

## 5.2 Working Principle of a Transformer

A transformer is a static device which transfers electric energy from one circuit to another circuit without changing the frequency of the system. It works on the electromagnetic induction principle. According to this principle, an emf is induced in a coil if it links a changing flux.

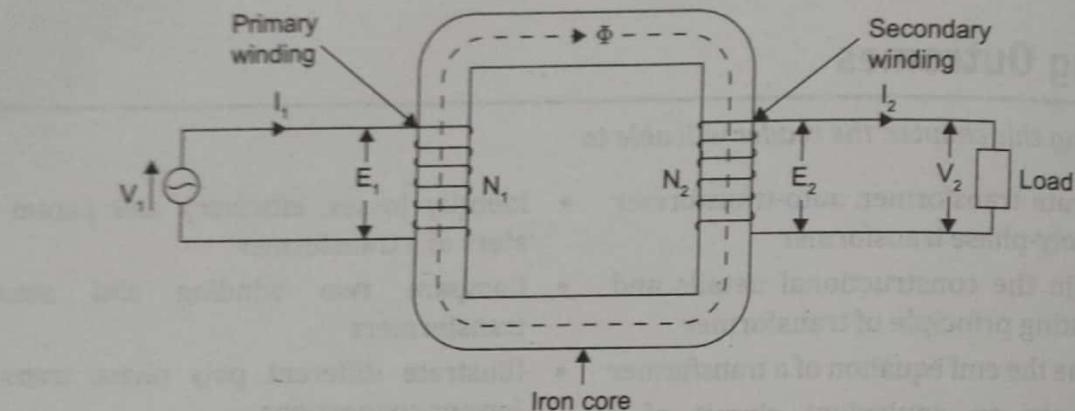


Figure (5.1): Arrangement of a simple transformer

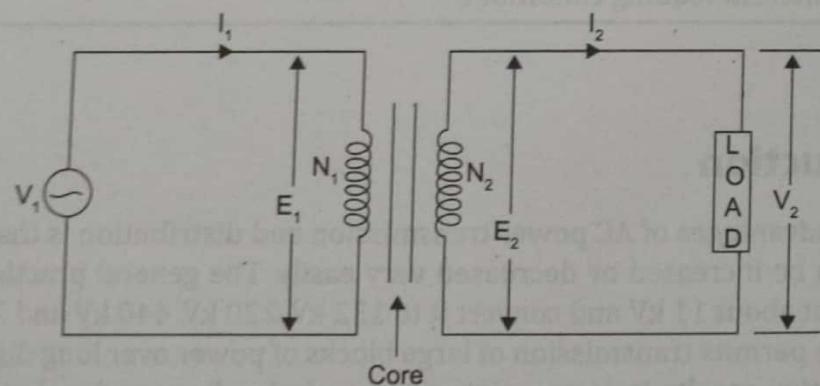


Figure (5.2): Symbolic representation

Consider two coils 1 and 2 wound on a simple magnetic circuit as shown in figure (5.1). These two coils are insulated from each other and there is no electrical connection, but magnetically coupled. The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil, in which it produces mutually induced emf according to Faraday's law of electromagnetic induction. If the secondary coil circuit is closed, a current flows in it and so an electrical energy transfers from the first coil to the second coil. Coil 1 which receives energy from the source of AC supply is called the primary coil or primary winding and coil 2, which is connected to the load and delivers energy to the load is called the secondary coil or secondary winding. The symbolic representation of a two winding transform is as shown in figure (5.2). The two vertical lines are used to represent magnetic core, which signify the tight magnetic coupling between the windings.

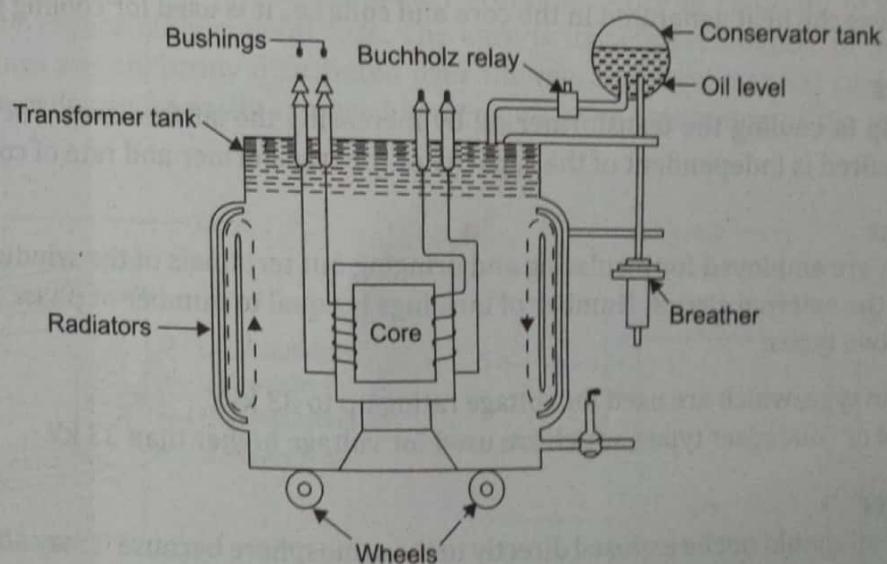
### 5.3 Transformer on DC Supply

A transformer cannot operate on DC supply. If rated DC supply is applied to the primary winding of a transformer, the flux produced in the transformer core will not vary but remain constant in magnitude and therefore, no emf will be induced in the primary and secondary windings. If there is no self induced emf in the primary winding, to oppose the applied voltage and since the resistance of the primary winding is low, therefore, a heavy current will flow through the primary winding which may result in damage of the winding. This is the reason that DC is never applied to the transformer.

### 5.4 Transformer Construction

A transformer is a static device and its construction is simple as there are no moving parts. The main parts of the transformer as shown in figure (5.3) are

- (i) Magnetic core
- (ii) Windings
- (iii) Conservator tank
- (iv) Transformer oil
- (v) Radiators
- (vi) Bushings
- (vii) Breather
- (viii) Container
- (ix) Buchholz relay



**Figure (5.3):** Various parts of a distribution transformer

#### (i) Magnetic Core

It is made of silicon steel or sheet steel with low reluctance. In addition to this, the sheets are laminated and are coated with an oxide layer to reduce the iron losses.

The thickness of lamination is 0.35 mm for 60 Hz operation and 5 mm for 25 Hz operation. It is the common part between the two windings. It helps both the windings to link through the magnetic flux.

### (ii) Windings

A conventional transformer has two windings. The winding which receives the electrical energy is called the primary winding and the winding which delivers the electrical energy to the load is known as secondary winding. The two windings are electrically separated but magnetically coupled through magnetic core. The windings are usually made of copper or aluminum with enamel coating. The low voltage winding carries high current so the wire diameter is more than the high voltage winding.

### (iii) Conservator Tank

It is an air tight cylindrical drum containing transformer oil, placed at the top of the transformer and connected to the transformer tank by a pipe. The main tank is completely filled with oil. The oil in the transformer tank expands due to increase in temperature and contracts when the temperature or load reduces. The function of the conservator tank is to take up contraction and expansion of oil without allowing it to come in contact with outside air and moisture.

### (iv) Transformer Oil

It is a mineral oil obtained by refining crude petroleum. It serves the following purposes.

- Acts as an insulating medium between windings and tank.
- Protects the tank from dirt and moisture.
- Carries away the heat generated in the core and coils i.e., it is used for cooling purpose.

### (v) Radiators

Radiators help in cooling the transformer oil by increasing the surface area. The number of radiators required is independent of the capacity of the transformer and rate of cooling.

### (vi) Bushings

The bushings are employed for insulating and bringing out terminals of the winding from the container to the external circuit. Number of bushings is equal to number of phases. These are generally of two types:

- (a) Porcelain type, which are used for voltage rating up to 33 kV.
- (b) Oil filled or condenser types, which are used for voltage higher than 33 kV.

### (vii) Breather

Transformer oil should not be exposed directly to the atmosphere because it may absorb moisture and dust from the environment and may lose its electrical properties in a very short time. To avoid this problem a breather is provided on the top of the conservator. It mainly consists of a silica gel. The silica gel absorbs the moisture content of air so that the oil contamination can be prevented. The silica gel which is blue in color turns pink when it absorbs moisture fully. It is replaced periodically as a routine maintenance.

**(viii) Container**

Cast iron or cast steel air tight containers are provided with radiators. The container contains the core windings and oil.

**(ix) Buchholz Relay**

It is a relay, provided in between conservator tank and transformer tank. It is used for protection of oil filled transformers from incipient faults below oil level.

It is a gas actuated relay. It operates on the generation of gases due to any internal fault of the transformer, to give an alarm and to disconnect the transformer from supply mains incase of severe internal fault.

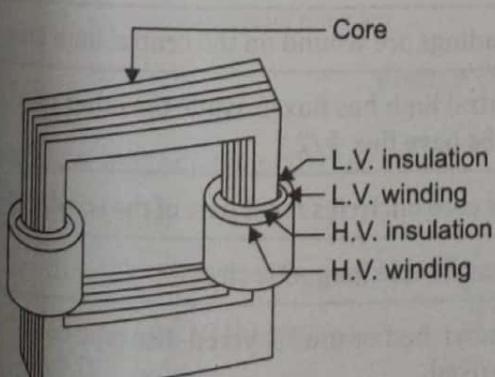
## 5.5 Types of Transformers

Depending upon the connection of winding, transformers are classified into two types, namely

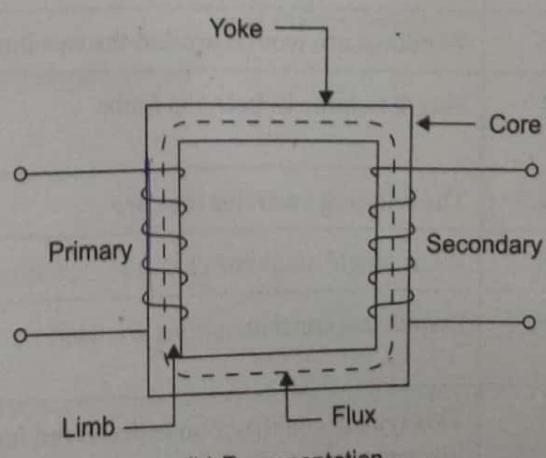
- (1) Core type transformer
- (2) Shell type transformer

### 5.5.1 Core Type Transformer

It has a single magnetic circuit. The core is rectangular, having two limbs. Windings are wound on the two limbs. In this type of transformer, windings are surrounded by the core as shown in figure (5.4). The two vertical portions are called limbs, each carry one half of the primary and one half of the secondary windings. Core is made up of silicon steel laminations to reduce eddy current loss. The core is usually rectangular or square type. As the windings are uniformly distributed over the two limbs the natural cooling is more effective. The coils can be easily removed by removing the lamination of the top yoke, for maintenance.



(a) Construction



(b) Representation

Figure (5.4): Core type transformer

### 5.5.2. Shell Type Transformer

It has a double magnetic circuit. The core has three limbs. Both the windings are wound on the center limb. In this type of a transformer the core is surrounded by the windings as shown in figure (5.5). Shell type transformers are preferred for high voltage applications. As the core is surrounded by the windings, the natural cooling does not exist. For removing any winding for maintenance, large numbers of laminations are required to be removed. The coils used are generally multilayer disc type or sandwich coils.

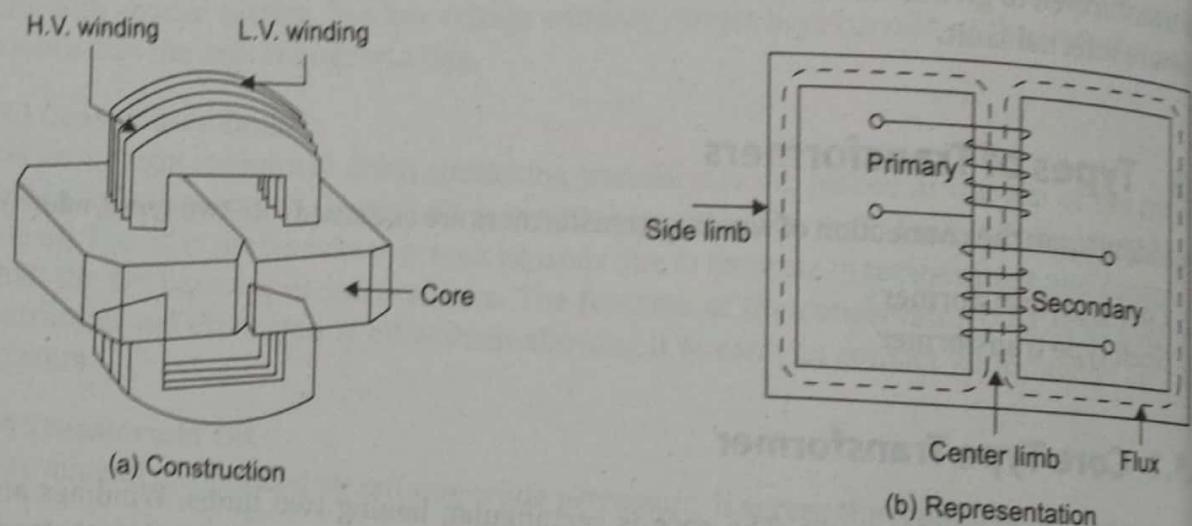


Figure (5.5): Shell type transformer

### 5.5.3 Comparison of Core & Shell Type Transformers

S. No.	Core Type Transformer	Shell Type Transformer
01	Two limbs for single phase transformer	Three limbs for single phase transformer
02	Windings are wound around the two limbs	Windings are wound on the central limb only
03	Flux $\Phi$ is same in both the limbs	Central limb has flux $\Phi$ , while the other two limbs have flux $\Phi/2$
04	The winding encircles the core	The core encircles most part of the windings
05	It has single magnetic circuit	It has double magnetic circuit
06	Cylindrical concentric coils are used	Sandwiched or multilayered disc type coils are used
07	This type of construction is preferred for low voltage transformers	This type of construction is preferred for very high voltage transformers
08	Rarely used	Widely used

## 5.6 Ideal Transformer

An ideal transformer is one that has

- (i) no winding resistance.
- (ii) no leakage flux i.e., the same flux links with both the windings.
- (iii) no iron losses (i.e., eddy current and hysteresis losses) in the core.

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical transformers have properties that come very close to an ideal transformer.

When the primary coil is connected to alternating voltage  $V_1$  a current  $I_m$  ( $I_0$ ) flows through it. Since the primary coil is purely inductive, the current  $I_m$  lags the applied voltage  $V_1$  by  $90^\circ$ . Due to current  $I_m$ , flux is produced in the primary winding and some of the flux is also linked with the secondary winding and hence emf's  $E_1$  and  $E_2$  are induced in the primary and secondary windings respectively. According to Lenz's law the induced emf opposes the cause producing it which is supply voltage  $V_1$ . Hence  $E_1$  is in antiphase with  $V_1$  but equal in magnitude. The induced emf  $E_2$  also opposes  $V_1$  hence in antiphase with  $V_1$  but its magnitude depends on  $N_2$ . Thus  $E_1$  and  $E_2$  are in phase. The phasor diagram of an ideal transformer is shown in the figure (5.6).

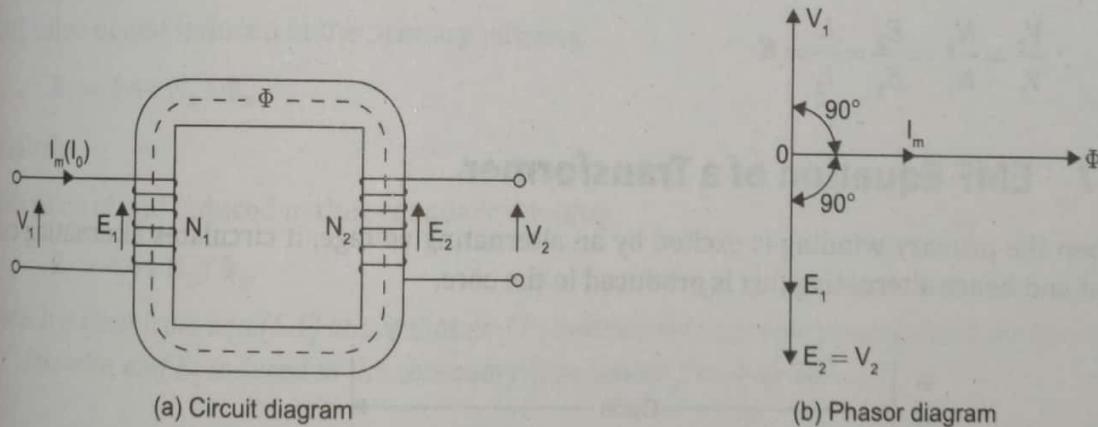


Figure (5.6): Ideal transformer

Here  $I_m$  = magnetizing current

Mathematically,

Let  $\Phi = \Phi_m \sin \omega t = \Phi_m \angle 0^\circ$  is taken as reference phasor

The induced emf in the primary

$$E_1 = -N_1 \frac{d\Phi}{dt} = -N_1 \frac{d}{dt}(\Phi_m \sin \omega t) = -N\Phi_m \omega \cos \omega t = N\Phi_m \omega \sin(\omega t - 90^\circ) = E_{1m} \sin(\omega t - 90^\circ)$$

$E_{1m} = N\Phi_m \omega$  = Max. value of induced e.m.f.

i.e.,  $E_1$  lags the flux,  $\Phi$  by an angle  $90^\circ$

$$V_1 = -E_1 = N_1 \frac{d\Phi}{dt} \quad \text{or} \quad V_1 = -E_1$$

i.e.  $V_1$  leads the flux,  $\Phi$  by an angle  $90^\circ$

The induced emf in the secondary

$$E_2 = -N_2 \frac{d\Phi}{dt} = -N_2 \frac{d}{dt}(\Phi_m \sin \omega t) = -N_2 \Phi_m \omega \cos \omega t = N_2 \Phi_m \omega \sin(\omega t - 90^\circ) = E_{2m} \sin(\omega t - 90^\circ)$$

$$E_{2m} = N_2 \Phi_m \omega = \text{Max. value of induced emf}$$

i.e.,  $E_2$  lags the flux,  $\Phi$  by an angle  $90^\circ$ , so  $E_1$  and  $E_2$  are phase.

In an ideal transformer there is no power loss

i.e., input VA = output VA

$$\Rightarrow E_1 I_1 = E_2 I_2$$

$$\Rightarrow V_1 I_1 = V_2 I_2 (\because V_1 = E_1, V_2 = E_2)$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{I_1}{I_2}$$

$$\therefore \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{I_1}{I_2} = K$$

## 5.7 EMF Equation of a Transformer

When the primary winding is excited by an alternating voltage, it circulates alternating current and hence alternating flux is produced in the core.

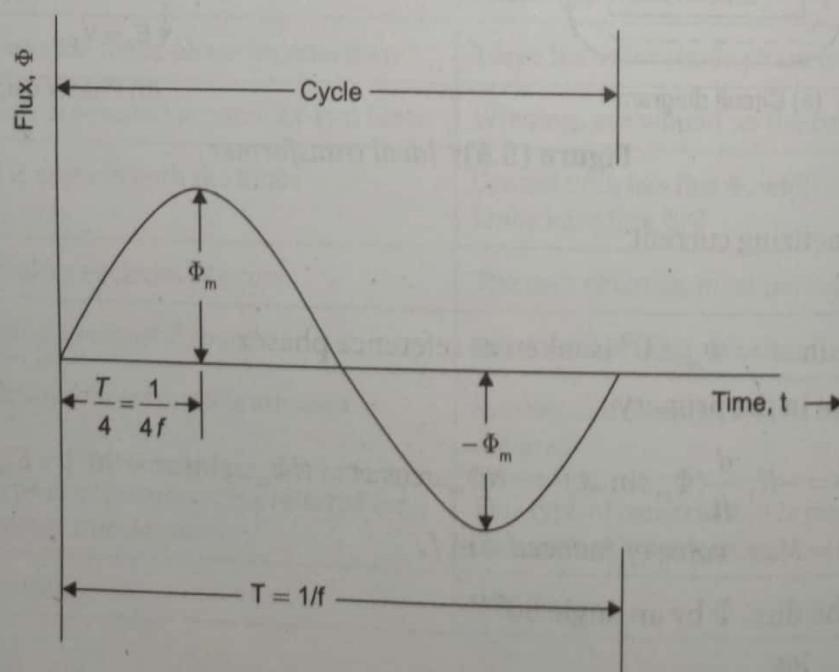


Figure (5.7): Sinusoidal flux

Let  $\Phi = \Phi_m \sin \omega t$

Where  $\Phi_m$  = Maximum value of flux

$f$  = Frequency of supply voltage

$N_1$  = Number of primary winding turns

$N_2$  = Number of secondary winding turns

$E_1$  = r.m.s value of the primary induced emf

$E_2$  = r.m.s. value of the secondary induced emf

According to Faraday's Law of electromagnetic induction, emf induced is given by

$$E = -N \frac{d\Phi}{dt} = -N \frac{d}{dt}(\Phi_m \sin \omega t) = -N\Phi_m \omega \cos \omega t = N\Phi_m \omega \sin(\omega t - 90^\circ) \quad (5.1)$$

It is clear from the above equation that maximum value of induced emf is

$$E_{max} = N\Phi_m \omega$$

The r.m.s. value induced emf is

$$E_{rms} = \frac{E_{max}}{\sqrt{2}} = \frac{N\omega\Phi_m}{\sqrt{2}} = \frac{2N\pi f\Phi_m}{\sqrt{2}} = 4.44 N f \Phi_m \quad (5.2)$$

RMS value of emf induced in the primary winding

$$E_1 = 4.44 N_1 f \Phi_m \quad (5.3)$$

Similarly

RMS value of emf induced in the secondary winding

$$E_2 = 4.44 N_2 f \Phi_m \quad (5.4)$$

Note: It is clear from eqn.(5.1) above that emf  $E_1$  induced in the primary lags behind the flux  $\Phi$  by  $90^\circ$ . Likewise, emf  $E_2$  induced in the secondary lags behind flux  $\Phi$  by  $90^\circ$ .

## 5.8 Voltage and Current Transformation Ratios

Voltage Ratio:

RMS value of emf induced in the primary winding

$$E_1 = 4.44 N_1 f \Phi_m$$

RMS value of emf induced in the secondary winding

$$E_2 = 4.44 N_2 f \Phi_m$$

$$\therefore \frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f \Phi_m \quad (5.5)$$

It means that the emf per turn is same in both primary and secondary.

In an ideal transformer, the voltage drops in primary and secondary windings are negligible.

Therefore, for an ideal transformer  $V_1 = E_1$  and  $V_2 = E_2$

The ratio of secondary voltage to primary voltage is same as the ratio of secondary turns to the primary turns. This ratio is known as transformation ratio, K

$$\text{Therefore, } K = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} \quad (5.6)$$

For step-up transformer  $V_2 > V_1 \Rightarrow K > 1$

For step-down transformer  $V_2 < V_1 \Rightarrow K < 1$

#### Current Ratio:

In an ideal transformer, the losses are negligible so that volt-ampere input to the primary is approximately equal to the volt-ampere output from the secondary

Input VA = Output VA on no load

$$\begin{aligned} V_1 I_1 &= V_2 I_2 \Rightarrow \frac{V_2}{V_1} = \frac{I_1}{I_2} \\ \therefore \frac{I_1}{I_2} &= \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K \end{aligned} \quad (5.7)$$

i.e. primary and secondary currents are inversely proportional to their respective turns.

**Solved Problem-1:** The maximum flux density in the core of a 250/3000V, 50Hz 1-ϕ, transformer is 1.2 Wb/m<sup>2</sup>. If the emf per turn is 8V, determine

- (i) Primary and secondary turns
- (ii) Area of the core

**Solution:** Given that

$$E_1 = 250V, E_2 = 3000V$$

$$B_m = 1.2 \text{ Wb/m}^2$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 8$$

- (i) Primary and secondary turns

$$\frac{E_1}{N_1} = 8 \Rightarrow N_1 = \frac{E_1}{8} = \frac{250}{8} = 32 \text{ turns}$$

$$\frac{E_2}{N_2} = 8 \Rightarrow N_2 = \frac{E_2}{8} = \frac{3000}{8} = 375 \text{ turns}$$

- (ii) Area of the core

$$E_1 = 4.44 f N_1 \Phi_m = 4.44 f N_1 B_m A$$

$$\Rightarrow A = \frac{E_1}{4.44 f N_1 B_m} = \frac{250}{4.44 \times 50 \times 32 \times 1.2} = 0.03 m^2$$

**Solved Problem-2:** A 2000/200V, 20kVA transformer has 66 turns in the secondary. Calculate the primary turns and full-load current in the primary and secondary, neglecting losses?

**Solution:** Given that

$$E_1 = 2000, E_2 = 200 \\ \text{kVA} = 20, N_2 = 66$$

$$K = \frac{E_2}{E_1} = \frac{200}{2000} = \frac{1}{10}$$

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{1}{10} \Rightarrow N_1 = 10 \times 66 = 660$$

Primary turns,  $N_1 = 660$

$$\text{kVA} = VI = EI$$

$$\Rightarrow E_1 I_1 = \text{kVA}$$

$$I_1 = \frac{\text{kVA}}{E_1} = \frac{20 \times 10^3}{2000} = 10A$$

$$I_2 = \frac{\text{kVA}}{E_2} = \frac{20 \times 10^3}{200} = 100A$$

Full-load current in the primary winding,  $I_1 = 10A$

Full-load current in the secondary winding,  $I_2 = 100A$

**Solved Problem-3:** A 1- $\phi$  transformer has 400 primary and 100 secondary turns. The net cross-sectional area of the core is  $60 \text{ cm}^2$ . If the primary winding is connected to a 50 Hz supply at 520V. Calculate:

- (i) Peak value of flux density in the core
- (ii) Voltage induced in the secondary winding
- (iii) Transformation ratio
- (iv) Emf induced per turn in both the windings

**Solution:** Given that

$$N_1 = 400, N_2 = 100$$

$$A = 60 \text{ cm}^2 = 60 \times 10^{-4} \text{ m}^2, f = 50 \text{ Hz}$$

$$E_1 = V_1 = 520 \text{ V}$$

$$(i) E_1 = 4.44 f \Phi_m N_1 = 4.44 f B_m A N_1$$

Peak value of flux density in the core

$$B_m = \frac{E_1}{4.44 f A N_1} = \frac{520}{4.44 \times 50 \times 60 \times 10^{-4} \times 400} = 0.9759 \text{ Tesla}$$

- (ii) Voltage induced in the secondary winding

$$E_2 = 4.44 f B_m A N_2 = 4.44 \times 50 \times 0.9759 \times 60 \times 10^{-4} \times 100 = 130 \text{ V}$$

- (iii) Transformation ratio,  $K = \frac{N_2}{N_1} = \frac{100}{400} = 0.25$

- (iv) Emf induced per turn

$$(a) \text{ in the primary winding} = E_1/N_1 = 520/400 = 1.3$$

$$(b) \text{ in the secondary winding} = E_2/N_2 = 130/100 = 1.3$$

## 5.9 Practical Transformer

A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) iron losses (ii) winding resistances and (iii) magnetic leakage, giving rise to leakage reactances.

(i) **Iron Losses:** Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc. It may be noted that the magnitude of iron losses is quite small in a practical transformer.

(ii) **Winding Resistances:** Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance  $R_1$  and secondary resistance  $R_2$  act in series with the respective windings as shown in figure (5.8). When current flows through the windings, there will be power loss as well as a loss in voltage due to IR drop. This will affect the power factor and  $E_1$  will be less than  $V_1$  while  $V_2$  will be less than  $E_2$ .

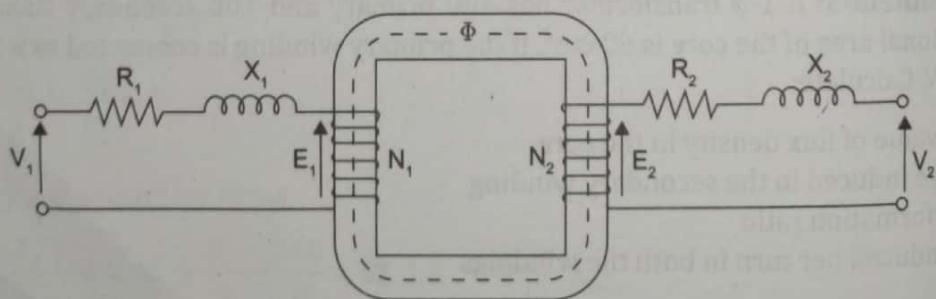
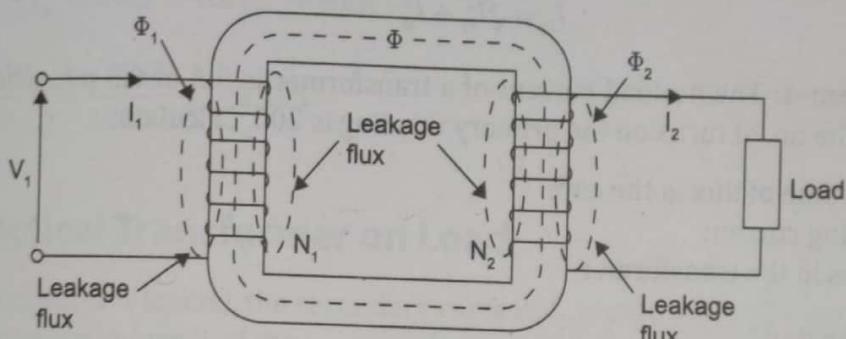


Figure (5.8): Practical transformer

(iii) **Leakage Reactances:** Both primary and secondary currents produce flux. The flux  $\Phi$ , which links both the windings is the useful flux and is called mutual flux. However, primary current would produce some flux  $\Phi$  which would not link the secondary winding shown in figure (5.9). Similarly, secondary current would produce some flux  $\Phi$  that would not link the primary winding. The flux such as  $\Phi_1$  or  $\Phi_2$  which links only one winding is

called leakage flux. The leakage flux paths are mainly through the air. The effect of these leakage fluxes would be the same as though inductive reactance were connected in series with each winding of transformer that had no leakage flux as shown in figure (5.8). In other words, the effect of primary leakage flux  $\Phi_1$  is to introduce an inductive reactance  $X_1$  in series with the primary winding as shown in figure (5.9). Similarly, the secondary leakage flux  $\Phi_2$  introduces an inductive reactance  $X_2$  in series with the secondary winding. There will be no power loss due to leakage reactance. However, the presence of leakage reactance in the windings changes the power factor as well, as there is voltage loss due to IX drop.

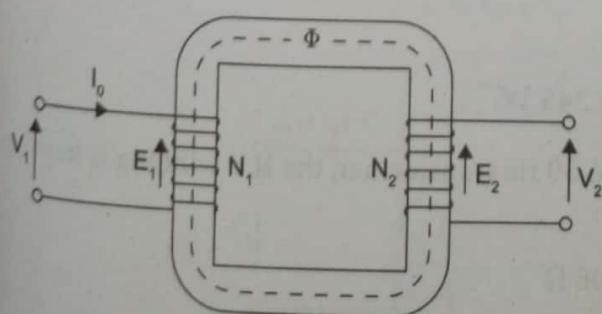


**Figure (5.9): Practical transformer with leakage fluxes**

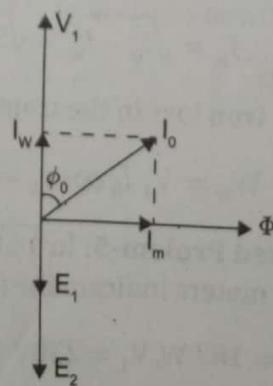
## 5.10 Practical Transformer on No-Load

In the case of ideal transformer, losses are neglected, but practically it is not possible. When the primary of a transformer is connected to AC supply source, and the secondary is open, the transformer is said to be on no-load.

When the transformer is on no-load, the primary current is not completely reactive. But it has to supply iron losses in the core and a very small amount of copper losses in primary (there being no copper loss in secondary as it is open). Hence, the no load primary input current  $I_0$  is not at  $90^\circ$  behind  $V_1$  but lags it by an angle  $\phi_0 < 90^\circ$ .



(a) Circuit diagram



(b) Phasor diagram

**Figure (5.10): Transformer on no-load**

No load input power of the transformer is

$$W_0 = V_1 I_0 \cos\phi_0 \quad (5.8)$$

The phasor diagram under no-load condition is drawn as shown in figure 5.10(b).

From the phasor diagram  $I_0$  has two components:

- (i) Working or active component,  $I_w = I_0 \cos\phi_0$ , which is in phase with  $V_1$
- (ii) Magnetizing component,  $I_m = I_0 \sin\phi_0$ , which is in quadrature with  $V_1$

$$\therefore I_0 = \sqrt{I_m^2 + I_w^2} \quad (5.9)$$

**Solved Problem-4:** The no-load current of a transformer is 5 A at 0.3 p.f., when supplied at 230V, 50 Hz. The no. of turns on the primary winding is 200. Calculate:

- (i) the max. value of flux in the core
- (ii) magnetizing current
- (iii) iron losses in the transformer

**Solution:** Given that

$$I_0 = 5 \text{ A}, \cos\phi_0 = 0.3$$

$$V_1 = E_1 = 230 \text{ V}, N_1 = 200$$

$$(i) E_1 = 4.44 f \Phi_m N_1$$

The maximum value of flux into the core

$$\Phi_m = \frac{E_1}{4.44 f N_1} = \frac{230}{4.44 \times 50 \times 200} = 5.18 \text{ mWb}$$

(ii) The magnetizing current

$$I_w = I_0 \cos\phi_0 = 5 \times 0.3 = 1.5$$

$$\therefore I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{5^2 - 1.5^2} = 4.77 \text{ A}$$

(iii) Iron loss in the transformer

$$W_0 = V_1 I_0 \cos\phi_0 = 230 \times 5 \times 0.3 = 345 \text{ W}$$

**Solved Problem-5:** In a 50 kVA, 2300/230 V, 50 Hz transformer, the H.V winding is kept open. The meters indicate the following readings:

$$W_0 = 187 \text{ W}, V_1 = 230 \text{ V}, I_0 = 6.5 \text{ A}, R_1 = 0.06 \Omega$$

Find (i) core loss (ii)  $I_w$  (iii)  $I_m$  (iv)  $\cos\phi_0$

**solution:** Given that

$$W_0 = 187 \text{ W}, V_1 = 230 \text{ V}, I_0 = 6.5 \text{ A}, R_1 = 0.06 \Omega$$

(i) Copper loss due to no-load current =  $I_0^2 R_1 = 6.5^2 \times 0.06 = 2.535 \text{ W}$   
 $\therefore$  Core loss =  $W_0 - \text{Copper loss} = 187 - 2.535 = 184.46 \text{ W}$

(ii)  $I_w = I_0 \cos \phi_0 = \frac{W_0}{V_1} = \frac{187}{230} = 0.813 \text{ A}$

(iii)  $I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{6.5^2 - 0.813^2} = 6.4 \text{ A}$

(iv)  $\cos \phi_0 = \frac{W_0}{V_1 I_0} = \frac{187}{230 \times 6.5} = 0.125$

## 5.11 Practical Transformer on Load

When the transformer is loaded, the secondary current,  $I_2$  is set up.  $I_2$  will be in phase with  $V_2$  if the load is resistive, it lags  $V_2$  if the load is inductive, it leads  $V_2$  if the load is capacitive. The secondary current  $I_2$  sets up its own MMF ( $= N_2 I_2$ ) and hence it produces flux  $\Phi_2$ , which is in opposition to the main primary flux  $\Phi_1$ , which is due to  $I_0$ . Secondary flux  $\Phi_2$  weakens the main flux  $\Phi$  momentarily and hence primary back e m f  $E_1$  tends to be reduced. For a moment  $V_1$  gains the upper hand over  $E_1$  and hence causes additional current  $I_2'$  to flow in primary and hence flux  $\Phi_2'$  (due to MMF  $N_2 I_2'$ ) which counter-balances the secondary flux  $\Phi_2$ .

$$\therefore \Phi_2' = \Phi_2$$

$$N_1 I_2' = N_2 I_2$$

$$\Rightarrow I_2' = \frac{N_2}{N_1} I_2 = K I_2 \quad (5.10)$$

Here  $I_2'$  is known as load component of primary current. This current is in anti-phase with  $I_2$ .

$$\therefore I_1 = I_0 + I_2' \quad (5.11)$$

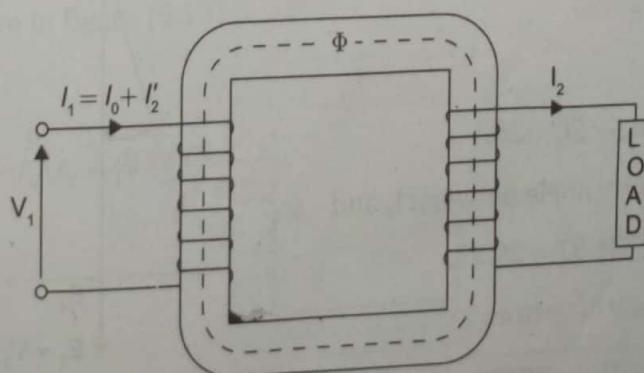


Figure (5.11): Transformer on load

The phasor diagram of a transformer under load condition can be drawn as shown in figure (5.12)

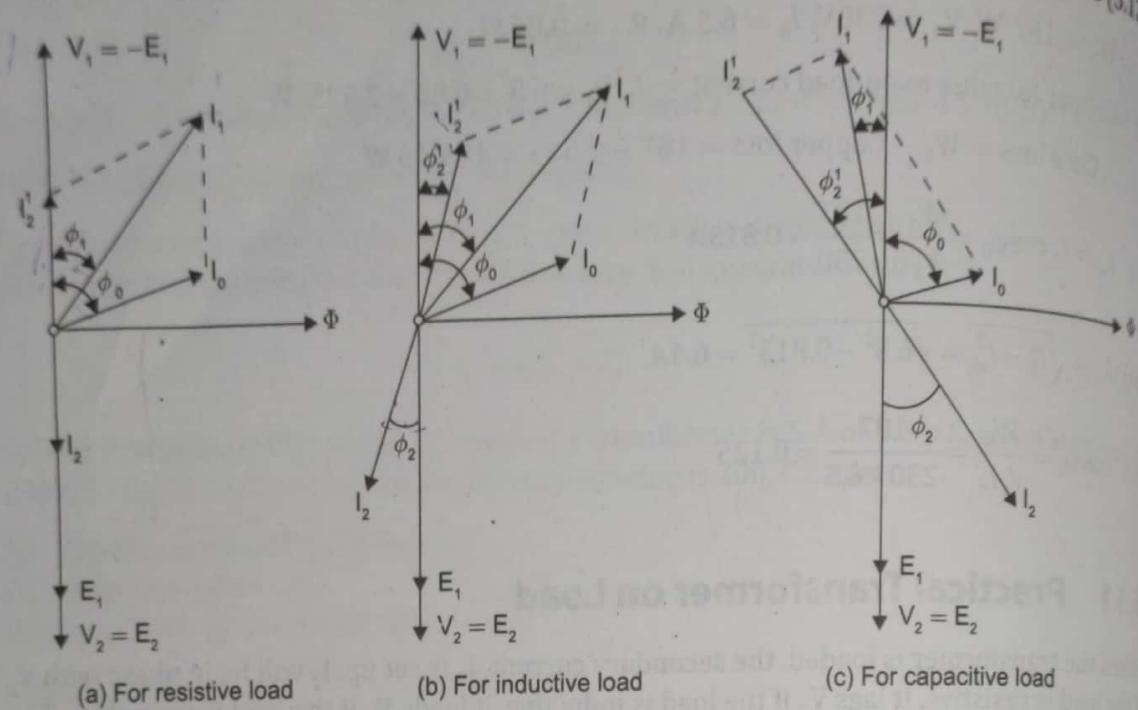


Figure (5.12): Phasor diagram of transformer on load

**Solved Problem-6:** The primary of a certain transformer takes 1A at a p. f. of 0.4 when it is connected across a 200 V, 50 Hz supply and the secondary is on open. The number of turns on the secondary is half of the primary. A load, taking 50 A at a lagging p.f. of 0.8 is now connected across the secondary. What is the new value of the primary current?

**Solution:** Given that

$$I_0 = 1 \text{ A}, f = 50 \text{ Hz}, V_1 = 200 \text{ V}$$

$$N_2 = \frac{1}{2} N_1, I_2 = 50 \text{ A}$$

$$\cos \phi_0 = 0.4 \Rightarrow \phi_0 = 66.42^\circ$$

$$\cos \phi_2 = 0.8 \Rightarrow \phi_2 = 36.87^\circ$$

$$K = \frac{N_2}{N_1} = 0.5$$

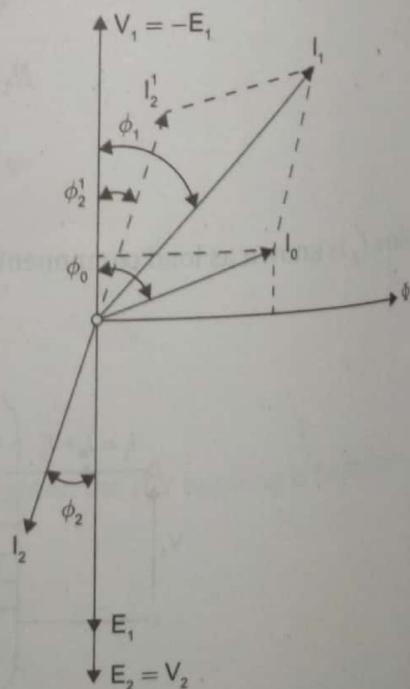
$$I_2' = K I_2 = 0.5 \times 50 = 25 \text{ A}$$

From the phasor diagram angle between  $I_0$  and

$$I_2' = \phi_0 - \phi_2 = 66.42 - 36.87 = 29.55^\circ$$

Using parallelogram law of vectors

$$I_1' = \sqrt{1^2 + 25^2 + 2 \times 1 \times 25 \cos 29.55} = 26.875 \text{ A}$$



**Solved Problem-7:** A single phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no-load current is 3A at a p.f. of 0.2 lagging when the secondary current is 280A at a p.f. of 0.8 lagging. Calculate the primary current and its power factor. Assume the voltage drop in the windings to be negligible.

**Solution:** Given that

$$N_1 = 1000, N_2 = 200$$

$$\text{No-load current, } I_0 = 3\text{A}$$

$$\text{Load or secondary current, } I_2 = 280\text{A}$$

$$K = \frac{N_2}{N_1} = \frac{200}{1000} = 0.2$$

$$\text{Load p.f., } \cos \phi_2 = 0.8 \text{ lagging} \Rightarrow \phi_2 = 36.86^\circ$$

$$\therefore \overline{I}_2 = 280 \angle -36.86^\circ$$

$$\text{No-load p.f., } \cos \phi_0 = 0.2 \text{ lagging} \Rightarrow \phi_0 = 78.46^\circ$$

$$\therefore \overline{I}_0 = 3 \angle -78.46^\circ = 0.6 - j2.94 \text{ A}$$

Load component of primary current

$$\overline{I}'_2 = K \overline{I}_2 = 0.2 \times 280 \angle -36.86^\circ = 56 \angle -36.86^\circ = 44.8 - j33.6$$

Primary supply current

$$\overline{I}_1 = \overline{I}'_2 + \overline{I}_0 = (44.8 - j33.6) + (0.6 - j2.94) = 45.39 - j36.54 = 58.27 \angle -38.83^\circ$$

$\therefore$  Primary current = 58.27A

$$\text{Primary p.f., } \cos \phi_1 = \cos(38.83) = 0.78 \text{ lagging}$$

## 5.12 Transformer with Resistance and Leakage Reactance

In actual (practical) transformers, the transformer windings consist of both resistance and reactance as shown in figure (5.13).

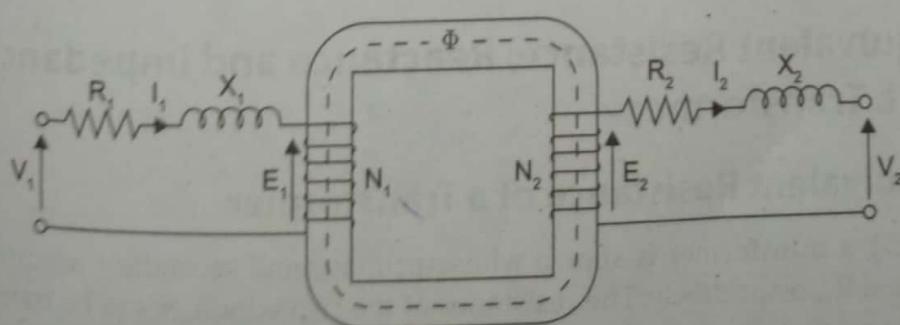


Figure (5.13): Transformer with winding resistance and reactance

From figure (5.13), we can write that

$$\text{Primary Impedance, } Z_1 = R_1 + jX_1 = \sqrt{R_1^2 + X_1^2}$$

$$\text{Secondary impedance, } Z_2 = R_2 + jX_2 = \sqrt{R_2^2 + X_2^2}$$

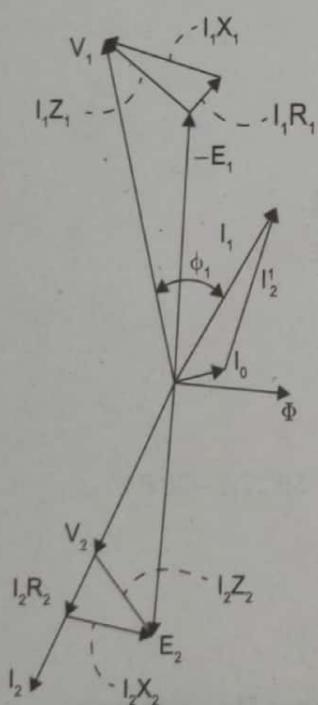
$$\text{Primary induced emf, } E_1 = V_1 - I_1(R_1 + jX_1) = V_1 - I_1Z_1$$

$$\text{or } V_1 = E_1 + I_1Z_1$$

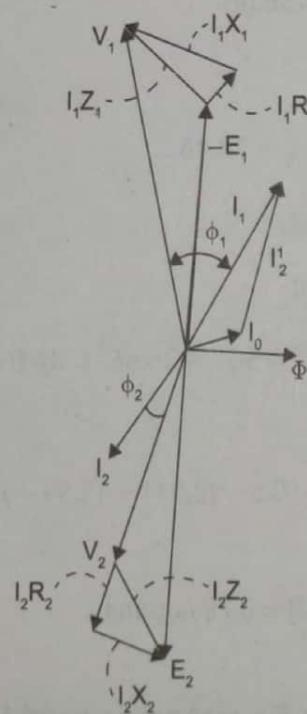
$$\text{Secondary terminal voltage, } V_2 = E_2 - I_2(R_2 + jX_2) = E_2 - I_2Z_2$$

$$\text{or } E_2 = V_2 + I_2Z_2$$

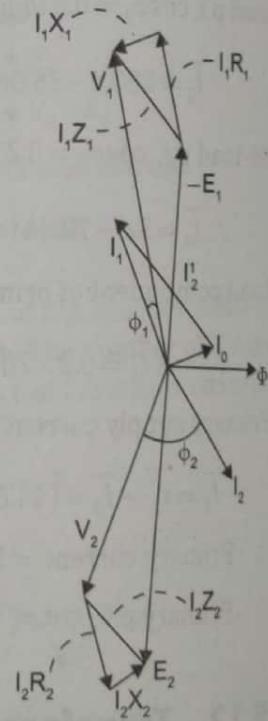
The phasor diagram of the transformer for resistive, inductive and capacitive load is as shown in figure (5.14).



(a) For resistive load



(b) For inductive load



(c) For capacitive load

Figure (5.14): Phasor diagram of a transformer

## 5.13 Equivalent Resistance, Reactance and Impedance of a Transformer

### 5.13.1 Equivalent Resistance of a Transformer

In figure (5.15), a transformer is shown whose primary and secondary windings have resistances of  $R_1$  and  $R_2$ , respectively. The resistance of the two windings can be transferred to any one side either primary or secondary without affecting the performance of the transformer. The transfer of the resistances on any one side is advantageous as it makes calculations very easy.

The total copper loss due to both resistances is given by

$$\text{Total losses} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 \left( R_1 + \frac{I_2^2}{I_1^2} R_2 \right) = I_1 \left( R_1 + \frac{R_2}{K^2} \right) \quad (5.12)$$

Now the equation 5.12 indicates that the total copper loss can be expressed as  $I_1^2 R_1 + I_1^2 \frac{R_2}{K^2}$ .

This means  $\frac{R_2}{K^2}$  is the resistance value of  $R_2$  shifted to primary side which causes same copper loss with  $I_1$  as  $R_2$  causes with  $I_2$ . This value of resistance  $\frac{R_2}{K^2}$  which is the value of  $R_2$  referred to

primary is called equivalent resistance of secondary when referred to primary. It is denoted as  $R'_2$ .

$$\therefore R'_2 = \frac{R_2}{K^2} = \text{Equivalent resistance of second when referred to primary.}$$

From eqn. 5.12, we can write

$$\text{Total copper loss} = I_1^2 (R_1 + R'_2) = I_1^2 R_{01} \quad (5.13)$$

Where  $R_{01} = R_1 + \frac{R_2}{K^2}$  = Equivalent resistance of the transformer when referred to primary.

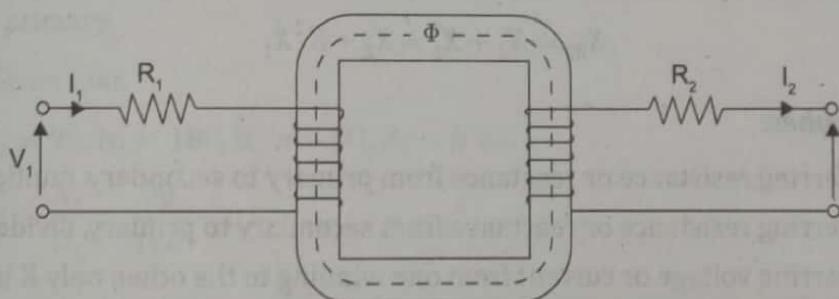


Figure (5.15)

Similarly

$$\begin{aligned} \text{Total copper loss} &= I_1^2 R_1 + I_2^2 R_2 = I_2^2 \left( \frac{I_1^2}{I_2^2} R_1 + R_2 \right) \\ &= I_2^2 (K^2 R_1 + R_2) = I_2^2 (R'_1 + R_2) = I_2^2 R_{02} \end{aligned} \quad (5.14)$$

where  $R'_1 = K^2 R_1$  = Equivalent resistance of the primary when referred to secondary

$$\therefore R_{02} = K^2 R_1 + R_2 = \text{Equivalent resistance of the transformer when referred to secondary}$$

### 5.13.2 Equivalent Reactance of the Transformer

Similar to the resistance, the leakage reactance can also be transferred from primary to secondary and vice versa. The relation through  $K^2$  remains the same for the transfer of reactance as it was studied earlier for the resistances.

Let  $X'_2$  be the reactance of the secondary winding referred to the primary side. For  $X'_2$  to produce the same effect in the primary side as  $X_2$  in the secondary side, each must absorb the same reactive volt amperes.

$$VAR = VI \sin \phi = IZ \times I \times \frac{X}{Z} = I^2 X$$

Equating the reactive volt-amperes consumed by  $X'_2$  and  $X_2$  gives

$$\begin{aligned} (I'_2)^2 X'_2 &= I_2^2 X_2 \\ \Rightarrow X'_2 &= \frac{I_2^2}{(I'_2)^2} X_2 = \frac{I_2^2}{(KI_2)^2} X_2 = \frac{X_2}{K^2} \\ \therefore X'_2 &= \frac{X_2}{K^2} \end{aligned}$$

Equivalent reactance of the transformer when referred to primary

$$X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K^2} \quad (5.15)$$

Equivalent reactance of the transformer when referred to secondary

$$X_{02} = X_2 + X'_1 = X_2 + K^2 X_1 \quad (5.16)$$

#### Points to Remember:

- When transferring resistance or reactance from primary to secondary, multiply it by  $K^2$ .
- When transferring resistance or reactance from secondary to primary, divide it by  $K^2$ .
- When transferring voltage or current from one winding to the other, only K is used.

#### 5.13.3 Equivalent Impedance of a Transformer

The transformer primary has resistance  $R_1$  and reactance  $X_1$ , while the transformer secondary has resistance  $R_2$  and reactance  $X_2$ . Thus, we can say that the total impedance of primary winding  $Z_1$  is given by

$$Z_1 = R_1 + jX_1 = \sqrt{R_1^2 + X_1^2}$$

And the total impedance of secondary winding  $Z_2$  is given by

$$Z_2 = R_2 + jX_2 = \sqrt{R_2^2 + X_2^2}$$

Similar to resistance and reactance, the impedance also can be referred to any one side.

So, equivalent impedance when referred to primary

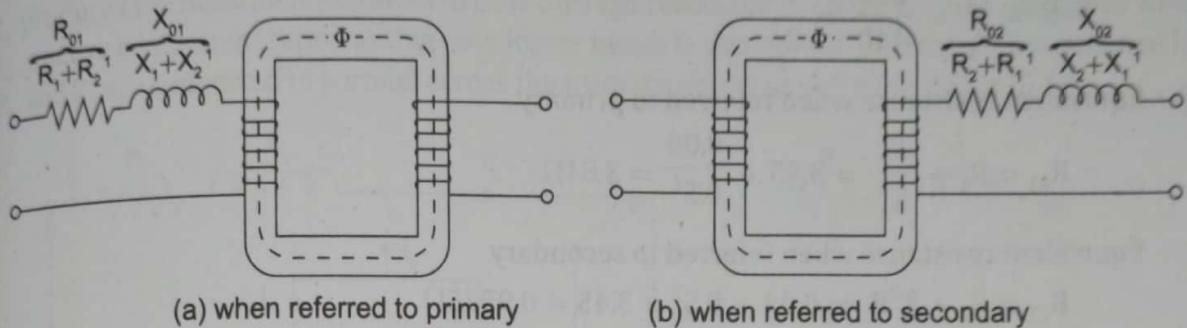
$$Z_{01} = R_{01} + jX_{01}$$

$$\text{Where } R_{01} = R_1 + R_2/K^2, X_{01} = X_1 + X_2/K^2$$

Equivalent impedance when referred to secondary

$$Z_{02} = R_{02} + j X_{02}$$

$$\text{Where } R_{02} = R_2 + K^2 R_1, X_{02} = X_2 + K^2 X_1$$



**Figure (5.16): Equivalent impedance of transformer**

**Solved Problem-8:** A 1- $\phi$  transformer has 90 and 180 turns respectively in its secondary and primary windings. The respective resistances are  $0.2\Omega$  and  $0.6\Omega$ . Calculate the equivalent resistance of (i) the primary in terms of the secondary winding (ii) the secondary in terms of the primary winding and (iii) also calculate the total resistance of the transformer when referred to primary.

**Solution:** Given that

$$N_2 = 90, N_1 = 180, R_1 = 0.6\Omega, R_2 = 0.2\Omega$$

$$K = \frac{N_2}{N_1} = \frac{90}{180} = 0.5$$

- (i) The equivalent resistance of the primary winding in terms of secondary winding

$$R'_1 = K^2 R_1 = 0.5^2 \times 0.6 = 0.15\Omega$$

- (ii) The equivalent resistance of the secondary winding in terms of the primary winding

$$R'_2 = \frac{R_2}{K^2} = \frac{0.2}{0.5^2} = 0.8\Omega$$

- (iii) The total resistance of the transformer when referred to primary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.6 + \frac{0.2}{0.5^2} = 0.6 + 0.8 = 1.4\Omega$$

**Solved Problem-9:** A 5kVA, 440/220V transformer has  $R_1 = 3.45\Omega$ ,  $R_2 = 0.09\Omega$ ,  $X_1 = 6.2\Omega$ ,  $X_2 = 0.015\Omega$ . Calculate for the transformer

- (i) Equivalent resistance when referred to both primary and secondary.
- (ii) Equivalent reactance when referred to both primary and secondary.
- (iii) Equivalent impedance when referred to both primary and secondary.

**Solution:** Given that

$$V_2 = 220, V_1 = 440$$

$$R_1 = 3.45\Omega, R_2 = 0.09\Omega$$

$$X_1 = 1.2\Omega, X_2 = 0.015\Omega$$

$$K = \frac{V_2}{V_1} = \frac{220}{440} = \frac{1}{2}$$

(i) Equivalent resistance when referred to primary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.09}{0.5^2} = 3.81\Omega$$

Equivalent resistance when referred to secondary

$$R_{02} = R_2 + K^2 R_1 = 0.09 + 0.5^2 \times 3.45 = 0.9525\Omega$$

(ii) Equivalent reactance when referred to primary

$$X_{01} = X_1 + \frac{X_2}{K^2} = 1.2 + \frac{0.015}{0.5^2} = 1.26\Omega$$

Equivalent reactance when referred to secondary

$$X_{02} = X_2 + K^2 X_1 = 0.015 + 0.5^2 \times 1.2 = 0.315\Omega$$

(iii) Equivalent impedance when referred to primary

$$Z_{01} = R_{01} + j X_{01} = 3.81 + j 1.26 = 4.013 \angle 18.3^\circ$$

Equivalent impedance as referred to secondary

$$Z_{02} = R_{02} + j X_{02} = 0.9525 + j 0.315 = 1 \angle 18.3^\circ$$

## 5.14 Equivalent Circuit of a Transformer

In transformers, the problems concerning voltages and currents can be solved by the use of phasor diagrams. However, it is more convenient to represent the transformer by an equivalent circuit. The term, equivalent circuit of a transformer, means the combination of fixed and variable resistances and reactances, which exactly simulates performance and working of the machine. If an equivalent circuit is available, the computations can be done by the direct application of the circuit theory. The transformer shown diagrammatically in figure (5.17) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the windings, whose function is only to transform the voltage.

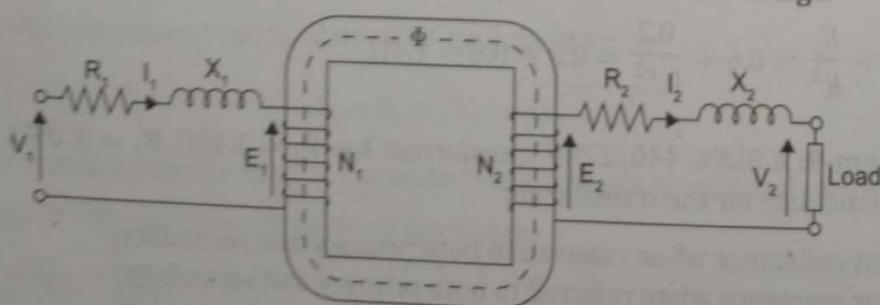


Figure (5.17): Practical transformer

For a transformer, no-load primary current  $I_0$  has two components

$$I_m = I_0 \sin \phi_0 = \text{Magnetizing component}$$

$$I_w = I_0 \cos \phi_0 = \text{Active component}$$

$I_m$  produces the flux and is assumed to flow through reactance  $X_0$  called no-load reactance while  $I_w$  is active component representing core losses hence is assumed to flow through resistance  $R_0$ .  $R_0$  and  $X_0$  are connected in parallel across the primary circuit as shown in figure (5.18).

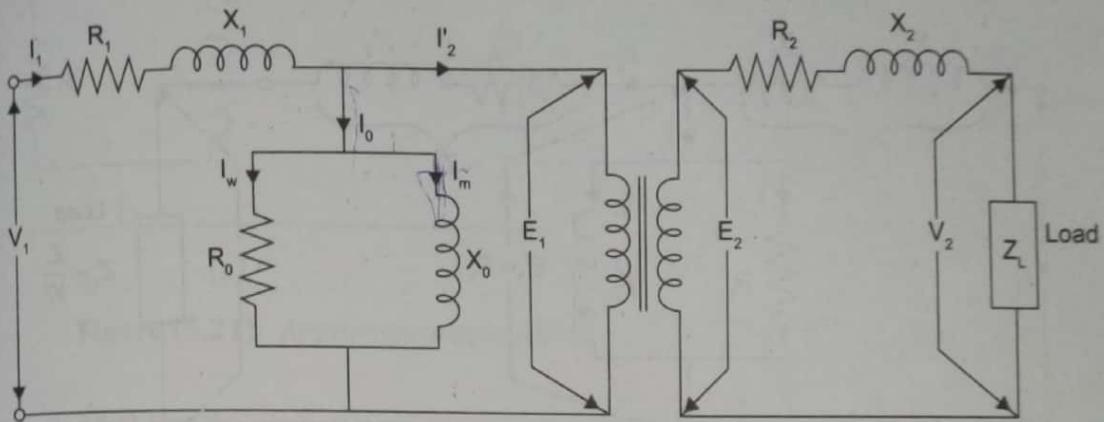


Figure (5.18): Equivalent circuit of a transformer

From the equivalent circuit, we can write

$$R_0 = \frac{E_1}{I_w} \quad \text{and} \quad X_0 = \frac{E_1}{I_m}$$

To make the calculations simpler, it is preferable to transfer voltage, current and impedance to either primary or secondary of transformer.

#### (i) Equivalent Circuit when Referred to Primary

Let us see how to transfer secondary values to primary side

$E'_2$  = The primary equivalent of secondary emf =  $E_2/K = E_1$

$V'_2$  = The primary equivalent of secondary terminal voltage =  $V_2/K$

$$\text{i.e., } I_1 V'_2 = I_2 V_2 \Rightarrow V'_2 = \left( \frac{I_2}{I_1} \right) V_2 = \frac{V_2}{K}$$

$I'_2$  = The primary equivalent of secondary current =  $K I_2$

$$\text{i.e., } V_1 I'_2 = V_2 I_2 \Rightarrow I'_2 = \left( \frac{V_2}{V_1} \right) I_2 = K I_2$$

$R'_2$  = The primary equivalent of secondary resistance =  $R_2/K^2$

$X'_2$  = The primary equivalent of secondary reactance =  $X_2/K^2$

$Z'_2$  = The primary equivalent of secondary impedance =  $Z_2/K^2$

$$R_{01} = R_1 + R_2/K^2, X_{01} = X_1 + X_2/K^2$$

$$Z_{01} = R_{01} + jX_{01}$$

Where  $K = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$  = Transformation ratio.

Thus, the equivalent circuit of the transformer when referred to primary is as shown in figure (5.19).

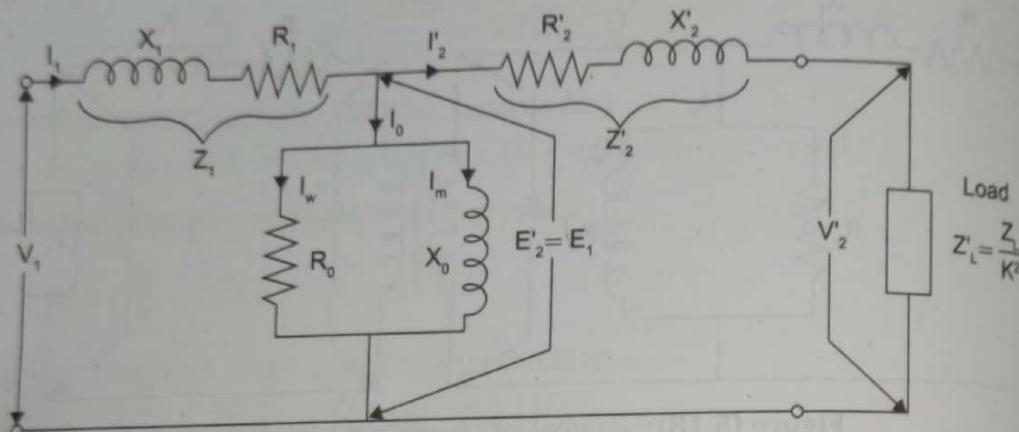


Figure (5.19): Equivalent circuit when referred to primary

#### Approximate Equivalent Circuit of a Transformer:

The no-load current  $I_0$  is usually less than 5% of the full load primary current. The voltage drop produced by  $I_0$  in  $(R_1 + jX_1)$  is negligible for practical purposes. Therefore, it is immaterial that the shunt branch  $R_0//X_0$  is connected before the primary series impedance  $(R_1 + jX_1)$  or after it. The currents  $I_m$  and  $I_w$  are not much affected. Therefore, the equivalent circuit can be further simplified by shifting the no load branch containing  $R_0$  and  $X_0$  to the supply terminals as shown in figure (5.20). By doing this, we are creating an error that the drop across  $R_1$  and  $X_1$  due to  $I_0$  is neglected. Hence, equivalent circuit is called approximate equivalent circuit.

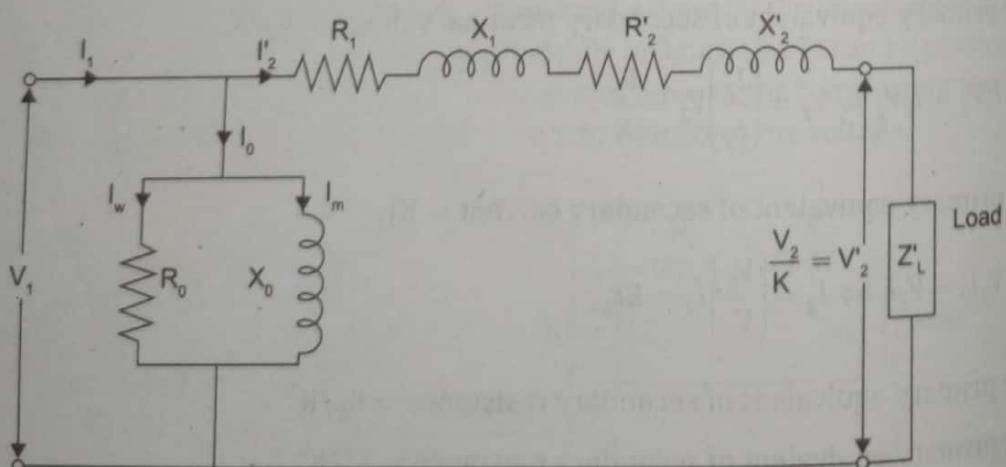
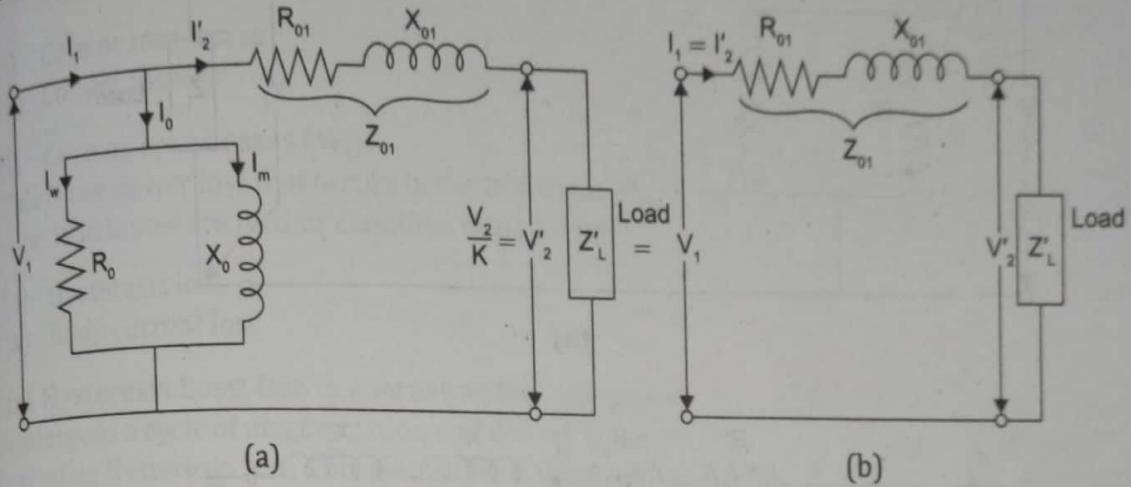


Figure (5.20): Approximate equivalent circuit when referred to primary

The equivalent circuit can be further simplified by combining parameters  $R_1$  and  $R'_2$  as  $R_{01}$  and  $X_1$  and  $X'_2$  as  $X_{01}$  where  $R_{01} = R_1 + R_2/K^2$ ,  $X_{01} = X_1 + X_2/K^2$ . The circuit is shown in the figure (5.21).



**Figure (5.21): Approximate equivalent circuit when referred to primary**

In a similar fashion, the approximate equivalent circuit when referred to secondary can also be obtained.

### (ii) Equivalent Circuit when Referred to Secondary

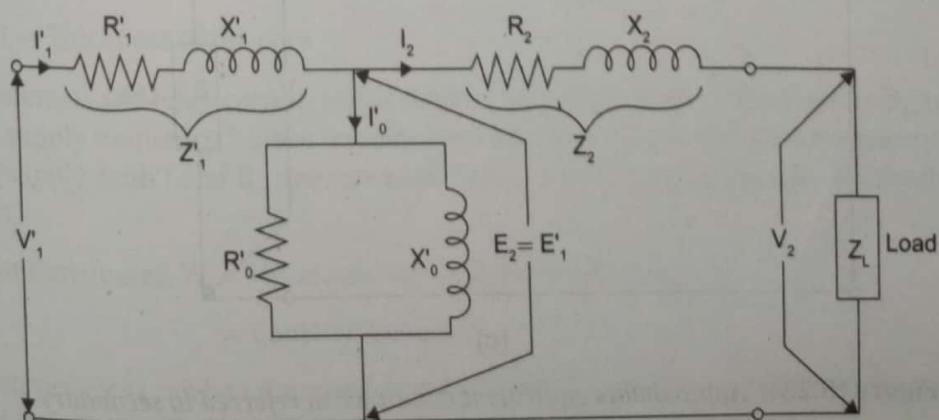
Similarly, if all the primary quantities are transferred to secondary, we get the equivalent circuit of transformer referred to secondary as shown in figure (5.22).

$$R'_1 = K^2 R_1, \quad X'_1 = K^2 X_1, \quad Z'_1 = K^2 Z_1$$

$$E'_1 = K E_1, \quad I'_1 = \frac{I_1}{K}, \quad I'_0 = \frac{I_0}{K}$$

$$R_{02} = R_2 + K^2 R_1, \quad X_{02} = X_2 + K^2 X_1, \quad Z_{02} = R_{02} + j X_{02}$$

Similarly, the exciting circuit parameters also get transformed to secondary as  $R'_0$  and  $X'_0$ . The equivalent circuit referred to secondary is as shown in figure (5.24).



**Figure (5.22): Equivalent circuit when referred to secondary**

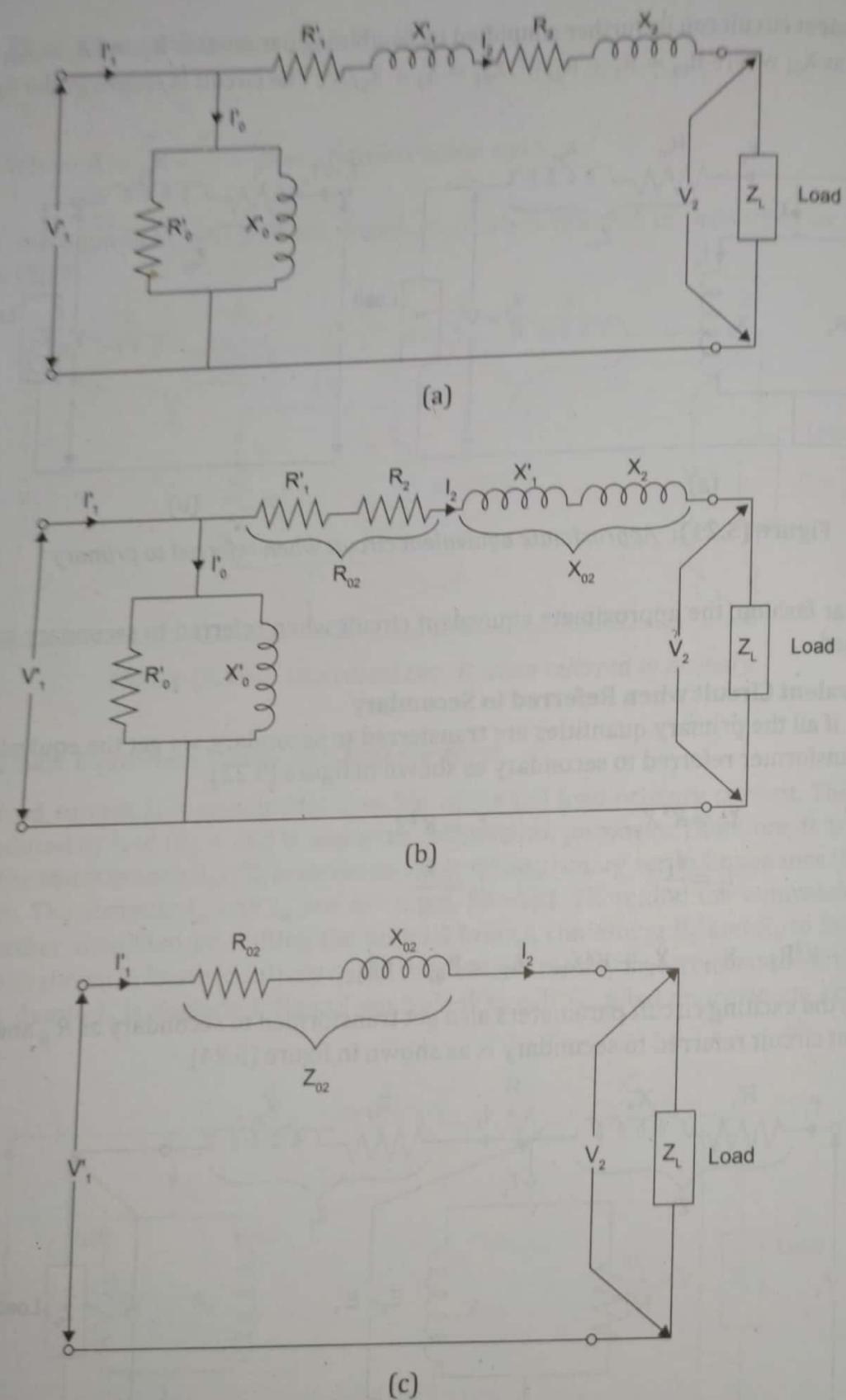


Figure (5.23): Approximate equivalent circuit when referred to secondary

## 5.15 Losses in a Transformer

Since the transformer is a static device, there are no friction and windage losses. Hence the losses occurring in transformer are:

- (i) Core or iron losses
- (ii) Copper losses

### (i) Core or Iron Losses ( $W_i$ )

This is the power loss that occurs in the iron part. This loss is due to the alternating flux in the core. Iron losses are further classified into two types.

- (a) Hysteresis loss
- (b) Eddy current loss

**(a) Hysteresis Loss:** Due to alternating flux setup in the magnetic core of the transformer, it undergoes a cycle of magnetization and demagnetization resulting in a loss of energy which is called as Hysteresis loss. This loss is directly proportional to the supply frequency. It is given by

$$\text{Hysteresis loss, } W_h = \eta B_{\max}^{1.6} \cdot f \cdot v \text{ watts} \quad (5.17)$$

Where  $\eta$  = Hysteresis constant, depends on material

$B_{\max}$  = Maximum flux density

$f$  = Frequency

$v$  = Volume of the core

**(b) Eddy Current Loss:** This power loss is due to the alternating flux linking the core, which will induce an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation, converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency. It is given by

$$\text{Eddy current loss, } W_e = K_e B_{\max}^2 f^2 t^2 \text{ watts/m}^3 \quad (5.18)$$

Where  $K_e$  = Eddy current constant

$t$  = Thickness of the core

Both hysteresis and eddy current losses depend upon (i) maximum flux density  $B_m$  in the core and (ii) supply frequency  $f$ . Since transformers are connected to constant frequency, constant voltage supply, both  $f$  and  $B_m$  are constant. Hence, core or iron losses are practically same at all loads.

$$\begin{aligned} \therefore \text{Iron or Core losses, } W_i &= \text{Hysteresis loss} + \text{Eddy current loss} \\ &= \text{Constant losses} \end{aligned}$$

The hysteresis loss can be minimized by using steel of high silicon content whereas eddy current loss can be reduced by using core of thin laminations.

### (ii) Copper Losses ( $W_{cu}$ )

Copper loss is the power ( $I^2R$ ) wasted in the form of heat due to resistance of the primary and secondary windings.

$$\text{Total losses} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02} \quad (5.19)$$

Obviously copper loss depends upon the load current and is proportion to the square of the load current. Thus, if copper losses are 800 W at a load current of 20 A, then they will be  $(1/2)^2 \times 800 = 200$  W at a load current of 10 A.

This loss is proportional to the square of the load hence it is called the variable loss where as the iron loss is called the constant loss as the supply voltage and frequency are constants.

Thus, for a transformer,

$$\text{Total losses in a transformer} = W_i + W_{cu} = \text{Constant losses} + \text{Variable losses}$$

It may be noted that in a transformer, copper losses account for about 90% of the total losses.

**Solved Problem-10:** A 1000/2000V transformer has 750 W hysteresis losses and 250 W eddy current losses. When the applied voltage is doubled and frequency is halved, find the new losses.

**Solution:** Given that

$$\text{Voltage rating} = 1000/2000\text{V}$$

$$\text{Hysteresis losses, } W_{h1} = 750\text{W}$$

$$\text{Eddy current losses, } W_{e1} = 250\text{W}$$

$$V_2 = 2V_1, f_2 = 0.5f_1$$

$$\text{Hysteresis loss, } W_h \propto B_{\max}^{1.6} \cdot f$$

$$\text{Eddy current loss, } W_e \propto B_{\max}^2 f^2$$

It is known that for a transformer

$$V = 4.44 N f \phi_m = 4.44 N f B_m A$$

For constant A and N,  $B_m \propto (V/f)$

$$\therefore W_h \propto B_{\max}^{1.6} \cdot f \propto \left(\frac{V}{f}\right)^{1.6} \times f \propto (V)^{1.6} (f)^{-0.6}$$

$$\Rightarrow \frac{W_{h2}}{W_{h1}} = \left(\frac{V_2}{V_1}\right)^{1.6} \times \left(\frac{f_2}{f_1}\right)^{-0.6}$$

$$\Rightarrow \frac{W_{h2}}{750} = \left(\frac{2V_1}{V_1}\right)^{1.6} \times \left(\frac{0.5f_1}{f_1}\right)^{-0.6} = (2)^{1.6} (0.5)^{-0.6} = 4.5948$$

$$\Rightarrow W_{h2} = 4.5948 \times 750 = 3446\text{W}$$

$$W_e \propto B_{\max}^2 f^2 \propto \left(\frac{V}{f}\right)^2 \times f^2 \propto V^2$$

$$\Rightarrow \frac{W_{e2}}{W_{e1}} = \left(\frac{V_2}{V_1}\right)^2$$

$$\Rightarrow \frac{W_{e2}}{250} = \left(\frac{2V_1}{V_1}\right)^2 = 4$$

$$\Rightarrow W_{e2} = 250 \times 4 = 1000W$$

## 5.16 Efficiency of a Transformer

Efficiency of a transformer at a given load and power factor is defined as the ratio of output power to the input power, both the quantities expressed in the same units.

$$\text{Efficiency, } \eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

$$= \frac{V_2 I_2 \times \text{power factor}}{V_2 I_2 \times \text{power factor} + I_1^2 R_1 + I_2^2 R_2 + W_i} \quad (5.20)$$

It may appear that efficiency can be determined by directly loading the transformer and measuring the input power and output power. However, this method has the following drawbacks:

- Since the efficiency of a transformer is very high, even 1% error in each wattmeter (output and input) may give ridiculous results. This test, for instance, may give efficiency higher than 100%.
- Since the test is performed with transformer on load, a considerable amount of power is wasted. For large transformers, the cost of power alone would be considerable.
- The test gives no information about the proportion of various losses.

Due to these drawbacks, direct loading method is seldom used to determine the efficiency of a transformer. In practice, the efficiency can be calculated by determining iron loss from open circuit test and copper loss from short circuit test.

### 5.16.1 Condition for Maximum Efficiency

Let us consider primary side

$$\text{Primary input} = V_1 I_1 \cos \phi_1$$

$$\text{Iron losses, } W_i = W_h + W_e$$

$$\text{Copper losses, } W_{cu} = I_1^2 R_{01} \text{ (or) } I_2^2 R_{02}$$

$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}}$$

$$\Rightarrow \eta = \frac{V_1 I_1 \cos \phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos \phi_1} = 1 - \frac{I_1 R_{01}}{V_1 \cos \phi_1} - \frac{W_i}{V_1 I_1 \cos \phi_1}$$

For ' $\eta$ ' to be maximum,  $\frac{d}{d I_1} (\eta) = 0$

$$\Rightarrow \frac{d}{d I_1} \left( 1 - \frac{I_1 R_{01}}{V_1 \cos \phi_1} - \frac{W_i}{V_1 I_1 \cos \phi_1} \right) = 0$$

$$\Rightarrow \frac{-R_{01}}{V_1 \cos \phi_1} + \frac{W_i}{V_1 I_1^2 \cos \phi_1} = 0$$

$$\Rightarrow \frac{R_{01}}{V_1 \cos \phi_1} = \frac{W_i}{V_1 I_1^2 \cos \phi_1}$$

$$\Rightarrow I_1^2 R_{01} = W_i$$

$\Rightarrow$  copper losses = iron losses

Hence the transformer efficiency will be maximum when copper losses are equal to iron losses.

$$\therefore I_1^2 R_{01} \text{ (or)} I_2^2 R_{02} = W_i \quad (5.21)$$

Current corresponding to maximum efficiency is

$$I_1 = \sqrt{\frac{W_i}{R_{01}}} \quad \text{(or)} \quad I_2 = \sqrt{\frac{W_i}{R_{02}}} \quad (5.22)$$

### 5.16.2 Efficiency at any Desired Load

In general, the efficiency at any desired load can be calculated as follows:

Let  $S$  = Full load kVA of the transformer

$W_{cu}$  = Full load copper loss

$\cos \phi$  = p.f of the load

$x$  = ratio of actual load to full load

For example, at  $\frac{1}{2}$  F.L,  $x = 1/2$  & at  $\frac{1}{4}$  F.L,  $x = 1/4$

$\therefore$  output of the transformer =  $xS \cos \phi$

Iron loss =  $W_i$

Copper loss =  $x^2 W_{cu}$

Therefore, the efficiency at any load is given by

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output power}}{\text{Output power} + \text{Losses}}$$

$$\eta_{\text{at any load}} = \frac{xS \cos \phi}{xS \cos \phi + W_i + x^2 W_{cu}} \quad (5.23)$$

Where  $x$  = Fraction of F.L. at which the transformer is working  
for maximum efficiency  
copper losses = iron losses

$$\begin{aligned} x^2 W_{cu} &= W_i \\ \Rightarrow x &= \sqrt{\frac{W_i}{W_{cu}}} = \sqrt{\frac{\text{Iron loss}}{\text{F.L. copper loss}}} \\ \Rightarrow \frac{kVA_{\max}}{\text{Full load kVA}} &= \sqrt{\frac{\text{Iron loss}}{\text{F.L. copper loss}}} \end{aligned} \quad (5.24)$$

$\therefore$  Load kVA corresponding to maximum efficiency is given by

$$kVA_{\max} = \text{full load kVA} \times \sqrt{\frac{W_i}{\text{full load copper loss}}} \quad (5.25)$$

It may be noted that the value of kVA at which the efficiency is maximum is independent of p.f. of the load.

**Solved Problem-11:** A 200 kVA, 1-phase transformer has an efficiency of 98% at full load. If the maximum efficiency occurs at  $\frac{3}{4}$  full load, calculate the

- (i) iron losses
- (ii) copper loss at F.L.
- (iii) efficiency at half -load

Assume a p.f. of 0.8 at all loads.

**Solution:** Given that

$$\text{kVA rating} = 200 \text{ kVA}$$

$$\eta_{FL} = 98\% \text{ at p.f.} = 0.8$$

$$\eta_{\max} \text{ occurs at } \frac{3}{4} \text{ full load}$$

$$\eta_{FL} = \frac{1 \times 200 \times 10^3 \times 0.8}{1 \times 200 \times 10^3 \times 0.8 + W_i + W_{cu}} = 0.98$$

$$\begin{aligned} \Rightarrow \frac{160 \times 10^3}{0.98} &= 160 \times 10^3 + W_i + W_{cu} \\ \Rightarrow W_i + W_{cu} &= 3265.3 \end{aligned} \quad (1)$$

$$\begin{aligned} \text{kVA}_{\max} &= \text{kVA}_{FL} \times \sqrt{\frac{W_i}{W_{cu}}} \\ \frac{3}{4} \times 200 \times 10^3 &= 200 \times 10^3 \times \sqrt{\frac{W_i}{W_{cu}}} \\ \Rightarrow \frac{W_i}{W_{cu}} &= \frac{9}{16} \\ \Rightarrow W_i &= 0.5625 W_{cu} \end{aligned}$$
(2)

From eqn. (1)  $0.5625 W_{cu} + W_{cu} = 3265.3$

$$\Rightarrow W_{cu} = \frac{3265.3}{1.5625} = 2090 \text{ W} = 2.09 \text{ kW}$$

From eqn. (2)  $W_i = 0.5625 \times 2.09 = 1.175 \text{ kW}$

- (i) Iron losses,  $W_i = 1.175 \text{ kW}$
- (ii) Copper loss at full load,  $W_{cu} = 2.09 \text{ kW}$
- (iii) Efficiency at half full load

$$\eta \text{ at } \frac{1}{2} \text{ F.L.} = \frac{\frac{1}{2} \times 200 \times 0.8}{\frac{1}{2} \times 200 \times 0.8 + W_i + \frac{1}{4} W_{cu}} \times 100 = \frac{80}{80 + \frac{2.09}{4} + 1.175} \times 100 = \frac{80 \times 100}{81.6975} = 97.9\%$$

**Solved Problem-12:** A 50kVA transformer on full load has a copper loss of 600W and iron loss of 500W. Calculate the maximum efficiency and the load at which it occurs. Assume load p.f. = 1.

**Solution:** Given that

$$\text{kVA rating} = 50$$

$$\text{Copper loss, } W_{cu} = 600 \text{ W}$$

$$\text{Iron loss, } W_i = 500 \text{ W}$$

At maximum efficiency,

$$\text{Copper loss} = \text{Iron loss} = 500 \text{ W}$$

$$\therefore x^2 W_{cu} = W_i$$

$$\Rightarrow x = \sqrt{\frac{W_i}{W_{cu}}} = \sqrt{\frac{500}{600}} = 0.9129$$

$$\therefore \text{Output at maximum efficiency} = xS \cos \phi = 0.9129 \times 50 \times 1 = 45.645 \text{ kW}$$

Therefore, maximum efficiency,

$$\eta = \frac{\text{Output power}}{\text{Output power} + \text{Losses}} = \frac{45.645}{45.645 + 0.5 + 0.5} \times 100 = 97.86\%$$

∴ Load kVA corresponding to maximum efficiency is given by

$$\text{kVA}_{\max} = \text{full load kVA} \times \sqrt{\frac{W_i}{\text{full load copper loss}}} = 50 \times \sqrt{\frac{500}{600}} = 45.645 \text{kVA}$$

**Solved Problem-13:** The efficiency of a 200 kVA, single phase transformer is 98% when operating at full load, 0.8 p.f. lagging. The iron loss in the transformer is 2000W. Calculate:

- (i) Full load copper loss    (ii) Half-full load copper loss

**Solution:** Given that

$$\text{kVA rating} = 200$$

$$\eta = 98\% \text{ at full load, at p.f.} = 0.8$$

$$\text{Iron loss in the transformer, } W_i = 2000 \text{W}$$

$$\text{Full load output} = xS \cos \phi = 1 \times 200 \times 0.8 = 160 \text{ kW}$$

$$\eta = \frac{\text{Output power}}{\text{Input power}} \Rightarrow \text{Input power} = \frac{160}{0.98} = 163.265$$

$$\text{Total losses} = \text{input power} - \text{output power} = 163.265 - 160 = 3.265 \text{ kW}$$

$$\text{i.e., } W_i + W_{cu} = 3265$$

$$W_{cu} = 3265 - W_i = 3265 - 2000 = 1265 \text{W}$$

$$\text{Therefore, Full load copper loss} = 1265 \text{W}$$

$$\text{At half full load, } x = 1/2 = 0.5$$

$$\text{Half - full load copper loss} = x^2 W_{cu} = 0.5^2 \times 1265 = 316.25 \text{W}$$

## 5.17 Regulation of a Transformer

When the transformer is loaded its terminal voltage falls from no load to full load. The change in secondary terminal voltage from no load to full load and expressed as secondary no load voltage is known as regulation down.

$$\therefore \% \text{ regulation down} = \left( \frac{{}_0V_2 - V_2}{{}_0V_2} \right) \times 100 \quad (5.26)$$

If the change in voltage is expressed as secondary full-load voltage then it is known as regulation up.

$$\% \text{ regulation up} = \left( \frac{{}_0V_2 - V_2}{V_2} \right) \times 100 \quad (5.27)$$

Where  ${}_0V_2$  = No-load secondary voltage = KV<sub>1</sub>

$V_2$  = Secondary voltage on load

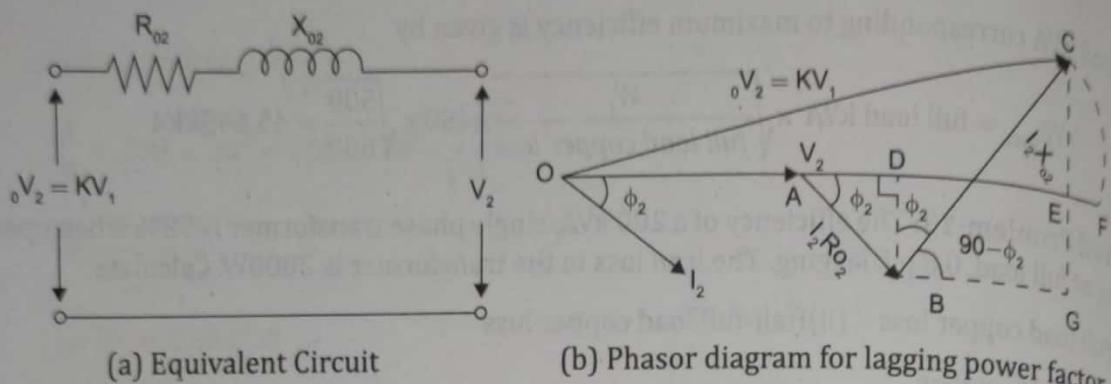


Figure (5.24)

Unless stated otherwise, regulation is to be taken as regulation down. Lesser the regulation better the transformer performance. It may be noted that percentage voltage regulation of the transformer will be the same whether primary or secondary side.

From the phasor diagram, considering  $OE \approx OF$  (by neglecting EF)

$$\therefore OC = OE = OA + AD + DE$$

$$_0V_2 = V_2 + AD + DE \quad (5.28)$$

$$\text{From } DADB \cos\phi_2 = \frac{AD}{AB}$$

$$\Rightarrow AD = AB \cos\phi_2$$

$$\Rightarrow AD = I_2 R_{02} \cos\phi_2 \quad (5.29)$$

$$\text{From } DBCG \cos(90 - \phi_2) = \frac{BG}{BC}$$

$$\Rightarrow \sin\phi_2 = \frac{DE}{BC}$$

$$\Rightarrow DE = BC \sin\phi_2$$

$$\Rightarrow DE = I_2 X_{02} \sin\phi_2 \quad (5.30)$$

From eqns (5.28), (5.29) & (5.30)

$$_0V_2 = V_2 + AD + DE$$

$$= V_2 + I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2$$

$$_0V_2 - V_2 = I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2$$

In general the approximate voltage drop is given by

$$\text{Voltage drop} = _0V_2 - V_2 = I_2 R_{02} \cos\phi_2 \pm I_2 X_{02} \sin\phi_2 \quad (5.31)$$

**Note:** +ve sign for lagging p.f. & -ve sign for leading p.f.

$$\therefore \% \text{ Voltage regulation} = \left( \frac{{V_2 - V_2}}{{V_2}} \right) \times 100 = \left( \frac{I_2 R_{02} \cos \phi_2 + I_2 X_{02} \sin \phi_2}{{V_2}} \right) \times 100 \quad (5.32)$$

### 5.17.1 Condition for Zero Regulation

The regulation is zero if

$$I_2 R_{02} \cos \phi_2 + I_2 X_{02} \sin \phi_2 = 0$$

$$I_2 R_{02} \cos \phi_2 = I_2 X_{02} \sin \phi_2$$

$$\Rightarrow \tan \phi_2 = -\frac{R_{02}}{X_{02}}$$

$$\Rightarrow \phi_2 = \tan^{-1} \left( -\frac{R_{02}}{X_{02}} \right)$$

The negative sign indicates that zero regulation occurs at leading power factors i.e., for capacitive loads.

### 5.17.2 Condition for Maximum Regulation

The regulation will be maximum if

$$\frac{d}{d\phi_2} (\text{regulation}) = 0$$

$$\Rightarrow \frac{d}{d\phi_2} \left( \frac{I_2 R_{02} \cos \phi_2 + I_2 X_{02} \sin \phi_2}{{V_2}} \right) = 0$$

$$\Rightarrow -I_2 R_{02} \sin \phi_2 + I_2 X_{02} \cos \phi_2 = 0$$

$$\Rightarrow I_2 R_{02} \sin \phi_2 = I_2 X_{02} \cos \phi_2$$

$$\Rightarrow \tan \phi_2 = \frac{X_{02}}{R_{02}} \Rightarrow \phi_2 = \tan^{-1} \left( \frac{X_{02}}{R_{02}} \right)$$

Thus, maximum regulation occurs at lagging power factor of the load. The lagging power factor angle of the load is equal to the angle of the equivalent impedance of the transformer.

**Solved Problem-14:** A 10 kVA single phase transformer for 2000/400V at no load, has  $R_1 = 4.5\Omega$ ,  $X_1 = 12\Omega$ ,  $R_2 = 0.2\Omega$ ,  $X_2 = 0.45\Omega$ . Determine the approximate value of the secondary voltage at full load, 0.8 power factor lagging when the primary applied voltage is 2000 V.

**Solution:** Given that

$$\text{kVA rating of transformer} = 10$$

$$E_1/E_2 = 2000/400 \text{ V}$$

$$R_1 = 4.5\Omega, X_1 = 12\Omega, R_2 = 0.2\Omega, X_2 = 0.45\Omega$$

$$K = \frac{V_1}{V_2} = \frac{400}{2000} = 0.2$$

$$R_{02} = R_2 + K^2 R_1 = 0.2 + 4.5 \times 0.2^2 = 0.38\Omega$$

$$X_{02} = X_2 + K^2 X_1 = 0.45 + 0.2^2 \times 12 = 0.93\Omega$$

$$\text{Full load secondary current, } I_2 = \frac{\text{kVA rating}}{V_2} = \frac{10 \times 1000}{400} = 25A$$

$$\cos\phi_2 = 0.8 \Rightarrow \sin\phi_2 = 0.6$$

$$_0V_2 = E_2 = KE_1 = KV_1 = 2000 \times 0.5 = 400V$$

Approximate voltage drop,

$$_0V_2 - V_2 = I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2$$

$$400 - V_2 = 25 \times 0.38 \times 0.8 + 25 \times 0.93 \times 0.6$$

$$\Rightarrow V_2 = 400 - 7.6 - 13.95 = 378.45V$$

**Solved Problem-15:** A 20kVA, 2500/500V, single phase transformer has the following parameters

HV winding:  $R = 8\Omega, X = 1.7\Omega$

LV winding:  $R = 0.3\Omega, X = 0.7\Omega$

Find the voltage regulation and secondary terminal voltage at full load for a p.f. of

- (i) 0.8 lag
- (ii) 0.8 lead

**Solution:** Given that

$$\text{kVA rating of transformer} = 20$$

$$E_1/E_2 = 2500/500V$$

$$K = \frac{E_2}{E_1} = \frac{500}{2500} = 0.2$$

$$R_1 = 8\Omega, X_1 = 1.7\Omega, R_2 = 0.3\Omega, X_2 = 0.7\Omega$$

$$R_{02} = R_2 + K^2 R_1 = 0.3 + 8 \times 0.2^2 = 0.62\Omega$$

$$X_{02} = X_2 + K^2 X_1 = 0.7 + 0.2^2 \times 1.7 = 0.768\Omega$$

$$\text{Full load secondary current, } I_2 = \frac{kVA \text{ rating}}{V_2} = \frac{20 \times 1000}{500} = 40A$$

$$_0V_2 = E_2 = KE_1 = KV_1 = 2500 \times 0.2 = 500V$$

$$\begin{aligned} \text{(i)} \quad & \cos\phi_2 = 0.8 \text{ lag} \\ & \Rightarrow \sin\phi_2 = 0.6 \end{aligned}$$

Approximate voltage drop,

$$_0V_2 - V_2 = I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2$$

$$500 - V_2 = 40 \times 0.62 \times 0.8 + 40 \times 0.768 \times 0.6 = 38.272$$

$$\Rightarrow V_2 = 500 - 38.272 = 461.73V$$

$$\% \text{ Voltage regulation} = \left( \frac{I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2}{_0V_2} \right) \times 100 = \frac{38.272}{500} \times 100 = 7.65\%$$

$$\text{(ii)} \quad \cos\phi_2 = 0.8 \text{ lead}$$

$$\Rightarrow \sin\phi_2 = 0.6$$

Approximate voltage drop,

$$_0V_2 - V_2 = I_2 R_{02} \cos\phi_2 - I_2 X_{02} \sin\phi_2$$

$$500 - V_2 = 40 \times 0.62 \times 0.8 - 40 \times 0.768 \times 0.6 = 1.408$$

$$\Rightarrow V_2 = 500 - 1.408 = 498.59V$$

$$\% \text{ Voltage regulation} = \left( \frac{I_2 R_{02} \cos\phi_2 - I_2 X_{02} \sin\phi_2}{_0V_2} \right) \times 100 = \frac{1.408}{500} \times 100 = 0.2816\%$$

## 5.18 Rating of the Transformer

An important factor in the design and operation of electrical machines is the relation between the life of the insulation and operating temperature of the machine. Therefore, temperature rise resulting from the losses is a determining factor in the rating of a machine. We know that copper loss in a transformer depends on current and iron loss depends on voltage. Therefore, the total loss in a transformer depends on the volt-ampere product only and not on the phase angle between voltage and current i.e., it is independent of load power factor. For this reason, the rating of a transformer is in kVA and not kW.

## 5.19 Distribution & Power Transformers

### 5.19.1 Distribution Transformers

Transformers used to step down the distribution voltage to a standard service voltage or from transmission voltage to distribution voltage are known as distribution transformers. They are kept in operation all the 24 hours a day whether they are carrying any load or not. They have a good voltage regulation and are designed for a small value of leakage reactance.

### 5.19.2 Power Transformers

Power Transformers are used in generating stations or substations at each end of a power transmission line for stepping up or stepping down the voltage. They are put in operation during load periods and are disconnected during light periods. They are designed to have maximum efficiency at or near full load. These are designed to have considerably greater leakage reactance. In these transformers, the voltage regulation is less important than the current limiting effect of higher leakage reactance.

### 5.19.3 Comparison between Distribution & Power Transformers

S. No.	Distribution Transformers	Power Transformers
01	Used to supply power for different consumers	Used in generating stations and substations
02	Always step-down transformers	Step-up or step-down transformers
03	The secondary of this transformer is usually star connected	The secondary of this transformer is usually delta connected
04	Kept in operation for all the 24 hours in a day	Operates mostly when load exists
05	Iron losses takes place always	Losses takes place when loaded
06	Designed to have maximum efficiency at about 50% of full load	Designed to have maximum efficiency at nearly full load
07	Usually ratings are upto 500kVA	Ratings will vary depending upon service
08	These have good voltage regulation	Voltage regulation is less important

### 5.20 Applications of Single Phase Transformers

Transformers are used in a number of applications:

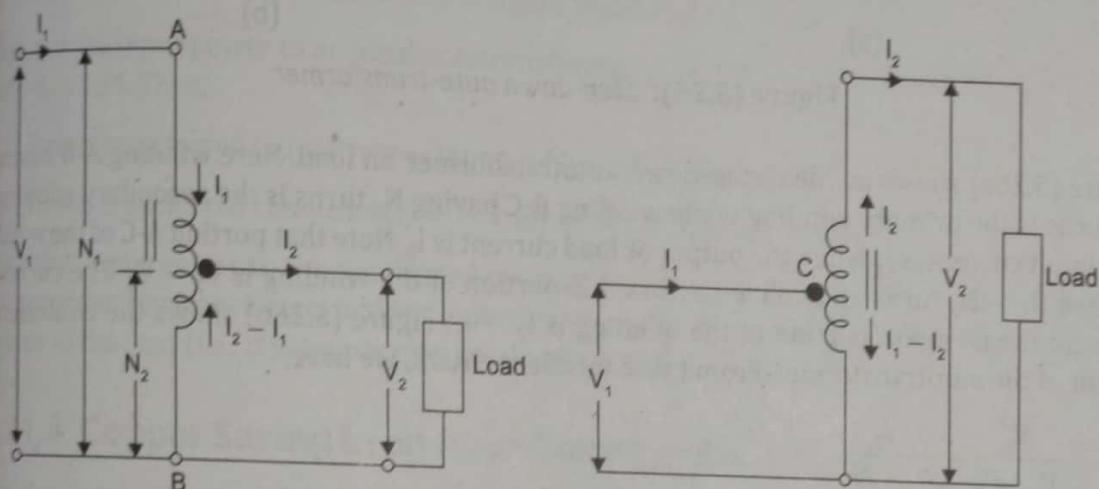
- (i) To change the level of voltage and current in electric power systems.  
Electric energy can be generated at the most economic level at ( $11kV - 33kV$ ). Stepping of generated voltage to high voltage, extra high voltage ( $>230kV$ ), or even ultra high voltage ( $\geq 750kV$ ) to suit the power transmission requirement to minimize losses and increase transmission capacity of lines. The transmission voltage is stepped down in many stages for distribution and utilization of domestic, commercial and industrial consumers.
- (ii) As an impedance matching device for maximum power transfer in low power electronic and control circuits.
- (iii) As a coupling device.
- (iv) To measure voltage and currents: these are known as instrument transformers.
- (v) Converting HVAC to HVDC in combined AC/DC power systems.

## 5.21 Auto-Transformer

A transformer, in which a part of the winding is common to both primary and secondary circuits, is called an auto-transformer. Obviously, in this transformer the primary and secondary are not electrically isolated from each other as is the case with a 2-winding transformer. But its theory and operation are similar to those of a 2-winding transformer. Because of one winding, it uses less copper and hence, is cheaper. It is used where transformation ratio differs little from unity.

### 5.21.1 Types of Auto-Transformers

An auto-transformer may step-down or step-up the voltage. The auto-transformers are divided as (i) Step-down auto-transformer and (ii) Step-up auto-transformer



(a) Step-down transformer

(b) Step-up transformer

Figure (5.25): Auto-transformer

- (i) **Step-Down Auto-Transformer:** In this case, the complete winding acts as primary winding while the tapped section of this winding works as secondary winding as shown in figure (5.25a). The current in section CB is vector difference of  $I_2$  and  $I_1$ . But as the two currents are practically in phase opposition, the resultant current is  $(I_2 - I_1)$  where  $I_2$  is greater than  $I_1$ .
- (ii) **Step-Up Auto-Transformer:** In this case, the whole winding works as a secondary winding and its portion performs the function of primary winding as shown in figure (5.25b). The current in section CB is vector difference of  $I_1$  and  $I_2$ . But as the two currents are practically in phase opposition, the resultant current is  $(I_1 - I_2)$  where  $I_2$  is less than  $I_1$ .

### 5.21.2 Transformation Ratio of an Auto-Transformer

Neglecting the losses, the leakage reactance and magnetizing current, the transformation ratio of an auto transformer can be obtained as

$$K = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

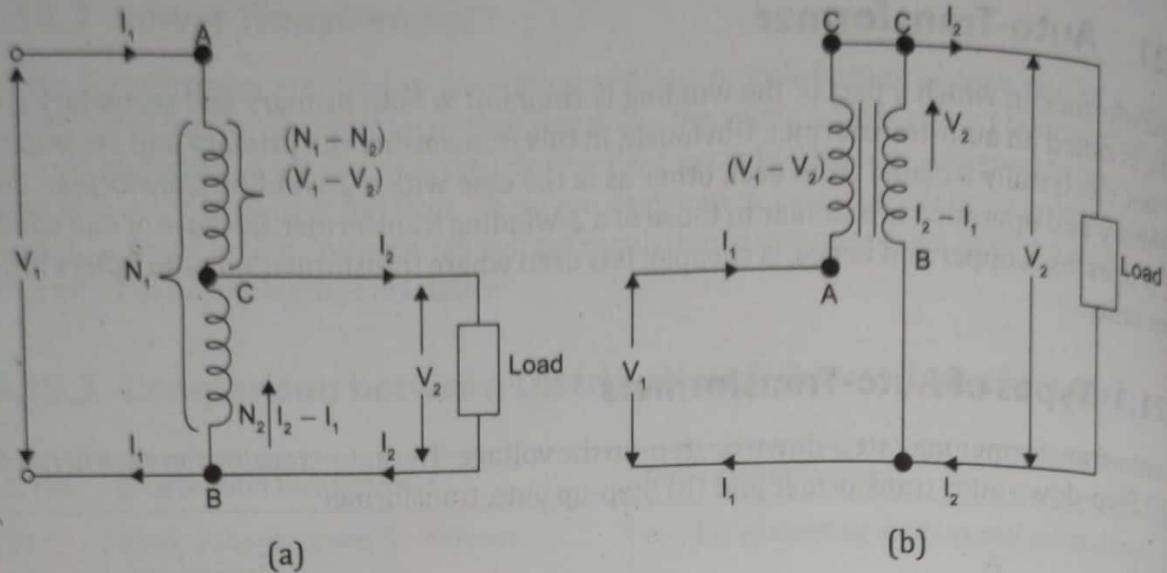


Figure (5.26): Step-down auto-transformer

Figure (5.26a) shows an ideal step-down autotransformer on load. Here winding A-B having  $N_1$  turns is the primary winding while winding B-C having  $N_2$  turns is the secondary winding. The input current is  $I_1$  while the output or load current is  $I_2$ . Note that portion A-C of the winding has  $N_1 - N_2$  turns and voltage across this portion of the winding is  $V_1 - V_2$ . The current through the common portion of the winding is  $I_2 - I_1$ . Figure (5.26b) shows the equivalent circuit of the autotransformer. From this equivalent circuit, we have,

$$\begin{aligned} \frac{V_2}{V_1 - V_2} &= \frac{N_2}{N_1 - N_2} \\ V_2(N_1 - N_2) &= N_2(V_1 - V_2) \\ \Rightarrow V_2 N_1 &= N_2 V_1 \\ \Rightarrow \frac{V_2}{V_1} &= \frac{N_2}{N_1} = K \end{aligned} \quad (5.33)$$

Also

$$\begin{aligned} I_1(V_1 - V_2) &= (I_2 - I_1)V_2 \\ \Rightarrow -V_2 I_1 + V_1 I_1 &= V_2 I_2 - V_2 I_1 \\ \Rightarrow V_1 I_1 &= V_2 I_2 \\ \Rightarrow \frac{V_2}{V_1} &= \frac{I_1}{I_2} = K \end{aligned} \quad (5.34)$$

From eqns.(5.33) & (5.34)

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K \quad (5.35)$$

Also  $V_2 I_2 = V_1 I_1$  (Input apparent power = Output apparent power)

### 5.21.3 Power Transfer in an Auto-Transformer

Since the primary and secondary windings of an autotransformer are connected magnetically as well as electrically, the power from primary is transferred to the secondary inductively (transformer action) as well as conductively (i.e., directly as windings are electrically connected).

$$\text{Output apparent power} = V_2 I_2$$

$$\begin{aligned}\text{Apparent power transferred inductively} &= V_2(I_2 - I_1) = V_2(I_2 - K I_2) \\ &= V_2 I_2 (1 - K) = V_1 I_1 (1 - K)\end{aligned}$$

$$\therefore \text{Power transferred inductively} = \text{input} \times (1 - K) \quad (5.36)$$

$$\text{Power transferred conductively} = \text{input} - \text{input} \times (1 - K) = K \times \text{input} \quad (5.37)$$

Suppose the input power to an ideal autotransformer is 500 W and its voltage transformation ratio  $K = 1/4$ . Then,

$$\text{Power transferred inductively} = \text{input} \times (1 - K) = 500(1 - 1/4) = 375\text{W}$$

$$\text{Power transferred conductively} = K \times \text{input} = 1/4 \times 500 = 125\text{W}$$

Note that input power to the autotransformer is 500 W. Out of this, 375 W is transferred to the secondary by transformer action (inductively) while 125 W is conducted directly from the source to the load (i.e., it is transferred conductively to the load).

### 5.21.4 Copper Saving in an Auto-Transformer

For the same output and voltage transformation ratio  $K (= N_2/N_1)$ , an autotransformer requires less copper than an ordinary 2-winding transformer. Figure (5.27a) shows an ordinary 2-winding transformer whereas figure 5.27(b) shows an autotransformer having the same output and voltage transformation ratio  $K$ .

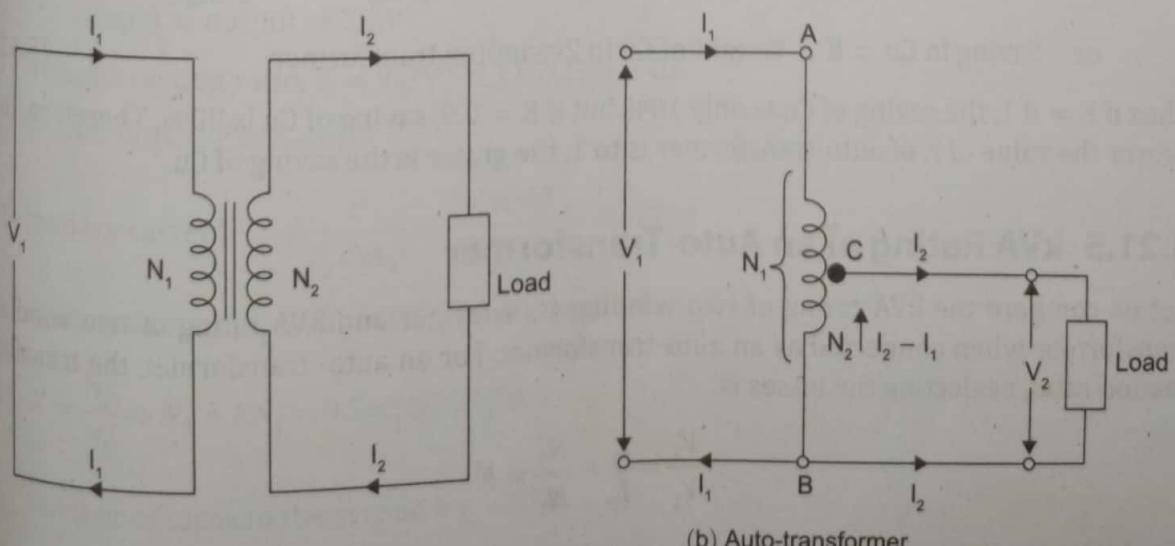


Figure (5.27)

The length of copper required in a winding is proportional to the number of turns and the area of cross-section of the winding wire is proportional to the current rating. Therefore, the volume and hence weight of copper required in a winding is proportional to current  $\times$  turns i.e.,

Weight of Cu required in a winding  $\propto$  current  $\times$  turns

2-winding transformer:

$$\text{Weight of Cu required in a winding} \propto (I_1 N_1 + I_2 N_2) \quad (5.38)$$

Auto-transformer:

$$\text{Weight of Cu required in section A-C} \propto I_1 (N_1 - N_2)$$

$$\text{Weight of Cu required in section C-B} \propto (I_2 - I_1) N_2$$

$$\therefore \text{Total weight of Cu required} \propto I_1 (N_1 - N_2) + (I_2 - I_1) N_2 \quad (5.39)$$

$$\begin{aligned} \frac{\text{Weight of Cu in auto-transformer}}{\text{Weight of Cu in 2-winding transformer}} &= \frac{I_1 (N_1 - N_2) + (I_2 - I_1) N_2}{(I_1 N_1 + I_2 N_2)} \\ &= \frac{I_1 N_1 - I_1 N_2 + I_2 N_2 - I_1 N_2}{(I_1 N_1 + I_2 N_2)} = \frac{I_1 N_1 + I_2 N_2 - 2I_1 N_2}{(I_1 N_1 + I_2 N_2)} \\ &= 1 - \frac{2I_1 N_2}{(I_1 N_1 + I_2 N_2)} = 1 - \frac{2I_1 N_2}{2I_1 N_1} \quad (\because I_1 N_1 = I_2 N_2) \\ &= 1 - \frac{N_2}{N_1} = 1 - K \end{aligned}$$

$$\therefore \text{Weight of Cu in auto-transformer} = (1 - K) \times \text{Weight of Cu in 2-winding transformer}$$

$$\text{or } W_{auto} = (1 - K) W_{2-wdg} \quad (5.40)$$

$$\therefore \text{Saving in Cu} = W_{2-wdg} - W_{auto} = W_{2-wdg} - (1 - K) W_{2-wdg} = K \times W_{2-wdg}$$

$$\text{or } \text{Saving in Cu} = K \times \text{Weight of Cu in 2-winding transformer} \quad (5.41)$$

Thus if  $K = 0.1$ , the saving of Cu is only 10% but if  $K = 0.9$ , saving of Cu is 90%. Therefore, the nearer the value of K of auto-transformer is to 1, the greater is the saving of Cu.

### 5.21.5 kVA Rating of an Auto-Transformer

Let us compare the kVA rating of two winding transformer and kVA rating of two winding transformer when connected as an auto-transformer. For an auto-transformer, the transformation ratio, neglecting the losses is

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$$

The kVA rating of an auto-transformer is

$$(kVA)_{auto} = V_1 I_1 = V_2 I_2$$

If the same transformer as shown in figure (5.27b) is used as a 2-winding transformer such that winding AC acts as the primary and winding BC as the secondary, then its kVA rating can be obtained as

$$(kVA)_{2-wdg} = (V_1 - V_2) I_1 = V_2 (I_2 - I_1)$$

$$\frac{(kVA)_{auto}}{(kVA)_{2-wdg}} = \frac{V_1 I_1}{(V_1 - V_2) I_1} = \frac{V_1 I_1}{V_1 I_1 (1 - V_2/V_1)} = \frac{1}{1 - V_2/V_1} = \frac{1}{1 - K}$$

$$\Rightarrow (kVA)_{auto} = \frac{1}{1 - K} (kVA)_{2-wdg}$$

Also

$$\frac{(kVA)_{auto}}{(kVA)_{2-wdg}} = \frac{V_2 I_2}{V_2 (I_2 - I_1)} = \frac{V_2 I_2}{V_2 I_2 (1 - I_1/I_2)} = \frac{1}{1 - I_1/I_2} = \frac{1}{1 - K}$$

$$\Rightarrow (kVA)_{auto} = \frac{1}{1 - K} (kVA)_{2-wdg} \quad (5.42)$$

**Solved Problem-16:** An auto-transformer supplies a load of 3 kW at 115V at a unity power factor. If the applied primary voltage is 230V, calculate:

- (i) transformation ratio
- (ii) secondary current
- (iii) primary current
- (iv) number of turns in the secondary if total number of turns is 500
- (v) power transformed
- (vi) power conducted directly from the supply mains to the load

**Solution:** Given that

$$V_1 = 230V, V_2 = 115V$$

$$\text{Input} = \text{output} = 3\text{kW}$$

$$(i) \text{ Transformation ratio, } K = V_2/V_1 = 115/230 = 0.5$$

$$(ii) \text{ Power output, } P = V_2^2 \cos\phi$$

$$\therefore \text{Secondary current, } I_2 = \frac{P}{V_2 \cos\phi} = \frac{3 \times 1000}{115 \times 1} = 26.087A$$

$$(iii) \text{ Primary current, } I_1 = K I_2 = 0.5 \times 26.087 = 13.04$$

$$(iv) K = \frac{N_2}{N_1} \Rightarrow N_2 = K N_1 = 0.5 \times 500 = 250$$

$$\text{Number of turns in the secondary} = 250$$

$$(v) \text{ Power transferred inductively} = \text{Input} \times (1 - K) = 3 \times (1 - 0.5) = 1.5\text{kW}$$

$$(vi) \text{ Power transferred conductively} = \text{Input} \times K = 3 \times 0.5 = 1.5\text{kW}$$

### 5.21.6 Conversion of Two-Winding Transformer Into Auto-Transformer

Any 2-winding transformer can be converted into an auto-transformer either step-down or step-up by connecting the two windings electrically in series. Figure (5.28a) shows such a transformer with its polarity markings. If we employ additive polarity between the high-voltage and low-voltage sides, we get a step-up auto-transformer. If, however, we use the subtractive polarity, we get a step-down auto-transformer.

Let us consider a conventional 20 kVA, 2400/240 V transformer to be connected in auto-transformer configuration.

#### (i) Additive Polarity

Figure (5.28b) shows the series connections of the windings with additive polarity. The circuit is redrawn in figure (5.28c) showing common terminal of the transformer at the top whereas figure (5.28d) shows the same circuit with common terminal at the bottom. Because of additive polarity,  $V_2 = 2400 + 240 = 2640$  V and  $V_1$  is 2400 V, the transformer acts as a step-up transformer. As shown in figure 5.28(d), common current flows towards the common terminal.

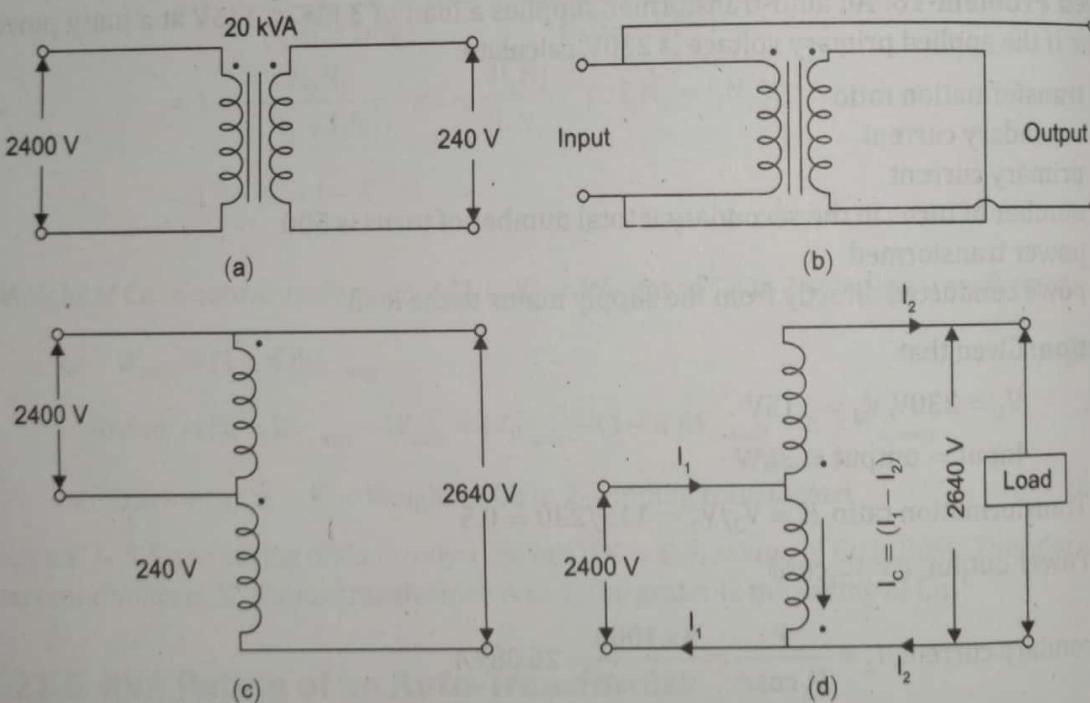
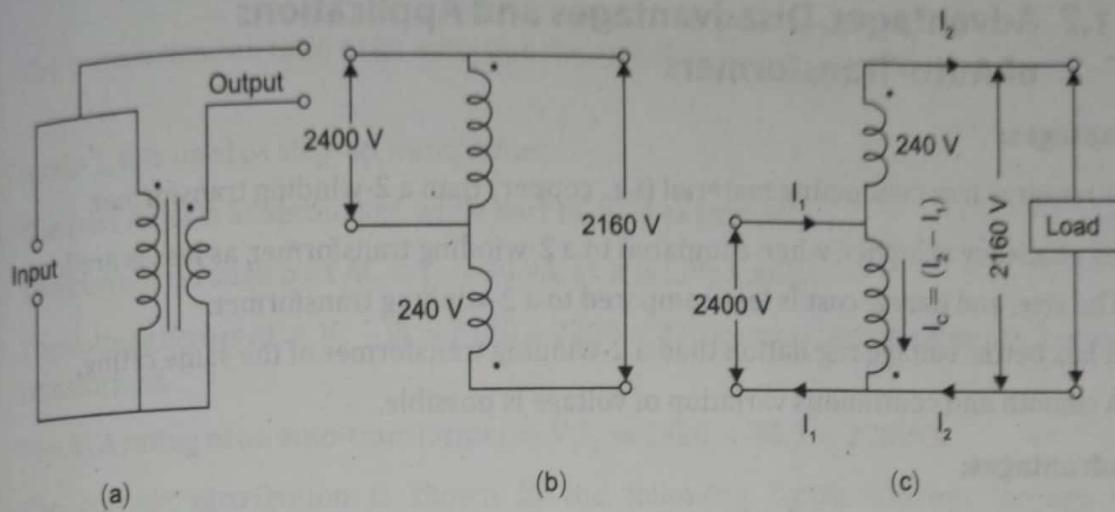


Figure (5.28) Conversion of 2-winding transformer into auto-transformer

#### (b) Subtractive Polarity:

Figure (5.29a) shows the series connections of the windings with subtractive polarity. The circuit has been redrawn with common polarity at top in figure 5.29(b) and at bottom in figure 5.29(c). Since the polarity is subtractive  $V_1 = 2400$  V and  $V_2 = 2400 - 240 = 2160$  V, the transformer acts as a step-down auto-transformer. The common current flows away from the common terminal.



**Figure (5.29)** Conversion of 2-siding transformer into auto-transformer

**Solved Problem-17:** The primary and secondary voltages of an auto-transformer are 500 V and 400 V, respectively. Show with the aid of diagram, the current distribution in the winding when the secondary current is 100 A and calculate the economy of Cu in this particular case.

**Solution:** Given that

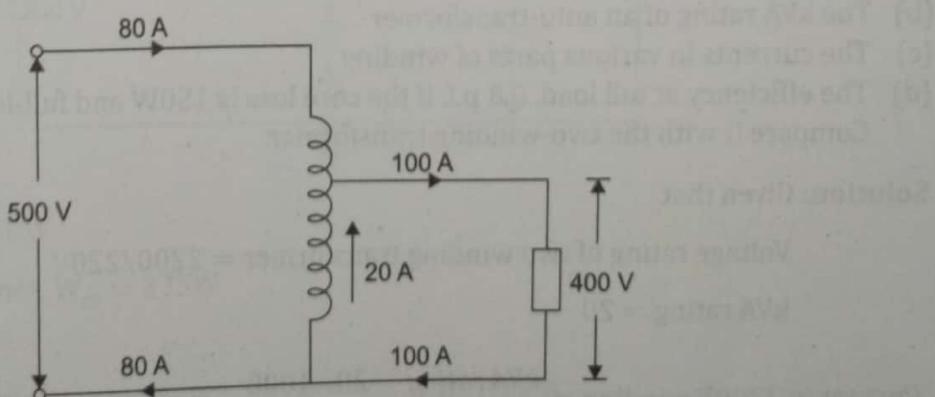
$$V_1 = 500 \text{ V}, V_2 = 400 \text{ V}$$

$$I_2 = 100 \text{ A}$$

$$\text{Transformation ratio} = V_2/V_1 = 400/500 = 0.8$$

$$\therefore I_1 = K I_2 = 0.8 \times 100 = 80 \text{ A}$$

The current distribution is shown in the following figure.



$$\text{Saving in copper} = K \times W_{2-wdg} = 0.8W_{2-wdg}$$

$$\therefore \text{Percentage saving} = 0.8 \times 100 = 80$$

### 5.21.7 Advantages, Disadvantages and Applications of Auto-Transformers

#### Advantages:

- It requires less conducting material (i.e., copper) than a 2-winding transformer.
- Its efficiency is higher, when compared to a 2-winding transformer, as losses are less.
- The size, and hence, cost is less compared to a 2-winding transformer.
- It has better voltage regulation than a 2-winding transformer of the same rating.
- A smooth and continuous variation of voltage is possible.

#### Disadvantages:

- The two windings are not electrically separate and in case of failure of insulation between the two, a severe shock may be felt on the low voltage side.
- The use of auto-transformer is more economical only when the transformation ratio K is nearer to unity.

#### Applications:

- Generally used for starting of Induction motors.
- These are used to compensate for voltage drops in transmission and distribution lines. When used for this purpose, they are known as booster transformers.
- Autotransformers are used for continuously variable supply.
- These are used in control equipment for 1-phase and 3-phase electrical locomotives.
- It is used as interconnecting transformers in 132 kV/330 kV system.

**Solved Problem-18:** A 2200/220V, 20kVA, two winding transformer is connected as an auto-transformer to transform 2200V to 2420V. Find:

- The transformation ratio of an auto-transformer
- The kVA rating of an auto-transformer
- The currents in various parts of winding
- The efficiency at full load, 0.8 p.f. if the core loss is 150W and full-load copper loss 275W. Compare it with the two-winding transformer.

**Solution:** Given that

$$\text{Voltage rating of two winding transformer} = 2200/220V$$

$$\text{kVA rating} = 20$$

$$\text{Current in } 2200\text{V winding} = \frac{\text{kVA rating}}{2200} = \frac{20 \times 1000}{2200} = 9.09A$$

$$\text{Current in } 220\text{V winding} = \frac{\text{kVA rating}}{220} = \frac{20 \times 1000}{220} = 90.9A$$

(a) The transformation ratio of an auto-transformer,  $K = \frac{V_2}{V_1} = \frac{2420}{2200} = 1.1$

As  $K > 1$ , it is used as step-up transformer

(b) The part AB acts as secondary, while part BC acts as primary.

The current through part AC =  $I_2 = 90.9A$ , as it is fully loaded.

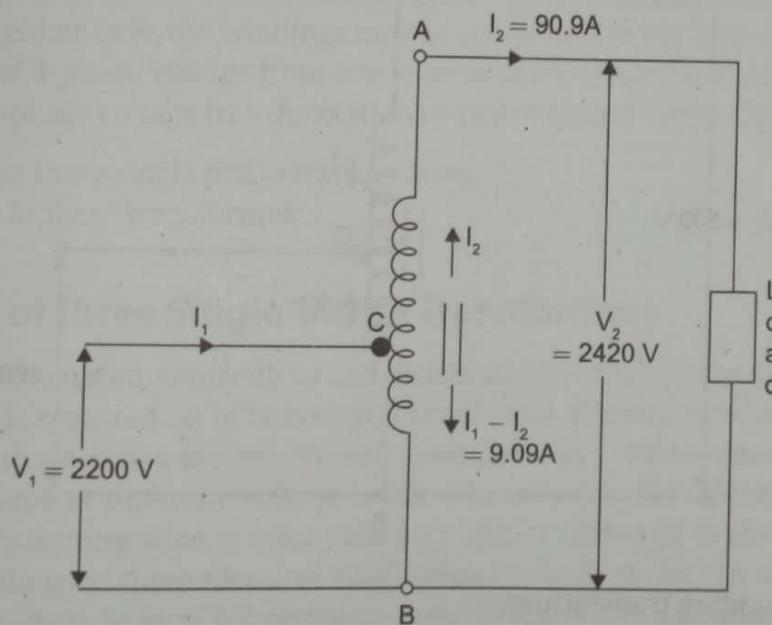
The voltage across AC =  $V_2 - V_1 = 2420 - 2200 = 220V$ , secondary winding of a 2-winding transformer.

The kVA rating of an auto-transformer =  $V_2 I_2 = 2420 \times 90.9 = 220\text{kVA}$

(c) The current distribution is shown in the following figure. Current through part AC =  $I_2 = 90.9A$

$$\text{Now } K = \frac{I_1}{I_2} = 1.1 \Rightarrow I_1 = 1.1 \times 90.9 = 100A$$

Current through part BC =  $I_1 - I_2 = 100 - 90.9 = 9.09A$



(d) Core loss,  $W_i = 150W$

Full-load copper loss,  $W_{cu} = 275W$

$$\text{p.f.} = \cos\phi = 0.8$$

$$\eta_{auto} = \frac{kVA \text{ rating} \times \text{p.f.}}{kVA \text{ rating} \times \text{p.f.} + W_i + W_{cu}} \times 100 = \frac{220 \times 0.8}{220 \times 0.8 + 0.15 + 0.275} \times 100 = 99.759\%$$

$$\eta_{2-wdg} = \frac{kVA \text{ rating} \times \text{p.f.}}{kVA \text{ rating} \times \text{p.f.} + W_i + W_{cu}} \times 100 = \frac{20 \times 0.8}{20 \times 0.8 + 0.15 + 0.275} \times 100 = 97.4\%$$

It can be seen that an auto-transformer gives more efficiency on full load as compared to a 2-winding transformer, with same excitation voltage and winding currents.

**Solved Problem-19:** A 400/100V, 10kVA, two winding transformer is connected as an auto-transformer to supply a 400V circuit from 500V source. When tested as a 2-winding transformer at rated load of 0.8 p.f. lagging, its efficiency is 96%. Determine its kVA rating and efficiency as an auto-transformer.

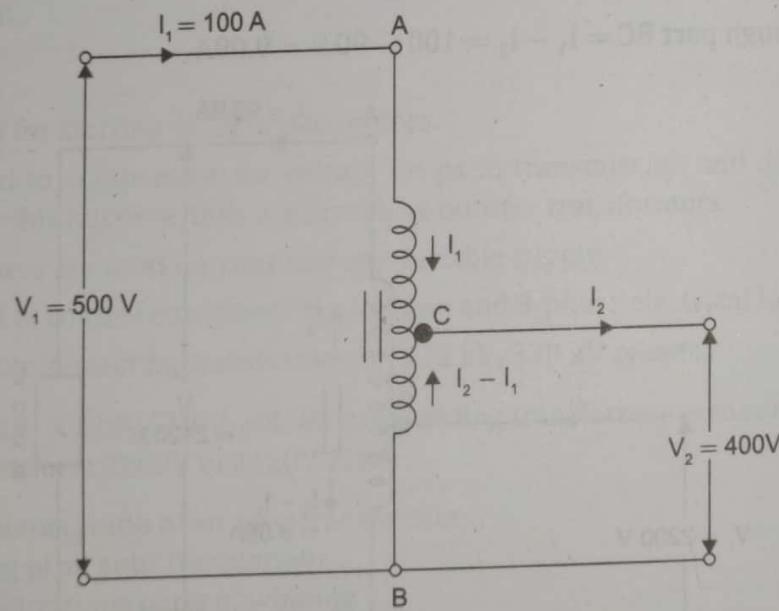
**Solution:** Given that

$$\text{Voltage rating of two winding transformer} = 400/100\text{V}$$

$$\text{kVA rating} = 10$$

$$\text{Current in } 400\text{V winding} = \frac{\text{kVA rating}}{400} = \frac{10 \times 1000}{400} = 25\text{A}$$

$$\text{Current in } 100\text{V winding} = \frac{\text{kVA rating}}{100} = \frac{10 \times 1000}{100} = 100\text{A}$$



When used as 2-winding transformer:

$$\eta_{2-\text{wdg}} = \frac{\text{kVA rating} \times \text{p.f.}}{\text{kVA rating} \times \text{p.f.} + W_i + W_{cu}} \times 100$$

$$96 = \frac{10 \times 0.8}{10 \times 0.8 + \text{total losses}} \times 100$$

$$\Rightarrow \text{total losses} = 333.3\text{W}$$

When used as an auto-transformer:

$$\eta_{auto} = \frac{\text{kVA rating} \times \text{p.f.}}{\text{kVA rating} \times \text{p.f.} + \text{Total losses}} \times 100$$

$$I_1 = \frac{kVA \text{ rating}}{100V} = \frac{10 \times 1000}{100} = 100A$$

$$\therefore kVA \text{ rating of auto-transformer} = \frac{V_1 I_1}{1000} = \frac{500 \times 100}{1000} = 50kVA$$

$$\eta_{\text{auto}} = \frac{kVA \text{ rating} \times p.f.}{kVA \text{ rating} \times p.f. + \text{Total losses}} \times 100 = \frac{50 \times 0.8}{50 \times 0.8 + 0.333} \times 100 = 99.17\%$$

## 5.22 Three-Phase Transformer

The generation of electrical power is usually three phase. To step-up the generated voltages for transmission purposes, and to step-down the transmission voltages for distribution purposes, it is necessary to transform the 3-phase voltage system to a higher or lower value. Three-phase circuits are the most economical for AC power transmission and distribution. As a consequence, 3-phase transformers are the most widely used in power systems. A 3-phase transformer may be a single unit (all windings wound around the same core, immersed in one tank) or it may be made up of three single-phase units. In practice the choice between one or another type is governed mainly by economical reasons, transportation, future expansion and reliability etc. In either case, the windings may be connected in Y-Y,  $\Delta$ - $\Delta$ , Y- $\Delta$  or  $\Delta$ -Y.

The transfer of 3-phase voltage from one level to another level is known as 3-phase transformation. The 3-phase voltage transformation can be obtained by the following two methods.

- (i) Using bank of three single phase transformers.
- (ii) Using single 3-phase transformer.

### 5.22.1 Bank of three Single-Phase Transformers

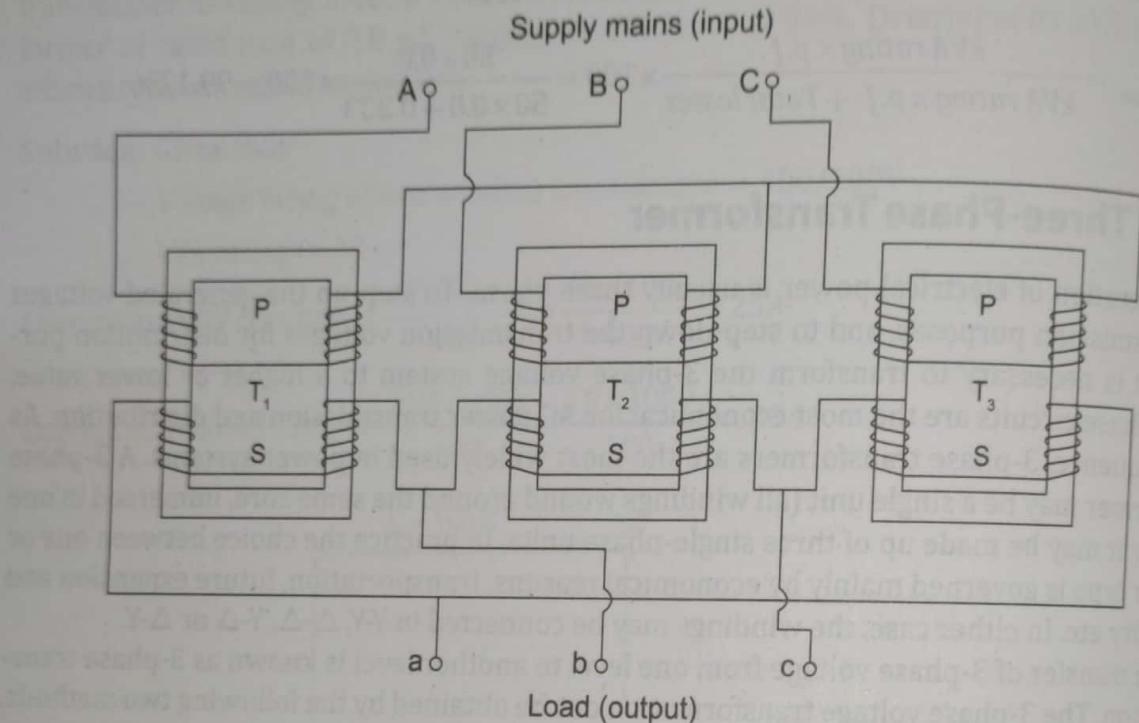
Electric power is generated, transmitted and distributed in three-phase form. Even where single-phase power is required, as in homes and small establishments, these are merely tapped off from a basic three-phase system. Transformers are, therefore, required to interconnect three phase systems at different voltage levels. This can be done using three single-phase transformers, constituting what is often called a transformer bank as shown in figure (5.30). The primary windings of three identical single-phase transformers can usually be connected either in star or in delta to form a 3-phase system. Similarly, the secondary windings can also be connected in star or delta. We have, therefore, four methods of interconnection of primary/secondary, viz., star/star, star/delta, delta/star and delta/delta. The primary and secondary windings shown parallel to each other belong to the same single-phase transformer.

#### Advantages:

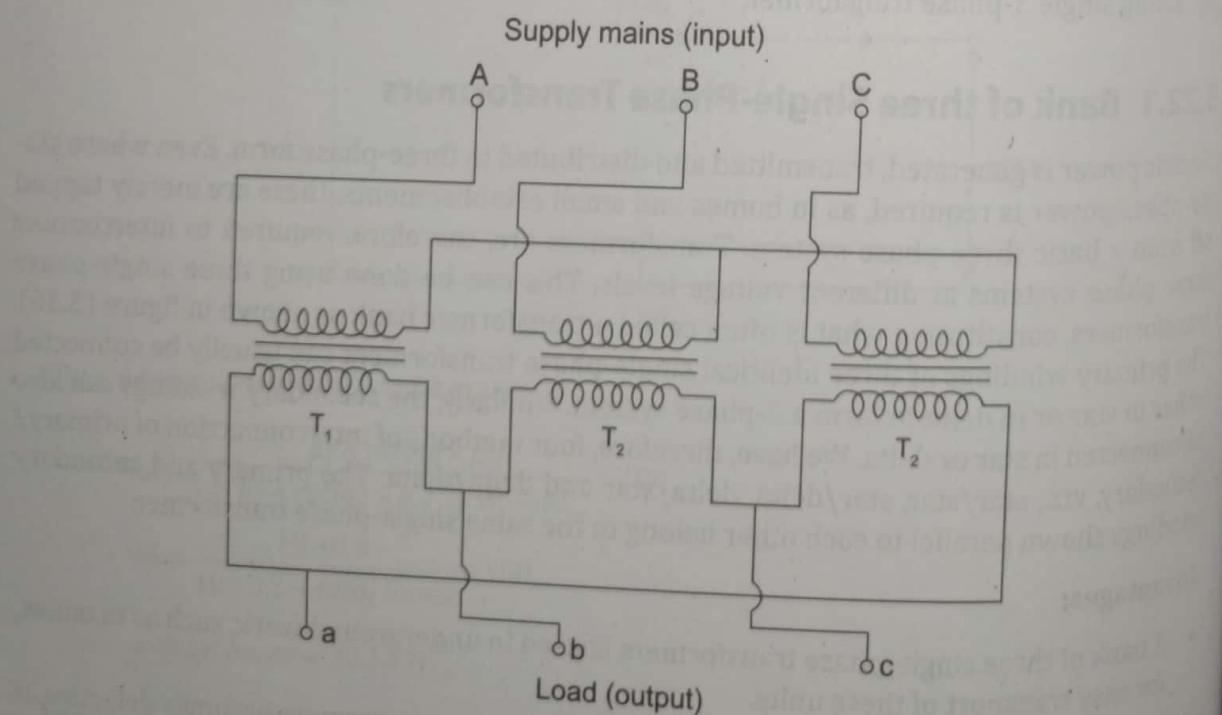
- A bank of three single-phase transformers is used in underground work, such as in mines, for easy transport of these units.
- When one transformer in a bank of three single-phase transformers becomes defective, it may be removed from service and the other two transformers may be reconnected in open delta to supply service at rated capacity on an emergency basis until repairs can be made.

**Disadvantages:**

- A bank of three single phase transformers costs around 20% more than single 3-phase transformer unit for the same rating.
- It occupies more space when compared to a single 3-phase transformer unit.



(a) Schematic diagram

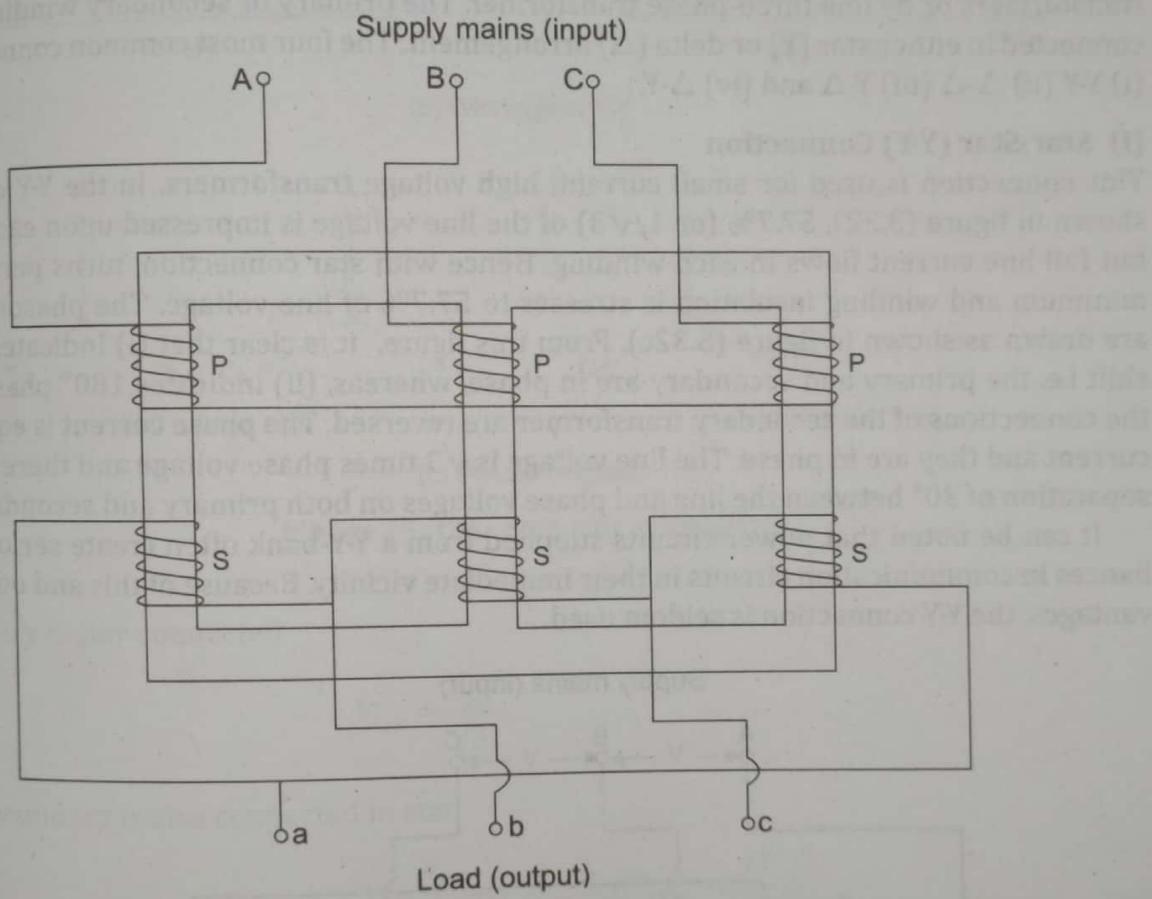


(b) Winding diagram

**Figure (5.30): Bank of three single phase transformers in star-delta**

### 5.22.2 Single Unit of Three-Phase Transformer

Instead of a bank of three separate single-phase transformers, each having its own separate iron core, a single transformer can be designed to serve the same function. Such a single unit, called a three-phase transformer, has three primary windings and three secondary windings as shown in figure (5.31). These primary and secondary windings can be connected in star or in delta. Such a transformer differs from the single-phase transformers in the design of the iron-core. In the single-phase transformer bank the fluxes associated with a particular phase utilize an iron core which serves only that phase, whereas in the three-phase transformer the iron-core couples different phases together. Because of this sharing of the iron-core by the three phases, such transformers can be built more economically.



**Figure (5.31): 3-phase transformer (core type) in star-delta**

#### Advantages:

A single 3-phase transformer unit has the following advantages:

- It occupies less space for the same rating, compared to a bank of three single-phase transformers.
- It weighs and costs less, too.
- Since only one unit is required to be handled, it is easy for the operator.
- The core is of a smaller size and hence less material is required.
- It can be transported very easily.

Because of these advantages, 3-phase transformers are in common use, especially for large power transformations.

**Disadvantages:**

- In a three-phase transformer unit, when one phase becomes defective, the entire three-phase unit must be removed from service.
- For very big transformers, it is impossible to transport large three phase transformers to the site.

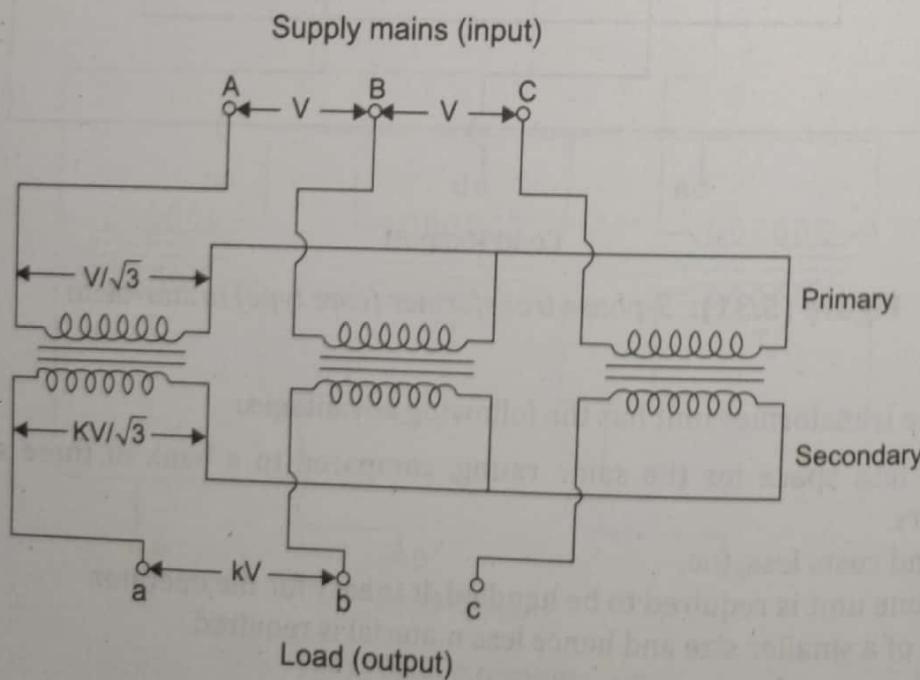
## 5.23 Three-Phase Transformer Connections

A three-phase transformer can be built by suitably connecting a bank of three single-phase transformers or by one three-phase transformer. The primary or secondary windings may be connected in either star ( $Y$ ) or delta ( $\Delta$ ) arrangement. The four most common connections are (i)  $Y-Y$  (ii)  $\Delta-\Delta$  (iii)  $Y-\Delta$  and (iv)  $\Delta-Y$ .

### (i) Star-Star ( $Y-Y$ ) Connection

This connection is used for small current, high voltage transformers. In the  $Y-Y$  connection shown in figure (3.32), 57.7% (or  $1/\sqrt{3}$ ) of the line voltage is impressed upon each winding but full line current flows in each winding. Hence with star connection, turns per phase are minimum and winding insulation is stressed to 57.7% of line voltage. The phasor diagrams are drawn as shown in figure (5.32c). From this figure, it is clear that (i) indicates  $0^\circ$  phase shift i.e. the primary and secondary are in phase, whereas, (ii) indicates  $180^\circ$  phase shift i.e. the connections of the secondary transformer are reversed. The phase current is equal to line current and they are in phase. The line voltage is  $\sqrt{3}$  times phase voltage and there is a phase separation of  $30^\circ$  between the line and phase voltages on both primary and secondary side.

It can be noted that power circuits supplied from a  $Y-Y$  bank often create serious disturbances in communication circuits in their immediate vicinity. Because of this and other disadvantages, the  $Y-Y$  connection is seldom used.



(a) Schematic diagram

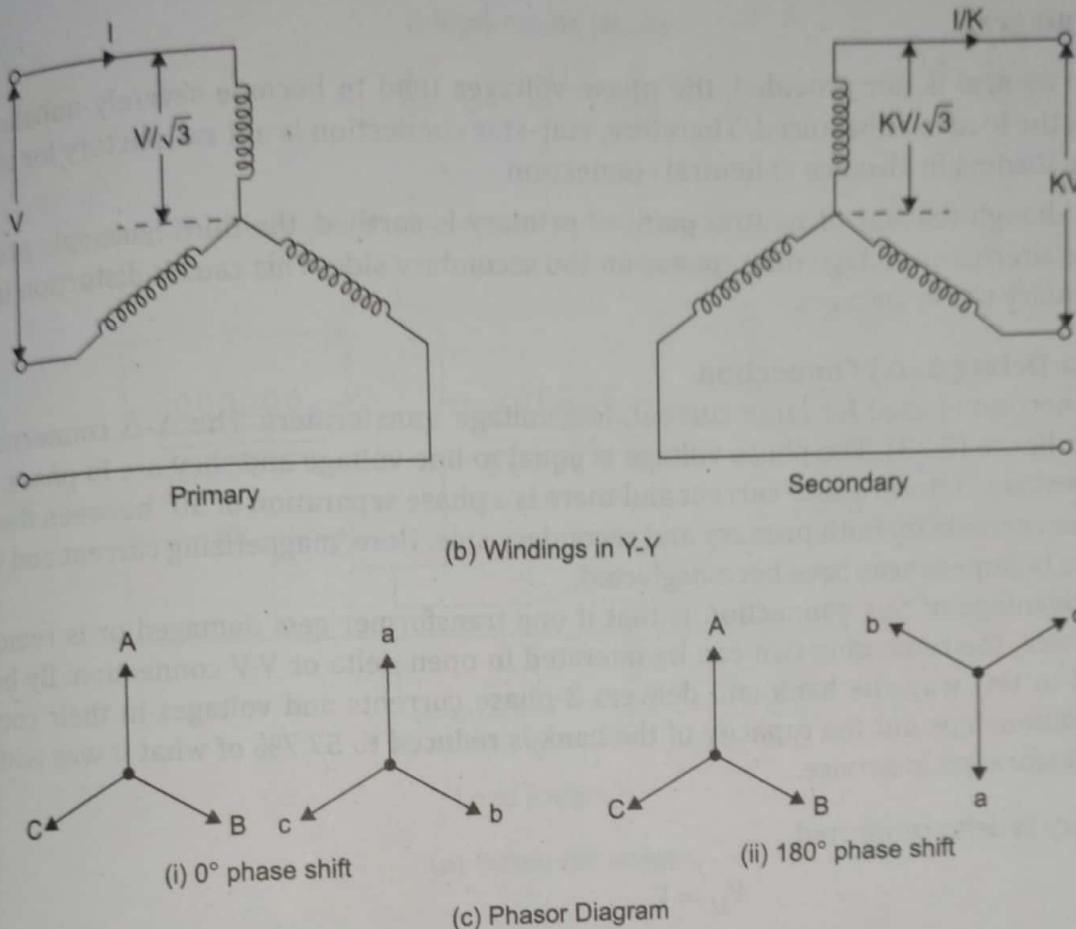


Figure (5.32): Star-star connection

As primary is star connected

$$V_{1ph} = \frac{V_{1L}}{\sqrt{3}}$$

Since secondary is also connected in star

$$K = \frac{V_{2ph}}{V_{1ph}} \Rightarrow V_{2ph} = KV_{1ph} = K\left(\frac{V_{1L}}{\sqrt{3}}\right)$$

$$V_{2L} = \sqrt{3}V_{2ph} = \sqrt{3}K\left(\frac{V_{1L}}{\sqrt{3}}\right) = KV_{1L} \quad (5.43)$$

#### Advantages:

- Due to star connection, phase voltage is  $\frac{1}{\sqrt{3}}$  times of line voltage. Hence less number of turns are required.
- There is no phase shift between the primary and secondary voltages.
- As the neutral is available, it is suitable for 3-phase 4-wire system.

### **Disadvantages:**

- If the neutral is not provided, the phase voltages tend to become severely unbalanced, when the load is unbalanced. Therefore, star-star connection is not satisfactory for unbalanced loading in absence of neutral connection.
- Even though the star or neutral point of primary is earthed, the third harmonic present in the alternator voltage may appear on the secondary side. This causes distortion in the secondary phase voltages.

### **(ii) Delta-Delta ( $\Delta$ - $\Delta$ ) Connection**

This connection is used for large current, low voltage transformers. The  $\Delta$ - $\Delta$  connection is shown in figure (5.33). The phase voltage is equal to line voltage and they are in phase. The line current is  $\sqrt{3}$  times phase current and there is a phase separation of  $30^\circ$  between the line and phase currents on both primary and secondary side. Here, magnetizing current and voltage drops in impedances have been neglected.

An advantage of this connection is that if one transformer gets damaged or is removed from service, the remaining two can be operated in open-delta or V-V connection. By being operated in this way, the bank still delivers 3-phase currents and voltages in their correct phase relationships but the capacity of the bank is reduced to 57.7% of what it was with all three transformers in service.

As primary is delta connected

$$V_{1L} = V_{1ph}$$

Since secondary is also connected in delta

$$K = \frac{V_{2ph}}{V_{1ph}} \Rightarrow V_{2ph} = KV_{1ph} = KV_{1L}$$

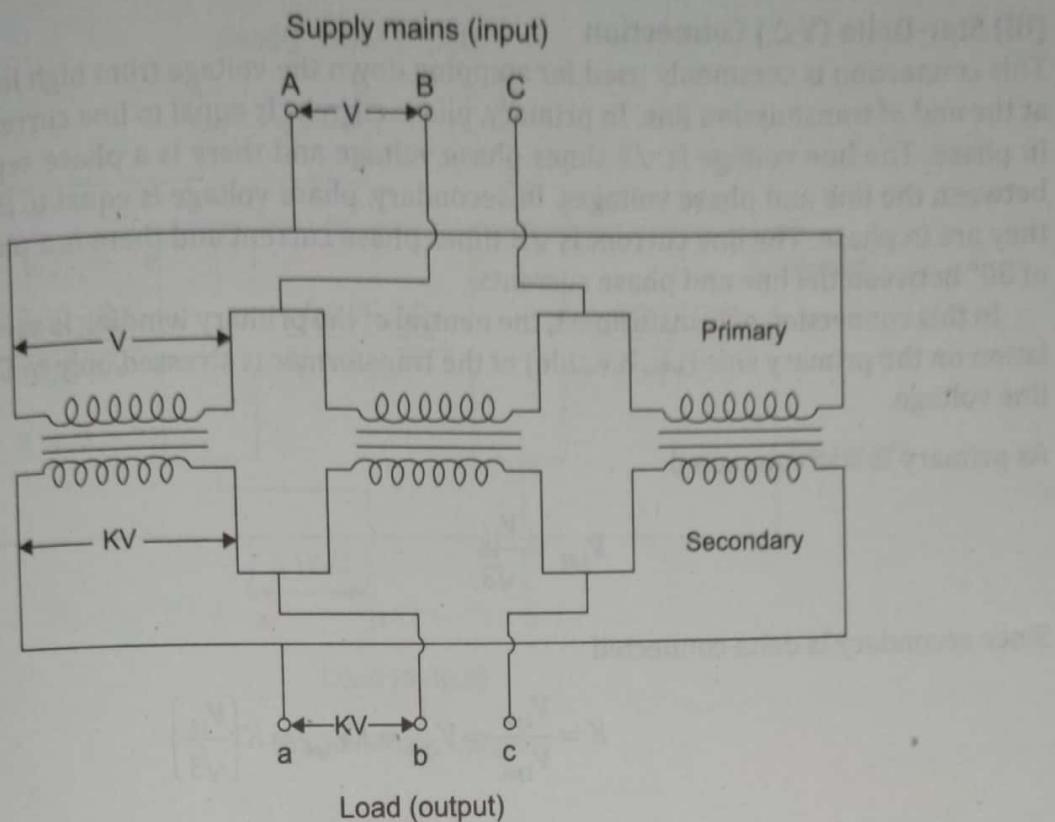
$$V_{2L} = V_{2ph} = KV_{1L} \quad (5.44)$$

### **Advantages:**

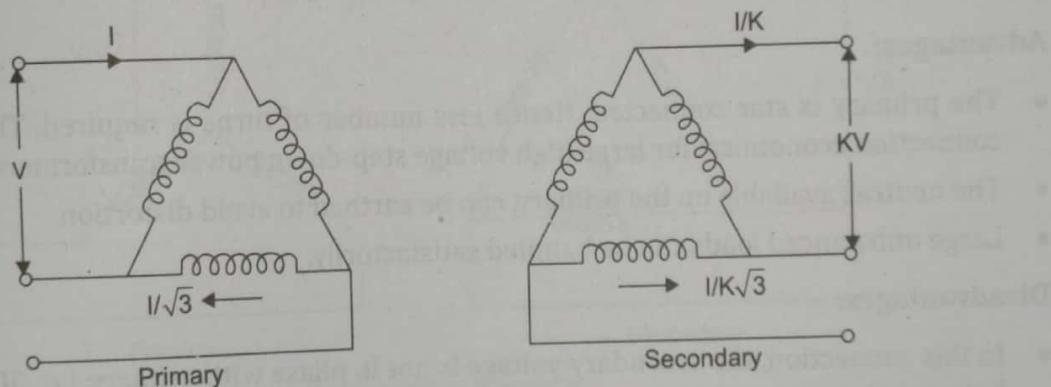
- This connection is satisfactory for both balanced and unbalanced loading.
- If a third harmonic is present, it circulates in a closed path and therefore does not appear in the output voltage wave.
- If one transformer gets damaged or is removed from service, the remaining two can be operated in open-delta or V-V connection.

### **Disadvantages:**

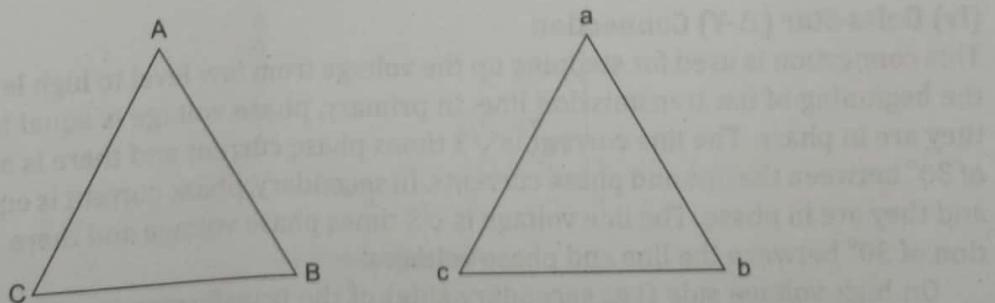
- As the neutral is not available, it is not suitable for 3-phase 4-wire system.
- More insulation is required due to more number of turns per phase when compared to star-star connection.



(a) Schematic diagram



(b) Windings in delta-delta



(c) Phasor Diagram

**Figure (5.33): Delta-delta ( $\Delta-\Delta$ ) connection**

### (iii) Star-Delta ( $\text{Y}-\Delta$ ) Connection

This connection is commonly used for stepping down the voltage from high level to low level at the end of transmission line. In primary, phase current is equal to line current and they are in phase. The line voltage is  $\sqrt{3}$  times phase voltage and there is a phase separation of  $30^\circ$  between the line and phase voltages. In secondary, phase voltage is equal to line voltage and they are in phase. The line current is  $\sqrt{3}$  times phase current and there is a phase separation of  $30^\circ$  between the line and phase currents.

In this connection of transformers, the neutral of the primary winding is earthed. The insulation on the primary side (i.e., h.v side) of the transformer is stressed only to 57.7% of line to line voltage.

As primary is star connected

$$V_{1ph} = \frac{V_{1L}}{\sqrt{3}}$$

Since secondary is delta connected

$$K = \frac{V_{2ph}}{V_{1ph}} \Rightarrow V_{2ph} = KV_{1ph} = K \left( \frac{V_{1L}}{\sqrt{3}} \right)$$

$$V_{2L} = V_{2ph} = K \left( \frac{V_{1L}}{\sqrt{3}} \right) = \left( \frac{K}{\sqrt{3}} \right) V_{1L} \quad (5.45)$$

#### Advantages:

- The primary is star connected. Hence less number of turns is required. This makes the connection economical for large high voltage step-down power transformers.
- The neutral, available on the primary, can be earthed to avoid distortion.
- Large unbalanced loads can be handled satisfactorily.

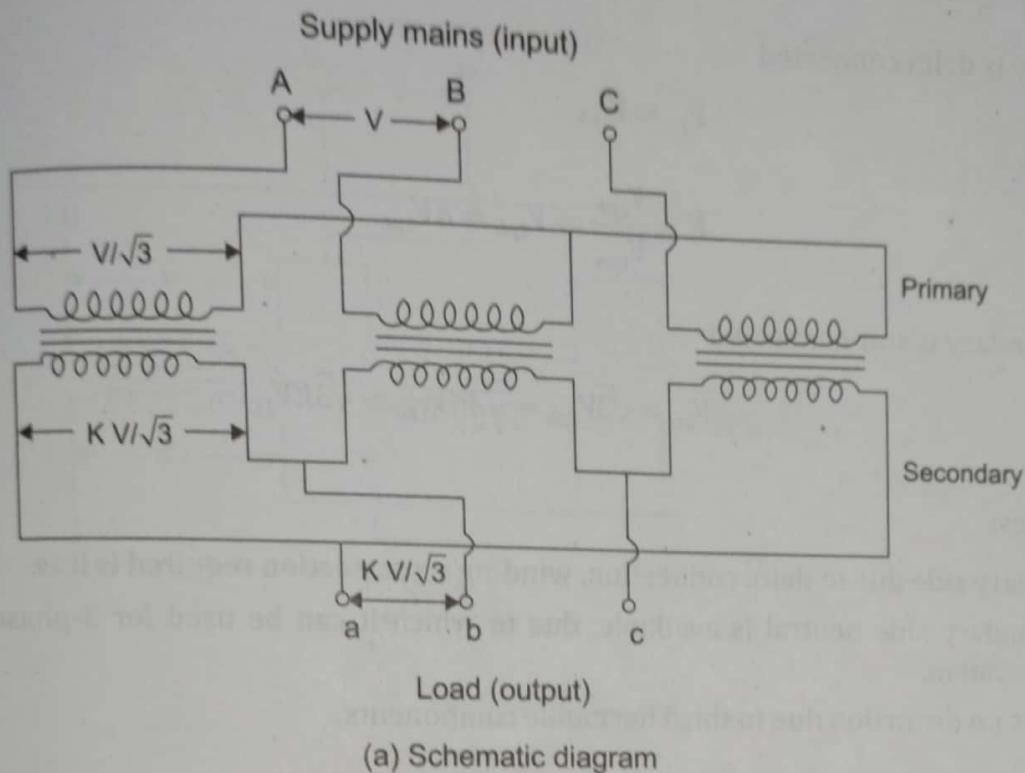
#### Disadvantages:

- In this connection, the secondary voltage is not in phase with primary. i.e.,  $30^\circ$  phase shift between the primary and the secondary voltages. Hence this connection cannot be paralleled with star-star or delta-delta connected transformers.

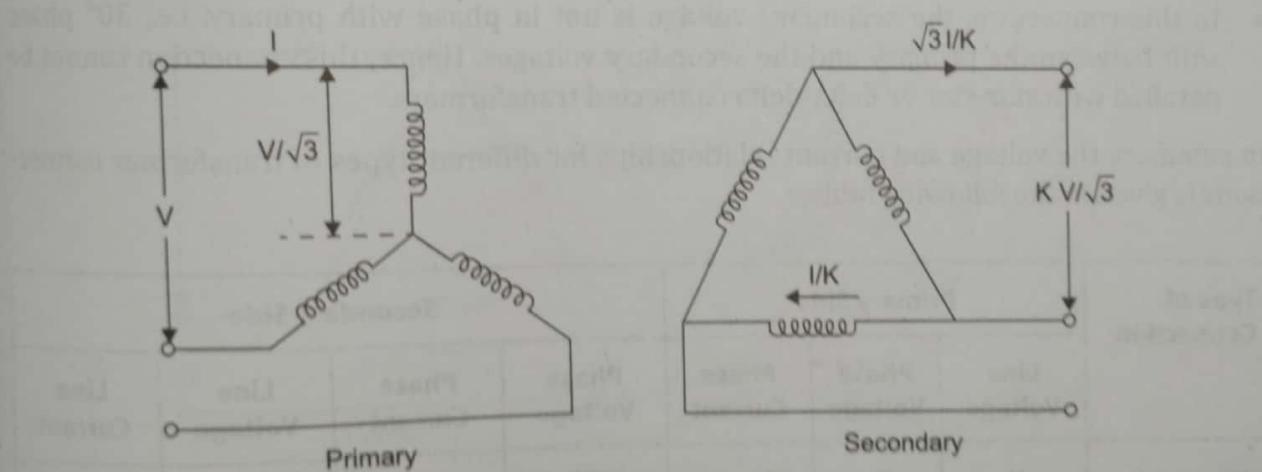
### (iv) Delta-Star ( $\Delta-\text{Y}$ ) Connection

This connection is used for stepping up the voltage from low level to high level, for example at the beginning of h.v. transmission line. In primary, phase voltage is equal to line voltage and they are in phase. The line current is  $\sqrt{3}$  times phase current and there is a phase separation of  $30^\circ$  between the line and phase currents. In secondary, phase current is equal to line current and they are in phase. The line voltage is  $\sqrt{3}$  times phase voltage and there is a phase separation of  $30^\circ$  between the line and phase voltages.

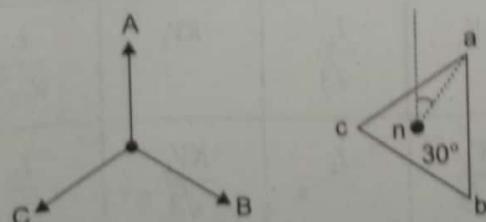
On high voltage side (i.e., secondary side) of the transformer, insulation is stressed only about 57.7% of line voltage. These transformers are also used as distribution transformers, for example 11kV/400V, used to distribute power to consumers by 3-phase, 4-wire system.



(a) Schematic diagram



(b) Windings in star-delta



(c) Phasor Diagram

Figure (5.34): Star-delta ( $Y-\Delta$ ) connection

As primary is delta connected

$$V_{1L} = V_{1ph}$$

$$K = \frac{V_{2ph}}{V_{1ph}} \Rightarrow V_{2ph} = KV_{1ph}$$

Since secondary is star connected

$$V_{2L} = \sqrt{3}V_{2ph} = \sqrt{3}KV_{1ph} = \sqrt{3}KV_{1L} \quad (5.46)$$

#### Advantages:

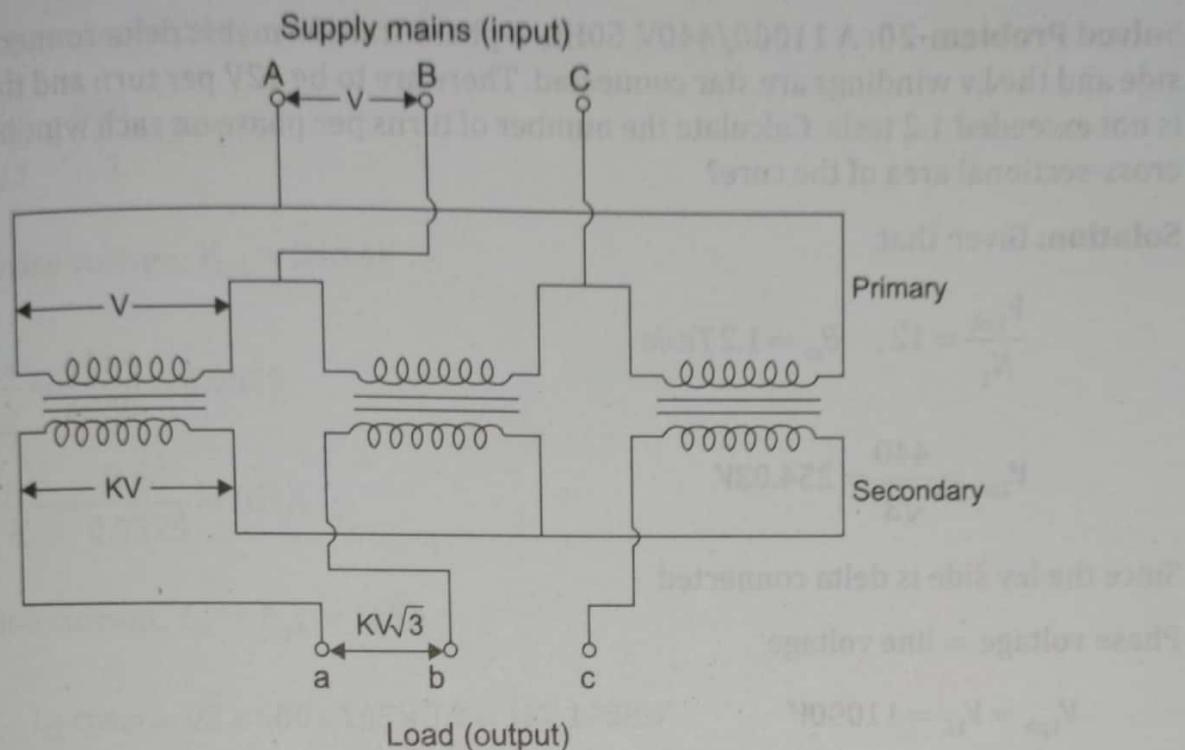
- In primary side due to delta connection, winding cross-section required is less.
- In secondary side neutral is available, due to which it can be used for 3-phase, 4-wire supply system.
- There is no distortion due to third harmonic components.

#### Disadvantages:

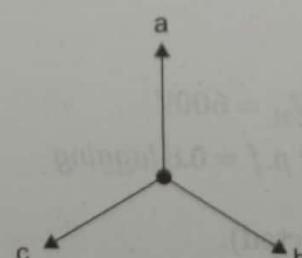
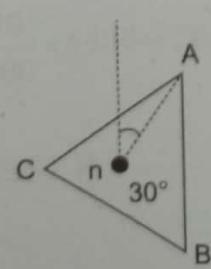
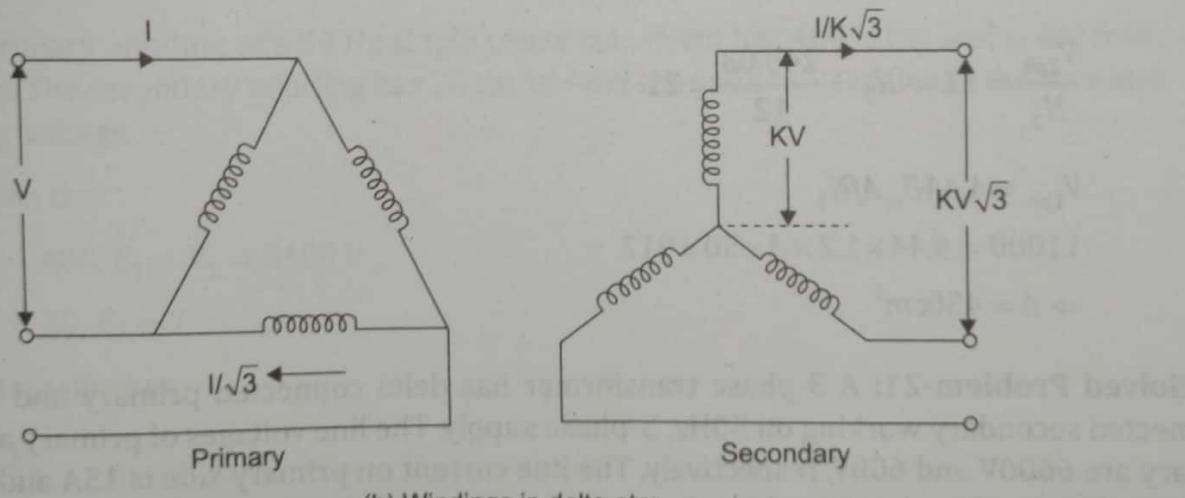
- In this connection, the secondary voltage is not in phase with primary. i.e.,  $30^\circ$  phase shift between the primary and the secondary voltages. Hence, this connection cannot be paralleled with star-star or delta-delta connected transformers.

In summary the voltage and current relationships for different types of transformer connections is given in the following table:

Type of Connection	Primary Side			Secondary Side			
	Line Voltage	Phase Voltage	Phase Current	Phase Voltage	Phase Current	Line Voltage	Line Current
Y-Y	$V_L$	$\frac{V_L}{\sqrt{3}}$	$I_L$	$\frac{KV_L}{\sqrt{3}}$	$\frac{I_L}{K}$	$KV_L$	$\frac{I_L}{K}$
$\Delta$ - $\Delta$	$V_L$	$V_L$	$\frac{I_L}{\sqrt{3}}$	$KV_L$	$\frac{I_L}{K\sqrt{3}}$	$KV_L$	$\frac{I_L}{K}$
Y- $\Delta$	$V_L$	$\frac{V_L}{\sqrt{3}}$	$I_L$	$\frac{KV_L}{\sqrt{3}}$	$\frac{I_L}{K}$	$\frac{KV_L}{\sqrt{3}}$	$\frac{\sqrt{3}I_L}{K}$
$\Delta$ -Y	$V_L$	$V_L$	$\frac{I_L}{\sqrt{3}}$	$KV_L$	$\frac{I_L}{K\sqrt{3}}$	$\sqrt{3}KV_L$	$\frac{I_L}{K\sqrt{3}}$



(a) Schematic diagram

Figure (5.35): Delta-star ( $\Delta$ -Y) connection

**Solved Problem-20:** A 11000/440V, 50Hz, 3-phase transformer is delta connected on the h.v side and the l.v windings are star connected. There are to be 12V per turn and the flux density is not exceeded 1.2 tesla. Calculate the number of turns per phase on each winding and the net cross-sectional area of the core?

**Solution:** Given that

$$\frac{V_{1ph}}{N_1} = 12, \quad B_m = 1.2 \text{ Tesla}$$

$$V_{2ph} = \frac{440}{\sqrt{3}} = 254.03V$$

Since the h.v side is delta connected

Phase voltage = line voltage

$$V_{1ph} = V_{1L} = 11000V$$

$$\therefore \frac{11000}{N_1} = 12 \Rightarrow N_1 = \frac{11000}{12} = 917$$

$$\frac{V_{2ph}}{N_2} = 12 \Rightarrow N_2 = \frac{254.03}{12} = 21$$

$$V_{1ph} = 4.44B_mAfN_1$$

$$11000 = 4.44 \times 1.2 \times A \times 50 \times 917$$

$$\Rightarrow A = 450 \text{ cm}^2$$

**Solved Problem-21:** A 3-phase transformer has delta connected primary and a star connected secondary working on 50Hz, 3-phase supply. The line voltages of primary and secondary are 6600V and 600V, respectively. The line current on primary side is 15A and secondary has a balanced load at 0.8 p.f. lagging. Calculate the secondary phase voltage, line current and output power?

**Solution:** Given that

$$V_{1L} = 6600V, \quad V_{2L} = 600V$$

$$I_{1L} = 15A, \quad \text{load p.f.} = 0.8 \text{ lagging}$$

Primary side(delta connected):

$$V_{1ph} = V_{1L} = 6600V$$

$$I_{1L} = 15A$$

$$I_{1ph} = \frac{I_{1L}}{\sqrt{3}} = \frac{15}{\sqrt{3}} = 8.666A$$

Secondary side (star connected):

$$V_{2ph} = \frac{V_{2L}}{\sqrt{3}} = \frac{600}{\sqrt{3}} = 346.4V$$

∴ Secondary phase voltage,  $V_{2ph} = 346.4V$

$$K = \frac{V_{2ph}}{V_{1ph}} = \frac{346.4}{6600} = 0.0525$$

$$I_{2ph} = \frac{I_{1ph}}{K} = \frac{8.66}{0.0525} = 165A$$

∴ Secondary line current,  $I_{2L} = I_{2ph} = 165A$

$$\text{Output} = \sqrt{3}V_{2L} I_{2L} \cos\varphi = \sqrt{3} \times 600 \times 165 \times 0.8 = 137.178kW$$

## Additional Solved Problems

**ASP-1:** The primary winding of a 50 Hz single phase transform has 480 turns and is fed from 6400 V supply. The secondary winding has 20 turns. Find the peak value of flux in the core and the secondary voltage.

**Solution:** Given that

$$N_1 = 480, E_1 = V_1 = 6400 V$$

$$N_2 = 20, E_2 = ?$$

For an ideal transformer

$$\frac{N_2}{N_1} = \frac{E_2}{E_1}$$

$$\therefore E_2 = \frac{N_2}{N_1} \times E_1 = \frac{20}{480} \times 6400 = 266.7V$$

Secondary voltage,  $E_2 = 266.7 V$

$$E_1 = 4.44 f N_1 \Phi_m$$

$$6400 = 4.44 \times 50 \times 480 \times \Phi_m$$

$$\Rightarrow \Phi_m = 60 \text{ mWb}$$

Peak value of flux in the core,  $\Phi_m = 60 \text{ mWb}$

**ASP-2:** The no. of turns on the primary and secondary windings of a single phase transformer are 350 and 35, respectively. If the primary is connected to 6.2 kV 50 Hz supply, determine the secondary voltage.

**Solution:** Given that

$$N_1 = 350, N_2 = 35$$

$$E_1 = V_1 = 6.2 \text{ kV}, V_2 = ?$$

For an ideal transformer

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{V_2}{V_1}$$

$$\Rightarrow V_2 = \frac{N_2}{N_1} \times V_1 = 6200 \times \frac{35}{350} = 620 \text{ V}$$

**ASP-3:** A 100 kVA 1-phase transformer has full load primary current of 400A and total resistance referred to primary is  $0.006\Omega$ . If the iron loss amounts to 500W, find the efficiency at full load and half full load at:

- (i) unity power factor
- (ii) 0.8 power factor

**Solution:** Given that

$$\text{kVA rating of transformer} = 100$$

$$I_1 = 400 \text{ A}, R_{01} = 0.006\Omega$$

$$\text{Iron loss, } W_i = 500 \text{ W}$$

$$(i) \text{ Full load copper loss, } W_{cu} = I_1^2 R_{01} = 400^2 \times 0.006 = 960 \text{ W}$$

$$\text{Total losses} = W_i + W_{cu} = 500 + 960 = 1460 \text{ W}$$

- (a) Efficiency at unity power factor

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}} \times 100 = \frac{100 \times 10^3 \times 1}{100 \times 10^3 \times 1 + 1460} \times 100 = 98.56\%$$

- (b) Efficiency at 0.8 power factor

$$\eta = \frac{100 \times 10 \times 0.8}{100 \times 10 \times 0.8 + 1460} \times 100 = 98.21\%$$

- (ii) At half full load, the copper losses are

$$W_{cu} = \left(\frac{I_1}{2}\right)^2 \times R_{01} = (200)^2 \times 0.006 = 240 \text{ W}$$

$$\text{Total losses} = W_i + W_{cu} = 500 + 240 = 740 \text{ W}$$

(a) Efficiency at unity power factor

$$\eta = \frac{50 \times 10^3 \times 1}{50 \times 10^3 \times 1 + 740} \times 100 = 98.54\%$$

(b) Efficiency at 0.8 power factor

$$\eta = \frac{50 \times 10^3 \times 0.8}{50 \times 10^3 \times 0.8 + 740} \times 100 = 98.18\%$$

**ASP-4:** A 150 kVA transformer has an iron loss of 1400W and full load copper loss of 1600W.

Find the efficiency of a transformer at 50% full load at (i) upf (ii) 0.8 p.f. lagging

**Solution:** Given that

kVA rating = 150

Iron loss,  $W_i = 1400\text{W} = 1.4\text{kW}$

Full load copper loss,  $W_{cu} = 1600\text{W} = 1.6\text{kW}$

Efficiency at any desired load can be found by using the following relation

$$\eta = \frac{x \times \text{full load kVA} \times \text{p.f.}}{x \times \text{full load kVA} \times \text{p.f.} + W_i + x^2 W_{cu}} \times 100$$

(i) Efficiency at half full load (50% full load) and at u.p.f

$$\eta = \frac{0.5 \times 150 \times 1}{0.5 \times 150 \times 1 + \frac{1}{4} \times 1.6 + 1.4} \times 100 = \frac{75}{76.8} \times 100 = 97.65\%$$

(ii) Efficiency at half full load (50% full load) and at 0.8 p.f. lagging

$$\eta = \frac{0.5 \times 150 \times 0.8}{0.5 \times 150 \times 0.8 + \frac{1}{4} \times 1.6 + 1.4} \times 100 = \frac{60}{61.8} \times 100 = 97.08\%$$

**ASP-5:** When a transformer is connected to a 1000V, 50Hz supply the core loss is 1000W, of which 650W is hysteresis and 350W is eddy current loss. If the applied voltage is raised to 2,000V and the frequency to 100Hz, find the new core losses?**Solution:** Given that

Here, both voltage and frequency are doubled, leaving the flux density unchanged.

**With 1000 V at 50 Hz:**

$W_h = Af \text{ or } 650 = 50A \Rightarrow A = 13$

$W_e = Bf^2 \text{ or } 350 = B \times 50^2 \Rightarrow B = 7/50$

**With 2000 V at 100 Hz:**

$W_h = Af = 13 \times 100 = 1300\text{W}$

$W_e = Bf^2 = (7/50) \times 100^2 = 1400\text{W}$

$\therefore$  New core loss =  $1300 + 1400 = 2700 \text{ W}$

**ASP-6:** A 4400V, 50Hz transformer has hysteresis loss of 1200W, eddy current loss of 1800W and full load copper loss of 4000W. If the transformer is supplied at 6600V, 75Hz what will be the losses? Assume that the full-load current remains the same.

**Solution:** Given that

$$\text{Hysteresis loss at } 50\text{Hz} = 1200\text{W}$$

$$\text{Eddy current loss at } 50\text{ Hz} = 1800\text{W}$$

$$\text{Full load copper loss} = 4000\text{W}$$

We know that

$$\text{Hysteresis loss, } W_h \propto f$$

$$\Rightarrow W_h = Af$$

$$\frac{W_{h2}}{W_{h1}} = \frac{f_2}{f_1} \Rightarrow W_{h2} = W_{h1} \times \frac{f_2}{f_1} = 1200 \times \frac{75}{50} = 1800 \text{W}$$

Hence, hysteresis loss at 6600V, 75Hz = 1800W

$$\text{Eddy current loss, } W_e \propto f^2 \Rightarrow W_e = Bf^2$$

$$\frac{W_{e2}}{W_{e1}} = \frac{f_2^2}{f_1^2} \Rightarrow W_{e2} = W_{e1} \times \frac{f_2^2}{f_1^2} = 1800 \times \frac{75^2}{50^2} = 4050 \text{W}$$

Hence, eddy current loss at 6600V, 75Hz = 4050W

Since the full load current remains the same, full load copper loss = 4000W (as before)

**ASP-7:** A 500kVA single phase transformer has full load copper losses of 7.5kW and iron losses of 6.5kW. Find the efficiency at half full load of a p.f. 0.8 lagging. Also find the maximum efficiency and the kVA load at which it occurs?

**Solution:** Given that

$$\text{kVA rating, } S = 500$$

$$\text{Iron losses, } W_i = 6.5 \text{ kW}$$

$$\text{Copper losses at full load, } W_{cu} = 7.5 \text{ kW}$$

$$\text{p.f.} = \cos \phi = 0.8 \text{ lagging}$$

At half full load i.e., ratio of actual load to full load,  $x = 1/2$

$$\eta_{\text{at any load}} = \frac{xS \cos \phi}{xS \cos \phi + W_i + x^2 W_{cu}} \times 100$$

$$\eta_{\text{at } \frac{1}{2} \text{ F.L.}} = \frac{0.5 \times 500 \times 0.8}{0.5 \times 500 \times 0.8 + 6.5 + 0.5^2 \times 7.5} \times 100 = \frac{200}{200 + 8.375} \times 100 = 95.98\%$$

At maximum efficiency,

$$\text{Iron losses} = \text{Copper losses}$$

$$\therefore x^2 W_{cu} = W_i$$

$$\Rightarrow x = \sqrt{\frac{W_i}{W_{cu}}} = \sqrt{\frac{\text{Iron loss}}{\text{F.L. copper loss}}} = \sqrt{\frac{6.5}{7.5}} = 0.8667$$

$$\text{Output at max. efficiency} = xS \cos \phi = 0.8667 \times 500 \times 0.8 = 346.67 \text{ kW}$$

$$\text{Total losses} = \text{Iron losses} + \text{Copper losses} = 6.5 + 6.5 = 13 \text{ kW}$$

$$\therefore \text{Max. efficiency, } \eta_{\max} = \frac{\text{output}}{\text{output} + \text{losses}} \times 100 = \frac{346.67}{346.67 + 13} \times 100 = 96.38\%$$

$$kVA_{\max.} = kVA_{F.L.} \times \sqrt{\frac{W_i}{W_{cu}}} = 0.8667 \times 500 = 433.35 \text{ kVA}$$

**ASP-8:** A 100kVA single phase transformer has iron and copper losses each equal to 6.5kW at a full load current. Find the efficiency at a load of 75kVA, 0.8 p.f. lagging.

**Solution:** Given that

$$\text{kVA rating, } S = 100$$

$$W_i = W_{cu} = 6.5 \text{ kW}$$

$$\text{p.f} = 0.8 \text{ lagging}$$

$$x = \frac{\text{actual load}}{\text{full load}} = \frac{75}{100} = 0.75$$

Efficiency at any desired load is given by

$$\begin{aligned} \eta_{\text{at any load}} &= \frac{xS \cos \phi}{xS \cos \phi + W_i + x^2 W_{cu}} \times 100 \\ &= \frac{0.75 \times 100 \times 0.8}{0.75 \times 100 \times 0.8 + 6.5 + (0.75)^2 \times 6.5} \times 100 = \frac{60}{60 + 2.344} \times 100 = 96.24\% \end{aligned}$$

**ASP-9:** It is proposed to transmit the power generated by a 200 MVA, 11 kV, 50 Hz, 3-phase generator to a three-phase 220 kV transmission line using a bank of three single-phase transformers. Find the turns ratio and the voltage and current ratings for each single-phase transformer when the connections are:

- (i) Star/Star   (ii) Star/Delta   (iii) Delta/Star and   (iv) Delta/Delta

**Solution:** Given that

$$V_{1L} = 11\text{kV}, V_{2L} = 220\text{kV}$$

MVA rating = 200

MVA rating per phase =  $200/3 = 66.67$

Type of connection	$V_{1ph}$ (kV)	$V_{2ph}$ (kV)	$\frac{N_2}{N_1} = \frac{V_{2ph}}{V_{1ph}}$	$I_{1ph} = \frac{66.67}{V_{1ph}}$ (kA)	$I_{2ph} = \frac{66.67}{V_{2ph}}$ (kA)
Y-Y	$\frac{V_{1L}}{\sqrt{3}} = 6.35$	$\frac{V_{2L}}{\sqrt{3}} = 127$	20	10.49	0.52
Y-Δ	$\frac{V_{1L}}{\sqrt{3}} = 6.35$	$V_{2L} = 220$	34.6	10.49	0.3
Δ-Y	$V_{1L} = 11$	$\frac{V_{2L}}{\sqrt{3}} = 127$	11.5	6.06	0.52
Δ-Δ	$V_{1L} = 11$	$V_{2L} = 220$	20	6.06	0.3

**ASP-10:** A 100kVA single phase transformer has iron and copper losses each equal to 4.5kW at a full load current. Find the efficiency at a load of 75kVA, 0.8 p.f. lagging.

**Solution:** Given that

$$\text{kVA rating, } S = 100$$

$$W_i = W_{cu} = 4.5\text{kW}$$

p.f = 0.8 lagging

$$x = \frac{\text{actual load}}{\text{full load}} = \frac{75}{100} = 0.75$$

Efficiency at any desired load is given by

$$\begin{aligned}\eta_{\text{at any load}} &= \frac{xS \cos \phi}{xS \cos \phi + W_i + x^2 W_{cu}} \times 100 \\ &= \frac{0.75 \times 100 \times 0.8}{0.75 \times 100 \times 0.8 + 2.5 + (0.75)^2 \times 2.5} \times 100 = \frac{60}{60 + 3.906} \times 100 = 93.9\%\end{aligned}$$

**ASP-11:** A 2300/230V, 20kVA, two winding transformer is connected as an auto-transformer, with constant source voltage of 2300V. At full load of unity power factor, calculate:

- (a) Power output
- (b) Power transferred and conducted
- (c) If the efficiency of the two winding transformer, at 0.8 p.f. is 95%, find the auto-transformer efficiency at the same p.f.

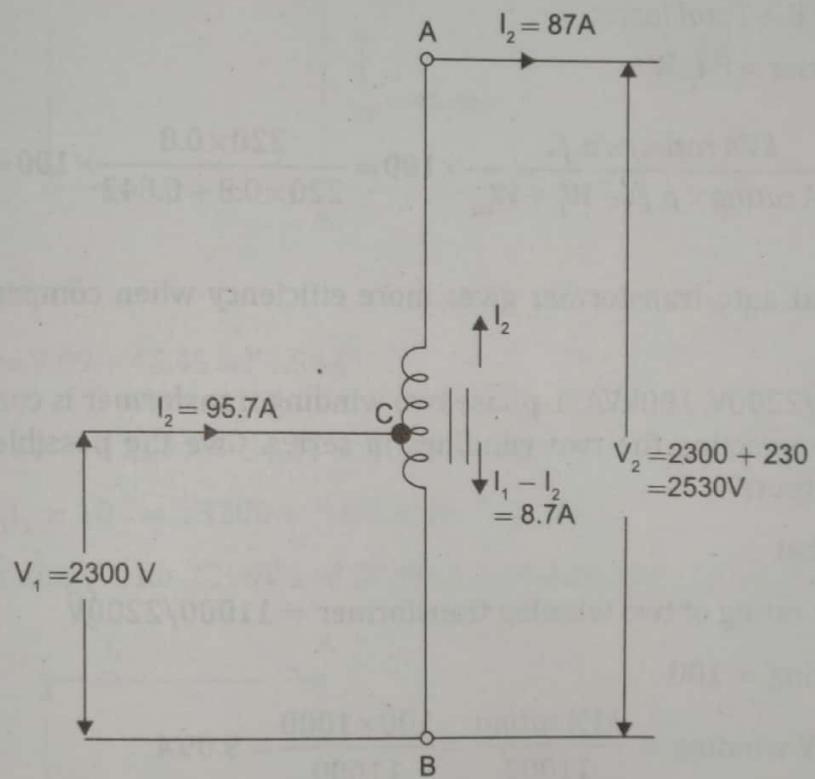
Solution: Given that

Voltage rating of two winding transformer = 2300/230V

kVA rating = 20

$$\text{Current in } 2300\text{V winding} = \frac{\text{kVA rating}}{2300} = \frac{20 \times 1000}{2300} = 8.7\text{A}$$

$$\text{Current in } 230\text{V winding} = \frac{\text{kVA rating}}{230} = \frac{20 \times 1000}{230} = 87\text{A}$$



The currents in the auto transformer are:

$$K = \frac{V_2}{V_1} = \frac{2530}{2300} = 1.1$$

Current through part AC =  $I_2 = 87\text{ A}$

The current distribution is shown in the above figure

$$\text{Now } K = \frac{I_1}{I_2} = 1.1 \Rightarrow I_1 = 1.1 \times 87 = 95.7\text{ A}$$

Current through part BC =  $I_1 - I_2 = 95.7 - 87 = 8.7\text{ A}$

(a) The kVA rating of an auto-transformer =  $V_2 I_2 = 2530 \times 87 = 220\text{kVA}$

The power output,  $P_{\text{out}} = 220 \times \text{p.f.} = 220 \times 1 = 220\text{kW}$

- (b) Part AC acts as a part in which power is transformed by transformer action

Power transferred inductively =  $I_2 \times$  voltage across AC =  $87 \times 230 = 20\text{kVA} = 20\text{kW}$  at p.f. = 1

Power transferred conductively =  $P_{out} - 20 = 220 - 20 = 200\text{kW}$

- (c) Now efficiency of 2-winding transformer is 95% at p.f. of 0.8

$$\eta_{2-wdg} = \frac{kVA \text{ rating} \times p.f.}{kVA \text{ rating} \times p.f. + \text{Total losses}} \times 100$$

$$95 = \frac{20 \times 0.8}{20 \times 0.8 + \text{Total losses}} \times 100$$

$$\Rightarrow \text{Total losses} = 842\text{W}$$

$$\eta_{auto} = \frac{kVA \text{ rating} \times p.f.}{kVA \text{ rating} \times p.f. + W_i + W_{cu}} \times 100 = \frac{220 \times 0.8}{220 \times 0.8 + 0.842} \times 100 = 99.52\%$$

It can be seen that auto-transformer gives more efficiency when compared to a 2-winding transformer.

**ASP-12:** A 11000/2200V, 100kVA, 1-phase two winding transformer is connected as an auto-transformer by connecting the two windings in series. Give the possible values of voltage ratios and kVA outputs?

**Solution:** Given that

Voltage rating of two winding transformer = 11000/2200V

kVA rating = 100

$$\text{Current in } 11000\text{V winding} = \frac{kVA \text{ rating}}{11000} = \frac{100 \times 1000}{11000} = 9.09\text{A}$$

$$\text{Current in } 2200\text{V winding} = \frac{kVA \text{ rating}}{2200} = \frac{100 \times 1000}{2200} = 45.45\text{A}$$

It is to be noted that if the windings of the 2-winding transformer are connected in series to form an auto-transformer, the rated currents are not exceeded.

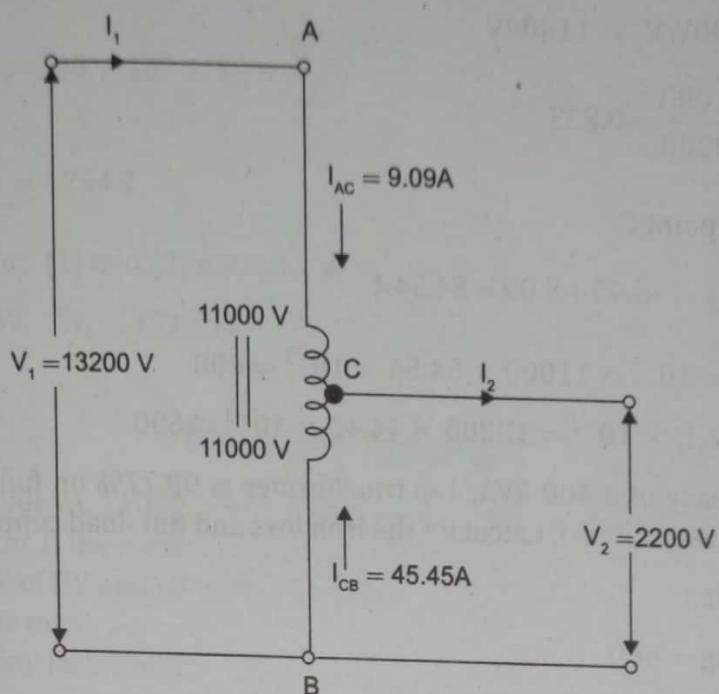
**Case-I:** Here the winding AC for 11000V and BC for 2200V as shown in figure. Therefore

$$V_{AC} = 11000\text{V}, V_{BC} = 2200\text{V}$$

$$\text{So } V_1 = V_{AC} + V_{BC} = 11000 + 2200 = 13200\text{V}$$

$$\therefore V_2 = 13200\text{V}, V_2 = 2200\text{V}$$

$$K = \frac{V_2}{V_1} = \frac{2200}{13200} = 0.1666$$



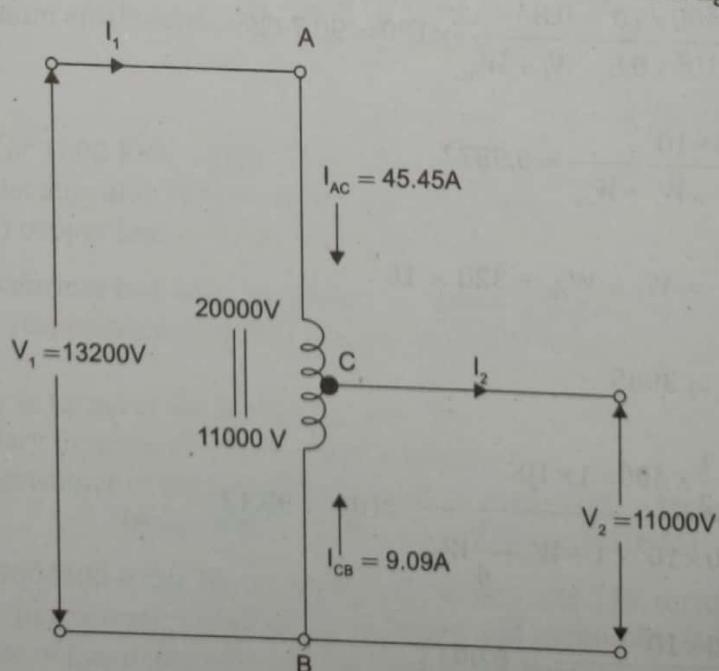
By applying KCL at point C

$$I_2 = I_{AC} + I_{CB} = 9.09 + 45.45 = 54.54 \text{ A}$$

$$\text{kVA output} = V_2 I_2 \times 10^{-3} = 2200 \times 54.54 \times 10^{-3} = 120$$

$$(\text{or}) \text{ kVA output} = V_1 I_1 \times 10^{-3} = 13200 \times 9.09 \times 10^{-3} = 120$$

**Case-II:** Here the winding AC for 2200V and BC for 11000V as shown in figure. Therefore



$$V_{AC} = 2200 \text{ V}, V_{BC} = 11000 \text{ V}$$

$$\text{So } V_1 = V_{AC} + V_{BC} = 11000 + 2200 = 13200 \text{ V}$$

$$\therefore V_1 = 13200V, V_2 = 11000V$$

$$K = \frac{V_2}{V_1} = \frac{11000}{13200} = 0.833$$

By applying KCL at point C

$$I_2 = I_{AC} + I_{CB} = 45.45 + 9.09 = 54.54A$$

$$\text{kVA output} = V_2 I_2 \times 10^{-3} = 11000 \times 54.54 \times 10^{-3} = 600$$

$$(\text{or}) \text{ kVA output} = V_1 I_1 \times 10^{-3} = 13200 \times 44.45 \times 10^{-3} = 600$$

**ASP-13:** The efficiency of a 400 kVA, 1- $\phi$  transformer is 98.77% on full load at 0.8 p.f. and 99.13% on half load at unity p.f. Calculate the iron loss and full-load copper loss.

**Solution:** Given that

$$\text{kVA rating} = 400$$

$$\eta = 98.77\% \text{ a full load, at p.f.} = 0.8$$

$$\eta = 99.13\% \text{ at } \frac{1}{2} \text{ load, at p.f.} = 1$$

Efficiency at any desired load is given by

$$\eta_{\text{at any load}} = \frac{xS \cos \phi}{xS \cos \phi + W_i + x^2 W_{cu}} \times 100$$

$$\Rightarrow \frac{1 \times 400 \times 10^3 \times 0.8}{1 \times 400 \times 10^3 \times 0.8 + W_i + W_{cu}} \times 100 = 98.77\%$$

$$\Rightarrow \frac{320 \times 10^3}{320 \times 10^3 + W_i + W_{cu}} = 0.9877$$

$$\Rightarrow \frac{320 \times 10^3}{0.9877} = W_i + W_{cu} + 320 \times 10^3$$

$$\Rightarrow W_i + W_{cu} = 3985$$

(1)

$$\eta \text{ at } \frac{1}{2} \text{ F.L.} = \frac{\frac{1}{2} \times 400 \times 1 \times 10^3}{\frac{1}{2} \times 400 \times 10^3 \times 1 + W_i + \frac{1}{4} W_{cu}} \times 100 = 99.13$$

$$\Rightarrow \frac{200 \times 10^3}{200 \times 10^3 + W_i + \frac{W_{cu}}{4}} = 0.9913$$

$$\Rightarrow \frac{200 \times 10^3}{0.9913} = 200 \times 10^3 + W_i + \frac{W_{Cu}}{4}$$

$$\Rightarrow W_i + \frac{W_{Cu}}{4} = 1754.2 \quad (2)$$

By solving above eqns. (1) and (2) we can get

$$W_{cu} = 2973 \text{ W}, \quad W_i = 1012 \text{ W}$$

### Exercise Problems

1. The core of 1000kVA, 11000/550 V, 50Hz, single phase transformer has a cross sections of  $20\text{cm} \times 20\text{cm}$ . If the maximum core density is 1.3 tesla, calculate

- (i) The number of HV and LV turns per phase
  - (ii) The emf per turn
- Assume a staking factor of 0.9.

[Ans:  $N_1 = 1059, N_2 = 53, \text{emf/turn} = 10.39\text{V}$ ]

2. A 3300/220V, 30kVA, 1-phase transformer takes no-load current of 1.5A when the low voltage winding kept open. The iron loss component is equal to 0.4A, find

- (i) Magnetizing component
- (ii) No. load i/p power
- (iii) Power factor of no load current

[Ans:  $I_\mu = 1.44\text{A}, P_0 = 1320\text{W}, \cos\phi_0 = 0.267$ ]

3. A 100kVA, 1-phase transformer has an iron loss of 1kW and full load copper loss of 1.5kW. Find the maximum efficiency at a power factor of 0.8 lagging and the corresponding kVA loading.

[Ans: 97.029%, 81.65 kVA]

4. The efficiency of 1000 kVA, 110/200V, 50Hz, 1-phase transformer is 98.5% at half full load at 0.8 p.f. leading and 98% at full load, upf. Determine

- (i) Iron loss (ii) copper loss (iii) Max. efficiency at upf

5. A 1-phase transformer has 180 and 300 turns, respectively, in its secondary and primary windings. The respective resistances are  $0.233 \Omega$  and  $0.067 \Omega$  calculate the equivalent resistance of

- (i) the primary in terms of the secondary winding
- (ii) the secondary in terms of the primary winding
- (iii) the total resistance of the transformer in terms of the primary

[Ans: (i)  $R'_1 = 0.0827\Omega$  (ii)  $R'_2 = 0.1891\Omega$  (iii)  $R_{01} = 0.256\Omega$ ]

6. A 100-kVA, 3300/400-V, 50 Hz, 1 phase transformer has 110 turns on the secondary. Calculate the approximate values of the primary and secondary full-load currents, the maximum value of flux in the core and the number of primary turns.

[Ans:  $I_1 = 30.3 \text{ A}, I_2 = 250 \text{ A}, \Phi_m = 16.4 \text{ mWb}, N_1 = 907$ ]

7. The no-load current of a transformer is 4.0 A at 0.25 p.f. when supplied at 250-V, 50 Hz. The number of turns on the primary winding is 200. Calculate  
 (i) the r.m.s. value of the flux in the core (assume sinusoidal flux)  
 (ii) the core loss  
 (iii) the magnetizing current.  
 [Ans: (i) 3.96 mWb (ii) 250 W (iii) 3.87 A]
8. A 400/200-V, 1-phase transformer is supplying a load of 50 A at a power factor of 0.866 lagging. The no-load current is 2 A at 0.208 p.f. lagging. Calculate the primary current and primary power factor.  
 [Ans: 26.4 A; 0.838 lag]
9. The following data apply to a single- phase transformer: output: 100 kVA, secondary voltage: 400V; Primary turns: 200; secondary turns: 40; Neglecting the losses, calculate:  
 (i) the primary applied voltage (ii) the normal primary and secondary currents (iii) the secondary current, when the load is 25 kW at 0.8 power factor.  
 [Ans: (i) 2000V (ii) 50A (iii) 78.125A]
10. A 30 kVA, 2400/120-V, 50-Hz transformer has a high voltage winding resistance of  $0.1\Omega$  and a leakage reactance of  $0.22\Omega$ . The low voltage winding resistance is  $0.035\Omega$  and the leakage reactance is  $0.012\Omega$ . Find the equivalent winding resistance, reactance and impedance referred to the (i) high voltage side and (ii) the low-voltage side.  
 [Ans: (i)  $Z_{01} = 15\Omega$  (ii)  $Z_{02} = 0.0374\Omega$ ]
11. In a transformer, the core loss is found to be 52W at 40 Hz and 90W at 60 Hz measured at same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.  
 [Ans: 45W, 25W]
12. At full-load, the Cu and iron losses in a 100-kVA transformer are each equal to 6.5 kW. Find the efficiency at a load of 65 kVA, power factor 0.8.  
 [Ans: 93.58%]
13. A single phase transformer with a ratio of 6600/400V takes a no load current of 0.7A at a p.f. of 0.24. If the secondary supplies a current of 120A at 0.8 lagging p.f. Calculate the primary current and p.f..  
 [Ans:  $I_1 = 7.828\text{ A}$ , p.f. = 0.765 lagging]
14. Full load efficiency of a 4000/400V, 40kVA, single phase transformer is 94%. Maximum efficiency occurs at 90% of full load. Find iron loss and full load copper loss of the transformer, the load p.f. being 0.8 lagging.  
 [Ans:  $W_i = 0.914\text{ kW}$ ,  $W_{cu} = 1.128\text{ kW}$ ]
15. A 600kVA single phase transformer when working at u.p.f. has an efficiency of 92% at full load and also at half full load. Determine its efficiency when it operates at u.p.f. and 75% of full load.  
 [Ans: 96.41%]

16. A transformer is connected to 2200 V, 40 Hz supply. The core-loss is 800 watts out of which 600 watts are due to hysteresis and the remaining, eddy current losses. Determine the core loss if the supply voltage and frequency are 3300 V and 60 Hz respectively.

[Ans: 1.350kW]

17. The efficiency of a 1000-kVA, 110/220 V, 50-Hz, single-phase transformer, is 98.5 % at half full-load at 0.8 p.f. leading and 98.8 % at full-load unity p.f. Determine (a) iron loss (b) full-load copper loss and (c) maximum efficiency at unity p.f.

[Ans: (a) 4.073 kW (b) 16.146 kW c) 98.9%]

18. A step-down transformer is connected to 3-phase, 6kV supply. The supply current is 12A. The ratio of turns per phase is 10. Determine the secondary line voltage, the line current and the output for the following connections:

(i) Star/Star (ii) Star/Delta (iii) Delta/Star and (iv) Delta/Delta

[Ans: (i) 600V, 120A, 124.7kVA (ii) 346.4V, 207.84A, 124.7kVA  
 (iii) 1039V, 69.3A, 124.7kVA (iv) 600V, 120A, 124.7kVA]

## Review Questions

- Explain why transformer cannot operate on DC supply.
- Transformer rating in kVA but not in kW like other machine. Justify.
- Explain how the primary current increase as the current on the secondary side of the transformer is increased.
- Obtain the equivalent circuit of a transformer when referred to primary from the fundamentals.
- Obtain the equivalent circuit of a transformer when referred to secondary from the fundamentals.
- Define a transformer. How is the energy transferred from one circuit to the other?
- Prove that the efficiency of a transformer is maximum when variable losses equal to constant losses.
- Explain why hysteresis and eddy current losses occur in a transformer.
- Prove that the EMF induced in the windings of the transformer will lag behind the flux by  $90^\circ$ .
- Explain how do you minimize the hysteresis and eddy current losses in a single phase transformer.
- Explain the functions of the following in a transformer.  
 (i) Breather (ii) Conservator (iii) Transformer oil
- Explain the following with respect to single phase transformer.  
 (i) Core (ii) Winding (iii) Methods of cooling (iv) Conservator and bushing
- Define voltage regulation of a transformer. What causes a change in secondary terminal voltage of a transformer, as it is loaded.
- Derive an expression for the saving of copper effected by using an auto-transformer instead of 2-winding transformer.

15. Why kVA rating of auto-transformer is more than the corresponding two winding transformer?
16. Explain how a two winding transformer can be converted into an auto-transformer? What is its new rating?
17. Derive the expression for voltage regulation of a transformer from the simplified approximate equivalent circuit and obtain condition for zero regulation.
18. List out the advantages and disadvantages of a bank of three 1-phase transformers to a single unit of 3-phase transformer.
19. What are the distinguish features of (i) Y-Y (ii)  $\Delta$ - $\Delta$  (iii) Y- $\Delta$  (iv)  $\Delta$ -Y 3-phase transformer connections? Compare their advantages and disadvantages.
20. Explain the following characteristics of an auto transformer with a two winding transformer.
  - (i) Rating (ii) Losses (iii) Impedance drop (iv) Voltage regulation

### Multiple Choice Questions

1. A transformer works on
  - (a) DC (b) AC (c) AC & DC both (d) Neither AC nor DC
2. The efficiency of a transformer is maximum when
  - (a) It runs at half full load (b) It runs at full load.
  - (c) Its Cu loss equals iron loss (d) It runs overload
3. The frequency of the secondary voltage of a transformer will be -----
  - (a) Less than the frequency of the primary voltage.
  - (b) Equal to the primary voltage.
  - (c) Greater than the frequency of the primary voltage.
  - (d) Very much greater than the frequency of the primary voltage
4. Transformer core is laminated to
  - (a) Reduce the copper losses (b) Reduce the core losses.
  - (c) Reduce the eddy current losses (d) Turn ratio is higher than voltage ratio
5. Which type of loss is not common to transformers and rotating machines?
  - (a) Eddy current loss (b) Copper loss
  - (c) Hysteresis loss (d) Windage loss
6. The voltage ratio of the transformer is given as
  - (a)  $V_s/V_p$  (b)  $T_s/T_p$  (c)  $T_p/T_s$  (d)  $V_p/V_s$
7. The dielectric strength of transformer oil should be
  - (a) 100V (b) 5 kV (c) 30 kV (d) 132 kV
8. Transformer cores are laminated with
  - (a) Low carbon steel (b) Silicon sheet steel
  - (c) Nickel alloy steel (d) Chromium sheet steel
9. Transformer oil provides
  - (a) Insulation and cooling (b) Cooling and lubrication
  - (c) Lubrication and insulation (d) Insulation, cooling and lubrication

10. Electric power is transformed upon one coil to other coil in a transformer  
(a) Electrically (b) Electro Magnetically (c) Magnetically (d) Physically
11. The basic function of a transformer is to change  
(a) the level of the voltage (b) the power level  
(c) the power factor (d) the frequency
12. In a transformer, electrical power is transferred from one circuit to another without change in  
(a) voltage (b) current (c) frequency (d) turns ratio
13. The efficiency of a power transformer is around  
(a) 50% (b) 65% (c) 80% (d) 95%
14. In a transformer operating at constant voltage, if the input frequency increases, the core loss  
(a) increases (b) decreases (c) remains constant (d) none
15. The inductive reactance of a transformer depends on  
(a) electromotive force (b) magneto motive force  
(c) magnetic flux (d) leakage flux
16. The generation voltage is usually  
(a) between 11 kV and 33 kV (b) between 132 kV and 400 kV  
(c) between 400 kV and 700 kV (d) None
17. The primary winding of a 220/6 V, 50 Hz transformer is energized from 110 V, 60 Hz supply. The secondary output voltage will be  
(a) 3.6 V (b) 6.5 V (c) 3.0 V (d) 6.0 V
18. The emf induced in the primary of a transformer  
(a) is in phase with the flux (b) lags behind the flux by  $90^\circ$ .  
(c) leads the flux by  $90^\circ$  (d) is in phase opposition to that of flux.
19. A 1:5 step-up transformer has 120V across the primary and 600 ohms resistance across the secondary. Assuming 100% efficiency, the primary current equals  
(a) 0.2 A (b) 5 A (c) 10 A (d) 20 A
20. The efficiency of a transformer is mainly dependent on  
(a) core losses (b) copper losses (c) stray losses (d) both (a) and (b)
21. A 220/440V, 50 Hz, 5kVA, single phase transformer operates on 220V, 40Hz supply with secondary winding open circuited. Then  
(a) Both eddy current and hysteresis losses decreases.  
(b) Both eddy current and hysteresis losses increases.  
(c) Eddy current loss remains the same but hysteresis loss increases.  
(d) Eddy current loss increases but hysteresis loss remains the same
22. No load current in a transformer  
(a) lags the applied voltage by  $90^\circ$   
(b) lags the applied voltage by somewhat less than  $90^\circ$   
(c) leads the applied voltage by  $90^\circ$   
(d) leads the applied voltage by somewhat less than  $90^\circ$

23. A transformer operates most efficiently at 3/4th full load. Its iron ( $W_i$ ) and copper loss ( $W_{cu}$ ) are related as  
 (a)  $W_i/W_{cu} = 16/9$  (b)  $W_i/W_{cu} = 4/3$  (c)  $W_i/W_{cu} = 9/16$  (d)  $W_i/W_{cu} = 3/4$
24. A step-up transformer increases  
 (a) voltage (b) current (c) power (d) frequency
25. The conservator is used in a transformer because  
 (a) it supplies oil to the transformer whenever needed  
 (b) protects the transformer from damage when oil expands due to rise in temperature  
 (c) it provides fresh air and cools down the oil  
 (d) none
26. Which of the following has the highest efficiency?  
 (a) DC shunt motor (b) transformer (c) induction motor (d) synchronous motor
27. Transformer is used -----  
 (a) to step-up the voltage (b) to step-down the voltage  
 (c) on DC (d) to step-up or step-down the voltage
28. A 200 kVA transformer has an iron loss of 1 kW and full-load Cu loss of 2kW. Its load kVA corresponding to maximum efficiency is  
 (a) 100 kVA (b) 141.4 kVA (c) 50 kVA (d) 200 kVA
29. If Cu loss of a transformer at 7/8th full load is 4900 W, then its full-load Cu loss would be-----  
 (a) 5600W (b) 6400W (c) 375W (d) 429W
30. The maximum efficiency of a 100kVA transformer having iron loss of 900 kW and F.L. Cu loss of 1600 W occurs at  
 (a) 56.3 kVA (b) 133.3 kVA (c) 75 kVA (d) 177.7 kVA
31. Transformers are rated in kVA instead of kW because  
 (a) load power factor is often not known  
 (b) kVA is fixed whereas kW depends on load p.f.  
 (c) total transformer loss depends on volt-ampere  
 (d) it has become customary
32. Which of the following connections is best suited for 3-phase, 4-wire service?  
 (a)  $\Delta$ - $\Delta$  (b) Y-Y (c)  $\Delta$ -Y (d) Y- $\Delta$
33. The secondary line to line voltage of a star-delta connected transformer is measured to be 400 V. If the turns ratio between the primary and secondary coils is 2:1, the applied line to line voltage in the primary is:  
 (a) 462 V (b) 346 V (c) 1386 V (d) 800 V
34. The secondary line to line voltage of a delta-delta connected transformer is measured to be 400 V. If the turns ratio between the primary and secondary coils is 2:1, the applied line to line voltage in the primary is:  
 (a) 462 V (b) 346 V (c) 1386 V (d) 800 V
35. The secondary line to line voltage of a delta-star connected transformer is measured to be 400 V. If the turns ratio between the primary and secondary coils is 2:1, the applied line to line voltage in the primary is:  
 (a) 800 V (b) 500 V (c) 1386 V (d) 462 V

36. The secondary line current of a star-delta connected transformer is measured to be 100A. If the turns ratio between the primary and secondary coils is 2:1, the line current in the primary is:  
 (a) 50 A (b) 28.9 A (c) 57.7 A (d) 60 A
37. The secondary line current of a delta-star connected transformer is measured to be 100A. If the turns ratio between the primary and secondary coils is 2:1, the line current in the primary is:  
 (a) 86.6 A (b) 50 A (c) 60 A (d) 57.7 A
38. A Y-Y connection of transformer does not work satisfactorily if the load is  
 (a) zero (b) balanced (c) unbalanced (d) equal to full load
39. A  $\Delta$ -Y connection of transformer is generally used when it is necessary to  
 (a) step-up the voltage (b) step-down the voltage  
 (c) maintains constant voltage (d) all

### Answers

1.b	2.c	3.b	4.c	5.d	6.a	7.c	8.b	9.d	10.c
11.a	12.c	13.d	14.a	15.d	16.a	17.c	18.c	19.a	20.d
21.a	22.b	23.c	24.a	25.b	26.b	27.d	28.b	29.b	30.c
31.c	32.b	33.c	34.d	35.d	36.b	37.a	38.c	39.a	