① U-I/2

Control Systems

Control Systems is an interconnection of the physical components to provide a desired function, involving some kind of controlling action in it.

or

The Control Systems is that means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner

Classification of Control Systems

The Control system may be classified in a number of ways depending on the purpose of classification

- Natural Control
- Manual Control
  - open loop
  - closed loop
- Automatic Control (depends on FB)
  - Open loop
  - closed loop
    - Position Control
    - Velocity Control
- unit VII
  - Modern Control
  - Conventional Control
- Lumped Parameter
- Distributed Parameter
- Deterministic Control (Predictable & repeatable)
- Stochastic control
- Linear
- Non Linear
- Analog Continuous time Control
- Discrete or Digital or µp based Control
- Time invariant
- Time varient — Motor, missile System
- Adaptive Control
- optimal Control

As per applications

→ Process Control
→ Temperature Control
→ Traffic Control
~~→ Lamp~~

As per i/p and o/p

- Single i/p Single o/p (SISO)
- Multiple i/p and multiple o/p (MIMO)

- Depending on the number of open loop poles of T/f function present at the origin of the s-plane,
  - Type 0
  - Type 1
  - Type 2 etc
- Depending on the order of the differential eqn used to describe the system,
  - First order
  - Second order etc
- Depending on the type of damping
  - undamped system
  - under damped system
  - critically damped system
  - over damped system

→

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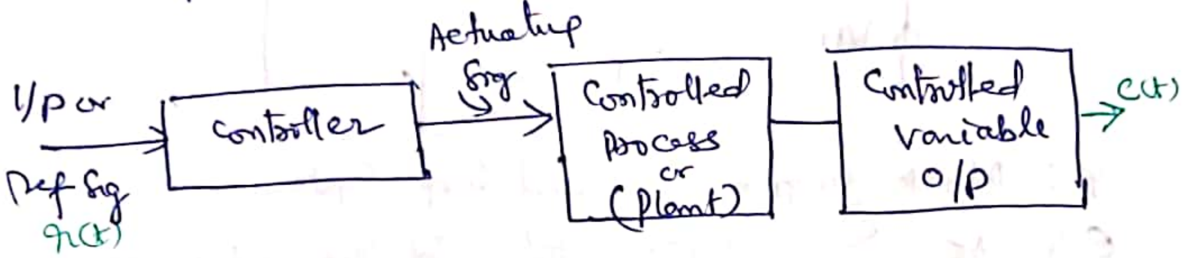
Open loop Control System :→

In the system o/p has no effect on the control action or i/p called open loop control system.

or,

If this control system i/p is independent. ~~on o/p~~ or, o/p not effect the i/p (no feed back used). Any control sys that operates on a time basis is open-loop



or

Fig 1: Block diagram of open loop control system

Example of open loop control system are:

i) washing machine
ii) Fan, heater, Traffic light etc

Advantage of open loop System

- Simple in construction
- Easy in maintenance
- No stability problem
- economical

Disadvantages

- inaccurate and unreliable
- Can't sense environmental changes and internal disturbance
- Poor accuracy, time-to-time ~~re~~ Calibration required

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Closed loop control systems: The feedback control system are often referred to as closed loop control systems.
or,
In closed loop control system o/p effect the I/p or I/p is depend on o/p.

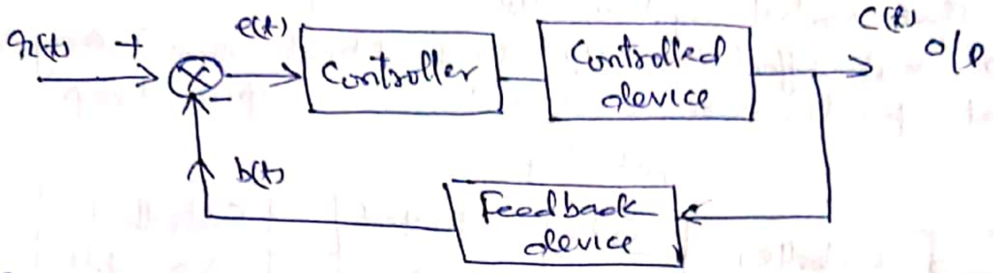

fig: Block diagram of closed loop system

e.g. AC, Regulated Power Supply, Tracking radar etc

Advantages

- accuracy is very high
- Such a system senses environmental changes, as well as internal disturbances and accordingly modified error signal
- Effect of non-linearity and distortion is less
- Bandwidth is high

Disadvantage

- Complicated system
- more costlier
- Due to feed, problem of stability
- Gain is reduced due to FB

Examples (may be open/closed loop system)

- Traffic light control
- Room heating system
- washing machine

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Comparison of open loop and closed loop control system

	open loop	closed loop
1	No feed back	Feed back used
2	o/p measurement Not required for operation of system	o/p measurement is necessary
3	Feed back element not required	required
4	No Error detector	required
5	inaccurate an unreliable	accurate and reliable
6	Highly sensitive to the disturbance and environmental changes	Less sensitive to disturbances and environmental changes
7	Small Band width	Large BW (due to FB)
8	Simple in construction & cheap	Complicated to design hence costly
9	Generally stable in nature	Stability is major problem while design
10	Highly affected by non linearities	Reduced effect of non linearities
11	Consume less Power	- Consume more Power - FB reduced over all gain $M = \frac{C}{R} = \frac{G}{1+GH}$

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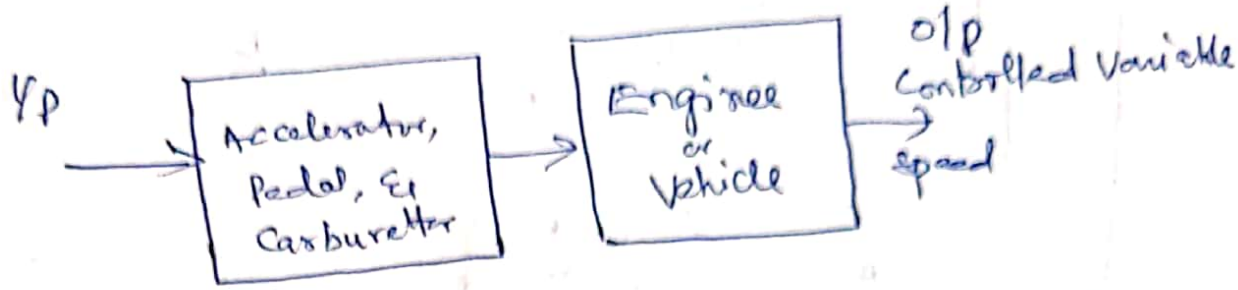
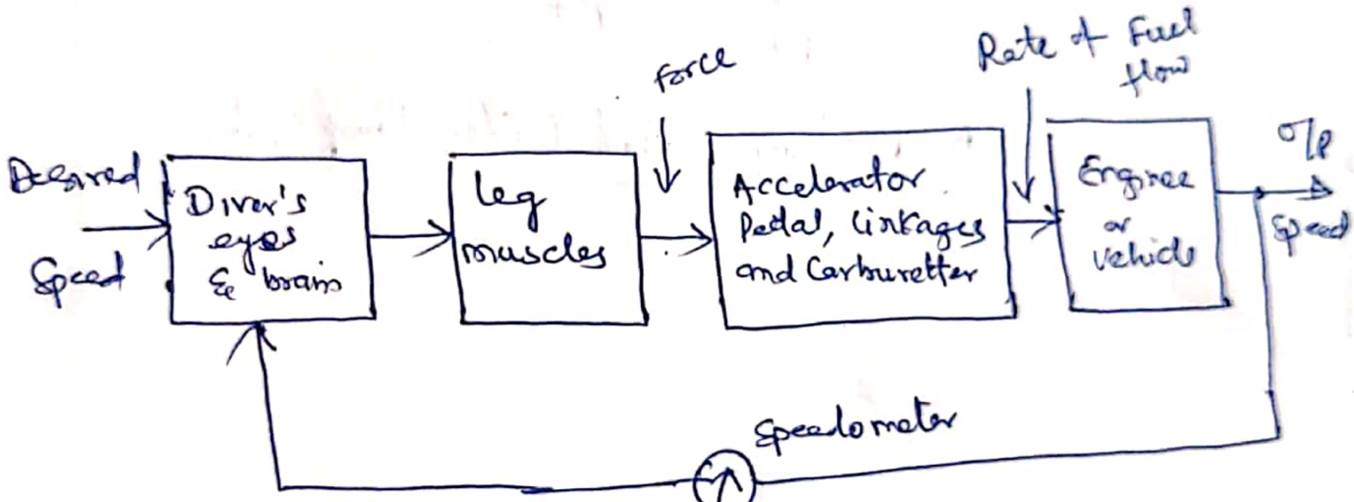
Unit - I

Control Systems Engineering



Basic control system (open loop)



Visual link from speedometer

Manually controlled closed loop system

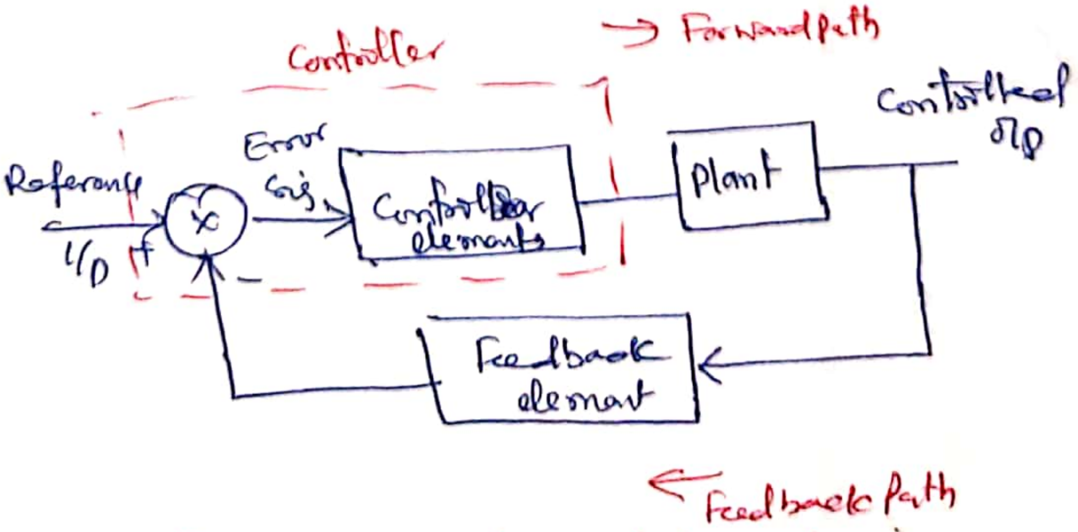


Fig Automatic control system

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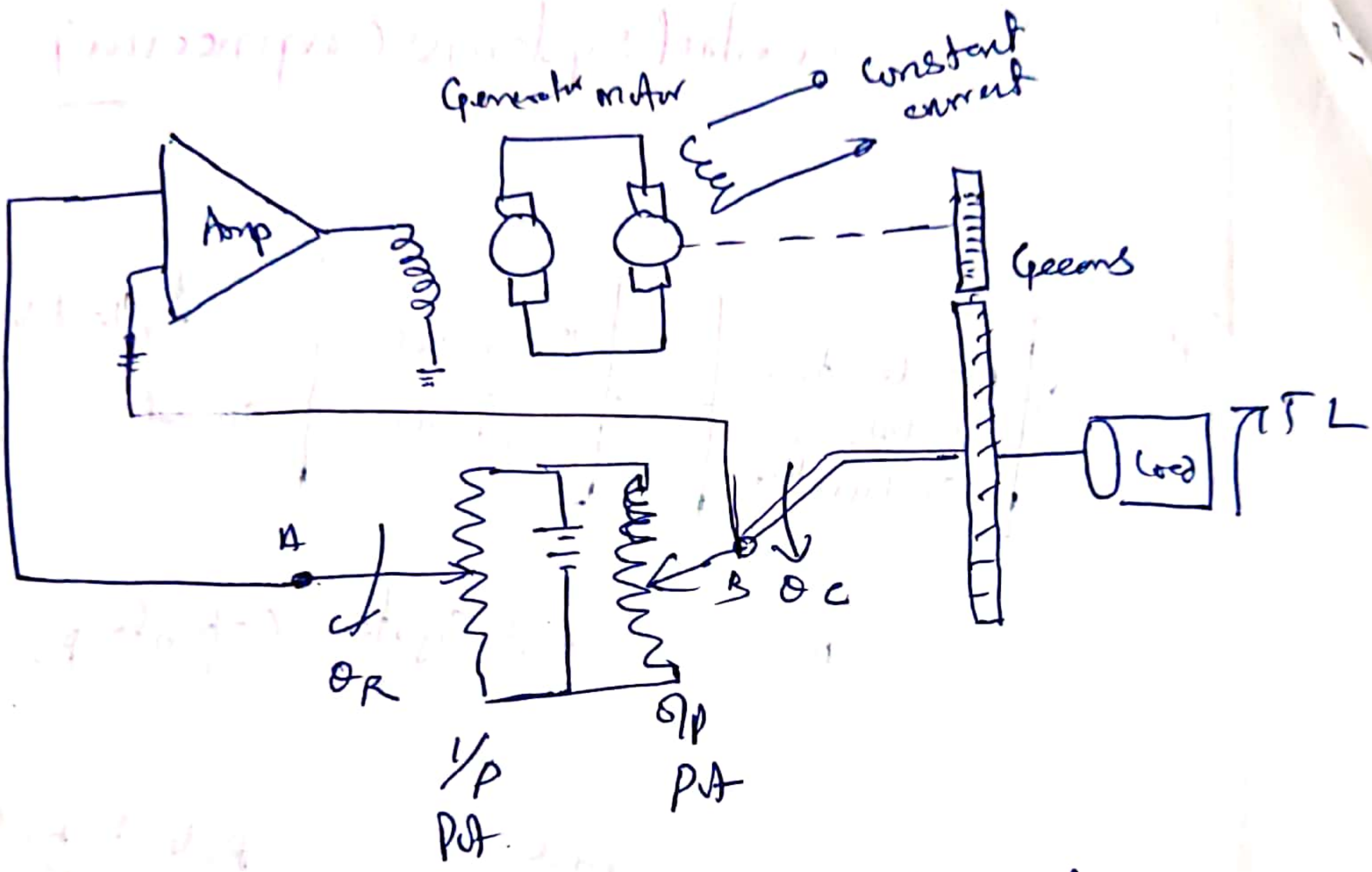

A Position Control (Closed loop system)

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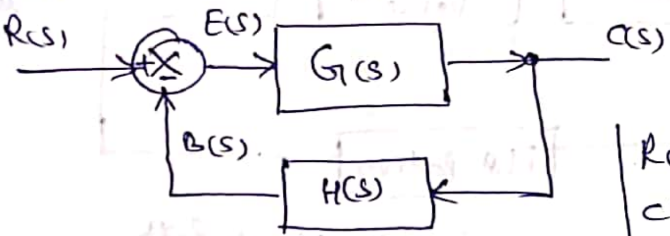
Feedback – A portion of output applied back to the input is known as feedback.

The purpose of feedback are:

- Reduce the system error
- Feedback also have the effect on:
  - Stability
  - Bandwidth
  - overall gain
  - disturbance
  - sensitivity → Contd. See U1/4(1)

Transfer function (TF) – It defined to be the ratio of the Laplace Transform of the O/P variable to the Laplace transform of I/p variable under assumption of that all initial conditions are zero.



$R(s) \rightarrow$ Reference I/p
$C(s) \rightarrow$ O/p Sig
$B(s) =$ Feedback Sig
$E(s) =$ actuating Sig
$G(s) \rightarrow$ Forward path TF
$H(s) \rightarrow$ Feed back element TF
$G(s)H(s) \rightarrow$ Loop transfer function
$T(s) = \frac{C(s)}{R(s)} \rightarrow$ closed loop TF

$$C(s) = G(s) \cdot E(s) \quad \text{—— (1)}$$

$$E(s) = R(s) - B(s) \quad \text{—— (2)}$$

$$= R(s) - C(s)H(s) \quad \text{—— (3)}$$

$$\{\because B(s) = C(s)H(s)\}$$

Now eliminating E(s) from eq (1) & (3) we get

$$C(s) = G(s)(R(s) - C(s) \cdot H(s))$$

$$C(s)[1 + G(s)H(s)] = G(s)R(s)$$

or $$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = T(s)}$$

→ Feedback affects the gain G of a factor $\underline{1 + G(s)H(s)}$

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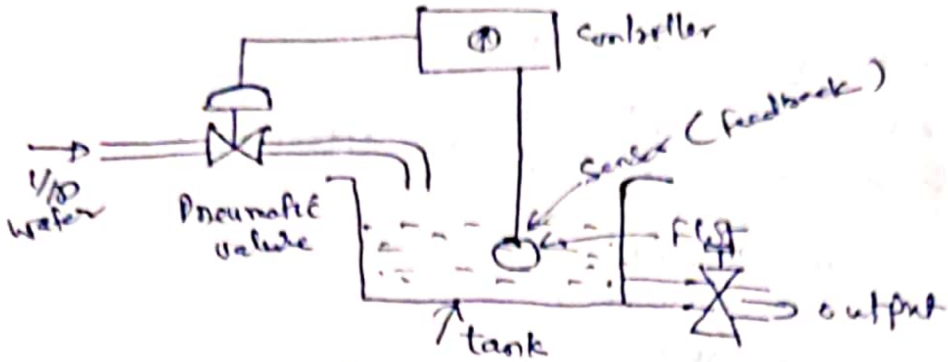
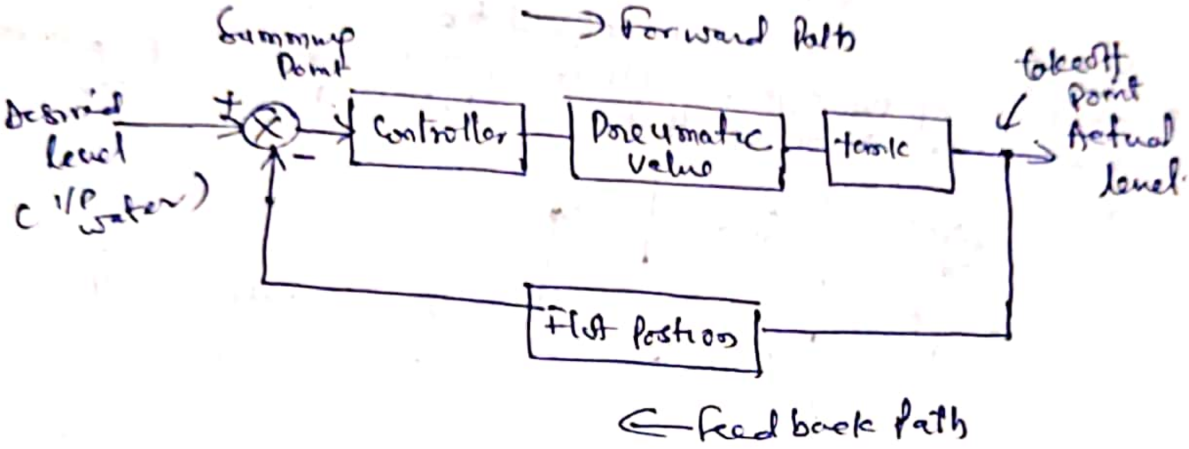
Block diagram representation





The Block diagram has following 5 basic Components

① Blocks — functional block
② TF of elements shown inside the blocks
③ Summing Point
④ take off Points
⑤ Arrows

Advantage of block diagram

- Very simple to construct the BD for complicated systems
- function of individual element can be visualised
- over all performance of System can be studied by using TF
- over all closed loop TF Can be easily Calculated

Disadvantage

→ Do Not include any information about physical construction of System
- Source of energy is generally not shown in the BD.
  It is not unique.

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Feedback effect on Control Systems

Sensitivity :- It is used to describe the relative variation in overall Transfer function $T(s) = C(s)/R(s)$ due to variation in $G(s)$ and is defined as,

$$\text{Sensitivity} = \frac{\text{Percentage change in } T(s)}{\text{Percentage change in } G(s)}$$

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{\partial T}{\partial G}\cdot\frac{G}{T} = \frac{1}{1+GH} \quad \text{— Closed loop}$$

$$\left\{ \because \frac{\partial T}{\partial G} = \frac{(1+GH)-GH}{(1+GH)^2}, \quad T = \frac{G}{1+GH}, \; G=G \right\}$$

$$S_G^T = 1 \;(\text{in a } ^{\text{this}}\text{case } T = G)$$

- Thus sensitivity of a closed loop system w.r.t variation in G is reduced by a factor $(1+GH)$ as compared to open loop system.

Sensitivity of T w.r.t H

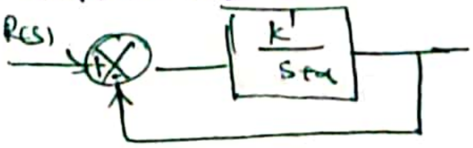
$$S_H^T = \frac{\partial T}{\partial H}\cdot\frac{H}{T} = -\frac{GH}{1+GH}$$

$$= G\left[\frac{-G}{(1+GH)^2}\right]\cdot\frac{H}{G/(1+GH)} = \frac{-GH}{1+GH}$$

- for large value of GH the sensitivity of feedback system w.r.t H approaches unity.

Effect of Feedback on bandwidth :- Control system is a low pass filter. A Large BW implies that the system responds accurately to higher frequencies.

Closed loop system increases the BW of systems. It responds to freq from dc upto a certain value ω_b at which gain drops to $1/\sqrt{2}$ of its dc value, thus ω_b is the BW



Simple Feedback system

open loop

$$\frac{C(S)}{R(S)} = G(s) = \frac{k'}{s+\alpha} \quad — ① \Rightarrow \frac{k'/\alpha}{S/\alpha + 1}$$

$$= \frac{k}{\tau s+1} \; ; \; k = k'/\alpha \; , \; \tau = 1/\alpha$$

$$G(0) = k = k'/\alpha \quad — ② \text{ (DC gain)}$$

Closed loop TF is,

$$\frac{C(s)}{R(s)} = \frac{k}{\tau s+(1+k)} = \frac{k'}{s+(\alpha+k')}$$

$$\left\{ \because \frac{C(s)}{R(s)} = \frac{\frac{k}{\tau s+1}}{1+\frac{k}{\tau s+1}} = \frac{k}{\tau s+1+k} \right.$$

$$= \frac{k/(1+k)}{\tau_c s+1} \; ; \; \tau_c = \frac{\tau}{1+k}$$

The bandwidth is determined as below

$$(\omega_b \tau)^2 + 1 = (\sqrt{2})^2 \quad \text{or} \quad \omega_b(OL) = 1/\tau$$

$$(\omega_b \tau_c)^2 + 1 = (\sqrt{2})^2 \quad \text{or} \quad \omega_b(CL) = 1/\tau_c$$

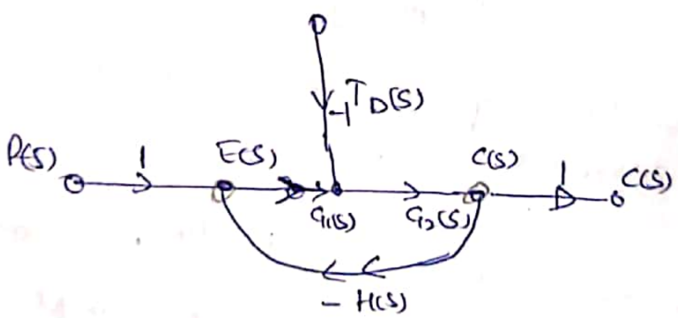
$$\frac{\omega_b(CL)}{\omega_b(OL)} = \frac{\tau}{\tau_c} = \underline{\underline{(1+k)}}$$

$$H(f) = \frac{1}{1+jf/f_2} \quad H(f) = 1/\sqrt{2}$$

at $f = f_2$

Thus the closed loop System have the bandwidth $(1+K)$ times the BW of open loop System.

Effect of Disturbance in feed back System



$$\frac{C(S)}{T_D(S)} = \frac{-G_2(S)}{1+G_1(S)\,G_2(S)\,H(S)}$$

if $G_1 G_2 H(S) >> 1$,

$$\frac{C(S)}{T_D(S)} = \frac{-1}{G_1(S)\,H(S)}$$

by making $G_1(S)$ high effect of disturbance can be reduced.

Mathematical models of Physical Systems

A physical system is a collection of physical objects connected together to serve an objective.

mathematical model of physical system is solved for various i/p conditions, The result represents the dynamic response of the system. The mathematical model of a system is linear if it obeys the principle of superposition and homogeneity

e.g The system is linear if.

$$\alpha_1 x_1(t) + \alpha_2 x_2(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

where $y_1(t)$ and $y_2(t)$ are responses of two input $x_1(t)$ & $x_2(t)$ respectively and α_1 and α_2 are constant

✓ mathematical models of most physical systems are characterised by differential equations.

Differential eqn of physical system :→

Mechanical Systems

Mass : $E = \frac{1}{2} M v^2 = KE$; motion energy

Inertia : $E = \frac{1}{2} J \omega^2 = KE$; " "

Spring (translatory) : $E = \frac{1}{2} K x^2 = PE$; (T) deformation energy

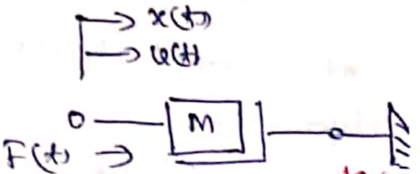
Spring (torsional) ; $E = \frac{1}{2} K \theta^2 = PE$ (T) ; " "

Damper is a dissipative element and power it consume is given as

$$P = f v^2 (\omega)$$
$$= f \omega^2 (\omega)$$

Ideal element in mechanical systems :-

Force – N (unit)

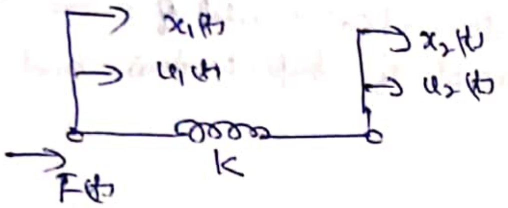


$$F = m\frac{du}{dt} = m\frac{dx^2}{dt}$$ $\left\{ L\frac{d^2q}{dt^2} \right\}$ $L\frac{di}{dt}$

property of a element that → store KE of translational motion

① Mass element

unit K → N/m

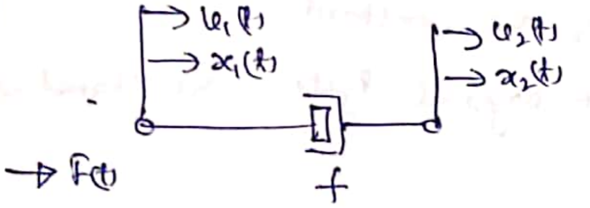


$$F = K(x_1 - x_2) = Kx = K\int_{-\infty}^{t}(u_1 - u_2)\,dt$$
$$= K\int_{-\infty}^{t} u\,dt$$

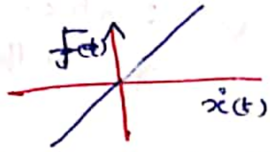
$\frac{1}{C}\int i\,dt$)

② The spring element – is element store PE (unit of K N/m)

units
x – m
u – m/s
F – N
M – Kg
K – N/m
f = N/m/s
(LR)



$$F = f(u_1 - u_2) = fu = f(\dot{x}_1 - \dot{x}_2)$$
$$= f\dot{x}$$

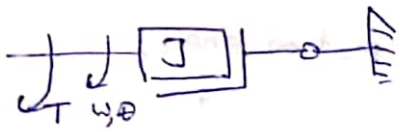


f → unit – f → N/m/sec

③ The damper element (viscus friction represents a retarding force that is a linear relationship between applied force and velocity)

Rotational element

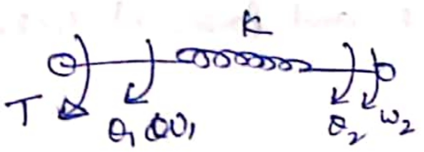
unit of in rotation motion
θ (rad)
ω (rad/sec)
J (Kg-m²)
T (N-m)
K (N-m/rad)
f = N-m/rad/s



$$T = J\frac{d\omega}{dt} = J\frac{d^2\theta}{dt^2}$$

④ The inertia element

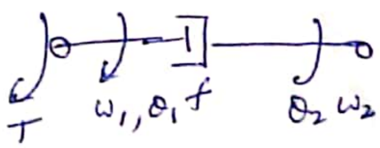
J – Kg-m²
T – N-M



$$T = K(\theta_1 - \theta_2) = K\theta = K\int_{-\infty}^{t}\omega_1 - \omega_2$$
$$= K\int_{-\infty}^{\infty}\omega\,dt$$

⑤ The torsional spring element

unit K – N-m/rad



$$T = f(\omega_1 - \omega_2) = f\omega = f(\dot{\theta}_1 - \dot{\theta}_2)$$
$$T = f\dot{\theta}$$

⑥ The Damper element

Fig: Ideal mechanical systems

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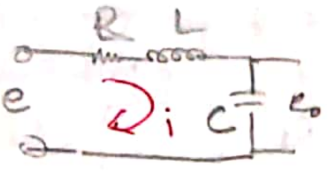
Analogous quantity in force (Torque) – Voltage Analogy

Mech Translational Sys	Mech. Rotational System	Electrical System
Force F	Torque T	Voltage e
mass M	Moment of inertia J	Inductance L
viscous friction coefficient f or B	viscous friction coefficient f	Resistance R
Spring stiffness K	Torsional spring coeff K	Capacitance $1/C$
Displacement x	Angular displacement θ	Charge q
velocity $\dot{x}$	Angular velocity $\dot{\theta}$	current i

Analogous Quantity in Force (Torque) current Analogy

Mechanical Translational	Mechanical Rotational Systems	Electrical Systems
Force F	Torque T	Current I
Mass M	Moment of inertia J	Capacitance C
viscous friction coeff 'f' 'B'	viscous friction coeff 'f'	Reciprocal of Resistance $1/R$
Spring stiffness 'k'	Torsional Spring coeff 'k'	Reciprocal of inductance $1/L$
Displacement x	Angular displacement θ	magnetic flux linkage 'λ' ϕ
velocity $\dot{x}$	angular velocity $\dot{\theta}$ $\dot{\omega} = \frac{d\theta}{dt}$	Voltage $e = \frac{d\phi}{dt}$



$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = e$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e$$

$$i = C\frac{de}{dt} + \frac{e}{R} + \frac{1}{L}\int_{-\infty}^{t} e\,dt$$

$$i = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi$$

$$\boxed{\begin{aligned}\phi &= \int e\,dt \\ q &= \int i\,dt\end{aligned}}$$

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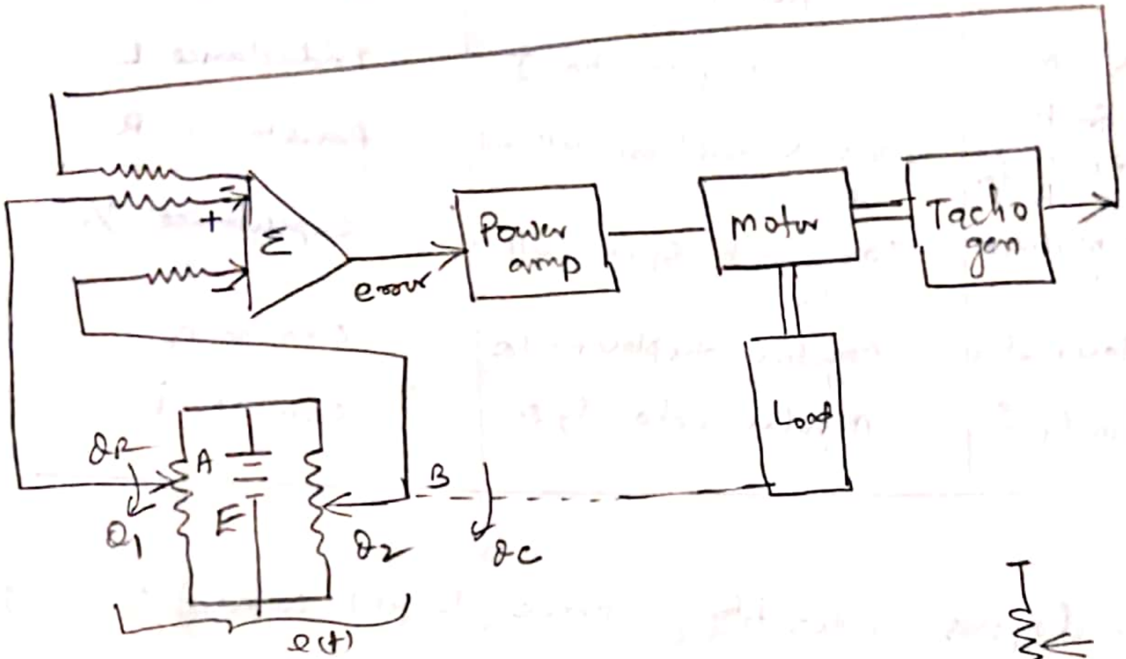
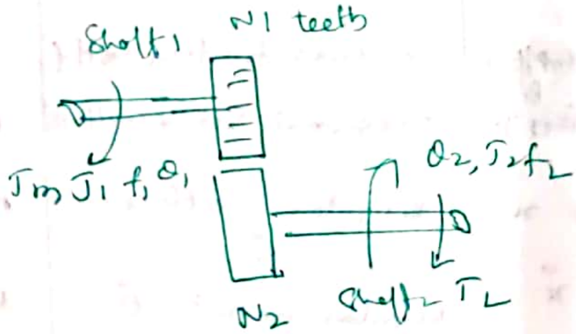


$$e(t) = K_s\,[\theta_R(t) - \theta_C(t)]$$

$$K_s = \frac{E}{2\pi N} \quad V/rad$$

$N \rightarrow$ No of turns of wire wound resistance

Gear Train



If r_1 is radius of gear 1 and r_2 is radius of gear 2. Since linear distance travelled along the surface of each gear is same, $\theta_1 r_1 = \theta_2 r_2$. The No of teeth on gear surface being proportional to gear radius, we obtain

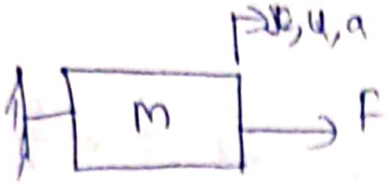
$$\frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} \quad \text{—— (1)} \qquad \boxed{\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1}} \Rightarrow \frac{\theta_2}{\theta_1} = \frac{r_1}{r_2}$$

Work done by gear 1 is equal to gear 2 in ideal condition

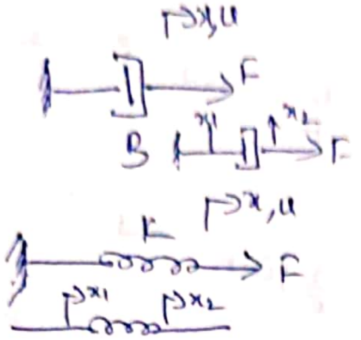
$$T_1\theta_1 = T_2\theta_2 \Rightarrow \boxed{\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1 r_1}{N_2 r_2}}$$

mechanical component relation M, B, & K

②



$F = m\frac{d^2x}{dt^2}$ or, $m\frac{du}{dt}$



$F = B\frac{dx}{dt}$ or Bu

$F = B\frac{d(x_1-x_2)}{dt}$ or $B(u_1-u_2)$

$F = Kx$ or $K\int u\,dt$

$F = K(x_1-x_2)$ or $K\int(u_1-u_2)\,dt$

If $F = e$ (Force voltage analogy) | Force current analogy

Items	Mech Systems: Translational	Rotational	electrical System $F = e$	electrical System $F = i$
I/p (independent variable)	Force (F)	T Torque	voltage e	current, i
O/p (dependent variable)	Velocity u or, displacement, x	ω / θ	Current, i / charge, q	voltage e / flux Φ
Dissipative element	Frictional coeff of dash pot 'B'	B	Resistance 'R'	conductance $\frac{1}{R}$ reciprocal of Resistance
Storage element	Mass m / Stiffness of spring k	J (J - MOI (moment of inertia))	Inductance L / inverse of capacitance $\frac{1}{C}$	Capacitance C / inverse of Inductance $\frac{1}{L}$
Physical law	newton law $\Sigma F = 0$		Kirchoff's law $\Sigma V = 0$	$\Sigma i = 0$
changing the level of independent variable	$\frac{F_1}{F_2} = \frac{l_1}{l_2}$ (lever)		Transformer $\frac{e_1}{e_2} = \frac{N_1}{N_2}$	Transformer $\frac{i_1}{i_2} = \frac{N_2}{N_1}$

T - Torque
J - moment of inertia
θ → Angular displacement
ω → " velocity

go to 3

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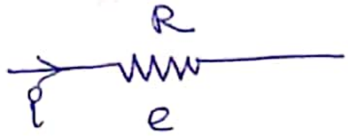
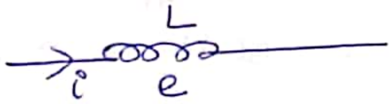
Current voltage relation R, L & C

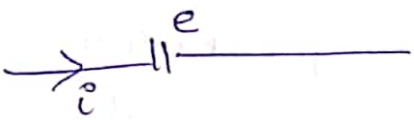
(1)



$e = iR$ and $i = \frac{e}{R}$



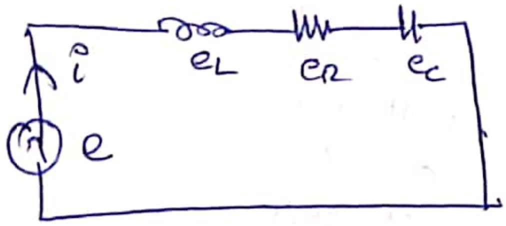
$e = L\frac{di}{dt}$ and $i = \frac{1}{L}\int e\,dt$



$e = \frac{1}{C}\int i\,dt$ and $i = C\frac{de}{dt}$

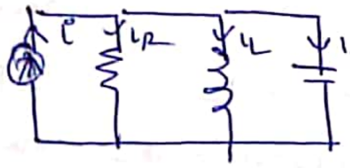
go to (2)

(3)



$e = e_L + e_R + e_C$

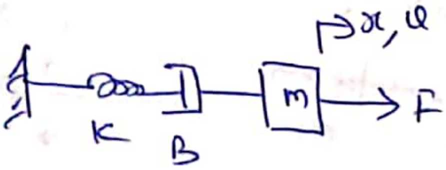
$$\boxed{e = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt} \quad \text{— (1)}$$



$i = i_R + i_L + i_C$

$= \frac{e}{R} + \frac{1}{L}\int e\,dt + C\frac{de}{dt}$

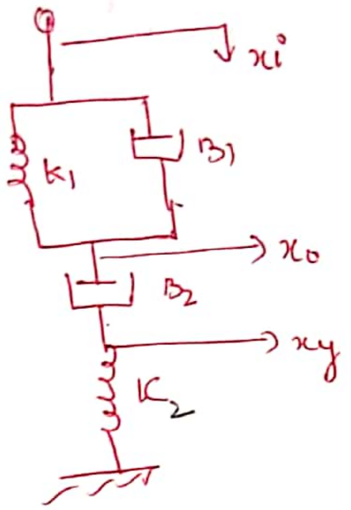
$$\boxed{i = \frac{e}{R} + C\frac{de}{dt} + \frac{1}{L}\int e\,dt} \quad \text{— (2)}$$

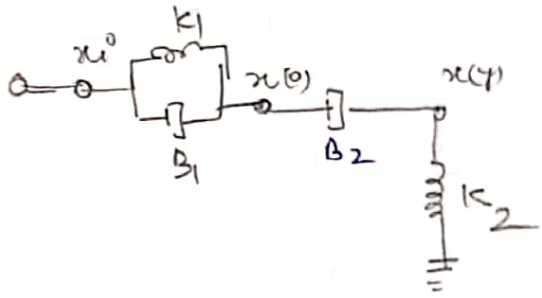


$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$

or, $$\boxed{F = m\frac{du}{dt} + Bu + K\int u\,dt} \quad \text{— (3)}$$

Find TF $\frac{X_o}{X_i}$ from Given fig





there are 3 displacement x_i, x_o and x_y. The I/p x_i and O/p x_o. So TF $x_o(s)/x_i(s)$, the equillibrium eqn are

At no x_o $\quad B_1\left(\frac{dx_o}{dt}-\frac{dx_i}{dt}\right)+k_1(x_o-x_i)+B_2\frac{d(x_o-x_y)}{dt}=0$ — ①

$B_2\left(\frac{dx_y}{dt}-\frac{dx_o}{dt}\right)+k_2 x_y=0$ — ②

taking Laplace TF of ①

$B_1[S\,X_o(S)-SX_i(S)]+K_1\,X_o(S)-K_1X_i(S)+B_2SX_o(S)-B_2SX_y(S)$

$B_1[S\,X_o(S)-SX_i(S)]+K_1\,X_o(S)-K_1X_i(S)+B_2SX_o(S)-B_2SX_y(S)$

or, $B_1SX_o(S)-B_1SX_i(S)+K_1X_o(S)-K_1X_i(S)+B_2SX_o(S)-B_2SX_y(S)$ — ③

or, $[B_1S+K_1+B_2S]X_o(S)-X_i(S)[K_1+B_1S]-B_2SX_y(S)$

Sm Laplace TF of ②

$B_2S\,X_y(S)-B_2SX_o(S)+K_2X_y(S)$

or, $(B_2S+K_2)X_y(S)-B_2SX_o(S) \Rightarrow X_y(S)=\frac{B_2S}{SB_2+K_2}X_o(S)$

Substituting this in ③ we get

$X_o(S)[B_1S+B_2S+K_1]-X_i(S)[B_1S+K_1]-B_2S\left[\frac{B_2S}{SB_2+K_2}\right]X_o(S)$

or, $X_o(S)\left[B_1S+B_2S+K_1-\frac{B_2^2S^2}{SB_2+K_2}\right]=X_i(S)[B_1S+K_1]$

$\frac{X_o(S)}{X_i(S)}=\frac{(B_1S+K_1)(SB_2+K_2)}{(B_1S+B_2S+K_1)(SB_2+K_2)-B_2^2S^2}$

$=(B_1S+K_1)(SB_2+K_2)/B_1B_2S^2+\cancel{B_2^2S^2}+K_1SB_2+B_1SK_2+B_2SK_2+K_1K_2-\cancel{B_2^2S^2}$

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$$\frac{X_o(s)}{X_i(s)} = \frac{K_1K_2\left[\left(1+s\frac{B_1}{K_1}\right)\left(1+\frac{sB_2}{K_2}\right)\right]}{K_1K_2\left[1+\frac{s^2B_1B_2}{K_1K_2}+s\frac{B_1}{K_1}+s\frac{B_2}{K_2}\right]+\frac{B_2}{K_1}s}$$

$$= \frac{\left(1+\frac{B_1}{K_1}s\right)\left(1+s\frac{B_2}{K_2}\right)}{\left(1+s\frac{B_1}{K_1}\right)\left(1+s\frac{B_2}{K_2}\right)+s\frac{B_2}{K_1}}$$

Force voltage analogy

From eqn ① & ②

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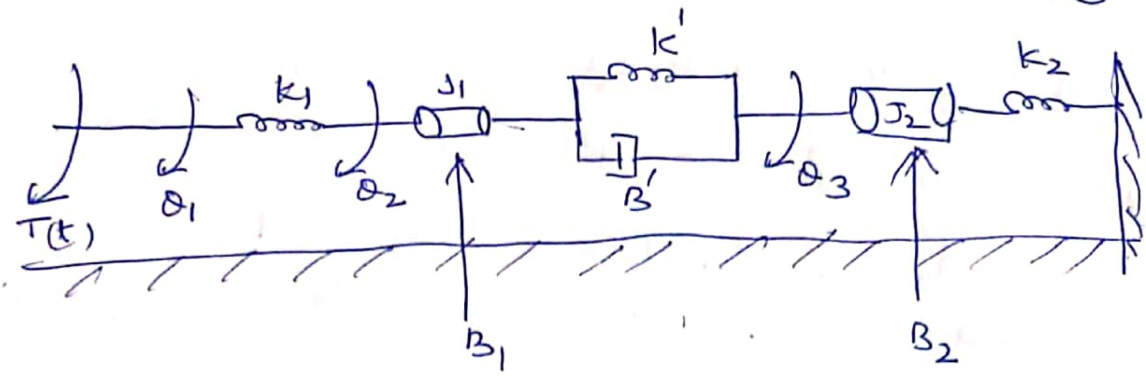
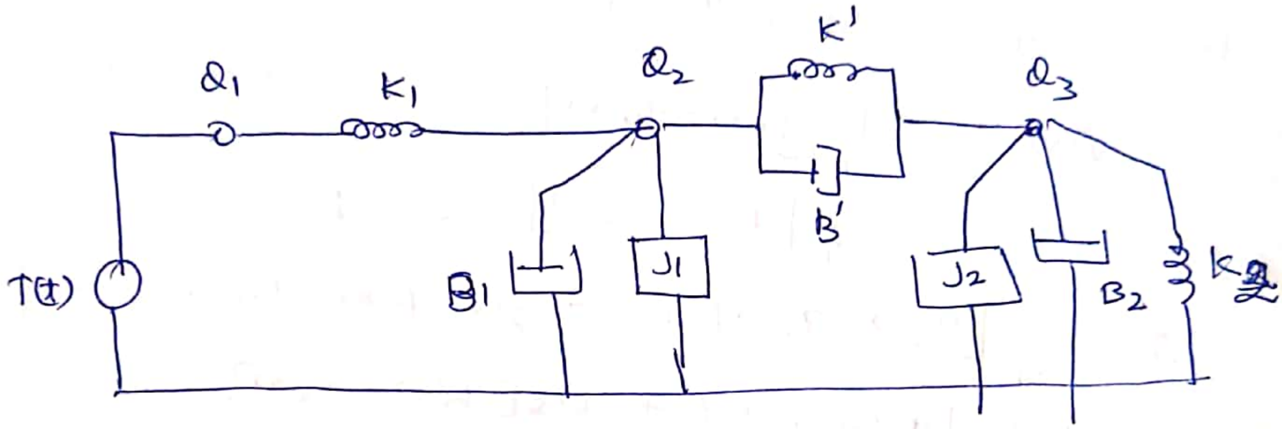
Q Obtain electrical (equivalent) analogous

(1)





Hence equations are

Node 1

$$T(t) = K_1(\theta_1(t) - \theta_2(t)) \quad \text{—— (1)}$$

Node 2

$$0 = B_1 \frac{d\theta_2(t)}{dt} + J_1 \frac{d^2\theta_2(t)}{dt} + K_1\left(\theta_2(t) - \theta_1(t)\right] + K'\left(\theta_2(t) - \theta_3(t)\right]$$
$$+ B' \frac{d\left[\theta_2(t) - \theta_3(t)\right]}{dt} \quad \text{—— (2)}$$

Node (3)

$$0 = J_2 \frac{d^2\theta_3(t)}{dt} + B_2 \frac{d^2\theta_3(t)}{dt} + K_2 \theta_3(t) + K'\left(\theta_3(t) - \theta_2(t)\right]$$
$$+ B' \frac{d\left[\theta_3(t) - \theta_2(t)\right]}{dt} \quad \text{—— (3)}$$

From eqs (1), (2) & (3) by takup Laplace Transform

$$T(S) = K_1\left[\theta_1(S) - \theta_2(S)\right] \quad \text{—— (4)}$$
$$0 = K_1\left[\theta_2(S) - \theta_1(S)\right] + J_1 S^2 \theta_2(S) + B_1 S \theta_2(S)$$
$$K_1\left(\theta_2(S) - \theta_1(S)\right] + B' S\left[\theta_2(S) - \theta_3(S)\right] \text{—— (5)}$$

$$0 = K'[\theta_3(s) - \theta_2(s)] + B's[\theta_3(s) - \theta_2(s)] + J_2 s^2 \theta_3(s)$$
$$+ B_2 s\theta_3(s) + K_2\theta_3(s) \quad \text{--- (6)}$$

<u>Now</u> for equivalent Force-voltage analogy

Elements in parallel in node based cct always connected in series of same cct is drawn base on loop basis.

$F \to V$, $m \to L$, $K \to 1/C$, $\theta(s) \to q(s)$

then eqn (4) can be written as

$$V(s) = \frac{1}{C_1}[q_1(s) - q_2(s)] \qquad sq_1 \to I_1$$

or,

$$V(s) = \frac{1}{sC_1}[I_1(s) - I_2(s)] \quad \text{--- (7)}$$

$$0 = \frac{1}{C_1}[q_2(s) - q_1(s)] + L_1 s^2 q_2(s) + R_1 s q_2(s)$$
$$+ \frac{1}{C'}[q_2(s) - q_3(s)] + R's[q_2(s) - q_3(s)]$$

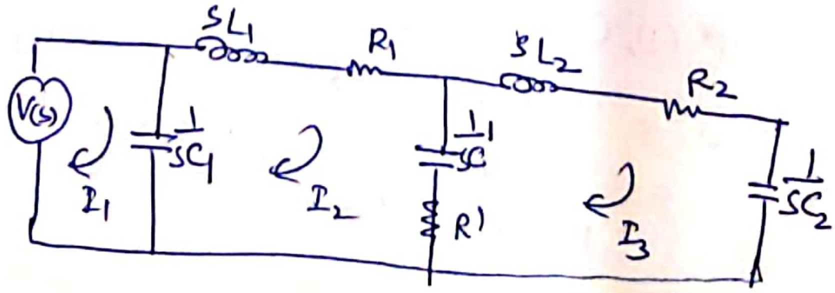
or,

$$0 = \frac{1}{sC_1}[I_2(s) - I_1(s)] + L_1 s I_2(s) + R_1 I_2(s) + \frac{1}{sC'}[I_2(s) - I_3(s)]$$
$$+ R'[I_2(s) - I_3(s)] \quad \text{--- (8)}$$

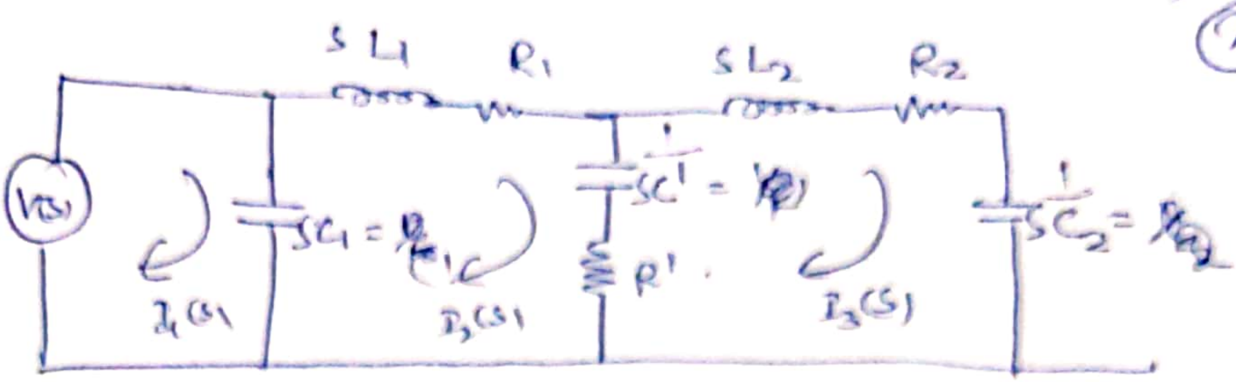
<u>Similarly</u>

$$0 = \frac{1}{C'}[q_3(s) - q_2(s)] + R's[q_3(s) - q_2(s)] + L_2 s^2 q_3(s)$$
$$+ R_2 s q_3(s) + \frac{1}{C_2} q_3(s)$$

or,
$$\frac{1}{sC'}[I_3(s) - I_2(s)] + R'[I_3(s) - I_2(s)] + L_2 s I_3(s)$$
$$+ R_2 I_3(s) + \frac{1}{sC_2} I_3(s) \quad \text{--- (9)}$$



②



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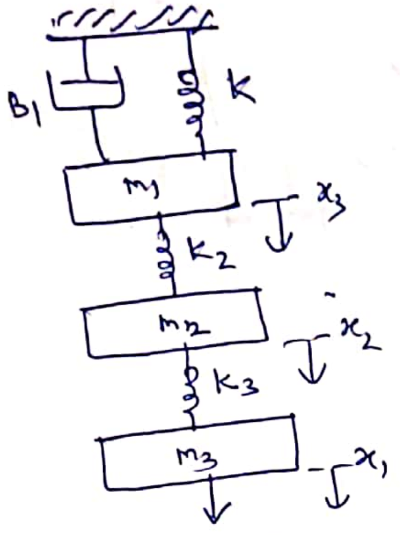
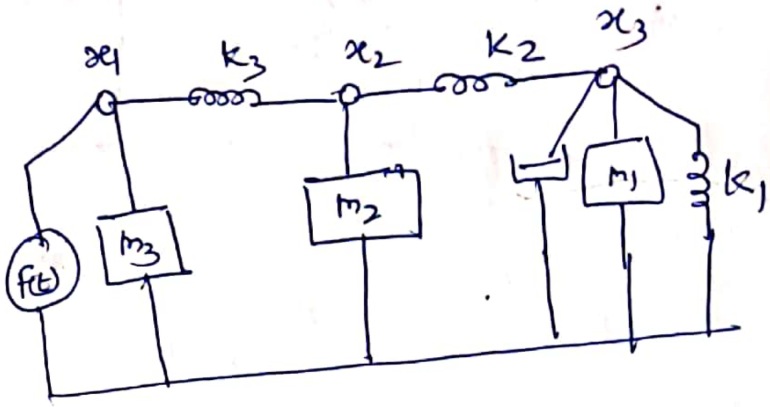
(5) Draw equivalent mech system of given structure and obtain electrical equivalent circuit.





$F = f\cos\omega t$

At node (1)

$$F(t) = m_2\frac{d^2x_1}{dt^2} + K_3(x_1 - x_2)$$

or $$F(t) = m_2\frac{dv_1}{dt} + k_2\int(v_1 - v_2)\,dt \quad \text{— (1)}$$

At node (2)

$$m_3\frac{d^2x_2}{dt^2} + k_3(x_2 - x_1) + k_2(x_2 - x_3)$$

or $$m_3\frac{dv_2}{dt} + k_3\int(v_2 - v_1)\,dt + k_2\int(v_2 - v_3)\,dt \quad \text{— (2)}$$

At node (3)

$$m_1\frac{d^2x_3}{dt^2} + k_2(x_3 - x_2) + B\frac{dx_3}{dt} + k_1x_3 \qquad \left\{ x = \int v\,dt \right.$$

$$m_1\frac{dv_3}{dt} + k_2\int(v_3 - v_2)\,dt + Bv_3 + k_1\int v_3\,dt \quad \text{— (3)}$$

for, Force $\rightarrow$ voltage $\quad F \rightarrow V$

velocity $\rightarrow$ Current, $\quad M \rightarrow L$, $\quad k = 1/C$, $\quad B \rightarrow R$

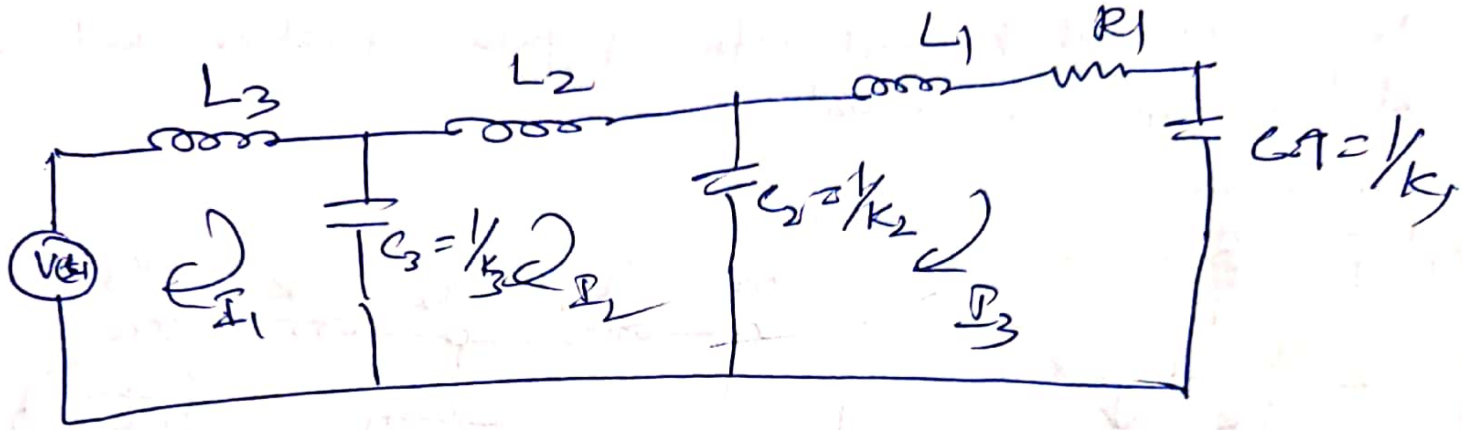
Substituting these in eqn (1), (2) & (3) we get

$$V(t) = L_2\frac{dI_1}{dt} + \frac{1}{K_2}\int(I_1 - I_2)\,dt \quad \text{— (4)}$$

$$0 = L_3\frac{dI_2}{dt} + \frac{1}{K_3}\int(I_2 - I_1)\,dt + \frac{1}{K_2}\int(I_2 - I_3)\,dt \quad \text{— (5)}$$

$$0 = L_1\frac{dI_3}{dt} + RI_3 + \frac{I_3}{C_1} + \frac{1}{C_2}\int(I_3 - I_2)\,dt \quad \text{— (6)}$$

— (7)



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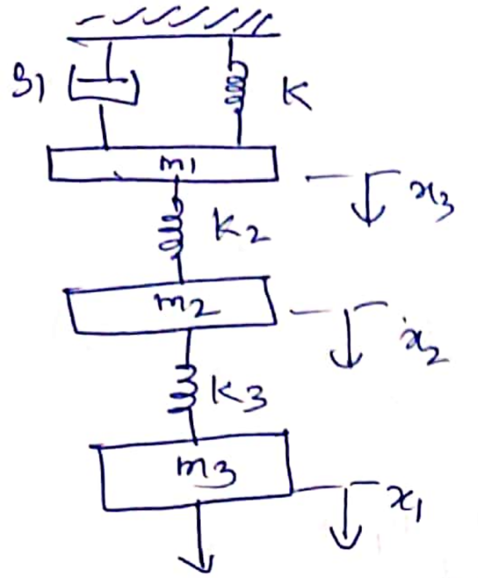
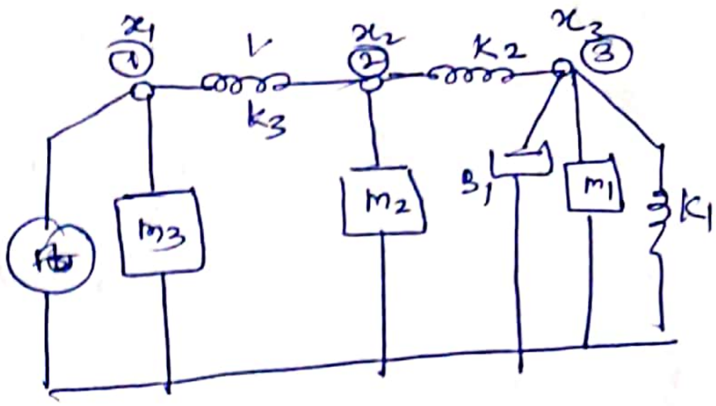
Draw equivalent mechanical system of given structure and obtain electrical equivalent ckt





$F = f\cos\omega t = f \cos\omega t$

At node 1

$$F(t) = m_3 \frac{d^2x_1}{dt^2} + K_3(x_1 - x_2) \quad \text{or,} \quad F(s) = m_3 s^2 X_1(s) + K_3(X_1 - X_2) \quad -①$$

At node 2

$$M_3 \frac{d^2x_2}{dt^2} + K_3(x_2 - x_1) + K_2(x_2 - x_3) \quad -②$$

or,

$$m_3 s^2 X_2 + K_3(X_2 - X_1) + K_2(X_2 - X_3)$$

At node 3

$$m_1 \frac{d^2x_3}{dt^2} + B_1 \frac{dx_3}{dt} + K_1 x_3 + K_2(x_3 - x_1)$$

or,

$$m_1 s^2 X_3 + B_1 s X_3 + K_1 X_3 + K_2(X_3 - X_1) \quad -③$$

$$F(s) = \frac{fs}{s^2 + \omega^2} \quad \text{where } f \to \text{constant}$$

Now Force - voltage analogy

$M \to L$ $\qquad x \to q$

$B \to R$

$K \to 1/C$

Then

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$$V(s) = L_3 s^2 q_1 + \frac{1}{C_3}(q_1 - q_2)$$

$$= L_3 s I_1(s) + \frac{1}{sC_3}[I_1(s) - I_2(s)]$$

$$V(s) = L_3 s I_1 - \frac{1}{sC_3}(I_1 - I_2) \quad \text{— (4)} \qquad I_1 = sq_1 = \frac{dq_1}{dt}$$

from (2)

$$0 = L_2 s^2 q_2 + \frac{1}{C_3}(q_2 - q_1) + \frac{1}{C_2}(q_2 - q_3)$$

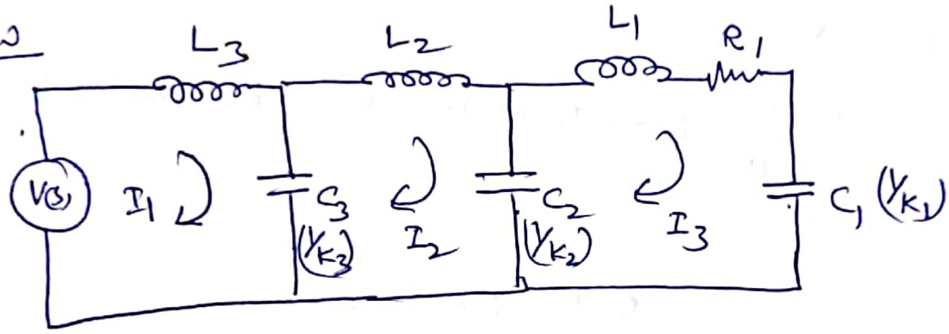
or, $$L_2 s I_2 + \frac{1}{sC_3}(I_2 - I_1) + \frac{1}{sC_2}(I_2 - I_3) \quad \text{— (5)}$$

from (3)

$$0 = L_1 s^2 q_3 + R_1 s q_3 + \frac{1}{C_1} q_1 + \frac{1}{C_2}(q_3 - q_2)$$

or, $$0 = L_1 s I_3 + R_1 I_3 + \frac{1}{sC_1} I_1 + \frac{1}{sC_2}(I_3 - I_2) \quad \text{— (6)}$$

Now



F-I analogy, $x = \phi$ $\quad \therefore V(s) = s\,\phi(s)$

(II) $m \to C$, $B = 1/R$, $k = 1/L$

$$I(s) = C_3 s^2 \phi_1 + \frac{1}{L_3}(\phi_1 - \phi_2)$$

$$0 = C_2 s^2 \phi_2 + \frac{1}{L_3}(\phi_2 - \phi_1) + \frac{1}{L_2}(\phi_2 - \phi_3)$$

$$0 = C_1 s^2 \phi_3 + \frac{1}{R_1} s\phi_3 + \frac{\phi_3}{L} + \frac{1}{L_2}(\phi_3 - \phi_2)$$

or $$0 = C_1 s V_3(s) + \frac{V_3}{R_1} + \frac{V_3}{sL} + \frac{1}{sL_2}(V_3 - V_2)$$

$$0 = sC_2 V_2 + \frac{(V_2 - V_1)}{sL_3} + \frac{1}{sL_2}(V_2 - V_3)$$

$$I(s) = sC_3 V_1 + \frac{1}{sL_3}(V_1 - V_2)$$

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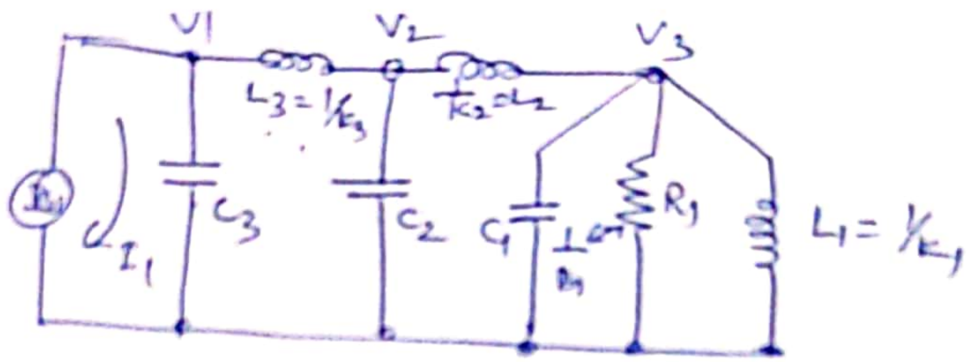

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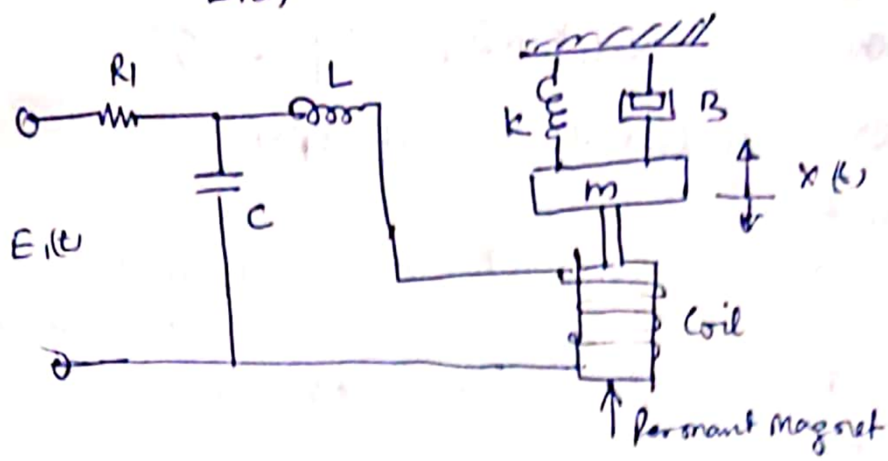
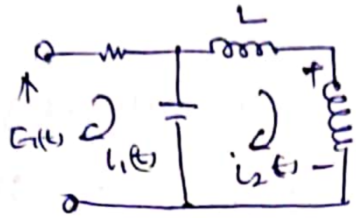
Find TF $\frac{X(S)}{E_1(S)}$ of the fig given below





Coil e_b = Back emf opposite direction to that of $E_1(t)$

$$E_1(t) = i_1 R + \frac{1}{C} \int (i_1 - i_2)\, dt$$

$$E_1(t) = i_1(t) R + \frac{1}{C} \int (i_1(t) - i_2(t))\, dt \quad \text{— (1)}$$

$$0 = L\frac{di_2(t)}{dt} + \frac{1}{C}\int [i_2(t) - i_1(t)]\, dt + e_b \quad \text{— (2)}$$

Taking Laplace TF

$$E_1(S) = I_1(S) R + \frac{1}{SC}[I_1(S) - I_2(S)] \quad \text{— (3)}$$

$$0 = SL I_2(S) + \frac{1}{SC}[I_2(S) - I_1(S)] + E_b(S) \quad \text{(4)}$$

Now $e_b \propto \frac{dx(t)}{dt}$

$$e_b(t) = K_b \frac{dx(t)}{dt}$$ where K_b is the back emf constant

$$E_b(S) = K_b S X(S) \quad \text{— (5)}$$

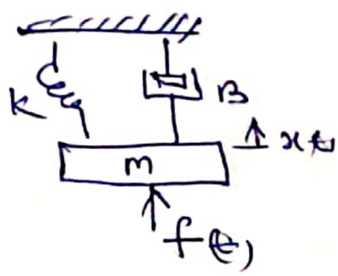
Now consider the mechanical System:

F(t) is produced because of flux due to current $i_2(t)$

$$F(t) \propto i_2(t)$$

or, $F(t) = K_i\, i_2(t)$ $\therefore K_i \to$ constant

or, $F(S) = K_i I_2(S)$ — (6)



$$F(t) = M\frac{d^2x(t)}{dt^2} + K x(t) + B\frac{dx(t)}{dt}$$

$$F(S) = MS^2 X(S) + K X(S) + BS X(S)$$

$$X(S)[MS^2 + BS + K] \quad \text{— (7)}$$

from eqn ⑥ & ⑦ we have

$$K_i \, I_2(s) = X(s)[ms^2+Bs+K] \quad \text{—— ⑧}$$

from ③ & ④ find out the value of $E_1(s)$

from eqn ④

$$0 = sLI_2(s) + \frac{I_2(s)}{sC} - \frac{I_1(s)}{sC} + K_b s X(s)$$

$$\therefore E_b(s) = K_b s X(s)$$

$$I_1(s) = s^2 LC I_2(s) + K_b s^2 C X(s) + I_2(s)$$

Substitute $I_1(s)$ in eqn ③ we have

$$E_1(s) = I_1(s)\left[R_1 + \frac{1}{sC}\right] - \frac{I_2(s)}{sC}$$

$$E_1(s) = \left[s^2 LC I_2(s) + K_b s^2 C X(s) + I_2(s)\right]\left(R_1 + \frac{1}{sC}\right) - \frac{I_2(s)}{sC}$$

$$E_1(s) = \left[s^2 LC R_1 I_2(s) + K_b s^2 C R_1 X(s) + I_2(s) R_1 + sL I_2(s) + K_b s X(s) + \cancel{\frac{I_2(s)}{sC}} - \cancel{\frac{I_2(s)}{sC}}\right]$$

$$E_1(s) = \left[s^2 LCR_1 + R_1 + sL\right] I_2(s) + K_b s^2 C R_1 X(s) + K_b s X(s)$$

$$= \left[s^2 LCR_1 + R_1 + sL\right] I_2(s) + K_b s X(s)[sCR_1 + 1]$$

$$\text{or, } I_2(s) = \frac{E_1(s) - K_b s X(s)[sCR_1 + 1]}{[s^2 LCR_1 + sL + R_1]} \quad \text{—— ⑨}$$

Substituting these value in eqn ⑧ we have

$$K_i \left[\frac{E_1(s) - K_b s X(s)[1 + sCR_1]}{[s^2 LCR_1 + sL + R_1]}\right] = X(s)\left[ms^2 + Bs + K\right]$$

$$K_i E_1(s) - K_i K_b s X(s)(1 + sCR_1] = X(s)[ms^2 + Bs + K]\cdot[s^2 LCR_1 + sL + R_1]$$

$$\text{or, } K_i E_1(s) = X(s)\left[(ms^2 + Bs + K)(s^2 LCR_1 + sL + R_1) + K_i K_b s(1 + sCR_1)\right]$$

$$\text{or, } \boxed{\frac{X(s)}{E_1(s)} = \frac{K_i}{(ms^2 + Bs + K)(s^2 LCR_1 + sL + R_1) + K_i K_b s(1 + sCR_1)}}$$

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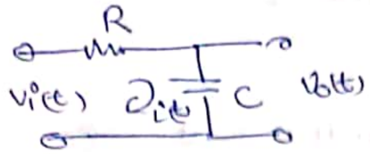
Find the TF of (2)

(a)



= $V_i(s) \rightarrow \boxed{\frac{1}{1+sCR}} \rightarrow V_o(s)$

We know using KVL

$$V_i(t) = i(t) \cdot R + \frac{1}{C}\int i(t)dt \quad —(1)$$

$$V_o(t) = \frac{1}{C}\int i(t)\,dt \quad —(2)$$

~~TF w.r.t~~

taking Laplace transform of eqn (1) & (2) we get

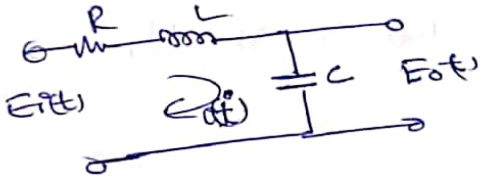
$$V_i(s) = I(s)R + \frac{1}{sC}I(s) \quad —(3)$$

$$V_o(s) = \frac{1}{sC}I(s) \quad —(4)$$

$$TF = \frac{V_o(s)}{V_i(s)} = \frac{1/sC\ \cancel{I(s)}}{\cancel{I(s)}[R+1/sC]}$$

$$\boxed{TF = \frac{1}{1+sCR}}$$

(b)



using KVL

$$E_i(t) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \quad —(5)$$

$$E_o(t) = \frac{1}{C}\int i(t)dt \quad —(6)$$

taking Laplace TF of (5) & (6) we get

$$E_i(s) = I(s)R + LS\,I(s) + \frac{I(s)}{SC} \quad —(7)$$

$$E_o(s) = \frac{I(s)}{SC} \quad —(8)$$

$$TF = \frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{SC}}{\left[R+SL+\frac{1}{SC}\right]}$$

$$= \frac{1}{SCR + S^2LC + 1}$$

$$E_i(S) \rightarrow \boxed{\frac{1}{S^2LC+SCR+1}} \rightarrow E_o(S)$$

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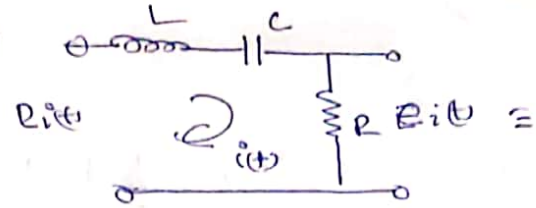
(c)



$$e_i(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt + R\,i(t) \quad \text{— (9)}$$

$$e_o(t) = R\,i(t) \quad \text{— (10)}$$

Laplace TF of eqn (9) & (10) are

$$E_i(S) = SL\,I(s) + \frac{I(s)}{SC} + R\,I(s) \quad \text{— (11)}$$

$$E_o(S) = I(s)\,R \quad \text{— (12)}$$

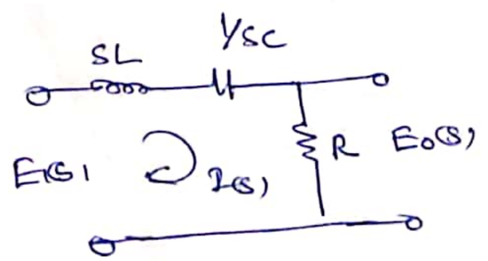
$$TF = \frac{E_o(s)}{E_i(s)} = \frac{SCR}{S^2LC + SCR + 1} \quad \text{— (13)}$$

Advantage of TF:

- It gives mathematical model of all system components
- uses Laplace TF to convert time domain to eqn into algebraic eqn
- once TF is known o/p response of I/P can be calculated
- It helps to test the stability
-

Disadvantage

- only applicable for linear system
- not providing physical structure of the system
- Effect of initial condition is neglected

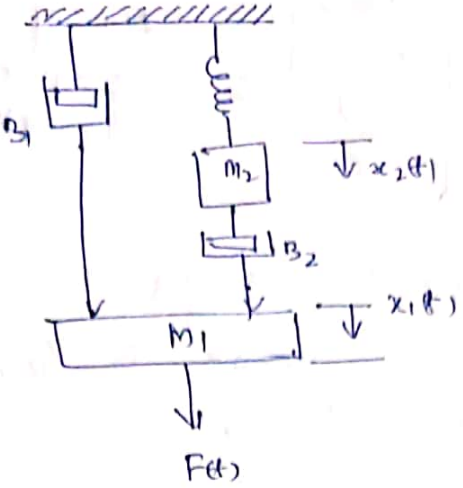


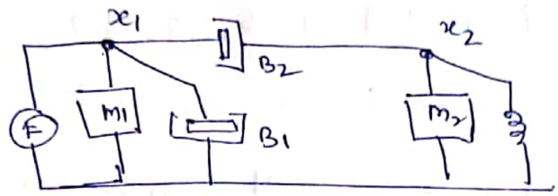
$$E_i(S) = \left(SL + \frac{1}{SC} + R\right) I_s \quad \text{— (1)}$$

$$E_o(S) = I_s\,R$$

$$TF = \frac{E_o(S)}{E_i(S)} = \frac{SCR}{S^2LC + SCR + 1}$$

Q Find the electrical equivalent of this





Two nodes x_1 and x_2, M_1, B_1 same displacement

$\Sigma F = 0$

At node 1 $\quad F - m_1 \frac{d^2x_1}{dt^2} - B_1\frac{dx}{dt} - B_2\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right)$ — (1)

~~Laplace~~

at node 2 $\quad 0 = M_2 \frac{d^2x_2}{dt^2} + Kx_2 + B_2\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right)$ — (2)

taking laplace transform

$$F = s^2M_1X_1 + B_1sX_1 + B_2s(X_1 - X_2) \quad \text{— (3)}$$

$$0 = s^2M_2X_2 + B_2s(X_2 - X_1) + KX_2 \quad \text{— (4)}$$

Now Force voltage Analogy

$M \to L$, $B \to R$, $K \to 1/C$, $F = V$, $X \to q$

Substitute these values in eq (3) & (4)

We get

$$V(s) = s^2L_1q_1 + R_1sq_1 + R_2s(q_1 - q_2) \quad \text{— (5)}$$

$$0 = s^2L_2q_2 + R_2s(q_2 - q_1) + \frac{1}{C}q_2 \quad \text{— (6)}$$

Replacing $\frac{I(s)}{s} = q(s) \Rightarrow \quad I(s) = sq(s)$ we get

$$V(s) = sL_1I_1(s) + R_1I(s) + R_2[I_1(s) - I_2(s)] \quad \text{— (7)}$$

$$0 - sL_2I_2(s) + R_2[I_2(s) - I_1(s)] + \frac{I_2(s)}{sC} \quad \text{— (8)}$$

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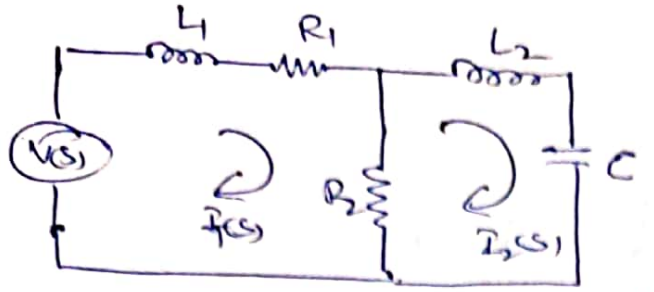
Hence



No of loop Currents is equal to number of displacement

(II) Force - Current analogy

Force - Current $m \to C$, $B - 1/R$, $k = 1/L$, $x - \phi$

Eqn (3) & (4) Can be written as

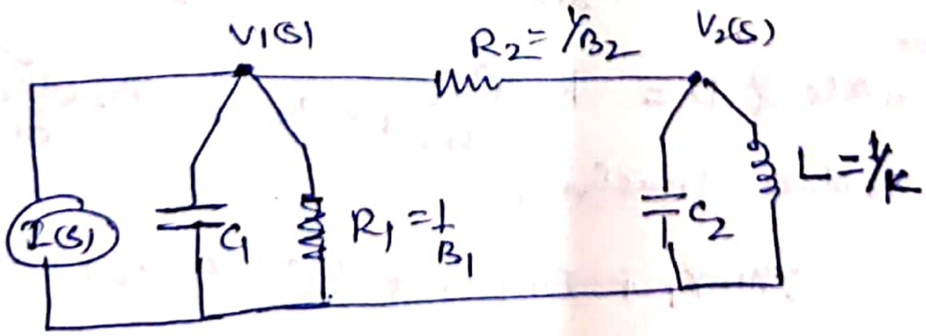
$$I(s) = C_1 s^2 \phi_1 + \frac{1}{R_1} s\phi_1 + \frac{1}{R_2} s(\phi_1 - \phi_2) \quad \text{— (9)}$$

$$0 = C_2 s^2 \phi_2 + \frac{1}{L}\phi_2 + \frac{s}{R_2}(\phi_2 - \phi_1) \quad \text{— (10)}$$

Here $s\phi(S) = V(S)$ So (9) & (10) Can be written as

$$I(S) = C_1 V_1(S) + \frac{V_1(S)}{R_1} + \frac{1}{R_2}[V_1(S) - V_2(S)] \quad \text{— (11)}$$

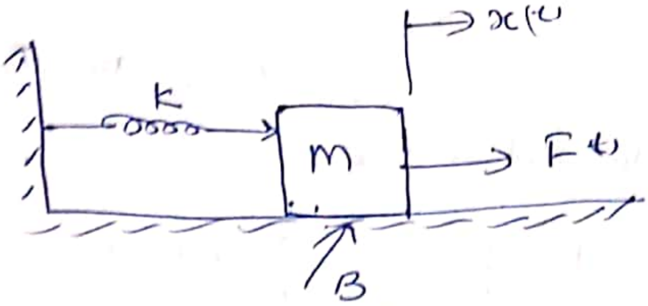
$$0 = C_2 V_2(S) + \frac{V_2 s}{sL} + \frac{1}{R_2}[V_2(S) - V_1(S)] \quad \text{— (12)}$$



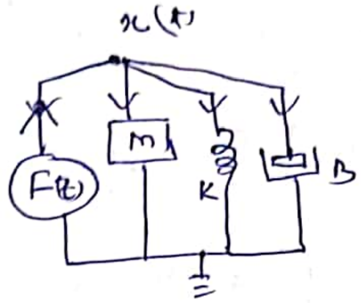
Number of node voltages equal to number of displacements

88

Q.



Find electrical equivalent



In equilibrium condition

$$\Sigma F = 0,$$

$$F(t) = M\frac{d^2x(t)}{dt} + K\,x(t) + B\frac{dx(t)}{dt} \quad — ①$$

taking Laplace TF

$$F(S) = MS^2 x(S) + K\,X(S) + B S\,X(S) \quad — ②$$

$$= X(S)\left[MS^2 + BS + K\right] \quad — ③$$

① Force-Voltage method

$F(S) = V(S)$, $X(S) = q(S)$, $M = L$, $B = R$, $K = 1/C$

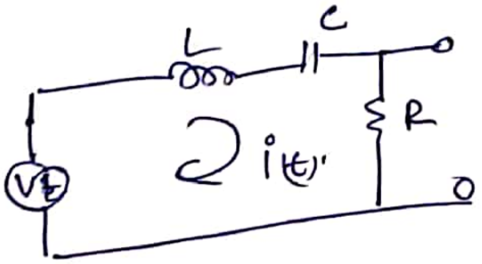
eqn ③ will be

$$V(S) = q(S)\left[LS^2 + RS + \frac{1}{C}\right]$$

$\therefore \frac{I(S)}{S} = \frac{q(S)}{S}$ replace $q(S)$ by $\frac{I_S}{S}$ we have

$$V(S) = I(S)\left[LS + R + \frac{1}{SC}\right] \quad — ④$$

$\rightarrow v(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt + i(t)R$



Force – Current analogy

Let $F(s) = I(s)$, $\phi s = X(s)$, $m = C$, $B = \frac{1}{R}$, $K = \frac{1}{L}$

Eqn ③ can be written as

$$I(s) = \phi(s)\left[s^2 C + \frac{s}{R} + \frac{1}{L}\right]$$

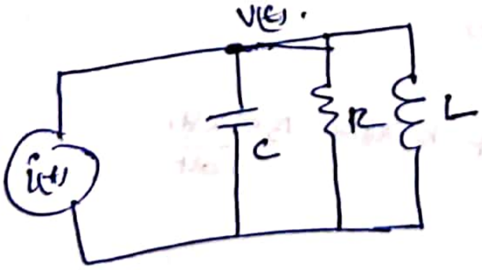
Replacing $\phi(s) = \frac{V(s)}{s}$

$$I(s) = \frac{V(s)}{s}\left[s^2 C + \frac{s}{R} + \frac{1}{L}\right]$$

$$= V(s)\left[sC + \frac{1}{R} + \frac{1}{sL}\right]$$

Inverse Laplace

$$i(t) = C\frac{dv(t)}{dt} + \frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt$$



Servo motors

Converts elect energy into mechanical o/p

→ All DC servo motors are separately excited type to ensure linear characteristics

DC
- → Series
- → Shunt
- → permanent Magnet

- Higher o/p than from AC motor of same size
- Linear characteristics easily achieved
- Easier speed control
- High Torque to inertia ratio
- quick response to control sig
- light weight
- low electrical & mech constant

AC → rugged

Adv – light in weight
- lower cost
- High efficiency
- less maintenance since brushes & commutator are not there.

disadv

characteristics are non linear
- not suitable for position control

- Application

Air craft, EM actuator, process control, robotics, mechanical tools etc

Application
- suitable for low power applications

- Features of field control DC Servo Motor
  - Preferred for small rated motors
  - Large Time constant
  - open loop system
  - control circuit is simple.

- Features of Armature control
  - Suitable for large rated motors
  - Small time constant hence fast response
  - Closed loop syst
  - Back emf provides internal damping which make operation more suitable
  - Efficiency & overall performance is better than field control motor

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Field Control DC Servomotor :-

In this motor, the armature is supplied with a constant current or voltage. When armature voltage is constant, the torque is directly proportional to the field flux. Field current is proportional to field flux, the torque of the motor is controlled by controlling the field current.

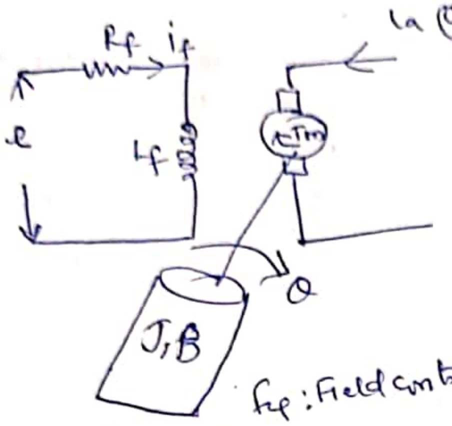
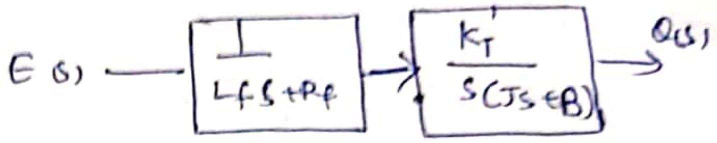


fig : Field control of DC motor



BD of Field control DC motor

In this system

$R_f \rightarrow$ field winding resistance (ohm)
L_f — " " inductance (Henrys)
e — field control voltage
i_f — field current
T_m — Torque developed by motor

J — equivalent MOI of motor and load refer to motor shaft Kg m2
f or B equivalent viscous friction N-m/rad/s
θ — angular displacement

In field control motor, Ia is constant, since motor is operated linear region of the magnetization curve, the flux is proportional to the field current, i.e,

$$\phi \propto I_f$$

$$\phi = K_f i_f \quad \text{where } K_f \text{ is constant}$$

The Torque developed by the motor

$$T_m \propto \phi I_a$$

$$\propto K_f i_f i_a$$

$$T_m = K_T K_f i_f i_a = K_T' i_f \quad \text{where } K_T' \text{ is constant}$$

(A)

The eqn of field current

$$L_f \frac{di_f}{dt} + R_f i_f = e \quad — (1)$$

The Torque eqn is

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} \pm T_m = K_T' i_f \quad — (2)$$

from (A)

taking Laplace TF of (1) & (2) we get

$$L_f S I_f(S) + R_f I_f(S) = E(S) \quad — (3)$$

$$I_f(S) = \frac{E(S)}{L_f S + R_f} \quad — (4)$$

Substitute value of $I_f(S)$ in (2) we get

$$(J S^2 + BS)\theta(S) = T_m(S) = K_T' I_f(S) = \frac{K_T' E(S)}{L_f S + R_f}$$

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$$\frac{\theta(s)}{E(s)} = \frac{k'_T}{s(L_f s + R_f)(Js + B)}$$

$$= \frac{\dfrac{k'_T}{R_f B}}{s\left[\dfrac{L_f s}{R_f} + 1\right]\left[\dfrac{Js}{B} + 1\right]}$$

$$\boxed{\frac{\theta(s)}{E(s)} = \frac{k_m}{s[\tau_f s + 1][\tau_m s + 1]}}$$

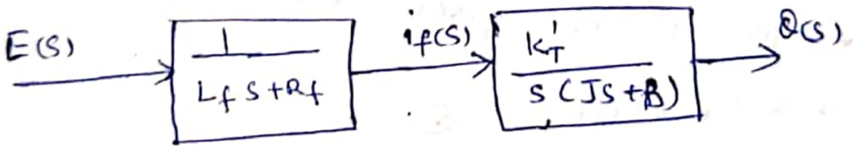
where

$k_m = \dfrac{k'_T}{R_f B}$ = motor gain constant

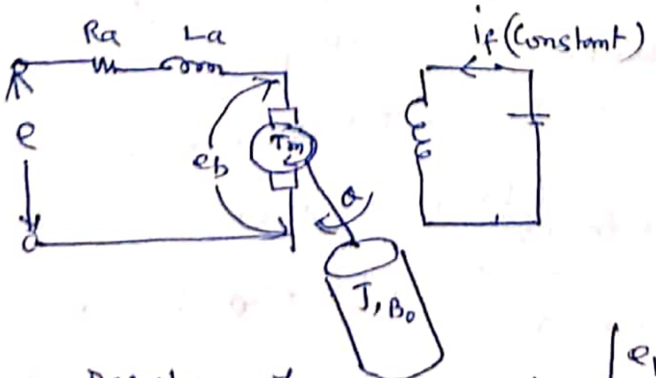
$\tau_f = \dfrac{L_f}{R_f} \rightarrow$ L/R time constant of field winding

$\tau_m = \dfrac{J}{B} \rightarrow$ mechanical time constant

TF can be represented by Block diagram.



Armature Control DC Servomotor → In this motor field current is constant. Speed of motor is directly proportional to armature current



Ra → Resistance of armature winding
La → Inductance of " "
ia – ~~current~~ armature Current
if – field current
e – applied armature voltage

e_b → back emf
Tm – Torque developed by motor
θ → angular displacement
J → equivalent MI of motor
Bo → equivalent viscous friction
→ slope of torque speed characteristics

In Servo motor applications, The DC motors are used in linear range of the magnetization curve. Therefore, the air gap flux ϕ is proportional to the field current, i.e.

$$\phi \propto i_f$$

$$\phi = k_f i_f \quad \text{where } k_f \text{ is constant}$$

The Torque T_m developed by the motor is porportional to the product of flux ϕ and armature current i_a, i.e

$$T_m \propto \phi i_a$$

$$\propto k_f i_f i_a$$

$$T_m = K_1 k_f i_f i_a \quad \text{——} \ (1)$$

in armature controlled DC motor i_f is constant, so the eqn of T_m can be written as

$$T_m = k_T i_a \quad \text{where } k_T \text{ motor torque constant}$$

The motor back emf is proportional to speed, i.e

$$e_b \propto \frac{d\theta}{dt}$$

$$e_b = k_b \frac{d\theta}{dt} \quad \text{——} \ (2) \quad \text{where } k_b \text{ is back emf constant}$$

The differential eqn of armature ckt is

$$e = L_a \frac{di_a}{dt} + R_a i_a + e_b \quad (3)$$ and Torque eqn

$$T_m = k_T i_a = J\frac{d^2\theta}{dt^2} + B_0 \frac{d\theta}{dt} \quad - (4)$$

taking Laplace TF we get

$$E_b = k_b s \theta(s) \quad \text{——} (5)$$

$$E(s) = sL_a I_a(s) + R_a I_a(s) + E_b(s) \quad - (6)$$

$$I_a(s)[L_a s + R_a] = E(s) - E_b(s)$$

$$I_a(s) = \frac{E(s) - E_b(s)}{[L_a s + R_a]} \quad \text{——} \ (7)$$

from eqn (4) Laplace TF

$$T_m(s) = k_T I_a(s) = Js^2 \theta(s) + B_a s\theta(s)$$

$$k_T I_a(s) = [Js^2 + B_a s]\theta(s)$$

Put value of I_a from (7)

$$k_T \left[\frac{E(s) - E_b(s)}{(L_a s + R_a)}\right] = [Js^2 + B_0 s]\theta(s)$$

$$k_T \frac{[E(s) - k_b s\theta(s)]}{(L_a s + R_a)} = [Js^2 + B_0 s]\theta(s)$$

$$= (Js + B_0) s\theta(s)$$

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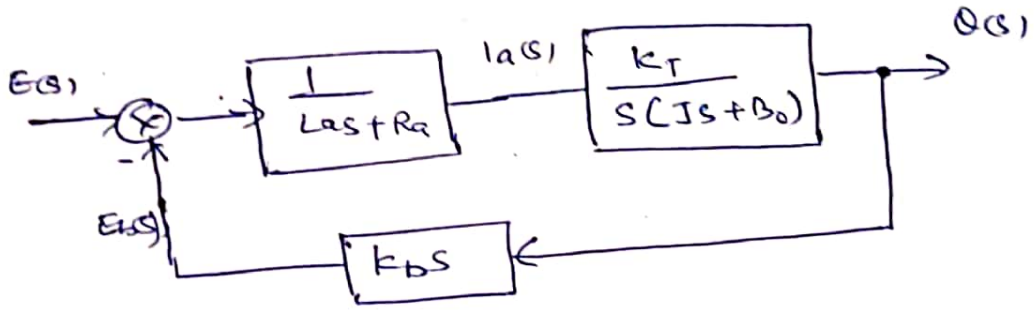
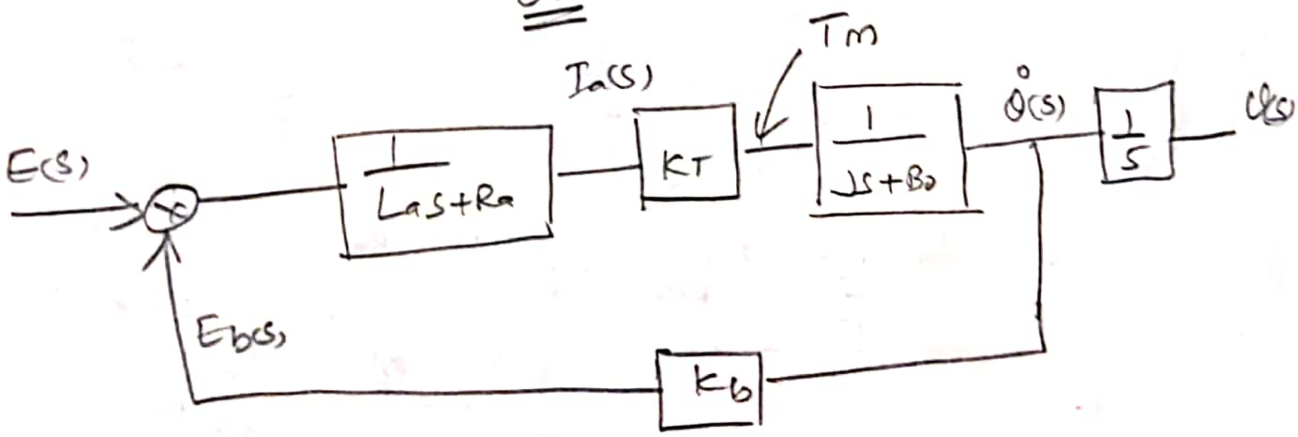
$$(L_a s + R_a)(Js + B_0)\, s\,\theta(s) = K_T\left[E(s) - K_b s\,\theta(s)\right]$$

$$s\left[(L_a s + R_a)(Js + B_0) + K_T K_b\right]\theta(s) = K_T E(s)$$

$$\frac{\theta(s)}{E(s)} = \frac{K_T}{s\left[(L_a s + R_a)(Js + B_0) + K_T K_b\right]}$$



or



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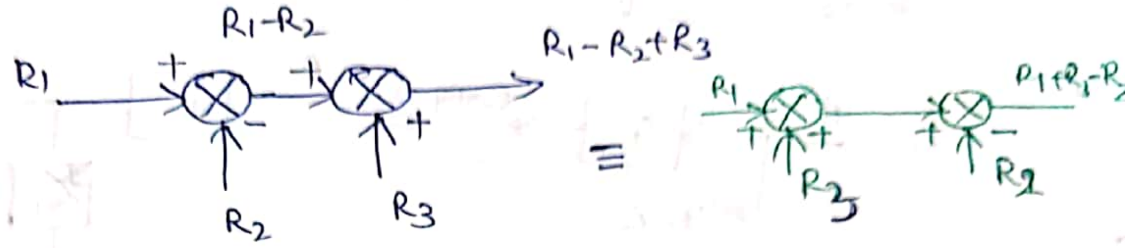
(3)

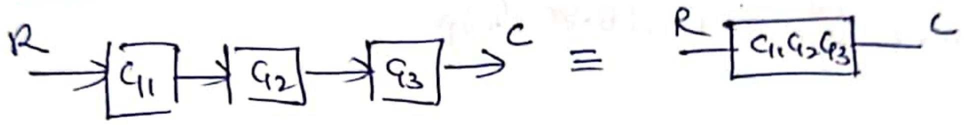
Rules for Block diagram reduction :-

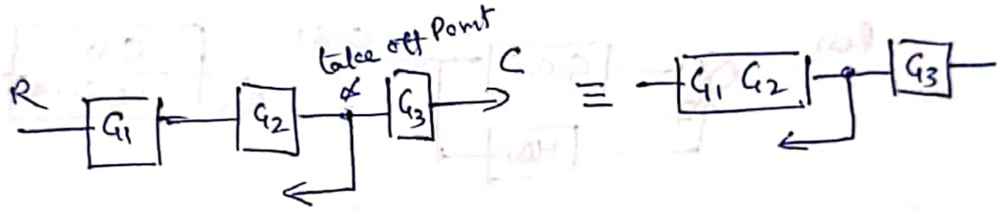
To bring BD into simple form it is necessary to reduce the BD using proper logic such that output of such system and value of any feedback signal not get distrubed.

Rule 1 Associative law

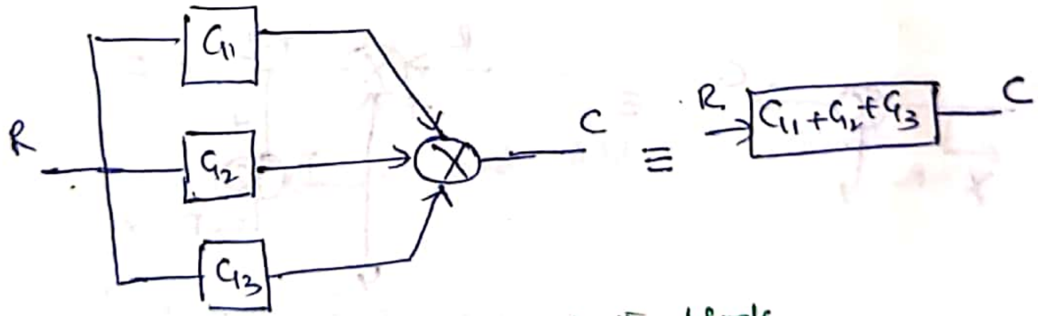


Series block

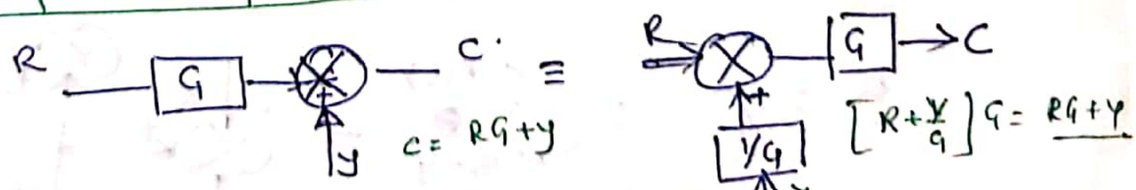




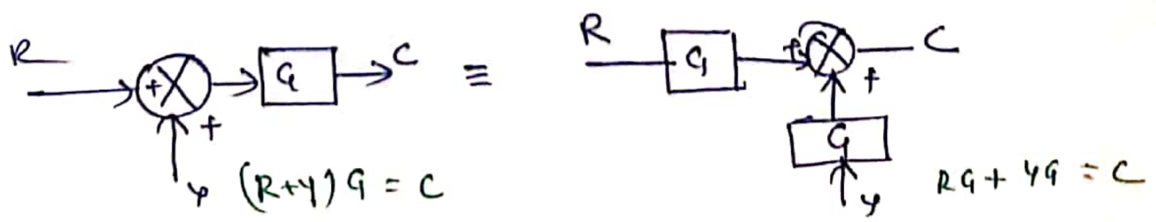
Parallel block



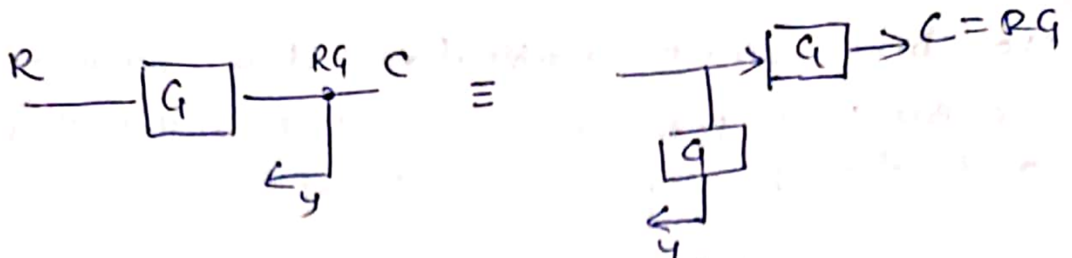
Shifting summing point behind the block



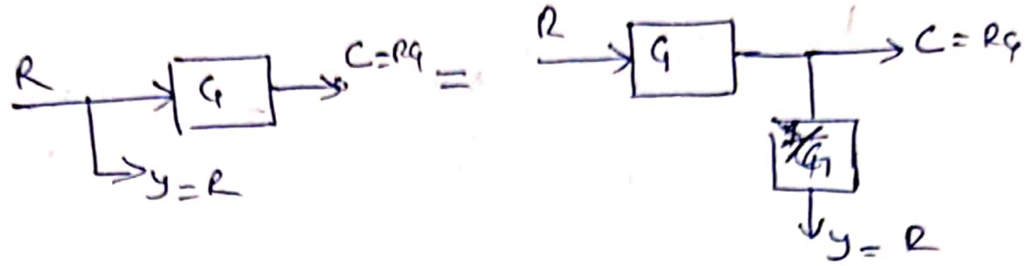
Shifting the summing point beyond the block



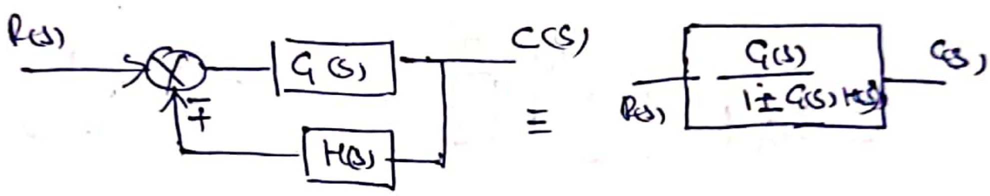
Shifting take off point behind the block



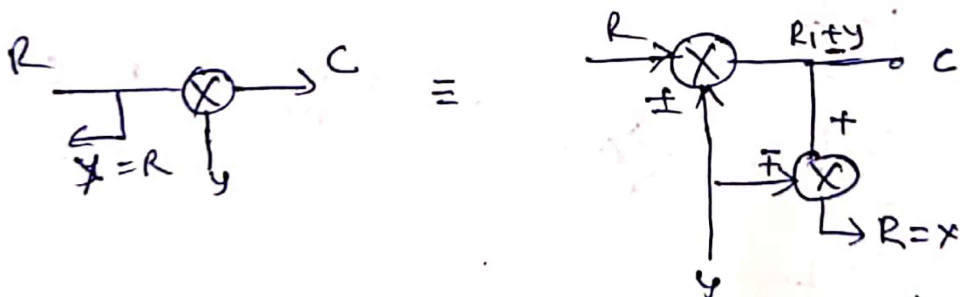
Shifting take off point beyond the block



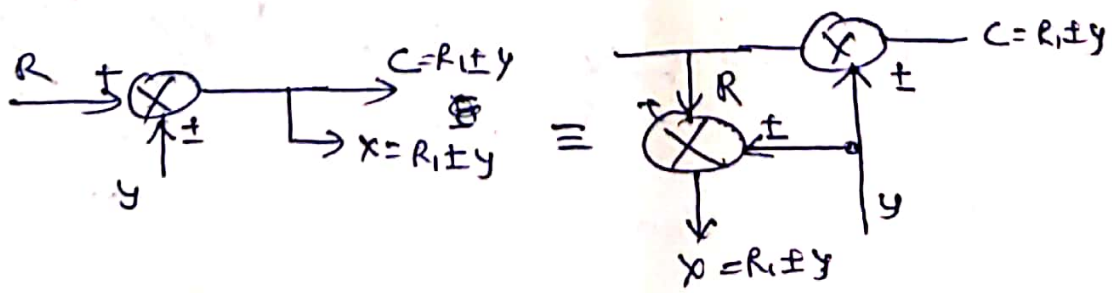
Removing minor feedback loop



Critical rule (Shifting take off after summing point) ~~after~~



Shifting the take off point before a summing point



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Block diagram reduction Problems

Procedure to solve Block diagram reduction Problem

(1)

Step 1: Reduce the blocks connected in series

Step 2: Reduced the block connected in Parallel

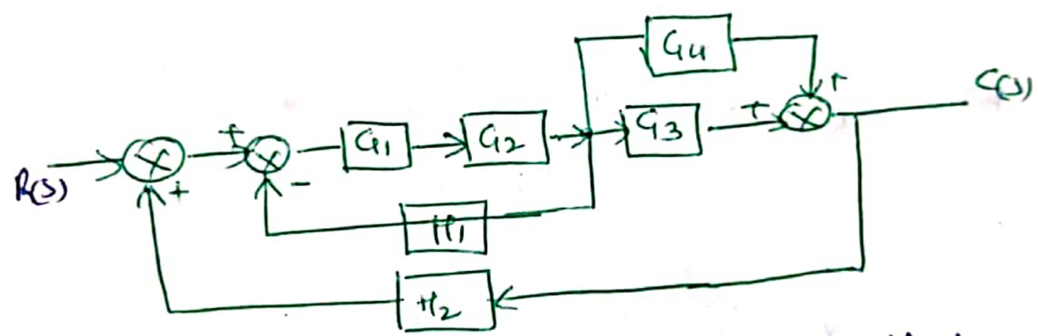
Step 3: Reduced the minor internal feedback loops

Step 4: as far as possible try to shift take off points towards right and summing point toward left.

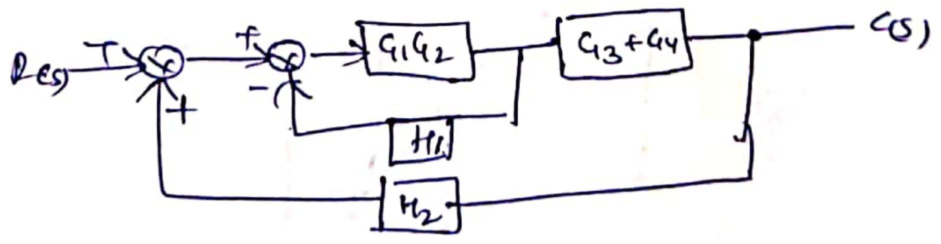
Step 5: Repeat steps 1–4 till simple form is obtained

Step 6: using std TF of simple closed loop system, obtain obtain the closed loop TF $\frac{C(s)}{R(s)}$ of the overall system

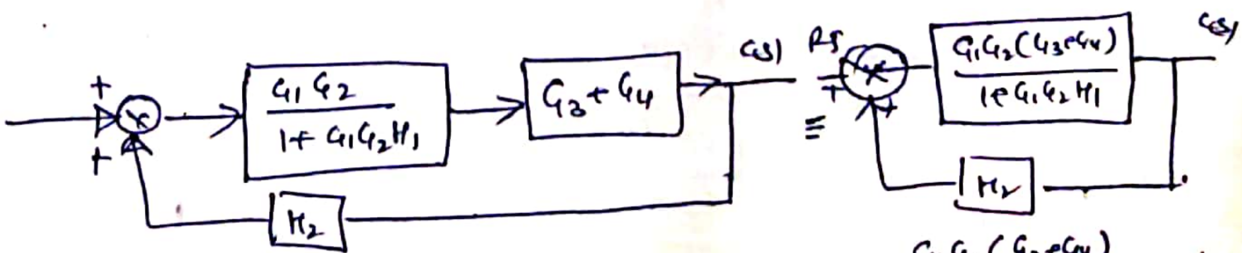
Q.1 find TF = $\frac{C(s)}{R(s)}$ of given fig



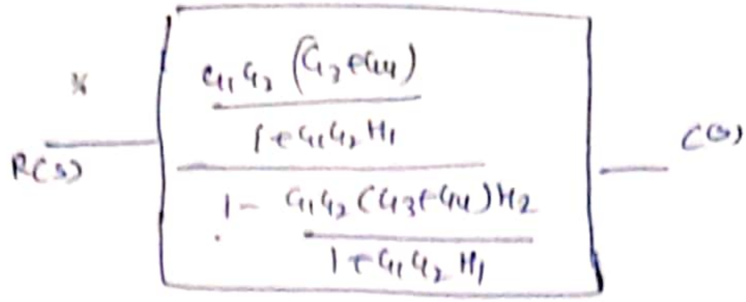
① Combine G_1 & G_2 block and Add G_3 & G_4 block



② Reduced minor feedback loop

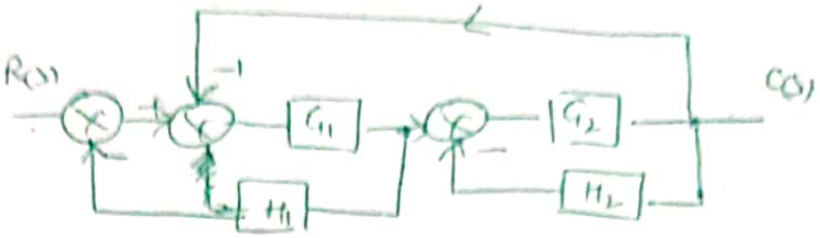


$$= \frac{C(s)}{R(s)} = \frac{G_1G_2(G_3+G_4)}{1+G_1G_2H_1 - G_1G_2(G_3+G_4)H_2}$$

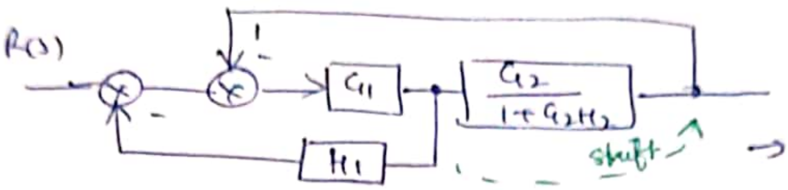


$$\frac{C(s)}{R(s)} = \frac{G_1G_2(G_3+G_4)}{1+G_1G_2H_1 - G_1G_2(G_3+G_4)H_2}$$

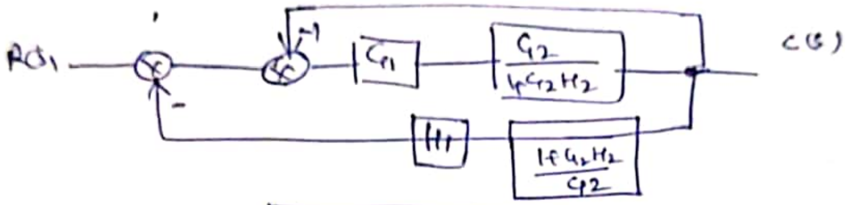
Q Find $\frac{C(s)}{R(s)}$

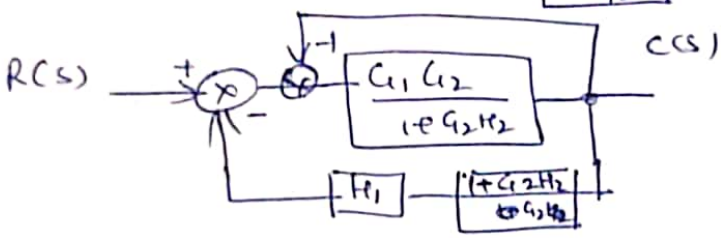


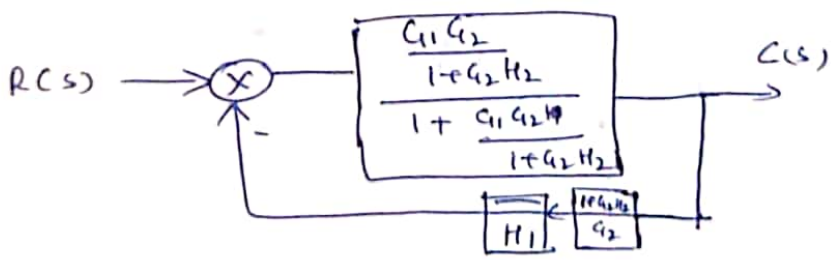
① Remove inner feedback loop

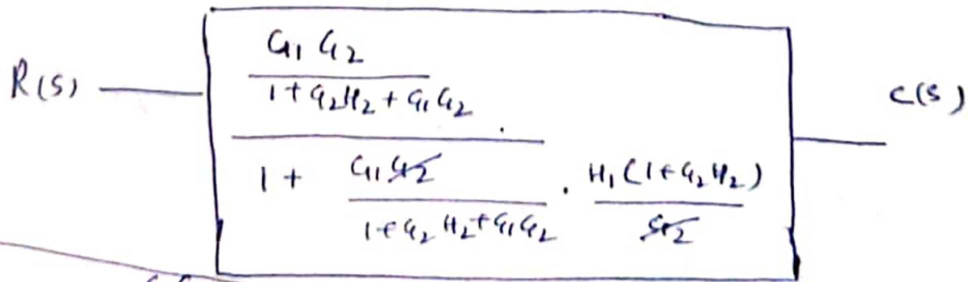


→ to shift divide by $\boxed{\frac{1+G_2H_2}{G_2}}$









$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_2H_2+G_1G_2+G_1H_1+G_1G_2H_1H_2}}$$

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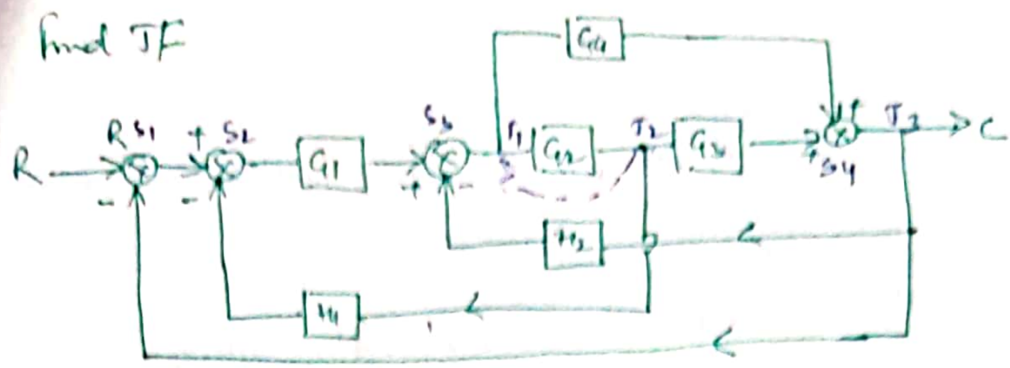
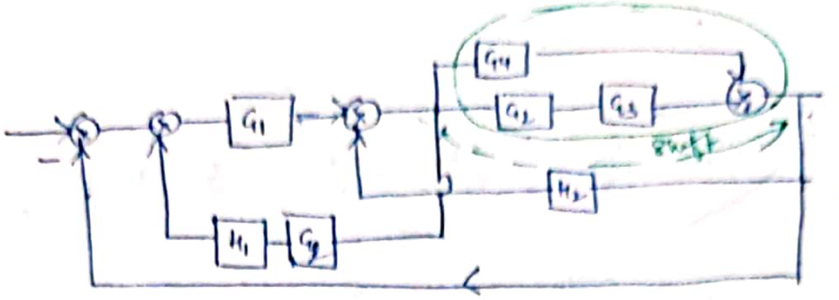
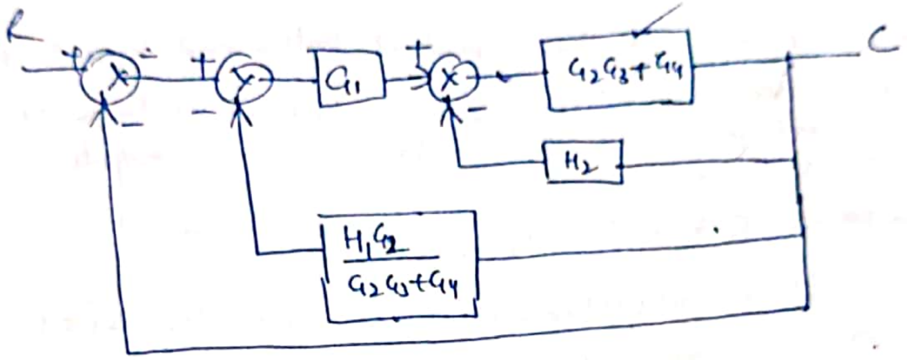
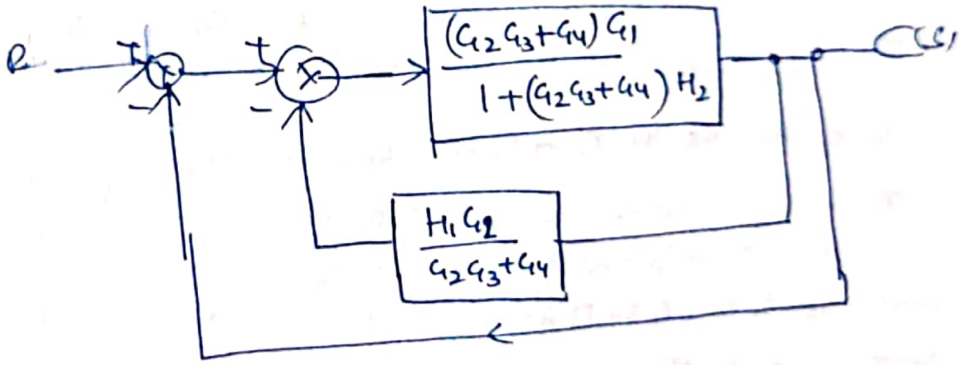
Find TF

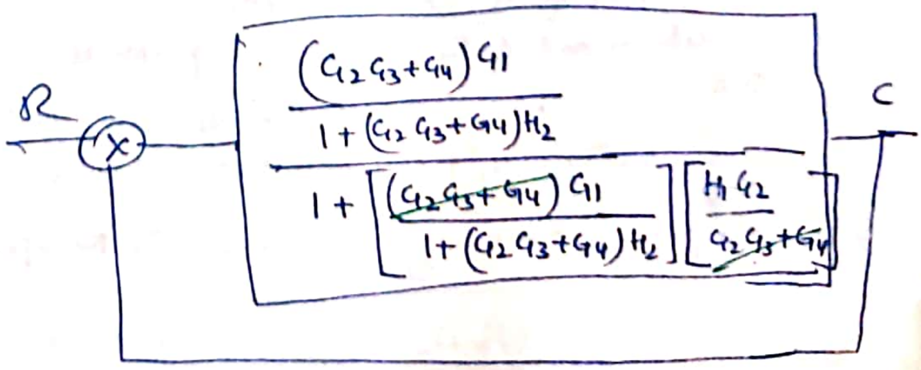




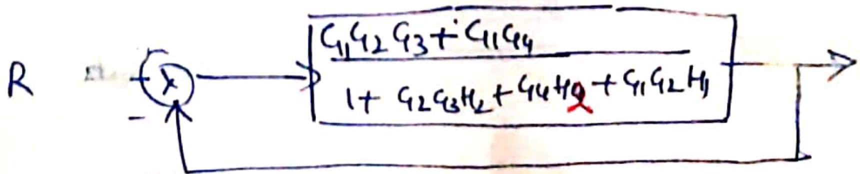
Here two step simplified inner loop and then shift take off point to right







$$\frac{\dfrac{(G_2G_3+G_4)G_1}{1+(G_2G_3+G_4)H_2}}{1+\left[\dfrac{(\cancel{G_2G_3+G_4})G_1}{1+(G_2G_3+G_4)H_2}\right]\left[\dfrac{H_1G_2}{\cancel{G_2G_3+G_4}}\right]}$$



$$\frac{G_1G_2G_3+G_1G_4}{1+G_2G_3H_2+G_4H_2+G_1G_2H_1}$$

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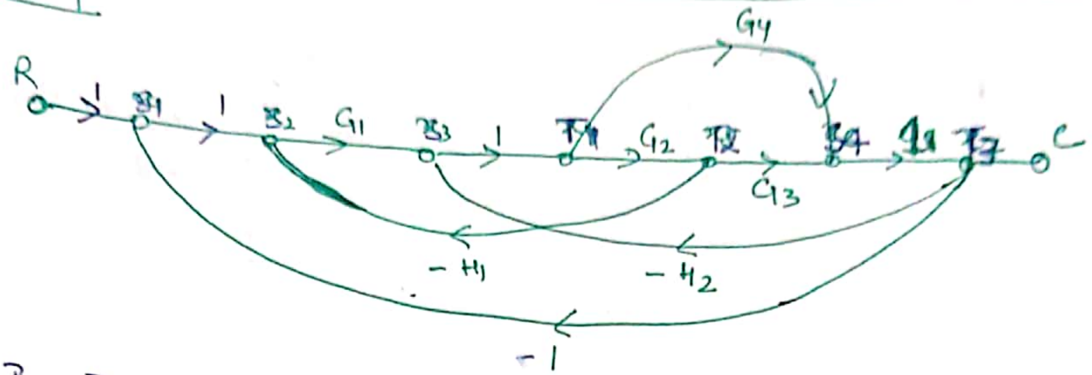
$$\frac{R}{} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4}$$

$$\boxed{TF = \frac{C}{R} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4}}$$

SFG



For This SFG there are two Forward paths and five loops and there are ~~two~~ two nontouching loops and all the loops are touching both the forward paths. The Forward path and gain associate with them are given as

Forward path 1 $R\ S_1\ S_2\ S_3\ T_1\ T_2\ S_4\ T_3\ C$,

$$P_1 = (1)(1)(G_1)(1)(G_2)(1)(G_3)(1) = G_1G_2G_3, \quad \Delta_1 = 1$$

Forward Path 2, $R\ S_1\ S_2\ S_3\ T_1 \cdot S_4 - T_3 - C$

$$P_2 = (1)(1)(G_1)(1)(G_4)(1) = G_1G_4, \quad \Delta_2 = 1$$

The loops and gains associated with them are as follows:

Loop $S_1 - S_2 - S_3 - T_1\ T_2\ S_4\ T_3\ S_1 = L_1 = -G_1G_2G_3$

Loop $S_2 - S_3 - T_1 - T_2 - S_2 = L_2 = -G_1G_2H_1$

Loop $S_3\ T_1\ T_2\ S_4\ T_3 - S_3 = L_3 = -G_2G_3H_2$

Loop $R - S_1\ S_2\ S_3\ T_1\ S_4\ T_3\ S_1 = L_4 = -G_1G_4$.

Loop $S_3\ T_1\ S_4\ T_3 - S_3 = L_5 = -G_4H_2$

the determinant of The Signal flow graph is

$$\Delta = 1 - (-G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_1G_4 - G_4H_2)$$

$$= 1 + G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2$$

Applying MASON's gain formula, The transfer function is

$$\boxed{\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2}}$$

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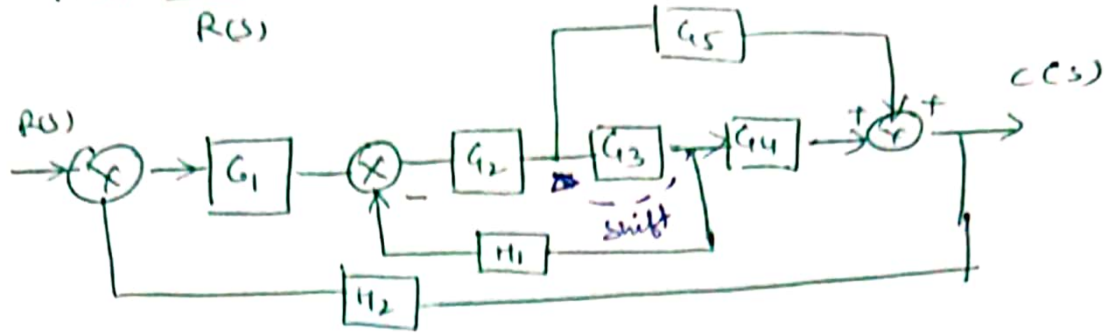
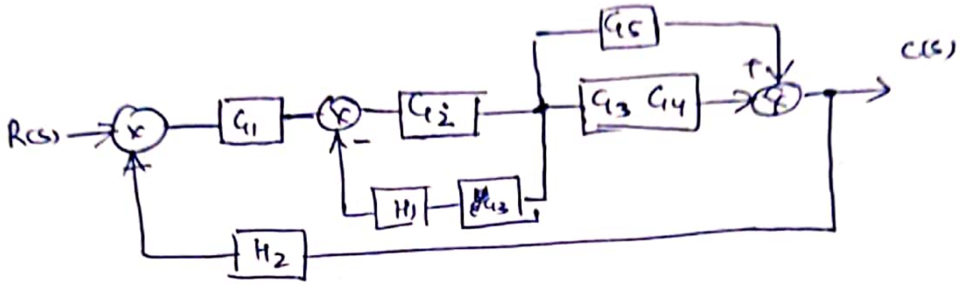
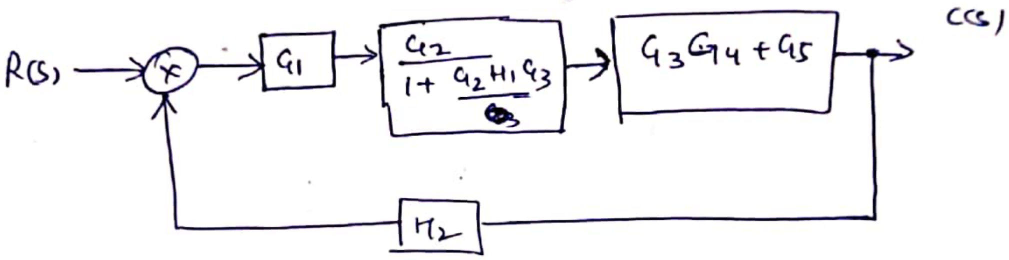
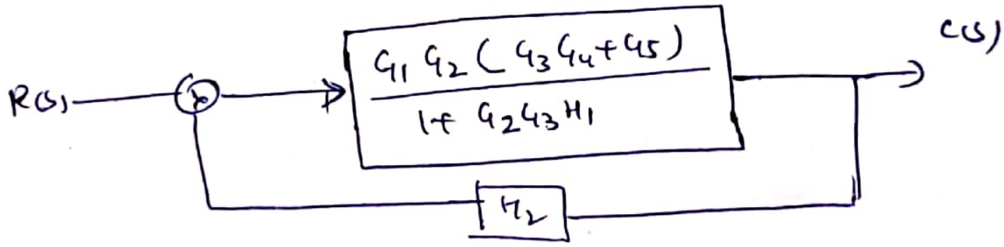
Q

find $\frac{C(s)}{R(s)}$









R(s) $\boxed{\dfrac{\dfrac{G_1G_2G_3G_4+G_1G_2G_5}{1+G_2G_3H_1}}{1+\dfrac{(G_1G_2G_3G_4+G_1G_2G_5)H_2}{1+G_2G_3H_1}}}$ C(s)

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4+G_1G_2G_5}{1+G_2G_3H_1+G_1G_2G_3G_4H_2+G_1G_2G_5H_2}.$$

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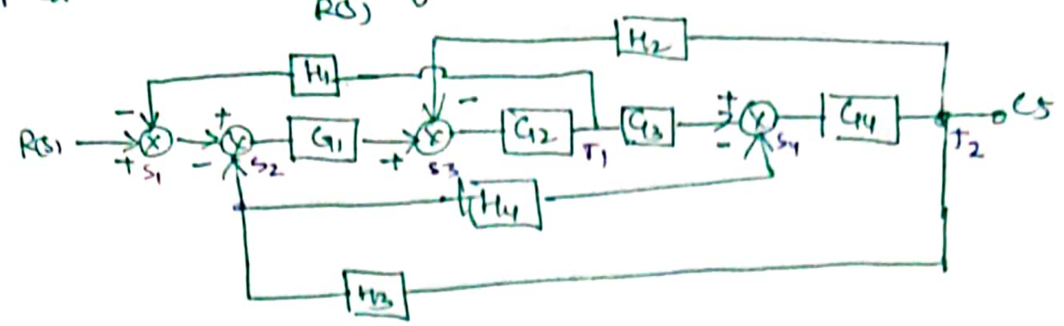
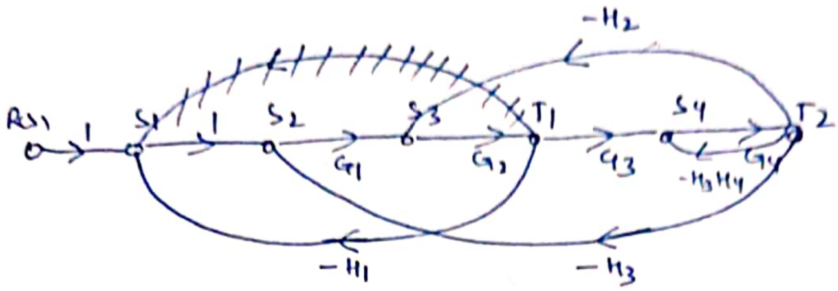
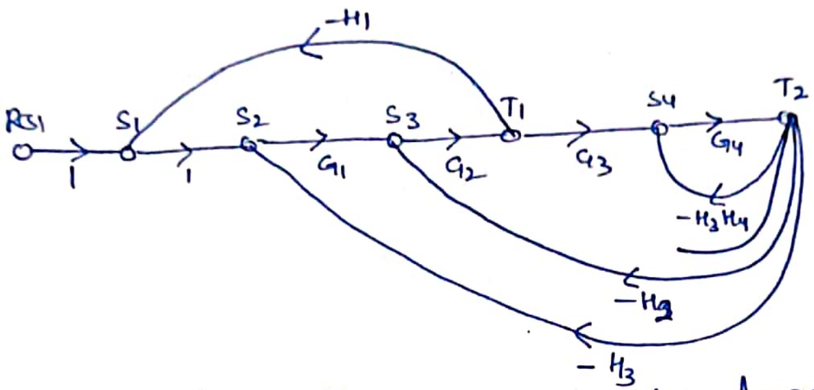
Q. Find the TF $\frac{C(s)}{R(s)}$ for the system shown using SFG.







Here only one forward path and four loops, there are one pair of two non touching loops, and the four loops are touching the forward path.

The forward path $R-S_1-S_2-S_3-T_1-S_4-T_2$.

$$P_1 = G_1G_2G_3G_4, \quad \Delta_1 = 1$$

Loops and gains associated with them

Loop1 $= S_1 - S_2\ S_3 - T_1 - S_1 = -G_1G_2H_1$

Loop2 $= S_2 - S_3\ T_1\ S_4\ T_2\ S_2 = -G_1G_2G_3G_4H_3$

Loop 3 $= S_3\ T_1\ S_4\ T_2\ S_3 = -G_2G_3G_4H_2$

Loop4 $= S_4T_2S_4 = -G_4H_3H_4$

the pairs of non touching loops is L_1 & L_4 = L_{14}

$L_{14} = S_1-S_2-S_3-T_1-S_1$ & $S_4T_2S_4 = G_1G_2H_1G_4H_3H_4$

$$\Delta = 1-(L_1+L_2+L_3+L_4)+L_{14}$$
$$= (1+G_1G_2H_1+G_1G_2G_3G_4H_3+G_2G_3G_4H_2+G_4H_3H_4+G_1G_2G_4H_1H_3H_4)$$

$$TF = \frac{M_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4}{\Delta}$$

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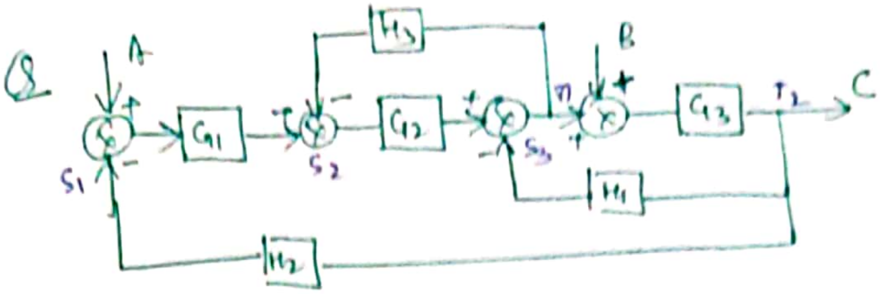
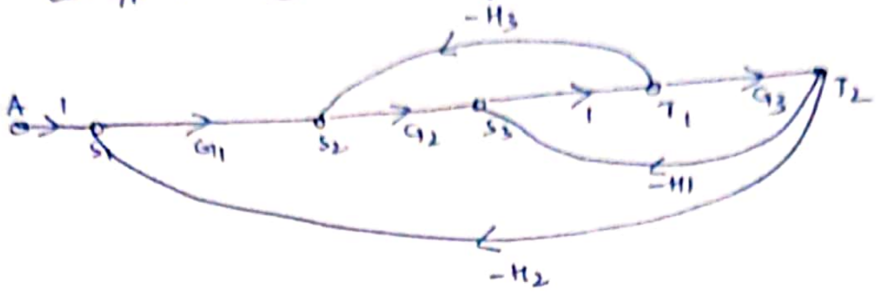
Find TF when the input R is at (a) station A & (b) at station B

Q



Let I/p is at (A) SFG neglect I/p B.



Forward Path one and no of loop 3.

Forward Path gain

$P_1 = G_1 G_2 G_3$, $\Delta_1 = 1$

Loops and gains associated with them

$L_1 = -G_1 G_2 G_3 H_2$

$L_2 = G_2 H_3$

$L_3 \quad G_3 H_1$

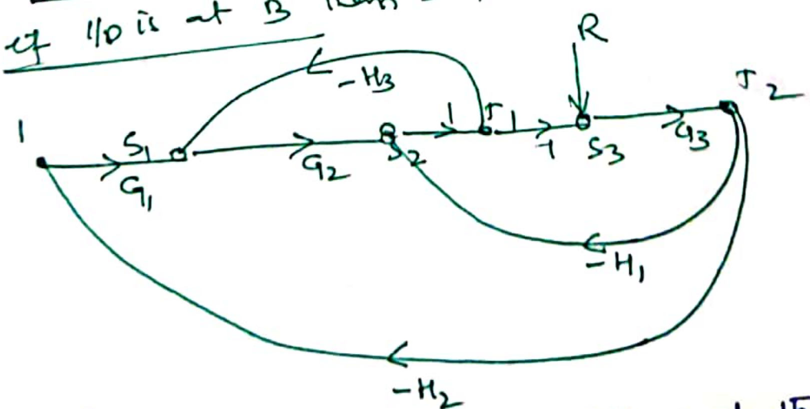
$\Rightarrow \Delta = 1 + G_1 G_2 G_3 H_2 + G_2 H_3 + G_3 H_1$

There are no nontouching loops then

C/R due to I/p at A. is

$$\boxed{TF = \frac{C}{R} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3}{1 + G_3 H_1 + G_2 H_3 + G_1 G_2 G_3 H_2}}$$

If I/p is at B then SFG



In this case one Forward Path and three loops.

$P_1 = G_3$, $\Delta_1 = 1 + G_2 H_3$

$\Delta = 1 + G_1 G_2 G_3 H_2 + G_3 H_1 + G_2 H_3$

$$TF = \frac{P_1 \Delta_1}{\Delta}$$

$$\boxed{TF = \frac{G_3(1 + G_2 H_3)}{1 + G_1 G_2 G_3 H_2 + G_3 H_1 + G_2 H_3}}$$

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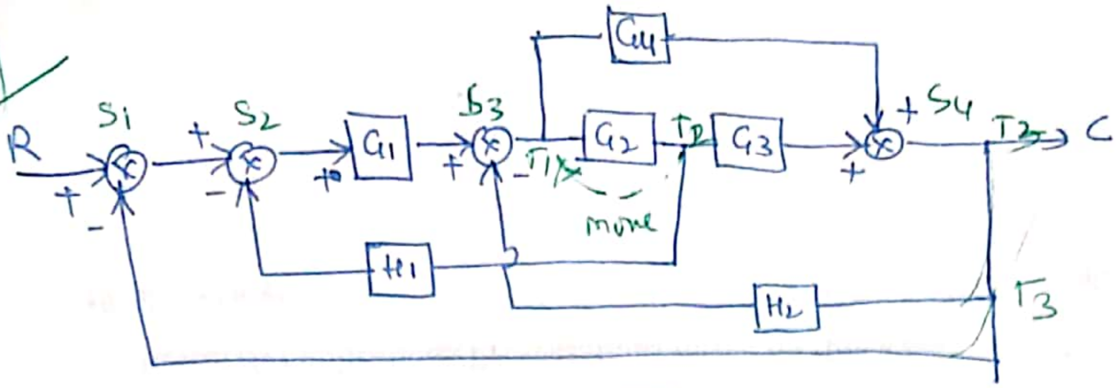
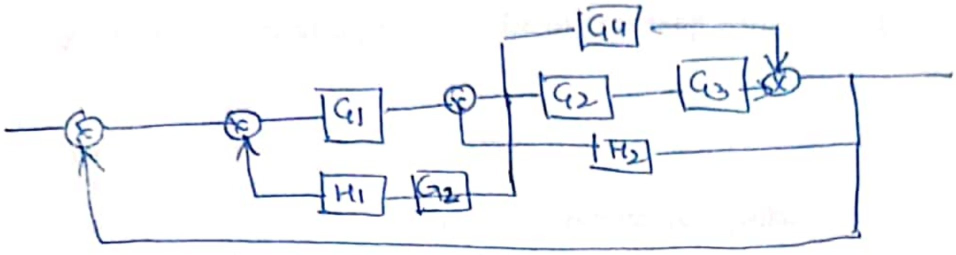
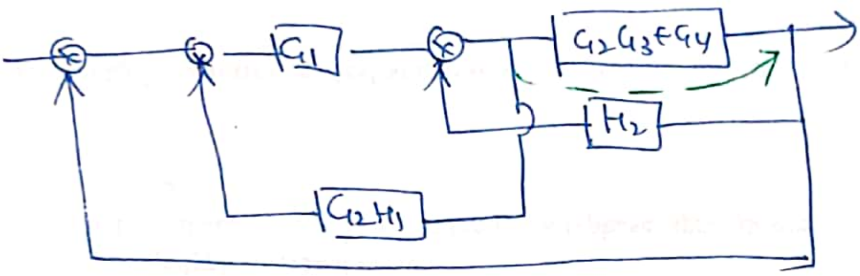
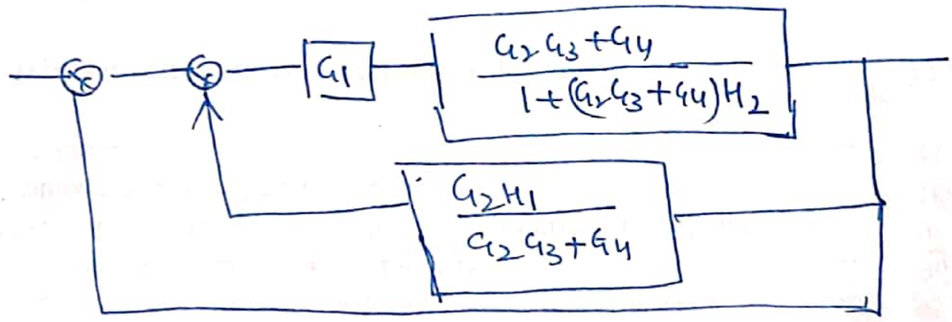
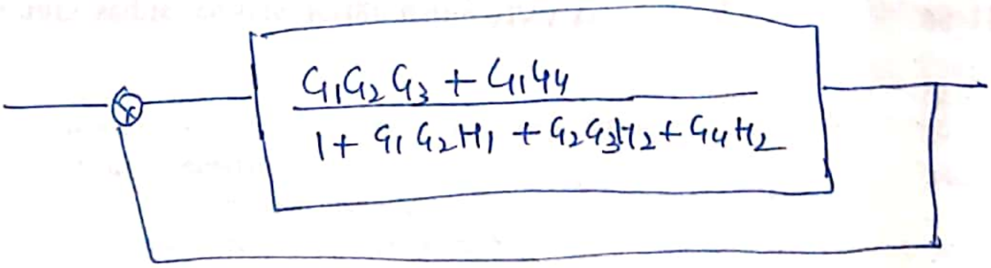
Q1

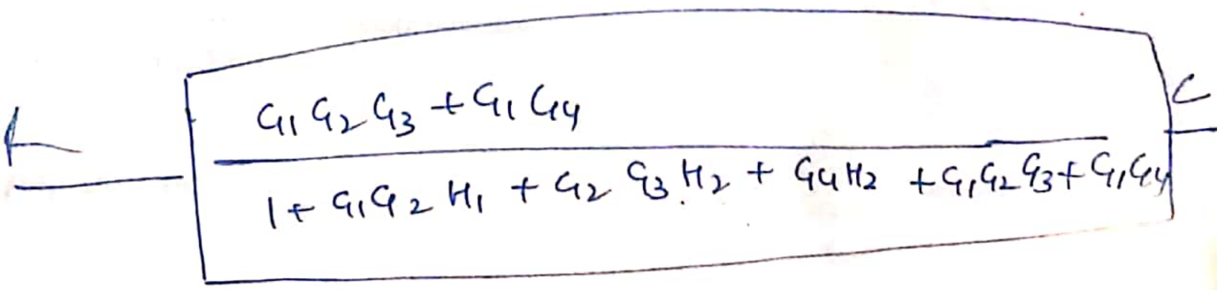












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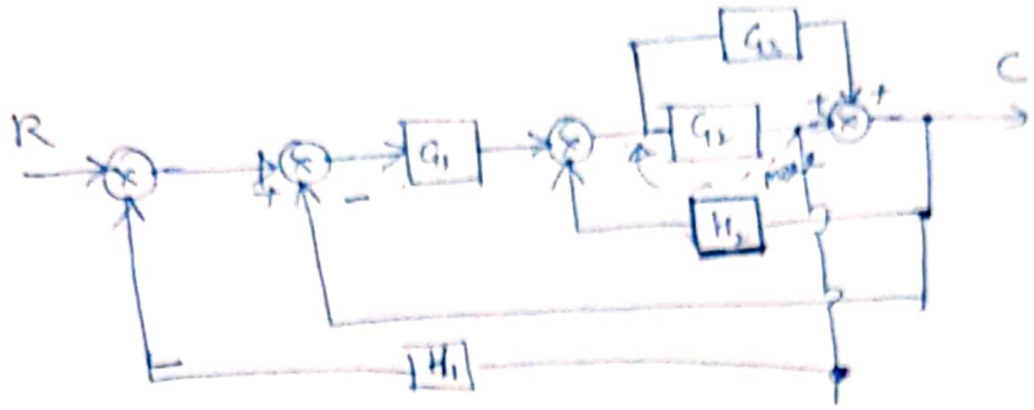
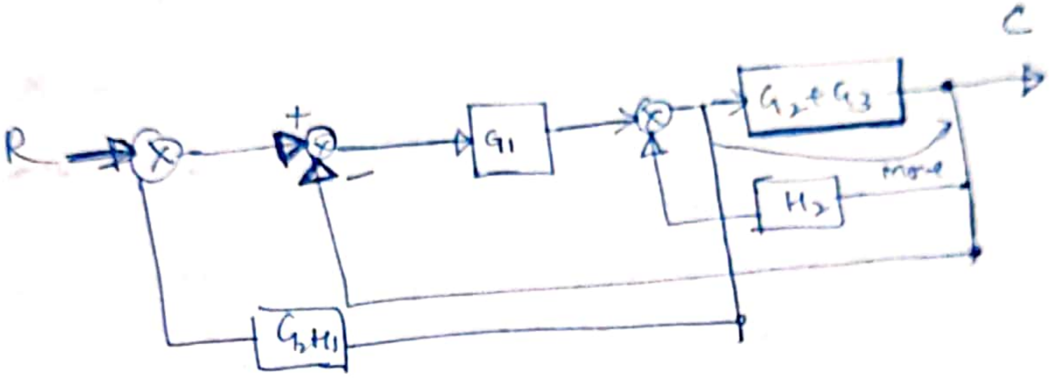
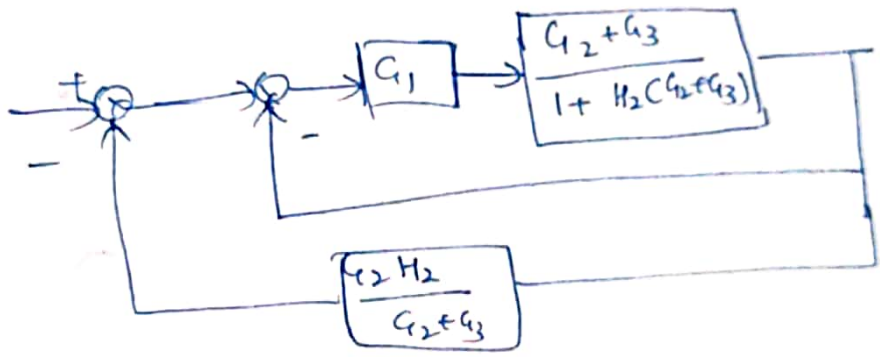
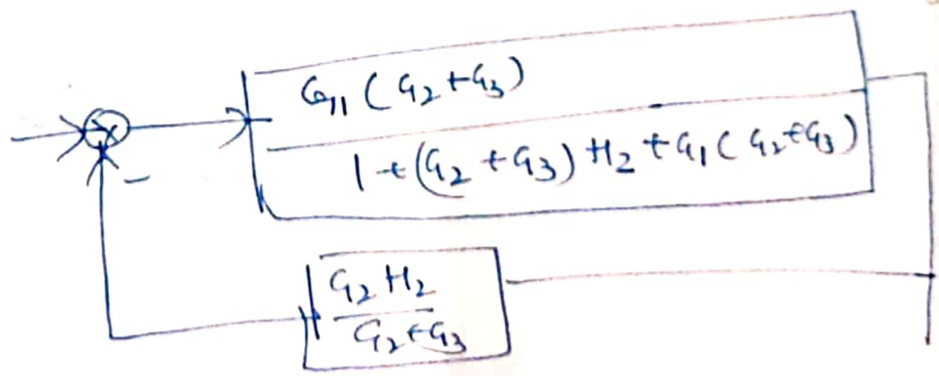
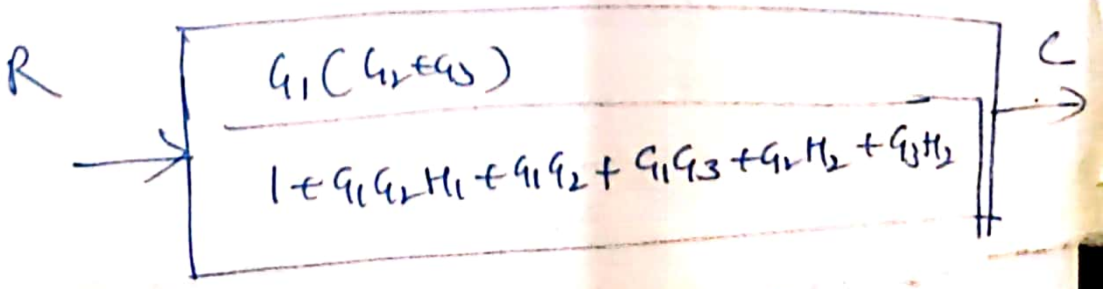
For Block diagram find $\frac{C}{R}$ ratio.











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Signal flow graphs : It is regarded as simplified version of block diagram. It is developed by S.J. Mason. Mason's gain formula is used to find (one step solution) Transfer of complex systems. Various terms used in formulation of SFG.

Non touching loops : Non touching loops are loops which donot posses any common node

Self loop contains single branch

branch → line segment joining two nodes of SFG.

Path gain : Product of branch gains encountered in traversing a path is called the path gain.

*

Properties of SFG :

1 - SFG applies only to linear systems

2. The eqn for which a SFG is drawn must be algebraic eqn in the form of cause - and - effect.

3. Nodes are used to represent variables.

4. Signals travel along branches only in the direction described by the arrows of the branches.

5. The branch directing from node x_k to x_j represents the dependence of x_j upon x_k but not reverse.

6. A signal x_k travelling along a branch between x_k and x_j is multiplied by the gain of the branch a_{kj} so that a signal $a_{kj}\, x_k$ is delivered at x_j.

7. For a given system SFG is not unique.

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Procedure can be followed to convert a BD into SFG

- In a given BD, assume node at I/P, O/P, at every summing point, at every branch point and between cascade blocks.
- Draw the nodes separately as big thick dots and number the dots in the order 1, 2, 3, ------.
- From the BD, find the gain between the nodes in the main forward path and connect all the corresponding nodes by directed straight line segments and mark the gain between the nodes on the segment.
- Draw the forward paths between various nodes and mark the gain between nodes on the directed branches
- Draw the feedback paths between various nodes and mark the gain of FB paths along with sign

SFG terminology

I/P nodes → A node which have only outgoing branches, represented by independent source/variables

O/P nodes → A node which have only incoming branches

Mixed nodes → not belong to either I/P or O/P is called a mixed node.

Path :- A path is traversal of branches connected by nodes in the direction of arrows. If a node is counted more than once, then path is called an open path.

Loop :- If the path ends the starting node and does not encounter any node more than once, it is called a loop

Path gain :- Product of the branch gains encountered in traversing the path.

Loop gain : Product of all branches gains encountered of the branches constituting the loops.

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MASON's Gain Formula

$$\text{Over all } TF = \frac{\sum_{K=1}^{N} P_K \Delta_K}{\Delta}$$

$N \rightarrow$ No of forward path between i/p & o/p

T_K – gain of k^{th} forward path between i/p & o/p

Δ_K – value of Δ for that part of the graph not touching the k^{th} forward path

$$\Delta = 1 - \sum_i L_{i1} + \sum_J L_{j2} - \sum_K L_{k3} + \cdots\cdots$$

L_{mr} = gain of product of the mth ($m = i, j, k, \ldots$) possible combination of r nontouching loops ($1 \le r \le L$)

Δ = 1 – (Sum of the gains of all individual loops) + Sum of products of gains of all possible combinations of two non touching loops) – (Sum of products of gains of all possible combinations of three non touching loops) + – – – .

Application of the gain formula between o/p nodes & non i/p nodes

Let y_{in} be an i/p and y_{out} be an o/p node of a SFG. The gain y_{out}/y_2, where y_2 is not an i/p, may be written as,

$$\frac{y_{out}}{y_2} = \frac{y_{out}/y_{in}}{y_2/y_{in}} = \frac{\sum P_K \Delta_K \big|_{\text{from } y_{in} \text{ to } y_{out}} / \Delta}{\sum P_K \Delta_K \big|_{\text{from } y_{in} \text{ to } y_2} / \Delta}$$

$$\frac{y_{out}}{y_2} = \frac{\sum P_K \Delta_K \big|_{\text{from } y_{in} \text{ to } y_{out}}}{\sum P_K \Delta_K \big|_{\text{from } y_{in} \text{ to } y_2}}$$

$\therefore$ Δ does not appear in the final eqn.

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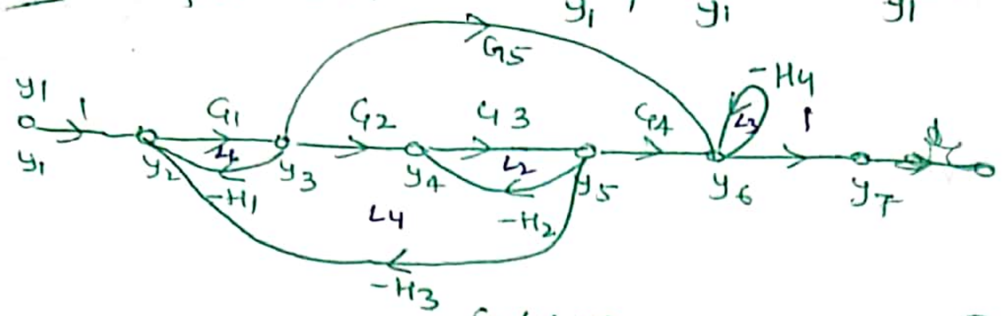
Q from SFG find $\frac{y_2}{y_1}$, $\frac{y_4}{y_1}$ and $\frac{y_6}{y_1}$ or y_7/y_1, $\frac{y_7}{y_2}$



Single loop — Two non touching loops gain products

$$\Delta = 1 + (G_1H_1 + G_3H_2 + H_4 + G_1G_2G_3H_3) + (\underbrace{G_1G_3H_1H_2}_{L12} + \underbrace{G_1H_1H_4}_{L13}$$
$$+ \underbrace{G_3H_2H_4}_{L23} + \underbrace{G_1G_2G_3H_3H_4}_{L34}) + \underbrace{G_1G_3H_1H_2H_4}_{L123})$$ three non Touching loops

$$\Delta = 1 + G_1H_1 + G_3H_2 + H_4 + G_1G_2G_3H_3 + G_1G_3H_1H_2 + G_1H_1H_4 +$$
$$G_3H_2H_4 + G_1G_2G_3H_3H_4 + G_1G_2H_1H_2H_4$$

Now. To find out

$$y_2/y_1 = \frac{1 + G_3H_2 + H_4 + G_3H_2H_4}{\Delta}$$

$T_K\Delta_K = 1\cdot(1 + G_3H_2 + H_4 + G_3H_2H_4)$ Δ_K.

$\{ \therefore \Delta_K = 1 + G_3H_2 + H_4 + G_3H_2H_4$, $G = 1$

$$\frac{y_4}{y_1} = \frac{G_1G_2(1 + H_4)}{\Delta}$$

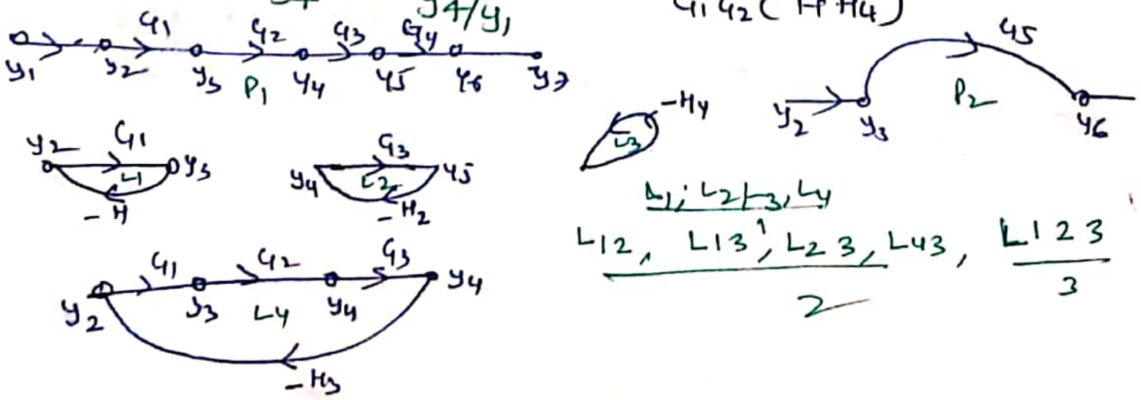
$\{ \therefore \Delta_K = (1 + H_4)$, $P_K = G_1G_2$

$$\frac{y_6}{1} = \frac{y_7}{y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1 + G_3H_2)}{\Delta}$$

For y_7/y_2

$$y_7/y_2 = \frac{y_7/y_1}{y_2/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1 + G_3H_2)}{(1 + G_3H_2 + H_4 + G_3H_2H_4)}$$

$$y_7/y_4 = \frac{y_7/y_1}{y_4/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1 + G_3H_2)}{G_1G_2(1 + H_4)}$$



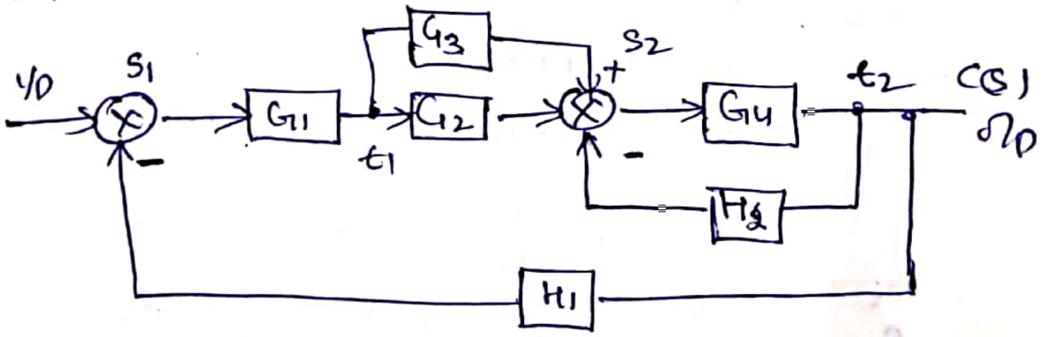
L_1; L_2, L_3, L_4

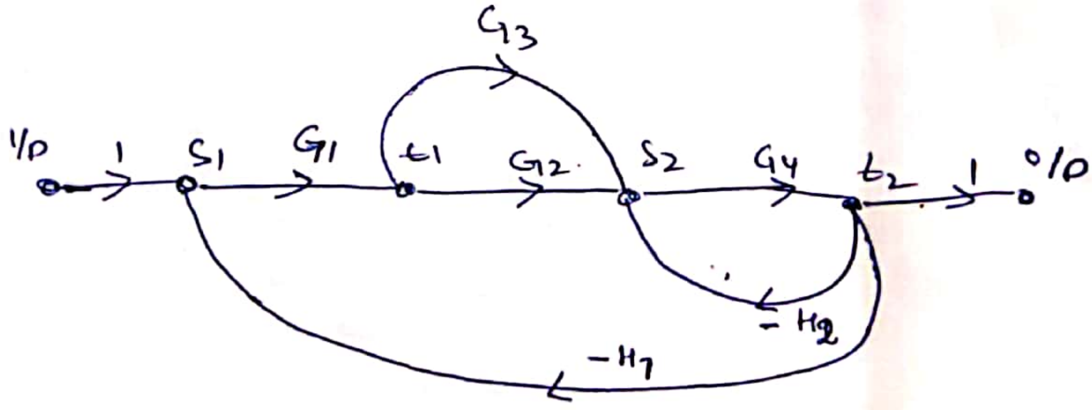
$\underbrace{L12, L13, L23, L43}_{2}$, $\underbrace{L123}_{3}$

From the given blocks:

1. Name all the summing points and take off points in a block diagram.
2. Represent each summing point and take off point by separate node in a signal flow graph
3. Connect them by the branches instead of blocks, indicating block transfer functions as the gains of the corresponding branches
4. Show the input and output nodes separately if required, to complete signal flow graph

Example





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Methods to obtain Signal Flow Graph:

① from the system eqns

① Represent each variable by a separate node

② use the property that value of the variable represented by a node is an algebraic sum of all the signals entering at that node, to simulate the eqns.

③ Coefficients of the variables in the eqns are to be represented as the branch gains, Joining the nodes in signal flow graph.

④ Show the i/p and o/p variables separately to complete signal flow graph.

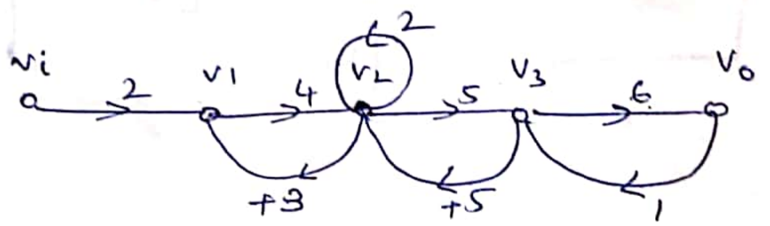
Example.

$$V_1 = 2V_i + 3V_2$$

$$V_2 = 4V_1 + 5V_3 + 2V_2$$

$$V_3 = 5V_2 + V_0$$

$$V_0 = 6V_3$$



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Q A System is described by the following set of linear eqn

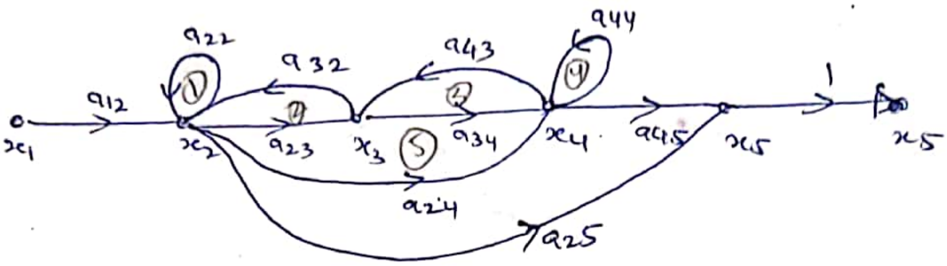
$x_2 = a_{12} x_1 + a_{22} x_2 + a_{32} x_3$ — ①

$x_3 = a_{23} x_2 + a_{43} x_4$ — ②

$x_4 = a_{24} x_2 + a_{34} x_3 + a_{44} x_4$ — ③

$x_5 = a_{25} x_2 + a_{45} x_4$ — ④

Draw SFG find x_5 / x_1 TF



Forward path ①

$x_1 - x_2 - x_3 - x_4 - x_5$

$T_1 = a_{12} a_{23} a_{34} a_{45}$,

$\Delta_1 = 1$

Forward path ②, $x_1 - x_2 - x_4 - x_5$

$T_2 = a_{12} a_{24} a_{45}$,

$\Delta_2 = 1$

Forward path ③ $x_1 - x_2 - x_5$

$T_3 = a_{12} a_{25}$,

$\Delta_3 \; 1 - a_{34} a_{43} - a_{44}$

The loop gains associated with them as follows:

Self loop ① $x_2 - x_2$, $\rightarrow L_1 = a_{22}$

loop ② $x_2 - x_3 - x_2 \rightarrow L_2 = a_{23} a_{32}$

Loop ③ $x_3 - x_4 - x_3 \rightarrow L_3 = a_{34} a_{43}$

Self loop ④ $\rightarrow x_4 - x_4 \rightarrow L_4 = a_{44}$

loop $x_2 - x_4 - x_3 - x_2 \rightarrow L_5 = a_{24} a_{43} a_{32}$

The pairs of two non touching loops and product of gains associated with them are as follows

Loop $L_1 \& L_3$ = $L_{13} = a_{22} a_{34} a_{43}$

$L_1 \& L_4$ = $L_{14} = a_{22} a_{44}$

$L_2 \& L_4$ = $L_{24} = a_{23} a_{32} a_{44}$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_{13} + L_{14} + L_{24})$

$$T = \frac{x_5}{x_1} = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3}{\Delta}$$

$$T = \frac{x_5}{x_1} = \frac{a_{12} a_{23} a_{34} a_{45} + a_{12} a_{24} a_{45} + a_{12} a_{25} (1 - a_{34} a_{43} - a_{44})}{1 - a_{22} - a_{23} a_{32} - a_{34} a_{43} - a_{44} - a_{24} a_{43} a_{32} + a_{22} a_{34} a_{43} + a_{22} a_{44} + a_{23} a_{32} a_{44}}$$

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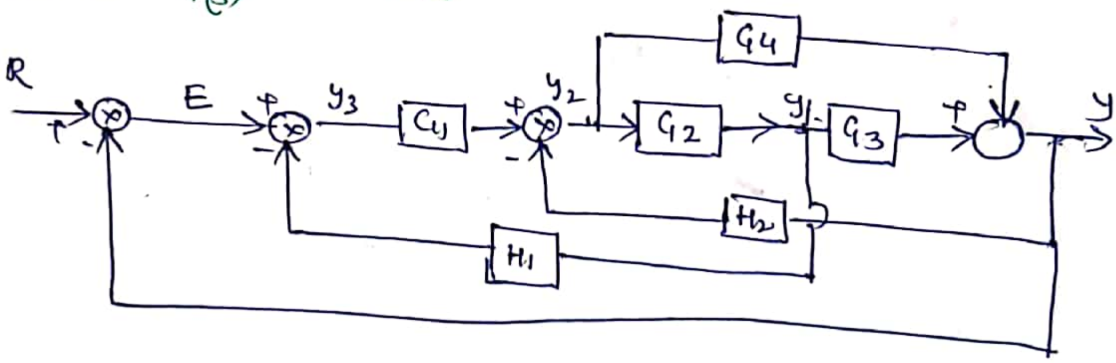
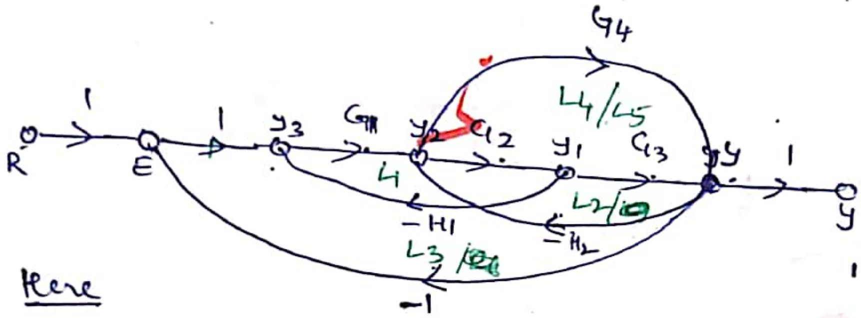
Find $\frac{E(s)}{R(s)}$ and $\frac{Y(s)}{E(s)}$ from fig





$L_1 = y_3 y_2 y_1 y_3$
$L_2 = y_2 y_1 y y_2$
$L_3 = E y_3 y_2 y_1 y E$
$L_4 = E y_3 y_2 y E$
~~$L_5 = E y_3 y_2 y$~~
$L_5 = y_2 y y_2$

Here

$$\Delta = 1 + \underbrace{G_1 G_2 H_1}_{L_1} + \underbrace{G_1 G_4}_{L_4} + \underbrace{G_4 H_2}_{L_5} + \underbrace{G_1 G_2 G_3}_{L_3} + \underbrace{G_2 G_3 H_2}_{L_2}$$

$$\frac{E(s)}{R(s)} = \frac{\sum T_k \Delta_k}{\Delta} = \frac{T_1 \Delta_1}{\Delta}$$

Here $T_1 = 1$,

$\Delta_1 = 1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2$

So,

$$\boxed{\frac{E(s)}{R(s)} = \frac{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2}{\Delta}}$$

We know that

$$\frac{Y(s)}{E(s)} = \frac{Y(s)}{R(s)} \Big/ \frac{E(s)}{R(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{\Delta} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

Here $\Delta_1 = \Delta_2 = 1$

So,

$$\boxed{\frac{Y(s)}{E(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_2}}$$