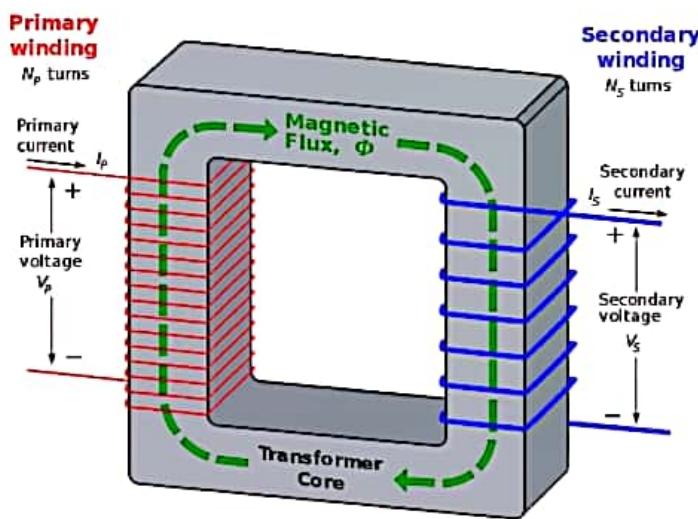


# EMF EQUATION OF A TRANSFORMER



- In a transformer, an alternating electrical source is applied to the primary winding and due to this, magnetizing current flows through the primary winding which produces alternating flux in the core of the transformer.
- This flux links with both primary and secondary windings. As this flux is alternating in nature, there must be a rate of change of flux.
- According to Faraday's Law of EMI, if any coil or conductor links with any changing flux, there must be an induced emf in it.

# EMF EQUATION OF A TRANSFORMER

- Let's say,  $T$  is number of turns in a winding,  $\Phi_m$  is the maximum flux in the core in Wb.
- As per Faraday's Law of EMF,

$$emf, e = -T \frac{d\phi}{dt}$$

- Where  $\phi$  is the instantaneous alternating flux and represented as,

$$\phi = \phi_m \sin 2\pi ft$$

$$Hence, e = -T \frac{d(\phi_m \sin 2\pi ft)}{dt}$$

$$\Rightarrow e = -T\phi_m \cos(2\pi ft) \times 2\pi f$$

$$\Rightarrow e = -T\phi_m \times 2\pi f \cos(2\pi ft)$$

# EMF EQUATION OF A TRANSFORMER

- As the maximum value of  $\cos 2\pi ft$  is 1, the maximum value of induced emf  $e$  is,

$$e_m = T\phi_m \times 2\pi f$$

- To obtain the rms value of induced counter emf, divide this maximum value of  $e$  by  $\sqrt{2}$ .

Then,  $E = \frac{2\pi}{\sqrt{2}} \times \phi_m f T$

$E = 4.44\phi_m f T$  Volts (Since  $\frac{2\pi}{\sqrt{2}} = 4.44$ )

- This is the EMF equation of transformer.

$E = 4.44\phi_m f N$

# EMF EQUATION OF A TRANSFORMER

- If  $E_1$  &  $E_2$  are primary and secondary emfs and  $T_1$  &  $T_2$  are primary and secondary turns then, **voltage ratio or turns ratio of transformer** is,

$$\frac{E_1}{E_2} = \frac{4.44\phi_{mf}T_1}{4.44\phi_{mf}T_2}$$

$$\Rightarrow \boxed{\frac{E_1}{E_2} = \frac{T_1}{T_2}} \quad \text{or} \quad \boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

- This above stated ratio is also known as **voltage ratio of transformer** if it is expressed as ratio of the primary and secondary voltages of transformer.
- As the voltage in primary and secondary of transformer is directly proportional to the number of turns in the respective winding, the transformation ratio of transformer is sometime expressed in ratio of turns and referred as **turns ratio of transformer**.

# EMF EQUATION OF A TRANSFORMER

- When a load is connected across the secondary winding, a current  $I_2$  flows.
- Then, input power = output power, or  $V_1 I_1 = V_2 I_2$ , i.e., the **primary and secondary volt-amperes are equal**.

- Thus,

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$P \cdot \text{mmf} = N \times i_1$$

- Combining the above equation with the previously derived voltage-turn ratio, we get

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

- The **rating** of a transformer is stated in terms of the volt-amperes that it can transform without overheating.
- The transformer rating is either  $V_1 I_1$  or  $V_2 I_2$ , where,  $I_2$  is the full load secondary current.

## VOLTAGE & AMPERE-TURNS RATIO - NUMERICAL

3. An ideal transformer, connected to a 240V mains, supplies a 12V, 150W lamp. Calculate the transformer turns ratio and the current taken from the supply.

$$V_1 = 240V ; P = 150W$$

$$V_2 = 12V \quad V_2 I_2 = 150$$

$$I_2 = \frac{150}{12} = 12.5A$$

$$V_1 I_1 = V_2 I_2$$
$$I_1 = \frac{V_2 I_2}{V_1} = 0.625A$$

$$\frac{N_1}{N_2} = \frac{V_1}{V_2}$$

## VOLTAGE & AMPERE-TURNS RATIO - NUMERICAL

4. A 5kVA single-phase transformer has a turns ratio of 10:1 and is fed from a 2.5 kV supply. Neglecting losses, determine (a) the full load secondary current, (b) the minimum load resistance which can be connected across the secondary winding to give full load kVA and (c) the primary current at full load kVA.

(a)  $\frac{N_1}{N_2} = \frac{10}{1}$  and  $V_1 = 2.5 \text{ kV} = 2500 \text{ V}$

Since  $\frac{N_1}{N_2} = \frac{V_1}{V_2}$ , secondary voltage

$$V_2 = V_1 \left( \frac{N_2}{N_1} \right) = 2500 \left( \frac{1}{10} \right) = 250 \text{ V}$$

The transformer rating in volt-amperes =  $V_2 I_2$  (at full load), i.e.  $5000 = 250 I_2$

$$\text{Hence full load secondary current } I_2 = \frac{5000}{250} = 20 \text{ A}$$

(b) Minimum value of load resistance,  $R_L = \frac{V_2}{I_2}$   
 $= \frac{250}{20}$   
 $= 12.5 \Omega$

(c)  $\frac{N_1}{N_2} = \frac{I_2}{I_1}$ , from which primary current,  
 $I_1 = I_2 \left( \frac{N_2}{N_1} \right)$   
 $= 20 \left( \frac{1}{10} \right)$   
 $= 2 \text{ A}$

# EMF EQUATION OF A TRANSFORMER - NUMERICAL

1. A 100kVA, 4000V/200V, 50Hz single-phase transformer has 100 secondary turns. Determine (a) the primary and secondary current, (b) the number of primary turns and (c) the maximum value of the flux.

$$V_1 = 4000 \text{ V}, V_2 = 200 \text{ V}, f = 50 \text{ Hz}, N_2 = 100 \text{ turns}$$

(a) Transformer rating =  $V_1 I_1 = V_2 I_2 = 100000 \text{ VA}$

$$\text{Hence primary current, } I_1 = \frac{100000}{V_1} = \frac{100000}{4000} \\ = 25 \text{ A}$$

$$\text{and secondary current, } I_2 = \frac{100000}{V_2} = \frac{100000}{200} \\ = 500 \text{ A}$$

(b) From equation (3),  $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\text{from which, primary turns, } N_1 = \left( \frac{V_1}{V_2} \right) (N_2) \\ = \left( \frac{4000}{200} \right) (100)$$

$$\text{i.e. } N_1 = 2000 \text{ turns}$$

(c) From equation (5),  $E_2 = 4.44 f \Phi_m N_2$

from which, maximum flux  $\Phi_m$

$$= \frac{E_2}{4.44 f N_2} = \frac{200}{4.44(50)(100)}$$

(assuming  $E_2 = V_2$ )

$$= 9.01 \times 10^{-3} \text{ Wh or } 9.01 \text{ mWh}$$

# EMF EQUATION OF A TRANSFORMER - NUMERICAL

2. A single-phase, 50Hz transformer has 25 primary turns and 300 secondary turns. The cross-sectional area of the core is 300 sq. cm. When the primary winding is connected to a 250V supply, determine (a) the maximum value of the flux density in the core, and (b) the voltage induced in the secondary winding.

$$\text{e.m.f. } E_1 = 4.44 f \Phi_m N_1 \text{ volts i.e.}$$

$$250 = 4.44(50)\Phi_m(25)$$

from which, maximum flux density,

$$\begin{aligned}\Phi_m &= \frac{250}{(4.44)(50)(25)} \text{ Wb} \\ &= 0.04505 \text{ Wb}\end{aligned}$$

However,  $\Phi_m = B_m \times A$ , where  $B_m$  = maximum flux density in the core and  $A$  = cross-sectional area of the core.

$$\text{Hence } B_m \times 300 \times 10^{-4} = 0.04505$$

from which, maximum flux density,

$$B_m = \frac{0.04505}{300 \times 10^{-4}} = 1.50 \text{ T}$$

(b)  $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ , from which,  $V_2 = V_1 \left( \frac{N_2}{N_1} \right)$

i.e. voltage induced in the secondary winding,

$$V_2 = (250) \left( \frac{300}{25} \right) = 3000 \text{ V or } 3 \text{ kV}$$

## EMF EQUATION OF A TRANSFORMER - NUMERICAL

3. A single-phase 500V/100V, 50Hz, transformer has a maximum core flux density of 1.5T and an effective core cross-sectional area of 50 sq. cm. Determine the number of primary and secondary turns.

$$\begin{aligned}\text{The e.m.f. equation for a transformer is } E &\equiv 4.44 f \Phi_m N \\ \text{and maximum flux, } \Phi_m &= B \times A = (1.5)(50 \times 10^{-4}) \\ &= 75 \times 10^{-4} \text{ Wb}\end{aligned}$$

$$\text{Since } E_1 = 4.44 f \Phi_m N_1$$

$$\begin{aligned}\text{then primary turns, } N_1 &= \frac{E_1}{4.44 f \Phi_m} \\ &= \frac{500}{4.44(50)(75 \times 10^{-4})} \\ &= 300 \text{ turns}\end{aligned}$$

$$\text{Since } E_2 = 4.44 f \Phi_m N_2$$

$$\begin{aligned}\text{then secondary turns, } N_2 &= \frac{E_2}{4.44 f \Phi_m} \\ &= \frac{100}{4.44(50)(75 \times 10^{-4})} \\ &= 60 \text{ turns}\end{aligned}$$

## EMF EQUATION OF A TRANSFORMER - NUMERICAL

4. A 4500V/225V, 50Hz single-phase transformer is to have an approximate e.m.f. per turn of 15V and operate with a maximum flux density of 1.4T. Calculate (a) the number of primary and secondary turns and (b) the cross-sectional area of the core.

(a) E.m.f. per turn  $= \frac{E_1}{N_1} = \frac{E_2}{N_2} = 15$

Hence primary turns,  $N_1 = \frac{E_1}{15} = \frac{4500}{15} = 300$

and secondary turns,  $N_2 = \frac{E_2}{15} = \frac{225}{15} = 15$

(b) E.m.f.  $E_1 = 4.44f\Phi_m N_1$

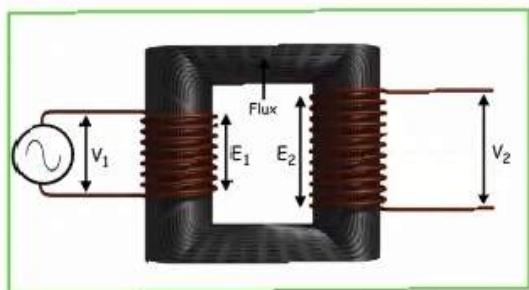
from which,  $\Phi_m = \frac{E_1}{4.44fN_1} = \frac{4500}{4.44(50)(300)} = 0.0676 \text{ Wb}$

Now flux  $\Phi_m = B_m \times A$ , where  $A$  is the cross-sectional area of the core, hence

area  $A = \frac{\Phi_m}{B_m} = \frac{0.0676}{1.4} = 0.0483 \text{ m}^2 \text{ or } 483 \text{ cm}^2$

# IDEAL TRANSFORMER

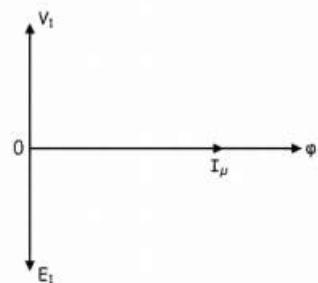
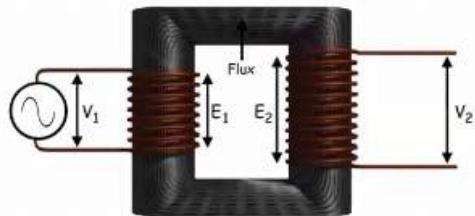
- An **ideal transformer** is a conceptual transformer which does not practically exist in reality. It has the following properties:



1. The power delivered by the transformer is equal to the power supplied to the transformer which means **100% efficiency**.
2. There is **no winding resistance & reactance**, which eliminates the copper losses inside a transformer.
3. The core has **infinite permeability** which means there are no core losses i.e. there is no hysteresis loss or eddy current loss.
4. There is **no leakage flux**; the entire magnetic flux is linked through the core.

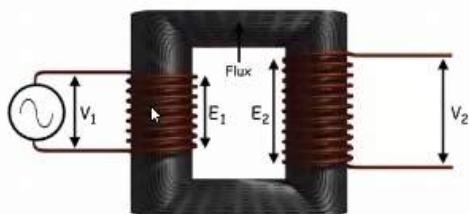
# IDEAL TRANSFORMER

- For developing counter emf  $E_1$  across the primary winding, it draws current from the source to produce required magnetizing flux.
- As the primary winding is purely inductive, that current lags  $90^\circ$  from the supply voltage. This current is called magnetizing current of transformer  $I_\mu$ .

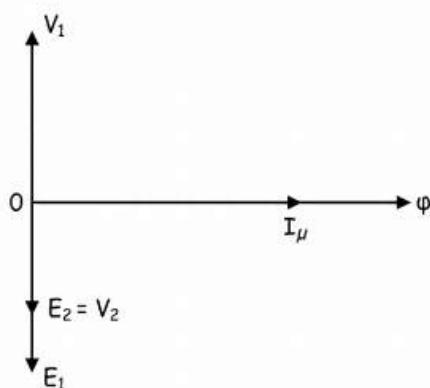


Abhishek Reddy Sheri's screen

# IDEAL TRANSFORMER

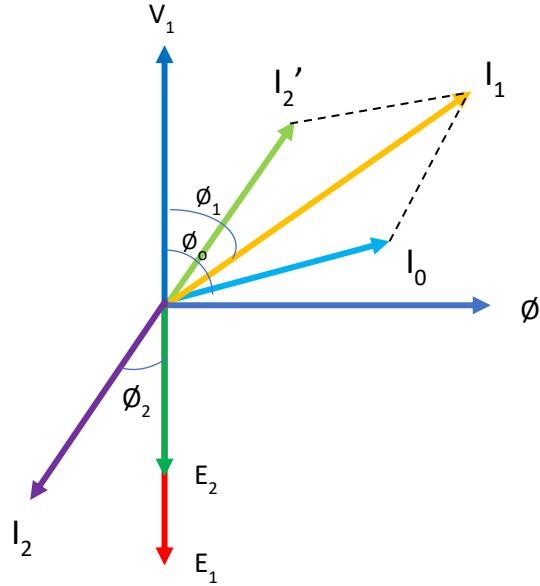


- This alternating magnetizing current  $I_\mu$  produces an alternating magnetizing flux  $\Phi$ .
- The flux is proportional to that current which producing it hence the flux would be in phase with the current.
- This flux also links the secondary winding through the core of the transformer.
- As a result, there would be another emf  $E_2$  induced across the secondary winding, and this is mutually induced emf as shown in the figure.



## IDEAL TRANSFORMER ON LOAD

### *INDUCTIVE LOAD*



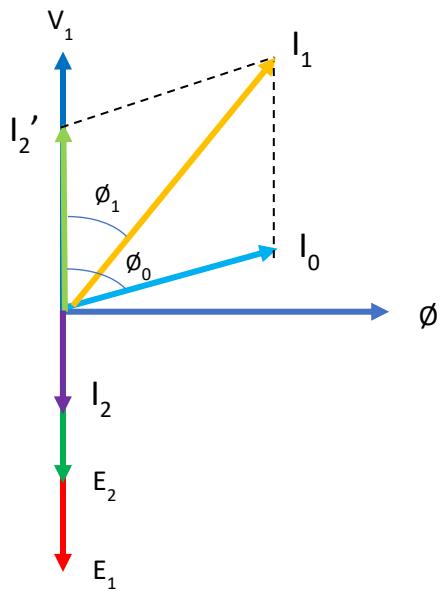
Phasor diagram of a transformer when load is inductive,

$I_2$ = secondary current lags  $E_2$  by  $\phi_2$ .

$I_2'$ = load component of primary current which is anti-phase with  $I_2$  and equal to it in magnitude

$I_1$ = Primary current which is vector sum of  $I_0$  and  $I_2'$  and lags behind  $V_1$  by angle  $\phi_1$ .

### *RESISTIVE LOAD*



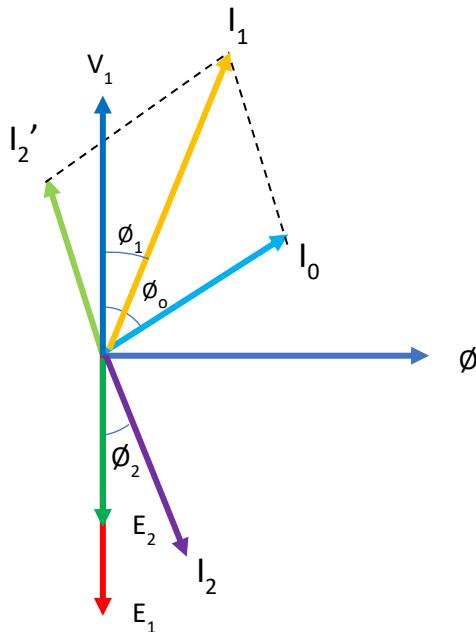
Phasor diagram of a transformer when load is resistive

$I_2$ = secondary current in phase with  $E_2= V_2$

$I_2'$ = load component of primary current which is anti-phase with  $I_2$  and equal to it in magnitude

$I_1$ = Primary current which is vector sum of  $I_0$  and  $I_2'$  and lags behind  $V_1$  by angle  $\phi_1$ .

### CAPACITIVE LOAD



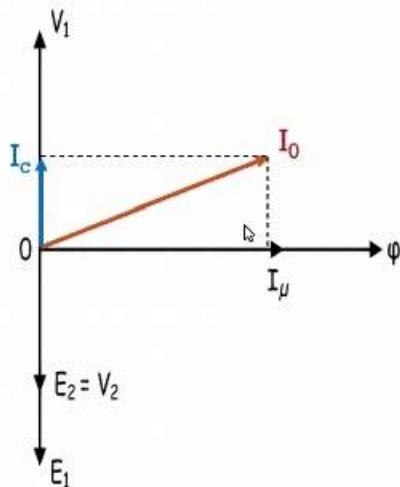
Phasor diagram of a transformer when load is capacitive,

$I_2$ = secondary current leads  $E_2$  by  $\phi_2$ .

$I_2'$ = load component of primary current which is anti-phase with  $I_2$  and equal to it in magnitude

$I_1$ = Primary current which is vector sum of  $I_0$  and  $I_2'$  and lags behind  $V_1$  by angle  $\phi_1$ .

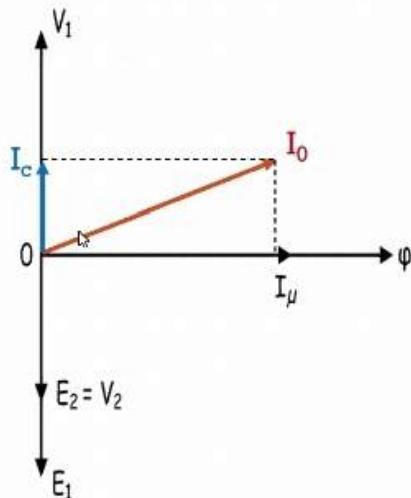
# PRACTICAL TRANSFORMER ON LOAD



- In a practical transformer, there are hysteresis and eddy current losses in transformer core.
- Magnetizing current in a practical transformer is a little bit greater than actual magnetizing current.
- Total current supplied from the source has two components,
  - i. Magnetizing current component ( $I_\mu$ ) which is merely utilized for magnetizing the core, and
  - ii. Core loss component ( $I_c$ ) of the source current is consumed for compensating the core losses in transformers.



# PRACTICAL TRANSFORMER ON LOAD

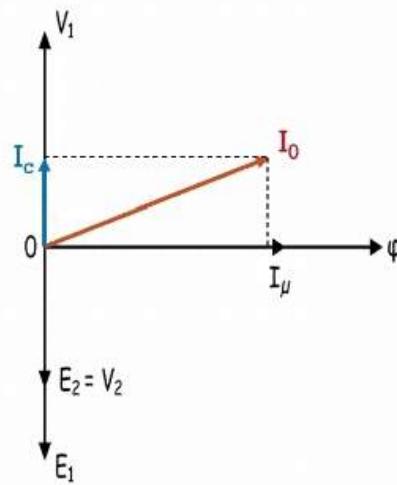


- Because of this core loss component, the source current in a transformer on no-load condition supplied from the source as source current is not exactly at  $90^\circ$  lags of the supply voltage, but it lags behind an angle  $\theta$  is less than  $90^\circ$ .
- If total current supplied from source is  $I_o$ , it will have one component in phase with supply voltage  $V_1$  and this component of the current  $I_c$  is core loss component.
- This component is taken in phase with the source voltage because it is associated with active or working losses in transformers.

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# PRACTICAL TRANSFORMER ON LOAD

- Another component of the source current is denoted as  $I_\mu$
- This component produces the alternating magnetic flux in the core, so it is watt-less; means it is a reactive part of the transformer source current.
- Hence  $I_\mu$  will be in quadrature with  $V_1$  and in phase with alternating flux  $\Phi$ .
- Hence, total primary current in a transformer on no-load condition can be represented as:



$$\vec{I}_o = \vec{I}_c + \vec{I}_\mu$$

where,  $\vec{I}_\mu = I_o \cos \theta$

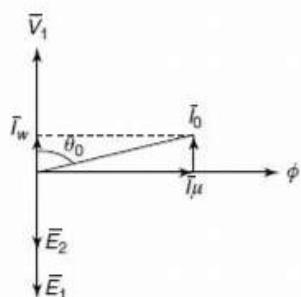
and  $\vec{I}_c \perp \vec{I}_\mu$

$$I_o = \sqrt{I_\mu^2 + I_c^2}$$

# NUMERICAL ON IDEAL TRANSFORMER

1. The no-load current of a 4,400/440 V, single-phase, 50 Hz transformer is 0.04. It consumes power 80 W at no load when supply is given to LV side and HV side is kept open. Calculate the following:

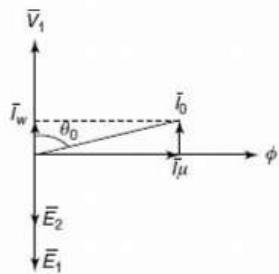
- i. Power factor of no-load current.
- ii. Iron loss component of current.
- iii. Magnetizing component of current.



# NUMERICAL ON IDEAL TRANSFORMER

1. The no-load current of a 4,400/440 V, single-phase, 50 Hz transformer is 0.04. It consumes power 80 W at no load when supply is given to LV side and HV side is kept open. Calculate the following:

- Power factor of no-load current.
- Iron loss component of current.
- Magnetizing component of current.



$$\begin{aligned}I_0 &= 0.04 \text{ A} \\W_0 &= 80 \text{ W} \\V_0 &= 4400 \text{ V} \\V_0 &= V_0 \cos \phi \\W_0 &= V_0 \sin \phi \\W_0 &= 4400 \times 0.45 \\W_0 &= 1980 \text{ W} \\&\text{Sir } \phi = 0^\circ\end{aligned}$$

# NUMERICAL ON IDEAL TRANSFORMER

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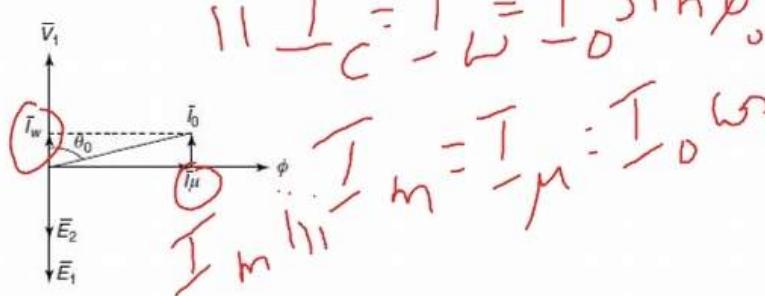
- i. Power factor of no-load current.
  - ii. Iron loss component of current.
  - iii. Magnetizing component of current.

$$I_1 = 0.04 \text{ A}$$

$$v = 80 \text{ m}$$

$$N_0 = 44^{\circ} \quad \text{and} \quad N_1 = 15^{\circ}$$

$$\omega_0 = \nu_0 \sin \phi_0$$

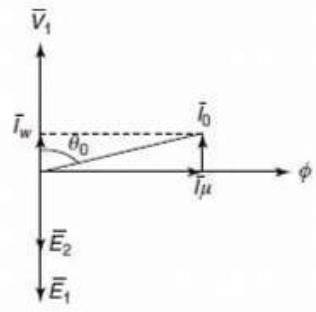


# NUMERICAL ON IDEAL TRANSFORMER

1. The no-load current of a 4,400/440 V, single-phase, 50 Hz transformer is 0.04. It consumes power 80 W at no load when supply is given to LV side and HV side is kept open. Calculate the following:

- i. Power factor of no-load current.
- ii. Iron loss component of current.
- iii. Magnetizing component of current.

$$W_0 = 80 \text{ W}, I_0 = 0.04 \text{ A}, V_1 = 4,400 \text{ V}$$



$$\cos \theta_0 = \frac{W_0}{V_1 I_0} = \frac{80}{4,400 \times 0.04} = 0.454 \quad \sin \theta_0 = \sqrt{1 - \cos^2 \theta_0} = 0.891$$

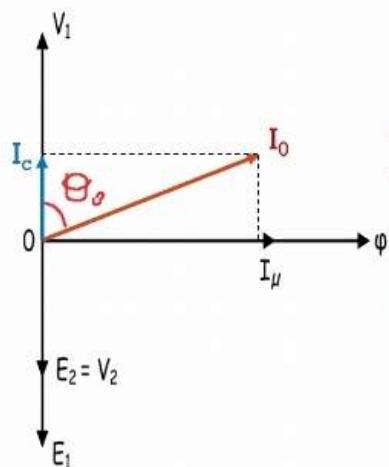
The no-load power factor is 0.454 (lagging).

$$I_W = I_0 \cos \theta_0 = 0.04 \times 0.454 = 0.0187 \text{ A}$$

$$\therefore I_\mu = I_0 \sin \theta_0 = 0.04 \times 0.891 = 0.0356$$



# PRACTICAL TRANSFORMER ON LOAD



The following points are most important:

- The no-load primary current is 1–5 % of full-load current.
- Since  $I_0$  is very small, the no-load copper loss is negligible. Hence, no-load input is practically equal to the iron loss in the transformer.
- Since core loss is solely responsible for shifting the current vector  $I_0$ , the angle  $\theta_0$  is known as hysteresis angle of advance.

# NUMERICAL ON IDEAL TRANSFORMER

2. A transformer takes a current of 0.8A when its primary is connected to a 240 volt, 50Hz supply, the secondary being on open circuit. If the power absorbed is 72 watts, determine (a) the iron loss current, (b) the power factor on no-load and (c) the magnetizing current.

$$I_0 = 0.8 \text{ A}, V_1 = 240 \text{ V}$$

$$\begin{aligned} \text{(a)} \quad \text{Power absorbed} &= \text{total core loss} = 72 \\ &= V_1 I_0 \cos \phi_0 \end{aligned}$$

$$\text{Hence } 72 = 240 I_0 \cos \phi_0$$

$$\begin{aligned} \text{and iron loss current, } I_C &= I_0 \cos \phi_0 = \frac{72}{240} \\ &= 0.30 \text{ A} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{Power factor at no load, } \cos \phi_0 &= \frac{I_C}{I_0} = \frac{0.30}{0.80} \\ &= 0.375 \end{aligned}$$

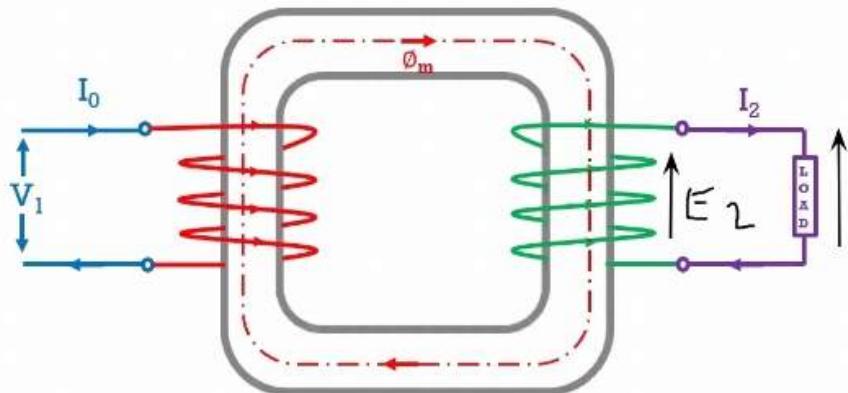
$$\text{(c)} \quad I_0^2 = I_C^2 + I_M^2,$$

from which magnetizing current,

$$\begin{aligned} I_M &= \sqrt{(I_0^2 - I_C^2)} = \sqrt{(0.80^2 - 0.30^2)} \\ &= 0.74 \text{ A} \end{aligned}$$

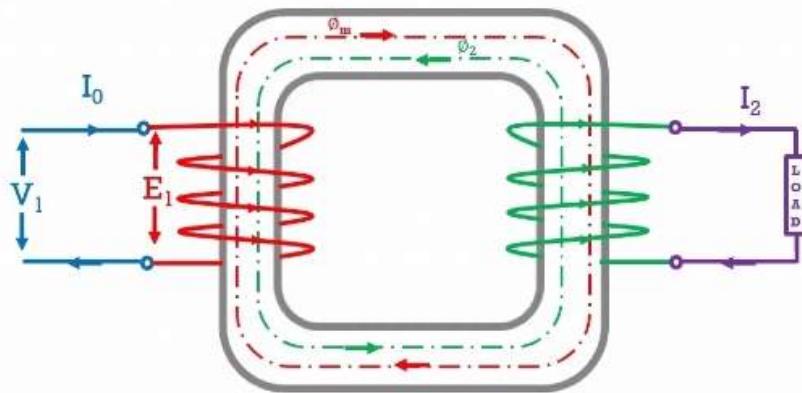


# TRANSFORMER ON LOAD



- When load is connected to the secondary winding of a transformer,  $I_2$  (secondary current) is set up in the secondary winding.
- The magnitude and phase of  $I_2$  with respect to  $V_2$  (secondary voltage) depends upon the characteristics of the load.
- Secondary current  $I_2$  is
  - i. *in phase with  $V_2$ , if load is resistive,*
  - ii. *lags  $V_2$  if load is inductive and*
  - iii. *it leads  $V_2$  if load is capacitive*

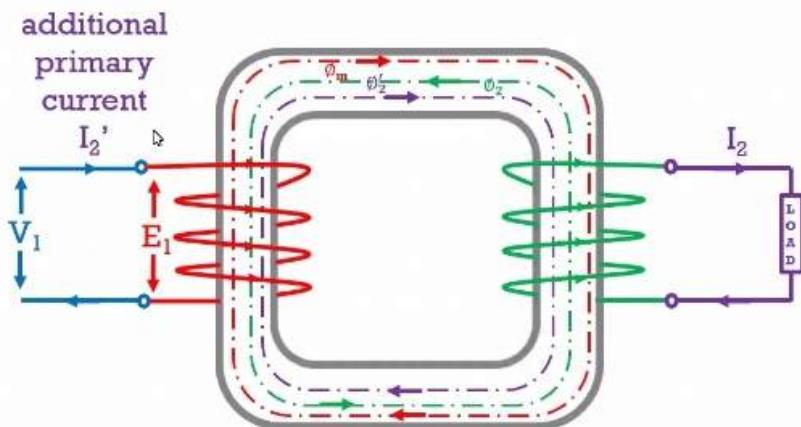
# TRANSFORMER ON LOAD



- This secondary current sets up m.m.f ( $=N_2 I_2$ ) and hence it produces magnetic flux  $\Phi_2$  which is in opposition to the main primary flux  $\Phi$  in accordance with Lenz's law.
- The secondary ampere-turns  $N_2 I_2$  are known as **demagnetizing amp-turns**.
- The opposing secondary flux  $\Phi_2$  **weakens** the primary flux  $\Phi$  **momentarily**, hence the primary winding back e.m.f  $E_1$  tends to be reduced.

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# TRANSFORMER ON LOAD



- For a moment  $V_1 > E_1$  and hence more current to flow in the primary winding .
- Let the additional primary current be  $I_2'$  which is known as load component of primary current.
- This current ( $I_2'$ ) is anti-phase with  $I_2$ .
- This load component of primary current( $I_2'$ ) sets up its own flux  $\Phi_2'$ , which is in opposition to secondary flux  $\Phi_2$ , but is in the same direction as primary flux  $\Phi$ .
- The flux  $\Phi_2'$  is equal to  $\Phi_2$ . Hence, the two flux cancel each other out.
- So, we can say that whatever the load conditions, the net flux passing through the core is approximately the same as at no-load.



**Note:** If a voltage or turns ratio is specified, it is always put in the order as, input : output which is primary : secondary. It is to be noted from eq. ⑫ that almost any desired voltage ratio can be obtained by adjusting the number of turns.

#### 4.8 STEP-UP AND STEP-DOWN TRANSFORMER

A transformer in which the output (secondary) voltage is greater than its input (primary) voltage is called a **step-up transformer**.

A transformer in which the output (secondary) voltage is lesser than its input (primary) voltage is called a **step-down transformer**.

A transformer may receive energy at one voltage and deliver it at the same voltage. Such a transformer is called a **one-to-one (1:1) transformer**. For a 1:1 T/F,  $N_1 = N_2$  and  $E_1 = E_2$ . Such a transformer is used to isolate two circuits.

#### 4.9 IDEAL TRANSFORMER

An ideal transformer is just a conceptual device and does not practically exist in reality. It has the following properties:

- (i) The primary and secondary resistances are negligible.
- (ii) The core has infinite permeability ( $\mu$ ) so that negligible mmf is required to establish the flux in the core.
- (iii) The leakage flux and leakage inductances are zero. The entire flux is confined to the core and windings linking them.
- (iv) There are no losses due to resistances, hysteresis and eddy currents. Thus the efficiency is 100%.

#### 4.9.1 Ideal Transformer on No-load

Consider an ideal transformer on no-load as shown in fig. 4.7.

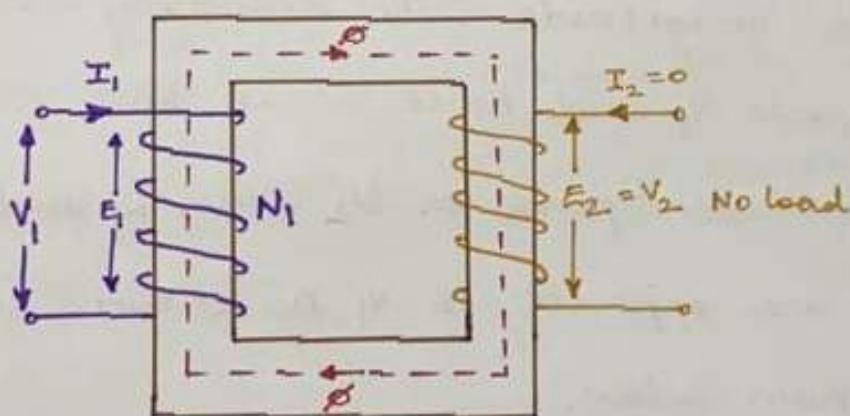


Fig. 4.7 Ideal Transformer on No-load

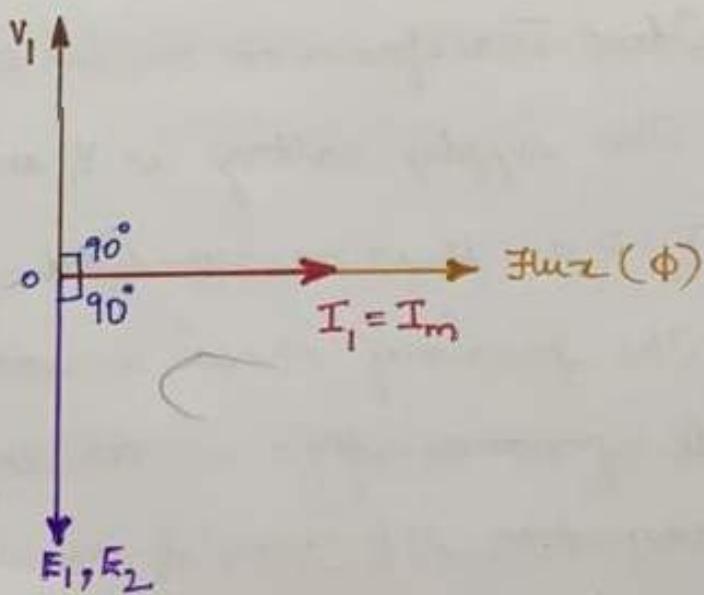
The supply voltage is  $V_1$  and the secondary current is  $I_2 = 0$  as it is on no-load.

The primary draws a current  $I_1$ , which is just necessary to produce flux in the core. As this current is used to magnetise the core, it is called magnetising current denoted by  $I_m$ .

As the transformer is ideal, the winding resistance is zero and is purely inductive in nature. The magnetising current  $I_m$  is very small and lags  $V_1$  by  $90^\circ$ , as the winding is purely inductive. This  $I_m$  produces an alternating flux  $\phi$  which is in phase with  $I_m$ .

The flux links with both the winding, producing induced emfs,  $E_1$  and  $E_2$ , in the primary and secondary winding respectively.

According to Lenz's law, the induced emf opposes the very cause producing it which is the supply voltage  $V_1$ . Hence  $E_1$  is in anti-phase with  $V_1$  but it is equal in magnitude. The secondary induced emf  $E_2$  also opposes  $V_1$  and hence is in anti-phase with  $V_1$  and its magnitude depends on  $N_2$ . Thus,  $E_1$  and  $E_2$  are in phase and are opposite to  $V_1$  as shown in the phasor diagram below.



Phasor diagram of ideal T/F on No-load.

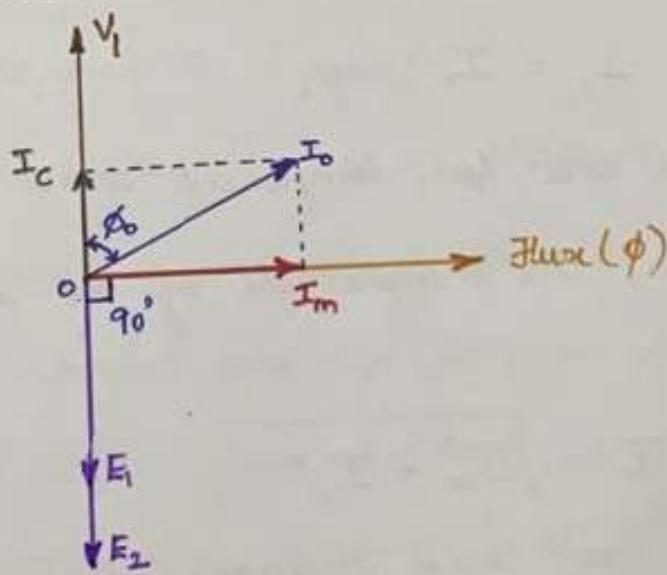
#### 4.10 PRACTICAL TRANSFORMER ON NO-LOAD

Actually in practical transformer, iron core causes eddy-currents and hysteresis losses as it is subjected to alternating flux.

Apart from these losses in a practical transformer the primary winding has some resistance and hence, small primary copper loss is present.

Thus, the primary current under no-load has to supply the iron losses i.e., hysteresis and eddy current losses and also a small amount of copper loss. This current is denoted as  $I_0$ .

In practical transformer, due to winding resistance, no-load current  $I_0$  is no longer at  $90^\circ$  w.r.t  $V_1$ , but it lags  $V_1$  by an angle  $\phi_0$  which is less than  $90^\circ$  as shown in the figure below.



Phasor diagram of a practical transformer on No-load.

From the phasor diagram it is clear that the no-load input current  $I_o$  has two components:

- (i) A purely reactive component  $I_m$  called the magnetising component of no-load current required to produce the flux. This is also called wattless component.
- (ii) An active component  $I_c$  which supplies total losses under no-load condition called the power component of no-load current. This is also called core-loss component.

Therefore, the total current on no-load,  $I_o$  is the vector sum of the above mentioned currents  $I_m, I_c$

$$\therefore I_o = I_m + I_c$$

From the phasor diagram,

$$I_m = I_o \sin \phi.$$

This is the magnetising component lagging  $V_1$  by exactly  $90^\circ$ . and,

$$I_c = I_o \cos \phi.$$

This is the core-loss component which is in phase with  $V_1$ .

Thus, the magnitude of no-load current is given by,

$$I_o = \sqrt{I_m^2 + I_c^2}$$

while,  $\phi_o \rightarrow$  no-load primary power factor angle.

It may be noted that, the no-load current  $I_0$  is very small, about 3% to 5% of the full-load current or rated current. Hence, primary copper loss is negligible and hence is called core loss or iron loss component.

Hence, power input  $W_0$  on no-load always represents the iron losses, as Cu loss are negligibly small. The iron losses are denoted as  $P_i$  and are constant for all the load conditions.

$$W_0 = V_1 I_0 \cos \phi_0 = P_i = \text{iron losses}$$

#### 4.11 TRANSFORMER ON LOAD

When the transformer is loaded, the current  $I_2$  flows through the secondary winding. The magnitude and phase of  $I_2$  is determined by the load.

- If the load is inductive,  $I_2$  lags  $V_2$
- If the load is capacitive,  $I_2$  leads  $V_2$
- If the load is resistive,  $I_2$  is in phase with  $V_2$ .

There exists a secondary m.m.f  $N_2 I_2$  due to which secondary current sets-up its own flux  $\phi_2$ . This flux opposes the main flux  $\phi$  which is produced in the core due to magnetising component of no-load current. Hence, mmf  $N_2 I_2$  is called demagnetising ampere turns as shown in fig. 4.8.

The flux  $\phi_2$  momentarily reduces the main flux  $\phi$  due to which the primary induced emf  $E_1$  also reduces. Hence, the vector difference  $(V_1 - E_1)$  increases due to which primary draws more current from the supply.

This additional current drawn is due to the load and hence is called load component of primary current denoted as,  $I_2'$

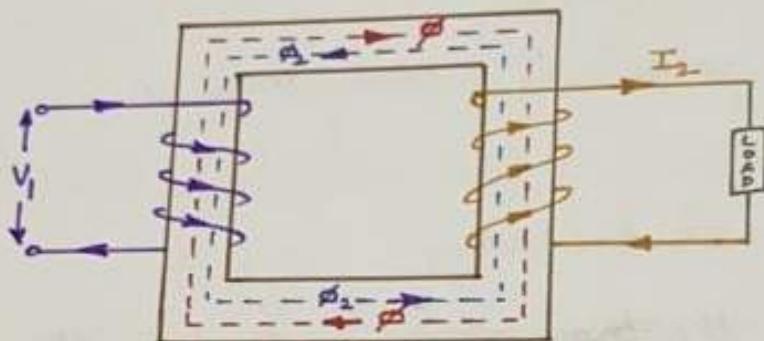


Fig. 4.8  $\phi_2$  flux opposing  $\phi$

The current  $I_2'$  is in anti-phase with  $I_2$ . The current  $I_2'$  sets up its own flux which opposes the flux  $\phi_2$ . The  $\phi_2'$  flux neutralises the flux  $\phi_2$  produced by  $I_2$  and helps the main flux  $\phi$ .

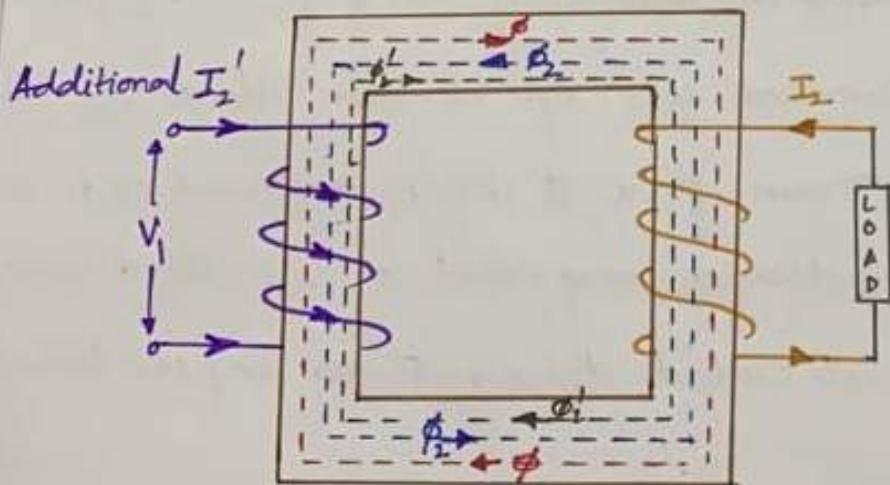


Fig. 4.9 Primary drawing more current  $I_2'$

# TRANSFORMER ON LOAD

- As the Ampere-turns are balanced,

$$\begin{aligned} N_2 I_2' &= N_1 I_2' \\ I_2' &= \frac{N_2}{N_1} I_2 \\ I_2' &= k \cdot I_2, \text{ where } k = \frac{N_2}{N_1} = \text{turns ratio} \end{aligned}$$

- Hence, when transformer is no load, the primary winding has two currents in it; one is  $I_0$  and the other is  $I_2'$
- The total primary current  $I_1$  is the vector sum of  $I_0$  and  $I_2'$

$$\vec{I}_1 = \vec{I}_0 + \vec{I}_2'$$

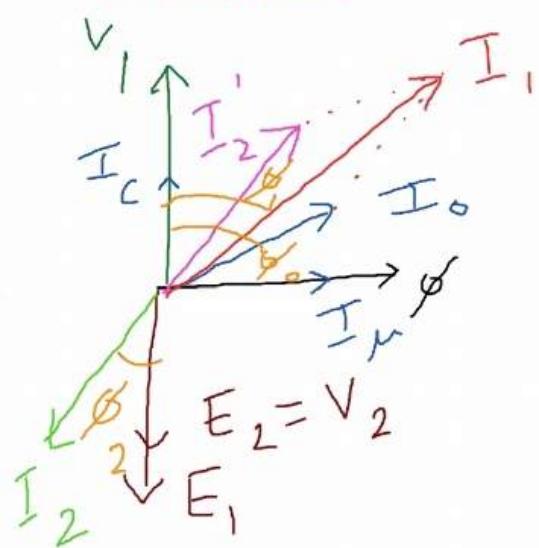
where,  $\vec{I}_0$  is the no-load current which lags  $V_1$  by an angle  $\phi$  which has two components  $I_\mu$  and  $I_c$

and  $\vec{I}_2'$  is the load component of primary current which is anti-phase with  $I_2$ . The phase of  $I_2$  is decided by the nature of load

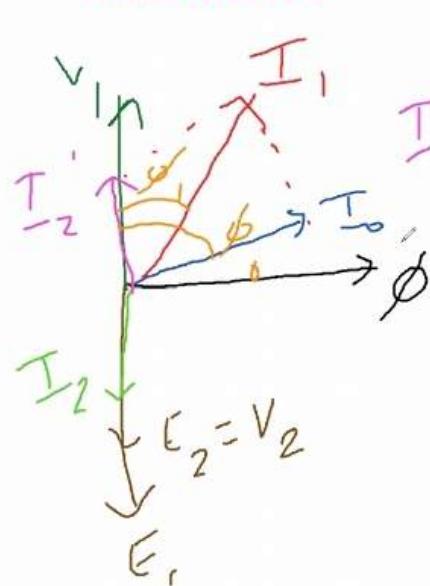


# TRANSFORMER ON LOAD – PHASOR DIAGRAMS

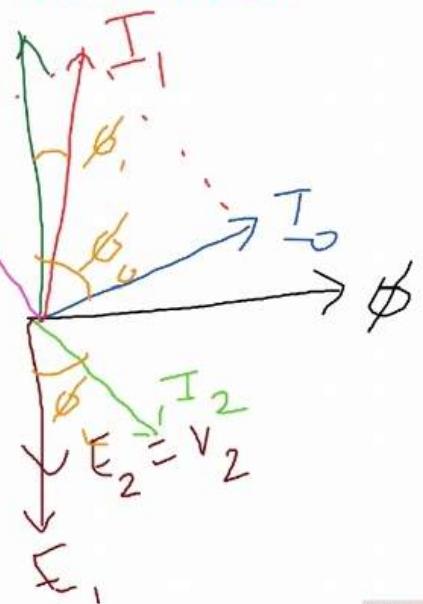
INDUCTIVE LOAD



RESISTIVE LOAD



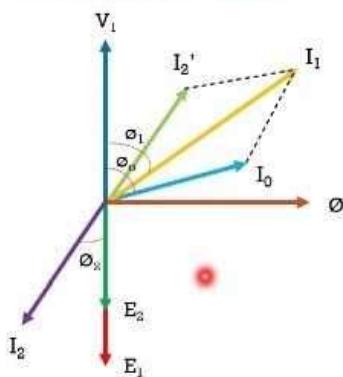
CAPACITIVE LOAD



k Reddy Sheri's screen

# TRANSFORMER ON LOAD – PHASOR DIAGRAMS

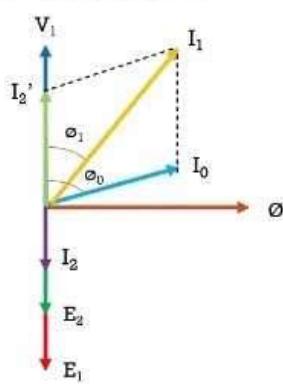
INDUCTIVE LOAD



Phasor diagram of a transformer when load is inductive,  
 $I_2$ = secondary current lags  $E_2$  by  $\varphi_2$ .

$I_2'$ = load component of primary current which is anti-phase with  $I_2$  and also equal to it in magnitude( as  $K=1$  )  
 $I_1$ = Primary current which is vector sum of  $I_0$  and  $I_2'$  and lags behind  $V_1$  by angle  $\varphi_1$ .

RESISTIVE LOAD

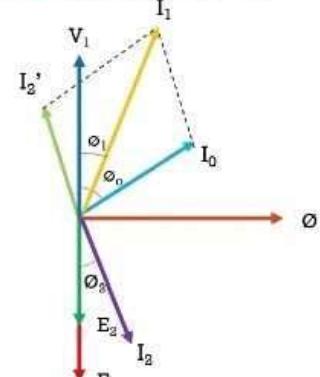


Phasor diagram of a transformer when load is resistive  
 $I_2$ = secondary current in phase with  $E_2=V_2$

$I_2'$ = load component of primary current which is anti-phase with  $I_2$  and also equal to it in magnitude

$I_1$ = Primary current which is vector sum of  $I_0$  and  $I_2'$  and lags behind  $V_1$  by angle  $\varphi_1$ .

CAPACITIVE LOAD



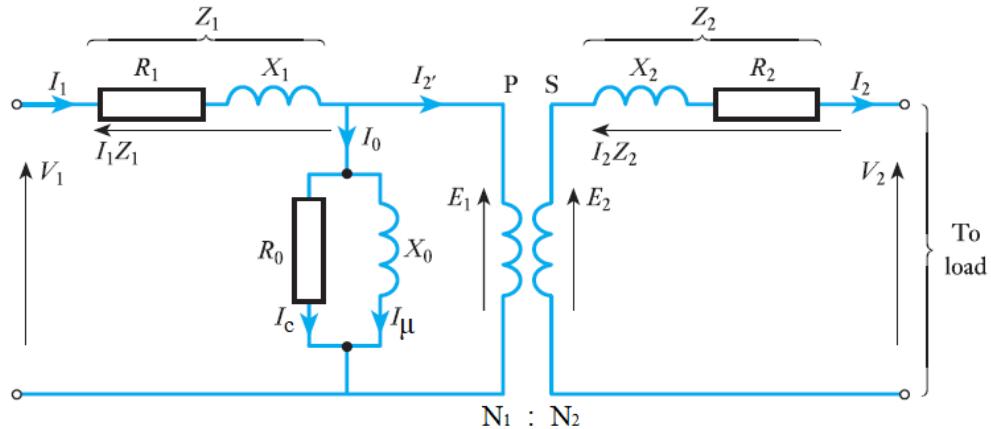
Phasor diagram of a transformer when load is capacitive,  
 $I_2$ = secondary current leads  $E_2$  by  $\varphi_2$ .

$I_2'$ = load component of primary current which is anti-phase with  $I_2$  and also equal to it in magnitude( as  $K=1$  )

$I_1$ = Primary current which is vector sum of  $I_0$  and  $I_2'$  and lags behind  $V_1$  by angle  $\varphi_1$ .

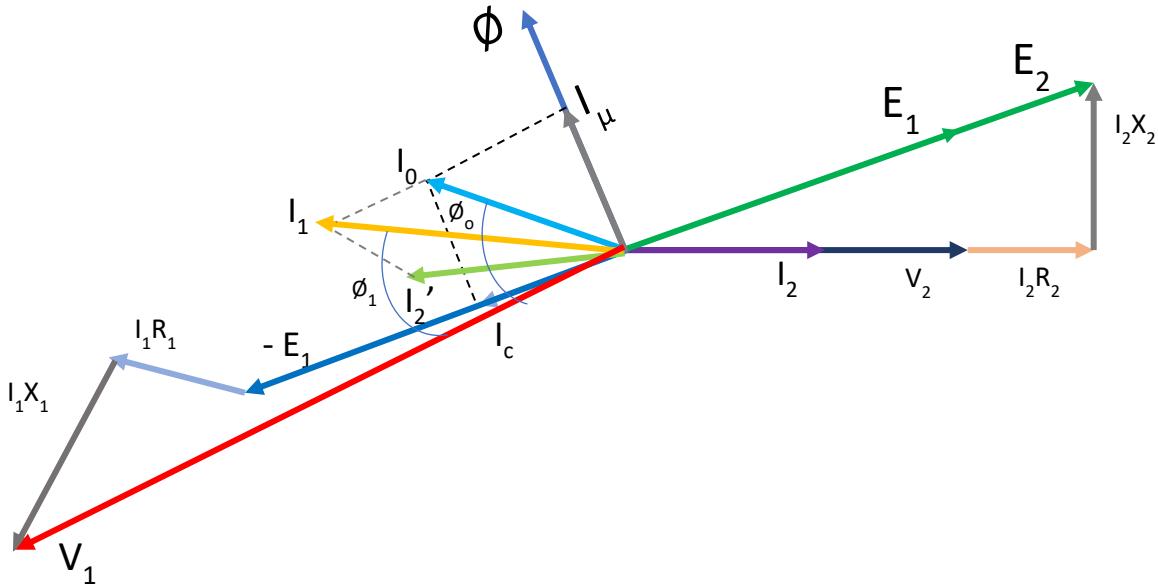
## TRANSFORMER ON LOAD

### Actual Transformer Equivalent circuit



- $V_1$  is the applied voltage to the transformer
- $V_2$  is the load voltage of the load connected to the transformer
- $E_1$  is the induced primary voltage in the transformer
- $E_2$  is the induced secondary voltage in the transformer
- $R_1$  and  $R_2$  are resistances of the primary and secondary windings of the actual transformer
- $X_1$  and  $X_2$  represent the reactance of the windings due to leakage flux in the actual transformer
- $Z_1$  and  $Z_2$  are impedances of the primary and secondary windings of the actual transformer
- $X_0$  is inductive reactance such that it takes a reactive current equal to the magnetizing current  $I_\mu$  of the actual transformer
- $R_0$  resistor for core losses due to hysteresis and eddy currents
- $I_\mu$  is magnetizing component of no-load current
- $I_c$  is core loss component of no-load current
- $I_0$  is no-load current
- $I_1$  is primary current
- $I_2'$  is load component of primary current
- $I_2$  is secondary current
- $N_1$  and  $N_2$  are the primary and secondary turns
- $I_1 Z_1$  is the series drop in the primary winding
- $I_2 Z_2$  is the series drop in the secondary winding

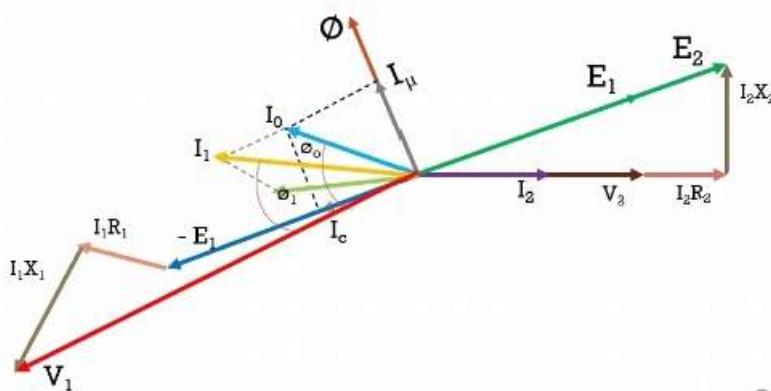
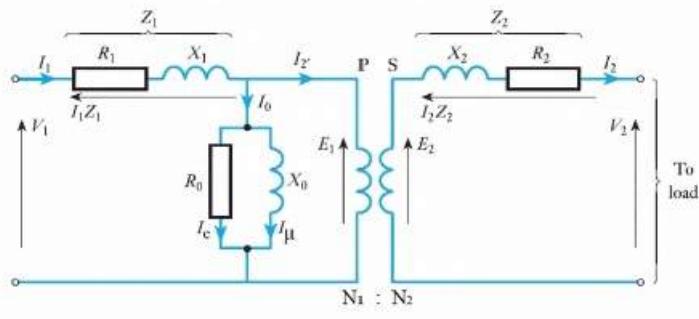
## Phasor Diagram



## Construction Steps

1. Let's begin from load voltage  $V_2$  and move towards the applied voltage  $V_1$
2.  $I_2$  will be in phase with  $V_2$  because in a resistive load current is in phase with the voltage
3. The resistive drop  $I_2R_2$  will be parallel to  $I_2$  which means it is in phase with  $V_2$  the reactive drop  $I_2X_2$  will be in quadrature, i.e., in  $90^\circ$  with the resistive drop  $I_2R_2$
4. Applying KVL in the secondary loop, we get the phasor  $E_2 = V_2 + I_2R_2 + I_2X_2$  as shown
5. From the EMF equation, we know that flux  $\phi$  is responsible induced emfs  $E_1$  and  $E_2$  and flux  $\phi$  lead the emf by  $90^\circ$  so we draw the  $\phi$  phasor perpendicular to  $E_2$
6. Same flux  $\phi$  is responsible for  $E_1$ , and if the no. of turns in the primary is less than secondary, the magnitude of  $E_1$  would be less than  $E_2$  shown by a phasor smaller in length compared to  $E_2$
7. Flux  $\phi$  is produced due to the magnetising current  $I_\mu$  so the current  $I_\mu$  will be drawn in phase with flux  $\phi$
8. The emf induced in the primary is in accordance with the Lenz's law which is  $-E_1$  which is drawn equal and opposite to  $E_1$ .  $-E_1$  is the voltage across resistance  $R_0$  and the current flowing through  $R_0$  is  $I_c$  so obviously  $I_c$  will be in phase with  $-E_1$
9. From the figure, the no-load current  $I_0$  is the phasor sum of  $I_\mu$  and  $I_c$  which gives the  $I_0$  phasor
10. Also, from the figure, the primary current  $I_1$  is the phasor sum of  $I_0$  and  $I_2'$  and the  $I_2'$  is in anti-phase with  $I_2$ . Hence,  $I_1$  can be drawn as shown
11. Current  $I_1$  flows through  $R_1$  and  $X_1$  which gives rise to two voltage drops, the resistive drop  $I_1R_1$  and the reactance drop  $I_1X_1$  where  $I_1R_1$  is in phase with  $I_1$  and drop  $I_1X_1$  is in quadrature with  $I_1R_1$  drop. Since, both are voltage drops they should be added to the voltage phasor  $-E_1$
12. Applying KVL in the primary loop, we get phasor  $V_1 = -E_1 + I_1R_1 + I_1X_1$  as shown
13.  $\phi_0$  is the no load phase angle due no load current  $I_0$  with respect to  $V_1$
14.  $\phi_1$  is the primary circuit phase angle due to primary current  $I_1$  with respect to  $V_1$ .

# ACTUAL TRANSFORMER ON RESISTIVE LOAD

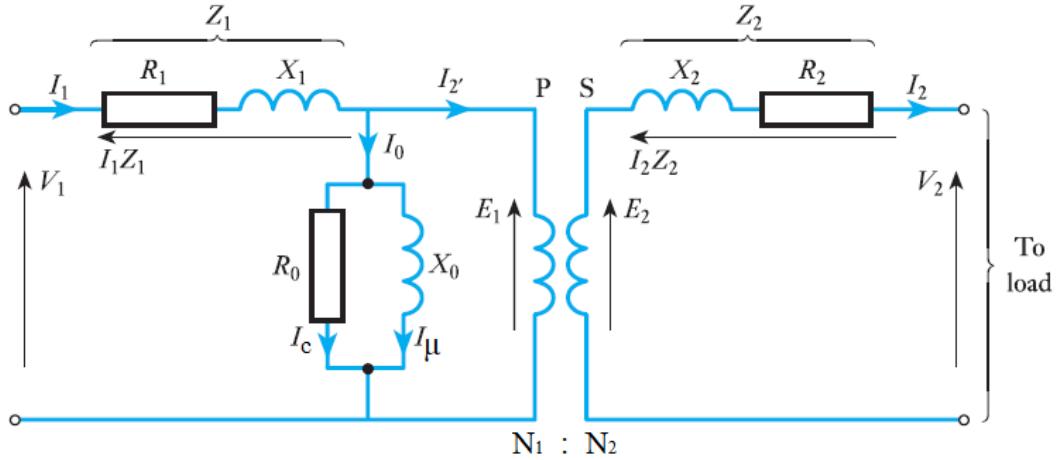


## Construction Steps

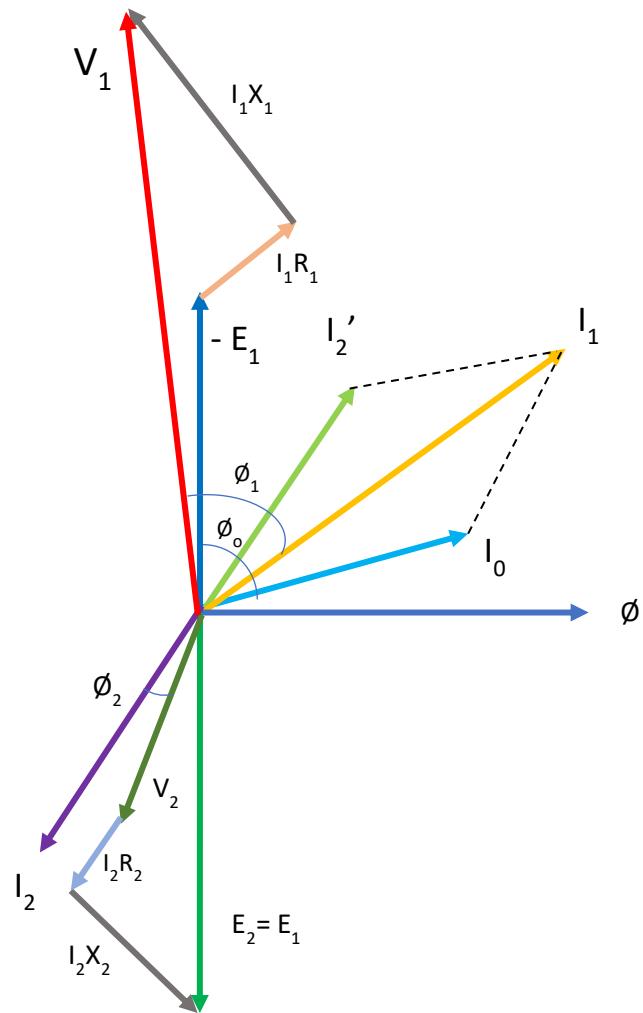
- Let's begin from load voltage  $V_2$  and move towards the applied voltage  $V_1$
- $I_2$  will be in phase with  $V_2$  because in a resistive load current is in phase with the voltage
- The resistive drop  $I_2R_2$  will be parallel to  $I_2$  which means it is in phase with  $V_2$  the reactive drop  $I_2X_2$  will be in quadrature, i.e., in  $90^\circ$  with the resistive drop  $I_2R_2$
- Applying KVL in the secondary loop, we get the phasor  $E_2 = V_2 + I_2R_2 + I_2X_2$  as shown
- From the EMF equation, we know that flux  $\Phi$  is responsible induced emfs  $E_1$  and  $E_2$  and flux  $\Phi$  lead the emf by  $90^\circ$  so we draw the  $\Phi$  phasor perpendicular to  $E_2$

## ACTUAL TRANSFORMER ON INDUCTIVE LOAD

### Equivalent Circuit



### Phasor Diagram

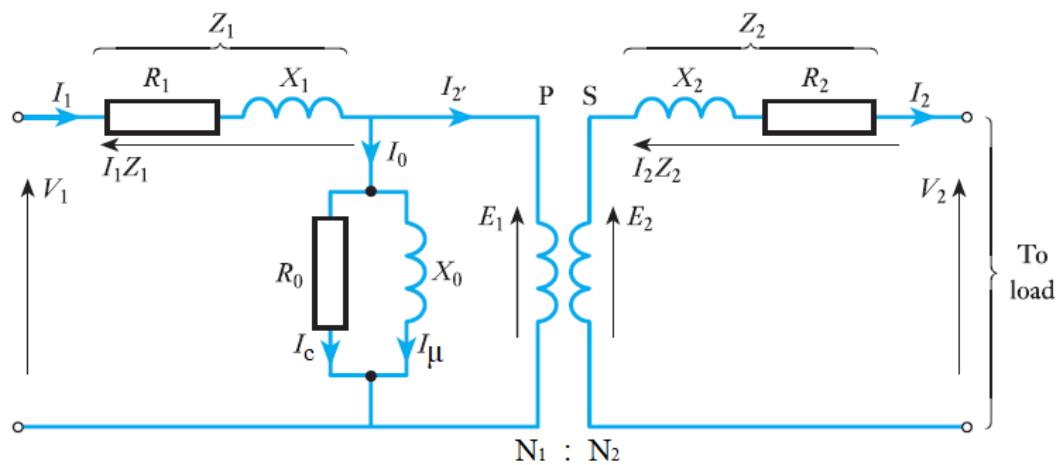


### *Construction Steps*

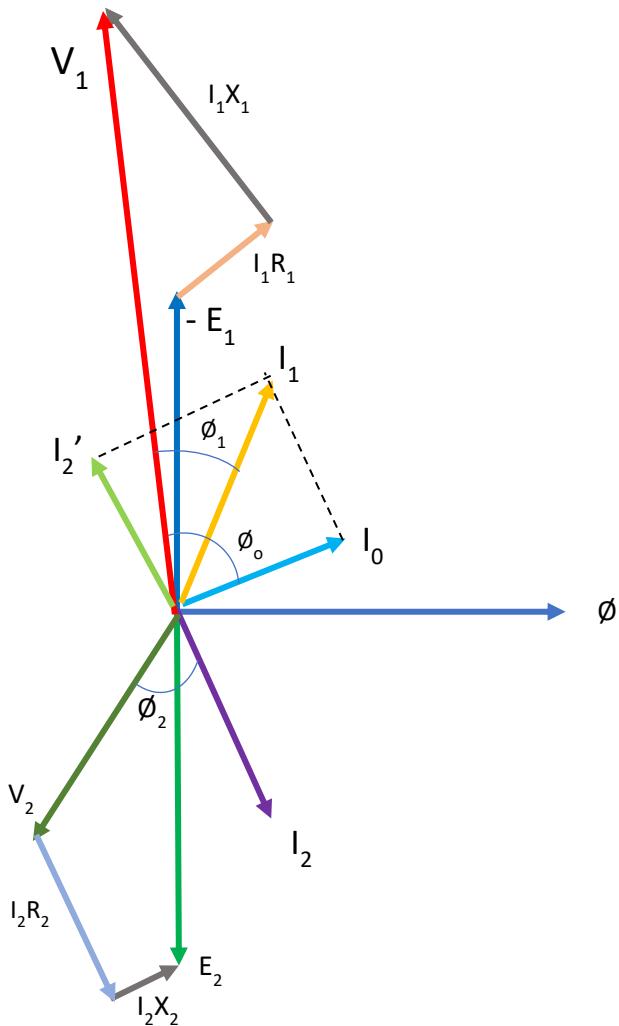
1. Take flux  $\phi$ , a reference
2. Induces emf  $E_1$  and  $E_2$  lags the flux by 90 degrees.
3. The component of the applied voltage to the primary equal and opposite to induced emf in the primary winding,  $-E_1$  (according to Lenz's Law)
4. Current  $I_0$  lags the voltage  $c$ .
5. The power factor of the load is lagging. Therefore, current  $I_2$  is drawn lagging  $V_2$  by an angle  $\phi_2$ .
6. The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage  $V_2$  is the phase difference of  $E_2$  and voltage drop.
7.  $V_2 + I_2 R_2 + I_2 X_2 = E_2$   
where,  $I_2 R_2$  is in phase with  $I_2$  and  $I_2 X_2$  is in quadrature with  $I_2$ .
8. The total current flowing in the primary winding is the phasor sum of  $I_1'$  and  $I_0$ .
9. Primary applied voltage  $V_1$  is the phasor sum of  $-E_1$  and the voltage drop in the primary winding.
10. Current  $I_1'$  is drawn equal and opposite to the current  $I_2$
11.  $V_1 = -E_1 + I_1 R_1 + I_1 X_1$   
where,  $I_1 R_1$  is in phase with  $I_1$  and  $I_1 X_1$  is in quadrature with  $I_1$ .
12. The phasor difference between  $V_1$  and  $I_1$  gives the power factor angle  $\phi_1$  of the primary side of the transformer.
13. The phasor difference between  $V_2$  and  $I_2$  gives the power factor angle  $\phi_2$  of the primary side of the transformer.
14. The phasor difference between  $V_1$  and  $I_0$  gives the no load power factor angle  $\phi_0$  of the primary side of the transformer.

## ACTUAL TRANSFORMER ON CAPACITIVE LOAD

## Equivalent Circuit



## Phasor diagram

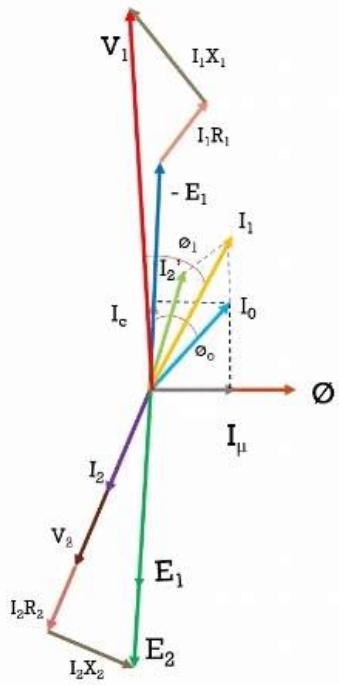


### *Construction Steps*

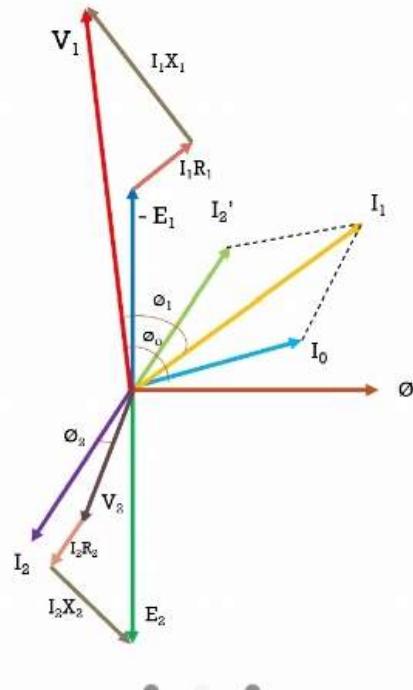
1. Take flux  $\phi$ , a reference
2. Induces emf  $E_1$  and  $E_2$  lags the flux by 90 degrees.
3. The component of the applied voltage to the primary equal and opposite to induced emf in the primary winding,  $-E_1$  (according to Lenz's Law)
4. Current  $I_0$  lags the voltage  $-E_1$ .
5. The power factor of the load is leading. Therefore, current  $I_2$  is drawn lagging  $V_2$  by an angle  $\phi_2$ .
6. The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage  $V_2$  is the phase difference of  $E_2$  and voltage drop.
7.  $V_2 - (I_2 R_2 + I_2 X_2) = E_2$   
where,  $I_2 R_2$  is in phase with  $I_2$  and  $I_2 X_2$  is in quadrature with  $I_2$ .
8. The total current flowing in the primary winding is the phasor sum of  $I_1'$  and  $I_0$ .
9. Primary applied voltage  $V_1$  is the phasor sum of  $-E_1$  and the voltage drop in the primary winding.
10. Current  $I_1'$  is drawn equal and opposite to the current  $I_2$
11.  $V_1 = -E_1 + I_1 R_1 + I_1 X_1$   
where,  $I_1 R_1$  is in phase with  $I_1$  and  $I_1 X_1$  is in quadrature with  $I_1$ .
12. The phasor difference between  $V_1$  and  $I_1$  gives the power factor angle  $\phi_1$  of the primary side of the transformer.
13. The phasor difference between  $V_2$  and  $I_2$  gives the power factor angle  $\phi_2$  of the primary side of the transformer.
14. The phasor difference between  $V_1$  and  $I_0$  gives the no load power factor angle  $\phi_0$  of the primary side of the transformer.

# ACTUAL TRANSFORMER ON VARIOUS LOADS

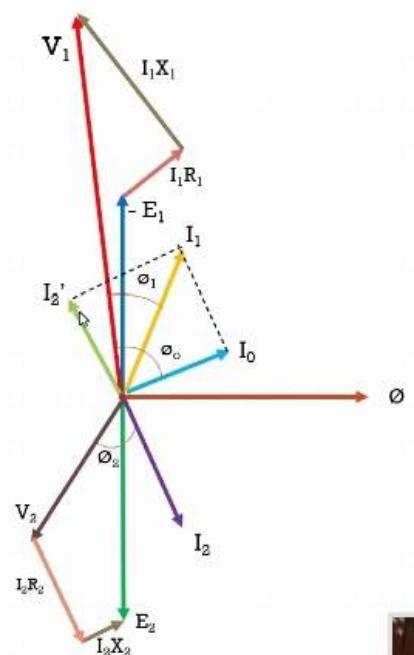
RESISTIVE LOAD



INDUCTIVE LOAD



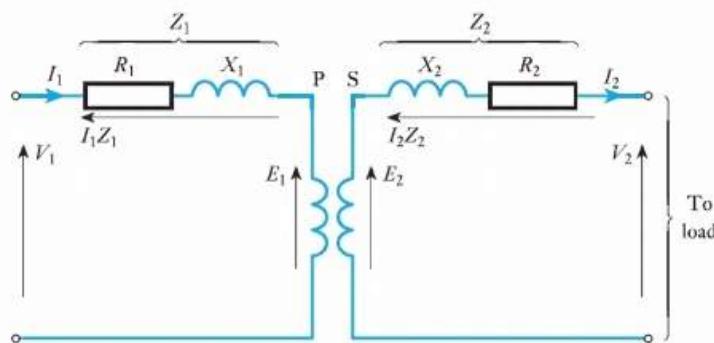
CAPACITIVE LOAD



Check Reddy Sheri's screen

## APPROXIMATE EQUIVALENT CIRCUIT OF A TRANSFORMER

- Since the no-load current of a transformer is only about 1– 3 % of the full-load primary current, we can omit the parallel circuits  $R_0$  and  $X_0$  from the equivalent circuit without introducing an appreciable error when we are considering the behaviour of the transformer on full load.
- Thus we have the simpler equivalent circuit



# REFERRING OF PARAMETERS

- It is theoretically possible to replace the resistance  $R_2$  of the secondary by inserting additional resistance  $R_2'$  in the primary circuit such that the power absorbed in  $R_2'$  when carrying the primary current is equal to that in  $R_2$  due to the secondary current, i.e.

$$I_1^2 R_{2'} = I_2^2 R_2$$

$$R_2' = R_2 \left( \frac{I_2}{I_1} \right)^2 \simeq R_2 \left( \frac{V_1}{V_2} \right)^2$$

- Hence if  $R_e$  is a single resistance in the primary circuit equivalent to the primary and secondary resistances of the actual transformer then,

$$R_e = R_1 + R_2' = R_1 + R_2 \left( \frac{V_1}{V_2} \right)^2$$

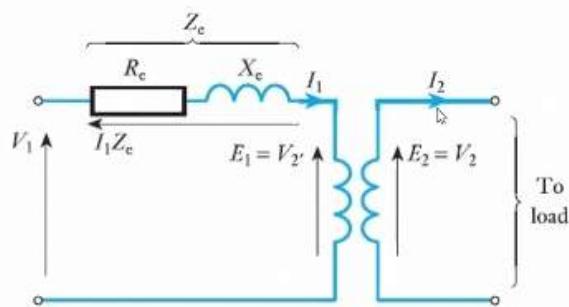


# REFERRING OF PARAMETERS

- If  $Z_e$  is the equivalent impedance of the primary and secondary windings referred to the primary circuit

$$Z_e = \sqrt{(R_e^2 + X_e^2)}$$

- The simplified equivalent circuit of the transformer after referring the secondary parameters to primary can be drawn as,



# REFERRING OF PARAMETERS

- Similarly, since the inductance of a coil is proportional to the square of the number of turns, the primary leakage reactance  $X_1$  can be replaced by an equivalent reactance  $X_1'$  in the secondary circuit, such that

$$X_1' = X_1 \left( \frac{N_2}{N_1} \right)^2 \simeq X_1 \left( \frac{V_2}{V_1} \right)^2$$

- If  $X_e$  is the single reactance in the secondary circuit equivalent to  $X_1$  and  $X_2$  of the actual transformer

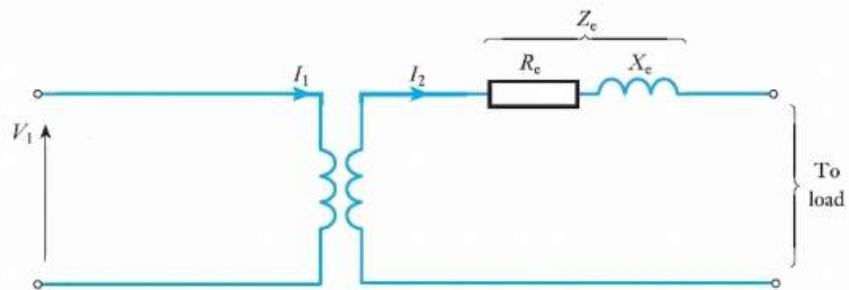
$$X_e = X_2 + X_1' = X_2 + X_1 \left( \frac{V_2}{V_1} \right)^2$$

# REFERRING OF PARAMETERS

- If  $Z_e$  is the equivalent impedance of the primary and secondary windings referred to the secondary circuit

$$Z_e = \sqrt{(R_e^2 + X_e^2)}$$

- The simplified equivalent circuit of the transformer after referring the secondary parameters to primary can be drawn as,



# LOSSES IN THE TRANSFORMER

## Copper Losses (Variable Losses)

- **Copper losses** are variable and result in a heating of the conductors, since they possess resistance.
- If  $R_1$  and  $R_2$  are the primary and secondary winding resistances, then the total copper loss is  $I_1^2 R_1 + I_2^2 R_2$

where,

$I_1^2 R_1$  correspond to the primary copper losses and

$I_2^2 R_2$  correspond to the secondary copper losses and

## Iron Losses (Constant Losses)

- **Hysteresis loss** is the heating of the core as a result of the internal molecular structure reversals which occur as the magnetic flux alternates. The loss is proportional to the area of the hysteresis loop and thus low loss nickel iron alloys are used for the core since their hysteresis loops have small areas.
- **Eddy current loss** is the heating of the core due to e.m.f.s being induced not only in the transformer windings but also in the core. These induced e.m.f.s set up circulating currents, called eddy currents.



# EFFICIENCY OF TRANSFORMER

$$\text{Efficiency, } \eta = \frac{\text{Output Power}}{\text{Input Power}}$$

$$\text{Efficiency, } \eta = \frac{\text{Output Power}}{\text{Output Power} + \text{losses}}$$

$$\text{Efficiency, } \eta = \frac{\text{Input Power} - \text{losses}}{\text{Input Power}}$$

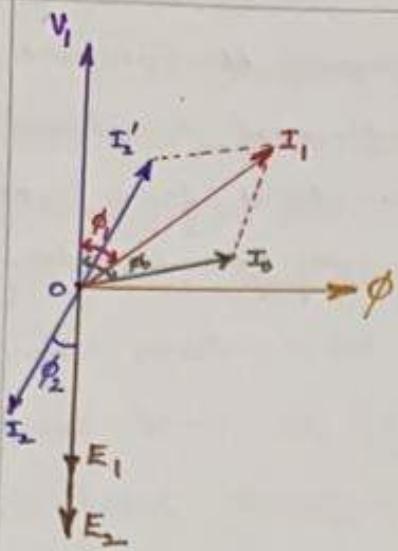
- Efficiency is usually **expressed as a percentage**. It is not uncommon for power transformers to have efficiencies of between 95% and 98%.

**Output power** =  $V_2 I_2 \cos\varphi_2$ , where  $\cos\varphi_2$  is the load power factor

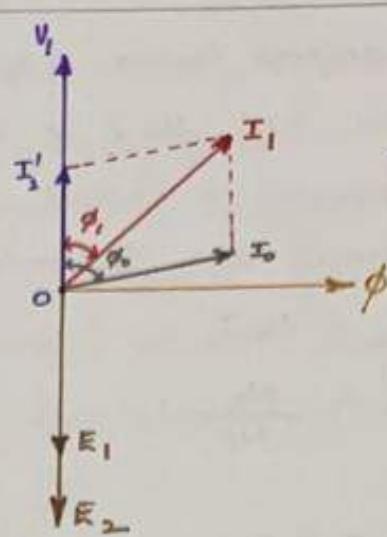
total losses = copper loss + iron losses

and input power = output power + losses

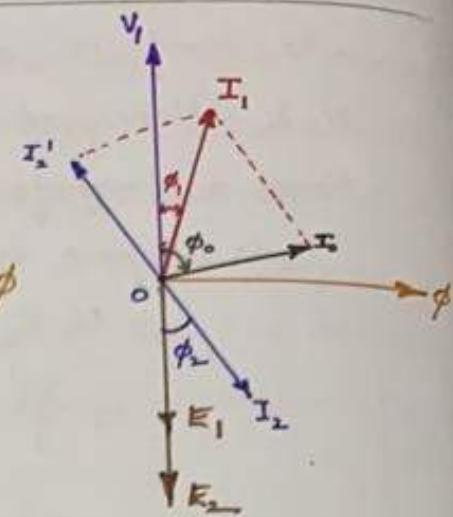




(a) Inductive load



(b) Resistive load

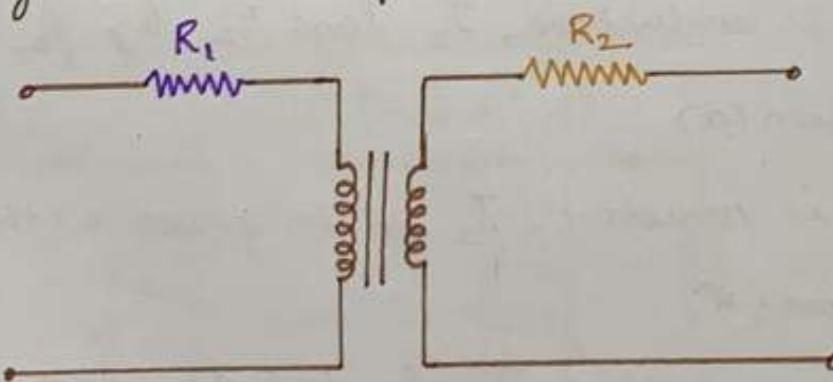


(c) Capacitive load.

#### 4.12 REFERRED VALUES

In order to simplify calculations, it is theoretically possible to transfer voltage, current and impedance of one winding to the other and combine them into single values for each quantity. Thus, we have to work in one winding only which is more convenient.

Let us transfer the resistance of secondary winding  $R_2$  to the primary side.



Suppose that  $R_2'$  is the resistance of the secondary winding referred or reflected to the primary winding. This reflected resistance  $R_2'$  should produce the same effect in primary as  $R_2$  produces in the secondary. Therefore, power consumed in  $R_2'$  when carrying the primary current is equal to the power consumed by  $R_2$  due to the secondary current.

That is,

$$I_2'^2 R_2' = I_2^2 R_2$$

$$R_2' = \left[ \frac{I_2}{I_2'} \right]^2 R_2$$

But,

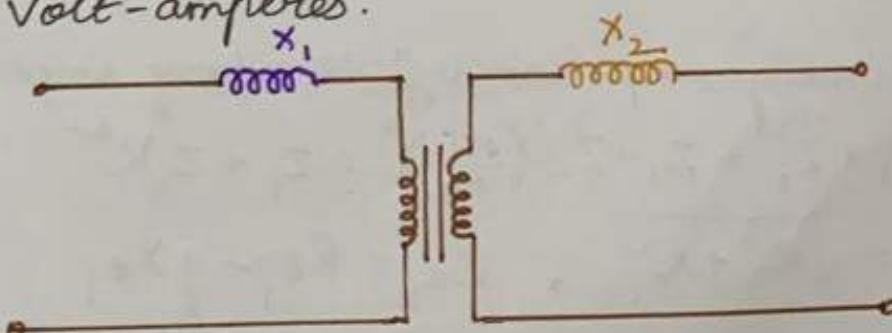
$$I_2 N_2 = I_2' N_1$$

$$\frac{I_2}{I_2'} = \frac{N_1}{N_2} = K$$

and

$$R_2' = \left( \frac{N_1}{N_2} \right)^2 R_2 = K^2 R_2$$

Let  $X_2'$  be the reactance of the secondary winding referred to the primary side. For  $X_2'$  to produce the same effect in the primary side as in the secondary side, each must absorb the same reactive volt-amperes.



$$\begin{aligned}VA_m &= VI \sin \phi \\&= I^2 \cdot I \cdot \frac{X}{Z} \\&= I^2 X\end{aligned}$$

Equating the reactive voltaamps consumed by  $x_2'$  and  $x_2$  gives,

$$(I_2')^2 x_2' = I_2^2 x_2$$

$$x_2' = \left[ \frac{I_2}{I_2'} \right]^2 x_2$$

$$x_2' = \left[ \frac{N_1}{N_2} \right]^2 x_2$$

$$\therefore x_2' = K^2 x_2$$

Let  $R_{e_1}$ ,  $X_{e_1}$  and  $Z_{e_1}$  represent the effective resistance, effective reactance and effective impedance respectively of the whole transformer referred to primary, then

$R_{e_1}$  = primary resistance + secondary resistance referred

$$\therefore R_{e_1} = R_1 + R_2' = R_1 + R_2 \left( \frac{N_1}{N_2} \right)^2 = R_1 + K^2 R_2 \text{ to primary.}$$

Similarly,

$X_{e_1}$  = Primary reactance + secondary reactance referred to primary

$$X_{e_1} = X_1 + X_2' = X_1 + X_2 \left( \frac{N_1}{N_2} \right)^2 = X_1 + K^2 X_2$$

Hence,

$Z_{e_1}$  = Primary impedance + secondary impedance referred to primary.

$$Z_{e_1} = Z_1 + Z_2' = Z_1 + Z_2 \left( \frac{N_1}{N_2} \right)^2 = Z_1 + Z_2 K^2$$

$$\text{Also, } Z_{e_1} = \sqrt{R_{e_1}^2 + X_{e_1}^2}, \quad Z_{e_1} = R_{e_1} + j X_{e_1}$$

## Equivalent values referred to secondary

The equivalent values referred to secondary can also be found in the same manner. If  $R_{e_2}$ ,  $X_{e_2}$  and  $Z_{e_2}$  denote the equivalent resistance, reactance and impedance respectively of the whole transformer referred to secondary, then

$R_{e_2}$  = Secondary Resistance + Primary resistance ref. to secondary

$$\therefore R_{e_2} = R_2 + R_1' = R_2 + R_1 \left( \frac{N_2}{N_1} \right)^2 = R_2 + \frac{R_1}{K^2}$$

Similarly,

$$X_{e_2} = X_2 + X_1' = X_2 + X_1 \left( \frac{N_2}{N_1} \right)^2 = X_2 + \frac{X_1}{K^2}$$

Hence,

$$Z_{e_2} = Z_2 + Z_1' = Z_2 + Z_1 \left( \frac{N_2}{N_1} \right)^2 = Z_2 + \frac{Z_1}{K^2}$$

Also,

$$Z_{e_2} = R_{e_2} + j X_{e_2} \quad Z_{e_2} = \sqrt{R_{e_2}^2 + X_{e_2}^2}$$

$$R_{e_1} = R_1 + R_2 \left( \frac{N_1}{N_2} \right)^2$$

$$R_{e_1} \left( \frac{N_2}{N_1} \right)^2 = R_1 \left( \frac{N_2}{N_1} \right)^2 + R_2 = R_{e_2}$$

$$\therefore R_{e_2} = R_{e_1} \left( \frac{N_2}{N_1} \right)^2 = \frac{R_{e_1}}{K^2}$$

Similarly,

$$X_{e_2} = X_{e_1} \left( \frac{N_2}{N_1} \right)^2 = \frac{X_{e_1}}{K^2}$$

and

$$Z_{e_2} = Z_{e_1} \left( \frac{N_2}{N_1} \right)^2 = \frac{Z_{e_1}}{K^2}$$

#### 4.13 REGULATION

Majority of the loads connected to the transformer secondary are designed to operate at practically constant voltage. However, as the current is taken through the transformer, the load terminal voltage changes because of the voltage drop in the internal impedance of the transformer. The term voltage regulation is used to identify the characteristic of the voltage change in a transformer with loading.

The voltage regulation of a transformer is defined as the arithmetic difference between the no-load ( $I_2 = 0$ ) and full-load ( $I_2 = I_{2\text{fl}}$ ) at a given power factor with the same value of primary voltage for both rated and no-load.

It is expressed generally in percentage voltage regulation.

So,

$$\text{Voltage Regulation} = \frac{\text{Secondary No-load Voltage} - \text{Secondary Load Voltage}}{\text{Secondary no-load voltage}} \times 100 \%$$

If

$E_2$  = no load voltage

$V_2$  = load voltage

$$\text{Then } \% \text{ Regulation} = \frac{E_2 - V_2}{E_2} \times 100 \%$$

#### 4.14 LOSSES IN A TRANSFORMER

The losses which occur in a transformer are (a) Core losses or Iron losses,  $P_i$   
(b) Copper losses or  $I^2R$  losses,  $P_c$

##### Iron or Core Losses ( $P_i$ )

Iron losses occur in the magnetic core of the transformer. This loss is the sum of hysteresis and eddy current losses,  $P_h$  and  $P_e$  respectively.

$$\therefore P_i = P_h + P_e$$

Hysteresis and Eddy current losses are given by,

$$P_h = k_h f B_m^x$$

$$P_e = k_e f B_m^2$$

where,

$k_h \rightarrow$  proportionality constant which depends on volume and quality of the core material and the units used.

$k_e \rightarrow$  proportionality constant whose value depends on volume & resistivity of the core material, thickness of lamination and units used.

$B_m \rightarrow$  Maximum flux density in the core

$f \rightarrow$  Frequency of alternating current

$x \rightarrow$  Steinmetz constant. Value ranges from 1.5 to 2.5 depending on the core material used.

### Copper loss or $I^2R$ Losses

Copper loss is the  $I^2R$  losses which take place in the transformer primary and secondary windings because of their winding resistances.

Total copper loss in the transformer ( $P_c$ ) = Primary winding copper loss ( $I_1^2 R_1$ ) + Secondary winding copper loss ( $I_2^2 R_2$ )

$$\therefore P_c = I_1^2 R_1 + I_2^2 R_2$$

Copper loss varies with the square of the load current.

As there are no moving parts in a transformer, there are no mechanical or rotational losses happening in the transformer.

### 4.15 EFFICIENCY OF A TRANSFORMER

Like all other machines, efficiency of the transformer is the ratio of output to the input.

$$\eta = \frac{\text{Output of the transformer}}{\text{Input of the transformer}}$$

where, input = output + losses

As discussed earlier, losses are iron and copper losses.

So efficiency is given as,

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Iron loss} + \text{Cu losses}}$$

If -

$V_2$  is the voltage across the load

$I_2$  is the full load current

$\cos\phi_2$  is the load power factor and

$P_i, P_c$  represents iron and Cu losses respectively

while  $R_2$  is the total resistance referred to secondary side, then efficiency at full load

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_c}$$

and  $P_c = I_2^2 R_2$  Watts

Hence, percentage efficiency at full load is,

$$\eta = \frac{VA \times p.f}{VA \times p.f + P_i + P_c} \times 100\%.$$

Efficiency at  $x\%$  full-load can be found as,

$$\eta_x = \frac{x \times \text{full load output}}{x \times \text{full load output} + P_i + x^2 P_c}$$

Since,

- (i) Iron losses are constant at all loading conditions
- (ii) Copper losses vary according to the square of the loading condition.

# VOLTAGE REGULATION OF A TRANSFORMER

- Voltage regulation is a measure of change in the voltage magnitude between the sending and receiving end of a component.
- It is commonly used in power engineering to describe the percentage voltage difference between no load and full load voltages
- The voltage regulation of a transformer is defined as the variation of the secondary voltage between no load and full load, expressed as either a per-unit or a percentage of the *no-load* voltage, the primary voltage being assumed constant, i.e.

$$\text{Voltage Regulation} = \frac{\text{No load voltage} - \text{full load voltage}}{\text{no load voltage}}$$

The equation for the **voltage regulation of transformer**, represented in percentage, is

$$\text{Voltage regulation}(\%) = \frac{E_2 - V_2}{V_2} \times 100\%$$

# TESTING OF TRANSFORMERS

- The efficiency and regulation of a transformer on any load condition and at any power factor condition can be predetermined by indirect loading method.
- In this method, the actual load is not used on transformer. But the equivalent circuit parameters of a transformer are determined by conducting two tests on a transformer which are,
  1. Open circuit test (O.C Test)
  2. Short circuit test (S.C.Test)
- The parameters calculated from these test results are effective in determining the regulation and efficiency of a transformer at any load and power factor condition, without actually loading the transformer.
- The advantage of this method is that without much power loss the tests can be performed and results can be obtained.

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Recording

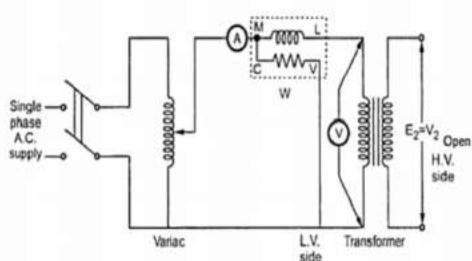
## TESTING OF TRANSFORMERS

- Suppose 400MVA, 20kV/400kV power transformer, to test this transformer directly by loading it, it will be really a monumental task. Firstly, finding a load for such high rating transformer is difficult.
- A load which will actually carry so much of current and it will dissipate so much of power, then to be able to cool it. This would waste so much money, time and energy.
- In open circuit test and short-circuit tests, we are essentially creating situations such that the transformer is put through such an operating condition which will emulate the normal loading condition, although not simultaneously.
- O.C. Test we apply rated voltage and low current
- S.C. Test we apply rated current and low voltage
- These two tests enable the efficiency and the voltage regulation to be calculated without actually loading the transformer and with an accuracy far higher than is possible by direct measurement of input and output powers and voltages.
- Also, the power required to carry out these tests is very small compared with the full-load output of the transformer.



Recording

## OPEN CIRCUIT (O.C.) TEST



- The circuit diagram for conducting the test is shown in figure.
- The **rated voltage** is applied to one winding and other winding is kept open (*usually LV winding is supplied, while HV is kept open for ease of testing and availability of supply*).
- Since the **no-load current** is a **very small percentage** of the full load current, which can be in the range of 0.2 to 2%.
- The input power measured by a wattmeter consists of the core loss and primary winding ohmic loss.
- If the no-load current is 1% of full load current, ohmic loss in primary winding resistance is just 0.01% of the load loss at rated current; the value of winding loss is negligible as compared to the core losses.
- Hence, the **entire wattmeter reading can be taken as the total core loss**.

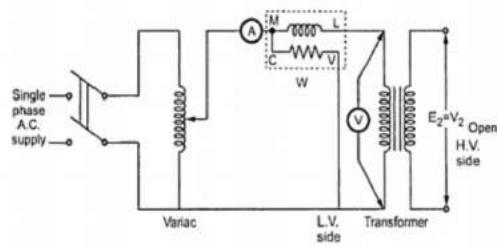
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# OPEN CIRCUIT (O.C.) TEST

recording

- Data obtained from O.C. Test



When the primary voltage is adjusted to its rated value with the help of variac, readings of ammeter and wattmeter are to be recorded.

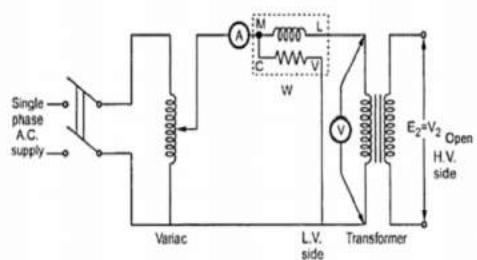
Voltage (V <sub>0</sub> )	Current (I <sub>0</sub> )	Power (W <sub>0</sub> )
Rated Voltage of the Primary or LV winding	Very small percentage of the full load current	Total Core loss

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# OPEN CIRCUIT (O.C.) TEST

Recording



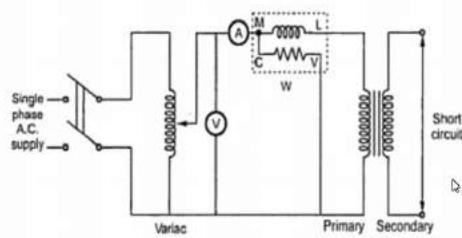
- The primary is excited by rated voltage, which is adjusted precisely with the help of a variac.
- The wattmeter measures input power.
- The ammeter measures input current.
- The voltmeter gives the value of rated primary voltage applied at rated frequency.

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# SHORT CIRCUIT (S.C.) TEST

Recording



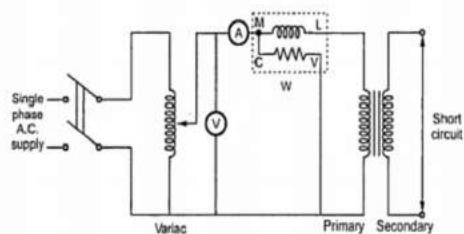
- A short circuit test is done to measure the load loss of a transformer.
- The secondary is short circuited with the help of thick copper wire or solid link.
- As high voltage side always has low current side, it is convenient to connect high voltage side to supply and shorting the low voltage side.
- As secondary is shorted, its resistance is very very small and on rated voltage it may draw very large current.
- Such large current can cause overheating and burning of the transformer.
- To limit this short circuit current, primary is supplied with low voltage which is just enough to cause rated current to flow through primary which can be observed on an ammeter.



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## SHORT CIRCUIT (S.C.) TEST

Recording



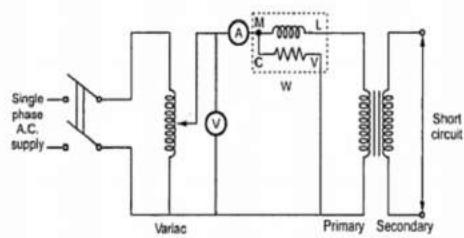
- The low voltage can be adjusted with the help of variac. Hence this test is also called low voltage test or reduced voltage.
- Now the **current flowing through the windings are rated current** hence the total copper loss is full load copper loss.
- Now the **voltage supplied is low** which is a small fraction of the rated voltage.
- The **iron losses are function of applied voltage**. So the iron losses in reduced voltage test are very **small**.
- Hence, the **wattmeter reading** is the power loss which is **equal to full load copper losses** as iron losses are very low.



# SHORT CIRCUIT (S.C.) TEST

Recording

- Data obtained from S.C. Test



The wattmeter reading as well as voltmeter, ammeter readings are recorded. The observation table is as follows,

Voltage ( $V_{sc}$ )	Current ( $I_{sc}$ )	Power ( $W_{sc}$ )
Small fraction of the rated voltage	Rated current	Total Copper loss



Recording

## CALCULATION OF EFFICIENCY FROM O.C. AND S.C. TESTS

- We know that,  
From O.C. test,  $W_o$  = Iron or Core losses =  $P_i$   
From S.C. test,  $W_{sc}$  = Copper losses =  $P_{cu}$
- Also, Efficiency,  $\eta = \frac{\text{Output Power}}{\text{Output Power} + \text{losses}}$

$$\therefore \% \text{ Efficiency}, \eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_{cu} + P_i} \times 100$$

Thus, for any p.f.  $\cos \phi_2$ , the efficiency can be predetermined. Similarly at any load which is fraction of full load then also efficiency can be predetermined as,

$$\therefore \% \text{ Efficiency at any load } x, \eta_x = \frac{x V_2 I_2 \cos \phi_2}{x V_2 I_2 \cos \phi_2 + x^2 P_{cu} + P_i} \times 100$$

where,  $x$  is the % of full load



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Recording

## O.C. & S.C. TEST - NUMERICAL

1. A 5 KVA, 500/250 V, 50 Hz, single phase transformer gave the following readings,  
O.C. Test : 500 V, 1 A, 50 W (L.V. side open)  
S.C. Test : 25 V, 10 A, 60 W (L.V. side shorted)  
Determine : i) The efficiency on full load, 0.8 lagging p.f.

ii) The efficiency on 60% of full load, 0.8 leading p.f.

From O.C. test,  $W_o =$  Iron or Core losses = 50 W =  $P_i$   
From S.C. test,  $W_{sc} =$  Copper losses = 60 W =  $P_{cu}$

i) The efficiency on full load, 0.8 lagging p.f.

$$V_2 I_2 = 50 \text{ kVA} = 50 \times 10^3 \text{ VA}$$

$$\cos\theta_2 = 0.8, P_i = 50 \text{ W}, P_{cu} = 60 \text{ W}$$

$$\therefore \% \text{ Efficiency, } \eta = \frac{5 \times 10^3 \times 0.8}{5 \times 10^3 \times 0.8 + 60 + 50} \times 100 = 97.32 \%$$

ii) The efficiency on full load, 60% of full load, 0.8 leading p.f.

$$\% \text{ Efficiency at any load } x, \eta_x = \frac{0.6 \times 5 \times 10^3 \times 0.8}{0.6 \times 5 \times 10^3 \times 0.8 + (0.6^2 \times 60) + 50} \times 100 = 97.103 \%$$



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Recording

## O.C. & S.C. TEST - NUMERICAL

2. The open circuit and short circuit tests on a 10 KVA, 125/250 V, 50 Hz, single phase transformer gave the following results :

O.C. test : 125 V, 0.6 A, 50 W (on L.V. side)  
S.C. test : 15 V, 30 A, 100 W (on H.V. side)

Calculate : i) copper loss on full load  
ii) full load efficiency at 0.8 leading p.f.  
iii) half load efficiency at 0.8 leading p.f.

i) Copper loss on full load

$$(I_2) \text{ F.L.} = \text{VA rating}/V_2 = (10 \times 10^3)/250 = 40 \text{ A}$$

In short circuit test,  $I_{sc} = 30 \text{ A}$  and not equal to full load value 40 A.

Hence  $W_{sc}$  does not give copper loss on full load

$$\therefore W_{sc} = P_{cu} \text{ at } 30 \text{ A} = 100 \text{ W}$$

Now

$$P_{cu} \propto I^2$$
$$(\text{P}_{cu} \text{ at } 30 \text{ A})/(\text{P}_{cu} \text{ at } 40 \text{ A}) = (30/40)^2$$

$$100/(\text{P}_{cu} \text{ at } 40 \text{ A}) = 900/1600$$

$$\text{P}_{cu} \text{ at } 40 \text{ A} = 177.78 \text{ W}$$

$$\therefore (\text{P}_{cu}) \text{ F.L.} = 177.78 \text{ W}$$

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## O.C. & S.C. TEST - NUMERICAL

- The open circuit and short circuit tests on a 10 KVA, 125/250 V, 50 Hz, single phase transformer gave the following results :

O.C. test : 125 V, 0.6 A, 50 W (on L.V. side)  
 S.C. test : 15 V, 30 A, 100 W (on H.V. side)

- Calculate : i) copper loss on full load  
 ii) full load efficiency at 0.8 leading p.f.  
 iii) half load efficiency at 0.8 leading p.f.

ii) Full load  $\eta$ ,  $\cos \Phi_2 = 0.8$

$$\% \text{ Efficiency}, \eta = \frac{250 \times 40 \times 0.8}{250 \times 40 \times 0.8 + 50 + 177.78} \times 100 = 97.23 \%$$

iii) Half load  $\eta$ ,  $\cos \Phi_2 = 0.8$

$$\% \text{ Efficiency at any load } x, \eta_x = \frac{0.5 \times 10 \times 10^3 \times 0.8}{0.5 \times 10 \times 10^3 \times 0.8 + (0.5^2 \times 177.78) + 50} \times 100 = 97.103 \%$$

# EQUIVALENT CIRCUIT PARAMETERS

## Open Circuit Test

$W_o$  = Iron losses

- The no load shunt parameters are calculated from the OC test as
  - ✓ The no load power factor,  $\cos \Phi_o = W_o / V_o I_o$
- Once the power factor is obtained, the no load component currents are determined as,
- ✓ Magnetizing component of no load current,  $I_m = I_o \sin \Phi_o$
  - ✓ Core loss component of no load current,  $I_m = I_o \cos \Phi_o$
  - ✓ Then, the magnetizing branch reactance,  $X_o = V_o / I_m$
  - ✓ Resistance representing core loss,  $R_o = V_o / I_c$

## Short Circuit Test

$W_{sc}$  = Full load copper losses

- ✓ Equivalent resistance referred to HV side,  $R_{e1} = W_{sc} / I_{sc}^2$
- ✓ Equivalent impedance referred to HV side,  $Z_{01} = V_{sc} / I_{sc}$
- ✓ Equivalent leakage reactance referred to HV side,  $X_{01} = \sqrt{(Z_{01}^2 - R_{e1}^2)}$
- ✓ And also short circuit power factor

$$\cos \Phi_{sc} = W_{sc} / V_{sc} I_{sc}$$

You

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## O.C. & S.C. TEST - NUMERICAL

- 3. The following results were obtained on a 50 kVA transformer. Opencircuit test – primary voltage, 3300 V; secondary voltage, 400 V; primary power, 430 W. Short-circuit test – primary voltage, 124 V; primary current, 15.3 A; primary power, 525 W; secondary current, full-load value. Calculate the efficiencies at full load and at half load for 0.7 power factor.

$$\text{Core loss} = 430 \text{ W}$$

$$I^2R \text{ loss on full load} = 525 \text{ W}$$

$$\therefore \text{Total loss on full load} = 955 \text{ W} = 0.955 \text{ kW}$$

$$\text{and Efficiency on full load} = \frac{50 \times 0.7}{(50 \times 0.7) + 0.955}$$
$$= \left(1 - \frac{0.955}{35.95}\right) = 0.973$$
$$= 97.3 \%$$

$$I^2R \text{ loss on half load} = 525 \times (0.5)^2 = 131 \text{ W}$$

$$\therefore \text{Total loss on half load} = 430 + 131 = 561 \text{ W} = 0.561 \text{ kW}$$

$$\text{and Efficiency on half load} = \frac{25 \times 0.7}{(25 \times 0.7) + 0.561}$$

$$= \left(1 - \frac{0.561}{18.06}\right) = 0.969$$
$$= 96.9 \%$$

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## Numericals

- Determine the efficiency of the transformer in the previous problem at half full-load and 0.8 power factor.

$$\% \eta = \frac{\pi V_2 I_2 \cos \phi_2}{\pi V_2 I_2 \cos \phi_2 + P_i + \pi^2 P_{cu}}$$
$$= \frac{0.5 \times 40 \times 10^3 \times 0.8}{0.5 \times 40 \times 10^3 \times 0.8 + 500 + (0.5)^2 \times 800} \times 100$$
$$\% \eta = 95.8 \%$$



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## THREE PHASE TRANSFORMERS

- If we take three single-phase transformers and connect their primary windings to each other and their secondary windings to each other in a fixed configuration, we can use the transformers on a three-phase supply.
- Three-phase, also written as 3-phase or  $3\varphi$  supplies are used for electrical power generation, transmission, and distribution, as well as for all industrial uses.
- Three-phase supplies have many electrical advantages over single-phase power and when considering three-phase transformers we have to deal with three alternating voltages and currents differing in phase-time by 120 degrees
- The three phase transformer consists three transformers either separate or combined with one core.
- The primary and secondary of the transformer can be independently connected either in star or delta.
- There are four possible connections for a 3-phase transformer bank.

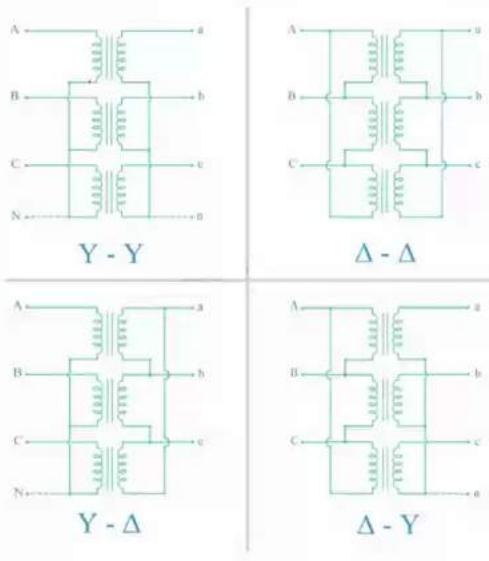
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## THREE PHASE TRANSFORMERS CONNECTIONS



- i. Δ - Δ (Delta – Delta) Connection
- ii. Y - Y (Star – Star) Connection
- iii. Δ - Y (Delta – Star) Connection
- iv. Y - Δ (Star – Delta ) Connection

▪ The choice of connection of three phase transformer depends on the various factors like the availability of a neutral connection for grounding protection or load connections, insulation to ground and voltage stress, availability of a path for the flow of third harmonics, etc.

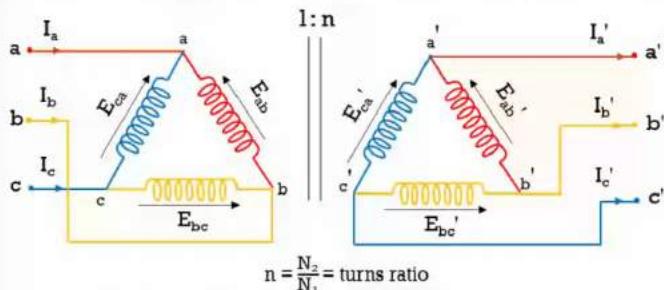
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## Δ – Δ (DELTA – DELTA) CONNECTION



Type equation here.

$I_a$ ,  $I_b$ ,  $I_c$  are Primary Line Currents

$E_{ab}$ ,  $E_{bc}$ ,  $E_{ca}$  are Primary Phase Voltages

$I_a'$ ,  $I_b'$ ,  $I_c'$  are Secondary Line Currents

$E_{a'b'}$ ,  $E_{b'c'}$ ,  $E_{c'a'}$  are Secondary Phase Voltages

a, b, c are Primary terminals connected to source

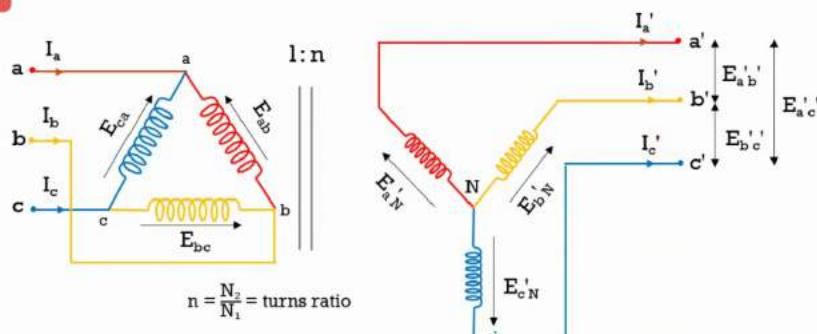
a', b', c' Secondary terminals connected to load

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$I_a$ ,  $I_b$ ,  $I_c$  are Primary Line Currents

$E_{ab}$ ,  $E_{bc}$ ,  $E_{ca}$  are Primary Phase Voltages

$I'_a$ ,  $I'_b$ ,  $I'_c$  are Secondary Line Currents

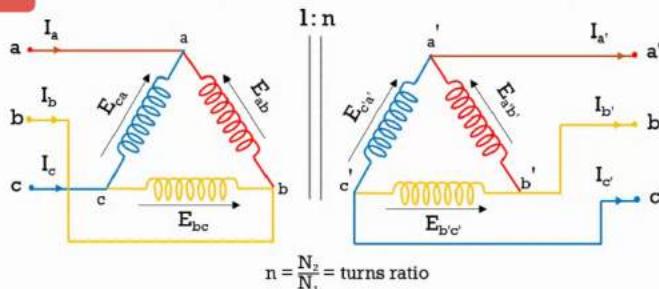
$E'_{aN}$ ,  $E'_{bN}$ ,  $E'_{cN}$  are Secondary Phase Voltages

$E'_{ab}$ ,  $E'_{bc}$ ,  $E'_{ca}$  are Secondary Line Voltages



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## Δ - Δ (DELTA - DELTA) CONNECTION



$I_a, I_b, I_c$  are Primary Line Currents

$E_{ab}, E_{bc}, E_{ca}$  are Primary Phase Voltages

$I_{a'}, I_{b'}, I_{c'}$  are Secondary Line Currents

$E_{a'b'}, E_{b'c'}, E_{c'a'}$  are Secondary Phase Voltages

a, b, c are Primary terminals connected to source

a', b', c' Secondary terminals connected to load

Let the line current in the primary side of the Δ connection be  $I$  Amps

So, the phase current for a Δ connection will be  $\frac{I}{\sqrt{3}}$  Amps

We know that,

$$N_1 I_1 = N_2 I_2$$

Substituting  $I_1 = \frac{I}{\sqrt{3}}$ , we get,

$$N_1 \times \frac{I}{\sqrt{3}} = N_2 I_2$$

$$I_2 = \frac{N_1}{N_2} \times \frac{I}{\sqrt{3}}$$

$$I_2 = \frac{I}{n\sqrt{3}}$$

$$\text{Also, } \frac{N_1}{V_1} = \frac{N_2}{V_2}$$

$$V_2 = \frac{N_2}{N_1} \times V_L$$

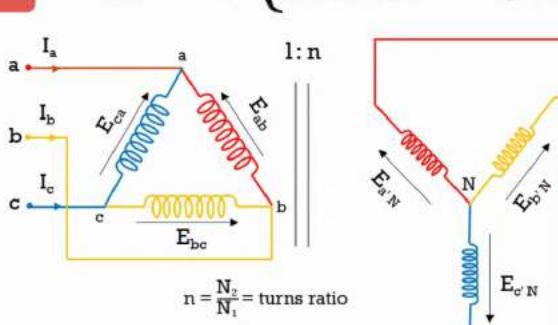
$$\Rightarrow V_2 = nV_L$$

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## Δ - Y (DELTA - STAR) CONNECTION



$$n = \frac{N_2}{N_1} = \text{turns ratio}$$

$I_a, I_b, I_c$  are Primary Line Currents

$E_{ab}, E_{bc}, E_{ca}$  are Primary Phase Voltages

$I_{a'}, I_{b'}, I_{c'}$  are Secondary Line Currents

$E_{a'N}, E_{b'N}, E_{c'N}$  are Secondary Phase Voltages

$E_{a'b'}, E_{b'c'}, E_{a'c'}$  are Secondary Line Voltages

For primary side ,

$$V_L = V_{ph} \text{ for } \Delta \text{ connection}$$

Let  $I_L = I$  Amps

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{I}{\sqrt{3}} \quad [ \text{Since } I_L = \sqrt{3} I_{ph} ]$$

From MMF equation,

$$N_1 I_1 = N_2 I_2$$

We also know that,

$$\frac{N_1}{N_2} = \frac{V_1}{V_2}$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$\frac{1}{n} = \frac{I_2}{I_1} \times n$$

$$\frac{1}{n} = I_2 \times \frac{\sqrt{3}}{I}$$

$$\therefore I_p = \frac{I}{n\sqrt{3}}$$

$$\frac{1}{n} = \frac{V_L}{V_2}$$

$$\therefore V_2 = n V_L$$

So  $V_{ph}$  on the secondary side =  $n V_L$

$V_L$  on the second

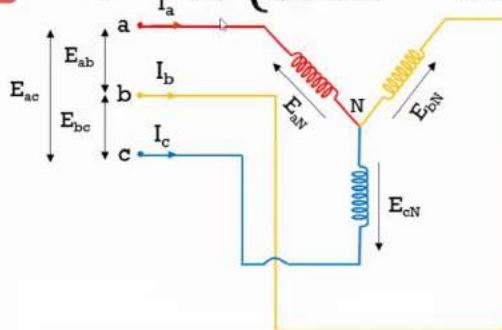
$$\therefore V_L :$$

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## Y - Δ (STAR - DELTA ) CONNECTION

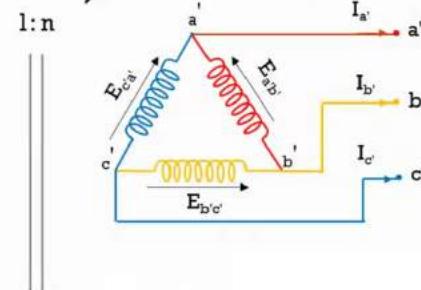


On the primary side  $I_L = I_{ph}$  and  $V_L = \sqrt{3} \times V_{ph}$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1} \Rightarrow \frac{1}{n} = \frac{I_2}{I_L}$$

On the secondary side,  $I_{ph} = \frac{I}{n}$ ;  $I_L = \sqrt{3} \times I_{ph}$

$$\therefore I_L = \frac{I\sqrt{3}}{n}$$



$$\text{Also, } \frac{N_1}{N_2} = \frac{V_1}{V_2} \Rightarrow \frac{1}{n} = \frac{V}{\sqrt{3}V_L}$$

$$V_P = \frac{n \times V}{\sqrt{3}}$$

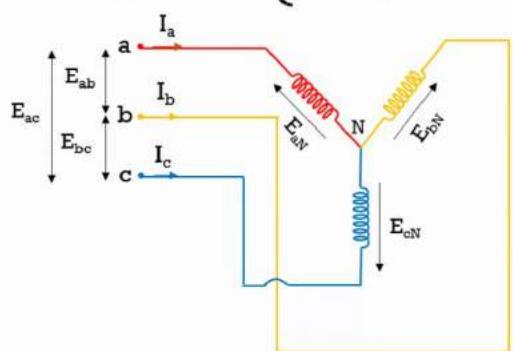
$$\therefore V_P = V_L = \frac{n \times V}{\sqrt{3}}$$

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## Y - Y (STAR - STAR) CONNECTION

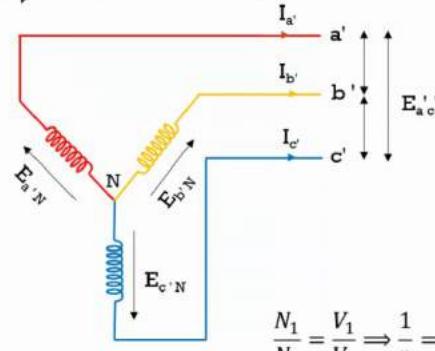


On the primary side  $I_L = I_{ph}$  and  $V_L = \sqrt{3} \times V_{ph}$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1} \Rightarrow \frac{1}{n} = \frac{I_2}{I_L}$$

On the secondary side,  $I_{ph} = \frac{I}{n}$ ;  $I_L = I_{ph}$

$$\therefore I_L = \frac{I}{n}$$



$$\frac{N_1}{N_2} = \frac{V_1}{V_2} \Rightarrow \frac{1}{n} = \frac{V}{\sqrt{3} V_2}$$

$$V_2 = \frac{n \times V}{\sqrt{3}} = V_p$$

For star connection,  $V_L = \dots$

$$\therefore V_L = \frac{n \times V \sqrt{3}}{\sqrt{3}} = nV$$

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# COMPARISON OF CONNECTIONS

Connection Type	PRIMARY SIDE					SECONDARY SIDE			
	Line Voltage	Phase Voltage	Line Current	Phase Current		Line Voltage	Phase Voltage	Line Current	Phase Current
Δ - Δ	$V_L$	$V_{ph} = V_L$	$I_L$	$I_{ph} = \frac{I_L}{\sqrt{3}}$		$V_L = nV_L$	$V_{ph} = nV_L$	$I_L = \frac{I}{n}$	$I_{ph} = \frac{I}{n\sqrt{3}}$
Δ - Y	$V_L$	$V_{ph} = V_L$	$I_L$	$I_{ph} = \frac{I_L}{\sqrt{3}}$	$V_L = \sqrt{3} nV_L$	$V_{ph} = nV_L$	$I_L = \frac{I}{n\sqrt{3}}$	$I_{ph} = \frac{I}{n\sqrt{3}}$	
Y - Δ	$V_L$	$V_{ph} = \frac{V_L}{\sqrt{3}}$	$I_L$	$I_{ph} = I$	$V_L = \frac{nV_L}{\sqrt{3}}$	$V_{ph} = \frac{nV_L}{\sqrt{3}}$	$I_L = \frac{\sqrt{3} I}{n}$	$I_{ph} = \frac{I}{n}$	
Y - Y	$V_L$	$V_{ph} = \frac{V_L}{\sqrt{3}}$	$I_L$	$I_{ph} = I$	$V_L = nV_L$	$V_{ph} = \frac{nV_L}{\sqrt{3}}$	$I_L = \frac{I}{n}$	$I_{ph} = \frac{I}{n}$	

# Transformers

$$\omega = 2\pi f$$

$$f = \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$(E_1)_{\text{rms}} = \sqrt{2} \pi f \phi_m N_1$$

$$= 4.44 \phi_m f N_1$$

In an ideal transformer  
Power is conserved.

$$P_1 = P_2$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\beta W = \omega_2 - \omega_1$$

$$\boxed{\beta W = \frac{R}{2L} + \frac{R}{2L}} \Rightarrow \boxed{\beta W = \frac{R}{L}}$$

For induced EMF in Transformer

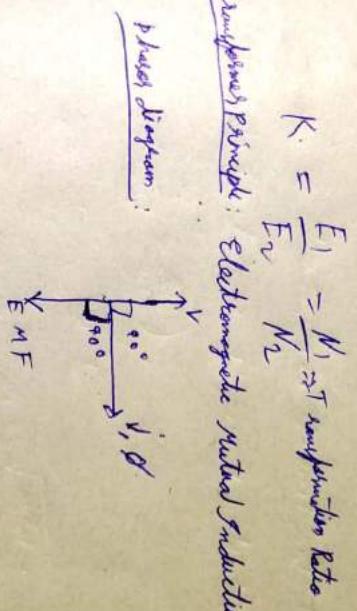
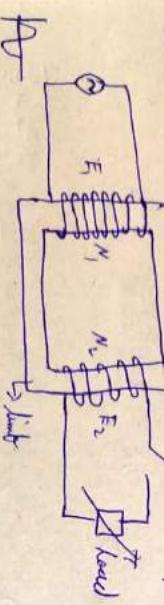
$$\phi = \phi_m \sin \omega t$$

$$e = \frac{d\phi}{dt} = \phi_m \omega \cos \omega t$$

$$\Rightarrow 2\pi f \phi_m \sin(\omega t - 90^\circ)$$

$$E_{\text{max}} \Rightarrow 2\pi f \phi_m$$

4.07/2010



$\therefore$  The Values EMF by  $180^\circ$

- 1) Losses must be zero.

- 2)  $N = \infty$

$$ex \frac{d\phi}{dt}$$

$$e_1 = N_1 \cdot \frac{d\phi}{dt} (\text{Max value})$$

$$e_1 = N_1 \omega_i \phi_m \sin \omega t$$

$$e_1 = 2\pi f \cdot \phi_m N_1 \sin(\omega t - 90^\circ)$$

$$(E_1)_{\text{max}} = 2\pi f \phi_m N_1$$

$$\begin{aligned} (E_2)_{\text{rms}} &= \sqrt{2} \pi f \phi_m N_2 \\ &= 4.44 \phi_m f N_2 \\ &= \omega_i \phi_m (N_2 \sin(\omega t - 90^\circ)) \\ \frac{V_2}{V_1} &= \frac{I_1}{I_2} \end{aligned}$$

4.07/2010

$N_2 > N_1 \rightarrow$  Step up voltage Transformer  
Step-down current Transformer

$N_2 < N_1 \rightarrow$  Step down voltage Transformer  
Step up current Transformer.

## 5.6 Ideal Transformer

An ideal transformer is one that has

- (i) no winding resistance.
- (ii) no leakage flux i.e., the same flux links with both the windings.
- (iii) no iron losses (i.e., eddy current and hysteresis losses) in the core.

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical transformers have properties that come very close to an ideal transformer.

When the primary coil is connected to alternating voltage  $V_1$ , a current  $I_m$  ( $I_0$ ) flows through it. Since the primary coil is purely inductive, the current  $I_m$  lags the applied voltage  $V_1$  by  $90^\circ$ . Due to current  $I_m$ , flux is produced in the primary winding and some of the flux is also linked with the secondary winding and hence emf's  $E_1$  and  $E_2$  are induced in the primary and secondary windings respectively. According to Lenz's law the induced emf opposes the cause producing it which is supply voltage  $V_1$ . Hence  $E_1$  is in antiphase with  $V_1$  but equal in magnitude. The induced emf  $E_2$  also opposes  $V_1$  hence in antiphase with  $V_1$  but its magnitude depends on  $N_2$ . Thus  $E_1$  and  $E_2$  are in phase. The phasor diagram of an ideal transformer is shown in the figure (5.6).

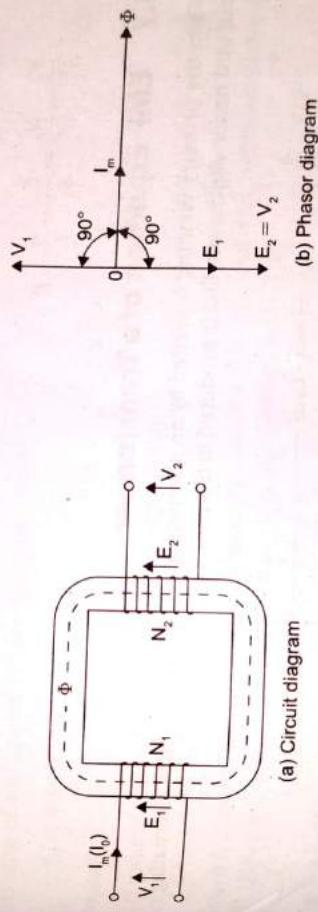


Figure (5.6): Ideal transformer

Here  $I_m$  = magnetizing current  
Mathematically,

Let  $\Phi = \Phi_m \sin \omega t = \Phi_m \angle 0^\circ$  is taken as reference phasor  
The induced emf in the primary

$$E_1 = -N_1 \frac{d\Phi}{dt} = -N_1 \frac{d}{dt}(\Phi_m \sin \omega t) = -N\Phi_m \omega \cos \omega t = N\Phi_m \omega \sin(\omega t - 90^\circ) = E_{1m} \sin(\omega t - 90^\circ)$$

i.e.,  $E_1$  lags the flux,  $\Phi$  by an angle  $90^\circ$

$$V_1 = -E_1 = N_1 \frac{d\Phi}{dt} \quad \text{or} \quad V_1 = -E_1$$

i.e  $V_1$  leads the flux,  $\Phi$  by an angle  $90^\circ$

The induced emf in the secondary

$$E_2 = -N_2 \frac{d\Phi}{dt} = -N_2 \frac{d}{dt}(\Phi_m \sin \omega t) = -N_2 \Phi_m \omega \cos \omega t = N_2 \Phi_m \omega \sin(\omega t - 90^\circ) = E_{2m} \sin(\omega t - 90^\circ)$$

$E_{2m} = N_2 \Phi_m \omega$  = Max. value of induced emf

i.e.,  $E_2$  lags the flux,  $\Phi$  by an angle  $90^\circ$ , so  $E_1$  and  $E_2$  are phase.

In an ideal transformer there is no power loss

i.e., input VA = output VA

$$\Rightarrow E_1 I_1 = E_2 I_2$$

$$\Rightarrow V_1 I_1 = V_2 I_2 (\because V_1 = E_1, V_2 = E_2)$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{I_1}{I_2}$$

$$\therefore \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{I_1}{I_2} = K$$

## 5.7 EMF Equation of a Transformer

When the primary winding is excited by an alternating voltage, it circulates alternating current and hence alternating flux is produced in the core.

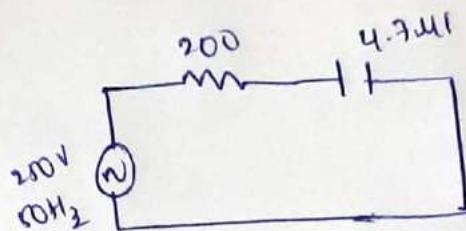
i) An RC circuit consisting of a  $4.7\mu F$  capacitor in series with a  $200\Omega$  resistor is connected to a  $200V, 50Hz$  supply. Determine. a) the current b) the power-factor and c) the values for true, apparent and reactive powers

$$R = 200\Omega$$

$$C = 4.7\mu F$$

$$V = 200$$

$$f = 50Hz$$



$$i) I = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + (X_C)^2}$$

$$= \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$

$$Z = \sqrt{200^2 + \left(\frac{1}{2\pi \times 3.14 \times 50 \times 4.7 \times 10^{-6}}\right)^2}$$

$$Z = 706.21$$

$$I = \frac{250}{706.21}$$

$$I = 0.354 A$$

ii) power factor

$$\cos \phi = \frac{R}{Z}$$

$$\cos \phi = \frac{200}{706.21} \Rightarrow \cos \phi = 0.283$$

$$\phi = 73.56^\circ$$

$$\text{iii) True power} : V I \cos \phi$$

$$= 250 \times 0.354 \times \cos(73.56)$$

$$= 25.04$$

$$\text{Apparent power} : V \times I$$

$$= 250 \times 0.354$$

$$= 88.5$$

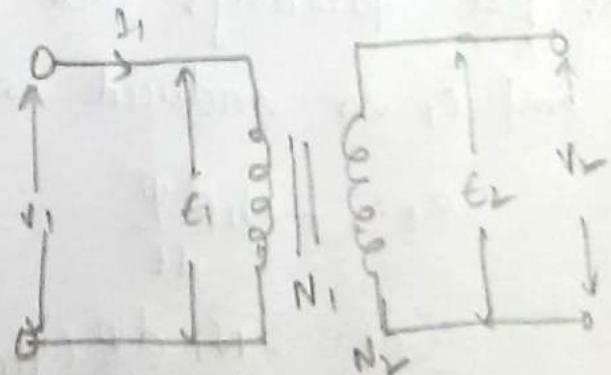
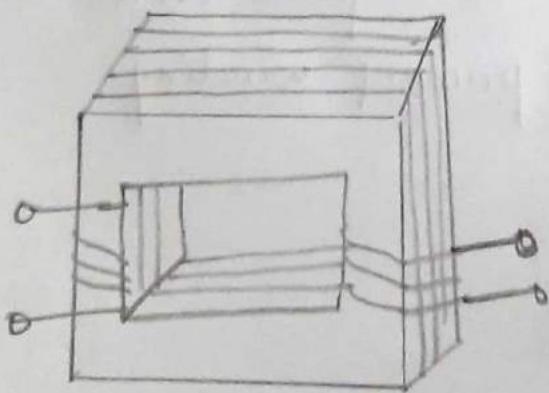
$$\text{Reactive power} : V I \sin \phi$$

$$= 250 \times 0.354 \times \sin(73.56)$$

$$= 84.88$$

- b) Explain the principle of operation of a single phase transformer.  
Derive the emf equation a single phase transformer.

When an alternating voltage  $V_1$  is applied to a primary winding, an alternating current  $I_1$  flows in it producing an alternating flux in the core. As per faradays law of electromagnetic induction, an emf  $e_1$  is induced in the primary winding



$$e_1 = -N_1 \frac{d\psi}{dt}$$

where  $N_1$  is the number of turns in the primary

Winding. The induced emf in the primary winding is nearly equal and opposite to the applied voltage  $v$ . Assuming leakage flux to be negligible, almost the flux produced in primary winding links with the secondary winding.

$$e_2 = -N_2 \frac{d\psi}{dt}$$

Where  $N_2$  is the no. of turns in the secondary winding. If the secondary circuit is closed through the load, a current  $I_2$  flows in secondary winding. The energy is transferred from the primary to secondary winding.

### EMF equation

As the primary winding is excited by a sinusoidal alternating voltage, an alternating current flows in the winding producing sinusoidally varying flux  $\phi$  in the core.

$$\phi = \Phi_m \sin \omega t$$

As per Faraday's law of electromagnetic induction an emf  $e_1$  is induced in the primary winding.

$$e_1 = -N_1 \frac{d\psi}{dt}$$

$$e_1 = -N_1 \frac{d\psi}{dt} (\Phi_m \sin \omega t)$$

$$e_1 = -N_1 \Phi_m \omega \cos \omega t$$

$$e_1 = -N_1 \Phi_m \sin(\omega t - 90^\circ)$$

$$= 2\pi f N_1 \Phi_m \omega \sin(\omega t - 90^\circ)$$

Maximum value of induced emf =  $2\pi f \Phi_m N_1$

Hence rms value of induced emf in primary winding is

$$\epsilon_1 = \frac{\epsilon_{\max}}{\sqrt{2}} = \frac{2\pi f N_1 \Phi_m}{\sqrt{2}}$$

$$\epsilon_1 = 4.44 f N_1 \Phi_m$$

In secondary winding

$$\epsilon_2 = \frac{\epsilon_{\max}}{\sqrt{2}} = \frac{2\pi f N_2 \Phi_m}{\sqrt{2}}$$

$$\epsilon_2 = 4.44 f N_2 \Phi_m$$

Emf per turn is same in primary and secondary winding and an equal emf is induced in each turn of primary and secondary winding.