Logic Gates & Boolean Algebra

properties of X-OR gate

① $A \oplus 1 = \overline{A}$

$A \oplus 1 = A \cdot \overline{1} + \overline{A} \cdot 1 \Rightarrow \overline{A}$

② $A \oplus 0 = A$

$A \oplus 0 = A \cdot \overline{0} + \overline{A} \cdot 0 = A$

③ $A \oplus A = 0$

$A \oplus A = A \cdot \overline{A} + \overline{A} \cdot A = 0$

④ $A \oplus \overline{A} = 1$

$A \cdot \overline{\overline{A}} + \overline{A} \cdot \overline{A} \Rightarrow A \cdot A + \overline{A}\,\overline{A} \Rightarrow A + \overline{A} = 1$

⑤ $AB \oplus AC = A(B \oplus C)$

Proof

$AB \cdot \overline{AC} + \overline{AB} \cdot AC$

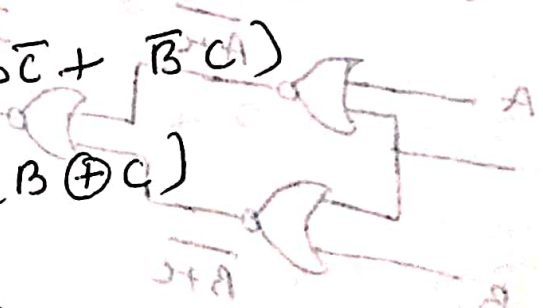
$AB(\overline{A} + \overline{C}) + (\overline{A} + \overline{B})AC$

$AB \cdot \overline{A} + AB\overline{C} + \overline{A} \cdot AC + \overline{B}AC$

$0 + AB\overline{C} + 0 + A\overline{B}C$

$A(B\overline{C} + \overline{B}C)$

$A(B \oplus C)$



(6) $A \oplus B = C$ then $A \oplus C = B$, $B \oplus C = A$,

$A \oplus B \oplus C = 0$

<u>Proof</u> : $A \oplus B \oplus C = (A \oplus B) \oplus (A \oplus B)$

$(A \oplus B) \cdot \overline{A \oplus B} + \overline{A \oplus B} \; A \oplus B$

$0 + 0 = 0$

<u>Proof</u> $A \oplus C = A \oplus (A \oplus B)$

$= A \cdot \overline{(A \oplus B)} + \bar{A} \cdot (A \oplus B)$

$= A \cdot (\bar{A}B + \bar{A}\bar{B}) + \bar{A}(\bar{A}B + A\bar{B})$

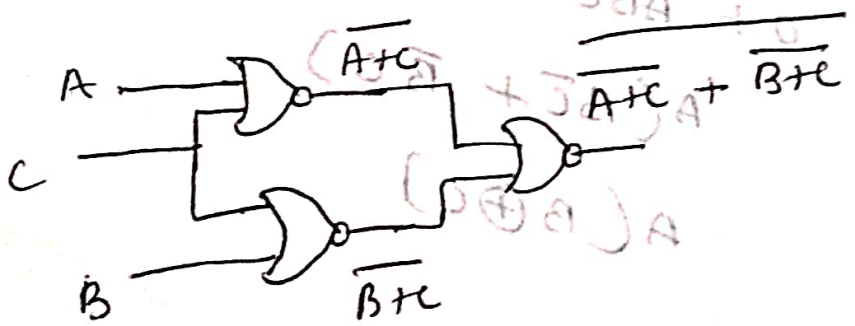
$= AB + 0 + \bar{A}B +$

$= B(A + \bar{A}) = B \cdot 1 = B$

(7) what is the minimum no. of gates required to implement the boolean function $(AB+C)$ if we have to use only 2-input NOR gates.

$f = AB + C$

using Distributive law $(A+C)(B+C)$



$\overline{\overline{A+C} + \overline{B+C}} = (A+C) \cdot (B+C)$

$= AB + BC + C$

$= AB + C(1+B)$

$AB + C$

Boolean Algebra

Boolean algebra is a system of mathematical logic. It is an algebraic system consisting of the set of elements (0, 1), two binary operators called OR, AND, and one unary operator NOT. It is basic mathematical tool in the analysis and synthesis of switching circuits. It is a way to express logic functions algebraically.

Axioms and laws of Boolean Algebra

Axioms (or) postulates of Boolean algebra are a set of logical expressions that we accept without proof and upon which we can build a set of useful theorems.

	AND operation	OR operation	NOT operation
Axiom 1	$0 \cdot 0 = 0$	$0 + 0 = 0$	$\overline{1} = 0$
Axiom 2	$0 \cdot 1 = 0$	$0 + 1 = 1$	$\overline{0} = 1$
Axiom 3	$1 \cdot 0 = 0$	$1 + 0 = 1$	
Axiom 4	$1 \cdot 1 = 1$	$1 + 1 = 1$	

Complementation Law

Law ① $\overline{0} = 1$

② $\overline{1} = 0$

③ If $A = 0$ then $\overline{A} = 1$

④ If $A = 1$ then $\overline{A} = 0$

⑤ $\overline{\overline{A}} = A$ (Double inversion)

AND Law

Law ① $A \cdot 0 = 0$ (Null law)

② $A \cdot 1 = A$ (Identity Law)

③ $A \cdot A = A$

④ $A \cdot \overline{A} = 0$

OR law

Law ① $A + 0 = A$ (Null law)

② $A + 1 = 1$ (Identity law)

③ $A + A = A$

④ $A + \overline{A} = 1$

Commutative Law

Law ① $A + B = B + A$

② $AB = BA$

Associative law

Law ① $(A+B)+C = A+(B+C)$

② $(AB)C = A.(BC)$

Distributive law

Law ① $A(B+C) = AB+AC$

② $A+BC = (A+B)(A+C)$

Redundant literal rule

① $A+\bar{A}B = A+B$

proof $A+\bar{A}B = (A+\bar{A})(A+B)$ (∵ Distributive law)

$= 1.(A+B)$

$= A+B$

② $A(\bar{A}+B) = AB$

proof $A(\bar{A}+B) = A.\bar{A}+AB$ (using Distributive law)

$= 0+AB$

$= AB$

Idempotence law

Law ① $A.A = A$

② $A+A = A$

Absorption law

Law ① $A + AB = A$

② $A \cdot (A+B) = A$

Consensus Theorem (Included factor Theorem)

Theorem 1 $AB + \bar{A}C + BC = AB + \bar{A}C$

LHS $= AB + \bar{A}C + BC \Rightarrow AB + \bar{A}C + BC(A+\bar{A})$

$= AB + \bar{A}C + ABC + \bar{A}BC$

$= AB(1+C) + \bar{A}C(1+B)$

$= AB + \bar{A}C$

$=$ RHS

Thus theorem can be extended to any no. of Variables.

$AB + \bar{A}C + BCD = AB + \bar{A}C$

Proof $AB + \bar{A}C + BCD$

$= AB + \bar{A}C + BC + BCD$

$= AB + \bar{A}C + BC$

$= AB + \bar{A}C$

Demorgan's theorem

Law ① $\overline{A+B} = \overline{A} \cdot \overline{B}$

② $\overline{AB} = \overline{A} + \overline{B}$

Examples

① Apply demorgans theorem to expression

$$f = \overline{\overline{AB}\,(CD + \overline{E}F)\,(\overline{AB} + \overline{CD})}$$

$$= \overline{\overline{AB}} + \overline{(CD + \overline{E}F)} + \overline{(\overline{AB} + \overline{CD})} \quad (\because \overline{AB} = \overline{A} + \overline{B})$$

$$= AB + \overline{CD} \cdot \overline{\overline{E}F} + \overline{\overline{AB}} \cdot \overline{\overline{CD}}$$

$$= AB + \overline{CD} \cdot \overline{\overline{E}F} + AB \cdot CD$$

$$= AB + [(\overline{C} + \overline{D})(E + \overline{F})] + AB \cdot CD$$

$$= AB + (\overline{C} + \overline{D})(E + \overline{F}) + ABCD$$

② Reduce the expression $f = \overline{\overline{AB} + \overline{A} + AB}$

$$f = \overline{\overline{AB} + \overline{A} + AB}$$

$$= \overline{\overline{AB}} \cdot \overline{\overline{A}} \cdot \overline{AB}$$

$$= AB \cdot A \cdot \overline{AB}$$

$$= 0$$

Duality theorem

In positive logic system more positive of two voltage levels represented as '1' and more negative by '0'.

In Negative logic system more positive of two voltage levels represented as '0' and more negative by '1'.

The distinction between positive logic and negative logic system is OR gate in positive logic becomes and AND gate, vice versa.

The duality theorem applies when we are changing one logic system to another.

Duality Theorem states that

'0' becomes 1
'1' becomes 0
'AND' becomes 'OR'
OR becomes 'AND'

Note The variables are not complemented in this process.

The dual of Ex-OR is equal to its Complement

$A \oplus B = \bar{A}B + A\bar{B}$ Dual is $(\bar{A} + B) \cdot (A + \bar{B})$

$\overline{A \oplus B} = \overline{\bar{A}B + A\bar{B}} \Rightarrow (\overline{\bar{A}B}) \cdot (\overline{A\bar{B}})$

$\Rightarrow (A + \bar{B})(\bar{A} + B)$

Dual logic or Inversion logic or (positive & Negative logic)

signal status	+ve logic	-ve logic
L	L	H
H	H	L
NOT	NOT	NOT
AND	AND	OR
OR	OR	AND
NAND	NAND	NOR
NOR	NOR	NAND
Ex-OR	Ex-OR	Ex-NOR
Ex-NOR	Ex-NOR	Ex-OR

The one and only NOT gate can not change its behaviour with applied inverted logic.

Example

AND truth table applied with

+ve logic			-ve logic		
A	B	AB	A	B	AB
L	L	L	H	H	H
L	H	L	H	L	H
H	L	L	L	H	H
H	H	H	L	L	L

Complement of a function

A simple procedure to find the Complement of a function is to take the dual of the function and Complement each literal

Example

① $f = AB + \bar{A}\,\bar{B}$, determine the dual of f

$f = AB + \bar{A}\,\bar{B}$

Dual of $f = (A+B)(\bar{A}+\bar{B})$

$= A\bar{A} + A\bar{B} + \bar{A}B + B\bar{B}$

$= A\bar{B} + \bar{A}B$

② $f = AB + \bar{A}\,\bar{B}$, determine the Complement of f

Dual of $f = (A+B)(\bar{A}+\bar{B}) \Rightarrow A\bar{B} + \bar{A}B$

Complement of each literal in Dual of f expression $= \bar{A}B + A\bar{B}$

Show that

① $f = A\left[B + \bar{C}\left(\overline{AB + A\bar{C}}\right)\right] = AB$

Solution

$$f = A\left[B + \bar{C}\left(\overline{AB + A\bar{C}}\right)\right]$$

$$= A\left[B + \bar{C}\left(\overline{AB} \cdot \overline{A\bar{C}}\right)\right]$$

$$= A\left[B + \bar{C}\left(\bar{A} + \bar{B} \cdot \bar{A} + \bar{\bar{C}}\right)\right]$$

$$= A\left[B + \bar{C}\left((\bar{A} + \bar{B})(\bar{A} + C)\right)\right]$$

$$= A\left[B + \bar{C}\left(\bar{A} + \bar{A}C + \bar{A}\bar{B} + \bar{B}C\right)\right]$$

$$= A\left[B + \bar{A}C + 0 + \bar{A}\bar{B}\bar{C} + 0\right]$$

$$= AB + 0 + 0$$

$$= AB$$

② $f = A + B\left[AC + (B + \bar{C})D\right] = A + BD$

$$= A + B\left[AC + BD + \bar{C}D\right]$$

$$= \cancel{AC + ABD + A\bar{C}D + ABC + BD + B\bar{C}D}$$

$$= AC(1 + A) + BD(1 + A) + \cancel{A\bar{C}D + B\bar{C}D}$$

$$= AC + BD + A\bar{C}D + B\bar{C}D$$

$$= A(C + \bar{C}D) + BD(1 + \bar{C})$$

$$= A(C + D) + BD$$

$$= AC + AD + BD$$

$$= A + ABC + BD + B\bar{C}D$$

$$= A(1 + BC) + BD(1 + \bar{C})$$

$$= A + BD$$

(3) $f = \overline{(A + \overline{BC})} \cdot (A\overline{B} + ABC) = 0$

(Ans) $f = (\overline{A} \cdot \overline{\overline{BC}})\ (A\overline{B} + ABC)$

$= (\overline{A} \cdot BC)\ (A\overline{B} + ABC)$

$= 0$

(4) $f = A\overline{B}C + B + B\overline{D} + AB\overline{D} + \overline{A}C = B + C$

$f = A\overline{B}C + \overline{A}C + B(\overline{D} + 1 + A\overline{D})$

$= A\overline{B}C + \overline{A}C + B$ $\quad \boxed{\because A + \overline{A}B = A + B}$

$= B + \overline{B}AC + \overline{A}C$

$= B + AC + \overline{A}C$

$= B + C(A + \overline{A})$

$= B + C$

<u>Boolean functions and Their representation</u>

<u>Sum of product form</u> also called Disjunctive normal form (DNF)

(1) $f(A, B, C) = \overline{A}B + \overline{B}C$

$\underbrace{\overline{A}B}_{\text{product terms}} + \underbrace{\overline{B}C}_{\text{product terms}} \rightarrow$ SOP

product of sums also called conjunctive normal form (CNF)

$f(A, B, C) = (A + \bar{B})(B + C)$

$(\bar{A} + \bar{B})(A + C)$ → POS

↓ ↓

sum terms

Standard sum of product

It is also called canonical SOP form (or) Disjunctive canonical form (DCF). In this function is sum of number of product terms where each product term contains all the variables of the function either in complemented (or) uncomplemented form.

Minterm :- A product term which contains all the variables of the function either in complemented (or) uncomplemented form called minterm.

n- variable function have 2^n minterms.

In minterms, each variable being primed if ~~cases~~ (complemented) if corresponding bit of binary number is '0' and unprimed (~~uncomplemented~~ (uncomplemented) if a 1.

A	B	minterm (standard product)
0	0	$\bar{A}\bar{B} = m_0$
0	1	$\bar{A}B = m_1$
1	0	$A\bar{B} = m_2$
1	1	$AB = m_3$

Note minterm designation m_j, where j denotes the decimal equivalent of the binary number

Standard POS

It is called canonical POS form (or) Conjunctive canonical form (CCF). In this ~~form~~ the function is a product of number of sum terms. where each sum term contains all the variables of the function either in complemented (or) uncomplemented form.

Maxterms (standard sum term):- A sum term contains all the variables of the function either in complemented (or) uncomplemented form called maxterm.

For a n-variable function, there are 2^n maxterms.

In maxterm, each variable being primed (complemented) if corresponding bit of binary number '1' and unprimed (uncomplemented) if a '0'

A	B	Maxterm (standard sum)
0	0	$A+B = M_0$
0	1	$A+\bar{B} = M_1$
1	0	$\bar{A}+B = M_2$
1	1	$\bar{A}+\bar{B} = M_3$

Note Maxterm designation M_j, where 'j' denotes the decimal equivalent of the binary number.

Maxterms	A B C	Minterms
$M_0 = A+B+C$	0 0 0	$\bar{A}\bar{B}\bar{C} = m_0$
$M_1 = A+B+\bar{C}$	0 0 1	$\bar{A}\bar{B}C = m_1$
$M_2 = A+\bar{B}+C$	0 1 0	$\bar{A}B\bar{C} = m_2$
$M_3 = A+\bar{B}+\bar{C}$	0 1 1	$\bar{A}BC = m_3$
$M_4 = \bar{A}+B+C$	1 0 0	$A\bar{B}\bar{C} = m_4$
$M_5 = \bar{A}+B+\bar{C}$	1 0 1	$A\bar{B}C = m_5$
$M_6 = \bar{A}+\bar{B}+C$	1 1 0	$AB\bar{C} = m_6$
$M_7 = \bar{A}+\bar{B}+\bar{C}$	1 1 1	$ABC = m_7$

① $f(A,B,C) = \Sigma m(0,2,4,6,7)$

$\overline{f(A,B,C)} = \Sigma m(1,3,5) \Rightarrow m_1 + m_3 + m_5$

$\overline{\overline{f(A,B,C)}} = f(A,B,C)$

$\overline{m_1 + m_3 + m_5} = \overline{m_1} \cdot \overline{m_3} \cdot \overline{m_5}$

$= \cancel{\overline{M_1} \cdot \overline{M_3} \cdot M_5}$

$M_1 \; M_3 \; M_5$

$= \Pi M(1,3,5)$

$\boxed{\overline{m_j} = M_j}$

② $f(A,B,C) = \Pi M(1,3,5)$

$\overline{f(A,B,C)} = \Pi M(0,2,4,6,7)$

$= M_0 \cdot M_2 \; M_4 \; M_6 \; M_7$

$\overline{\overline{f(A,B,C)}} = M_0 \; M_2 \; M_4 \; M_6 \; M_7$

$= \overline{M_0} + \overline{M_2} + \overline{M_4} + \overline{M_6} + \overline{M_7}$

$= m_0 + m_2 + m_4 + m_6 + m_7$

$= \Sigma m(0,2,4,6,7)$

$\boxed{\overline{M_j} = m_j}$

Conversion from SOP form to POS form

1. Simplify SOP equation
2. Invert SOP equation
3. Multiply and simplify the invert equation
4. Invert the result.

Sum of product form can be implemented by using two level gate network NAND - NAND logic.

NAND - NAND realisation is same as AND - OR

product of sums form can be implemented by using two level gate network NOR - NOR logic

NOR - NOR realisation is same as OR - AND

If the signals are propagating through two stages of gates, then it is called two level gate network.

① Determine the Complement of the following function

$$f_1 = \bar{A}\,\bar{B}C + A\bar{B}\bar{C}$$

<u>Solution</u>

$$f_1 = \bar{A}\,\bar{B}C + A\bar{B}\bar{C}$$

Dual of f_1 is $f_D = (\bar{A}+\bar{B}+C)(A+\bar{B}+\bar{C})$

Complement of f_1 is $\bar{f}_D = (A+B+\bar{C})(\bar{A}+B+\bar{C})$

$\boxed{\text{Complement of each literal in Dual of } f_1}$

<u>For Verification</u>

$$f_1 = \bar{A}\,\bar{B}C + A\bar{B}\bar{C}$$

$$\bar{f}_1 = \overline{(\bar{A}\,\bar{B}C + A\bar{B}\bar{C})}$$

$$\bar{f}_1 = \overline{\bar{A}\,\bar{B}C} \cdot \overline{A\bar{B}\bar{C}}$$

$$= \bar{\bar{A}}+\bar{\bar{B}}+\bar{C} \cdot \bar{A}+\bar{\bar{B}}+\bar{\bar{C}}$$

$$= (A+B+\bar{C})(\bar{A}+B+C)$$

(2) Convert the SOP equation $F = A\bar{B}C + \bar{A}B\bar{C}$ into it's POS form.

(Ans) <u>step 1</u> :- simplify SOP equation, if possible

$$F = A\bar{B}C + \bar{A}B\bar{C}$$

<u>step 2</u> :- Invert the SOP equation.

$$\bar{F} = \overline{A\bar{B}C + \bar{A}B\bar{C}}$$

$$= \overline{A\bar{B}C} \cdot \overline{\bar{A}B\bar{C}}$$

$$\bar{F} = (\bar{A}+B+\bar{C}) \cdot (A+\bar{B}+C)$$

<u>step 3</u> :- multiply and simplify the invert equation

$$\bar{F} = (\bar{A}+B+\bar{C})\ (A+\bar{B}+C)$$

$$\bar{F} = \bar{A}\bar{B} + \bar{A}C + AB + BC + A\bar{C} + \bar{B}\bar{C}$$

$$=$$

<u>step 4</u> :- Invert the result of step 3

$$\bar{F} = \bar{A}\bar{B} + \bar{A}C + AB + BC + A\bar{C} + \bar{B}\bar{C}$$

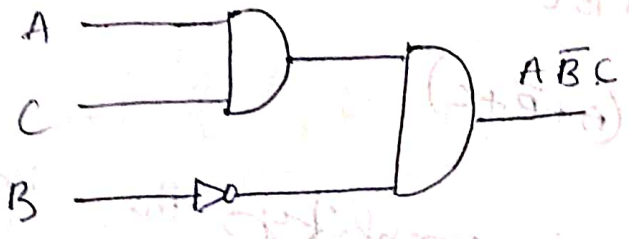
$$\bar{\bar{F}} = \overline{\bar{A}\bar{B} + \bar{A}C + AB + BC + A\bar{C} + \bar{B}\bar{C}}$$

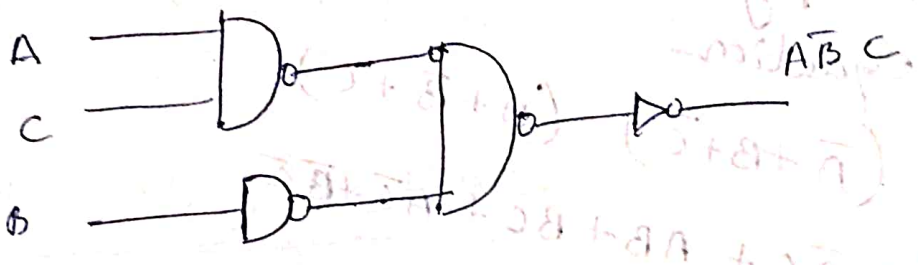
$$= \overline{\bar{A}\bar{B}} \cdot \overline{\bar{A}C} \cdot \overline{AB} \cdot \overline{BC} \cdot \overline{A\bar{C}}\ \overline{\bar{B}\bar{C}}$$

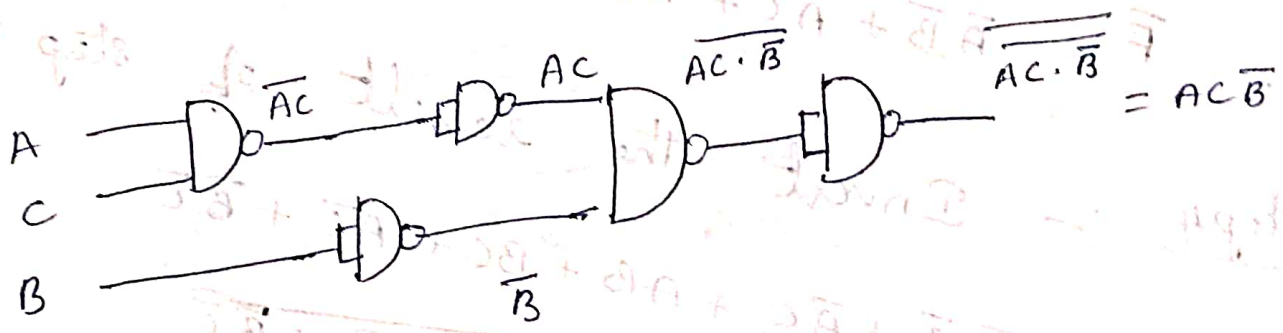
$$= (\bar{A}+\bar{B})\ (A+C)\ (\bar{A}+\bar{B})\ (\bar{B}+\bar{C})\ (\bar{A}+C)\ (B+C)$$

① Design $A\bar{B}C$ using minimum no. of NAND gates.

(Ans) $AC\bar{B}$







~~The EX-OR gate is~~

EX-OR

The o/p of an EX-OR gate is modulo sum of its two inputs. It is also called

$$A \oplus B = \bar{A}B + A\bar{B}$$

EX-NOR

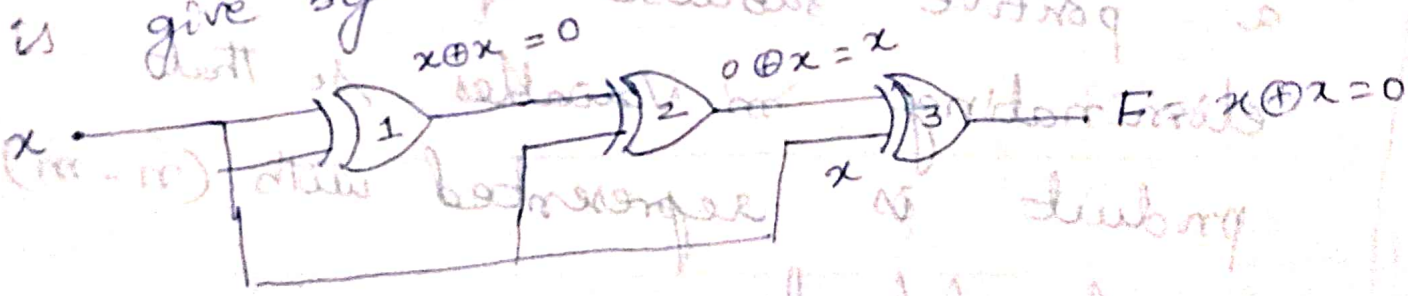
The o/p of an EX-NOR gate is '1' only when the two inputs are equal

(6) which of the following expressions does not represent exclusive NOR of x and y?

(a) $xy + x'y'$ (b) $x \oplus y$

(c) $x' \oplus y$ (d) $x' \oplus y'$

(7) For the circuit shown below the output F is give by



(8) Rules to simplify K-maps

(a) At the time of grouping the adjacent cells containing 1's always use maximum possible group.

(b) All the cell's containing 1's must be covered at least once in any group.

(c) At the time of grouping don't care (X) values can be taken as 1's.

(d) All don't care's values need not be covered [If boolean expression is reduced using don't cares then use don't cares otherwise don't care it.

(i) If subcube has 2^m cells $\longrightarrow$ Elimination of 'm' variables

(ii) In a function of n-variables, consider a positive subcube of 2^m cells, is eliminating 'm' variables so that product is represented with (n-m) no. of literals

No. of Variables	No. of cells containing 1's grouped	No. of variables eliminated	No. of literals present in the resulting term
2	4	2	0
	2	1	1
	1	0	2
3	8	3	0
	4	2	1
	2	1	2
	1	0	3
4	16	4	0
	8	3	1
	4	2	2
	2	1	3
	1	0	4

Karnaugh map

1. 'n' Variable K-map will have 2^n cells (or) square
2. Adjacent cells differ by 1 bit (To maintain adjacency property gray code sequence is used)
3. Each cell represents a term of ~~two~~ n literals
4. Grouping of adjacent squares can be in the order of 2^m where $m = 0, 1, 2, \ldots$
4. Grouping of two adjacent (pair) squares containing 1's represents a term of $(n-1)$ literals
5. Grouping of four adjacent (pair) sequences containing 1's represents a term of $(n-2)$ literals.
6. Grouping of Eight adjacent (pair) sequences containing 1's represent a term of $(n-3)$ literals.
7. Grouping of 'p' adjacent sequences $(p = 2^m)$ containing 1's represent a term of $(n-m)$ literals

* sum of all minterms is equal to one
* product of all maxterms is equal to zero

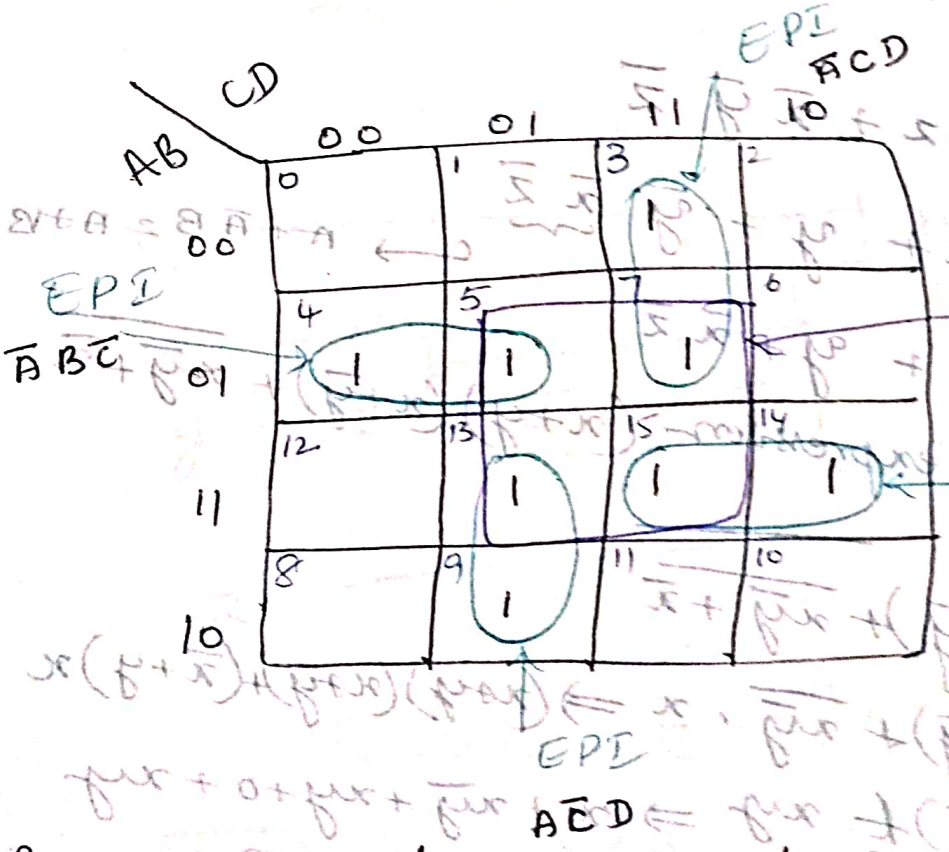
Prime Implicant

Each square (or) rectangle (or) cell made up of the bunch of adjacent minterms is called a sub cube. Each of these sub cubes is called a prime implicant (PI)

Essential prime implicant :-

The prime implicant which contains at least one '1', which can not be covered by any other prime implicant is called essential prime Implicant (EPI)

$$f(A,B,C,D) = \Sigma m(3,4,5,7,9,13,14,15)$$



Prime implicants

$\bar{A}B\bar{C}$, $A\bar{C}D$

ABC, $\bar{A}CD$

BD

Essential prime implicants

$\bar{A}B\bar{C}$

$A\bar{C}D$

ABC

$\bar{A}CD$

The minimized expression by covering all 1's

$F = \bar{A}B\bar{C} + A\bar{C}D + ABC + \bar{A}CD$, It is unique expression

(1) Consider the Boolean function $f(w, x, y, z)$
$wy + xy + \bar{w}xyz + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}$
Determine the complete set of essential prime implicants.

(A)

$$wy + \underbrace{xy + \bar{w}xyz}_{} + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}$$

$$\underbrace{wy} + xy + \underbrace{\bar{w}\bar{x}y} + xz + \bar{x}\bar{y}\bar{z}$$

$$y(w + \bar{w}\bar{x}) + xy + xz + \bar{x}\bar{y}\bar{z}$$

$$wy + \underbrace{\bar{x}y + xy}_{} + xz + \bar{x}\bar{y}\bar{z}$$

$$\underbrace{wy + y}_{} + xz + \bar{x}\bar{y}\bar{z}$$

$$y + xz + \bar{x}\bar{y}\bar{z}$$

$$\cancel{y}\, xz + y + \bar{y}\,\underbrace{\bar{x}\bar{z}}_{} \longrightarrow A + \bar{A}B = A + B$$

$$xz + y + \bar{x}\bar{z}$$

(2) The Boolean expression $(x+y)(x+\bar{y}) + \overline{x\bar{y} + \bar{x}}$ simplifies to

$$(x+y)(x+\bar{y}) + \overline{x\bar{y} + \bar{x}}$$

$$(x+y)(x+\bar{y}) + \overline{x\bar{y}} \cdot x \Rightarrow (x+y)(x+\bar{y}) + (\bar{x}+y)x$$

$$(x+y)(x+\bar{y}) + xy \Rightarrow x + x\bar{y} + xy + 0 + xy$$

$$x + xy + x\bar{y} \Rightarrow x(1 + y + \bar{y}) = x$$

(3) For an n-variable Boolean function, the maximum number of prime implicantes is $\frac{(n-1)}{2}$

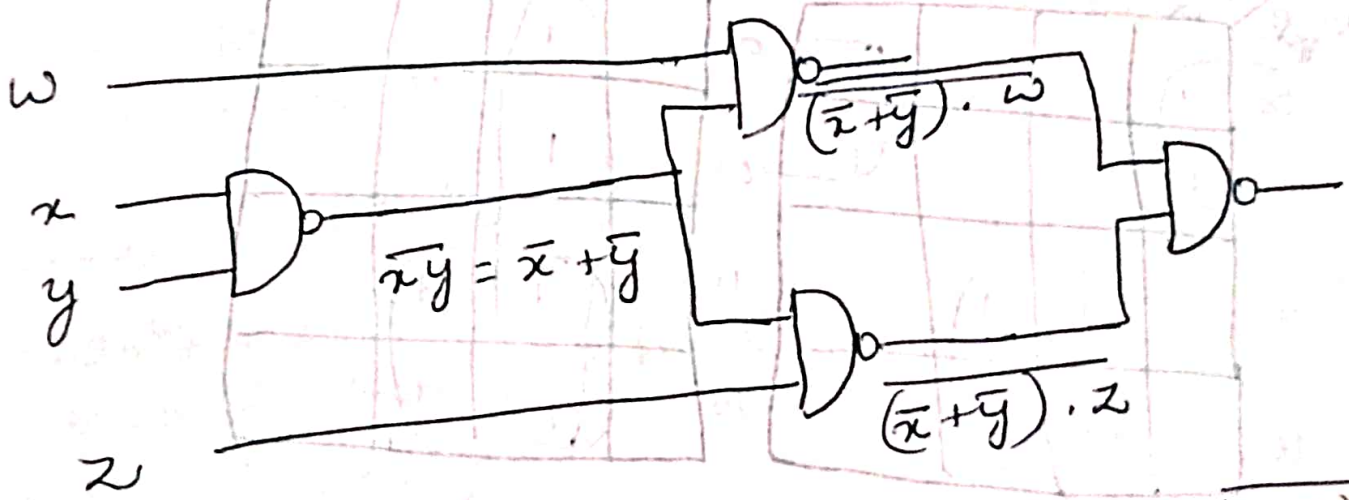
(4) Karnaugh map is used to minimize the no. of gates and fan in of a digital circuit.

(5) minimum no. of 2-input NAND gates required to implement the function $F = (\bar{x}+\bar{y})(z+w)$ is 4

$$= (\bar{x}+\bar{y})(w+z) \Rightarrow \bar{x}w + \bar{x}z + \bar{y}w + \bar{y}z$$

$$= \cancel{\bar{x}(w+z) + \bar{y}(w+z)}$$

$$= z(\bar{x}+\bar{y}) + w(\bar{x}+\bar{y})$$



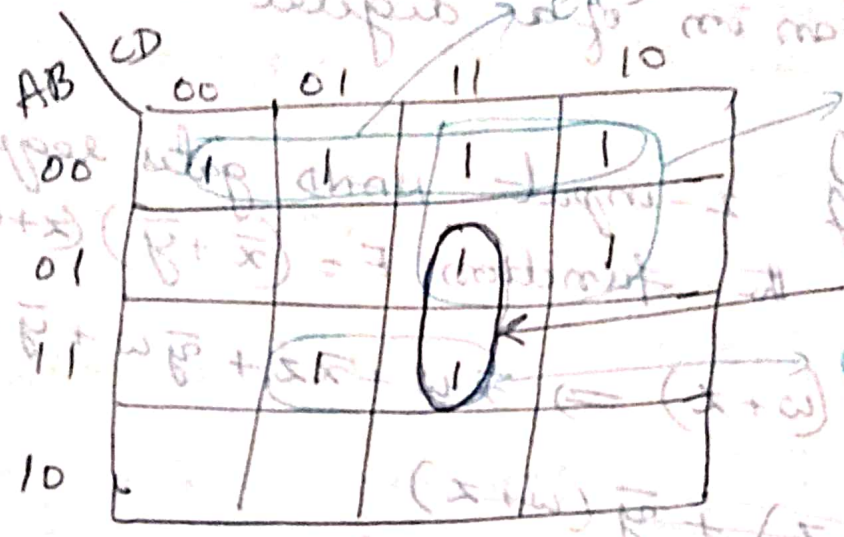
$$\overline{\overline{(\bar{x}+\bar{y})w} \cdot \overline{(\bar{x}+\bar{y})z}} \Rightarrow \cancel{\overline{(\bar{x}+\bar{y})w + (\bar{x}+\bar{y})z}}$$

$$\cancel{\overline{(\bar{x}+\bar{y})} + \bar{w} + \bar{x} + y} \quad (\bar{x}+\bar{y})w + (\bar{x}+\bar{y})z$$

$$= (\bar{x}+\bar{y})(z+w)$$

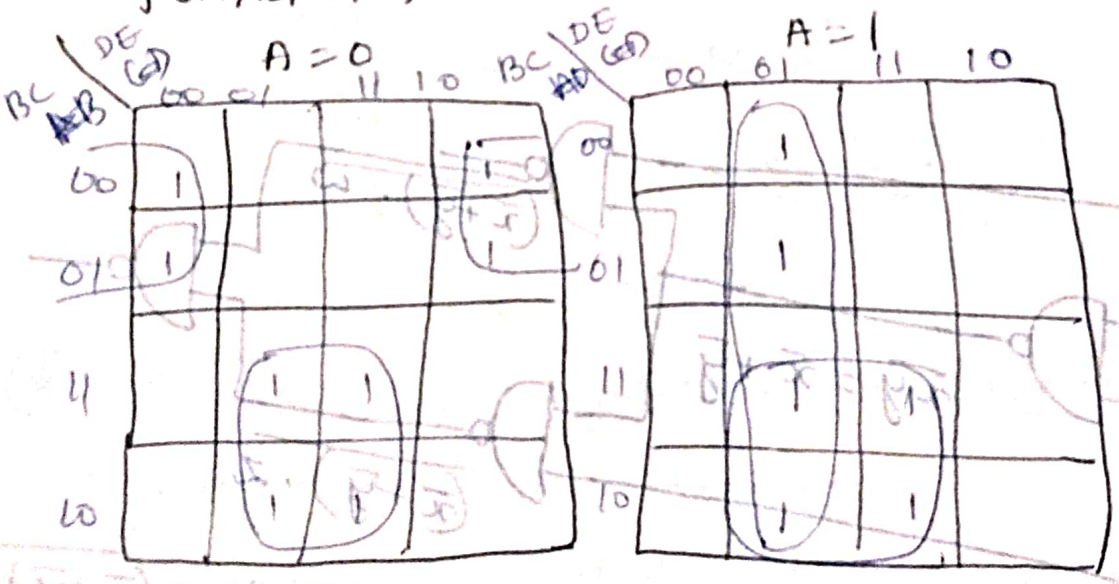
① Reduce the following expression using K-map and identify prime implicant and essential prime implicant

$f(A,B,C,D) = \Sigma m(0,1,2,3,6,7,13,15)$



② Simplify the boolean function.

$f(A,B,C,D,E) = \Sigma m(0,2,4,6,9,11,13,15,17,21,25,27,29,31)$



$\overline{A}\,\overline{B}E + BE + A\overline{D}E$

Number systems

① Find possible bases of $23+44+14+32 = 223$

$2b+3+4b+4+1b+4+3b+2 = 2b^2+2b+3$

$2b^2-18b-10=0$

$2b^2-10b+2b-10=0$

$(b-5)(2b+2)=0$

$b=5$, since base must be +ve

② $\sqrt{41} = 5$

$\sqrt{4b+1} = 5$

$4b+1 = 25$

$4b = 24$

$b = 6$

③ $(16)_{10} = (100)_b$

$1\times10+6 = b^2$

$b^2 = 16 \Rightarrow b=4$

④ $(1204)_b = (292)_{10}$

$b^3+2b^2+4 = 292$

$b^2(b+2) = 288 \Rightarrow b=6$

⑤ $(211)_b = (152)_8$

$2b^2+b+1 = 8^2+5\times8+2$

$2b^2+b-105=0 \Rightarrow b = \frac{-1\pm\sqrt{1+4\times2\times105}}{2\times2}$

$b = \frac{28}{4}$ or $\frac{-30}{4} \Rightarrow b=7$ (must be +ve)

(6) $(73)_x = (54)_y$

$7x + 3 = 5y + 4$ then $x = 8$, and $y = 11$

(7) $\frac{302}{20} = 12.1$

$\frac{3b^2+2}{2b} = b + 2 + \frac{1}{b}$

$b^2 - 4b = 0$

$b(b-4) = 0$

$b = 0$ (or) $b = 4$

Since 'b' must be greater than 0

(8) $\frac{312}{20} = 13.1$

$\frac{3b^2+b+2}{2b} = b + 3 + \frac{1}{b}$

$\frac{3b^2+b+2}{2b} = \frac{b^2+3b+1}{b}$

$3b^3 + b^2 + 2b = 2b^3 + 6b^2 + 2b$

$b^3 - 5b^2 = 0$

$b^2(b-5) = 0$

(9) Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is 3

(Ans) $1 \times 5^2 + 2 \times 5 + 3 = 38$

$(38)_{10} = (x8)_y \Rightarrow xy + 8 = 38$

$xy = 30$

possible solutions are

① 3, 10 ② 5, 6 ③ 6, 5

(10) sign extension is the step in converting a signed integer from one size to another.

(11) The number of 1's in 8 bit representation of −127 in 2's complement form is m and that in 1's complement form is n. what is the value of m ; n is 2 ; 1

+127 0111 1111

−127 1's ⇒ 1000 0000

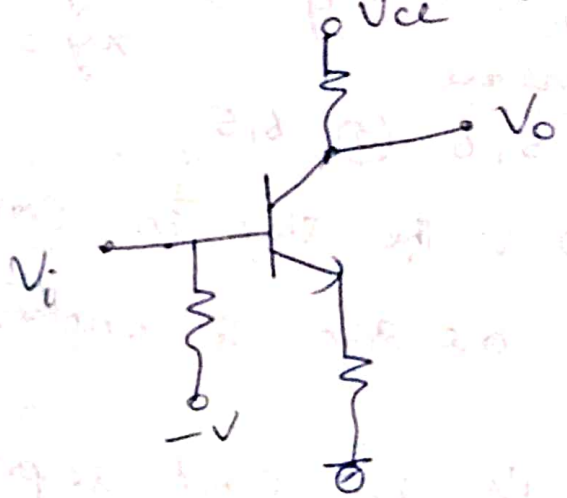
1

−127 in 2's ⇒ 1000 0001

(12) A number in 4-bit 2's complement representation is $x_3\ x_2\ x_1\ x_0$. This number when stored in 8 bits will be $x_3\ x_3\ x_3\ x_3\ x_3\ x_2\ x_1\ x_0$

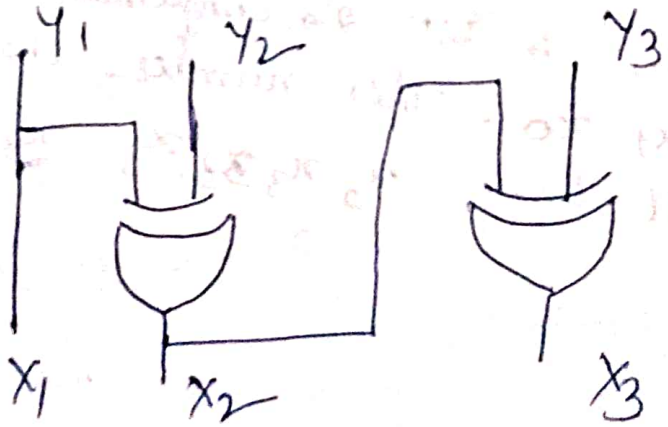
① The o/p of a logic gate is '1' when all its inputs are at logic '0' the gate is either <u>NOR</u> or EX-NOR

② The gate shown in figure is <u>NOT</u>



③ when signed numbers are used in binary arithmetic, then which of the following notations would have unique representation for zero <u>2's Complement</u>

④ The logic circuit given below converts a binary code $Y_1\ Y_2\ Y_3$ in to <u>Gray code</u>



(5) Venn diagram representing the boolean expression $A + (\bar{A} \cdot B)$

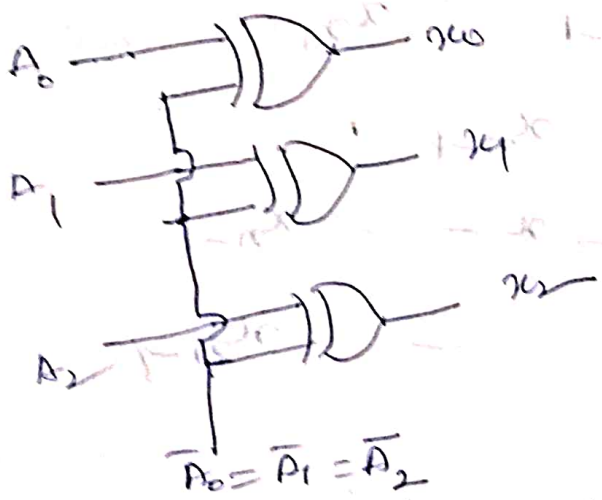
(6) Boolean expression for shaded area

$A\bar{B}C + A\bar{B}C + \bar{A}BC$

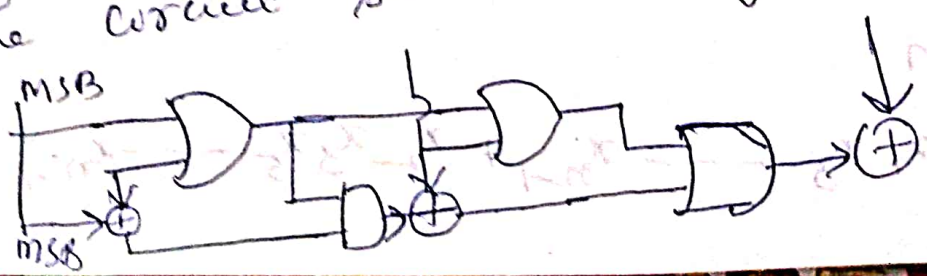
(7) Boolean expression for shaded area

$\bar{A}\bar{B}C + AB$

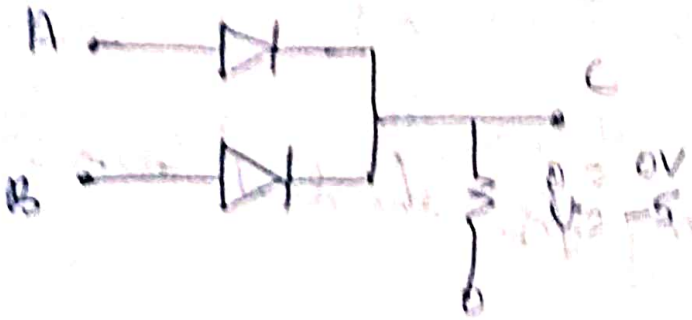
(8) In the figure shown below $x_2\ x_1\ x_0$ will be 1's complement of $A_2\ A_1\ A_0$ if $\underline{Y = 1}$



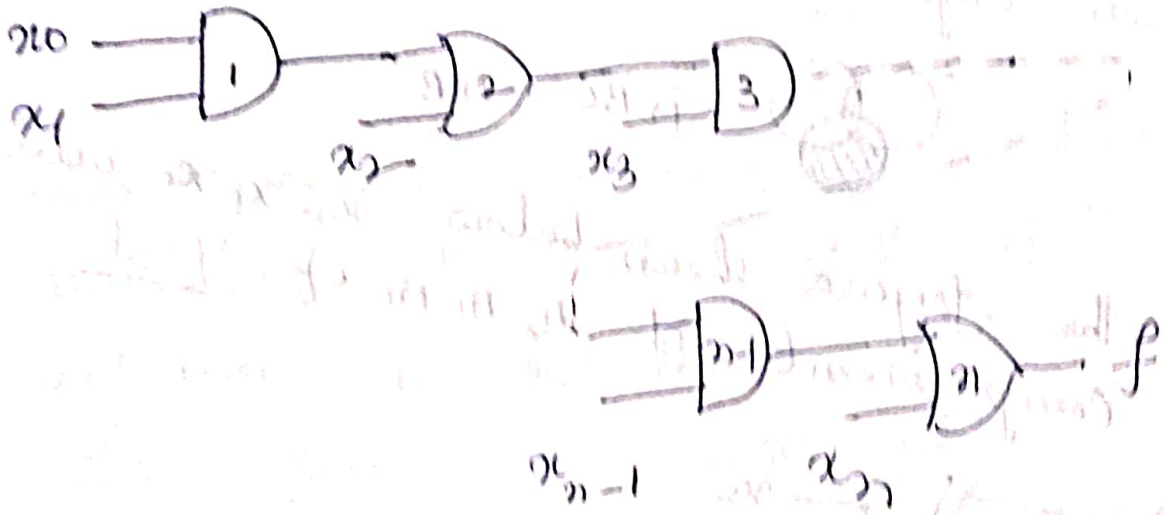
(9) The circuit shown in fig. Converts Gray to BCD



(10) If negative logic is used, diode gate shown in the given figure will represent an



(11) In the given network of AND and OR gates f can be written as ——



$x_0 x_1 x_3 \text{ - - - - - } x_{n-1}$

$+ x_2 x_3 x_5 \text{ - - - } x \text{ - - - } x_{n-1}$

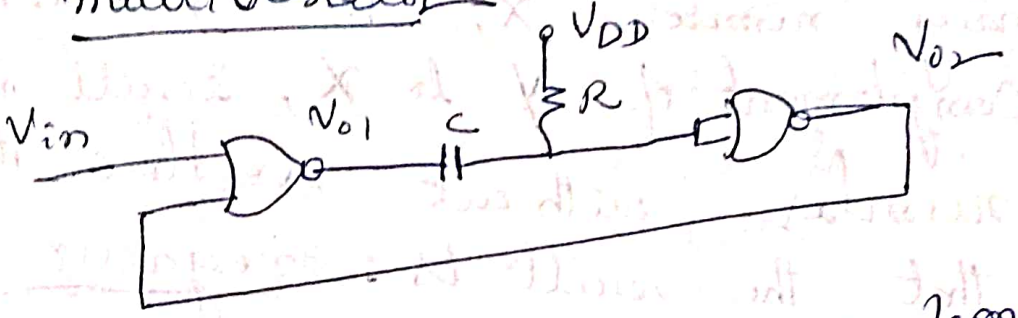
$+ x_4 x_5 x_7 \text{ - - - - - - } x_{n-1}$

$+ x_{n-2} x_{n-1}$

$+ x_n$

<u>Ans</u> $x_0 x_1 x_3 \text{ - - - } x_{n-1} + x_2 x_3 x_5 \text{ - - - } x_{n-1} + \text{ - - - } x_{n-2} x_{n-1} + x_n$

(12) The figure given below shows the circuit of which one of the following is monostable multivibrator



(13) A full adder can be implemented with half adders and OR gate. A 4 bit parallel full adder without any initial carry requires 7 half adders and 3 OR gates

(14) A four variable switching function has minterms m_6 and m_9. If literals in these minterms are complemented, then corresponding minterm numbers are m_9 and m_6.

(5) The 2's Complement representation of -17 is $\underline{101111}$

$+17 \Rightarrow 010001$

1's complement of $+17$: 101110

2's complement: $101110 + 1 = 101111$

$2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$101111 \Rightarrow -32 + 1\times8 + 1\times4 + 1\times2 + 1\times1$

$\Rightarrow -32 + 8 + 4 + 2 + 1$

$\Rightarrow -32 + 15 \Rightarrow -17$

(6) 4 bit 2's Complement representation of a decimal number is 1000. The number is

options

(a) ~~+8~~ (a) $+8 \Rightarrow 01000 \Rightarrow 8\times1 + 4\times0 + 2\times0 + 1\times0$

(b) $0 \Rightarrow 0000$

(c) $-7 \Rightarrow$

$+7 \Rightarrow 0111$

1's complement: 1000

$-7 \Rightarrow 1001$

$2^3\ 2^2\ 2^1\ 2^0$

1001

$-8\times1 + 1\times1 \Rightarrow -7$

(d) -8

$+8 \Rightarrow 01000$

1's complement: 10111

$\Rightarrow 1000 \Rightarrow -8$

(7) The range of signed decimal numbers that can be represented by 6 bit 1's Complement number is -31 to $+31$

Range of signed numbers $-2^{n-1}-1$ to $+2^{n-1}-1$

$n = 6$ bit

$-2^{6-1}-1$ to $+2^{6-1}-1 \Rightarrow -2^5-1$ to $+2^5-1$

Range is $-(31)_{10}$ to $+(31)_{10}$

(8) Decimal 43 in Hexadecimal and BCD number systems is respectively $(2B)_{16}$ and 0100 0011

$16 \,|\, 43$
$\quad 2 - B \uparrow \Rightarrow (2B)_{16}$

(9) $x = 01110$ and $y = 11001$ are two 5-bit binary numbers represented in 2's Complement format. The sum of x and y represented in 2's Complement format using 6 bits is

000111

$x = 01110 \Rightarrow +14$

$y = 11001 \Rightarrow -16\times1 + 8\times1 + 2\times1 \Rightarrow -16+9 \Rightarrow -7$

$x+y \Rightarrow +14-7 \Rightarrow +7$

$000111 \Rightarrow 2^2 + 2^1\times1 + 2^0\times1 \Rightarrow +7$

(10) 11001, 1001 and 111001 correspond to the 2's Complement representation of which one of the following sets of number?

(Ans)

$$\begin{array}{ccccc} 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 1 & 0 & 0 & 1 \end{array}$$

$\rightarrow$

$$-2^4 \times 1 + 1 \times 2^3 + 1 \times 2^0 \Rightarrow -16 + 8 + 1 \Rightarrow -16 + 9 \Rightarrow -7$$

$$\begin{array}{cccc} 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 0 & 0 & 1 \end{array}$$

$\rightarrow$

$$\Rightarrow -2^3 \times 1 + 2^0 \Rightarrow -8 + 1 \Rightarrow -7$$

$$\begin{array}{cccccc} 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 1 & 0 & 0 & 1 \end{array}$$

$\rightarrow$

$$\Rightarrow -2^5 \times 1 + 2^4 \times 1 + 2^3 \times 1 + 2^0 \times 1$$

$$\Rightarrow -32 + 16 + 8 + 1 \Rightarrow -32 + 25 = -7$$

$$\begin{array}{cc} 2^1 & 2^0 \end{array}$$

~~111001~~

(H) 1001 $\Rightarrow$ −7 in 2's Complement form

$$\left.\begin{array}{l} 11001 \\ 111001 \end{array}\right\} -7 \quad \text{(sign extension bits)}$$

sign extension

mag

(2) A new Binary Coded pentary (BCP) number system is proposed in which every digit of a base-5 number is represented by its corresponding 3-bit binary code. For example, the base 5 number 24 will be represented by its BCP code 010100. In this numbering system, the BCP code 100010011001 corresponds to the following number in base-5 systems.

$$(24)_5 \Rightarrow \underbrace{010}_{2}\,\Big|\,\underbrace{100}_{4}$$

$$\begin{array}{c|c|c|c} 100 & 010 & 011 & 001 \\ 4 & 2 & 3 & 1 \end{array} \Rightarrow 4231$$

Boolean Algebra

(1) The number of Boolean functions that can be generated by 'n' variable is equal to 2^{2^n}

(2) Let $n=2$ then $2^{2^2} = 16$ Boolean functions

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

(2) Two 2's Complement numbers having sign bits x and y are added and the sign bit of the result is z. Then the occurence of overflow is indicated by the boolean function.

$\therefore$ overflow $= \bar{x}\,\bar{y}\,z + x\,y\,\bar{z}$

→ occurence of overflow changes the true sign of the result.

→ No overflow is occured while adding one +ve and ~~or~~ one −ve date.

3

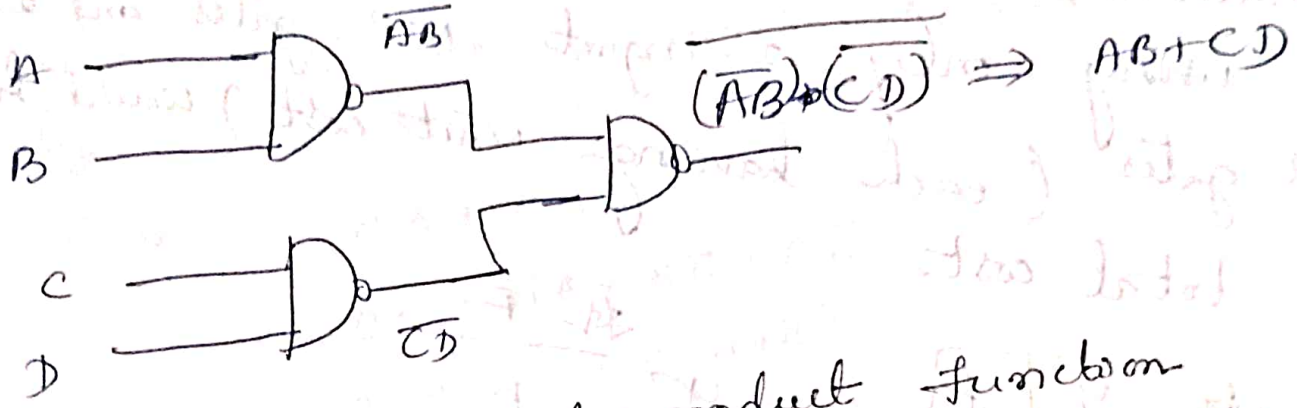
x	y	z	overflow
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(5) The number of distinct Boolean expressions of 4 Variables is 65536

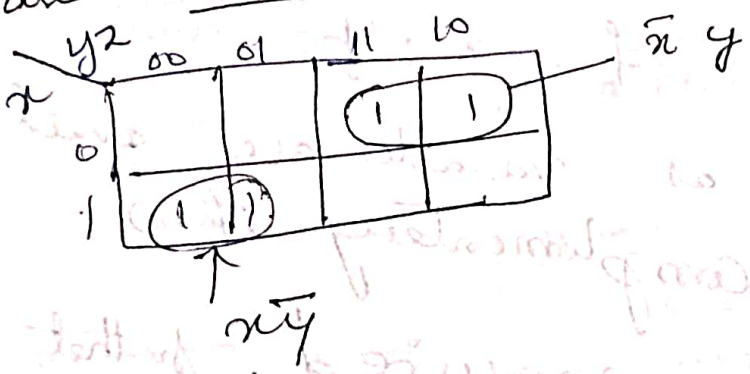
$2^{2^n} \Rightarrow 2^{2^4} \rightarrow 2^{16} \rightarrow 65536$

(8) The Boolean function $Y = AB + CD$ is to be realized using only 2-input NAND gates. The minimum no. of NAND gates required is

3



(9) ~~If~~ In the sum of product function $f(x,y,z) = \Sigma m(2,3,4,5)$, the prime implicants are $\bar{x}y$, $x\bar{y}$



(11) For an n-variable Boolean function the maximum number of prime implicants is $2^{(n-1)}$

All The number of ungrouped cells in the given k-map is only maximum no. of prime implicants.