6.21 SOLVED PROBLEMS

PROBLEM 1

An alphabet consists of 3 letters A, B and C with respective probabilities of transmission 1/3, 1/6 and 1/2. Find the average amount of information associated with the transmission of a letter.

Solution:

The average amount of information is the entropy

$$H = -\left[\frac{1}{3}\log_2\frac{1}{3} + \frac{1}{6}\log_2\frac{1}{6} + \frac{1}{2}\log_2\frac{1}{2}\right]$$

$$= \frac{1}{3}\log_2 3 + \frac{1}{6}\log_2 6 + \frac{1}{2}\log_2 2$$

$$= 1.459 \text{ bits/symbol.}$$

PROBLEM 2

An independent discrete source transmitts letters from an alphabet consisting of A and B with respective probabilities of 0.6 and 0.4.
If the consecutive letters are statistically independent and two symbol words are transmitted find the probabilities for all the two letter words possible and find the entropy of such scheme.

Solution:

The possible two letter words from the given alphabet set are :

AA, AB, BA and BB

The corresponding probabilities are

$$P(AA) = 0.6 \times 0.6 = 0.36$$

$$P(AB) = 0.6 \times 0.4 = 0.24$$

$$P(BA) = 0.6 \times 0.4 = 0.24$$

$$P(BB) = 0.4 \times 0.4 = 0.16$$

Entropy of the above system is

$$-[\,0.36 \log 0.36 + 0.24 \log 0.24 + 0.24 \log 0.24 + 0.16 \log 0.16\,]$$

$$= 1.942 \textit{ bits/word}$$

PROBLEM 3

Consider that a pair of dice are thrown. Find the average amount of information contained in the message "The sum of the faces should be 8".

Solution:

The sample space consists of the events

{(2,6) (6,2), (3,5) (5,3), (4,4)} each with a probability of 1/36

The average amount of information contained by the above message is

$$-\left[5 \times \frac{1}{36} \log_2 \frac{1}{36}\right] = \frac{-5}{36} \log_2 36$$

$$= 0.718 \textit{ bits}$$

PROBLEM 4

PROBLEM 11

A black and white TV picture consists of 3×10^6 picture elements and 16 different brightness levels. Pictures are repeated at the rate of 32 per second. All picture elements are assumed to be independent and all levels have equal likelihood of occurrence. Find the average rate of information conveyed by this TV picture source.

Solution:

Since all the 16 levels are of equal probability, probability of each level $\frac{1}{16}$

$$H(X) = \log_2 16 = 4 \ ^{bits}/_{element}$$

$$\text{Symbol rate} = 3 \times 10^6 \times 32 = 96 \times 10^6 \ ^{elements}/_{sec}$$

$\therefore$ Information rate $R = r \cdot H(X)$

$$= 96 \times 10^6 \times 4$$

$$= 384 \ ^{Mb}/_{sec}$$

PROBLEM 13

A card is drawn at random from ordinary deck of 52 playing cards. Find the information in bits that you receive when you are total that the card is a heart, a face card, and a heart face card. [210402, Set – 2]

Solution:

$$P(\text{card to be heart}) = \frac{13}{52} = \frac{1}{4}$$

The corresponding $I = \log \frac{1}{P} = 2$ bits

$$P(\text{card to be a face card}) = \frac{12}{52}$$

Corresponding $I = \log \frac{1}{P} = 2.11$ bits

$$P(\text{card to be heart face card}) = \frac{3}{52}$$

Corresponding $I = 4.11$ bits

PROBLEM 14

A source generates 8 symbols A, B, C, D, E, F, G and H with probabilities 0.5, 0.2, 0.1, 0.05, 0.05, 0.05, 0.03 and 0.02. Find the entropy and information rate if the symbols are generated at a rate of 1000/sec. {210402, Set – 1]

Solution:

Entropy
$$H = -[0.5 \log 0.5 + 0.2 \log 0.2 + 0.1 \log 0.1 + 0.15 \log 0.05 + 0.03 \log 0.03 + 0.02 \log 0.02]$$
$$= 2.209 \text{ bits/symbol}$$

Information rate
$$= R$$
$$= r \cdot H$$
$$= (1000)\,(2.209)$$
$$= 2209 \text{ bits/sec}$$