RESONANCE

1. Introduction:-

In many of the electrical circuits, the resonance is a very important phenomenon. The study of resonance is very useful, particularly in the area of communications. For example, the ability of radio receiver to select a certain frequency, transmitted by a station and to eliminate frequencies from other stations is based on the principle of resonance.

Resonance is defined as a phenomenon in which applied voltage and resulting current are in phase. An a.c circuit is said to be in resonance if it exhibits unity power factor condition, that means applied voltage and resulting current are in phase.

Thus, in an a.c circuit, under the condition of resonance, the reactance gets cancelled if the inductive and capacitive reactances are in series (or) the succeptance gets cancelled if the inductive and capacitive reactances are in parallel and the complex impedance of a.c circuit consists only the real resistive part.

Resonance in series circuits is referred as series resonance (or) simply resonance. Resonance in parallel circuits is referred as parallel resonance (or) anti resonance.

Series Resonance:-

In a series RLC circuit, the current lags behind the voltage (or) leads the applied voltage depending up on the values of X_L and X_C. X_L causes the total current to lag behind the applied voltage, while X_C causes the total current to lead the applied voltage. When $X_L > X_C$, the circuit is predominantly inductive, and when $X_C > X_L$, the circuit is predominantly capacitive. However, if one of the parameters of the series RLC circuit is varied in such a way that the current in the circuit is in phase with applied voltage and then the circuit is said to be in resonance.

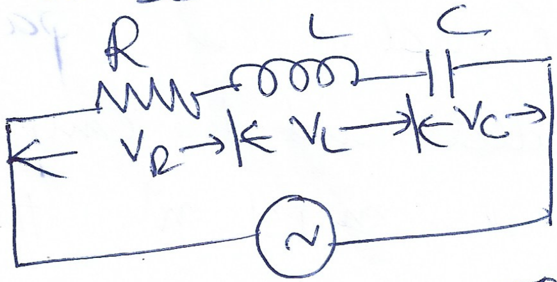


Fig ①

Consider the series RLC circuit shown in Fig ①.

The Total impedance of the series RLC circuit is $Z = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$

It is clear from the circuit that the current $I = V_s/2$. 3-2

The ckt is said to be in resonance if the current is in phase with the applied voltage. In a series RLC circuit, series resonance occurs when reactance part in impedance is equal to zero.

$$X_L - X_C = 0 \Rightarrow X_L = X_C$$

The frequency at which the resonance occurs is called the resonant frequency.

Since $X_L = X_C$, the impedance in a series RLC circuit is purely resistive. At the resonant frequency f_r, the voltages across capacitance and inductance are equal in magnitude. Since they are 180° out of phase with each other, they cancel each other and hence, zero voltage apperars across LC combination.

At resonance $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$

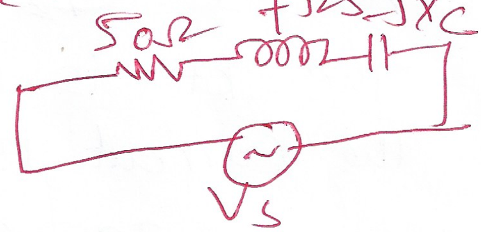
$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{(2\pi)^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

In a series RLC circuit, the resonance may be produced by varying the frequency, keeping L & C constant. otherwise, resonance may be produced by varying either L (or) C for fixed frequency.

① . Determine the value of Capacitive reactance & impedance at resonance.



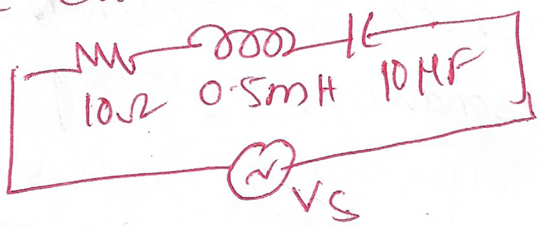
$X_L = X_C$

$X_L = 25\,\Omega$

$X_C = 25\,\Omega = \frac{1}{\omega C}$

$\underline{\underline{Z = R = 50\,\Omega}}$

② . Determine the resonant frequency for the ckt shown in fig.



The resonant frequency

$f_r = \frac{1}{2\pi\sqrt{LC}} = 2.25\,kHz.$

③ . In a series RLC circuit driven with a sinusoidal a.c. voltage source, determine value of 'C' required to achieve resonance in circuit at 5kHz if value of resistance & inductance are 2Ω & 1Ω respectively.

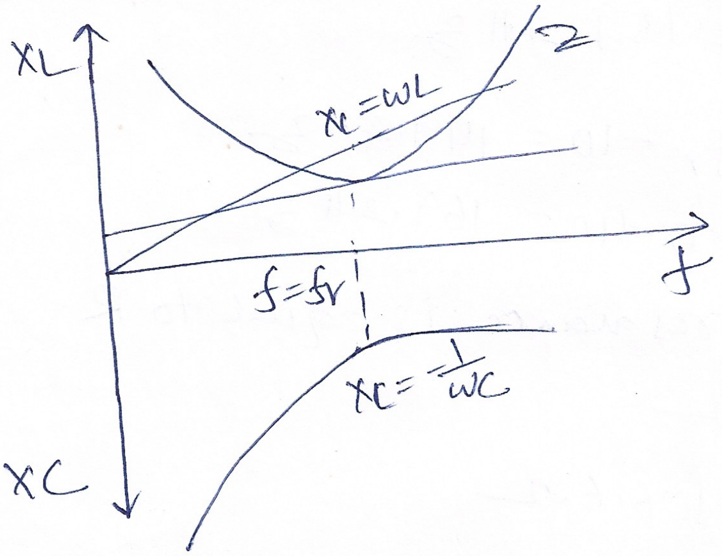
$R = 2\,\Omega, \; L = 1\,mH \qquad f_0 = 5\,kHz$

$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{(2\pi f_0)^2 L} = 1.0132\,\mu F.$

Impedance & phase angle of a series resonant Circuit:-

The impedance of a series RLC circuit is $|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

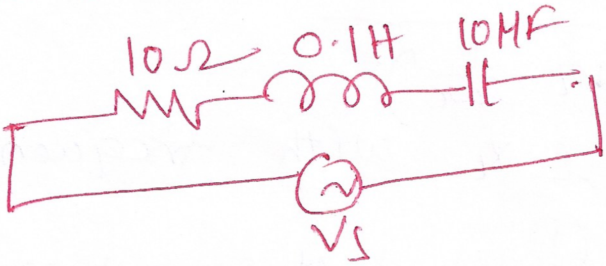
The variation of X_C & X_L with frequency is shown in fig. 1.



At zero frequency, both X_C and Z are infinitely large, and X_L is zero because at zero frequency the capacitor acts as an open circuit and the inductor acts as a short circuit. As the frequency increases, X_C decreases & X_L increases. Since X_C is larger than X_L, at frequencies below the resonant frequency f_r, Z decreases along with X_C. At resonant f_r, $X_C = X_L$ and $Z = R$. At frequencies above the resonant frequency f_r, X_L is larger than X_C causing Z to increase.

The phase angle as a function of frequency is shown in fig 2. At frequency below the resonant frequency, current leads the source voltage because the capacitive reactance is greater than the inductive reactance. The phase angle decreases as the frequency approaches the resonant value, and is 0 at resonance ... at frequencies above resonance

Determine the impedance at resonant frequency 10 Hz above resonant frequency, and 10 Hz below resonant frequency.



$$f_r = \frac{1}{2\pi\sqrt{LC}} = 159.2 \, Hz$$

At 10 Hz below $= f_r - 10 = 149.2 \, Hz$

At 10 Hz above $= f_r + 10 = 169.2 \, Hz$

Impedance at resonance is equal to R

$$Z = 10 \, \Omega$$

at 149.2 Hz $X_{C_1} = \frac{1}{\omega_1 C} = 106.6 \, \Omega$

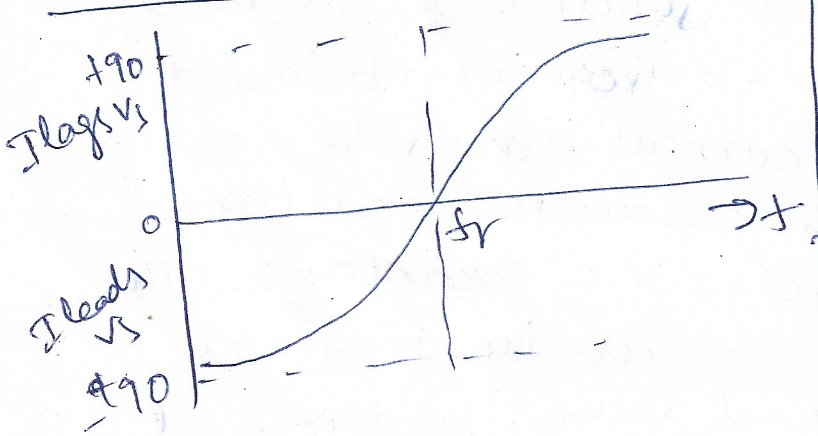
at 169.2 Hz $X_{C_2} = \frac{1}{\omega_2 C} = 94.06 \, \Omega$

at 149.2 $X_{L_1} = \omega_1 L = 93.75 \, \Omega$

at 169.2 $X_{L_2} = \omega_2 L = 106.31 \, \Omega$

at 149.2 $Z = \sqrt{R^2 + (X_{L_1} - X_{C_1})^2} = 16.28 \, \Omega$ $X_{C_1} > X_{L_1}$ Capacitive

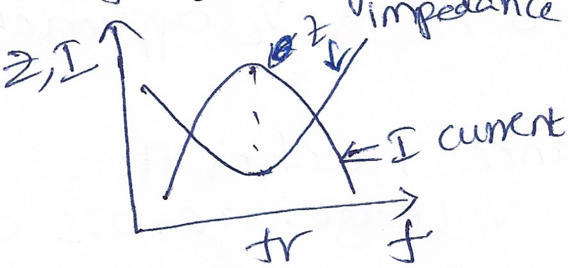
at 169.2 $Z = \sqrt{R^2 + (X_{L_2} - X_{C_2})^2} = 15.81 \, \Omega$ $X_{C_2} < X_{L_2}$ Inductive.



the current lags behind the source voltage, because the inductive reactance is greater than capacitive reactance. As the frequency goes higher, the phase angle approaches 90°.

VOLTAGES & CURRENT IN A SERIES RESONANT CIRCUIT :-

The variation of impedance and current with frequency is shown in fig (1).



At resonant frequency, the capacitive reactance is equal to inductive reactance, and hence the impedance is minimum. Because of the minimum impedance, maximum current flows through the circuit.

The voltage drops across resistance, inductance and capacitance also varies with frequency. At $f=0$, the capacitor acts as an open circuit and blocks the current. The complete source voltage appears across the capacitor. As the frequency increases, X_C decreases and X_L increases, causing total reactance $X_C - X_L$ to decrease. As a result, the impedance decreases and the current increases. As the current increases V_R also increases and both V_C & V_L increase.

When the frequency reaches it resonant value f_r, the impedance is equal to R & hence, the current reaches its maximum value and V_R is at its maximum value.

As the frequency is increased above resonance, X_L continues to increase and X_C continues to decrease, causing the total reactance, $(X_L - X_C)$ to increase. As a result there is an increase in impedance and a decrease in current. As the current decreases, V_R also decreases & V_C and V_L decrease. As the frequency becomes very high, the current approaches zero, both V_R and V_C approaches zero, and V_L approaches V_S.

The drop across resistance reaches its maximum when $f = f_r$. Voltage across C and voltage across L is not maximum at resonant frequency. At resonant frequency f_r, the voltages V_C & V_L are equal in magnitude but opposite in phase. The voltage V_C is maximum at frequency f_C which is less than f_r and voltage V_L is maximum at frequency f_L which is greater than f_r.

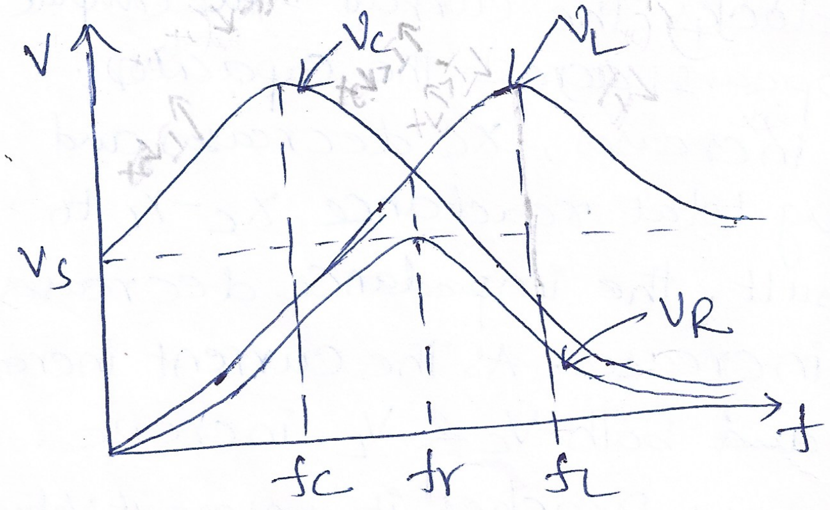


fig ②

Frequencies for maximum voltages across L and C:

Consider that voltage across capacitor is V_C and it is given by,

$$V_C = I\left(\frac{1}{\omega C}\right), \quad \text{but } I = \frac{V}{Z}$$

$$V_C = \frac{V}{\omega C\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \rightarrow (1)$$

To find frequency at which V_C is maximum we have to differentiate w.r.t 'ω' and equate it to zero. But first removing radical sign by squaring expression. Then equating $\frac{dV_C^2}{d\omega}=0$ since when V_C^2 is maximum, V_C is maximum.

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By squaring eqn ①, we have

$$V_C^2 = \frac{V^2}{\omega^2C^2\left(R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right)} \Rightarrow V_C^2 = \frac{V^2}{\omega^2C^2\left[R^2+\frac{(\omega^2LC-1)^2}{\omega^2C^2}\right]}$$

$$V_C^2 = \frac{V^2}{\omega^2R^2C^2+(\omega^2LC-1)^2}$$

Now differentiate & equate to zero.

$$\frac{dV_C^2}{d\omega} = \frac{V^2\left[R^2C^2\,2\omega+2(\omega^2LC-1)(2\omega LC)\right]}{\left[\omega^2R^2C^2+(\omega^2LC-1)^2\right]^2}=0$$

equating only numerator term to zero, we have

$$V^2\left[2\omega R^2C^2+2(\omega^2LC-1)(2\omega LC)\right]=0$$

But V is input which cannot be zero.

$$2\omega R^2C^2+2(\omega^2LC-1)(2\omega LC)=0$$

$$2\omega R^2C^2+4\omega^3L^2C^2-4\omega LC=0$$

$$4\omega^3L^2C^2=4\omega LC-2\omega R^2C^2$$

$$\omega^2=\frac{4\omega LC}{4\omega L^2C^2}-\frac{2\omega R^2C^2}{4\omega L^2C^2}$$

$$\omega=\sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}} \text{ rad/sec}$$

The frequency at which capacitor voltage V_C is maximum, is given by

$$f_C=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}$$

$$f_C=\frac{1}{2\pi\sqrt{LC}}\sqrt{1-\frac{R^2C}{2L}}=f_r\sqrt{1-\frac{R^2C}{2L}}$$

The voltage across inductor is V_L is given by

$$V_L = I\cdot(\omega L), \text{ but } I = \frac{V}{Z}$$

$$V_L = \frac{V(\omega L)}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} \rightarrow (2)$$

squaring eqn (2) $\quad V_L^2 = \dfrac{V^2\omega^2L^2}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$

$$V_L^2 = \frac{V^2\omega^2L^2}{R^2+\frac{(\omega^2LC-1)^2}{\omega^2C^2}} = \frac{V^2\omega^4L^2C^2}{\omega^2R^2C^2+(\omega^2LC-1)^2}$$

By differentiating V_L^2 with respect to ω and equating only numerator term to zero

$$\frac{dV_L^2}{d\omega} = \frac{\left[\omega^2R^2C^2+(\omega^2LC-1)^2\right]4\omega^3V^2L^2C^2 - V^2\omega^4L^2C^2\left[2\omega R^2C^2+2(\omega^2LC-1)2\omega LC\right]}{\left[\omega^2R^2C^2+(\omega^2LC-1)^2\right]^2} = 0$$

$$V^2\omega^3L^2C^2\left[4\omega^2R^2C^2+4(\omega^2LC-1)^2-2\omega^2R^2C^2-4\omega^2LC(\omega^2LC-1)\right]$$

$$2\omega^2R^2C^2+4\omega^4L^2C^2+4-2\omega^2LC - 4\omega^4L^2C^2+4\omega^2LC = 0.$$

$$2\omega^2LC-\omega^2R^2C^2-2=0$$

$$\omega^2(2LC-R^2C^2)=2 \Rightarrow \omega^2 = \frac{2}{2LC-R^2C^2}$$

$$\omega^2 = \frac{1}{LC-\frac{R^2C^2}{2}}$$

$$\omega = \sqrt{\frac{1}{LC-\frac{R^2C^2}{2}}} \text{ rad/sec.}$$

$$f_L = \frac{1}{2\pi\sqrt{LC-\frac{R^2C^2}{2}}} = \frac{1}{2\pi\sqrt{LC}\sqrt{1-\frac{R^2C^2}{2LC}}}$$

$$= \frac{1}{2\pi\sqrt{LC}\sqrt{1-\frac{R^2C}{2L}}}$$

$$= f_r\Big/\sqrt{1-\frac{R^2C}{2L}}$$

① A series RLC circuit with resistance of 10Ω, inductance of 0.1H and capacitance of 50μF is supplied by voltage source of 50V. Find resonant frequency and the frequencies at which maximum voltage appears across L and C. 3-6

$R = 10\Omega, \quad L = 0.1H, \quad C = 50\mu F$

(i) Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}} = 71.18 Hz$.

(ii) Frequency f_C at which maximum voltage appears across capacitor is given by.

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = 70.28 Hz$$

(iii) Frequency f_L at which maximum voltage appears across inductor is given by.

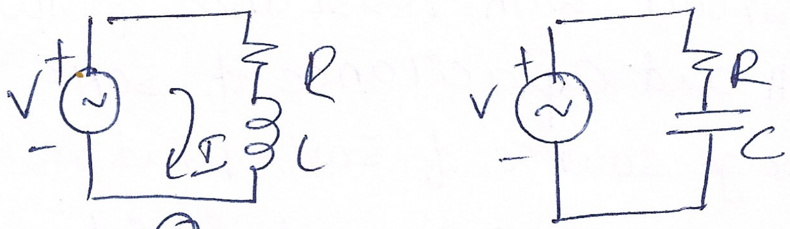
$$f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2C^2}{2}}} = 72.08 Hz$$

Q-factor (or) Figure of Merit

The resonance phenomenon is observed in ac circuits consisting reactive elements such as inductor and capacitor. These two elements are basic passive elements of energy storing type. It is found convenient to express the efficiency with which these elements store energy. It is found simpler to compare various inductors and capacitors in terms of efficiency while designing such circuits. Such efficiency is measured as quality factor. It is also called figure of merit.

The figure of merit (or) Q-factor is defined as

$$Q = 2\pi \times \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}}$$



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Consider the sinusoidal voltage 'V' is applied to an inductor with leakage resistance in series.

→ The maximum energy stored per cycle is given by $W_L = \frac{1}{2} L I_m^2$

where I_m = peak (or) maximum value of current in the circuit.

→ The average power dissipated in inductor is given by $P = \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_m^2 R$.

The total energy dissipated per cycle is given by

Energy dissipated per cycle $= \frac{P}{f} = \frac{\frac{1}{2} I_m^2 R}{f}$

where f = frequency of operation.

The quality of inductor is given by

$$Q = 2\pi \times \frac{\frac{1}{2} L I_m^2}{\frac{1}{2} I_m^2 R / f} = \frac{(2\pi f) L}{R} = \frac{\omega L}{R}.$$

Now consider 'V' is applied to a capacitor with a small resistance R in series.

The maximum energy stored per cycle is given by

$W_c = \frac{1}{2} C V_m^2 = \frac{1}{2} C \left[\frac{I_m}{\omega C}\right]^2 = \frac{1}{2} \frac{I_m^2}{\omega^2 C}$ where V_m = peak (or) max value of applied voltage

Everage dissipated / cycle $= \frac{P_D}{f} = \frac{\frac{1}{2} I_m^2 R}{f}$

$$Q = 2\pi \times \frac{\frac{1}{2} I_m^2 / \omega^2 C}{\frac{1}{2} I_m^2 R / f} = \frac{2\pi f}{\omega^2 R C} = \frac{1}{\omega R C}$$

Series Resonant Circuit as Voltage amplifier-

Voltage across L and C under Resonance:-

Under resonance, the current in the circuit is of maximum value and is given by.

$$I_0 = \frac{V}{Z_0} = \frac{V}{R}$$

. Under resonance, Voltage across L is given by

$$V_L = (I_0)(jX_L) = \frac{V}{R}(j\omega_0 L) = j\left(\frac{\omega_0 L}{R}\right)V.$$

$$V_L = jQ_0 V = Q_0 V \angle +90° \text{ Volts.}$$

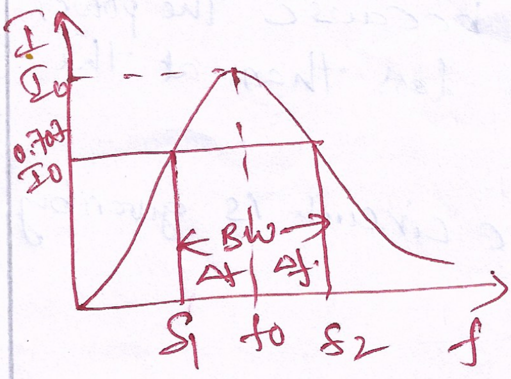
Under resonance, voltage across C is given by

$$V_C = I_0(-jX_C) = \frac{V}{R}\left(\frac{-j}{\omega_0 C}\right) = -j\left(\frac{1}{\omega_0 C R}\right)V$$

$$V_C = -jQ_0 V = Q_0 V \angle -90° \text{ Volts.}$$

From the expressions of voltages V_L & V_C it is clear that the magnitudes of V_L & V_C are same but they are out of phase, hence both cancel out each other under resonance. Moreover the magnitudes of V_L & V_C are Q_0 times applied voltage V. But Q_0 is a number greater than unity. Thus under resonance voltages developed across L & C are greater than the applied voltage V. In other words, under resonance, series resonant circuit acts as voltage amplifier where Q_0 is amplification (or) magnification factor.

Bandwidth & selectivity of series resonant circuit:-



Bandwidth of series RLC circuit is defined as the band of frequencies over which the power in the circuit is half of its maximum value.

At frequency of resonance, the maximum current I_0 is given by $I_0 = V/R$. This current is maximum, as impedance is minimum at resonance. Hence at resonant frequency, power in the circuit is maximum & given by

$P_0 = P_{max} = I_0^2 R$, half of maximum power is given by

$P' = \frac{P_0}{2} = \frac{I_0^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 . R$.

So at the frequencies, where the power in the circuit is half of its maximum value, current becomes $\frac{1}{\sqrt{2}}$ times (or) 0.707 times of its maximum value. The frequencies at which power in the circuit is half of its maximum value are called half power frequencies.

At f_r, $P_0 = P_{max} = I_0^2 . R$

At frequency f_1, power in circuit is half & it is given by $P' = \frac{1}{2} I_0^2 R$.

At frequency f_2, power in ckt is half & is given by $P' = \frac{1}{2} I_0^2 . R$.

f_1 is called lower half-power frequency and f_2 is called upper half power frequency

Bandwidth $= (f_2 - f_1)$ Hz

The half-power frequencies are also referred as 3dB frequencies (or) 3dB points because the power at these frequencies is 3dB less than at the resonance.

The current in a series RLC circuit is given by

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At half power point,

$$I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{V}{R}$$

$$\frac{1}{\sqrt{2}}\frac{\cancel{V}}{R} = \frac{\cancel{V}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

squaring on both sides.

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2 \Rightarrow \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R \rightarrow (1)$$

* At half-power frequencies f_1 & f_2, the reactive part of impedance of series RLC circuit is equal to resistive part of impedance.

eqn (1) is quadratic in ω, which gives two values of ω as ω_1 & ω_2

we can write.

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \rightarrow (2)$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \rightarrow (3)$$

Add eqn 2 & 3 $\Rightarrow (\omega_1 + \omega_2)L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = 0$

$$(\omega_1 + \omega_2)L - \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C} = 0$$

$$(\omega_1 + \omega_2)L = \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C} \Rightarrow \omega_1 \omega_2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_1 \omega_2 = \omega_0^2 \Rightarrow f_1 f_2 = f_0^2 \Rightarrow f_0 = \sqrt{f_1 f_2} \rightarrow (4)$$

eqn (4) shows that the resonant frequency is the geometric mean of two half power frequencies.

subtract eqn (3) from (2)

$$(\omega_2-\omega_1)L+\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right)\frac{1}{C}=2R$$

$$(\omega_2-\omega_1)+\left(\frac{\omega_2-\omega_1}{\omega_1\omega_2}\right)\frac{1}{LC}=\frac{2R}{L}$$

$$\therefore\ \omega_1\omega_2=\frac{1}{LC}$$

$$(\omega_2-\omega_1)+(\omega_2-\omega_1)=\frac{2R}{L}$$

$$\omega_2-\omega_1=R/L$$

$$f_2-f_1=\frac{R}{2\pi L}$$

$$B\cdot W=f_2-f_1=\frac{R}{2\pi L}=2\Delta f \qquad \Delta f=\frac{R}{4\pi L}$$

Selectivity of resonant circuit is defined as the ability of circuit to discriminate (or) distinguish between desired & undesired frequencies.

selectivity is also defined as the ratio of resonant frequency to the bandwidth of resonant circuit.

$$\text{selectivity}=\frac{f_r}{BW}=\frac{f_0}{f_2-f_1}=\frac{f_0}{R/2\pi L}=\frac{2\pi f_0 L}{C}=\frac{\omega_0 L}{R}=Q_0$$

Selectivity of series resonant circuit is directly proportional to the quality factor of circuit at resonant frequency.

If Q_0 is very high, amplitude response curve becomes sharper effectively decreasing B.W

$$Q_0=\frac{f_r}{BW}=\frac{f_r}{f_2-f_1}$$

$$BW=\frac{f_r}{Q_0}=f_2-f_1$$

Band width of series resonant circuit is inversely proportional to the quality factor Q_0 of the circuit at resonant frequency

① A series resonant ckt has a bandwidth of 100Hz & Contains a 20mH inductance & a 2μF Capacitance. Determine (i) f_0 ② Q (iii) Z_{in} at resonance (iv) f_2

$$BW = f_2 - f_1 = 100\,Hz$$

$$L = 20\,mH$$

$$C = 1\,\mu F$$

(i) $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

(ii) $Q = \dfrac{f_0}{BW}$

$BW = \dfrac{R}{2\pi L} \Rightarrow$ (iii) $R = BW \cdot 2\pi L$

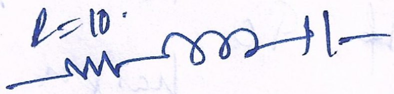
(iv) $f_2 = f_0 + \Delta f = f_0 + \dfrac{R}{4\pi L}$

2. Design a series RLC circuit that will have an impedance of $10\,\Omega$ at the resonant frequency of $\omega_0 = 100$ rad/s & Quality factor of 80. Find the Bandwidth.

$\omega_0 = 100$ rad/s. $Z = R = 10\,\Omega$

$Q = 80$

$BW = \dfrac{f_0}{Q} = \dfrac{2\pi f_0}{2\pi Q} = \dfrac{\omega_0}{2\pi Q}$



$BW = \dfrac{R}{2\pi L}$

$L = \dfrac{R}{2\pi \cdot BW}$

$f_r = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow C = \dfrac{1}{f_r^2 (2\pi)^2 L}$

Expression for Bandwidth

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \rightarrow (1)$$

At half power points $I = \frac{I_0}{\sqrt{2}}$

$$I_0 = \frac{V}{R}$$

$$I = \frac{V}{\sqrt{2}R} \rightarrow (2)$$

from eqn (1) & (2)

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

Squaring on both sides.

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0.$$

$$\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0.$$

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

for low values of R, the term $\left(\frac{R}{4L}\right)^2$ can be neglected in comparision with term

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}}$$

The resonant frequency for this circuit

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \pm \frac{R}{2L} + \omega_0$$ Considering only +ve signs of ω_0

$$\omega_1 = \omega_0 - \frac{R}{2L} \Rightarrow f_1 = f_0 - \frac{R}{4\pi L}$$

$$\omega_2 = \omega_0 + \frac{R}{2L} \qquad f_2 = f_0 + \frac{R}{4\pi L}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L}$$

$$\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}$$

$$\text{selectivity} = \frac{f_r}{\text{Bandwidth}} = \frac{1/\cancel{2\pi}\sqrt{LC}}{R/\cancel{2\pi} L} = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

$$= \frac{\omega_0 L}{R} = Q.$$

Quality factor:- It is measure of voltage magnification in the ~~XX~~ series resonant circuit. It is also measure of selectivity (or) sharpness of series resonant circuit.

$$Q_0 = \frac{\text{Voltage across Inductor (or) capacitor}}{\text{Voltage at resonance}} = \frac{V_{L0}}{V}, \frac{V_{C0}}{V}$$

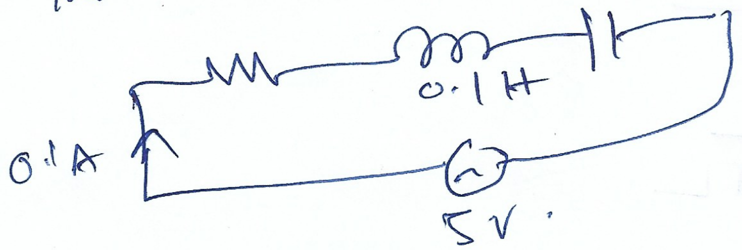
$$Q_0 = \frac{I_0 X_{L0}}{I_0 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

$$Q_0 = \frac{\frac{1}{\sqrt{LC}} \times L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

1. An RLC series circuit with a resistance of 10Ω inductance of 0.2H and a capacitance of 40μF is supplied with a 100V supply at variable frequency. Find the following w.r.t. the series resonant circuit.

(a) resonant frequency (b) current (c) power (d) power factor.
(e) voltage across R, L & C at resonant freq
(f) quality factor (g) half-power points.
(h) resonant curve & phasor diagram at reso

2. A minimum current of 0.1 A flows through the cir when the capacitor is at 5MF with a fixed frequency at a voltage of 5V. Determine the frequency at which the circuit resonates; the Band width, quality factor Q_o & the values of resistance at resonant fre



$I_o = 0.1A$

$R = \frac{V}{I_o} = \frac{5}{0.1} = 50\Omega.$

$f_o = \frac{1}{2\pi\sqrt{LC}}$ $\quad Q_o = \frac{1}{R}\sqrt{\frac{L}{C}}$

$BW = \frac{R}{2\pi L} =$

3. A series RLC Ckt has $R = 10\Omega$ & $L = 60mH$. At a frequency of 25Hz, the PF of the circuit is 45° lead. At what frequency will the ckt resonant

$\tan\phi = \frac{X_C - X_L}{R}$

$\tan 45 =$

$X_C = 19.42\Omega$

$X_C = \frac{1}{2\pi f C}$

$C = 327.82\mu F$

$f_o = \frac{1}{2\pi\sqrt{LC}}$

1. A series RLC circuit consists of a resistance of $1k\Omega$ and an inductance of 100mH in series with capacitance of 10pF. If 100V is applied as input across the combination determine, (i) The resonant frequency (ii) maximum current in the circuit (iii) Q-factor of the ckt (iv) The half-power frequencies. 3-8

$R = 1k\Omega$ $L = 100mH$ $C = 10pF$

(i) $f_r = \frac{1}{2\pi\sqrt{LC}} = 159.15KHz$

(ii) Max current $I_0 = \frac{V}{R} = 0.1A$.

(iii) $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = 100$

(iv) Bandwidth $= 2\Delta f = f_2 - f_1 = \frac{R}{2\pi L}$

$\Delta f = \frac{R}{4\pi L} = 795.77Hz$

(v) Lower half power frequency $f_1 = f_r - \Delta f$
$= 158.35kHz$

Upper half power frequency $= f_2 = f_r + \Delta f$
$= 159.95kHz$.

2. A series RLC circuit consists of $R = 100\Omega$, $L = 100mH$ & $C = 10nF$. The applied voltage across circuit is 100V. (i) find resonant frequency w_0 (ii) quality factor Q_0 (iii) Two half power frequencies w_1 & w_2 (iv) Band width Δw.

(i) $w_0 = \frac{1}{\sqrt{LC}} = 31.6277 \text{ krad/se}$

(ii) $Q_0 = \frac{w_0 L}{R} = 31.6277$

(iii) Band width $(\Delta\omega) = \omega_2 - \omega_1 = \frac{R}{L} = 1000$ rad/sec

(iv) upper half power frequ. $\omega_2 = \omega_0 + \frac{\Delta\omega}{2}$

$= 32.1227$ Krad/se

$\omega_1 = \omega_0 - \frac{\Delta\omega}{2}$

$= 31.1227$ Kr/s

3. A series RLC circuit with 8Ω resistance should be designed to have a bandwidth of 50Hz; Determine the values of L & C so that the circuit resonates at 250Hz.

$R = 8\Omega \quad f_2 - f_1 = 50Hz \quad f_0 = 250Hz$

(i) $Q_0 = \frac{f_0}{f_2 - f_1} = \frac{250}{50} = 5$

$Q_0 = \frac{\omega_0 L}{R}$

$C = \frac{8 Q_0}{2\pi \times 250} = 25.46mH$

(ii) $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$

$C = \frac{L}{R^2 Q_0^2} = 15.9125\mu F$

~~C1, C2, C5, C6, D7, D8, E1, E4, E5~~

~~E9, F3, G2, G8, H1, H6,~~

~~25, 30, 34, 35,~~

3. A series connected RLC circuit has $R = 4$ & $L = 25mH$

(a) Calculate the value of C with a $Q = 50$.

(b) ω_1, ω_2 & B.W.

(c) average power at $\omega = \omega_0$, ω_1, ω_2 Take V_m = 100

(a) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow C = \frac{L}{Q^2 R^2}$

(b) $BW = \frac{R}{2\pi L}$ $\Delta f = \frac{R}{9\pi L}$

$f_r = \frac{1}{2\pi\sqrt{LC}} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$

$\omega_1 = \omega_r - \frac{R}{L}$ $\omega_2 = \omega_r + R/L$

(c) At ω_0 $P_0 = \frac{V^2}{R} = \frac{(V_m/\sqrt{2})^2}{R}$

at ω_1 & ω_2, power is half of the power at resonant frequency

$P' = \frac{P_0}{2}$

$V_{rms} = \frac{V_m}{\sqrt{2}}$

4. Given a series RLC ckt with $R = 100\,\Omega$, $L = 0.5H$, $C = 40\mu F$. Calculate f_r, f_1 & f_2.

$f_r = \frac{1}{2\pi\sqrt{LC}}$ $\Delta f = \frac{R}{4\pi L}$

$f_1 = f_r - \Delta f$

$f_2 = f_r + \Delta f$