Dot Conventions:-

The sign of mutually induced voltage depends on direction of winding of the coils. But it is very inconvenient to supply the information about winding direction of the coils. Hence Dot Conventions are used for purpose of indicating direction of winding.

The dot conventions are interpreted as follows:-

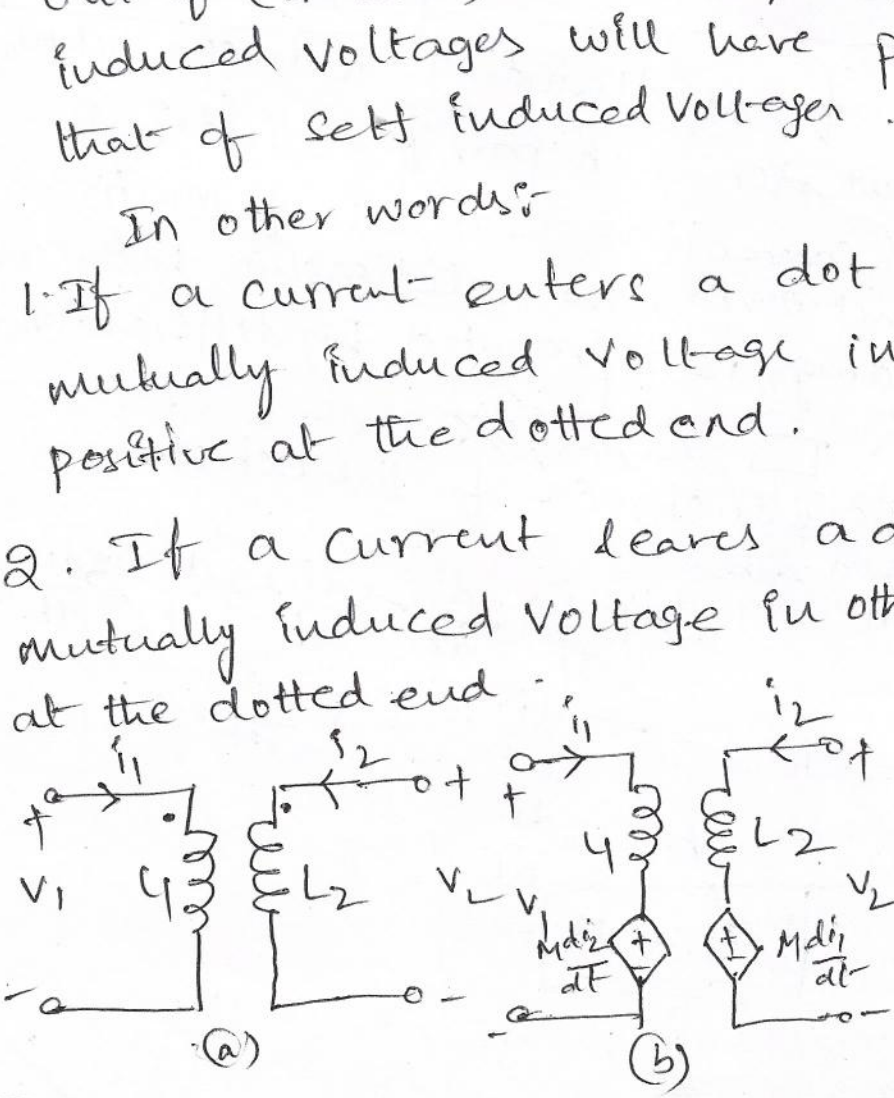
1. If a positive current enters into the dots of both the coils (or) out of dots of both the coils, then mutually induced voltages for both the coils add to the self induced voltages hence mutually induced voltages will have same polarity as that of self induced voltages.

2. If a positive current enter into (or out of) the dot in one coil and in other coil current flows out of (or into) the dot, then the mutually induced voltages will have polarity opposite to that of self induced voltages.

In other words:-

1. If a current enters a dot in one coil, then mutually induced voltage in other coil is positive at the dotted end.

2. If a current leaves a dot in one coil, then mutually induced voltage in other coil is negative at the dotted end.

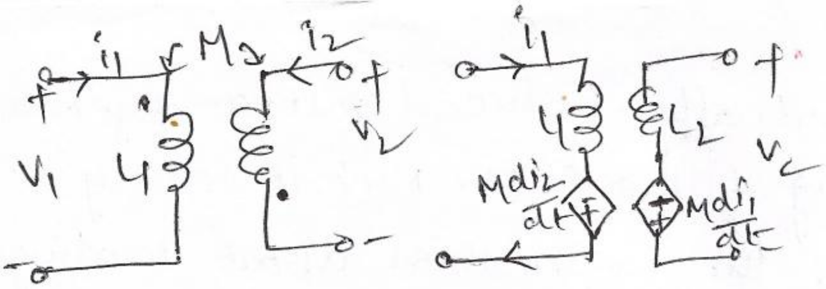


$$V_1 - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$

$$V_2 - L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$

$$\boxed{V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \quad (1)}$$

$$\boxed{V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \quad (2)}$$



(a) (b)

$$V_1 - L_1\frac{di_1}{dt} + M_1\frac{di_2}{dt} = 0$$

$$V_2 - L_2\frac{di_2}{dt} + M_2\frac{di_2}{dt} = 0$$

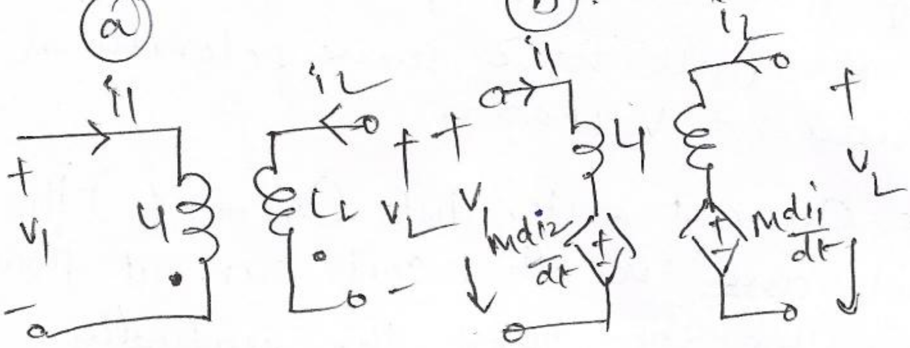
$$\boxed{\begin{aligned} V_1 &= L_1\frac{di_1}{dt} - M_1\frac{di_2}{dt} \\ V_2 &= L_2\frac{di_2}{dt} - M_2\frac{di_2}{dt} \end{aligned}}$$

(a) (b)

both are same

$$V_1 - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$

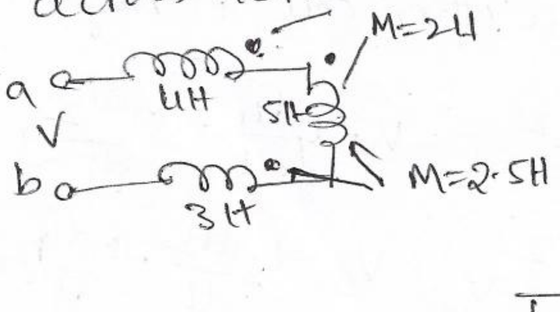
$$V_2 - L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$



$$\boxed{\begin{aligned} V_1 &= L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ V_2 &= L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \end{aligned}}$$

The analysis of multiwinding inductor networks can be carried out for each pair of windings using same dot convention. In case of multi winding inductor networks, the relationship between each pair of windings is represented by different forms of the dots such as □, △, O, ☆ etc.

1. calculate effective inductance of the ckt shown across terminals a & b



$$V - 4\frac{di}{dt} + 2\frac{di}{dt} - 5\frac{di}{dt} + 2\frac{di}{dt} - 2.5\frac{di}{dt} - 3\frac{di}{dt} - 2.5\frac{di}{dt} = 0$$

$$V = 13\frac{di}{dt}$$

$$\boxed{L_{eff} = 13}$$

Inductive coupling in series:-

When two inductors having self inductances L_1 & L_2 are coupled in series, mutual inductance M exists between them. Two kinds of series connection are possible as follows.

Series Aiding:-

In this connection, two coils are connected in series such that their induced fluxes (or) voltages are additive in nature.

Hence currents i_1 & i_2 nothing but current i which is entering dots for both the coils.

Self induced voltage in coil 1 $(V_1) = -L_1\frac{di_1}{dt} = -L_1\frac{di}{dt}$

" " " coil 2 $(V_2) = -L_2\frac{di_2}{dt} = -L_2\frac{di}{dt}$

Mutually induced voltage in coil 1 due to change in current in coil 2 $= V_1' = -M\frac{di}{dt}$

" " " " coil 2 " " " " coil 1 $= V_2' = -M\frac{di}{dt}$.

Total Induced voltage $= V_1 + V_2 + V_1' + V_2'$

$= -\left(L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}\right)$

$= -(L_1 + L_2 + 2M)\frac{di}{dt}$

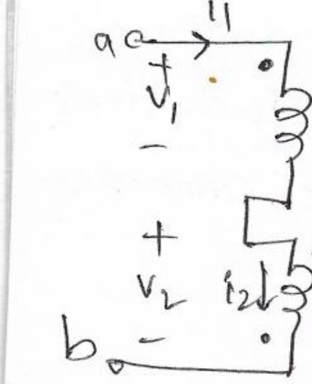
if 'L' is equivalent inductance across terminals a-b then total induced voltage in single inductance would be equal to $-L_{eff}\frac{di}{dt}$.

Comparing two voltages

$$\boxed{L_{eff} = L_1 + L_2 + 2M}$$

the coils are connected in series, & the currents through the two coils are ... the same direction ... in order to

Series opposing :-



In this connection, two coils are connected in such a way that, the induced fluxes (or) voltages are of opposite polarities.

Here i_1 & i_2 is same series current (i) which is entering dot for coil L_1 and leaving dot for coil L_2.

Self induced voltage in coil(1) $= -L_1\frac{di}{dt}$

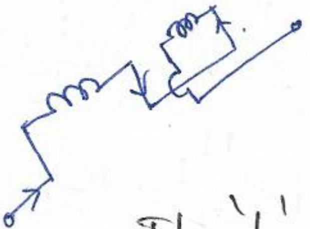
" " " coil(2) $= -L_2\frac{di}{dt}$

Mutually induced voltage in coil 1 due to change in current in coil 2 $= v_1' = +M\frac{di}{dt}$

" " " " coil 2 " " " "

" in coil 2 $= v_2' = +M\frac{di}{dt}$

Total induced voltage $= v_1+v_2+v_1'+v_2'$



$$= -L_1\frac{di}{dt} - L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$= -(L_1+L_2-2M)\frac{di}{dt}$$

If 'L' is equivalent inductance across terminals a & b then total induced voltage in single inductance would be equal to $-L_{eff}\frac{di}{dt}$.

Comparing two voltages,

$$L_{eff} = L_1 + L_2 - 2M.$$

Inductive coupling in parallel :-

When two inductors having self inductances L_1 & L_2 are coupled in parallel, we have two kinds of connection as follows.

parallel Aiding :-

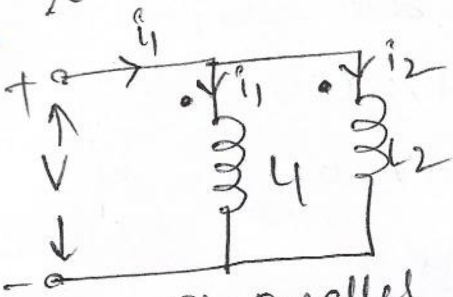


Fig (1) parallel coupling

Applying Kirchhoff's voltage law to both loops, we get

$$-j\omega L_1 i_1 - j\omega M i_2 + V = 0$$

$$-j\omega L_2 i_2 - j\omega M i_1 + V = 0$$

$$V = j\omega L_1 i_1 + j\omega M i_2 \rightarrow (1)$$

$$V = j\omega L_2 i_2 + j\omega M i_1 \rightarrow (2).$$

equating (1) & (2)

$$j\omega L_1 i_1 + j\omega M i_2 = j\omega L_2 i_2 + j\omega M i_1$$

$$i = i_1 + i_2 \Rightarrow i_2 = i - i_1$$

putting value of i_2 in above eqn, we get

$$j\omega L_1 i_1 + j\omega M(i - i_1) = j\omega L_2 (i - i_1) + j\omega M i_1$$

$$j\omega i_1 (L_1 + L_2 - 2M) = j\omega i (L_2 - M)$$

$$i_1 = \left[\frac{L_2 - M}{L_1 + L_2 - 2M}\right] i$$

$$i_2 = \left[\frac{L_1 - M}{L_1 + L_2 - 2M}\right] i$$

putting values of i_1 & i_2 in eqn (1), we get

$$V = j\omega\left[\frac{L_1(L_2 - M)}{L_1 + L_2 - 2M} + \frac{M(L_1 - M)}{L_1 + L_2 - 2M}\right] i$$

$$V = j\omega\left[\frac{L_1L_2 - L_1M + L_1M - M^2}{L_1 + L_2 - 2M}\right] i$$

$$V = j\omega\left[\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right] i \rightarrow (3)$$

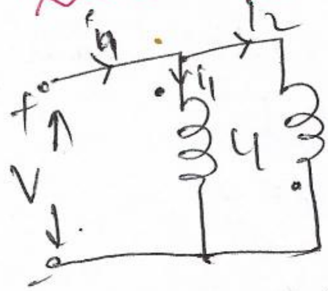
if L is effective inductance of parallel combination, then

$$V = j\omega L_{eff}\, i \rightarrow (4)$$

comparing two eqns (3) & (4)

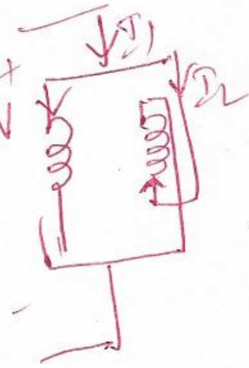
$$\boxed{L_{eff} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}} \rightarrow 5.$$

parallel opposing:



Applying KVL to both sids, we get

$$-j\omega L_1 i_1 + j\omega M i_2 + V = 0$$
$$-j\omega L_2 i_2 + j\omega M i_1 + V = 0$$



$$j\omega L_1 i_1 - j\omega M i_2 = V \rightarrow (1)$$
$$j\omega L_2 i_2 - j\omega M i_1 = V \rightarrow (2)$$

equate (1) & (2)

$$j\omega L_1 i_1 - j\omega M i_2 = j\omega L_2 i_2 - j\omega M i_1$$
$$i = i_1 + i_2$$
$$i_2 = i - i_1$$

Substituting value of i_2 in above eqn we have

$$j\omega L_1 i_1 + j\omega M(i - i_1) = j\omega L_2 (i - i_1) - j\omega M i_1$$
$$j\omega i_1 (L_1 + L_2 + 2M) = j\omega i (L_2 + M)$$
$$i_1 = \left[\frac{L_2 + M}{L_1 + L_2 + 2M}\right] i$$
$$i_2 = \left[\frac{L_1 + M}{L_1 + L_2 + 2M}\right] i$$

putting values of i & i_2 in eqn (1) we get

$$V = j\omega \left[\frac{L_1(L_2 + M)}{L_1 + L_2 + 2M} - \frac{M(L_1 + M)}{L_1 + L_2 + 2M}\right] i$$
$$= j\omega \left[\frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}\right] i$$

if 'L' is effective inductance of parallel combination

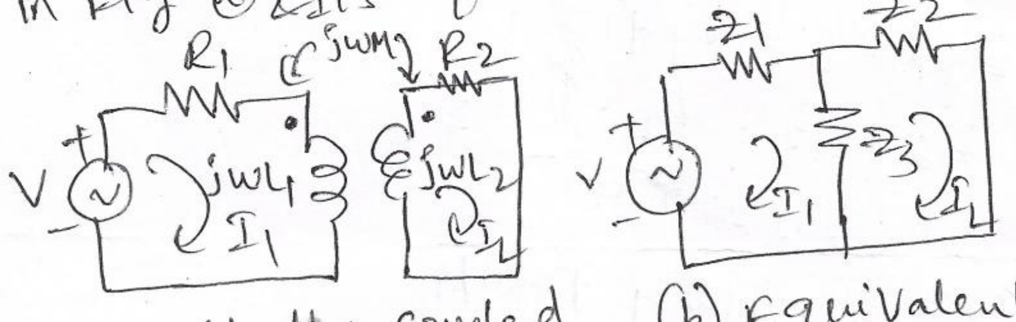
$$V = j\omega L_{eff}\, i$$

$$\boxed{L_{eff} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

Two coils connected in parallel such that fluxes produced by the coils act in the opposite direction. such a connection is known as parallel differential coupling.

Conductively Coupled Equivalent Circuit:

From the point of view of easy circuit analysis, it is desirable to replace magnetically coupled circuit with an equivalent network called conductively coupled circuit. In this circuit no magnetic couplings are involved. The dot convention is also not needed in conductively coupled circuit and can be analyzed by general network simplification techniques such as mesh analysis, nodal analysis etc. Consider magnetically coupled circuit as shown in fig ⓐ & its equivalent T-section fig ⓑ.



ⓐ Magnetically coupled circuit   (b) Equivalent T section.

For ckt ⓐ
Applying KVL to loop 1, we get:

$$-R_1 I_1 - j\omega L_1 I_1 + j\omega M I_2 + V = 0$$

$$V = (R_1 + j\omega L_1) I_1 - (j\omega M) I_2 \rightarrow (1)$$

Applying KVL to loop 2, we get

$$j\omega M I_1 - R_2 I_2 - j\omega L_2 I_2 = 0$$

$$-j\omega M I_1 + (R_2 + j\omega L_2) I_2 = 0 \rightarrow (2)$$

writing eqns in matrix form.

$$\begin{bmatrix} (R_1 + j\omega L_1) & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix} \rightarrow (3)$$

Consid fig (b)

Applying KVL to loop 1, we get

$\cancel{-Z_1 I_1 = Z_3 I_4}$ Using inspect.

$$(Z_1+Z_3)I_1 - Z_3 I_2 = V \rightarrow (4)$$

Apply KVL to loop 2, we get

$$-Z_3 I_1 + (Z_2+Z_3)I_2 = 0 \rightarrow (5)$$

writing equs in matrix form (4) & (5)

$$\begin{bmatrix} Z_1+Z_3 & -Z_3 \\ -Z_3 & Z_2+Z_3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix} \rightarrow (6)$$

comparing (4) & (6).

$$Z_1+Z_3 = R_1 + j\omega L_1$$

$$Z_2+Z_3 = R_2 + j\omega L_2$$

$$Z_3 = j\omega M.$$

$\cancel{Z_1+Z_3 = R_1 + j\omega L_1 + j\omega M}$

$$Z_1 = R_1 + j\omega L_1 - Z_3 = R_1 + j\omega L_1 - j\omega M$$

$$= R_1 + j\omega(L_1 - M)$$

$$Z_2 = R_2 + j\omega L_2 - Z_3 = R_2 + j\omega L_2 - j\omega M$$

$$= R_2 + j\omega(L_2 - M)$$

$$Z_3 = j\omega M$$

The conductively coupled equivalent circuit of magnetically coupled circuit is shown in fig (c).

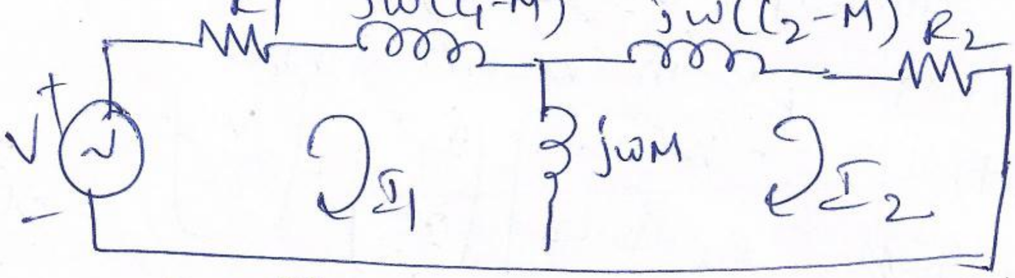
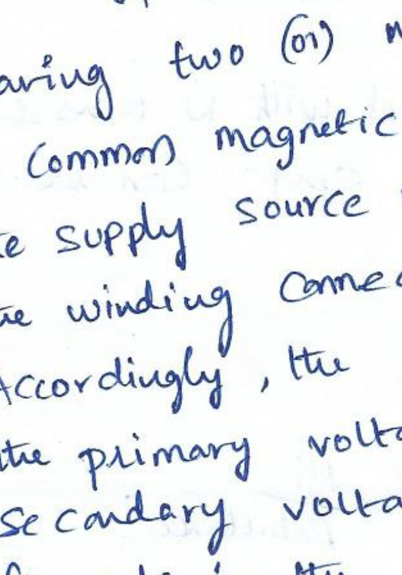


Fig (c).

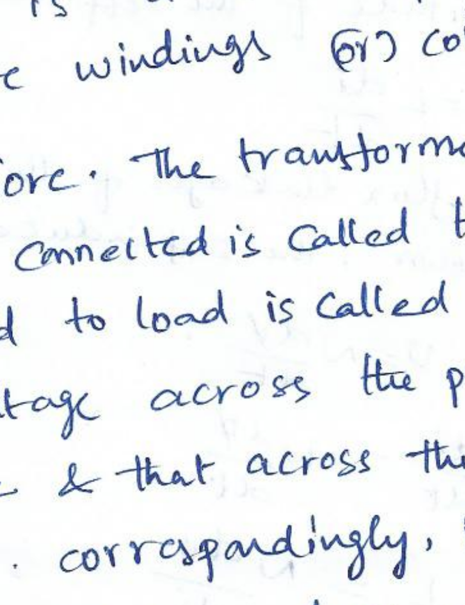
Ideal Transformer:-

Transfer of energy from one ckt to another circuit through mutual induction is widely utilised in power systems. This purpose is served by transformers. Most often, They transform energy at one voltage (or current) into energy at some other voltage (or current).

A transformer is a static piece of apparatus, having two (or) more windings (or) coils arranged on a common magnetic core. The transformer winding to which the supply source is connected is called the primary, while the winding connected to load is called the secondary. Accordingly, the voltage across the primary is called the primary voltage & that across the secondary, the secondary voltage. correspondingly, i_1 & i_2 are the currents in the primary & secondary windings.



fig(1)(a)



fig(b)

The vertical lines between the coils represent the iron core; the currents are assumed such that the mutual inductance is positive.

An ideal transformer characterized by assuming

(i) Zero power dissipation in the primary & secondary windings, i.e, resistances in the coils are assumed to be zero.

(ii) The self inductances of the primary & secondary are extremely large in camparision with the load impedance

(iii) The coefficient of coupling is equal to unity, i.e, the ~~coils are~~ coils are tightly coupled without having any leakage flux. If the flux produced by the current flowing in a coil links all the turns, the self inductance of either the primary (or) secondary coil is proportional to the square of the number of turns of the coil.

The magnitude of the self induced emf is given by

$$v = L\frac{di}{dt}.$$

If the flux linkages of the coil with N turns & current are known, the self induced emf can be expressed as

$$v = N\frac{d\phi}{dt}$$

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$L = N\frac{d\phi}{di} \qquad \because \phi = \frac{Ni}{\text{Reluctance}}.$$

$$L = N\frac{d}{di}\left[\frac{Ni}{\text{reluctance}}\right]$$

$$L = \frac{N^2}{\text{reluctance}}$$

$$L \propto N^2$$

from the above relation, it follows that

$$\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = a^2$$

where $a = N_2/N_1$ is called the turns ratio of the transformer. The turns ratio a can also be expressed in terms of the primary & secondary voltages. If the magnetic permeability of the core is infinitely large then the flux would be confined to the core. If ϕ is the flux through a single turn coil on the core & N_1, N_2 are the no of turns of the primary & secondary respectively. then the total flux through windings 1 & 2 respectively are

$\phi_1 = N_1 \phi$; $\phi_2 = N_2 \phi$

Also, we have $V_1 = \frac{d\phi_1}{dt}$ & $V_2 = \frac{d\phi_2}{dt}$.

$$\frac{V_2}{V_1} = \frac{N_2 \frac{d\phi_1}{dt}}{N_1 \frac{d\phi_1}{dt}} = \frac{N_2}{N_1}$$

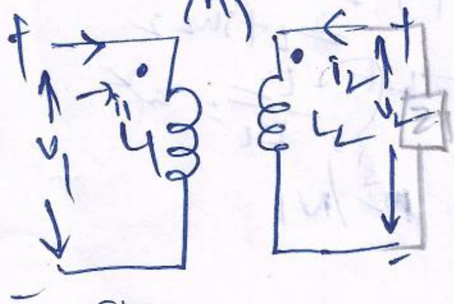


fig 2

Fig (2) shows an ideal transformer to which the secondary is connected to a load impedance Z_L. The turns ratio $\frac{N_2}{N_1} = a$.

The ideal transformer is very useful model to circuit calculations, because with few additional elements like R, L & C, the actual behaviour of the physical transformer can be accurately represented.

When the excitations are sinusoidal voltages (or) currents, the steady state response will also be sinusoidal. We can use phasors for representing these voltages & currents. The input impedance of the transformer can be determined by writing mesh equations for the ckt shown in fig (2).

$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \rightarrow (1)$$

$$0 = -j\omega M I_1 + (Z_L + j\omega L_2) I_2 \rightarrow (2)$$

where V_1, V_2 are the voltage phasors & I_1, I_2 are the current phasors in the two windings. $j\omega L_1$ & $j\omega L_2$ are the impedances of the self inductances & $j\omega M$ is the impedance of the mutual inductance, ω is the angular frequency.

$$I_2 = \frac{j\omega M I_1}{Z_L + j\omega L_2} \rightarrow (3)$$

Substitute eqn (3) in eqn (1)

$$V_1 = I_1 j\omega L_1 + \frac{I_1 \omega^2 M^2}{Z_L + j\omega L_2}$$

The input impedance $Z_{in} = \frac{V_1}{I} = j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2}$

When the coefficient of coupling is assumed to be equal to unity

$M = \sqrt{L_1 L_2}$ $\therefore Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{Z_L + j\omega L_2}$

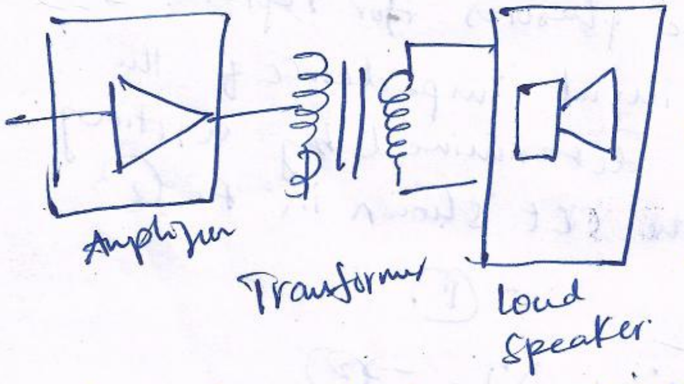
we have already established that $\frac{L_2}{L_1} = a^2$

where a is the turns ratio N_2/N_1

$$Z_{in} = j\omega L_1 + \frac{\omega^2 L_1^2 a^2}{(Z_L + j\omega L_2)}$$

$$Z_{in} = \frac{(Z_L + j\omega L_2) j\omega L_1 + \omega^2 L_1^2 a^2}{Z_L + j\omega L_2} = \frac{j\omega L_1 Z_L}{Z_L + j\omega L_2}$$

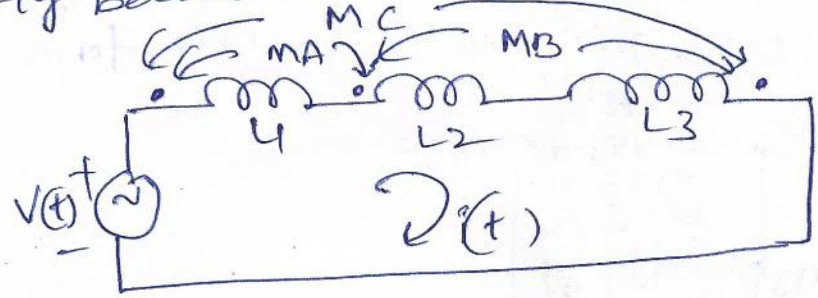
As L_2 is assumed to be infinitely large compared to Z_L, $Z_{in} = \frac{j\omega L_1 Z_L}{j\omega a^2 L_1} = \frac{Z_L}{a^2} = \left(\frac{N_1}{N_2}\right)^2 Z_L$.



The ideal transformers change the impedance of a load and can be used to match circuits with different impedance in order to achieve maximum power transfer.

For example, the input impedance of a loudspeaker is usually very small say 3 Ω to 12 Ω for connecting directly to amplifier. The transformer with proper turns ratio can be placed between the o/p of the amplifier & the input of the loud speaker to match the impedance problems

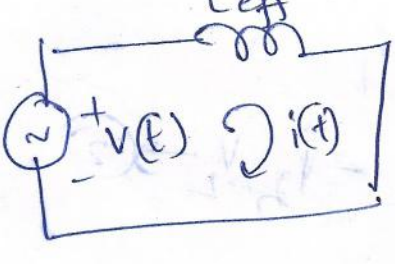
1. write voltage equation for the network shown in fig below and determine the effective inductance.



$$-L_1\frac{di(t)}{dt} - M_A\frac{di(t)}{dt} + M_C\frac{di(t)}{dt} - L_2\frac{di(t)}{dt} - M_A\frac{di(t)}{dt} + M_B\frac{di(t)}{dt}$$

$$-L_3\frac{di(t)}{dt} + M_B\frac{di(t)}{dt} + M_C\frac{di(t)}{dt} + v(t) = 0$$

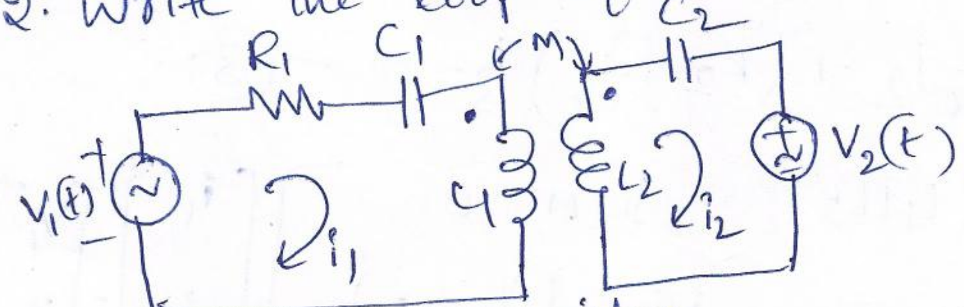
$$v(t) = \left[L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C\right]\frac{di(t)}{dt}$$

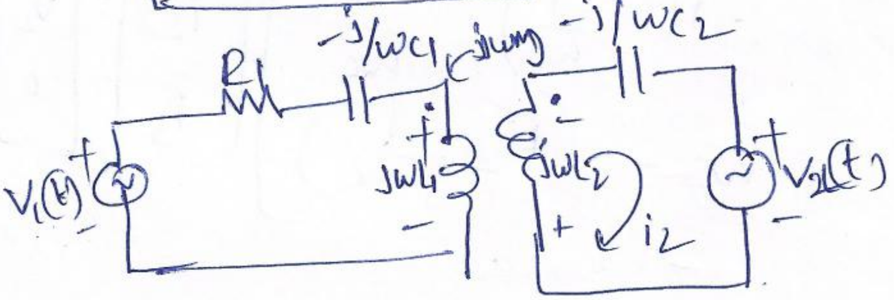


$$v(t) = L_{eff}\frac{di(t)}{dt}$$

$$L_{eff} = L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C \ H$$

2. write the loop equations for the network below





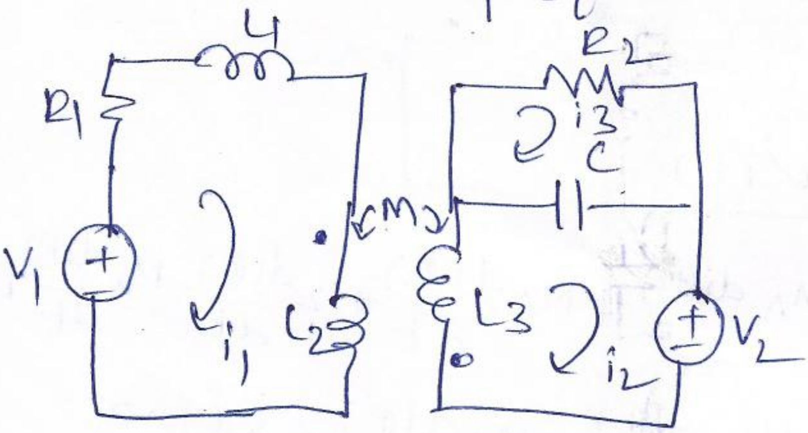
$$-R_1 i_1 - \left(\frac{-j}{\omega C_1}\right) i_1 - j\omega L_1 i_1 + j\omega M i_2 + v_1(t) = 0$$

$$v_1(t) = \left(R_1 + j\omega L_1 - \frac{j}{\omega C_1}\right) i_1 - j\omega M i_2 \rightarrow (1)$$

$$-\left(-\frac{j}{\omega C_2}\right) - v_2(t) - j\omega L_2 + j\omega M i_1 = 0$$

$$V_2(t) = j\omega M i_1 - \left(j\omega L_2 + \frac{j}{\omega C_2}\right) i_2 \rightarrow (2)$$

(3). For the coupled network shown in below fig, write down the loop equations in matrix form.



$$-R_1 i_1 - j\omega L_1 i_1 - j\omega L_2 i_1 - j\omega M i_2 + V_1 = 0$$

$$R_1 + j\omega(L_1+L_2) i_1 + j\omega M i_2 = V_1 \rightarrow (1)$$

$$-\left(-\frac{j}{\omega C}\right)(i_2 - i_3) - V_2 - j\omega L_3 i_2 - j\omega M i_1 = 0$$

$$-j\omega M i_1 \mp j\left(\omega L - \frac{1}{\omega C}\right) i_2 - \frac{j}{\omega C} i_3 = V_2 \rightarrow (2)$$

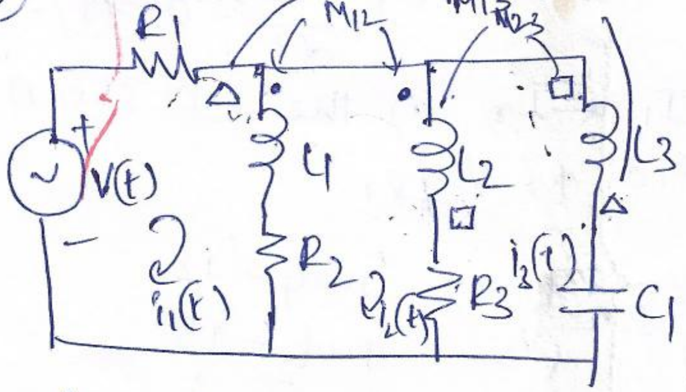
$$-R_2 i_3 + \left(\frac{-j}{\omega C}\right)(i_3 - i_2) = 0$$

$$+\frac{j}{\omega C} i_2 + \left(R_2 - \frac{j}{\omega C}\right) i_3 = 0 \rightarrow (3)$$

$$\begin{bmatrix} R_1 + j\omega(L_1+L_2) & j\omega M & 0 \\ -j\omega M & -j\left(\omega L - \frac{1}{\omega C}\right) & \frac{-j}{\omega C} \\ 0 & \frac{j}{\omega C} & \left(R_2 - \frac{j}{\omega C}\right) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix}$$

(4) obtain equivalent 'T' for magnetically coupled circuit as shown in fig below. explained in pg No:

(5). write the loop equations for the coupled CKt below



loop 1 :-

$$-R_1 i_1(t) - L_1 \frac{d(i_1(t)-i_2(t))}{dt} - M_{12}\frac{d(i_2(t)-i_3(t))}{dt} + M_{13}\frac{di_3(t)}{dt} - R_2(i_1(t)-i_2(t)) + V(t) = 0$$

$$(R_1+R_2)\, i_1(t) + L\frac{di_1(t)}{dt} - (L_1 - M_{12})\frac{di_2(t)}{dt}$$
$$- R_2 i_2(t) - M_{13}\frac{di_3(t)}{dt} = V(t) \rightarrow (1)$$

Applying KVL to loop 2:-

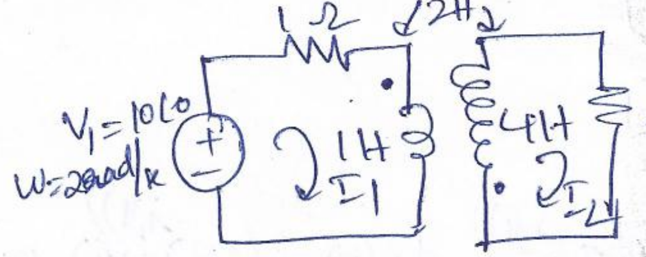
$$-L_2 \frac{d\, i_2(t)-i_3(t)}{dt} - R_3[i_2(t)-i_3(t)] - R_2(i_2(t)-i_1(t))$$
$$-L_1\frac{d(i_2(t)-i_1(t))}{dt} + M_{12}\frac{d}{dt}[i_2(t)-i_3(t)] + M_{23}\frac{di_3(t)}{dt}$$
$$+M_{12}\frac{d}{dt}(i_2(t)-i_3(t)) - M_{13}\frac{di_3(t)}{dt} = 0$$

$$R_2 i_1(t) - (R_2+R_3)i_2(t) + R_3 i_3(t) + [L_1 - M_{12}]\frac{di_1(t)}{dt}$$
$$-(L_2+L_1-2M_{12})\frac{di_2(t)}{dt} + [L_2+M_{23}-M_{12}-M_{13}]\frac{di_3(t)}{dt} = 0 \rightarrow (2)$$

$$-L_3\frac{di_3(t)}{dt} - \frac{1}{C_1}\int i_3(t)dt - R_3[i_3(t)-i_2(t)] - L_2\frac{d}{dt}(i_3(t)-i_2(t))$$
$$-M_{23}\frac{d}{dt}[i_3(t)-i_2(t)] + M_{13}\frac{d}{dt}(i_1(t)-i_2(t)) - M_{23}\frac{d}{dt}i_3(t)$$
$$+M_{12}\frac{d}{dt}(i_1(t)-i_2(t)) = 0$$

$$(M_{12}+M_{13})\frac{di_1(t)}{dt} + R_3 i_2(t) + [L_2 + M_{23} - M_{13} - M_{12}]\frac{di_2(t)}{dt}$$
$$-\frac{1}{C}\int i_3(t)\,dt - [L_2 + L_3 + 2M_{23}]\frac{di_3(t)}{dt} = 0 \rightarrow (3).$$

(7) solve for currents I_1 & I_2 in the ckt shown below. Also find ratio of V_2/V_1



$j\omega L_1 = j2$
$j\omega L_2 = j8$
$j\omega M = j4\ \Omega$

Applying KVL to loop 1:-

$$-1I_1 - 1\frac{dI_1}{dt} - 2\frac{dI_2}{dt} + 10\angle 0 = 0$$

$$I_1 + \frac{dI_1}{dt} + 2\frac{dI_2}{dt} = 10\angle 0 \rightarrow (1)$$

$$-I_1 - 2jI_1 - j4I_2 + 10\angle 0 = 0$$

$$I_1(1+2j) + j4I_2 = 10\angle 0 \rightarrow (1)$$

Applying KVL to loop 2

$$-2I_2 - 8jI_2 - 4jI_1 = 0$$

$$j4I_1 + (2+8j)I_2 = 0 \rightarrow (2)$$

Solving eqn (1) & (2).

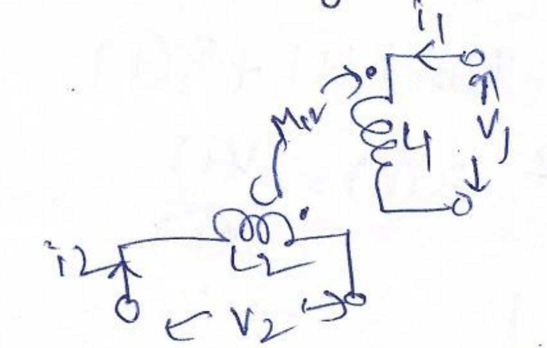
$$I_2 = 1.6828\angle -75.38^\circ A$$

$$I_1 = 1.7345\angle -89.41^\circ A.$$

Voltage ratio $\frac{V_2}{V_1} = \frac{2I_2}{V_1} = \frac{2\times 1.6828\angle -75.38}{10\angle 0}$

$$= 0.3366\angle -75.38$$

(8). Write the voltage equation for the following network showing coupled coil equivalent circuit



Applying KVL to loop1

$$-L_1\frac{di_1}{dt}+M_{12}\frac{di_2}{dt}+V_1=0$$

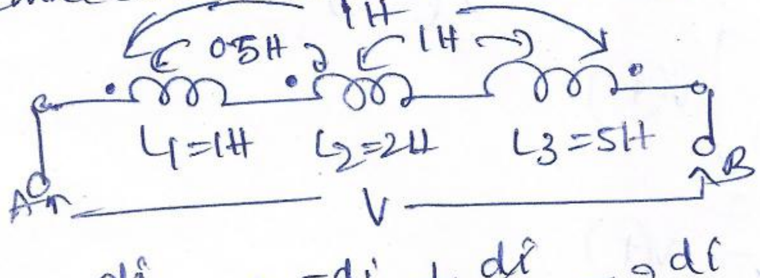
$$V_1=L_1\frac{di_1}{dt}-M_{12}\frac{di_2}{dt} \rightarrow (1)$$

Applying KVL to loop 2

$$-L_2\frac{di_2}{dt}+M_{12}\frac{di_1}{dt}+V_2=0$$

$$V_2=-M_{12}\frac{di_1}{dt}+L_2\frac{di_2}{dt} \rightarrow (2).$$

(9) Find the total inductance of three series connected coupled coils shown in below figure.



$$-\cancel{\frac{di}{dt}}-0.5\frac{di}{dt}+\cancel{\frac{di}{dt}}-2\frac{di}{dt}-0.5\frac{di}{dt}+\frac{di}{dt}$$

$$-5\frac{di}{dt}+1\frac{di}{dt}+\frac{di}{dt}+V=0$$

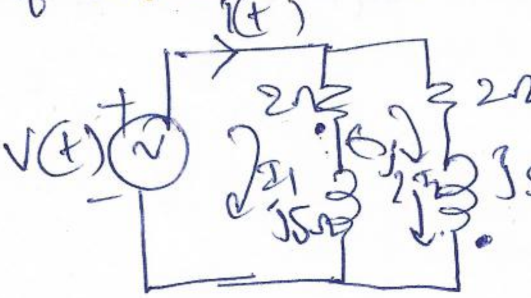
$$-5\frac{di}{dt}+V=0$$

$$V=5\frac{di}{dt}$$

$$V=L_{eff}\frac{dI}{dt}$$

$$L_{eff}=5H$$

(10). Determine the equivalent input impedance Z_{AB} of the network shown in figure



$I(t) = I_1(t) + I_2(t)$

$Z(t) = \frac{V(t)}{I(t)}$

Apply KVL to loop 1

$-2(I_1 - I_2) - j5(I_1 - i_2) + j2I_2 + V(t) = 0$

$(2+j5)I_1 - (2-j3)I_2 = V(t) \rightarrow (1)$

Apply KVL to loop 2

$-2I_2 - j5I_2 + j2(I_1) - j5(I_2 - I_1) - j5I_2 - 2(I_2 - I_1) = 0$

$(2+j7)I_1 + (-4-17j)I_2 = 0 \rightarrow (2)$

$$\begin{bmatrix} (2+j5) & -(2-j3) \\ (2+j7) & -(4+17j) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V(t) \\ 0 \end{bmatrix}$$

$I_1 = \left(\frac{2+j7}{-17+j20}\right) V(t)$

$I_2 = \frac{2+j7}{-17+j20} V(t)$

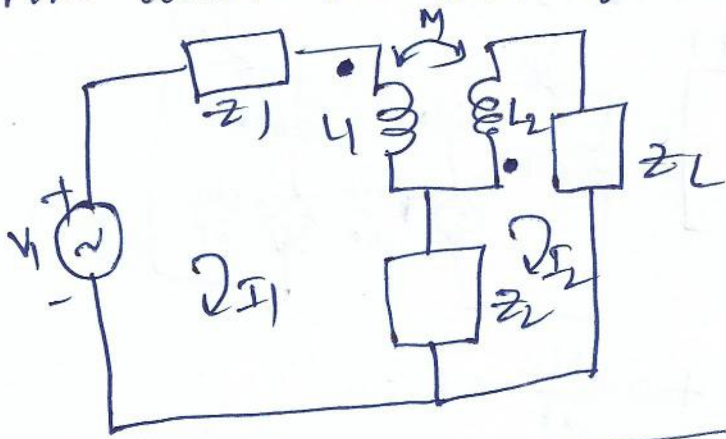
$I(t) = \frac{4+j14}{-17+j20} V(t)$

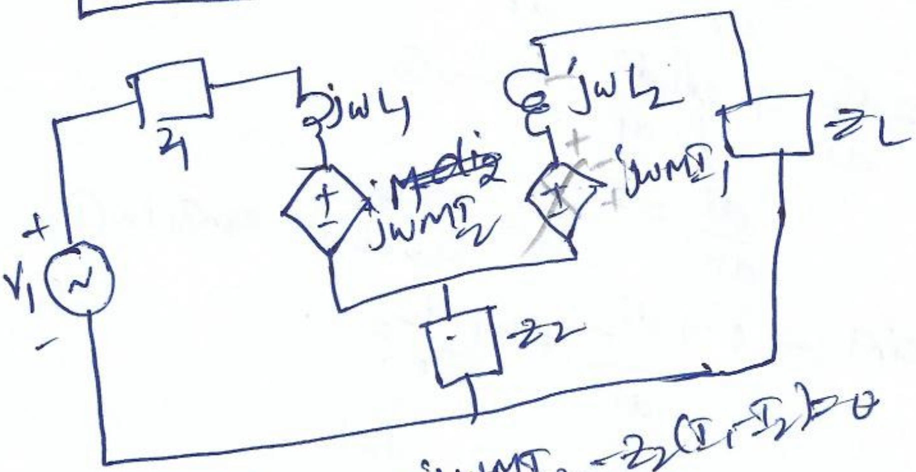
$Z_{AB} = \frac{V(t)}{I(t)} = \frac{-17+j20}{4+j14} = 1.8027\angle 56.31^\circ \Omega$

$= 0.9999 + j1.4999 \Omega$

$Z_{AB} = (1+j1.5)\Omega$

Write down the mesh equations for the N/W shown below.





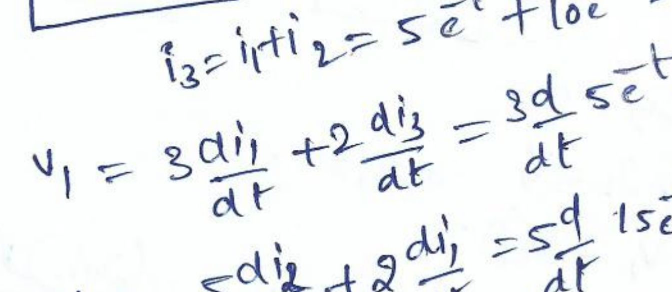
$V_1 - Z_1 I_1 - j\omega L_1 I_1 - j\omega M I_2 - Z_2(I_1 - I_2) = 0$

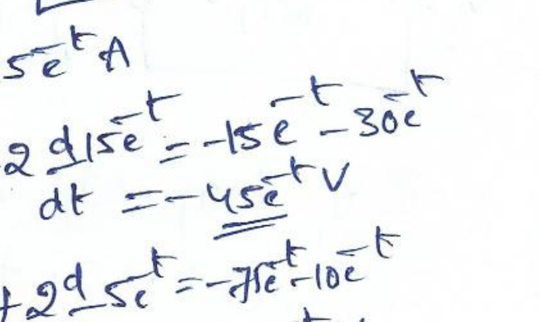
$(Z_1 + j\omega L_1 + Z_2) I_1 - (Z_2 - j\omega M) I_2 = V_1$ — (I)

$-Z_2(I_2 - I_1) + j\omega M I_1 - j\omega L_2 I_2 - Z_L I_2 = 0$

$-(Z_2 - j\omega M) I_1 + (Z_2 + j\omega L_2 + Z_L) I_2 = 0$ — (II)

Find the voltages V_1 & V_2 in the network below.



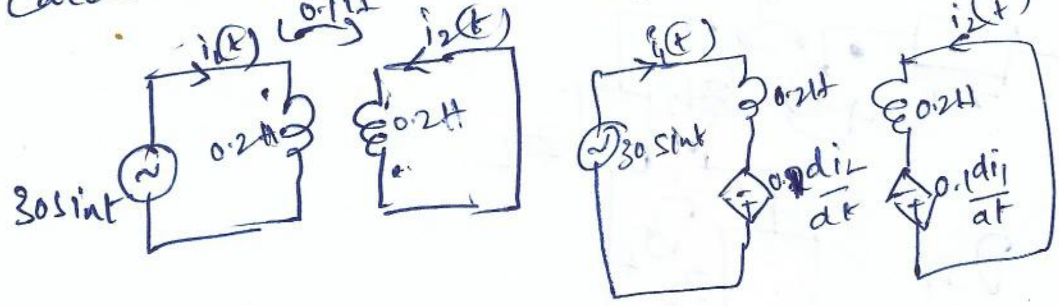


$i_3 = i_1 + i_2 = 5e^{-t} + 10e^{-t} = 15e^{-t}$ A

$V_1 = 3\frac{di_1}{dt} + 2\frac{di_3}{dt} = 3\frac{d}{dt}5e^{-t} + 2\frac{d}{dt}15e^{-t} = -15e^{-t} - 30e^{-t} = \underline{\underline{-45e^{-t}\ V}}$

$V_2 = 5\frac{di_2}{dt} + 2\frac{di_1}{dt} = 5\frac{d}{dt}15e^{-t} + 2\frac{d}{dt}5e^{-t} = -75e^{-t} - 10e^{-t} = \underline{\underline{-85e^{-t}\ V}}$

Calculate the current $i_L(t)$



$$30\sin t - 0.2\frac{di_1}{dt} + 0.1\frac{di_2}{dt} = 0 \rightarrow (1)$$

$$-0.2\frac{di_2}{dt} + 0.1\frac{di'}{dt} = 0 \rightarrow (2)$$

$$\frac{di'}{dt} = 2\frac{di_2}{dt} \rightarrow (3)$$ substitute eqn(3) in (1)

$$30\sin t - 0.4\frac{di_2}{dt} + 0.1\frac{di_2}{dt} = 0.$$

$$0.3\frac{di_2}{dt} = 30\sin t$$

$$\frac{di_2}{dt} = 100\sin t$$

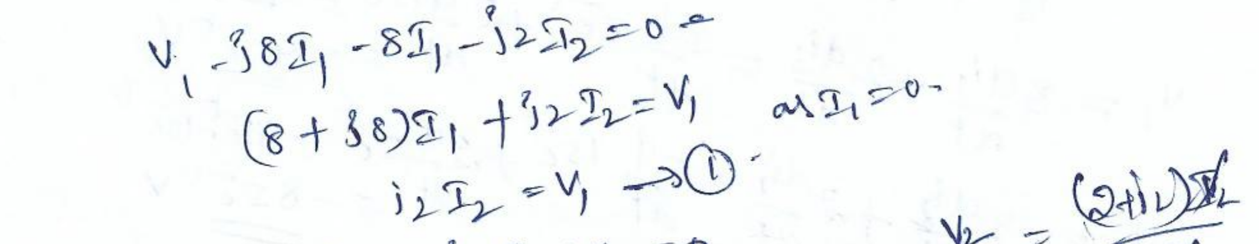
$$di_2 = 100\sin t\,dt$$

$$i_2(t) = 100\int_0^t \sin t\,dt.$$

$$= 100\left[-\cos t\right]_0^t$$

$$= 100(1-\cos t)$$

Determine the ratio $\frac{V_2}{V_1}$ in the circuit if $I_1 = 0$.



$$V_1 - j8I_1 - 8I_1 - j2I_2 = 0$$

$$(8+j8)I_1 + j2I_2 = V_1 \quad \text{as } I_1 = 0.$$

$$j2I_2 = V_1 \rightarrow (1).$$

$$-2I_2 - j2I_2 - j2I_1 + V_2 = 0$$

$$-j2I_1 + (2+j2)I_2 = V_2$$

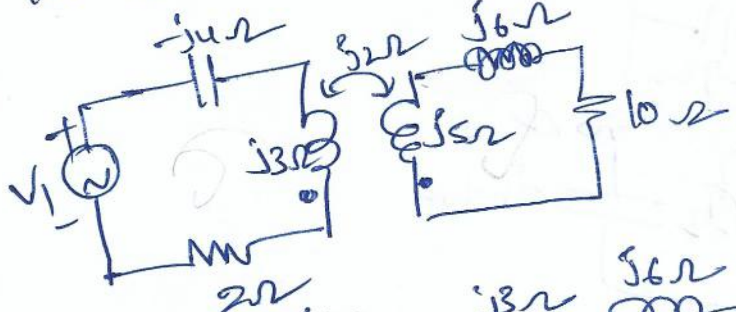
$$(2+j2)I_2 = V_2$$

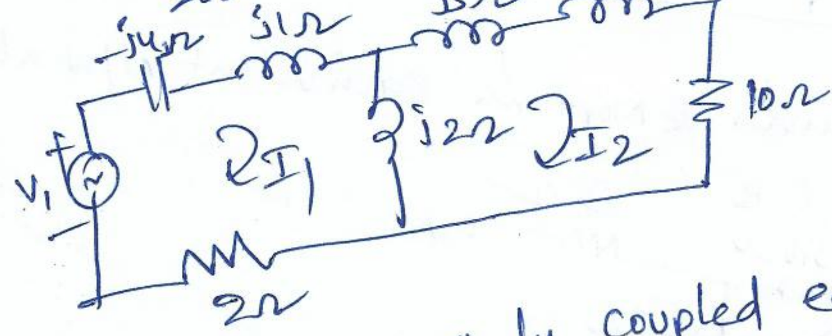
$$\frac{V_2}{V_1} = \frac{(2+j2)\cancel{I_2}}{j2\cancel{I_2}}$$

$$= \frac{2\sqrt{2}\angle 45}{2\angle 90^\circ}$$

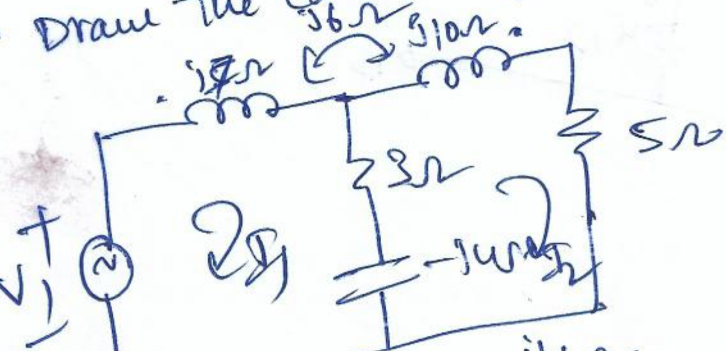
$$= 1.41\angle -45\,V$$

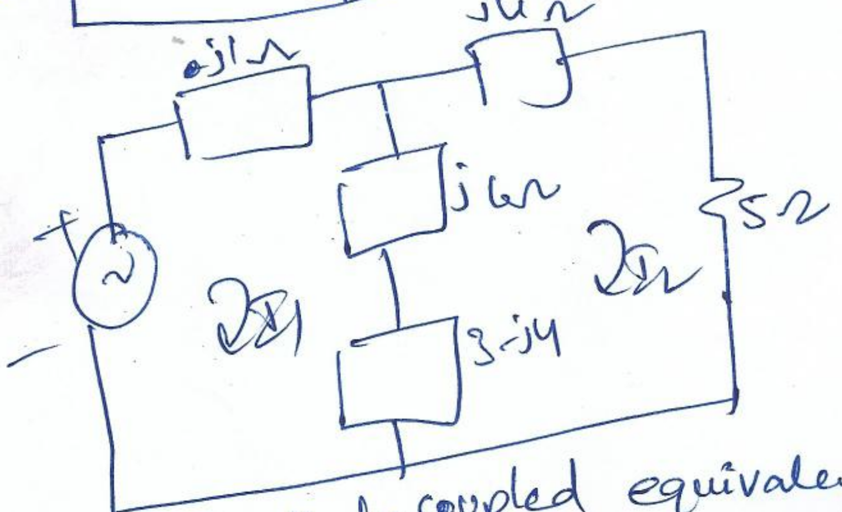
1. Find the conductively coupled equivalent circuit for the network shown below.



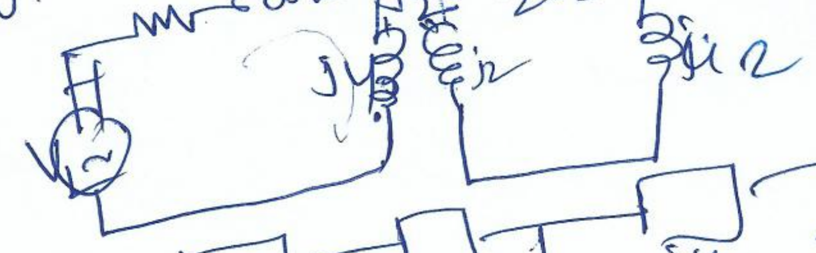


2. Draw the conductively coupled equivalent

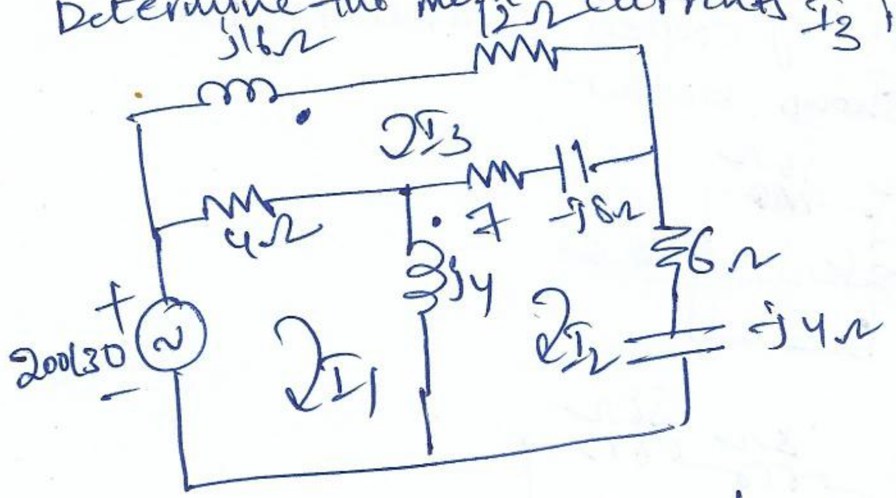




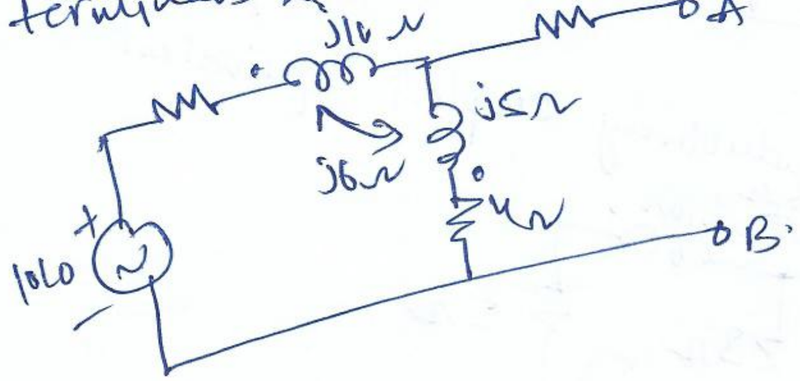
3. Find the conductively coupled equivalent Ckt for the N/W.



Determine the mesh currents I_3 in the N/W.



Find Thevenin's & Norton's equivalent N/W at terminals A & B.



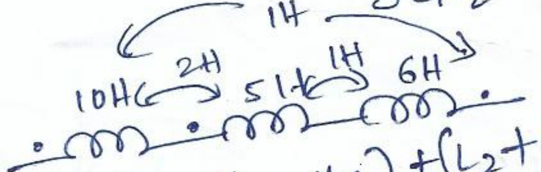
Subtract ① & ②

$4M = 0.5$

$M = 0.125\,H$

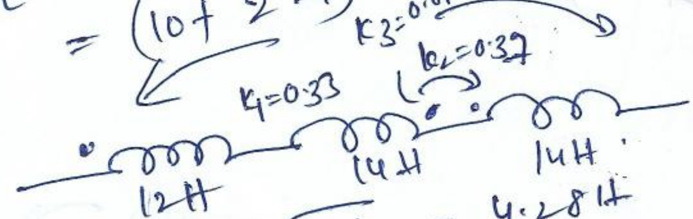
$$k = \frac{M}{\sqrt{L_1L_2}} = \frac{0.125}{\sqrt{0.2\times0.15}} = 0.72.$$

(1)



$$L = (L_1 + M_{12} - M_{13}) + (L_2 + M_{21} - M_{23}) + (L_3 - L_{31} - L_{32})$$
$$= (10 + 2 - 1) + (5 + 2 - 1) + (6 - 1 - 1) = 21H.$$

(2)



$M_{12} = M_{21} = K_1\sqrt{L_1L_2} = 4.28H$

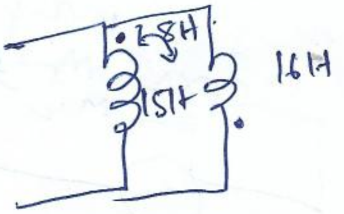
$M_{23} = M_{32} = k_2\sqrt{L_2L_3} = 5.18H$

$M_{31} = M_{13} = K_3\sqrt{L_3L_1} = 8.42H.$

$L = (L_1 - M_{12} + M_{13}) + (L_2 - M_{21} + M_{23}) + (L_3 + M_{31} - L_{M2})$

$= 37.92H.$

(3)

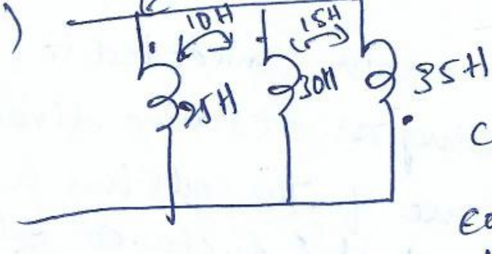


$L_A = L_1 - M_{12} = 7H$

$L_B = L_2 - M_{12} = 8H$

$\frac{1}{L} = \frac{1}{L_A} + \frac{1}{L_B} = 3.73H$

(4)



coil A $L_A = L_1 + M_{12} - M_{13} = 25 + 10 - 10 = 25H$

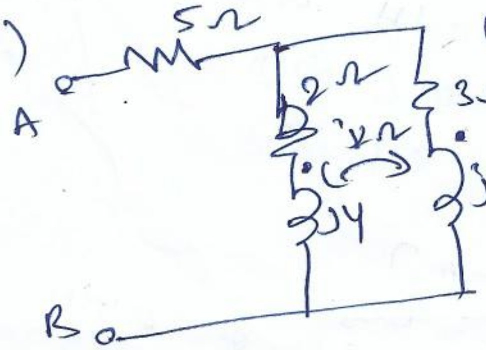
coil B $L_B = L_2 - M_{23} + M_{21} = 35 - 15 + 10 = 25H$

coil $L_C = L_3 - M_{32} - M_{31} = 35 - 15 - 10 = 10H.$

$$\frac{1}{L} = \frac{1}{L_A} + \frac{1}{L_B} + \frac{1}{L_C} = \frac{9}{50}$$

$L = 50/9 = 5.55H.$

(5)



$Z_1 = 5\Omega$ $Z_2 = (2 + j4)\,\Omega$ $Z_3 = (3 + j3)\,\Omega$

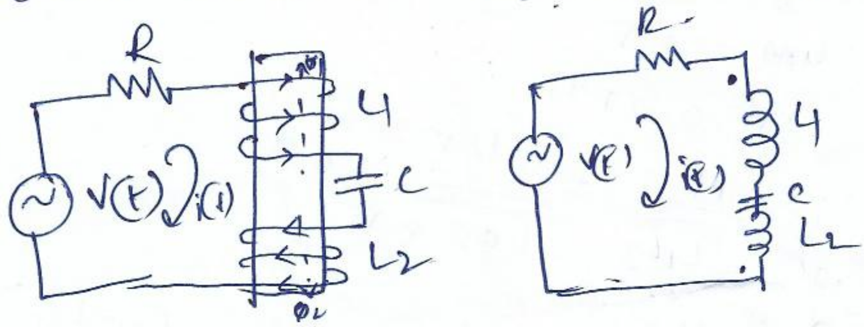
$Z_M = j2\,\Omega$

$$Z = Z_1 + \frac{Z_2Z_3 - Z_M^2}{Z_2 + Z_3 - 2Z_M}$$

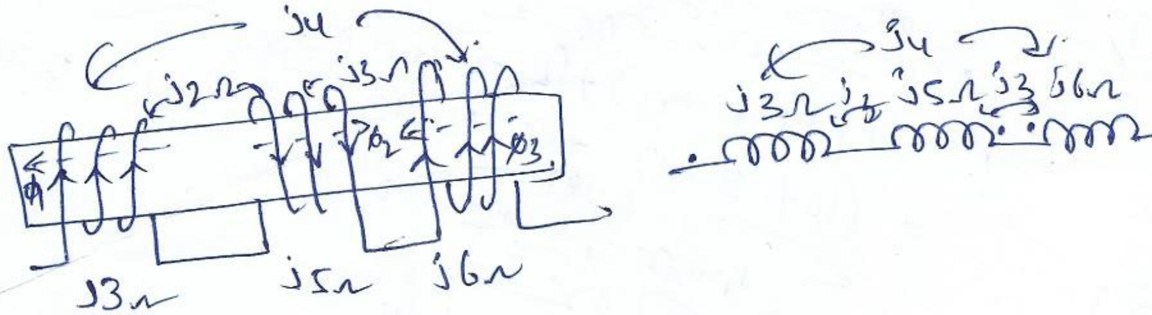
$= 6.9\angle 24.16°\,\Omega$

1. obtain the dotted equivalent circuit for following fig

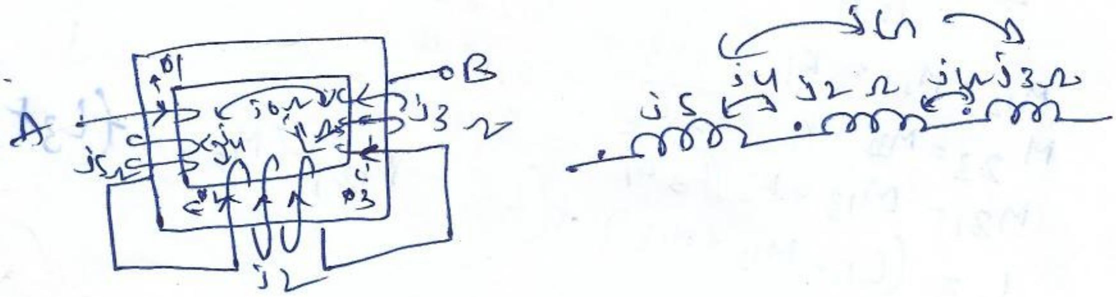
①



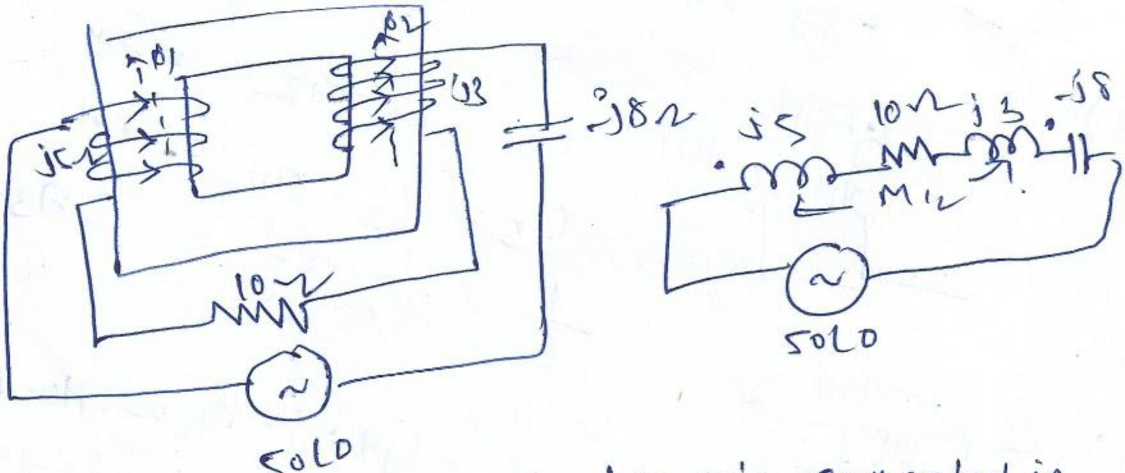
②



③



④



1. The combined inductance of two coils connected in series is 0.6 H (or) 0.1 H depending on relative direction of currents in the two coils. If one of the coils has a self-inductance of 0.2 H, find ⓐ mutual inductance & ⓑ coefficient of coupling.

$L_{diff} = 0.1H \quad L_{com} = 0.6H$

$L_1 = 0.2H$

$L_{com} = L_1 + L_2 + 2M = 0.6 \rightarrow ①$

$L_{diff} = L_1 + L_2 - 2M = 0.1 \rightarrow ②$

Add ① & ②

$2(L_1 + L_2) = 0.7$

$L_1 + L_2 = 0.35$

$L_2 = 0.35 - 0.2 = 0.15H$

Two coils with a coefficient of coupling of 0.6 between them are connected in series so as to magnetise in (a) same direction (b) opposite direction. The total inductance in the same direction is 1.5H and in the opposite direction is 0.5H. Find the self-inductance of the coils.

$K = 0.6 \quad L_{cum} = 1.5H \quad L_{diff} = 0.5H$

$L_{cum} = L_1 + L_2 + 2M \rightarrow (1)$

$L_{diff} = L_1 + L_2 - 2M \rightarrow (2)$

Subtract eqn (2) from (1)

$1 = 4M$

$M = 0.25H$

Add eqn (1) & (2)

$2(L_1 + L_2) = 2$

$L_1 + L_2 = 1 \rightarrow (3)$

$k = \frac{M}{\sqrt{L_1 L_2}}$

$L_1 L_2 = 0.1736 \rightarrow (4)$

$(L_1 + L_2)^2 = L_1^2 + L_2^2 + 2L_1L_2$

$1^2 = L_1^2 + L_2^2 + 2 \times 0.1736$

Solve eqn (3) & (4)

$L_1 = 0.22H \quad L_2 = 0.78H$

Two coils having self-inductances of 4mH & 7mH respectively are connected in parallel. If the mutual inductance between them is 5mH, find the equivalent inductance.

$L_1 = 4mH \quad L_2 = 7mH \quad M = 5mH$

for parallel aiding $L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = 3mH$

for parallel opposing $L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = 0.143\,mH$

Two inductors are connected in parallel. Their equivalent inductance when the mutual inductance aids the self-inductance is 6mH & it is 2mH when the mutual inductance opposes the self-inductance. If the ratio of the self-inductances is 1:3 and the mutual inductance between the coils is 4mH. find the self-inductances.

$L_{com} = 6mH \quad L_{diff} = 2mH \quad \frac{L_1}{L_2} = 1:3 \quad M = 4mH$

$$L_{com} = \frac{L_1L_2 - M^2}{L_1+L_2-2M} \qquad L_{diff} = \frac{L_1L_2-M^2}{L_1+L_2+2M}$$

$$6 = \frac{L_1L_2-16}{L_1+L_2-8} \rightarrow (1) \qquad 2 = \frac{L_1L_2-16}{L_1+L_2+8} \rightarrow (2)$$

from eqn (1) & (2).

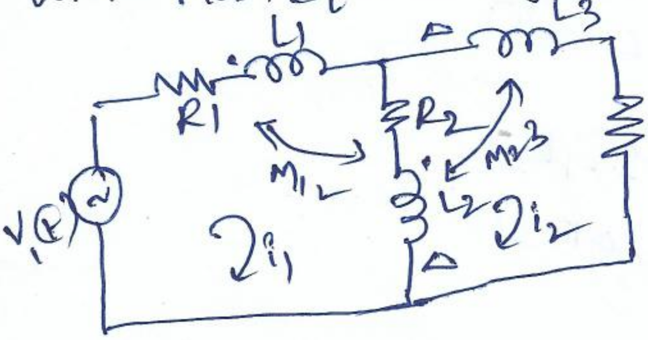
$$2(L_1+L_2+8) = 6(L_1+L_2-8)$$

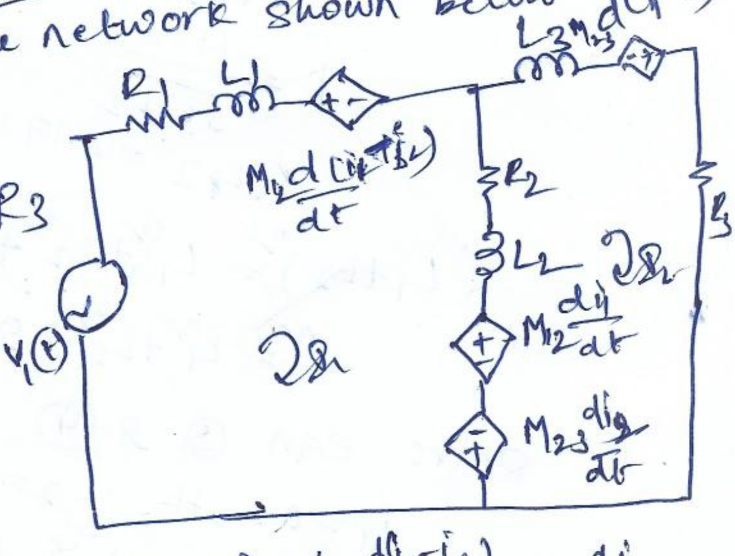
$$L_1+L_2 = 16$$

$$L_1/L_2 = 1:3$$

$$\underline{L_2 = 6.95\,mH}, \quad \underline{L_1 = 9.035\,mH}$$

Write Mesh equations for the network shown below.





$$V_1(t) - I_1R_1 - L_1\frac{di_1}{dt} - M_{12}\frac{d}{dt}(i_1-i_2) - R_2(i_1-i_2) - L_2\frac{d(i_1-i_2)}{dt} - M_{12}\frac{di_1}{dt} + M_{23}\frac{di_2}{dt} = 0.$$

$$(R_1+R_2)i_1 + \frac{di_1}{dt}(L_1 + \cancel{M_{12}} + 2M_{12}) + R_2 i_2 \mp (L_2+M_{12}-M_{23})\frac{di_2}{dt} = V_1(t) \rightarrow (1).$$

$$-L_3\frac{di_2}{dt} + M_{23}\frac{d}{dt}(i_1-i_2) - R_3 i_2 - M_{23}\frac{di_2}{dt} + M_{12}\frac{di_1}{dt} - L_2\frac{d(i_2-i_1)}{dt} - R_2(i_2-i_1) = 0$$

$$-R_2 i_1 - (L_2+M_{12}+M_{23})\frac{di_1}{dt} + (R_2+R_3)i_2 + (L_2+L_3+2M_{23})\frac{di_2}{dt} = 0 \rightarrow (2)$$

1. An ideal transformer has $N_1 = 10$ turns & $N_2 = 100$ turns what is the value of the impedance referred to as the primary, if a 1000Ω resistor is placed across the secondary.

Turns ratio $a = \frac{100}{10} = 10$

$$Z_{in} = \frac{Z_L}{a^2} = \frac{1000}{10^2} = 10\Omega.$$

b) The primary & secondary currents can also be expressed in terms of turns ratio.

$$I_1 j\omega M = I_2 (Z_L + j\omega L_2)$$

$$\frac{I_1}{I_2} = \frac{Z_L + j\omega L_2}{j\omega M}$$

L_2 is large compared to Z_L

$$\frac{I_1}{I_2} = \frac{j\omega L_2}{j\omega M} = \frac{L_2}{M}$$

$$M = \sqrt{L_1 L_2}$$

$$\frac{I_1}{I_2} = \frac{L_2}{\sqrt{L_1 L_2}} = \sqrt{\frac{L_2}{L_1}} = a = \frac{N_2}{N_1}$$

2. An Amplifier with an output impedance of 1936Ω is to feed a loud speaker with an impedance of 4Ω

(a) Calculate the desired turns ratio for an ideal transformer to connect the two system.

(b) An rms current of 20mA at 500Hz is flowing in the primary. Calculate the rms value of current in the secondary at 500Hz.

(c) what is the power delivered to the load?

$$Z_{in} = \frac{Z_L}{a^2} \qquad \frac{N_2}{N_1} = \frac{1}{22}$$

$$a = \frac{1}{22}$$

(b) $\frac{I_1}{I_2} = a$

$I_2 = 0.44A.$

(c) power delivered to the load (speaker)

$= (0.44)^2 \times 4 = \underline{\underline{0.774W}}$