System Compensation
(Linear Control System Design)

Compensation :→

It is a Corrective network introduced into the System to compensate for the deficiency in the performance of the Plant and achieve desired Specifications

Outline

- Introduction to compensation design
- Phase Lead Compensation
- Phase Lag Compensation
- Phase Lead-lag Compensation
- PID Control

Type of Compensation :->

- Series Compensation
- Parallel Compensation
- Series - Parallel compensation

Question: What is system compensation?

Given the control plant, the procedure of controller design to satisfy the requirement is called system compensation.

Question: Why to compensate?

The closed-loop system has the function of self-tunning. By selecting a particular value of the gain *K*, some single performance requirement may be met.

Is it possible to meet more than one performance requirement?

Sometimes, it is not possible.

Something new has to be done to the system in order to make it perform as required.

1.Control system design and compensation

- **Design ： Need to design the whole controller to satisfy the system requirement.**
- **Compensation ： Only need to design part of the controller with known structure.**

2. Three elements for compensation

Original part of the system

Performance requirement

Compensation device

Introduction to Compensation Design

Performance Requirement

1. Time domain criteria（step response）

- Overshoot, settling time, rising time, steady-state error

2. Frequency domain criteria

- Open-loop frequency domain criteria：
  crossover frequency, phase margin, gain margin
- Closed-loop frequency domain criteria ：
  maximum value M_r , resonant frequency, bandwidth

Correlation between Phase Margin and damping ratio

Let

$$G(s)H(s) = \frac{K}{s(\tau s+1)} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

where $\omega_n = \sqrt{\frac{K}{\tau}}$, $2\xi\omega_n = \frac{1}{\tau}$

Sinusoidal TF

$$G(j\omega)H(j\omega) = \frac{\omega_n^2}{j\omega(j\omega+2\xi\omega_n)}$$

at gain crossover freq $\omega=\omega_1$ Ra $|G(j\omega)H(j\omega)|=1$

$$|G(j\omega)H(j\omega)| = \frac{\omega_n^2}{\omega_1\sqrt{\omega_1^2+4\xi^2\omega_n^2}} = 1$$

or,

$$\omega_1^4 + 4\xi^2\omega_n^2\omega_1^2 - \omega_n^4 = 0$$

which yield

$$\left(\frac{\omega_1}{\omega_n}\right)^2 = \sqrt{4\xi^4+1} - 2\xi^2$$

The Phase Margin of the system is given by

$$\phi = -90^\circ - \tan^{-1}\left(\frac{\omega_1}{2\xi\omega_n}\right) + 180^\circ$$

Substitute ω_1

$$= -90^\circ - \tan^{-1}\left[\frac{1}{2\xi}\left\{(4\xi^2+1)^{1/2} - 2\xi^2\right\}^{1/2}\right]$$

$$= \tan^{-1}\left[\frac{2\xi}{\sqrt{(4\xi^2+1)^{1/2} - 2\xi^2}}\right]$$

$$\boxed{\phi_m = \tan^{-1}\left[\frac{2\xi}{\sqrt{\left\{\sqrt{4\xi^2+1} - 2\xi^2\right\}}}\right]}$$

Frequency domain and time domain criteria

Resonant peak

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

Resonant frequency

$$\omega_r = \omega_n \sqrt{1-2\xi^2}$$

Bandwidth

$$\omega_b = \omega_n \sqrt{1-2\xi^2 + \sqrt{(1-2\xi^2)^2 + 1}}$$

Gain crossover frequency

Phase margin

$$\Phi_m \; (\gamma) = arctg \frac{2\xi}{\sqrt{\sqrt{4\xi^4 + 1} - 2\xi^2}}$$

arctg $= \tan^{-1}$

Percentage overshoot

Mp

$$\sigma\% = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$$

Settling time

Frequency domain and time domain criteria

Resonant peak $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

Resonant frequency $\omega_r = \omega_n\sqrt{1-2\xi^2}$

Bandwidth $\omega_b = \omega_n\sqrt{1-2\xi^2+\sqrt{(1-2\xi^2)^2+1}}$

Gain crossover frequency $\omega_c = \omega_n\sqrt{\sqrt{4\xi^4+1}-2\xi^2}$

Phase margin $\gamma = arctg\frac{2\xi}{\sqrt{\sqrt{4\xi^4+1}-2\xi^2}}$

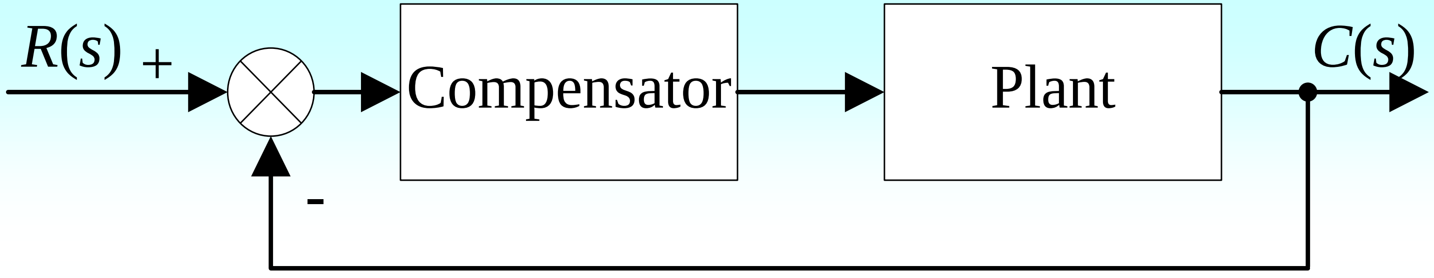
arc tg $= \tan^{-1}$

Percentage overshoot $\sigma\% = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\times 100\%$

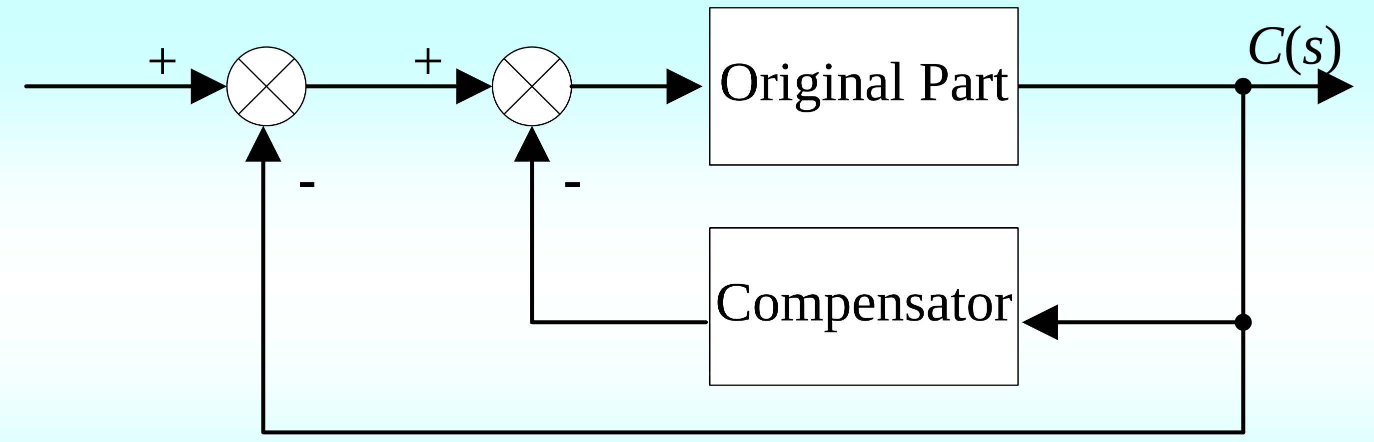
Settling time $t_s = \frac{3}{\xi\omega_n}, \frac{4}{\xi\omega_n}$

Structure of Compensator

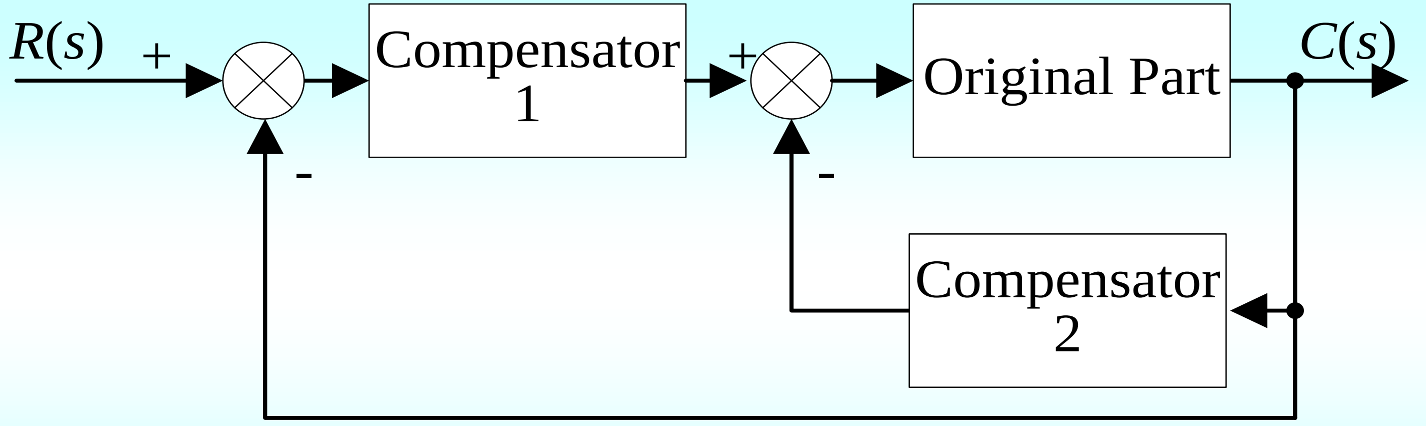
- **According to the way of compensation, the compensator can be classified into following categories:**



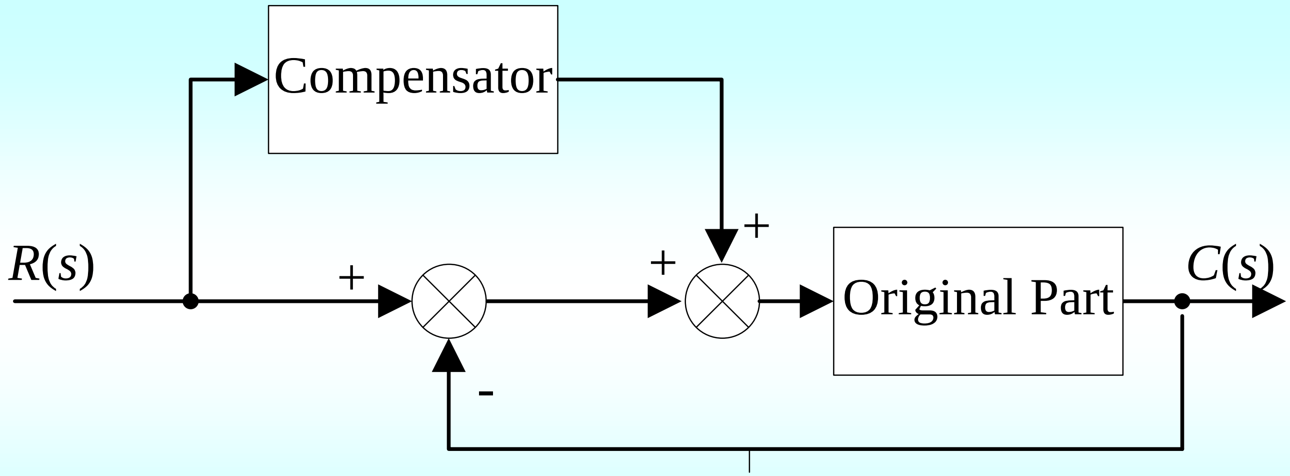
(a) Cascade Compensation



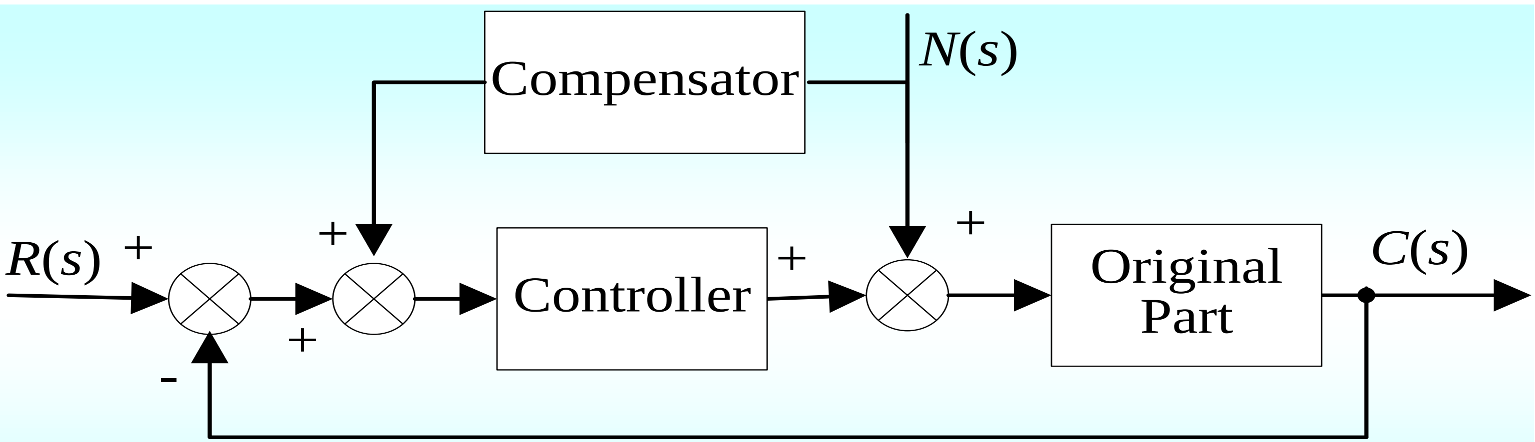
(b) Feedback compensation



(c)Cascade and feedback compensation



(d) Feed-forward compensation



(e) Disturbance compensation

Remark：

- **Cascade compensation and feedback compensation are inside the feedback loop.**
- **Feed-forward compensation and disturbance compensation are outside the feedback loop.**

Methods for Compensator Design

1. Frequency Response Based Method

Main idea: By inserting the compensator, the Bode diagram of the original system is altered to achieve performance requirements.

Original open-loop Bode diagram + Bode diagram of compensator + alteration of gain = open-loop Bode diagram with compensation

2. Root Locus Based Method

Main idea: Inserting the compensator introduces new open-loop zeros and poles to change the closed-loop root locus to satisfy the requirement.

Cascade Compensation

- **Frequency response based compensation**

  Phase lead compensation

Phase lag compensation

Phase lead-lag compensation

- **Fundamental rule for control design：PID control**

Each requirement relates to a different region of the frequency axis in the Bode diagram.

1. **The steady-state error relates to the magnitude at low frequency.**
2. **The transient response requirement relates to the gain crossover frequency, which usually occurs at higher frequencies.**

Three design rules for cascade compensator:

1. **The system is stable with satisfactory steady-state error, but dynamic performance is not good enough.**

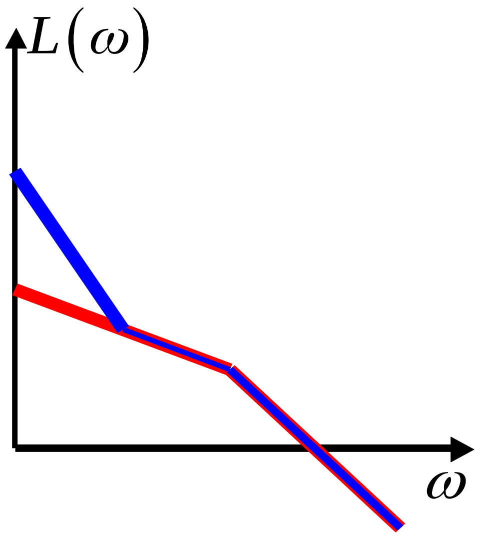
   Compensator is used to change medium and high frequency parts to change crossover frequency and phase margin.

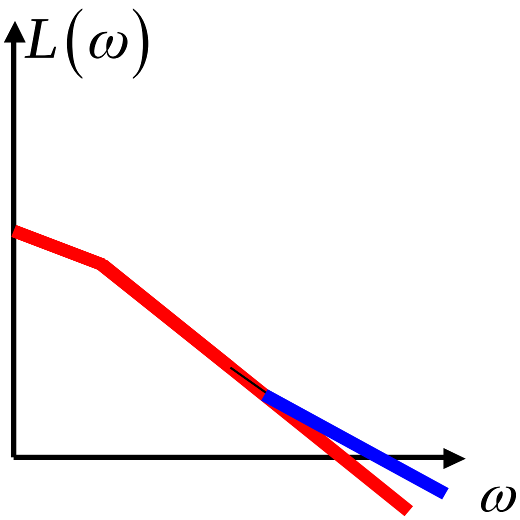
2. **The system is stable with satisfactory transient performance, but the steady-state error is large.**

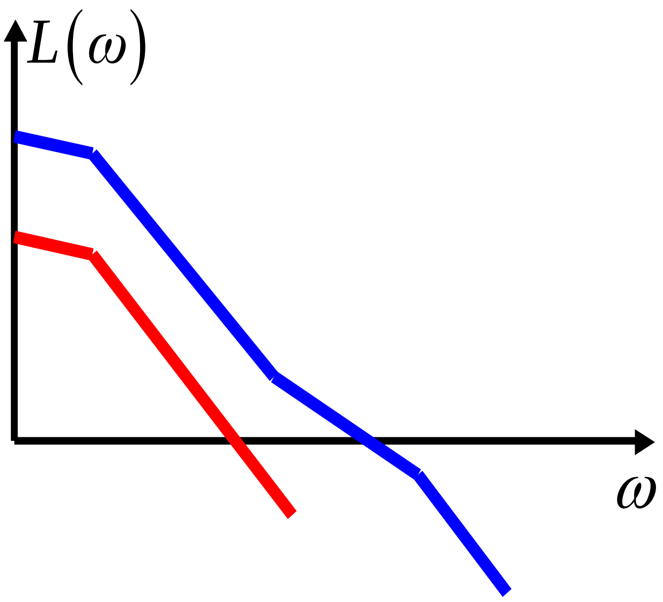
   Compensator is used to increase gain and change lower frequency part, but keep medium and higher frequency parts unchanged.

3. **If the steady-state and transient performance are either unsatisfactory, the compensator should be able to increase gain of the lower frequency part and change the medium and higher frequency parts.**

Change of Bode Diagram







a)change of lower frequency part

b)change of medium and higher frequency parts

b)change of lower, medium and higher frequency parts

Lead Compensation :→

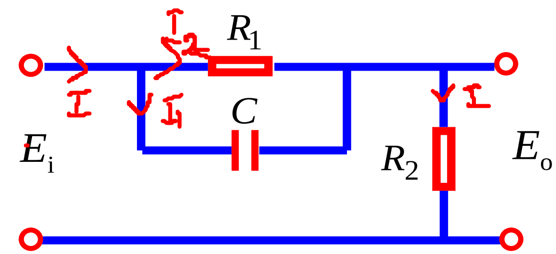
A Lead Compensator speed up the transient response and increase the margin of stability of a system

It also helps to increase the system error constant through the limited range.

→ Compensation required
— to stabilize unstable system
— to obtain desired result

From fig $I_1 + I_2 = I$

$$C\frac{d(E_i - E_o)}{dt} + \frac{(E_i - E_o)}{R_1} = \frac{E_o}{R_2}$$



Passive Phase Lead Network

taking Laplace Transform

$$sC[E_i(s) - E_o(s)] + \frac{E_i(s)}{R_1} - \frac{E_o(s)}{R_1} = \frac{E_o}{R_2}$$

$$E_i(s)\left[sC + \frac{1}{R_1}\right] = E_o(s)\left[sC + \frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{R_1 R_2}{R_1 + R_2 + R_1 R_2 sC} \cdot \frac{1 + sCR_1}{R_1} = \frac{s + \frac{1}{R_1 C}}{s + \frac{R_1 + R_2}{R_1 R_2 C}}$$

$$= \frac{s + \frac{1}{R_1 C}}{s + \left[\frac{1}{\frac{R_2}{R_1 + R_2}}\right] R_1 C} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

$$\left\{ \begin{array}{l} \because T = R_1 C \\ \alpha = \frac{R_2}{R_1 + R_2} \end{array} \right.$$

This is generally expressed as

Transfer function Lead Compensation Network

$$G_c = \frac{E_o(s)}{E_i(s)} = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

where $T = R_1 C$

$$\alpha = \frac{R_2}{R_1+R_2} < 1$$

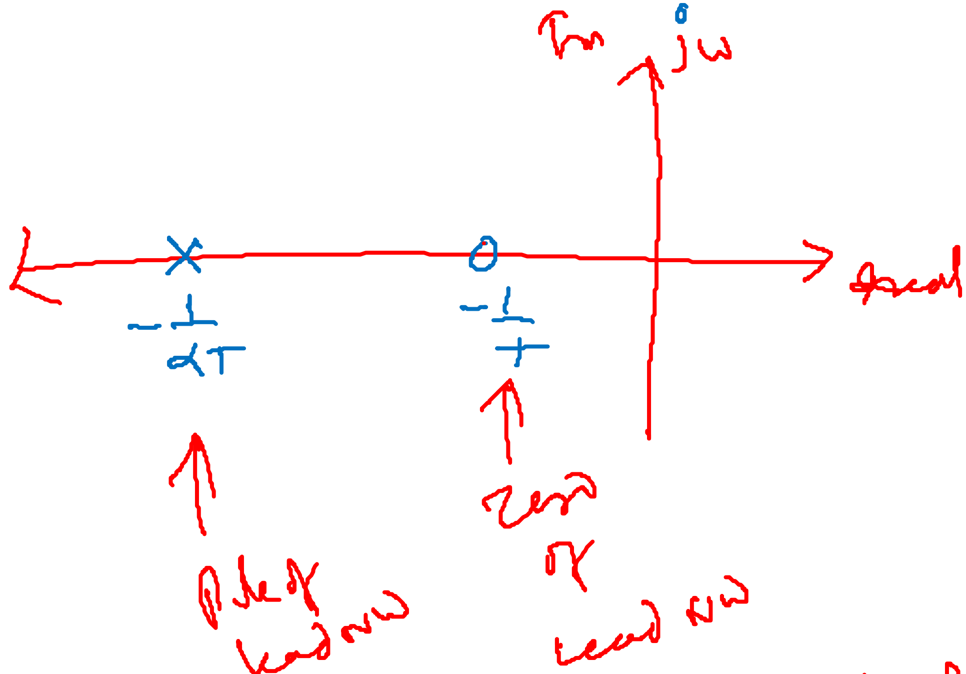


fig: Pole-Zero diagram of lead NW

Maximum lead angle (ϕ_m)

Let maximum lead angle which provide by compensator is (ϕ_m), and at what freq it provides this angle

$$\frac{E_o(s)}{E_i(s)} = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$

Replace $s \to j\omega$

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{\alpha(1+j\omega T)}{1+j\omega\alpha T}, \quad \phi = \frac{\tan^{-1}\left(\frac{\omega T}{1}\right)}{\tan^{-1}(\alpha\omega T)}$$

$$\phi = \tan^{-1}(\omega T) - \tan^{-1}(\alpha\omega T)$$

$$\left|\frac{E_o(j\omega)}{E_i(j\omega)}\right| = \frac{\alpha\sqrt{1+\omega^2 T^2}}{\sqrt{1+\omega^2\alpha^2 T^2}} = M \quad \text{--- (1)}$$

Phase $\phi = \tan^{-1}(\omega T) - \tan^{-1}\omega\alpha T$ — (2)

let find ϕ_m, using $\frac{d\phi}{d\omega} = 0$,

$$\therefore \frac{d}{d\omega}\left[\tan^{-1}\omega T - \tan^{-1}\omega\alpha T\right] = 0$$

$$\therefore \frac{\frac{1}{T}}{\omega^2+\left(\frac{1}{T}\right)^2} - \frac{\frac{1}{\alpha T}}{\omega^2+\left(\frac{1}{\alpha T}\right)^2} = 0$$

$$\therefore \frac{T}{1+\omega^2T^2} - \frac{\alpha T}{1+\alpha^2\omega^2T^2} = 0$$

$\omega_m \rightarrow$ Geometric mean of two corner freq $\omega_{c_1} = \frac{1}{T}$. $\omega_{c_2} = \frac{1}{\alpha T}$

$$\omega^2\alpha T^2(\alpha-1) = -(1-\alpha)$$

$$\omega^2\alpha T^2 = 1$$

$$\omega^2 = \frac{1}{\alpha T^2} \Rightarrow \omega_m = \frac{1}{T\sqrt{\alpha}} = \sqrt{\frac{1}{T}\cdot\frac{1}{\alpha T}} \quad \text{— (3)}$$

where $\omega_m \rightarrow$ freq at which we achieve max lead angle.

$W_m \rightarrow$ freq at which ϕ_m is Maximum

W_m is geometric mean of two corner frequencies of compensator which are $W_{c_1} = \frac{1}{T}$, $W_{c_2} = \frac{1}{\alpha T}$

taking tan both side

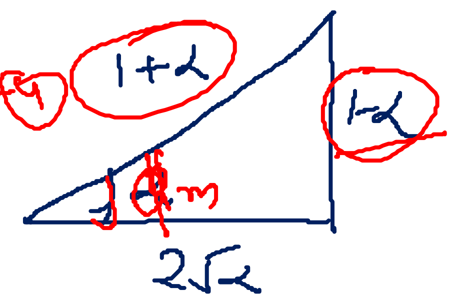
$$\tan\phi = \tan\left[\tan^{-1} WT - \tan^{-1}\alpha WT\right] \qquad \because W = \frac{1}{T\sqrt{\alpha}}$$

$$\tan\phi = \left(\frac{WT - \alpha WT}{1 + WT\alpha WT}\right) = \frac{WT(1-\alpha)}{1+W^2T^2\alpha} = \frac{\frac{\cancel{T}}{\cancel{T}\sqrt{\alpha}}(1-\alpha)}{1+\frac{\cancel{T^2}}{\cancel{T^2}\cancel{\alpha}}\cdot\cancel{\alpha}}$$

at $W = W_m = \frac{1}{T\sqrt{\alpha}}$

$$\tan\phi_m = \frac{1-\alpha}{\sqrt{\alpha}(1+1)} = \frac{1-\alpha}{2\sqrt{\alpha}} \quad -(4)$$

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha} \quad -(5)$$



for $\phi_m = \frac{1-\alpha}{1+\alpha}$

Polar Plot of lead Compensator :-

$$m = \frac{\alpha\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2\alpha^2T^2}}$$

$$m = \frac{\alpha\sqrt{\frac{1}{T^2}+\omega^2}}{\sqrt{\frac{1}{T^2}+\alpha^2\omega^2}}$$

When $\omega = 0$, $M = \alpha$ and $\phi = 0^\circ$

$\omega = \infty$, $m = 1$ and $\phi = 0^\circ$

$$\phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T$$

$\boxed{\alpha < 1}$ angle ϕ will be +ve leading

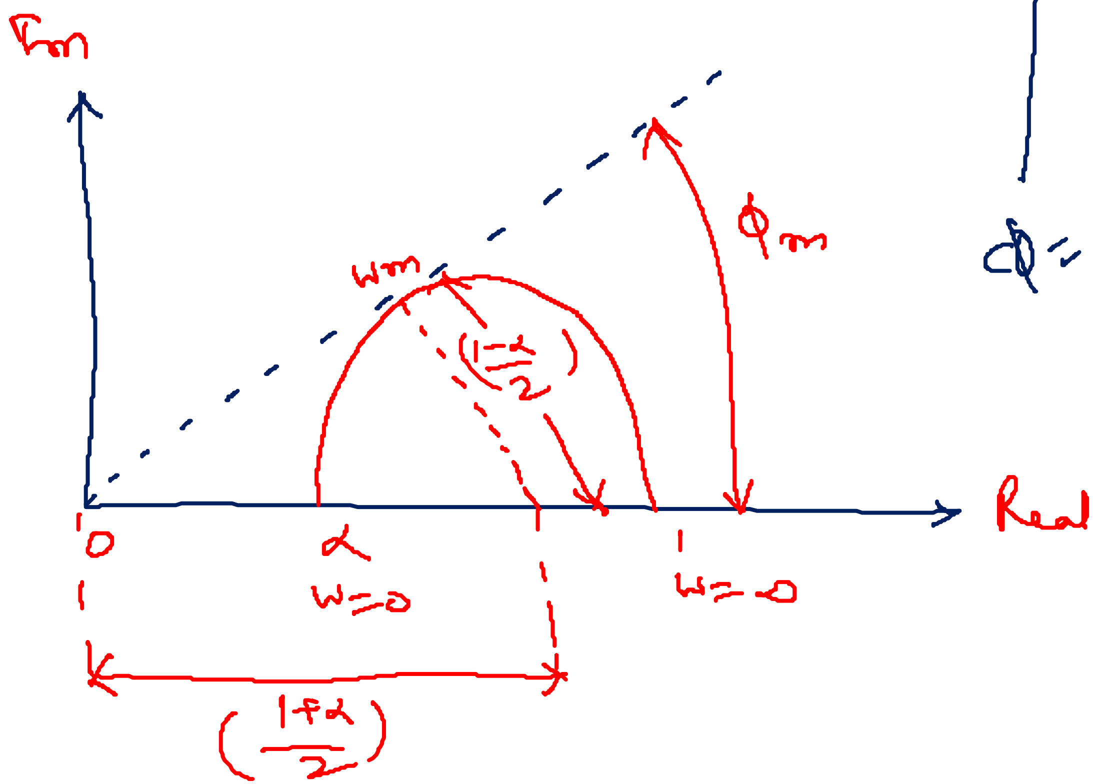


Fig: Polar plot of Lead Compensator

Bode Plot of lead Compensator

Con

Corner freq of the lead compensator

$W_{c_1} = \frac{1}{T}$ (For a zero at $S = -\frac{1}{T}$)

$W_{c_2} = \frac{1}{\alpha T}$ (for a pole at $S = -\frac{1}{\alpha T}$)

and $k_c = \alpha$

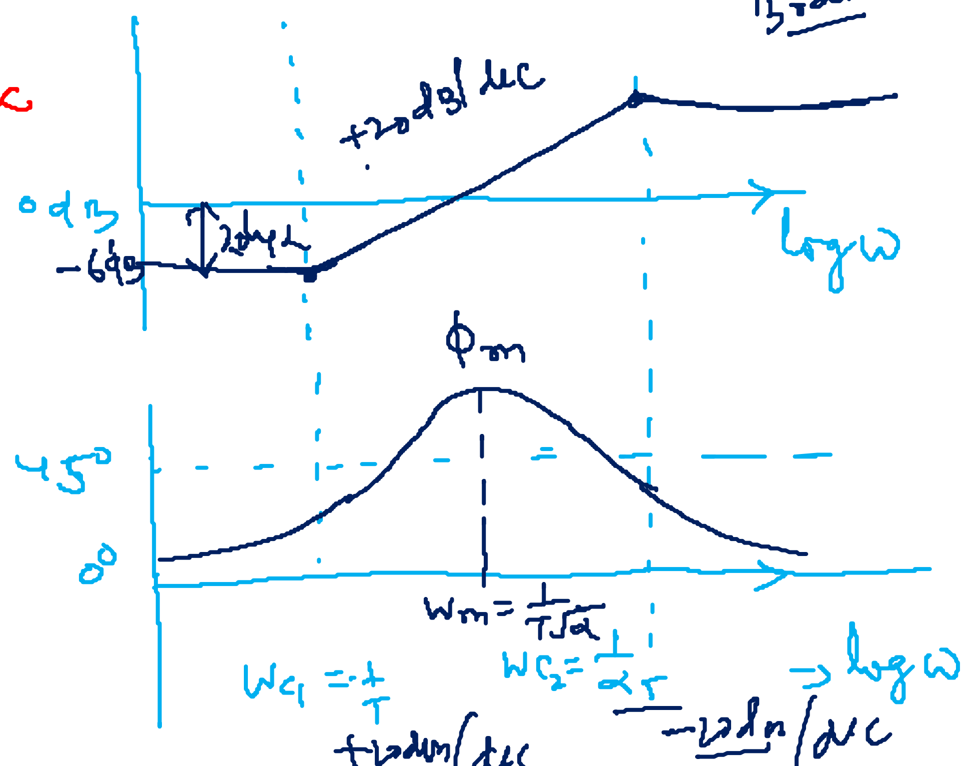
$$G_c(j\omega) = \frac{\alpha(1 + j\omega T)}{(1 + j\omega \alpha T)}$$

at $W = W_m$

$$m = \frac{\alpha\sqrt{1 + W_m^2 T^2}}{\sqrt{1 + W_m^2 \alpha^2 T^2}} = \sqrt{\alpha}$$

$$\phi = \tan^{-1} w_1 - \tan^{-1} \alpha w_1$$

Bode Plot



Bode Plot of lead compensator

Effect of lead Compensator :→

- lead compensator add dominant zero and pole
- increase the damping of closed loop system
- increase damping means → less overshoot
  - less rise time
  - less settling time

  ] Improves Transient time resp

- Improve Phase Margin of the System (Closed loop System)
- Improve Gain Margin, Improve relative stability.
- It Improve BW of closed loop System result faster response.

Limitation of lead compensation :→

- Lead compensation require additional gain means need extra NW.
- More BW some time not desirable
- Compensated system has larger undershoot than overshoot
- Maximum lead angle available from single network is about 60°. If lead angle need 70° to 90° need multistage NW.

- **Constraints for application of lead compensation：**
- **Constraint 1: The system is stable.**

  If it is unstable, the phase need to compensate is too big. The noise takes severe effects on the system.
- **Constraint 2: The phase cannot reduce very fast around the gain crossover frequency.**

  The phase lead compensation can only provide less than 60^{o} extra phase margin.

Phase Lag Compensation

Transfer function:

$E_i = I(R_1 + R_2 + \frac{1}{cs})$ —(1)

$E_o = I \times (R_2 + \frac{1}{sc})$ —(2)

$G_c(s) = \frac{E_o}{E_i}$

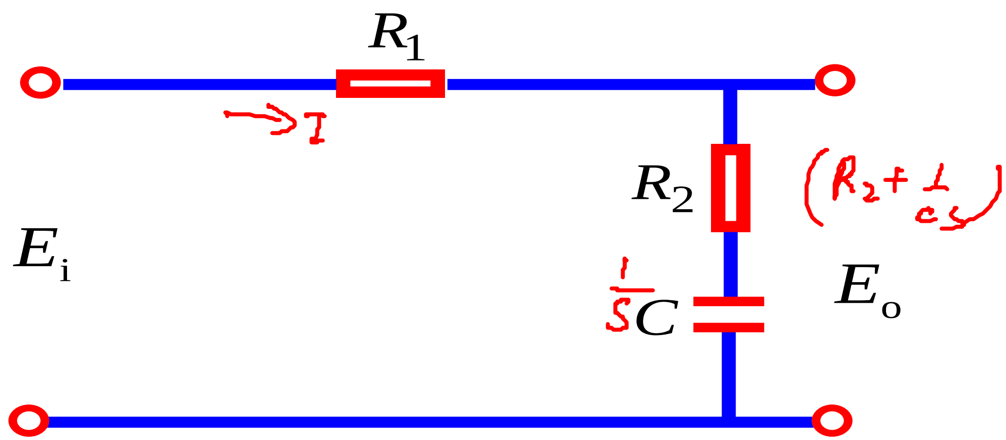
$$G_c(s) = \frac{R_2 + \dfrac{1}{CS}}{R_1 + (R_2 + \dfrac{1}{CS})}$$

$$= \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1} = \frac{s + \frac{1}{R_2 C}}{\left(\frac{R_1 + R_2}{R_2}\right) s + \frac{1}{R_2 C}}$$

$$= \frac{s + \frac{1}{R_2 C}}{\left(\frac{R_1 + R_2}{R_2}\right) S + \frac{1}{R_2 C}}$$

$$= \frac{S + \frac{1}{T}}{\beta s + \frac{1}{T}} = \frac{1 + sT}{1 + \beta sT}$$

$$= \frac{S + \frac{1}{R_2 C}}{\left(\frac{R_1 + R_2}{R_2}\right) S + \frac{1}{R_2 C}} = \frac{S + \frac{1}{T}}{\beta S + \frac{1}{T}} = \frac{1 + ST}{1 + \beta ST}$$



Passive phase lag network

Phase Lag Compensation

Transfer function:

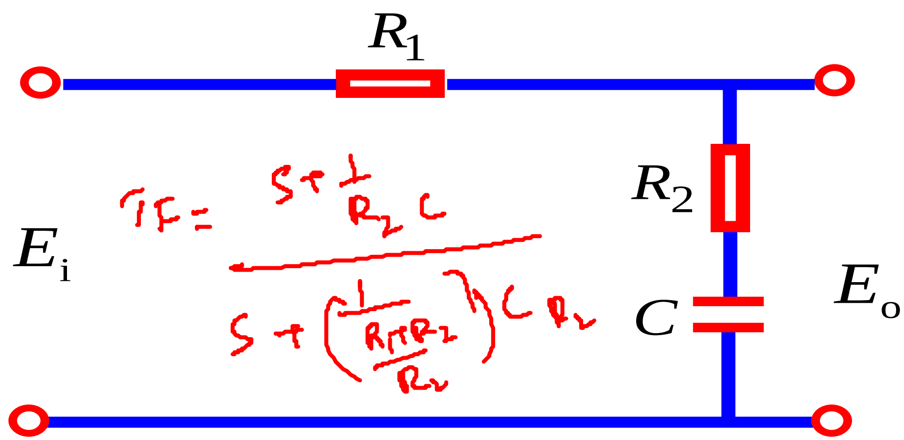
$$G_c(s)=\frac{R_2+\frac{1}{CS}}{R_1+(R_2+\frac{1}{CS})}$$

$$=\frac{\overset{T}{R_2C}s+1}{(R_1+R_2)Cs+1}$$

$$=\frac{Ts+1}{\beta Ts+1}$$

where $T = R_2C$

$\beta = \frac{R_1+R_2}{R_2} > 1$



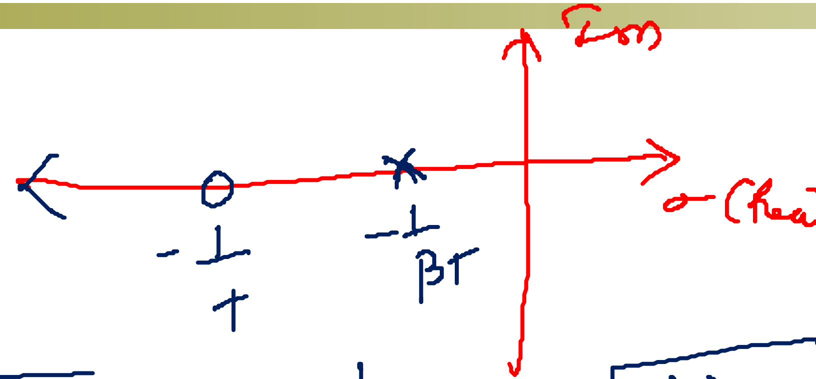
$TF = \frac{s+\frac{1}{R_2C}}{s+\left(\frac{1}{\frac{R_1+R_2}{R_2}}\right)CR_2}$

Passive phase lag network

A Lag Compensation Improve the Steady State performance of a System while nearly preserving its transient response.

$$TF = G_c = \frac{1}{\beta}\,\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

$$G_c = \frac{1}{\beta} \cdot \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$$



$$\boxed{\beta > 1}$$

Maximum Lag angle and β

$$\frac{d\phi}{d\omega} = 0 ,$$

$$M = |G(j\omega)| = \left| \frac{1 + j\omega T}{1 + j\omega \beta T} \right|$$

$$M = \frac{\sqrt{1 + \omega^2 T^2}}{\sqrt{1 + \omega^2 \beta^2 T^2}}$$

$$\phi = \tan^{-1}(\omega T) - \tan^{-1}(\omega \beta T)$$

$\omega \beta T > \omega T$

$\phi \rightarrow \frac{-ve}{\text{lag}}$

$$\frac{d}{d\omega}\left[\tan^{-1}(\omega T) - \tan^{-1}(\omega \beta T)\right]$$

After solving $\omega_m = \frac{1}{T\sqrt{\beta}} = \sqrt{\left(\frac{1}{T} \cdot \frac{1}{T\beta}\right)}$

$\omega_m \rightarrow$ freq at which lag angle is max

and it G.M. of two corner freq

$$W_{C_1} = \frac{1}{T},$$

$$W_{C_2} = \frac{1}{\beta T}.$$

Note :→ The Primary function Lag Comp is to Provide attenuation in the high freq range to give a System Sufficient Phase Margin

The Phase lag angle does not Play a role in the lag Compensation

SS performa → Low freq

Tr → High freq range

Polar Plot of Lag Compensator

$$\boxed{\phi = \tan^{-1}\omega T - \tan^{-1}\beta\omega T}$$
$$90^\circ - 90^\circ$$

$$m = \frac{\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2\beta^2T^2}} = \frac{\cancel{\omega T}\sqrt{\frac{1}{\omega^2T^2}+1}}{\cancel{\omega T}\sqrt{\frac{1}{\omega^2T^2}+\beta^2}} = \frac{\sqrt{1}}{\sqrt{\beta^2}} = \frac{1}{\beta} \checkmark$$

$\Downarrow$

$\omega = \infty$

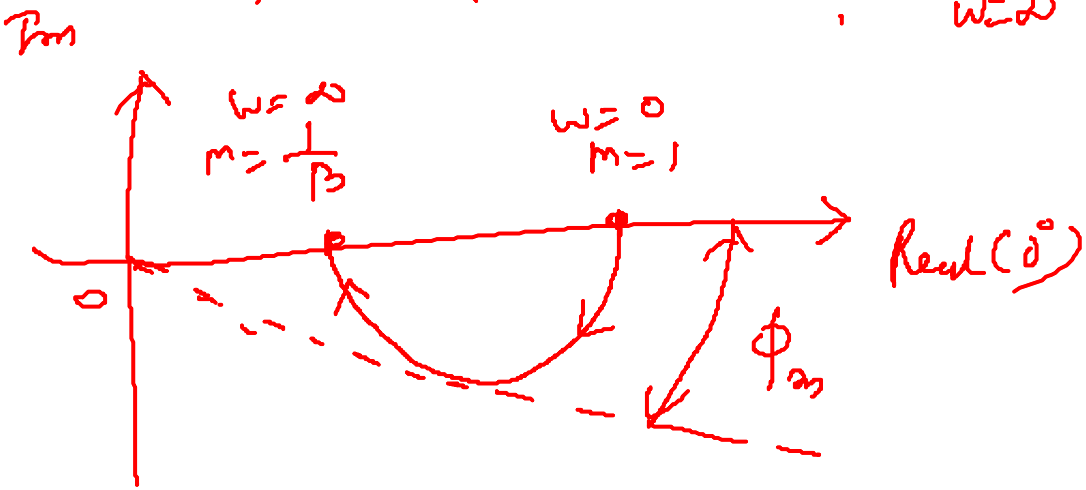
$\omega = 0, \quad m = 1, \quad \phi = 0^\circ$

$\omega = \infty, \quad m = \frac{1}{\beta}, \quad \phi = 0^\circ$

$\underline{\beta > 1} \qquad \frac{1}{\beta} \checkmark$

$$\boxed{\omega\beta T > \omega T}$$

$$\boxed{\phi = -ve}$$



Bode Plot of Lag Compensator

$$M = \frac{\sqrt{1+(\omega T)^2}}{\sqrt{1+(\omega \beta T)^2}}$$

$$20\log M = 20\log\sqrt{1+(\omega T)^2} - 20\log\sqrt{1+(\omega\beta T)^2}$$

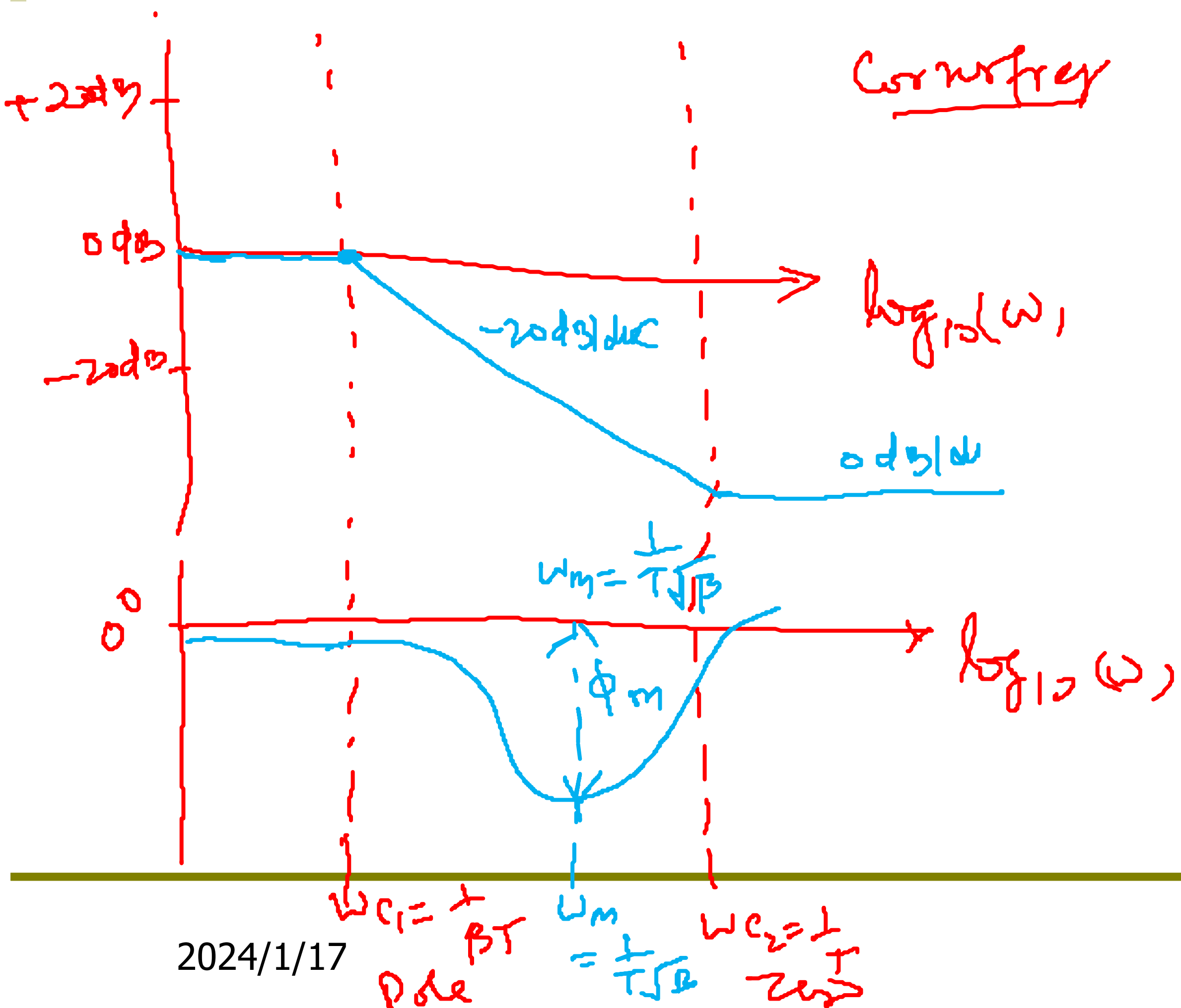
$$\phi = \tan^{-1}\omega T - \tan^{-1}\beta\omega T$$

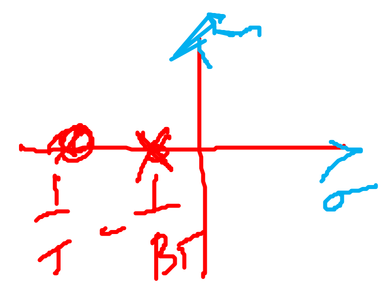
Corner freq

$$\omega_{c_2} = \frac{1}{T}$$

$$\omega_{c_1} = \frac{1}{\beta T}$$

$\boxed{\beta > 1}$





Cor f	$\beta>1$ Slope	Netw
$\omega_c = \frac{1}{\beta T}$	−20	−20
$\omega_{c_2} = \frac{1}{T}$	+20	0

Effect and limitation of Lag Compensator :->

- It allow high gain at low freq. Thus basically it is a Low Pass filter, hence Improve Steady
→ State performance
  ↳ attenuation characteristics is used for compensation

  Phase lag characteristics is of no use in comp.

→ It shift the gain cross over freq (ω_{gc}) to lower freq point. Thus BW of system reduced.

→ BW ↓ → System become slower t_r ↑, t_s ↑.
  transient lasts for longer time

→ System become more Sensitive to Parameters variations

→ It act as Prop + integral controller
  Thus System will be less stable.

Comments on phase lag compensation:

1、Phase lag compensator is a low-pass filter. It changes the low-frequency part to reduce gain crossover frequency. The phase is of no consequence around the gain crossover frequency.

2、Be able to amplify the magnitude of low-frequency part, and thus reduce the steady-sate error.

3、The slope around gain crossover freuency is -20dB/dec.

Resonance peak is reduced, and the system is more stable.

4、Reduce the gain crossover frequency, and then reduce the bandwidth. The rising time is increased. The system response slows down.

- **Applicable for the following systems：**

（1）The transient performance is satisfactory, but the steady-state performance is desired to be improved.

（2）High requirement for noise attenuation.

- **Drawback：The system response is slow down.**

Comparison of phase lead and lag compensation

	Phase lead compensation	Phase lag compensation
Main Idea	**Improve transient performance by using phase lead characteristics**	**Improve the steady-state performance by using magnitude attenuation at the high-frequency part**
Effect	**(1) Around ω_c, the absolute value of slope is reduced. Phase margin γ and gain margin GM are increased. (2) Increase the bandwidth (3) With bigger γ, overshoot is reduced. (4) Take no effect on the steady-state performance.**	**(1) Keep relative stability unchanged, but reduce the steady-state error. (2) Reduce ω_c and then closed-loop bandwidth (3) For specific open-loop gain, γ, GM and resonant peak M_r are all improved due to magnitude attenuation around ω_c**
Weakness	**(1) Broad bandwidth reduces the filtering for noise. (2) For passive network implementation, need an extra amplifier.**	**Narrow Bandwidth increase the response time.**
Application	**(1) Extra phase lead compensation is less than 55^0. (2) Require broad bandwidth and fast response (3) No matter the noise at high-frequency part.**	**(1) The phase lag of the uncompensated system is fast around ω_c. (2) Bandwidth and transient response are satisfactory. (3) Require attenuation of noise (4) The phase margin can be satisfied at the low frequency.**

Lag-Lead Compensator :→

Transfer function of Lag-Lead Compensator

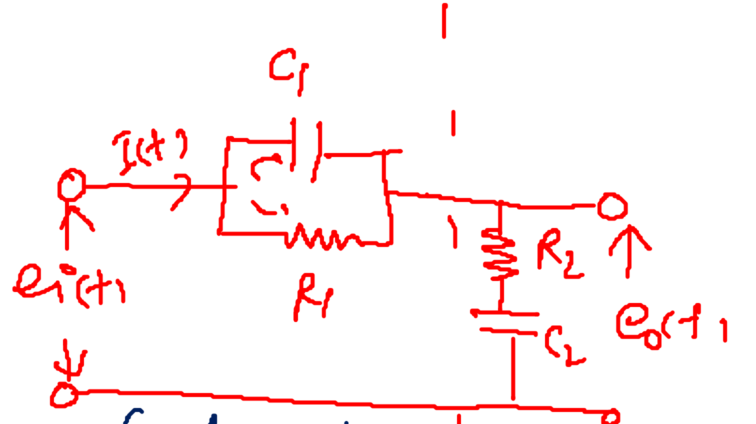


fig Lag-Lead Compensator

$$\frac{e_i - e_o}{R_1} + C_1 \frac{d(e_i - e_o)}{dt} = i(t) \quad \text{—} (1)$$

taking Laplace Transform

$$\frac{1}{R_1} E_i(s) - \frac{1}{R} E_o(s) + sC_1 E_i(s) - sC_1 E_o(s) = I(s) \quad \text{—} (2)$$

and o/p eqn

$$i(t) R_2 + \frac{1}{C_2} \int i\,dt = e_o(t) \quad \text{—} (3)$$

taking Lap Transform

$$I(S)\left[R_2 + \frac{1}{SC_2}\right] = E_0(S) \quad \text{— (4)}$$

Substituting $I(S)$ in (3) and (4) we get

$$E_1(S)\left[\frac{1}{R_1} + SC_1\right] - E_0(S)\left[\frac{1}{R_1} + SC_1\right] + \left[\frac{1}{SC_2} + R_2\right] E_0(S) \quad \text{— (5)}$$

$$\frac{E_1(S)\left[(1+SR_1C_1)(1+SR_2C_2)\right]}{SR_1C_2} = E_0(S)\left[\frac{(1+SR_1C_1)(1+SR_2C_2)}{SR_1C_2}\right]$$

$$\frac{E_0(S)}{E_1(S)} = \frac{(1+SR_1C_1)(1+SR_2C_2)}{S^2 R_1R_2C_1C_2 + S\left[R_1C_1 + R_2C_2 + R_1C_2\right] + 1}$$

$$= \frac{R_1R_2C_1C_2\left(S + \frac{1}{R_1C_1}\right)\left(S + \frac{1}{R_2C_2}\right)}{R_1R_2C_1C_2\left[S^2 + S\left[\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right]\right] + \frac{1}{R_1R_2C_1C_2}}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{\beta}{T_2}\right)\left(s+\frac{1}{\beta T_1}\right)} = \frac{\left(s+\frac{1}{T_2}\right)\left(s+\frac{1}{T_1}\right)}{\left(s+\frac{1}{\alpha T_2}\right)\left(s+\frac{1}{\beta T_1}\right)}$$

lead lag

It can be expressed

$$\frac{(1+T_1 s)(1+T_2 s)}{\left(1+\frac{T_2}{\beta}s\right)(1+T_1\beta s)}$$

$$\boxed{\frac{(s+z_{c_1})(s+z_{c_2})}{(s+p_{c_1})(s+p_{c_2})}}$$

where $R_1C_1 = T_1$

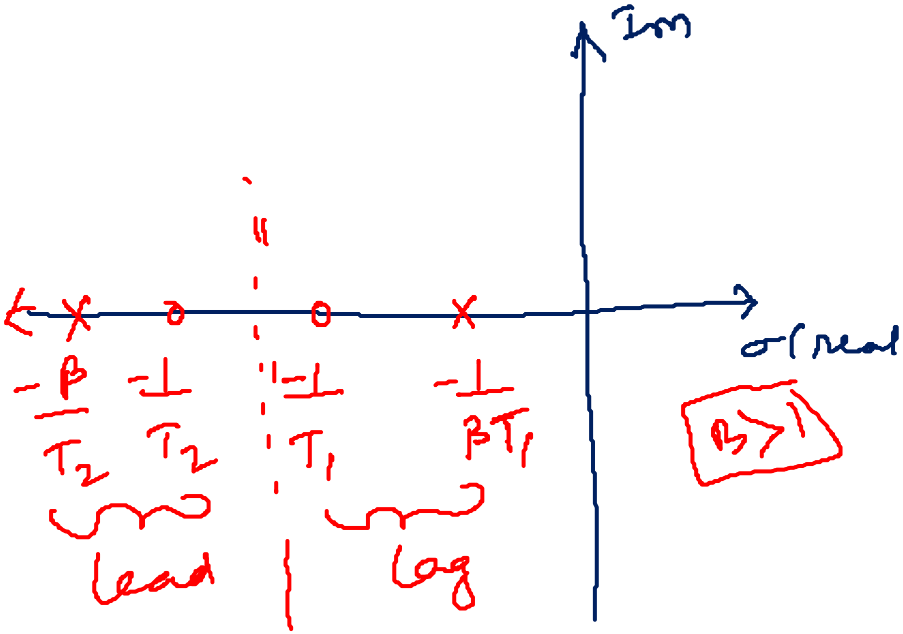
$R_2C_2 = T_2$

$$\frac{\beta}{T_1} + \frac{1}{\beta T_2} = \frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}$$

or, $\alpha\beta T_1 T_2 = R_1R_2C_1C_2$

or, $\alpha\beta = 1$ $\boxed{\beta = \frac{1}{\alpha}}$

$\boxed{\beta > 1}$ $\boxed{\beta = \frac{z_{c_1}}{p_{c_1}} =}$ $\frac{p_{c_2}}{z_{c_2}} > 1$



$\boxed{\beta > 1}$

Poles are $-\frac{\beta}{T_1}$ and $-\frac{1}{\beta T_2}$

and zeros are

$-\frac{1}{T_1}$ and $-\frac{1}{T_2}$

$$\frac{E_0(s)}{E_i(s)} = \frac{(1+T_1 j\omega)(1+T_2 j\omega)}{(1+\beta T_1 j\omega)(1+j\omega T_2/\beta)}$$

P. Cas M/A

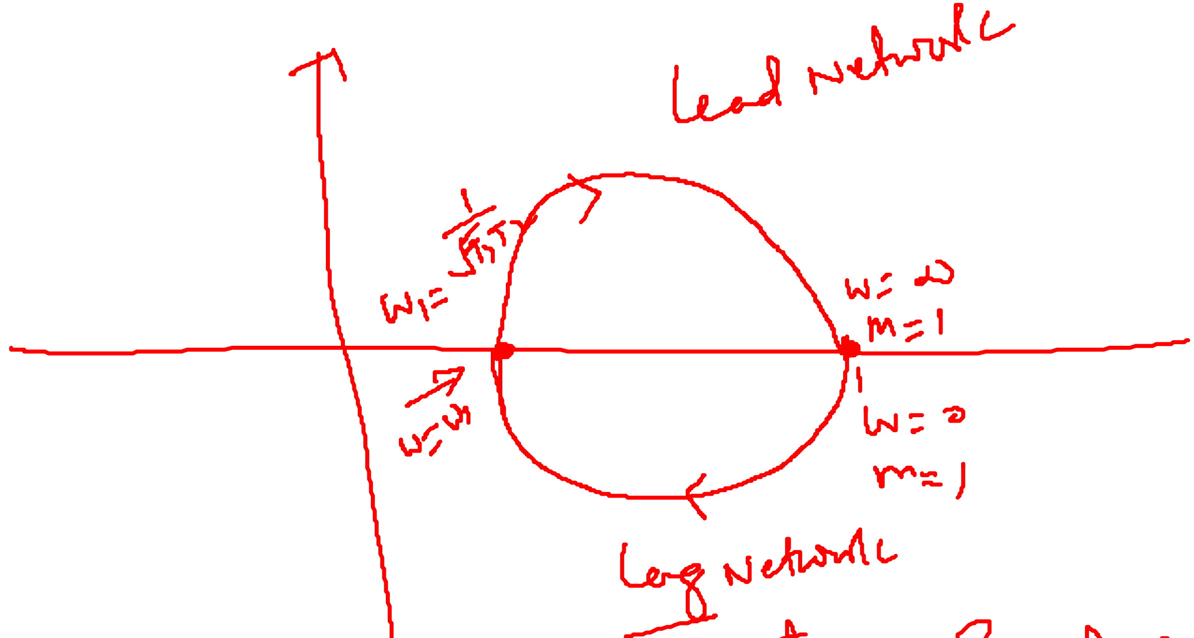
$\omega = 0$	$M = 1$	$\phi = 0^\circ$
$\omega = \infty$	$M = 1$	$\phi = 0^\circ$

$$M = \frac{\sqrt{1+\omega^2 T_1^2}\,\sqrt{1+\omega^2 T_2^2}}{\sqrt{1+\omega^2 \beta^2 T_1^2}\,\sqrt{1+\omega^2 T_2^2/\beta^2}}$$

$180^\circ - 90^\circ$
$= 90^\circ$

$$\phi = \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\frac{T_1\omega}{\beta} - \tan^{-1}\beta T_2\omega$$

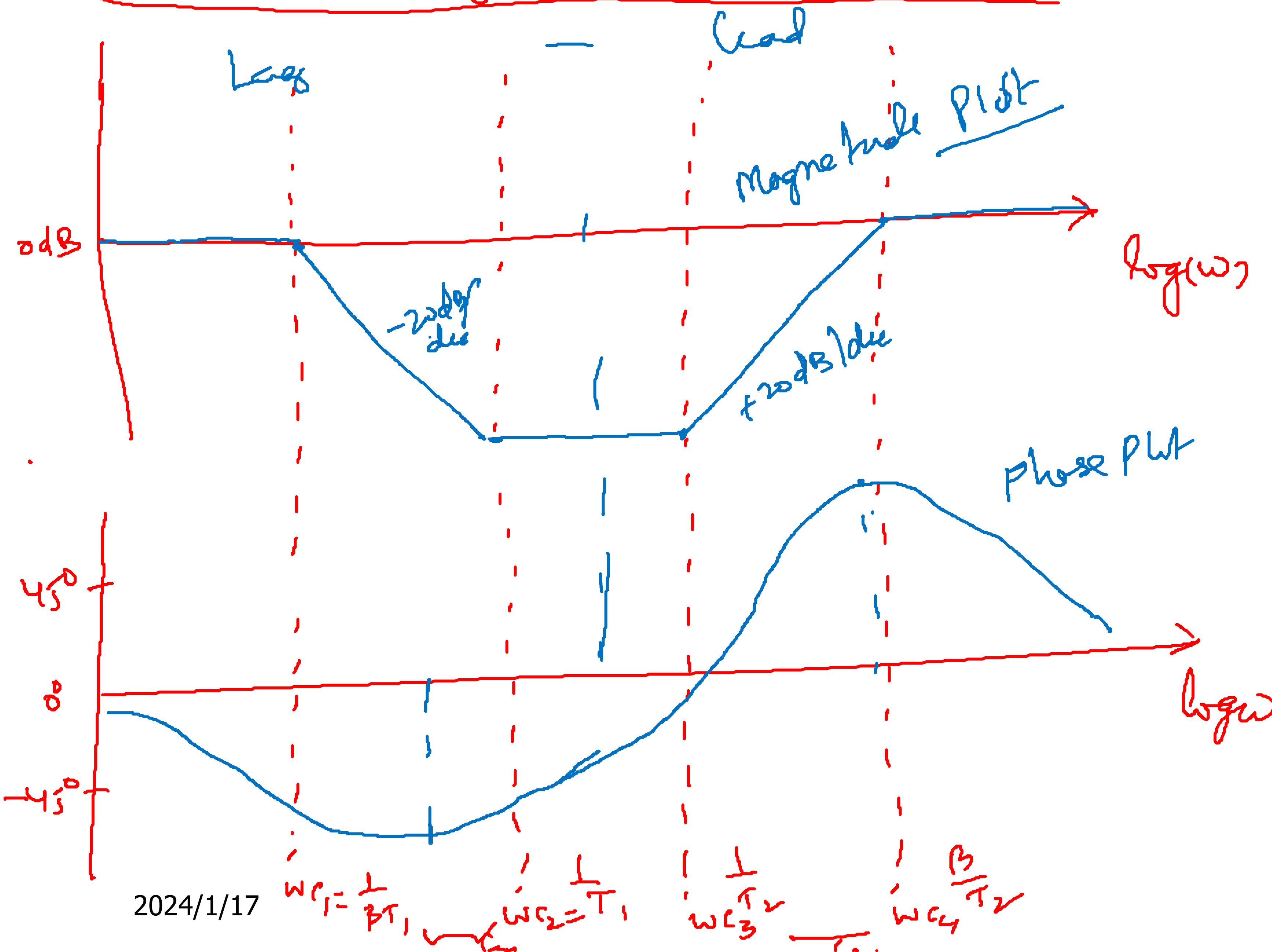
$M = \sqrt{1+\omega^2 T_1^2}\ \sqrt{}$



Starting Point and terminating Point Same

$\boxed{M = 1,\ \phi = 0^\circ}$

Bode Plot of Lag-Lead Compensator



Effect of Lag-Lead Compensator

① It is used when both fast response & good static accuracy are desired. Use Lag-lead compensator.

→ It increases low freq gain which improve BW & system response is fast. In general lead compensator is used to improve BW and Lag compensator is used to improve damping

PID Controller

P + I + D

P → Proportional

I → Integral

D → derivative

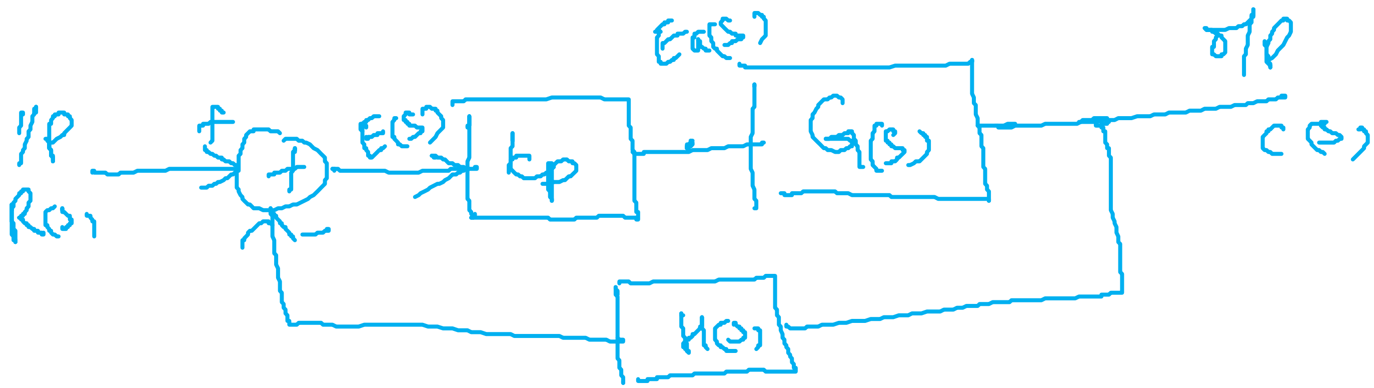


Fig A Proportional control.

PID Controller

P + I + D

Proportional

Integral

derivative

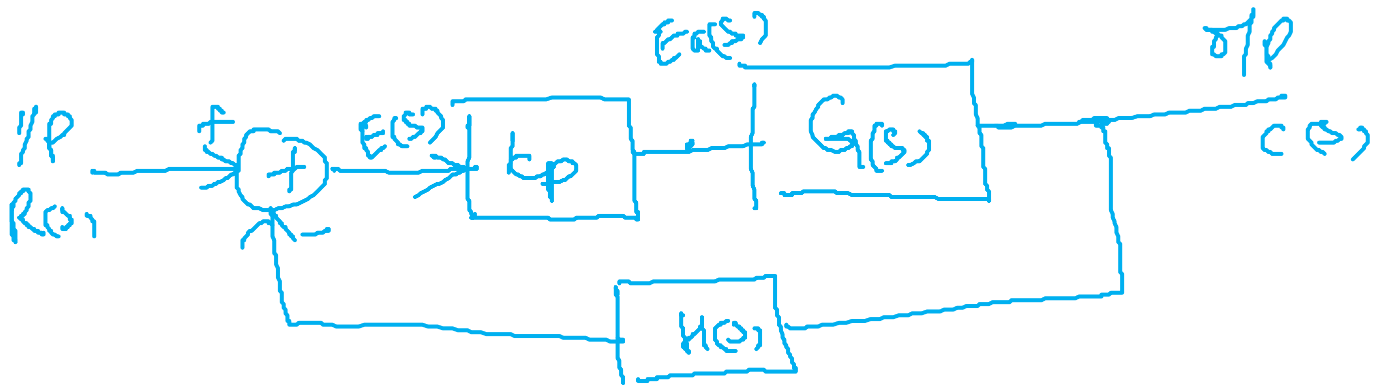


Fig A Proportional control.

PID Controller

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P → Proportional

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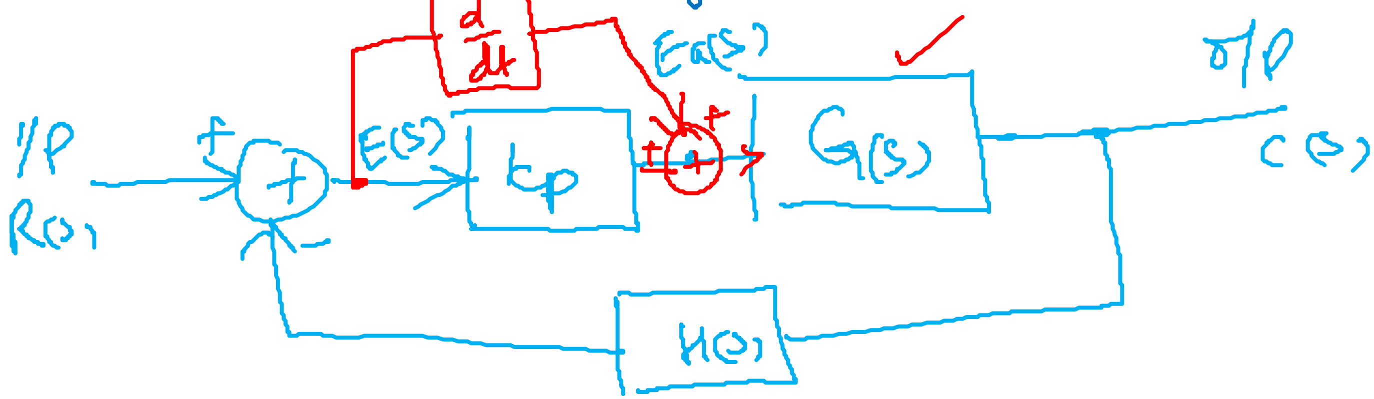
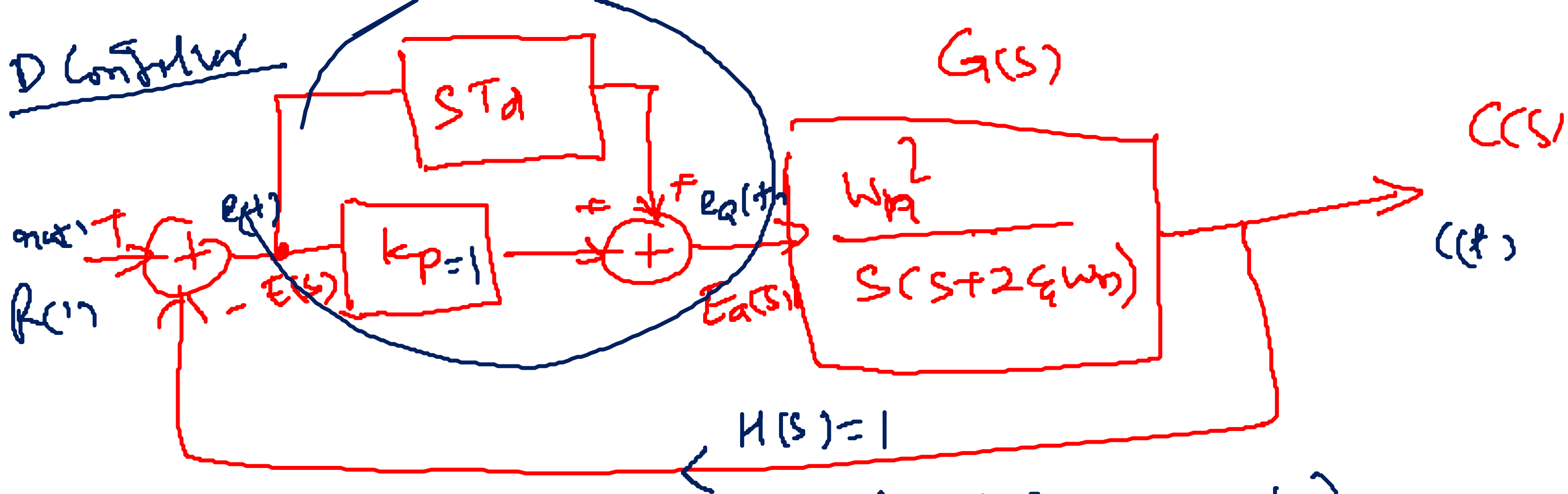


Fig of Proportional control.

Proportional Derivative (PD Control)

PD Controller



Let $kp = 1$ ($kp \rightarrow$ Proportional gain)

$$e_a(t) = e(t) + T_d \frac{d}{dt} e(t) \quad \text{—— (1)}$$

$$E_a(S) = E(S) + ST_d\, E(S)$$

$$= E(S)\left[1 + ST_d\right] \quad \text{—— (2)}$$

$$\frac{C(S)}{R(S)} = \frac{(1+ST_d)\,\omega_n^2 / S(S+2\xi\omega_n)}{1 + (1+ST_d)\left(\frac{\omega_n^2}{S(S+2\xi\omega_n)}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2(1+sT_d)}{s^2+2\xi\omega_n s+\omega_n^2+sT_d\omega_n^2}$$

sT_d

P control

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$(1+sT_d)$ $\to$ $\boxed{s=-\frac{1}{T_d}}$

change

PD controller

$$\frac{\omega_n^2(1+sT_d)}{s^2+(2\xi\omega_n+\omega_n^2T_d)s+\omega_n^2} \Rightarrow \equiv$$

$$s^2+(2\xi\omega_n+\omega_n^2T_d)s+\omega_n^2=0$$

$$2\xi'\omega_n = 2\xi\omega_n + \omega_n^2 T_d$$

$$\xi' = \frac{2\xi\omega_n+\omega_n^2 T_d}{2\omega_n}$$

$r(t)=t$

ramp $R(s)=1/s^2$

~~step~~ response

$$\boxed{\xi' = \xi + \frac{\omega_n T_d}{2}}$$

PD controller

$$e_{ss} \quad \lim_{s\to 0} sE(s)$$

$$\to \lim_{s\to 0} s\cdot\frac{1}{s^2}\cdot\frac{s(s+2\xi\omega_n)}{s^2+(2\xi\omega_n+\omega_n^2T_d)s+\omega_n^2}$$

$$\boxed{e_{ss} = \frac{2\xi}{\omega_n}}$$

Steady State error of PD Controller P-controller

is same

$t_r =$

$t_p =$

$t_s =$

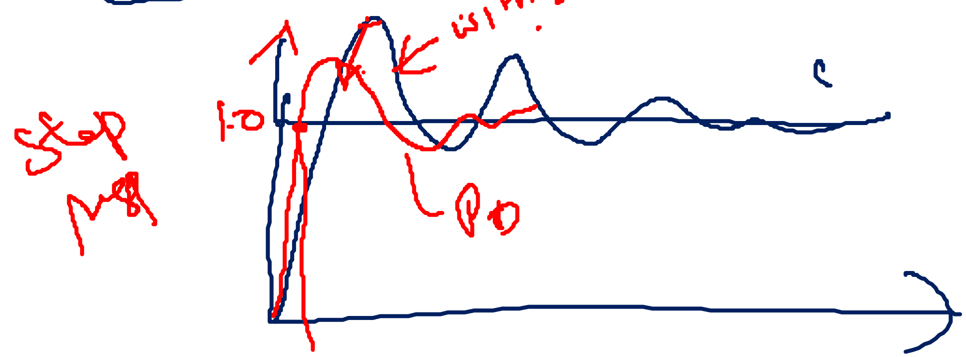
$M_p =$

ξ

= lead compensator

$$\boxed{S = \frac{1}{T_d}}$$

rise t_r t_p



$$K_p = \lim_{s \to 0} G(s)\, H(s) = \infty$$

$$K_v = \lim_{s \to 0} s\, G(s)\, H(s) = \frac{\omega_n}{2\xi}$$

As there is no change in coefficients, error also will remain same. Hen controller has following effects on system.

i) It increases damping ratio.

ii) 'ω_n' for system remains unchanged.

iii) 'TYPE' of the system remains unchanged.

iv) It reduces peak overshoot.

v) It reduces settling time.

vi) Steady state error remains unchanged.

In general it improves transient part without affecting steady state.

In PI Controller

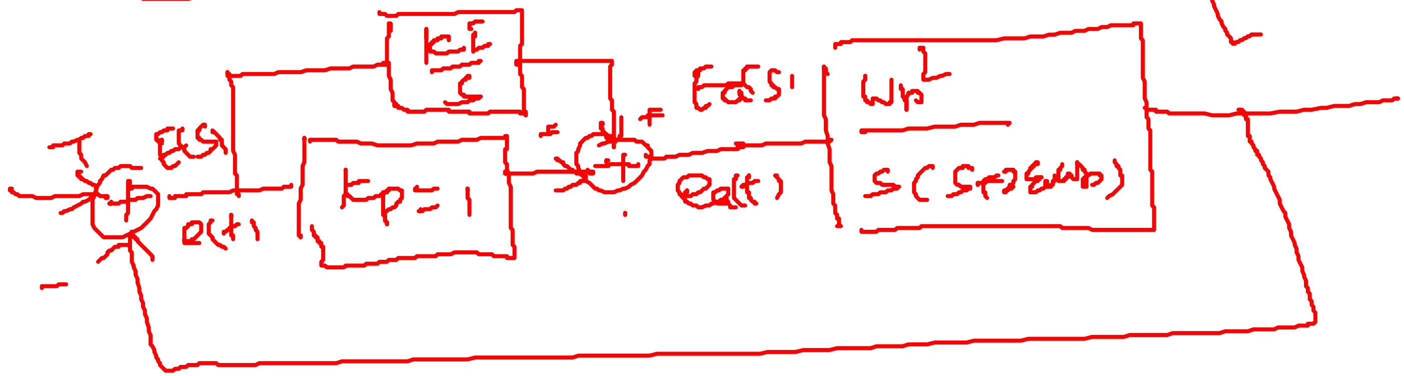
Proportional + Integral Controller

P + I

$\int e(t)\,dt = \frac{K_I}{S}$

$K_I \int e(t)\,dt$

$\frac{1}{s} = \int dt$



$$e_a(t) = e(t) + K_i \int e(t)\,dt$$

$$E_a(s) = E(s)\left(1 + \frac{K_i}{s}\right) \quad \text{——— (1)}$$

$$\frac{C(s)}{R(s)} = \frac{\left(1 + \frac{K_i}{s}\right)\left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s}\right)}{1 + \left(1 + \frac{K_i}{s}\right)\left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{(s+k_i)(\omega_n^2)}{s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + K_I\omega_n^2} \quad \text{--- ③}$$

third order

$$s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + K_I\omega_n^2 = 0$$

order of system increases by 1

let $r(t) = (t)$ $\quad R(s) = 1/s^2$

ess

$$E(s) = R(s) \cdot \frac{s^2(s+2\xi\omega_n)}{s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + K_I\omega_n^2}$$

$$e_{ss} = \lim_{s \to 0} s E(s) = s \cdot \frac{1}{\cancel{s^2}} \frac{\cancel{s^2}(s+2\xi\omega_n)}{(\quad)} = 0$$

I/P Parabolic $R(s) = 1/s^3$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^3} \cdot \frac{s^2 (s + 2\xi\omega_n)}{s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + K_i \omega_n^2} = \frac{2\xi\omega_n}{K_i \omega_n^2}$$

$$\boxed{e_{ss} = \frac{2\xi}{K_i \omega_n}}$$

PI Controller Improves

Steady-State performance

→ order of System is increase by 1

PID = lag-lead

PI = lag

PD = lead

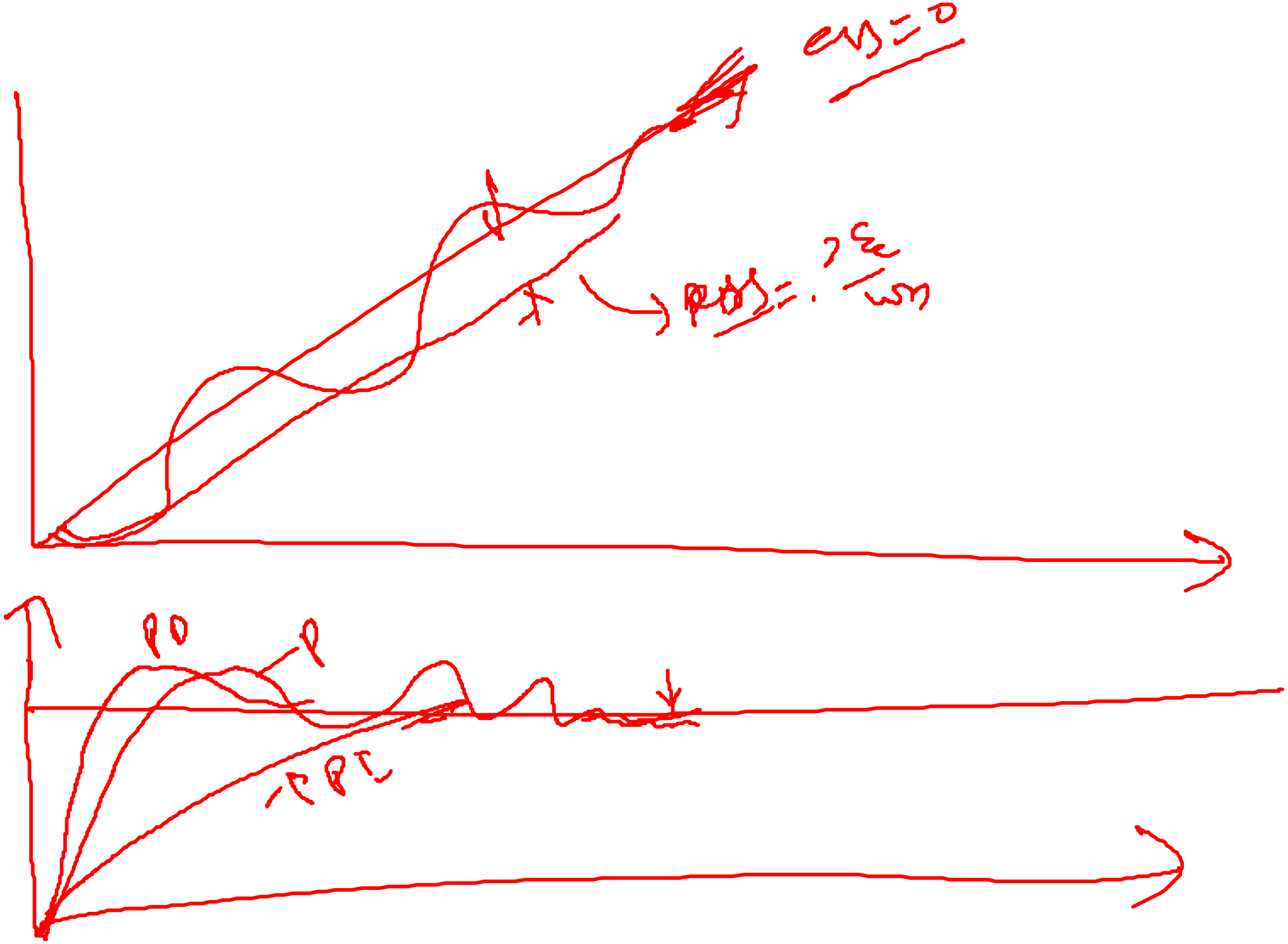
Hence PI controller has following effects :

i) It increases order of the system.

ii) It increases TYPE of the system.

iii) Design of K_i must be proper to maintain stability of system. So it makes syste relatively less stable.

iv) Steady state error reduces tremendously for same type of inputs. i.e. in gene this controller improves steady state part affecting the transient part.


ess=0
PD
P

PID Controller

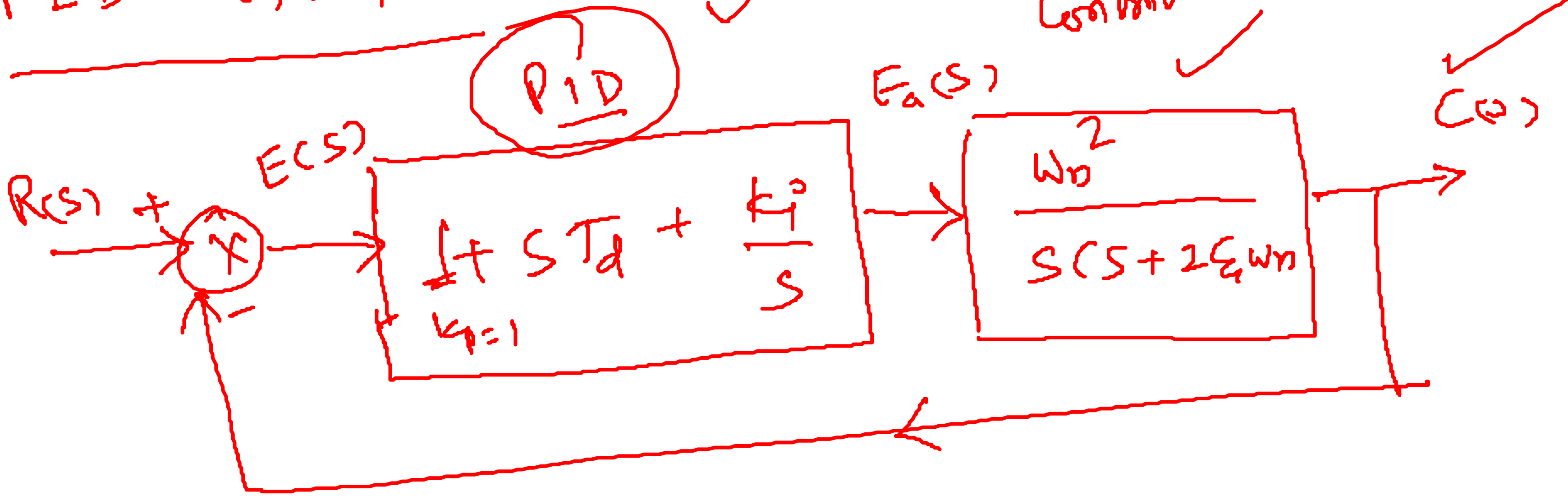
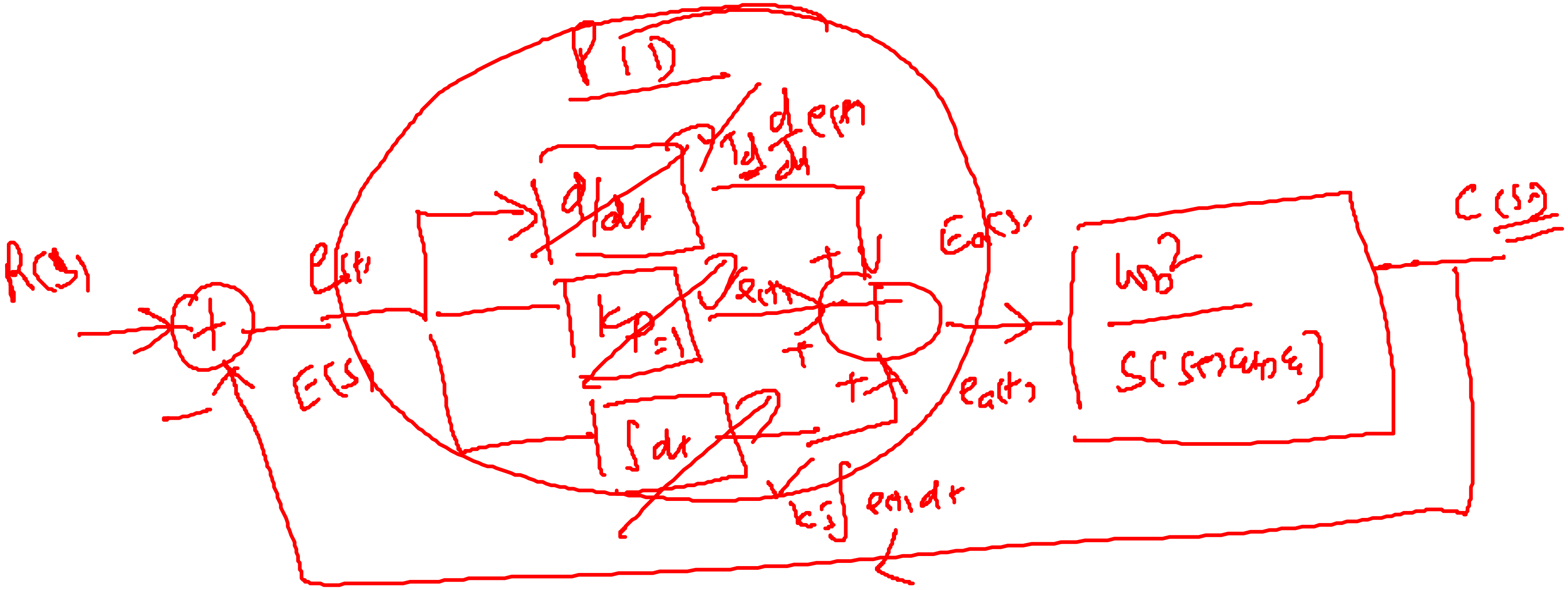


Fig: BD of PID Controller

The actuator

- $K_p \rightarrow$ Proportional Gain
- $K_I \rightarrow$ Integral Gain
- $K_d \,/\, T_d \rightarrow$ derivative gain



$$e_a(t) = e(t) + T_d \frac{de(t)}{dt} + K_I \int e(t)\,dt \quad \text{— (1)}$$

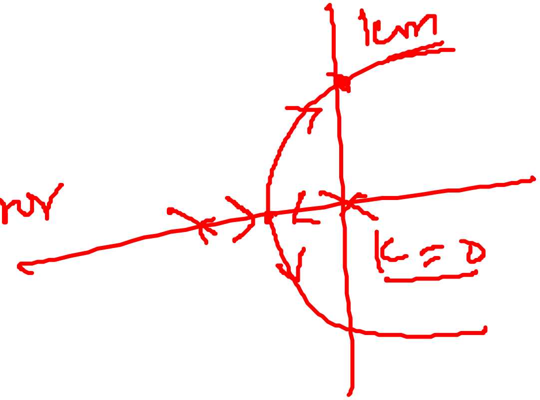
$$E_a(s) = E(s) + T_d s E(s) + \frac{K_I}{s} E(s)$$

$$E_a(s) = E(s)\left[1 + T_d s + \frac{K_I}{s}\right]$$

K_p → Prop controller → Improve e_{ss} tracking accuracy

$K \uparrow$ → May be unstable

↓ lead to constant steady state error



$$\boxed{\downarrow e_{ss} = \frac{1}{1+K_p \uparrow}}$$

Integral Controller (K_i) → It reduces steady state error

drawback → lead oscillatory response

() system

Derivative (P+D)

↳ effective during transient period

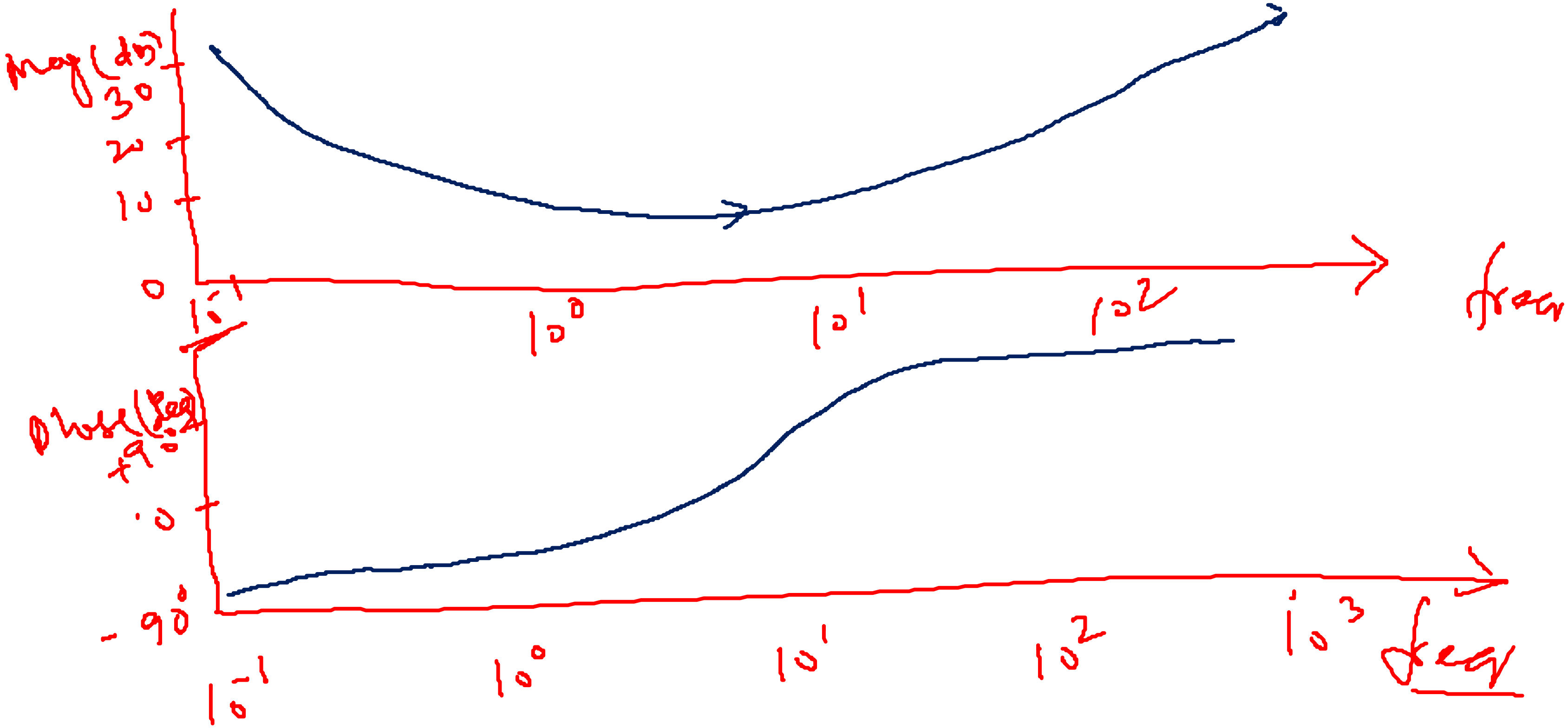
$\boxed{P+I+D}$

PID → include all three controller P + I + D

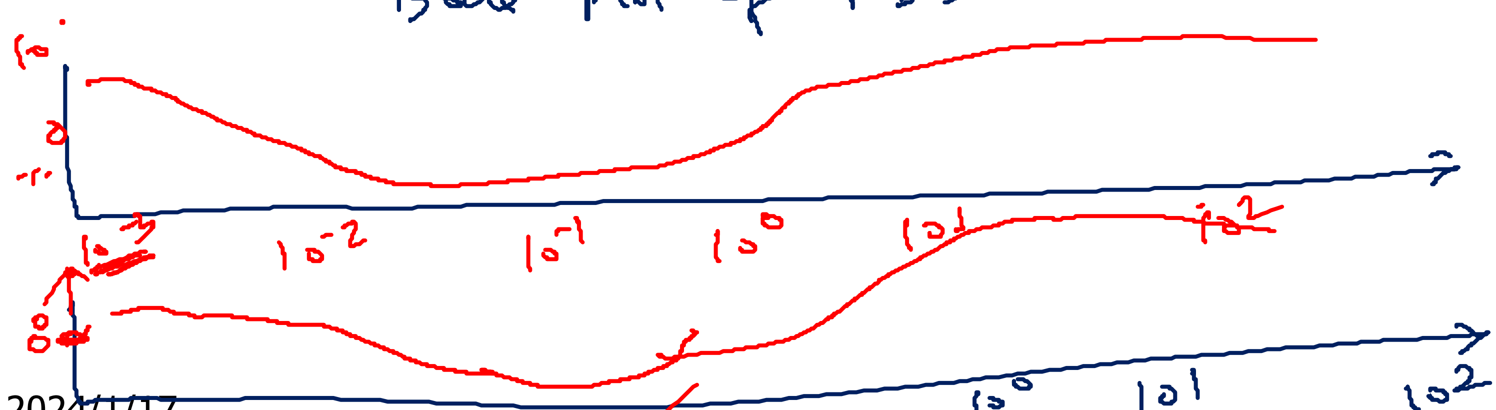
↳ stabilize the gain

reduces steady state

reduces rate of change errors



Bode Plot of PID Controller



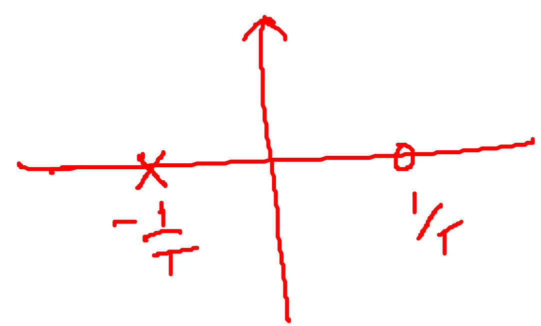
Lag-Lead Compensator

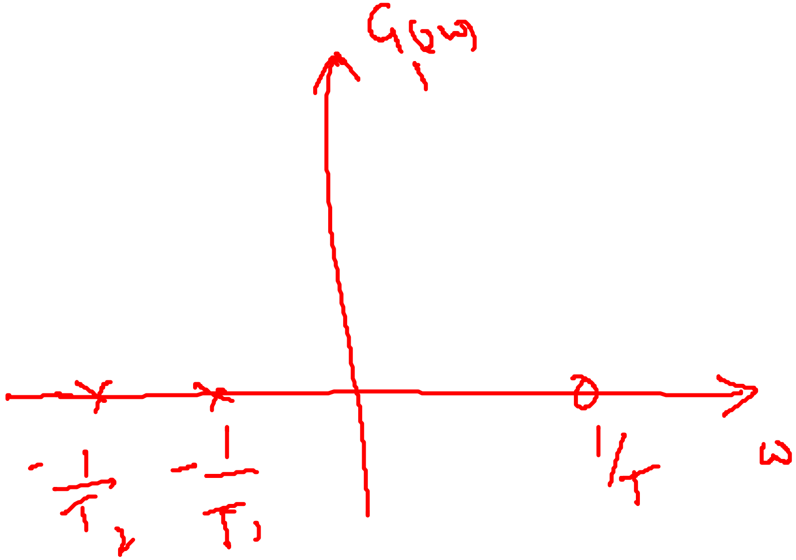
All Pass and minimum phase System

We have considered minimum phase TF so far i.e, those with all poles and zeros in the LHS

Consider a TF having pole-zero pattern which is antisymmetric about Im axis i.e for every pole in the LHS, there is a zero in mirror image position. e.g

$$G(j\omega) = \frac{1 - j\omega T}{1 + j\omega T}$$





Fig(a) Non minimum Phase Function

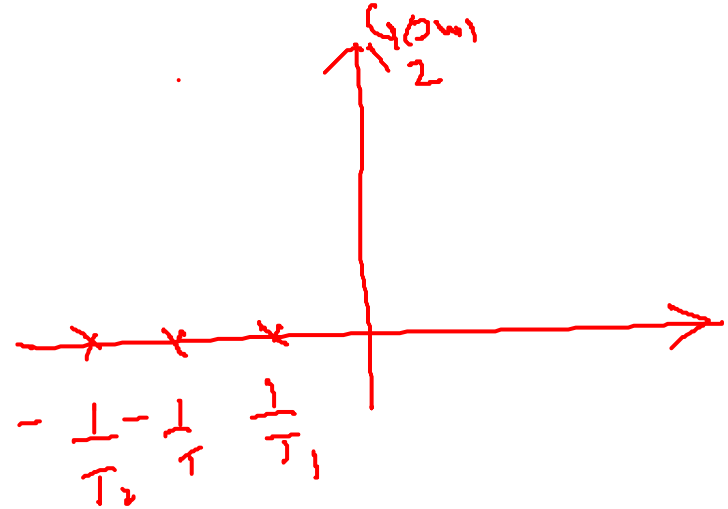


Fig (b) minimum phase function

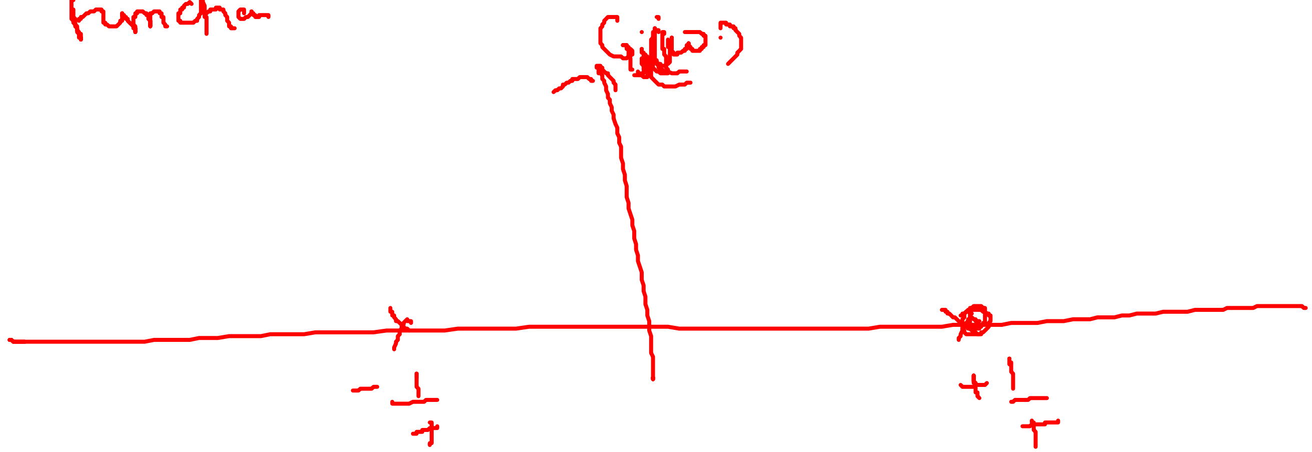
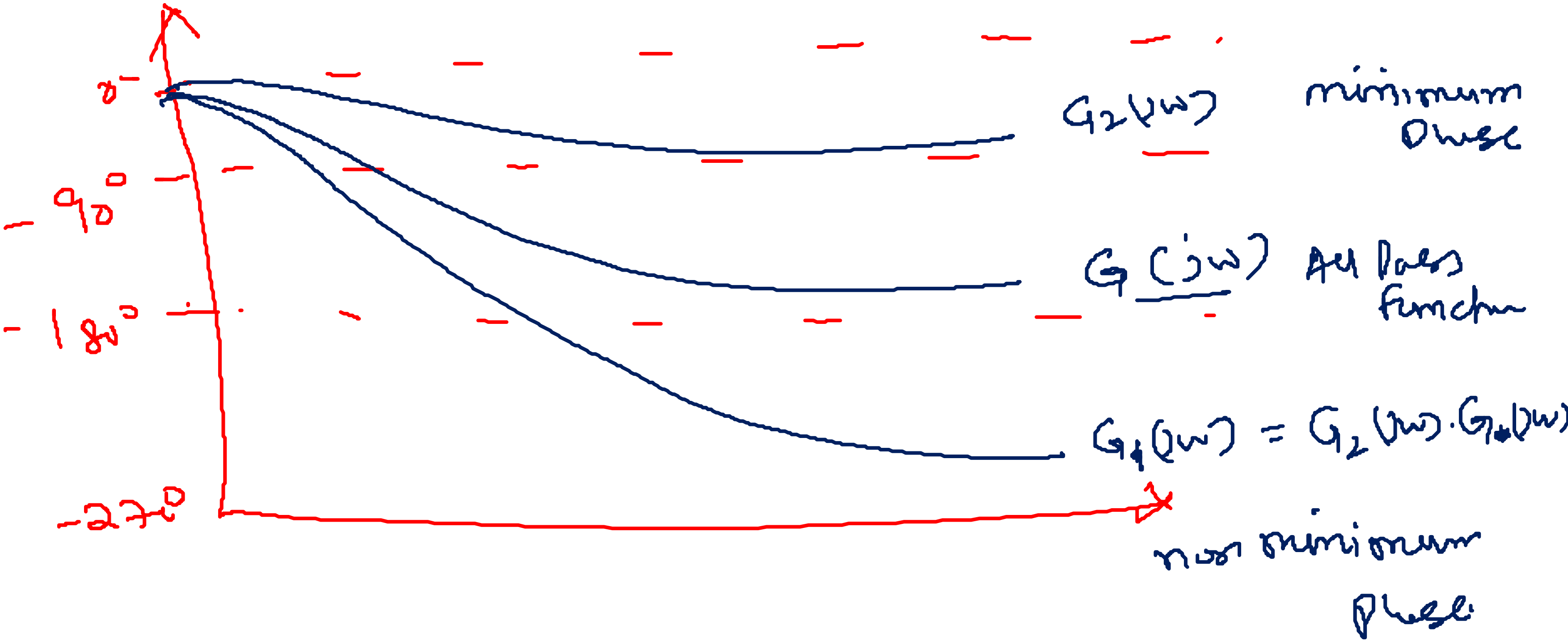


Fig (c) All Pass Function



All pass function :⇒ Magnitude unity all freq

Phase = $-2\tan^{-1}(\omega T)$

$0°$ to $180°$

$\boxed{\omega \to 0 \text{ to } \infty}$

→ A TF which has one or more zeros in RHS is known as nonminimum phase TF

→ Larger the phase lags present in the system

Common example Non minimum phase element

$$\boxed{G(j\omega) = e^{-j\omega T}}$$

$1 \angle -\omega T$ rad.

$1 \angle -57.3\,\omega T^{\circ}$ (deg.)

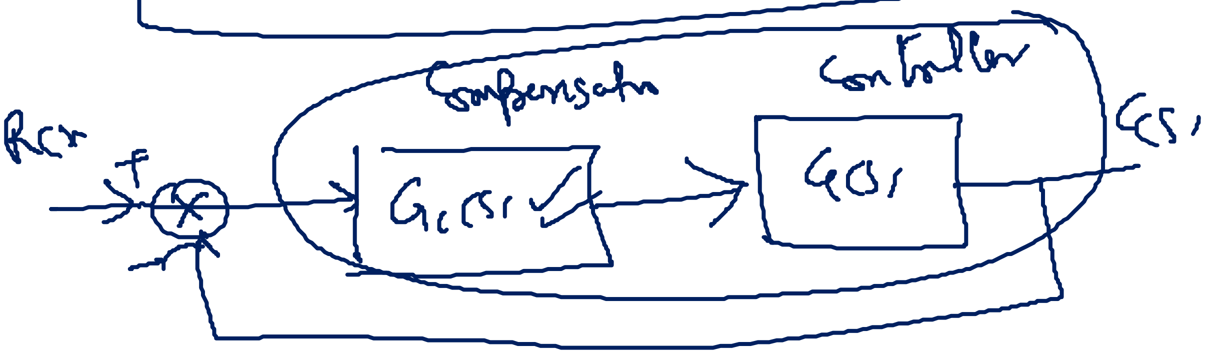
Lead compensator design.

Step I at zero freq Lead comp handle the gain α, $(\alpha<1)$ + amplifier has gain K_c

$$G_c(s) = K_c\alpha\left(\frac{1+Ts}{1+\alpha Ts}\right)$$

where $K_c\alpha$ = dc gain K

$$\boxed{G_c(s) = \frac{K(1+Ts)}{1+\alpha Ts}} \quad \text{—} \; (1)$$



$$G_c(s)\cdot G(s) = \frac{K(1+Ts)}{(1+\alpha Ts)}\cdot G(s)$$

$$= \frac{KG(s)\,(1+Ts)}{(1+\alpha Ts)} = G_1(s)\frac{(1+Ts)}{(1+\alpha Ts)}$$

where

$$\boxed{G_1 = KG(s)}$$

Step: 2 Generally in design problem error Constant

→ using value of k determined above and draw a Bode of $G_1(j\omega)$ → uncompensated Bode

Step 3 Generally ϕ_m specified

Let ϕ_s = ϕ_m specified

ϕ_1 → ϕ_m obtained from Step 2

Determine Phase lead $\phi_m = \phi_s - \phi_1 + \epsilon$

ϵ → Safety of Margin $= 5^\circ$ to 15°

Step 4 Using eqn

$\sin(\phi_m) = \frac{1-\alpha}{1+\alpha}$ determine 'α'

Step 5 Determine the freq w_{m} at which the magnitude of the uncompensated system is $-10\log\left(\frac{1}{\alpha}\right)$ dB. Select this freq as new gain crossover freq (w_{gc}) — (3)

This freq is

$$w_m = \frac{1}{T\sqrt{\alpha}} \quad \text{—— (4)}$$

as α is known determine $\frac{1}{T}$,

Step 6 Determine two corner freq

$$w_{c_1} = \frac{1}{T} \text{ and } w_{c_2} = \frac{1}{\alpha T}$$

Step 7 $K = K_c \alpha \Rightarrow$ determine value of K_c

Step 8: Sketch Bode & PM of Compensated System

Check Gm, Pm - - - if satisfy - - -

Q. Design a lead compensator such that the closed loop system shown below will satisfy the following specification

$K_v = \lim_{s \to 0} s\,G(s)H(s)$

① Static velocity constant = 20 sec
② $\phi_m = 50^\circ$
③ $G_m \geq 10\,dB$

Step 1 Assume lead compensator as

$$G_c(s) = K_c \alpha \frac{(1+Ts)}{(1+\alpha Ts)} = \frac{K(1+Ts)}{1+\alpha Ts} \quad \text{—— ①}$$

$$G_1(s) = K\,G(s) \qquad G(s) = \frac{10}{s(s+1)}$$

$$G_1(s) = \frac{10K}{s(s+1)} \quad \text{—— ②}$$

$$K_v = 20 = \lim_{s \to 0} s \cdot \frac{10K}{s(s+1)} \cdot \frac{(1+Ts)}{(1+\alpha Ts)} = \frac{10K}{}$$

$$K = \frac{20}{10} = 2 \qquad \boxed{K = 2}$$

$$G_1(s) = \frac{2 \times 10}{s(s+1)} = \frac{20}{s(s+1)} \quad \text{— (3)}$$

ω	$\frac{1}{j\omega}$	$-\tan^{-1}(\omega_1)$	ϕ_R
0.1	$-90°$	$-5.71°$	$-95.71°$
1	$-90°$	$-45°$	$-135°$
2	$-90°$	$-63.4°$	$-153.4°$
10	$-90°$	$-84.2°$	-174.2
∞	$-90°$	$-90°$	$-180°$

$$G_1(j\omega) = \frac{20}{j\omega(1+j\omega)} \quad \text{— (4)}$$

$$20\log G_1(j\omega)_m = 20\log 20 - 20\log \omega - 20\log|1+j\omega| \quad \text{— (5)}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}(\omega)$$

$$\phi = -90° - \tan^{-1}\omega$$

Corner freq $\boxed{\omega c_1 = 1}$

factor	Corner freq (rad)	Slope contribution (db/dec)	Net Slope	Asymptotic Plot
				$20\log 20 = +26\,dB$ — Start line
$K = 20$	—	—	—	-20 db/dec line which intersects 26 db $\omega = 1$
$\frac{1}{j\omega}$	—	-20	-20	$sp \to \omega c_1 = 1$
$\frac{1}{1+j\omega}$	$\omega c_1 = 1$	-20	-40	$\omega c_1 = 1$ to ∞

$Pm \approx 15$ $\quad Gm = \infty \quad \omega gc = 4$ rad

$\phi_m = \phi_1 = 15^\circ$, $\downarrow 5^\circ$

$\boxed{\omega_{gc} = 4\,\text{rad}}$

$G_m = \infty \rightarrow$ uncompensated Bode plot

$\phi_s = 50^\circ$

Safety of margin $\boxed{\epsilon = 5^\circ}$

$\phi_m = \phi_s - \phi + \epsilon$

$= 50^\circ - 15^\circ + 5^\circ = \underline{\underline{40^\circ}}$

$\sin \phi_m = \dfrac{1-\alpha}{1+\alpha}$

$\sin(40^\circ) = \dfrac{1-\alpha}{1+\alpha} = 0.6427 \Rightarrow \boxed{\alpha = 0.2174}$

$\boxed{\text{Choose } \alpha = 0.21}$

$\underline{\text{step}}$ $\quad -10\log\left(\dfrac{1}{\alpha}\right) = -10\log\left(\dfrac{1}{0.21}\right) = \underline{\underline{-6.78\,\text{dB}}}$

Find freq at which uncompensated system have gain $= \underline{-6.78\,\text{dB}}$

$$W_m = \frac{1}{T\sqrt{\alpha}} \qquad \rightarrow T = \frac{1}{2.7495} = \underline{0.3637}$$

$$6 = \frac{1}{T\sqrt{0.21}} \Rightarrow \frac{1}{T} = 2.7495 \text{ rad/sec} = \underline{\underline{W_{C_1}}}$$

Next Corner $= W_{C_2} = \dfrac{1}{\alpha T} = \boxed{13.09}$ W_{C_2}

Compensator N/W $\underline{K_c\alpha} \dfrac{(1+TS)}{(1+\alpha TS)}$

$\underline{K} = K_c\alpha$

$2 = K_c \times 0.21 = \boxed{K_c = 9.523}$

$\boxed{\alpha T = \underline{0.0763}}$

$$\boxed{G_c(s) = 9.521 \times 0.21 \frac{(1+0.3637S)}{(1+0.0763S)}}$$

<u>Design System</u> $G_c(S).G(S)$

$$2\text{dB} \quad = \frac{2(1+0.3637S)}{(1+0.0763S)} \cdot \frac{10}{S(S+1)} = \frac{20(1+0.3637S)}{\underline{S}(S+1)(S+0.0763S)}$$

$$|G(j\omega)G_c(j\omega)| = 20\log 20 + 20\log(1+0.367j\omega) - 20\log\left|\frac{1}{j\omega}\right| - 20\log\left|\frac{1}{1+j\omega}\right| - 20\log|1+j0.0076\omega|$$

$$\phi = \tan^{-1}(0.367\omega) - 90^\circ - \tan^{-1}(\omega) - \tan^{-1}(0.076\omega)$$

$20\log|G(j\omega)G_c(j\omega)|$

$= 20\log 20 + 20\log|1+0.3637\omega|$

$- 20\log|j\omega| - 20\log|1+j\omega|$

$- 20\log|1+j0.076\omega|$

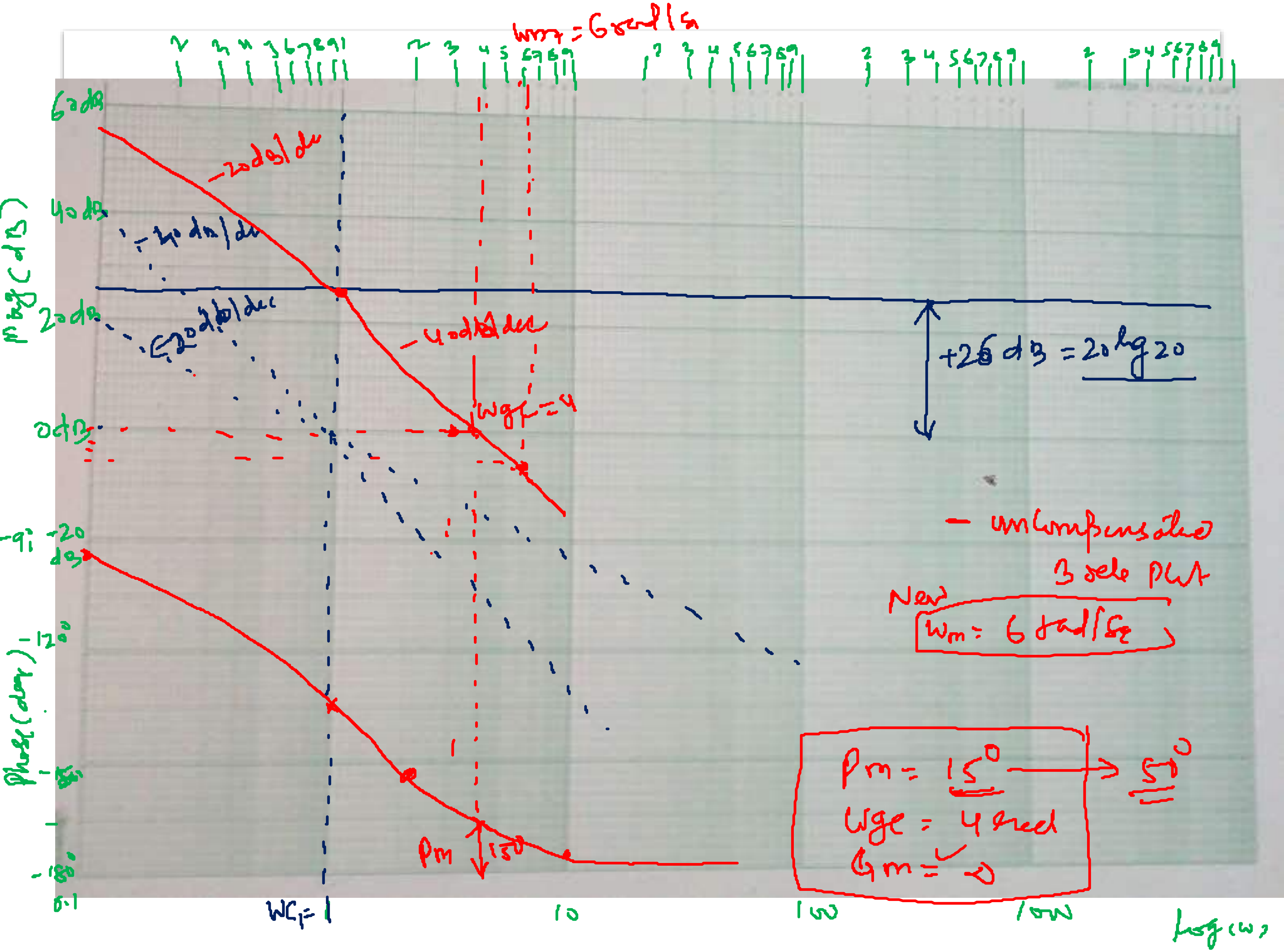
w	$\frac{1}{j\omega}$	$-\tan^{-1}(\omega)$	$+\tan^{-1}(0.367)$	$-\tan^{-1}(0.0763)$	ΦR
0.1	$-90°$	-5.71	$+2.08$	$-0.43°$	-94.0
1	$-90°$	$-45°$	$+20$	$-4.36°$	-119.36
2	$-90°$	-63.4	$+36°$	-8.67	-126.07
10	$-90°$	-84.2	$+74$	$-37.3°$	-137.5
100	$-90°$	-89.4	$+88°$	$-82.53°$	$-173.9°$

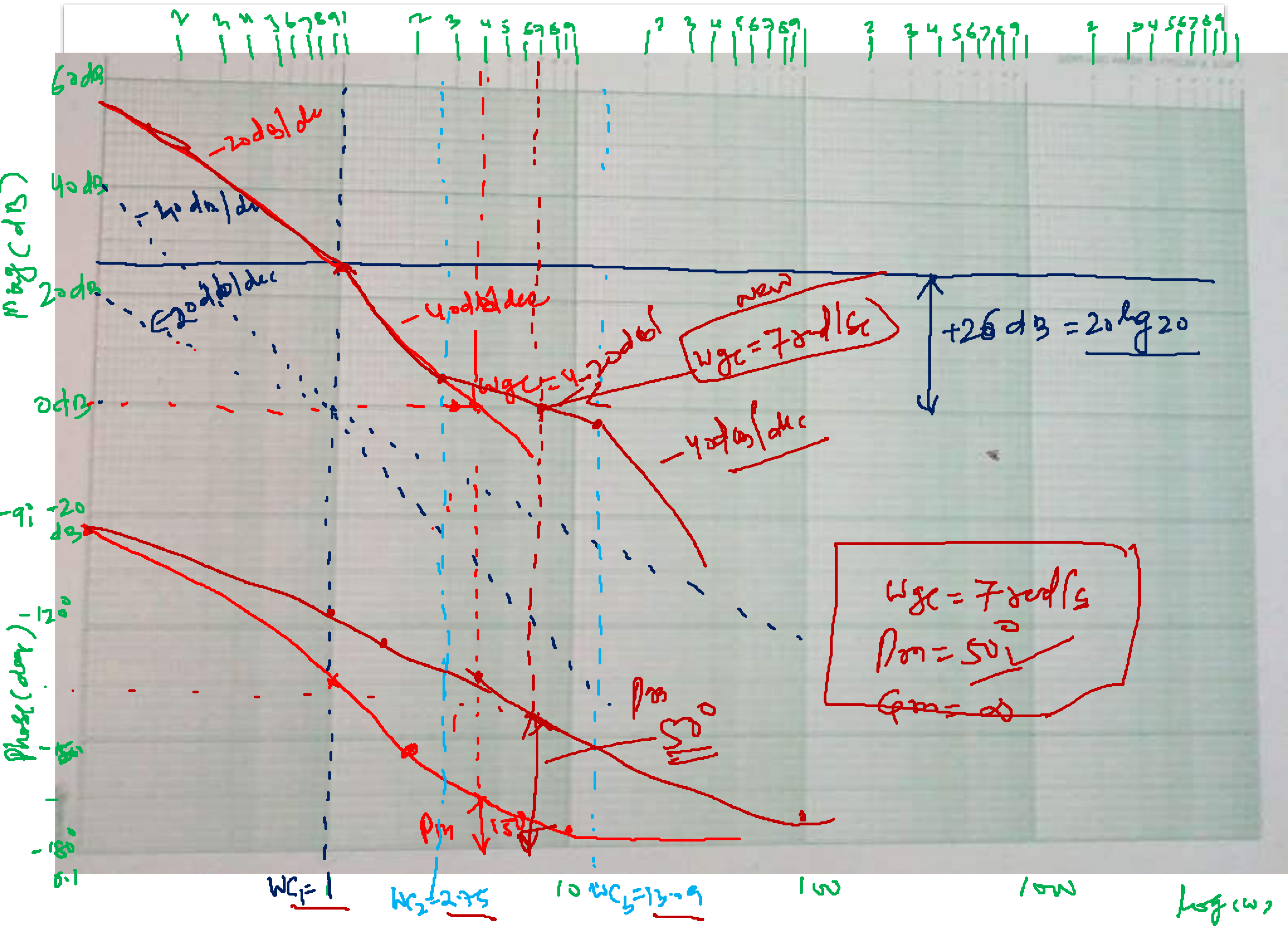
Cor freq $\omega c_1 = 1$ rad/sec → (Pole)

$\omega c_2 = 2.75$ rad/sec (zero)

$\omega c_3 = 13.09$ rad/sec (Pole)

factor	Cor freq	Slope con db/dec	Net slope db/dec	Asymptotic plot
k=20	—	-	-	$20\log 20 = +26$ dB Straight line
$\frac{1}{j\omega}$	—	-20	-20	-20 db/dec slope - - - $\omega=1$ ($\boxed{\text{sp} - \omega=1}$)
$\frac{1}{1+j\omega}$	$\omega c_1 = 1$	-20	-40	$\omega = 1 - 2.75$
$(1+j0.3637\omega)$	$\omega c_2 = 2.75$	$+20$	-20	$2.75 - 13.09$
$\frac{1}{(1+j\omega 0.076)}$	$\omega c_3 = 13.09$	-20	-40	$13.09 - \infty$


wm = 6 rad/s
60 dB
40 dB
20 dB
0 dB
-20 dB
Mag (dB)
-20 dB/dec
-40 dB/dec
-20 dB/dec
-40 dB/dec
wgc = 4
+26 dB = 20 log 20
- uncompensated Bode plot
New wm = 6 rad/sec
Phase (deg)
-120°
-180°
Pm
15°
Pm = 15° → 50°
Wgc = 4 rad
Gm = ∞
0.1
Wc1 = 1
10
100
1000
log(ω)


60 dB
40 dB
20 dB
0 dB
-20 dB
Mag (dB)
Phase (deg)
-20 dB/dec
-40 dB/dec
-20 dB/dec
-40 dB/dec
-20 dB/dec
-40 dB/dec
$\omega_{gc1} = 4.2$
new
$\omega_{gc} = 7$ rad/s
+26 dB = 20 log 20
$\omega_{gc} = 7$ rad/s
Pm = 50°
Gm = ∞
Pm
50°
Pm
15°
0.1
$\omega c_1 = 1$
$\omega c_2 = 2.75$
10
$\omega c_3 = 13.9$
100
1000
log(ω)

Lag Compensator design

Step 1 Assume lag compensator N/W

$$G_c(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = \frac{1+Ts}{1+\beta Ts} \quad \text{--- (1)}$$

$$G_1(s) = K\,G(s) \quad \text{--- (2)}$$

from given error constant find value of K

Step 2 Using K determine $G_1(s) \rightarrow$ Sketch Bode Plot

obtain ϕ_1 = Pm (uncompensated)

Step 3 Generally Pm specified Let ϕ_s = Pm specified

$$\phi_2 = \phi_s + \epsilon \quad \text{--- (3)}$$

$\epsilon = 5^\circ$ to 15°

Safety of margin

Step 4 find freq ω_2 corresponding to ϕ_m of ϕ_2 degree

$-180^\circ + \phi_2$ choose it new gain crossover freq

Step 3: To have ω_2 as new ω_{gc}, determine attenuation necessary to shift the magnitude curve up or down to 0 dB. This shift is due to contribution of 'β' which is $20\log\left(\frac{1}{\beta}\right)$.

Shift to have ω_2 as new gain crossover

$$= 20\log\frac{1}{\beta} = \underline{-20\log\beta}$$

Note → down shift must be taken negative
and up shift " " " +ve

Step 6 choose upper corner freq $\frac{1}{T}$ which is $\frac{1}{2}$ or $\frac{1}{10}$ below ω_2 determine in step 4

$$\omega c_2 = \frac{1}{T} = \frac{\omega_2}{2} \text{ or } \frac{\omega_2}{10}.$$

Thus determine 'T'
find other Corner freq $\omega_{c1} = \frac{1}{\beta T}$.

Step 7 → Draw the Compensated System Bode Plot and check the Specifications.

Q Design a Lag compensator for a System to give

$$G(s) = \frac{0.025}{s(1+0.5s)(1+0.05)}$$

① Velocity error constant $= 20\,sec$

② $\phi_m = 40^\circ$

Sol

Assume $G_1(s) = K\,G(s)$

$$= \frac{0.025K}{s(1+0.5s)(1+0.05s)}$$

$$K_v \rightarrow \lim_{s\to 0} s \cdot G_1(s)\, G_c(s) = 20$$

$$\frac{\cancel{s} \cdot 0.25K}{\cancel{s}(1+0.5s)(1+0.05s)} \cdot \frac{(1+Ts)}{(1+\beta Ts)} = 20$$

$$\boxed{K = 800}$$

$$G_1(s) = \frac{0.025 \times 800}{s(1+0.5s)(1+0.05s)} = \frac{20}{s(1+0.5s)(1+0.05s)}$$

Step 2 Draw a Bode plot of $G_1(j\omega)$

$$G_1(j\omega) = \frac{20}{j\omega(1+j0.5\omega)(1+j0.05\omega)}$$

Corner freq $\omega_{C_1} = \frac{1}{0.5} = 2$ rad/sec

$\omega_{C_2} = \frac{1}{0.05} = 20$ rad/sec

$$20\log|G_1(j\omega)| = 20\log(20) - 20\log|j\omega| - 20\log|1+j0.5\omega| - 20\log|1+j0.05\omega|$$

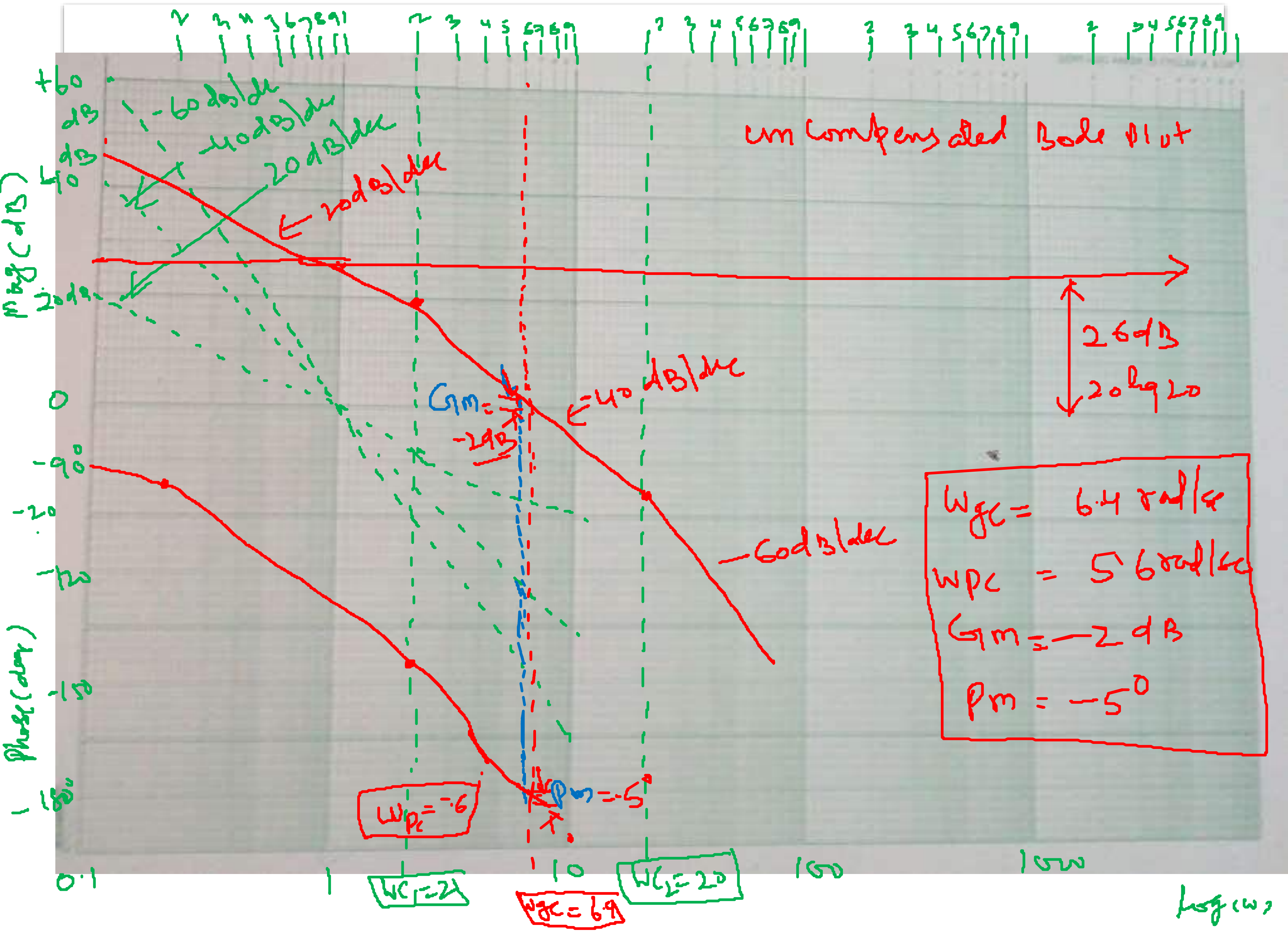
$$\text{Phase angle } \phi = \angle G_1(j\omega) = \frac{\angle 20^\circ}{\angle j\omega \, \angle(1+j0.5\omega) \, \angle(1+j0.05\omega)}$$

$$\phi_R = 0^\circ - 90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.05\omega)$$

Factor	Corner freq (rad/sec)	Slope Contributed (dB/dec)	Net Slope (dB/dec)	asymptotic Plot
$K=20$	–	–	–	$20\log 20 = +26\,dB$ Straight line
$\frac{1}{j\omega}$	–	-20	-20	$SP \rightarrow 2$
$\frac{1}{(1+j0.5\omega)}$	2	-20	-40	$2 \rightarrow 20$
$\frac{1}{(1+j0.05\omega)}$	20	-20	-60	$20 \rightarrow \infty$

Phase ϕ_R

ω	$\frac{1}{j\omega}$ (deg)	$-\tan^{-1}(0.5\omega)$ (deg)	$-\tan^{-1}(0.05\omega)$ (deg)	ϕ_R (deg)
0.2	$-90°$	-5.71	-0.57	$-96.28°$
2	$-90°$	$-45°$	-5.71	-140.70
5	$-90°$	-68.19	-14.03	$-172.22°$
10	$-90°$	$-78.69°$	-26.56	$-195.25°$


uncompensated Bode plot
-60 dB/dec
-40 dB/dec
20 dB/dec
-20 dB/dec
-40 dB/dec
-60 dB/dec
26 dB
20 log 20
Gm = -2 dB
Pm = -5°
$W_{pc} = .6$
$W_{c_1} = 2$
$W_{gc} = 6.9$
$W_{c_2} = 20$
$W_{gc} = 6.4$ rad/sec
$W_{pc} = 5.6$ rad/sec
Gm = -2 dB
Pm = -5°
Mag (dB)
Phase (deg)
log(w)
0.1
1
10
100
1000

Uncompensated System is unstable

$W_{gc} = 6.4$ rad/sec,
$W_{pc} = 5.6$ rad/sec
$Pm = -5^\circ$
$Gm = -2$ dB

Step:3 Required $Pm = 40^\circ$
$\epsilon = 5^\circ$

$\phi_2 = \phi_s + \epsilon = 40^\circ + 5^\circ = \underline{\underline{45^\circ}}$

Step4 From Bode Plot Find W_2 which gives
$Pm = \underline{40^\circ} \Rightarrow W_2 = 1.5$ rad/sec

Step:5 Let $W_2 =$ New $W_{gc} = 1.5$ rad/sec
For this it is necessary to bring magnitude curve -23 dB down

$$-20\log\beta = -23\,dB$$

$\beta = 14.12$ Let $\boxed{\beta = 14}$

Choose $\omega_{c2} = \dfrac{\omega_2}{10} = \dfrac{1.5}{10} = 0.15$ rad/sec

$$\omega_{c1} = \frac{1}{T} = 0.15 \Rightarrow \boxed{T = 6.66\ \text{sec}}$$

and

$$\omega_{c1} = \frac{1}{\beta T} = \frac{1}{14\times 6.66} = 0.0107\ \text{rad/sec}$$

$$\boxed{\beta T = 93.46}$$

Thus lag compensator is

$$G_c(s) = \frac{1+6.66s}{(1+93.46s)} \qquad \left\{ \left(\frac{1+sT}{1+\beta Ts} \right) \right.$$

Thus compensated system is

$$G_c(s)G(s) = \frac{20(1+6.66s)}{s(1+0.5s)(1+0.05s)(1+93.46s)}$$

New Cornerfreq

$$\omega_{c_1} = 0.01 \text{ rad/s} \left(\frac{1}{\beta T} \right)$$

$$\omega_{c_2} = \frac{1}{T} = 0.15 \text{ rad/s}$$

$$\omega_{c_3} = \frac{1}{0.5} = 2 \text{ rad/sec}$$

$$\omega_{c_4} = \frac{1}{0.05} = 20 \text{ rad/sec}$$

System is

$$G_c(\omega) G(\omega) = \frac{20(1+j6.66\omega)}{j\omega(1+0.5j\omega)(1+0.05j\omega)(1+93.46j\omega)}$$

$$M = 20\log|G_c(j\omega)G(j\omega)| = 20\log 20 + 20\log|1+j6.66\omega| - 20\log|j\omega| - 20\log|1+0.5j\omega|$$
$$- 20\log|1+0.05j\omega| - 20\log|1+93.46j\omega|$$

$$\phi = \angle G_c(j\omega)G(j\omega) = \frac{\angle 20 \; \angle(1+j6.66)}{\angle j\omega \; \angle(1+0.5j\omega) \; \angle(1+0.05j\omega) \; \angle(1+93.46j\omega)}$$
$$= \tan^{-1}(6.66\omega) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}(0.5\omega) - \tan^{-1}(0.05\omega)$$
$$- \tan^{-1}(93.46\omega)$$

Factor	Corner freq (rad/sec)	Slope cont	Net slope	Asymptotic Plot
K=20	-	-	-	20 log 20 = +26 dB Straight line
$\frac{1}{j\omega}$	-	-20	-20	-20 dB/dec Slope line from Sp - 0.01 rad/sec
$\frac{1}{1+93.46j\omega}$	0.01	-20	-40	-40 dB/dec " " 0.01 to 0.15 rad/s
$(1+j6.66j\omega)$	0.15	+20	-20	-20 " " " " 0.15 → 2 rad/s
$\frac{1}{(1+0.5j\omega)}$	2	-20	-40	-40 " " " 2 - 20 rad/s
$\frac{1}{(1+0.05j\omega)}$	20	-20	-60	-60 dB/dec " " 20 to ∞

Phase ϕ

ω	1/jω	$-\tan^{-1}(93.46)$	$+\tan^{-1}(6.66\omega)$	$-\tan^{-1}(0.5\omega)$	$-\tan^{-1}(0.05\omega)$	ϕR
0.01	-90°	-43.06	3.81°	-0.28°	-0.028°	-129.5°
0.05	-90°	-77.92°	18.41°	-1.43°	-0.14°	-151.08°
0.1	-90°	-84°	33.66°	-2.86°	-0.28°	-143.48°
1	-90°	-89.38	81.46°	-26.56°	-2.86°	-127.34
2	-90	-89.7	+85.7°	-45°	-5.71°	-144.7°
10	-90°	-90°	89°	-78.1°	-26.56	-195.5°

From Fig

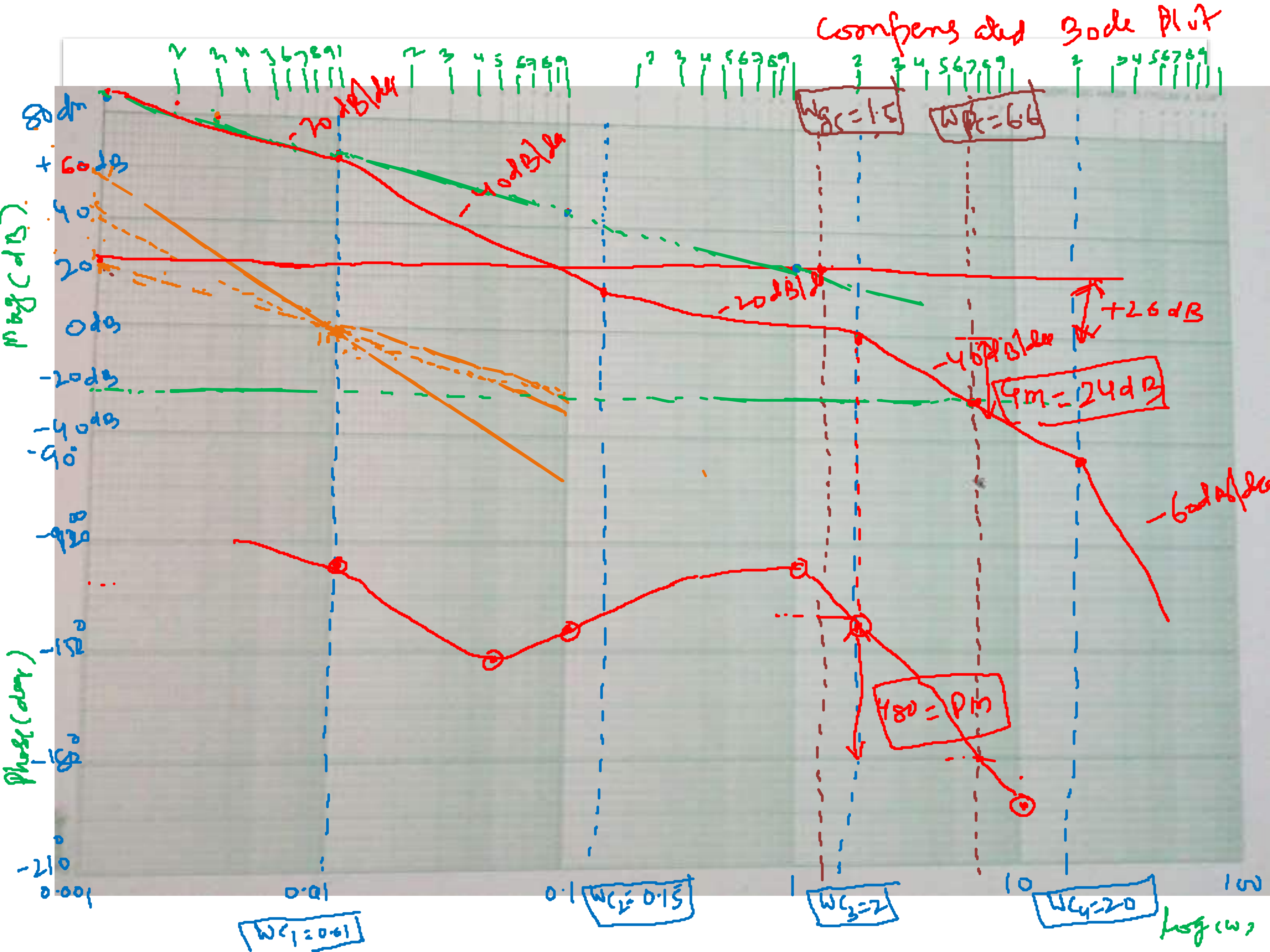
$\omega_{gc} = 1.3$ rad/sec

$\omega_{pc} = 6.6$ rad/s

$Gm = +24$ dB

$Pm \quad 48^\circ$

Thus the compensated system satisfies the specifications



Compensated Bode Plot
80 dB
+60 dB
40
20
0 dB
-20 dB
-40 dB
-90°
-120°
-150°
-180°
-210°
Mag (dB)
Phase (deg)
-20 dB/dec
-40 dB/dec
-20 dB/dec
-40 dB/dec
-60 dB/dec
ωgc = 1.5
ωpc = 6.6
+26 dB
Gm = 24 dB
180 = PM
0.001
0.01
0.1
1
10
100
ωc1 = 0.01
ωc2 = 0.15
ωc3 = 2
ωc4 = 20
log(ω)

13.4.4 Steps to Design Lead Compensator

Step 1 : At zero frequency the lead compensator has gain α. But as $\alpha < 1$ it provides an attenuation. To cancel this attenuation, the practical lead compensator is realised with an amplifier having gain K_C in series with basic lead network. Hence the practical transfer function of a lead compensator from the design point of view is assumed to be,

$$G_C(s) = K_c \alpha \frac{(1+Ts)}{(1+\alpha Ts)}$$

where $K_c \alpha$ = d.c. gain = K ...(7)

$$\therefore \quad G_c(s) = \frac{K(1+Ts)}{(1+\alpha Ts)} \quad \text{...(8)}$$

The open loop transfer function of the compensated system thus becomes,

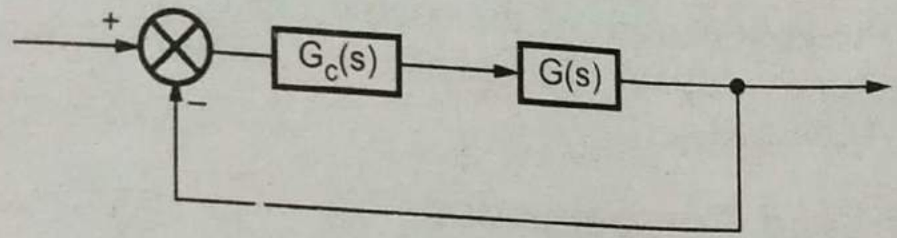


Fig. 13.8

$$G_C(s)\,G(s) = \frac{K(1+Ts)}{(1+\alpha Ts)}\cdot G(s) = \frac{(1+Ts)}{(1+\alpha Ts)}\cdot K\,G(s) = \frac{(1+Ts)}{(1+\alpha Ts)}\,G_1(s)$$

where $G_1(s) = KG(s)$

Generally in such design problems one of the error constant is given as specification. From the above result, determine the value of K satisfying the given error constant.

Step 2 : Using the value of K determined above, draw the Bode plot of $G_1(j\omega)$. This is the Bode plot of, gain adjusted but uncompensated system. Obtain the phase margin.

Step 3 : Generally P.M. is specified for the design problem.

Let ϕ_s = P.M. specified

ϕ_1 = P.M. obtained in the step 2

Determine necessary phase lead ϕ_m required to be added. For this use the relation,

$$\phi_m = \phi_s - \phi_1 + \varepsilon$$

where ε = margin of safety as cross-over frequency may shift due to compensation

$= 5° \text{ to } 15°$

Step 4: Using the equation,

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

determine the value of α.

Step 5: Determine the frequency ω_m at which the magnitude of the uncompensated system is $-10\ \text{Log}\left(\frac{1}{\alpha}\right)$ dB. Select this frequency as new gain crossover frequency. This frequency ω_m is,

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

as α is known, determine $\frac{1}{T}$.

Step 6: Determine the two frequencies of the lead compensator.

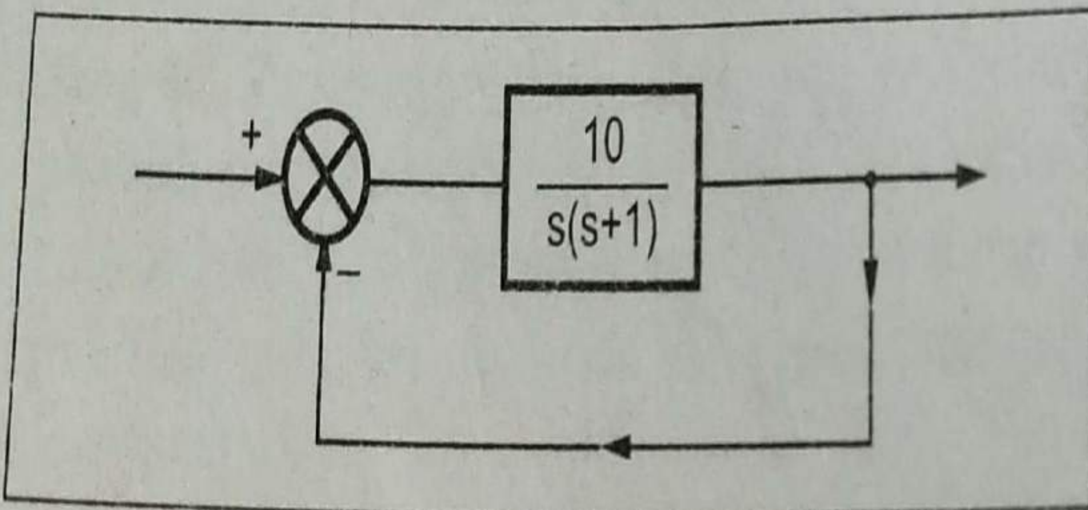
$$\omega_{C1} = \frac{1}{T} \text{ and } \omega_{C2} = \frac{1}{\alpha T}$$

Step 7: As $K = K_C\,\alpha$,

determine the value of K_C.

lead of more than 70° to 90° is required ... compensators are required.

Ex. 13.1 *For the system shown in the Fig. 13.9, design a lead compensator such that the closed loop system will satisfy the following specifications :*



Static velocity error constant = 20 sec

Phase margin = 50°

Gain margin ≥ 10 dB

Fig. 13.9

Sol. : **Step 1 :** Assume a lead compensator as,

$$G_c(s) = K_c\alpha \frac{(1+Ts)}{(1+\alpha Ts)} = \frac{K(1+Ts)}{(1+\alpha Ts)}$$

and $G_1(s) = KG(s) = \dfrac{10K}{s(s+1)}$

$$K_v = 20 = \lim_{s \to 0} sG(s)\,H(s)$$

$$\therefore \quad 20 = \lim_{s \to 0} s \,.\, \frac{10K(1+Ts)}{s(s+1)\,(1+\alpha Ts)}$$

$$\therefore \quad 20 = 10K$$

$$\therefore \quad K = 2$$

$$\therefore \quad G_1(s) = \frac{20}{s(s+1)}$$

Step 2 : Sketch the Bode plot of $G_1(s)$ which is shown in the Fig. 13.10.

Factors : 20 Log 20 = 26 dB

1 pole at origin

1 simple pole with corer frequency $\omega c = 1$.

Thus line of slope – 20 dB/dec till $\omega c = 1$ and line of slope –40 dB/dec from 1 onwards

Phase angle table : $G_1(j\omega) = \dfrac{20}{j\omega\,(1+j\omega)}$

ω	$\frac{1}{j\omega}$	$-\tan\omega$	ϕ_R
0.1	$-90°$	$-5.71°$	$-95.71°$
1	$-90°$	$-45°$	$-135°$
2	$-90°$	$-63.4°$	$-153.4°$
10	$-90°$	$-84.2°$	$-174.2°$
∞	$-90°$	$-90°$	$-180°$

From the Fig. 13.10,

$$\phi_1 = \text{P.M.} = 15°, \quad \omega_{gc} = 4 \text{ rad/sec}, \quad \text{G.M.} = +\infty \text{ dB}$$

Step 3 :

$$\phi_s = 50°$$

$$\therefore \quad \phi_m = \phi_s - \phi_1 + \varepsilon, \qquad \text{let } \varepsilon = 5°$$

$$= 50° - 15° + 5°$$

$$= 40°$$

Step 4 :

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\therefore \quad \sin 40° = \frac{1-\alpha}{1+\alpha} = 0.6427$$

$$\therefore \quad 1-\alpha = 0.6427\,(1+\alpha)$$

$$\therefore \quad \alpha = 0.2174$$

Choose $\alpha = 0.21$

Phase angle table : $G_1(j\omega) = \dfrac{20}{j\omega\,(1 + j\omega)}$

ω	$\dfrac{1}{j\omega}$	$-\tan\omega$	ϕ_R
0.1	$-90°$	$-5.71°$	$-95.71°$
1	$-90°$	$-45°$	$-135°$
2	$-90°$	$-63.4°$	$-153.4°$
10	$-90°$	$-84.2°$	$-174.2°$
∞	$-90°$	$-90°$	$-180°$

om the Fig. 13.10,

$$\phi_1 = \text{P.M.} = 15°, \quad \omega_{gc} = 4 \text{ rad/sec}, \quad \text{G.M.} = +\infty \text{ dB}$$

:

$$\phi_s = 50°$$

$$\therefore \quad \phi_m = \phi_s - \phi_1 + \varepsilon, \qquad \text{let } \varepsilon = 5°$$

$$= 50° - 15° + 5°$$

$$= 40°$$

	-90°	-90°	-180°

From the Fig. 13.10,

$$\phi_1 = \text{P.M.} = 15^\circ, \quad \omega_{gc} = 4 \text{ rad/sec}, \quad \text{G.M.} = +\infty \text{ dB}$$

Step 3 :

$$\phi_s = 50^\circ$$

$$\therefore \quad \phi_m = \phi_s - \phi_1 + \varepsilon, \quad \text{let } \varepsilon = 5^\circ$$

$$= 50^\circ - 15^\circ + 5^\circ$$

$$= 40^\circ$$

Step 4 :

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\therefore \quad \sin 40^\circ = \frac{1-\alpha}{1+\alpha} = 0.6427$$

$$\therefore \quad 1-\alpha = 0.6427\,(1+\alpha)$$

$$\therefore \quad \alpha = 0.2174$$

Choose $\alpha = 0.21$

$$-10 \log\left(\frac{1}{\alpha}\right) = -6.78 \text{ dB for } \alpha$$

Refer Fig. 13.10 and find frequency at which gain of the uncompensated system -6.78 dB, this is ω_m.

From Fig. 13.10, $\omega_m = 6$ rad/sec

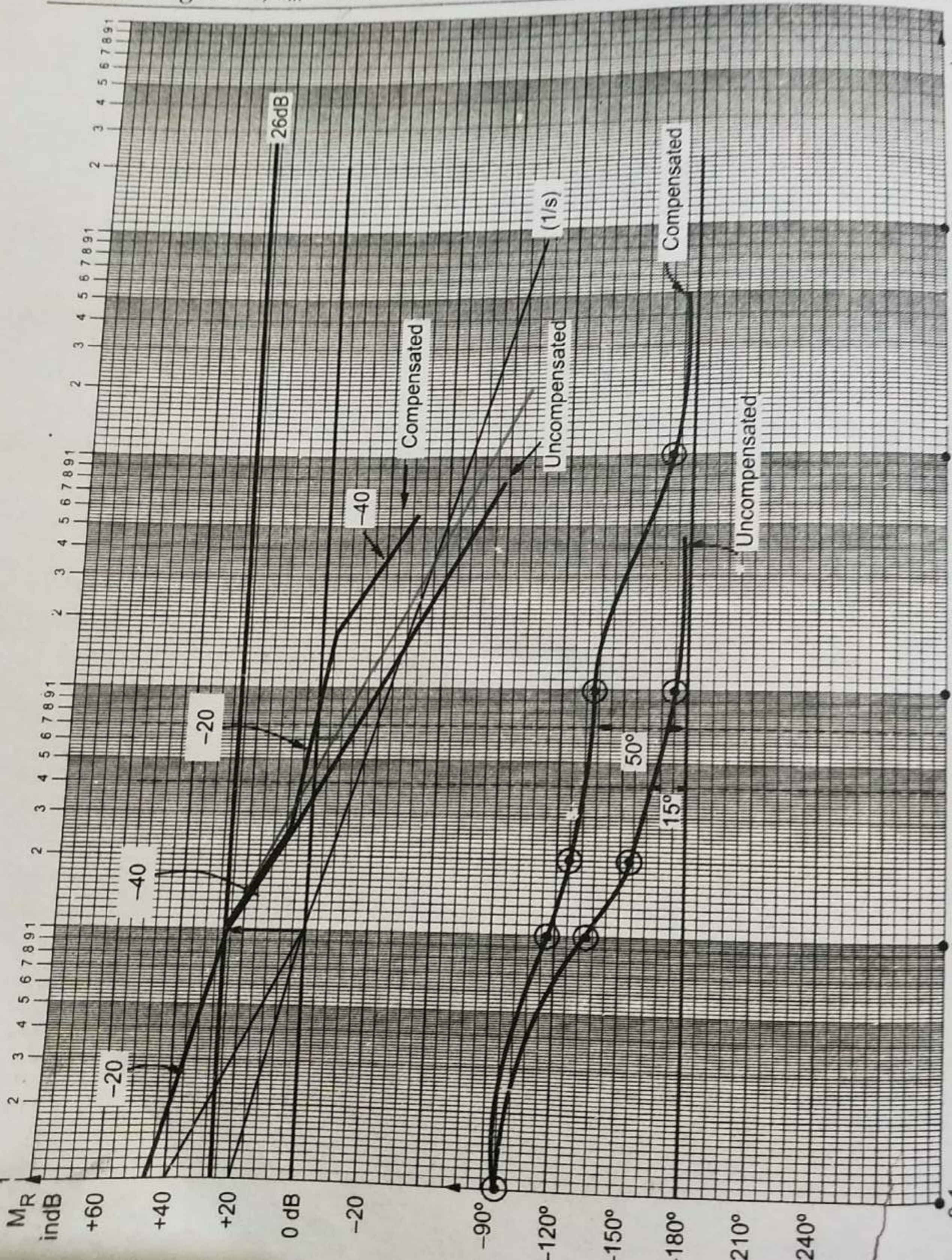


FIG. 13.10

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

$$\therefore \quad \frac{1}{T} = 2.7495$$

Step 6 : Two corner frequencies of the lead compensator are,

$$\omega_{c1} = \frac{1}{T} = 2.7495 \text{ and } \omega_{c2} = \frac{1}{\alpha T} = 13.09$$

Step 7 :

$$K = K_c\,\alpha$$

$$\therefore \quad K_c = \frac{K}{\alpha} = \frac{2}{0.21} = 9.523$$

Step 8 :

$$G_c(s) = 9.523 \times 0.21 \frac{(1+0.3637\,s)}{(1+0.0763\,s)}$$

$$= \frac{2(1+0.3637\,s)}{(1+0.0763\,s)}$$

This is the designed lead compensator.

$$\therefore \quad G_c(s)\,G(s) = \frac{20\,(1+0.3637\,s)}{s(1+s)\,(1+0.0763\,s)}$$

Draw the Bode plot for this transfer function and obtain the values of G.M. and P.M. The plot is drawn on the same semilog paper shown in the Fig. 13.10.

Phase angle table for the compensated system :

Phas... ...ensated system :

ω	$\frac{1}{j\omega}$	$-\tan\omega$	$+\tan 0.3637\,\omega$	$-\tan 0.1763\,\omega$	ϕ
0.1	− 90°	− 5.71°	+ 2.08°	− 0.43°	− 94.06°
1	− 90°	− 45°	+ 20°	− 4.36°	− 119.36°
2	− 90°	− 63.4°	+ 36°	− 8.67°	− 126.07°
10	− 90°	− 84.2°	+ 74°	− 37.3°	− 137.5°
100	− 90°	− 89.4°	+ 88°	− 82.53°	− 173.9°

For magnitude plot, $K = 20$

$$\therefore \quad 20 \text{ Log } 20 = 26 \text{ dB}$$

One pole at origin, straight line of slope − 20 dB/dec.

$\omega_{c1} = 1$, slope becomes − 40 dB/dec due to simple pole.

$\omega_{c2} = \dfrac{1}{T} = 2.75$, slope becomes − 20 dB/dec due to simple zero.

$\omega_{c3} = \dfrac{1}{\alpha T} = 13.09$, slope becomes − 40 dB/dec due to simple pole.

From the Fig. 13.10, for compensated system

$$\omega_{gc} = 7 \text{ rad/sec}$$

$$\text{P.M.} = +50^\circ$$

$$\text{G.M.} = +\infty \text{ dB}$$

Thus the compensated system satisfies all the specifications.

13.5 Lag Compensator

...pensator is illustrated with an example.

Ex. 13.3 *Consider the unity feedback system whose open loop transfer function is,*

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

Design suitable lag-lead compensator so as to achieve,
static velocity error constant = 10 sec
Phase margin=50°
Gain margin ≥ 10 dB

Sol. : Let the transfer function of the compensator be,

$$G_c(s) = \frac{(1+T_1 s)(1+T_2 s)}{\left(1+\frac{T_1}{\beta}s\right)(1+\beta T_2 s)}$$

$$K_v = \lim_{s\to 0} s\, G_c(s)\, G(s)$$

$$\therefore \quad 10 = \lim_{s\to 0} \frac{s\,.\,(1+T_1 s)(1+T_2 s)}{\left(1+\frac{T_1}{\beta}s\right)(1+\beta T_2 s)} \cdot \frac{K}{s(s+1)(s+2)}$$

$$\therefore \quad 10 = \frac{K}{2}$$

$$\therefore \quad K = 20$$

The uncompensated system is,

$$G_1(s) = \frac{20}{s(s+1)(s+2)} = \frac{10}{s(s+1)(1+0.5s)}$$

The Bode plot of uncompensated system is shown in the Fig. 13.21. From it the vario[us] specifications can be obtained.

Factors : 20 Log 10 = 20 dB

1 pole at origin, – 20 dB/dec

$$\omega_{c1} = 1, \text{ simple pole, } -20 \text{ dB/dec}$$

$$\omega_{c2} = 2, \text{ simple pole, } -20 \text{ dB/dec}$$

$$G_1(j\omega) = \frac{10}{j\omega(1+j\omega)(1+0.5j\omega)}$$

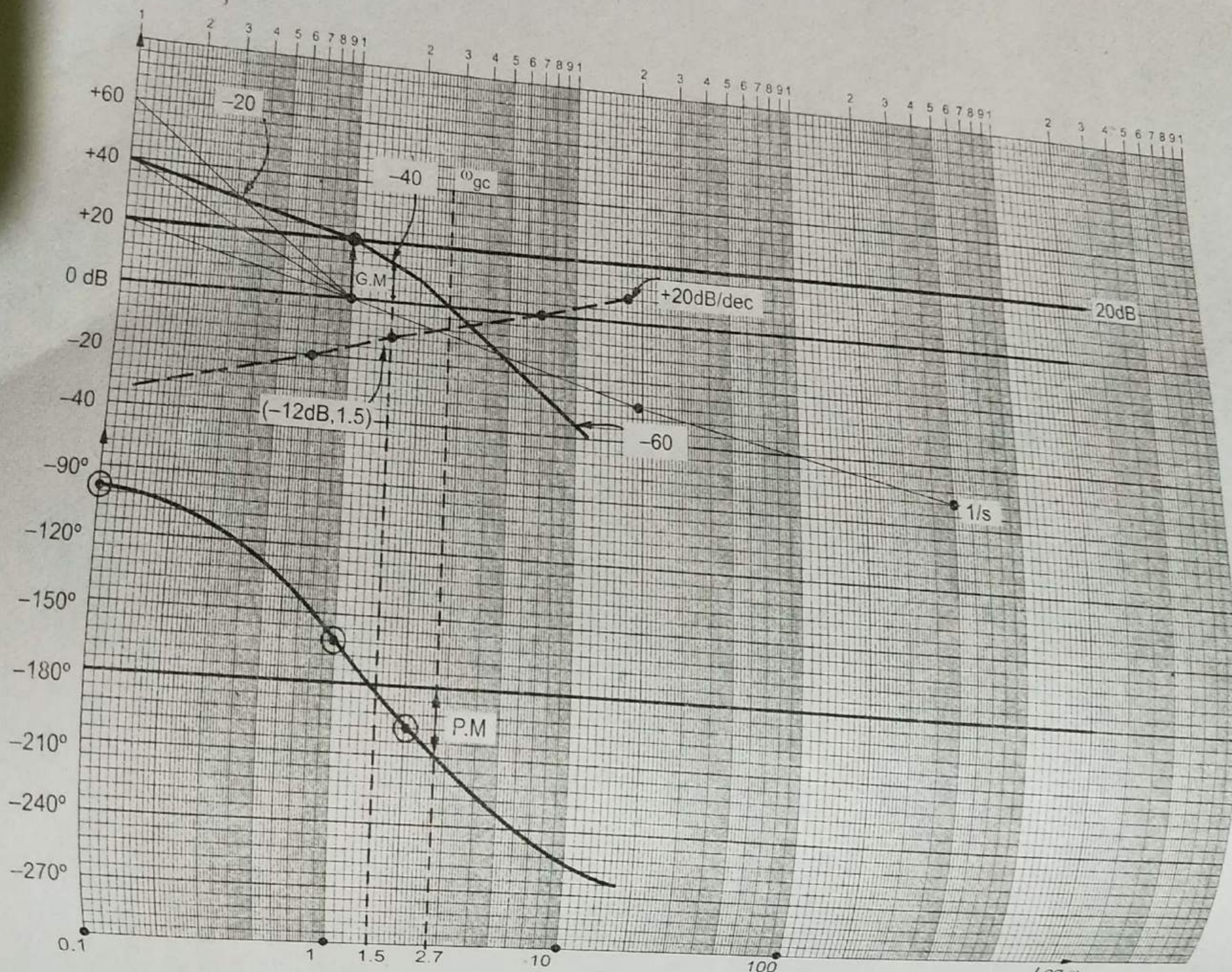


Fig. 13.21 Bode plot of uncompensated system in Ex.3.

Phase angle table :

ω	$\frac{1}{j\omega}$	$-\tan\omega$	$-\tan 0.5\omega$	ϕ
0.1	$-90°$	$-5.71°$	$-2.86°$	$-98.5°$
1	$-90°$	$-45°$	$-26.56°$	$-161.5°$
2	$-90°$	$-63.4°$	$-45°$	$-198.4°$

From Fig. 13.21, the various values are,

$$\omega_{gc} = 2.7 \text{ rad/sec}$$
$$\omega_{pc} = 1.5 \text{ rad/sec}$$
$$\text{G.M.} = -12 \text{ dB}$$
$$\text{P.M.} = -36°$$

The uncompensated system is unstable in nature.

For uncompensated system, $\omega_{pc} = 1.5$ i.e. $\angle G(j\omega)$ at $\omega = 1.5$ is $-180°$.

$$\text{P.M.} = -36°$$

The uncompensated system is unstable in nature.

For uncompensated system, $\omega_{pc} = 1.5$ i.e. $\angle G(j\omega)$ at $\omega = 1.5$ is $-180°$.

To have P.M. of 50°, it is necessary to add phase lead of 50° at $\omega = 1.5$ ar
$\omega = 1.5$ must be the new gain cross-over frequency.

Choose new gain crossover frequency as 1.5 rad/sec. The angle of ph
$+50°$.

$$\phi_m = 50°$$

Now
$$\sin\phi_m = \frac{1-\alpha}{1+\alpha} \quad \text{and} \quad \alpha\beta = 1$$

$$\therefore \quad \alpha = \frac{1}{\beta}$$

$$\therefore \quad \sin\phi_m = \frac{1-\dfrac{1}{\beta}}{1+\dfrac{1}{\beta}} = \frac{\beta-1}{\beta+1}$$

$$\therefore \quad 0.766 = \frac{\beta-1}{\beta+1}$$

$$\therefore \quad \beta+1 = 1.3054\,(\beta-1)$$

$$\therefore \quad \beta = 7.54$$

choosing $\beta = 10$ as practically minimum β is generally 10.

As the new gain cross-over frequency is 1.5, the corner frequency c
generally $\frac{1}{10}$th of the gain crossover frequency.

As T_2 is known, the another corner frequency is,

$$\text{Corner frequency } \omega = \frac{1}{\beta T_2} = 0.015 \text{ rad/sec}$$

Hence the transfer function of the phase lag portion is,

$$\frac{(1+T_2 s)}{(1+\beta T_2 s)} = \frac{(1+6.67\, s)}{(1+66.67\, s)}$$

The phase lead portion can be obtained as below :

From the Fig. 13.21, it can be observed that $|G(j\omega)|$ at $\omega = 1.5$ is $+12$ dB.

So to have $\omega = 1.5$ as new gain crossover frequency, the lag-lead compensator mu contribute -12 dB at $\omega = 1.5$ rad/sec. Thus draw a straight line from the point havin co-ordinates (1.5 rad/sec, -12 dB) of slope $+20$ dB/dec. Its intersection with 0 dB line is at rad/sec and its intersection with -20 dB line is at 0.7 rad/sec. These are the corne frequencies for the lead portion. Hence the transfer function of the lead portion is,

$$\frac{(1+T_1 s)}{\left(1+\frac{T_1}{\beta} s\right)} = \frac{(1+1.43\, s)}{(1+0.143\, s)}$$

as

$$T_1 = \frac{1}{\omega_{c_1}} = \frac{1}{0.7} = 1.43$$

and T_1 ... 1.43

as $\qquad \omega_{c_1} \qquad 0.7$

and
$$\frac{T_1}{\beta} = \frac{1.43}{10} = 0.143$$

Hence the transfer function of lag-lead compensator is,

$$G_c(s) = \frac{(1+1.43\,s)\,(1+6.67\,s)}{(1+0.143\,s)\,(1+66.67\,s)}$$

Thus the transfer function of the compensated system is,

$$G_c(s)\,G(s) = \frac{10(1+1.43\,s)\,(1+6.67\,s)}{s(1+0.143\,s)\,(1+66.67\,s)(1+s)\,(1+0.5s)}$$

Let us obtain the Bode plot of this compensated system and check the specifications

Factors : K = 10, 20 Log 10 = 20 dB

1 Pole at origin pole, – 20 dB/dec

ω_{c1} = 0.015, simple pole, – 20 dB/dec

ω_{c2} = 0.15, simple zero, + 20 dB/dec

ω_{c3} = 0.7, simple zero, + 20 dB/dec

ω_{c4} = 1, simple pole, – 20 dB/dec

ω_{c5} = 2, simple pole, – 20 dB/dec

ω_{c6} = 7, simple pole, – 20 dB/dec

$$G_c(j\omega)\,G(j\omega) = \frac{10(1+1.43\,j\omega)\,(1+6.67\,j\omega)}{j\omega(1+0.143\,j\omega)(1+66.67\,j\omega)(1+j\omega)\,(1+0.5j\omega)}$$

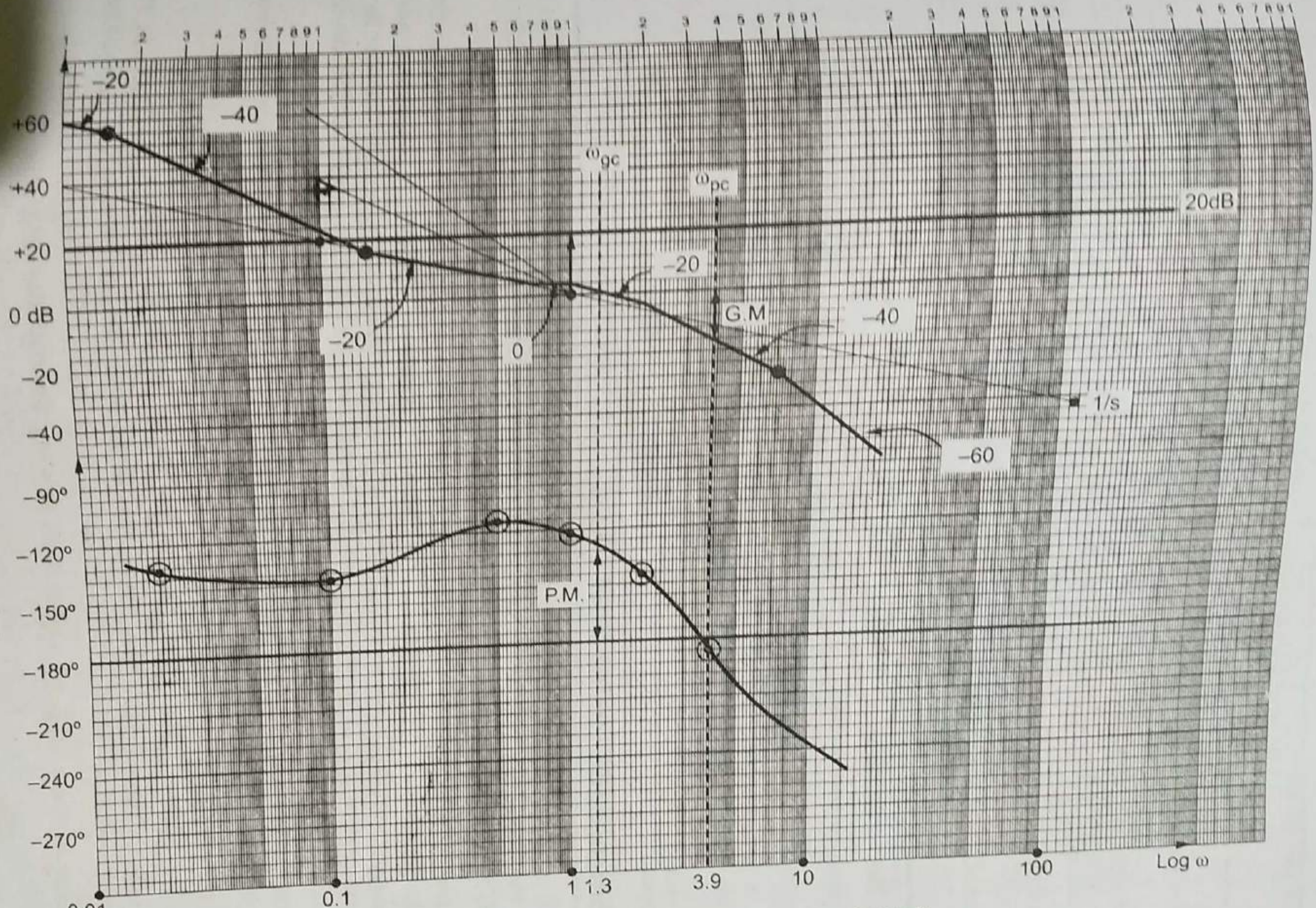


Fig. 13.22 Bode plot of compensated system in Ex.3

Phase angle table for the compensated system :

ω	$\frac{1}{j\omega}$	$-\tan^{-1}66.67\,\omega$	$+\tan^{-1}6.67\,\omega$	$+\tan^{-1}1.43\,\omega$	$-\tan^{-1}\omega$	$-\tan^{-1}0.5\,\omega$	$-\tan^{-1}0.143\,\omega$	ϕ_R
0.02	$-90°$	$-58.13°$	$+7.6°$	$+1.63°$	$-1.1°$	$-0.57°$	$-0.16°$	$-135.7°$
0.1	$-90°$	$-81.46°$	$+33.7°$	$+8.13°$	$-5.7°$	$-2.86°$	$-0.81°$	$-139°$
0.5	$-90°$	$-88.2°$	$+73.3°$	$+35.5°$	$-26.56°$	$-14.03°$	$-4.08°$	$-114.1°$
1	$-90°$	$-89.1°$	$+81.47°$	$+55°$	$-45°$	$-26.56°$	$-8.13°$	$-122.3°$
2	$-90°$	$-89.5°$	$+85.7°$	$+70.1°$	$-63.4°$	$-45°$	$-15.96°$	$-148°$
4	$-90°$	$-89.7°$	$+87.8°$	$+80°$	$-75.9°$	$-63.4°$	$-29.8°$	$-181°$

The Bode plot of compensated system is shown in the Fig. 13.22.

The specifications are

$$\omega_{gc} = 1.3 \text{ rad/sec}$$
$$\omega_{pc} = 3.9 \text{ rad/sec}$$
$$\text{G.M.} = +16 \text{ dB}$$
$$\text{P.M.} = +50°$$

Thus compensated system satisfies the specifications.

13.4.4 Steps to Design Lead Compensator

Step 1 : At zero frequency the lead compensator has gain α. But as $\alpha < 1$ i provides an attenuation. To cancel this attenuation, the practical lead compensator i realised with an amplifier having gain K_C in series with basic lead network. Hence th practical transfer function of a lead compensator from the design point of view i assumed to be,

$$G_C(s) = K_c \alpha \frac{(1+Ts)}{(1+\alpha Ts)}$$

where $K_c \alpha$ = d.c. gain = K ...(7)

$$\therefore \quad G_c(s) = \frac{K(1+Ts)}{(1+\alpha Ts)} \quad ...(8)$$

The open loop transfer function of the compensated system thus becomes,

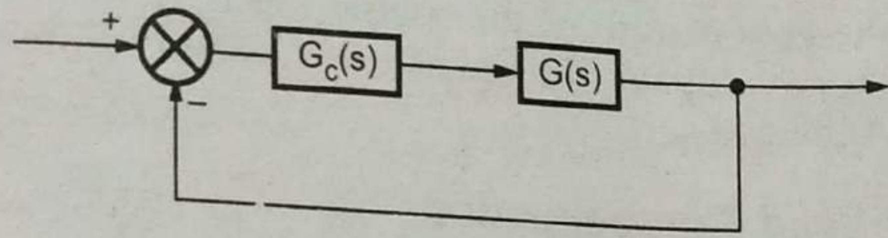


Fig. 13.8

$$G_C(s)\,G(s) = \frac{K(1+Ts)}{(1+\alpha Ts)}\cdot G(s) = \frac{(1+Ts)}{(1+\alpha Ts)}\cdot K\,G(s) = \frac{(1+Ts)}{(1+\alpha Ts)}\,G_1(s)$$

where $G_1(s) = KG(s)$

Generally in such design problems one of the error constant is given as specification. From the above result, determine the value of K satisfying the given error constant.

Step 2 : Using the value of K determined above, draw the Bode plot of $G_1(j\omega)$. This is the Bode plot of, gain adjusted but uncompensated system. Obtain the phase margin.

Step 3 : Generally P.M. is specified for the design problem.

Let ϕ_s = P.M. specified

ϕ_1 = P.M. obtained in the step 2

Determine necessary phase lead ϕ_m required to be added. For this use the relation,

$$\phi_m = \phi_s - \phi_1 + \varepsilon$$

where ε = margin of safety as cross-over frequency may shift due to compensation

$= 5°$ to $15°$

$= 5$ to 15°

Step 4 : Using the equation,

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

determine the value of α.

Step 5 : Determine the frequency ω_m at which the magnitude of the uncompensated system is $-10 \text{ Log} \left(\frac{1}{\alpha}\right)$ dB. Select this frequency as new gain crossover frequency. This frequency ω_m is,

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

as α is known, determine $\frac{1}{T}$.

Step 6 : Determine the two frequencies of the lead compensator.

$$\omega_{C1} = \frac{1}{T} \text{ and } \omega_{C2} = \frac{1}{\alpha T}$$

Step 7 : As $K = K_C \alpha$,

determine the value of K_C.

13.5.4 Steps to Design Lag Compensator

Step 1 : Assume a lag compensator having a transfer function,

$$G_C(s) = \frac{1}{\beta}\frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\beta T}\right)} = \frac{(1+Ts)}{(1+\beta Ts)}$$

Assume $G_1(s) = K\,G(s)$

From the given error constant, determine the value of K which satisfies the steady state performance.

Step 2 : Using the value of K determined above, draw the Bode plot of $G_1(j\omega)$. Obtain the phase margin. This is say ϕ_1.

Step 3 : Generally phase margin is specified.

Let ϕ_s = P.M. specified

Then determine, $\phi_2 = \phi_s + \varepsilon$

where ε = margin of safety = 5° to 15°.

The ε compensates for the phase lag of the lag compensator.

Step 4 : Find the frequency ω_2 corresponding to phase margin of ϕ_2 degrees i.e. the frequency at which phase angle of open loop transfer function is $-180° + \phi_2$. Choose this as new gain crossover frequency.

Step 5 : To have ω_2 as the new gain crossover frequency, determine th attenuation necessary to shift the magnitude curve up or down to 0 dB. This shift i due to the contribution of β which is $20 \text{ Log } \frac{1}{\beta}$.

$$\therefore \text{ Shift to have } \omega_2 \text{ as new gain crossover} = 20 \text{ Log } \frac{1}{\beta} = -20 \text{ Log } \beta$$

Down shift must be taken negative while up shift positive. Hence determine the value o β.

Step 6 : Choose the upper corner frequency $\frac{'1'}{T}$ which is $\frac{1}{2}$ or $\frac{1}{10}$ below the determined in step 4.

$$\therefore \qquad \omega_{c2} = \frac{1}{T} = \frac{\omega_2}{2} \text{ or } \frac{\omega_2}{10}$$

Thus determine the value of T.

The other corner frequency for the lag compensator is,

$$\omega_{c1} = \frac{1}{\beta T}$$

Step 7 : Thus once transfer function of the lag compensator is known, draw the Bode plot of compensated system and check the specifications. If specifications are no

Design of Lead Compensator using Root Locus :→

Step 1 From the given specifications find the desired locations of the dominant closed loop poles

2. Assume the lead compensator as

$$G_c(s) = K_c \alpha \frac{(1+Ts)}{(1+\alpha Ts)}, \quad \alpha < 1$$

K_c is determined from the requirement of open loop gain.

3. Find the sum of the angles at the desired locations of one of the dominant closed loop poles with the open loop poles and zeros of original system. This angle must be odd multiple of 180°. If it is not calculate necessary angle ϕ to be added to get the sum as an odd multiples of 180°

this ϕ must be contributed by lead compensator. If

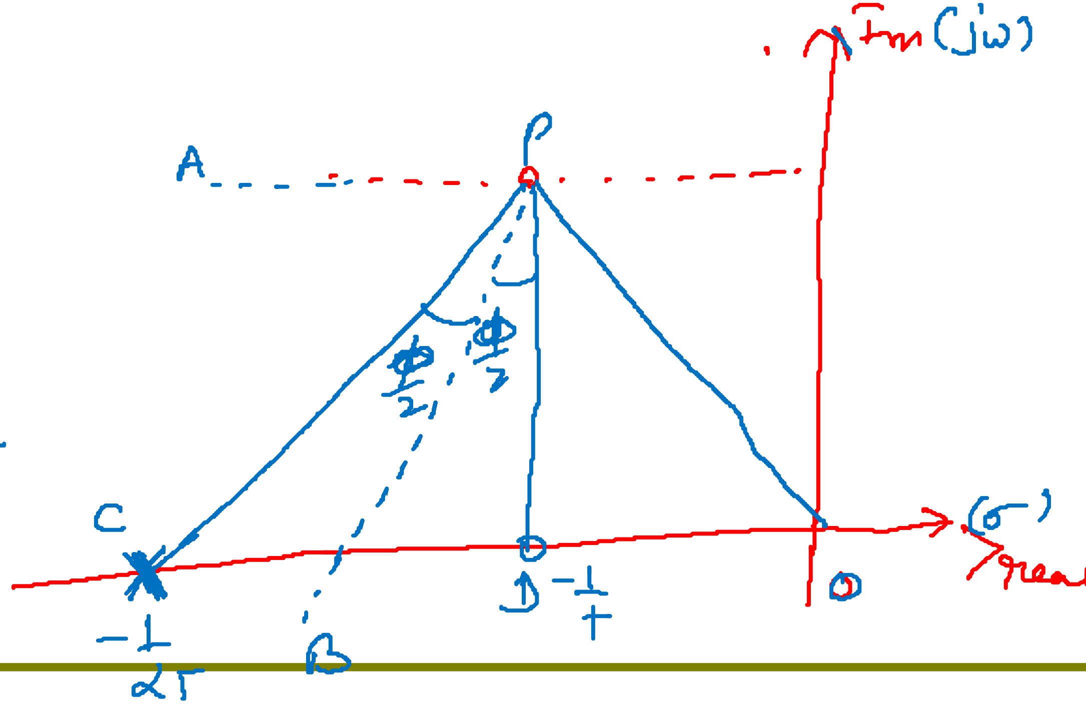
ϕ is more than 60° then two or more lead networks required.. the ϕ help to determine value of α and T.

(5) To determine α and T for known ϕ, draw, the horizontal line from one of the closed loop dominant pole say 'P'

join origin to P

Bisect the angle between line PA and PO

Draw the two line PC and PD, that makes angle $\pm\frac{\phi}{2}$ with the bisector PB.



The intersection of PC and PD with the negative real axis gives the necessary pole and zero compensator

(5) The open loop gain can be determined by applying the magnitude conditions at point 'P'.

(6) check that the compensated system satisfies all the specifications. If not, adjust the pole zero of compensator till specification is satisfied.

Q Design a suitable lead compensator for a system with unity feedback and having open loop TF

$$G(s) = \frac{K}{s(s+1)(s+4)}$$

to meet the specifications

① Damping ratio $\xi = 0.5$
② undamped natural freq (ω_n) = 2 rad/sec

Sol Sketch RL for uncompensated system

Here Poles = 3 $s = 0, -1, -4,$
Zeros = 0 —

$P - Z \Rightarrow 3 - 0 \Rightarrow 3$ branches lead to ∞

Starting of RL at $s = 0, -1, -4,$ and

Terminating at ∞, ∞, ∞

Angle of asymptote $\phi_A = \dfrac{\pm(2q+1)\pi}{P-Z} = \dfrac{(2q+1)\pi}{3}$; $q = 0, 1, 2,$

Centroid $(-\sigma) = \dfrac{0-1-4}{3} = -1.67.$

Break Point $\dfrac{dK}{ds} = 0,$

$$1 + G(s)H(s) = 0,$$

$$1 + \frac{K}{s(s+1)(s+4)} = 0,$$

$$s^3 + 5s^2 + 4s + K = 0$$

$$K = -s^3 - 5s^2 - 4s$$

$$\frac{dK}{ds} = -3s^2 - 10s - 4 = 0$$

$$s = -0.464, -2.86$$

The Point -0.464 is valid Point.

Intersection Point

$$s^3+5s^2+4s+K=0$$

s^3	1	4
s^2	5	K
s^1	$\frac{20-K}{5}$	
s^0	K	

Auxiliary eqn

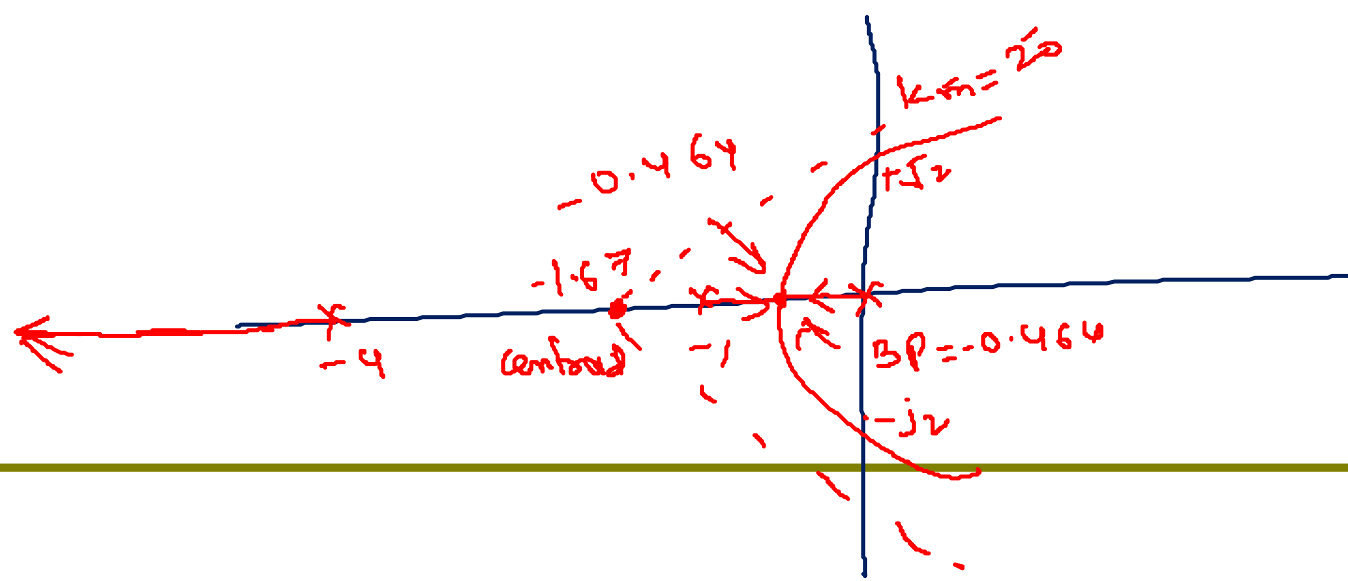
$\Rightarrow 5s^2+K=0$ or $5s^2+20=0$

$\Rightarrow \boxed{K_m < 20}$

$s^2=-4$

$s=\sqrt{(-4)}=\pm j2$

$\boxed{s=\pm j2}$



Step 1 $\xi = 0.5$ and $\omega_n = 2$

The desired dominant Poles are at

$$-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$s_1, s_2 = -1 \pm j1.73$$

It can be seen that dominant Poles are not at Root Locus

Step 2 Assume Compensation NW

$$G_c(s) = K_c\alpha\frac{(1+Ts)}{(1+\alpha Ts)} \quad \text{—— (1)}$$

Step 3 $\angle G(s)H(s)$ at dominant Poles

$$\angle \frac{K}{s(s+1)(s+4)}\Bigg|_{s=-1\pm j1.73} \quad \text{is}$$

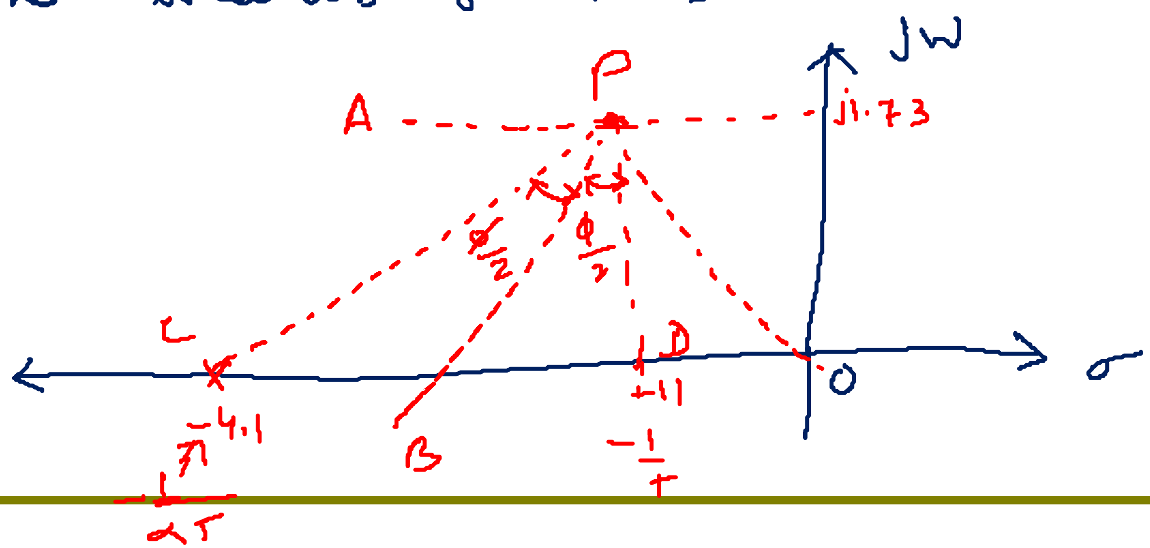
$$\phi = \frac{\angle K}{\angle(-1+j1.73)\,\angle(-1+j1.73+1)\,\angle(-1+j1.73+4)}$$

$$\phi = \frac{\angle K}{\angle(-1+j1.73)\,\angle j(1.73)\,\angle 3+j1.73} = \frac{0^\circ}{120^\circ \cdot 90^\circ \cdot 29.97^\circ} = -240^\circ$$

Angle to be contributed by lead angle is

$$-180^\circ - (-240^\circ) = 60^\circ$$

<u>Step 4:</u> Find location of Poles and zero.



- as To determine α and T for know Φ, draw the Horizontal from one of dominant Pole $(-1+j1.73)$ say Point 'P'

join origin to 'P'. Bisect the angle between PA and PO. Draw the two line PC and PD that make angle $\pm\frac{\Phi}{2}$ with PB

the intersection of PC and PD with negative real axis.

$\therefore$ zero at $S=-1$ will cancel the Pole at $S=-1$ So, we select Pole at $\underline{S=-1.2}$ (Slightly away from $S=-1$)

and Pole at $S=-4.1$ which is very close to $S=-4$, Hence select at $\underline{S=-4.6}$

So,

$$\frac{1}{T} = 1.2 \Rightarrow T = 0.833$$

$$\frac{1}{\alpha T} = 4.6 \Rightarrow \alpha T = 0.2173, \alpha = 0.261 < 1$$

The compensated System T F

$$G_c(s)\, G(s) = \frac{K(1+0.833s)}{s(s+1)(s+4)(1+0.217s)}$$

Step 5 use magnitude condition find K value at $s=-1+j1.73$

$$\left| G_c(s)\,G(s) \right|_{s=-1+j1.73} = 1$$

$$\frac{K\,|1+0.833(-1+j1.73)|}{|(-1+j1.73)|\,|j1.73|\,|3+j1.73|\,|1+0.2173(-1+j1.73)|} = 1$$

$$\frac{K(1.449)}{1.998 \times 1.73 \times 3.46 \times 0.8682} = 1$$

$$\boxed{K = 7.166}$$

$$G_c(s)G(s) = \frac{7.166\,(1+0.8333s)}{s(1+s)(s+4)(1+0.2173s)}$$

$$G_c(s)G(s) = \frac{27.47\,(s+1.2)}{s(s+1)(s+4)(s+4.6)}$$

Compensated Root Locus