Boolean Function Representation

The use of switching devices like transistors give rise to a special case of the Boolean algebra called as switching algebra. In switching algebra, all the variables assume one of the two values which are 0 and 1.

A Boolean function is an algebraic form of Boolean expression. A Boolean function of n-variables is represented by f(A,B,C,D). By using Boolean laws and theorems, we can simplify the Boolean functions of digital circuits. A brief note of different ways of representing a Boolean function is shown below.

- Sum-of-Products (SOP) Form
- Product-of-sums (POS) form
- Canonical forms

There are two types of canonical forms:

- Sum-of-min terms or Canonical SOP or Standard SOP
- Product-of- max terms or Canonical POS or Standard POS

AND = product
OR = sum

Sum of Product (SOP) Form

The sum-of-products (SOP) form is a method (or form) of simplifying the Boolean expressions of logic gates. In this SOP form of Boolean function representation, the variables are operated by AND (product) to form a product term and all these product terms are ORed (summed or added) together to get the final function.

$$f(A,B,C) = \boxed{\bar{A}B + B\bar{C}}$$

Sum term (OR terms)

Sum of product (SOP)

Product terms (AND terms)

Product of Sums (POS) Form

The product of sums form is a method (or form) of simplifying the Boolean expressions of logic gates. In this POS form, all the variables are ORed, i.e. written as sums to form sum terms.

$$f(A,B,C) = (B+\bar{C})(A+C)$$

product (AND) term

product of sum (POS)

Sum terms (OR term

Canonical Form (Standard SOP and POS Form)

Any Boolean function that is expressed as a sum of minterms or as a product of max terms is said to be in its "canonical form".

Sum of minterms

$$f(A,B,C) = A\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}C$$

Minterm ✓

3 variables so I get max 2^3 minterms

Product of maxterm

$$f(A,B,C) = \underbrace{(A+\bar{B}+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)}_{\text{Maxterm}}$$

3 Variable function can get 2^3 maxterms

n Variable function can contain 2^n minterms & 2^n maxterms ✓

It mainly involves in two Boolean terms, "minterms" and "maxterms".

When the SOP form of a Boolean expression is in canonical form, then each of its product term is called ‘minterm’. So, the canonical form of sum of products function is also known as “minterm canonical form” or Sum-of-minterms or standard canonical SOP form.

Similarly, when the POS form of a Boolean expression is in canonical form, then each of its sum term is called ‘maxterm’. So, the canonical form of product of sums function is also known as “maxterm canonical form or Product-of sum or standard canonical POS form”.

Min terms

A minterm is defined as the product term of n variables, in which each of the n variables will appear once either in its complemented or un-complemented form. The min term is denoted as mi where i is in the range of $0 \le i < 2^n$.

Max terms

A max term is defined as the product of n variables, within the range of $0 \le i < 2^n$. The max term is denoted as Mi. In max term, each variable is complimented, if its value is assigned to 1, and each variable is un-complimented if its value is assigned to 0.

standard SOP (or) Sum of minterms (or) Cannonical SOP

$f(A,B,C,D) = \underbrace{\bar{A} B \bar{C} D}_{\text{minterm}} + \underbrace{A B \bar{C}}_{\text{product term}} \rightarrow$ 'D' is missing

$f(A,B,C,D) = \bar{A} B \bar{C} D + A B \bar{C} (D + \bar{D})$ $D + \bar{D} = 1$

$= \boxed{\bar{A} B \bar{C} D + A B \bar{C} D + A B \bar{C} \bar{D}}$

minterms ✓

standard SOP

standard SOP

sum of minterm

Standard Maxterms

$f(A, B, C) = \underbrace{(A+\bar{B}+C)}_{\text{maxterm}} \; (A+\bar{C}) \rightarrow$ sum terms

not maxterm

missing variable as 'B'

$= (A+\bar{B}+C)\;(A+\bar{C}+\underbrace{B\bar{B}}_{0}) \qquad \boxed{\because B\cdot\bar{B}=0}$

$= (A+\bar{B}+C)\;(A+B+\bar{C})\;(A+\bar{B}+\bar{C})$

maxterms

standard POS (or)

- product of maxterms

2 variables function

decimal	A	B	minterm
0	0	0	$\bar{A}\,\bar{B} = m_0$
1	0	1	$\bar{A}\,B = m_1$
2	1	0	$A\,\bar{B} = m_2$
3	1	1	$A\,B = m_3$

minterm

Normal variable = 1 = A

Complement = 0 = $\bar{A}$

2 variable = 2^2 minterm

$m_j = j = 0, 1, 2, 3$

$f(A, B) = \Sigma(0, 1, 2, 3)$

$= \bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB$

$= \bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB$

3 Variable function

Decimal	A	B	C	minterm	
0	0	0	0	$\bar{A}\bar{B}\bar{C}$	$= m_0$
1	0	0	1	$\bar{A}\bar{B}C$	$= m_1$
2	0	1	0	$\bar{A}B\bar{C}$	$= m_2$
3	0	1	1	$\bar{A}BC$	$= m_3$
4	1	0	0	$A\bar{B}\bar{C}$	$= m_4$
5	1	0	1	$A\bar{B}C$	$= m_5$
6	1	1	0	$AB\bar{C}$	$= m_6$
7	1	1	1	ABC	m_7

3 Variable $= 2^3$ minterm
Normal Variable $= A$
Complement $= \bar{A}$

$$f(A,B,C) = \Sigma m(0,1,2,3,4,5,6,7)$$

2 Variable maxterm function

decimal	A	B	Maxterm	
0	0	0	$A+B$	$= M_0$
1	0	1	$A+\bar{B}$	$= M_1$
2	1	0	$\bar{A}+B$	$= M_2$
3	1	1	$\bar{A}+\bar{B}$	$= M_3$

Maxterm
Normal Variable $= 0 = A$
Complement Variable $= 1 = \bar{A}$
2 Variable function $= 2^2$ Maxterm

$$f(A,B) = \Pi M(0,1,2,3)$$
$$= \underbrace{(A+B)}_{(M_0)}\ \underbrace{(A+\bar{B})}_{(M_1)}\ \underbrace{(\bar{A}+B)}_{(M_2)}\ \underbrace{(\bar{A}+\bar{B})}_{(M_3)}$$

3 Variable function

decimal	A	B	C	Maxterm
0	0	0	0	$A+B+C = M_0$
1	0	0	1	$A+B+\bar{C} = M_1$
2	0	1	0	$A+\bar{B}+C = M_2$
3	0	1	1	$A+\bar{B}+\bar{C} = M_3$
4	1	0	0	$\bar{A}+B+C = M_4$
5	1	0	1	$\bar{A}+B+\bar{C} = M_5$
6	1	1	0	$\bar{A}+\bar{B}+C = M_6$
7	1	1	1	$\bar{A}+\bar{B}+\bar{C} = M_7$

$f(A,B,C) = \Pi M(0,1,2,3,4,5,6,7)$

$f(A,B,C) = A\bar{B} + A\bar{C}$, in sum of minterm

SOP → standard SOP

'C' missing 'B' missing

minterm
1 = A
0 = $\bar{A}$

$= A\bar{B}(C+\bar{C}) + A\bar{C}(B+\bar{B})$

$= \underbrace{A\bar{B}C}_{\underbrace{101}_{5}} + \underbrace{A\bar{B}\bar{C}}_{\underbrace{100}_{4\ (1)}} + \underbrace{AB\bar{C}}_{\underbrace{110}_{6}} + \underbrace{A\bar{B}\bar{C}}_{\underbrace{100}_{4\ \cancel{(2)}}}$

$= \Sigma m(4,5,6)$ ✓

$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$ ✓

$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$ ✓

$f(A,B) = (A+\bar{B})(\bar{A}+C) \rightarrow$ POS

convert POS in to standard POS (or) product of maxterms

$f(A,B) = (A+\bar{B})(\bar{A}+C)$

$(A+\bar{B}) \downarrow$ 'C' missing

$(\bar{A}+C) \downarrow$ B missing

$= (A+\bar{B}+C\bar{C})(\bar{A}+C+B\bar{B})$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+C+B)(\bar{A}+C+\bar{B})$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

$\underbrace{0\ 1\ 0}_{2} \quad \underbrace{0\ 1\ 1}_{3} \quad \underbrace{1\ 0\ 0}_{4} \quad \underbrace{1\ 1\ 0}_{6}$

$\prod M(2,3,4,6)$ ✓

(3) $f(A,B,C) = \underset{1\,0\,1}{A\bar{B}C} + \underset{1\,1\,1}{ABC} \rightarrow$ sum of minterm

$f(A,B,C) = \Sigma m(5,7)$

$f(A,B,C) = \Sigma m(5,7)$

$$\overline{f(A,B,C)} = \overline{A\overline{B}C + ABC}$$

$\because \overline{A+B} = \overline{A}\cdot\overline{B}$

$\overline{AB} = \overline{A}+\overline{B}$

$$= \overline{A\overline{B}C} \cdot \overline{ABC}$$

$$= (\overline{A}+\overline{\overline{B}}+\overline{C})\cdot(\overline{A}+\overline{B}+\overline{C})$$

$$= (\overline{A}+B+\overline{C})\cdot(\overline{A}+\overline{B}+\overline{C})$$

$$= (\overline{A}\cdot\overline{A} + \overline{A}\cdot\overline{B} + \overline{A}\cdot\overline{C})(\overline{A}B + B\cdot\overline{B} + B\overline{C})(\overline{A}\,\overline{C} + \overline{B}\,\overline{C} + \overline{C}\,\overline{C})$$

$\Sigma m(5,7)$

$$\overline{f(A,B,C)} = \Sigma m(0,1,2,3,4,6)$$

$$\overline{\overline{f(A,B,C)}} = \Pi M(0,1,2,3,4,6)$$

$$f(A,B,C) = \Sigma m(5,7)$$

$$f(A,B,C) = \Pi M(0,1,2,3,4,6)$$

① $f(A,B,C) = \Sigma m(5,7)$

$\overline{f(A,B,C)} = \Sigma m(0,1,2,3,4,6)$

$\overline{\overline{f(A,B,C)}} = \Pi M(0,1,2,3,4,6)$

$f(A,B,C)$ = 3 Variables

$8 = 2^3$ minterms

$\Sigma m(0,1,2,3,4,5,6,7)$

② $f(A,B,C) = \Sigma m(0,3,5,6)$

$f(A,B,C) = \Pi M(1,2,4,7)$

① Convert SOP form to POS form for the given Boolean function.

$$f(A,B,C) = \underbrace{\bar{A}\bar{B}C + \bar{A}BC} + \underbrace{\bar{A}BC + ABC}$$

Step 1 Simplify the given boolean function

$$\bar{A}C(\bar{B}+B) + BC(\bar{A}+A)$$

$$\boxed{\because A+\bar{A}=1 \quad B+\bar{B}=1}$$

$$\bar{A}C + BC$$

Step 2 Invert the step 1 boolean equation

$$\overline{\bar{A}C + BC}$$

$$\boxed{\because \overline{A+B} = \bar{A}\cdot\bar{B}}$$

$$\because \overline{AB} = \bar{A}+\bar{B}$$

$$\overline{\bar{A}C}\cdot\overline{BC}$$

$$(\bar{\bar{A}}+\bar{C})\cdot(\bar{B}+\bar{C})$$

$$(A+\bar{C})(\bar{B}+\bar{C})$$

Step 3 multiply and simplify the inverted Boolean expression from step 2

$$(A\bar{B}) + A\bar{C} + \bar{B}\bar{C} + \bar{C}\cdot\bar{C}$$

$$\boxed{\bar{C}\cdot\bar{C} = \bar{C} \quad 1+A+\bar{B}=1}$$

$$A\bar{B} + \underbrace{A\bar{C} + \bar{B}\bar{C} + \bar{C}}$$

$$A\bar{B} + \bar{C}(A+\bar{B}+1)$$

$$A\bar{B} + \bar{C}$$

$A\overline{B} + \overline{C}$

<u>Step 4</u> : Invert above step 3 boolean

$$\overline{A\overline{B} + \overline{C}}$$

$$\overline{A\overline{B}} \cdot \overline{\overline{C}}$$

$$(\overline{A}+B) \cdot C \longrightarrow \underline{\underline{POS}}$$