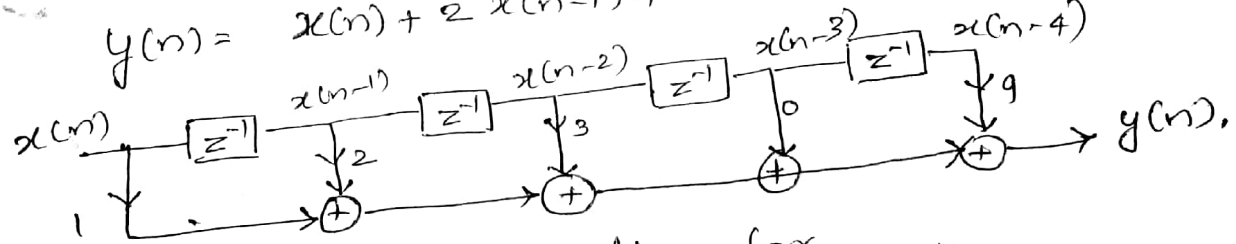
. It needs M multipliers, (M-1) delay elements, (M-1) adders.

Cascade form:

$$H(z) = H_1(z) H_2(z).$$

28. Realize FIR filter using direct form and cascade form. $H(z) = 1 + 2z^{-1} + 3z^{-2} + 9z^{-4}$.

sol: $y(n) = x(n) + 2x(n-1) + 3x(n-2) + 9x(n-4)$



29. Obtain cascade realization for

$$H(z) = 1 + \frac{3}{4} z^{-1} + \frac{17}{8} z^{-2} + \frac{3}{4} z^{-3} + z^{-4}$$

$$= \left(1 + \frac{1}{2} z^{-1} + z^{-2}\right)\left(1 + \frac{1}{4} z^{-1} + z^{-2}\right)$$

$$= H_1(z) . H_2(z)$$

$$H_1(z) = 1 + \frac{1}{2} z^{-1} + z^{-2}$$

$$\Rightarrow y_1(n) = x_1(n) + \frac{1}{2} x_1(n-1) + x_1(n-2)$$

$$H_2(z) = 1 + \frac{1}{4} z^{-1} + z^{-2}$$

$$y_2(n) = x_2(n) + \frac{1}{4} x_2(n-1) + x_2(n-2)$$

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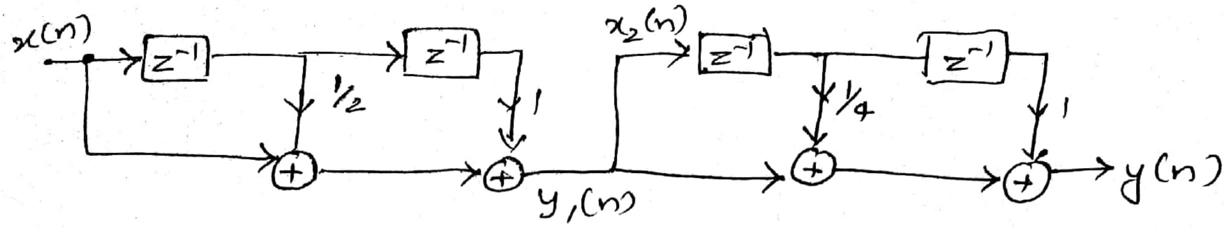

Inverse Z-transform methods:

1) Partial fraction expansion
2) Long division method
3) Convolution method
4) Residue method.

1. Partial fraction method:

First make the given funⁿ proper. i.e. the power in the num should be less than the power in the den.

28. Find the inverse z-transform of

$$X(z) = \frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}} \quad a)\ |z| > 2.\ b)\ |z| < 1 \quad c)\ 1 < |z| < 2$$

Sol:

$$X(z) = \frac{z(z+3)}{z^2+3z+2} = \frac{z(z+3)}{(z+1)(z+2)}$$

$$\Rightarrow \frac{X(z)}{z} = \frac{(z+3)}{(z+1)(z+2)} = \frac{C_1}{z+1} + \frac{C_2}{z+2}$$

$$C_1 = (z+1)\frac{X(z)}{z}\Big|_{z=-1} = \frac{z+3}{z+2}\Big|_{z=-1} = 2$$

$$C_2 = (z+2)\frac{X(z)}{z}\Big|_{z=-2} = \frac{z+3}{z+1}\Big|_{z=-2} = -1$$

$$\therefore X(z) = \frac{2z}{z+1} - \frac{z}{z+2}$$

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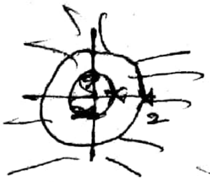
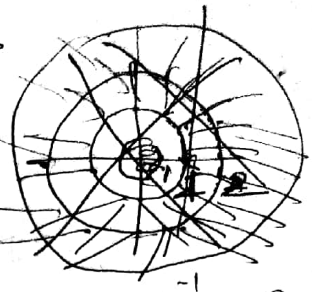
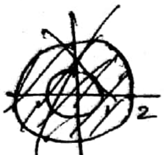




a) ROC $|z| > 2$

$\Rightarrow x(n) = 2\,u(n)(-1)^n - (-2)^n u(n)$.

b) ROC $|z| < 1$



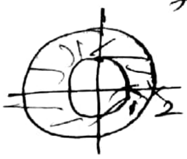
$\Rightarrow x(n) = -2(-1)^n u(-n-1) + (-2)^n u(-n-1)$

$\frac{z}{z-a} \overset{z^{-1}}{\longleftrightarrow} a^n u(n)$ for $|z| > a$.

$\frac{z}{z-a} \overset{z^{-1}}{\longleftrightarrow} -(a)^n u(-n-1)$ for $|z| < a$

c) ROC $1 < |z| < 2$

$\Rightarrow x(n) = \underbrace{2\,u(n)(-1)^n}_{\Downarrow} + \underbrace{(-2)^n u(-n-1)}_{\Downarrow}$



$|z| > 1$ — Causal

$|z| < 2$ — Non-causal

29. Determine the causal signal $x(n)$ whose z-transform

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol:-

$$X(z) = \frac{z^3}{(z-2)(z-1)^2}$$

$$\Rightarrow \frac{X(z)}{z} = \frac{z^2}{(z-2)(z-1)^2} = \frac{C_1}{z-2} + \frac{C_2}{z-1} + \frac{C_3}{(z-1)^2}$$

$$C_1 = (z-2)\frac{X(z)}{z}\Big|_{z=2} = \frac{z^2}{(z-1)^2}\Big|_{z=2} = \frac{4}{1}$$

$$C_2 = \frac{d}{dz}\left((z-1)^2 \frac{X(z)}{z}\right)\Big|_{z=1} = \frac{d}{dz}\left[\frac{\cancel{(z-1)^2}\, z^2}{(z-2)\cancel{(z-1)^2}}\right]_{z=1}$$

$$= \frac{(z-2)\times 2z - z^2 \times 1}{(z-2)^2}\Big|_{z=1} = \frac{z^2-4}{(z-2)^2}\Big|_{z=1} = -3$$

$$C_3 = (z-1)^2 \frac{X(z)}{z}\Big|_{z=1} = \frac{z^2}{z-2}\Big|_{z=1} = \frac{1}{-1} = -1$$

$$\Rightarrow \frac{X(z)}{z} = \frac{4}{z-2} - \frac{3}{z-1} - \frac{1}{(z-1)^2}$$

$$= 4(2)^n u(n) - 3(1)^n u(n) - (1)^n n\,u(n).$$

$\left[\because \frac{z}{(z-a)^2} \overset{z^{-1}}{\longleftrightarrow} n a^{n-1} u(n) \text{ for } |z| > a\right]$

2. Long-division method:

i) Express the numerator & denominator in the descending powers of 'z' if $|z| > a$ & then perform division

ii) Express them in ascending powers of z if $|z| < a$ & then perform division.

30. Find the inverse z-transform of $X(z) = \frac{1}{1-az^{-1}}$, $|z| > a$.

Sol:- $X(z) = \frac{z}{z-a}$ (descending powers)

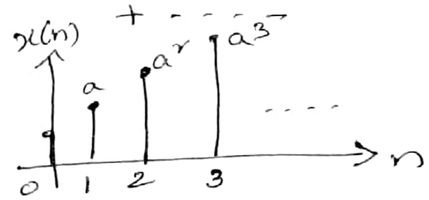
$$\begin{array}{l} \quad 1 + \frac{a}{z} + \frac{a^2}{z^2} + \cdots \\ z-a \,\big)\, z \\ \quad \cancel{z} - a \\ \quad \underline{- \quad +} \\ \qquad a \\ \qquad \cancel{a} - \frac{a^2}{z} \\ \qquad \underline{- \quad +} \\ \qquad\quad \frac{a^2}{z} \\ \qquad\quad \cancel{\frac{a^2}{z}} - \frac{a^3}{z^2} \\ \qquad\quad \underline{- \quad +} \\ \qquad\qquad \frac{a^3}{z^2} \end{array}$$

$\Rightarrow X(z) = 1 + \frac{a}{z} + \frac{a^2}{z^2} + \frac{a^3}{z^3} + \cdots$

Inverse z-transform

$x(n) = \delta(n) + a\delta(n-1) + a^2\delta(n-2) + \cdots$

$\therefore x(n) = a^n u(n)$ for $n \geq 0$



31. Find I Z-T of $X(z) = \frac{z}{2z^2-3z+1}$, $|z| < \frac{1}{2}$

Sol:- $X(z) = \frac{z}{1-3z+2z^2}$ (ascending powers)

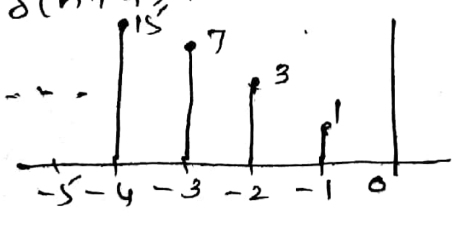
$$\begin{array}{l} \qquad z + 3z^2 + 7z^3 + 15z^4 + \cdots \\ 1-3z+2z^2 \,\big)\, z \\ \qquad \cancel{z} - 3z^2 + 2z^3 \\ \qquad \underline{- \quad + \quad -} \\ \qquad\quad 3z^2 - 2z^3 \\ \qquad\quad \cancel{3z^2} - 9z^3 + 6z^4 \\ \qquad\quad \underline{- \quad + \quad -} \\ \qquad\qquad 7z^3 - 6z^4 \\ \qquad\qquad \cancel{7z^3} - 21z^4 + 14z^5 \\ \qquad\qquad \underline{- \quad + \quad -} \\ \qquad\qquad\quad 15z^4 - 14z^5 \\ \qquad\qquad\quad \cancel{15z^4} - 45z^5 + 30z^6 \\ \qquad\qquad\quad \underline{- \quad + \quad -} \\ \qquad\qquad\qquad 31z^5 - 30z^6 \end{array}$$

$X(z) = z + 3z^2 + 7z^3 + 15z^4 \cdots$

$(\because \delta(n-1) = z^{-1}$

$\delta(n+1) = z)$

$x(n) = \delta(n+1) + 3\delta(n+2) + 7\delta(n+3) + 15\delta(n+4) + \cdots$



$x(n) = (2^{-n} - 1)\, u(-n-1)$

$n < 0$

3. Convolution method :

i) Represent $X(z)$ in the form of $X_1(z)\,X_2(z)$

ii) Obtain $x_1(n)$ & $x_2(n)$

iii) Find convolution of $x_1(n)$ & $x_2(n)$ & obtain $x(n)$.

32. Find I-Z Transform of $X(z) = \frac{z^2}{(z-2)(z-3)}$ using convolution method.

Sol:-

i) $X(z) = \frac{z^2}{(z-2)(z-3)}$; let $X_1(z) = \frac{z}{z-2}$; $X_2(z) = \frac{z}{z-3}$

ii) $x_1(n) = 2^n u(n)$ $\quad x_2(n) = 3^n u(n)$

iii) $x_1(n) * x_2(n) = 2^n u(n) * 3^n u(n)$

$$= \sum_{k=-\infty}^{\infty} (2)^k u(k) \cdot 3^{n-k} u(n-k)$$

$$= (3)^n \sum_{k=-\infty}^{\infty} \left(\frac{2}{3}\right)^k u(k)\, u(n-k)$$

$$= (3)^n \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k \times 1.$$

$$= 3^n \left[1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots\right]$$

$\boxed{\frac{a(1-r^{n+1})}{1-r} \quad (r<1)}$

$$\therefore x(n) = 3^n \left[\frac{\left(1-\left(\frac{2}{3}\right)^{n+1}\right)}{1-\frac{2}{3}}\right] = 3^{n+1}\left[1-\left(\frac{2}{3}\right)^{n+1}\right] u(n).$$

33. $X(z) = \frac{z^2}{\left(z-\frac{1}{4}\right)^2}$; $X_1(z) = \frac{z}{z-1/4}$; $X_2(z) = \frac{z}{z-1/4}$

$$\Rightarrow x_1(n) = x_2(n) = \left(\frac{1}{4}\right)^n u(n).$$

$$x(n) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{4}\right)^k u(k) \left(\frac{1}{4}\right)^{n-k} u(n-k)$$

$$= \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} 1 . = \left(\frac{1}{4}\right)^n (n+1)$$

$$\therefore x(n) = \left(\frac{1}{4}\right)^n (n+1)\, u(n)$$

4. Residue method:

The residues of the poles are found by using cauchy residue theorem.

The residues of the poles are given by

$x(n) = \sum$ [residues of $X(z) z^{n-1}$ at the poles inside the circle]

$|z| > r$

for $|z| < r$ apply $-$ sign.

34. Use residue method to find the IZT of

$X(z) = \dfrac{z}{(z-2)(z-3)}$ a) $|z| < 2$ b) $2 < |z| < 3$. c) $|z| > 3$

sol:- a) As ROC $|z| < 2$ the sequence is non-causal.

The two poles are at $z = 3$ & 2.

$x(n) = -\sum$ residues of $X(z) z^{n-1}$ at poles $z = 2$ & $z = 3$.

$$= -\left[\frac{\cancel{(z-2)} \cdot z (z^{n-1})}{\cancel{(z-2)} (z-3)} \Bigg|_{z=2} + \frac{\cancel{(z-3)}\, z (z^{n-1})}{(z-2)\cancel{(z-3)}} \Bigg|_{z=3} \right]$$

$= -[-(2)^n + (3)^n] = 2^n - 3^n$.

for $n<0$ (As the sequence is non-causal)

$x(n) = (2^n - 3^n)\, u(-n-1)$

b) $x(n) = -\sum$ residues of $X(z) z^{n-1}$ at pole $z = 3$ for $n<0$

$= +\sum$ residues of $X(z) z^{n-1}$ at pole $z = 2$ for $n \geq 0$

for $n<0$ $x(n) = -3^n$

$n \geq 0$ $x(n) = -2^n$

$\therefore x(n) = -2^n u(n) - 3^n u(-n-1)$.

c) $x(n) = -2^n u(n) + 3^n u(n)$.

Assignment - 1

1. For a causal discrete time Invariant signal $x(n) = (\frac{1}{3})^n u(n) - \frac{1}{5}(\frac{1}{3})^{n-1} u(n-1)$, when the o/p is $y(n) = (\frac{1}{2})^n u(n)$. cal. the transfer funⁿ, impulse response & check the stability.

2. ~~Det~~ Find ~~the~~ the response of the System described by its Differen~~tial~~ce eqⁿ ~~using unilateral z-transform~~
$$y(n) - \frac{7}{12} y(n-1) + \frac{1}{2} y(n-2) = x(n)$$ with initial Condⁿs
$y(-1) = 2$, $y(-2) = 4$ & $x(n) = (\frac{1}{5})^n u(n)$.

3. The step response of an LTI system is $(\frac{1}{3})^{n-2} u(n+2)$ find its impulse response.

4) Realize the filter transfer funⁿ in direct form
$$H(z) = (1 - z^{-1}) / (1 + 2z^{-1} - 3z^{-2})$$

5. Check the stability of $H(z) = \dfrac{z^2 - z + 1}{z^2 - z + 1/2}$

6. Discuss classification of Signals and systems.

7. Obtain parallel form realization for $H(z) = \dfrac{(1+z^{-1})(1+3z^{-1})}{(1+\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})(1+\frac{1}{4}z^{-1})}$

8. obtain the Cascade form realization for the system.
$$y(n) = -0.1 y(n-1) + 0.2 y(n-2) + 3x(n) + 3.6 x(n-1) + 0.6 x(n-2).$$

9) Discuss the concept of stability and Causality with examples.

10) Check the following filters for time-invariant, causal and linear.
i) $y(n) = (n-1) x^2(n+1)$
ii) $y(n) = n^2 x(n-2)$.