Exercise 2(F)

1. Evaluate $\oint_C \frac{z^2}{(z-1)^2 (z+2)} dz$ where C: $|z| = 3$.

2. Evaluate $\oint_C \frac{dz}{z^4+1}$ where C is the circle $x^2 + y^2 = 2x$.

3. Evaluate $\int_C \frac{dz}{\sinh z}$ where C: $|z| = 2$.

4. Evaluate $\int_C \frac{z^3}{(z-1)^2 (z-3)} dz$ where C: $|z| = 2$.

5. Evaluate $\oint_C \tanh z \, dz$ where C: $|z| = 2$.

Answers

1. $2\pi i$

2. $\frac{-\pi i}{\sqrt{2}}$

3. πi

4. $\frac{-7\pi i}{2}$

5. $4\pi i$

SUMMARY

- Arc is considered as a set of all points of a closed finite interval under continuous mapping.
- **Closed curve:** Let $z(t) = x(t) + iy(t), a \le t \le b$ be a continuous curve or arc, then the point $z(a)$ is called *initial point* and $z(b)$ the *terminal point* of curve C is called a *simple closed curve* or *Jordan arc*.
- **Smooth Curve:** A continuous differentiable curve (arc) is said to be a smooth curve. Geometrically, a smooth curve has a tangent at every point whose direction is determined by arg $z'(t)$.
- **Contour:** A piecewise smooth closed curve is called *contour*.
- **Simply connected regions:** A region R in which every closed curve contained in that region contains only those points that lie inside R.

  Geometrically, a simply connected domain has no holes inside for if a simple closed curve should surround a hole, then the curve could not be shrunk beyond the hole.
- **Line integral**

  The generalization of a real integral to the definite integral of complex function over a real integral. If $f(z)$ is an analytic function and $f'(z)$ is continuous at each point within and on a simple closed curve C, then $\oint_C f(z)dz = 0$.
- **Cauchy–Groursat theorem:** Let $f(z)$ be analytic in a simply connected domain D and let C be any closed curve contained in D, then $\oint_C f(z)dz = 0$.

  If $f(z)$ is analytic in a region R and A and B are two points in that region, then $\int_A^B f(z)dz$ is independent of the path joining P and Q lying entirely in R.
- **Morera's theorem:** Let $f(z)$ be continuous in simply connected domain D and $\oint_C f(z)dz = 0$ where C is a simple closed curve, then $f(z)$ is analytic in D.
- **Fundamental theorem of integral calculus:** Let $f(z)$ be an analytic function in a simply connected domain D and $G(z)$ is the integral function of $f(z)$, i.e., $g^1(z) = f(z)$,

  then $\int_a^b f(z)\, dz = \int_a^b g'(z)\, dz = [g(z)]_a^b = g(b) - g(a)$, where a and b are in D.
- **Cauchy's integral formula:** If $f(z)$ is analytic within and on a closed curve C and a is any point within C, then $f(a) = \dfrac{1}{2\pi i}\oint_C \dfrac{f(z)}{z-a}dz.$

  Let $f(z)$ be analytic in a simply connected domain D, a be any point in D, and C be any simple closed curve in D enclosing a point $z = a$, then $f(z)$ has derivatives of all order in D which are also analytic in D. Further, $f^{(n)}(a) = \dfrac{n!}{2\pi i}\oint_C \dfrac{f(z)}{(z-a)^{n+1}}dz$
- **Louville's theorem:** If $f(z)$ is analytic and bounded for all values of z, then $f(z)$ must be constant.
- **Taylor's theorem:** Let $f(z)$ be analytic at all points within a circle C with centre z_0 and radius r. Then for every point z within C, we have

$$f(z) = f(z_0) + f'(z_0)(z - z_0) + \frac{(z - z_0)^2}{2!} f''(z_0) + \ldots + \frac{f^n(z_0)}{n!}(z - z_0)^n + \ldots$$
$$= f(z_0) + \sum_{n=1}^{\infty} \frac{(z - z_0)^n}{n!} f^n(z_0)$$

- **Laurent's series:** If $f(z)$ is analytic inside and on the boundary of a ring shaped region R bounded by two concentric circles C_1 and C_2 of radii r_1 and r_2 $(r_1 > r_2)$ respectively having centre at a, then for all z in region R

$$f(z) = a_0 + a_1(z - a) + a_2(z - a)^2 + \ldots + a_{-1}(z - a)^{-1} + a_{-2}(z - a)^{-2} + \ldots$$
$$= \sum_{n=0}^{\infty} a_n (z - a)^n + \sum_{n=1}^{\infty} a_{-n}(z - a)^{-n}$$

where
$$a_n = \frac{1}{2\pi i} \oint_{C_1} \left(\frac{f(z)}{(z - a)^{n+1}}\right) dz \qquad n = 0, 1, 2, \ldots$$
$$a_{-n} = \frac{1}{2\pi i} \oint_{C_2} \left(\frac{f(z)}{(z - a)^{-n+1}}\right) dz \qquad n = 1, 2, 3, \ldots$$

- **Singularity:** Singularity of a function $f(z)$ is a point at which the function ceases to be regular (analytic).
  A zero of an analytic function $f(z)$ is a value of z such that $f(z) = 0$.
  Isolated singularity: If $z = a$ is the only singular point of $f(z)$ within the neighbourhood of the point $z = a$, it is called *isolated singularity*.
  Non-isolated singularity: If more than one singular point exist within the neighbourhood of $z = a$, then $z = a$ is said to be *non-isolated singularity* of $f(z)$.
  Removable singularity: If the principal part of Laurent's series contains no terms, then the point $z = z_0$ is called *removable singularity*.
  Essential singularity: If the principal part of Laurent's series contains infinite number of terms of $(z - z_0)$, then $z = z_0$ is called *essential singularity*.
  Isolated essential singularity: If $z = a$ is an essential singular point and $z = a$ is the limit point of zeros of $f(z)$, then $z = a$ is called *isolated essential singularity*.
  Non-isolated essential singularity: If $z = a$ is an essential singular point of $f(z)$ and $z = a$ is the limit point of poles, then $z = a$ is called *non-isolated essential singularity*.
- **Poles of $f(z)$:** If $Lt_{z \to z_0} f(z) = \infty$, then $z = a$ is a pole of $f(z)$.
  A pole of order one is called *simple pole*.
  A pole of order two is called *double pole*.
- **Residues:** The coefficient of $(z - z_0)^{-1}$ in the Laurent's expansion of $f(z)$ about an isolated singularity of $z = a$ is called *residue* of $f(z)$ at $z = a$.
- **Cauchy's residue theorem:** Let C be a simple closed curve and $f(z)$ be analytic on and inside C except at a finite number of singularities $z_1, z_2, \ldots, z_n$ lying inside C, then
  $\oint f(z) dz = 2\pi i \sum_{i=1}^{n} Res(z_k) = 2\pi i$ (sum of residues of $f(z)$ at its singular points)

OBJECTIVE QUESTIONS

1. Expand $\frac{1}{z(z-2)}$ when $|z|<2$.

(A) $\frac{-1}{2z}\left(1-\frac{z}{2}+\frac{z^2}{4}-\frac{z^3}{8}+\ldots\right)$

(B) $\frac{-1}{2z}(1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\ldots)$

(C) $\frac{1}{2z}(1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\ldots)$

(D) $\frac{-1}{2z}(1+\frac{2z}{2}+\frac{3z^2}{4}+\frac{4z^3}{8}+\ldots)$

2. Expand the function $f(z) = \frac{z-1}{z}$ as Laurent's series for $|z-1|>1$.

(A) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(z+1)^n}$

(B) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(z-1)^{n+1}}$

(C) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(z-1)^n}$

(D) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2z-1)^n}$

3. Expand ze^z by Taylor's series about $z = 1$.

(A) $ez[1+(z-1)+\frac{(z-1)^2}{2!}+\frac{(z-1)^3}{3!}+\ldots]$

(B) $ez\left[1+(z+1)+\frac{(z+1)^2}{2!}+\frac{(z\mp1)^3}{3!}+\ldots\right]$

(C) $ez\left[1+(z-1)+\frac{(z-1)^2}{2}+\frac{(z-1)^3}{3}+\ldots\right]$

(D) $ez[1+(z+1)+\frac{(z+1)^2}{2}+\frac{(z+1)^3}{3}+\ldots]$

4. Obtain the Taylor's series expansion of $f(z)=\frac{1}{z}$ about the point $z = 1$.

(A) $1+(z-1)+(z-1)^2+(z-1)^3+\ldots$

(B) $1-(z+1)+(z+1)^2-(z+1)^3+\ldots$

(C) $1-(z-2)+(z-3)^2-(z-4)^3+\ldots$

(D) $1-(z-1)+(z-1)^2-(z-1)^3+\ldots$

5. The sum function of the series $\sum_{1}^{\infty}\frac{z^n}{n!}$ is

(A) exponential function

(B) logarithmic function

(C) sine function

(D) cosine function

6. Expand $f(z)=log\ (1+z)$ in a Taylor's series about $z = 0$.

(A) $1+z+z^2+z^3+\ldots$

(B) $z+\frac{z^2}{2}+\frac{z^3}{3}+\ldots$

(C) $1-2z+3z^2-4z^3+\ldots$

(D) $-1-z-z^2-z^3-\ldots$

7. A necessary condition such that the series $\sum u_n$ is convergent is

(A) $\underset{n\to0}{Lt}\ u_n=0$

(B) $\underset{n\to\infty}{Lt}\ u_n=\infty$

(C) $\underset{n\to1}{Lt}\ u_n=0$

(D) $\underset{n\to0}{Lt}\ u_n=0$

8. If the limit of a sequence exists, then it is unique and known as a

(A) divergent sequence

(B) infinite sequence

(C) convergent sequence

(D) finite sequence

9. Expand $\dfrac{1}{(z-2)}$ when $|z|<1$.

(A) $\dfrac{-1}{2z}\left(1-\dfrac{z}{2}+\dfrac{z^2}{4}-\dfrac{z^3}{8}+\ldots\right)$

(B) $\dfrac{-1}{2}\left(1+\dfrac{z}{2}+\dfrac{z^2}{4}+\dfrac{z^3}{8}+\ldots\right)$

(C) $\dfrac{1}{2}\left(1-\dfrac{z}{2}+\dfrac{z^2}{4}-\dfrac{z^3}{8}+\ldots\right)$

(D) $1+\dfrac{2z}{2}+\dfrac{3z^2}{4}+\dfrac{4z^3}{8}+\ldots$

10. Laurent's series of the function $f(z)=\dfrac{e^z}{(z-1)^2}$ about $z = 1$ is

(A) $f(z)=e\left(\dfrac{1}{(z+1)^2}+\dfrac{1}{z+1}\dfrac{1}{2!}+\ldots\right)$

(B) $f(z)=e\left(\dfrac{1}{(z-1)^2}+\dfrac{1}{z-1}\dfrac{1}{2!}+\ldots\right)$

(C) $f(z)=\left(\dfrac{1}{(z-1)^2}+\dfrac{1}{z-1}\dfrac{1}{2!}+\ldots\right)$

(D) $f(z)=\left(\dfrac{1}{(z+1)^2}+\dfrac{1}{z+1}\dfrac{1}{2!}+\ldots\right)$

11. Find the Laurent's series for $\dfrac{z}{(z+1)(z+2)}$ about $z = -2$.

(A) $\dfrac{2}{z+1}+1+(z+2)+(z-1)^2+\ldots$

(B) $\dfrac{2}{z+1}+1+(z-2)+(z-1)^2+\ldots$

(C) $\dfrac{2}{z+1}+1+(z+2)+(z+1)^2+\ldots$

(D) $\dfrac{2}{z-1}+1+(z+2)+(z+1)^2+\ldots$

12. The value of z for which $f(z) = 0$ is called

(A) pole of $f(z)$

(B) zero of $f(z)$

(C) singular point of $f(z)$

(D) isolated singular point of $f(z)$

13. The zero of $f(z) = \dfrac{(z-1)^3}{z^2}$ is

(A) $z = 0$

(B) $z = 1$

(C) $z = \infty$

(D) $z = 2$

14. The function $f(z) = \dfrac{1}{z-1}$ at $z = \infty$ is known as

(A) pole of $f(z)$

(B) zero of $f(z)$

(C) singular point of $f(z)$

(D) isolated singular point of $f(z)$

15. The zeros of sin z are

(A) $\pm n\pi,\ n\in z$

(B) $(2n+1)\pi/2,\ n\in z$

(C) $\pm n\pi,\ n\in z.$

(D) $\pm 2n\pi,\ n\in z$

16. The zeros of e^z are

(A) no zeros

(B) ∞

(C) 0

(D) 1

17. The isolated singular points of $\dfrac{e^z}{z^2+1}$ are

(A) $z = 0$ and 1

(B) $z = -1$ and 1

(C) $z = i$ and $-i$

(D) $z = 0$ and i

18. $f(z) = e^{1/z}$ at $z = 0$ is called
    (A) pole of $f(z)$
    (B) zero of $f(z)$
    (C) essential singular point of $f(z)$
    (D) isolated essential singular point of $f(z)$
19. If the principal part of Laurent's series contains finite number of points say m is called
    (A) pole of order m
    (B) zero of order m
    (C) essential singular point of $f(z)$
    (D) isolated singular point of $f(z)$
20. A pole of order one is called
    (A) removable singularity
    (B) simple pole
    (C) essential singular point of $f(z)$
    (D) isolated singular point of $f(z)$
21. The function $f(z)$ is not defined at $z = a$ but $Lt_{z \to a} f(z)$ exists, then $z = a$ is called
    (A) removable singularity
    (B) simple pole
    (C) essential singular point of $f(z)$
    (D) isolated singular point of $f(z)$
22. If $f(z) = \begin{cases} \dfrac{\sin z}{z}, z \neq 0 \end{cases}$ and $f(0) = 0$, then $z = 0$ is called
    (A) removable singularity
    (B) simple pole
    (C) essential singular point of $f(z)$
    (D) isolated singular point of $f(z)$
23. $f(z) = z^3$ at $z = \infty$ is called
    (A) simple pole
    (B) pole of order 3
    (C) essential singularity
    (D) isolated singular point
24. $f(z) = e^z$ at $z = \infty$ is called
    (A) simple pole
    (B) pole of order 3
    (C) essential singularity
    (D) isolated singular point
25. $f(z) = \dfrac{1 - \cos z}{z}$ at $z = 0$ is called
    (A) removable singularity
    (B) simple pole
    (C) essential singular point of $f(z)$
    (D) isolated singular point of $f(z)$
26. $\sec\dfrac{1}{z}$ at $z = 0$ is
    (A) removable singularity
    (B) simple pole
    (C) non-isolated essential singular point
    (D) isolated essential singular point
27. $\sin\dfrac{1}{1-z}$ at $z = 1$ is
    (A) removable singularity
    (B) simple pole
    (C) non-isolated essential singular point
    (D) essential singular point
28. The poles of $f(z) = \dfrac{z}{(z+1)(z+2)}$ are
    (A) $z = -1$ and -2 are simple poles
    (B) $z = 1$ and -2 are simple poles
    (C) $z = -1$ and 2 are simple poles
    (D) $z = 1$ and 2 are simple poles
29. The poles of $\dfrac{1 - e^{2z}}{z^4}$ are
    (A) $z = 0$ is a simple pole
    (B) $z = 0$ is a pole of order 2
    (C) $z = 0$ is a pole of order 3
    (D) $z = 0$ is a pole of order 4

30. The pole of $\dfrac{e^{2z}}{z^2+\pi^2}$ are

(A) $z = \pi i$
(B) $z = -\pi i$
(C) $z = \pm\pi i$
(D) $z = 0$

31. If the power series $\sum a^n z^n$ converges for $|z| < R$ and diverges for $|z| > R$, then R is called

(A) radius of convergence
(B) circle of convergence
(C) limit of convergence
(D) boundary of convergence

32 If the power series $\sum a^n z^n$ converges for $|z| < R$ and diverges for $|z| > R$, then $|z| = R$ is called

(A) radius of convergence
(B) circle of convergence
(C) limit of convergence
(D) boundary of convergence

33. The Taylor's series approximation that exists for $f(z)$ is

(A) analytic
(B) singular
(C) analytic everywhere except origin
(D) non-analytic

34. Taylor's series expansion of $f(z) = \dfrac{1}{z}$ about the point $z = 1$ is valid for

(A) $|z-1| < 2$
(B) $|z-1| < 1$
(C) $|z-1| < 3$
(D) $|z+1| < 2$

35. The expansion $\sum (-1)^n \dfrac{z^{2n+1}}{(2n+1)!}, |z| < \infty$ represents

(A) sin z
(B) cos z
(C) sinh z
(D) cosh z

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(A) sin z
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(A) sin z
(B) cos z
(C) sinh z
(D) cosh z

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(A) sin z
(B) cos z
(C) sinh z
(D) cosh z

39. The expansion $1 + \sum (n+1)|z+1|^n =$ represents for $|z+1| < 1$

(A) sin z
(B) z
(C) $\dfrac{1}{z}$
(D) $\dfrac{1}{z^2}$

40. The expansion $1 + \sum (n+1)|z-2|^n =$ represents for $|z-2| < 1$

(A) $\frac{1}{z^2}$ about $z = 2$

(B) $\frac{1}{z^2}$ about $z = -2$

(C) $\frac{1}{z^2}$ about $z = 1$

(D) $\frac{1}{z^2}$ about $z = -1$

41. The Taylor's series expansion for $f(z) = \sinh z$ about $z = \pi i$ is

(A) $\sin(z - \pi i)$

(B) $\sinh(z - \pi i)$

(C) $\cos(z - \pi i)$

(D) $\text{coh}(z - \pi i)$

42. The Taylor's series expansion for $f(z) = \cosh z$ about $z = \pi i$ is

(A) $\sin(z - \pi i)$

(B) $\sinh(z - \pi i)$

(C) $\cos(z - \pi i)$

(D) $-\cosh(z - \pi i)$

43. Expand $f(z) = log\ (1 - z)$ as a Taylor's series about $z = 0$.

(A) $1 + z + z^2 + z^3 + \ldots$

(B) $-z + z^{2/2} + z^{3/3} + \ldots$

(C) $1 - 2z + 3z^2 - 4z^3 + \ldots$

(D) $-1 - z - z^2 - z^3 - \ldots$

44. The expansion $\frac{1}{\sqrt{2}}\left[1 + \left(z - \frac{\pi}{4}\right) - \frac{1}{2}\left(z - \frac{\pi}{4}\right)^2 - \frac{1}{6}\left(z - \frac{\pi}{4}\right)^3 + \ldots..\right]$

(A) $\sin z$ about $z = \frac{\pi}{4}$

(B) $\sin z$ about $z = \frac{\pi}{2}$

(C) $\sin z$ about $z = \frac{\pi}{3}$

(D) $\sin z$ about $z = \pi$

45. The expansion $\left[(z-1) - \frac{1}{2}(z-1)^2 + \frac{1}{3}(z-1)^3 + \ldots\right] =$

(A) $\log z$ about $z = 0$

(B) $\log z$ about $z = 1$

(C) $\log z$ about $z = -1$

(D) $\log z$ about $z = 2$

46. The expansion $\left[(z+1) - \frac{1}{2}(z+1)^2 + \frac{1}{3}(z+1)^3 + \ldots..\right] =$

(A) $\log z$ about $z = 0$

(B) $\log z$ about $z = 1$

(C) $\log z$ about $z = -1$

(D) $\log z$ about $z = 2$

47. The Laurent's expansion for $f(z) = \frac{(z-1)}{z}$, where $|z - 1| > 1$ is

(A) $\sum \frac{(-1)^n}{(z-1)^n}$

(B) $\sum \frac{(-1)^{2n}}{(z-1)^n}$

(C) $\sum \frac{(1)}{(z-1)^n}$

(D) $\sum \frac{(-1)^n}{(z+1)^n}$

48. The series $1 - z + z^2 - z^3 + \ldots$ is equal to

(A) $\frac{1}{1-z}$

(B) $\frac{1}{1+z}$

(C) $\frac{1}{z}$

(D) $\frac{1}{z+2}$

49. The series $1 + z + z^2 + z^3 + \dots$ is equal to

(A) $\frac{1}{1-z}$ (B) $\frac{1}{1+z}$

(C) $\frac{1}{z}$ (D) $\frac{1}{z+2}$

50. The residue of $f(z)$ at $z = a$ is given by

(A) $\frac{1}{2\pi i}\int_C f(z)dz$ (B) $\int_C f(z)dz$

(C) $2\pi i\int_C f(z)dz$ (D) $\frac{1}{2\pi}\int_C f(z)dz$

51. The residue of $e^{1/z}$ at $z = 0$ is

(A) 0 (B) 1

(C) –1 (D) does not exist

52. The poles of the function $\frac{z}{\cos z}$ are

(A) $\pm n\pi, n \in Z$ (B) $(2n+1)\pi/2, n \in Z$

(C) $\pm n\pi, n \in Z$ (D) $\pm 2n\pi, n \in Z$

53. The poles of the function $\cot z$ are

(A) $\pm n\pi, n \in Z$ (B) $(2n+1)\pi/2, n \in Z$

(C) $\pm n\pi, n \in Z$ (D) $\pm 2n\pi, n \in Z$

54. The poles of the function $\tan z$ are

(A) $\pm n\pi, n \in Z$ (B) $(2n+1)\pi/2, n \in Z$

(C) $\pm n\pi, n \in Z$ (D) $\pm 2n\pi, n \in Z$

55. The residue of $\cot z$ at $z = n\pi$ is

(A) 0 (B) 1

(C) –1 (D) n

56. The residue of $f(z) = \frac{z}{(z-1)(z+2)}$ at $z = 1$ is

(A) $z = \frac{-1}{3}$ (B) $z = \frac{1}{3}$

(C) $z = \frac{2}{3}$ (D) $z = \frac{-2}{3}$

57. Residue of $f(z) = \frac{z}{(z+1)(z+2)}$ at $z = -1$ is

(A) 0 (B) 1

(C) –1 (D) 2

58. Residue of $f(z) = \frac{z}{(z+1)(z+2)}$ at $z = -2$ is

(A) 0 (B) 1

(C) –1 (D) 2

59. The residue of $\frac{1-e^{2z}}{z}$ at $z = 0$ is

(A) 0
(B) 1
(C) -1
(D) 2

60. The residue of $\dfrac{e^{2z}}{z^2+\pi^2}$ at $z=-\pi i$ is

(A) $z=\pi i$
(B) $z=-\pi i$
(C) $z=\pm\pi i$
(D) $z=\dfrac{i}{2\pi}$

61. The residue of $\dfrac{e^{2z}}{z^2+\pi^2}$ at $z=\pi i$ is

(A) $z=\dfrac{i}{\pi}$
(B) $z=\dfrac{-i}{\pi}$
(C) $z=\dfrac{i}{2\pi}$
(D) $z=\dfrac{-i}{2\pi}$

62. The residue of $\dfrac{1-e^{2z}}{z^4}$ at $z=0$ is

(A) $-4/3$
(B) $4/3$
(C) $3/4$
(D) $-3/4$

63. The residue of $e^z z^{-5}$ at $z=0$

(A) $-1/24$
(B) $1/24$
(C) $2/24$
(D) $-3/4$

64. Residue of $\dfrac{ze^z}{(z-3)^2}$ is

(A) $4e^2$
(B) $4e^3$
(C) $2e^2$
(D) $5e^2$

65. The residue of $\dfrac{e^{iz}}{z^2+1}$ at $z=-i$ is

(A) $z=$ ie/2
(B) $z=$-ie/2
(C) $z=$ ie/3
(D) $z=$ ie/4

66. The residue of $z\cos\dfrac{1}{z}$, at $z=0$ is

(A) $-4/3$
(B) $4/3$
(C) $1/2$
(D) $-1/2$

67. The residue of $\dfrac{z-\sin z}{z^2}$ at $z=0$ is

(A) 0
(B) 1
(C) -1
(D) 2

68. Residue of $\dfrac{1-e^{2z}}{z^4}$ at $z=0$ is

(A) 0
(B) 1
(C) $\dfrac{-4}{3}$
(D) 2

69. $\displaystyle\int_C \frac{1}{(z^2+4)^2}dz$, where $C:|z|=1$ is

(A) 0 (B) 1

(C) $\frac{-4}{3}$ (D) 2

70. The value of $\int_C \frac{z}{z(z-1)(z-2)} dz$ over the circle $|z| = 3$ is

(A) 0 (B) 1

(C) −1 (D) 2

71. The value of $\int_C \frac{1}{z} e^z dz$ over the circle $|z| = 3$ is

(A) $2\pi i$ (B) $-\pi i$

(C) πi (D) $-2\pi i$

72. The value of $\int_C \frac{2e^z}{z(z-3)} dz$ over the circle $|z| = 2$ is

(A) $2\pi i/3$ (B) $-\pi i/3$

(C) $4\pi i/3$ (D) $-4\pi i/3$

73. The value of $\int_c \frac{e^z}{(z-3)^2} dz$ over the circle $|z-1| = 1$ is

(A) $2\pi i/3$ (B) 0

(C) $4\pi i/3$ (D) $-4\pi i/3$

74. The value of $\int_c \frac{z+1}{z(z-2)} dz$ over the circle $|z| = 1.5$ is

(A) $2\pi i$ (B) $-\pi i$

(C) $4\pi i$ (D) $-4\pi i$

75. Residue of ze^z at $z = 0$ is

(A) 0 (B) 1

(C) $\frac{-1}{2}$ (D) $\frac{1}{2}$

76. Residue of $\tan z$ at $z = \frac{\pi}{2}$ is

(A) 0 (B) 1

(C) −1 (D) 2

77. Residue of $\frac{\cos z}{z}$ at $z = 0$

(A) 0 (B) 1

(C) −1 (D) 2

78. Residue of $\frac{e^z}{\sin z + z\cos z}$ at $z = 0$ is

(A) $\frac{2}{3}$ (B) 1

(C) $\frac{-1}{2}$ (D) $\frac{1}{2}$

79. The limit point of the poles of $f(z)$ is

(A) removable singularity
(B) simple pole
(C) non-isolated essential singular point
(D) isolated essential singular point

Answer

1. B	2. C	3. A	4. D	5. A	6. B	7. D	8. C
9. B	10. B	11. A	12. A	13. B	14. B	15. A	16. A
17. C	18. C	19. A	20. B	21. A	22. A	23. B	24. C
25. A	26. C	27. D	28. A	29. C	30. C	31. A	32. B
33. A	34. B	35. A	36. C	37. D	38. B	39. D	40. A
41. B	42. D	43. B	44. A	45. B	46. C	47. A	48. B
49. A	50. A	51. B	52. B	53. A	54. B	55. B	56. B
57. C	58. D	59. A	60. D	61. C	62. A	63. B	64. B
65. A	66. D	67. A	68. C	69. A	70. A	71. A	72. D
73. B	74. B	75. A	76. C	77. B	78. D	79. C	