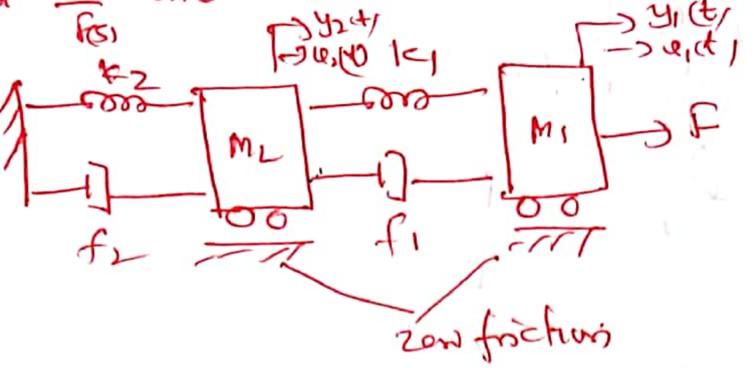
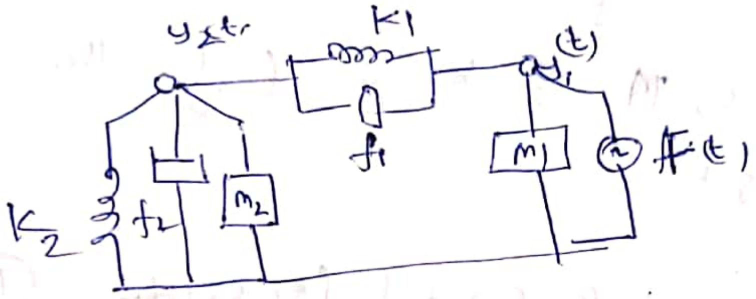
Q Find $\frac{X(s)}{F(s)}$ and find electrical equivalent ckt





At Node $y_1(t)$

$$F(t) = m_1 \frac{d^2 y_1}{dt^2} + f_1 \cancel{\frac{dy_1(t) - dy_2(t)}{dt}} \; f_1 \frac{d(y_1 - y_2)}{dt} + K_1(y_1 - y_2)$$

$$F(t) = m_1 \frac{d^2 y_1}{dt^2} + f_1 \frac{d}{dt}(y_1 - y_2) + K_1(y_1 - y_2) \quad \text{— (1)}$$

At Node $y_2(t)$

$$m_2 \frac{d^2 y_2}{dt} + f_2 \frac{dy_2}{dt} + K_2 y_2 + K_1(y_2 - y_1) + f_1 \frac{d}{dt}(y_2 - y_1) = 0 \quad \text{— (2)}$$

eq (1) & (2) can written in term of velocity

$$F(t) = m_1 \frac{dv_1}{dt} + f_1(v_1 - v_2) + K_1 \int (v_1 - v_2)\,dt \quad \text{— (3)}$$

$$m_2 \frac{dv_2}{dt} + f_2 v_2 + K_2 \int v_2\,dt + K_1 \int (v_2 - v_1)\,dt + f_1(v_2 - v_1) = 0 \quad \text{— (4)}$$

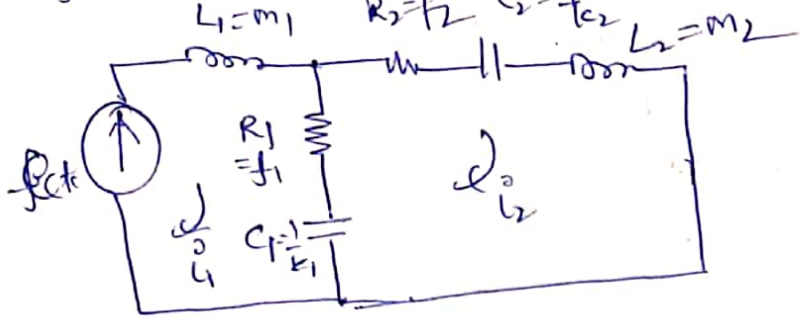
In Force-Voltage analogy $v \to i$

$F \to e$, $m \to L$, $K = 1/C$, $f \to R$ Substitute in (3) & (4)

$$e = L_1 \frac{di_1}{dt} + R_1(i_1 - i_2) + \frac{1}{C_1} \int (i_1 - i_2)\,dt \quad \text{— (5)}$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2\,dt + \frac{1}{C_1} \int (i_2 - i_1)\,dt + R_1(i_2 - i_1) \quad \text{— (6)}$$

from m (5)



__Force – Current analogy__

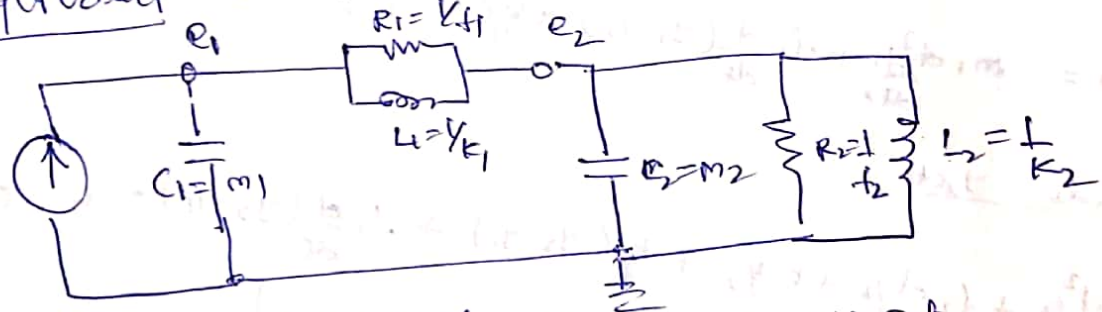
$F \to i, \quad v \to e, \quad M = C, \quad k = \frac{1}{L} \quad f = \frac{1}{R}$ then from

fr (3) & (4) we get

$$i(t) = C_1 \frac{de_1}{dt} + \frac{1}{R_1}(e_1 - e_2) + \frac{1}{L_1}\int (e_1 - e_2)\,dt \quad \text{—(7)}$$

$$\frac{1}{R_1}(e_2 - e_1) + \frac{1}{L_1}\int (e_2 - e_1)\,dt + \frac{e_2}{R_0} + \frac{1}{L_2}\int e_2\,dt + C_2\frac{de_2}{dt} = 0$$
—(8)

__equivalent electrical circuit__



to obtain TF taking Laplace TF (1) & (2) we get

$$F(s) = m_1 s^2 Y_1(s) + f_1 s\,[Y_1(s) - Y_2(s)] + k\,[Y_1(s) - Y_2(s)] \quad \text{—(9)}$$

$$F(s) = [M_1 s^2 + f_1 s + k]\,Y_1(s) - [sf_1 + k_1]\,Y_2(s) \quad \text{—(9)}$$

$$m_2 s^2 Y_2(s) + f_2 s\,Y_2(s) + k_2 Y_2(s) + sf_1[Y_2(s) - Y_1(s)] + k[Y_2(s) - Y_1(s)] = 0$$

$$[m_2 s^2 + (f_2 + f_1)s + (k_1 + k_2)]\,Y_2(s) - (F_1 s + k]\,Y_1(s) = 0 \quad \text{—(10)}$$

$$Y_2(s) = \frac{(F_1 s + k)\,Y_1(s)}{m s^2 + (f_2 + f_1)s + (k_1 + k)} \quad \text{—(11)}$$

Substituting $Y_2(s)$ in (9) & (10) we

$$\frac{Y_1(s)}{F(s)} = m_2 s^2 + (f_1 + f_2)s + (k_1 + k_2) \Big/ (m_1 s^2 + f_1 s + k_1)[m_2 s^2 + (f_2 + f_1)s + (k_2 + k)] - (f_1 s + k_1)^2$$

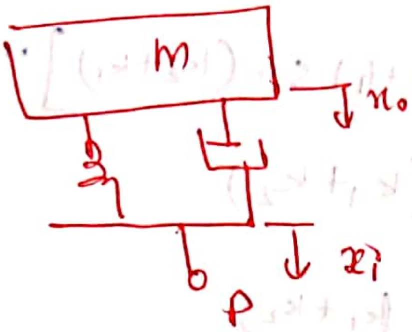
$$F(s) = (m_1 s^2 + f_1 s + k_1)\, Y_1(s) - \frac{(f_1 s + k)(f_1 s + k)\, Y_1(s)}{m_2 s^2 + (f_2 + f_1)s + (k_2 + k_1)}$$

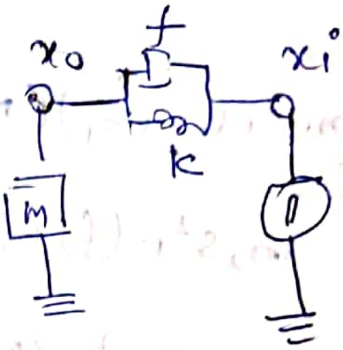
$$F(s) = \frac{\left[(m_1 s^2 + f_1 s + k_1)(m_2 s^2 + (f_2 + f_1)s + (k_2 + k_1)\right] Y_1(s) - (f_1 s + k)^2}{m_2 s^2 + (f_2 + f_1)s + (k_1 + k_2)}$$

$$\underline{TF} = \frac{Y_1(s)}{F(s)} = \frac{m_2 s^2 + (f_2 + f_1)s + (k_1 + k_2)}{(m_1 s^2 + f_1 s + k_1)(m_2 s^2 + (f_2 + f_1)s + (k_2 + k_1)) - (f_1 s + k_1)^2}$$

$$= \frac{m_2 s^2 + (f_2 + f_1)s + k_1 + k_2}{m_1 m_2 s^4 + (m_1 f_1 + m_1 f_2 + m_2 f_1)s^3 + (m_1 k_2 + m_1 k_1 + m_2 k_1 + f_1 f_2)s^2 + (f_1 k_2 + f_2 k_1)s - k_1^2}$$

Q find the TF $\frac{X_o(s)}{X_i(s)}$





$$m\frac{d^2x_o}{dt^2} + k(x_o - x_i) + f\left(\frac{dx_o}{dt} - \frac{dx_i}{dt}\right) = 0$$

taking Laplace Transform

$$ms^2 X_o(s) + k(X_o(s) - X_i(s)) + fs(X_o(s) - X_i(s)) = 0$$

$$ms^2 X_o(s) + kX_o(s) + fs X_o(s) = k X_i(s) + fs X_i(s)$$

$$(ms^2 + fs + k) X_o(s) = (fs + k) X_i(s)$$

$$\boxed{\frac{X_o(s)}{X_i(s)} = \frac{fs + k}{ms^2 + fs + k}}$$

Steps to Solve Problems on Analogous System

① Identify all displacements due to the applied force. The element & spring and friction between two moving surface cause change in displacement

② Draw equivalent diagram based on node basis. The elements under same displacement will get connected in parallel under that node. Each displacement represented by seperate node. Element causing change in displacement is always between two nodes.

③ Write equilibrium equations. At each node algebraic sum of all the forces acting at the node is zero.

④ In F-V analogy, use following replacement and rewrite eqn

$F \rightarrow V$, $M \rightarrow L$, $B - R$; $K = 1/C$ $x - q$, $\dot{x} \rightarrow i$

⑤ Simulate the eq using loop method. No of displacement is equal to no of loop currents.

⑥ In $F \rightarrow I$ analogy

$F \rightarrow I$, $M \rightarrow C$, $B \rightarrow 1/R$, $K \rightarrow 1/L$, $x \rightarrow \phi$, $\dot{x} = e$

⑦ Simulate the eqn using node basis. No of displacement equal to numbers of node voltages. Infact the system will be exactly same as equivalent mechanical system obtained in step 2 with appropriate replacements

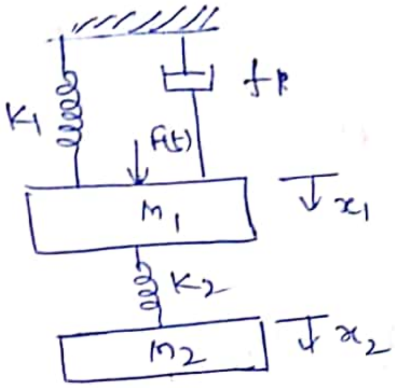
Analogous System

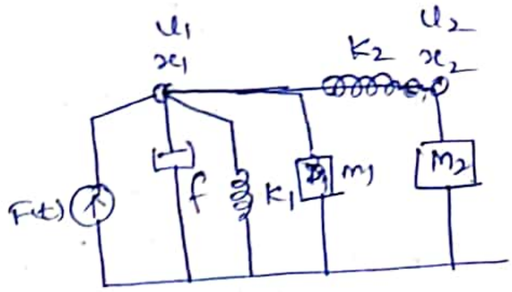
Force – voltage analogy → Direct analogy
Force – current analogy → inverse "

Force-voltage analogy, Force-current analogy

	Mechanical System		Electrical System	
	Translational	Rotational	Voltage analogy	Current analogy
I/P Variable	Force (F)	Torque (T)	Voltage (e)	Current (i)
Energy storage	Mass (M)	Inertia (J)	Inductance (L)	Capacitance (C)
Dissipation	Friction (B)	Torsional friction (B)	Resistance (R)	$1/R$
Energy storing element	Spring constant K (N/m)	Torsional spring constant (K) (Nm/rad)	$1/C$	$1/L$
			charge (q)	flux (ϕ)
O/P Variable	Displacement x	θ (angle)		
	velocity $v = \frac{dx}{dt}$	$\dot{\theta} = \frac{d\theta}{dt} = \omega$	current $i = \frac{dq}{dt}$	$e = \frac{d\phi}{dt}$

Q. obtain the analogous electrical circuits based on on a) Force - current analogy (b) Force - voltage analogy also find the transfer function





The differential eqn at node x_1

$$F(t) = m_1 \frac{d^2 x_1}{dt^2} + f\frac{dx_1}{dt} + K_1 x_1 + K_2(x_1 - x_2) \quad -(1)$$

or $$F(t) = m_1 \frac{dv_1}{dt} + f v_1 + K_1\int v_1 dt + K_2\int (v_1 - v_2) dt$$

At Node 2

$$m_2 \frac{d^2 x_2}{dt^2} + K_2(x_2 - x_1) = 0$$

or $$m_2 \frac{dv_2}{dt} + K_2\int (v_2 - v_1) dt = 0 \quad -(2)$$

The analogous electrical network on Force - Current analogy is

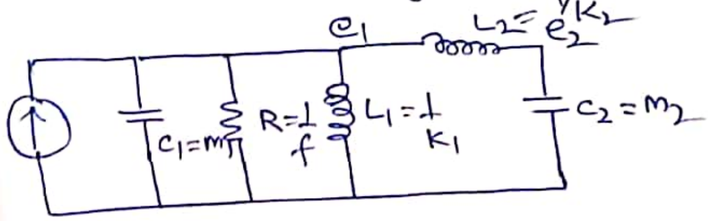
we know $F(t) \equiv i(t)$, $v = e$

$m = C$, $K = \frac{1}{L}$ $f = 1/R$

from eqn (1) & (2) Substitute analogous value

$$i(t) = C_1 \frac{de_1}{dt} + \frac{e_1}{R_f} + \frac{1}{L_1}\int e_1 dt + \frac{1}{L_2}\int (e_1 - e_2) dt \quad -(3)$$

$$C_2 \frac{de_2}{dt} + \frac{1}{L_2}\int (e_2 - e_1) dt \quad -(4)$$



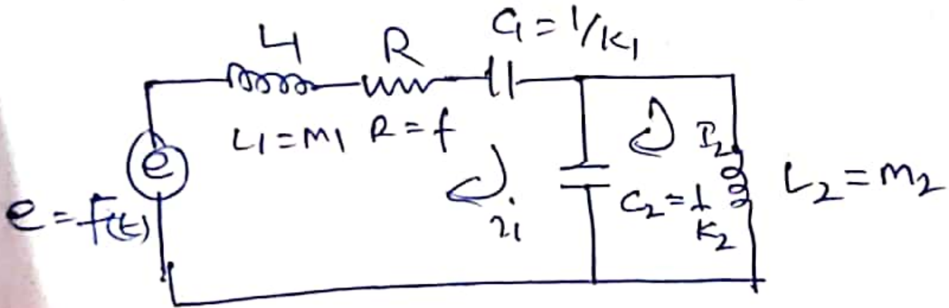
Force – voltage analogy – we know that

Force – $F \rightarrow e$, $u - i$, $m = L$, $K = 1/C$, $f = R$

from eqn ① & ② we get

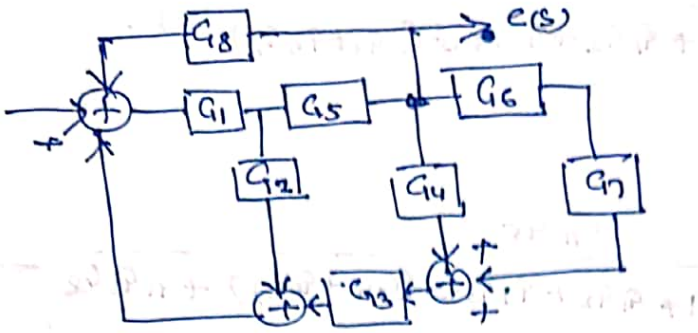
$$e = L_1 \frac{di}{dt} + iR + \frac{1}{C_1}\int i_1 dt + \frac{1}{C_2}\int (i_1 - i_2)\, dt \quad \text{—} (5)$$

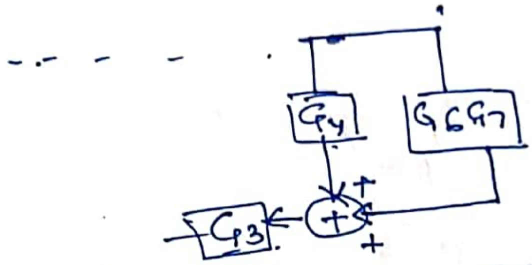
$$L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_1)\, dt \quad \text{—} (6)$$

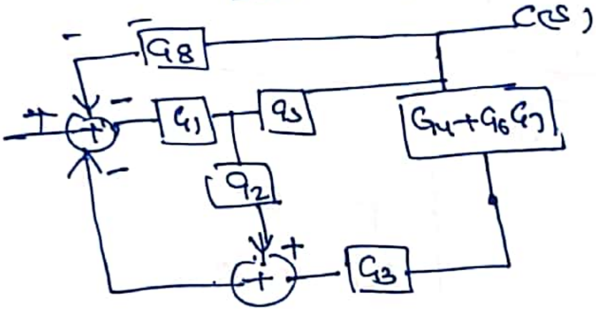


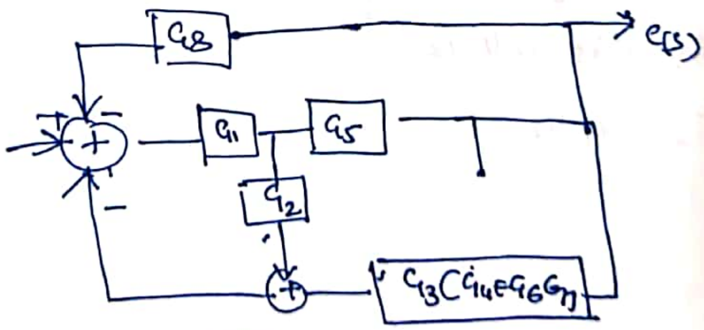
③ $\frac{X_1(s)}{F(s)}$

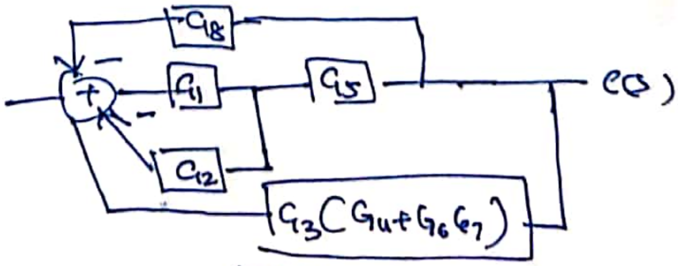
1 find C(s)/R(s) by block Reduction

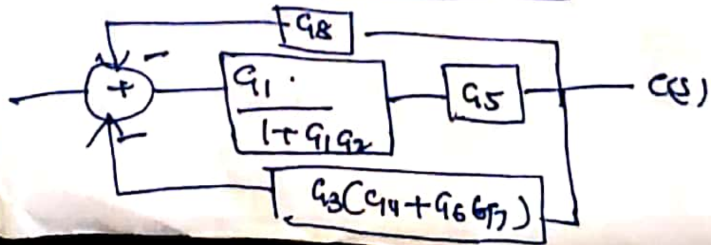


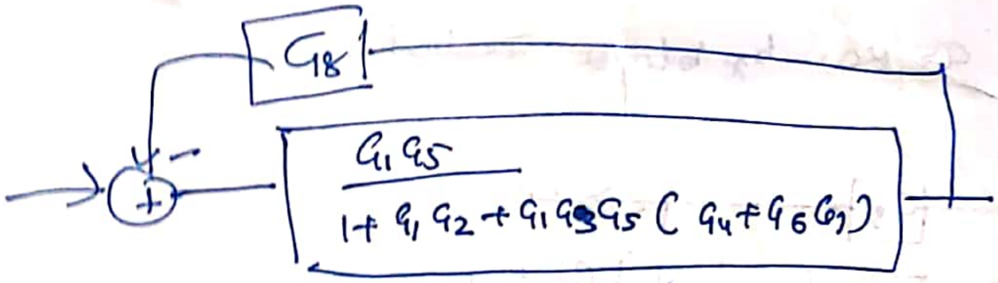


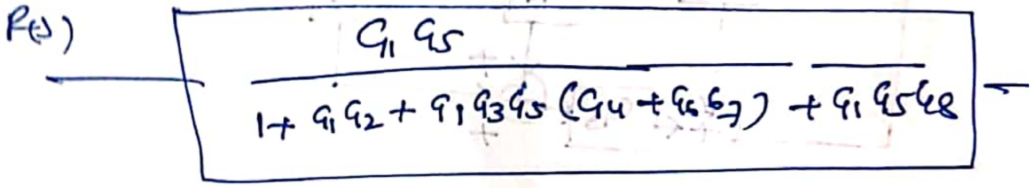




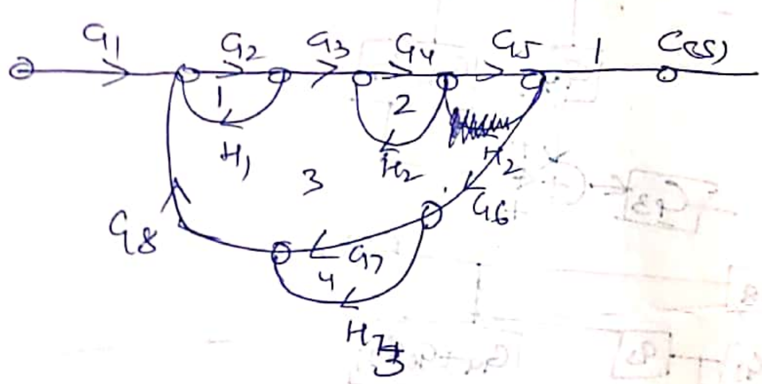








Q find $\frac{C(s)}{R(s)}$



$L_1 = G_2H_1$

$L_2 = G_4H_2$

$L_3 = G_1G_2G_3G_4G_5G_6G_7G_8$

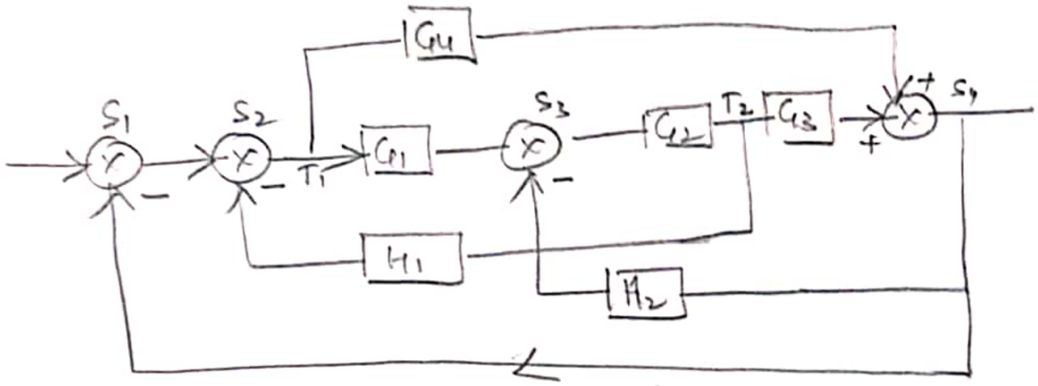
$L_4 = G_2G_3G_4G_5G_6H_3G_8$

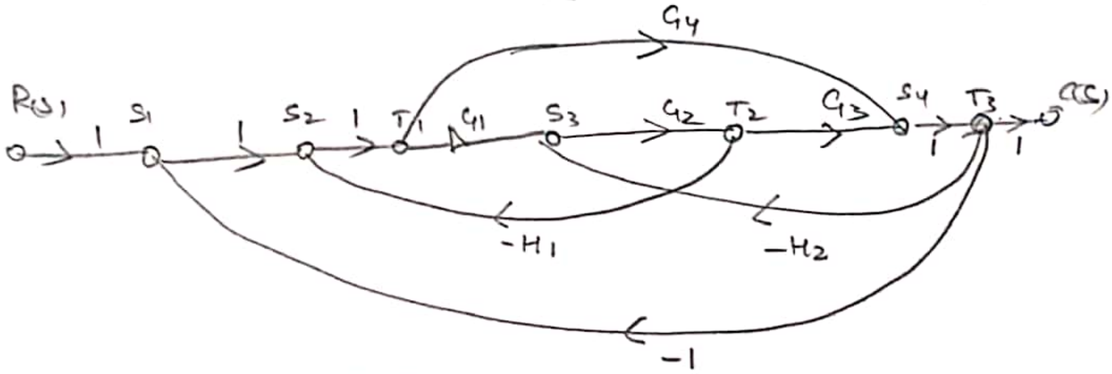
$L_2 \& L_3$ $G_2G_4H_1H_2$

$P_1 = G_1G_2G_3G_4G_5$

$\Delta_1 = 1-0 = 1$

Q





No of Forward Path 2 $N=2$

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{k=1}^{2} P_k \Delta_k = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

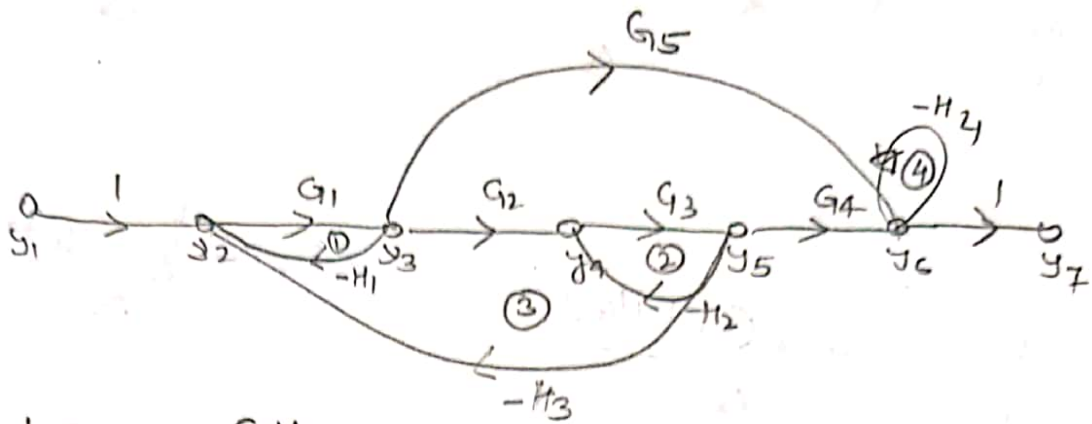
$P_1 = G_1 G_2 G_3$, $P_2 = G_4$

$L_1 = -G_1 G_2 H_1$, $L_2 = -G_2 G_3 H_2$, $L_3 = -G_1 G_2 G_3$

$L_4 = -G_4$, $L_5 = G_4 H_2 H_1 G_2$

$-H_1$

Find y_2/y_1, $\frac{y_4}{y_1}$, $\frac{y_6}{y_1}$ & y_7/y_2



$L_{11} = -G_1H_1$

$L_{21} = -G_3H_2$

$L_{31} = -G_1G_2G_3H_3$

$L_{41} = -H_4$

Pairs of two Non touching loop $[(1,2)\ (1,4)\ (3,4)\ (2,4)]$

$L_{12} = L_{11}$ & $L_{21} \Rightarrow G_1H_1G_3H_2 = G_1G_3H_1H_2$

$L_{22} = L_{11}$ & $L_{41} \rightarrow G_1H_1H_4$

$L_{33} = L_{31}$ & $L_{41} \rightarrow G_1G_2G_3H_3H_4$

$L_{3\cancel{4}} = L_{21}$ & $L_{41} \rightarrow G_3H_2H_4$

Pairs of three non touching loops

$L_{31} = L_{11}$ & L_{21} & L_{41} $= G_1G_3H_1H_2H_4$

$\Delta = 1 - (L_{11} + L_{21} + L_{31} + L_{41}) + (L_{12} + L_{22} + L_{23} + L_{24}) - (L_{31})$

$= 1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_1G_3H_1H_2 + G_1H_1H_4$

$+ G_1G_2G_3H_3H_4 + G_3H_2H_4 + G_1G_3H_1H_2H_4$

$P_1 = G_1G_2G_3G_4$ and $\Delta_1 = 1$

$P_2 = G_1G_5$ and $\Delta_2 = 1 + G_3H_2$

for y_2/y_1: $P_1 = 1$, $\Delta_1 = 1 + G_2H_3 + H_4 + \underset{\text{Pairs}}{G_3H_2H_4}$

$\frac{y_4}{y_1}$, $P_1 = G_1G_2$ $\quad \Delta_1 = 1 + H_4$

$$\frac{y_2}{y_1} = \frac{1+G_3H_2+H_4+G_3H_2H_4}{\Delta} = \frac{\sum P_k\Delta_k\Big|_{\text{from } y_1 \text{ to } y_2}}{\Delta}$$

SFG

$$\frac{y_4}{y_1} = \frac{G_1G_2(1+H_4)}{\Delta} = \frac{\sum P_k\Delta_k\Big|_{\text{from } y_1 \text{ to } y_4}}{\Delta}$$

$$\frac{y_6}{y_1} = \frac{y_7}{y_1} = \frac{G_1G_2G_3G_4+G_1G_5(1+G_3H_2)}{\Delta} = \sum P_k\Delta_k\Big|_{y_1 \text{ to } y_7}$$

$$\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1} = \frac{\sum P_k\Delta_k\Big|_{\text{for } y_1 \text{ to } y_7}/\Delta}{\sum P_k\Delta_k\Big|_{\text{for } y_1 \text{ to } y_2}/\Delta}$$

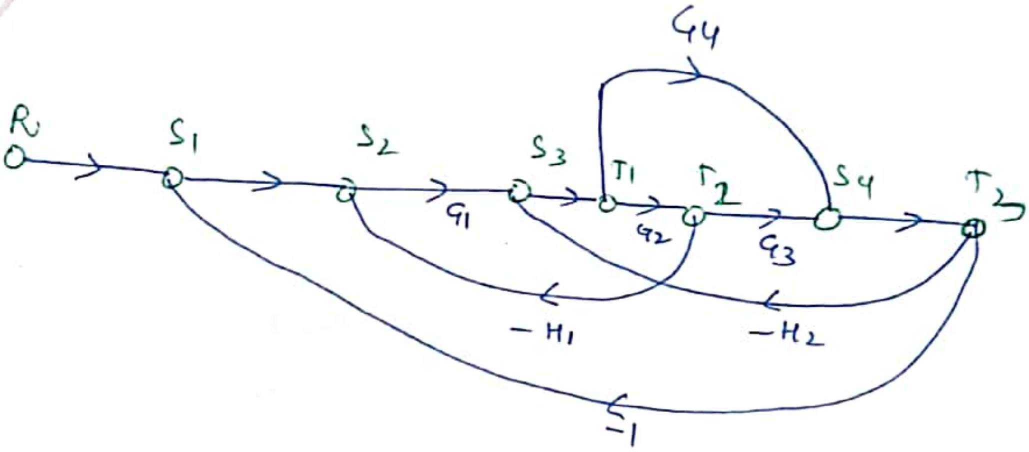
$$= \frac{\sum P_k\Delta_k\Big|_{\text{for } y_1 \text{ to } y_7}}{\sum P_k\Delta_k\Big|_{\text{for } y_1 \text{ to } y_2}}$$

$$\frac{y_7}{y_2} = \frac{G_1G_2G_3G_4+G_1G_5(1+G_3H_2)}{1+G_3H_2+H_4+G_3H_2H_4}$$

$-H_1$

SFG

$$\frac{C}{R} = \sum \frac{P_k \Delta_k}{\Delta}$$



Forward path P_1 $R \Rightarrow S_1 \Rightarrow S_2 \Rightarrow S_3 \Rightarrow T_1 \rightarrow T_2 \Rightarrow S_4 \Rightarrow T_3$

$$P_1 = G_1 G_2 G_3, \quad \Delta_1 = 1$$

Forward path P_2. $R - S_1 - S_2 - S_3 - T_1 - S_4 - T_3$

$$P_2 = G_1 G_4, \quad \Delta_2 = 1$$

loops ~~gam~~ and gain associated with them

Loop 1, $L_1 = S_1 - S_2 - S_3 - T_1\ T_2 - S_4 - T_3 - S_1$

$$L_1 = -G_1 G_2 G_3$$

Loop 2 - $S_2 - S_3 - T_1 - T_2 - S_2$, $L_2 = -G_1 G_2 H_1$

Loop 3 - $S_3 - T_1 - T_2 - S_4 - T_3 - S_3 = L_3 = -G_2 G_3 H_2$

Loop 4 - $S_3 - T_1 - S_4 - T_3 - S_3 \Rightarrow$ ~~L4=G4H1~~ $L_4 = -G_4 H_2$

Loop 5 - $S_1 - S_2 - S_3 - T_1 - S_4 - T_3 - S_1 \Rightarrow L_5 = -G_1 G_4$

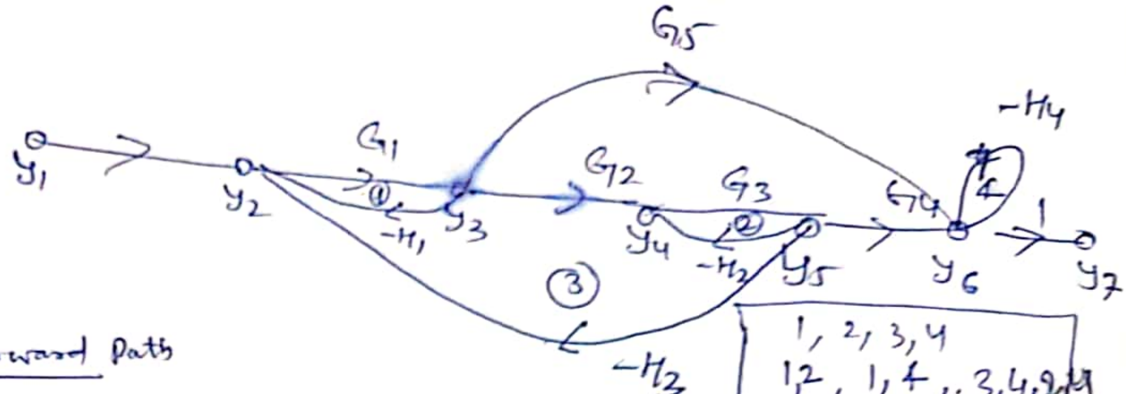
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$$

$$= 1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 H_3 H_2 + G_4 H_2 + G_1 G_4$$

$$\frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 + G_1 G_4}{\Delta}$$

$-H_1$

Find y_2/y_1, y_4/y_1, y_6/y_1, y_7/y_2



1, 2, 3, 4
1,2, 1,4, 3,4, 2,4
1, 2, 4

Forward path

$P_1 = G_1 G_2 G_3 G_4$, $\Delta_1 = 1$

$P_2 = G_1 G_5$, $\Delta_1 = 1 + G_3 H_2$

$\Delta = 1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_1H_1H_4 + G_3H_2H_4 + G_1G_3H_1H_2H_4 + G_1G_2G_3H_3H_4 + G_1G_3H_1H_2$

(1) y_2/y_1

$P_1 = 1$, $\Delta_1 = 1 + G_3H_2 + H_4 + G_3H_2H_4$

$$y_2/y_1 = \frac{1+G_3H_2+H_4+G_3H_2H_4}{\Delta}$$

(2) $\frac{y_6}{y_1}$, Forward path $P_1 = G_1G_2G_3G_4$, $\Delta_1 = 1$

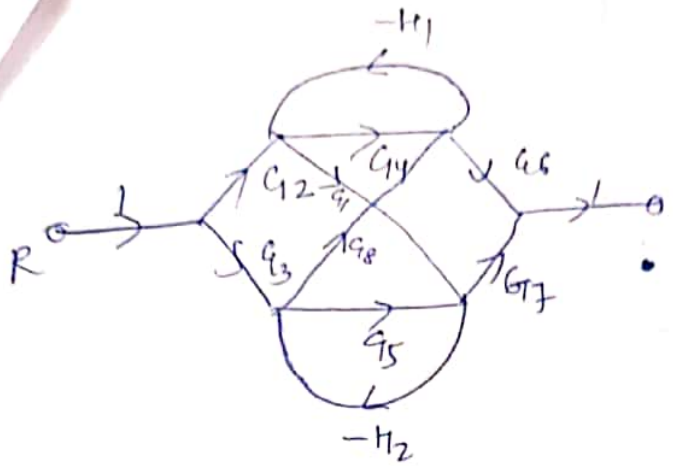
$P_2 = G_1G_5$, $\Delta_2 = 1+G_3H_2$

$$\frac{y_7}{y_1} = \frac{y_6}{y_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{\Delta}$$

(3) $\frac{y_4}{y_1} = \ldots =$, $P_1 = G_1G_2$, $\Delta_1 = 1+H_4$

$$= \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2(1+H_4)}{\Delta}$$

$$\boxed{\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1+G_3H_2+H_4+G_3H_2H_4}}$$



Forward Path

$P_1 = G_2 G_4 G_6$, $\Delta_1 = 1 + G_5 H_2$

$P_2 \; G_3 G_5 G_7$, $\Delta_2 = 1 + G_4 H_1$

$P_3 = G_2 G_1 G_7$, $\Delta_3 = 1$

$P_4 = G_3 G_8 G_6$ $\Delta_4 = 1$

$P_5 = -G_2 G_1 H_2 G_8 G_6$ $\Delta_5 = 1$

$P_6 = -G_3 G_8 H_1 G_1 G_7$ $\Delta_6 = 1$

Probe loops

$L_1 = G_4 H_1$

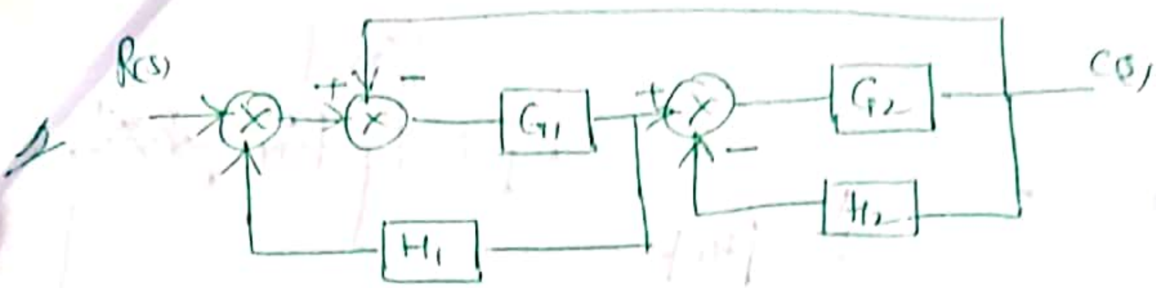
$L_2 \; G_5 H_2$

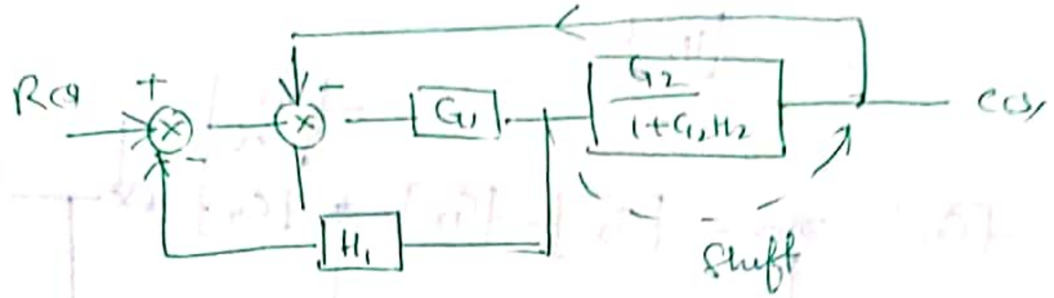
$L_3 = H_1 G_1 H_2 G_8$

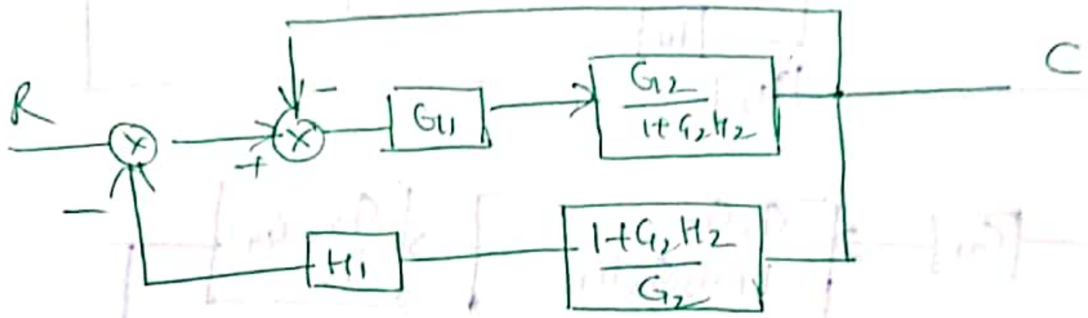
Two non touchy loops

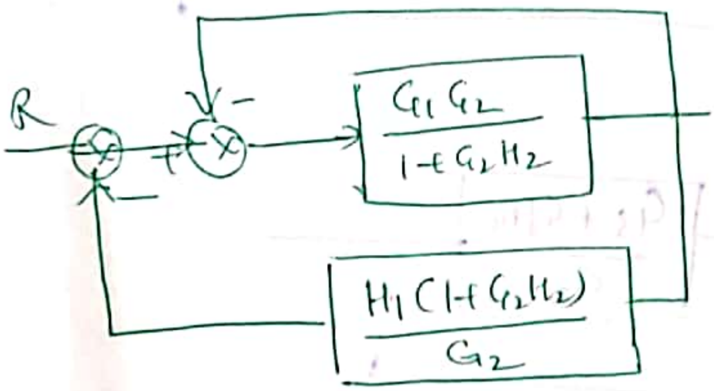
$L_1 L_2 \quad G_4 G_5 H_1 H_2$

$\Delta = 1 + (L_1 + L_2 + L_3) + L_1 L_2$

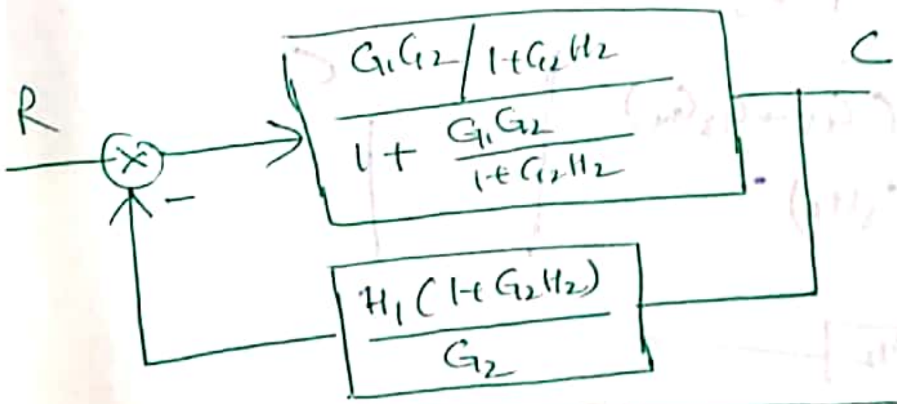


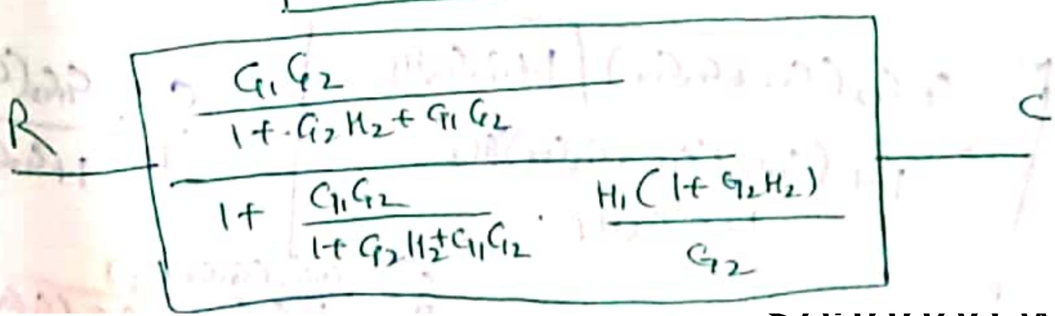




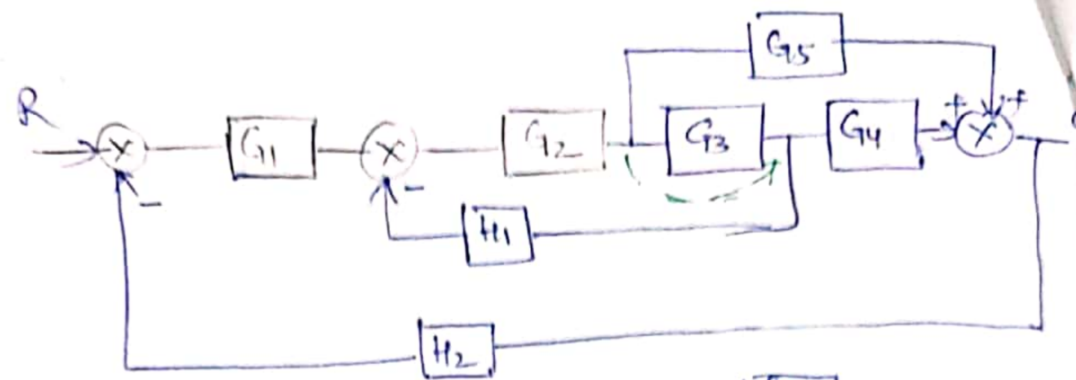


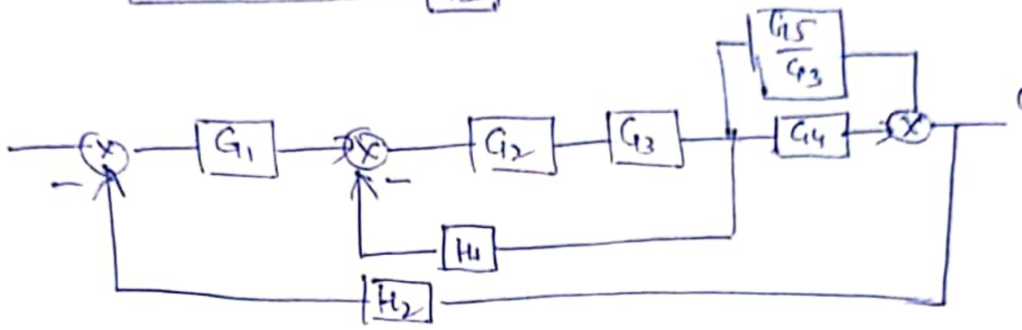
$$\frac{C}{R} = \frac{G_1G_2}{1+G_1G_2+G_2H_2+G_1G_1+G \ldots + G_1G_2H_1H_2}$$

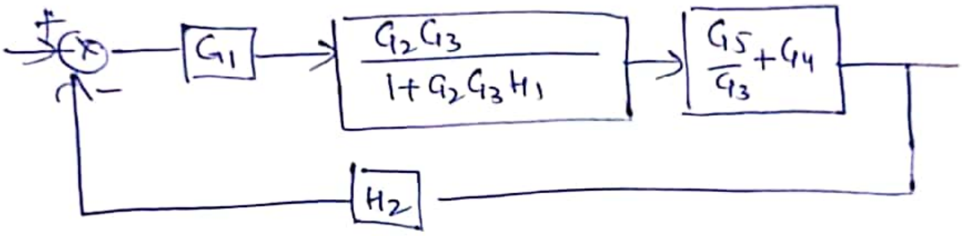


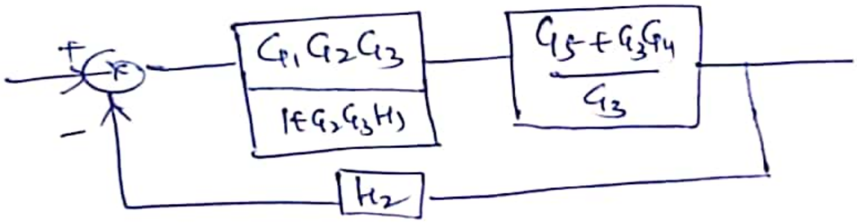


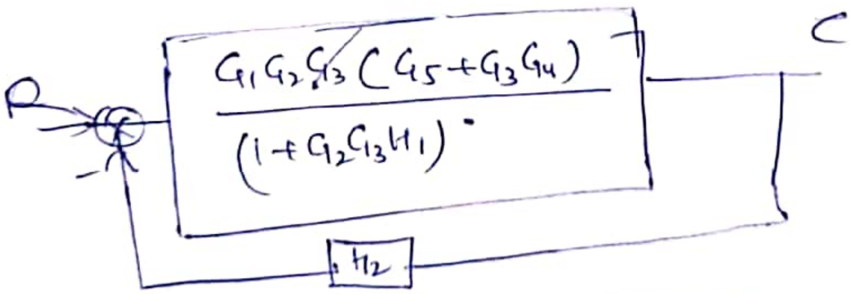
Q

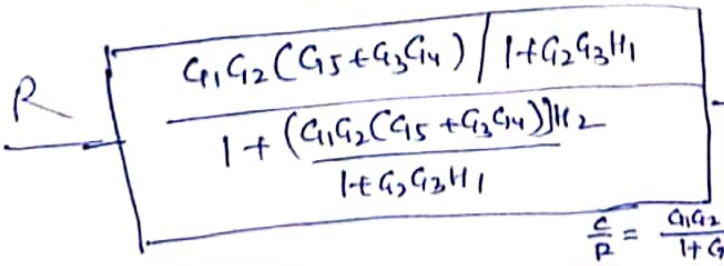






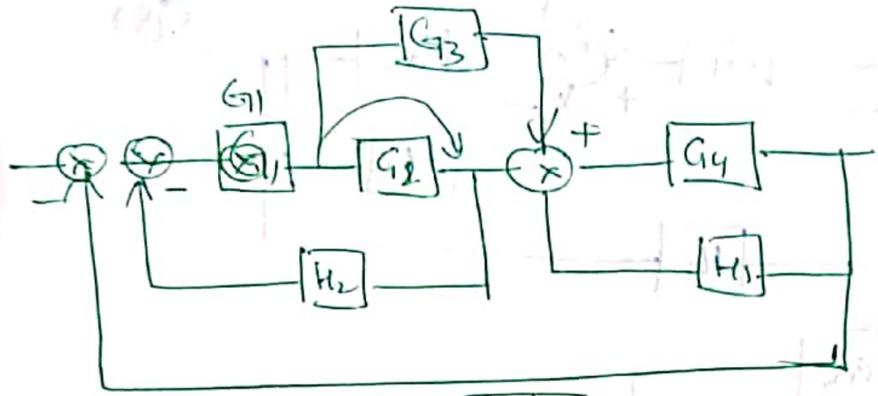


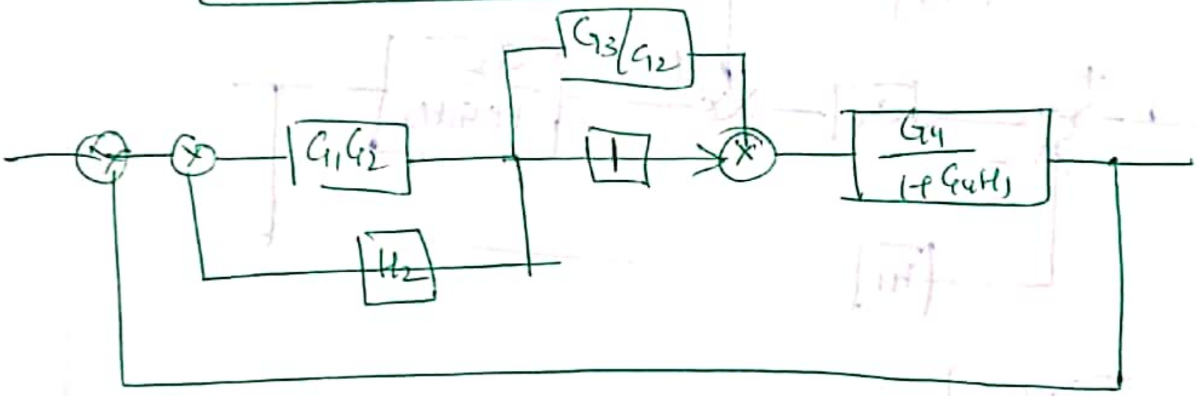


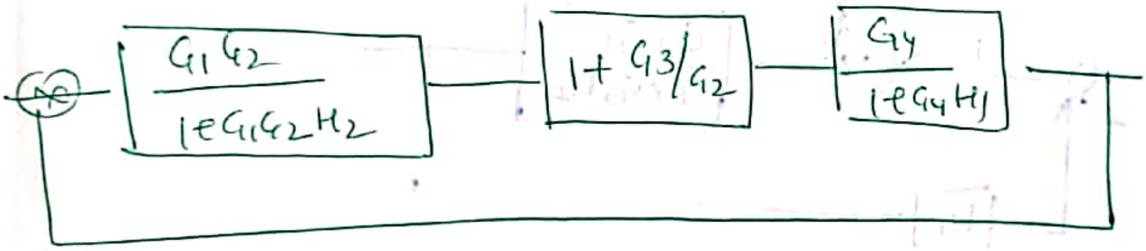


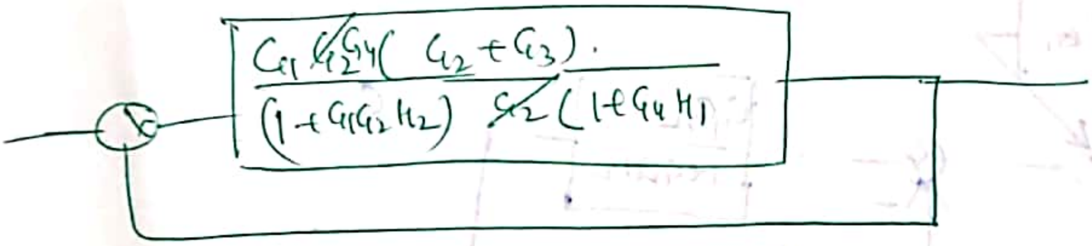
$$\frac{C}{R} = \frac{G_1G_2(G_3G_4+G_5)}{1+G_2G_3H_1+G_1G_2(G_3G_4+G_5)H_2}$$

$$\frac{C}{R} = \frac{G_1G_2(G_3G_4+G_5)}{1+G_2G_3H_1+G_1G_2(G_3G_4+G_5)H_2}$$





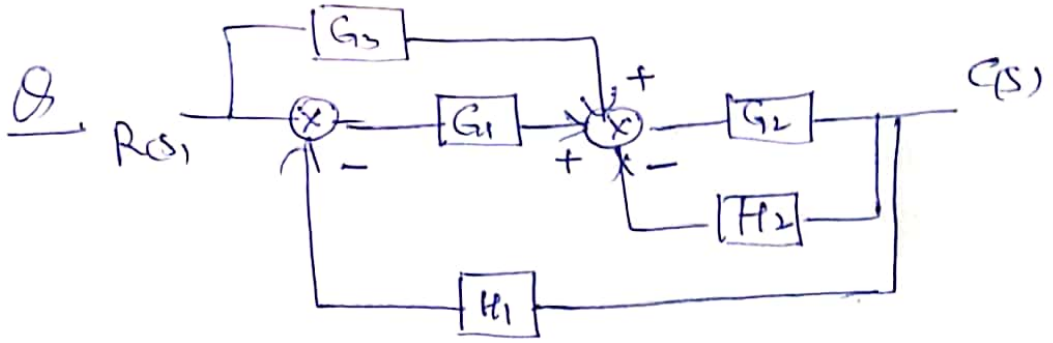


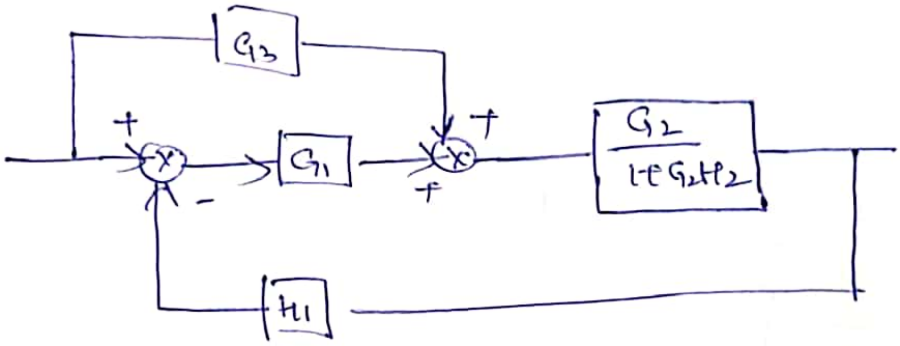


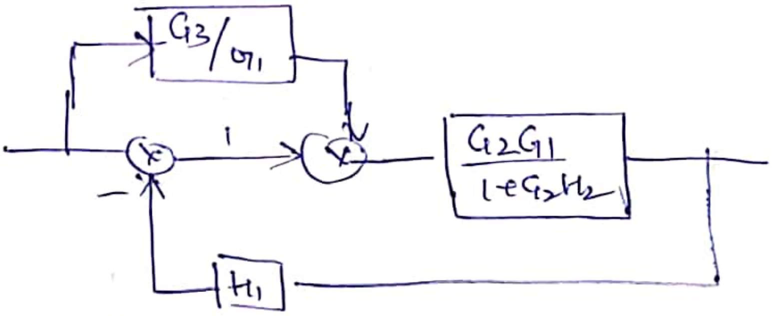
$$\boxed{\dfrac{\dfrac{G_1G_4(G_2+G_3)}{(1+G_1G_2H_2)(1+G_4H_1)}}{1+\dfrac{G_1G_4(G_2+G_3)}{(1+G_1G_2H_2)(1+G_4H_1)}}}$$

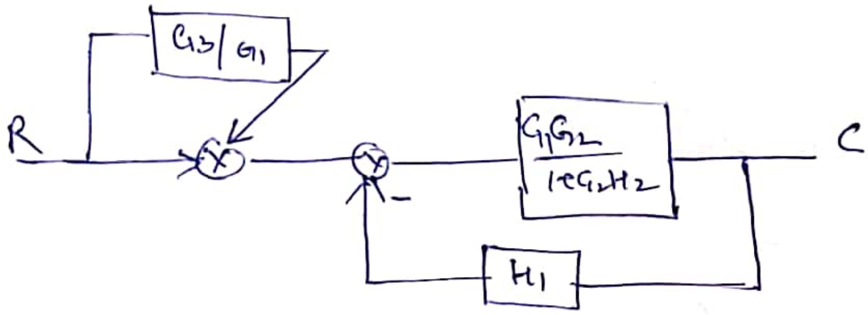
$$= \frac{G_1G_4(G_2+G_3)}{(1+G_1G_2H_2)(1+G_4H_1)+G_1G_4(G_2+G_3)}$$

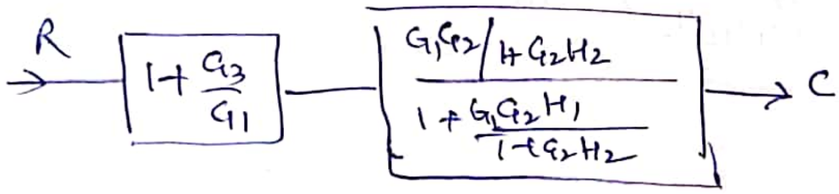
Q











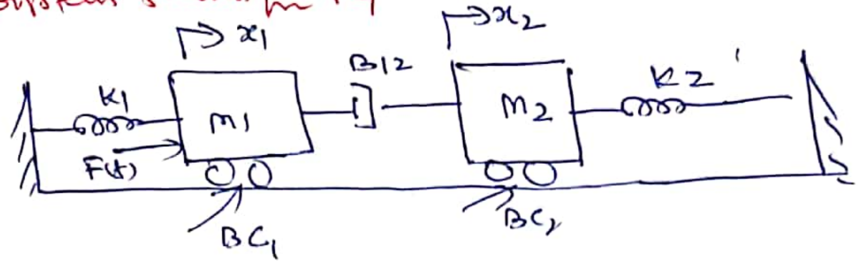
$$R \rightarrow \left(\frac{G_1+G_3}{\cancel{G_1}}\right)\left(\frac{\cancel{G_1}G_2}{1+G_1G_2H_1+G_2H_2}\right) \rightarrow C$$

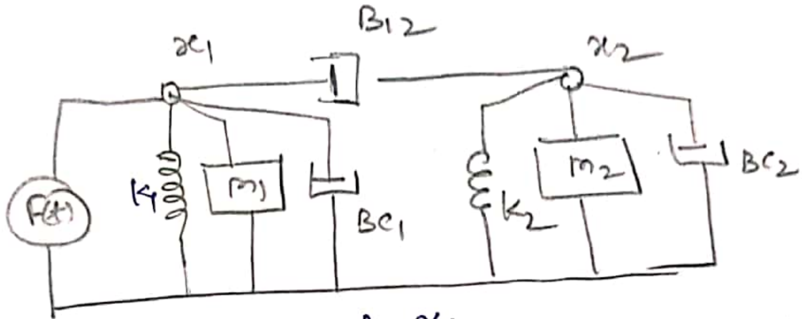
$$\boxed{\frac{C}{R} = \frac{(G_1+G_3)G_2}{1+G_1G_2H_1+G_2H_2}}$$

Solution of Assignment-1 ①

Q1 Derive the System eqn & find the value of $X_2(s)/F(s)$ for the system shown in fig

②





equilibrium eqn at node x_1

$$F(t) = m_1\frac{d^2x_1}{dt^2} + BC_1\frac{dx_1}{dt} + B_{12}\frac{d}{dt}(x_1-x_2) + k_1x_1 \quad —①$$

$$k_2x_2 + m_2\frac{d^2x_2}{dt^2} + BC_2\frac{dx_2}{dt} + B_{12}\frac{d}{dt}(x_2-x_1) = 0 \quad —②$$

taking Laplace transform of eqn ① & ② we get

$$F(s) = m_1s^2X_1(s) + BC_1\,s\,X_1(s) + B_{12}[X_1(s) - X_2(s)] + k_1X_1(s) \quad —③$$

$$B_{12}s\,X_1(s) = k_2X_2(s) + m_2s^2X_2(s) + BC_2X_2(s) + B_{12}X_2(s) \quad ④$$

$$X_1(s) = \frac{X_2(s)[s^2m_2 + B_{12}s + s\,BC_2 + k_2]}{B_{12}s} \quad —⑤$$

Put the value of $X_1(s)$ in ③ we get

$$F(s) = X_1(s)[s^2m_1 + sB_{12} + sBC_1 + k] - B_{12}\,s\,X_2(s) \quad —③$$

$$F(s) = \frac{X_2(s)[s^2m_2 + B_{12}s + sBC_2 + k][s^2m_1 + sB_{12} + sBC_1 + k] - B_{12}\,s\times s}{B_{12}s}$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{B_{12}s}{(s^2m_2 + sB_{12} + sBC_1 + k)(s^2m_1 + sB_{12} + sB_c + k) - B_{12}^2s^2}}$$

Q2 S.T the system shown in fig (1) & (2) are analogous

②

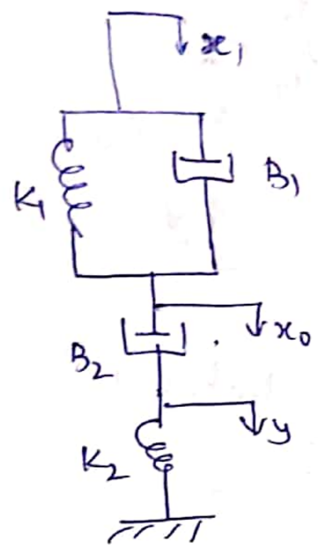


fig (1) mechanical system

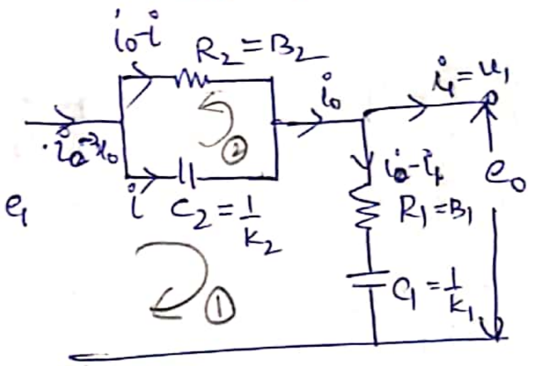
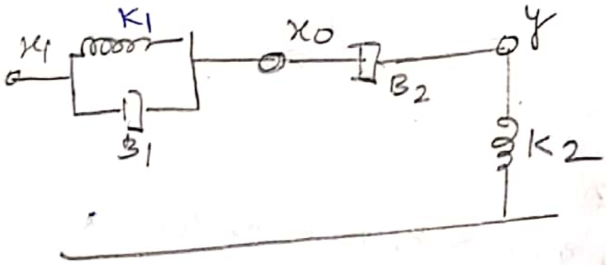


fig (2) Force voltage analogous



let $\frac{dx_0}{dt} = u_0$, $\frac{dx_1}{dt} = u_1$, $\frac{dy}{dt} = u$

At node x_0:

$$B_1 \frac{d(x_0 - x_1)}{dt} + K_1 (x_0 - x_1) + B_2 \frac{d(x_0 - y)}{dt} = 0$$

$$B_1 (u_0 - u_1) + K_1 \int (u_0 - u_1)\, dt + B_2 (u_0 - u) = 0 \quad \text{①}$$

$u_0 \rightarrow i_0$, $u_1 \rightarrow i_1$, $u \Rightarrow i$

At node y: $B_2 \frac{d(y - x_0)}{dt} + K_2 y = 0$

$$B_2 (u - u_0) + K_2 \int u\, dt \quad \text{②}$$

Force-voltage analogy $F \rightarrow e$, $L = M$, $u \rightarrow i$, $K = 1/C$, $B = R$

substitute in eqn ① & ② we get

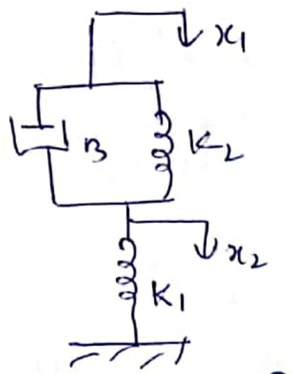
$$R_1 (i_0 - i_1) + \frac{1}{C_1} \int (i_0 - i_1)\, dt + R_2 (i_0 - i) \quad \text{③}$$

$$R_2 (i - i_0) + \frac{1}{C_2} \int i\, dt \quad \text{④}$$

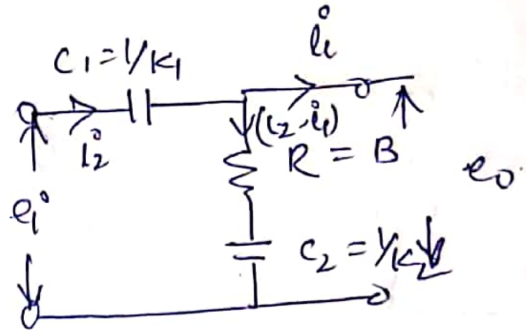
$$R_2 (i - i_0) + K_2 \int i\, dt \quad \text{④}$$

if you plot this eqn we get fig ②.

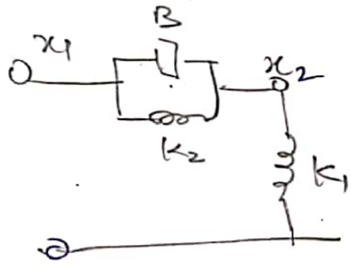
Q 3 Find electrical analogous (F-V) of mechanical system and derive TF = $X_2(S)/X_1(S)$



mechanical ①



F-V - electrical ②



At node x_2

$$B\frac{d}{dt}(x_2 - x_1) + K_1 x_2 + K_2(x_2 - x_1) \quad — ①$$

or, ~~B(...)~~ taking Laplace Transform we have

$$BS(X_2(S) - X_1(S)) + K_1 X_2(S) + K_2[X_2(S) - X_1(S)] \quad K_2 = \frac{1}{C_2},\ B = R\ K_1 = 1/C_1$$

$$[K_1 + K_2 + BS]\, X_2(S) = (K_2 + BS)\, X_1(S)$$

$$\boxed{TF = \frac{X_2(S)}{X_1(S)} = \frac{K_2 + BS}{K_1 + K_2 + BS}}$$

check $\dfrac{\frac{1}{C_2} + RS}{1/C_1 + 1/C_2 + RS} = \dfrac{(1 + RC_2S)C_1}{C_1 + C_2 + C_1C_2RS}$

for Force-voltage analogy $F \to e$, $\frac{dx_2}{dt}\ u \to i$, $M = L$, $K = 1/C$, $B = R$

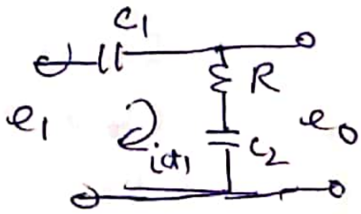
from ① we get

$$R_p(i_2 - i_1) + \frac{1}{C_1}\int i_2\,dt + 1/C_2 \int (i_2 - i_1)\,dt \quad — ②$$

The electrical equivalent cct is shown fig ②

$$e_i = \frac{1}{C_1}\int i\,dt + 1/C_2\int i\,dt + Ri \quad — ③$$

$$e_0 = Ri + 1/C_2 \int i\,dt \quad — ④$$



taking Laplace TF

$$E_1(S) = \frac{I(S)}{C_1S} + \frac{I(S)}{C_2S} + R\,I(S) \quad — ⑤$$

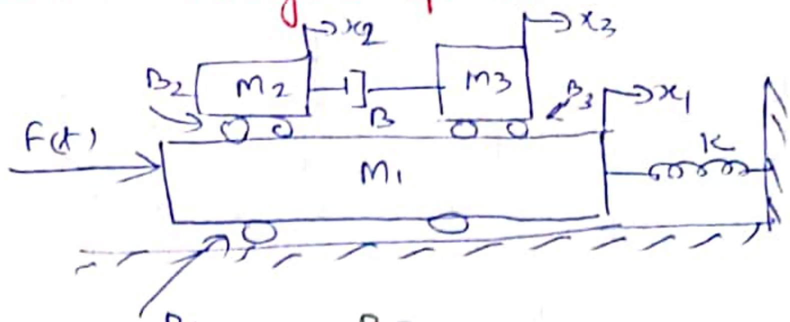
$$E_0(S) = R\,I(S) + 1/C_2S \cdot I(S) \quad — ⑥$$

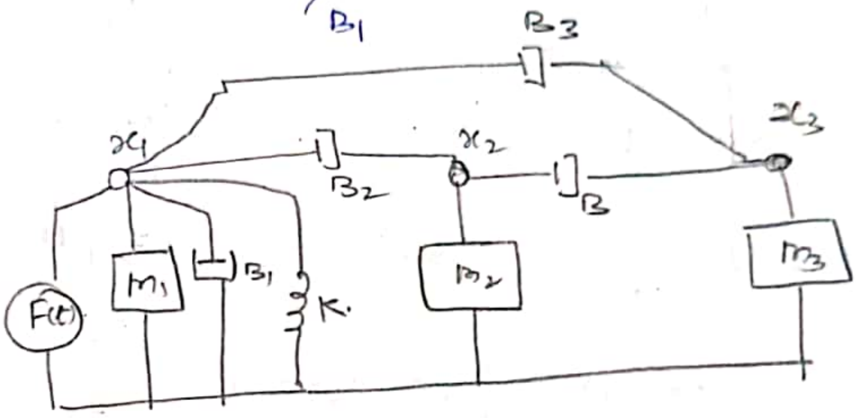
from ⑤ & ⑥

$$\boxed{TF = \frac{E_0(S)}{E_1(S)} = \frac{(1 + RC_2S)C_1}{C_1 + C_2 + C_1C_2RS}}$$

Q.1

Find Electrical analogous of the mechanical system





At node x_1

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K x_1 + B_2 \frac{d(x_1 - x_2)}{dt} + B_3 \frac{d(x_1 - x_3)}{dt}$$

or $$F(t) = M_1 \frac{du_1}{dt} + B_1 u_1 + K \int u_1 \, dt + B_2 (u_1 - u_2) + B_3 (u_1 - u_3) \quad \text{— (1)}$$

At node x_2

$$M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{d(x_2 - x_1)}{dt} + B \frac{d(x_2 - x_3)}{dt}$$

$$M_2 \frac{du_2}{dt} + B_2 (u_2 - u_1) + B (u_2 - u_3) \quad \text{—— (2)}$$

At node x_3

$$M_3 \frac{d^2 x_3}{dt^2} + B \frac{d(x_3 - x_2)}{dt} + B_3 \frac{d(x_3 - x_1)}{dt}$$

$$M_3 \frac{du_3}{dt} + B (u_3 - u_2) + B_3 (u_3 - u_1) \quad \text{—— (3)}$$

Force – voltage analogy substitute $F \to e$, $u \to i$, $M = L$, $K = 1/C$, $B = R$

then eqn (1), (2) & (3) will be

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C} \int i_1 \, dt + R_2 (i_1 - i_2) + R_3 (i_1 - i_3) \quad \text{—— (4)}$$

$$L_2 \frac{di_2}{dt} + R_2 (i_2 - i_1) + R (i_2 - i_3) \quad \text{—— (5)}$$

$$L_3 \frac{di_3}{dt} + R (i_3 - i_2) + R_3 (i_3 - i_1) \quad \text{—— (6)}$$

equivalenct cct in force-voltage analogy

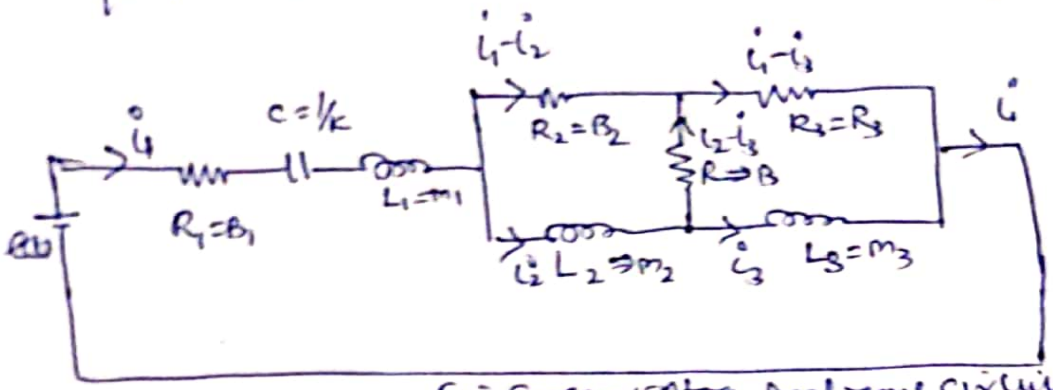


fig Force-voltage Analogous circuit

__Force force-current analogy__

$F\rightarrow i(t)$, $u\rightarrow e$, $M=C$, $B\rightarrow 1/R$, $K=1/L$, substitute in eqn ①②③

we get

__Node e_1__

$$i(t) = c_1\frac{de_1}{dt} + \frac{e_1}{R_1} + \frac{1}{R_2}(e_1-e_2) + \frac{1}{R_3}(e_1-e_3) + \frac{1}{L}\int e_1 dt \quad \text{— ⑦}$$

__node e_2__

$$c_2\frac{de_2}{dt} + \frac{1}{R_2}(e_2-e_1) + \frac{1}{R}(e_2-e_3) = 0 \quad \text{— ⑧}$$

__Node e_3__

$$c_3\frac{de_3}{dt} + \frac{1}{R_3}(e_3-e_1) + \frac{1}{R}(e_3-e_2) \quad \text{— ⑨}$$

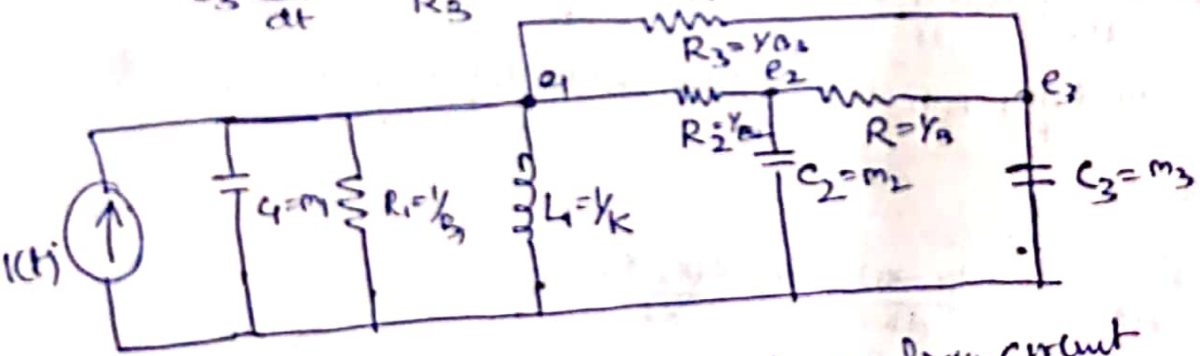
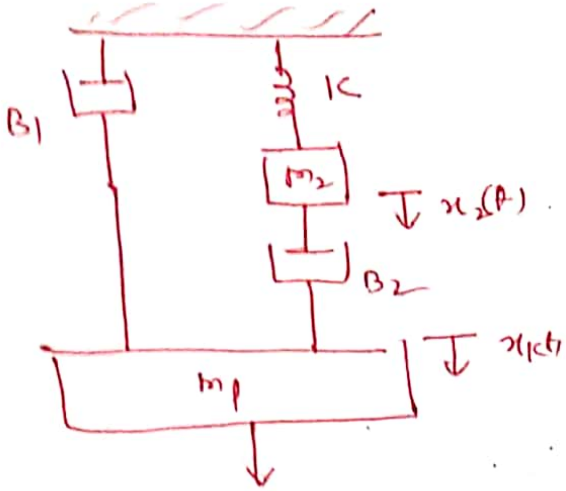
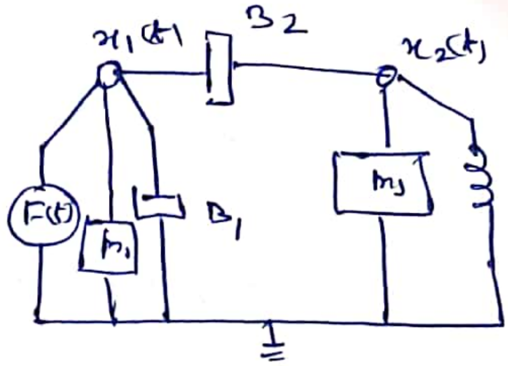


fig Force Current analogy circuit

Q2





equilibrium eqn

At node 1

$$F(t) = m_1 \frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B_2\left[\frac{dx_1}{dt} - \frac{dx_2}{dt}\right] \quad — (1)$$

$$\text{or } F(t) = m_1\frac{du_1}{dt} + B_1 u_1 + B_2(u_1 - u_2) \quad — (2)$$

$u = \frac{dx}{dt}$

At node 2

$$m_2\frac{d^2x_2}{dt} + Kx_2 + B_2\left[\frac{dx_2}{dt} - \frac{dx_1}{dt}\right] \quad — (3)$$

$$m_2\frac{du_2}{dt} + K\int u_2\,dt + B_2(u_2 - u_1) \quad — (4)$$

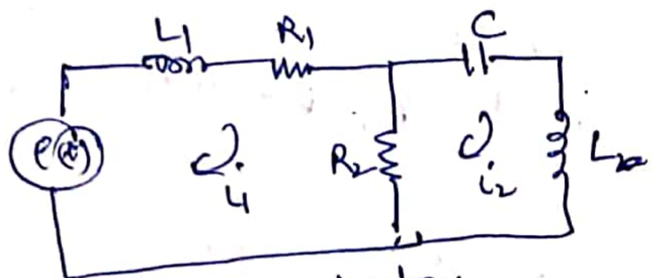
Force - voltage analogy

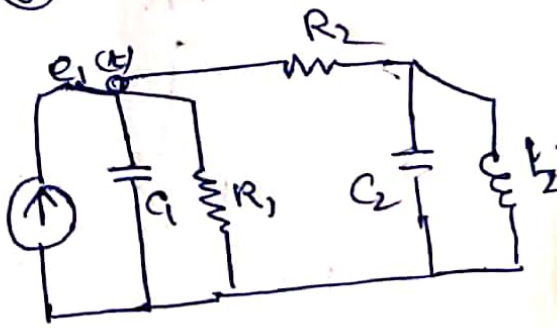
$F \to e$, $M \to L$, $K = 1/C$, $x - q$, $u = i$

from eqn (2) & (4)

$$e(t) = L_1\frac{di_1}{dt} + R_1 i_1 + R_2(i_1 - i_2) \quad — (5)$$

$$L_2\frac{di_2}{dt} + 1/C\int i_2\,dt + R_2(i_2 - i_1) \quad — (6)$$





Force - Current Analogy

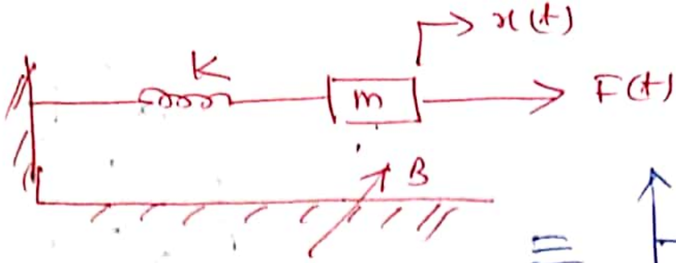
$F \to i$, $m = C$, $K = 1/L$, $B = 1/R$, $x = \phi$, $u = e$

from (2) (4)

$$i(t) = C_1\frac{de}{dt} + e_1/R_1 + \frac{e_1 - e_2}{R_2} \quad — (7)$$

$$C_2\, de_2/dt + 1/L\int e_2\,dt + e_2 - e_1/R_2 \quad — (8)$$

Ex



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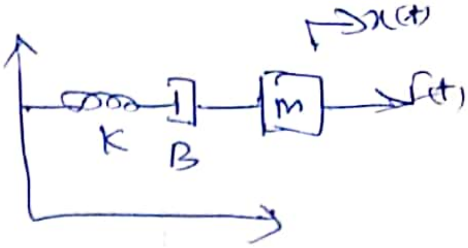
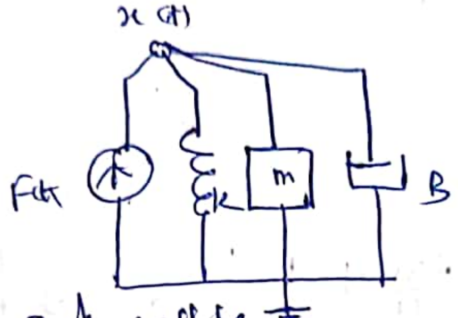


The equivalent system



equilibrium eqn will be

$$F(t) = m\frac{d^2x(t)}{dt} + Kx(t) + B\frac{dx(t)}{dt} \quad \text{—— (1)}$$

By F–V analogy replacement mech → elec

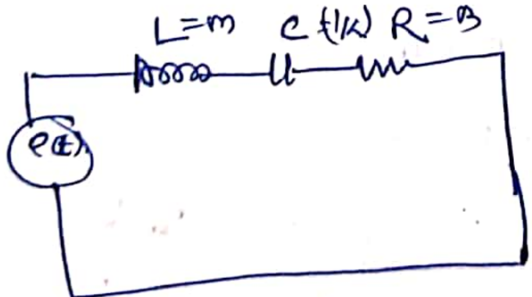
$F(t) \to e(t)$, $m = L$, $K = 1/C$, $B = R$, $x(t) = q$, $\dot{x} = \dot{i}$

$u(t) = \dot{i}$

$$F(t) = m\frac{du(t)}{dt} + K\int u\,dt + Bu(t) \quad \text{—— (2)}$$

By replacing

$$e(t) = L\frac{di}{dt} + 1/C\int i\,dt + Ri(t) \quad \text{—— (3)}$$



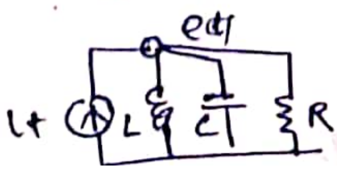
Current → Force analogy

$F(t) = i(t)$, $m = C$, $1/K = L$, $B = 1/R$, $e(t) = u(t)$, $x = \phi$

$K = 1/L$

from En (2)

$$i(t) = C\frac{de(t)}{dt} + \frac{1}{L}\int u(t)\,dt + \frac{e(t)}{R} \quad \text{—— (4)}$$



$L = 1/K$

$R = 1/B$