SIGNALS AND SYSTEMS

II B.TECH I SEM

II ECE-2

- Name of the Faculty: Mr.A.BALA RAJU, Assistant.Professor
- Name of the Course: Signals and Systems.
- Class: II B.Tech-ECE 2-I sem.
- Subject Code: EC 304PC.
- Number of Lectures hours/Week: 4
- Number of Tutorial hours/Week: 1
- Number of Credits: 4

COURSE OBJECTIVES

- This gives the basics of Signals and Systems required for all Electrical Engineering related course
- To understand the behaviour of signal in time and frequency domain
- To understand the characteristics of LTI systems
- This gives concepts of Signals and Systems and its analysis using different transform techniques.

COURSE OUTCOMES

Upon completing this course, the student will be able to

- Differentiate various signal functions.
- Represent any arbitrary signal in time and frequency domain.
- Understand the characteristics of linear time invariant systems.
- Analyse the signals with different transform technique
- Understand different sampling techniques and comparison of signals

SYLLABUS

UNIT - I

- **Signal Analysis:** Analogy between Vectors and Signals, Orthogonal Signal Space, Signal approximation using Orthogonal functions, Mean Square Error, Closed or complete set of Orthogonal functions, Orthogonality in Complex functions, Classification of Signals and systems, Exponential and Sinusoidal signals, Concepts of Impulse function, Unit Step function, Signum function

SYLLABUS

UNIT – II

- **Fourier series:** Representation of Fourier series, Continuous time periodic signals, Properties of Fourier Series, Dirichlet's conditions, Trigonometric Fourier Series and Exponential Fourier Series, Complex Fourier spectrum.
- **Fourier Transforms**: Deriving Fourier Transform from Fourier series, Fourier Transform of arbitrary signal, Fourier Transform of standard signals, Fourier Transform of Periodic Signals, Properties of Fourier Transform, Fourier Transforms involving Impulse function and Signum function, Introduction to Hilbert Transform.

SYLLABUS

UNIT – III

- **Signal Transmission through Linear Systems:** Linear System, Impulse response, Response of a Linear System, Linear Time Invariant(LTI) System, Linear Time Variant (LTV) System, Transfer function of a LTI System, Filter characteristic of Linear System, Distortion less transmission through a system, Signal bandwidth, System Bandwidth, Ideal LPF, HPF, and BPF characteristics, Causality and Paley-Wiener criterion for physical realization, Relationship between Bandwidth and rise time, Convolution and Correlation of Signals, Concept of convolution in Time domain and Frequency domain, Graphical representation of Convolution.

SYLLABUS

UNIT – IV

- **Laplace Transforms:** Laplace Transforms (L.T), Inverse Laplace Transform, Concept of Region of Convergence (ROC) for Laplace Transforms, Properties of L.T, Relation between L.T and F.T of a signal, Laplace Transform of certain signals using waveform synthesis.
- **Z–Transforms:** Concept of Z- Transform of a Discrete Sequence, Distinction between Laplace, Fourier and Z Transforms, Region of Convergence in Z-Transform, Constraints on ROC for various classes of signals, Inverse Z-transform, Properties of Z-transforms.

SYLLABUS

UNIT – V

- **Sampling theorem:** Graphical and analytical proof for Band Limited Signals, Impulse Sampling, Natural and Flat top Sampling, Reconstruction of signal from its samples, Effect of under sampling – Aliasing, Introduction to Band Pass Sampling.
- **Correlation:** Cross Correlation and Auto Correlation of Functions, Properties of Correlation Functions, Energy Density Spectrum, Parsevals Theorem, Power Density Spectrum, Relation between Autocorrelation Function and Energy/Power Spectral Density Function, Relation between Convolution and Correlation, Detection of Periodic Signals in the presence of Noise by Correlation, Extraction of Signal from Noise by Filtering.

TEXT BOOKS:

- 1. Signals, Systems & Communications - B.P. Lathi, 2013, BSP.
- 2. Signals and Systems - A.V. Oppenheim, A.S. Willsky and S.H. Nawabi, 2 Ed.

REFERENCE BOOKS:

- 1. Signals and Systems – Simon Haykin and Van Veen, Wiley 2 Ed.,
- 2. Signals and Systems – A. Rama Krishna Rao, 2008, TMH
- 3. Fundamentals of Signals and Systems - Michel J. Robert, 2008, MGH International Edition.
- 4. Signals, Systems and Transforms - C. L. Philips, J.M.Parr and Eve A.Riskin, 3 Ed., 2004, PE.
- 5. Signals and Systems – K. Deergha Rao, Birkhauser, 2018.

FOURIER SERIES REPRESENTATION OF CONTINUOUS TIME PERIODIC SIGNALS

UNIT-II

Signals can have dual personalities

Time Domain perspective:

i) Depicts its wave form

ii)Signal width(duration)

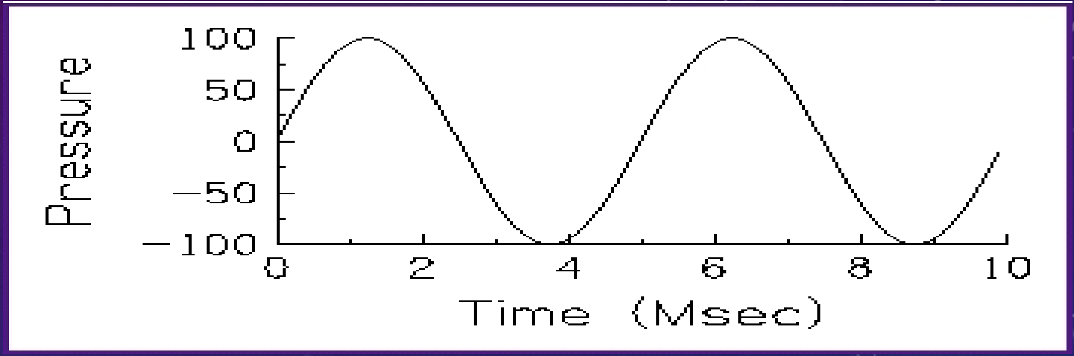
iii) Rate at which the waveform decays

Frequency Domain perspectives:

i) In terms its sinusoidal components

ii) their relative amplitudes and phases

TIME DOMAIN AND FREQUENCY DOMAIN



Time Domain:

i) Depicts its wave form

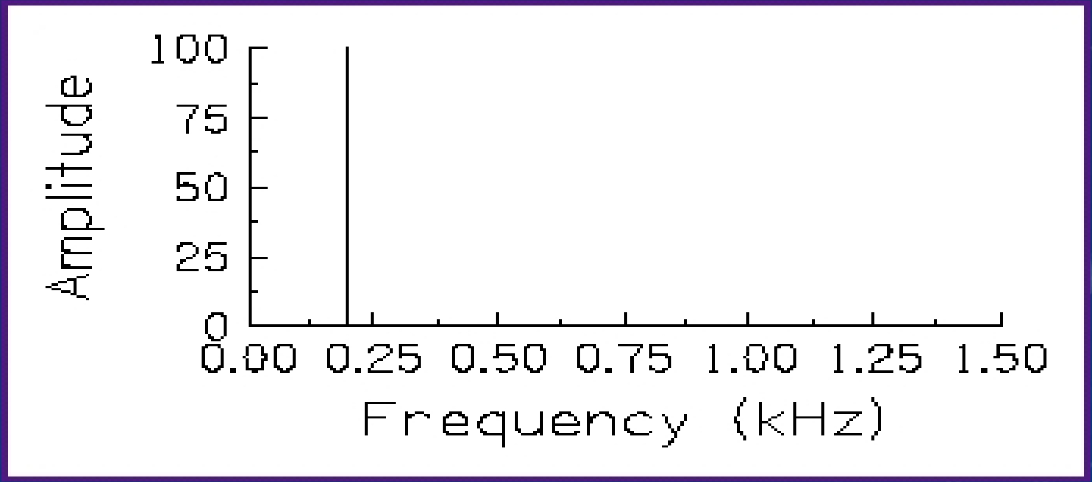
ii)Signal width(duration)

iii) Rate at which the waveform decays

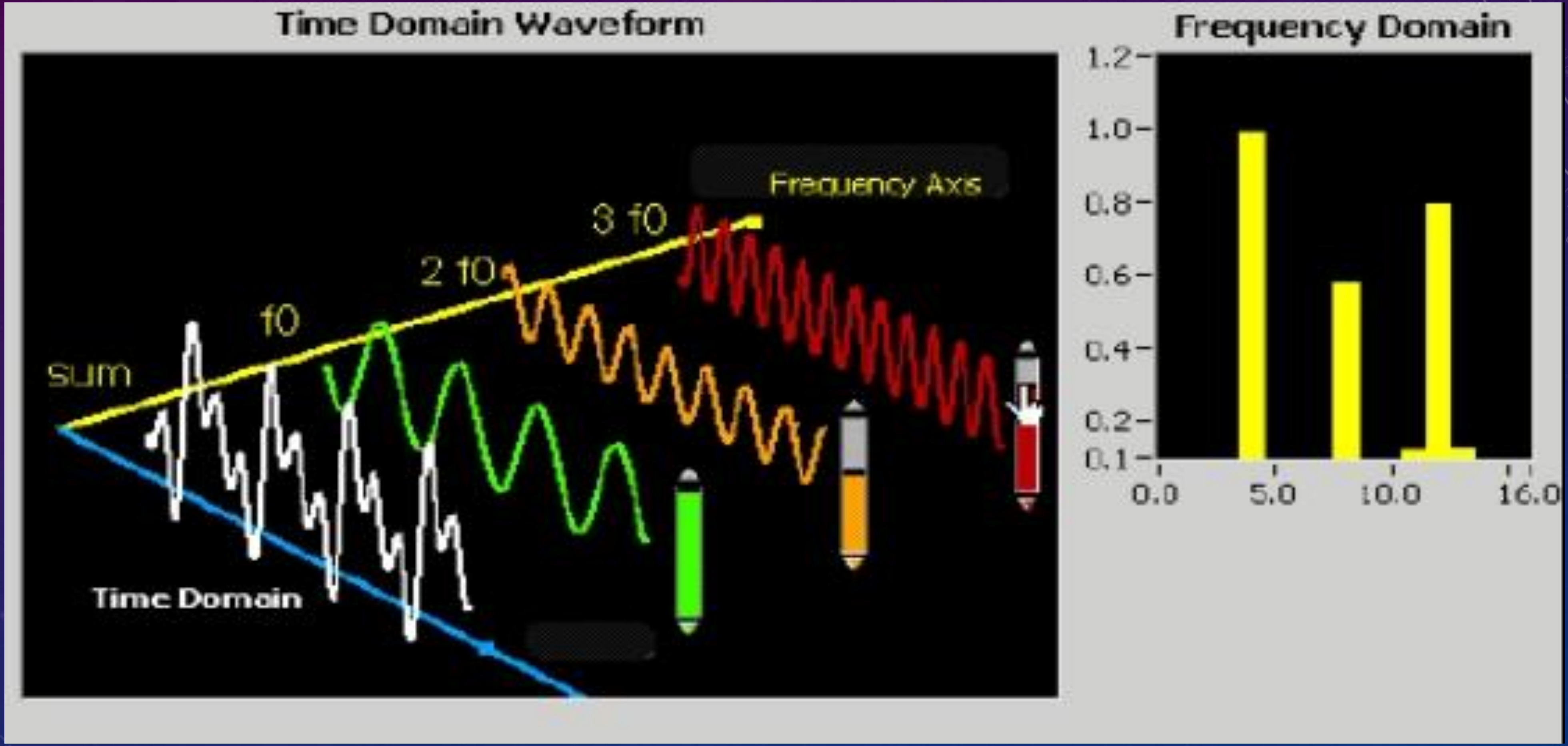
- Amplitude = 100
- Frequency = number of cycles in one second = 200 Hz

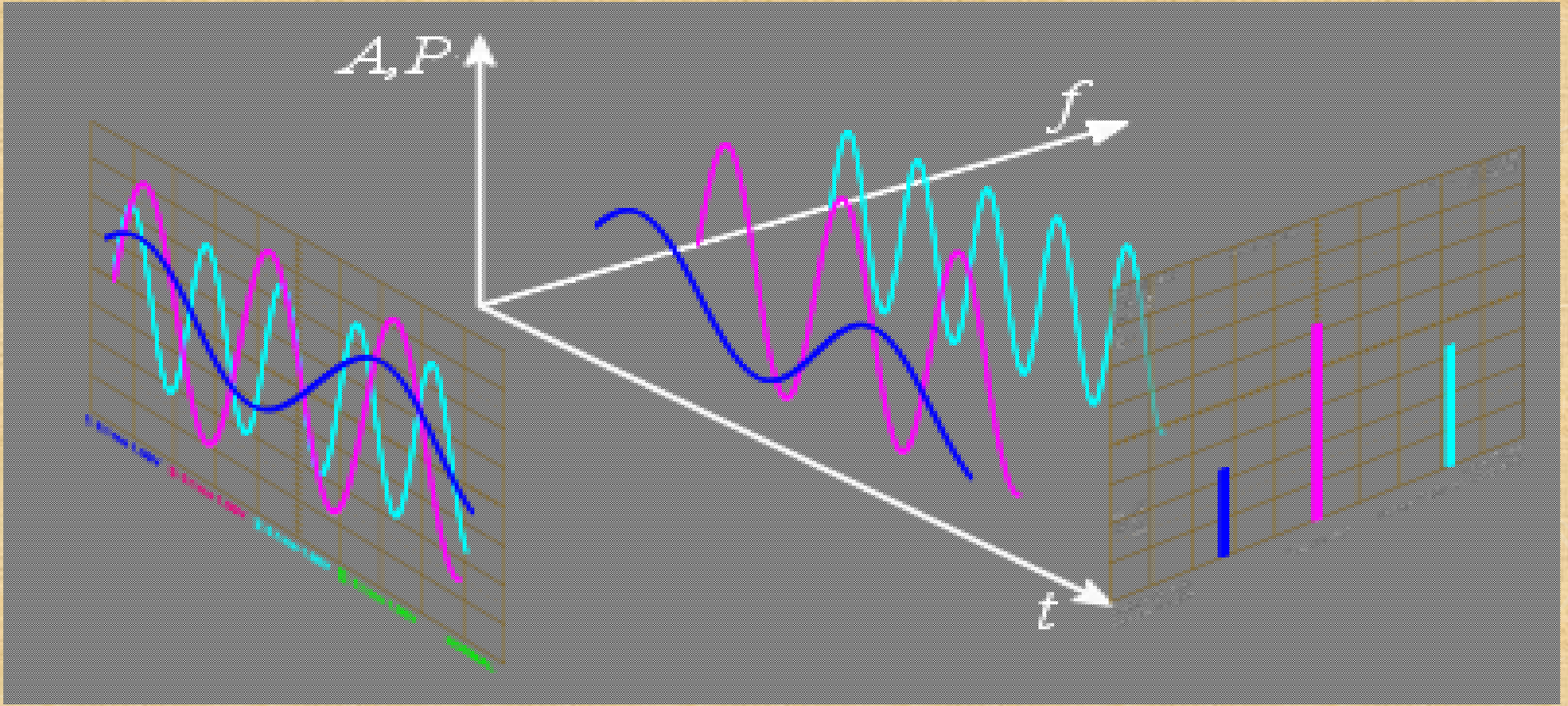
TIME DOMAIN AND FREQUENCY DOMAIN

- Frequency domain:
  - Tells us how properties (amplitudes) change over frequencies:



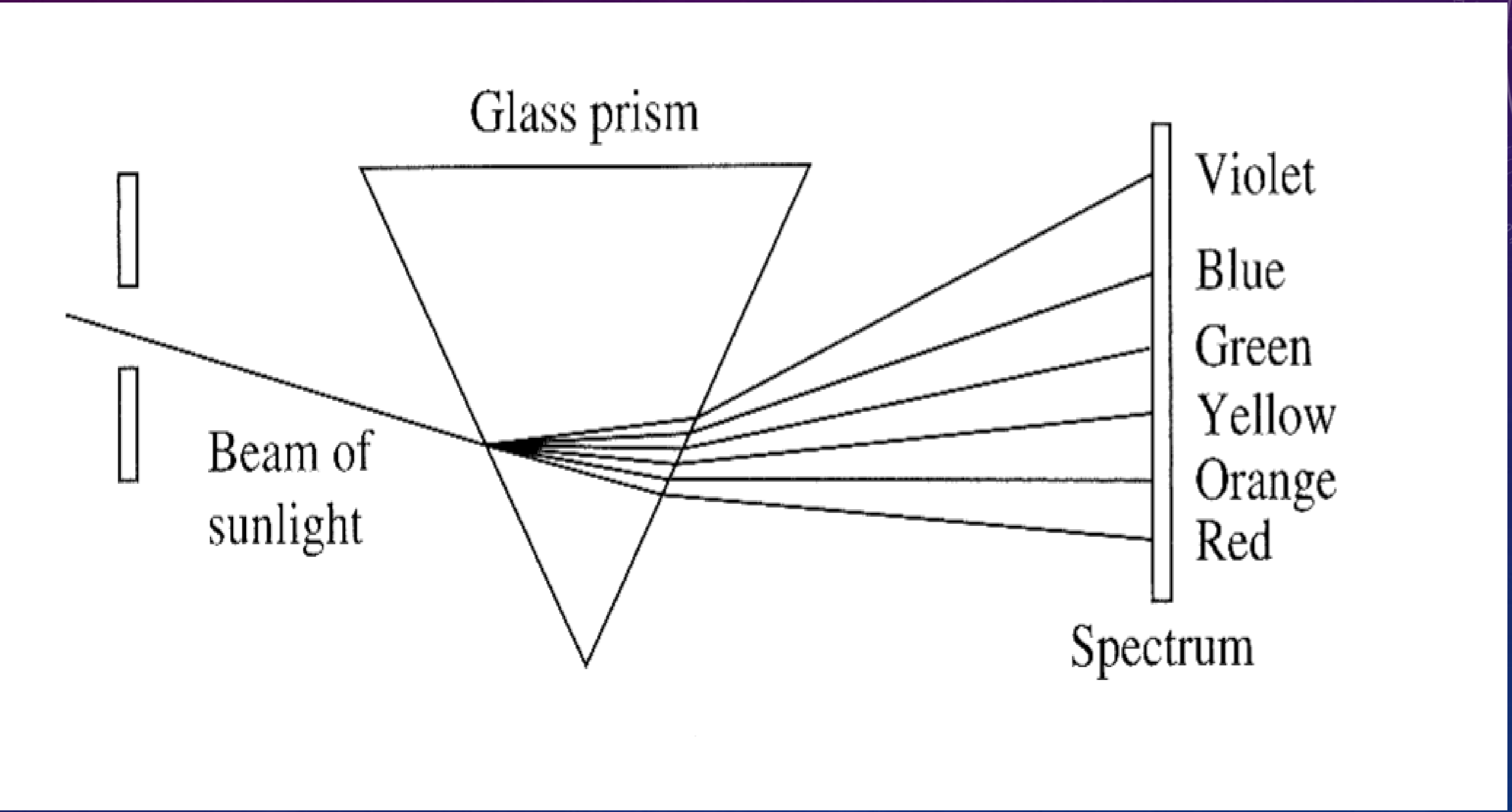
TIME DOMAIN AND FREQUENCY DOMAIN



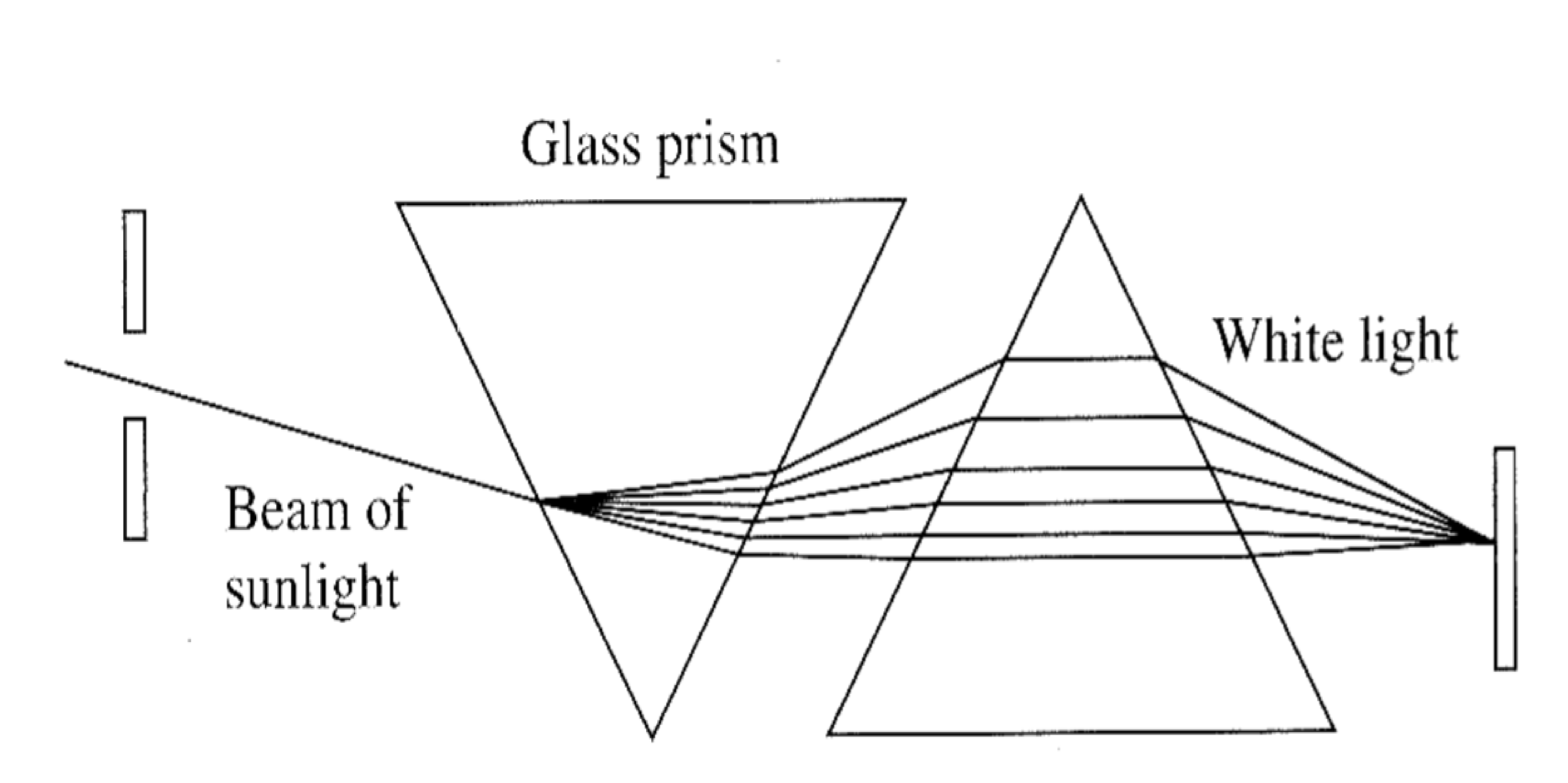

A,P
f
t

FOURIER SERIES REPRESENTATION

- Resolving the signal into its frequency (sinusoids) components is called as Frequency Analysis of the signal
- The Decomposition of the signals in-terms of sinusoidal or complex exponential components.
- With Such a Decomposition, a signal is said to be represented in the Frequency Domain



Glass prism
Beam of
sunlight
Violet
Blue
Green
Yellow
Orange
Red
Spectrum


Glass prism
White light
Beam of
sunlight

FOURIER SERIES REPRESENTATION

- In our treatment of frequency analysis , we will develop the proper mathematical tools (Prism) for the decomposition of the signals (light) into sinusoidal frequency components(colours)
- The basic motivation for developing the frequency analysis tools is to provide a mathematical and pictorial representation for the frequency components that are contained in any given signal

FOURIER SERIES

- The Basic Mathematical representation of periodic signals is the Fourier Series
- Which is a linear weighted sum of harmonically related sinusoids or complex exponentials
- The Decomposition of the Periodic signals into sinusoids or complex exponentials

FOURIER TRANSFORMS

- Such a Decomposition of the Non-Periodic/ energy signals is called Fourier Transform representation

FOURIER SERIES REPRESENTATION OF CONTINUOUS-TIME PERIODIC SIGNALS

- A signal is periodic if, for some positive value of T, $x(t) = x(t + T)$ for all t.
- The fundamental period of $x(t)$ is the minimum positive, nonzero value of T for which the above eq. is satisfied, and the value $\omega_0 = 2\pi/T$ is referred to as the fundamental frequency or the angular frequency
- we also introduced two basic periodic signals,
- the sinusoidal signal $x(t) = \cos \omega_0 t$ and
- the periodic complex exponential $x(t)=e^{j\omega 0t}$
- Both of these signals are periodic with fundamental frequency ω_0 and fundamental period $T = 2\pi / \omega_0$

FOURIER SERIES REPRESENTATION OF CONTINUOUS-TIME PERIODIC SIGNALS

- Let
$$\phi_k(t) = e^{jk\omega_0 t} = e^{jk(2\pi/T)t}, \quad k = 0, \pm 1, \pm 2, \ldots .$$

is the set of *harmonically related* complex exponentials

- Each of these signals has a fundamental frequency that is a multiple of ω_0, and therefore, each is periodic with period *T.*
- Thus, a linear combination of harmonically related complex exponentials

  of the form
$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

FOURIER SERIES REPRESENTATION OF CONTINUOUS-TIME PERIODIC SIGNALS

- is also periodic with period *T*
- The term *for k* = 0 is a constant.
- The terms for $k = +1$ and $k = -1$ both have fundamental frequency equal to ω_0 and are collectively referred to as the *fundamental components* or the *first harmonic components.*
- The two terms for $k = +2$ and $k = -2$ are periodic with half the period (or, equivalently, twice
- the frequency) of the fundamental components and are referred to as the *second harmonic*
- *components.*
- More generally, the components for $k = +N$ and $k = -N$ are referred to as the *Nth* harmonic components.

FOURIER SERIES REPRESENTATION

- Thus, a linear combination of harmonically related complex exponentials of the form

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

- The representation of a periodic signal in the form of eq. is referred to as the *Fourier series* representation.

DIRICHLET’S CONDITIONS

Condition 1. *x*(*t*) is *absolutely integrable* over one period, i.e.

$$\int_T |x(t)|dt < \infty$$

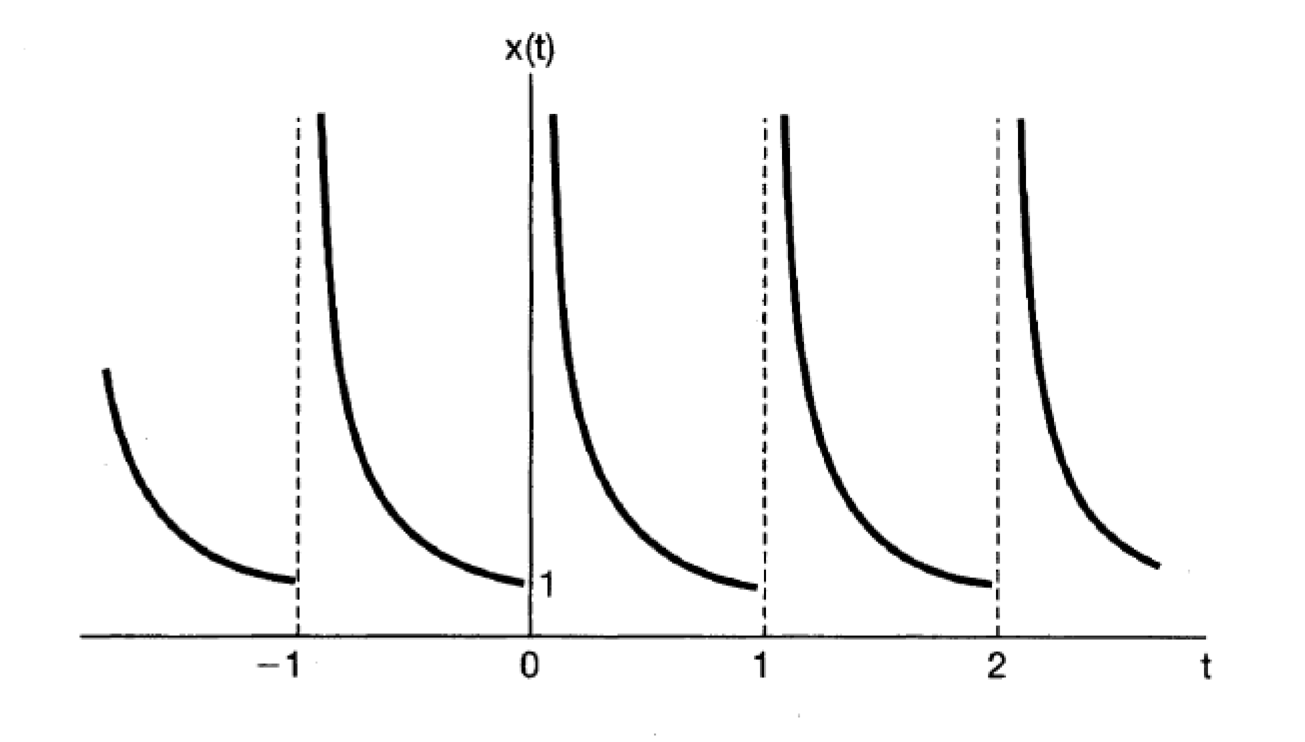
Condition 2. In a finite time interval, *x*(*t*) has a *finite* number of maxima and minima.

Condition 3. In a finite time interval, *x*(*t*) has only a *finite* number of discontinuities.

DIRICHLET'S CONDITIONS

- A periodic signal that violates the first Dirichlet condition is

$$x(t) = \frac{1}{t}, \quad 0 < t \le 1;$$

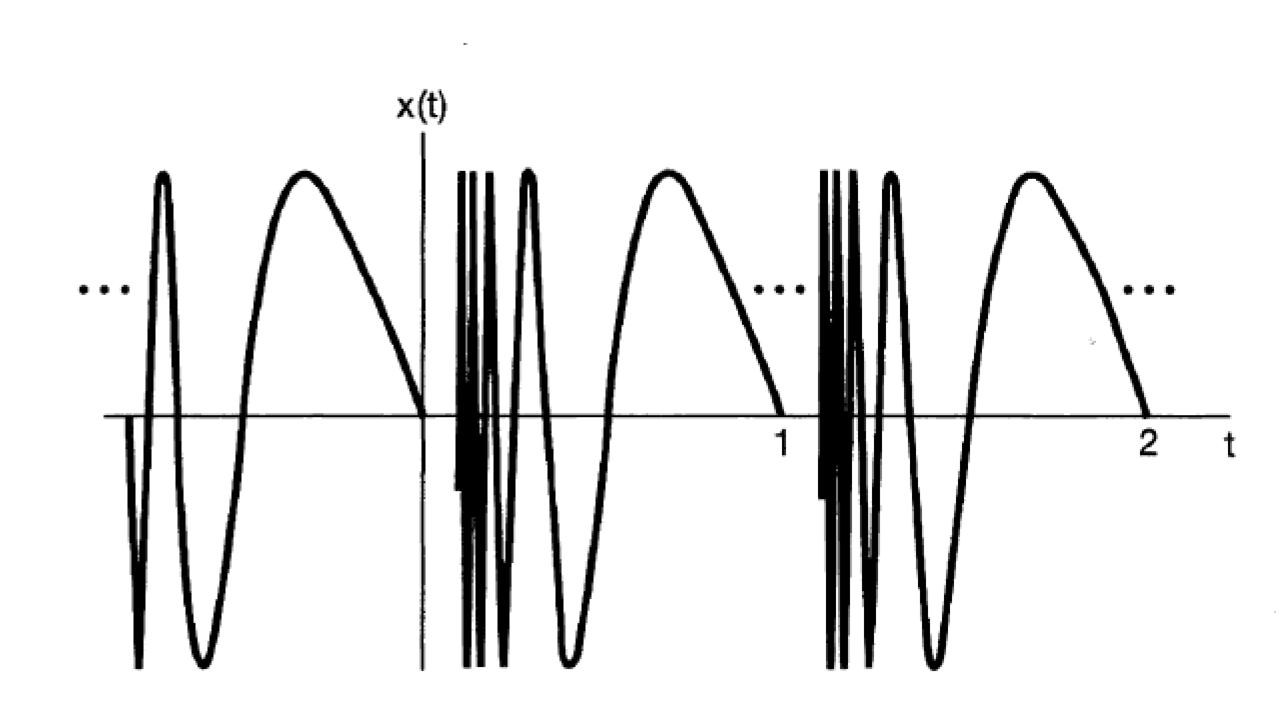


DIRICHLET'S CONDITIONS

An example of a function that meets Condition 1 but not Condition 2 is

$$x(t) = \sin\left(\frac{2\pi}{t}\right) \qquad 0 < t \leq 1$$

an infinite number of maxima and minima in the interval



DIRICHLET'S CONDITIONS

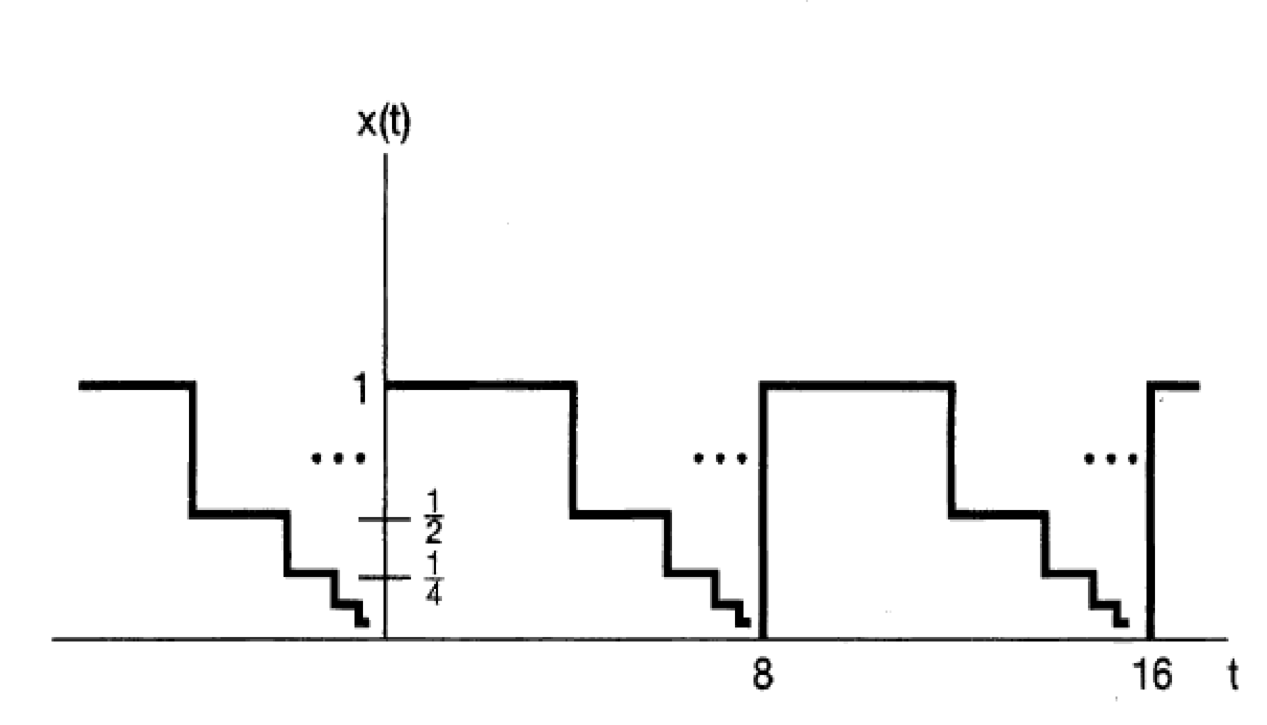
A signal periodic with period 8 that violates the third Dirichlet condition [for $0 < t < 8$ the value of $x(t)$ decreases by a factor of 2 whenever the distance from t to 8 decreases by a factor of 2

$$x(t) = 1, \; 0 \le t < 4$$

$$x(t) = 1/2 \quad 4 \le t < 6$$

$$x(t) = 1/4, \; 6 \le t < 7$$

$$x(t) = 1/8, \; 7 \le t < 7.5$$



TYPES OF FOURIER SERIES REPRESENTATIONS

- Two Different ways of expressing the same periodic function

They are

i) Trigonometric Fourier Series (TFS) Representation

ii) Exponential Fourier Series (EFS) Representation

TRIGONOMETRIC FOURIER SERIES (TFS) REPRESENTATION

- We have already shown that functions sin ωot, sin2ωot ,………. etc., form an orthogonal set over any interval (to, to+T ,T=2π/ωo).
- This set, however, is not complete.
- This is evident from the fact a function cos ωot is orthogonal to sin ωot over the same interval.
- Hence to complete the set , we must include cosine as well as sine functions.
- It can be shown that the composite set of functions consisting of a set cos nωot and sin nωot for (n=0, l, 2,...) forms a complete orthogonal set

TRIGONOMETRIC FOURIER SERIES (TFS) REPRESENTATION

- Note that for n =0, sin nωot is zero, but cos nωot =1. Thus we have a completed orthogonal set represented by functions 1, cos ωot, cos 2ωot,……....cos nωot,.. . ; sin ωot, sin 2ωot,..., sin nωot,..., etc.
- It therefore follows that any function f(t) can be represented in terms of these functions over any interval (to, to+T ,T=2π/ωo).). Thus

TRIGONOMETRIC FOURIER SERIES (TFS) REPRESENTATION

$$f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \cdots + a_n \cos n\omega_0 t + \cdots$$
$$+ b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \cdots + b_n \sin n\omega_0 t + \cdots$$
$$(t_0 < t < t_0 + 2\pi/\omega_0)$$

For convenience we shall denote $2\pi/\omega_0$ by T. The preceding equation can be expressed as

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \qquad (t_0 < t < t_0 + T) \tag{1.46}$$

Equation 1.46 is the trigonometric Fourier series representation of $f(t)$ over an interval $(t_0, t_0 + T)$.

$$a_n = \frac{\int_{t_0}^{(t_0+T)} f(t) \cos n\omega_0 t \, dt}{\int_{t_0}^{(t_0+T)} \cos^2 n\omega_0 t \, dt} \tag{1.47a}$$

and

$$b_n = \frac{\int_{t_0}^{(t_0+T)} f(t) \sin n\omega_0 t \, dt}{\int_{t_0}^{(t_0+T)} \sin^2 n\omega_0 t \, dt} \tag{1.47b}$$

If we let $n = 0$ in Eq. 1.47a, we get

$$a_0 = \frac{1}{T} \int_{t_0}^{(t_0+T)} f(t) \, dt \tag{1.48a}$$

We also have

$$\int_{t_0}^{(t_0+T)} \cos^2 n\omega_0 t \, dt = \int_{t_0}^{(t_0+T)} \sin^2 n\omega_0 t \, dt = \frac{T}{2}$$

- Therefore

$$a_n = \frac{2}{T}\int_{t_0}^{(t_0+T)} f(t)\cos n\omega_0 t\, dt$$

$$b_n = \frac{2}{T}\int_{t_0}^{(t_0+T)} f(t)\sin n\omega_0 t\, dt$$

The constant term ao in the series is given by

$$a_0 = \frac{1}{T}\int_{t_0}^{(t_0+T)} f(t)\, dt$$

The constant term ao is the average value of f(t) over the interval (to, to+T)
Thus ao is the d-c component of f(t) over this interval.

EXPONENTIAL FOURIER SERIES (EFS) REPRESENTATION

It can be shown easily that a set of exponential functions $\{e^{jn\omega_0 t}\}$, $(n = 0, \pm 1, \pm 2, \ldots)$ is orthogonal over an interval $(t_0, t_0 + 2\pi/\omega_0)$ for any value of t_0. Note that this is a set of complex functions. We can demonstrate the orthogonality of this set by considering the integral

$$I = \int_{t_0}^{t_0+2\pi/\omega_0} (e^{jn\omega_0 t})(e^{jm\omega_0 t})^* \, dt = \int_{t_0}^{t_0+2\pi/\omega_0} e^{jn\omega_0 t} e^{-jm\omega_0 t} \, dt$$

If $n = m$, the integral I is given by

$$I = \int_{t_0}^{t_0+2\pi/\omega_0} dt = \frac{2\pi}{\omega_0}$$

If $n \neq m$, the integral I is given by

$$I = \frac{1}{j(n-m)\omega_0} e^{j(n-m)\omega_0 t} \Big|_{t_0}^{t_0+2\pi/\omega_0}$$

$$= \frac{1}{j(n-m)\omega_0} e^{i(n-m)\omega_0 t_0}[e^{i2\pi(n-m)} - 1]$$

Since both n and m are integers, $e^{j2\pi(n-m)}$ is equal to unity, and hence the integral is zero $I=0$

Thus

$$\int_{t_0}^{t_0+2\pi/\omega_0} e^{jn\omega_0 t}(e^{jn\omega_0 t})^* \, dt = \begin{cases} \dfrac{2\pi}{\omega_0} & m = n \\ 0 & m \neq n \end{cases} \tag{1.54}$$

As before, let

$$\frac{2\pi}{\omega_0} = T$$

It is evident from Eq. 1.54 that the set of functions

$$\{e^{jn\omega_0 t}\} \qquad (n = 0, \pm 1, \pm 2, \ldots)$$

is orthogonal over the interval $(t_0, t_0 + T)$ where $T = 2\pi/\omega_0$. Further, it can be shown that this is a complete set. It is therefore possible to represent an arbitrary function $f(t)$ by a linear combination of exponential functions over an interval $(t_0, t_0 + T)$

A.

this is a complete set. It is therefore possible to represent an arbitrary function $f(t)$ by a linear combination of exponential functions over an interval $(t_0, t_0 + T)$:

$$f(t) = F_0 + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t} + \cdots + F_n e^{jn\omega_0 t} + \cdots$$
$$+ F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + \cdots + F_{-n} e^{-jn\omega_0 t} + \cdots$$

for

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \qquad (t_0 < t < t_0 + T)$$

known as exponential fourier series representation of f (t) over the interval (to, to + T).

- The Various Coefficients in series is given by

$$F_n = \frac{\int_{t_0}^{t_0+T} f(t)(e^{jn\omega_0 t})^* \, dt}{\int_{t_0}^{t_0+T} e^{jn\omega_0 t}(e^{jn\omega_0 t})^* \, dt}$$

$$= \frac{\int_{t_0}^{t_0+T} f(t) e^{-jn\omega t_0} \, dt}{\int_{t_0}^{t_0+T} e^{jn\omega_0 t} e^{-jn\omega_0 t} \, dt}$$

$$= \frac{1}{T} \int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} \, dt$$

EXPONENTIAL FOURIER SERIES (EFS) REPRESENTATION

Summarizing the results: Any given function $f(t)$ may be expressed as a discrete sum of exponential functions $\{e^{jn\omega_0 t}\}$, $(n = 0, \pm 1, \pm 2, \ldots)$ over an interval $t_0 < t < t_0 + T$, $(\omega_0 = 2\pi/T)$.

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \qquad (t_0 < t < t_0 + T)$$

where

$$F_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{-jn\omega_0 t}\, dt$$

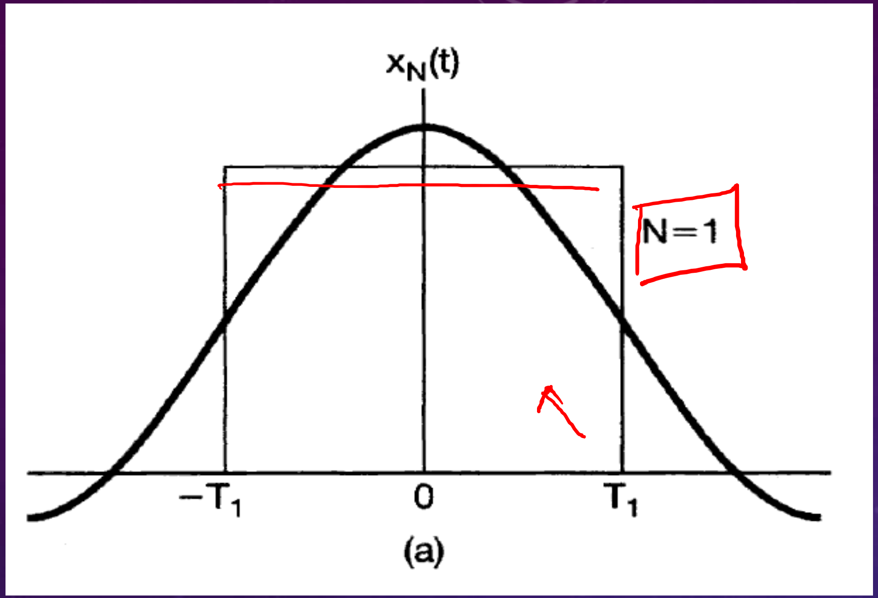
RELATIONSHIP BETWEEN TFS & EFS REPRESENTATIONS

It should be noted that the trigonometric and the exponential Fourier series are not two different types of series but two different ways of expressing the same series. The coefficients of one series can be obtained from those of the other.

$$a_0 = F_0$$
$$a_n = F_n + F_{-n}$$
$$b_n = j(F_n - F_{-n})$$

and

$$F_n = \tfrac{1}{2}(a_n - jb_n)$$



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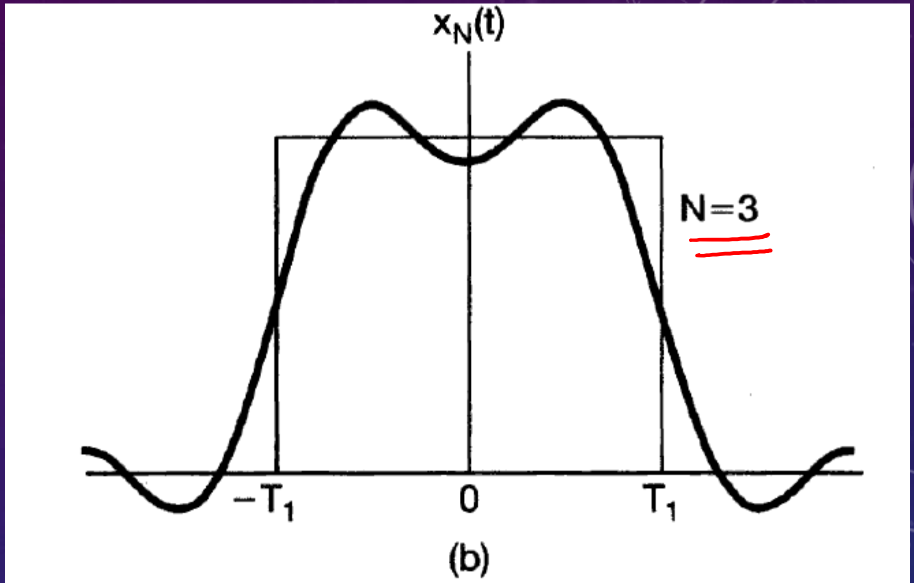

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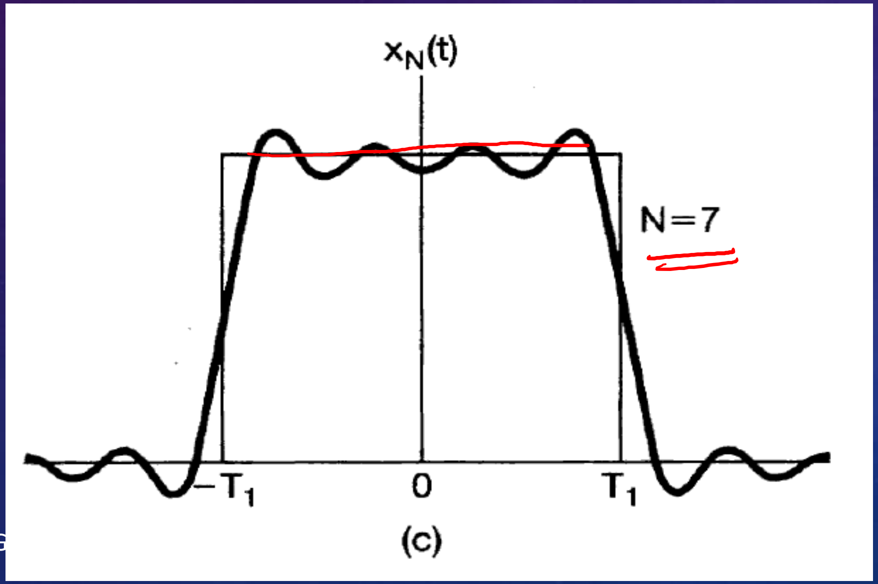

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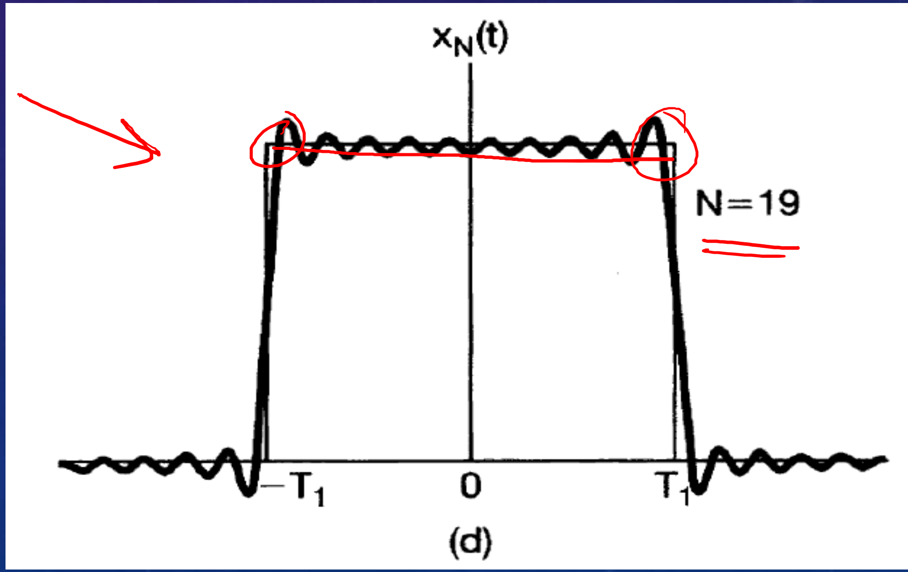
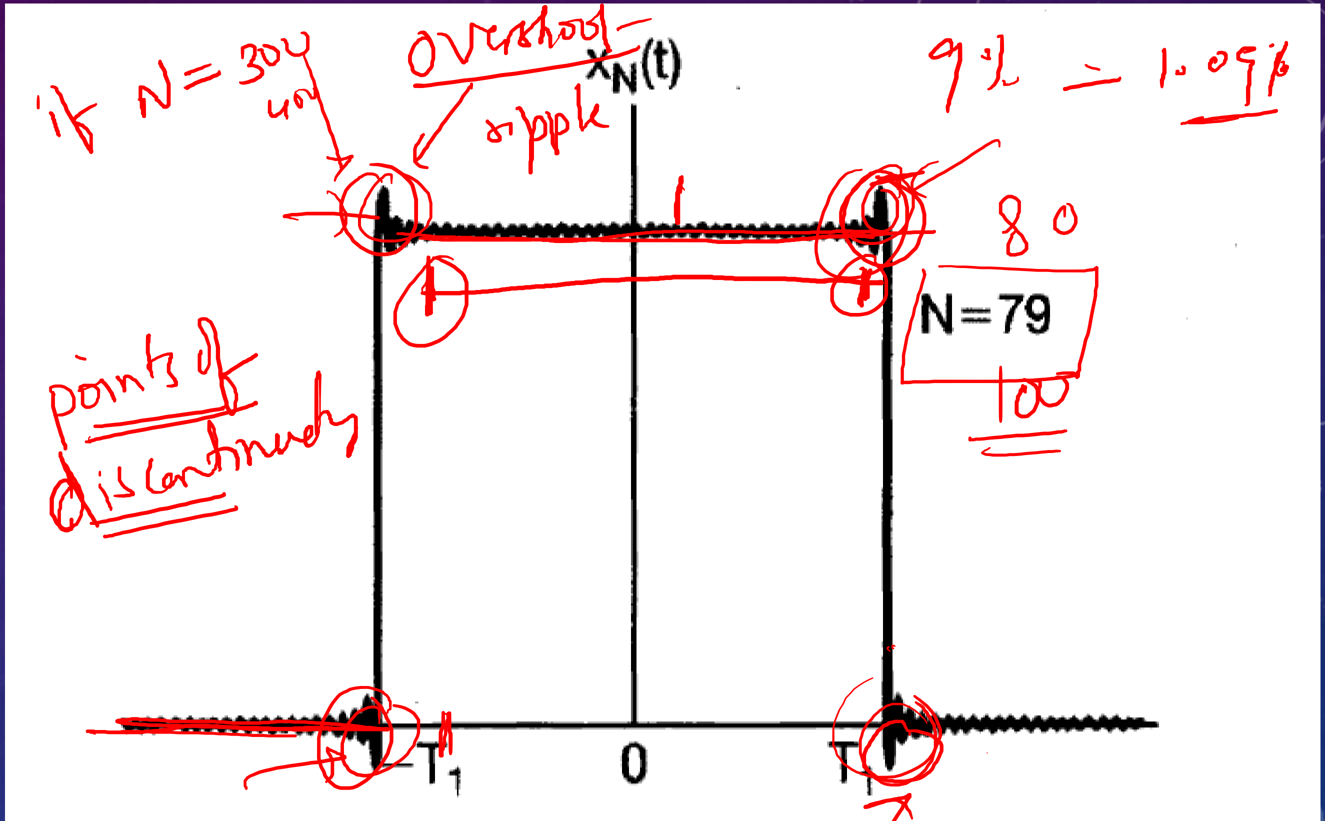


(d)

GIBB'S PHENOMENON

Convergence of the Fourier series representation of a square wave



(e)

FOURIER SERIES REPRESENTATION

FOURIER SERIES REPRESENTATION

classification → F.S → Two types — (i) TFS (ii) EFS

what is F.S

→ (i) Set of mutually orthogonal fns → forms a complete then any fn (Periodic / Non-Periodic) can be expressed interms of these orthogonal fns

→ Sin nω₀t, Cos nω₀t — orthogonal
Sin Sin "
Cos nω₀t Cos mω₀t "

only Sin nω₀t — not complete
only cos ✗
Sin nω₀t, Cos mω₀t — complete

FOURIER SERIES REPRESENTATION

TFS

$t_0+T-t_0 = T$

$$f(t) = a_0 + \sum_{n=1}^{\alpha}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right) \text{ over } (t_0, t_0+T)$$

Periodic

a_0, a_n & b_n — F.S. Coefficients

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \text{avg value} = \text{d.c. Value of } f(t)$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos n\omega_0 t\,dt$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin n\omega_0 t\,dt$$

Exponential Fourier Series (EFS)

→ Complex exponential fn = $e^{j\omega_0 t}$

set of " = $e^{j\omega_0 t}, e^{j2\omega_0 t}, \ldots e^{jn\omega_0 t}$

2nd

nth

$n = [0, \pm 1, \pm 2 \ldots]$

$= e^{-j\omega_0 t}, e^{-j2\omega_0 t} \ldots e^{-jn\omega_0 t}$

Whether = Orthogonal (or) Not

$\phi_k(t) = e^{\pm jn\omega_0 t}$ →

TFS

Sin, Cos → forms Complete

$\sin n\omega_0 t$ ✓

$\sin n\omega_0 t$ – Set

Check for orthogonality Complex exp $\quad I = \int_{t_1}^{t_2} f_1(t) f_2^*(t)\,dt = 0$

$$= \int_{t_1}^{t_2} f_1^*(t) f_2(t)\,dt$$

$f_1(t) = e^{jn\omega_0 t}$, $\quad f_2 = e^{jm\omega_0 t}$

$f_2^*(t) = \quad a \pm ib \quad$ ($i \neq 0$)

$f_2^*(t) = e^{-jm\omega_0 t}$

$I =$

$$I = \int_{t_0}^{t_0+\frac{2\pi}{\omega_0}} e^{jn\omega_0 t} \cdot e^{-jm\omega_0 t}\,dt \Rightarrow \int_{t_0}^{t_0+\frac{2\pi}{\omega_0}} e^{j(n-m)\omega_0 t}\,dt$$

$$\frac{e^{j(n-m)\omega_0 t}}{(n-m)\omega_0}\Bigg|_{t_0}^{t_0+\frac{2\pi}{\omega_0}}$$

$$= \frac{1}{(n-m)\omega_0}\left(e^{j(n-m)\omega_0\left(t_0+\frac{2\pi}{\omega_0}\right)} - e^{+j(n-m)\omega_0 t_0}\right)$$

n, m are Integers

n−m – is also Int

$e^{j2\pi K}$

$\underline{\cos 2\pi k}_{1} + j\,\underline{\sin 2\pi K}_{0}$

$$= \frac{1}{(n-m)\omega_0}\left[\underline{e^{j(n-m)\omega_0 t_0}} \cdot e^{j(n-m)2\pi} - e^{j(n-m)\omega_0 t_0}\right]$$

$$I = \frac{1}{(n-m)\omega_0}\left[e^{j(n-m)\omega_0 t_0}\left(\frac{e^{j(n-m)2\pi}}{1} - 1\right)\right]$$

$\boxed{I = 0}$

$$\int_{t_1}^{t_2} f_1(t)\, f_2^*(t)\, dt = 0, \quad \left.\begin{matrix} e^{jn\omega_0 t} \\ e^{jm\omega_0 t} \end{matrix}\right\} \rightarrow \text{orthogonal fns}$$

(i) orthogonal (ii) Complete $\rightarrow e^{jn\omega_0 t} \rightarrow n = 0, \pm 1, \pm 2, \pm 3 \ldots$

$e^{jn\omega_0 t} = \underline{\cos n\omega_0 t} + j\underline{\sin n\omega_0 t} \Rightarrow$ Complete set

(iii) any fn can be expressed by these set of mutually orthogonal complex exponential fns $\rightarrow$ EFS

sinusoids TFS

$$f(t) \cong F_0 + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t} + \ldots F_n e^{jn\omega_0 t} + \ldots$$

$$+ F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + \ldots F_{-n} e^{-jn\omega_0 t}$$

$$f(t) \cong F_0 + \sum_{n=-\alpha}^{\alpha} F_n e^{jn\omega_0 t} \quad (n \neq 0)$$

$$\boxed{f(t) = \sum_{n=-\alpha}^{\alpha} F_n e^{jn\omega_0 t}}$$ — EFS of $f(t)$ $\left(t_0,\ t_0+\frac{2\pi}{\omega_0}\right)$

Over one Period

$$F_n = \frac{\int_{t_0}^{t_0+\frac{2\pi}{\omega_0}} f(t) \cdot e^{-jn\omega_0 t} \cdot dt}{\int_{t_0}^{t_0+\frac{2\pi}{\omega_0}} e^{jn\omega_0 t} \cdot e^{-jn\omega_0 t} \cdot dt}$$

$$C_2 = \frac{\int f_1(t) f_2^*(t)}{f_2(t) f_2^*(t)}$$

$e^0 = \int 1 \cdot dt$

$t \Big|$

$= T$

$$F_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,e^{-jn\omega_0 t}\,dt \rightarrow \text{E F.S. coefficients}$$

$$n=0 \Rightarrow F_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = a_0 = \text{avg value of } f(t)$$

d.c. value

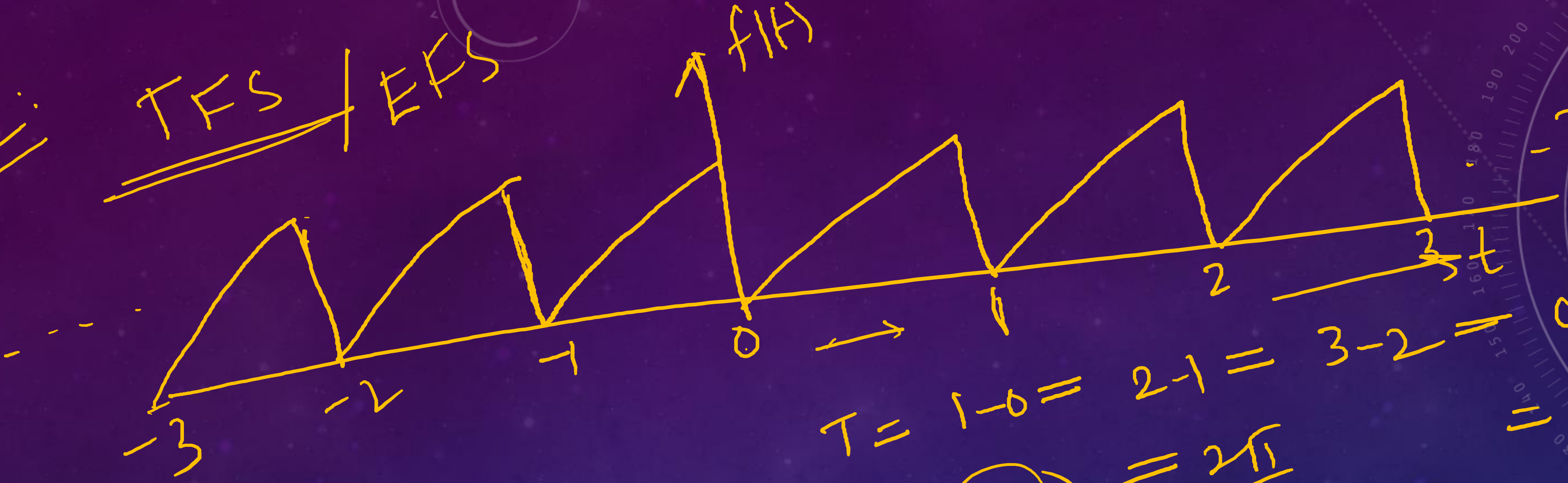
$$EFS \rightarrow f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \rightarrow \text{where } F_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\, e^{-jn\omega_0 t}\, dt$$

periodic

$$TFS = f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right)$$

f(t) — Same — TFS, EFS

Problem: TFS / EFS



$f(t)$ - Periodic

$T = 1-0 = 2-1 = 3-2 = 0-(-1)$

$= -1-(-2)$

$T = 1 = \frac{2\pi}{\omega_0}$

$(0, 1)$

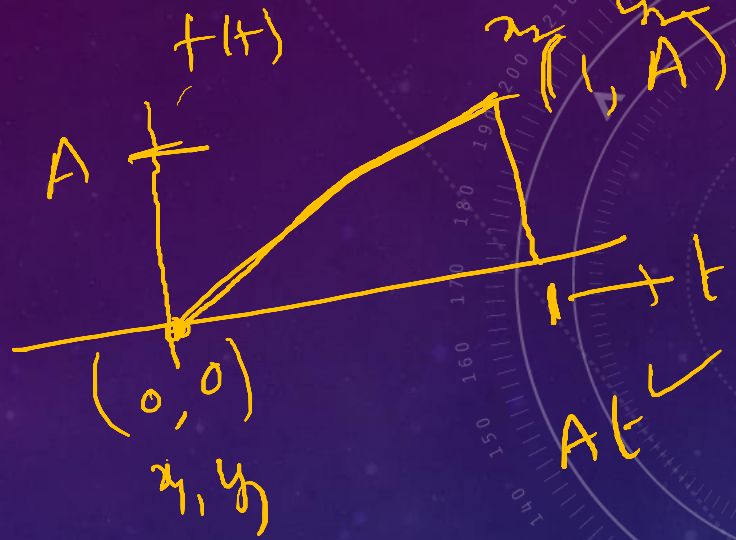
Sol: TFS = $f(t) \cong a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right)$ —(i)

$a_0 = \frac{1}{T} \int_0^T f(t)\, dt =$

$y - y_1 = m(x - x_1)$

$x \to t$, $y \to f(t)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \text{slope}$



$f(t) - 0 = \dfrac{A - 0}{1 - 0}(t - 0)$

$\boxed{f(t) = At}$ for $(0 \le t \le 1)$

$a_0 = \dfrac{1}{1}\int_0^1 At \cdot dt = \left.\dfrac{At^2}{2}\right|_0^1$

$\boxed{a_0 = \dfrac{A}{2}}$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\, dt$$

$$T = \frac{2\pi}{\omega_0} \Rightarrow \frac{2\pi}{T} = \omega_0$$

$$= \frac{2}{1}\int_0^1 \underset{u}{At} \cdot \underset{v}{\cos(n2\pi t)}\, dt$$

$$\int uv = u\int v - \int u' \int v$$

$$\int uv = u\int v - \int u' \int v$$

$$= 2A\left[t\,\frac{\sin 2\pi n t}{2\pi n} - \int 1 \cdot \frac{\sin 2\pi n t}{2\pi n}\, dt \right]$$

$$= 2A\left[t\,\frac{\sin 2\pi n t}{2\pi n} + \frac{\cos 2\pi n t}{(2\pi n)^2} \right]_0^1$$

$$a_n = 2A\left[\frac{1}{(2\pi n)^2} - \frac{1}{(2\pi n)^2}\right] = 0$$

$$\omega_0 = 2\pi$$
$$T = 1$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

$$= 2\int_0^1 At\,.\,\sin 2\pi n t\,dt$$

$$= 2A\left[-t\,.\,\frac{\cos 2\pi n t}{2\pi n} + \int 1.\,\frac{\cos 2\pi n t}{(2\pi n)}\,.\,dt\right]$$

$$= 2A\left[-\frac{t\cos 2\pi n t}{2\pi n} + \frac{\sin 2\pi n t}{(2\pi n)^2}\right]_0^1$$

$$2A\left[-\frac{1}{2\pi n} - 0 - 0\right]$$

$$= \frac{A}{\pi n}$$

$$\boxed{b_n = \frac{A}{\pi n}}$$

$a_0 = \frac{A}{2}$, $a_n = 0$, $\boxed{b_n = \frac{A}{\pi n}}$ $= -\frac{A}{\pi n}$

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{A}{\pi n} \sin(2\pi n t)$$

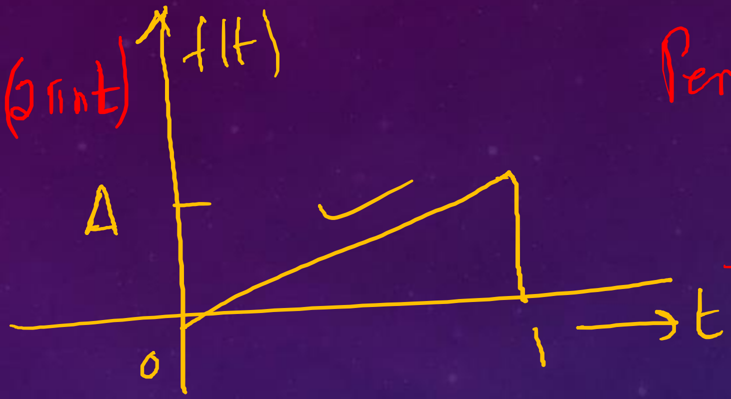
for $(0 \le t \le 1)$

$$\boxed{f(t) = \frac{A}{2} - \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{\sin 2\pi n t}{n}}$$

a_0 b_n

TFS

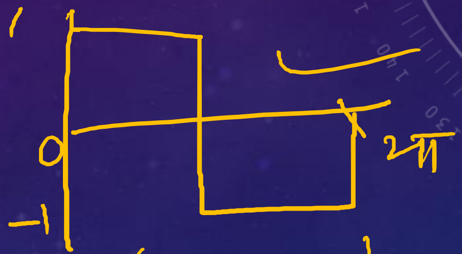
$$f(t) = \frac{A}{2} + \sum_{n=-\infty}^{\infty} \left(\frac{A}{-\pi n}\right) \sin(2\pi n t)$$



Periodic — with $T = 1$

TFS

$$f(t) = \frac{A}{2} - \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2\pi n t)}{n}$$



'Sin t'

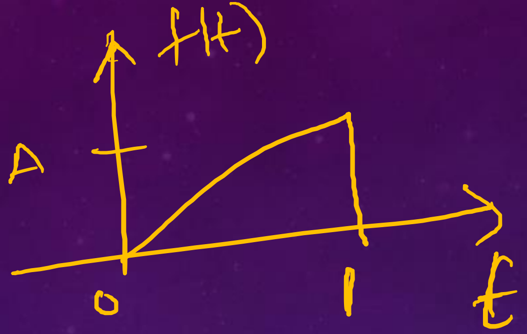
$$= \frac{A}{2} - \frac{A}{\pi}\sin(2\pi t) - \frac{A}{2}\sin(4\pi t) - \frac{A}{3}\sin(6\pi t) - \dots$$

$$\frac{4}{\pi}\sin t + \frac{4}{3\pi}\sin 3t + \dots \frac{4}{n\pi}\sin nt$$

TFS

$a_n = 0$

EFS !



$$\boxed{f(t) = At \quad \text{for } 0 \le t \le 1}$$

$T = 1$

$$f(t) = \sum_{n=\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$\omega_0 = \frac{2\pi}{T} = 2\pi$$

F_0 & $F_n = ?$

$$F_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{1}\int_0^1 At.dt = \frac{A}{2} = \underline{\underline{a_0}}$$

$$F_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}.dt$$

$$F_n = \frac{1}{1}\int_0^1 At \cdot e^{-j(n2\pi t)}\,dt$$

$$= A\left(t \cdot \frac{e^{-j2\pi nt}}{-j2\pi n} - \int 1 \cdot \frac{e^{-j2\pi nt}}{(-j2\pi n)}\,dt \right]$$

$$= A\left| \frac{-t \cdot e^{-j2\pi nt}}{2j\pi n} - \frac{e^{-j2\pi nt}}{(j2\pi n)^2} \right]_0^1$$

$$= A\left| \frac{-1}{j2\pi n} + \cancel{e^{-j}}\frac{1}{(2\pi n)^2} - 0 + 1\,\frac{-1}{(2\pi n)^2} \right]$$

$$= A\left| -\frac{1}{j2\pi n} + \cancel{\frac{1}{(2\pi n)^2}} - \cancel{\frac{1}{(2\pi n)^2}} \right]$$

$$e^{-j2\pi n} = 1$$

$$= 1$$

$$F_n = -\frac{A}{j2\pi n}$$

$$f(t) = \frac{A}{2} - \frac{A}{2j}\sum_{n=-\infty}^{\infty} \frac{e^{-j2\pi nt}}{\pi n}$$

$$f(t) = \frac{A}{2} - \frac{A}{j2} \sum_{n=-\infty}^{\infty} \frac{e^{-j2\pi n t}}{\pi n}$$

F_0

EFS $f(t)$

$$f(t) = \frac{A}{2} + \frac{A}{2} \sum_{n=1}^{\infty} \frac{\sin 2\pi n t}{\pi n}$$

TFS

a_0

$$F_n = \frac{A}{-j2\pi n}$$

Relationship between TFS & EFS

EFS $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$ where

$F_n = \frac{1}{T}\int_0^T f(t) e^{-jn\omega_0 t}.dt$ — (1)

$F_{-n} = \frac{1}{T}\int_0^T f(t) e^{jn\omega_0 t}.dt$ — (2)

$\boxed{F_n = F_{-n}^*}$ ✓

$F_n = \alpha_n + j\beta_n$

$F_1 = \alpha_1 + j\beta_1$

$F_{-1} = \alpha_{-1} + j\beta_{-1}$

$$f(t) = F_0 + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t} + \cdots F_n e^{jn\omega_0 t} +$$

$$F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + \cdots F_{-n} e^{-jn\omega_0 t}$$

$F_n = \alpha_n + j\beta_n$

$\boxed{F_{-n} = \alpha_n - j\beta_n}$

$$= F_0 + (\alpha_1 + j\beta_1) e^{j\omega_0 t} + (\alpha_2 + j\beta_2) e^{j2\omega_0 t} + \cdots (\alpha_n + j\beta_n) e^{n\omega_0 t}$$

$$+ (\alpha_1 - j\beta_1) e^{-j\omega_0 t} + (\alpha_2 - j\beta_2) e^{-j2\omega_0 t} + \cdots (\alpha_n - j\beta_n) e^{-jn\omega_0 t}$$

$$= F_0 + \alpha_1 \left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right) + \alpha_n \left(e^{j2\omega_0 t} + e^{-j2\omega_0 t} \right) + \cdots$$

$$+ j\beta_1 \left(e^{j\omega_0 t} - e^{-j\omega_0 t} \right) + j\beta_2 \left(e^{j2\omega_0 t} - e^{-j2\omega_0 t} \right) + \cdots$$

$$= F_0 + \alpha_1 2\cos\omega_0 t + \alpha_2 2\cos 2\omega_0 t + \cdots \alpha_n 2\cos n\omega_0 t + \cdots$$

$$+ j\beta_1 2j\sin\omega_0 t + j\beta_2 2j(\sin 2\omega_0 t) + \cdots (j\beta_n)(2j)(\sin n\omega_0 t)$$

$$= F_0 + \sum_{n=1}^{\infty} \left(2\alpha_n \cos n\omega_0 t - \underline{\underline{2\beta_n}} \sin n\omega_0 t \right) \text{ — } \underline{\underline{\text{EFS}}} \text{ — } 3$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right) \text{ — } 4$$

$$a_0 = F_0, \quad a_n = 2\alpha_n$$

$$b_n = -2\beta_n$$

$$F_n = \alpha_n + j\beta_n$$

$$F_{-n} = \alpha_n - j\beta_n$$

$$F_n + F_{-n} = \underline{\underline{2\alpha_n}}$$

$$\boxed{a_0 = F_0}$$

$$a_n = 2\alpha_n = F_n + F_{-n}$$

$$b_n = -2\beta_n = j(F_n - F_{-n})$$

$$F_n = \alpha_n + j\beta_n$$

$$F_{-n} = \alpha_n - j\beta_n$$

$$F_n - F_{-n} = 2j\beta_n$$

$$j(F_n - F_{-n}) = -2\beta_n$$

$F_n, F_{-n}, F_0 \longrightarrow$

$$a_n = F_n + F_{-n}$$

$$a_0 = F_0$$

$$b_n = j(F_n - F_{-n})$$

$$F_n = -\frac{A}{j\pi n}$$

$$F_n = -\frac{A}{j2\pi n} + \frac{A}{j2\pi n}$$

$a_0, a_n, b_n \longrightarrow$

$$F_n = \frac{1}{2}(a_n - jb_n)$$

$$F_{-n} = \frac{1}{2}(a_n + jb_n)$$

$$F_n = \frac{-A}{j2\pi n},$$

$$F_{-n} = \frac{-A}{j2\pi(-n)} = \frac{A}{j2\pi n} \Rightarrow \boxed{F_n + F_{-n} = 0 = a_n}$$

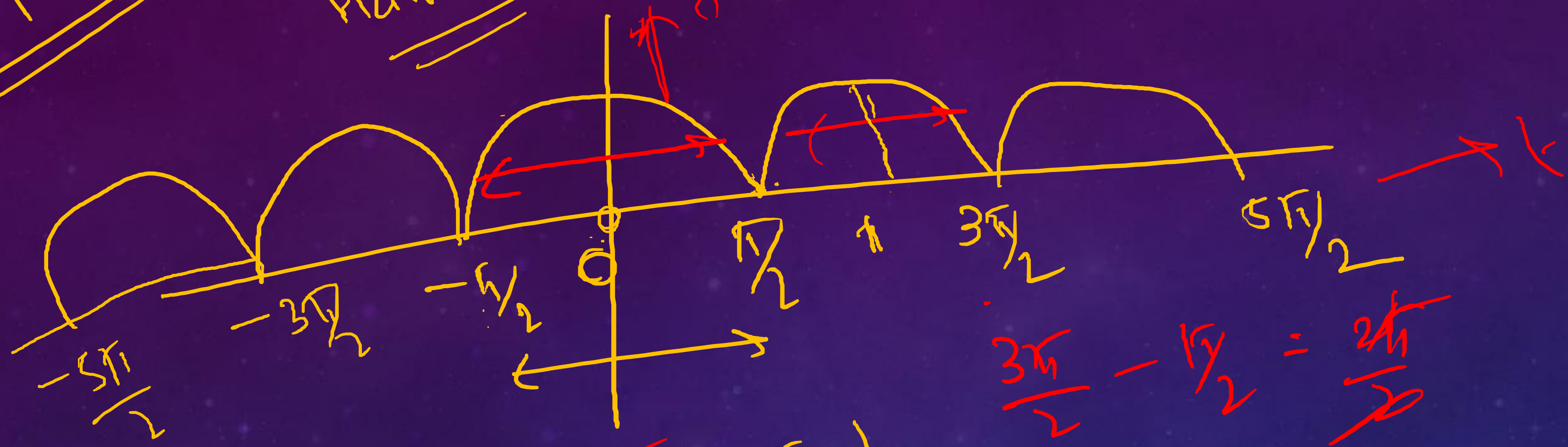
$$j(F_n - F_{-n}) = j\left(\frac{-A}{j2\pi n} - \frac{A}{j2\pi n}\right)$$

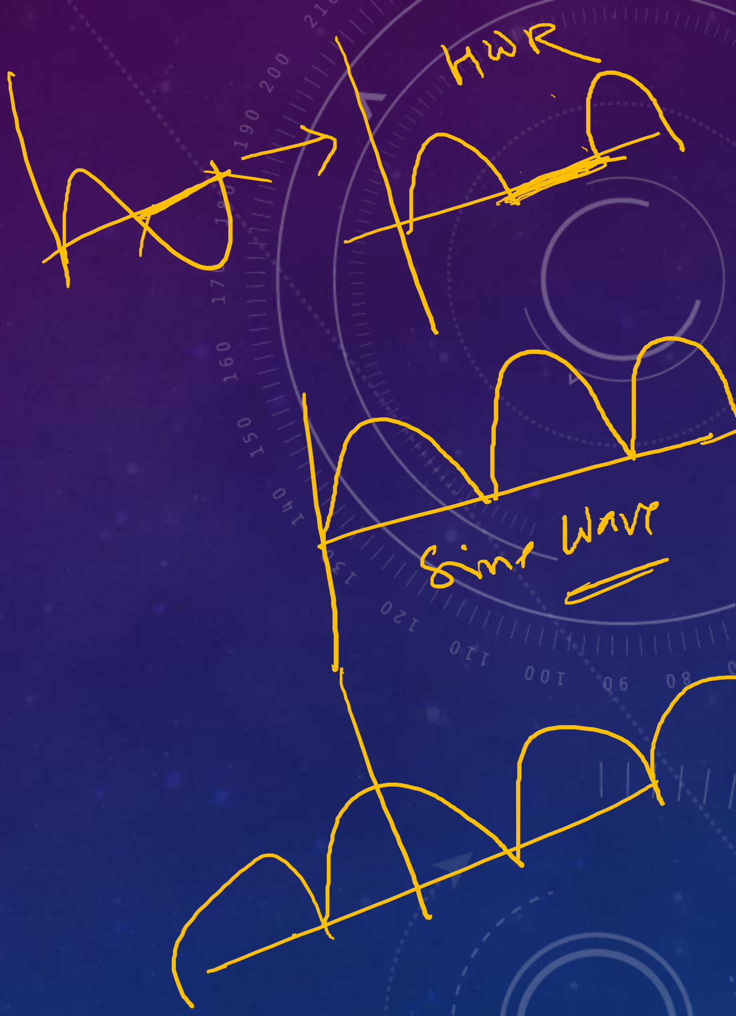
$$= j\left(\frac{-2A}{j2\pi n}\right) = \boxed{\frac{-A}{\pi n} = b_n}$$

TFS

Full wave rectified Cosine function

Half-wave





$$\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$

$$-\frac{\pi}{2} + \frac{3\pi}{2} = \frac{2\pi}{2}$$

$$\frac{3\pi}{2} - \frac{\pi}{2} = \frac{2\pi}{2}$$

$$T = \pi$$

$$f(t) = \cos t$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos t\,dt = \frac{1}{\pi}\sin t\Big|_{-\pi/2}^{\pi/2}$$

$$\omega_0 = \frac{2}{\pi}\left(\sin \pi/2 - \sin(-\pi/2)\right) =$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$$

$$a_n = \frac{1}{T}\int_{-\pi/2}^{\pi/2} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos t \quad \cos(2nt)\,dt$$

$$= \quad 2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$\frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \frac{\cos(1+2n)t + \cos(2n-1)t}{2} \cdot dt \qquad \underline{\cos t \cos 2nt}$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos(1+2n)t \, dt + \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos(2n-1)t \, dt$$

$$-\pi/2 \le t \le \pi/2$$

$$a_n = \frac{2}{\pi}\left(\frac{(-1)^n}{(2n+1)} + \frac{(-1)^{n+1}}{(2n-1)}\right]$$

$$b_n = 0$$

$$f(t) = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{(-1)^n}{2n+1} + \frac{(-1)^{n+1}}{2n-1}\right] \cos 2nt$$

Fourier Series → ?

types — (i) TFS (ii) EFS

Justify the following statements

(i) odd fns have only sine terms ($a_n = 0$, $b_n \neq 0$)

(ii) even " " " cosine " ($a_n \neq 0$, $b_n = 0$)

(iii) fns with half wave symmetry have only odd harmonics ($a_n \neq 0$, $b_n \neq 0$ but $n = $ odd)

Definition → even fn → $f(t) = f(-t)$ → Cosine

$f(-t) = f(t)$ → Symmetrical about Vertical axis

odd → $f(t)$ → $f(-t) = -f(t)$ → Anti-Symmetric about Vertical axis → sine

$$\boxed{\begin{array}{l} \text{even} \times \text{even} = \text{even} \\ \text{even} \times \text{odd} = \text{odd} \\ \text{odd} \times \text{odd} = \text{even} \end{array}}$$

We know that → $\cos n\omega t$ → is an even fn
→ $\sin n\omega t$. is an odd fn.

f(t) → Periodic
↓
why F.S

$$I = \boxed{\int_{-T}^{T} f_e(t)\,dt} = \int_{-T}^{0} f_e(t)\,dt + \int_{0}^{T} f_e(t)\,dt$$

$t = -t$ ⟹

$$= \int_{0}^{T} f_e(t)\,dt + \int_{0}^{T} f_e(t)\,dt = 2\int_{0}^{T} f_e(t)\,dt$$

$I =$ $f(t)$ = Periodic , $f_o(t)$

$$I = \int_{-T}^{T} f_o(t)\,dt$$

$$I = \int_{-T}^{0} f_o(t)\,dt + \int_{0}^{T} f_o(t)\,dt$$

$$I = \int_{-T}^{0} f_o(-t)\,dt + \int_0^T f_o(t)\,dt$$

$$= -\int_0^T f_o(t)\,dt + \int_0^T f_o(t)\,dt$$

$$\boxed{I = 0}$$

even $= \int_{-T}^{T} f_e(t)\,dt = 2\int_0^T f_e(t)\,dt$

odd $= \int_{-T}^{T} f_o(t)\,dt = 0$

$f(t)$ = even function = $f_e(t) \rightarrow$

$$f_e(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right) \rightarrow \text{TFS}$$

(ii) $$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f_e(t) \cos n\omega_0 t \cdot dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t)\,dt$$

even × even = even; even

$$= \frac{2}{T} 2 \int_0^{T/2} f(t)$$

$$\boxed{a_n = \frac{4}{T} \int_0^{T/2} f_e(t) \cdot \cos n\omega_0 t \cdot dt}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f_e(t)\sin n\omega_0 t \, dt$$

even × odd = odd

$$I = \int_{-T/2}^{T/2} \text{odd fn} = 0$$

$$\boxed{b_n = 0}$$

i.e $f_e(t)$ - for even fn — only Cosine terms are existing

(i) $f(t) = f_0(t)$ — Periodic — TFS | EFS

$$f_0(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f_o(t)\cos n\omega_0 t\,dt = 0 = \text{zero}$$

$$\underbrace{\text{odd} \times \text{even}}_{\text{odd}}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f_o(t)\sin n\omega_0 t\,dt = \frac{4}{T}\int_{0}^{T/2} f_o(t)\sin n\omega_0 t\,dt$$

$$\underbrace{\text{odd} \times \text{odd}}_{\text{even}}$$

for odd fns have only sine terms ($b_n \neq 0$, $a_n = 0$)

Functions with half-wave Symmetry

even = $f(-t) = f(t)$

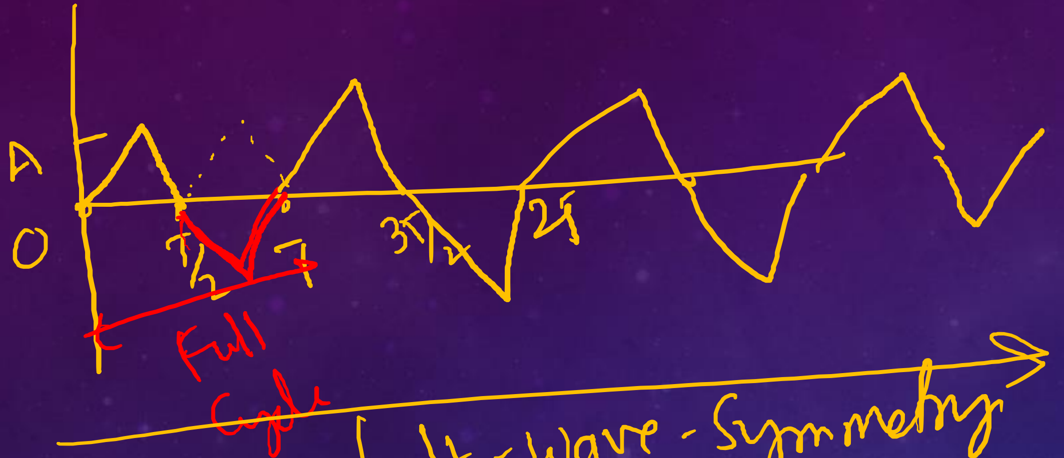
odd = $f(-t) = -f(t)$

half-wave Symmetry

$$f(t \pm T/2) = -f(t)$$

$$\boxed{f(t) = -f(t \pm T/2)}$$

$(T/2 + T/2)$



half-wave-Symmetry

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\, dt$$

$$a_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\, dt + \frac{2}{T}\int_{T/2}^{T} f(t)\cos n\omega_0 t \cdot dt$$

$t + T/2$

$$= \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt + \int_{T/2}^{T} f\left(t+\frac{T}{2}\right)\cos n\omega_0\left(t+\frac{T}{2}\right)dt$$

$t = t + T/2$

$T/2 = t + T$

$= 0$

$t = T/2$

$$f\left(t \pm \frac{T}{2}\right) = -f(t)$$

$$a_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt - \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\cos n\pi\,dt$$

$n = \text{even} = I = 0$

$$n = \text{odd} = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt$$

$= \text{odd harmonics}$

$\cos n\omega_0 t \cdot \cos n\omega_0 \frac{T}{2}$

$T = \frac{2\pi}{\omega_0} \Rightarrow \omega_0 T = 2\pi$

$\cos n\omega_0 t \quad \cos(\pi n t)$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\, dt \Rightarrow \int_0^{T/2} + \int_{T/2}^{T}$$

$$f(t+T/2) = -f(t)$$

$b_n = 0$ for n = even

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t\, dt = \text{odd values of } 'n'$$

half wave symmetry

$a_n = 0$ for n = even

$= \frac{4}{T} \int_0^{T/2} f(t) \cos n\omega_0 t \, dt$. for n = odd

$b_n = 0$ for n = even

$= \frac{4}{T} \int_0^{T/2} f(t) \sin n\omega_0 t \, dt$. for n = odd

have only odd harmonics.

even — Cosine terms ($a_n \neq 0$) $b_n = 0$

odd — Sine terms ($b_n \neq 0$)

EFS coefficients → $\boxed{f(t) = \sin n\omega_0 t}$

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

(t_0, t_0+T)

TFS → a_0, a_n, b_n coefficients

EFS = F_n coefficients

$n = (-2 \text{ to } 2)$

$$= \dots\dots F_{-2} e^{-j2\omega_0 t} + F_{-1} e^{-j\omega_0 t} + F_0 + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t}$$

F_0 → dc value

avg value

F_2 → 2nd harmonic

F_1 → 1st harmonic

F_1 → EFS

F_{-1}

F_0 — d.c. Component, $e^{j\omega_0 t}, e^{-j\omega_0 t}$ → 1st harmonic

$|F_1|$ → EFS coefficient

$\underline{e^{2j\omega_0 t}, e^{-j2\omega_0 t}}$ → 2nd harmonic

F_2, F_{-2} → EFS coeff

$\underline{e^{jn\omega_0 t}, e^{-jn\omega_0 t}}$ → nth harmonics

F_n, F_{-n}

$|F_n|$

$|F_{-n}|$

Same in magnitude

different phase

$$f(t) = \sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t}$$

$$\Downarrow$$

$$F_1 = \frac{1}{2j}, \quad F_{-1} = \frac{-1}{2j}$$

$\checkmark\ F_1 = \frac{1}{2j}$, $\quad F_0 = 0 = dc$

$\checkmark\ F_{-1} = \frac{1}{2j}$ $\quad \underbrace{\left(\begin{matrix} F_2 & \cdots & F_n \\ F_{-2} & & F_{-n} \end{matrix} \right]}_{\text{zero}}$

$$|F_1| = |F_{-1}| = \frac{1}{2j}$$

(x) $\boxed{f(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos(2\omega_0 t + \pi/4)}$

$$1 + \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + \cancel{2}\frac{(e^{j\omega_0 t} + e^{-j\omega_0 t})}{\cancel{2}} + \frac{e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}}{2}$$

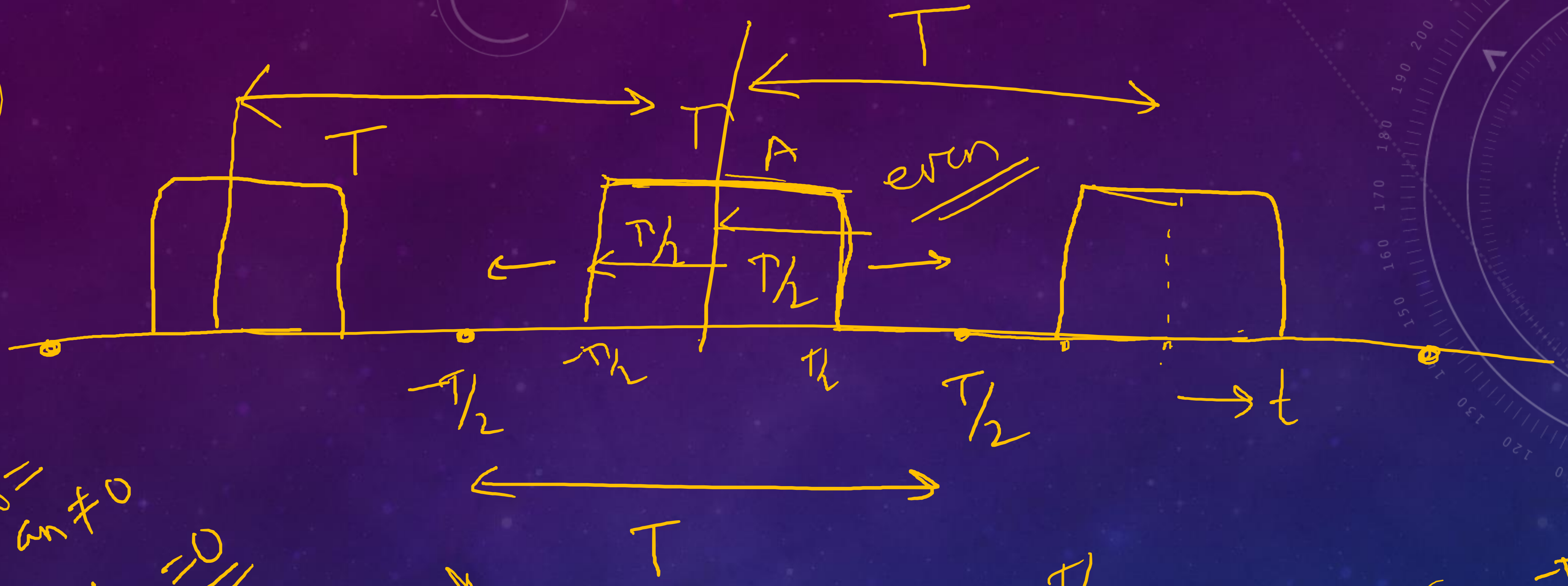
$$1 + \left(1+\frac{1}{j}\right)e^{jw_0t} + \left(1-\frac{1}{j}\right)e^{-jw_0t} + \frac{1}{2}e^{j2w_0t}\cdot e^{j\pi/4}$$

$$+ \frac{1}{2}e^{-j2w_0t}\cdot e^{-j\pi/4}$$

$$\Rightarrow F_0 = 1, \quad F_1 = 1+\frac{1}{j}, \quad F_{-1} = \left(1-\frac{1}{j}\right)$$

$$F_2 = \frac{1}{2}e^{j\pi/4}, \quad F_{-2} = \frac{1}{2}e^{-j\pi/4}$$

$$= \frac{1}{2}\frac{(1+j)}{\sqrt{2}}, \quad F_{-2} = \frac{1}{2}\frac{(1-j)}{\sqrt{2}}$$



$a_n \neq 0$

$b_n = 0$

Fourier Coefficients

TFS

EFS fn.

$\int_{-T/2}^{T/2}$

$f(t) = A \quad -T/2 < t < T/2$

$= 0$ other

TFS

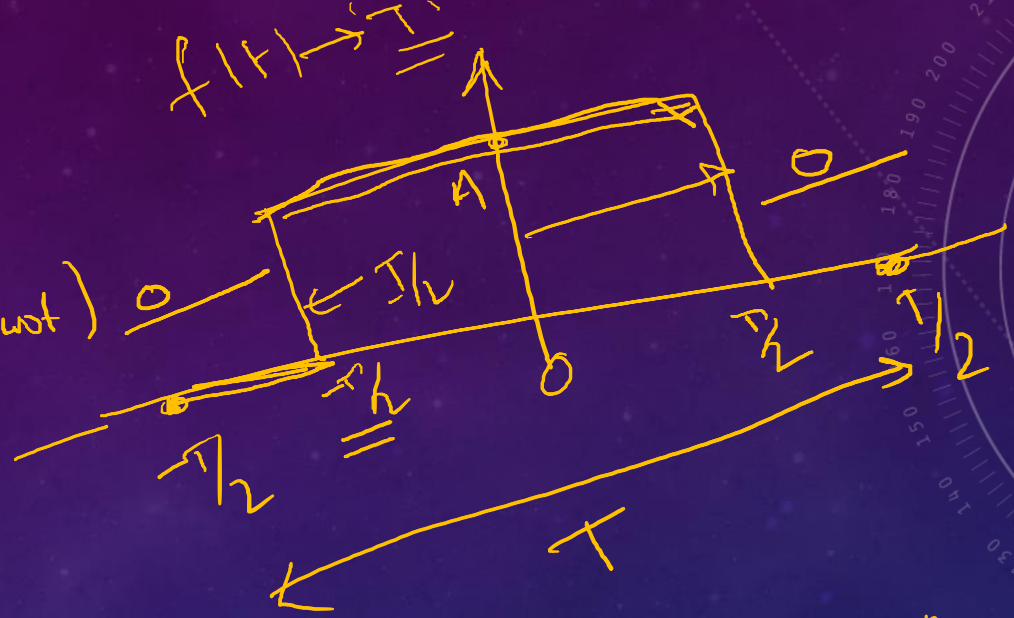
$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

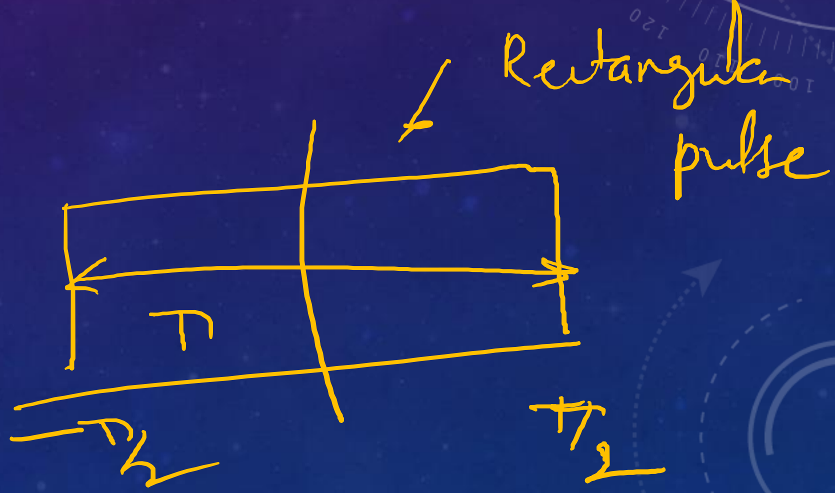
$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$f(t) = \boxed{A} \quad (-T/2 \text{ to } T/2)$$

$$-T/2 < t < T/2$$

$$= 0 \quad \text{otherwise}$$





$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} A\,dt = \frac{At}{T}\Big|_{-T/2}^{T/2} = \frac{A\tau}{T} = a_0 \checkmark$$

$(\tau = \text{width}$

$T = \text{Period})$

$$\omega_0 = \frac{2\pi}{T}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T}\int_{-T/2}^{T/2} A \cdot \cos n\omega_0 t$$

$$= \frac{2A}{T}\left(\frac{\sin n\omega_0 t}{n\omega_0}\right]_{-T/2}^{T/2} = \frac{2A}{T}\left(\frac{\sin n\omega_0 T/2 + \sin n\omega_0 T/2}{n\omega_0}\right]$$

$$= \frac{4A}{T}\,\frac{\sin n\omega_0 T/2}{n\omega_0}$$

$$a_n = \frac{2}{T}\,\cancel{x}\,\frac{\sin(n\omega_0 T/2)}{(n\omega_0)T/2} \times \cancel{T/2}$$

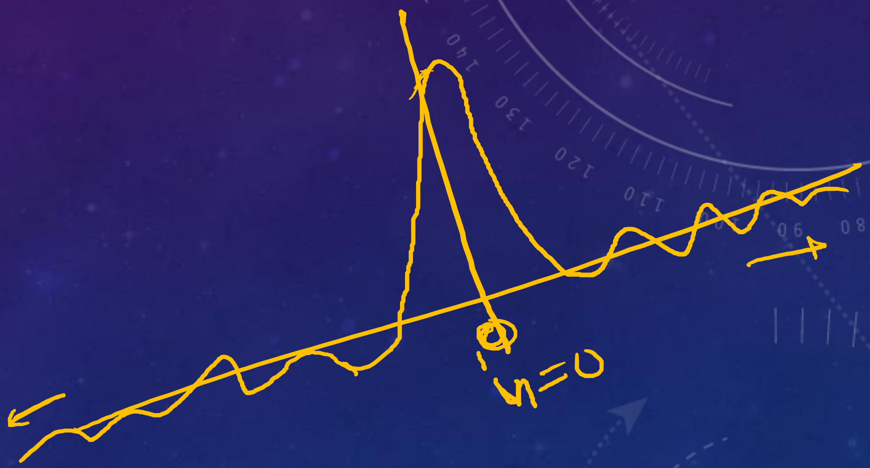
$$= \frac{2T}{T}\,\frac{\sin(n\omega_0 T/2)}{(n\omega_0 T/2)}$$

$$a_n = \frac{2T}{T}\,Sa(n\omega_0 T/2)$$

$$\boxed{b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega_0 t\,dt = 0}$$

$$Sa(x) = \frac{\sin x}{x}$$

$\sin(x)\uparrow$



$$f(t) = \underbrace{\frac{AT}{T}}_{a_0} + \sum_{n=1}^{\infty} \underbrace{\frac{2T}{T}}_{a_n} \; Sa(n\omega_0 T/2) \cdot \cos(n\omega_0 t)$$

TFS

$$\frac{\sin x}{x} = Sa(x)$$

$$EFS = f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$F_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_0 t} \cdot dt$$

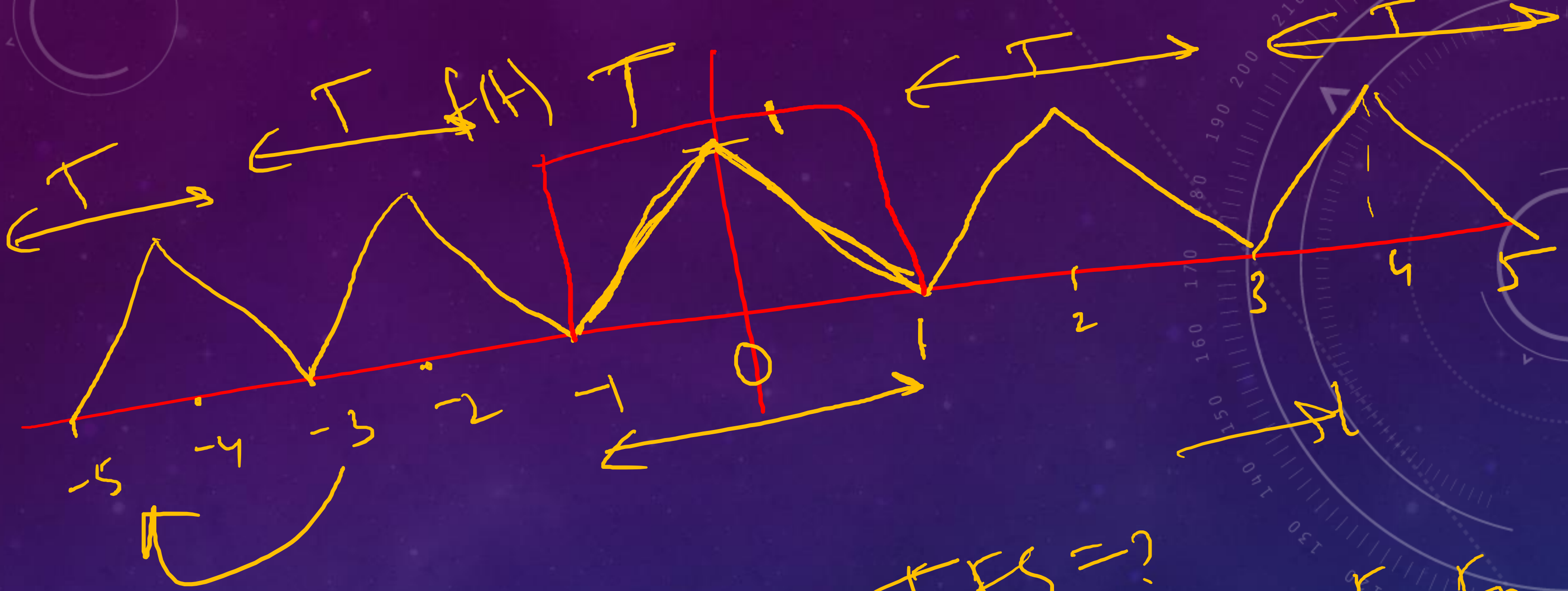
$$A \cdot e^{-jn\omega_0 t}$$

$$F_n = \frac{AT}{T} \; Sa(n\omega_0 T/2)$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{AT}{T} Sa(n\omega_0 T/2) e^{jn\omega_0 t}$$

EFSR

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$



TFS = ?

EFS = ? → F_0, F_n

a_0

a_n

b_n

$-3-(-5)$

$= -3+5$

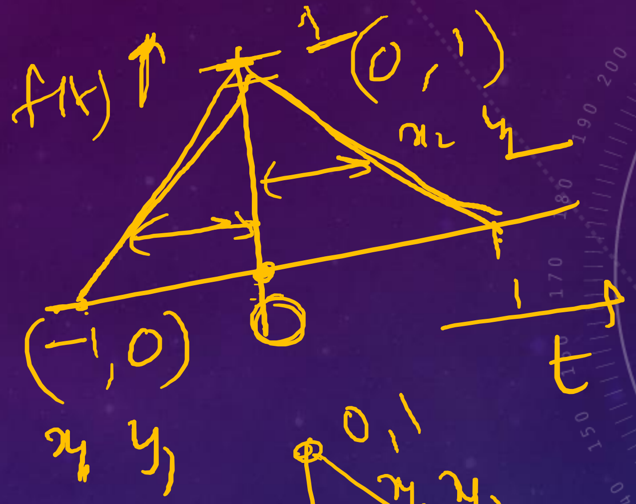
$= 2$

$1-(-1)$

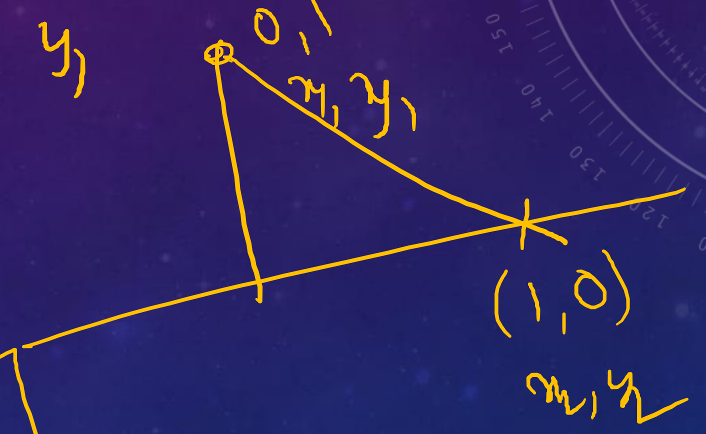
$= 2$

eqn for $f(t) =$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{f(t)-0}{t+1} = \frac{1-0}{0+1}$$



$$f(t) = 1+t \qquad -1<t<0$$



$$\frac{f(t)-1}{t-0} = \frac{0-1}{1-0}$$

$$f(t)-1 = -t$$

$$f(t) = 1-t \rightarrow \boxed{0<t<1}$$

$$f(t) = \begin{cases} 1-t & \text{for } 0<t<1 \\ 1+t & -1<t<0 \end{cases}$$

$$= \boxed{1-|t|}$$

$$= f(t)$$

$|t| = -t$ for $t<0$

$|t| = t$ for $t>0$

$$a_0 = \frac{1}{T}\int_{-1}^{1} f(t)\,dt$$

$$a_n = \frac{2}{T}\int_{-1}^{1} f(t)\cos n\omega_0 t\,dt$$

$$b_n = \frac{2}{T}\int_{-1}^{1} f(t)\sin n\omega_0 t\,dt$$

$\sin n\omega_0 t$

$$a_n = \int_{-1}^{0}(1+t)\cos n\omega_0 t + \int_{0}^{1}(1-t)\cos n\omega_0 t$$

$F_0 = \frac{1}{2}, \quad F_n = \left[\frac{2}{(n\pi)^2} - \frac{(-1)^n}{}\right]$

$$F_n = \frac{1}{n^2\pi^2}\left[1-(-1)^n\right]$$

Eq 2

$$\boxed{f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n^2\pi^2}\left[1-(-1)^n\right] e^{jn\omega_0 t}}$$

π

$\omega_0 = \frac{2\pi}{T}$

$T = 2$

Properties of Fourier Series

EFS

$$f(t) = \sum_{n=-\alpha}^{\alpha} F_n e^{jn\omega_0 t}$$

$$f(t) \xleftrightarrow{F.S} F_n$$

$$\boxed{F_n = \frac{1}{T}\int_0^T f(t) e^{-jn\omega_0 t}\, dt}$$

$$\boxed{f(t) \xleftrightarrow{F.S} F_n}$$

If $f(t)$ — Periodic 'T'

$$f(t) = a_0 + \sum_{n=1}^{\alpha} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

TFS

$$\boxed{f(t) = \sum_{n=-\alpha}^{\alpha} F_n e^{jn\omega_0 t}}$$

Linearity Property of F.S

Linearity → two Prop → additivity & homogenity (or) Scaling

$x(t) \xleftrightarrow{F.S} a_n$

$y(t) \xleftrightarrow{F.S} b_n$

$x(t)+y(t) \xrightarrow{F.S} a_n + b_n$?

additivity $A\,x(t) + B\,y(t) \rightarrow A \cdot a_n + B\,b_n$

$x(t) \xleftrightarrow{F.S} a_n$

Scalar 'A' $x(t) \xrightarrow{F.S} A \cdot a_n$

$A\,x(t)$

If $x(t) \xleftrightarrow[T]{F.S} a_n$, $y(t) \xleftrightarrow[T]{F.S} b_n$ then

$z(t) = \underline{A\,x(t) + B\,y(t)} \longrightarrow \underline{A\,a_n + B\,b_n}$

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}, \qquad F_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt$$

Let $z(t) = A\,x(t) + B\,y(t) \longleftrightarrow C_n$

$z(t) \longleftrightarrow F_n$

$$C_n = \frac{1}{T}\int_0^T \underset{y(t)}{\overset{x(t)}{z(t)}}\, e^{-jn\omega_0 t}\, dt$$

an

bn

$$C_n = \frac{1}{T}\int_0^T \big(A\,x(t) + B\,y(t)\big)\, e^{-jn\omega_0 t}\, dt$$

$$C_n = \frac{1}{T}\int_0^T A\,x(t)\,e^{-jn\omega_0 t}\,dt + \frac{1}{T}\int_0^T B\,y(t)\,e^{-jn\omega_0 t}\,dt$$

$$= A\,\frac{1}{T}\int_0^T x(t)\,e^{-jn\omega_0 t} + B\,\frac{1}{T}\int_0^T y(t)\,e^{-jn\omega_0 t}\,dt$$

$$C_n = A\,a_n + B\,b_n$$

$$z(t) \longleftrightarrow C_n$$

$$A\,x(t) + B\,y(t) \xleftrightarrow{FS} A\,a_n + B\,b_n$$

Linearity FS

I–II Exams

Semester – may–24 APR] June, July, Aug, Sep

4-M

4-months

Aug-10th – I–II yr → 1½ month

Assign – middle – Current sem

7th – 7/10/20 – away from subject

21/10/20 – I–II – Regular

Starting

Most

ASAP →

no exams

4½ – m

Class I–II

Exam

stop – current concepts

I–I–Supply

(1) →

23/9 – 14 days – supply

21/10 – Regular

21/10/20 — Regular - (I-II)

30%

Stop - Current Semester

Start — I - II

No change

$4 \times 3 = 12$

PDF

Phone

Book | NB

Start Prep

Practice — SS

AICTE

JNTUH

Start Preparation

Properties of Fourier Series

1. Linearity Prop.

$x(t) \overset{F.S}{\underset{T}{\longleftrightarrow}} a_n$

$y(t) \longleftrightarrow b_n$

$A\,x(t) + B\,y(t) \longrightarrow A\,a_n + B\,b_n$

$A = B = 1$

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t} \quad (1)$$

$$a_n = \frac{1}{T}\int_T x(t)\, e^{-jn\omega_0 t} \quad (2)$$

analysis $\rightarrow$ $x(t) \leftarrow f_1, f_2, f_3, f_4$

synthesis $\rightarrow$ Combining all the freq. $x(t)$

eqn (1) Synthesis

eqn (2) Analysis

a_1, a_2, a_3

② time shifting Prop of F.S

if $x(t) \rightarrow$ Periodic with 'T' $\xrightarrow{F.S} a_n$ then if $x(t)$ is shifted by t_0 seconds $\rightarrow x(t-t_0) \longrightarrow$?

$$\boxed{a_n = \frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}.dt}$$

$$\boxed{b_n \rightarrow x(t-t_0)} \Rightarrow b_n = \frac{1}{T}\int_T x(t-t_0)e^{-jn\omega_0 t}.dt$$

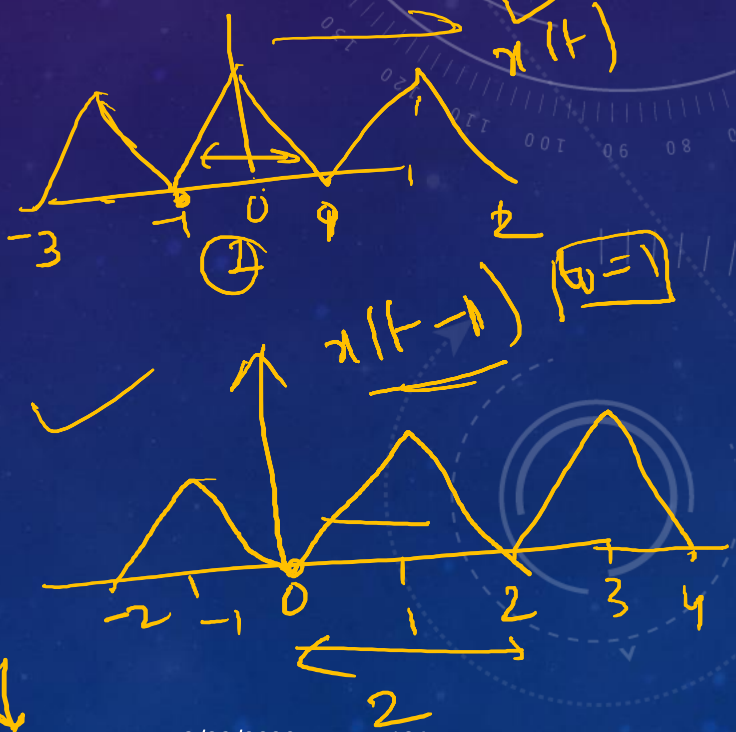
Let $t-t_0=\tau \Rightarrow t=(t_0+\tau)$

$dt = d\tau$

$$b_n = \frac{1}{T}\int_T x(\tau)\, e^{-jn\omega_0(t_0+\tau)}\, d\tau$$

$x(t) \xrightarrow{F.S} a_n$

$x(t-t_0) \rightarrow$?



phase →

$$b_n = \frac{1}{T}\int_T x(\tau)\, e^{-jn\omega_0 t_0} \cdot e^{-jn\omega_0 \tau} \cdot d\tau$$

$$= e^{-jn\omega_0 t_0} \underbrace{\frac{1}{T}\int x(\tau)\, e^{-jn\omega_0 \tau}\, d\tau}_{a_n}$$

$$\boxed{b_n = e^{-jn\omega_0 t_0}\, a_n}$$

both $x(t)$ & $x(t-t_0)$ — same mag

but $x(t-t_0)$ — phase change

$e^{-jn\omega_0 t_0}$

$$x(t) \xrightarrow{F.S} a_n$$

$$x(t-t_0) \longrightarrow \boxed{e^{-jn\omega_0 t_0}}\, a_n$$

mag. $x(t-t_0)$ $\quad \left|e^{-jn\omega_0 t_0}\right| \left|a_n\right|$

$\left|e^{j\theta}\right| =$ $\quad 1 \cdot a_n = a_n$

mag $|x(t)| =$ $\quad |a_n| =$

(*) Time Reversal

$x(t) \rightarrow T \rightarrow a_n$ then

$x(-t) \rightarrow T \rightarrow a_{-n}$

$$x(t) = \sum_{n=-\alpha}^{\alpha} a_n e^{+jn\omega_0 t}$$

let us Replace $t \rightarrow$ '$-t$'

$$x(-t) = \sum_{n=\alpha} a_n e^{-jn\omega_0 t}$$

let $\boxed{-n=m} \Rightarrow n=-m$

$$x(-t) = \sum_{m=\alpha}^{\alpha} a_{-m} e^{jm\omega_0 t}$$

$$\boxed{x(-t) \rightarrow a_{-n}}$$

$\rightarrow$ If $x(t) = \boxed{\text{even}} \Rightarrow x(-t) = x(t)$

$\therefore \boxed{a_{-n} = a_n}$

$x(t) = \text{odd} \Rightarrow x(-t) = -x(t)$

$a_{-n} = -a_n$

$\boxed{-a_{-n} = a_n}$

Conjugation & Conjugation Symmetry

$x(t) \rightarrow T \rightarrow a_n$ then

$x^*(t) \rightarrow T \rightarrow ?$

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$$

take the Conjugation both sides

$$x^*(t) = \sum_{n=-\infty}^{\infty} a_n^* e^{-jn\omega_0 t}$$

$$= \sum_{m=-\infty}^{\infty} a_{-m}^* e^{jm\omega_0 t}$$

$$x^*(t) = \sum_{n=-\infty}^{\infty} a_{-n}^* e^{jn\omega_0 t}$$

$$\boxed{x^*(t) \longrightarrow a_{-n}^*}$$

if $x(t) =$ real $\Rightarrow$

both $\boxed{x(t) = x^*(t)}$

$$\boxed{a_n = a_{-n}^*} \rightarrow a_n^* = a_{-n}$$

$(e^{j\theta} = \cos\theta + j\sin\theta$

$\hookrightarrow e^{-j\theta} = \cos\theta - j\sin\theta$

$\boxed{y = b}$

$x = a + jb$

$x^* = a - jb$

$y^* = b$

if $x(t) \to$ real $=$ $a_n = a_{-n}^{*}$ $\left(x(t) = x^{*}(t) \right)$

$\Rightarrow$ $\boxed{a_n^{*} = a_{-n}}$ $\rightarrow$ Conjugate Symmetric

$x = a + jb$

$x^{*} = a - jb$

$\sqrt{a^2 + b^2}$

$|a_n| = |a_{-n}^{*}| =$

$x(-t) = x(t)$

if $x(t) \to$ real + even $\Rightarrow$ $a_{-n} = a_n$

$\boxed{a_n = a_n^{*}}$ $\boxed{= a_{-n}}$

Parseval's Relation of Periodic Signals

$x(t) \rightarrow T \rightarrow a_n$

Parseval's $\rightarrow$ Periodic – Power Signals

avg Power $x(t) = \frac{1}{T}\int_0^T |x(t)|^2 dt \rightarrow$

avg Power in time domain

Periodic $\rightarrow$ F.S $\rightarrow a_n$

$$P_{avg} = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$= \frac{1}{T}\int_0^T x(t)\, x^*(t)\, dt$$

W.K.T

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$$

$$\boxed{x^*(t) = \sum_{n=-\infty}^{\infty} a^*(n)\, e^{-jn\omega_0 t}}$$

$$P_{avg} = \frac{1}{T}\int_T x(t)\left(\sum_{n=\alpha}^{\alpha} a_n^* e^{-jn\omega_0 t}\right) dt$$

$$x(t) = \sum_{n=\alpha}^{\alpha} a_n e^{jn\omega_0 t}$$

$$a_n = \frac{1}{T}\int_T x(t) e^{-jn\omega_0 t}$$

$$= \sum_{n=\alpha}^{\alpha} a_n^* \frac{1}{T}\int_T x(t) e^{-jn\omega_0 t} dt$$

$$= \sum_{n=\alpha}^{\alpha} a_n^* \cdot a_n = \sum_{n=-\alpha}^{\alpha} |a_n|^2 = |a_0|^2 + |a_1|^2 + |a_2|^2 + \cdots |a_n|^2$$

$$\frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{n=\alpha}^{\alpha} |a_n|^2$$

→ avg Power in the Periodic Signal equals to the Sum of the avg Powers in all of its harmonic Components

$$\frac{1}{T}\int_0^t |x(t)|^2 dt = \sum_{n=\alpha}^{\alpha} |a_n|^2$$

Parseval's Relation

Time Scaling

$x(t) \to T \to a_n$

$x(\alpha t) \to$ Time Scaling $\to$?

$$x(t) = \sum_{n=\alpha}^{\alpha} a_n e^{jn\omega_0 t}$$

$$x(\alpha t) = \sum_{n=\alpha}^{\alpha} a_n e^{+jn\omega_0(\alpha t)}$$

$$x(\alpha t) = \sum_{n=\alpha}^{\alpha} a_n e^{+jn(\alpha\omega_0)t}$$

$$x(\alpha t) = \sum a_n e^{jn(\alpha\omega_0)t}$$

Time Scaling → No change a_n

But 'T' = will be changing

$$T = \frac{2\pi}{\alpha\omega_0}$$

$$= \boxed{\text{Period} = T/\alpha}$$

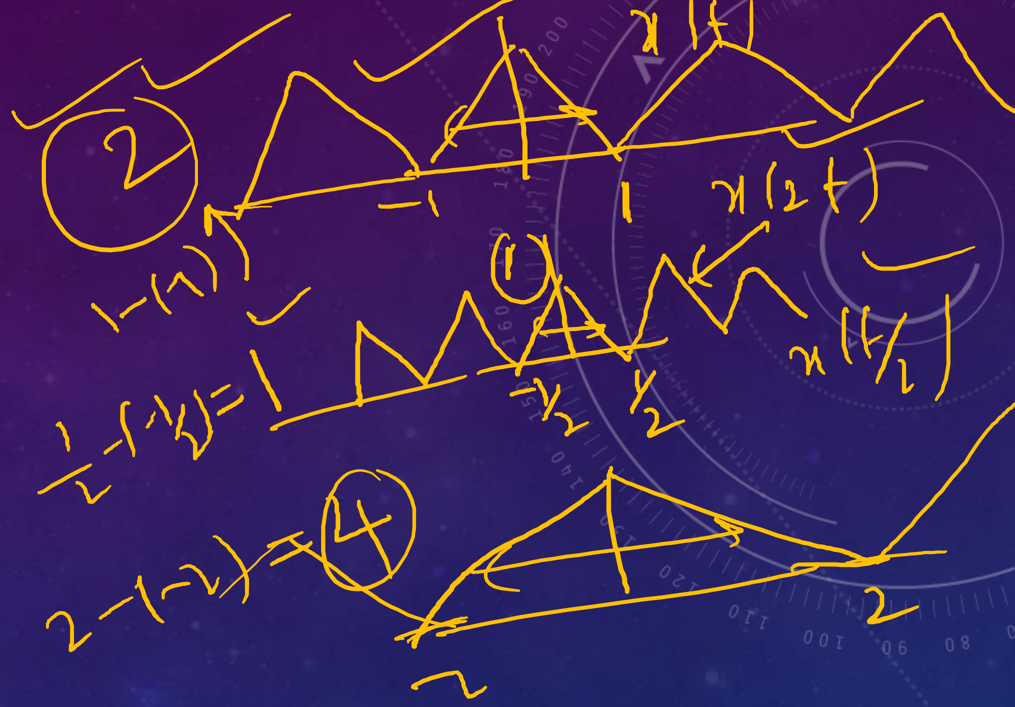
$$T = \frac{2\pi}{\alpha\omega_0}$$

$$T/\alpha$$

$$\omega_0 \rightarrow \alpha\omega_0$$

$\omega_0 \downarrow T$

$\alpha\omega_0 \downarrow T/\alpha$



time Sc

Properties of Fourier Series $x(t)$ & $y(t)$ are $\rightarrow$ 'T'

① Linearity Prop $\rightarrow$ $A\,x(t) + B\,y(t) \longrightarrow A\,a_n + B\,b_n$

② Time Reversal $\rightarrow$ $x(t) \longrightarrow a_n \Rightarrow x(-t) \rightarrow a_{-n}$

③ Time Shifting $\longrightarrow$ $x(t) \longrightarrow a_n \longrightarrow x(t-t_0) \longrightarrow e^{-jn\omega t_0} \cdot a_n$

④ Time Scaling $\longrightarrow$ $x(t) \longrightarrow a_n \Rightarrow x(\alpha t) \longrightarrow a_n$

⑤ Conjugation $\rightarrow$ $x(t) \rightarrow a_n \Rightarrow x^*(t) \longrightarrow a^*_{-n}$

if $x(t)$ - real $\Rightarrow x(t) = x^*(t) \Rightarrow a_n = a^*_{-n} \Rightarrow \boxed{a_n = a_n^*}$

$x(t)$ = real & even $\Rightarrow x(t) = x(-t) \Rightarrow \boxed{a_n = a_{-n} = a_n^*}$

Parseval's Relation → Explains the Relation in Power Signal in time domain & freq Domain (F.S. coefficient)

$x(t)$ — Periodic — Power Signals ⇒ Power Relation

$x(t) \rightarrow a_n$ ⇒ $$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |a_n|^2$$

$$= |a_0|^2 + |a_1|^2 + \cdots |a_n|^2$$

DC   1st f   nth freq

avg. Power = Sum of the Power of all freq Components

Gibb's phenomenon

$f(t) / x(t)$

$$f(t) = \sum_{n=-\alpha}^{\alpha} a_n e^{jn\omega_0 t}$$

Periodic — approximated by Sinusoids / complex Expo —

If the no. of terms (fns) in the appro — ↑

MSE ↓

the occurrence of overshoot / Ripple at the points of discontinuities in the approx — of F.S is called as Gibbs phenome —

Concept of Negative freq

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} = \underset{dc}{F_0} + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t} + \dots + F_n e^{jn\omega_0 t}$$
$$+ F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + F_{-n} e^{-jn\omega_0 t}$$

F_1 = F.S. coff of 1st freq. Compon. ω_0

F_{-1} " " " ω_0

F_2 " 2nd

F_{-2} " "

F_n | F_{-n}

$$\boxed{\begin{aligned} F_1 &= \omega_0 \\ F_{-1} &= \omega_0 \end{aligned}}$$

Sunlight $f(t)$

F.S

$F_1 = a + jb$

$F_{-1} = a - jb$

$|F_1| = |F_{-1}|$

phasors are diff / opp) in direction but magnitudes are Same

freq = no. of cycles / sec. = 2

→ Negative freq = 2 — but phase opp.

$e^{j\omega_0 t} + e^{-j\omega_0 t}$

$(e^{j\omega_0 t} + e^{-j\omega_0 t}) = \cancel{2} \cos \omega_0 t$

Positive freq → Construct Real X Signal

In the Analysis → +ve & -ve freq / Comp

Mathematical model

$-\omega$

Complex Fourier Spectrum

Spectrum → Set of freq.

Fourier → F.S | F.T

Complex → the F.S. coefficients of any Periodic Signal is Complex

Freq. Present, 0, $\omega_0, -\omega_0, 2\omega_0, -2\omega_0$ - - - - $n\omega_0, -n\omega_0$

$$f(t) \quad \boxed{\omega_0 = \frac{2\pi}{T}}$$

$$f(t) = F_0 + F_1 e^{j\omega_0 t} + F_2 e^{+j2\omega_0 t} + \cdots F_n e^{jn\omega_0 t} + \cdots$$

$$F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + \cdots F_n e^{-jn\omega_0 t} + \cdots$$

d.c

'0'

d.c – Component – freq = 0

1st freq " $= \omega_0, -\omega_0$ → F_1, F_{-1}

2nd " " $= 2\omega_0, -2\omega_0$ → F_2, F_{-2}

nth freq - - $= n\omega_0, -n\omega_0$ → F_n, F_n

$\omega_0 = 0 = F_0$

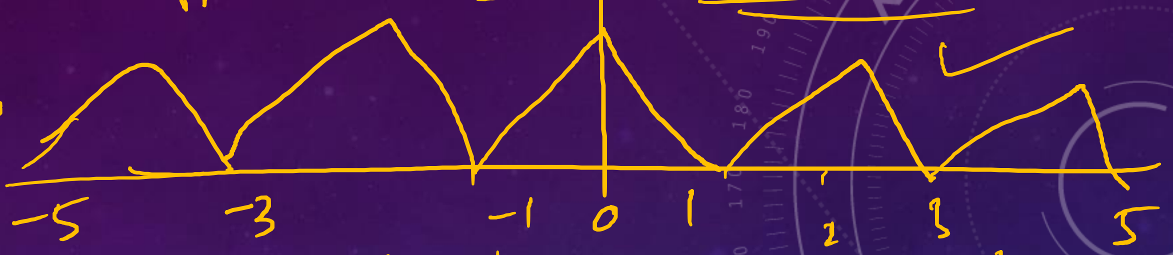
$\omega = \omega_0 = F_1, F_{-1}$

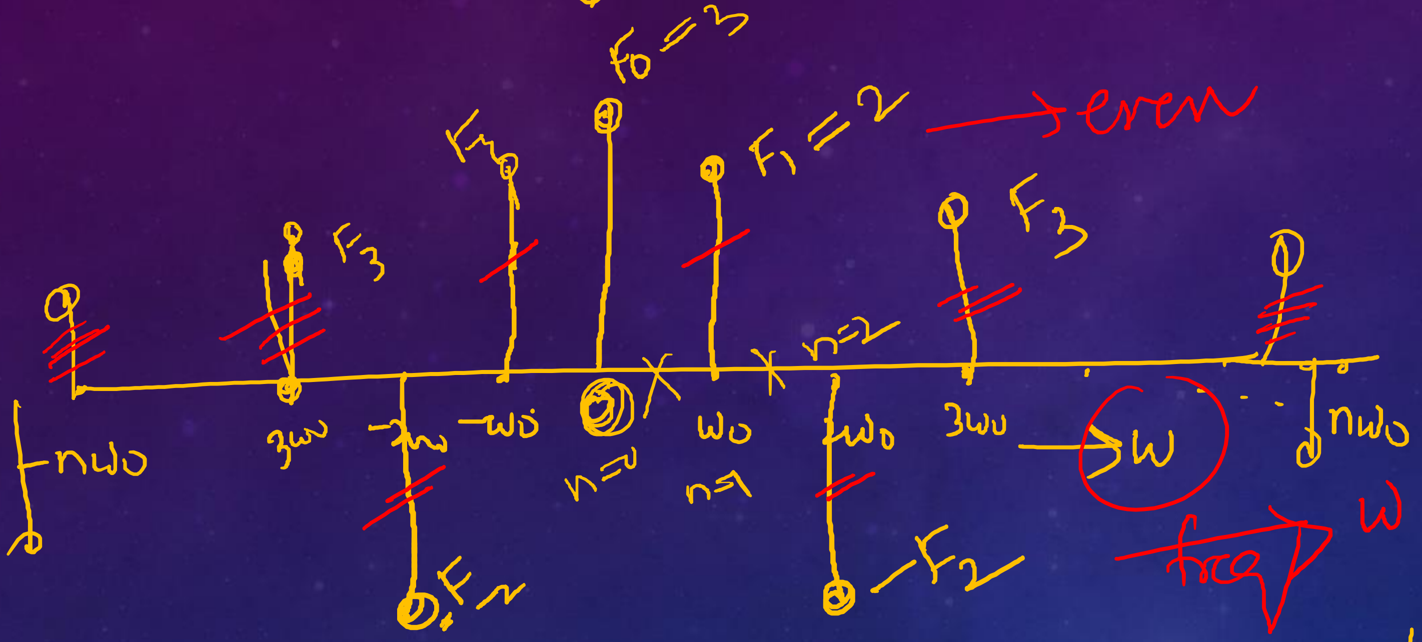
$= 2\omega_0 = F_2, F_{-2}$

F_n, F_{-n}

f(t) → Periodic → time domain →
↳ Freq. Domain Repⁿ

time domain | waveform





$$F_n = \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|\,|e^{-jn\omega_0 t}|\,dt$$

1

$$F_{-n} = \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|\,|e^{jn\omega_0 t}|\,dt$$

→ magnitude of Various freq Components

freq Vs Magn — Spectrum

magnitude

$|F_n| = |F_{-n}| =$ Same

$\int |f(t)|\,dt$

→ Complex Fourier Spectrum

$n = 0, \pm 1, \pm 2, \pm 3 \ldots$ $n\omega_0$

only discrete freq $\Rightarrow$ $\omega_0, 2\omega_0, 3\omega_0 \ldots$

discrete Spectrum

Line Spectrum

magnitude spectrum

$$F_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$F_{-n} = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{+jn\omega_0 t}\, dt$$

$|F_n| = |F_{-n}| =$

Same

$F_n = |F_n|\, e^{-j\theta}$

$F_{-n} = |F_n|\, e^{+j\theta}$

By Nature F_n = Complex Quantity $= a + jb$

$a_0 = 1, \quad a_1 = 1 - \frac{1}{2}j$

$a_{-1} = 1 + \frac{1}{2}j$

$a_2 = \frac{\sqrt{2}}{4}(1-j)$

$a_{-2} = \frac{\sqrt{2}}{4}(1+j)$

Complex Nature of F_n → two Spectrum

freq Vs magnitude = Mag - Spectrum

freq Vs Phase = phase Spectrum

$F_n = |F_n| e^{-j\theta}$

$F_n = |F_n| e^{+j\theta}$

$F_n = a_n + j b_n$

$|F_n| = \sqrt{a_n^2 + b_n^2}$

$\theta = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

magnitude Spectrum → Symmetric about the Vertical axis → even fn of freq. 'ω'

phase

phase Spectrum → Anti-Symmetric odd fn of freq 'ω'

Summary Fourier Series

① → $f(t) \to$ Periodic → as sum of complex exponentials / sinusoidal signals

$e^{jn\omega_0 t}$ — complete

$\sin n\omega_0 t$ & $\cos n\omega_0 t$ — complete set of ortho fns

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right) \to \text{TFS}$$

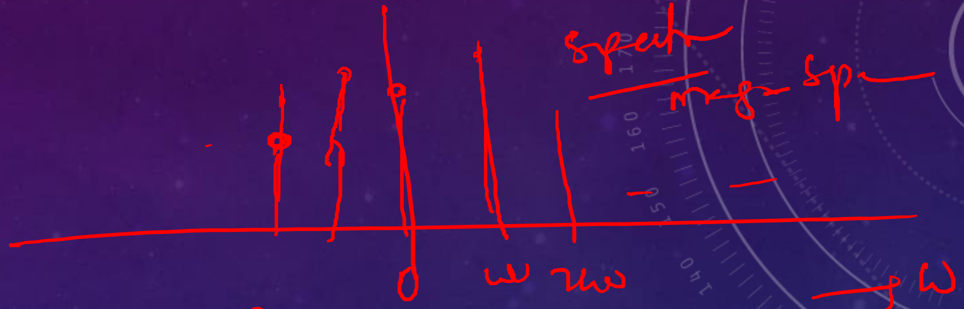
$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt, \quad a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t\,dt, \quad b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega_0 t\,dt$$

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}, \quad F_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,e^{-jn\omega_0 t}\,dt \to \text{EFS}$$

Relationship b/n TFS & EFS

→ Express / find

Complex F. Spectrum

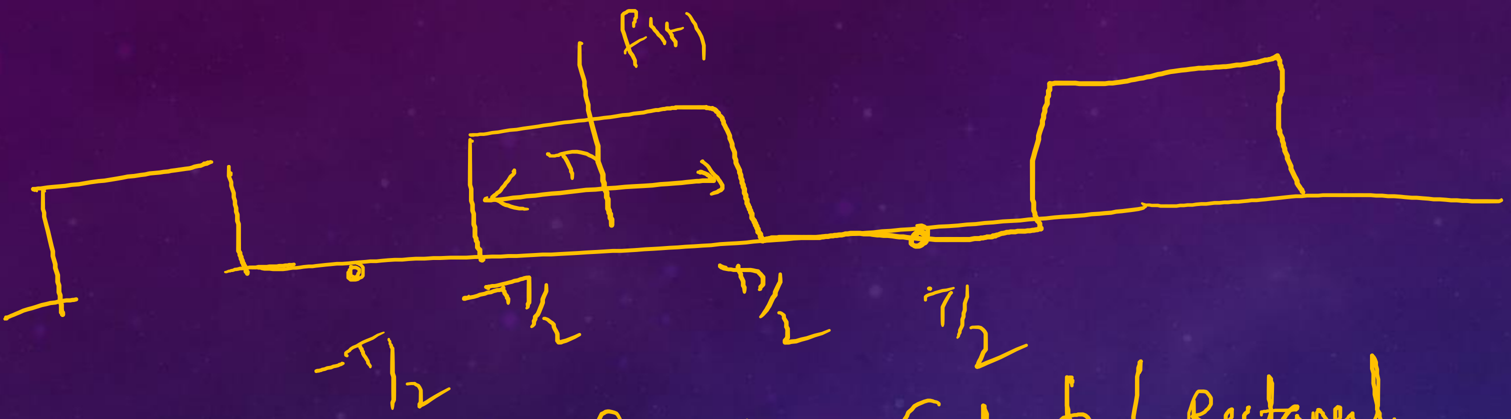


— Negative freq → $e^{j\omega_0 t}$, $e^{-j\omega_0 t}$

— Gibbs Ph

Properties of F.S

Fourier Transform Representation



$f(t)$ = Periodic fn = T — Gate fn / Rectangular pulse

EFS: $f(t) = \sum_n F_n e^{jn\omega_0 t}$

$$F_n = \frac{A\tau}{T} Sa\left(\frac{n\omega_0\tau}{2}\right)$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{A\tau}{T} Sa\left(\frac{n\omega_0\tau}{2}\right) e^{jn\omega_0 t}$$

$$Sa(x) = \frac{\sin x}{x}$$

$Sa(0) = 1$ — 1

$$\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2} = Sa\left(\frac{n\omega_0\tau}{2}\right)$$

$$\frac{\sin n\omega_0 T/2}{n\omega_0 T/2} = 0 \Rightarrow \quad \sin\frac{n\omega_0 T}{2} = 0 = \quad \sin\frac{n\omega_0 T}{2} = \sin(\pm n\pi)$$

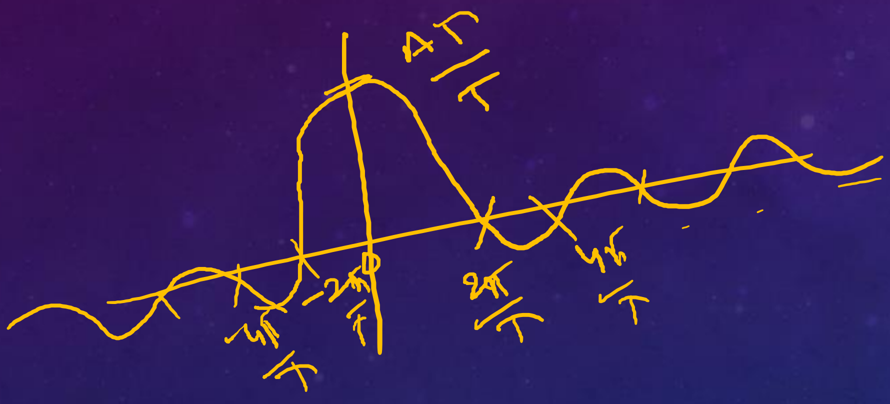
$$\frac{n\omega_0 T}{2} = \pm n\pi \Rightarrow \quad n\omega_0 T = \pm 2n\pi/T$$

$$n\omega_0 = \pm\frac{2n\pi}{T}$$

$$= \frac{2\pi}{T}, \frac{4\pi}{T}, \frac{6\pi}{T} \ldots \frac{2n\pi}{T}$$

T-width

T=Period



$$f(t) = \sum \frac{AT}{T} Sa\left(\frac{n\omega_0 T}{2}\right) e^{jn\omega_0 t}$$

$$n=0 \quad Sa(0) = 1 \Rightarrow \quad \frac{AT}{T}, \quad \frac{AT}{T} Sa\left(\frac{\omega_0 T}{2}\right), \quad \frac{AT}{T} Sa\left(\frac{2\omega_0 T}{2}\right),$$

$\frac{A\tau}{T} \rightarrow$ magnitude of Sampling fn $\rightarrow$ T = Period of the Pulse

τ = width of the pulse

Ex:- $\tau = \frac{1}{20}$ = width

$T = \frac{1}{4}$ = Period

$\frac{\tau}{T} = \frac{\frac{1}{20}}{\frac{1}{4}} = \frac{1}{5}$.

Zeros $\Rightarrow$ at $\pm \frac{2n\pi}{\tau}$

$= \frac{2n\pi}{\frac{1}{20}}$

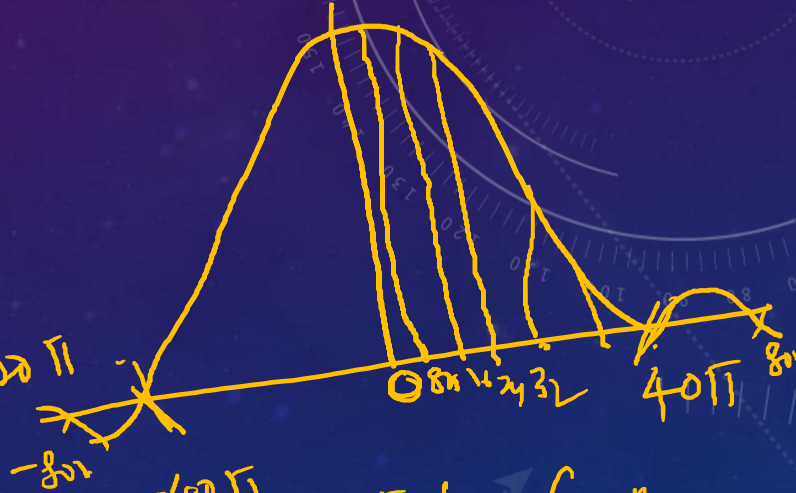
$\pm 40n\pi$

$\frac{n\omega_0\tau}{2} = \pm n\pi$

$n\omega_0\tau = \pm 2n\pi$

$n\omega_0 = \pm 40n\pi$

$= \pm 40\pi, \pm 80\pi, \pm 120\pi$



$\omega_0 = \frac{2\pi}{T}$

$= \pm 8\pi$, $\pm 16\pi$, $\pm 24\pi$, $\pm 32\pi$, $\pm 40\pi$

$T = \frac{1}{20}$ = width

$T = \frac{1}{2}$, zeros - at , $\pm 40\pi, \pm 80\pi, 120\pi$. —

$n\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1/2} = n4\pi$

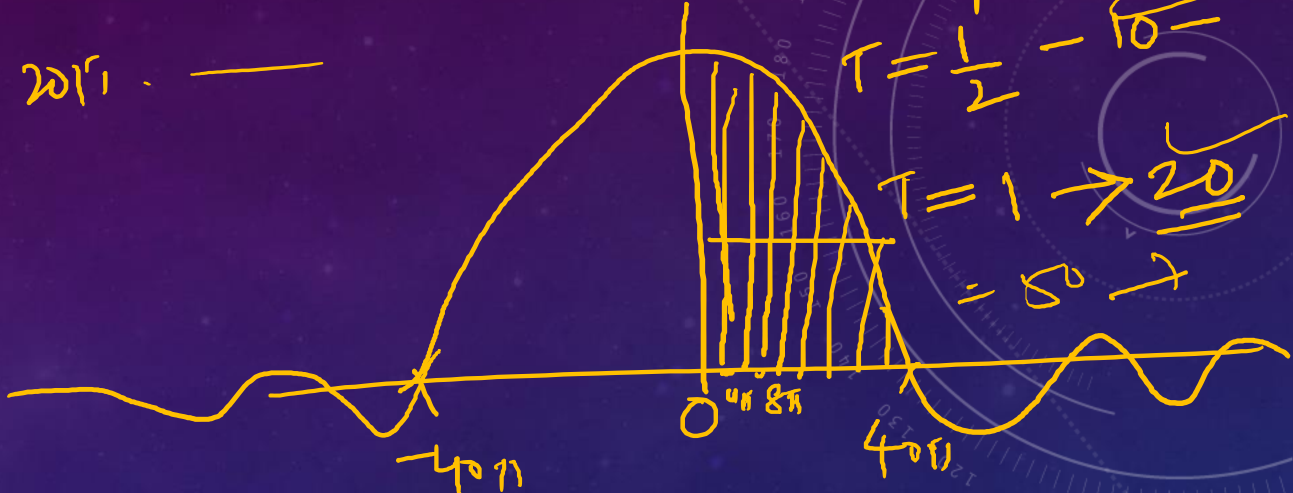
$4\pi, 8\pi, 12\pi, 16\pi, 20\pi, 24\pi, 28\pi, 32\pi, 36\pi, 40\pi$

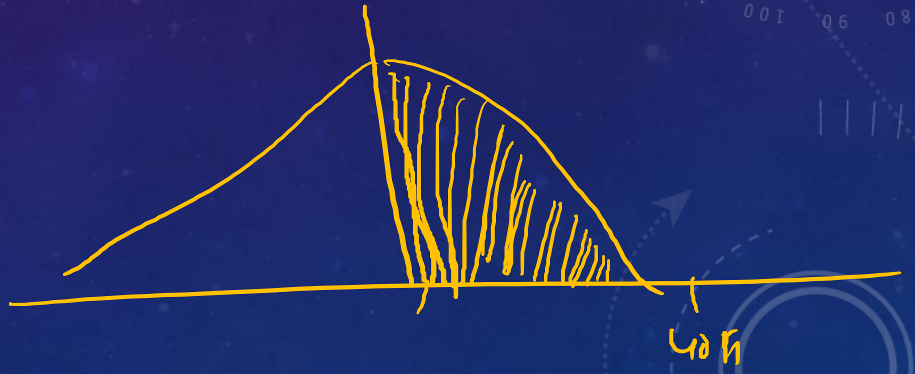
$T = \frac{1}{4}$ — 5 —

$T = \frac{1}{2}$ — 10 —

$T = 1 \rightarrow 20$

$= 80 \rightarrow$





$T = 1 \Rightarrow T = 1/20$

$\omega_0 = \frac{2\pi}{T} = 2\pi$

$T = 50 \Rightarrow \frac{2\pi}{50} = \omega = \cancel{0.1256}$

$0.02 \times 2\pi$

$T \uparrow \Rightarrow$ Spectrum

$T = \text{large} = \uparrow$ close to infinity

$\omega = \frac{2\pi}{T} = 0$

$\frac{1}{4} = 5$

$\frac{1}{2} = 10$

$1 = 20$

there No spacing b/n Successive freq Components

$\Rightarrow$ freq = Continuous

$T = 2$

$f(t) = \cancel{\sum_n} F_n e^{j\omega t}$

$\int_{-\infty}^{\infty}$ $\rightarrow \omega$

more freq Presents

F.S $\rightarrow f(t) = \sum_{n=-\alpha}^{\alpha} F_n e^{jn\omega_0 t}$ $\rightarrow$ discrete Sum of Complex Expon-

Freq = finite

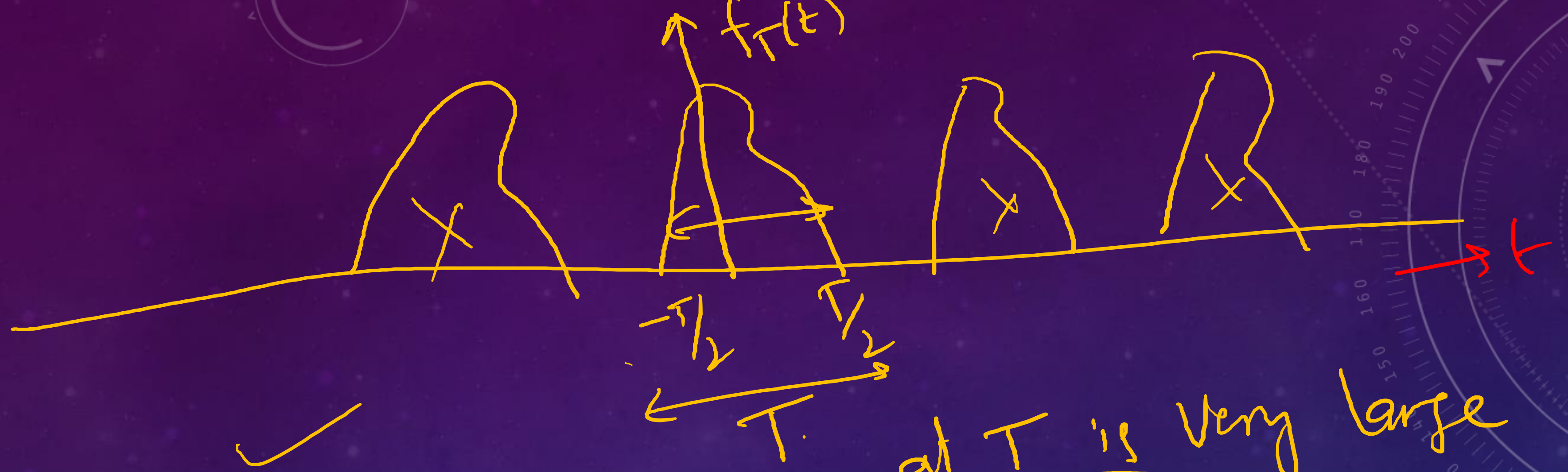
Spectrum $\rightarrow$ discrete in Nature

F.T $\rightarrow$ Freq = infinite

Spectrum $\rightarrow$ Continuous $\rightarrow$ $\cancel{\Sigma}$ — Replaced with $\int_{-\alpha}^{\alpha}$

Represnting $\rightarrow$ fn- over the entire Interval

$(-\alpha, \alpha)$



$f_T(t)$ = Periodic = F.S

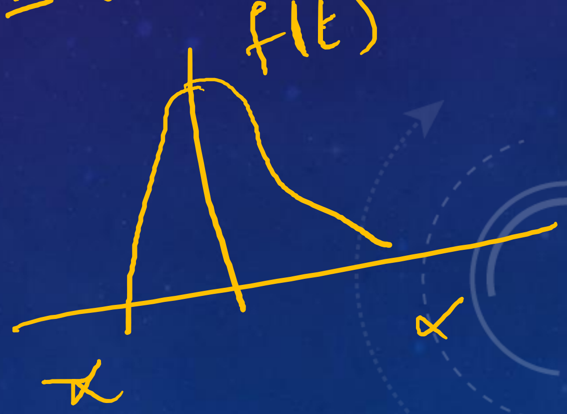
$$f_T(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$F_n = \frac{1}{T}\int_{-T/2}^{T/2} f_T(t)\, e^{-jn\omega_0 t}\, dt$$

at T is very large

$T = \infty$

Periodic = $T = \infty$ means



$$\boxed{\lim_{T\to\infty} f_T(t) = f(t)}$$

$$f_T(t) = \sum_n F_n e^{jn\omega_0 t}$$

$$F_n = \frac{1}{T}\int_{-T/2}^{T/2} f_T(t)\, e^{-jn\omega_0 t}\, dt$$

$$T F_n = \int_{-T/2}^{T/2} f_T(t)\, e^{-jn\omega_0 t}\, dt$$

Let $F(\omega_n) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega_n t}\, dt$

$$T F_n = F(n\omega_0) \Rightarrow$$

$$F_n = \frac{1}{T} F(n\omega_0)$$

$$f_T(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} F(n\omega_0)\, e^{jn\omega_0 t}$$

$T = \frac{2\pi}{\omega_0} \rightarrow T = \infty$

$$\lim_{T\to\infty} f_T(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} F(n\omega_0)\, e^{jn\omega_0 t}\, \omega_0$$

as $T \to \infty \Rightarrow \omega_0 = \text{small} = \Delta\omega$

freq spacing $= 0 \Rightarrow$ close to each other

in the range $\Rightarrow$ more freq

$$f(t) = \frac{1}{2\pi}$$

$$\lim_{T\to\alpha} f_T(t) = \frac{1}{2\pi} \sum_{n=-\alpha}^{\alpha} F(n\cancel{\Delta\omega_0})\, e^{j(n\cancel{\Delta\omega_0})t} \cdot \cancel{\Delta\omega_0}$$

Non aperiodic

$$f(t) = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} F(\omega)\, e^{j\omega t}\, d\omega \rightarrow IFT\{F(\omega)\}$$

$$F(\omega) = \int_{-\alpha}^{\alpha} f(t)\, e^{-j\omega t}\cdot dt \rightarrow FT[f(t)] = F(\omega)$$

FS → Periodic

FT → Non Periodic/Periodic

the FT[f(t)] denoted F(ω)

$$F(\omega) = \int_{-\alpha}^{\alpha} f(t) e^{-j\omega t} . dt \quad \text{--- } (1)$$

$$I\,FT[F(\omega)] = f(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} F(\omega) e^{j\omega t} . d\omega \quad \text{--- } (2)$$

$$f(t) \xleftrightarrow{F.T} F(\omega)$$

f(t) - Periodic - FS

f(t) Non - " - FT

Freq. Domain

Thank you