CHAPTER-4

Time-harmonic fields: the field varies periodically (sinusoidal) with time.

- If $\mathbf{A}$ is any vector, then its phasor form can be $\mathbf{A}_S(x, y, z)$

$$\mathbf{A}_S(x, y, z) = A_o e^{j\theta x} \mathbf{a}_y$$

- The time-harmonic form

$$\mathbf{A} = \Re e\{\mathbf{A}_S e^{j\omega t}\}$$

$\mathbf{A} = A_o \cos(\omega t - \beta x)\, \mathbf{a}_y$ can be written as $\mathbf{A} = \Re e\{\mathbf{A}_o e^{-j\beta x} \mathbf{a}_y\, e^{j\omega t}\}$

- $\frac{\partial \mathbf{A}}{\partial t} \rightarrow j\omega \mathbf{A}_S$
- $\int \mathbf{A}\, \partial t \rightarrow \frac{\mathbf{A}_S}{j\omega}$

Differential (Point Form)	Integral Form
$\nabla . \mathbf{D}_s = \rho_{vs}$	$\oint_S \mathbf{D}_s . d\mathbf{S} = \int_v \rho_{vs}\, dv$
$\nabla . \mathbf{B}_s = 0$	$\oint_S \mathbf{B}_s . d\mathbf{S} = 0$
$\nabla \times \mathbf{E}_s = -j\omega \mathbf{B}_S$	$\oint_L \mathbf{E}_s . d\mathbf{l} = -j\omega \int_S \mathbf{B}_s . d\mathbf{S}$
$\nabla \times \mathbf{H}_s = \mathbf{J}_s + j\omega \mathbf{D}_s$	$\oint_L \mathbf{H}_s .\ d\mathbf{l} = \int_S (\mathbf{J}_s + j\omega \mathbf{D}_s) . d\mathbf{S}$

- A wave is a mean of transporting energy/power/information.
- Free space ($\sigma = 0, \epsilon = \epsilon_o, \mu = \mu_o$)
- Lossless dielectrics ($\sigma = 0, \epsilon = \epsilon_o \epsilon_r, \mu = \mu_o \mu_r, \sigma \ll \omega\epsilon$)
- Lossy dielectrics ($\sigma \neq 0, \epsilon = \epsilon_o \epsilon_r, \mu = \mu_o \mu_r$)
- Good conductors ($\sigma \cong \infty, \epsilon = \epsilon_o, \mu = \mu_o \mu_r, \sigma \gg \omega\epsilon$)

Wave/ Plane wave Equations

- A homogeneous medium is one for which ϵ, μ, and σ are constants throughout the medium.
- In an isotropic medium, ϵ is a scalar and **D** and **E** have everywhere the same direction.
- *Wave eqns. in lossy dielectrics*
- Lossy dielectrics ($\sigma \neq 0, \epsilon = \epsilon_o \epsilon_r, \mu = \mu_o \mu_r$) and charge free $\rho_v = 0$.
- $\nabla . \mathbf{E}_s = 0$
- $\nabla . \mathbf{H}_s = 0$
- $\nabla \times \mathbf{E}_s = -j\omega\mu \mathbf{H}_s$
- $\nabla \times \mathbf{H}_s = (\sigma + j\omega\epsilon) \mathbf{E}_s$

- $\nabla \times \nabla \times \mathbf{E}_s = -j\omega\mu(\nabla \times \mathbf{H}_s)$
- $\nabla \times (\nabla \times \mathbf{A}) \quad = \nabla(\nabla . \mathbf{A}) - \nabla^2 \mathbf{A}$

$$\left.\begin{aligned} \nabla^2 \mathbf{E}_s - \gamma^2 \mathbf{E}_s &= 0 \\ \nabla^2 \mathbf{H}_s - \gamma^2 \mathbf{H}_s &= 0 \end{aligned}\right]$$

Helmholtz's equations or vector wave equations

- $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$ ----→ Propagation constant
- $\gamma = \alpha + j\beta$ → α: attenuation constant ; β: Phase constant

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\epsilon}\right]^2} - 1\right]}$$

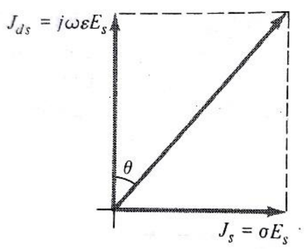
$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\epsilon}\right]^2} + 1\right]}$$

- $\mathbf{E}_s = E_{xs}(z)\mathbf{a}_x$
- $(\nabla^2 - \gamma^2)E_{xs}(z) = 0$
- $E_{xs}(z) = E_o e^{-\gamma z} + E_o' e^{\gamma z}$
- $\mathbf{E}(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$
- $\mathbf{H}(z,t) = H_o e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_y$
- $H_o = c$
- η--→ intrinsic impedance (in ohms)
- $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon)}} = |\eta|\angle\theta_\eta = |\eta|e^{j\theta_\eta}$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}, \qquad \tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}$$

$$\mathbf{H} = \frac{E_0}{|\eta|}e^{-\alpha z}\cos\left(\omega t - \beta z - \theta_\eta\right)\mathbf{a}_y$$

- $e^{-\alpha z}$ → the amount of attenuation.
- $\alpha - -\rightarrow$ measured in nepers/m (Np/m)
- $\beta - -\rightarrow$ measured in radians/m
- $u = \frac{\omega}{\beta} \qquad \lambda = \frac{2\pi}{\beta}$
- $\frac{|\mathbf{J}_{cs}|}{|\mathbf{J}_{ds}|} = \frac{|\sigma\mathbf{E}_s|}{|j\omega\epsilon\mathbf{E}_s|} = \frac{\sigma}{\epsilon\omega} = \tan\theta$
- Loss tangent $\qquad \tan\theta = \frac{\sigma}{\omega\epsilon}$



- We know

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon} \qquad \tan\theta = \frac{\sigma}{\omega\epsilon}$$

$$\theta = 2\theta_\eta$$

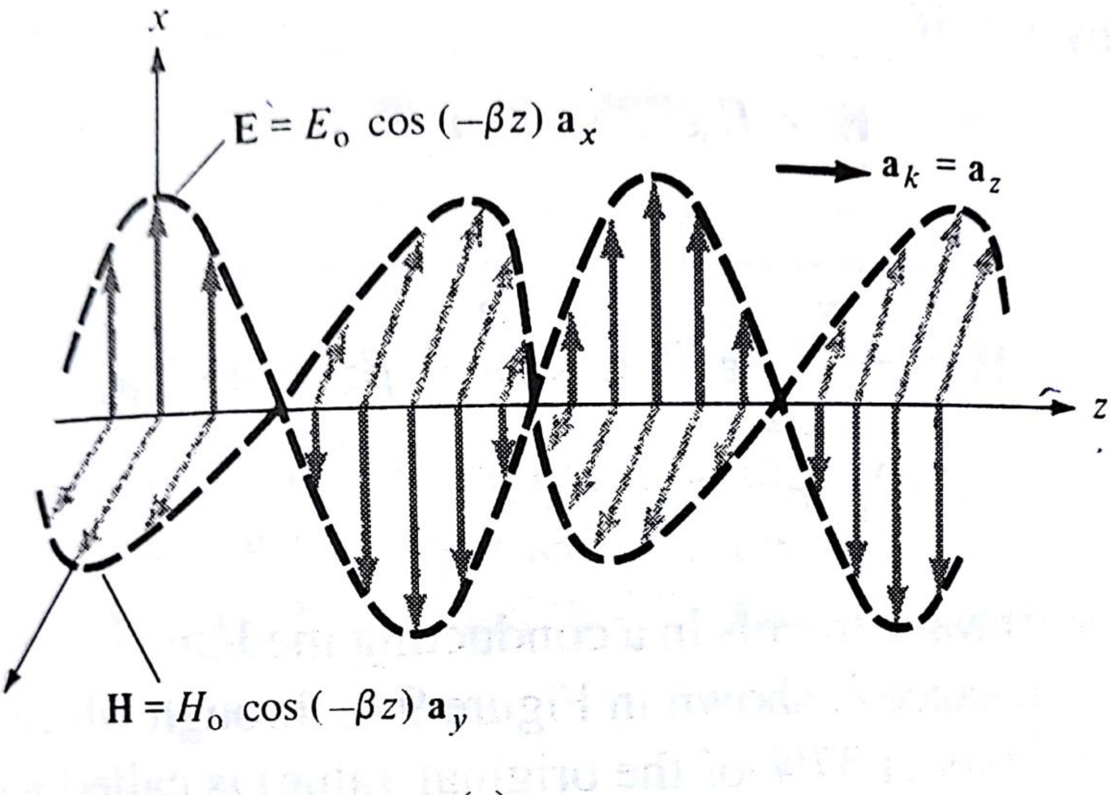
$$\nabla \times \mathbf{H}_s = (\sigma + j\omega\epsilon)\mathbf{E}_s = j\omega\epsilon\left[1 - \frac{j\sigma}{\omega\epsilon}\right] = j\omega\epsilon_c\mathbf{E}_s$$

$$\epsilon_c = \epsilon\left[1 - \frac{j\sigma}{\omega\epsilon}\right]$$

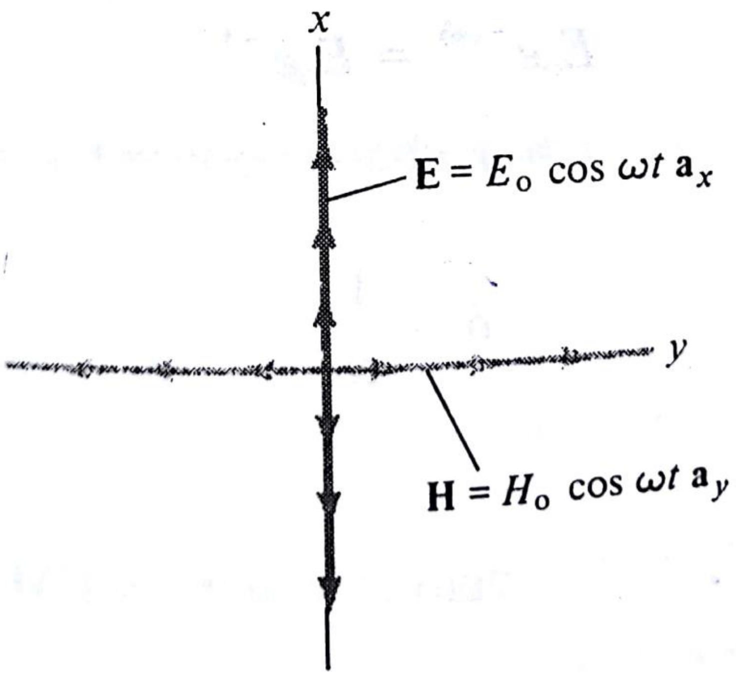
$$\epsilon_c = \epsilon' - j\epsilon''$$

- ϵ_c ----------→ complex permittivity of the medium
- $\epsilon' = \epsilon$ and $\epsilon'' = \sigma/\omega$
- $\tan\theta = \frac{\epsilon''}{\epsilon'} = \sigma/\omega\epsilon$
- ***<u>Wave eqns. in lossless dielectrics</u>***
- $\sigma \ll \omega\epsilon, \qquad \sigma = 0, \epsilon = \epsilon_o\epsilon_r, \mu = \mu_o\mu_r$
- $\alpha = 0, \beta = \omega\sqrt{\mu\epsilon}$
- $u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$
- $\eta = \sqrt{\frac{\mu}{\epsilon}}\angle 0°$
- $\lambda = 2\pi/\beta$

- ***<u>Wave eqns. in lossless free space</u>***
- $\sigma = 0, \epsilon = \epsilon_o, \mu = \mu_o$
- $\alpha = 0, \beta = \omega\sqrt{\mu_o\epsilon_o} = \frac{\omega}{c}$
- $u = \frac{1}{\sqrt{\mu_o\epsilon_o}} = c$
- $\lambda = 2\pi/\beta$
- $\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = 120\pi = 377\Omega$
- $\mathbf{E} = E_o\cos(\omega t - \beta z)\mathbf{a}_x$
- $\mathbf{H} = H_o\cos(\omega t - \beta z)\mathbf{a}_y = \frac{E_o}{\eta_o}\cos(\omega t - \beta z)\mathbf{a}_y$
- ***<u>Uniform plane wave</u>***
- A uniform plane wave has both **E** & **H** components with the same magnitude throughout any transverse plane.
- Both **E** & **H** are everywhere normal to the direction of wave propagation.
- The EM wave has no electric and magnetic components along the direction of wave propagation. Such a wave is called a *transverse electromagnetic* (TEM) wave.
- The direction in which the electric field points is the *polarization* of a TEM wave.
- ***<u>Wave eqns. in lossless free space</u>***
- $\sigma \gg \omega\epsilon, \qquad \sigma \cong 0, \qquad \epsilon = \epsilon_o \qquad, \mu = \mu_o\mu_r$
- $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$
- $u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}, \qquad \lambda = \frac{2\pi}{\beta}$
- $\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45°$



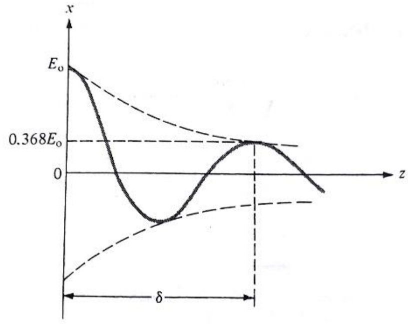
(a)



- $\mathbf{E} = E_o\, \mathrm{e}^{-\alpha z} \cos(\omega t - \beta z)\mathbf{a}_x$
- $\mathbf{E} = \frac{E_o}{\sqrt{\frac{\omega\mu}{\sigma}}} \mathrm{e}^{-\alpha z} \cos(\omega t - \beta z - 45°)\mathbf{a}_y$
- As the EM wave travels in a conducting medium, the amplitude is attenuated by the factor $\mathrm{e}^{-\alpha z}$. The distance δ through which the wave amplitude decreases to a factor e^{-1} (about 37% of the original value) is called skin depth or penetration depth of the medium.

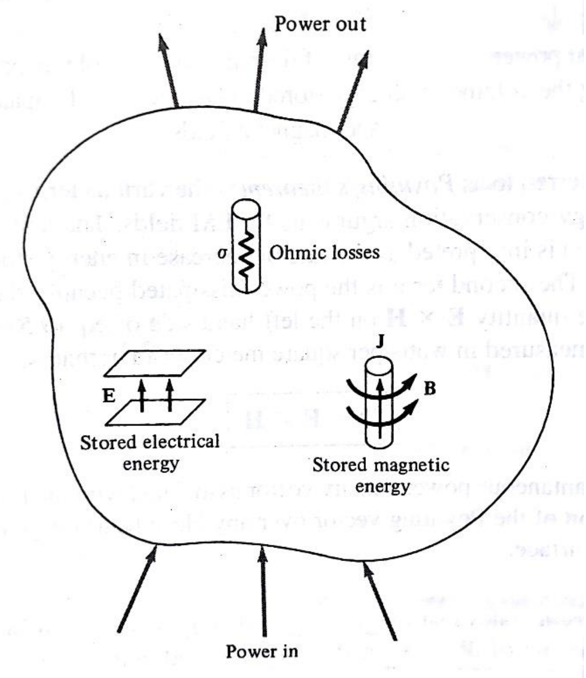
$$E_o e^{-\alpha\delta} = E_o e^{-1}$$
$$\delta = 1\backslash\alpha$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha}$$

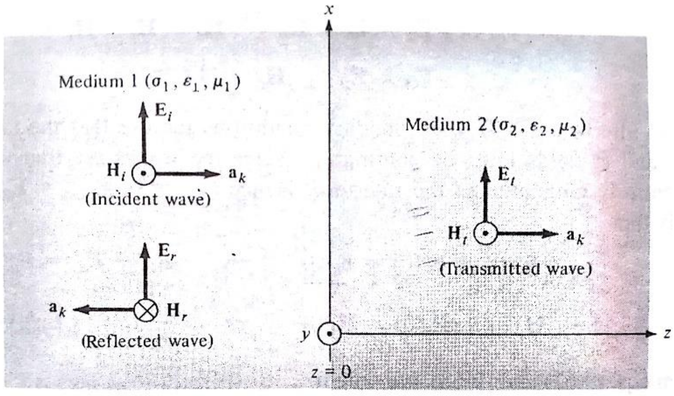


Power and the Poynting Vector

- $\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$
- $\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}$
- $\mathbf{E}.(\nabla \times \mathbf{H}) = \mathbf{E}.\left(\sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}\right)$
- $\mathbf{H}.(\nabla \times \mathbf{E}) = \mathbf{H}.\left(-\mu \frac{\partial \mathbf{H}}{\partial t}\right)$
- $-\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \nabla.(\mathbf{E} \times \mathbf{H}) = \sigma E^2 + \frac{1}{2}\epsilon \frac{\partial E^2}{\partial t}$
- $\int_v \nabla.(\mathbf{E} \times \mathbf{H})\, dv = -\frac{\partial}{\partial t}\int_v \left[\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right] dv - \int_v \sigma E^2\, dv$
- $\oint_S (\mathbf{E} \times \mathbf{H}).d\mathbf{S} = -\frac{\partial}{\partial t}\int_v \left[\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right] dv - \int_v \sigma E^2\, dv$
- Total power leaving the volume = rate of decrease in energy stored in electric and magnetic fields – Ohmic power dissipated
- This is referred to as Poynting's Theorem
- $\mathcal{P} = \mathbf{E} \times \mathbf{H}$ is the Poynting vector measured in watts/m^2 (W/m^2)
- $\boldsymbol{\mathcal{P}}_{ave}(z) = \frac{1}{2}\mathrm{Re}(\mathbf{E}_s \times \mathbf{H}_s^*)$---------→ Average power
- $\boldsymbol{\mathcal{P}}_{ave}(z) = \frac{E_o^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta \mathbf{a}_z$
- $P_{ave} = \int_S \boldsymbol{\mathcal{P}}_{ave} . d\mathbf{S}$ → total time-average power crossing a given surface S



Reflection of a Plane Wave at Normal Incidence



- Incident wave is travelling along $+\mathbf{a}_z$ in medium 1.
- $\mathbf{E}_{is}(z) = E_{io}e^{-\gamma_1 z}\mathbf{a}_x$
- $\mathbf{H}_{is}(z) = H_{io}e^{-\gamma_1 z}\mathbf{a}_y$
- $\mathbf{H}_{is}(z) = \frac{E_{io}}{\eta_1}e^{-\gamma_1 z}\mathbf{a}_y$
- Incident wave is travelling along $-\mathbf{a}_z$ in medium 1
- $\mathbf{E}_{rs}(z) = E_{ro}e^{-\gamma_1 z}\mathbf{a}_x$
- $\mathbf{H}_{rs}(z) = \frac{E_{ro}}{\eta_1}e^{-\gamma_1 z}\mathbf{a}_y$
- Incident wave is travelling along $+\mathbf{a}_z$ in medium 2
- $\mathbf{E}_{ts}(z) = E_{to}e^{-\gamma_2 z}\mathbf{a}_x$
- $\mathbf{H}_{ts}(z) = \frac{E_{to}}{\eta_1}e^{-\gamma_2 z}\mathbf{a}_y$
- In medium 1

  $\mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_r \qquad \mathbf{H}_1 = \mathbf{H}_i + \mathbf{H}_r$
- $\mathbf{E}_2 = \mathbf{E}_t \qquad \mathbf{H}_2 = \mathbf{H}_t$
- At the interface $z = 0$, the boundary conditions require that the tangential components of **E** and **H** fields must be continuous.
- $\mathbf{E}_{1t} = \mathbf{E}_{2t} \qquad \mathbf{H}_{1t} = \mathbf{H}_{2t}$
- $\mathbf{E}_i(0) + \mathbf{E}_r(0) = \mathbf{E}_t(0) \qquad E_{io} + E_{ro} = E_{to}$
- $\mathbf{H}_i(0) + \mathbf{H}_r(0) = \mathbf{H}_t(0) \qquad \frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2}$
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