Unit 1

1. (i) <u>Radiation resistance</u>:

From circuit point of view, antenna can be replaced with equivalent impedance. Real part of this impedance is called Radiation resistance.

The component of antenna resistance that accounts for the power radiated into space and is equal in ohms to the radiated power in watts divided by the square of the effective current in amperes. The other component is the loss resistance

(ii) <u>Beam Area (Ω_A)</u>

is defined as

$$\Omega_A = \int_{\substack{\text{complete} \\ \text{solid angle}}} P_n(\theta,\phi)\, d\Omega \qquad \text{(Units — radians)}$$

where $P_n(\theta,\phi) = P(\theta,\phi) / [P(\theta,\phi)]_{max}$

and $d\Omega = \sin\theta \, d\theta \, d\phi$

Alternately $\Omega_A = \int_{\theta}\int_{\phi} P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi$

Significance — It is the solid angle over which an equivalent isotropic antenna would radiate same total power as the actual antenna with $P(\theta,\phi)_{max}$ radiated in all directions

iii. Radiation intensity: $U(\theta,\phi)$

is defined as the power radiated per unit solid angle

$$U(\theta,\phi) = P(\theta,\phi)\,\frac{P_{total}}{4\pi}$$

$$P_{total} = \int_\theta\int_\phi P(\theta,\phi)\sin\theta\,d\theta\,d\phi$$

Units — Watts / radian

(iv) Directivity: (D)

is a theoritically quantity defined as the ratio of maximum radiated power to the average value

$$D = \frac{P_{max}(\theta,\phi)}{P_{avg}}$$

Average value over complete space (4π steradians) is given by

$$P_{avg} = \frac{P_{total}}{4\pi}$$

(no units)

$$D = \frac{P_{max}}{\left[\int_\theta\int_\phi P(\theta,\phi)\sin\theta\,d\theta\,d\phi\right]/4\pi}$$

$$= \frac{4\pi}{\int_\theta\int_\phi \frac{P(\theta,\phi)}{P_{max}}\sin\theta\,d\theta\,d\phi}$$

$\frac{P(\theta,\phi)}{P_{max}} = P_n(\theta,\phi)$ and so the denominator is simply beam area, Ω_A

$$\therefore \; D = \frac{4\pi}{\Omega_A}$$

Significance – Larger the value is, sharper is the pattern

(v) <u>Gain</u> (G)

Directivity computation doesn't take loss resistance, presented by antenna material and its radome, into account~~ed~~. In real we get a lesser value and this value is referred to as Gain of the antenna.

Mathematically, $G = KD$

where K is the antenna efficiency.

$(0 < K < 1)$

(vi) <u>Isotropic radiator</u>:

is one which radiates equal power in all directions.

It satisfies $P_n(\theta, \phi) = 1$

i.e., $\Omega_A = 4\pi$ and $D = 1$

(E plane & H plane patterns, both are ~~circular~~ circles)

(vii) <u>Hertzian dipole</u>:

is a dipole ~~which~~ (hypothetical one) which satisfies the condition that the phase of current at all points on the wire is same

$i(z) = I_0$

(viii) Antenna efficiency : (K)

is the fraction of fed power that is radiated.

$$K = \frac{R_{rad}}{R_{rad} + R_{loss}}$$

(ix) Field and power patterns :-

Plot of E or H as a function of θ and ϕ is called field pattern

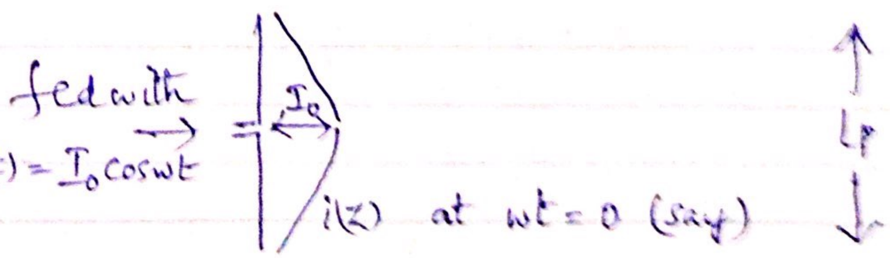
Plot of P as a function of θ and ϕ is called Power pattern

Each one is a 3-D pattern. On paper, we describe the same using two 2-D pattern cuts called E plane and H plane pattern cuts

[Ex : $E(\theta, \phi)$ cut by the surface ϕ = a constant gives 2-D pattern cut, $E(\theta)$]

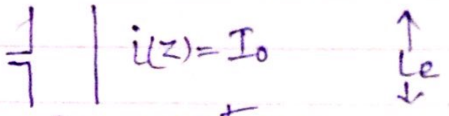
(x) Effective length :- (l_e)

Consider a wire antenna that carries $i(z)$ as shown below at certain time instant.



It effective length is given by the length of an equivalent antenna (imaginary one) carrying current I_0

at all points on it and that it radiates same field in all directions as the actual antenna.



(imaginary antenna

When transmitting,

$$l_e = \frac{[i(z)]_{avg}}{[i(z)]_{max}} \cdot l_p$$

$$[i(z)]_{avg} = \frac{1}{l_p} \int_{-l_p/2}^{l_p/2} i(z)\, dz$$

When receiving,

$$l_e = \frac{\text{O.C. voltage at base terminals}}{\text{Incident } E} = \frac{V_{OC}}{E}$$

(xi) Effective aperture : (A_e)

is the ratio of received power to the power density at the location of the antenna

$$A_e = \frac{P_r}{S}$$

(xii) Omni directional antenna :

is an antenna which gives isotropic kind of behavior in one plane (E or H plane or pattern cut). i.e., one of the two 2-D plots is a circle

Ex: Dipole antenna

(xiii) <u>HPBW</u> : (Ω_{HP})

is the beamwidth between half power points

$$\Omega_{HP} = \theta_{HP}\phi_{HP}$$

where θ_{HP} and ϕ_{HP} are the half power angles in the two planes

<u>FNBW</u> : (Ω_{FN})

is the beamwidth/solid angle between the first nulls

$$\Omega_{FN} = \theta_{FN}\phi_{FN}$$

(xiv) <u>Resolution</u> :

Resolution of an antenna with beam area Ω_A refers to the maximum number of clearly differen-tiable signals coming from all directions in space.

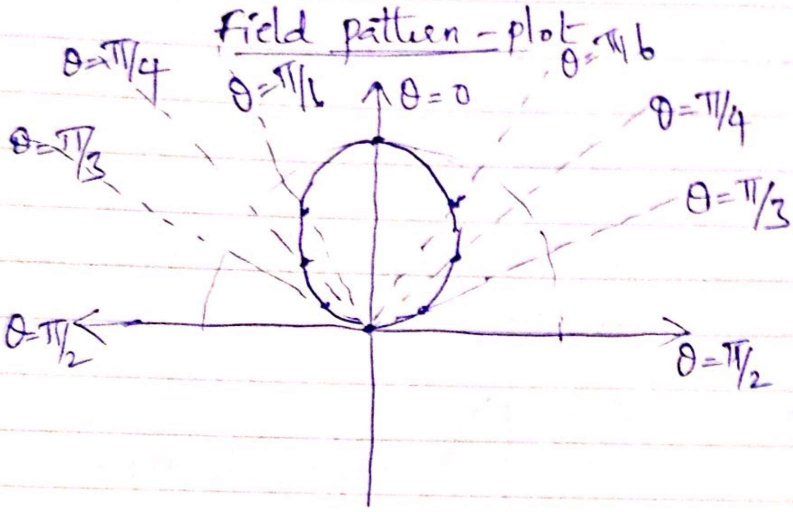
$$\text{The number} = \frac{4\pi}{\Omega_A}$$

is numerically equal to Directivity.

2. Given $E_n(\theta) = \cos^2\theta, \quad 0 \le \theta \le \pi/2$

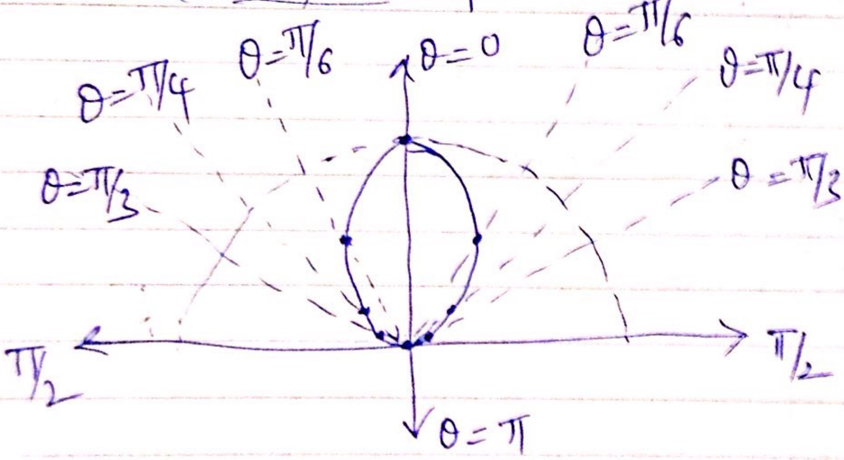
$P_n(\theta) = E_n^2(\theta) = \cos^4\theta, \quad 0 \le \theta \le \pi/2$

Field pattern-plot



θ	$E_n(\theta)$	$P_n(\theta)$
0	1	1
$\pi/6$	$3/4$	$9/16$
$\pi/4$	$1/2$	$1/4$
$\pi/3$	$1/4$	$1/16$
$\pi/2$	0	0

Power pattern plot



HPBW :- Equate $P_n(\theta)$ to $\frac{1}{2}$

$\cos^4\theta = \frac{1}{2}$

$\Rightarrow \theta = 32.75^\circ$

$\therefore \theta_{HP} = 2\theta = 65.5^\circ$ (or) 1.143 rad

NOTE : Ω_{HP} can't be found here since $E_n(\phi)$ isn't given

FNBW :-

Equate $P_n(\theta)$ to zero

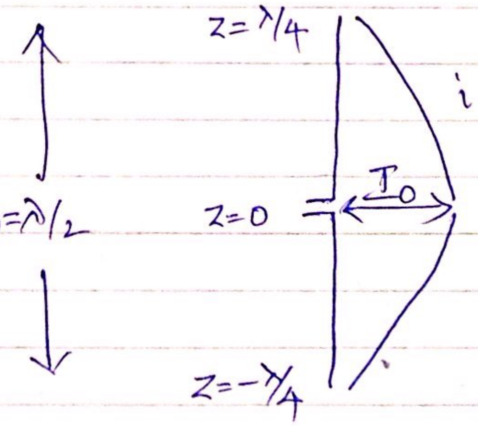
$$\cos^4\theta = 0$$

$$\Rightarrow \theta = \pi/2$$

$$\therefore \theta_{FN} = 2\theta = \pi \text{ rad}$$

3. find the effective length of half wave dipole antenna. Show that its Directivity is 1.64

Ans: In the transmitting case



$$i(Z) = I_0 \cos\beta Z$$

$$l_e = \frac{1}{I_0}\int_{-L_p/2}^{L_p/2} i(Z)\,dZ$$

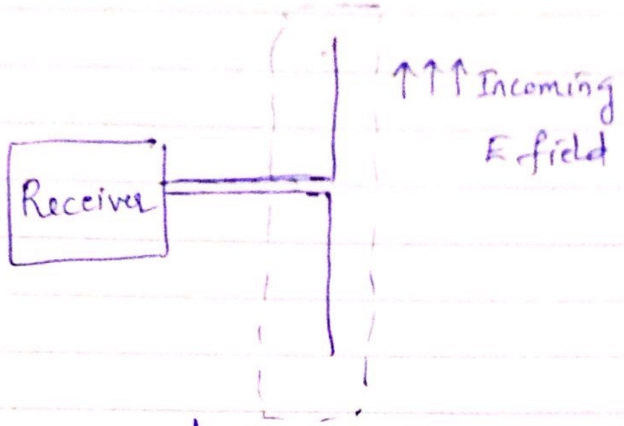
(Refer Q. 1(x))

$$= \frac{1}{I_0}\int_{-\lambda/4}^{\lambda/4} I_0 \cos\beta Z\,dZ$$

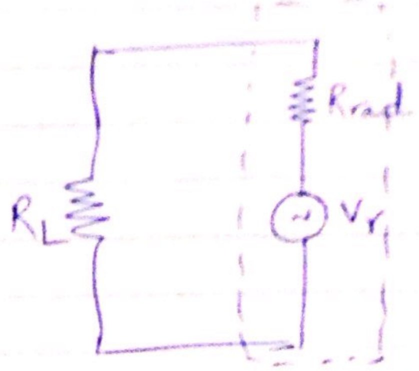
$$l_e = \frac{\lambda}{\pi} \text{ units} \quad \text{——— eq ①}$$

To get the Directivity, we consider receiving case and get its equivalent

The received voltage is transferred to receiver circuitry the receiver's input voltage becomes load to the antenna. Let Vr be the received voltage



$\lambda/2$ dipole Antenna



Equivalent circuit of the dipole

Under matching conditions ($R_L = R_{rad}$), power delivered to the load is

$$P_r = \left(\frac{V_r}{R_L + R_{rad}}\right)^2 R_L$$

i.e., $$P_r = \frac{V_r^2}{4 R_{rad}} \quad \text{—— eq } ②$$

V_r can be expressed interms of incoming E-field strength at the location of the antenna

as $$V_r = E l_e \quad \text{—— eq } ③$$

P_r can be alternately expressed as

$$P_r = S A_e \quad \text{—— eq } ④$$

where S is the power density (Poynting vector) at the location of the antenna and A_e is the equivalent effective aperture.

$$S = \frac{E^2}{\eta} \quad \text{—— eq } ⑤$$

where $\eta = 120\pi\ \Omega$ —— eq ⑥

Also $R_{rad} = 73\ \Omega$ —— eq ⑦

From eqs (2) and (4) we get

$$\frac{V^2}{4R_{rad}} = SA_e$$

$$\Rightarrow A_e = \frac{V^2}{4SR_{rad}}$$

Using eqs (1),(3), (5) and (6) and (7)

$$A_e = \frac{(\cancel{E} l_e)^2}{4\left(\frac{\cancel{E}^2}{\eta}\right)(73)}$$

$$= \frac{(\lambda/\pi)^2}{4\left(\frac{1}{120\pi}\right)(73)}$$

$$= 0.131\lambda^2$$

Directivity, $D = \frac{4\pi A_e}{\lambda^2}$

$$= \frac{4\pi(0.131\lambda^2)}{\lambda^2}$$

$$= 1.64$$

In decibels, $D(dB) = 10\log 1.64$

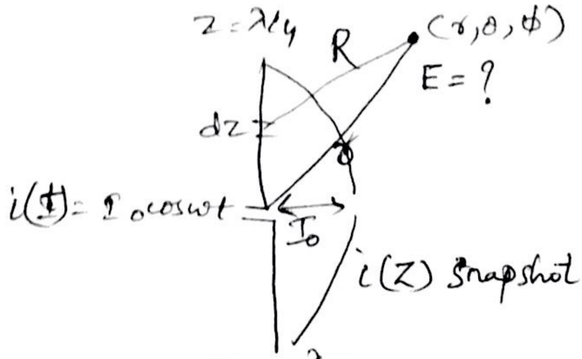
$$= 2.15 \text{ dB}$$

Q) Compute the radiation resistance of an half wave dipole when fed ~~alternating current~~ with sin current obtain field expression? ~~element (Starting from vector potential~~ expression)?

Ans

Current element, $R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

Tangent drawn to horizontal



$$A = \frac{\mu}{4\pi} \int \frac{i\,dl}{r}$$

$$= \frac{\mu}{4\pi} \frac{i(t')\,dl}{r}$$

$$= \frac{\mu}{4\pi} \frac{i(t')\,dz}{R}$$

$$i(Z) = I_0 \cos\left(\frac{2\pi}{\lambda} z\right), \quad -\lambda/4 \le z \le \lambda/4$$

A delayed signal is represented as $f(t\ t_0)$ or $f(t)e^{-j\beta r}$

$$dA = \frac{\mu}{4\pi} \frac{i(t)\,dz}{r - z\cos\theta} \underbrace{e^{-j\beta R}}_{\text{delay}}$$

$r - z\cos\theta \simeq r$

$$dA = \frac{\mu}{4\pi} \frac{i(t)\,dz}{r} e^{-j\beta(r - z\cos\theta)}$$

$$A = \int dA \quad \text{complete wire length}$$

$$= \int_{-\lambda/4}^{\lambda/4} \frac{\mu\, i(t)\,dz}{4\pi r} e^{-j\beta(r - z\cos\theta)}$$

$$A_z = \frac{\mu\, i(t)\, e^{-j\beta r}}{2\pi\beta r} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta} \quad \text{--- (1)}$$

$H = ?$ $\quad \frac{1}{\mu} \nabla \times A$

$$H_\phi = \frac{j\, i(t)\, e^{-j\beta r}}{2\pi r} \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} \right] \quad \text{--- (2)}$$

Tangent drawn to horizontally drawn imaginary circle is a_ϕ

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

$P_{avg} \propto \frac{1}{r^2} \longrightarrow$ direction :: a_r

$P_r = (E_\theta H_\phi)_{avg} \neq 0$

$(P_\theta)_{avg} = (-E_r H_\phi)_{avg} = 0$

$\left(\frac{E}{H} = \eta\right)$

$E_\theta = \eta H_\phi$

$$= \eta \frac{j\, i(t)\, e^{-j\beta r}}{2\pi r} \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} \right] \quad \text{--- (3)}$$

$P = E \times H$

$$P_r = \frac{\eta\, i^2(t)}{4\pi^2 r^2} \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta} \quad \text{--- (4)}$$

P_{r} ~~/lax~~ $[i^2(t)]_{avg} = [I_0^2 \cos^2\omega t]_{avg} = \frac{I_0^2}{2}$ — 4

9

$$(P_r)_{avg} = \frac{\eta I_0^2 \cos^2(\pi/2 \cos\theta)}{8\pi^2 r^2 \sin^2\theta} \quad — (5)$$

$$P_{total} = \int_0^{8\pi} (P)_{avg}\, dr \quad \text{where } dr = 2\pi r^2 \sin\theta\, d\theta$$

$$= \int_{\theta=0}^{\pi} \frac{\eta I_0^2 \cos^2(\pi/2 \cos\theta)}{8\pi^2 r^2 \sin^2\theta} (2\pi r^2 \sin\theta\, d\theta)$$

$$P_{total} = \frac{0.609\,\eta I_0^2}{2\pi} \quad — (6)$$

$$\left(\begin{array}{l} P = I_{eff}^2 R \\ I_{eff} = \frac{I_0}{\sqrt{2}} \end{array} \right)$$

$$\boxed{R_{rad} = 73\,\Omega}$$

Scanned by CamScanner

Prove that the Effective length of an antenna when used for receiving is same as when used for transmitting.

The Effective length (l_{eff}) is defined as the length of an Equivalent linear antenna which has current $I(0)$ along its length at all points radiating the field strength in direction perpendicular to the length same as actual antenna.

For transmitting antenna,

$$I(0)\, l_{eff}(trans) = \int_{-l/2}^{+l/2} I(z)\,dz.$$

i.e $$l_{eff}(trans) = \frac{1}{I(0)} \int_{-l/2}^{+l/2} I(z)\,dz. \quad —(1)$$

For Receiving antenna Effective length can be defined as the ratio of the open circuit voltage developed at the antenna terminals to the given received field strength for receiving antenna l_{eff} is given as

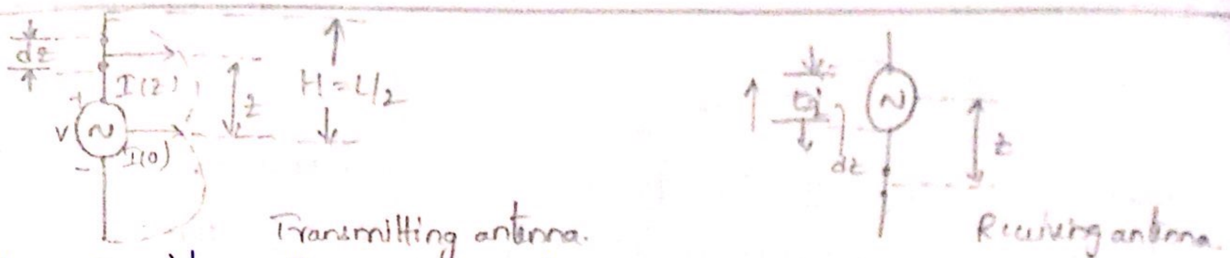
$$l_{eff}(rec) = \frac{-V_{oc}}{E}. \quad —(2)$$

To show equality of both transmitting and receiving antennas Apply reciprocity theorem.

First consider an antenna used as transmitting antenna let Z_a be the antenna impedance measured at antenna terminals. Assume voltage 'V' is applied at antenna terminals then current produced at the antenna terminal is given by

$$I(0) = V/Z_a.$$

Similarly current at any point z along the antenna is $I(z)$.



Now consider the same antenna is used as receiving antenna Electromagnetic field E_z^i is incident on it. This induces voltage $E_z^i dz$ in the element dz. when antenna terminals are short circuited, The ideal generator ($E_z^i dz$) produces current dI_{sc} in the antenna.

According to reciprocity theorem,

$$\frac{V}{I(z)} = \frac{E_z^i dz}{dI_{sc}} \quad \text{ie} \quad dI_{sc} = \frac{E_z^i dz}{V} I(z)$$

According to superposition theorem, total short circuited current any antenna terminals is sum of currents produced by all the differential voltages along entire antenna length.

$$I_{sc} = \frac{1}{V} \int E_z^i I(z) dz.$$

But, According to thevenins, open circuit voltage is

$$V_{oc} = -I_{sc} Z_a.$$

$$V_{oc} = \frac{-Z_a}{V} \int E_z^i I(z) dz.$$

$$V_{oc} = -\frac{1}{I(0)} \int E_z^i \, I(z) dz.$$

$$V_{oc} = \frac{-E_z}{I(0)} \int I(z) \, dz \qquad (E_z^i = E_z)$$

$$\frac{-V_{oc}}{E_z} = \frac{1}{I(0)} \int I(z) dz.$$

from -(2)

$$l_{eff\,(rec)} = \frac{1}{I(0)} \int I(z) dz. \quad —— (3)$$

from 1 and 3 Effective length of transmitting antenna is equal to receiving antenna.

Small Loop Antennas:

Loop antennas may take many different forms such as circle, square, rectangle etc. Loop antennas are generally classified into two categories viz, electrically small and electrically large antennas. Electrically small antennas are those whose overall length is less than one tenth is number of terms in the loop times the circumference of the loop. Here we shall keep our discussion confined to small loop antennas only.

Small loops are usually not used as transmitting antennas as they have radiation resistance smaller compared to short dipoles. However many unintentional sources of radiation such as transformers, inductor, printed circuit boards etc essentially behave as small loop antennas. A small loop of current is also called a magnetic dipole and its magnetic dipole moment is equal to the product of the area with the current it carries. Thus for these types of small current loops, the shape of the loop is not important. For a given current, it is the area of the loop that determines the magnitude of the radiated fields.

Fig. 7.11 shows a small current loop of radius ρ_0 placed on the xy plane with its axis oriented in the z direction. The loop carries a current I_0.

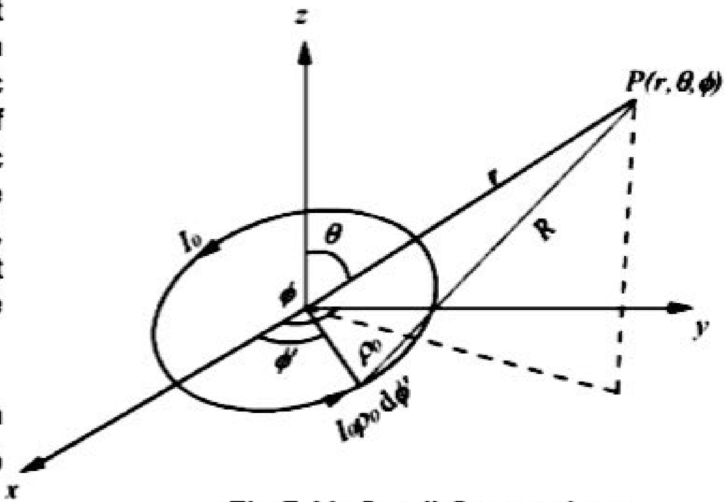


Fig 7.11: Small Current loop

ɔr $\rho_0 << \lambda_0$, the loop may be treated as a point source. As shown in Fig 7.11, the elementary current element placed ϕ' has a vector orientation $\hat{a}_{\phi}' = -\hat{a}_x \sin\phi' + \hat{a}_y \cos\phi'$. For this current element the vector potential by equation '.21)

$$d\vec{A} = \frac{\mu_0}{4\pi} I_0 \rho_0 d\phi' \left(-\hat{a}_x \sin\phi' + \hat{a}_y \cos\phi'\right) \frac{e^{-jk_0 R}}{R} \quad \text{.........(7.50)}$$

'here

$e^{-jk_0}\left(r-\rho_0 \sin\theta\cos(\phi-\phi')\right)d\phi'$

$1 + jk_0\rho_0 \sin\theta\cos(\phi-\phi')\big)d\phi$

$\rho_0 \sin\theta\cos(\phi-\phi')$ since $k_0\rho_0 = \dfrac{2\pi\rho_0}{\lambda_0}$ is very small as $\rho_0 << \lambda_0$)

$\Big) + jk_0\rho_0 \sin\theta\left(-\hat{a}_x \sin\phi' + \hat{a}_y \cos\phi'\right)(\cos\phi'\sin\phi' + \sin\phi\sin\phi')\Big]d\phi'$

$\phi\,\hat{a}_x\, 2\pi\Big]$

(7.54)

Using (7.37c) and (7.37e), the radiated field components can be written as

$$\vec{E} = \omega k_0 \frac{\mu_0}{4\pi} M \frac{e^{-jk_0 r}}{r} \sin\theta \hat{a}_\phi \text{ ....(7.52a)}$$

$$\vec{H} = -\frac{\omega k_0}{\eta_0} \frac{\mu_0}{4\pi} M \frac{e^{-jk_0 r}}{r} \sin\theta \hat{a}_\theta \text{ ....(7.52b)}$$

where $M = \left|\vec{M}\right|$, $M = \pi \rho_0^2 I_0 \hat{a}_z$ is the dipole moment of the loop.

The field radiated by a small loop antenna is dual of that small dipole antenna, i.e., a short current filament, the role of electric and magnetic fields are interchanged.

From (7.52) ,

$$\vec{E} \times \vec{H}^* = \frac{\omega^2 k_0^2}{\eta_0} \left(\frac{\mu_0}{4\pi}\right)^2 M^2 \frac{\sin^2\theta}{r^2} \hat{a}_r$$

erefore the radiated power

$$= \frac{1}{2}\mathrm{Re}\left[\int_0^{2\pi}\int_0^{\pi}\frac{\omega^2 k_0^2}{\eta_0}\left(\frac{\mu_0}{4\pi}\right)^2 M^2 \frac{\sin^2\theta}{r^2} r^2 \sin\theta \, d\theta \, d\phi\right]$$

$$= \frac{M^2\omega^2 k_0^2}{2\eta}\left(\frac{\mu_0}{4\pi}\right)^2 2\pi\int_0^{\pi}\sin^3\theta \, d\theta$$

$$= \frac{M^2\eta_0 k_0^4}{12\pi}$$

ıe radiation resistance of a loop antenna can be found be

$$, = 320\pi^6\left(\frac{\rho_0}{\lambda_0}\right)^4$$

the antenna consists of N number of turns; the radiation resistance increases by a factor of N^2.

nall loop antennas are often used as receiving antennas.

7. Given $P_n(\phi) = \cos^2 2\phi \ , \ -\pi/4 \le \phi \le \pi/4$

Beam area, $\Omega_A = \iint_{\theta\ \phi} P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi$

But $P_n(\theta)$ is not given. Take $P_n(\theta) = 1 \ , \ 0 \le \theta \le \pi$ (Omnidirectional)

$$\therefore \Omega_A = \int_{\theta=0}^{\pi} \int_{\phi=-\pi/4}^{\pi/4} \cos^2 2\phi \sin\theta \, d\theta \, d\phi$$

$=$

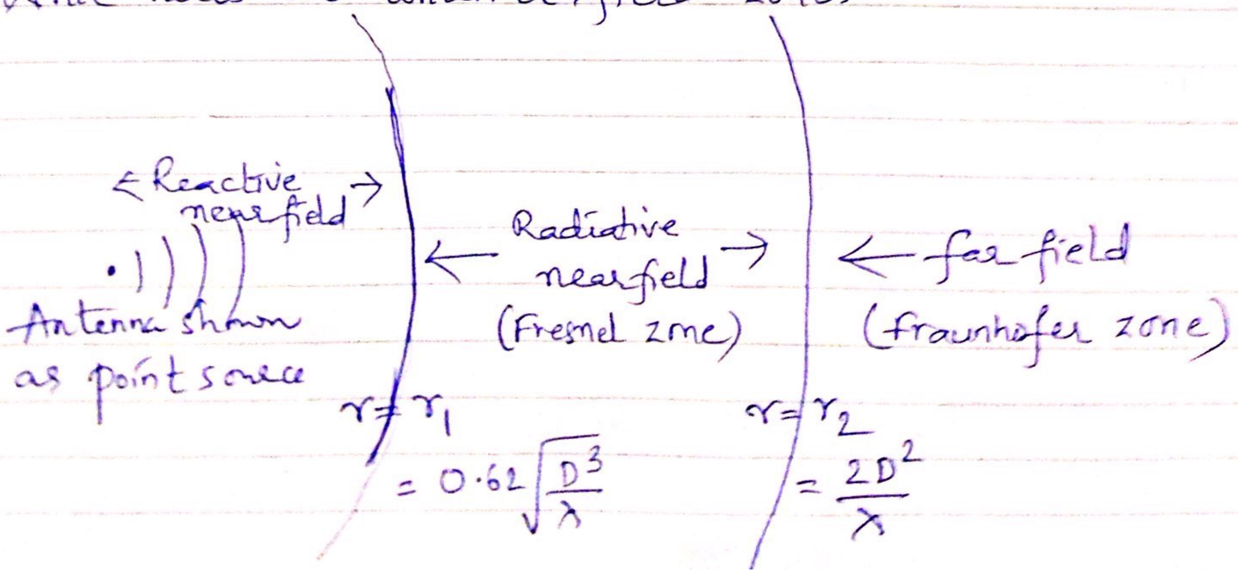
$$D = \frac{4\pi}{\Omega_A}$$

$=$

· Since we took $P_n(\theta) = 1$, the pattern is omnidirectional (neither unidirectional nor bidirectional)

NOTE : A half wave dipole gives omnidirectional pattern (neither bidirectional nor unidirectional) — donut shape

8. Write notes on antenna field zones.



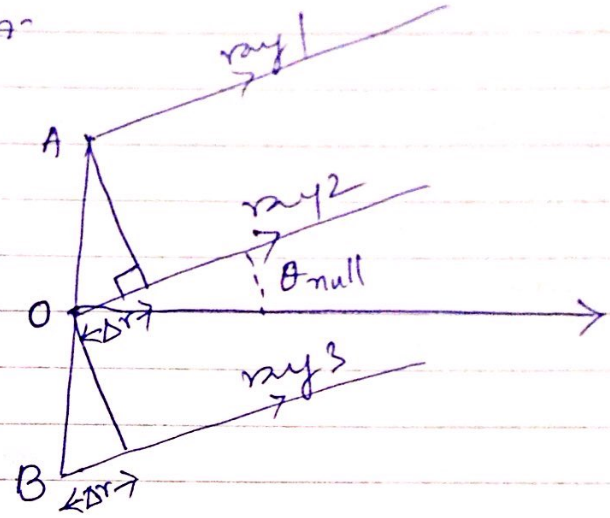
9. Derive the relationship between D and A_e.

Ans.

Consider a far field communication case where an aperture antenna is ~~con~~ assumed transmitting.

(AB, shown, is the dimension in one plane)

AB = a units (say)



Consider 3 rays leaving AB (on leaves from the centres and the other two from the ends) and moving towards far off receiving antenna.

~~Their propagation direction makes angle θ with horizontal axis.~~

Consider the direction of propagation, θ_{null} shown where null (zero field) is seen.

Null exists along the direction θ_{null} when path diff Δr (see the diagram) equals $\lambda/2$

From geometry, $\cos(90 - \theta_{null}) = \dfrac{\Delta r}{a/2}$

$$\Rightarrow \sin\theta_{null} = \frac{\lambda/2}{a/2}$$

For small θ_{null} values, $\sin\theta_{null} \simeq \theta_{null}$

$$\therefore \theta_{null} = \frac{\lambda}{a}$$

θ_{FN} in this plane is given by $2\theta_{null}$

$$\theta_{FN} = \frac{2\lambda}{a}$$

and $\theta_{HP} \simeq \frac{\theta_{FN}}{2} = \frac{\lambda}{a}$ —— eq (1)

Working out in the same way for the other dimension of the aperture, we can get

$$\phi_{HP} = \frac{\lambda}{b} \quad \text{—— eq (2)}$$

where b is the second dimension of the aperture

Directivity of this aperture antenna is obtained as follows.

$$D = \frac{4\pi}{\Omega_A}$$

$$\simeq \frac{4\pi}{\theta_{HP}\,\phi_{HP}}$$

$$= \frac{4\pi}{\left(\frac{\lambda}{a}\right)\left(\frac{\lambda}{b}\right)}$$

$$= \frac{4\pi\,(ab)}{\lambda^2}$$

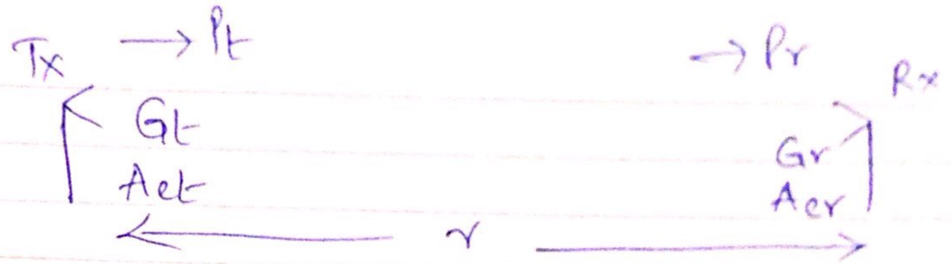
But ab is the physical area of the rectangular aperture.

$$A_p = ab = A_{e(max)}$$

$$\therefore\ D = \frac{4\pi A_{e(max)}}{\lambda^2}$$

10. Frii's transmission equation :-

It gives the relationship between transmitted power, received power, both antenna gains, separation distance and operating wavelength.



Consider a wireless comm. link as shown in the diagram. (& Give sufficient description)

When power P_t watts is transmitted towards the receiver, power density at its location is given by

$$P = \frac{P_t}{4\pi r^2} G_t \quad \text{—— eq ①}$$

(Power density, $P = \dfrac{P_t}{4\pi r^2}$ in the ideal case)

Power absorbed by the receiving antenna, with effective aperture A_{er} is

$$P_r = P \cdot A_{er}$$

$$P_r = \frac{P_t G_t}{4\pi r^2} A_{er} \quad \text{—— eq ②}$$

Also we have $G_r = \dfrac{4\pi A_{er}}{\lambda^2}$ —— eq ③

Solving eqs ①, ② and ③ we get

$$P_r = \frac{P_t G_t G_r}{[4\pi r/\lambda]^2}$$

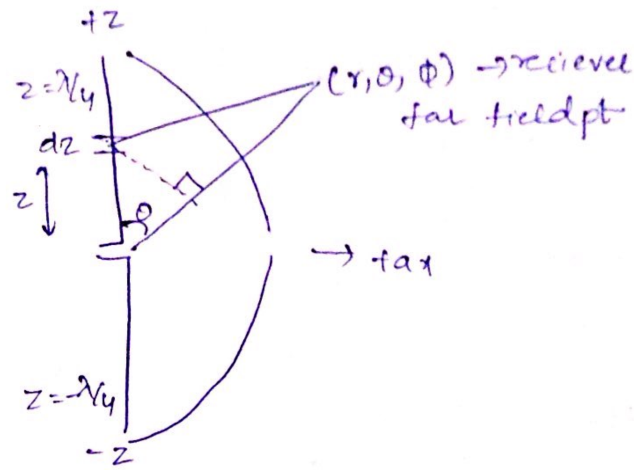
In decibel form

$$P_r(dB) = P_t(dB) + G_t(dB) + G_r(dB) - 10 \log \left[\frac{4\pi r}{\lambda}\right]^2$$

The last term in RHS represents path loss.

Compute radiation resistance of AC element (starting from vector potential expression)?

Ans



An infinitely small element carrying alternate current is reffered to as alternate current element also reffered to as oscillating electric dipole.

Since the elements is infinitely small, the current distribution can be considered to be uniform.

Step 1 Get expression for magnetic vector potential

we have

$$A(r,t) = \frac{\mu}{4\pi}\int \frac{J(r,t)}{r}\,dv$$

$$dA = \frac{\mu}{4\pi}\oint \frac{J(t')}{r}\,dv$$

$$\mu H = \nabla \times A$$

$$H = \frac{1}{4\pi}\int \frac{I\,dl \times \hat{n}}{r^2} = \frac{1}{4\pi}\int \frac{J_s\,da \times \hat{n}}{r^2}$$

$$= \frac{1}{4\pi}\int \frac{J_v\,dv \times \hat{n}}{r^2}$$

$$i(t) = I_0 \cos\omega t$$

$$dA = \frac{\mu}{4\pi}\frac{i(t)\,dl}{r} \Rightarrow dA = \frac{\mu}{4\pi}\frac{i(t)\,dz}{R}$$

$$R = r - z\cos\theta$$

$dA_x = dA_y = 0$

$dA_z \neq 0$

$dA_r = dA_z \cos\theta$

$dA_\theta = -dA_z \sin\theta$

$dA_\phi = 0$

$\mu H = \nabla \times A$

$B = \nabla \times A$

$H = \frac{1}{\mu} \nabla \times A$

$$\nabla \times A = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r a_\theta & r\sin\theta\, a_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & r A_\theta & r\sin\theta\, A_\phi \end{vmatrix}$$

$\partial H_r = \partial H_\theta = 0$

$$\partial H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$\cancel{\nabla \times H} = \quad \mu H = \frac{1}{\mu} \nabla \times A$

$$H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$$H_\phi = \frac{I\,dz \sin\theta}{4\pi}\left[\frac{-\omega \sin\omega t'}{rc} + \frac{\cos\omega t'}{r^2}\right] - (4)$$

$$\left(t' = t - \frac{r}{c}\right) - v$$

$E = ?$

$\nabla \times H = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t}$

H has only component

$$E_r = \frac{2 I dz \cos\theta}{4\pi \epsilon_0}\left[\frac{\cos\omega t'}{r^2 c} + \frac{\sin\omega t'}{\omega r^3}\right] - (5)$$

Ste $E = ?$

$$\nabla \times H = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t}$$

$H_r - H_\theta = 0$, $(H_\phi \neq 0)$

H has only component

$$E_r = \frac{2 i dz \cos\theta}{4\pi\epsilon_0}\left[\frac{\cos\omega t'}{r^2 c} + \frac{\sin\omega t'}{\omega r^3}\right] \quad \text{—(5)}$$

$$E_\theta = \frac{i dz \sin\theta}{4\pi\epsilon_0}\left[\frac{-\omega \sin\omega t'}{r c^2} + \frac{\cos\omega t'}{r^2 c} + \frac{\sin\omega t'}{\omega r^3}\right] \quad \text{—(6)}$$

$(i = I_0 \cos\omega t)$

$i(t) = I_0 \cos\omega t$

$$\frac{dq}{dt} = I_0 \cos\omega t$$

$$q = \frac{I_0 \sin\omega t}{\omega}$$

$$E_r = \frac{2q\, dz \cos\theta}{4\pi\epsilon_0 r^3}$$

$$E_\theta = \frac{q\, dz \sin\theta}{4\pi\epsilon_0 r^3}$$

$P = E \times H$

$$E \times H = \begin{vmatrix} a_r & a_\theta & a_\phi \\ E_r & E_\theta & \cancel{E_\phi} \\ \cancel{H_r} & \cancel{H_\theta} & H_\phi \end{vmatrix}$$

$P_r = E_\theta H_\phi$

$$P_r = \frac{i^2(t)\, dz^2 \sin^2\theta}{16\pi^2 \epsilon}\left[\frac{\sin 2\omega t'}{2\omega r^5} + \frac{\cos 2\omega t'}{r^2 c} - \frac{\omega \sin 2\omega t'}{r^3 c^2}\right]$$

$P_\theta = -E_r H_\phi$

$$= \frac{i^2(t)\, dz^2 \sin 2\theta}{16\pi^2 \epsilon_0}\left[\frac{-\cos 2\omega t'}{r^4 c} - \frac{\sin 2\omega t'}{2\omega r^5} + \frac{\cos 2\omega t'}{2 r^3 c^2}\right] \quad \text{—(8)}$$

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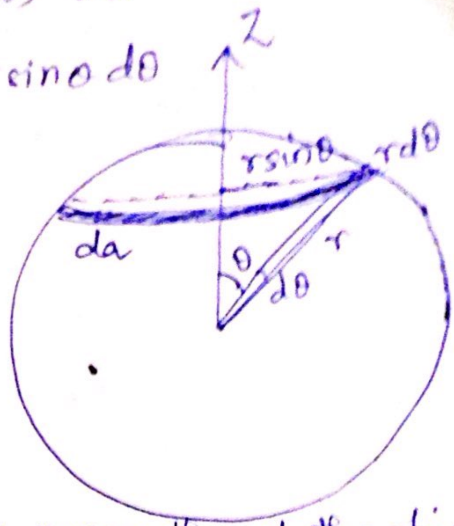
$P_{total} = \int \vec{p} \cdot d\vec{a}$

circular strip; $da = 2\pi (r\sin\theta)\, r\,d\theta$

$= 2\pi r^2 \sin\theta\, d\theta$



$$(P_r)_{avg} = \frac{\omega^2 I_0^2 dz^2 \sin^2\theta}{32\pi^2 \epsilon_0 r^2 c^3}$$

$(P_\theta)_{avg} = 0$

$(P_{total})_{avg} = \oint_{sphere} P_r \cdot da$ power passing through the strip

$$= \int_{\theta=0}^{\pi} \frac{\omega^2 I_0^2 dz^2 \sin^2\theta}{32\pi^2 r^2 c^3 \epsilon_0} \left[2\pi r^2 \sin\theta\, d\theta\right]$$

$$= \frac{\omega^2 I_0^2 dz^2}{12\pi c^3 \epsilon_0}$$

$$I_{eff} = I_{rms} = \frac{I_0}{\sqrt{2}}$$

$\omega = 2\pi f = 2\pi \cancel{c}/\lambda$

$$(P_{total})_{avg} = 4\pi \frac{c^2}{\lambda^2} \frac{I_0^2 dz^2}{12\pi \epsilon_0 c^3}$$

$$P = \frac{\pi I_0^2 dz^2}{3\lambda^2 \epsilon_0 c}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$P_{total} = \frac{\pi I_0^2 dz^2}{3\lambda^2 \epsilon_0 \left(\frac{1}{\sqrt{\mu_0\epsilon_0}}\right)}$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0 = 120\pi$$

$$(P_{total})_{avg} = \frac{\eta \pi I_0^2 dz^2}{3\lambda^2}$$

$$= \frac{2(120)\pi I_{eff}^2 dz^2}{3\lambda^2}$$

$$P_{(total)\,avg.} = 80\pi^2 \left(\frac{dz}{\lambda}\right)^2 I_{eff}^2 \quad \text{—— (10)}$$

$$P = I_{eff}^2 R \quad \text{—— (11)}$$

$\therefore$ compare (10) & (11)

$$\boxed{R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2}$$

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11. (a) $P_{total} = 5KW$

Since the antenna is isotropic, power density at a point r units from it is

$$P = \frac{P_{total}}{4\pi r^2}$$

with $r = 12 Km$, $P = \frac{5\times10^3}{4\pi(12\times10^3)^2}$ watts/m^2

$=$

(b) $P_t = 4KW$

Gain of the antenna $G(dB) = 24$ dB

$\Rightarrow G = 10^{2.4} = 251.19$

Power density at 10Km distance is given by

$$P = \frac{P_t}{4\pi r^2}\cdot G$$

$$= \frac{4\times10^3}{4\pi(10\times10^3)^2}\times 251.19 \text{ W/m}^2$$

13. (a) $R_{loss} = 7\Omega$

$R_{rad} = 90\Omega$

$D(dB) = 17dB$

$\Rightarrow D = 10^{1.7} = 50.12$

$G = KD$ where $K = \frac{R_{rad}}{R_{loss}+R_{rad}}$

$$G = \frac{90}{90+7} \times 50.12 = 46.5$$

$$G\,(dB) = 10\log(46.5) = 16.67\,dB$$

(b) Radiation resistance of short dipoles is given by

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \quad (l < \lambda/10)$$

For $l = \lambda/15$

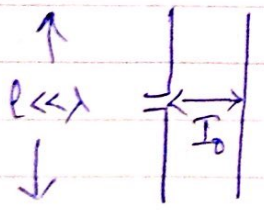
$$R_{rad} = 80\pi^2\left(\frac{\lambda/15}{\lambda}\right)^2 \Omega$$

For $l = \lambda/20$

$$R_{rad} = 80\pi^2\left(\frac{\lambda/20}{\lambda}\right)^2 \Omega$$

14. <u>Directivity of short dipole :</u>

The current distribution along the length of a short dipole is assumed to be nearly constant.



$$i(z) = I_0$$

Its effective length, $l_e = \frac{I_{avg}}{I_{max}} \cdot l_p$

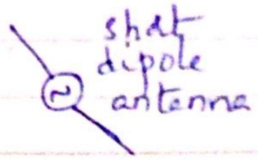
$$= \frac{\cancel{I_0}}{\cancel{I_0}}\, l_p$$

Proceeding the same way as done in solution for Q.(3) ~~we get~~ with $l_e = l_p$, we get $D = 1.5$

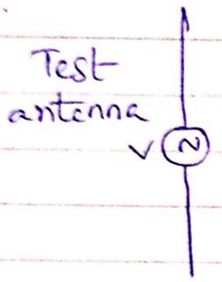
In decibels, D (short dipole) = 1·76 dB

15. Equality of directional patterns (Antenna theorem):

Statement: The directional pattern of a receiving antenna is identical with its directional pattern as a transmitting antenna



Proof:



The directional pattern of a transmitting antenna is the polar characteristic that indicates the strength of the radiated field at a fixed distance in different directions in space.

The directional pattern of a receiving antenna is the polar characteristic that indicates the response of the antenna to unit field strength from different directions

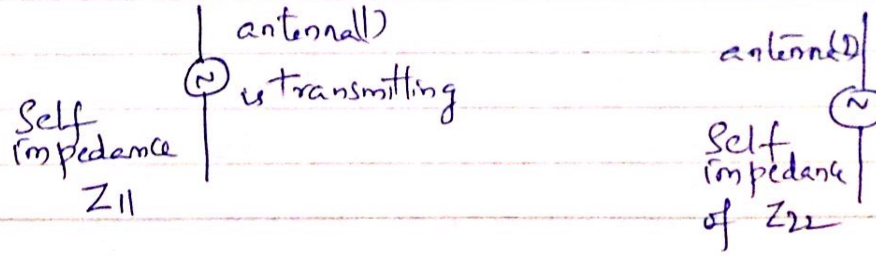
A voltage V applied to the test antenna results in a current, say, I flowing in the short dipole, which will be a measure of the electric field at the position of the dipole. If then the voltage V is applied to the dipole and the test antenna current is measured, the receiving pattern of the test antenne can be obtained.

But by reciprocity theorem, for every location of the dipole, the ratio of V to I is the same as before. Therefore the directional pattern of a receiving antenna is identical with its directional pattern as transmitting antenna.

Equality of transmitting and receiving antennas :-

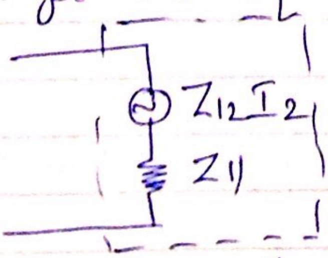
Statement : The impedance of an isolated antenna when used for receiving is the same as when used for transmitting.

Proof :



If antenna(2) is far from antenna (1), ~~the~~ the ~~self~~ impedance of antenna(1) is equal to Z_{11}, its self impedance. Z_{12} may be ignored when antenna (1) is transmitting

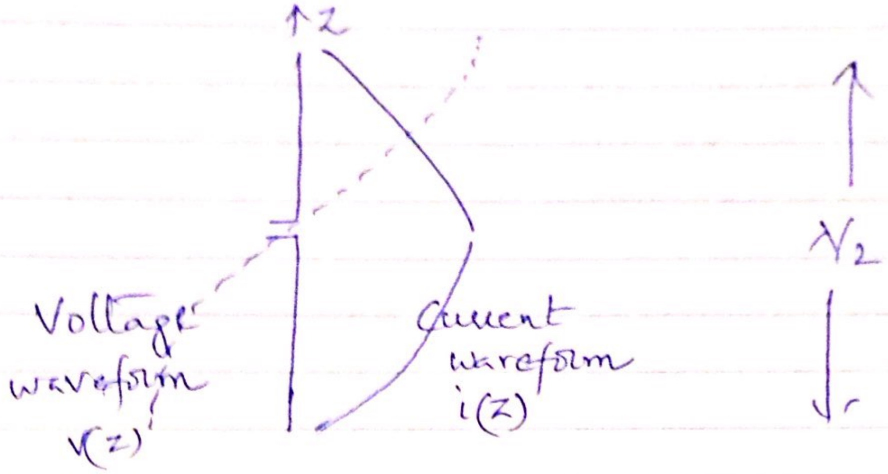
When antenna(1) is used for receiving, Z_{12} is the only coupling between the antennas. For this case, one can get the equivalent circuit,

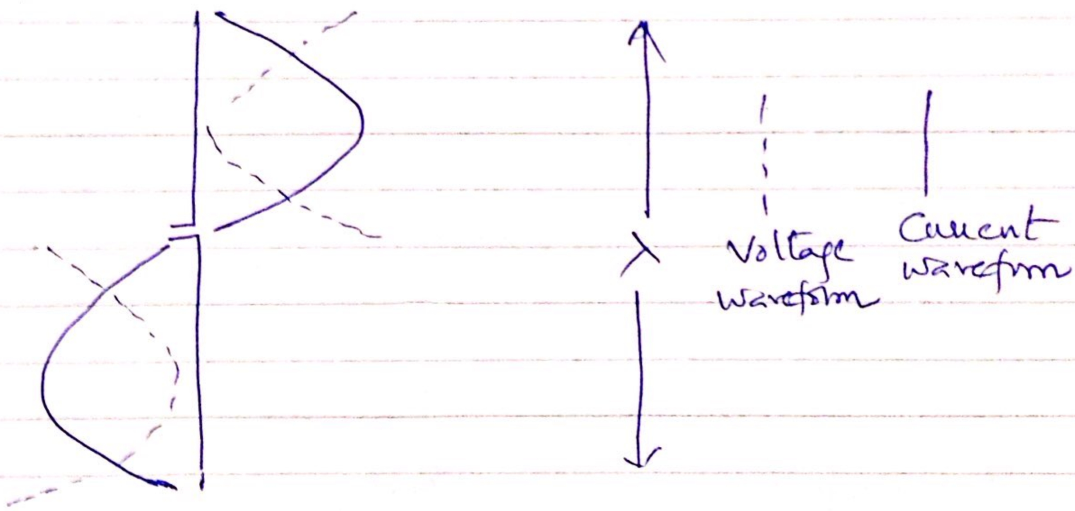


The antenna exhibits terminal behavior of a generator with internal impedance Z_{11} and thus the receiving impedance is the same as before. Hence proved.

16. Draw the current and voltage distributions along the length of $\lambda/2$ and λ units long center-fed dipoles.

Ans.





17. Compare fields from a short dipole and a small loop.

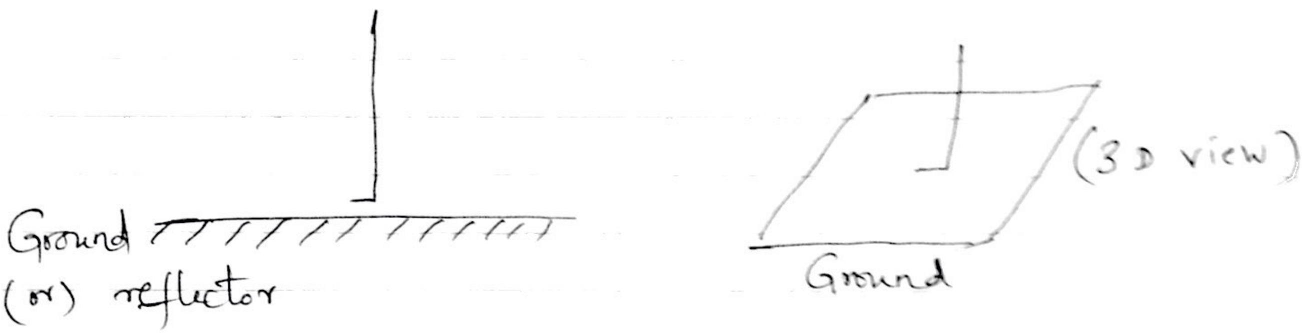
Ans.

The small horizontal loop, shown, antenna may be regarded as the magnetic counterpart of a short vertical dipole. Both have identical field distribution with E and H field expressions interchanged

A horizontal loop and a vertical dipole give vertically polarized field. Similarly, a vertical loop and a horizontally place dipole give horizontal polarization.

Both loop and dipole give same Directivity equal to 1.5

18. Monopole antenna :-



In applications where mounting of antenna onto the surface of working body (eg, a vehicle), monopole is one of the preferred antennas. The body itself can act as ground or reflector.

The single pole is put in place above ground and is excited to get radiation. The antenna is called monopole antenna. Radiation characteristics of this antenne can be obtained by using method of images. Resultant pattern is seen only in the upper hemisphere.

The pattern is identical with the pattern a dipole antenna gives but with the difference, power being radiated only in one hemisphere

$R_{rad} = 36.5\ \Omega$