Analog & Digital Communications

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UNIT- II

Angle Modulation

Contents

Basic concepts of Phase Modulation, Frequency Modulation: Single tone frequency modulation, Spectrum Analysis of Sinusoidal FM Wave using Bessel functions, Narrow band FM, Wide band FM, Constant Average Power, Transmission bandwidth of FM Wave - Generation of FM Signal- Armstrong Method, Detection of FM Signal: Balanced slope detector, Phase locked loop, Comparison of FM and AM., Concept of Pre-emphasis and de-emphasis.

Basic Definitions

- Let $\Theta_i(t)$ Denotes the angle of a modulated sinusoidal carrier, which is a function of the message. we express the resulting angle-modulated wave as

$$s(t) = A_c \cos[\Theta_i(t)] \qquad \ldots\ldots\ldots\ldots..1$$

Where Ac is the carrier amplitude.

- A complete oscillation occurs whenever $\Theta_i(t)$ changes by 2Π radians. If $\Theta_i(t)$ increases monotonically with time, the Average frequency in Hz. Over an interval from ‘t’ to ‘t+Δt’, is given by

$$f_{\Delta t}(t) = \frac{\Theta i(t+\Delta t) - \Theta i(t)}{2\Pi\Delta t}$$

We can now define instantaneous frequency of the angle modulated wave s(t) by

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$f_i(t) = \lim_{\Delta t \to 0} \left(\frac{\Theta i(t+\Delta t) - \Theta i(t)}{2\Pi\Delta t}\right)$$

$$f_i(t) = \frac{1}{2\Pi}\,\frac{d\Theta i(t)}{dt}$$

According to $s(t) = A_c \mathrm{Cos}[\Theta i(t)]$ Angle Modulated wave s(t) is a rotating phasor of length A_c and Angle $\Theta i(t)$. The Angular velocity of such a phasor is $d\Theta i(t)/dt$, In the simple case of an unmodulated carrier. The angle $\Theta i(t)$ is

$$\Theta_i(t) = 2\,\Pi f_c\, t + \varphi_c$$

And the corresponding phasor rotates with a constant angular velocity equal to $2\,\Pi f_c$. The constant φ_c is the value of $\Theta_i(t)$ at t=0

We can have Two kinds of Angle Modulations

1) Frequency Modulation
2) Phase Modulation

Phase Modulation

- Phase Modulation (PM) is that form of angle modulation in which the angle $\Theta_i(t)$ is varied Linearly with the baseband signal m(t),as shown by

$$\Theta_i(t) = 2\Pi f_c t + K_p m(t)$$

The term $2\Pi f_c t$ represents Unmodulated Carrier, K_p represents Phase sensitivity of the modulator expressed in **Radians/Volt**

$$\boxed{\mathbf{s(t) = A_c Cos[2\Pi f_c t + K_p m(t)]}}$$

…… Equation of Phase Modulation.

Frequency Modulation

- Frequency Modulation (FM) is that form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the baseband signal m(t), as shown by

$$f_i(t) = f_c + K_f\, m(t)$$

The term f_c represents the frequency of unmodulated Carrier and the constant K_f Represents the frequency sensitivity of the modulator, expressed in **Hertz/Volt**

➢ **We Know that**

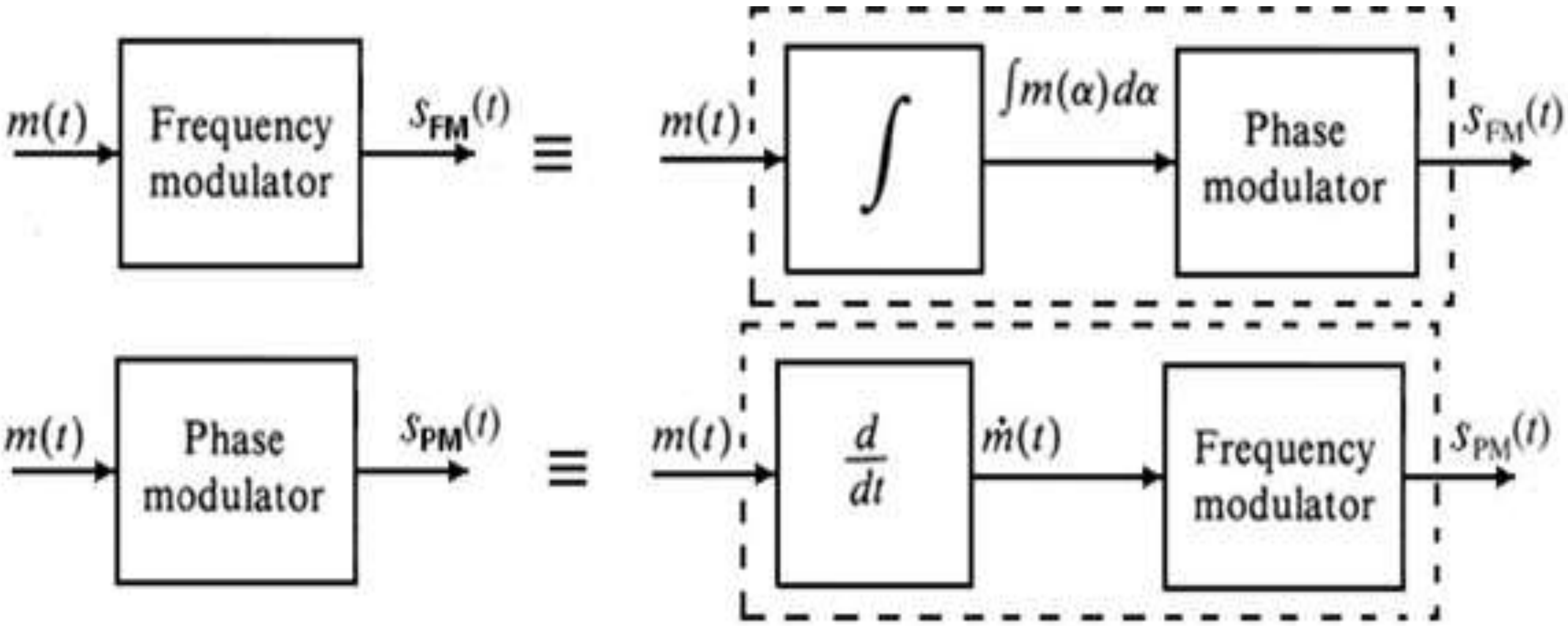
$$f_i(t) = \frac{1}{2\Pi} \frac{d\Theta_i(t)}{dt}$$

- $\theta_i(t) = 2\Pi f_c t + 2\Pi K_f \int_0^t m(t)dt$

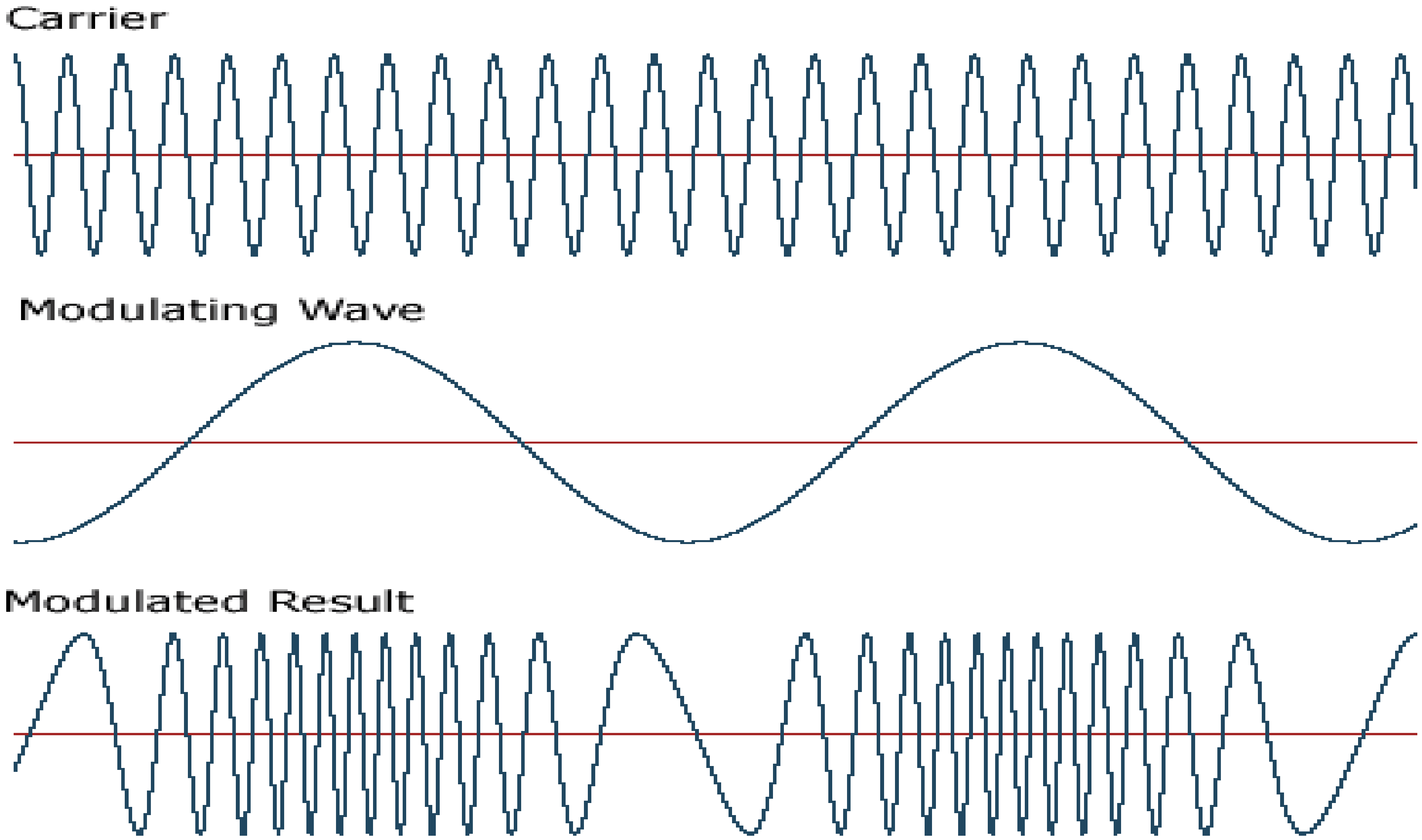
The Frequency Modulated wave therefore described by

$$\boxed{\mathbf{s(t) = A_c \,Cos[2\,\Pi f_c\, t + 2\,\Pi\, K_f \int_0^t m(t)dt]}}$$ ……Equation of Frequency Modulation.

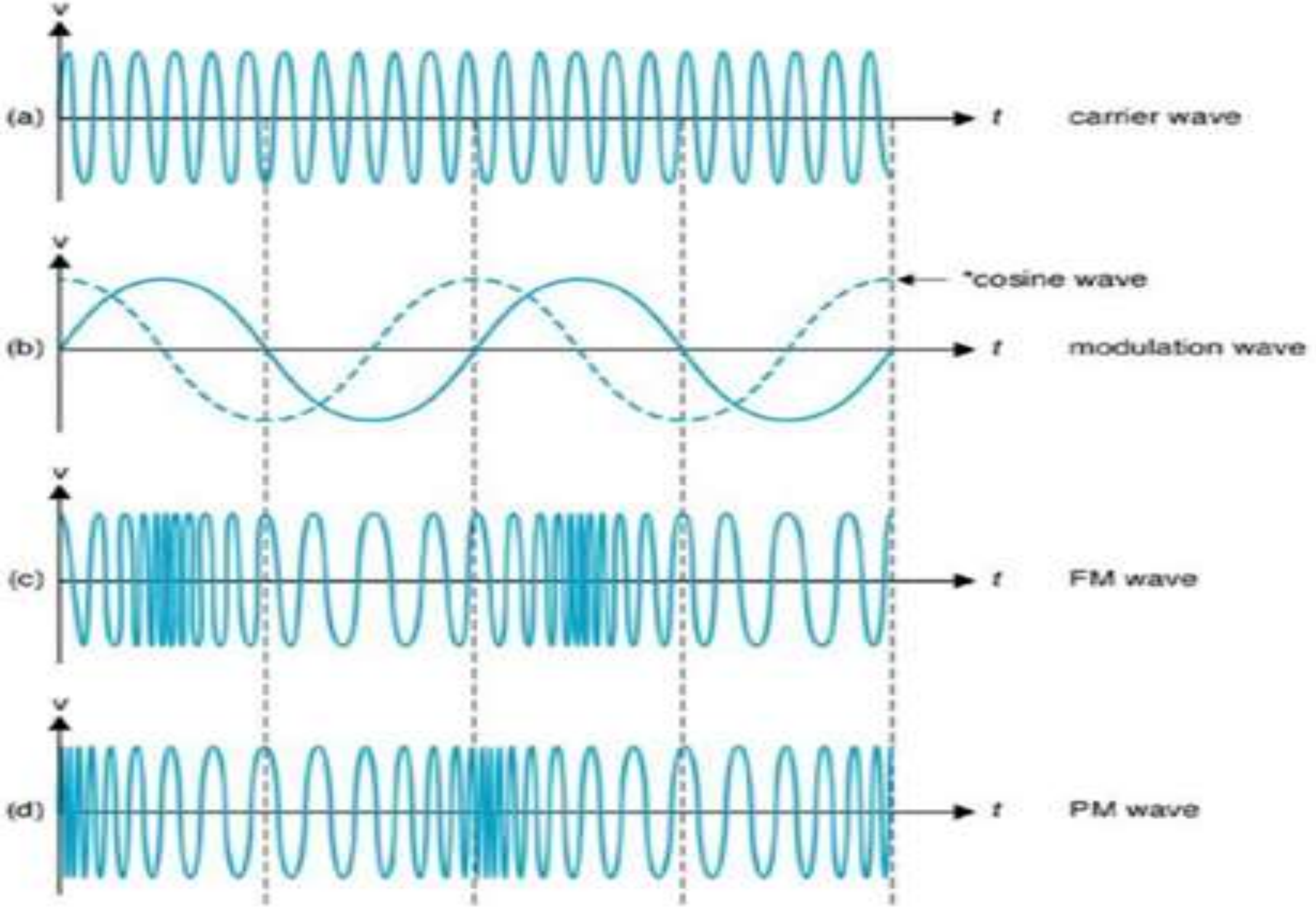
- $s(t) = A_c \text{Cos}[2\Pi f_c t + K_p m(t)]$ Equation of Phase Modulation.
- $s(t) = A_c \text{Cos}[2\,\Pi f_c t + 2\,\Pi\, K_f \int_0^t m(t)\,dt]$ Equation of Frequency Modulation.



FM wave



FM vs PM Wave



v
(a)
t
carrier wave
v
*cosine wave
(b)
t
modulation wave
v
(c)
t
FM wave
v
(d)
t
PM wave

Single Tone Angle Modulation (PM)

Let a message signal $m(t) = A_m \cos 2\pi f_m t$

The phase modulated signal is represented by

$$s_{PM}(t) = A_c \cos\left[\omega_c t + k_p m(t)\right]$$

For a single-tone modulating signal, the PM wave is represented by

$$s_{PM}(t) = A_c \cos\left[\omega_c t + k_p A_m \cos 2\pi f_m t\right]$$

$$= A_c \cos\left[\omega_c t + \beta_p \cos 2\pi f_m t\right]$$

where $\beta_p = k_p A_m$ is called phase modulation index.

Phase deviation $\Delta\theta = k_p \ \max\{m(t)\} = k_p A_m$.

Phase modulation index $\beta_p = \Delta\theta = k_p A_m$.

Single -Tone modulation of Frequency Modulation

$s(t) = A_c \cos[2\Pi f_c t + 2\Pi K_f \int_0^t m(t)dt]$ $\quad where \ \ m(t) = A_m \cos 2\pi f_m t$

$s(t) = A_c \cos[\Theta_i(t)]$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t)dt$$

$$= 2\pi f_c t + 2\pi k_f \frac{A_m}{2\pi f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$$\therefore \theta(t) = 2\pi f_c t + \beta_f \sin(2\pi f_m t)$$

i

Where

$$\beta_f = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

where $\beta_f = \dfrac{k_f A_m}{f_m} = \dfrac{\Delta f}{f_m}$ is called frequency modulation index, and where again $\Delta f = k_f A_m$ is known as frequency deviation.

$$S_{FM}(t) = A_c \cos[2\Pi f_c t + \beta_f \sin(2\Pi f_m t)]$$

β is Modulation Index

- When it is Small or less than 1 Radian then it is called as **Narrow Band FM**
- When it is Greater or More than 1 Radian then it is called as **Wide Band FM**

Brief Summary

Phase Modulation: **P.M** : $A_c \cos\left[\omega_c t + \beta_p \cos 2\pi f_m t\right]$

Phase Modulation index $\beta_p = k_p A_m$;

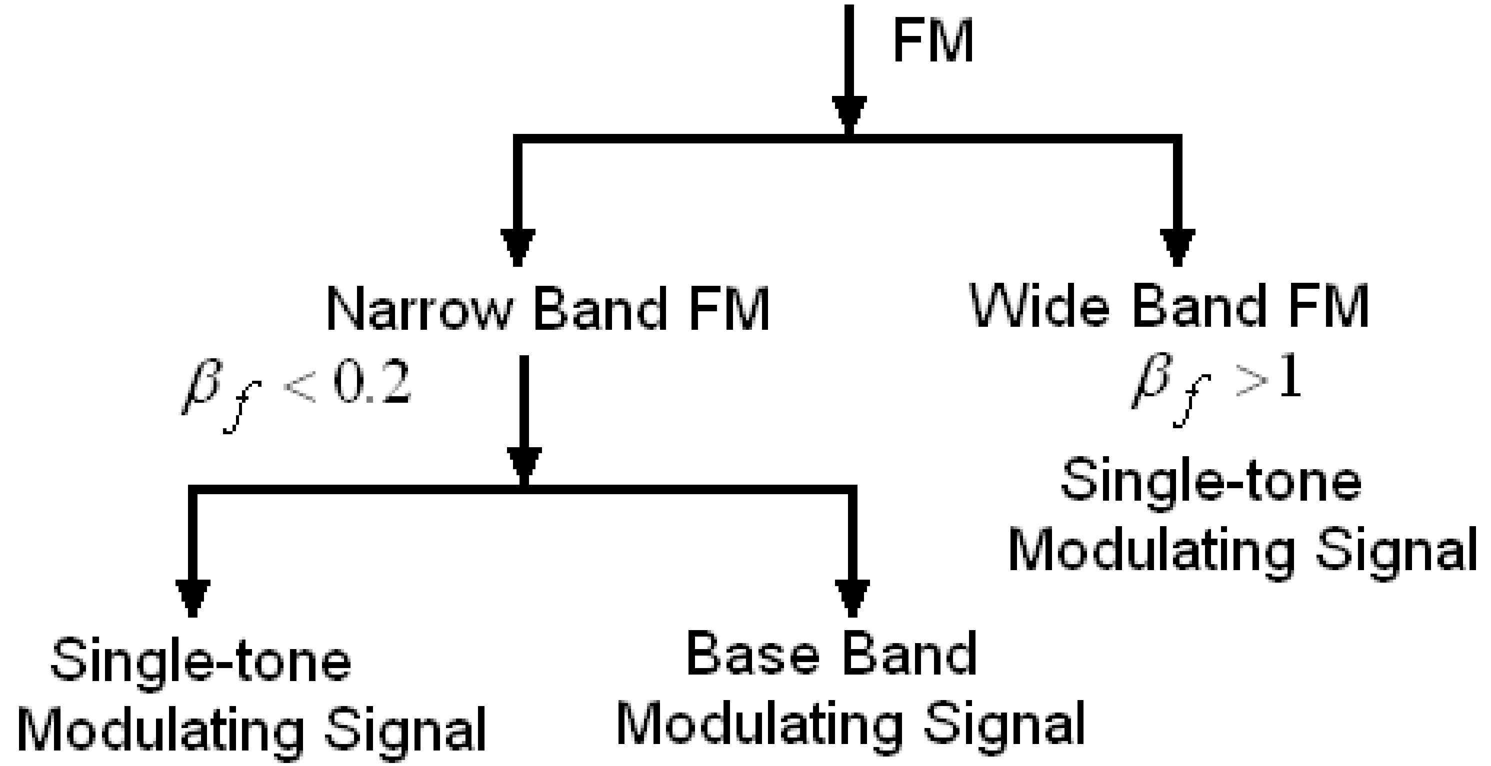
Phase deviation $\Delta\theta = k_p A_m$

Frequency Modulation: **F.M** : $A_c \cos\left[\omega_c t + \beta_f \cos 2\pi f_m t\right]$

Frequency Modulation index $\beta_f = \dfrac{k_f A_m}{f_m} = \dfrac{\Delta f}{f_m}$;

Frequency deviation $\Delta f = k_f A_m$

$$S_{FM}(t) = A_c \cos[2\Pi f_c t + \beta_f \sin(2\Pi f_m t)]$$



$$s_{PM}(t) = A_C \cos\left[\omega_C t + k_P m(t)\right]$$

$$s_{FM}(t) = A_C \cos\left[\omega_C t + 2\pi k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

Narrowband F.M

Narrowband F.M.

$$\beta_f << \pi / 2 \; (\beta_f < 0.2)$$

$$s_{FM}(t) = A_C \cos\left[\omega_C t + \beta_f \sin 2\pi f_m t\right]$$
$$= A_C\left[\cos\omega_C t \cdot \cos(\beta_f \sin 2\pi f_m t) - \sin\omega_C t \cdot \sin(\beta_f \sin 2\pi f_m t)\right]$$

For $\beta_f << \pi / 2$, $\cos(\beta_f \sin 2\pi f_m t) = 1$ and

$$\sin(\beta_f \sin 2\pi f_m t) = \beta_f \sin 2\pi f_m t \, .$$

Therefore, the narrowband FM is described by

$$s_{NBFM}(t) = A_C\left[\cos\omega_C t - \sin\omega_C t \cdot \beta_f \sin 2\pi f_m t\right]$$
$$= A_C \cos\omega_C t - A_C \beta_f \sin\omega_C t \cdot \sin 2\pi f_m t$$
$$= A_C \cos\omega_C t - \frac{A_C\beta_f}{2}\left[\cos(\omega_C - \omega_m) - \cos(\omega_C + \omega_m)\right]$$
$$= A_C \cos\omega_C t - \frac{A_C\beta_f}{2} \cos(\omega_C - \omega_m) + \frac{A_C\beta_f}{2}\cos(\omega_C + \omega_m)$$

$$s_{AM}(t) = A_C \cos\omega_C t + \frac{\mu A_C}{2}\cos(\omega_C - \omega_m)t + \frac{\mu A_C}{2}\cos(\omega_C + \omega_m)t$$

Frequency spectrum of NBFM

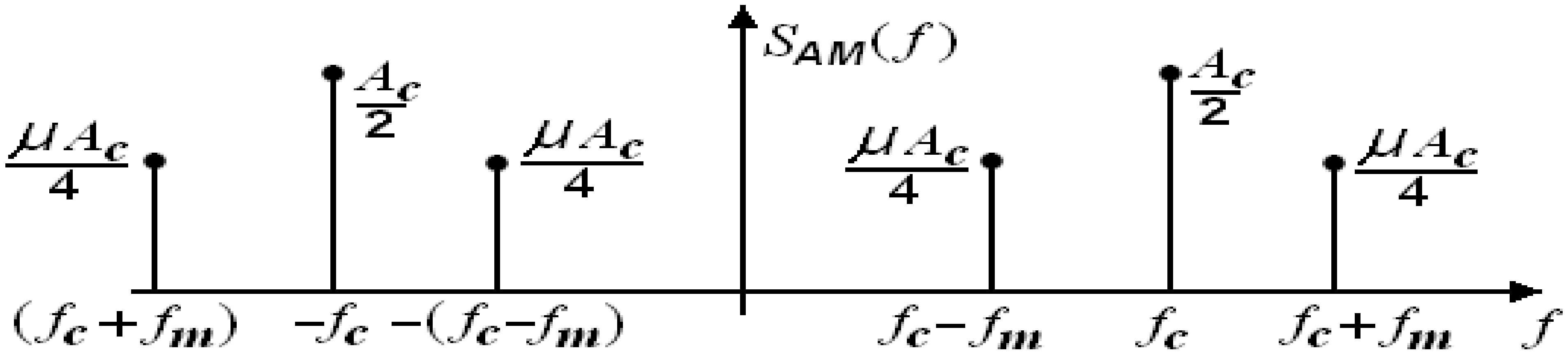
The time domain expression for NBFM is

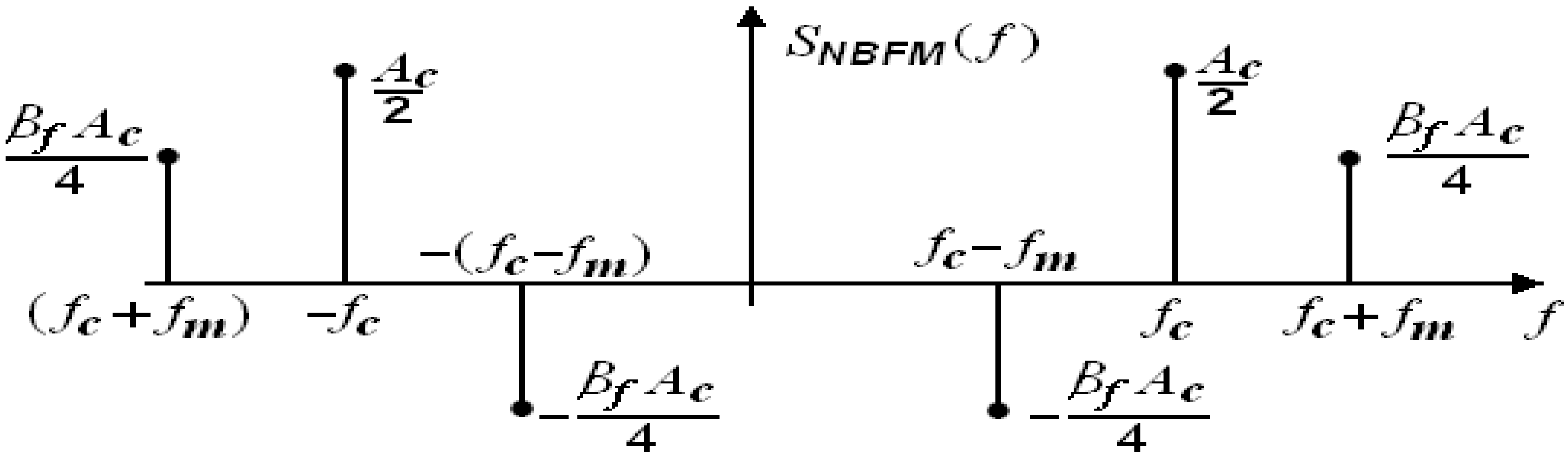
$$s_{NBFM}(t) = A_C \cos \omega_C t - \frac{A_C \beta_f}{2} \cos(\omega_C - \omega_m) + \frac{A_C \beta_f}{2} \cos(\omega_C + \omega_m)$$

The frequency spectrum of *Narrow Band Frequency Modulation* is represented by

$$S_{FM}(f) = \frac{A_C}{2}[\delta(f - f_C) + \delta(f + f_C)] - \frac{A_C \beta_f}{4}[\delta(f - f_C + f_m) + \delta(f + f_C - f_m)]$$
$$+ \frac{A_C \beta_f}{4}[\delta(f - f_C - f_m) + \delta(f + f_C + f_m)]$$

Spectrum of AM and NBFM





Bandwidth of NBFM

The time domain expression for NBFM is

$$s_{NBFM}(t) = A_C \cos\omega_C t - \frac{A_C\beta_f}{2}\cos(\omega_C - \omega_m) + \frac{A_C\beta_f}{2}\cos(\omega_C + \omega_m)$$

It resembles the AM signal and hence bandwidth is twice of the message signal frequency

BW= 2fm

Power of NBFM

$$s_{FM}(t) = A_C \cos\left[\omega_C t + \beta_f \sin 2\pi f_m t\right]$$
$$= A_C\left[\cos\omega_C t \cdot \cos(\beta_f \sin 2\pi f_m t) - \sin\omega_C t \cdot \sin(\beta_f \sin 2\pi f_m t)\right]$$

For $\beta_f << \pi / 2$, $\cos(\beta_f \sin 2\pi f_m t) = 1$ and

$\sin(\beta_f \sin 2\pi f_m t) = \beta_f \sin 2\pi f_m t$.

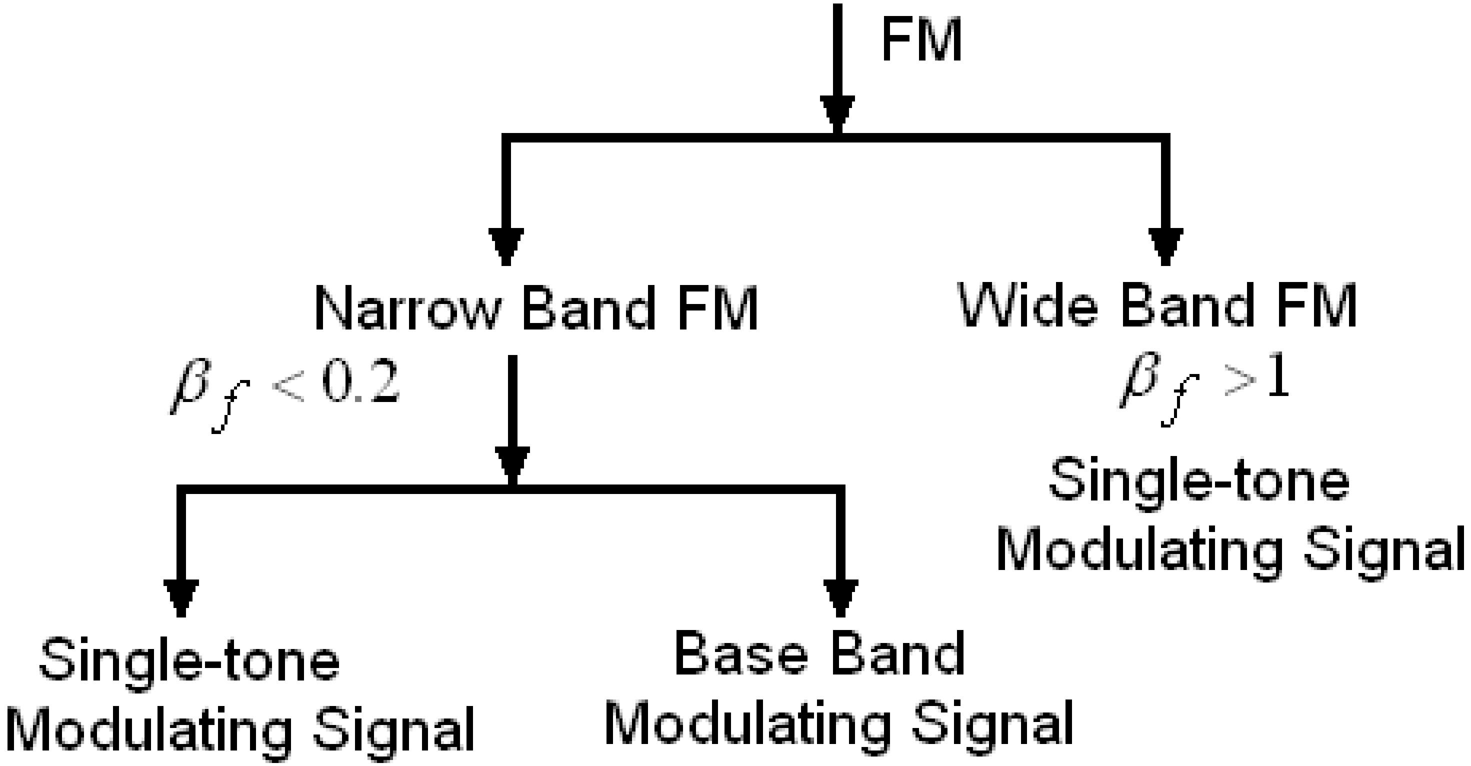
Therefore, the narrowband FM is described by

$$s_{NBFM}(t) = A_C\left[\cos\omega_C t - \sin\omega_C t \cdot \beta_f \sin 2\pi f_m t\right]$$
$$= A_C \cos\omega_C t - A_C \beta_f \sin\omega_C t \cdot \sin 2\pi f_m t$$
$$= A_C \cos\omega_C t - \frac{A_C \beta_f}{2}\left[\cos(\omega_C - \omega_m) - \cos(\omega_C + \omega_m)\right]$$
$$= A_C \cos\omega_C t - \frac{A_C \beta_f}{2} \cos(\omega_C - \omega_m) + \frac{A_C \beta_f}{2} \cos(\omega_C + \omega_m)$$

- Pt = Pc + Power of USB + Power of LSB   (as in AM)

- $\text{Pt} = \frac{Ac^2}{2} + \frac{\beta^2 Ac^2}{8} + \frac{\beta^2 Ac^2}{8}$
- $\text{Pt} = \frac{Ac^2}{2} + \frac{\beta^2 Ac^2}{4}$
- $\text{Pt} = Pc + \frac{Pc\beta^2}{2}$

- $Pt = Pc(1 + \frac{\beta^2}{2})$



$$s_{PM}(t) = A_C \cos\left[\omega_C t + k_p m(t)\right]$$

$$s_{FM}(t) = A_C \cos\left[\omega_C t + 2\pi k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

NBFM for Base Band Signal

$$s_{FM}(t) = A_c \cos[\omega_c t + k_f \int_{-\infty}^{t} m(t)\,dt]$$

$$= A_c \cos\omega_c t \cdot \cos\left\{k_f \int_{-\infty}^{t} m(t)\,dt\right\}$$

$$- A_c \sin\omega_c t \cdot \sin\left\{k_f \int_{-\infty}^{t} m(t)\,dt\right\}$$

For $|k_f \int_{-\infty}^{t} m(t)\,dt| << 1$

$$\cos\left\{k_f \int_{-\infty}^{t} m(t)\ dt\right\} \simeq 1$$ **and** $$\sin\left\{k_f \int_{-\infty}^{t} m(t)\ dt\right\} \simeq k_f \int_{-\infty}^{t} m(t)\ dt$$

$$S_{NBFM}(t) = A_c\left[\cos\omega_c\,t - k_f\left(\int_{-\infty}^{t} m(t)\ dt\right)\ \sin\omega_c\,t\right]$$

$$s_{NBFM}(t) = A_c \cos\omega_c t - k_f A_c m'(t) \sin\omega_c t$$ **where** $m'(t) = \int_{-\infty}^{t} m(t)\,dt$

$$S_{NBFM}(f) = \frac{A_c}{2}[\delta(f-f_c)+\delta(f+f_c)] + \frac{A_c}{2}k_f\ \frac{[M'(f-f_c)+M'(f+f_c)]}{2j}$$

NBPM for Base Band Signal

$$s_{PM}(t) = A_c \cos[\omega_c t + k_p m(t)]$$

$$s_{PM}(t) = A_c \left[\cos\omega_c t \cdot \cos\left\{k_p m(t)\right\} - \sin\omega_c t \cdot \sin\left\{k_p m(t)\right\} \right]$$

For $|k_p m(t)| << 1$ **,** $\cos\{|k_p m(t)|\} \simeq 1$ **and** $\sin\{|k_p m(t)|1\} \simeq k_p m(t)$

$$s_{NBPM}(t) = A_c \left[\cos\omega_c t - k_p m(t) \sin\omega_c t \right]$$

$$= A_c \cos\omega_c t - A_c k_p m(t) \sin\omega_c t$$

$$S_{NBPM}(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c}{2} k_p \frac{[M'(f - f_c) + M'(f + f_c)]}{2j}$$

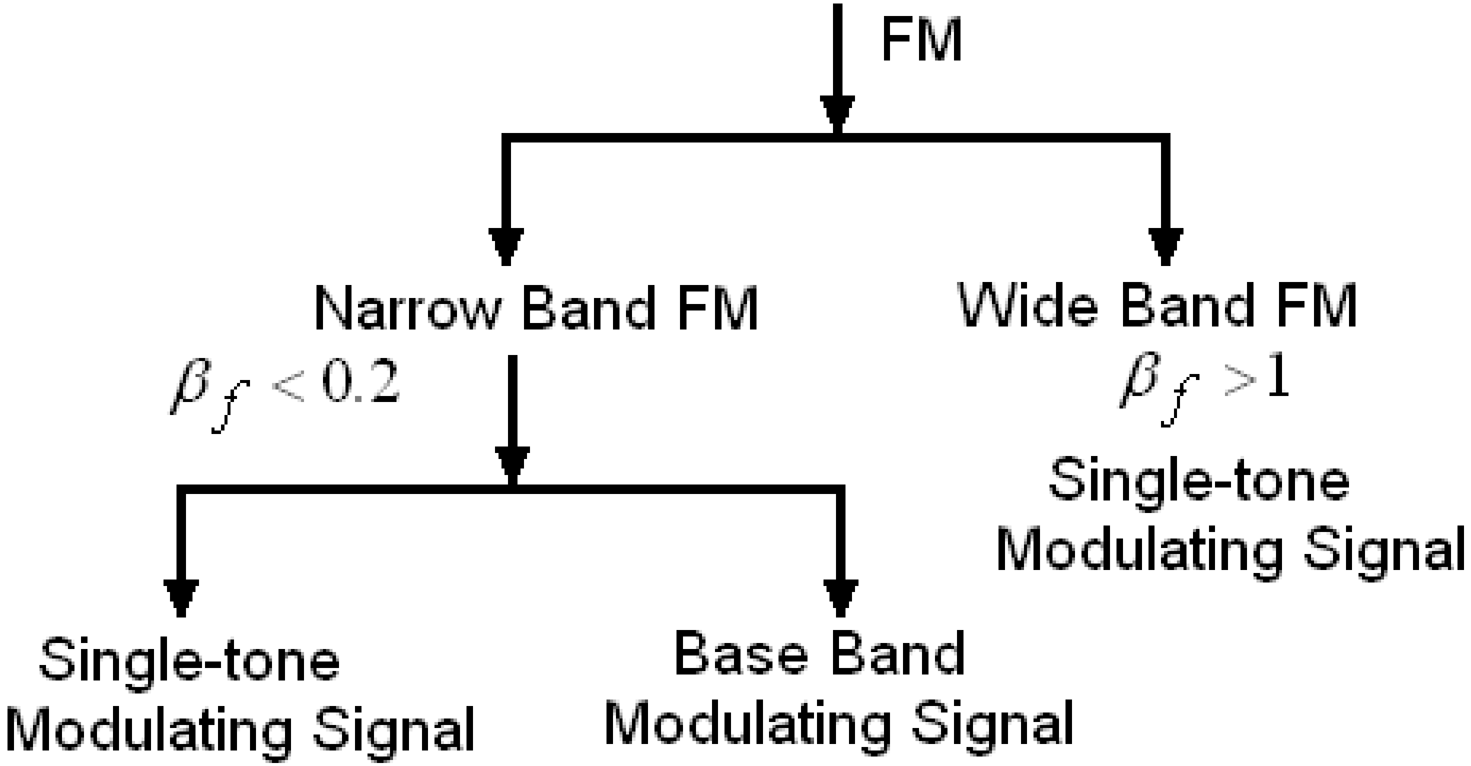
Comparison

Similarities:

- Both have the same modulated bandwidth $2W$, where W is the highest modulating signal frequency.
- The sideband spectrum for FM has a phase shift of $\mathbf{90^0}$ with respect to the carrier, whereas that of AM is in-phase with the carrier.

Differences:

- In an AM signal, the oscillation frequency is constant, and the amplitude varies with time, whereas in an FM signal, the amplitude stays constant, and frequency varies with time.



$$s_{PM}(t) = A_C \cos\left[\omega_C t + k_p m(t)\right]$$

$$s_{FM}(t) = A_C \cos\left[\omega_C t + 2\pi k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

Wideband FM

$$\beta_f > 1$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta_f \sin(2\pi f_m t)\right]$$

The above Eq. can be rewritten as

$$s_{FM}(t) = \mathrm{Re}\{A_c e^{j2\Pi f_c t} \quad e^{j\beta_f \sin(2\,\Pi\, f_m t)}\}$$

For simplicity, the modulation index of FM has been considered as β instead of β_f. Since $\sin(2\,\Pi\, f_m t)$ is a periodic signal with fundamental period $T = 1/f_m$, the complex expontial $e^{j\beta_f \sin(2\,\Pi\, f_m t)}$ is also periodic with the same fundamental period. Therefore, this complex exponential can be expanded in Fourier series representation as $e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta_f \sin(2\pi f_m t)\right]$$

The above Eq. can be rewritten as

$$s_{FM}(t) = \mathrm{Re}\{A_c e^{j2\Pi f_c t} e^{j\beta_f \sin(2\Pi f_m t)}\}$$

$$S_{FM}(t) = A_c \mathrm{Re}\{\acute{S}(t)\, e^{j2\Pi f_c t}\}$$

Modulation index of FM has been considered as β instead of β_f.

Since $\sin(2\,\Pi\, f_m t)$ is a periodic signal with fundamental period $T = 1/f_m$, the complex expontial $e^{j\beta_f \sin(2\,\Pi\, f_m t)}$ is also periodic with the same fundamental period.

Therefore, this complex exponential can be expanded in Fourier series representation as

$$\acute{S}(t) = e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} e^{j\beta \sin(2\pi f_m t)} e^{-j2\pi n f_m t} \, dt$$

Where $T = 1/f_m$

In the Above Equation Substitute Let $2\pi f_m t = x$.

$$t = \frac{x}{2\Pi fm}; \quad dt = \frac{dx}{2\Pi fm}$$

$$if\ 't'\ is\ -\frac{T}{2} \qquad x = -\Pi$$

$$if\ 't'\ is\ +\frac{T}{2} \qquad x = \Pi$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin(x)} e^{-jnx} dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin(x) - nx)} dx$$

The integral on the right-hand side is known as the n[th] order Bessel function of the first kind and is denoted by $J_n(\beta)$. Therefore, $c_n = J_n(\beta)$ and Eq. (4.23) can be written as

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} \tag{5.26}$$

By substituting

$$s_{FM}(t) = \mathrm{Re}\left\{A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}\, e^{j2\pi f_c t}\right\}$$
$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

Taking Fourier transform of Eq. (5.27), we get

$$S(f) = \frac{1}{2} A_c \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)]$$

From the spectral analysis we see that there is a carrier component and a number of side-frequencies around the carrier frequency at $\pm n f_m$.

$$s_{FM}(t) = A_C \sum_{n=-\infty}^{\infty} J_n(\beta_f)\cos(\omega_C + nf_m)t$$

Spectrum of WBFM

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta_f)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$

The carrier with amplitude $\frac{A_C}{2} J_0(\beta_f)$

Wideband FM…

$$s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta_f) \cos 2\Pi (f_c + n f_m) t \;\; ;$$

$$S_{FM}(f) = \frac{A_C}{2} \sum_{n=-\infty}^{\infty} J_n(\beta_f) \left[\delta(f - f_C t - n f_m) + \delta(f + f_C t + n f_m) \right]$$

The carrier with amplitude = $\frac{A_C}{2} J_0(\beta_f)$

$$\begin{aligned} s_{FM}(t) = A_c [& J_0(\beta_f) \cos 2\Pi f_c\, t \\ & - J_1(\beta_f)\{\cos 2\Pi (f_c - f_m) - \cos 2\Pi (f_c + f_m)\} \\ & + J_2(\beta_f)\{\cos 2\Pi (f_c - 2f_m) - \cos 2\Pi (f_c + 2f_m)\} \\ & - J_3(\beta_f)\{\cos 2\Pi (f_c - 3f_m) - \cos 2\Pi (f_c + 3f_m)\} \\ & + \ldots\ldots\ldots] \end{aligned}$$

- $J_0(\beta_f)$ is Carrier Component
- $J_1(\beta_f)$ First Set of Side Frequencies
- $J_2(\beta_f)$ Second Set of Side Frequencies
- $J_3(\beta_f)$ Third Set of Side Frequencies and So on……….

The carrier with amplitude $\frac{A_c}{2} J_0(\beta_f)$

A set of side frequencies specified simultaneously on the either side of the carrier at a frequency separation of

$$f_m, 2f_m, \ldots, nf_m$$

Properties of Bessel Functions

- 1) $$J_n(\beta_f) = \begin{cases} J_{-n}(\beta_f) & for\ n - even \\ -J_{-n}(\beta_f), & for\ n - odd \end{cases}$$

- 2) $$J_n(\beta_f) = \frac{\beta^n}{2^n n!}$$

$for\ small\ Values\ of\ \beta$

$J_0(\beta_f) \sim 1$

$J_1(\beta_f) \sim \beta/2$

$J_n(\beta_f) \sim 0$

- 3) $\sum_{n=-\infty}^{\infty} J_n^2(\beta_f) = 1$

The Bessel functions

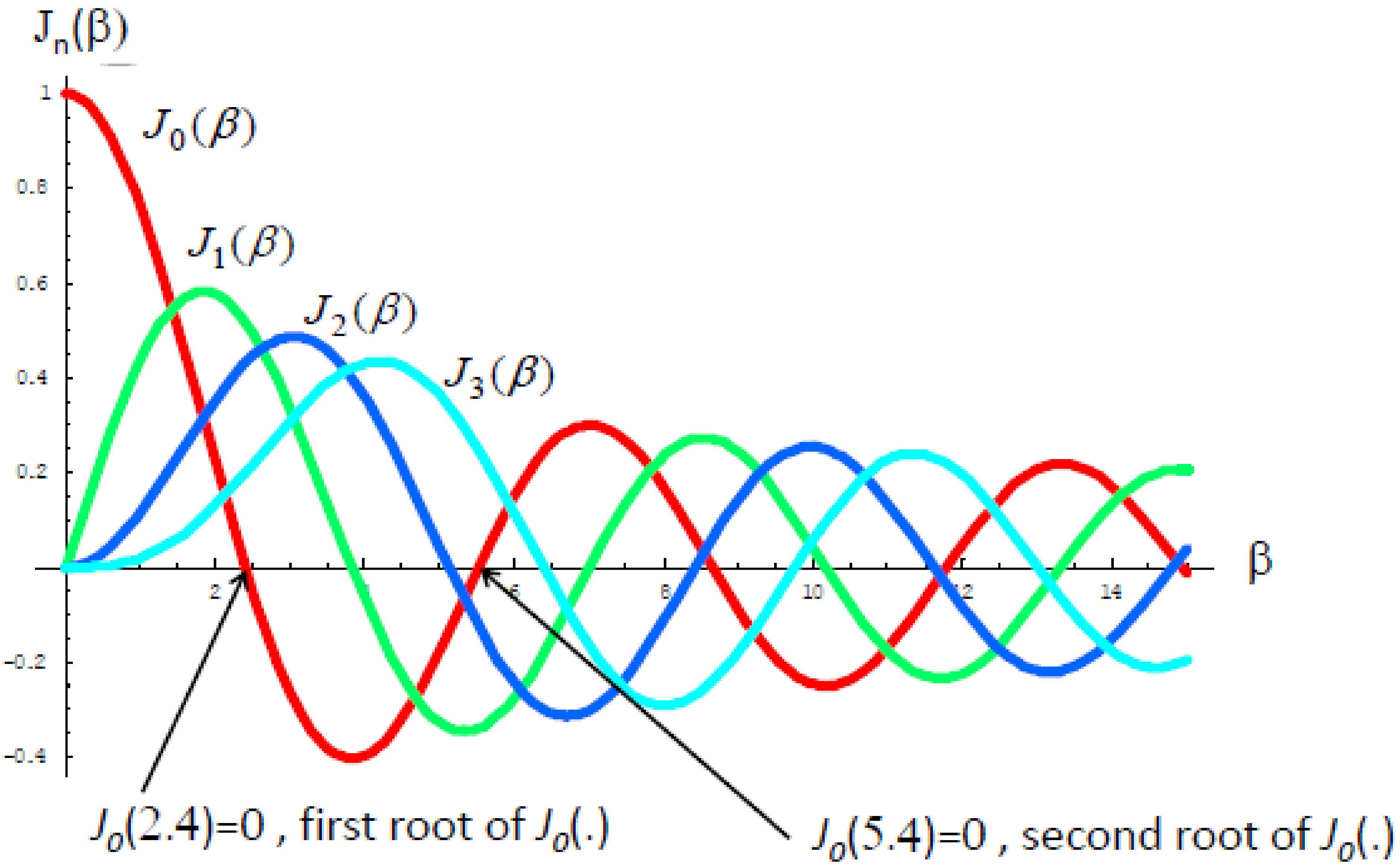


Table of Bessel functions of the first kind

Modulation index	Carrier	Side frequency pairs								
β_f	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
0.00	1.00	---	---	---	---	---	---	---	---	---
0.25	0.98	0.12	---	---	---	---	---	---	---	---
0.5	0.94	0.24	0.03	---	---	---	---	---	---	---
1.0	0.77	0.44	0.11	0.02	---	---	---	---	---	---
1.5	0.51	0.56	0.23	0.06	0.01	---	---	---	---	---
2.0	0.22	0.58	0.35	0.13	0.03	---	---	---	---	---
2.4	0	0.52	0.43	0.20	0.06	0.02	---	---	---	---
2.5	-0.05	0.50	0.45	0.22	0.07	0.02	0.01	---	---	---
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	---	---	---
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	---	---
5.0	-0.18	-0.13	0.05	0.36	0.39	0.26	0.13	0.05	0.02	---
5.45	0	-0.34	-0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta_f)[\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m))]$$

n=0 $S_{FM}(f) = \frac{A_c}{2} \; J_0(\beta_f)[\delta(f-f_c) + \delta(f+f_c)]$

n=1 $S_{FM}(f) = \frac{A_c}{2} \; J_1(\beta_f)[\delta(f-(f_c+f_m)) + \delta\,(f+(f_c+f_m))]$

n=-1 $S_{FM}(f) \; = \; \frac{A_c}{2} \; J_{-1}(\beta_f)[\delta(f-(f_c-f_m)) + \delta\,(f+(f_c-f_m))]$

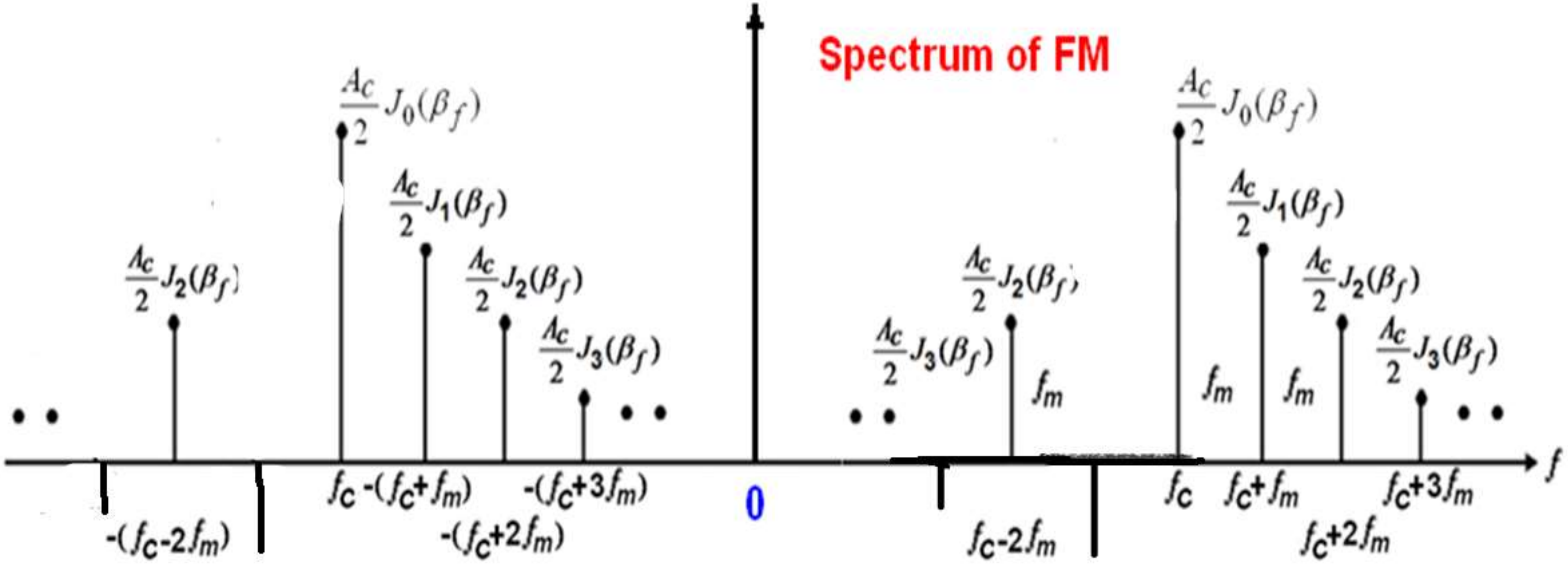
$= -\frac{A_c}{2} \; J_1(\beta_f)[\delta(f-(f_c-f_m)) + \delta\,(f+(f_c-f_m))]$

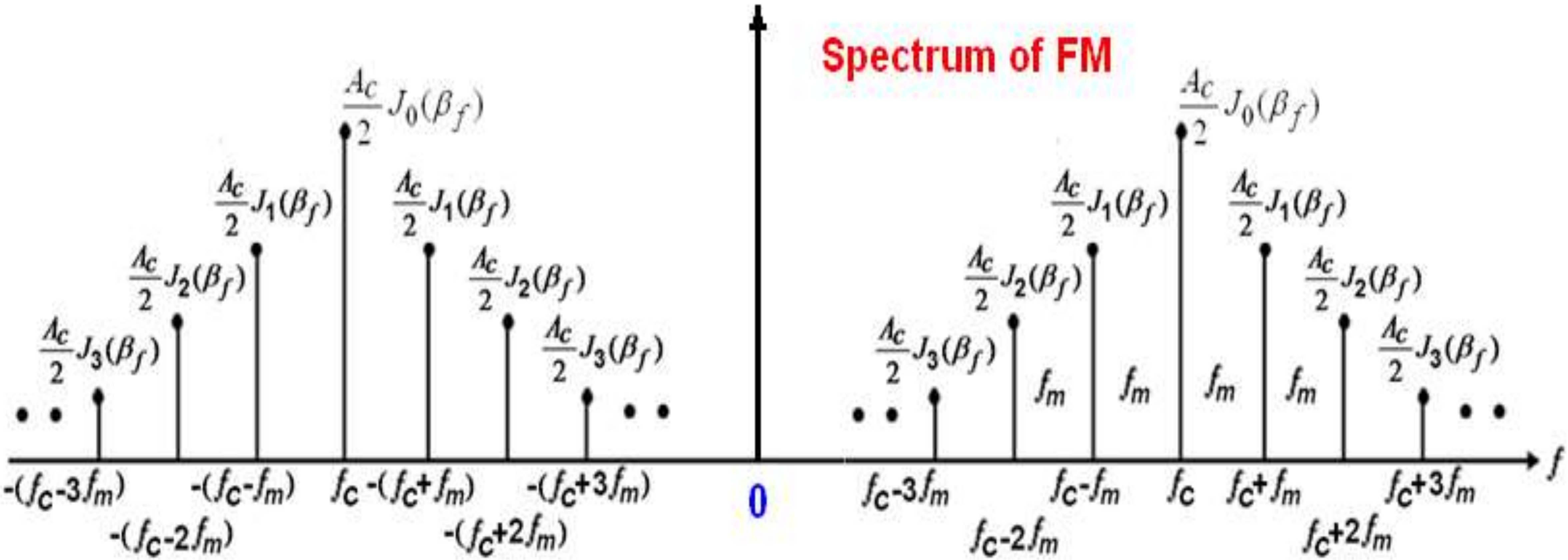
n=2 $S_{FM}(f) = \frac{A_c}{2} \; J_2(\beta_f)[\delta(f-(f_c+2f_m)) + \delta\,(f+(f_c+2f_m))]$

n=-2 $S_{FM}(f) = \frac{A_c}{2} \; J_{-2}\left(\beta_f\right)[\delta(f-(f_c-2f_m)) + \delta\,(f+(f_c-2f_m))]$

$= \frac{A_c}{2} \; J_2(\beta_f)[\delta(f-(f_c-2f_m)) + \delta\,(f+(f_c-2f_m))]$

and So On……………….


Spectrum of FM
$\frac{A_c}{2}J_0(\beta_f)$
$\frac{A_c}{2}J_1(\beta_f)$
$\frac{A_c}{2}J_2(\beta_f)$
$\frac{A_c}{2}J_3(\beta_f)$
f_m
f
0
$-(f_c-2f_m)$
f_c
$-(f_c+f_m)$
$-(f_c+2f_m)$
$-(f_c+3f_m)$
f_c-2f_m
f_c+f_m
f_c+2f_m
f_c+3f_m



Spectrum of FM
$\frac{A_c}{2}J_0(\beta_f)$
$\frac{A_c}{2}J_1(\beta_f)$
$\frac{A_c}{2}J_2(\beta_f)$
$\frac{A_c}{2}J_3(\beta_f)$
f_m
$-(f_c-3f_m)$
$-(f_c-2f_m)$
$-(f_c-f_m)$
f_c
$-(f_c+f_m)$
$-(f_c+2f_m)$
$-(f_c+3f_m)$
0
f_c-3f_m
f_c-2f_m
f_c-f_m
f_c
f_c+f_m
f_c+2f_m
f_c+3f_m
f

Spectrum Analysis

1) Spectrum consists of Carrier and Infinite No. of Side bands.

2) Theoretical BW of FM is Infinite.

3) The Spacing between the spectral components is equal to ‘fm’ message frequency.

4) In the Spectrum (fc+fm) and (fc-fm) are called as 1st order side bands (fc+2fm) and (fc-2fm) are called as 2nd order side bands and so on…. Hence Spectrum contains infinite order of side bands.

5) The Magnitude of spectrum components depends on the Bessel function values. But these values gradually decreases as ‘n’ increases. So, the magnitude of higher order frequencies are negligible.

Spectrum Analysis contd…..

6) The Carrier magnitude in the spectrum depends on the modulation index.

The Bessel function $J_0(\beta_f) = 0$ for $\beta = 2.4, 5.5, 8.6, 11.8 \ldots \ldots$

7) For the Above Values of $'\beta'$ the carrier magnitude in the spectrum is Zero and the efficiency is 100%

i.e. Carrier power is suppressed and the Total power is given to side bands only.

$$\beta_f = \frac{\Delta f}{f_m} = \frac{kf\,Am}{f_m}$$

Case-1 : Case 1: Fix f_m and vary A_m

Let Kf = 1Khz/v of the Modulator & fm = 1Khz

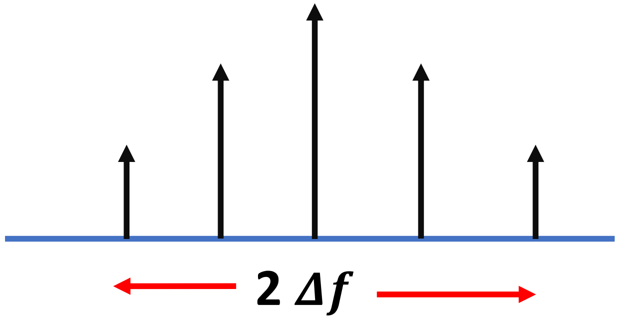
If Am= 1v $\beta_f = \frac{\Delta f}{f_m} = \frac{\frac{1Khz}{v} * 1v}{1Khz} = 1$ **; 0.77(fc), 0.44(fc+fm), 0.11(fc+2fm)**

-

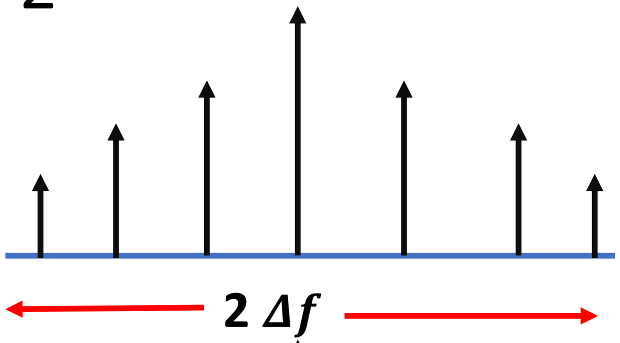
If Am= 2v $\beta_f = \frac{\Delta f}{f_m} = \frac{\frac{1Khz}{v} * 2v}{1Khz} = 2$ **; 0.22(fc), 0.57(fc $\pm$ fm), 0.35(fc $\pm$ 2fm), 0.12(fc $\pm$ 3fm)**

If Am= 3v $\beta_f = \frac{\Delta f}{f_m} = \frac{\frac{1Khz}{v} * 3v}{1Khz} = 3$ **; fc, (fc $\pm$ fm), (fc $\pm$ 2fm), (fc $\pm$ 3fm), (fc $\pm$ 4fm), (fc $\pm$ 5fm),**

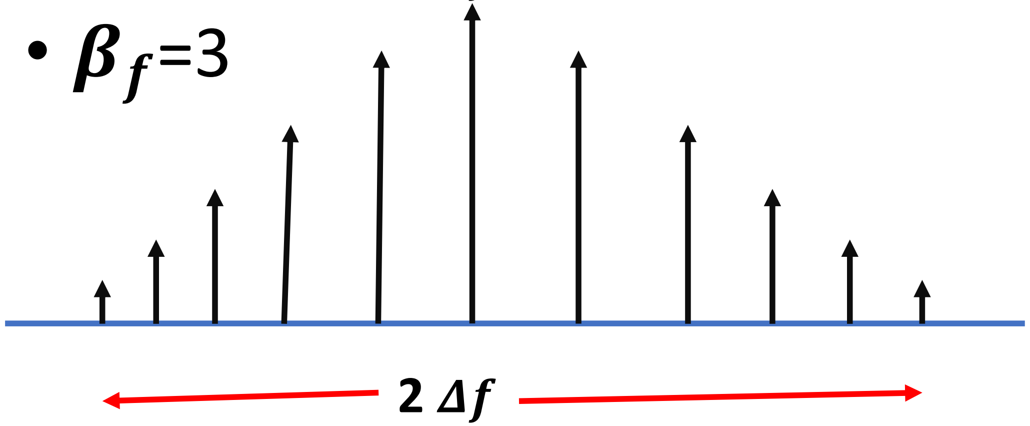
- β_f=1



- β_f=2



- β_f=3



- **Case-2 :** Fix A_m and vary f_m

$$\boldsymbol{\beta_f = \frac{\Delta f}{f_m} = \frac{kfAm}{f_m}}$$

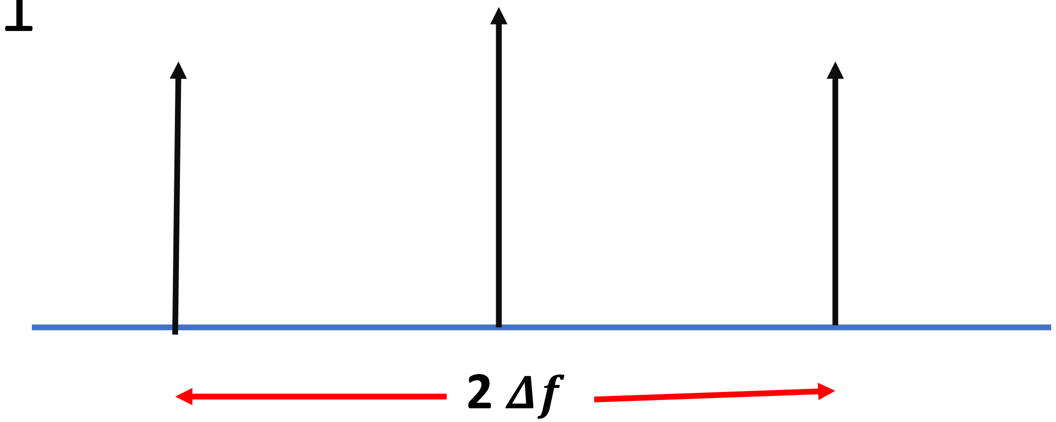
- **Let Kf = 1Khz/v of the Modulator & fm = 1Khz**

- **If** $\boldsymbol{\beta_f = 1}$ $\boldsymbol{f_m = \Delta f}$ **; (fc), (fc+fm),**

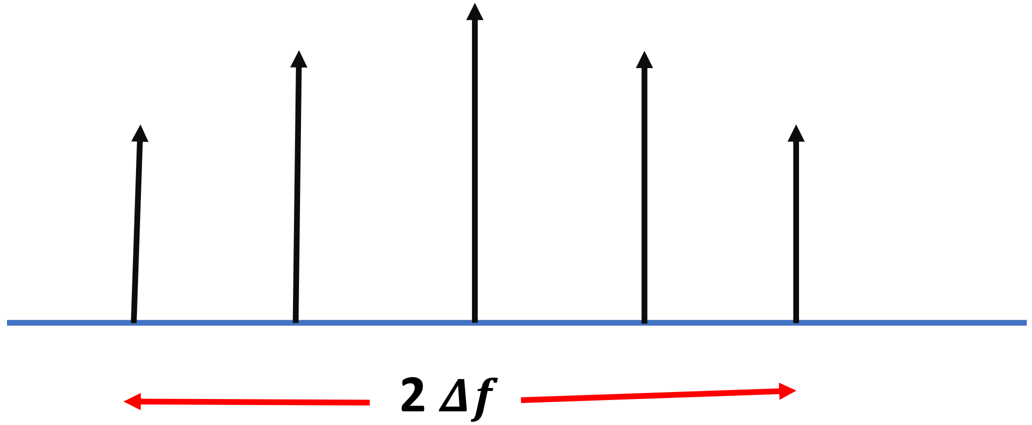
- **If** $\boldsymbol{\beta_f = 2}$ $\boldsymbol{2 = \frac{\Delta f}{f_m}}$ $\boldsymbol{(\frac{\Delta f}{2} = f_m)}$ **; (fc), (fc** $\pm$ **500Hz), (fc** $\pm$ **1Khz)**

- **If** $\boldsymbol{\beta_f = 5}$ $\boldsymbol{5 = \frac{\Delta f}{f_m}}$ $\boldsymbol{(\frac{\Delta f}{5} = f_m)}$ **; fc, (fc** $\pm$ **200Hz), (fc** $\pm$ **400Hz), (fc** $\pm$ **600Hz), (fc** $\pm$ **800Hz), (fc** $\pm$ **1KHz),**

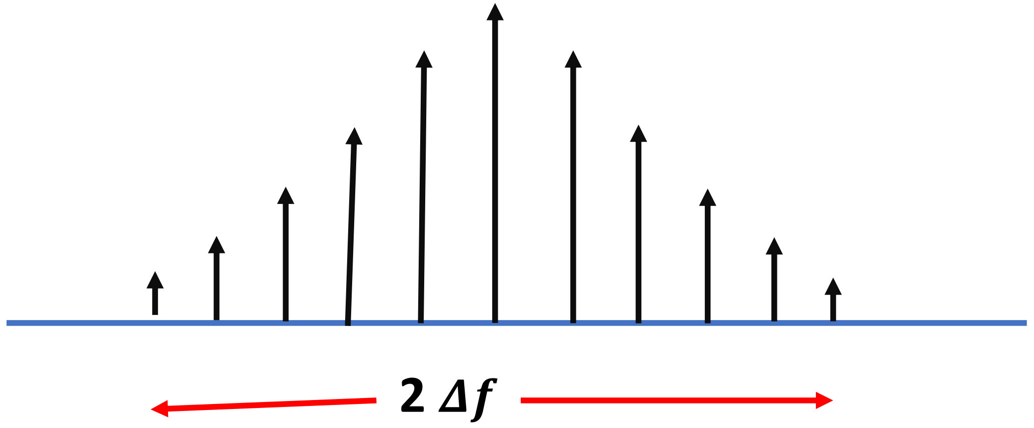
- β_f=1



- β_f=2



- β_f=5



Carson's Rule

The Carson rule state that the approximate bandwidth necessary to transmit an angle modulated wave as twice the sum of the frequency deviation and the highest modulating signal frequency.

$$\text{B.W}_{FM} = 2nf_m \quad \text{n}= 1+ \beta_f$$

$$\text{B.W}_{FM} = 2f_m\left(1 + \beta_f\right) = 2\ (\Delta f + f_m) = 2\Delta f\,(1 + \frac{1}{\beta_f})$$

Approximates the 98% of the Total power & Carson's BW is less than actual Tx BW

Universal Curve for FM Transmission Bandwidth

Carson's rule is simple but unfortunately it does not always provide a good estimate of the transmission bandwidth, in particular, for the wideband frequency modulation.

TABLE 4.2 *Number of Significant Side-Frequencies of a Wide-Band FM Signal for Varying Modulation Index*

Modulation Index β	*Number of Significant Side-Frequencies* $2n_{max}$
0.1	2
0.3	4
0.5	4
1.0	6
2.0	8
5.0	16
10.0	28
20.0	50
30.0	70

Total Power of WBFM

- $s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta_f) \cos 2\Pi(f_c + nf_m)t$;

$$\text{Pt} = \left(\frac{A_c \sum_{n=-\infty}^{\infty} J_n(\beta_f)}{\sqrt{2}}\right)^2$$

$$\text{Pt} = \frac{Ac^2}{2R} {J_0}^2 \left(\beta_f\right) + \frac{Ac^2}{2R} {J_1}^2 \left(\beta_f\right) + \frac{Ac^2}{2R} {J_1}^2 \left(\beta_f\right) + \frac{Ac^2}{2R} {J_2}^2 \left(\beta_f\right) + \frac{Ac^2}{2R} {J_2}^2 \left(\beta_f\right) + \cdots$$

$$\text{Pt} = \frac{Ac^2}{2R} {J_0}^2 \left(\beta_f\right) + 2 * \frac{Ac^2}{2R} {J_1}^2 \left(\beta_f\right) + 2 * \frac{Ac^2}{2R} {J_2}^2 \left(\beta_f\right) + \cdots$$

Carrier Power **1^{st} Order Side Bands Power** **2^{nd} Order Side Bands Power**

Pt = $\frac{Ac^2}{2R}\left[J_0{}^2\left(\beta_f\right)+J_1{}^2\left(\beta_f\right)+J_1{}^2\left(\beta_f\right)+J_2{}^2\left(\beta_f\right)+J_2{}^2\left(\beta_f\right)+\cdots\right]$

- Pt = $\frac{Ac^2}{2R}\ \left(\sum_{n=-\infty}^{\infty} J_n{}^2\left(\beta_f\right)\right)$

- Pt = $\frac{Ac^2}{2R}$

- Note:

1) Same as Unmodulated Carrier Power
2) **Independent of Modulation Index** But in AM & DSB-SC, SSB-Sc & NBFM it is **Dependent** on Modulation Index

Observations

- In AM: Carrier and first two sidebands.

  In WBFM: Carrier and infinite number of sidebands.

- The *J* coefficients represent the amplitude of a particular pair of sidebands.
  Thus, the modulation index determine the number of sideband components have significant amplitudes.

- As No. of sidebands increases

$$\beta_f = \frac{\Delta f}{fm}, \quad fm \downarrow \Rightarrow \beta_f \uparrow$$

- In AM $\quad \mu \uparrow \Rightarrow P_{SB} \uparrow \Rightarrow, P_t \uparrow$

  but in FM as $\beta_f \uparrow$ Total power is constant, but B.W $\uparrow$

Observations…

- In AM, the BW = 2 f_m , but in FM the BW is determined by f_m and β_f
- In FM, the amplitude of the carrier component is constant, whereas in AM, the carrier component does not remain constant. The J coefficients J_0 is a function of β_f. The overall amplitude of the FM wave remains constant.
- The carrier component of the FM wave disappear completely at β_f = 2.4, 5.5, 8.6, 11.8 etc.

First carrier null, second carrier null . . .

Band Width of a Sinusoidally Modulated FM Signal

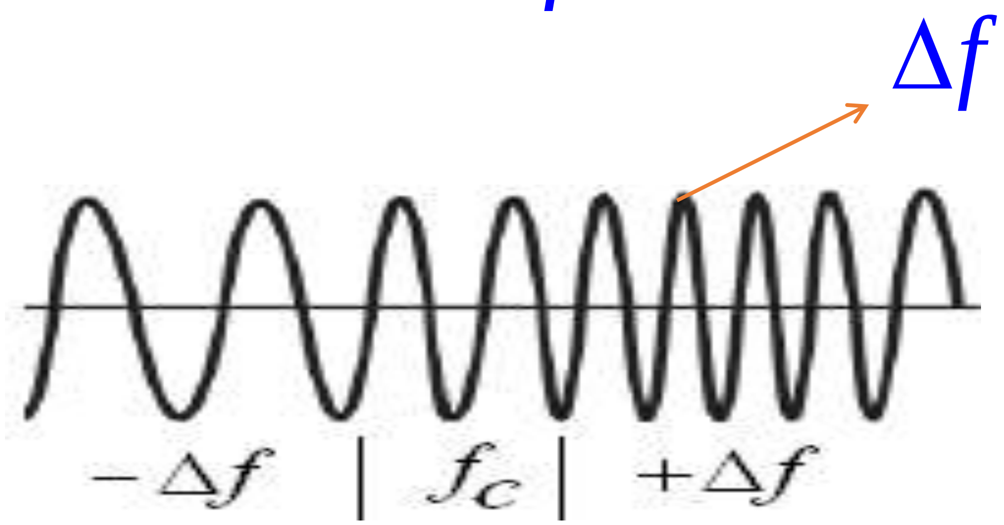
NBFM: B.W = $2f_m$

WBFM: The bandwidth of FM wave depends on both *modulating frequency* and *modulation index.*

$$\beta_f = \frac{\Delta f}{fm}, \quad fm \downarrow \Rightarrow \beta_f \uparrow$$ No. of sidebands increases

hence BW increases.

The BW depends on . BW = 2 Δf Hz



Effect of the Modulation Index on Bandwidth

$$\beta_f = \frac{\Delta f}{fm}$$

$$\beta_f < 0.2 \qquad \mathrm{B.W}_{NBFM} = 2\,f_m$$

$$\beta_f > 1 \qquad \mathrm{BW} = 2\Delta f = 2\beta_f\, fm$$

$$\mathrm{B.W} = 2(nf_m)$$

The Carson's rule

$$\mathrm{B.W}_{FM} = 2f_m\left(1+\beta_f\right) = 2\,(\Delta f + f_m) = 2\Delta f\,(1+\frac{1}{\beta_f})$$

Generation of NBFM

$$s_{FM}(t) = A_C \cos\left[\omega_C t + \beta_f \sin 2\pi f_m t\right]$$

$$= A_C\left[\cos\omega_C t \cdot \cos(\beta_f \sin 2\pi f_m t) - \sin\omega_C t \cdot \sin(\beta_f \sin 2\pi f_m t)\right]$$

For $\beta_f << \pi/2$, $\cos(\beta_f \sin 2\pi f_m t) = 1$ and

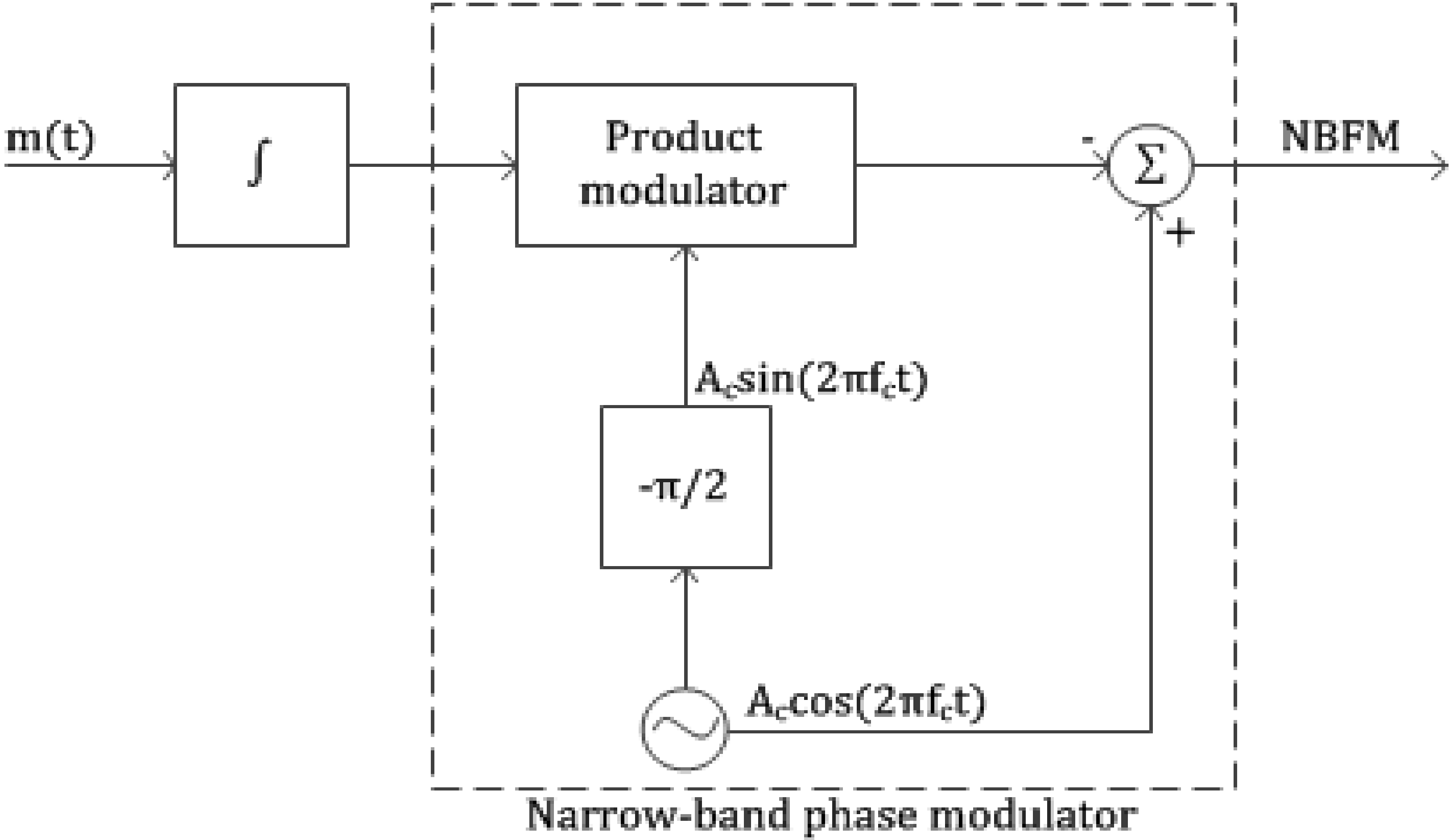
$$\sin(\beta_f \sin 2\pi f_m t) = \beta_f \sin 2\pi f_m t .$$

Therefore, the narrowband FM is described by

$$s_{NBFM}(t) = A_C\left[\cos\omega_C t - \sin\omega_C t \cdot \beta_f \sin 2\pi f_m t\right]$$

$$= A_C \cos\omega_C t - A_C \beta_f \sin\omega_C t \cdot \sin 2\pi f_m t$$

Block Diagram for generation of NBFM



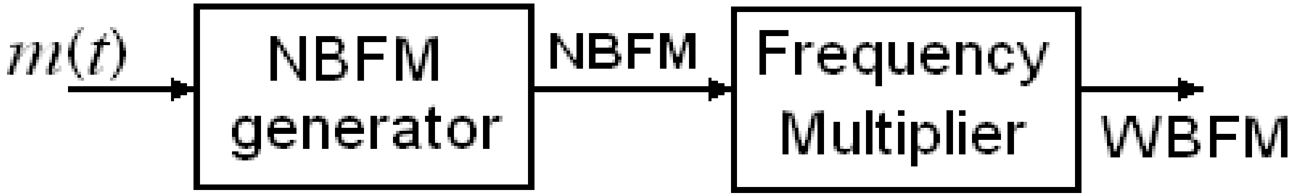
Generation of Wide Band FM

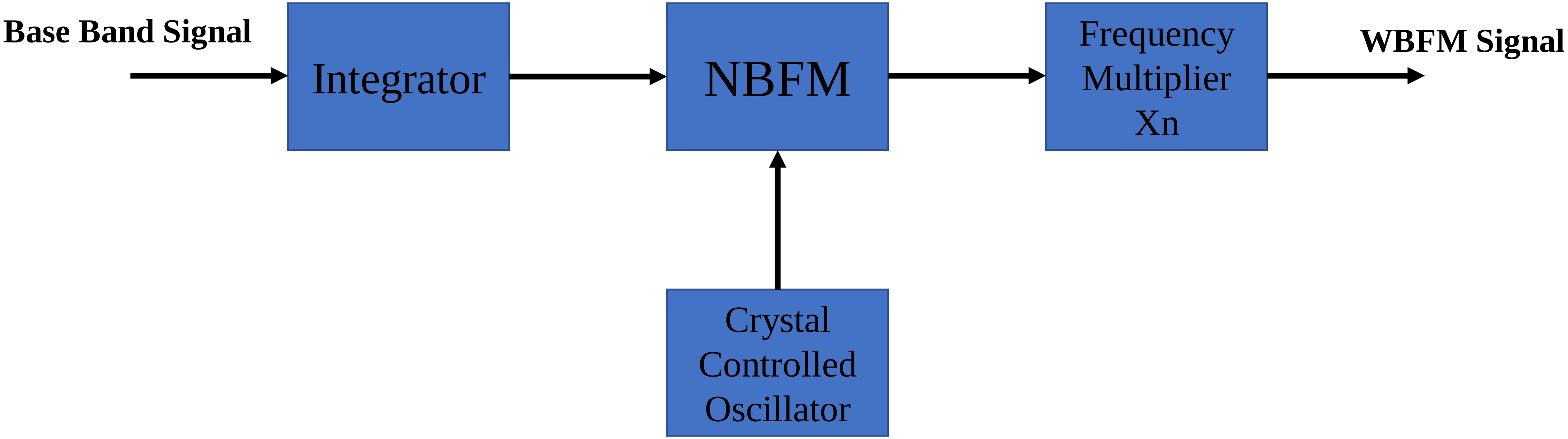
- Direct Method {Not there in Syllabus}
- Indirect Method (or) Armstrong Method

Detection of FM

- Single Slope
- Balanced Slope
- Phase Locked Loop(PLL)***

Indirect Method





$$s_1(t) = A_c \cos\left[2\pi f_1 t + 2\pi k_f \int_0^t m(t)\ dt\right]$$

Output of NBFM

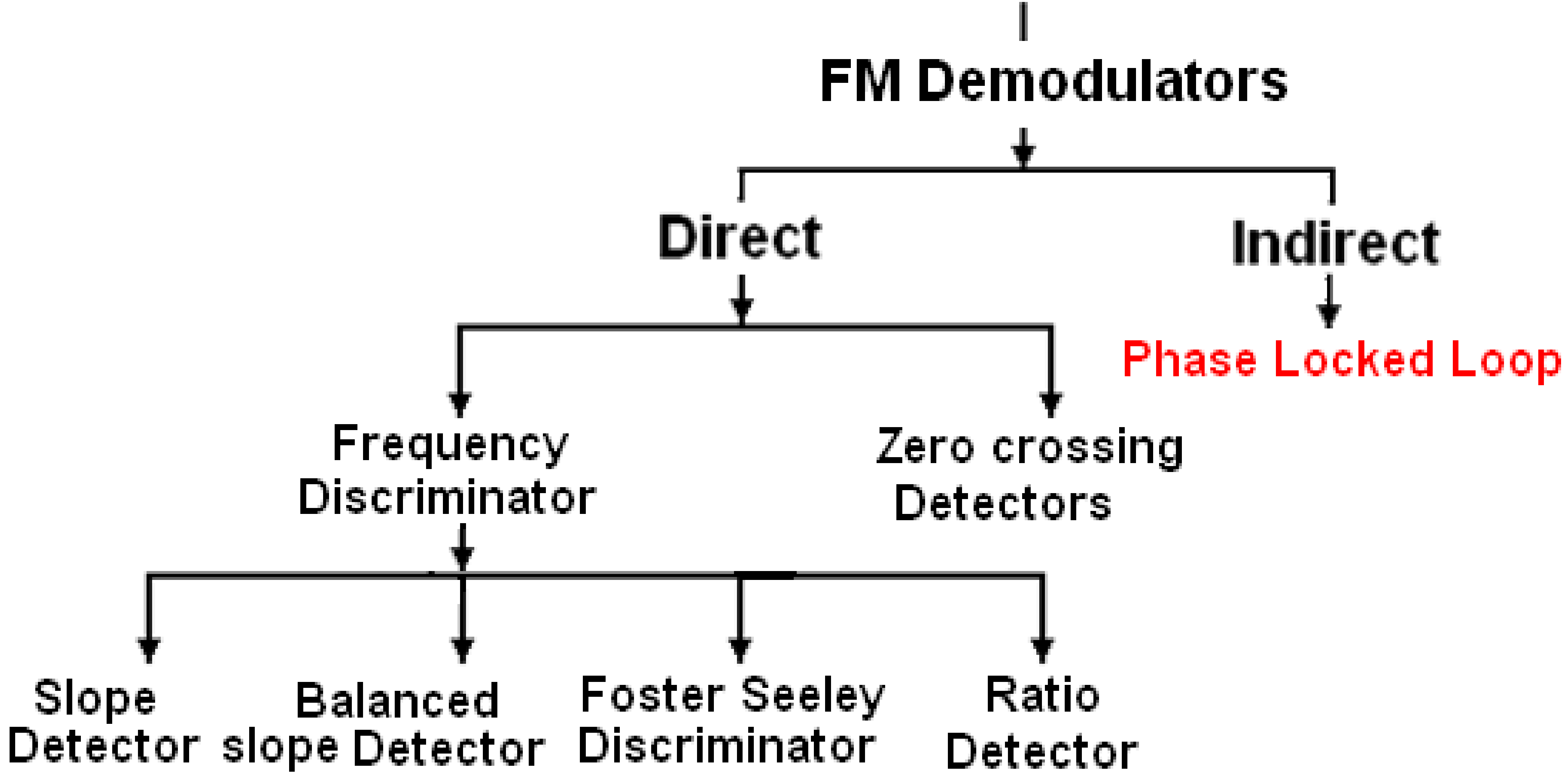
$$s_1(t) = A_c \cos[2\pi f_1 t + \beta_f \sin(2\Pi f_m t)] \qquad (\beta f < 0.3)$$

Output of Frequency Multiplier

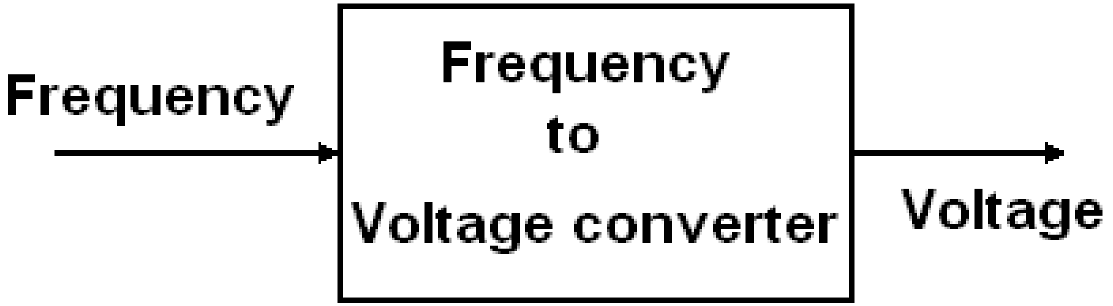
$$s_1(t) = A_c \cos[2\pi n f_1 t + n\beta_f \sin(2\Pi f_m t)]$$

$$s_1(t) = A_c \cos[2\pi f_c t + \beta_f \sin(2\Pi f_m t)] \qquad (\beta f > 1)$$

FM Demodulators



Basic Idea



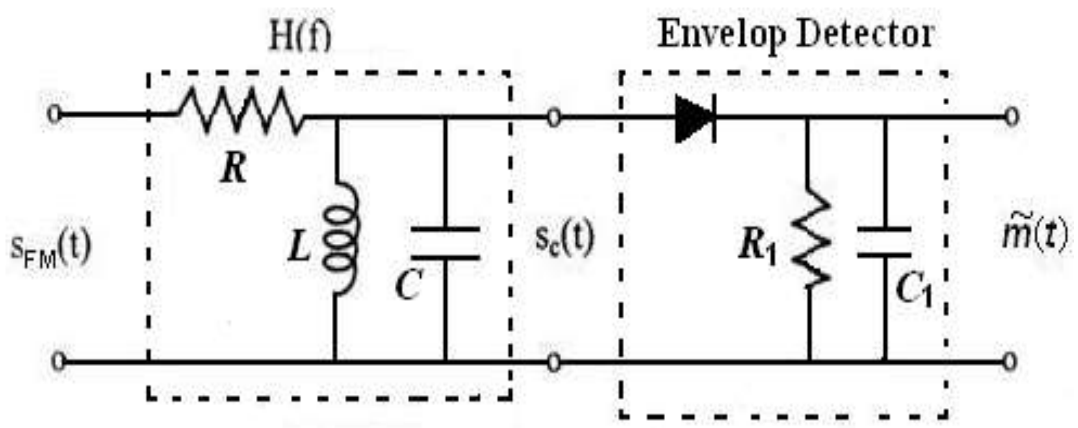
$$V_{out}(t) = k_d \, \Delta f$$

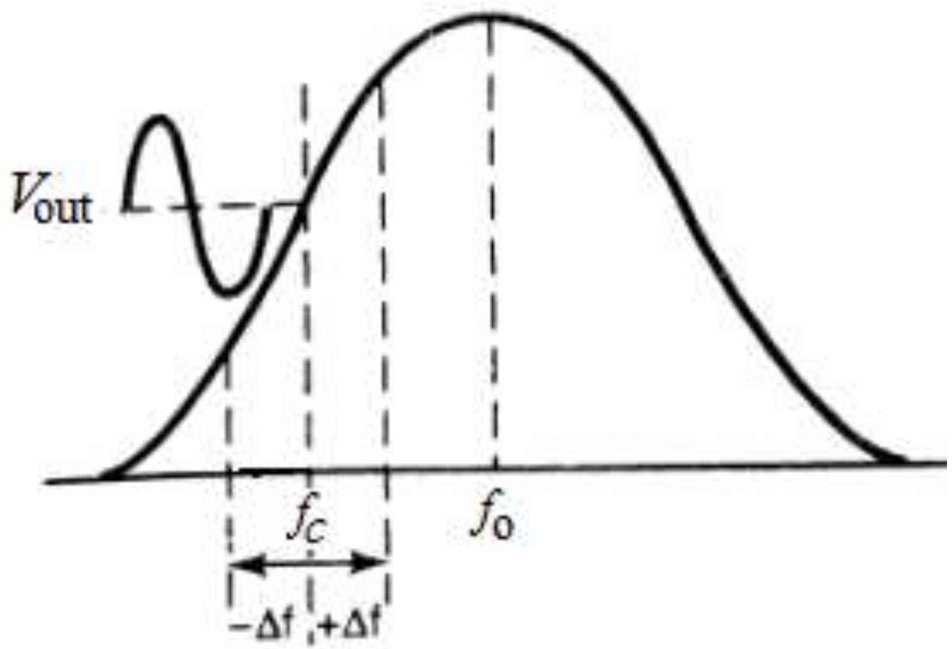
where $V_{out}(t)$ = demodulated output signal (Volts)

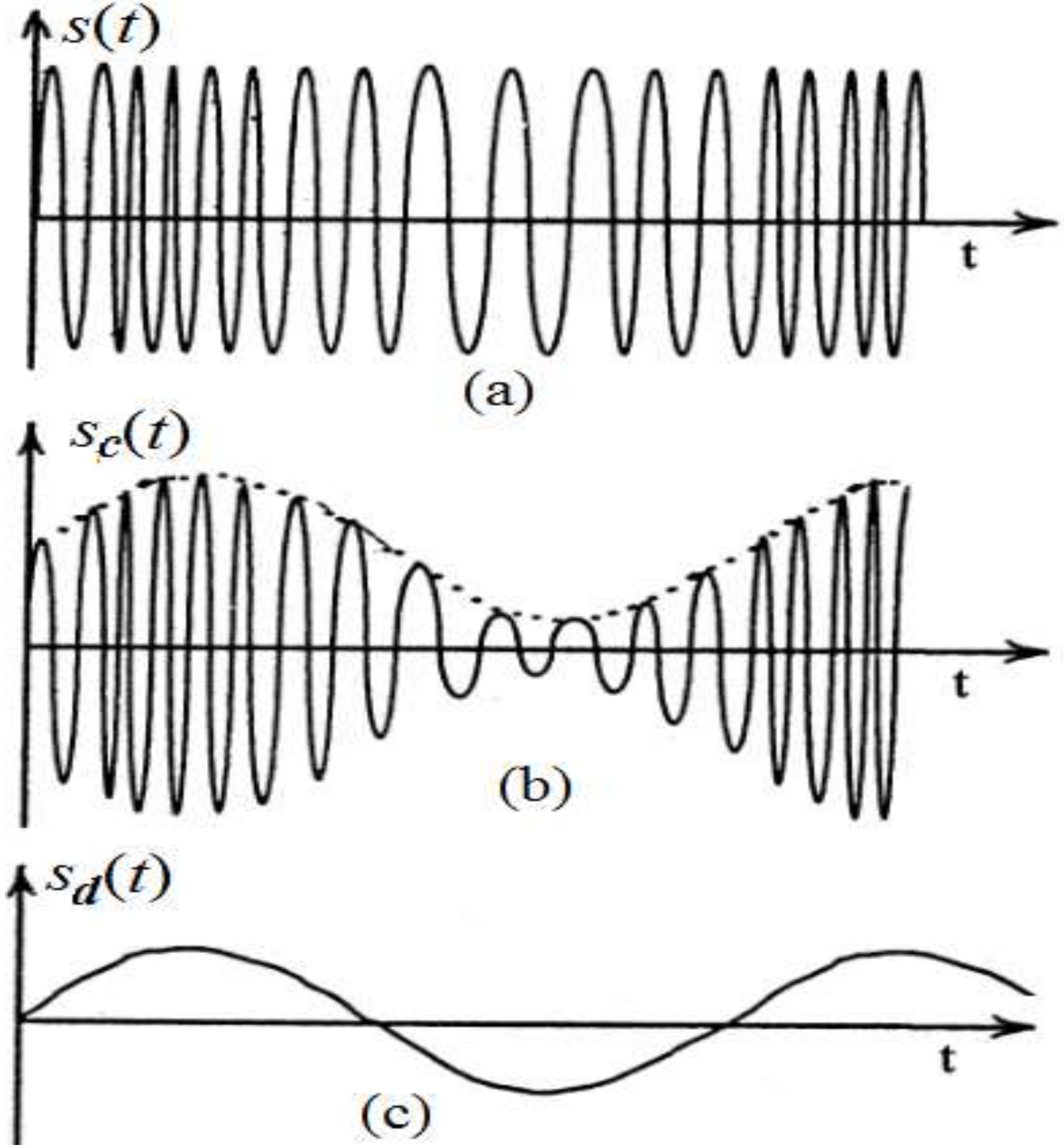
k_d = demodulator transfer function (Volts per Hertz)

Δf = difference between the input frequency and the center frequency of the demodulator (Hertz).

1. Slope Detector







(a)
(b)
(c)

Slope Detector . . .

***Advantages*:** The only advantage of the basic slope detector circuit is its simplicity.

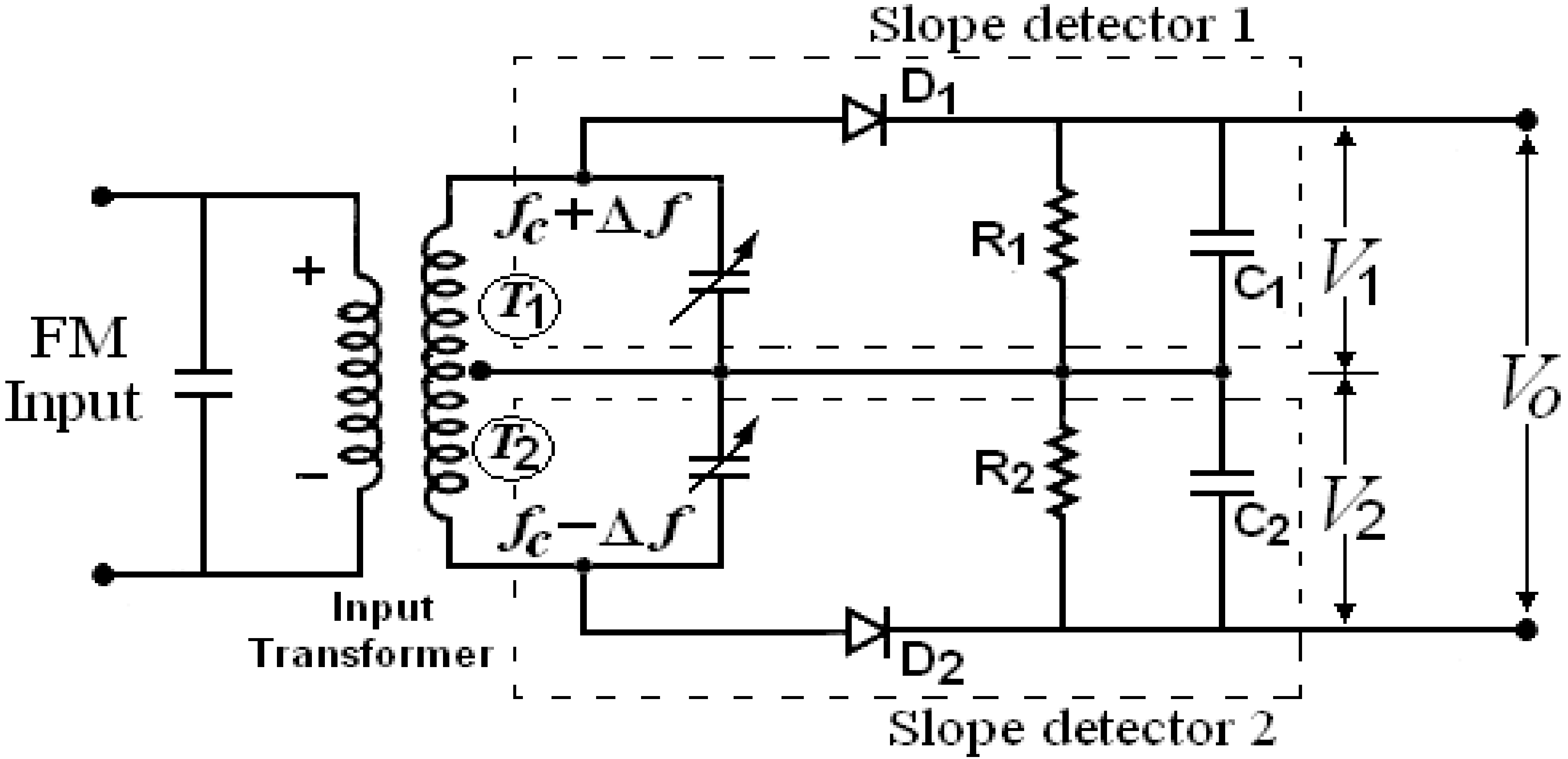
***Limitations*:**

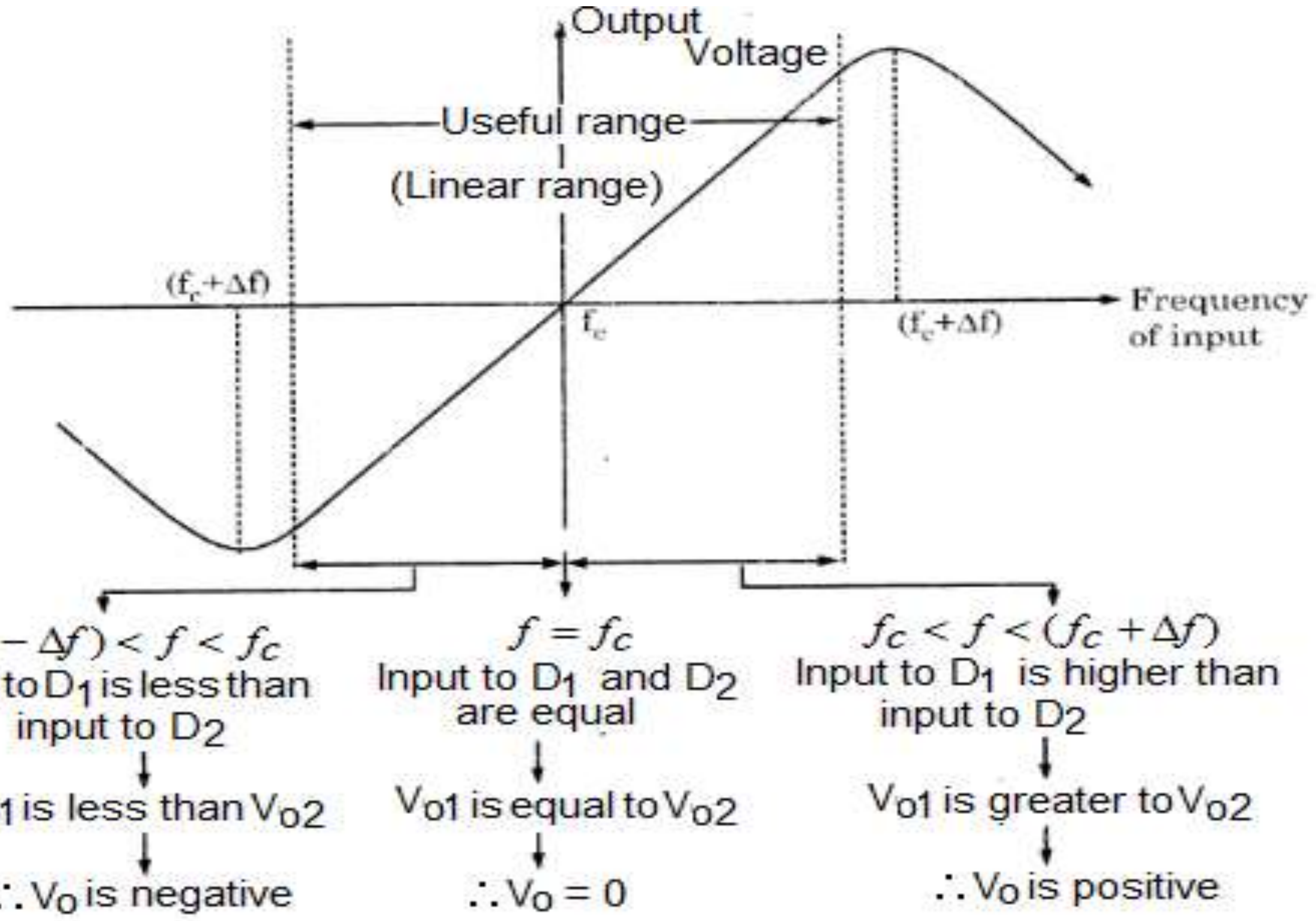
(i) The range of linear slope of tuned circuit is quite small.

(ii) The detector also responds to spurious amplitude variations of the input FM.

These drawbacks are overcome by using *balanced slope detector*.

2. Balanced Slope Detector





Output Voltage
Useful range
(Linear range)
$(f_c+\Delta f)$
f_c
$(f_c+\Delta f)$
Frequency of input
$(f_c - \Delta f) < f < f_c$
Input to D_1 is less than input to D_2
V_{o1} is less than V_{o2}
$\therefore V_o$ is negative
$f = f_c$
Input to D_1 and D_2 are equal
V_{o1} is equal to V_{o2}
$\therefore V_o = 0$
$f_c < f < (f_c + \Delta f)$
Input to D_1 is higher than input to D_2
V_{o1} is greater to V_{o2}
$\therefore V_o$ is positive

Advantages:

(i) This circuit is more efficient than simple slope detector.

(ii) It has better linearity than the simple slope detector.

Limitations:

(i) Even though linearity is good, it is not good enough.

(ii) This circuit is difficult to tune since the three tuned circuits are to be tuned at different frequencies, and

(iii) Amplitude limiting is not provided.

Limitations of direct methods of FM generation

- Difficult to obtain a high order of stability in carrier frequency because tank circuit consists of *L* and *C*.

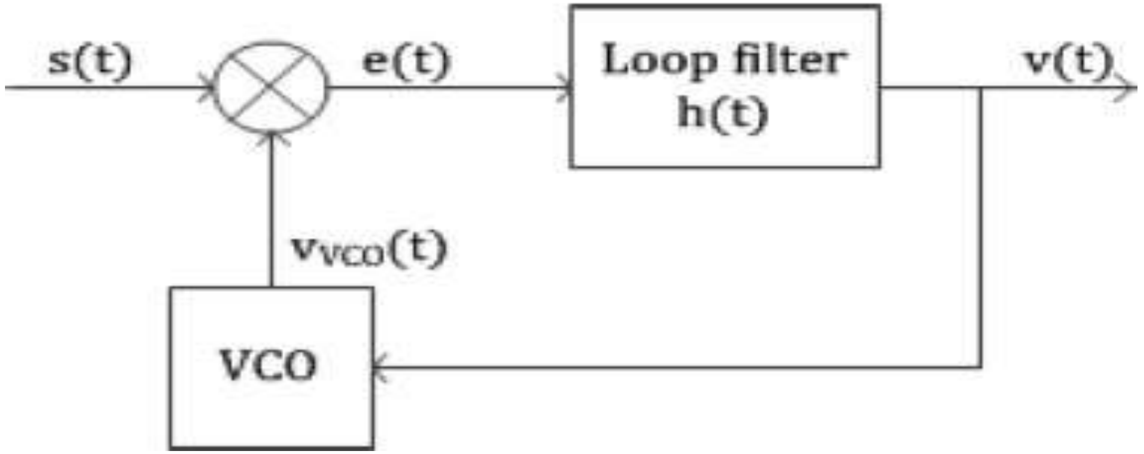
  The *crystal oscillator* can be used for *carrier frequency* stability, but *frequency deviation* is limited.

- The *non linearity* produces a *frequency variation* due to *harmonics of the modulating signal* hence there are distortions in the output FM signal.

Phase-Locked Loop (PLL) as FM Demodulator

- $s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(t)dt\right]$
- $s(t) = Ac \cos[2\pi fct + \emptyset 1(t)]$

$$\emptyset_1(t) = 2\pi k_f \int_0^t m(t)dt$$



PLL Demodulator

Let the VCO output be defined by

$$r_{vco}(t) = A_v \mathrm{Sin}\left[2\pi f_c t + 2\pi Kv \int_0^t v(t)dt\right]$$

where $\emptyset 2(t) = 2\pi Kv \int_0^t v(t)dt$

Kv is the Frequency Sensitivity of the VCO in Hz/V

We Assume,

1) The Frequency of the VCO Output is Precisely set at Unmodulated Carrier Frequency fc
2) VCO Output has 90˚ Phase shift w.r.t the Unmodulated Carrier Frequency

Here k_v is the frequency sensitivity of the VCO measured in hertz per volt. The multiplication of $s(t)$ and $v_{VCO}(t)$ results

$$s(t)v_{VCO}(t) = A_c \cos[2\pi f_c t + \emptyset_1(t)]\, A_v \sin[2\pi f_c t + \emptyset_2(t)]$$

$$= \frac{A_c A_v}{2} \sin[4\pi f_c t + \emptyset_1(t) + \emptyset_2(t)] + \frac{A_c A_v}{2} \sin[\emptyset_2(t) - \emptyset_1(t)]$$

FM Wave s(t) and VCO Output r(t) is applied to the Multiplier Producing two Components

1) High Frequency component

$$k_m A_c A_v \sin[4\pi f_c t + \varphi_1(t) + \varphi_2(t)]$$

2) Low Frequency component is Obtained at the Output of Multiplier

$$e(t) = k_m A_c A_v \sin[\varphi_1(t) - \varphi_2(t)]$$

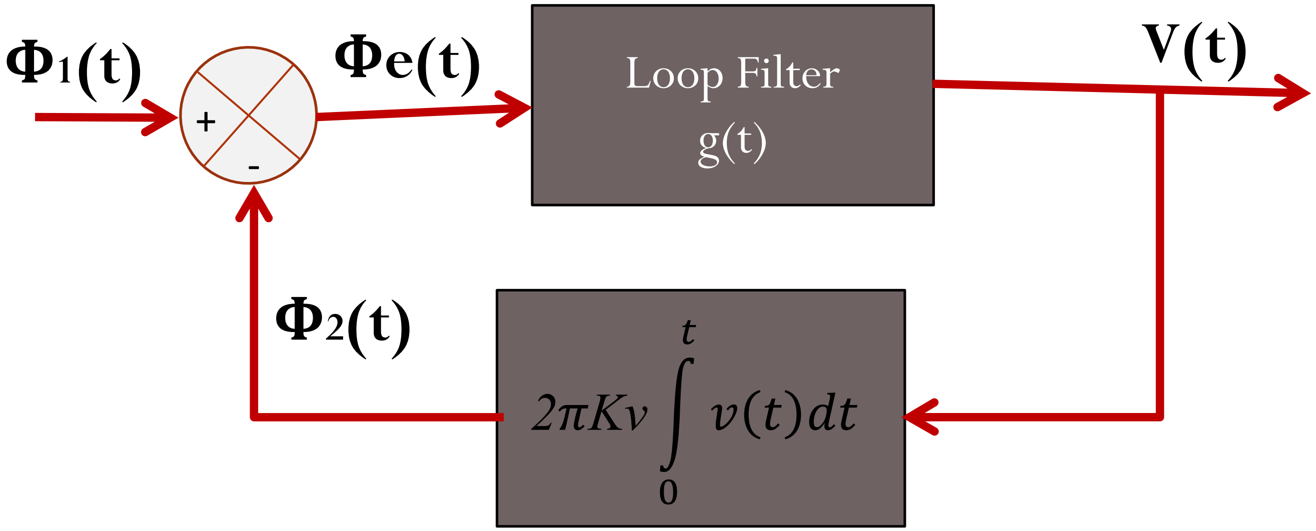
$$e(t) = k_m A_c A_v \sin[\varphi_e(t)] = K \sin[\varphi_e(t)]$$

where, $\varphi_e(t) = [\varphi_1(t) - \varphi_2(t)]$ (Phase Error)

Because $\varphi_e(t)$ is Small, $\sin \varphi_e(t) = \varphi_e(t)$

$$\varphi_e(t) = \left[\varphi_1(t) - 2\pi K v \int_0^t v(t)dt\right]$$

where, $\varphi_e(t) = [\varphi_1(t) - \varphi_2(t)]$ (Phase Error)



$$\varphi_e(t) = \varphi_1(t) - 2\pi Kv \int_0^t v(t)dt$$

$$\varphi_e(t) + 2\pi Kv \int_0^t v(t)dt = \varphi_1(t) \ldots\ldots \text{Eq1}$$

Differentiating ………Eq1

$$\frac{d\varphi_e(t)}{dt} + 2\pi Kv\, v(t) = \frac{d\varphi_1(t)}{dt} \ldots\ldots Eq2$$

where, $v(t) = \int_{-\propto}^{\propto} \varphi_e(\tau) g(t-\tau) d\,\tau = \varphi_e(t) * g(t)$ in T.D.

$V(f) = \varphi_e(f)\, G(f)$ in F.D.

Finding the F.T. of the Equation-2

$$j2\pi f\, \varphi_e(f) + 2\pi Kv\, \varphi_e(f)\, G(f) = j2\pi f\, \varphi_1(f)$$

$$\mathrm{jf}\varphi_e(f) + Kv\, \varphi_e(f)\, \mathrm{G}(f) = \mathrm{jf}\, \varphi_1(f)$$

$$\mathrm{jf}\,[1+\frac{Kv}{jf}\mathrm{G}(f)]\, \varphi_e(f) = \mathrm{jf}\, \varphi_1(f)$$

$$\varphi_e(f) = \frac{\varphi_1(f)}{1+\frac{Kv}{jf}G(f)}$$

Equation of the Controlled Voltage to the VCO is $v(f) = \varphi_e(f)G(f)$

Now the Design of G(f) is Such That Mod. Of $\frac{Kv}{jf}G(f)\ if \gg 1$

$$v(f) = \frac{jf2\pi}{2\pi kv}\varphi_1(f) \ldots eq3$$

We Know, $\emptyset 1(\mathrm{t}) = 2\pi K_f \int_0^t m(t)dt$

$$\frac{d}{dt}\varphi_1(t) = 2\pi k_f\, m(t)$$

Taking Inverse F.T of Eq3

$$v(t) = \frac{1}{2\pi kv}\frac{d}{dt}\varphi_1(t)$$

$$v(t) = \frac{1}{2\pi Kv} 2\pi Kf\, m(t)$$

$$\boxed{v(t) = \frac{Kf}{Kv} m(t)}$$

Since The Control voltage of the VCO is Proportional to the Message Signal ,Hence V(t) is the Demodulated Signal.

PREEMPHASIS AND DEEMPHASIS NETWORKS

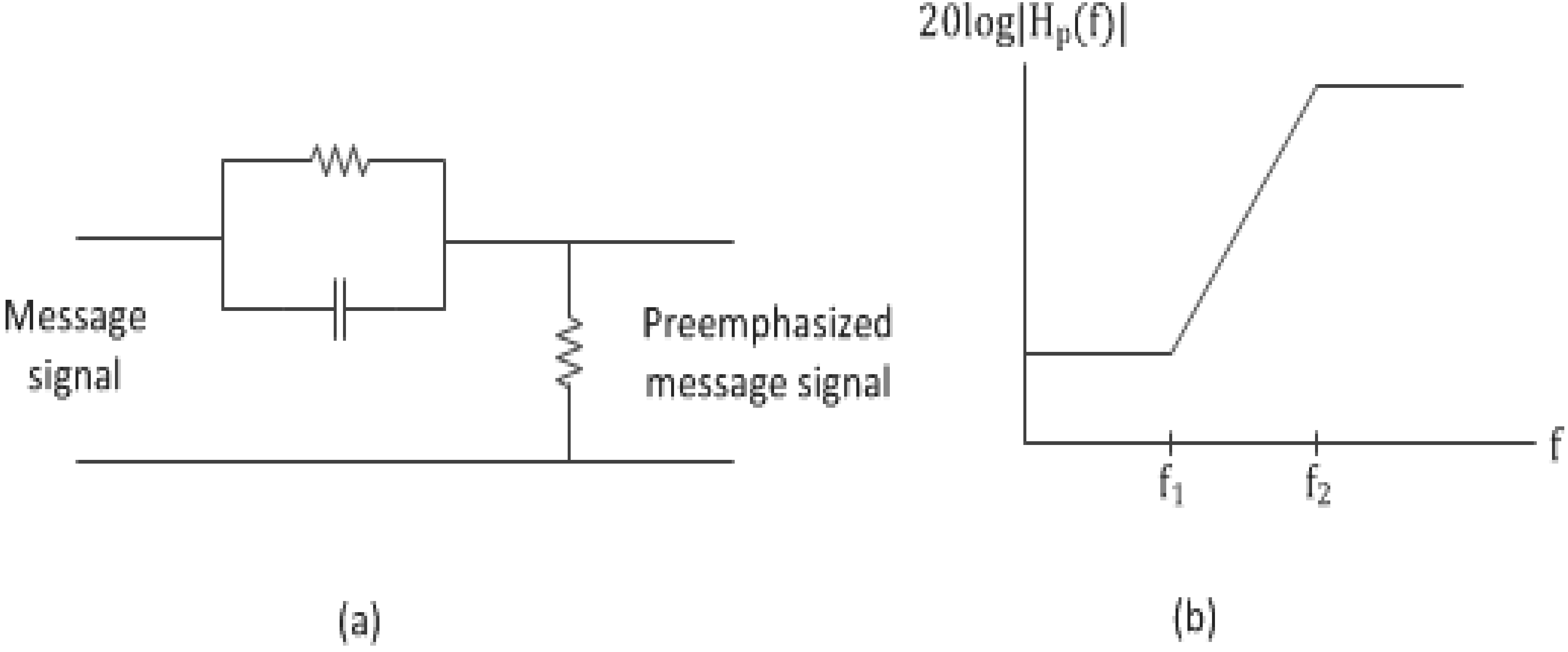


Figure ;(a) Pre emphasis network. (b) Frequency response of preemphasis network

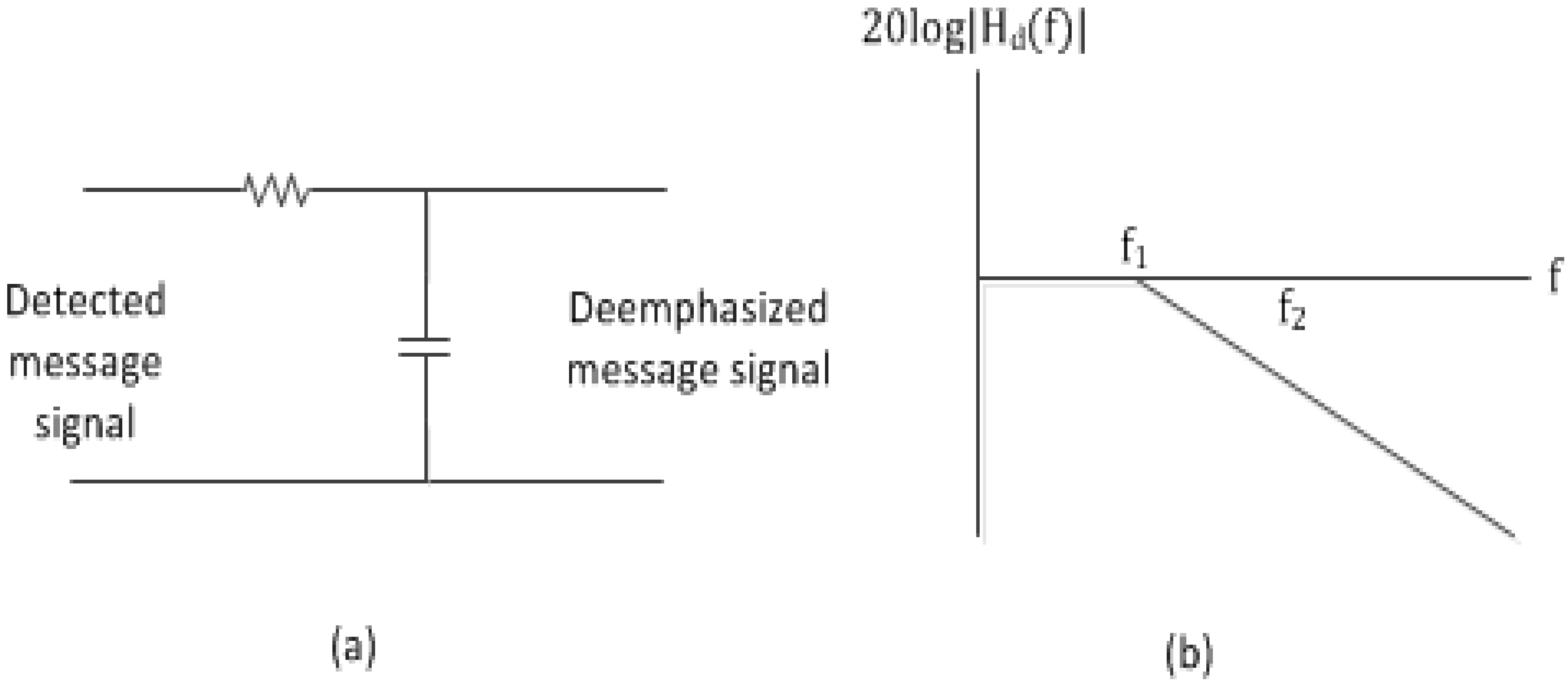


Figure (a) Deemphasis network. (b) Frequency response of Deemphasis network.

Comparison of AM and FM:

S.No	AM	FM
1	Pt= Pc$(1+\frac{\mu^2}{2})$	Pt= Pc
2	AM Requires More Power	Less Power
3	Power Varies with Modulation Index	Independent of Modulation Index
4	33.33% (Max. Efficiency)	33.33% (Max. Efficiency) when β= 2.4,5.5,8.6,11.2..
5	BW = 2fm	BW = 2(1+β)fm
6	BW is Low	Very High
7	BW is Independent of Mod. Index	BW Varies with Modulation Index
8	AM Receiver is Less Complex	More Complex

Comparison of AM and FM contd…

S.No	AM	FM
9	Effect of Noise is More in AM	Effect of Noise is Less in FM
10	550KHz to 1650KHz	88MHz to 108MHz
11	Intermediate Frequency =455KHz	Intermediate Frequency =10.7MHz
12	Practical BW = 10KHz	Practical BW = 200KHz
13	$\mu = 1$	B= 5
14	fm = 5KHz	fm = 15KHz
15	Ionospheric Propagation(NON-LOS)	Line of Sight Propagation(LOS)
16	Area of Coverage is More	Limited Because of LOS
17	Frequency Reuse is Not Possible	Frequency Reuse is Possible

1) A sinusoidal modulating waveform of amplitude 5 V and a frequency of 2 KHz is applied to FM generator, which has a frequency sensitivity of 40 Hz/volt. Calculate the frequency deviation, modulation index, and bandwidth.

Given, Am=5V , fm=2KHz , kf=40Hz/volt

$\Delta f = kf * Am$

$\Delta f = 40 \times 5 = 200Hz$

$\beta = \Delta f / fm$; $\beta = 200/2000 = 0.1$

Here, the value of **modulation index**, β is 0.1, which is less than one. Hence, it is Narrow Band FM.

$BW = 2fm = 2 \times 2K = 4KHz$

Therefore, the **bandwidth** of Narrow Band FM wave is 4KHz.

2) An FM wave is given by $s(t)=20\cos(8\pi\times10^6t+9\sin(2\pi\times10^3t))$. Calculate the frequency deviation, bandwidth, and power of FM wave.

Sol: Given, the equation of an FM wave as $s(t)=20\cos(8\pi\times10^6t+9\sin(2\pi\times10^3t))$.

We know the standard equation of an FM wave as

- $s(t)=Ac\cos(2\pi fct+\beta\sin(2\pi fmt))$ comparing the above two equations.

$Ac=20V$,$fc=4\times10^6Hz=4MHz$,$fm=1\times10^3Hz=1KHz$, $\beta=9$

Here, the value of modulation index is greater than one. Hence, it is **Wide Band FM**.

We know the formula for modulation index as $\beta = \Delta f / fm$

$\Delta f = \beta fm \quad = \Delta f = 9 \times 1KHz = 9KHz$

The formula for Bandwidth of Wide Band FM wave is $BW = 2(\beta + 1)fm$

$BW = 2(9+1)1K = 20KHz$

Formula for power of FM wave is $Pc = Ac^2/2R$

Assume, $R = 1\Omega$ and substitute Ac value in the above equation.

$P = (20)^2/2(1) = 200W$

Therefore, the **power** of FM wave is 200 **watts**.

3) Consider a Narrow band Signal $S(t)=10\cos(2\pi\times10^6t+0.4\sin(2\pi\times10^3t))$ is passed through Two Frequency Multipliers connected in cascade and $N_1= 2$ and $N_2= 5$. Determine the carrier frequency. Modulation index & BW at the output of 1st and 2nd Multipliers.

Solution:

NBFM	NBFM	WBFM
fc= 1MHz	fc= 2MHz	fc= 10MHz
β= 0.4	β= 0.8	β= 4
fm=1KHz	fm=1KHz	fm=1KHz
	BW= 2KHz	BW= 10KHz

4) Consider a Angle Modulated Signal $S(t)=10\cos(2\pi\times10^6 t+ 8 \sin(4\pi\times10^3 t))$. Assuming the given signal as FM

i) Determine the Modulation Index, Frequency Deviation, BW & Power

ii) Repeat the above calculations if the message signal is Doubled.

Solution:

a) $\Delta f = \beta f_m = 8\times2\text{KHz} = 16\text{KHz}$

b) $BW=2(\beta+1)f_m = 2(8+1)2\text{KHz} = 36\text{KHz}$

c) $P=(10)^2/2(1)=50\text{W}$

a) fm is Doubled ,Hence Mod. Index is Halved

b) $\beta= \Delta f/f_m = 16/4\text{KHz} = 4$; b) $BW=2(\beta+1)f_m = 2(4+1)4\text{KHz} = 40\text{KHz}$

c) $P=(10)^2/2(1)=50\text{W}$

5) In an FM System the audio frequency is 500Hz and Audio Voltage is 2.4V, the Deviation is 4.8KHz a) find the Expression.

b) If the audio frequency voltage is increased to 7.2V. What is the new Deviation

c) If the Audio frequency voltage is raised to 10V, While the audio frequency is dropped to 200Hz. What is the deviation and also calculate Modulation Index for All the above cases and write the expression for Modulated waveform.

- Solution:

a) fm= 500Hz ; Δf= 4.8KHz ; Am= 2.4V; β=?

$\beta = \Delta f / fm$ = 4.8KHz/ 500Hz

=9.6

s(t)=AcCos(2π×fct+9.6sin(2π×500t)).

b) $A_m = 7.2V$; $f_m = 500Hz$; $\Delta f = ?$ $\beta = ?$;

K_f is Constant for a Modulator (from (a))

$K_f = \Delta f / f_m = 4.8KHz / 500Hz = 2KHz$

$\Delta f = K_f * f_m = 2KHz * 7.2V = 14.4KHz$

$\beta = 14.4KHz/500Hz = 28.8$

$$\mathbf{s(t)=AcCos(2\pi \times fct+28.8sin(2\pi \times 500t))}$$

c) $A_m = 10V$; $f_m = 200Hz$; $\Delta f = ?$ $\beta = ?$; $K_f = 2KHz$

$\Delta f = K_f * f_m = 2KHz * 10V = 20KHz$

$\beta = 20KHz/200Hz = 100$

$$\mathbf{s(t)=AcCos(2\pi \times fct+100sin(2\pi \times 200t))}$$

6) A FM Transmitter is Operated at a carrier of 100MHz with a carrier voltage of 8V the modulating signal has an amplitude of 3V and frequency 6KHz resulting in a frequency deviation of 60KHz. Write the voltage equations for the following Conditions.

a) Original Values

b) Audio Amplitude increased to 4V

c) Audio Changed to 2V and 3KHz

Solution:

a) Ac = 8V; fc= 100MHz ; Am= 3V; fm=6KHz; Δf=60KHz

$\beta = \Delta f / fm = 60KHz / 6KHz = 10$

$$\mathbf{s(t)= 8Cos(2\pi \times 100*10^6 t+10sin(2\pi \times 6*10^3 t)).}$$

b) $A_c = 8V$; $f_c = 100MHz$; $A_m = 3V$; $f_m=6KHz$; $\Delta f=60KHz$

$\Delta f = k_f*A_m = 20KHz*4 = 80KHz$

$\beta = \Delta f / f_m = 80KHz / 6KHz = 13.33$

$$\mathbf{s(t)= 8Cos(2\pi\times100*10^{6}t+13.33sin(2\pi\times6*10^{3}t)).}$$

b) $A_c = 8V$; $f_c = 100MHz$; $A_m = 3V$; $f_m=6KHz$; $\Delta f=60KHz$

$\Delta f = k_f*A_m = 20KHz*2 = 40KHz$

$\beta = \Delta f / f_m = 40KHz / 3KHz = 13.33$

$$\mathbf{s(t)= 8Cos(2\pi\times100*10^{6}t+13.33sin(2\pi\times6*10^{3}t)).}$$

7) An FM Wave is described by $s(t)=20\cos(3\pi\times10^8 t+10\cos(2000\pi t))$. Find the Approximate Bandwidth of an FM Wave.

Solution:

$\beta = 10$ $\quad$ $f_m = 1KHz$;

$BW = 2(1+\beta)f_m = 2(1+10)1KHz = 22KHz$

8) Find the number of Channels that can be accommodated in the FM Band with frequency deviation of 75KHz and message signal frequency equal to 15KHz.

Solution:

$f_m = 15KHz$; $\Delta f = 75KHz$; **(FM Range: 88MHz- 108MHz)**

Total BW= (108MHz-88MHz)=20MHz

Number of Signals = Total BW/Signal BW = 20MHz/180KHz = 110

Signal BW= $2(\Delta f + f_m) = 2(75KHz+15KHz) = 180KHz$

9) For an FM modulator with peak frequency deviation 10 KHz, a modulating signal frequency 10 KHz, peak modulating signal amplitude 10 V, and a 500 KHz carrier, determine

a) Actual minimum bandwidth from the Bessel table.
b) Plot the output frequency spectrum for the Bessel approximation.
c) Approximation minimum bandwidth using Carson’s rule.
d) Comment the results.

- From the Bessel table for $\beta_f = 1,\ J_0 = 0.77,\ J_1 = 0.44,\ J_2 = 0.11,$ and $J_3 = 0.02$

Solution:

Given that $\Delta f = 10$ KHz, $f_m = 10$ KHz, $A_m = 10$, and $f_c = 500$

(a) Modulation index $\beta_f = \dfrac{\Delta f}{f_m} = \dfrac{10\text{KHz}}{10\text{KHz}} = 1$

From the table, $\beta_f = 1$ yields three sets of significant sidebands.

Therefore the $\text{B.W} = 2(nf_m) = 2(3\text{X}10\text{KHz}) = 60$ KHz

(b) For $\beta_f = 1$, $J_0 = 0.77, J_1 = 0.44, J_2 = 0.11$, and $J_3 = 0.02$

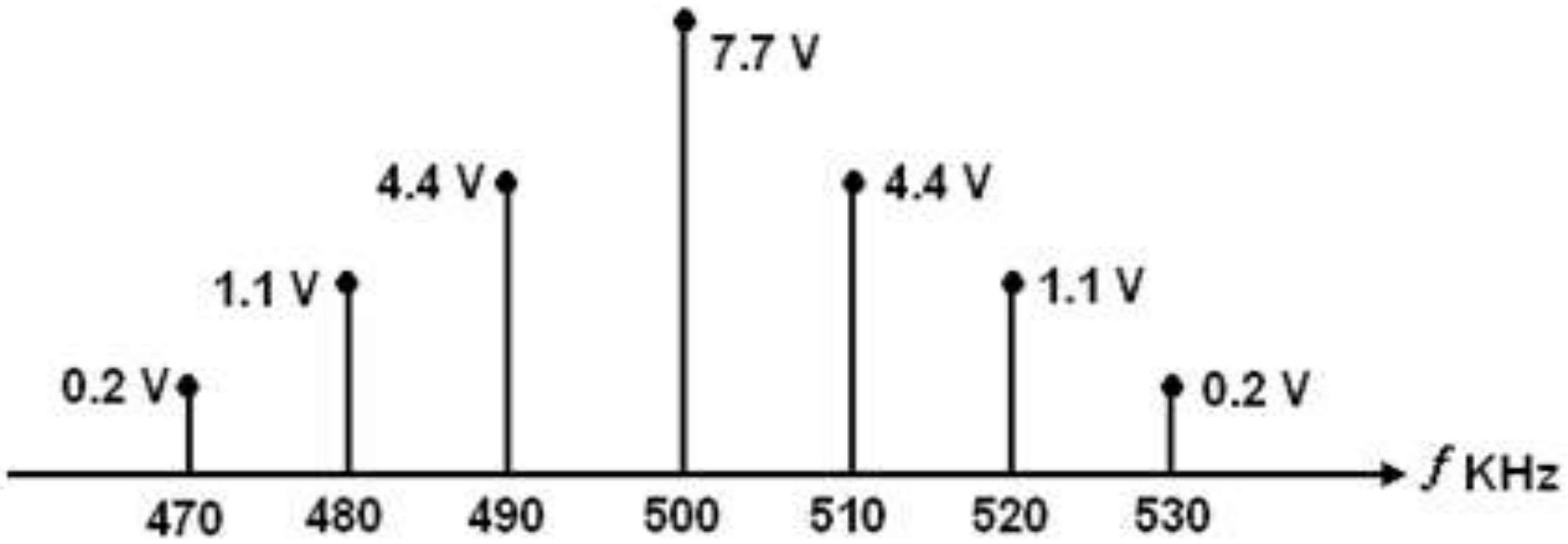
The relative amplitudes of carrier and sidebands are

$V_{J_0} = 0.77\text{X}10 = 7.7\,V$; $V_{J_1} = 0.44\text{X}10 = 4.4\,V$

$V_{J_2} = 0.11\text{X}10 = 1.1\,V$; $V_{J_3} = 0.02\text{X}10 = 0.2\,V$

The corresponding spectrum is shown in figure

The corresponding spectrum is shown in figure



(c) The Carson's rule for FM bandwidth is given by

$$B.W = 2(\Delta f + f_m) = 2(10\text{KHz} + 10\text{KHz}) = 40\text{KHz}$$

(d) **Comments:** The bandwidth from Carson's rule is less than the actual minimum bandwidth required to pass all the significant sideband sets as defined by the Bessel table.

Therefore, a system that was design using Carson's rule would have a narrower bandwidth and their poor performance than a system designed using the Bessel table.

For modulation indices above 5, Carson's rule is a close approximation to the actual bandwidth required.

10) (a) Determine the unmodulated carrier power for FM modulator and conditions given in Ex1. Assume the load resistance 50 Ω.

(b) Determine the total power in the angle modulated wave.

(c) Comment the results.

Ans: (a) $P_C = \dfrac{\left(A_C/\sqrt{2}\right)^2}{R} = \dfrac{A_C^2}{2R} = \dfrac{10^2}{2(50)} = 1$ watts

(b) The total power in the modulated signal is given by

$$P_t = \frac{7.7^2}{2(50)} + 2\frac{4.4^2}{2(50)} + 2\frac{1.1^2}{2(50)} + 2\frac{0.2^2}{2(50)}$$
$$= 0.5929 + 0.3872 + 0.0242 + 0.0008 = 1.0051$$

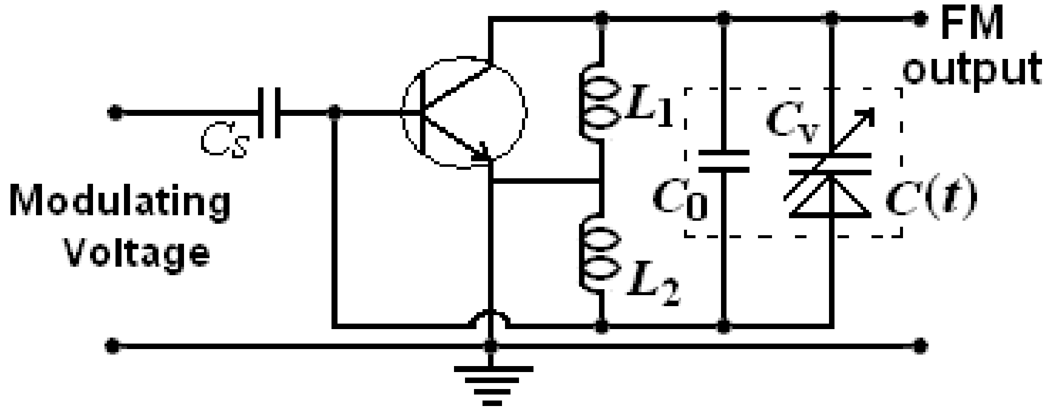
(c) The results of (a) and (b) are not exactly equal because the values given in the Bessel table have been rounded off. However, the results are close enough to illustrate that the power in the modulated wave and the unmodulated carrier are equal.

Beyond the Syllabus

Direct Generation of WBFM

Varactor Diode Modulator (Direct Method)

The capacitance of a varactor diode is inversely proportional to the reversed biased voltage amplitude.



$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}}$$

where $C(t) = C_0 + C_v$

For $m(t)$, the capacitance $C(t) = C_0 - k_c m(t)$

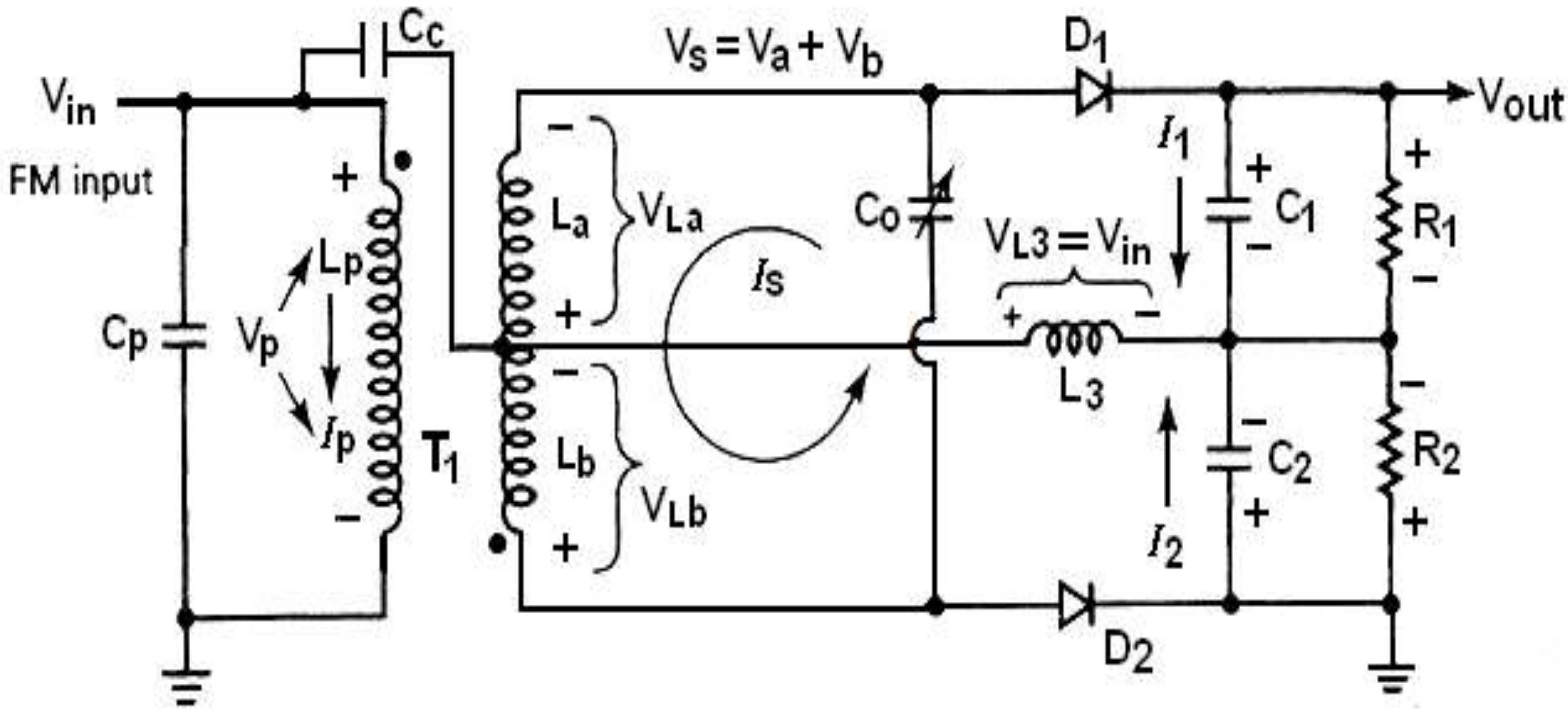
$$f_i(t) = \frac{f_0}{\sqrt{1 - \frac{k_c}{C_0} m(t)}}$$

where $f_0 = \frac{1}{2\pi\sqrt{C_0(L_1+L_2)}}$

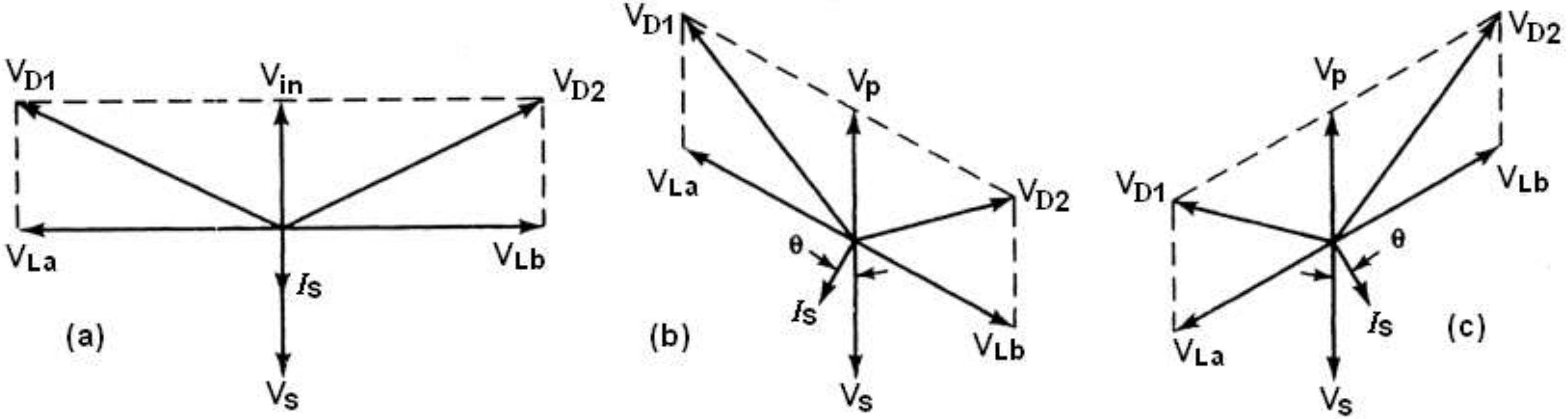
$$f_i(t) = f_0\left[1 + \frac{k_c}{2C_0} m(t)\right]$$

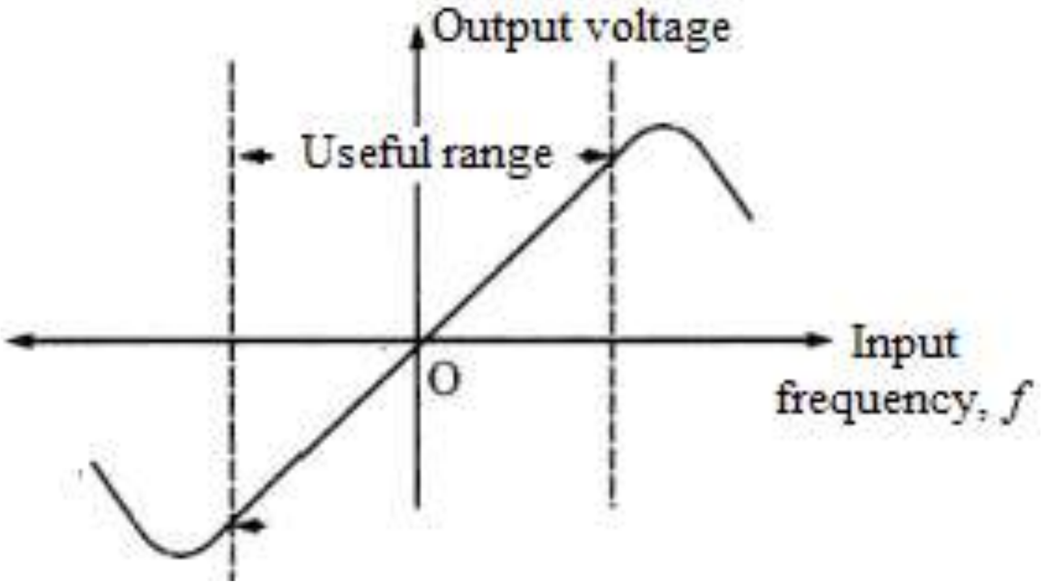
$$f_i(t) = f_0\left[1 + k_f m(t)\right]$$

3. Foster-Seeley Discriminator (Phase Discriminator)



$f_{in} = f_o$ $f_{in} > f_o$ $f_{in} < f_o$





***Advantages*:**

- Tuning procedure is simpler than balanced slope detector, because it contains only two tuned circuits, and both are tuned to the same frequency .
- Better linearity, because the operation of the circuit is dependent more on the primary to secondary phase relationship which is very much linear.

***Limitations*:**

It does not provide amplitude limiting. So, in the presence of noise or any other spurious amplitude variations, the demodulator output respond to them and produce errors.

4. Ratio Detector

Similar to the Foster-Seeley discriminator .

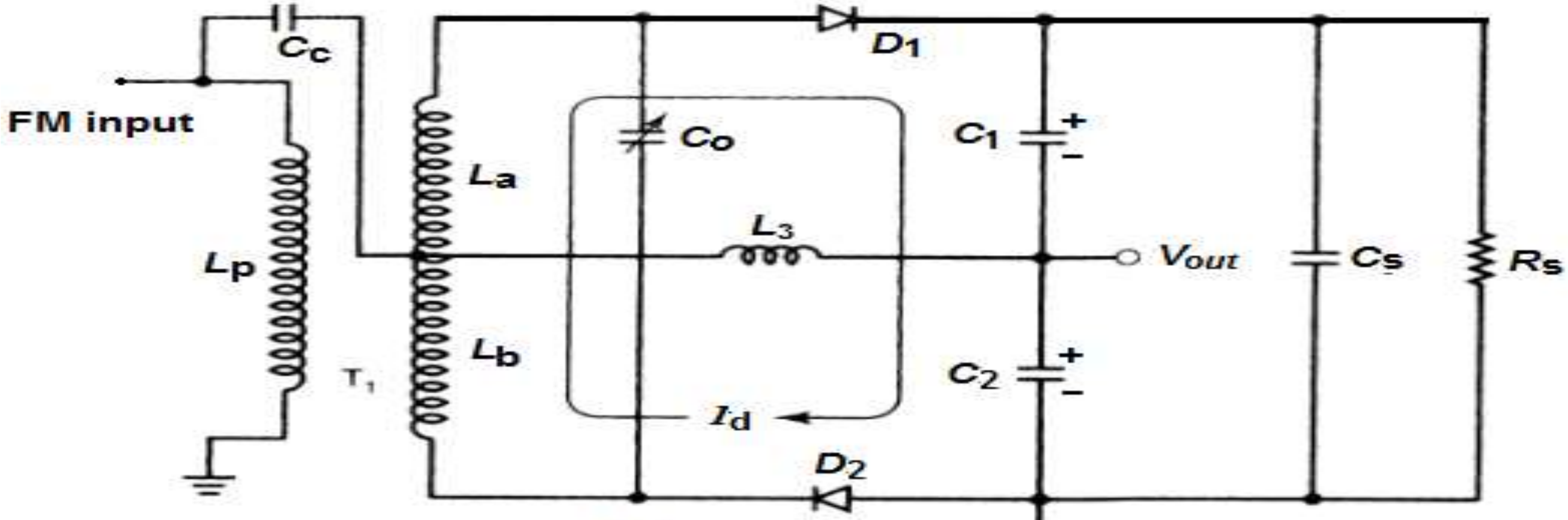
(i) The direction of diode is reversed.

(ii) A large capacitance C_s is included in the circuit.

(iii) The output is taken different locations.

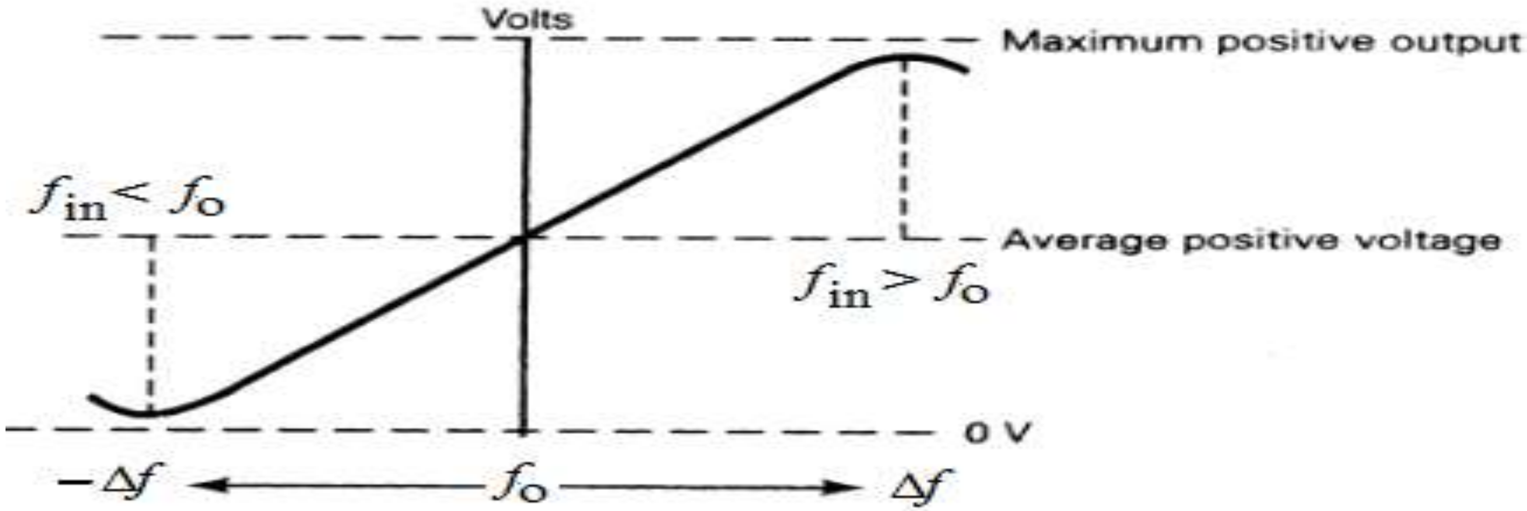
Advantages:

- Easy to align.
- Good linearity due to linear phase relationship between primary and secondary.
- Amplitude limiting is provided inherently. Hence additional limiter is not required.



C_c
D_1
FM input
C_o
C_1
+
−
L_a
L_3
L_p
V_{out}
C_s
R_s
L_b
C_2
+
−
T_1
I_d
D_2

(a)



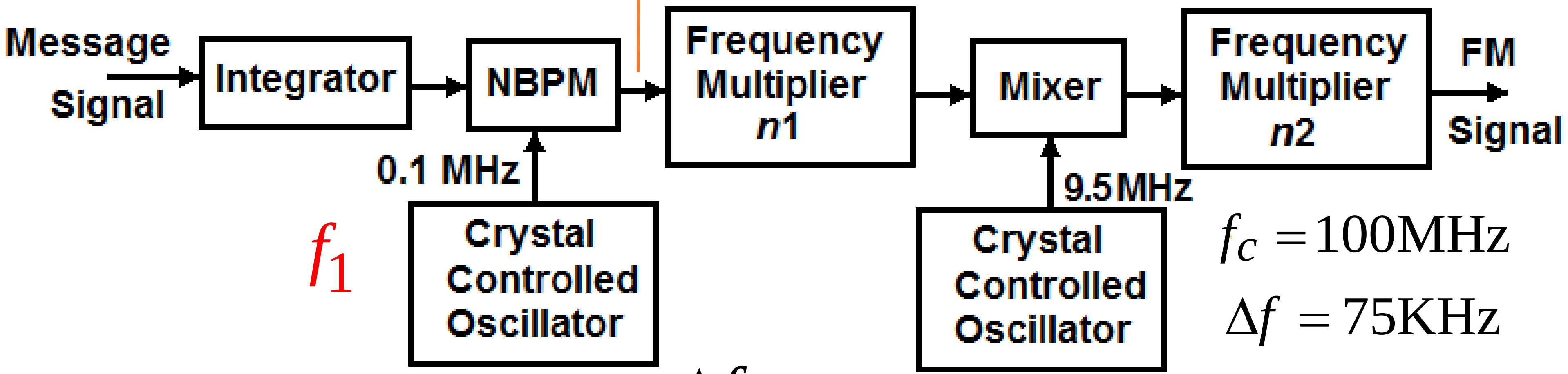
Volts
Maximum positive output
$f_{in} < f_o$
Average positive voltage
$f_{in} > f_o$
0 V
$-\Delta f$
f_o
Δf

(b)

Example

Audio $100\text{Hz} - 15\text{KHz}$

$$f_1 = 0.1\,\text{MHz}$$
$$\Delta f_1 = 20\,\text{Hz}$$



f_1

$f_c = 100\text{MHz}$

$\Delta f = 75\text{KHz}$

Let $\beta_1 = 0.2 \qquad \beta_1 = \dfrac{\Delta f_1}{f_m},$

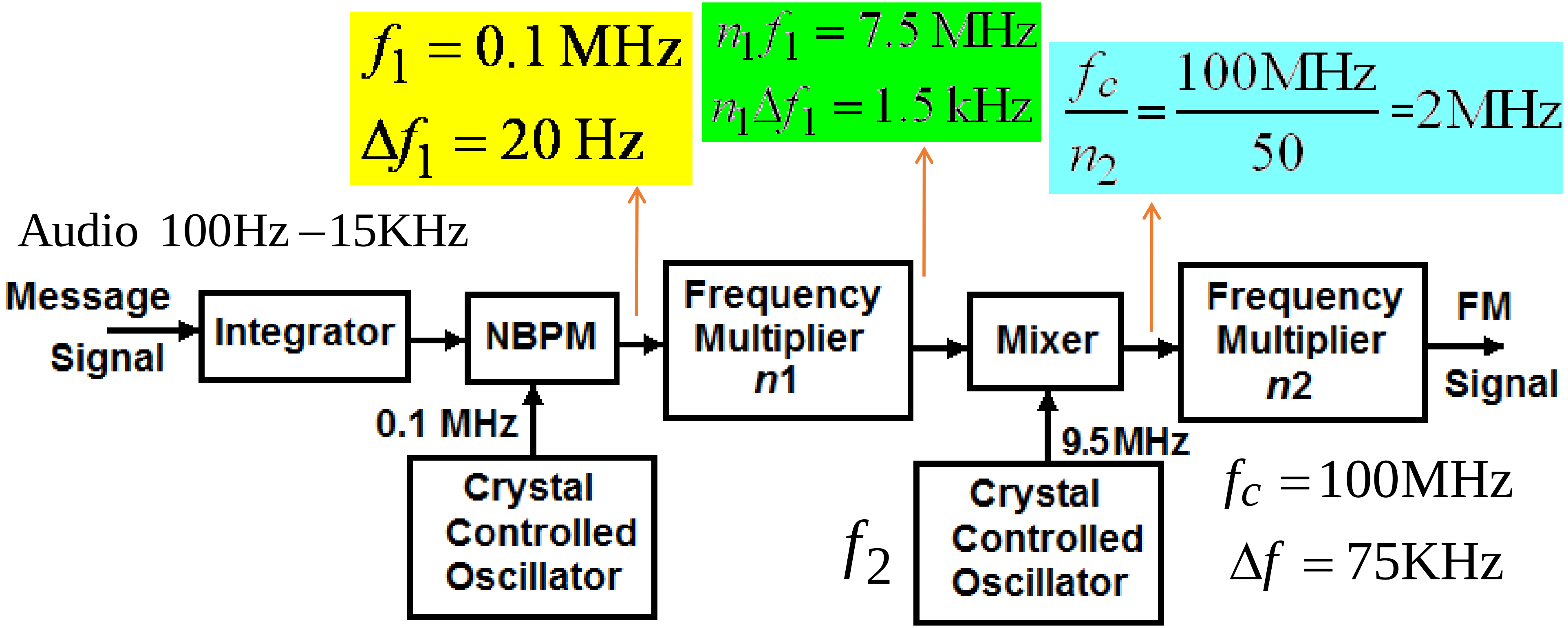
Then $f_m = 100\text{Hz}$

$\Delta f_1 = 20\text{Hz}$

To produce $\Delta f = 75\text{KHz}$

Frequency Multiplication $\dfrac{\Delta f}{\Delta f_1} = \dfrac{75\text{KHz}}{20\text{Hz}} = 3750$

Two stages $n_1 n_2 = 3750$



Two stages $n_1 n_2 = 3750 \quad f_2 - n_1 f_1 = \frac{f_c}{n_2} \Rightarrow n_1 = 75,\ n_2 = 50$

Parameter	At the Phase Modulator output	At the first frequency multiplier output	At the mixer output	At the second frequency multiplier output
Carrier frequency	0.1 MHz	7.5 MHz	2.0 MHz	100 MHz
Frequency deviation	20 Hz	1.5 KHz	1.5 KHz	75 KHz

Thank You