Assignment - 3

① Distinguish between Stationary and Non-stationary process.

stationary process :- A random process is said to be stationary if all its statistical properties, such as mean, variance, moments do not change with time.

whereas in ~~stat~~ non-stationary process, all its statistical properties, such as mean, variance, moments change with time.

② State the properties of Auto correlation function.

1) The mean square value of Random process $x(t)$ is $E[(x(t))^2] = R_{xx}(0)$.

It is equal to the process of two process.

<u>proof</u>: we know that,

$$R_{xx}(\tau) = E[x(t)\,x(t+\tau)]$$

if $\tau = 0$

$$R_{xx}(\tau) = E[x(t)\,x(t)]$$

$$R_{xx}(0) = E[(x(t))^2]$$

2) The auto correlation function is maximum at the origin i.e; $|R_{xx}(\tau)| \leq R_{xx}(0)$

3) $R_{xx}(\tau)$ is even function of τ i.e.

$$R_{xx}(-\tau) = R_{xx}(\tau)$$

<u>proof</u>: we know that

$$R_{xx}(\tau) = E[x(t) \cdot x(t+\tau)]$$

let, $\tau = -\tau$

$$R_{xx}(-\tau) = E[x(t) x(t-\tau)]$$

let $t-\tau = u$

$$R_{xx}(-\tau) = E[x(u+\tau) \cdot x(u)]$$

$$= R_{xx}(\tau)$$

4) If $x(t)$ has a periodic, then its auto correlation function is also periodic.

5) If the random process $z(t)$ is a sum of two random processes $x(t)$ & $y(t)$ i.e.

$z(t) = x(t) * y(t)$ then

$$R_{zz}(\tau) = R_{xx}(\tau) + R_{yy}(\tau) + R_{xy}(\tau) + R_{yx}(\tau).$$

<u>Proof</u>: Consider

$$R_{zz}(\tau) = E[z(t) \cdot z(t+\tau)]$$

$$= E[(x(t) + y(t))(x(t+\tau) + y(t+\tau))]$$

$$= E[x(t) x(t+\tau) + x(t) y(t+\tau) + y(t) x(t+\tau) + y(t) y(t+\tau)]$$

$= R_{xx}(\tau) + R_{xy}(\tau) + R_{yx}(\tau) + R_{yy}(\tau)$.

3) write about the following ergodic process
a) Mean Ergodic process
b) Auto correlation Ergodic process
c) Cross correlation Ergodic process.

A Random process which satisfies the Ergodic theorem is called ergodic process.

a) <u>Mean Ergodic process</u>:

A random process $X(t)$ is said to mean Ergodic (or) Ergodic in mean of the time average of any sample function $x(t)$ is equal to the statistical average $\bar{X}$.

$$E[X(t)] = \bar{X} = A(x(t)) = \bar{x}$$

b) <u>Auto correlation Ergodic process</u>

The Random process $X(t)$ is said to be time Auto correlation in Ergodic (or) Ergodic in Auto correlation, if the time auto correlation function of any sample function $x(t)$ is equal to the statistical auto correlation function of Random process $X(t)$.

i.e. $A[x(t)\, x(t+\tau)] = E[x(t)\, x(t+\tau)]$

$$R_{xx}(\tau) = R_{XX}(\tau)$$

$$\square_{XX}(\tau) = R_{XX}(\tau)$$

c) Cross Correlation Ergodic Process :

Two random processes $X(t)$ & $Y(t)$ are said to be cross correlation ergodic (or) Ergodic in Cross Correlation, if the time cross correlation function of sample functions of $x(t)$ & $y(t)$ is equal to the statistical cross correlation function of two random process $X(t)$ & $Y(t)$.

i.e. $A[x(t)\, y(t+\tau)] = E[X(t)\, Y(t+\tau)]$

$$R_{xy}(\tau) = R_{XY}(\tau)$$

$$\square_{xy}(\tau) = R_{XY}(\tau)$$

(4) Random process is defined by $Y(t) = X(t)\cos(\omega_0 t + \theta)$ where $X(t)$ is a loss random process that amplitude modulates a carrier of constant angular freq. 'ω' with a random phase 'θ' independent of $X(t)$ and uniformly distributed on $(-\pi, \pi)$.

a) Find $E(Y(t))$ b) Find auto correlation fun. of $Y(t)$.

c) Is $y(t)$ coss or not?

Sol. Given, $Y(t) = x(t)\cos(\omega t + \theta)$

since the phase θ is uniformly distributed on $(-\pi, \pi)$, then the density function f is,

$$f_\theta(\theta) = \begin{cases} \frac{1}{2\pi}, & -\pi \le \theta \le \pi \\ 0, & \text{elsewhere.} \end{cases}$$

a) Mean value of $E[Y(t)] = E[x(t)\cos(\omega t + \theta)]$

sin θ & $x(t)$ are independent.

$$= E[x(t)] \cdot E(\cos(\omega t + \theta))$$

$$E(\cos(\omega t + \theta)) = \int_{-\pi}^{\pi} \cos(\omega t + \theta)\, f_\theta(\theta)\, d\theta$$

$$= \int_{-\pi}^{\pi} \frac{\cos(\omega t + \theta)}{2\pi}\, d\theta$$

$$= \frac{1}{2\pi}\left(\frac{\sin(\omega t + \theta)}{1}\right)_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\left[\sin(\omega t + \pi) - \sin(\omega t - \pi)\right]$$

$$= \frac{1}{2\pi}\left(-\sin(\omega t) + \sin(\omega t)\right)$$

$$= 0$$

$$E(Y(t)) = E[x(t)] \cdot 0 = 0$$

b) $R_{YY}(\tau) = E[Y(t)\, Y(t+\tau)]$ = auto correlation

$$= E[x(t)\cos(\omega t + \theta) \cdot x(t+\tau)\cos(\omega(t+\tau) + \theta)]$$

$$= E[x(t)\cos(\omega t + \theta) \cdot x(t+\tau)\cos(\omega(t+\tau) + \theta)]$$

$$= E\left[x(t)\,x(t+\tau)\right] E\left[\cos(\omega t+\theta)\cos(\omega(t+\tau)+\theta)\right]$$

$$= R_{xx}(\tau) \cdot \int_{-\pi}^{\pi} \frac{\cos(\omega t+\theta)\cos(\omega(t+\tau)+\theta)}{2\pi}\, d\theta$$

$$= R_{xx}(\tau) \cdot \int_{-\pi}^{\pi} \frac{\cos(2\omega t+\omega\tau+2\theta)+\cos(\omega\tau)}{2(2\pi)}\, d\theta$$

$$= R_{xx}(\tau) \cdot \left[\frac{\frac{\sin(2\omega t+\omega\tau+2\theta)}{2} + \cos(\omega\tau)\,\theta}{4\pi}\right]_{-\pi}^{\pi}$$

$$= R_{xx}(\tau) \cdot \left[\frac{\sin(2\omega t+\omega\tau+2\pi) - \sin(2\omega t+\omega\tau-2\pi)}{8\pi} + \frac{\pi\cos(\omega\tau)+\pi\cos(\omega\tau)}{4\pi}\right]$$

$$= R_{xx}(\tau) \cdot \left[\frac{-\sin(2\omega t+\omega\tau) + \sin(2\omega t+\omega\tau)}{8\pi} + \frac{2\pi\cos(\omega\tau)}{4\pi}\right]$$

$$= R_{xx}(\tau) \cdot \frac{\cos(\omega\tau)}{2} = \frac{1}{2} R_{xx}(\tau)\cos(\omega\tau).$$

[It is a function of τ only but independent of time 't']

c) $y(t)$ mean is constant and

$R_{yy}(\tau)$ is independent of time 't' and function of τ only.

So $y(t)$ is WSS i.e. wide sense stationary

(5) The process has an auto - correlation function given by $R_{xx}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$. Find the mean value, mean square value and variance of the process.

* Mean $\Rightarrow \underset{\tau \to \infty}{Lt} R_{xx}(\tau) = \underset{\tau \to \infty}{Lt} \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$

$$\bar{X}^2 = \underset{\tau \to \infty}{Lt} \frac{25 + 36/\tau^2}{6.25 + 4/\tau^2} = \frac{25}{6.25}$$

$$\bar{X}^2 = 4$$

$$\text{Mean} = \bar{X} = 2.$$

* Mean Square $\Rightarrow E[X^2(t)] = \underset{\tau \to \infty}{Lt} R_{xx}(0)$

$$E[X^2(t)] = \frac{36}{4} = 9$$

* Variance $\Rightarrow \sigma^2 = E[X^2(t)] - [E(X(t))]^2$

$$= (9) - (2)^2 = 9 - 4 = 5$$