CHAPTER-5
Waveguides

Course Name : Electromagnetic Fields and Waves

Course Code: EC402PC

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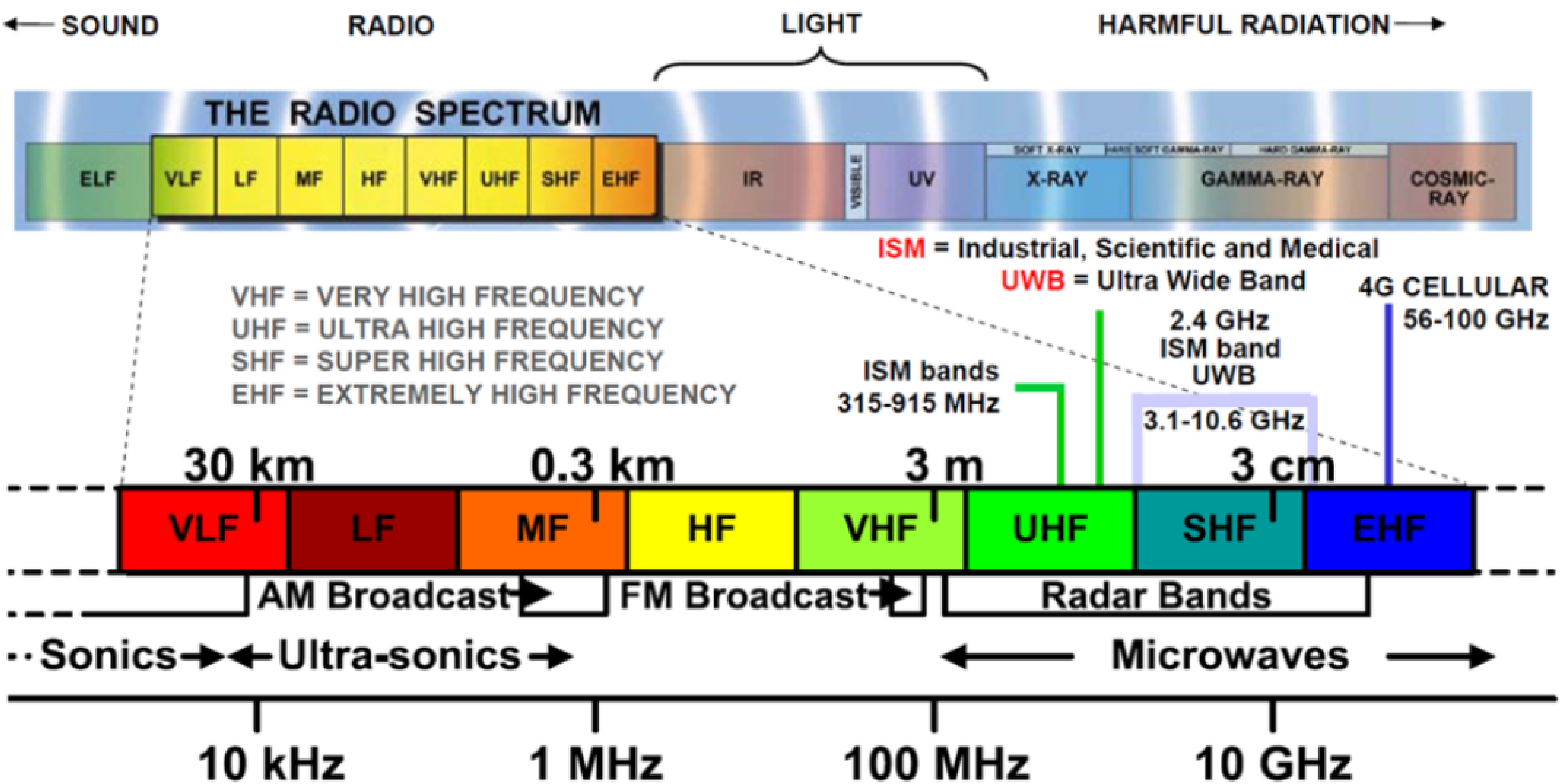
Contents

- Electromagnetic spectrum and bands
- Rectangular waveguides
- TE/TM Modes
- Calculation of velocities, wavelengths, impedances
- Cut-off frequencies, characteristic equation
- Power transmission
- Microstrip lines

Electromagnetic Spectrum



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Waveguides

- Waveguides guide the EM energy from generator to the load.
- A transmission line may regard as a waveguide.
- Waveguide supports many possible field configurations.
- Waveguides use at (3-300 GHz) frequencies to obtain larger bandwidth and lower attenuation.
- Waveguide can operate only above a certain frequency called cut-off frequency (HPF).
- Waveguide may take any arbitrary shape but uniform cross section.
- Common waveguides are either rectangular or circular.

Waveguides

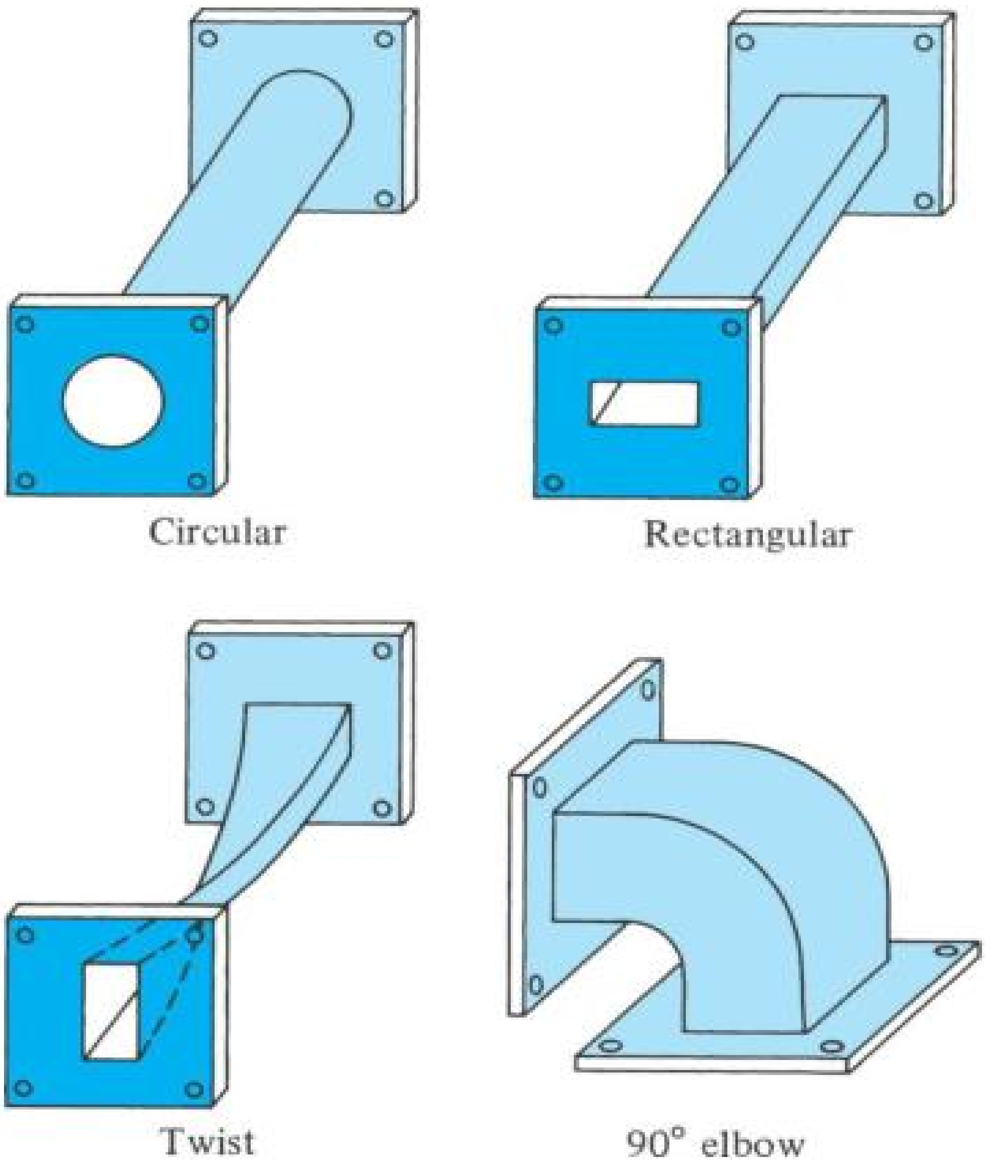

Fig. 5.1: Typical Waveguides

Rectangular Waveguides

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Rectangular Waveguides

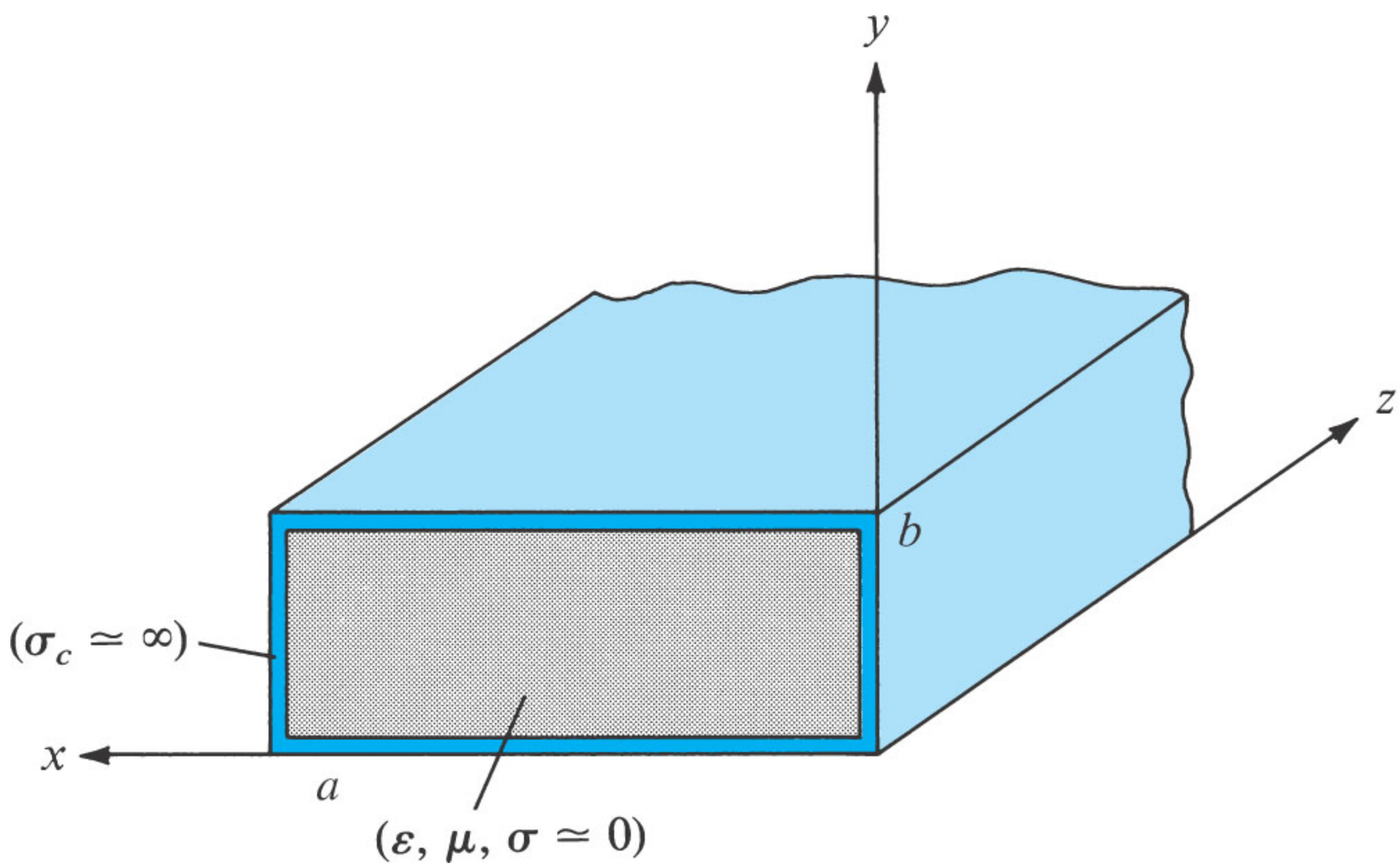

Fig. 5.2: Rectangular Waveguide

Rectangular Waveguides

RECTANGULAR WAVEGUIDES:

- Fig. 5.1: shows a rectangular WG with perfectly conducting walls and filled with a lossless material.

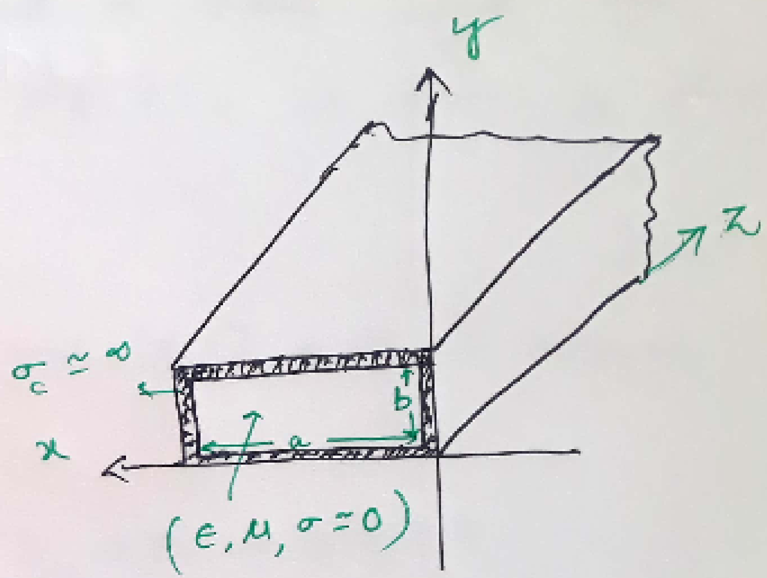

Fig. 5.1: Rectangular Waveguide

- From the wave equations (plane wave equations), we know that

$$\left.\begin{aligned} \nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s &= 0 \\ \nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s &= 0 \end{aligned}\right\}$$

where

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

Rectangular Waveguides

Similarly, for a lossless medium, the phasor form equations may be assumed as

$$\left.\begin{aligned} \nabla^2 \bar{E}_s + k^2 \bar{E}_s = 0 \\ \nabla^2 \bar{H}_s + k^2 \bar{H}_s = 0 \end{aligned}\right\}$$

where $\rightarrow (5.1)$

$$k^2 = \omega\sqrt{\mu\epsilon} \rightarrow (5.2)$$

If we let $\bar{E}_s = (E_{xs}, E_{ys}, E_{zs})$ and

$$\left.\begin{aligned} \bar{E}_s = (E_{xs}, E_{ys}, E_{zs}) \\ \bar{H}_s = (H_{xs}, H_{ys}, H_{zs}) \end{aligned}\right\}$$

Helmholtz eqns.

Rectangular Waveguides

✓ In order to obtain $\bar{E}$ and $\bar{H}$ fields inside the rectangular waveguide in Fig 5.1, we have to solve eqns (5.1) and (5.2).

✓ For the z-component, eqns (5.1) & (5.2) becomes

$$\frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0 \longrightarrow (5.3)$$

lly
$$\frac{\partial^2 H_{zs}}{\partial x^2} + \frac{\partial^2 H_{zs}}{\partial y^2} + \frac{\partial^2 H_{zs}}{\partial z^2} + k^2 H_{zs} = 0 \longrightarrow (5.4)$$

✓ Let us solve eqn (5.3), which is a partial DE.

✓ This can be solved by separation of variables, so let

$$E_{zs}(x, y, z) = X(x)\, Y(y)\, Z(z) \longrightarrow (5.5)$$

Rectangular Waveguides

⇒ where $X(x)$, $Y(y)$, and $Z(z)$ are functions of x, y, and z, respectively.

Substituting (5.5) in (5.3), we get

$$\Rightarrow Y(y)Z(z)\frac{\partial^2 X(x)}{\partial x^2} + X(x)Z(z)\frac{\partial^2 Y(y)}{\partial y^2} + X(x)Y(y)\frac{\partial^2 Z(z)}{\partial z^2}$$

$$= -k^2\left[X(x)\,Y(y)\,Z(z)\right]$$

$$\Rightarrow \frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2} + \frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2} + \frac{1}{Z(z)}\cdot\frac{\partial^2 Z(z)}{\partial z^2} = -k^2$$

Rectangular Waveguides

$$\Rightarrow \frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2 \longrightarrow (5.6)$$

- Since the variables are independent, each term in (5.6) must be a constant, and can be written as

$$-k_x^2 - k_y^2 + \gamma^2 = -k^2 \longrightarrow (5.7)$$

- where $-k_x^2$, $-k_y^2$, and γ^2 are separation constants.

$$\Rightarrow \frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k_x^2 - k_y^2 + \gamma^2$$

Rectangular Waveguides

comparing x, y, z terms on both sides,

$$\Rightarrow \quad \frac{X''}{X} = -k_x^2 \qquad\Big|\qquad X'' + k_x^2 X = 0$$

$$\frac{Y''}{Y} = -k_y^2 \qquad\Big|\qquad Y'' + k_y^2 Y = 0$$

$$\frac{Z''}{Z} = \gamma^2 \qquad\Big|\qquad Z'' - \gamma^2 Z = 0$$

$$\longrightarrow (5.8)$$

γ is chosen for z-component due to the fact that the guided wave propagate along the guide axis 'z' in the positive or negative direction, and the propagation may result in E_{zs} and H_{zs} that approach zero as $z \longrightarrow \pm\infty$.

Rectangular Waveguides

- The solution to eqns in (5.8) can be written as

$$\left.\begin{aligned} X(x) &= C_1 \cos K_x x + C_2 \sin K_x x \\ Y(y) &= C_3 \cos K_y y + C_4 \sin K_y y \\ Z(z) &= C_5 e^{\gamma z} + C_6 e^{-\gamma z} \end{aligned}\right\} \longrightarrow (5.9)$$

- Substituting (5.9) into (5.5) gives

$$E_{zs}(x, y, z) = (C_1 \cos K_x x + C_2 \sin K_x x)(C_3 \cos K_y y + C_4 \sin K_y y)(C_5 e^{\gamma z} + C_6 e^{-\gamma z}) \rightarrow (5.10)$$

- Since the direction of wave propagation in Fig 5.1 is $+z$-axis, the terms $e^{\gamma z}$ becomes zero.

Rectangular Waveguides

$$\Rightarrow \quad E_{zs}(x,y,z) = \left(C_1 C_6 \cos k_x x + C_2 C_6 \sin k_x x\right)\left(C_3 C_6 \cos k_y y + C_4 C_6 \sin k_y y\right) e^{-\gamma z}$$

$$\Rightarrow \quad E_{zs}(x,y,z) = \left(A_1 \cos k_x x + A_2 \sin k_x x\right)\left(A_3 \cos k_y y + A_4 \sin k_y y\right) e^{-\gamma z} \longrightarrow (5.11)$$

where $A_1 = C_1 C_6$, $A_2 = C_2 C_6$, $A_3 = C_3 C_6$, $A_4 = C_4 C_6$

Uly we get

$$H_{zs}(x,y,z) = \left(B_1 \cos k_x x + B_2 \sin k_x x\right)\left(B_3 \cos k_y y + B_4 \sin k_y y\right) e^{-\gamma z} \longrightarrow (5.12)$$

where $B_1 = d_1 d_6$, $B_2 = d_2 d_6$, $B_3 = d_3 d_6$, $B_4 = d_4 d_6$.

Rectangular Waveguides

- we use Maxwell's eqns to solve for E_{xs}, E_{ys}, H_{xs}, and H_{ys} from E_{zs} and H_{zs}.

we know $\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s \rightarrow (5.13)$

and $\nabla \times \bar{H}_s = j\omega\epsilon \bar{E}_s \rightarrow (5.14)$

from (5.13)

$$\nabla \times \bar{E}_s = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{xs} & E_{ys} & E_{zs} \end{vmatrix}$$

$$\nabla \times \bar{H}_s = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{xs} & H_{ys} & H_{zs} \end{vmatrix}$$

Rectangular Waveguides

$$\Rightarrow \left(\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z}\right)\bar{a}_x + \left(\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x}\right)\bar{a}_y + \left(\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y}\right)\bar{a}_z$$

$$= -j\omega\mu\left(H_{xs} + H_{ys} + H_{zs}\right)$$

$$\Rightarrow \left(\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z}\right)\bar{a}_x + \bar{a}_y\left(\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x}\right.$$

$$+ \left(\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y}\right)\bar{a}_z$$

$$= j\omega\epsilon\left(E_{xs} + E_{ys} + E_{zs}\right)$$

Rectangular Waveguides

Comparing (x, y, z) on both sides

$$\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} = -j\omega\mu H_{xs}$$

$$\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x} = -j\omega\mu H_{ys}$$

$$\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} = -j\omega\mu H_{zs}$$

$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} = j\omega\epsilon E_{xs}$$

$$\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} = j\omega\epsilon E_{zs}$$

$$\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} = j\omega\epsilon E_{ys}$$

$\longrightarrow$ (5.15)

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✓ We will now express E_{xs}, E_{ys}, H_{xs}, and H_{ys} in terms of E_{zs}, H_{zs}.

Let's take

$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} = j\omega\epsilon E_{xs}$$

we know $H_{ys} = \frac{1}{j\omega\mu}\left(\frac{\partial E_{zs}}{\partial x} - \frac{\partial E_{xs}}{\partial z}\right)$

substituting H_{ys} in the above eqn

$$\Rightarrow \frac{\partial H_{zs}}{\partial y} - \frac{\partial}{\partial z}\left\{\frac{1}{j\omega\mu}\left(\frac{\partial E_{zs}}{\partial x} - \frac{\partial E_{xs}}{\partial z}\right)\right\} = j\omega\epsilon E_{xs}$$

$$\Rightarrow \frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\left[\frac{\partial^2 E_{xs}}{\partial z^2} - \frac{\partial^2 E_{zs}}{\partial x \partial z}\right] = j\omega\epsilon E_{xs} \longrightarrow (5.16)$$

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from eqns (5.11) and (5.12) we know that all field components vary with z according to $e^{-\gamma z}$, that is,

$$E_{zs} = K_1 e^{-\gamma z} \qquad E_{xs} = K_2 e^{-\gamma z}$$

$$\frac{\partial E_{zs}}{\partial z} = -\gamma\left(K_1 e^{-\gamma z}\right) \qquad \frac{\partial E_{xs}}{\partial z} = -\gamma K_2 e^{-\gamma z}$$

$$= -\gamma E_{zs}$$

$$\frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 K_2 e^{-\gamma z}$$

$$= \gamma^2 E_{xs}$$

Substituting these in (5.16)

$$\Rightarrow \frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\left[\gamma^2 E_{xs} + \gamma \frac{\partial E_{zs}}{\partial x}\right] = j\omega\epsilon E_{xs}$$

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Rearranging the terms in the eqn ∴

$$\Rightarrow \frac{\partial H_{zs}}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_{zs}}{\partial x} = j\omega\epsilon E_{xs} - \frac{\gamma^2}{j\omega\mu} E_{xs}$$

$$= \frac{-1}{j\omega\mu}\left(\gamma^2 E_{xs} + j^2\omega^2\mu\epsilon E_{xs}\right)$$

$$= \frac{-1}{j\omega\mu}\left(\gamma^2 + \omega^2\mu\epsilon\right) E_{xs}$$

$$h^2 = \gamma^2 + \omega^2\mu\epsilon = \gamma^2 + k^2$$

$$\Rightarrow E_{xs} = \frac{-j\omega\mu}{h^2}\left(\frac{\partial H_{zs}}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_{zs}}{\partial x}\right)$$

$$= \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

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$$\therefore \left.\begin{aligned}
E_{xs} &= \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y} \\
E_{ys} &= \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} \\
H_{xs} &= \frac{j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial x} \\
H_{ys} &= \frac{-j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial y}
\end{aligned}\right\} \longrightarrow (5.17)$$

we know $h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2$ $\{\because (5.7)$

Rectangular Waveguides

TRANSVERSE MAGNETIC (TM) MODES:

- the magnetic field H has its components transverse (normal) to the direction of wave propagation.
- we set $H_z = 0$ and determine E_x, E_y, E_z, H_x, and H_y.
- First let us determine E_z from

$$E_{zs}(x,y,z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)\, e^{-\gamma z} \longrightarrow (5.11)$$

- At the walls (perfect conductor) of the rectangular waveguide, the tangential components of E be ~~y~~ continuous:

Rectangular Waveguides

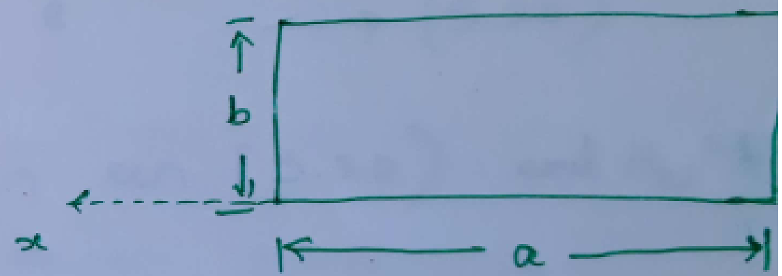
- At the walls (perfect conductor) of the rectangular waveguide, the tangential components of E be continuous:

at $y = 0$; $E_{zs} = 0$

at $y = b$; $E_{xs} = 0$

at $x = 0$; $E_{zs} = 0$

at $x = a$; $E_{zs} = 0$



- This is due to the boundary is a perfect conductor and there will not be any field exists on the other side of the boundary.

This is due to the boundary is a perfect conductor and there will not be any field exists on the other side of the boundary.

therefore, in eqn (5.11), $A_1 = 0 = A_3$

then
$$E_{zs}(x,y,z) = (A_2 \sin k_x x)(A_4 \sin k_y y)\, e^{-\gamma z}$$
$$= E_0 \sin k_x x \sin k_y y\, e^{-\gamma z} \longrightarrow (5.18)$$

here $E_0 = A_2 A_4$.

Rectangular Waveguides

Also $E_{zs} = 0$ at $y = b$ and $x = a$.

then $E_{zs}(x, y, z) = E_0 \sin k_x x \sin k_y y \, e^{-\gamma z}$

$= E_0 \sin k_x(a) \sin k_y(b) \, e^{-\gamma z} = 0$

i.e., $\sin k_x a = 0$ and $\sin k_y b = 0$

$\Rightarrow k_x a = m\pi$, $m = 1, 2, 3, \ldots$

$\Rightarrow k_y b = n\pi$; $n = 1, 2, 3, \ldots$

$k_x = \frac{m\pi}{a}$ and $k_y = \frac{n\pi}{b}$ $\longrightarrow$ (5.19)

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Substituting these values in eqn (5.18), we get

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \rightarrow (5.20)$$

now by taking E_{zs} as given in eqn (5.20) and $H_{zs}=0$,

we can define other components of E & H as

$$E_{xs} = \frac{-\gamma}{h^2} \frac{\partial}{\partial x}\left[E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}\right]$$

$$= \frac{-\gamma}{h^2} \left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_{xs} = \frac{-\gamma}{h^2} \left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \rightarrow (5.21)$$

$$E_{ys} = \frac{-\gamma}{h^2} \left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \rightarrow (5.22)$$

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$$H_{xs} = \frac{j\omega\epsilon}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z} \longrightarrow (5.22)$$

$$H_{ys} = \frac{-j\omega\epsilon}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z} \longrightarrow (5.23)$$

we know that $h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2$

$$\therefore\ h^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (5.24)$$

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✓ Since $h^2 = \gamma^2 + k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$$\Rightarrow \gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2$$

$$\Rightarrow \gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2} \longrightarrow (5.25)$$

where $k = \omega\sqrt{\mu\epsilon}$ and $\gamma = \alpha + j\beta$.

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Case (i):

when $\gamma = 0$ i.e., $\alpha = 0 = \beta$

$$\Rightarrow \quad k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega^2 \mu \epsilon$$

$$\Rightarrow \quad \omega^2 = \frac{1}{\mu\epsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\Rightarrow \quad \omega = \sqrt{\frac{1}{\mu\epsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$= \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Rectangular Waveguides

the value of 'ω' that causes $\gamma = 0$ $\{\alpha = \beta = 0\}$ is called the cut-off frequency 'ω_c'.

$$\therefore \quad \omega_c = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \longrightarrow (5.26)$$

that means no propagation takes place at 'ω_c'.

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Case (ii):

if $k^2 = \omega^2\mu\epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

and $\gamma = \alpha$; $\beta = 0$,

in this case, there is no wave propagation and these nonpropagating modes are said to be evanescent.

Case (iii):

If $k^2 = \omega^2\mu\epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$,

$\gamma = j\beta$ and $\alpha = 0$

then, eqn (5.25) becomes

$$j\beta = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2\right]^{1/2}$$

Squaring on both the sides

$$-\beta^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2$$

$$\beta = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \longrightarrow (5.27)$$

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Therefore, the cut-off frequency can be obtained (for a high-pass filter)

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} \longrightarrow (5.28)$$

where $u' = \frac{1}{\sqrt{\mu\epsilon}}$ is the phase velocity of the uniform planewave in the lossless dielectric medium ($\sigma = 0, \mu, \epsilon$) filling the waveguide.

The cut-off wavelength λ_c is given by

$$\lambda_c = \frac{u'}{f_c} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}} \longrightarrow (5.29)$$

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from (5.28) and (5.29) we can state that TM_{11} has the lowest cut-off frequency and the longest cut-off wavelength of the TM modes.

now $\beta = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$ and

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c \big/ f = \frac{1}{2\pi f\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

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$$\Rightarrow \left(f_c/f\right)^2 = \frac{1}{\omega^2\mu\epsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega^2\mu\epsilon\left(f_c/f\right)^2$$

Substituting this in β

$$\therefore \quad \beta = \sqrt{k^2 - \left[\omega^2\mu\epsilon\left(f_c/f\right)^2\right]} \qquad \text{where } k^2 = \omega^2\mu\epsilon$$

$$= \sqrt{\omega^2\mu\epsilon - \omega^2\mu\epsilon\left(f_c/f\right)^2}$$

$$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(f_c/f\right)^2} \longrightarrow (5.30)$$

$$\beta = \beta'\sqrt{1 - \left(f_c/f\right)^2} \longrightarrow (5.31)$$

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where $\beta' = \omega\sqrt{\mu\epsilon} = \omega/u'$ is the phase constant of the uniform plane wave in the dielectric medium.

✓ The propagation constant γ for evanescent mode can be defined as $\gamma = \alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$

$k^2 = \omega^2\mu\epsilon$ and $\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega^2\mu\epsilon\left(f_c/f\right)^2$

$$\gamma = \alpha = \sqrt{\omega^2\mu\epsilon\left(f_c/f\right)^2 - \omega^2\mu\epsilon} = \omega\sqrt{\mu\epsilon}\sqrt{\left(f_c/f\right)^2 - 1}$$

$$\alpha = \beta'\sqrt{\left(f_c/f\right)^2 - 1} \longrightarrow (5.32)$$

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✓ the phase velocity (u_p) and the wavelength in the guide are

$$u_p = \frac{\omega}{\beta}, \quad \lambda = \frac{2\pi}{\beta} = \frac{u_p}{f}$$

✓ The intrinsic wave impedance of the mode is

$$E_{xs} = \frac{-\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$\cancel{H_{xs} = \frac{j\omega\epsilon}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}}$$

$$H_{ys} = \frac{-j\omega\epsilon}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

Rectangular Waveguides

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$$= \frac{-\gamma}{h^2} \cdot \left(\frac{-h^2}{j\omega\epsilon} \right) = \frac{\gamma}{j\omega\epsilon}$$

$$\eta_{TM} = \frac{\gamma}{j\omega\epsilon} \quad ; \quad \text{here } \gamma = j\beta$$

$$\eta_{TM} = \frac{\beta}{\omega\epsilon} = \frac{\cancel{\omega}\sqrt{\mu\epsilon}}{\cancel{\omega}\epsilon} \sqrt{1-(f_c/f)^2}$$

$$= \sqrt{\frac{\mu}{\epsilon}} \sqrt{1-(f_c/f)^2}$$

$$\eta_{TM} = \eta' \sqrt{1-(f_c/f)^2} \longrightarrow (5.33)$$

η': intrinsic impedance of Uniform plane wave in the medium.

Rectangular Waveguides

TRANSVERSE ELECTRIC (TE) MODES:

- The electric field is transverse (or normal) to the direction of wave propagation.
- We set $E_z = 0$ and determine other field components E_x, E_y, H_x, H_y, and H_z.
- When $E_{zs} = 0$,

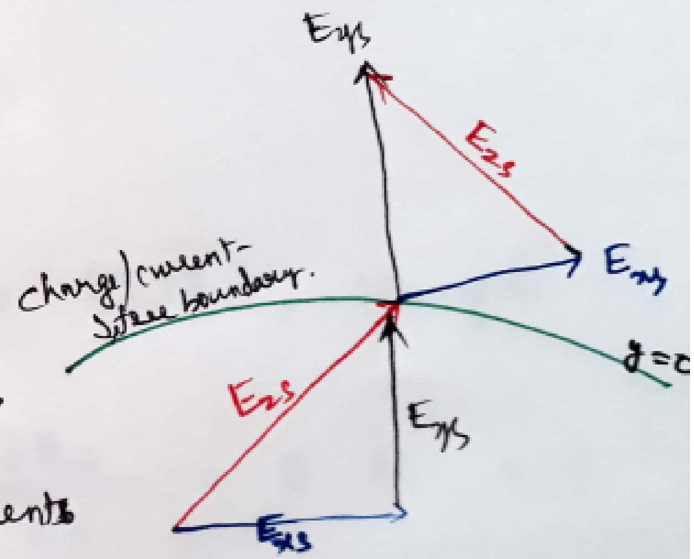
$$\left.\begin{aligned} E_{xs} &= \frac{-j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y} & H_{xs} &= \frac{-\gamma}{h^2}\frac{\partial H_{zs}}{\partial x} \\ E_{ys} &= \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} & H_{ys} &= \frac{-\gamma}{h^2}\frac{\partial H_{zs}}{\partial y} \end{aligned}\right\} \quad (5.17)$$

Rectangular Waveguides

when boundary conditions are applied, the tangential components of the electric field be continuous at the walls of the waveguide.

at $y=0$, (xz-plane),

when E_{zs} is the resulting vector (direction of wave propagation), E_{xs} will be it's tangential components and E_{ys} will be it's normal component.



Rectangular Waveguides

- Since the tangential components (E_{xs}) is continuous at the boundary,

$$E_{1t} = E_{xs} = E_{2t} = E_{xs}.$$

- but the boundary walls are made up of the perfect conductors, $E_{2t} = 0$.

$$\therefore \quad E_{1t} = E_{xs} = 0 \quad \text{at} \quad y = 0$$

$$\text{Illy} \quad E_{xs} = 0 \quad \text{at} \quad y = b$$

- at $x = 0$. (yz-plane), the tangential component will be E_{ys} and the normal component be E_{xs}.

Rectangular Waveguides

applying boundary components.

at $x=0$; $E_{ys}=0$

at $x=a$; $E_{ys}=0$

Substituting these values in (5.17), we get

$$\frac{\partial H_{zs}}{\partial y}=0 \quad \text{at } \cancel{x}\,y=0 \qquad\qquad \frac{\partial H_{zs}}{\partial x}=0 \quad \text{at } x=0$$

$$\frac{\partial H_{zs}}{\partial y}=0 \quad \text{at } y=b \qquad\qquad \frac{\partial H_{zs}}{\partial x}=0 \quad \text{at } x=a$$

Rectangular Waveguides

we know that

$$H_{zs}(x,y,z) = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)e^{-\gamma z} \rightarrow (5.12)$$

$$\frac{\partial H_{zs}}{\partial y} = (B_1 \cos k_x x + B_2 \sin k_x x)(-B_3 k_y \sin k_y y + B_4 k_y \cos k_y y)e^{-\gamma z} \longrightarrow (5.37)$$

at $y=0 \Rightarrow \frac{\partial H_{zs}}{\partial y} = 0$

$$\Rightarrow \frac{\partial H_{zs}}{\partial y} = (B_1 \cos k_x x)(B_4)e^{-\gamma z} = 0$$

- In this case, B_1, $\cos k_x x$, and $e^{-\gamma z}$ are not equal to zero.
- So, ~~that~~ in order to satisfy the boundary conditions B_4 must be zero.

Rectangular Waveguides

Similarly, B_2 must be zero to satisfy $(x=0)$.

Now the eqn (5.12) becomes

$$H_{zs}(x, y, z) = (B_1 \cos k_x x)(B_3 \cos k_y y)\, e^{-\gamma z}$$

$$= B_1 B_3 \cos k_x x \cos k_y y\, e^{-\gamma z}$$

$$\therefore \quad H_{zs}(x, y, z) = H_0 \cos k_x x \cos k_y y\, e^{-\gamma z} \rightarrow (5.38)$$

Since (5.36) $\frac{\partial H_{zs}}{\partial x} = 0$ at $x = a$

$$\Rightarrow \frac{\partial H_{zs}}{\partial x} = H_0 k_x (-\sin k_x x) \cos k_y y\, e^{-\gamma z}$$

Rectangular Waveguides

at $x = a$,

$$\Rightarrow \frac{\partial H_{zs}}{\partial x} = -H_0 K_x \sin(K_x a) \cos k_y y e^{-\gamma z} = 0$$

$$\Rightarrow \sin k_x a = 0 \quad \Rightarrow \quad K_x a = m\pi$$

$$\Rightarrow \quad K_x = \left(\frac{m\pi}{a}\right), \quad m = 1, 2, 3, ---$$

$$\longrightarrow (5.39)$$

Since (5.36) $\frac{\partial H_{zs}}{\partial y} = 0$ at $y = b$

$$\Rightarrow \frac{\partial H_{zs}}{\partial y} = -H_0 k_y \cos K_x x \sin k_y y e^{-\gamma z}$$

at $y = b$,

$$\Rightarrow \frac{\partial H_{zs}}{\partial y} = -H_0 k_y \cos K_x x \sin k_y b e^{-\gamma z} = 0$$

Rectangular Waveguides

$$\Rightarrow \quad \sin K_y b = 0 \quad \Rightarrow \quad K_y b = n\pi$$

$$\Rightarrow \quad K_y = \frac{n\pi}{b}, \quad n = 1, 2, 3, \ldots \longrightarrow (5.40)$$

✓ Substituting (5.39) and (5.40) into (5.38) gives us

$$H_{zs}(x, y, z) = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \longrightarrow (5.41)$$

Now by taking H_{zs} as given in (5.41) and $E_{zs} = 0$ (TE),
we can define other components of $\bar{E}$ & $\bar{H}$ as ($\because$ 5.34)

$$E_{xs} = \frac{-j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

$$= \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[H_0 \cos k_x x \cos k_y y \, e^{-\gamma z}\right]$$

$$= \frac{-j\omega\mu}{h^2} H_0 \cos k_x x \left[-k_y \sin k_y y\right] e^{-\gamma z}$$

$$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_{ys} = \frac{-j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

Rectangular Waveguides

$$H_{xs} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{ys} = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$\longrightarrow (5.42)$$

now, $h^2 = \gamma^2 + k^2 = K_x^2 + K_y^2$

and $h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (5.43)$

which is similar to TM mode.

and $\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2} \longrightarrow (5.44)$$

Rectangular Waveguides

✓ This implies that the lowest mode can be TE_{10} and TE_{01} depending on the values of a and b (the dimensions of the waveguide).

✓ It is standard practice to have $a > b$ so that $1/a^2 < 1/b^2$ in dimensions of the guide.

since $f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$TE_{10} \Rightarrow f_{c_{TE_{10}}} = \frac{u'}{2}\left(\frac{1}{a}\right) = \frac{u'}{2a}$

$TE_{01} \Rightarrow f_{c_{TE_{01}}} = \frac{u'}{2}\left(\frac{1}{b}\right) = \frac{u'}{2b}$.

Rectangular Waveguides

✓ Since $a > b$, $f_{c_{TE_{01}}} > f_{c_{TE_{10}}}$. Thus TE_{10} mode is the lowest mode and called the dominant mode of the waveguide.

✓ The dominant mode cut-off frequency

$$f_{c_{TE_{10}}} = \frac{u'}{2a} \longrightarrow (5.43)$$

and the cut-off wavelength

$$\lambda_{c_{TE_{10}}} = 2a. \longrightarrow (5.44)$$

✓ The cut off frequency for TM_{11} is

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{u'}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

Rectangular Waveguides

$$f_{c_{TM11}} = \frac{u' (a^2+b^2)^{1/2}}{2ab} \longrightarrow (5.45)$$

$f_{c_{TM11}} > f_{c_{TE10}}$, hence, TM_{11} cannot be regarded as the dominant mode.

* the dominant mode is the mode with the lowest cut-off frequency (& longest cut-off wavelength).

Rectangular Waveguides

✓ Also any EM wave with frequency $f < f_{c_{10}}$ (or $\lambda > \lambda_{c_{10}}$) will not be propagated in the guide.

✓ The intrinsic impedance for TE mode is

for $\gamma = j\beta$ case,

$$\eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

from eqn (5.42)

$$E_x = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_y = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$j\omega\mu$

Rectangular Waveguides

$$\eta_{TE} = \frac{E_x}{H_y} = \frac{j\omega\mu}{\cancel{h^2}} \cdot \frac{\cancel{h^2}}{\gamma} = \frac{j\omega\mu}{\gamma} = \frac{\cancel{j}\omega\mu}{\cancel{j}\beta}$$

$$\eta_{TE} = \frac{\omega\mu}{\beta} \longrightarrow (5.46)$$

for TM Mode since eqn (5.33a)

$$\eta_{TM} = \frac{\beta}{\omega\epsilon}$$

{ η_{TE} and η_{TM} both are not the same, eventhough f_c, λ_c, λ, are the same for both TE and TM }.

Rectangular Waveguides

$$\eta_{TE} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{1}{\sqrt{1-(f_c/f)^2}}$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-(f_c/f)^2}} \longrightarrow (5.47)$$

from eqns (5.33b) and (5.47) it can be noted that both η_{TM} and η_{TE} are purely resistive and vary with frequency.

also $\eta_{TE}\,\eta_{TM} = \eta'^2 \longrightarrow (5.48)$

Power Transmission And Attenuation

- We use Poynting theorem to calculate the power.
- The average Poynting vector

$$P_{avg} = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\} \rightarrow (5.49)$$

we assume the wave propagation is in $+z$ direction.

Hence,

$$P_{avg} = \frac{1}{2} Re\{E_{xs} H_{ys}^* - E_{ys} H_{xs}^*) \bar{a}_z$$

$$= \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta} \bar{a}_z \rightarrow (5.50)$$

$\{ \because H_{ys} = \frac{E_{xs}}{\eta}$

$\{ \text{lly } H_x = \frac{H_{xs}}{\eta}$

- $\eta = \eta_{TE}$ for TE modes

Rectangular Waveguides

- $\eta = \eta_{TE}$ for TE modes

$\eta = \eta_{TM}$ for TM modes.

$$\left\{ \because \; H_{ys} = \frac{E_{xs}}{\eta} \right.$$

$$\left\{ \begin{array}{l} \text{|ly} \quad H_x = \frac{H_{xs}}{\eta} \\ H_{xs} = \frac{E_{ys}}{\eta}. \end{array} \right.$$

- The total average power transmitted across the cross section of the WG.

$$P_{avg} = \int \bar{P}_{avg} \cdot d\bar{s}$$

$$= \int_{x=0}^{a} \int_{y=0}^{b} \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta} \, dy \, dx \longrightarrow (5.51)$$

- This is used to calculate the attenuation in a lossy waveguide.

✓ Attenuation describes the phenomena of reduction of power intensity according to the law

$$P_{avg} = P_0 e^{-2\alpha z} \longrightarrow (5.52)$$

$$\alpha = \alpha_c + \alpha_d$$

α_c : attenuation due to conduction loss (or) ohmic loss.

α_d : dielectric loss.

✓ We know that

$$\gamma = \alpha_d + j\beta_d$$

$$= \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$$

here $k = \omega^2 \mu \epsilon_c \longrightarrow (5.53)$

$$\epsilon_c = \epsilon' - j\epsilon'' = \epsilon - j\frac{\sigma}{\omega} \longrightarrow (5.54)$$

Rectangular Waveguides

$$\gamma^2 = (\alpha_d + j\beta_d)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\left(\epsilon - j\frac{\sigma}{\omega}\right)$$

$$= \alpha_d^2 - \beta_d^2 + 2j\alpha_d\beta_d = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon + j\omega\mu\sigma$$

equating real and imaginary parts.

$$\alpha_d^2 - \beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

$$2\alpha_d\beta_d = \omega\mu\sigma \Rightarrow \alpha_d = \frac{\omega\mu\sigma}{2\beta_d} \longrightarrow (5.55)$$

* assuming $\alpha_d^2 << \beta_d^2$, $\alpha_d^2 - \beta_d^2 \simeq -\beta_d^2$

$$\therefore -\beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

Rectangular Waveguides

$$\therefore \; -\beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

$$\beta_d = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$= \omega\sqrt{\mu\epsilon}\,\sqrt{1-\left(f_c/f\right)^2} \longrightarrow (5.56)$$

now

$$\alpha_d = \frac{\cancel{\omega}\mu\sigma}{2\cancel{\omega}\sqrt{\mu\epsilon}}\left(\sqrt{1-\left(f_c/f\right)^2}\right)^{-1}$$

$$= \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} \cdot \frac{1}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-\left(f_c/f\right)^2}}$$

where $\eta' = \sqrt{\mu/\epsilon}$

$\longrightarrow (5.57)$

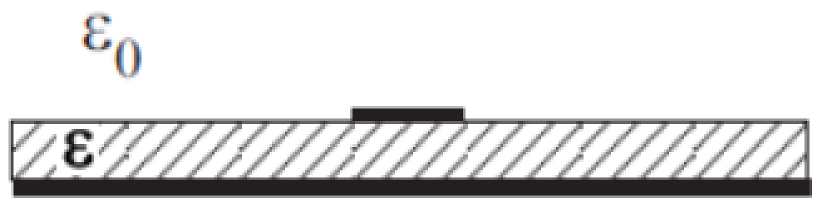
Rectangular Waveguides

$$\alpha_c\Big|_{TE} = \frac{2R_s}{b\eta'\sqrt{1-\left(f_c/f\right)^2}}\left[\left(1+\frac{b}{a}\right)\left(f_c/f\right)^2 + \frac{\frac{b}{a}\left(\frac{b}{a}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\left(1-\left(\frac{f_c}{f}\right)^2\right)\right]$$

$$\alpha_c\Big|_{TM} = \frac{2R_s}{b\eta'\sqrt{1-\left(f_c/f\right)^2}}\;\frac{\left(b/a\right)^3 m^2+n^2}{\left(b/a\right)^2 m^2+n^2}$$

Microstrip Transmission Lines

- ✓ Transmission Lines with conductors embedded in an inhomogeneous dielectric medium can not support TEM.



- ✓ Part of the field is in the air and part in the dielectric between the strip conductor and ground.
- ✓ In practice, the dielectric substrate is electrically thin ($h \ll \lambda$).
- ✓ The electric field is transverse and is called quasi-TEM.
- ✓ The line capacitance

$$C = \frac{Q}{V_0}$$

- ✓ The capacitance of the line without the dielectric is C_{air}.

Microstrip Transmission Lines

$$LC_{air} = \mu_0 \epsilon_0$$

- ✓ The velocity in the vacuum filled line with a TEM field distribution is identical to that in free space, where

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

- ✓ Thus L becomes

$$L = \frac{1}{c^2 C_{air}}$$

- ✓ L is not affected by dielectric properties of the medium.
- ✓ The characteristic impedance

$$Z_0 = \sqrt{\frac{L}{C}}$$

Microstrip Transmission Lines

$$Z_0 = \frac{1}{c}\frac{1}{\sqrt{CC_{air}}}$$

- ✓ The phase velocity is given by

$$v_p = \frac{1}{\sqrt{LC}} = c\sqrt{\frac{C_{air}}{C}}$$

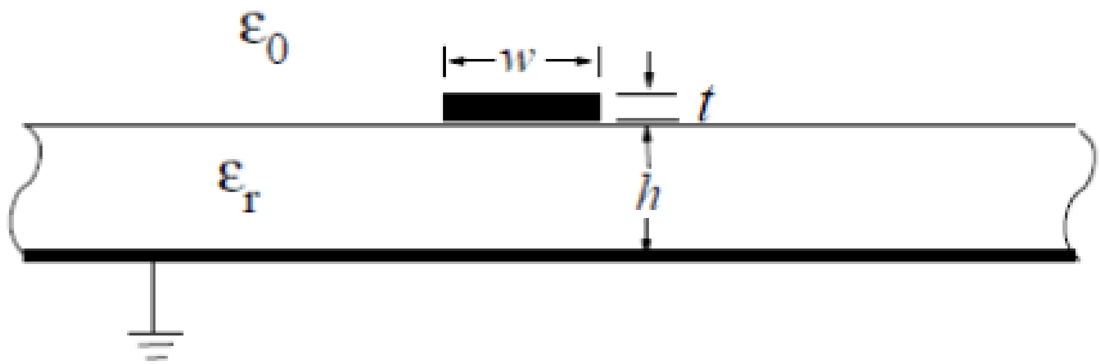
- ✓ Since the field is distributed in the homogeneous medium and in free space, the effective permittivity of the microstrip line is

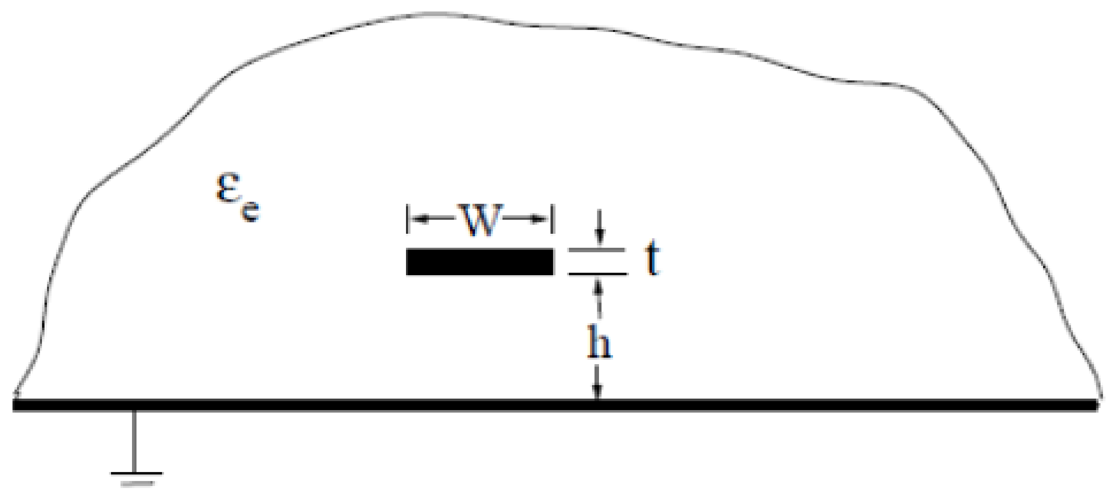
$$\sqrt{\epsilon_e} = \frac{c}{v_p}$$

$$\epsilon_e = \frac{C}{C_{air}}$$

- $1 < \epsilon_e < \epsilon_r$

Microstrip Transmission Lines





Microstrip Transmission Lines

Effective Dielectric Constant and Characteristic Impedance

- Hammerstad formula for effective dielectric constant is

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left(1 + \frac{10h}{w}\right)^{-a\cdot b}$$

$$a(u)\big|_{u=w/h} = 1 + \frac{1}{49}\ln\left[\frac{u^4 + \{u/52\}^2}{u^4 + 0.432}\right] + \frac{1}{18.7}\ln\left[1 + \left(\frac{u}{18.1}\right)^3\right]$$

$$b(\epsilon_r) = 0.564\left[\frac{\epsilon_r - 0.9}{\epsilon_r + 3}\right]^{0.053}$$

Microstrip Transmission Lines

Effective Dielectric Constant and Characteristic Impedance

$$Z_0 = \frac{Z_{01}}{\sqrt{\epsilon_e}}$$

$$Z_{01} = Z_0\big|_{(\epsilon_r=1)} = 60\ln\left[\frac{F_1 h}{w} + \sqrt{1 + \left(\frac{2h}{w}\right)^2}\right]$$

$$F_1 = 6 + (2\pi - 6)\exp\left\{-(30.666h/w)^{0.7528}\right\}$$

Textbooks & Reference books

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