Unit-II DFT

DFT is a powerful computation tool to evaluate fourier transform on a digital Computer.

$$\text{F.T. of } x(t) \Rightarrow X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{2\pi} x(t)\, e^{-j\omega t}\, dt.$$

Similarly Discrete Fourier transform is defined as

$$DFT[x(n)] \Rightarrow X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N} \quad \text{for } k = 0, 1, \ldots N-1.$$

$$= \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \quad \text{where } W_N = e^{-j2\pi/N} \rightarrow \text{Twiddle factor}$$

$$IDFT[X(k)] = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}} \quad \text{for } n = 0, 1, \ldots N-1$$

→ Drawback of DTFT is the spectrum is continuous

To overcome this we go for DFT.

→ Sample the spectrum of DTFT to get DFT.

→ Prove that $W_N^{r} = W_N^{r \pm N} = W_N^{r \pm 2N}$ (Periodicity)

FFT

$$W_N = e^{-j2\pi/N} \qquad W_N^{r} = e^{-(j2\pi/N)r}$$

$$W_N^{r \pm N} = e^{-\left(j\frac{2\pi}{N}\right)r \pm N} = e^{-j\frac{2\pi r}{N}} \cdot e^{\mp j\frac{2\pi N}{N}}$$

$$= e^{-j2\pi r/N}\left(\cos 2\pi \pm j\sin 2\pi\right)$$

$$\therefore W_N^{r \pm N} = e^{-j\frac{2\pi r}{N}} = W_N^{r}.$$

∴ Twiddle factor is periodic w.r.to "N".

→ Prove $W_N^{r \pm N/2} = -W_N^{r}$ (symmetric property)

$$W_N^{r \pm N/2} = e^{-\frac{j2\pi}{N}\left(r \pm \frac{N}{2}\right)} = e^{-\frac{j2\pi r}{N}} \cdot e^{\pm j\pi}$$

$$= -e^{-j\frac{2\pi r}{N}} = -W_N^{r}.$$

$$\therefore W_N^{r \pm N/2} = -W_N^{r}$$

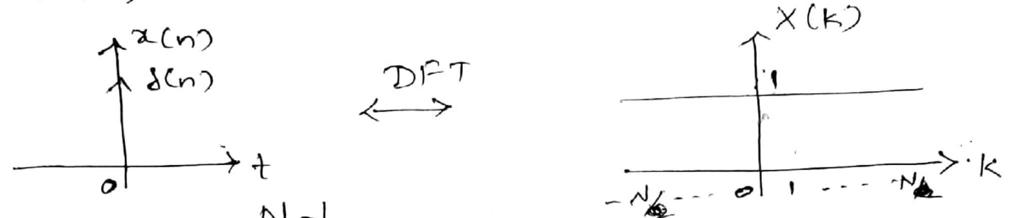
Problems:

1. Find the N-point DFT of i) $x(n) = \delta(n)$ ii) $\delta(n-n_0)$.

<u>sol:-</u> i). $$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \quad \text{for } k = 0, 1, \ldots N-1$$

$$= \sum_{n=0}^{N-1} \delta(n) e^{-j2\pi nk/N}$$

$$\therefore X(k) = 1 \cdot e^{0} = 1 \quad \text{for all } k.$$



ii) $$X(k) = \sum_{n=0}^{N-1} \delta(n-n_0) e^{-j2\pi nk/N}$$

$$= 1 \cdot e^{-j\frac{2\pi n_0 k}{N}}$$

2. Find 4-point DFT of a sequence $x(n) = [1\ 1\ 0\ 0]$ & find its IDFT.

<u>sol:-</u> $N = 4$.

DFT: $$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \quad \text{for } k = 0, 1, \ldots N-1$$

$$X(0) = \sum_{n=0}^{3} x(n) e^{-j2\pi nk/N}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0 = 2.$$

$$X(1) = \sum_{n=0}^{3} x(n) e^{-j2\pi nk/N}$$

$$= x(0) + x(1) e^{-j\pi/2} + x(2) e^{-j\pi} + x(3) e^{-j3\pi/2}$$

$$= 1 + \cos\pi/2 - j\sin\pi/2 = 1 - j$$

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j2\pi nk/N}$$

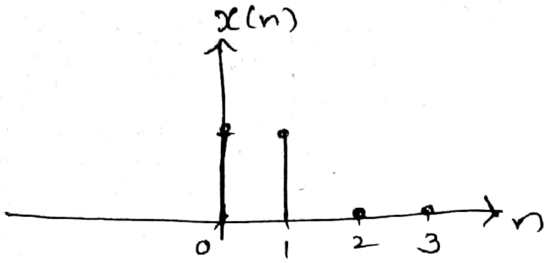
$$= x(0) + x(1) e^{-j\pi} + x(2) e^{-j2\pi} + x(3) e^{-j3\pi}$$

$$= 1 + \cos\pi - j\sin\pi = 0$$

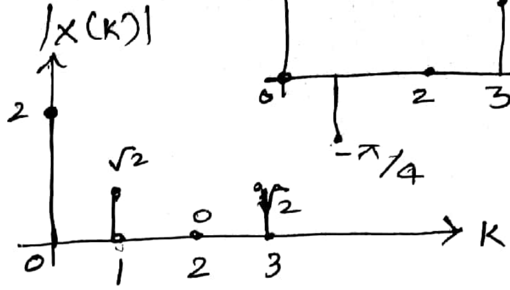
$$X(3) = x(0) + x(1) e^{-j3\pi/2} + x(2) e^{-j3\pi} + x(3) e^{-j9\pi/2}$$

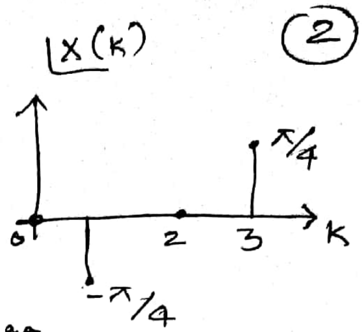
$$= 1 + \cos\tfrac{3\pi}{2} - j\sin 3\pi/2 = 1 + j$$

$\therefore X(k) = [2 \quad 1-j \quad 0 \quad 1+j]$



DFT $\longleftrightarrow$





IDFT of $X(k) = [2 \quad 1-j \quad 0 \quad 1+j]$

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\, e^{j2\pi nk/N} \quad \text{for } n = 0, 1, \ldots N-1$$

$$y(0) = \frac{1}{4}\sum_{k=0}^{3} Y(k) = \frac{1}{4}(2 + 1 - \cancel{j} + 0 + 1 + \cancel{j}) = 1.$$

$$y(1) = \frac{1}{4}\sum_{k=0}^{3} Y(k)\, e^{j2\pi k/N}$$

$$= \frac{1}{4}\left[Y(0) + Y(1)e^{j\pi/2} + Y(2)e^{j\pi} + Y(3)e^{+j3\pi/2}\right]$$

$$= \frac{1}{4}\left[2 + (1-j)(j) + 0 + (1+j)(-j)\right]$$

$$= \frac{1}{4}\left[2 + \cancel{j} - j^2 - \cancel{j} - j^2\right] = 1$$

$$y(2) = \frac{1}{4}\sum_{k=0}^{3} Y(k)\, e^{j4\pi k/N}$$

$$= \frac{1}{4}\left[Y(0) + Y(1)e^{j\pi} + Y(2)e^{j2\pi} + Y(3)e^{j3\pi}\right]$$

$$= \frac{1}{4}\left[2 + (1-j)(-1) + 0 + (1+j)(-1)\right]$$

$$= \frac{1}{4}\left[2 + (-1 + \cancel{j}) + (-1 - \cancel{j})\right] = 0$$

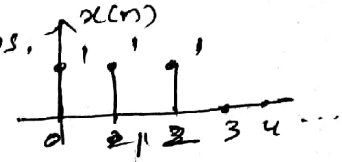
$$y(3) = \frac{1}{4}\left[Y(0) + Y(1)e^{+j3\pi/2} + Y(2)e^{+j3\pi} + Y(3)e^{+j9\pi/2}\right]$$

$$= \frac{1}{4}\left[2 + (1-j)(-j) + 0 + (1+j)(+j)\right]$$

$$= \frac{1}{4}\left[2 - \cancel{j} + j^2 + 0 + \cancel{j} + j^2\right] = 0.$$

$\therefore y(n) = [1 \quad 1 \quad 0 \quad 0] = x(n).$

3. Find the DFT of a sequence $x(n) = 1$ for $0 \leq n \leq 2$
$= 0$ otherwise.
for i) $N=4$ ii) $N=8$. Plot $|X(k)|$ & $\angle X(k)$.



Sol^n i) $N = 4$. Append N-L no. of zeros.

$x(n) = [1 \ \ 1 \ \ 1 \ \ 0]$

For $N=8$

$x(n) = [1 \ \ 1 \ \ 1 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]$.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \qquad k = 0 \text{ to } N-1.$$

$$X(0) = \sum_{n=0}^{3} x(n) \cdot 1 = x(0)+x(1)+x(2)+x(3) = 3.$$

$$X(1) = \sum_{n=0}^{3} x(n) \cdot e^{-j\frac{2\pi n}{N}} = x(0) + x(1)e^{-j\pi/2} + x(2)e^{-j\pi} + 0$$

$$= 1 + 1.(-j) + (-1) = -j$$

$$X(2) = \sum_{n=0}^{3} x(n) \cdot e^{-j\frac{4\pi n}{4}} = x(0) + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + 0$$

$$= 1 - 1 + 1 = 1$$

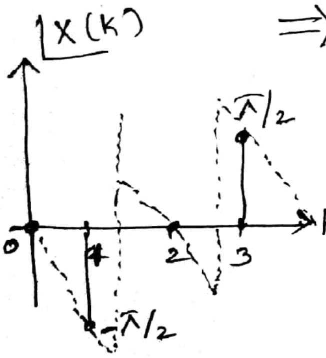
$$X(3) = \sum_{n=0}^{3} x(n) e^{-j\frac{6\pi n}{4}} = x(0) + x(1)e^{-j3\pi/2} + x(2)e^{-j3\pi} + 0$$

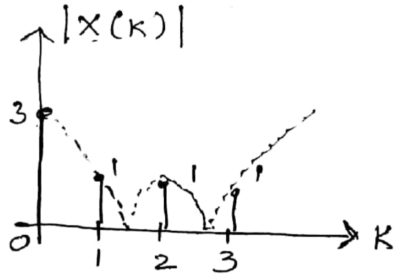
$$= 1 + j - 1 = j$$

$$\Rightarrow X(k) = [3 \ \ -j \ \ 1 \ \ j].$$

$$|X(k)| = [3 \ \ 1 \ \ 1 \ \ 1]$$

$$\angle X(k) = [0 \ \ -\pi/2 \ \ 0 \ \ \pi/2]$$





ii) $N=8$; $x(n) = [1 \ \ 1 \ \ 1 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]$

$$X(0) = \sum_{n=0}^{7} x(n) \cdot 1 = 3.$$

$$X(1) = \sum_{n=0}^{7} x(n) e^{-j\frac{\pi n}{4}} = x(0) + x(1)e^{-j\pi/4} + x(2)e^{-j\pi/2}$$

$$= 1 + \left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) - j$$

$$= 1.707 - j1.707$$

$$X(2) = \sum_{n=0}^{7} x(n) e^{-j\pi n/2} = x(0) + x(1) e^{-j\pi/2} + x(2) e^{-j3\pi}$$
$$= 1 + 1(-j) + 1(-1) = -j$$

$$X(3) = \sum_{n=0}^{7} x(n) e^{-j\frac{\pi 3n}{4}} = x(0) + x(1) \cdot e^{-j3\pi/4} + x(2) e^{-j3\pi/2}$$
$$= 1 + 1(-0.707 - j\,0.707) + (0-j)(-1)$$
$$= 1 - 0.707 - j0.707 + j$$
$$= .293 + 0.293j$$

$$X(4) = \sum_{n=0}^{7} x(n) e^{-j\pi n} = x(0) + x(1) e^{-j\pi} + x(2) e^{-j2\pi}$$
$$= 1 + 1(-1) + 1(1) = 1$$

$$X(5) = \sum_{n=0}^{7} x(n) e^{-j\frac{\pi n 5}{4}} = x(0) + x(1) e^{-j5\pi/4} + x(2) e^{-j5\pi/2}$$
$$= 1 + (-0.707 + j0.707) + 1(0-j)(1)$$
$$= 0.293 - j\,0.293$$

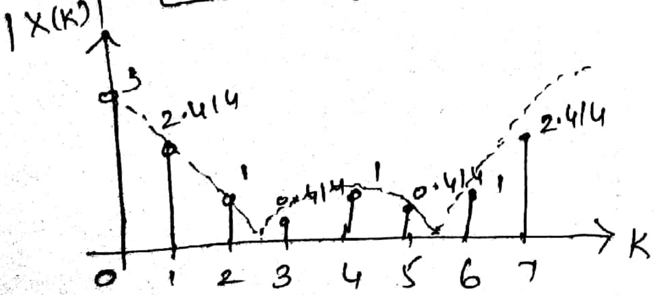
$$X(6) = \sum_{n=0}^{7} x(n) e^{-j\frac{\pi n 3}{2}} = x(0) + x(1) e^{-j3\pi/2} + x(2) e^{-j3\pi}$$
$$= 1 + 1(j) + 1(-1) = j$$

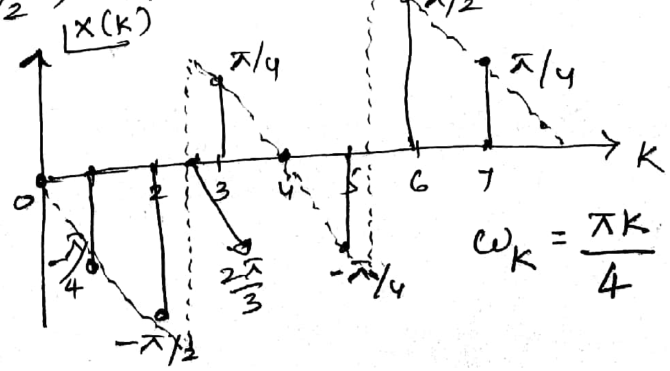
$$X(7) = \sum_{n=0}^{7} x(n) e^{-j\frac{7\pi n}{4}} = x(0) + x(1) \times e^{-j\frac{7\pi}{4}} + x(2) e^{-j7\pi/2}$$
$$= 1 + 1(0.707 + j0.707) + 1(0+j)$$
$$= 1.707 + j(1.707)$$

$$\therefore X(K) = [3, (1.707 - j1.707), -j, (0.293 + j\,0.293), 1, (0.293 - j0.293), j, (1.707 + j1.707)]$$

$$|X(K)| = [3, 2.414, 1, 0.414, 1, 0.414, 1, 2.414]$$

$$\angle X(K) = [0, -\pi/4, -\pi/2, \pi/4, 0, -\pi/4, \pi/2, \pi/4]$$





$$\omega_K = \frac{\pi K}{4}$$

If $N=4$, it is difficult to analyse the entire spectrum. Hence the resol^n of the spectrum is poor. Inorder to increase the resol^n increase the size of N.
The display of the results is better if $N=8$.

4. Find 8-point DFT of $x(n) = [1, 1, 1, 1, 1, 1, 0, 0]$

Sol: $X(K) = \{6, -0.707-j1.707, 1-j, 0.707+j0.293, 0, 0.707-j0.293, 1+j, -0.707+j1.707\}$

5. Find IDFT of the sequence $X(K) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\}$.

Sol:- $x(n) = \{1, 0.75, 0.5, 0.25, 1, 0.75, 0.5, 0.25\}$.

Relationship of DFT to other transforms:

a) Relationship to the Fourier Transform

F.T is given by

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$$

where $X(e^{j\omega})$ is continuous fun^n of ω.

DFT is

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \qquad K=0,1 \ldots N-1$$

DFT is sampled version of F.T.

$$X(K) = X(e^{j\omega})\Big|_{\omega=\frac{2\pi K}{N}} \qquad K=0,1 \ldots N-1$$

b) Relationship to the z-transform:

$$X(z) = \sum_{n=0}^{N-1} x(n)\, z^{-n}$$

where $x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(k)\, e^{j2\pi kn/N}$

$$X(z) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j2\pi kn/N} z^{-n}\right] = \frac{1}{N}\sum_{K=0}^{N-1} X(K) \sum_{n=0}^{N-1}\left(e^{\frac{j2\pi K}{N}} z^{-1}\right)^n$$

$$= \frac{1-z^{-N}}{N}\sum_{K=0}^{N-1} \frac{X(K)}{1-e^{j2\pi K/N} z^{-1}}$$

Properties of DFT :-

1. Periodicity : $x(n)$ & $X(K)$ are periodic w.r.to 'N'.

$$x(n+N) = x(n) \quad \& \quad X(K+N) = X(K).$$

Proof:- $$X(K+N) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n(K+N)}{N}}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}} \left(\cos 2\pi n - j\sin 2\pi n\right)$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}} = X(K)$$

$$\therefore X(K+N) = X(K)$$

$$x(n+N) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi (n+N)k}{N}}$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi nk}{N}} \cdot e^{j2\pi K}$$

$$\therefore x(n+N) = x(n).$$

2. Linearity : $a_1 x_1(n) + a_2 x_2(n) \overset{DFT}{\longleftrightarrow} a_1 X_1(K) + a_2 X_2(K)$.

3. Time reversal : $x(-n)_N \overset{DFT}{\longleftrightarrow} X(-K)_N$

$$x(N-n) \overset{DFT}{\longleftrightarrow} X(N-K).$$

Proof :

$$DFT\left(x(N-n)\right) = \sum_{n=0}^{N-1} x(N-n)\, e^{-j\frac{2\pi kn}{N}}$$

let $N-n = m$

$n = N-m$

$$= \sum_{m=0}^{N-1} x(m)\, e^{-j\frac{2\pi k(N-m)}{N}}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{j\frac{2\pi km}{N}} \cdot e^{-j2\pi k} \quad e^{j\frac{2\pi Nm}{N}}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{j\frac{2\pi km}{N}} \times e^{-j\frac{2\pi Nm}{N}}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{-j\frac{2\pi m}{N}(N-K)} \times 1.$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n(N-K)}{N}} = X(N-K)$$

$$\therefore DFT\left(x(N-n)\right) = X(N-K).$$

4. Circular freq. shift : $x(n)\, e^{\frac{j2\pi l n}{N}} \xleftrightarrow{DFT} X(k-l)_N = X(N+k-l)$

Proof :- $D.F.T.\left[x(n)\, e^{\frac{j2\pi l n}{N}}\right] = \sum_{n=0}^{N-1} x(n)\, e^{\frac{j2\pi l n}{N}}\, e^{-j\frac{2\pi k n}{N}}$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n}{N}(k-l)}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n}{N}(k-l+N)} \cdot e^{j\frac{2\pi n N}{N}}$$

$$= X(N+k-l) = X(k-l)_N .$$

5. Circular time shift: $x(n-m)_N \xleftrightarrow{DFT} e^{-j\frac{2\pi k m}{N}} X(k).$

$$x(n-m+N) \xleftrightarrow{DFT} e^{-j\frac{2\pi k m}{N}} X(k).$$

Proof :- $D.F.T\left[x(n-m+N)\right] = \sum_{n=0}^{N-1} x(n-m+N)\, e^{-j\frac{2\pi k n}{N}}$

$$= \sum_{n=0}^{m-1} x(n-m+N)\, e^{-j\frac{2\pi k n}{N}} + \sum_{n=m}^{N-1} x(n-m+N)\, e^{-j\frac{2\pi k n}{N}}$$

let $n-m+N = l$

If $n=0 \Rightarrow l = N-m$ If $n=m \Rightarrow l = N$

If $n=m-1 \Rightarrow l = N-1$ If $n = N-1 \Rightarrow l = 2N-m-1$ $(N-m-1)$

$$= \sum_{l=N-m}^{N-1} x(l)\, e^{-j\frac{2\pi k (l+m-N)}{N}} + \sum_{l=N}^{2N-m-1} x(l)\, e^{-j\frac{2\pi k}{N}(l+m-N)}$$

$$= \sum_{l=0}^{N-1} x(l)\, e^{-j\frac{2\pi k l}{N}} \cdot e^{-j\frac{2\pi k m}{N}} \cdot e^{+j\frac{2\pi k N}{N}}$$

$$\therefore DFT\left[x(n-m+N)\right] = X(k) \cdot e^{-j\frac{2\pi k m}{N}} .$$

6. Circular Convolution: $x_1(n) \circledN x_2(n) \xleftrightarrow{DFT} X_1(k) \cdot X_2(k).$

Proof :- $DFT\left[x_1(n) \circledN x_2(n)\right] = \sum_{n=0}^{N-1} x_1(n) \circledN x_2(n)\, e^{-j\frac{2\pi k n}{N}}$

$$= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} x_1(m)\, x_2(n-m+N)\, e^{-j\frac{2\pi k n}{N}}$$

$$= \sum_{m=0}^{N-1} x_1(m)\, e^{-j\frac{2\pi k m}{N}} \cdot \sum_{n=0}^{N-1} x_2(n-m+N)\, e^{-j\frac{2\pi k (n-m+N)}{N}}$$

let $n-m+N = p$.

$$= \sum_{m=0}^{N-1} x_1(m)\, e^{-j\frac{2\pi k m}{N}} \sum_{p=0}^{N-1} x_2(p)\, e^{-j\frac{2\pi k p}{N}} = X_1(k)\, X_2(k)$$

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7. Mult^n of two sequences: $x_1(n)\,x_2(n) \xleftrightarrow{DFT} \frac{1}{N}\left(X_1(k) \text{ (N) } X_2(k)\right)$

8. Complex Conjugate: If $x(n)$ is complex,

$$x^*(n) \xleftrightarrow{DFT} X^*(-k)_N = X^*(N-k).$$

Proof:-

$$DFT\left[x^*(n)\right] = \sum_{n=0}^{N-1} x^*(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$= \left[\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n}{N}(N-k)} \times \underbrace{e^{j\frac{2\pi n \cancel{N}}{\cancel{N}}}}_{1}\right]^*$$

$$= \left[\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n(N-k)}{N}}\right]^* = X^*(N-k).$$

9. Parseval's theorem:

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2$$

Proof:-

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n)\, x^*(n).$$

$$= \sum_{n=0}^{N-1} x(n)\left[\frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi kn}{N}}\right]^*$$

$$= \sum_{n=0}^{N-1} x(n)\left(\frac{1}{N}\sum_{k=0}^{N-1} X^*(k)\, e^{-j\frac{2\pi kn}{N}}\right)$$

$$= \frac{1}{N}\sum_{k=0}^{N-1}\left(\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}\right) \cdot X^*(k).$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k) \cdot X^*(k) = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2$$

Circular Convolution using DFT & IDFT:

$$x_1(n) \text{ (N) } x_2(n) \xleftrightarrow{DFT} X_1(k)\, X_2(k).$$

Problem:

6. Find circular convolution of $x_1(n) = \{1, 1, 2, 1\}$ & $x_2(n) = \{1, 2, 3, 4\}$ using DFT & IDFT approach.

Sol:-

$X_1(k) = \{5, -1, 1, -1\}$

$X_2(k) = \{10, -2+2j, -2, -2-2j\}$

$X(k) = \{50, 2-2j, -2\ 2+2j\}$

Find IDFT $\{X(k)\}$

$x(n) = \{13, 14, 11, 12\}$.

Sectioned Convolution:

If one of the sequences in convolution is very longer than the other, then it is difficult to perform linear convolution.

i) The entire sequence must be available ~~before convolution~~ to carry out convolution. This makes long delay.

ii) Large amt of mem is req^d to store the sequences.

Two methods to filter sectioned data.

i) Overlap add method

ii) Overlap save method.

Problem:

1. Find the o/p $y(n)$ of a filter whose $h(n) = \{1, 1, 1\}$ & $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using overlap add method.

Sol:-

$h(n) = \{1, 1, 1\}$.

$L_s \rightarrow$ Length of section $= 3$.

$M \rightarrow$ Length of $h(n) = 3$.

$N = L_s + M_s - 1 = 3 + 3 - 1 = 5$.

Divide the longer length sequence into sections of length $L_s = 3$.

$x_1(n) = \{3, -1, 0, 0, 0\}$

$x_2(n) = \{1, 3, 2, 0, 0\}$

$x_3(n) = \{0, 1, 2, 0, 0\}$

$x_4(n) = \{1, 0, 0, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0\}$

length of the o/p sequence $= L + M - 1$

$= 10 + 3 - 1 = 12$

$$y_1(n) = x_1(n) \textcircled{N} h(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ -1 \\ 0 \end{bmatrix}$$

$$y_2(n) = x_2(n) \textcircled{N} h(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 5 \\ 2 \end{bmatrix}$$

$y_3(n) = [0 \ 1 \ 3 \ 3 \ 2]$ $\quad y_4(n) = [1 \ 1 \ 1 \ 0 \ 0]$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$y_1(n)$	3	2	2	-1	0									
$y_2(n)$				1	4	6	5	2						
$y_3(n)$							0	1	3	2	2	1	0	0
$y_4(n)$										1	1	1	0	0
$y(n)$	3	2	2	0	4	6	5	3	3	4	3	1		

Verification:

	3	-1	0	1	3	2	0	1	2	1
1	3	-1	0	1	3	2	0	1	2	1
1	3	-1	0	1	3	2	0	1	2	1
1	3	-1	0	1	3	2	0	1	2	1

$y(n) = [3 \ 2 \ 2 \ 0 \ 4 \ 6 \ 5 \ 3 \ 3 \ 4 \ 3 \ 1]$.

8. Find linear convolution of $x(n) = \{1, 2, -1, 2, 3, -2, -1, 1, 1, 2, -1\}$ & $h(n) = \{1, 2\}$ using overadd method.

sol:- Let $L_s = 3$ & $M_s = 2 \Rightarrow N = L_s + M_s - 1 = 4$

Length of the o/p sequence $= L + M - 1 = 11 + 2 - 1 = 12$

$\therefore x_1(n)=\{1,2,-1,0\}$ $x_2(n)=\{2,3,-2,0\}$ $h(n)=\{1,2,0,0\}$

$x_3(n)=\{-1,1,1,0\}$ $x_4(n)=\{2,-1,0,0\}$

$$y_1(n)=h(n)\; Ⓝ \; x_1(n)=\begin{bmatrix}1&0&0&2\\2&1&0&0\\0&2&1&0\\0&0&2&1\end{bmatrix}\begin{bmatrix}1\\2\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\4\\3\\-2\end{bmatrix}$$

sp → 0

$$y_2(n)=h(n)\; Ⓝ \; x_2(n)=\begin{bmatrix}1&0&0&2\\2&1&0&0\\0&2&1&0\\0&0&2&1\end{bmatrix}\begin{bmatrix}2\\3\\-2\\0\end{bmatrix}=\begin{bmatrix}2\\7\\+4\\-4\end{bmatrix}$$

sp → 3

$$y_3(n)=h(n)\; Ⓝ \; x_3(n)=\begin{bmatrix}1&0&0&2\\2&1&0&0\\0&2&1&0\\0&0&2&1\end{bmatrix}\begin{bmatrix}-1\\1\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\-1\\3\\2\end{bmatrix}$$

sp = 6

$$y_4(n)=h(n)\; Ⓝ \; x_4(n)=\begin{bmatrix}1&0&0&2\\2&1&0&0\\0&2&1&0\\0&0&2&1\end{bmatrix}\begin{bmatrix}\cancel{4}\,2\\-1\\0\\0\end{bmatrix}=\begin{bmatrix}2\\3\\-2\\0\end{bmatrix}$$

sp = 9

n	0	1	2	3	4	5	6	7	8	9	10	11	12
$y_1(n)$	1	4	3	-2									
$y_2(n)$				2	7	+4	-4						
$y_3(n)$							-1	-1	3	2			
$y_4(n)$										2	3	-2	0
$y(n)$	1	4	3	0	7	+4	~~3~~ -5	-1	3	4	3	-2	0

$y(n)=\{1, 4, 3, 0, 7, +4, -5, -1, 3, 4, 3, -2\}$

Verification:

h(n) \ x(n)	1	2	-1	2	3	-2	-1	1	1	2	-1
1	1	2	-1	2	3	-2	-1	1	1	2	-1
2	2	4	-2	4	6	-4	-2	2	2	4	-2

$\Rightarrow y(n)=\{1,4,3,0,7,4,-5,-1,3,4,3,-2\}$

Scanned by CamScanner

9. Find the o/p $y(n)$ whose impulse response is $h(n) = \{1, 1, 1\}$ & the i/p sequence is $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using overlap save method.

Sol:- Let $L = 3$ & $M = 3$.

Add $(M-1)$ no. of zeros prior to the first section. & take $(M-1)$ data points from the previous section.

$x_1(n) = \{0, 0, 3, -1, 0\}$ $\quad$ $x_2(n) = \{-1, 0, 1, 3, 2\}$

$x_3(n) = \{3, 2, 0, 1, 2\}$ $\quad$ $x_4(n) = \{1, 2, 1, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0\}$

$$y_1(n) = x_1(n) \textcircled{N} h(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

$sp = -2$

$$y_2(n) = x_2(n) \textcircled{N} h(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 4 \\ 6 \end{bmatrix}$$

$sp = 1$

$$y_3(n) = x_3(n) \textcircled{N} h(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 5 \\ 3 \\ 3 \end{bmatrix}$$

$sp = 4$

$$y_4(n) = x_4(n) \textcircled{N} h(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

$sp = 7$

n	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
$y_1(n)$	-1	0	3	2	2									
$y_2(n)$				4	1	0	4	6						
$y_3(n)$							6	7	5	3	3			
$y_4(n)$										1	3	4	3	1
$y(n)$			3	2	2	0	4	6	5	3	3	4	3	1

10. Perform the linear convolution of $h(n) = \{1, 1, 2, 1\}$ & $x(n) = \{1, -1, 1, 2, 1, 0, 1, -4, 3, 2, 1, 0, 1, 1\}$ using overlap add & save methods.

sol: $L = 3$; $M = 4$; $L_s + M - 1 = 3 + 4 - 1 = 6$

Total length $= L + M - 1 = 14 + 4 - 1 = 17$

$y(n) = \{1, 0, 2, 2, 4, 6, 5, -2, 1, -2, 5, 8, 5, 3, 3, 3, 1\}$

Fast Fourier Transform (FFT).

Fourier transform

1. Gives the freq info for an aperiodic signal
2. Freq spectrum is continuous

Fourier series

1. Gives the harmonic content of a periodic signal
2. spectrum is discrete.

DTFT

1. Sampling is performed only in time domain
2. Spectrum is continuous

DFT

1. Sampling is performed in both time & freq domains.
2. spectrum is discrete

FFT reduces the computation time required to compute a DFT and improves the performance.

Computations required in DFT:

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK} \quad \text{where } W_N = e^{\frac{-j2\pi}{N}}$$

$$= x(0) W_N^0 + x(1) W_N^k + x(2) W_N^{2k} + x(3) W_N^{3K} \quad (\text{for } n = 0 \text{ to } 3)$$

for $K = 0$ to 3

Each $X(K)$ $\begin{cases} \text{complex mult}^{ns} = 4 \\ \text{complex add}^{ns} = 3 \end{cases}$

$\therefore$ DFT requires $4 \times 4 = N^2 \rightarrow CM$

$3 \times 4 = (N-1) N \rightarrow CA$

CM $-$ $(a+jb)(c+jd) = ac + jad + jbc - bd$

$= 4$ real mult^{ns}

$= (ac - bd) + j(ad + bc) = 2$ real add^{ns}

CA $-$ $(a+jb) + (c+jd) = a + c + j(b+d) = 2$ real add^{ns}.

$\therefore$ Total RM $= 4N^2$

RA $= \underbrace{2N^2}_{\text{from CM}} + \underbrace{2N(N-1)}_{\text{from CA}}$.

FFT reduces the no. of mult^{ns} & add^{ns}. It uses the two basic properties of the twiddle factor.

Symmetry prop: $W_N^{K+N/2} = -W_N^K$

Periodicity prop: $W_N^{K+N} = W_N^K$.

The no. of complex multiplications reduces from N^2 to $\frac{N}{2}\log_2 N$.

Two algorithms of FFT:

1. Radix-2 DITFFT (Decimation-in-Time)
2. Radix-2 DIFFFT (Decimation-in Freq).

Radix-2 DITFFT: No. of o/p points N can be expressed as a power of 2 i.e. $N = 2^M$.

$W_N^1 = e^{-j2\pi/N}$

$x(0)$, $x(1)$, $x(2)$, $x(3)$, $x(4)$, $x(5)$, $x(6)$, $x(7)$

$\left.\begin{matrix} x(0) \\ x(2) \\ x(4) \\ x(6) \end{matrix}\right\} x_e(n) = x(2n)$

$\left.\begin{matrix} x(1) \\ x(3) \\ x(5) \\ x(7) \end{matrix}\right\} x_o(n) = x(2n+1)$

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nK} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)K}$$

$$X(K) = \sum_{n=0}^{\frac{N}{2}-1} x_e(n) W_N^{2nK} + \sum_{n=0}^{\frac{N}{2}-1} x_o(n) W_N^{2nK} W_N^K$$

$$W_N^2 = \left(e^{-j\frac{2\pi}{N}}\right)^2 = e^{-\frac{j2\pi}{N/2}} = W_{N/2}^1$$

$$\Rightarrow X(K) = \sum_{n=0}^{N/2-1} x_e(n) W_{N/2}^{nK} + \sum_{n=0}^{\frac{N}{2}-1} x_o(n) W_{N/2}^{nK} W_N^K$$

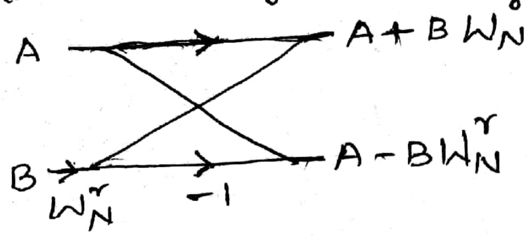
$$= \sum_{n=0}^{\frac{N}{2}-1} x_e(n) W_{N/2}^{nK} + W_N^K \sum_{n=0}^{\frac{N}{2}-1} x_o(n) W_{N/2}^{nK}.$$

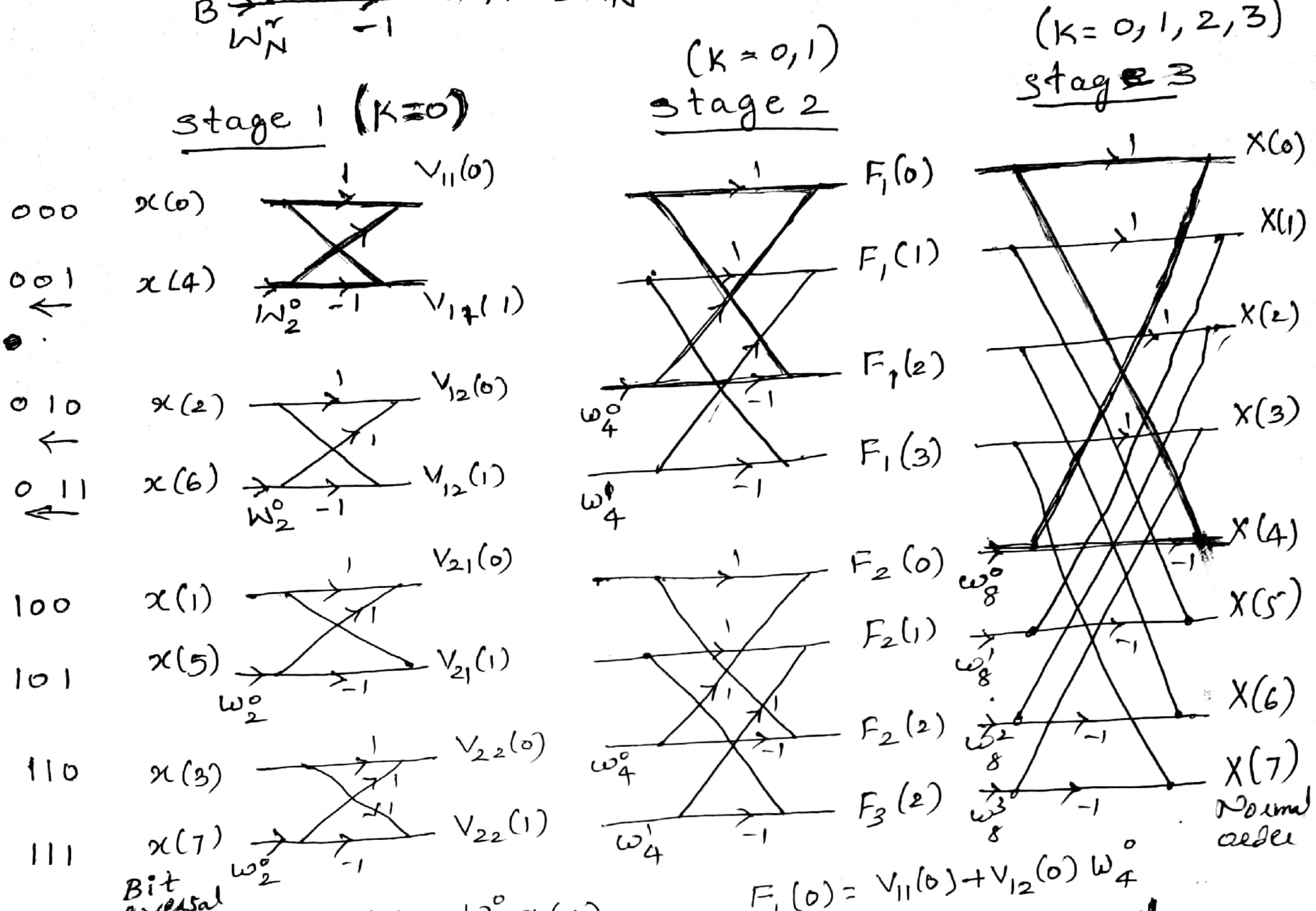
$$X(k) = X_e(K) + W_N^K X_o(K).$$

$$X\left(k+\frac{N}{2}\right) = X_e(K) + W_N^{K+N/2} X_o(K). \text{ (symm. prop).}$$

$$= X_e(K) - W_N^K X_o(k). \quad \left(\because W_N^{N/2} = e^{-j\frac{2\pi}{N}\times\frac{N}{2}} = e^{-j\pi} = -1\right)$$

Basic butterfly diagram of DITFFT.





$V_{11}(0) = x(0) + W_2^0\, x(4)$

$V_{11}(1) = x(0) - W_2^0\, x(4)$

$V_{12}(0) = x(2) + W_2^0\, x(6)$

$V_{12}(1) = x(2) - W_2^0\, x(6)$

$V_{21}(0) = x(1) + W_2^0\, x(5)$

$V_{21}(1) = x(1) - W_2^0\, x(5)$

$V_{22}(0) = x(3) + W_2^0\, x(7)$

$V_{22}(1) = x(3) - W_2^0\, x(7)$

← stage -1

$F_1(0) = V_{11}(0) + V_{12}(0)\, W_4$

$F_1(1) = V_{11}(1) + V_{12}(1)\, W_4^1$

$F_1(2) = V_{11}(0) - V_{12}(0)\, W_4^0$

$F_1(3) = V_{11}(1) - V_{12}(1)\, W_4^1$

$F_2(0) = V_{21}(0) + W_4^0\, V_{22}(0)$

$F_2(1) = V_{21}(1) + W_4^1\, V_{22}(1)$

$F_2(2) = V_{21}(0) - W_4^0\, V_{22}(0)$

$F_2(3) = V_{21}(1) - W_4^1\, V_{22}(1).$

Stage - 2 →

stage-3

$X(0) = F_1(0) + W_8^0\, F_2(0)$

$X(1) = F_1(1) + W_8^1\, F_2(1)$

$X(2) = F_1(2) + W_8^2\, F_2(2)$

$X(3) = F_1(3) + W_8^3\, F_2(3)$

$X(4) = F_1(0) - W_8^0\, F_2(0)$

$X(5) = F_1(1) - W_8^1\, F_2(1)$

$X(6) = F_1(2) - W_8^2\, F_2(2)$

$X(7) = F_1(3) - W_8^3\, F_2(3)$

Steps of radix-2 DIT-FFT algorithm.

1. No. of i/p samples. $\boxed{N = 2^M}$, M is an integer.
2. The input sequence is shuffled thro' bit-reversal.
3. No. of stages in the flowgraph $\boxed{M = \log_2 N}$
4. Each stage consists of $\boxed{\frac{N}{2}}$ butterflies.
5. Inputs/o/ps for each butterfly are separated by $\boxed{2^{m-1}}$ samples, where m represents the stage index, i.e. for first stage m=1, second stage m=2.
6. No. of sets of butterflies needed are $\boxed{2^{M-m}}$.

No. of complex multiplications in each stage are $\frac{N}{2}$.

→ No. of complex multiplications

∵ $W_N^k \, x_0(2n+1)$ is added & subtracted in each butterfly. ∴ Total no. of complex mult^{n}s are $\frac{N}{2} \times \log_2 N$.

No. of complex add^{n}s in each stage are N.

→ No. of complex add^{n}s

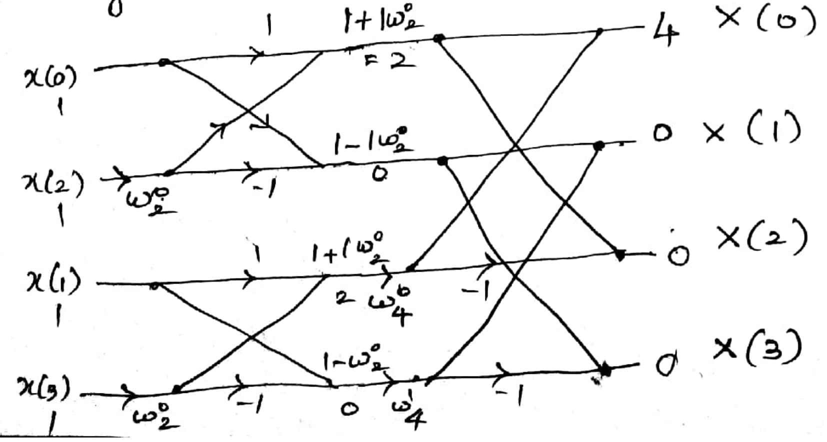
Total no. of complex add^{n}s are $N \log_2 N$.

Comparison of DFT & FFT

No. of points N	No. of complex Mult^{n}s using DFT = N^2	FFT = $\frac{N}{2}\log_2 N$	Speed improvement factor $\frac{N^2}{(N/2)\log_2 N}$
4	16	4	4
8	64	12	5.33
16	256	32	8

11. Compute 4-point DFT of the sequence $x(n) = \{1,1,1,1\}$ using radix-2 DITFFT algorithm.

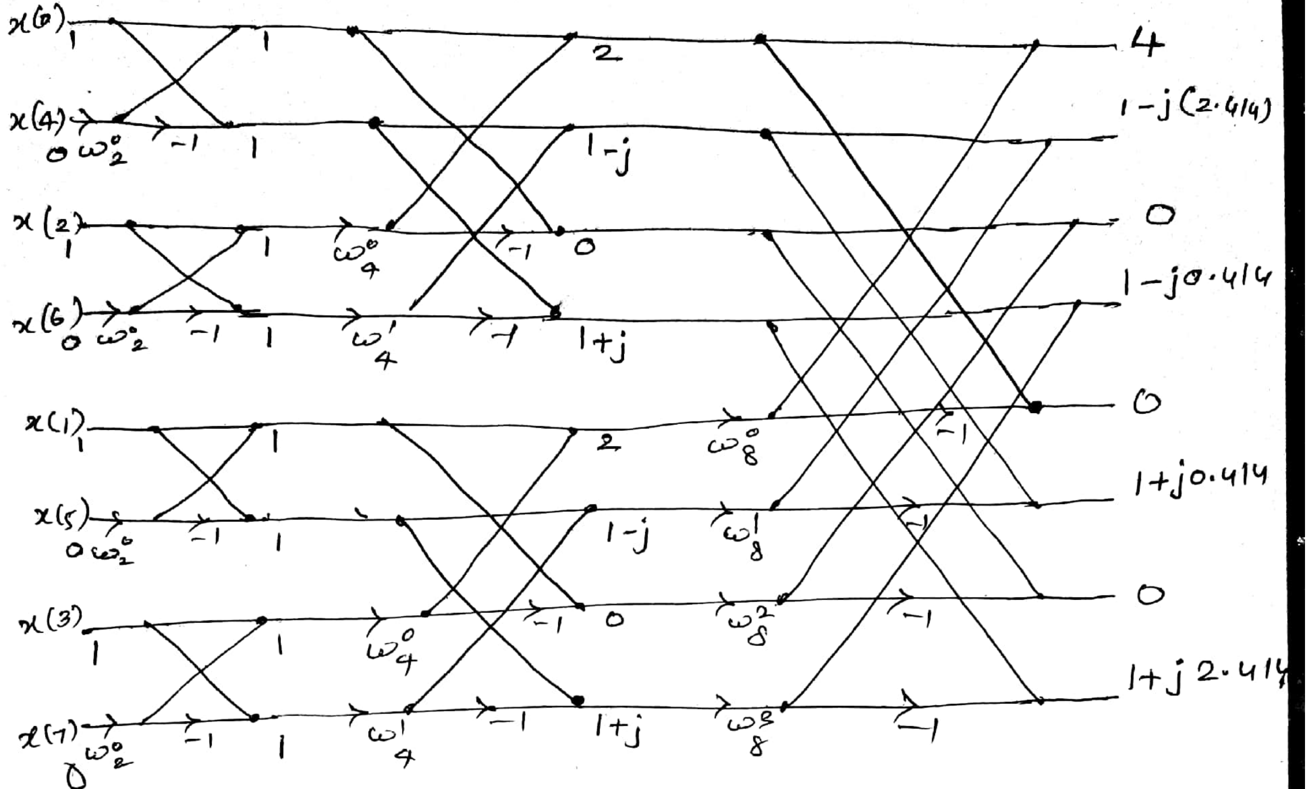
Sol:



∴ X(K) = {4, 0, 0, 0}.

12. Find 8-point DFT of $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$ using radix-2 DITFFT algorithm.

Sol:-



$\therefore X(K) = \{4, 1-2.414j, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414\}$.

$W_8^0 = 1$ $\qquad$ $W_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

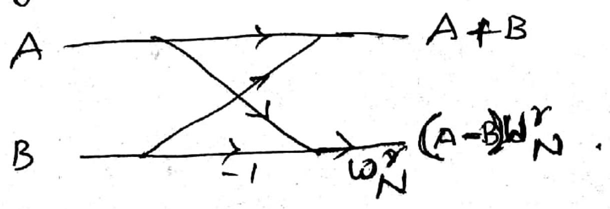
$W_8^2 = -1$ $\qquad$ $W_8^3 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

13. Find 8-point DFT of $x(n) = \{1, 1, 1, 1, 1, 1, 1, 1\}$ using radix-2 DIT FFT algorithm.

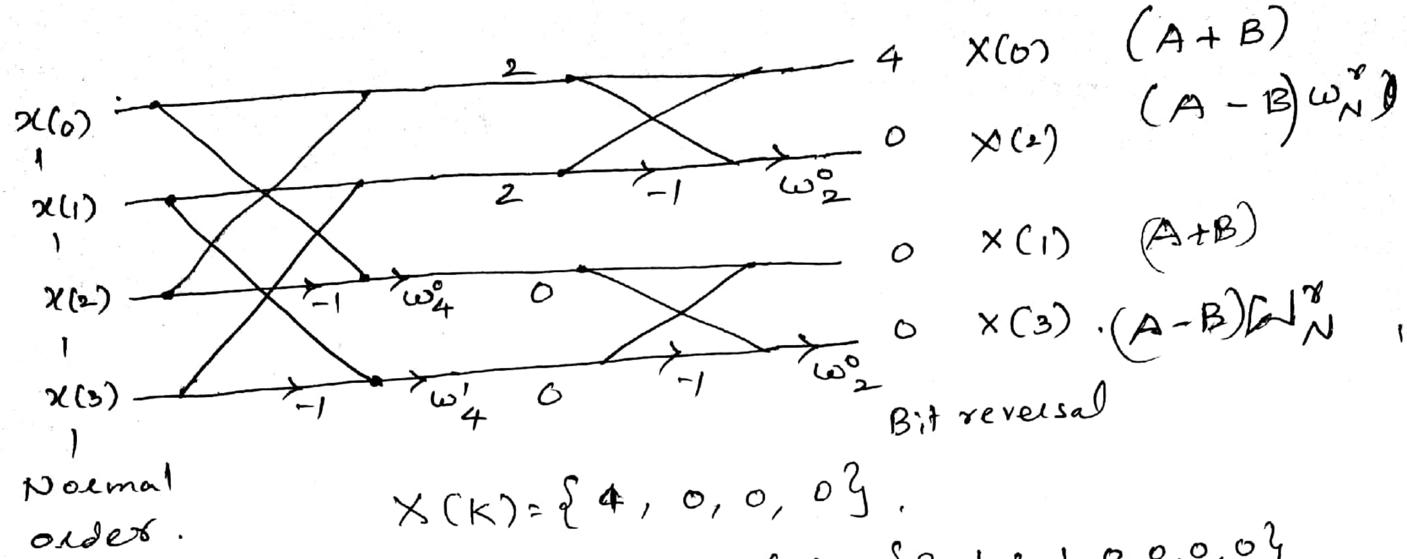
Sol:- $X(K) = \{8, 0, 0, 0, 0, 0, 0, 0\}$.

Radix-2 DIF FFT (Decimation-in freq FFT)

1) The inputs are in normal order but o/ps are in bit-reversal order for DIF FFT.
2) For N=8, stage 3 of DITFFT will become first stage of DIFFFT.
3) Basic butterfly diagram is



14. Find 4-point DFT of $x(n) = \{1, 1, 1, 1\}$ using radix-2 DIF FFT algorithm.



$X(K) = \{4, 0, 0, 0\}$.

15. Find 8-point DFT of $x(n) = \{2, 1, 2, 1, 0, 0, 0, 0\}$ using a) DIT FFT b) DIF FFT.

Sol:- $X(K) = \{6, 2 - j3.414, 0, 2 + j0.586, 2, 2 - j0.586, 0, 2 + j3.414\}$

16. Find 4-point DFT of $x(n) = \{1, 2, 3, 4\}$ using a) DIT FFT b) DIF FFT algorithms.

Sol:- $X(K) = \{10, -2+2j, -2, -2-2j\}$.

17. Draw 16-point radix-2 DIT FFT butterfly diagram.

18. Find IDFT of $X(K) = \{10, -2+2j, -2, -2-2j\}$ using a) DIT IFFT b) DIF IFFT methods

Sol:- $x(n) = \{1, 2, 3, 4\}$.

Mind-map for Unit I (Chapter 1)

Time-domain analysis of signals

— classification of signals

— operations on signals

Time-domain analysis of systems (To find response of the system).

— classification of systems

Freq-domain analysis of signals (To find freq of the signal)
(z-Transform: To find ROC)

— DTFT, Power & Energy spectrums.

Freq-domain analysis of systems (To find B.W. of the system)

— Diff. eqns of first & second order systems
(z-Transform: To find stability)

— Resonator.

Linear & circular convolutions.

Mind-map for Unit I (Chaper 2)

Z-transform for signals.

— ROC, Relation b/w F.T & Z.T

— properties, appns

(z-transform of system) System function

— poles & zeros of it.

— stability criterion.

Realization of digital filters. — Direct form-I, Direct form-II, Transform direct form, Cascade, parallel

— Recursive (IIR)

— Non-recursive (FIR)

Inverse z-transform methods. — Long Division, Partial fraction, Convolution, Residue.

Mind-map for Unit-II

DFT, IDFT

— properties

— Relationship with other transforms

Circular convolution using DFT & IDFT

sectioned convolution — overlap add
— overlap save

FFT, IFFT

— DIT FFT

— DIF FFT

FFT with general Radix

1) Develop Radix-3 DITFFT for $N=9$.

N pt seq is divided into 3 sequences of length 3 samples.

$$X(K) = \sum_{n=0}^{8} x(n)\, W_N^{nK}$$

$x(3n)$ $x(3n+1)$ $x(3n+2)$

$$\Rightarrow X(K) = \sum_{n=0}^{2} x(3n)\, W_N^{3nk} + \sum_{n=0}^{2} x(3n+1)\, W_N^{3nk} \cdot W_N^{k} + \sum_{n=0}^{2} x(3n+2)\, W_N^{3nK} \cdot W_N^{2K}$$

$$X(K) = X_1(K) + W_9^{K} X_2(K) + W_9^{2K} X_3(K)$$

$$\underline{X_1(K)} = \sum_{n=0}^{2} x(3n)\, W_9^{3nK}$$

$$X_1(0) = \sum_{n=0}^{2} x(3n)\, W_N^{0} = x(0) + x(3) + x(6)$$

$$X_1(1) = \sum_{n=0}^{2} x(3n)\, W_9^{3n} = x(0)\, W_9^{0} + x(3)\, W_9^{3} + x(6)\, W_9^{6}$$

$$X_1(2) = \sum_{n=0}^{2} x(3n)\, W_9^{6n} = x(0)\, W_9^{0} + x(3)\, W_9^{6} + x(6)\, W_9^{12}$$

$$\underline{X_2(K)} = \sum_{n=0}^{2} x(3n+1)\, W_9^{3nK}$$

$$X_2(0) = \sum_{n=0}^{2} x(3n+1)\, W_9^{0} = x(1) + x(4) + x(7)$$

$$X_2(1) = \sum_{n=0}^{2} x(3n+1)\, \omega_9^{3n} = x(1)\, \omega_9^{0} + x(4)\, \omega_9^{3} + x(7)\, \omega_9^{6}$$

$$X_2(2) = \sum_{n=0}^{2} x(3n+1)\, \omega_9^{6n} = x(1)\, \omega_9^{0} + x(4)\, \omega_9^{6} + x(7)\, \omega_9^{12}$$

$$\underline{X_3(K)} = \sum_{n=0}^{2} x(3n+2)\, \omega_9^{3nK}$$

$$X_3(0) = \sum_{n=0}^{2} x(3n+2) \cdot \omega_N^{0} = x(2) + x(5) + x(8)$$

$$X_3(1) = \sum_{n=0}^{2} x(3n+2)\, \omega_9^{3n} = x(2)\, \omega_9^{0} + x(5)\, \omega_9^{3} + x(7)\, \omega_9^{6}$$

$$X_3(2) = \sum_{n=0}^{2} x(3n+2)\, \omega_9^{6n} = x(2)\, \omega_9^{0} + x(5)\, \omega_9^{6} + x(7)\, \omega_9^{12}$$

$$\Rightarrow X(K) = X_1(K) + W_9^K X_2(K) + W_9^{2K} X_3(K)$$

$$X(0) = X_1(0) + W_9^0 X_2(0) + W_9^0 X_3(0)$$
$$= X_1(0) + X_2(0) + X_3(0).$$

$$X(1) = X_1(1) + W_9^1 X_2(1) + W_9^2 X_3(1)$$

$$X(2) = X_1(2) + W_9^2 X_2(2) + W_9^4 X_3(2).$$

$$X(3) = X_1(3) + W_9^3 X_2(3) + W_9^6 X_3(3) = X_1(0) + W_9^3 X_2(0) + W_9^6 X_3(0).$$

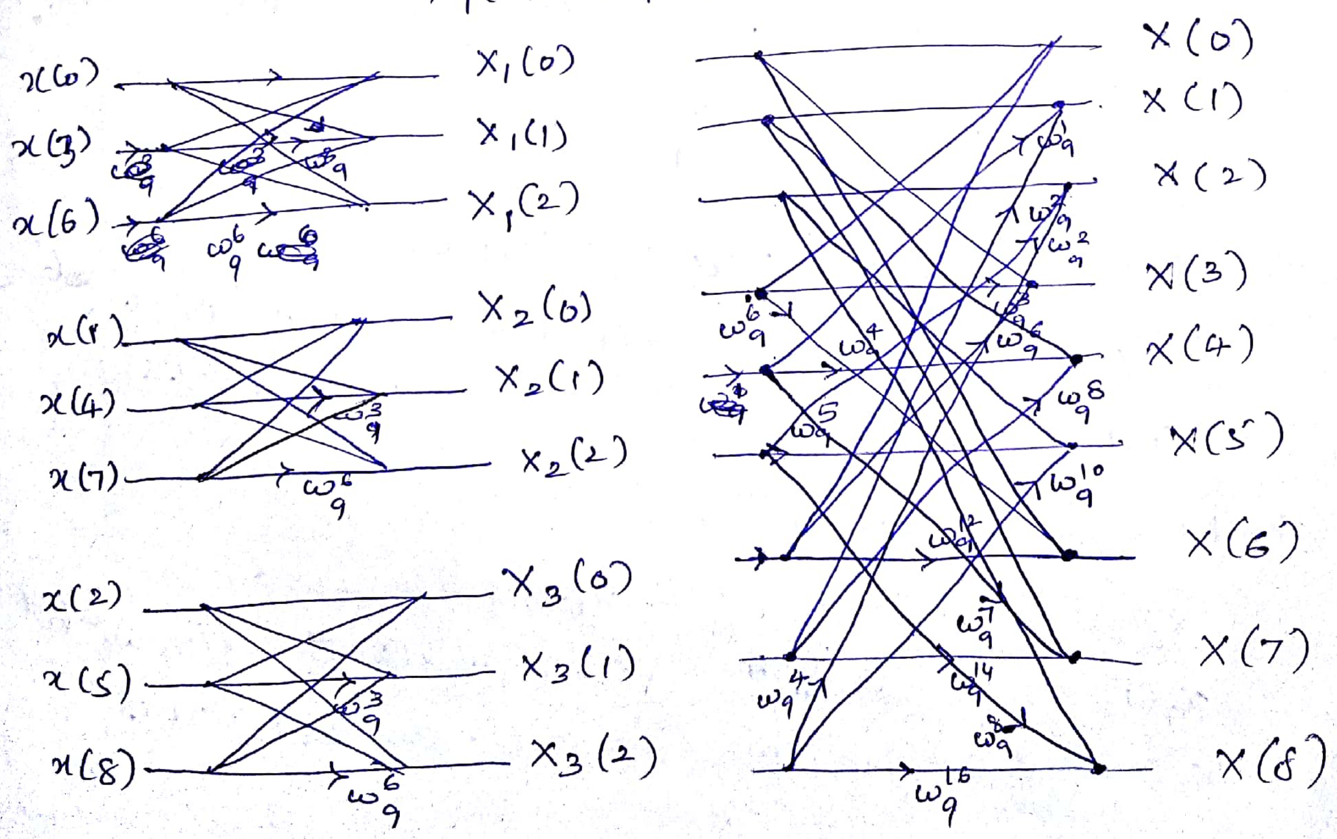
$$X(4) = X_1(4) + W_9^4 X_2(4) + W_9^8 X_3(4)$$
$$= X_1(1) + W_9^4 X_2(1) + W_9^8 X_3(1)$$

$$X(5) = X_1(5) + W_9^5 X_2(5) + W_9^{10} X_3(5)$$
$$= X_1(2) + W_9^5 X_2(2) + W_9^{10} X_3(2)$$

$$X(6) = X_1(6) + W_9^6 X_2(6) + W_9^{12} X_3(6)$$
$$= X_1(0) + W_9^6 X_2(0) + W_9^{12} X_3(0)$$

$$X(7) = X_1(7) + W_9^7 X \cdot X_2(7) + W_9^{14} X_3(7)$$
$$= X_1(1) + W_9^7 X_2(1) + W_9^{14} X_3(1)$$

$$X(8) = X_1(8) + W_9^8 X_2(8) + W_9^{16} X_3(8)$$
$$= X_1(2) + W_9^8 X_2(2) + W_9^{16} X_3(2)$$



Assignment - II DFT & FFT

1. Determine 8-point DFT of the sequence.

$$x(n) = \begin{cases} 1 ; & -4 \leq n \leq 3 \\ 0 ; & \text{otherwise} \end{cases}$$

2. Compute 4-point DFT of a sequence $x(n) = \{0, 1, 2, 3\}$ using DIT algorithm.

3. Find IDFT of the sequence using DIF algorithm

$$X(K) = \{10, -2-2j, -2, -2+2j\}$$

4. State and prove the properties of DFT.

5. Compute linear convolution of $x(n) = \{1, 3, 5, -4, -6, +8, 7, 12, 2, 4, -8\}$ and $h(n) = \{2, 1\}$ using overlap add and save methods.

6. Explain the number of computations required for N-point DFT and FFT.

7. Explain Radix-2 Decimation-in-Time algorithm.

8.