Generally any network may be represented by a rectangular box.

A pair of terminals at which a signal may be enter (or) leave a network is called a port.

A port is defined as any pair of terminals into which energy is supplied (or) from which energy is withdrawn (or) where network variables may be measured.

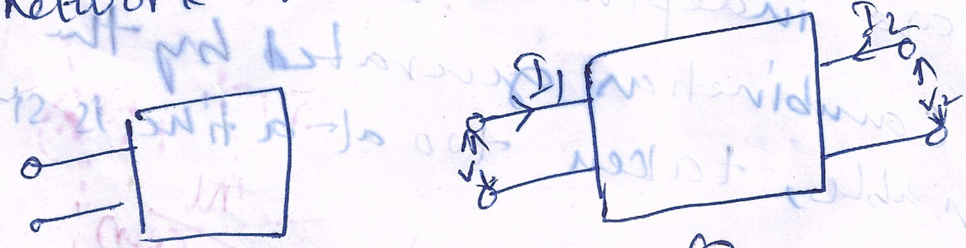


fig 1(b)

A two port network is simply a N/w inside a black box & the N/w has only two pairs of accessible terminals, usually one pair represent the i/p & the other represents the o/p. Such a N/w is common in electronic, communication & tx & distribution system. fig 1.(b) is a two-port (or) two-terminal pair N/w, in which the four terminals have been paired into ports 1-1' & 2-2'. Two ports containing no sources in their branches are called passive ports among them are power transmission lines & transformers.

Two ports containing sources in their branches are called active ports. A voltage (or) current assigned to each of the two ports.

The voltage & current at the I/p terminals are V_1 & I_1, where as V_2 & I_2 are specified at the o/p port. It is also assumed that currents are entering into the N/w at the upper terminals 1 & 2 respectively. The variables of the two port N/w are V_1, V_2, I_1 & I_2. Two of them are dependent variable, the other two are independent variable. The no of possible combinations generated by the four variables, taken two at a time is six $({}^4C_2)$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^nP_r = \frac{n!}{(n-r)!}$$

Open circuit Impedance (Z) parameters:-

A general two-port N/w which doesnot contain any independent sources is shown in fig ①.

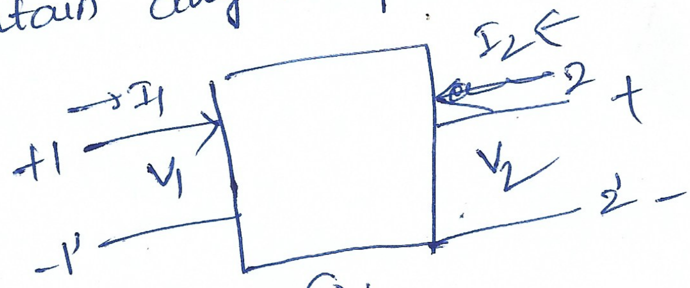


fig ①.

The Z parameters of two-port for the positive directions of voltages & currents may be defined by expressing the port voltages V_1 & V_2 in terms of the current I_1 & I_2.

Here V_1 & V_2 are dependent variables & I_1, I_2 are independent variables.

The voltage at port $1-1'$ is the response produced by the two currents I_1 & I_2. Thus.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow ①$$

Similarly $V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow ②$.

Z_{11}, Z_{12}, Z_{21} & Z_{22} are the network functions and are called impedance (Z) parameters and are defined by eqns ① & ②. These parameters can be represented by matrices.

The matrix equation $[V] = [Z][I]$

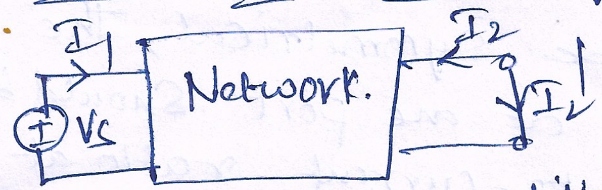
V is the column matrix $= \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Z is the square matrix $= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

(I) in the column matrix $= \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Condition for Reciprocity :-



N/w for deriving condition for reciprocity.

A N/w is said to be reciprocal if the ratio of excitation at one port to response at the other port same if excitation & response are interchanged.

(a) Voltage 'V_s' is applied at the i/p port with the output port short-circuited.

i.e., $V_1 = V_s$

$V_2 = 0$

$I_2 = -I_2'$

From the z-parameter equation

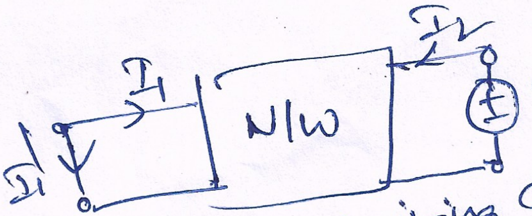
$$V_s = z_{11} I_1 - z_{12} I_2'$$

$$0 = z_{21} I_1 - z_{22} I_2' \Rightarrow I_1 = \frac{z_{22} I_2'}{z_{21}}$$

$$V_s = \frac{z_{11} z_{22} I_2'}{z_{21}} - z_{12} I_2'$$

$$\frac{V_s}{I_2'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{21}}$$

(b) Voltage V_s is applied at the o/p port with the input port short-circuited.



N/w for deriving conditions for reciprocity.

i.e., $V_2 = V_s$

$V_1 = 0$

$I_1 = -I_1'$

From the z-parameter equation

$$0 = -z_{11} I_1' + z_{12} I_2$$

$$V_s = -z_{21} I_1' + z_{22} I_2$$

$$I_2 = \frac{z_{11}}{z_{12}} I_1'$$

$$V_s = -z_{21} I_1' + z_{22} \frac{z_{11}}{z_{12}} I_1'$$

$$\frac{V_s}{I_1'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{12}}$$

For the N/w to be reciprocal

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'} \Rightarrow z_{12} = z_{21}$$

Condition for symmetry:-

For a network to be symmetrical, the voltage to current ratio at one port should be the same as the voltage-to-current ratio at the other port with one of the ports open-circuited.

ⓐ when the output port is open-circuited i.e, $I_2 = 0$ from the Z-parameter equation.

$$V_s = Z_{11} I_1$$

$$\frac{V_s}{I_1} = Z_{11}$$

ⓑ when the input port is open-circuited i.e $I_1 = 0$.
from the Z-parameter equation

$$V_s = Z_{22} I_2 \Rightarrow \frac{V_s}{I_2} = Z_{22}$$

For the network to be symmetrical

$$\frac{V_s}{I_1} = \frac{V_s}{I_2} \Rightarrow \boxed{Z_{11} = Z_{22}}$$

problems on z-parameters

1. Test results of a 2-port N/w are

(a) $I_1 = 0.1\angle 0\,A$, $V_1 = 5.2\angle 50\,V$, $V_2 = 4.1\angle -25$, with port 2 open circuit

(b) $I_2 = 0.1\angle 0$ $V_1 = 3.1\angle -80\,V$, $V_2 = 4.2\angle 60\,V$, with port 1 open circuit

Find z parameters.

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{5.2\angle 50}{0.1\angle 0} = 52\angle 50\,\Omega$$

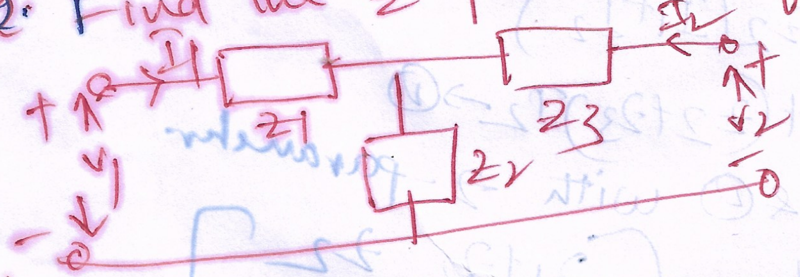
$$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{4.1\angle -25}{0.1\angle 0} = 41\angle -25$$

$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{3.1\angle -80}{0.1\angle 0} = 31\angle -80$$

$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{4.2\angle 60}{0.1\angle 0} = 42\angle 60$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 52\angle 50 & 31\angle -80 \\ 41\angle -25 & 42\angle 60 \end{bmatrix}$$

2. Find the z-parameters of the N/w below



Method 1:-

Case 1:- when the o/p port is open circuited $I_2 = 0$

Apply KVL to mesh I

$$V_1 = (Z_1 + Z_2) I_1$$

$$z_{11} = \frac{V_1}{I}\Big|_{I_2=0} = Z_1 + Z_2$$

$$V_2 = Z_2 I_1$$

$$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_2$$

Case 2: when the input port is open circuited ie $I_1=0$

Applying KVL to mesh 2

$$V_2=(z_2+z_3)I_2$$

$$z_{22}=\frac{V_2}{I_2}\Big|_{I_1=0}=z_2+z_3$$

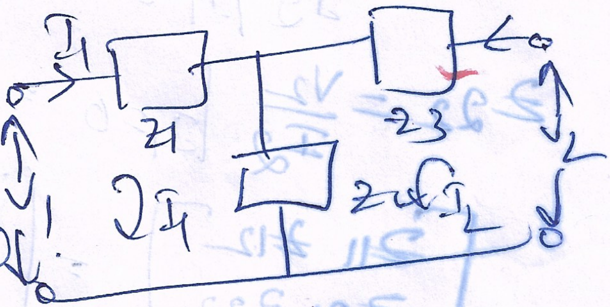
$$V_1=z_2 I_2$$

$$z_{12}=\frac{V_1}{I_2}\Big|_{I_1=0}=z_2$$

Hence, The z-parameters are

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_1+z_2 & z_2 \\ z_2 & z_2+z_3 \end{bmatrix}$$

Second method:-



KVL to mesh I

$$V_1=z_1 I_1+z_2(I_1+I_2)$$

$$V_1=(z_1+z_2)I_1+z_2 I_2 \rightarrow (1)$$

KVL to mesh II

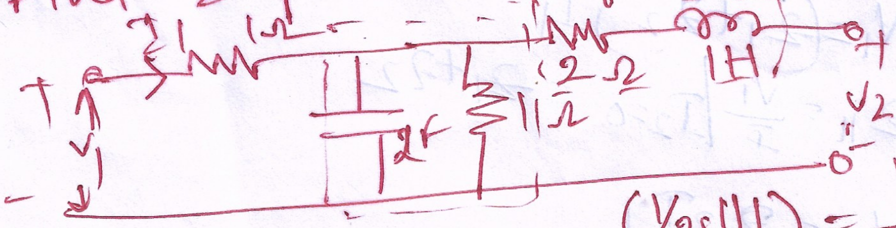
$$V_2=z_3 I_2+z_2(I_1+I_2)$$

$$=z_2 I_1+(z_2+z_3)I_2 \rightarrow (2)$$

Compare eqn (1) & (2) with z-parameter

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_1+z_2 & z_2 \\ z_2 & z_2+z_3 \end{bmatrix}$$

3. Find z-parameters of the Network below.



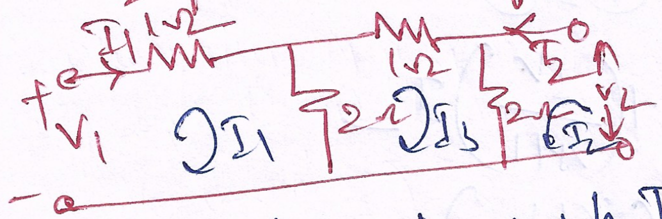
$$z_1=1 \quad z_2=(1/2s \| 1)=\frac{1/2s \times 1}{1/2s+1}=\frac{1}{2s+1} \quad z_3=s+2$$

$$z_{11}=\frac{V_1}{I_1}\Big|_{I_2=0}=z_1+z_2=1+\frac{1}{2s+1}=\frac{2s+2}{2s+1}$$

$$z_{12}=z_{21}=z_2=\frac{1}{2s+1}$$

$$z_{22}=\frac{V_2}{I_2}\Big|_{I_1=0}=z_2+z_3=\frac{1}{2s+1}+s+2=\frac{2s^2+5s+3}{2s+1}$$

4. Find z parameters of the network.



Apply KVL to mesh I

$$V_1 = 3I_1 - 2I_3 \rightarrow (1)$$

Apply KVL to mesh 2

$$V_2 = 2I_2 + 2I_3 \rightarrow (2)$$

Apply KVL to mesh 3

$$-2I_1 + 2I_2 + 5I_3 = 0$$

$$I_3 = \frac{2I_1}{5} - \frac{2I_2}{5} \rightarrow (3)$$

Substitute eqn (3) in (1) & (2).

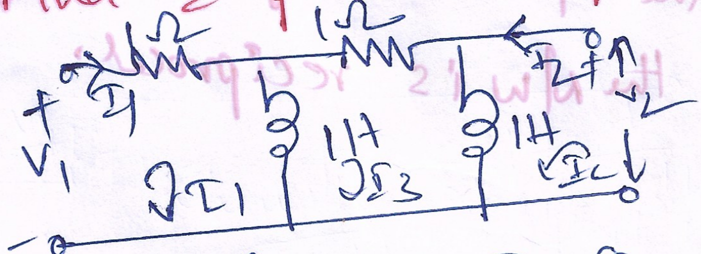
$$V_1 = \frac{11}{5}I_1 + \frac{4}{5}I_2 \rightarrow (4)$$

$$V_2 = \frac{4}{5}I_1 + \frac{6}{5}I_2 \rightarrow (5)$$

Compare eqn 4 & 5 with z param

$$\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} = \begin{bmatrix} 11/5 & 4/5 \\ 4/5 & 6/5 \end{bmatrix}$$

5. Find z-parameters of the network



$$V_1 = (s+1)I_1 - sI_3 \rightarrow (1)$$

$$V_2 = sI_2 + sI_3 \rightarrow (2)$$

$$-sI_1 + sI_2 + (2s+1)I_3 = 0$$

$$I_3 = \frac{sI_1}{2s+1} - \frac{sI_2}{2s+1} \rightarrow (2)$$

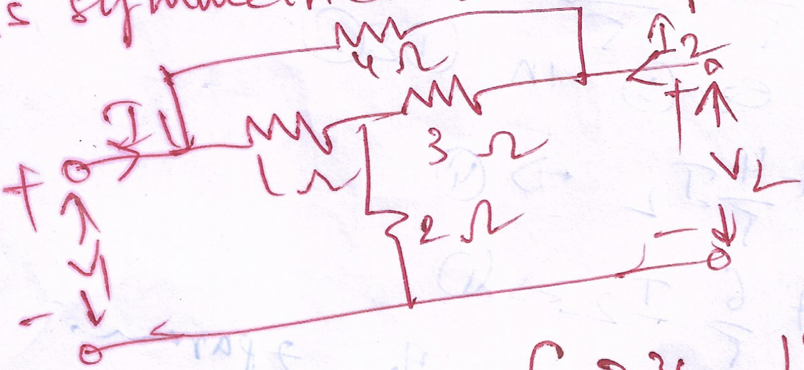
Substitute eqn ③ in ① & ②

$$V_1 = \left(\frac{s^2+3s+1}{2s+1}\right) I_1 + \left(\frac{s^2}{2s+1}\right) I_2$$

$$V_2 = \left(\frac{s^2}{2s+1}\right) I_1 + \left(\frac{s^2+s}{2s+1}\right) I_2$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{s^2+3s+1}{2s+1} & \frac{s^2}{2s+1} \\ \frac{s^2}{2s+1} & \frac{s^2+s}{2s+1} \end{bmatrix}$$

6. Find the open circuit impedance parameters for the N/W. Determine whether the network is symmetrical and reciprocal.



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 23/8 & 19/8 \\ 19/8 & \frac{31}{8} \end{bmatrix}$$

$Z_{11} \neq Z_{22}$, the N/W is not symmetrical.

$Z_{12} = Z_{21}$, the N/W is reciprocal.

SHORT-CIRCUIT ADMITTANCE PARAMETERS (Y PARAMETERS)

The Y parameters of two-port network may be defined by expressing the two-port currents I_1 & I_2 interms of the two-port Voltages V_1 & V_2 (Independent Variables)

$$(I_1, I_2) = f(V_1, V_2)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2.$$

In matrix form, we can write

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

The individual Y parameters for a given N/w can be defined by setting each of the port Voltages equal to zero (short circuit the ports)

Case 1:- when the o/p port short circuited i.e, $V_2 = 0$.

$$\boxed{Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}}$$

where Y_{11} is the driving point admittance with output-port short circuited. It is also called short-circuit input admittance.

Similarly

$$\boxed{Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}}$$

where Y_{21} is the transfer admittance with o/p port short-circuited. It is also called short-circuit forward transfer admittance.

Case 2:-

when the i/p port is short-circuited i.e, $V_1 = 0$

$$\boxed{Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}}$$

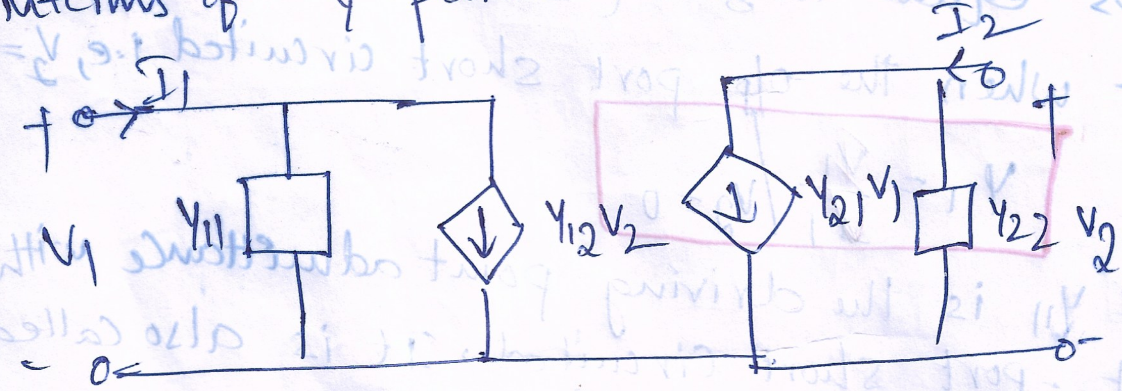
Where Y_{12} is the transfer admittance with i/p port Short-circuited. It is also called short-circuit reverse transfer admittance.

Similarly, $\boxed{Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}}$

where Y_{22} is the short-circuit driving point admittance with input port short-circuited. It is also called the short circuit output admittance.

As these admittance parameters are measured with either input (or) output port short-circuited, these are called short-circuit admittance parameters.

The equivalent circuit of the two port N/w interms of Y parameters is shown in Fig.



Fig① Equivalent CKt of the two port Network interms of Y-parameters.

Condition for Reciprocity:-

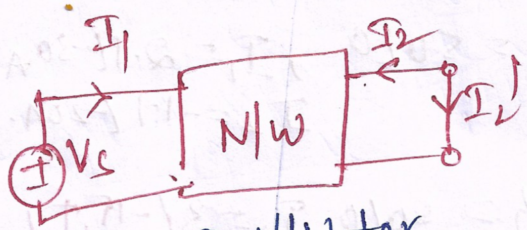


Fig 2ⓐ N/W for deriving ~~for~~ condition for reciprocity

ⓐ. voltage V_s is applied at input port with output port short-circuited

$V_1 = V_s \quad I_2 = -I_2'$

$V_2 = 0$

From Y parameters $-I_2' = Y_{21} V_s$

$$\frac{I_2'}{V_s} = -Y_{21}$$

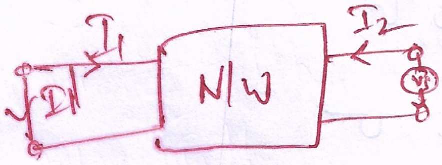


Fig 2ⓑ N/W for deriving condition for reciprocity.

ⓑ. Voltage V_s is applied at output port with the i/p port Short-circuited i.e.

$V_2 = V_s, \; I_1 = -I_1', \; V_1 = 0.$

from the Y-parameters: $-I_1' = Y_{12} V_s$

$$\frac{I_1'}{V_s} = -Y_{12}$$

for the N/w to be reciprocal

$$\frac{I_2'}{V_s} = \frac{I_1'}{V_s} \implies \boxed{Y_{12} = Y_{21}} *$$

Condition for Symmetry:-

ⓐ when the o/p port short circuited i.e, $V_2 = 0$.

from Y paramet

$I_1 = Y_{11} V_s$

$$\frac{V_s}{I_1} = \frac{1}{Y_{11}}$$

ⓑ when the i/p port short circuited i.e $V_1 = 0$.

from Y-parameters.

$I_2 = Y_{22} V_s$

$$V_s / I_2 = \frac{1}{Y_{22}}$$

for the N/w to be symmetrical $\frac{V_s}{I_1} = \frac{V_s}{I_2}$

$\boxed{Y_{11} = Y_{22}}$ *

Problems on Y-parameters

1. Test results of 2-port N/W are

(a) port-2 short-circuited: $V_1 = 50\angle 0$, $I_1 = 2.1\angle -30$ A, $I_2 = -1.1\angle -20$ A

(b) port-1 short-circuited: $V_2 = 50\angle 0$, $I_2 = 3\angle -15$, $I_1 = -1.1\angle -20$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = 0.042\angle -30$ ℧

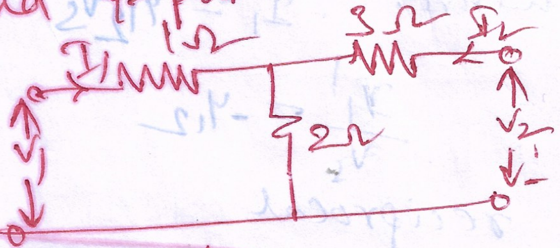
$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = -0.022\angle -20$ ℧

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = -0.022\angle -20$ ℧

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = 0.06\angle -15$ ℧

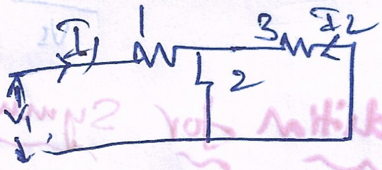
The Network is reciprocal ~~but~~ not symmetric.

2. Find Y-parameters for the N/w



First method:-

Case 1:- When the o/p port short-circuited $V_2 = 0$



$Req = (3\|2) + 1 = \frac{11}{5}$

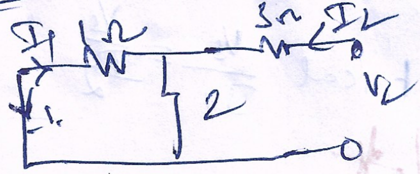
$V_1 = \frac{11}{5} I_1$ $\quad Y_{11} = \frac{I_1}{V_1} = \frac{5}{11}$ ℧

$I_2 = -\frac{2}{2+3} I_1 = -\frac{2}{5} I_1 = -\frac{2}{5} \times \frac{5}{11} V_1$

$I_2 = \frac{-2}{11} V_1$

$Y_{21} = \frac{I_2}{V_1} = -\frac{2}{11}$ ℧

Case 2:- when the i/p port short circuited



$Req = (1\|2) + 3 = \frac{11}{3}$ Ω

$V_2 = \frac{11}{3} I_2 \Rightarrow Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{3}{11}$ ℧

$$I_1 = -I_2 \times \frac{2}{2+1} = \frac{-3V_2}{11} \times \frac{2}{3} = \frac{-2V_2}{11}$$

$$Y_{12} = \frac{I_1}{V_2} = \frac{-2}{11} \, \mho$$

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{bmatrix} 5/11 & -2/11 \\ -2/11 & 3/11 \end{bmatrix}$$

Second method:-

$$I_1 = \frac{V_1 - V_3}{1} = V_1 - V_3 \longrightarrow (1)$$

$$I_2 = \frac{V_2 - V_3}{3} = \frac{V_2}{3} - \frac{V_3}{3} \longrightarrow (2)$$

KCL at node (3)

$$I_1 + I_2 = \frac{V_3}{2} \longrightarrow (3)$$

$$V_1 - V_3 + \frac{V_2}{3} - \frac{V_3}{3} = \frac{V_3}{2}$$

$$V_3 = \frac{6}{11} V_1 + \frac{2}{11} V_2 \longrightarrow (4)$$

Sub (4) in (1)

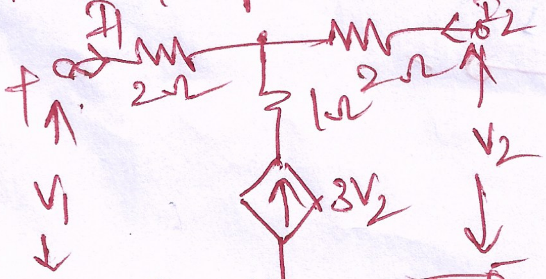
$$I_1 = \frac{5}{11} V_1 - \frac{2}{11} V_2$$

Sub (4) in (2) $I_2 = \frac{-2}{11} V_1 + \frac{3}{11} V_2$

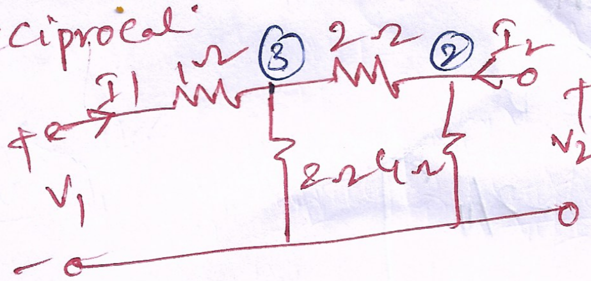
Compare eqns (5) & (6) with Y-parameters.

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{bmatrix} 5/11 & -2/11 \\ -2/11 & 3/11 \end{bmatrix}$$

3. Find Y-parameters of the N/W shown below



4. Determine Y-parameters for the N/w shown below. Determine whether the N/w is symmetrical and reciprocal.



$$I_1 = \frac{V_1 - V_3}{1} = V_1 - V_3 \rightarrow (1)$$

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_2}{2} \rightarrow (2)$$

$$V_1 - V_3 = V_3 - \frac{V_2}{2}$$

$$V_3 = \frac{V_1}{2} + \frac{V_2}{4} \rightarrow (3)$$

sol

$$I_2 = \frac{V_2}{4} + \frac{V_2 - V_3}{2} \rightarrow (4)$$

substitute (3) in (4)

$$= \frac{V_2}{4} + \frac{V_2}{2} - \frac{1}{2}\left[\frac{V_1}{2} + \frac{V_2}{4}\right]$$

$$= \cancel{\frac{3}{4}V_2} - \frac{V_1}{4} + \frac{5V_2}{8} \rightarrow (4)(ii)$$

substitute 3 in in (1).

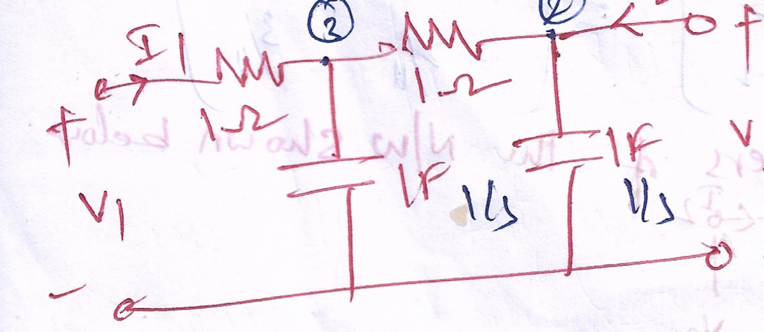
$$I_1 = V - \frac{V_1}{2} - \frac{V_2}{4} = \frac{V_1}{2} - \frac{V_1}{4} \rightarrow (i)$$

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 \\ -1/4 & 5/8 \end{pmatrix}$$

$Y_{11} \neq Y_{22}$ the N/w is not symmetrical

$Y_{12} = Y_{21}$ " " reciprocal.

5. Determine the short-circuit admittance parameters of the N/w below.



$$I_1 = \frac{V_1 - V_3}{1} = V_1 - V_3 \rightarrow (1).$$

$$I_1 = \frac{V_3}{1/s} + \frac{V_3 - V_2}{1} \rightarrow (2)$$

equate (1) & (2)

$$V_1 - V_3 = sV_3 + V_3 - V_2$$

$$V_3(2+s) = V_1 + V_2$$

$$V_3 = \frac{V_1 + V_2}{s+2} = \frac{V_1}{s+2} + \frac{V_2}{s+2} \rightarrow (3)$$

$$I_2 = \frac{V_2}{1/s} + \frac{V_2 - V_3}{1} \rightarrow (4)$$

Substitute eqn (3) in 4

$$I_2 = sV_2 + V_2 - \frac{V_1}{s+2} - \frac{V_2}{s+2}$$

$$= \frac{-V_1}{s+2} + V_2\left[s+1 - \frac{1}{s+2}\right]$$

$$= \frac{-V_1}{s+2} + \frac{s^2+3s+1}{s+2} V_2 \rightarrow (ii)$$

Substitute eqn (3) in (1).

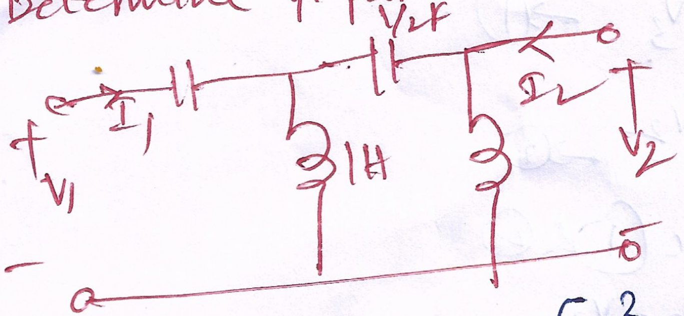
$$I_1 = V_1 - \frac{V_1}{s+2} - \frac{V_2}{s+2}.$$

$$= \frac{s+1}{s+2} V_1 - \frac{1}{s+2} V_2$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{s+1}{s+2} & -1/s+2 \\ -\frac{1}{s+2} & \frac{s^2+3s+1}{s+2} \end{bmatrix}$$

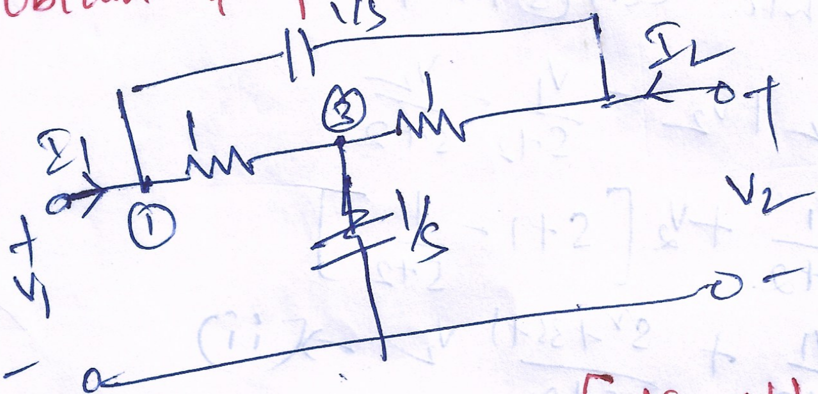
circuit is reciprocal.

(6). Determine Y-parameter for the N/w below



Ans:- $\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{bmatrix} \dfrac{s^3+2s}{4(s^2+1)} & \dfrac{-s^3}{4(s^2+1)} \\ \dfrac{-s^3}{4(s^2+1)} & \dfrac{s^4+6s^2+4}{4s(s^2+1)} \end{bmatrix}$

(7). Obtain Y-parameters of the N/w



Ans:- $\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{s+2} & \dfrac{-(s^2+2s+1)}{s+2} \\ \dfrac{-(s^2+s+1)}{s+2} & \dfrac{s^2+3s+1}{2} \end{bmatrix}$