CHAPTER-1

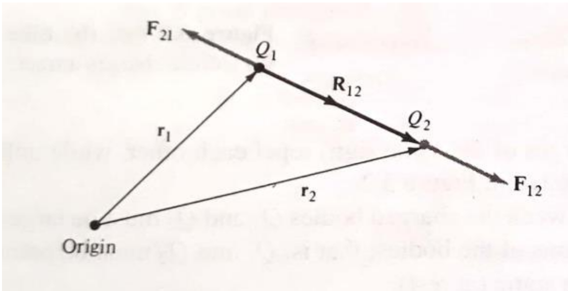
- **Coulomb's Law:** States that the force F between two point charges Q_1 and Q_2 is:
  - Along the line joining them
  - Directly proportional to the product Q_1Q_2 of the charges
  - Inversely proportional to the square of the distance R between them

$$F = \frac{kQ_1Q_2}{R^2}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \cong \frac{10^{-9}}{36\pi} \text{ F/m}$$

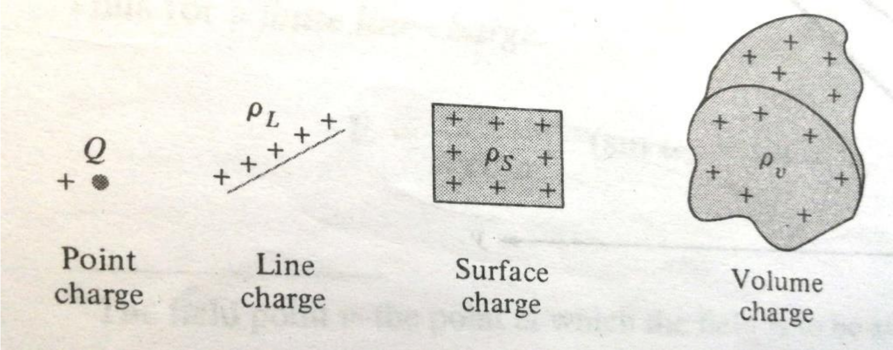
- ϵ_0 is the permittivity of the free space.
- F measured in newton (N)



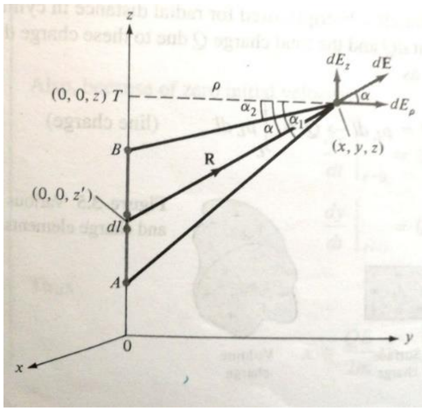
- $\boldsymbol{F}_{12} = \frac{Q_1Q_2}{4\pi\epsilon_o R^2}\boldsymbol{a}_{R_{12}}$
- $\boldsymbol{R}_{12} = \boldsymbol{r}_2 - \boldsymbol{r}_1$
- $R = |\boldsymbol{R}_{12}|$
- $\boldsymbol{a}_{R_{12}} = \frac{\boldsymbol{R}_{12}}{R}$
- $\boldsymbol{F}_{21} = |\boldsymbol{F}_{12}|\boldsymbol{a}_{R_{21}} = |\boldsymbol{F}_{12}|(-\boldsymbol{a}_{R_{12}})$
- $\boldsymbol{F}_{21} = -\boldsymbol{F}_{12}$
- If you have more than two point charges, we can use the **principle of superposition** to determine the force on a particular charge.
- $\boldsymbol{F} = \frac{Q}{4\pi\epsilon_o}\sum_{k=1}^{N}\frac{Q_k(\boldsymbol{r}-\boldsymbol{r}_k)}{|\boldsymbol{r}-\boldsymbol{r}_k|^3}$
- The electric field intensity (or electric field strength) **E** is the force per unit charge when placed in an electric field.
- $\boldsymbol{E} = \frac{\boldsymbol{F}}{Q}$

- $\boldsymbol{E} = \frac{Q}{4\pi\epsilon_o R^2}\boldsymbol{a}_R = \frac{Q(\boldsymbol{r}-\boldsymbol{r}')}{4\pi\epsilon_o|\boldsymbol{r}-\boldsymbol{r}'|^3}$

- **<u>Electric fields due to continuous charge distributions</u>**



- $dQ = \rho_L dl \rightarrow Q = \int_L \rho_L dl$ (line charge)
- $dQ = \rho_S dS \rightarrow Q = \int_S \rho_S dS$ (surface charge)
- $dQ = \rho_v dv \rightarrow Q = \int_v \rho_v dv$ (volume charge)
- $\rho_L \rightarrow \frac{C}{m}$
- $\rho_S \rightarrow C/m^2$
- $\rho_v \rightarrow C/m^3$
- $\boldsymbol{E} = \int_{\boldsymbol{L}} \frac{\rho_L dl}{4\pi\epsilon_o R^2}\boldsymbol{a}_R$ (line charge)
- $\boldsymbol{E} = \int_{\boldsymbol{S}} \frac{\rho_S dS}{4\pi\epsilon_o R^2}\boldsymbol{a}_R$ (surface charge)
- $\boldsymbol{E} = \int_{\boldsymbol{v}} \frac{\rho_v dv}{4\pi\epsilon_o R^2}\boldsymbol{a}_R$ (volume charge)
- Line Charge

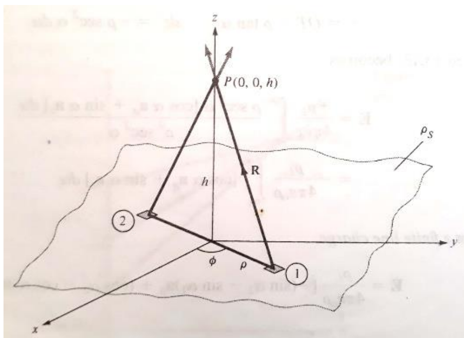


$$\boldsymbol{E} = \frac{\rho_L}{4\pi\epsilon_o\rho}[-(\sin\alpha_2 - \sin\alpha_1)\boldsymbol{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\boldsymbol{a}_z]$$

infinite line charge

$$\boldsymbol{E} = \frac{\rho_L}{2\pi\epsilon_o\rho}\boldsymbol{a}_\rho$$

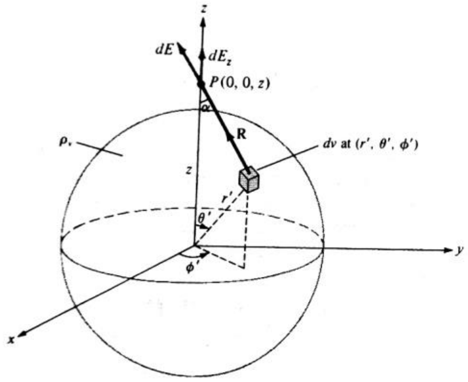
- Surface Charge



$$\boldsymbol{E} = \frac{\rho_S}{2\epsilon_o}\boldsymbol{a}_n$$

- Volume Charge

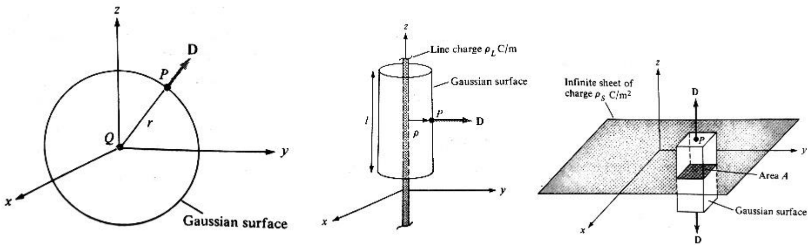
$$\boldsymbol{E} = \frac{Q}{4\pi\epsilon_o r^2}\boldsymbol{a}_r$$

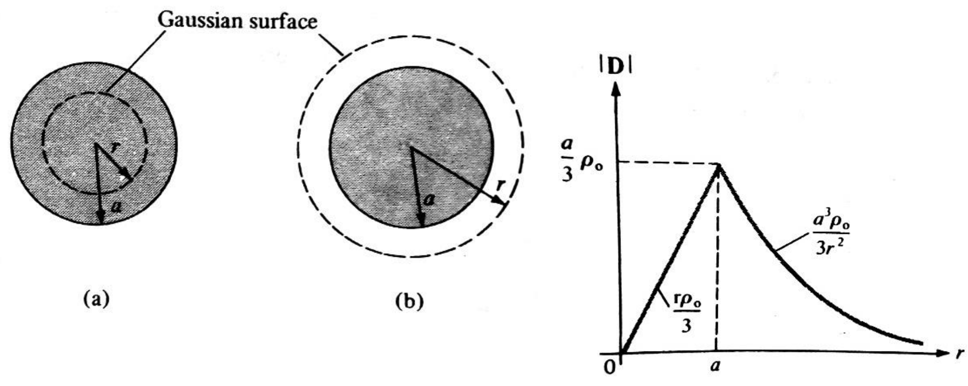


- Electric Flux Density (electrical displacement)

$$\boldsymbol{D} = \epsilon_o \boldsymbol{E}$$

- $\boldsymbol{D}$ is measured in C/m^2
- The electric flux $\psi = \int_S \boldsymbol{D}.d\boldsymbol{S} = \int_S \epsilon_o \boldsymbol{E}.d\boldsymbol{S}$
- **<u>Gauss's Law:</u>** States that the total electric flux ψ through any closed surface is equal to the total charge enclosed by that surface.
- $\psi = Q_{enc}$
- $\psi = \oint_S \boldsymbol{D}.d\boldsymbol{S} = \int_v \rho_v dv$
- $\rho_v = \nabla.\boldsymbol{D}$ -------→ **Maxwell's equation for electrostatic fields**





- $$\mathbf{D} = \begin{cases} \frac{r}{3}\rho_o \mathbf{a}_r & 0 < r \le a \\ \frac{a^3}{3r^2}\rho_o \mathbf{a}_r & r \ge a \end{cases}$$
- Electric Potential $V = \frac{Q}{4\pi\epsilon_o r}$, measured in volts (V)
- $V_{AB} = -\int_A^B \boldsymbol{E}.d\boldsymbol{l}$

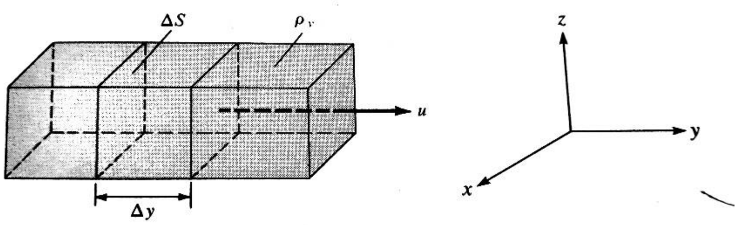
- $\oint_L \; \boldsymbol{E}.d\boldsymbol{l} = 0$
- $\oint_L \; \boldsymbol{E}.d\boldsymbol{l} = \int_S \; (\nabla \times \boldsymbol{E}).d\boldsymbol{S} = 0$
- $\nabla \times \boldsymbol{E} = 0$ ----------------→ **Second Maxwell's Eqn**.
- $\mathbf{E} = -\boldsymbol{\nabla} V$ -------→ Conservation of electric field
- $V = -\int_{\infty}^{r} \boldsymbol{E}.d\boldsymbol{l}$
- $V(r) = \frac{1}{4\pi\epsilon_o}\int_L \; \frac{\rho_L(\boldsymbol{r}')d\mathbf{l}'}{|\boldsymbol{r}-\boldsymbol{r}\prime|}$ (Line Charge)
- $V(r) = \frac{1}{4\pi\epsilon_o}\int_S \; \frac{\rho_S(\boldsymbol{r}')d\mathbf{S}'}{|\boldsymbol{r}-\boldsymbol{r}\prime|}$ (Surface Charge)
- $V(r) = \frac{1}{4\pi\epsilon_o}\int_v \; \frac{\rho_v(\boldsymbol{r}')dv\prime}{|\boldsymbol{r}-\boldsymbol{r}\prime|}$ (Volume Charge)
- **<u>Energy density in electrostatic fields</u>**
- Energy present in an assembly of charges

  $W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k$ (in joules)
- $W_E = \frac{1}{2}\int_L \; \rho_L V dl$ (line charge)
- $W_E = \frac{1}{2}\int_S \; \rho_S V dS$ (surface charge)
- $W_E = \frac{1}{2}\int_v \; \rho_v V dv$ (volume charge)
- $W_E = \frac{1}{2}\int_v \; (\boldsymbol{D}.\boldsymbol{E})dv = \frac{1}{2}\int_v \; \epsilon_o E^2 dv$
- Energy density w_E in (J/m^3)

$$w_E = \frac{dW_E}{dv} = \frac{1}{2}(\boldsymbol{D}.\boldsymbol{E}) = \frac{1}{2}\epsilon_o E^2 = \frac{D^2}{2\epsilon_o}$$

<u>Convection and Conduction Currents</u>

- The current through a given area is the electric charge passing through the area per unit time
- $I = \frac{dQ}{dt}$
- Current density J (A/m^2)
- $J = \frac{\Delta I}{\Delta S}$
- $I = \int_S \; \boldsymbol{J}.d\boldsymbol{S}$



- $\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \, \Delta S \, \frac{\Delta y}{\Delta t} = \rho_v \, \Delta S \, u_y$
- $J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y$
- $\mathbf{J} = \rho_v \boldsymbol{u}$
- I: Convection current (A)
- J: convection current density (A/m^2)
- The force on the electron charge when an electric field is applied
  $\mathbf{F} = -e\mathbf{E}$ E=F/Q ……. F=ma
- If an electron with mass m is moving in an electric field **E** with an average drift velocity **u**, according to Newton's law, the average change in the momentum of the free electron must be matched with the applied force

$$\frac{m\boldsymbol{u}}{\tau} = -e\mathbf{E}$$

$$\boldsymbol{u} = -\frac{e\tau}{m}\mathbf{E}$$

- If there are n electrons per unit volume, the electronic charge density

$$\rho_v = -ne$$

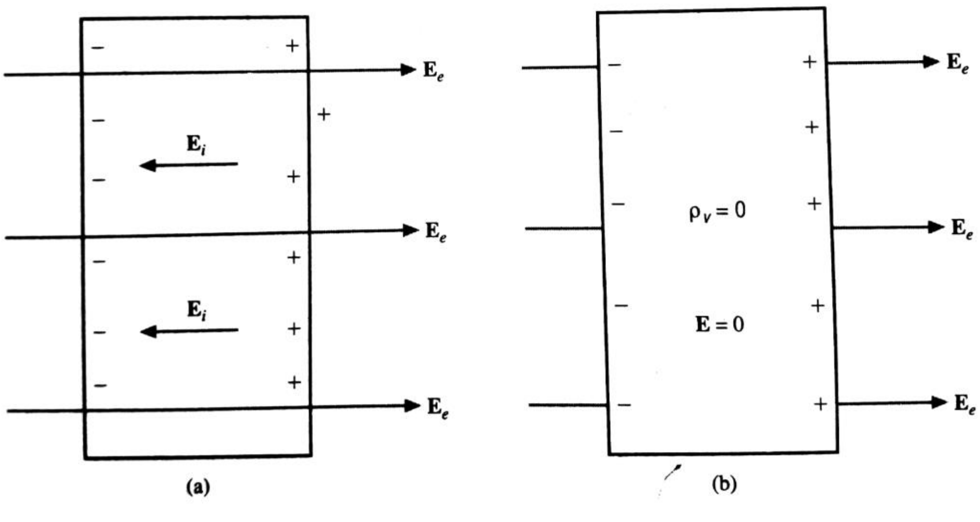
- The conduction density is

  $\mathbf{J} = \rho_v \boldsymbol{u} = \frac{ne^2\tau}{m}\mathbf{E} = \sigma\mathbf{E}$ ----→ **Ohm's law**

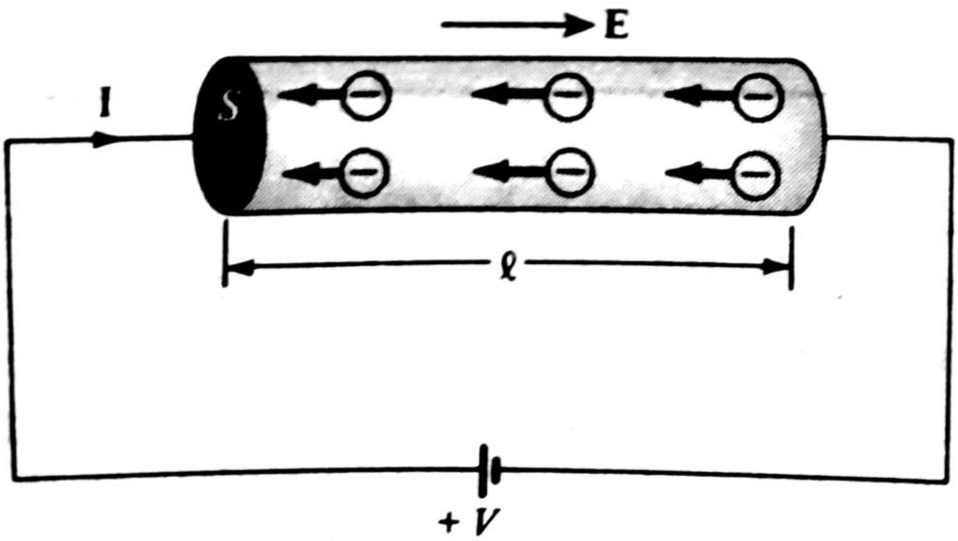
  $\sigma = \frac{ne^2\tau}{m}$ is the conductivity of the conductor ($\mho/m$ or S/m)

  Conductors

- A perfect conductor ($\sigma = \infty$) cannot contain an electrostatic field within it.
- $\mathbf{E} = 0, \quad \rho_v = 0, \quad V_{ab} = 0$ inside a conductor



- The **E** field in a conductor of uniform cross section



$E = \frac{V}{l}$ $J = \sigma E$ $J = \frac{I}{S}$

$$\frac{I}{S} = \sigma E = \frac{\sigma V}{l}$$

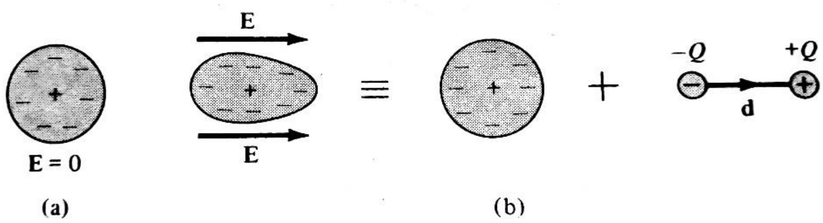
$$R = \frac{V}{I} = \frac{l}{\sigma S} = \frac{\rho_C l}{S}$$

- $\rho_C = 1/\sigma$ is the resistivity of the material (Ω/m)
- $R = \frac{V}{I} = \frac{\int_v \mathbf{E.dl}}{\int_S \sigma\mathbf{E}.d\mathbf{S}}$
- $P = \int_v \rho_v dv\, \mathbf{E}.\mathbf{u} = \int_v \mathbf{E}.\rho_v \mathbf{u}\, dv$
- $P = \int_v \mathbf{E}.\mathbf{J}\, dv$ ------------------------→ Joule's Law
- The power density (W/m^3)

$$w_P = \frac{dP}{dv} = \mathbf{E}.\mathbf{J} = \sigma|\mathbf{E}|^2$$

Polarization: Dipole moment per unit volume of the dielectric. Measured in C/m^2

- $\mathbf{p} = Q\mathbf{d}$



- Polarised volume and surface charge densities

$$\rho_{pv} = \mathbf{P}.\mathbf{a}_n$$

$$\rho_{ps} = -\nabla . \mathbf{P}$$

- The total volume charge density

$$\rho_t = \rho_v + \rho_{pv} = \nabla . \epsilon_o \mathbf{E}$$
$$\rho_v = \nabla . \epsilon_o \mathbf{E} - \rho_{pv}$$
$$= \nabla . (\epsilon_o \mathbf{E} + \mathbf{P})$$
$$= \nabla . \mathbf{D}$$
$$\mathbf{D} = \epsilon_o \mathbf{E} + \mathbf{P}$$

- $\mathbf{P} = \chi_e \epsilon_o \mathbf{E}$
- χ_e is the electric susceptibility of the material, is a measure of how susceptible (or sensitive) a given dielectric is to electric fields.
- $\mathbf{D} = \epsilon_o (1 + \chi_e) \mathbf{E} = \epsilon_o \epsilon_r \mathbf{E}$
- $\mathbf{D} = \epsilon \mathbf{E}$
- $\epsilon = \epsilon_o \epsilon_r$
- $\epsilon_r = 1 + \chi_e = \frac{\epsilon}{\epsilon_o}$
- The ***dielectric constant*** or ***relative permittivity*** ϵ_r is the ratio of the permittivity of the dielectric to that of free space.
- ***Dielectric breakdown*****:** the phenomenon of dielectric conducting
- ***Dielectric strength*****:** the maximum electric field that a dielectric can tolerate or withstand without electrical breakdown.

Continuity Equation and Relaxation time:

- ***Principle of charge conservation:*** the time rate of decrease of charge within a given volume is equal to the net outward current flow through the surface of the volume

$$I_{out} = \oint \mathbf{J} . d\mathbf{S} = -\frac{dQ_{\text{in}}}{dt}$$

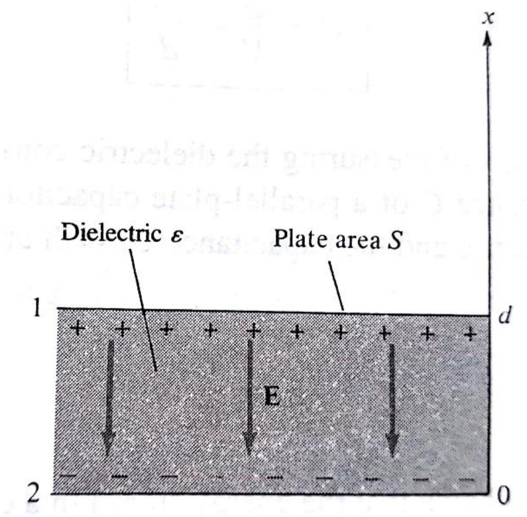
$$\oint_S \mathbf{J} . d\mathbf{S} = \int_v \nabla . \mathbf{J} \, dv$$

$$-\frac{dQ_{\text{in}}}{dt} = -\frac{d}{dt} \int_v \rho_v \, dv = -\int_v \frac{\partial \rho_v}{\partial t} \, dv$$

$$\int_v \nabla . \mathbf{J} \, dv = -\int_v \frac{\partial \rho_v}{\partial t} \, dv$$

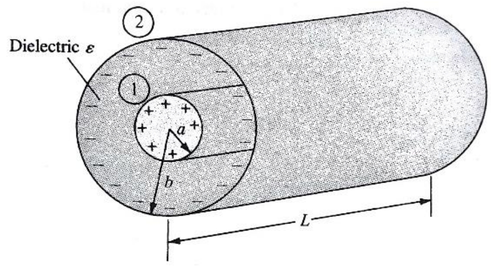
$$\nabla . \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$$

- $\mathbf{J} = \sigma\mathbf{E}$ $\qquad\qquad$ $\nabla.\mathbf{E} = \rho_v/\epsilon$
- $\nabla.\sigma\mathbf{E} = \frac{\sigma\rho_v}{\epsilon} = -\frac{\partial\rho_v}{\partial t}$
- $\frac{\partial\rho_v}{\partial t} + \frac{\sigma\rho_v}{\epsilon} = 0$
- $\rho_v = \rho_{vo}e^{-t/T_r}$
- $T_r = \frac{\epsilon}{\sigma}$
- Relaxation time is the time it takes a charge placed in the interior of a material to drop to e^{-1} ($= 36.8\%$) of its initial value.



Poisson's and Laplace's Equations

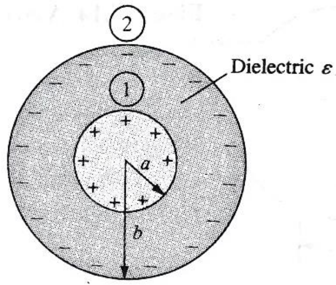
- $\nabla.\mathbf{D} = \nabla.\epsilon\mathbf{E} = \rho_v$
- $\mathbf{E} = -\nabla\mathrm{V}$
- $\nabla.(-\epsilon\nabla V) = \rho_v$
- $\nabla^2 V = -\frac{\rho_v}{\epsilon}$ $\qquad$ Poisson's Eqn. (for an inhomogeneous medium)



- $\nabla^2 V = 0$ $\qquad$ Laplace's Eqn.

<u>Resistance and Capacitance</u>

- $R = \frac{V}{I} = \frac{\int \mathbf{E}.d\mathbf{l}}{\oint \sigma\mathbf{E}.d\mathbf{S}}$
- $C = \frac{Q}{V} = \frac{\epsilon\oint \mathbf{E}.d\mathbf{S}}{\int \mathbf{E}.d\mathbf{l}}$



- Parallel –plate Capacitor:

$$C = \frac{Q}{V} = \frac{\epsilon S}{d}$$

Energy stored in a parallel plate capacitor

$$W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

- Coaxial Capacitor:

$$C = \frac{Q}{V} = \frac{2\pi\epsilon L}{\ln\frac{b}{a}}$$

- Spherical Capacitor

$$C = \frac{Q}{V} = \frac{4\pi\epsilon}{\frac{1}{\mathrm{a}} - \frac{1}{b}}$$

- $RC = \frac{\epsilon}{\sigma}$