Analog & Digital Communications

By
KASULA RAGHU
Assistant Professor
Dept. of E.C.E.
MGIT

- **Subject Code: EC403PC**
- **Google Class Room Code: hglitlo**
- **Google Class Link: meet.google.com/boa-dzzc-oxj**

Course Objectives

- To develop ability to analyse system requirements of analog and digital communication systems.

- To understand the generation, detection of various analog and digital modulation techniques.

- To acquire theoretical knowledge of each block in AM, FM transmitters and receivers.

- To understand the concepts of baseband transmissions.

Course Outcomes

Upon completing this course, the student will be able to

- Analyse and design of various continuous wave and angle modulation and demodulation techniques
- Understand the effect of noise present in continuous wave and angle modulation techniques.
- Attain the knowledge about AM , FM Transmitters and Receivers
- Analyse and design the various Pulse Modulation Techniques.
- Understand the concepts of Digital Modulation Techniques and Baseband transmission.

What we will Learn?

Analog Communications

UNIT – I Amplitude Modulation

UNIT – II Angle Modulation

UNIT – III Transmitters

UNIT – IV Pulse Modulation (PAM,PWM,PPM) one Line

Digital Communications

UNIT – IV Pulse Code Modulation

UNIT – V Digital Modulation Techniques

UNIT - I

- **Amplitude Modulation**

Need for modulation, Amplitude Modulation - Time and frequency domain description, single tone modulation, power relations in AM waves, Generation of AM waves - Switching modulator, Detection of AM Waves - Envelope detector, DSBSC modulation - time and frequency domain description, Generation of DSBSC Waves - Balanced Modulators, Coherent detection of DSB-SC Modulated waves, COSTAS Loop, SSB modulation - time and frequency domain description, frequency discrimination and Phase discrimination methods for generating SSB, Demodulation of SSB Waves, principle of Vestigial side band modulation.

Fourier Transforms & Inverse Fourier Transform Formula

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Representation

- **F.T. of**

$x(t) = X(f)$	**General Notation**
$m(t) = M(f)$	Used for **Message Signal(KHz)**
$c(t) = C(f)$	Used for **Carrier Signal(MHz)**
$s(t) = S(f)$	Used for **Modulated Signal(MHz)**

I.F.T. of

$X(f) = x(t)$

$G(f) = g(t)$

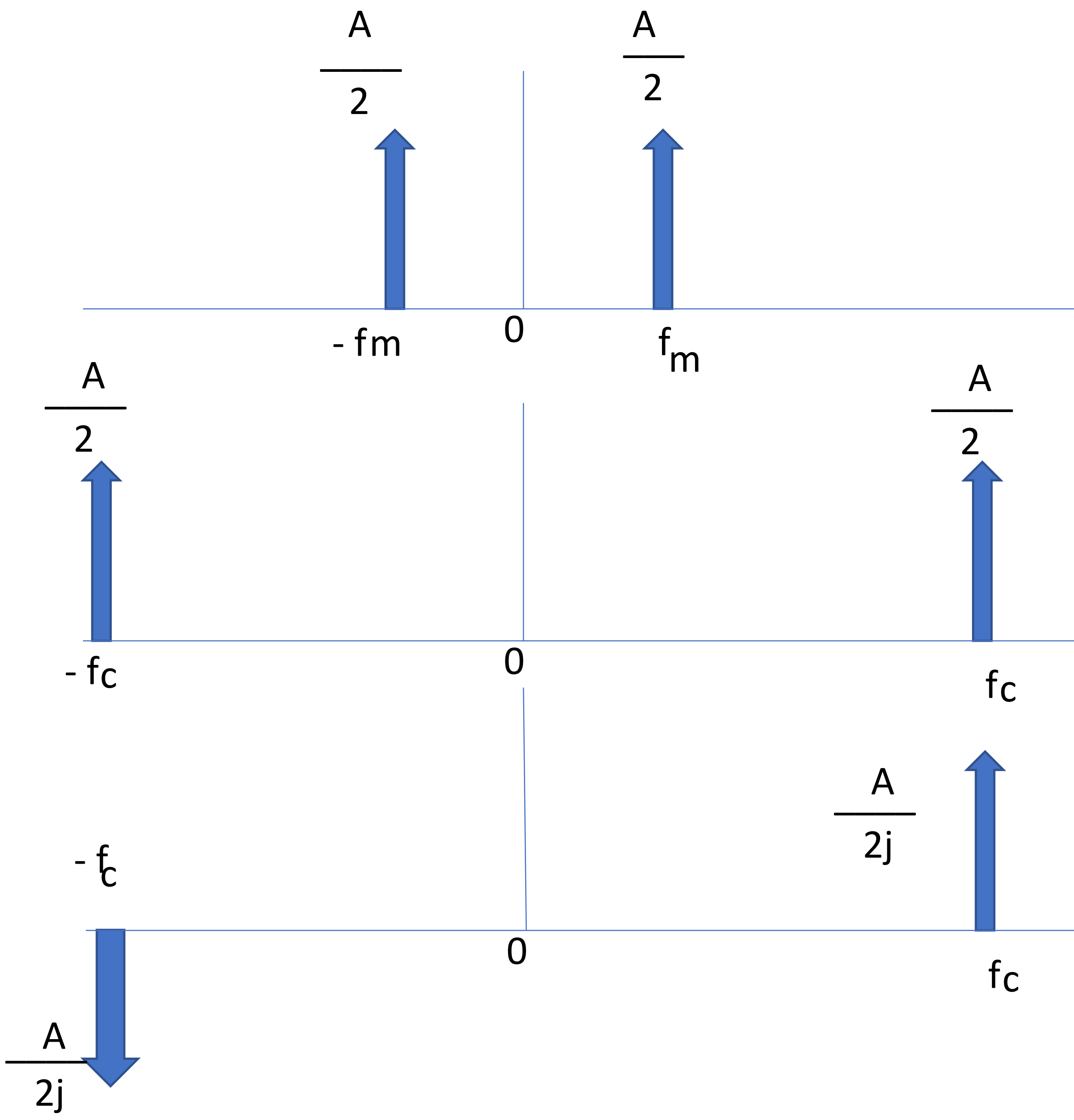
$M(f) = m(t)$

F.T. of Cos & Sin

$$A \cos(2\pi f_m t) = \frac{A}{2}[\delta(f - f_m) + \delta(f + f_m)]$$

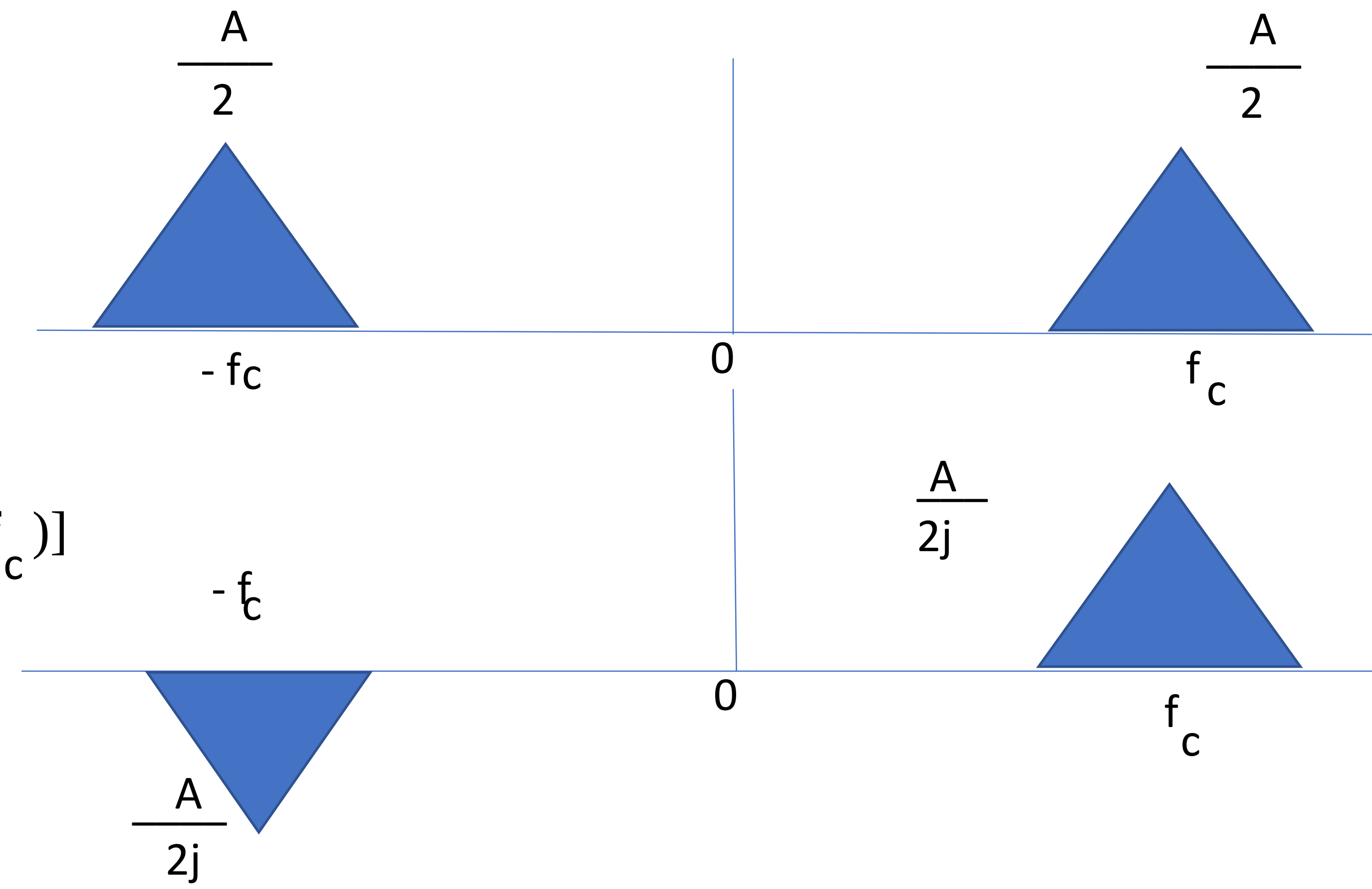
$$A \cos(2\pi f_c t) = \frac{A}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

$$A \sin(2\pi f_c t) = \frac{A}{2j}[\delta(f - f_c) - \delta(f + f_c)]$$

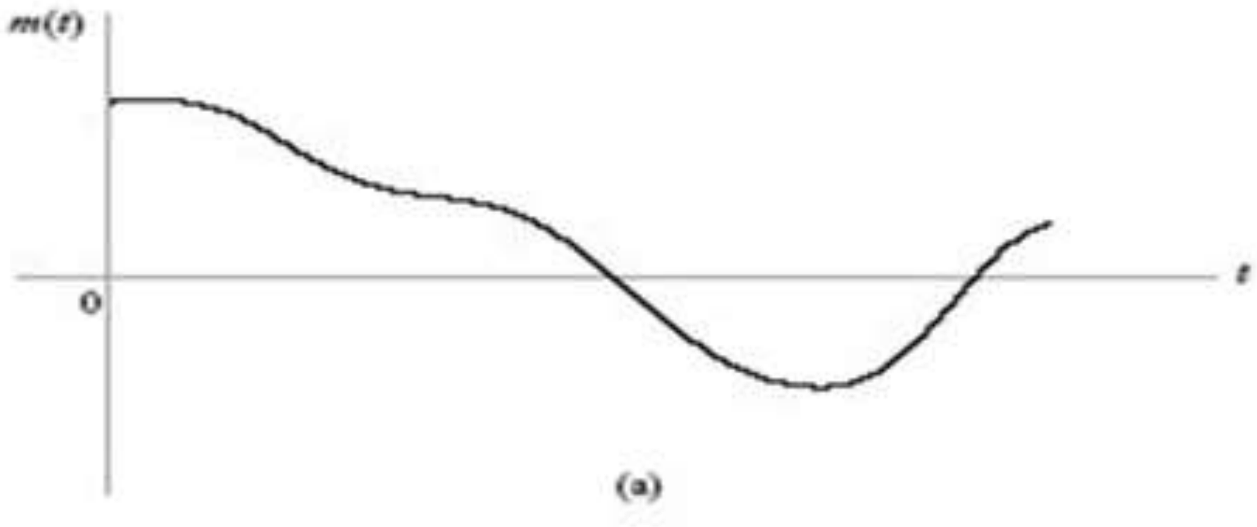


$$m(t)\,A\cos(2\pi f_c t) = \frac{A}{2}[M(f - f_c) - M(f + f_c)]$$

$$m(t)\,A\sin(2\pi f_c t) = \frac{A}{2j}[M(f - f_c) - M(f + f_c)]$$



In Time Domain



Message Signal (or)
Base Band Signal (or)
Modulating Signal (or)
Information Signal

In Frequency Domain

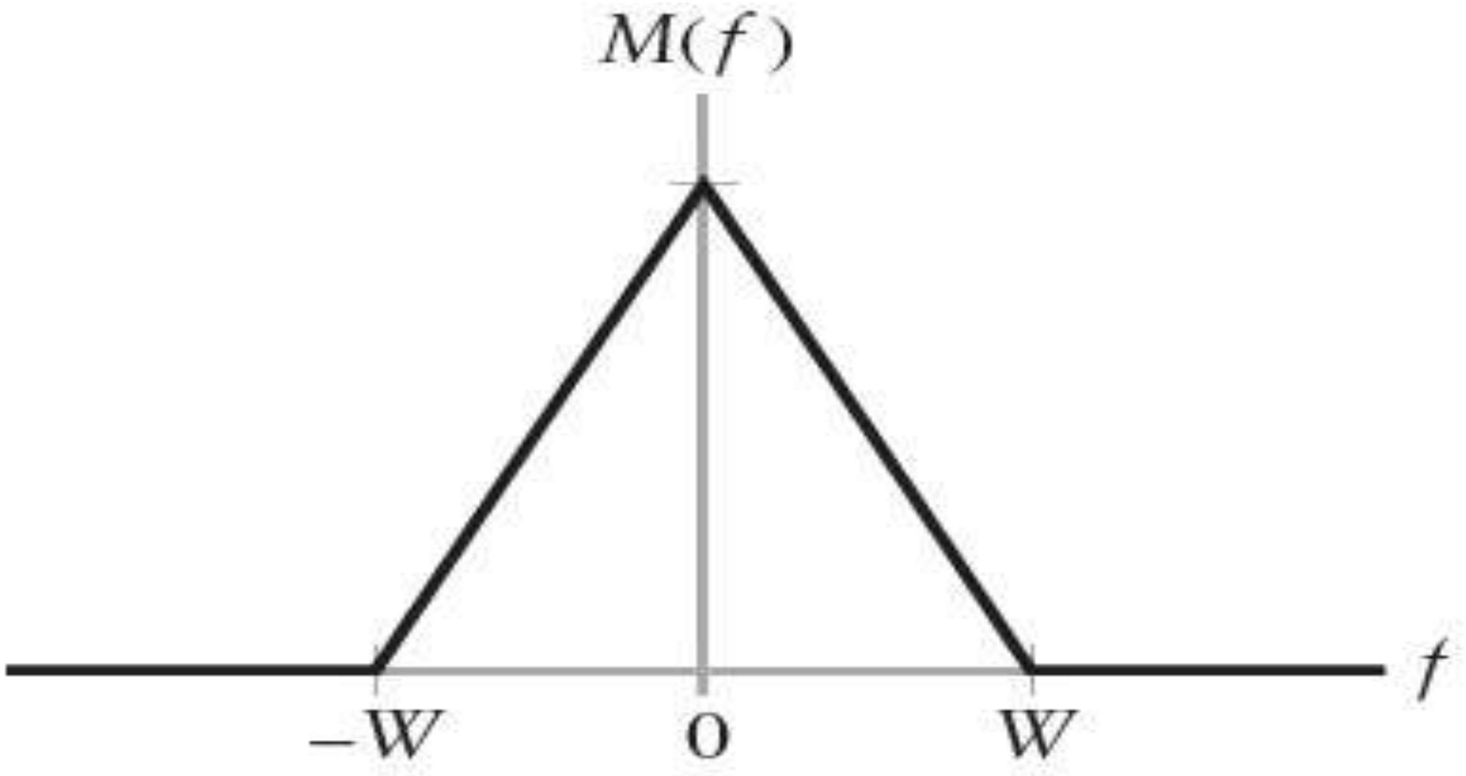


FIGURE 3.31

Band width = (W - 0) Hz = W Hz

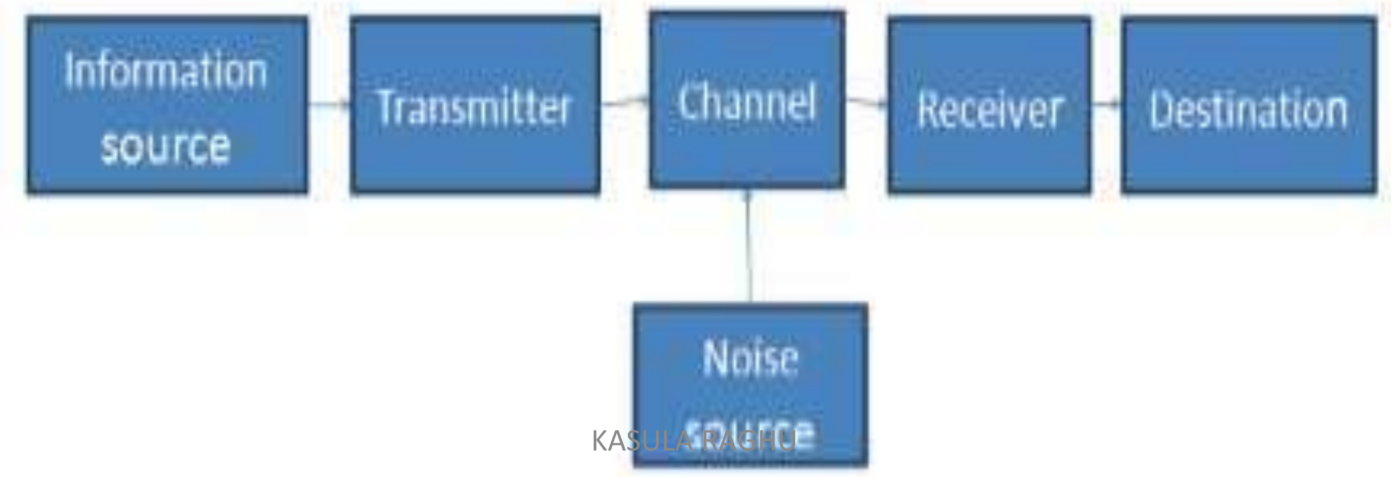
Introduction

Elements of Communication System:

Communication: It is the process of conveying or transferring information from one point to another.

(Or)

It is the process of establishing connection or link between two points for information exchange.



Elements of Communication System

Information source :

The message or information to be communicated originates in information source. Message can be words, group of words, code, data, symbols, signals etc.

Transmitter :

The objective of the transmitter block is to collect the incoming message signal and modify it in a suitable fashion (if needed), such that, it can be transmitted via the chosen channel to the receiving point.

Elements of Communication System:

Channel :

Channel is the physical medium which connects the transmitter with that of the receiver. The physical medium includes copper wire, coaxial cable, fibre optic cable, wave guide and free space or atmosphere.

Receiver :

The receiver block receives the incoming modified version of the message signal from the channel and processes it to recreate the original (non- electrical) form of the message signal.

Signal, Message, Information

Signal:

It is a physical quantity which varies with respect to time or space or independent or dependent variable.

(Or)

It is electrical waveform which carries information.

Ex: $m(t) = A\cos(\omega t+\phi)$

Where, A= Amplitude or peak amplitude(Volts)

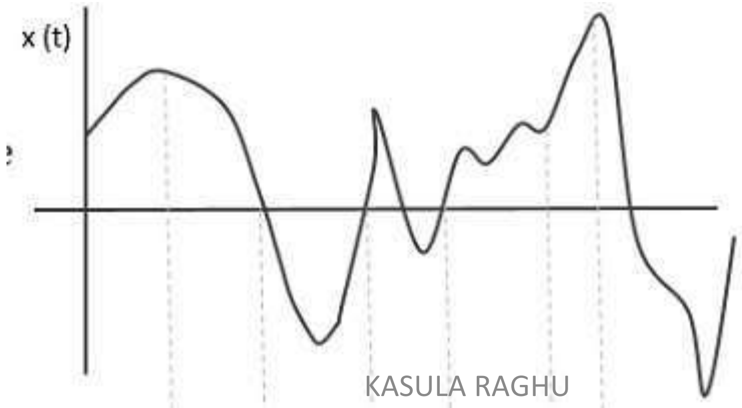
w = Frequency (rad/sec)

ϕ = Phase (rad)

Types of Signals

- Analog or Continuous Signal
- Digital Signal

Analog or Continuous Signal: If the amplitude of signal continuously varies with respect to time or if the signal contains infinite number of amplitudes, it is called Analog or continuous signal.



Types of Signals

Digital Signal: If the signal contains only two discrete amplitudes , then it is called digital signal.

With respect to communication , signals are classified into,

- Baseband signal
- Bandpass signal

Baseband signal:

If the signal contains zero frequency or near to zero frequency, it is called baseband signal.

Ex: Voice, Audio, Video, Bio-medical signals etc.

Types of Signals

Bandpass signal: If the signal contains band of frequencies far away from base or zero, it is called bandpass signal.

Ex: AM, FM signals.

Message: It is sequence of symbols. Ex: Happy New Year 2021.

Information: The content in the message is called information. It is inversely proportional to probability of occurrence of the symbol.

Information is measured in bits, decits, nats.

Limitations of Communication System

- **Technological Problems**:

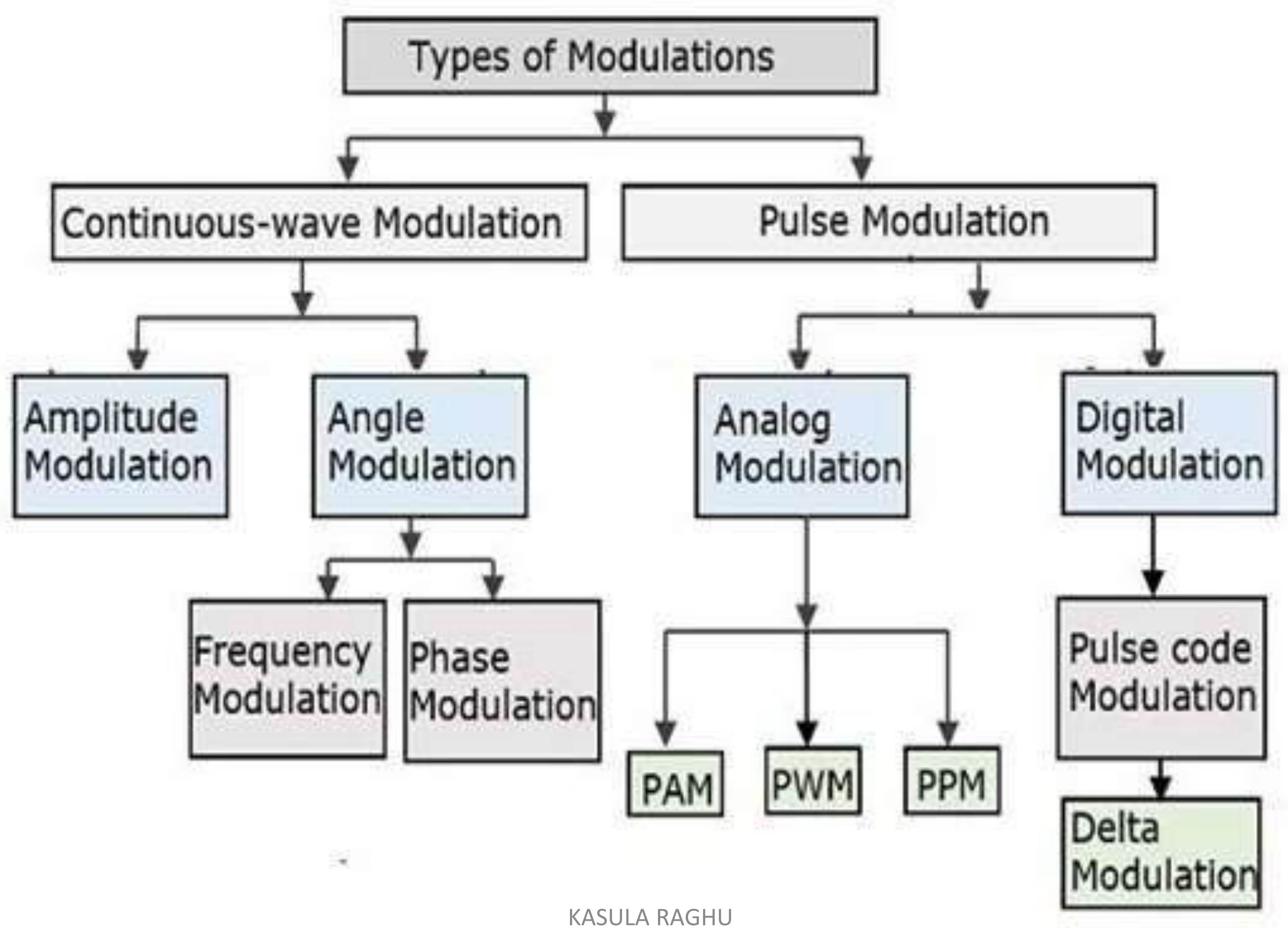
  To implement communication systems, Tx, Rx, channel are required which requires hardware. Communication system is expensive and complex.

- **Bandwidth & Noise**:

  The effect of noise can be reduced by providing more bandwidth to stations but due to this less number of stations can only be accommodated.

- **Signal to Noise Ratio (SNR)**: Noise should be low to increase channel capacity but it is an unavoidable aspect of communication system.

Types of Modulation



Modulation : Any Low Frequency/Message Signal m(t) is Multiplied by a High Frequency/Carrier Signal c(t) then the signal get shifted to Right side and Left side to the Frequency of Carrier signal (i. e M Hz).

Different Modulations

$$c(t) = A_c \cos(2\pi f_c t + \phi)$$

S.NO	Modulation	What Changed	Constant	Constant
1	AM	Amplitude	Frequency	Phase
2	FM	Frequency	Amplitude	Phase
3	PM	Phase	Amplitude	Frequency

*****w. r. t Amplitude of Message Signal

- Modulation
  - The process by which some characteristic of a carrier wave is varied in accordance with an information-bearing signal.
  - Continuous-wave modulation
    - *Amplitude modulation*
    - *Frequency modulation*
- AM modulation family
  - Amplitude modulation (AM)
  - Double sideband-suppressed carrier (DSB-SC)
  - Single sideband (SSB)
  - Vestigial sideband (VSB)

Modulation

It is the process of varying the characteristics of high frequency carrier in accordance with instantaneous values of modulating or message or baseband signal.

(Or)

It is a **frequency translation** technique which converts **baseband/low frequency signal** to **band pass/high frequency signal**.

Modulation is used at the transmitter. **(Filter Used ?)**
Demodulation is used at the Receiver. **(Filter Used ?)**

Types of Modulation

- **Amplitude Modulation**: Amplitude of the carrier is varied in accordance with the instantaneous values of modulating signal.

- **Frequency Modulation**: Frequency of the carrier is varied in accordance with the instantaneous values of modulating signal.

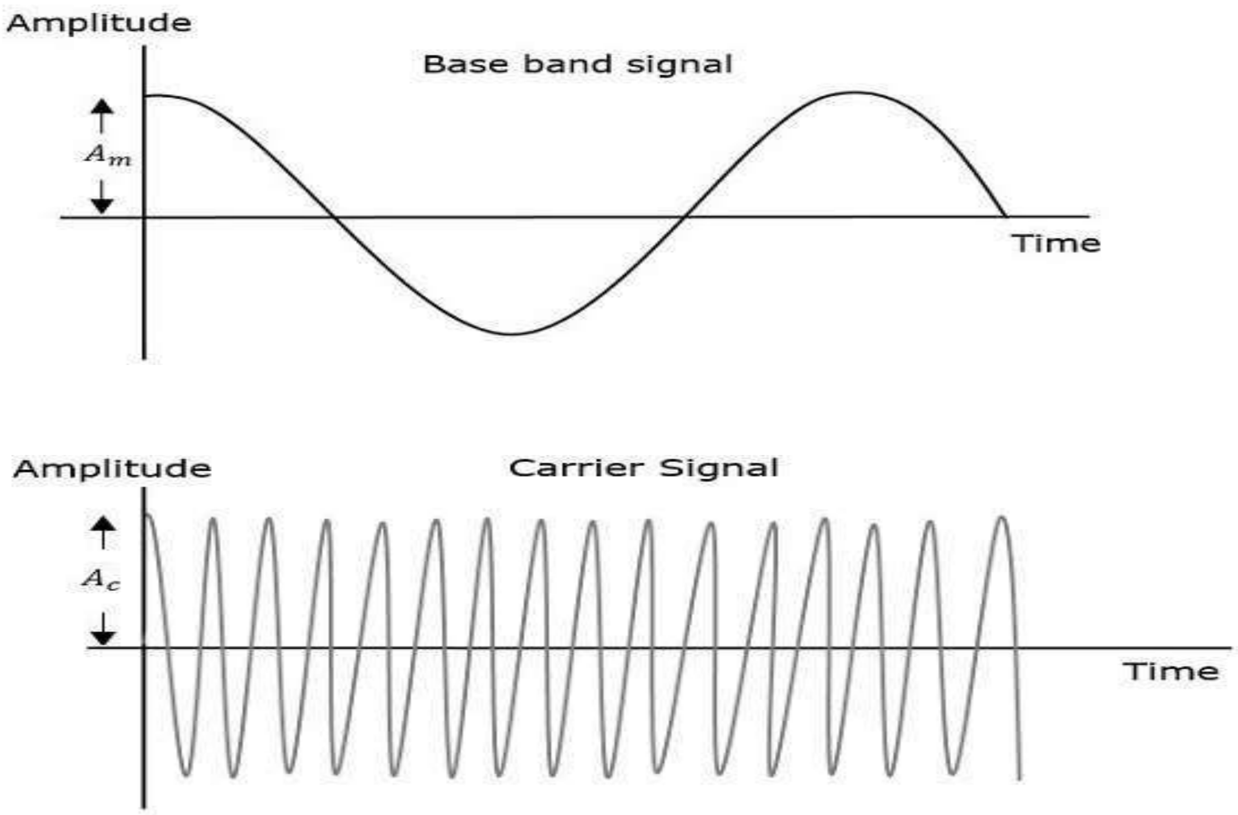
- **Phase Modulation**: Phase of the carrier is varied in accordance with the instantaneous values of modulating signal.

Need for Modulation

- To Reduce the height of an antenna
- For Multiplexing
- For Wideband Signal to Narrow banding
- To reduce noise effects
- To avoid equipment limitation or to reduce the size of the equipment.

Amplitude Modulation

The amplitude of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.



Carrier & Message Signals

The carrier signal is given by,

$$c(t) = A_c \, Cos2\pi f_c t$$

Where, A_c = Maximum amplitude of the carrier signal.

f_c = Frequency of the carrier signal.

Modulating or baseband signal is given by,

$$m(t) = A_m \, Cos2\pi f_m t$$

Where, A_m = Amplitude of the baseband signal.

$c(t) = A_c \cos 2\pi f_c t$ Carrier Wave

$S(t) = [A_C + m(t)] \cos 2\pi f_c t$

$= A_c [1 + K_a\, m(t)] \cos 2\pi f_c t$ **Time Domain Equation of AM**

K_a = Amplitude Sensitivity of the Modulator

when $m(t)$ = Zero

then $s(t) = c(t)$ which is called as **Unmodulated Carrier**

Before Modulation , Magnitude is **: A_c**

After Modulation , Magnitude is **: $A_c [1 + K_a\, m(t)]$**

Amplitude Modulation

- The envelope of s(t) has essentially the same shape as the message signal m(t) provided that two conditions are satisfied :
  - *The amplitude of* $k_a m(t)$ *is always less than unity*

$$\mu = m = |k_a m(t)| < 1, \qquad \text{for all} \quad t$$

$$\mu = m = K_a[\text{Maximum Voltage of Message signal}] = K_a[m(t)]_{max}$$

  - *The carrier frequency* f_c *is much greater than the highest frequency component W of the message signal*

$$f_c >> W$$

- Envelope detector
  - *A device whose output traces the envelope of the AM wave acting as the input signal*

$$S(t) = A_c\,[1 + K_a\, m(t)]\, \text{Cos}2\pi f_c t$$

$$S(t) = A_c\, \text{Cos}2\pi f_c t + A_c K_a\, m(t)\, \text{Cos}2\pi f_c t$$

Note: In A.M. Carrier is also Transmitting along with the Modulated signal which is used at the Receiver for Demodulation

- The Fourier transform or spectrum of the AM wave *s(t)*

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

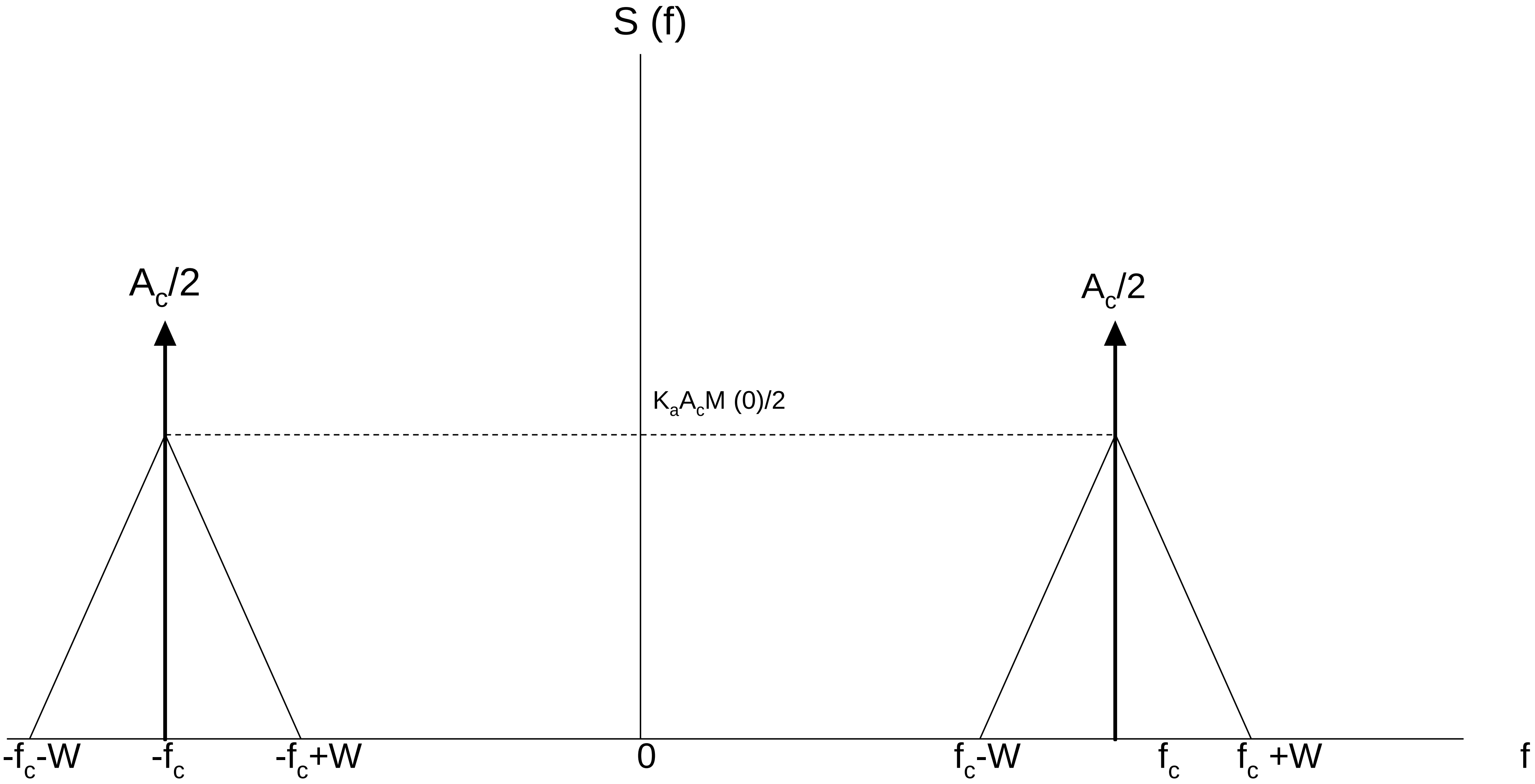


Fig : Spectrum of AM signal

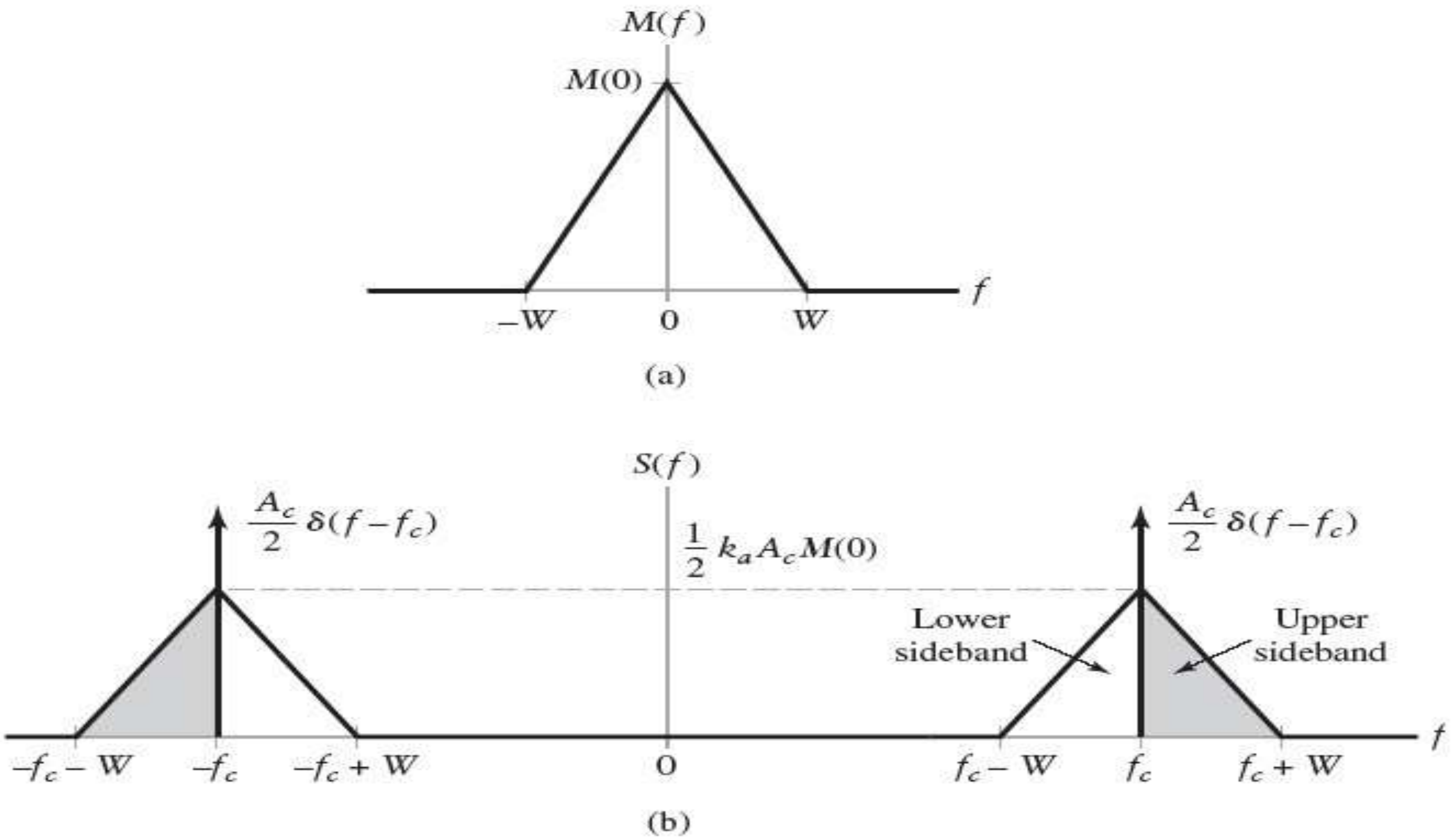


FIGURE 3.2 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of AM wave $s(t)$.

BW of AM = ?

For positive frequencies, the highest frequency component of the AM wave equals f_c+W, and the lowest frequency component equals f_c-W. The difference between these two frequencies defines the transmission bandwidth B_T of the AM wave, which is exactly twice the message bandwidth W

$$B_T = 2W$$

AM Wave Contains

Carrier Component at f_c

LSB from f_c-W to f_c

USB from f_c to f_c+W

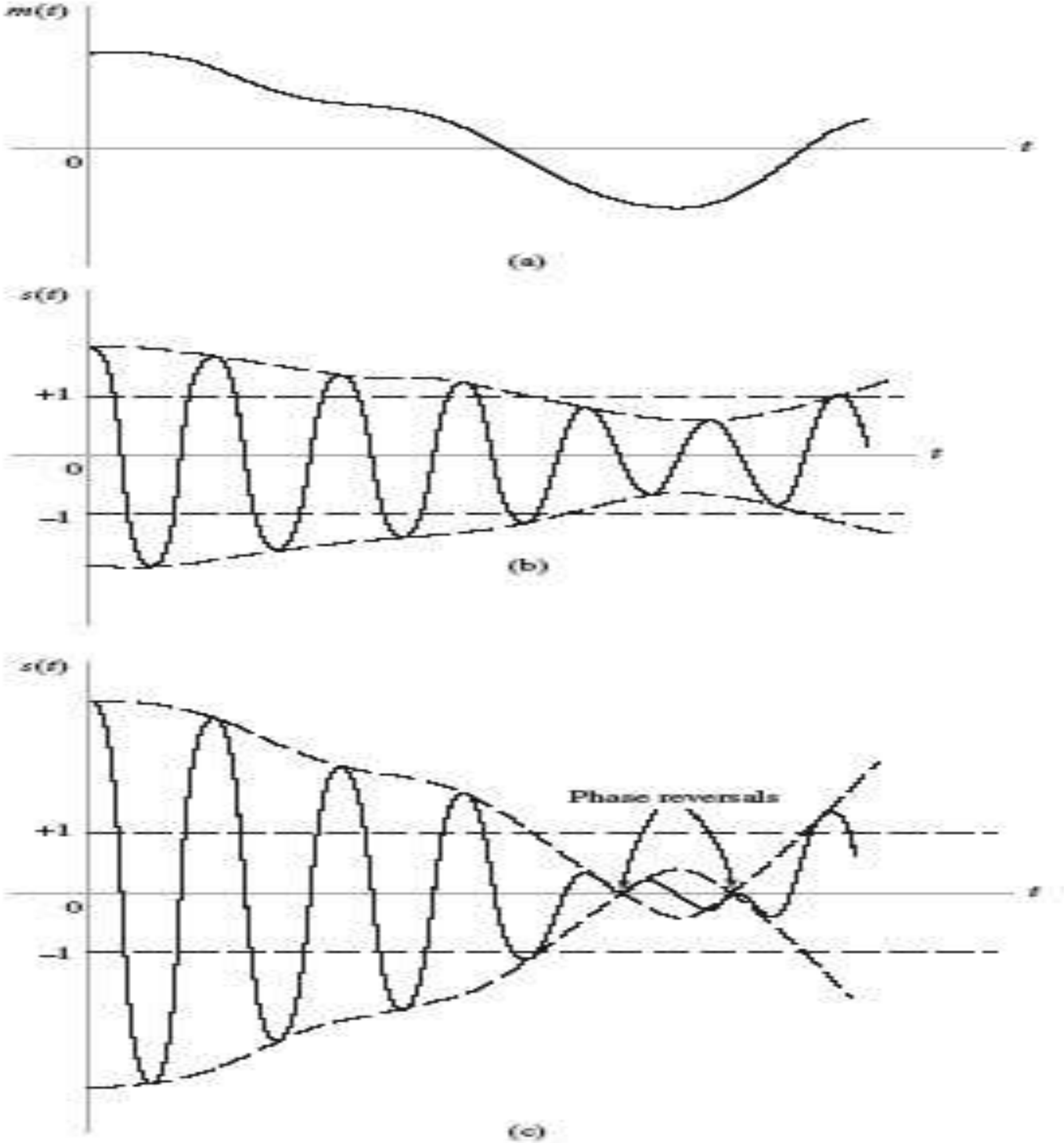
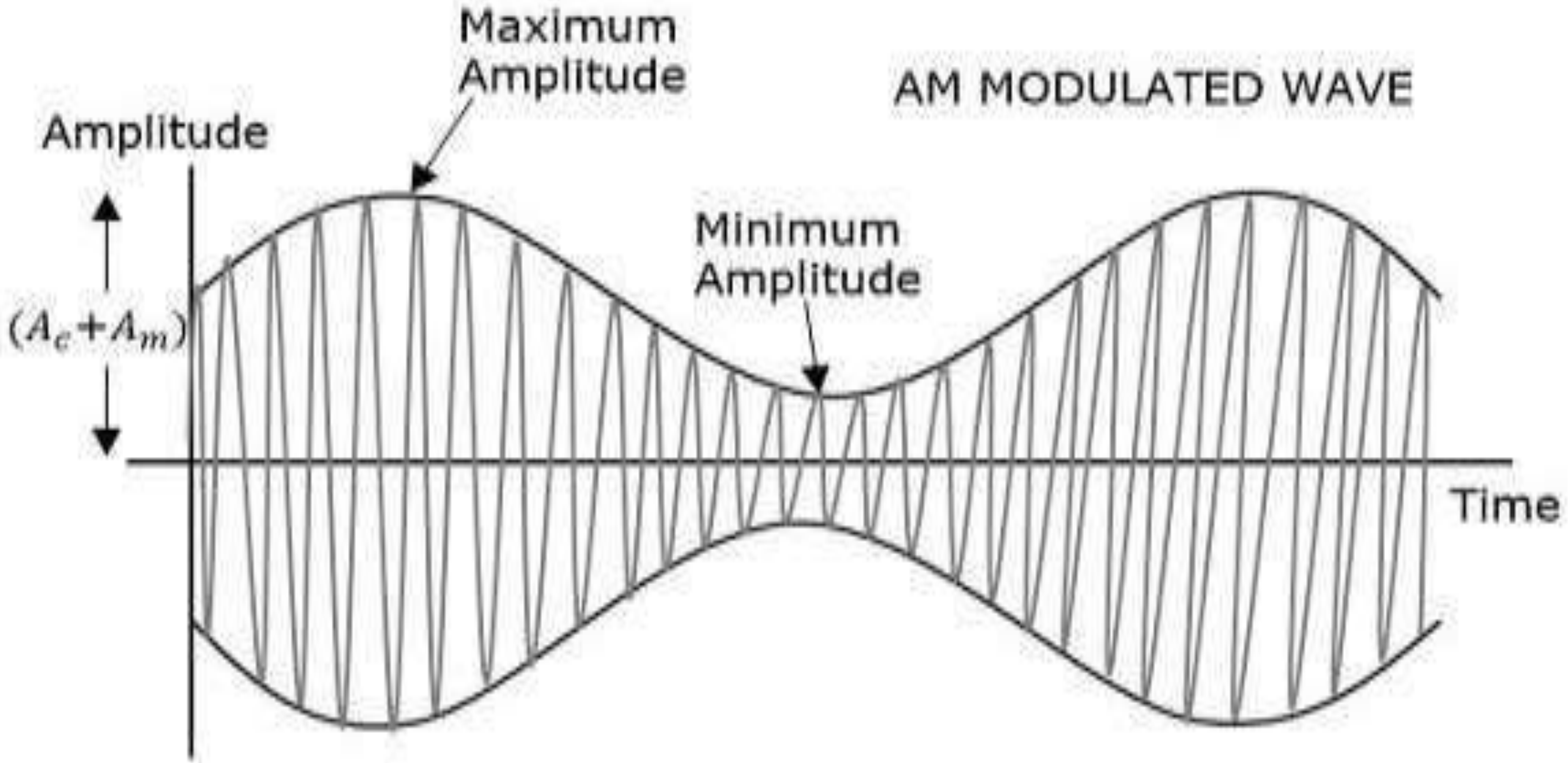


Figure 3.1 Illustration of the amplitude modulation process. (*a*) Message signal $m(t)$. (*b*) AM wave for $k_a m(t) < 1$ for all t. (*c*) AM wave for $|k_a m(t)| > 1$ for some t.


AM MODULATED WAVE
Maximum Amplitude
Amplitude
Minimum Amplitude
(A_c+A_m)
Time

Modulation Index

Modulation index or depth of modulation is given by,

$$\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = A_m/A_c$$

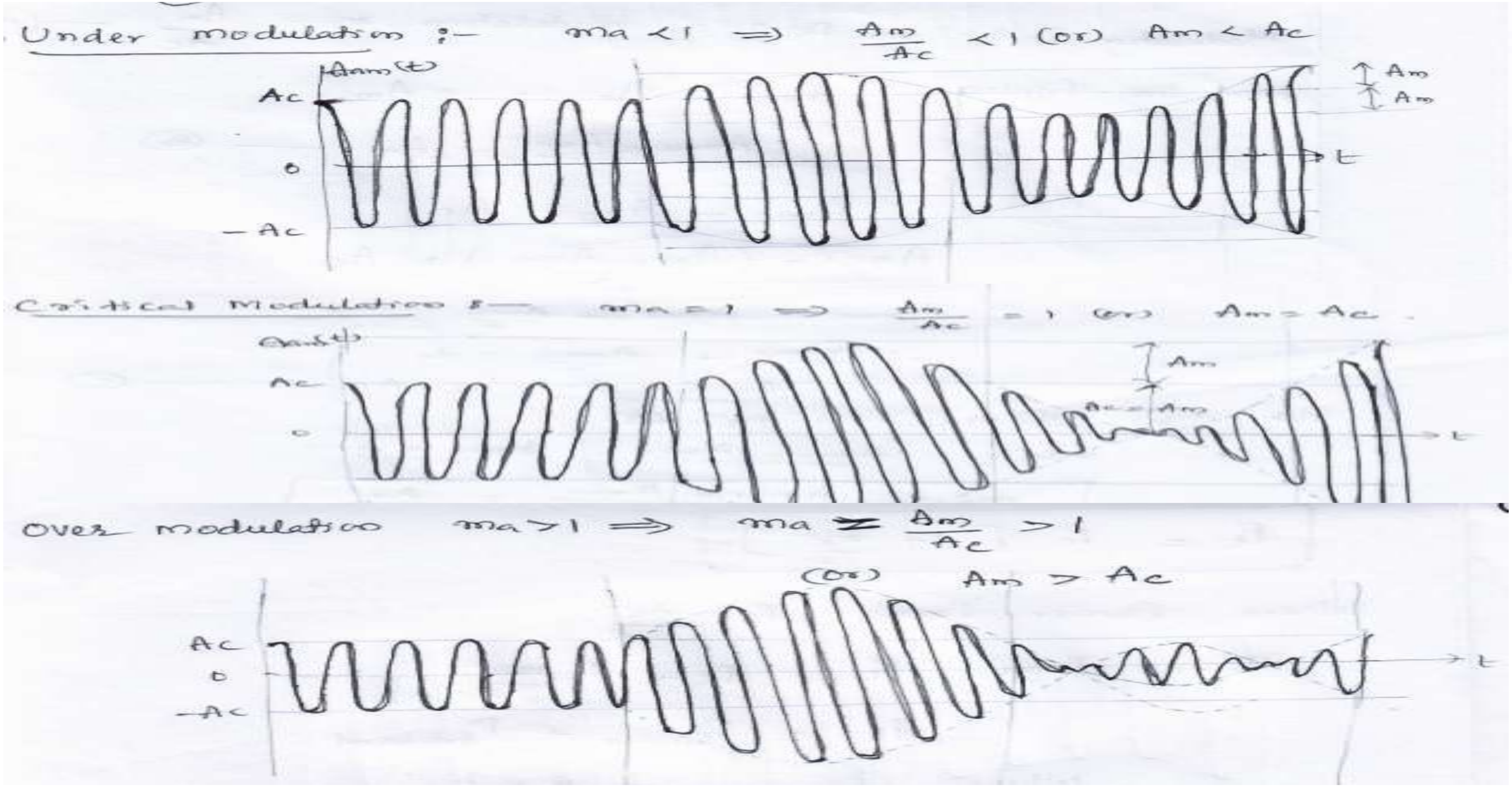
Percentage of modulation index is,

$$\% \mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} X100 = [A_m/A_c] X100$$

Types of AM with respect to modulation index:

- Under Modulation (μ <1)
- Critical Modulation (μ =1)
- Over Modulation (μ >1)

Types of AM


Under modulation :- ma < 1 ⇒ Am/Ac < 1 (or) Am < Ac
Ac
0
-Ac
Am
Am
t
Critical Modulation :- ma = 1 ⇒ Am/Ac = 1 (or) Am = Ac
Ac
0
Am
t
Over modulation ma > 1 ⇒ ma = Am/Ac > 1
(or) Am > Ac
Ac
0
-Ac
t

Single Tone Modulation of A.M.

$$S(t) = A_c\,[1 + K_a\, m(t)]\, \cos 2\pi f_c t$$

$$= A_c\,[1 + K_a A_m \cos 2\pi f_m t]\, \cos 2\pi f_c t$$

$$S(t) = A_c\,[1 + \mu \cos 2\pi f_m t]\, \cos 2\pi f_c t$$ **Standard Form of A.M.**

$$\mu = A_m K_a = A_m / A_c = \text{Modulation Index}$$

$$s(t) = Ac \cos 2\pi f_c t + Ac\,\mu \cos 2\pi f_c t \cos 2\pi f_m t$$

$$S(t) = Ac\cos 2\pi f_c t \;+\; \mu Ac/2\cos[2\pi(f_c+f_m)]t \;+\; \mu Ac/2\cos[2\pi(f_c-f_m)]t$$

I term II term III term

I term: Carrier signal with amplitude Ac and frequency fc.

II.term: Amplitude= $\mu Ac/2$, frequency= f_c+f_m , Upper sideband frequency

III.term: Amplitude= $\mu Ac/2$, frequency= f_c-f_m , Lower sideband frequency

Expanding the equation (2), we get

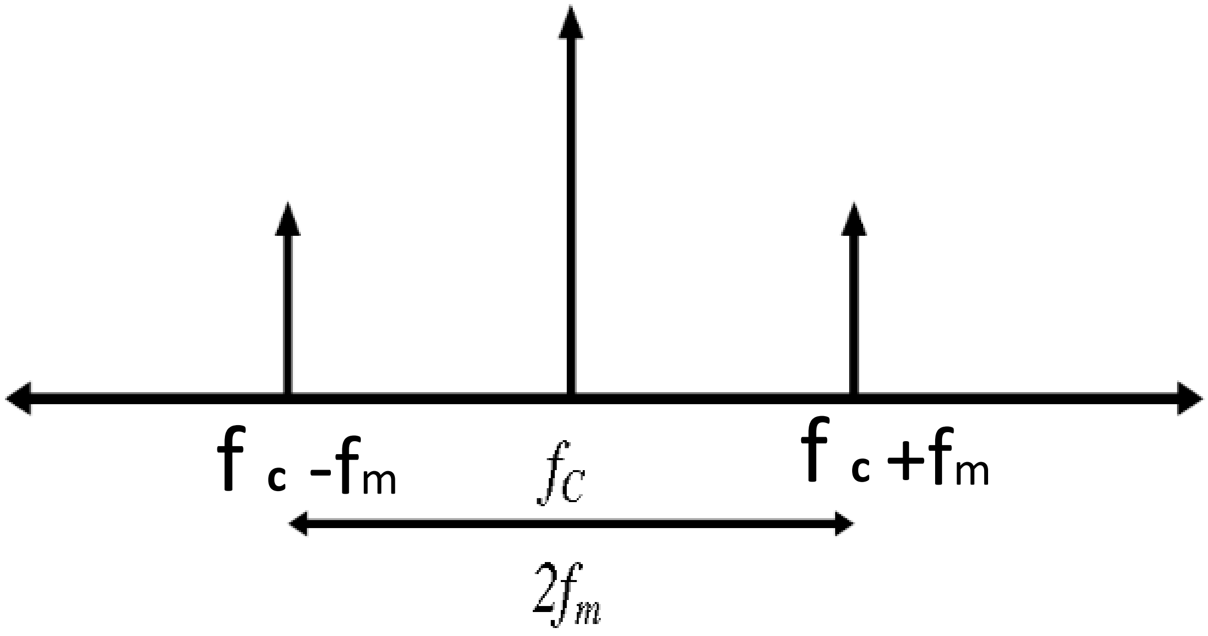
$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} m A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of $s(t)$ is obtained as follows.

$$s(f) = \frac{1}{2} A_c [\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{4} m A_c [\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$$

$$+ \frac{1}{4} m A_c [\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$$

Thus the spectrum of an AM wave, for the special case of sinusoidal modulation consists of delta functions at $\pm f_c$, $f_c \pm f_m$, and $-f_c \pm f_m$. The spectrum for positive frequencies is as shown in figure



Frequency Domain characteristics of single tone AM

S.No	Message	FT	BW of Message	BW of AM Signal
1	m(t)	M(f)	*W*	*2W*
2	$\cos 2\pi f_m t$	$0.5\,[\delta(f - fm) + \delta(f + fm)]$	*fm*	*2fm*

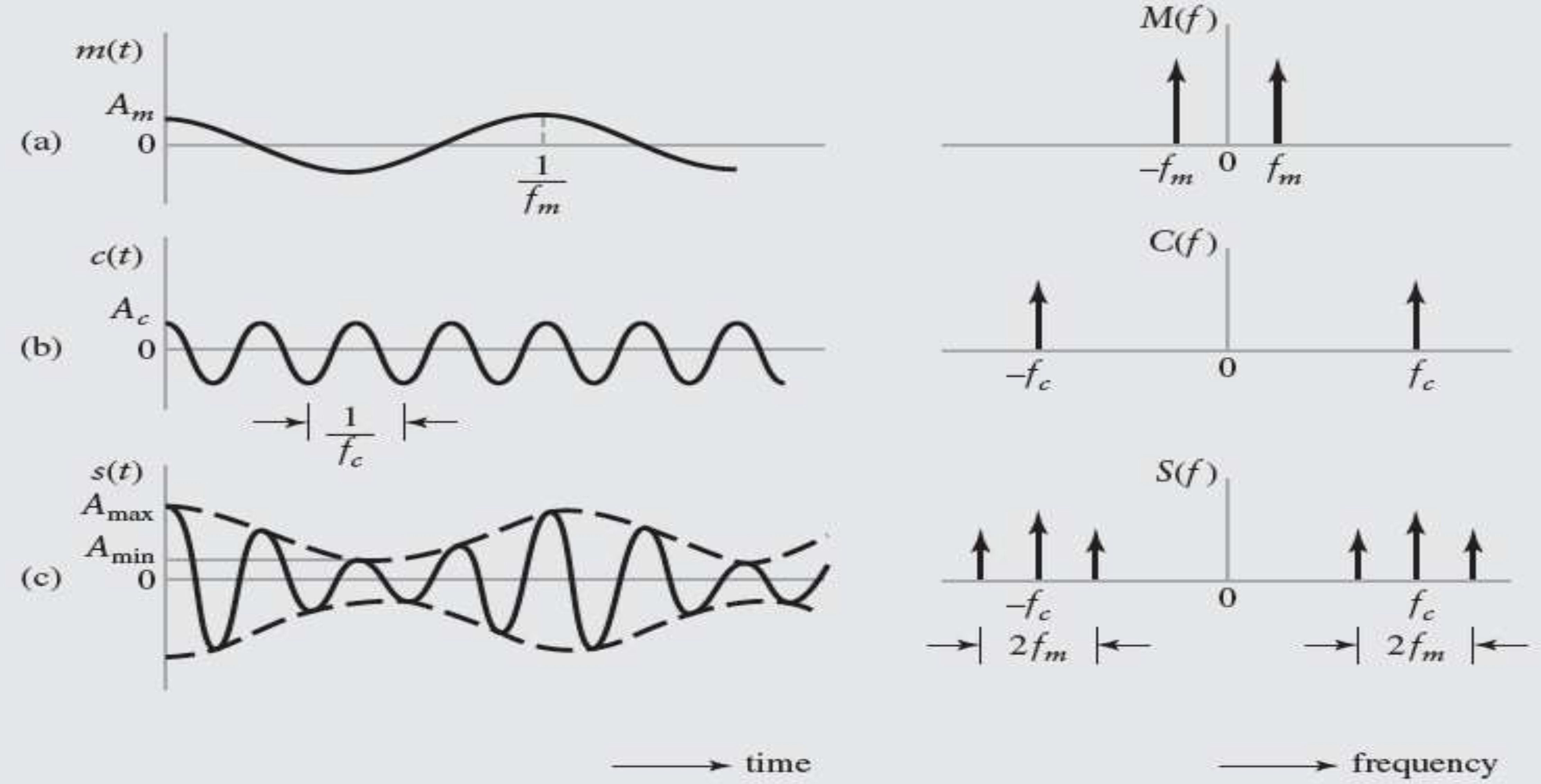


FIGURE 3.3 Illustration of the time-domain (on the left) and frequency-domain (on the right) characteristics of amplitude modulation produced by a single tone. (*a*) Modulating wave. (*b*) Carrier wave. (*c*) AM wave.

Power Calculation of AM Wave

AM Wave Contains

Total Power = Carrier Power + USB Power + LSB Power

- **Power relations in AM waves:**

Consider the expression for single tone/sinusoidal AM wave

$$s(t) = A_c Cos(2\pi f_c t) + \frac{1}{2} m A_c Cos[2\pi(f_c + f_m)t] + \frac{1}{2} m A_c Cos[2\pi(f_c - f_m)t] \quad \text{.................}(1)$$

This expression contains three components. They are carrier component, upper side band and lower side band. Therefore Average power of the AM wave is sum of these three components.

Therefore the total power in the amplitude modulated wave is given by

$$Pt = \frac{V^2_{car}}{R} + \frac{V^2_{LSB}}{R} + \frac{V^2_{USB}}{R} \quad \text{......................}(2)$$

Where all the voltages are rms values and R is the resistance, in which the power is dissipated.

$$P_C = \frac{V^2_{car}}{R} = \frac{\left(A_c/\sqrt{2}\right)^2}{R} = \frac{A_c^{\,2}}{2R}$$

$$P_{LSB} = \frac{V_{LSB}^{\,2}}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c^{\,2}}{8R} = \frac{m^2}{4} P_c$$

$$P_{USB} = \frac{V_{USB}^{\,2}}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c^{\,2}}{8R} = \frac{m^2}{4} P_c$$

Therefore total average power is given by

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$P_t = P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c$$

$$P_t = P_c\left(1 + \frac{m^2}{4} + \frac{m^2}{4}\right)$$

$$P_t = P_c\left(1 + \frac{m^2}{2}\right) \qquad \text{..............................(3)}$$

- $\mu = 0$ $P_t = P_c$
- $\mu = 0.5$ $P_t = 1.125 P_c$
- $\mu = 1$ $P_t = 1.5\, P_c$

Note : When μ is increased from 0 to 1 Power Increased by 50%

Relationship Between Carrier Power & Side Band Power

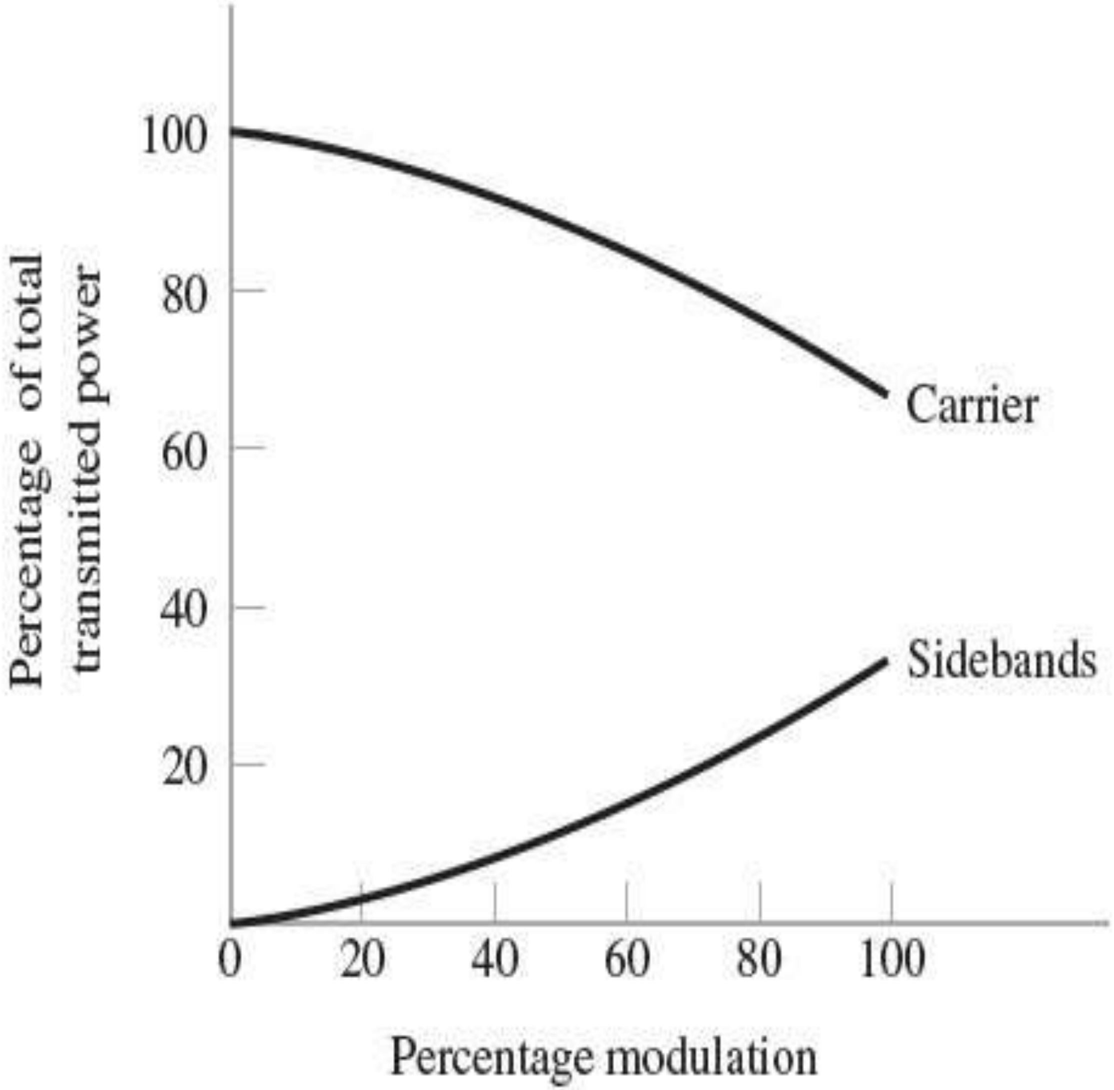


FIGURE 3.4 Variations of carrier power and total sideband power with percentage modulation in amplitude modulation.

Power Efficiency or Modulation Efficiency

It is the ratio of Utilized Power to the total power in the modulated wave.

$$\frac{P_{SB}}{P_t} = \frac{P_c\left(m^2/2\right)}{P_c\left(1+m^2/2\right)}$$

$$\frac{P_{SB}}{P_t} = \frac{m^2}{2+m^2} \qquad \text{..........................(4)}$$

$\mu = 1 \qquad \eta = 33.33\%$

$\mu = 0.75 \qquad \eta = 22.22\%$

$\mu = 0.5 \qquad \eta = 11.11\ \%$

Exercise for Multi Tone Modulation

$BW = 2f_{m2}$ if $(f_{m2} > f_{m1})$

$BW = 2f_{m1}$ if $(f_{m1} > f_{m2})$

Multi Tone Modulation

- $S(t) = Ac\,[1+ K_a\, m(t)]\, Cos2\pi f_c t$

  Where $m(t) = A_{m1}\, Cos2\pi f_{m1}\, t + A_{m2}\, Cos2\pi f_{m2}\, t$ (for multi-tone) ($f_{m2} > f_{m1}$)

  $= Ac\,[1+ K_a\, A_{m1}\, Cos2\pi f_{m1}\, t + Ka\, A_{m2}\, Cos2\pi f_{m2}\, t\,]\, Cos2\pi f_c t$

- $S(t) = Ac\, Cos2\pi f_c t + Ac\, \mu 1\, Cos2\pi f_c t\, Cos2\pi f_{m1}\, t + Ac\, \mu 2\, Cos2\pi f_c t\, Cos2\pi f_{m2}\, t\,]$

  where $\mu = Ka\, Am$; $\mu_1 = Ka\, Am_1$; $\mu_2 = Ka\, Am_2$

Case2: Multitone Modulation

Let a message signal consists of three sinusoidal signals of different frequencies.

$$m(t) = A_1 \cos 2\pi f_1 t + A_2 \cos 2\pi f_2 t + A_3 \cos 2\pi f_3 t$$

$$\begin{aligned}
s(t) &= A_c[1 + k_a m(t)]\cos 2\pi f_c t \\
&= A_c[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t + \mu_3 \cos 2\pi f_3 t]\cos 2\pi f_c t \\
&= A_c \cos 2\pi f_c t + \left[\frac{\mu_1 A_c}{2}\cos 2\pi(f_c - f_1)t + \frac{\mu_1 A_c}{2}\cos 2\pi(f_c + f_1)t\right] + \\
&\quad \left[\frac{\mu_2 A_c}{2}\cos 2\pi(f_c - f_2)t + \frac{\mu_2 A_c}{2}\cos 2\pi(f_c + f_2)t\right] + \\
&\quad \left[\frac{\mu_3 A_c}{2}\cos 2\pi(f_c - f_3)t + \frac{\mu_3 A_c}{2}\cos 2\pi(f_c + f_3)t\right]
\end{aligned}$$

Multi Tone Amplitude Modulation

$$S_{AM}(t) = A_c\left(1 + \mu_1 \cos 2\pi f_{M_1} t + \mu_2 \cos 2\pi f_{M_2} t\right)\cos 2\pi f_c t$$

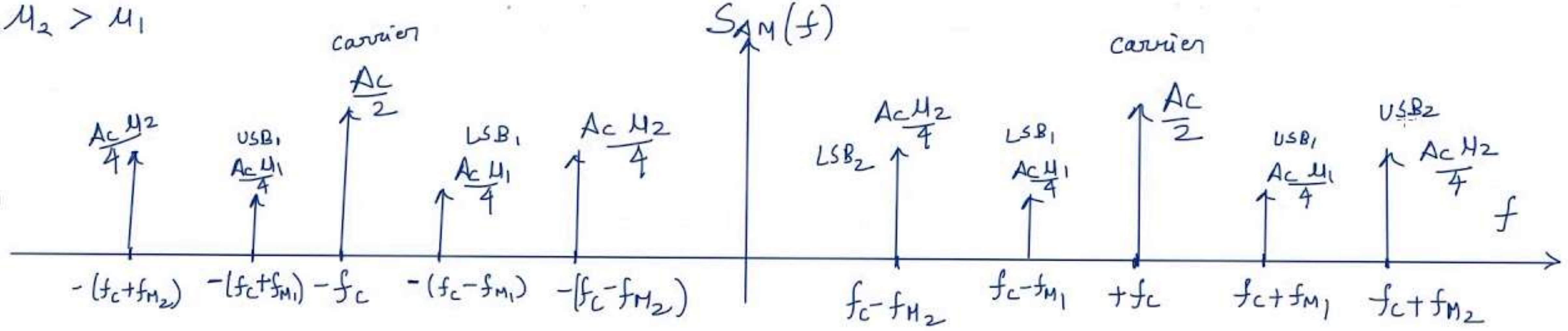
$$S_{AM}(t) = A_c \cos 2\pi f_c t + A_c \mu_1 \cos 2\pi f_{M_1} t \cos 2\pi f_c t + A_c \mu_2 \cos 2\pi f_{M_2} t \cos 2\pi f_c t$$

$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu_1}{2} \cos(2\pi(f_c + f_{M_1})t) + \frac{A_c \mu_1}{2} \cos(2\pi(f_c - f_{M_1})t)$$

$$+ \frac{A_c \mu_2}{2} \cos(2\pi(f_c + f_{M_2})t) + \frac{A_c \mu_2}{2} \cos(2\pi(f_c - f_{M_2})t)$$

$f_{M_2} > f_{M_1}$

$\mu_2 > \mu_1$



Power of Multi - Tone AM Signal

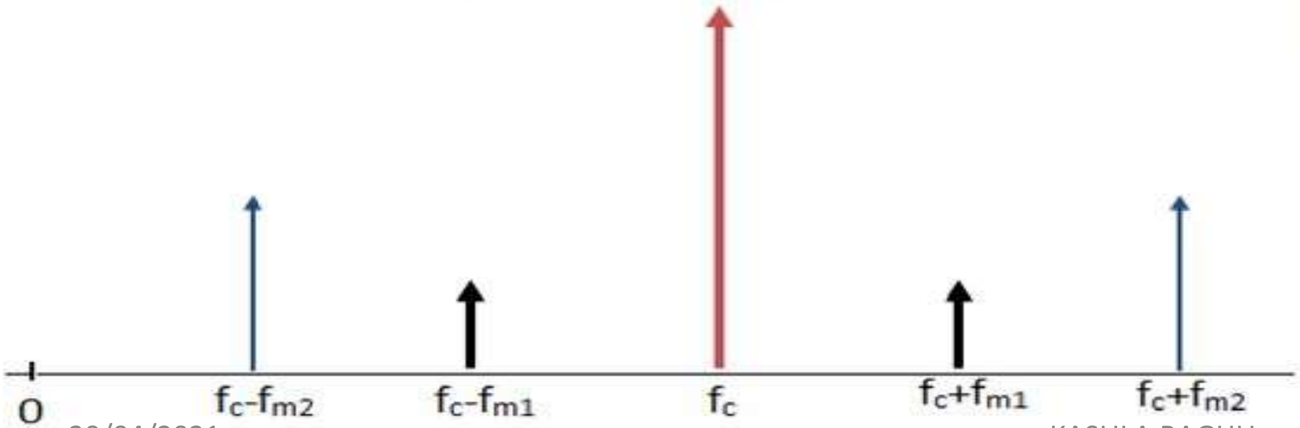
$$P_t = P_c + P_{SB}$$

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$P_t = P_c + P_{USB_1} + P_{USB_2} + P_{LSB_1} + P_{LSB_2}$$

$$P_t = P_c\left[1 + \frac{\mu_t^2}{2}\right] \quad where\ \mu_t^2 = \sqrt{\mu_1^2 + \mu_2^2}$$

$$= total\ modulation\ index$$



Generation of AM waves

- Square Law Modulator
- Switching Modulator

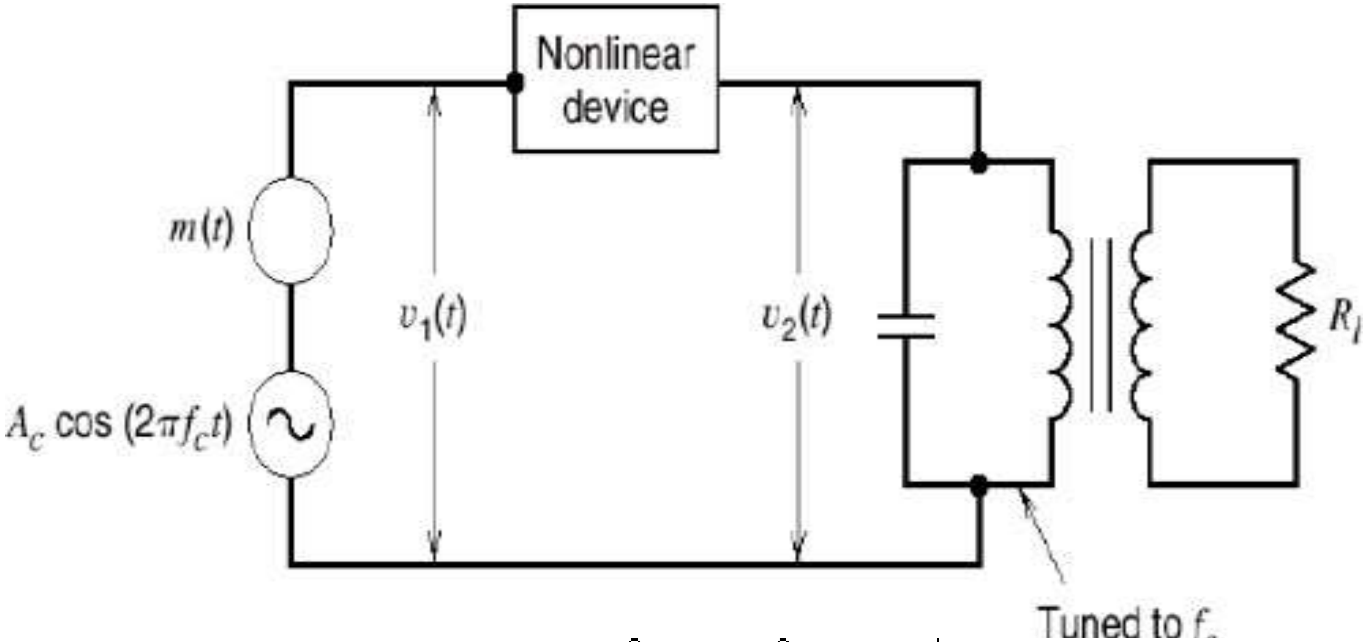
Detection of AM Wave

- Envelop Detector

Generation of AM Wave

Square Law modulator:

- Contains 1) non-linear device ,2) Band pass filter, 3) Carrier source and modulating signal



$$V_0 = a_0 + a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 + a_4 V_{in}^4 + \ldots\ldots$$

When the in put is very small, the higher power terms can be neglected. Hence the output is approximately given by $V_0 = a_O + a_1 V_{in} + a_2 V_{in}^2$

$$V_{in} = c(t) + m(t)$$

$$V_{in} = A_c \cos 2\pi f_c t + m(t)$$

As the level of the input is very small, the output can be considered up to square of the input, *i.e.*, $V_0 = a_O + a_1 V_{in} + a_2 V_{in}^2$

$$V_0 = a_0 + a_1[A_c \cos 2\pi f_c t + m(t)] + a_2[A_c \cos 2\pi f_c t + m(t)]^2$$

$$V_0 = a_0 + a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2}(1 + \cos 4\pi f_c t) + a_2[m(t)]^2 + 2a_2 m(t) A_c \cos 2\pi f_c t$$

$$V_0 = a_0 + a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2}\cos 4\pi f_c t + a_2 m^2(t) + 2a_2 m(t) A_c \cos 2\pi f_c t$$

Taking Fourier transform on both sides, we get

$$V_0(f) = (a_0 + \frac{a_2 A_c^2}{2})\delta(f) + \frac{a_1 A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + a_1 M(f) +$$

$$\frac{a_2 A_c^2}{4}\left[\delta(f - 2f_c) + \delta(f + 2f_c)\right] + a_2 M(f) + a_2 A_c\left[M(f - f_c) + M(f + f_c)\right]$$

Frequency band centered at f_c with a deviation of $\pm W$, Hz.

The required AM signal with a carrier frequency f_c can be separated using a band pass filter at the out put of the square law device. The filter should have a lower cut-off frequency ranging between 2W and (f_c-W) and upper cut-off frequency between (f_c+W) and 2 f_c

Therefore the filter out put is

$$s(t) = a_1 A_c \cos 2\pi f_c t + 2a_2 A_c m(t) \cos 2\pi f_c t$$

$$s(t) = a_1 A_c \left[1 + 2\frac{a_2}{a_1} m(t)\right] \cos 2\pi f_c t$$

If $m(t) = A_m \cos 2\pi f_m t$, we get

$$s(t) = a_1 A_c \left[1 + 2\frac{a_2}{a_1} A_m \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

Comparing this with the standard representation of AM signal,

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

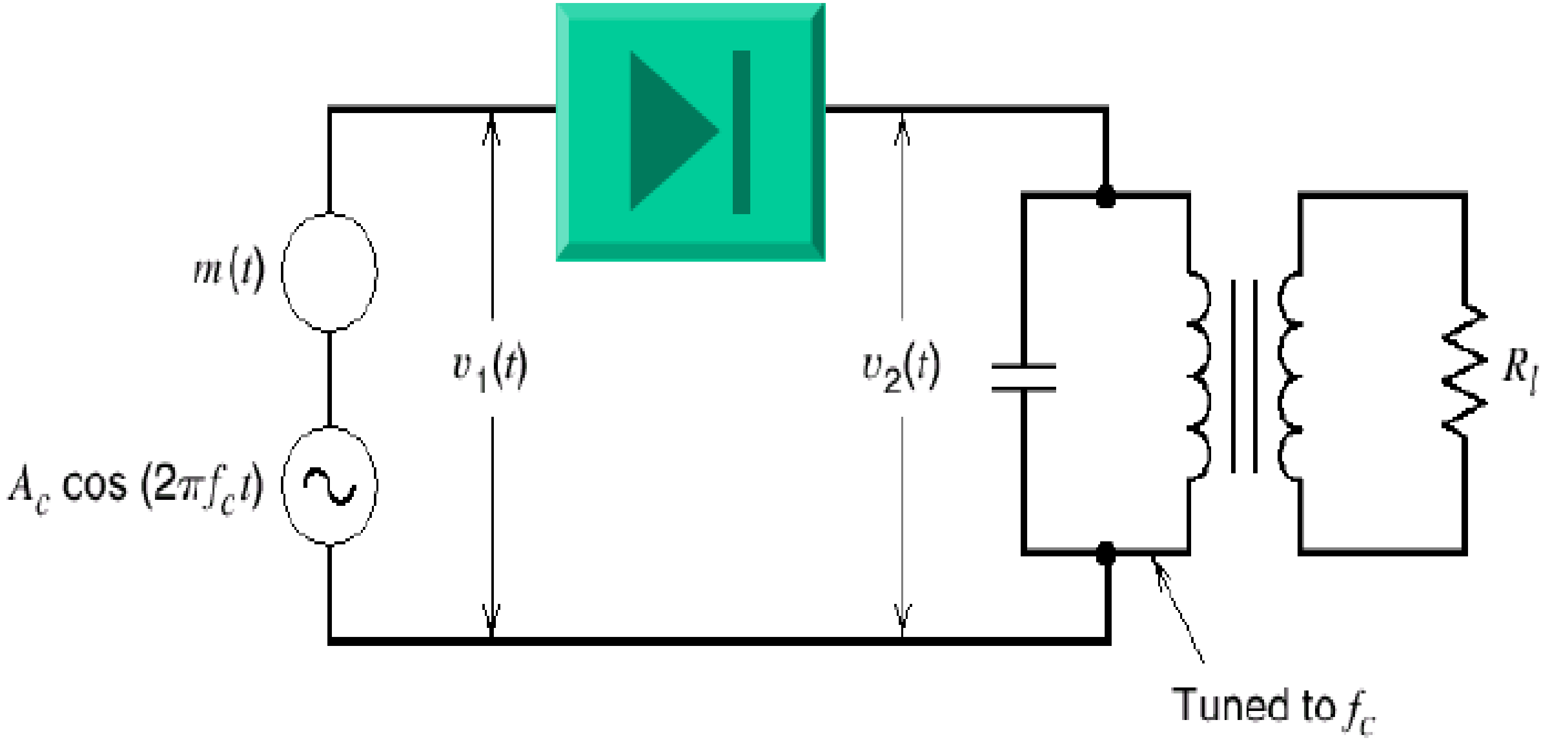
Therefore modulation index of the output signal is given by

$$m = 2\frac{a_2}{a_1} A_m$$

The output AM signal is free from distortion and attenuation only when $(f_c - W) > 2W$ or $f_c > 3W$.

Switching Modulator

Consider a semiconductor diode used as an ideal switch to which the carrier signal $c(t) = A_c \cos(2\pi f_c t)$ and information signal $m(t)$ are applied simultaneously as shown figure



The total input for the diode at any instant is given by

$$v_1 = c(t) + m(t)$$

$$v_1 = A_c \cos 2\pi f_c t + m(t)$$

When the peak amplitude of c(t) is maintained more than that of information signal, the operation is assumed to be dependent on only c(t) irrespective of m(t).

When c(t) is positive, v2=v1 since the diode is forward biased.
Similarly,
when c(t) is negative, v2=0 since diode is reverse biased.

Based upon above operation, switching response of the diode is periodic rectangular wave with an amplitude unity and is given by

$$p(t)=\frac{1}{2}+\frac{1}{\pi}\sum_{n=-\infty}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))$$

$$p(t)=\frac{1}{2}+\underset{n=0,1}{\frac{2}{\pi}}\cos(2\pi f_c t)-\underset{n=-1,+2}{\frac{2}{3\pi}}\cos(6\pi f_c t)+-$$

Therefore the diode response V_o is a product of switching response $p(t)$ and input v_1.

$$v_2=v_1 * p(t)$$

$$V_2=\left[A_c\cos 2\pi f_c t+m(t)\right]\left[\frac{1}{2}+\frac{2}{\pi}\cos 2\pi f_c t-\frac{2}{3\pi}\cos 6\pi f_c t+-+-\right]$$

Applying the Fourier Transform, we get

$$V_2(f)=\frac{A_c}{4}\left[\delta(f-f_c)+\delta(f+f_c)\right]+\frac{M(f)}{2}+\frac{A_c}{\pi}\delta(f)$$
$$+\frac{A_c}{2\pi}\left[\delta(f-2f_c)+\delta(f+2f_c)\right]+\frac{1}{\pi}\left[M(f-f_c)+M(f+f_c)\right]$$

$$-\frac{A_c}{6\pi}\left[\delta(f-4f_c)+\delta(f+4f_c)\right]-\frac{A_c}{3\pi}\left[\delta(f-2f_c)+\delta(f+2f_c)\right]$$

$$-\frac{1}{3\pi}\left[M(f-3f_c)+M(f+f_c)\right]$$

The diode output v_2 consists of

a dc component at $f=0$.

Information signal ranging from 0 to w Hz and infinite number of frequency bands centered at f, $2f_c$, $3f_c$, $4f_c$, ---------

The required AM signal is centered at fc can be separated using band pass filter.

The lower cut off-frequency for the band pass filter should be between w and fc-w and the upper cut-off frequency between fc+w and 2fc.

The filter output is given by the equation

$$S(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi}\frac{m(t)}{A_c}\right]\cos 2\pi f_c t$$

For a single tone information, let $m(t) = A_m \cos(2\pi f_m t)$

$$S(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi}\frac{A_m}{A_c}\cos 2\pi f_m t\right]\cos 2\pi f_c t$$

Therefore modulation index, $m = \frac{4}{\pi}\frac{A_m}{A_c}$

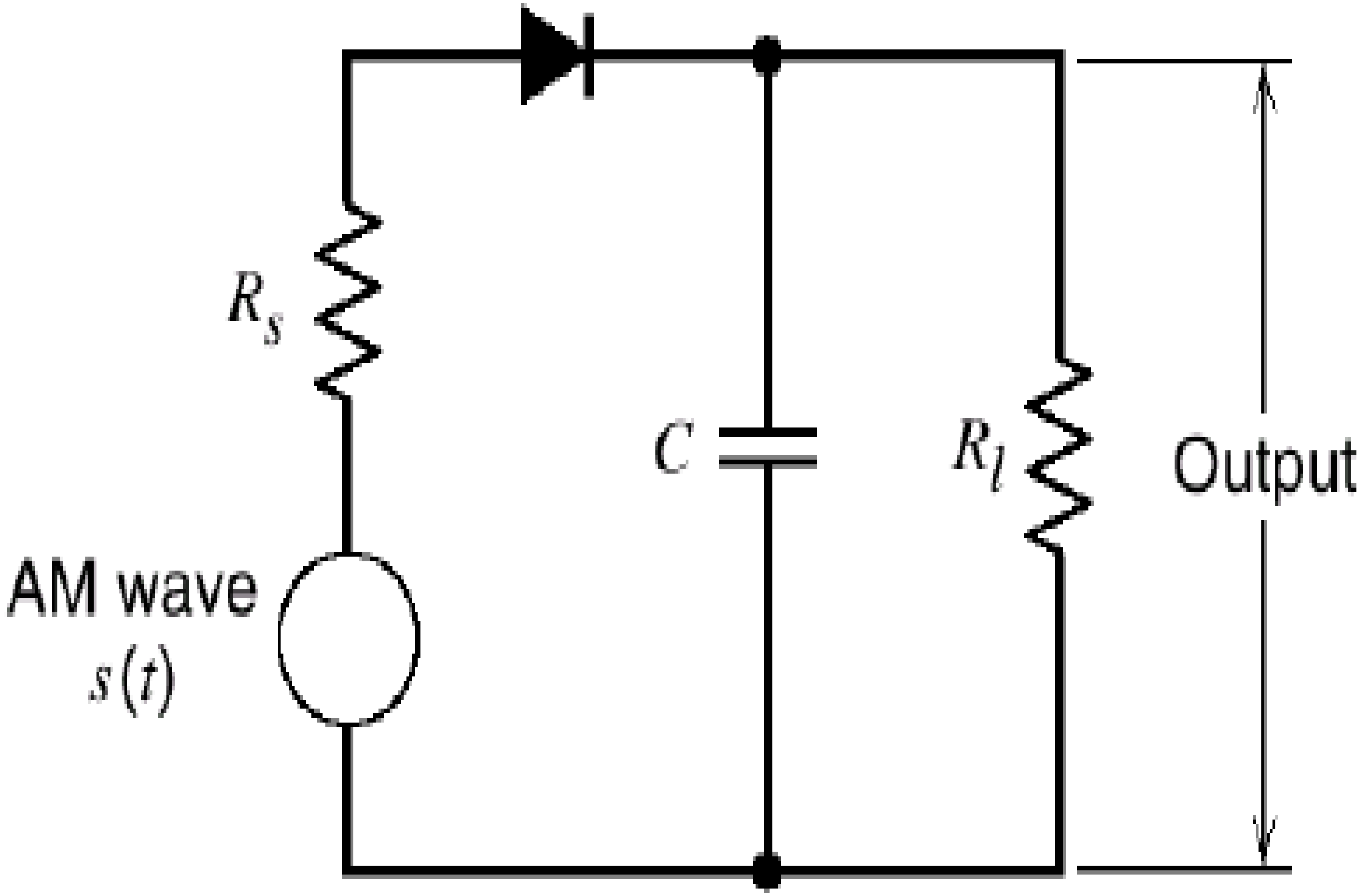
The output AM signal is free from distortions and attenuations only when $fc\text{-}w$>w or $fc>2w$.

Envelope Detector

- **Note:** In AM the Peak Amplitude of the carrier which is also called as the Envelop is varied according to the message signal. So, the envelop of the AM Signal represents the message. E.D is used to track the peak amplitude of the signal

i/p	o/p
$A\cos(2\pi f_c t)$	A
$10\sin(2\pi f_c t)$	10
$e^{-t}\cos(2\pi f_c t)$	e^{-t}
$A\cos(2\pi f_c t) + B\sin(2\pi f_c t)$	$\sqrt{(A^2 + B^2)}$

Envelope Detector



- The charge time constant ($r_f + R_s$) C must be short compared with the carrier period

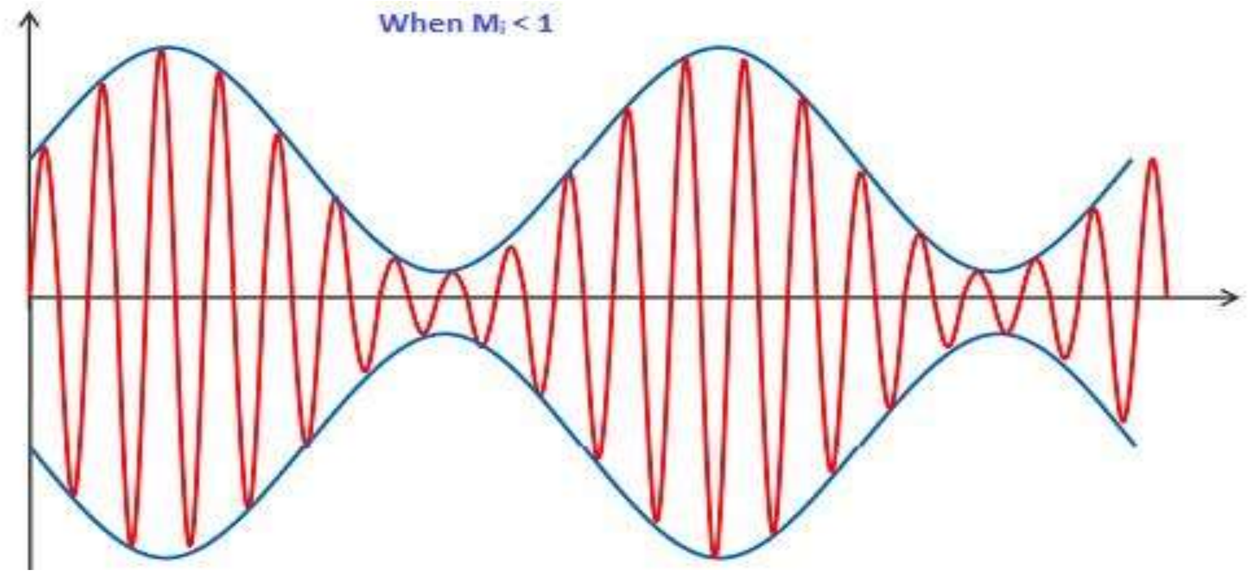
$$\left(r_f + R_s\right)C << \frac{1}{f_c}$$

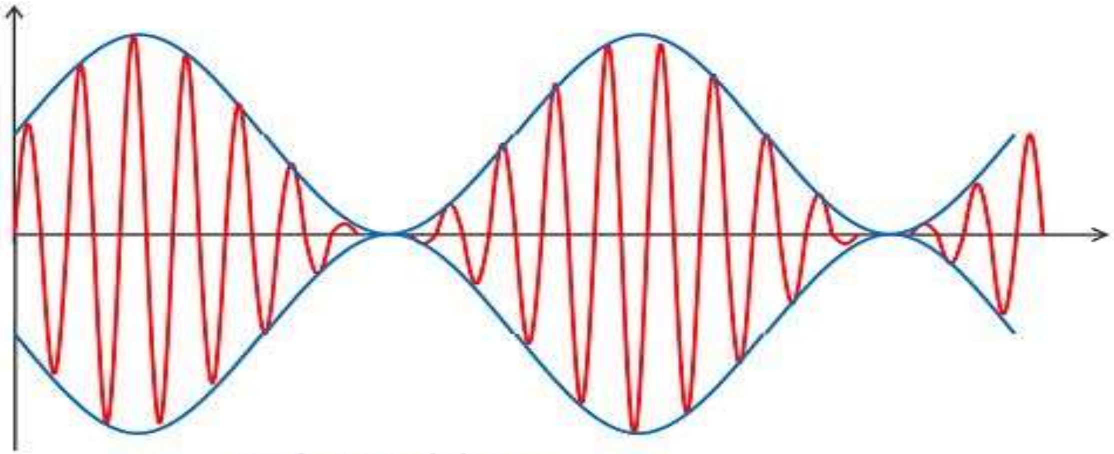
On the other hand, the discharging time-constant $R_L C$ must be long enough to ensure that the capacitor discharges slowly through the load resistor R_L between the positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave.

That is the discharge time constant shall satisfy the condition,

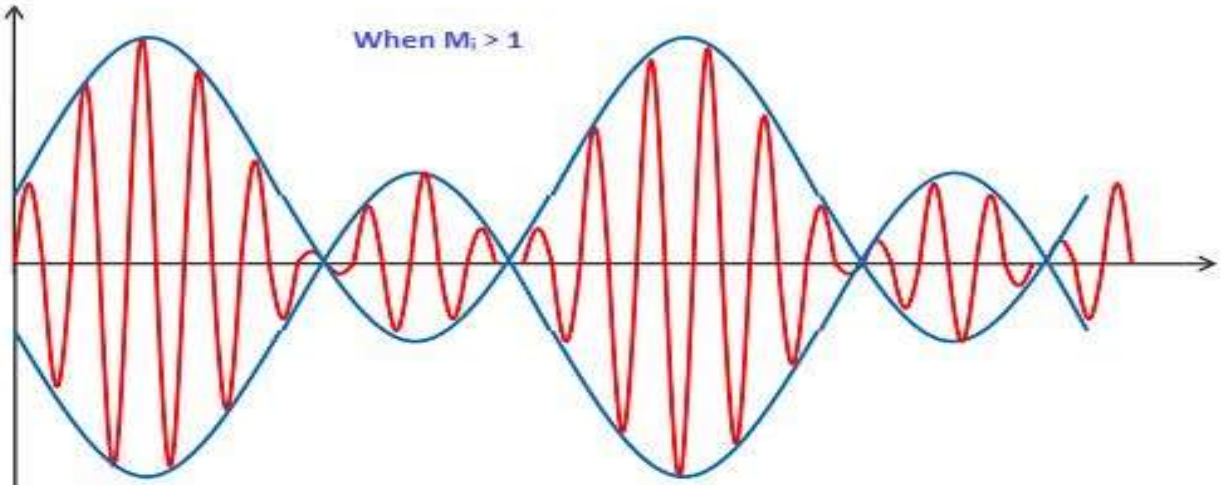
$$\frac{1}{f_c} << R_L C << \frac{1}{W}$$

Sketch the Output of the Below Signals when Passed through Envelop Detector





Perfect Modulation



Over Modulation

- **Advantages**

  1) Generation and demodulation of AM wave are easy

  2) One Tx & Many Rx

- **Disadvantages**

  1) More Power taken by Carrier is 66.66%

  2) BW = 2W Hz of 2fm

Ex2: In an AM transmission the carrier signal $5\cos 2\pi(10^5)t$ is modulated with a message signal $2\cos 2\pi(100)t$

(a) What is the modulation index? (b) What are the frequency components are available in AM signal? (c) Determine the Carrier power, sideband power and total power. (d) What is the efficiency of the AM system?

(a) $\mu = 2/5 = 0.4$ or 40 %

(b) Carrier frequency = 100000 Hz.;

$\text{LSB} = 100000 - 100 = 99900 \text{ Hz}$

$\text{USB} = 100000 + 100 = 100100 \text{ Hz}$

(c) Determine the Carrier power, sideband power and total power.

(c) Carrier power $P_c = \frac{A_c^2}{2} = \frac{25}{2} = 12.5$ W

$$P_{LSB} = P_{USB} = \frac{\mu^2 A_c^2}{8} = \frac{(0.4)^2 X25}{8} = 0.5 \text{ W}$$

Total side band power $= P_{SB} = P_{LSB} + P_{USB} = 1$ W;

Total power $P_t = P_c + P_{SB} = 12.5 + 1 = 13.5$ W;

(d) What is the efficiency of the AM system?

(d) Efficiency $\eta = \frac{\text{Total sideband power}}{\text{Total power}}$

$$= \frac{P_{SB}}{P_t} = \left(\frac{\mu^2}{2+\mu^2}\right) = \frac{(0.4)^2}{2+(0.4)^2} = 7.4\%$$

Ex3: A 400W carrier is modulated to a depth of 75%. Calculate the total power in the modulated wave. Also determine the total sideband power

Ans: The total power $P_t = P_c\left(1+\dfrac{\mu^2}{2}\right)$

$$= 400\left(1+\frac{0.75^2}{2}\right) = 512W$$

The sideband power = $P_{SB} = P_t - P_c$

$$= 512 - 400 = 112\,W$$

Ex4: A broadcast radio transmitter radiates 10 KW, when the modulation percentage is 60. Determine the carrier, lower sideband, upper sideband and total sideband power.

Ans: The carrier power $P_c = \dfrac{P_t}{\left(1+\dfrac{\mu^2}{2}\right)} = \dfrac{10}{1+\dfrac{0.6^2}{2}} = 8.47\ \text{KW}$

The total sideband power $= P_{SB} = P_t - P_c =$

$$10 - 8.47\text{KW} = 1.53\ \text{KW}$$

The lower sideband power $= P_{LSB} = \dfrac{P_{SB}}{2} = 765\text{W}$

The upper sideband power $= P_{USB} = \dfrac{P_{SB}}{2} = 765\text{W}$

Ex5: The antenna current of an AM transmitter is 8 A, when only the carrier is sent, but increases to 8.93 A, when the carrier is modulated by a single sine wave. Find the percentage of modulation. Determine the antenna current when the modulation index is 0.8.

Ans: (a) Given that the carrier current and total currents are

$$I_C = 8A \quad \text{and} \quad I_t = 8.93A$$

The total current and carrier currents are related by

$$I_t = I_C\sqrt{1+\frac{\mu^2}{2}} \Rightarrow \mu = 70.1\% \quad \text{or} \quad 0.701$$

(b) For $\mu = 0.80$

$$I_t = I_C\sqrt{1+\frac{\mu^2}{2}} = 9.19A$$

Ex6: A certain transmitter radiates 9 KW power with the carrier un-modulated and 10.125 KW, when the carrier is simultaneously modulated. Calculate the modulation index. If another sine wave corresponding to 40% modulation is transmitted simultaneously, determine the resultant modulation index and total radiated power.

Ans: (a) Given that $P_c = 9$ KW and $P_t = 10.125$ KW.

$$P_t = P_c\left(1+\frac{\mu_1^2}{2}\right) \Rightarrow \mu_1 = 0.50 \text{ or } 50\%.$$

(b) The second modulating sine wave modulation index is given by $\mu_2 = 40\%$ or 0.40.

Then the total modulation index

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.5^2 + 0.4^2} = 0.64 \text{ or } 64\%$$

(C) The total radiated power

$$P_t = P_c\left(1+\frac{\mu_t^2}{2}\right) = 10.84 \text{ KW}.$$

Double Side Band- SC Modulation

DSB-SC MODULATION

If $m(t)$ is the message signal and $c(t) = A_c \cos(2\pi f_c t)$ is the carrier signal, then

DSBSC modulated wave $s(t)$ is given by

$$s(t) = c(t)\, m(t)$$

$$s(t) = A_c \cos(2\pi f_c t) m(t)$$

The Advantage of Suppressing the Carrier is Power Saved (66.66%)

Note : We are not Transmitting the Carrier along with Modulated Signal as in AM.

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

For the case when base band signal $m(t)$ is limited to the interval $-W<f<W$ as shown in figure below, we find that the spectrum $S(f)$ of the DSBSC wave $s(t)$ is as illustrated below. Except for a change in scaling factor, the modulation process simply translates the spectrum of the base band signal by f_c. The transmission bandwidth required by DSBSC modulation is the same as that for AM.

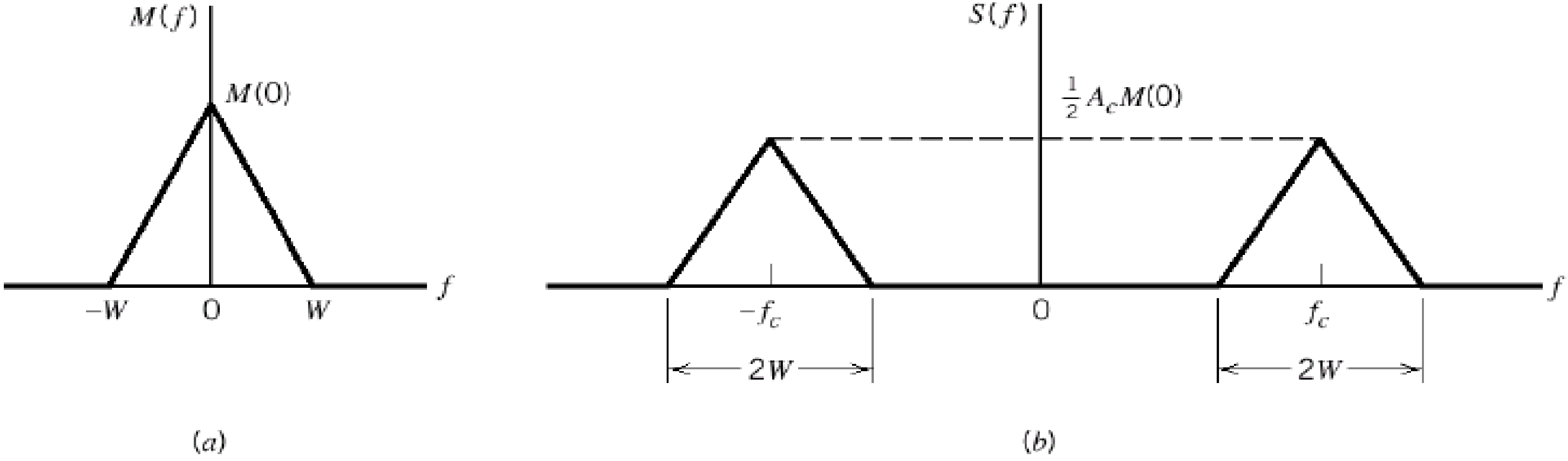


Figure: Message and the corresponding DSBSC spectrum

- **What about Bandwidth & Power of DSB-SC Wave?**

Carrier Power is Saved.

BW Remains Same

Single Tone Modulation of DSB-SC

$$S(t) = A_c\,[1 + K_a\, m(t)]\, \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + A_c K_a A_m \cos 2\pi f_c t \cos 2\pi f_m t$$

$$= A_c K_a A_m \cos 2\pi f_c t \cos 2\pi f_m t = c(t)\, m(t)$$

$$S(t) = A_c\, \mu \cos 2\pi f_c t \cos 2\pi f_m t$$

$$\mu = A_m K_a = A_m / A_c = \text{Modulation Index}$$

$$s(t) = Ac\ \mu \cos 2\pi f_c t \cos 2\pi f_m t$$

$$S(t) = Ac\mu/2 \cos[2\pi(f_c+f_m)]t + Ac\mu/2 \cos[2\pi (f_c-f_m)]t$$

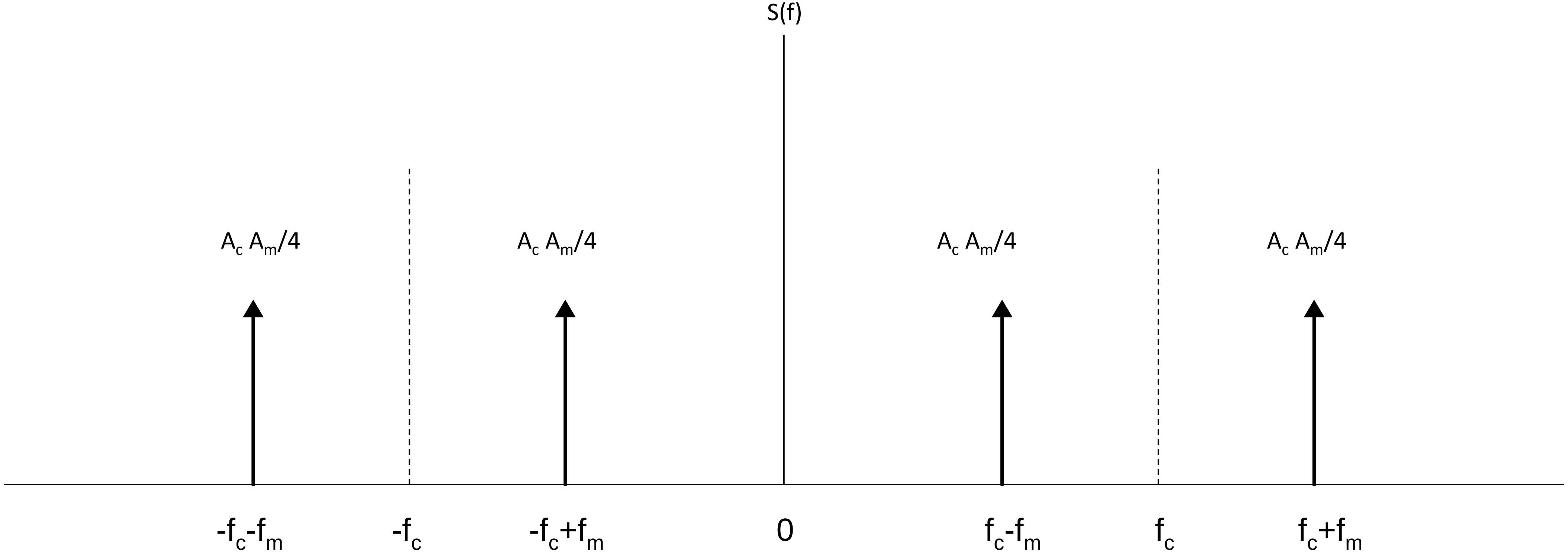
I term II term

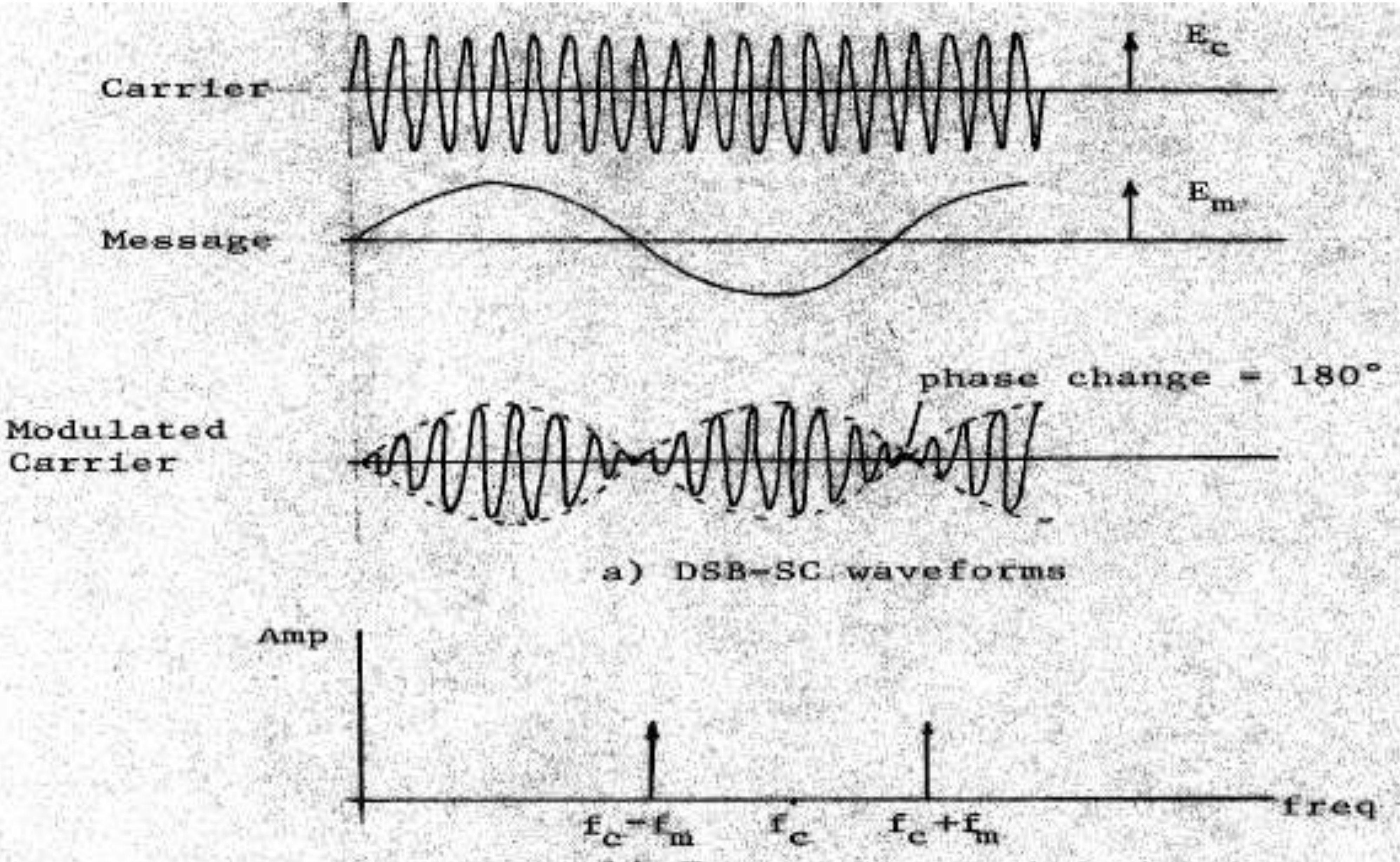
$$= Ac\mu/4\,[\delta(f-(f_c+f_m))+\delta(f+(f_c+f_m))] + Ac\mu/4\,[\delta(f-(f_c-f_m))+\delta(f+(f_c-f_m))]$$

I term: Amplitude= $\mu Ac/2$, frequency= f_c+f_m , Upper sideband frequency

II term: Amplitude= $\mu Ac/2$, frequency= f_c-f_m , Lower sideband frequency

Spectrum of Single tone Modulation





a) DSB-SC waveforms

b) Frequency spectrum

Total Power Required for DSB-SC Wave

- $S(t) = A_c\, \mu \, Cos2\pi f_c t \, Cos2\pi f_m t$

  $= A_c\mu/2 Cos[2\pi(f_c+f_m)]t \; + \; A_c\mu/2 Cos[2\pi (f_c-f_m)]t$

  Total Power = Power in LSB + Power in USB

  $= (A_c\mu/2\sqrt{2})^2 \; + (A_c\mu/2\sqrt{2})^2$

  $= A_c^2\mu^2/4$

  $= P_c\mu^2/2$

  Power Efficiency of Modulation Efficiency = 100%

Generation of DSBSC Waves

Balanced Modulator (Product Modulator)

Ring Modulator

Detection of DSB-SC waves

Coherent Detection or Synchronous Detection or Heterodyne Detection

Costas Receiver

Balanced Modulator (Product Modulator)

$$s_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$
$$s_2(t) = A_c[1 - k_a m(t)]\cos(2\pi f_c t)$$

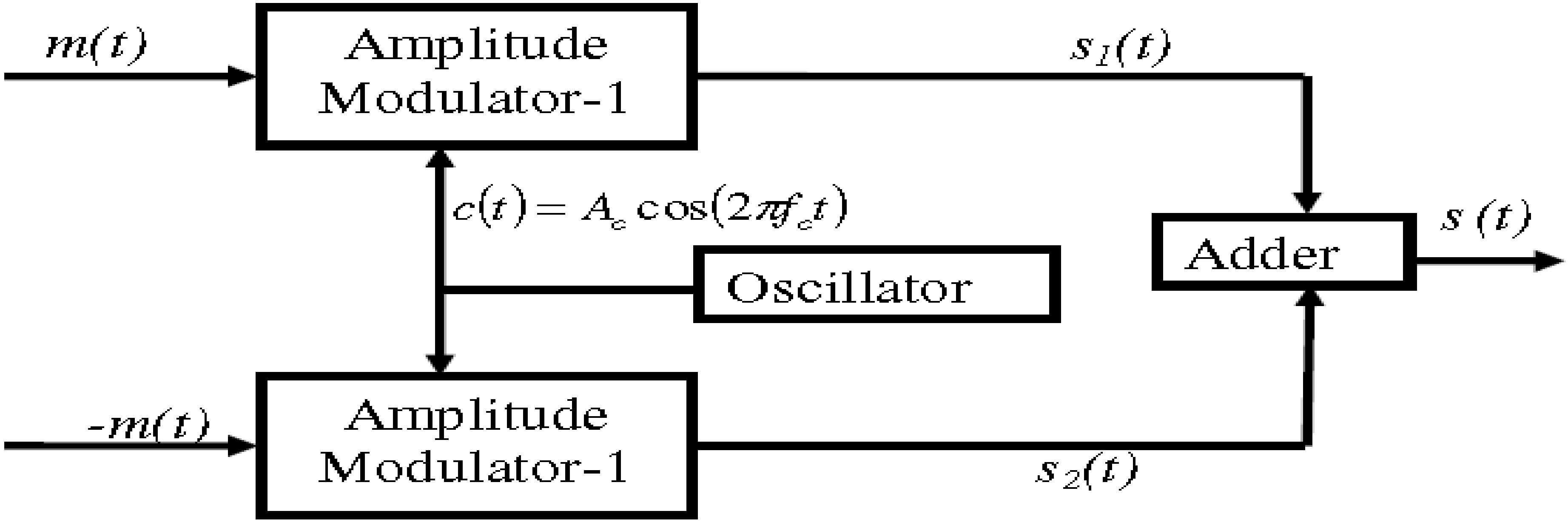


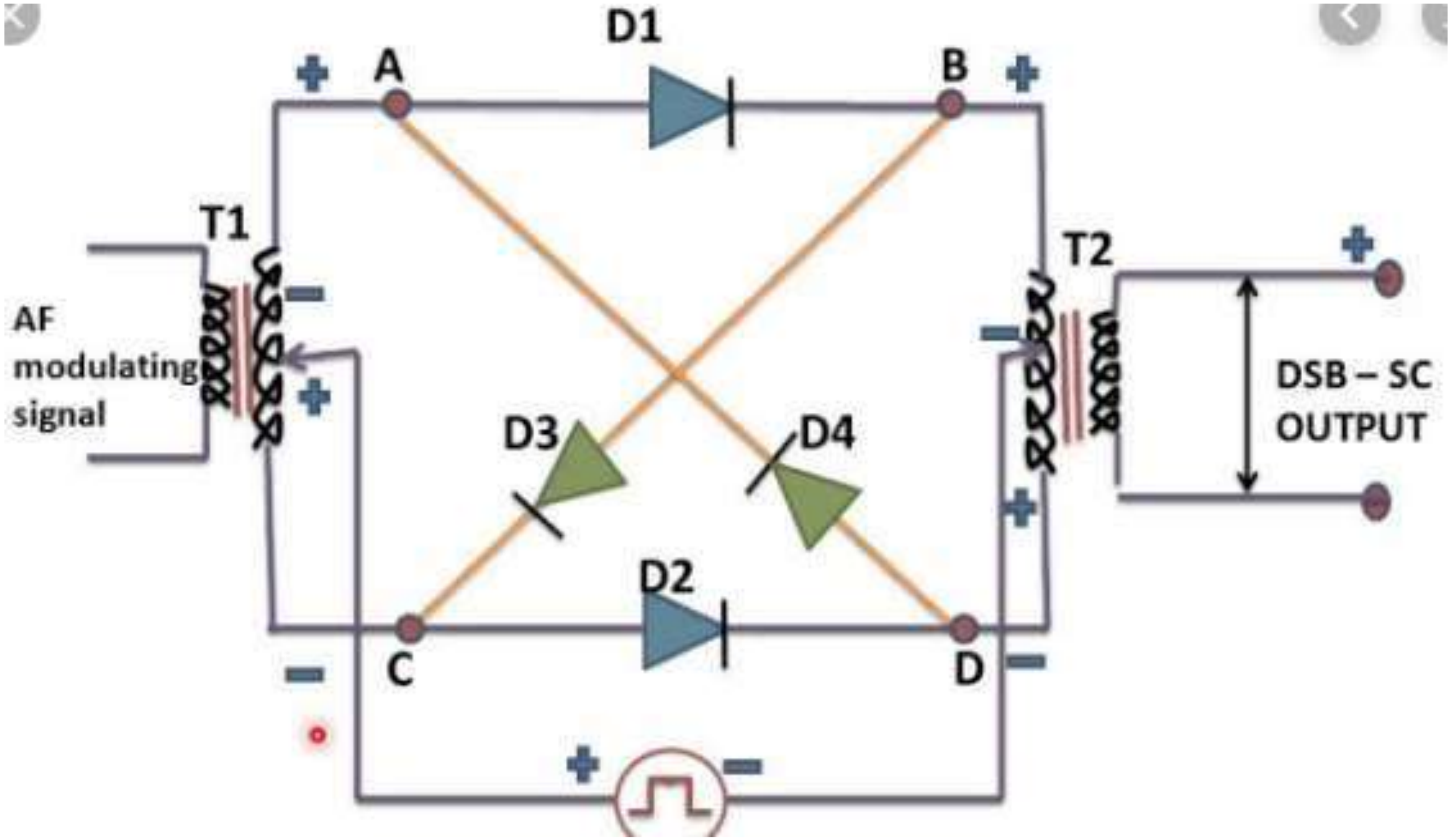
Fig .3 : Balanced modulator

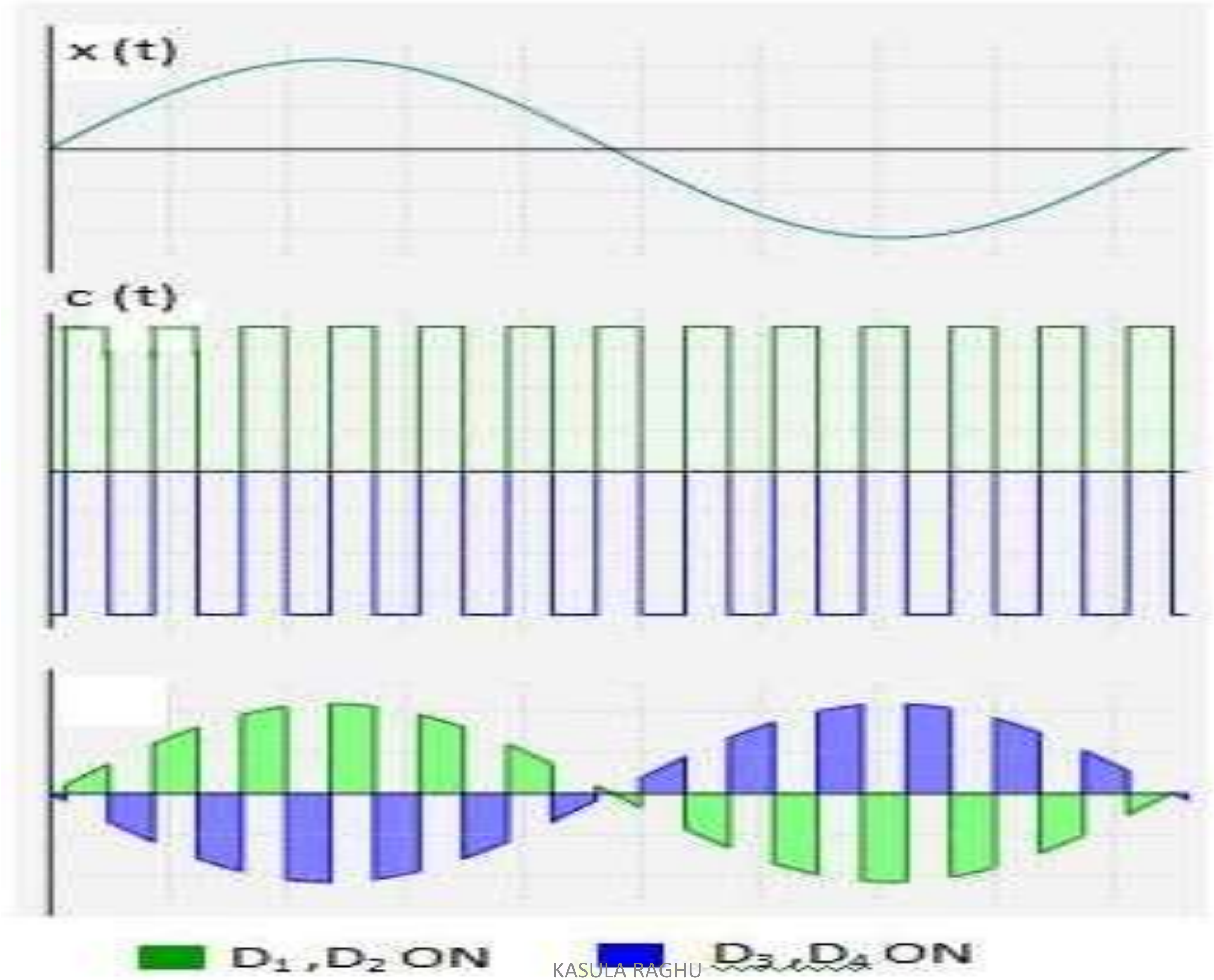
Subtracting $s_2(t)$ from $s_1(t)$,

$$s(t) = s_1(t) - s_2(t)$$

$$s(t) = 2k_a m(t) A_c \cos(2\pi f_c t)$$

Ring Modulation




x (t)
c (t)
D_1 , D_2 ON
D_3 , D_4 ON

Thus the ring modulator in its ideal form is a product modulator for square wave carrier and the base band signal m(t). The square wave carrier can be expanded using Fourier series as

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t(2n-1))$$

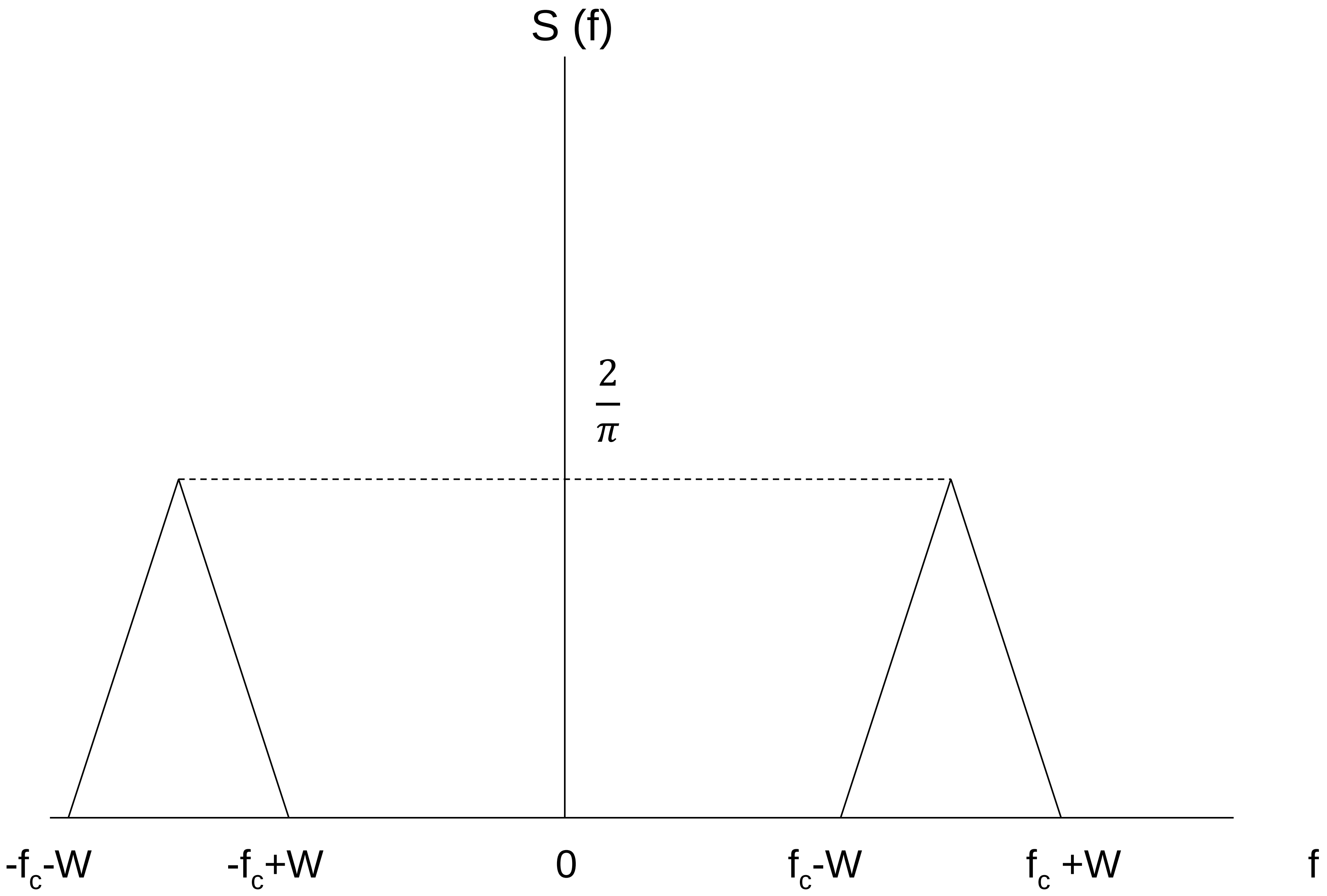
Therefore the ring modulator out put is given by

$$s(t) = m(t)c(t)$$

$$s(t) = m(t)\left[\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t(2n-1))\right]$$

$$s(t) = \frac{4}{\pi} m(t)\text{Cos}2\pi f_c t - \frac{4}{3\pi} m(t)\text{Cos}2\pi(3f_c)t + \ldots\ldots..$$

$$S(f) = \frac{2}{\pi} [M(f - fc) + M(f + fc)] - \frac{2}{3\pi} [M(f - 3fc) + M(f + 3fc)] + \ldots\ldots$$


S (f)
$\frac{2}{\pi}$
$-f_c$-W
$-f_c$+W
0
f_c-W
f_c +W
f

Coherent or Synchronous or Heterodyne Detection

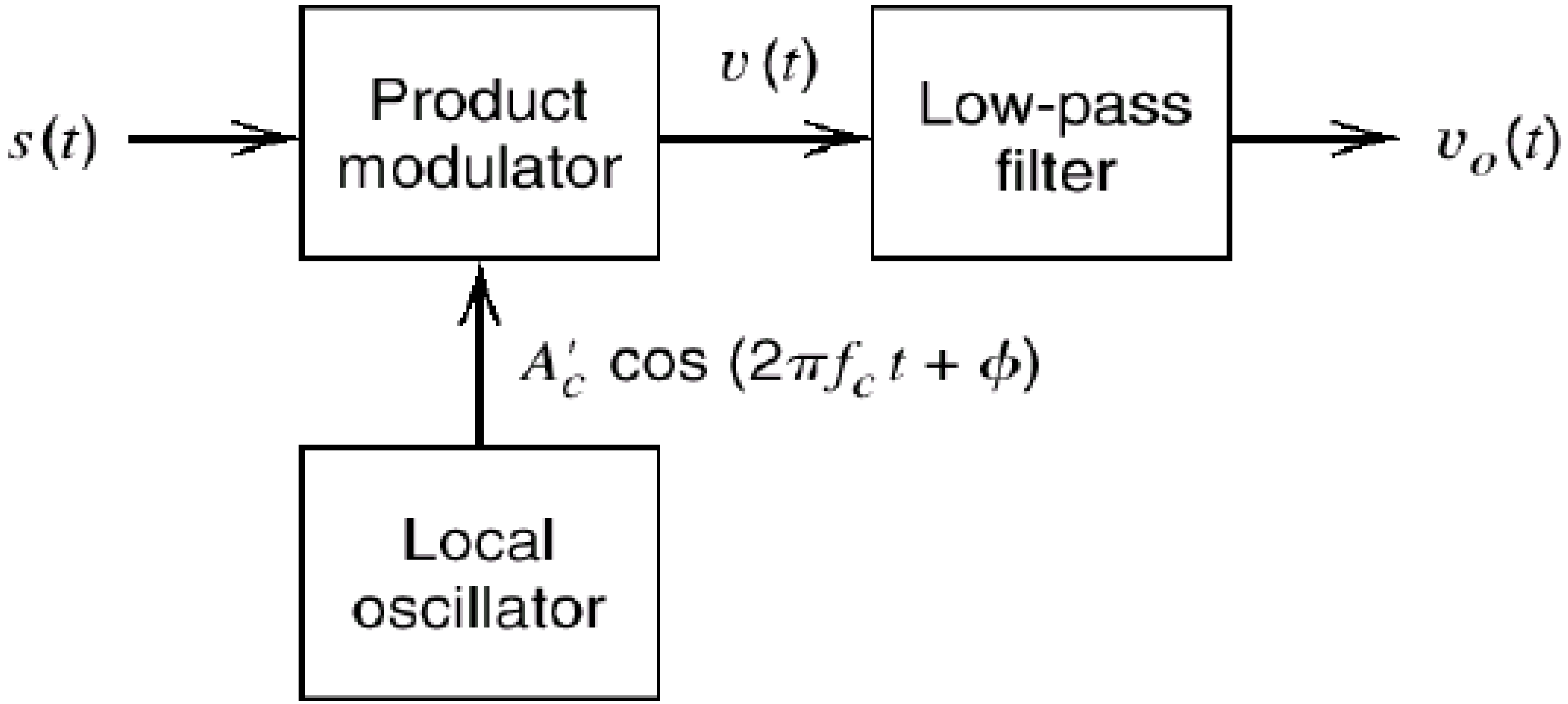


Fig.5 : Coherent detector

Let $A_c^1 \cos(2\pi f_c t + \phi)$ be the local oscillator signal, and $s(t) = A_c \cos(2\pi f_c t) m(t)$ be the DSBSC wave. Then the product modulator output $v(t)$ is given by

$$v(t) = A_c A_c^1 \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t)$$

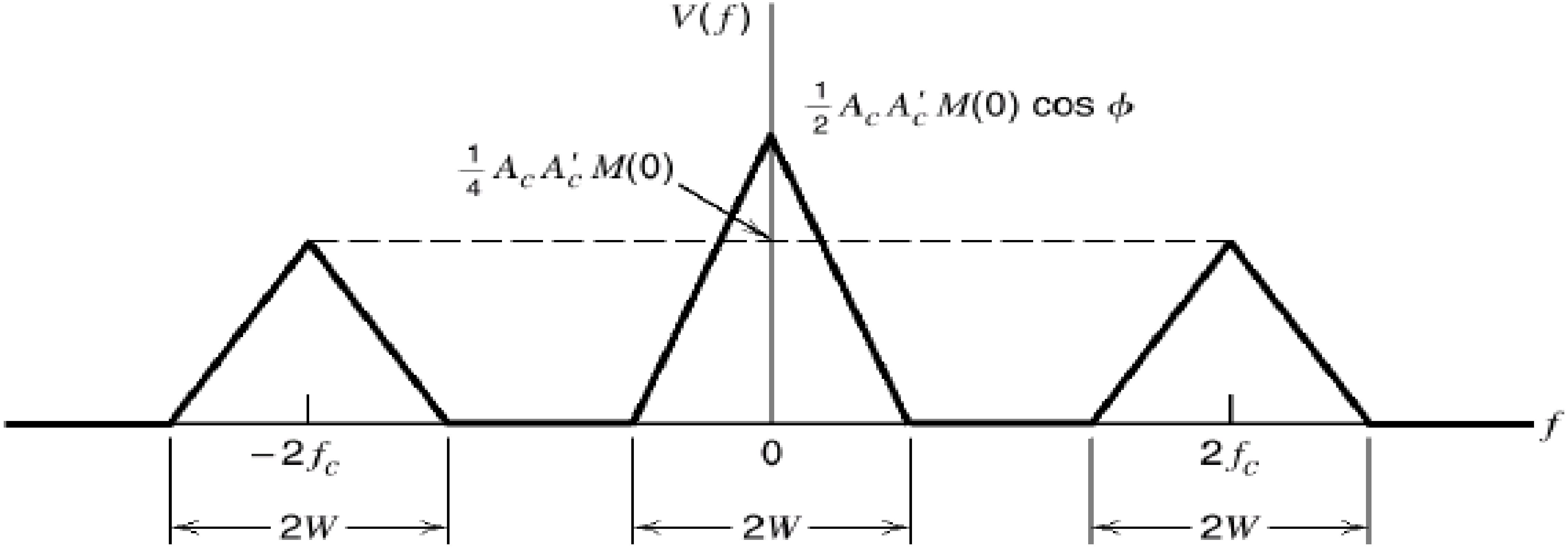
$$v(t) = \frac{A_c A_c^1}{4} \cos(4\pi f_c t + \phi) m(t) + \frac{A_c A_c^1}{2} \cos(\phi) m(t)$$

The first term in the above expression represents a DSBSC modulated signal with a carrier frequency $2f_c$, and the second term represents the scaled version of message signal. Assuming that the message signal is band limited to the interval $-w < f < w$, the spectrum of $v(t)$ is plotted as shown below.

From the spectrum, it is clear that the unwanted component (first term in the expression) can be removed by the low-pass filter, provided that the cut-off frequency of the filter is greater than W but less than 2fc-W. The filter output is given by

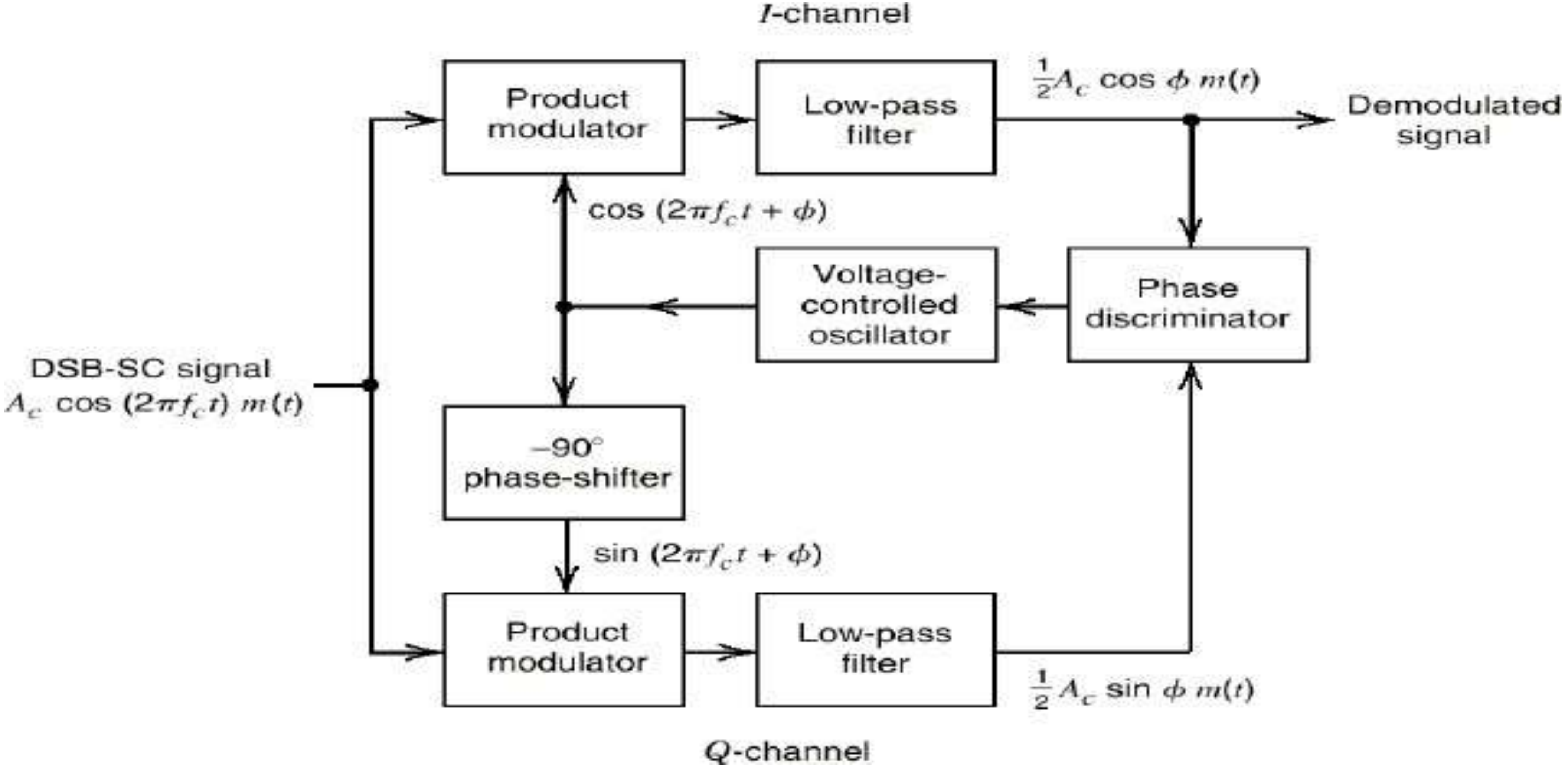
$$v_o(t) = \frac{A_c A_c^1}{2} \cos(\phi) m(t)$$

- The quadrature null effect
  - The zero demodulated signal, when occurs for Φ=±π/2
  - The phase error Φ in the local oscillator causes the detector output to be attenuated by a factor equal to cos Φ



$V(f)$
$\frac{1}{2}A_c A_c' M(0) \cos\phi$
$\frac{1}{4}A_c A_c' M(0)$
f
$-2f_c$
0
$2f_c$
$2W$
$2W$
$2W$

Costas Receiver



- **Advantages**

  One to One Communications (Walkie Talkie)

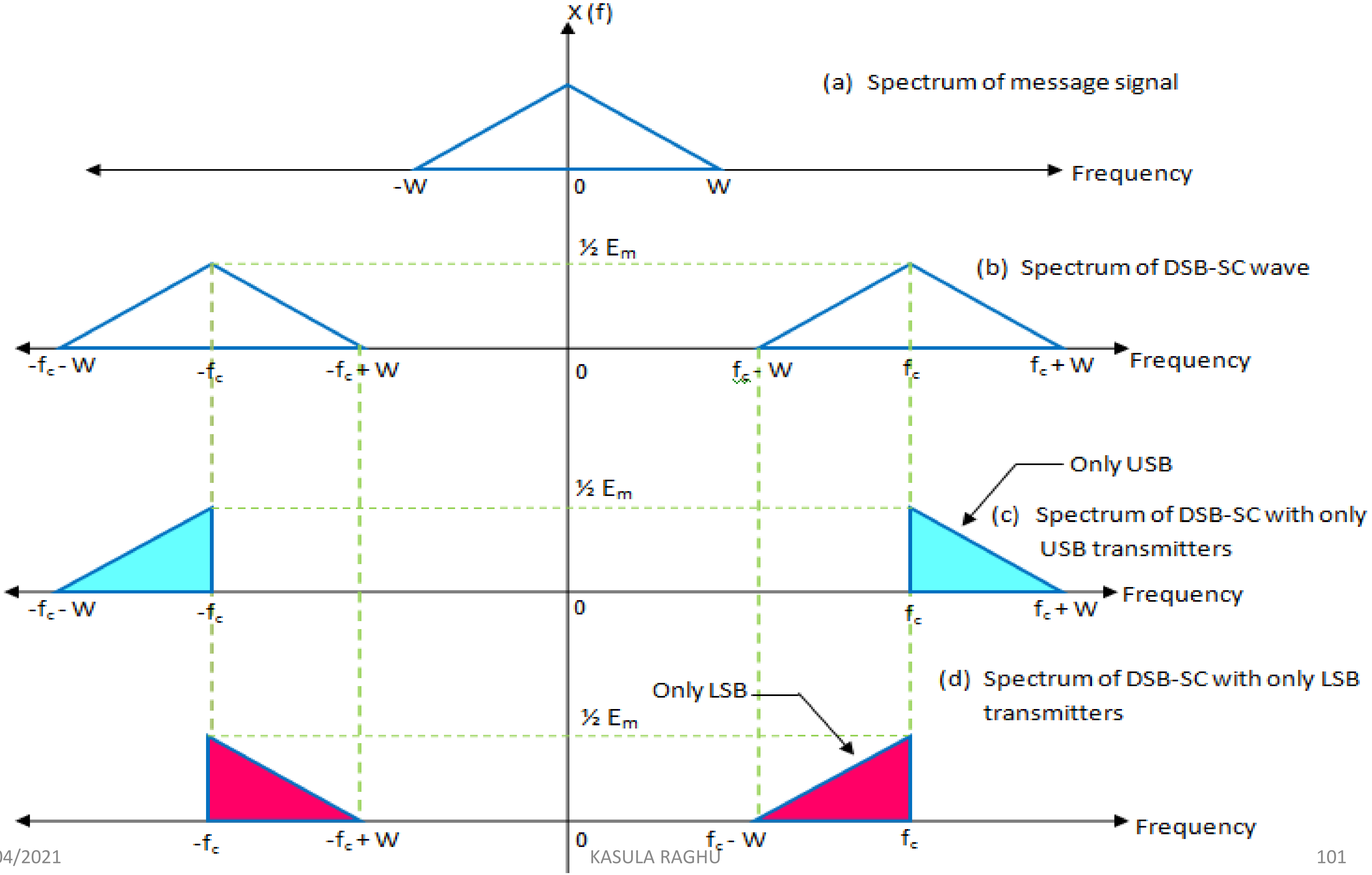
- **Disadvantages**

  Less Power than AM

  100% Modulation efficiency

  BW = AM = DSC-SC = 2W Hz

Single Side Band- SC Modulation



X (f)
(a) Spectrum of message signal
Frequency
-W
0
W
½ E_m
(b) Spectrum of DSB-SC wave
Frequency
$-f_c - W$
$-f_c$
$-f_c + W$
0
$f_c - W$
f_c
$f_c + W$
Only USB
½ E_m
(c) Spectrum of DSB-SC with only USB transmitters
Frequency
$-f_c - W$
$-f_c$
0
f_c
$f_c + W$
(d) Spectrum of DSB-SC with only LSB transmitters
Only LSB
½ E_m
Frequency
$-f_c$
$-f_c + W$
0
$f_c - W$
f_c

Single-Sideband Modulation

- Single-Sideband Modulation
  - Suppress one of the two sideband in the DSB-SC modulated wave
- Theory
  - A DSB-SC modulator using the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

  - The resulting DSB-SC modulated wave is

$$\begin{aligned} S_{DSB}(t) &= c(t)m(t) \\ &= A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) \\ &= \frac{1}{2}A_c A_m \cos[2\pi(f_c + f_m)t] + \frac{1}{2}A_c A_m \cos[2\pi(f_c - f_m)t] \end{aligned}$$

  - Suppressing the second term in the above Eq. the upper and lower SSB modulated wave are

$$S_{USSB}(t) = \frac{1}{2}A_c A_m \cos[2\pi(f_c + f_m)t] \quad (3.14)$$

$$S_{USSB}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) - \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t)$$

$$S_{LSSB}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) + \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t)$$

- A sinusoidal SSB modulated wave

$$S_{SSB}(t) = \frac{1}{2} A_c A_m \cos(2\pi f_c t) \cos(2\pi f_m t) + \frac{1}{2} A_c A_m \sin(2\pi f_c t) \sin(2\pi f_m t)$$

$$\boxed{S_{SSB}(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) \pm \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)}$$

Equation of SSB-SC

Total Power & Power or Modulation efficiency

Total Power = Power in LSB or

= Power in USB

= $(A_c\mu/2\sqrt{2})$

= $A_c\mu /8$

= $P_c\mu /4$

Power Efficiency of Modulation Efficiency = 100%

Generation of SSB-SC Wave

Frequency Description Method

Phase Description Method

Detection of SSB-SC waves

Coherent Detection

Frequency Description Method

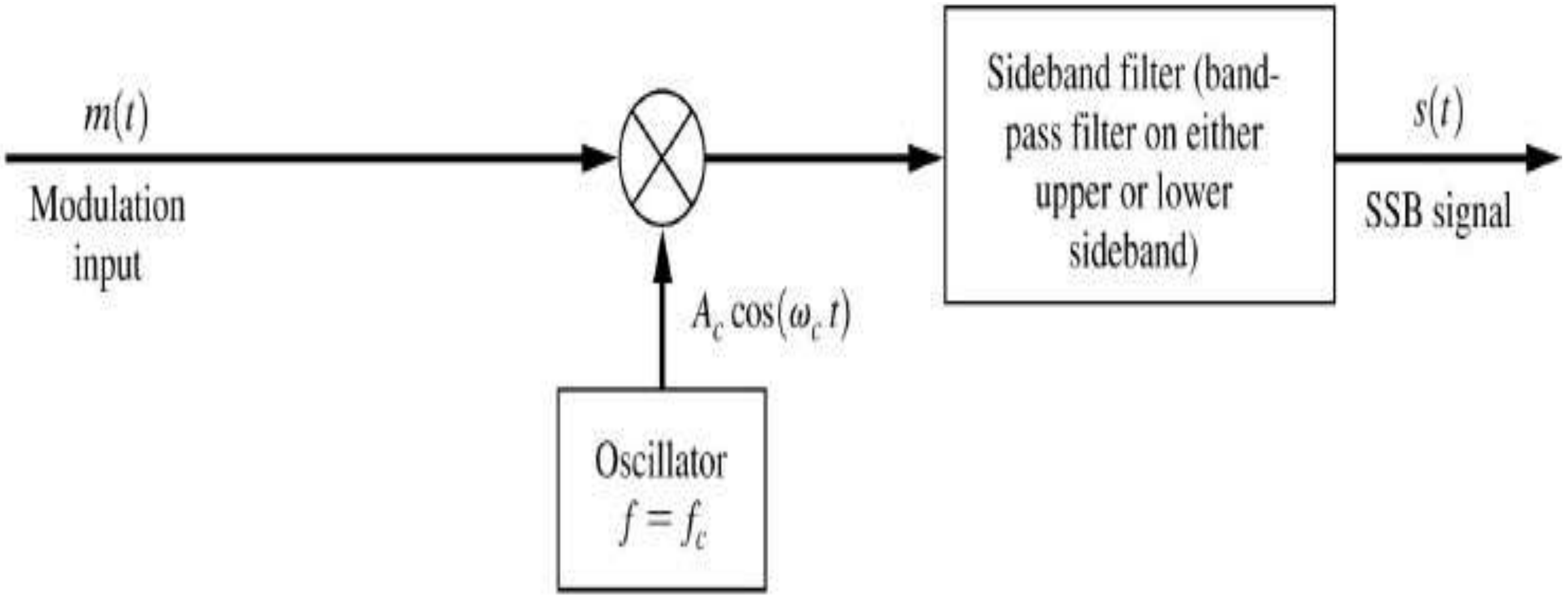


Figure .8 : Frequency discriminator to generate SSBSC wave

• Upto 500Khz	Mechanical Filter
• Upto 20 MHz	RC Filter
• Greater Than 20 MHz	Crystal filter

Phase Description Method

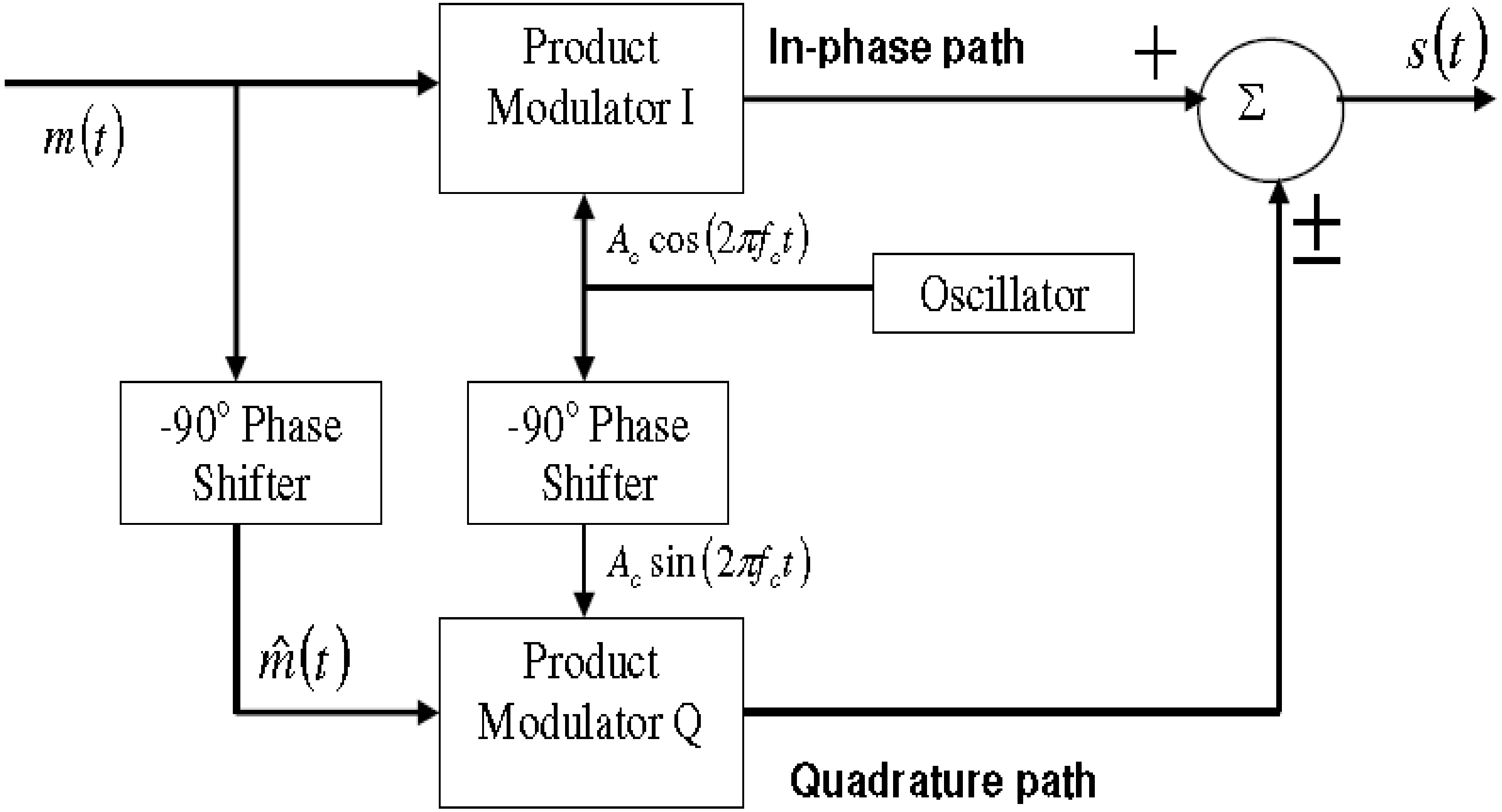


Figure 15 : Block diagram of phase discriminator

Coherent Detection

Demodulation of SSBSC wave using coherent detection is as shown in 16 . The SSB wave *s(t)* together with a locally generated carrier $c(t) = A_c{}^{1}\cos(2\pi f_c t + \phi)$ is applied to a product modulator and then low-pass filtering of the modulator output yields the message signal.

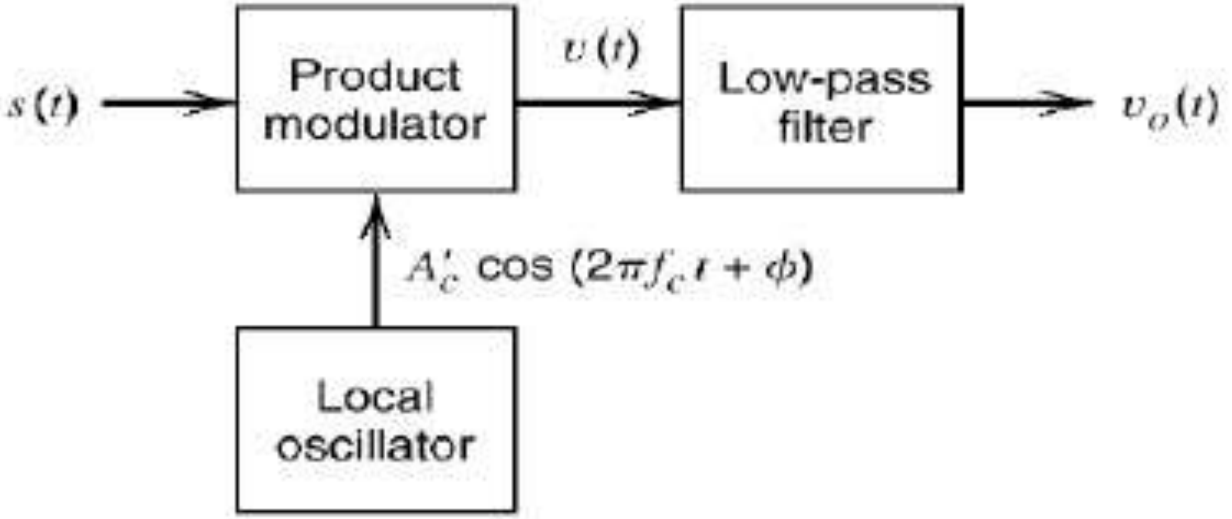


Figure 16 : Block diagram of coherent detector for SSBSC

The product modulator output *v(t)* is given by

$$v(t) = A_c^{1}\cos(2\pi f_c t + \phi)s(t), \quad \text{Put } \phi = 0$$

$$v(t) = \frac{1}{2}A_c\cos(2\pi f_c t)[m(t)\cos(2\pi f_c t) \pm \hat{m}(t)\sin(2\pi f_c t)]$$

$$v(t) = \frac{1}{4}A_c m(t) + \frac{1}{4}A_c[m(t)\cos(4\pi f_c t) \pm \hat{m}(t)\sin(4\pi f_c t)] \quad \text{..............(1)}$$

The first term in the above equation 1 is desired message signal. The other term represents an SSB wave with a carrier frequency of $2f_c$ as such; it is an unwanted component, which is removed by low-pass filter.

$$s(t) = \frac{Ac}{2}\left(m(t)\,\mathrm{Cos}2\pi f_c t + \hat{m}(t)\mathrm{Sin}2\pi f_c t\right)$$

Output of Product Modulator is S(t) x (LO)o

$$v(t) = \left[\frac{Ac}{2}\left(m(t)\,\mathrm{Cos}2\pi f_c t + \hat{m}(t)\mathrm{Sin}2\pi f_c t\right)\right]\mathrm{Cos}(2\pi f_c t + \varphi)$$

$$v(t) = \left[\frac{Ac}{2}\left(m(t)\,\mathrm{Cos}2\pi f_c t + \hat{m}(t)\mathrm{Sin}2\pi f_c t\right)\right]\mathrm{Cos}(2\pi f_c t + \varphi)$$

When Passed through LPF

$$V_o(t) = \frac{Ac}{2}m(t)\cos\varphi + \frac{Ac}{2}\hat{m}(t)\sin\varphi$$

Time Domain Equation of SSB -SC

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \qquad \text{--------------------} (1)$$

where $S_I(t)$ is the in-phase component of the SSB wave and $S_Q(t)$ is its quadrature component. The in-phase component $S_I(t)$ except for a scaling factor, may be derived from $S(t)$ by first multiplying $S(t)$ by $\cos(2\pi f_c t)$ and then passing the product through a low-pass filter. Similarly, the quadrature component $S_Q(t)$, except for a scaling factor, may be derived from $s(t)$ by first multiplying $s(t)$ by $\sin(2\pi f_c t)$ and then passing the product through an identical filter.

The Fourier transformation of $S_I(t)$ and $S_Q(t)$ are related to that of SSB wave as follows, respectively.

$$S_I(f) = \begin{cases} S(f - f_c) + S(f + f_c), -w \le f \le w \\ 0, elsewhere \end{cases} \qquad \text{-------------------} (2)$$

$$S_Q(f) = \begin{cases} j[S(f-f_c) - S(f+f_c)], -w \le f \le w \\ 0, elsewhere \end{cases} \quad \text{------------------}(\ 3\ )$$

where $-w < f < w$ defines the frequency band occupied by the message signal $m(t)$.

Consider the SSB wave that is obtained by transmitting only the upper side band, shown in figure 10 . Two frequency shifted spectra $S(f-f_c)$ and $S(f+f_c)$ are shown in figure 11 and figure 12 respectively. Therefore, from equations 2 and 3 , it follows that the corresponding spectra of the in- phase component $S_I(t)$ and the quadrature component $S_Q(t)$ are as shown in figure 13 and 14 respectively.

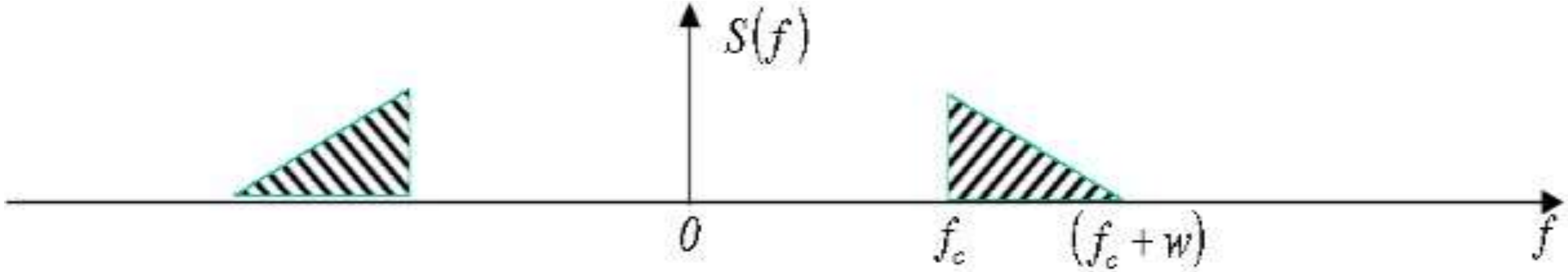


Figure 10 : Spectrum of SSBSC-USB

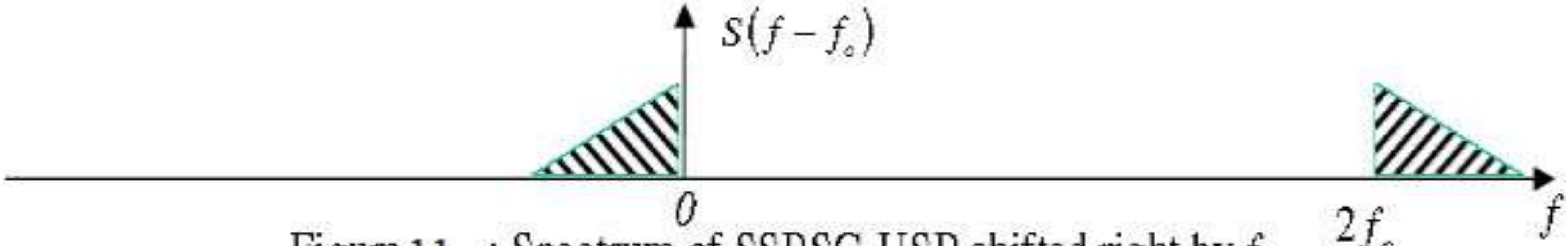


Figure 11 : Spectrum of SSBSC-USB shifted right by f_c

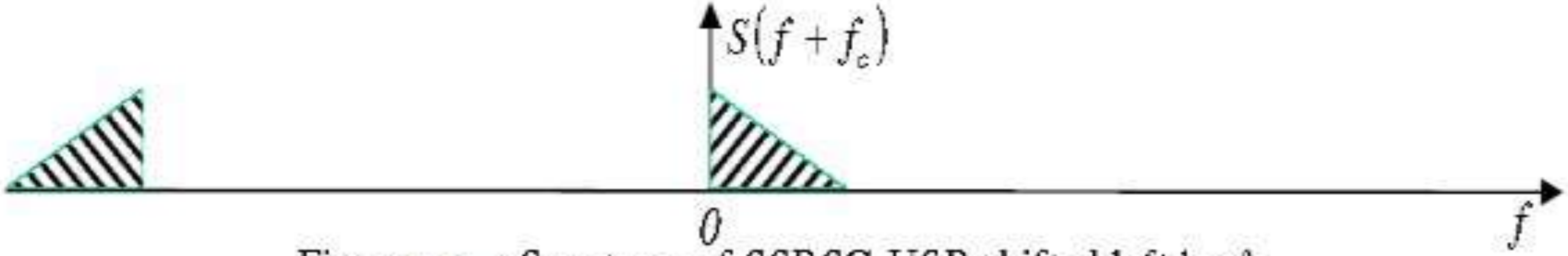


Figure 12 : Spectrum of SSBSC-USB shifted left by f_c

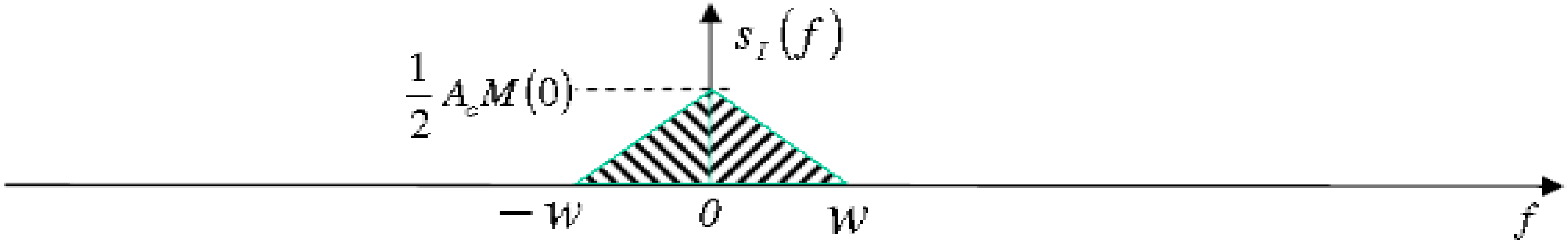


Figure 13 : Spectrum of in-phase component of SSBSC-USB

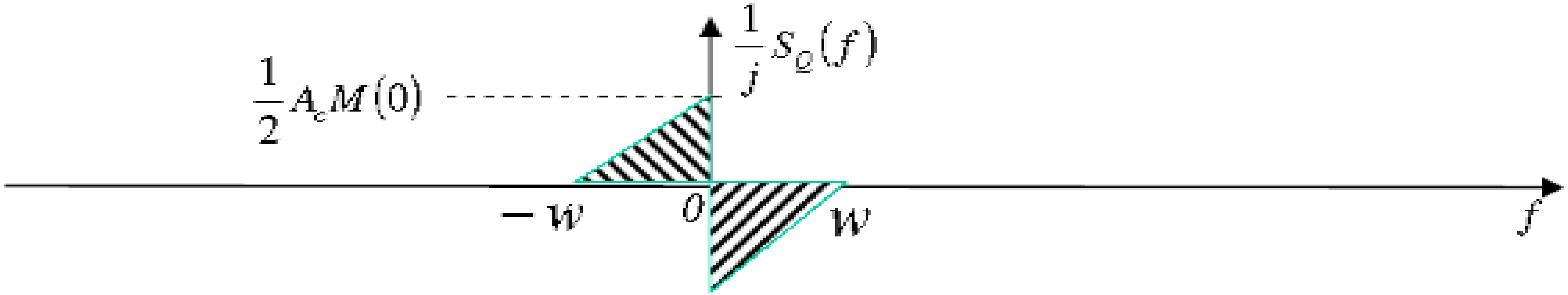


Figure 14 : Spectrum of quadrature component of SSBSC-USB

From the figure 13 , it is found that

$$S_I(f) = \frac{1}{2}A_cM(f)$$

where $M(f)$ is the Fourier transform of the message signal $m(t)$. Accordingly in-phase component $S_I(t)$ is defined by equation 4

$$\boxed{s_I(t) = \frac{1}{2} A_c m(t)} \qquad \text{-------------------- (4)}$$

Now on the basis of figure14 , it is found that

$$S_Q(f) = \begin{cases} \frac{-j}{2} A_c M(f), f > 0 \\ 0, f = 0 \\ \frac{j}{2} A_c M(f), f < 0 \end{cases}$$

$$S_Q(f) = \frac{-j}{2} A_c \operatorname{sgn}(f) M(f) \qquad \text{-------------------- (5)}$$

where $\operatorname{sgn}(f)$ is the Signum function.

But from the discussions on Hilbert transforms, it is shown that

$$-j\,\mathrm{sgn}(f)M(f)=\hat{M}(f) \qquad \text{------------------- (6)}$$

where $\hat{M}(f)$ is the Fourier transform of the Hilbert transform of *m(t)*. Hence the substituting equation (6) in (5), we get

$$S_Q(f)=\frac{1}{2}A_c\hat{M}(f) \qquad \text{-------------------------- (7)}$$

Therefore quadrature component $s_Q(t)$ is defined by equation 8

$$\boxed{s_Q(t)=\frac{1}{2}A_c\hat{m}(t)} \qquad \text{-------------------------- (8)}$$

Therefore substituting equations (4) and (8) in equation in (1), we find that canonical representation of an SSB wave *s(t)* obtained by transmitting only the upper side band is given by the equation 9

$$\boxed{s_U(t)=\frac{1}{2}A_c m(t)\cos(2\pi f_c t)-\frac{1}{2}A_c\hat{m}(t)\sin(2\pi f_c t)} \qquad \text{-------------- (9)}$$

- Following the same procedure, we can find the canonical representation for an SS wave
- s(t) obtained by transmitting only the lower side band is given by

$$s_L(t) = \frac{1}{2} A_c m(t)\cos(2\pi f_c t) + \frac{1}{2} A_c \hat{m}(t)\sin(2\pi f_c t) \qquad \text{-------------} \ (10)$$

- **Advantages**

  Voice Communications

- **Disadvantages**

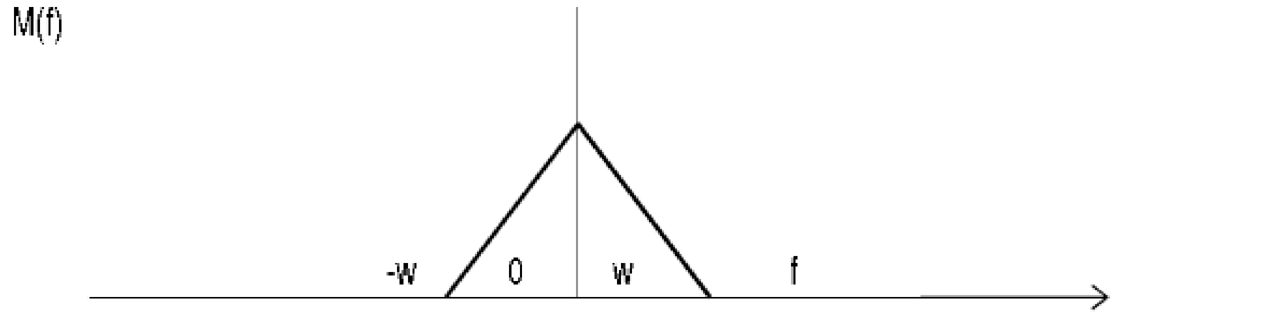
  Sharp Cut-off Frequency Filters are not Available Practically

  Less Power than AM & DSB-SC

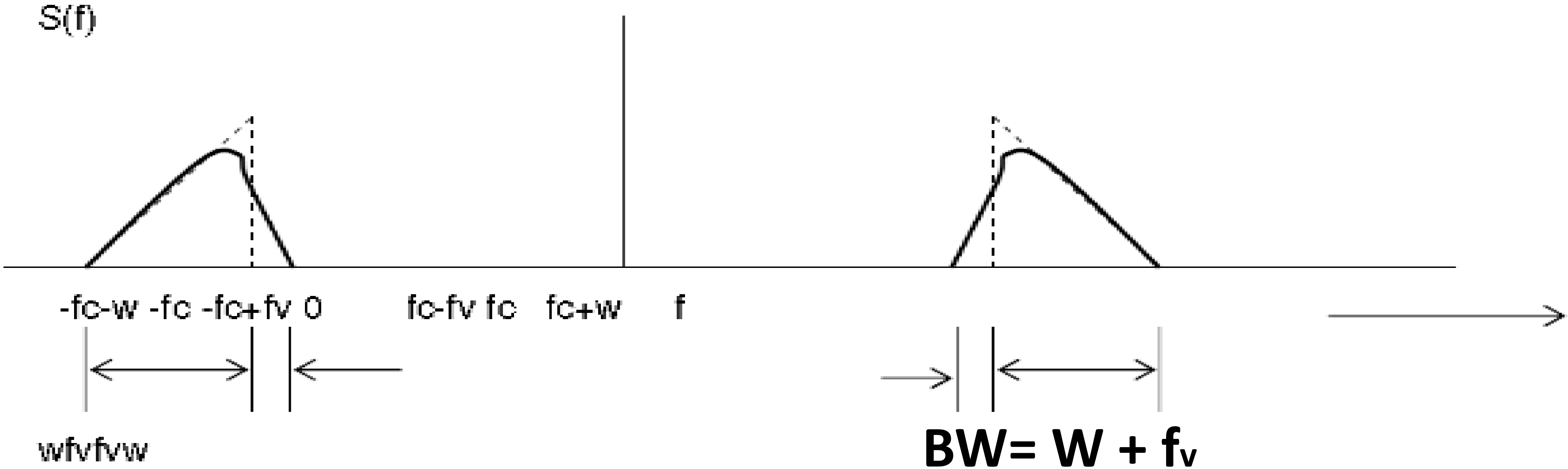
  100% Modulation efficiency

  BW = WHz or f_m Hz

Vestigial Side Band Modulation



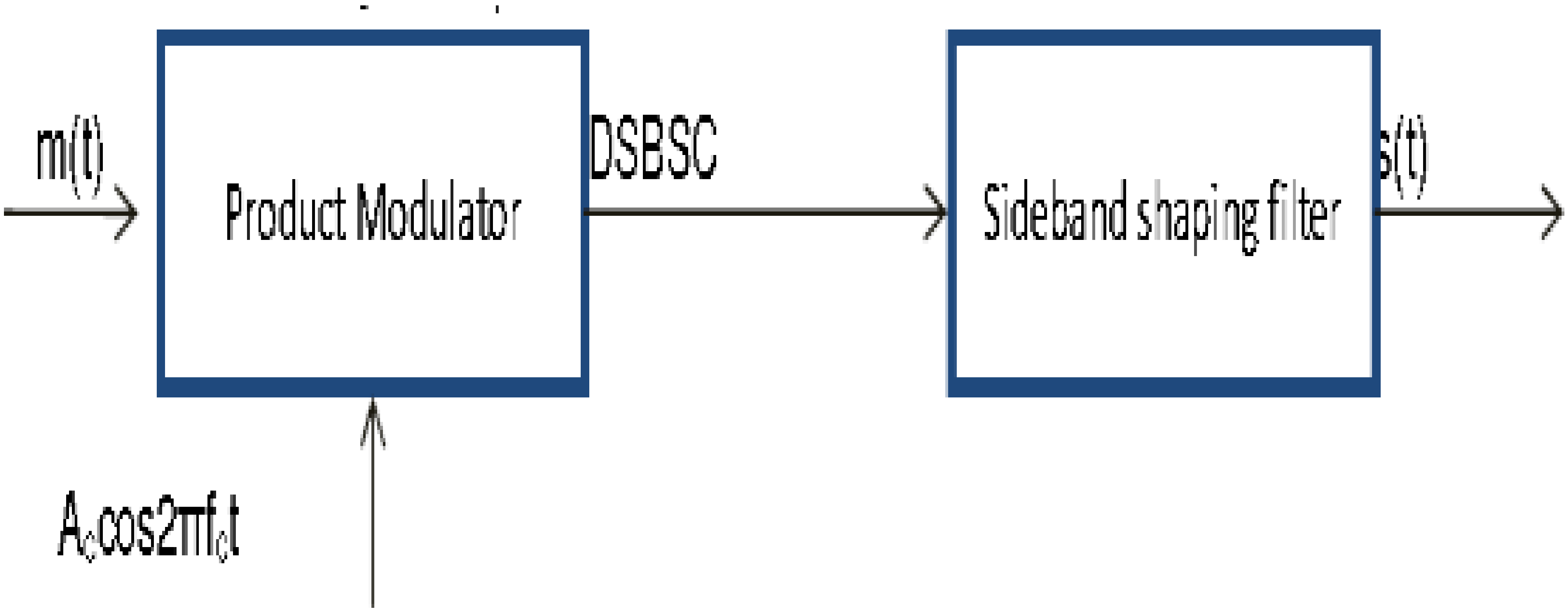
Fig(a) Spectrum of message signal



BW= W + f_v

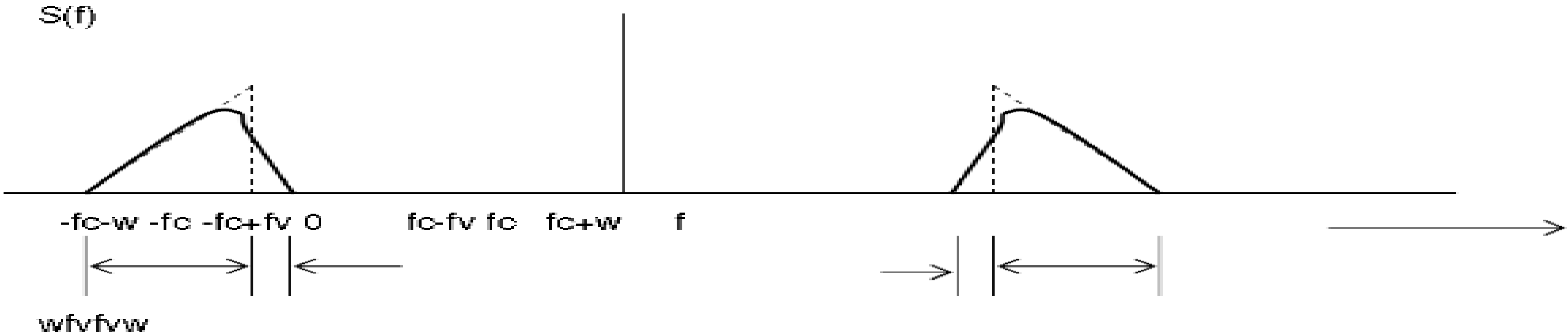
Fig(b) Spectrum of VSB wave containing vestige of the Lower side band

Generation of VSB Modulated Wave

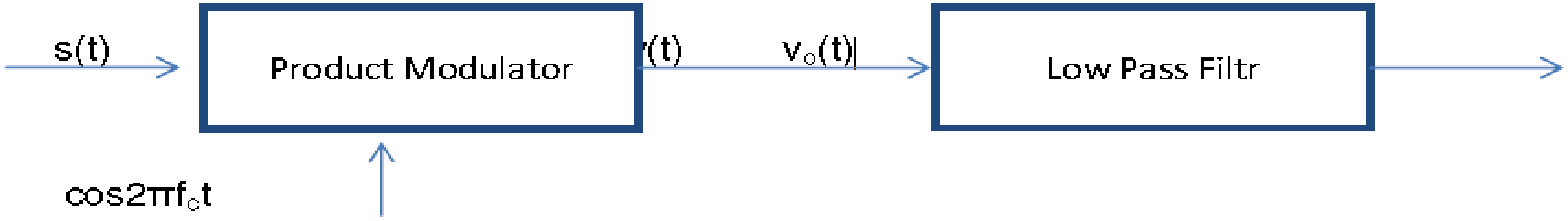


$$s(t) = [A_c\, m(t) \text{Cos}2\pi f_c\, t]\, h(t)$$

$$S(f) = \frac{Ac}{2} [M(f - fc) + M(f + fc)]\, H(f)$$



Fig(b) Spectrum of VSB wave containing vestige of the Lower side band



Fig(b). Block diagram of VSB Demodulator

Then, $v(t) = s(t).\ \cos 2\pi f_c t$-----------------------------------(2)

In frequency domain Eqn (2) becomes,

$V(f) = \frac{1}{2}\,[S(\,f - f_c\,) + S(\,f + f_c\,)]$ ------------------------------------(3)

Substitution of Eqn (1) in Eqn (3) gives

$$V(f) = \tfrac{1}{2}[A_c\,/2\ [M\ (f - f_c - f_c) + M(f - f_c + f_c)]H(f - f_c\,)$$

$$+ \tfrac{1}{2}[A_c\,/2\ [M\ (f + f_c - f_c) + M(f + f_c + f_c)]H(f + f_c\,)$$

$$V(f) = \tfrac{1}{2}[A_c\,/2\ [M\ (f - 2\,f_c) + M(f\,)]H(f - f_c\,)$$

$$+ \tfrac{1}{2}[A_c\,/2\ [M\ (f\,) + M(f + 2f_c)]H(f + f_c\,)$$

$V(f) = A_c /4 \; M(f)[H(f - f_c) + H(f + f_c)]$

$+ A_c /4 \; [M(f - 2f_c) H(f - f_c) + M(f + 2f_c) H(f + f_c)]$ --------------------------------(4)

The spectrum of V(f) as shown in fig below,

V(f)

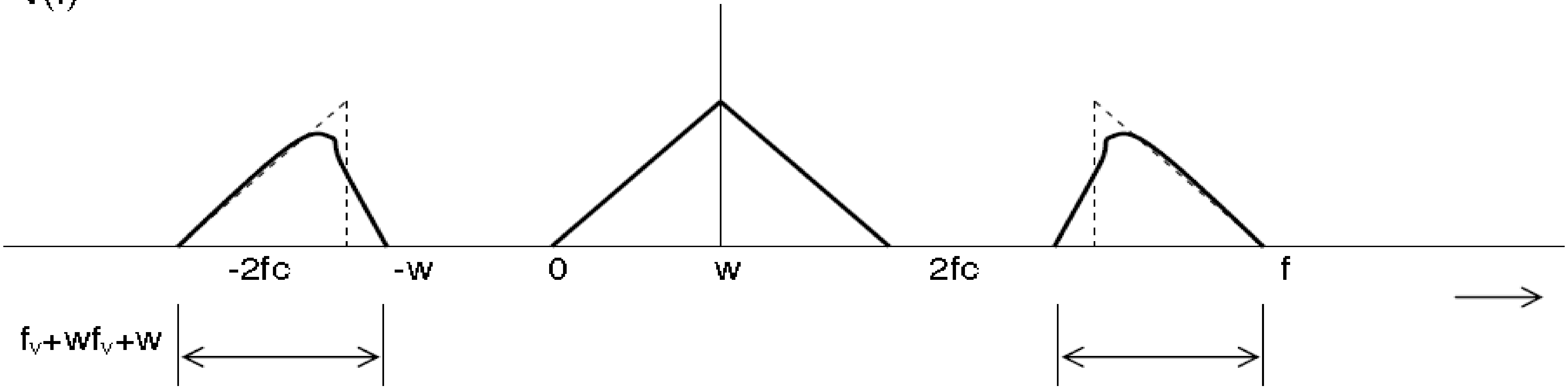


Fig ©. Spectrum of the product modulator output v(t)

Pass v(t) to a Low pass filter to eliminate VSB wave corresponding to $2f_c$.

$V_o(f) = A_c /4 \; M(f)[H(f - f_c) + H(f + f_c)]$ --(5)

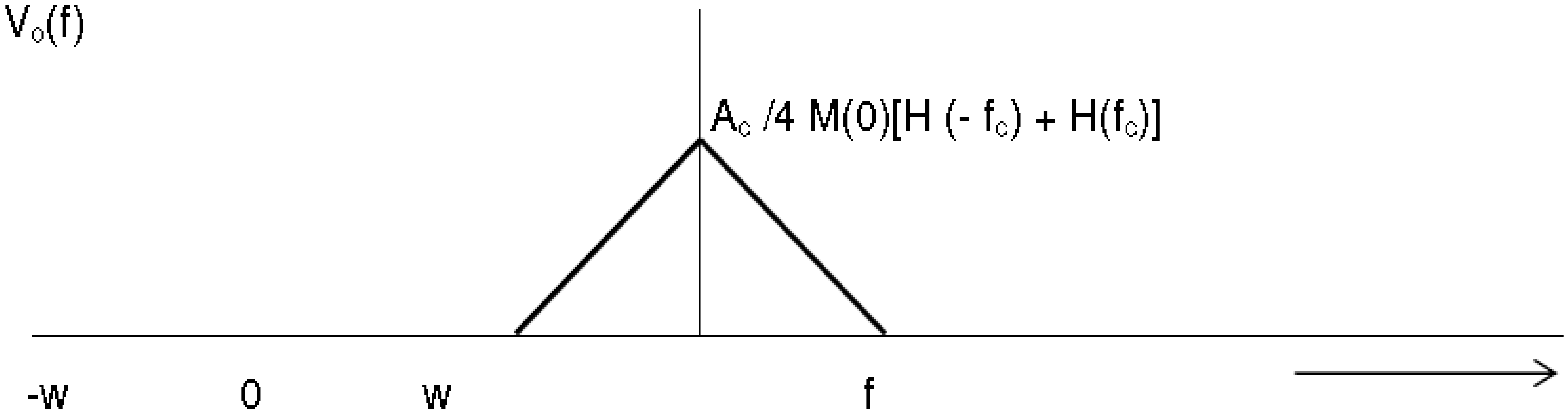
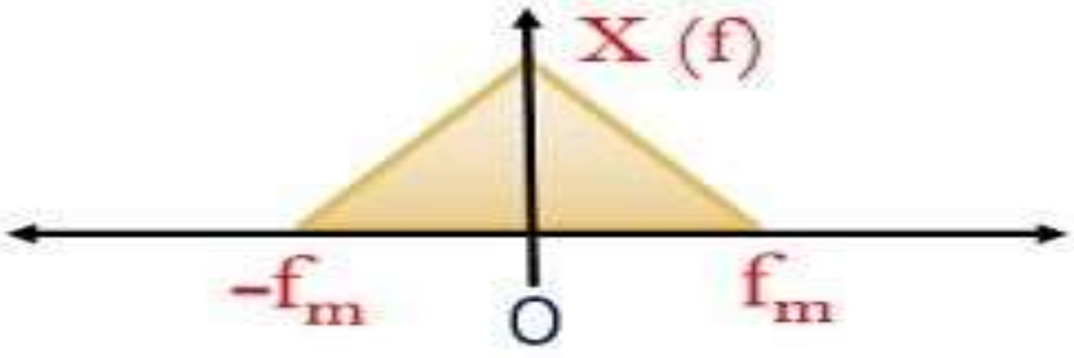


Fig (d). Spectrum of the demodulated Signal $v_0(t)$.

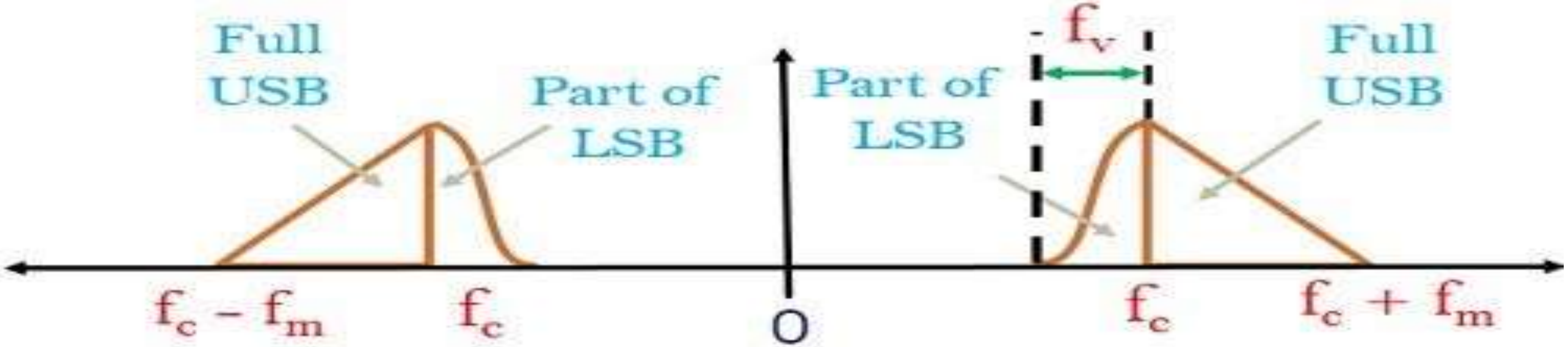
For a distortion less reproduction of the original signal m(t), $V_0(f)$ to be a scaled version of M(f). Therefore, the transfer function H(f) must satisfy the condition

$$H(f - f_c) + H(f + f_c) = 2H(f_c) \text{------------------} (6)$$

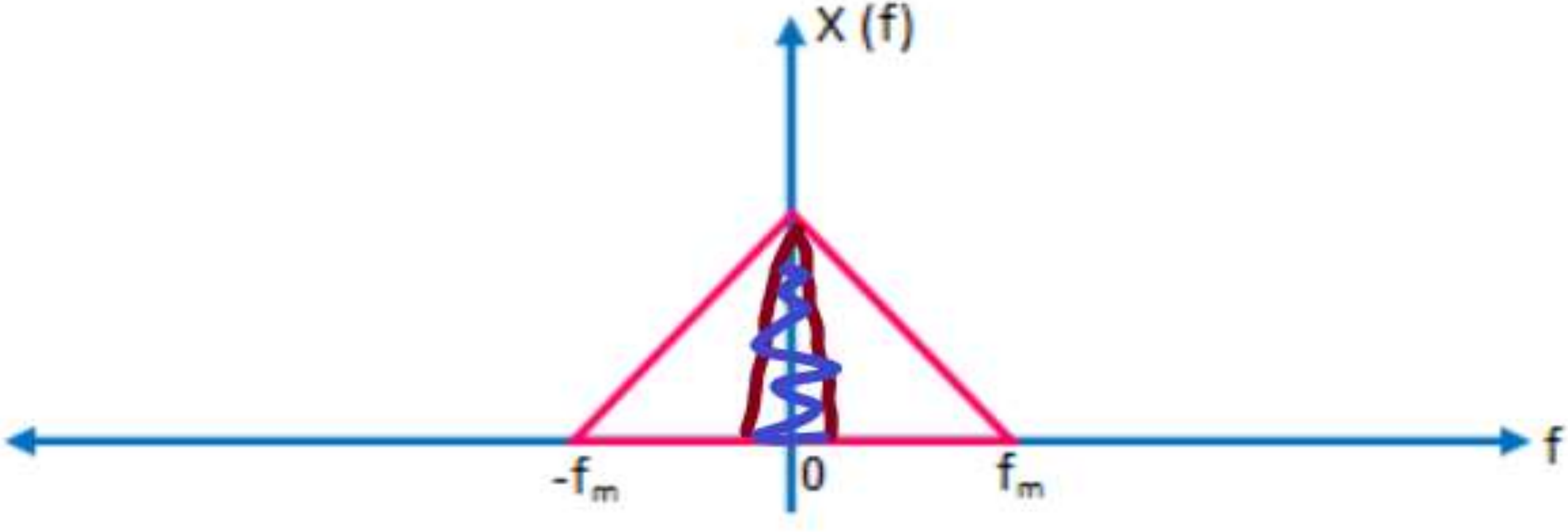
Where $H(f_c)$ is a constant



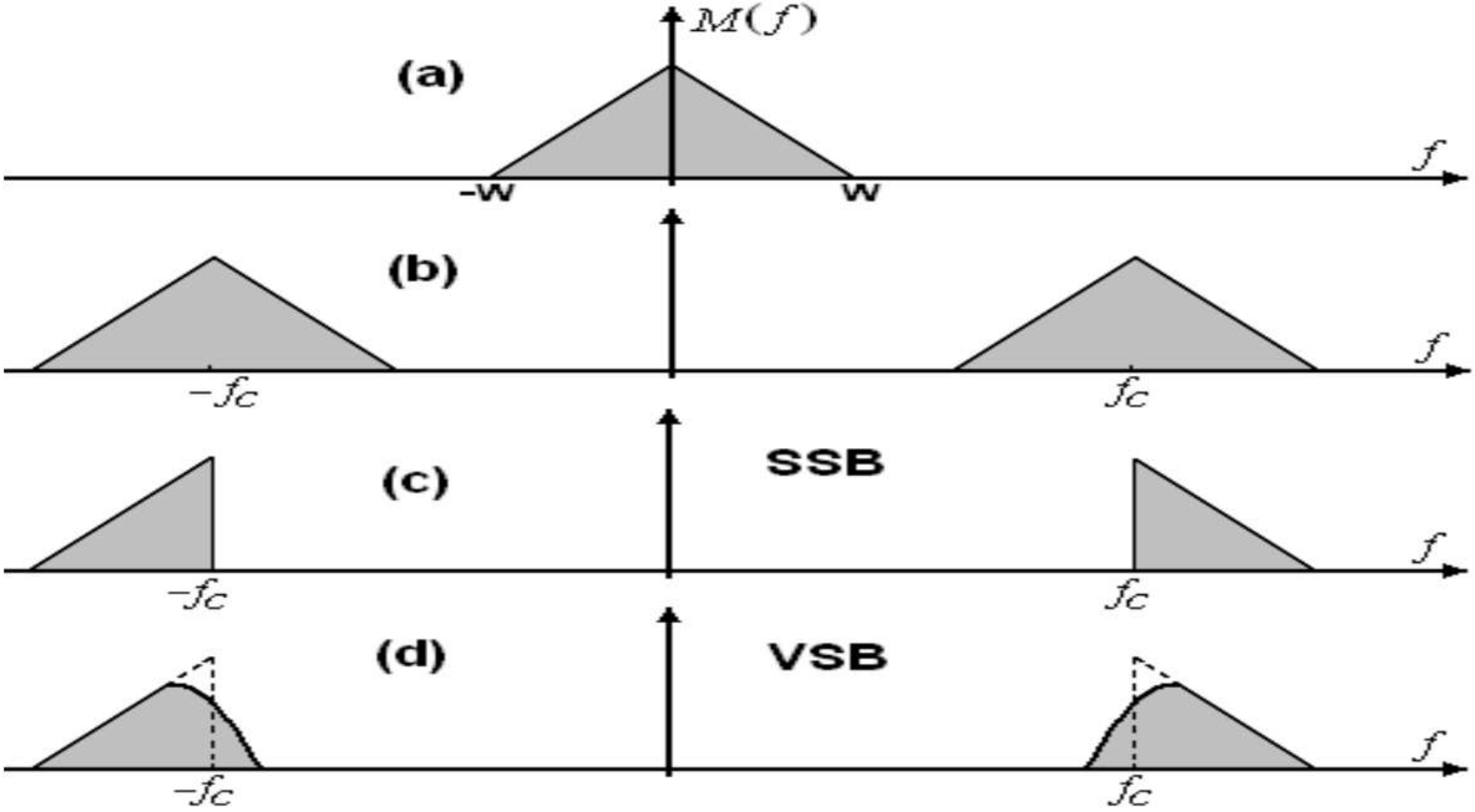
Spectrum of message signal



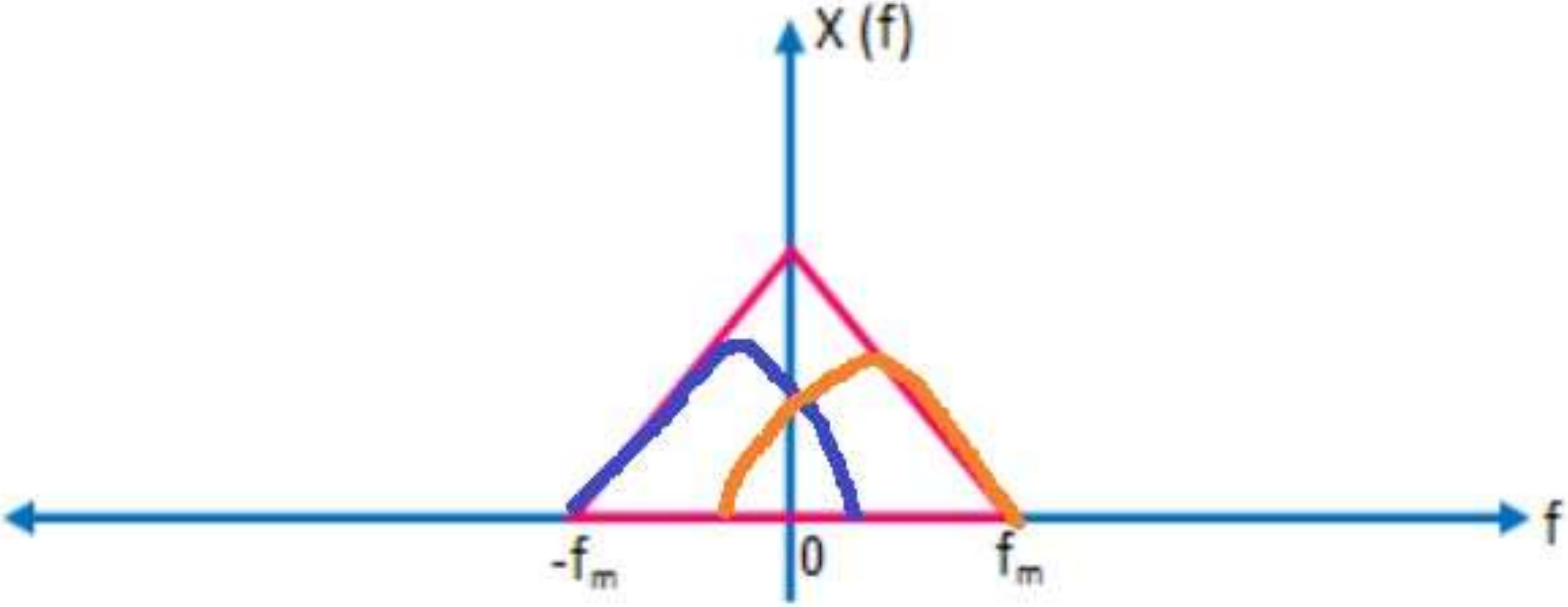
Spectrum of VSB signal



Distorted Output



$M(f)$
(a)
-W
W
f
(b)
$-f_C$
f_C
f
SSB
(c)
$-f_C$
f_C
f
VSB
(d)
$-f_C$
f_C
f



Perfect Output (Loss is Compensated with the Gain)

Envelope detection of a VSB Wave plus Carrier

To make demodulation of VSB wave possible by an envelope detector at the receiving end it is necessary to transmit a sizeable carrier together with the modulated wave. The scaled expression of VSB wave by factor k_a with the carrier component $A_c \cos(2\pi f_c t)$ can be given by

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c}{2} k_a m(t) \cos(2\pi f_c t)$$

$$- \frac{A_c}{2} k_a m_Q(t) \sin(2\pi f_c t)$$

$$= A_c \left[1 + \frac{k_a}{2} m(t)\right] \cos(2\pi f_c t) - \frac{A_c k_a}{2} m_Q(t) \sin(2\pi f_c t) \quad \text{................(1)}$$

where k_a is the modulation index; it determines the percentage modulation.

When above signal s(t) is passed through the envelope detector, the detector output is given by,

$$a(t) = A_c \left[\left(1 + \frac{k_a}{2} m(t)\right)^2 + \left(\frac{k_a}{2} m_Q(t)\right)^2\right]^{1/2}$$

$$= A_c\left[1+\frac{k_a}{2}m(t)\right]\left(1+\left[\frac{\frac{1}{2}k_a\ m_Q(t)}{1+\frac{1}{2}k_a\ m(t)}\right]^2\right)^{1/2} \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

The detector output is distorted by the quadrature component $m_Q(t)$ as indicated by equation (2).

Methods to reduce distortion

- Distortion can be reduced by reducing percentage modulation, k_a.
- Distortion can be reduced by reducing $m_Q(t)$ by increasing the width of the vestigial sideband.

Comparisons of **AM,DSBSC,SSB-SC,VSB**

S.NO	Parameters	AM	DSB-SC	SSB	VSB
1	General Equation				
2	Singletone Equation				
3	General BW				
4	Singletone BW				
5	Total Power				

*****Complete the Following**

S.NO	Parameters	AM	DSB-SC	SSB	VSB
6	Generation Methods				
7	Detection Methods				
8	Power or Modulation efficiency				
9	Applications				

*******Complete the Following**

Thank You