Block Diagram Algebra

and

Reduction technique

Review: Block Diagram of a System

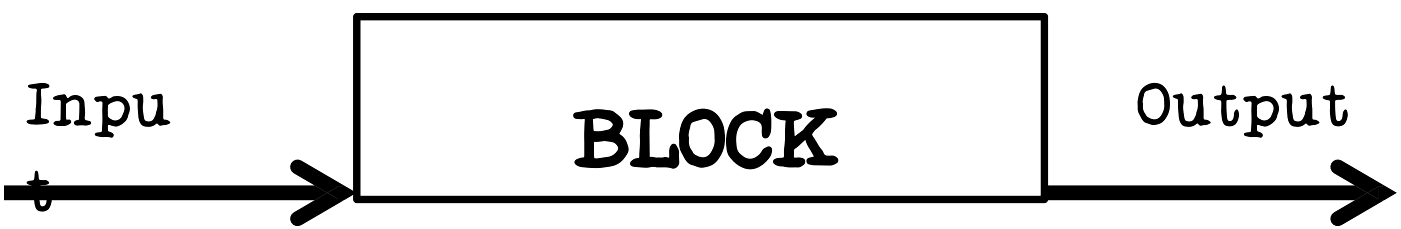
- It is a short hand pictorial representation of the system which depicts
  - Each functional component or sub-system and
  - Flow of signals from one sub-system to another
- Components of a block diagram:
  - Blocks to represent components
  - Arrows to indicate direction of signal flow
  - Summing points to show merging signals
  - Take off points to indicate branching of signals

Block Diagram Reduction

- Block diagram reduction refers to simplification of block diagrams of complex systems through certain rearrangements
- Simplification enables easy calculation of the overall transfer function of the system
- Simplification is done using certain rules called the 'rules of block diagram algebra'
- All these rules are derived by simply algebraic manipulations of the equations representing the blocks

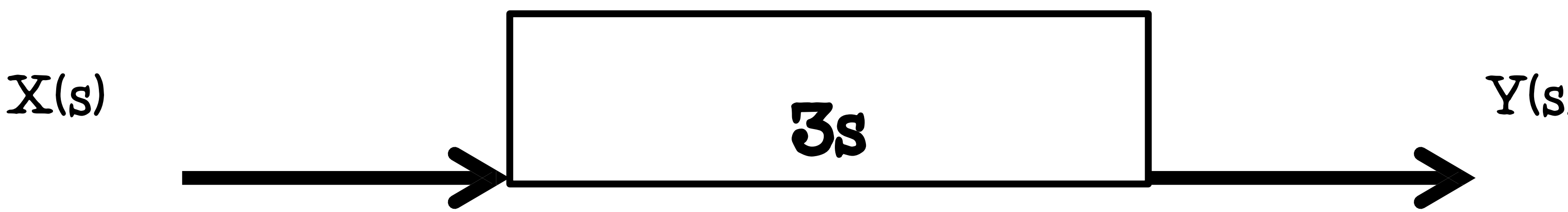
Block Diagram Fundamentals

- **Block Diagram:** It is shorthand, pictorial representation of the cause and effect relationship between input and output of a physical system.



Block Diagram Fundamentals

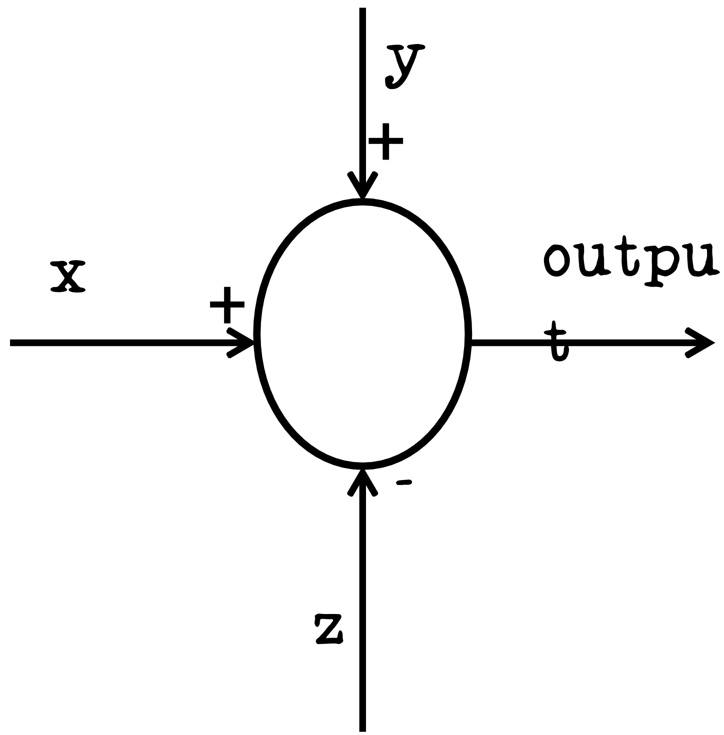
- **<u>Output:</u>** The value of theinput ismultiplied to the value of block gain to get the output.



Output $Y(s) = 3s.\, X(s)$

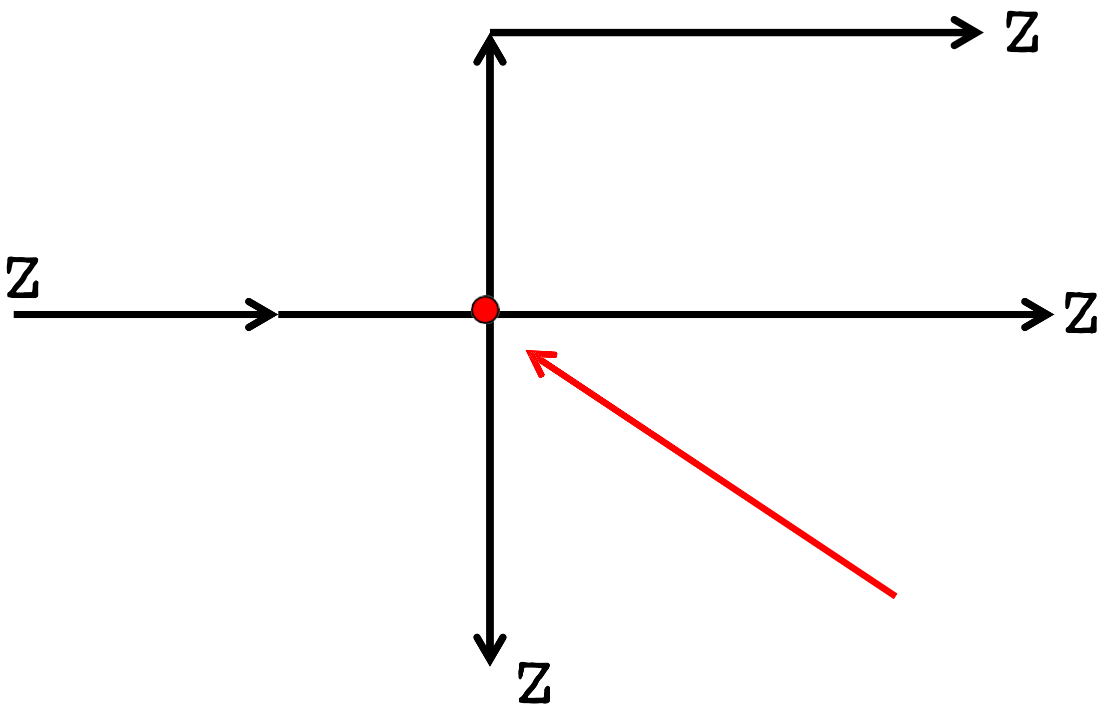
Block Diagram Fundamentals

- **Summing Point:** Two or moresignalscan be added/ substracted at summing point.



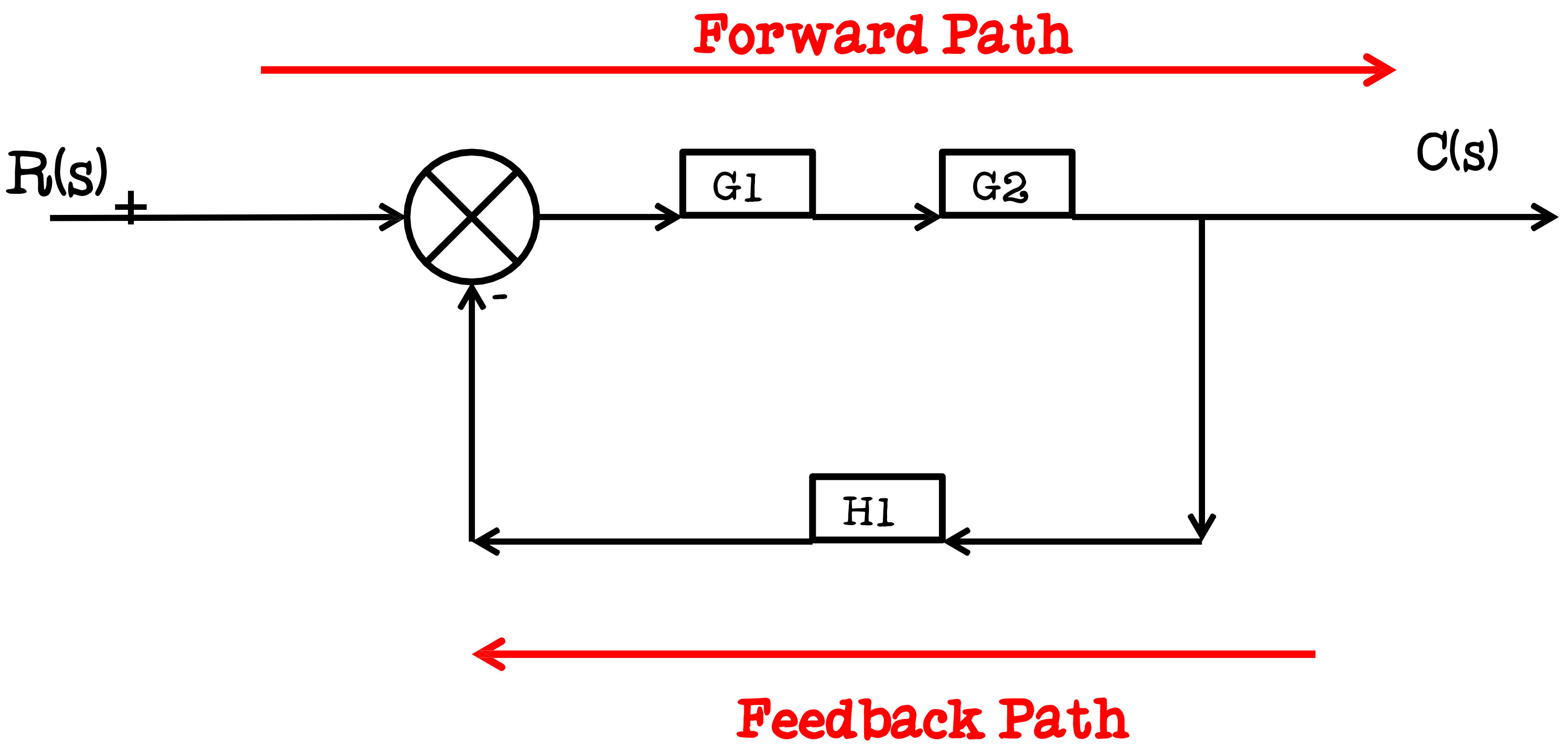
Output =x+y-z

- **Take off Point:** The output signal can be applied to two or more points from a take off point.



Block Diagram Fundamentals

- **Forward Path:** The direction of flow of signal is from input to output

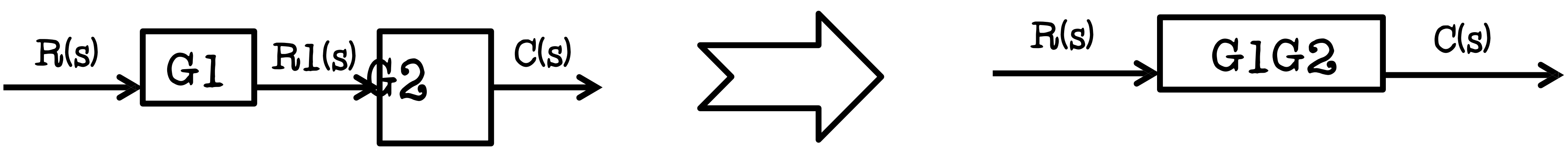


- **Feedback Path:** The direction of flow of signal is from output to input

Block Diagram Reduction Techniques

Rule 1: For blocks in cascade

Gain of blocks connected in cascade gets multiplied with each other.

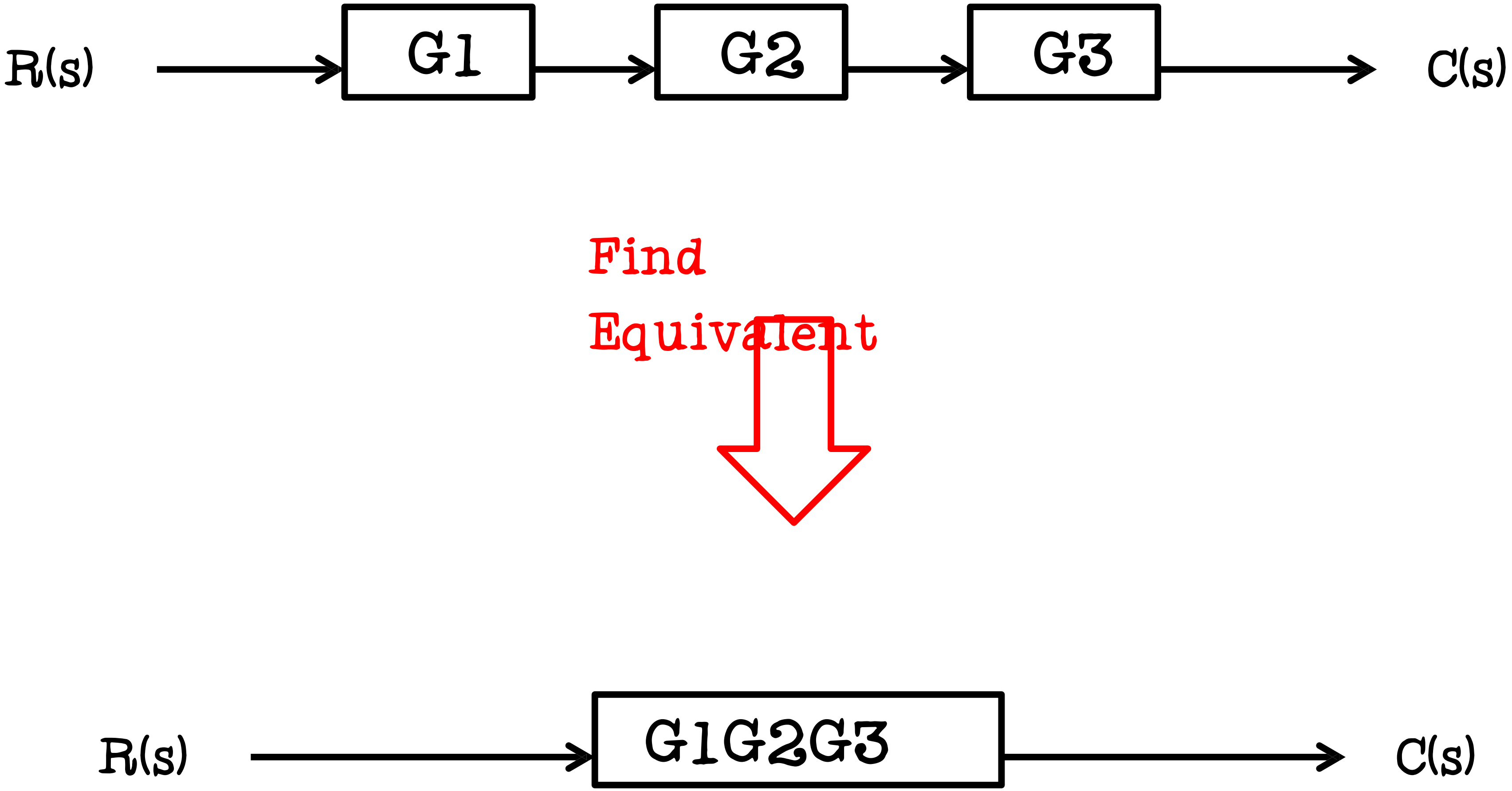


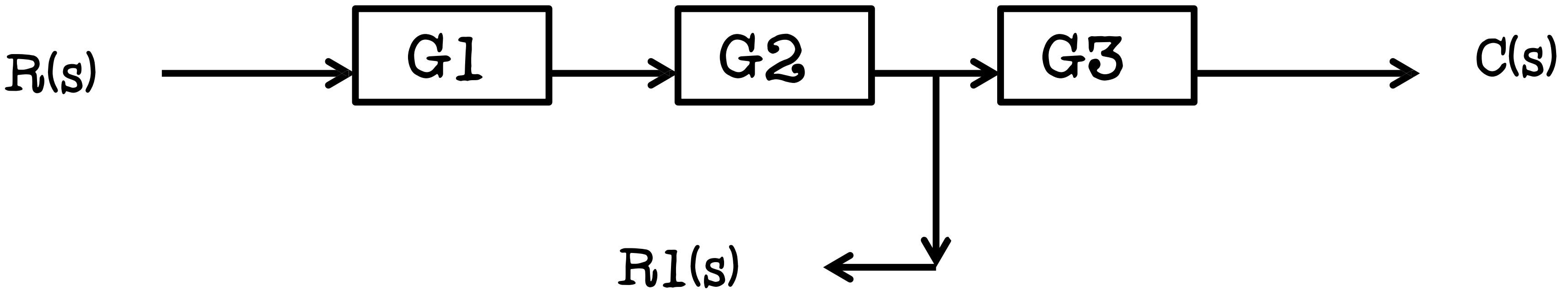
R1(s)=G1 R(s)

C(s)= G1 G2 R(s)

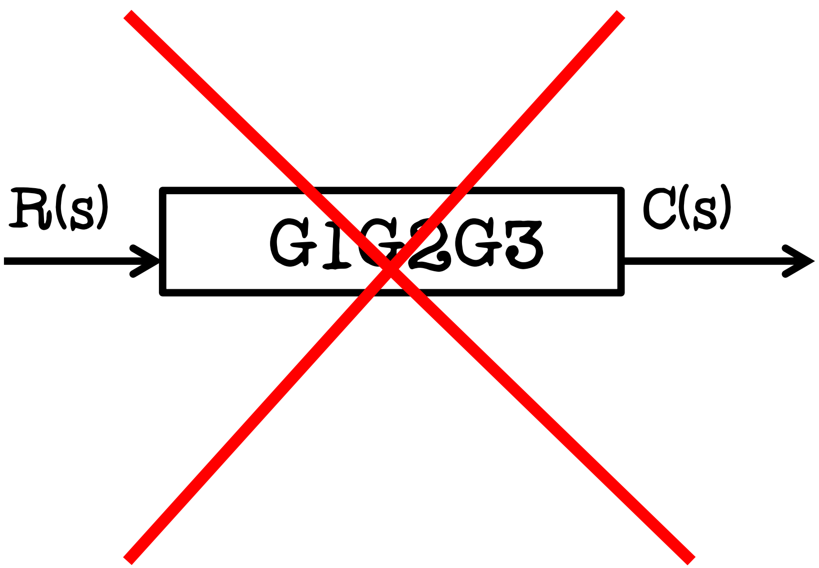
C(s) =G2 R1(s)

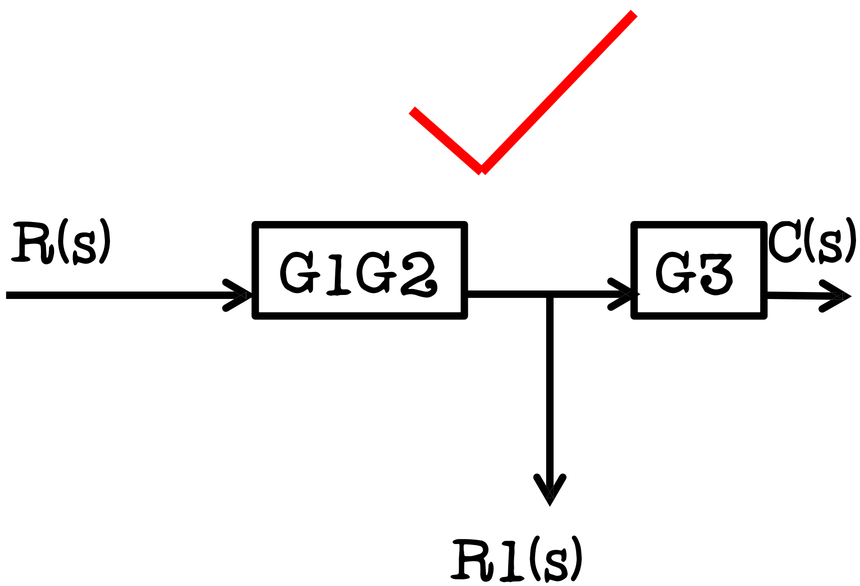
=G1 G2 R(s)


R(s)
G1
G2
G3
C(s)
Find
Equivalent
R(s)
G1G2G3
C(s)



Find Equivalent

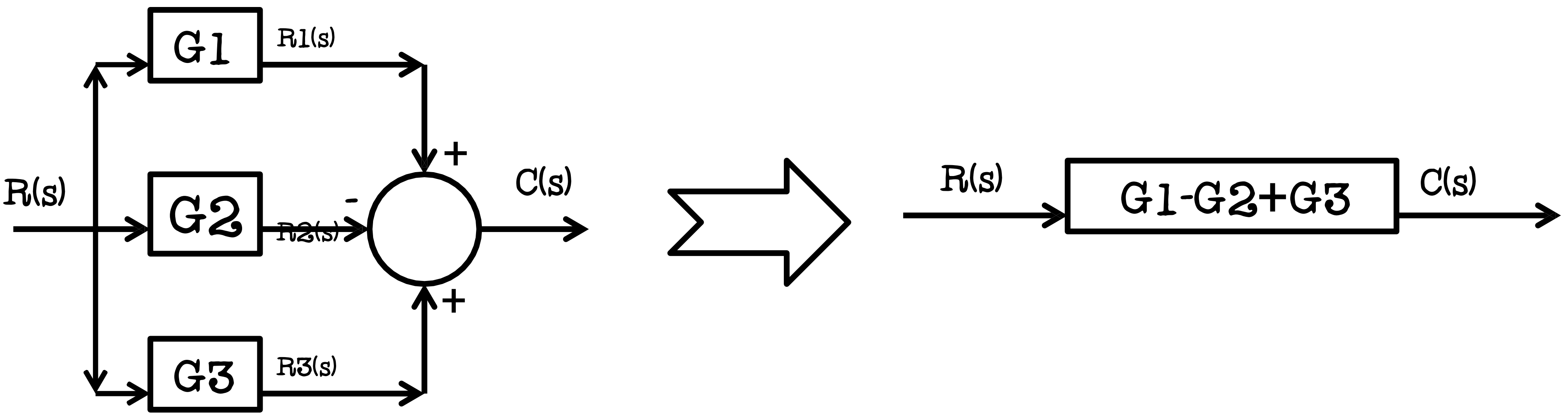




Block Diagram Reduction Techniques

Rule 2: For blocks in Parallel

Gain of blocks connected in parallel gets added algebraically.



$C(s) = R1(s) - R2(s) + R3(s)$

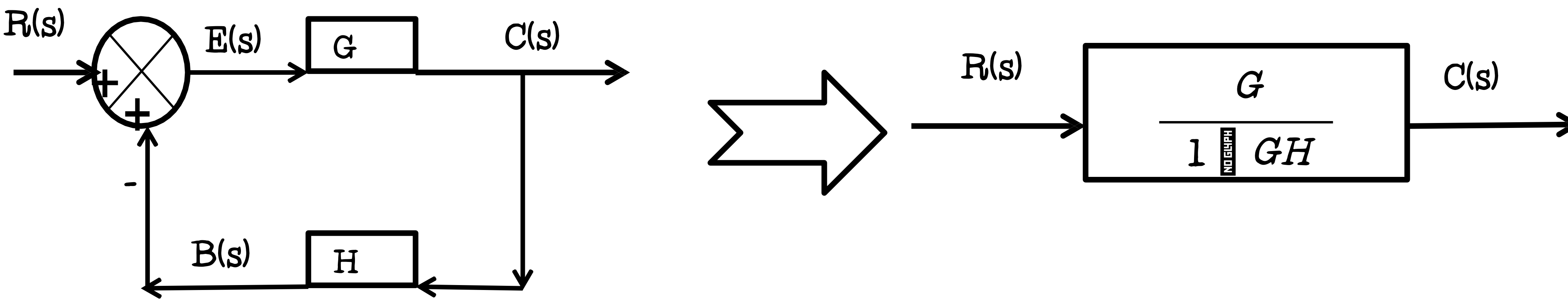
$= G1R(s) - G2R(s) + G3R(s)$

$C(s) = (G1 - G2 + G3)\ R(s)$

$C(s) = (G1 - G2 + G3)\ R(s)$

Block Diagram Reduction Techniques

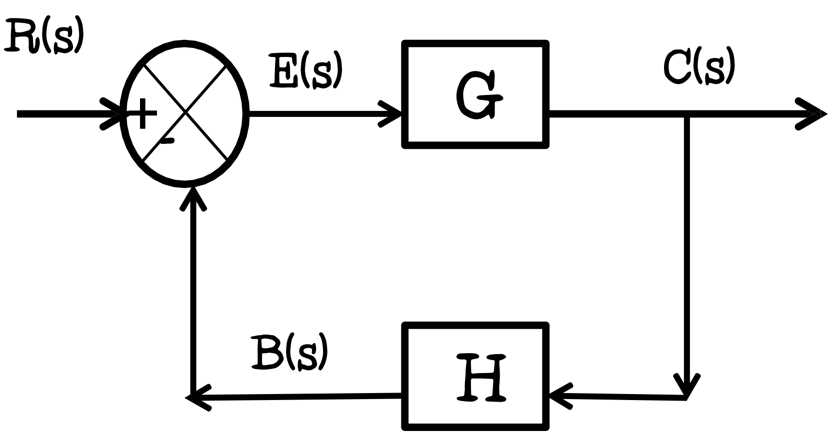
Rule 3: Eliminate Feedback Loop



$$\frac{C(s)}{R(s)} = \frac{G}{1 \mp GH}$$

In General

From Shown Figure,



$$E(s) = R(s) - B(s)$$

and

$$C(s) = G.E(s)$$
$$= G[R(s) - B(s)]$$
$$= GR(s) - GB(s)$$

But

$$B(s) = H.C(s)$$

$$\Rightarrow C(s) = G.R(s) - G.H.C(s)$$

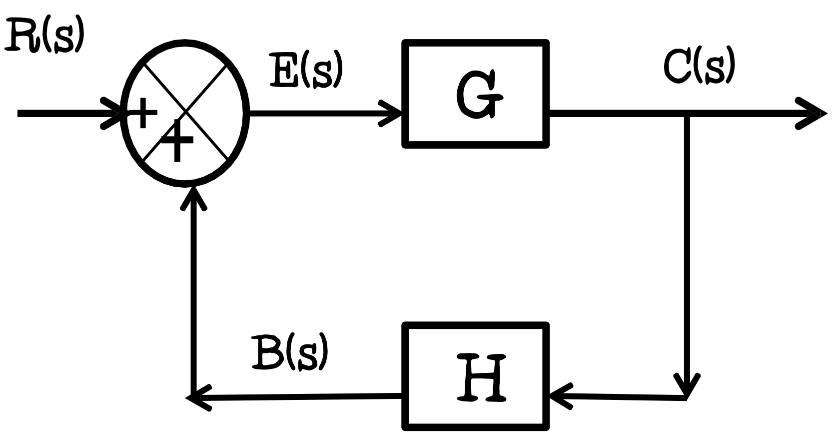
$$C(s) + G.H = GR(s)$$

$$\Rightarrow C(s)\{1 + G.H\} = G.R(s)$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G}{1 + GH}$$

For Negative Feedback

From Shown Figure,



$$E(s) = R(s) + B(s)$$

and

$$C(s) = G.E(s)$$

$$= G[R(s) + B(s)]$$

$$= GR(s) + GB(s)$$

But

$$B(s) = H.C(s)$$

$$\therefore C(s) = G.R(s) + G.H.C(s)$$

$$C(s) - G.H = GR(s)$$

$$\therefore C(s)\{1 - G.H\} = G.R(s)$$

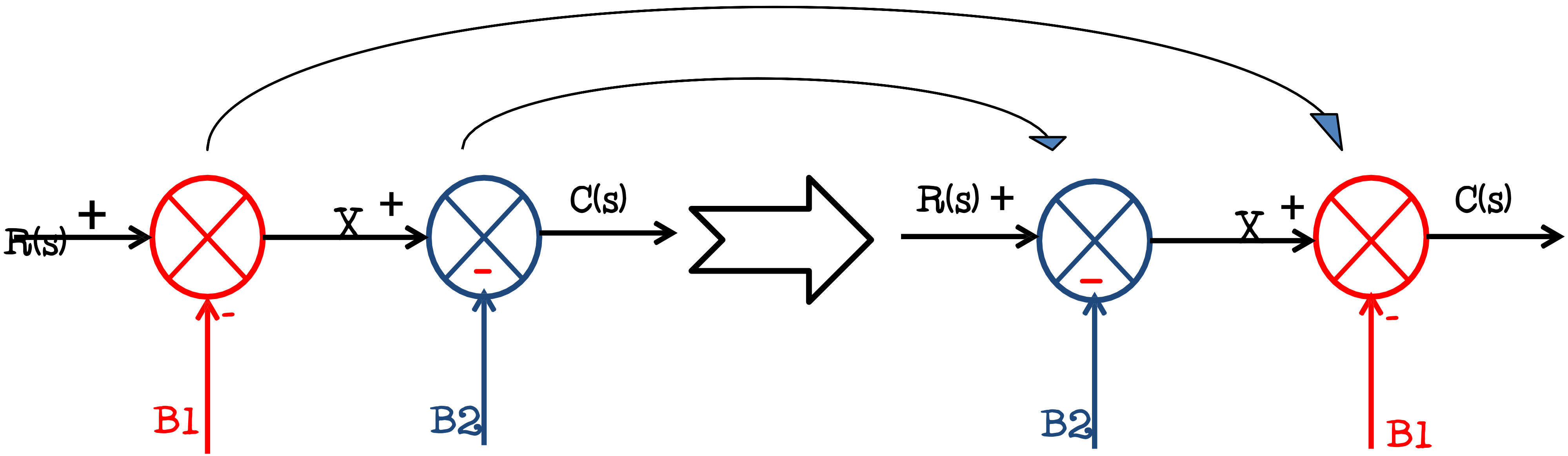
$$\therefore \frac{C(s)}{R(s)} = \frac{G}{1 - GH}$$

For Positive Feedback

Block Diagram Reduction Techniques

Rule 4: Associative Law for Summing Points

The order of summing points can be changed if two or more summing points are in series

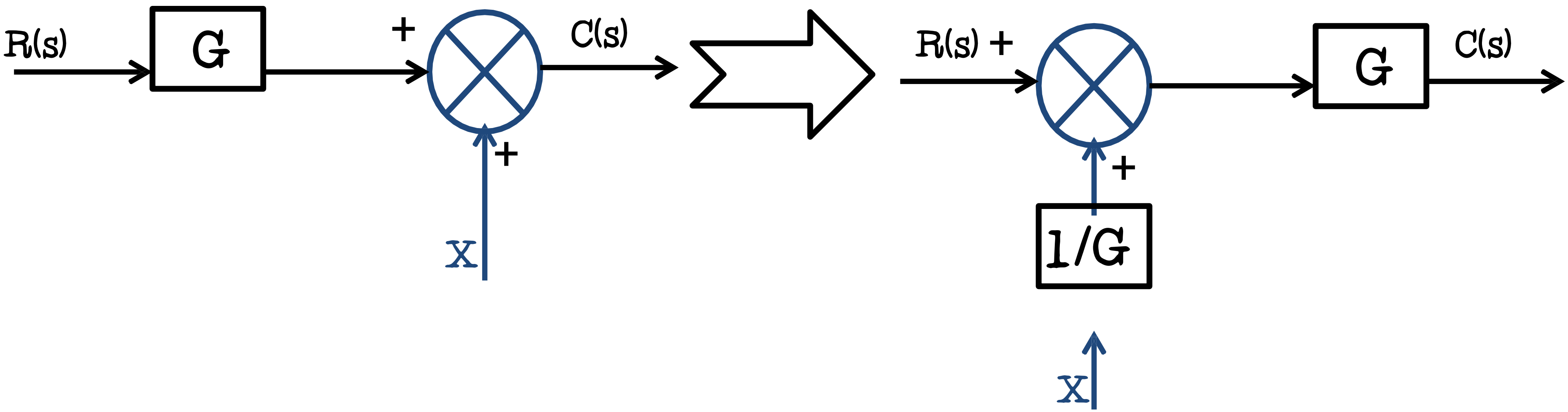


X=R(s)-B1
C(s)=X-B2
C(s)=R(s)-B1-B2

X=R(s)-B2
C(s)=X-B1
C(s)=R(s)-B2-B1

Block Diagram Reduction Techniques

Rule 5: Shift summing point before block

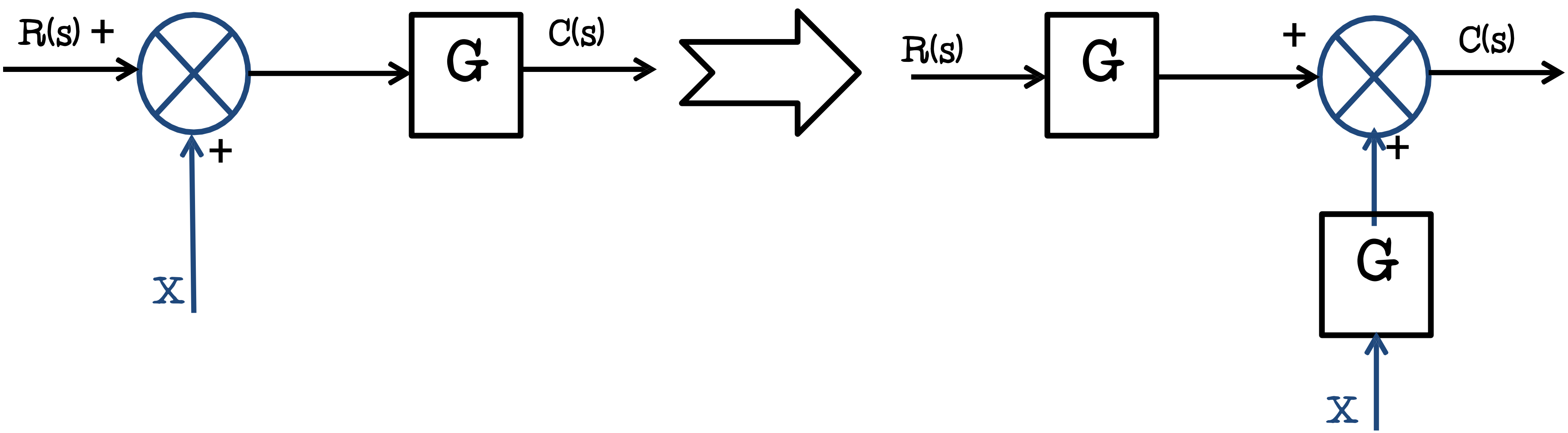


C(s)=R(s)G+X

C(s)=G{R(s)+X/G}
=GR(s)+X

Block Diagram Reduction Techniques

Rule 6: Shift summing point after block



$C(s)=G\{R(s)+X\}$

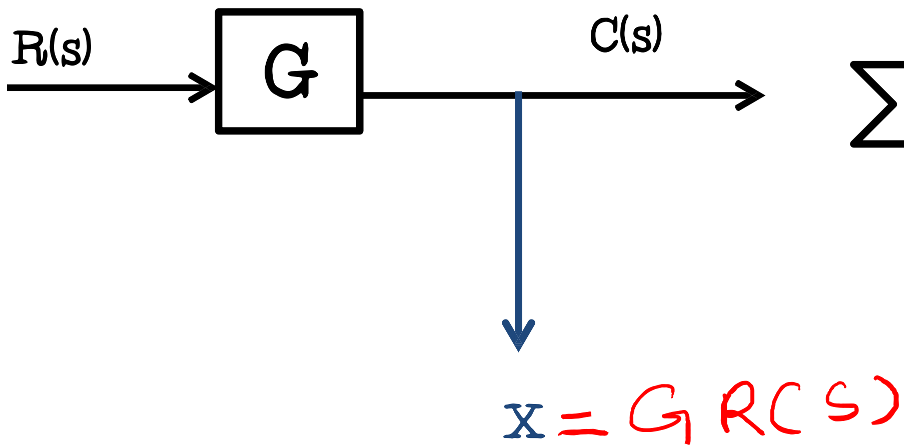
$=GR(s)+GX$

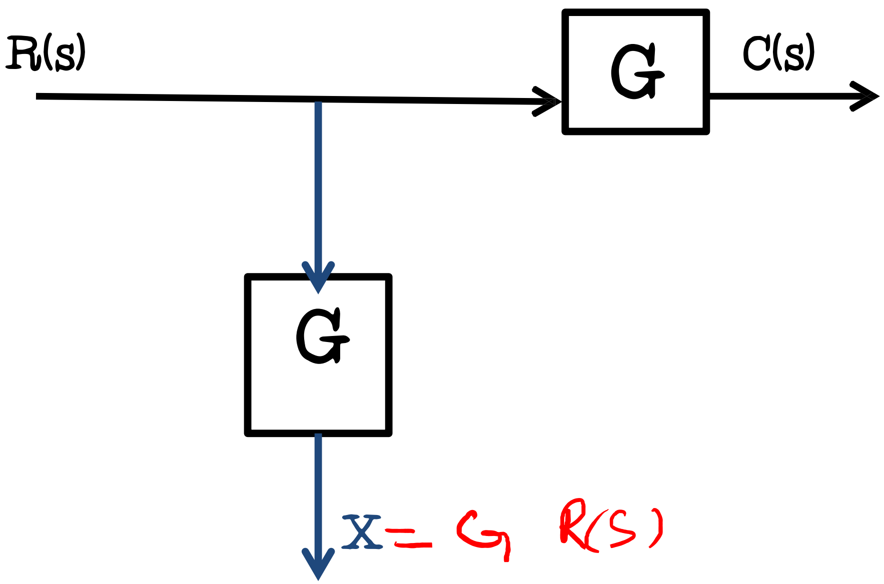
$C(s)=GR(s)+XG$

$=GR(s)+XG$

Block Diagram Reduction Techniques

Rule 7: Shift a take off point before block





$C(s)=GR(s)$

and

$X=C(s)=GR(s)$

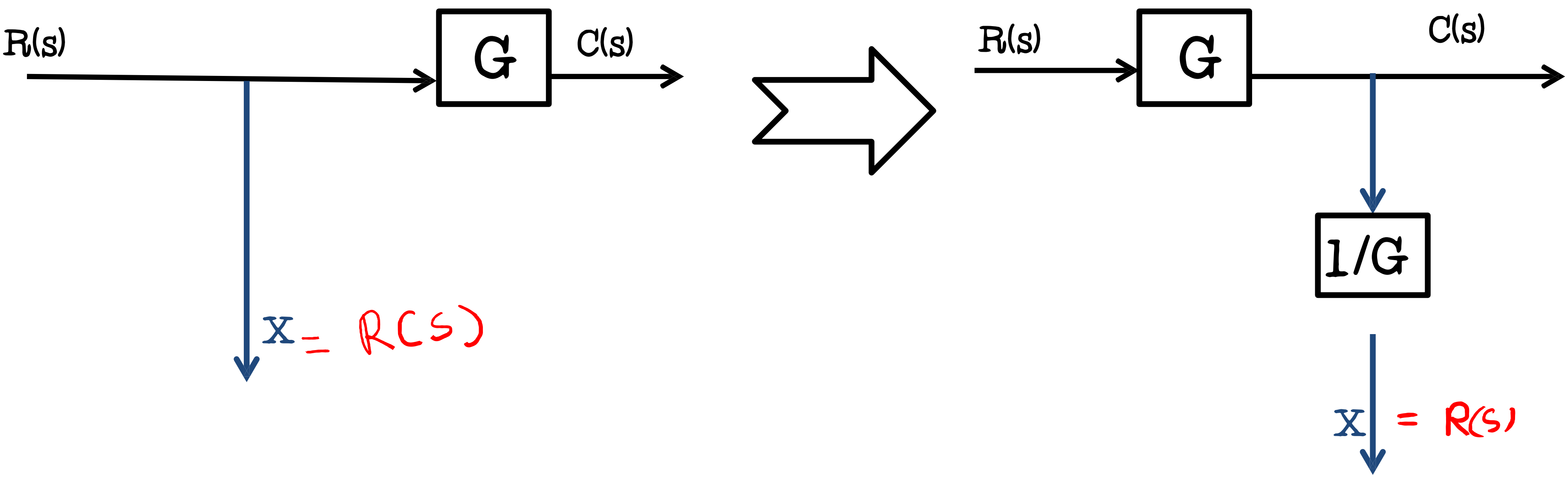
$C(s)=GR(s)$

and

$X=GR(s)$

Block Diagram Reduction Techniques

Rule 8: Shift a take off point after block



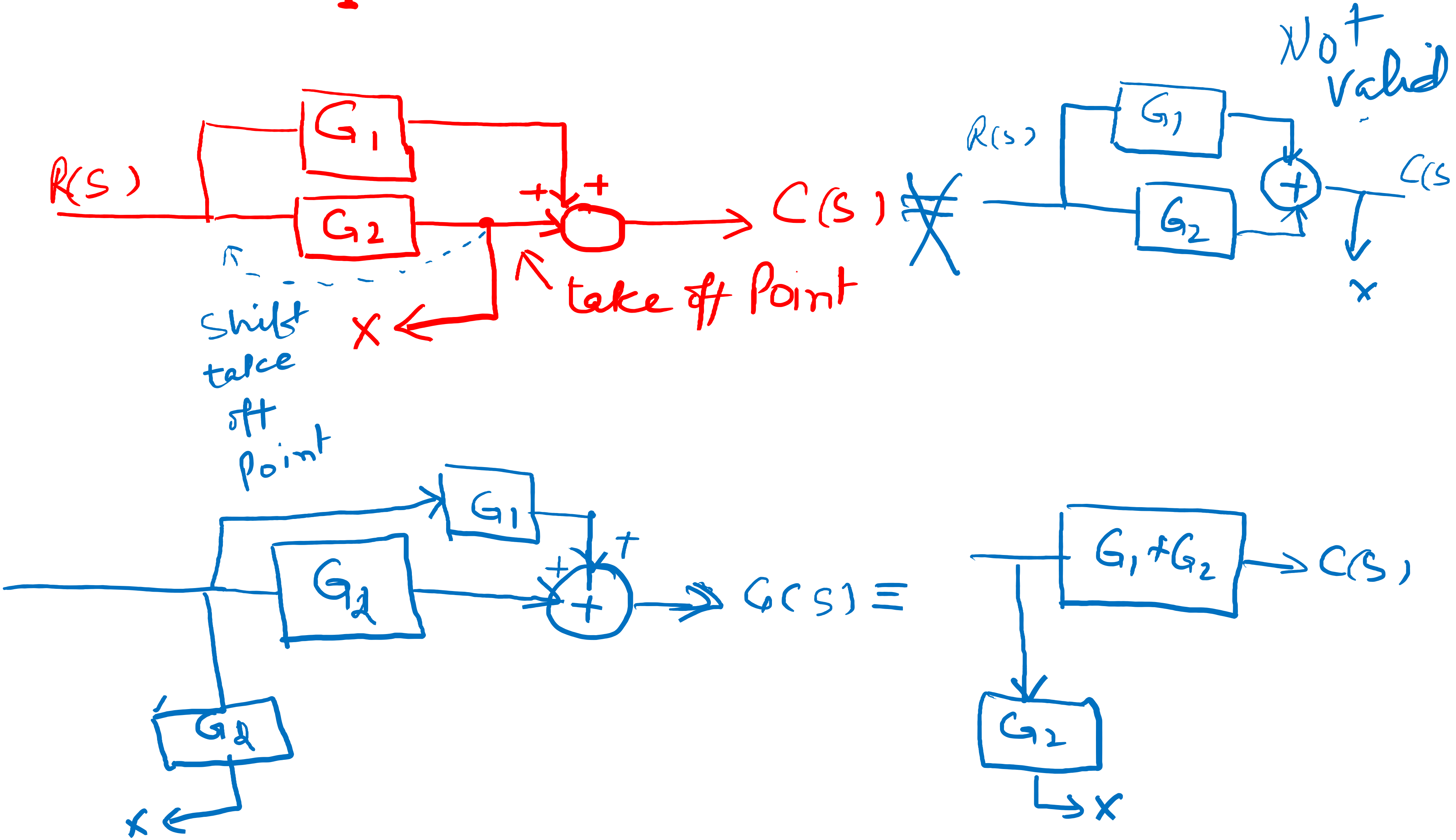
$C(s)=GR(s)$ and $X=R(s)$

$C(s)=GR(s)$ and $X=C(s).\{1/G\}$

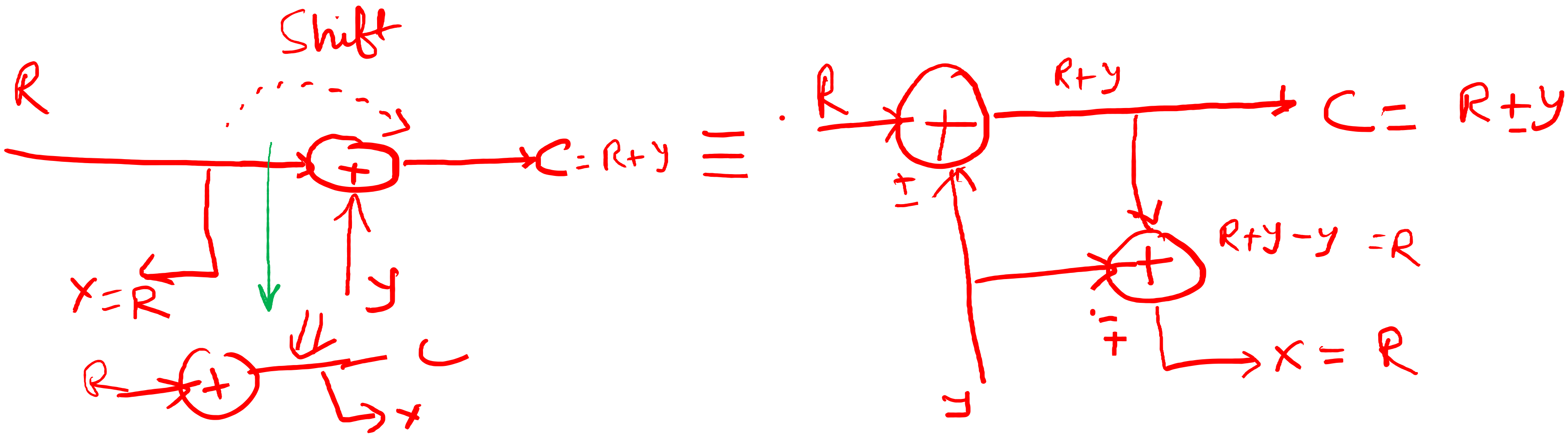
$=GR(s).\{1/G\}$

$= R(s)$

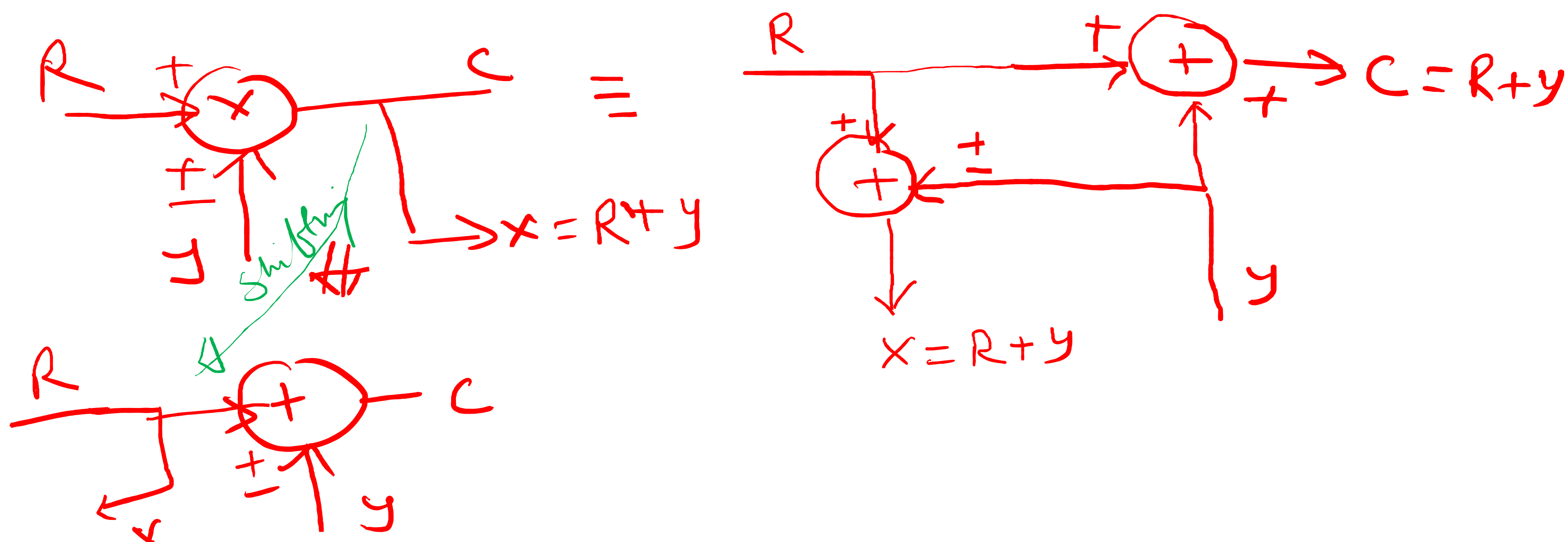
Take of point



Crital Rules (1) Shifting take off Point after Summing Point



② Shifting take off Point before summing Point



Block Diagram Reduction Techniques

- While solving block diagram for getting single block equivalent, the said rules need to be applied. After each simplification a decision needs to be taken. For each decision we suggest preferences as

Block Diagram Reduction Rules

Rule 1 – Check for the blocks connected in series and simplify.

Rule 2 – Check for the blocks connected in parallel and simplify

Rule 3 – Check for the blocks connected in feedback loop and simplify. (minor feedback loop)

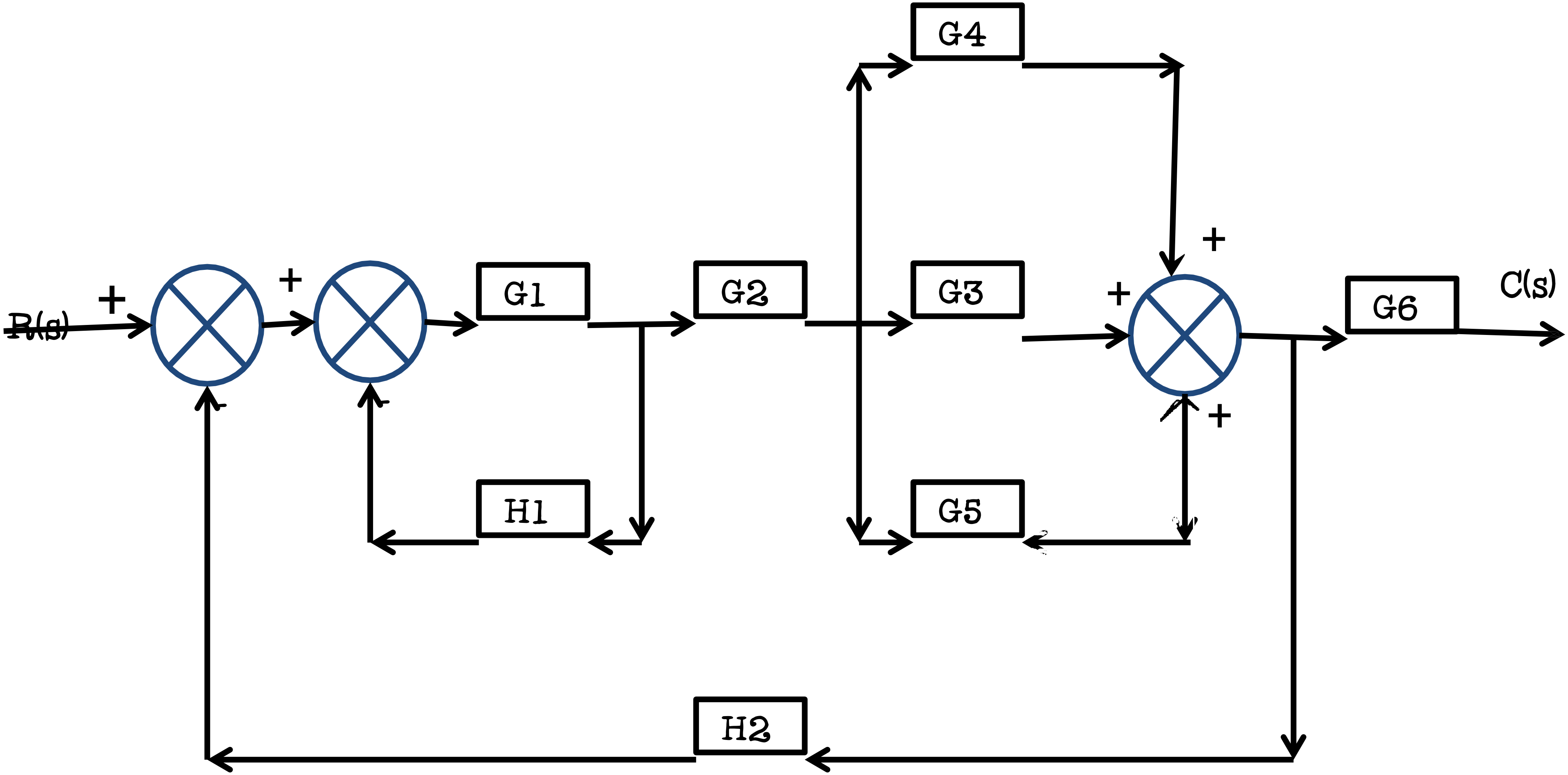
Rule 4 – If there is difficulty with take-off point while simplifying, shift it towards right as far as possible

Rule 5 – If there is difficulty with summing point while simplifying, shift it towards left as far as possible .

Rule 6 – Repeat the above steps till you get the simplified form, i.e., single block.

Note – The **transfer function present in this single block** is the transfer function of the overall block diagram.

Example 1

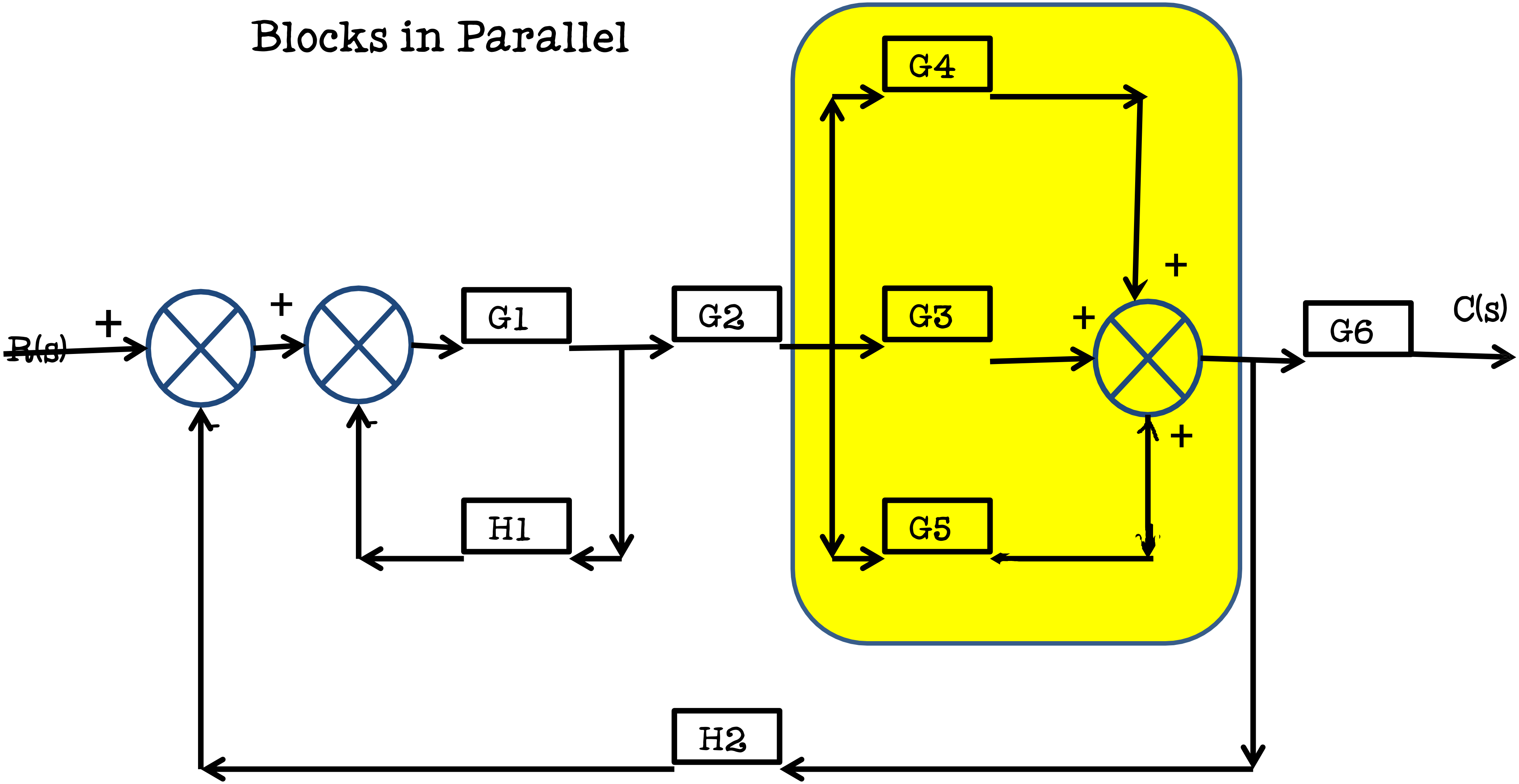


- Rule 1 cannot be usedas there are no immediate series blocks.
- Hence Rule 2 can be applied to G4, G3, G5 in parallel to get an equivalent of G3+G4+G5

Example 1 cont....

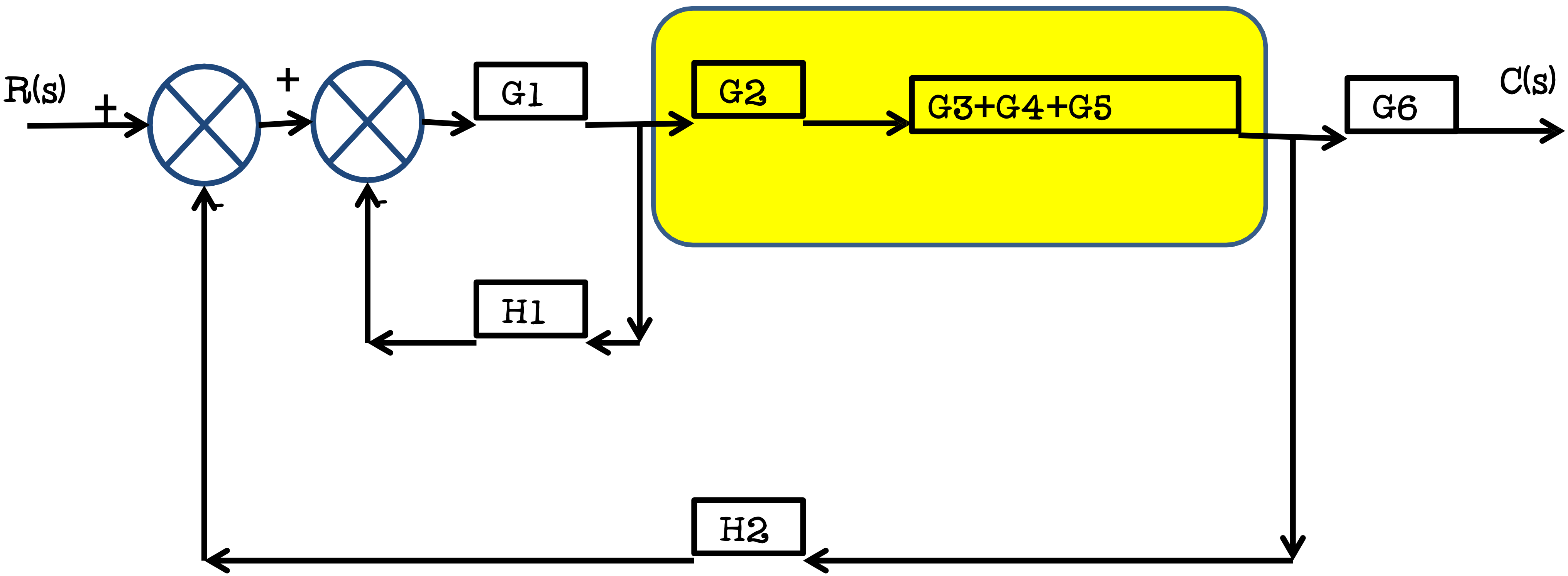
Apply Rule 2

Blocks in Parallel



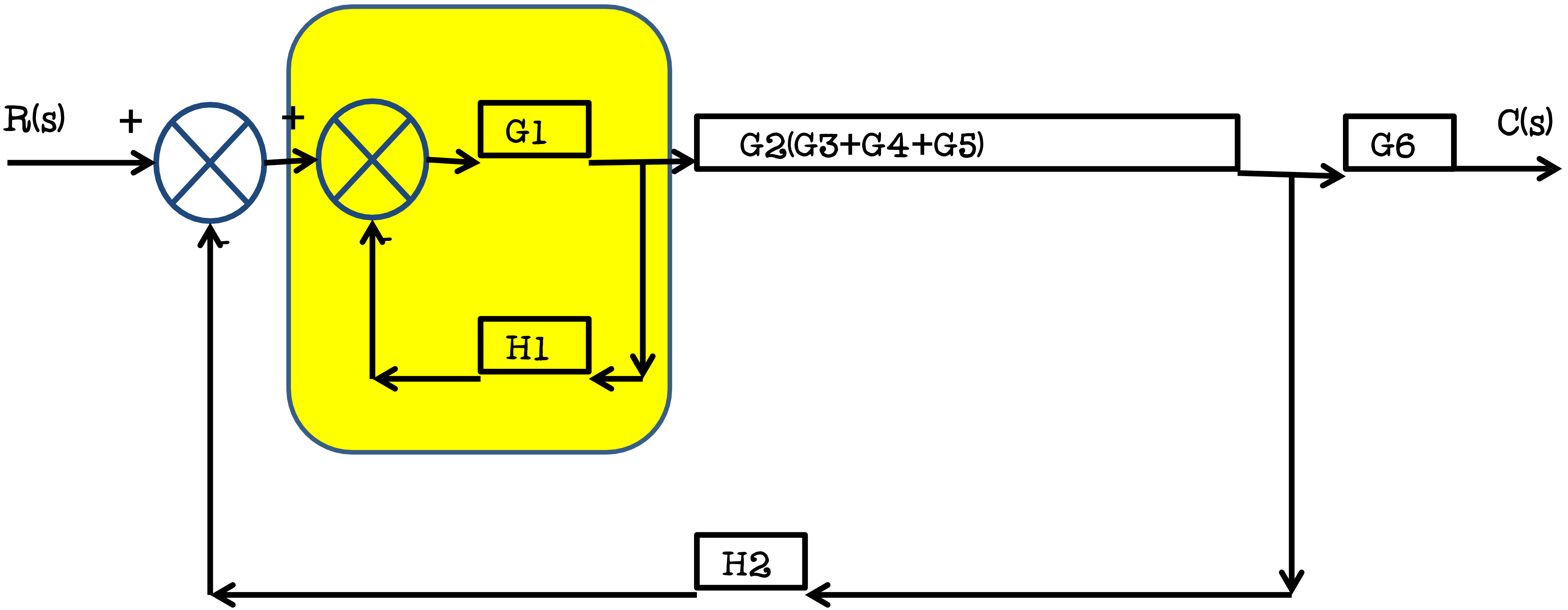
Example 1 cont....

Apply Rule 1 Blocks in series



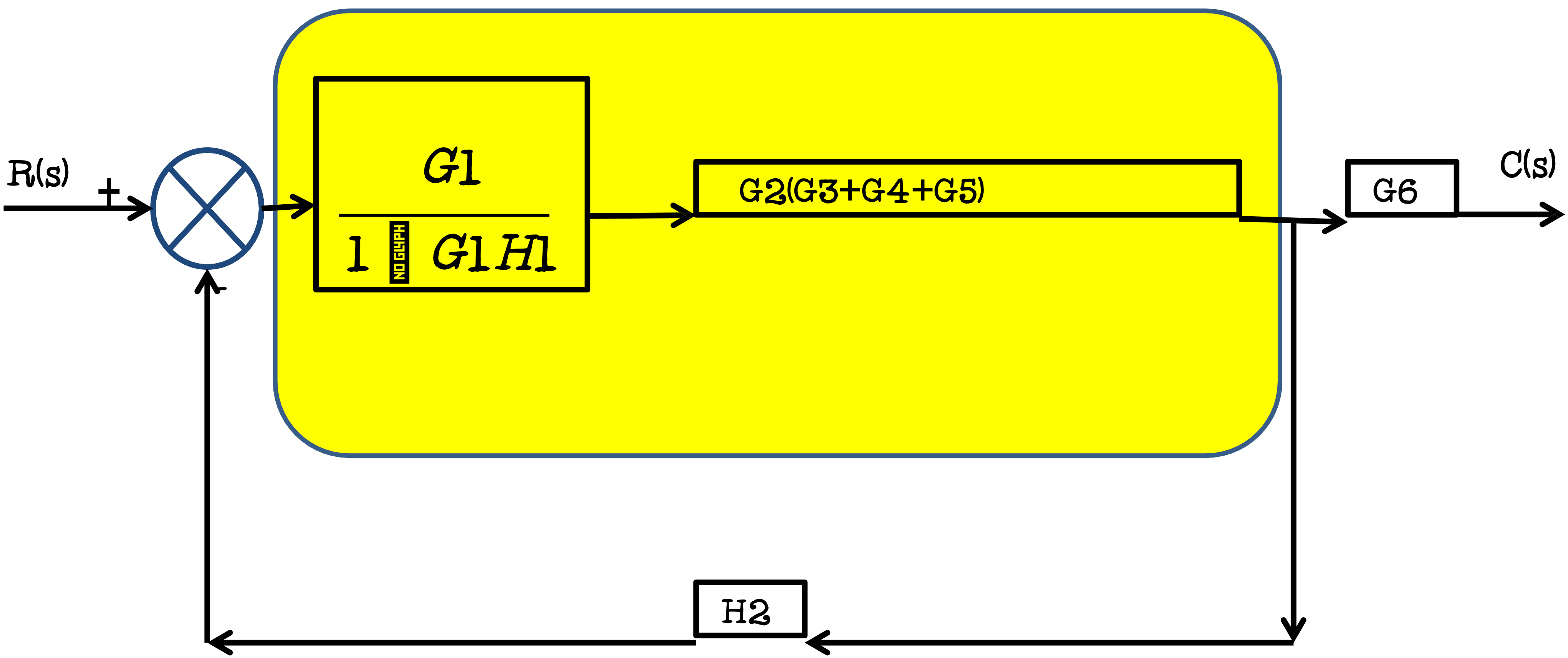
Example 1 cont....

Apply Rule 3 Elimination of feedback loop



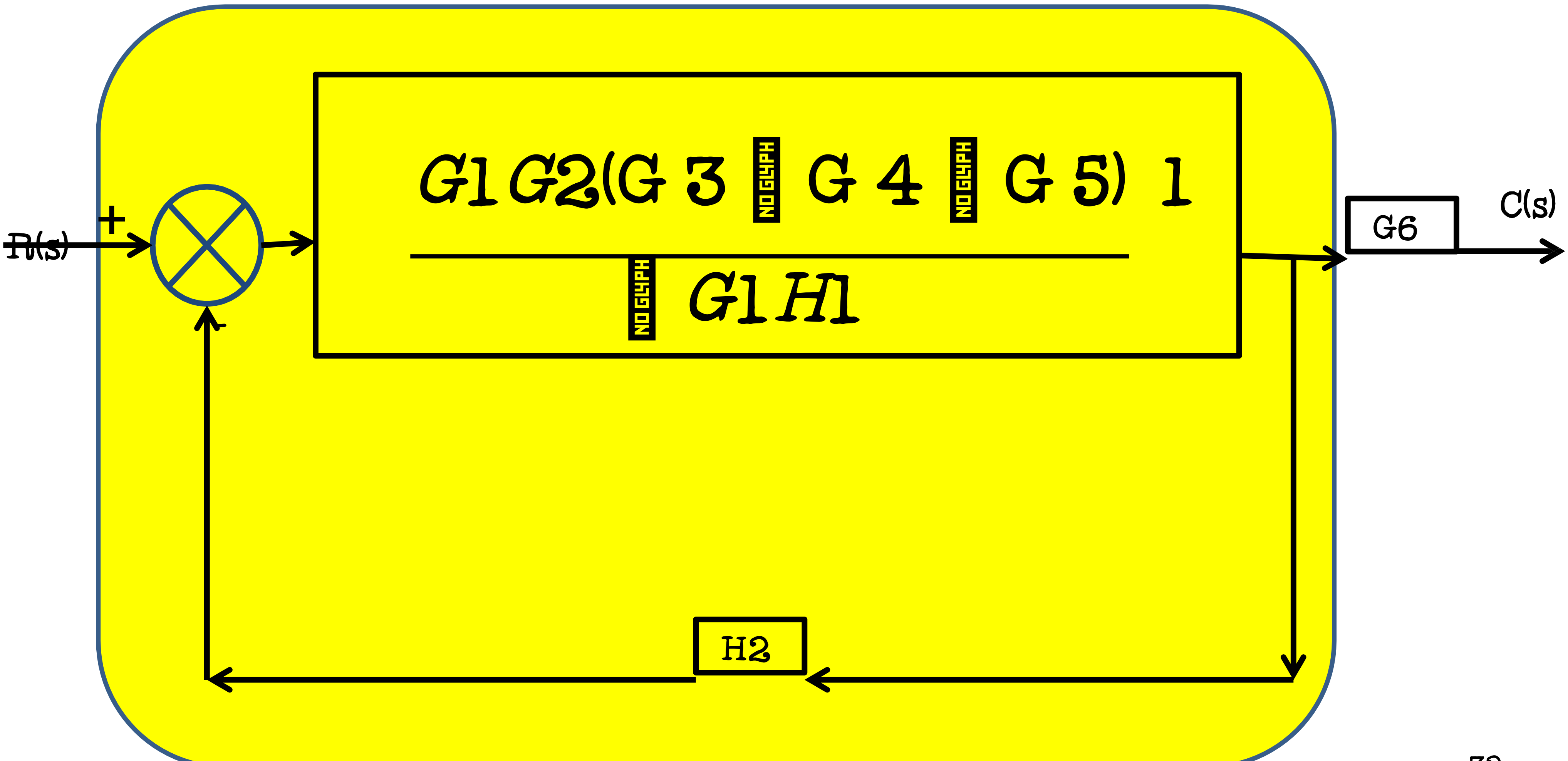
Example 1 cont....

Apply Rule 1 Blocks in series



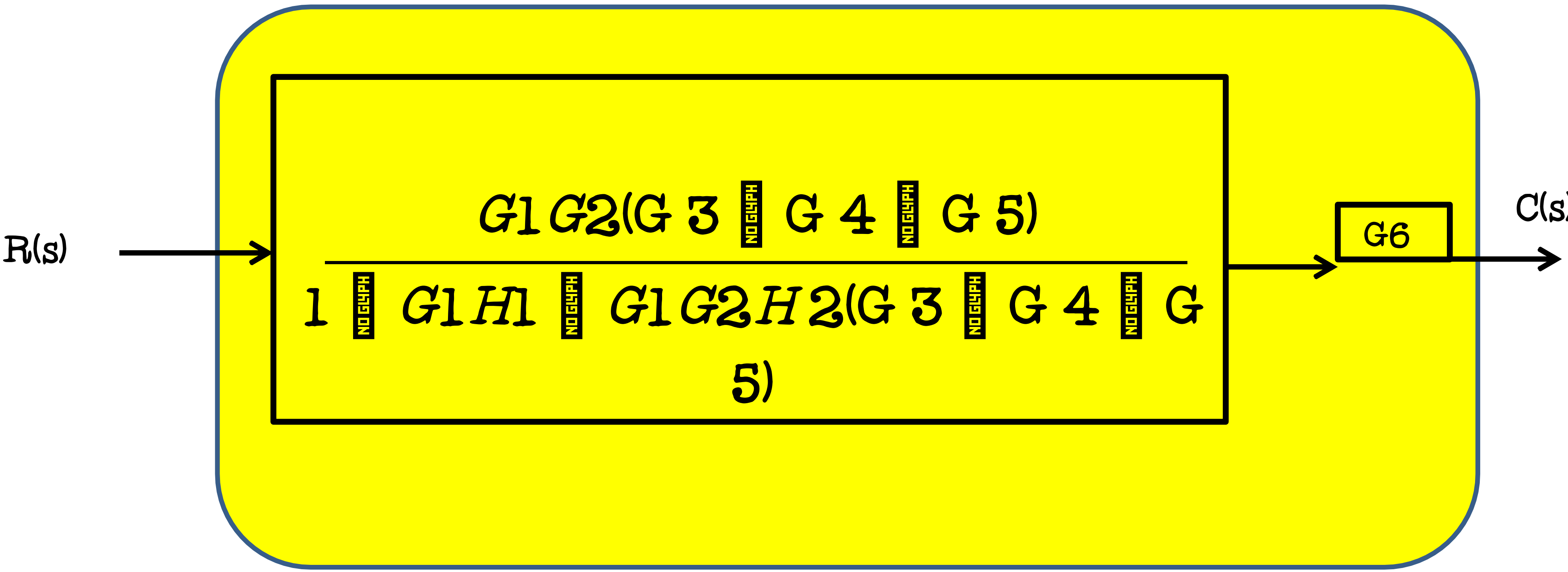
Example 1 cont....

Apply Rule 3 Elimination of feedback loop

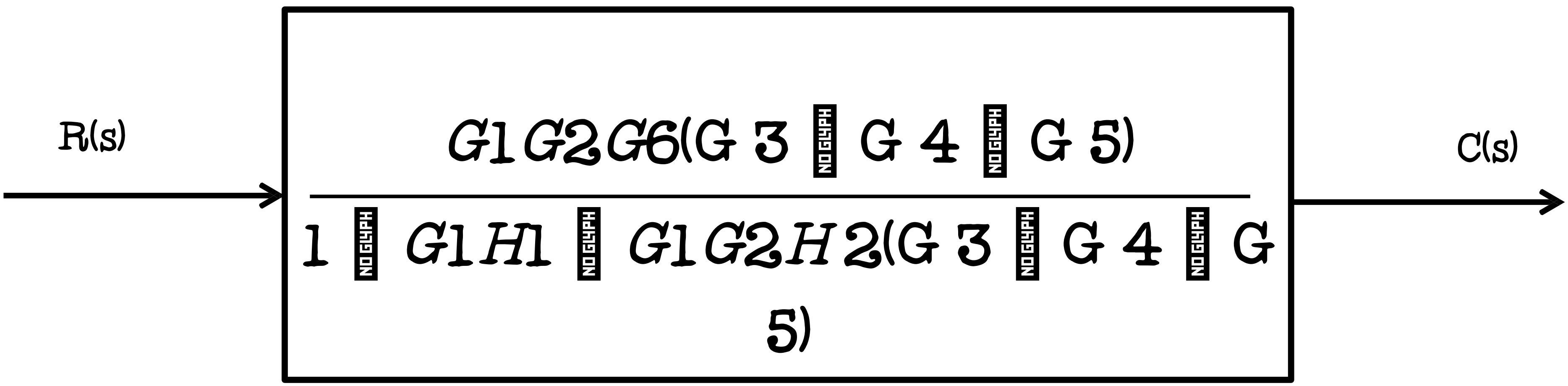


Example 1 cont....

Apply Rule 1 Blocks in series



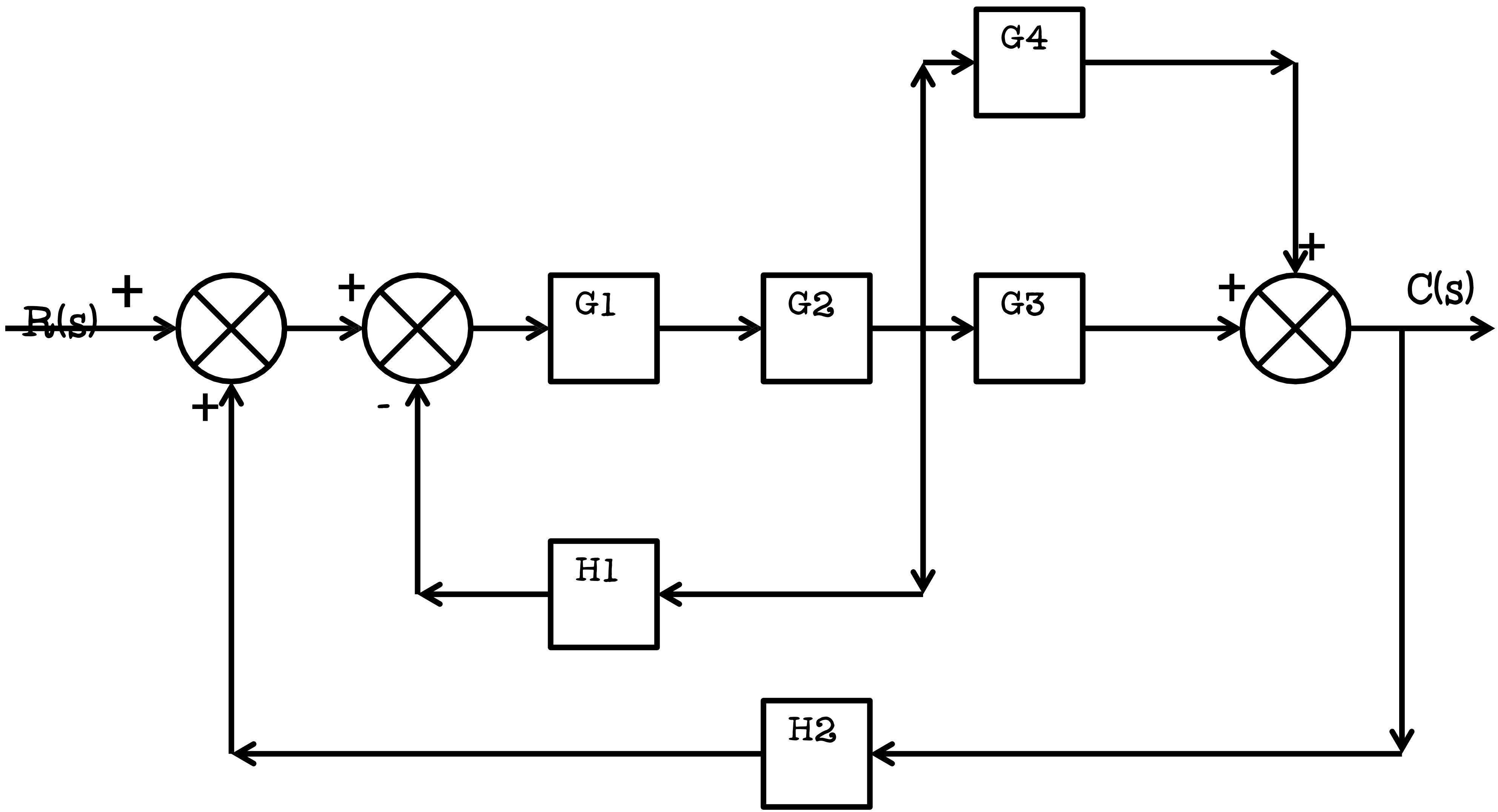
Example 1 cont….



Example 1 cont....

$$\frac{C(s)}{R(s)} \square \frac{G1\,G2\,G6(G3 \square G4 \square G5)}{1 \square G1\,H1 \square G1\,G2\,H2(G3 \square G4 \square G5)}$$

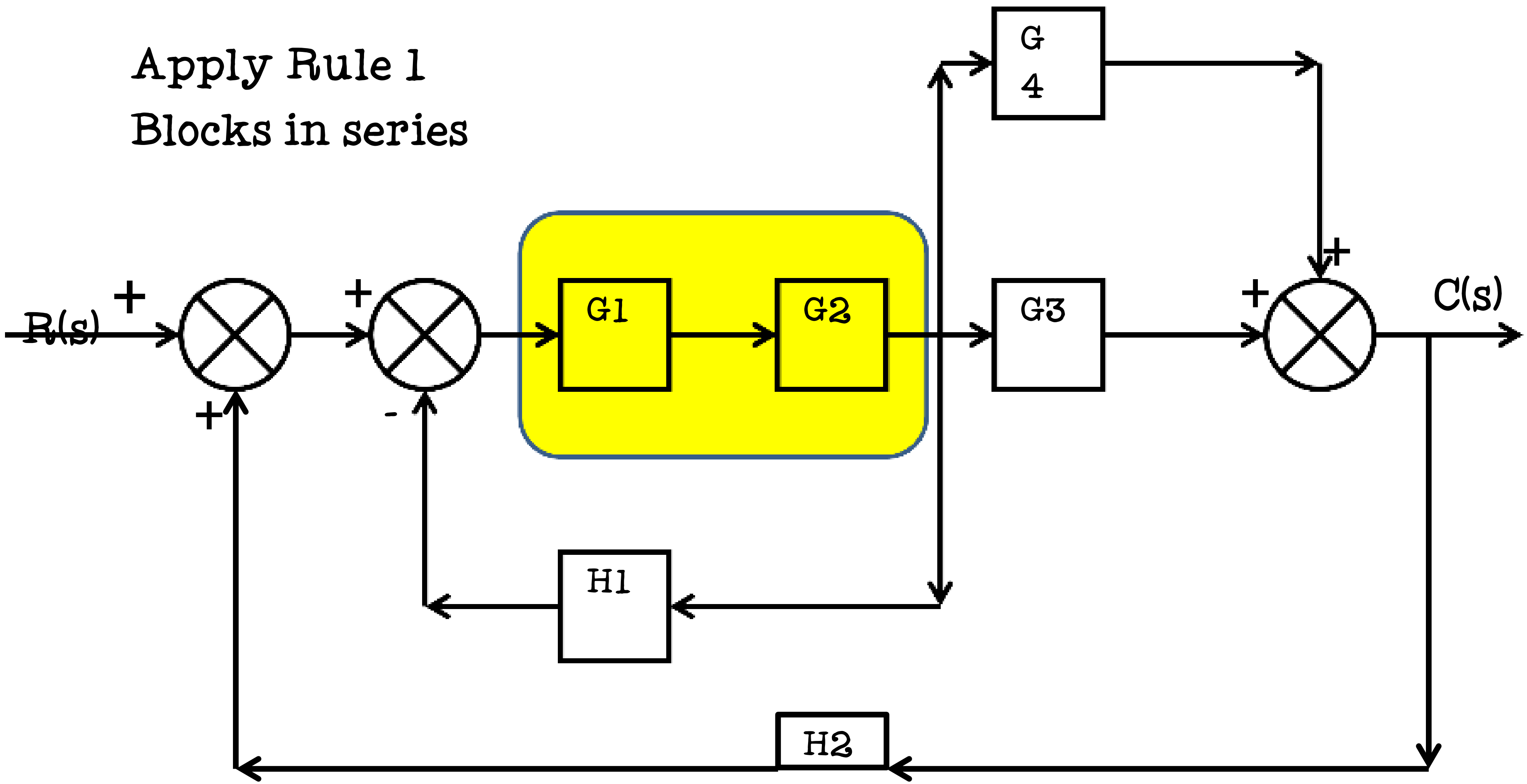
Example 2



Example 2 cont....

Apply Rule 1

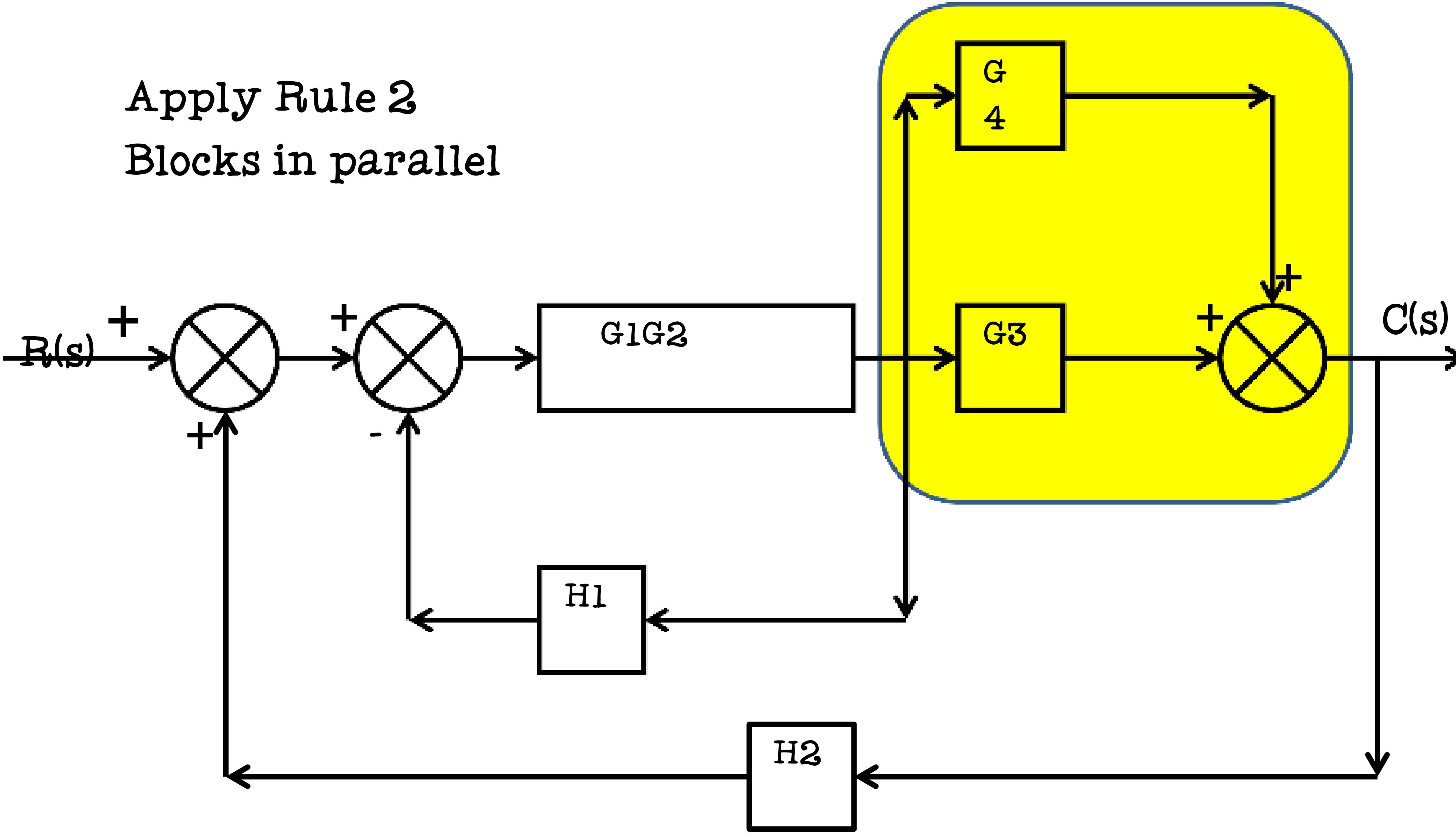
Blocks in series



Example 2 cont….

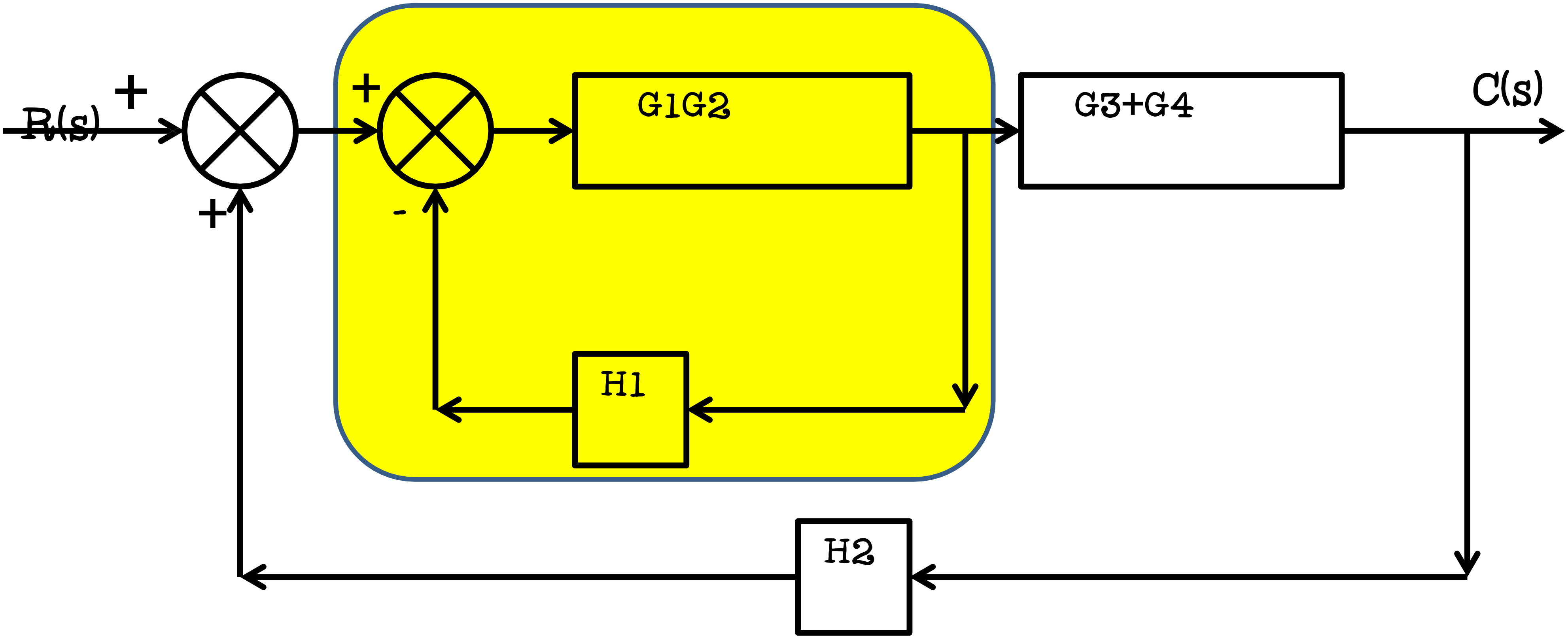
Apply Rule 2

Blocks in parallel



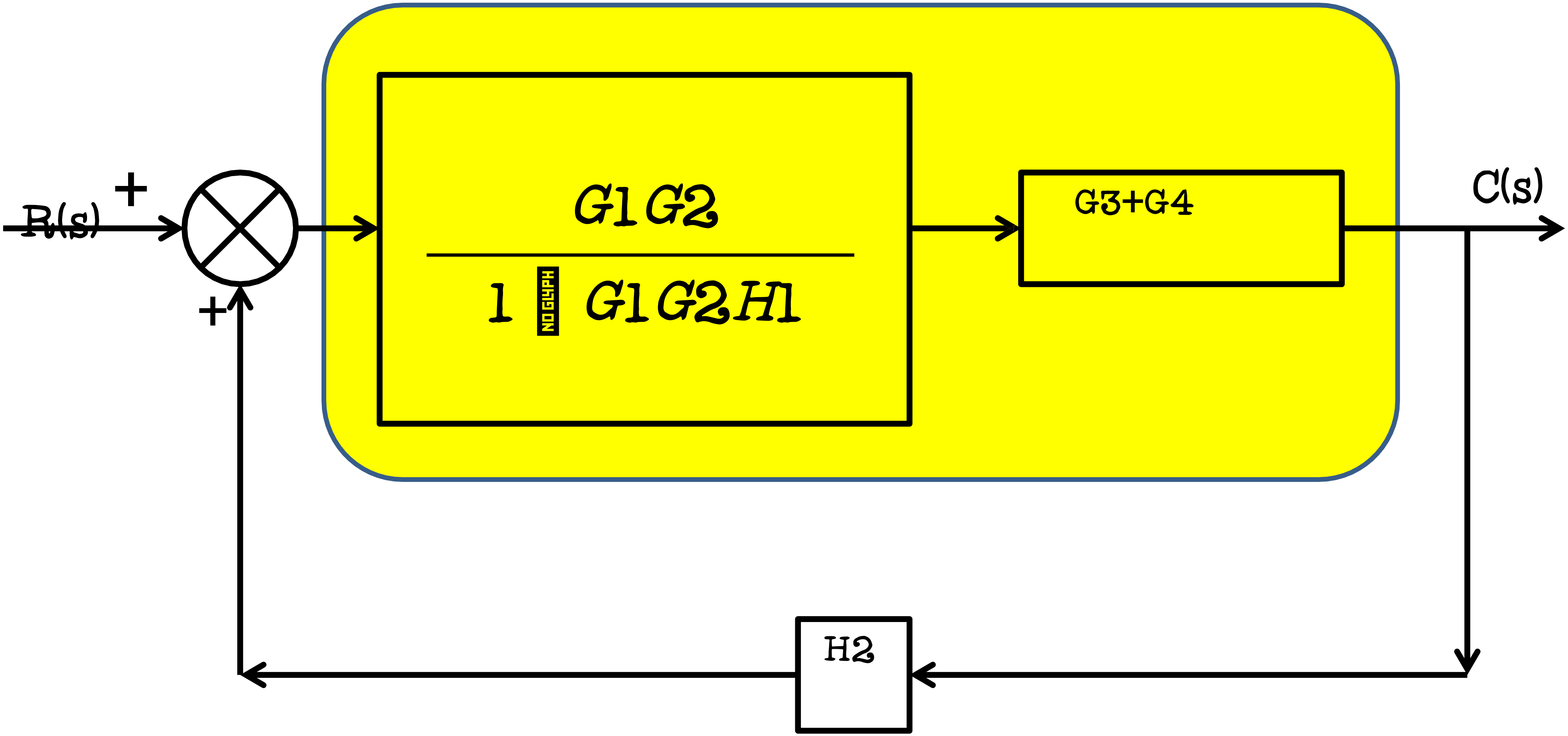
Apply Rule 3

Elimination of feedback loop



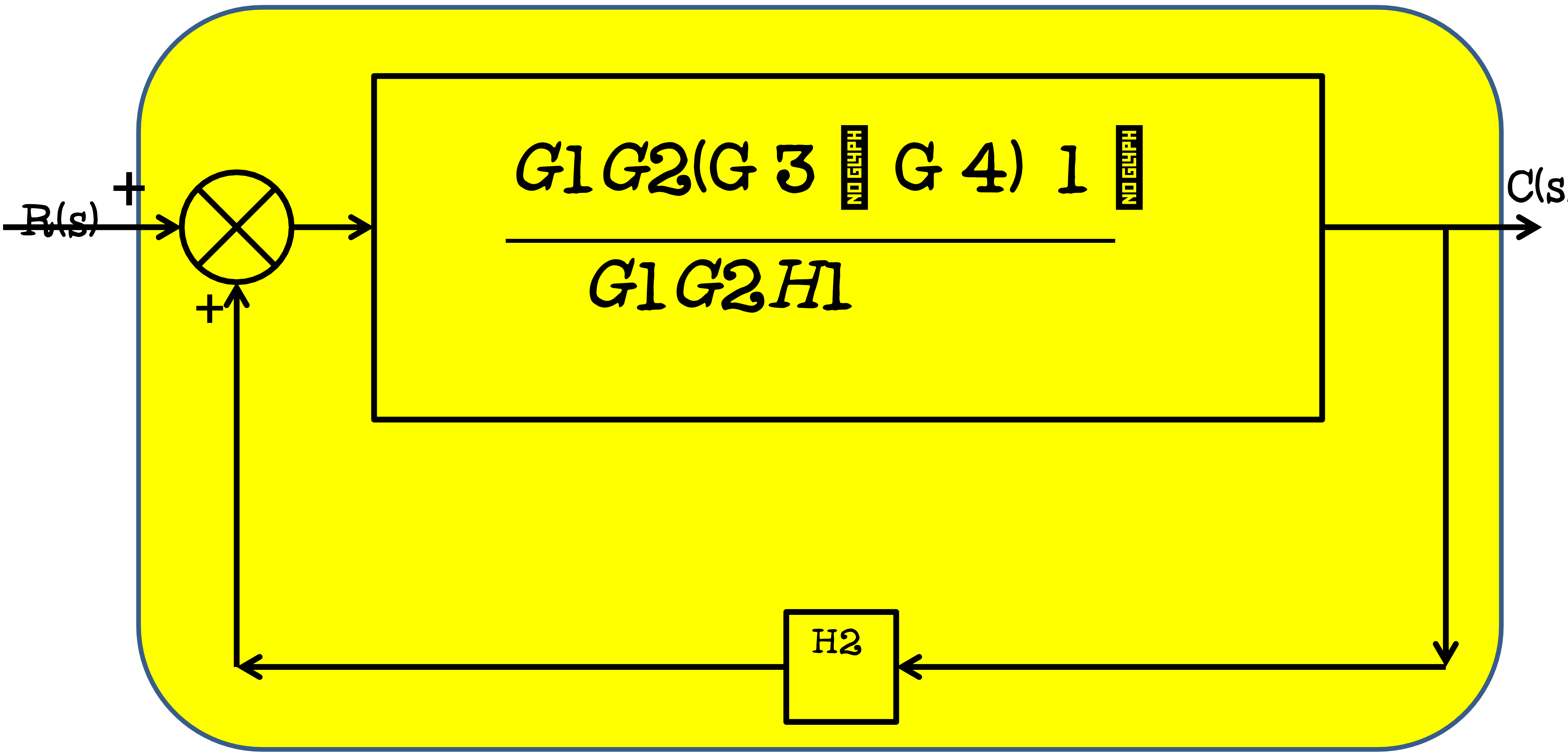
Example 2 cont….

Apply Rule 2 Blocks in series



Example 2 cont….

Apply Rule 3 Elimination of feedback loop

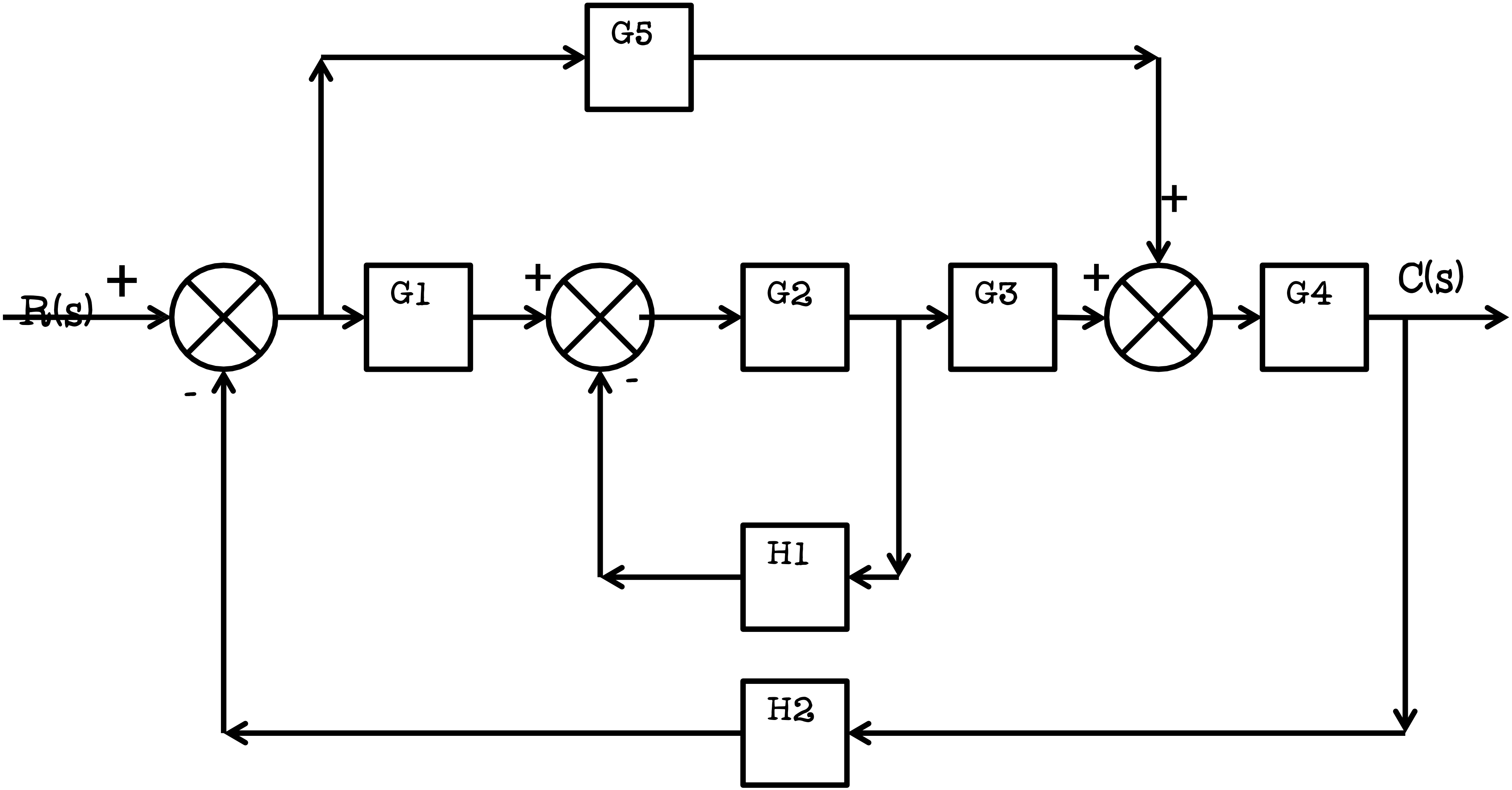


Example 2 cont....

Example 2 cont....

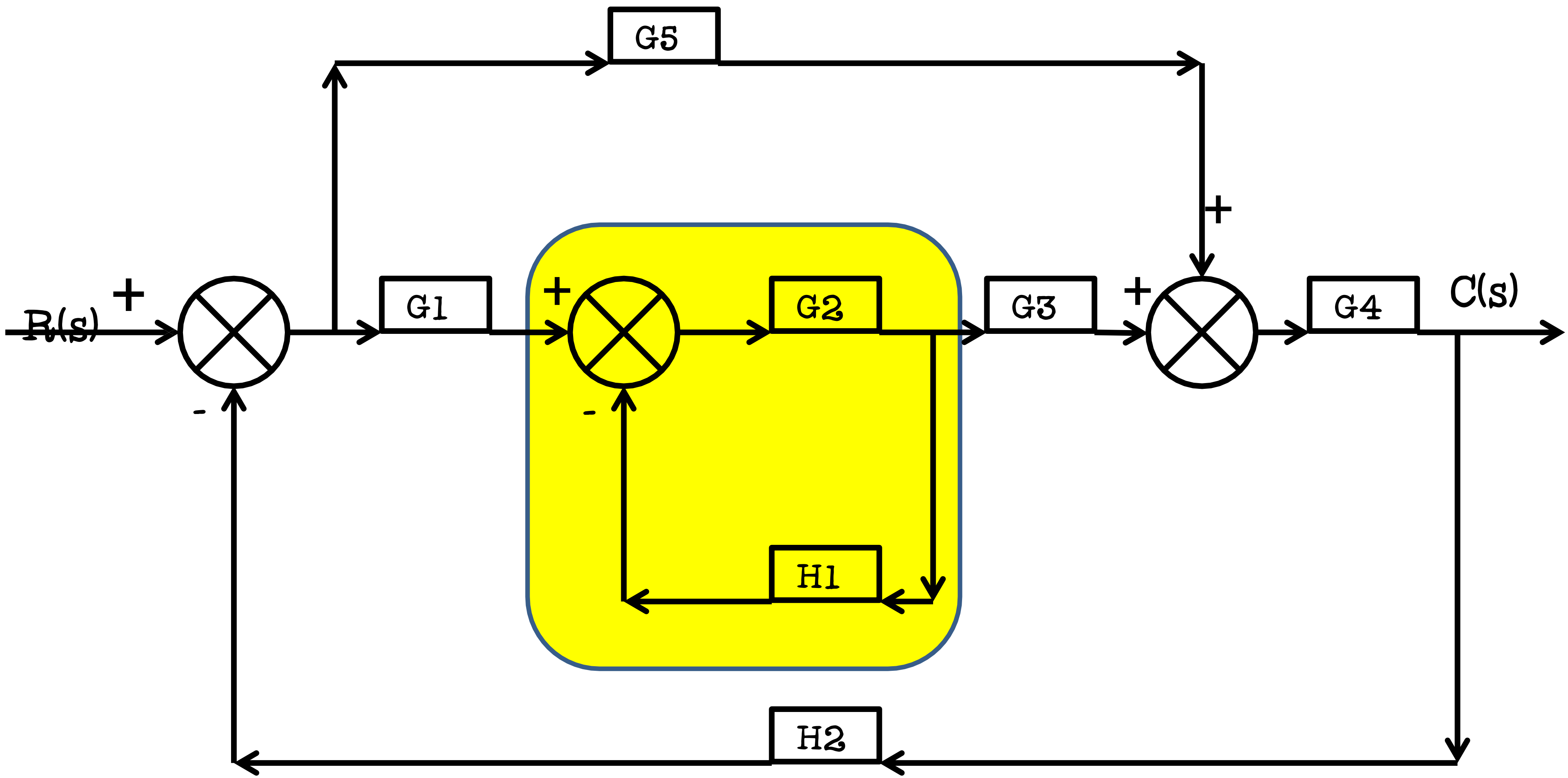
$$\frac{C(s)}{R(s)} \square \frac{G1G2(G3 \square G4)}{1 \square G1G2H1 \square G1G2G3H2 \square G1G2G4H2}$$

Example 3



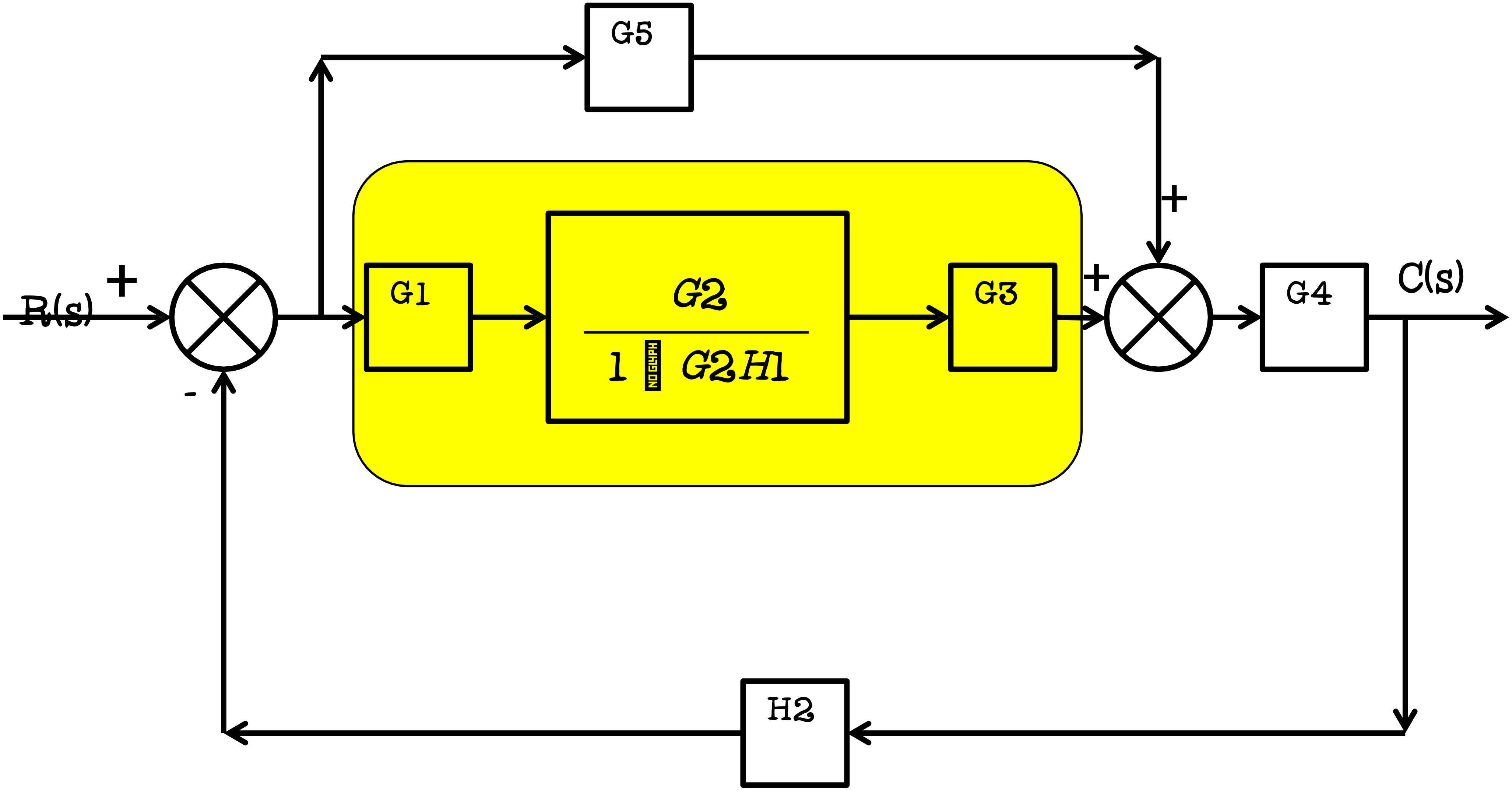
Example 3 cont…

Apply Rule 3 Elimination of feedback loop



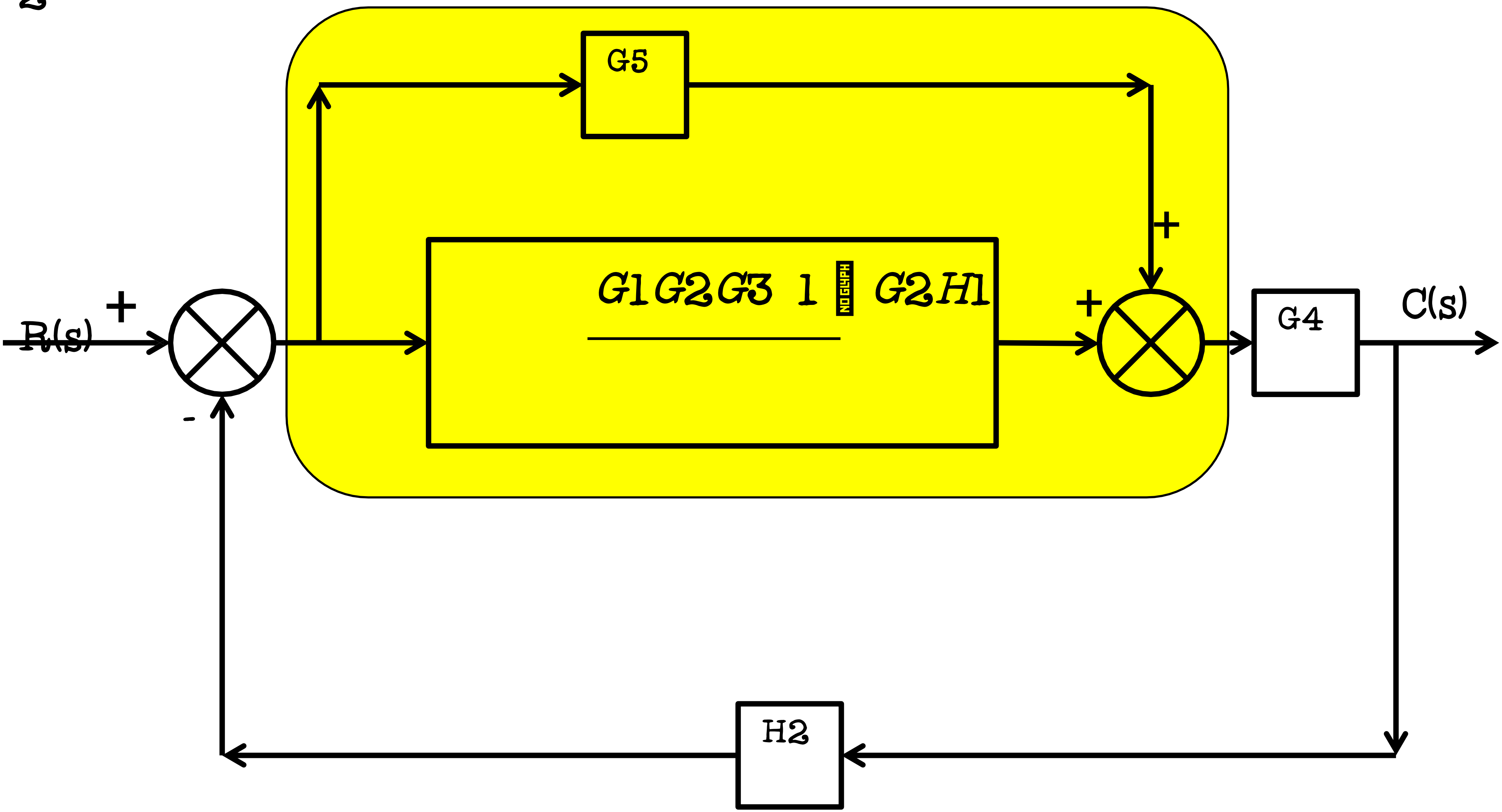
Example 3 cont....

Apply Rule 1 Blocks in series

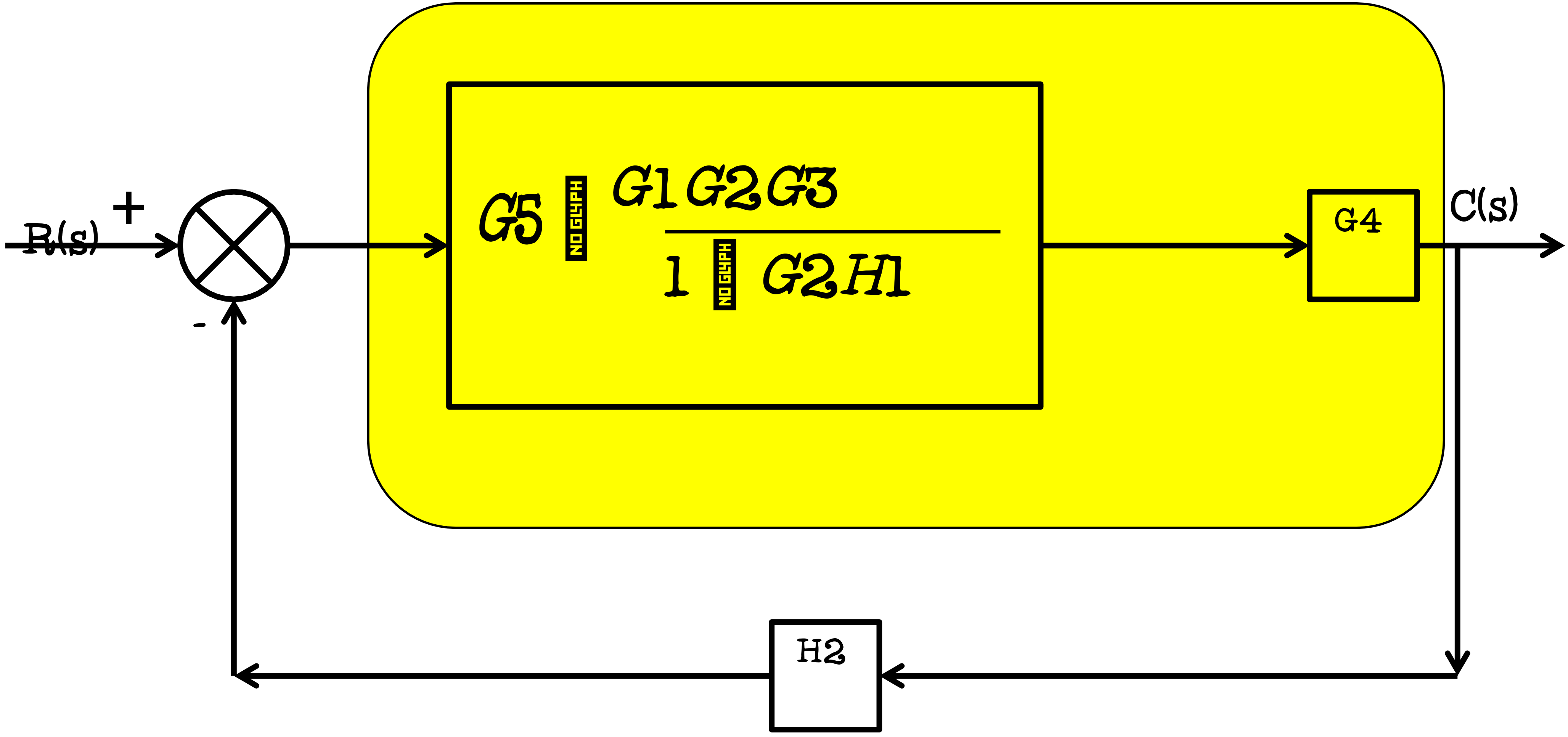


Example 3 cont...

Apply Rule 2 Blocks in parallel

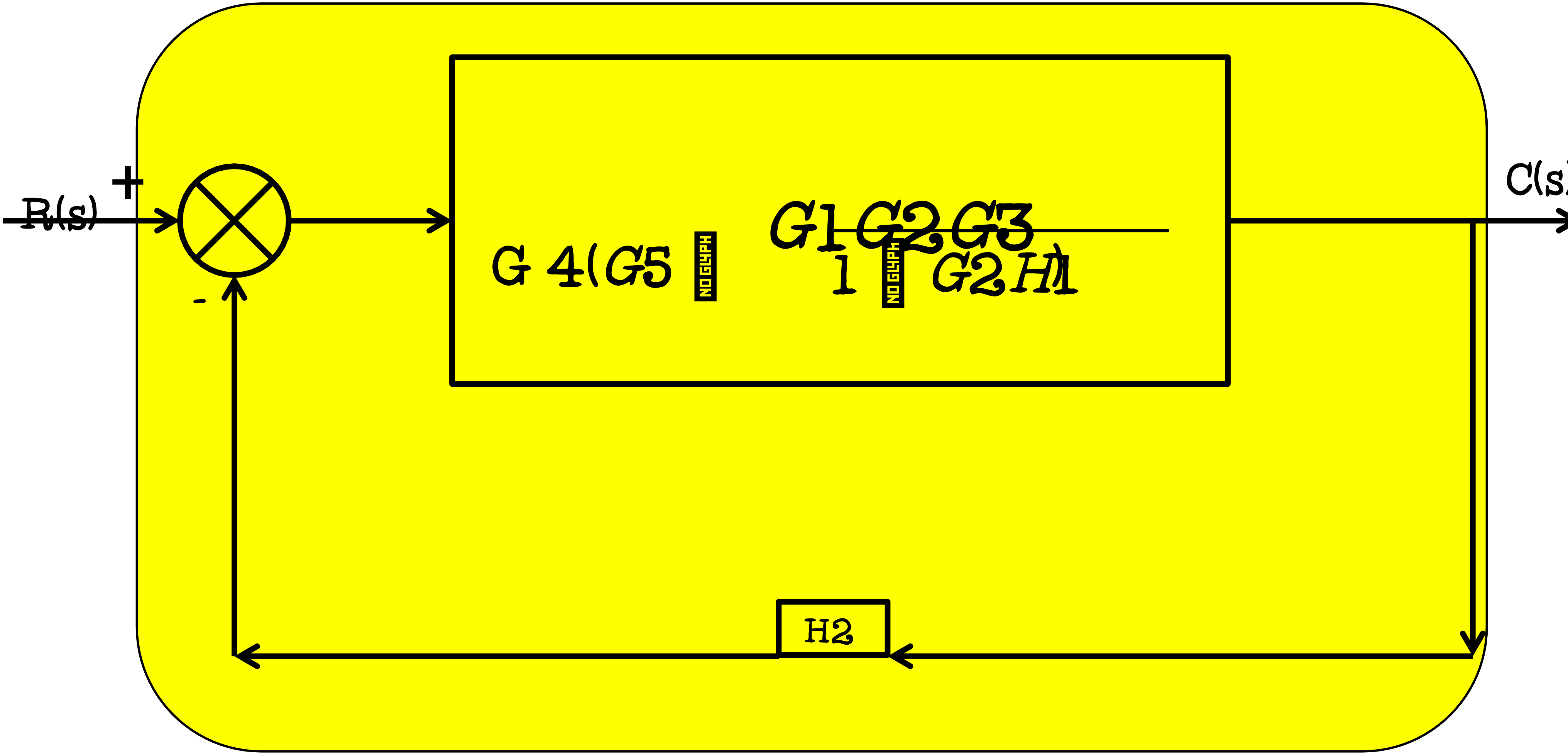


Apply Rule 1 Blocks in series



Example 3 cont....

Apply Rule 3 Elimination of feedback loop



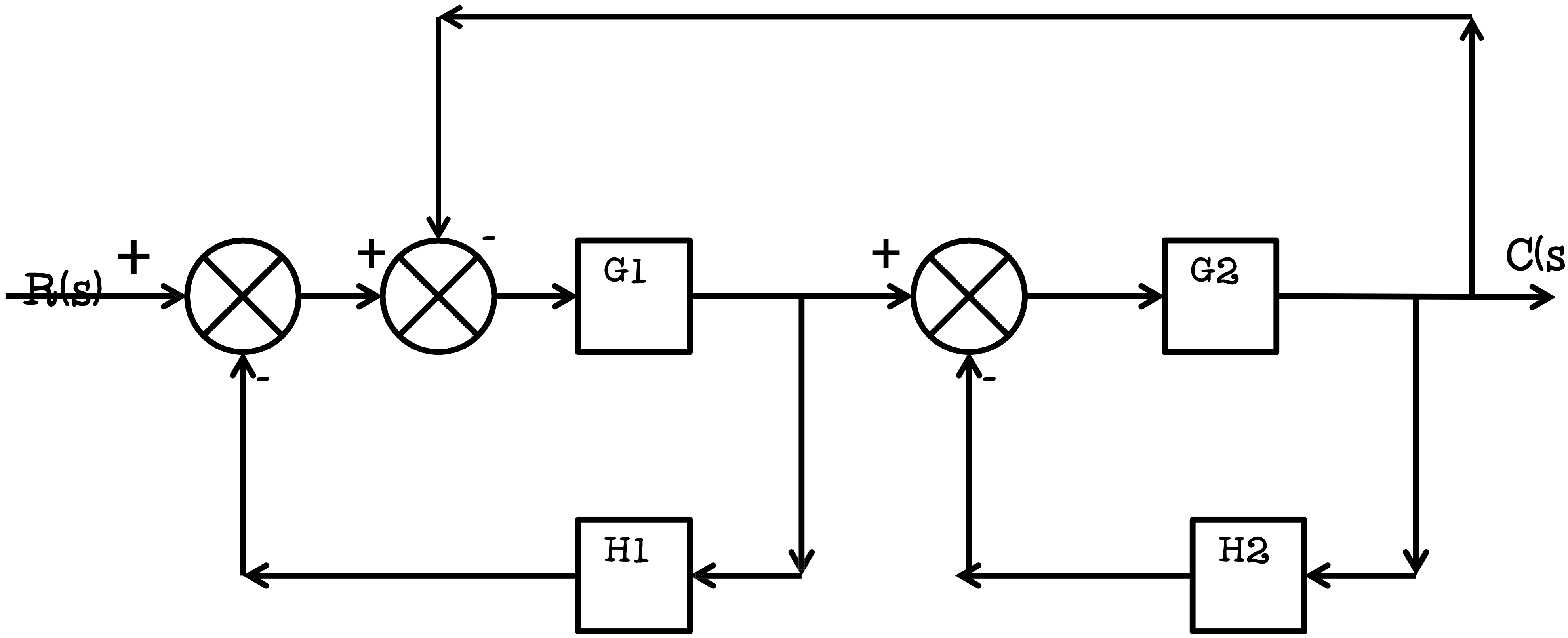
Example 3 cont….

$$R(s) \longrightarrow \boxed{\dfrac{G4G5 \text{[NO GLYPH]} G2G4G5H1 \text{[NO GLYPH]} G1G2G3G4}{1 \text{[NO GLYPH]} G2H1 \text{[NO GLYPH]} G4G5H2 \text{[NO GLYPH]} G2G4G5H1H2 \text{[NO GLYPH]} G1G2G3G4H2}} \longrightarrow C(s)$$

Example 3 cont….

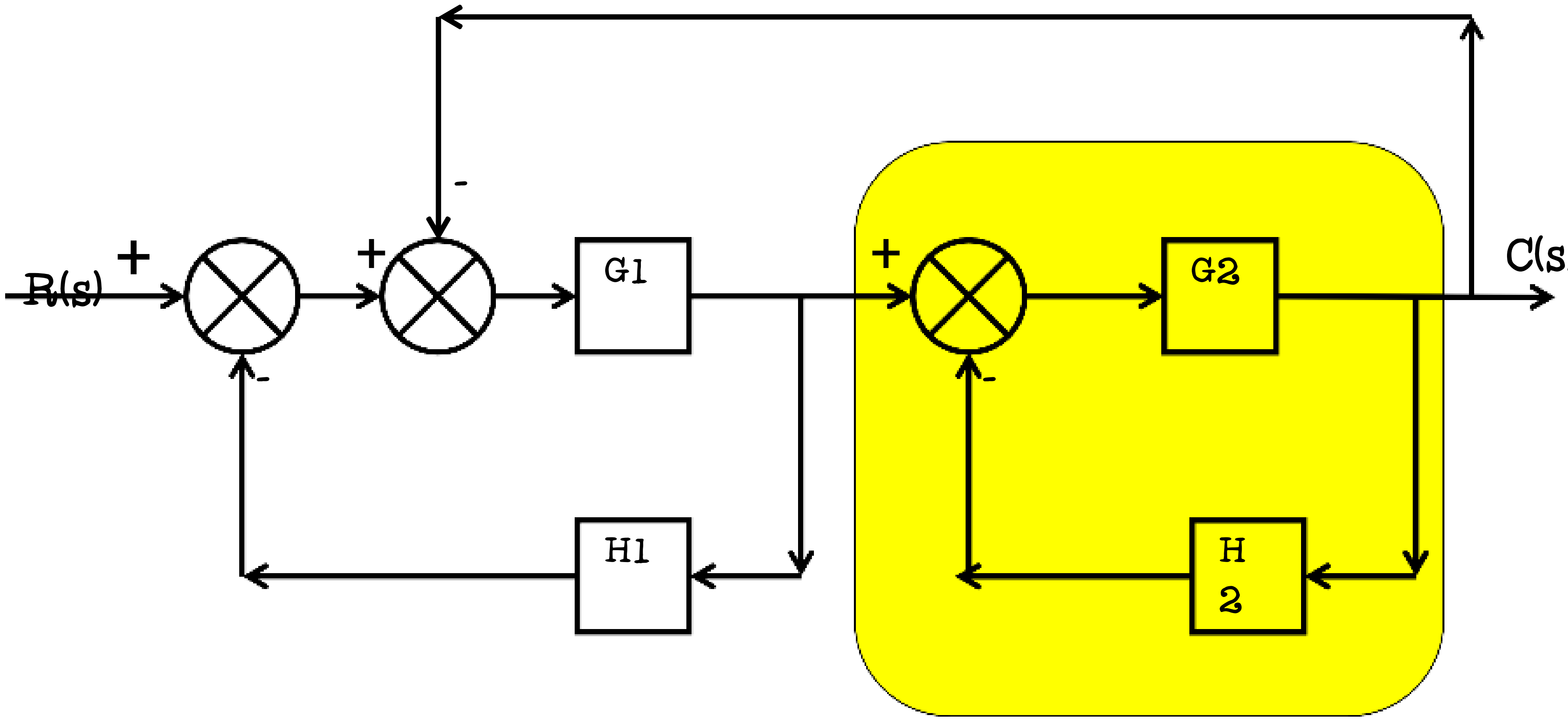
$$\frac{C(s)}{R(s)} \square \; \frac{G4G5 \square G2G4G5H1 \square G1G2G3G4}{1 \square G2H1 \square G4G5H2 \square G2G4G5H1H2 \square G1G2G3G4H2}$$

Example 4

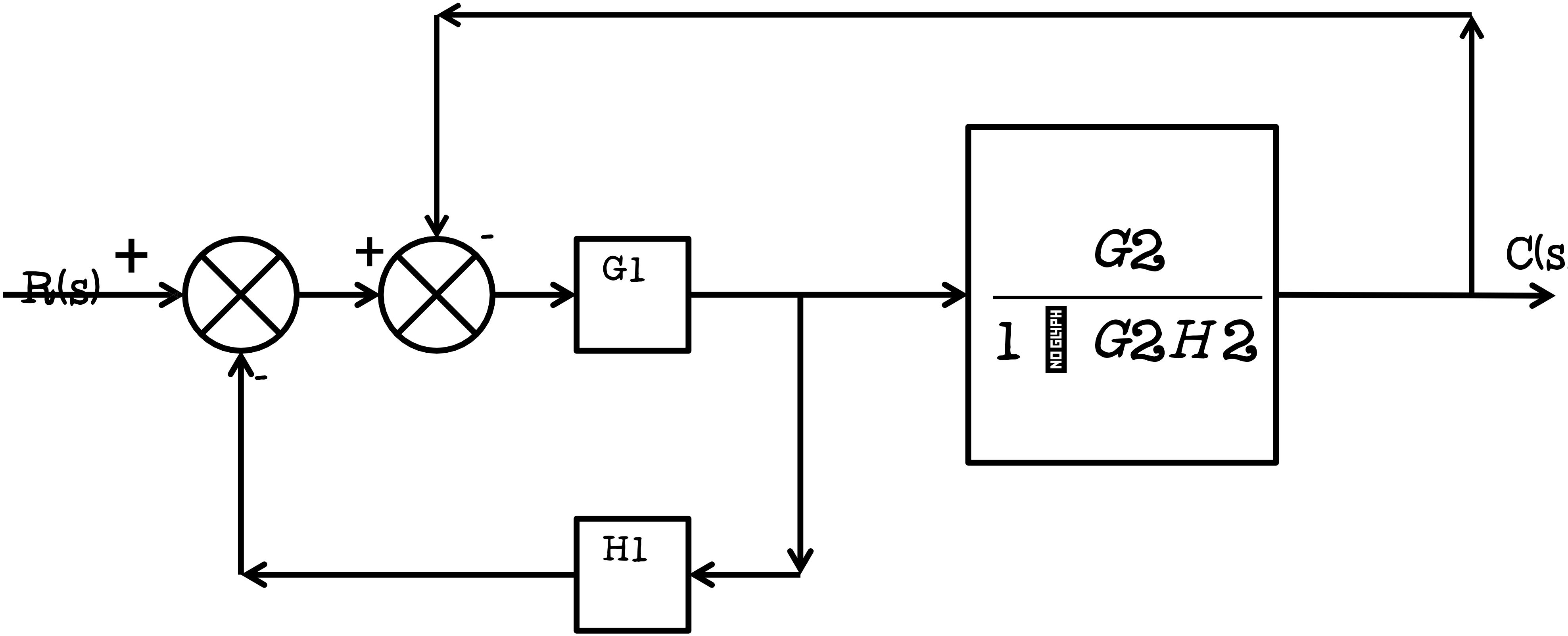


Example 4 cont…

Apply Rule 3 Elimination of feedback loop



Example 4 cont....



- Now Rule 1, 2 or 3 cannot be used directly.
- There are possible ways of going ahead.

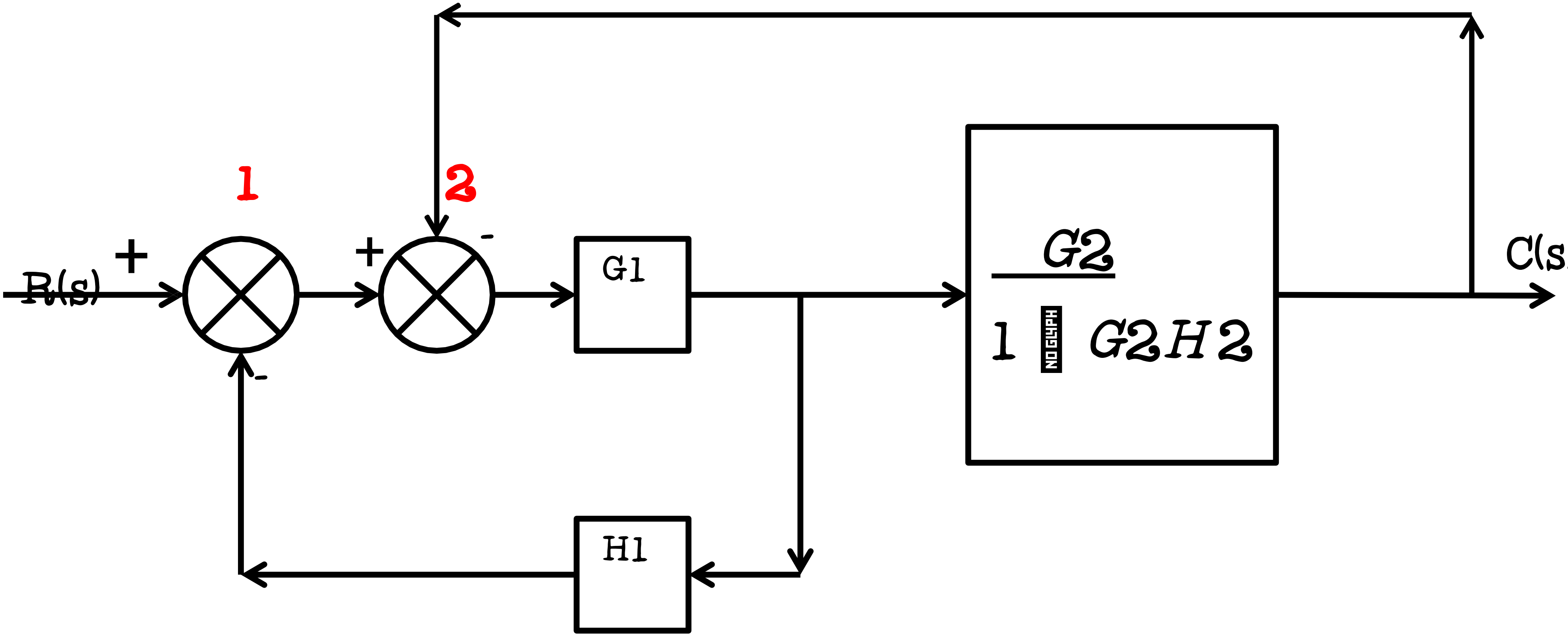
   a. Use Rule 4 & interchange order of summing so that Rule 3 can be used on G.H1 loop.

   b. Shift take off point after $\frac{G2}{1 \text{ [illegible] } G2H2}$ block reduce by Rule 1, followed by Rule 3.

<u>Which option we have to use????</u>

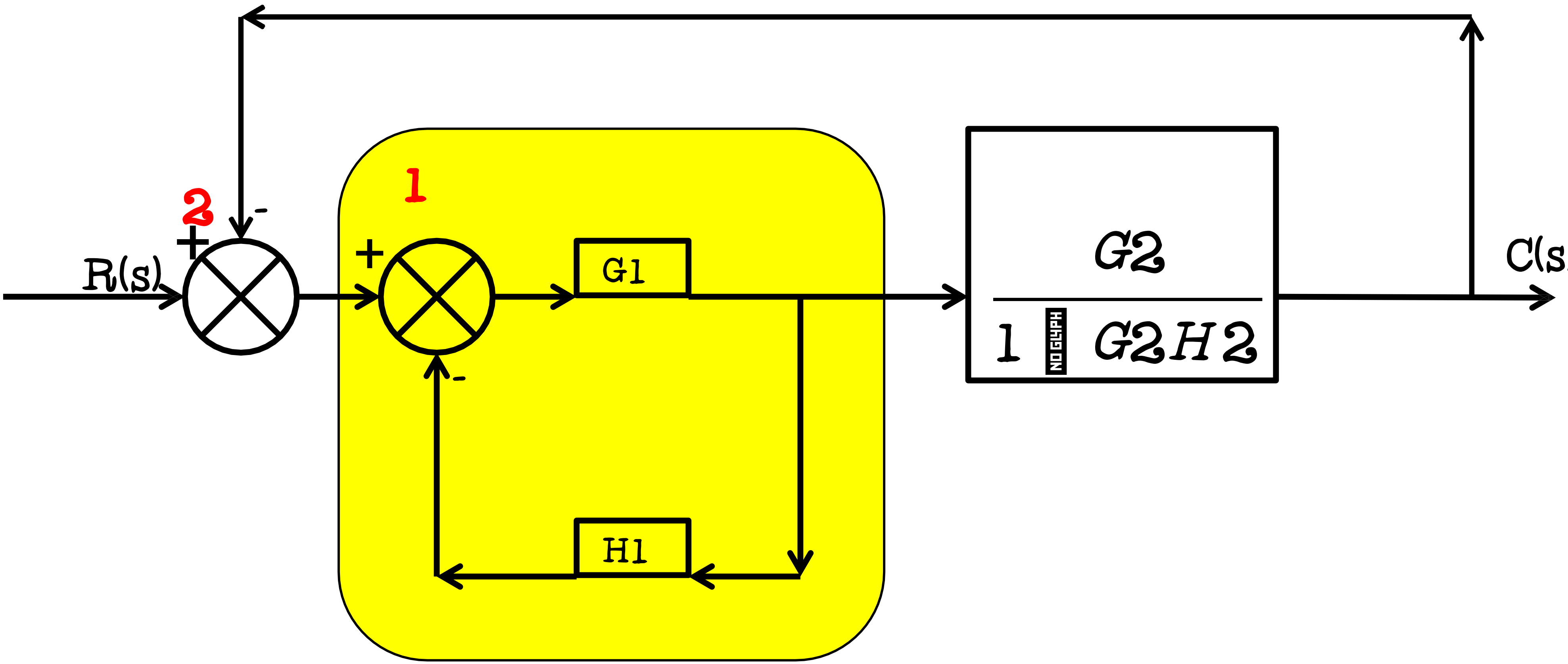
Example 4 cont…

Apply Rule 4 Exchange summing order



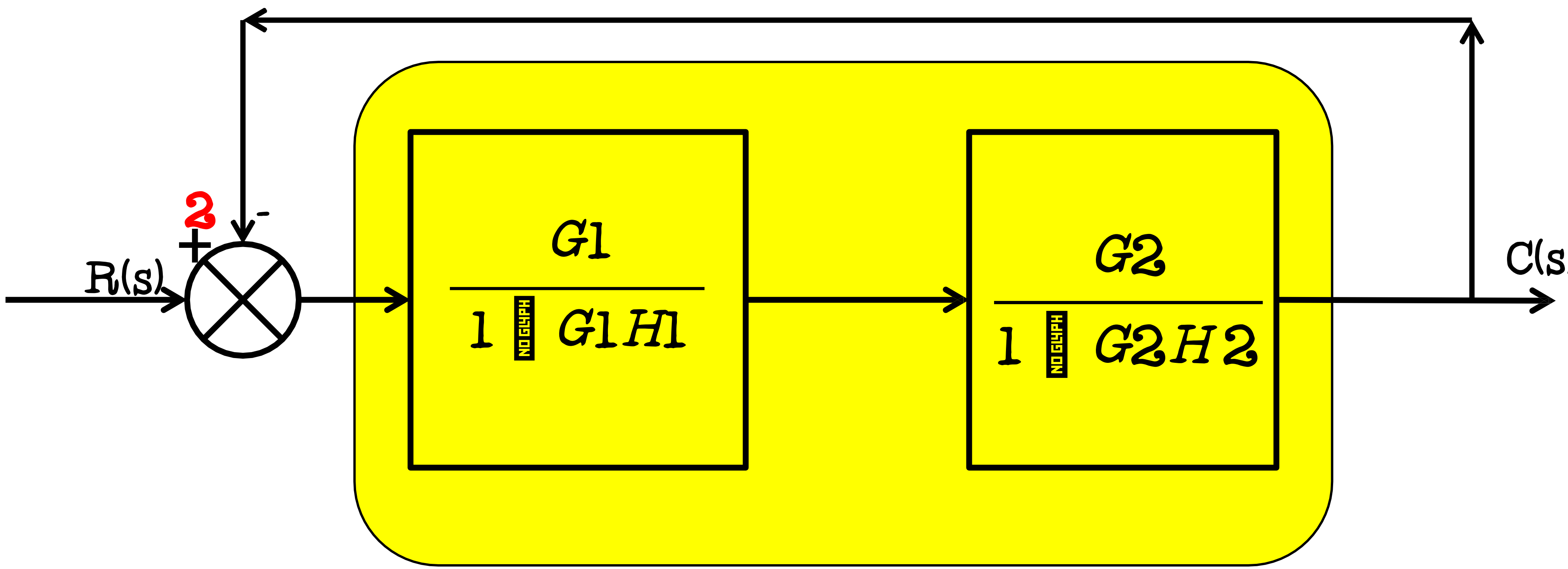
Example 4 cont….

Apply Rule 3 Elimination feedback loop

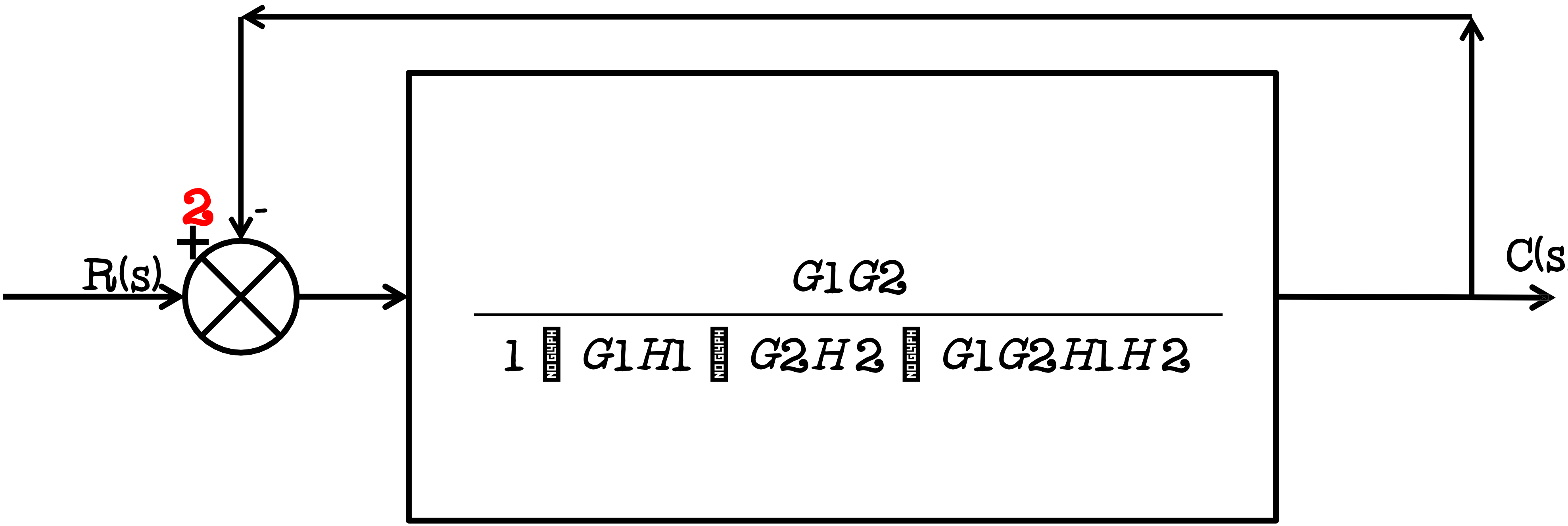


Example 4 cont….

Apply Rule 1 Bocks in series



Example 4 cont....



Now which Rule will be applied

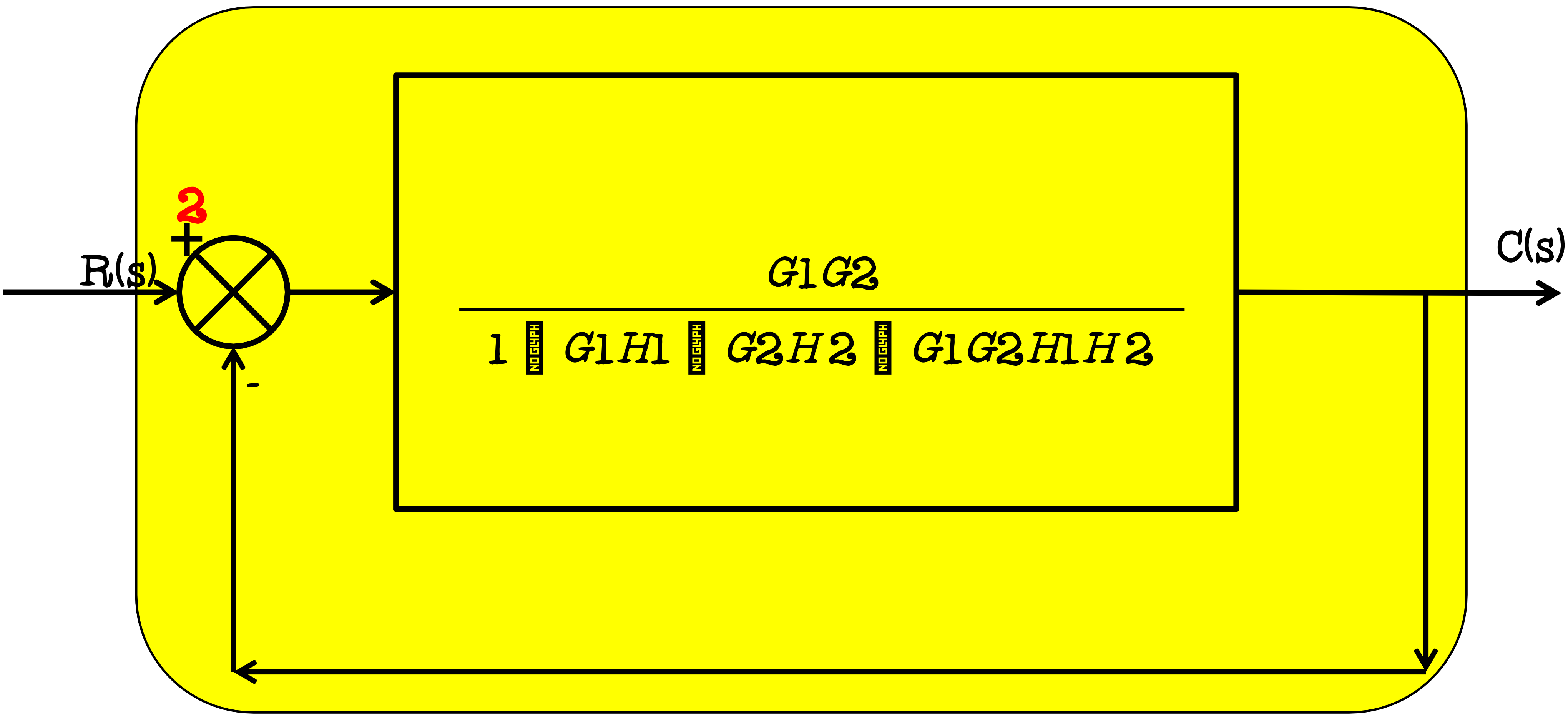
-------It is blocks in parallel **OR**

-------It is feed back loop

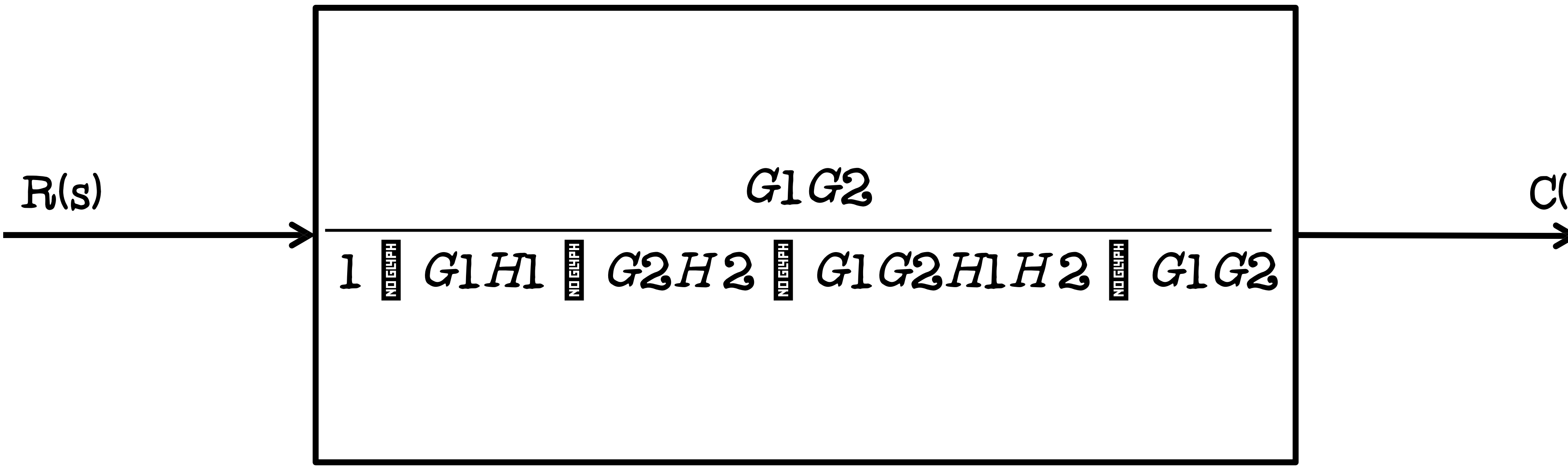
Example 4 cont....

Let us rearrange the block diagram to understand

Apply Rule 3 Elimination of feed back loop



Example 4 cont....

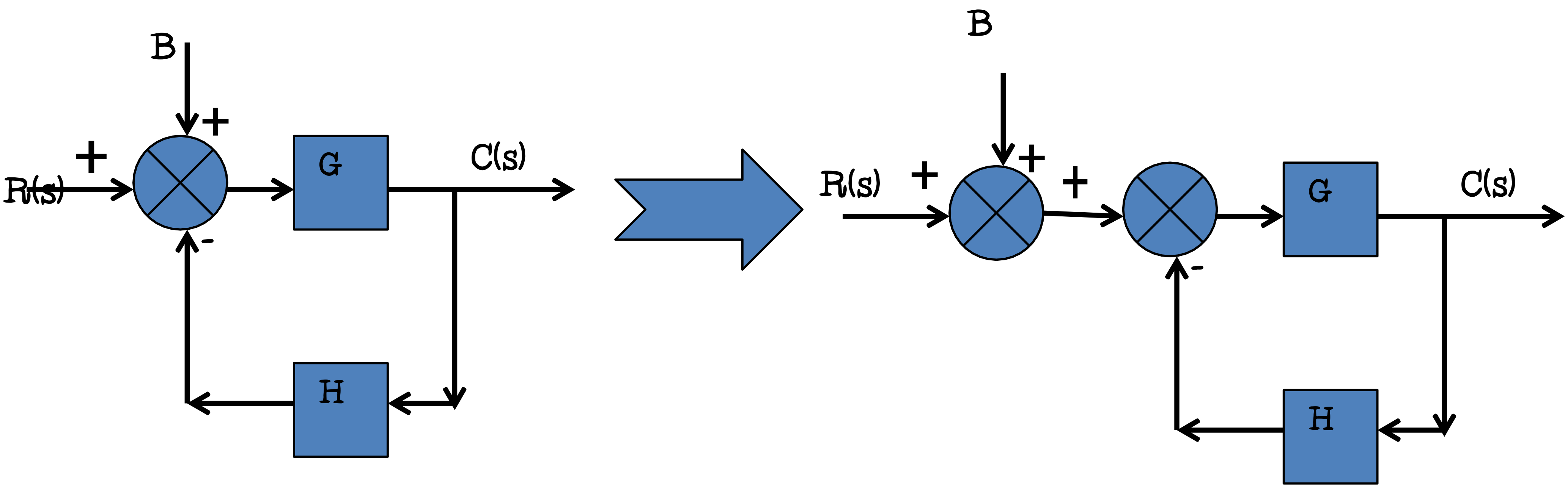


Example 4 cont....

$$\frac{C(s)}{R(s)} \square \frac{G1G2}{1 \square G1H1 \square G2H2 \square G1G2H1H2 \square G1G2}$$

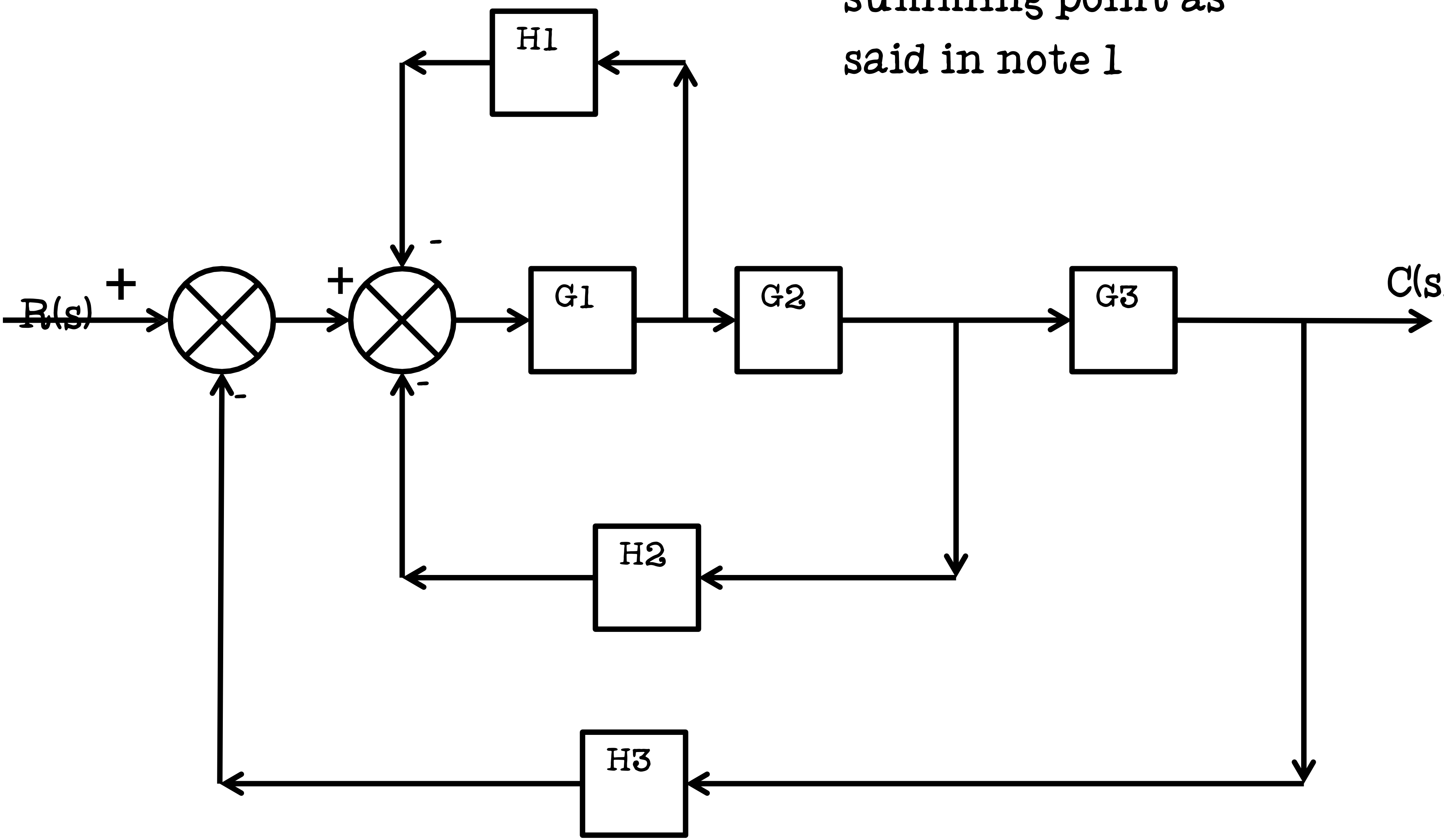
Note 1: According to Rule 4

- By corollary, one can split a summing point to two summing point and sum in any order



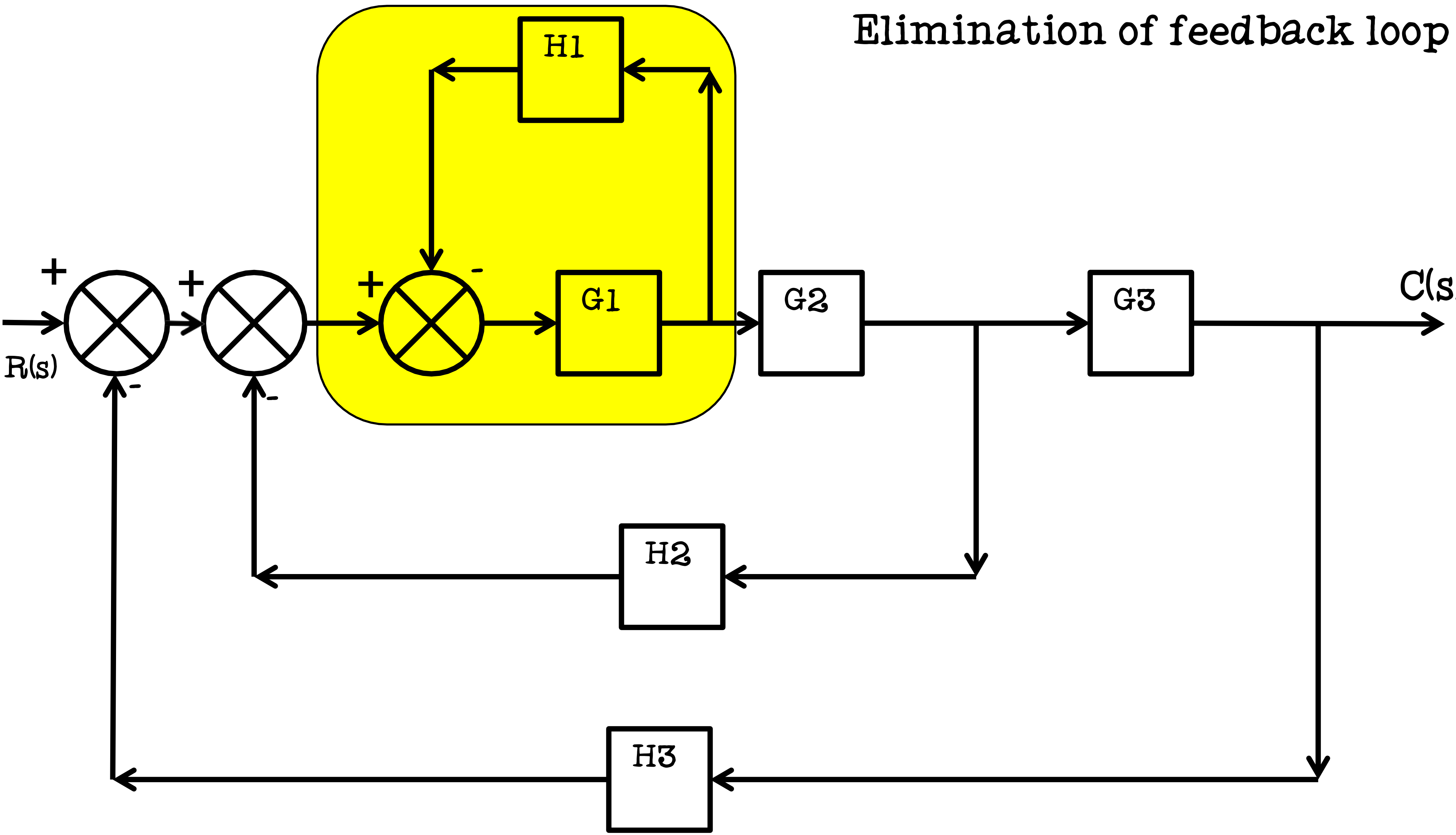
Example 5

Simplify, by splitting second summing point as said in note 1



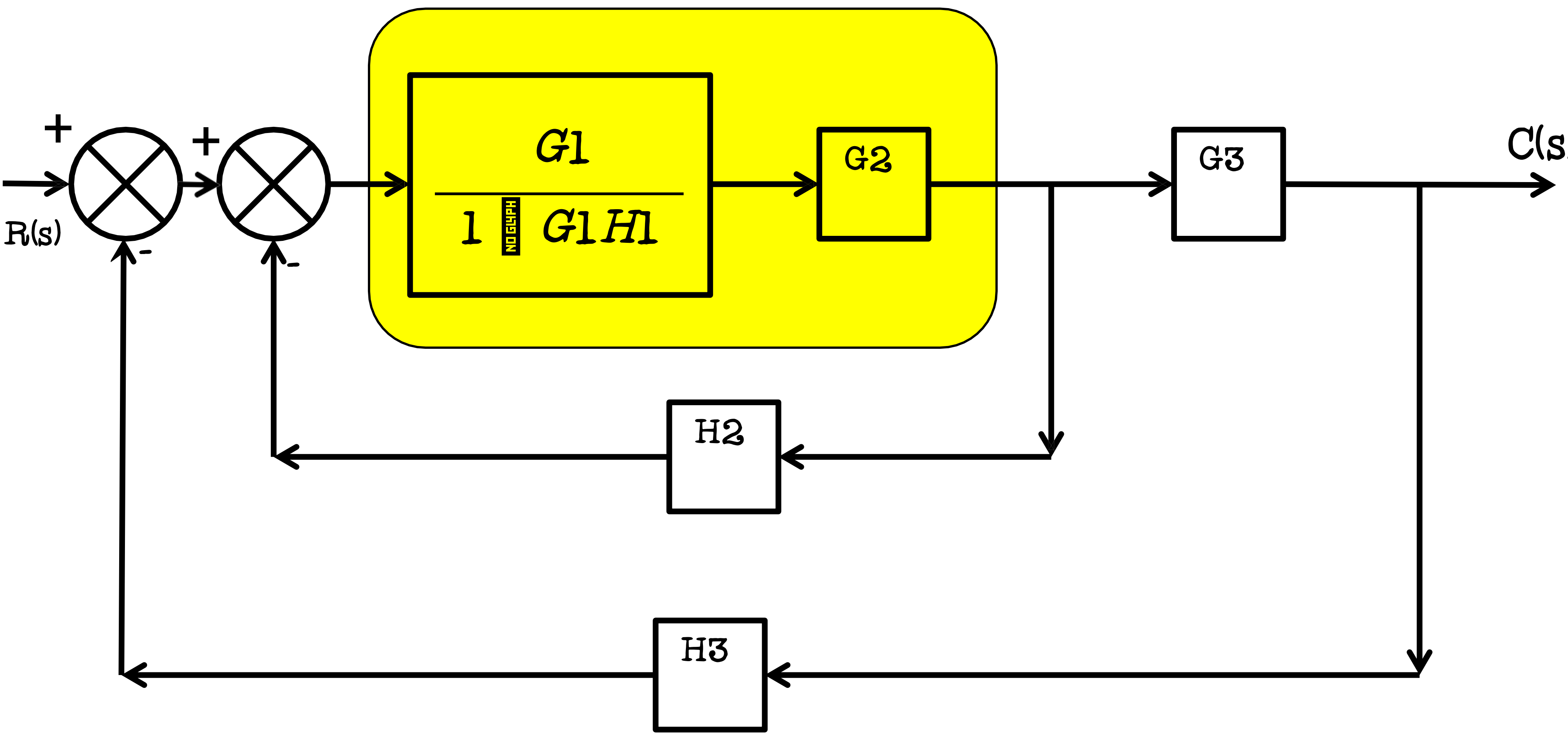
Apply rule 3

Elimination of feedback loop



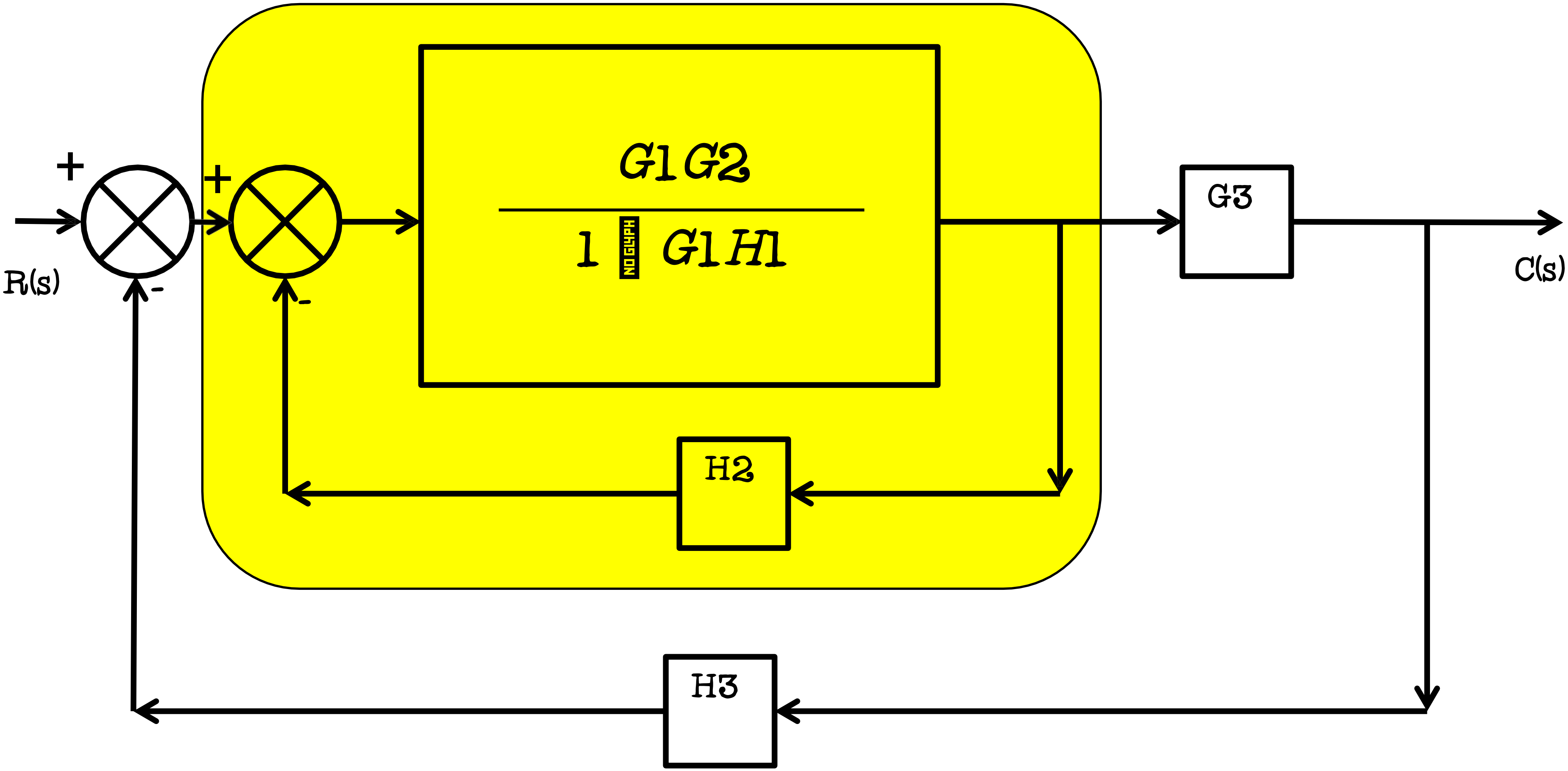
Example 5 cont….

Apply rule 1 Blocks in series



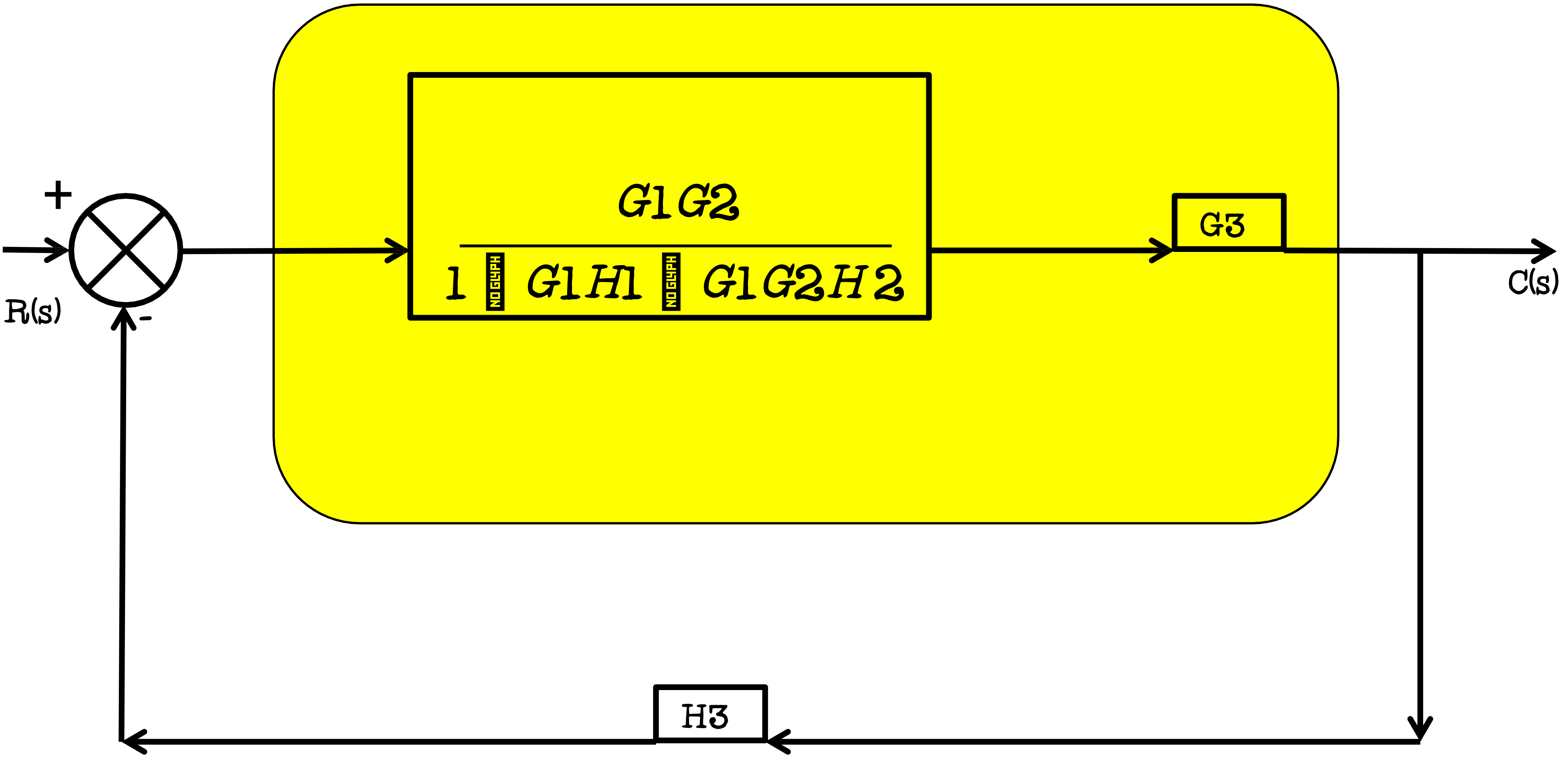
Example 5 cont…

Apply rule 3 — Elimination of feedback loop



Example 5 cont...

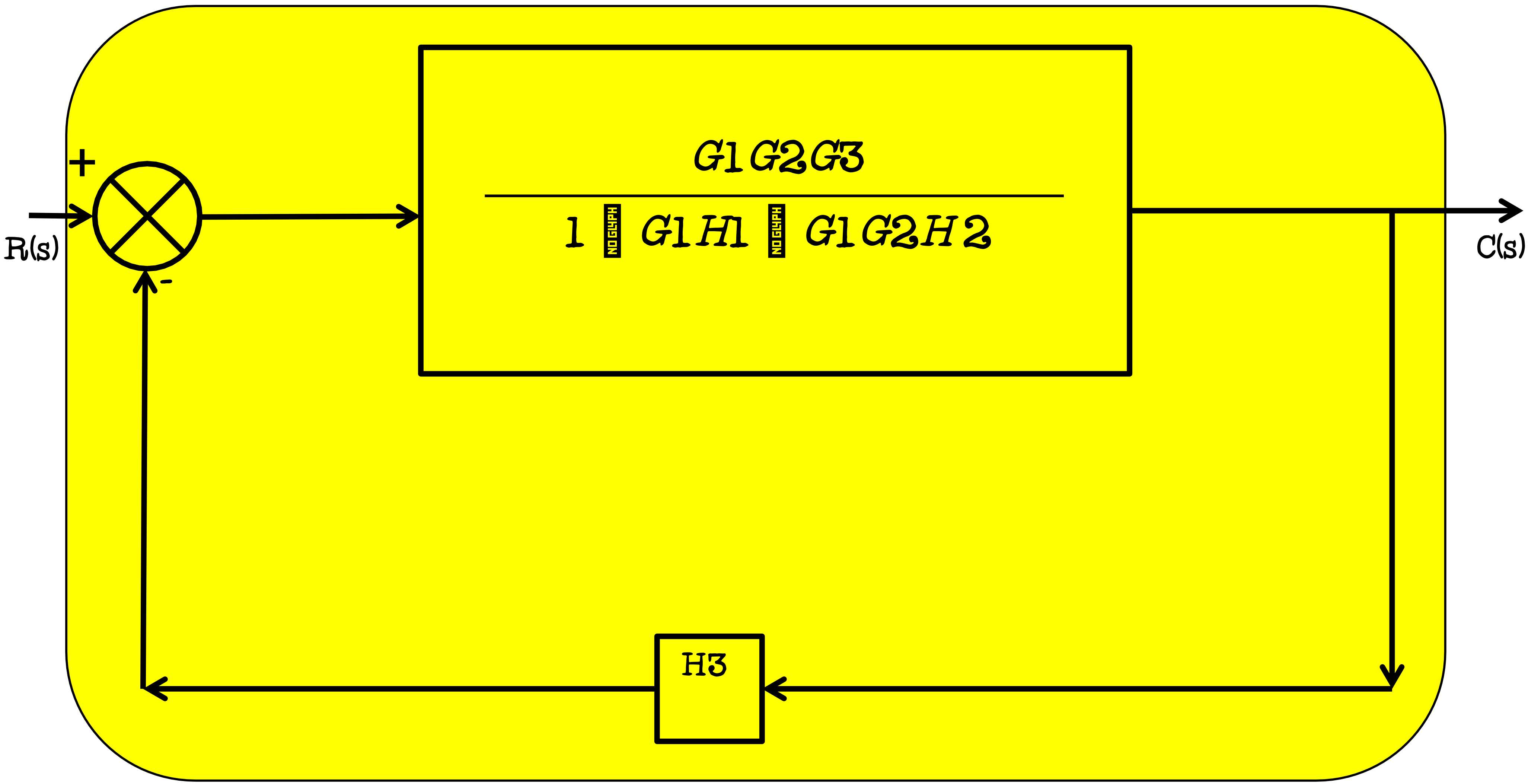
Apply rule 1 Blocks in series



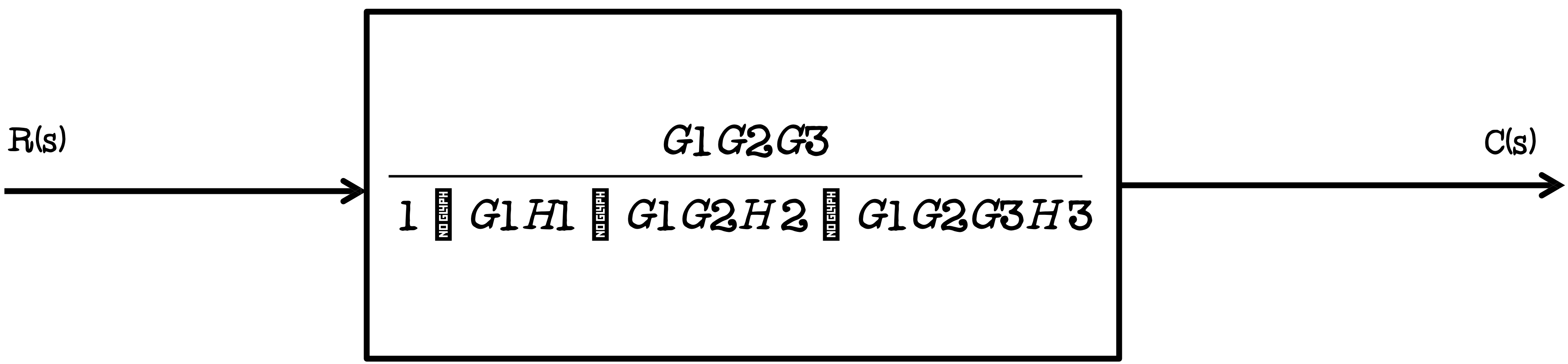
Example 5 cont....

Apply rule 3

Elimination of feedback loop



Example 5 cont....



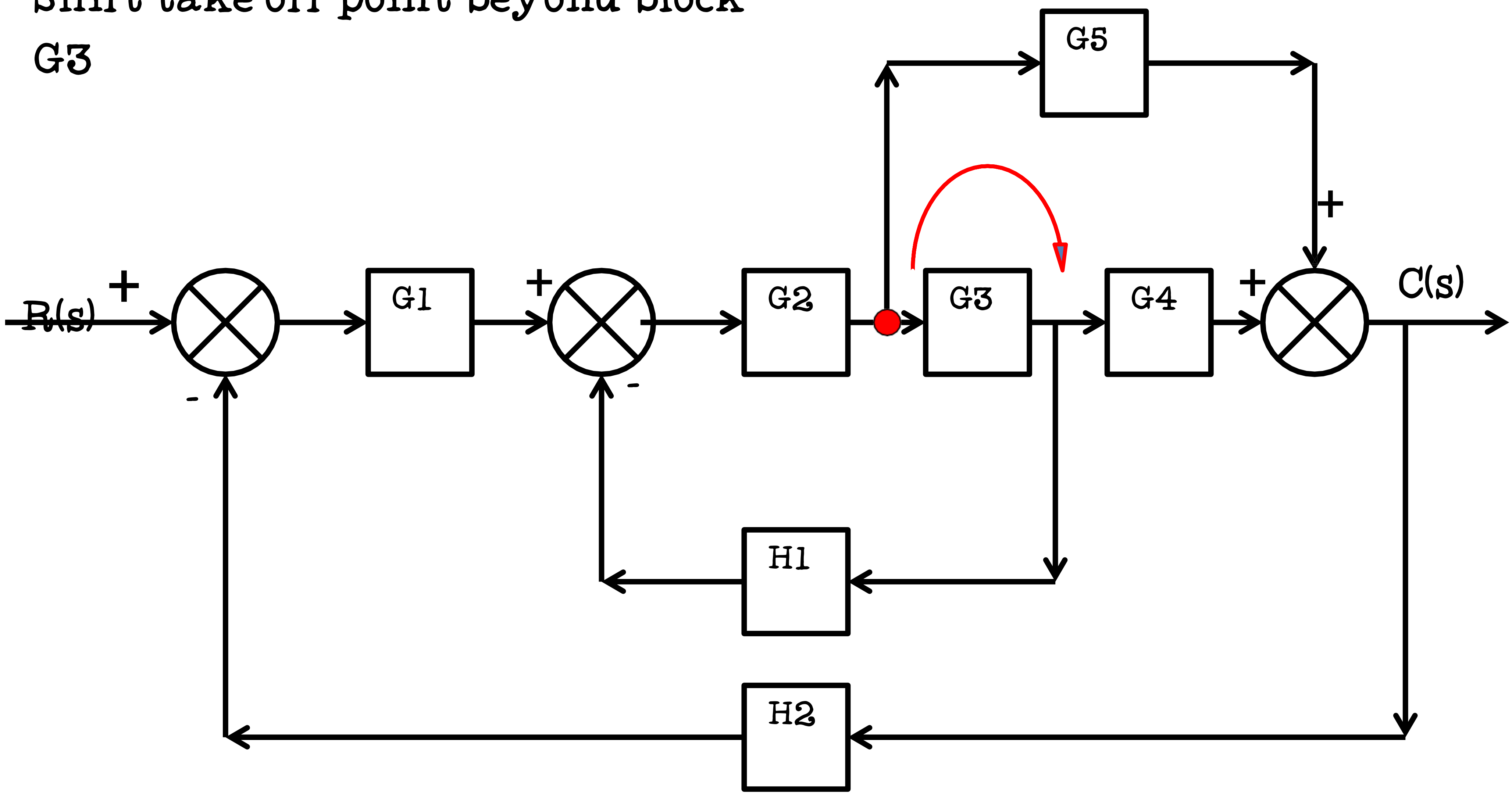
Example 5 cont....

$$\frac{C(s)}{R(s)} \text{NO GLYPH} \frac{G1G2G3}{1 \text{ NO GLYPH } G1H1 \text{ NO GLYPH } G1G2H2 \text{ NO GLYPH } G1G2G3H3}$$

Example 6

Apply rule 8

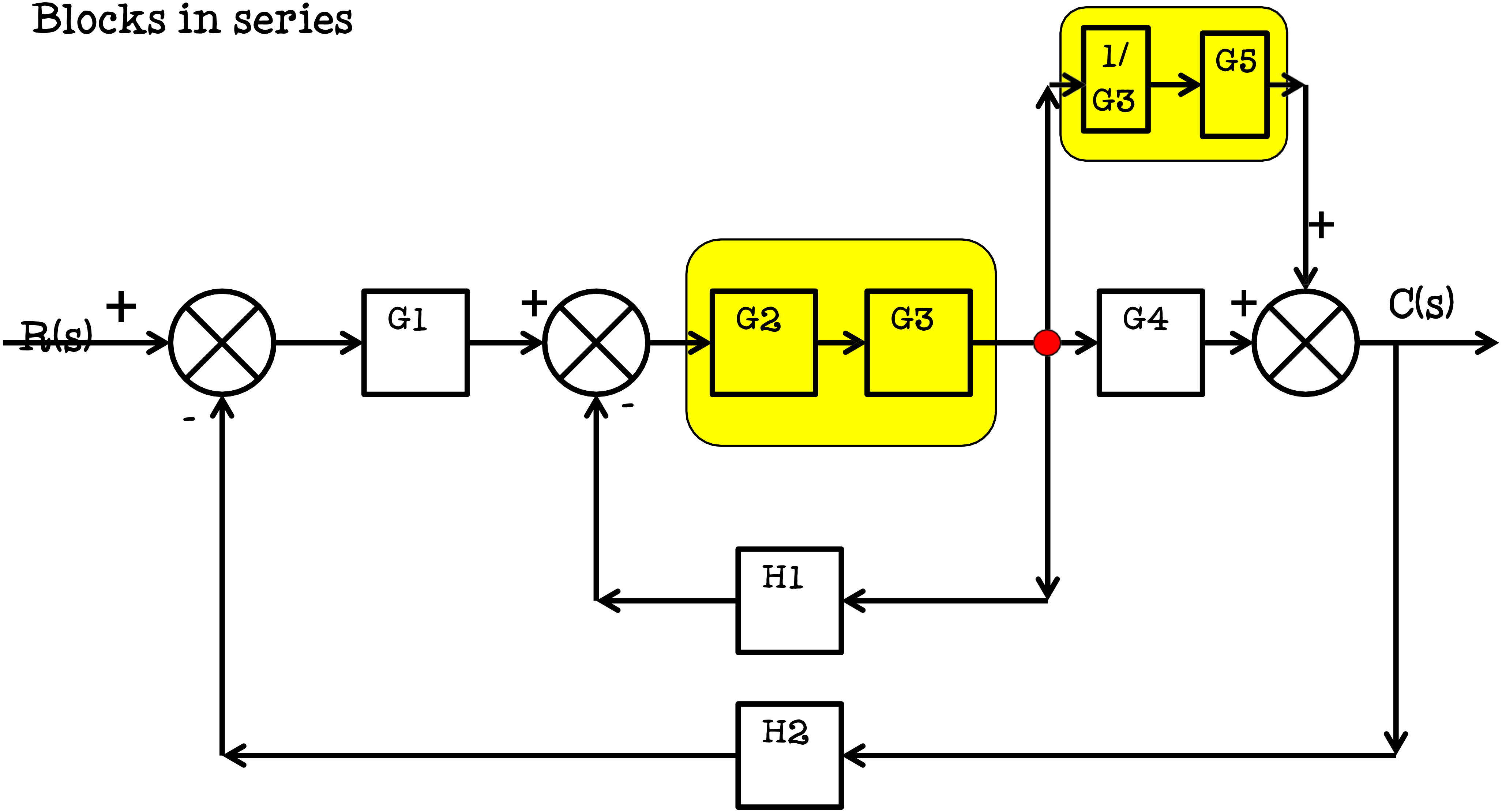
Shift take off point beyond block G3



Example 6 cont....

Apply rule 1

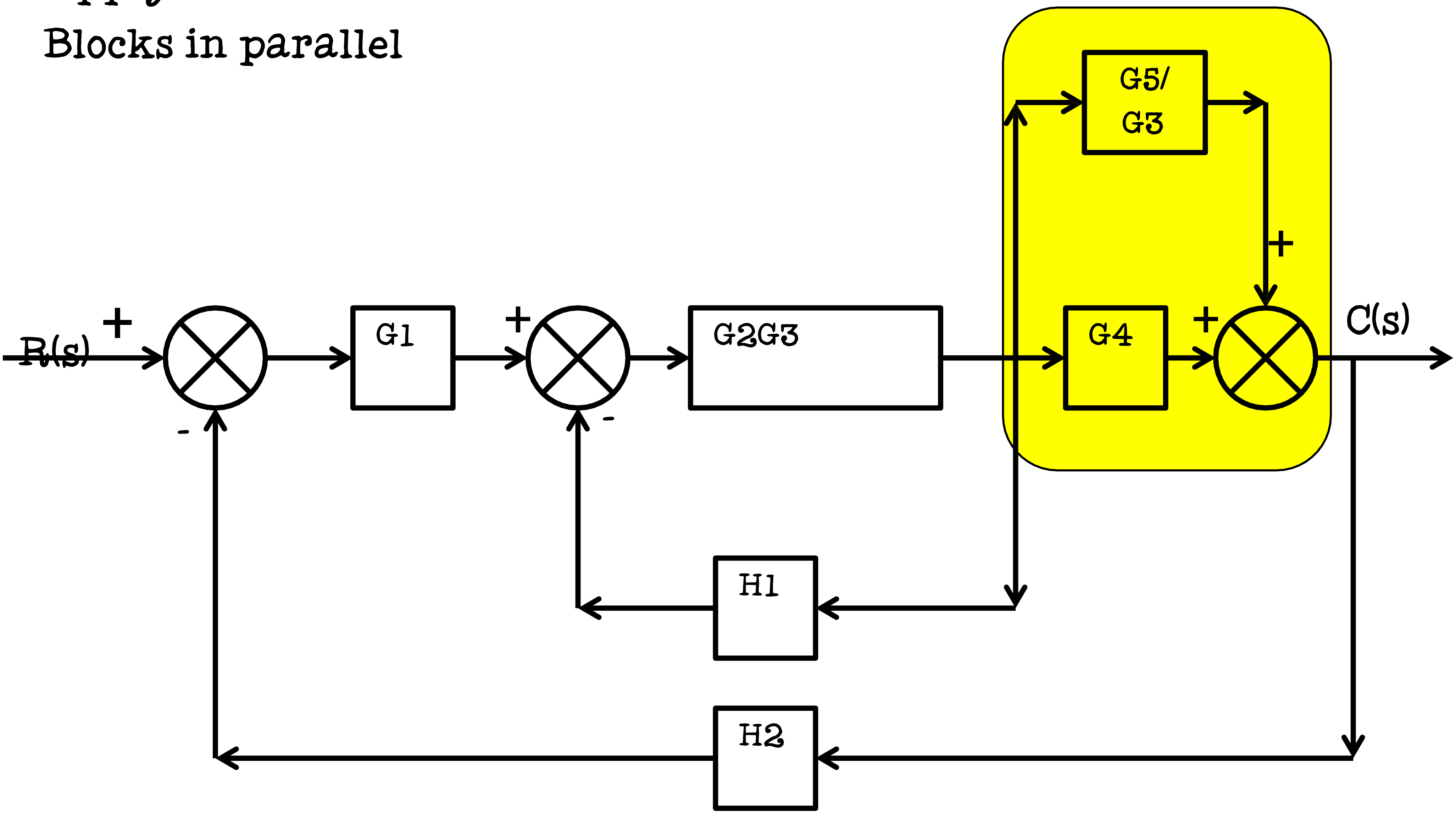
Blocks in series



Example 6 cont…

Apply rule 2

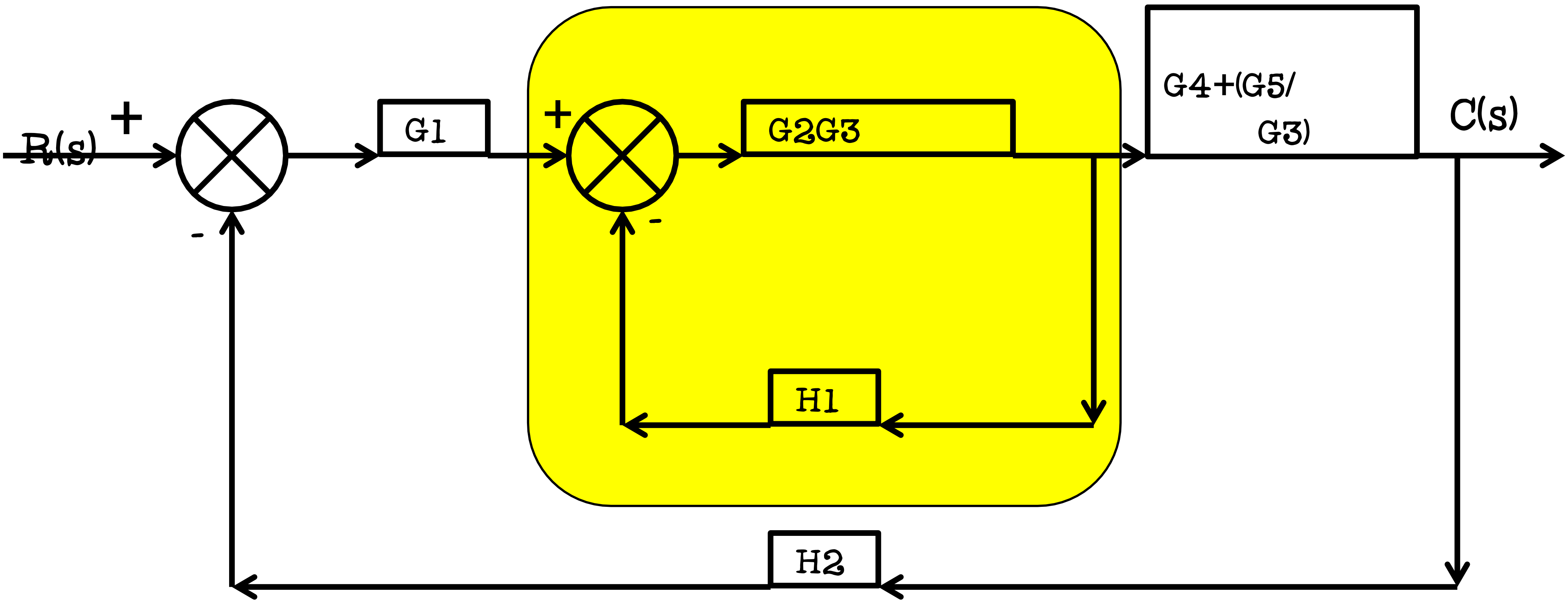
Blocks in parallel



Example 6 cont....

Apply rule 3

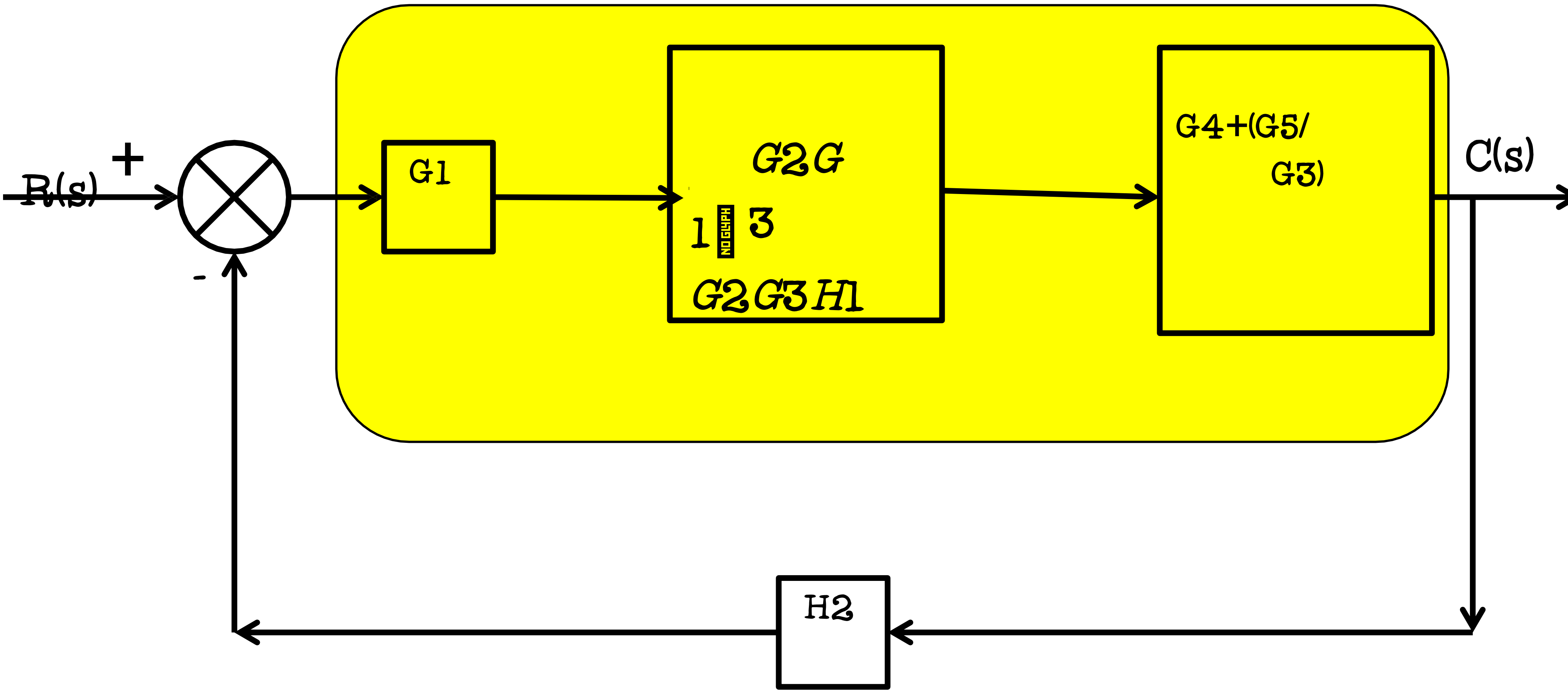
Feedback loop



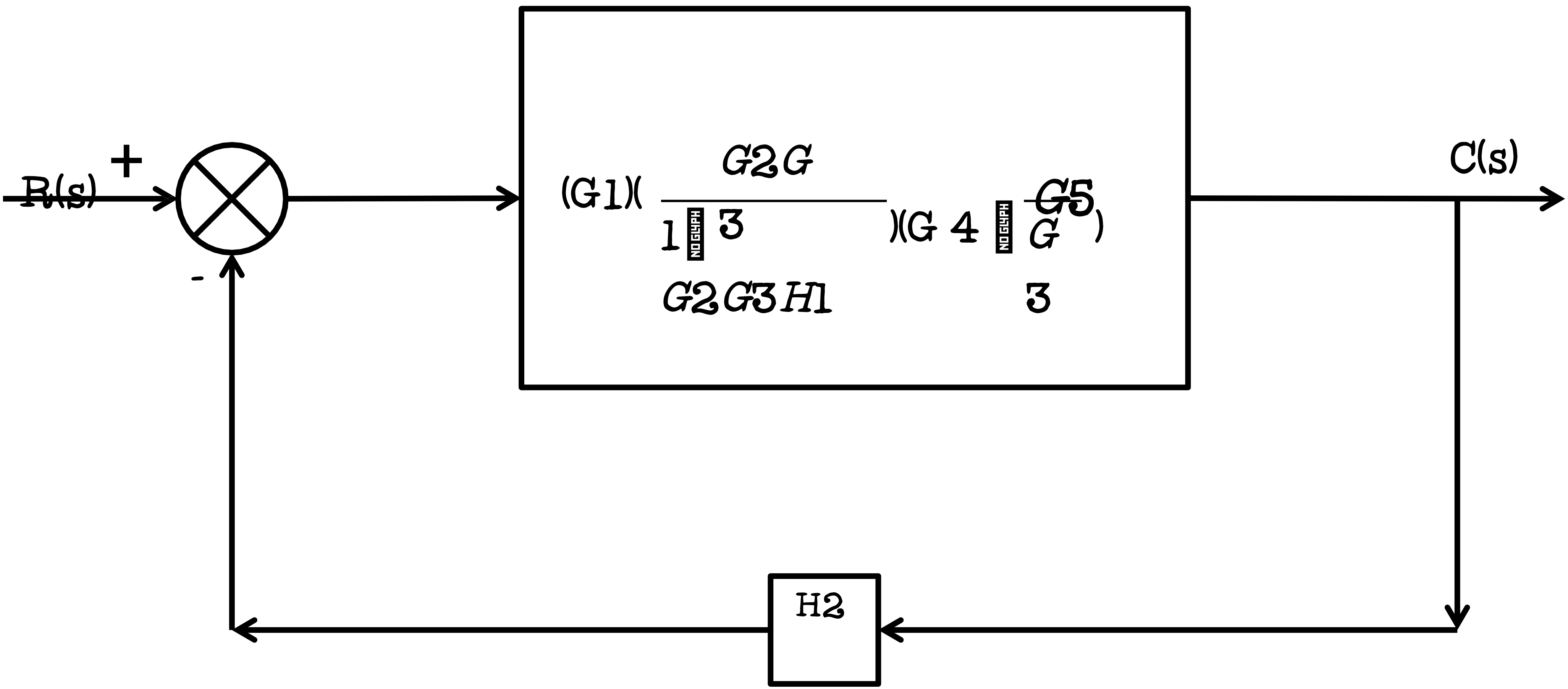
Example 6 cont....

Apply rule 1

Blocks in series



Example 6 cont....



Example 6 cont….

$$\text{NO GLYPH}\,(G1)\left(\frac{G2G3}{1\;\text{NO GLYPH}\;G2G3H1}\right)\left(G4\;\text{NO GLYPH}\;\frac{G5}{G3}\right)$$

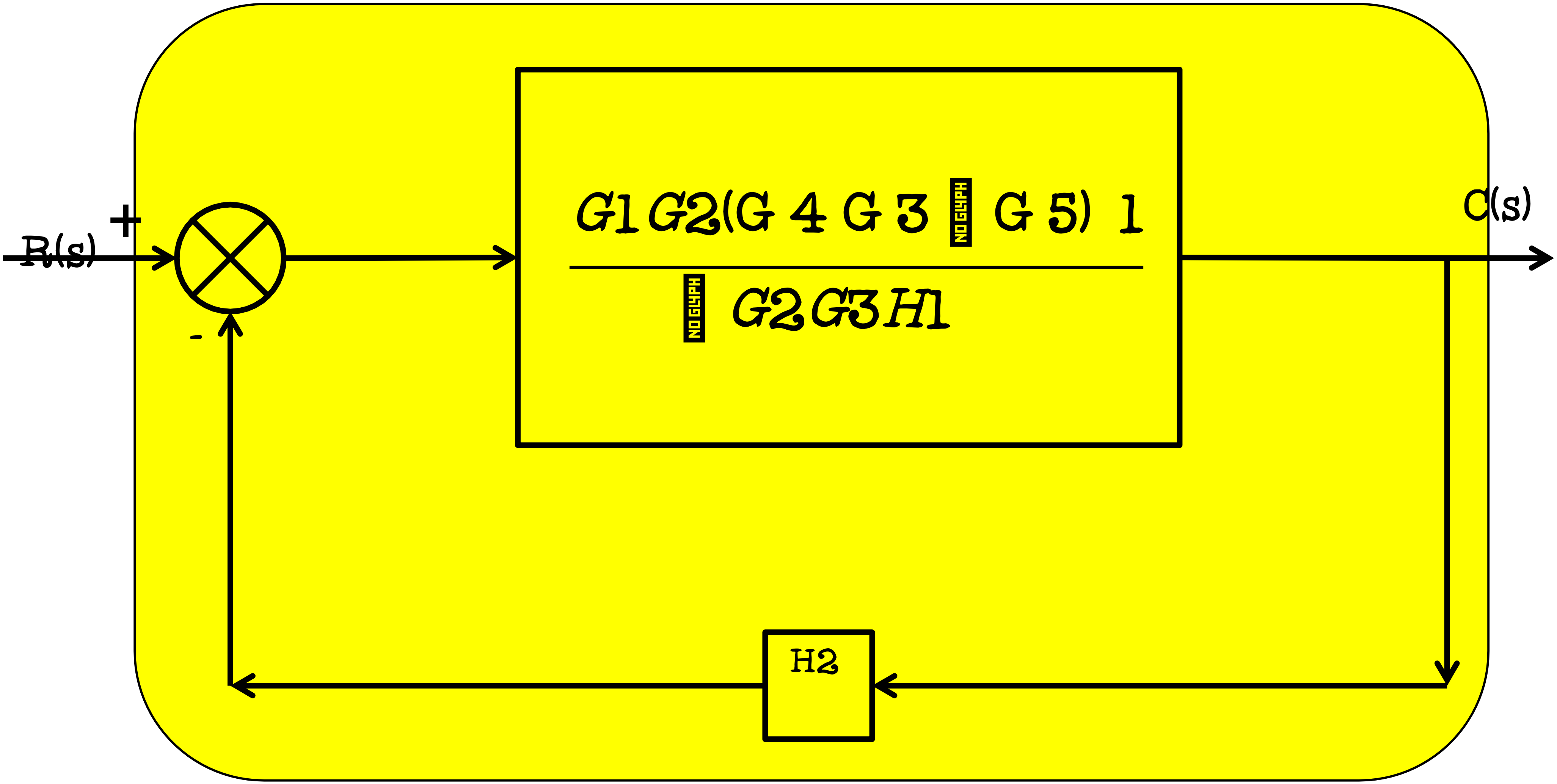
$$\text{NO GLYPH}\,(G1)\left(\frac{G2\cancel{G3}}{1\;\text{NO GLYPH}\;G2G3H1}\right)\left(\frac{G4G3\;\text{NO GLYPH}\;G5}{\cancel{G3}}\right)$$

$$\text{NO GLYPH}\;\frac{G1G2(G4G3\;\text{NO GLYPH}\;G5)\;1}{\text{NO GLYPH}\;G2G3H1}$$

Example 6 cont....

Apply rule 3

Feedback loop



Example 6 cont....

$$R(s) \longrightarrow \boxed{\dfrac{G1\,G2(G\,4\;G\,3 \;\text{NO GLYPH}\; G\,5)}{1 \;\text{NO GLYPH}\; G2\,G3\,H1 \;\text{NO GLYPH}\; G1\,G2\,H\,2(G\,3G\,4 \;\text{NO GLYPH}\; G\,5)}} \longrightarrow C(s)$$

Example 6 cont....

$$\frac{C(S)}{R(S)} \square \frac{G1\,G2(G\,4\;G\,3 \square G\,5)}{1 \square G2\,G3\,H1 \square G1\,G2\,H\,2(G\,3G\,4 \square G\,5)}$$

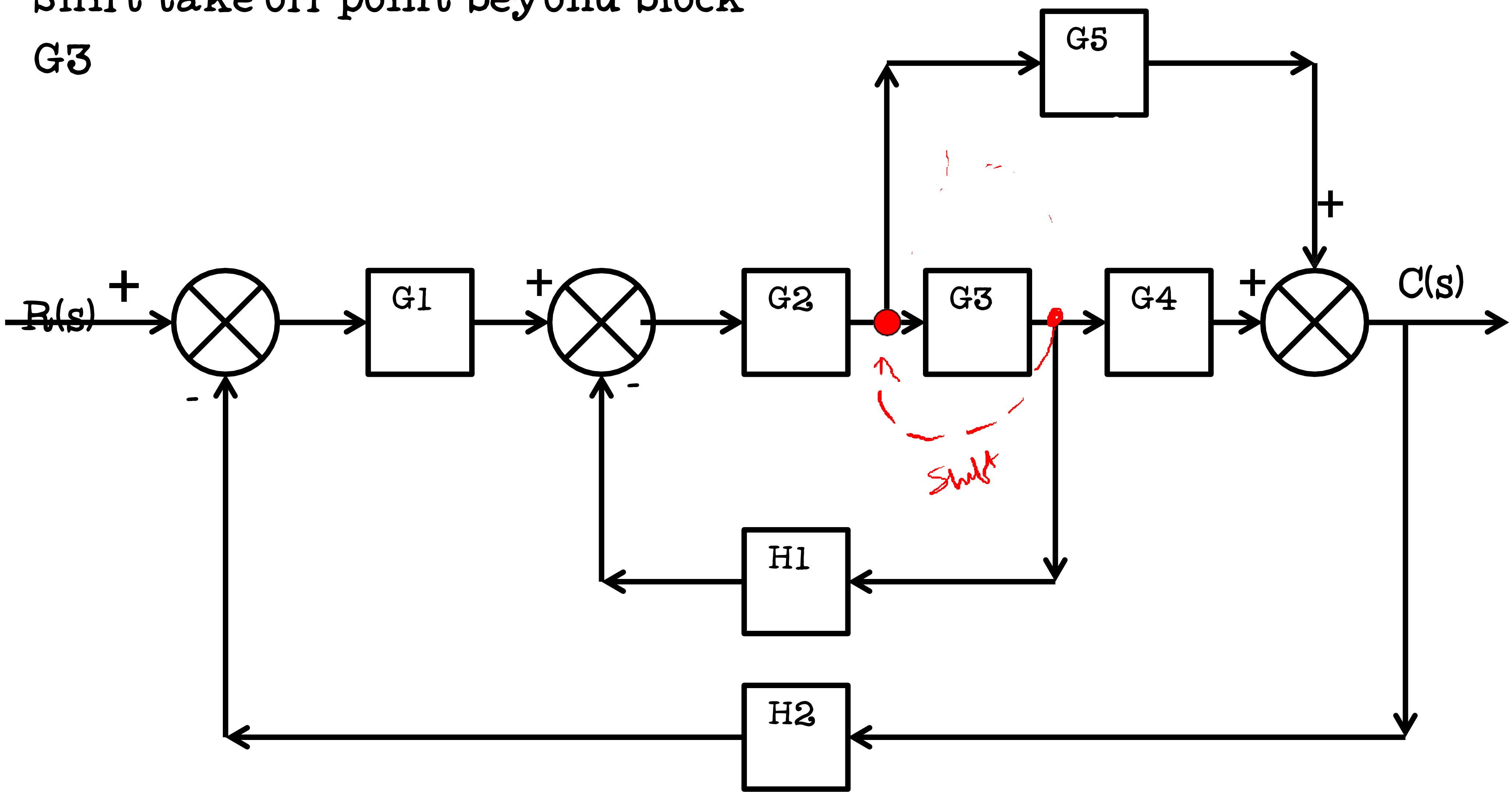
Example 6

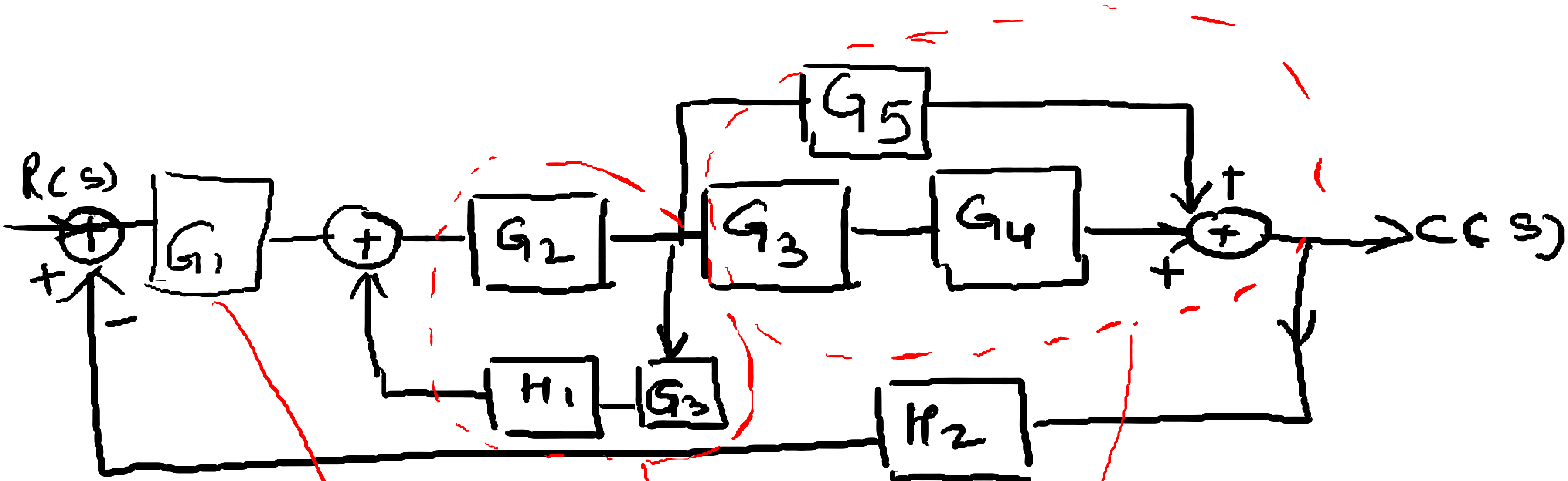
Apply rule 8

before

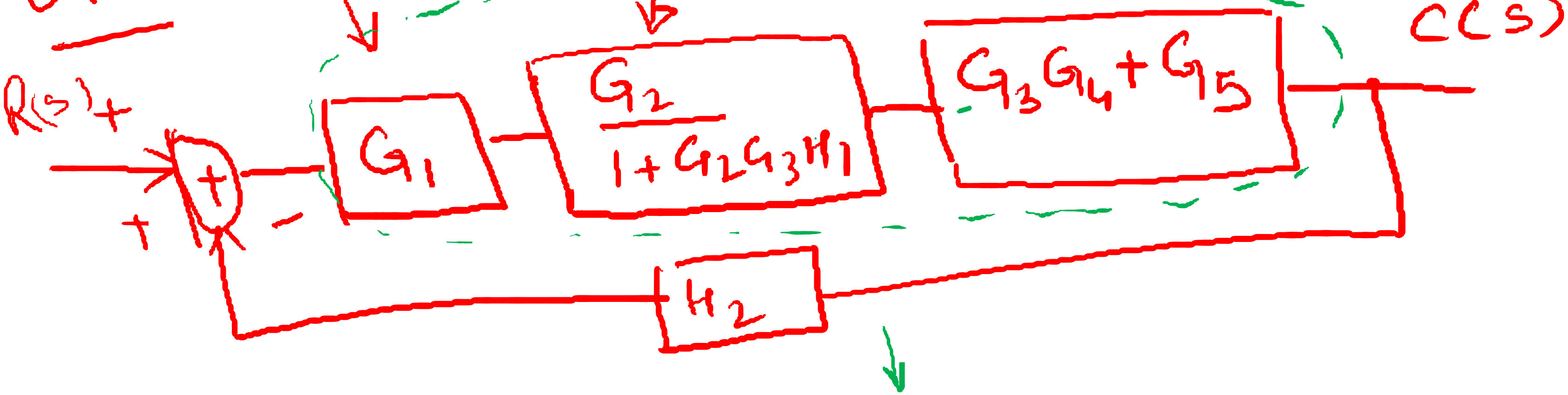
Shift take off point ~~beyond~~ block

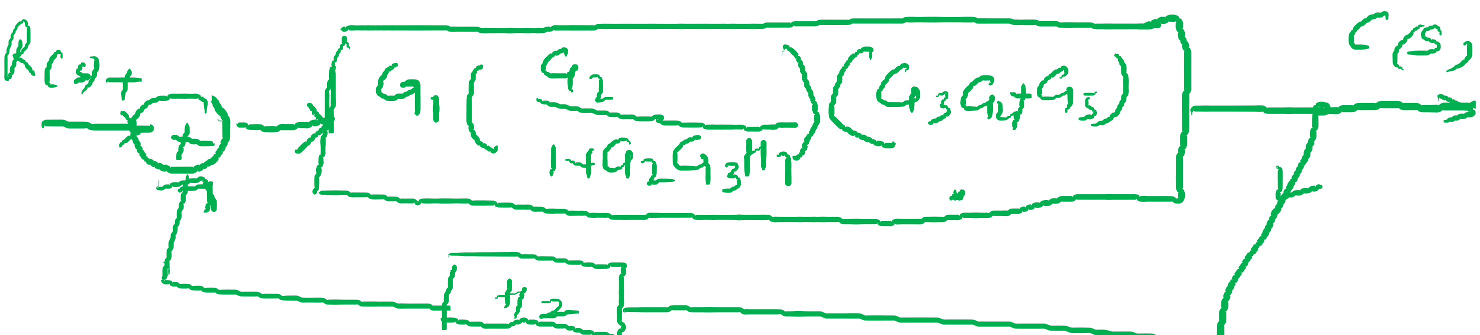
G3




R(s)
G1
G2
G3
G4
G5
H1
G3
H2
C(s)

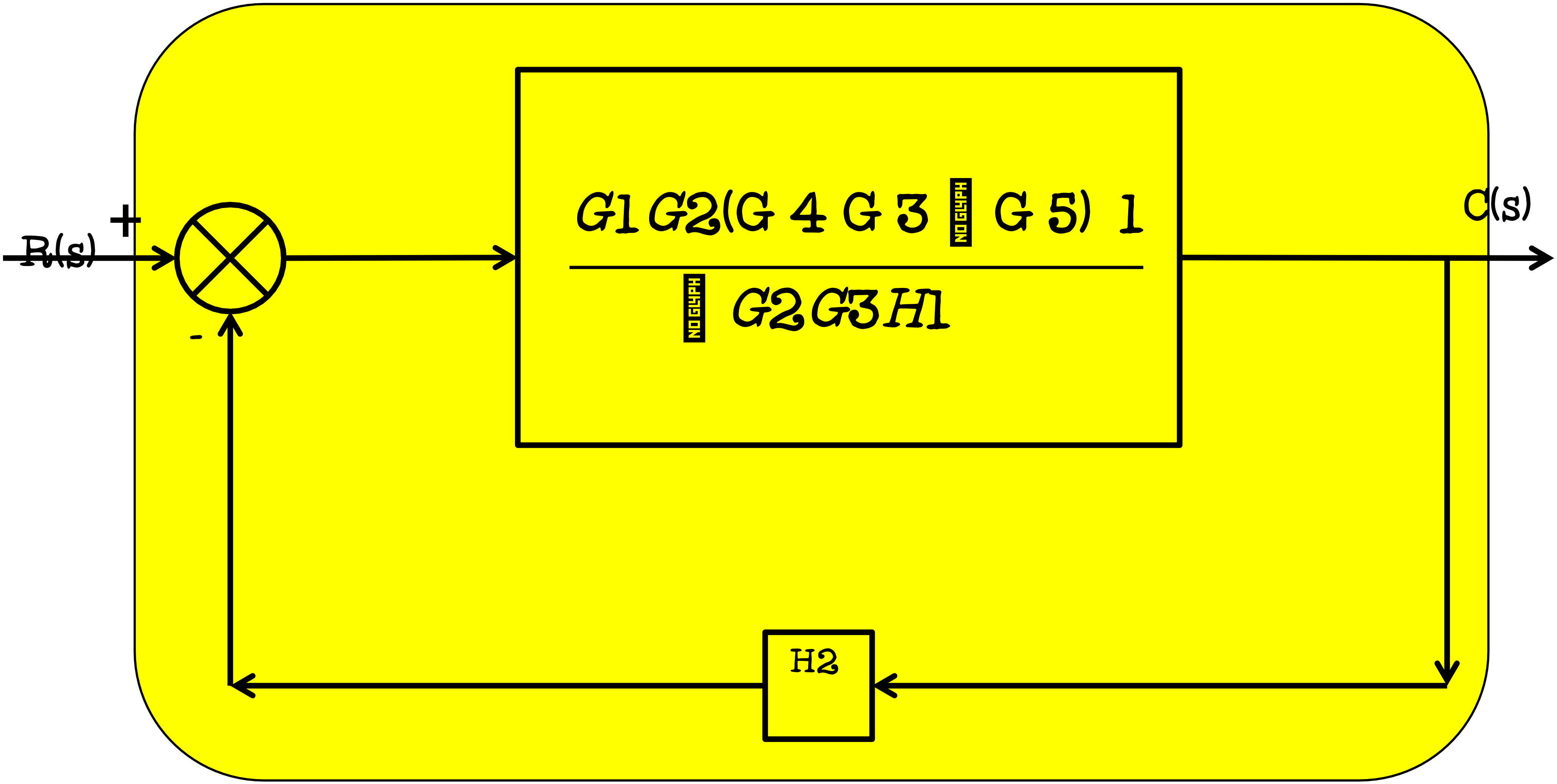
OR


R(s)
G1
$\frac{G_2}{1+G_2G_3H_1}$
$G_3G_4+G_5$
H2
C(s)

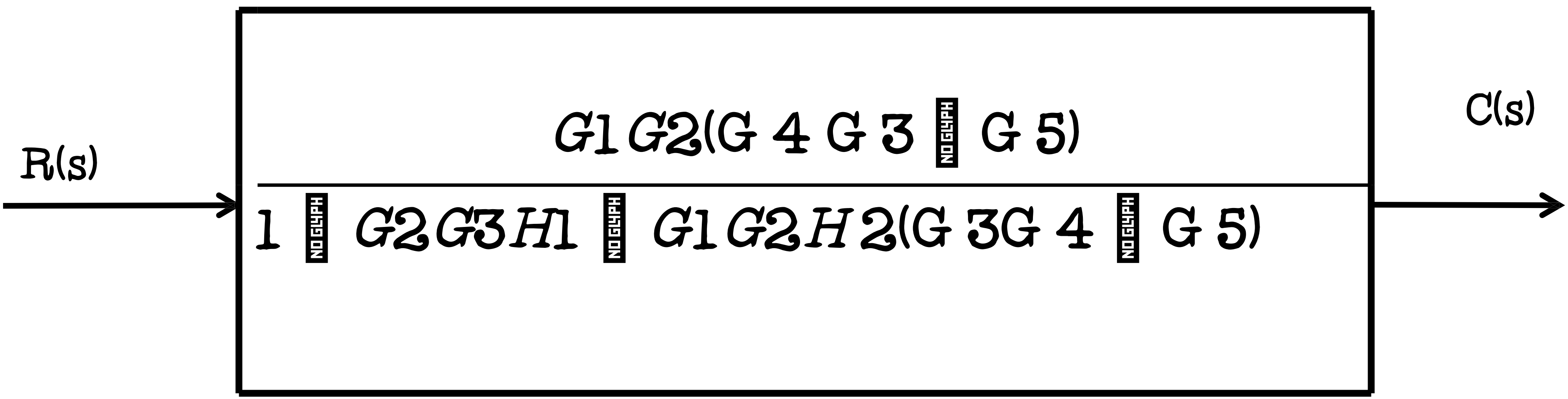

R(s)
$G_1\left(\frac{G_2}{1+G_2G_3H_1}\right)(G_3G_4+G_5)$
H2
C(s)

Apply rule 3

Feedback loop



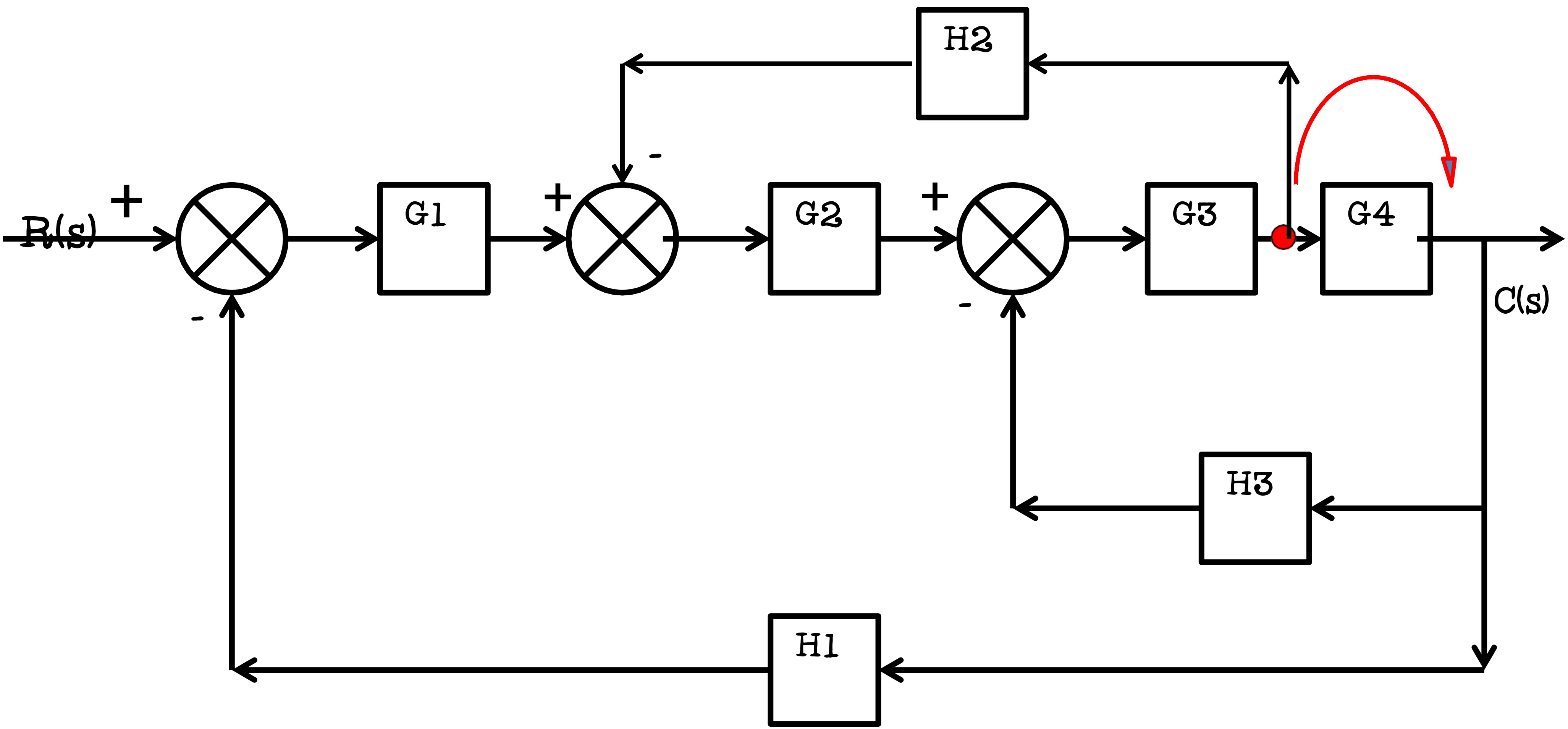
Example 6 cont....



$$\frac{C(S)}{R(S)} \square \frac{G_1 G_2 (G_4 G_3 \square G_5)}{1 \square G_2 G_3 H_1 \square G_1 G_2 H_2 (G_3 G_4 \square G_5)}$$

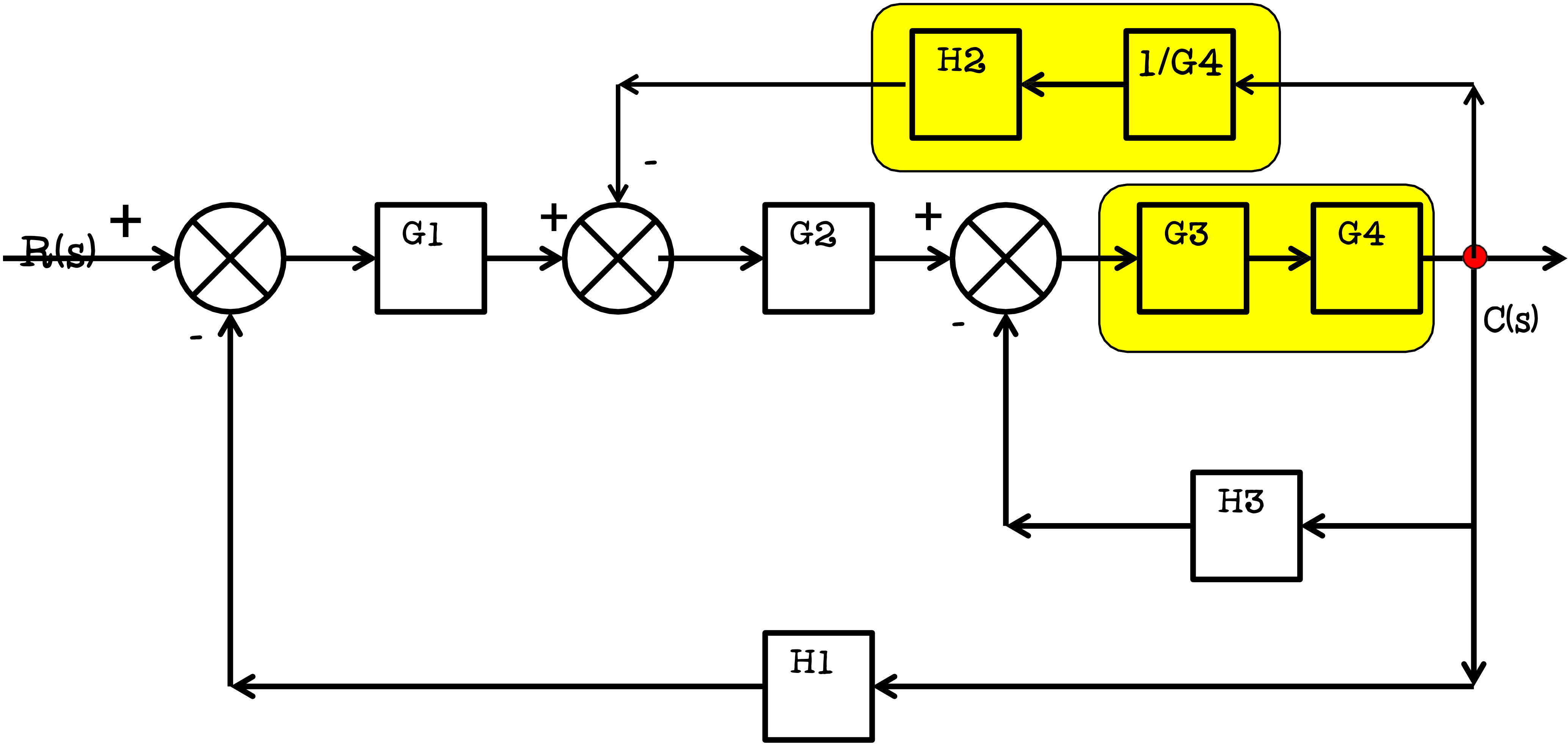
Example 7

Apply rule 8 Shift take off point after block G4



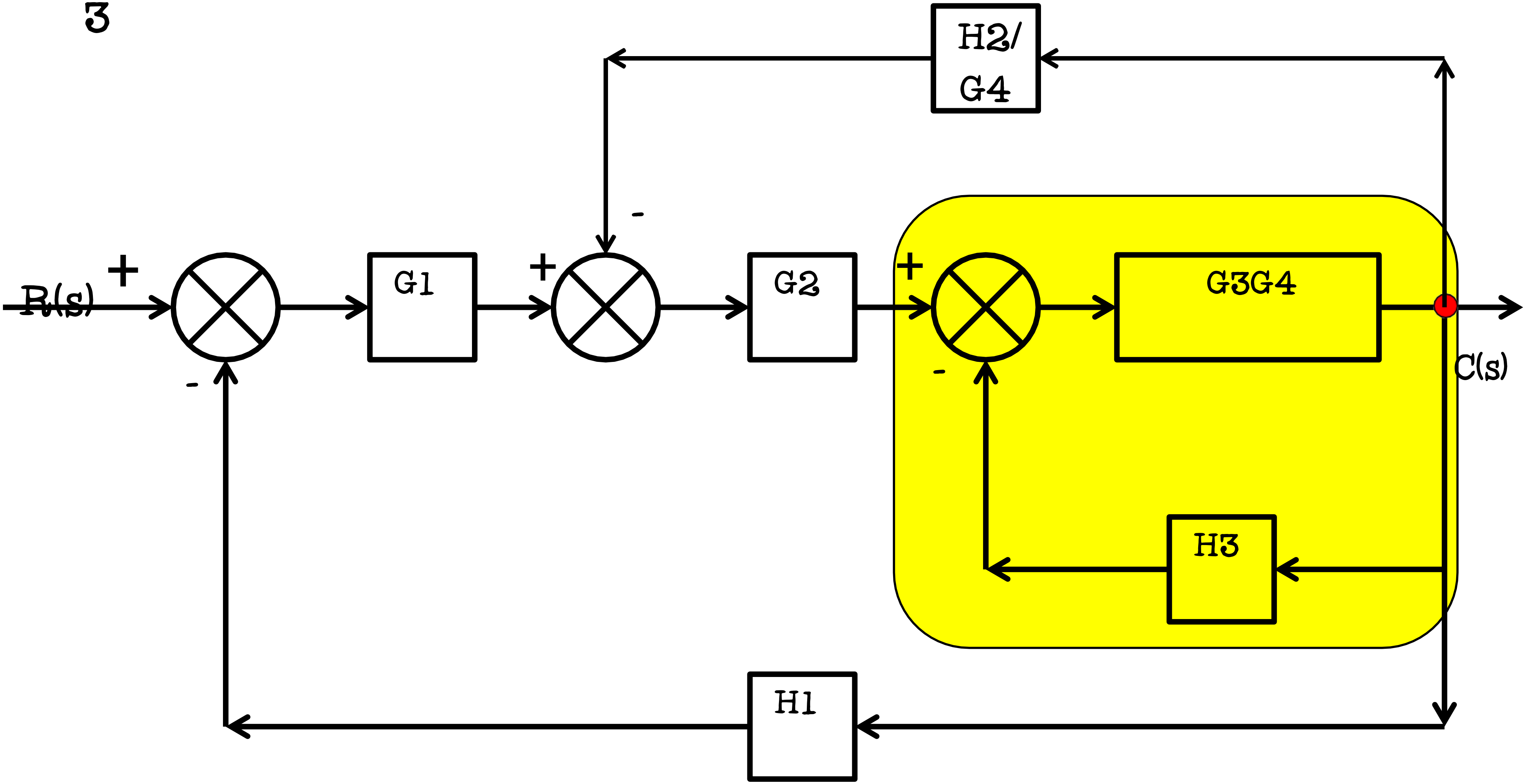
Example 7 cont....

Apply rule 1 Blocks in series



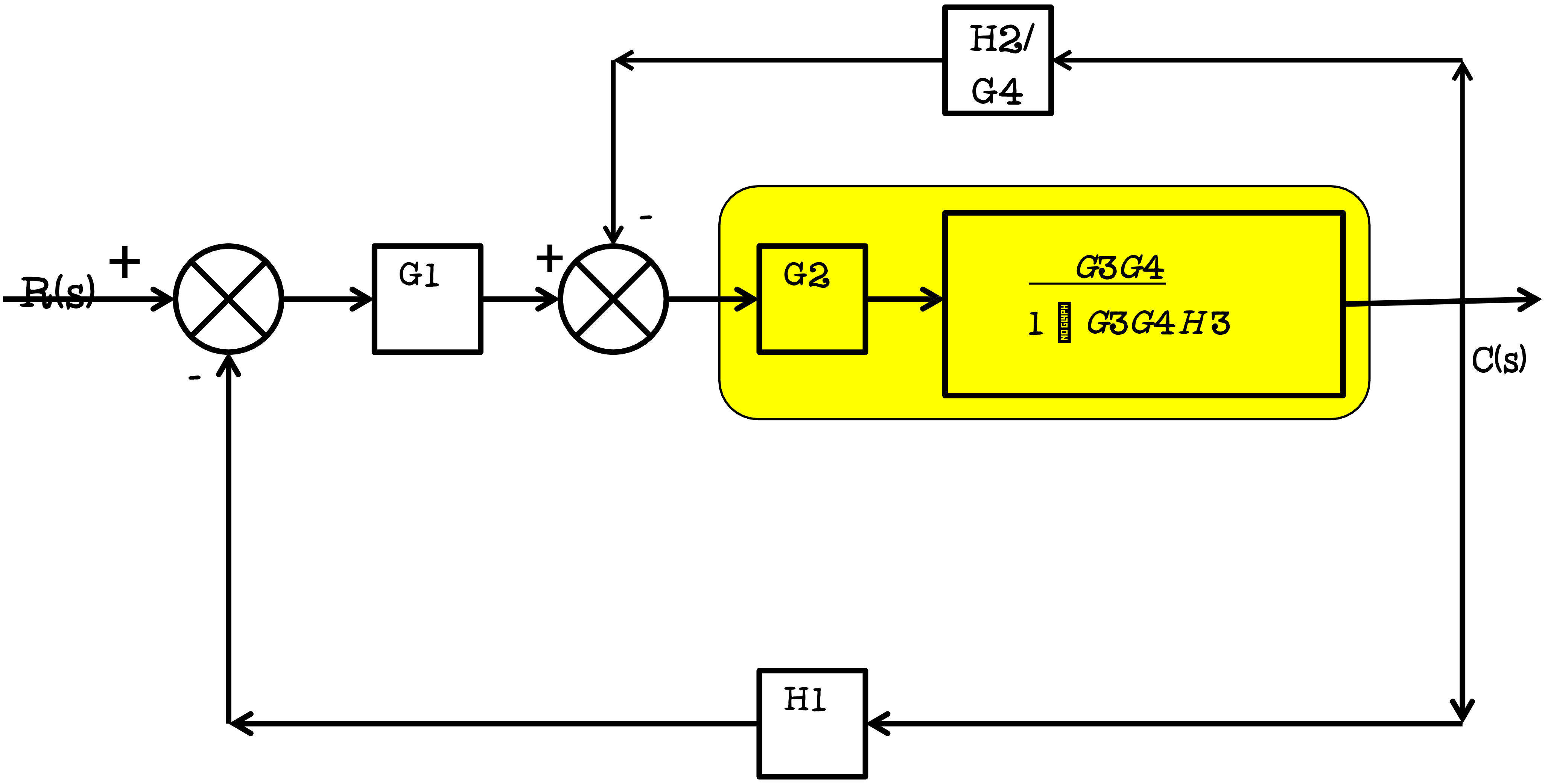
Example 7 cont....

Apply rule 3 Feedback loop



Example 7 cont…

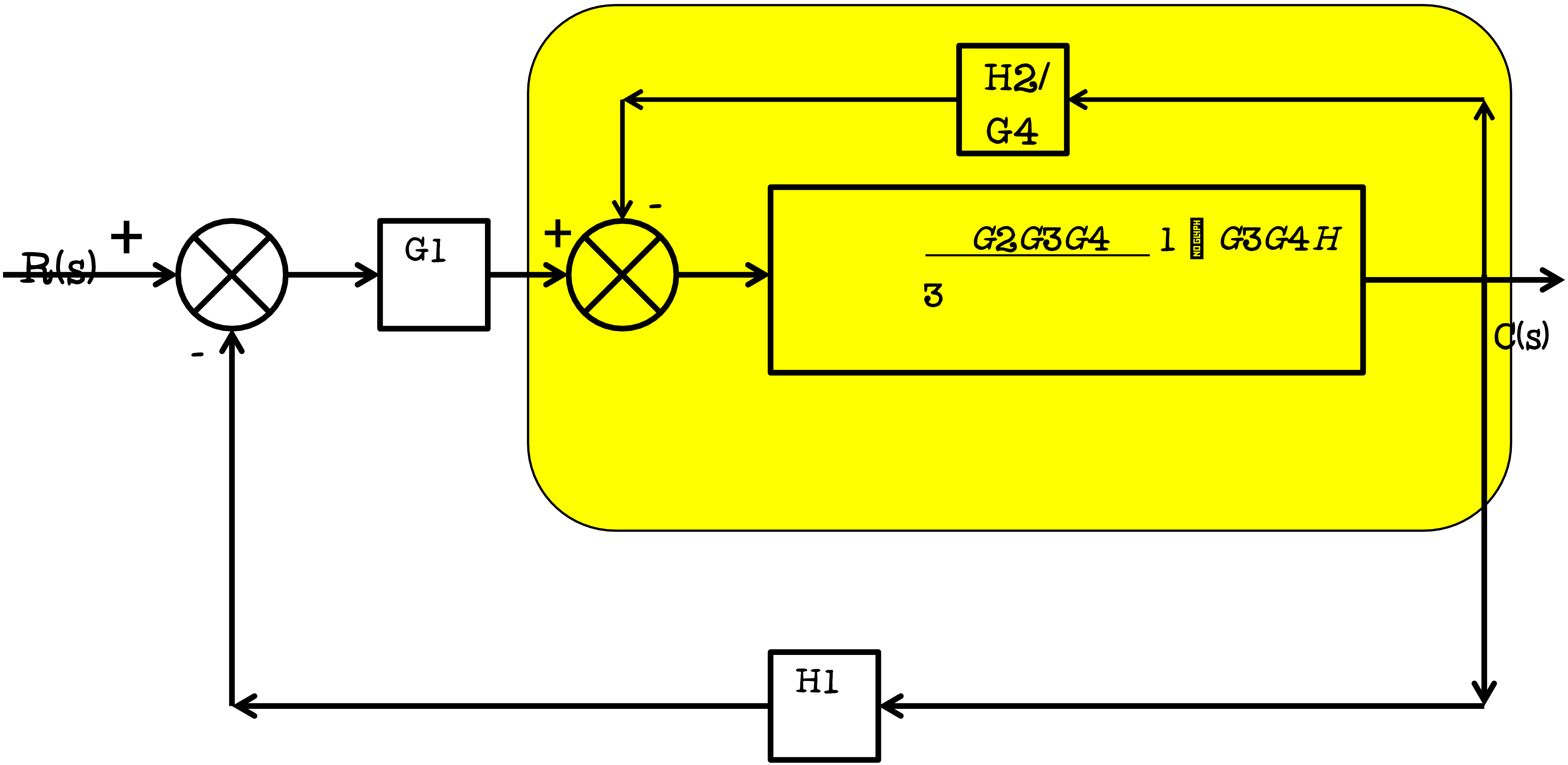
Apply rule 1 Blocks in series



Example 7 cont…

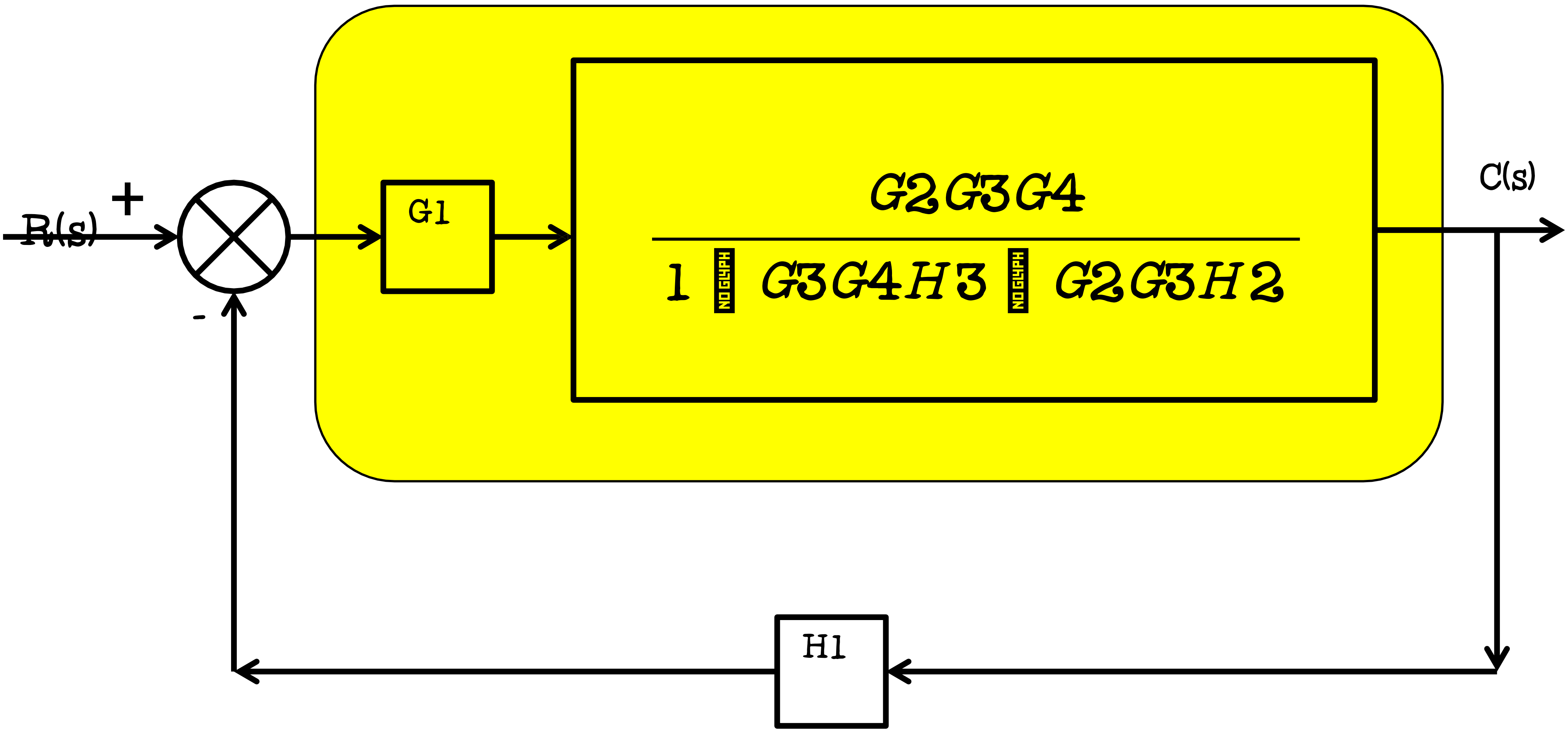
Apply rule 3

Feedback loop



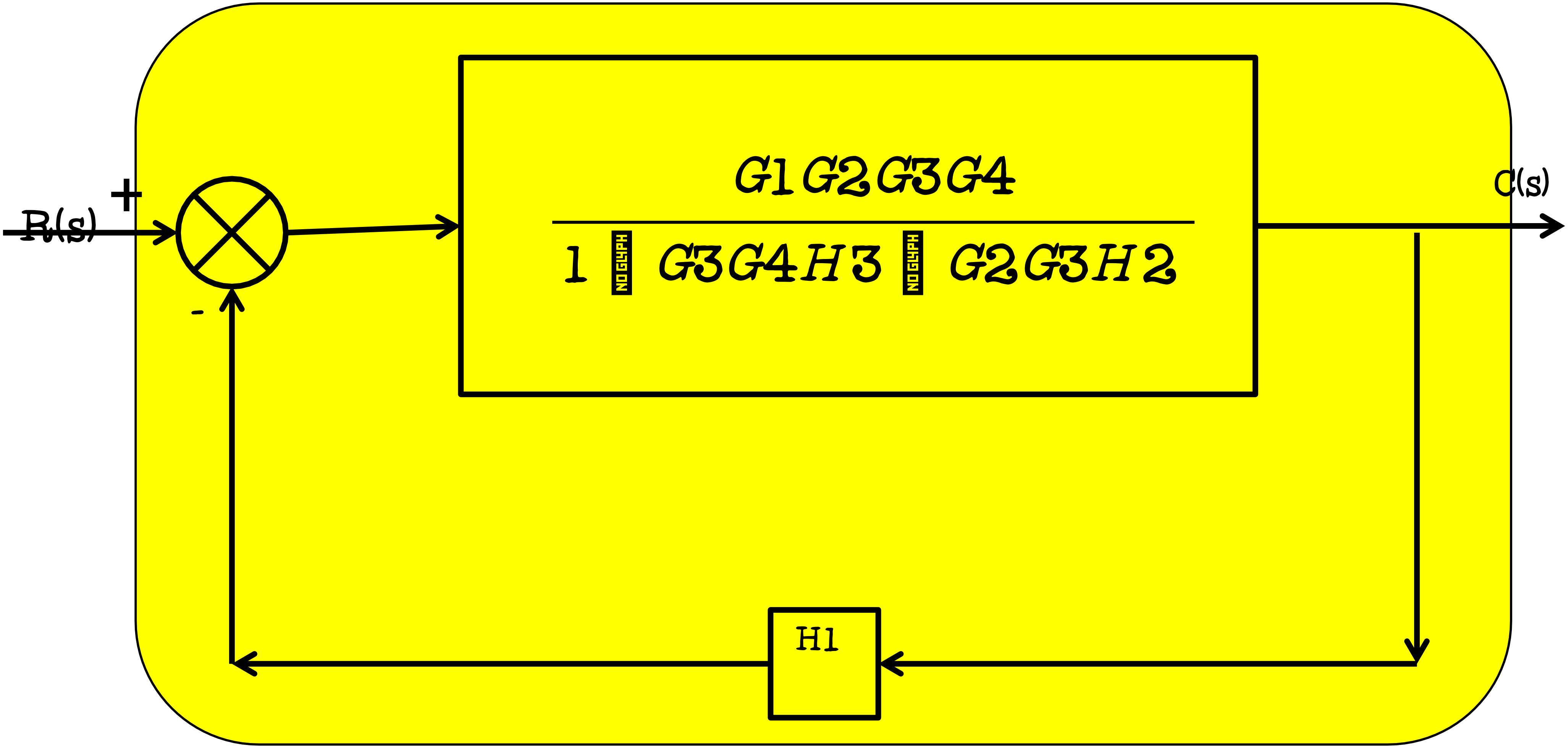
Example 7 cont…

Apply rule 1 Blocks in series



Example 7 cont....

Apply rule 3 Feedback loop



Example 7 cont....

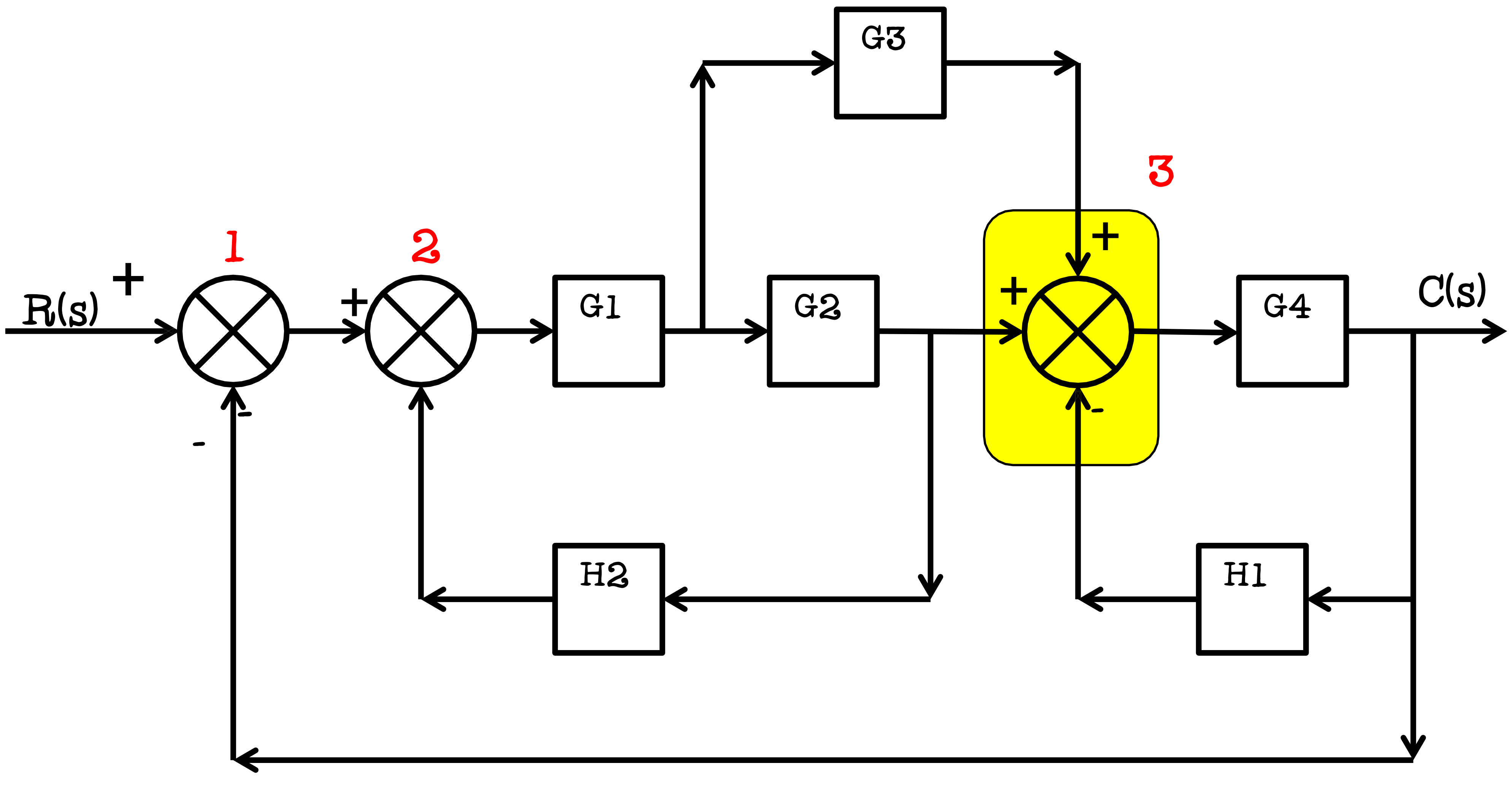
$$R(s) \longrightarrow \boxed{\dfrac{G1\,G2\,G3\,G4}{1 \;\text{NO GLYPH}\; G3G4H3 \;\text{NO GLYPH}\; G2G3H2 \;\text{NO GLYPH}\; G1G2G3G4H1}} \longrightarrow C(s)$$

Example 7 cont....

$$\frac{C(S)}{R(S)} \,\text{NO GLYPH}\, \frac{G1\,G2\,G3\,G4}{1 \,\text{NO GLYPH}\, G3\,G4\,H3 \,\text{NO GLYPH}\, G2\,G3\,H2 \,\text{NO GLYPH}\, G1\,G2\,G3\,G4\,H1}$$

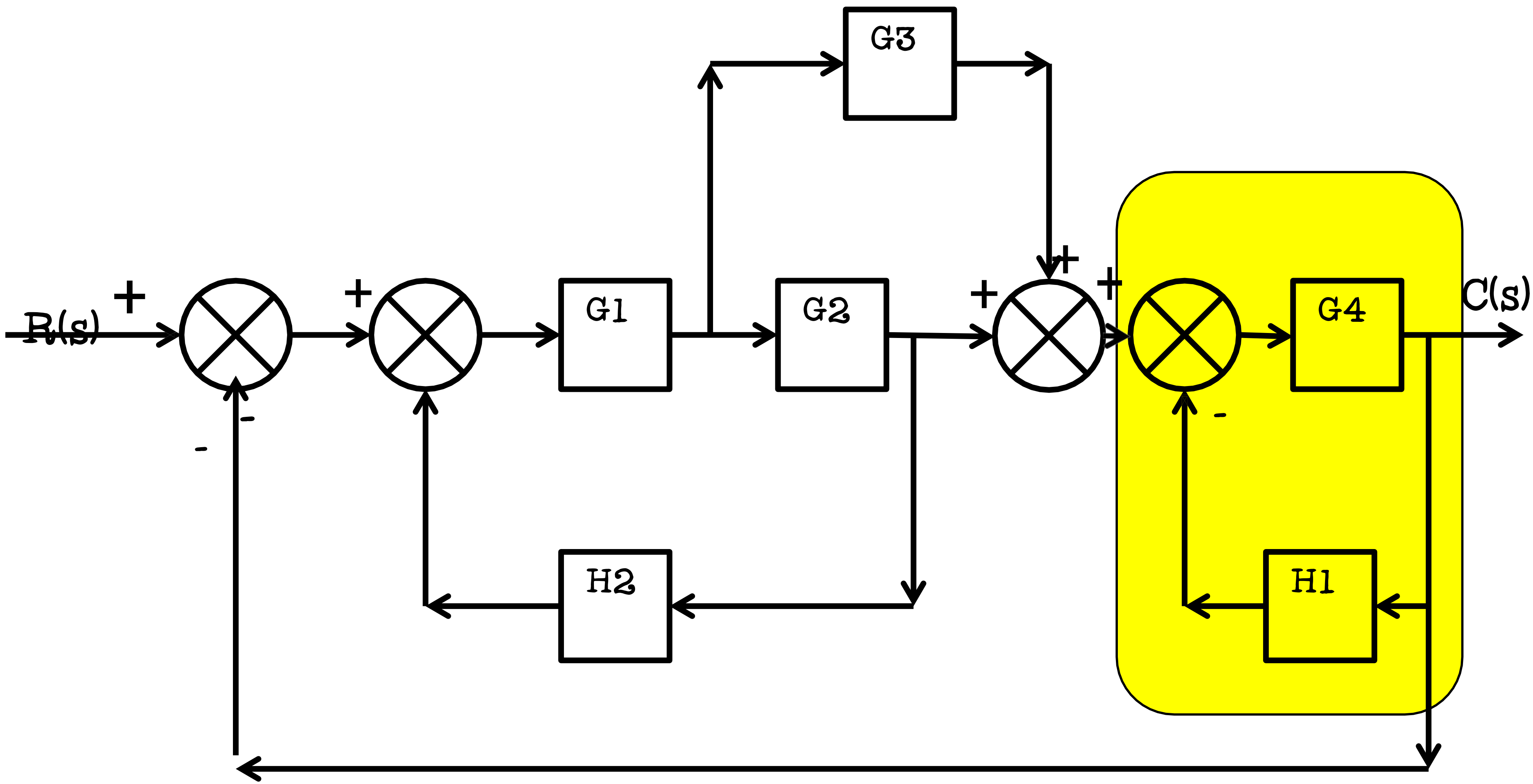
Example 8

Simplify, by splitting 3rd summing point as given in Note 1



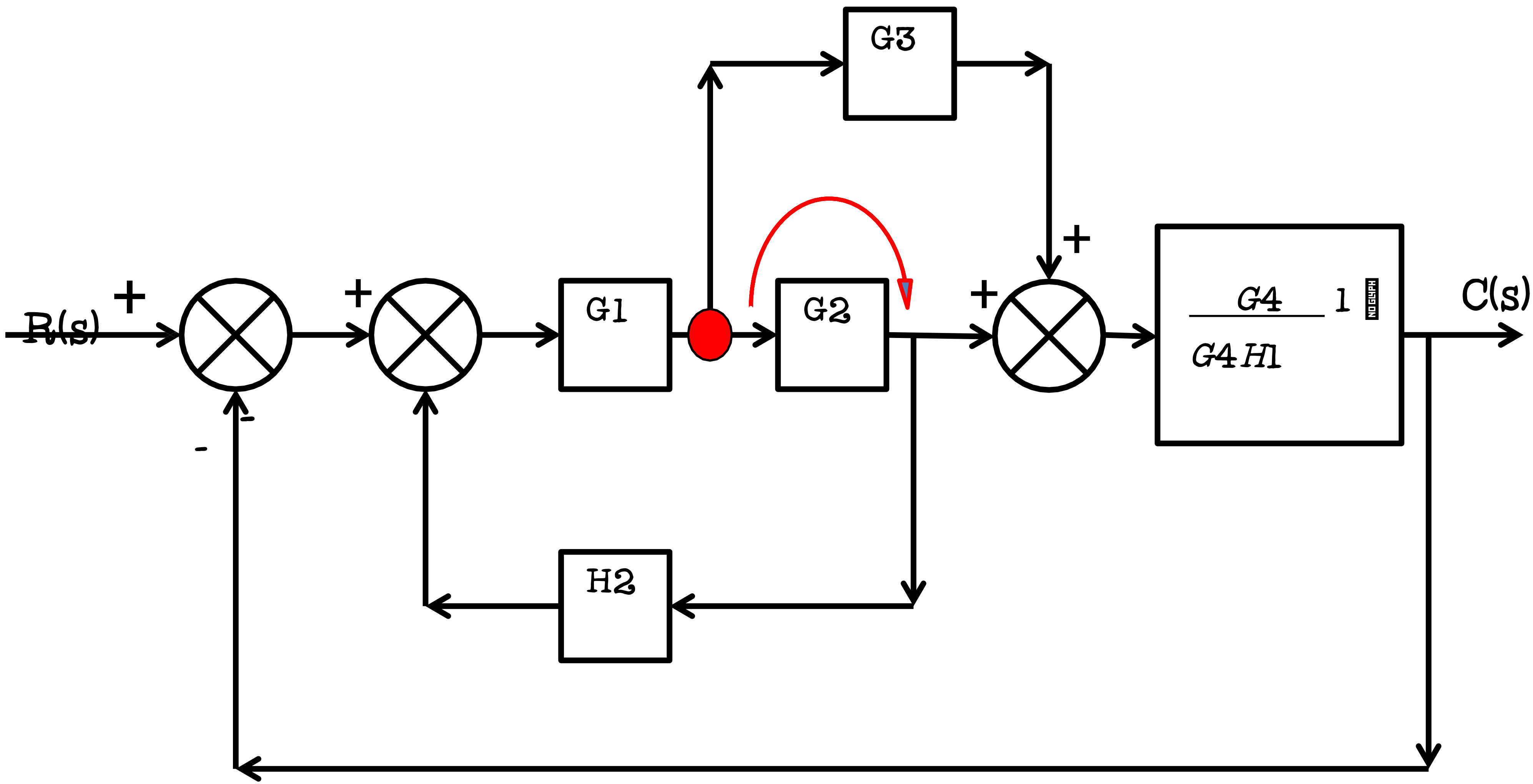
Example 8 cont…

Apply Rule 3 Elimination of Feedback loop



Example 8 cont….

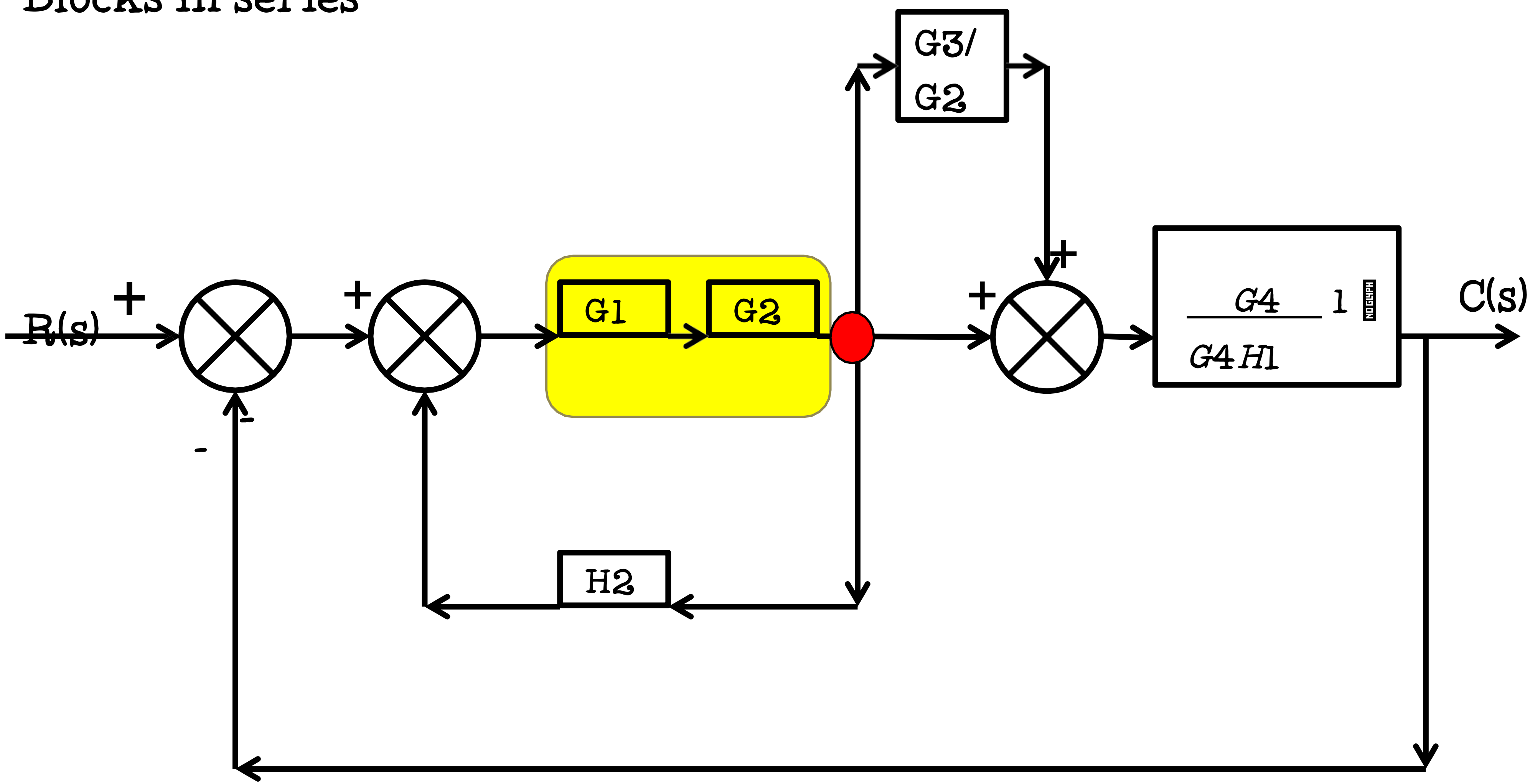
Apply Rule 8 Shift take off point after block



Example 8 cont....

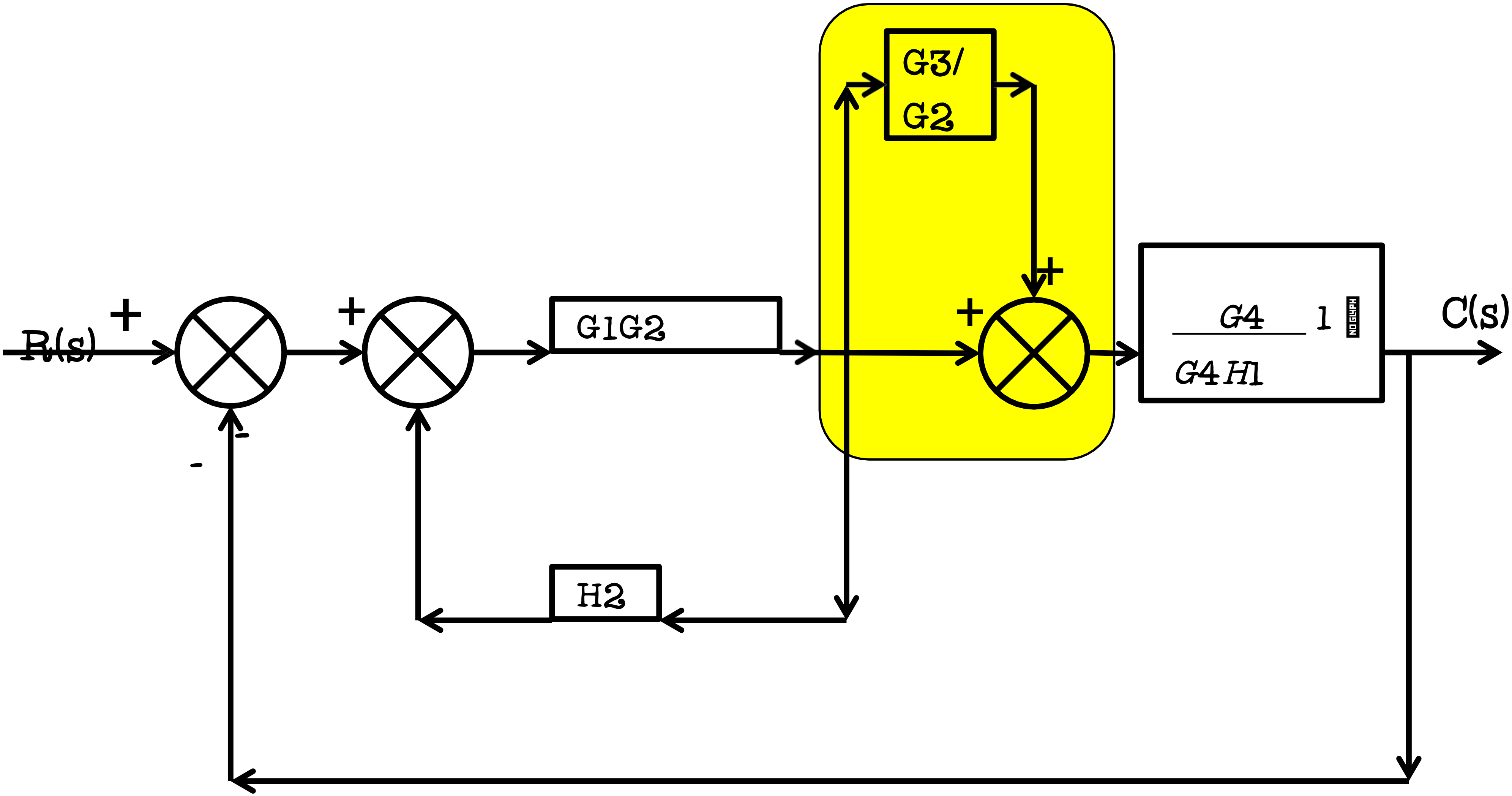
Apply Rule 1

Blocks in series



Example 8 cont....

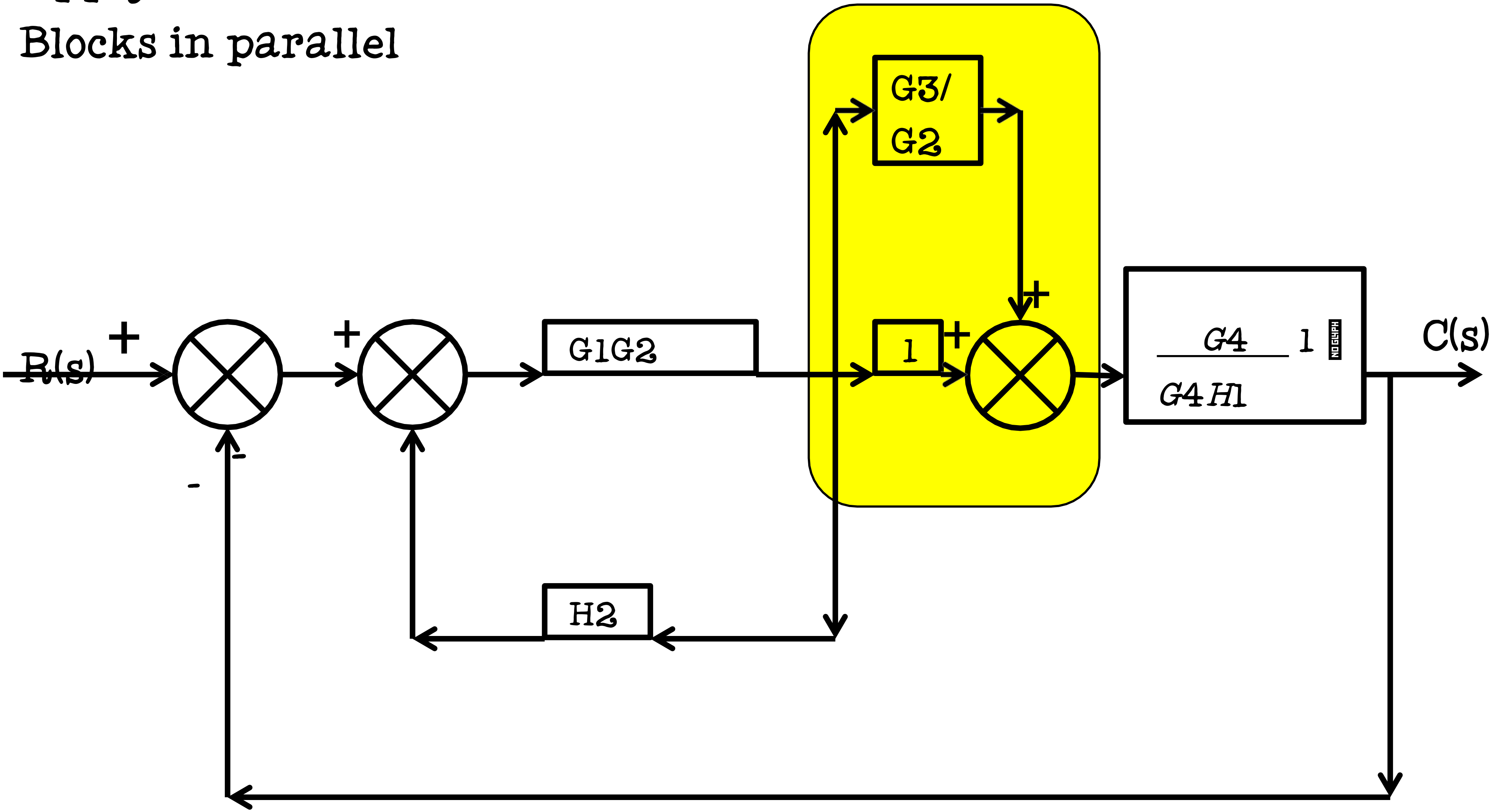
Now which rule we have to use?



Example 8 cont...

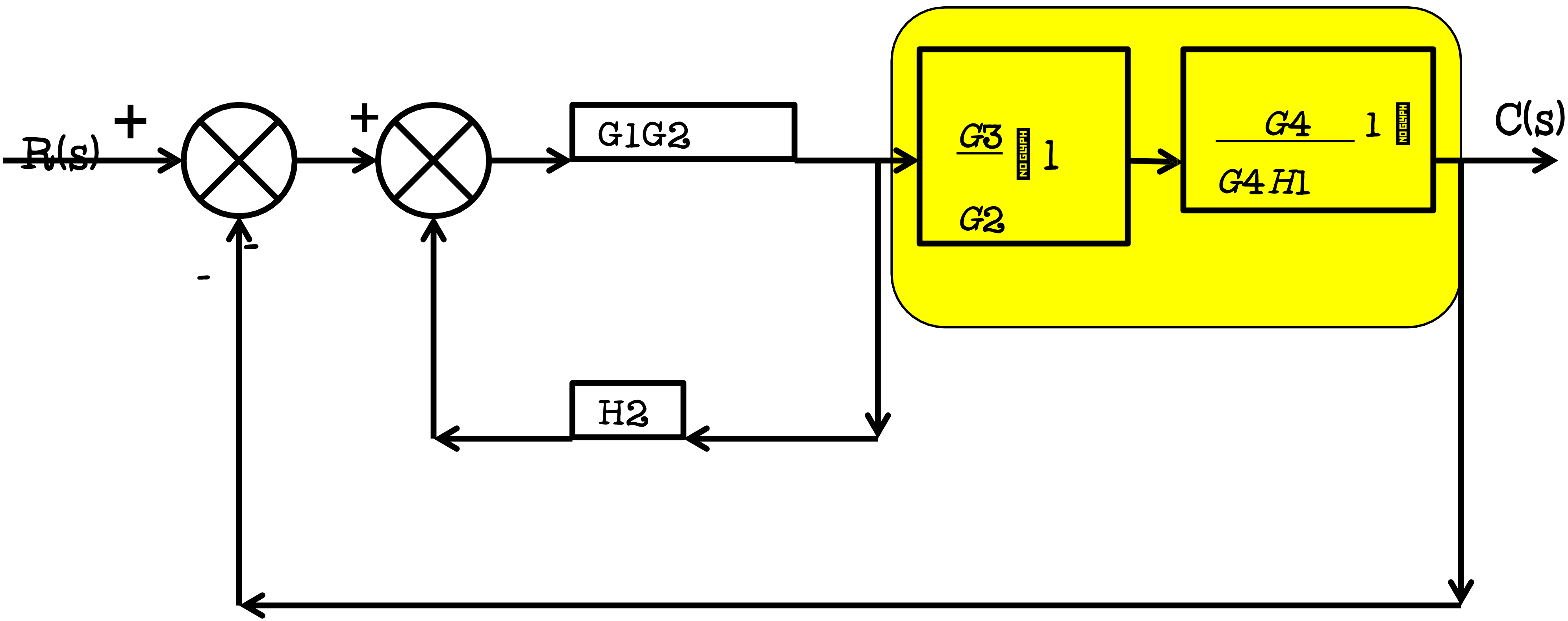
Apply Rule 2

Blocks in parallel



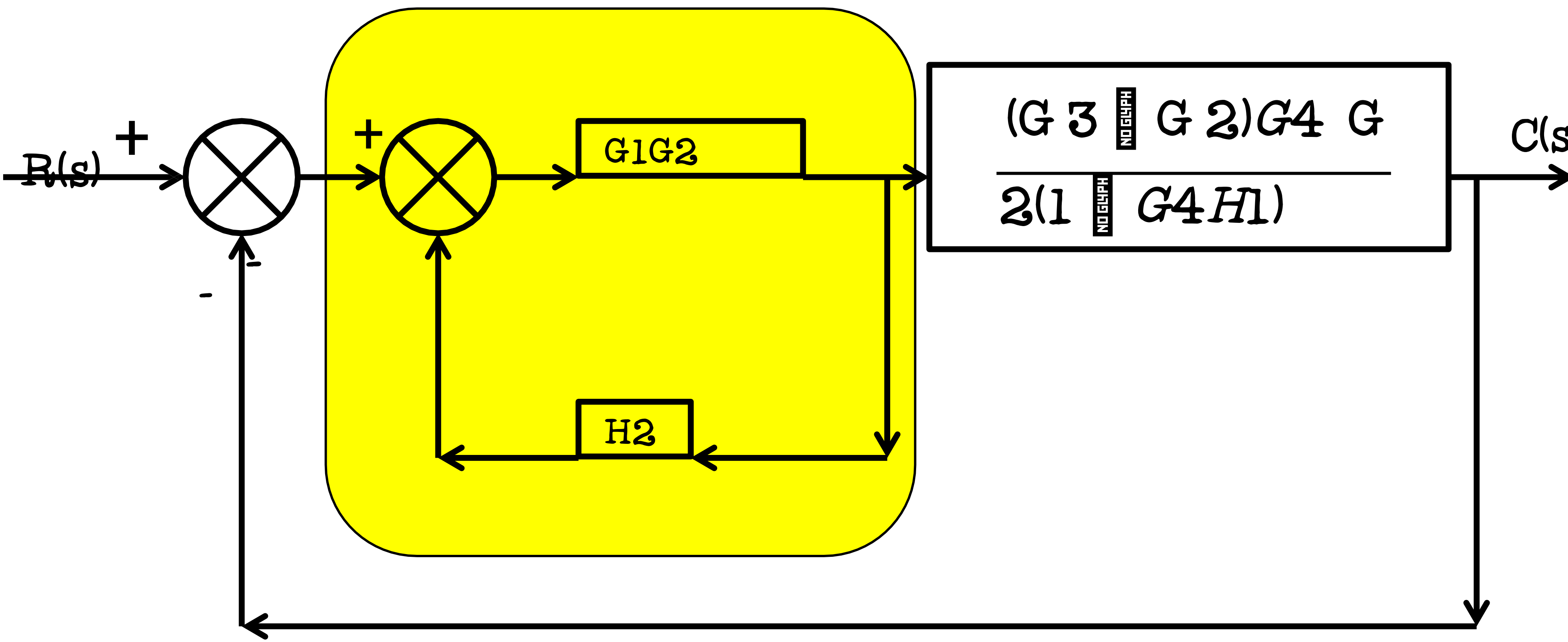
Example 8 cont...

Apply Rule 1 Blocks in series



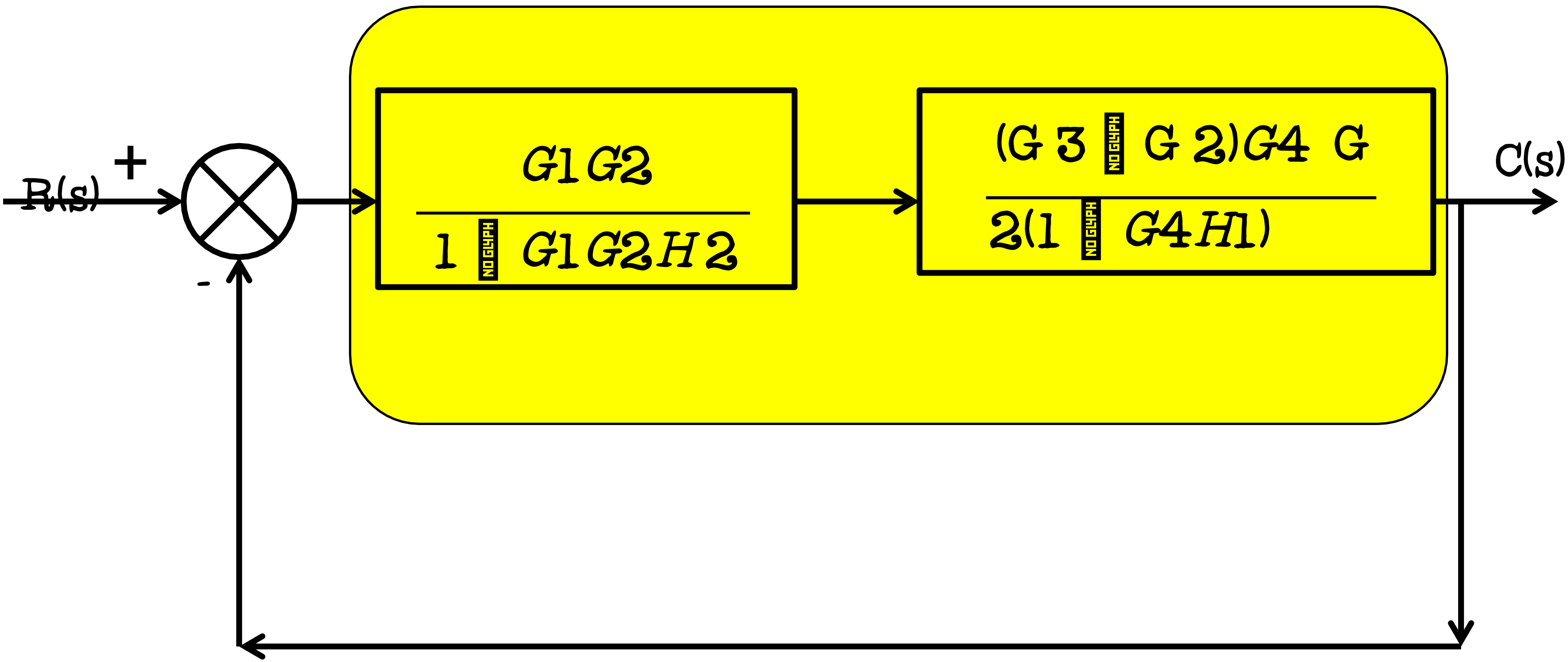
Example 8 cont….

Apply Rule 3 Elimination of Feedback Loop

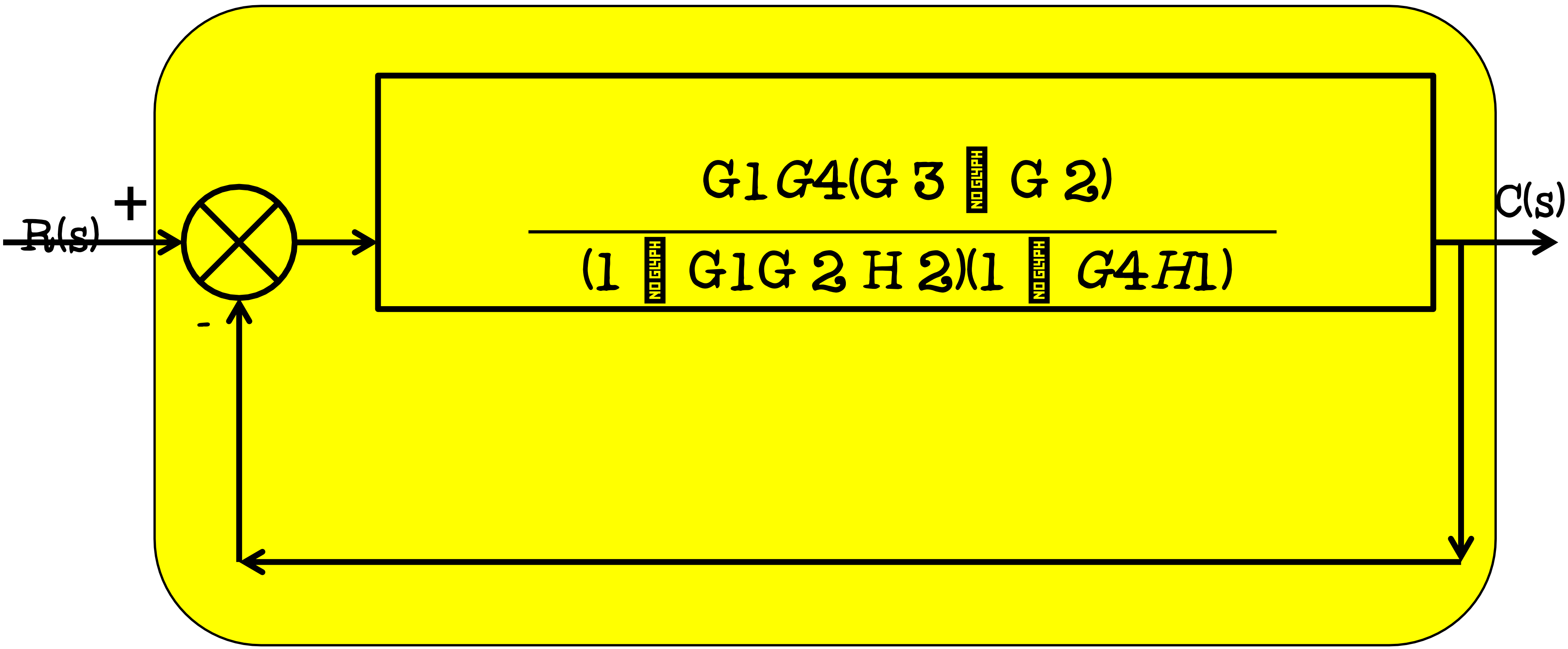


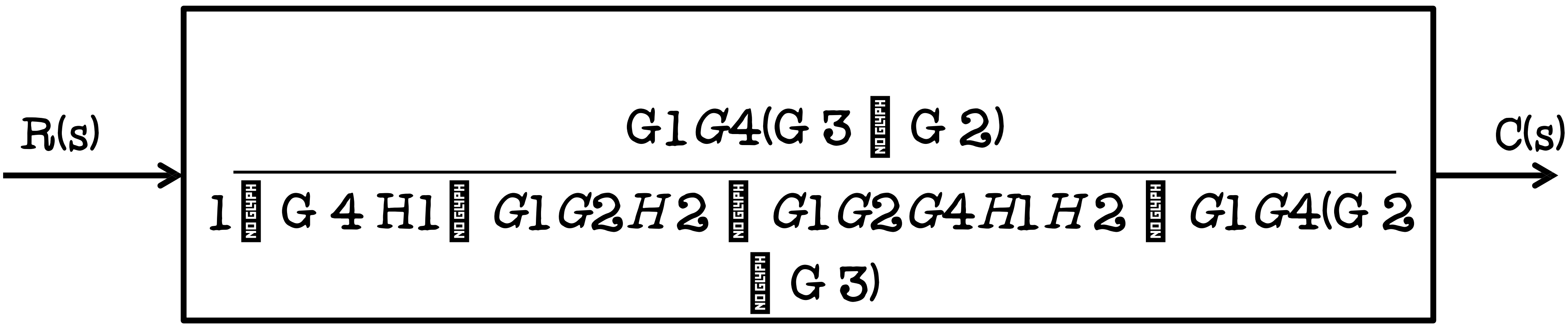
Example 8 cont….

Apply Rule 1 Blocks in series



Apply Rule 3 Elimination of Feedback loop



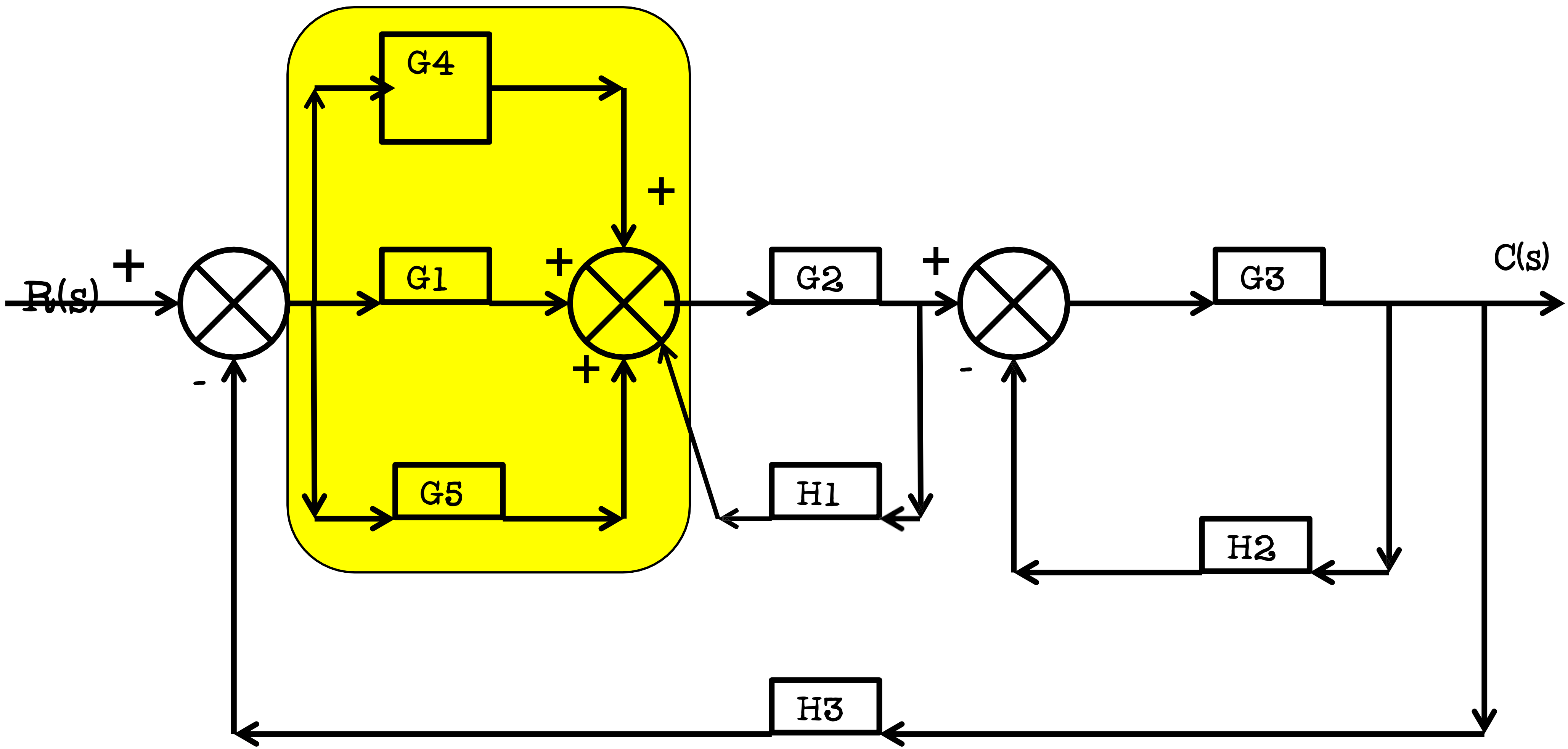

R(s)
G1 G4(G 3 [illegible] G 2)
1 [illegible] G 4 H1 [illegible] G1 G2 H 2 [illegible] G1 G2 G4 H1 H 2 [illegible] G1 G4(G 2 [illegible] G 3)
C(s)

Example 8 cont….

$$\frac{C(s)}{R(s)} \square \frac{G_1 G_4 (G_3 \square G_2)}{1 \square G_4 H_1 \square G_1 G_2 H_2 \square G_1 G_2 G_4 H_1 H_2 \square G_1 G_4 (G_2 \square G_3)}$$

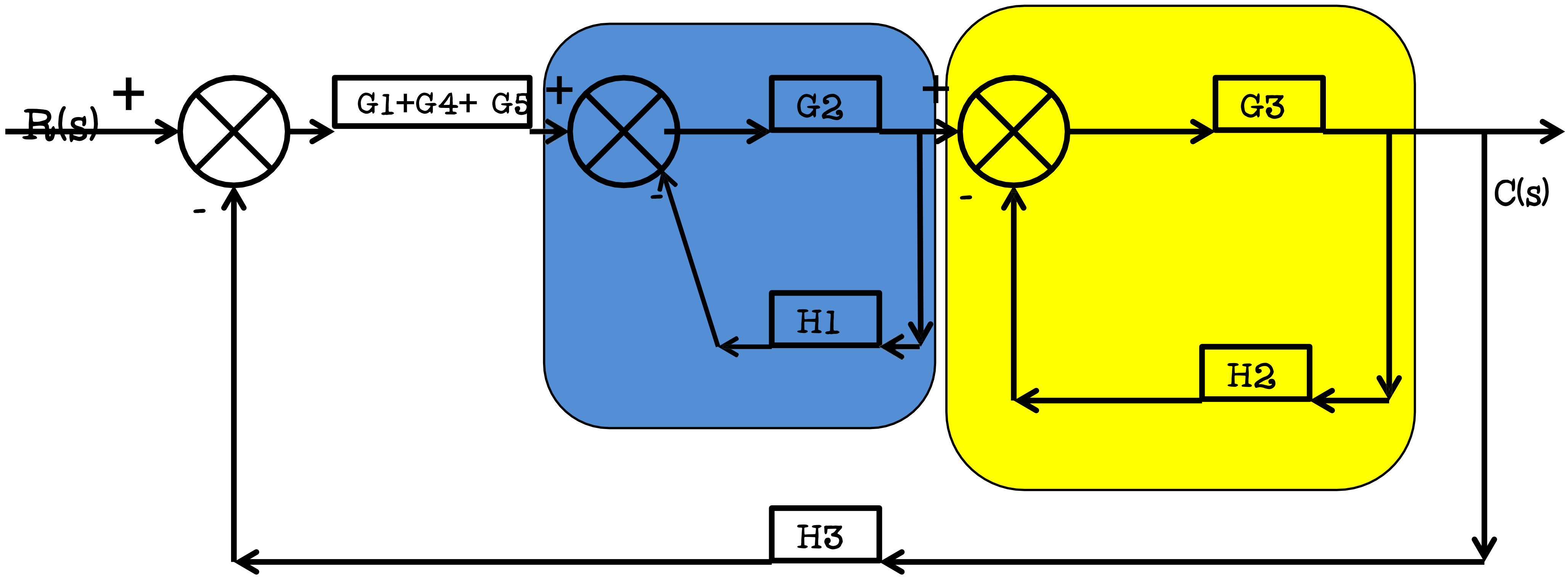
Example 9

Apply rule 2 Blocks in Parallel



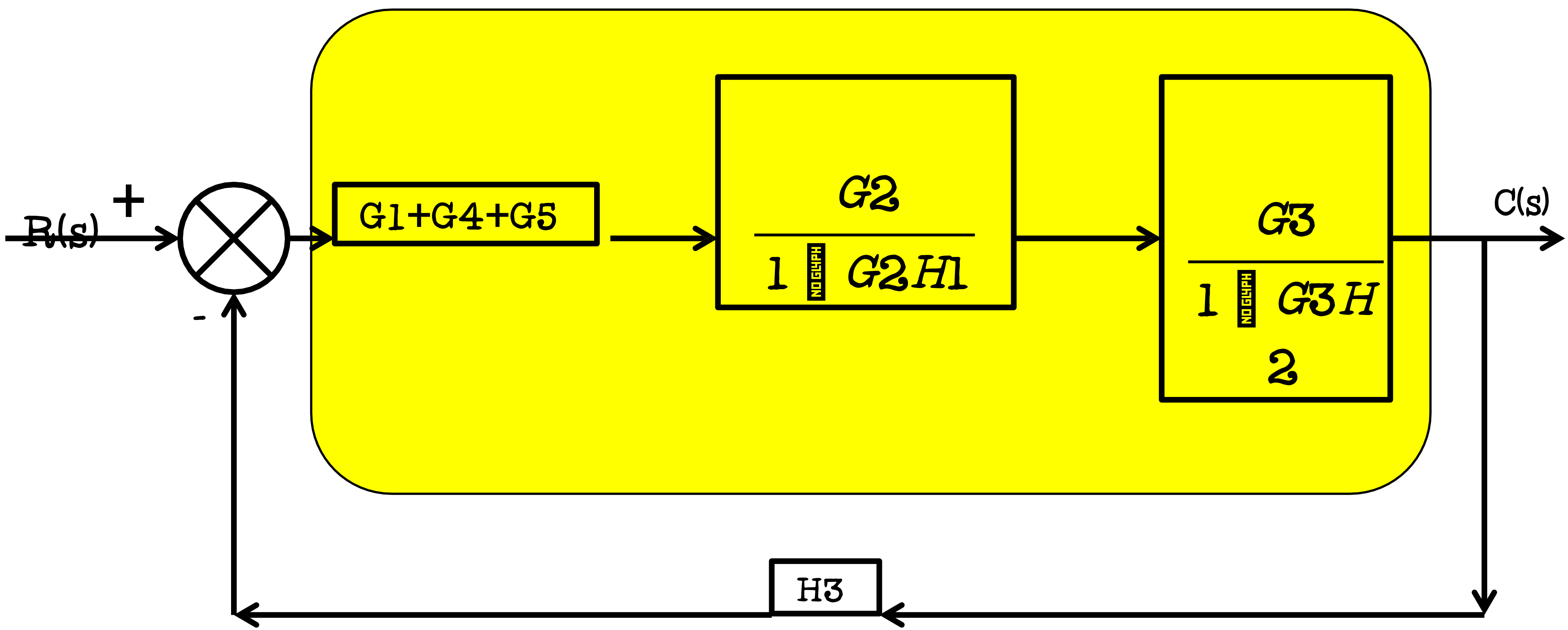
Example 9 cont…

Apply rule 3 Elimination of Feedback Loop



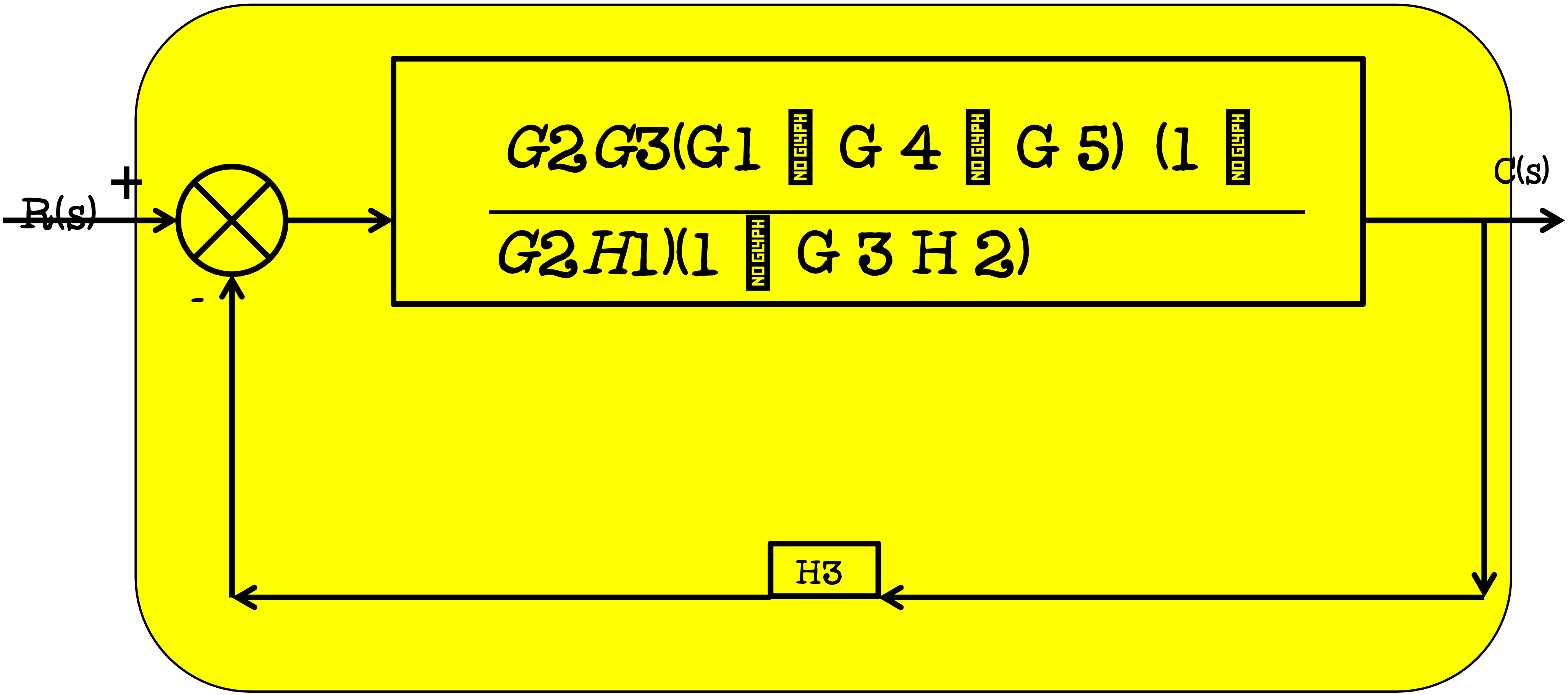
Example 9 cont....

Apply rule 1 Blocks in Series

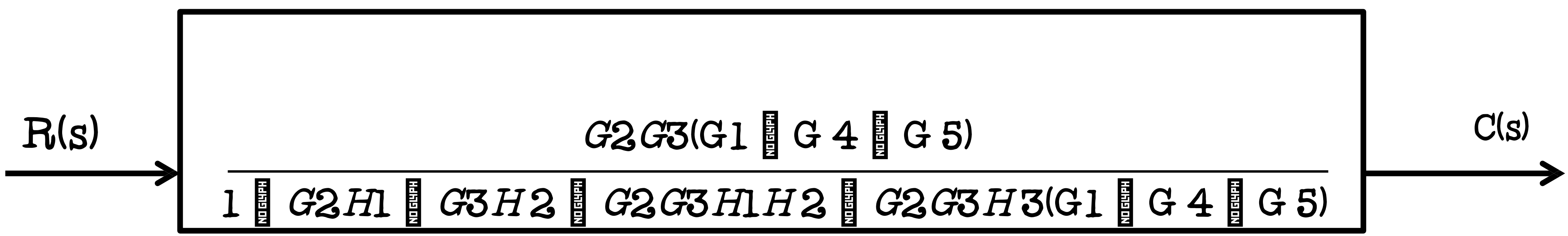


Example 9 cont...

Apply rule 3 Elimination of Feedback loop



Example 9 cont....

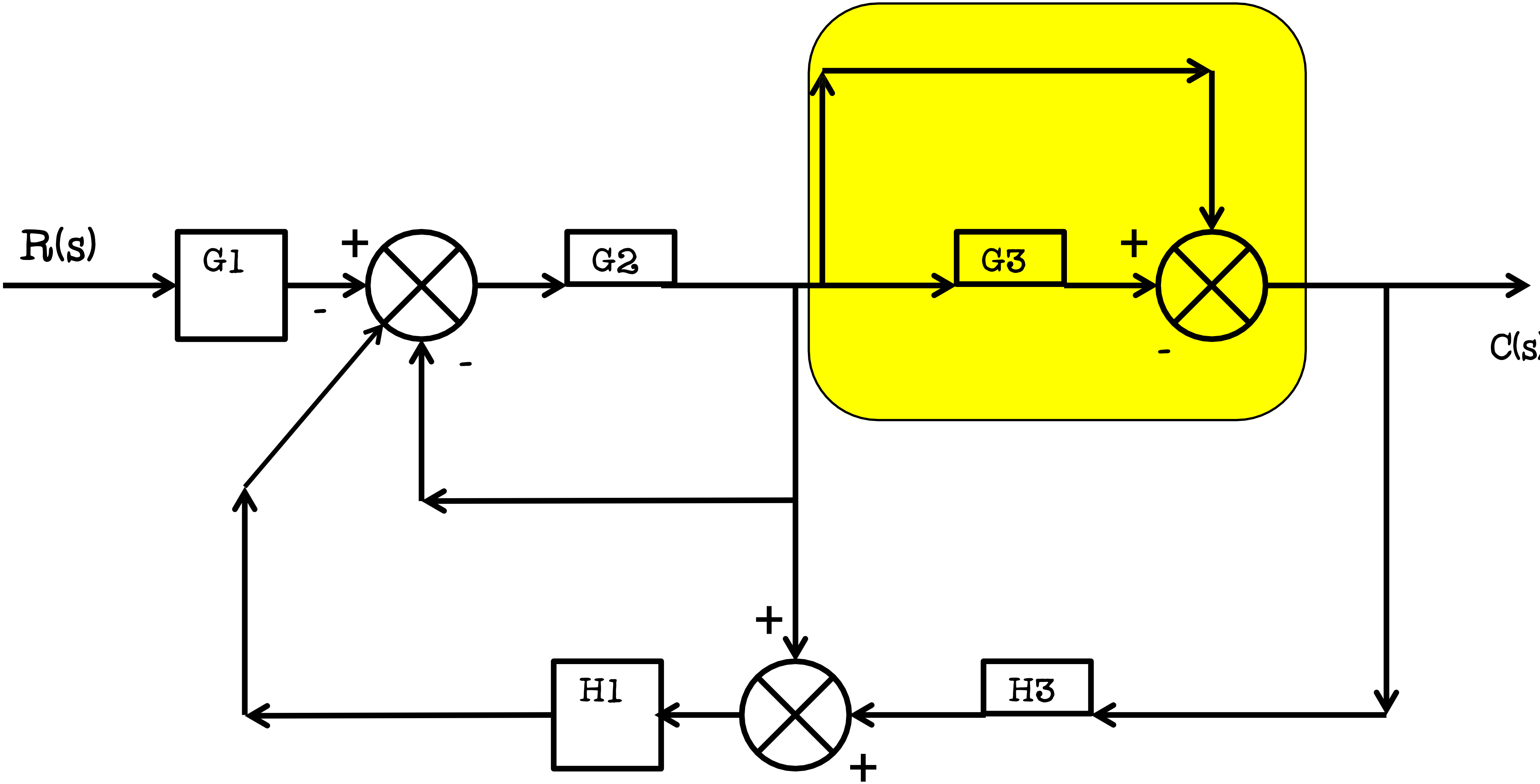


Example 9 cont....

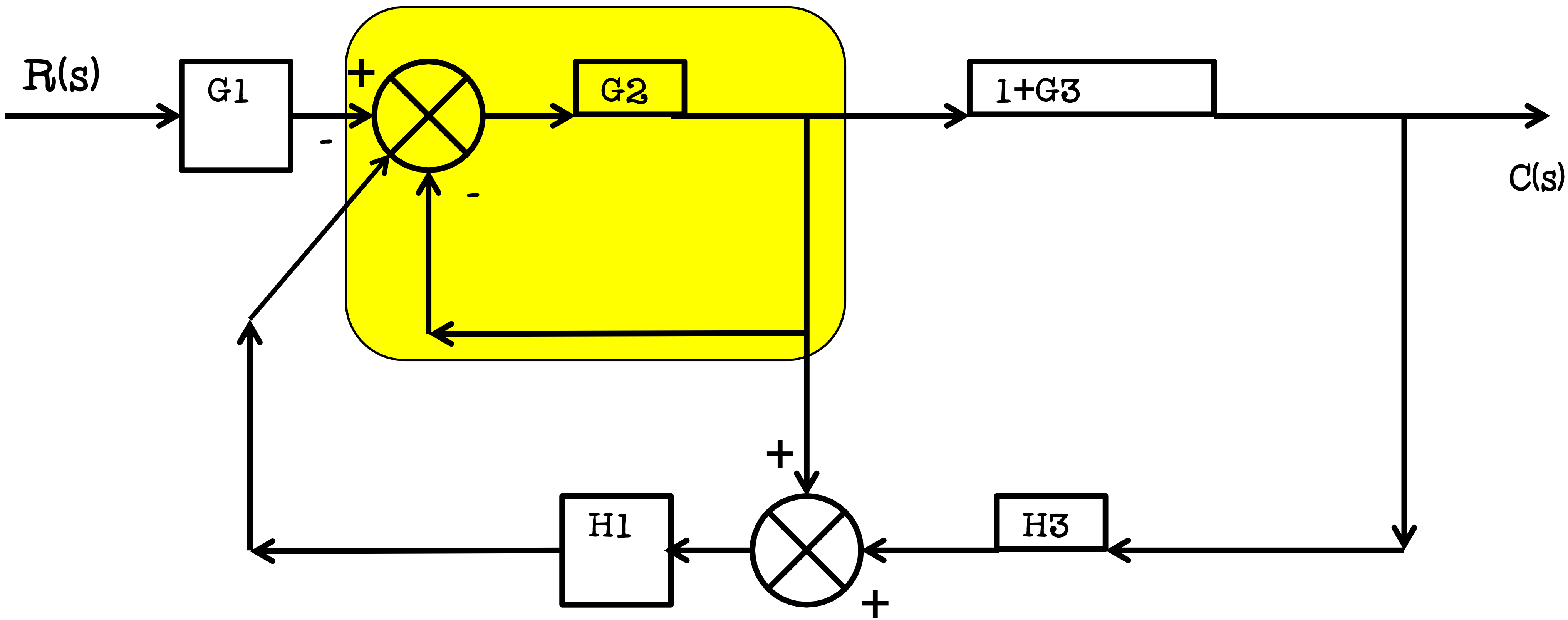
$$\frac{C(s)}{R(s)} \square \frac{G_2 G_3 (G_1 \square G_4 \square G_5)}{1 \square G_2 H_1 \square G_3 H_2 \square G_2 G_3 H_1 H_2 \square G_2 G_3 H_3 (G_1 \square G_4 \square G_5)}$$

Example 10

Apply rule 2 Blocks in Parallel

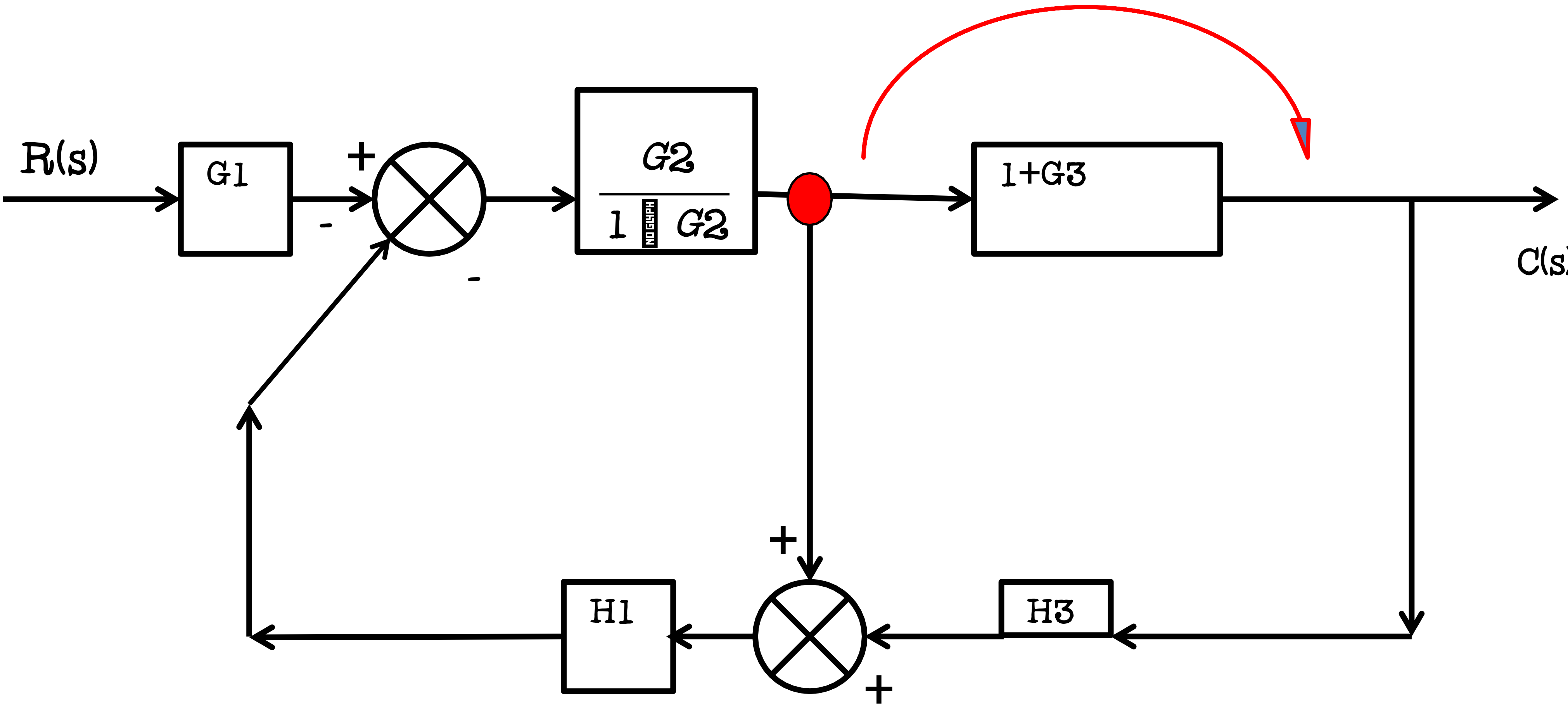


Apply rule 3   Elimination of Feedback Loop



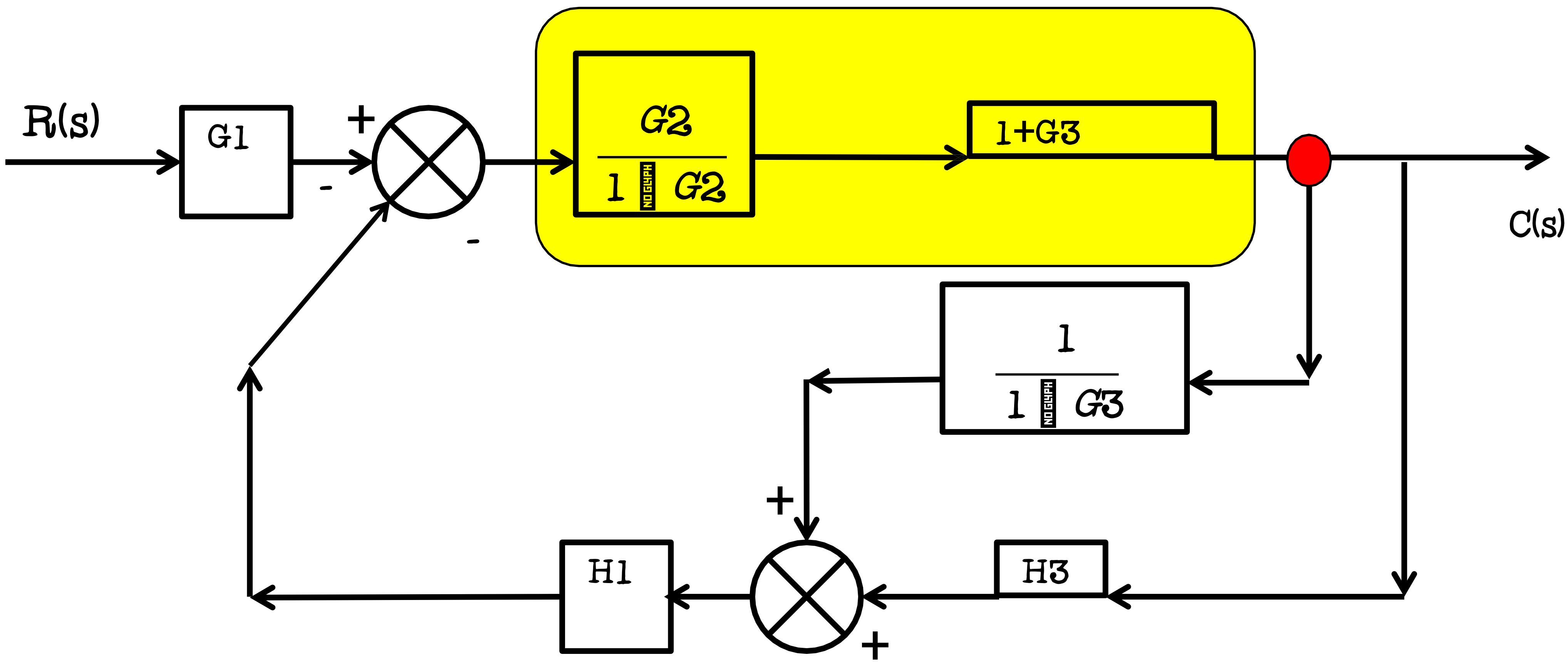
Example 10 cont....

Apply rule 8 Shift take off point after block



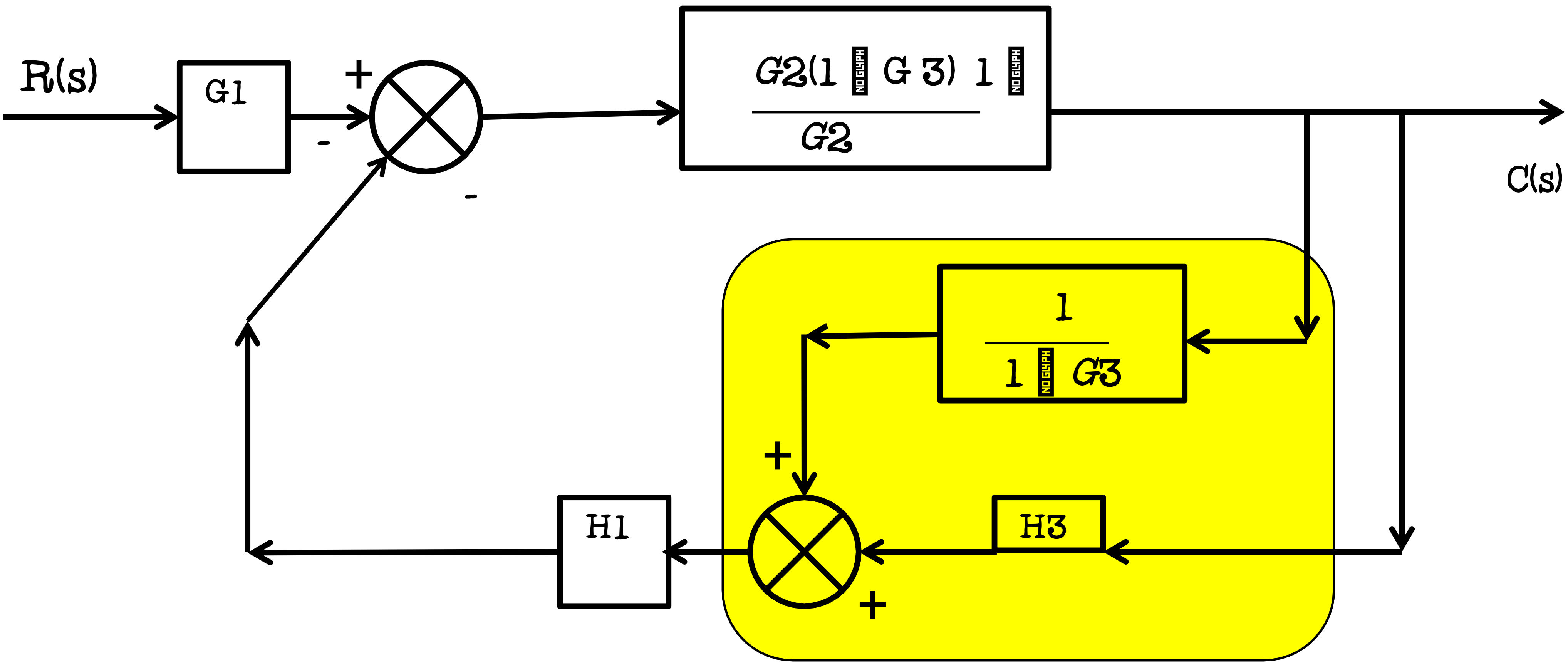
Example 10 cont….

Apply rule 1 Blocks in series



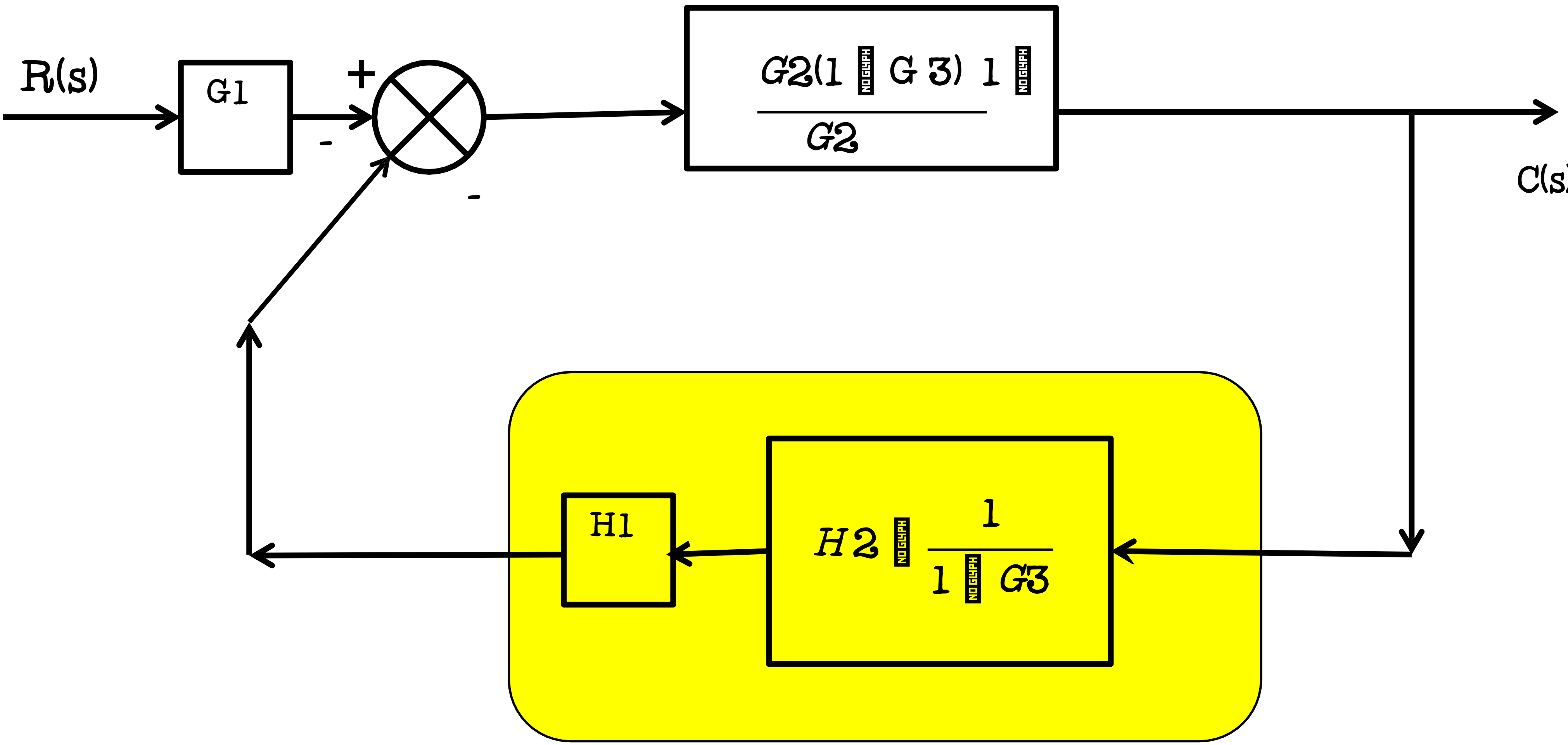
Example 10 cont....

Apply rule 2 Blocks in Parallel



Example 10 cont….

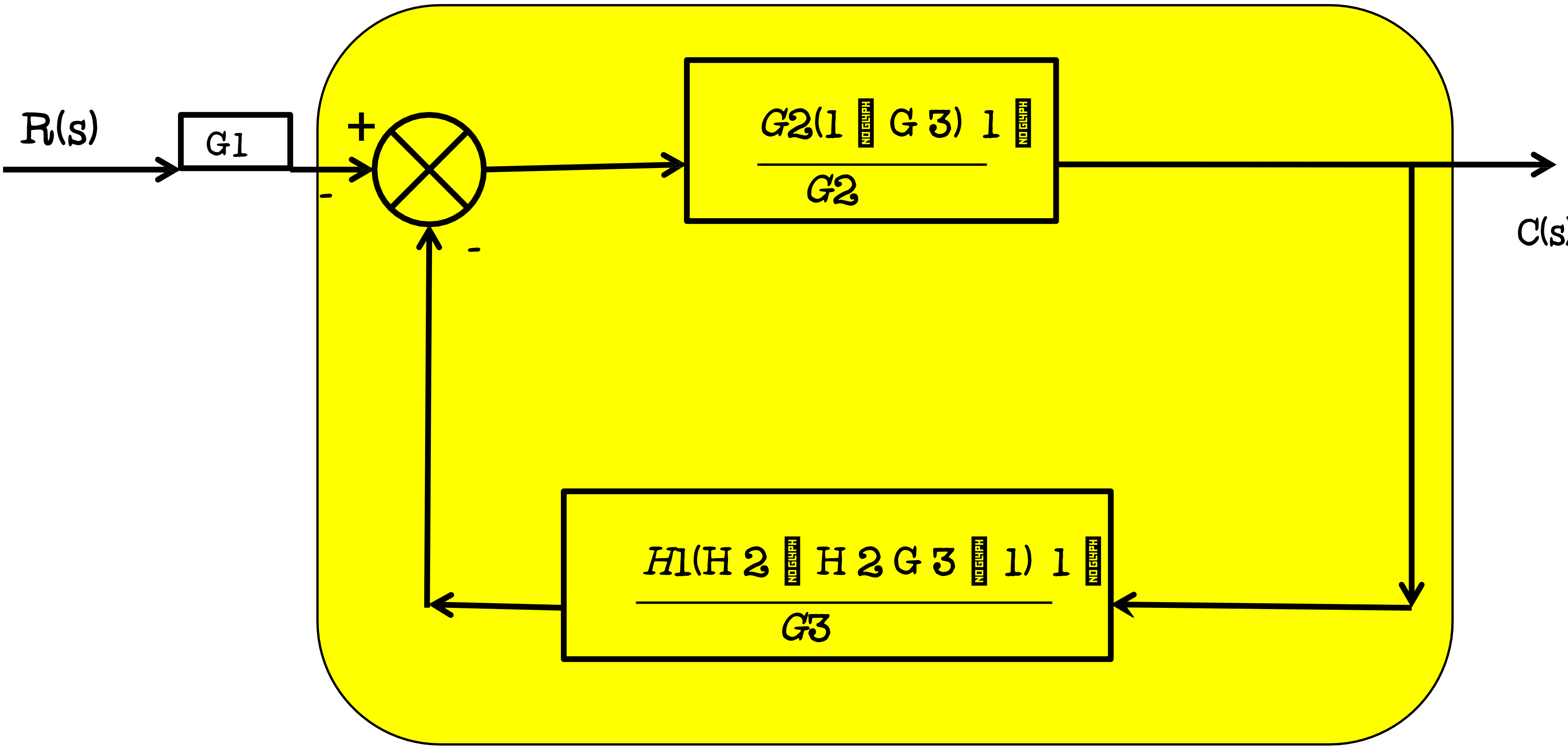
Apply rule 1 Blocks in Series



Example 10 cont....

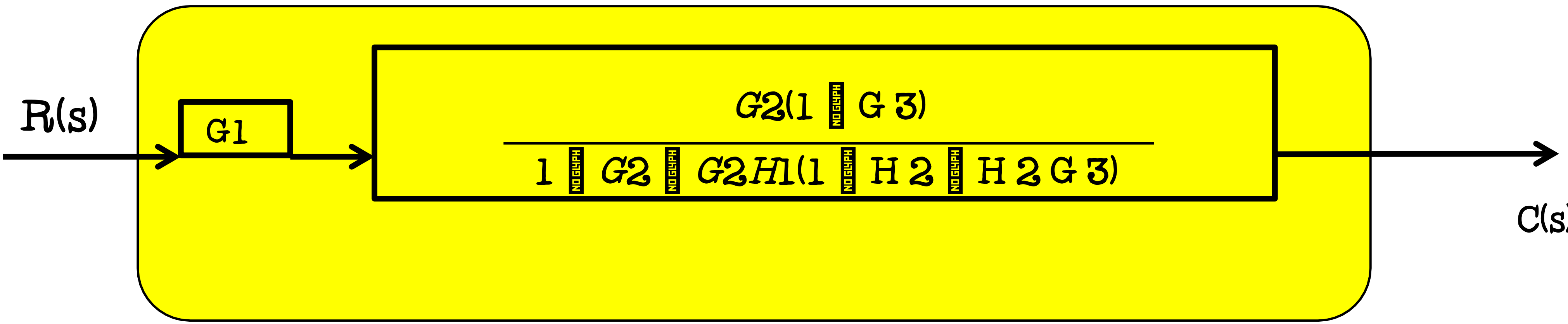
Apply rule 3

Elimination of Feedback loop



Example 10 cont….

Apply rule 1 Blocks in series

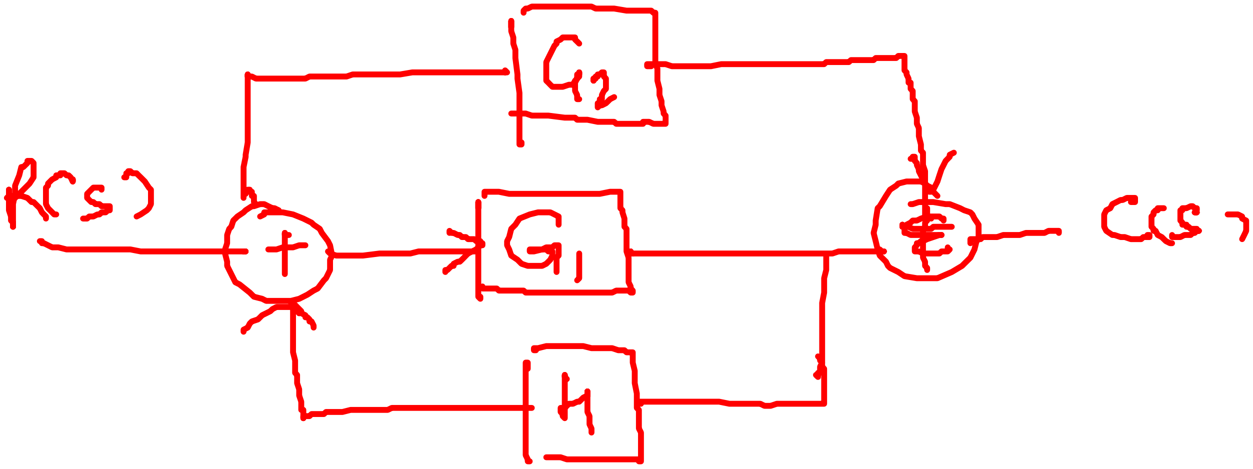



R(s)
G1G2(1 G3)
1 G2 G2H1(1 H2 H2G3)
C(s)

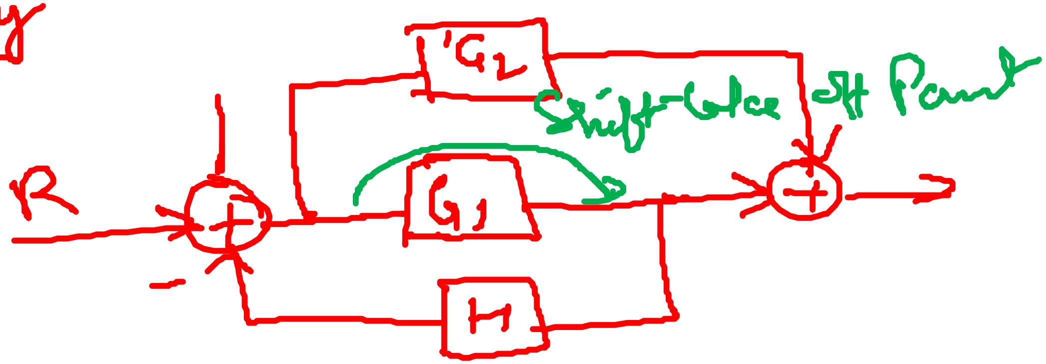
Example 10 cont....

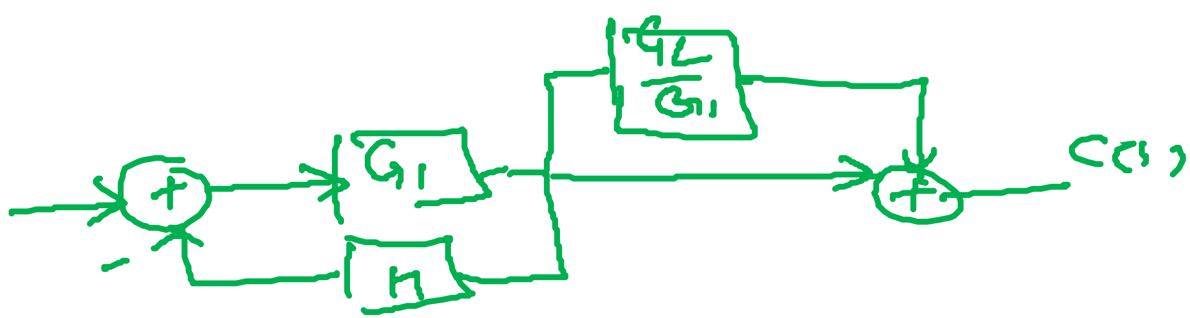
$$\frac{C(s)}{R(s)} \square \frac{G1\,G2(1 \square G3)}{1 \square G2 \square G2H1(1 \square H2 \square H2G3)}$$

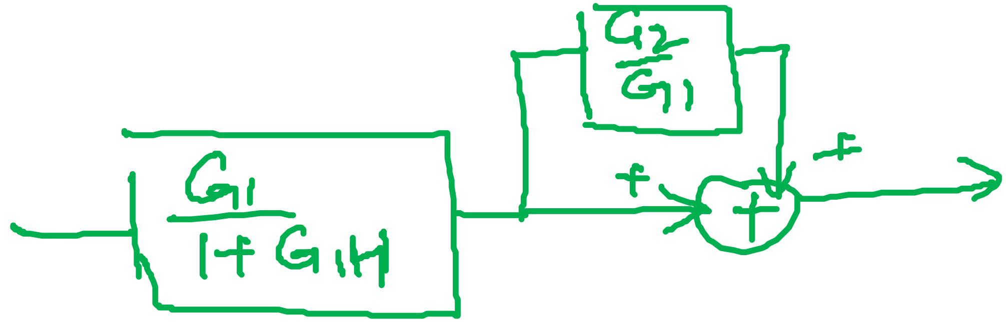
Q Gate Problem

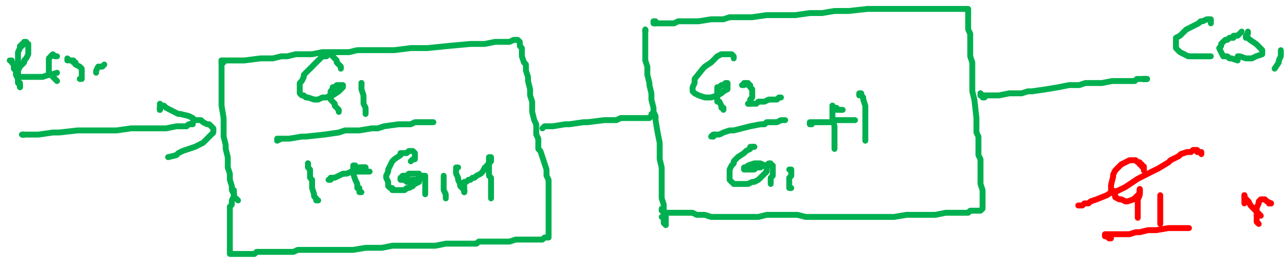


Simplify

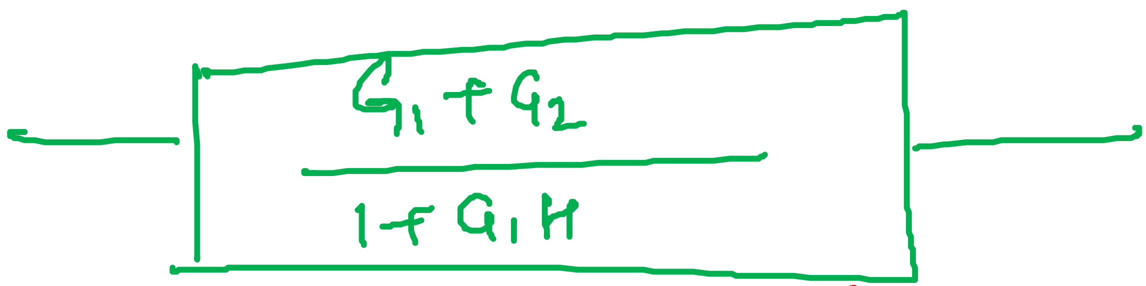








$$\frac{\cancel{G_1}}{1+G_1H} \times \frac{G_2}{\cancel{G_1}} + \frac{G_1}{1+G_1H}$$



$$\frac{G_2}{1+G_1H} + \frac{G_1}{1+G_1H}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1+G_2}{1+G_1H}}$$

Thank You