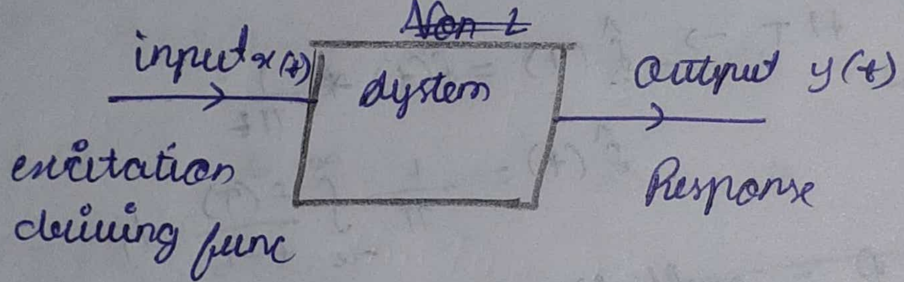
Unit 3: Signal Transmission through linear system.

Signal: Varying w.r.t time (I/p variable)

System: Physical device / Hardware / circuit / Software to perform some basic operations



ex: Filter, ~~Interrogator~~ Integrators, differentiators, Amplifiers, Attention

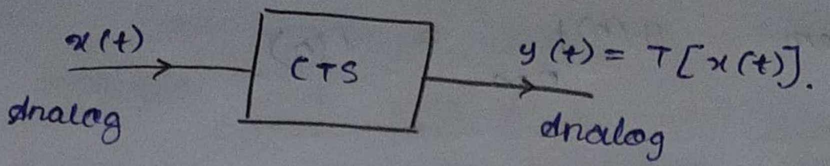
h(t) → Impulse Response. → which describes behaviour
The characteristics of response
↳ The i/p, o/p relation.

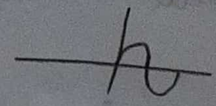
Purpose → change some prop of the signal
→ extract some feature
→ Change the spectrum → LF → HF
↓
speed video

we need a system

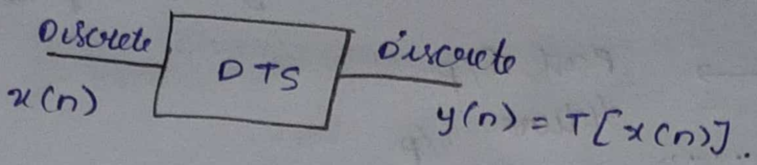
System → can be → CTS
→ DTS

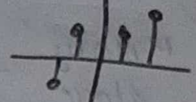
CTS ⇒ i/p is continuous / Analog & o/p → is also Analog/cont.





DTS ⇒ It process only digital signals
discrete





Classification of systems : Both CTS & DTS

(i) Linear & non-linear system

(ii) Time variant (shift variant) & time invariant (shift invariant)

(iii) static and dynamic system

(iv) Causal and Non-causal system

(v) stable and unstable system

(1) static & dynamic systems :

$x(t)$ & $y(t) \longrightarrow$ CTS

$x(n)$ & $y(n) \longrightarrow$ DTS

when i/p $x(t) \rightarrow y(t)$ = response

n or t the time at which o/p is obtained

static system : No memory.

A CTS or DTS is said to be static only if the o/p at any instant of time depends only at that instant of time.

$$y(t)\big|_{t=10} = x(t)\big|_{t=10}$$

t = present i/p = n

t+1 = future i/p = n+1

t−1 = past i/p = n−1

circuits only with resistors is an example of static system $y(n)\big|_{n=0} = x(n)$

Dynamic system: system with memory

$\Rightarrow$ o/p may depends not only on present i/p.

memory store $\longrightarrow$ Past i/p, future i/p

circuits with Inductors or capacitors

2. **Linear & non-linear systems:**

superposition $\rightarrow$ principle satisfied by a system then linear system

i/p $x(t)$ $\longrightarrow$ o/p $y(t)$ response.
complex signal

$x(t), x(n) \longrightarrow$ CTS/DTS $\longrightarrow y(t), y(n)$

$x(t) \rightarrow x_1(t), x_2(t), x_3(t)$ dividing into smaller for which the value obtained easily.

$$x(t) = x_1(t) + x_2(t) + \dots + x_n(t)$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$y(t) = y_1(t) + y_2(t) + \dots + y_n(t)$$

superposition : (i) additivity :$\rightarrow$ $x(t) \rightarrow y(t)$

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

when $\underbrace{x_1(t) + x_2(t)}_{x(t)} \longrightarrow \underbrace{y_1(t) + y_2(t)}_{y(t)}$

(ii) scaling / homogenity : If $x(t) \rightarrow y(t)$ then

$a\,x(t) \longrightarrow a\,y(t)$

If superposition not satisfied → Non-linear system.

3. Time Variant and Time Invariant systems:

Time Invariant : The system characters / behaviours does not depend on time

i/p - o/p characteristics of the system do not vary w.r.t time

also called shift invariant

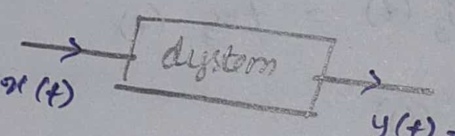
otherwise time variant / shift variant

at $t = 0$ & $t = 10$ we get same o/p but time at which response occurs may differ

LTI = Linear Time Invariant system.

1. $y(t) = 2x(t) + 3$ ⟹ i/p - o/p relation

$x(t) = $ i/p , $y(t) = $ o/p



Check the system for linearity.

Sol. If obeys superposition theorem = linear

$x(t) \longrightarrow y(t) = 2x(t) + 3$

$x_1(t)$, $x_2(t)$

↓ ↓

$y_1(t)$, $y_2(t)$

$y_1(t) = 2x_1(t) + 3$

$y_2(t) = 2x_2(t) + 3$

Let $ax_1(t) + bx_2(t) = x_3(t) \longrightarrow y_3(t) = 2x_3(t) + 3$

↓

$ay_1(t) + by_2(t)$

$y_3(t) = 2x_3(t) + 3 = 2(ax_1(t) + bx_2(t)) + 3$

$= 2ax_1(t) + 2bx_2(t) + 3$

$ay_1(t) = a(2x_1(t) + 3) = 2ax_1(t) + 3a$

$by_2(t) = b(2x_2(t) + 3) = 2bx_2(t) + 3b$

$ay_1(t) + by_2(t) = 2ax_1(t) + 3a + 2bx_2(t) + 3b \rightarrow ①$

$ax_1(t) + bx_2(t) = 2ax_1(t) + 2bx_2(t) + 3. \rightarrow ②$

$\therefore$ ① $\neq$ ②

$\therefore$ Non-linear system.

2. $y(t) = t\,x(t)$ check for linearity

Sol. if $x_1(t) \longrightarrow y_1(t) = t\,x_1(t) \rightarrow ①$

$x_2(t) \longrightarrow y_2(t) = t\,x_2(t) \rightarrow ②$

$ax_1(t) + bx_2(t) = x_3(t)$

if $x_3(t) \longrightarrow y_3(t) = t x_3(t)$

$y_3(t) = t(ax_1(t) + bx_2(t))$

$= at\,x_1(t) + bt\,x_2(t) \rightarrow ③$

$ay_1(t) + by_2(t) = a\,t\,x_1(t) + b\,t\,x_2(t) \longrightarrow ④$

$ax_1(t) + bx_2(t) \longrightarrow ay_1(t) + by_2(t)$

③ = ④ $\Rightarrow \therefore$ linear system

3. $y(t) = x^2(t)$

$y_1(t) = x_1^2(t)$, $y_2(t) = x_2^2(t)$, $y_3(t) = x_3^2(t) = (ax_1(t) + bx_2(t))^2$

$y_3(t) = ay_1(t) + by_2(t) = ax_1^2(t) + bx_2^2(t) \rightarrow ①$

$y_3(t) = x_3^2(t) = a_1^2 x_1^2(t) + b^2 x_2^2(t) + 2ab\, x_1(t) x_2(t) \rightarrow ②$

$\therefore$ Non lineas.

4. $y(t) = e^{x(t)}$

$y_1(t) = e^{x_1(t)}$ $\quad y_2(t) = e^{x_2(t)}$

$y_3(t) = e^{x_3(t)} = e^{a x_1(t) + b x_2(t)} = e^{a x_1(t)} \cdot e^{b x_2(t)} \rightarrow ①$

$y_3(t) = a y_1(t) + b\, y_2(t) = a e^{x_1(t)} + b e^{x_2(t)} \rightarrow ②$

$\therefore$ Non - lineas

5. $y(t) = x(t)\, x(t-4)$.

$y_1(t) = x_1(t)\, x_1(t-4)$, $\quad y_2(t) = x_2(t)\, x_2(t-4)$

$y_3(t) = x_3(t)\, x_3(t-4)$

$x_3(t) = a x_1(t) + b x_2(t)$

$x_3(t-4) = a x_1(t-4) + b x_2(t-4)$

$y_3(t) = (a x_1(t) + b x_2(t))(a x_1(t-4) + b x_2(t-4)) \rightarrow ①$

$y_3(t) = a y_1(t) + b y_2(t)$

$y_3(t) = a x_1(t) x_1(t-4) + b x_2(t) x_2(t-4) \rightarrow ②$

$\therefore$ Non - lineas

6 =) $y(n) = 2x(n) + 5$

when i/p $= x(n) = 0$ =) $y(n) = 5 =$ o/p

2) $y(t) = e^{x(t)}$ =) if $x(t) = 0$ =) then $y(t) = 1$

} Non - lineas

2) when i/p $= 0$ and o/p $= 0$ $\longrightarrow$ lineas.

Time Invariance:

Step 1: Let $y_1(t)$ - is the o/p of system when ~~to~~ t_0 delay is introduced in $x(t)$

If $x(t) \longrightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$

$y_1(t) = T[x(t-t_0)] \longrightarrow (1)$

Step 2: let $y_2(t)$ is o/p of system when to delay is introduced at the o/p.

$y_2(t)$ i.e $y(t-t_0)$

$y_1(t) = y(t-t_0) \longrightarrow (2)$

$(1) = (2) \Rightarrow y_1(t) = y_2(t)$ then time invariant system.

1. $y(n) = 2x(n)+5$ check for time invariance.

1) introduce n_0 time delay in the i/p signal

$y(n) = 2x(n)+5$

$y_1(n) = T[x(n-n_0)]$

$y_1(n) = 2x(n-n_0)+5$

2) Introduce the same n_0 time delay in the o/p

$y_2(n) = y(n-n_0)$

~~$x(n) \Rightarrow y(n)$~~

~~$x(n-n_0) \rightarrow y(n-n_0)$~~

~~$\hookrightarrow y_1(n)$ $\hookrightarrow y_2(n)$~~

$y(n-n_0) = 2x(n-n_0)+5 = y_2(n)$

$y_1(n) = y_2(n)$

$\therefore$ TIVS.

2. $y(t) = t\,x(t)$

$y_1(t) = T[x(t-t_0)]$

$y_2(t) = y(t-t_0)$

If $y_1(t) = y_2(t)$ ⇒ Time invariant system

$y_1(t) = (t \cancel{-t_0})\, x(t-t_0)$

↙

o/p of system when t_0 delay is applied in the i/p signal

$y_2(t) = y(t-t_0)$, $y(t) = t\,x(t)$

$y(t-t_0) = (t-t_0)\,x(t-t_0) = y_2(t)$

$y_1(t) \neq y_2(t)$

∴ Time variant system

3. $y(t) = \sin 6t \cdot x(t)$

$y_1(t) = \sin 6t \cdot x(t-t_0)$, $y_2(t) = \sin 6(t-t_0)\, x(t-t_0)$

∴ $y_1(t) \neq y_2(t)$ ⇒ TVS

4. $y(t) = \sin(x(t))$

$y_1(t) = \sin(x(t-t_0))$, $y_2(t) = \sin(x(t-t_0))$

∴ $y_1(t) = y_2(t)$ ⇒ TIVS

5. $y(t) = e^{x(t)}$

$y_1(t) = e^{x(t-t_0)}$, $y_2(t) = e^{x(t-t_0)}$

∴ $y_1(t) = y_2(t)$ ⇒ TIVS

6. $y(t) = x(3t)$

$y_1(t) = x(3t-t_0)$ $y_2(t) = x(3(t-t_0)) = x(3t-3t_0)$

∴ $y_1(t) \neq y_2(t)$ ⇒ TVS

Time Response of a linear time Invariant system :

LTI $\longrightarrow$ linearity + Time invariance $\longrightarrow$ satisfied

linearity $\downarrow$ superposition

Time invariance $\downarrow$ response will not change wherever the i/p is applied

Time response $\rightarrow$ o/p of LTI system in time domain

$f(t)/x(t)$ $\longrightarrow$ [LTI system] $\longrightarrow$ $r(t)/y(t)$

driving function   response

$f(t) \longrightarrow r(t)$

$\downarrow$

divide into parts

$f_1(t) \longrightarrow r_1(t)$

$f_2(t) \longrightarrow r_2(t)$

$\vdots$

$f_n(t) \longrightarrow r_n(t)$

LTI system response $\rightarrow$ Impulse response.

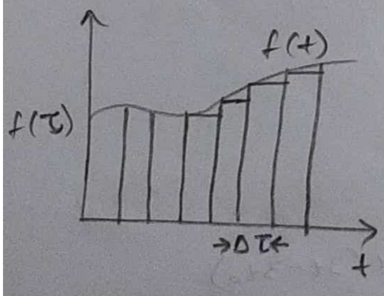
It describes the behaviour /characteristics of the system in time domain.

Time response of LTI system $\longrightarrow$ Impulse response $\rightarrow h(t)$

Impulse $\downarrow$ $\delta(t)$ response $\downarrow$ o/p

~~when~~ It is the response of the system when Impulse func ($\delta(t)$) (or) delta func is applied at the i/p of the system then it is called Impulse response

If $x(t) = \delta(t)$ $= \text{Impulse}$ then $y(t) = h(t)$ $= \text{Impulse response}$



staircase wave.

$f(t) \longrightarrow$ [LTI] $\longrightarrow r(t) = r_1(t) + r_2(t)$

$\downarrow$

$f_1 + f_2$

$f(t) = \sum_{i=1}^{n} f_i(t) \longrightarrow r(t) = \sum r_i(n)$

as ΔT width of the pulse is minimum → better the approximation

as $\Delta T \to 0$ ⟹ pulse → ↑ Impulse   at $t=0$, $\delta(t)=1$

at $t = T$, $\Delta T \cdot f(T)$ = weight of Impulse $= \delta(t-T)$

$\delta(t-T)$ — Impulse occurring at $t = T$; ΔT — width; $f(T)$ — height

$x(t)$ → [LTI] → $y(t)$ $r(t) = h(t)$

$x(t) = \delta(t)$ → $h(t)$

$\delta(t-T)$ → $h(t-T)$

$$r(t) = \lim_{\Delta T \to 0} \sum_{T=-\infty}^{\infty} f(T)\,\Delta T\, h(t-T).$$

$$f(t) \begin{cases} f_T \longrightarrow h(t-T) \\ f_2 \longrightarrow h(t-2) \\ f_1 \longrightarrow h(t-1) \end{cases}$$

$$\boxed{r(t) = \int_{-\infty}^{\infty} f(T)\, h(t-T)\, dT}$$

$r(t) = f(t) * h(t)$ → Impulse response

($f(t)$ — i/p; $*$ — convolution)

2) Response of LTI system in time domain = Convolution of i/p & impulse response.

⟹ $h(t)$ → describes the characteristics of LTI system.

⟹ $f(t)$ → $f_1(t) \to r_1(t)$
→ $f_2(t) \to r_2(t)$
⋮
→ $f_n(t) \to r_n(t)$

Total response for $f(t)$

$f(t) = f_1(t) + f_2(t) + \dots + f_n(t)$

$r(t) = r_1(t) + r_2(t) + \dots + r_n(t)$

If $f(t) = \delta(t)$ then $y(t) = h(t)$.

$y(t) = r(t) = f(t) * h(t)$

Transfer function of the LTI system:

$h(t)$ = Impulse Response → will describe chars / behaviours of LTI system in time domain

Response in freq domain → Transfer function.

↓ Freq domain counterpart of Impulse Resp.

$H(\omega)$ = Transfer function = $F.T[h(t)]$

w.k.t $\Rightarrow y(t) = r(t) = f(t) * h(t) \rightarrow$ convolution in time domain.

Apply F.T on both sides.

$f(t) \longleftrightarrow F(\omega)$

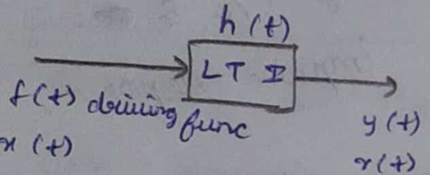
$h(t) \longleftrightarrow H(\omega)$, $r(t) \longleftrightarrow R(\omega)$

$F.T[r(t)] = F.T[f(t) * h(t)]$

w.k.t $\rightarrow$ convolution in time domain gives multiplication in freq. domain

$R(\omega) = F(\omega) \cdot H(\omega)$. $\Rightarrow H(\omega) = \dfrac{R(\omega)}{F(\omega)} = \dfrac{Y(\omega)}{F(\omega)}$

$H(\omega) = \dfrac{F.T[\text{Response or O/P}]}{F.T[\text{i/p}]}$ = Transfer function.
(with initial conditions = zero)



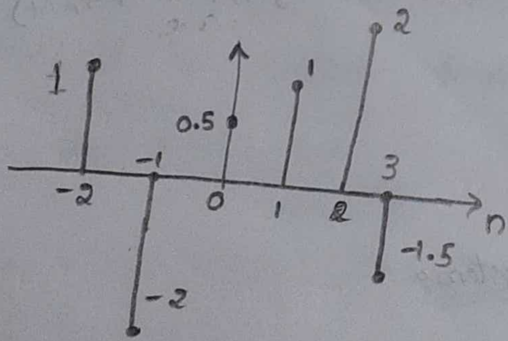
$F(\omega) / X(\omega)$

$= f(t) * h(t) \rightarrow$ Time domain representation

$R(\omega) = F(\omega) \cdot H(\omega)$

$H(\omega) = \dfrac{R(\omega)}{F(\omega)} \rightarrow$ freq domain representation

~~Representation~~ Representation of arbitrary sequence:



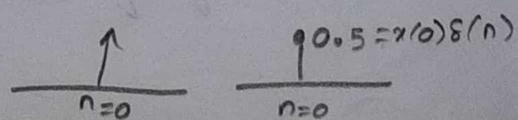
$x(n) = [1, -2, 0.5, 1, 2, -1.5]$
↳ express this using impulse

Impulse $= \delta(t) = 1$, $t = 0$
$= 0$, $t \neq 0$

Impulse sequence $\Rightarrow \delta(n) = 1$ for $n = 0$
$= 0$ $n \neq 0$

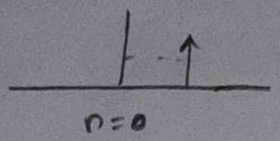
write $x(0) = 0.5$ using impulse

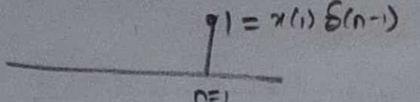
$x(0)\, \delta(n) = 0.5(1) = 0.5 = x(0)$

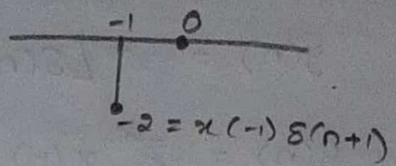


$x(0)\, \delta(n-1) = 0$

$x(1) = x(1)\,\delta(n-1)$, $x(2) = x(2)\,\delta(n-2)$, $x(-1) = x(-1)\,\delta(n+1)$

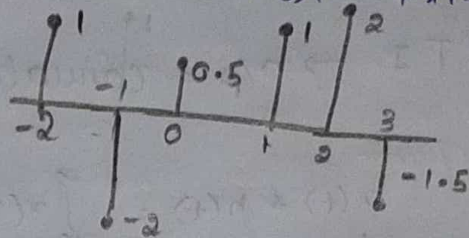






$x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + x(3)\,\delta(n-3) + \cdots + x(-1)\,\delta(n+1)$

$+ x(-2)\,\delta(n+2) + \cdots\cdots =$



$$\boxed{x(n) = \underbrace{\sum_{K=-\infty}^{\infty} x(K)}_{\text{Amplitude/strength of impulse}}\ \underbrace{\delta(n-K)}_{\text{location of impulse}}}$$

$= 0.5\,\delta(n) + \delta(n-1) + 2\delta(n-2) - 1.5\,\delta(n-3)$

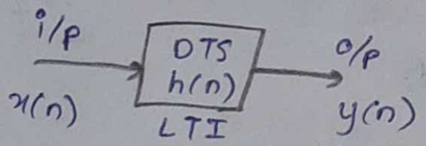
$\quad - 2\,\delta(n+1) + \delta(n+2)$

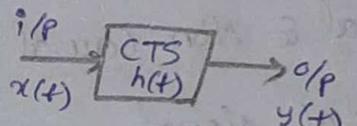
Convolution sum:

Convolution Integral $= y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$\downarrow$

O/P for CTS





The O/P of the LTI system $= y(n) = T[x(n)]$

↳ Transformation

$$y(n) = T\left[\sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)\right]$$

$y(n) =$ O/P of DT LTI system.

$$= T\left[\sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)\right]$$

$$= \sum_{K=-\infty}^{\infty} x(K)\, T[\delta(n-K)]$$

($\because x(K)$ is scalar like const) let $x(n) = \delta(n-K)$

~~y(n) =~~ If $x(n) = \delta(n)$ then $y(n) = h(n)$

= impulse = Impulse response

If $x(n) = \delta(n-k)$ then $y(n) = h(n-k)$

$y(n) = T[\delta(n-k)] = h(n-k)$

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, T(\delta(n-k))$

$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

Any arbitrary $= x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$

LTI $\rightarrow$ o/p = convolution $= x(n) * h(n)$

$= x(t) * h(t)$

$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Properties: (i) Commutative : $x(t) * h(t) = h(t) * x(t)$

$x(n) * h(n) = h(n) * x(n)$

$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{n=-\infty}^{\infty} h(k)\, x(n-k) = h(n) * x(n)$

(ii) Associative : $(x(t) * h_1(t)) * h_2(t) = x(t) * (h_1(t) * h_2(t))$

(iii) Distributive : $x(n) * (h_1(n) + h_2(n)) = x(n) * h_1(n) + x(n) h_2(n)$

Causal & Non-causal systems:

Causal signal $\rightarrow x(t) = 0 \quad t<0$ (only +ve)

$x(t) = 0 \quad t>0 \rightarrow$ Anti causal (-ve)

$x(t) \neq 0 \quad (t<0 \;\&\; t>0) \rightarrow$ Non-causal (Both +ve & -ve)

Causal:

If the response of the system depends on present & past values of the i/p but not on future i/p

Non-Causal:

Response depends on present, past & future i/p's

$y(t) = 2x(t+2) + x(t$

at $t = 0 \Rightarrow y(0) =$

$y(n) = x(n) + x(n-$

$y(0) = x(0) + x(-3$

$y(1) = x(1) + x(-$

causal systems $\rightarrow$

the de

Non-causal system

If $y(n) = \sum_{k=-\infty}^{\infty} x($

$x(n)$ = present i/p , $h(n)$

w.r.t $y(n) = \sum_{k=-\infty}^{\infty} x(k$

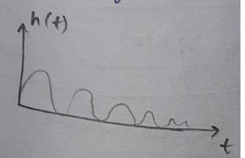
$= \dots\dots\dots$

for $-ve$ 'k' $\Rightarrow$ th

do don't conside

$\therefore h(n) = $

Condition for caus



Causal

If $x(n) = \delta(n-k)$ then $y(n) = h(n-k)$

$y(n) = T[\delta(n-k)] = h(n-k)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, T(\delta(n-k))$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Any arbitrary $= x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$

LTI $\rightarrow$ o/p = convolution $= x(n) * h(n)$

$= x(t) * h(t)$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

Properties: (i) Commutative: $x(t) * h(t) = h(t) * x(t)$

$x(n) * h(n) = h(n) * x(n)$

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) = \sum_{n=-\infty}^{\infty} h(k) x(n-k) = h(n) * x(n)$$

(ii) Associative: $(x(t) * h_1(t)) * h_2(t) = x(t) * (h_1(t) * h_2(t))$

(iii) Distributive: $x(n) * (h_1(n) + h_2(n)) = x(n) * h_1(n) + x(n) h_2(n)$

Causal & Non-Causal systems:

Causal signal $\longrightarrow x(t) = 0 \quad t < 0$ (only +ve)

$x(t) = 0 \quad t > 0 \rightarrow$ Anti causal (−ve)

$x(t) \neq 0 \quad (t < 0 \,\&\, t > 0) \rightarrow$ Non-causal (Both +ve & −ve)

Causal:

If the response of the system depends on present & past values of the i/p but not on future i/p

Non-Causal:

Response depends on present, past & future i/p's

$y(t) = 2x(t+2) + x(t) + x(t-4)$

at $t = 0 \Rightarrow y(0) = \underset{\text{future}}{2x(2)} + \underset{\text{present}}{x(0)} + \underset{\text{past}}{x(-4)}$ = Non-causal ~~signal~~ system

$y(n) = x(n) + x(n-3)$

$$\left.\begin{array}{l} y(0) = x(0) + x(-3) \\ y(1) = x(1) + x(-2) \end{array}\right\} \text{causal system}$$

causal systems → Non-anticipative

They can't predict the response of the system prior to the application of i/p

Non-causal system → anticipative.

If $y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

$x(n)$ = present i/p, $h(n)$ = Impulse response → how to find causality

w.r.t $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$$= \ldots\ldots h(-2)\underbrace{x(n+2)}_{\text{future i/p}} + h(-1)\, x(n+1) + h(0)\underbrace{x(n)}_{\text{present i/p}}$$

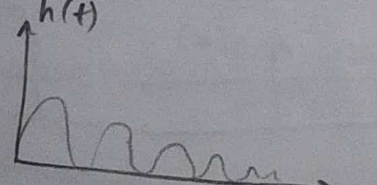
$$+ h(1)\underbrace{x(n-1)}_{\text{past i/p}} + h(2)x(n-2) + \ldots$$

for -ve 'k' ⇒ the response depended on future i/p's

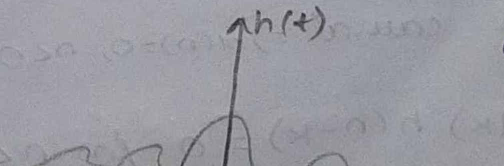
so don't consider for -ve 'k' for causal system.

$\therefore h(n) = 0$ for $n < 0$

Condition for causality = Impulse response = $h(n) = 0$ for $n<0$



Causal



Non-causal

W.K.t the response of the LTI system, from the convolution integral sum.

$y(t) = f(t) * h(t) =$

$$y(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, f(t-\tau)\, d\tau$$

Let the i/p $f(t)$ is bounded $|f(\tau)| = \text{finite} = M$

$$|y(t)| = \int_{-\infty}^{\infty} |h(\tau)\, f(t-\tau)\, d\tau| \le \int_{-\infty}^{\infty} |h(\tau)||f(t-\tau)|\, d\tau$$

$$|y(t)| \le \int_{-\infty}^{\infty} |h(\tau)|\, M$$

for the stability of the system = Bounded o/p $= \int_{-\infty}^{\infty} h(\tau)\, d\tau = \text{finite}$

$h(t) \rightarrow$ I.R $\Rightarrow \int_{-\infty}^{\infty} h(t)\, dt$ = absolutely integrable = o/p = bounded

condition for stability : The I R Must be absolutely integrable

$\int_{-\infty}^{\infty} h(t)\, dt = \text{finite}$ $\qquad \sum_{n=-\infty}^{\infty} h(n) = \text{finite}$

absolutely summable.

causality $\longrightarrow h(t) = 0, \ t < 0$

Convolution : This will give in the i/p - o/p relation response of an LTI system.

$$y(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$$

Let $f_1(t)$ & $f_2(t)$ are the i/p's then the conv of $f_1(t)$ & $f_2(t)$ denoted by $f(t) = f_1(t) * f_2(t)$

$$= \int_{-\infty}^{\infty} f_1(\tau)\, f_2(t-\tau)\, d\tau$$

time convolution.

2. By using frequency convolution.

$$f_1(t).\,f_2(t) \Rightarrow f_1(t).\,f_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\lambda)F_2(\omega-\lambda)\,d\lambda$$

$$= \frac{1}{2\pi}\left[F_1(\omega) * F_2(\omega)\right]$$

$y(t) = f(t) * h(t)$ Apply F.T $\Rightarrow \int_{-\infty}^{\infty} f(\tau)\,h(t-\tau)\,d\tau$

$Y(\omega) = F(\omega).\,H(\omega)$

$$H(\omega) = \frac{Y(\omega)}{F(\omega)} = \frac{\text{F.T of the o/p}}{\text{F.T of the i/p}} = \text{Transfer function.}$$

F.T of Impulse response = Transfer function

$\text{F.T}\left[h(t)\right] = H(\omega).$

Inverse F.T of transfer function = $h(t)$ = Impulse response

1. Find the response $y(t)$ of the LTI system when $h(t) = IR = u(t-1)$ & $f(t) = e^{-2t}u(t)$

Sol.

$f(t) = e^{-2t}u(t)$ → LTI system → $y(t) = ?$

$h(t) = u(t-1)$

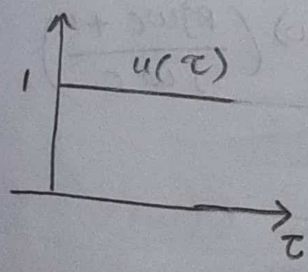
Analytical Approach $\Rightarrow y(t) = f(t) * h(t)$

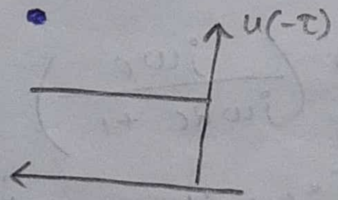
$$= \int_{-\infty}^{\infty} f(\tau)\,h(t-\tau)\,d\tau.$$

W.K.T $u(t) = 1,\ t \geq 0$

$u(t) = 0,\ t < 0$

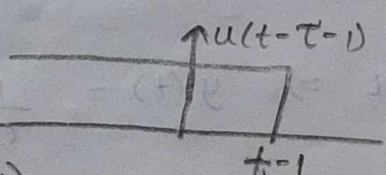
$$= \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)\,u(t-\tau-1)\,d\tau.$$

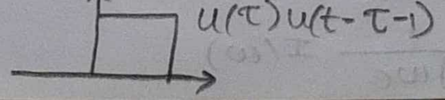




$t-1 = +ve$, to right

$t-1 = -ve$ to left





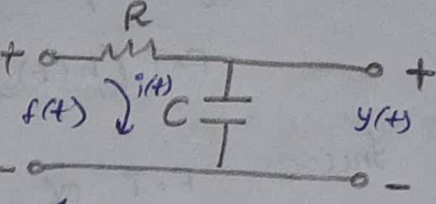
$$y(t) = \int_{-\infty}^{\infty} e^{-2\tau} u(\tau)\, u(t-\tau-1)\, d\tau.$$

$$= \int_{0}^{t-1} e^{-2\tau} d\tau = \left(\frac{e^{-2\tau}}{-2}\right)_0^{t-1} = \frac{-1}{2}\left(e^{-2(t-1)} - 1\right)$$

$$= \frac{-1}{2}\left(e^{-2t}e^{2} - 1\right)$$

$$e^{-2t}u(t) * u(t-1) = \frac{1}{2}\left(1 - e^{-2(t-1)}\right)$$

2. Find the transfer function and impulse response of the ~~RLC~~ RC N/w



Low pass filter.

Sol $H(\omega) = \dfrac{F.T\,[o/p]}{F.T\,[i/p]}$

$h(t) = F^{-1}[H(\omega)]$

$V_L = L\dfrac{di(t)}{dt}$, $V_C = \dfrac{1}{C}\int i(t)\,dt$, $V_R = IR$

KVL $\Rightarrow$ $f(t) - i(t)R - \dfrac{1}{C}\int i(t)\,dt = 0$

$f(t) = i(t)R + \dfrac{1}{C}\int i(t)\,dt.$

Apply F.T

$F.T\,[f(t)] = F.T\left[R\,i(t) + \dfrac{1}{C}\int i(t)\,dt\right]$

$F(\omega) = R\,I(\omega) + \dfrac{1}{C}\dfrac{1}{j\omega}I(\omega)$

$F(\omega) = I(\omega)\left[R + \dfrac{1}{j\omega C}\right] = I(\omega)\left(\dfrac{Rj\omega C + 1}{j\omega C}\right)$

$I(\omega) = F(\omega)\left(\dfrac{j\omega C}{j\omega RC + 1}\right)$

KVL $\Rightarrow$ $y(t) = \dfrac{1}{C}\int i(t)\,dt$

$Y(\omega) = \dfrac{1}{j\omega C}I(\omega)$

~~I(ω)~~ = $Y(\omega) = \frac{1}{j\omega C} \cdot \frac{F(\omega)\, \cancel{j\omega C}}{1 + j\omega RC}$

$$\frac{Y(\omega)}{F(\omega)} = \frac{1}{1+j\omega RC} = H(\omega) = \text{Transfer function}$$

$$h(t) = F^{-1}[H(\omega)] = F^{-1}\left(\frac{1}{1+j\omega RC}\right)$$

w.r.t $e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$.

$$h(t) = F^{-1}\left(\frac{1}{RC(\frac{1}{RC}+j\omega)}\right) = \frac{1}{RC} e^{-\frac{1}{RC}t} u(t)$$

= Impulse Response.

3. $H(\omega) = \frac{1}{1+j\omega RC}$, if $R = 1\Omega$, $C = 1F$ find the response of RC N/W for the i/p ~~x(t)~~ $x(t) = e^{-2t}u(t)$

Sol. Impulse response $= h(t) = \frac{1}{RC} e^{-t/RC} u(t)$

Input signal $= x(t) = e^{-2t}u(t)$

output response $= y(t) = x(t) * h(t)$

Apply F.T

$$Y(\omega) = X(\omega) . H(\omega)$$

$x(t) = e^{-2t}u(t) \Rightarrow X(\omega) = \frac{1}{2+j\omega}$

$h(t) = \frac{1}{RC} e^{-t/RC} u(t) \Rightarrow H(\omega) = \frac{1}{1+j\omega RC} = \frac{1}{1+j\omega}$

$$Y(\omega) = \frac{1}{2+j\omega} \cdot \frac{1}{1+j\omega} \quad \cancel{= \frac{1}{2+3j\omega-\omega^2}}$$

$$Y(\omega) = \frac{-1}{2+j\omega} + \frac{1}{1+j\omega}.$$

$$y(t) = F^{-1}[Y(\omega)] = F^{-1}\left(\frac{-1}{2+j\omega} + \frac{1}{1+j\omega}\right)$$

$$y(t) = -e^{-2t}u(t) + e^{-t}u(t)$$

$$y(t) = u(t)\left(e^{-t} - e^{-2t}\right)$$

(03)

$$y(t) = x(t) * h(t)$$

$$H(\omega) = \frac{1}{1+j\omega RC}, \quad H(\omega) = \frac{1}{1+j\omega}$$

$$h(t) = e^{-t}u(t)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)\, e^{-(t-\tau)}\, u(t-\tau)\, d\tau$$

$$u(\tau)\, u(t-\tau) = 0 \longrightarrow t$$

$$y(t) = \int_0^t e^{-2\tau} e^{-t} e^{\tau}\, d\tau = e^{-t}\int_0^t e^{-\tau}\, d\tau$$

$$= e^{-t}\left(\frac{e^{-\tau}}{-1}\right)_0^t$$

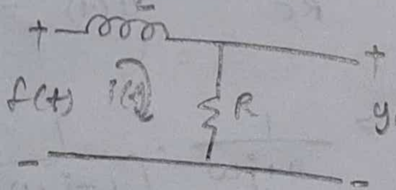
$$= -e^{-t}\left(e^{-t} - 1\right)$$

$$= -e^{-2t} + e^{-t}$$

4. Find the impulse response of the N/w.

Sol.

$$H(\omega) = \frac{F.T[O/P]}{F.T[i/P]}$$



$$h(t) = F^{-1}[H(\omega)]$$

$$V_L = L\frac{di(t)}{dt} \;\&\; V_R = i(t)R$$

$$KVL \Rightarrow \quad f(t) - L\frac{di(t)}{dt} - i(t)R = 0$$

$$f(t) = L\frac{di(t)}{dt} + i(t)R$$

$$F(\omega) = j\omega L\, I(\omega) + I(\omega)R$$

$$= I(\omega)(j\omega L + R)$$

$$I(\omega) = \frac{F(\omega)}{R + j\omega L}$$

$$KVL \Rightarrow y(t) = i(t)R \quad , \quad Y(\omega) = I(\omega)\cdot R$$

$$Y(\omega) = \frac{F(\omega)}{R + j\omega L} R$$

$$H(\omega) = \frac{Y(\omega)}{F(\omega)}$$

$$H(\omega) = \frac{R}{R + j\omega L}$$

$$h(t) = F^{-1}[H(\omega)] = F^{-1}\left[\frac{R}{R + j\omega L}\right]$$

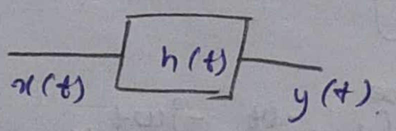
$$= F^{-1}\left[\frac{1}{1 + j\omega L/R}\right]$$

$$\cancel{= e^{-L/R\, t} u(t)} = F^{-1}\left[\frac{1}{L/R(R/L + j\omega)}\right]$$

$$h(t) = \frac{R}{L} e^{-R/L\, t} u(t). = \text{Impulse Response.}$$

5. Consider the system with Impulse response $h(t) = \frac{1}{RC} e^{-t/RC} u(t)$ find the step response.

Sol



If $x(t) = \delta(t) =$ Impulse then $y(t) = h(t) = IR$

$x(t) = u(t) =$ step then $y(t) = s(t) =$ step response

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \frac{1}{RC} e^{-\tau/RC} u(\tau)\, u(t-\tau)\, d\tau$$

$$= \int_{0}^{t} \frac{1}{RC} e^{-\tau/RC}\, d\tau = \frac{1}{RC} \frac{\left(e^{-\tau/RC}\right)_0^t}{-1/RC}$$

$$= -\left(e^{-t/RC} - 1\right)$$

step response $= y(t) = 1 - e^{-t/RC}$

6. A system has the response $v_o(t) = (e^{-2t} + e^{-3t})u(t)$ for the i/p ~~at t~~ $v_i(t) = \delta(t)$. Find the excitation (i/p) driving function to produce $v_o(t) = t e^{-2t} u(t)$

Sol. $x(t) = v_i(t) = \delta(t)$ if ~~$y(t) =$~~ $h(t) = v_o(t) = (e^{-2t} + e^{-3t})u(t)$

$x(t) = ?$ if $y(t) = te^{-2t} u(t)$

$y(t) = x(t) * h(t)$

$Y(\omega) = X(\omega) \cdot H(\omega)$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)}$$

$$x(t) = F^{-1}[X(\omega)] = F^{-1}\left(\frac{Y(\omega)}{H(\omega)}\right)$$

$Y(\omega) = F^{-1}\left[t e^{-2t} u(t)\right]$

$$Y(\omega) = \int_{0}^{\infty} t\, e^{-2t} e^{-j\omega t}\, dt$$

$$Y(\omega) = \int_0^\infty t\, e^{-(2+j\omega)t}\, dt = \frac{1}{(2+j\omega)^2}$$

$$H(\omega) = F^{-1}\left[(e^{-2t} + e^{-3t})\, u(t)\right] = \frac{1}{2+j\omega} + \frac{1}{3+j\omega}$$

$$= \frac{5+2j\omega}{(2+j\omega)(3+j\omega)}$$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{\frac{1}{\cancel{(2+j\omega)^2}}}{\frac{5+2j\omega}{\cancel{(2+j\omega)}(3+j\omega)}} = \frac{3+j\omega}{(2+j\omega)(5+2j\omega)}$$

$$x(t) = F^{-1}[X(\omega)] = F^{-1}\left[\frac{1}{2+j\omega} - \frac{1}{5+2j\omega}\right]$$

$$= e^{-2t}u(t) - \frac{1}{2}e^{-5/2\, t}\, u(t)$$

Graphical Method of computing convolution:

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)\, f_2(t-\tau)\, d\tau$$

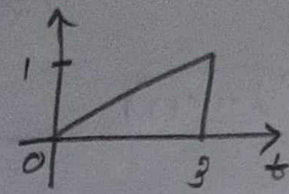
$$f_1(t) \cdot f_2(t) = \frac{1}{2\pi}\left[F_1(\omega) * F_2(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\lambda) \cdot F_2(\omega-\lambda)\, d\lambda$$

Steps :

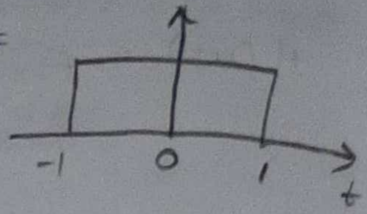
1. Replace t by τ (dummy) to get $f_1(\tau)$ & $f_2(\tau)$
2. ~~by~~ Time reverse $f_2(\tau) = f_2(-\tau)$ by folding $f_2(\tau)$ along vertical axis
3. Shift $f_2(-\tau)$ by 't' to get $f_2(t-\tau)$ if $t = +ve$ then shift $f_2(-\tau)$ in the $+ve, \tau$ direction. If $t = -ve$ then shift $f_2(-\tau)$ in the $-ve$ τ direction
4. Find the area under product of $f_1(\tau) f_2(t-\tau)$ for a particular t i.e $\int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau)\, d\tau$
5. Repeat step 4 for all the other values of t_0

1. When two $f_1(t)$ & $f_2(t)$ are known graphically

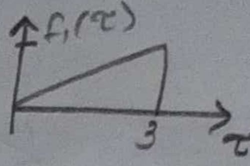
$f_1(t) =$

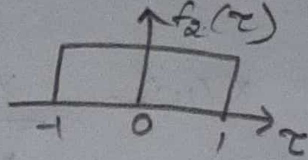


$f_2(t) =$



Sol. 1. Replace t by τ

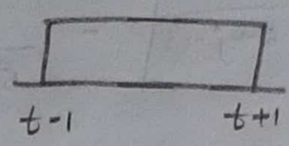


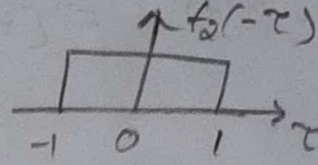


2. $f_2(-\tau)$ by folding $f_2(\tau)$ about vertical axis.

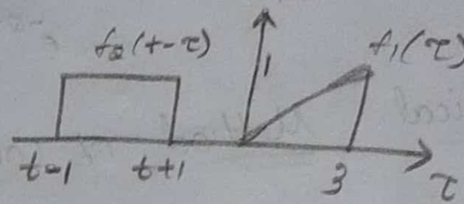
3. Shift $f_2(-\tau)$ by 't'

$f_2(t-\tau)$



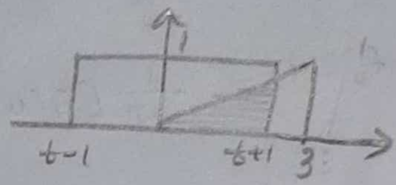


4. As long as $t+1<0$ we get No common product.



$\therefore$ The area under the product curve $=0$

$\therefore$ $t+1$ should be >0.



$$f_1(\tau) - 0 = \frac{1-0}{3-0}(\tau - 0)$$

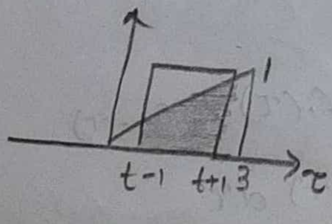
$$f_1(\tau) = \frac{\tau}{3} \qquad 0 \le \tau \le 3.$$

$t-1<0,\ t+1>0 \implies -1<t<1$

$$\int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau)\, d\tau = \int_0^{t+1} \tau/3 \cdot 1 \cdot d\tau$$

$$= \frac{1}{3}\left(\frac{\tau^2}{2}\right)_0^{t+1}$$

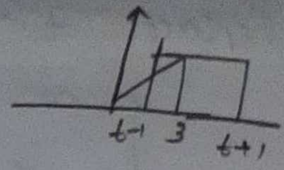
$$= \frac{(t+1)^2}{6} \cdot \text{ for } -1<t<1$$



$$\int_{t-1}^{t+1} \frac{\tau}{3}\, d\tau = \frac{1}{3}\left(\frac{\tau^2}{2}\right)_{t-1}^{t+1}$$

for $t-1 \ge 0 \implies t \ge 1$, $t+1 \le 3 \implies t \le 2$ $\implies 1 \le t \le 2$ $= \frac{1}{6}(t^2+2t+1-t^2+2t-1) = \frac{2}{3}t$

$$y(t) = \int_{t-1}^{3} f_1(\tau) f_2(t-\tau)\, d\tau = \int_{t-1} \frac{\tau}{3} \cdot 1 \cdot d\tau$$



$$= \frac{1}{6} (\tau^2)_{t-1}^{3} = \frac{1}{6} (9 - t^2 + 2t - 1)$$

$$= \frac{-t^2 + 2t + 8}{6}$$

for $2 \leq t \leq 4$

for $t-1 < 3 = t < 4$

$t+1 > 3 = t > 2$

for $t-1 > 3$ $\Rightarrow$ No common product

$f(t) = f_1(t) * f_2(t)$

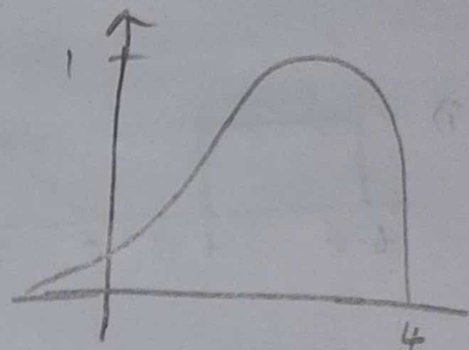
$= 0 \qquad t < -1$

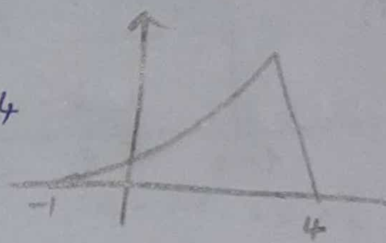
$= \frac{1}{6}(t+1)^2 \qquad -1 \leq t < 1$

$= \frac{2}{3} t \qquad 1 \leq t < 2$

$= \frac{1}{6}(-t^2 + 2t + 8) \qquad 2 \leq t \leq 4$

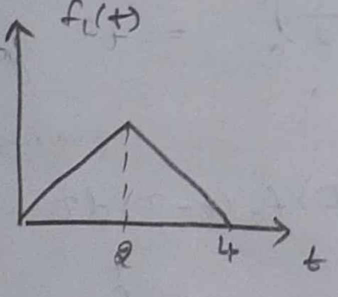
$= 0 \qquad t > 4.$

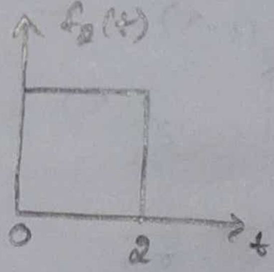




Plot by substituting values

2.





Find convolution using graphical Method -

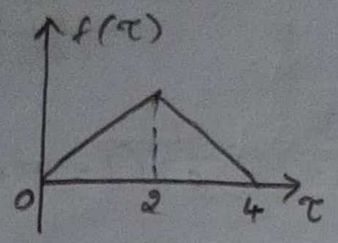
$f_1(t) - 0 = \frac{2-0}{2-0}(t-0)$

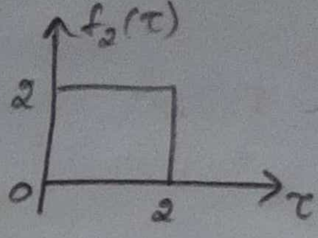
$f_2(t) = 2$ for $0 \leq t \leq 2$

$f_1(t) = t, \quad 0 \leq t \leq 2$

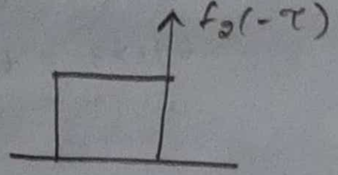
$f_1(t) = 4 - t, \quad 2 \leq t \leq 4$

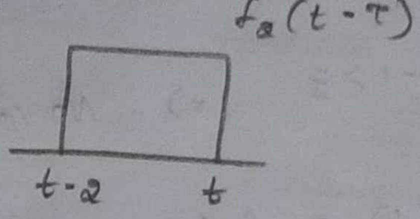
1) Replace t by τ.





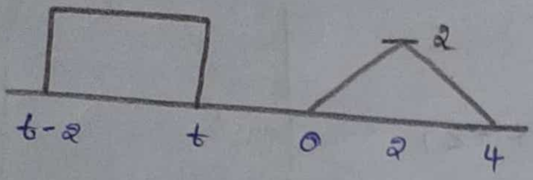
2) Reverse $f_2(\tau)$ and shift by t





3)

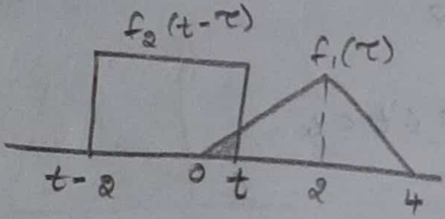
(i)



$f_1(t) * f_2(t) = 0$

for $t \le 0$.

(ii)

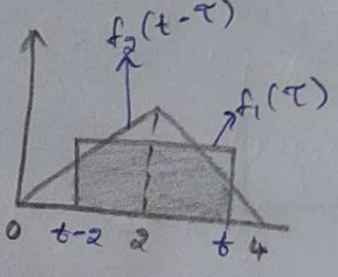


$$\int_0^t f_1(\tau) f_2(t-\tau)\, d\tau$$

$$= \int_0^t \tau \cdot 2 \, d\tau$$

$$= 2\left(\frac{\tau^2}{2}\right)_0^t = t^2 \quad \text{for } 0 \le t \le 2$$

(iii)



$$= \int_{t-2}^{2} f_1(\tau) f_2(t-\tau)\, d\tau + \int_2^t f_1(\tau) f_2(t-\tau)\, d\tau$$

$t > 2$

$t-2 < 2$

$t < 4$

$2 \le t \le 4$

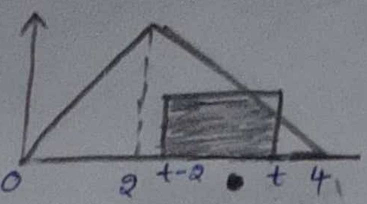
$$= \int_{t-2}^{2} \tau \cdot 2\, d\tau + \int_2^t (4-\tau)\, 2\, d\tau$$

$$= (\tau^2)_{t-2}^{2} + (8\tau - \tau^2)_2^t$$

$$= \cancel{4} - t^2 + 4t - \cancel{4} + 8t - t^2 - 16 + 4.$$

$$= -2t^2 + 12t - 12. \quad \text{for } 2 \le t \le 4$$

(iv)



$$f(t) = \int_{t-2}^{t} f_1(\tau)\, f_2(t-\tau)\, d\tau$$

$$= \int_{t-2}^{t} (4-\tau)\, 2\, d\tau$$

$t < 4$

$t-2 \le 4$

$t < 6$

$4 \le t \le 6$

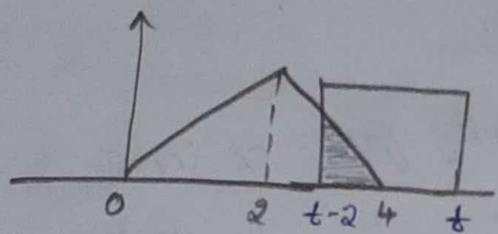
$$= 8(\tau)_{t-2}^{t} - 2\left(\frac{\tau^2}{2}\right)_{t-2}^{t}$$

$$= 8(t-t+2) - (t^2-(t-2)^2)$$

$$= 16 - t^2 + t^2 - 4t + 4$$

$$= -4t + 20.$$

(v)



$$f(t) = \int_{t-2}^{4} f_1(\tau)\, f_2(t-\tau)\, d\tau$$

$$= \int_{t-2}^{4} (4-\tau)\, 2\, d\tau$$

$t-2 \le 4$, $t-2 \ge 2$

$t \le 6$, $t \ge 4$

$4 \le t \le 6$

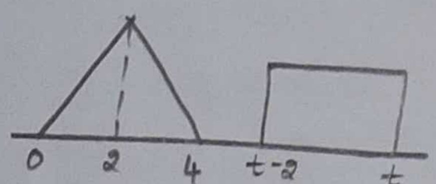
$$= 8(\tau)_{t-2}^{4} - (\tau^2)_{t-2}^{4}$$

$$= 8(4-t+2) - (16 - t^2 + 4t - 4)$$

$$= 48 - 8t - 16 + t^2 - 4t + 4$$

$$= t^2 - 12t + 36$$

(vi)



$f_1(t) * f_2(t) = 0$

for $t \ge 6$

$t-2 \ge 4$

$t \ge 6$

$$f(t) = 0 \qquad t < 0$$

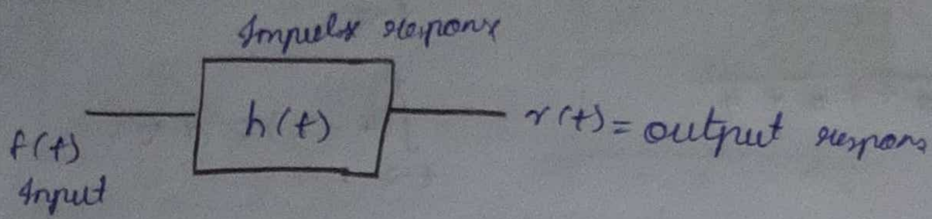
$$= t^2 \qquad 0 \le t \le 2$$

$$= -2t^2 + 12t - 12 \qquad 2 \le t < 4$$

$$= t^2 - 12t + 36 \qquad 4 \le t \le 6$$

$$= 0 \qquad t \ge 6$$

Filter characteristics of linear system:



=) LTI → behaves as a filter.

h(t) = which describes the system behaviour in time domain.

$y(t) = r(t) = f(t) * h(t) \rightarrow (1)$

$f(t) \xleftrightarrow{F.T} F(\omega)$ = spectral density func. of input

$h(t) \xleftrightarrow{F.T} H(\omega)$ = " " " " system

$y(t) \xleftrightarrow{F.T} Y(\omega)$ = " " " " output.

Apply F.T to eq (1).

F.T [Convolution in time domain] = Multiplication in frequency domain

$Y(\omega) = F(\omega) \cdot H(\omega)$

$Y(\omega) = K \cdot F(\omega)$ → Amplitude Factor

If $K > 1$ = amplified version

$K < 1$ = Attenuation = some components lost

$K = 1$ = Buffer

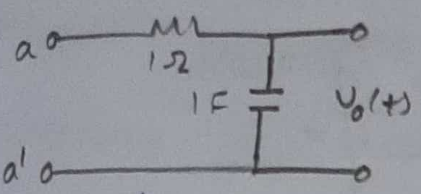
No change or some may undergo some phase shift.

⇒ The spectral density func. of the i/p $F(\omega)$ is modified by the system to $Y(\omega) = F(\omega)H(\omega)$

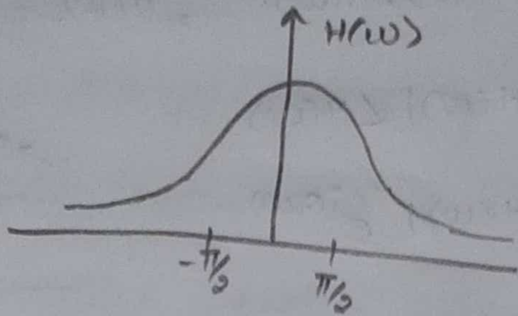
$H(\omega) \rightarrow$ Representing the system → performs the desired operation on the applied i/p signal

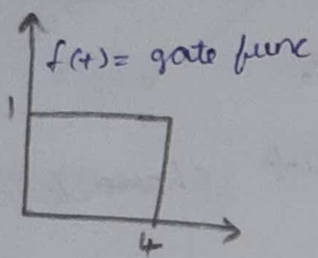
⇒ an LTI system behaves as a filter.

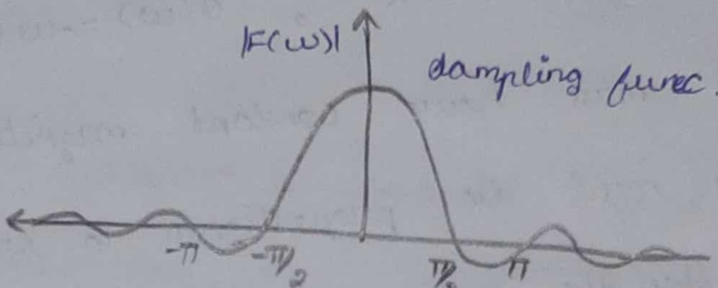
⇒ Filter = Freq. domain operation.

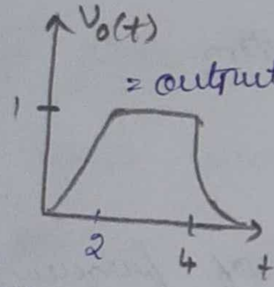


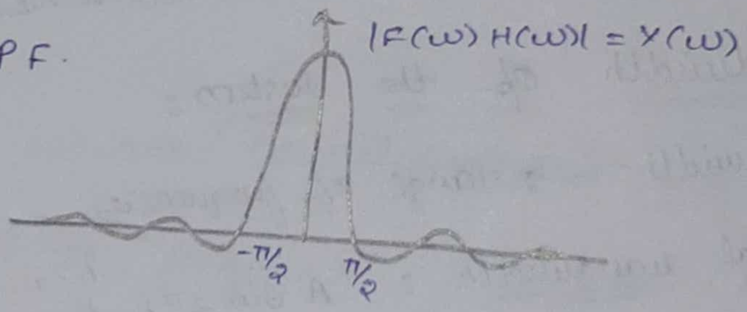
Low pass filter





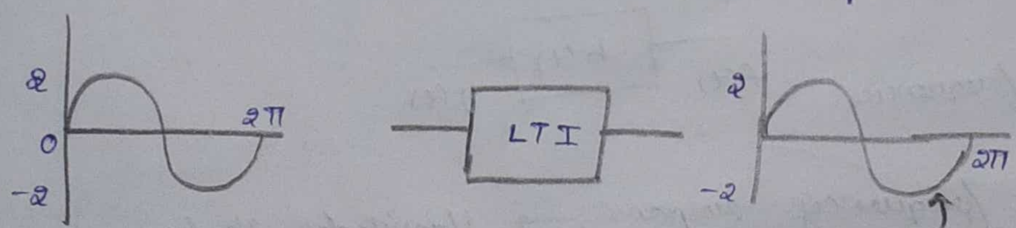


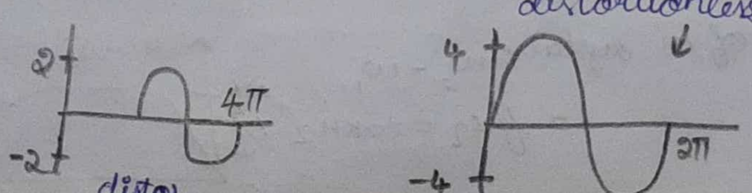




allows only lower freq's & rejects higher freq's

Distortionless transmission : No change in o/p





(i) o/p is the exact replica of the i/p

(ii) " " " amplified version

(iii) " " " shifted version

$y(t) = Kf(t - t_0)$ = distortionless transmission

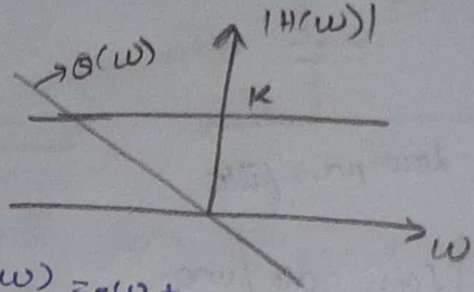
Apply F.T

$Y(\omega) = KF(\omega)\, e^{-j\omega t_0}$

w.k.t, $Y(\omega) = F(\omega) \cdot H(\omega)$, $H(\omega) = Ke^{-j\omega t_0}$

$H(\omega) = |H(\omega)| \angle H(\omega)$

$= |H(\omega)|\, e^{j\theta(\omega)}$



$\therefore |H(\omega)| = K$, $\theta(\omega) = -\omega t_0$

(i) system must have constant magnitude characteristics

(ii) phase must be proportional to freq.

then the system is distortionless system.

<u>Bandwidth</u> <u>of</u> <u>the</u> <u>system</u>:

Bandwidth → range of frequencies – Band of frequencies

<u>Signal</u> <u>bandwidth</u> : $A \sin 2\pi f_m t$

only one freq.

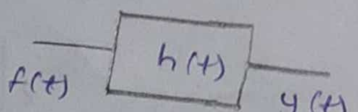
f_m = B.W of the signal

voice signal → 4KHz ⇒ B.W of a 4KHz

<u>System</u> <u>bandwidth</u> :

↙

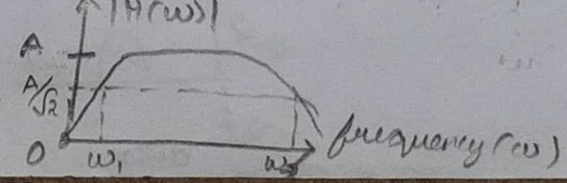
Range of frequencies



Plot the frequency response → Magnitude Vs frequency

Bandwidth of a system: $\omega_2 - \omega_1$

= freq = 20KHz



i/p signal < 20KHZ ⇒ processed & No distortion

> 20KHZ ⇒ distortion.

Physical signal - for distortionless transmission - infinite B.W

⇒ Inpractice - system with ∞ B.W is not practical.

↳ fairly large - distortionless transmission

Physical → as freq ↑ = energy content of signal ↓

Necessary to construct a system that will transmit the freq components which contains most of the energy of the signal.

⇒ Attenuation of HF components would tend to very small distortion.

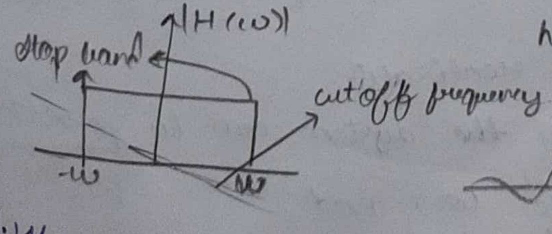
<u>Ideal filters</u>:

⇒ which removes unwanted (signal/freq/phase/noise)

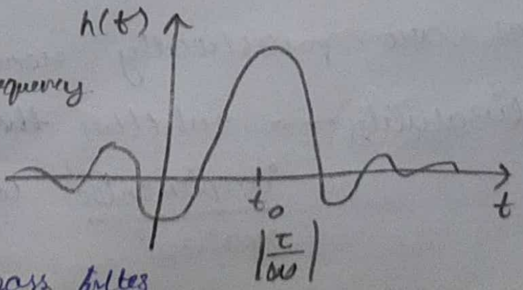
⇒ selection of particular frequencies

⇒ frequency selective filters.

- → low pass filter
- → High " "
- → Band " "
- → Band stop " "
- → All " "



B.W = 2W



Ideal low pass filter

Low pass filter allows low freqs rejects High freq.

Gate function $= G_{2W}(\omega)$

$$H(\omega) = |H(\omega)| e^{j\theta(\omega)}$$

$$= G_{2W}(\omega) e^{-j\omega t_0}$$

$H(\omega) = T/F \Rightarrow$ Impulse Response $= h(t)$

$$h(t) = F^{-1}[H(\omega)]$$

$$= F^{-1}[G_{2W}(\omega) e^{-j\omega t_0}]$$

$$h(t) = \frac{W}{\pi} Sa(W(t-t_0))$$

The impulse response of ideal LPF is existing even before applying the driving func/i/p/ excitation which is not possible in practice

∴ Ideal systems are non causal system

causality $\rightarrow h(t) = 0, \ t<0$

∴ Ideal systems are not practically realizable

⇒ All ideal systems are practically not realizable

Causality, physical realizability & paley - wiener criterion:

causality → Response of the system is a func. of present & past values of i/ps
→ Impulse response of the causal system $h(t) = 0, t<0$

Causal systems are practically realizable

Physical realizability → whether the system can be practically physically implemented (or) not realised

The Impulse response of a physically realizable system is ~~equal~~ $h(t) = 0$, $t<0$. → causality condition

physical realizable

↓

Time domain representation of causality.

Causality condition in freq. domain → paley-wiener criteria which is freq domain counterpart of $h(t)=0, t<0$.

$$I = \int_{-\infty}^{\infty} \frac{\ln|H(\omega)|}{1+\omega^2}\, d\omega = \text{finite.}$$

$$\int_{-\infty}^{\infty} |H(\omega)|^2 d\omega \leq \infty \ (\text{finite}) \rightarrow H(\omega) \text{ must be square integrable}$$

I = finite → Paley wiener criteria is satisfied then the system is causal system → physically realizable.

$h(t) = 0, t<0$ ⇒ Time domain condition for causal

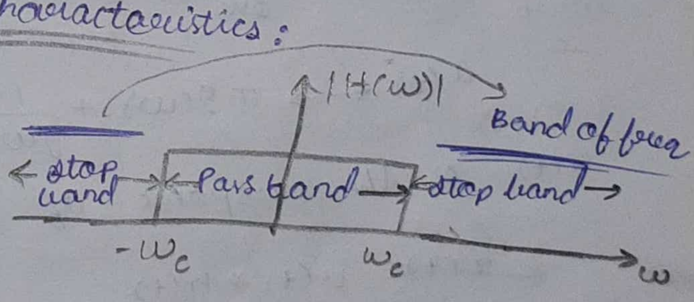
$$I = \int_{-\infty}^{\infty} \frac{\ln|H(\omega)|}{1+\omega^2}\, d\omega = \text{finite} \Rightarrow \text{freq. domain condition for causal}$$

If $H(\omega) = 0$ ⇒ then $\ln|H(\omega)| = \infty$ = paley-wiener criteria is violated then Non causal then not practically realizable.

<u>Ideal</u> ~~to~~ <u>LPF</u> <u>magnitude</u> characteristics:

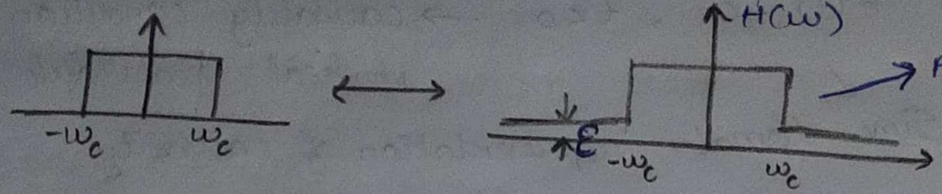
$|H(\omega)| = 1$ for $-\omega_c \leq \omega \leq \omega_c$

$= 0$ for $\omega > \omega_c$ & $\omega < -\omega_c$



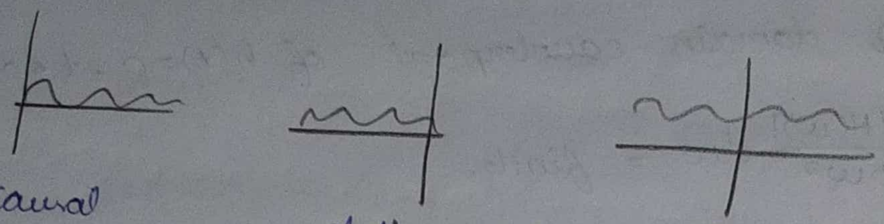
⇒ The system must not have zero magnitude characteristics over a band of frequencies. but it can have zero magnitude characteristics at a particular value of freq.

Non causal to Practically realisable system.



→ PW criteria is satisfied

~~Relationship between Bandwidth~~



Causal

Anti causal

Non-causal

Relationship between Bandwidth & Rise time:

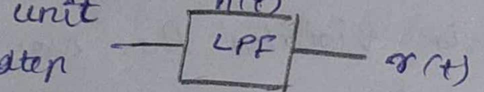
Rise time → time taken for system to reach from initial value to final value.

→ 10% of its value to 90% Max value.

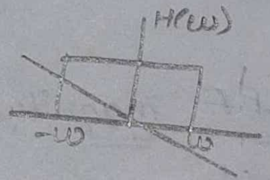
Rise time $\propto \frac{1}{\text{Bandwidth}}$ $\Rightarrow t_r \propto \frac{1}{B \cdot W}$.

$\Rightarrow t_r = \frac{2\pi}{W}$ for a LPF $W = B \cdot W$.

W_c = cutt off freq / B.W



$H(\omega) = G_{2W}(\omega)\, e^{-j\omega t_0}$



$u(t) \longleftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$

$r(t)$ is the response of LPF to $u(t)$ then

$r(t) = u(t) * h(t)$

$R(\omega) = U(\omega)\, H(\omega)$

$= \left(\pi\delta(\omega) + \frac{1}{j\omega}\right) H(\omega)$

$R(\omega) = \pi\delta(\omega)\, H(\omega) + \frac{1}{j\omega} H(\omega)$

$$R(\omega) = \pi \underset{\text{(}\because\text{ Impulse func)}}{\delta(\omega) H(0)} + \frac{1}{j\omega} G_{2W}(\omega) e^{-j\omega t_0}$$

$$= \pi \delta(\omega) + \frac{1}{j\omega} G_{2W}(\omega) e^{-j\omega t_0}$$

$$r(t) = F^{-1}[R(\omega)] = F^{-1}[\pi\delta(\omega)] + F^{-1}\left[\frac{1}{j\omega} G_{2W}(\omega) e^{-j\omega t_0}\right]$$

w.k.t

$$A \longleftrightarrow 2\pi A \delta(\omega)$$

$$1 \longleftrightarrow 2\pi \delta(\omega)$$

$$\tfrac{1}{2} \longleftrightarrow \pi \delta(\omega)$$

$$r(t) = \frac{1}{2} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{j\omega} e^{-j\omega t_0} e^{j\omega t} \, d\omega.$$

$$= \frac{1}{2} + \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega(t-t_0)} \cdot \frac{1}{j\omega} \, d\omega$$

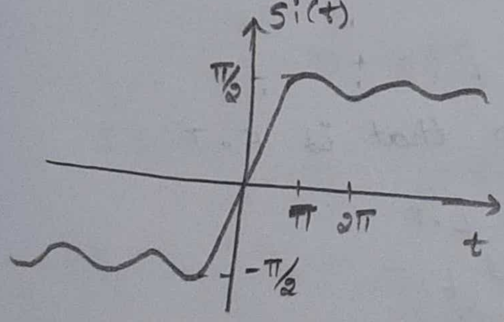
$$= \frac{1}{2} + \frac{1}{2\pi}\left(\int_{-W}^{W} \frac{\cos\omega(t-t_0)}{j\omega} d\omega + \cancel{j}\int_{-W}^{W} \frac{\sin\omega(t-t_0)}{\cancel{j}\omega} d\omega\right)$$

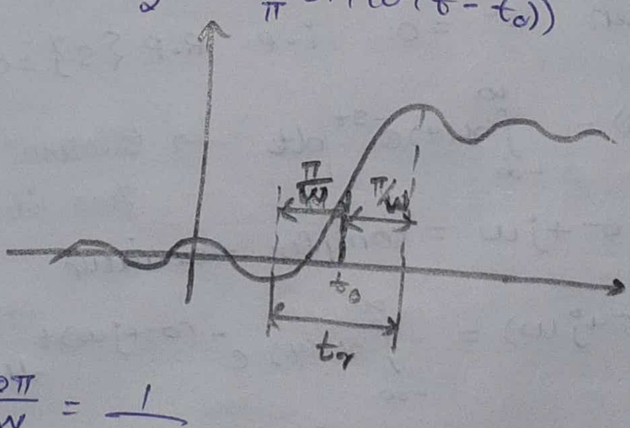
$$= \frac{1}{2} + \frac{1}{2\pi}\left(0 + 2\int_{0}^{W} \frac{\sin\omega(t-t_0)}{\omega} d\omega\right)$$

$$= \frac{1}{2} + \frac{1}{\pi}\int_{0}^{W(t-t_0)} \frac{\sin x}{x} dx$$

$$= \frac{1}{2} + \frac{1}{\pi}\int_{0}^{W(t-t_0)} Sa(x)\, dx = \frac{1}{2} + \frac{1}{\pi} Si(x)$$

$$= \frac{1}{2} + \frac{1}{\pi} Si(W(t-t_0))$$





$$\therefore t_r = \frac{2\pi}{W} = \frac{1}{B.W}$$