Unit - 04 Laplace Transforms & Z- Transforms

Laplace Transforms (is the extension of fourier transform)

w.k.t., $F.T[f(t)] = F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$ is the fourier transform representation of function $f(t)$

→ Existense conditions of F.T (i.e., dirichlet's conditions) :-

(i) $\int_{-\infty}^{\infty} |f(t)| dt < \infty$ = absolutely integrable.

functions like $u(t)$, $\delta(t)$, $sgn(t)$, $\sin\omega t$ are not absolutely integrable.

∴ for this functions i.e, functions which are not absolutely integrable cannot be analyzed by fourier transforms.

But, conditions are assumed i.e., an indirect method is used for analyzing the functions.

This is the main disadvantage of F.T ie, some of the functions which are not absolutely integrable cannot be analyzed by F.T.

To overcome this problem, Laplace transforms are used.

→ Laplace transform

Let $x(t)$ be a signal whose laplace transform is $X(s)$

i.e., $x(t) \xleftrightarrow{L.T} X(s)$

$$\Rightarrow \boxed{X(s) = LT[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-st} dt} \rightarrow (1)$$

where, 's' is a complex variable which is defined as, $s = \underbrace{\sigma}_{\text{Real part of the 's'}} + \underbrace{j\omega}_{\text{imaginary part of the 's'}}$

W.K.T, $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = F.T[x(t)] \rightarrow (2)$

From (1) and (2) we can say,

if σ = R.P of $\{s\} = 0$, then it is known as F.T

→ Let us have, $s = (\sigma + j\omega)$ be a complex variable

then $X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t) e^{-(\sigma + j\omega)t} dt$

$$X(s) = \int_{-\infty}^{\infty} \underbrace{x(t) e^{-\sigma t}} e^{-j\omega t} dt$$

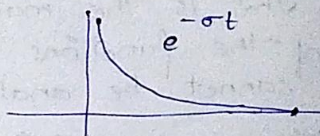
$$X(s) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = F.T$$

$x(t)$ is a function which is absolutely not integrable

$e^{-\sigma t}$ is a real exponent.

if $\sigma > 0$, $e^{-\sigma t}$ is a decaying exponential

$\Rightarrow \int_{-\infty}^{\infty} |x(t)| dt = \infty \times 0 = 0$



$\therefore$ $x(t) e^{-\sigma t}$ is absolutely integrable.

$\therefore$ F.T is converging.

$$\boxed{X(\omega) = F.T[x(t)] = X(s)\Big|_{\sigma = 0}}$$

If $x(t)$ is a multiplied by a real exponential with decreasing amplitude $\Rightarrow \int_{-\infty}^{\infty} |x(t) e^{-\sigma t}| dt < \infty$

$\therefore$ Most of the functions can be analyzed by Laplace transform.

→ By using F.T / L.T, we can analyse CTS.
i.e., one of the application of laplace transform is, it is used analyse continuous time system.

Q) Problems on L.T

* Let $x(t) = e^{-at}u(t)$ then find $L.T[x(t)] = X(s)$

w.k.t., $X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$

$$= \int_{-\infty}^{\infty} e^{-at} u(t) e^{-st} dt \qquad u(t) = \begin{cases} 1, & \text{for } t>0 \\ 0, & \text{for } t<0 \end{cases}$$

$$= \int_{0}^{\infty} e^{-(a+s)t} dt$$

$$= \left[\frac{e^{-(a+s)t}}{-(a+s)} \right]_0^{\infty} = \frac{-1}{(s+a)} \left[e^{-\infty} - e^{0} \right]$$

$$\boxed{X(s) = \frac{1}{(s+a)}}$$

i.e., $$\boxed{e^{-at}u(t) \xleftrightarrow{L.T} \frac{1}{(s+a)}}$$

Let us consider, $e^{-(s+a)t}$

for $t=\infty$, if $s+a<0$, then $e^{-(s+a)t} = \infty$

$\therefore$ $s+a$ must be >0

i.e., $s > -a$

$$e^{-(s+a)t} = e^{-\infty} = 0$$

i.e., $\int \left| e^{-at} e^{-\sigma t} \right| \cancel{\left| e^{-j\omega t} \right|} dt < \infty$

($e^{-j\omega t}$ has no role $\because |e^{-j\omega t}| = 1$)

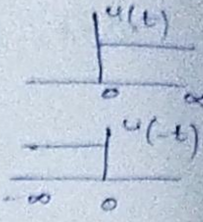
if $Re\{s\} > -a$, then only the integral value is converging.

$$\therefore \quad e^{-at}u(t) \xleftrightarrow{L.T} \frac{1}{s+a} \quad \text{is valid for } s > -a \ \text{(or)} \ s+a > 0$$

Q. $x(t) = -e^{-at}u(t)$, find $X(s) = ?$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} -e^{-at}\, u(t)\, e^{-st}\, dt$$

$$= -\int_{-\infty}^{0} e^{-at}\, e^{-st}\, dt$$



$$= -\int_{-\infty}^{0} e^{-(a+s)t}\, dt$$

$$= + \left[\frac{e^{-(a+s)t}}{(a+s)}\right]_{-\infty}^{0}$$

$$= \frac{1}{(s+a)}\left[0 - \infty\right] = \infty \quad \text{for } s+a>0$$

$$= \frac{1}{(s+a)} \quad \text{for } s+a<0 \ \text{(or)} \ s<-a$$

$$\therefore -e^{-at}u(-t) \xleftrightarrow{L.T} \frac{1}{(s+a)} \quad \text{is valid for } s<-a \ \text{(or)} \ s+a<0$$

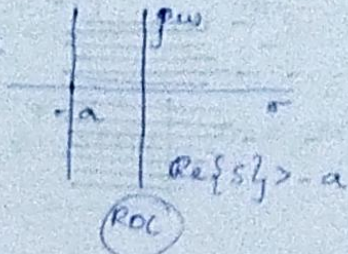
(i) $e^{-at}u(t) \longleftrightarrow \frac{1}{s+a}$ for $Re\{s\} > -a$

(ii) $-e^{-at}u(-t) \longleftrightarrow \frac{1}{s+a}$ for $Re\{s\} < -a$

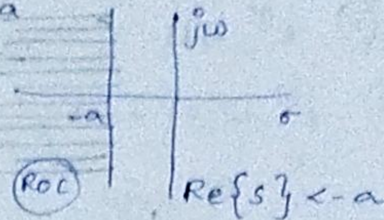
The values for which the L.T ~~at~~, converges is known as region of convergence.

Region of convergence (ROC):- The set of values of 's' for which the L.T converges is known as ROC.

$e^{-at}u(t) \xleftrightarrow{L.T} \frac{1}{s+a}$, $Re\{s\} > -a$



$-e^{-at}u(-t) \xleftrightarrow{L.T} \frac{1}{s+a}$, $Re\{s\} < -a$



(Q) Find the L.T of $3e^{-2t}u(t) - 2e^{-t}u(t)$

let $x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

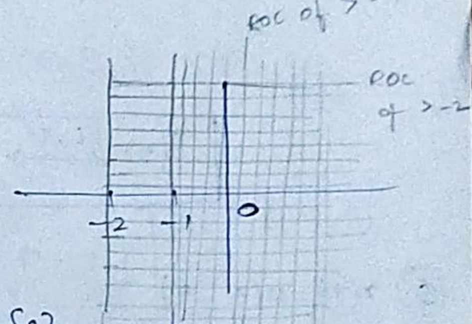
$$= \int_{-\infty}^{\infty} \left(3e^{-2t} - 2e^{-t}\right) u(t)\, e^{-st}\, dt$$

$$= 3\int_{0}^{\infty} e^{-(s+2)t}\, dt - 2\int_{0}^{\infty} e^{-(s+1)t}\, dt$$

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}$$

for $\frac{3}{s+2}$, ROC is $Re\{s\} > -2$

for $\frac{2}{s+1}$, ROC is $Re\{s\} > -1$



The ROC of $X(s)$ will be intersection of $Re\{s\} > -2$ and $Re\{s\} > -1$ (i.e., common part)

$\therefore X(s) = \frac{3}{s+2} - \frac{2}{s+1}$ for $Re\{s\} > -1$

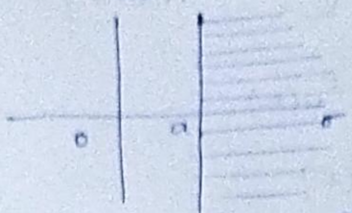
$\left[\because Re\{s\} > -2 \cap Re\{s\} > -1 = Re\{s\} > -1\right.$

W.K.T, $e^{-at}u(t) \xleftrightarrow{L.T} \frac{1}{s+a}$

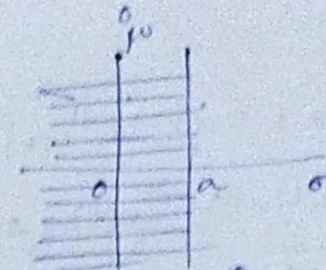
for $a=0$, $\boxed{u(t) \xleftrightarrow{L.T} \frac{1}{s}}$ for $Re\{s\} > 0$

Illy (i) $e^{at}u(t) \longleftrightarrow \frac{1}{s-a}$, ROC = $Re\{s\} > a$

(ii) $-e^{at}u(-t) \longleftrightarrow \frac{1}{s-a}$, ROC = $Re\{s\} < a$



ROC of (i)



ROC of (ii)

Q Find the L.T of an impulse function $\delta(t)$

w.k.t., $\delta(t) = \begin{cases} 1, & \cancel{t \neq 0}\ t=0 \\ 0, & t \neq 0 \end{cases}$

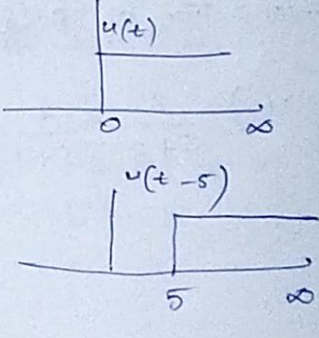
$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = \int_{-\infty}^{\infty} \delta(t) e^{-st} dt$$

$$= e^{-st}\Big|_{t=0} = 1$$

$$\boxed{\delta(t) \xleftrightarrow{L.T} 1.}$$

Q $x(t) = u(t-5)$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st} dt$$



$$= \int_{5}^{\infty} e^{-st} dt$$

$$= \left(\frac{e^{-st}}{-s} \right)_{5}^{\infty}$$

$$= -\frac{1}{s}\left(0 - e^{-5s}\right)$$

$$x(s) = \frac{e^{-5s}}{s}$$

$$\boxed{u(t-5) \xleftrightarrow{L.T} \frac{1}{s}e^{-5s}}$$

Q) $x(t) = \sin\omega_0 t \, u(t)$, $X(s) = ?$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt \qquad u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$= \int_{-\infty}^{\infty} \sin\omega_0 t \, u(t) e^{-st}\,dt$$

Replace $\sin\omega_0 t = \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$

$$X(s) = \int_0^{\infty}\left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right)e^{-st}\,dt$$

$$= \frac{1}{2j}\left[\int_0^{\infty} e^{-(-j\omega_0 + s)t}\,dt - \int_0^{\infty} e^{-(s+j\omega_0)t}\,dt\right]$$

$$= \frac{1}{2j}\left[\left(\frac{e^{-(s-j\omega_0)t}}{-(s-j\omega_0)}\right)_0^{\infty} - \left[\frac{e^{-(s+j\omega_0)t}}{-(s+j\omega_0)}\right]_0^{\infty}\right]$$

$$= \frac{1}{2j}\left[\left(\frac{-1}{(s-j\omega_0)}\left[0-1\right]\right) + \frac{1}{s+j\omega_0}\left(0-1\right)\right]$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega_0} - \frac{1}{s+j\omega_0}\right]$$

$$= \frac{1}{2j}\left[\frac{\cancel{s} + j\omega_0 - \cancel{s} + j\omega_0}{(s-j\omega_0)(s+j\omega_0)}\right] = \frac{1}{\cancel{2j}}\left[\frac{\cancel{2}j\omega_0}{s^2 - (j^2\omega_0^2)}\right]$$

$$X(s) = \frac{\omega_0}{s^2+\omega_0^2}$$

$$\boxed{\sin\omega_0 t\, u(t) \xleftrightarrow{L.T} \frac{\omega_0}{s^2+\omega_0^2}}$$ for $Re\{s\} > 0$

(9) $x(t) = \cos\omega_0 t\, u(t)$, $X(s) = ?$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} \cos\omega_0 t\, u(t)\, e^{-st}\, dt.$$

Replace $\cos\omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$$= \frac{1}{2}\int_0^{\infty} \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) e^{-st}\, dt$$

$$= \frac{1}{2}\left[\int_0^{\infty} e^{-(s-j\omega_0)t}\, dt + \int_0^{\infty} e^{-(s+j\omega_0)t}\, dt\right]$$

$$= \frac{1}{2}\left[\left(\frac{e^{-(s-j\omega_0)}}{-(s-j\omega_0)}\right)_0^{\infty} + \left(\frac{e^{-(s+j\omega_0)}}{-(s+j\omega_0)}\right)_0^{\infty}\right]$$

$$= \frac{1}{2}\left[\frac{-1}{(s-j\omega_0)}(0-1) - \frac{1}{(s+j\omega_0)}(0-1)\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right]$$

$$= \frac{1}{2}\left(\frac{s+\cancel{j\omega_0}+s-\cancel{j\omega_0}}{s^2-(j\omega_0)^2}\right) = \frac{s}{s^2+\omega_0^2}$$

$$\boxed{\cos\omega_0 t\, u(t) \xleftrightarrow{L.T} \frac{s}{s^2+\omega_0^2}}$$

(8) $x(t) = e^{-at}\cos\omega_0 t\, u(t)$

$$x(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt \qquad u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$= \int_{-\infty}^{\infty} e^{-at}\cos\omega_0 t\, u(t)\, e^{-st}\, dt$$

Replace $\cos\omega_0 t$ by $\dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$$x(s) = \int_0^{\infty} e^{-(a+s)t} \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right) dt$$

$$= \frac{1}{2}\left[\int_0^{\infty} e^{-(s+a-j\omega_0)t}\, dt + \int_0^{\infty} e^{-(s+a+j\omega_0)t}\, dt \right]$$

$$= \frac{1}{2}\left[\left(\frac{e^{-(s+a-j\omega_0)}}{-(s+a-j\omega_0)} \right)_0^{\infty} + \left(\frac{e^{-(s+a+j\omega_0)}}{-(s+a+j\omega_0)} \right)_0^{\infty} \right]$$

$$= \frac{1}{2}\left[\frac{+1}{(s+a-j\omega_0)} + \frac{1}{(s+a+j\omega_0)} \right]$$

$$= \frac{1}{2}\left[\frac{s+a+\cancel{j\omega_0} + s+a-\cancel{j\omega_0}}{(s+a)^2 - (j\omega_0)^2} \right]$$

$$x(s) = \frac{s+a}{(s+a)^2 + \omega_0^2}$$

$$\boxed{e^{-at}\cos\omega_0 t\, u(t) \xleftrightarrow{L.T} \frac{s+a}{(s+a)^2 + \omega_0^2}, \quad Re\{s\} \geq a}$$

(9) $x(t) = e^{at}\cos\omega_0 t\, u(t)$, $x(s) = ?$

$$\boxed{e^{at}\cos\omega_0 t\, u(t) \xleftrightarrow{L.T} \frac{s-a}{(s-a)^2 + \omega_0^2}} \quad Re\{s\} > a$$

$$\text{lly} \quad \boxed{\begin{array}{l} e^{-at}\sin\omega_0 t\, u(t) \xleftrightarrow{L.T} \dfrac{\omega_0}{(s+a)^2+\omega_0^2},\ Re\{s\} > -a \\ e^{at}\sin\omega_0 t\, u(t) \xleftrightarrow{L.T} \dfrac{\omega_0}{(s-a)^2+\omega_0^2},\ Re\{s\} > a \end{array}}$$

Q) $x(t) = \delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$, $X(s) = ?$

$$X(s) = \underset{Re(s)>0}{\underset{\downarrow}{1}} - \frac{4}{3}\underset{Re\{s\}>-1}{\underset{\downarrow}{\left(\frac{1}{s+1}\right)}} + \frac{1}{3}\underset{Re\{s\}>2}{\underset{\downarrow}{\left(\frac{1}{s-2}\right)}}$$

$$X(s) = 1 - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2}, \quad Re\{s\} > 2$$

Q) $x(t) = t\, u(t)$

ILATE

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt = \int_0^{\infty} t\, e^{-st}\,dt$$

$$= \left(t\frac{e^{-st}}{-s}\right)_0^{\infty} - \int_0^{\infty}\frac{e^{-st}}{-s}\,dt$$

$$= \cancel{\frac{t}{s}(0-1)} + \frac{1}{s}\left(\frac{e^{-st}}{-s}\right)_0^{\infty}$$

$$= \cancel{\frac{t}{s}} - \frac{1}{s^2}(0-1)$$

$$= \cancel{\frac{t}{s}} + \frac{1}{s^2}$$

$$\boxed{t\, u(t) \xleftrightarrow{L.T} \frac{1}{s^2}}$$

Q) $x(t) = t^2 u(t)$

$$X(s) = \int_{-\infty}^{\infty} t^2 u(t)\, e^{-st}\,dt$$

$$= \int_0^{\infty} t^2 e^{-st}\,dt$$

$$= \left(\cancel{t^2 \frac{e^{-st}}{-s}}\right)_0^{\infty} - \int_0^{\infty} 2t \frac{e^{-st}}{-s} dt$$

$$= \frac{2}{s}\left[\left(\cancel{t \frac{e^{-st}}{-s}}\right)_0^{\infty} - \int_0^{\infty} 1 \frac{e^{-st}}{-s}\right]$$

$$= \frac{2}{s}\left[\frac{1}{s}\left(\frac{e^{-st}}{-s}\right)_0^{\infty}\right]$$

$$= -\frac{2}{s^3}(0-1) = \frac{2}{s^3}$$

$$\boxed{t^2 u(t) \xleftrightarrow{L.T} \frac{2}{s^3}}$$

9) $x(t) = t\, e^{at} u(t)$

$$X(s) = \int_{-\infty}^{\infty} t\, e^{at} u(t)\, e^{-st}\, dt.$$

$$= \int_0^{\infty} t\, e^{-(s-a)t}\, dt$$

$$= \left(t \frac{e^{-(s-a)t}}{-(s-a)}\right)_0^{\infty} - \int_0^{\infty} \frac{e^{-(s-a)t}}{-(s-a)} dt$$

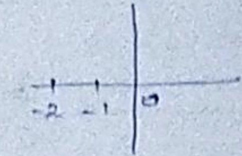
$$= \frac{1}{s-a} \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^{\infty}$$

$$= \frac{-1}{(s-a)^2}(0-1) = \frac{1}{(s-a)^2}$$

$$\boxed{t\, e^{at} u(t) \xleftrightarrow{L.T} \frac{1}{(s-a)^2}}$$

Q $x(t) = e^{-2t}u(t) + e^{-t}\cos 3t\, u(t)$

$$X(s) = \underbrace{\frac{1}{s+2}}_{1} + \underbrace{\frac{(s+1)}{(s+1)^2+(3)^2}}_{2}$$



1: $Re\{s\} > -2$ 2: $Re\{s\} > -1$

$$X(s) = \frac{1}{s+2} + \frac{(s+1)}{(s+1)^2+9}, \quad Re\{s\} > -1$$

Q $x(t) = \sinh\omega_0 t\, u(t)$, $X(s) = ?$

$$\sinh\omega_0 t = \frac{e^{\omega_0 t} - e^{-\omega_0 t}}{2}$$

$$\boxed{\sinh\omega_0 t\, u(t) \xleftrightarrow{L.T} \frac{\omega_0}{s^2-\omega_0^2}}$$

Q $x(t) = \sin at \cos bt\, u(t)$, $X(s) = ?$

$$X(s) = \int_0^\infty \sin at \cos bt\, e^{-st}\, dt$$

$$= \frac{1}{2}\int_0^\infty \left(\sin(a+b)t + \sin(a-b)t\right) e^{-st}\, dt$$

$$\cancel{= \frac{1}{2}\int_0^\infty e} = \frac{1}{2}\left[\frac{(a+b)}{s^2+(a+b)^2} + \frac{(a-b)}{s^2+(a-b)^2}\right]$$

Q $x(t) = \cos^3 t$

W.k.t., $\cos 3t = 4\cos^3 t - 3\cos t$

$$\Rightarrow \cos^3 t = \frac{1}{4}(\cos 3t + 3\cos t)$$

$$X(s) = \frac{1}{4}\frac{s}{s^2+3^2} + \frac{3}{4}\frac{s}{s^2+1}$$

Q) $x(t) = 4 - 6t + 3t^2 + t^3, \quad X(s) = ?$

$$X(s) = \frac{4}{s} - \frac{6}{s^2} + 3 \cdot \frac{2}{s^3} + \frac{3!}{s^4}$$

$$X(s) = \frac{4}{s} - \frac{6}{s^2} + \frac{6}{s^3} + \frac{6}{s^4}$$

Q) Find the L.T of $x(t) = e^{-b|t|}$

$$x(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

w.r.t, if $t>0$, $|t| = t$
if $t<0$, $|t| = -t$

$$= \int_{-\infty}^{\infty} e^{-b|t|}\, e^{-st}\, dt$$

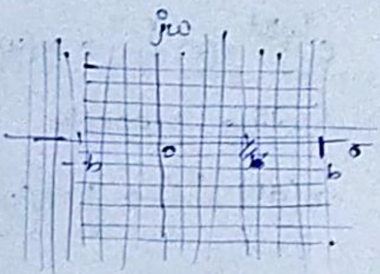
$$= \int_{-\infty}^{\infty} e^{bt} u(t) e^{-st}\, dt + \int_{-\infty}^{\infty} e^{-bt} \cdot \underset{u(t)}{e^{-st}}\, dt$$

$$\left[\because e^{-b|t|} = e^{-bt} u(t) + e^{bt} u(-t)\right]$$

$$= \int_{-\infty}^{0} e^{-(s-b)t}\, dt + \int_{0}^{\infty} e^{-(s+b)t}\, dt$$

$$x(s) = -\frac{1}{s-b} + \frac{1}{s+b}$$

$\operatorname{Re}\{s\} < b$ (R_1) $\quad \operatorname{Re}\{s\} > -b$ (R_2)



$$R = R_1 \cap R_2$$

$$\therefore X(s) = \frac{1}{s+b} - \frac{1}{s-b}, \quad \text{for } -b < \operatorname{Re}\{s\} < b$$

Properties of laplace transforms :-

→ $x(t) \xleftrightarrow{L.T} X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$ is known as bilateral L.T or two sided L.T ($-\infty$ to $+\infty$)

$X(s) = \int_{0}^{\infty} x(t) e^{-st} dt$ is known as unilateral L.T or one sided L.T (0 to ∞)

Properties of F.T	Properties of L.T
(i) Linearity property $a x_1(t) + b x_2(t) \xleftrightarrow{F.T} a X_1(\omega) + b X_2(\omega)$	(i) Linearity property If $x_1(t) \xleftrightarrow{L.T} X_1(s)$ and $x_2(t) \xleftrightarrow{L.T} X_2(s)$. Then, $a x_1(t) + b x_2(t) \xleftrightarrow{L.T} a X_1(s) + b X_2(s)$
(ii) Time shifting property $x(t) \xleftrightarrow{F.T} X(\omega)$ $x(t-t_0) \xleftrightarrow{F.T} e^{-j\omega t_0} X(\omega)$	(ii) Time shifting property $x(t) \xleftrightarrow{L.T} X(s)$ $x(t-t_0) \xleftrightarrow{L.T} e^{-st_0} X(s)$
(iii) Frequency shifting property $e^{j\omega_0 t} x(t) \xleftrightarrow{F.T} X(\omega - \omega_0)$	(iii) Frequency shifting property $e^{+s_0 t} x(t) \xleftrightarrow{L.T} X(s - s_0)$
(iv) Scaling property $x(at) \xleftrightarrow{F.T} \frac{1}{a} F\left(\frac{\omega}{a}\right)$	(iv) Scaling property $x(at) \xleftrightarrow{L.T} \frac{1}{a} F\left(\frac{s}{a}\right)$
(v) Integration in time domain $\int_{-\infty}^{t} x(\tau) d\tau = \frac{1}{j\omega} X(\omega)$	(v) Integration in time domain $\int_{-\infty}^{t} x(\tau) d\tau = \frac{1}{s} X(s)$
(vi) Convolution in time domain $x_1(t) * x_2(t) \xleftrightarrow{F.T} X_1(\omega) \cdot X_2(\omega)$	(vi) Convolution in time domain $x_1(t) * x_2(t) \xleftrightarrow{L.T} X_1(s) X_2(s)$
(vii) Convolution in frequency domain $x_1(t) \cdot x_2(t) = \frac{1}{2\pi} \left[X_1(\omega) * X_2(\omega)\right]$	(vii) Convolution in frequency domain $x_1(t) \cdot x_2(t) \xleftrightarrow{L.T} \frac{1}{2\pi} \left[X_1(s) * X_2(s)\right]$

(viii) Differentiation in time domain	(viii) Differentiation in time domain
$\frac{d}{dt} x(t) \xleftrightarrow{F.T} j\omega\, X(\omega)$	$\frac{d}{dt} x(t) \xleftrightarrow{L.T} s\, X(s)$
(ix) $jt\, x(t) \xleftrightarrow{F.T} \frac{d}{d\omega} X(\omega)$	(ix) $-t\, x(t) \xleftrightarrow{L.T} \frac{d}{ds} X(s)$
↓ Differentiation in frequency domain	↓ Differentiation in frequency domain
	$(-1)^n t^n x(t) \longleftrightarrow \frac{d^n}{ds^n} X(s)$

→ S & S by A.V oppenheim & schaffer (T.B)

(x) <u>Initial value theorem property</u>

→ The value of the function at $t=0$ is the initial value

for a function $x(t)$, initial value is $x(t)\big|_{at\ t=0} = x(0)$

→ This theorem is applied for purely unilateral L.T

<u>STATEMENT</u> :- If $x(t)$ and its derivative $x'(t)$ are laplace transformable then,

$$\lim_{t\to 0} x(t) = x(0) = \lim_{s\to\infty} s.X(s)$$

<u>PROOF</u> :- w.k.t., $L.T[x(t)] = X(s)$

let $L.T[x'(t)] = s.X(s)$., This is true, when there are no initial conditions.

If there exists any initial condition then,

$$L.T[x'(t)] = s.X(s) - x(0)$$

let, $\int_0^\infty x'(t) e^{-st}\, dt = s\, X(s) - x(0)$

applying limits on b.s.

$$\Rightarrow \lim_{s\to\infty} \int_0^\infty x'(t) e^{-st} dt = \lim_{s\to\infty} (s\,X(s) - x(0))$$

$$\Rightarrow \int_0^\infty x'(t)\,dt \lim_{s\to\infty} e^{-st} = \lim_{s\to\infty} s.X(s) - \lim_{s\to\infty} x(0)$$

$$\Rightarrow 0 = \lim_{s\to\infty} s\,X(s) - x(0)$$

$$\Rightarrow \boxed{x(0) = \lim_{s\to\infty} s.X(s)}$$ is known as initial value theorem of L.T

Final value theorem property

Final value is the value of the function at $t = \infty$

i.e., $\lim_{t\to\infty} x(t) = x(\infty)$

STATEMENT :- If $x(t)$ and its derivative $x'(t)$ are laplace transformable then,

$$x(\infty) = \lim_{s\to 0} s.X(s)$$

PROOF :- let, $L.T[x(t)] = X(s)$

$$L.T[x'(t)] = s.X(s) - x(0)$$

$$\int_0^\infty x'(t) e^{-st} dt = s\,X(s) - x(0)$$

Applying limits on b.s

$$\lim_{s\to 0} \int_0^\infty x'(t) e^{-st} dt = \lim_{s\to 0} (s\,X(s) - x(0))$$

$$\int_0^\infty x'(t)\,dt.1 = \lim_{s\to 0} s\,X(s) - x(0)$$

$$x(t)\Big|_0^\infty = \lim_{s\to 0} s\,X(s) - x(0)$$

$$x(\infty) - \cancel{x(0)} = \lim_{s\to 0} s\,X(s) - \cancel{x(0)}$$

$\boxed{x(\infty) = \lim_{s \to 0} s\,X(s)}$ is known as final value theorem of L.T

(Q) Find the L.T of $t e^{-at} u(t)$ by using properties

let $x(t) = e^{-at} u(t)$

Then, $X(s) = \frac{1}{s+a}$

Using property of differentiation in s-domain,

$$t\,x(t) \longleftrightarrow -\frac{d}{ds} X(s)$$

$$t\,e^{-at} u(t) \longleftrightarrow -\frac{d}{ds}\left(\frac{1}{s+a}\right)$$

$$t\,e^{-at} u(t) \longleftrightarrow -\left(\frac{-1}{(s+a)^2}\right)$$

$$t\,e^{-at} u(t) \longleftrightarrow \frac{1}{(s+a)^2}$$

(Q) Find the L.T of $f(t) = e^{-t} u(t) * \sin 3\pi t\, u(t)$.

Using convolution in time domain property we have,

$$f_1(t) * f_2(t) \longleftrightarrow F_1(s)\, F_2(s)$$

let $f_1(t) = e^{-t} u(t)$ and $f_2(t) = \sin 3\pi t\ u(t)$

Then, $F_1(s) = \frac{1}{s+1}$ $\qquad F_2(s) = \frac{3\pi}{s^2 + 9\pi^2}$.

$$L.T\left[e^{-t} u(t) * \sin 3\pi t\, u(t)\right] = \frac{1}{s+1} \cdot \frac{3\pi}{s^2 + 9\pi^2}$$

(Q) Find L.T of $t \cdot \frac{d}{dt}\left[e^{-t} \sin t\, u(t)\right]$

$$\sin t \longleftrightarrow \frac{1}{s^2 + 1^2}$$

$$e^{-t} \sin t \longleftrightarrow \frac{1}{(s+1)^2 + 1^2}$$

$$\frac{d}{dt}\left[x(t)\right] = s\,X(s)$$

$$\frac{d}{dt}\left[e^{-t}\sin t\right] = \frac{s}{(s+1)^2+1}$$

$$t\, f(t) \longleftrightarrow -\frac{d}{ds}X(s)$$

$$\therefore t\frac{d}{dt}\left[e^{-t}\sin t\, u(t)\right] \longleftrightarrow -\frac{d}{dt}\left[\frac{s}{(s+1)^2+1}\right]$$

$$= -\left[\frac{((s+1)^2+1).1 - s.2(s+1).1}{((s+1)^2+1)^2}\right]$$

$$= -\left[\frac{(s+1)^2+1-2s^2-2s}{[(s+1)^2+1]^2}\right]$$

$$= -\left[\frac{s^2+\cancel{2s}+2-2s^2-\cancel{2s}}{[(s+1)^2+1]^2}\right]$$

$$= \frac{s^2-2}{[(s+1)^2+1]^2}$$

Q) L.T of $t\sin at\, u(t)$

$$\sin at \longleftrightarrow \frac{a}{s^2+a^2}$$

$$t\sin at \longleftrightarrow -\frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)$$

$$t\sin at\, u(t) \longleftrightarrow \frac{2as}{(s^2+a^2)}$$

(8) L.T of $t e^{-t} u(t-T) = ?$

$x(t) = u(t) \longleftrightarrow \frac{1}{s} = X(s)$

$x(t-T) \longleftrightarrow e^{-sT} X(s)$ [Time shifting]

$= \frac{e^{-sT}}{s}$

$e^{+s_0 t} x(t) \longleftrightarrow X(s-s_0)$ [Frequency shifting in s-domain]

$= X(s+1) \; (s_0 = -1)$

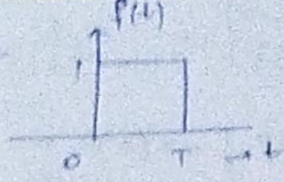
$e^{-t} u(t-T) \longleftrightarrow \frac{e^{-(s+1)T}}{(s+1)}$

$t e^{-t} u(t-T) \Leftrightarrow -\frac{d}{ds}\left[\frac{e^{-(s+1)T}}{(s+1)}\right]$

$\cancel{- \left\{(s+1) e^{-(s+1)T}(-(s+1))\right.}$

$t e^{-t} u(t-T) \longleftrightarrow \frac{(sT+T+1) e^{-(s+1)T}}{(s+1)^2}$

(Q) Find the L.T of

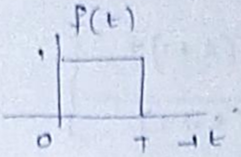


$f(t) = 1$, for $0 \leq t \leq T$

$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} \, dt$

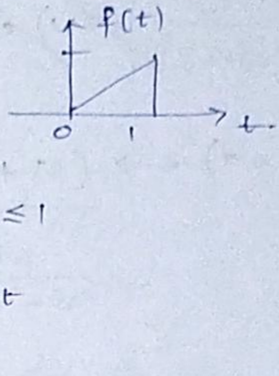
$= \int_0^T 1 \cdot e^{-st} \, dt = \left(\frac{e^{-st}}{-s} \right)_0^T = \frac{-1}{s}\left(e^{-sT} - e^{-0} \right)$

$= \frac{1 - e^{-sT}}{s}$



$\longleftrightarrow \frac{1 - e^{-st}}{s}.$

(Q) Find the L.T of



$f(t) = t, \quad 0 \leq t \leq 1$

$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} \, dt$

$= \int_0^1 t \, e^{-st} \, dt$

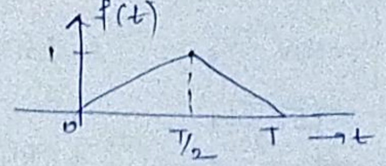
$= \left(t \frac{e^{-st}}{-s} \right)_0^1 - \int_0^1 1 \, \frac{e^{-st}}{-s} \, dt$

$= \frac{e^{-s}}{-s} + \frac{1}{s} \left(\frac{e^{-st}}{-s} \right)_0^1$

$= -\frac{1}{s} e^{-s} - \frac{1}{s^2}\left(e^{-s} - 1 \right)$

$F(s) = \frac{-1}{s} e^{-s} - \frac{1}{s^2} e^{-s} + \frac{1}{s^2}$

(Q) find the L.T of



$$f(t) = \begin{cases} \frac{2}{T}t & \text{for } 0 \le t \le T/2 \\ 2 - \frac{2}{T}t & \text{for } T/2 \le t < T \end{cases}$$

$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_0^{T/2} \frac{2}{T} t\, e^{-st}\, dt + \int_{T/2}^{T} \left(2 - \frac{2}{T}t\right) e^{-st}\, dt$$

$$= \frac{2}{T}\int_0^{T/2} t\, e^{-st}\, dt + 2\int_{T/2}^{T} e^{-st}\, dt - \frac{2}{T}\int_{T/2}^{T} t\, e^{-st}\, dt$$

$$= \frac{2}{T}\left[\left(t\frac{e^{-st}}{-s}\right)_0^{T/2} - \int_0^{T/2} \frac{e^{-st}}{-s}\, dt\right] + 2\left(\frac{e^{-st}}{-s}\right)_{T/2}^{T} - \frac{2}{T}\left[\left(t\frac{e^{-st}}{-s}\right)_{T/2}^{T} - \int_{T/2}^{T} \frac{e^{-st}}{-s}\, dt\right]$$

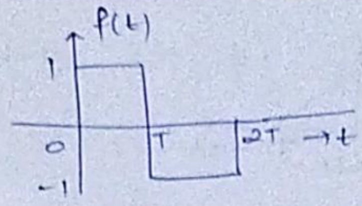
$$= \frac{2}{T}\left[\frac{-T}{2s} e^{-sT/2} + \frac{1}{s}\left(\frac{e^{-st}}{-s}\right)_0^{T/2}\right] - \frac{2}{s}\left(e^{-sT} - e^{-sT/2}\right) - \frac{2}{T}\left[\left(-\frac{T}{s}e^{-sT} + \frac{T}{2s}e^{-sT/2}\right) + \frac{1}{s}\left(\frac{e^{-st}}{-s}\right)_{T/2}^{T}\right]$$

$$= \frac{2}{\cancel{T}}\left(-\frac{\cancel{T}}{2s}e^{-sT/2}\right) - \frac{2}{Ts^2}\left(e^{-sT/2} - 1\right) - \frac{2}{s}e^{-sT} + \frac{2}{s}e^{-sT/2} - \frac{2}{T}\left(-\frac{T}{s}e^{-sT} + \frac{T}{2s}e^{-sT/2}\right) + 2/Ts^2\left(e^{-sT} - e^{-sT/2}\right)$$

$$= \cancel{-\frac{1}{s}e^{-sT/2}} - \frac{2}{Ts^2}e^{-sT/2} + \frac{2}{Ts^2} - \cancel{\frac{2}{s}e^{-sT}} + \cancel{\frac{2}{s}e^{-sT/2}} + \cancel{\frac{2}{s}e^{-sT}} - \cancel{\frac{1}{s}e^{-sT/2}} + \frac{2}{Ts^2}e^{-sT} - \frac{2}{Ts^2}e^{-sT/2}$$

$$F(s) = \frac{2}{Ts^2} - \frac{4}{Ts^2}e^{-sT/2} + \frac{2}{Ts^2}e^{-sT}$$

(Q) $f(t) =$



$$f(t) = \begin{cases} 1, & 0 \le t < T \\ -1, & T \le t \le 2T \end{cases}$$

$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_0^T 1\, e^{-st}\, dt - \int_T^{2T} 1\, e^{-st}\, dt$$

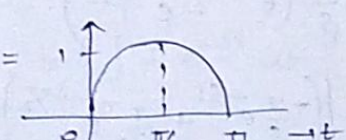
$$= \left(\frac{e^{-st}}{-s}\right)_0^T - \left(\frac{e^{-st}}{-s}\right)_T^{2T}$$

$$= -\frac{1}{s}\left(e^{-sT} - 1\right) + \frac{1}{s}\left(e^{-2sT} - e^{-sT}\right)$$

$$= \frac{-e^{-sT}}{s} + \frac{1}{s} + \frac{e^{-2sT}}{s} - \frac{e^{-sT}}{s}.$$

$$F(s) = \frac{e^{-2sT}}{s} - \frac{2e^{-sT}}{s} + \frac{1}{s}.$$

(Q) $f(t) =$



$$f(t) = \begin{cases} \sin t, & 0 \le t \le \pi \\ 0, & \text{otherwise} \end{cases}$$

$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt = \int_0^{\pi} \sin t \cdot e^{-st}\, dt$$

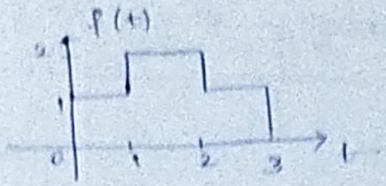
We have, $\displaystyle\int_0^{\pi} \sin at\, e^{-bt}\, dt = \frac{1}{a^2+b^2}\left[-b \sin at\, e^{-bt} - e^{-bt} a \cos at\right]$

Here, $a = 1$, $b = s$.

$$= \frac{1}{1+s^2}\left[-s \sin t \, e^{-st} - e^{-st} \cos t\right]$$

$$F(s) = \frac{-s \sin t \, e^{-st} - e^{-st} \cos t}{1+s^2}$$

(6) $f(t) =$



$$f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 2, & 1 \le t < 2 \\ 1, & 2 \le t < 3 \end{cases}$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} \, dt$$

$$= \int_0^1 1 \, e^{-st} \, dt + \int_1^2 2 \, e^{-st} \, dt + \int_2^3 1 \, e^{-st} \, dt$$

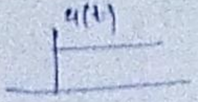
$$= \left(\frac{e^{-st}}{-s}\right)_0^1 + 2\left(\frac{e^{-st}}{-s}\right)_1^2 + \left(\frac{e^{-st}}{-s}\right)_2^3$$

$$= -\frac{1}{s}\left(e^{-s} - 1\right) - \frac{2}{s}\left(e^{-2s} - e^{-s}\right) - \frac{1}{s}\left(e^{-3s} - e^{-2s}\right)$$

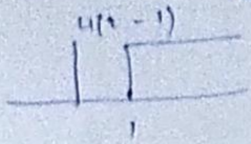
$$= -\frac{e^{-s}}{s} + \frac{1}{s} - \frac{2}{s}e^{-2s} + \frac{2}{s}e^{-s} - \frac{1}{s}e^{-3s} + \frac{1}{s}e^{-2s}$$

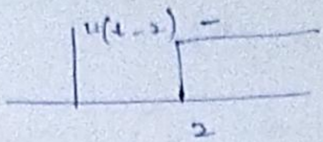
$$F(s) = -\frac{1}{s}e^{-3s} - \frac{1}{s}e^{-2s} + \frac{1}{s}e^{-s} + \frac{1}{s}$$

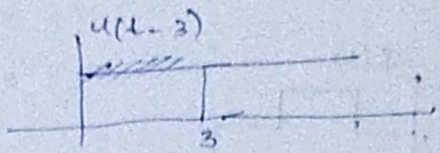
$$F(s) = \frac{1 + e^{-s} - e^{-2s} - e^{-3s}}{s}$$

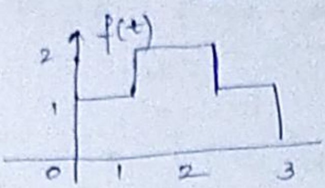
→ 

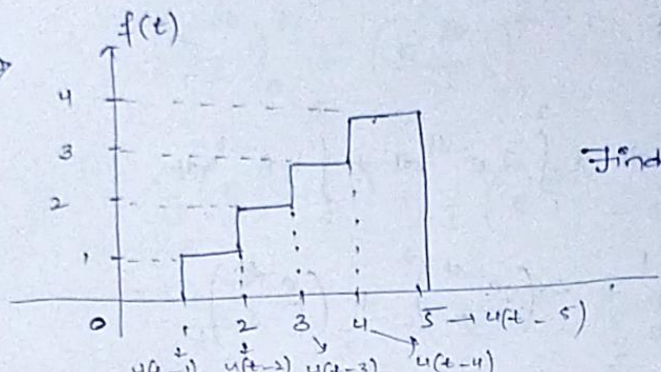
$$\boxed{f(t) = u(t) + u(t-1) - u(t-2) - u(t-3)}$$









→ 

Find the L.T of $f(t)$

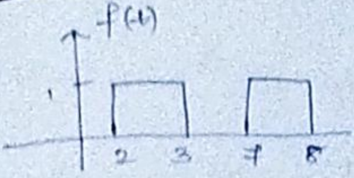
$$f(t) = \underbrace{u(t-1)}_{1} + u(t-2) + u(t-3) + u(t-4) - 4u(t-5)$$

or $$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st}\, dt$$

$$= \int_0^1 0 e^{-st}\, dt + \int_1^2 1 e^{-st}\, dt + \int_2^3 2 e^{-st}\, dt + \int_3^4 3 e^{-st}\, dt + \int_4^5 4 e^{-st}\, dt$$

from this,

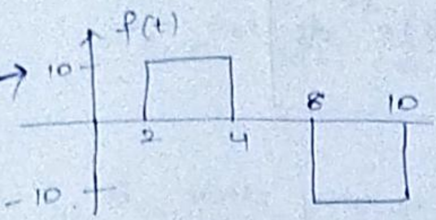
$$F(s) = \frac{1}{s} e^{-s} + \frac{1}{s} e^{-2s} + \frac{1}{s} e^{-3s} + \frac{1}{s} e^{-4s} - \frac{4}{s} e^{-5s}$$



→ find the L.T of f(t)?

$f(t) = u(t-2) - u(t-3) + u(t-7) - u(t-8)$

$F(s) = \frac{1}{s}e^{-2s} - \frac{1}{s}e^{-3s} + \frac{1}{s}e^{-7s} - \frac{1}{s}e^{-8s}$

→


$f(t) = 10\,u(t-2) - 10\,u(t-4) - 10\,u(t-8) + 10\,u(t-10)$

$F(s) = \frac{10}{s}e^{-2s} - \frac{10}{s}e^{-4s} - \frac{10}{s}e^{-8s} + \frac{10}{s}e^{-10s}$

→ Find the L.T of $f(t) = e^{-at}\left[A\cos bt + \frac{B-Aa}{b}\sin bt\right]u(t)$

$f(t) = A\,e^{-at}\cos bt\,u(t) + \frac{B-Aa}{b}e^{-at}\sin bt\,u(t)$

$F(s) = \frac{A.(s+a)}{(s+a)^2+b^2} + \frac{B-Aa}{\cancel{b}}\,\frac{\cancel{b}}{(s+a)^2+b^2}$

$F(s) = \frac{A(s+a)}{(s+a)^2+b^2} + \frac{B-Aa}{(s+a)^2+b^2}$

$= \frac{As + \cancel{Aa} + B - \cancel{Aa}}{(s+a)^2+b^2}$

$F(s) = \frac{As+B}{(s+a)^2+b^2}$

Inverse laplace transforms

W.K.T, $LT[f(t)] = F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$

Then, Inverse laplace transform $= L^{-1}[F(s)] = ?$

$$\boxed{L^{-1}[F(s)] = f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{st} ds}$$

→ If any function $F(s)$ is in such a way that it is a rational function.

i.e., $F(s) = \frac{A(s)}{B(s)}$

Then, one method of finding Inverse L.T is

→ express this $F(s)$ in terms of partial functions

i.e., $F(s) = \frac{A_1}{s-a_1} + \frac{A_2}{s-a_2} + \frac{A_3}{s-a_3} + \dots + \frac{A_K}{s-a_K}$

then, $L^{-1}[F(s)] = L^{-1}\left(\frac{A_1}{s-a_1}\right) + L^{-1}\left(\frac{A_2}{s-a_2}\right) + L^{-1}\left(\frac{A_3}{s-a_3}\right) + \dots L^{-1}\left(\frac{A_K}{s-a_K}\right)$

where, $a_1, a_2, \dots a_K$ are known as the poles of the transfer function

! poles are the values of the function at which $F(s)$ will become infinite.

and llly zeros are the values at which $F(s)$ will becomes zero.

Simply, poles are roots of denominator

zeros " " " numerator.

W.k.t, $e^{at}u(t) \longleftrightarrow \frac{1}{s-a}$, $R = Re\{s\} > a$, RHS sequence

Illy $-e^{at}u(-t) \longleftrightarrow \frac{1}{s-a}$, $R = Re\{s\} < a$, LHS "

Now assuming RHS sequence,

$$A_k e^{a_k t} u(t) \longleftrightarrow \frac{A_k}{s-a_k}, \quad ROC = Re\{s\} > a$$

Illy assuming LHS sequence,

$$-A_k e^{a_k t} u(-t) \longleftrightarrow \frac{A_k}{s-a_k}, \quad ROC = Re\{s\} < a$$

Q) $F(s) = \dfrac{-5s-7}{(s+1)(s-1)(s+2)}$, find $f(t)$?

If ROC is not given, then by default assume it as an RHS sequence

$$F(s) = \frac{A_1}{s+1} + \frac{A_2}{s-1} + \frac{A_3}{s+2}$$

By applying partial functions

$A_1 = 1, \quad A_2 = -2, \quad A_3 = 1$

$$\Rightarrow F(s) = \frac{1}{s+1} - \frac{2}{s-1} + \frac{1}{s+2}$$

$$L^{-1}[F(s)] = L^{-1}\left(\frac{1}{s+1}\right) - 2L^{-1}\left(\frac{1}{s-1}\right) + L^{-1}\left(\frac{1}{s+2}\right)$$

$$f(t) = e^{t}u(t) - 2e^{-t}u(t) + e^{2t}u(t)$$

Q) find the ILT of $F(s) = \frac{2s+9}{s^2+4s+29}$

$$F(s) = \frac{2s+4+5}{(s+2)^2+5^2}$$

$$= \frac{2(s+2)+5}{(s+2)^2+5^2}$$

$$F(s) = \frac{2(s+2)}{(s+2)^2+5^2} + \frac{5}{(s+2)^2+5^2}$$

$$f(t) = 2e^{-2t}\cos 5t + 5e^{-2t}\sin 5t$$

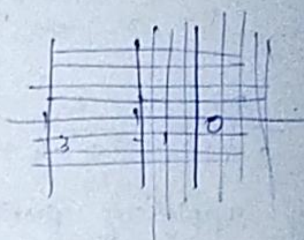
Q) $F(s) = \frac{2(s+2)}{s^2+4s+3}$, find the inverse L.T.?

$$F(s) = \frac{A}{s+1} + \frac{B}{s+3}$$

$$A = +1, \quad B = -1$$

$$\Rightarrow F(s) = \frac{1}{s+1} - \frac{1}{s+3}$$

$$\underbrace{R > -1 \qquad R > -3}_{\cap \text{ is } R > -1}$$

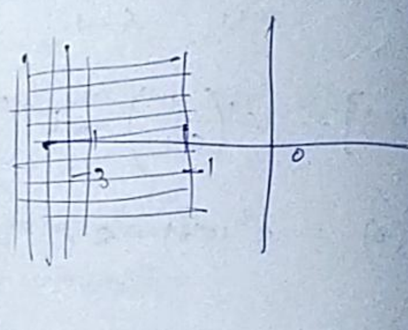


Case (i) Assuming RHS sequence

$$f(t) = e^{-t}u(t) - e^{-3t}u(t).$$

Case (ii) Assuming LHS sequence

$\cap$ is $R < -3$.



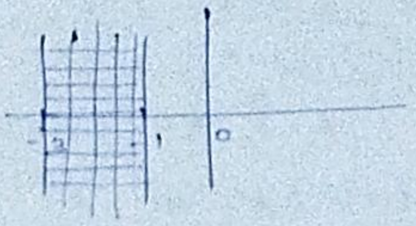
$$F(s) = \frac{1}{s+1} - \frac{1}{s+3}$$

$$\underbrace{R < -1 \cap R < -3}_{R < -3}$$

$$f(t) = -e^{-t}u(-t) + e^{-3t}u(-t)$$

Case (iii) $\dfrac{1}{s+1} - \dfrac{1}{s+3}$

$$\begin{array}{cc} \dfrac{1}{s+1} & \dfrac{1}{s+3} \\ \downarrow & \downarrow \\ \text{LHS} & \text{RHS} \\ \downarrow & \downarrow \\ R<-1 & R>-3 \end{array}$$



$R<-1 \cap R>-3 = -3<R<-1$

$f(t) = -e^{-t}u(-t) - e^{-3t}u(t)$

(Whenever we assume LHS sequence, ROC is left of the left most pole)

|||^{ly} (for RHS sequence, ROC is right of the right most pole)

Q) Find ILT of $\dfrac{s^2+2s-1}{s^3+3s^2+2s}$

Q) Find the ILT of $F(s) = \log\left(\dfrac{s+a}{s+b}\right)$

$f(t) \xleftrightarrow{LT} F(s)$

$\dfrac{d}{ds}F(s) \longleftrightarrow -t\,f(t)$

$F(s) = \log\left(\dfrac{s+a}{s+b}\right)$

$-\dfrac{d}{ds}F(s) = \dfrac{-1}{\left(\dfrac{s+a}{s+b}\right)}\left(\dfrac{s+b-(s+a)}{(s+b)^{\cancel{2}}}\right)$

$= \dfrac{a-b}{(s+a)(s+b)} = \dfrac{1}{s+b} - \dfrac{1}{s+a}$

$t\,f(t) = -e^{-at}u(t) + e^{-bt}u(t)$

$f(t) = \dfrac{1}{t}\left(e^{-bt}u(t) - e^{-at}u(t)\right)$

Q) $F(s) = s\log\left(\dfrac{s+a}{s+b}\right) = s\log(s+a) - s\log(s+b)$

$f(t) = \dfrac{1}{t}\left(ae^{-at} - be^{-bt} + e^{-at} - e^{-bt}\right)u(t)$

Q $F(s) = \dfrac{s^2+2s+5}{(s+3)(s+5)^2} = \dfrac{A}{s+3} + \dfrac{B}{s+5} + \dfrac{C}{(s+5)^2}$, $Re\{s\} > -3$

$A = 2, \quad B = -1, \quad C = 10.$

$f(t) = 2e^{-3t}u(t) - e^{-5t}u(t) + 10te^{-5t}u(t)$

Q $F(s) = \dfrac{s^2+6s+7}{s^2+3s+2}$, $Re\{s\} > -1$

$= 1 + \dfrac{3s+5}{s^2+3s+2} = 1 + \dfrac{(3s+5)}{(s+1)(s+2)}$

$= 1 + \dfrac{2}{s+1} + \dfrac{1}{s+2}$

$\therefore f(t) = \delta(t) + 2e^{-t}u(t) + e^{-2t}u(t)$

Q $F(s) = \dfrac{2 + 2se^{-2s} + 4e^{-4s}}{s^2+4s+3}$

$= \dfrac{2}{(s+1)(s+3)} + \dfrac{2s\,e^{-2s}}{(s+1)(s+2)} + \dfrac{4e^{-4s}}{(s+1)(s+2)}$

$F_1(s) = \dfrac{2}{(s+1)(s+3)} = \dfrac{1}{s+1} - \dfrac{1}{s+3}.$

$f_1(t) = (e^{-t} - e^{-3t})u(t)$

$F_2(s) = \dfrac{2s}{(s+1)(s+3)} = \dfrac{-1}{s+1} + \dfrac{3}{s+3}$

$f_2(t) = -e^{-t}u(t) + 3e^{-3t}u(t)$

$F_3(s) = \dfrac{4}{(s+1)(s+3)} = \dfrac{2}{s+1} - \dfrac{2}{s+3}$

$f_3(t) = 2e^{-t}u(t) - 2e^{-3t}u(t)$

$u(t) \longleftrightarrow \dfrac{1}{s}$

$u(t-5) \longleftrightarrow \dfrac{e^{-5s}}{5}$