Signal flow graph : It is regarded as simplified version of block diagram. It is developed by S.J. Mason. Mason's gain formula is used to find (one step solution) Transfer of Complex System

Various terms used in formulation of SFG.

Non touching loops : Non touching loops are loops which donot posses any common node

Self loop contains single branch

branch → line segment joining two nodes of SFG.

Path gain : Product of branch gains encountered in traversing a path is called the path gain.

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Properties of SFG :

1 - SFG applies only to linear systems

2. The eqn for which a SFG is drawn must be algebraic eqn in the form of cause - and - effect.

3. nodes are used to represent variables.

4. signals travel along branches only in the direction described by the arrows of the branches.

5. the branch directing from node x_k to x_j represents the dependence of x_j upon x_k but not reverse.

6. A signal x_k travelling along a branch between x_k and x_j is multiplied by the gain of the branch a_{kj} so the a signal a_{kj} x_k is delivered at x_j.

7. For a given system SFG is not unique.

Procedure can be follo

- In a given BD, assume node at I/P, O/P, at every summing point, at every branch point and between cascade blocks.
- Draw the nodes separately as big thick dots and number the dots in the order 1, 2, 3, - - - - -.
- From the BD, find the gain between the nodes in the main forward path and connect all the corresponding nodes by directed straight line segments and mark the gain between the nodes on the segment.
- Draw the forward paths between various nodes and mark the gain between nodes on the directed branches
- Draw the Feedback paths between various nodes along with sign and mark the gain of FB paths

SFG terminology

I/P nodes → A node which have only outgoing branches, represented by independent Source / variables

O/P nodes → A node which have only incoming branches

Mixed nodes → not belong to either I/P or O/P is called a mixed node.

Path :- A path is traversal of branches connected by nodes in the direction of arrows. If a node is counted more than once, then path is called an open path.

Loop :- If the path ends the starting node and does not encounter any node more than once, it is called a loop

Path gain :- Product of the branch gains encounted in traversing the path.

Loop gain : Product of all branches gains encounted of the branches constituting the loops.

Procedure can be followed to convert a BD into SFG

- In a given BD, assume node at I/P, O/P, at every summing point, at every branch point and between cascade blocks.
- Draw the nodes separately as big thick dots and number the dots in the order 1, 2, 3, - - - -.
- From the BD, find the gain between the nodes in the main forward path and connect all the corresponding nodes by directed straight line segments and mark the gain between the nodes on the segment.
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MASON's Gain Formula

$$\text{Over all } TF = \frac{\sum_{K=1}^{N} P_K \Delta_K}{\Delta}$$

$N \rightarrow$ no of forward path between i/p & o/p

T_K - gain of k^{th} forward path between i/p & o/p

Δ_K - value of Δ for that part of the graph not touching the k^{th} forward path

$$\Delta = 1 - \sum_i L_{i1} + \sum_j L_{j2} - \sum_k L_{k3} + \cdots\cdots$$

L_{mr} = gain of product of the mth ($m = i, j, k, ---$) possible combination of r nontouching loops ($1 \leq r \leq L$)

Δ = 1 - (Sum of the gains of all individual loops) + Sum of products of gains of all possible combinations of two non touching loops) - (Sum of products of gains of all possible combinations of three non touching loops) + - - - -

Application of the gain formula between o/p nodes & non i/p nodes

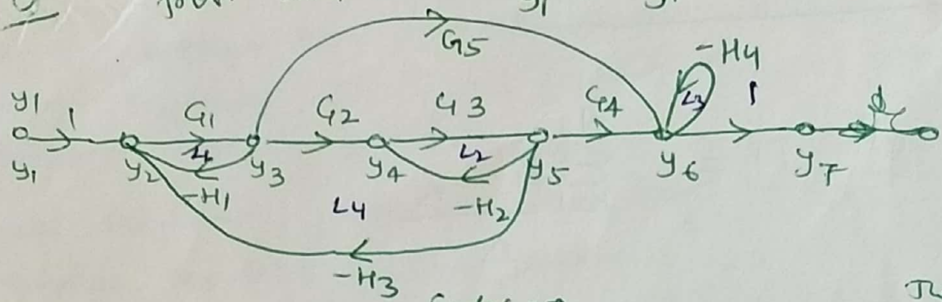
Let y_{in} be an i/p and y_{out} be an o/p node of a SFG. The gain y_{out}/y_2, where y_2 is not an i/p, may be written as,

$$\frac{y_{out}}{y_2} = \frac{y_{out}/y_{in}}{y_2/y_{in}} = \frac{\sum P_K \Delta_K \big|_{\text{from } y_{in} \text{ to } y_{out}} / \Delta}{\sum P_K \Delta_K \big|_{\text{from } y_{in} \text{ to } y_2} / \Delta}$$

$$\frac{y_{out}}{y_2} = \frac{\sum P_K \Delta_K \big|_{\text{from } y_{in} \text{ to } y_{out}}}{\sum P_K \Delta_K \big|_{\text{from } y_{in} \text{ to } y_2}}$$

$\therefore$ Δ does not appear in the final eqn.

Q. from SFG find $\frac{y_2}{y_1}$, $\frac{y_4}{y_1}$ and $\frac{y_6}{y_1}$ or y_7/y_1, $\frac{y_7}{y_2}$



Single loop — Two non touching loops gain products

$$\Delta = 1 + (G_1H_1 + G_3H_2 + H_4 + G_1G_2G_3H_3) + (\underbrace{G_1G_3H_1H_2}_{L12} + \underbrace{G_1H_1H_4}_{L13}$$

$$+ \underbrace{G_3H_2H_4}_{L23} + \underbrace{G_1G_2G_3H_3H_4}_{L34}) + \underbrace{G_1G_3H_1H_2H_4}_{L123})$$ → three non Touching loops

$$\Delta = 1 + G_1H_1 + G_3H_2 + H_4 + G_1G_2G_3H_3 + G_1G_3H_1H_2 + G_1H_1H_4 +$$
$$G_3H_2H_4 + G_1G_2G_3H_3H_4 + G_1G_2H_1H_2H_4$$

Now To find out

$$y_2/y_1 = \frac{1 + G_3H_2 + H_4 + G_3H_2H_4}{\Delta}$$

$$T_K\Delta_K = 1 \cdot (1 + G_3H_2 + H_4 + G_3H_2H_4) \qquad \Delta_K.$$

$$\left\{ \begin{array}{l} \therefore \Delta_K = 1 + G_3H_2 + H_4 + G_3H_2H_4 \\ G = 1 \end{array} \right.$$

$$\frac{y_4}{y_1} = \frac{G_1G_2(1+H_4)}{\Delta}$$

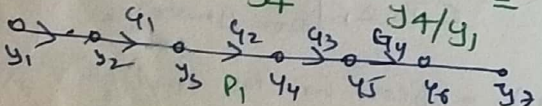
$$\left\{ \begin{array}{l} \therefore \Delta_K = (1+H_4) \\ P_K = G_1G_2 \end{array} \right.$$

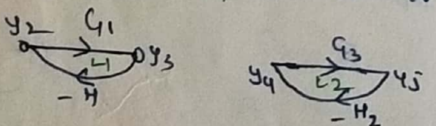
$$\frac{y_6}{1} = \frac{y_7}{y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{\Delta}$$

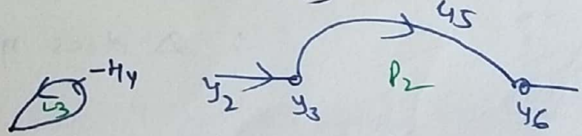
For y_7/y_2

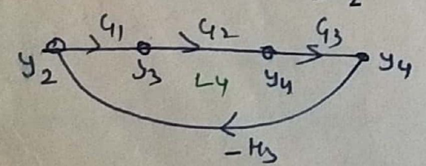
$$y_7/y_2 = \frac{y_7/y_1}{y_2/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{(1 + G_3H_2 + H_4 + G_3H_2H_4)}$$

$$y_7/y_4 = \frac{y_7/y_1}{y_4/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{G_1G_2(1+H_4)}$$









L1, L2, L3, L4

L12, L13, L23, L43, L123

2 — 3

From the given blocks:

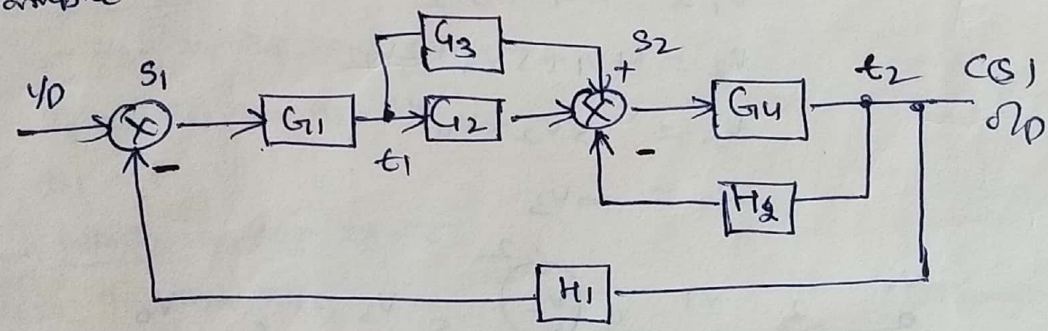
① Name all the summing points and take off points in a block diagram.

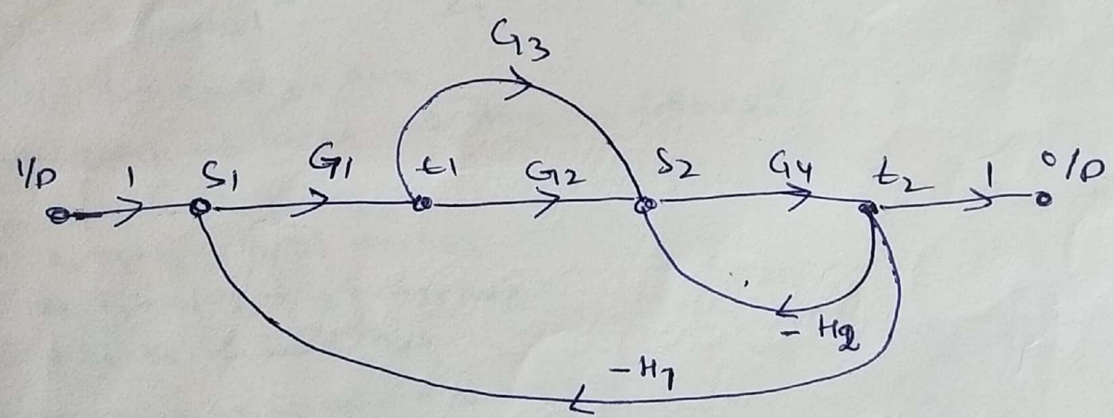
② Represent each summing point and take off point by separate node in a signal flow graph

③ Connect them by the branches instead of blocks, indicating block transfer functions as the gains of the corresponding branches

④ Show the input and output nodes separately if required, to complete signal flow graph

Example





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Methods to obtain Signal Flow Graph:

(1) from the system eqns

1. Represent each variable by a separate node
2. use the property that value of the variable represented by a node is an algebraic sum of all the signals entering at that node, to simulate the eqns.
3. Coefficients of the variables in the eqns are to be represented as the branch gains, joining the nodes in signal flow graph.
4. Show the i/p and o/p variables separately to complete signal flow graph.

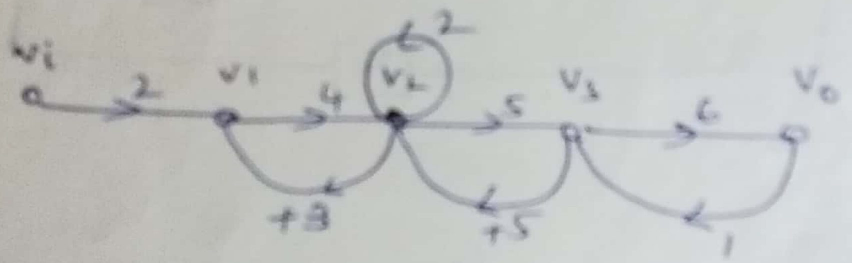
Example.

$$V_1 = 2V_i + 3V_2$$

$$V_2 = 4V_1 + 5V_3 + 2V_2$$

$$V_3 = 5V_2 + V_o$$

$$V_o = 6V_3$$



A System is described by the following set of linear eqn

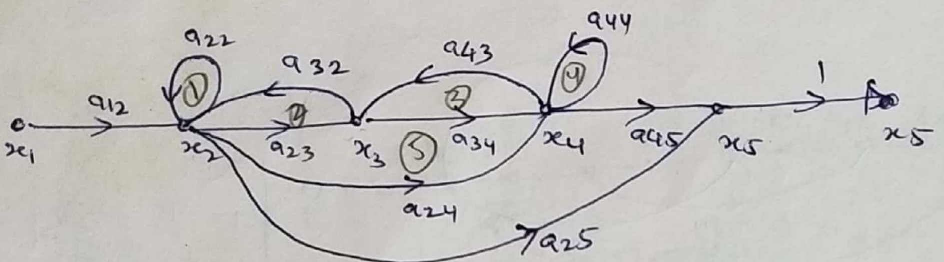
$x_2 = a_{12}x_1 + a_{22}x_2 + a_{32}x_3$ — ①

$x_3 = a_{23}x_2 + a_{43}x_4$ — ②

$x_4 = a_{24}x_2 + a_{34}x_3 + a_{44}x_4$ — ③

$x_5 = a_{25}x_2 + a_{45}x_4$ — ④

Draw SFG find x_5/x_1 TF



Forward path ①

$x_1 - x_2 - x_3 - x_4 - x_5$

$T_1 = a_{12}\,a_{23}\,a_{34}\,a_{45}$,

$\Delta_1 = 1$

Forward path ②, $x_1 - x_2 - x_4 - x_5$

$T_2 = a_{12}\,a_{24}\,a_{45}$,

$\Delta_2 = 1$

Forward path ③ $x_1 - x_2\ x_5$

$T_3 = a_{12}\,a_{25}$,

$\Delta_3 \quad 1 - a_{34}a_{43} - a_{44}$

The Loop gains associated with them as follows:

Self loop ① $x_2 - x_2, \rightarrow L_1 = a_{22}$

Loop ② $x_2 - x_3 - x_2 \rightarrow L_2 = a_{23}a_{32}$

Loop ③ $x_3 - x_4 - x_3 \rightarrow L_3 = a_{34}\,a_{43}$

Self Loop ④ $\rightarrow x_4 - x_4 \rightarrow L_4 = a_{44}$

Loop $x_2 - x_4 - x_3 - x_2 \rightarrow L_5 = a_{24}\,a_{43}\,a_{32}$

The pairs of two non touching loops and product of gains associated with them are as follows

Loop $L_1 \& L_3 = \quad L_{13} = a_{22}\,a_{34}\,a_{43}$

$L_1 \& L_4 = \quad L_{14} = a_{22}\,a_{44}$

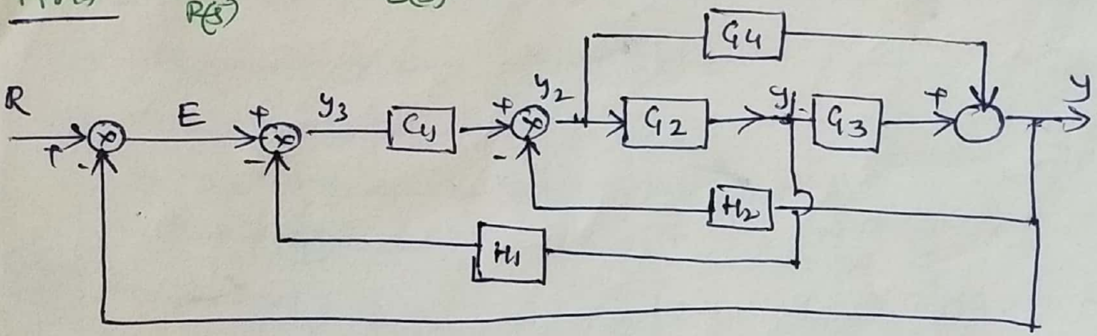
$L_2 \& L_4 = \quad L_{24} = a_{23}\,a_{32}\,a_{44}$

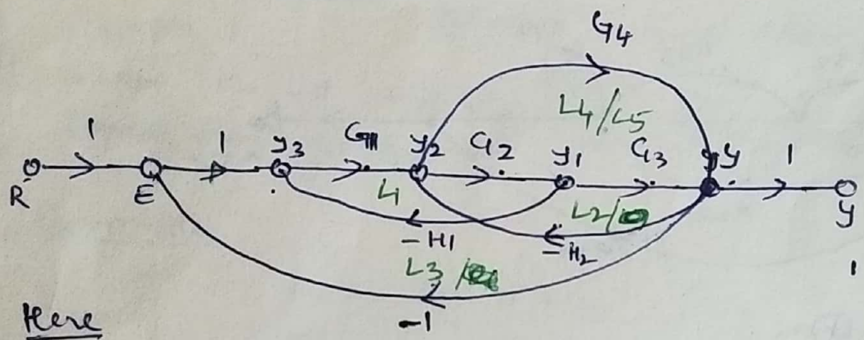
$T = \frac{x_5}{x_1} = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3}{\Delta}$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_{13} + L_{14} + L_{24})$

$$T = \frac{x_5}{x_1} = \frac{a_{12}a_{23}a_{34}a_{45} + a_{12}a_{24}a_{45} + a_{12}a_{25}(1 - a_{34}a_{43} - a_{44})}{1 - a_{22} - a_{23}a_{32} - a_{34}a_{43} - a_{44} - a_{24}a_{43}a_{32} + a_{22}a_{34}a_{43} + a_{22}a_{44} + a_{23}a_{32}a_{44}}$$

Find $\frac{E(s)}{R(s)}$ and $\frac{Y(s)}{E(s)}$ from fig





$L_1 = y_3 y_2 y_1 y_3$

$L_2 = y_2 y_1 y\, y_2$

$L_3 = E\, y_3 y_2 y_1 y\, E$

$L_4 = E\, y_3 y_2 y\, E$

~~$L_5 = E\, y_3 y_2\, y$~~

$L_5 = y_2 y\, y_2$

Here

$$\Delta = 1 + \underbrace{G_1 G_2 H_1}_{L_1} + \underbrace{G_1 G_4}_{L_4} + \underbrace{G_4 H_2}_{L_5} + \underbrace{G_1 G_2 G_3}_{L_3} + \underbrace{G_2 G_3 H_2}_{L_2}$$

$$\frac{E(s)}{R(s)} = \frac{\sum T_k \Delta_k}{\Delta} = \frac{T_1 \Delta_1}{\Delta}$$

Here $T_k = 1$,

$\Delta_k = 1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2$

So, $$\boxed{\frac{E(s)}{R(s)} = \frac{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2}{\Delta}}$$ ✓

We know that

$$\frac{Y(s)}{E(s)} = \frac{Y(s)}{R(s)} \Big/ \frac{E(s)}{R(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{\Delta} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

Here $\Delta_1 = \Delta_2 = 1$

So, $$\boxed{\frac{Y(s)}{E(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2}}$$ ✓