Assignment - 5

① Define Noise ? write short notes on different types of noises ?

In the context of signals, noise may be defined as an undesired & unwanted electrical signal, which is accompained by the desired signal. Noise is an unintentional fluctuations that tend to disturb transmission and reproduction of transmitted signals. Noise signals may or may not be predictable in nature.

Noise is classified into two broad groups depending on the source

1. External noise
2. Internal noise

Different types of noise can be categorized on the basis of their statistical properties in time and frequency domain.

Noise whose sources are external to the receiver / system is called external noise such as atmospheric noise, extraterrestrial noise etc.

The noise created within a device or a system is called internal noise. This noise is also called function noise.

② what is thermal noise ?

In any conducting material, electrons move randomly and the noise produced is called thermal noise. Each free electron inside of a conducting medium is in motion due to temperature.

Thermal noise power is proportional to the temperature in degree kelvin and the band width of the system.

(3) write short notes on Johnson's noise.

Thermal noise is also called Johnson noise. Each free electron inside of a conducting medium is in motion due to temperature. Thermal noise power is proportional to the temperature in °K and the bandwidth of the system. At 0°K there is no motion of electrons and hence noise is zero.

$$P_n \propto TB$$

T = temp. in °K , B = Bandwidth in Hz

Noise power is $P_n = kTB$ watts

k = Boltzmans constant given by

$$k = 1.38 \times 10^{-23} \ J/°K$$

The power density spectrum of noise voltage contributing the thermal noise is given by

$$S_v(\omega) = \frac{2kTR}{1+\left(\frac{\omega}{a}\right)^2}$$

(4) Derive the mathematical description of noise figure?

Noise figure gives the amount of noise internally generated by the system. It is the ratio of power density of the total noise available at the output of the network to the power density at the output only due to the input noise source. It gives the measure of system performance of the noise.

$$F = \frac{S_{no}(\omega)}{S'_{no}(\omega)} = \frac{S'_{no}(\omega) + S''_{no}(\omega)}{S'_{no}(\omega)} = 1 + \frac{S''_{no}(\omega)}{S'_{no}(\omega)}$$

where, $S_{no}(\omega)$ = the total noise power spectral density at the output.

$S'_{no}(\omega)$ = noise power spectral density at the output due input noise and.

$S''_{no}(\omega)$ = noise power spectral density at the output due to the noise generated internally by the system.

If for noiseless system, $S''_{no}(\omega) = 0$

and $S_{no}(\omega) = S'_{no}(\omega)$

Then $F = 1$.

If $F > 1$, the system is said to be noisy system.

The range of F is $1 < F < \infty$, as f increases, the system becomes noisy.

⑤ Define average noise figure.

We define the Noise Figure which is constant with respect to frequency. It is also called spot noise figure. But in noise practical cases the Noise Figure depends on frequency. So, Average Noise Figure of the system should be considered.

The average Noise Figure $\bar{F}$ is defined as the ratio of the total output available noise power N_o to the total output available noise power N due to the source alone.

$$\bar{F} = \frac{N_o}{N_{so}}$$

where, $N_o = N_{so} + N_{sys}$

$$N_{so} = \frac{1}{\pi} \int_0^\infty G_a S_{ni}(\omega)\, d\omega$$

But, $S_{ni}(\omega) = \frac{KT_e}{2}$

$$= \frac{k}{2\pi} \int_0^\infty T_e G_a\, d\omega$$

$$N_o = \frac{k}{2\pi} \int_0^\infty F T_e G_a\, d\omega$$

$$\therefore \bar{F} = \frac{\frac{k}{2\pi} \int_0^\infty F T_e G_a\, d\omega}{\frac{k}{2\pi} T_e G_a\, d\omega}$$

$$\bar{F} = \frac{\int_0^\infty F T_e G_a\, d\omega}{\int_0^\infty T_e G_a\, d\omega}$$

If T_e is constant then,

Average noise figure is $\bar{F} = \frac{\int_0^\infty F G_a \, d\omega}{\int_0^\infty G_a \, d\omega}$.

(6) Derive the expression for noise figure of n number of cascaded amplifiers & derive expression for friis formula.

Consider two amplifiers are cascaded

G_{a_1} - Power gain of amplifier 1.

G_{a_2} - Power gain of amplifier 2.

T_{e_1} - Equivalent input noise temperature of amplifier 1.

T_{e_2} - Equivalent input noise temperature of amplifier 2.

F_1 - Noise Figure of amplifier 1.

F_2 - Noise Figure of amplifier 2.

The overall gain is

$$G_a = G_{a_1} * G_{a_2} \quad \text{——— (1)}$$

The total output noise power available is

$$N_o = N_{o_1} + N_{o_2} + N_{o_3}$$

where N_{o_1} = output noise power due to input noise power N_i.

$$\therefore N_{o_1} = G_{a_1} G_{a_2} N_i = G_a N_i \quad \text{——— (2)}$$

N_{o_2} = output noise power due to noise power generated internally by first amplifier.

$$N_{sys1} = G_{a_1} N_i (F_1 - 1) G_{a_2}$$

$$N_{o_2} = G_a N_i (F_1 - 1) \quad \text{——— (3)}$$

N_{O_3} = output noise power due to noise power generated by second amplifier.

$$N_{O_3} = G_{a_2} N_i (F_2 - 1) \longrightarrow (4)$$

we know that overall Noise Figure is,

$$F = \frac{N_o}{G_a N_i}$$

or $N_o = G_a N_i F$

Now $N_o = N_{O_1} + N_{O_2} + N_{O_3}$

$$G_a N_i F = G_a N_i + G_a N_i (F_1 - 1) + G_{a_2} N_i (F_2 - 1) \longrightarrow (5)$$

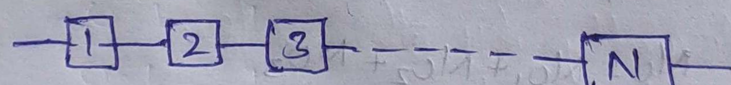
$$F = 1 + F_1 - 1 + \frac{(F_2 - 1) G_{a_2}}{G_{a_1} G_{a_2}}$$

$$F = F_1 + \frac{F_2 - 1}{G_{a_1}}$$

similarly for 3 amplifier cascaded

$$F = F_1 + \frac{F_2 - 1}{G_{a_1}} + \frac{F_3 - 1}{G_{a_1} G_{a_2}}$$

For N amplifiers cascaded,



The overall Noise Figure is,

$$F = F_1 + \frac{F_2 - 1}{G_{a_1}} + \frac{F_3 - 1}{G_{a_1} G_{a_2}} + \cdots + \frac{F_n - 1}{G_{a_1} G_{a_2} \cdots G_{a_n}}$$

This equation is called Fris's Formula. It shows that the contribution to overall Noise Figure is mainly by the first stage.

(7) Define the terms Information, Entropy and information rate.

Informations: Information is something carried out by a probable event. The message associated with the event with least probability of occurance, consists of maximum information.

In a particular communication system, if the allowable messages are $m_1, m_2, m_3 \cdots$ with respective probability of occurance as $P_1, P_2, P_3, \cdots$

Then, $P_1 + P_2 + P_3 + \cdots = 1$.

If the transmitter selects a message, m_k having a probability P_k and assuming that the receiver has correctly recognized the message. Then the amount of an information, I_k is given by,

$$I_k = \log_2\left(\frac{1}{P_k}\right).$$

Entropy:

The total amount of an information conveyed per message is called entropy, denoted by H.

$$H = \frac{I_{total}}{L}$$

$$H = \sum_{K=1}^{M} P_k \log_2\left(\frac{1}{P_k}\right).$$

Information Rate :-

If the source of the message generates at the rate 'r' messages/sec, then the information rate is defined to be $R = r \cdot H$.

$=$ Average no. of bits of information/sec.

(8) write the mathematical expression for channel capacity.

$$C = I(X,Y)_{max} = H(X)_{max} = H(X/Y)$$

(9) Five symbols of the alphabet of discrete memory less source and their probabilities are given below.

$S = \{S0, S1, S2, S3, S4\}$, $P(S) = (0.4, 0.2, 0.2, 0.1, 0.1)$

write the code words of the symbols and calculate code efficiency using Shannon fano coding and Huffman coding.

$$\left.\begin{array}{l} S0 \rightarrow 0.4 \\ S1 \rightarrow 0.2 \\ S2 \rightarrow 0.2 \\ S3 \rightarrow 0.1 \\ S4 \rightarrow 0.1 \end{array}\right\} \rightarrow \text{decreasing order.}$$

using Shannon fano coding

S0	0.4	0
S1	0.2	1
S2	0.2	1
S3	0.1	1
S4	0.1	1

S1	0.2	1	0
S2	0.2	1	1
S3	0.1	1	1
S4	0.1	1	1

S2	0.2	1	1	0
S3	0.1	1	1	1
S4	0.1	1	1	1

S3	0.1	1 1 1 0
S4	0.1	1 1 1 1

S0	0	1
S1	10	2
S2	110	3
S3	1110	4
S4	1111	4

} length of code word

Average code word length

$L = 1\times0.4 + 2\times0.2 + 3\times0.2 + 4\times0.1 + 4\times0.1$

$= 0.4 + 0.4 + 0.6 + 0.4 + 0.4 = 2.2$ bits (code word)

Entropy $H = -(0.4\log_2 0.4 + 0.2\log_2 0.2 + 0.2\log_2 0.2 + 0.1\log_2 0.1 + 0.1\log 0.1)$

$= -(-2.122) = 2.122$

$\eta = \frac{H}{L} \times 100\%$

code efficiency $(\eta) = \frac{2.122}{2.2} \times 100\% = 96.45\%$

Using Huffman code :

S0	S1	S2	S3	S4
0.4	0.2	0.2	0.1	0.1

0.4	1	0.41	0.41	0.6 0
0.2	10	0.210	0.4 00	0.4 1
0.2	000	0.2000	0.2 10	
0.1	0100	0.2100		
0.1	1100			

$\text{length} = \sum_{i=1}^{M} P_i(L)$

$= 0.4(1) + 0.2(2) + 0.2(3) + 0.1(4) + 0.1(4)$

$= 2.2$

$$H = \sum_{i=1}^{n} P_i \log_2\left(\frac{1}{P_i}\right) = -\sum_{i=1}^{n} P_i \log_2 (P_i)$$

$$= -(0.4\log 0.4 + 0.2\log 0.2 + 0.2\log 0.2 + 0.1\log 0.1 + 0.1\log 0.1)$$

$$= 2.122$$

$$\eta = \frac{2.122}{2.2} \times 100 = 96.45\%$$