Karnaugh map

Gray code

0	0	0	0	$\rightarrow$	0
0	0	0	1	$\rightarrow$	1
0	0	1	1	$\rightarrow$	3
0	0	1	0	$\rightarrow$	2
0	1	1	0		6
0	1	1	1		7
0	1	0	1		5
0	1	0	0		4
1	1	0	0		12
1	1	0	1		13
1	1	1	1		15
1	1	1	0		14
1	0	1	0		10
1	0	1	1		11
1	0	0	1		9
1	0	0	0		8

2 Variable K-map

2 Variables $\Rightarrow 2^2$ minterms

$\Rightarrow 2^2$ maxterms

$$f(A,B) = \Sigma m(0,1,2,3)$$

$$f(A,B) = \Pi M(0,1,2,3)$$

A \ B	0	1
0	00 m_0	01 m_1
1	10 m_2	11 m_3

A	B	minterms
0	0	$\bar{A}\,\bar{B} = m_1$
0	1	$\bar{A}\,B = m_2$
1	0	$A\,\bar{B} = m_3$
1	1	$A\,B = m_4$

A \ B	0	1
0	00 M_0	01 M_1
1	10 M_2	11 M_3

A	B	maxterms
0	0	$A + B = M_0$
0	1	$A + \overline{B} = M_1$
1	0	$\overline{A} + B = M_2$
1	1	$\overline{A} + \overline{B} = M_3$

Simplify given boolean expression using K-map

$f(A, B) = \Sigma m(0, 2, 3)$

A \ B	0	1
0	$_0$ 1	$_1$
1	$_2$ 1	$_3$ 1

$\overline{B}$

A

Grouping $2^0, 2^1, 2^2, 2^3$, $2^4, 2^5, 2^n$

2, 4, 8, 16, 32 minterms

$f(A, B) = A + \overline{B}$

② $f(A, B) = \Sigma m(1, 2, 3)$

A \ B	0	1
0	$_0$	$_1$ 1
1	$_2$ 1	$_3$ 1

B

A

$A + B$

③ $f(A, B) = \pi M(1, 2, 3)$ determine, simplified form of boolean expression using k-map

A \ B	0	1
0		0
1	0	0

$\rightarrow \bar{B}$

$\rightarrow \bar{A}$

$\bar{A}\,\bar{B}$ ✓

③ 3- Variable k- map

$f(A, B, C) = \Sigma m(0, 1, 2, 3, 4, 5, 6, 7, \ )$

3 Variable map $= 2^3$ cells

A \ BC	00	01	11	10
0	0 $\bar{A}\bar{B}\bar{C}$	1 $\bar{A}\bar{B}C$	3 $\bar{A}BC$	2 $\bar{A}B\bar{C}$
1	4 $A\bar{B}\bar{C}$	5 $A\bar{B}C$	7 ABC	6 $AB\bar{C}$

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

$f(A, B, C) = \pi M(0, 1, 2, 3, 4, 5, 6, 7)$

A \ BC	00	01	11	10
0	0: $A+B+C$	1: $A+B+\bar{C}$	3: $A+\bar{B}+\bar{C}$	2: $A+\bar{B}+C$
1	4: $\bar{A}+B+C$	5: $\bar{A}+B+\bar{C}$	7: $\bar{A}+\bar{B}+\bar{C}$	6: $\bar{A}+\bar{B}+C$

maxterms

Given maxterms are represented by zero in the corresponding cell

Simplify given boolean expression using K-map

$$f(A,B,C) = \Sigma m(0,2,3,6,7)$$

A \ BC	00	01	11	10
0	0: 1	1:	3: 1	2: 1
1	4:	5:	7: 1	6: 1

$\bar{A}\bar{C}$

B

$$B + \bar{A}\bar{C}$$

$$f(A,B,C) = \Pi M(0,2,3,6,7)$$

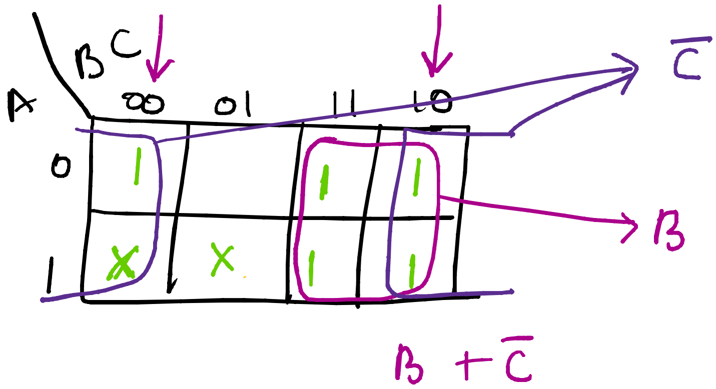
A \ BC	00	01	11	10
0	0		0	0
1			0	0

$(A+C)$

$\bar{B}$

$$(\bar{B})(A+C)$$

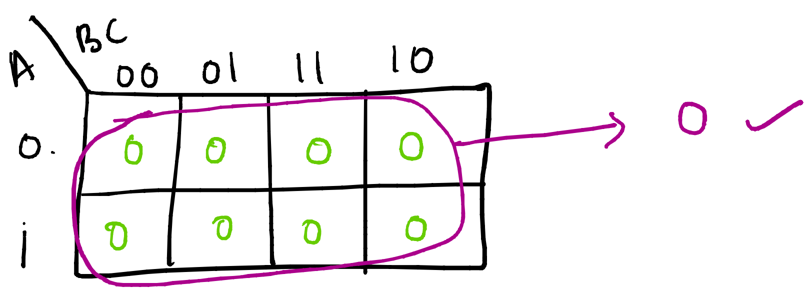
$$f(A,B,C) = \Sigma m(0,2,3,6,7) + \Sigma d(4,5)$$



2^2 minterms

2 literals will be reduced, we could represent expression in terms of single

$f(A,B,C) = \Pi M(0,1,2,3,4,5,6,7)$

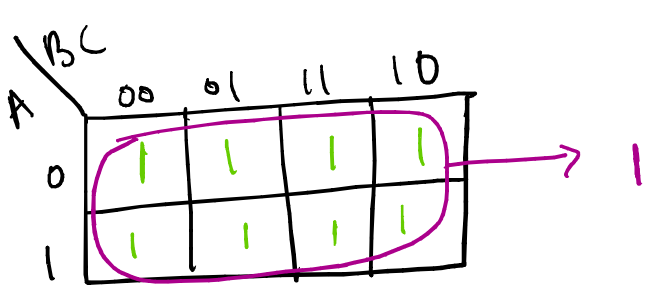


3 Variables → 8 max as group

2^3 → literals

maxterm as a group ↓ reduced

$f(A,B,C) = \Sigma m(0,1,2,3,4,5,6,7)$



2^3 → reduced

terms a group

4-Variable K-map $\Rightarrow 2^4$ cells

4 - Variable K-map $\Rightarrow 2^4$ cells

$$f(A,B,C,D) = \sum m(0,2,4,6,8,10,12,14)$$

AB \ CD	00	01	11	10
00	0: 1	1	3	2: 1
01	4: 1	5	7	6: 1
11	12: 1	13	15	14: 1
10	8: 1	9	11	10: 1

$\overline{D}$

8 minterms (2^{3}) → Variable reduced

$$f(A,B,C,D) = \sum m(0,1,2,3,8,9,10,11,13,15) + \sum d(5,7)$$

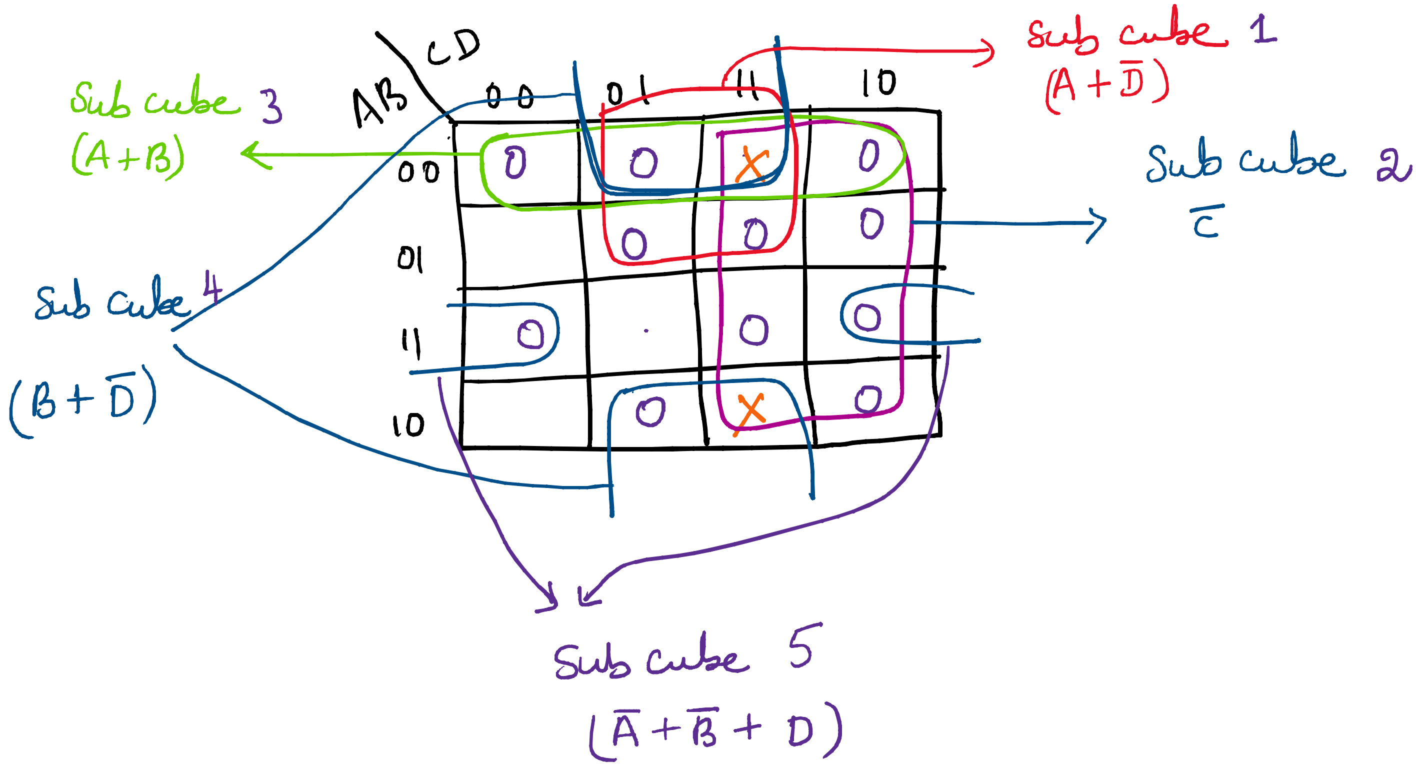
AB \ CD	00	01	11	10
00	1	1	1	1
01		X	X	
11		1	1	
10	1	1	1	1

$\overline{B}$

D

$\overline{B} + D$ ✓

$$f(A,B,C,D) = \prod M(0,1,2,5,6,7,9,10,12,14,15) + \prod d(3,11)$$

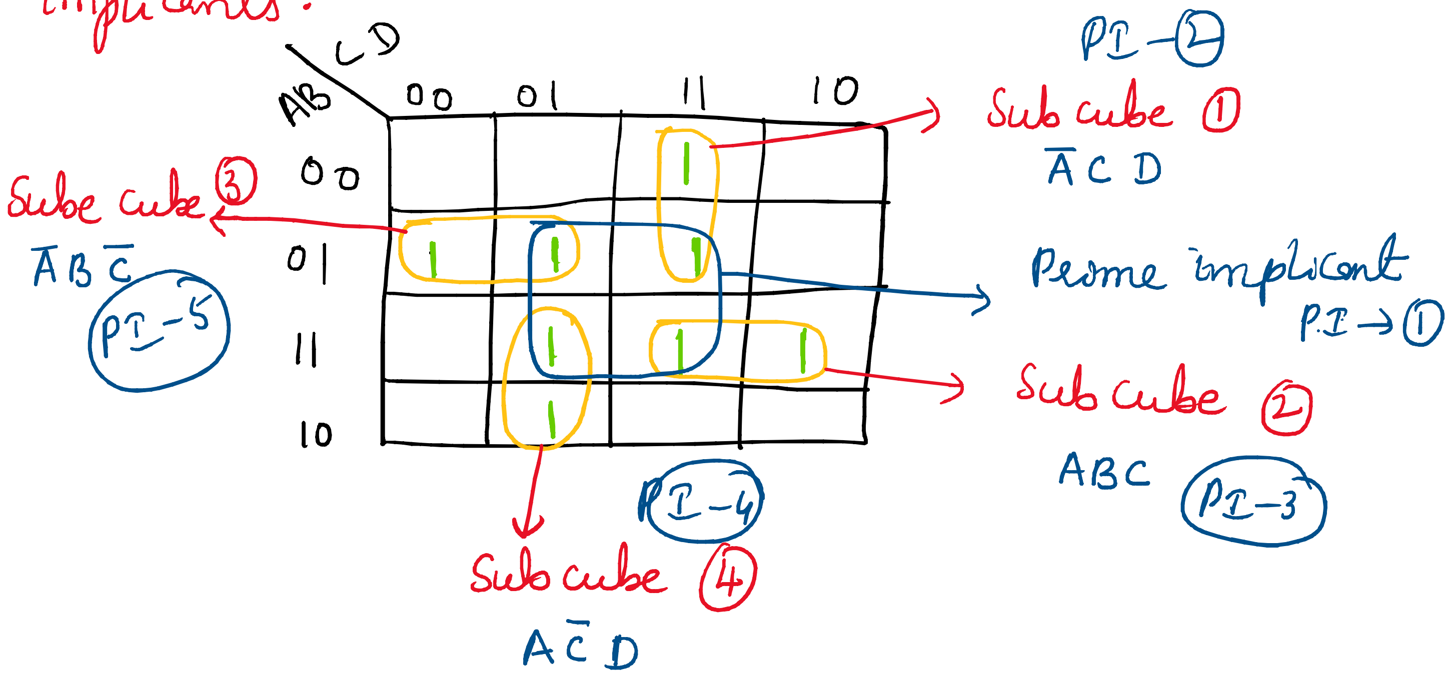


$$(A+\bar{D})\,(\bar{C})\,(A+B)\,(B+\bar{D})\,(\bar{A}+\bar{B}+D)$$

① If subcube has 2^m cells → Elimination of m variables

② In a function of 'n' variables, consider a positive subcube of 2^m cells, is eliminating 'm' variables so that product is represented with $(n-m)$ no. of literals.

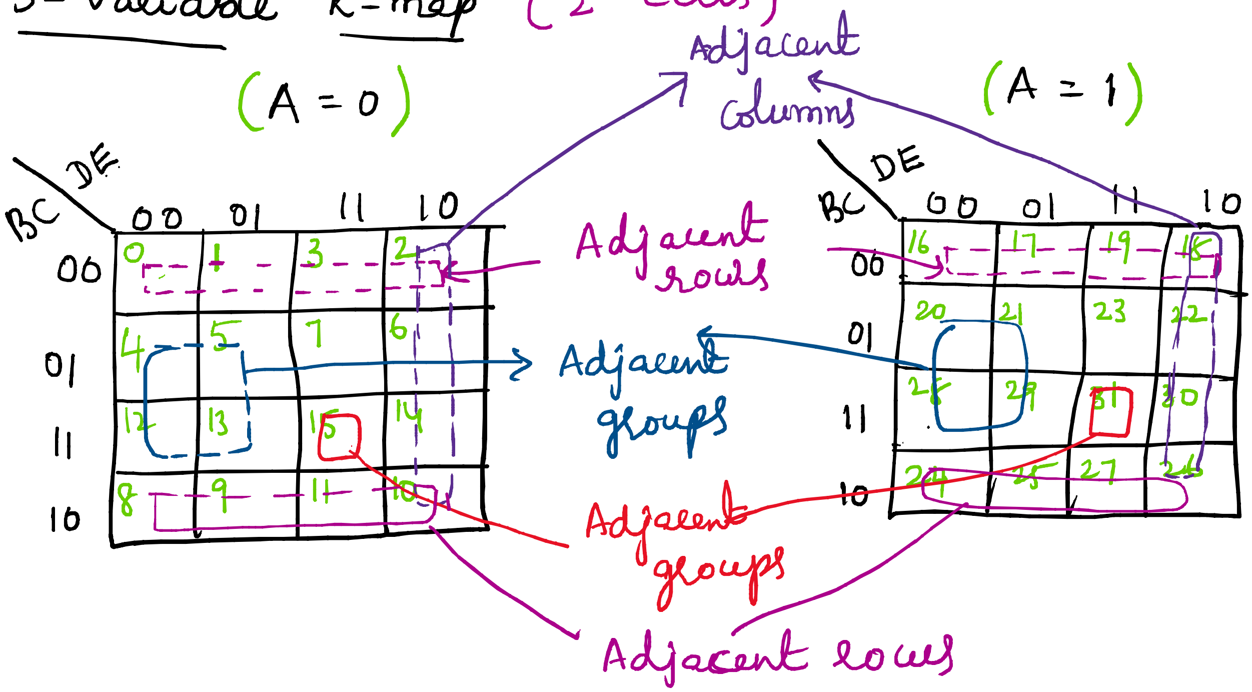
$f(A,B,C,D) = \Sigma m(3,4,5,7,9,13,14,15)$, determine prime implicants and essential prime implicants.



Essential prime implicants → $\bar{A}CD$, ABC, $\bar{A}B\bar{C}$ & $A\bar{C}D$

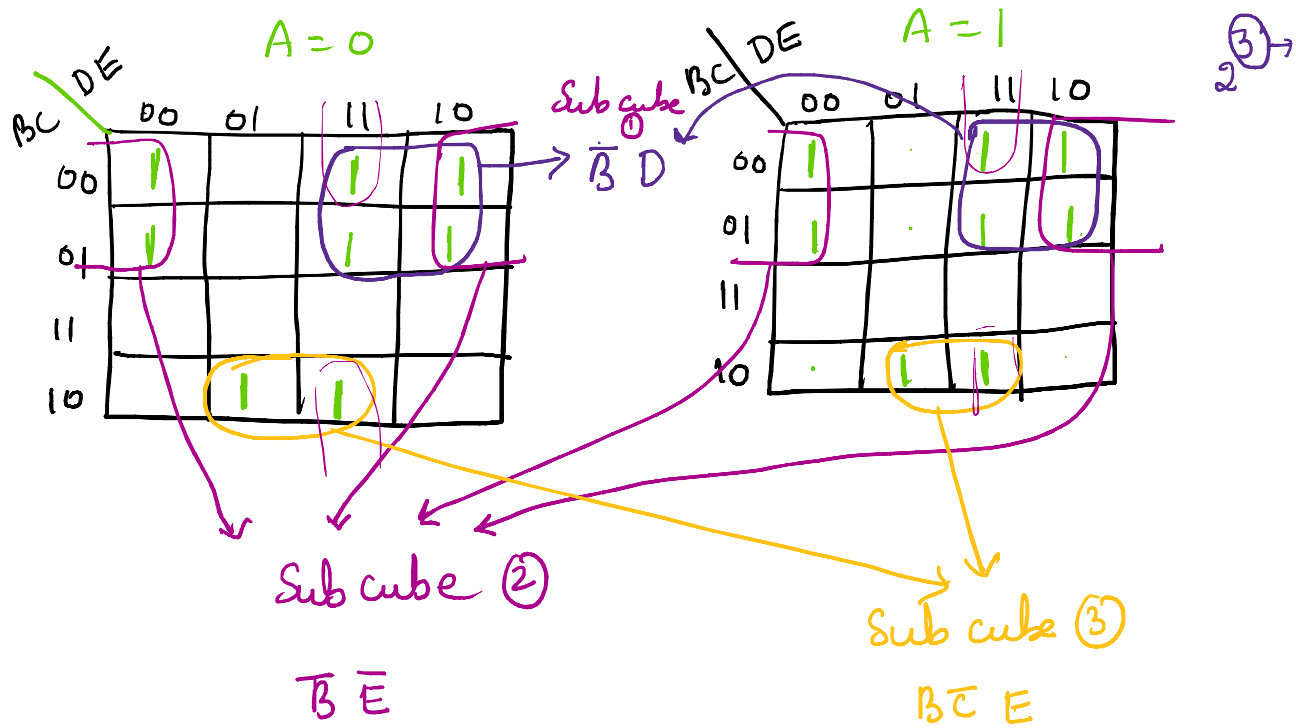
Prime implicants → BD, $\bar{A}CD$, ABC, $\bar{A}B\bar{C}$ & $A\bar{C}D$ ✓

5-Variable K-map (2^5 cells)



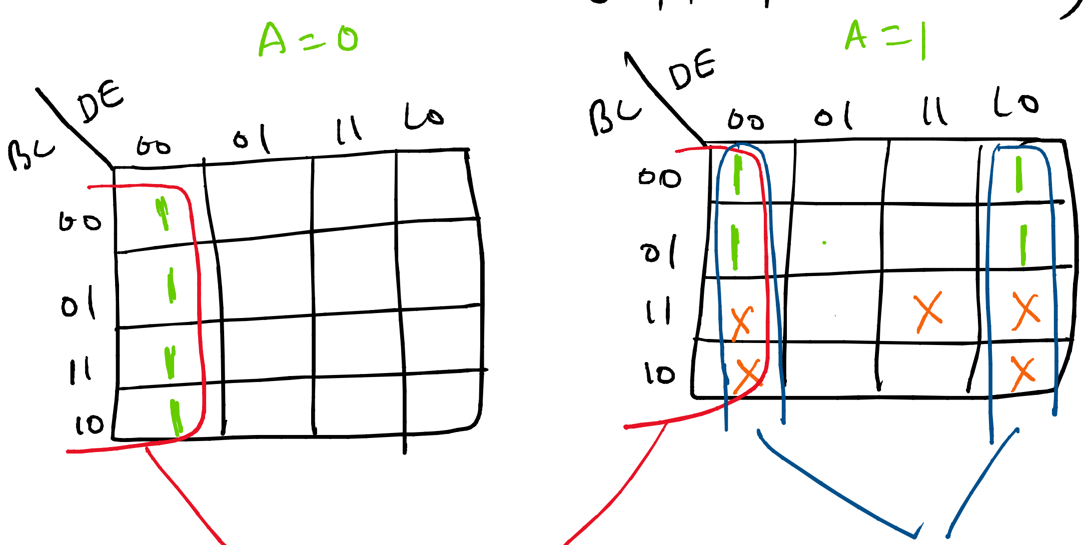
Simplify following Boolean expression using k-map

$f(A, B, C, D, E) = \Sigma m(0, 2, 3, 4, 6, 7, 9, 11, 16, 18, 19, 20, 22, 23, 25, 27)$



$\bar{B}D + \bar{B}\bar{E} + B\bar{C}E$ → Essential prime

$$f(A,B,C,D,E) = \Sigma m(0,4,8,12,16,18,20,22) + \Sigma d(24,26,28,30,31)$$



Sub cube ①

$\bar{D}\,\bar{E}$

Sub cube ②

$A\bar{E}$

$$\bar{D}\bar{E} + A\bar{E}$$ ✓

$$f(A,B,C,D,E) = \Sigma m(0,4,7,8,9,10,11,16,24,25,26,27,29,31)$$