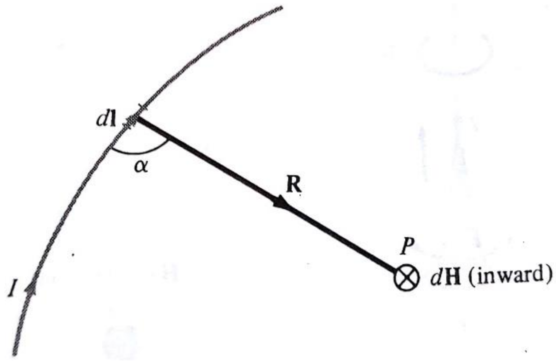
CHAPTER-2

- **Biot-Savart's Law:** States that the differential magnetic field intensity dH produced at a point P by the differential current element $I\ dl$ is
    - proportional to the product $I\ dl$
    - proportional sin of the angle α between the element and the line joining P to the element
    - Inversely proportional to the square of the distance R between P and the element



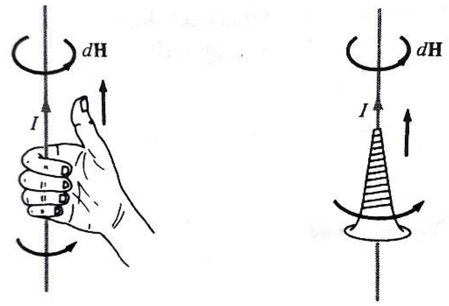
$$dH \propto \frac{I\ dl \sin\alpha}{R^2}$$

$$dH = \frac{kI\ dl \sin\alpha}{R^2}$$

$$k = \frac{1}{4\pi}$$

$$dH = \frac{I\ dl \sin\alpha}{4\pi R^2}$$

$$d\mathbf{H} = \frac{I\ d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\ d\mathbf{l} \times \mathbf{R}}{4\pi R^3} \qquad \text{A/m}$$

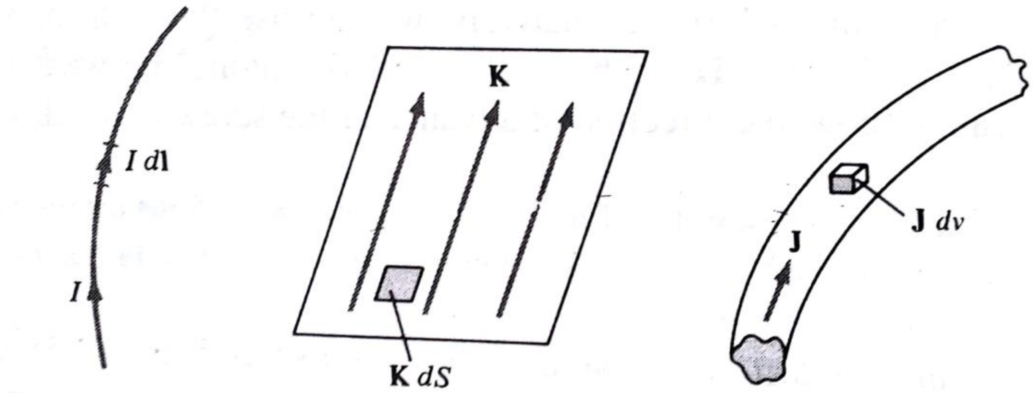


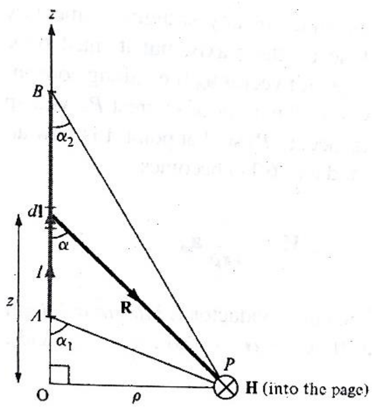
- $I\ d\mathbf{l} \equiv \mathbf{K}\ dS \equiv \mathbf{J}\ dv$

- $\mathbf{H} = \int_L \frac{I\ d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2}$ (Line Current)

- $\mathbf{H} = \int_S \frac{\mathbf{K}\ dS \times \mathbf{a}_R}{4\pi R^2}$ (Surface Current)

- $\mathbf{H} = \int_v \frac{\mathbf{J}\ dv \times \mathbf{a}_R}{4\pi R^2}$ (Volume Current)





- Magnetic field due to line current
- Field due to finite length current element

$$\mathbf{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi$$

- Field due to semi- infinite length element

$$\mathbf{H} = \frac{I}{4\pi\rho}\mathbf{a}_\phi$$

- Field due to infinite length current element

$$\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi$$

- $\mathbf{a}_\phi = \mathbf{a}_l \times \mathbf{a}_\rho$

Ampere's Circuit Law

- States that the line integral of **H** around a closed path is the same as the net current I_{enc} enclosed by the path.

$$\oint \mathbf{H}.d\mathbf{l} = I_{enc}$$

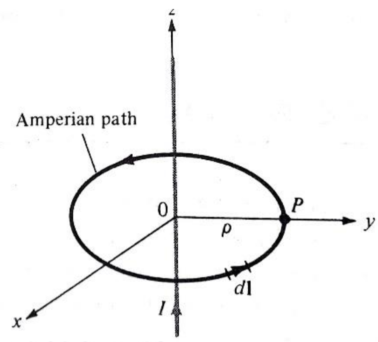
- Applying the Stoke's theorem,

$$I_{enc} = \oint_L \mathbf{H}.d\mathbf{l} = \int_S (\nabla \times \mathbf{H}).d\mathbf{S}$$

- By the definition, $I_{enc} = \int_S \mathbf{J}.d\mathbf{S}$
- $\nabla \times \mathbf{H} = \mathbf{J}$ ---------→ **Maxwell's Eqn. for magnetostatic fields**
- This also says magnetostatic field is not conservative.
- **H** due to infinite line current:

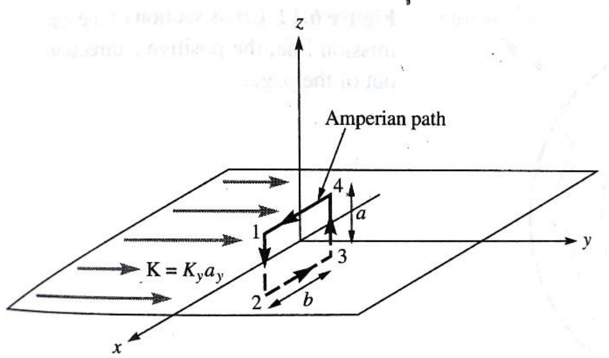
$$I = \int H_\phi \mathbf{a}_\phi \,.\, \rho d\phi \mathbf{a}_\phi = H_\phi \int \rho\, d\phi$$

$$\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi$$



- **H** due to infinite sheet current:

$$\oint_L \mathbf{H}.d\mathbf{l} = I_{enc} = k_y b$$

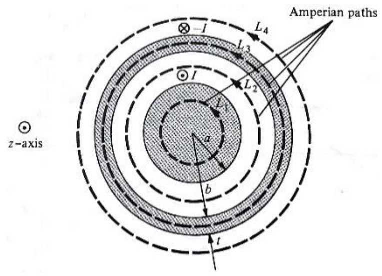


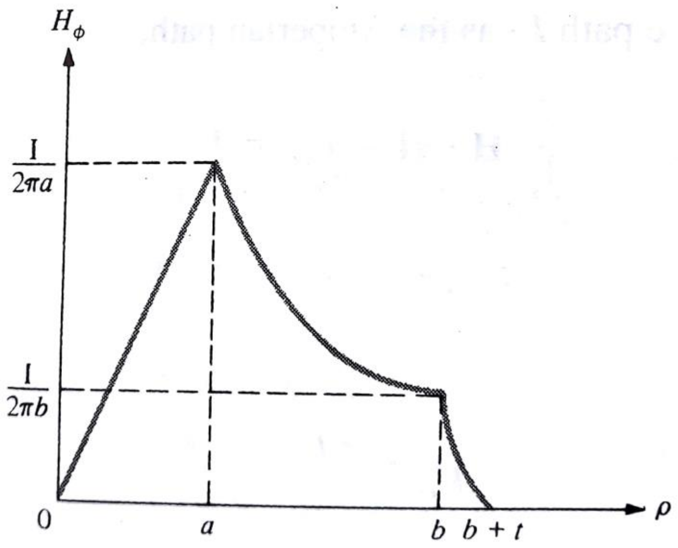
$$\mathbf{H} = \begin{cases} \frac{1}{2} k_y \mathbf{a}_x, & z > 0 \\ -\frac{1}{2} k_y \mathbf{a}_x, & z < 0 \end{cases}$$

$$\mathbf{H} = \frac{1}{2} \mathbf{K} \times \mathbf{a}_n$$

- **H** due to infinitely long coaxial transmission line:

$$\mathbf{H} = \begin{cases} \frac{I\rho}{2\pi a^2} \mathbf{a}_\phi, & 0 \le \rho \le a \\ \frac{I}{2\pi\rho} \mathbf{a}_\phi, & a \le \rho \le b \\ \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right] \mathbf{a}_\phi, & b \le \rho \le b + t \\ 0, & \rho \ge b + t \end{cases}$$



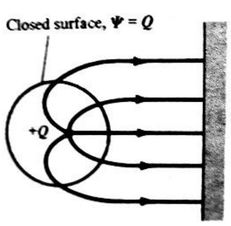


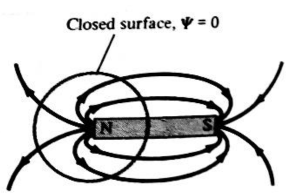
Magnetic Flux Density:

- $\mathbf{B} = \mu_o \mathbf{H}$
- μ_o:permiability of free space$= 4\pi \times 10^{-7}$H/m
- The magnetic flux through a surface S is given by

$$\psi = \int_S \mathbf{B}.\, d\mathbf{S}$$

ψ: Magnetic flux in webers (Wb)
B: Magnetic flux density in webers/m^2 or tesla (T)





- $\psi = \oint_S \mathbf{B}.d\mathbf{S} = 0$ -→An isolated magnetic charge does not exist.
- $\nabla.\mathbf{B} = 0$

Differential (Point Form)	Integral Form	Remarks
$\nabla.\mathbf{D} = \rho_v$	$\oint_S \mathbf{D}.d\mathbf{S} = \int_v \rho_v dv$	Gauss's Law
$\nabla.\mathbf{B} = 0$	$\oint_S \mathbf{B}.d\mathbf{S} = 0$	Nonexistence of magnetic monopole
$\nabla \times \mathbf{E} = 0$	$\oint_L \mathbf{E}.d\mathbf{l} = 0$	Conservative nature of electrostatic field
$\nabla \times \mathbf{H} = \mathbf{J}$	$\oint_L \mathbf{H}.d\mathbf{l} = \int_S \mathbf{J}.d\mathbf{S}$	Ampere's law

Magnetic Scalar and Vector Potentials

- $\mathbf{H} = -\nabla V_m$ if $\mathbf{J} = 0$
- $\mathbf{B} = \nabla \times \mathbf{A}$
- $V = \int \frac{dQ}{4\pi\epsilon_o r}$
- $\mathbf{A} = \int_L \frac{\mu_o I d\mathbf{l}}{4\pi R}$ (Line Current)
- $\mathbf{A} = \int_S \frac{\mu_o \mathbf{K} dS}{4\pi R}$ (Surface Current)
- $\mathbf{A} = \int_v \frac{\mu_o \mathbf{J} dv}{4\pi R}$ (Volume Current)
- $\mathbf{B} = \frac{\mu_o}{4\pi} \int_L \frac{I d\mathbf{l}' \times \mathbf{R}}{R^3}$
- $\mathbf{B} = \nabla \times \int_L \frac{\mu_o I d\mathbf{l}'}{4\pi R}$
- $\mathbf{A} = \int_L \frac{\mu_o I d\mathbf{l}'}{4\pi R}$
- $\psi = \oint_S \mathbf{B}.d\mathbf{S} = \oint_S \nabla \times \mathbf{A}.d\mathbf{S}$
- $\psi = \oint_L \mathbf{A}.d\mathbf{l}$

Forces due to Magnetic Fields

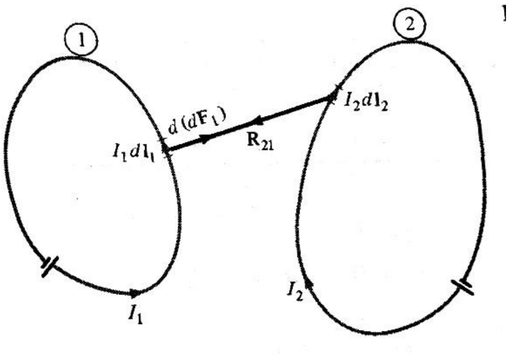
Force on a charged particle:

- $\mathbf{F}_e = Q\mathbf{E}$
- $\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$
- $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$

Force on a charged particle:

- $\mathbf{J} = \rho_v \mathbf{u}$
- $\mathbf{F} = \oint_L \ I d\mathbf{l} \times \mathbf{B}$
- $\mathbf{F} = \int_S \ \mathbf{K}\, dS \times \mathbf{B}$
- $\mathbf{F} = \int_v \ \mathbf{J}\, dv \times \mathbf{B}$
- $\mathbf{B}$: Force per unit current element

Force between two Current Elements:



- $d(d\mathbf{F}_1) = I_1 d\mathbf{l}_1 \times d\mathbf{B}_2$

  $d\mathbf{B}_2 = \frac{\mu_o I_2 d\mathbf{l}_2 \times \mathbf{a}_{R_{21}}}{4\pi R_{21}^2}$

  $d(d\mathbf{F}_1) = \frac{\mu_o I_1 d\mathbf{l}_1 \times (I_2 d\mathbf{l}_2 \times \mathbf{a}_{R_{21}})}{4\pi R_{21}^2}$

$$\mathbf{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\mathbf{l}_1 \times (d\mathbf{l}_2 \times \mathbf{a}_{R_{21}})}{R_{21}^2}$$

- This is also known as Ampere's Force Law
- $\mathbf{F}_2 = -\mathbf{F}_1$