Unit-3 Microwave Tubes

- O-Type (original type) (Linear-Beam Tubes)
- M-Type (Crossed-field tubes)

→ μWave Tubes are used as signal sources at μwave frequencies.

→ In Linear-Beam Tubes, 2-Cavity klystron and reflex klystron are mostly used.

→ The helix traveling-wave Tube (TWT), the coupled-Cavity TWT, the forward-wave amplifier (FWA), & backward-wave amplifier & oscillator are the other O-type tubes that are available.

→ Twystron is a hybrid amplifier that uses combinations of klystron and TWT components.

→ TWT are suitable for amplification purpose.

→ Klystron and TWT amplifiers can deliver a peak power output upto 30MW at the frequency of 10 GHz.

→ The gain of these tubes is on the order of 30 to 50 dB.

→ Microwave Tubes are the choice for high-power transmitter outputs.

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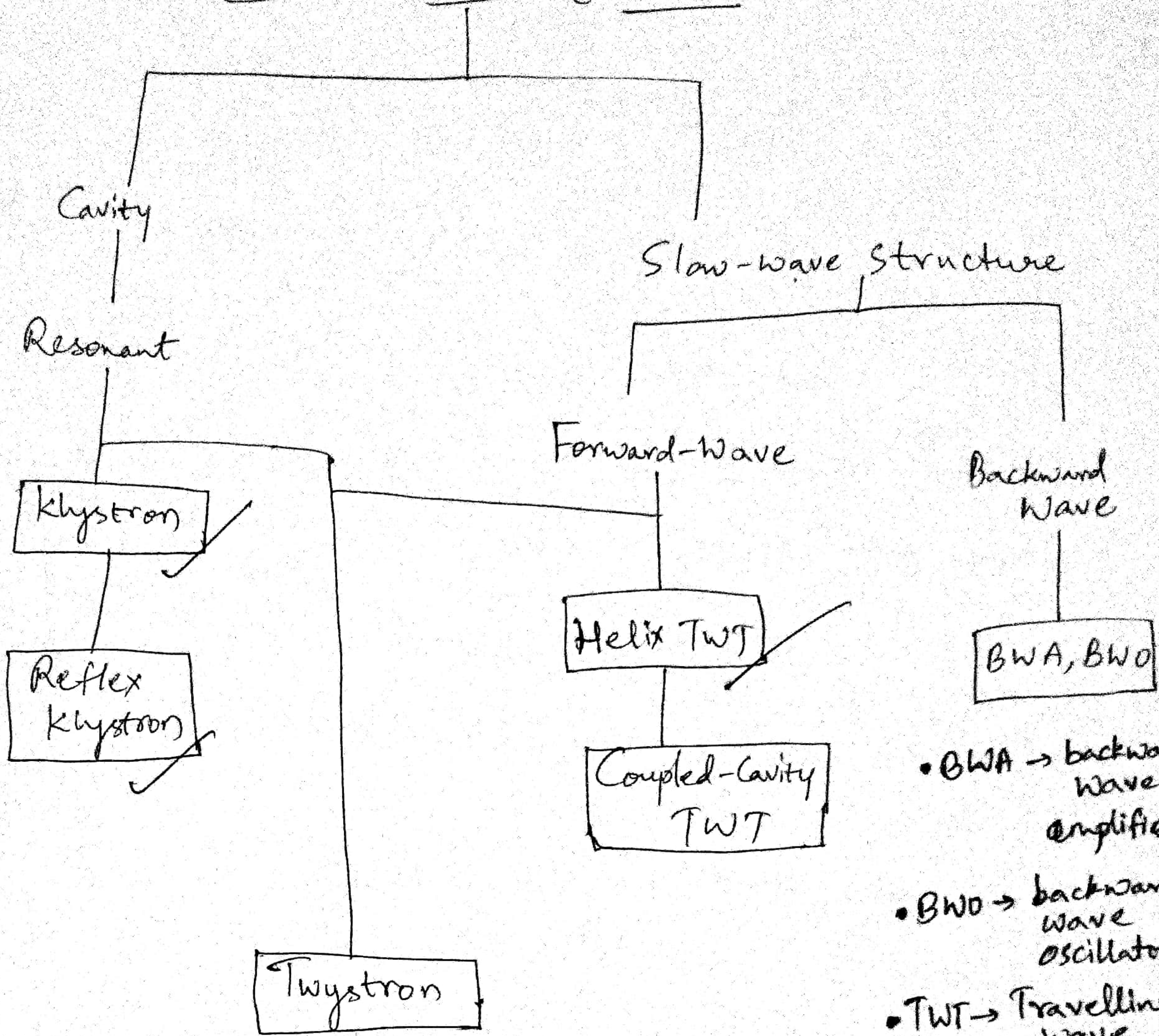
Page 2

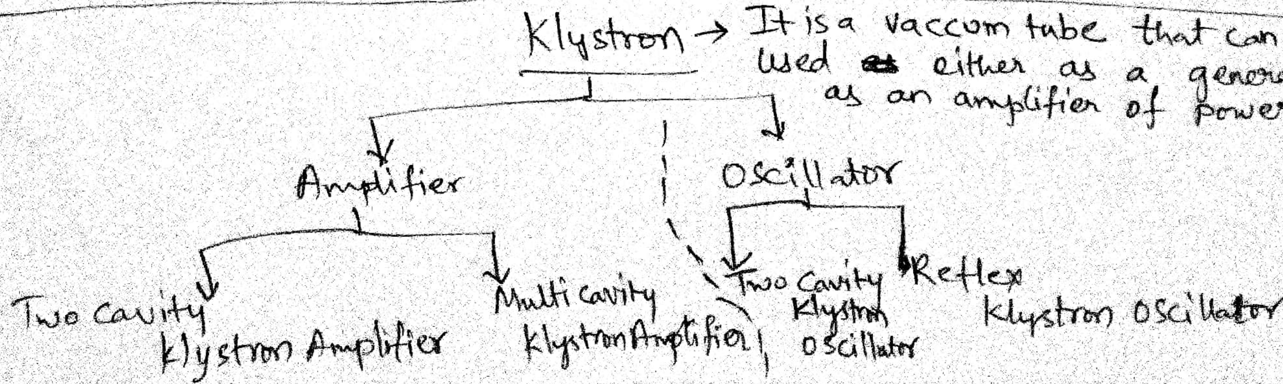
Classification of Linear Beam Tubes (O-type)



- BWA → backward wave amplifier
- BWO → backward wave oscillator.
- TWT → Travelling wave Tube.

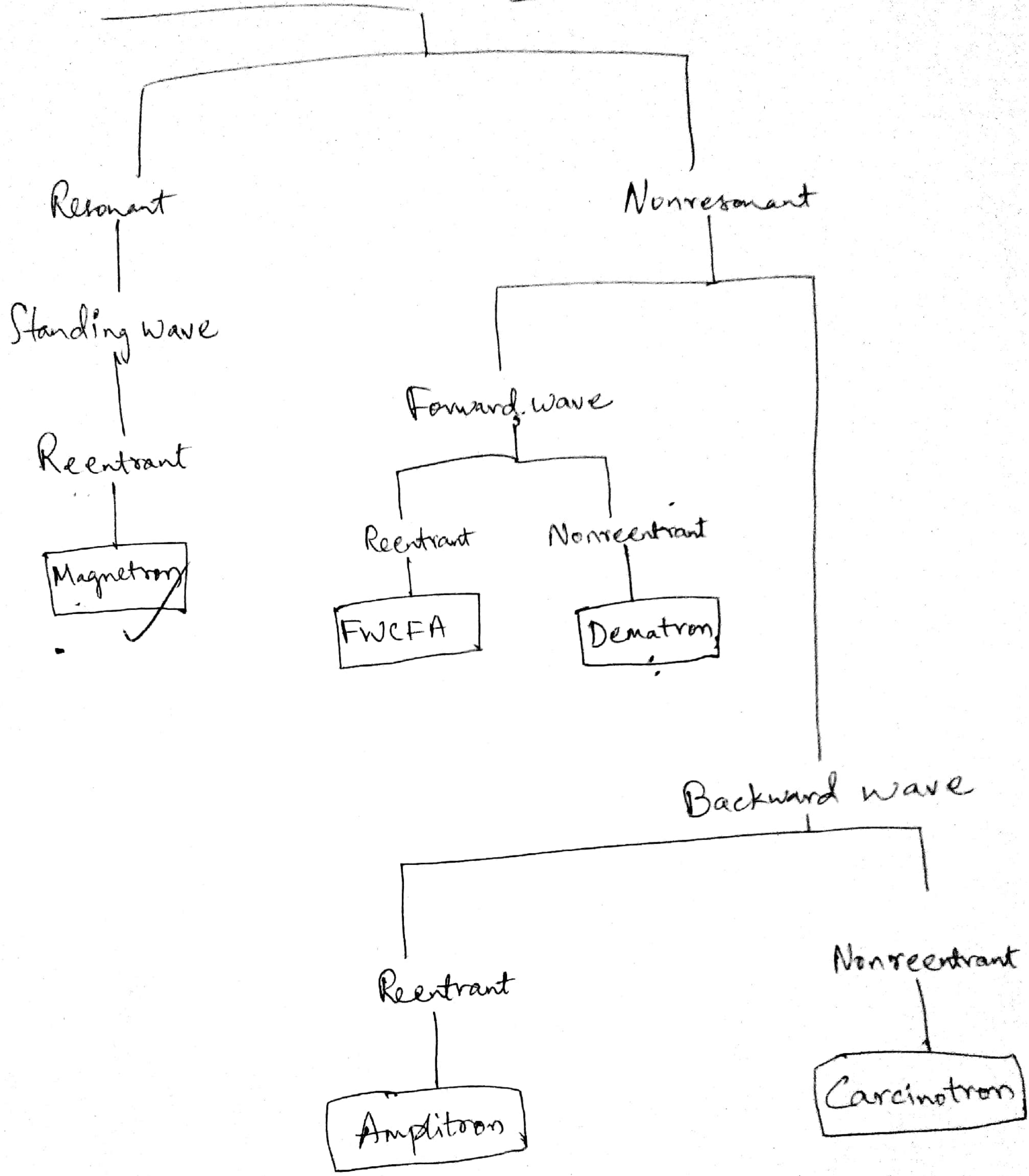
→ In Linear beam tubes, the dc magnetic field and dc electric field ~~od be~~ are parallel to each other whereas in Crossedfield tubes, they are $\perp^{lar}$ to each other.

Klystron → It is a vaccum tube that can be used ~~as~~ either as a generator or as an amplifier of power.



Page 3

Classification of Crossed-field Tubes (M-Type) 2



FWCFA → Forward wave crossed-field amplifiers.

Amplitron (or) BWCFA → Backward wave crossed-field amplifier

Carcinotrons (or) BWCFO → Backward wave crossed-field oscillators

8.5.1 Two Cavity Klystron Amplifier

A two cavity klystron amplifier is shown in Fig. 8.9 which is basically a velocity modulated tube. Here a high velocity electron beam is formed, focussed and sent down along a glass tube through an input cavity (buncher), a field free drift space and an output cavity (catcher) to a collector electrode/anode. The anode is kept at a positive potential with respect to cathode. The electron beam passes through a gap '*A*' consisting of two grids of the buncher cavity separated by a very small distance and two other grids of the catcher cavity with a small gap '*B*'. The input and output are taken from the tube via resonant cavities with the aid of coupling loops.

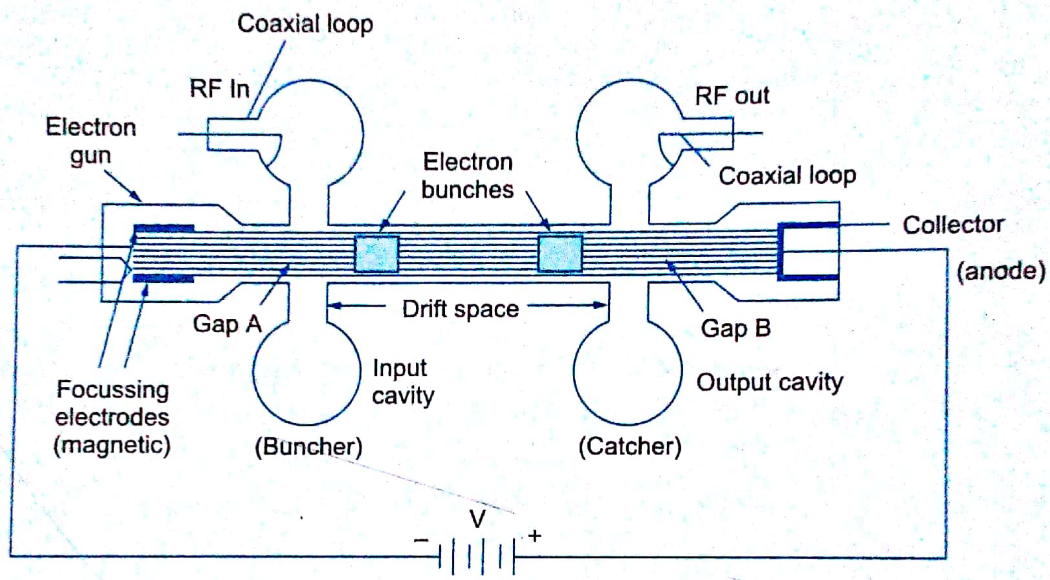


Fig. 8.9 Two cavity klystron amplifier.

Operation

The RF signal to be amplified is used for exciting the input buncher cavity thereby developing an alternating voltage of signal frequency across the gap *A*.

Let us now consider the effect of this gap voltage on the electron beam passing through gap *A*. The situation is best explained by means of an Applegate diagram shown in Fig. 8.10. At point *B*' on the input *RF* cycle, the alternating voltage is zero and going positive. At this instant, the electric field across gap *A* is zero and an electron which passes through gap *A* at this instant is unaffected by the *RF* signal. Let this electron be called the reference electron e_R which travels with an unchanged velocity $v_o = \sqrt{2\,eV/m}$ where *V* is the anode to cathode voltage.

At point '*C*' of the input *RF* cycle an electron which leaves gap *A* later than reference electron e_R, called the late electron e_l is subjected to maximum positive *RF* voltage and hence travels towards gap *B* with an increased velocity ($v > v_o$) and this electron tries to overtake the reference electron e_R.

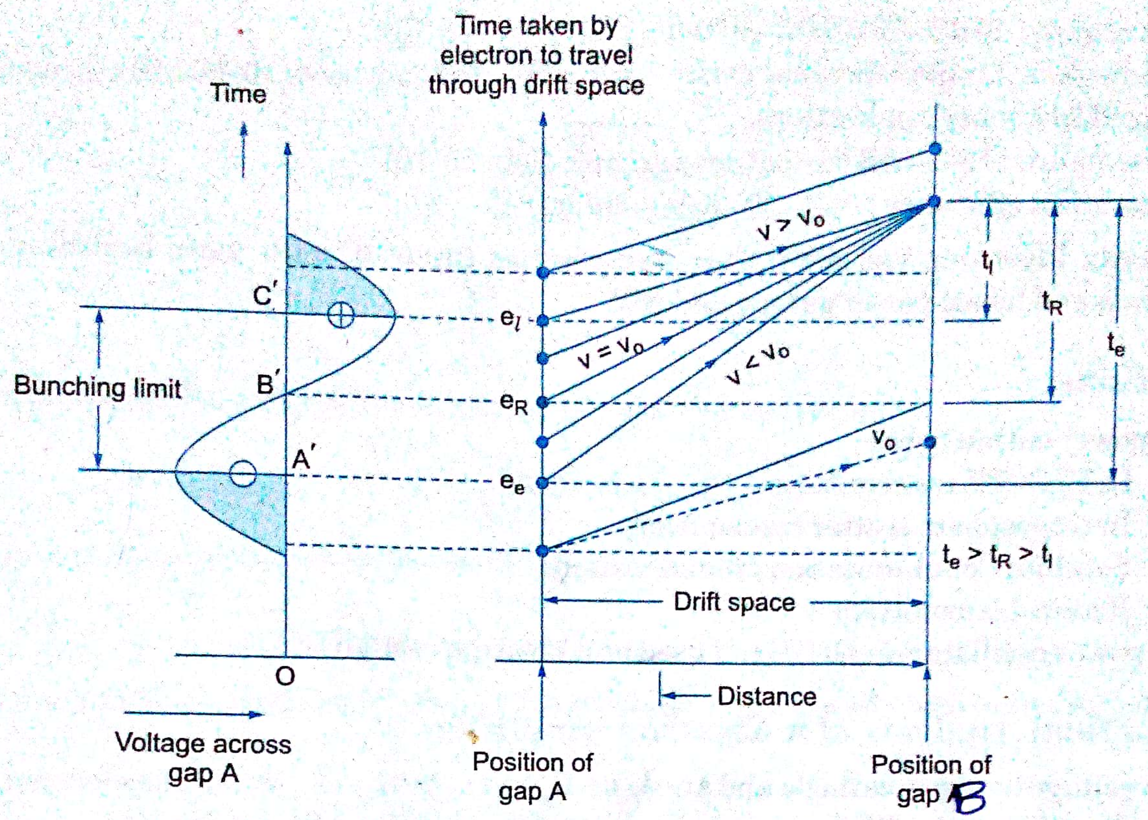


Fig. 8.10 Apple gate diagram of a klystron amplifier.

Similarly an early electron e_e that passes the gap 'A' slightly before the reference electron e_R is subjected to a maximum negative field. Hence this early electron is decelerated and travels with a reduced velocity v_o. This electron e_e falls back and reference electron e_R catches up with the early electron.

Therefore the velocity of electron varies in accordance with *RF* input voltage, resulting in *velocity modulation* of the electron beam.

As a result of these actions, the electrons in the bunching limit (between points '*A*″ and '*C*″) gradually bunch together as they travel down the drift space, from gap *A* to gap *B*. The pulsating stream of electrons pass through gap *B* and excite oscillations in the output cavity (catcher). The density of electrons passing the gap *B* vary cyclically with time, that is the electron beam contains an ac current and is current modulated. The drift space converts the velocity modulation into current modulation.

Bunching occurs only once per cycle centered around the reference electron. With proper design (optimum gap widths, anode to cathode voltage, drift space length etc), a little *RF* power applied to the buncher cavity results in large beam currents at the catcher cavity with a considerable power gain.

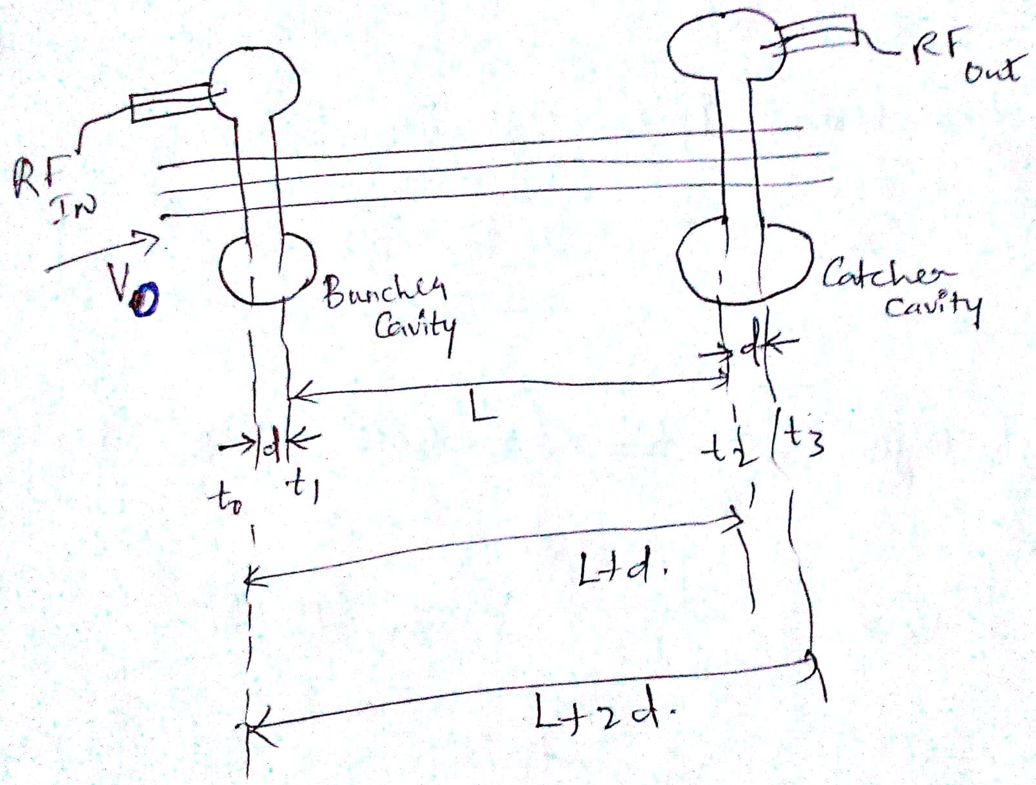
Performance Characteristics

1. Frequency : 250 MHz to 100 GHz. (60 GHz nominal)
2. Power : 10 kW–500 kW (CW) 30 MW (pulsed)

- Amplifier
  - Two cavity Klystron Amplifier
  - Multi Cavity Klystron Amplifier
- Oscillator
  - Two Cavity Klystron Oscillator
  - Reflex Klystron Oscillator

Mathematical Analysis of Two cavity klystron Amplifier :

(a) Velocity Modulation Process :



→ When electrons are first accelerated by the high dc voltage 'V_0' before entering the buncher grids (Gap A), their velocity will be

$$v_0 = \sqrt{\frac{2eV_0}{m}} \quad — (1)$$

→ When a RF input signal is applied to the input terminal, the gap voltage between the buncher grids appears as

$$V_s = V_1 \sin\omega t \quad — (2)$$

Where, $\boxed{V_1 << V_0}$

→ Transit time at the buncher gap distance 'd' is

$$\tau = \frac{d}{v_0} = t_1 - t_0 \quad — (3)$$

→ Average gap transit angle θ_g is given as

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0} \quad — (4)$$

→ Average voltage in the buncher gap is given as.

$$\langle V_s \rangle = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin\omega t \, dt \quad — (5)$$

$$= \frac{V_1}{\tau}\int_{t_0}^{t_1} \sin\omega t \, dt$$

$$\langle V_s \rangle = \frac{V_1}{T}\left[-\frac{1}{\omega}\cos\omega t\right]_{t_0}^{t_1}$$

$$= \frac{-V_1}{\omega T}\left[\cos\omega t_1 - \cos\omega t_0\right]$$

$$\langle V_s \rangle = \frac{V_1}{\omega T}\left[\cos\omega t_0 - \cos\omega t_1\right]$$

W.K.T.

$$\omega t_1 - \omega t_0 = \frac{\omega d}{v_0} \Rightarrow \underline{\omega t_1} = \omega t_0 + \frac{\omega d}{v_0} \quad \text{— (6)}$$

$$\Rightarrow \langle V_s \rangle = \frac{V_1}{\omega T}\left[\cos\omega t_0 - \cos\left(\omega t_0 + \frac{\omega d}{v_0}\right)\right]$$

Replace

$$\left[\frac{\omega d}{v_0} = \frac{\omega d}{2v_0} + \frac{\omega d}{2v_0}\right]$$

$$\theta_g = \frac{\theta_g}{2} + \frac{\theta_g}{2}$$

$$= \frac{V_1}{\omega T}\left[\cos\left(\omega t_0 + \frac{\theta_g}{2} - \frac{\theta_g}{2}\right) - \cos\left(\omega t_0 + \frac{\theta_g}{2} + \frac{\theta_g}{2}\right)\right]$$

$$\boxed{\begin{array}{l} A \rightarrow \omega t_0 + \frac{\theta_g}{2} \\ B \rightarrow \frac{\theta_g}{2} \end{array}}$$

$$\cos(A-B) - \cos(A+B) = 2\sin A \sin B.$$

$$\therefore \langle V_s \rangle = \frac{V_1}{\omega T}\left[2\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\sin\frac{\theta_g}{2}\right]$$

(or)

$$\langle V_s \rangle = \frac{V_1}{\omega\tau}\left[2\sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right)\sin\left(\frac{\omega d}{2v_0}\right)\right]$$

$$\langle V_s \rangle = \frac{V_1}{\left(\frac{\omega d}{2v_0}\right)}\left[\sin\left(\frac{\omega d}{2v_0}\right)\sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right)\right]$$

$$\langle V_s \rangle = \frac{V_1}{\frac{\theta_g}{2}}\left[\sin\left(\frac{\theta_g}{2}\right)\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$\langle V_s \rangle = V_1\left[\frac{\sin\left(\frac{\theta_g}{2}\right)}{\left(\frac{\theta_g}{2}\right)}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$\boxed{\beta_i = \frac{\sin(\theta_g/2)}{(\theta_g/2)}} \rightarrow$$ Beam-coupling coefficient of input cavity gap.

$$\langle V_s \rangle = V_1\left[\beta_i \sin(\omega t_0 + \theta_g/2)\right]$$

Now, the exit velocity from buncher gap is given by

$$v_0(t_1) = \sqrt{\frac{2e}{m}\left(V_0 + \langle V_s \rangle\right)}$$

$$v_0(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$v_0(t_1) = \sqrt{\frac{2e}{m}V_0\left[1 + \frac{\beta_i V_1}{V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$\boxed{\frac{\beta_i V_1}{V_0} \rightarrow \text{depth of velocity modulation.}}$$

W.K.T., $V_1 << V_0$ & by using binomial expansion under assumption of $(\beta_i V_1 << V_0)$

we get,

$$v_0(t_1) = \sqrt{\frac{2eV_0}{m}}\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

(neglecting higher order terms)

$$\boxed{(1+x)^{1/2} = 1 + \frac{x}{2} + \frac{x^2}{2!} + \cdots}$$

$$\therefore v_0(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

→ equation of velocity modulation

$$(1+x)^{1/2} = 1 + \frac{x}{2} + \frac{x^2}{2!} + \cdots$$

Velocity Modulation :. (Another form)

→ Average voltage in buncher gap,

$$\langle V_s \rangle = \frac{V_1}{\omega \tau}\left[\cos \omega t_0 - \cos \omega t_1\right]$$

$$\rightarrow \omega t_1 - \omega t_0 = \frac{\omega d}{v_0}$$

$$\Rightarrow \underline{\omega t_1} = \omega t_0 + \frac{\omega d}{v_0}$$

But, $\omega t_0 = \omega t_1 - \frac{\omega d}{v_0}$

$$\Rightarrow \langle V_s \rangle = \frac{V_1}{\omega \tau}\left[\cos\left(\omega t_1 - \frac{\omega d}{v_0}\right) - \cos \omega t_1\right]$$

$$\Rightarrow \langle V_s \rangle = \frac{V_1}{\omega \tau}\left[\cos\left(\omega t_1 - \frac{\theta_g}{2} - \frac{\theta_g}{2}\right) - \cos\left(\omega t_1 - \frac{\theta_g}{2} + \frac{\theta_g}{2}\right)\right]$$

Replace

$$\frac{\omega d}{v_0} = \frac{\omega d}{2v_0} + \frac{\omega d}{2v_0}$$

$$\theta_g = \frac{\theta_g}{2} + \frac{\theta_g}{2}$$

Using,

$$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$$

$A \rightarrow \omega t_1 - \frac{\theta_g}{2}$

$B \rightarrow \frac{\theta_g}{2}$

we get,

$$\Rightarrow \langle V_s \rangle = \frac{V_1}{\omega \tau}\left[2 \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \sin \frac{\theta_g}{2}\right]$$

$$= \frac{V_1}{\frac{\theta_g}{2}}\left[\sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \sin \frac{\theta_g}{2}\right]$$

$$\Rightarrow \langle V_s \rangle = V_1 \left[\beta_i \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

Now, exit velocity from buncher gap is

$$v_0(t_1) = \sqrt{\frac{2e[V_0 + \langle V_s \rangle]}{m}}$$

$$v_0(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]}$$

$$v_0(t_1) = \sqrt{\frac{2eV_0}{m}\left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]}$$

$$\boxed{v_0 = \sqrt{\frac{2eV_0}{m}}}$$

$$v_0(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]^{1/2}$$

By using binomial expansion and neglecting higher order terms,

$$\Rightarrow v_0(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$\boxed{(1+x)^{1/2} = 1 + \frac{x}{2} + \frac{x^2}{2!} + \dots}$$

Bunching Process :

→ Once the electron leaves the buncher cavity, they drift with a velocity,

$$v_0(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \quad \text{I}$$

$(V_1 << V_0)$

along in the field-free space between the two cavities.

→ The effect of velocity modulation produces bunching of the electron beam or current modulation.

$\omega t = \frac{\pi}{2}$

$t = \frac{\pi}{2\omega}$

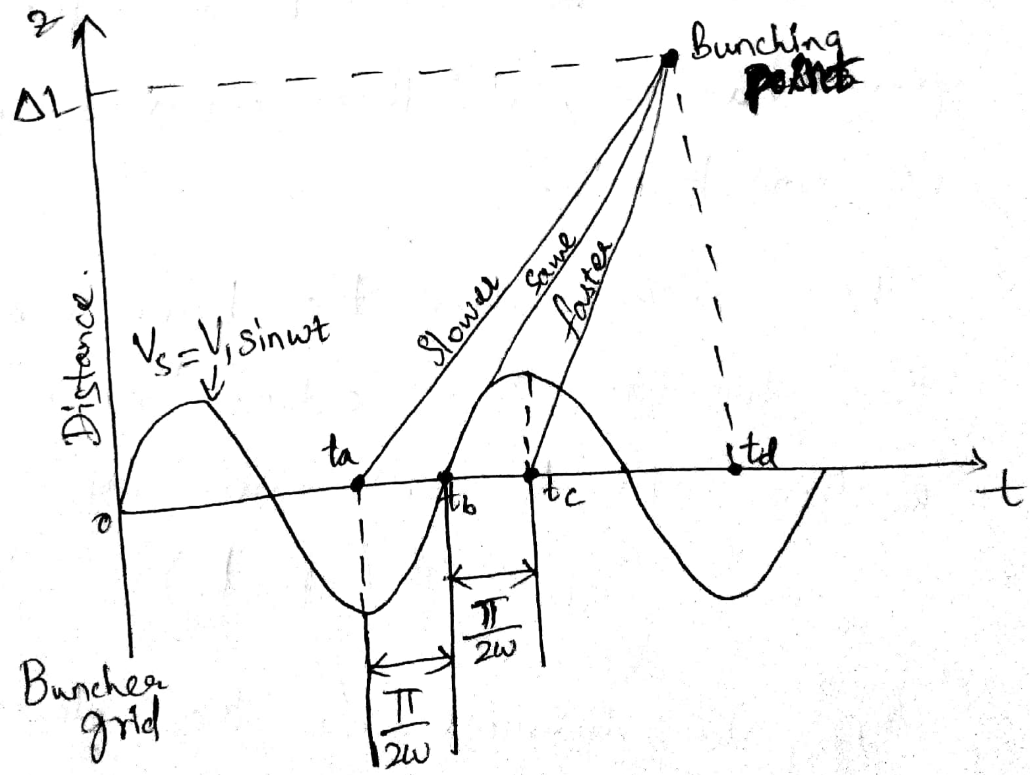


fig.1 Bunching distance.

→ The electrons that pass the buncher at $V_s = 0$ travel through with velocity v_0. Those electrons that pass the buncher during positive half cycles of V_s travel faster than the electrons that passed the buncher when $V_s = 0$. Similarly, during negative half cycle of V_s, some electrons will travel slower than the electrons that passed the buncher when $V_s = 0$.

→ Fig 1. shows the trajectories of minimum, zero & maximum electron acceleration.

→ Now at a distance of ΔL along the drift space from buncher, all these electrons form electron bunch.

→ The distance from the buncher grid to the location of electron bunch for that electron at t_b is given by,

$$\Delta L = v_0 (t_d - t_b) \quad \text{—— (1)}$$

→ Illy, the distances for the electrons at t_a & t_c are

$$\Delta L = v_{min} (t_d - t_a) \quad \text{— (2)}$$

$$\Delta L = v_{max}(t_d - t_c) \quad - (3)$$

→ from the fig1.,

$$\left.\begin{aligned} t_a &= t_b - \frac{\pi}{2\omega} \\ &\& \\ t_c &= t_b + \frac{\pi}{2\omega} \end{aligned}\right\} (4)$$

→ from (2), (3) & (4)

$$\Delta L = v_{min}\left(t_d - t_b + \frac{\pi}{2\omega}\right) \quad - (5)$$

$$\Delta L = v_{max}\left(t_d - t_b - \frac{\pi}{2\omega}\right) \quad - (6)$$

→ Now, finding out v_{max} & v_{min} from (I)

$$v_{max} = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\right] \quad \text{[at positive half cycle } \pi/2\text{]} \quad - (7)$$

$$v_{min} = v_0\left[1 - \frac{\beta_i V_1}{2V_0}\right] \quad - (8) \quad \text{[at -ve half cycle } -\pi/2\text{]}$$

→ Now,

$$\Delta L = v_{max}\left(t_d - t_b - \frac{\pi}{2\omega}\right)$$

$$= v_0\left[1 + \frac{\beta_i V_1}{2V_0}\right]\left(t_d - t_b - \frac{\pi}{2\omega}\right)$$

$$\Delta L = \left(v_0\left[t_d - t_b - \frac{\pi}{2\omega}\right]\right)$$

$$+\left(\left[v_0 \frac{\beta_i V_1}{2V_0}\right]\left[t_d - t_b - \frac{\pi}{2\omega}\right]\right)$$

$$\Delta L = \left[v_0 (t_d - t_b)\right] + \left[-\frac{v_0 \pi}{2\omega} + \frac{v_0 \beta_i V_1}{2V_0}(t_d - t_b)\right.$$
$$\left.-\frac{v_0 \pi}{2\omega}\frac{\beta_i V_1}{2V_0}\right] \quad -(9)$$

$\rightarrow$ |||y, $\Delta L = v_{min}\left(t_d - t_b + \frac{\pi}{2\omega}\right)$

$$= v_0\left[1 - \frac{V_1 \beta_i}{2V_0}\right]\left(t_d - t_b + \frac{\pi}{2\omega}\right)$$

$$\Delta L = \left[v_0 (t_d - t_b)\right] + \left[\frac{v_0 \pi}{2\omega} - \frac{v_0 \beta_i V_1}{2V_0}(t_d - t_b)\right.$$
$$\left.-\frac{v_0 \pi}{2\omega}\frac{\beta_i V_1}{2V_0}\right] \quad -(10)$$

from eq. 9 & 10

$\rightarrow$ The necessary condition for those electrons at t_a, t_b & t_c to meet at the same distance ΔL is ~~from eqn 9 & from eqn 10~~

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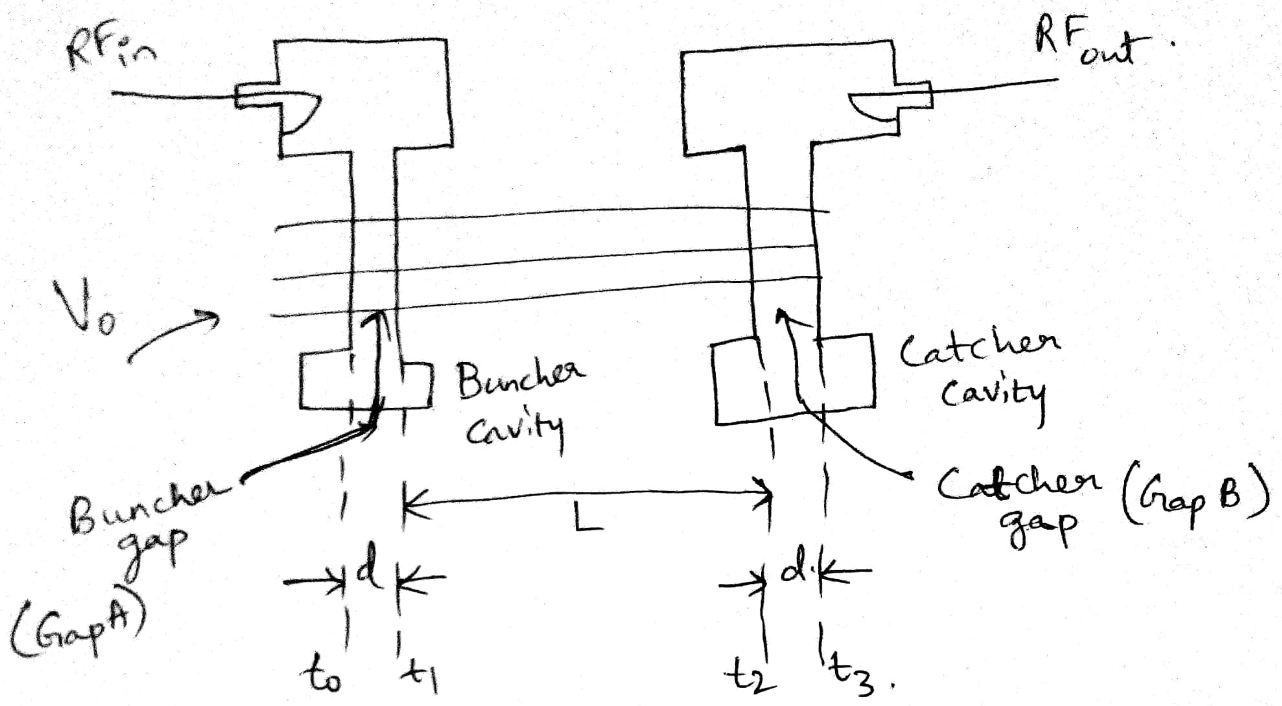
(2)

Consequently, $t_d - t_b \approx \cancel{\quad} \dfrac{\pi V_0}{\omega_{Bi} V_1}$

$$\therefore \boxed{\Delta L = v_0 \frac{\pi V_0}{\omega_{Bi} V_1}}$$

Current at catcher gap :-



→ The transit time for an electron to travel a distance 'L' as shown in fig. above is

$$T = t_2 - t_1 = \frac{L}{v_0(t_1)}$$

$$\Rightarrow T = \frac{L}{v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$T = \frac{L}{v_0}\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]^{-1}$$

By using binomial expansion of $(1+x)^{-1} = 1 - x + x^2 - \dots$

& neglecting higher order terms,

we get,

$$t_2 - t_1 = T = \frac{L}{v_0}\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \quad \text{--- (1)}$$

$$\Rightarrow T = t_2 - t_1 = T_0\left[1 - \beta_i V_1/2V_0 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad \left[T_0 = \frac{L}{v_0}\right]$$

→ In terms of radians, eq. (1) can be written as

$$\omega T = \omega(t_2 - t_1) = \frac{\omega L}{v_0}\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

Here, $\frac{L}{v_0} = T_0$ → dc transit time.

$$\frac{\omega L}{v_0} = \omega T_0 = \theta_0 \rightarrow \text{dc transit angle.}$$

$$\Rightarrow \omega T = \omega t_2 - \omega t_1 = \theta_0 - \frac{\theta_0 \beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

Here, $\frac{\theta_0 \beta_i V_1}{2V_0} = X$ → Bunching parameter of Klystron.

$$\Rightarrow \omega T = \omega t_2 - \omega t_1 = \theta_0 - X\sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \quad \text{--- (2)}$$

③

W.K.T.

$\rightarrow \tau = t_1 - t_0 = \frac{d}{v_0}$

$\rightarrow \omega\tau = \theta_g = \omega t_1 - \omega t_0 = \frac{\omega d}{v_0}$

$\Rightarrow \omega t_1 = \omega t_0 + \frac{\omega d}{v_0} = \omega t_0 + \theta_g = \omega t_0 + \frac{\theta_g}{2} + \frac{\theta_g}{2}$.

$\Rightarrow \tau = t_1 - t_0 \Rightarrow \omega\tau = \omega t_1 - \omega t_0$ — (i)

$\Rightarrow \theta_g = \omega t_1 - \omega t_0$

$\Rightarrow \omega t_1 = \frac{\theta_g}{2} + \frac{\theta_g}{2} + \omega t_0$

from ② & (i)

$$\omega t_2 - \left(\omega t_0 + \frac{\theta_g}{2} + \frac{\theta_g}{2}\right) = \theta_0 - X \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

$$\Rightarrow \omega t_2 - \left(\theta_0 + \frac{\theta_g}{2}\right) = \left(\omega t_0 + \frac{\theta_g}{2}\right) - X \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

Where, $\left(\omega t_0 + \frac{\theta_g}{2}\right) \rightarrow$ Buncher cavity departure angle.

$\left[\omega t_2 - \left(\theta_0 + \frac{\theta_g}{2}\right)\right] \rightarrow$ Catcher cavity arrival angle.

from ①

$$t_2 = t_1 + T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

w.k.T,

$$\tau = t_1 - t_0 \Rightarrow t_1 = \tau + t_0$$

$$\Rightarrow t_2 = \tau + t_0 + T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \quad \text{--- ③}$$

Differentiate. eq. ③ w.r.t t_0,

$$\Rightarrow \frac{dt_2}{dt_0} = 0 + 1 + 0 - \frac{\omega T_0 \beta_i V_1}{2V_0}\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

$$\Rightarrow \frac{dt_2}{dt_0} = 1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right) \quad \text{--- ④}$$

→ At buncher gap a charge 'dQ_0' passing through at a time interval 'dt_0' is given as. $\boxed{dQ_0 = I_0 dt_0}$

$I_0 \rightarrow$ dc current.

→ From principle of conservation of charges this same amount of charge 'dQ_0' also passes the catcher gap at a later time interval 'dt_2'. Hence, $\boxed{I_0 dt_0 = I_2 dt_2}$

~~from ④ & ⑤~~

$$\Rightarrow \frac{dt_2}{dt_0} = \frac{I_0}{I_2} \quad \text{--- ⑤}$$

from (4) & (5) (7)

$$\boxed{I_2 = \frac{I_0}{1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)}}$$

using relation,

$$t_2 = t_0 + \tau + T_0$$

$$\omega t_2 = \omega t_0 + \omega\tau + \omega T_0$$

$$\Rightarrow \omega t_2 = \omega t_0 + \theta_g + \theta_0 = \omega t_0 + \frac{\theta_g}{2} + \frac{\theta_g}{2} + \theta_0$$

$$\Rightarrow \omega t_0 + \frac{\theta_g}{2} = \omega t_2 - \theta_0 - \frac{\theta_g}{2}.$$

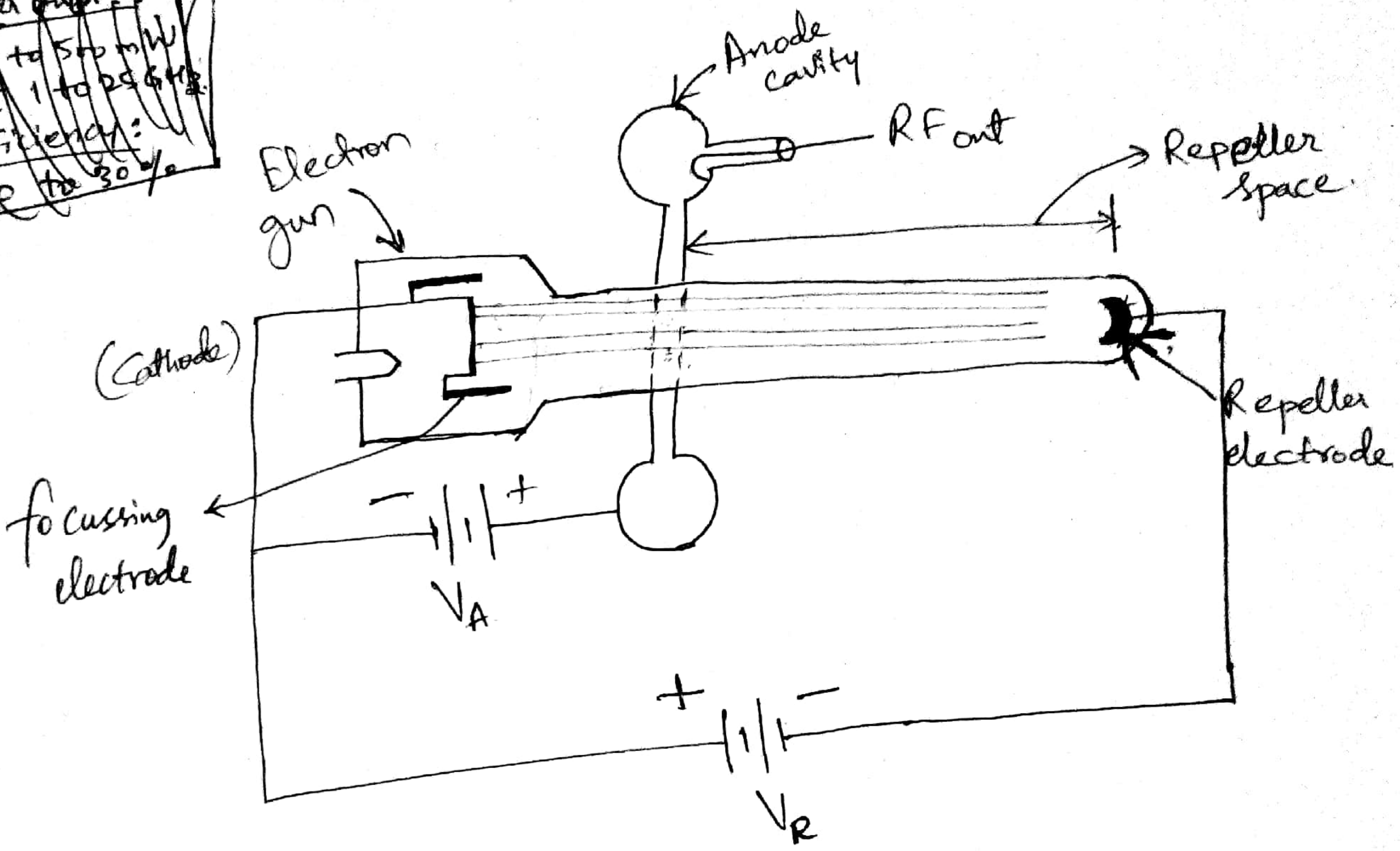
in terms of t_2, the current is.

$$\boxed{I_2 = \frac{I_0}{1 - X\cos\left(\omega t_2 - \theta_0 - \frac{\theta_g}{2}\right)}}$$

Reflex Klystron Oscillator :-

~~Power outputs 10 to 500 mW at 1 to 25 GHz. Efficiency: 20 to 30%~~

fig1. Reflex klystron Oscillator

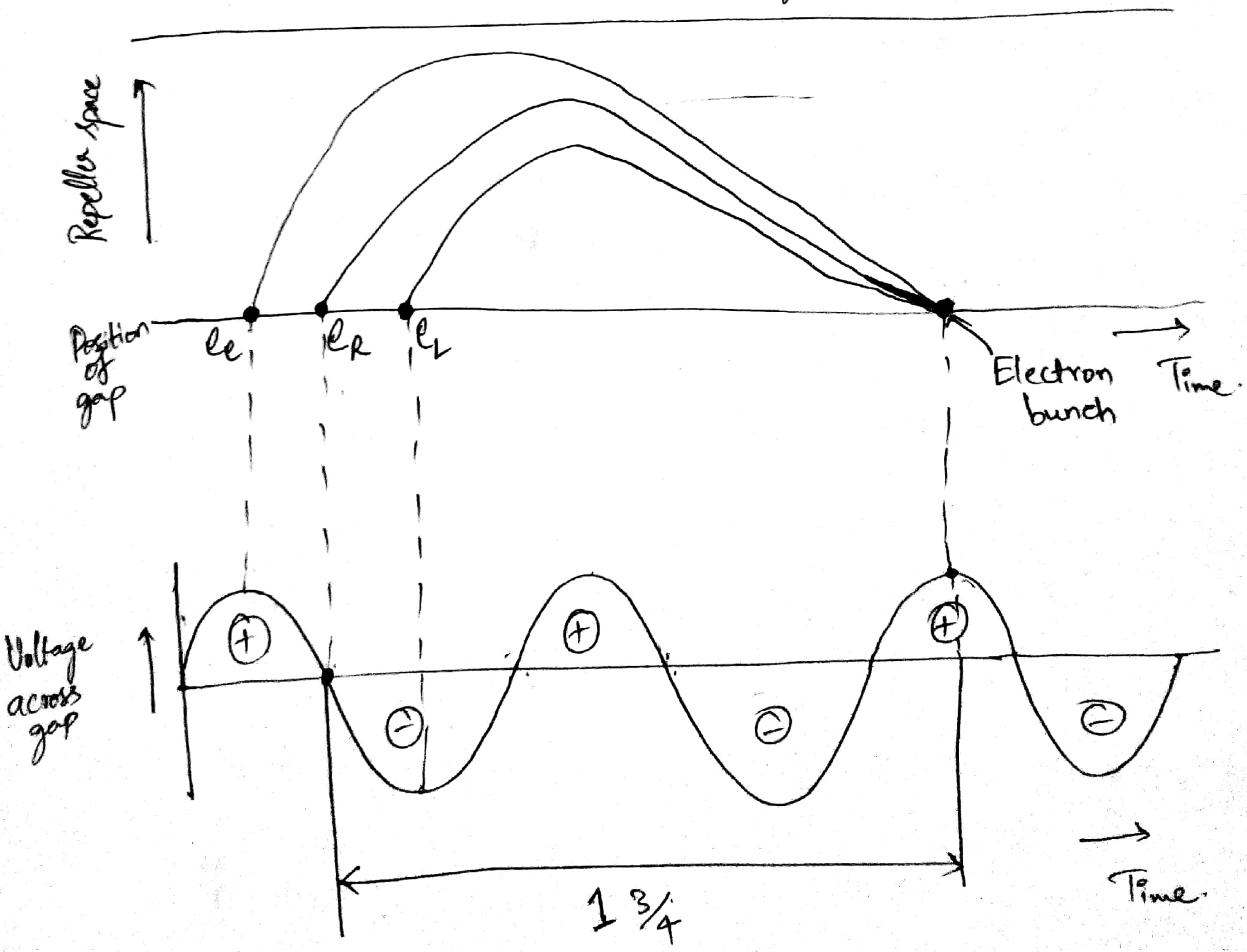


1. It consists of an electron gun, a filament surrounded by Cathode and a focussing electrode at cathode potential as shown in fig 1.
2. The electron beam is accelerated towards the anode Cavity.
3. After passing the Cavity gap, electrons travel towards a repeller electrode, which is at negative potential V_R.
4. The electrons never reach the repeller because of the negative field and are returned back towards the gap.

(5) Under suitable conditions, the electrons give more energy to the gap than they took from the gap on their forward journey and oscillations are sustained.

Operation :-

(1) Initially inside the cavity there will be oscillations which will be sustained later by device operation. This can be explained by applegate diagram.



(10) To general, the optimum transit time is given as $T = n + \frac{3}{4}$; where 'n' is any integer.
i.e., $1\frac{3}{4}, 2\frac{3}{4}, 3\frac{3}{4} \ldots$

② The RF voltage that is produced across the gap by the cavity oscillations act on the electrons to cause velocity modulation.

③ 'e_R' is the reference electron that passes through gap when gap voltage is zero.

④ 'e_e' is the early electron that passes through gap before 'e_R' and is accelerated by positive voltage across gap. Since it penetrates deep into repeller space and ~~has~~ ~~more~~ also takes more time to return back to cavity gap.

⑤ 'e_l' is the late electron that passes through gap after 'e_R' and is deaccelerated by negative voltage across gap. Since its penetration into repeller space is less and also takes less time to return back to cavity gap.

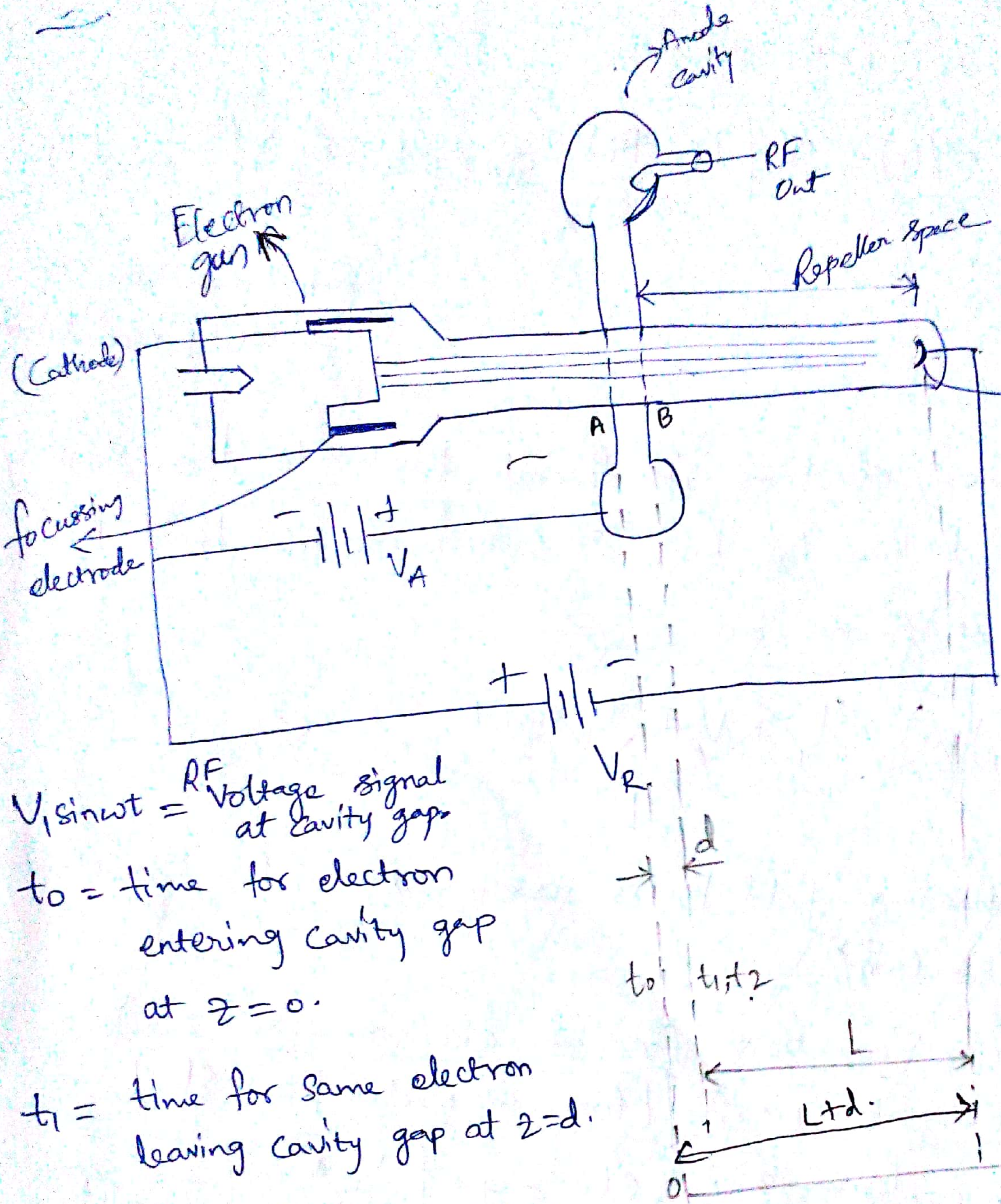
⑥ All these electrons moves towards the repeller and gets reflected by the negative voltage ~~of~~ the repeller.

⑦ Again they come back towards cavity gap and fall on the walls of the cavity and forms as an electron bunch.

⑧ Bunches occur once per cycle and these bunches transfer maximum energy to the gap to get sustained oscillations.

⑨ For ~~of~~ sustained oscillations, i.e, to give maximum energy to cavity gap, the time taken by electrons to travel into repeller space & back to gap (called transit time) must have an optimum value.

Mathematical Analysis of Reflex Klystron Oscillator ;



$V_1 << V_A$

Z is a direction along Repeller space. nothing but z-ending distance

$V_1 \sin\omega t$ = RF Voltage signal at cavity gap.

t_0 = time for electron entering cavity gap at $z=0$.

t_1 = time for same electron leaving cavity gap at $z=d$.

t_2 = time for same electron returned by retarding field to $z=d$ and collected on walls of cavity. → time for same electron to reach repeller electrode and come back to cavity gap at $z=d$.

→ The electron entering the cavity gap from cathode at $z=0$ & time t_0 is assumed to have uniform velocity

$$v_0 = \sqrt{\frac{2eV_A}{m}} = 0.593 \times 10^6 \sqrt{V_A} \text{ m/s} \quad \text{—(1)}$$

→ The same electron leaves the cavity gap at $z=d$ at time t_1 with velocity,

$$v_0(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_A}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \quad —②$$

[from equation of velocity modulation]

→ The same electron is forced back to the cavity to $z=d$ and time t_2 by retarding electric field E, which is given by,

$$E = \frac{V_R + V_A + V_1 \sin \omega t}{L} \quad — (3)$$

$$\underline{|V_1 \sin \omega t| <<< (V_R + V_A)}$$

→ Force equation,

mass × acceleration

$$m\left(\frac{d^2 z}{dt^2}\right) = -eE$$

$$m\left(\frac{d^2 z}{dt^2}\right) = -e\left[\frac{V_R + V_A}{L}\right] \quad — (4)$$

Integrating eq. (4) over interval $[t, t_1]$

$$\frac{dz}{dt} = \frac{-e}{mL}(V_R + V_A)[t]_{t_1}^{t} + K_1$$

$$\frac{dz}{dt} = \frac{-e}{mL}(V_R + V_A)(t - t_1) + K_1 \quad — (5)$$

At $t = t_1 \Rightarrow \frac{dz}{dt} = v_0(t_1)$, — (6)

from (5) & (6)

$F = m \cdot a$

Velocity (speed) $= m/s$ = distance / time

$a = m/s^2$

$$v_0(t_1) = \frac{-e}{mL}(V_R + V_A)(t_1 - t_1) + K_1$$

$$\Rightarrow \boxed{K_1 = v_0(t_1)} \quad - (7)$$

from (5) & (7)

$$\frac{dz}{dt} = \frac{-e}{mL}(V_R + V_A)(t - t_1) + v_0(t_1) \quad - (8)$$

Integrating eq. (8) over interval $[t, t_1]$

$$z = \frac{-e}{mL}(V_R + V_A)\int_{t_1}^{t}(t - t_1)\,dt + v_0(t_1)\int_{t_1}^{t} dt$$

$$z = \frac{-e}{mL}(V_R + V_A)\left[\frac{(t - t_1)^2}{2}\right] + v_0(t_1)\left[t - t_1\right] + K_2 \quad - (9)$$

$$\int_{t_1}^{t}(t - t_1)\,dt = \left[\frac{t^2}{2}\right]_{t_1}^{t} - t_1\left[t\right]_{t_1}^{t}$$

$$= \left[\frac{t^2 - t_1^2}{2}\right] - t_1\left[t - t_1\right]$$

$$= \frac{t^2 - t_1^2 - 2t_1 t + 2t_1^2}{2} = \frac{(t - t_1)^2}{2}$$

Now, at $t = t_1 \Rightarrow z = d$ — (10)

from (9) & (10)

$$d = 0 + 0 + k_2$$

$$\Rightarrow \boxed{k_2 = d} \quad — (11)$$

from (9) & (11)

$$\Rightarrow z = \frac{-e}{2mL}(V_R + V_A)(t - t_1)^2 + v_0(t_1)(t - t_1) + d \quad — (12)$$

Now, at $t = t_2 \Rightarrow z = d$ — (13)

from (12) & (13)

$$d = \frac{-e}{2mL}(V_R + V_A)(t_2 - t_1)^2 + v_0(t_1)(t_2 - t_1) + d.$$

$$\Rightarrow \frac{e}{2mL}(V_R + V_A)(t_2 - t_1) = v_0(t_1)$$

$$\Rightarrow \boxed{(t_2 - t_1) = \frac{[v_0(t_1)](2mL)}{e(V_R + V_A)}} \quad — (14)$$

$\therefore \omega(t_2 - t_1) = \theta_0' + x' \cdot \sin(\omega t_1 - \theta_g/2)$

→ Round-trip transit time in repeller region is given by

$$T' = t_2 - t_1 = \frac{2mL}{e[V_R + V_A]} \cdot v_0(t_1)$$

$$T' = \frac{2mL}{e(V_R + V_A)}\left[v_0\left(1 + \frac{\beta_i V_1}{2V_A}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right)\right]$$

$$T' = \frac{2mLv_0}{e(V_R + V_A)}\left[1 + \frac{\beta_i V_1}{2V_A}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$T' = T_0'\left[1 + \frac{\beta_i V_1}{2V_A}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \quad (15)$$

T_0' → round-trip dc transit time → $\frac{2mLv_0}{e(V_R + V_A)}$

Multiply eq. (15) by radian frequency (ω)

$$\omega T' = \omega T_0'\left[1 + \frac{\beta_i V_1}{2V_A}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$\omega(t_2 - t_1) = \theta_0' + \frac{\beta_i V_1 \theta_0'}{2V_A}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

$\theta_0' \rightarrow \omega T_0'$ → round-trip dc transit angle.

$x' = \frac{\beta_i V_1 \theta_0'}{2V_A}$ → Bunching Parameter of reflex klystron.