Network Analysis and transmission Lines

~~Network~~ — 3 units — 2 units

- Interconnection of circuit elements (active or passive)

N.A - To find out voltage across any element in circuit.

ex: ohm's, KVL, KCL, Mesh, nodal

Network Topology: An element in N.A becomes branch in N.T — geometry, graph.

Node will be Node only.

- for Complicated Networks
- doesn't distinguish basic elements - node, branch, loop (x not clear)
- In N.T. only geometrical structure is considered.

Elements: 3 or more elements — principal node.

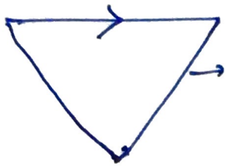
- **Node:** part at which 2 or more elements have common connection
- **Branch:** line connectiong a pair of nodes, The line representing a single element and series of connected elements.
- **loop:** When there is more than 1 (one) path b/w 2 nodes there will be circuit or loop.
- **Mesh:** single loop / loop which don't have any other loops in it.

Linear graph: Collection of nodes and branches. The nodes are joined together by branches.

Graphs:

1) oriented graph: If every branch of a graph has a direction or each element in the connected graph

oriented – directions given / directed graph
unoriented - no directions given / undirected



→ un.O

2) planar - If graph can be drawn on 2D / planar such that no 2 branches cross each other.

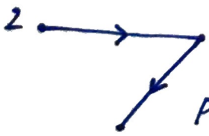
3) Non-planar: graph on 2D such that 2 or more branches intersect at some point other than a node.

 → planar because it can be drawn as 

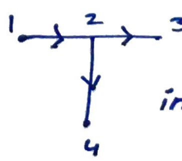
even if you drag it there is intersection then non-planar

4) Sub-graph: Sub of branches and nodes.

→ If Sub. G contains B & N in less no then proper S.G.
→ If Sub. G " " " " " & all nodes present - improper s graph. → Branches less nodes same.



proper

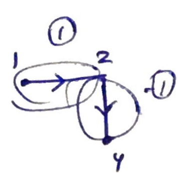


improper

compared to actual graph

5) Path: path is improper S.Graph.

At terminating node , only 1 branch is incident.
at remaining nodes, 2 " " "

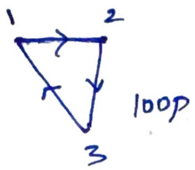


ex: 1, 4 terminating node have only one branch
at 2 has 2 branches 2 connections.

6) **Connected graph:** if there exists a path b/w any pair of nodes.

unconnected: No connection b/w all.

7) loop or circuit: It is a connected subgraph of connected graph at each node of which are incident exact at 2 branches.



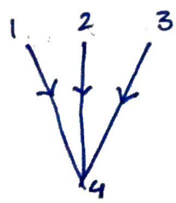
- If 2 terminals of path are made to coincide it results a loop of circuit.

- no.of branches incident on the node = degree of node.
- atleast 2 branches in loop
- There are exactly 2 path b/w any pair of nodes in loop.
- Max no.of possible branches = no.of modes.
- Nodes can be incident to 1 or more elements.
- A node & branch are incident if the node is a terminal of branch.

8) **Tree:** connected s.g of Network has all nodes of Original graph but no closed path.

Improper s.G but ↗

- has all no nodes so there exists only 1 path b/w pair of nodes.



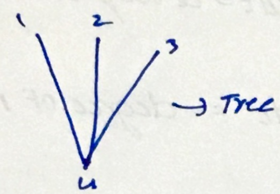
- If n = no.of nodes then they are n-1 branches in tree.
- Tree doesn't contain any loop
- Every C.Graph has atleast 1 tree.

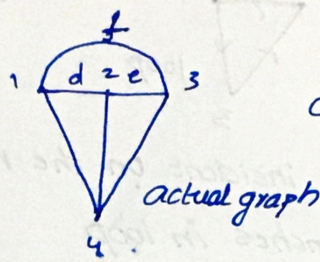
- The min. terminal nodes in a tree are two.

- tree is set of branches with every node connected to every other node in such a way that removal of any branch destroys the property.

9) Branch of tree/ twig : n nodes = n-1 twigs

10) Cotree: The left over branches after forming a tree





def- co-tree

- Branch of cotree - Chord/link
- No.of branches in co tree is $b-(n-1)$ b = no. of branches of actual G
- set of " forming a compliment of tree is co-tree
- If n = no.of nodes then rank is $r = n-1$.

- Incidence Matrix:

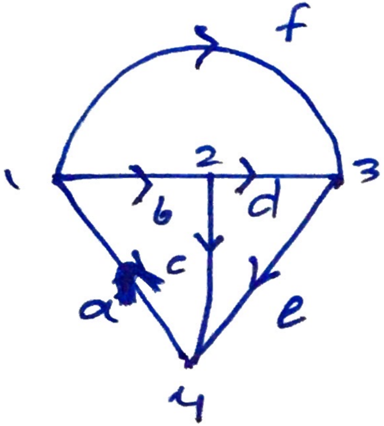
- Oriented graph can be explained.
- gives info abt
  - branches incident on which nodes
  - orientation at the node(towards/away from nodes)

Types:

i) complete incidence (A_a)

ii) Reduced " (A)

- Size of matrix : n/b



+1 → leaving

−1 → entering

$$\begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline 1 & -1 & +1 & 0 & 0 & 0 & -1 \\ 2 & 0 & -1 & +1 & +1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 & +1 & -1 \\ 4 & +1 & 0 & -1 & 0 & -1 & 0 \end{array}$$

no. of nodes forms rows

" branches " colums

NETWORK ANALYSIS AND TRANSMISSION LINES

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ECE - 01
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Unit - 1 Assignment (CO - 1)

1. Define the following terms in Network Topology.

a) **Node** : The point at which two or more elements have a common connection is called a 'node.'

b) **Branch** : The line connecting a pair of nodes by representing a single element or a series of connected elements is called a 'branch.'

c) **Loop** : A 'loop' is a closed path in a circuit where two nodes are not traversed twice except the initial point, which is also the final one.

d) **Mesh** : A 'mesh' is a loop with no other paths inside it (or) A loop with no other loops inside it is called a 'mesh.'

e) **Graph** : If each element or a branch of a network is represented on a diagram by a line irrespective of the characteristics of the elements, we get a 'graph.'

f) **Oriented graph** : If every branch of the graph has a direction or each element in the graph is assigned a direction, then it is called an 'oriented graph.'

g) **Unoriented graph** : If the branches and elements of the graph aren't assigned any direction, then it is called an 'unoriented graph.'

h) Planar graph : A graph is said to be 'planar' if it can be drawn on a plane surface such that no two branches cross each other.

i) Non-planar graph : If a graph drawn on a two-dimensional plane has two or more branches intersecting at a point other than node on a graph, then it is called a 'non-planar' graph.

j) Connected graph : A graph is said to be 'connected' if there exists a path between any pair of nodes.

k) Disconnected graph : If any pair of nodes has no path between them, then it is called a 'disconnected graph.'

l) Subgraph : The subset of nodes or branches of a graph is called a 'subgraph.'

Proper subgraph : It contains branches and nodes less in number than the original graph.

Improper subgraph : It contains all the nodes but branches are less in number compared to the original graph.

m) Tree : A 'tree' is a connected subgraph of a network which consists of all the nodes of the original graph but no closed paths.

Properties of a tree :

i) It contains all the nodes of the original graph.

ii) It does not contain any closed loops.

iii) It has exactly one and only one path between any two nodes.

iv) The tree has exactly $(n-1)$ branches where 'n' is the number of nodes.

v) The minimum terminal nodes in a tree are two.

n) Co-tree : A set of branches forming the compliment of a tree is called a 'co-tree'. In other words, the left over branches in the original graph after it forms a tree is called 'co-tree'.

o) Rank of graph : The 'rank' of a connected graph is defined as $(n-1)$ where, n is the number of nodes in the graph.

p) Degree of node : The no. of branches connected to a node is called 'degree of node'.

q) Complete incidence matrix : It gives us information about all the branches incident on the nodes and their orientations at the each node, whether towards or away from the node. A complete incidence matrix is a rectangular matrix A_a with 'n' rows and 'b' columns where, n is no. of nodes and b is no. of branches. $(i,j)^{th}$ element of A_a is

$$\begin{cases} +1 & \text{if branch 'j' is incident at node 'i' and is directed away from it} \\ -1 & \text{if branch 'j' is incident at node 'i' and is directed towards it} \\ 0 & \text{if branch 'j' is not incident at node 'i'} \end{cases}$$

→ Properties of complete incidence matrix :

i) Sum of entries in any column is zero.

ii) Rank of the matrix is $(n-1)$.

iii) Determinent of any loop is always zero.

r) Reduced incidence matrix : When any one row is eliminated from complete incidence matrix by mathematical manupulation, then it is called 'reduced incidence matrix', denoted by A with $(n-1)$ rows and 'b' ~~b.~~ colums.

The no. of possible trees in a linear graph is $|AA^T|$.

s) Loop matrix : For a given directed graph, it is always possible to state which branches are involved in the formation of loops / circuits. A loop matrix is represented by B_a. For a graph with 'n' nodes and 'b' branches, loop matrix B_a is a rectangular matrix with 'b' columns and as many rows as the loops.

If branch 'k' is in loop 'h' and orientation of the loop coincides then, $b_{hk} = +1$, if opposite then $b_{hk} = -1$ and 0 if branch 'k' is not in loop 'h'.

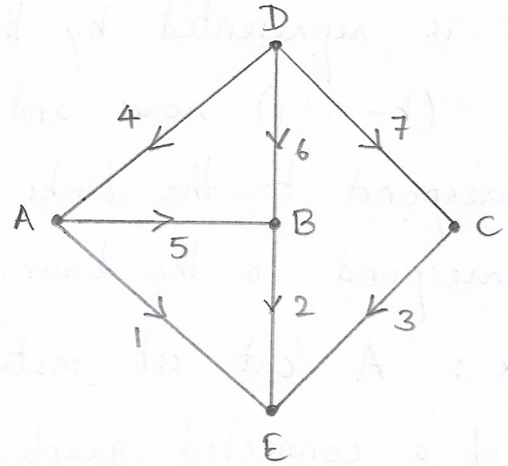
t) **Basic tie-set matrix**: In a basic tie-set matrix, the no. of fundamental loops are equal to the no. of links. It is represented by the letter 'B' and consists of $(b-n+1)$ rows and 'b' columns where, rows correspond to the links of co-tree and columns correspond to the branches.

u) **Cut-set matrix**: A cut-set matrix is a minimal set of branches of a connected graph such that the removal of these branches causes the graph to be cut into exactly two parts.

v) **Basic cut-set matrix**: A basic cut-set matrix consists of exactly one and only one branch of the network tree, together with any links which must be cut to divide the network into two parts. It is represented by the letter 'Q' and consists of $(n-1)$ rows and b columns. The conventional direction for the fundamental cut-set is taken to be same as the direction of the tree branch which defines the particular cutset.

② a) Write complete incidence matrix and reduced incidence matrix for the given graphs below. Also write KCL equations and Branch voltages interms of node voltages. Find the no. of possible trees.

(i)



Sol. → Complete Incidence Matrix :

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
A_a = \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} +1 & 0 & 0 & -1 & +1 & 0 & 0 \\ 0 & +1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & +1 & 0 & +1 & +1 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}_{5 \times 7}
\end{array}
$$

→ Reduced Incidence Matrix :

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
A = \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} +1 & 0 & 0 & -1 & +1 & 0 & 0 \\ 0 & +1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & +1 & 0 & +1 & +1 \end{bmatrix}_{4 \times 7}
\end{array}
$$

→ KCL equations :

$$A\, I_b = 0 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}_{4\times 7} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}_{7\times 1} = 0$$

$$\Rightarrow \left.\begin{aligned} i_1 - i_4 + i_5 &= 0 \\ i_2 - i_5 - i_6 &= 0 \\ i_3 - i_7 &= 0 \\ i_4 + i_6 + i_7 &= 0 \end{aligned}\right\} \text{KCL eq.'s}$$

→ Branch Voltages in terms of node voltages :

$$V_b = A^T V_n \Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}_{7\times 4} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix}_{4\times 1}$$

$$\Rightarrow \left.\begin{aligned} V_1 &= V_A \\ V_2 &= V_B \\ V_3 &= V_C \\ V_4 &= -V_A + V_D \\ V_5 &= V_A - V_B \\ V_6 &= -V_B + V_D \\ V_7 &= -V_C + V_D \end{aligned}\right\} \text{Branch voltages}$$

→ No. of possible trees $= |A \cdot A^T|$

$$\therefore A \cdot A^T = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}_{4\times7} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & +1 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}_{7\times4}$$

$$= \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}_{4\times4}$$

$$\therefore |A \cdot A^T| = \begin{vmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{vmatrix}$$

$$= (+1)(0) + (-1)(0) + (1)(2) \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} +$$

$$(-1)(-1) \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ 0 & 0 & -1 \end{vmatrix}$$

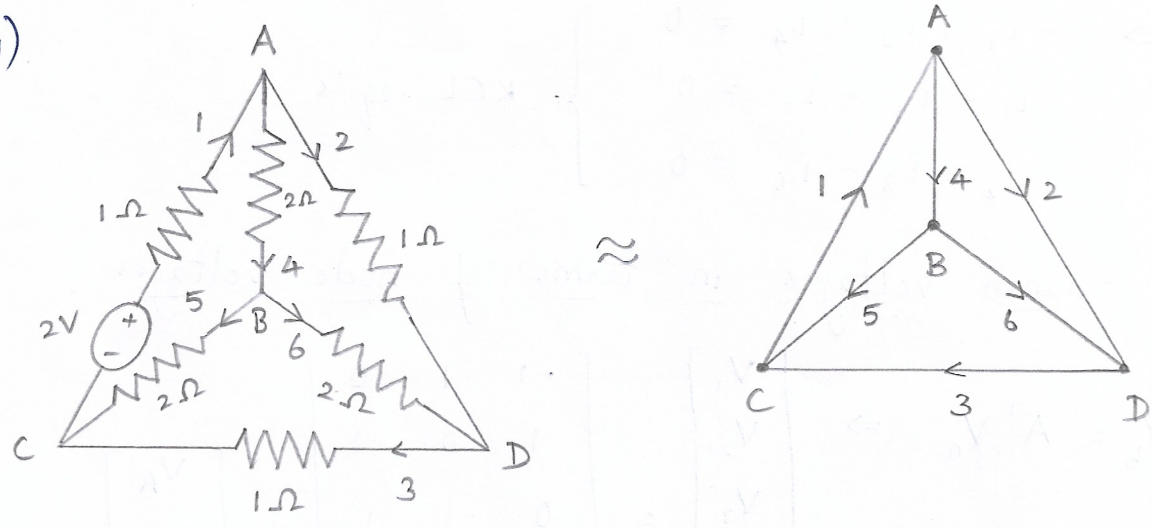
$$= 2[24 + 4 - 4] + 1[-9 + 1 - 0]$$

$$= 32 - 8$$

$$= 24$$

$\therefore$ No. of possible trees $= 24$

(ii)



→ Complete Incidence Matrix :

$$A_a = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & +1 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & -1 & +1 & +1 \\ +1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & +1 & 0 & 0 & -1 \end{bmatrix}_{4\times 6} \end{array}$$

→ Reduced Incidence Matrix :

Obtaining RIM by removing the row B.

$$A = \begin{array}{c} \\ A \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & +1 & 0 & +1 & 0 & 0 \\ +1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & +1 & 0 & 0 & -1 \end{bmatrix}_{3\times 6} \end{array}$$

→ KCL equations :

$$A I_b = 0 \Rightarrow \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \end{bmatrix}_{3\times 6} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{aligned} -i_1 + i_2 + i_4 &= 0 \\ i_1 - i_3 - i_5 &= 0 \\ -i_2 + i_3 - i_6 &= 0 \end{aligned}\right\} \text{KCL eq.'s}$$

→ Branch voltages in terms of node voltages :

$$V_b = A^T V_n \Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}_{6\times 3} \begin{bmatrix} V_A \\ V_C \\ V_D \end{bmatrix}_{3\times 1}$$

$$\Rightarrow \left.\begin{aligned} V_1 &= -V_A + V_C \; ; & V_4 &= V_A \\ V_2 &= V_A - V_D \; ; & V_5 &= -V_C \\ V_3 &= -V_C + V_D \; ; & V_6 &= -V_D \end{aligned}\right\} \text{Branch voltages}$$

→ No. of possible trees $= |A \cdot A^T|$

$$\therefore A \cdot A^T = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \end{bmatrix}_{3\times 6} \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}_{6\times 3}$$

$$= \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}_{3\times 3}$$

$$\therefore |A \cdot A^T| = \begin{vmatrix} \overset{+}{3} & \overset{-}{-1} & \overset{+}{-1} \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$$\Rightarrow 3[8] - (-1)[-4] - 1[4]$$

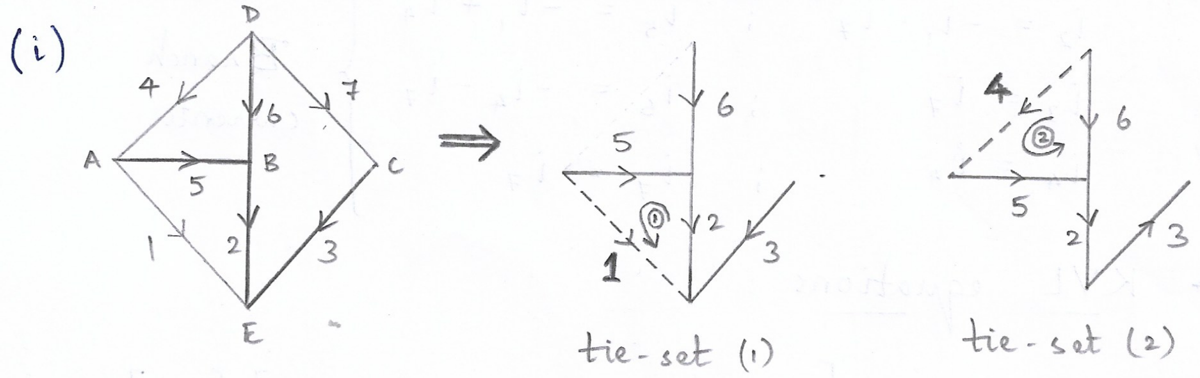
$$\Rightarrow 24 - 4 - 4$$

$$\Rightarrow 16$$

$\therefore$ No. of possible trees = 16

② b) Find the basic tie-set matrix for the graphs below. Also write branch currents in terms of loop currents along with KVL equations.

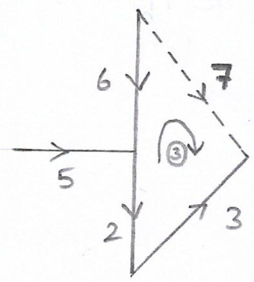
(i)



tie-set (1) tie-set (2)

→ Basic Tie-set matrix

$$B = \begin{array}{c} \\ \text{tieset }(1,2,5) \\ \text{tieset }(4,5,6) \\ \text{tieset }(7,2,3,6) \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} +1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & +1 & +1 & -1 & 0 \\ 0 & -1 & +1 & 0 & 0 & -1 & +1 \end{bmatrix}_{3\times 7} \end{array}$$



tie-set (3)

B_t : B_L

$$\begin{array}{c} \begin{array}{ccccccc} 2 & 3 & 5 & 6 & 1 & 4 & 7 \end{array} \\ \begin{bmatrix} -1 & 0 & -1 & 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & -1 & 0 & +1 & 0 \\ -1 & +1 & 0 & -1 & 0 & 0 & +1 \end{bmatrix} \end{array} \Rightarrow [B_t : U]$$

Unit matrix

→ Branch currents in terms of loop currents :

$$i_b = B^T i_L \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}_{7\times 3} \begin{bmatrix} i_1 \\ i_4 \\ i_7 \end{bmatrix}_{3\times 1}$$

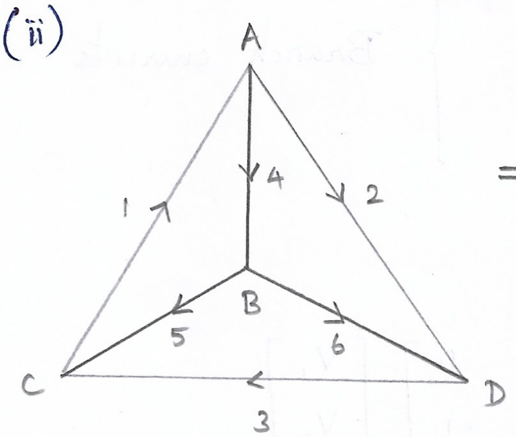
$$\Rightarrow \left.\begin{array}{ll} i_1 = i_1 & \\ i_2 = -i_1 - i_7 \; ; & i_5 = -i_1 + i_4 \\ i_3 = i_7 \; ; & i_6 = -i_4 - i_7 \\ i_4 = i_4 \; ; & i_7 = i_7 \end{array}\right\} \text{Branch currents}$$

→ KVL equations :

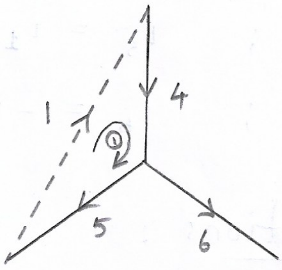
$$BV_b = 0 \Rightarrow \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{array}{l} V_1 - V_2 - V_5 = 0 \\ V_4 + V_5 - V_6 = 0 \\ -V_2 + V_3 - V_6 + V_7 = 0 \end{array}\right\} \text{Branch voltages}$$

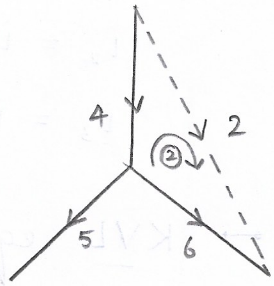
(ii)



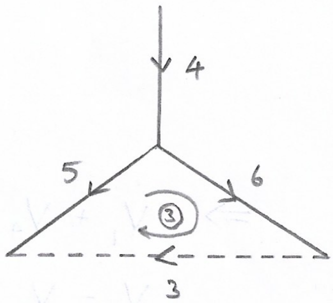
$\Longrightarrow$



tie-set (1)



tieset (2)



tie-set (3)

→ Basic tie-set matrix

$$B = \begin{array}{l} \\ \text{tieset }(1,4,5) \\ \text{tieset }(2,4,6) \\ \text{tieset }(3,5,6) \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & +1 & +1 & 0 \\ 0 & +1 & 0 & -1 & 0 & -1 \\ 0 & 0 & +1 & 0 & -1 & +1 \end{bmatrix} \end{array}$$

$$\begin{array}{c} B_t \qquad : \qquad B_L \\ \begin{array}{cccccc} 4 & 5 & 6 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} +1 & +1 & 0 & +1 & 0 & 0 \\ -1 & 0 & -1 & 0 & +1 & 0 \\ 0 & -1 & +1 & 0 & 0 & +1 \end{bmatrix} \end{array} = [B_t : U]$$

→ Branch currents in terms of loop currents :

$$i_b = B^T i_L \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}_{6\times 3} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_{3\times 1}$$

$$\Rightarrow \quad i_1 = i_1 \; ; \quad i_4 = i_1 - i_2$$
$$i_2 = i_2 \; ; \quad i_5 = i_1 - i_3$$
$$i_3 = i_3 \; ; \quad i_6 = -i_2 + i_3$$
Branch currents

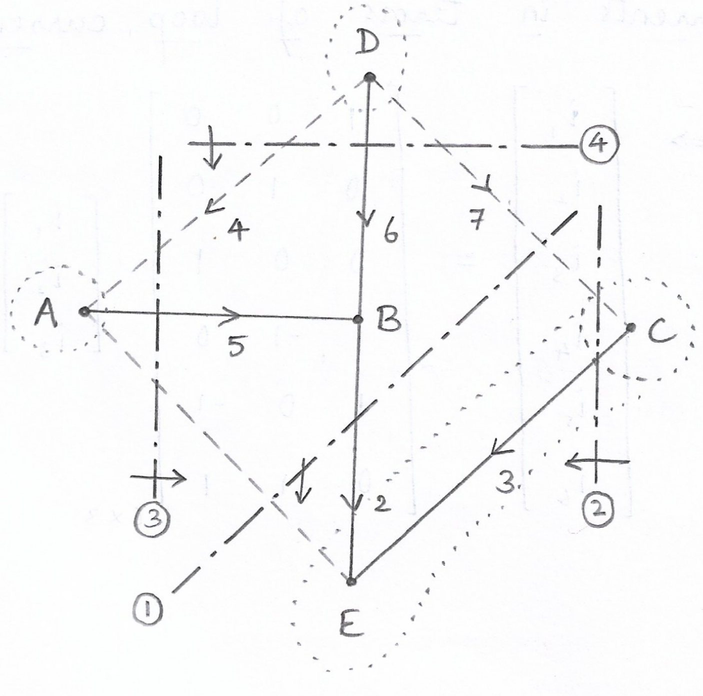
→ KVL equations :

$$BV_b = 0 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{aligned} V_1 + V_4 + V_5 &= 0 \\ V_2 - V_4 - V_6 &= 0 \\ V_3 - V_5 + V_6 &= 0 \end{aligned}\right\} \text{KVL eq's}$$

② c) Write basic cut-set matrix for the graphs below. Also write branch voltages in terms of twig voltages along with KCL equations.

(i)



→ Basic Cut-set matrix :

$$Q = \begin{array}{c} \\ \text{cutset } (2,1,7) \\ \text{cutset } (3,7) \\ \text{cutset } (5,1,4) \\ \text{cutset } (6,4,7) \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} +1 & +1 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 \\ +1 & 0 & 0 & -1 & +1 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & +1 & +1 \end{bmatrix} \end{array}$$

$$Q_L \quad : \quad Q_T$$

$$\begin{array}{ccccccc} 1 & 4 & 7 & 2 & 3 & 5 & 6 \end{array}$$

$$\begin{bmatrix} +1 & 0 & +1 & +1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & +1 & 0 & 0 \\ +1 & -1 & 0 & 0 & 0 & +1 & 0 \\ 0 & +1 & +1 & 0 & 0 & 0 & +1 \end{bmatrix} \Rightarrow [\,Q_L : U\,]$$

→ Branch voltages in terms of twig voltages :

$$V_b = Q^T V_t \Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 \end{bmatrix}_{7\times 4} \begin{bmatrix} V_2 \\ V_3 \\ V_5 \\ V_6 \end{bmatrix}_{4\times 1}$$

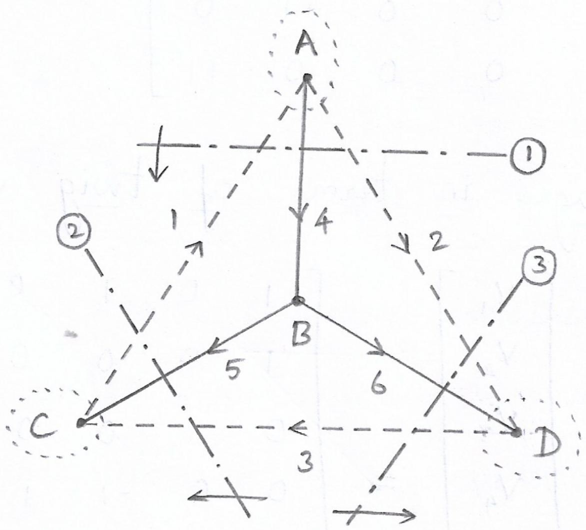
$$\Rightarrow \left.\begin{array}{lll} V_1 = V_2 + V_5 & & \\ V_2 = V_2 & ; & V_5 = V_5 \\ V_3 = V_3 & ; & V_6 = V_6 \\ V_4 = -V_5 + V_6 & ; & V_7 = V_2 - V_3 + V_6 \end{array}\right\} \text{Branch voltages}$$

→ KCL equations :

$$Q\,I_b = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{aligned} & i_1 + i_2 + i_7 = 0 \\ & i_3 - i_7 = 0 \\ & i_1 - i_4 + i_5 = 0 \\ & i_4 + i_6 + i_7 = 0 \end{aligned}\right\} \text{KCL eq's}$$

(ii)



→ Basic Cut-set matrix :

$$Q = \begin{array}{r} \\ \text{cutset } (4,1,2) \\ \text{cutset } (5,1,3) \\ \text{cutset } (6,2,3) \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & +1 & 0 & +1 & 0 & 0 \\ -1 & 0 & +1 & 0 & +1 & 0 \\ 0 & +1 & -1 & 0 & 0 & +1 \end{bmatrix} \end{array}$$

$$\begin{array}{cccccc} & Q_L & & : & Q_T & \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$\begin{bmatrix} -1 & +1 & 0 & +1 & 0 & 0 \\ -1 & 0 & +1 & 0 & +1 & 0 \\ 0 & +1 & -1 & 0 & 0 & +1 \end{bmatrix} \Rightarrow [Q_L : U]$$

→ Branch voltages in terms of twig voltages :

$$V_b = Q^T V_t \Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{6\times 3} \begin{bmatrix} V_4 \\ V_5 \\ V_6 \end{bmatrix}_{3\times 1}$$

$$\Rightarrow \left.\begin{array}{lll} V_1 = -V_4 - V_5 & ; & V_4 = V_4 \\ V_2 = V_4 + V_6 & ; & V_5 = V_5 \\ V_3 = V_5 - V_6 & ; & V_6 = V_6 \end{array}\right\} \text{Branch voltages}$$

→ KCL equations :

$$Q I_b = 0 \Rightarrow \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{array}{l} -i_1 + i_2 + i_4 = 0 \\ -i_1 + i_3 + i_5 = 0 \\ i_2 - i_3 + i_6 = 0 \end{array}\right\} \text{KCL eq:'s}$$

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06/08/17