UNIT-1 DC CIRCUITS

→ Electric Current :-

It is defined as a flow of charge per unit time. Denoted by 'I' or 'i'.

i.e $i = Q/t$ Coulombs/sec (or) Amperes

→ Electric Potential :-

It is the capacity of an element or conductor to do some work per unit charge.

Denoted with V.

i.e $V = W/Q$ Joules/Coulombs

The difference between the potentials at both the ends of a conductor is known as Potential difference (or) Voltage. Or

Units for voltage are volts (or) Joules/Coulombs.

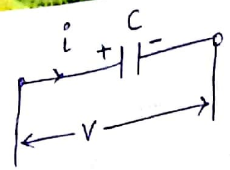
→ Electric Resistance :-

It is defined as the opposition offered by the conductor to the ~~folow~~ of charge. Denoted by R.

$R \propto \frac{l}{a}$ l → length, a → Cross sectional area

$$\boxed{R = \frac{\rho l}{a}}$$

Units : OHMS (Ω)



$$i = C\frac{dv}{dt}$$

$$dv = \frac{1}{C}\, i\,dt$$

$$\int_0^t dv = \frac{1}{C}\int_0^t i\,dt$$

$$v(t) - v(0) = \frac{1}{C}\int_0^t i\,dt$$

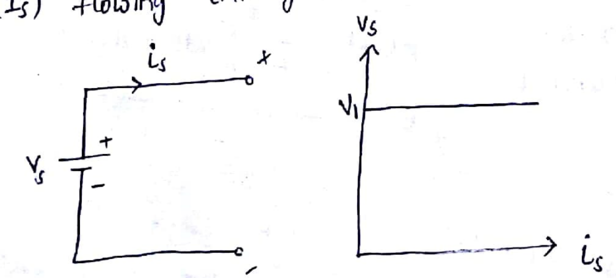
Expression for voltage.

$$\boxed{v(t) = \frac{1}{C}\int_0^t i\,dt + v(0)}$$

× The energy stored in a Capacitor $\boxed{W = \frac{1}{2} CV^2}$

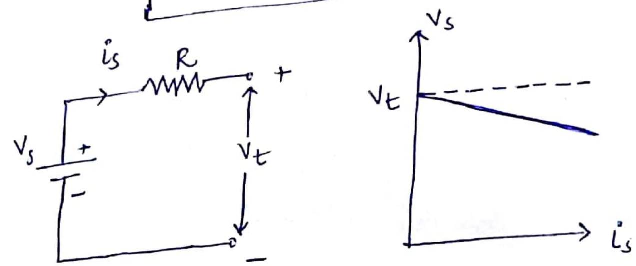
Types of Sources

An ideal voltage source is an element in which the voltage (V_s) is completely independent of the current (I_s) flowing through it's terminals

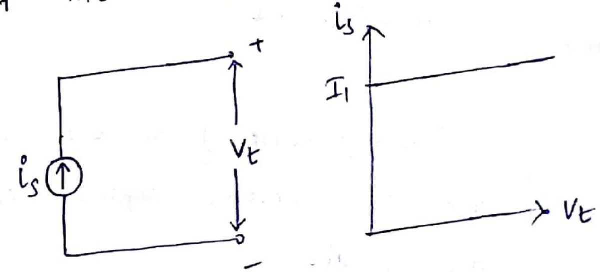


In practical case their exists an internal resistance in series with the voltage source, due to which the voltage across the terminals reduces as the current is increases.

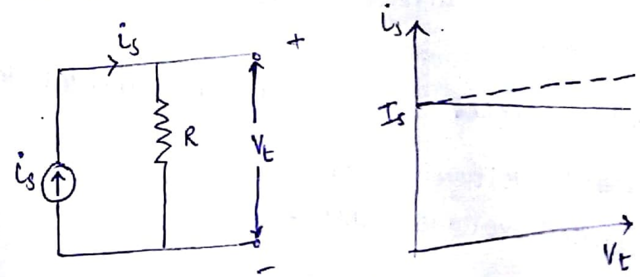
i.e., $\boxed{V_t = V_s - i_s R}$



→ An ideal current source is an element in which the source current i_s is completely independent of the voltage across it's terminals.



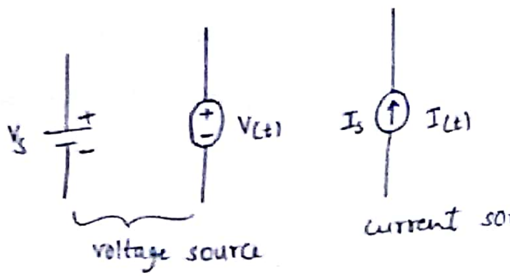
In practical case their exists an internal resistance in parallel with the current source due to which the current through its terminals reduces as the voltage is increased.



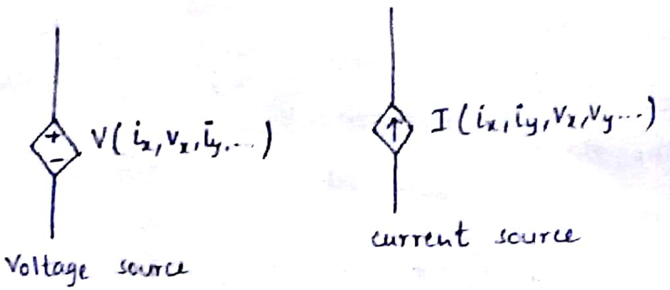
The voltage / current sources are sub-divided into two types

1. Independent sources and
2. Dependent sources

→ An Independent source (voltage/current) is the one in which the voltage or current is completely independent of the other variables in the circuit.



→ A dependent source (voltage/current) is the one in which the voltage or current depends on other variables in the circuit

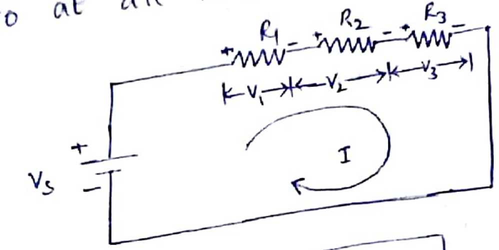


→ The dependent sources are further divided into

1. Voltage controlled voltage source
2. Current controlled voltage source.
3. Voltage controlled current source
4. Current controlled current source.

Kirchoff's VOLTAGE LAW :- (KVL)

KVL states that the algebraic sum of the branch voltages around a closed path is equal to zero at all instants of time



$$\boxed{V_s - V_1 - V_2 - V_3 = 0}$$

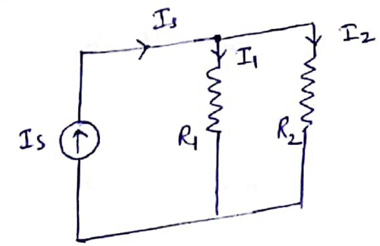
$$\Rightarrow V_s = V_1 + V_2 + V_3$$

KIRCHOFF'S CURRENT LAW (KCL) :-

KCL states that the algebraic sum of the currents at a node (or) junction is equal to zero.

(or)

KCL states that the sum of the currents entering into a node is equal to the sum of the currents leaving that node.



$$I_s - I_1 - I_2 = 0$$

$$\Rightarrow I_s = I_1 + I_2$$

→ Resistance in series ⟶ $R_T = R_1 + R_2 + R_3 + \dots\dots$

Resistance in parallel ⟶ $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots$

Q) Two lamps A and B takes current of 0.8A and 0.9A respectively when connected across 110V supply. If the lamps are connected in series across a 220V supply. Find the circuit current and the voltage across each lamp. Assume the filament resistance is unaltered.

Sol:

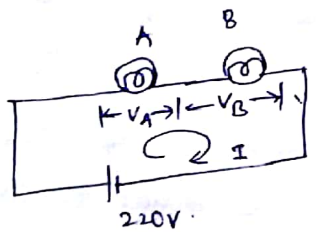
(A) ↓ 0.8A ↓ 110V ↓

$R_A = \frac{110}{0.8}$

$= 137.5\,\Omega$

(B) ↓ 0.9A ↓ 110V ↓

$R_B = \frac{110}{0.9} = 122.2\,\Omega$



$i = V/R = \frac{220}{259.7} = 0.84.$

$V_A = 115.5V \quad V_B = 102.6V$

Q. Two lamps of 250V, 100W and 200V, 100W are connected in series across a 250V supply calculate the circuit current and the voltage across each lamp.

Sol

(A)

$P = i^2R = \frac{V^2}{R}$

$100 = \frac{250 \times 250}{R}$

$\boxed{R_A = 625}$

(B)

$P = V^2/R$

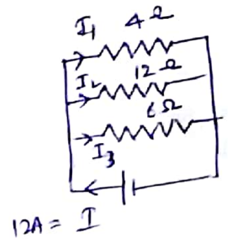
$100 = \frac{200 \times 200}{R_B}$

$\boxed{R_B = 400}$

$i = V/R = \frac{250}{1025} = 0.244A$

$V_A = 152.5V \quad V_B = 97.5$

Q. 3 resistors 4Ω, 12Ω and 6Ω are connected in parallel if the total current taken is 12A Find the current through each resistor



$\frac{1}{R_T} = \frac{1}{4} + \frac{1}{12} + \frac{1}{6}$

$\boxed{R_T = 2\Omega}$

$i = V/R_T \Rightarrow 12 = \frac{V}{2}$

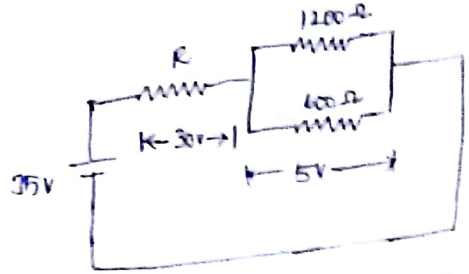
$\boxed{V = 24V}$

$i_1 = \frac{24}{4} = 6A$

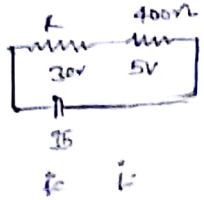
$i_2 = \frac{24}{12} = 2A$

$i_3 = \frac{24}{6} = 4A$

Q. In the circuit shown find the value of R.



$\Rightarrow$



$i = \frac{V}{R} = i = \frac{5}{400} = 0.0125$

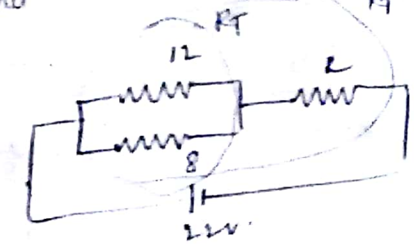
~~5 = i × 400~~

$0.0125 = \frac{30}{R}$

$\boxed{R = 2400\Omega}$

Q. A resistance 'R' is connected in series with a parallel circuit comprising 2 resistors 12 Ω and 8 Ω respectively. The total power dissipated in the circuit is 70 W when the applied voltage is 22 V. Find the value of R

Sol:



$\frac{1}{R_T'} = \frac{1}{12} + \frac{1}{8}$

$= \frac{2+3}{24}$

$R_T = \frac{24}{5}$

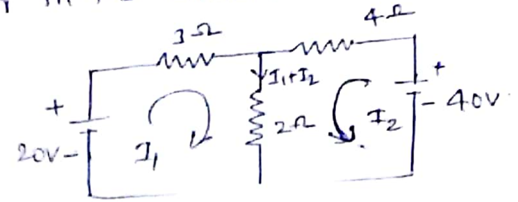
$P = i^2 R = \frac{V^2}{R}$

$70 = \frac{22 \times 22}{R_T'}$

$R_T' = 6.914\,\Omega$

$\boxed{R = 2.11\Omega}$

Q. Using Kirchoff's law find the current through 2 Ω resistor in the circuit shown



KVL for loop 1: $-3i_1 - 2(i_1 + i_2) = 20$

$-3i_1 - 2i_1 - 2i_2 = 20$

$+5i_1 + 2i_2 = 20$ —(1)

KVL for loop 2

$-4i_2 - 2(i_1 + i_2) = 40$

$-4i_2 - 2i_1 - 2i_2 = 40$

$-2i_1 - 6i_2 = 40$

$+i_1 + 3i_2 = 20$ —(2)

$(2) \times 5 = +5i_1 + 15i_2 = 100$ —(3)

Solve for i_1 & i_2.

~~$13i_2 = 80$~~

~~$i_2 = 6.15$~~

~~$i_1 = +55$~~

	i_1	i_2	1
2	-20	5	2
3	-20	1	3

$\frac{i_1}{-40+60} = \frac{i_2}{-20+100} = \frac{1}{15-2}$

$\frac{i_1}{20} = \frac{i_2}{80} = \frac{1}{13}$

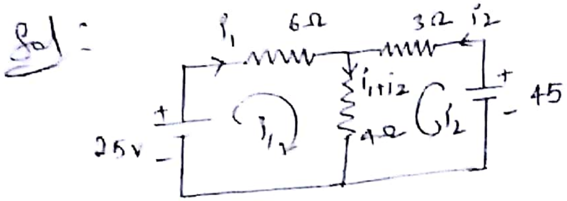
$\boxed{i_1 = 1.53}$

$\boxed{i_2 = 6.15}$

$\boxed{I = 7.68}$

Q. Find the current through all the resistors in the circuit shown by using KVL

Sol :



loop 1 : $25-6i_1-4(i_1+i_2)=0$

$25-10i_1-4i_2=0$ —①

loop 2 : $45-3i_2-4(i_1+i_2)=0$

$45-4i_1-7i_2=0$ —②

i_1	i_2	1	
-4	25	-10	-4
-7	45	-4	-7

$$\frac{i_1}{-180+175} = \frac{i_2}{-160+450} = \frac{1}{70-16}$$

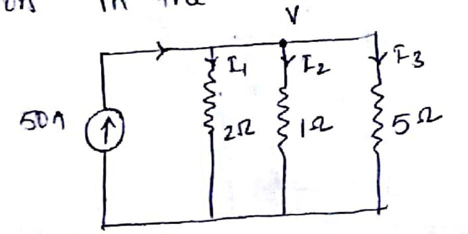
$$\frac{i_1}{-5} = \frac{i_2}{350} = \frac{1}{54}$$

$i_1 = -0.092$ A

$i_2 = 6.481$ A

$i_1+i_2 = 6.389$ A

Q. Find the current flowing through all the resistors in the circuit shown



$50 = I_1+I_2+I_3$ —①

$I_1 = \frac{V}{2}$, $I_2 = \frac{V}{1}$, $I_3 = \frac{V}{5}$.

$50 = \frac{V}{2} + V + \frac{V}{5}$

$50 = \frac{5V+10V+2V}{10} = \frac{17V}{10}$

$500 = 17V$

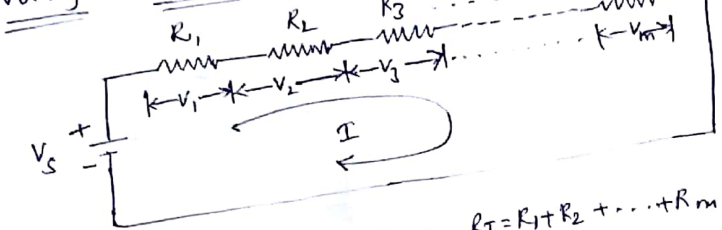
$\boxed{V = 29.411}$

$i_1 = 14.70$ A

$i_2 = 29.41$ A

$i_3 = 5.882$ A.

Voltage division principle :-



$I = \frac{V_s}{R_T}$ where $R_T = R_1+R_2+\dots+R_m$

$V_1 = IR_1$

$V_2 = IR_2$

$\vdots$

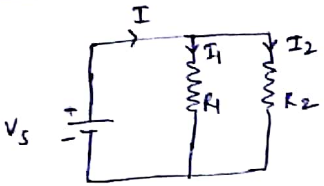
$V_m = IR_m$

$V_m = \cancel{R_m} \frac{V_s}{R_T} \times R_m$

$$\boxed{V_m = \frac{R_m}{R_T} \times V_s}$$

i.e in a series circuit the voltage across any n^{th} resistor is equal to the ratio of it's resistance to the total resistance multiplied by the source voltage.

Current division principle :-



$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V_s}{R_1} \Rightarrow V_s = I_1 R_1$$

$$I_2 = \frac{V_s}{R_2} \Rightarrow V_s = I_2 R_2$$

$$I = \frac{V_s}{R_T} = \frac{V_s}{R_1 R_2}(R_1+R_2)$$

$$= \frac{I_1 \cancel{R_1}(R_1+R_2)}{\cancel{R_1} R_2}$$

$$\Rightarrow I = \frac{I_1(R_1+R_2)}{R_2}$$

$$\Rightarrow \boxed{I_1 = \frac{R_2}{R_1+R_2} \times I}$$

Similarly, $\Rightarrow \boxed{I_2 = \frac{R_1}{R_1+R_2} \times I}$

Generalised Equation $\boxed{I_j = \frac{R_p}{R_j+R_p} \times I}$

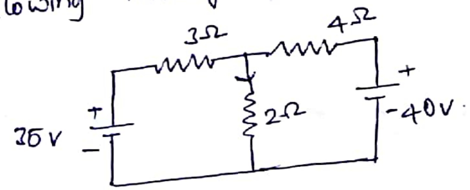
R_p = resultant of opp. resistor in parallel

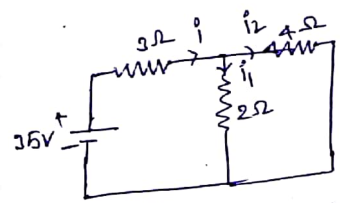
i.e In a parallel circuit the current flowing through any j^{th} resistor is equal to the ratio of it's opposite parallel resistance to the sum of it's all the resistance and opposite resistance multiplied by the total current entering into the circuit.

Superposition THEOREM :-

Superposition theorem states that, in a linear circuit with two or more sources, the response in any element is equal to the algebraic sum of the responses with individual sources acting alone, while the other sources are non-operative.

Q. By using superposition theorem find the current following through 2Ω resistor in the circuit shown.





$R_T = \frac{4}{3}+3 = \frac{13}{3}$

$i = \frac{35}{13} \times 3 = 8.075$

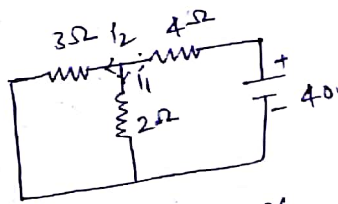
$\cancel{x} = \frac{i_1}{i_2} = \frac{R_2}{R_1}$

$\frac{x}{8.075-x} = \frac{\cancel{4}\,2}{\cancel{2}}$

$x = 16.15 - 2x$

$3x = 16.15$

$\boxed{x = 5.38A}$



$R_T = \frac{6}{5}+4 = \frac{26}{5}$

$i = \frac{40}{26} \times 5 = 7.69$

$\frac{i_1}{i_2} = \frac{R_2}{R_1}$

$\frac{x}{7.69-x} = \frac{3}{2}$

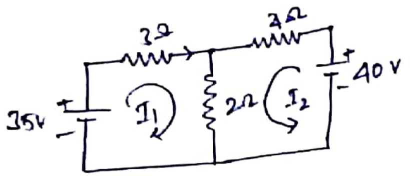
$2x = 23.07 - 3x$

$5x = 23.07$

$\boxed{x' = 4.61A}$

i through 2Ω = 5.38 + 4.61 = 9.99A

Verify superposition theorem for the above circuit



$35-3i_1-2(i_1+i_2)=0$

~~40~~ $35-5i_1-2i_2=0$ —①

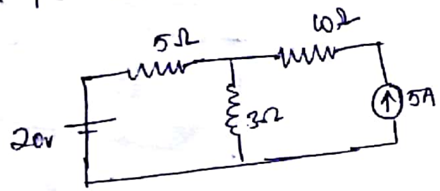
$40-4i_2-2(i_1+i_2)=0$

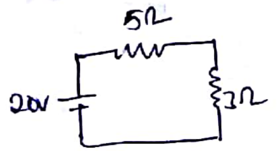
$40-2i_1-6i_2=0$ —②

$\boxed{i_1=5A}$ $\boxed{i_2=5A}$

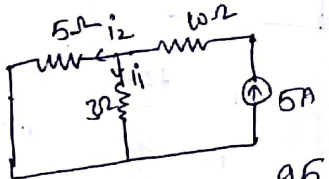
$\boxed{i_1+i_2=10A}$

Q. Find the current through 3Ω resistor by using super. posn theorem.





$i = \frac{20}{8} = 2.5A$



$R_T = \frac{15}{8} + 10 = \frac{96}{8}$

$\frac{i_1}{i_2} = \frac{R_2}{R_1}$

$\frac{x}{5-x} = \frac{5}{3}$

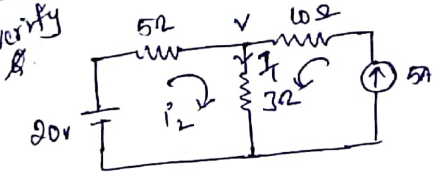
$3x = 25-5x$

$8x = 25$

$x = 3.125A$

Ans:- $I = 5.625A$

verify



$\frac{\cancel{5}}{5+3} \times 5 = \frac{25}{8} = 3.125A$

$i = \frac{20}{8} =$

$5 = V/3 + \frac{V-20}{5}$

$5 = \frac{5V+3V-60}{15}$

$75 = 8V-60$

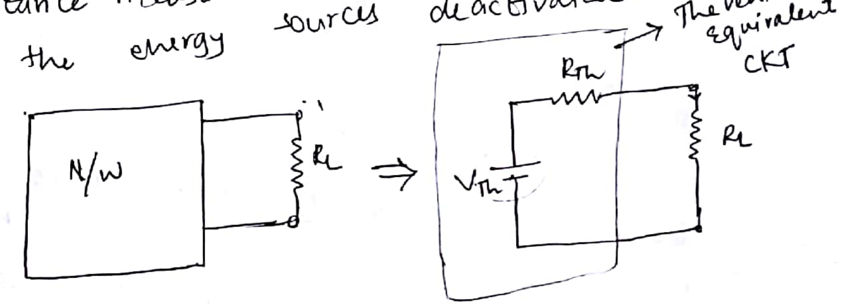
$135 = 8V$

$\boxed{V = 16.875}$

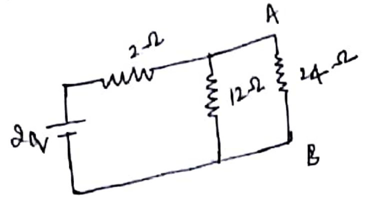
$I = V/3 = \frac{16.875}{3} = 5.625$

THEVENIN'S THEOREM.

Thevenin's theorm states that linear circuit with two terminals having no. of voltage, current sources and resistors can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a single resistance, where the voltage source value is equal to the open circuit voltage measured across the terminals and resistance value is equal to the equivalent resistance measured through the terminals with all the energy sources deactivated.

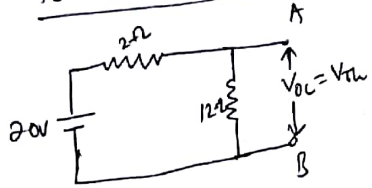


Q. Find the thevenin's equivalent circuit across AB for the circuit shown.



Sol

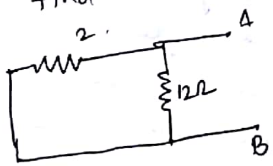
To find V_{Th}



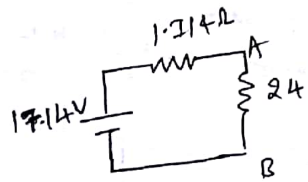
$i = \frac{20}{14} = 1.428$

$\cancel{20 - 2.856}$ $\boxed{V_{Th} = 17.136}$ $\simeq$ 17.14 V

To find R_{Th}



$\boxed{R_{Th} = 1.714\Omega}$.

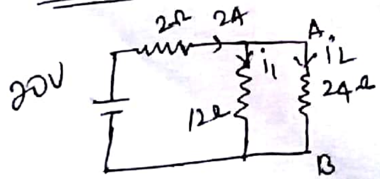


$\Rightarrow I_L' = \frac{17.14}{25.714}$

$= 0.66$.

Q. Verify Thevenin's theorem for the above circuit :-

$R_{eff} = 10$



$i = \frac{20}{10} = 2A$.

$\frac{i_1}{i_L} = \frac{\cancel{24}}{12} = 2$

$\frac{i_1}{2 - i_1} = 2$

$i_1 = 4 - 2i_1$

$3i_1 = 4$

$\boxed{i_1 = 4/3}$

$\frac{2 - i_2}{i_2} = 2$

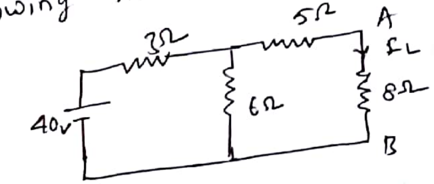
$2i_2 = 2 - i_2$

$3i_2 = 2$

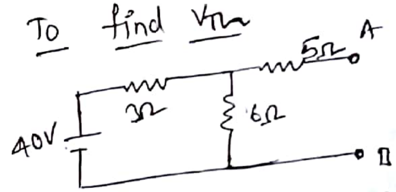
$\boxed{i_2 = 2/3}$

$\boxed{I_L = 0.66A}$

Q. By using thevenin's theorem find the current flowing in 8Ω resistor



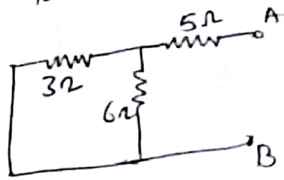
To find V_{Th}



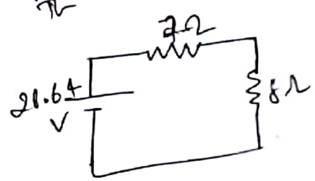
$i = \frac{40}{9} = 4.44A$

$V_{Th} = 26.64V$

To find R_{Th}

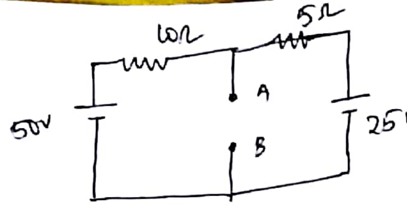


$R_{Th} = 7\Omega$



$\boxed{I_L = 1.776A}$

Q. Determine thevenin's equivalent circuit across AB for the circuit shown below.



To find V_{Th}

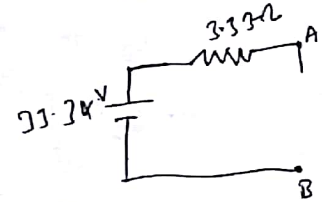
$50 - 10i_1 - 5i_1 - 25 = 0$

$25 = 15i$

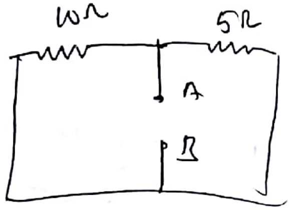
$i = \frac{25}{15} = 1.66A.$

$V_{oc} = 50 - 16.61$

$V_{Th} = 33.34V.$

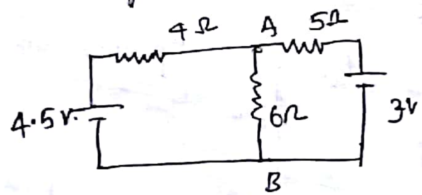
$\Rightarrow$ 

To find R_{Th}

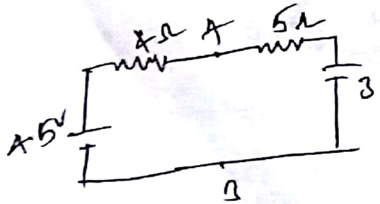


$\boxed{R_{Th} = 3.33\,\Omega}$

Q. By using thevenin's theorem find the current flowing through 6Ω resistance



To find V_{th}



$i = \frac{1.5}{9}$

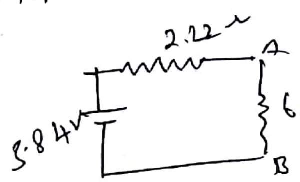
$= 0.166$

$V_{th} = 4.5 - 0.66$

$= 3.84.$

To find R_{Th}

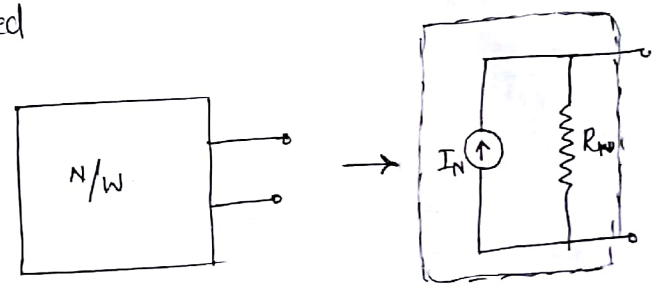
$R_{Th} = 2.22\,\Omega$



$i = \frac{3.84}{8.22} = 0.467A$

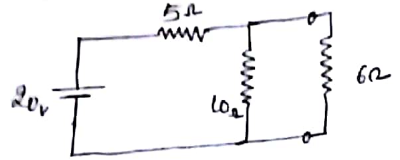
NORTON'S THEOREM :-

Norton's theorem states that any linear circuit with no. of voltage, current sources and resistors can be replaced by an equivalent circuit consisting of single current source in parallel with a single resistor, where the current source value is equal to the short circuit current flowing through the terminals and the resistance value is equal to the equivalent resistance measured through the terminals with all the energy sources deactivated

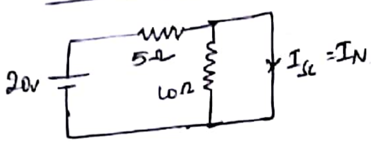


↳ Norton's Equivalent ckt

Q. Find the norton's equivalent ckt across AB for the ckt shown.

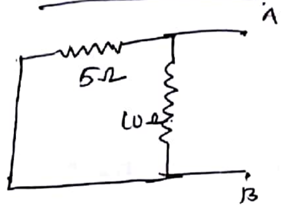


To find I_N

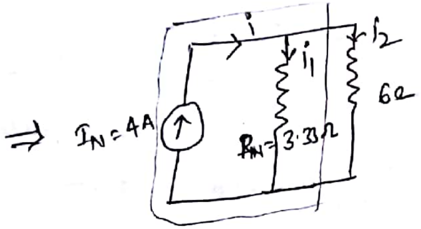


$I_N = \frac{20}{5} = 4A.$

To find R_N



$R_N = \frac{10 \times 5}{15} = 3.33\Omega$



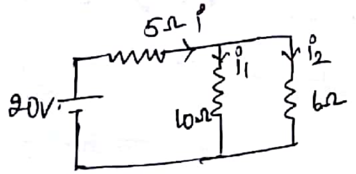
$\frac{i_1}{i_2} = \frac{6}{3.33}$

$\frac{4-x}{x} = \frac{6}{3.33}$

$13.32 - 3.33x = 6x$

$\boxed{x = 1.42A}$

Q Verify norton's theorem for the above circuit



$i = \frac{20}{8.75} = 2.285$

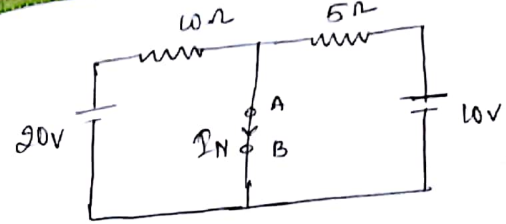
$\frac{2.285 - x}{x} = \frac{6}{10}$

$22.85 - 10x = 6x$

$16x = 22.85$

$\boxed{x = 1.428A}$

Q Determine the Norton's equivalent circuit at terminals AB for the circuit shown.



To find I_N.

$I_1 = \frac{20}{10} = 2A$

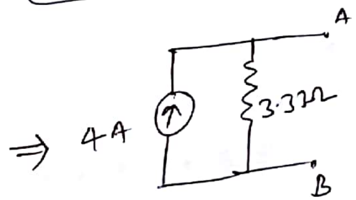
$I_2 = \frac{10}{5} = 2A$

$\boxed{I_N = 4A}$

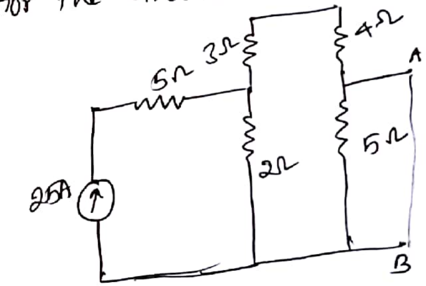
To find R_N

$\frac{10 \times 5}{10+5} = 3.33\Omega$

$\Rightarrow$



Q Determine the norton's equivalent circuit across AB for the circuit shown.

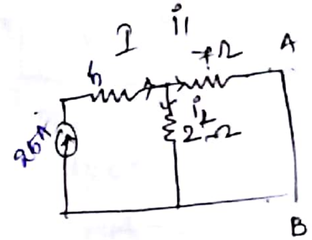


To find I_N

~~$I_N = 1.455$~~

$\boxed{I_N = 5.55A}$

$I_{sc} = I_N$



$\frac{i_1}{i_2} = \frac{2}{7}$

$\frac{x}{25-x} = \frac{2}{7}$

$7x = 50 - 2x$

$9x = 50$

$x = 5.55$

~~$\frac{i_1}{i_2} = \frac{2}{7}$~~

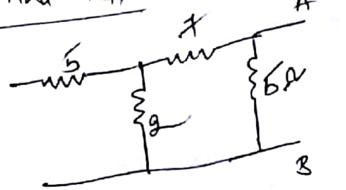
~~$\frac{x}{6.55 - x} = \frac{2}{7}$~~

~~$7x = 13.1 - 2x$~~

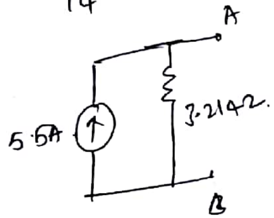
~~$9x = 13.1$~~

~~$x = 1.455$~~

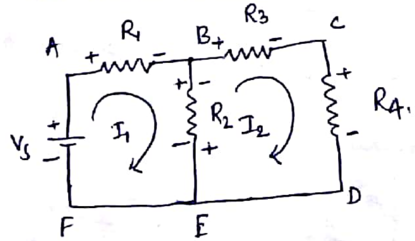
To find R_N



$$R_N = \frac{9\times5}{14} = 3.214\Omega$$



Loop (or) Mesh Analysis :- for voltage sources (KVL)



KVL → ABEFA

$V_s - I_1R_1 - (I_1 - I_2)R_2 = 0$

$-I_1(R_1+R_2) + I_2R_2 + V_s = 0$ —①

KVL → BCDEB

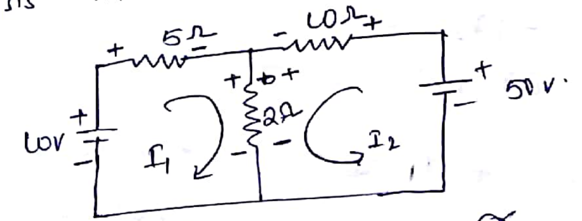
$-I_2R_3 - R_4I_2 - (I_2 - I_1)R_2 = 0$

$-I_2R_3 - R_4I_2 - I_2R_2 + I_1R_2 = 0$

$-I_1R_2 + I_2(R_2+R_3+R_4) = 0$ —②

By solving equ^n's ① & ② the currents I_1 and I_2 can be determined.

Q. Find the currents I_1 & I_2 by using mesh analysis in the circuit shown.



$\boxed{\text{Calc. } ax+by=c}$

$10 - 5I_1 - 2(I_1+I_2) = 0$ —①

$\cancel{10} + 7I_1 + 2I_2 - 10 = 0$ —①

$50 - 10I_2 - 2(I_1+I_2) = 0$

$+2I_1 + 12I_2 - 50 = 0$ —②

$I_1 = +1/4 \quad I_2 = +33/8$

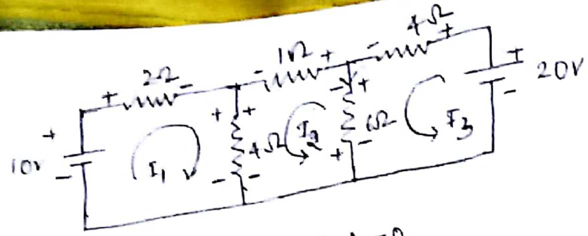
I_1	I_2	1	
+7	-10	7	2
12	-50	12	12

$$\frac{I_1}{-100+120} = \frac{I_2}{-20+350} = \frac{1}{84-4}$$

$$\frac{I_1}{20} = \frac{I_2}{330} = \frac{1}{80}$$

$I_1 = 1/4 \quad I_2 = 33/8$ //

Q. Calculate the current through 6Ω resistor by using loop analysis.



Loop 1 $10 - 2I_1 - 4(I_1 + I_2) = 0$

$-6I_1 - 4I_2 + 10 = 0$

$6I_1 + 4I_2 = 10$ — (1)

Loop 2 $-I_2 - 4(I_1 + I_2) - 6(I_2 - I_3) = 0.$

$-I_2 - 4I_1 - 4I_2 - 6I_2 + 6I_3 = 0$

$-4I_1 - 11I_2 + 6I_3 = 0$ — (2)

$\cancel{-11I_2 + 2I_3 = 0}$

$\cancel{2I_3 = 11I_2}$

Loop 3. $20 - 4I_3 - 6(I_3 - I_2) = 0$

$+6I_2 - 10I_3 + 20 = 0$

$-6I_2 + 10I_3 = 20$ — (3)

$\cancel{-6I_2 + 10 \times \frac{11I_2}{2} = 20}$

$\cancel{-6I_2 + 55I_2 = 20}$

$\cancel{49I_2 = 20}$

$\cancel{I_2 = \frac{20}{49}}$

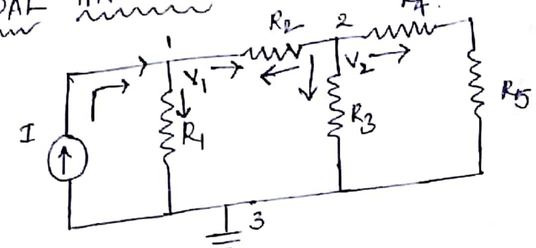
$I_1 = \frac{65}{71} = 0.915A$

$I_2 = \frac{80}{71} = 1.126A$

$I_3 = \frac{190}{71} = 2.676A$

$I_3 - I_2 = 1.55A$

NODAL ANALYSIS :- {KCL}



KCL → NODE 1

$$I = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}.$$

$$\Rightarrow V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = I \quad (1)$$

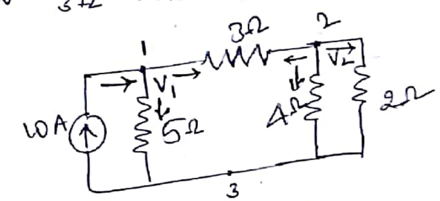
KCL → Node 2.

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0$$

$$\Rightarrow -V_1\left(\frac{1}{R_2}\right) + V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right) = 0 \quad (2)$$

By solving equations (1) & (2) the node voltages V_1 & V_2 can be determined.

Q. By using nodal analysis find the current flowing through 3Ω resistance in the circuit shown



KCL at node 1

$$10 = \frac{V_1}{5} + \frac{V_1 - V_2}{3}$$

$$10 = \frac{V_1}{5} + \frac{V_1}{3} - \frac{V_2}{3}$$

$$10 = \frac{8V_1}{15} - \frac{V_2}{3} \quad (1)$$

node 2

$$\frac{V_2 - V_1}{3} + \frac{V_2}{4} + \frac{V_2}{2} = 0$$

$$\frac{V_2}{3} + \frac{V_2}{4} + \frac{V_2}{2} - \frac{V_1}{3} = 0$$

$$\frac{4V_2 + 3V_2 + 6V_2}{12} - \frac{V_1}{3} = 0$$

$$\frac{13V_2}{\cancel{12}_{4}} = \frac{V_1}{\cancel{3}} \Rightarrow 13V_2 = 4V_1$$

$$10 = \frac{8V_1}{15} - \frac{4V_1}{13\times3}$$

$$10 = \frac{8V_1}{15} - \frac{4V_1}{39} = V_1(0.53 - 0.10)$$

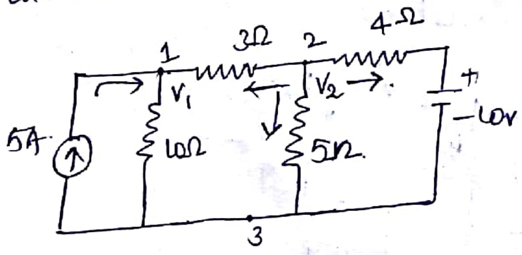
$$V_1 = \frac{10}{0.431}$$

$$\boxed{V_1 = 23.201V}$$

$$V_2 = 7.139V$$

$$\boxed{I = 5.354A}$$

Q. Write the node voltage equation and determine the current through 3Ω resistance.



KCL at 1:

$$5 = \frac{V_1}{10} + \frac{V_1 - V_2}{3}$$

$$5 = \frac{V_1}{10} + \frac{V_1}{3} - \frac{V_2}{3}$$

$$5 = \frac{13V_1}{30} - \frac{V_2}{3} \quad (1)$$

At node 2

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{4} = 0$$

$$\frac{V_2}{3} + \frac{V_2}{5} + \frac{V_2}{4} - \frac{V_1}{3} - \frac{\cancel{10}^{5}}{\cancel{4}_{2}} = 0$$

$$\frac{47V_2}{60} - \frac{V_1}{3} = 5/2 \quad (2)$$

$$= \frac{47V_2 - 20V_1}{\cancel{60}\ 30} = \frac{5}{\cancel{2}}$$

$$\frac{13V_1 - 10V_2}{30} = 5$$

$$13V_1 - 10V_2 = 150 \quad (1)$$

$$-20V_1 + 47V_2 = 150$$

$$20V_1 - 47V_2 = -150 \quad (2)$$

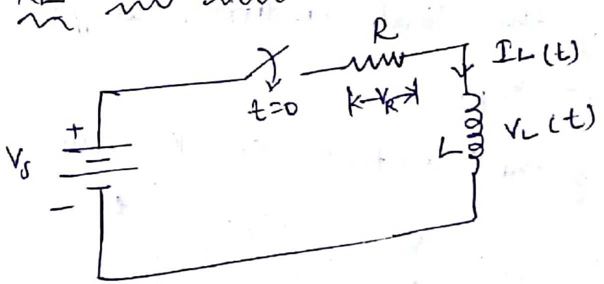
$$\boxed{V_1 = 20.802V}$$

$$\boxed{V_2 = 12.043V}$$

$$\boxed{I = 2.919A}$$

Time domain Analysis :-

RL series circuit :-



KVL → $-V_S + V_R + V_L(t) = 0$

$$\Rightarrow -V_s + I_L(t)R + L\frac{dI_L(t)}{dt} = 0$$

$$\Rightarrow \frac{dI_L(t)}{dt} + \frac{R}{L}I_L(t) = \frac{V_s}{L}$$

$$I_L(t) = K_1 + K_2 e^{-t/\tau}$$

find K_1 & K_2 by using initial & final conditions

$$\therefore I_L(t)(0^-) = I_L(t)(0^+) = 0$$

The solution for the above eqn is

$$I_L(t) = K_1 + K_2 e^{-t/\tau}$$

At the moment of switching the inductor the current does not change

i.e., $I_L(0^-) = I_L(0^+) = 0$.

Substituting the initial condn's we get

$$0 = K_1 + K_2$$

$$\Rightarrow \boxed{K_2 = -K_1}$$

when $t \to \infty$ the inductor acts as short circuit

and $I_L(\infty) = V_s/R$.

Substituting the final conditions we get

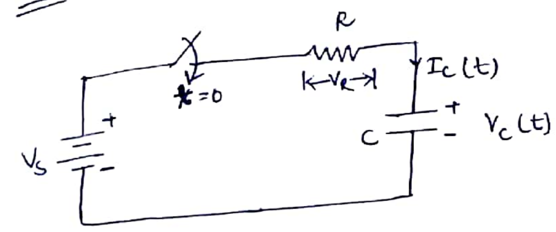
$$K_1 = V_s/R$$

Substituting the constants K_1 & K_2 values

$$\boxed{I_L(t) = \frac{V_s}{R}(1 - e^{-t/\tau})} \quad *$$

where $\tau = \frac{L}{R}$

RC SERIES CKT



KVL → $-V_s + V_R + V_c = 0$

$$\Rightarrow -V_s + I_c(t) \times R + V_c(t) = 0$$

$$\Rightarrow -V_s + RC \times \frac{dV_c(t)}{dt} + V_c(t) = 0$$

The solution for above equation is $V_c(t) = A + Be^{-t/\tau}$

At $t(0^+)$, $V_c(t) = 0$

$$0 = A + B \Rightarrow B = -A$$

At $t \to \infty$, $V_c(t) = V_s$

$$\therefore V_s = A.$$

Substituting the constant values A and B

$$\boxed{V_c(t) = V_s(1 - e^{-t/\tau})}$$

where $\tau = RC$. (time constant.)

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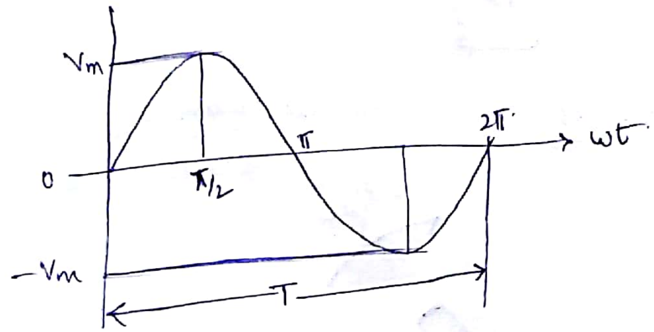
UNIT-II **AC CIRCUITS** 12/09/18

Alternating Quantity :-

An alternating quantity changes in magnitude and alternates in the direction at regular intervals of time. ~~So any quantity~~.



The terms related to an alternating quantity are

① **Amplitude :-**

It is defined as the maximum value attained by the alternating quantity.
It is also known as peak value or maximum value.

② **Time period :-**

The time taken for an alternating quantity to complete one cycle is known as it's Time period.

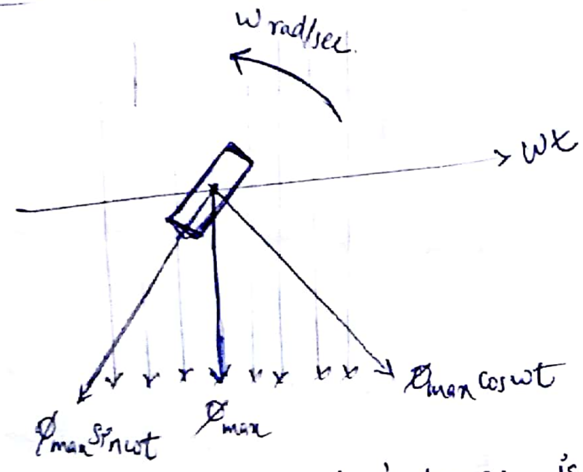
③ Frequency :-

It is defined as the no.of cycles of an alternating quantity in one second is known as it's Frequency.

Units are Hertz (Hz) or cycles/second.

$$\boxed{T = \frac{1}{f}}$$

Representation of a Sinusoidal wave :-



Consider a conductor with 'N' turns is rotating at ω rad/sec in a stationary magnetic field.

$$e \text{ (or) } v = -N\frac{d\phi}{dt}$$

$$e \text{ (or) } v = -N\frac{d}{dt}(\phi_{max}\cos\omega t)$$

$$e \text{ (or) } v = +N \times \phi_{max} \times \omega \sin\omega t$$

$$\Rightarrow \boxed{v = V_m \sin\omega t}, \quad V_m = N\phi_{max}\omega$$

The maximum flux ϕ_{max} can be divided into two components, $\phi_{max}\sin\omega t$ which acts in parallel to the conductor and $\phi_{max}\cos\omega t$ which is in $\perp^r$ to the conductor and which is responsible to induce an emf in the conductor.

i.e. v or $e \propto \phi$

i.e the emf induced in a current carrying conductor placed in a stationary magnetic field is a sinusoidal wave. Hence the currents induced will also have a sinusoidal in shape, which is represented by $\boxed{i = I_m \sin\omega t}$

AVERAGE VALUE :-

The arithametic sum of all the values of an alternating quantity over a period of cycle is known as its Average Value.

i.e, Average value $= \frac{\text{Area of the wave over full cycle}}{\text{BASE}}$

Since a symmetrical alternating quantity has an average value of zero, only half of the cycle is considered while measuring the averge value of an alternating quantity.

i.e Average value $= \frac{\text{Area of the wave over half cycle}}{\text{Base}}$.

(5) For a sinusoidal wave $V_{Avg} = \frac{1}{\pi}\int_0^{\pi} v\, d(\omega t)$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\, d(\omega t)$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin\omega t\, d(\omega t)$$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= \frac{V_m}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$\Rightarrow \frac{V_m}{\pi}\left[+1+1\right]$$

$$\boxed{V_{Avg} = \frac{2V_m}{\pi}}$$

$$\Rightarrow \boxed{V_{Avg} = 0.636\, V_m}$$

$||^{ly}$ $\boxed{I_{Avg} = 0.636\, I_m}$

RMS VALUE :-

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t\, d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}}\sqrt{\int_0^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}}\sqrt{\left[\frac{1}{2}(2\pi - 0) - \left(\frac{\sin 2\omega t}{4}\right)_0^{2\pi}\right]}$$

$$\boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}} = 0.707\, V_m.$$

$$\boxed{V_{RMS} = 0.707\, V_m}$$

$||^{ly}$ $\boxed{I_{RMS} = 0.707\, I_m}$

Form factor :-

It is defined as the ratio of RMS value to the average value of an alternating quantity

* For a sinusoidal wave form factor $= \frac{0.707\, V_m}{0.636\, V_m}$

$$= 1.11$$

Peak factor :-

It is defined as the ratio of the maximum value to the rms value of an alternating quantity.

* For a sinusoidal wave peak factor $= \frac{V_m}{0.707\, V_m}$

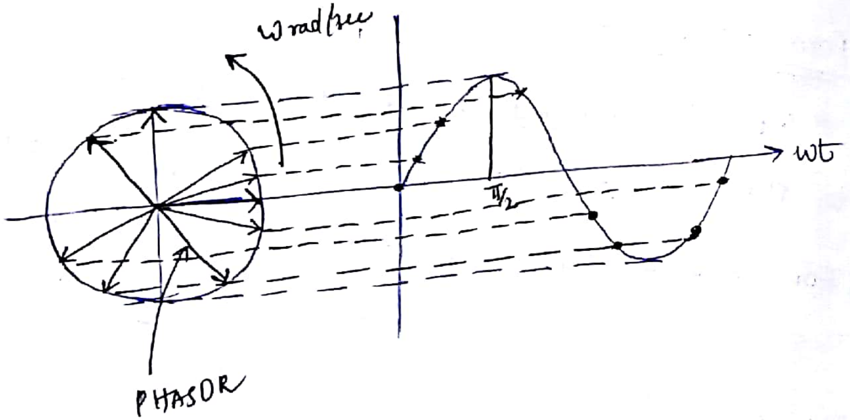
$$= 1.414$$

PHASE :-

The phase of an alternating quantity is the advancment of the wave from the starting point (or) zero (or) reference point.

PHASOR :-

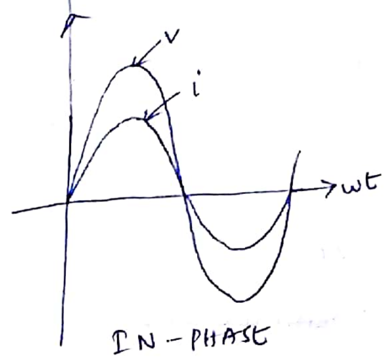
A phasor is straight line with fixed length rotating in anti clock-wise direction with a constant angular velocity.

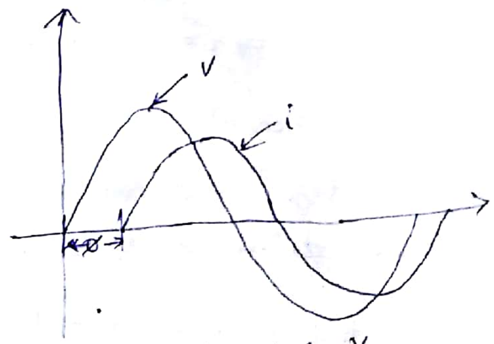


→ When two wave forms with same frequency have same starting points, then they are said to be in phase.

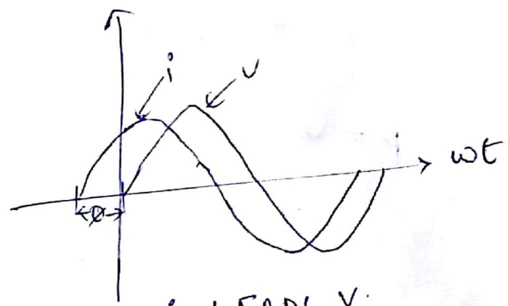
→ When the two wave forms with same frequency has different starting points, then they are said to be out of phase.

→ The difference in the phase of the two wave forms starting at different points is known as Phase difference.



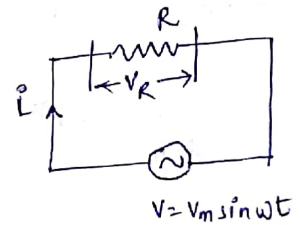


OUT OF PHASE



ANALYSIS OF AC CIRCUITS :-

1. With Pure resistance (R) :



Consider $\boxed{v = V_m \sin\omega t}$ —(1) as reference.

The current flowing in the circuit $i = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$

$$\Rightarrow i = I_m \sin\omega t \quad \text{—(2)} \qquad , I_m = \frac{V_m}{R}$$

From equations (1) & (2) in a pure resistive circuit the voltage and current are in phase (or) the phase difference is zero.

The instantaneous power $p = vi$

$$= V_m \sin\omega t \, I_m \sin\omega t$$

$$= V_m I_m \sin^2\omega t$$

$$= V_m I_m \left(\frac{1 - \cos 2\omega t}{2}\right)$$

$$p = \underbrace{\frac{V_m I_m}{2}}_{\text{Constant power}} - \underbrace{\frac{V_m I_m}{2}\cos 2\omega t}_{\text{fluctuating power}}$$

The average power over a full cycle ~~is~~

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$

$$= \frac{1}{2\pi}\left[\frac{V_m I_m}{2}[2\pi] - \frac{V_m I_m}{2}\left(\frac{\sin 2\omega t}{2}\right)_0^{2\pi}\right]$$

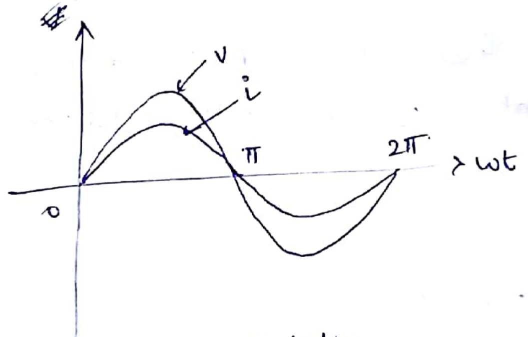
$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{8\pi}(\sin 4\pi - \sin 0)$$

$$= \frac{V_m I_m}{2}$$

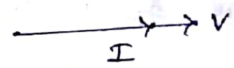
$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$\boxed{P_{avg} = V_{RMS} \times I_{RMS}}$$

i.e In a pure resistive circuit the average power is equal to the product of the RMS voltage and RMS current.

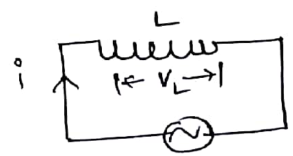


wave representation



phasor representation

2. with Pure Inductance :-



$v = V_m \sin\omega t$

$v = V_m \sin\omega t$ —(1)

$v = V_m \sin\omega t = V_L$

$= L\frac{di}{dt}$

$$\Rightarrow V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m \sin\omega t}{L} dt$$

Integrating on both sides

$$i = \frac{V_m}{L}\left[-\frac{\cos\omega t}{\omega}\right]$$

$$\Rightarrow i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$$

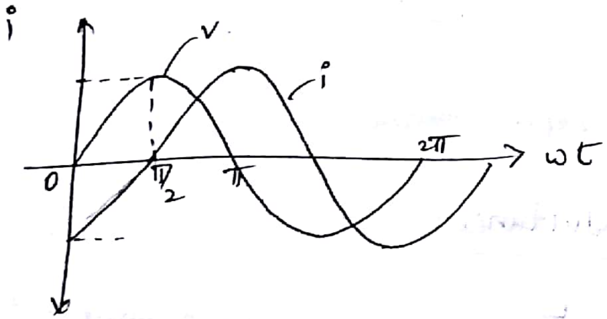
$$i = I_m \sin(\omega t - \pi/2) \quad \text{—} (2)$$

where $I_m = \frac{V_m}{\omega L}$

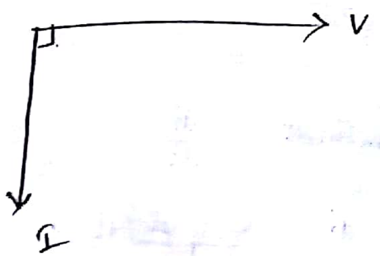
$$I_m = \frac{V_m}{\omega L} = \cancel{\omega}\ \frac{V_m}{X_L}$$

where X_L is known as inductive reactance.

from equations (1) & (2) in a pure inductance circuit, the current lags the voltage by an angle $\pi/2$ or $90°$



Phasor diagram :-



The instantaneous power $'p' = v \times i$

$$= V_m \sin\omega t \times I_m \sin(\omega t - \pi/2)$$

$$= V_m \times I_m \sin\omega t\,(-\cos\omega t)$$

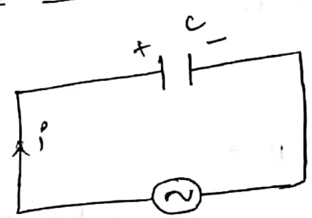
$$= -\frac{V_m I_m}{2}\sin 2\omega t$$

The average power $P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \times \sin 2\omega t\, dt$

$$= 0$$

i.e In a pure inductive circuit the power consumed is zero

3. With Pure Capacitance :-



Consider $V = V_m \sin\omega t$ —(1) as preference

$\therefore Q = C.V = C\,V_m \sin\omega t$

$$i = \frac{dq}{dt} = \frac{d}{dt}(C V_m \sin\omega t)$$

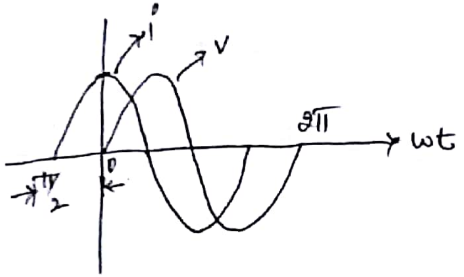
$$\Rightarrow i = C.V_m \cos\omega t \times \omega$$

$$\Rightarrow i = \omega C V_m \cos\omega t$$

$$\Rightarrow i = I_m \sin(\omega t + \pi/2).$$

where $I_m = \omega C V_m = \frac{V_m}{X_C}$, $X_C = \frac{1}{\omega C}$

where X_C is knows as capacitive reactance.



from equations ① & ② , in a pure capacitive circuit the current always leads the voltage by an angle $\pi/2$.

The instantaneous power $p = Vi$

$$= V_m \sin\omega t \times I_m \sin(\omega t + \tfrac{\pi}{2})$$

$$= V_m I_m \sin\omega t \times \cos\omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t.$$

The average power $P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t$

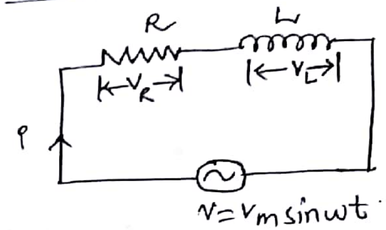
$$= \frac{1}{2\pi} \times \frac{V_m I_m}{2\times 2}\Big[-\cos\omega t\Big]_0^{2\pi}$$

$$= \frac{1}{2\pi}\frac{V_m I_m}{4}\Big[-\cos 2\pi + \cos 0\Big]$$

$$= 0$$

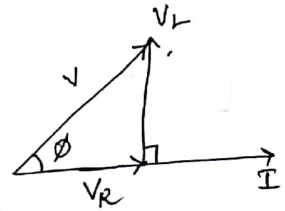
i.e. The power consumed in a pure capacitive circuit is always zero.

R-L series circuit :-



Let $V_R = IR \longrightarrow$ In phase with I

$V_L = I X_L \longrightarrow$ leads I by $\pi/2$.



From the phasor diagram the resultant voltage.

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$\boxed{V = IZ}$$

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR}$$

$$= \frac{X_L}{R}$$

$$\tan\phi = \frac{\omega L}{R}$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

The instantaneous power $p = vi = V_m \sin\omega t \times I_m \sin(\omega t - \phi)$

$= V_m I_m \sin\omega t [\sin\omega t \cos\phi - \cos\omega t \sin\phi]$

$= V_m I_m [\sin^2\omega t \cos\phi - \sin\omega t \cos\omega t \sin\phi]$

$= V_m I_m \left[\frac{1-\cos 2\omega t}{2} \times \cos\phi - \frac{\sin 2\omega t}{2} . \sin\phi\right]$

$= \frac{V_m I_m}{2} [\cos\phi - \cos 2\omega t \cos\phi - \sin 2\omega t \sin\phi]$

$= \frac{V_m I_m}{2} [\cos\phi - \cos(2\omega t - \phi)]$

The average power $P = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} [\cos\phi - \cos(2\omega t - \phi)] d\omega t$

$= \frac{1}{2\pi} \frac{V_m I_m}{2} \left[\cos\phi - \int_0^{2\pi} [\cos(2\omega t - \phi)]\right]$

$= \frac{V_m I_m}{4\pi} \left[\cos\phi \, \omega t \Big|_0^{2\pi} - \frac{\sin(2\omega t - \phi)}{2} \Big|_0^{2\pi}\right]$

$= \frac{V_m I_m}{4\pi} \left[2\pi \cos\phi - \left[\frac{\sin(4\pi - \phi)}{2} - \frac{\sin(-\phi)}{2}\right] \times 0\right]$

$= \frac{V_m I_m}{4\pi} [2\pi \cos\phi + 0 + 0]$

$= \frac{V_m I_m}{2} \cos\phi$

$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$

$$\boxed{P = VI \underbrace{\cos\phi}_{\text{power factor}}}$$

From the phasor diagram

$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$

$\cos\phi = R/Z.$

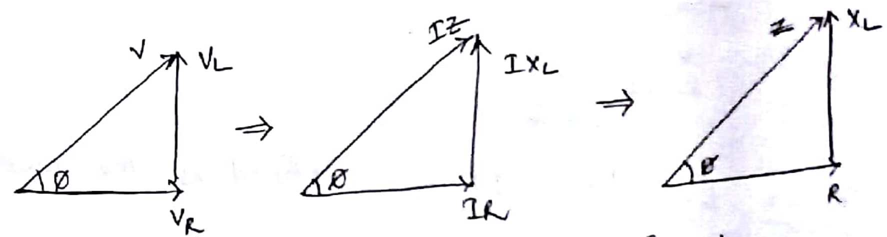
$\therefore \quad P = IZ \times I \times \frac{R}{Z}$

$$\Rightarrow \boxed{P = I^2 R}$$

i.e In an R-L series circuit the power consume-d is equal to I^2R, since the power consumed in an inductor is zero.

Power Factor :- (no units)

It is defined as the cosine of the phase angle between the voltage and current in an A-C circuit.



Impedance Triangle

$\boxed{\cos\phi = \frac{R}{Z}}$

~~where~~ $\tan\phi = \frac{X_L}{R} \Rightarrow \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

i.e power factor ($\cos\phi$) can also be defined as the ratio of resistance to the impedance of the A-C circuit.

Power in an A-C circuit :-

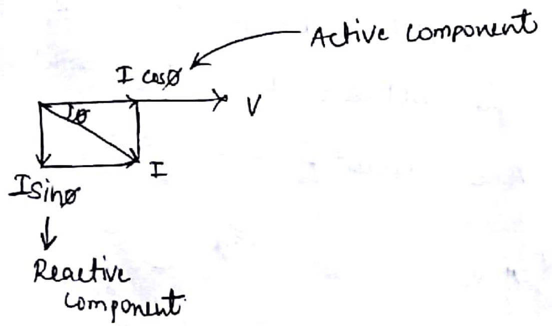
The power in an A-C circuit consists of an active (or) real power and reactive power.

⇒ The power which do some useful work in the circuit is known as "active power."

⇒ The power which doesn't do any useful work is known as "Reactive power".

T

⇒ The total power available in an A-C circuit is known as "Apperarent power".



Active power(P):- It is defined as the power in an A-C circuit due to the active component of the current. Units are watts.

i.e $P = VI\cos\phi$ WATTS

$\Rightarrow V = IZ$

$P = IZ \times I \times \frac{R}{Z} = I^2R$ WATTS

Reactive power (Q) :-

It is defined as the power in an A-C circuit due to ~~their~~ reactive component of the current.

Units are VARs

i.e $Q = VI\sin\phi$ VARs

$= IZ \times I \times \frac{X_L}{Z} = I^2X_L$ VARs.

Apparent Power (S) :-

It is defined as the total power available in an A-C circuit. Units are VA.

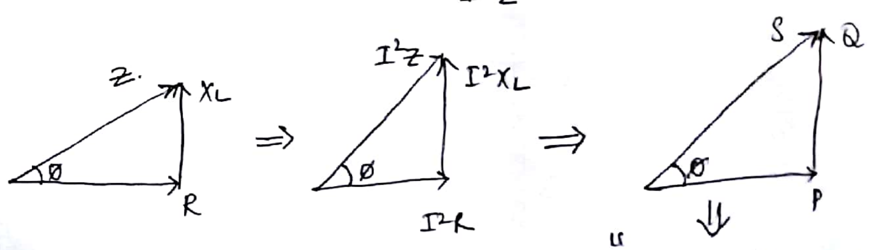
i.e $S = \sqrt{P^2+Q^2}$

$P^2 = (I^2R)^2$, $Q^2 = (I^2X_L)^2$

$S = \sqrt{(I^2R)^2 + (I^2X_L)^2}$

$= I^2\sqrt{R^2+X_L^2}$

$S = I^2Z$ VA



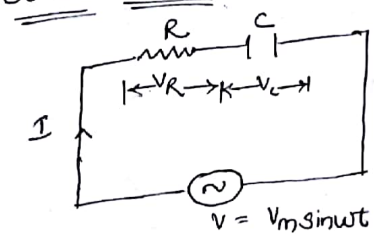
"Power Triangle"

from power triangle, ~~cos ø~~

$\cos\phi = P/S = \dfrac{\text{real power / Active}}{\text{apparent power}}$ No units

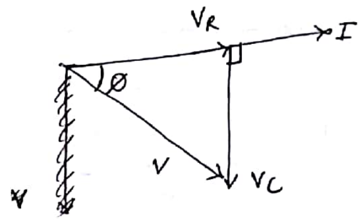
i.e Power factor can also be defined as the ratio of Active / Real power to the apparent power in an A-C circuit.

RC series circuit :-



let $V_R = IR$ In phase with I

$V_C = I X_C$ lags current by $\pi/2$.



from the phasor diagram the resultant is

$$V = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

$$\boxed{V = IZ}$$

$Z = \sqrt{R^2 + X_C^2}$

$Z \rightarrow$ impedance.

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{\omega CR}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)}$$

The instantaneous power $P = VI$.

$= V_m \sin\omega t \, I_m \sin(\omega t + \phi)$

$= V_m I_m \sin\omega t\,[\sin\omega t \cos\phi + \cos\omega t \sin\phi]$

$= V_m I_m\,[\sin^2\omega t \cos\phi + \sin\omega t \cos\omega t \sin\phi]$

$= V_m I_m\left[\frac{1-\cos 2\omega t}{2}\cos\phi + \frac{\sin 2\omega t}{2}\sin\phi\right]$

$= \frac{V_m I_m}{2}\left[\cos\phi - \cos\phi\cos 2\omega t + \sin\phi \sin 2\omega t\right]$

$$\boxed{P = \frac{V_m I_m}{2}\left[\cos\phi - \cos(\phi + 2\omega t)\right]}$$

→ The average power $P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t+\phi)\right] d\omega t$

$= \frac{V_m I_m}{2} \times \frac{1}{2\pi}\left[\cos\phi\, \omega t - \frac{\sin(2\omega t+\phi)}{2}\right]_0^{2\pi}$

$= \frac{V_m I_m}{2} \times \frac{1}{2\pi}\left[\cos\phi\, 2\pi\right]$

$= \frac{V_m I_m}{2}\cos\phi$

$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi$.

$$\boxed{P_{avg} = V_{rms} I_{rms} \cos\phi}$$

Pure resistor $\phi = 0$ $\boxed{P = VI}$ max

Pure capacitor/Inductor $\phi = 90°$ $\boxed{P = 0}$.

from phasor diagram

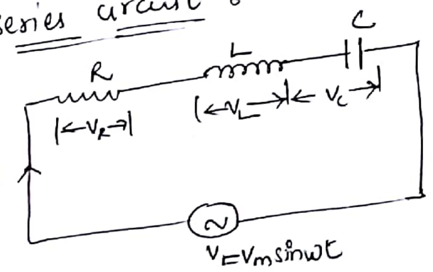
$\cos\phi = \frac{V_R}{V}$

$= \frac{IR}{IZ} = \frac{R}{Z}$

$P = IZ \times I \times R/Z$

$\boxed{P = I^2 \times R}$

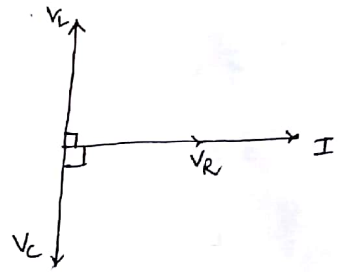
RLC series circuit :-



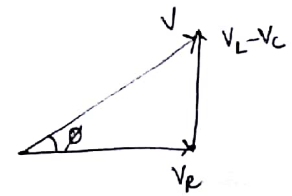
Let $V_R = IR$ → In phase with I

$V_L = IX_L$ → Leads current by $\pi/2$

$V_C = IX_C$ → Lags I by $\pi/2$



Case (i) :- Inductor DOMINANT ($X_L > X_C$)



$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

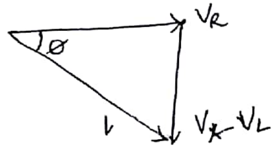
$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = IZ, \quad \boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$

Case (ii) 'C' DOMINANT ($X_C > X_L$)



$$V = \sqrt{V_R^2 + (V_X - V_L)^2}$$

$$V = \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = IZ, \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{V_C - V_L}{V_R} = \frac{X_C - X_L}{R}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)}$$

Case (iii) if $X_L = X_C$, The total impedance of the circuit will be equal to the resistance R and the circuit acts like a pure resistive circuit, in which the voltage and current are in phase.

Q. A coil having a resistance of $7\,\Omega$ and an inductance of 31.8 mH is connected to 230 V 50Hz ac supply. Calculate

(i) the circuit current

(ii) Phase angle

(iii) Power factor

(iv) Power consumed.

$R = 7\,\Omega \quad X_L = \omega L = \frac{~~314~~ \; 9909.26}{1000}\,\Omega$

$= 9.99026$

(i) $i = \frac{230}{~~9.99~~ \; ~~30.072~~ \; 12.19}$

$= ~~7.66~~\,A \quad 18.86\,A$

(ii) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = 54.98^\circ$

(iii) $\cos\phi = 0.57386$

(iv) $P = 2489.89$ WATTS

Q A 200V 50Hz inductive circuit takes a current of 10A lagging by 30°. Find

(i) Resistance

ii, reactance

iii, inductance of the coil

$$\tan\phi = \frac{X_L}{R} \Rightarrow \tan 30^\circ = \frac{X_L}{R} = \frac{1}{\sqrt{3}} \Rightarrow R = \sqrt{3}X_L$$

$$i = \frac{V}{\sqrt{R^2+X_L^2}} \Rightarrow 10 = \frac{200}{\sqrt{R^2+X_L^2}}$$

$R^2+X_L^2 = 400$

$3X_L^2 + X_L^2 = 400$ $\quad 4X_L^2 = 400$ $\quad \boxed{X_L = 10}$

reactance = 10

$\omega L = 10$ $\quad 2\pi f \times L = 10$

$$L = \frac{10}{2\pi f} = 0.031\text{H}$$

$\boxed{R = 10\sqrt{3}}$

Q. A series RLC circuit containing a resistance of 12Ω, 0.15H and a capacitor of $100\,\mu F$ are connected in series across a 100V 50Hz supply calculate

i, the circuit impedance

ii, the circuit current

iii, power factor.

Also draw the voltage phasor diagram.

$R = 12\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{\frac{\pi}{100}} = 31.83\Omega$$

$X_L = \omega L = 47.12\,\Omega$

$$Z = \sqrt{(12)^2 + (47.12 - 31.83)^2}$$

$\boxed{Z = 19.43\Omega}$

$$i = \frac{V}{Z} = \frac{100}{19.43} = 5.146\,A$$

$$\text{Power factor} = \cos\phi = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

$$= \frac{12}{19.43}$$

$= 0.617$

Q. An ac circuit composed of a series connect of $R = 50\Omega$, $L = 0.3H$ and $C = 15\,\mu F$. The circuit is connected to an ac voltage source of 25V and 60Hz. Find the current in the circuit & phase difference b/w voltage and current.

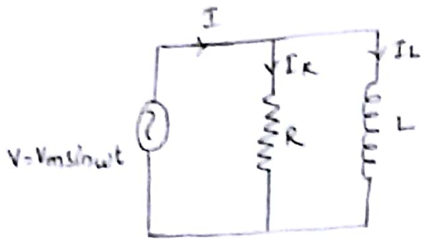
$X_L = 94.24\Omega$ $\quad X_C = 212.31\,\Omega$

$$i = \frac{25}{\sqrt{50^2 + (212.31 - 94.24)^2}} = 0.1949\,A.$$

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{212.31 - 94.24}{50}$$

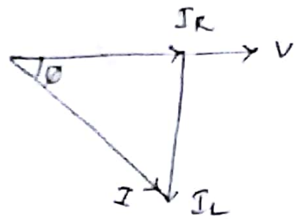
$\boxed{\phi = 67.04^\circ}$

RL parallel circuit :-



let $I_R = \frac{V}{R}$ In phase with V

$I_L = \frac{V}{X_L}$ LAGS V by 90°



$\boxed{I = I_R + I_L}$

From the phasor diagram

$$I = \sqrt{I_R^2 + I_L^2}$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L}\right)^2}$$

$$= V\sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}$$

$$\boxed{Z = \frac{V}{I} = \frac{1}{\left[\frac{1}{R^2} + \frac{1}{X_L^2}\right]^{1/2}}}$$

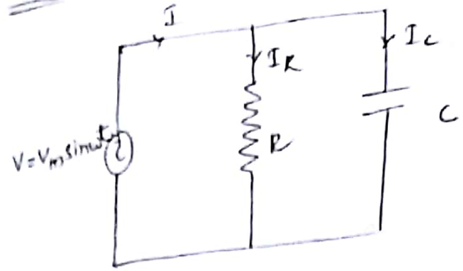
$$\phi = \tan^{-1}\left(\frac{I_L}{I_R}\right) = \tan^{-1}\left(\frac{R}{X_L}\right) = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

$$\cos\phi = \frac{I_R}{I} = \frac{\frac{V}{R}}{\frac{V}{Z}} = \frac{Z}{R}$$

$$\cos\phi = \frac{Z}{R}$$

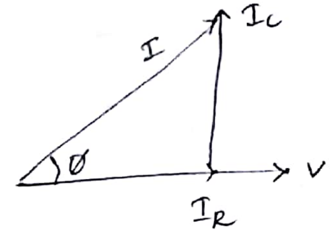
The instantaneous power $P = VI = V_m \sin\omega t \, I$

RC parallel circuit :-



let $I_R = \frac{V}{R}$ In phase with V

$I_C = \frac{V}{X_C}$ leads V by $\pi/2$



$I = I_R + I_C$

From the phasor diagram

$$I = \sqrt{I_R^2 + I_C^2}$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_C}\right)^2}$$

$$= V\sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}}$$

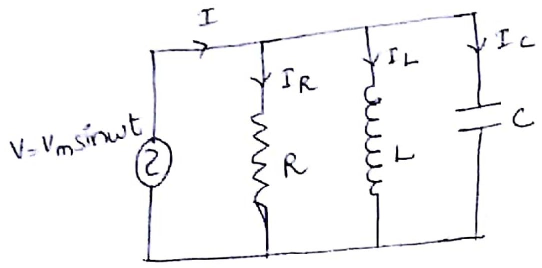
$$\frac{V}{I} = Z = \frac{1}{\left[\frac{1}{R^2} + \frac{1}{X_C^2}\right]^{1/2}}$$

$$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right) = \tan^{-1}\left(\frac{R}{X_C}\right) = \tan^{-1}(R\omega C)$$

$\phi = \cos^{-1}(I_R/I)$ $\phi = \cos^{-1}(Z/R)$

$\boxed{\cos\phi = Z/R}$

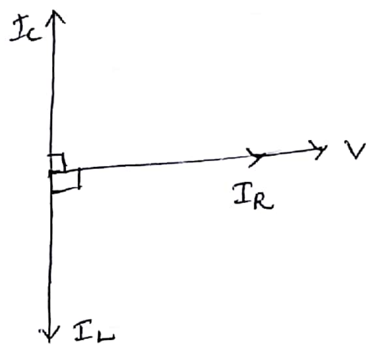
RLC parallel circuit :-



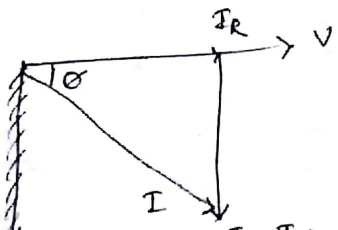
Let $I_R = \frac{V}{R}$ In pase with V

$I_L = \frac{V}{X_L}$ Lags v by $\pi/2$

$I_C = \frac{V}{X_C}$ Leads V by $\pi/2$



Case (i) Inductor dominant ($X_L < X_C$)



$I_L - I_C$

$I = \sqrt{I_R^2 + (I_L - I_C)^2}$

$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2}$

$I = V\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$

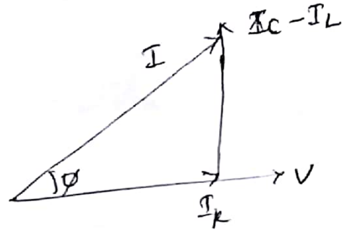
$\frac{V}{I} = Z = \frac{1}{\left[\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2\right]^{1/2}}$

$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right) = \tan^{-1}\left(\frac{\frac{1}{X_L} - \frac{1}{X_C}}{\frac{1}{R}}\right)$

$= \tan^{-1}\left(\frac{(X_C - X_L)R}{X_C X_L}\right)$

$= \tan^{-1}\left[R\left(\frac{1}{X_L} - \frac{1}{X_C}\right)\right]$

Case (ii) Capacitor dominant ($X_C < X_L$)



$I = \sqrt{I_R^2 + (I_C - I_L)^2}$

$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_C} - \frac{V}{X_L}\right)^2}$

$I = V\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$

$\frac{V}{I} = Z = \frac{1}{\left[\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2\right]}$

$\phi = \tan^{-1}\left(\frac{I_C - I_L}{R}\right) = \tan^{-1}\left(\frac{X_L - X_C}{X_C X_L} R\right)$

$= \tan^{-1}\left[R\left(\frac{1}{X_C} - \frac{1}{X_L}\right)\right]$

Case (iii) :- If $X_L = X_C$, The total impedance of the circuit will be equal to the resistance R and the circuit acts like a pure resistive circuit, in which the voltage and current are in phase.

Three Phase Balanced System :-

The three phase balanced system (voltage/current) is a combination of 3 single phase systems (voltage or current) of which the three voltages (or) currents differ in phase by 120° electrical degrees from each other in a particular sequence.

Advantages :-

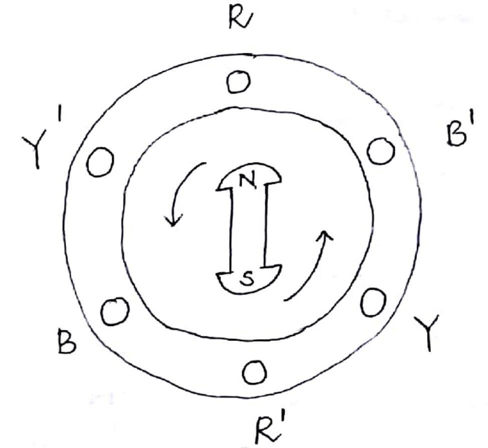
(i) The three phase power supplied to a three phase balanced circuit is constant at every instant of time and hence three phase motors have uniform torque.

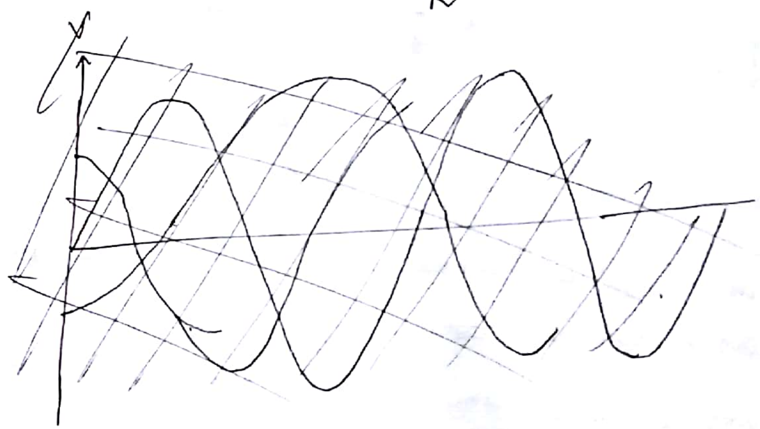
(ii) To transmit a given amount of power over a given length a three phase transmission circuit requires less conductor material than a single phase circuit.

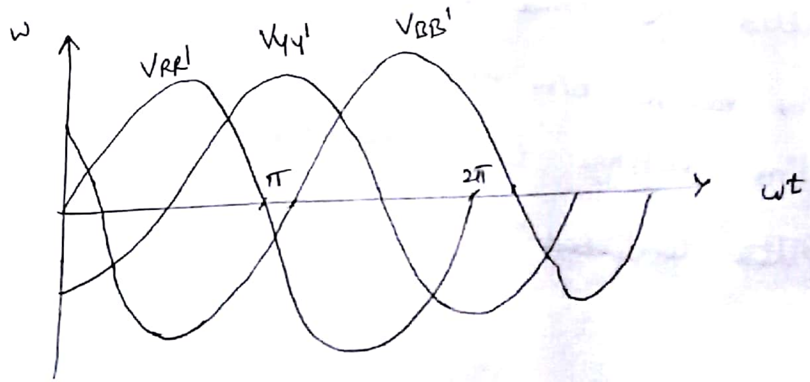
(iii) In a given frame size a three phase motor (or) generator produces more output than it's single phase counter part.

(iv) Three phase motors can be easily started.

(v) Three phase machines control equipment are smaller, cheaper, lighter in weight and more efficient.



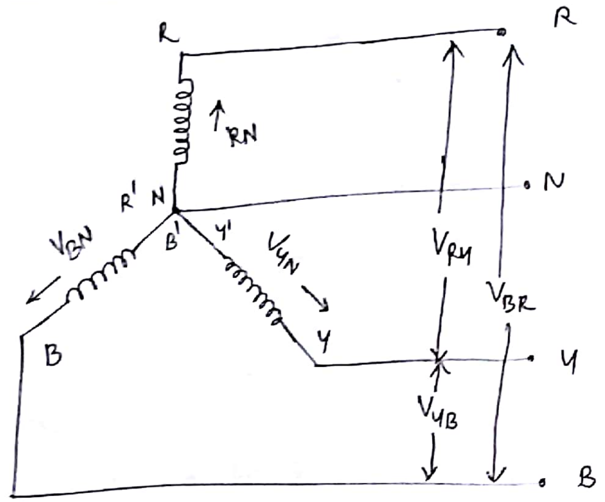




A three phase balanced system can be represented by

1. Star conne~~d~~cted system
2. Delta connected system.

Star connected System:-



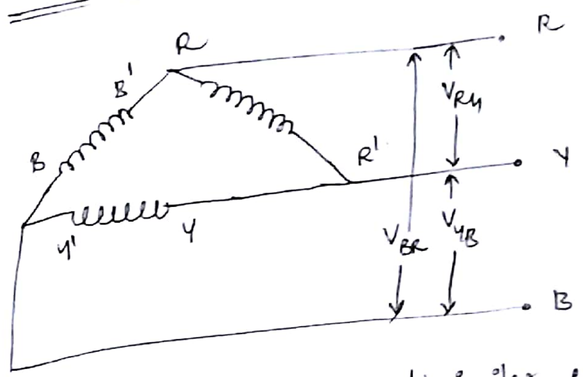
→ In this system similar ends of the three phases are joined together. ~~The~~

→ The voltage b/w any line and the neutral point is called The Phase voltage (V_{RN}, V_{YN}, V_{BN})

→ The voltage b/w any two lines is called the line voltage (V_{RY}, V_{BY}, V_{BR}).

~~Delta connected~~

Delta connected System:-



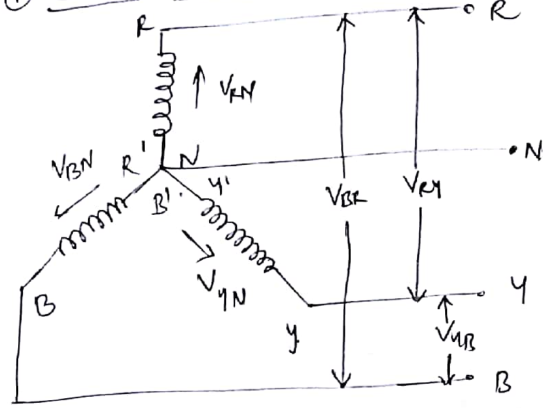
→ In this system the dissimilar ends of the windings are joined together i.e R' to Y, Y' to B and B' to R.

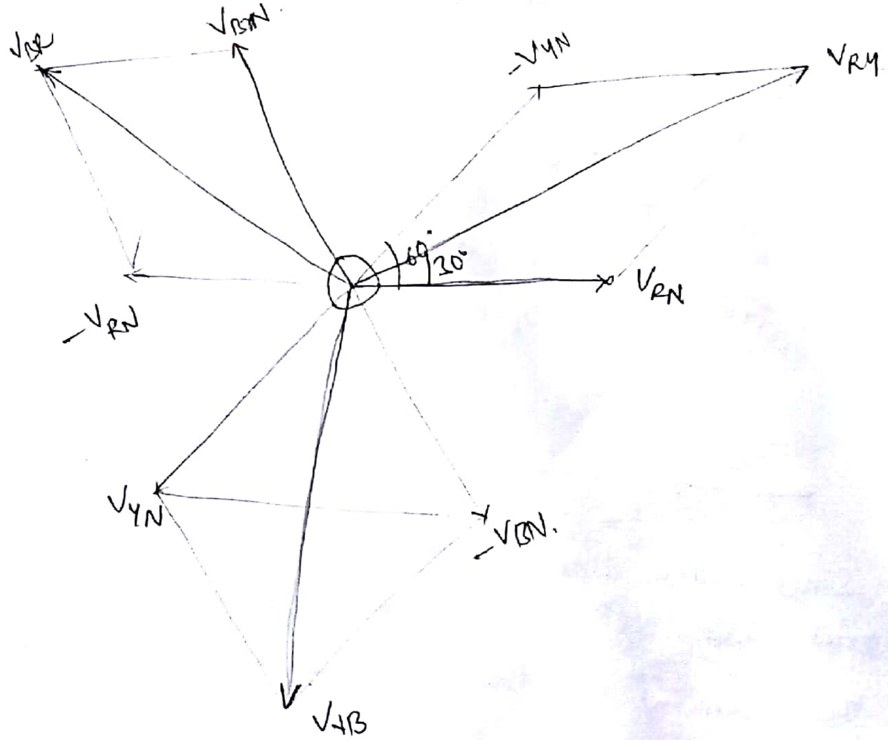
→ Since there is no common terminal, only three line voltages V_{RY}, V_{YB}, & V_{BR} are available and these are also refered as phase voltages in delta connected system.

~~Filling of d-orbital &~~ Crystal ~~field~~ stabilisation ~~energy~~

Voltage and Current Relations in Three phase balanced Systems :

① Star Connected System :-





$|V_{RN}| = |-V_{YN}| = V_{ph}$

$V_{RY} = 2 V_{ph} \times \cos 30°$

$\Rightarrow \boxed{V_{RY} = \sqrt{3} V_{ph}}$

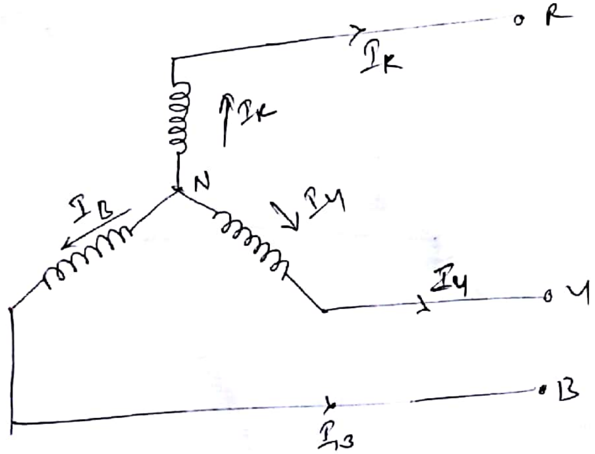
Similarly $V_{YB} = \sqrt{3} V_{ph}$ and

$V_{BR} = \sqrt{3} V_{ph}$

ie in a balanced star connected System

i, line voltage = $\sqrt{3}$ Phase voltage

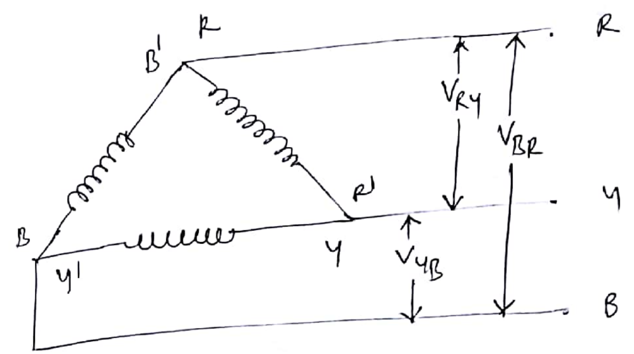
ii, All line voltages are equal in magnitude and displaced by 120°



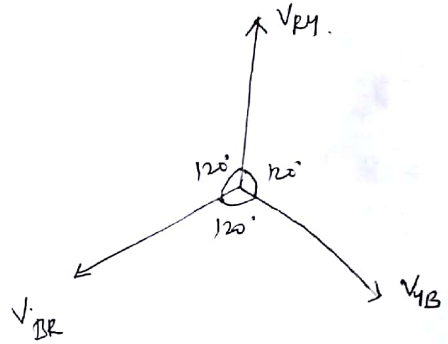
Since each line conductor is connected in series with it's individual phase winding, the current in the line conductor is same as that in the phase to which it is connected.

i.e $\boxed{I_L = I_{ph} = I_R = I_Y = I_B}$

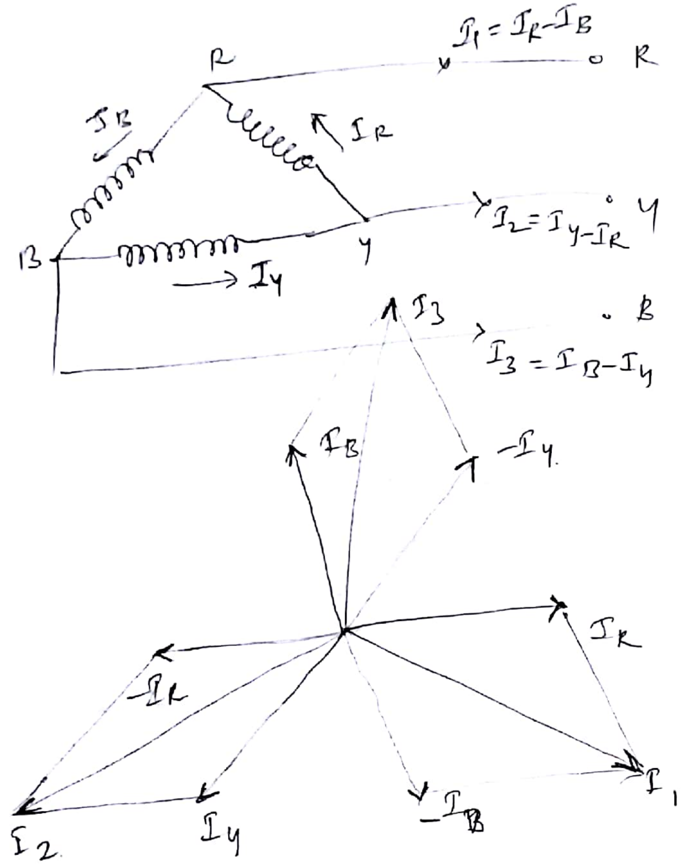
② Delta connected System :-



Since the system is balanced all the phase voltages are equal in magnitude, displaced by 120° and are equal to the line voltages



$\boxed{V_L = V_{ph} = V_{RY} = V_{YB} = V_{BR}}$



$|I_R| = |-I_B| = I_{ph}$

$I_1 = 2\, I_{ph} \times \cos 30°$

$$\boxed{I_1 = \sqrt{3}\, I_{ph}}$$

||ly $I_3 = \sqrt{3}\, I_{ph}$

$I_2 = \sqrt{3}\, I_{ph}$

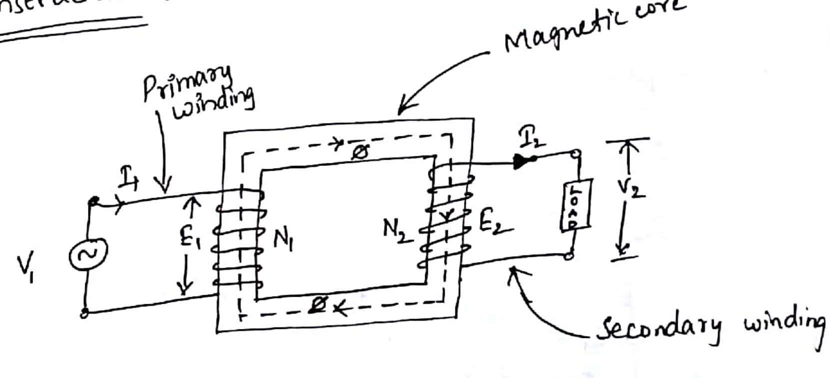
i.e In a balanced three phase delta connected system. $I_L = \sqrt{3}\, I_{ph}$.

UNIT-III TRANSFORMER

16/10/18

A transformer is a static device which converts the power from one level to another level without any change in the frequency.

Construction :-



→ A transformer consists of a magnetic core on which two windings namely primary and secondary windings with N_1 & N_2 terus respectively.

→ The primary winding is exited with an Alternating voltage V_1, alowing a primary current I_1 to flow through the primary winding.

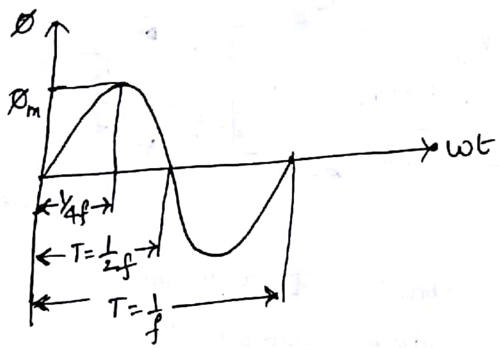
→ The change in current I_1 developes a magnetic flux Ø in the primary winding which in turn induces an emf E_1 in the primary.

→ The magnetic flux links with the secondary winding inducing an emf E_2. I_2 & V_2 are the load current and voltage.

Working principle :-

A transformer works on the principle of mutual Induction, which states that the change in current In a coil or winding induces an emf in its neighbouring coil, when the two coils are inductively coupled with each other.

EMF Equation :-



Average EMF per turn $= \frac{d\phi}{dt}$ $[\because N=1]$

$= \frac{\phi_m - 0}{1/4f} = 4f\phi_m$ VOLTS

Since the flux is alternating ~~flux~~, the emf induced will also be alternating in nature.

Therfore, for a sinusoidal wave form-factor

$FF = \frac{V_{RMS}}{V_{AVG}} = 1.11$

$\Rightarrow V_{RMS} = 1.11 \times V_{AVG}$

Therefore the RMS value of the emf induced

$= 1.11 \times 4f\phi_m$

$= 4.44 f\phi_m$ VOLTS

$\therefore$ EMF Induced in the primary winding

$\boxed{E_1 = N_1 \times 4.44 f\phi_m}$ VOLTS

Similarly $\boxed{E_2 = N_2 \times 4.44 f\phi_m}$ VOLTS

$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = K}$ VOLTAGE TRANSFORMATION RATIO

$\therefore E_2 = K \cdot E_1$ & $\frac{N_2}{N_1} = K$

(i) If $N_2 > N_1$, $K > 1$, $E_2 > E_1 \rightarrow$ 'STEP-UP' T/F

(ii) If $N_2 < N_1$; $K < 1$; $E_2 < E_1 \rightarrow$ 'STEP-DOWN' T/F

(iii) If $N_2 = N_1$; $K = 1$; $E_2 = E_1 \rightarrow$ 'Isolation' T/F

Ideal Transformer :- ~~no load~~ :

A transformer is said to be ideal if it satisfies the following conditions.

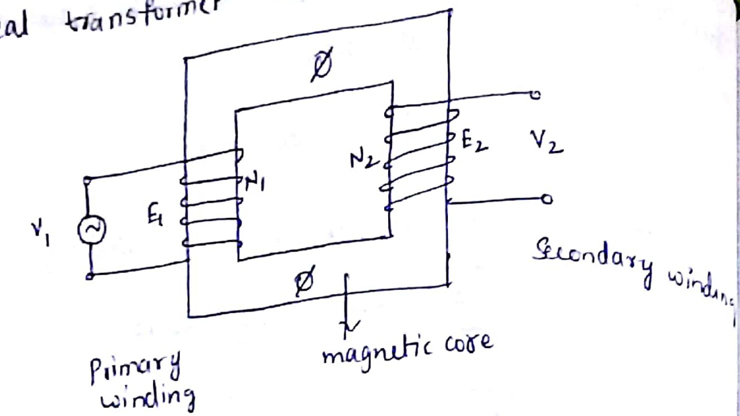
(i) No losses $E_1 = V_1$ & $E_2 = V_2$

(ii) winding resistance is zero. $V_1 I_1 = V_2 I_2$.

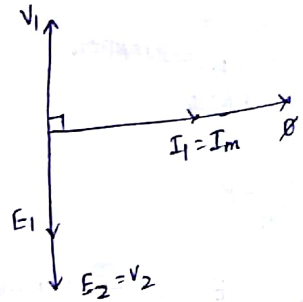
(iii) leakage flux is zero.

(iv) permeability of the magnetic core is high

Ideal transformer - no load.



Phasor diagram:-



since, the primary winding current I_1 is responsible for magnetising the core it is also known as magnetising current (I_m)

i.e $I_1 = I_m$

Since the transformer is ideal, the winding is purely inductive in nature and hence the primary current I_1 lags V_1 by 90°.

According to lenz law, the emf. Induced E_1 in the primary winding acts exactly opposite to the cause of producing it.

i.e E_1 i.e in anti phase with V_1 but equal in magnitude.

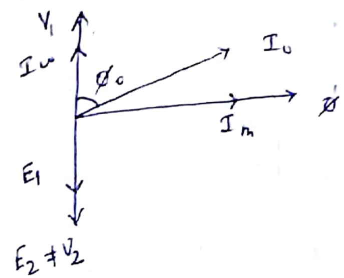
The secondary induced emf E_2 is also in anti phase with V_1 but the magnitude depends on N_2

Since there are no losses $E_2 = V_2$

E_1 and E_2 are in phase.

* Practical transformer - no load (same as previous diagram)

phase diagram:-



In a practical transformer with no load, the primary winding current (I_1) is termed as no load current (I_0).

I_0 is responsible to produce the losses in the transformer and to magnetise the core

∴ The no load current is split into the components: I_m - magnetising component

I_w - working component.

i.e $\vec{I_0} = \bar{I_m} + \bar{I_w}$

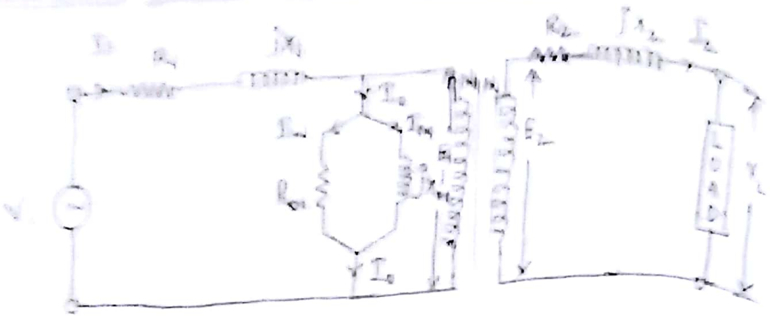
Since the winding has some resistance the no load current I_0 lags V_1 by an angle Ø (Ø<90°)

Since there are losses the primary induced emf E_1 will never be equal to V_1 and $|||^{ly}$ $E_2 \neq V_2$

$\cos Ø$ is known as no load power factor from the phasor diagram,

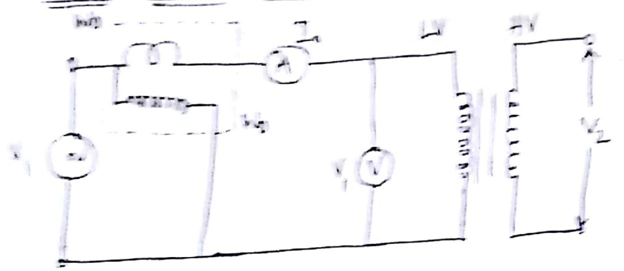
$$\boxed{\begin{aligned} I_m &= I_0 \sin Ø_0 \\ I_w &= I_0 \cos Ø_0 \end{aligned}}$$

Equivalent circuit of a Transformer :-



Open circuit test :- (or) No Load Test :-

To find Iron losses or Constant Losses



- In this test, the rated voltage corresponding to the primary side (LV) is applied as the input voltage.
- Watt meter reading (W_0) = Iron losses (P_i)
- Ammeter reading (I_0) = Primary no load current
- Voltmeter reading = Primary applied voltage V_1

Since $W_0 = V_1 I_0 \cos\phi_0$, the no load power factor ⇒ $\boxed{\cos\phi_0 = \frac{W_0}{V_1 I_0}}$

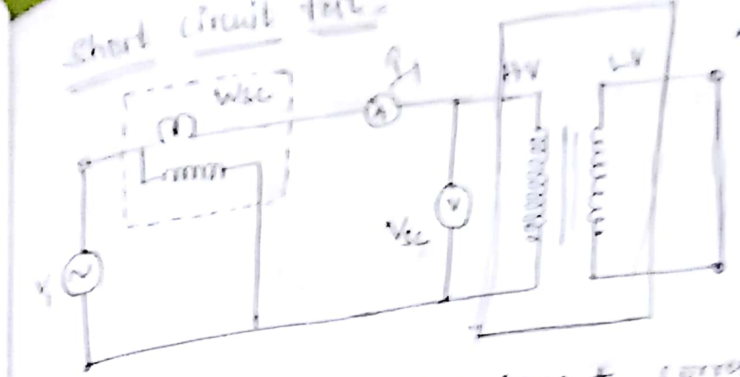
$I_m = I_0 \sin\phi_0$

$I_w = I_0 \cos\phi_0$

Short circuit test :-

To find copper losses

Copper Losses



In this test the rated current corresponding to the primary side (HV) is applied on the primary side.

- Watt meter reading (W_{sc}) = copper losses (P_c)
- Ammeter reading = Primary rated current (I_1)
- Voltmeter reading = Short circuit voltage (V_{sc})

Since $P_c = I_1^2 R_{01}$

$\rightarrow R_{01} = \frac{P_c}{I_1^2}$

Impedance referred to primary side

$Z_{01} = \frac{V_{sc}}{I_1}$

$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$

∴ Reactance of primary side $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$

Efficiency of a Transformer :-

The output of a transformer $= V_2 I_2 \cos\phi_2$

The input of the transformer = Output + losses

$$I/p = V_2 I_2 \cos\phi_2 + P_i + P_c$$

$$\therefore \text{ \% efficiency} = \frac{O/p}{I/p}$$

VA rating of T/F

$$= \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_c} \times 100$$

$$\text{\% Efficiency} = \frac{\text{VA Rating} \times \cos\phi_2}{\text{VA Rating} \times \cos\phi_2 + P_i + P_c} \times 100$$

let $n = \dfrac{\text{ACTUAL LOAD}}{\text{FULL LOAD}}$

$\therefore \; I_2 = n I_2$ and

$$P_c = (nI_2)^2 R_2$$

$$= n^2 I_2^2 R_2$$

$$P_c = n^2 P_c$$

$$\boxed{\therefore \text{ \% Efficiency} = \frac{n \times \text{VA RATING} \times \cos\phi_2}{n \times \text{VA RATING} \times \cos\phi_2 + P_i + P_c n^2} \times 100}$$

Condition for maximum efficiency :-

For max η. $\eta = \dfrac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_c}$

$$\frac{d\eta}{dI_2} = 0 \Rightarrow (V_2 I_2 \cos\phi_2 + P_i + P_c)\frac{d}{dI_2}(V_2 I_2 \cos\phi_2)$$

$$- V_2 I_2 \cos\phi_2 \frac{d}{dI_2}(V_2 I_2 \cos\phi_2 + P_i + P_c) = 0$$

$\Downarrow$ $I_2^2 R_2$

$$(V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_2)(V_2 \cos\phi_2) = V_2 I_2 \cos\phi_2 \times V_2 \cos\phi_2 + 2 I_2 R_2 \times V_2 \cos\phi_2$$

$$\cancel{V_2^2 \cos^2\phi_2 I_2} + P_i V_2 \cos\phi_2 + I_2^2 R_2 V_2 \cos\phi_2 = \cancel{V_2^2 \cos^2\phi_2 I_2} + 2 I_2 R_2 V_2 \cos\phi_2$$

$$P_i \cancel{V_2 \cos\phi_2} + I_2^2 R_2 \cancel{V_2 \cos\phi_2} = 2 I_2^2 R_2 \cancel{V_2 \cos\phi_2}$$

$$P_i + P_c = 2 I_2^2 R_2.$$

$$P_i + P_c = 2P_c$$

$$\boxed{P_i = P_c}$$

$\Rightarrow$ To obtain the maximum efficiency of transformer, the iron losses should be equal to copper losses.

Q. A 150 KVA transformer has iron losses of 3000 WATTS and full load copper losses of 800 W. Find the efficiency of the transformer at 75% full load

(i) for unity power factor

(ii) 0.8 power factor lagging.

$$n = 3/4$$

(i)

$$\eta = \frac{\frac{3}{4} \times 150000 \times 1}{\frac{3}{4} \times 150000 \times 1 + 3000 + 800 \times (3/4)^2} \times 100$$

$$= \cancel{\frac{1125}{1163} \times 100} \quad \cancel{0.961 \times 100} \quad 97.024\%$$

$$= \cancel{96.1\%}$$

(ii)

$$\eta = \frac{\frac{3}{4} \times 150000 \times 0.8}{\frac{3}{4} \times 150000 \times 0.8 + 3000 + 800 \times (3/4)^2} \times 100$$

$$= 96.30\%$$

Q. A 2KV transformer has iron losses 50 W and full load upper losses 200W, what will be efficiency at half of the full load and also determine its maximum efficiency.

$\cos\phi_2 = 0.8$

$$n = 1/2.$$

$$\eta = \frac{\frac{1}{2} \times 2000 \times 0.8}{\frac{1}{2} \times 2000 \times 0.8 + 50 + 200 \times (\frac{1}{2})^2} \times 100$$

$$= \frac{8}{9} \times 100$$

$$= 88.88\%$$

$$P_i = P_c$$

$$50 = 200n^2$$

$$\boxed{n = 1/2}$$

$\therefore$ max $\eta = 88.88\%$.

Q. A 4KVA, 200/400 Volts, 50Hz single phase transformer has the following test results.

OC Test - 200V, 0.8 A, 50W

SC Test - 17.5V, 9A, 50W

Calculate the full load efficiency at 0.8 power factor lagging and unity power factor.

$$\eta = \frac{1 \times 4000 \times 0.8}{4000 \times 0.8 + 50 + 50} \times 100$$

$$= 96.96\%$$

$$\eta = \frac{1 \times 4000}{4000 + 100} \times 100$$

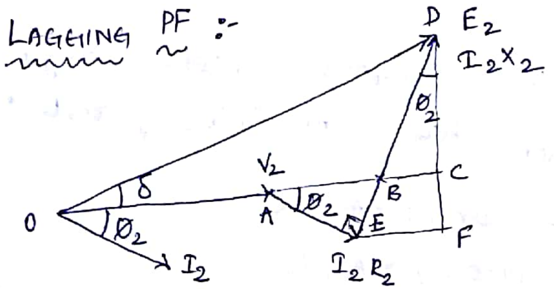
$$= 97.56\%$$

Voltage regulation of a Transformer :-

Voltage regulation is defined as the percentage of difference in noload and full load voltage of a transformer with respect to its full load voltage.

i.e, $\boxed{\%\text{ Regulation} = \frac{E_2 - V_2}{V_2} \times 100}$

→ For LAGGING PF :-



Let ϕ_2 is phase angle b/w the secondary voltage (V_2) and secondary current (I_2).

Since the angle (δ) b/w OD and OC is negligibly small

$$OD \simeq OC.$$

$$\therefore \quad E_2 = OC = OA + AB + BC$$

$$\Rightarrow E_2 = V_2 + I_2R_2\cos\phi_2 + I_2X_2\sin\phi_2$$

$$\therefore \%R = \frac{E_2 - V_2}{V_2} \times 100$$

$$= \frac{\cancel{V_2} + I_2R_2\cos\phi_2 + I_2X_2\sin\phi_2 - \cancel{V_2}}{V_2} \times 100$$

$$\boxed{\%R = \frac{I_2R_2\cos\phi_2 + I_2X_2\sin\phi_2}{V_2} \times 100}$$

Similarly for leading power factors,

$$\boxed{\%R = \frac{I_2R_2\cos\phi_2 - I_2X_2\sin\phi_2}{V_2} \times 100}$$

Voltage regulation corresponding to the primary side is expressed as

$$\boxed{\%R = \frac{\cancel{(I_1R_{01} + I_1X_{01})}\; I_1R_{01}\cos\phi_2 + I_1X_{01}\sin\phi_2}{V_1} \times 100}$$

Q. A 15KVA, 450/120 V (H.P / L.V) 50Hz transformer gave the following test results.

OC	120V	4.2A	80W.
SC	9.65V	22.2A	120W

Use only this SC result for voltage regul.

Find the voltage regulation at 0.8 PF lagging

$$R_{01} = \frac{P_c}{I_1^2} = \frac{120}{(22.2)^2} = 0.243\Omega \quad \cancel{0.535\Omega} \quad \cancel{0.802\Omega}$$

$$Z_{01} = \frac{V_{sc}}{I_1} = \frac{9.65}{22.2} = 0.4346\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = 0.359\ \Omega$$

$$I_1 = \frac{15 \times 1000}{450} = 33.3A.$$

$$\%R = \frac{I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi}{V_1} \times 100$$

$$= \frac{33.3 \times 0.24 \times 0.8 + 33.3 \times 0.36 \times 0.6}{450} \times 100$$

$$= 3.019\%.$$

Q. The primary and secondary winding resistances of 40KVA, 6600/250 V single phase transformer are 10 Ω and 0.02 Ω respectively. The equivalent leakage reactance as reffered to primary winding is 35 Ω. Find the full load voltage regulation for PF 0.8 lagging and leading.

$X_{01} = 35\,\Omega$

$$\boxed{R_{01} = R_1 + \frac{R_2}{K^2}}$$

$$K = \frac{V_2}{V_1} \text{ or } \frac{N_2}{N_1}$$

$$K = \cancel{\frac{6600}{25}}\ \frac{250}{6600}$$

$$= 0.037$$

$$R_{01} = 10 + \frac{0.02}{(0.037)^2}$$

$$= 24.609\,\Omega$$

$$I_1 = \frac{40 \times 1000}{6600} = 6.06A$$

for lagging

$$\%R = \frac{6.06 \times 24.6 \times 0.8 + 6.06 \times 35 \times 0.6}{6600} \times 100$$

$$= 3.735\%.$$

for leading

$$\%R = -0.1212\%.$$

Q. The following test data were taken on 110KVA 4400/440 V 50HZ transformer

OC Test : 440V, 2A, 1200W

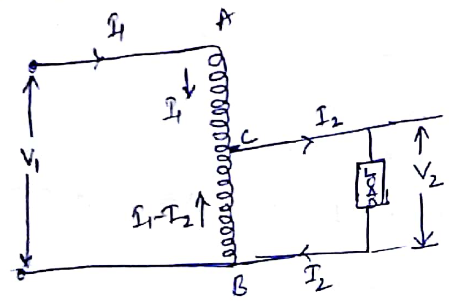
SC Test : 18V 200A, 2000W

Find the efficiency and voltage regulation at

i) unity power factor

(ii) 0.8 PF lagging

Auto Transformers :-



An auto transformer shares the common single winding as both primary and secondary.

AB → primary winding

BC → secondary winding

Let N_1 & N_2 are No. of turns of primary and secondary winding.

If V_1 is voltage applied across primary,

The voltage per turn $= \frac{V_1}{N_1}$

∴ The voltage across secondary

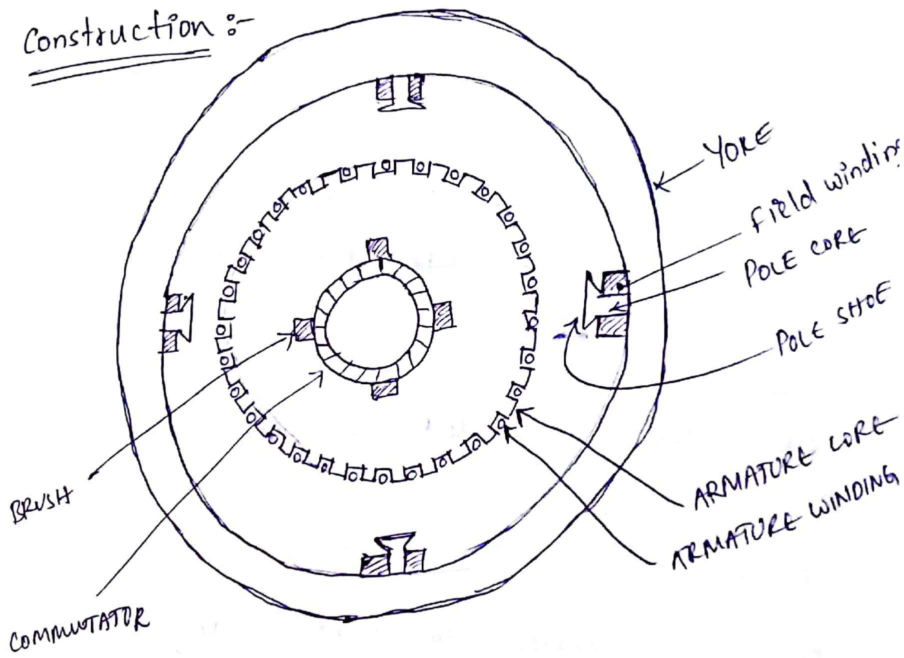
$$V_L = \frac{V_1}{N_1} \times N_2 = V_2$$

$$\Rightarrow \boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = K}$$

☆ UNIT-4 ☆

D.C – GENERATORS

Construction :-



→ The main parts of a dc generator are :

(i) YOKE
(ii) POLES
(iii) FIELD WINDING
(iv) ARMATURE
(v) COMMUTATOR
(vi) BRUSHES

YOKE :-

It serves the purpose of outermost cover of the dc generator so that the machine gets protected from moisture, dust and gases.

→ It also provides mechanical support to the poles.

→ It is made by cast Iron which provides a low reluctance path for a magnetic flux

POLE :-

Each pole is divided into two parts

i, Pole core

ii, POLE SHOE

→ Pole core carries the field winding which is used to produce the flux.

→ Pole shoe enlarges the area of armature core to come across the flux which produces the induced emf.

FIELD WINDING :-

The field winding is placed on the pole core which behaves as an electromagnet producing the necessary flux.

→ It is made up of a conducting material such as aluminium (or) copper.

Armature :-

Armature consists of

(i) Armature core

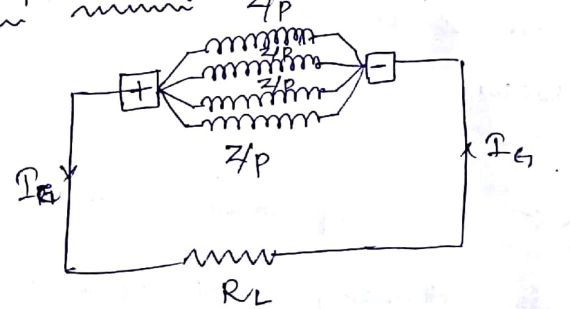
(ii) Armature winding

→ Armature core is cylindrical in shape consisting of slots on its periphery and air ~~bags~~ ducts which permits air flow through armature for cooling purpose. It is made up of cast Iron (or) cast steel.

→ The armature winding is an inter connection of armature conductors placed in the slots provided on the armature core periphery. It is made up of copper and the generation of emf takes place in it.

→ The armature winding is of two types

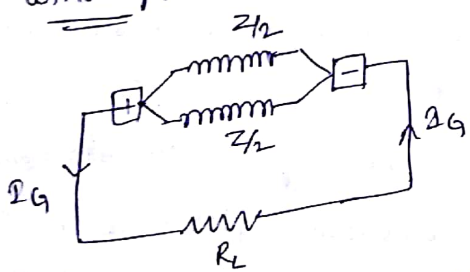
(i) Lap winding.



In this type the armature coils / conductors are connected in series through commutator segments in such a way that the armature winding is divided into as many parallel paths as the no. of poles of the machine.

→ If there are 'Z' conductors and 'P' poles, then there will be 'P' no. of parallel paths each containing Z/P conductors in series.

Wave winding :-



In this type the armature conductors are connected in series through commutator segments in such a way that the armature winding is divided into two parallel paths irrespective of the no. of poles of the machine.

→ If there are 'Z' conductors then their will be Z/2 conductors in series in each parallel path.

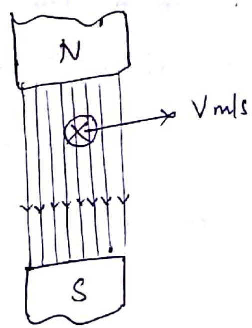
COMMUTATOR :- The emf induced in the armature winding is alternating, which needs rectification into direct emf, which is done by the commutator

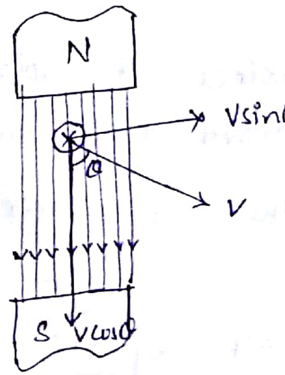
BRUSHES :-

These are used to collect the emf from the commutator segments and is made available to the stationary external loads. These are made of copper.

WORKING PRINCIPLE :-

The working principle of a dc generator is based on the dynamically induced emf or, Faraday's law of electromagnetic induction, which states that "when the flux linking the conductor or coil changes an emf is induced in it".





Consider a conductor of length 'l' mts moving in a uniform magnetic field with a velocity of v m/s.

→ let the conductor moves through a distance of dx in dt secs, the area swept the conductor is equal to $l dx$.

→ Therefore the magnetic flux cut by the conductor $= B \cdot l dx$

where B = magnetic flux density.

→ Therefore, according to faraday's law

$$e = N \frac{d\phi}{dt}$$

$$e = \frac{B l dx}{dt} \qquad \because N = 1, \ d\phi = B l dx$$

$$\Rightarrow e = B l \frac{dx}{dt}$$

$$\Rightarrow e = B l v \text{ volts}$$

Since the velocity with which the conductor moves across the magnetic field is $v \sin\theta$, the emf induced in the conductor $= B l v \sin\theta$

$$\boxed{e = B l v \sin\theta} \text{ volts}$$

EMF Equation:-

let ϕ = magnetic flux per pole in webres.

Z = total no.of armature conductors.

P = No.of poles of the generator

A = No.of parallel paths.

N = Speed of armature in Rpm.

The magnetic flux cut by one conductor in one revolution of the armature

$$d\phi = P \times \phi \quad Wb$$

Time taken for one revolution (dt) $= \frac{60}{N}$ sec.

Therefore the emf induced in one conductor in one revolution $e = \overset{1}{N} \frac{d\phi}{dt}$

$$e = \frac{P \times \phi}{60/N}$$

$$\Rightarrow \boxed{e = \frac{P \phi N}{60} \text{ VOLTS}}$$

→ The total emf of the generator = emf per conductor X no.of conductors in series per parallel path.

ie $$E_g = \frac{P \times \phi \times N}{60} \times \frac{Z}{A}$$

$$\boxed{E_g = \frac{\phi Z N P}{60A} \text{ VOLTS}}$$

For LAP WINDING,

A = P

$$\therefore \boxed{E_g = \frac{\phi Z N}{60}} \text{ VOLTS}$$

FOR WAVE WINDING, $A=2$

$$\boxed{E_g = \frac{\phi Z N P}{120}} \text{ VOLTS}$$

Q. Calculate the emf generated by a four pole wave wound DC generator having 65 slots with 12 conductors per slot when driven at 1200 rpm. The flux per pole is 0·02 wb

$Z = 65 \times 12$ $\qquad \phi = 0.02$

$P = 4$

$N = 1200$ rmp

$A = 2$

$$E_g = \frac{0.02 \times 65 \times 12 \times 1200 \times 4}{60 \times 2}$$

$$= 624 \text{ VOLTS}$$

Q. An 8 pole wave connected armature has 800 conductors and the flux per pole is 0·04 wb. At what speed it must be driven to generate 400 volts.

$Z = 800 \times 8$ $\qquad \phi = 0.04$

$P = 8$ $\qquad E = 400$

$N = ?$

$A = 2$

$$400 = \frac{0.04 \times 800 \times 8 \times N \times 8}{60 \times 2}$$

$$\boxed{N = 187.5 \text{ rpm}}$$

Q. A six pole lap wound DC generator has 600 conductors on its armature. The flux per pole is 0·02 wb. Calculate

(i) The speed at which the generator must run to generate 300 volts

(ii) what would be the speed of the generator if it is wave wound.

(i) $E_g = \frac{\phi Z N}{60} \Rightarrow 300 = \frac{0.02 \times 600 \times 6 \times 6}{60\,30}$

$$\boxed{N = 1500 \text{ rpm}}$$

(ii) $E_g = \frac{\phi Z N P}{120} \Rightarrow E = \frac{0.02 \times 600 \times N \times 6}{120\ 20\ 10} = 300$

$$\boxed{N = 500 \text{ rpm}}$$

Q. An 8 pole lap wound armature rotated at 350 rpm is required to generate 260 volts. The magnetic flux per pole is 0·05 wb. If the armature has 120 slots find the no. of conductors per slot.

$Z = 120 \times x$ $\qquad \phi = 0.05$

$P = 8$ $\qquad E = 260$ volts

$N = 350$

$$\cancel{260}\ 120 = \frac{0.05 \times \cancel{120}x \times 8 \times 350}{\cancel{60} \times 8}$$

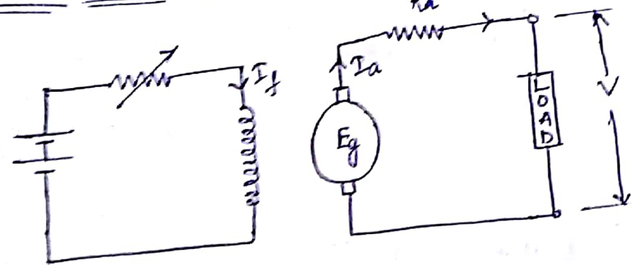
$x = 7.4$

$\Rightarrow$ 7 conductors

Types of DC generators :-

- DC GENERATOR
  - SEPERATELY EXCITED
  - SELF EXCITED
    - SERIES
    - SHUNT
    - COMPOUND
      - CUMMULATIVE
      - DIFFERENTIAL

Seperately excited DC-generator :-



→ The DC generator whose field winding is excited from an independent external DC source is called Seperately excited dc-generator.

→ Armature current $\boxed{I_a = I_L}$.

→ Terminal voltage $\boxed{V = E_g - I_a R_a}$

→ Power developed by the generator $\boxed{P_g = E_g \times I_a}$

$$P_g = (V + I_a R_a) I_a.$$

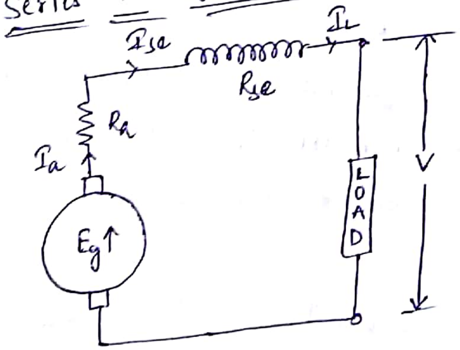
→ Power supplied to the load $P_L = V I_a$ (or) $V I_L$

$$P_L = (E_g - I_a R_a) I_a$$

SELF EXCITED DC GENERATOR :-

The DC generator whose field winding is excited by the current from the output of the generator itself, is called as self excited DC-generator.

(1) Series DC - generator :-



→ In a series dc-generator the field winding is connected in series with the armature winding so that the whole armature current flows through the field winding as well as the load.

$$I_a = I_{se} = I_L$$

$$V = E_g - I_a R_a - I_{se} R_{se}$$

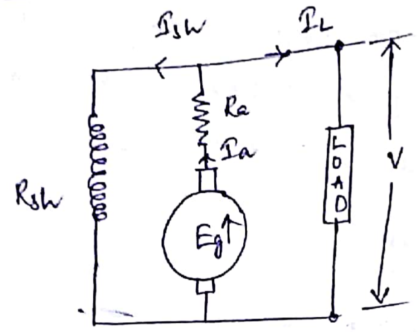
$$= E_g - I_a (R_a + R_{se})$$

$$P_g = E_g I_a = E_g [V + I_a (R_a + R_{se})] I_a$$

$P_L = VI_L$

$= [E_g - I_a(R_a + R_{se})] I_L$

② **SHUNT DC generator :-**



In a DC-shunt generator the field winding is connected in parallel with the armature winding so that the terminal voltage is supplied across it.

$I_a = I_{sh} + I_L$, $I_{sh} = \frac{V}{R_{sh}}$

$V = E_g - I_a R_a = I_{sh} R_{sh}$

$P_g = E_g I_a = (V + I_a R_a) I_a$

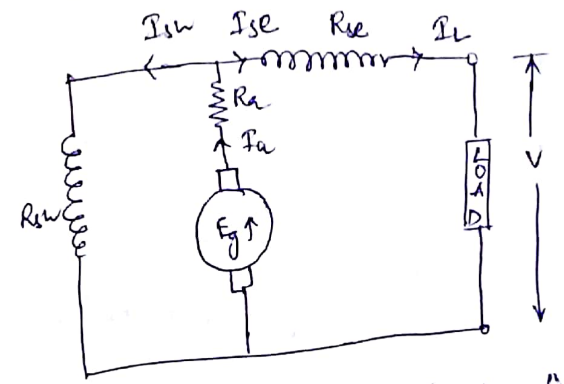
$= \cancel{E_{sh}}$

$P_L = VI_L = (E_g - I_a R_a) I_L = I_{sh} R_{sh} I_L$

③ **COMPOUND GENERATOR :-**

In a DC-compound generator there are two sets of field winding on each pole one in series and the other in parallel with the armature winding.

(i) **Cummulative compound DC generator :-**



In this type only shunt field winding is in parallel with the armature

$I_a = I_{sh} + I_{se}$, $I_{sh} = \frac{V + I_{se} R_{se}}{R_{sh}}$

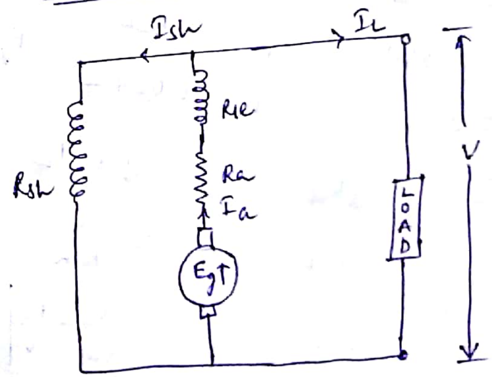
$I_{se} = I_L$

$V = E_g - I_a R_a - I_{se} R_{se} = I_{sh} R_{sh}$

$P_g = E_g I_a = (V + I_a R_a + I_{se} R_{se}) I_a$

$P_L = V I_L = (E_g - I_a R_a - I_{se} R_{se}) I_L$

(ii) **Differential compound DC generator :-**



In this type the shunt field winding is in parallel with both series field and armature

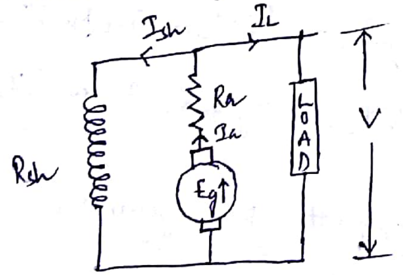
$$I_a = I_{se} = I_{sh} + I_L \quad , \quad I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_g - I_a(R_a + R_{se})$$

$$P_g = E_g I_a = [V + I_a(R_a + R_{se})] I_a$$

$$P_L = V I_L = [E_g - I_a(R_a + R_{se})] I_L.$$

Q. A 100KW 240V DC shunt generator has field resistance of 55Ω and armature resistance of 0.067 Ω. Find full load generated voltage.



$P_L = 100KW$ $\quad R_{sh} = 55\Omega$ $\quad E_g = ?$

$V = 240V$ $\quad R_a = 0.067\,\Omega$

$V = E_g - I_a R_a$ $\quad I_a = I_{sh} + I_L$

$\cancel{V=} E_g = V + I_a R_a$ $\quad I_{sh} = \frac{V}{R_{sh}}$

$= 240 + 28.2$ $\quad \boxed{I_a = \frac{V}{R_{sh}} + \frac{P_L}{V}}$

$\boxed{E_g = 268.2\,V}$ $\quad I_a = 421\,A$

Q. A 30KW 300V DC shunt generator has armature and field resistances of 0.05 Ω and 100 Ω resp. Calculate the total power developed by the armature when it delivers full load output.

$R_a = 0.05$ $\quad R_{sh} = 100$

$P_L = 30KW$ $\quad V = 300$

$$P_g = E_g I_a$$

$$I_a = \frac{V}{R_{sh}} + \frac{P_L}{V} = 103$$

$$E_g = V + I_a R_a$$

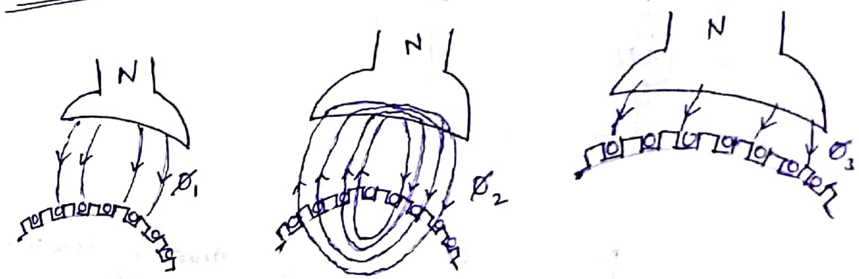
$$= 300 + 103 \times 0.05$$

$$E_g = \cancel{330.45}\ 305.15$$

$$P_g = \cancel{330.45}\ 305.15 \times 103$$

$$= \cancel{855}\ 31430.45\ W$$

ARMATURE REACTION :-



In a dc machine the current flowing through armature conductors creates a magnetic flux known as Armature flux which distorts and weakens the magnetic flux (main flux) from the poles. This action of armature flux over the main flux is known as Armature reaction.

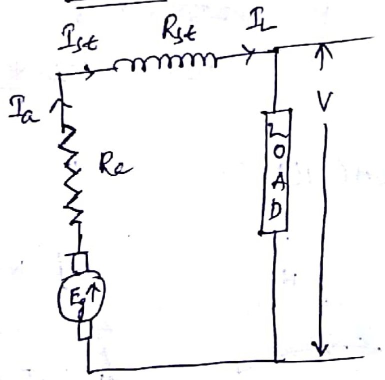
DC generator Characteristics :-

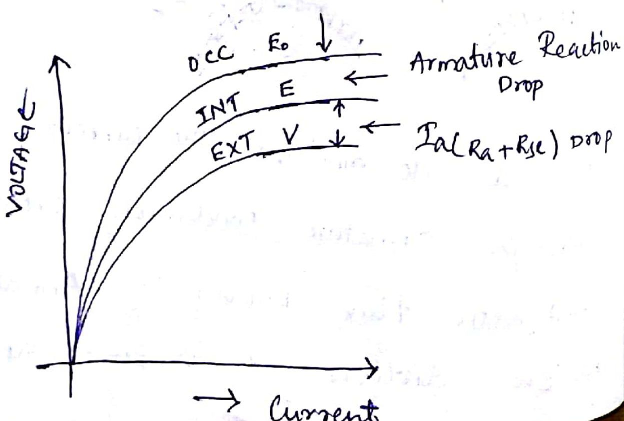
(i) Open ckt characteristics (E_0 vs I_f)

(ii) Internal characteristics (E vs I_a)

(iii) External characteristics (V vs I_a)

① Series generator characteristics :-





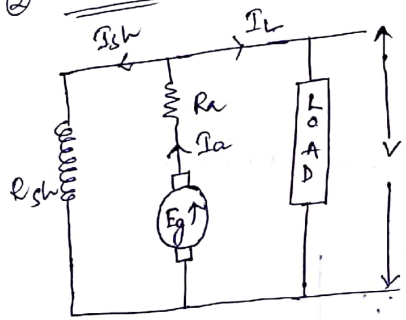
→ OCC is obtained by disconnecting the field winding and exciting the machine from a separate dc source.

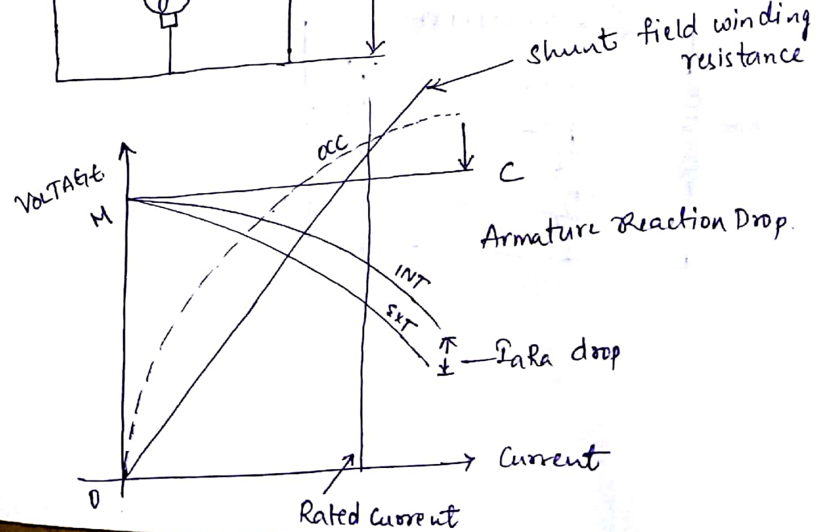
→ The emf generated (E) under load conditions will be less than Emf (E_0) under no load conditions due to the armature reaction.

Therefore the internal characteristic lies below OCC.

→ Since $V = E - I_aR_a - I_aR_{se}$, the external characteristic lies below the internal by an amount equal to $I_a(R_a+R_{se})$, known as ohmic drop.

② SHUNT generator characteristics :-





→ When the generator is run at normal speed it will build up voltage OM which will be constant represented by line MC.

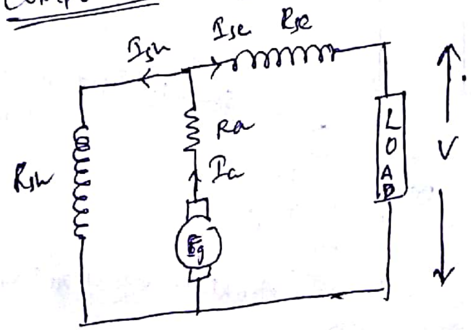
The OCC of a shunt generator is similar to that of series generator

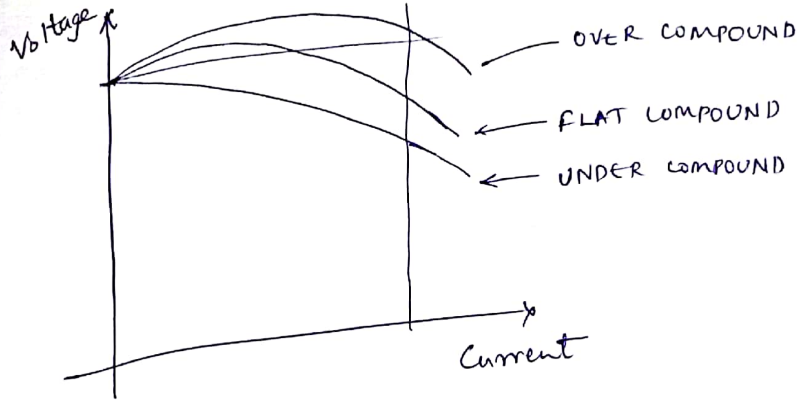
→ When the generator is loaded the flux per pole is reduced due to the armature reaction and hence the emf generated E on Load is less that the emf generated at no load.

(1)

→ Since $V = E_g - I_a R_a$ the external characteristic lies below the internal by an amount equal to $I_a R_a$ drop.

(3) COMPOUND generator characteristics :-





Star Delta Connection

$S = \frac{70.7 \times 70.7}{509.9} = 9.802$ volt-Ampere

$V_R \propto I$

$V_R = I R$

$\frac{70.7}{509.9} \times 500 = 69.327$

Find current at 2 seconds

$I = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = \sqrt{2} I$

$= \sqrt{2} \times \frac{70.7}{509.9}$

$i = \frac{100}{509.9} \cdot \sin(1000t + 41.309) = -0.172037$ Amp

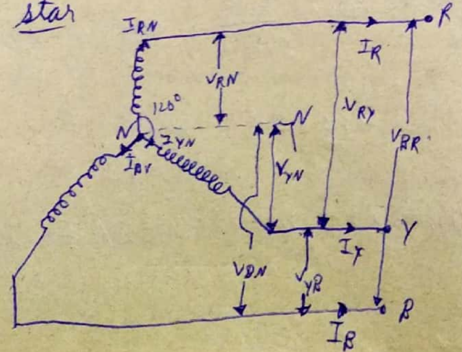
For parallel

for capacitor $z = -jX_C$

For inductor $z = +jX_L$

Three Phase Circuits :

star



V_{RN}, V_{YN}, V_{BN} are phase voltages ✓

V_{RY}, V_{YB}, V_{BR} are line voltages ✓

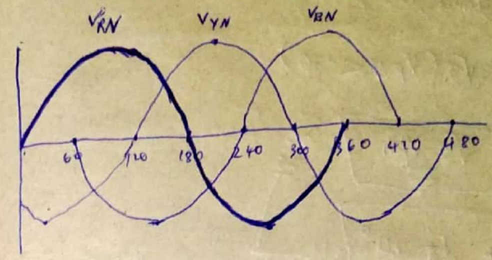
$V_{RN} = V_m \sin\omega t. = V\angle 0$

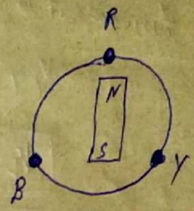
$V_{YN} = V_m \sin(\omega t + 120) = V\angle -120$

$V_{BN} = V_m \sin(\omega t - 120) = V\angle 120$ (or) $V\angle -240$

I_{RN}, I_{YN}, I_{BN} are phase currents.

I_R, I_Y, I_B are line currents.





Apply KCL across load 'a'

$\boxed{I_{RN} = I_R}$ → Line current will be equal to phase current.

Here all the 'I' taken are RMS values of currents.

Apply KVL across V_{YN}, V_{RN}, V_{R-Y}

$-V_{RN} + V_{RY} + V_{YN} = 0$

$\boxed{V_{RY} = V_{RN} - V_{YN}}$

$V_{RY} = V_m \sin\omega t - V_m \sin(\omega t - 120)$

$= V_m(\sin\omega t - \sin(\omega t - 120))$

$= V_m \times 2\cos\left(\frac{\omega t + (\omega t - 120)}{2}\right)\sin\left(\frac{\omega t - (\omega t - 120)}{2}\right)$

$\Rightarrow V_m \times 2\cos(\omega t - 60)\sin 60$

$\Rightarrow V_m \times 2 \times \cos(\omega t - 60) \times \frac{\sqrt{3}}{2}$

$\Rightarrow \sqrt{3} V_m \cos(\omega t - 60)$

$= \sqrt{3} V_m \cos(\omega t - 60 + 90 - 90)$

~~$V_m(1\angle 0)$~~ $V_{RY} = \sqrt{3}\, V_m \sin(\omega t + 30)$

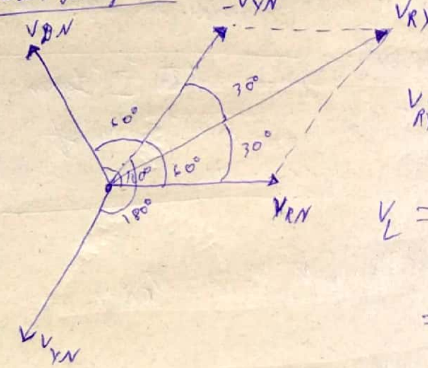
Line current is $\sqrt{3}$ times the phase current

$$V_{RY} = \sqrt{3}(V_{RN} \angle +30)$$

$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$

$\boxed{V_L \text{ leads } V_{ph} \text{ by } 30^\circ}$

Phasor Diagrams $\boxed{V_{RY}}$



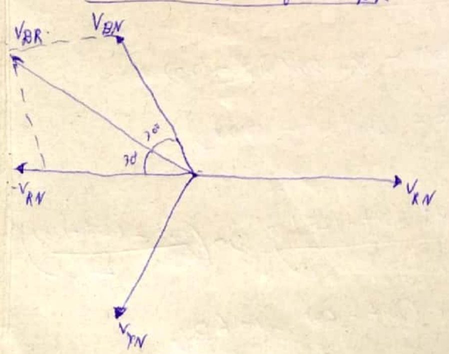
$$V_{RY} = \sqrt{(V_{RN})^2 + (V_{YN})^2 + 2 V_{RN} V_{YN} \cos 60^\circ}$$

$$V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2 \times V_{RN} \cdot \frac{V_{YN}}{2}}$$

$$\Rightarrow \sqrt{V_{ph}^2 + V_{ph}^2 + V_{ph}^2}$$

$$= \sqrt{3}\, V_{ph}$$

Phasor Diagram for V_{BR}

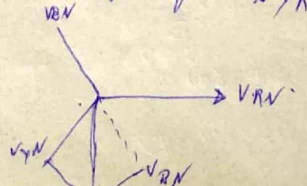


$$V_{BR} = \sqrt{(V_{BN})^2 + (V_{RN})^2 + 2 V_{BN} V_{RN} \cos 60}$$

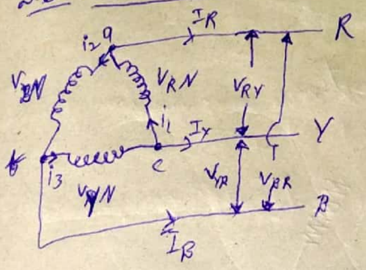
$$= \sqrt{}$$

$$= \sqrt{3}\, V_{ph}$$

Phasor Diagram for V_{YR}



Delta Conversion:



V_{RN}, V_{YN}, V_{BN} → phase voltages

i_1, i_2, i_3 are phase currents

V_{RY}, V_{YB}, V_{BR} are line voltages

I_R, I_Y, I_B are line currents

$V_{RN} = V_m \sin \omega t$

$V_{YN} = V_m \sin(\omega t - 120)$

$V_{BN} = V_m \sin(\omega t - 240)$

$i_1 = I_m \sin \omega t$

$i_2 = I_m \sin(\omega - 120^\circ)$

$i_3 = I_m \sin(\omega t - 240^\circ)$

Applying KVL

$$-V_{RN} + V_{RY} = 0$$

$$V_{RY} = V_{RN}$$

$$\boxed{V_L = V_{ph}}$$

Applying KCL at node a

$$i_1 = I_R + i_2$$

$$I_R = i_1 - i_2$$

$$I_m \sin \omega t - I_m \sin(\omega t - 240)$$

$$= I_m \cdot 2\cos\left(\frac{\omega t + \omega t - 240}{2}\right) \cdot \sin\left(\frac{\omega t - (\omega t - 240)}{2}\right)$$

$$\Rightarrow I_m\, 2\cos(\omega t - 120) \sin(120)$$

$$\Rightarrow \frac{\sqrt{3}}{2} \cdot 2 \cdot I_m \cos(\omega t - 120)$$

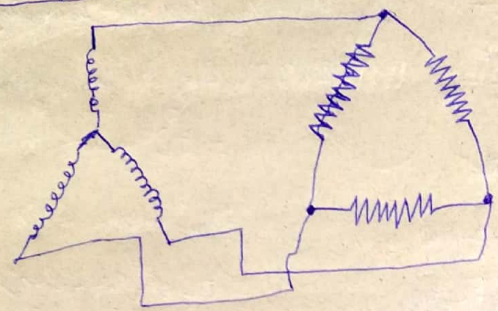
$$i_1 \Rightarrow \sqrt{3} \cdot I_m \cos(\omega t - 120)$$

$$i_1 \Rightarrow \sqrt{3}\, I_m \sin(\omega t - 30^\circ) \rightarrow x$$

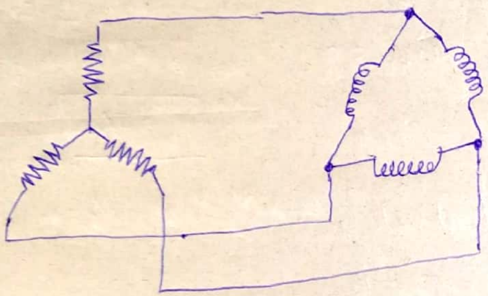
$$\boxed{i_1 = \sqrt{3}\, I_m \sin(\omega t - 30^\circ)}$$

line current lags phase current by 30°

Phasor Diagrams : +30° connection



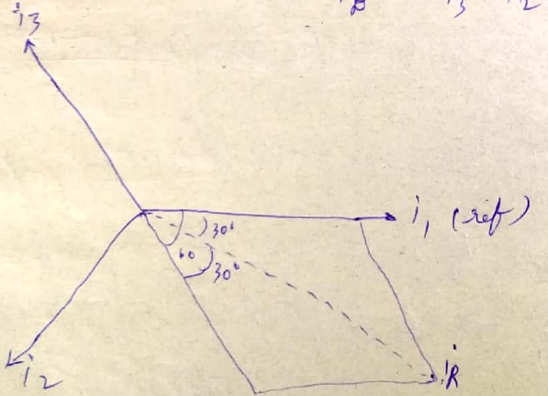
−30° connection



$i_1 = I\angle 0$ $\quad$ $i_R = i_1 - i_3$

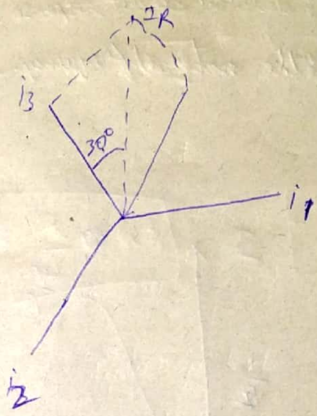
$i_2 = I\angle -120$ $\quad$ $i_Y = i_2 - i_1$

$i_3 = I\angle -240$ $\quad$ $i_B = i_3 - i_2$



$I_R = \sqrt{i_1^2 + i_3^2 + 2 i_1 i_3 \cos(60)}$

$= \sqrt{I_{ph}^2 + I_{ph}^2 + I_{ph}^2} \Rightarrow I_L = \sqrt{3}\, I_{ph}$



For i_2 and i_3.

---

3 phase power = $\sqrt{3} V_L I_L \cos\phi$

per.

$i_{3-\phi} = 3 P_{1-\phi}$

$= 3 (V_{ph})(I_{ph}) \cos\phi$

$= 3 \times \frac{V_L}{\sqrt{3}} \cdot I_L \cos\phi \Rightarrow \sqrt{3} I_L V_L \cos\phi$

For star:

$3\text{-}\phi = 3 Q_{1\text{-}\phi}$

$3 V I \sin\phi$

$= \sqrt{3} \cdot V_L I_L \cdot \sin\phi$

Delta:

$V_L = V_{ph}$ / $I_L = \sqrt{3} I_{ph}$

$I_{ph} = I_L / \sqrt{3}$

Subtract the phase angle of the R-phase from all the phase angles

$\angle V_R{}' = 27° - 27° = 0°$

$\angle V_Y{}' = 27° - 147° = -120°$

$\angle V_B{}' = 27° + 93° = 120°$

Therefore, the phase sequence is 'RYB'

(ii) First, convert the cosine waveforms into phasors

$v_{an} = 200 \cos(\omega t + 10°) \Rightarrow V_{an} = 200\angle 10°$

$v_{bn} = 200 \cos(\omega t - 230°) \Rightarrow V_{an} = 200\angle -230°$

$v_{cn} = 200 \cos(\omega t - 110°) \Rightarrow V_{an} = 200\angle -110°$

Subtract the phase angle of the a-phase from all the phase angles

$\angle V_{an}{}' = 10° - 10° = 0°$

$\angle V_{bn}{}' = 10° + 230° = 240°$

$\angle V_{cn}{}' = 10° + 110° = 120°$

Therefore, the phase sequence is 'abc'

4.5 Three Phase Balanced System Connections

4.5.1. Star Connection

When three similar polarity terminals of three sources are connected at a single point as shown in Figure (4.5) it is known as a star connection. The point of connection is known as source neutral and represented by '*N*'. Similarly, three loads are joined at a single point known as load neutral and represented by '*n*'. The wire joining source neutral point '*N*' and load neutral point '*n*' is known as neutral wire or simply neutral. After connecting three similar terminals at the neutral point, the remaining three free terminals of the source are joined to the load by means of individual condcutors known as lines R, Y and B as shown in figure (4.5). Three lines carry the currents from the source to the load and neutral carries all the return currents from load to source. Therefore neutral current, I_N is the sum of three currents.

$$\therefore I_N = I_R + I_Y + I_B \tag{4.4}$$

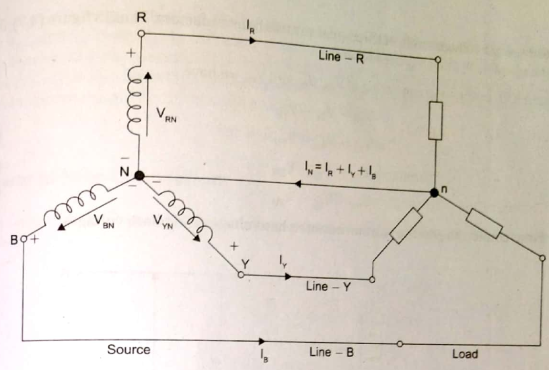


Figure (4.5): *Star connection*

In 3-phase balanced system if the loads in all the 3-phases are balanced, then three currents I_R, I_Y, I_B constitute balanced system of currents.

$$\therefore\ I_R = I\angle 0°,\ I_Y = I\angle -120°, I_B = I\angle 120°$$

$$I_N = I_R + I_Y + I_B = I\angle 0° + I\angle -120° + I\angle 120° = 0$$

From the above calculations, it is clear that neutral current I_N is zero. Hence, in a balanced system neutral wire is not required. Such a system is known as 3-phase 3-wire system.

Line Voltages and Phase Voltages: Phase voltage is a voltage measured between any line conductor and the neutral.Thus, in figure (4.6), three phase voltages V_{RN}, V_{YN} and V_{BN} are shown.

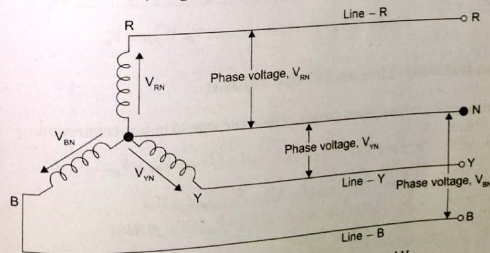


Figure (4.6): *Three phase voltages V_{RN}, V_{YN} and V_{BN}*

Line voltage is a voltage measured between any two line conductors. Thus in figure (4.7), three line voltages V_{RY}, V_{YB} and V_{BR} are shown.

Applying KVL to the loop consisting of V_{RN}, V_{RY} and V_{YN} we have

$$V_{RN} - V_{RY} - V_{YN} = 0$$

$$\Rightarrow V_{RY} = V_{RN} - V_{YN} \quad (4.5)$$

$$\text{Similarly, } V_{YB} = V_{YN} - V_{BN} \quad (4.6)$$

$$V_{BR} = V_{BN} - V_{RN} \quad (4.7)$$

The above equations give the relation between line voltages and phase voltages.

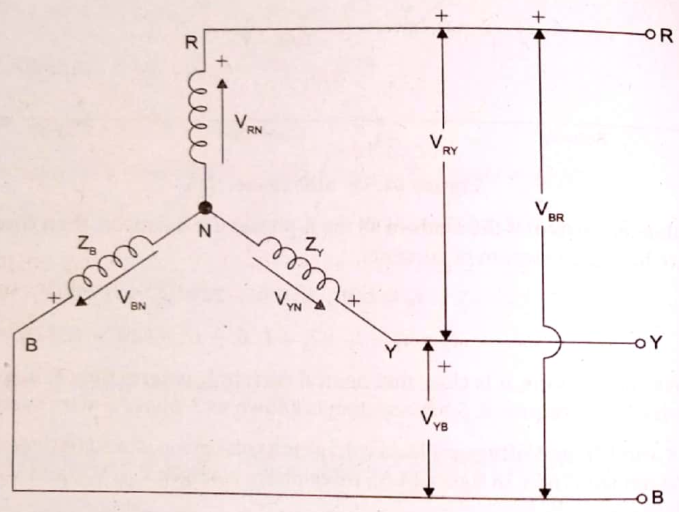


Figure (4.7): *Three line voltages V_{RY}, V_{YB} and V_{BR}*

Relation between Line and Phase Values:

Let $V_{RN} = V_{YN} = V_{BN}$ be the phase voltage of balanced system in star connection.

$$\therefore V_{RN} = V_{ph}\angle 0^\circ, \quad V_{YN} = V_{ph}\angle -120^\circ, \quad V_{BN} = V_{ph}\angle +120^\circ$$

$$\begin{aligned}\therefore V_{RY} &= V_L = V_{RN} - V_{YN} = V_{ph}\angle 0^\circ - V_{ph}\angle -120^\circ \\ &= V_{ph}[1\angle 0^\circ - 1\angle -120^\circ] = V_{ph}[1.5 + j0.866] \\ &= 1.732\angle 30^\circ \times V_{ph}\end{aligned}$$

$$\Rightarrow V_L = \sqrt{3}V_{ph} \quad (4.8)$$

Therefore, the magnitude of the line voltage, V_{RY} is $\sqrt{3}$ times the magnitude of the phase voltage V_{RN}, and V_{RY} leads V_{RN} by 30°. Same is the case with the other two line voltages. From the star connected 3-phase system, it is clearly observed that whatever current flows through the lines R, Y and B also flows through the respective phase windings or coils. Hence in a star connected system the phase currents and line currents are equal.

$$\text{For star connected } I_L = I_{ph} \quad (4.9)$$

Power in Star-Connection:

In a balanced star connected system, the power in each phase is same. Therefore 3-phase power

$$P = 3 \times \textit{Per phase power} = 3 \times V_{ph} I_{ph} \cos\phi$$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi = \sqrt{3} V_L I_L \cos\phi$$

$$P = \sqrt{3} V_L I_L \cos\phi \quad (4.10)$$

4.5.2 Delta or Mesh Connection

In a delta or mesh connection, the dissimilar ends of the 3-phase windings are joined together i.e. the starting end of one phase is joined to the finishing end of other phase and so on as shown in figure(4.8).

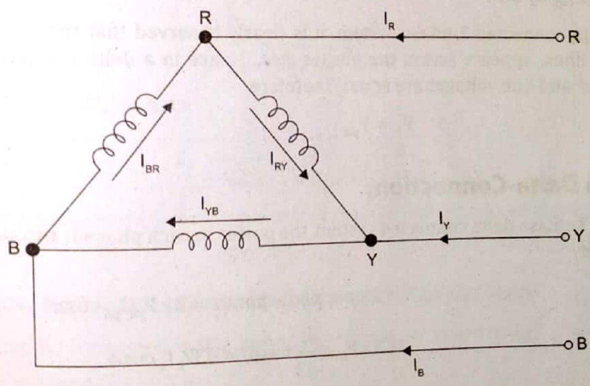


Figure (4.8): *Delta connection*

In the above figure, I_{RY}, I_{YB} and I_{BR} represents phase values of currents and I_R, I_Y and I_B represents line values of currents.

From the above figure (4.8), by applying KCL at node R

$$-I_{BR} - I_R + I_{RY} = 0$$

$$\Rightarrow I_R = I_{RY} - I_{BR} \tag{4.11}$$

$$\text{Similarly } I_Y = I_{YB} - I_{RY} \tag{4.12}$$

$$I_B = I_{BR} - I_{YB} \tag{4.13}$$

Relation Between Line and Phase Values:

Let $I_{RY} = I_{YB} = I_{BR} = I_{ph}$ be the phase current of balanced system in delta connection.

$$\therefore I_{RY} = I_{ph}\angle 0°, \quad I_{YB} = I_{ph}\angle -120°, \quad I_{BR} = I_{ph}\angle +120°$$

Line current flowing through the R-phase

$$I_R = I_L = I_{RY} - V_{BR} = I_{ph}\angle 0° - I_{ph}\angle 120°$$

$$= I_{ph}[1\angle 0° - 1\angle 120°] = I_{ph}[1.5 - j0.866]$$

$$= 1.732\angle -30° \times I_{ph}$$

$$\therefore \quad I_L = \sqrt{3} I_{ph} \tag{4.14}$$

From the above equation, it is clear that the line current I_L is $\sqrt{3}$ times the phase current and current I_R lags I_{RY} by 30°.

From the delta connected 3-phase system, it is clearly observed that the voltage that exists between the lines, appears across the phases also.. Hence in a delta connected system the phase voltages and line voltages are equal. Therefore

$$V_L = V_{ph} \tag{4.15}$$

Power in a Delta-Connection:

In a balanced 3-phase delta connected system, the power in each phase is the same. Therefore 3-phase power

$$P = 3 \times per\ phase\ power = 3 \times V_{ph} I_{ph} \cos\phi$$

$$= 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi = \sqrt{3} V_L I_L \cos\phi$$

$$P = \sqrt{3} V_L I_L \cos\phi \tag{4.16}$$

From eqns. (4.10) and (4.16), it is clear that irrespective of star connection or delta connection

$$\text{3-phase power, } P = \sqrt{3} V_L I_L \cos\phi \tag{4.17}$$

4.5.3 Balanced Star/Delta and Delta/Star Conversions

Any balanced star connected system can be converted into an equivalent delta connected system and vice versa.

For a balanced star connected load, let

Line voltage = V_L
Line current = I_L
Impedance per phase = Z_Y

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$Z_Y = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\sqrt{3} I_L} \tag{4.18}$$

For an equivalent delta connected system, the line voltages and currents must have the same values as in the star connected system, i.e.,

Line voltage = V_L
Line current = I_L
Impedance per phase = Z_Δ

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$Z_\Delta = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{I_L / \sqrt{3}} = \frac{\sqrt{3} V_L}{I_L} \tag{4.19}$$

From eqns. (4.18) & (4.19), we have

$$Z_Y = \frac{V_L}{\sqrt{3} I_L} = \frac{Z_\Delta}{\sqrt{3} \times \sqrt{3}} = \frac{Z_\Delta}{3} \tag{4.20}$$

Thus, when three equal impedances are connected in delta, the equivalent star impedance is one third of the delta impedance.

4.5.4 Relation between power in Delta and Star systems

Let a balanced load be connected in star having impedance per phase as Z_{ph}.

For a star connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{\sqrt{3} Z_{ph}}$$

$$I_L = I_{ph} = \frac{V_L}{\sqrt{3}Z_{ph}}$$

Now $P_Y = \sqrt{3}V_L I_L \cos\phi = \sqrt{3}V_L \times \frac{V_L}{\sqrt{3}Z_{ph}} \times \cos\phi = \frac{V_L^2}{Z_{ph}} \times \cos\phi$

$$\Rightarrow P_Y = \frac{V_L^2}{Z_{ph}} \times \cos\phi \quad (4.2$$

For a delta connected load,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}}$$

$$I_L = \sqrt{3}I_{ph} = \sqrt{3} \times \frac{V_L}{Z_{ph}}$$

Now $P_\Delta = \sqrt{3}V_L I_L \cos\phi = \sqrt{3}V_L \times \sqrt{3}\frac{V_L}{Z_{ph}} \times \cos\phi = 3\frac{V_L^2}{Z_{ph}} \times \cos\phi$

$$\Rightarrow P_\Delta = 3\frac{V_L^2}{Z_{ph}} \times \cos\phi \quad (4.2$$

From eqns. (4.21) & (4.22), we have

$$P_Y = \frac{P_\Delta}{3} \Rightarrow P_\Delta = 3P_Y \quad (4.2$$

Thus power consumed by a balanced delta connected load is three times that in the case star connected load.

4.5.5 Comparison between Star and Delta Connections

S. No	Star Connection	Delta Connection
1	Similar ends are joined	Dissimilar ends are joined
2	$V_L = \sqrt{3}V_{ph}$ & $I_L = I_{ph}$	$V_L = V_{ph}$ & $I_L = \sqrt{3}I_{ph}$
3	Possible to carry neutral current	Neutral wire is not available
4	Provides 3-phase 4-wire arrangement	Provides 3-phase 3-wire arrangement
5	The phasor sum of all the phase currents is zero	The phasor sum of all the phase voltages is zero
6	Can be used for lighting as well as power loads	Can be used for power loads only