I. Microwave Transmission Lines-II

Introduction:- Micro waves are electromagnetic waves whose frequencies range from 1GHz to 1000GHz. These are so called since they defined in terms of their wavelengths in the sense that micro refers to tiny ness In some way all frequencies above 1000GHz are called as microwave frequencies.

Electro Magnetic spectrum:-

VLF – ultra Low frequency

ELF – Extra low

VF/SLF – voice Frequency

VLF – very low

LF – Low

MF – Medium

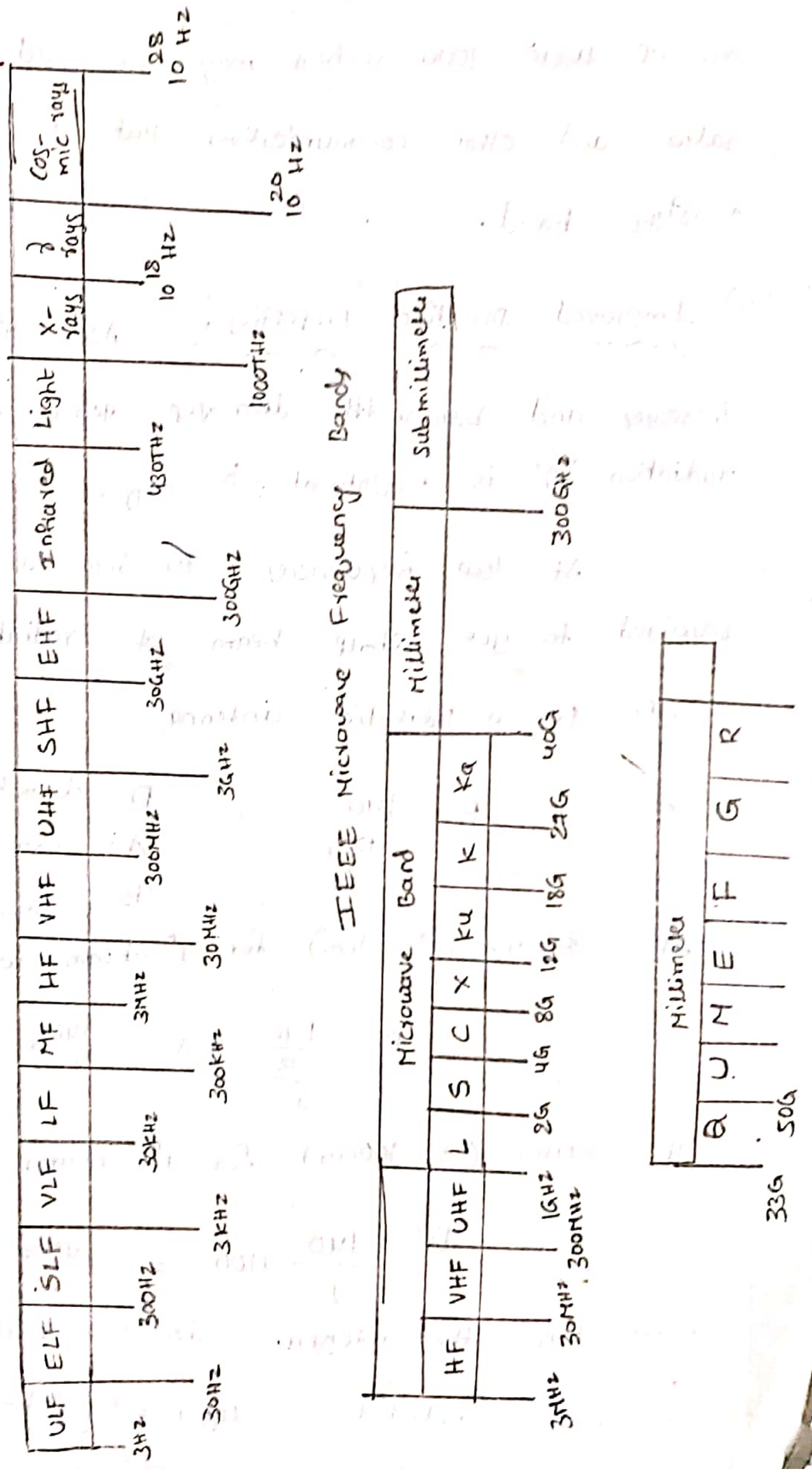
HF – High

VHF – very High

UHF – ultra High

SHF – Super High

EHF – Extreme High



Millimeter band has

Q – 33G – 50GHz

U – 40GHz – 60GHz

M – 50GHz – 75GHz

E – 60G – 90G

F – 90G – 140G

G – 140G – 220G

R – 220G – 300G

Submillimeter is >330GHz

Advantages of Microwaves :-

1) Large Bandwidth :- Micro waves have large band width (10^9 - 10^{12}...)
The advantage of large bandwidths is that the frequency range information channels will be a small percentage of the carrier frequency and more information can be transmitted in these frequencies.

→ It consists of 1000 sections of the frequency band $0-10^9$ Hz so one of these 1000 sections may be used to transmit all the TV, radio and other communications that is presently transmitted by the $0-10^9$ Hz band.

(2) Improved Directive Properties :- As frequency increases, directivity increases and beamwidth decreases. Hence the ~~Bandwidth~~ Beamwidth of radiation 'θ' is proportional to λ/D.

At low frequencies the size of antenna is large if it is required to get sharp beam of radiation.

Ex:- For a Parabolic antenna,

$$B = \frac{140^\circ}{D/\lambda}$$

D = diameter of antenna in cm
λ = wavelength in cm
B = Beamwidth in degrees

At 30GHz (λ = 1cm) for 1° beam width

$$D = \frac{140}{B} \times \lambda = \frac{140}{1} \times 1 = 140\text{cm}$$

At 300MHz (λ = 100cm) for 1° beam width

$$D = \frac{140}{1} \times 100 = 14000 = 14\text{m}$$

From this the antenna size is small for microwave frequencies

Power radiated $= \mu_0 \pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2$

l = length

I_o = ac current carried.

As frequency ↑, λ ↓ hence power radiated and gain increases

As gain (power) is $\propto \frac{1}{\lambda^2}$, high gain achieves at microwave frequencies.

→ High gain & directive antennas can be designed easily at these frequencies but it is impracticable at lower frequency bands.

(3) Less Fading and Reliability :- Fading effect due to variation in the transmission medium is more effective at low frequency. Due to "Line of Sight (LOS)" propagation and high frequencies there is less fading effect and hence microwave communication is more reliable

(4) Low power Requirements.

(5) Transparency property of Microwave :- The frequencies ranging from 300MHz – 10GHz are capable of freely propagating through the ionized layers surrounding the earth as well as through the atmosphere. The presence of such a transparent window in a microwave band facilitates the study of microwave radiation from the sun and stars in radio astronomical research of space. It also makes it possible for duplex communication and exchange of information between ground stations and space vehicles.

Disadvantages of Microwaves :-

① Transit time effects :- Electron transit angle between electrodes is the major limiting factor in the application of conventional microwave devices. The electron transit angle is

$$\theta_g = \omega\tau = \frac{\omega d}{v_0}$$

$$\tau = \frac{d}{v_0}$$

d = distance between anode and cathode

v_0 = velocity of electron.

At low frequencies transit time is negligible but at microwave frequencies this ~~effect~~ transit time is large compared to time period of the signal. This effect results in the reduction of the operating efficiency of the tube drastically.

(2) Line-of-Sight Technology (LOS) :- Due to this LOS technology microwaves does not pass through objects like mountains, buildings, airplanes. This drawback limits microwave communication systems to line-of-sight operating distances.

(3) Lead Inductance and Inter-Electrode Capacitance Effects :- The performance of microwave devices are adversely affected by parasitic circuit reactance because the circuit capacitances between tube electrodes and the circuit inductance of the lead wire are too large for a microwave resonant circuit. As the frequency increases the real part of the input admittance increases significantly to cause a over load of the input circuit and there by reduce the efficiency of conventional tubes and making them inoperable.

(4) Higher Radiation losses in transmission lines and connecting wires.

(5) Lumped elements such as Resistors, Capacitors, and inductors can't be used.

Communication :- Microwave is used in broadcasting and telecommunication transmissions.

Remote Sensing :- RADAR & SONAR. RADAR is used to illuminate an object by using a transmitter and receiver to detect its position and velocity. Radiometry is also one of the Remote Sensing Applications.

Heating :- We uses Microwave oven to bake and cook food. Microwave oven works based on the vibration of electrons present in the food particles.

Medical Science :- Microwaves heating properties are also used in Medical Science. It is used in diagnosis and various therapies. These are also used in drying, precooking and moisture leveling.

Electronic Warfare :- ECM (Electronic Counter Measure), ECCM (Electronic Counter counter measure) systems, spread spectrum systems.

-> Identifying objects (or) personnel by non-contact method.

-> Drying Machines

-> Rubber industry | plastics | chemical | forest product industries.

-> Mining | public works, breaking rock, tunnel boring, drying | breaking up concrete, breaking up coal seams, curing of cement.

-> Drying inks, drying | sterlising grains, drying | sterlising Pharmaceuticals, drying textiles leather, tobacco, Power transmission.

(6) As the wavelength is smaller, the attenuation during adver weather conditions is higher.

Applications of Microwaves:- Microwaves have a broad range of applications in modern technology.

Applications of Microwave Bands:-

L-Band:- wavelength in free space is 15cm to 30cm. These are used in navigations, GSM mobile phones, and in military applications. They can be used to measure soil moisture, of rain forests

S-Band:- wavelength is 7.5cm to 15cm. These are used in navigation beacons, optical communications, and wireless networks.

C-Band:- wavelength is 3.75cm to 7.5cm. These band microwaves penetrate clouds, smoke, dust, snow and rain to reveal the earth's surface. These are used in long distance radio telecommunications.

X-Band:- wavelength is 25mm to 37.5mm. These are used in satellite communications, broad band communications, radars, space communications and amateur radio signals.

KU-Band:- The wavelength is 16.7mm to 25mm. KU refers to Quartz-under. These are used in satellite communications for measuring the changes in the energy of the microwave pulses and they can determine speed and direction of wind near costal areas.

K-Band and Ka-Band:- For K-Band wavelength is 11.3mm to 16.7mm. For Ka band 5mm to 11.3mm. These are used in satellite communications, astronomical observations, and radars.

V-Band:- This band stays for a high attenuation. Radar applications are limited for a short range applications. Wavelength 4mm to 6mm.

...aveguides:- transporting wave...

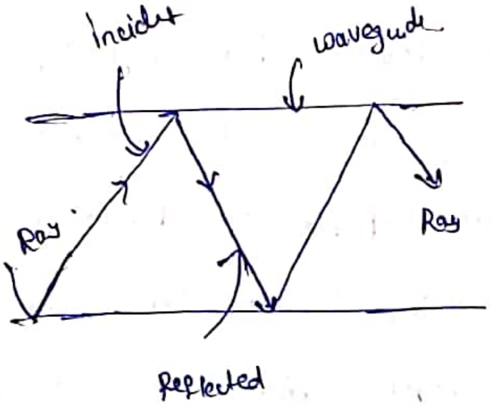
Waveguides :- waveguides are basically a device ("a guide") for transporting electromagnetic energy from one region to another. Waveguides are hallow metal tube. They are capable of directing power precisely to where it is needed, can handle large amounts of power and function as a high pass filter. waveguides are used at microwave frequencies.

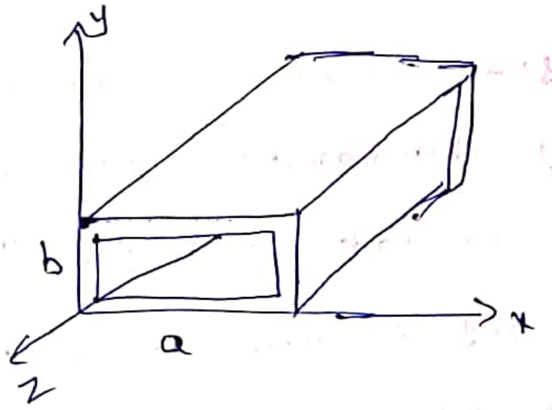
The tube wall provides distributed inductance, while the empty space between the tube walls provides distributed capacitance. Waveguides can't be used as electrical transmission lines for frequencies below cutoff frequency. The cutoff frequency increases as dimensions of the waveguide decrease.

Properties of waveguides :-

1) Electromagnetic transmission line generally used with in the building.
2) By construction, it is a hallow metal tube.
3) Inner walls of waveguide are coated with gold (or) silver for smooth finish.
4) Acts as high pass filter.
5) useful for frequencies above 1GHz.
6) Rectangular and circular waveguides are popular.
7) TEM mode does not exist in waveguides.
8) Mode of propagation is either TE (or) TM.

Rectangular waveguide :- A rectangular waveguide is a hallow metallic tube with rectangular crossection. a & b are the crossectional dimensions of the waveguide.





Modes of propagation:- In lossless waveguides the modes may be classified as

1) TEM mode
2) TE mode
3) TM mode

TEM Mode:-
All electromagnetic waves consists of electric and magnetic fields perpendicular to each other but propagating in the direction of travel. Along the length of normal transmission line, both electric and magnetic fields are pependicular (transverse) to the direction of wave travel. This is known as principal mode (or) TEM (Transverse Electric & Magnetic) mode.

This Mode of wave propagation can exist only where there are two conductors.

TEM mode Can't exist in the waveguides.

Consider a wave travelling in z-direction. For TEM wave

$E_z = 0$ & $H_z = 0$.

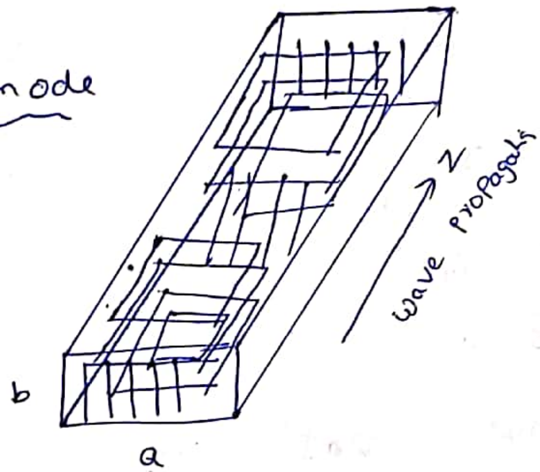
TE Mode:- When the EM wave propagates down a hallow tube such as waveguides, only one of the fields - either electric (or) magnet will actually be transverse to the waves propagation.

In TE modes the electric field is always transverse ($\perp r$) to the direction of propagation.

In TE mode no electric line is in direction of propagation that is $E_z = 0$, $H_z \neq 0$.

TM:-

TE mode

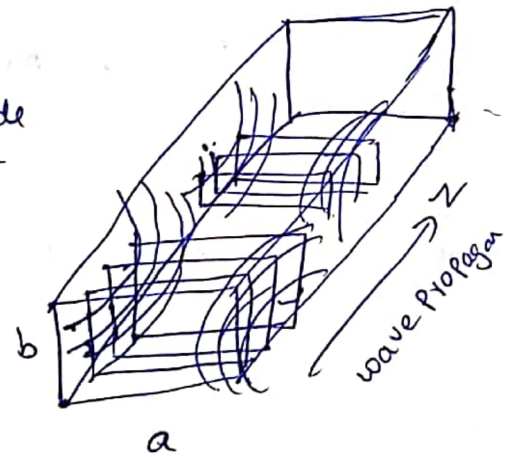


closed loops represents magnetic fields and open line represents electric field

So TE mode Electric field lines are $\perp r$ to wave propagation direction (z-direction).

TM mode :- The magnetic field always transverse to the direction of propagation and is called Transverse Magnetic (TM) mode. In TM modes $E_z \neq 0$; $H_z = 0$

TM mode



closed loops are magnetic lines and curved lines are electric field lines.

These are the TE & TM modes in Rectangular waveguides.

$\frac{\partial H_z}{\partial x_2}$

Maxwell's Equations:- These equations govern the principles guiding and propagation of electromagnetic energy and provide the foundations of all EM phenomena and their applications.

	Standard Form (Time Domain)	Time-Harmonic (Freq-Domain)
Faraday's law	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\nabla \times \vec{E} = -j\omega \vec{B}$
Ampere's law	$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t}$	$\nabla \times \vec{H} = (\sigma + j\omega\epsilon)\vec{E}$
Gauss' law	$\nabla \cdot \vec{D} = \rho_v$	$\nabla \cdot \vec{D} = \rho_v$
	$\nabla \cdot \vec{B} = 0$	$\nabla \cdot \vec{B} = 0$
and also we have	$\nabla \cdot \vec{J} = -\partial \rho / \partial t$	$\nabla \cdot \vec{J} = -j\omega\rho$

Wave equations for EM waves:-

Statement :-

$$\nabla^2 H = -\omega^2 \mu \epsilon H$$
$$\nabla^2 E = -\omega^2 \mu \epsilon E$$

are called as wave equations (or) Helmholtz wave equations

Proof :-

we know

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

For perfect dielectric $\sigma = 0$ (conductivity)

$$B = \mu H \qquad \frac{\partial}{\partial t} = j\omega$$

$\nabla \cdot D = \rho_v = 0$

$\nabla \cdot \epsilon E = 0 \Rightarrow \nabla \cdot E = 0$

$$\nabla \times \vec{E} = -j\omega\mu \vec{H}$$
$$\nabla \times \vec{H} = j\omega \epsilon \vec{E}$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times (-j\omega\mu \vec{H}) = -j\omega\mu (\nabla \times \vec{H}) = -j\omega\mu (j\omega\epsilon \vec{E})$$

$$\nabla \cdot \vec{E} - \nabla^2 \vec{E} = \omega^2 \mu \epsilon \vec{E} \quad \text{--- (1)}$$

$\nabla \cdot D = \rho_v = 0$ & $\nabla \cdot \epsilon \vec{E} = \epsilon (\nabla \cdot \vec{E}) = 0 \Rightarrow \nabla \cdot \vec{E} = 0$

nciples
provide

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$$ for ~~TE~~ wave.

$$\nabla \times \vec{H} = j\omega\epsilon \vec{E}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \vec{H_x} & \vec{H_y} & \vec{H_z} \end{vmatrix} = j\omega\epsilon\left[\hat{i}\vec{E_x} + \hat{j}\vec{E_y} + \hat{k}\vec{E_z}\right]$$

Replacin $\frac{\partial}{\partial z} = -\gamma$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ \vec{H_x} & \vec{H_y} & \vec{H_z} \end{vmatrix} = j\omega\epsilon\left[\hat{i}\vec{E_x} + \hat{j}\vec{E_y} + \hat{k}\vec{E_z}\right]$$

$$\frac{\partial \vec{H_z}}{\partial y} + \gamma \vec{H_y} = j\omega\epsilon \vec{E_x} \quad — ①$$

$$\frac{\partial \vec{H_z}}{\partial x} + \gamma \vec{H_x} = -j\omega\epsilon \vec{E_y} \quad — ②$$

$$\frac{\partial \vec{H_y}}{\partial x} - \frac{\partial \vec{H_x}}{\partial y} = j\omega\epsilon \vec{E_z} \quad — ③$$

lly

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \vec{E_x} & \vec{E_y} & \vec{E_z} \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ \vec{E_x} & \vec{E_y} & \vec{E_z} \end{vmatrix} = -j\omega\mu\left[\hat{i}\vec{H_x} + \hat{j}\vec{H_y} + \hat{k}\vec{H_z}\right]$$

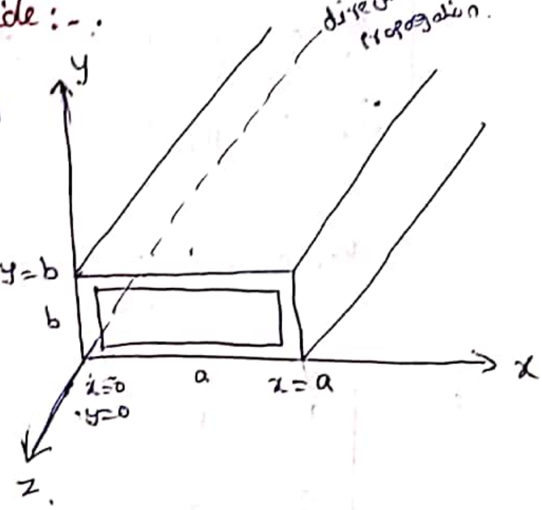
$$\frac{\partial \vec{E_z}}{\partial y} + \gamma \vec{E_y} = -j\omega\mu\vec{E_x} \quad — ④ \;; \quad \frac{\partial \vec{E_z}}{\partial x} + \gamma \vec{E_x} = j\omega\mu\vec{H_y} \quad — ⑤$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu \vec{H_z} \quad — ⑥$$

$$-\nabla^2 \vec{E} = \omega^2 \mu \varepsilon \vec{E}$$

$$\boxed{\begin{array}{c} \nabla^2 \vec{E} = -\omega^2 \mu \varepsilon \vec{E} \\ \hline \nabla^2 \vec{H} = -\omega^2 \mu \varepsilon \vec{H} \end{array}}$$

Propogation of waves in Rectangular waveguide :-



Consider a rectangular waveguide situated in the rectangular co-ordinate system with its breadth along x-axis, width along y-axis and the wave is propagating along the z-direction. waveguide is filled with air as dielectric.

The wave equations for TE and TM waves are

$$\nabla^2 \vec{H}_z = -\omega^2 \mu \varepsilon \vec{H}_z \quad \text{for TE wave } (E_z = 0)$$

$$\nabla^2 \vec{E}_z = -\omega^2 \mu \varepsilon \vec{E}_z \quad \text{for TM wave } (H_z = 0)$$

$$\nabla^2 \vec{E_z} = \frac{\partial^2 \vec{E}_z}{\partial x^2} + \frac{\partial^2 \vec{E}_z}{\partial y^2} + \frac{\partial^2 \vec{E}_z}{\partial z^2} = -\omega^2 \mu \varepsilon \vec{E}_z$$

since the wave is propagating in the z-direction we have the operator

$$\frac{\partial^2}{\partial z^2} = \gamma^2 \qquad \left\{ \because \frac{\partial}{\partial z} = -\gamma \right\}$$

$$\frac{\partial^2 \vec{E}_z}{\partial x^2} + \frac{\partial^2 \vec{E}_z}{\partial y^2} + \gamma^2 \vec{E}_z = -\omega^2 \mu \varepsilon \vec{E}_z$$

$$\frac{\partial^2 \vec{E}_z}{\partial x^2} + \frac{\partial^2 \vec{E}_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) \vec{E}_z = 0$$

Let $\gamma^2 + \omega^2 \mu \varepsilon = h^2$

$$\frac{\partial^2 \bar{E}_z}{\partial x^2} + \frac{\partial^2 \bar{E}_z}{\partial y^2} + h^2 \vec{E}_z = 0 \quad \text{for TM wave}$$

Combining equations ① and ⑤

From equation ⑤ we can write

$$H_y = \frac{1}{j\omega\mu}\frac{\partial \vec{E_z}}{\partial x} + \frac{\gamma}{j\omega\mu}\vec{E_x}$$

Substitute above equation in equation ① then

$$\frac{\partial \vec{H_z}}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial \vec{E_z}}{\partial x} + \frac{\gamma^2}{j\omega\mu}\vec{E_x} = j\omega\varepsilon\vec{E_x}$$

(a)

$$j\omega\mu\left\{\vec{E_x}\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right]\right\} = \gamma\frac{\partial \vec{E_z}}{\partial x} + j\omega\mu\frac{\partial \vec{H_z}}{\partial y}$$

$$\vec{E_x}\left(-\omega^2\mu\varepsilon - \gamma^2\right) = \gamma\frac{\partial \vec{E_z}}{\partial x} + j\omega\mu\frac{\partial \vec{H_z}}{\partial y}$$

$$\vec{E_x}\left(-h^2\right) = \gamma\frac{\partial \vec{E_z}}{\partial x} + j\omega\mu\frac{\partial \vec{H_z}}{\partial y} \qquad \left[\because \gamma^2 + \omega^2\mu\varepsilon = h^2\right]$$

$$\vec{E_x} = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial \vec{H_z}}{\partial y}$$

Ily

$$\vec{E_y} = -\frac{\gamma}{h^2}\frac{\partial \vec{E_z}}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\vec{H_x} = -\frac{\gamma}{h^2}\frac{\partial \vec{H_z}}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial \vec{E_z}}{\partial y}$$

$$\vec{H_y} = -\frac{\gamma}{h^2}\frac{\partial \vec{H_z}}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial \vec{E_z}}{\partial x}$$

These equations give a general relationship for field components with in a waveguide.

Propagation of TM waves in Rectangular Waveguides:-

For a TM wave $H_z = 0$, $E_z \neq 0$

The wave equation of a TM wave is

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \quad \text{—} (1)$$

Let us assume $E_z = XY$

X is a function of 'x' & Y is a function of 'y'

X & Y are independent variables

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 (XY)}{\partial x^2} = Y \frac{\partial^2 X}{\partial x^2}$$

$$\frac{\partial^2 E_z}{\partial y^2} = \frac{\partial^2 (XY)}{\partial y^2} = X \frac{\partial^2 Y}{\partial y^2}$$

Now

(1) ⇒ $$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

Let $\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -B^2$ —(2) $\quad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -A^2$ —(3)

$-B^2 - A^2 + h^2 = 0 \Rightarrow h^2 = A^2 + B^2$

Equations (2) & (3) are 2nd order differential equations, the solutions are

$X = C_1 \cos Bx + C_2 \sin Bx$

$Y = C_3 \cos Ay + C_4 \sin Ay$

C_1, C_2, C_3, C_4 are constants can be calculated by using boundary conditions.

The complete solution is $E_z = XY$

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \quad \text{—} (4)$$

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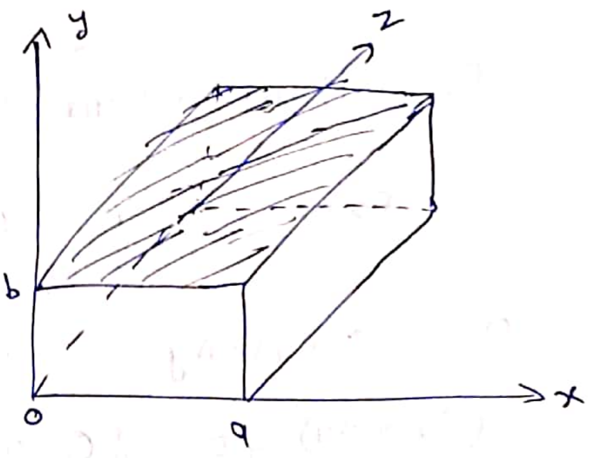
Boundary conditions:- The entire surface of a rectangular waveguide acts as a short circuit (or) ground for electric field, $E_z = 0$ all along the boundary walls of the waveguide



1st boundary condition:-

Bottom Plane (or) wall

we know $E_z = 0$, all along the bottom wall

that is $E_z = 0$ at $y = 0$ $\forall$ $x \to 0$ to a

2nd boundary condition:

Left side Plane (or) wall

$E_z = 0$ at $x = 0$ $\forall$ $y \to 0$ to b.

3rd boundary condition:- TOP Plane (or) wall

$E_z = 0$ at $y = b$ $\forall$ $x \to 0$ to a

4th boundary condition:- Right side plane (or) wall

$E_z = 0$ at $x = a$ $\forall$ $y \to 0$ to b.

Substituting 1st boundary in eqn 4 then

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$$

$E_z = 0$ at $y = 0$ $\forall$ $x \to 0$ to a

$$0 = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos 0 + C_4 \sin 0)$$

$$0 = C_3 (C_1 \cos Bx + C_2 \sin Bx)$$

$x \to 0$ to a so $(C_1 \cos Bx + C_2 \sin Bx) \neq 0$

then $\boxed{C_3 = 0}$

Now

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_4 \sin Ay) \quad \text{--- (5)}$$

Now apply ② nd boundary in eq^n ⑤ then

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_4 \sin Ay)$$

$E_z = 0$ at $x = 0$ $y \to 0$ to b

$$0 = C_1 C_4 \sin Ay$$

$(C_4 \sin Ay) \neq 0$, $\boxed{C_1 = 0}$

Now

$$E_z = C_2 C_4 \sin Bx \sin Ay \quad —— ⑥$$

Sub ③ rd condition in ⑥ then

$E_z = 0$, at $y = b$ $\forall$ $x \to 0$ to a

⑥ ⇒ $0 = C_2 C_4 \sin Bx \sin Ab$

$\sin Bx \neq 0$, $C_4 \neq 0$, $C_2 \neq 0$ $\sin Ab = 0$

$\sin Ab = 0$

Ab = a multiple of π

$Ab = n\pi$

$$\boxed{A = \frac{n\pi}{b}}$$

Sub ④ th condition in eq^n ⑥

$E_z = 0$ at $x = a$, $\forall$ $y \to 0$ to b

⑥ ⇒ $0 = C_2 C_4 \sin Ba \sin Ay$

$C_2 \neq 0$, $C_4 \neq 0$ $\sin Ay \neq 0$

$\sin Ba = 0$

$Ba = m\pi$

$$\boxed{B = \frac{m\pi}{a}}$$

The Complete Solution using eqn 6 is

$$E_z = C_2 C_4 \sin Bx \sin Ay$$

$$E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \quad e^{-\gamma z} e^{j\omega t}$$

$e^{-\gamma z}$ = Propagation along z-direction.

$e^{j\omega t}$ = Sinusoidal Variation w.r.t 't'.

Let $C = C_2 C_4$

$$\boxed{E_z = C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \quad e^{j\omega t - \gamma z}}$$

We know

$$\vec{E_x} = -\frac{\gamma}{h^2}\frac{\partial \vec{E_z}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial \vec{H_z}}{\partial y}$$

For TM wave $\vec{H_z} = 0$

$$\vec{E_x} = \frac{-\gamma}{h^2}\frac{\partial \vec{E_z}}{\partial x}$$

$$= \frac{-\gamma}{h^2} C \frac{\partial}{\partial x}\left(\sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\right) e^{j\omega t - \gamma z}$$

$$\boxed{\vec{E_x} = \frac{-\gamma}{h^2} C \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \quad e^{j\omega t - \gamma z}}$$

$$\vec{E_y} = \frac{-\gamma}{h^2}\frac{\partial \vec{E_z}}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial \vec{H_z}}{\partial x} = \frac{-\gamma}{h^2}\frac{\partial \vec{E_z}}{\partial y}$$

$$\boxed{\vec{E_y} = \frac{-\gamma}{h^2} C \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \ e^{j\omega t - \gamma z}}$$

$$\vec{H_x} = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial \vec{E_z}}{\partial y} = \frac{j\omega\varepsilon}{h^2}\frac{\partial \vec{E_z}}{\partial y}$$

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$$\boxed{\vec{H_x} = \frac{j\omega\varepsilon}{h^2}\, c\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\; e^{j\omega t - \gamma z}}$$

$$\vec{H_y} = \frac{-\gamma}{h^2}\frac{\partial \vec{H_z}}{\partial y} + \frac{j\omega\varepsilon}{h^2}\frac{\partial \vec{E_z}}{\partial x}$$

$$\boxed{\vec{H_y} = \frac{j\omega\varepsilon}{h^2}\, c\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\; e^{j\omega t - \gamma z}}$$

TM modes in Rectangular waveguides :-

Depending on the value of m and n we have various modes in TM waves.

Various TM_{mn} modes

① TM_{00} mode :- m=0 & n=0.

If m=0, n=0 are substituted in E_x, E_y, H_x, H_y all will become '0'

so TM_{00} mode does not exist.

② TM_{01} :- m=0, n=1 $E_x=0, E_y=0, H_x=0, H_y=0$ so TM_{01} also Can't exist.

③ TM_{10} ! m=1, n=0 also Can't exist.

④ TM_{11} :- m=1, n=1

TM_{11} exists and for all higher values of m and n, the components exist.

Means all higher order modes exists.

Propagation of TE waves in Rectangular Waveguides :— The TE_{mn} modes in rectangular waveguides are characterized by $E_z = 0$.

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \epsilon H_z$$

$$\left\{ \because \frac{\partial^2}{\partial z^2} = \gamma^2 \right\}$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu \epsilon H_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$$

Assume a solution $H_z = XY$

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

Let

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -B^2, \quad \& \quad \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = -A^2$$

$$-B^2 - A^2 + h^2 = 0$$

$$h^2 = A^2 + B^2$$

solving for X & Y by separation of variable method

$$X = C_1 \cos Bx + C_2 \sin Bx$$

$$Y = C_3 \cos Ay + C_4 \sin Ay$$

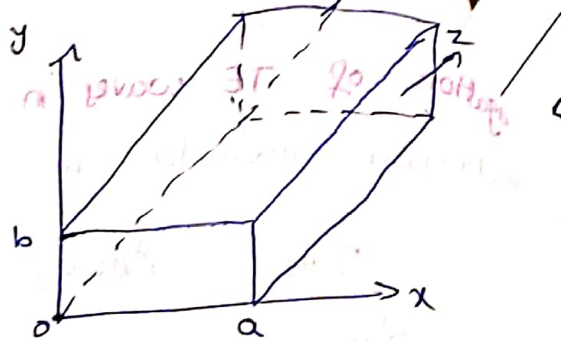
$$H_z = XY$$

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \quad \text{—— (A)}$$

C_1, C_2, C_3, C_4 are constants which can be evaluated by applying boundary conditions.

Boundary conditions :— For TE waves also we have four boundary conditions. For TE waves $E_z = 0$ but we have components along x & y

directions



$E_x = 0$ all along bottom and top walls of the waveguide

$E_y = 0$ all along left and right walls of the waveguide.

① $E_x = 0$ at $y = 0$ $\forall$ $x \to 0$ to a (bottom wall)

② $E_x = 0$ at $y = b$ $\forall$ $x \to 0$ to a (top wall)

③ $E_y = 0$ at $x = 0$ $\forall$ $y \to 0$ to b (left side wall)

④ $E_y = 0$ at $x = a$ $\forall$ $y \to 0$ to b (Right wall)

Apply ① in eqn Ⓐ ~~then~~

we know $\vec{E_x} = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} - \frac{\gamma^2}{h^2}\frac{\partial E_z}{\partial x}$

Since $E_z = 0$ for TE

$$\vec{E_x} = \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[(C_1\cos Bx + C_2\sin Bx)(C_4\sin Ay + C_3\cos Ay)\right]$$

$$\vec{E_x} = \frac{-j\omega\mu}{h^2}(C_1\cos Bx + C_2\sin Bx)(-AC_3\sin Ay + AC_4\cos Ay)$$

Now substitute ① boundary

$$0 = \frac{-j\omega\mu}{h^2}(C_1\cos Bx + C_2\sin Bx)(AC_4)$$

$(C_1\cos Bx + C_2\sin Bx) \neq 0$ $\quad A \neq 0$ $\quad \boxed{C_4 = 0}$

$$H_z = (C_1\cos Bx + C_2\sin Bx)(C_3\cos Ay)$$

we know $E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$

$E_z = 0$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}(C_1\cos Bx + C_2\sin Bx)(C_3\cos Ay)$$

$$E_y = \frac{j\omega\mu}{h^2}\left(-BC_1 \sin Bx + BC_2 \cos Bx\right) C_3 \cos Ay$$

substitute ③rd boundary $E_y = 0$ at $x=0$

$$0 = \frac{j\omega\mu}{h^2}\left(BC_2\right) C_3 \cos Ay$$

$\cos Ay \neq 0$, $B \neq 0$ $C_3 \neq 0$

$$\boxed{C_2 = 0}$$

$$H_z = C_1 C_3 \cos Bx \cos Ay$$

we know $E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$

$E_z = 0$

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}\left(C_1 C_3 \cos Bx \cos Ay\right)$$

$$E_x = \frac{j\omega\mu}{h^2} C_1 C_3 A \cos Bx \sin Ay$$

Substitute 2nd boundary $E_x = 0$ at $y = b$

$$0 = \frac{j\omega\mu}{h^2} C_1 C_3 A \cos Bx \sin Ab$$

$\cos Bx \neq 0$, $C_1 \neq 0$, $C_3 \neq 0$

$\sin Ab = 0 \Rightarrow Ab = n\pi \Rightarrow A = \frac{n\pi}{b}$

we know

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$E_z = 0$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}\left(C_1C_3 \cos Bx \cos Ay\right)$$

$$E_y = \frac{-j\omega\mu}{h^2} C_1C_3 B \sin Bx \cos Ay$$

Substitute ④th boundary

$$0 = \frac{-j\omega\mu}{h^2} C_1 C_3 B \sin Ba \cos Ay$$

$\cancel{x=}a$

$$C_1 \neq 0, \quad C_3 \neq 0 \quad \cos Ay \neq 0$$

$$\sin Ba = 0$$

$$Ba = m\pi$$

$$B = \frac{m\pi}{a}$$

$$H_z = C_1C_3 \cancel{A} \cos Bx \cos Ay$$

$$H_z = C_1C_3 \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$C_1C_3 = C = \text{Constant}$$

$$\boxed{H_z = C \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{(j\omega t - \gamma z)}}$$

Field Components

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}\left(C \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}\right)$$

$$\boxed{E_x = \frac{j\omega\mu}{h^2} C \cdot \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \, e^{(j\omega t - \gamma z)}}$$

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$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\boxed{E_y = \frac{-j\omega\mu}{h^2} c\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}}$$

||ly

$$H_x = \frac{\gamma}{h^2} c\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}$$

$$H_y = \frac{+\gamma}{h^2} c\left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x \; \sin\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}$$

TE modes in Rectangular waveguides :-

(a) TE_{00} mode :- $m=0, n=0$

$E_x = 0, \; E_y = 0, \; H_x = 0, \; H_y = 0$

All field components vanish, this mode can't exist.

(b) TE_{01} mode :- $m=0, n=1$

$E_y = 0, \; H_x = 0$ E_x & H_y exist

(c) TE_{10} mode :- $m=1, n=0$

$E_x = 0, \; H_y = 0$, E_y & H_x exist.

(d) TE_{11} mode :- $m=1, n=1$ This also exist.

Characteristics of a Rectangular waveguide :-

Cutoff frequency (waveguide as a HPF) :-

we know that a Rectangular waveguide acts as a High Pass Filter (HPF)

we know $h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$ —

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$ —

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} = \alpha + j\beta$$

At lower frequencies, $\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$ then 'γ' becomes real and positive and equal to attenuation constant 'α' means the wave is completely attenuated and no phase change. So the wave can't propagate.

At higher frequencies

$$\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

then γ becomes imaginary. There will be phase change 'β' and hence the wave propagates. At the transition $\gamma = 0$ and the propagation just starts.

Hence R.W.G acts as a H.P.F.

Cutoff frequency :- The frequency at which γ becomes '0' is called cutoff frequency (or) threshold frequency f_c.

At $f = f_c$, $\gamma = 0$, $\omega = 2\pi f = 2\pi f_c = \omega_c$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

at $f = f_c \Rightarrow \omega = \omega_c$, $\gamma = 0$ then

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2\mu\varepsilon$$

$$\omega_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega_c^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = 2\pi f_c$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

we know $c = \frac{1}{\sqrt{\mu\epsilon}}$ for free space

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \frac{c\pi}{2\pi}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

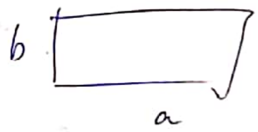
$$\boxed{f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

TE_{10}

$a =$

$b =$

Cutoff wavelength (λ_c):-



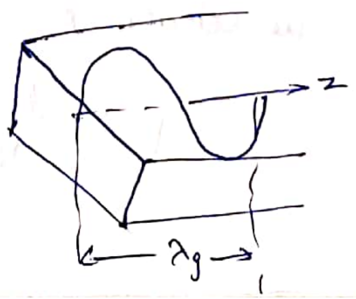
$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}}$$

$$\lambda_{c_{mn}} = \frac{2}{\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right)^{1/2}} = \frac{2}{\sqrt{\frac{m^2b^2+n^2a^2}{a^2b^2}}}$$

$$\boxed{\lambda_{c_{m,n}} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}}$$

All the wavelengths greater than $\lambda_{c_{m,n}}$ are attenuated and those less than λ_c are allowed to propagate inside the waveguide.

Guide wavelength (λ_g):- It is defined as the distance travelled by the wave in order to undergo a phase shift of 2π radians.



$$\lambda_g = \frac{2\pi}{\beta}$$

wave length in the free space $= \lambda_0$

Cutoff wavelength $= \lambda_c$

guide wave length $= \lambda_g$

$\lambda_0, \lambda_c, \lambda_g$ are related as $\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$$

If $\lambda_0 << \lambda_c$ the denominator is nearly '1' and $\lambda_g = \lambda_0$

When $\lambda_0 = \lambda_c \Rightarrow \lambda_g = \infty$

$\lambda_0 > \lambda_c \Rightarrow \lambda_g$ is imaginary means no propagation.

Phase velocity (V_p):- It is the velocity at which the phase of the wave changes along the length of the guide. It is more than (or) equal to the velocity of wave in free space.

$$V_p = \lambda_g f = \frac{2\pi f \cdot \lambda_g}{2\pi} = \frac{2\pi f}{2\pi/\lambda_g}$$

$$\boxed{V_p = \frac{\omega}{\beta}} \quad \boxed{V_p = \frac{c}{\sin\theta}} \quad \begin{array}{l}\omega = 2\pi f \\ \beta = 2\pi/\lambda_g\end{array}$$

Group velocity (V_g):- It is defined as the velocity with which the wave (energy) propagates through the waveguide and is always less than the velocity of wave in free space.

$$\boxed{V_g = \frac{d\omega}{d\beta}} \quad \boxed{V_g = c\sin\theta}$$

Expression for Vp :-

$$V_p = \frac{\omega}{\beta}$$

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \alpha + j\beta$$

For wave Propagation $\alpha = 0$

$$\gamma = j\beta \Rightarrow \gamma^2 = j^2\beta^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$

At $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2\mu\varepsilon$$

$$\omega_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = -\beta^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$

$$-\beta^2 = \omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon$$

$$\beta^2 = \omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon = \mu\varepsilon\left[\omega^2 - \omega_c^2\right]$$

$$\beta = \sqrt{\mu\varepsilon}\sqrt{\omega^2 - \omega_c^2}$$

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu\varepsilon}\sqrt{\omega^2 - \omega_c^2}} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{\left(1 - \frac{\omega_c^2}{\omega^2}\right)}}$$

$$\boxed{V_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

we know $f = c/\lambda_0$ & $f_c = c/\lambda_c$

$$\frac{f_c}{f} = \frac{c/\lambda_c}{c/\lambda_0} = \lambda_0/\lambda_c$$

$$\boxed{V_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

Expression for V_g :-

$$V_g = \frac{d\omega}{d\beta}$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\beta = \sqrt{\mu\varepsilon(\omega^2 - \omega_c^2)}$$

$$\frac{d\beta}{d\omega} = \frac{1}{\cancel{2}\sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}}(\cancel{2}\omega\mu\varepsilon) = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$V_g = \frac{d\omega}{d\beta} = \frac{\sqrt{1-(f_c/f)^2}}{\sqrt{\mu\varepsilon}}$$

$$V_g = c\sqrt{1-(f_c/f)^2} \qquad \left\{\frac{1}{\sqrt{\mu\varepsilon}} = c\right\}$$

$$\boxed{V_g = c\cdot\sqrt{1-(\lambda_0/\lambda_c)^2}} \qquad \left\{\because \frac{f_c}{f} = \lambda_0/\lambda_c\right\}$$

Product of V_p & V_g :-

$$V_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \qquad V_g = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$V_p \cdot V_g = \frac{c}{\cancel{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}} \cdot c\cancel{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\boxed{V_p \cdot V_g = c^2}$$

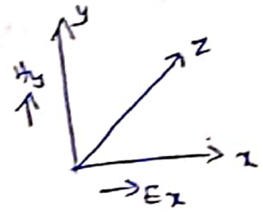
Relation between λ_g, λ_0 & λ_c :- $V_p = \lambda_g \cdot f = \frac{\lambda_g}{\lambda_0} \cdot c$

$$V_p = \frac{\cancel{c}}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \Rightarrow \frac{\lambda_g}{\lambda_0}\cdot\cancel{c}$$

$$\boxed{\lambda_g = \lambda_0 / \sqrt{1-(\lambda_0/\lambda_c)^2}}$$

Wave impedance (or) characteristic impedance :- It is defined as the ratio of the strength of electric field in one transverse direction to the strength of the magnetic field along the other transeverse direction

$$Z = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$



(or)

$$Z_z = \frac{\sqrt{E_x^2 + E_y^2}}{\sqrt{H_x^2 + H_y^2}}$$

(1) $$Z_{TM} = \frac{E_x}{H_y} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

For TM wave $H_z = 0$ and $\gamma = j\beta$

$$Z_{TM} = \frac{\cancel{+}\frac{\gamma}{\cancel{h^2}}\cancel{\frac{\partial E_z}{\partial x}}}{\cancel{+}\frac{j\omega\varepsilon}{\cancel{h^2}}\cancel{\frac{\partial E_z}{\partial x}}} = \frac{\gamma}{j\omega\varepsilon} = \frac{\cancel{j}\beta}{\cancel{j}\omega\varepsilon}$$

$$Z_{TM} = \frac{\beta}{\omega\varepsilon}$$

we know $\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$

$$Z_{TM} = \frac{\sqrt{\omega^2 - \omega_c^2}\sqrt{\mu\varepsilon}}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\boxed{Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - (\lambda_0/\lambda_c)^2}}$$

$$\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

For air $= 120\pi = 377\Omega = \eta$

η is the intrinsic impedance of free space

$$Z_{TM} = \eta\sqrt{1-(\lambda_0/\lambda_c)^2}$$

λ_0 always less than λ_c for wave propagation $Z_{TM} < \eta$

(2)

$$Z_{TE} = \frac{E_x}{H_y} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

For TE $\quad E_z = 0 \quad \gamma = j\beta$

$$Z_{TE} = \frac{+\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{+\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y}} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{j\beta} = \frac{\omega\mu}{\beta}$$

$$Z_{TE} = \frac{\omega\mu}{\sqrt{\varepsilon\mu}\sqrt{\omega^2-\omega_c^2}} = \frac{\eta}{\sqrt{1-(\omega_c/\omega)^2}} = \frac{\eta}{\sqrt{1-(f_c/f)^2}}$$

$$\boxed{Z_{TE} = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$$

$Z_{TE} > \eta$ as $\lambda_0 < \lambda_c$ for wave propagation. This means wave impedance for a TE wave is always greater than free space impedance.

(3) For TEM waves between any lines the cutoff frequency is '0' and wave impedance for TEM wave is the free space impedance.

$$\boxed{Z_{TEM} = \eta}$$

→ When the waveguide has a dielectric other than air with dielectric constant ε_r, then the behaviour changes

For air

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For air dielectric

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

waveguide with dielectric constant ε_r

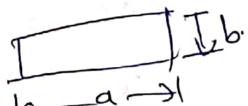
$$\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_r-(\lambda_0/\lambda_c)^2}} \qquad \lambda_{g\,dielectric} = \frac{\lambda_{g\,air}}{\sqrt{\varepsilon_r}}$$

Since $\varepsilon_r > 1$ $\lambda_{dielectric} < \lambda_{air}$ hence frequencies less than cutoff values can pass through the same guide.

Dominant mode:- It is the mode among TE/TM modes having lowest value of cutoff frequency and maximum cutoff wave-length in a given type of waveguide.

The walls of the waveguides are nearly perfect conductor so the boundary conditions require that the electric field be normal (perpendicular) to the waveguide walls and magnetic field must be tangential (parallel) to the wall. So only TE_{01}, TE_{10}, TE_{20} modes will exist but TM_{01}, TM_{10}, TM_{20} etc. modes does not exist.

Consider the dimensions of waveguide $a>b$



then $$\lambda_{c\,m,n} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}} \quad \text{for TE}$$

$m=0,\ n=1$

$$TE_{01} = \frac{2ab}{\sqrt{a^2}} = \frac{2\cancel{a}b}{\cancel{a}} = 2b$$

$m=1,\ n=0$

$$TE_{10} = \frac{2ab}{\sqrt{b^2}} = 2a = \text{maximum cut off wavelength}$$

$m=1, n=1$

$$TE_{11} = \frac{2ab}{\sqrt{a^2+b^2}}$$

So TE_{10} mode is having maximum cutoff wavelength

So it is the dominant mode.

Degenerate Modes:— Higher order modes having same cutoff frequency for a given waveguide called degenerate modes.

For Rectangular waveguide $TE_{mn} | TM_{mn}$ modes for which both $m \neq 0$, $n \neq 0$ will always be degenerate mode.

For a square waveguide in which $a=b$, all TE_{pq}, TE_{qp}, TM_{pq}, TM_{qp} modes are degenerate modes.

For ~~TE_{10}~~ ~~λ_{c10}~~ =

For TE mode $\lambda_{c_{10}} = \frac{2ab}{\sqrt{b^2}} = 2a$

For TE $\lambda_{c_{30}} = \frac{2ab}{\sqrt{9b^2}} = \frac{2a\cancel{b}}{3\cancel{b}} = \frac{2a}{3}$

If $a/b = 3 \Rightarrow a = 3b$

$\lambda_{c_{10}} = 2(3b) = 6b$

$\lambda_{c_{30}} = \frac{2(\cancel{3}b)}{\cancel{3}} = 2b$

$TE_{20} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$

$= \frac{2ab}{\sqrt{4b^2}} = \frac{2a\cancel{b}}{2\cancel{b}} = a = 2b$

$TE_{01} = \frac{2a\cancel{b}}{\cancel{a}} = 2b$

Ex:- Let $a=2b$ then TE_{20} & TE_{01} are degenerative modes.

TE_{20} $\lambda_c = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}} = a = 2b$

TE_{01} $\lambda_c = 2b$

Power Transmission in Rectangular waveguide :- The Power transmitted through a waveguide and the power loss in the guide walls can be calculated by means of Complex Poynting theorem. Assume that the waveguide is terminated in such a way that there is no reflection from the receiving end (or) that the waveguide is infinitely long as compared with its wavelength.

The Power transmitted P_{tr}, through a ~~wavelength~~ waveguide given by

$$P_{tr} = \int p \cdot ds = \int \frac{1}{2} (E \times H^*) \cdot ds$$

For a lossless dielectric, the time average power flow through a rectangular waveguide is

$$P_{tr} = \frac{1}{2Z_z} \int_a |E|^2 da = \frac{Z_z}{2} \int_a |H|^2 da.$$

where Z_z = impedance in z-direction.

$$Z_z = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

For TM_{mn} mode, the average power transmitted through a rectangular waveguide of dimensions 'a' and 'b' is

$$P_{tr} = \frac{1}{2\eta\sqrt{1-(\lambda_0/\lambda_c)^2}} \int_0^b \int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\, dy.$$

For TE_{mn} modes

$$Z_z = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

For TM_{mn} modes

$$Z_z = \eta\sqrt{1-(\lambda_0/\lambda_c)^2}$$

$$P_{tr} = \frac{\sqrt{1-(\lambda_0/\lambda_c)^2}}{2\eta} \int_0^b \int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\, dy.$$

$$P_{tr} = \frac{\sqrt{1-(f_c/f)^2}}{2\eta} \int_0^b \int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\, dy$$

$$P_{max} = 27\left(\frac{E_d}{f_{max}}\right)\sqrt{1-(f_c/f)^2}$$

E_d = dielectric strength of the insulating material in ~~microwave~~ waveguide in V/m

f_{max} = maximum frequency in MHz.

f = operating frequency

f_c = cut off frequency.

Power Loss in waveguide :- In a waveguide the wave is propagated by reflections from the walls. The tangential components of electric field and normal component of magnetic field develops losses in the walls. Due to this the average Power in the waveguide is dissipated.

The attenuation in the waveguide is α_g

$$\alpha_g = \frac{P_L}{2P_{tr}}$$

P_L = Power loss per unit length

P_{tr} = Power transmitted through waveguide.

Power Loss in waveguide:- As the electromagnetic wave propagates through a waveguide, the wave intensity gets attenuated because of losses in the waveguides. There are three types of losses in the waveguide which causes attenuation of transmitted signal.

1) Power loss in dielectric filling
2) Power loss in waveguide walls
3) Misaligned waveguide sections.

1) Power loss in dielectric filling:- when a guide is completely filled with a low loss dielectric ($\sigma \ll \omega\varepsilon$), the attenuation constant 'α' in the guide due to dielectric loss is

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

But instrinsic impedance in an unbounded dielectric $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

for free space

$$\alpha = \frac{\sigma\eta}{2}$$

The attenuation in waveguide α_g for TE_{mn} & TM_{mn} mode is

For TE_{mn}

$$\alpha_g = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\alpha}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}} \quad \text{Np/m}$$

For TM_{mn} mode

$$\alpha_g = \frac{\sigma\eta}{2}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \alpha\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$$

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2) Power Loss in waveguide walls :-

In waveguide the wave is propagated by reflections from walls. The tangential component of electric field and normal component magnetic field develops losses in the walls. Due to this the average power in the waveguide dissipated.

$$\alpha_g = \frac{P_L}{2P_{tr}}$$

P_L = Power loss per unit length.

P_{tr} = Power transmitted through the guide

(3) Misaligned waveguide sections:

When the waveguide sections are joined and if the joint is not proper (or) misaligned there will be loss due to reflection.

→ At frequencies below the cut-off frequency ($f < f_c$) the γ will be real and have only attenuation α.

$$\beta = \frac{2\pi}{\lambda g}$$

$$\lambda g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

$$\beta = \frac{2\pi}{\lambda}\sqrt{1-(f_c/f)^2} = j\frac{2\pi}{\lambda}\sqrt{\left(\frac{f_c}{f}\right)^2-1} = j\frac{2\pi f}{c}\sqrt{\left(\frac{f_c}{f}\right)^2-1}$$

$$= \frac{j2\pi}{c}\sqrt{\left(\frac{f_c^2-f^2}{\cancel{f^2}}\right)f^2} = \frac{j2\pi f_c}{c}\sqrt{1-\left(\frac{f}{f_c}\right)^2} = \frac{j2\pi f_c}{c}\sqrt{1-(f/f_c)^2}$$

$$\beta = j\alpha_c$$

$$\alpha_c = \frac{2\pi f_c}{c}\sqrt{1-\left(\frac{f}{f_c}\right)^2} \text{ Np/m} = \text{cutoff attenuation}$$

This is the stop band attenuation of the wave guide high pass filter.

$$\alpha_c = \frac{54.6}{\lambda_c}\sqrt{1-(f/f_c)^2} \quad \text{dB/ length.}$$

Conducting loss for TE_{mn} :-

$$\alpha_{cond} = \frac{2R_s}{b\eta}\left[\left(1+\frac{b}{a}\right)\left(\frac{f_c}{f}\right)^2 + \frac{b^2m^2+abn^2}{b^2m^2+a^2n^2}\left(1-\left(\frac{f_c}{f}\right)^2\right)\right] \quad NP/m$$

$$\eta = 377\sqrt{\frac{\mu_r}{\epsilon_r}} \qquad R_s = \text{sheet Resistivity in } \Omega/m^2$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \frac{1}{\sigma \delta_s} \qquad \delta_s = \text{skin depth} = \frac{1}{\sqrt{\pi f \mu_r \mu_0 \sigma}}$$

Conducting loss for TM_{mn} :-

$$\alpha_c = \frac{2R_s}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[\frac{b^3m^2+a^3n^2}{ab^2m^2+a^3n^2}\right] \quad NP/m.$$

Comparison of TE and TM modes :-

	TE Mode	TM mode
1.	Transverse electric mode	Transverse magnetic mode.
2.	Electric field component in z-direction is '0' $E_z=0$	Magnetic field component in z-direction is '0' $H_z=0$
3.	Electromagnetic energy is transmitted by H_z component	Electromagnetic energy is transmitted by E_z component.
4.	TE_{01} & TE_{10} modes exist	TM_{01} & TM_{10} does not exist
5.	cut off freq of dominant mode is $2a$ (TE_{10})	$\frac{2ab}{\sqrt{a^2+b^2}}$ (TM_{11})

6 1) A Rectangular waveguide is filled by dielectric material of $\varepsilon_r = 9$ has internal dimensions of 7cm x 3.5cm for dominant TE_{10} mode. Then 1) find f_c (2) V_p in the guide at freq of 2GHz (c) λ_g at 2GHz.

For dominant mode TE_{10} $m = 1, n = 0$

$a = 7 \times 10^{-2}\,m$ $\quad b = 3.5 \times 10^{-2}\,m$

$\varepsilon_r = 9$

(1)

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$\varepsilon = \varepsilon_0 \varepsilon_r \Rightarrow \varepsilon_0 = 8.854 \times 10^{-12}\ F/m$

$\varepsilon = 8.854 \times 10^{-12} \times 9$

$\mu_0 = 4\pi \times 10^{-7}\ H/m$

$$f_c = \frac{1}{2\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12} \times 9}} \sqrt{\left(\frac{1}{7 \times 10^{-2}}\right)^2 - 0}$$

$$\boxed{f_c = 713.79\ MHz}$$

(2)

$\lambda_c = c/f = \frac{3 \times 10^8}{2 \times 10^9}$ $\qquad f = 2GHz$

$\lambda_c = 0.15m$

$$V_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{0.15}{0.42}\right)^2}} = 3.21 \times 10^8\ m/s$$

$$\boxed{V_p = 3.21 \times 10^8\ m/s}$$

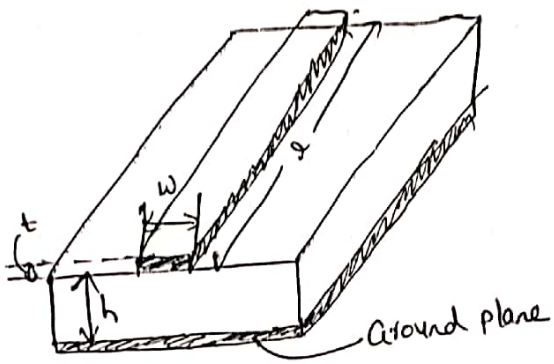
(3)

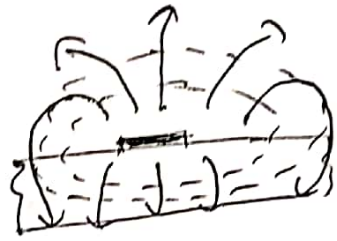
$$V_p = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$= \frac{0.15}{\sqrt{1 - \left(\frac{0.15}{0.42}\right)^2}} = 0.16m$$

$$\boxed{\lambda_g = 0.16m}$$

Introduction:- Microstrip line is one of the most popular types of planar transmission lines primarily because it can be fabricated by photolithographic processes and is easily miniaturized and integrated with both passive and active microwave devices. This is the major advantage of microstrip lines over striplines.





A conductor of width 'w' is printed on a thin grounded dielectric substrate of thickness 't' and relative permittivity ϵ_r. If the dielectric substrate were not present ($\epsilon_r = 1$), we have two wire line consisting of a flat strip conductor over a ground plane, embedded in a homogeneous medium (air). This constitute simple TEM line with $v_p = c$ & $\gamma = \beta = k_0$.

Microstrip ~~lines~~ has some of its field lines in the dielectric region between the strip conductor and the ground plane and some are in air region above the substrate. For this reason microstrip line cannot support a pure TEM wave. Since the phase velocity of TEM fields in the dielectric region is $v_p = c/\sqrt{\epsilon_r}$ but TEM fields in air is 'c' so a phase matching at dielectric-air interface is impossible.

8 Parameters of Microstrip lines:-

① Characteristic impedance:-

Microstrip lines are used extensively to interconnect high speed logic circuits in digital computers.

The characteristic impedance of a microstrip line is a function of the strip line width, thickness, the distance between the line and the ground plane, and homogeneous dielectric constant of the board material.

Z_0 is function of ratio of height to the width (w/h) of a line.

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_e}} \ln\left(\dfrac{8h}{w} + \dfrac{w}{4h}\right) \Omega & \text{for } \dfrac{w}{h} \leq 1 \\[2ex] \dfrac{120\pi}{\sqrt{\varepsilon_e}\left\{\dfrac{w}{h} + 1.393 + \dfrac{2}{3}\ln\left(\dfrac{w}{h} + 1.444\right)\right\}} \Omega & \text{for } \dfrac{w}{h} \geq 1 \end{cases}$$

For a given characteristic impedance Z_0 and dielectric constant ε_r, the $\frac{w}{h}$ ratio can be

$$\frac{w}{h} = \begin{cases} \dfrac{8e^{A}}{e^{2A} - 2} & \text{for } \dfrac{w}{h} < 2 \\[2ex] \dfrac{2}{\pi}\left[B - 1 - \ln(2B-1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left\{\ln(B-1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right\}\right] & \text{for } \dfrac{w}{h} > 2 \end{cases}$$

where

$$A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

$$B = 377\pi / 2Z_0\sqrt{\varepsilon_r}$$

② Effective dielectric constant ε_{re}:- In a microstrip line, part of the electric energy is stored in this conductor configuration is in the air and some is in dielectric, the effective dielectric constant for the wave on the line will lie some where between that of the air and that of the dielectric.

$$\varepsilon_e = \begin{cases} \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[\frac{1}{\sqrt{1+12(h/w)}} + 0.04\left(1-\left(\frac{w}{h}\right)^2\right)\right] & \text{for } \frac{w}{h} < 1 \\ \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[\frac{1}{\sqrt{1+12(h/w)}}\right] & \text{for } \frac{w}{h} \geq 1 \end{cases}$$

This calculation ignores strip thickness and frequency dispersion but their effects are usually negligible.

Because some fields are in air and some are in dielectric region the effective dielectric constant satisfies the relation

$$\boxed{1 < \varepsilon_e < \varepsilon_r}$$

③ wavelength :-

$$\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_e}} \quad (or) \quad \lambda_g = \frac{300}{f(GHz)\sqrt{\varepsilon_{re}}} \text{ mm}$$

ε_e = effective dielectric constant.

④ Phase velocity! -

$$v_p = c/\sqrt{\varepsilon_e}$$

Capacitance per unit length! :-

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{8h}{w} - \frac{w}{4h}\right)}$$

⑤ Propagation constant, -

$$\gamma = \beta = \omega\sqrt{\varepsilon_e \varepsilon_0 \mu_0} = \frac{2\pi}{\lambda_0} = \sqrt{\varepsilon_{re}}\,\frac{2\pi}{\lambda}$$

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⑥ Cutoff frequency :- In order to prevent higher order F the thickness of the microstrip substrate is restricted to 10% of a wavelength.

Examples :- ① 15 mil alumina is good up to 25 GHZ

② 4 mil GaAs 82 GHZ

③ 5 mil Quartz 121 GHZ.

Losses in Microstrip lines :- Assume that the dielectric substrate to be non-magnetic, the two factors that attenuate microwave signals are (i) non-zero conductivity of the dielectric

(2) Finite conductivity of both stripline and ground plane

These non ideal conductivities cause ohmic dissipation. Again we are having propagation losses and open ends of the strip behaves as antenna leading radiation losses.

These losses may be reduced by selecting the substrate of a high dielectric constant.

$$P = \frac{1}{2} V I^* = \frac{1}{2} (V^+ e^{-\alpha z} I^+ e^{-\alpha z})$$

$$= \frac{1}{2} \frac{|V^+|^2}{Z_0} e^{-2\alpha z} = P_0 e^{-2\alpha z}$$

$P_0 = \frac{|V^+|^2}{2Z_0}$ is the power at $z=0$

The attenuation constant $\alpha = \frac{-dP/dz}{2P(z)} = \alpha_d + \alpha_c$ —— ①

$\alpha_d =$ dielectric attenuation constant

$\alpha_c =$ conductor (or) ohmic attenuation constant

The gradient of power in the z-direction in ① is

$$-\frac{dP(z)}{dz} = -\frac{d}{dz}\,\frac{1}{2}(VI^*)$$

$$= P_c + P_d$$

σ = conductivity of the dielectric substrate.

$$\alpha_d \simeq \frac{P_d}{2P(z)} \quad NP/cm$$

$$\alpha_c \simeq \frac{P_c}{2P(z)} \quad NP/cm.$$

① Dielectric losses:- when the conductivity of a dielectric can't be neglected, the electric and magnetic fields in the dielectric are no longer in time phase.

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \quad NP/cm$$

This dielectric constant can be expressed in terms of dielectric loss tangent as

$$\tan\theta = \frac{\sigma}{\omega\varepsilon}$$

$$\alpha_d = \frac{\omega}{2}\sqrt{\mu\varepsilon}\,\tan\theta \quad NP/cm$$

Since the microstrip line is a nonmagnetic mixed dielectric system, the upper dielectric above the microstrip ribbon is air, in which no loss occur.

$$\boxed{\alpha_d = 4.34\,\frac{q\sigma}{\sqrt{\varepsilon_{re}}}\sqrt{\frac{\mu_0}{\varepsilon_0}}}$$

$\{\because 1NP = 8.686\,dB\}$

$$\alpha_d = 1.634\times10^3\,\frac{q\sigma}{\sqrt{\varepsilon_{re}}} \quad dB/cm$$

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q = dielectric filling factor

$$q = \frac{\epsilon_{re} - 1}{\epsilon_r - 1}$$

Attenuation constant per wavelength ay

$$\boxed{\alpha_d = 27.3 \left(\frac{q \epsilon_r}{\epsilon_{re}}\right) \frac{\tan\theta}{\lambda_g} \quad dB/\lambda_g}$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{re}}} = \frac{c}{f\sqrt{\epsilon_{re}}}$$

(2) Ohmic losses :- In a microstrip line over a low-loss dielectric substrate, the predominant sources of losses at micro wave frequencies are the non perfect conductors. The current dens in the conductors of a microstrip line is concentrated in the She that is nearly a skin depth thick inside the conductor surface and exposed to electric field (1)

Conductor attenuation (or) Ohmic losses may be calculated by finding the current distribution in the microstrip and ground conductor, and the power loss caused by these current densities.

The total distributed normalized resistance of microstrip structure R is the sum of the normalized resistance ~~and~~ of microstrip (R_1) and the normalized resistance of the ground plan (R_2).

$$R = R_1 + R_2$$

$$R_1 = \frac{R_s}{w}\left[\frac{0.95}{\pi} + \frac{0.132}{\pi}\frac{w}{h} - \frac{0.006}{\pi}\left(\frac{w}{h}\right)^2 + \frac{1}{\pi^2}\ln\frac{4\pi w}{t}\right]$$

$$R_2 = \frac{R_S}{W}\left[\frac{1}{1+3.8(h/w)+0.03(h/w)^2}\right]$$

For $w/h \leq 0.5$

$$R_1 = \frac{R_S}{W}\left(\frac{1}{\pi}+\frac{1}{\pi^2}\ln\left(\frac{4\pi W}{t}\right)\right)$$

$$R_2 = \frac{R_S}{W}\left[\frac{1}{1+3.8(h/w)+0.03(h/w)^2}\right]$$

where

R_S = Skin effect resistance

$$R_S = \sqrt{\frac{\omega\mu}{\sigma}}$$ (2)

t = thickness of the strip

The conductor attenuation (or) ohmic attenuation

$$\alpha_c = \frac{R_S}{\omega Z_0} \quad NP/m$$

ω = width of the strip

Z_0 = Characteristic impedance.

For thinner strips

$$\alpha_c = \frac{(R_1+R_2)}{2Z_0} \quad NP/m.$$

③ Radiation Losses :- A microstrip behaves as an open-circuited line since its dimensions are comparable to the wavelength of micro-waves used hence it ends behave as antenna and cause radiat loss

$$\frac{P_{rad}}{P_t} = \frac{R_r}{Z_0}$$

R_r = radiation resistance

$$R_r = 240\pi^2 \left(\frac{h}{\lambda_0}\right)^2 F \ \Omega$$

F = radiation factor

$$F = \frac{\varepsilon_e + 1}{\varepsilon_e} - \frac{\varepsilon_e - 1}{2(\varepsilon_e)^{3/2}} \ln\left(\frac{\sqrt{\varepsilon_e + 1}}{\sqrt{\varepsilon_e - 1}}\right)$$

The characteristic impedance increases, radiation losses decreases.

Quality Factor Q of Microstrip lines :- Many microwave integrated circuits require very high quality resonant circuits. The quality factor Q of a microstrip line is very high, but it is limited by the radiation losses of the substrates and with low dielectric constant.

$$\alpha_c = \frac{8.686 R_s}{Z_0 w} \ \text{dB/cm}$$ (3)

$$Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\varepsilon}} = \frac{377}{\sqrt{\varepsilon_r}} \frac{h}{w} \ \Omega$$

The wavelength $\lambda_g = \frac{30}{f\sqrt{\varepsilon_r}}$ cm

f in GHz.

Q_c is related to the conductor attenuation constant by

$$Q_c = \frac{27.3}{\alpha_c}$$

Where α_c is in dB/λ_g, Q_c of a wide line

$$Q_c = 39.5 \left(\frac{h}{R_s}\right) f_{GHz}$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = 2\pi\sqrt{\frac{f}{\sigma}} \ \Omega/\text{square}$$

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the quality factor Q_c of a wide strip line is

$$Q_c = 0.63h\sqrt{\sigma f_{GHz}}$$

σ is the conductivity of the dielectric substrate

For a cu $\sigma = 5.8 \times 10^7$ ℧/m

$$Q_{cu} = 4780h\sqrt{f_{GHz}}$$

For 25-mil alumina at 10GHz, the maximum Q_c achievable from wide microstrip lines is 954.

$$Q_d = \frac{27.3}{\alpha_d}$$

where α_d is in dB/λ_g

(4)

$$Q_d = \frac{\lambda_0}{\sqrt{\epsilon_{re}}\tan\theta} \simeq \frac{1}{\tan\theta}.$$

λ_0 is free space wavelength in cm.

$$\boxed{Q = \frac{Q_d Q_c}{Q_d + Q_c}}$$

Capacitance per unit length of a microstrip line:-

$$C = \begin{cases} \dfrac{2\pi\epsilon_0}{\ln\left(\dfrac{8h}{\omega} + \dfrac{\omega}{4h}\right)} & \dfrac{\omega}{h} \le 1 \\[2ex] \epsilon_0\left[\dfrac{\omega}{h} + 1.393 + 0.667\ln\left(\dfrac{\omega}{h} + 1.44\right)\right] & \dfrac{\omega}{h} > 1 \end{cases}$$