Steady state & Transient Response :-

A circuit having constant source is said to be in steady state if the current & voltages do not change with time. Thus, ckts with currents & voltages having constant amplitude & constant frequency, sinusoidal functions are also considered to be in a steady state. That means that the amplitude (or) frequency of a sinusoid never changes in steady state circuit.

In a N/W containing energy storage elements with change in excitation, the currents & voltages change from one state to another state. The behavior of the voltage (or) current when it is changed from one state to another is called the transient state. The time taken for ckt to change from one steady state to another steady state is called the transient time.

The application of KVL & KCL to circuits containing energy storage elements results in differential rather than algebraic equations. When we consider a ckt containing storage elements which are independent of the sources, the response depends upon the nature of the circuit and is called natural response.

Storage elements deliver their energy to the resistances. Hence, the response changes with time, gets saturated after some time, and is referred to as the transient response.

When we consider sources acting on a circuit, the response depends upon the nature of the source or ... this response is called forced response.

→ In otherwords, the complete response of ckt consists of two parts: The forced response and the transient transient response.

When we consider a differential eqn, The complete solution consists of two parts: the complementary function & particular solution. The complementary function dies out after a short interval and is referred to as the transient response (or) source free response. The particular solution is the steady state response (or) the forced response. The first step in finding the complete solution of ckt is to form a differential equation for the circuit. By obtaining the differential equation, several methods can be used to find out the complete solution.

* whenever a network containing energy storage elements such as inductor (or) capacitor is switched from one condition to another, either by change in applied source (or) change in network elements, the response current & voltage change from one state to the other state. The time taken to change from an initial steady state to the final steady state is known as the transient period (time). This response is known as transient response (or) transients. The response of the N/W after its attains a final steady value is independent of time & is called the steady-state response. The complete response of the N/W is determined by with the help of a differential equation.

Initial conditions :-

In solving the differential equations in the N/W, we get some arbitrary constant. Initial conditions are used to determine these arbitrary constants. It helps us to know the behaviour of elements at the instant of switching.

To differentiate between the time immediately before and immediately after the switching, the signs '−' & '+' are used. The conditions existing just before switching are denoted as $i(0^-)$, $v(0^-)$ etc. Conditions just after switching are denoted as $i(0^+)$, $v(0^+)$

Some times conditions at $t=\infty$ are used to In the evaluation of arbitrary constants. These are known as final condition.

In solving the problems for initial conditions in the network, we divide the time period in the following ways.

1. just before switching (from $t=-\infty$ to $t=0^-$)
2. just after switching (at $t=0^+$)
3. ~~...~~ after switching (for $t>0$)

If the N/W remains in one condition for a long time without any switching action, it is said to be under steady-state condition.

1. Initial Condition for the Resistor:-

For a resistor, current and voltage are related by $v(t)=Ri(t)$. The current through a resistor will change instantaneously if the voltage changes instantaneously. Similarly the voltage will change instantaneously if the current changes instantaneously.

2. Initial conditions for the Inductor :-

For an inductor, current & voltage are related by

$$\boxed{v(t) = L\frac{di}{dt}}$$

Voltage across the inductor is proportional to the rate of change of current. It is impossible to change the current through an inductor by a finite amount in zero time. This requires an infinite voltage across the inductor. So, an inductor doesnot allow sudden change in the current through it.

The current through the inductor is given by

$$i(t) = \frac{1}{L}\int_0^t v(t)\,dt + i(0)$$

where $i(0)$ is the initial current through the inductor.

→ If there is no current flowing through the inductor at $t=0^-$, the inductor will act as an open circuit at $t=0^+$.

→ If a current of value I_0 flows through the inductor at $t=0^-$, the inductor can be regarded as a current source of I_0 ampere at $t=0^+$.

3. Initial Conditions for the Capacitor:-

For the capacitor, current and voltage are related by $i(t) = C\frac{dv(t)}{dt}$

→ Current through a capacitor is proportional to the [rat]e of change of voltage. It is impossible to change [the] voltage across a capacitor by a finite amount [in] zero time. This requires an infinite current [through the c]apacitor. So, a capacitor does not allow an abrupt [chan]ge in voltage across it.

The voltage across the capacitor is given by,

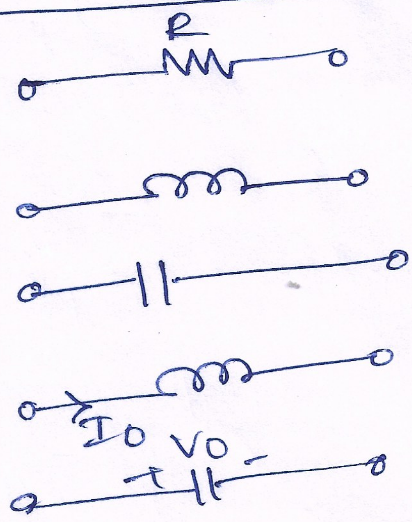
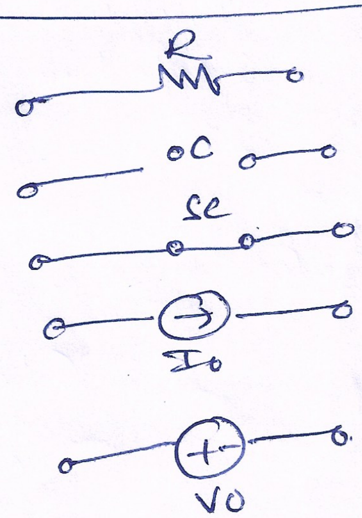
$$v(t) = \frac{1}{C}\int_0^t i(t)\,dt + v(0)$$

where $v(0)$ is the initial voltage across the capacitor.

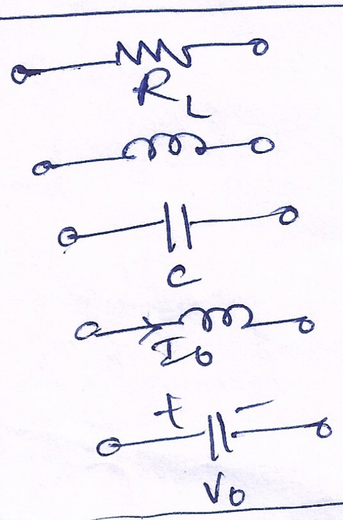
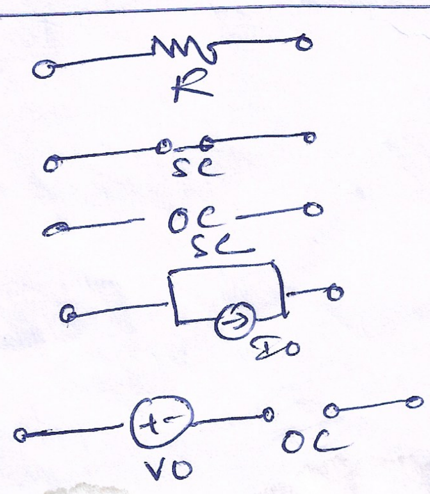
* → If there is no voltage across the capacitor at $t=0^-$, the capacitor will act as a short circuit at $t=0^+$.

* → If the capacitor is charged to a voltage (V_0) at $t=0^-$, it can be regarded as a voltage source of (V_0) volts at $t=0^+$.

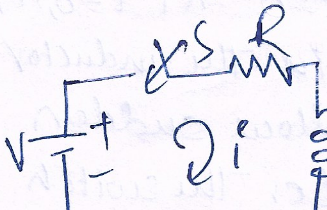
Initial Conditions

Element with initial conditions	Equivalent circuit at $t=0^+$
	

Final Conditions

Element with initial conditions	Equivalent circuit at $t=\infty$
	

DC Response of an R-L circuit:-



Fig(1)

Consider a circuit consisting of a resistance and inductance as shown fig(1). The inductor in the circuit is in initially uncharged ~~and~~ un-energised and in series with resistor.

when the switch 's' is closed, we can find the complete solution for the current.

Application of Kirchoff's voltage law to the CKt results in the following differential equation.

$$V = Ri + L\frac{di}{dt}$$

$$V = L\left(\frac{R}{L}i + \frac{di}{dt}\right) \Rightarrow \cancel{V} \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \rightarrow (1)$$

→ In the above eqn, the current is the solution to be found & 'V' is the applied constant voltage.

The voltage V is applied to the circuit only when the switch 's' is closed. The above eqn is a linear differential equation of first order.

→ Comparing it with a non-homogeneous differential equation $\frac{dx}{dt} + Px = K \rightarrow (2)$

Where P is constant and K may be a function of the independent variable t (or) a constant.

→ The solution of the above eqn is given by:

$$x(t) = e^{-Pt}\int K e^{+Pt} dt + ce^{-Pt} \rightarrow (3)$$

where c is an arbitrary constant & its value is obtained by putting t=0 in eqn x(t).

→ In a similar way, we can write the current eqn

$$i = ce^{-(R/L)t} + e^{-(R/L)t}\int \frac{V}{L}e^{(R/L)t} dt.$$

$$i = ce^{-(R/L)t} + \frac{V}{R} \rightarrow (4)$$

To determine the value of c in eqn (4), we use initial conditions. The switch 's' is closed at $t=0$. At $t=0^-$, i.e., just before closing the switch the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at $t=0^+$ just after the switch is closed, the current remains zero at $t=0^+$, just after the switch is closed

$t=0^+$, $i=0$, substitute above condition eqn in eqn (4).

$$0 = c + \frac{V}{R} \Rightarrow c = -\frac{V}{R}$$

Substituting the value of 'c' in eqn (4), we get

$$i = \frac{V}{R} - \frac{V}{R} e^{-(R/L t)} = \frac{V}{R}\left(1 - e^{-(R/L t)}\right) \rightarrow (5)$$

The eqn (5) consists of two parts, the steady state part V/R, and the transient part $V/R\, e^{-(R/L)t}$. When the switch 's' is closed, the response reaches a steady state value after a time interval as shown in fig (2).

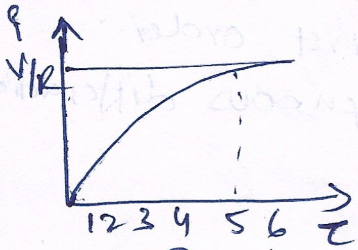


fig (1)

Here, the transition period is defined as the time taken for the current to reach its final (or) steady state value from its initial value. In the transient part of the solution, the quantity L/R is important in describing the curve since L/R is the time required for the current to reach from its initial value of zero to the final value V/R. The time constant of a function $\frac{V}{R} e^{-(R/L)t}$ is the time at which the exponent of e is unity, where 'e' is the base of natural logarithm. The term L/R is called time constant & is denoted by τ.

At one time constant, the current reaches 63.2% of its final value V/R.

$i(\tau) = 0.632 \frac{V}{R}$, $i(2\tau) = 0.865 \frac{V}{R}$, $i(3\tau) = 0.950\, V/R$

$i(5\tau) = 0.993\, V/R$.

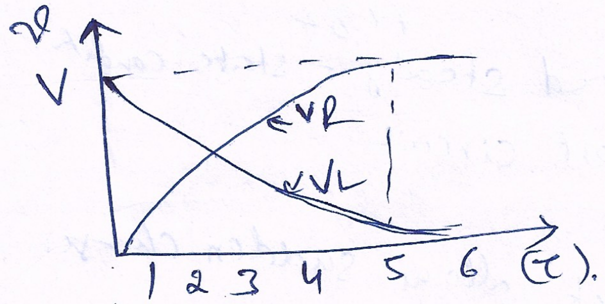
After 5τ, the current reaches 99.33 % of its final value.

The voltage across the resistor is

$$V_R = Ri = R\times\frac{V}{R}\left[1-e^{(-R/L\,t)}\right] = V\left[1-e^{(-R/L\,t)}\right]$$

The voltage across the inductor is

$$V_L = L\frac{di}{dt} = L\,\frac{V}{R}\times\frac{R}{L}\,e^{-(R/L\,t)} = V e^{(-R/L\,t)}$$



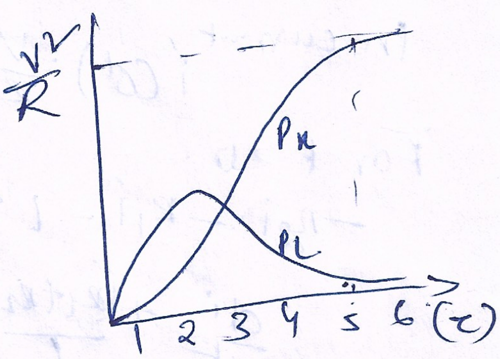
The power in the resistor is

$$P_R = v_R\, i = V\left[1-e^{(-R/L\,t)}\right]\frac{V}{R}\left[1-e^{(-R/L\,t)}\right]$$

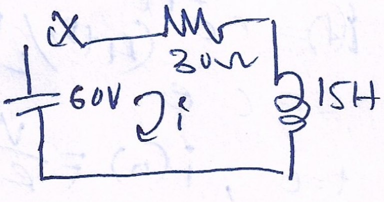
$$= \frac{V^2}{R}\left[1+e^{(-2R/L\,t)} - 2e^{(-R/L\,t)}\right]$$

$$P_L = V_L\, i = V e^{(-R/L\,t)}\,\frac{V}{R}\left[1-e^{(-R/L\,t)}\right]$$

$$= \frac{V^2}{R}\left[e^{(-R/L\,t)} - e^{(-2R/L\,t)}\right]$$



A series RL circuit with $R = 30\Omega$ & $L = 15H$ has a constant voltage $V = 60V$ applied at $t=0$. Determine the current i, the voltage across the resistor & the voltage across the inductor.



$$15\frac{di}{dt} + 30i = 60$$

$$\frac{di}{dt} + 2i = 4$$

$$i = c\,e^{-pt} + e^{-pt}\int k e^{pt}\,dt$$

$$i = c\,e^{-2t} + e^{-2t}\int 4 e^{2t}\,dt$$

$$i = c\,e^{-2t} + 2$$

At $t=0$, the switch is closed the inductor never allows sudden changes in currents, at $t=0^+$, the current in the circuit is zero.

$\therefore\ t=0^+ \quad i=0$

$\therefore\ 0 = c+2 \Rightarrow \therefore\ c = -2.$

$$i = 2 + (-2)e^{-2t}$$

$$= 2(1-e^{-2t})\,A.$$

$$V_R = iR = 2(1-e^{-2t})\times 30$$

$$= 60(1-e^{-2t})\,V$$

$$V_L = L\frac{di}{dt} = 15\times 2\times 2e^{-t}$$

$$= 60\,e^{-t}\,V.$$

2. In the network of Fig 1, the switch is initially at the position 1. On the steady state having reached, the switch is changed to the position 2. Find current $i(t)$.

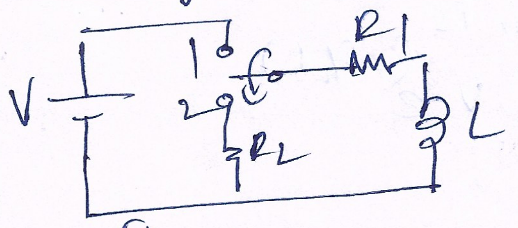


Fig 1

At $t=0^-$, the N/W shown in Fig 2:

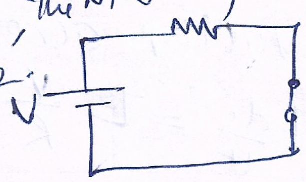


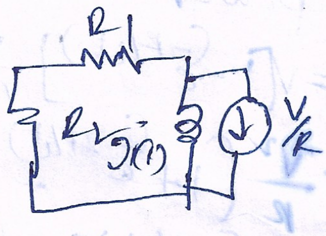
Fig 2

At $t=0^-$, the N/W has attained steady-state condition. Hence inductor acts as a short circuit.

$$i(0^-) = \frac{V}{R_1}$$

Since the inductor does not allow sudden change in current,

$$i(0^+) = \frac{V}{R_1}$$



For $t>0$.

$$-R_2 i - R_1 i - L\frac{di}{dt} = 0.$$

$$\frac{di}{dt} + \frac{(R_1+R_2)}{L} i = 0$$

$$\frac{di}{dt} + Pi = 0$$

$$P = \frac{R_1+R_2}{L}.$$

$$i(t) = ce^{-Pt}$$

$$= c\,e^{-(R_1+R_2/L\,t)}.$$

At $t=0$, $i(0) = \frac{V}{R_1}$

$$\frac{V}{R_1} = ce^0 = c$$

$$i(t) = \frac{V}{R_1} e^{-(R_1+R_2/L\,t)}.$$

In fig 1, the switch is closed at $t=0$, a steady state having previously been attained. Find the current $i(t)$.

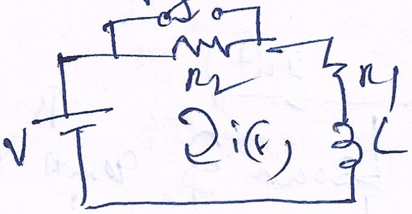


fig (1)

At $t=0^-$, the N/W is in steady-state & the equivalent ckt is fig (2).

$$i(0^-) = \frac{V}{R_1+R_2}.$$

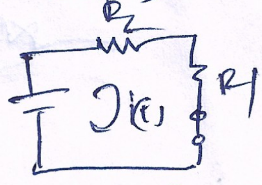
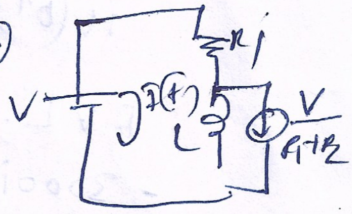


fig (2)

$$i(0^+) = \frac{V}{R_1+R_2}$$ since the inductor cannot accept sudden change instantaneously.

For $t>0$, the N/W is shown in fig (3)



$$V - R_1 i - L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R_1}{L} i = \frac{V}{L}$$

$$\frac{dx}{dt} + Px = K \qquad x(t) = e^{-Pt}\int K e^{Pt}\,dt + Ce^{-Pt}$$

$$P = \frac{R_1}{L}, \quad K = V/L$$

$$i(t) = e^{-R_1/L\,t}\int \frac{V}{L} e^{R_1/L\,t}\,dt + Ce^{-R_1/L\,t}$$

$$i(t) = \frac{V}{R_1} + Ce^{\left(-R_1/L\,t\right)}$$

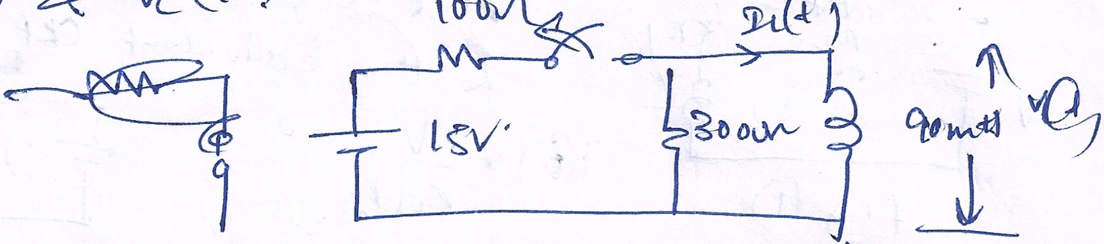
at $(t=0^+)$ $\quad i(t) = \frac{V}{R_1+R_2}$

$$\frac{V}{R_1+R_2} = \frac{V}{R_1} + C$$

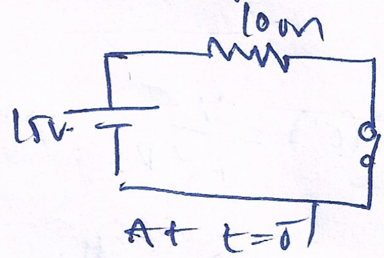
$$C = \frac{V}{R_1+R_2} - \frac{V}{R_1} = \frac{VR_1 - VR_1 - VR_2}{R_1(R_1+R_2)} = \frac{-VR_2}{R_1(R_1+R_2)}$$

$$i(t) = \frac{V}{R_1} - \frac{VR_2}{R_1(R_1+R_2)} e^{\left(-R_1/L\,t\right)} = \frac{V}{R_1}\left(1 - \frac{R_2}{R_1+R_2} e^{-R_1/L\,t}\right)$$

39. For the N/W shown in fig, steady state is reached with the switch closed. The switch is opend at $t=0$. Find $i_L(t)$ & $V_L(t)$

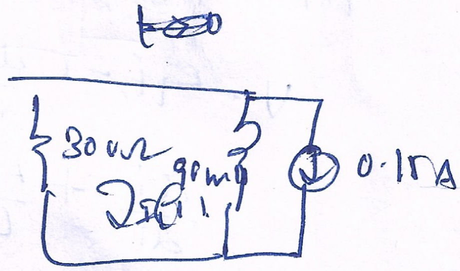


$$i_L(0^-) = \frac{15}{100} = 0.15A$$



$$i_L(0^+) = 0.15A.$$

KVL



$$-3000 i_L - 90\times10^{-3}\frac{di_L}{dt} = 0.$$

$$\frac{di_L}{dt} + 33.333\times10^{3}\, i_L = 0$$

$$\frac{di}{dt} + Pi = 0.$$

$$P = 33.33\times10^{3}$$

$$i_L(t) = k e^{-Pt} = k e^{-33.33\times10^{3}t}.$$

At $t=0^+$, $i_L(0) = 0.15A$.

$$k = 0.15$$

$$i_L(t) = 0.15\, e^{-33.33\times10^{3}t} \text{ for } t>0.$$

$$V_L(t) = L\frac{di}{dt}$$

$$= 90\times10^{-3} \times 0.15\, e^{-33.33\times10^{3}t} \times -33.33\times10^{3}$$

$$= -450\, e^{-33.33\times10^{3}t} \text{ for } t>0$$

5. The switch is moved from 1 to 2 at $t=0$. Determine $i(t)$.

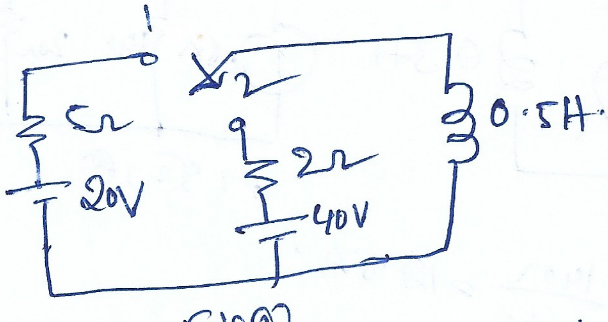


Fig ①

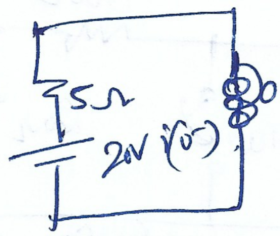


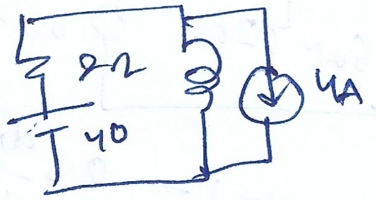
Fig ②

At $t=0^-$, the n/w is shown in Fig ②.

$i(0^-) = \frac{20}{5} = 4A$

$i(0^+) = 4A$ ∵ Current through the inductor cannot change instantaneously.

For $t>0$, the N/W is shown in Fig ③.



$40 - 2i - 0.5\frac{di}{dt} = 0$

$\frac{di}{dt} + 4i = 80$

$\frac{dx}{dt} + Px = Q$. $P=4$, $Q=80$

$i(t) = e^{-Pt}\int Q e^{Pt} dt + ke^{-Pt}$

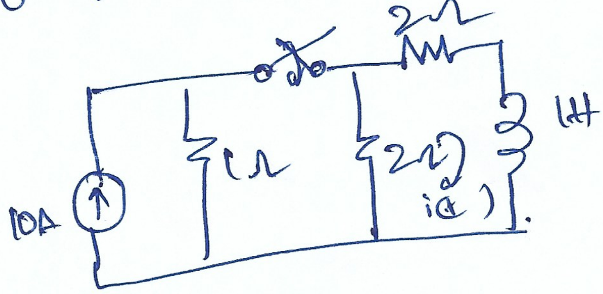
$= e^{-4t}\int 80 e^{4t} dt + ke^{-4t}$

$= \frac{80}{4} + ke^{-4t} = 20 + ke^{-4t}$

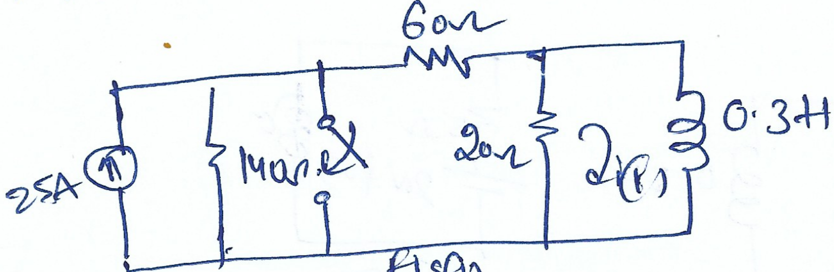
At $t=0$ $i(0) = 4A$ $k = -16$

$i(t) = 20 - 16e^{-4t}$ for $t>0$.

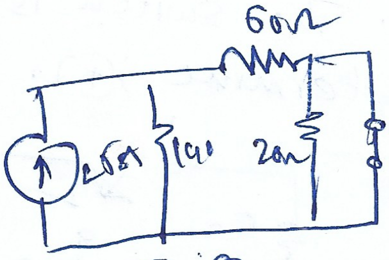
6. The switch is closed at $t=0$. Find $i(t)$ for $t>0$.



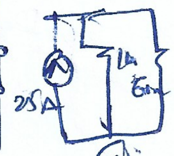
7. Find the current $i(t)$ for $t>0$.



Fig(1)



Fig(2)(a)



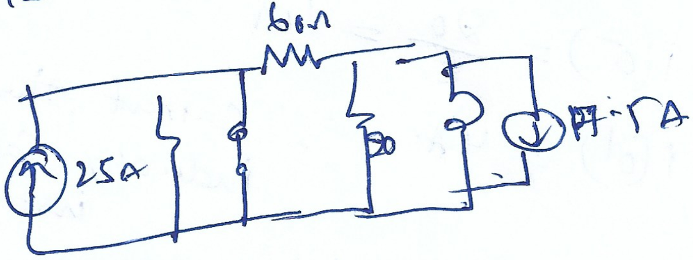
Fig(2)(b)

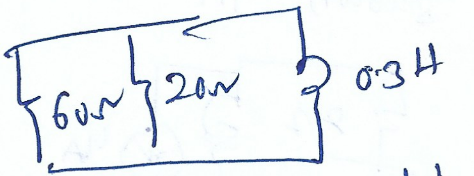
$t=0^-$

$$i(0^-) = 25 \times \frac{140}{140+60} = 17.5\,A.$$

$$i(0^+) = 17.5\,A.$$

for $t>0$





$$-15i - 0.3\frac{di}{dt} = 0.$$

$$\frac{di}{dt} + 50i = 0, \quad p = 50$$

$$i(t) = ke^{-pt} = ke^{-50t}$$

At $t=0$ $\quad i(0) = 17.5\,A$

$$i(t) = 17.5\,e^{-50t} \text{ for } t>0$$

DC Response of An R-C circuit:-

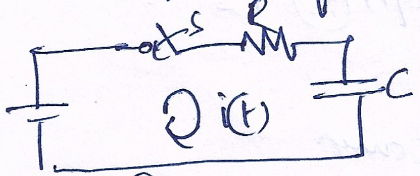


fig 1.

Consider a circuit consisting of resistance and capacitance as shown in fig 1.

The capacitor in the circuit is initially uncharged and is in series with a resistor. When the switch 's' is closed at $t=0$, we can determine the complete solution for the current. Application of the KVL to the ckt results in the following differential equation.

$$V = Ri + \frac{1}{C}\int i\,dt \rightarrow (1)$$

By differentiating the above eqn, we get

$$0 = R\frac{di}{dt} + \frac{i}{C}$$

$$\frac{di}{dt} + \frac{i}{RC} = 0 \rightarrow (2)$$

eqn (2) is a linear differential equation with only the complementary function. The particular solution for the above equation is zero.

→ comparing it with a homogeneous equation

$$\frac{dx}{dt} + Px = 0 \rightarrow (3)$$, where P is constant.

The solution of this equation is given by

$$x(t) = Ce^{-Pt} \rightarrow (4)$$, The value of C is obtained by putting $t=0$ in eqn $x(t)$

→ In a similar way, we can write the current eqn as.

$$i(t) = Ce^{(-t/RC)} \rightarrow (5)$$

To find the value of C, we use the initial conditions. In the circuit, the switch 's' is closed at $t=0$. Since the capacitor never allows sudden changes in voltage, it will act as a short circuit at $t=0^+$.

So, the current in the circuit at $t=0^+$ is V/R.

substituting this current in eqn (5) is

$$\frac{V}{R} = c$$

The current equation becomes

$$i = \frac{V}{R} e^{-t/RC} \rightarrow (6)$$

When the switch 's' is closed, the response decays with time as shown in Fig (2).

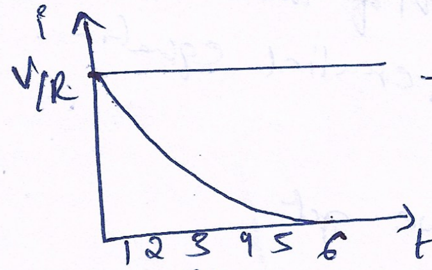


Fig (2)

In the solution, the quantity RC is the time constant, and is denoted by τ.

where $\tau = RC$ seconds.

After 5τ, the curve reaches 99% of its final value.

Voltage across the resistor is

$$V_R = Ri = R \times \frac{V}{R} e^{-t/RC} = Ve^{-t/RC} \rightarrow (7)$$

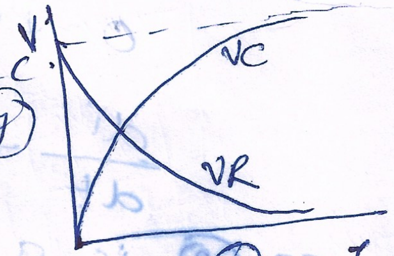


Fig (3)

Voltage across the capacitor is

$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int \frac{V}{R} e^{-t/RC} dt$$

$$= -\left(\frac{V}{RC} \times RCe^{-t/RC}\right) + c = -Ve^{-t/RC} + c$$

At $t=0$ voltage across capacitor is zero.

$$c = V$$

$$V_C = V(1 - e^{-t/RC}) \rightarrow (8)$$

power in the resistor

$$P_R = V_R i = Ve^{-t/RC} \times \frac{V}{R} e^{-t/RC} = \frac{V^2}{R} e^{-2t/RC}$$

power in the capacitor

$$P_C = V_C i = V(1 - e^{-t/RC}) \times \frac{V}{R} e^{-t/RC}$$

$$= \frac{V^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)$$

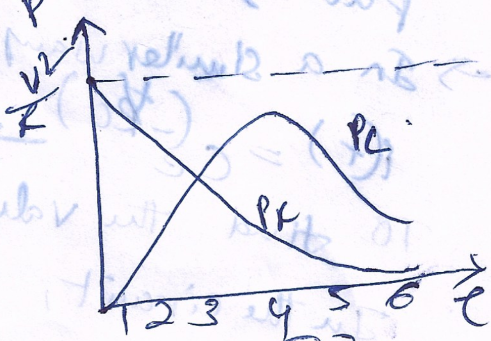
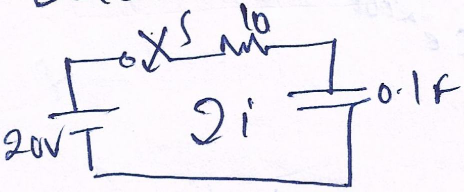


Fig (4)

1. A series RC circuit consists of 10Ω & a capacitor of 0.1 F as shown in fig ①. A constant voltage of 20V is applied to the circuit at $t=0$. Obtain the current equation. Determine the voltage across the resistor & the capacitor.



By applying KVL.

$$10i + \frac{1}{0.1}\int i\,dt = 20.$$

Differentiate with respect to t, we get.

$$10\frac{di}{dt} + \frac{i}{0.1} = 0 \Rightarrow \frac{di}{dt} + i = 0$$

The solution for the above eqn is $i = Ce^{-t}$

At $t=0$, the switch 's' is closed. Since the capacitor does not allow sudden changes in the voltage, the current in the ckt is $i = V/R = 20/10 = 2$ A.

At $t=0$, $i = 2A$

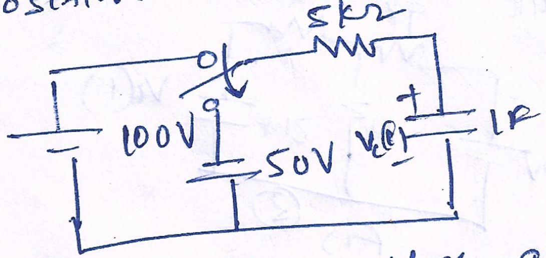
The current equation $i = 2e^{-t}$

Voltage across the resistor is $V_R = i \times R = 2e^{-t} \times 10$
$= 20e^{-t}$ V.

voltage across the capacitor is $V_C = V(1 - e^{-t/RC})$

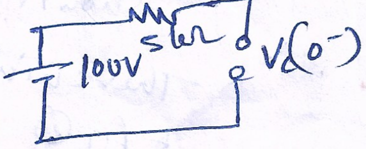
$$= 20(1 - e^{-t/RC})\,V$$

2. The switch in the circuit of fig moved from the position 1 to 2 at $t=0$. Find $V_C(t)$.



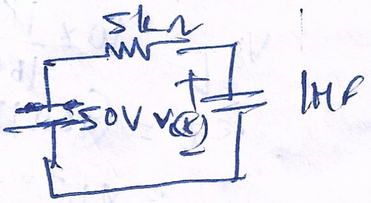
At $t=0^-$, the N/w has attained steady state condition. Hence, the capacitor acts as an open circuit.

$V_C(0^-) = 100V$.



Since the voltage across the capacitor cannot change instantaneously,

$V_C(0^+) = 100V$



For $t>0$,

writing the KCL eqn for $t>0$

$$1\times10^{-6}\frac{dV_C}{dt} + \frac{V_C + 50}{5000} = 0$$

$$\frac{dV_C}{dt} + 200V_C = 10^4$$

Comparing with the differential eqn $\frac{dv}{dt} + Pv = Q$.

$P = 200, Q = 10^4$

$$V_c(t) = e^{-Pt}\int Q e^{Pt}\,dt + k e^{-Pt}$$

$$= e^{-200t}\int 10^4 e^{200t}\,dt + k e^{-200t}$$

$$= -50 + k e^{-200t}$$

At $t = 0$, $V_c(0) = 100\,V$ $\quad 100 = -50 + k$

$k = 150$

$$V_c(t) = -50 + 150 e^{-200t}$$

3. In the network, the switch closes at $t=0$. The capacitor is initially uncharged. Find $V_c(t)$ & $i(t)$.

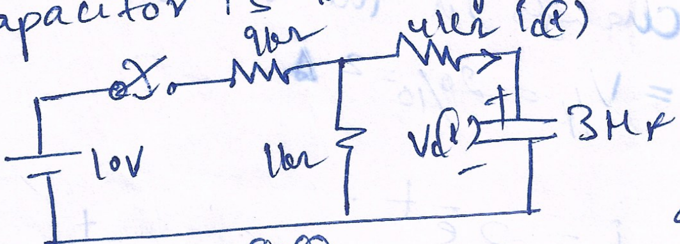


Fig ①

At $t = 0^-$, the capacitor is uncharged. Hence, it acts as short circuit

$V_c(0) = 0$, $i_c(0) = 0$.

At $t = 0^+$, the N/W is shown in Fig ②.

$V_c(0^+) = 0$.

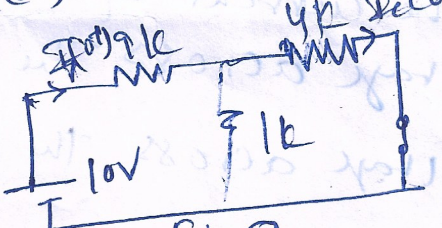


Fig ②

At $t = 0^+$ $\quad i_T(0^+) = \frac{10}{\frac{4\times1}{4+1} + 9k} = 1.02\,mA$

$$i_c(0^+) = I_T(0^+) \times \frac{1k}{1k + 4k} = 0.204\,mA$$

for $t > 0$, ~~representing~~ N/W shown in Fig ③.

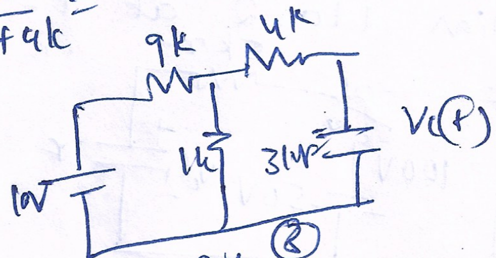
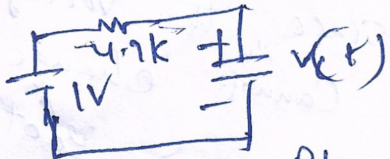


Fig ③

Thevenin's equivalent ckt of Fig ③ is Fig ④



$$V_{eq} = 10 \times \frac{1k}{1k + 9k} = 1\,V$$

$$R_{eq} = (9k \| 1k) + 4k = 4.9\,k\Omega$$

$$3\times10^{-6}\frac{dV_c}{dt} + \frac{V_c - 1}{4.9k} = 0.$$

$$\frac{dV_c}{dt} + 68.02 V_c = 68.02$$

$P = 68.02$ $\quad Q = 68.02$

$$V_c(t) = e^{-Pt}\int Q e^{Pt}\,dt + k e^{-Pt}$$

$$= 1 + k e^{-68.02t}.$$

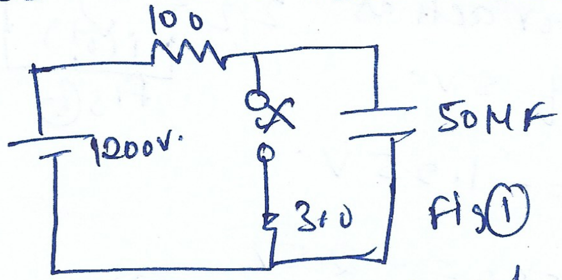
At $t=0$ $V_c(0) = 0$

$0 = 1 + k \Rightarrow k = -1$

$V_c(t) = 1 - e^{-68.02t}$

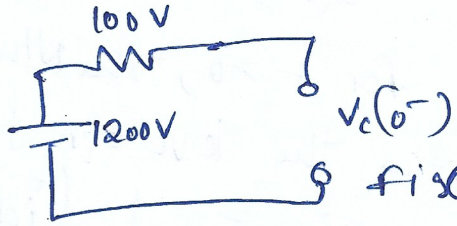
$\frac{d}{dt} V_c(t) = e^{-68.02t} \times 204.06 \times 10^{-6}$

(4). For the N/w, the switch is open for a long time & closes at t=0. Determine $V_c(t)$



Fig①

At $t=0^-$, the N/w is shown in fig ②.



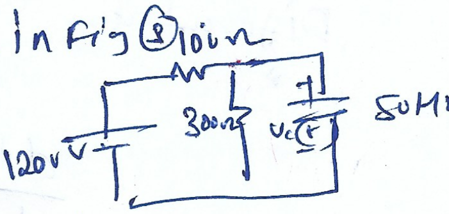
fig②.

$V_c(0^-) = 1200V$.

∴ the voltage across the capacitor cannot change instantaneously

$V_c(0^+) = 1200V$.

For $t>0$, the N/w is shown in fig ③



$$50\times10^{-6}\frac{dV_c}{dt} + \frac{V_c}{300} + \frac{V_c-1200}{100} = 0$$

$$\frac{dV_c}{dt} + 266.67\, V_c = 0.24\times10^{6}$$

$$\frac{dv}{dt} + Pv = Q \qquad P = 266.67 \quad Q = 0.24\times10^{6}$$

$$V_c(t) = e^{-Pt}\int Q e^{Pt}\,dt + ke^{-Pt} = e^{-266.67t}\int 0.24\times10^{6}\, e^{266.67t}\,dt + ke^{-266.67t}$$

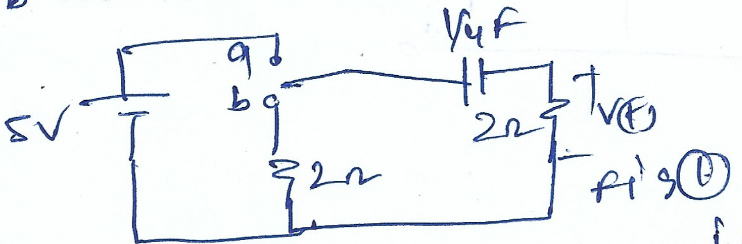
$$= 900 + ke^{-266.67t}$$

At $t=0$ $V_c(0) = 1200V$

$1200 = 900 + k \Rightarrow k = 300$

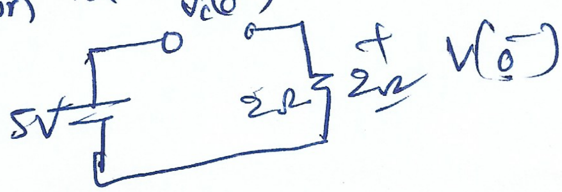
$V_c(t) = 900 + 300\,e^{-266.67t}$ for $t>0$

(5) In the N/w, the switch is shifted to position b at t=0. Find $v(t)$ for $t>0$



fig①

At $t=0^-$, the N/w shown in the fig ②.



$V_c(0^-) = 5V$

$v(0^-) = 0V$

At $t=0^+$, the N/w shown in fig

At $t=0^+$, the capacitor acts as a voltage source of 5V.

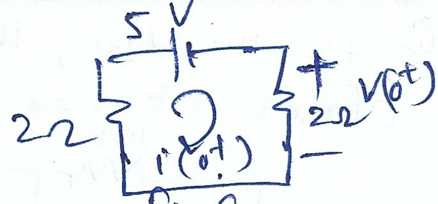
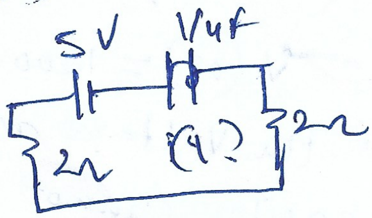


fig (b)

$$i(0^+) = \frac{-5}{4} = -1.25\,V.$$

$$V(0^+) = -1.25 \times 2 = -2.5\,V$$

for $t>0$, the N/w shown in fig (c)



writing the KVL eqn for $t>0$,

$$-2i - 5 - \frac{1}{1/4}\int_0^t i\,dt - 2i = 0. \quad \to (1)$$

Differentiating eqn (1)

$$-4\frac{di}{dt} - 4i = 0$$

$$\frac{di}{dt} + i = 0$$

$$P = 1$$

$$i(t) = ke^{-Pt} = ke^{-t}$$

At $t=0$, $i(0) = -1.25\,A.$

$$k = -1.25.$$

$$i(t) = -1.25\,e^{-t} \quad \text{for } t>0$$

$$v(t) = 2i(t) = -2.5\,e^{-t} \quad \text{for } t>0$$

6. The switch is open for a long time and at $t=0$, it is closed. Determine $V_2(t)$