UNIT-1

① Define probability & Axioms of probability.

Ans probability is defined as the ratio of Number of favourable outcomes to the Total number of outcomes.

$$P(A) = \frac{\text{number of favourable outcomes}}{\text{Total number of outcomes}}$$

Axioms of probability :-

1. $P(A) \geq 0$
2. $P(S) = 1$.

Probability of sample space is equal 1.

* $P(S) = 1 \Rightarrow$ then S is called certain event
* $P(\phi) = 0 \Rightarrow$ where ϕ is the null set then the event is called impossible event.

3. If N events defined as $A_1, A_2, A_3 \dots A_N$ on a sample space (S) then the probability function is given by $P\left(\bigcup_{i=1}^{N} A_i\right) = \sum_{i=1}^{N} P(A_i)$ if $A_m \cap A_n = \phi$

The probability of the event equal to the union of any number of mutually exclusive events is equal to the sum of individual probability.

② Joint probability:-

* The probability $P(A \cap B)$ is called as joint probability for 2 events A and B which intersect in the sample space.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B).$$

$$P(A \cup B) \leq P(A) + P(B).$$

* For mutually exclusive event the joint probability is equal to 0.

Conditional probability:-

* If A and B are events and $P(A) > 0$ then the probability of event B under condition A is $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$.

* The conditional probability satisfy the 3 axioms of probability.

* The 2 events A and B are said to be independent if and only if $P(A \cap B) = P(A) \cdot P(B)$.

Properties of Conditional probability:-

① P(A) is a probability measure overs condition on A $\Rightarrow P(A/A) = 1$.

$$P(S/A) = \frac{P(S \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1.$$

② If $P(A \cap B) = 0$, $A \cap B = \phi$ then $P\left(\frac{B}{A}\right) = 0$

③ $P\left((B \cap C)/A\right) = P\left(\frac{B}{(A \cap C)}\right) \cdot P(C/A)$.

if $P(A) > 0$ & $P(A \cap C) > 0$.

③ Total probability :-

Probability $p(A)$ of an event A is defined on a sample space 'S' can be expressed interms of conditional probabilities i.e,

$$p(A) = \sum_{n=1}^{N} p(A/B_n)\, p(B_n)$$

Baye's theorem :-

* The definition of conditional probability can be applied to any 2 events,

* Let B_n be any one of the event in the intersection on Total probability.

$$p(B_n/A) = \frac{p(B_n \cap A)}{p(A)} \quad p(A) > 0 \rightarrow ①$$

$$p(A/B_n) = \frac{p(A \cap B_n)}{p(B_n)} \quad p(B_n) > 0 \rightarrow ②$$

* One of the form of Baye's theorem is obtained by equating ① & ②.

$$p(B_n/A) = \frac{p(A/B_n)\, p(B_n)}{p(A)}$$

$$= \frac{p(A/B_n)\, p(B_n)}{\sum_{n=1}^{N} p(A/B_n)\, p(B_n)}$$

④ Total number of balls = 500

$n(s) = 500.$

Event for getting black colour ball $n(e) = 75$

" " " green " " $n(e) = 150$

" " " red " " $n(e) = 175$

" " " white " " $n(e) = 70$

" " " blue " " $n(e) = 30$

Probability for ~~selecting~~ getting black colour ball $P(b) = \frac{n(e)}{n(s)} = \frac{75}{500}$

" " " green " " $P(g) = \frac{n(e)}{n(s)} = \frac{150}{500}$

" " " red " " $P(r) = \frac{n(e)}{n(s)} = \frac{175}{500}$

" " " white " " $P(w) = \frac{n(e)}{n(s)} = \frac{70}{500}$

" " " blue " " $P(b) = \frac{n(e)}{n(s)} = \frac{30}{500}$

⑤ Total number of cards = 52

$n(s) = 52.$

a) Event for getting Jack card $\Rightarrow n(e) = 4$

probability that the card is Jack $P(J) = \frac{n(e)}{n(s)} = \frac{4}{52}.$

b). Event for getting the card 5 or smaller $n(e) = 16$ $[2, 3, 4, 5]$.

Probability that the card is 5 or smaller

$P(A) = \frac{n(e)}{n(s)} = \frac{16}{52} = \frac{4}{13}$

c) Event for getting the card is red 10

$n(e) = \underline{2}$

Probability that the card is red 10

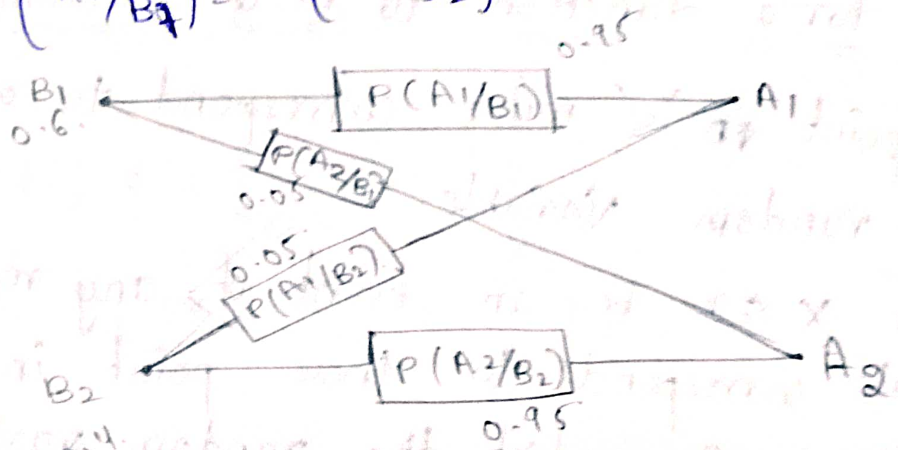
$$P(r) = \frac{n(e)}{n(s)} = \frac{2}{52} = \frac{1}{26}$$

⑥ given data,

$P(B_1) = 0.6$, $P(B_2) = 0.4$

$$P\left(\frac{A1}{B_1}\right) = P\left(\frac{A2}{B2}\right) = 0.95$$

$$P\left(A2/B_1\right) = P\left(A_1/B_2\right) = 0.05$$



From total probability,

$$P(A) = \sum_{n=1}^{N} P\left(A/B_n\right) P(B_n)$$

$$P(A_1) = P\left(A1/B_1\right) P(B_1) + P\left(A1/B_2\right) P(B_2)$$

$$= 0.95 \times 0.6 + 0.05 \times 0.4$$

$$P(A_1) = 0.59$$

$$P(A_2) = P\left(A2/B_1\right) P(B_1) + P\left(A2/B_2\right) P(B_2)$$

$$= 0.05 \times 0.6 + 0.95 \times 0.4$$

$$P(A_2) = 0.41$$

⑦ Random Variable :-

→ It is defined as a real function of element of sample space s.

→ random Variable is represented by capital letters & Elements of random variable by small letters.

→ An given experiment defined by a sample space SE with elements s, we assign to every s a real number according to some rule (or) function is random variable.

Conditions for a function to be a random Variable

① Every point in 's' must correspond to only one value of random Variable.

② The set $X \leq x$ be an event of any real number x the set corresponds to those points in the sample space for which the random variables X (s) doesnot exceed the number X.

③ The random variable must be within the limits of '$-\infty$ to $+\infty$' → $P(x = -\infty) = 0$

$P(x = +\infty) = 0$

(8) The probability density function is the derivative of probability distribution function.

i.e. $f_X(x) = \frac{d}{dx} F_X(x)$.

→ for discrete random variable,

$$f_X(x) = P(X = x).$$

Properties of probability density function:

1. $f_X(x) \geq 0$ for all x.
2. $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$.
3. $F_X(x) = \int_{-\infty}^{x} f_X(\xi)\,d\xi$
4. $P[x_1 < X < x_2] = \int_{x_1}^{x_2} f_X(x)\,dx$.

(9) given,

$$f_X(x) = k(1-x^2) \quad 0 < x \leq 1$$
$$= 0 \quad \text{else where.}$$

given that the function $f_X(x)$ is a valid pdf. so,

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1.$$

$$\int_0^1 k(1-x^2)\,dx = 1.$$

$$k\int_0^1 (1-x^2)\,dx = 1.$$

$$k\left[x - \frac{x^3}{3}\right]_0^1 = 1.$$

$$K\left[1-\frac{1}{3}\right]=1$$

$$K\left[\frac{2}{3}\right]=1$$

$$\boxed{K=\frac{3}{2}}$$

$\therefore$ value of K is $\frac{3}{2}$.

(10) Gaussian density function:-

* A random variable is said to be Gaussian random variable if it has density function in the form

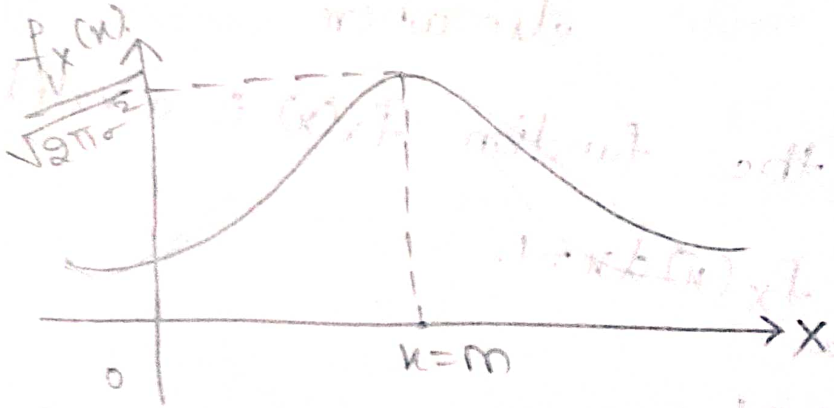
$$f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-m)^2}{2\sigma^2}}$$

where,

m = mean of the random variable.,

σ^2 = variance.

* graphical representation of gaussian density function is shown :-



* It is bell shaped curve.

* when $x=m$, the gaussian density will be maximum and it is equal to

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(0-m)^2}{2\sigma^2}}$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}$$

* for $m=0$, $\sigma^2=1$.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2}.$$

which is called as standardized gaussian density function.

Properties of guassian density function:-

① The guassian density function is also called normal density function.

② The fourier transform of guassian density is guassian density only.

③ At $x=m$, it is having the maximum probability which is equal to $\frac{1}{\sqrt{2\pi\sigma^2}}$

④ The curve obtained is like bell shaped curve and curve never goes below x-axis.

Poisson density function:-

* The poisson density function is given by

$$f_X(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k).$$

where $b>0$ is a real number.

~~$b = \lambda T$.~~

application:-

① In a wide variety of counting application.

② It describes the number of defective units in sample taken from a production.

(11) $a+3a+5a+7a+9a+11a+13a+15a+17a=1$

$81a=1$

$\boxed{a=\frac{1}{81}}$

$P(x \geq 3) = P(x=3)+P(x=4)+P(x=5)+P(x=6)+P(x=7)+P(x=8)$

$= \frac{7}{81}+\frac{9}{81}+\frac{11}{81}+\frac{13}{81}+\frac{15}{81}+\frac{17}{81}$

$= \frac{72}{81} = \frac{8}{9}.$

(12) Conditional density function:-

→ Conditional density function of the random variable X is the derivative of the conditional distibution function.

→ It is denoted by $f_X(x/B)$.

→ $$f_X(x/B) = \frac{dF_X(x/B)}{dx}$$

Properties of conditional density function:-

1. $f_X(x/B) \geq 0$
2. $\int_{-\infty}^{\infty} f_X(x/B)\, dx = 1$
3. $F_X(x/B) = \int_{-\infty}^{x} f_X(\xi/B)\, d\xi$
4. $P\{x_1 < X \leq x_2/B\} = \int_{x_1}^{x_2} f_X(x/B)\, dx$.