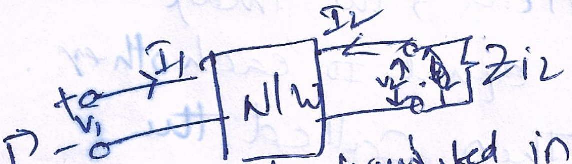
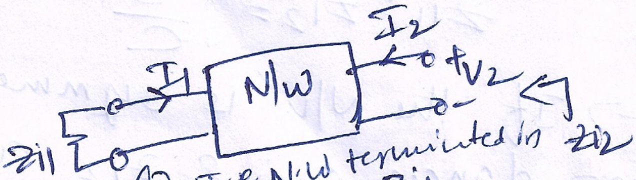
IMAGE PARAMETERS:-

The image impedance Z_{i1} & Z_{i2} of two port N/w are two values of impedance such that, if the port 1-1' of the network is terminated in Z_{i1}, the input impedance of the port 2-2' is Z_{i2}; and if the port 2-2' is terminated in Z_{i2}, the input (or driving point) impedance at the port 1-1' is Z_{i1}. Z_{i1} & Z_{i2} are also known image parameters



Z_{i1} (a) T.P. N/w terminated in Z_{i2}



(b) T.P. N/w terminated in Z_{i2} Z_{i1}

The image parameters can be expressed in terms of ABCD parameters.

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

if the N/w is terminated in Z_{i2} at 2-2'

$V_2 = -I_2 Z_{i2}$

$$\frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{-AI_2 Z_{i2} - BI_2}{-CI_2 Z_{i2} - DI_2}$$

$$Z_{i1} = \frac{-AZ_{i2} - B}{-CZ_{i2} - D} = \frac{AZ_{i2} + B}{CZ_{i2} + D} \longrightarrow (1)$$

Similarly if the N/w is terminated in Z_{i1} at 1-1'

$V_1 = -I_1 Z_{i1}$

$\frac{V_2}{I_2} = Z_{i2}$

$$-Z_{i1} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{AI_2 Z_{i2} - BI_2}{CI_2 Z_{i2} - DI_2}$$

$$-Z_{i1} = \frac{AZ_{i2} - B}{CZ_{i2} - D} \Longrightarrow -CZ_{i1}Z_{i2} + DZ_{i1} = AZ_{i2} - B.$$

$$Z_{i2}[A + CZ_{i1}] = B + DZ_{i1}$$

$$\boxed{Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + A}} \longrightarrow (2)$$

Substitute Z_{i1} in the above equation.

$$\cancel{Z_{i2}\left[C\left(\frac{-AZ_{i2}+B}{CZ_{i2}-D}\right)+A\right] = D\left[\frac{-AZ_{i2}+B}{CZ_{i2}-D}\right]+B}$$

Solving eqn ① & ②.

$$Z_{i1} = \sqrt{\frac{AB}{CD}} \quad \& \quad Z_{i2} = \sqrt{\frac{BD}{AC}} \rightarrow ③$$

If the network is symmetrical, then $A = D$.

$$Z_{i1} = Z_{i2} = \sqrt{\frac{B}{C}} \rightarrow ④$$

→ If the N/w is symmetrical, the image impedances Z_{i1} & Z_{i2} are equal to eachother. The image impedance is then called the characteristic impedance (or) the iterative impedance i.e., if a symmetrical network is terminated in Z_L, its input impedance will also be Z_L (or) its impedance transformation ratio is unity

→ since a reciprocal network can be described by two independent parameters, the image parameters Z_{i1} & Z_{i2} are sufficient to characterise reciprocal symmetric networks. Z_{i1} & Z_{i2}, the two image parameters do not completely define a network. A third parameter called image transfer constant (γ) is also used to describe reciprocal networks. This parameter may be obtained from the voltage & current ratios.

If the image impedance Z_{i2} is connected across the port 2-2', Then

$$V_1 = AV_2 - BI_2 \rightarrow ⑤$$

$$V_2 = -I_2 Z_{i2} \Rightarrow -I_2 = \frac{V_2}{Z_{i2}} — ⑥$$

$$V_1 = \left[A + \frac{B}{Z_{i2}}\right]V_2 — ⑦$$

$$I_1 = CV_2 - DI_2 \rightarrow (8)$$

$$I_1 = -[CZ_{i2} + D] I_2 \rightarrow (9)$$

from (7)

$$\frac{V_1}{V_2} = \left[A + \frac{B}{Z_{i2}}\right] = A + B\sqrt{\frac{AC}{BD}}$$

$$\frac{V_1}{V_2} = A + \frac{\sqrt{ABCD}}{D} = \frac{AD + \sqrt{ABCD}}{D} \rightarrow (10)$$

From (9)

$$\frac{-I_1}{I_2} = [CZ_{i2} + D] = D + C\sqrt{\frac{BD}{AC}}$$

$$= D + \frac{\sqrt{ABCD}}{A} = \frac{AD + \sqrt{ABCD}}{A} \rightarrow (11)$$

multiply eqn (10) & (11)

$$-\frac{V_1}{V_2} \times \frac{I_1}{I_2} = \frac{AD + \sqrt{ABCD}}{D} \times \frac{AD + \sqrt{ABCD}}{A}$$

$$-\frac{V_1}{V_2} \times \frac{I_1}{I_2} = \frac{(AD)^2 + ABCD + 2AD\sqrt{ABCD}}{AD}$$

$$= AD + BC + 2\sqrt{ABCD} = [\sqrt{AD} + \sqrt{BC}]^2$$

$$\sqrt{AD} + \sqrt{BC} = \sqrt{\frac{-V_1}{V_2} \times \frac{I_1}{I_2}}$$

$$\boxed{\because AD - BC = 1}$$

$$\sqrt{AD} + \sqrt{AD-1} = \sqrt{\frac{-V_1}{V_2} \times \frac{I_1}{I_2}}$$

Let $\cosh\phi = \sqrt{AD}$ $\sinh\phi = \sqrt{AD-1}$

$$\tanh\phi = \frac{\sqrt{AD-1}}{\sqrt{AD}} = \sqrt{\frac{BC}{AD}}$$

$$\phi = \tanh^{-1}\sqrt{\frac{BC}{AD}} \rightarrow e^{\phi} = \cosh\phi + \sinh\phi = \sqrt{\frac{-V_1}{V_2} \times \frac{I_1}{I_2}}$$

$$\phi = \log_e \sqrt{\frac{-V_1}{V_2} \times \frac{I_1}{I_2}} = \frac{1}{2}\log_e\left(\frac{-V_1}{V_2} \times \frac{I_1}{I_2}\right)$$

since $V_1 = Z_{i1} I_1$ $V_2 = -I_2 Z_{i2}$

$$\phi = \frac{1}{2}\log_e\left(\frac{Z_{i1} I_1}{Z_{i2} I_2} \times \frac{I_1}{I_2}\right) \Rightarrow \phi = \frac{1}{2}\log_e \frac{Z_{i1}}{Z_{i2}} + \log_e \frac{I_1}{I_2}$$

for symmetrical reciprocal n/w

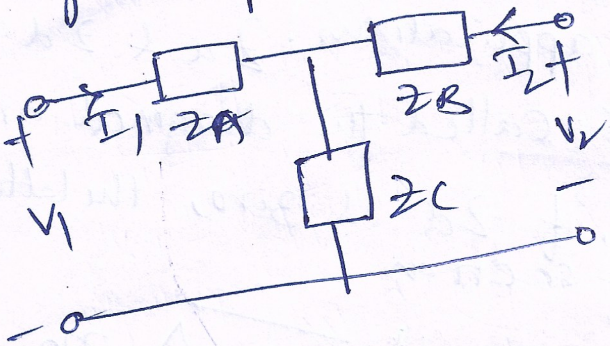
$Z_{i1} = Z_{i2}$

$$\phi = \log_e\left[\frac{I_1}{I_2}\right] = \gamma$$

... propagation

T-Network :-

Any two port N/w can be represented by an equivalent T network. The elements of the equivalent T network may be expressed in terms of Z-parameter.



$V_1 = (Z_A + Z_C) I_1 + Z_C I_2$

$V_2 = Z_C I_1 + (Z_B + Z_C) I_2$

$Z_{11} = Z_A + Z_C \quad Z_{12} = Z_C$

$Z_{21} = Z_C \quad Z_{22} = Z_B + Z_C$

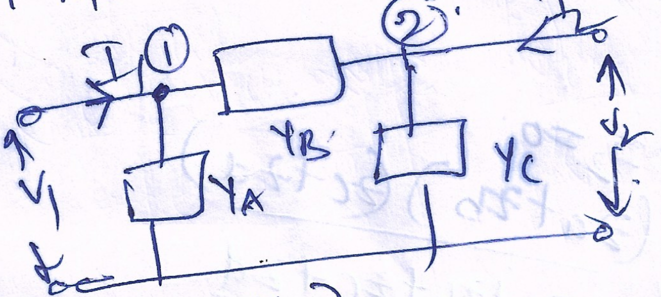
$Z_A = Z_{11} - Z_C = Z_{11} - Z_{12} = Z_{11} - Z_{21}$

$Z_C = Z_{12} = Z_{21}$

$Z_B = Z_{22} - Z_C = Z_{22} - Z_{21} = Z_{22} - Z_{12}$

Π-Network :-

Any two port N/w can be represented by an equivalent Π N/w



$I_1 = Y_A V_1 + Y_B (V_1 - V_2)$

$I_1 = (Y_A + Y_B) V_1 - Y_B V_2$ KCL at node 1

$I_2 = -Y_B V_1 + (Y_B + Y_C) V_2$ KCL at node 2.

$Y_{11} = Y_A + Y_B$

$Y_{12} = Y_{21} = -Y_B$

$Y_{22} = Y_B + Y_C$

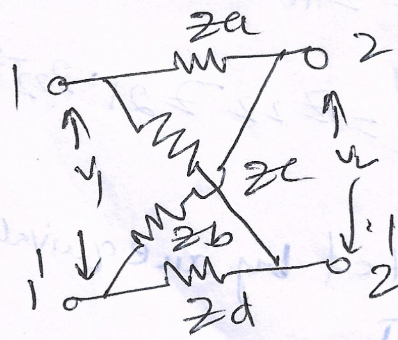
$Y_A = Y_{11} - Y_B = Y_{11} - Y_{12} = Y_{11} - Y_{21}$

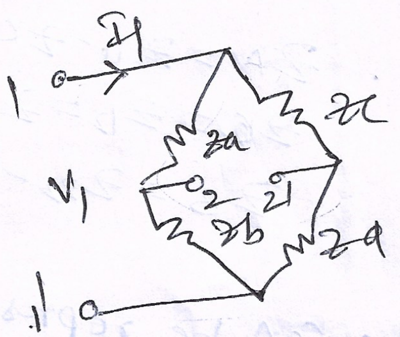
$Y_B = Y_{12} = Y_{21}$

$Y_C = Y_{22} - Y_B = Y_{22} + Y_{12} = Y_{22} + Y_{21}$

Lattice Network:-

one of the Common four-terminal two-port N/w is the lattice (or) bridge N/w. Lattice N/ws are used in filter sections and are also used as attenuators. Lattice structures used in preference to ladder structures in some special application. Z_a & Z_d are series arms & Z_b & Z_c are called the diagonal arms. It can be observed that, if Z_d is zero, the lattice structure becomes a Π-section.





$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$V_1 = I_1 \frac{(Z_a+Z_b)(Z_c+Z_d)}{Z_a+Z_b+Z_c+Z_d}$$

$$\boxed{Z_{11} = \frac{V_1}{I_1} = \frac{(Z_a+Z_b)(Z_c+Z_d)}{Z_a+Z_b+Z_c+Z_d}}$$ if N/w is symmetric

i.e, $Z_a = Z_d$, $Z_b = Z_c$

$$\boxed{Z_{11} = \frac{Z_a+Z_b}{2}}$$

when $I_2=0$, V_2 is the voltage across 2-2'

$$V_2 = V_1\left[\frac{Z_b}{Z_b+Z_a} - \frac{Z_d}{Z_c+Z_d}\right]$$

$$= I_1 \frac{\cancel{(Z_a+Z_b)}\cancel{(Z_c+Z_d)}}{Z_a+Z_b+Z_c+Z_d}\left\{\frac{Z_b(Z_c+Z_d)-Z_d(Z_b+Z_a)}{\cancel{(Z_a+Z_b)}\cancel{(Z_c+Z_d)}}\right.$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{Z_bZ_c - Z_aZ_b}{\ldots}$$

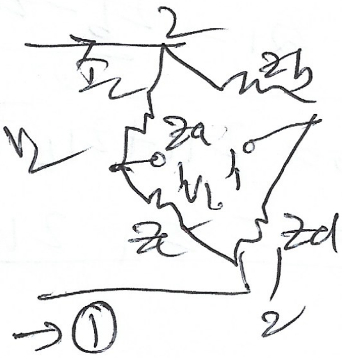
if ~~the~~ N/w is symmetric $Z_a = Z_d$, $Z_b = Z_c$

$$Z_{21} = \frac{Z_b^2 - Z_a^2}{2(Z_a+Z_b)} = \frac{(Z_b - Z_a)(\cancel{Z_a+Z_b})}{2(\cancel{Z_a+Z_b})}$$

$$\boxed{Z_{21} = \frac{Z_b - Z_a}{2}}$$

when the i/p port is open, $I_1 = 0$.

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$



$$V_1 = V_2\left[\frac{Z_c}{Z_c+Z_a} - \frac{Z_d}{Z_d+Z_b}\right] \rightarrow (1)$$

$$V_2 = I_2 \frac{(Z_a+Z_c)(Z_b+Z_d)}{Z_a+Z_b+Z_c+Z_d} \rightarrow (2)$$

substitute the (2) in (1).

$$V_1 = I_2\left[\frac{Z_c(Z_b+Z_d) - Z_d(Z_c+Z_a)}{(\cancel{Z_a+Z_c})(\cancel{Z_d+Z_b})}\right]\left[\frac{(\cancel{Z_a+Z_c})(\cancel{Z_b+Z_d})}{Z_a+Z_b+Z_c+Z_d}\right]$$

$$= I_2\left(\frac{Z_cZ_b + \cancel{Z_cZ_d} - \cancel{Z_cZ_d} - Z_aZ_d}{Z_a+Z_b+Z_c+Z_d}\right)$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{Z_cZ_b - Z_aZ_d}{Z_a+Z_b+Z_c+Z_d}$$

If the Network is symmetric

$Z_a = Z_d$ & $Z_b = Z_c$

$$\frac{V_1}{I_2} = \frac{Z_b^2 - Z_a^2}{2(Z_a+Z_b)} = \frac{[Z_b - Z_a][\cancel{Z_b+Z_a}]}{2[\cancel{Z_a+Z_b}]} = \frac{Z_b - Z_a}{2} = \frac{Z_c - Z_d}{2}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_2=0}$$

$$\frac{V_2}{I_2} = \frac{(Z_a+Z_c)(Z_b+Z_d)}{Z_a+Z_b+Z_c+Z_d}.$$

If the Network is symmetric i.e., $Z_a = Z_d$; $Z_b = Z_c$

$$Z_{22} = \frac{(Z_a + Z_b)(Z_a + Z_b)}{2(\cancel{Z_a + Z_b})} = \frac{Z_a + Z_b}{2}.$$

From the above equation.

$$Z_{11} = Z_{22} = \frac{Z_a + Z_b}{2}$$

$$Z_{12} = Z_{21} = \frac{Z_b - Z_a}{2}.$$

$$\boxed{\begin{aligned} Z_b &= Z_{11} + Z_{12} \\ Z_a &= Z_{11} - Z_{12} \end{aligned}}$$

Terminated Two-port N/ws:-

(a) Driving-point impedance at the input port of a load-terminated Network:-

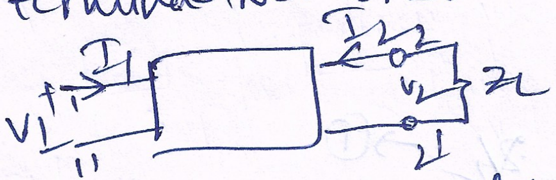


fig (1) Two port network

Fig shows a 2-port N/w connected to an ideal generator at the i/p port and to a load impedance at the output port. The input impedance of this N/w can be expressed in terms of parameters of the 2-port N/w

Interms of Z-parameters:-

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow (2)$$

The load at the o/p port 2-2' imposes the following constraints on the port Voltage & current.

$$V_2 = -Z_L I_2 \rightarrow (3)$$

substitute (3) in (2)

$$-I_2 Z_L = Z_{21}I_1 + Z_{22}I_2$$

$$I_2 = \frac{-Z_{21}}{Z_{22}+Z_L} I_1 \rightarrow (4)$$

substitute eqn (4) in (1)

$$Z_{in} = \frac{V_1}{I_1} = Z_{11} + Z_{12}\left(\frac{-Z_{21}}{Z_{22}+Z_L}\right)$$

$$= Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}+Z_L} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21} + Z_{11}Z_L}{Z_{22}+Z_L}$$

if the output port is open-circuited i.e, $Z_L = \infty$

$$Z_{in} = \frac{Z_L\left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_L} + Z_{11}\right]}{Z_L\left(\frac{Z_{22}}{Z_L} + 1\right)} = Z_{11}$$

if the output port is short-circuited i.e $Z_L = 0$

$$Z_{in} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} = \frac{1}{Y_{11}}$$

Interms of Y-parameters:-

If a load admittance Y_L is connected across the the output port

$$V_2 = -Z_L I_2$$

$$I_2 = -\frac{V_2}{Z_L} = -Y_L V_2 \rightarrow (1)$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (2)$$

$$I_2 = Y_{21} + Y_{22}V_2 \rightarrow (3)$$

substitute eqn (1) in (3).

$$-V_2 Y_L = Y_{21}V_1 + Y_{22}V_2$$

$$V_2 = -\left(\frac{Y_{21}}{Y_L + Y_{22}}\right) V_1 \rightarrow (4)$$

substitute eqn (4) in (2)

$$I_1 = Y_{11}V_1 - \frac{Y_{12}Y_{21}V_1}{Y_L + Y_{22}}$$

$$\frac{I_1}{V_1} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}} = \frac{Y_{11}Y_L + Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_L + Y_{22}}$$

$$\boxed{Z_{in} = \frac{V_1}{I_1} = \frac{Y_L + Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21} + Y_{11}Y_L}} *$$

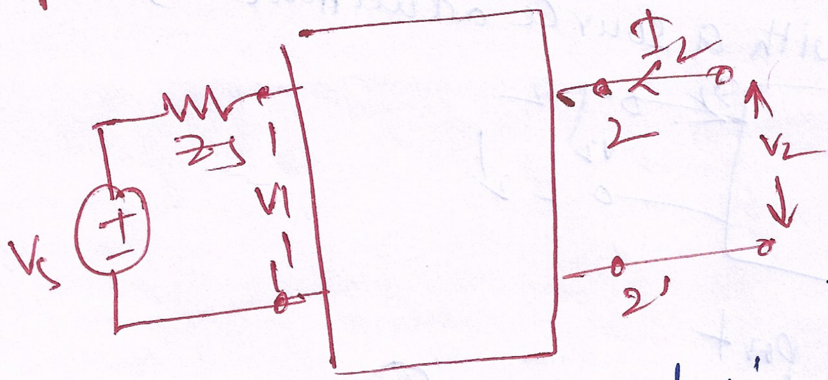
when o/p port is open-circuited. i.e $Y_L = 0$

$$Z_{in} = \frac{Y_{22}}{Y_{11}Y_{22} - Y_{21}Y_{12}} = \frac{Y_{22}}{\Delta Y} = Z_{11}$$

when o/p port is short-circuited i.e $Y_L = \infty$.

$$Z_{in} = \frac{Y_L\left[1 + \frac{Y_{22}}{Y_L}\right]}{Y_L\left[\frac{\Delta Y}{Y_L} + Y_{11}\right]} = \frac{1}{Y_{11}}$$

(B) Driving point impedance at the output port with source impedance at the input port :-



Consider a 2-port N/w connected to a generator at the i/p port with a source impedance Z_s.

In terms of Z parameters :-

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow (2)$$

$$V_1 = V_s - I_1 Z_s \rightarrow (3)$$

equate (1) & (3)

$$Z_{11}I_1 + Z_{12}I_2 = V_s - I_1 Z_s$$

$$-I_1 = \frac{Z_{12}I_2 - V_s}{Z_s + Z_{11}} \rightarrow (4)$$

substitute eqn (4) in (2) we get

$$V_2 = -Z_{21}\left[\frac{Z_{12}I_2 - V_s}{Z_s + Z_{11}}\right] + Z_{22}I_2$$

with no source voltage at the i/p port 1-1', i.e if the source V_s is short-circuited,

$$V_2 = \frac{-Z_{21}Z_{12}}{Z_s + Z_{11}} I_2 + Z_{22}I_2$$

the driving point impedance at the port 2-2' $= \frac{V_2}{I_2}$

$$\frac{V_2}{I_2} = \frac{Z_{22}Z_s + Z_{22}Z_{11} - Z_{21}Z_{12}}{Z_s + Z_{11}} \Rightarrow \frac{\Delta Z + Z_{22}Z_s}{Z_s + Z_{11}}$$

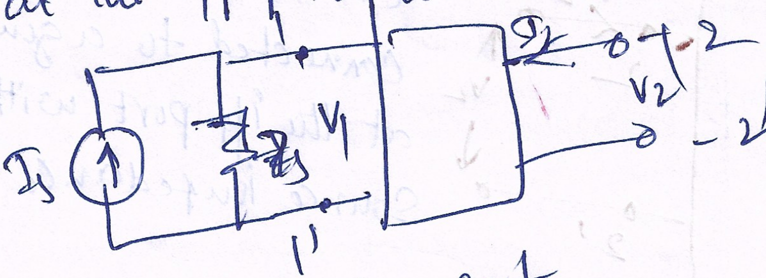
if the i/p port is open $Z_s \rightarrow \infty$

$$\frac{V_2}{I_2} = \frac{\frac{\Delta Z}{Z_s} + Z_{22}}{1 + \frac{Z_{11}}{Z_s}} = Z_{22}$$

If the source impedance is zero with a short-circuited i/p port, the driving point impedance at the output port is given by $\frac{V_2}{I_2} = \frac{\Delta Z}{\ldots} = \frac{1}{Y_{22}}$

In terms of Y-parameters:-

let us consider a two-port N/W conected to a current at the i/p port with a source admittance Y_s



I_1 at the port

$$I_1 = I_s - \frac{V_1}{Z_s} = I_s - V_1 Y_s \rightarrow (1)$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (2)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (3)$$

substitute eqn (1) & (2)

$$I_s - V_1 Y_s = Y_{11}V_1 + Y_{12}V_2$$

$$-V_1 = \frac{Y_{12}V_2 - I_s}{Y_s + Y_{11}} \rightarrow (4)$$

substitute eqn (4) in (3)

$$I_2 = -Y_{21}\left[\frac{Y_{12}V_2 - I_s}{Y_s + Y_{11}}\right] + Y_{22}V_2$$

with no source current at 1-1' ie if current source is open-circuited

$$I_2 = -\frac{Y_{21}Y_{12}V_2}{Y_s + Y_{11}} + Y_{22}V_2$$

Hence the driving point admittance at the o/p port is given as

$$\frac{I_2}{V_2} = \frac{Y_{22}Y_s + Y_{22}Y_{11} - Y_{21}Y_{12}}{Y_s + Y_{11}} = \frac{\Delta Y + Y_{22}Y_s}{Y_s + Y_{11}}$$

If the input is open circuited ie if the source admittance is zero. $(Z_s = \infty) \rightarrow Y_s = 0$

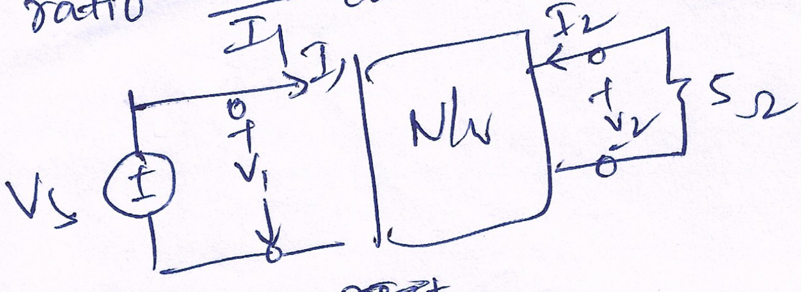
$$\frac{V_2}{I_2} = \frac{Y_{11}}{\Delta Y} = Z_{22}$$

If the input is short circuited ie $Y_s = \infty$, $Z_s = 0$.

$$Z_{22} = \frac{V_2}{I_2} = \frac{Y_s(1 + Y_{11}/Y_s)}{Y_s(\Delta Y/Y_s + Y_{22})} = \frac{1}{Y_{22}}$$

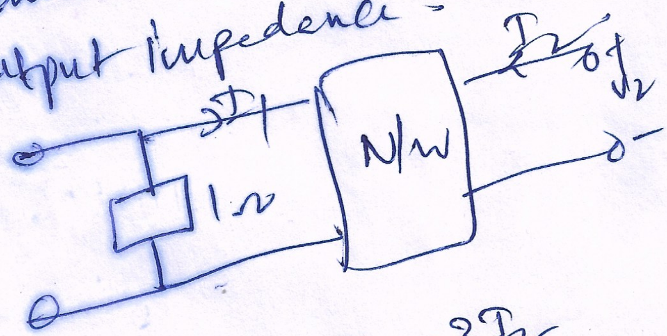
problems on Terminated 2-port N/w.

1. The Z-parameters are $Z_{11}=2\Omega$, $Z_{12}=1\Omega$, $Z_{21}=2\Omega$, $Z_{22}=5\Omega$. Calculate the voltage ratio $\frac{V_2}{V_s}$, current ratio $\frac{-I_2}{I_1}$ and input impedance $\frac{V_1}{I_1}$



~~$V_1 = 2I_1+$~~

2. The Y-parameters for a 2-port N/w $Y_{11}=4\mho$, $Y_{22}=5\mho$, $Y_{12}=Y_{21}=4\mho$. If a resistor of 1Ω is connected across port-1 of the N/w then find the output impedance.



3. The $V_1 = 8V_2 - 3I_2$
$I_1 = 6V_2 - 2I_2$
is connected across a load resistance of 5Ω the o/p port. Calculate the i/p impedance