SIGNALS AND SYSTEMS

II B.TECH I SEM

II ECE-2

- Name of the Faculty: Mr.A.BALA RAJU, Assistant.Professor
- Name of the Course: Signals and Systems.
- Class: II B.Tech-ECE 2-I sem.
- Subject Code: EC 304PC.
- Number of Lectures hours/Week: 4
- Number of Tutorial hours/Week: 1
- Number of Credits: 4

COURSE OBJECTIVES

- This gives the basics of Signals and Systems required for all Electrical Engineering related course
- To understand the behaviour of signal in time and frequency domain
- To understand the characteristics of LTI systems
- This gives concepts of Signals and Systems and its analysis using different transform techniques.

COURSE OUTCOMES

Upon completing this course, the student will be able to

- Differentiate various signal functions.
- Represent any arbitrary signal in time and frequency domain.
- Understand the characteristics of linear time invariant systems.
- Analyse the signals with different transform technique
- Understand different sampling techniques and comparison of signals

SYLLABUS

UNIT - I

- **Signal Analysis:** Analogy between Vectors and Signals, Orthogonal Signal Space, Signal approximation using Orthogonal functions, Mean Square Error, Closed or complete set of Orthogonal functions, Orthogonality in Complex functions, Classification of Signals and systems, Exponential and Sinusoidal signals, Concepts of Impulse function, Unit Step function, Signum function

SYLLABUS

UNIT – II

- **Fourier series:** Representation of Fourier series, Continuous time periodic signals, Properties of Fourier Series, Dirichlet's conditions, Trigonometric Fourier Series and Exponential Fourier Series, Complex Fourier spectrum.
- **Fourier Transforms**: Deriving Fourier Transform from Fourier series, Fourier Transform of arbitrary signal, Fourier Transform of standard signals, Fourier Transform of Periodic Signals, Properties of Fourier Transform, Fourier Transforms involving Impulse function and Signum function, Introduction to Hilbert Transform.

SYLLABUS

UNIT – III

- **Signal Transmission through Linear Systems:** Linear System, Impulse response, Response of a Linear System, Linear Time Invariant(LTI) System, Linear Time Variant (LTV) System, Transfer function of a LTI System, Filter characteristic of Linear System, Distortion less transmission through a system, Signal bandwidth, System Bandwidth, Ideal LPF, HPF, and BPF characteristics, Causality and Paley-Wiener criterion for physical realization, Relationship between Bandwidth and rise time, Convolution and Correlation of Signals, Concept of convolution in Time domain and Frequency domain, Graphical representation of Convolution.

SYLLABUS

UNIT – IV

- **Laplace Transforms:** Laplace Transforms (L.T), Inverse Laplace Transform, Concept of Region of Convergence (ROC) for Laplace Transforms, Properties of L.T, Relation between L.T and F.T of a signal, Laplace Transform of certain signals using waveform synthesis.
- **Z–Transforms:** Concept of Z- Transform of a Discrete Sequence, Distinction between Laplace, Fourier and Z Transforms, Region of Convergence in Z-Transform, Constraints on ROC for various classes of signals, Inverse Z-transform, Properties of Z-transforms.

SYLLABUS

UNIT – V

- **Sampling theorem:** Graphical and analytical proof for Band Limited Signals, Impulse Sampling, Natural and Flat top Sampling, Reconstruction of signal from its samples, Effect of under sampling – Aliasing, Introduction to Band Pass Sampling.
- **Correlation:** Cross Correlation and Auto Correlation of Functions, Properties of Correlation Functions, Energy Density Spectrum, Parsevals Theorem, Power Density Spectrum, Relation between Autocorrelation Function and Energy/Power Spectral Density Function, Relation between Convolution and Correlation, Detection of Periodic Signals in the presence of Noise by Correlation, Extraction of Signal from Noise by Filtering.

TEXT BOOKS:

- 1. Signals, Systems & Communications - B.P. Lathi, 2013, BSP.
- 2. Signals and Systems - A.V. Oppenheim, A.S. Willsky and S.H. Nawabi, 2 Ed.

REFERENCE BOOKS:

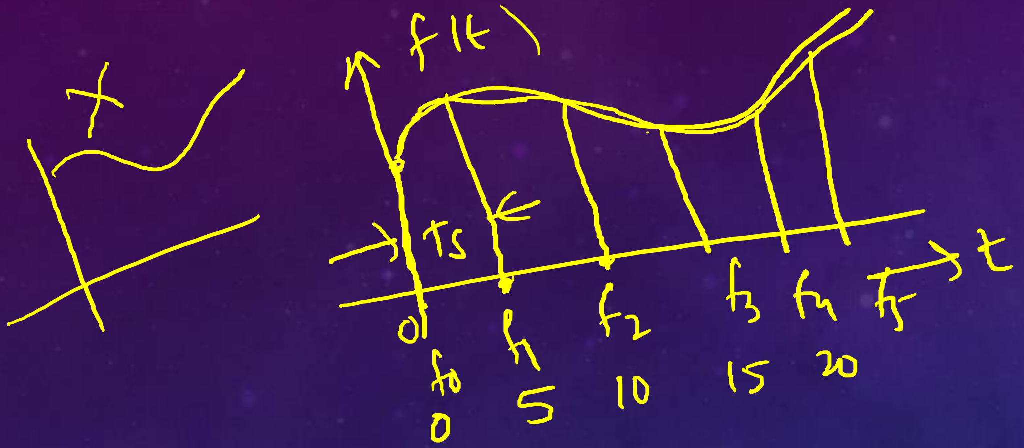
- 1. Signals and Systems – Simon Haykin and Van Veen, Wiley 2 Ed.,
- 2. Signals and Systems – A. Rama Krishna Rao, 2008, TMH
- 3. Fundamentals of Signals and Systems - Michel J. Robert, 2008, MGH International Edition.
- 4. Signals, Systems and Transforms - C. L. Philips, J.M.Parr and Eve A.Riskin, 3 Ed., 2004, PE.
- 5. Signals and Systems – K. Deergha Rao, Birkhauser, 2018.

SAMPLING THEOREM & CORRELATION

UNIT-V-I

Sampling theorem – Unit-V

→ Sampling is the Process of converting a continuous time Signal (CTS) into a discrete time Signal (DTS)



$f_0, f_1, f_2, f_3, \ldots$ are samples of CTS

T_s = Sampling interval.

= Spacing between two successive samples.

Sampling can be → uniform → Samples can be taken at equal interval of time

↳ Non-uniform – at Nonuniform intervals

Why Sampling → Is required to convert CTS to digital signal.

CTS → Sampling → discrete Time signal (DTS) → Quantization → Digital signal

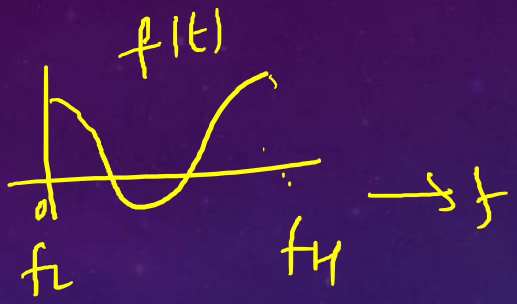
① ∴ to convert a CTS to a digital — Sampling is the first optn to be performed and then Quantization is performed.

→ then the Digital signal can be Processed by Processor.

→ ② Instead of the entire CTS, few of the samples can be transmitted which contains the same information. → BW constraint

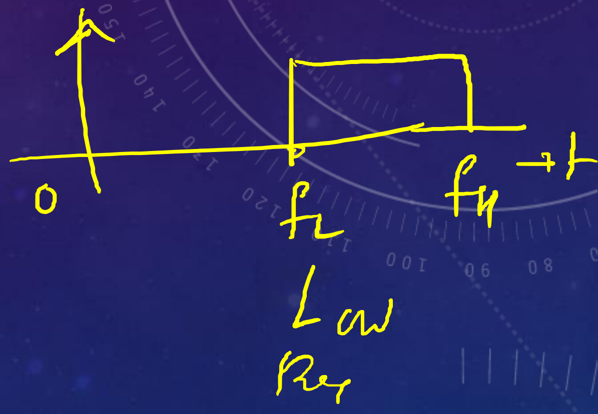
Sampling theorem for Bandlimited Signals & Band Pass Signals.

Bandlimited Signal : $f_L = 0$ & $f_H \neq 0$



f_H = High freq Compon

Band Pass Signal :- $f_L \neq 0$ & $f_H \neq 0$



f_H → there is no freq Component above f_H

① Sampling theorem for Bandlimited signals

$x(t) = 2\sin 2\pi \cancel{10} t + 4\cos 2\pi \cancel{20} t + 7\sin 2\pi 50 t$

$= \sin(2\pi f_m t)$

$\sin(W_m t)$

$\boxed{f_m = 50\,Hz}$

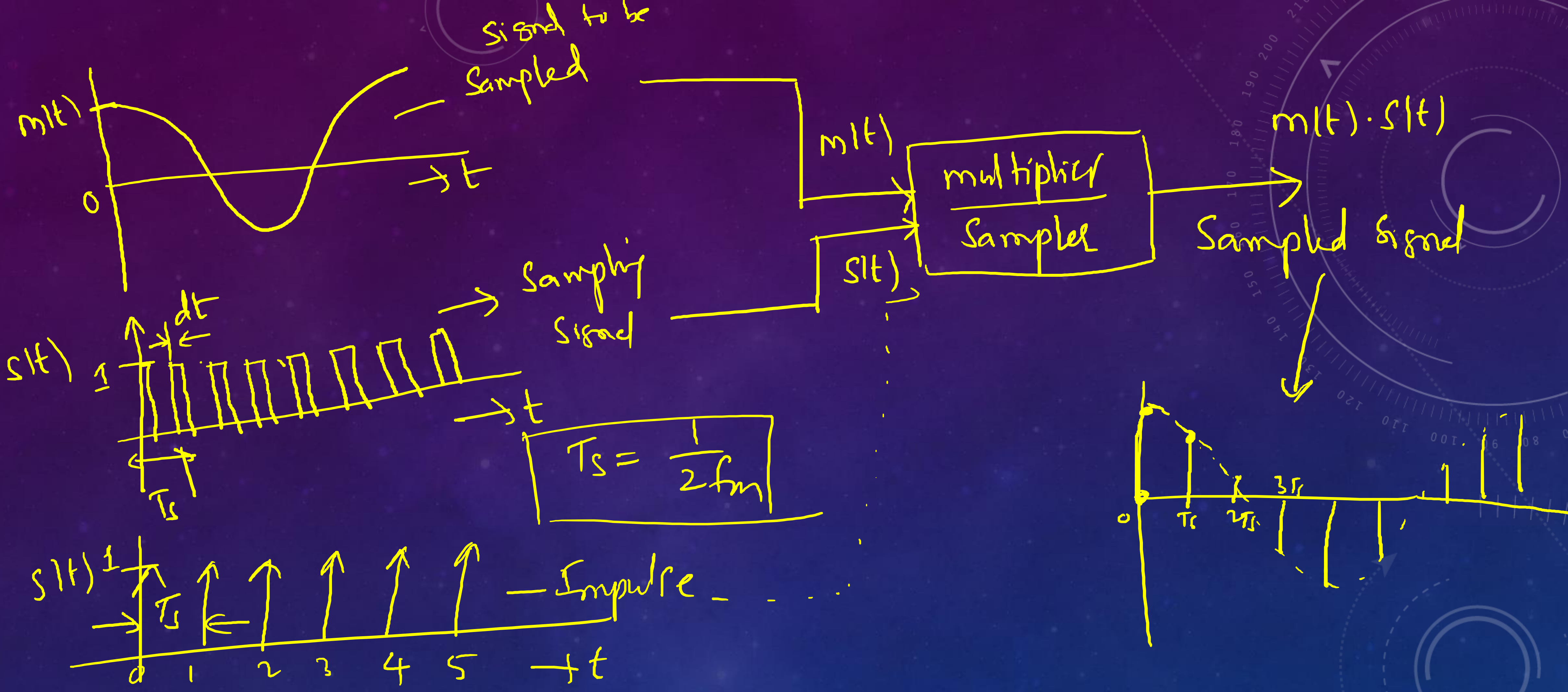
↳ highest freq component is f_m Hz –

↳ No freq. components above f_m Hz – then the signal is said to be bandlimited to f_m Hz

→ achieved by multiplying the signal to be sampled with the sampling signal → then called as sampled signal.

– Let $m(t)$ [$f(t)$] – is the information signal to be sampled

$s(t)$ – is the sampling signal.



dt → Narrow width of periodic pulse Train

Period = T_s

Statement: Any Bandlimited Signal $m(t)$ whose highest cut-off freq is f_m Hz, can be ~~determined~~ Recovered/Reconstructed from it's samples if the Sampling Interval $\boxed{T_s \le \frac{1}{2f_m}}$ (or) $\boxed{f_s \geq 2f_m}$

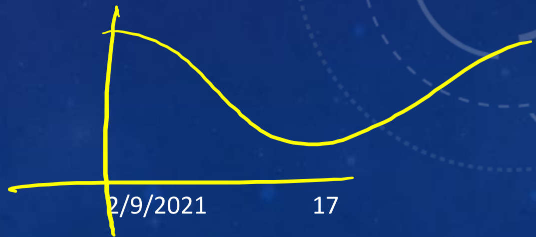
T_s = Sampling Interval

f_s = Sampling rate

$=$

$T_s \le \frac{1}{2f_m}$

$\Rightarrow$



Graphical Approach

Periodic Train of Pulses of width 'dt' & Spacing (Period) T_s $\boxed{T_s = \frac{1}{f_s}}$

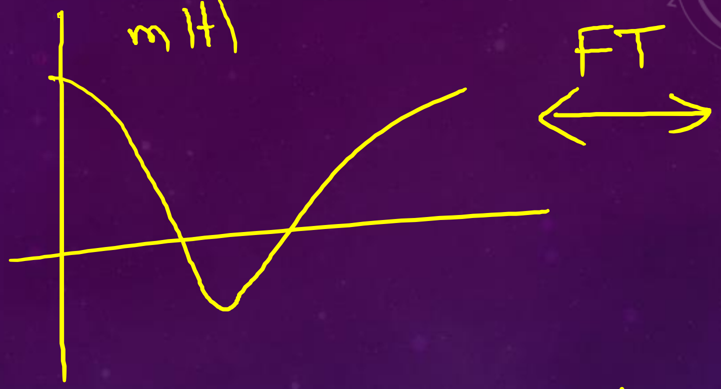
$$S(t) = \text{Fourier Series} = \frac{dt}{T_s} + \frac{2dt}{T_s}\left(\cos \frac{2\pi}{T_s} \times t + \cos 2\pi \times \frac{2}{T_s} \times t + \cos 2\pi \frac{\times 3 \times t}{T_s} + \cdots\right)$$

Sampled output $= s(t) \cdot m(t)$ → Information / message signal

$$= m(t)\frac{dt}{T_s} + \frac{2dt}{T_s}\left(m(t)\cos 2\pi f_s t + m(t)\cos 2\pi\, 2f_s t + m(t)\cos 2\pi\, 3f_s t + \cdots\right)$$

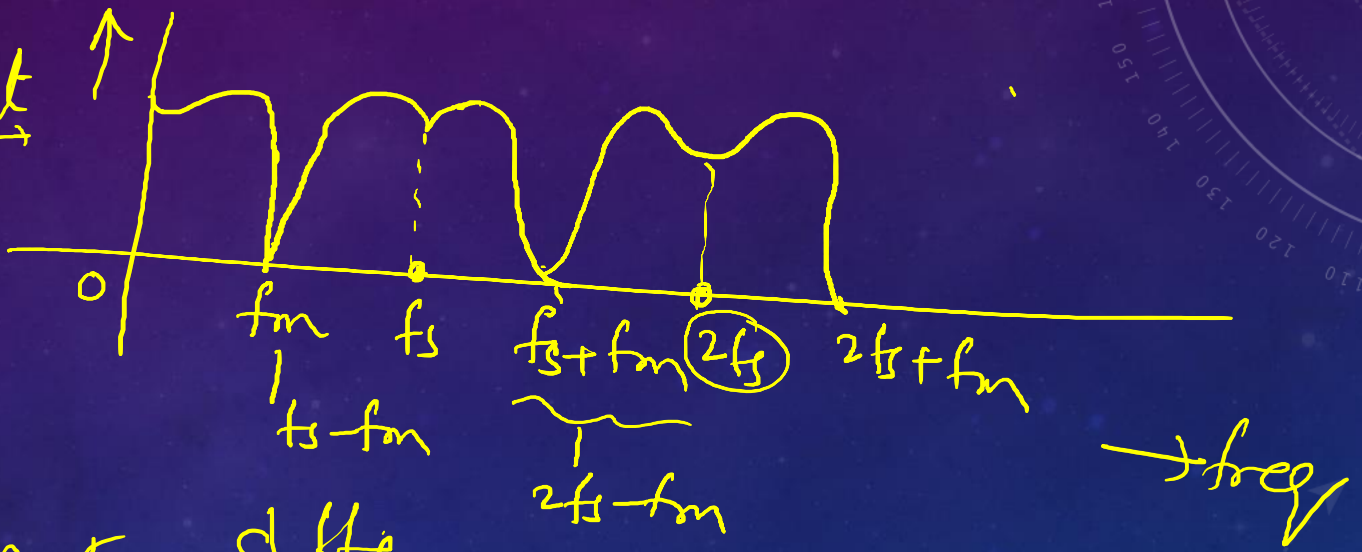
$\therefore$ dt - const & T_s = const

$f_m \Rightarrow f_s \pm f_m, \quad 2f_s \pm f_m$



— spectrum of the m(t)

spectrum of Sampled output



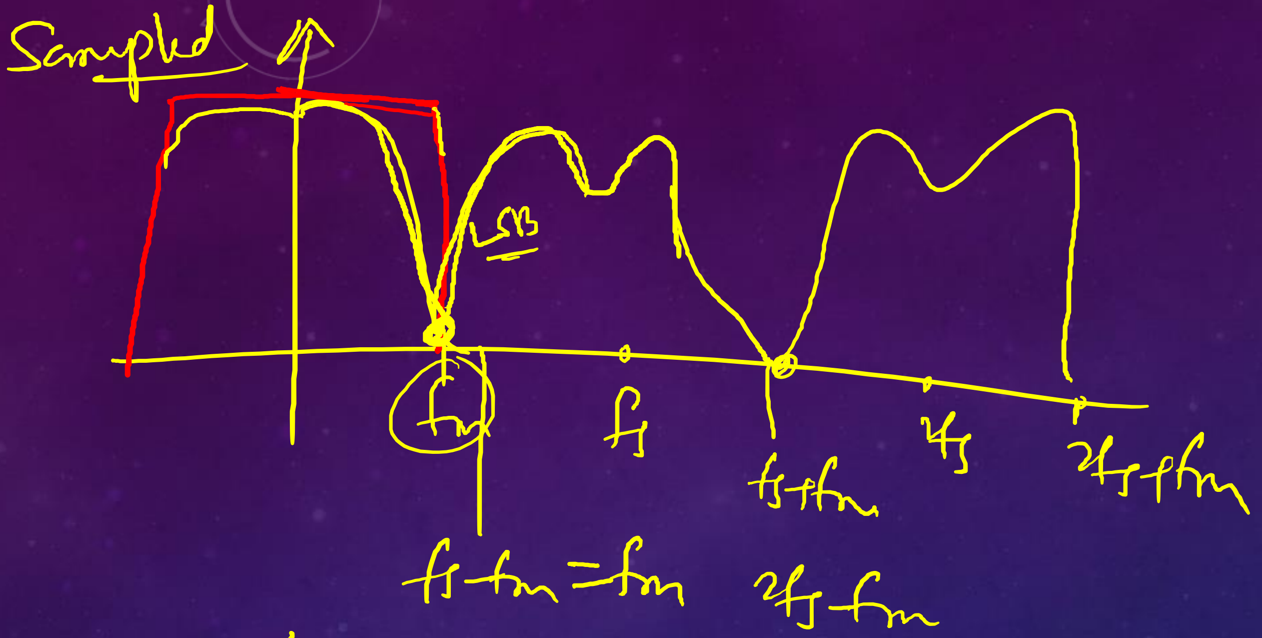
Modulation Prop

$m(t)\cos 2\pi f_s t$

fm Freq Shifting

$$m(t)\frac{\left(e^{j\omega_s t}+e^{-j\omega_s t}\right)}{2} = m(t)\,e^{j\omega_s t} \longleftrightarrow M(\omega-\omega_s)$$

$$(f-f_s) + M(f+f_s)$$



$f_s \pm f_m$

$2f_s \pm f_m$

Side bands

= If an Ideal Low Pass Filter, recover the spectrum of msg signal

(i) $\boxed{f_s = 2 f_m}$ ⇒ sampling rate is twice the highest cut-off freq 'fm'

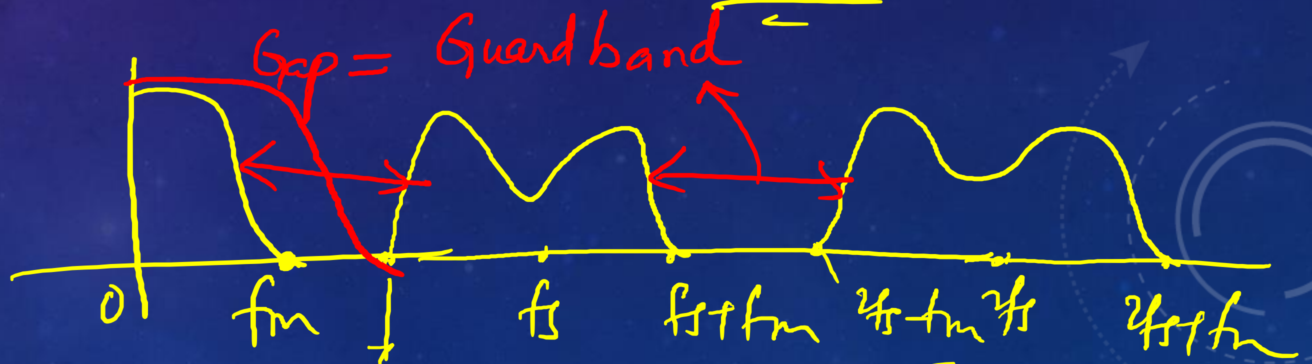
when $f_s = 2f_m \Rightarrow f_s - f_m = 2f_m - f_m = f_m$

with the help of Ideal LPF with cut-off freq 'fm' the spectrum of the Info— Signal can be Recoved / Reconstructed

⇒ But designing Ideal Filter is Practically Not possible

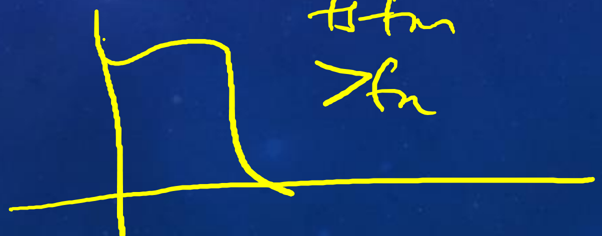
(ii) $f_s > 2f_m$

$f_s - f_m => f_m$



$f_s - f_m > f_m$

Recovery



with the help of Practical LPF, the spectrum of msg / Info Signal Can be Recovered w/o any distortion / Attenuation

→ the Gap between f_m & $f_s - f_m$ is called as guard band. which helps us in designing the filter

$f_s > 2f_m \Rightarrow$ the Recovery is Possible

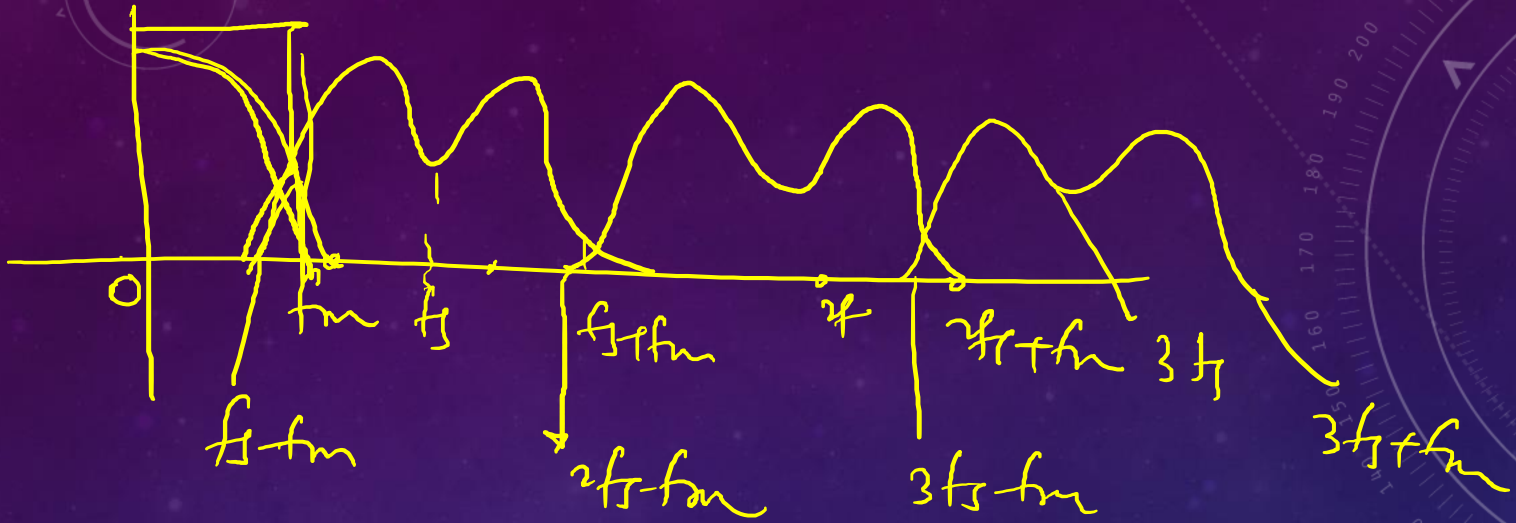
(iii) $f_s < 2f_m$ then $f_s - f_m$

(i) $2f_m - f_m = f_m$

(ii) $>2f_m - f_m = > f_m$

(iii) $< 2f_m \quad = \quad < f_m$

$f_s < 2f_m$



aliasing effect — overlapping of HF terms into msg frequencies

the Recovery is not Possible w/o distortion

(i) $f_s = 2f_m$ = Ideal LPF = Recovery ✓

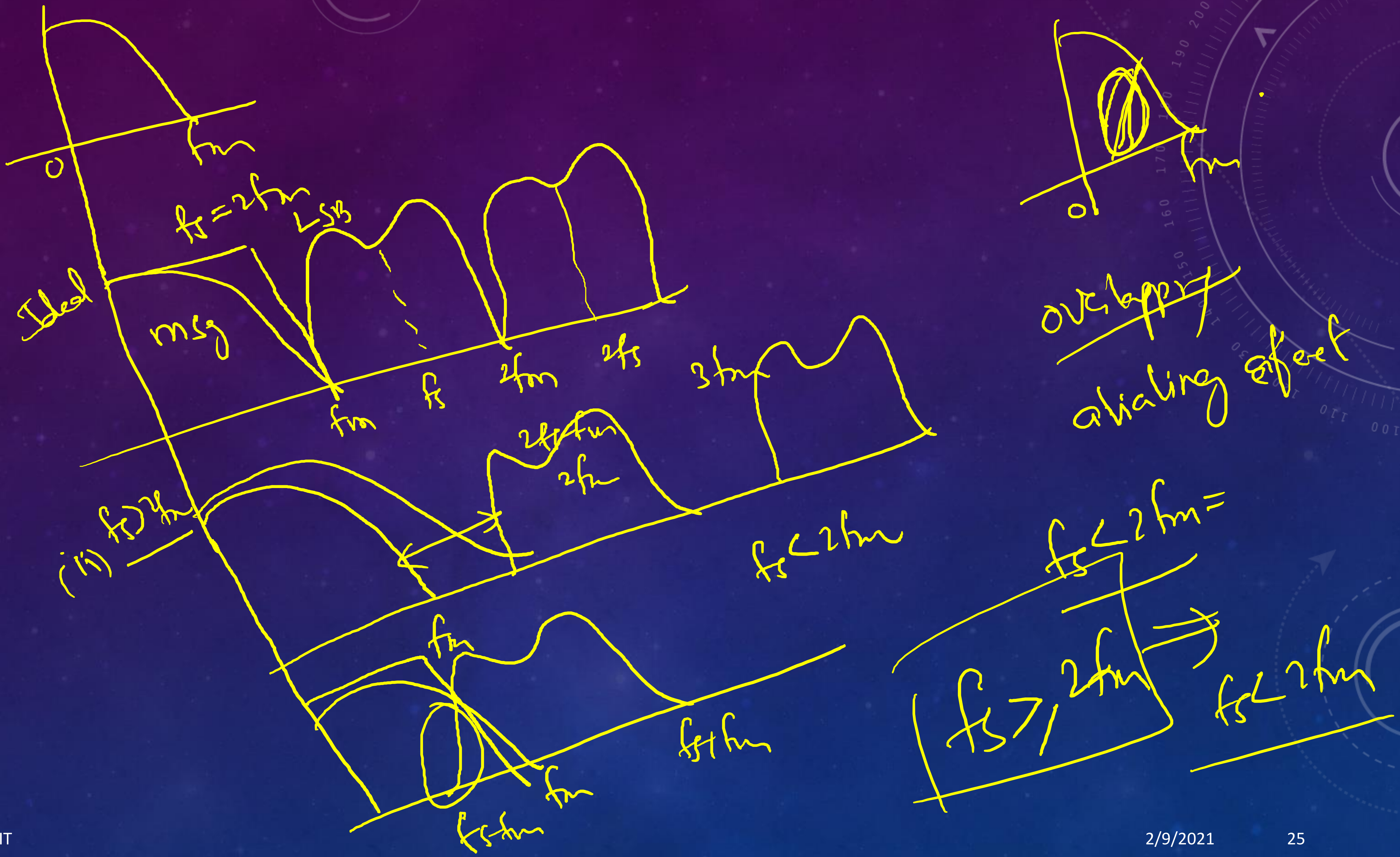
(ii) $f_s > 2f_m$ = Practical LPF = Recovery ✓

$f_s \geq 2f_m$ = Nyquist Rate of Sample

(iii) $f_s < 2f_m$ = Ideal / Pract. = Recovery not Possible = overlapping existing

$$\boxed{T_s \leq \frac{1}{2f_m}} \longrightarrow \text{Recovery/Reconstruction is possible}$$

$$\boxed{T_s > \frac{1}{2f_m}} \longrightarrow \text{Recovery not Possible}$$


0
fm
Ideal
fs=2fm
LSB
msg
fm
fs
2fm
2fs
3fm
(ii) fs>2fm
2fs+fm
2fm
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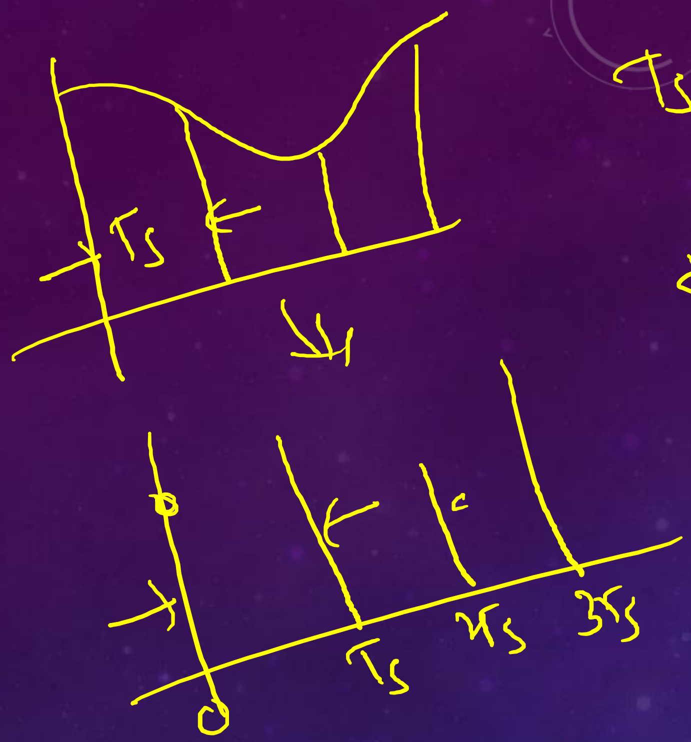


$T_s \le \frac{1}{2f_m}$

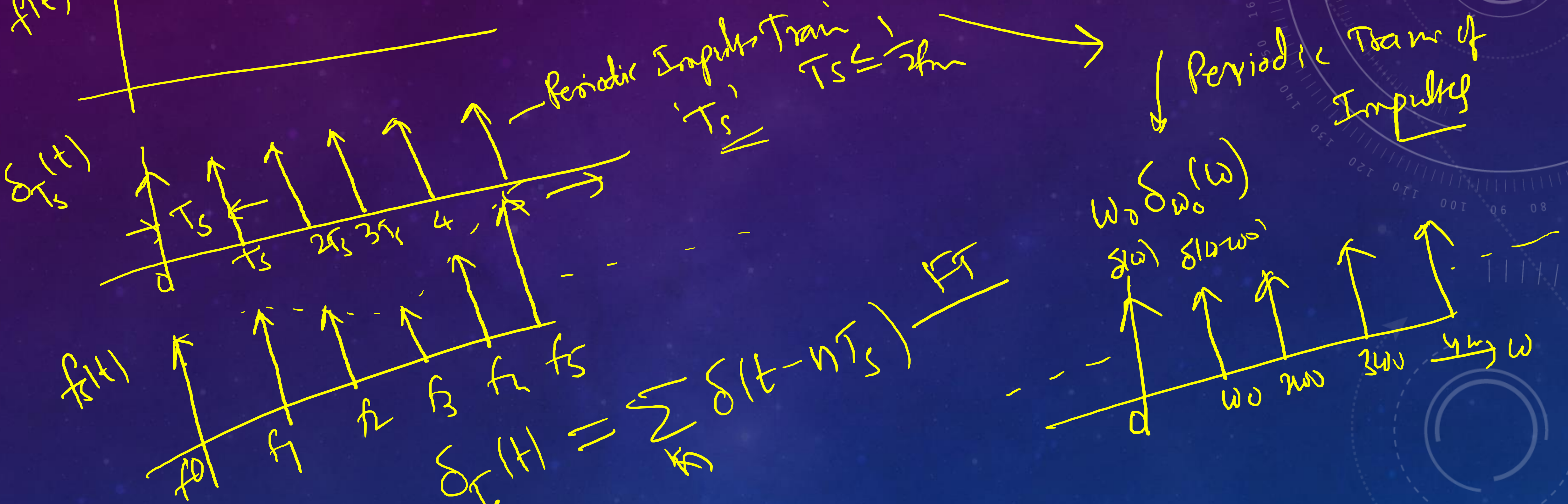
$T_s > \frac{1}{2f_m}$ (or) $f_s < 2f_m$

Graphical Method of Proving Sampling theo

Recovery

Analytical Method of Sampling theorem

$$\boxed{\omega_m = 2\pi f_m}$$



$$\delta_{T_s}(t) = \sum_{n} \delta(t - nT_s) \xrightarrow{FT}$$

$f_s(t)$ = Sampled Signal = $f(t)\delta_{T_s}(t)$ = multiplication in time domain equivalent to Conv- In Freq/ Domain

$f(t) \longleftrightarrow F(\omega)$

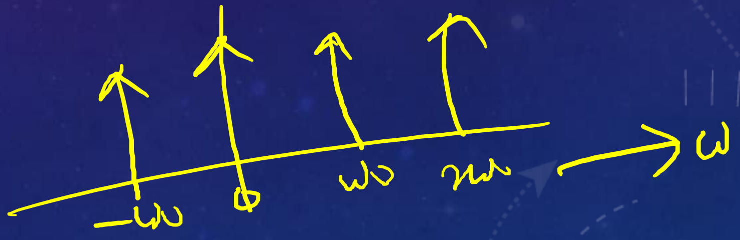
$\delta_{T_s}(t) \longleftrightarrow \omega_0 \delta_{\omega_0}(\omega)$

$f_s(t) \longleftrightarrow F_s(\omega)$

$f_s(t) = f(t)\delta_{T_s}(t)$

$$F_s(\omega) = \frac{1}{2\pi}\left[F(\omega) * \omega_0\delta_{\omega_0}(\omega)\right]$$

$$= \frac{1}{T_s}\left[F(\omega) * \delta_{\omega_0}(\omega)\right] \qquad \boxed{T_s = \frac{2\pi}{\omega_0}}$$



$\delta_{\omega_0}(\omega)$ = Periodic Train of Impulses

$$\delta_{\omega_0}(\omega) = \delta(\omega) + \delta(\omega-\omega_0) + \delta(\omega-2\omega_0) + \dots + \delta(\omega+\omega_0) + \delta(\omega+2\omega_0) + \dots$$

$$\delta_{\omega_0}(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$F_s(\omega) = \frac{1}{T_s}\left[F(\omega) * \delta_{\omega_0}(\omega)\right]$$

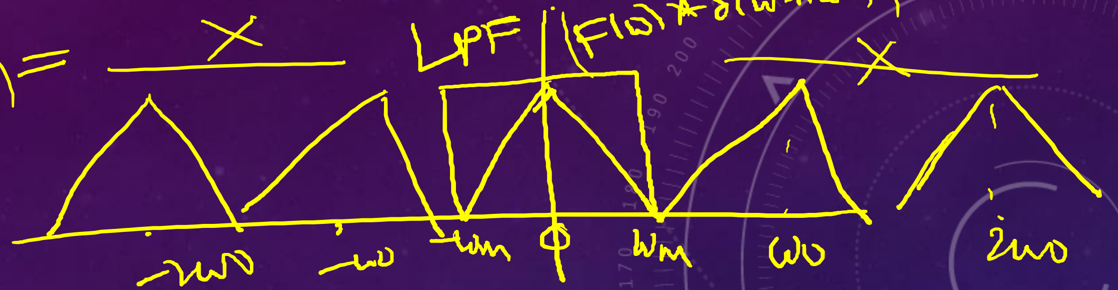
$$\delta_{\omega_0}(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

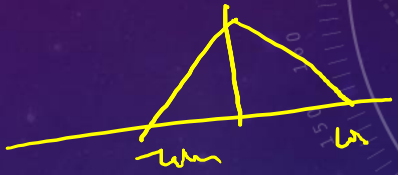
$$F_s(\omega) = \frac{1}{T_s}\left[F(\omega) * \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)\right]$$

Convolution of a fn with an Impulse give you the function itself

$$F_s(\omega) = \frac{1}{T_s}\left[\sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)\right]$$

$F_s(\omega) =$





$$f(t) * \delta(t) = f(t)\big|_{t=0}$$

$$f(t) * \delta(t - t_0) = f(t)\big|_{t=t_0}$$

$$F(\omega) + F(\omega - \omega_0) + F(\omega - 2\omega_0)$$

$$F_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} F(\omega - n\omega_0) = \text{after passing through an } \underline{\text{LPF}}$$

Ideal

Recover the msg spectrum

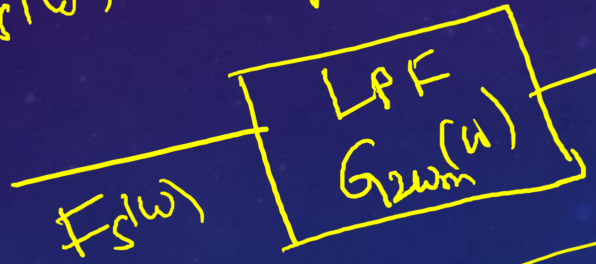
Recovery of f(t) from its Samples

$$\omega_0 = \frac{2\pi}{T_s} = 2\pi(2f_m)$$

$$F_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$$

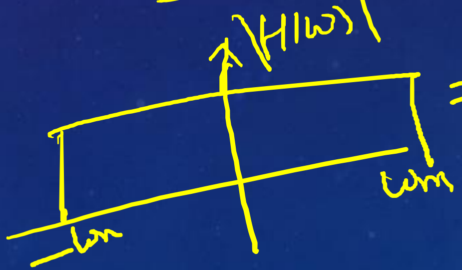
$$\boxed{\frac{1}{T_s} = 2f_m}$$

$F_s(\omega)$ multiplied with Ideal LPF $= \underline{\underline{F(\omega)}}$



$F(\omega) = F_s(\omega) \cdot G_{2\omega m}(\omega)$

$$\boxed{\omega_0 = 2\omega_m}$$



$= G_{2\omega m}(\omega)$ Gate fn

$$\boxed{\frac{\omega_m}{\pi} Sa(\omega_m t)} \Leftrightarrow$$

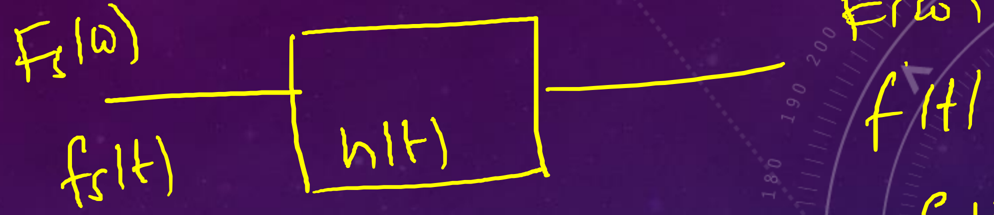
$$F_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$$

$$T_s F_s(\omega) = \sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$$

$$T_s F_s(\omega)\, G_{2\omega_m}(\omega) = F(\omega)$$

$$F(\omega) = F_s(\omega) . T_s G_{2\omega_m}(\omega)$$

$$\boxed{f(t) = f_s(t) * T_s \frac{\omega_m}{\pi} Sa(\omega_m t)}$$



$$f(t) = f_s(t) * h(t)$$

$$F(\omega) = F_s(\omega) H(\omega)$$

$$= F_s(\omega) . T_s G_{2\omega_m}(\omega)$$

$$H(\omega) = T_s G_{2\omega_m}(\omega)$$

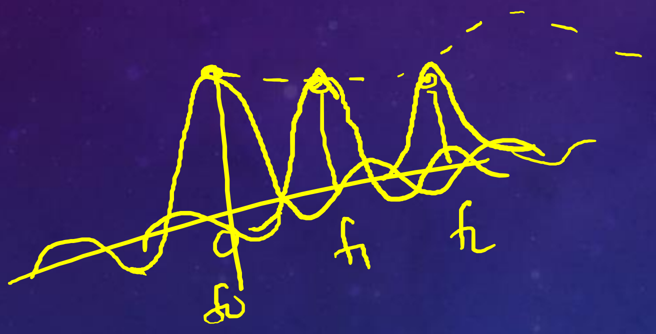
$$h(t) = T_s \frac{\omega_m}{\pi} Sa(\omega_m t)$$

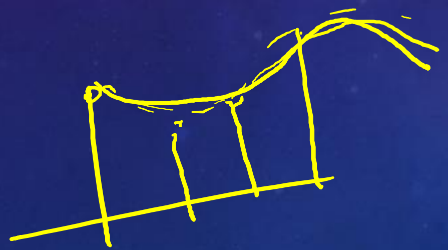
$$f(t) = f_s(t) * T_s \cdot \frac{\omega_m}{T_1} Sa(\omega_m t)$$

$$f(t) = \sum_n f_n \delta(t - nT_s) * \frac{1}{\cancel{2f_m \cdot T_1}} \cdot \cancel{\omega_m} Sa(\omega_m t)$$

$$f(t) = \sum_n f_n Sa(\omega_m t - nT_s)$$

$f(t)$ ↑ original





$$f_s(t) = \sum_n f_n \delta(t - nT_s)$$

$$\boxed{x(n) = \sum_k x(k)\delta(n-k)}$$

$$T_s = \frac{1}{2f_m}$$

$m(t) \rightarrow f_m \Rightarrow$ Sampling Interval $T_s \leq \frac{1}{2f_m}$ = Spacing b/n two Samples

CTS Reconstruct / Recover msg / Info Signal from its samples

If $T_s > \frac{1}{2f_m}$ (or) $f_s < 2f_m$ then, there is overlapping of HF components into msg spectrum

$\therefore$ Aliasing effect.

$\therefore$ Recovery is Not Possible w/o distortion

Calculate the Nyquist rate & Nyquist Interval

$f_s \geq 2f_m$

$T_s \leq \frac{1}{2f_m}$

$\boxed{\cos 2\pi f_m t}$

$m(t) = 3\cos 500\pi t + 15\sin 200\pi t + 5\cos 100\pi t$

$= 3\cos 2\pi 250 t + 15\sin 2\pi 100 t + 5\cos 2\pi 50 t$

$f_1 = 250$ $\quad f_2 = 100$ $\quad f_3 = 50$

Nyquist Rate = $f_s \geq 2f_m$ Highest freq Component

$f_s = 2 * f_1 = 500$ sec $\Rightarrow$ Nyq. Rate

$T_s = \frac{1}{f_s} = \frac{1}{500} = 2\,ms$

$$m(t) = \sin\left(\frac{1000\pi t}{\pi t}\right) = \sin\left(\frac{2\pi\, 500 t}{\pi t}\right)$$

$$f_m = 500\text{ Hz} \Rightarrow T_s = \frac{1}{2f_m} = \frac{1}{1000} = 1\text{ ms}$$

$$f_s = 2f_m = 1000\text{ Hz}$$

$$m(t) = \frac{\sin^2 50\pi t}{\pi t} = \frac{1}{\pi t}\left(\frac{1-\cos 2(50\pi t)}{2}\right)$$

$$f_m = 50 \Rightarrow f_s = 2f_m = 100$$

$$T_s = \frac{1}{100} = 0.1\text{ ms}$$

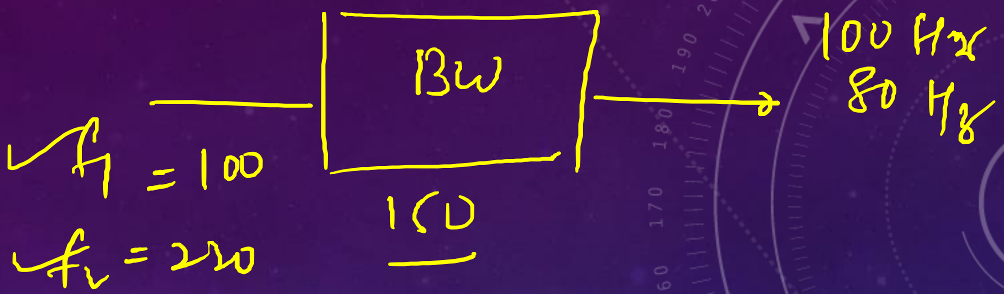
$$m(t) = \cos 2\pi\, 100t + \cos 2\pi\, 220t, \quad f_s = 300\text{ Hz}$$

Passed – LPF $BW = 150\text{ Hz}$

$$f_1 = 100, \quad f_2 = 220$$

$f_1 = 100, \; f_2 = 220$

$f_s = 300\,Hz$ & $BW = 150\,Hz$



$f_s = 300$

$f_r(t) = m(t) + m(t)\cos 2\pi f_r t + m(t)\cos 2\pi\, 2 f_s t + \_\_$

$\cos 2\pi f_m t$ → $\boxed{f_1 = 100}$

$\cos 2\pi f_m(t)$ $\cos 2\pi f_r(t)$

$\cos 2\pi (2f_s + f_m) + \cos 2\pi (2f_s - f_m)$

$\cos 2\pi (f_s + f_m)t + \cos 2\pi (f_s - f_m)t$

$f_s + f_m = 300 + 100 = 400 > 150 = \times$

$f_s - f_m = 300 - 100 = 200 > 150 \;\; \times$

$f_s + f_m = \times$

$f_s - f_m = 300 - 220 = 80\,Hz$

Sampling theorem for Band pass signal

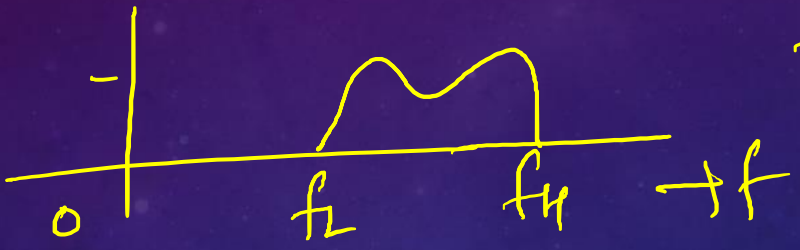
Bandlimited =

$\boxed{f_s \geq 2f_m}$

$f_L = 0 \Rightarrow$ Bandwidth $= (f_H - f_L)$

$f_H \neq 0$ BW $= f_H \Rightarrow \boxed{f_s = 2f_H}$

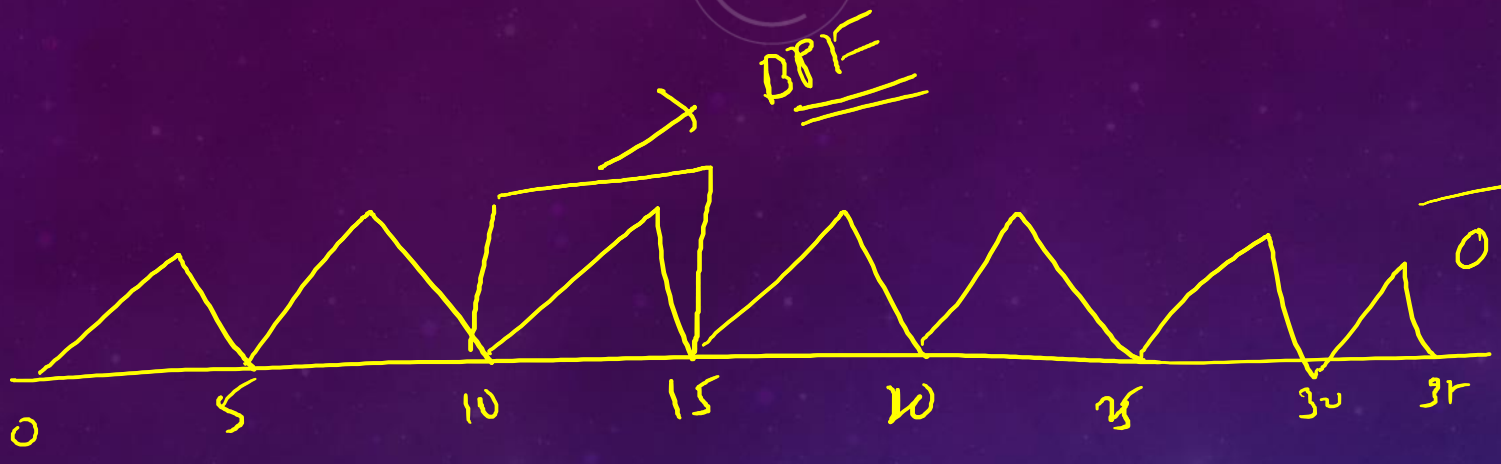
Bandpass

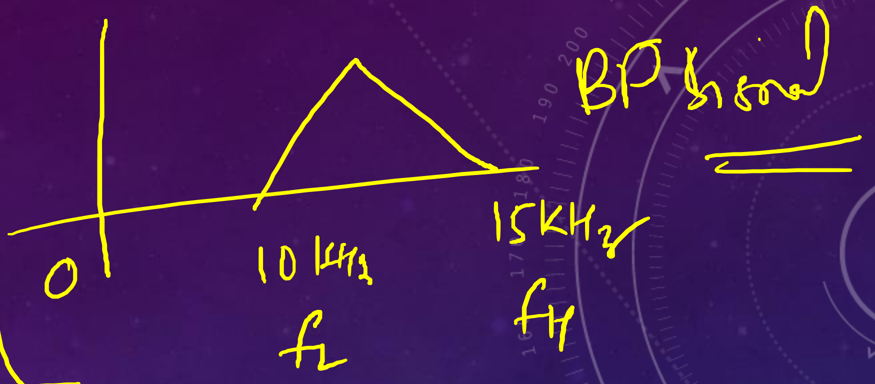


$f_L \neq 0$ Bandwidth

$f_H \neq 0$ $= (f_H - f_L)$

Sampling freq $= 2(f_H - f_L)$





$BW = f_H - f_L = 5KHz$

$f_s = 2(f_H - f_L) = 10KHz$

$f_L = 1 * f_s$, $f_H = 1.5 f_H$

Integer multiple of f_s

$f_s \pm f_L = 10+10 \quad 10-10 = 0$

$\qquad\qquad 15+10 \quad 15-10 = 5$

$f_s \pm f_H$

$2f_s \pm f_L$

$2f_s \pm f_H$

$3f_s \pm f_L$

$3f_s \pm f_H$

No overlapping

Recovery is possible

$f_s \pm f_L =$ 10+15 10−15 = 25, 5

$f_s \pm f_H =$ 20+10 20−10 30 10

$2f_s \pm f_L$ No overlapping

$2f_s \pm f_H$ Recovery Possible

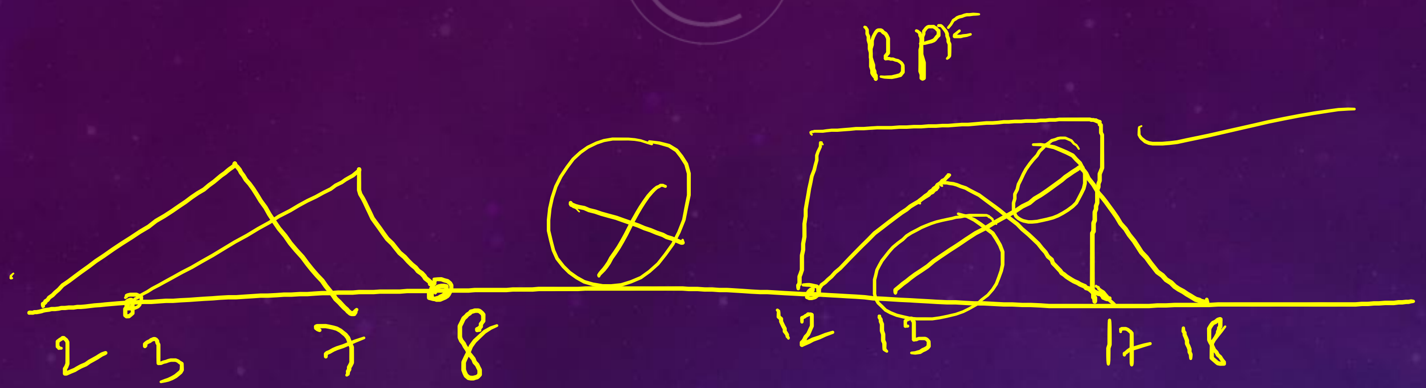
BW = 20 − 15 = $f_H - f_L$

= 5KHz

f_s = 2×5KHz = 10KHz

$f_L = 1.5 f_s$, $f_H = 2 \times f_s$

Integer multiple of f_s



BW = 5 kHz

$f_s = 2 \times 5 = 10$ kHz

$$f_L - f_s = 12 - 10 = 2$$
$$f_H - f_s = 17 - 10 = 7$$
$$2f_s - f_H = 20 - 17 = 3$$
$$2f_s - f_L = 20 - 12 = 8$$

$$3f_s - f_H = 30 - 17 = 13$$
$$3f_s - f_L = 30 - 12 = 18$$

Overlapping

Recovery X

$$\begin{cases} f_L = 1.2 * f_s \\ f_H = 1.7 * f_s \end{cases}$$

not an integer multiple

→ If either of f_L (or) f_H is an integer multiple of the $f_s = 2(f_H - f_L)$, Recovery of the Band Pass signal using BPF is Possible

→ If either of f_L (or) f_H is not an integer multiple of $2(f_H - f_L)$ ⇒ then with this Sampling freq, Recovery is not possible

∴ It is possible to Recover

$$\left.\begin{array}{l} 3f_s - f_H \geq f_H \\ 2f_s - f_L \leq f_L \end{array}\right\}$$

$$N f_s - f_H \geq f_H$$

$$N f_s \geq 2 f_H \Rightarrow \boxed{f_s \geq \frac{2 f_H}{N}}$$

$$(N-1)f_s - f_L \le f_L \Rightarrow (N-1)f_s \le 2f_L$$

$$f_s \le \frac{2f_L}{(N-1)} \Rightarrow f_s = \left(\frac{2f_L}{N-1}\right)$$

$$f_s = \frac{2f_H}{N}$$

$$\frac{\cancel{2}f_L}{N-1} = \frac{\cancel{2}f_H}{N}$$

$$BW = f_H - f_L$$

$$\boxed{f_L = f_H - BW}$$

$$N(f_H - BW) = (N-1)f_H$$

$$N\cancel{f_H} - B\cdot W * N = N\cancel{f_H} - f_H$$

$$\boxed{N = \frac{f_H}{B\cdot W}}$$

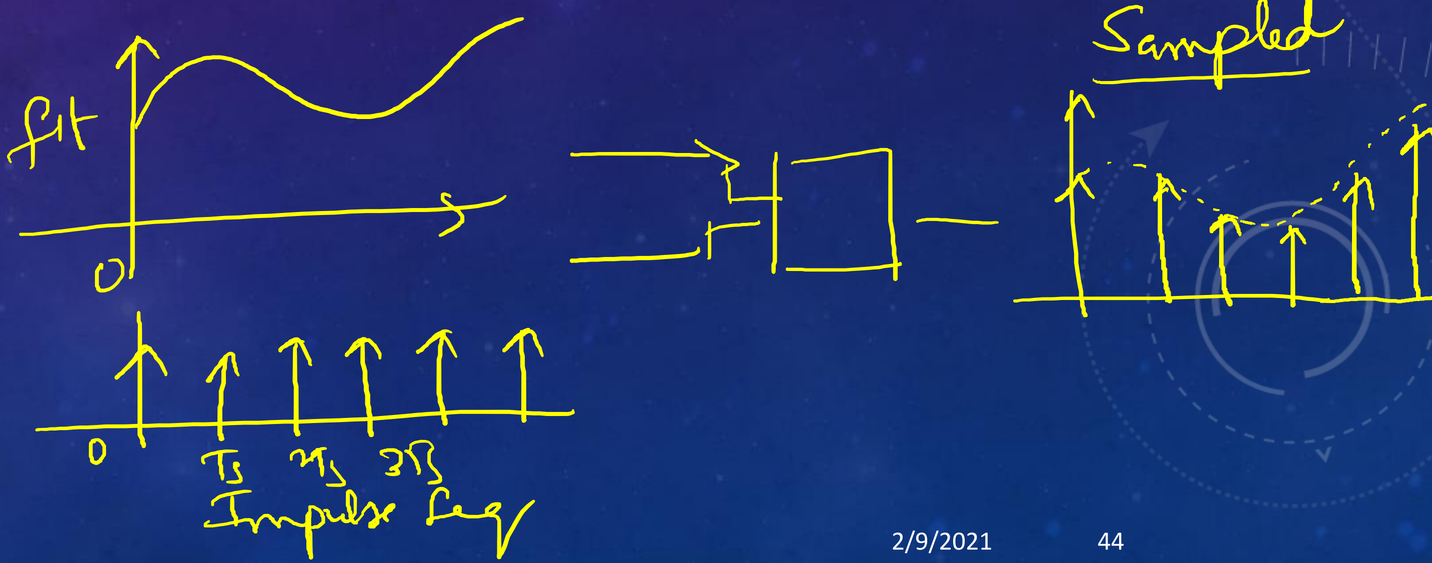
for Band limited signals : all the freq $f_s \geq 2f_m$ is possible to Recover

Bandpass = all the sampling freq $f_s \geq 2(f_H - f_L)$ are not usable

$\Rightarrow$ A Bandpass signal $m(t)$ having an upper freq limit f_H & Bandwidth 'B' can be specified in terms of its samples taken at a rate of $\boxed{f_s(min) = \frac{2f_H}{N}}$, where N is the largest integer not exceeding $\frac{f_H}{B}$. ∴ All the sampling freq greater than $f_s(min)$ are not usable unless they exceed $2f_H$

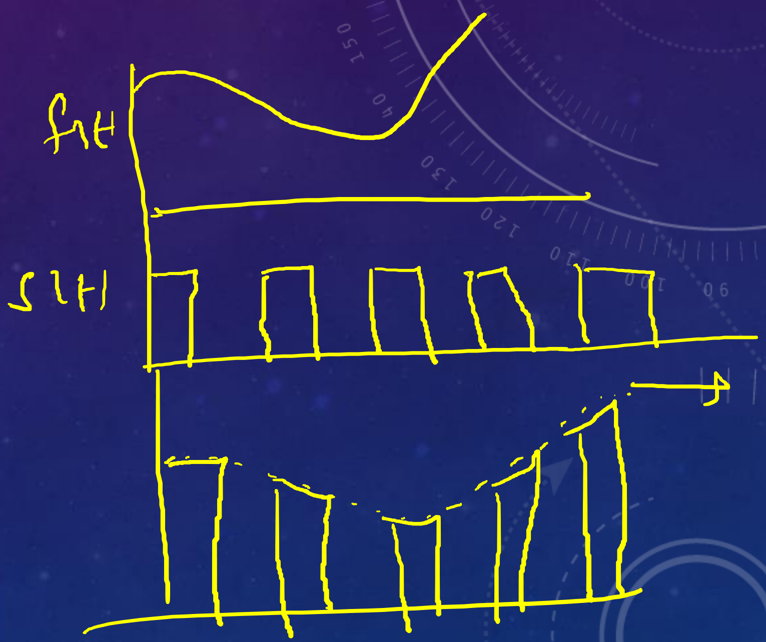
Types of Sampling → Instantaneous Sampling
→ Natural "
→ Flat Top "

Instantaneous Sampling:- the sampling Process in which the Sampling Signal is a true impulse sequence is known as the Instantaneous Sampling (or) Impulse Sampling



strength of impulse signal = width * height

— width of each impulse is increased to increase the strength called as <u>Natural Sampling</u>. i.e In Natural Sampling, the sampling signal is a pulse train of width T and unit height.



Natural Sampling

<u>Features of Natural Sampling</u>.

(i) strength is more ∵ width is more.

(ii) Top of each pulse will follow the envelope of base band signal.

<u>disadv</u> (1) strength is not uniform throughout the pulse

(2) Difficult to maintane the pulse shape during the transmission

Flat Top Sampling

Features

(1) Top of the pulse is flat instead of following base band signal

(2) width is more ∴ more strength

(3) Pulse shape is maintained

Thank you