Analog & Digital Communications

By

KASULA RAGHU
Assistant Professor
Dept. of E.C.E.,MGIT

UNIT- IV

Pulse Modulation
&
Pulse Code Modulation

Contents

- **Pulse Modulation:** Types of Pulse modulation- PAM, PWM and PPM. Comparison of FDM and TDM.
- **Pulse Code Modulation:** PCM Generation and Reconstruction, Quantization Noise, Non-Uniform, Quantization and Companding, DPCM, Adaptive DPCM, DM and Adaptive DM, Noise in PCM and DM.

ANALOG PULSE MODULATION SCHEMES

Pulse Modulation:

The process of transmitting the signals in the form of pulses by using some special techniques.

There are two types of pulse modulation systems,

1. Pulse Amplitude Modulation.
2. Pulse Width Modulation or Pulse Duration Modulation.
3. Pulse Position Modulation

Sampling theorem

Statement: A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal. i. e. $fs \geq 2fm$

(or)

- An analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.

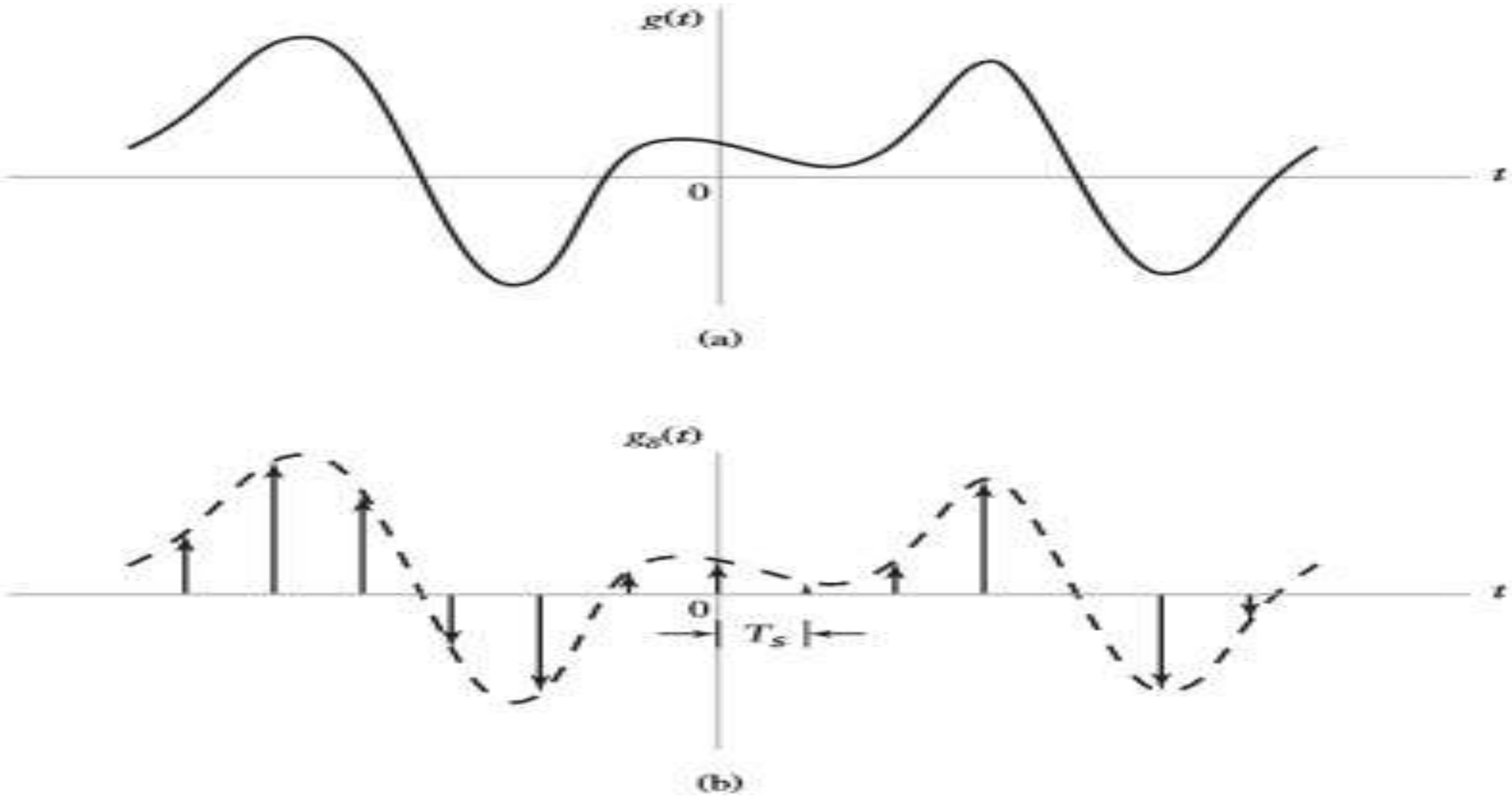
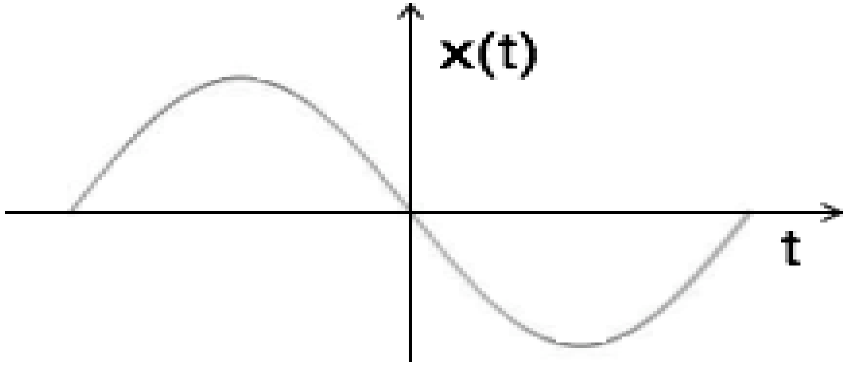
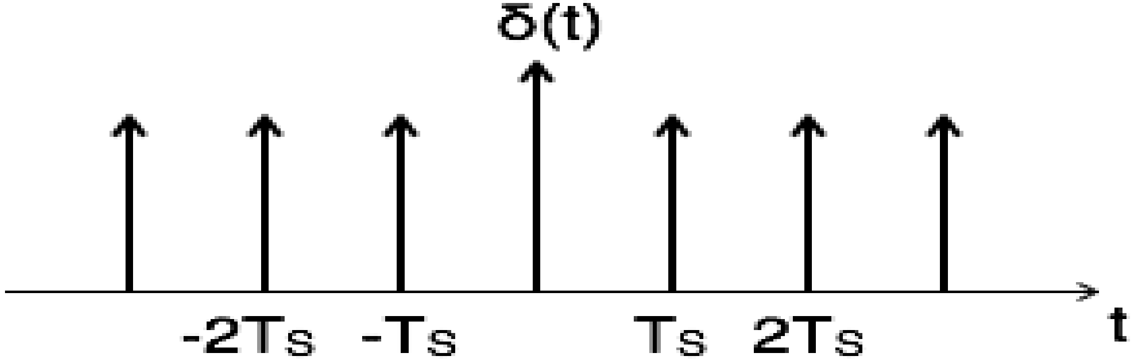
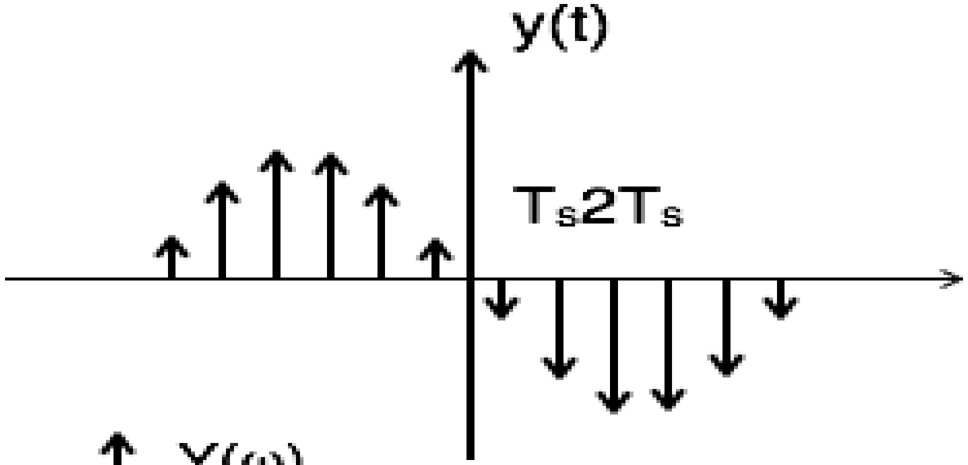


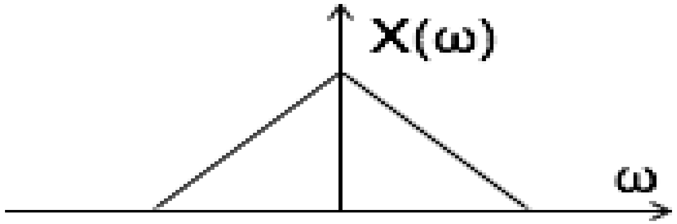
Figure: Illustration of sampling process. (a) Analog waveform (b) Instantaneously sampled representation of the analog Signal

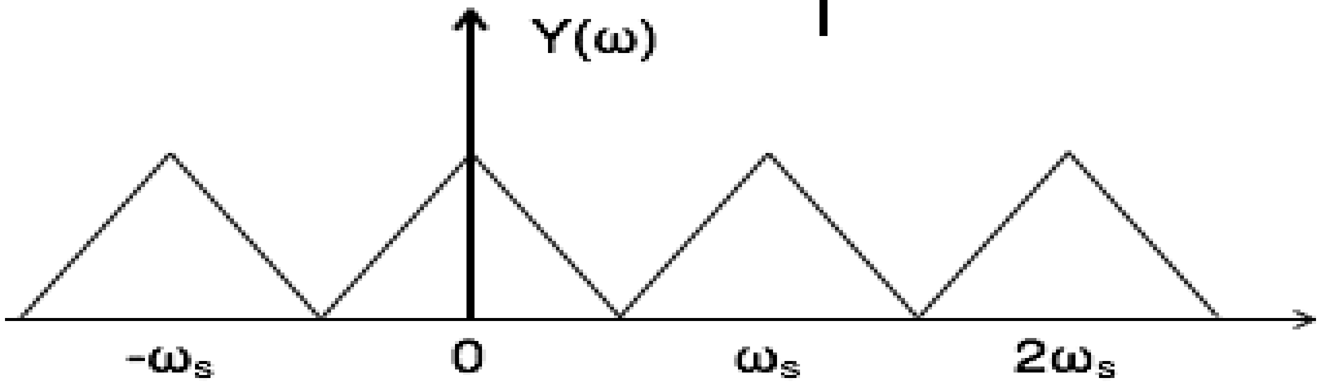

$x(t)$
t


$x(t)$
$\delta(t)$
multiplier
$y(t)$


$\delta(t)$
$-2T_S$
$-T_S$
T_S
$2T_S$
t


$y(t)$
$T_s 2T_s$


$X(\omega)$
ω


$Y(\omega)$
$-\omega_s$
0
ω_s
$2\omega_s$

$$g_\delta(t) = g(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

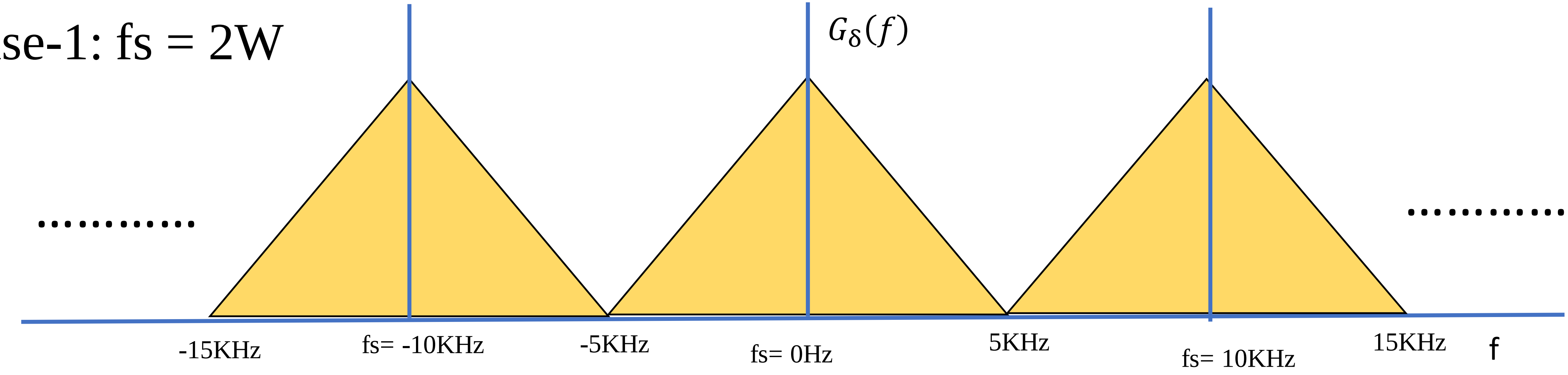
$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$$

$$G_\delta(f) = G(f) \circledast \frac{1}{T_s}\sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T_s})$$

We Know, $G(f) \circledast \delta(f - \frac{n}{T_s}) = G(f - \frac{n}{T_s})$

$$G_\delta(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(f - \frac{n}{T_s})$$

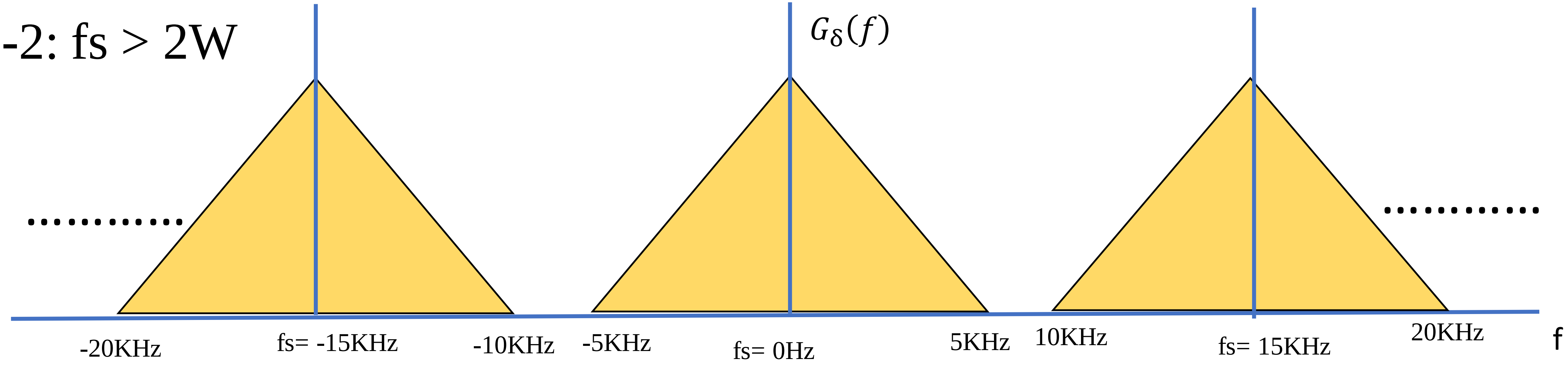
Case-1: fs = 2W



Let W= 5KHz Then Sampling Frequency will be equal to Greater than fs = 2W

Let fs= 10KHz (10,000 Samp./Sec)

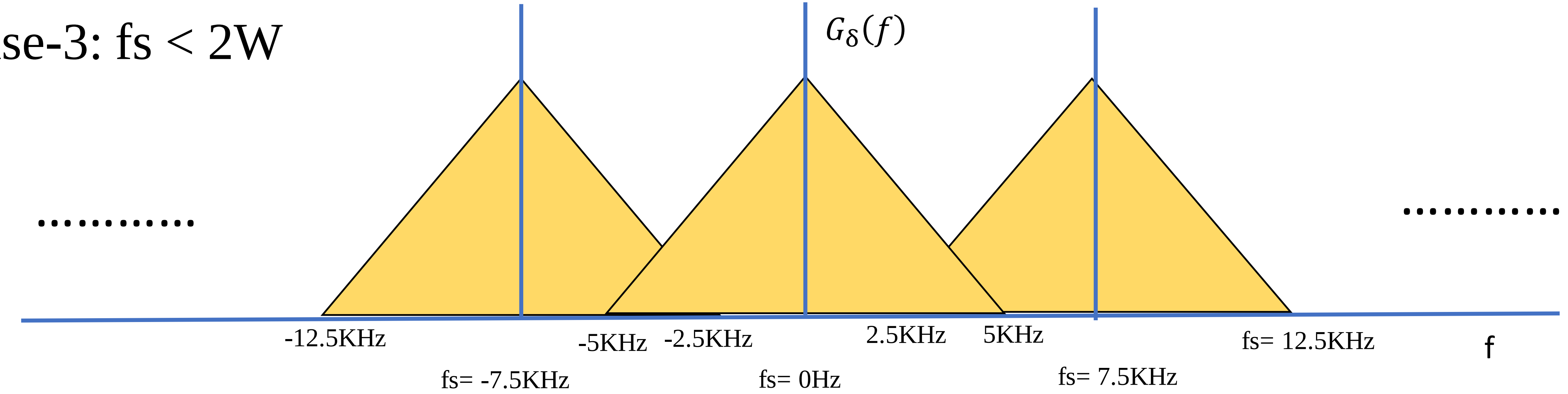
Case-2: fs > 2W



Let W= 5KHz Then Sampling Frequency will be equal to Greater than fs= 10KHz

Let fs= 15KHz (15,000 Samp./Sec)

Case-3: fs < 2W



Let W= 5KHz   Then Sampling Frequency will be equal to Greater than fs < 2W

Let fs= 7.5KHz   (7,500 Samp./Sec)

Drawback : Aliasing Effect

Corrective Measures for Aliasing

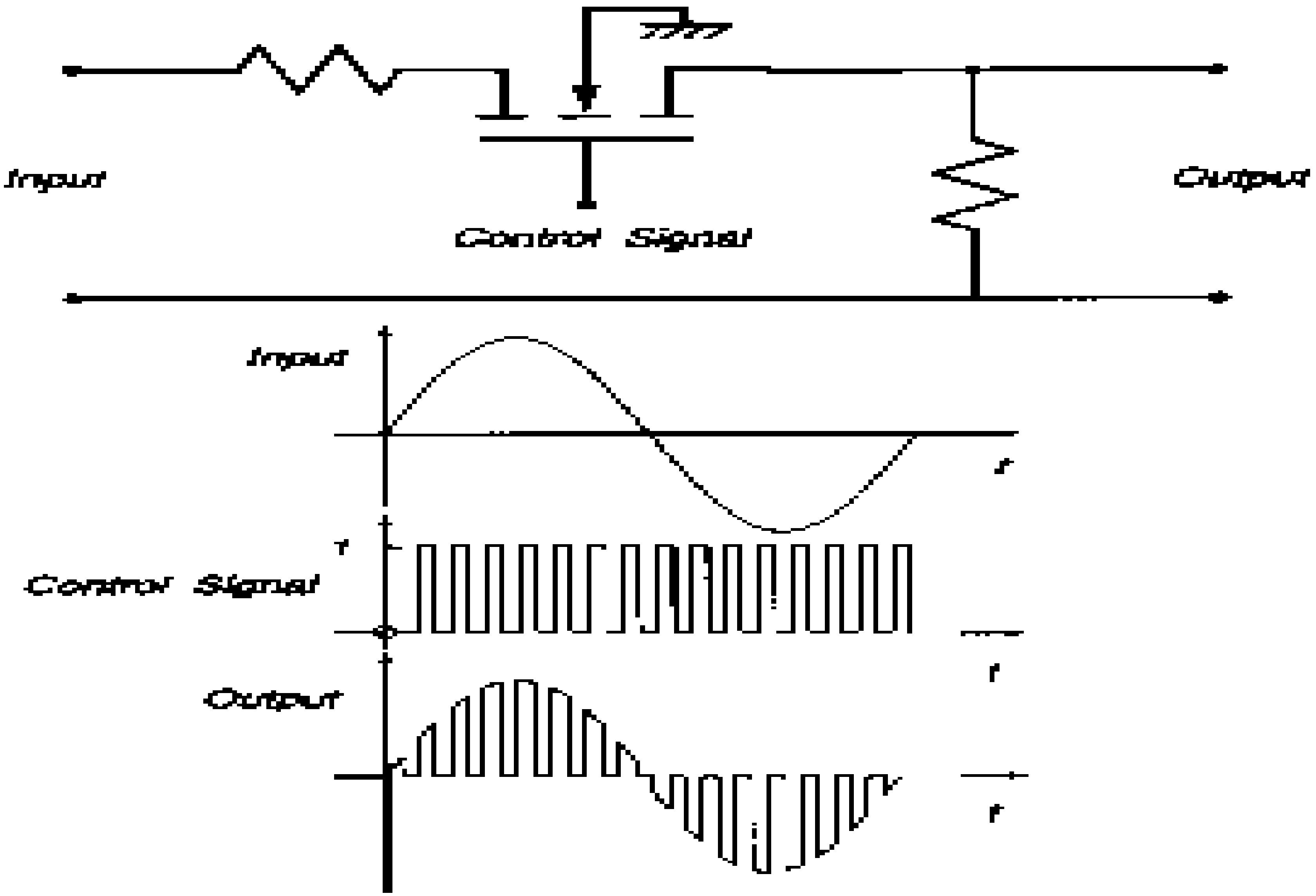
1. Prior to sampling, a low-pass anti-aliasing filter is used to attenuate those high frequency components of the signal that are not essential to the information being conveyed by the signal.
2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate. Also, it has the beneficial effect of easing the design of the reconstruction filter used to recover the original signal from its sampled version.

PULSE AMPLITUDE MODULATION(PAM)

In Pulse amplitude modulation, the amplitude of pulses of carrier pulse train is varied in accordance with the modulating signal.

In PAM , the pulses can be flat top type or natural type or ideal type. Out of these, flat top PAM is widely used because of easy noise removal.

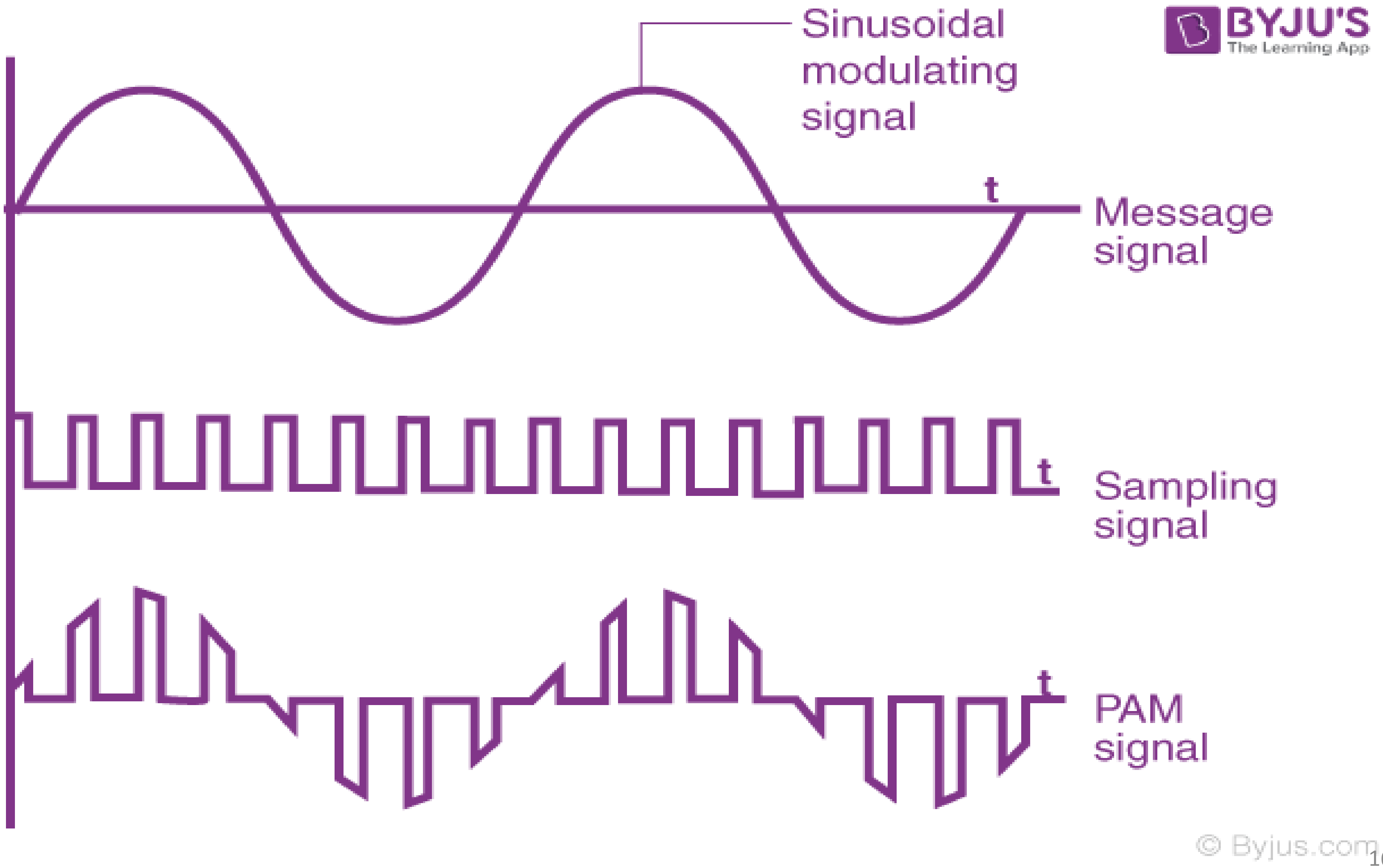
Natural Sampling



Natural Sampling

- The sample and hold circuit consists of two FETs and a capacitor.
- The sampling switch is closed for a short duration by a short pulse applied to the gate G1 of transistor.
- During this charged period, the Capacitor is quickly to a voltage equal to instantaneous sample value of incoming signal x(t).
- When the sampling switch is opened for next half of the duration capacitor is discharged to zero volts.
- Hence the output of circuit consists of a sequence of natural samples.

PAM GENERATION



$$s(t) = c(t)g(t)$$

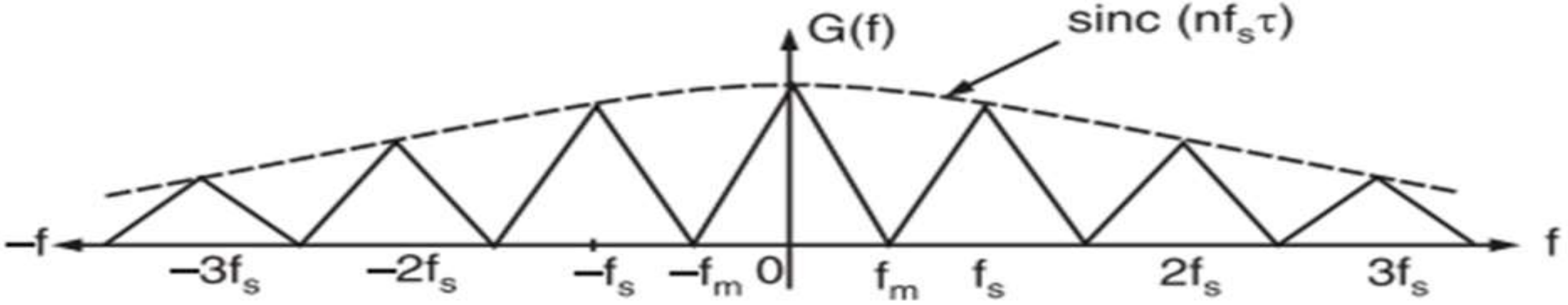
$$c(t) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} sinc\left(\frac{nT}{T_s}\right) \exp\left(\frac{j2\pi nt}{T_s}\right)$$

$$s(t\ ) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} sinc\left(\frac{nT}{T_s}\right) \exp\left(\frac{j2\pi nt}{T_s}\right) g(t)$$

Taking F.T on both sides

$$S(f) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} sinc\left(\frac{nT}{T_s}\right) G(f - \frac{n}{T_s})$$

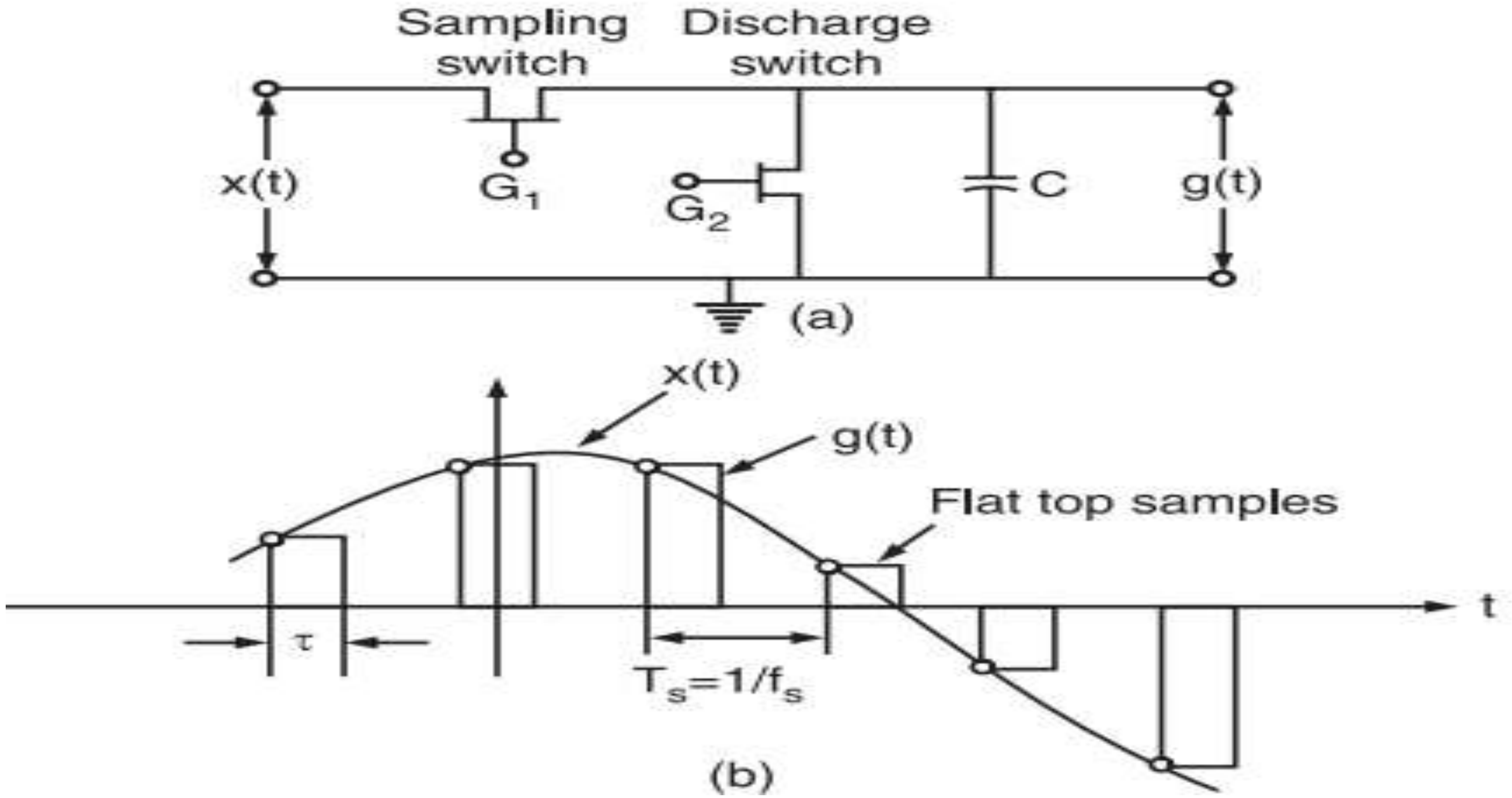
Spectrum of the Natural Sampled Signal



Note1: we Assumed that g(t) contains no frequencies outside –W to W, and sampling rate $\frac{1}{T_s}$ is greater than the Nyquist rate 2W, so that there is no Aliasing effect. We see that finite duration of the sampled pulses is to multiply with the nth Lobe of the spectrum S(f) by $\frac{AT}{T_s}$ $sinc\left(\frac{nT}{T_s}\right)$. The Original signal g(t) can be recovered from s(t) by passing through ideal LPF.

Note2: If AT=1 , so that each Rectangular pulse has Unit Area i.e. Pulse duration T approaches to Zero, then $s(f)$ approaches $G_\delta(f)$. **(Ideal Sampling)**

Flat-Top Sampling



$$s(t) = \sum_{n=-\infty}^{\infty} g(nT_s)h(t - nT_s)$$

The h(t) is a Rectangular Pulse of unit Amplitude and duration T. defined as below

$$h(t)= \begin{cases} 1 & 0 < t < T \\ \frac{1}{2} & t = 0, t = T \\ 0 & Otherwise \end{cases}$$

The instantaneous sampled version of g(t) is given by

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$$

Convolving the $g_\delta(t)$ *with pulse* $h(t)$, *we get*

$$g_\delta(t) * h(t) = \int_{-\infty}^{\infty} g_\delta(\tau)\, h(t - \tau)d\tau$$

$$g_\delta(t) * h(t) = \int_{-\infty}^{\infty} g_\delta(\tau)\, h(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t-nT_s)\, h(t-\tau)d\tau$$

$$= \sum_{n=-\infty}^{\infty} g(nT_s) \int_{-\infty}^{\infty} \delta(t-nT_s)h(t-\tau)d\tau$$

Using Shifting Property of Delta Function, we Obtain

$$g_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, h(t-nT_s)$$

Which is Mathematically Equivalent to the Convolution of instantaneous sampled signal version of g(t) and the pulse h(t).

$$s(t) = g_\delta(t) * h(t)$$

Taking F.T on Both Sides we Get,

$$S(f) = G_\delta(f) \cdot H(f)$$

$$S(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G\left(f - \frac{n}{T_s}\right) . H(f)$$

Finally, g(t) is a band limited signal with Sampling rate $\frac{1}{T_s}$ Greater than the Nyquist rate. Then s(t) is passed through LPF having TF in Frequency domain H(f) = T sinc(fT)exp(-jΠfT).

By Flat top Samples we have introduced Amplitude Distortion as well as delay of T/2. This effect is called as **Aperture effect.** The Distortion may be corrected by connecting an **Equalizer** in cascade with Low pass reconstruction filter. The Amplitude response of the equalizer is given by $\frac{1}{|H(f)|} = \frac{1}{T\ \mathrm{sinc}(fT)} = \frac{1}{T}\frac{\Pi f T}{\sin(\Pi f T)}$

Transmission bandwidth of PAM

In PAM signal the pulse duration τ is assumed to be very small compared to time period Ts (i.e τ < Ts)

If the maximum frequency in the modulating signal x(t) is fm then sampling frequency fs is given by fs $\geq$2fm Or Ts $\leq \frac{1}{2fm}$

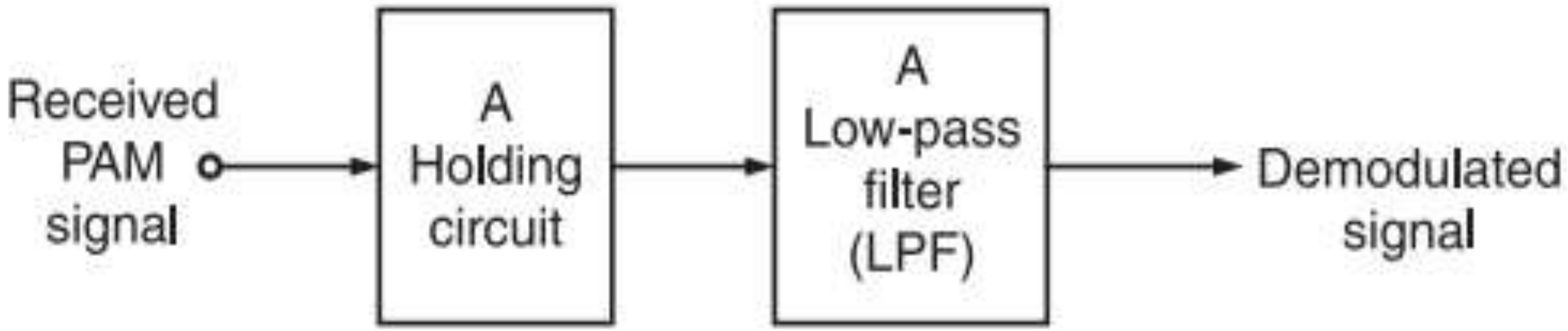
Therefore, $\tau \leq Ts \leq \frac{1}{2fm}$

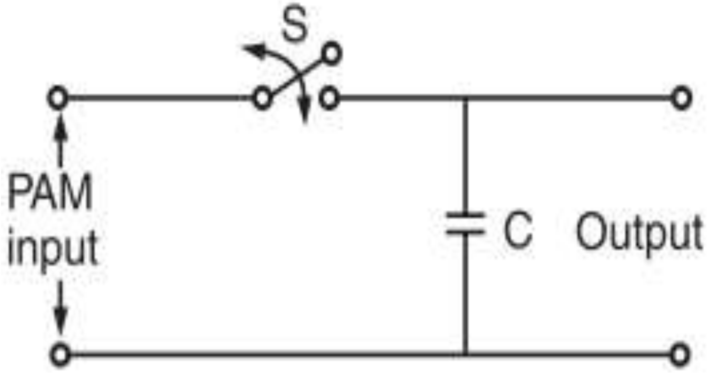
If ON and OFF time of PAM pulse is equal then maximum frequency of PAM pulse will be fmax = $\frac{1}{\tau+\tau} = \frac{1}{2\tau}$

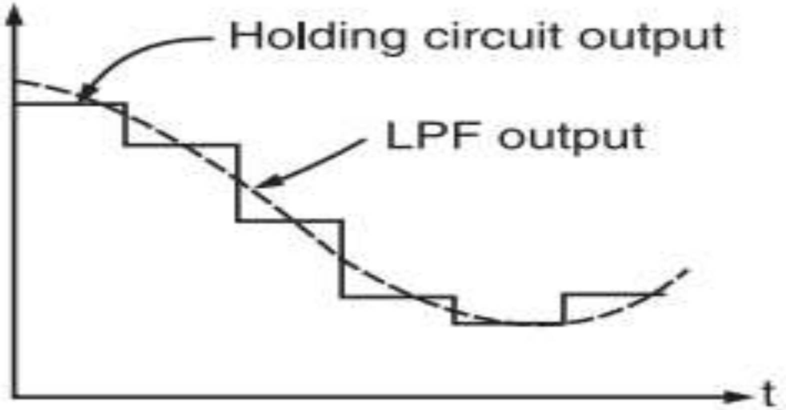
Therefore, Transmission bandwidth $= \frac{1}{2\tau} = \frac{1}{2\frac{1}{2fm}}$ = fm

Demodulation of PAM

Demodulation : It is the reverse process of getting message signal from the modulated signal.







Demodulation of PAM

- For PAM signals, demodulation is done using a holding circuit.
- The received PAM signal is first passed through a holding circuit and then through a lowpass filer.
- Switch S is closed during the arrival of the pulse and is opened at the end of the pulse.
- Capacitor C is charged to pulse amplitude value and holds this value during the interval between two pulses.
- Holding circuit output is then passed through a low pass filter to extract the original signal.

Advantages, Disadvantages of PAM

Advantages:

- It is the simple process for modulation and demodulation
- Transmitter and receiver circuits are simple and easy to construct.

Disadvantages:

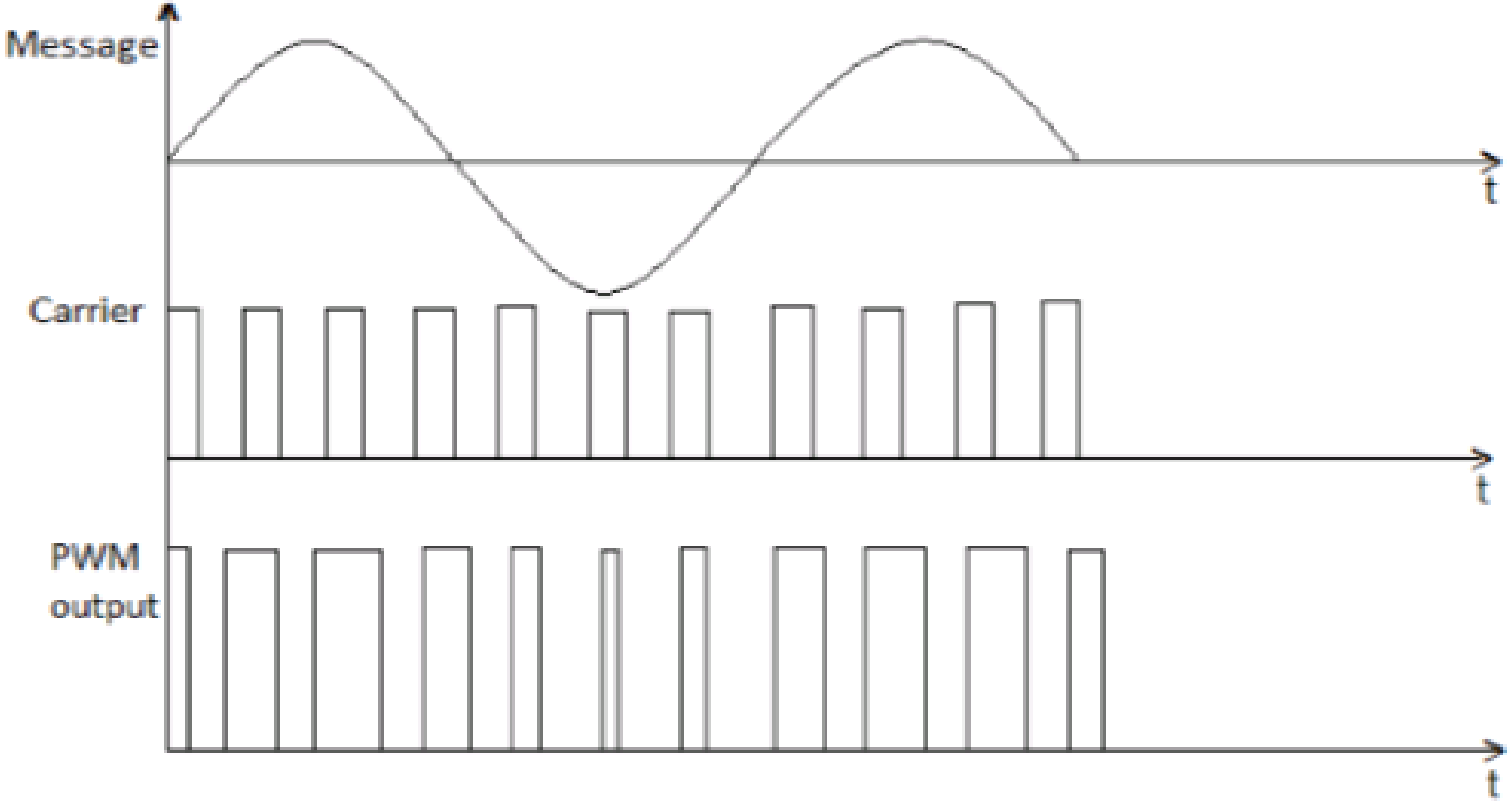
- Bandwidth requirement is high
- Interference of noise is maximum
- Power requirement is high

Applications:

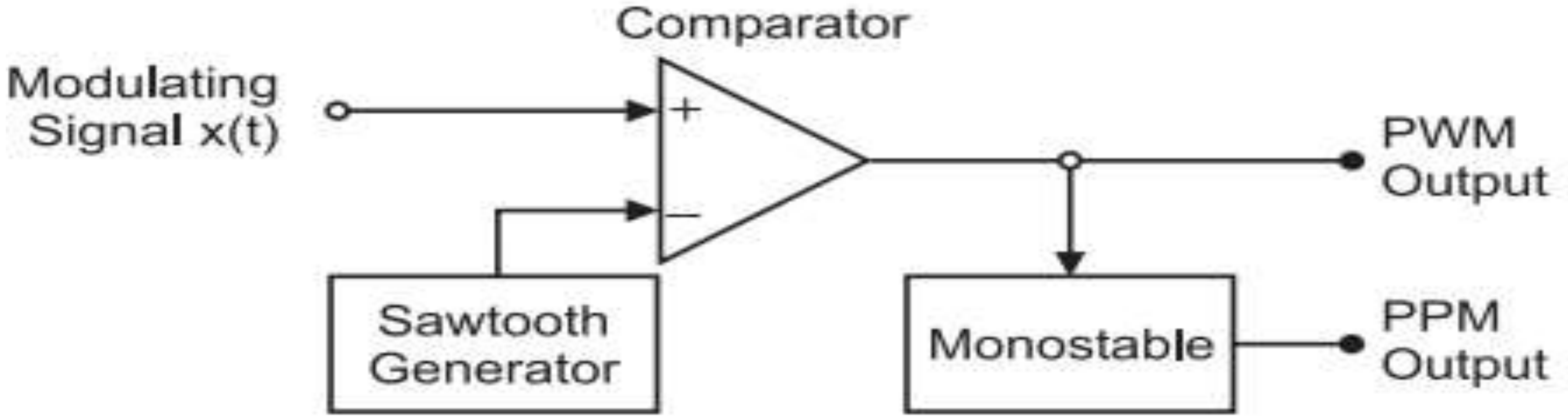
- Used in microcontrollers for generating control signals
- Used as electronic driver for LED lighting

Pulse Width Modulation(PWM)

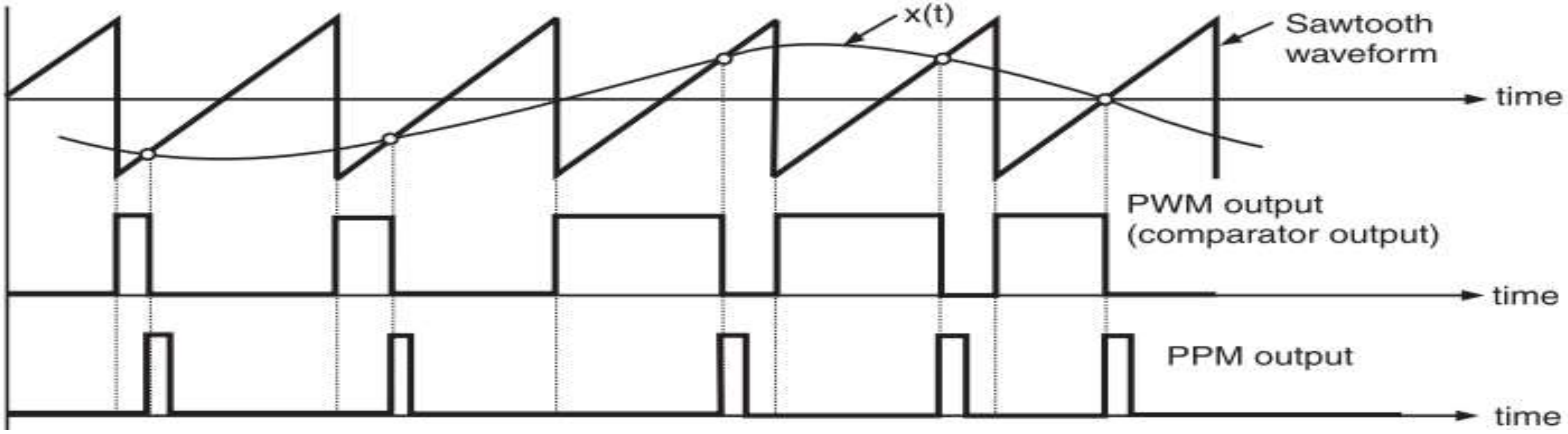
In PWM, the width of pulses of carrier pulse train is varied in proportion with amplitude of modulating signal.



GENERATION of PWM & PPM



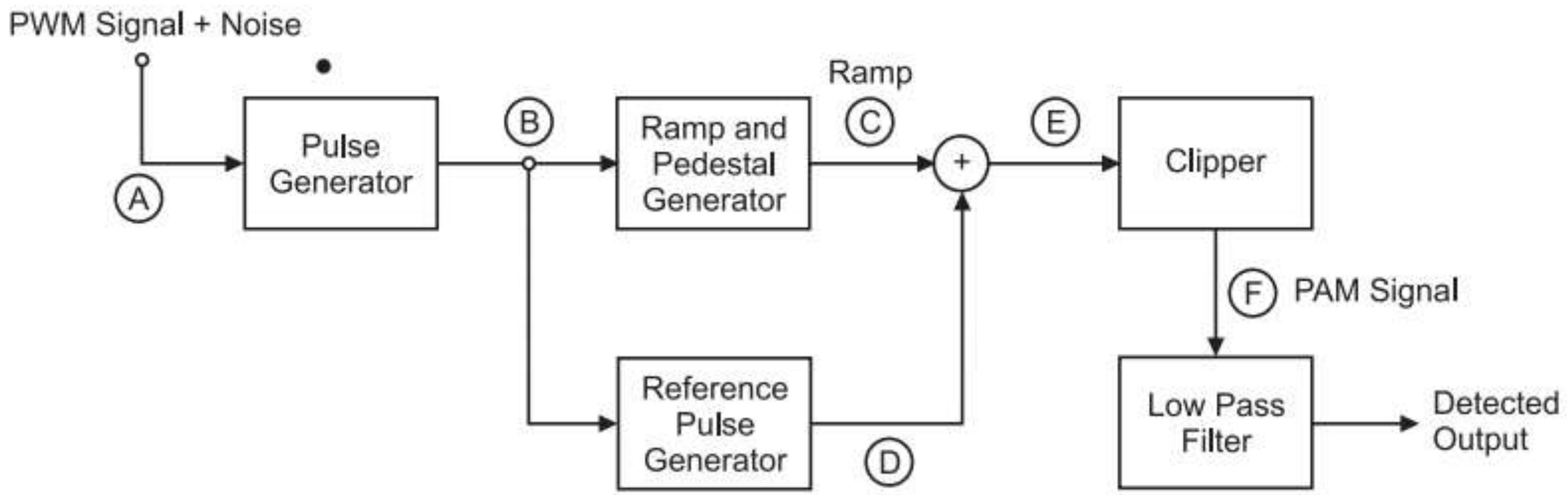
- A sawtooth generator generates a sawtooth signal of frequency f_s, and this sawtooth signal in this case is used as a sampling signal. It is applied to the inverting terminal of a comparator.
- The modulating signal x (t) is applied to the non-inverting terminal of the same comparator.
- The comparator output will remain high as long as the instantaneous amplitude of x (t) is higher than that of the ramp signal.
- This gives rise to a PWM signal at the comparator output as shown in fig.2 .



Here, it may be noted that the leading edges of the PWM waveform coincide with the falling edges of the ramp signal. Thus, the leading edges of PWM signal are always generated at fixed time instants.

However, the occurrence of its trailing edges will be dependent on the instantaneous amplitude of x(t). Therefore, this PWM signal is said to be trail edge modulated PWM.

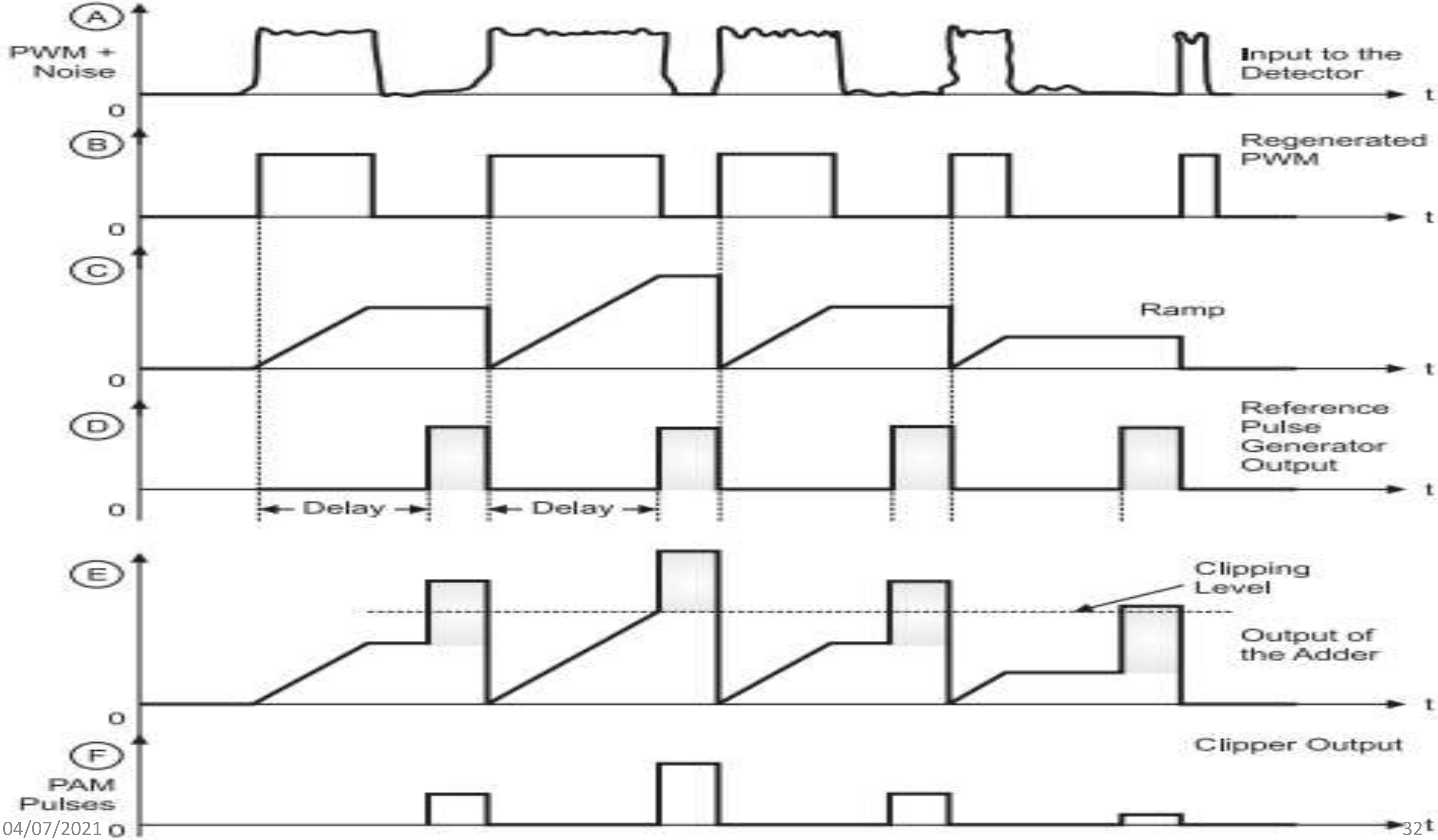
Detection of PWM



<u>The working operation of the circuit may be explained as under:</u>

- The PWM signal received at the input of the detection circuit is contaminated with noise. This signal is applied to pulse generator circuit which regenerates the PWM signal.
- Thus, some of the noise is removed and the pulses are squared up.

- The regenerated pulses are applied to a reference pulse generator. It produces a train of constant amplitude, constant width pulses.
- These pulses are synchronized to the leading edges of the regenerated PWM pulses but delayed by a fixed interval.
- The regenerated PWM pulses are also applied to a ramp generator. At the output of it, we get a constant slope ramp for the duration of the pulse. The height of the ramp is thus proportional to the width of the PWM pulses.
- At the end of the pulse, a sample and hold amplifier retains the final ramp voltage until it is reset at the end of the pulse.
- The constant amplitude pulses at the output of reference pulse generator are then added to the ramp signal.
- The output of the adder is then clipped off at a threshold level to generate a PAM signal at the output of the clipper.
- A low pass filter is used to recover the original modulating signal back from the PAM signal. The waveforms for this circuit have been shown in fig.4.


A
PWM +
Noise
0
Input to the
Detector
t
B
Regenerated
PWM
0
t
C
Ramp
0
t
D
Reference
Pulse
Generator
Output
0
t
Delay
Delay
E
Clipping
Level
Output of
the Adder
0
t
F
Clipper Output
PAM
Pulses
0
t

- **Advantages of PWM**

1. Less effect of noise i.e., very good noise immunity.
2. Synchronization between the transmitter and receiver is not essential(Which is essential in PPM).
3. It is possible to reconstruct the PWM signal from a noise, contaminated PWM, as discussed in the detection circuit. Thus, it is possible to separate out signal from noise (which is not possible in PAM).

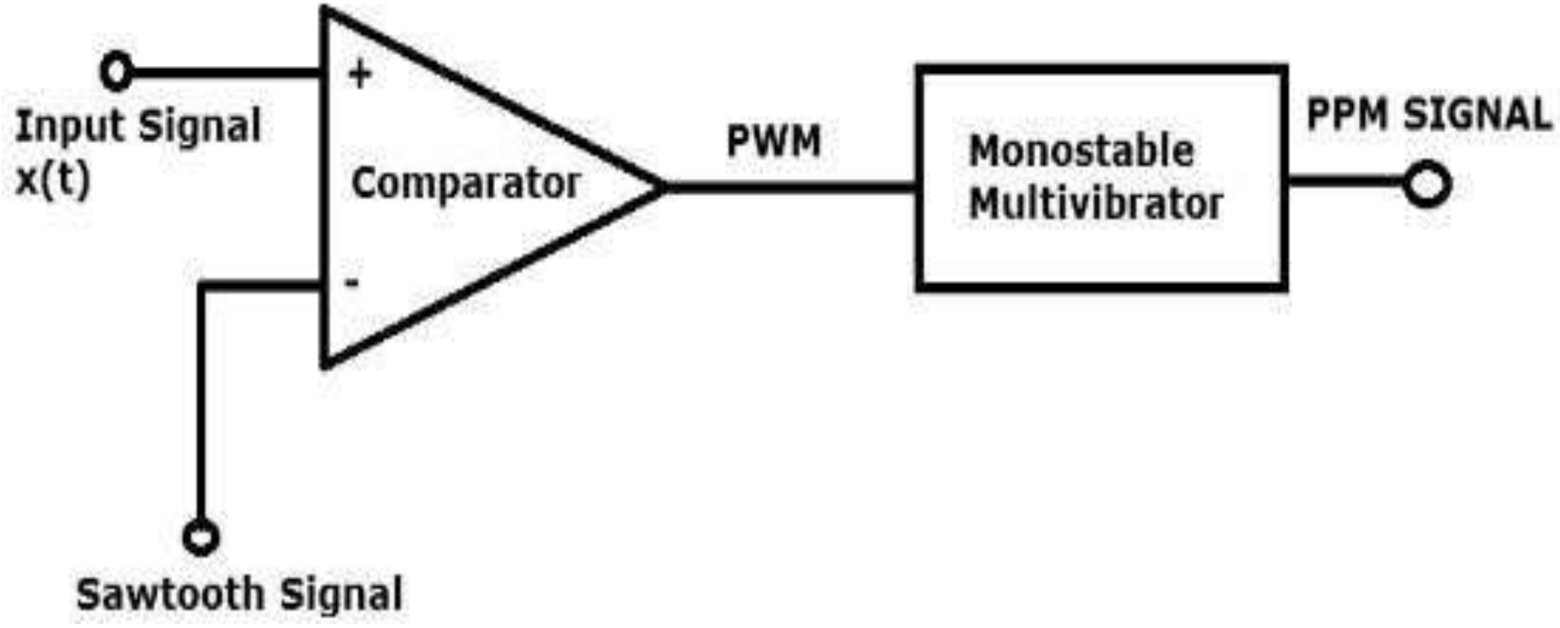
- **Disadvantages of PWM**

1. Due to the variable pulse width, the pulses have variable power contents. Hence, the transmission must be powerful enough to handle the maximum width, pulse, though the average power transmitted can be as low as 50% of this maximum power.
2. In order to avoid any waveform distortion, the bandwidth required for the PWM communication is large as compared to bandwidth of PAM.

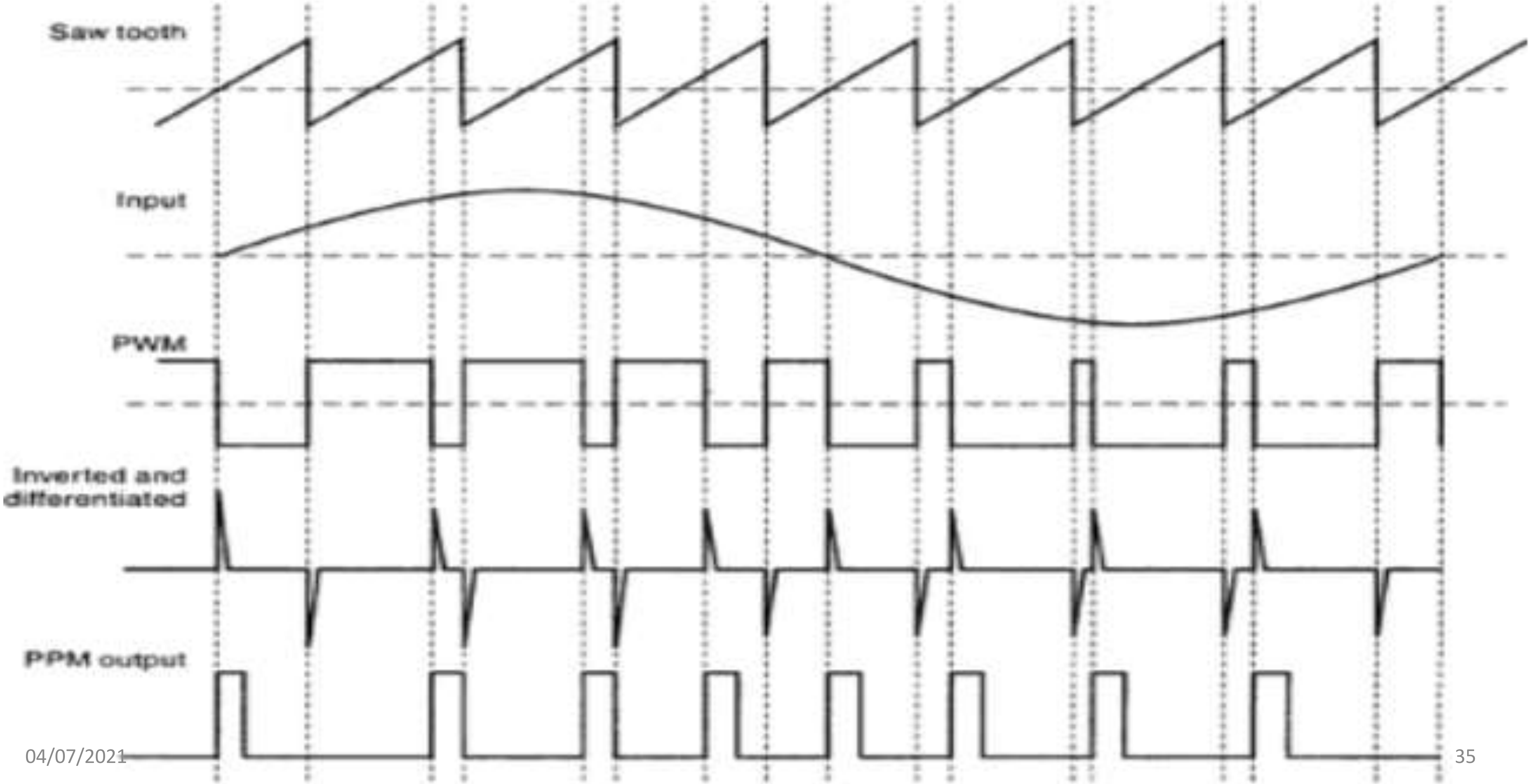
PULSE POSITION MODULATION(PPM)

Modulation technique in which position of pulses of carrier pulse train is varied in accordance with amplitude of modulating signal.

Generation:

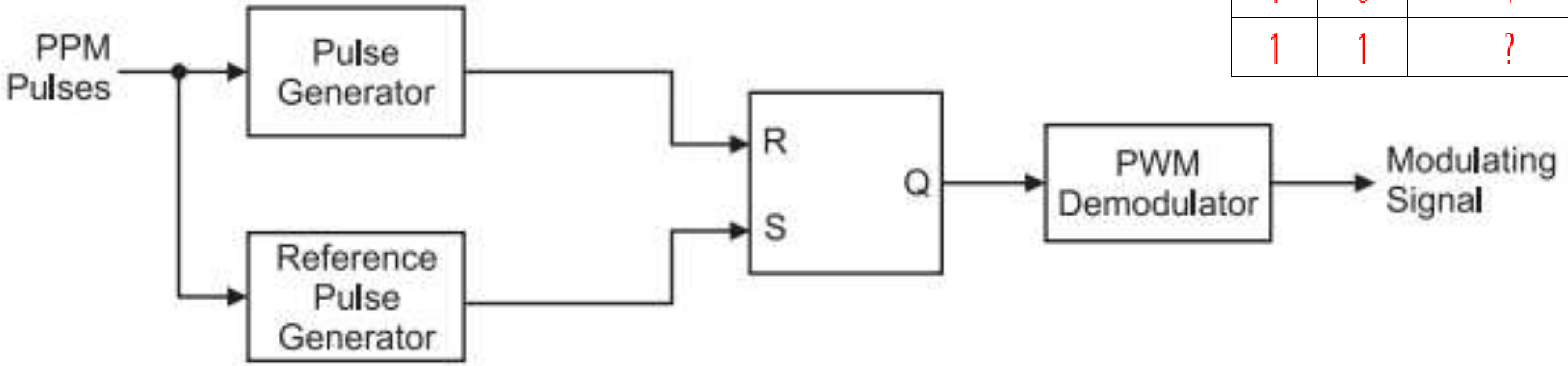


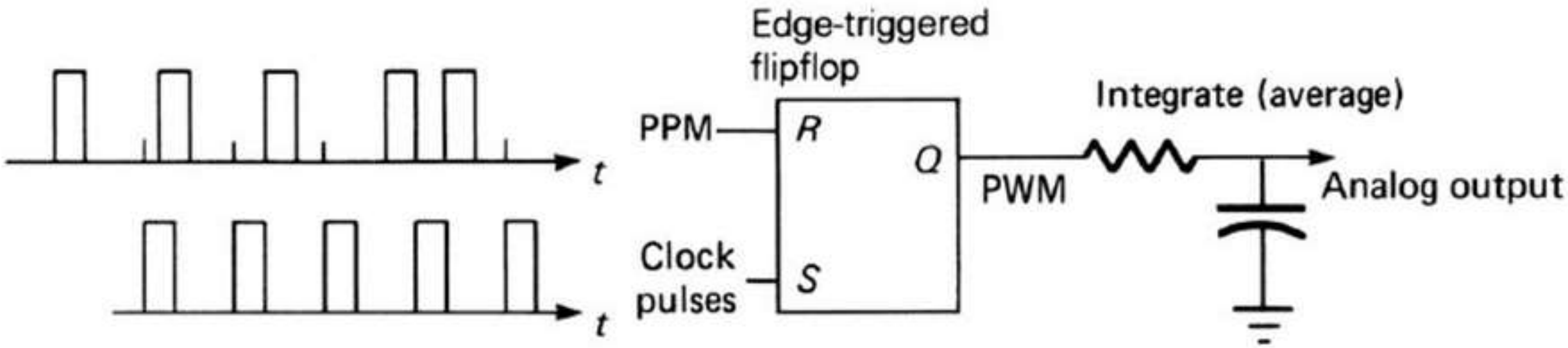
PPM GENERATION



DETECTION OF PPM

S	R	Q	STATE
0	0	PREVIOUS STATE	NO CHANGE
0	1	0	RESET
1	0	1	SET
1	1	?	FORBIDDEN

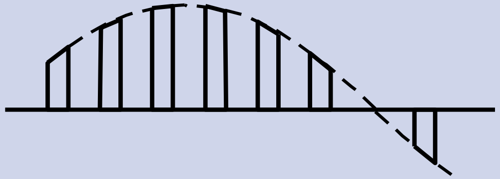
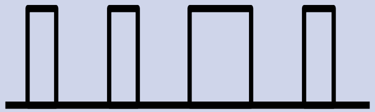
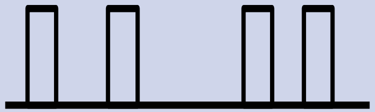




DETECTION OF PPM

- The circuit consists of S-R flipflop which is set or gives high output when reference pulses arrive.
- Reference pulses are generated by a reference pulse generator.
- Flip-flop circuit is reset and gives low output at the leading edge of PPM signal.
- The process repeats and we get PWM pulses at the output of flip-flop.
- PWM pulses are then demodulated in a PWM demodulator to get original modulating signal.

Comparison between PAM, PWM and PPM

S.No	8PAM	PWM	PPM
1	Amplitude is varied	Width is varied	Position is varied
2	Bandwidth depends on the width of the pulse	Bandwidth depends on the rise time of the pulse	Bandwidth depends on the rise time of the pulse
3	Instantaneous transmitter power varies with the amplitude of the pulses	Instantaneous transmitter power varies with the amplitude and the width of the pulses	Instantaneous transmitter power remains constant with the width of the pulses
4	System complexity is high	System complexity is low	System complexity is low
5	Noise interference is high	Noise interference is low	Noise interference is low
6	It is similar to amplitude modulation	It is similar to frequency modulation	It is similar to phase modulation
7	Output Waveform of PAM 	Output Waveform of PWM 	Output Waveform of PPM 

Multiplexing

1)Frequency Division Multiplexing (FDM)

2)Time Division Multiplexing (TDM)

3)Code Division Multiple Access(CDMA)

4)Orthogonal Frequency Division Multiple Access (OFDMA)

Advantages of multiplexing

1. Multiple signals can be sent simultaneously over a single communication channel.
2. Effective use of channel bandwidth
3. Multiplexing reduces cost
4. Multiplexing reduces circuit complexity

Applications of Multiplexing

1. Communication system
2. Computer memory
3. Telephone systems
4. TV broadcasting
5. Telemetry
6. Satellites

Frequency Division Multiplexing

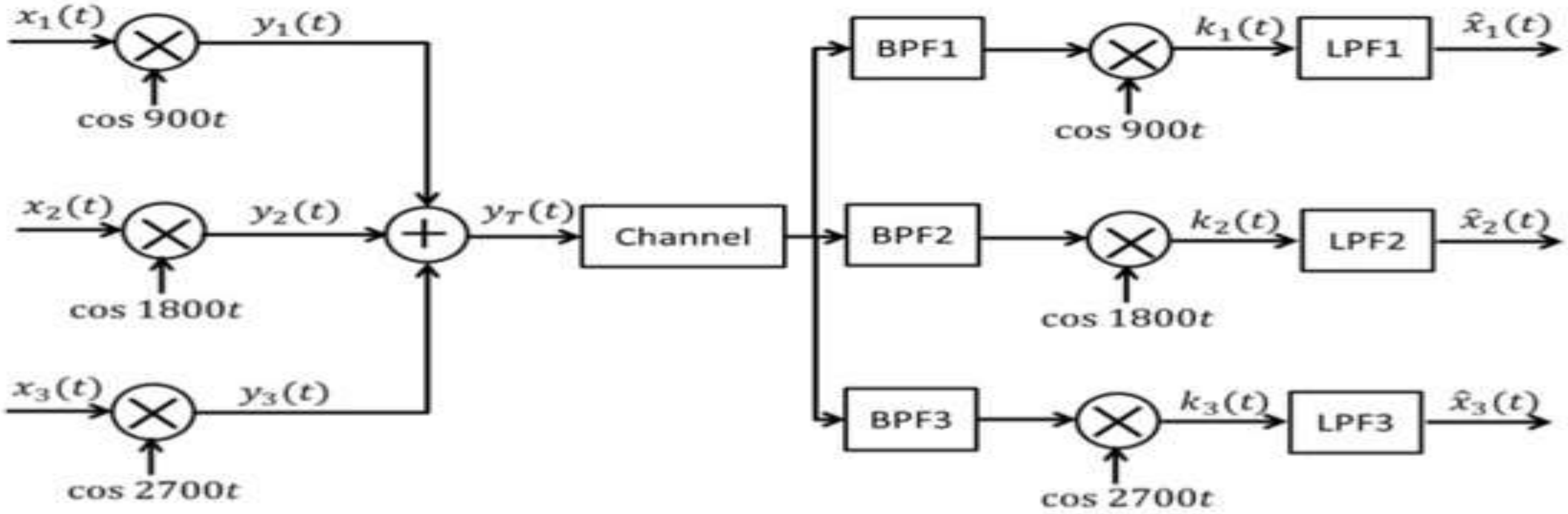


FIGURE Q2a

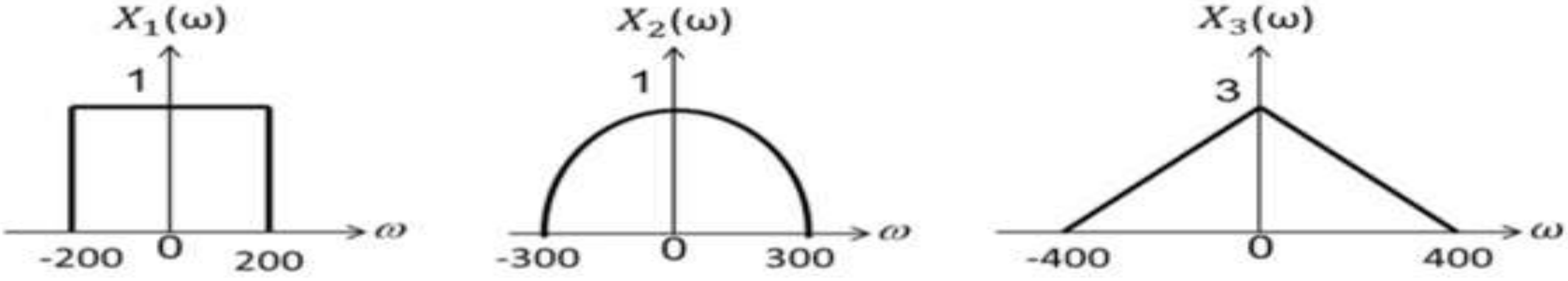
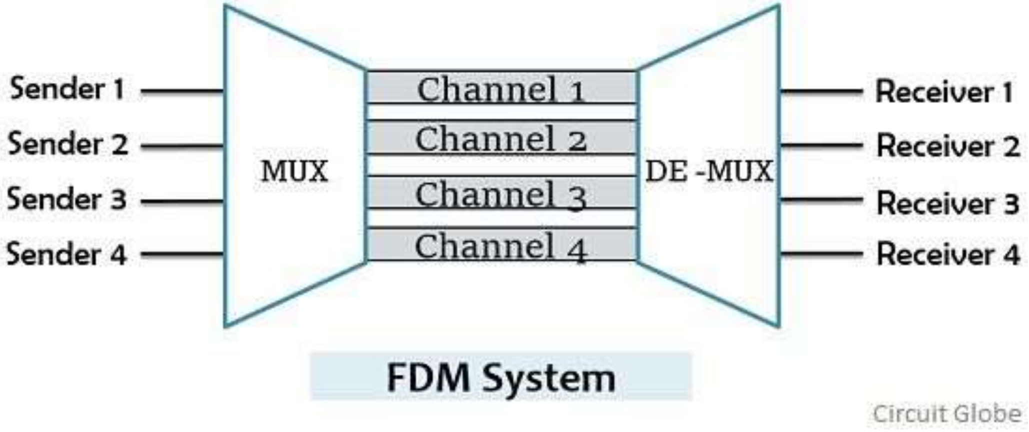
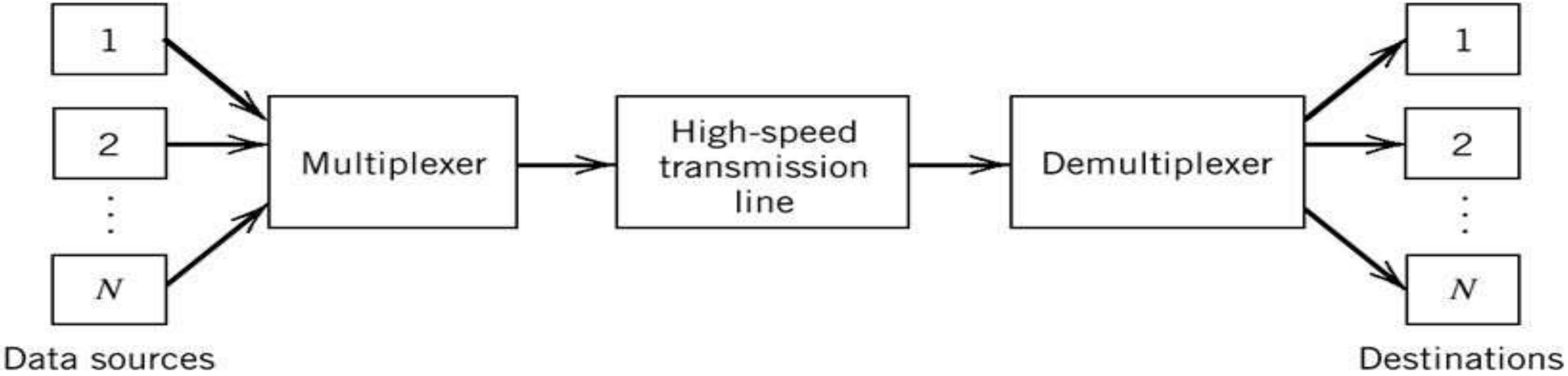


FIGURE Q2b

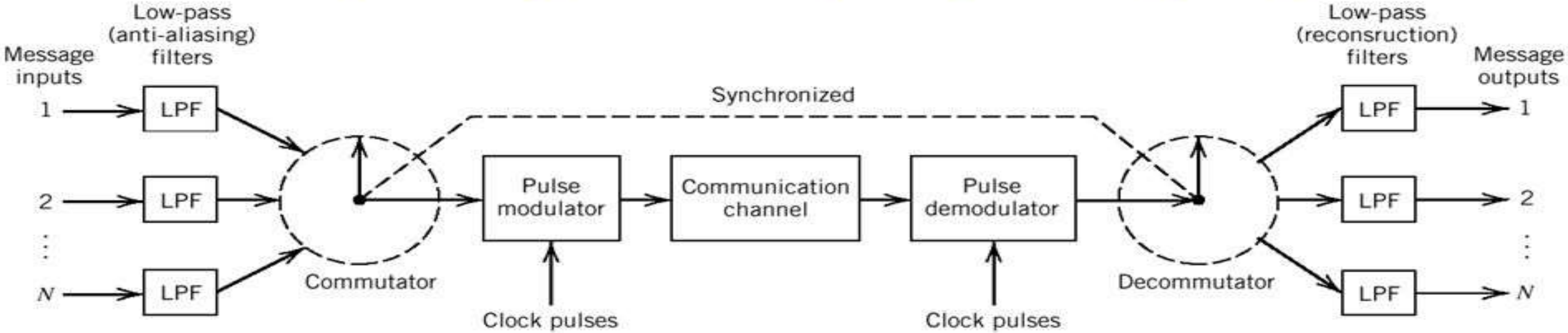


- The frequency division multiplexing divides the bandwidth of a channel into several logical sub-channels (individual signal frequencies (or) set of frequency bands).
- Each logical sub-channel is allotted for a different signal frequency. The individual signals are filtered and then modulated (frequency is shifted), in order to fit exactly into logical sub-channels.
- In this technique, each logical sub-channel (individual signal frequency) is allotted to each user. In other words, each user owns a sub-channel.
- Each logical sub-channel is separated by an unused bandwidth called Guard Band to prevent overlapping of signals. In other words, there exists a frequency gap between two adjacent signals to prevent signal overlapping. A guard band is a narrow frequency range that separates two signal frequencies.

Time Division Multiplexing



Conceptual diagram of multiplexing-demultiplexing.



PAM TDM System

Advantages of Frequency Division Multiplexing (FDM)

1. It transmits multiple signals simultaneously.
2. In frequency division multiplexing, the demodulation process is easy.
3. It does not need Synchronization between transmitter and receiver.

Disadvantages of Frequency Division Multiplexing (FDM)

1. It needs a large bandwidth communication channel.

Applications of Frequency Division Multiplexing (FDM)

1. Frequency division multiplexing is used for FM and AM radio broadcasting.
2. It is used in first generation cellular telephone.
3. It is used in television broadcasting.

Comparison of FDM and TDM

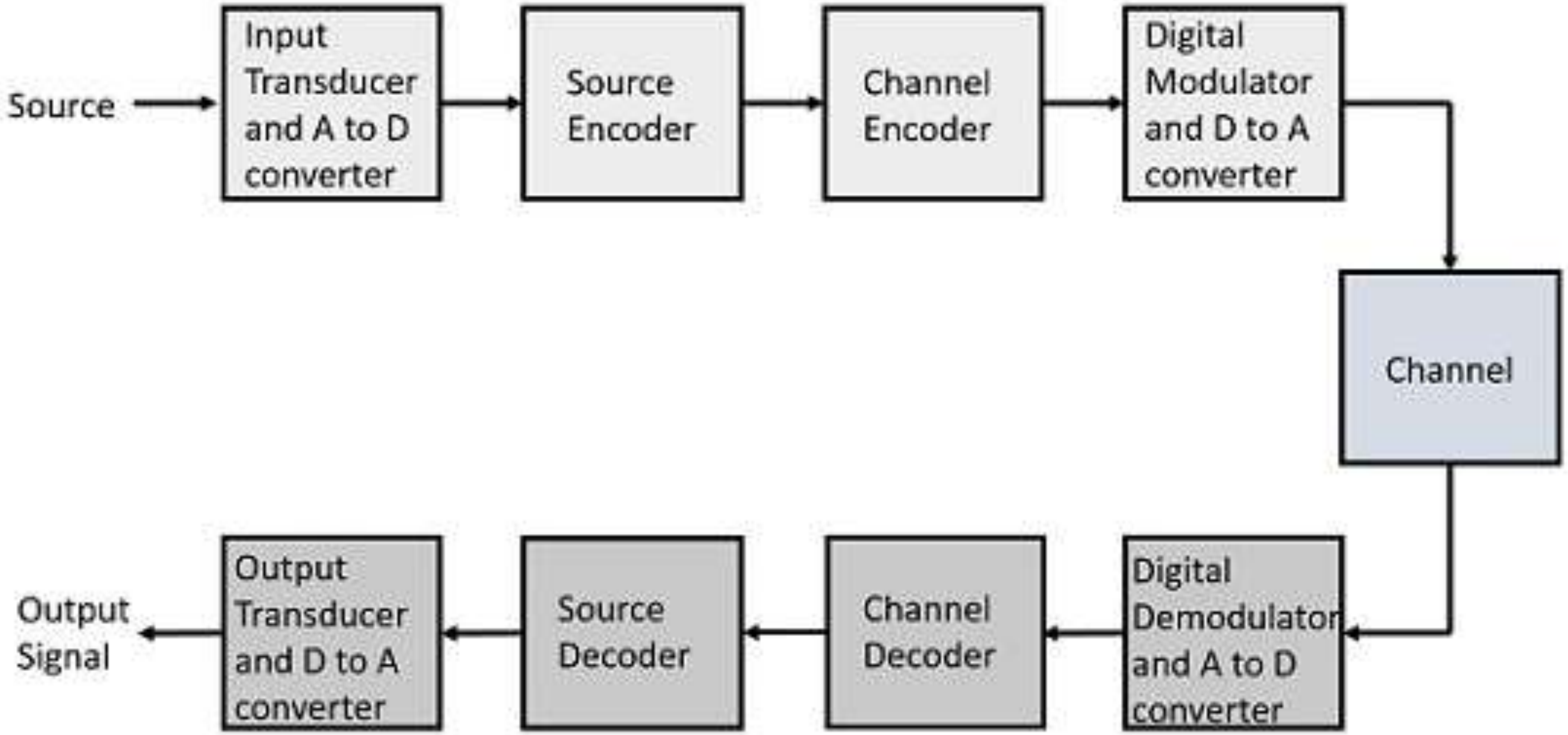
S. No	FDM	TDM
1	FDM stands for Frequency Division Multiplexing.	TDM stands for Time Division Multiplexing.
2	FDM is an Analog technique	TDM is a Digital technique.
3	The communication channel is divided by frequency	The communication channel is divided by time.
4	All signals of different frequencies are transmitted simultaneously.	All signals operate with the same frequency are transmitted at different times.
5	Synchronization is not required	Synchronization is required.
6	The bandwidth of the communication channel should be greater than the combined bandwidth of individual signals.	The bandwidth capacity of the communication channel should be greater than the multiple input signals.
7	FDM requires complex circuitry at the transmitter and receiver.	TDM does not require complex circuitry.
8	In FDM, the problem of crosstalk is severe.	In TDM, the problem of crosstalk is not severe.
9	The channel bandwidth is effectively used.	The channel bandwidth is wasted.
10	FDM requires Guard bands for its operation.	TDM requires sync pulse for its operation.
11	FDM is used in TV and RADIO broadcasting	TDM is used in Pulse code modulation

Digital Communications

Introduction

- Digital communication is a mode of communication where the information or the thought is encoded digitally as discreet signals and electronically transferred to the recipients.
- n digital communication information flows in a digital form and the source is generally the keyboard of the computer. A single individual is capable of digital communication and thus it also saves wastage of manpower and is one of the cheapest modes of communication.
- Digital communication is also a really quick way to communicate. The information can reach the recipient within a fraction of a second. An individual no longer has to wait to personally meet the other individual and share his information.

Basic Digital Communication System



Basic Elements of a Digital Communication System

Advantages

- The Digital Communication's it provides us added security to our information signal.
- The digital Communication system has more immunity to noise and external interference.
- Digital information can be saved and retrieved when necessary while it is not possible in analog.
- Digital Communication is cheaper than Analog Communication.
- The configuring process of digital communication system is simple as compared to analog communication system. Although, they are complex.
- In Digital Communication System, the error correction and detection techniques can be implemented easily.

Disadvantages

- **Disadvantages of digital communication:**

1) Generally, more bandwidth is required than that for analog systems.

2) Synchronization is required.

3) High power consumption (Due to various stages of conversion).

4) Complex circuit, more sophisticated device making is also drawbacks of digital system.

5) Introduce sampling error

6) As square wave is more affected by noise, That’s why while communicating through channel we send sin waves but while operating on device we use squire pulses.

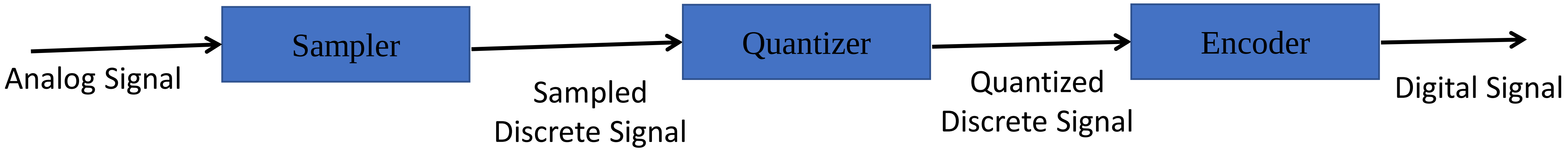
Shannon–Hartley theorem

- The Shannon–Hartley theorem states the channel capacity C, meaning the theoretical tightest upper bound on the information rate of data that can be communicated at an arbitrarily low error rate using an average received signal power S through an analog communication channel subject to additive white Gaussian noise of power N

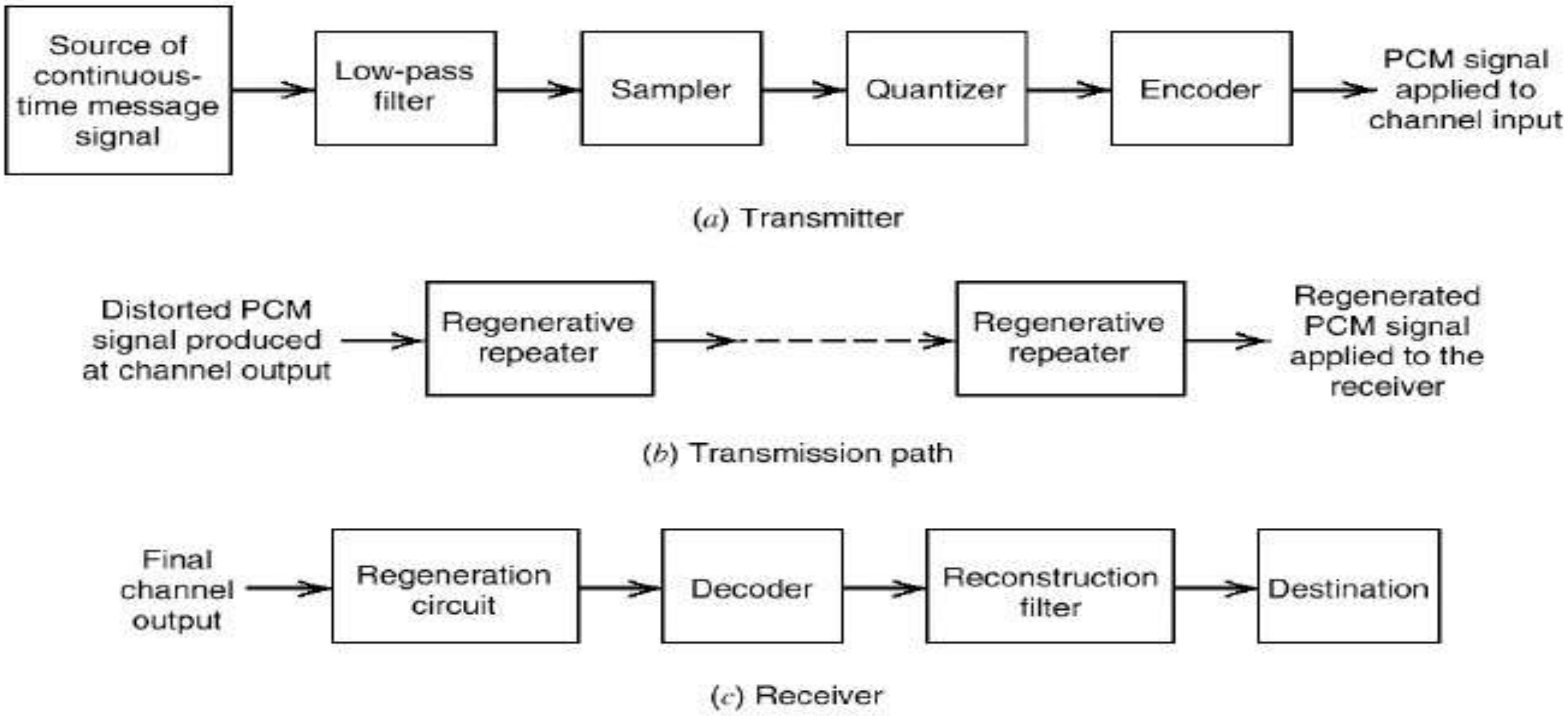
$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

- Where C is the channel capacity in bits per second, a theoretical upper bound on the net bit rate (information rate, sometimes denoted I) excluding error-correction codes;
- B is the bandwidth of the channel in hertz (passband bandwidth in case of a bandpass signal);
- S is the average received signal power over the bandwidth (in case of a carrier-modulated passband transmission, often denoted C), measured in watts (or volts squared);
- N is the average power of the noise and interference over the bandwidth, measured in watts (or volts squared); and
- S/N is the signal-to-noise ratio (SNR) or the carrier-to-noise ratio (CNR) of the communication signal to the noise and interference at the receiver (expressed as a linear power ratio, not as logarithmic decibels).

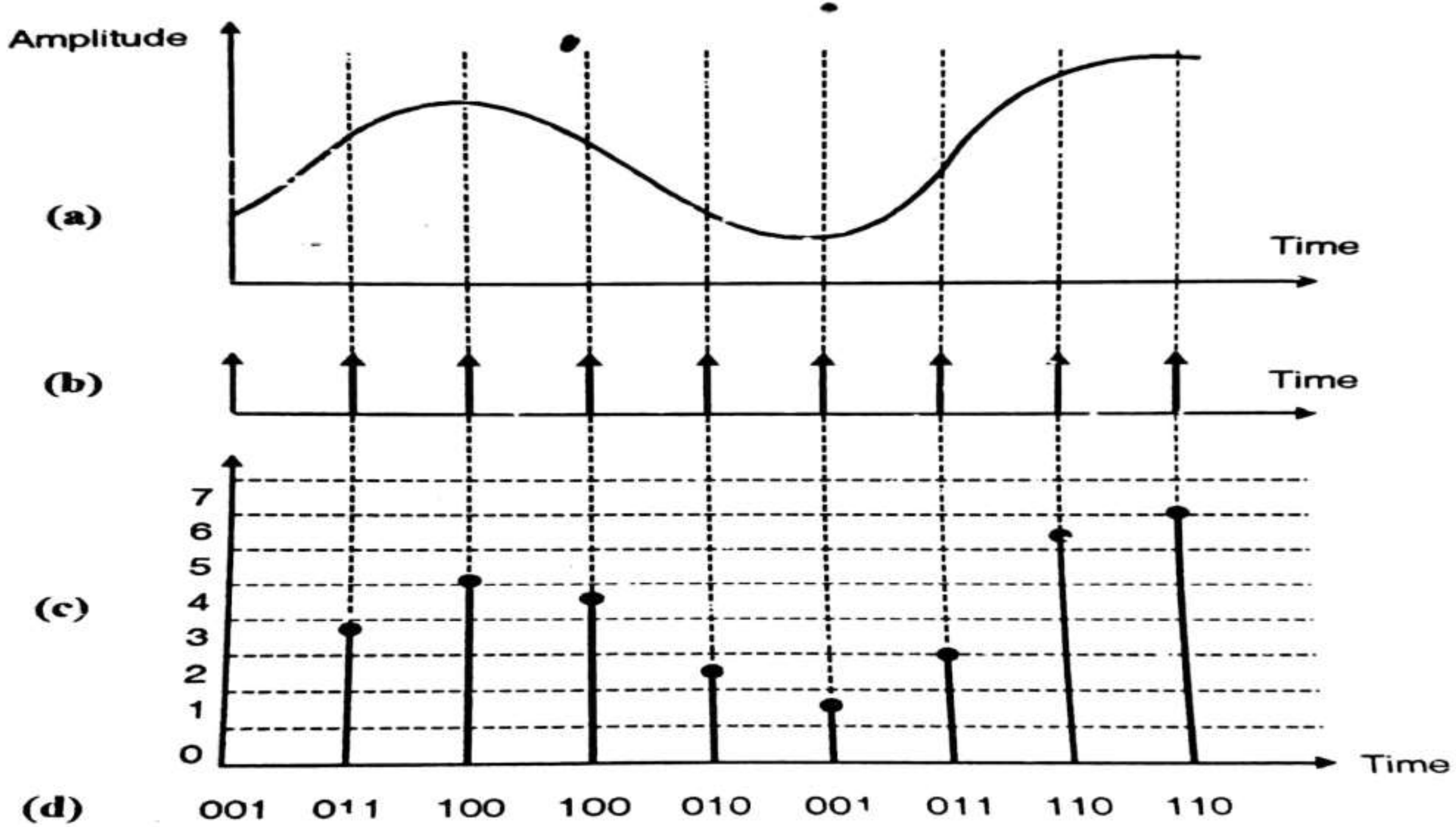
Analog to Digital Conversion



Pulse Code Modulation(PCM)



Sampler



Amplitude
(a)
Time
(b)
Time
(c)
7
6
5
4
3
2
1
0
Time
(d)
001 011 100 100 010 001 011 110 110

PCM encoding example

Level	Code word
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Levels are encoded using this table

Table: Quantization levels with belonging code words

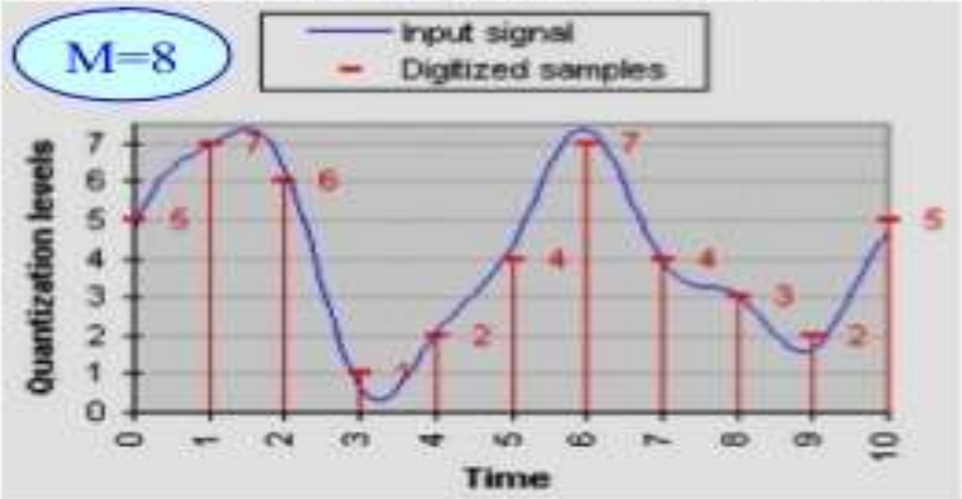


Chart 1. Quantization and digitalization of a signal.

Signal is quantized in 11 time points & 8 quantization segments.

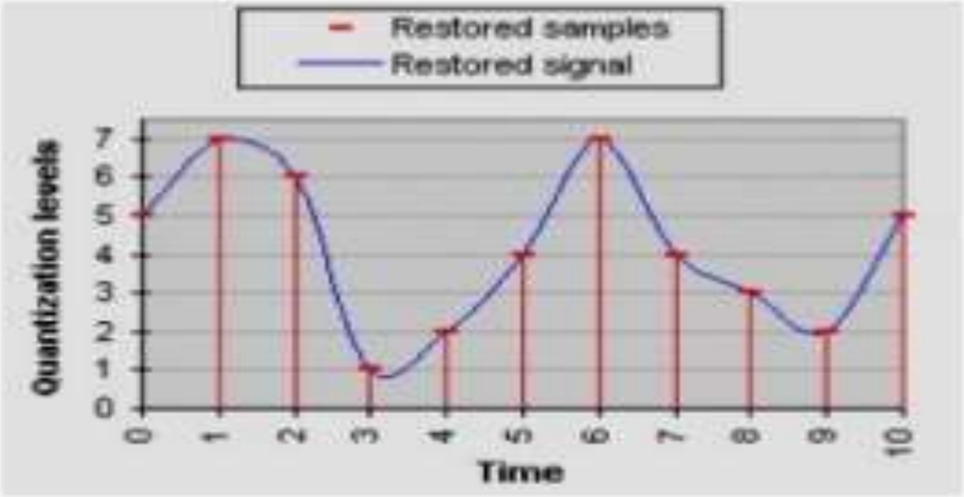


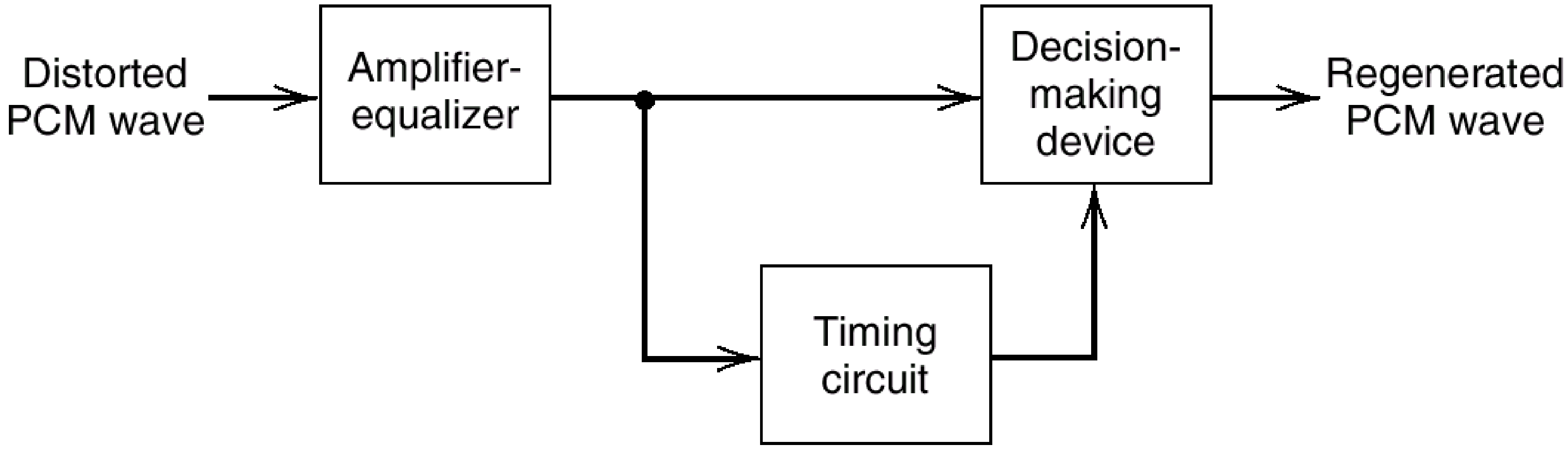
Chart 2. Process of restoring a signal.

PCM encoded signal in binary form:

101 111 110 001 010 100 111 100 011 010 101

Total of **33** bits were used to encode a signal

Block diagram of regenerative repeater



Regenerative Repeater

A regenerative repeater (see Figure 3.18) consists of (1) an equalizer, (2) a timing circuit, and (3) a decision-making device. The equalizer is used to undo the effect of the transmission channel to get back the pulses in their original shape before transmission. The timing circuit is used to recover the clock of the transmitted symbols (pulses), which is then used in the decision-making process. The function of the decision-making device is to detect the different pulses based on some threshold information.

The purpose of a regenerative repeater is to clean the PCM signal during its transmission through a channel.

Differential Pulse Code Modulation

DPCM

The samples of a signal are highly corrected with each other. This is because any signal does not change fast. That is its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with little difference. When these samples are encoded by standard PCM system, the resulting encoded signal contains redundant information.

Fig. shows a continuous time signal $x(t)$ by dotted line. This signal is sampled by flat top sampling at intervals $T_s, 2T_s, 3T_s \ldots nT_s$. The sampling frequency is selected to be higher than nyquist rate. The samples are encoded by using 3 bit (7 levels) PCM. The sample is quantized to the nearest digital level as shown by small

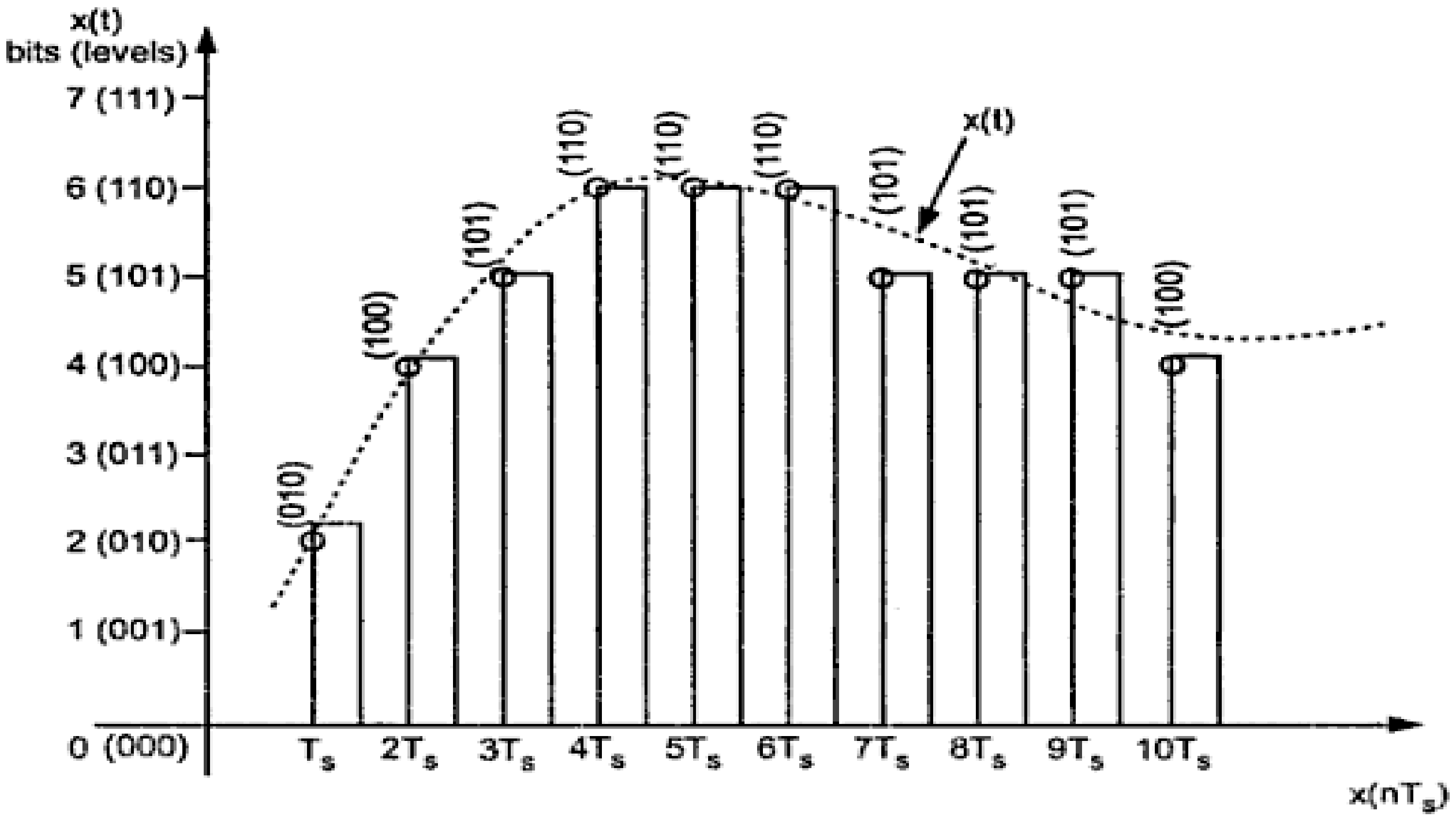


Fig. Redundant information in PCM

Introduction DPCM

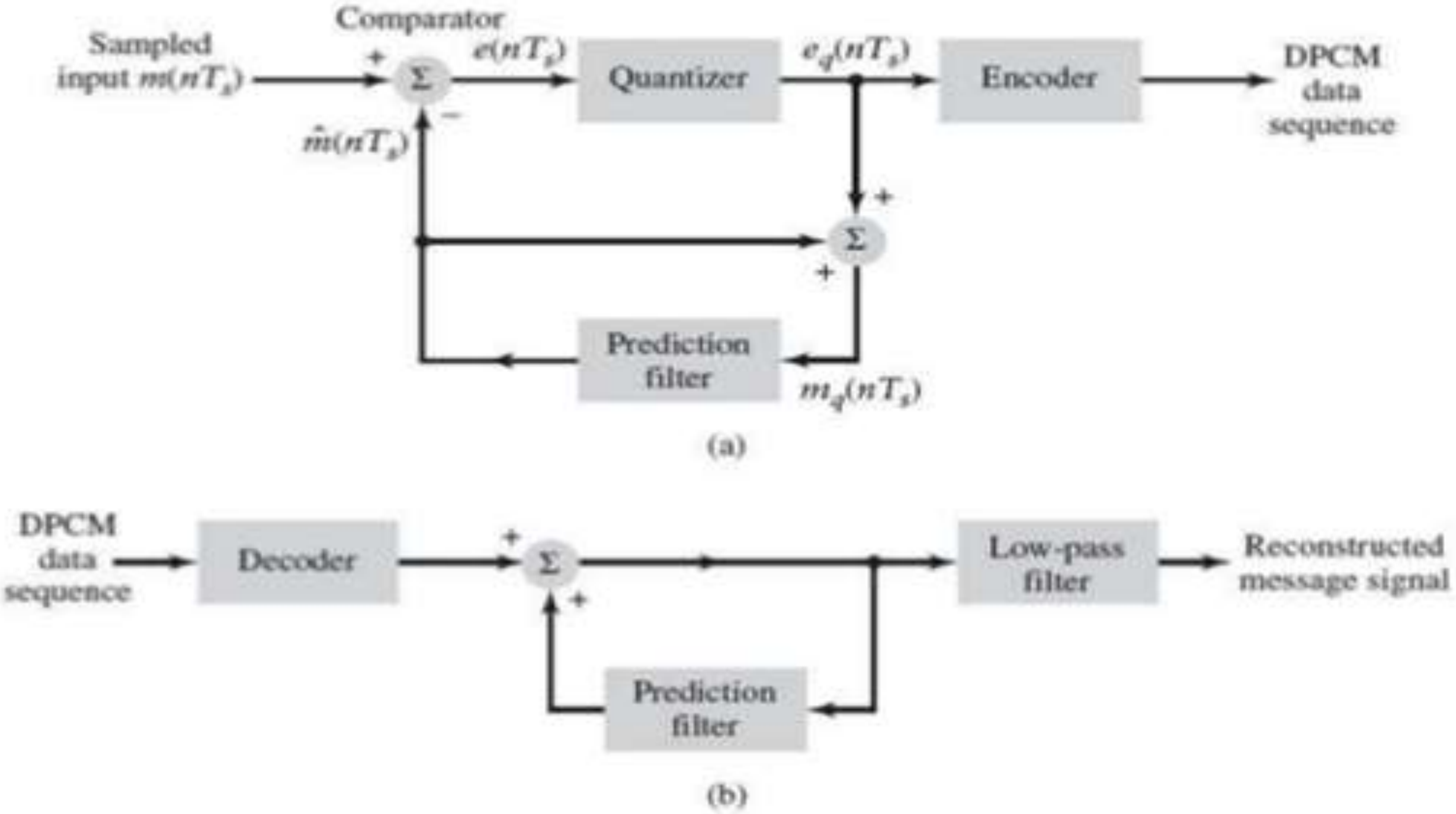


FIGURE 5.18 DPCM system: (*a*) Transmitter and (*b*) receiver.

$$e(nTs) = m(nTs) - \hat{m}(nTs)$$

It is the difference between unquantized input sample $m(nTs)$ and a prediction of it $\hat{m}(nTs)$.Prediction value is obtained by using Prediction filter. The difference signal $e(nTs)$ is called prediction error.

The Quantizer output is represented as $e_q(nTs) = e(nTs) + q_e(nTs)$

Here, $q_e(nTs)$is called the Quantization error. The Quantizer output $e_q(nTs)$ is added to the predicted value $\hat{m}(nTs)$ to produce the prediction filter input.

$$m_q(nTs) = \hat{m}(nTs) + e_q(nTs)$$

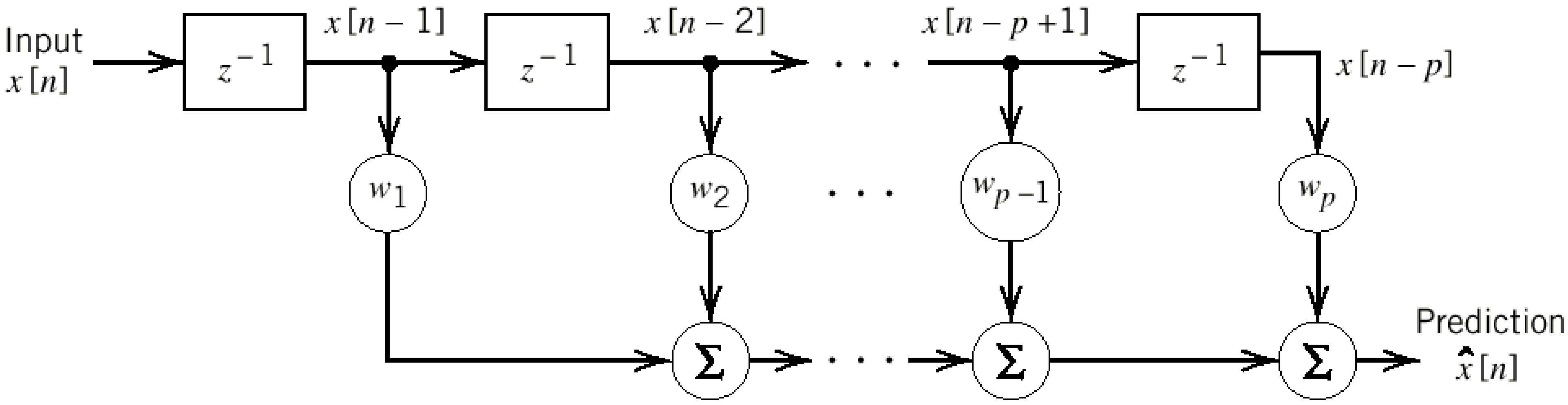
$$m_q(nTs) = \hat{m}(nTs) + e(nTs) + q_e(nTs)$$

Here $\hat{m}(nTs) + e(nTs)$ is equal to the input signal $m(nTs)$

$$m_q(nTs) = m(nTs) + q_e(nTs)$$

which represents the Quantized version of the input signal $m(nTs)$. If Prediction is Good Variance of the prediction error $e(nTs)$ *willl be smaller than the variance of the* $m(nTs)$.

Prediction Filter



$$\hat{x}(n) = \quad \widehat{m}(nTs) = \sum_{k=i}^{p} w_k m_q(nTs - kTs)$$

$$\mathrm{e}(nTs) = m(nTs) - \sum_{k=i}^{p} w_k m_q(nTs - kTs)$$

The Weights w_k should be adjusted Properly to have Good Prediction.

ADVANTAGES OF DPCM

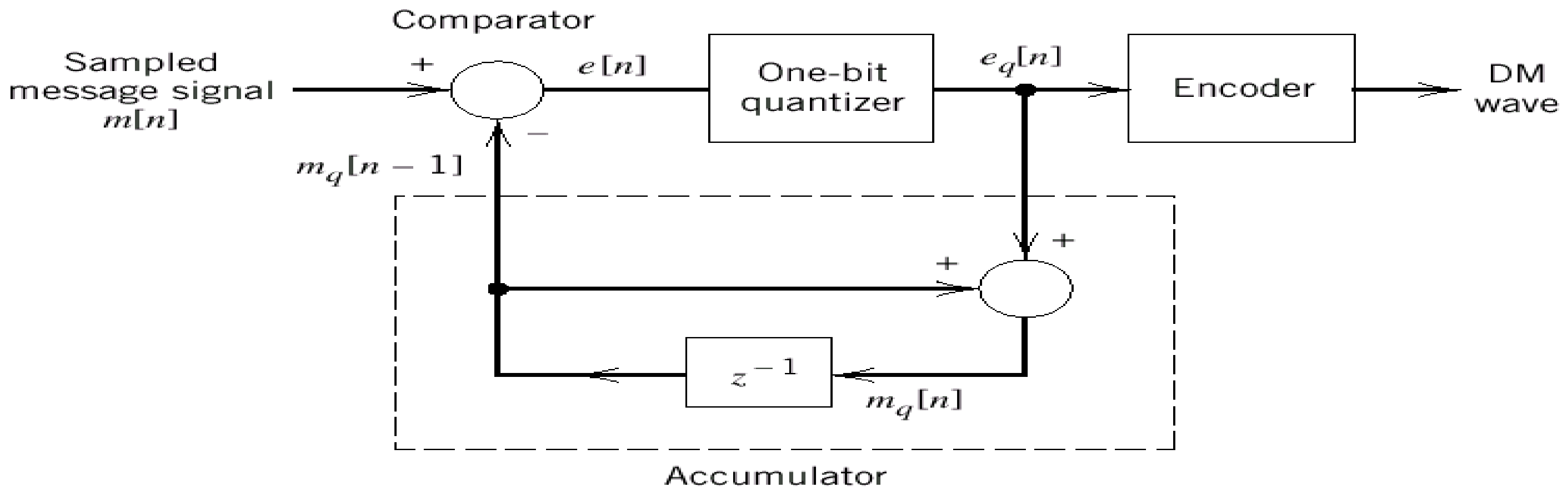
1) BANDWIDTH REQUIREMENT OF **DPCM** IS LESS COMPARED TO **PCM**.

2) QUANTIZATION ERROR IS REDUCED BECAUSE OF PREDICTION FILTER.

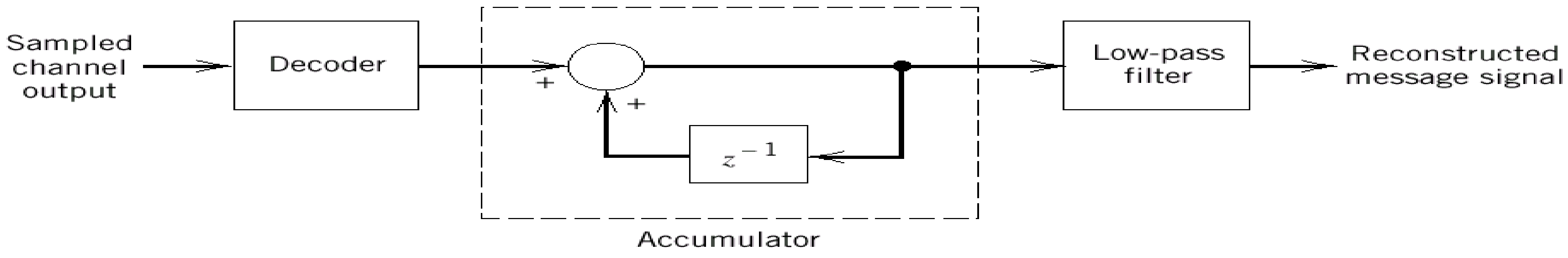
3) NUMBERS OF BITS USED TO REPRESENT . ONE SAMPLE VALUE ARE ALSO REDUCED COMPARED TO **PCM**

Delta Modulation / One Bit Modulation

DM system. (*a*) Transmitter. (*b*) Receiver.



(*a*)



(*b*)

Delta Modulation (DM)

- In DM, the message signal is over-sampled to purposely increase correlation between adjacent samples.

- The DM provides a staircase approximation to the message signal $m(t)$ as shown in Figure 3.22.

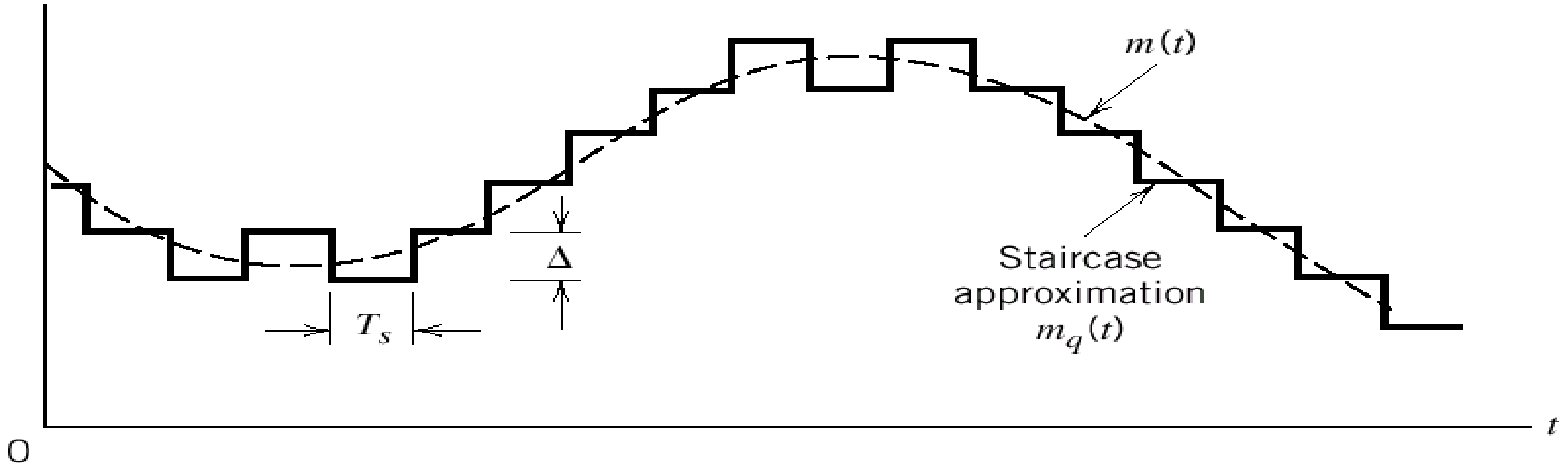
- The difference $e[nT_s]=m[nT_s]-m_q[(n-1)T_s]$ is quantized into only two levels $\pm\Delta$.
- The error $e[nT_s]$ is quantized to give

  $$e_q= \Delta \mathrm{sgn}(e[nT_s]).$$

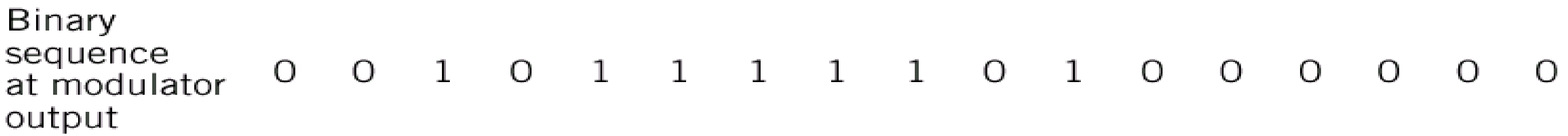
  The quantity e_q is then used to compute the new quantized level

  $$m_q[nT_s]=m_q[(n-1)T_s]+e_q[nT_s]$$
- In DM the quantization levels are represented by two symbols: 0 for $-\Delta$ and 1 for $+\Delta$. In fact the coding process is performed on e_q.
- The main advantage of DM is its simplicity as shown by Figure 3.23.

. Illustration of delta modulation

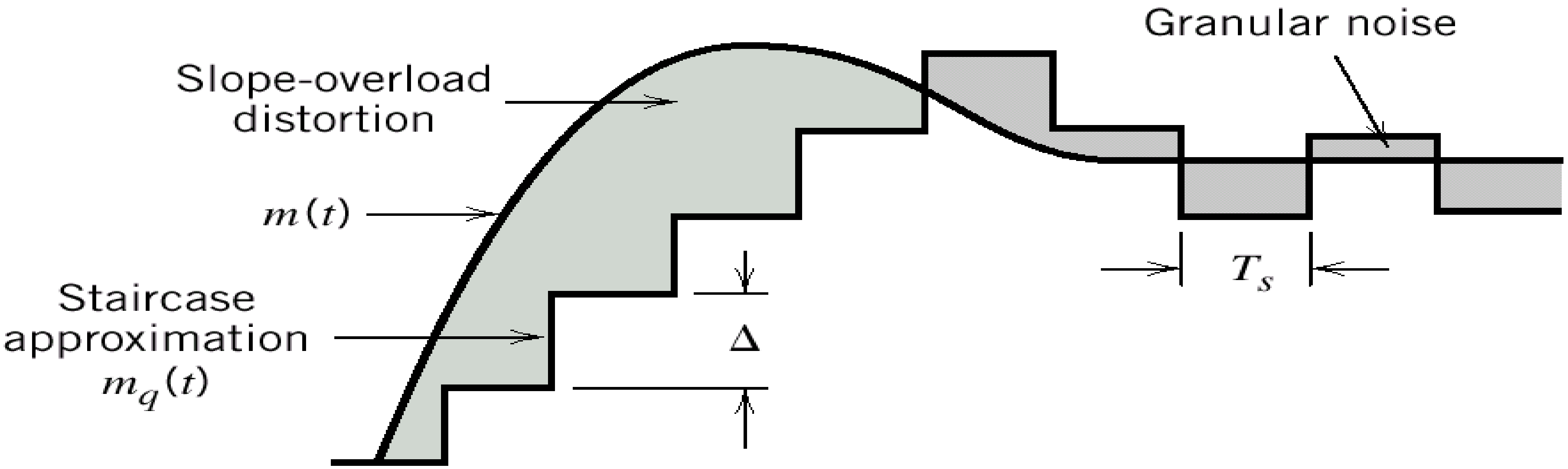


(a)



(b)

Illustration of the two different forms of quantization error in delta modulation.



Delta Modulation (Cont’d)

- The transmitter of a DM system (Figure 3.23a) is given by a comparator, a one-bit quantizer, an accumulator, and an encoder.
- The receiver of a DM system (Figure 3.23b) is given by a decoder, an accumulator, and a low-pass filter.
- DM is subject to two types of quantization error: Slope overload distortion and granular noise (see Figure 3.24).
- Slope overload distortion is due to the fact that the staircase approximation $m_q(t)$ can't follow closely the actual curve of the message signal $m(t)$. In order for $m_q(t)$ to follow closely $m(t)$, it is required that

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

be satisfied. Otherwise, step-size Δ is too small for the staircase approximation $m_q(t)$ to follow $m(t)$.

Delta Modulation (Cont’d)

- In contrast to slope-overload distortion, granular noise occurs when Δ is too large relative to the local slope characteristics of $m(t)$. granular noise is similar to quantization noise in PCM.
- It seems that a large Δ is needed for rapid variations of $m(t)$ to reduce the slope-overload distortion and a small Δ is needed for slowly varying $m(t)$ to reduce the granular noise. The optimum Δ can only be a compromise between the two cases.
- To satisfy both cases, an adaptive DM is needed, where the step size Δ can be adjusted in accordance with the input signal $m(t)$.

Condition for Slope overload distortion occurrence

Let the sine wave be represented as,

$$x(t) = A_m \sin(2\pi f_m t)$$

Slope of $x(t)$ will be maximum when derivative of $x(t)$ with respect to 't' will be maximum. The maximum slope of delta modulator is given

$$\text{Max. slope} = \frac{\text{Step size}}{\text{Sampling period}}$$

$$= \frac{\delta}{T_s} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Slope overload distortion will take place if slope of sine wave is greater than slope of delta modulator i.e.

$$\max\left|\frac{d}{dt}x(t)\right| > \frac{\delta}{T_s}$$

$$\max\left|\frac{d}{dt}A_m \sin(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$\max\left|A_m\, 2\pi f_m \cos(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$A_m\, 2\pi f_m > \frac{\delta}{T_s}$$

or $$\boxed{A_m > \frac{\delta}{2\pi f_m T_s}} \qquad \text{.................(2)}$$

SQNR of PCM

Derivation of Quantization Error/Noise or Noise Power for Uniform (Linear) Quantization

Step 1 : Quantization Error

Because of quantization, inherent errors are introduced in the signal. This error is called *quantization error*. We have defined quantization error as,

$$\varepsilon = x_q(nT_s) - x(nT_s) \quad \text{------------------------- (1)}$$

Step 2 : Step size

Let an input $x(nT_s)$ be of continuous amplitude in the range $-x_{max}$ to $+x_{max}$.

Therefore the total amplitude range becomes,

$$\begin{aligned} \text{Total amplitude range} &= x_{max} - (-x_{max}) \\ &= 2x_{max} \end{aligned} \quad \text{----------------------(2)}$$

If this amplitude range is divided into 'q' levels of quantizer, then the step size 'δ' is given as,

$$\delta = \frac{x_{max} - (-x_{max})}{q}$$

$$= \frac{2x_{max}}{q} \quad \text{-----------------(3)}$$

If signal $x(t)$ is normalized to minimum and maximum values equal to 1, then

$$\begin{aligned} x_{max} &= 1 \\ -x_{max} &= -1 \end{aligned} \quad \text{----------------------------(4)}$$

Therefore step size will be,

$$\delta = \frac{2}{q} \quad \text{(for normalized signal)} \quad \text{----------------------------(5)}$$

Step 3 : Pdf of Quantization error

If step size 'δ' is sufficiently small, then it is reasonable to assume that the quantization error 'ε' will be uniformly distributed random variable. The maximum quantization error is given by

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right| \quad \text{--------------------------(6)}$$

i.e. $$-\frac{\delta}{2} \geq \varepsilon_{max} \geq \frac{\delta}{2} \quad \text{---------------------------(7)}$$

Thus over the interval $\left(-\frac{\delta}{2}, \frac{\delta}{2}\right)$ quantization error is uniformly distributed random variable.

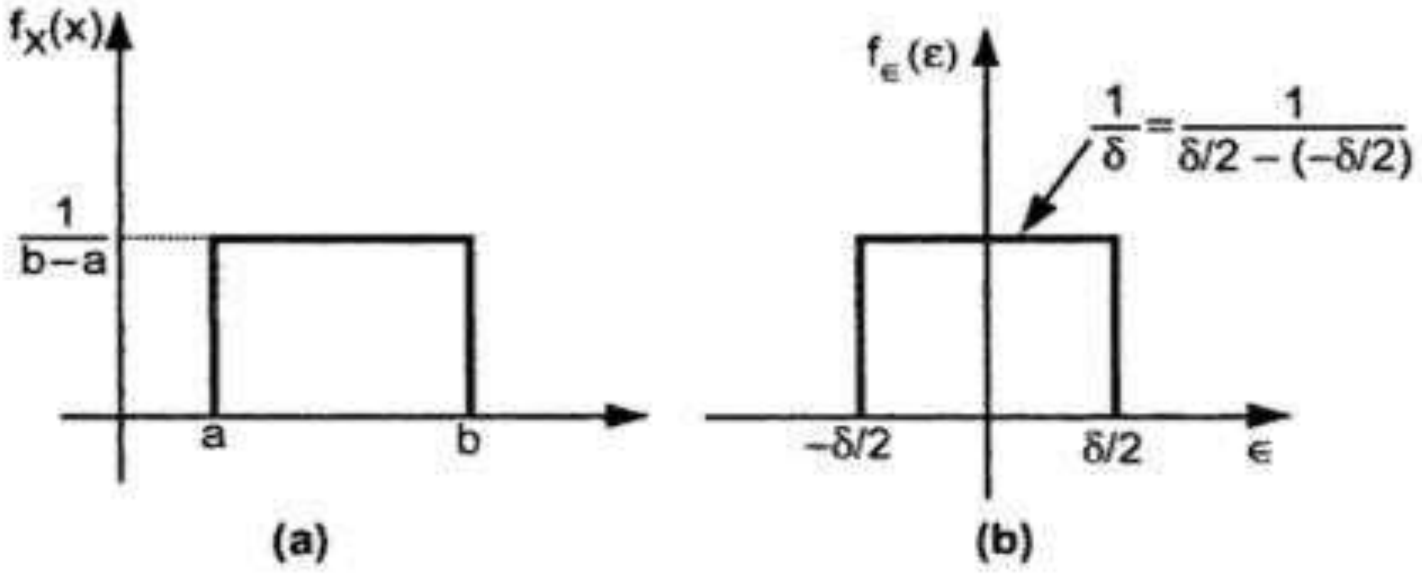


Fig. 10 (a) Uniform distribution
(b) Uniform distribution for quantization error

In above figure, a random variable is said to be uniformly distributed over an interval (a, b). Then PDF of 'X' is given by, (from equation of Uniform PDF).

$$f_X(x) = \begin{cases} 0 & \text{for} \quad x \le a \\ \dfrac{1}{b-a} & \text{for} \quad a < x \le b \\ 0 & \text{for} \quad x > b \end{cases} \qquad \text{.....................(8)}$$

Thus with the help of above equation we can define the probability density function for quantization error 'ε' as,

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & \text{for} \quad \varepsilon \le \dfrac{\delta}{2} \\ \dfrac{1}{\delta} & \text{for} \quad -\dfrac{\delta}{2} < \varepsilon \le \dfrac{\delta}{2} \\ 0 & \text{for} \quad \varepsilon > \dfrac{\delta}{2} \end{cases} \qquad \text{.............................(9)}$$

Step 4 : Noise Power

quantization error 'ε' has zero average value. That is mean 'm_ε' of the quantization error is zero.

The signal to quantization noise ratio of the quantizer is defined as,

$$\frac{S}{N} = \frac{\text{Signal power (normalized)}}{\text{Noise power (normalized)}} \qquad \cdots 10$$

If type of signal at input i.e., $x(t)$ is known, then it is possible to calculate signal power.

The noise power is given as,

$$\text{Noise power} = \frac{V_{noise}^2}{R} \qquad \cdots (11)$$

Here V_{noise}^2 is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF $f_\varepsilon(\varepsilon)$, its mean square value is given as,

$$\text{mean square value} = E[\varepsilon^2] = \bar{\varepsilon}^2 \qquad \cdots (12)$$

The mean square value of a random variable 'X' is given as,

$$\bar{X}^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx \quad \text{By definition} \qquad \cdots (13)$$

Here $$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\,d\varepsilon \qquad \cdots (14)$$

From equation 9 we can write above equation as,

$$E[\varepsilon^2] = \int_{-\delta/2}^{\delta/2} \varepsilon^2 \times \frac{1}{\delta}\,d\varepsilon$$

$$= \frac{1}{\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\delta/2}^{\delta/2} = \frac{1}{\delta}\left[\frac{(\delta/2)^3}{3} + \frac{(\delta/2)^3}{3}\right]$$

$$= \frac{1}{3\delta}\left[\frac{\delta^3}{8} + \frac{\delta^3}{8}\right] = \frac{\delta^2}{12} \qquad \cdots (15)$$

∴ From equation 1.8.25, the mean square value of noise voltage is,

$$V_{noise}^2 = \text{mean square value} = \frac{\delta^2}{12}$$

When load resistance, $R = 1$ ohm, then the noise power is normalized i.e.,

$$\text{Noise power (normalized)} = \frac{V_{noise}^2}{1} \qquad [\text{with } R = 1 \text{ in equation } 11 \quad]$$

$$= \frac{\delta^2 / 12}{1} = \frac{\delta^2}{12}$$

Thus we have,

Normalized noise power

or Quantization noise power $= \dfrac{\delta^2}{12}$ **; For linear quantization.**

or Quantization error (in terms of power) ... (16)

Derivation of Maximum Signal to Quantization Noise Ratio for Linear Quantization:

signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

$$= \frac{\text{Normalized signal power}}{(\delta^2 / 12)} \qquad \cdots (17)$$

The number of bits 'v' and quantization levels 'q' are related as,

$$q = 2^v \qquad \cdots (18)$$

Putting this value in equation (3) we have,

$$\delta = \frac{2\,x_{max}}{2^v} \qquad \cdots (19)$$

Putting this value in equation 1.8.30 we get,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\left(\frac{2\,x_{max}}{2^v}\right)^2 + 12}$$

Let normalized signal power be denoted as 'P'.

$$\frac{S}{N} = \frac{P}{\frac{4\,x_{max}^2}{2^{2v}} \times \frac{1}{12}} = \frac{3P}{x_{max}^2} \cdot 2^{2v}$$

This is the required relation for maximum signal to quantization noise ratio. Thus,

$$\boxed{\textbf{Maximum signal to quantization noise ratio : } \frac{S}{N} = \frac{3P}{x_{max}^2} \cdot 2^{2v}} \quad \cdots (20)$$

This equation shows that signal to noise power ratio of quantizer increases exponentially with increasing bits per sample.

If we assume that input $x(t)$ is normalized, i.e.,

$$x_{max} = 1 \quad \cdots (21)$$

Then signal to quantization noise ratio will be,

$$\frac{S}{N} = 3 \times 2^{2v} \times P \quad \cdots (22)$$

If the destination signal power 'P' is normalized, i.e.,

$$P \leq 1 \quad \cdots (23)$$

Then the signal to noise ratio is given as,

$$\frac{S}{N} \leq 3 \times 2^{2v} \quad \cdots (24)$$

Since $x_{max} = 1$ and $P \leq 1$, the signal to noise ratio given by above equation is normalized.

Expressing the signal to noise ratio in decibels,

$$\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB \quad \text{since power ratio.}$$

$$\leq 10 \log_{10} [3 \times 2^{2v}]$$

$$\leq (4.8 + 6v)\ dB$$

Thus,

Signal to Quantization noise ratio
for normalized values of power $: \left(\frac{S}{N}\right) dB \leq (4.8 + 6v)\ dB$
'P' and amplitude of input $x(t)$

$\cdots (25)$

$$\max\left|A_m\, 2\pi f_m \cos(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$A_m\, 2\pi f_m > \frac{\delta}{T_s}$$

or $$\boxed{A_m > \frac{\delta}{2\pi f_m T_s}} \qquad \text{...............(2)}$$

SNR for DM System

To obtain signal power :

slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

Here A_m is peak amplitude of sinusoided signal

δ is the step size

f_m is the signal frequency and

T_s is the sampling period.

From above equation, the maximum signal amplitude will be,

$$A_m = \frac{\delta}{2\pi f_m T_s} \qquad \text{..................}(1)$$

Signal power is given as,

$$P = \frac{V^2}{R}$$

Here V is the rms value of the signal. Here $V = \frac{A_m}{\sqrt{2}}$. Hence above equation becomes,

$$P = \left(\frac{A_m}{\sqrt{2}}\right)^2 / R$$

Normalized signal power is obtained by taking R = 1. Hence,

$$P = \frac{A_m^2}{2}$$

Putting for A_m from equation 1

$$P = \frac{\delta^2}{8\pi^2 f_m^2 T_s^2} \quad \text{.................(2)}$$

This is an expression for signal power in delta modulation.

(ii) To obtain noise power

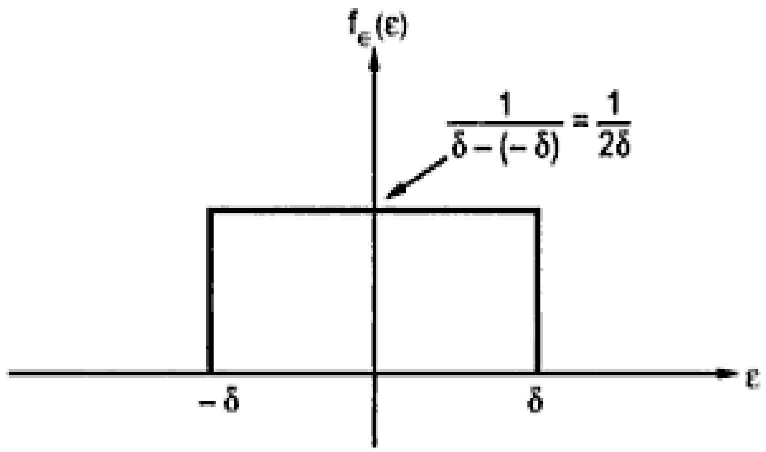


Fig. **Uniform distribution of quantization error**

We know that the maximum quantization error in delta modulation is equal to step size 'δ'. Let the quantization error be uniformly distributed over an interval $[-\delta, \delta]$. This is shown in Fig. From this figure the PDF of quantization error can be expressed as,

$$f_\epsilon(\varepsilon) = \begin{cases} 0 & for \quad \varepsilon < \delta \\ \dfrac{1}{2\delta} & for \quad -\delta < \varepsilon < \delta \\ 0 & for \quad \varepsilon > \delta \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

The noise power is given as,

$$\text{Noise power} = \frac{V_{noise}^2}{R}$$

Here V_{noise}^2 is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF $f_\epsilon(\varepsilon)$, its mean square value is given as,

$$\text{mean square value} = E[\varepsilon^2] = \overline{\varepsilon^2}$$

mean square value is given as,

From equation 3 ∞

$$E[\varepsilon^2] = \int_{-\delta}^{\delta} \varepsilon^2 \cdot \frac{1}{2\delta}\, d\varepsilon$$

$$= \frac{1}{2\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\delta}^{\delta}$$

$$= \frac{1}{2\delta}\left[\frac{\delta^3}{3} + \frac{\delta^3}{3}\right] = \frac{\delta^2}{3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Hence noise power will be,

$$\text{noise power} = \left(\frac{\delta^2}{3}\right) / R$$

Normalized noise power can be obtained with R = 1. Hence,

$$\text{noise power} = \frac{\delta^2}{3} \qquad \ldots\ldots\ldots\ldots\ldots(5)$$

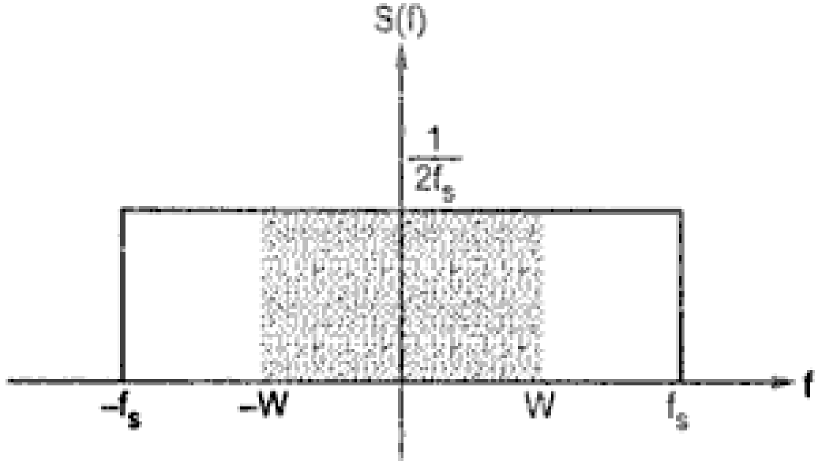


Fig. PSD of noise

This noise power is uniformly distributed over $-f_s$ to f_s range. This is illustrated in Fig. At the output of delta modulator receiver there is lowpass reconstruction filter whose cutoff frequency is 'W'. This cutoff frequency is equal to highest signal frequency. The reconstruction filter passes part of the noise power at the output as Fig. From the geometry of Fig. output noise power will be,

$$\text{Output noise power} = \frac{W}{f_s} \times \text{noise power} = \frac{W}{f_s} \times \frac{\delta^2}{3}$$

We know that $f_s = \frac{1}{T_s}$, hence above equation becomes,

$$\text{Output noise power} = \frac{W T_s \delta^2}{3} \qquad \text{..................(6)}$$

(iii) To obtain signal to noise power ratio

Signal to noise power ratio at the output of delta modulation receiver is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

From equation 2. and equation 6

$$\frac{S}{N} = \frac{\dfrac{\delta^2}{8\pi^2 f_m^2 T_s^2}}{\dfrac{W T_s \delta^2}{3}}$$

$$\boxed{\frac{S}{N} = \frac{3}{8\pi^2 W f_m^2 T_s^3}} \qquad \text{..............(7)}$$

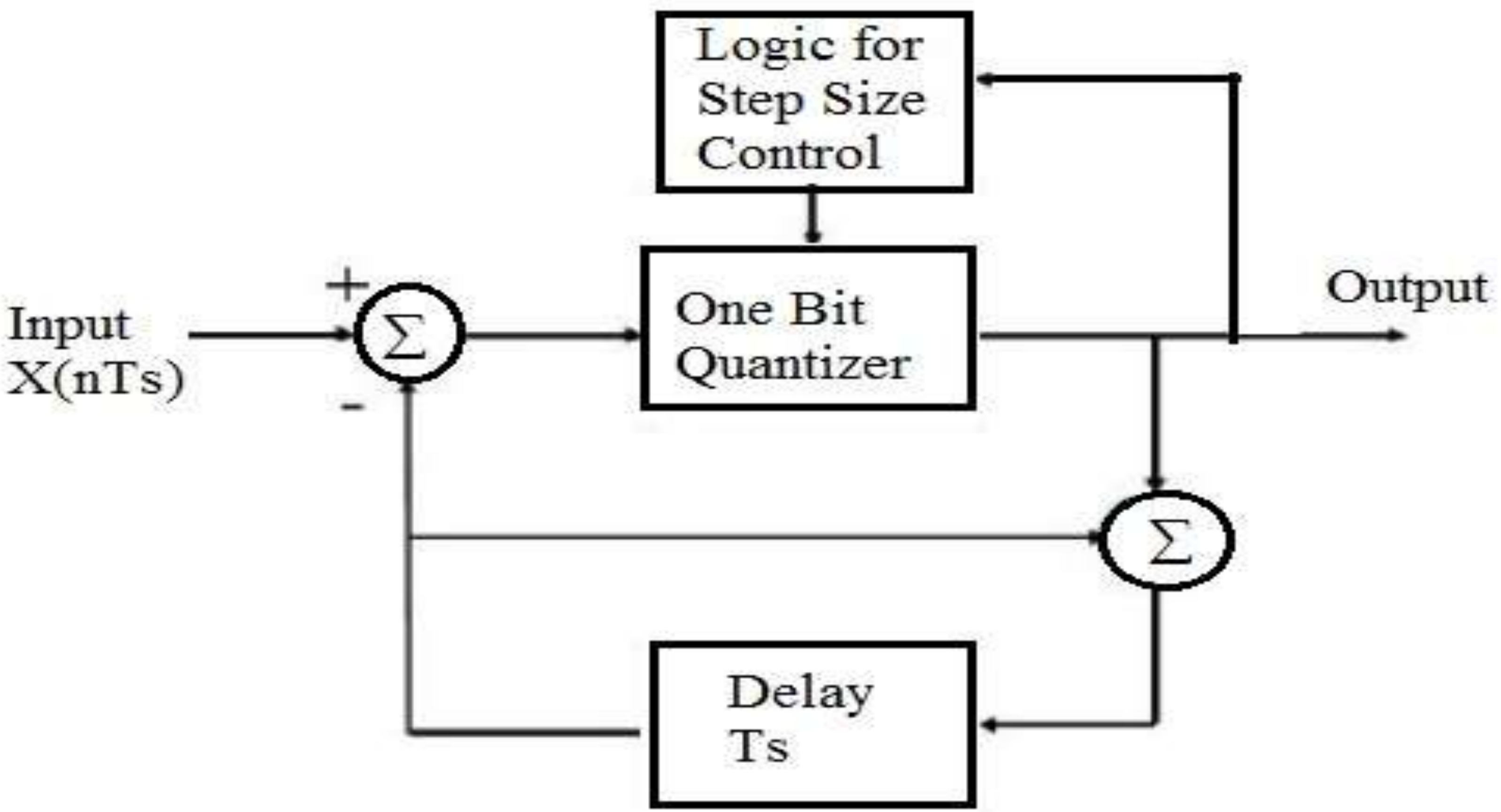
This is an expression for signal to noise power ratio in delta modulation.

Adaptive Delta Modulation (ADM)

NEED FOR ADAPTIVE DELTA MODULATION

- To overcome the quantization errors due to slope overload and granular noise, the **step size (Δ) is made adaptive** to variations in the input signal x(t).
- In the steep segment of the signal, the step size is increased.
- On the other hand if the input is varying slowly, the step size is reduced. This method is known as Adaptive Delta Modulation (ADM)
- The adaptive delta modulators can take **continuous changes in step size** or **discrete changes in step size.**

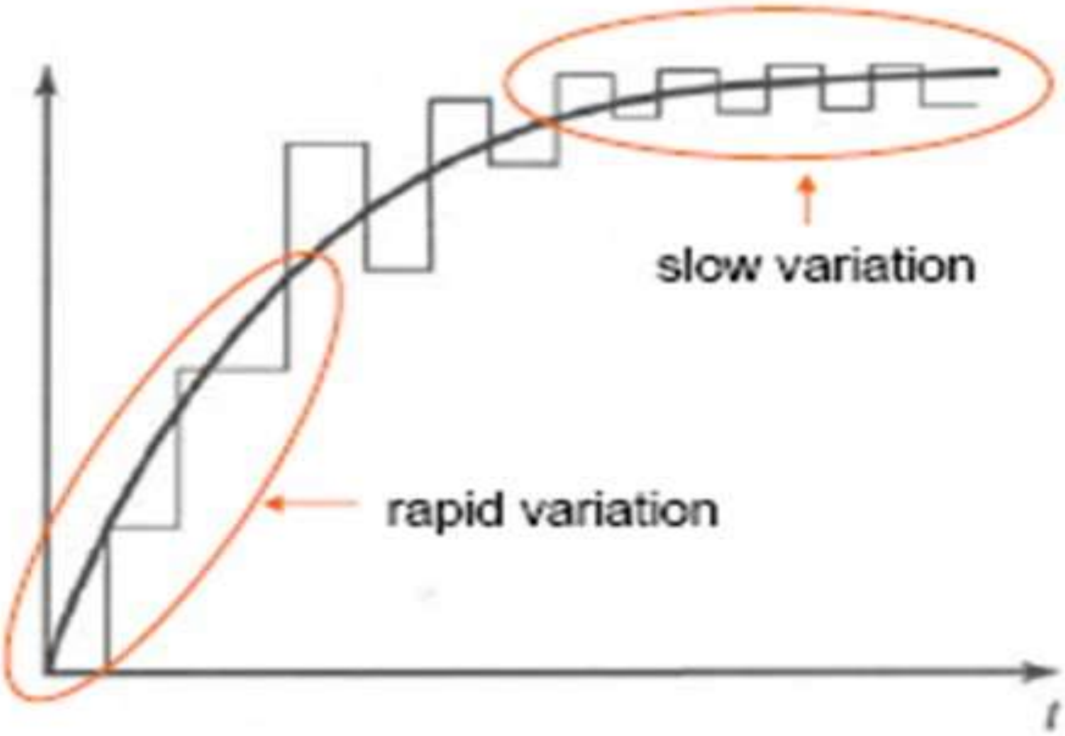
Block Diagram of Adaptive Delta Modulation



Adaptive Delta Modulation (ADM)

- Main idea: change step size according to changes in the input signal.
- If the input changes rapidly -> large step size. If the input changes slowly -> small step size.

- How to implement step size change?
- Simple solution: if two successive outputs have the same sign -> increase step size; if they are of opposite sign -> decrease step size.



Adaptive delta Modulation

- Adaptive delta modulation (ADM) is a modification of DM, in which the step size is adapted to the slope (variation) of the message signal.
- If successive errors are of opposite polarity, then the delta modulator is operating in the granular mode; in such a case it is advantageous to use reduced step size.
- If successive errors are of the same polarity, then the delta modulator is operating in its slope-overload mode; in this case, the step size should be increased.
- The algorithm used for adaptive DM with step size increase/decrease of 50% is

$$\Delta(k)=|\Delta(k\text{-}1)|(m_q(k)+0.5\mathrm{m}_q(k\text{-}1))/m_q(k), \qquad \text{if } \Delta(k\text{-}1)\geq\Delta_{\min}$$

$$\Delta(k)=\Delta_{\min}, \qquad \text{if } \Delta(k\text{-}1)<\Delta_{\min}$$

- where if $\Delta(k)$ is the step size at iteration k and $m_q(k)$ is the one-bit quantizer output that is equal to ±1.

Adaptive delta modulation system: (*a*) Transmitter. (*b*) Receiver

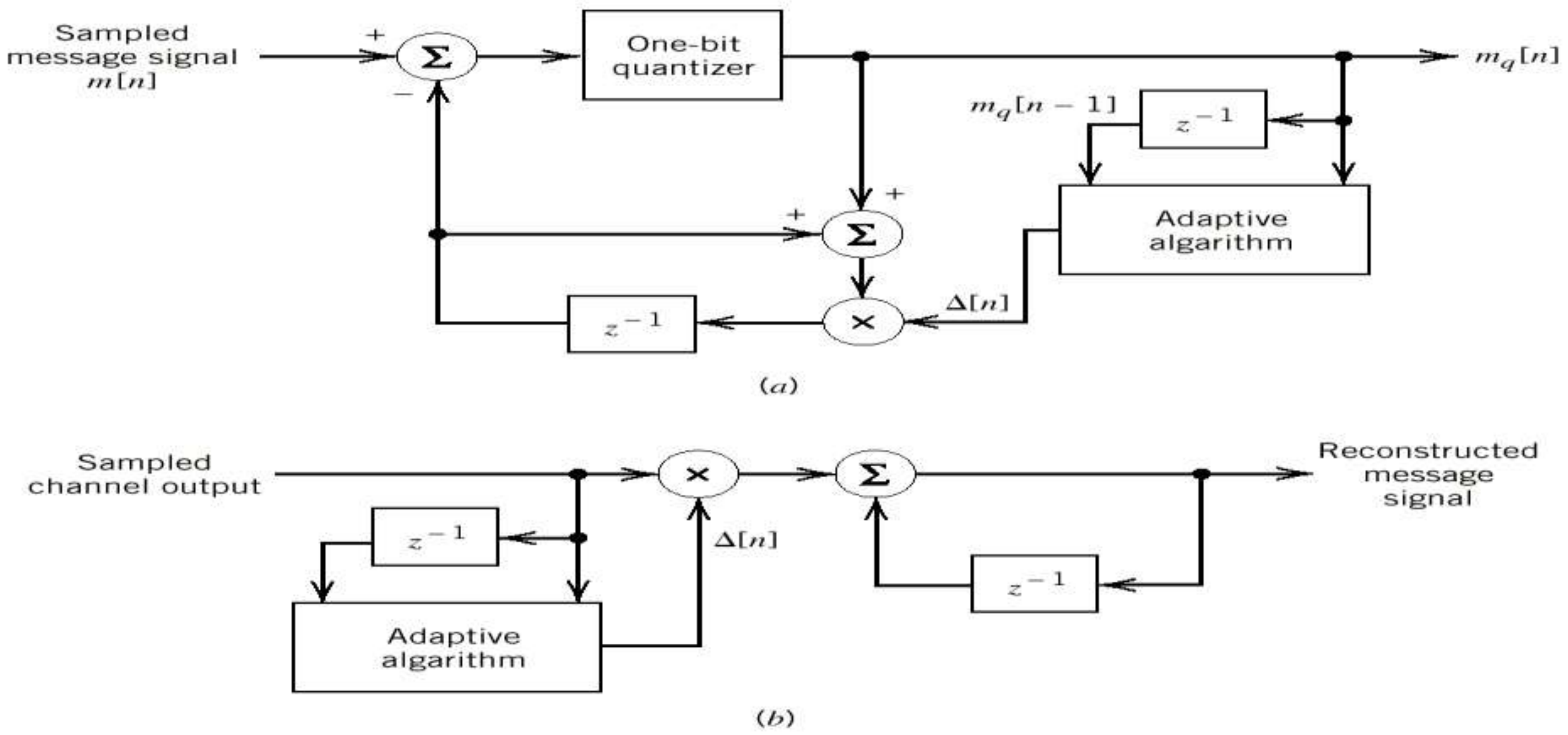
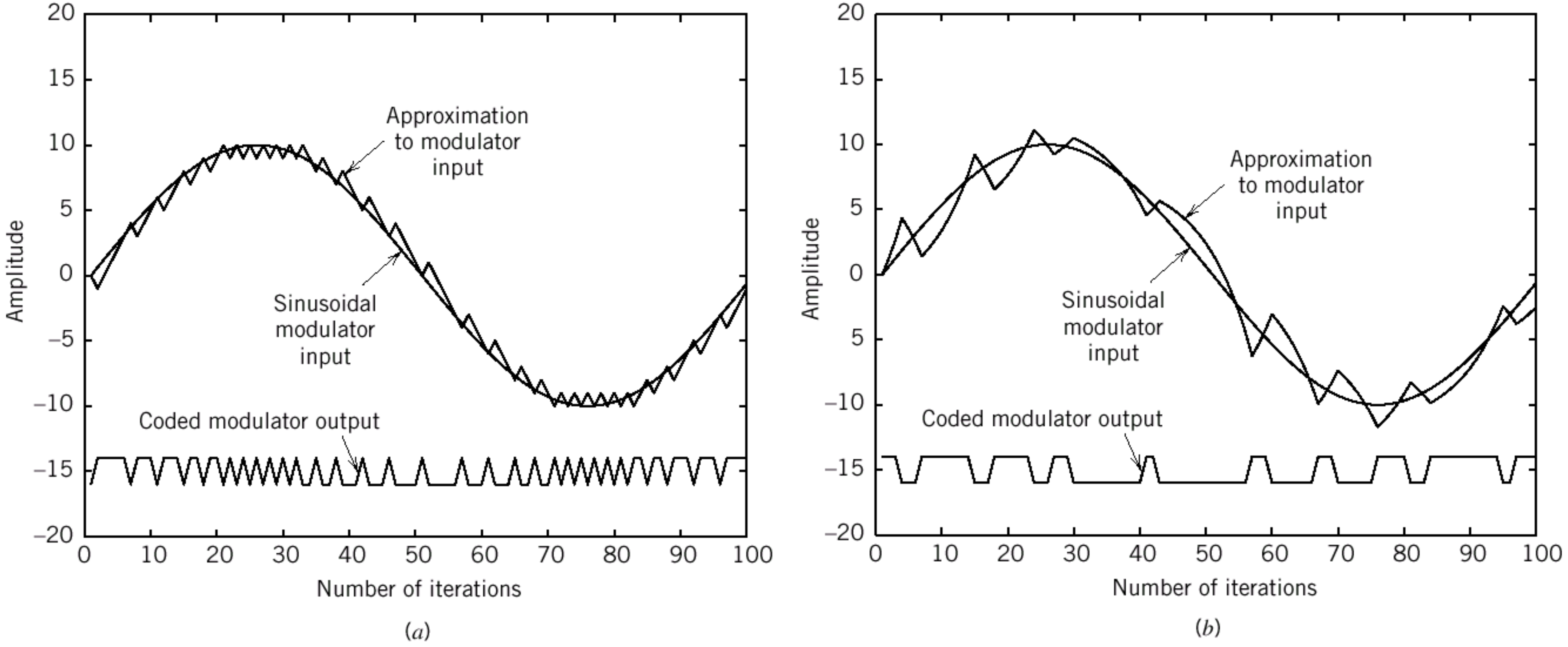
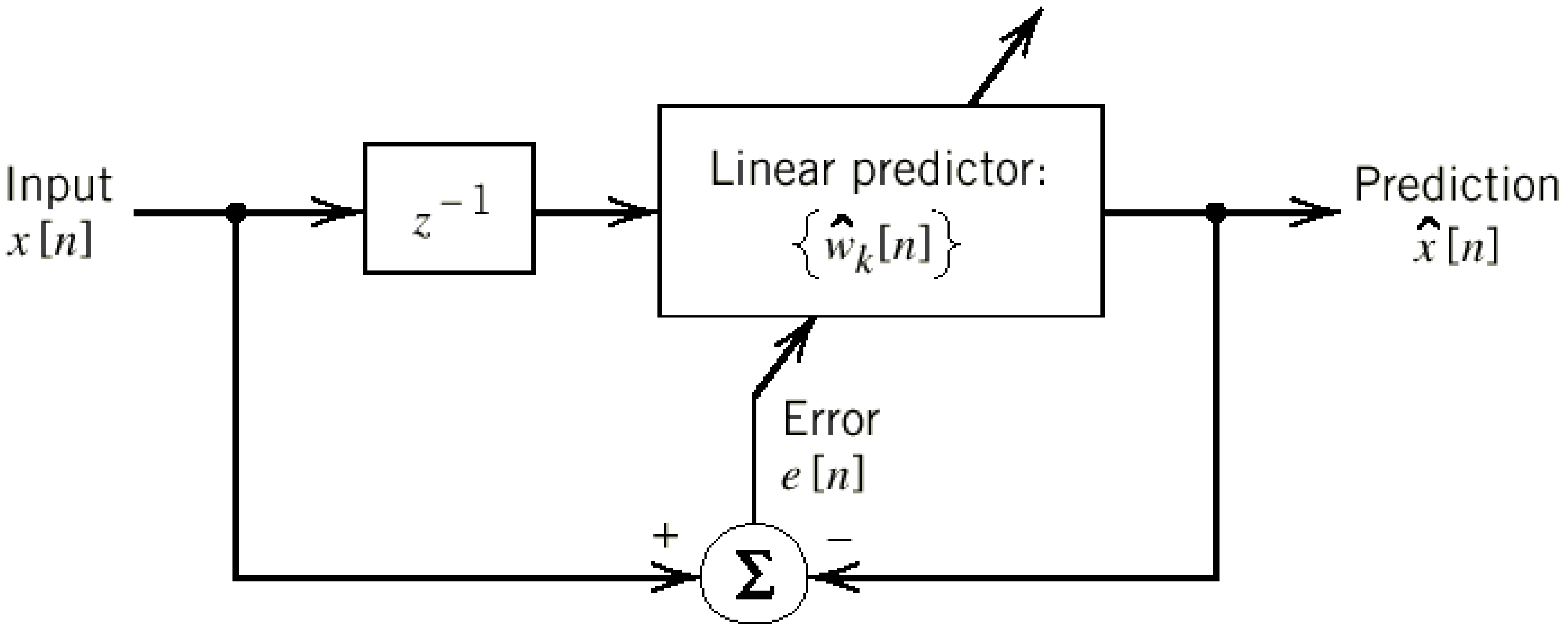


Figure 3.32

Waveforms resulting from the computer experiment on delta modulation: (*a*) Linear delta modulation. (*b*) Adaptive delta modulation.



Block diagram illustrating the linear adaptive prediction process.



Differential PCM (DPCM)

- Voice and video signals represented in PCM exhibit high correlation, which means that PCM signals contain redundant information. The result is an inefficient coding.
- By removing the PCM information redundancy a more efficient coded signal may be obtained. This is done using DPCM.
- In DPCM a linear prediction is performed on samples of a message signal m(kT_s)=$m(k)$, then the prediction error $e(k) = m(k) - \hat{m}(k)$ is computed and fed to a quantizer to obtain the quantized value $e_q(k)=e(k)+q(k)$, as shown by Figure 3.28a. $q(k)$ is the quantization error.
- The input of the linear predictor of Figure 3.28a is $m_q(k) = \hat{m}(k) + e_q(k) = m(k) + q(k)$, which represents a quantized version of the input sample $m(k)$.
- If the prediction is well performed, then the variance of $e(k)$ will be much smaller than the variance of $m(k)$, which results into a smaller number of levels to quantize $e(k)$.
- The receiver as given by Figure 3.28b, consists of a decoder which produces $e_q(k)$, that is added to the output of a prediction filter identical to the one used in the transmitter. The result is the quantized message signal $m_q(k)$.

Differential PCM (Continued)

- DPCM includes DM as a special case, where the prediction filter is a simple delay element. Simply put, DM is a one-bit version of DPCM.
- The problem of slope-overload distortion may also arise in DPCM, whenever the slope of the message signal changes too rapidly for the prediction filter to track it.
- The noise performance of DPCM is measured, as in other digital modulation systems, by the output signal-to-quantization noise, given by

$$(SNR)_o = \frac{\sigma_M^2}{\sigma_Q^2} = \left(\frac{\sigma_M^2}{\sigma_E^2}\right)\left(\frac{\sigma_E^2}{\sigma_Q^2}\right) = G_p (SNR)_Q$$

where $\sigma_M^2, \sigma_Q^2, \text{and}\, \sigma_E^2$ are the variances of $m(k)$, $q(k)$, and $e(k)$, respectively.

- The factor G_p is the processing gain produced by the DPCM quantization scheme. When G_p>1, which is the case most of the time, it represents the gain in SNR obtained by using DPCM compared to PCM.
- The receiver as given by Figure 3.28b, consists of a decoder which produces $e_q(k)$, that is added to the output of a prediction filter identical to the one used in the transmitter. The result is the quantized message signal $m_q(k)$.

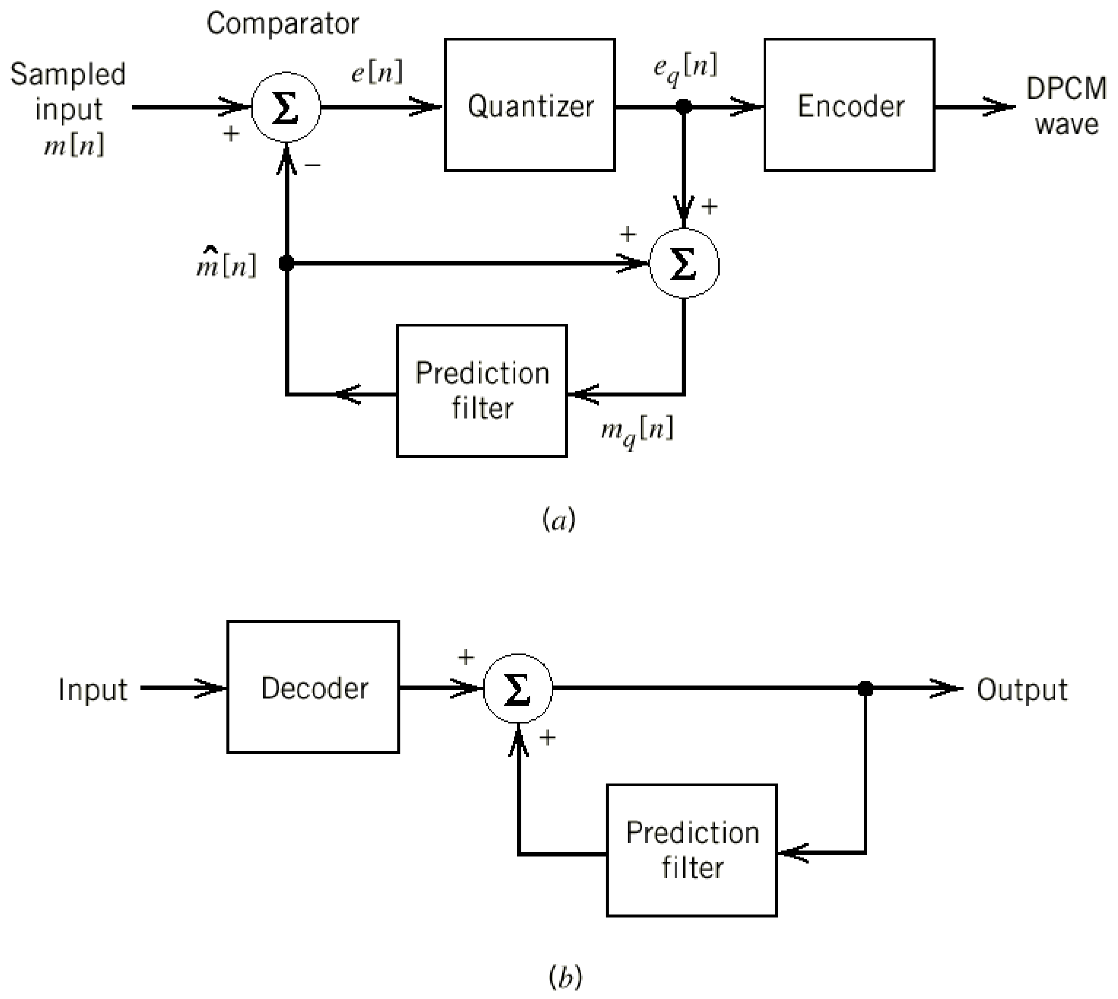


Figure 3.28

DPCM system. (*a*) Transmitter. (*b*) Receiver.

Adaptive DPCM

- In PCM, the standard bit rate is 64 kbits/s. The aim of all the variants of PCM is to reduce the number of bits used in the encoding process by removing redundancies.
- Adaptive DPCM (ADPCM) is a scheme that permits the coding of speech (voice) signals at 32 kbits/s through the combined use of adaptive quantization and adaptive prediction.
- Adaptive quantization refers to a quantizer that operates with a time-varying step-size $\Delta(k) = \phi\hat{\sigma}_M(k)$ and adaptive prediction filter refers to a filter with time-varying coefficients. ϕ is a constant and $\hat{\sigma}_M(k)$ is an estimate of the standard deviation of $m(k)$.
- In ADPCM adaptive quantization can be performed using *adaptive quantization with forward estimation* (AQF) or *adaptive quantization with backward estimation* (AQB).

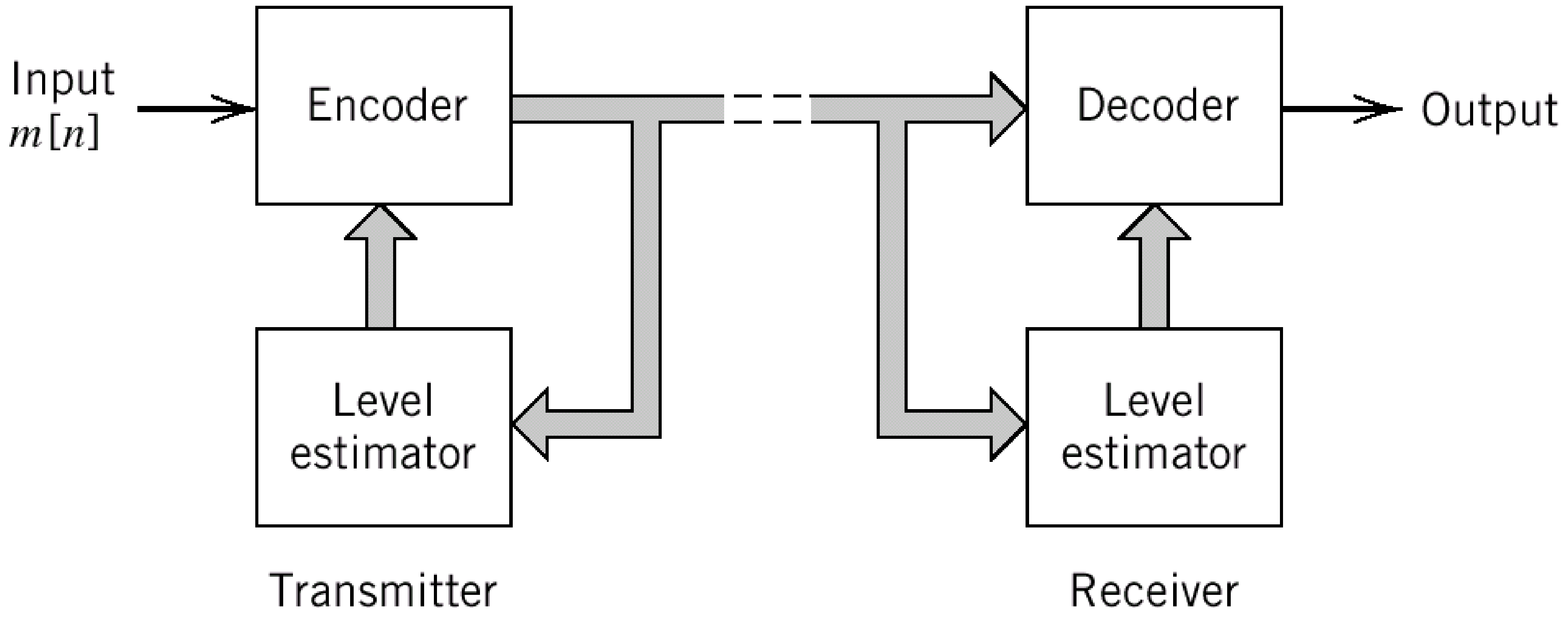
Adaptive DPCM (Continued)

- In ADPCM adaptive prediction can be performed using *adaptive prediction with forward estimation* (APF) or *adaptive prediction with backward estimation* (APB).

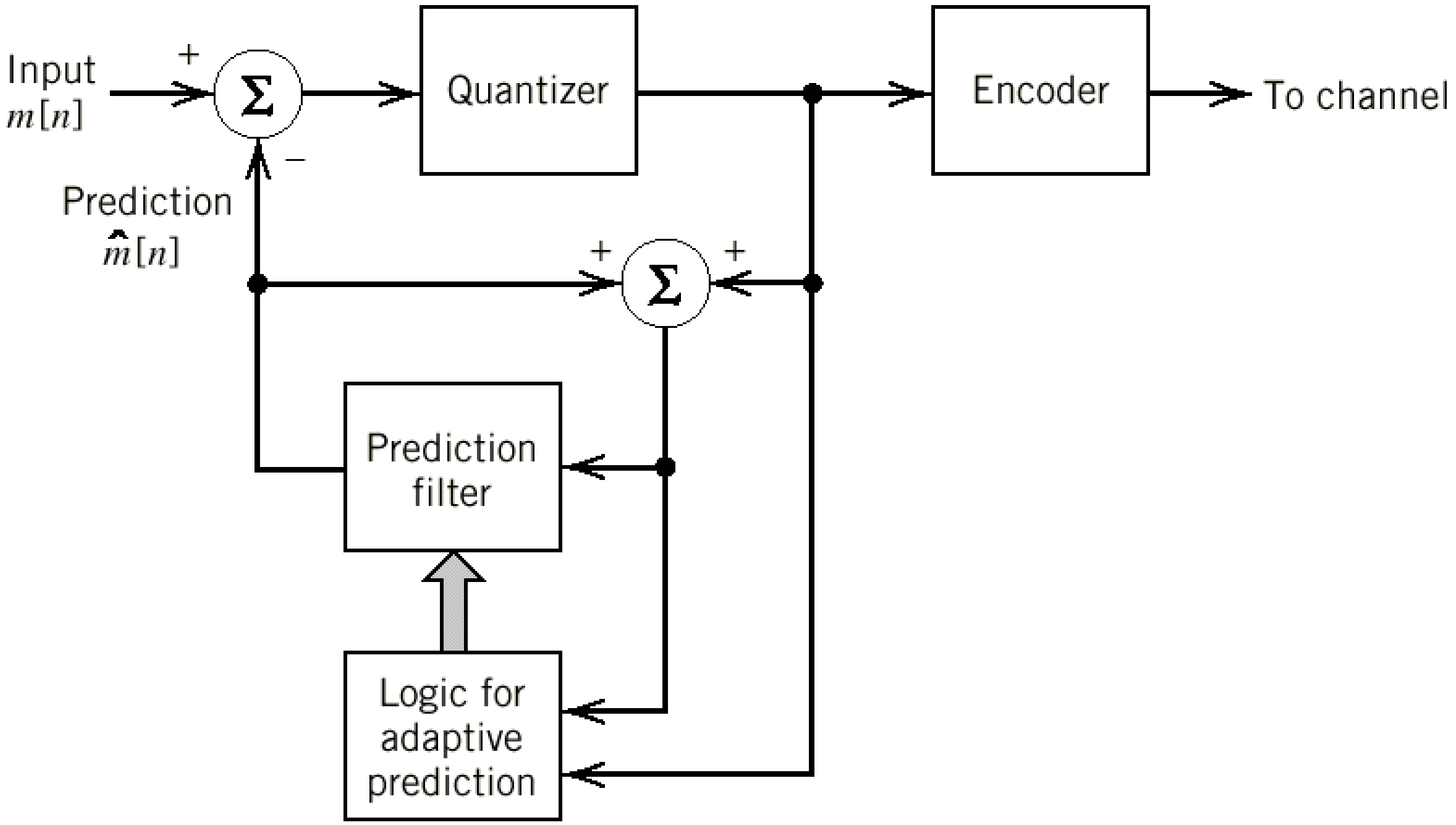
- AQF and APF use unquantized samples of the input message signal to estimate σ_M and the predictor coefficients **w**, respectively.
- AQB and APB use quantized samples of the input message signal to estimate σ_M and the predictor coefficients **w**, respectively.

- Both AQF and APF suffer from the same disadvantages, which are the buffering, the side (extra) information to be transmitted, and the delay. But by using AQB and APB these disadvantages are eliminated.
- Figure 3.29 shows the AQB scheme and Figure 3.30 shows the APB scheme.

Adaptive quantization with backward estimation (AQB).



Adaptive prediction with backward estimation (APB).



Line Encoding Techniques

Digital Data, Digital Signals

[the technique used in a number of LANs]

- Digital signal – is a sequence of discrete, discontinuous voltage pulses.
- Bit duration : the time it takes for the transmitter to emit the bit.
- Issues
  - Bit timing
  - Recovery from signal
  - Noise immunity

ON-OFF Coding

1 ⇔ Positive Voltage

0 ⇔ NO Voltage

Let Data: 01101

Bipolar Signaling /Non Return to Zero

Equal Positive & Negative Amplitude Voltage are represented

1 ⇔ Positive Voltage

0 ⇔ Negative Voltage

Return to Zero (RZ)

1 ⇔ Half Pulse width Rectangle is used for symbol ‘1’

0 ⇔ No Voltage

Split Phase or Manchester encoding

1 ⇔ **For Symbol ‘1’ with Positive pulse followed by negative pulse with both pulses being of equal amplitude and half-symbol wide.**

0 ⇔ **For Symbol ‘0’ the Polarities of these pulses are reversed. (Negative pulse followed by Positive pulse)**

Differential Encoding

1 ⇔ **No Transition for Symbol ‘1’**

0 ⇔ **Transition for Symbol ‘0’ based on the Reference Bit at Starting.**

Bit rate and Baud Rate

- Bit rate (R) : It is number of bits per second
- Baud Rate (r): It is number of Symbols per second

  If ‘n’ indicates number of bits/symbol $r = R/n$

  The total number of symbols $L = 2^n$
- Band width = 1/ Minimum Pulse Width Possible Band width = $1/T_b = R_b$

Prob1 :

An analog signal carries 4 bits/symbol elements. If 1000 signal elements are sent per second, Find bit rate and total number of elements.

Sol: n=4 bits/symbol elements

r= 1000 signal elements /second Bit Rate R= nr

=4x1000 =4kbps

The total number of elements L= $2^n=2^4$ =16

Prob2 :

An analog signal has a bit rate of 8000bps and a baud rate of 1000 baud. How many data elements are carried by each signal element? How many signal elements do we need?

Sol: R= 8000bps r= 1000

baud n=?

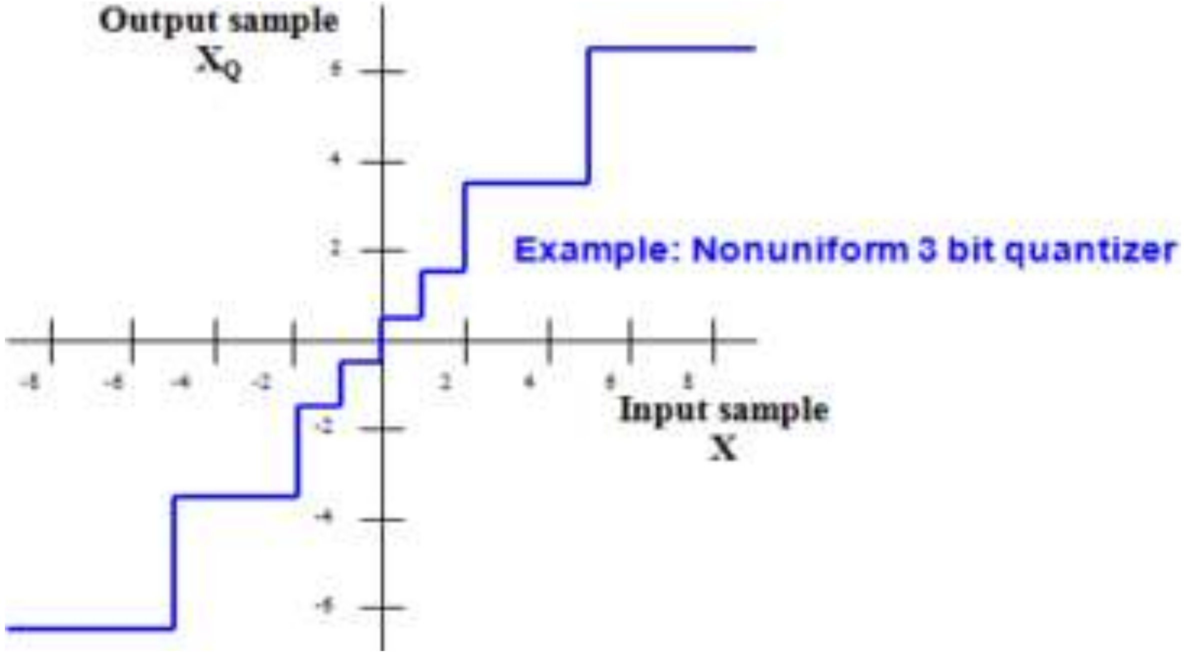
L=?

n= R/r = 8000/1000 =8 bits/element

L= $2^n=2^8$=256

Non-Uniform Quantization

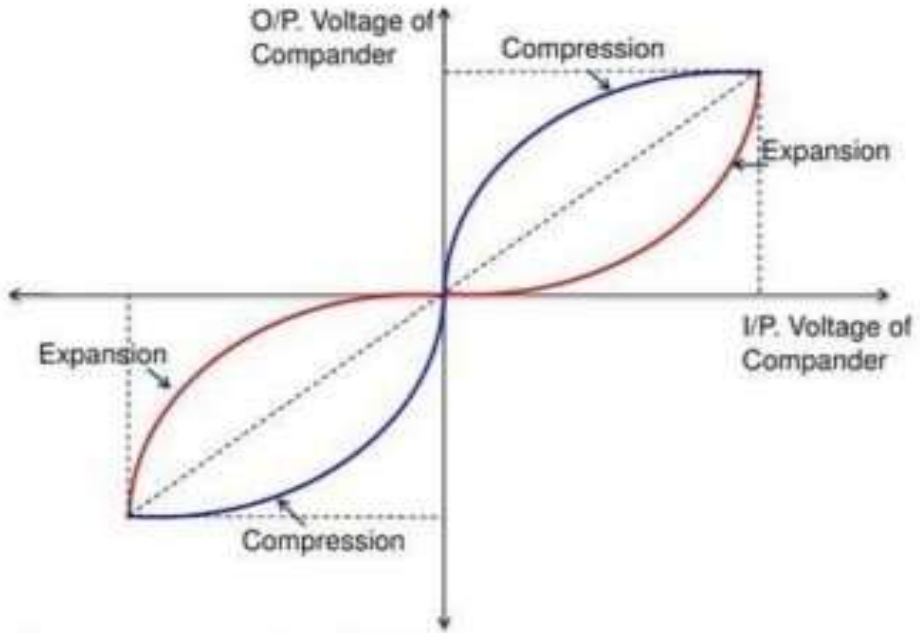
- In non-uniform quantization, the step size is not fixed. It varies according to certain law or as per input signal amplitude. The following fig shows the characteristics of Nonuniform quantizer.



In this figure observe that step size is small at low input signal levels. Hence quantization error is also small at these inputs. Therefore signal to quantization noise power ratio is improved at low signal levels. Stepsize is higher at high input levels. Hence signal to noise power ratio remains almost same throughout the dynamic range of quantizer.

Companding PCM System

- Non-uniform quantizers are difficult to make and expensive.
- An alternative is to first pass the speech signal through nonlinearity before quantizing with a uniform quantizer.
- The nonlinearity causes the signal amplitude to be compressed.
- At the receiver, the signal is expanded by an inverse to the nonlinearity.
- The process of compressing and expanding is called Companding.

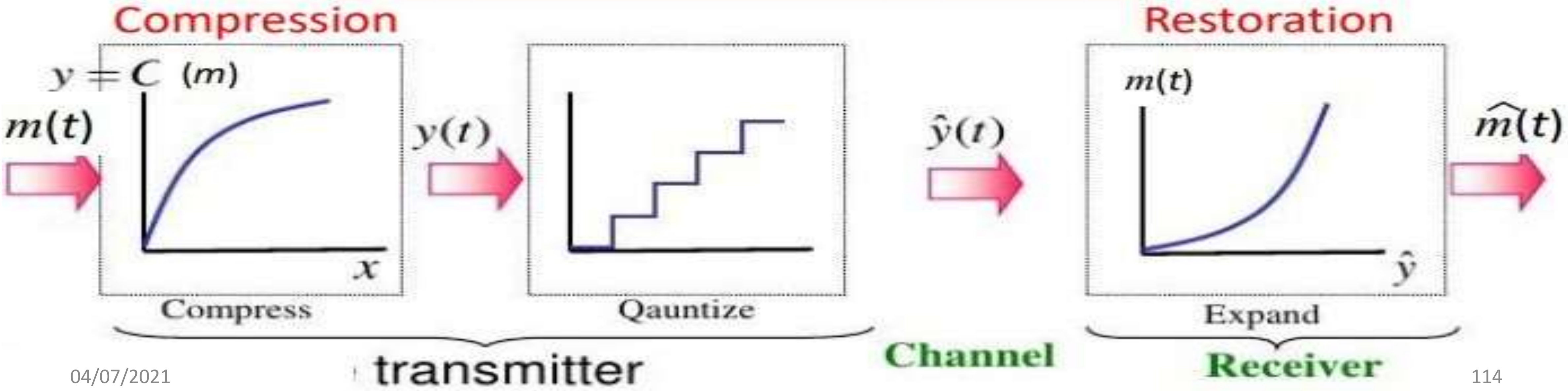


Use Compression and Expansion → Companding

Non-uniform quantization….process

- At the transmitter Uniformly quantizing the “compressed” signal.
- At the receiver, an inverse compression/expansion characteristic, called “expansion” is employed to avoid signal distortion.

$\boxed{\text{compression+expansion} \Rightarrow \text{companding}}$



Companding

- This is a non-linear technique / Companding used in PCM which compresses the data at the transmitter and expands the same data at the receiver. The effects of noise and crosstalk are reduced by using this technique.

- There are two types of Companding techniques. They are

  1) A-law Companding Technique

  2) μ-law Companding Technique

❖ µ-law Companding Technique.

- ❖ µ-law Companding is continuous in nature
- ❖ Uniform quantization is achieved at **$\mu = 0$**, where the characteristic curve is linear and no compression is done.
- ❖ µ-law has mid-tread at the origin.
- ❖ µ-law companding is used for speech and music signals.
- ❖ µ-law is used in North America and Japan.

❖ A-law Companding Technique

- ❖ Uniform quantization is achieved at **A=1**, where the characteristic curve is linear and no compression is done.
- ❖ A-law has mid-rise at the origin.
- ❖ A-law companding is used for PCM telephone systems.

μ-Law Companding:

Normally for speech and music signals a μ - law compression is used.

The Input and Output Relationship is given by

$$\boxed{|y| = \frac{\log[1 + \mu|x|]}{\log[1 + \mu]}}$$

A- L aw Companding:

The A law provides piecewise compressor characteristic. It has linear segment for low level inputs and logarithmic segment for high level inputs.

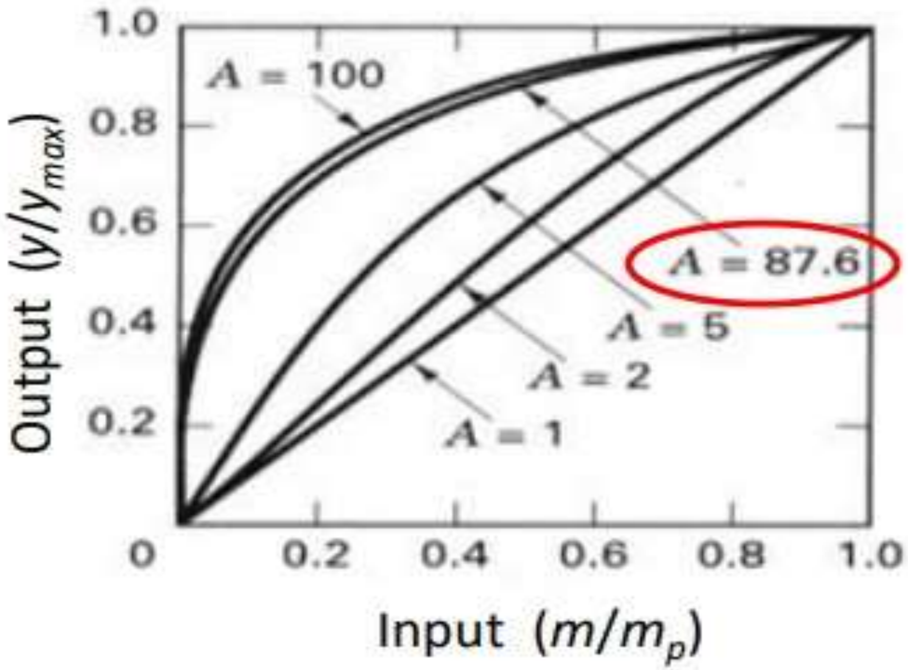
The Input and Output Relationship is given by

$$|y| = \begin{cases} \dfrac{A|x|}{1 + \log A}; & 0 \leq |x| \leq \dfrac{1}{A} \\[2mm] \dfrac{1 + \log[A|x|]}{1 + \log A}; & \dfrac{1}{A} \leq |x| \leq 1 \end{cases}$$

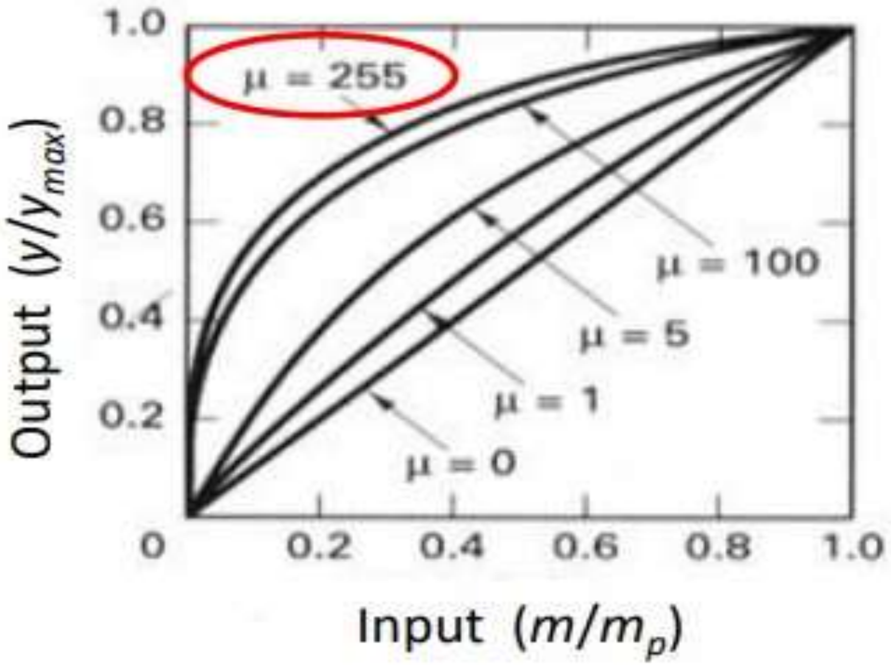
When A = 1, we get uniform quantization. The practical value for A is 87.56. Both A-law and μ-law companding is used for PCM telephone systems.

Companding Laws

A-Law Companding (Europe)



μ-Law Companding (North America)



$$y = \frac{A}{1+\log_e A}\left(\frac{m}{m_p}\right) \quad for \quad 0 \le \frac{m}{m_p} \le \frac{1}{A}$$

$$y = \frac{A}{1+\log_e A}\left(1+\log_e\left(\frac{A\,m}{m_p}\right)\right) \quad for \quad \frac{1}{A} \le \frac{m}{m_p} \le 1$$

$$\boxed{y = \frac{1}{\log_e(1+\mu)}\log_e\left(1+\frac{\mu\, m}{m_p}\right) \quad for \quad 0 \le \frac{m}{m_p} \le 1}$$

❖ μ-law Companding Technique

For given value of μ ,reciprocal slope of compression curve defines Quantum steps

$$\frac{dx}{dy} = \frac{\log(1+\mu)}{\mu} \, (1+\mu|x|)$$

❖ A-law Companding Technique:

For given value of A ,reciprocal slope of compression curve defines Quantum steps

$$\frac{dx}{dy} = (1+\log A)/A \qquad 0 \le |x| \le 1/A$$

$$= (1+\log A)|x| \qquad 1/A \le |x| \le 1$$

Thank You