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192

-: HAND WRITTEN NOTES:-  
OF

Electronic & communication  
Mechanical ENGG.

(ECE)

(I)

Electronic & Communication  
ENGG.

-: SUBJECT:-

# CONTROL SYSTEM

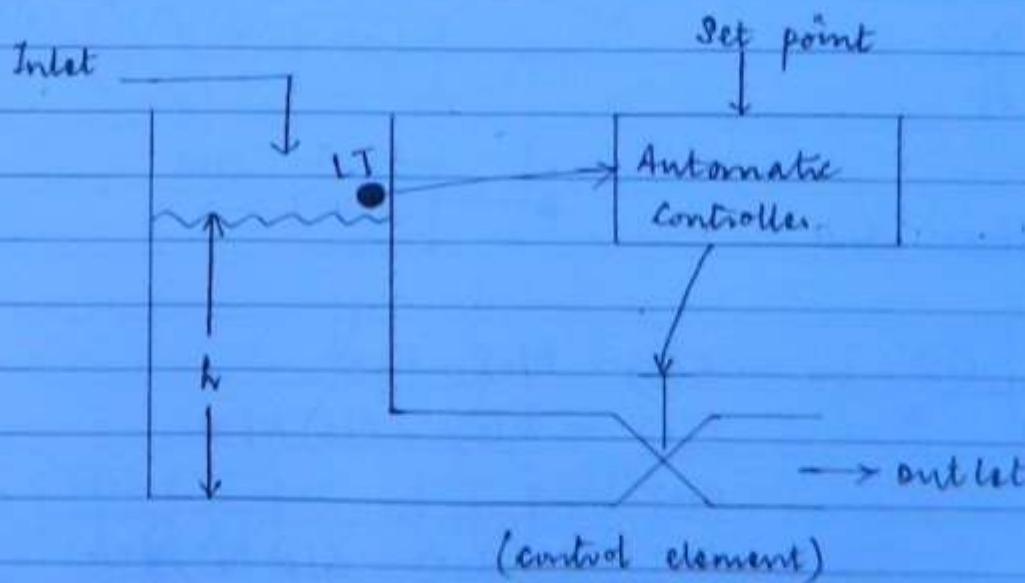
②

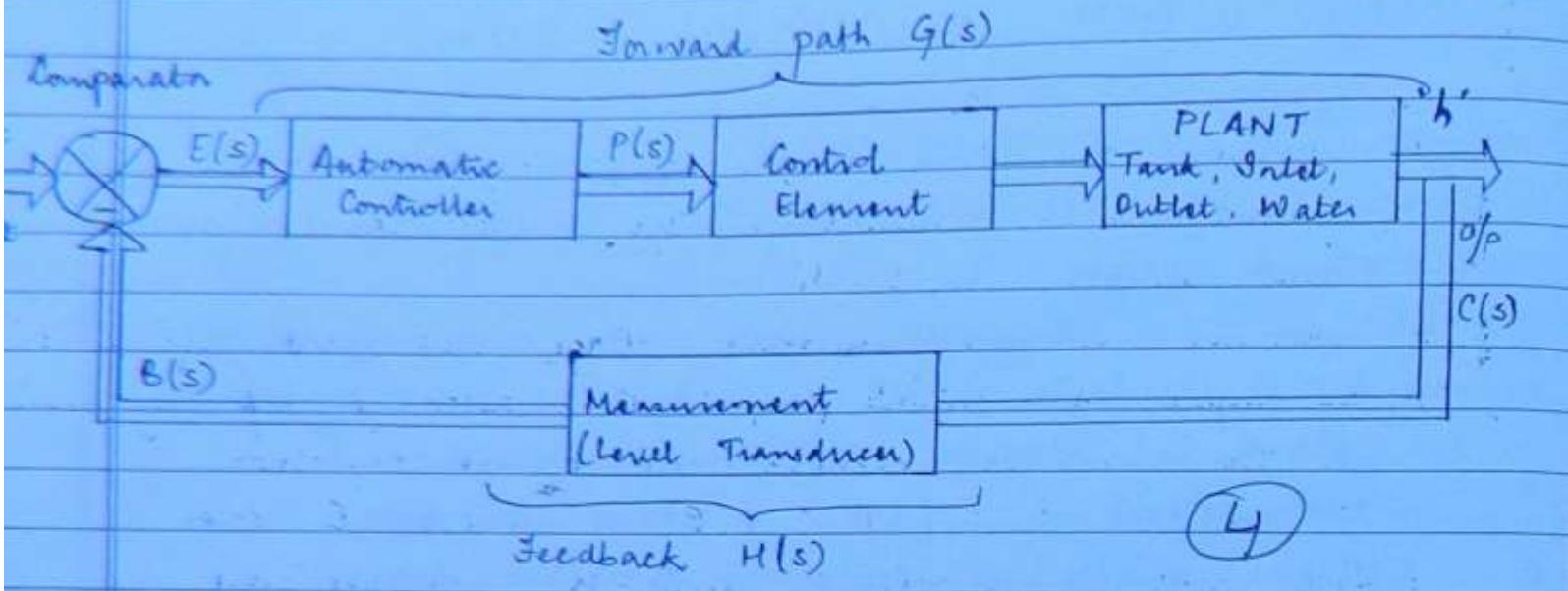
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# INTRODUCTION TO CONTROL SYSTEMS

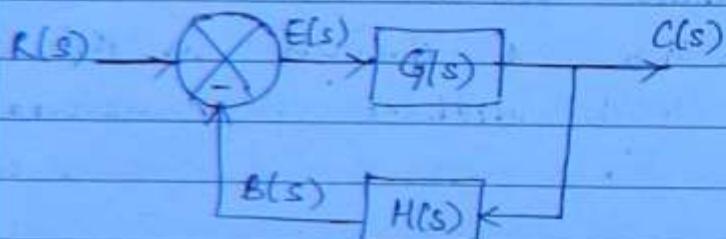
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1. Consider a liquid level control system whose control objective is to maintain the water level in the tank at a height 'h'
2. Controller is an automatic device with error signal  $E(s)$  as input & controller output  $P(s)$  affecting the dynamics on the plant to achieve the control objective.  
∴ Controller output  $P = f(E)$  where  $E = \text{error}$
3. The different modes of controller operation are proportional, proportional + integral & proportional + derivative
4. There are 2 basic control loop configurations
  - i) Closed loop (or) Feedback control system
    - In this configuration, the changes in the output are measured through feedback & compared with the input or "set point" to achieve control objective
    - Feedback implies measurement (sensors or transducers)





→ Control Canonical Form



→ Eq. Mathematical Form

$$\begin{array}{c|c|c} R(s) & G(s) & C(s) \\ \hline & 1 + H(s)G(s) & \end{array}$$

$$E(s) = R(s) - B(s)$$

$$\underline{C(s)} = R(s) - C(s) H(s)$$

$$G(s)$$

$$B(s) = G(s)R(s) - G(s)H(s) \cancel{C(s)}$$

$$C(s) - [H(s)G(s) + 1] = G(s)R(s)$$

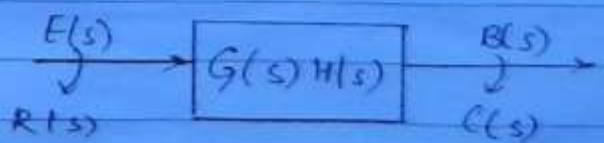
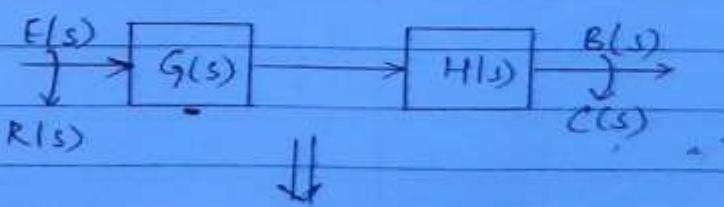
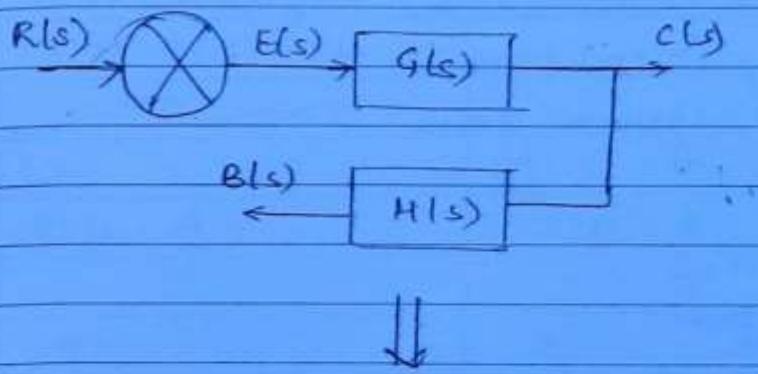
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stable subject to disturbance about his

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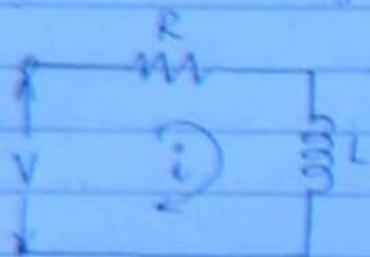
## ii) Open Loop Control Systems -

- In this configuration, the feedback or measurement is not connected to the forward path or controller (open loop)
- These systems are conditional control systems, formulated under the basic condition that the system is not subject to any type of disturbance.
- Feedback in an open loop system except for displaying the information about the output has no major significance. This insignificance of feedback is termed as elimination / removal of feedback.
- Open Loop systems are more stable than closed loop systems & hence do not require any performance analysis.
- The application of Open Loop control is limited because they are conditional control systems.



## CONCEPT OF TRANSFER FUNCTIONS -

It is a mathematical model representing a control system relating input & output in the form of the ratio i.e.  $\frac{\text{Output}}{\text{Input}}$



$$V = Ri + L \frac{di}{dt}$$

Applying L.T.

$$\begin{aligned} V(s) &= I(s)R + LsI(s) \\ &= I(s)[R + Ls] \end{aligned}$$

$$\underline{I(s)} = \underline{1}$$

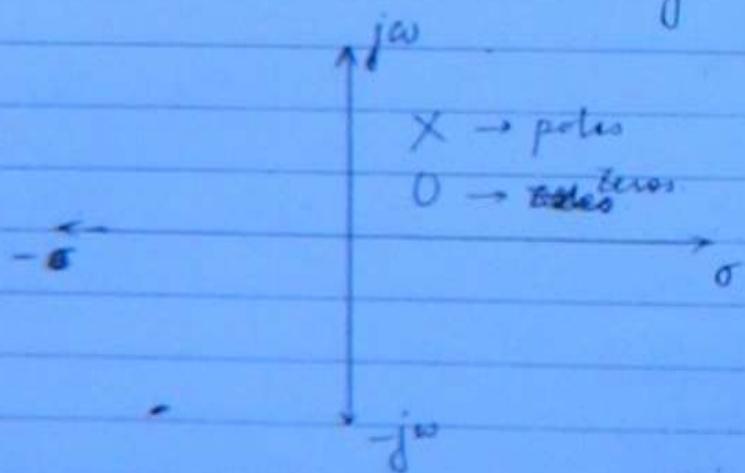
$$\underline{V(s)} = \underline{R + Ls}$$

$$\underline{I(s)} = \underline{\frac{1}{R + Ls}}$$

$$\underline{V(s)} = \underline{\left( s + \frac{R}{L} \right)}$$

$$\text{TF} = F(s) = \frac{N(s)}{D(s)} = \frac{C(s)}{R(s)} = \frac{k(s + z_1)(s + z_2)}{(s + p_1)(s + p_2)} \quad (1)$$

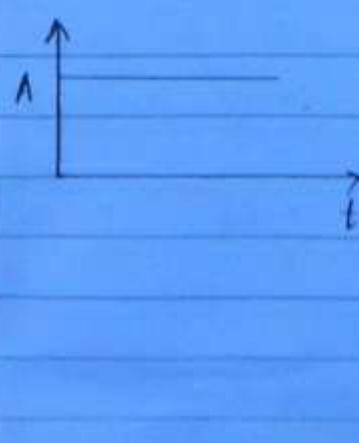
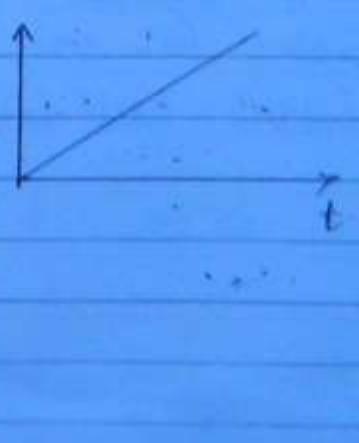
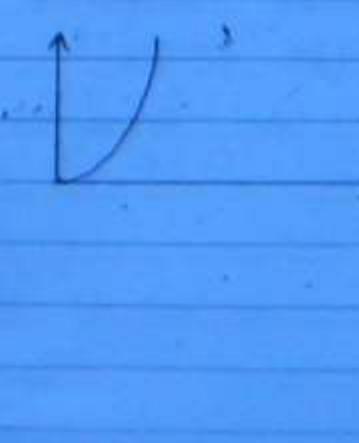
$k$  = system gain



$$\begin{aligned} s &= \sigma + j\omega \\ (\text{s-plane}) \end{aligned}$$

- \* From eq ①, transfer function of LTI system may be defined as ratio of Laplace transform of output to Laplace transform of input under the assumption that systems initial conditions are zero
- \* Poles & zeros are those critical frequencies which make the transfer function infinity or zero. Poles & zeros represent system parameters only.

Singularity Functions of Transfer Functions -  
the standard time domain test signals are also known as singularity functions which define the transfer function of LTI system

Impulse signal	Step Signal	Ramp Signal	Parabolic Signal
$u(t) = 1 \text{ for } t=0$ $= 0 \text{ for } t \neq 0$	$u(t) = A u(t)$ $= 0 \text{ for } t < 0$ $= 1 \text{ for } t > 0$	$\eta(t) = At \quad t > 0$ $= 0 \quad t < 0$	$\eta(t) = At^2/2 \quad t > 0$ $= 0 \quad t < 0$
L.T	<u>L.T.</u>	<u>L.T.</u>	<u>L.T.</u>
$(s) = 1$	$R(s) = A$ $s$	$R(s) = \frac{A}{s^2}$	$R(s) = \frac{A}{s^3}$
			

$$T.F = F(s) = \frac{C(s)}{R(s)}$$

(8)

$$C(s) = F(s) \times R(s)$$

Let  $R(s) = \text{Impulse signal} = 1$

$C(s) = \text{Impulse Response}$

$$= F(s) \times 1$$

$$\therefore T.F = F(s) \rightarrow ②$$

$\underline{L}(\text{Impulse Response}) = T.F.$

[WEIGHTING FUNCTION]

$\frac{d}{dt}(\text{Parabolic Response}) = \text{Ramp response}$

$\frac{d}{dt}(\text{Ramp Response}) = \text{Step response}$

$\frac{d}{dt}(\text{Step Response}) = \text{Impulse Response}$

$d(\text{Impulse Response}) = T.F.$

The impulse response of a system is

$$C(t) = -te^{-t} + 2e^{-t} \quad (t > 0)$$

Its open loop transfer function will be

a)  $\frac{2s+1}{(s+1)^2}$

b)  $\frac{2s+1}{s+1}$

c)  $\frac{2s+1}{s^2}$

d)  $\frac{2s+1}{s}$

$$C(t) = -te^{-t} + 2e^{-t}$$

$$T.F. = C(s) = \frac{-1}{(s+1)^2} + \frac{2}{s+1} = \frac{2s+1}{(s+1)^2}$$

$$TF = \frac{2s+1}{(s+1)^2}$$

(9)

$$\frac{G(s)}{1 + H(s)G(s)} = \frac{2s+1}{(s+1)^2}$$

$$H(s)G(s) = ?$$

$$\text{ Ideally } H(s) = 1$$

$$\frac{G(s)}{1 + G(s)} = \frac{2s+1}{(s+1)^2}$$

$$\Rightarrow G(s)[(s+1)^2] = [1 + G(s)](2s+1)$$

$$\Rightarrow G(s)[(s+1)^2 - (2s+1)] = 2s+1$$

$$\Rightarrow G(s) = \frac{2s+1}{s^2} \quad (0)$$

Shortcut

$$TF = \frac{2s+1}{(s+1)^2}$$

$$G(s) = \frac{2s+1}{(s+1)^2 - (2s+1)} \quad \text{only for } H(s) = 1 \quad \frac{N}{D-N}$$

$$G(s) = \frac{2s+1}{s^2}$$

CWB

chapter 2

Q8. What is the open loop DC gain of a unity negative feedback system having closed loop TF  $\frac{s+4}{s^2 + 7s + 13}$

- a)  $\frac{4}{13}$    b)  $\frac{4}{9}$    c) 4   d) 13

10)

$$TF = \frac{s+4}{s^2 + 7s + 13}$$

$$\frac{G(s)H(s)}{1 + G(s)H(s)} = \frac{s+4}{s^2 + 7s + 13 - (s+4)} = \frac{s+4}{s^2 + 6s + 9}$$

$$G(s) = \frac{s+4}{s^2 + 6s + 9}$$

$$H(s) = 1$$

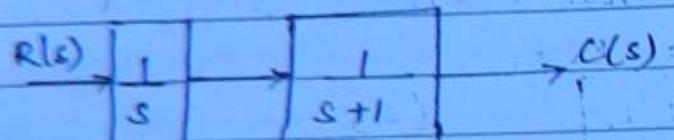
(10)

$$s > 0$$

$$G(s) = \frac{0+4}{0+0+9}$$

$$G(s) = \frac{4}{9} \quad \text{(b)}$$

Q3. Find the impulse response of the system



$$\text{Impulse response} = L^{-1}(\text{TF})$$

$$= L^{-1}\left[\frac{1}{s(s+1)}\right] \rightarrow \text{partial fractions}$$

$$= L^{-1}\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

$$= 1 - e^{-t} \quad \text{Ans}$$

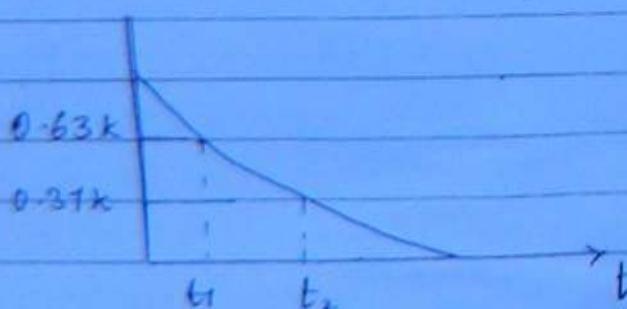
→ X →

Q4. The Impulse Response of the system having

$$\text{T.F. } C(s) = K$$

$$R(s) = \frac{K}{(s+\alpha)}$$

is shown in figure



The value of  $\alpha$  will be

- a)  $t_1$    b)  $\frac{1}{t_1}$    c)  $t_2 - t_1$    d)  $\frac{1}{t_2}$

$$\text{Sol} \quad \text{Imprvse Response} = L^{-1}(T.F) = L^{-1}\frac{K}{s+a} = \frac{Ke^{-at}}{s+a} \quad (1)$$

At  $t = t_2$

$$Ke^{-at_2} = 0.37 K$$

$$e^{-at_2} = 0.37$$

$$ke^{-at_2} = ke^{-t}$$

$$at_2 = 1$$

$a = \frac{1}{t_2}$	Answ
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Q5. A certain CS. has input  $R(t)$  &  $C(t)$  output  
If the input is first passed through a block having  
TF  $e^{-s}$  and then applied to the system, the  
modified output will be

- a)  $C(t-1) u(t)$
- b)  $C(t) u(t-1)$
- c)  $C(t-1) u(t-1)$
- d) None

Sol (c) By Laplace transformation theorem

$$R(s) \rightarrow [F(s)] \rightarrow C(s)$$

$$C(s) = R(s) F(s)$$

$$R(s) \xrightarrow{e^{-s}} [F(s)] \rightarrow C_m(s)$$

$$C_m(s) = R(s) e^{-s} F(s)$$

$$C_m(s) = C(s) e^{-s}$$

$$L^{-1} F(s) e^{-as} = \int (t-a) u(t-a) \cdot$$

$$L^{-1} C(s) e^{-s} = C(t-1) u(t-1) \rightarrow \text{Ans}$$

8. The Unit Step Response of the system is

$$Y(t) = te^{-t} \quad (t > 0)$$

So T.F will be

(12)

- a)  $\frac{1}{(s+1)^2}$
- b)  $\frac{1}{(s+1)}$
- c)  $\frac{s}{(s+1)^2}$
- d)  $\frac{s}{(s+1)}$

b) T.F =  $\frac{\mathcal{L}(te^{-t})}{R(s)} C(s)$

$$= \frac{1}{s^2} \cdot \frac{s}{(s+1)^2} - (C)$$

Using (3)  $\Rightarrow (1) \times \frac{(s+1)^2}{s^2} \Rightarrow (s+1)^2$

general result a repeated root  $\frac{1}{s^2}$  is required all the  
and we have to change constant value in the denominator

(or)

$\frac{d}{dt}$  (Step Response) = Impulse Response

$$\frac{d}{dt} Y(t) = Y'(t) = -te^{-t} + e^{-t}$$

~~$t \neq 0$~~

$$\mathcal{L}(Y'(t)) = T.F = \frac{-1}{(s+1)^2} + \frac{1}{(s+1)}$$

$$T.F = \frac{s}{(s+1)^2}$$

Q. The step response of the system is  
 $C(t) = 5 - \frac{4}{8}e^{-2t} + 8e^t \quad (t > 0)$

(13)

The gain of the T.F. in time constant form will be  
 a) -7.5 b) -7 c) 7.5 d) 7.

so)  $T.F. = \frac{C(s)}{R(s)}$

$\frac{d}{dt}$  (Step response) = Impulse Response

$$\begin{aligned}\frac{d}{dt}(C(t)) &= C'(t) = \frac{-4}{8}(-2)e^{-2t} + 8(-1)e^{-t} \\ &= e^{-2t} - 8e^{-t}\end{aligned}$$

$$\begin{aligned}L(C'(t)) &= T.F. = \frac{1}{(s+2)} - \frac{8}{(s+1)} \\ &= \frac{(s+1) - 8(s+2)}{(s+2)(s+1)} \\ &= -\frac{7s - 15}{(s+2)(s+1)} \\ &= -\frac{7(s + 15/7)}{(s+2)(s+1)}\end{aligned}$$

Time constant form  $(1 \pm Ts)$

$$\begin{aligned}T.F. &= -7 \times \frac{15}{7} \left[ 1 + \frac{7s}{15} \right] \\ &= \frac{2[1 + 0.5s]}{(1 + 0.5s)(1 + s)} \\ &= -7.5 \left( 1 + \frac{7s}{15} \right)\end{aligned}$$

→ depend only on poles  
 known as natural time constant (GATE)

$K = -7.5$	(a)
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Introduction to COMPENSATORS

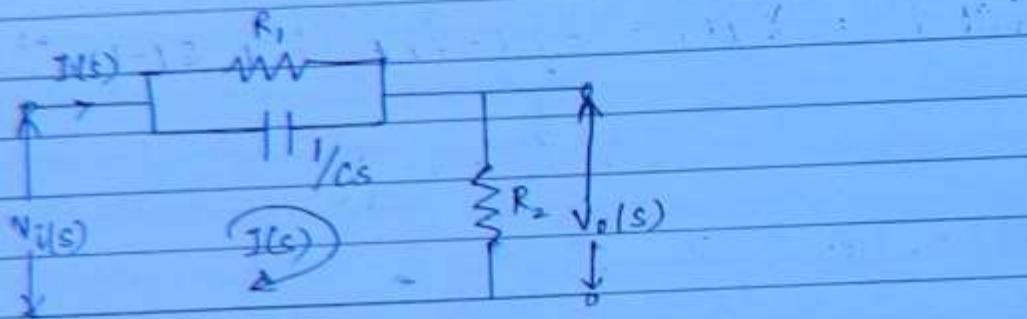
(14)

Compensators in control Systems are used for improving the performance specifications i.e transient & steady state response characteristics.

They are of 3 types -

### 1. Lead Compensator

It is used for improving the transient state or speed of response of the system.



$$V_o(s) = I(s) \left[ \frac{R_1 + R_2}{R_1 C_s + 1} \right]$$

$$= I(s) \left[ \frac{R_1 + R_2 + R_1 R_2 C_s}{R_1 C_s + 1} \right]$$

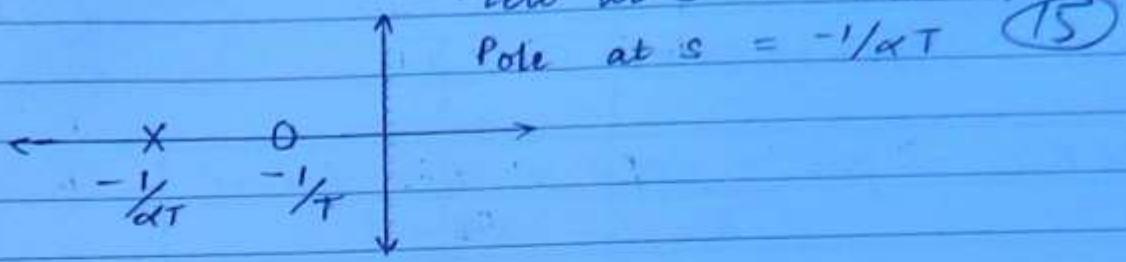
$$V_o(s) = I(s) R_2$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 (R_1 C_s + 1)}{R_1 + R_2 + R_1 R_2 C_s}$$

$T = R_1 C$	$\alpha = \text{Attenuation Gain} = \frac{R_2}{R_2 + R_1} \quad (\alpha < 1)$
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$$= \frac{R_2 (R_1 C_s + 1)}{R_1 + R_2 \left[ 1 + \frac{R_1 R_2 C_s}{R_1 + R_2} \right]}$$

$V_o(s) = \frac{\alpha (1 + TS)}{(1 + \alpha TS)}$
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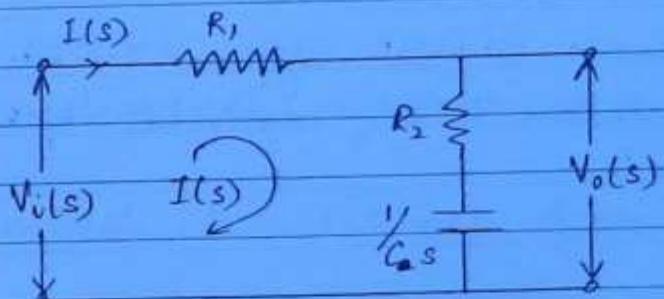


NOTE → The frequency nearest to origin is a zero in a LEAD compensator. Therefore its effect is not dominating.

If zero is added to a system TF it will represent behavior as a LEAD compensator.

## II. Lag compensator

It is used for improving the steady state response characteristics of the system i.e. elimination of steady state error b/w output & input.



$$V_i(s) = I(s) \times \left[ R_1 + R_2 + \frac{1}{C_s s} \right]$$

$$= I(s) \left[ \frac{R_1 C_s + R_2 C_s + 1}{C_s s} \right]$$

$$V_o(s) = I(s) \left[ R_2 + \frac{1}{C_s s} \right]$$

$$= I(s) \left[ \frac{R_2 C_s + 1}{C_s s} \right]$$

(16)

$$V_o(s) = \frac{R_2 Cs + 1}{R_1 Cs + R_2 Cs + 1}$$

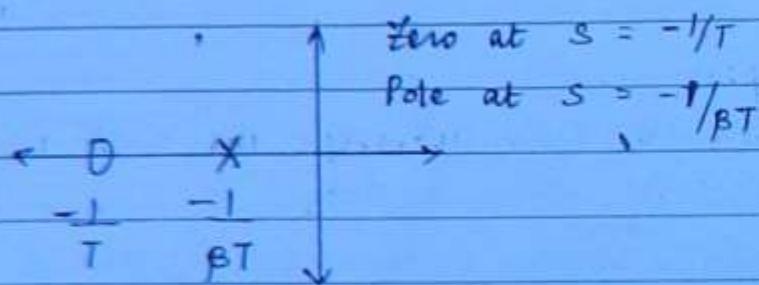
$$T = R_2 C \quad \beta = \frac{R_1 + R_2}{R_2} \approx 1 \quad (\beta > 1)$$

$$= R_2 Cs + 1$$

$$\frac{R_2 Cs}{R_2} \frac{(R_1 + R_2) + 1}{R_2}$$

$$V_o(s) = \frac{1 + Ts}{1 + \beta Ts}$$

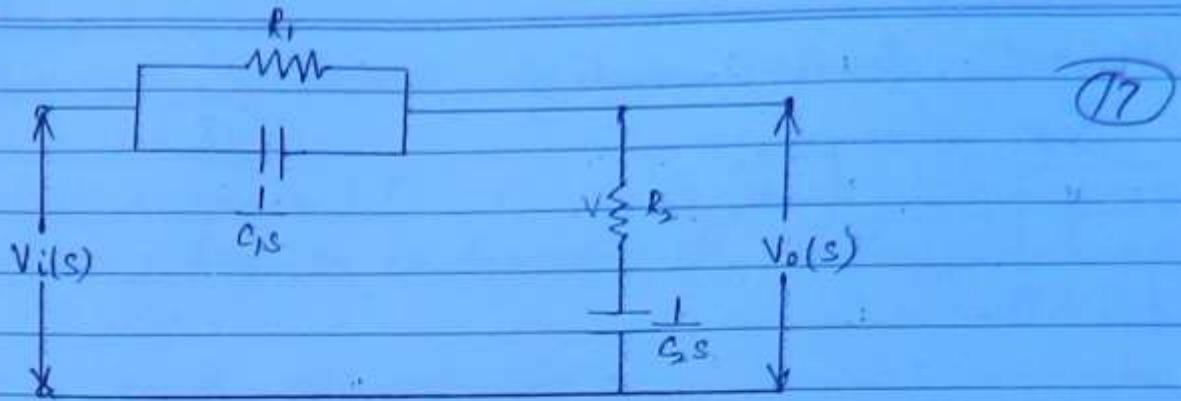
$$V_o(s) = \frac{1}{1 + \beta T s}$$



NOTE → If a pole is added to a system T.F in terms of compensation, it represents a LAG compensator.

### III LEAD LAG (a) LAG-LEAD compensators

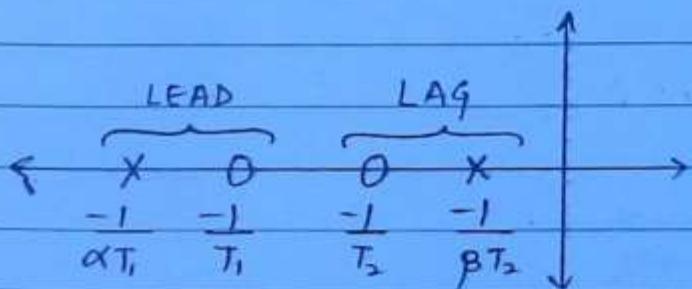
- It is used for improving both transient & steady state response characteristics
- It exhibits both LEAD & LAG characteristics in its frequency response



$$V_o(s) = \alpha(1 + T_1 s) (1 + T_2 s)$$

$$V_i(s) = (1 + \alpha T_1 s) (1 + \beta T_2 s)$$

$$T_1 = R_1 C_1 \quad / \quad T_2 = R_2 C_2$$



### MECHANICAL SYSTEMS -

All mechanical systems are classified into two types -

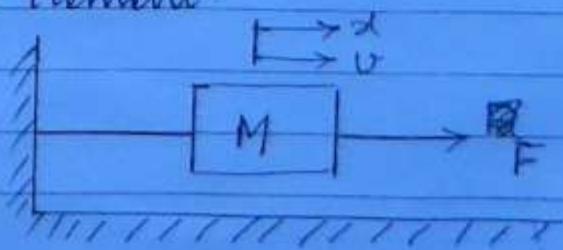
#### I. Mechanical Translational Systems -

Input = Force (F)

Output = Linear displacement (x) or. Linear Velocity (v)

The 3 ideal elements are -

#### I. Mass Element

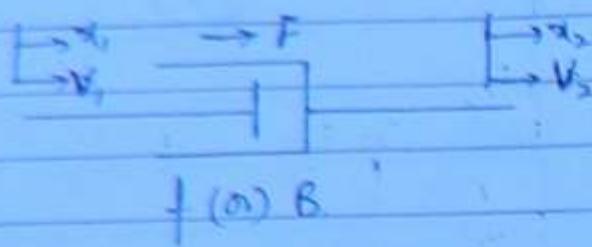


$$F = M \frac{dv}{dt}$$

$$F = M \frac{d^2x}{dt^2}$$

## 2. Damping Element (Friction)

(18)

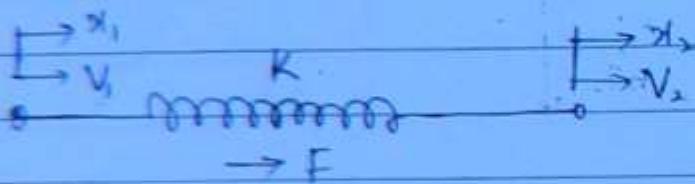


$$F = f(V_1 - V_2) = fV \quad (V = V_1 - V_2)$$

$$F = f \frac{d(x_1 - x_2)}{dt} = f \frac{dx}{dt}$$

$$(x = x_1 - x_2)$$

## 3. Spring Element (Stiffness)



$$F = k \int (V_1 - V_2) dt = k \int V dt \quad (V = V_1 - V_2)$$

$$F = k(x_1 - x_2) = kx \quad (x = x_1 - x_2)$$

## II Mechanical Rotational System

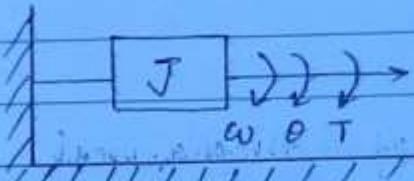
(79)

Input = Torque ( $T$ )

Output = Angular displacement ( $\theta$ ) (or) Angular Velocity ( $\omega$ )

The 3 ideal elements are -

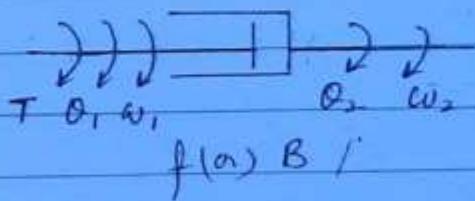
### 1. Inertia Element.



$$T = J \frac{d\omega}{dt}$$

$$T = J \frac{d^2\theta}{dt^2}$$

### 2. Torsional Damper Element (Friction)

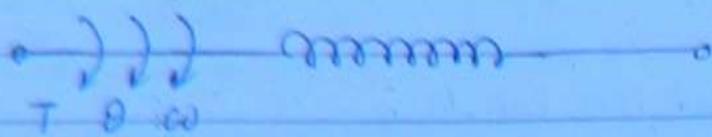


$$T = f(\omega_1 - \omega_2) = f\omega$$
$$(\omega = \omega_1 - \omega_2)$$

$$T = f \frac{d}{dt} (\theta_1 - \theta_2) = f \frac{d\theta}{dt}$$

$$(\theta = \theta_1 - \theta_2)$$

### 3 Torsional Spring Element (Stiffness)



(20)

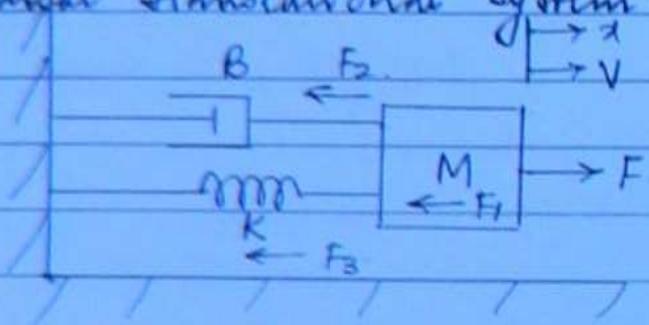
$$T = k \int w \, dt$$

$$T = k \theta$$

#### ANALOGOUS SYSTEMS -

The electrical equivalents of mechanical elements are known as analogous systems.

#### I. Mechanical Translational System

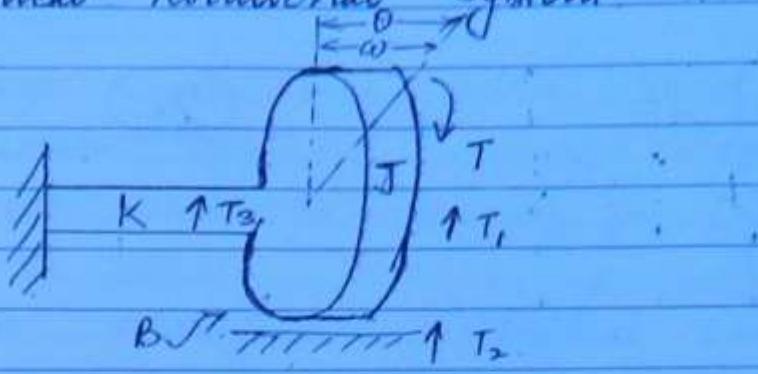


$$F = F_1 + F_2 + F_3$$

$$F = M \frac{dv}{dt} + BV + K \int v \, dt \quad ] \quad (1)$$

$$F = M \frac{d^2x}{dt^2} + B \frac{dx}{dt} + Kx \quad ]$$

## II. Mechanical Rotational System



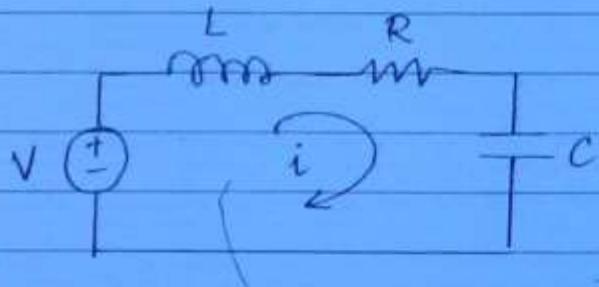
(21)

$$T = T_1 + T_2 + T_3$$

$$T = \left. \begin{aligned} & J d\omega + B\omega + K \int \omega dt \\ & dt \end{aligned} \right\} - (2)$$

$$T = \left. \begin{aligned} & J d^2\theta + B \frac{d\theta}{dt} + K\theta \\ & dt^2 \end{aligned} \right\}$$

## III Electrical System

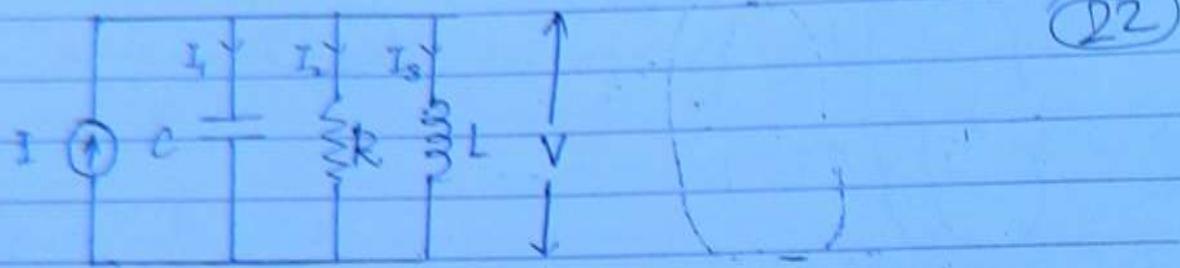


$$V = V_1 + V_2 + V_3$$

$$V = \left. \begin{aligned} & L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt \\ & dt \end{aligned} \right\} [i = \frac{dq}{dt} \quad (q = \text{charge})] - (3)$$

$$V = \left. \begin{aligned} & L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + Cq \end{aligned} \right\} - (3)$$

## IV Electrical Systems -



$$I = I_1 + I_2 + I_3$$

$$I = C \frac{dv}{dt} + \frac{v}{R} + \frac{1}{L} \int v dt$$

$$v = \frac{d\phi}{dt} \quad \phi = \text{flux}$$

$$I = C \frac{d^2\phi}{dt^2} + \frac{1}{R} \frac{d\phi}{dt} + \frac{\phi}{L}$$

- (4)

Comparing (1)  $\rightarrow$  (4)

1 F - T - V - Analogy

2 F - T - I - Analogy

Mass/Inertia element & spring element are known  
as "CONSERVATIVE ELEMENTS"

$$F - T - V - I$$

$$M - J - L - C$$

$$B - B - R - 1/R$$

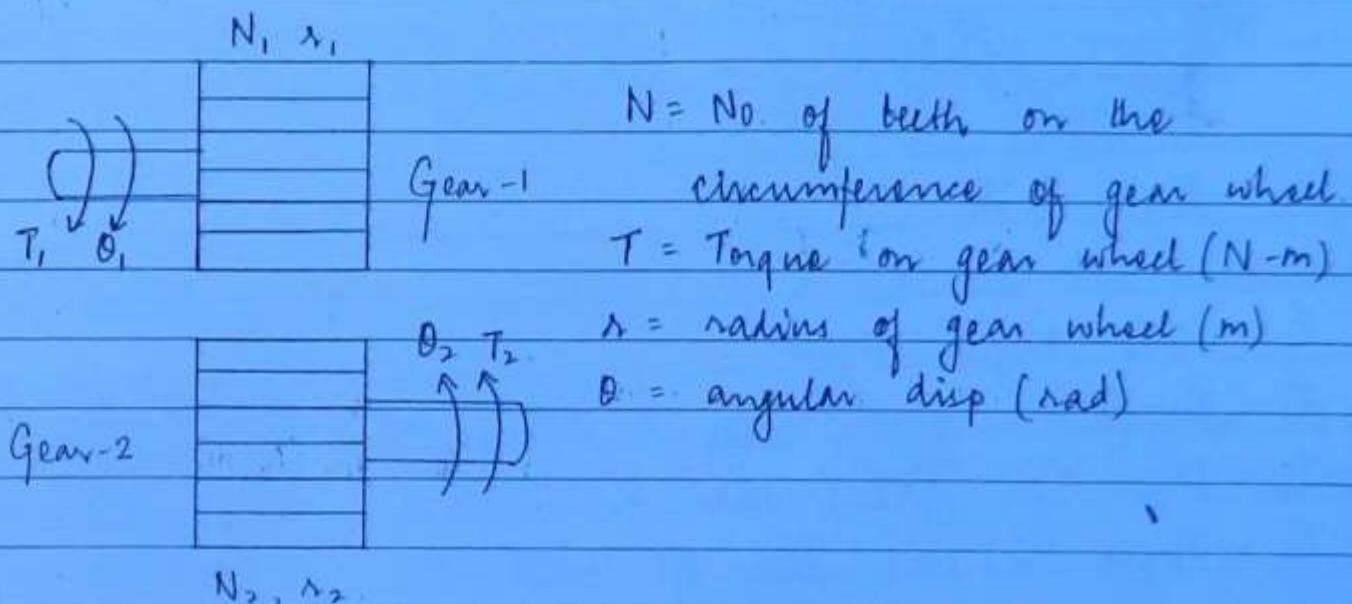
$$K - K - 1/C - 1/L$$

$$V - w - i - V$$

$$\alpha - \theta - q - \phi$$

## GEARS -

- \* Gears are mechanical devices which are used as intermediate elements between electrical motor & load.
- \* They are used for stepping up / stepping down either torque or speed.
- \* They are analogous to electrical transformers.



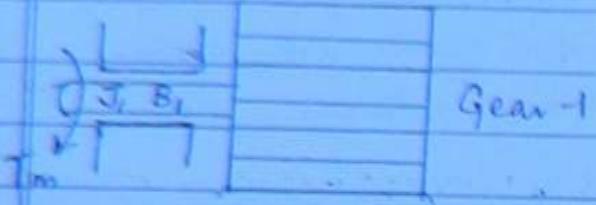
$$\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

For 'n' gear wheels -

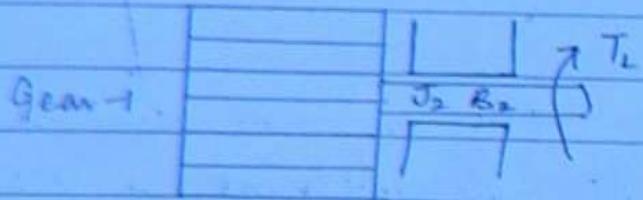
$$\boxed{\frac{N_x}{N_y} = \frac{T_x}{T_y} = \frac{r_x}{r_y} = \frac{\theta_y}{\theta_x} = \frac{\omega_y}{\omega_x} = \frac{\alpha_y}{\alpha_x}}$$

# Dynamics of Gear connected b/w Motor & Load -

$N_1, \alpha_1, T_1, \theta_1$



(24)



$N_2, \alpha_2, T_2, \theta_2$

$T_m$  = Motor torque ( $N\cdot m$ )

$T_1$  = Torque on gear-1 due to  $T_m$  ( $N\cdot m$ )

$T_2$  = Torque on gear-2 due to  $T_1$  ( $N\cdot m$ )

$T_L$  = Torque on load due to  $T_2$  ( $N\cdot m$ )

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + T_1$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} \Rightarrow T_1 = \left(\frac{N_1}{N_2}\right) T_2$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + T_L \left(\frac{N_1}{N_2}\right) T_2$$

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} \Rightarrow \ddot{\theta}_2 = \left(\frac{N_1}{N_2}\right) \ddot{\theta}_1 \quad \text{or} \quad \ddot{\theta}_1 = \left(\frac{N_2}{N_1}\right) \ddot{\theta}_2$$

$$\Rightarrow \dot{\theta}_2 = \left(\frac{N_1}{N_2}\right) \dot{\theta}_1 \quad \text{or} \quad \dot{\theta}_1 = \left(\frac{N_2}{N_1}\right) \dot{\theta}_2$$

## I Equivalent Inertia & Friction for Motor Side Gear (Gear -1)

$$T_M = \frac{J_1 d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)^2 J_2 \frac{d^2\theta_1}{dt^2} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L \quad (25)$$

$$T_M = \left[ J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2 \right] \frac{d^2\theta_1}{dt^2} + \left[ B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2 \right] \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$J_{eq_1} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2$$

$$B_{eq_1} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2$$

## II Equivalent Inertia & Friction for Load Side Gear (Gear -2)

$$T_M = \left(\frac{N_2}{N_1}\right) J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right) B_1 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

multiplying both sides by  $\frac{N_2}{N_1}$

$$\frac{N_2}{N_1} T_M = \left(\frac{N_2}{N_1}\right)^2 J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right)^2 B_1 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\frac{N_2}{N_1} T_M = \left[ \left(\frac{N_2}{N_1}\right)^2 J_1 + J_2 \int \frac{d^2\theta_2}{dt^2} \left[ \left(\frac{N_2}{N_1}\right)^2 B_1 + B_2 \right] \frac{d\theta_2}{dt} \right] + T_L$$

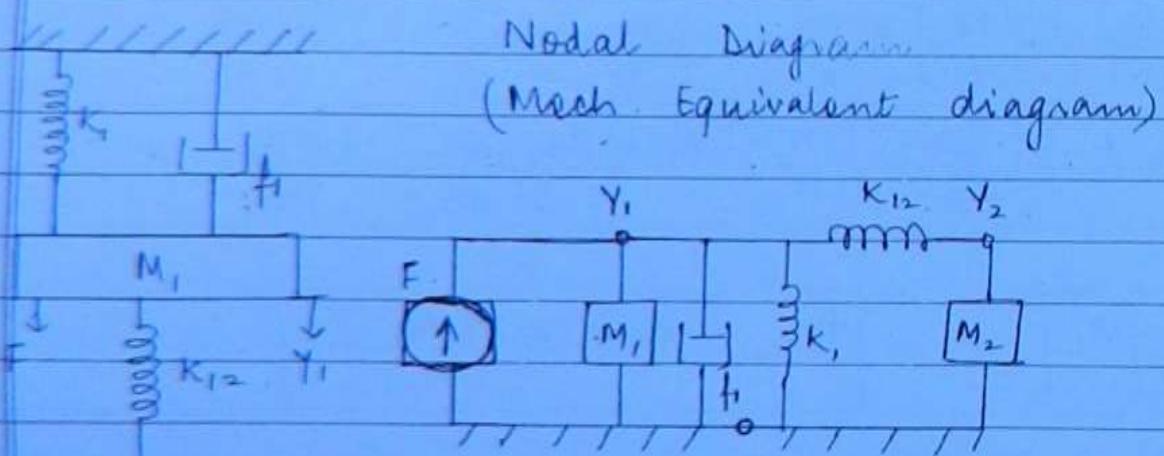
$$J_{eq_2} = J_1 + \left(\frac{N_2}{N_1}\right)^2 J_2$$

$$B_{eq_2} = B_1 + B_2 \left(\frac{N_2}{N_1}\right)^2$$

## NODAL METHOD -

(26)

- \* No. of nodes = No. of displacements
- \* Take an additional node which is a reference node.
- \* Connect mass or inertial mass elements between the principal node and reference only.
- \* Connect the spring and damping elements either between the principal nodes or between principal node & reference depending on their position.
- \* Obtain the nodal diagram & write the describing differential equations at each node.



Let's write  $y_1$

$$F = M_1 \frac{d^2 y_1}{dt^2} + f_1 \frac{dy_1}{dt} + K_1 y_1 + K_{12} (y_1 - y_2)$$

$f_1 = -\omega_1^2$

$$0 = M_2 \frac{d^2 y_2}{dt^2} + K_{12} (y_2 - y_1)$$

Transfer function  $\frac{Y_1(s)}{F(s)}$

(27)

$$F(s) = (M_1 s^2 + f_1 s + k_1 + K_{12}) Y_1(s) - K_{12} Y_2(s)$$

$$0 = (M_2 s^2 + k_{12}) Y_2(s) - K_{12} Y_1(s)$$

$$Y_2(s) = \begin{bmatrix} K_{12} \\ M_2 s^2 + k_{12} \end{bmatrix} Y_1(s)$$

$$F(s) = \left\{ (M_1 s^2 + f_1 s + k_1 + K_{12}) - \frac{K_{12}^2}{M_2 s^2 + k_{12}} \right\} Y_1(s)$$

$$F(s) = \left\{ (M_1 s^2 + f_1 s + k_1 + K_{12})(M_2 s^2 + k_{12}) - K_{12}^2 \right\} \frac{Y_1(s)}{M_2 s^2 + k_{12}}$$

$$Y_1(s) = \frac{M_2 s^2 + k_{12}}{(M_1 s^2 + f_1 s + k_1 + K_{12})(M_2 s^2 + k_{12}) - K_{12}^2}$$

$$F(s) = \frac{M_2 s^2 + k_{12}}{(M_1 s^2 + f_1 s + k_1 + K_{12})(M_2 s^2 + k_{12}) - K_{12}^2}$$

1 Mass element  $\rightarrow$  order -2

2 Mass element  $\rightarrow$  order -4

3 Mass element  $\rightarrow$  order -6

!  $n$  Mass elements  $\rightarrow$  order - $2n$

# F-V ANALOGY

(28)

$$V = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{C_1} + \frac{q_1 - q_2}{C_{12}}$$

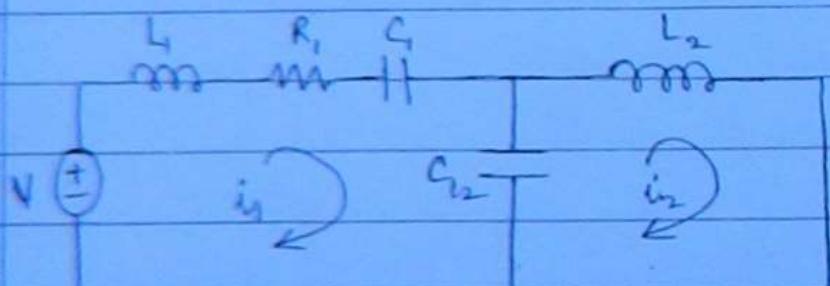
$$0 = L_2 \frac{d^2 q_2}{dt^2} + \frac{q_2 - q_1}{C_{12}}$$

$$i_1 = \frac{dq_1}{dt} \Rightarrow q_1 = \int i_1 dt$$

$$i_2 = \frac{dq_2}{dt} \Rightarrow q_2 = \int i_2 dt$$

$$V = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + \frac{1}{C_{12}} \int (i_1 - i_2) dt$$

$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_{12}} \int (i_2 - i_1) dt$$



# F - I ANALOGY

(29)

$$I = C_1 \frac{d^2 \phi_1}{dt^2} + \frac{1}{R_1} \frac{d\phi_1}{dt} + \frac{\phi_1}{L_1} + \frac{\phi_1 - \phi_2}{L_{12}}$$

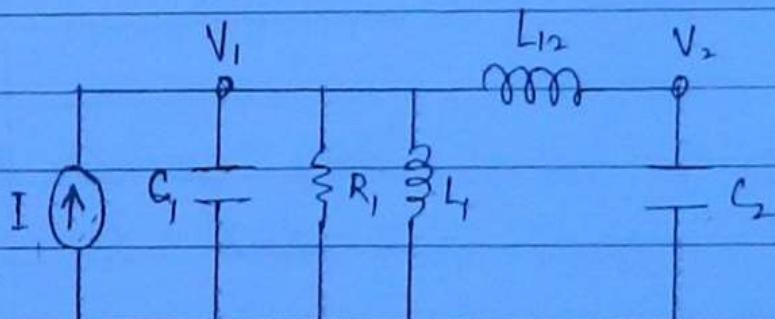
$$0 = C_2 \frac{d^2 \phi_2}{dt^2} + \frac{\phi_2 - \phi_1}{L_{12}}$$

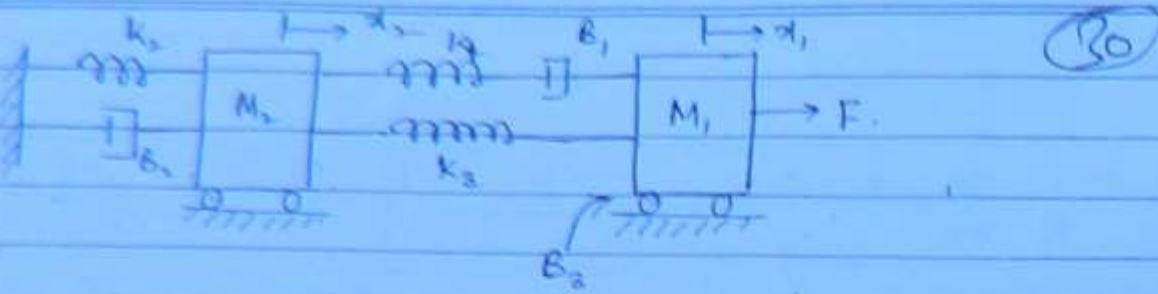
$$V_1 = \frac{d\phi_1}{dt} \Rightarrow \phi_1 = \int V_1 dt$$

$$V_2 = d\phi_2 \Rightarrow \phi_2 = \int V_2 dt$$

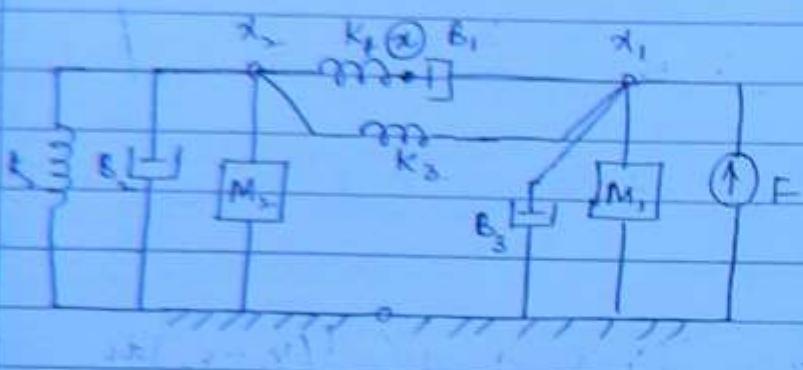
$$I = C_1 \frac{dV_1}{dt} + V_1 + \frac{1}{R_1} \int V_1 dt + \frac{1}{L_1} \int (V_1 - V_2) dt$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_{12}} \int (V_2 - V_1) dt$$





Nodal Diagram :-



At node  $x_1$ ,

$$F = M_1 \frac{d^2x_1}{dt^2} + B_2 \frac{dx_1}{dt} + k_3(x_1 - x_2) + B_1 \frac{d(x_1 - x)}{dt}$$

At dummy node  $x$ ,

$$0 = B_1 \frac{d(x - x_1)}{dt} + k_1(x - x_2)$$

At node  $x_2$ ,

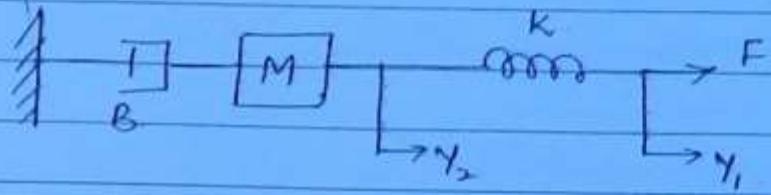
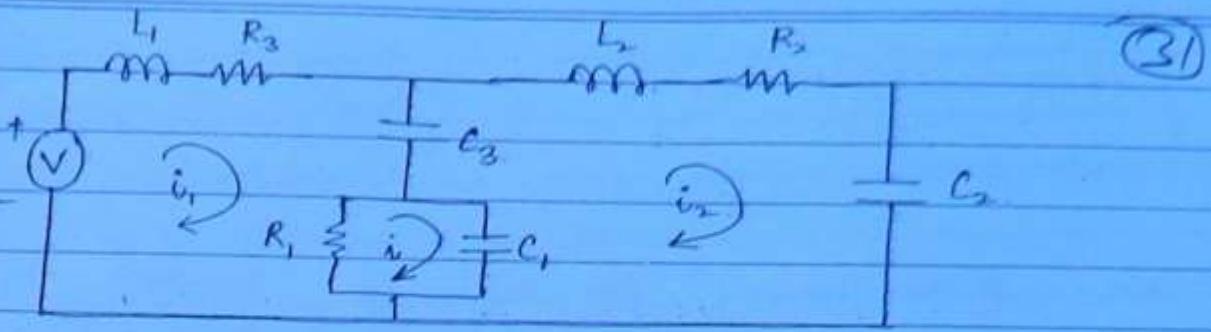
$$0 = M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + k_2 x_2 + k_3(x_2 - x_1) + k_1(x_2 - x)$$

F-V ANALOGY

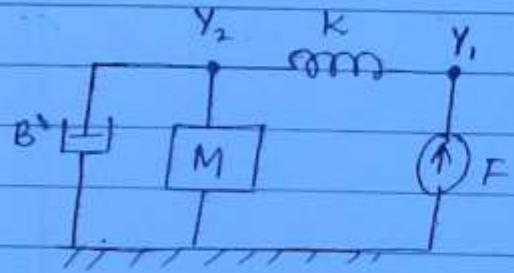
$$V = L_1 \frac{di_1}{dt} + R_3 i_1 + \frac{1}{C_3} \int (i_1 - i_2) dt + R_1 (i_1 - i)$$

$$0 = R_1 (i - i_1) + \frac{1}{C_1} \int (i - i_2) dt$$

$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2 dt + \frac{1}{C_2} \int (i_2 - i_1) dt + \frac{1}{C_1} \int (i_2 - i) dt$$



Nodal diagram -



Only one node as  
only one mass is  
present or only one  
displacement  
 $\therefore y_1$  is dummy node

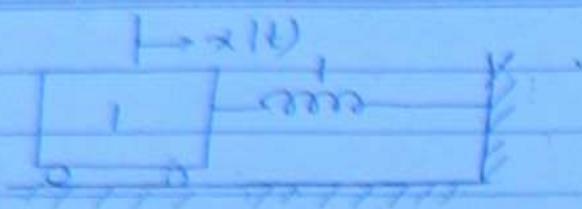
At dummy node  $y_1$

$$F = K(y_1 - y_2)$$

At node  $y_2$

$$0 = M \frac{d^2 y_2}{dt^2} + B \frac{dy_2}{dt} + K(y_2 - y_1)$$

$$F = K(y_1 - y_2) = M \frac{d^2 y_2}{dt^2} + B \frac{dy_2}{dt}$$



(32)

For unit impulse force the eq<sup>n</sup> for resulting vibration will be -

- a)  $\sin \omega t$
- b)  $\sin \omega t +$
- c)  $\sqrt{2} \sin \omega t$
- d)  $\sin \sqrt{t}$

$$F = M \frac{d^2x}{dt^2} + Kx$$

$\downarrow dt^2$        $\downarrow$   
|            |

$$F(s) = (s^2 + 1) X(s)$$

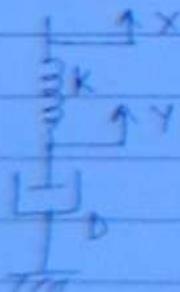
$$X(s) = \frac{F(s)}{s^2 + 1}$$

$F(s)$  = Impulse force = 1

$$X(s) = \frac{1}{s^2 + 1}$$

$$x(t) = \sin t \rightarrow (a)$$

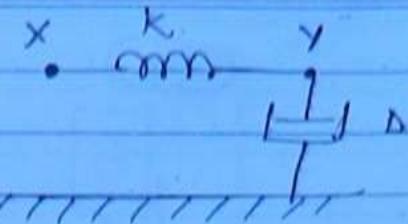
Q. Find the pole of mechanical system -



a)  $-\frac{K}{D}$       b)  $0, -\frac{K}{D}$

c)  $-KD$       d)  $-\frac{D}{K}$

sol.



(B3)

At mode Y

$$D = D \frac{dy}{dt} + k(y - x)$$

$$D \frac{dy}{dt} + ky = kx$$

$$[Ds + k] Y(s) = k X(s)$$

Dummy T.F

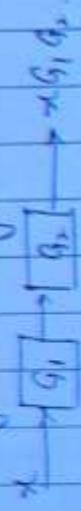
$$\frac{Y(s)}{X(s)} = \frac{k}{Ds + k}$$

$$\text{pole } \Rightarrow Ds + k = 0 \\ s = \frac{-k}{D} \quad -(a)$$

## # SIGNAL FLOW GRAPH

Equivalent diagram

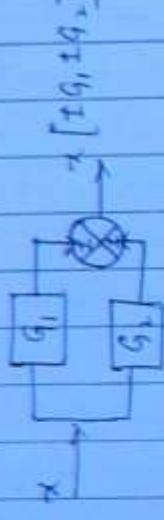
Original Diagram



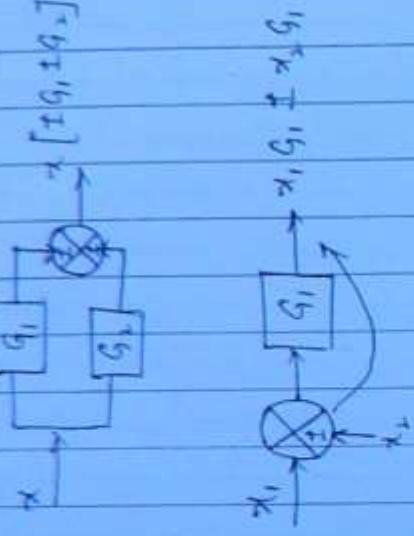
Rules

1. Combining blocks in series
2. Combining blocks in parallel
3. Shifting the summing element after the block
4. Shifting the summing element before the block
5. Shifting the take off point after the block
6. Shifting the take off point before the block

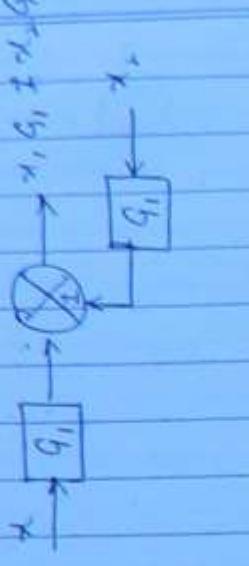
1. Combining blocks in series



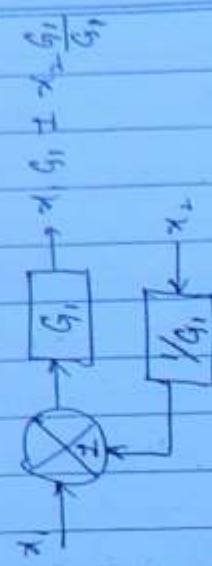
2. Combining blocks in parallel



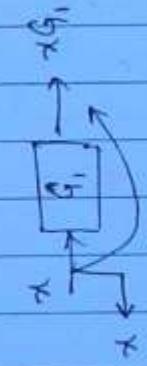
3. Shifting the summing element after the block



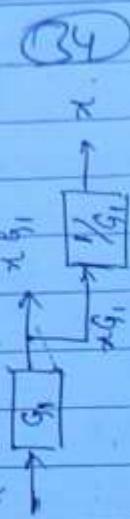
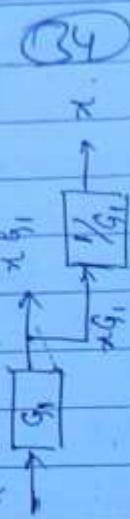
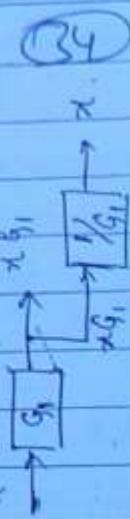
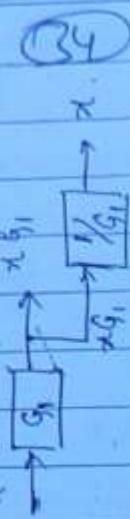
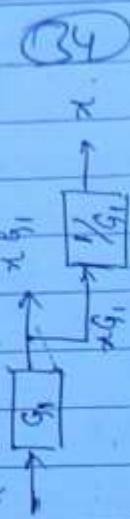
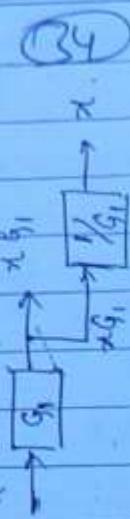
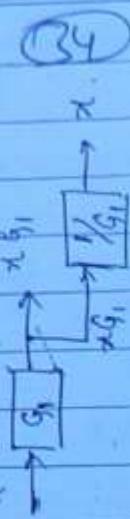
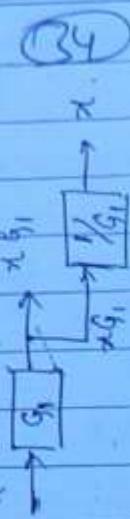
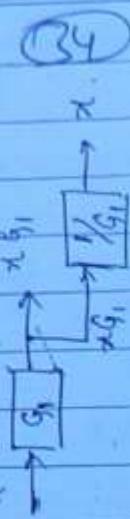
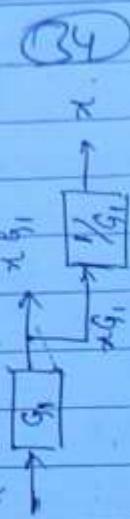
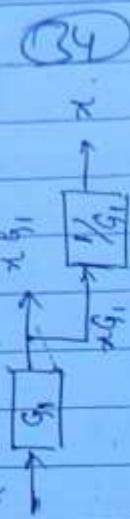
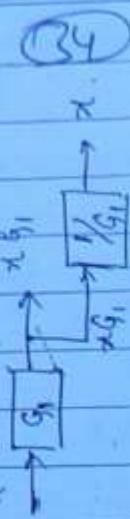
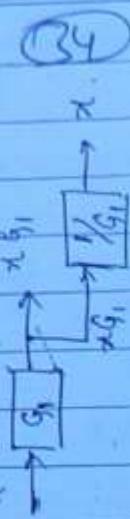
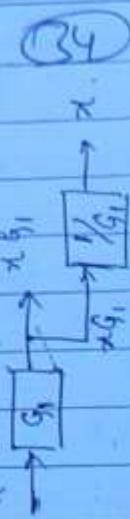
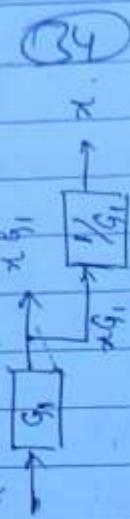
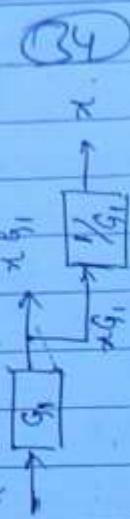
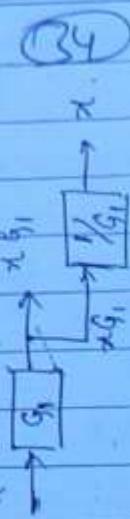
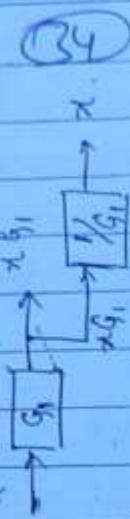
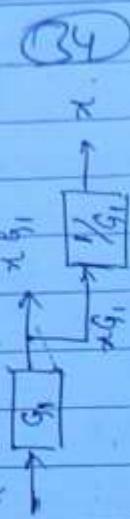
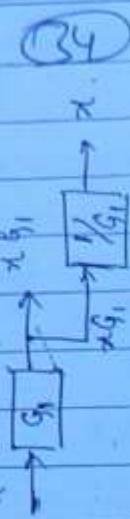
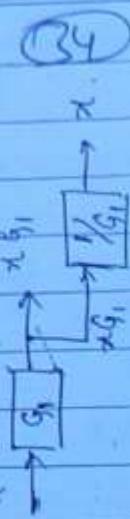
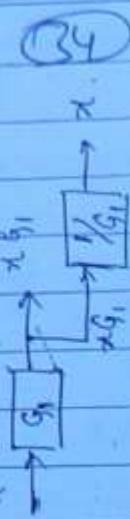
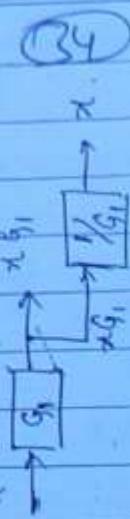
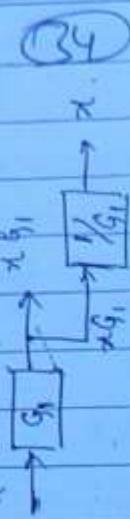
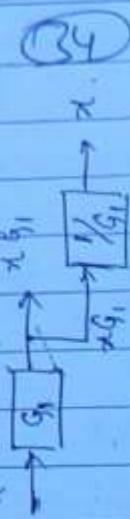
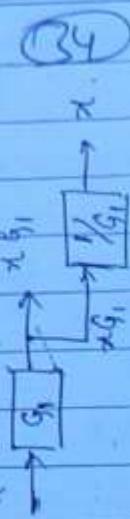
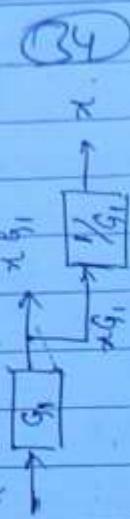
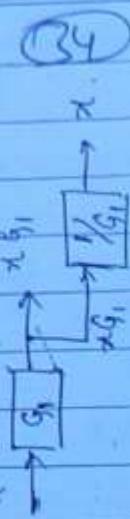
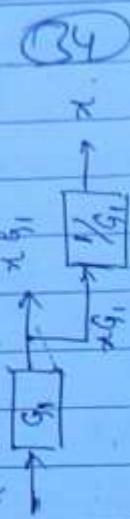
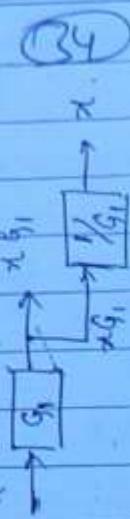
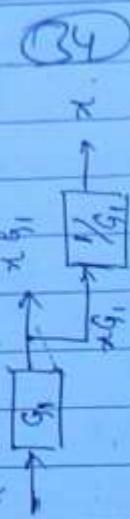
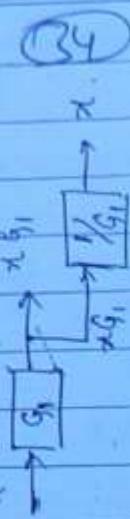
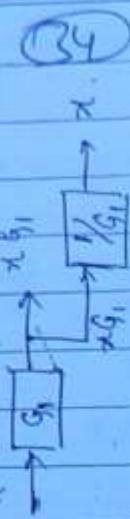
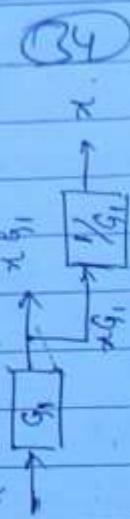
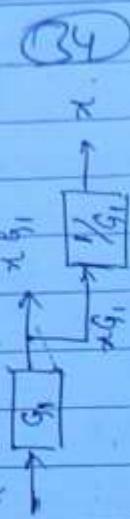
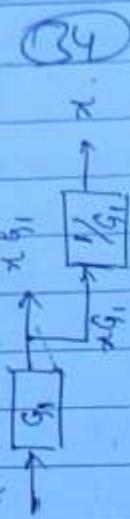
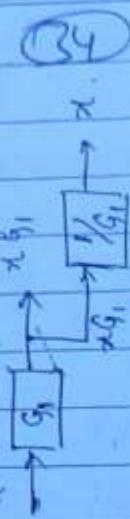
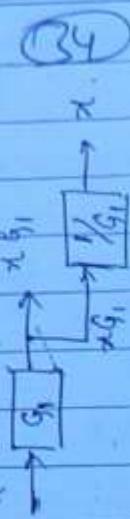
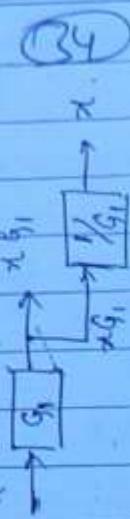
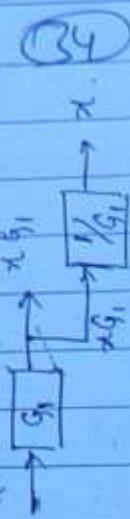
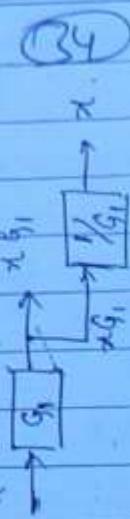
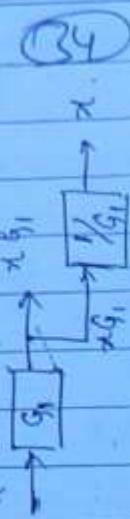
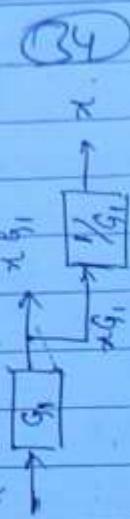
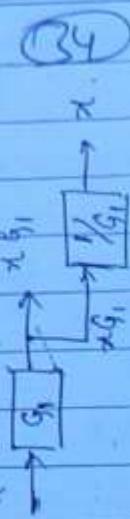
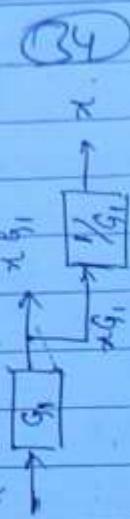
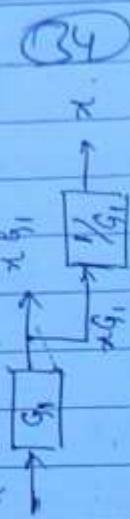
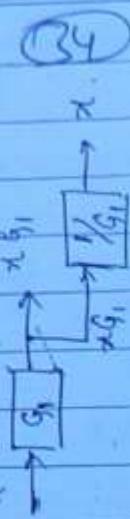
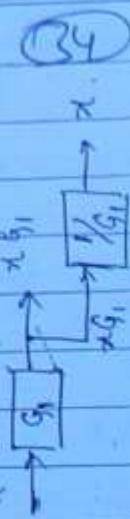
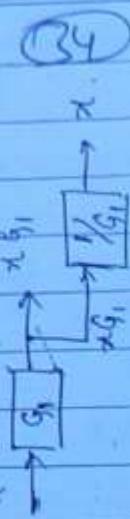
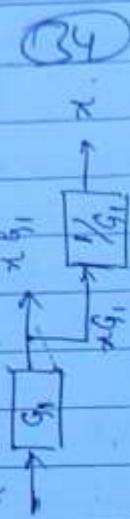
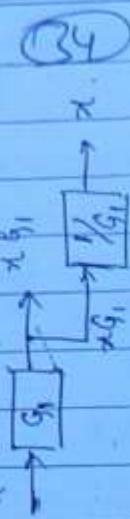
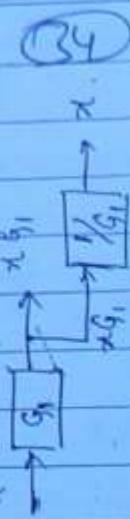
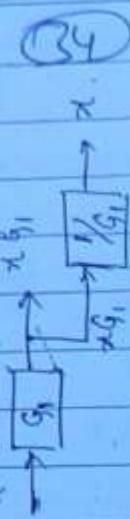
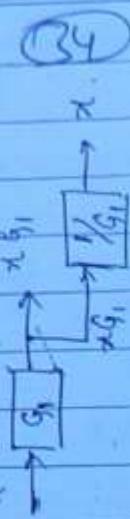
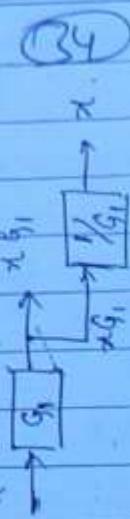
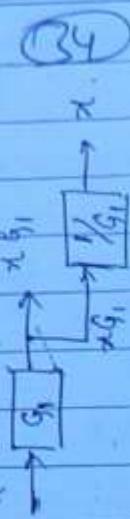
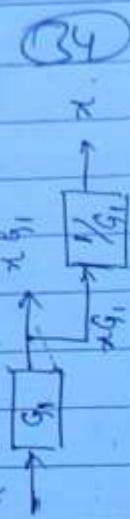
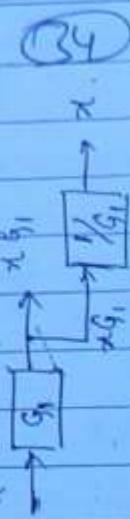
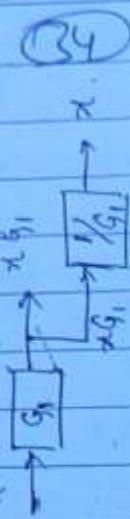
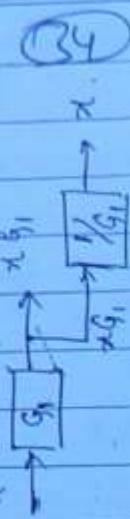
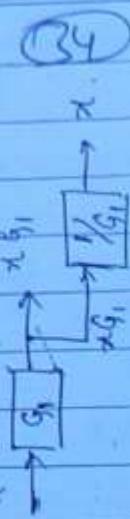
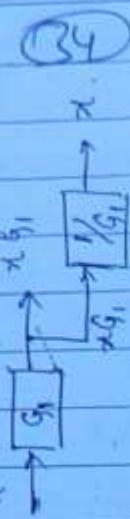
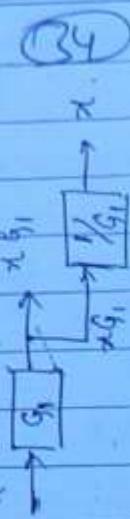
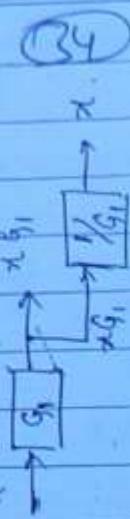
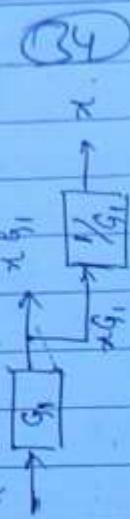
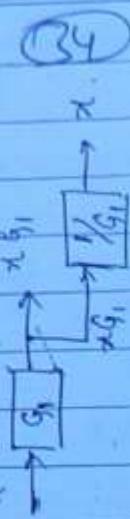
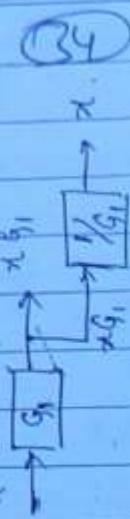
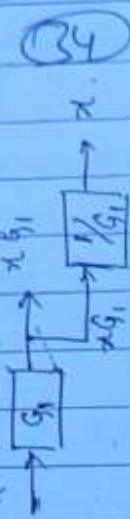
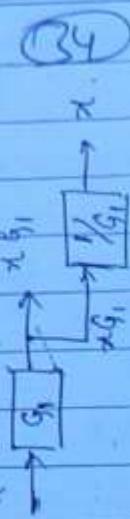
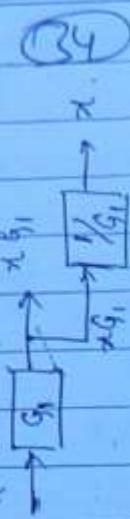
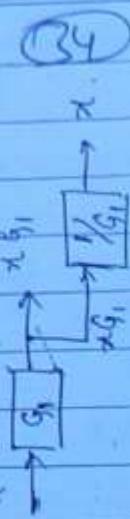
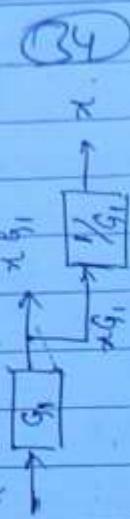
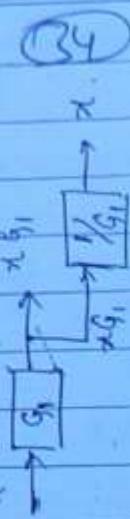
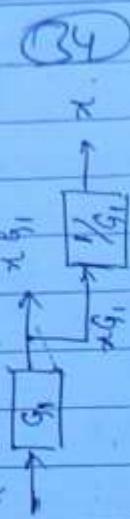
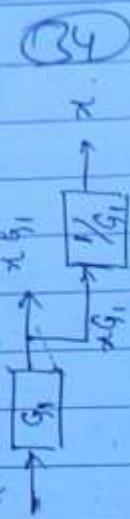
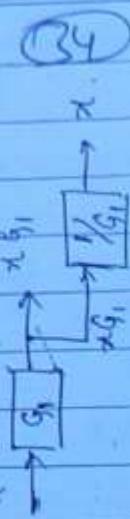
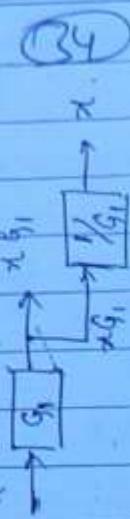
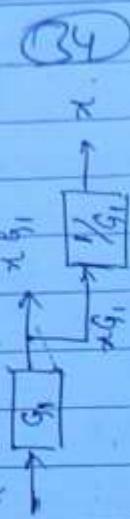
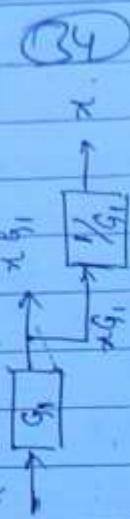
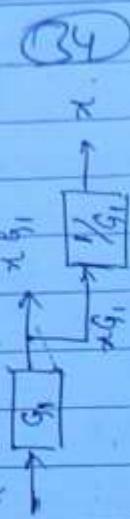
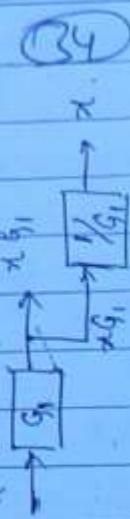
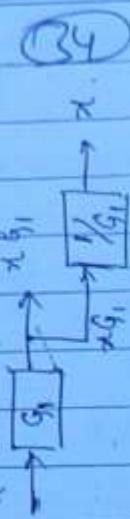
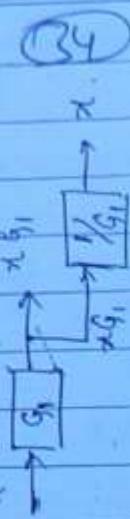
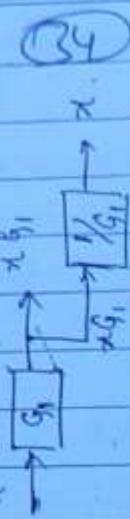
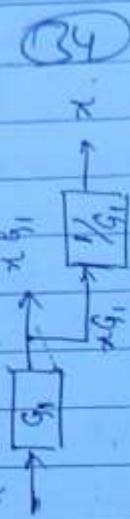
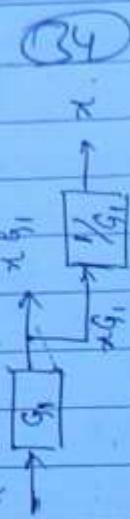
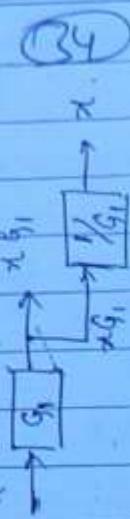
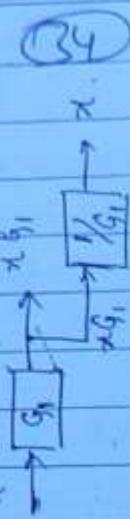
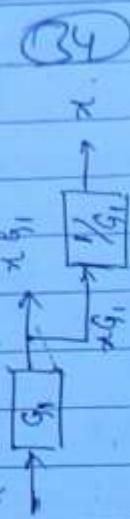
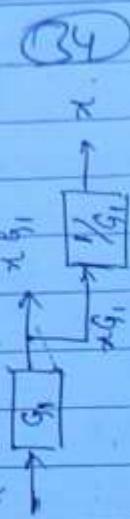
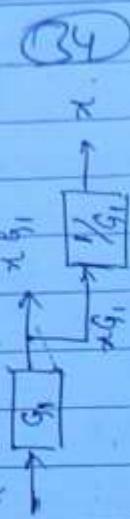
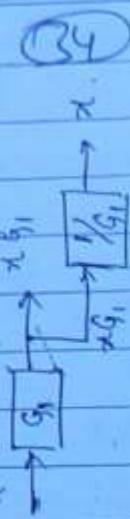
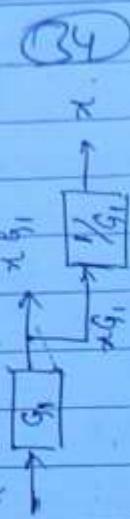
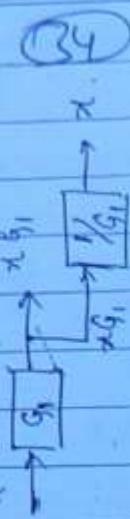
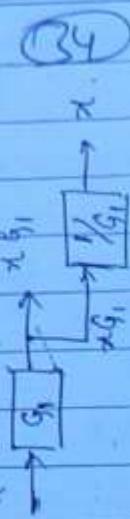
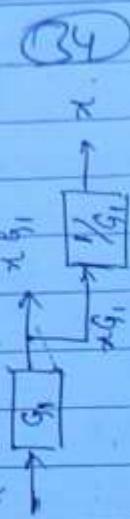
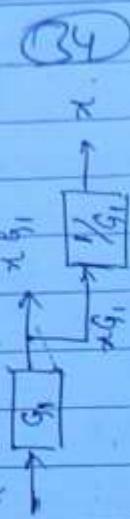
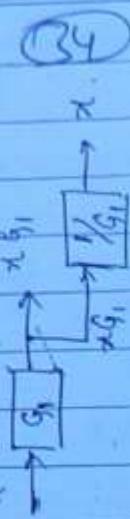
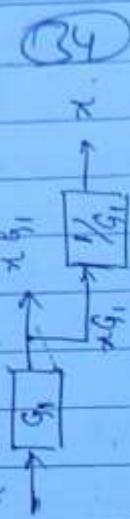
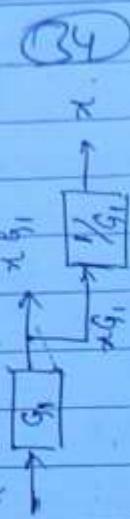
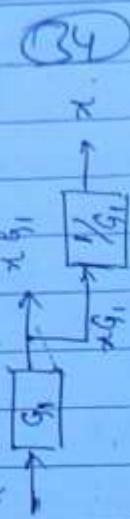
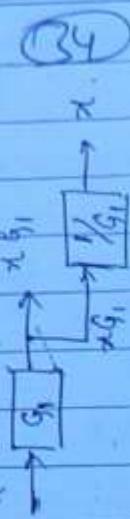
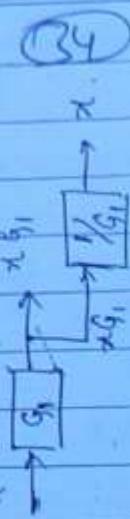
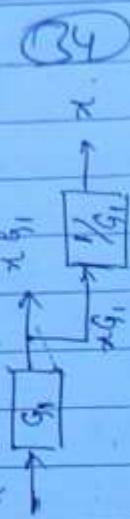
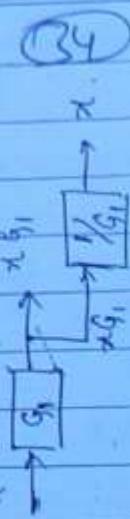
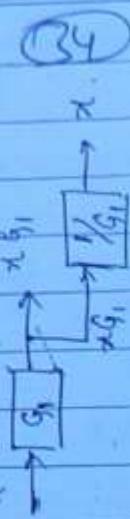
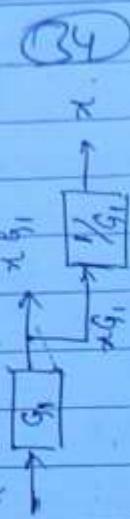
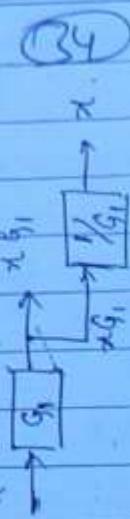
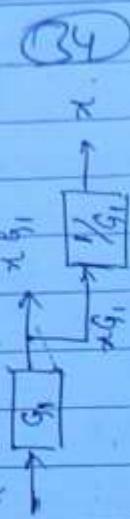
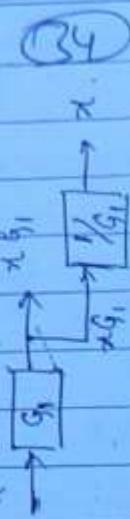
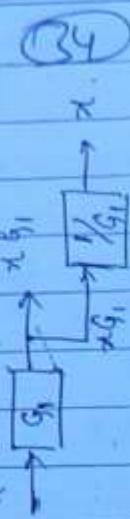
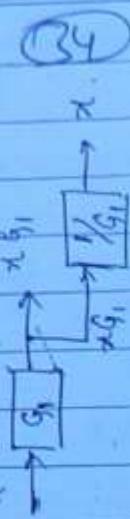
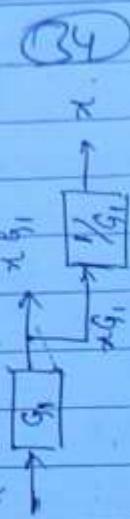
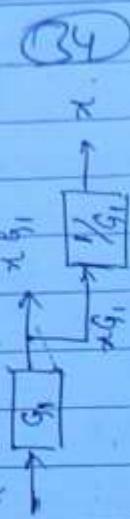
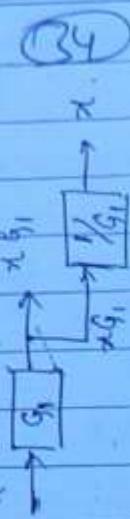
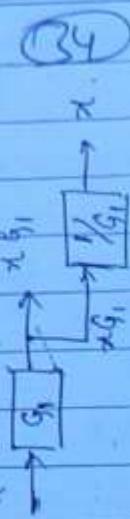
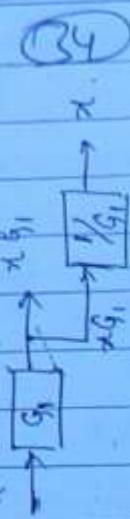
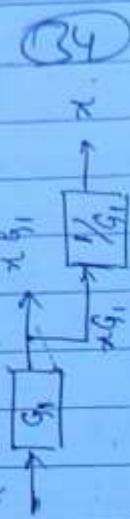
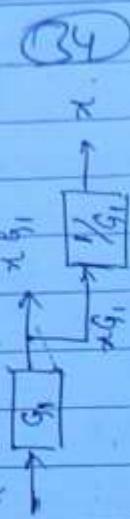
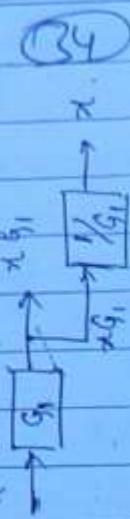
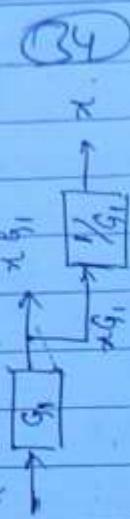
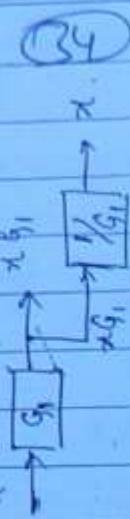
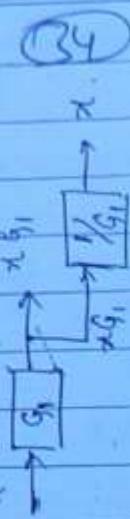
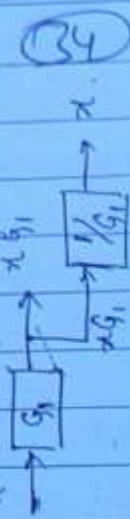
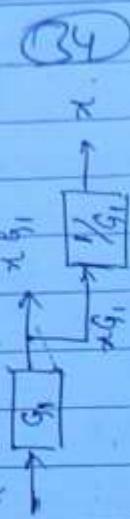
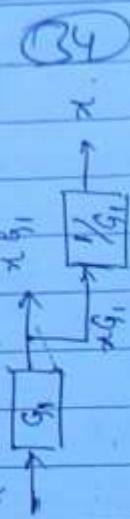
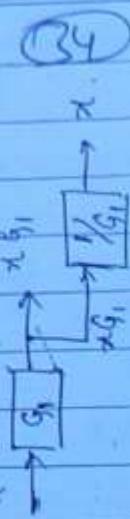
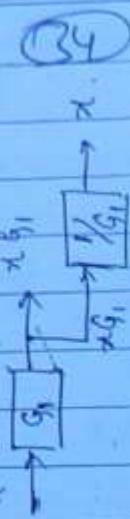
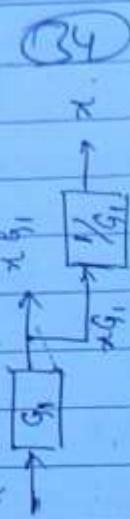
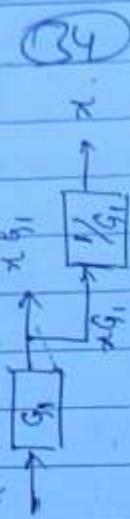
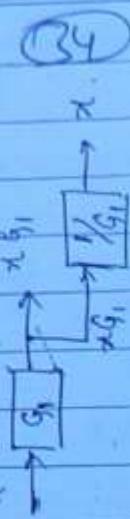
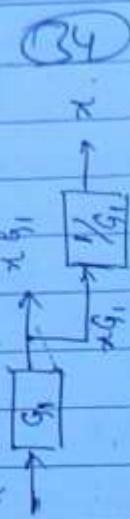
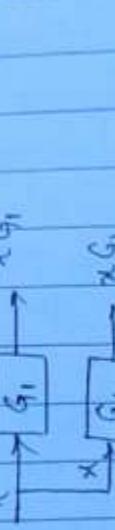
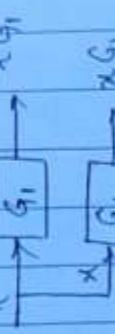
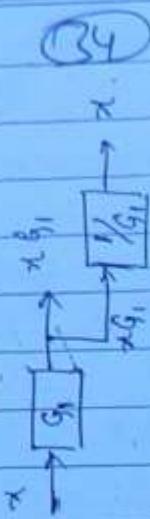
4. Shifting the summing element before the block



5. Shifting the take off point after the block



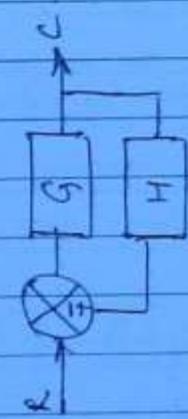
5. Shifting the take off point after the block



7. Transfer function of closed loop control system

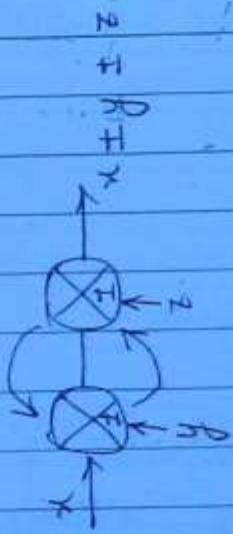


8. Block diagram transformation



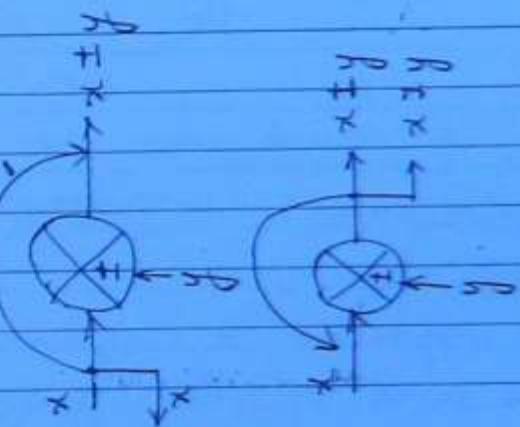
$$\frac{C}{R} = \frac{1}{H} \times \frac{G}{1+GH} = \frac{G}{1+GH}$$

9. Interchanging summing elements

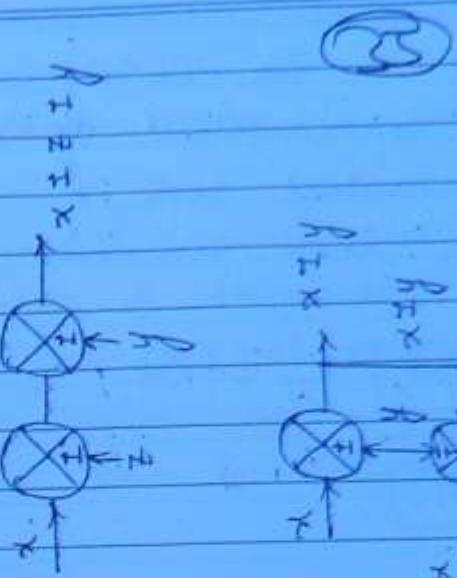


CRT 1: RULES

10. Shifting the take-off point after summing element



Shifting the take-off point before summing element

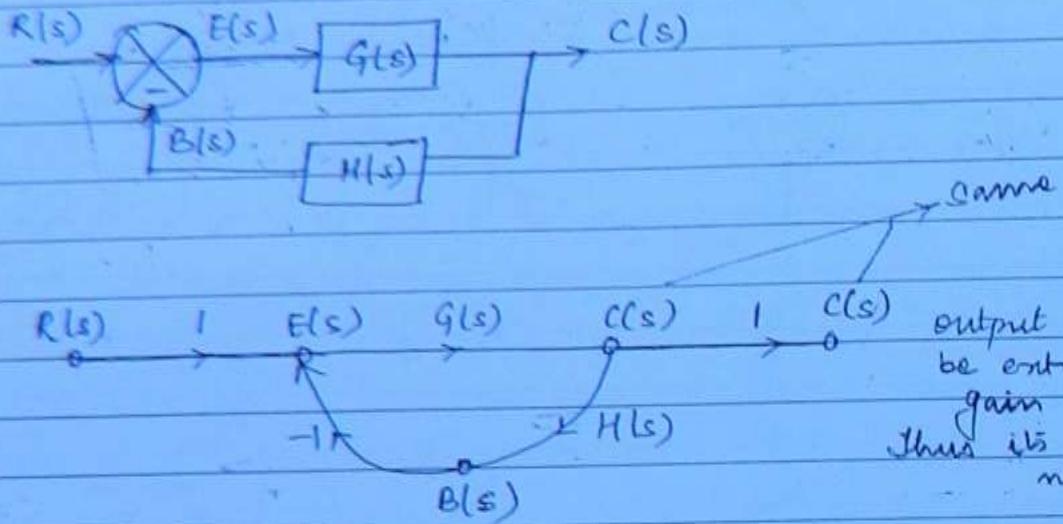


(35)

## (36)

### SIGNAL FLOW GRAPH for CLCS -

It's a graphical representation of control system in which nodes representing each of the system variables are connected by direct branches.



### TERMINOLOGY OF SFG'S.

#### 1. NODE -

It represents a system variable  $f$  is equal to the sum of all incoming signals at it.

#### 2. INPUT NODE or SOURCE NODE -

It's a node having only outgoing branches.

#### 3. OUTPUT NODE or SINK NODE -

It's a node having only incoming branches.

#### 4. MIXED or CHAIN NODE -

It's a node having both incoming & outgoing branches.

#### 5. PATH -

It is the traversal of the connected branches in the direction of branch arrow such that no node is traversed more than once.

## 6. FORWARD PATH -

It's a path from input node to output node

## 7. LOOP -

It's a path which originates & terminates at the same node

Self loop at defined input is not considered valid

Self loop at ~~not~~ defined output node is valid  
cuz output node can be extended

## NOTE -

Self loops on defined input nodes are not valid loops.

Self loops / loops on output nodes are valid loops

## 8. NON TOUCHING LOOPS -

Two or more loops are said to be non-touching if they do not have a common node

## MASON'S GAIN FORMULA -

$$\text{The overall gain or T.F} = \frac{\sum P_k \Delta_k}{\Delta}$$

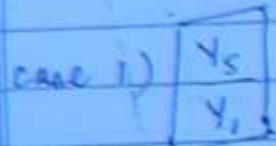
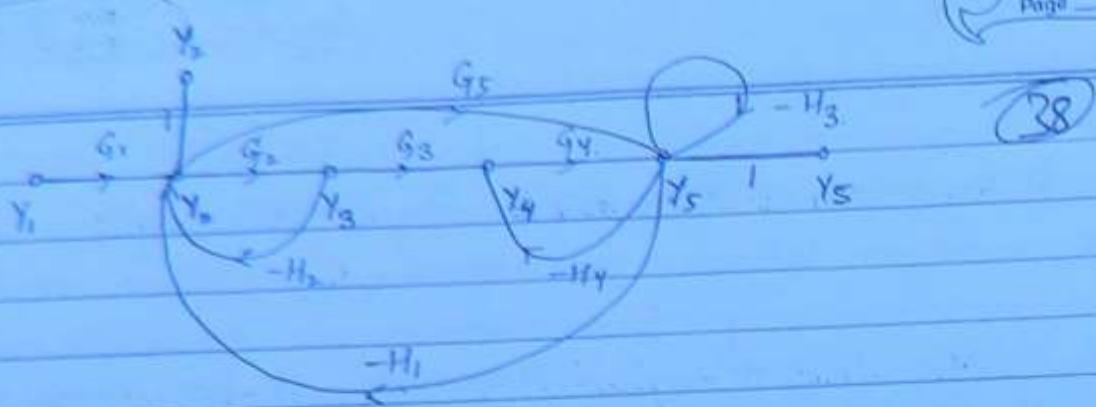
$$= P_1 \Delta_1 + P_2 \Delta_2 + \dots + P_k \Delta_k$$

where

$P_k$  = Path gain of  $k^{\text{th}}$  forward path

$$\Delta = 1 - \left[ \begin{array}{l} \text{Sum of loop gains} \\ \text{of all individual} \\ \text{loops} \end{array} \right] + \left[ \begin{array}{l} \text{Sum of gain} \\ \text{products of two} \\ \text{non touching loops} \end{array} \right] - \left[ \begin{array}{l} \text{Sum of gain products} \\ \text{of three non touching} \\ \text{loops} \end{array} \right]$$

$\Delta_k$  = It is that value of " $\Delta$ " obtained by removing all the loops touching  $k^{\text{th}}$  forward path.



1) Forward paths -

$$P_{1234} = G_1 G_2 G_3 G_4$$

$$P_{23} = G_2 G_3$$

2) To find  $\Delta$ .

i) Individual loops.

$$I_1 = -G_2 H_2 \quad I_4 = -G_2 G_3 G_4 H_1$$

$$I_2 = -G_4 H_4 \quad I_5 = -G_5 H_1$$

$$I_3 = -H_1$$

ii) Two NTL's.

$$L_1 = I_1 I_2 = G_2 H_2 G_4 H_4$$

$$L_2 = I_4 I_3 = G_2 H_2 H_3$$

3) To find  $\Delta_1$  &  $\Delta_2$

~~$$\Delta_1 = \Delta_2 = 1$$~~

$$\Delta = 1 - \begin{bmatrix} -G_2 H_2 - G_4 H_4 - H_3 \\ -G_2 G_3 G_4 H_1 - G_5 H_1 \end{bmatrix} + \begin{bmatrix} G_2 H_2 G_4 H_4 \\ G_2 H_2 H_3 \end{bmatrix}$$

3) To find  $\Delta_1$  &  $\Delta_2$

$$\Delta_1 = \Delta_2 = 1$$

$$\frac{Y_5}{Y_1} = P_1 \Delta_1 + P_2 \Delta_2$$

$$\frac{Y_5}{Y_1} = G_1 G_2 G_3 G_4 + G_1 G_5 - ①$$

case 2)  $\begin{bmatrix} Y_5 \\ Y_2 \end{bmatrix} = \begin{bmatrix} Y_5/Y_1 \\ Y_2/Y_1 \end{bmatrix}$

To find  $\begin{bmatrix} Y_2 \\ Y_1 \end{bmatrix}$

1) To find path

$$P_1 = G_1$$

2) To find  $\Delta$

$\Delta$  is same as above

[ $\Delta$  is independent of FP]

3) To find  $\Delta_1$

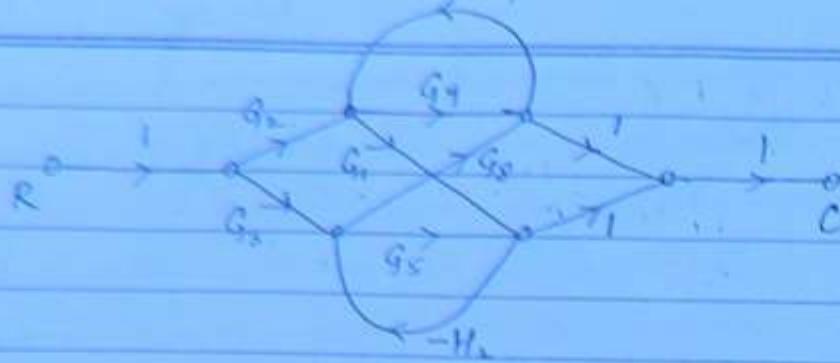
$$\Delta_1 = 1 - [-G_4 H_4 - H_3]$$

$$\frac{Y_2}{Y_1} = P_1 \Delta_1 = G_1 [1 + G_4 H_4 + H_3] - ②$$

$$\frac{Y_5}{Y_2} = \frac{Y_5/Y_1}{Y_2/Y_1} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{G_1 [1 + G_4 H_4 + H_3]}$$

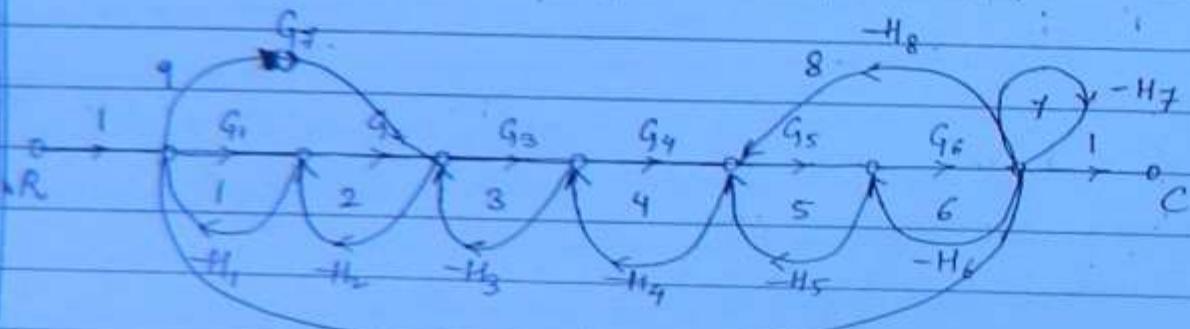
$$\frac{Y_5}{Y_2} = \frac{G_2 G_3 G_4 + G_5}{1 + G_4 H_4 + H_3} - ③$$

ONLY  
89



$$\frac{C}{R} = \frac{G_4 G_3 [1 + G_5 H_2] + G_3 G_5 [1 + G_4 H_1] + G_2 G_1 [1] + G_3 G_8 [1] + G_2 G_1 (-H_2) G_8 [1]}{1 - [-G_4 H_1 - G_5 H_2 + G_1 G_8 H_1 H_2] + [G_4 H_1 G_5 H_2]}$$

Q) For S.F.G find No. of forward paths & feedback paths -



- a) 3, 8
- b) 3, 10
- c) 3, 9
- d) 3, 11

$$L_{10} = G_8 (-H_8) (-H_7) (-H_3) (-H_2) (-H_1)$$

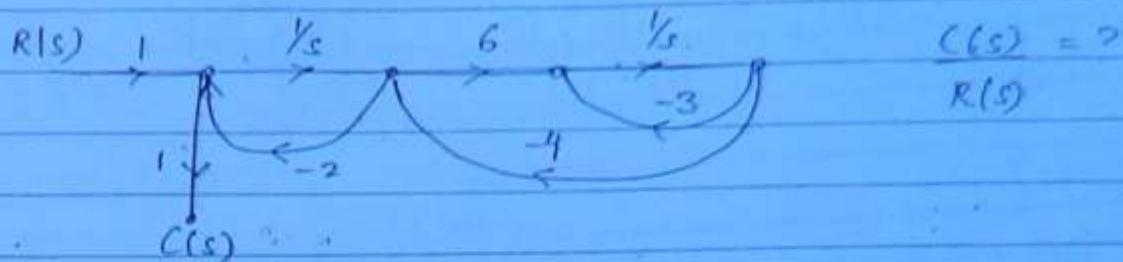
$$L_{11} = G_8 (-H_6) (-H_5) (H_4) (-H_3) (-H_2) (-H_1)$$

Ans (d)

CWR chapter -1

(Y1)

(5)



$$\frac{C(s)}{R(s)} = ?$$

sd

$$\frac{C}{R} = \frac{1 + \left\{ 1 - \left( -\frac{3}{s} - \frac{24}{s^2} \right) \right\}}{1 - \left[ \frac{-2}{s} - \frac{3}{s} - \frac{24}{s^2} \right] + \frac{6}{s^2}}$$

$$= \frac{s+27}{s^2+29s+6}$$

(d)

(4)

$$V_i(s) = Z_1(s) I_1(s) + Z_3(s) [I_1(s) - I_2(s)]$$

$$= Z_1 I_1 + Z_3 I_1 - Z_3 I_2$$

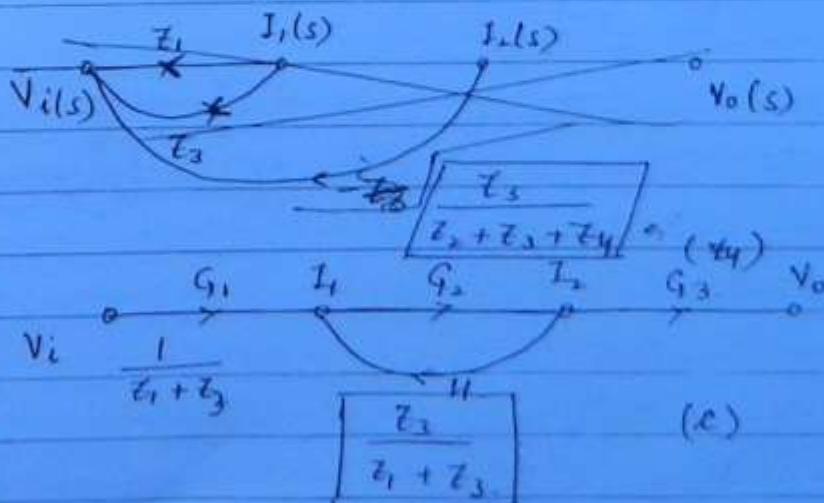
$$I_1 = \frac{V_i}{Z_1 + Z_3} + I_2 \cdot \frac{Z_3}{Z_1 + Z_3}$$

$$V_o(s) = Z_3 I_2$$

$$Z_2 I_2 + Z_4 I_2 + Z_3 (I_2 - I_1) = 0$$

$$Z_2 I_2 + Z_4 I_2 + Z_3 I_2 - Z_3 I_1 = 0$$

$$I_2 = \frac{Z_3}{Z_2 + Z_3 + Z_4} I_1$$



(c)



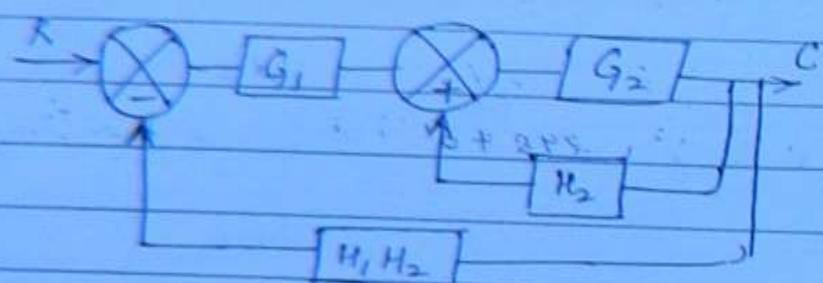
(42)

$$C = G_1 G_2 [1]$$

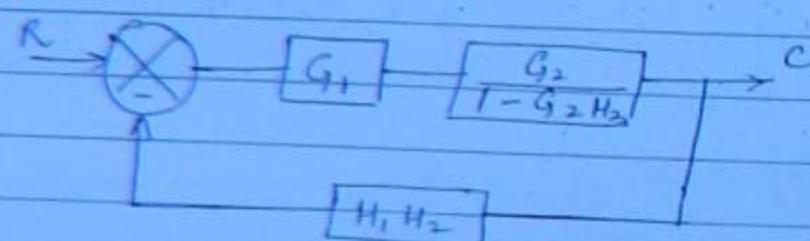
$$R = 1 - [G_1 H_1 + G_2 H_2 - G_1 G_2 H_1 H_2]$$

(c)

OR



↓



↓

$$C = G_1 G_2$$

$$R = \frac{1}{1 - G_2 H_2}$$

$$1 + \frac{G_1 G_2 (H_1 H_2)}{1 - G_2 H_2}$$

$$1 - G_2 H_2$$

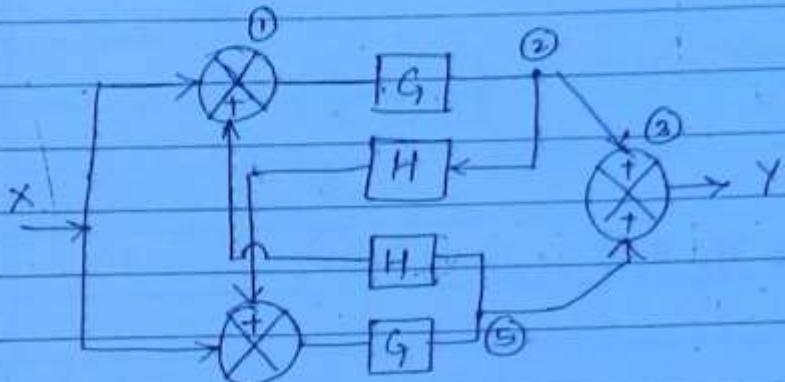
$$\frac{C}{R} = \frac{G_1 G_2}{1 - G_2 H_2}$$

$$1 - G_2 H_2 + G_1 G_2 H_1 H_2$$

Q 4 equals  
X

(4B)

- a)  $\frac{G}{1-GH}$    b)  $\frac{\partial G}{1-GH}$    c)  $\frac{GH}{1-GH}$    d)  $\frac{\partial^2 GH}{1-GH}$



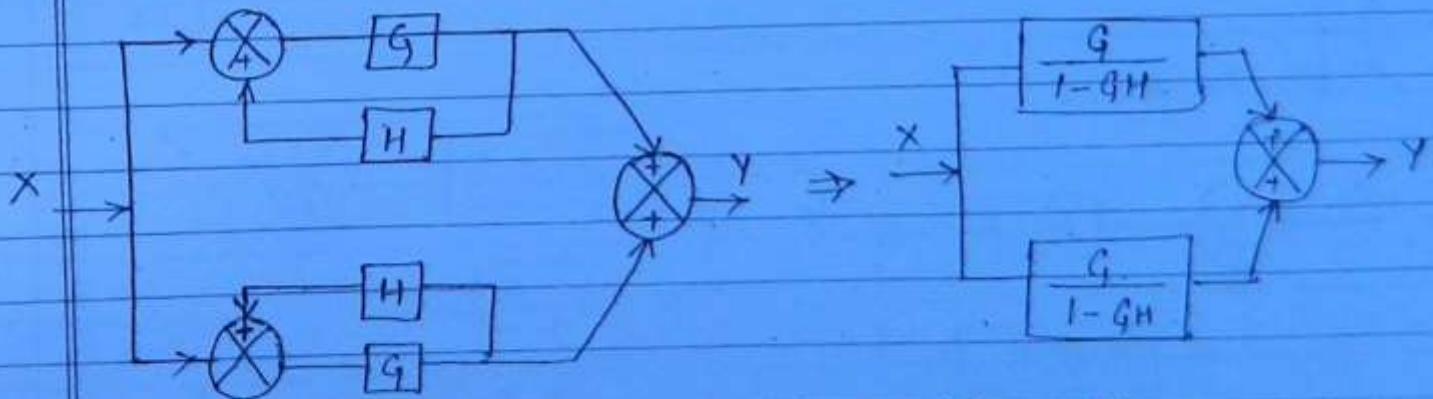
Sol.

$$\frac{Y}{X} = \frac{G[1] + G[1] + G^2H[1] + G^2H[1]}{1 - [G^2H^2]}$$

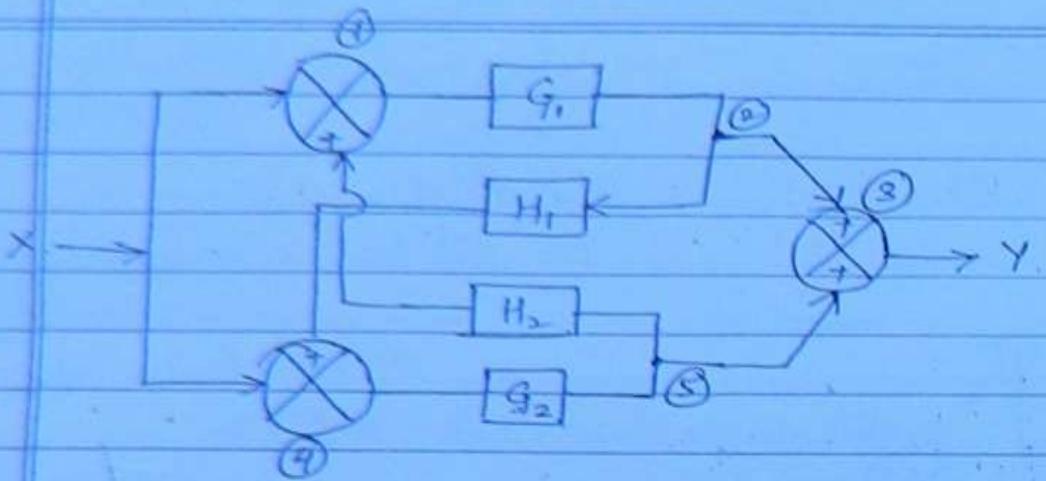
①②③      ④⑤③      ①②④⑤③      ④⑤①②③

$$\frac{Y}{X} = \frac{\partial G + \partial G^2 H}{1 - G^2 H^2} = \frac{\partial G [1 + GH]}{(1 - GH)(1 + GH)} = \frac{\partial G}{1 - GH} \quad -(b)$$

OR.



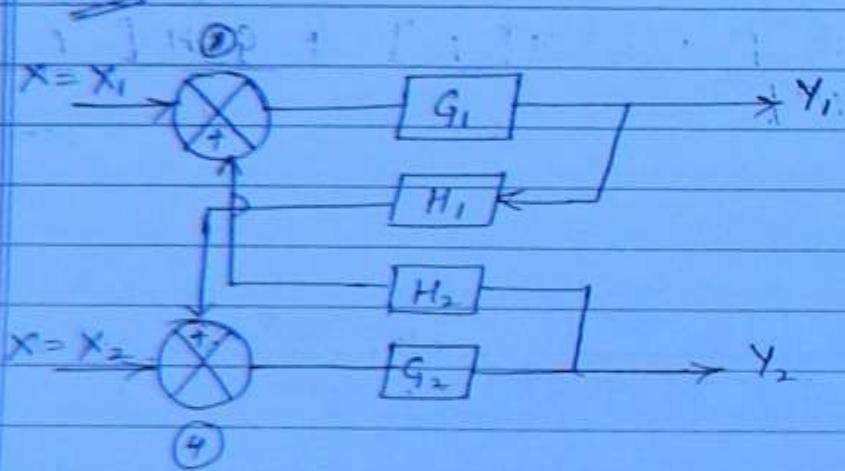
$$\frac{Y}{X} = \frac{\partial G}{1 - GH}$$



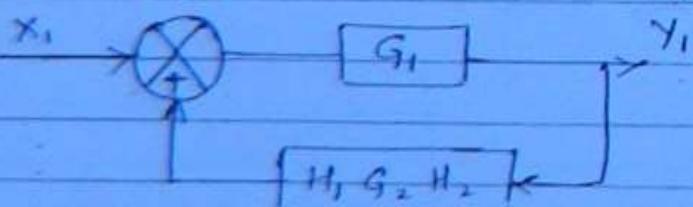
(44)

$$\frac{Y}{X} = \frac{G_1 + G_2 + G_1 H_1 G_2 + G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

OR



$$\rightarrow \frac{Y_1}{X_1} \Big|_{Y_2 = X_2 = 0}$$

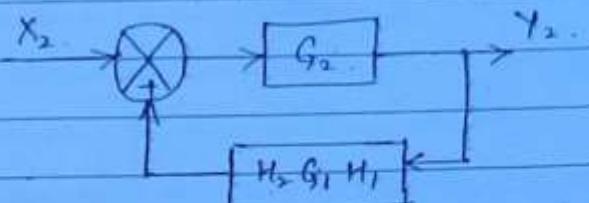


$$\frac{Y_1}{X_1} = \frac{G_1}{1 - G_1 H_1 G_2 H_2}$$

case 2  $\rightarrow$

$$\begin{array}{c|c} Y_2 \\ \hline X_2 & Y_1 = X_1 = 0 \end{array}$$

(45)

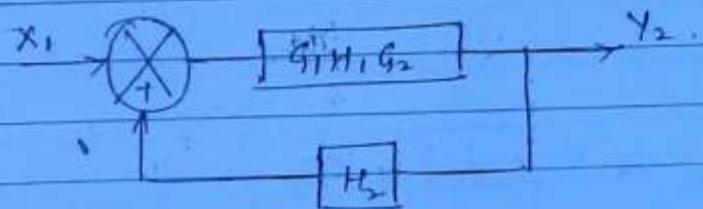


$$Y_2 = G_2$$

$$X_2 = 1 - G_1 H_1 G_2 H_2$$

case 3

$$\begin{array}{c|c} Y_2 \\ \hline X_1 & Y_1 = X_2 = 0 \end{array}$$

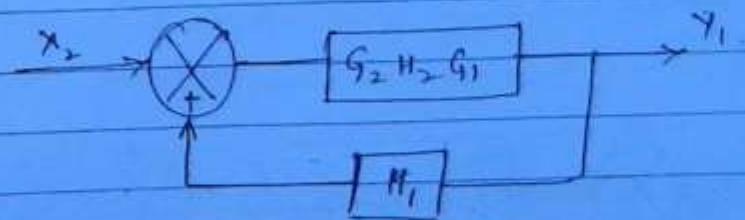


$$Y_2 = G_1 H_1 G_2$$

$$X_1 = 1 - G_1 H_1 G_2 H_2$$

case 4  $\rightarrow$

$$\begin{array}{c|c} Y_1 \\ \hline X_2 & Y_2 = X_1 = 0 \end{array}$$



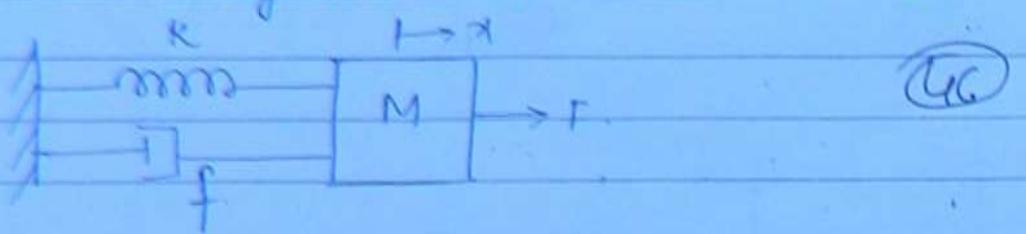
$$Y_1 = G_2 H_2 G_1$$

$$X_2 = 1 - G_1 H_1 G_2 H_2$$

$$\frac{Y}{X} = \frac{Y_1}{X_1} + \frac{Y_2}{X_2} + \frac{Y_1}{X_2} + \frac{Y_2}{X_1} = G_1 + G_2 + G_1 H_1 G_2 + G_2 H_2 G_1$$

$$1 - G_1 H_1 G_2 H_2$$

Q5. Draw an integrator based electronic ckt?

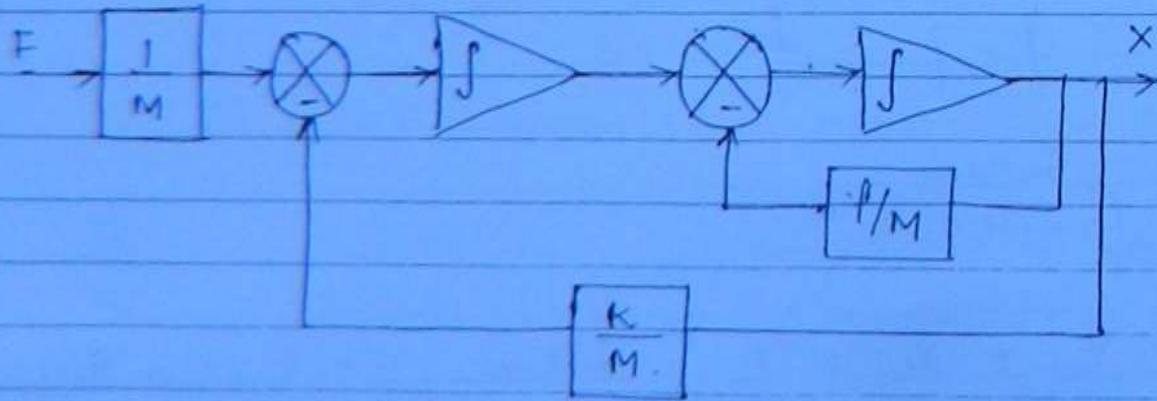
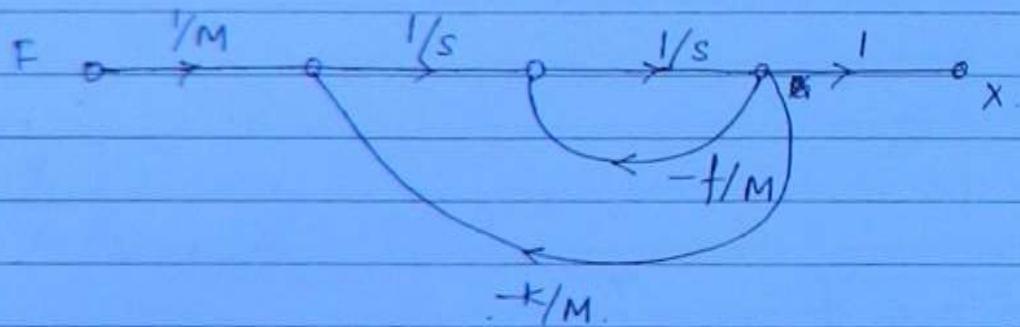


$$\text{Ans} \quad F = M \frac{d^2x}{dt^2} + f \frac{dx}{dt} + Kx.$$

$$F(s) = (Ms^2 + fs + K) X(s)$$

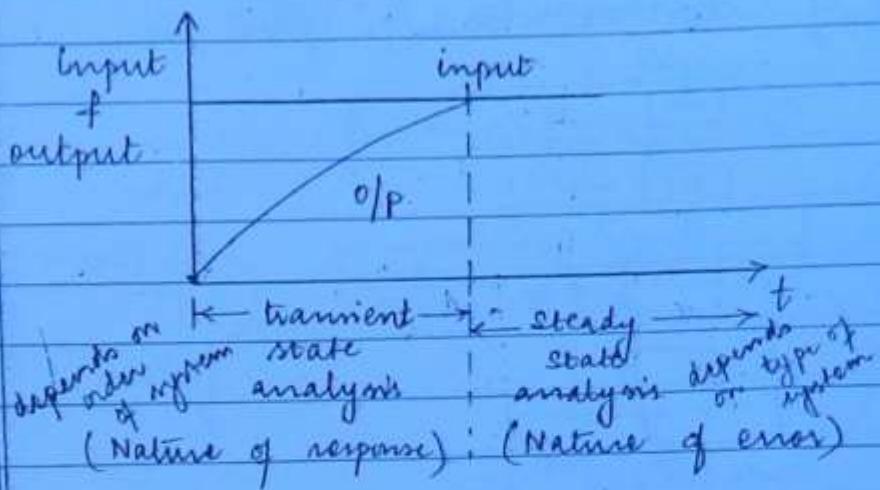
$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + fs + K} = \frac{1/M}{s^2 + \frac{f}{M}s + \frac{K}{M}}$$

$$\frac{x}{F} = \frac{1/M}{s^2} = \frac{1/M}{s^2} \cdot \frac{1 + \frac{f/M}{s} + \frac{K/M}{s^2}}{1 - \left[ \frac{-f/M}{s} - \frac{K/M}{s^2} \right]}$$



# TIME DOMAIN ANALYSIS

(47)



## STANDARD TEST SIGNALS -

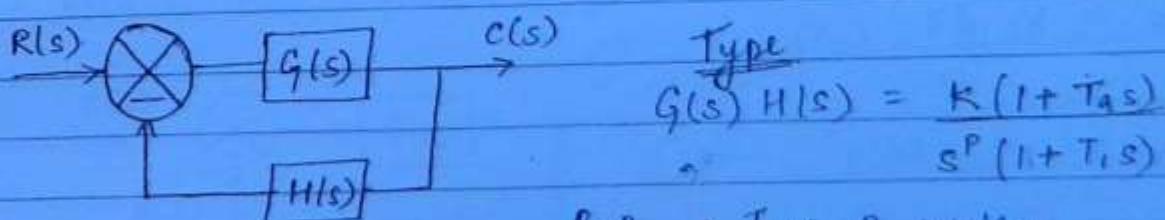
1. Sudden input  $\rightarrow$  step signal
  2. Velocity type input  $\rightarrow$  Ramp signal.
  3. Acceleration type input  $\rightarrow$  Parabolic signal
  4. Sudden shocks  $\rightarrow$  Impulse signals
- } Time domain Analysis
- } T.D.
- } stability Analysis

Signals ① & ④  $\Rightarrow$  Bounded inputs  $\rightarrow$  we use only step signal for transient analysis

Signals ② & ③  $\Rightarrow$  Un-bounded inputs

## TYPE & ORDER OF A SYSTEM

Every T.F. representing a control system has certain type & order



$p=0 \Rightarrow$  Type - 0 system

$p=1 \Rightarrow$  Type - 1 system

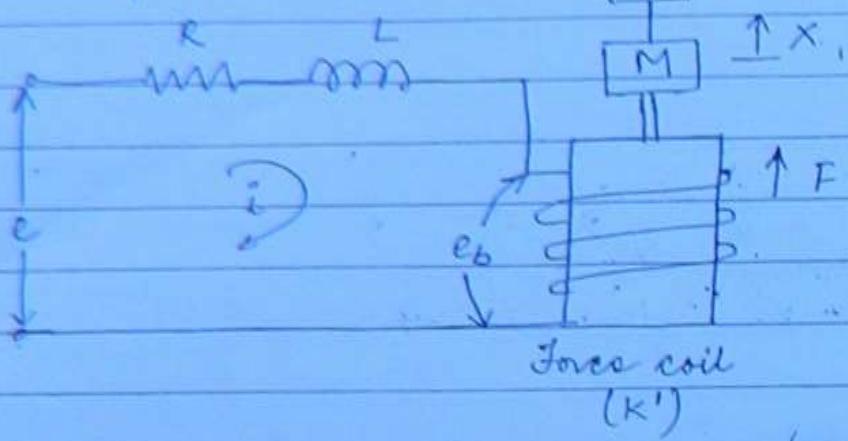
$\dots p=n \Rightarrow$  Type - n system

The no. of open loop poles at origin gives type of CS

$$\text{Order} - 1 + G(s) H(s) = 0$$

(48)

The highest power of characteristic eq<sup>n</sup> gives the order of the C.S.



$\rightarrow e = \text{input voltage}$

$\rightarrow e_b = \text{transduced voltage}$   
( $\propto$  speed)

Electrical system -

$$e = iR + L \frac{di}{dt} + e_b$$

$$e - e_b = iR + L \frac{di}{dt}$$

$$E(s) - E_b(s) = I s [R + L s] \rightarrow (1)$$

$\rightarrow e_b \propto \text{speed}$

$$e_b \propto \frac{dx}{dt}$$

$$e_b = k_b \frac{dx}{dt}$$

$$E_b(s) = k_b s X(s) \rightarrow (2)$$

$\rightarrow$  Force coil  $F \propto i \Rightarrow F = k' i$

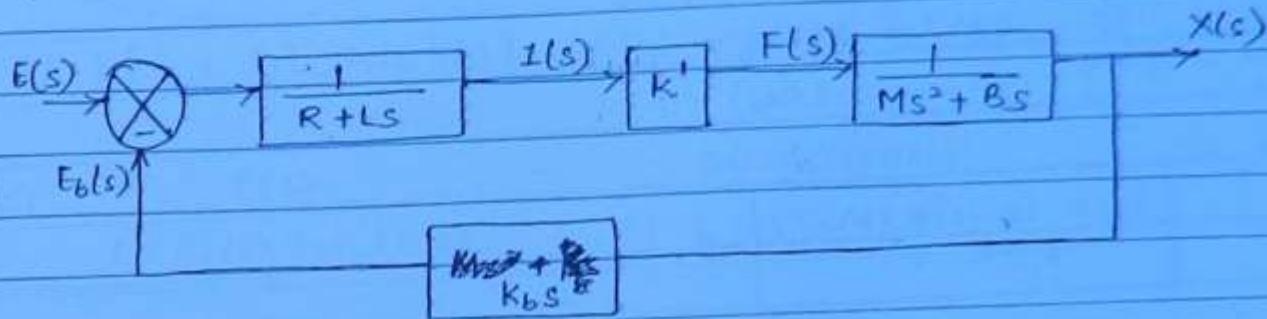
$$\Rightarrow F(s) = k' I(s) \rightarrow (3)$$

→ Mechanical system

$$F = M \frac{d^2x}{dt^2} + B \frac{dx}{dt}$$

$$\rightarrow F(s) = [Ms^2 + Bs] X(s) \quad (4)$$

(19)



$$G(s)H(s) = \frac{K' K_b \beta}{(R+Ls)(Ms^2+Bs)} = \text{Type 0 system}$$

$$1 + G(s)H(s) = 0 \Rightarrow 1 + \frac{K' K_b}{(R+Ls)(Ms+B)} = 0$$

$$\Rightarrow (R+Ls)(Ms+B) + K' K_b = 0$$

Order 2 system

### STEADY STATE RESPONSE ANALYSIS -

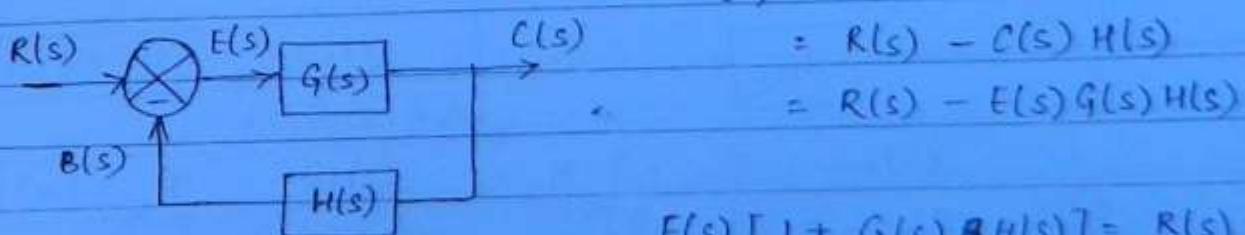
It deals with the estimation of magnitude of steady state error between input and output & depends on type of the control system.

To obtain an expression for error

$$E(s) = R(s) - B(s)$$

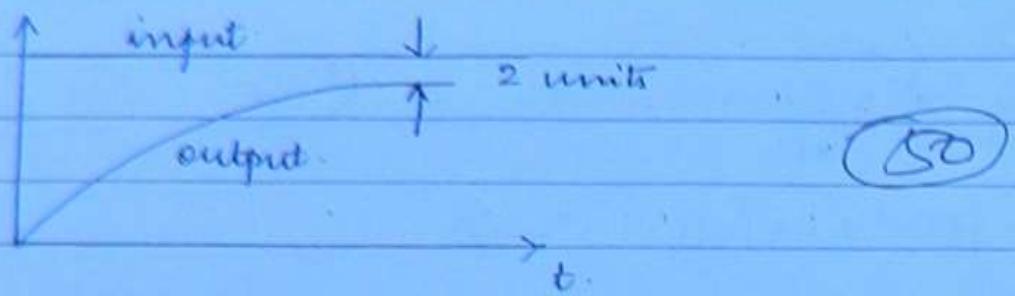
$$= R(s) - C(s) H(s)$$

$$= R(s) - E(s) G(s) H(s)$$



$$E(s) [1 + G(s) H(s)] = R(s)$$

$$E(s) = \frac{R(s)}{1 + H(s) G(s)}$$



$$\lim_{t \rightarrow \infty} e(t) = 2, \text{ units}$$

(e\_{ss})

$e_{ss} = \lim_{t \rightarrow \infty} e(t)$
---

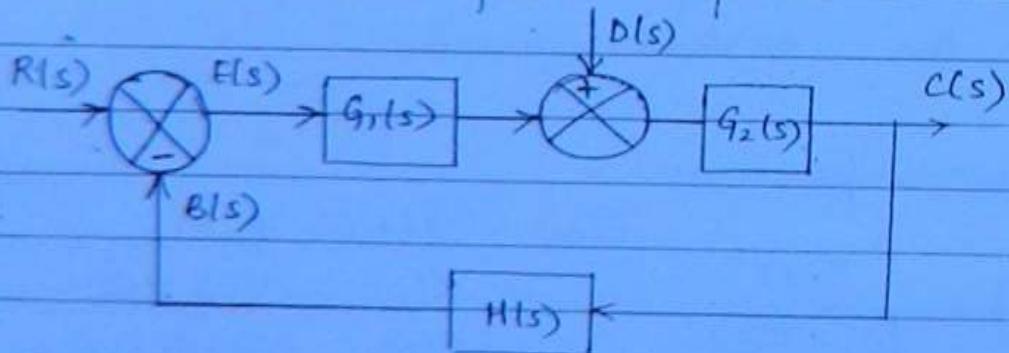
Applying final value theorem:

$e_{ss} = \lim_{s \rightarrow 0} s \cdot E(s)$
--

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s \cdot R(s)}{1 + G(s) H(s)}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{\{s \cdot R(s)\}}{1 + \lim_{s \rightarrow 0} \{H(s) G(s)\}}$$

→ To obtain an expression for error with disturbances -



$$E(s) = R(s) - B(s)$$

$$e(s) = R(s) - C(s) H(s)$$

$$C(s) = [E(s) G_1(s) + D(s)] G_2(s)$$

$$C(s) = E(s) G_1(s) G_2(s) + D(s) G_2(s)$$

(51)

$$E(s) = R(s) - \underbrace{E(s) G_1(s) G_2(s) H(s)}_{\rightarrow D(s) G_2(s) H(s)}$$

$$E(s) [1 + G_1(s) G_2(s) H(s)] = R(s) - D(s) G_2(s) H(s)$$

$E(s) =$	$\frac{R(s)}{1 + G_1(s) G_2(s) H(s)}$	$- \frac{D(s) G_2(s) H(s)}{1 + G_1(s) G_2(s) H(s)}$
----------	---------------------------------------	---

$$ess = \lim_{s \rightarrow 0} s \cdot E(s)$$

$ess = \lim_{s \rightarrow 0} s \cdot \frac{R(s)}{1 + G_1(s) G_2(s) H(s)} - \lim_{s \rightarrow 0} s \cdot \frac{D(s) G_2(s) H(s)}{1 + G_1(s) G_2(s) H(s)}$	Error due to $R \rightarrow$ put $D(s) = 0$
---	---

CWB chapter 2

Error due to  $D \rightarrow$  put  $R(s) = 0$   
Error due to comparison  
 $\rightarrow$  neither = 0

(5)

$$R = 0 \quad (\text{like given})$$

$$ess = - \lim_{s \rightarrow 0} s \cdot \frac{D(s) G_2(s) H(s)}{1 + G_1(s) G_2(s) H(s)}$$

$$ess = - \lim_{s \rightarrow 0} s \cdot \frac{G_2(s)}{1 + G_1(s) G_2(s)}$$

$$|ess| = \lim_{s \rightarrow 0} \frac{G_2(s)}{1 + G_1(s) G_2(s)}$$

$$|ess| = \lim_{s \rightarrow 0} \frac{G_2(s)}{G_1(s) G_2(s)} = \lim_{s \rightarrow 0} \left[ \frac{1}{G_1(s)} \right] \rightarrow (c)$$

Steady state error for all types of inputs -

a) Step input -

$$R(s) = \frac{A}{s}$$

(52)

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot A$$

$$= \frac{1 + G(s)H(s)}{1 + G(s)H(s)}$$

$$= \frac{A}{1 + G(s)H(s)}$$

$$= \frac{A}{1 + k_p}$$

$k_p$  = position error constant

$$\lim_{s \rightarrow 0} (1 + G(s)H(s))$$

$$= 1 + k_p$$

$$e_{ss} = \frac{A}{1 + k_p}$$

b) Ramp input -

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{A}{s^2}$$

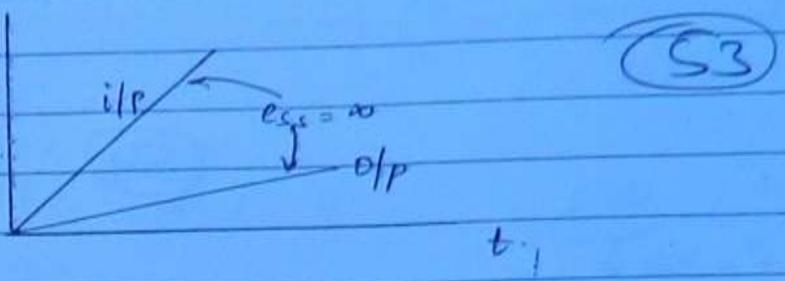
$$= \frac{1 + G(s)H(s)}{1 + G(s)H(s)}$$

$$= \frac{A}{\frac{1 + s \cdot G(s)H(s)}{s \cdot G(s)H(s)}} = \frac{A}{k_v}$$

$k_v$  = Velocity error constant

$$= \lim_{s \rightarrow 0} s \cdot G(s)H(s)$$

$$e_{ss} = \frac{A}{k_v}$$

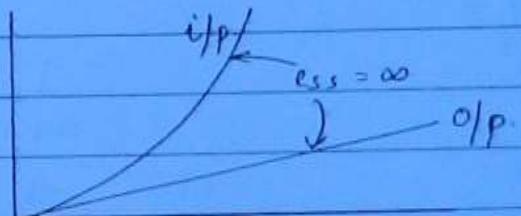


c) Parabolic Input -

$$R(s) = A$$

$$s^3$$

$$\frac{e_{ss}}{s \rightarrow 0} = \frac{A}{s^3} = \frac{A}{s \rightarrow 0} = \frac{At^2 + 0t^2 \cdot k(1+Tas)}{s \rightarrow 0} = \frac{At^2}{1+T_1 s} = \frac{A = 0}{0}$$



	Step Input	Ramp Input	Parabolic Input
TYPE - 0	$\frac{A}{1+K}$	$\infty$	$\infty$
	$K_p = K$	$K_V = 0$	$K_A = 0$
TYPE - 1	0	$\frac{A}{K}$	0
	$K_p = \infty$	$K_V = K$	$K_A = 0$
TYPE - 2	0	0	$\frac{A}{K}$

## Observations -

$$1. \quad e_{ss} \propto \frac{1}{K}$$

(54)

as  $K \uparrow \therefore e_{ss} \downarrow$

2. The maximum type number of linear control system is 2. Beyond type -2 the system exhibits non-linear characteristics more dominantly.

CWB chapter 3.

(Q1)

$$r(t) = 2 + 3t + 4t^2$$

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{4}{s^3}$$

$$= \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$$

$$G(s) = \frac{10}{s^2(4+s)}$$

$$e_{ss} = 0 + 0 + \frac{A}{K}$$

$$A = 8$$

$$k = k_A = \lim_{s \rightarrow 0} \frac{s^2 \times 10}{s^2(4+s)} = 10/4$$

$$e_{ss} = \frac{8 \times 4}{10} = 3.2 \text{ units} \quad -(d)$$

(18)

$$\text{Type -1} \quad e_{ss} = \frac{1}{K} \rightarrow \frac{5}{100} = \frac{1}{K}$$

(a5)

Type - D

$$e_{ss} = \frac{1}{1+k}$$

/s.  
Integrator

TYPE - I

$$e_{ss} = \frac{1}{K} = \frac{1}{4} = 0.25 \text{ units}$$

(d)

$$0.25 = \frac{1}{1+k} \Rightarrow k = 4$$

(a3)

$$n(t) = [1-t^2] 3u(t)$$

$$R(s) = \frac{3}{s} - \frac{6}{s^3}$$

$$e_{ss} = \frac{A}{1+k} - \frac{A}{K}$$

$$e_{ss} = \frac{3}{1+k_p} - \frac{6}{k_a} \quad (d)$$

(4)

$$e_{ss} = \frac{1}{1+k}$$

$$k = k_p = \lim_{s \rightarrow 0} \frac{10}{s^2 + 14s + 50}$$

$$k = \frac{10}{50}$$

$$e_{ss} = \frac{1}{1+k} = \frac{1}{1+0.2} = 0.83$$

$$e_{ss} = 0.83 \text{ units} \quad (b)$$

## ERROR SERIES-

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

(56)

$$\text{let } F(s) = \frac{1}{1 + G(s)H(s)}$$

$$E(s) = R(s)F(s)$$

$$L^{-1} E(s) = L^{-1} [R(s)F(s)]$$

$$e(t) = \int_0^\infty f(T) \nu(t-T) dT$$

Expanding  $\nu(t-T)$  using Taylor series -

$$\nu(t-T) = \nu(t) - T \dot{\nu}(t) + T^2 \ddot{\nu}(t) - \frac{T^3}{2!} \dddot{\nu}(t) + \dots$$

$$\begin{aligned} e(t) &= \nu(t) \int_0^\infty f(T) dT - \dot{\nu}(t) \int_0^\infty T f(T) dT + \frac{\ddot{\nu}(t)}{2!} \int_0^\infty T^2 f(T) dT \\ &\quad - \frac{\dddot{\nu}(t)}{3!} \int_0^\infty T^3 f(T) dT \dots \end{aligned}$$

Dynamic "Dynamic Error Constants"

$$K_0 = \int_0^\infty f(T) dT \quad K_1 = - \int_0^\infty T f(T) dT$$

$$K_2 = \int_0^\infty T^2 f(T) dT$$

$$K_3 = - \int_0^\infty T^3 f(T) dT$$

$$e(t) = K_0 \nu(t) + K_1 \overset{\circ}{\nu}(t) + K_2 \overset{''}{\nu}(t) + \frac{K_3 \overset{'''}{\nu}(t)}{3!} + \dots$$

error series

$$ess = \lim_{t \rightarrow \infty} e(t)$$

(S7)

To find Dynamic Error Constants -

$$\mathcal{L} f(T) = F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$\lim_{s \rightarrow 0} F(s) = \lim_{s \rightarrow 0} \int_0^{\infty} f(t) e^{-st} dt = \int_0^{\infty} f(t) dt \Rightarrow k_0$$

$$\rightarrow \frac{d}{ds} F(s) = \lim_{s \rightarrow 0} - \int_0^{\infty} T f(t) e^{-st} dt \Rightarrow - \int_0^{\infty} T f(t) dt \Rightarrow k_1$$

$$k_0 = \lim_{s \rightarrow 0} F(s)$$

$$k_1 = \lim_{s \rightarrow 0} \frac{d F(s)}{ds}$$

$$k_2 = \lim_{s \rightarrow 0} \frac{d^2 F(s)}{ds^2}$$

$$\text{where } F(s) = \frac{1}{1 + G(s) H(s)}$$

Relationship between static & dynamic error constants

$$G(s)H(s) = \frac{100}{s(s+2)}$$

(58)

I. Static Error Constants -

$$K_p = \lim_{s \rightarrow 0} \frac{100}{s(s+2)} = \infty$$

$$K_v = \lim_{s \rightarrow 0} s \cdot \frac{100}{s(s+2)} = 50$$

$$K_A = \lim_{s \rightarrow 0} s^2 \frac{100}{s(s+2)} = 0$$

II. Dynamic Error Constants -

$$F(s) = \frac{1}{1 + G(s)H(s)} = \frac{1}{1 + \frac{100}{s(s+2)}}$$

$$K_d = \lim_{s \rightarrow 0} F(s) = \lim_{s \rightarrow 0} \frac{1}{1 + \frac{100}{s(s+2)}} = \frac{1}{1 + \infty} = 0$$

$$K_d = \boxed{\frac{1}{1 + K_p}}$$

$$K_i = \lim_{s \rightarrow 0} \frac{d}{ds} F(s)$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \frac{1}{1 + \frac{100}{s(s+2)}} = \frac{d}{ds} \left[ \frac{s(s+2)}{s^2 + 2s + 100} \right]$$

$$= \frac{(s^2 + 2s + 100)(2s+2) - s(s+2)(2s+2)}{(s^2 + 2s + 100)^2}$$

$$K_1 = \lim_{s \rightarrow 0} \frac{d}{ds} F(s)$$

(59)

$$= \frac{100(2) - 0}{(0+0+100)^2} = \frac{200}{100 \times 100} = \frac{1}{50}$$

$$K_1 = \frac{1}{K_V}$$

$$K_2 = \frac{1}{K_A}$$

NOTE :

Static & Dynamic error constants  
need not always be direct reciprocal values.

Q. Find ess for  $\eta(t) = 5 + 2t$

1. Error Series -

$$ess = \lim_{t \rightarrow \infty} [K_0 \eta(t) + K_1 \dot{\eta}(t) + K_2 \ddot{\eta}(t) + \dots]$$

$$\eta(t) = 5 + 2t \Rightarrow K_0 = 0$$

$$\dot{\eta}(t) = 2 \Rightarrow K_1 = \frac{1}{50}$$

$$\ddot{\eta}(t) = 0$$

$$ess = \lim_{t \rightarrow \infty} [0 \times (5 + 2t) + \frac{1 \times 2}{50}]$$

$$ess = \frac{2}{50} \text{ units.}$$

2. Error Ratio

$$ess = \lim_{s \rightarrow 0} \frac{s \cdot R(s)}{1 + G(s) H(s)}$$

$$\eta(t) = 5 + 2t$$

$$R(s) = \frac{5}{s} + \frac{2}{s^2} = \frac{5s+2}{s^2}$$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{5s+2}{s^2}$$

(6)

$$1 + \frac{100}{s(s+2)}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{5s+2}{s + \cancel{s} \cdot 100} = \frac{2}{50} \text{ units}$$

### 3. Short cut method

$$r(t) = 5 + 2t$$

$$R(s) = \frac{5}{s} + \frac{2}{s^2}$$

$$G(s) = \frac{100}{s(s+2)}$$

$$e_{ss} = D + \frac{A}{k}$$

$$A = 2$$

$$k = k_v = \lim_{s \rightarrow 0} s \cdot \frac{100}{s(s+2)} = 50$$

$$e_{ss} = 2 \text{ units}$$

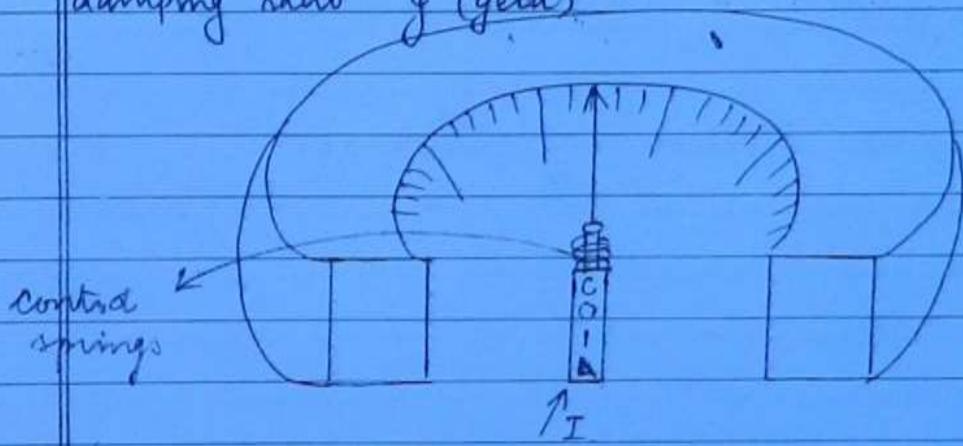
## TRANSIENT STATE ANALYSIS -

(G)

It deals with the nature of response of a system & depends on the order of the system.

The response of a 2<sup>nd</sup> order or higher order system exhibits continuous & sustained oscillations about the steady state value of input with a frequency known as undamped natural frequency  $\omega_n$  rad/s.

Those oscillations in the response are damped to the steady state value of input using appropriate damping methods with damping expressed mathematically as the damping ratio  $\zeta$  (beta).



Input = Deflecting torque ( $T_d$ )

Output = Angular deflection of pointer ( $\theta$ )

$J, B, K$

$$T_d = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} + K\theta$$

$$T_d(s) = (J s^2 + B s + K) \theta(s)$$

$$\frac{\theta(s)}{T_d(s)} = \frac{1}{J s^2 + B s + K} = \frac{s^2 + \frac{B}{J} s + \frac{K}{J}}{s^2 + \frac{B}{J} s + \frac{K}{J}}$$

$$s^2 + \frac{B}{J} s + \frac{K}{J} = s^2 + 2 \zeta \omega_n s + \omega_n^2$$

$$\omega_n = \sqrt{\frac{K}{J}} \text{ rad/s}$$

(62)

$$2G\sqrt{\frac{K}{J}} = \frac{B}{J} \Rightarrow \boxed{g = \frac{B}{2\sqrt{KJ}}} \\ g \propto B.$$

Undamped Natural frequency  $\omega_n$  rad/s.

$\therefore$  Damping Ratio  $\zeta$  [Gebet]

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2G\omega_n s + \omega_n^2}$$

Effect of Damping on Nature of Response -

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2G\omega_n s + \omega_n^2}$$

$$\begin{aligned} s^2 + 2G\omega_n s + \omega_n^2 &= 0 \\ &= -2G\omega_n s \pm \sqrt{4G^2\omega_n^2 - 4\omega_n^2} \\ &= -G\omega_n \pm \omega_n \sqrt{G^2 - 1} \end{aligned}$$

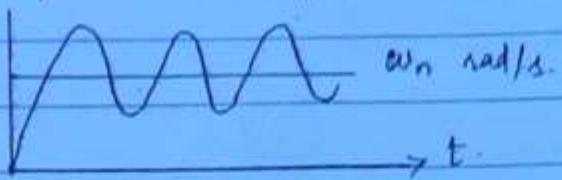
$$D = G^2 - 1 = 0 \Rightarrow \zeta = 1$$

$$D = G^2 - 1 < 0 \Rightarrow \zeta < 1$$

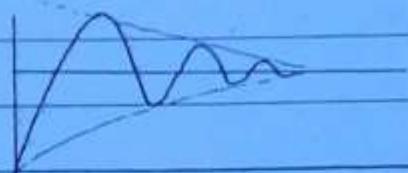
$$D = G^2 - 1 > 0 \Rightarrow \zeta > 1$$

case 1  $\Rightarrow$  Undamped case ( $\zeta = 0$ )

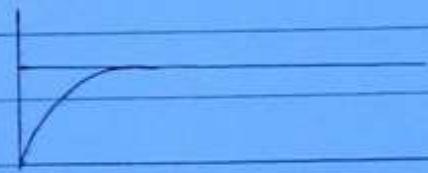
(G3)



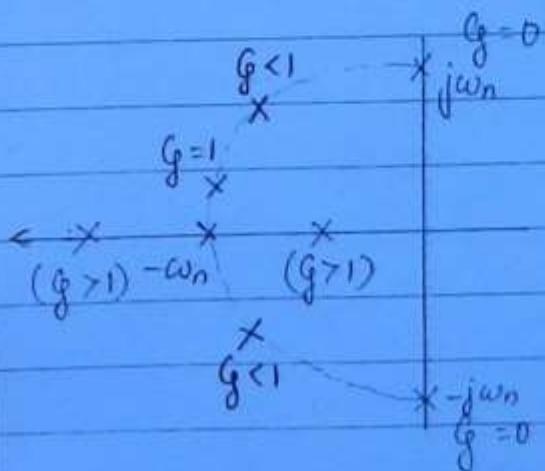
case 2  $\Rightarrow$  Underdamped case [ $0 < \zeta < 1$ ]



case 3  $\Rightarrow$  Critically damped case [ $\zeta = 1$ ]



case 4  $\Rightarrow$  Overdamped case [ $\zeta > 1$ ]

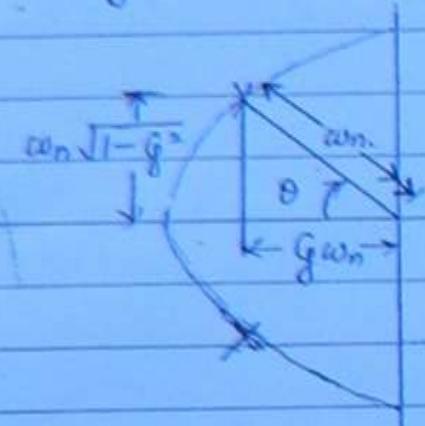


Most of the control systems are defined for  $\zeta < 1$  because the response can be analysed using more number of specifications

## Characteristics of Underdamped systems -

$$- \text{ gain } \geq \omega_n \sqrt{g^2 - 1}$$

(64)



$$(1) \cos \theta = \frac{g\omega_n}{\omega_n} \quad \theta = \cos^{-1} g$$

(2) Damping co-efficient

(a) Damping factor

(b) Actual damping

$$\alpha = g\omega_n$$

(3) Time constant of under damped response

$$T = \frac{1}{\alpha} = \frac{1}{g\omega_n}$$

(4) Damped Natural frequency

$$\omega_d = \omega_n \sqrt{1 - g^2} \text{ rad/s}$$

$$(5) s^2 + 2g\omega_n s + \omega_n^2 = (s + g\omega_n)^2 + \omega_d^2$$

(6) Damping ratio = Actual damping / Critical damping

$$\text{Actual damping} = g\omega_n$$

# TRANSIENT ANALYSIS (underdamped response)

(GS)

$$\text{Let } R(s) = \frac{1}{s}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$C(s) = \frac{1}{s} - \frac{(s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{(s + 2\zeta\omega_n)}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{(s + \zeta\omega_n)}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2} \cdot \frac{\zeta\omega_n}{\omega_n \sqrt{1 - \zeta^2}}$$

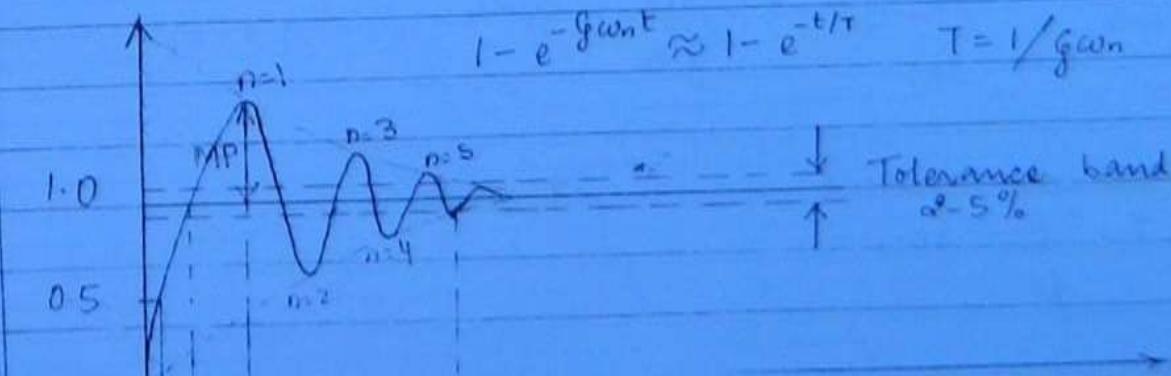
$$C(t) = 1 - e^{-\zeta\omega_n t} \cos \omega_d t - e^{-\zeta\omega_n t} \sin \omega_d t \cdot \frac{\zeta}{\sqrt{1 - \zeta^2}}$$

$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \left[ \frac{\sqrt{1 - \zeta^2}}{B} \cos \omega_d t + \frac{\zeta}{A} \sin \omega_d t \right]$$

$$A \sin \omega_d t + B \cos \omega_d t = \sqrt{A^2 + B^2} \sin [\omega_d t + \tan^{-1} B/A]$$

$$C(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin \left[ \omega_d t + \tan^{-1} \frac{\zeta}{\sqrt{1 - \zeta^2}} \right]$$

↓      ↓  
 Steady state      transient



Time required by response to reach

1. Delay time ( $t_d$ ) 50% of final value

$$t_d = \frac{1 + 0.7G}{\omega_n} \text{ secs. for unit step } i/p$$

2. Rise time ( $t_r$ ) 10 to 90% of final value

$$t_r = \frac{\pi - \theta}{\omega_n} \quad [\theta = \tan^{-1} \sqrt{1 - g^2}]$$

(66)

3. Peak time ( $t_p$ )

$$t_p = \frac{n\pi}{\omega_n} \text{ secs.}$$

4. Settling time ( $t_s$ )

$$8\% \text{ T.B.} \rightarrow 4T \rightarrow 4/g_{an} \text{ secs.}$$

$$5\% \text{ T.B.} \rightarrow 3T \rightarrow 3/g_{wn} \text{ secs.}$$

5. Maximum peak overshoot ( $M_p$ )

$$M_p = e^{-8\pi/\sqrt{1-g^2}} \approx e^{-8\pi\pi/\sqrt{1-g^2}}$$

6. Number of cycles

$$\omega_n = \frac{g\pi}{f_d} \Rightarrow f_d = \frac{\omega_n}{g\pi} \left[ \frac{\text{cycles}}{\text{sec}} \right]$$

$$8\% \text{ T.B.} \rightarrow t_s \times f_d \rightarrow \frac{4}{g_{wn}} f_d \text{ cycles}$$

$$5\% \text{ T.B.} \rightarrow t_s \times f_d \rightarrow \frac{3}{g_{wn}} f_d \text{ cycles}$$

7. Time interval / period of sample,  $T$

$$T = \frac{1}{f_d} \text{ secs.}$$

$f_d$

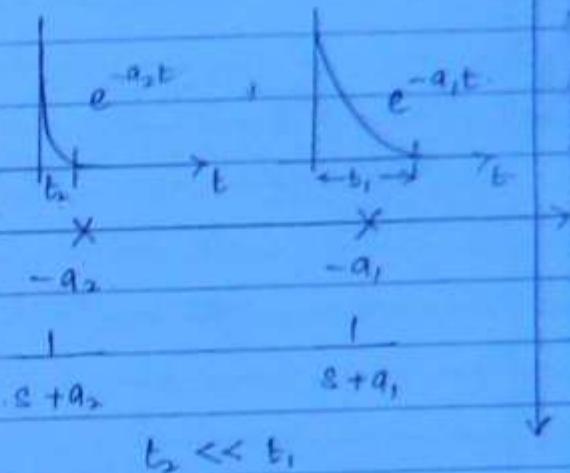
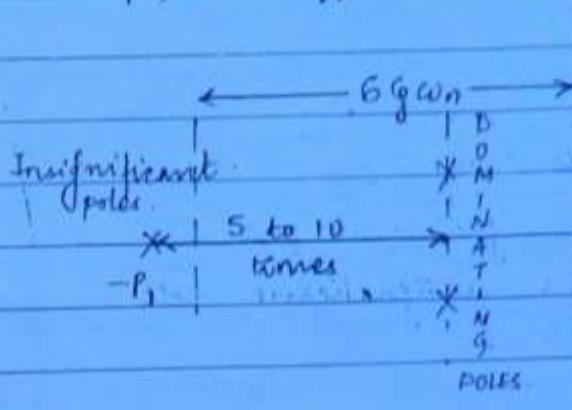
# TIME RESPONSE ANALYSIS FOR HIGHER ORDER SYSTEM -

Consider a 3<sup>rd</sup> order characteristic equation

$$s^3 + ps^2 + qs + k = 0$$

$$(s + p_1)(s^2 + q_1s + k_1) = 0$$

(67)



The time response of higher order system is obtained by approximating to second order system w.r.t dominant poles. The time domain specifications obtained for approximated second order system are valid for the higher order system also.

all values remain same for higher order & second order systems

CWB chapter 7

(5) (a)

chapter 6

$$\begin{aligned} \text{(22)} \quad T(s) &= \frac{5}{(s+5)(s^2+s+1)} = \frac{1}{(1+s/5)(s^2+s+1)} \\ &= \frac{1}{(s^2+s+1)} \quad (\text{d}) \end{aligned}$$

first convert to time constant form  $(1/T_s)$

then can remove insignificant pole

Chapter 3.

(14)

$$G(s) = \frac{100}{(s+1)(s+100)}$$

(168)

One pole is at  $s = -1$ .

Other pole is 100 times away.

After 10 times its insignificant pole.

So  $-(s+100)$  is an insignificant pole.

$$\begin{aligned} &= \frac{1}{(1+s)(1+\cancel{s+100})} \\ &= \frac{1}{1+s} = \frac{1}{1+Ts} \\ T &= 1 \text{ sec.} \end{aligned}$$

$$\begin{aligned} t_s &= 4T \Rightarrow 4 \times 1 \text{ sec} \\ &= 4 \text{ secs} \quad \text{Ans} \end{aligned}$$

Ques.

$$G(s) = \frac{K(s+2)}{s^3 + \alpha s^2 + 4s + 1}$$

$$1 + G(s) = 0$$

$$1 + \frac{K(s+2)}{s^3 + \alpha s^2 + 4s + 1} = 0$$

$$s^3 + \alpha s^2 + s(4+K) + (1+2K) = 0$$

$$(s+a)(s^2 + bs + c) = 0$$

$$\Rightarrow C = \omega_n^2 = 9$$

$$b = \omega_n \alpha = 2 \times 0.2 \times 3 = 1.2$$

$$\Rightarrow (s+a)(s^2 + 1.2s + 9)$$

$$s^3 + 1.2s^2 + 9s + as^2 + 1.2as + 9a = 0$$

$$s^3 + s^2(1.2+a) + s(9+1.2a) + 9a = 0$$

(69)

$$\alpha = 1.2 + a$$

$$4 + K = 9 + 1.2a \quad \square$$

$$2K + 1 = 9a$$

$$\underline{K = 7} \quad \underline{\alpha = 2.7}$$

CONN 4.

$$G(s) = \frac{K}{s(st+1)} \quad H(s) = 1$$

$$TF = \frac{G(s)}{1 + H(s)G(s)} = \frac{K}{s(st+1)} \quad TS^2 + s + K = 0$$
$$\frac{s^2 + s}{T} + \frac{K}{T} = 0$$
$$\omega_n = \sqrt{\frac{K}{T}} \text{ rad/s}$$

$$2G\sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$G = \frac{1}{2\sqrt{KT}}$$

case 1 %  $M_p = 40\%$

$$M_p = 0.4$$

$$e^{-g\pi/\sqrt{1-g^2}} = 0.4$$

$$g = g_1 = 0.18$$

let  $K = k_1$

$$g_1 = \frac{1}{2\sqrt{K_1 T}}$$

$$g_2 = \frac{1}{2\sqrt{K_2 T}}$$

$$\left[ \frac{0.28}{0.16} \right]^2 = \frac{k_2}{k_1}$$

$$K_2 = 3K_1$$

case 2 %  $M_p = 60\%$   $M_p = 0.6$

$$e^{-g\pi/\sqrt{1-g^2}} = 0.6$$

## SENSITIVITY ANALYSIS -

Let  $\alpha$  = A variable that changes its value.

$\beta$  = A parameter that changes the value of  $\alpha$ .

$$S_p^\alpha = \frac{\% \text{ change in } \alpha}{\% \text{ change in } \beta} = \frac{\partial \alpha / \alpha}{\partial \beta / \beta}$$

(170)

$$S_p^\alpha = \frac{\beta}{\alpha} \cdot \frac{\partial \alpha}{\partial \beta}$$

## Open Loop Control System -

$$R(s) \rightarrow G(s) H(s) \rightarrow C(s)$$

$$\text{Let } M(s) = O.L.C.S$$

$$\alpha = M(s) [O.L.C.S]$$

$$\beta = G(s)$$

$$S_{G(s)} = \frac{M(s)}{G(s)} \cdot \frac{\partial M(s)}{\partial G(s)}$$

$$M(s) = G(s) H(s)$$

$$G(s) = 1$$

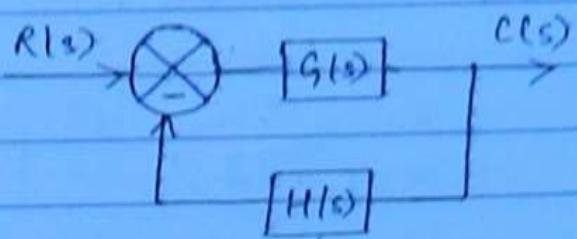
$$M(s) = H(s)$$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)} G(s) H(s) = H(s)$$

$$S_{G(s)} = \frac{H(s)}{H(s)} \times H(s) = 1$$

# Closed Loop Control System

(71)



$$\text{let } M(s) = \text{C.L.C.S.}$$

$$M(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$\text{Let } \alpha = M(s)$$

$$\beta = G(s)$$

$$S_{g(s)}^{M(s)} = \frac{G(s)}{M(s)} \frac{\partial M(s)}{\partial G(s)}$$

$$M(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$G(s) = 1 + G(s)H(s)$$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)} \left[ \frac{G(s)}{1 + G(s)H(s)} \right]$$

$$\frac{1 + G(s)H(s) - G(s)H(s)}{(1 + G(s)H(s))^2} = \frac{1}{(1 + H(s)G(s))^2}$$

$$S_{g(s)}^{M(s)} = |1 + G(s)H(s)| \times \frac{1}{(1 + G(s)H(s))^2}$$

$$S_{g(s)}^{M(s)} = \frac{1}{|1 + G(s)H(s)|}$$

$ 1 + G(s)H(s) $	= Noise Reduction factor
------------------	--------------------------

Open Loop Systems are highly sensitive to internal or external disturbances compared to closed loop systems.

CWB chapter 2

(172)

② FA  $\rightarrow$  disturbance in fwd path

$$\text{return diff} = 1 + G(s)H(s) \\ = 1 + AB \quad (b)$$

-ve means feedback.

③ B

$$10 \times 10 \times 10 = 100 \quad \frac{g(s)}{1+H(s)G(s)} \Rightarrow \frac{10^3}{1+10^3B} = 10 \\ 1+10^3 \times B \quad B = 9 \times 10^{-3} \quad (b)$$

B  $\rightarrow$  freq dependent & diff for diff freq ranges.

④

$$G(s) = \frac{k}{s(s+a)}$$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{1}{s+a} = \frac{a}{k}$$
$$\frac{1}{1+\frac{a}{k}} = \frac{k}{s(s+a)}$$

$$I \quad S_k^{e_{ss}} = \frac{k}{e_{ss}} \frac{\partial e_{ss}}{\partial k}$$

$$e_{ss} = \frac{a}{k}$$

$$e_{ss} = \frac{a}{k^2}$$

$$\frac{\partial \text{ess}}{\partial K} = \frac{\partial}{\partial K} \left[ \frac{a}{K^2} \right] = -\frac{a}{K^2}$$

(173)

$$S_K^{\text{ess}} = \frac{K^2}{a} \times \frac{-a}{K^2} = -1$$

$$\text{II } S_a^{\text{ess}} = \frac{\partial a}{\text{ess}} \cdot \frac{\partial \text{ess}}{\partial a}$$

$$\text{ess} = a \\ K.$$

$$a = K.$$

ess

$$\frac{\partial \text{ess}}{\partial a} = \frac{\partial}{\partial a} \left[ \frac{a}{K} \right] = \frac{1}{K}$$

$$S_a^{\text{ess}} = K \times \frac{1}{K} = 1$$

$$\text{Ans} = -1, 1 \quad (\text{b})$$

\* Sensitivity when it comes to comparison is a mock value.

In this ques  
sensitivity w.r.t both  $K$  &  $a$  is same  
for the given system.

6

$$G(s) = \frac{K}{s(s+2)}$$

$$R(s) = \frac{1}{s}$$

(74)

$$1 + \frac{s_2}{s(s+2)} = 0 \quad \text{for } k=32,$$

$$s^2 + 2s + 32 = 0$$

$$\omega_n = \sqrt{32} = 5.65 \text{ rad/s.}$$

$$sg \times 5.65 = 2.$$

$$g = 0.176$$

$$\omega_d = \omega_{d_1} = \omega_n \sqrt{1-g^2} \\ = 5.65 \times \sqrt{1-(0.176)^2} \\ = 5.56 \text{ rad/s.}$$

$$1 + \frac{16}{s(s+2)} = 0 \quad \text{for } k=16$$

$$s^2 + 2s + 16 = 0$$

$$\omega_n = \sqrt{16} = 4 \text{ rad/sec}$$

$$sg \times 4 = 2$$

$$g = 0.25$$

$$\omega_d = \omega_{d_2} = 4 \sqrt{1-(0.25)^2} \\ = 3.86 \text{ rad/s.}$$

$$\omega_{d_1} = \frac{5.56}{3.86} = 1.44 \quad \underline{\text{Ans}}$$

$$\omega_{d_2} = 3.86$$

chapter 3.

(125)

(5)  $L\{C(t)\} = TF = \frac{12.5 \times 8}{(s+5)^2 + 8^2} = \frac{100}{s^2 + 10s + 89}$

$$s^2 + 10s + 89 = s^2 + 2Gw_n s + w_n^2$$

$$w_n = \sqrt{89} = 9.5 \text{ rad/s}$$

$$2g \times 9.5 = 10 \Rightarrow g = 0.52 \quad (a)$$

(6)  $G(s) = -k$

$s(s+4)$

$$s^2 + 4s + 1 + \frac{k}{s(s+4)} = 0$$

$$s^2 + 4s + k = 0$$

$$w_n = \sqrt{k} \text{ rad/s}$$

$$2g\sqrt{k} = 4$$

given  $g = 0.5$

$$0.5 \times 0.5 \sqrt{k} = 4$$

$$\boxed{k = 16} \quad (b)$$

(7)  $H(s) = Y(s) = \frac{s+1}{s^2 + 2s + 1}$

$$X(t) = e^{-t} u(t) \quad X(s) = \frac{1}{s+1}$$

$$Y(s) = \frac{s+1}{(s+1)(s^2 + 2s + 2)}$$

$$Y(s) = \frac{1}{s^2 + 2s + 2}$$

$$Y(s) = \frac{1}{(s+1)^2 + 1^2}$$

$$Y(t) = [e^{-t} \sin t] u(t)$$

10

$$H(s+c)$$

$$(s+a)(s+b)$$

76

$$1) u(t) \rightarrow a + De^{-t} + Ee^{-3t}$$

$$2) e^{-2t} u(t) \rightarrow Fe^{-t} + Ge^{-3t}$$

$$\begin{aligned} 1) \frac{H(s+c)}{(s+a)(s+b)} &= \frac{k_1}{s} + \frac{k_2}{s+a} + \frac{k_3}{s+b} \\ &= 2 + De^{-t} + Ee^{-3t} \\ a = 1 & \quad b = 3 \end{aligned}$$

$$\lim_{s \rightarrow 0} \frac{H(s+c)}{s(s+a)(s+b)} = \lim_{t \rightarrow \infty} [2 + De^{-t} + Ee^{-3t}]$$

$$\frac{HC}{ab} = 2 \Rightarrow \frac{HC}{1 \times 3} = 2 \Rightarrow HC = 6$$

$$a) H(s+c)$$

$$\begin{array}{|c|c|} \hline (s+2)(s+a)(s+b) & \\ \hline C=2 & (a) \\ H=3 & \\ \hline \end{array}$$

3 terms are given here.  
only 2 constants are present in  $u(t)$   
so one has to cancel.  
obv.  $C = 2$ .

11

$$Y(s) = 1 - e^{-5t} - 5te^{-5t}$$

$\frac{d}{dt}$  (step response) = impulse response.

$$\begin{aligned} \text{Impulse response} &= 0 + 5e^{-5t} - 5[-te^{-5t} + e^{-5t}] \\ &= 5e^{-5t} + 25te^{-5t} - 5e^{-5t} \\ &= -15 + te^{-5t} \end{aligned}$$

$$TF = \frac{ds}{dt} = \frac{ds}{dt}$$

$$(16) \quad H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

$$X(t) = 10u(t) \quad Y(s) = \frac{10}{s}$$

$$Y(s) = \frac{10}{s(s+2)} = \frac{5}{s} - \frac{5}{s+2}$$

$$y(t) = 5[1 - e^{-2t}]$$

$$\frac{99}{100} \times 5 = 4.95$$

$$4.95 = 5[1 - e^{-2t}]$$

$$\ln e^{-2t} = \ln(0.01) \Rightarrow -2t = -4.6 \\ t = 2.3 \text{ sec.}$$

$$(17) \quad \frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = x(t)$$

$$(s^2 + 3s + 2)y(s) = X(s)$$

$$X(t) = 2u(t) \quad X(s) = \frac{2}{s}$$

$$Y(s) = \frac{2/s}{s^2 + 3s + 2} = \frac{2}{s(s+2)(s+1)} = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}$$

$$y(t) = (1 - 2e^{-t} + e^{-2t})u(t) \quad (\text{a})$$

$$(20) \quad \frac{4d^2c(t)}{dt^2} + \frac{8dc(t)}{dt} + 16c(t) = 16u(t)$$

$$(4s^2 + 8s + 16)c(s) = 16u(s)$$

$$\frac{c(s)}{u(s)} = \frac{16}{4s^2 + 8s + 16} = \frac{4}{s^2 + 2s + 4}$$

$$\boxed{\omega_n = 2 \text{ rad/s}} \quad \boxed{\alpha_d = 2} \\ \boxed{G = 0.5} \quad (\text{b})$$

(19)

$$G(s) = \frac{100 (K_p + K_D s)}{s(s+10)}$$

$$K_V = 1000$$

$$f = 0.5$$

(78)

$$1000 = \lim_{s \rightarrow 0} s \cdot 100 (K_p + K_D s)$$

$$K_p = 100$$

$$1 + G(s) = 0$$

$$1 + \frac{100 (K_p + K_D s)}{s(s+10)} = 0$$

$$s^2 + 10s + 100 K_p + 100 K_D s = 0$$

$$s^2 + s(10 + 100 K_D) + 100 K_p = 0$$

$$s^2 + s(10 + 100 K_D) + 100 \times 100 = 0$$

$$\omega_n = 100 \text{ rad/s}$$

$$2\zeta K_p = 10 + 100 K_D$$

$$\zeta = 0.5 \quad (\text{given})$$

$$2 \times 0.5 \times 100 = 10 + 100 K_D$$

$$K_D = 0.9 \quad (\text{b})$$

$$y(t) = \frac{1}{6} e^{-0.8t} \sin(0.6t) = \text{Impulse response}$$

$$\text{Impulse response} = \frac{1}{6} [ -0.8 e^{-0.8t} \sin 0.6t + e^{-0.8t} \cos 0.6t \cdot 0.6 ]$$

$$Y(s) = \frac{1}{6} \frac{(0.6)^s}{(s + 0.8)^2 + (0.6)^2} = 0$$

$$= \frac{0.1}{s^2 + 1.6s + 1}$$

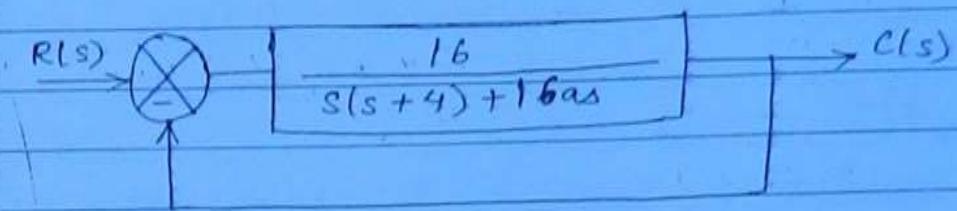
$$\omega_n = 1$$

$$2\zeta K_p = 1.6$$

$$K_p = 0.8 \quad (\text{b})$$

$$\frac{16}{s(s+4)} + \frac{16}{s(s+4) + 16as}$$

(79)



a)  $1 + \frac{16}{s(s+4) + 16as} = 0$

$$s^2 + s(4 + 16a) + 16 = 0$$

$$\omega_n = 4 \text{ rad/s}$$

$$dG \times 4 = 4 + 16a$$

Given % Mp = 1.5%

$$Mp = 0.015$$

$$e^{-g \wedge / \sqrt{1-g^2}} = 0.015$$

$$g = 0.8$$

$$g \times 0.8 \times 4 = 4 + 16a$$

$$a = 0.15$$

b)  $e_{ss}$  for unit ramp input without a [a=0]

$$G(s) = \frac{16}{s(s+4)}$$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{1}{s^2} = \frac{4}{16} = 0.25 \text{ mils}$$

$$1 + \frac{16}{s(s+4)}$$

With  $a = 0.15$

$$G(s) = \frac{16}{s(s+4) + 16 \times 0.155} = \frac{16}{s(s+6.4)} \quad (80)$$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{1}{s^2 + \frac{16}{s}} = \frac{6.4}{16} = 0.4 \text{ units}$$

(c)  $G(s) = K$

$$s(s+4) + K \times 0.155$$

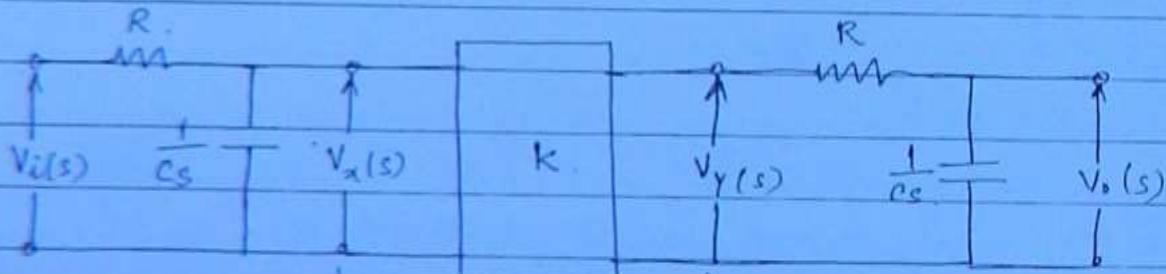
$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{1}{s^2} = \frac{1}{K}$$

$$s(s+4) + 0.155 \times K$$

$$0.95 = 4 + 0.15K$$

$$\boxed{K = 40}$$

(d)



$$\frac{V_x(s)}{V_i(s)} = \frac{1}{RCs + 1} \quad | \quad K = \frac{V_y(s)}{V_x(s)} \quad | \quad \frac{V_o(s)}{V_y(s)} = \frac{1}{RCs + 1}$$

$$\frac{V_i(s)}{\boxed{\frac{1}{RCs + 1}}} \rightarrow \boxed{V_x(s)} \xrightarrow{K} \boxed{V_y(s)} \xrightarrow{\frac{1}{RCs + 1}} \boxed{V_o(s)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{RCs + 1} \rightarrow \text{critically damped (d)}$$

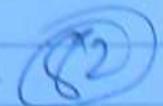
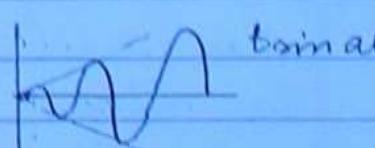
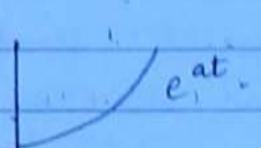
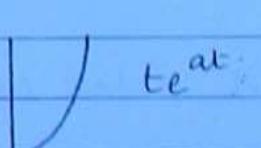
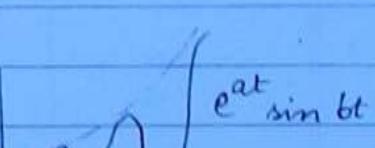
# STABILITY IN TIME DOMAIN

(81)

The stability of LTI system may be defined as when the system is subjected to bounded input the output should be bounded.

BIBO  $\rightarrow$  implies the impulse response of the system should tend to 0 as time approaches infinity. The stability of the system depends on roots of the characteristic equation  $1 + H(s)G(s) = 0$  ie closed loop poles.

C.L. Pole locations	Stability criteria	Impulse Response
1. $-x$ $-a$	$F(s) = \frac{1}{s+a}$ Absolutely stable	$e^{-at}$
2. $x$ $-a$	$F(s) = \frac{1}{(s+a)^2}$ "	$te^{-at}$
3. $*$ $-a$ $-jb$ $-jb$	$F(s) = \frac{1}{(s+a)^2+b^2}$ "	$e^{-at} \sin bt$
4. $*$	$F(s) = \frac{1}{s}$ Marginally (or) critically stable	$1$
5. $*ja$ $-ja$	$F(s) = \frac{1}{s^2+a^2}$ "	$\sin at$

6.	$F(s) = \frac{1}{s^2}$	unstable		
7.	$\begin{array}{c} ja \\ -ja \end{array}$	$F(s) = \frac{1}{(s^2 + a^2)^2}$	"	
8.	$\begin{array}{c} a \\ -a \end{array}$	$F(s) = \frac{1}{s-a}$	"	
9.	$\begin{array}{c} a \\ -a \end{array}$	$F(s) = \frac{1}{(s-a)^2}$	"	
10.	$\begin{array}{c} jb \\ -jb \end{array}$	$F(s) = \frac{1}{(s-a)^2 + b^2}$	"	

### Q8-1-13 ROUTH HURWITZ METHOD -

g)  $P(s) = s^4 + 8s^3 + 16s^2 + 16s + 5 = 0$

ROUTH ARRAY	$s^4$	1		
	$s^3$	1	18	5
	$s^2$	8	<del>16</del>	0
	$s^1$	$b_1 = 16$	$b_2 = 5$	0
	$s^0$	$c_1 = 13.5$	$c_2 = 0$	0
		$d_1 = 5$	0	0

$$b_1 = 8 \times 18 - 1 \times 16 = 16$$

(83)

$$b_2 = 8 \times 5 - 1 \times 0 = 5$$

8 → since denominator always has coefficients from the 1st column,

$$c_1 = 16 \times 16 - 8 \times 5 = 13.5$$

16 : . . .

the sign changes of only the first column are considered

$$c_2 = 13.5 \times 5 - 16 \times 0 = 5$$

13.5 : . . .

CWB chapter 4

(ii)  
\*

	$s^5$	1	2	3
	$s^4$	1	2	15
	$s^3$	<del><math>2E</math></del> $\rightarrow -12$	0	
	$s^2$	<del><math>2E+12</math></del> $\rightarrow$	-15	0
	$s^1$	<del><math>-24E-144-15E</math></del>	0	0
	$s^0$	15	0	0

$E \rightarrow$  small +ve no.

To check for sign changes

$$(i) \text{ If } \frac{2E+12}{E} = \frac{2(0)+12}{0} = +\infty$$

$$(ii) \text{ If } \frac{-24E-144-15E^2}{2E+12} = \frac{-144}{12} = -12$$

Two sign changes

$$+\infty \rightarrow -12$$

$$-12 \rightarrow +15$$

⇒ Two C.L poles in RHS of  $s$ -plane (unstable) (b)

(84)

Difficulty - 1

When the first element of any row is zero while the rest of the row has atleast one non-zero term, in such cases substitute a small tiny number ' $\epsilon$ ' in place of zero & evaluate the rest of the Routh Array in terms of  $\epsilon$ . Check for sign changes in the first column by taking limit  $\epsilon \rightarrow 0$  to comment on stability.

• (2)

$$\begin{array}{ccccc} s^4 & 1 & 8 & 20 & 16 \\ & | & & & \\ & 1 & & & \end{array}$$

$$\begin{array}{ccccc} s^5 & 2 & 12 & 16 & 0 \\ & | & & & \\ & 1 & & & \end{array}$$

$$\begin{array}{ccccc} s^6 & 2 & 12 & 16 & 0 \\ & | & & & \\ & 1 & & & \end{array}$$

$$\begin{array}{ccccc} s^3 & 8 & 24 & 0 & 0 \\ & | & & & \\ & 1 & & & \end{array}$$

→ Abrupt end  
(System now can be  
either marginally  
stable or unstable)

$$\begin{array}{ccccc} s^2 & 6 & 16 & 0 & 0 \\ & | & & & \\ & 1 & & & \end{array}$$

$$\begin{array}{ccccc} s^1 & 4.6 & 1 & 0 & 0 \\ & | & & & \\ & 1 & & & \end{array}$$

$$\begin{array}{ccccc} s^0 & 16 & 1 & 0 & 0 \\ & | & & & \\ & 1 & & & \end{array}$$

$$\text{Imp. eq}^{\circ} \rightarrow \underline{A(s)} = 2s^4 + 12s^2 + 16\epsilon$$

$$\frac{d}{ds} A(s) = 8s^3 + 24s +$$

The roots of  $A(s) = 0$  poles lying in left half plane

$$\text{Roots of } A(s) = -\frac{1}{2} \pm \sqrt{144 - 8 \times 16}{4}$$

$$= -2, -4$$

$$(s^2 + 2)(s^2 + 4) = 0$$

$$s = \pm j\sqrt{4}, \pm j^2$$

(85)

$\therefore$  marginally stable (as no roots are repeating)

(10)	$s^5$	$\ell$	4	$\omega$
*	$s^4$	1	$\ell$	1
	$s^3$	$\cancel{0}4$	$\cancel{0}9$	0
	$s^2$	1	1	0
	$s^1$	$\cancel{0}2$	0	0
	$s^0$	1	0	0

$$\rightarrow A_1(s) = s^4 + \omega s^2 + 1$$

$$\frac{d}{ds} A_1(s) = 4s^3 + 4s$$

$$\rightarrow A_2(s) = s^2 + 1$$

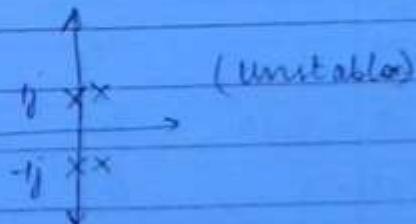
$$\frac{d}{ds} A_2(s) = 2s$$

for stability check only  
from notes of  
 $A_1(s)$

$$\text{Roots of } A_1(s) \Rightarrow \frac{-\pm\sqrt{4-4}}{2} = -1, -1$$

$$(s^2 + 1)(s^2 + 1) = 0$$

$$s = \pm j, \pm j$$

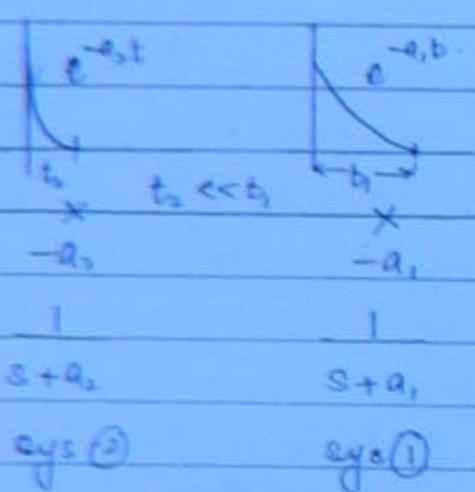


Difficulty - 2

(86)

When Routh - Array ends abruptly, construct an auxiliary equation  $A(s)$ , differentiate it to get new co-efficients and evaluate the root of the Routh Array. Check for multiplicity of auxiliary equation roots on jw axis to comment on stability.

### RELATIVE STABILITY ANALYSIS USING ROUTH ARRAY -



→ Both sys ① & ② are absolutely stable systems.  
→ Sys ② is said to be relatively more stable than sys ① because  $t_2 \ll t_1$ .

$$P(s) = s^3 + 7s^2 + 25s + 39 = 0$$

To check whether the roots are lying more -vely w.r.t -1?

1

$$s+1 = z$$

1

$$s = z - 1$$

$$P(z) = (z-1)^3 + 7(z-1)^2 + 25(z-1) + 39 = 0$$

$$P(z) = z^3 + 4z^2 + 14z + 20 = 0$$



-1

$z^3$	1	14
$z^2$	4	20
$z^1$	9	0
$z^0$	20	0

## Stereotypes for control problems

(89)

Difficulty - 3 -

Relative stability analysis using Routh array is not feasible for higher order polynomials because it involves shifting of origin of  $s$ -plane more negatively.

Shortcut

Substitute the given pt. (eg  $s+1=0$  means  $s=-1$ )

in the polynomial assuming the pt makes it stable

→ If LHS  $> 0$  all roots lie on the LS of  $s$ -plane

→ If LHS = 0 one root lies on the shifted axis line & rest of the roots to the left.

e.g. in prev ques  $s=-1$  in  $P(s)$

gives 00

∴ All 3 roots lie on LS of  $s$ -plane

Conditionally Stable Systems -

(6)

$s^4$	1	3	$K$	1) $4-2K > 0$
$s^3$	$\alpha$	$\alpha$	0	$\alpha^2$
$s^2$	$\alpha$	$K$	0	$K < 2$
$s^1$	$\frac{4-2K}{\alpha}$	0	0	2) $K > 0$
$s^0$	$K$	0	0	$0 < K < 2$

$$\text{At } K = K_{\max} = 2$$

$$\therefore s^1 \text{ now } = 0$$

$$A(s) = \alpha s^2 + K$$

$$-\alpha s^2 + \alpha = 0$$

$$\alpha = \frac{1}{s^2}$$

(8)

$$G(s) = \frac{K}{(s^2 + 2s + 2)(s + 2)}$$

(88)

$$1 + G(s) = 0$$

$$1 + \frac{K}{(s^2 + 2s + 2)(s + 2)} = 0$$

$$(s^2 + 2s + 2)(s + 2) + K = 0$$

$$s^3 + 4s^2 + 6s + (4 + K) = 0$$

$$\begin{array}{ccc} s^3 & 1 & 6 \\ s^2 & 4 & 4+K \\ s^1 & -4 & 0 \\ s^0 & 4+K & 0 \end{array}$$

$$1) \frac{-4 - (4+K)}{4} > 0$$

$$K < 20$$

$$2) 4 + K > 0 \Rightarrow K > -4$$

$$\boxed{-4 < K < 20}$$

$$k = K_{\max} = 20$$

$$A(s) = 4s^2 + (4+k)$$

$$= 4s^2 + (4+20) = 0$$

$$s^2 = -6$$

$$s = \pm j\sqrt{6}$$

$$\omega = \omega_{\text{max}} = \sqrt{6} \approx 2.45$$

Short cut method - (only for cubic polynomials)

$$s^3 + 4s^2 + 6s + (4+k) = 0 \quad (87)$$

$$1(4+k) < 6(4)$$

$$1(4+k) < 24$$

$$k < 20$$

EXTERNAL < INTERNAL (Stable)  
PRODUCTS PRODUCTS

EXTERNAL = INTERNAL (Marginally Stable)  
PRODUCTS PRODUCTS

EXTERNAL > INTERNAL (Unstable)  
PRODUCTS PRODUCTS

$$K = K_{max} = 20$$

$$A(s) = 4s^2 + (4+k) = 0 \quad [ \text{EVEN PART OF POLYNOMIAL} ]$$

$$4s^2 + (4+20) = 0$$

$$4s^2 + 24 = 0$$

$$s^2 = 1 \downarrow \sqrt{6}$$

$$\omega = \omega_{max} = \sqrt{6} \approx 2.4$$

$$(12) \quad G(s) = \frac{1(s+1)}{s^3 + as^2 + 2s + 1}$$

$$1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$$s^3 + as^2 + 2s + 1$$

$$s^3 + as^2 + 2s + 1 + K(s+1) = 0$$

$$s^3 + as^2 + s(2+k) + (1+k) = 0$$

$$1(1+k) < a(2+k)$$

stable

$$A(s) = a s^2 + (k+1) = 0$$

$$s^2 = -\frac{(k+1)}{a}$$

Q10

$$s^2 = -\frac{(k+1)(k+2)}{(k+1)}$$

$$s = \pm j\sqrt{k+2}$$

$$\omega = \sqrt{k+2}$$

$$\omega = \sqrt{k+2}$$

$$\Rightarrow k = 2$$

$$a = 2+1 = \frac{3}{4}$$

(b)

~~CONV~~ 1  $1 + \frac{10(k_p s + k_I)}{s(s^2 + s + 20)} = 0$

$$s^3 + s^2 + 20s + 10(k_p + k_I) = 0$$

$$s^3 + s^2 + s(20 + 10k_p) + 10k_I = 0$$

$s^3$	1	$20 + 10k_p$
$s^2$	1	$10k_I$
$s^1$	$20 + 10k_p - 10k_I$	0
$s^0$	$10k_I$	0

1)  $10k_I > 0$   
 $k_I > 0$

$k_p$  &  $k_I$  are dependent on each other so we're to

2)  $20 + 10k_p - 10k_I > 0$   
 $k_p > 10k_I - 20$   
10

choose those values of  $k_p$  &  $k_I$  for which there is no sign change in 1<sup>st</sup> column of relation table is satisfied

conv  
on  
of  
variables

$$G(s) = \frac{k(s+2)^2}{s(s^2+1)(s+4)}$$

$$s(s^2+1)(s+4)$$

91

$$1 + \frac{k(s+2)^2}{s(s^2+1)(s+4)} = 0$$

$$s(s^2+1)(s+4) + k(s+2)^2 = 0$$

$$s^4 + 4s^3 + s^2(1+k) + s(4+4k) + 4k = 0$$

$s^4$	1	$(1+k)$	$4k$
$s^3$	4	$(4+4k)$	0
$s^2$	$\cancel{0} \epsilon$	$4k$	0
$s^1$	$(4+4k)\epsilon - 16k$	0	0
$s^0$	$4k$	0	0

$$1) \quad 4k > 0 \Rightarrow \boxed{k > 0}$$

can't put  $\epsilon \rightarrow 0$

$$2) \quad \frac{(4+4k)\epsilon - 16k}{\epsilon} > 0 \Rightarrow \boxed{\epsilon > \frac{16k}{4+4k}}$$

here as there is a condition on  $\epsilon$  for stability.

### LIMITATIONS -

- All the co-efficients of the polynomials must be real.
- The sign changes in 1<sup>st</sup> column of Routh Array determines poles of RHS in s-plane but not their locations.

dy (5)

C-L system

$$G(s) = \frac{1}{s^3 + 1.5s^2 + s - 1}$$

(92)

$$s^3 + 1.5s^2 + s - 1$$

~~$$s^3 + 1.5s^2 + s - 1 = 0$$~~

~~$$s^3 + 1.5s^2 + s - 1, \cancel{s+1} = 0$$~~

$s^3$	1	1
$s^2$	1.5	-1
$s^1$	$\cancel{0.5}/1.5$	0
$s^0$	-1	0
	1	

Unstable

C-L system

$$H(s) = 20(s+1) \Rightarrow$$

$$\text{C-L pole} = 1 + H(s) G(s) = 0$$

$$1 + \frac{20(s+1)}{s^3 + 1.5s^2 + s - 1} = 0$$

$$s^3 + 1.5s^2 + s(1+20) + 19 = 0$$

$$s^3 + 1.5s^2 + 21s + 19 = 0$$

$s^3$	1	21
$s^2$	1.5	19
$s^1$	8.3	
$s^0$	19	

Stable

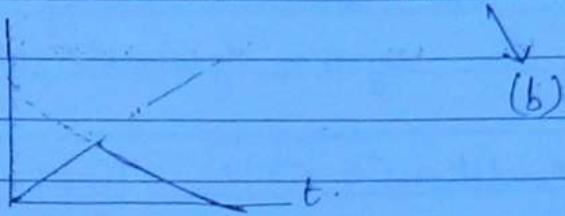
(C)

$$(5) \text{ Impulse response} = \frac{d}{dt} [1 - e^{-t}(1+t)]$$

(93)

$$= 0 + e^{-t} - [-te^{-t} + e^{-t}]$$

$$= te^{-t} \quad \text{stable.}$$



$$(9) 1 + \frac{k(s-2)^2}{(s+2)^2} = 0$$

$$G(s) = k \times \frac{s-2}{(s+2)^2} \quad H(s) = s-2$$

$$(s+2)^2 + k(s-2)^2 = 0$$

$$s^2 + 4 + 4s + k(s^2 + 4 - 4s) = 0$$

$$s^2(1+k) + s(4-4k) + (4+4k) = 0$$

$$\begin{matrix} s^2 & 1+k & 4+4k \\ s^1 & 4-4k & 0 \\ s^0 & 4+4k & 0 \end{matrix}$$

$$1) 1+k > 0 \Rightarrow k > -1$$

$$2) 4-4k > 0 \Rightarrow k < 1$$

$$\boxed{-1 < k < 1} \quad \boxed{0 \leq k < 1} \quad \text{Ans (C)}$$

## ROOT LOCUS

(94)

The root locus is defined as locus of closed loop poles obtained when system gain  $K$  is varied from 0 to  $\infty$ . It determines the relative stability of the system w.r.t. variations in system gain  $K$ .

### Angle & Magnitude Conditions

Angle condition  $\rightarrow$  is used for checking whether any point is lying on root locus or not. If also the validity of root locus shape for closed loop poles.

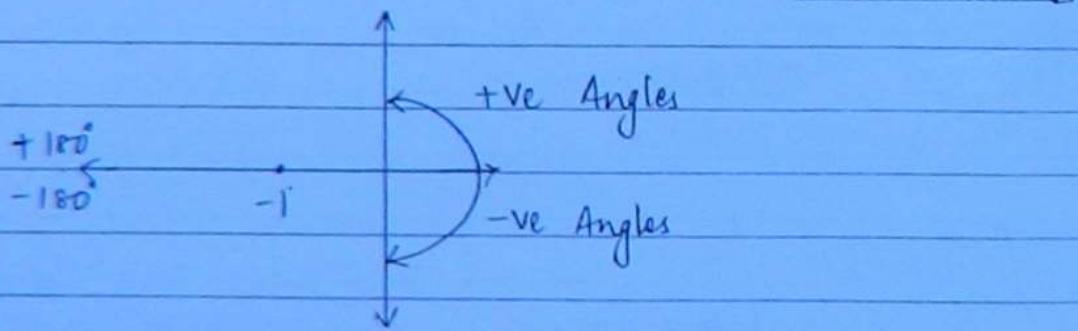
In negative feedback systems

$$1 + G(s) H(s) = 0$$

$$G(s) H(s) = -1$$

$$= -1 + 0j$$

$$= 180^\circ - \tan^{-1} \left[ \frac{0}{1} \right] = 180^\circ \approx \pm 180^\circ \\ \approx [2q+1] 180^\circ$$



Angle condition may be stated as, for a point to lie on root locus, the angle evaluated at that point must be an odd multiple of  $\pm 180^\circ$

Magnitude condition  $\rightarrow$  is used for finding the magnitude of system gain  $K$  at any point on the root locus.

(95)

$$|1 + G(s)H(s)| = D$$

$$G(s)H(s) = -1$$

$$|G(s)H(s)| = \sqrt{1^2 + 0^2} = 1$$

$$\boxed{|G(s)H(s)| = 1}$$

CWB chapter 5.

(b)  $s_1 = -3 + 4j$        $s_2 = -3 - 2j$   
 $G(s)H(s) = \frac{K}{(s+1)^4}$

$$|G(s)H(s)| = \frac{K}{|-3+4j+1|^4} = \frac{K}{[-2+4j]} = \frac{K}{[116^\circ] \times 4} = -464^\circ$$

$$|G(s)H(s)| = \frac{K}{|-3-2j+1|^4} = \frac{K}{[-2-2j]} = \frac{K}{[-135^\circ] \times 4} = +540^\circ = (3 \times 180^\circ)$$

odd multiple of  $180^\circ$

$s_2$  lies on root locus,  $s_1$  does not (b)

(9)

$$G(s) = K$$

$$s(s^2 + 7s + 12)$$

$$s = -1 + j$$

(96)

$$|G(s)| = K$$

$$\begin{aligned} s = -1 + j & \quad (-1+j)[(-1+j)^2 + 7(-1+j)+12] \\ & = 0^\circ \end{aligned}$$

$$= K + 0j$$

$$(-1+j)(s+5j) \\ = -180^\circ$$

$$(135^\circ)(45^\circ)$$

$$\begin{aligned} |G(s)|_{s=-1+j} &= 1 \Rightarrow \sqrt{K^2 + 0^2} = 1 \Rightarrow K = 1 \\ &\sqrt{(-1)^2 + 1^2} \sqrt{s^2 + 5^2} \\ &\Rightarrow K = \sqrt{100} \Rightarrow K = 10 \quad (d) \end{aligned}$$

Rule of  
RL are  
applicable  
only to  
but final  
RL  
is applicable  
to TF

### CONSTRUCTION RULES OF ROOT LOCUS -

Rule 1 → The root locus is symmetrical about real axis ( $G(s)H(s) = -1$ )

Rule 2 → Let

$$P = \text{no. of open loop poles}$$

$$Z = \text{no. of open loop zeros}$$

$$\text{and } P > Z$$

Then,

$$\text{No. of branches of RL} = P$$

$$\text{No. of branches terminating at zeros} = Z$$

$$\text{No. of branches terminating at infinity} = P - Z$$

we assume there's a zero at infinity.

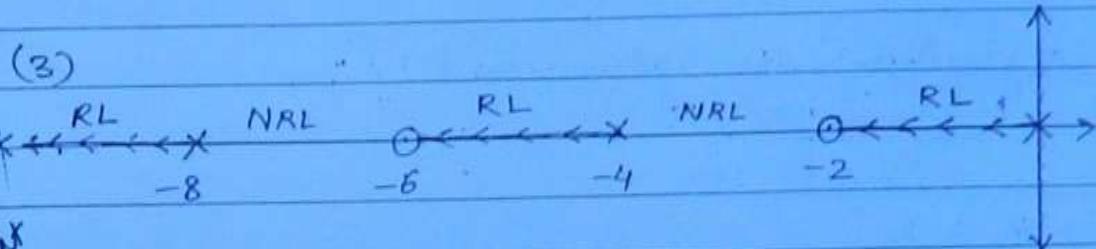
Rule 3 → A point on real axis is said to be

on root locus if to the right side of this point the sum of open loop poles and zeros is odd

eg.  $G(s) = \frac{k(s+2)(s+6)}{s(s+4)(s+8)}$

(Q7)

(2) :  $P = 3$        $Z = 2$        $P - Z = 1$



Closed Loop T.F.

$G(s) = \frac{k(s+2)(s+6)}{s(s+4)(s+8)}$

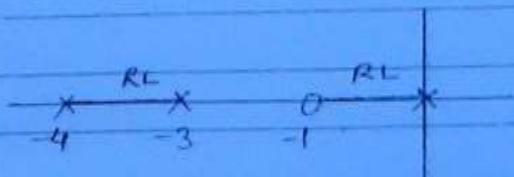
$$\frac{1 + G(s)}{1 + \frac{k(s+2)(s+6)}{s(s+4)(s+8)}} = \frac{s(s+4)(s+8) + k(s+2)(s+6)}{s(s+4)(s+8) + k(s+2)(s+6)}$$

$$= \frac{k(s+2)(s+6)}{s(s+4)(s+8) + k(s+2)(s+6)}$$

C.L. poles =  $s(s+4)(s+8) + k(s+2)(s+6) = 0$   
when  $k = 0$

C.L. poles = 0, -4, -8

(5)



(c)

Rule 4 → Angle of Asymptotes

(98)

The P-Z branches will terminate at infinity along certain straight lines known as asymptotes of root locus. Therefore, number of asymptotes = P-Z

Angle of asymptotes is given by :

$$\theta = \frac{[2q+1]180^\circ}{P-Z}$$

$$q = 0, 1, 2, 3, \dots$$

e.g.

$$P-Z = 2$$

$$\theta_1 = \frac{[2(0)+1]180^\circ}{2} = 90^\circ$$

$$\theta_2 = \frac{[2(1)+1]180^\circ}{2} = 270^\circ$$

(12)

$$\underbrace{s(s+4)(s^2+2s+1)}_{1+K(s+1)} + K(s+1) = 0$$

$$1+K(s+1) = 0$$

$$s(s+4)(s^2+2s+1)$$

$$1+G(s)H(s) = 0$$

$$G(s)H(s) = \frac{K(s+1)}{s(s+4)(s^2+2s+1)}$$

$$P=4 \quad Z=1 \quad P-Z=3$$

$$\theta_1 = \frac{[2(0)+1]180^\circ}{3} = 60^\circ \quad \leftarrow \quad \rightarrow \quad \frac{2\pi}{P-Z} = \frac{2\pi}{3} = \underline{120^\circ}$$

$$\theta_2 = \frac{[2(1)+1]180^\circ}{3} = 180^\circ \quad \leftarrow \quad \rightarrow$$

$$\theta_3 = \frac{[2(2)+1]180^\circ}{3} = 300^\circ \quad \leftarrow$$

Rule 5  $\rightarrow$  Centroid

(99)

It is the intersection point of asymptotes on the real axis. It may or may not be a part of RL.

$$\text{Centroid} = \sum \text{Real part of open loop poles} - \sum \text{Real part of open loop zeros}$$

$P - Z$

(10)

$$s^3 + 5s^2 + (k+6)s + k = 0$$

$$s^3 + 5s^2 + sk + 6c + k = 0$$

$$\underbrace{s^3 + 5s^2 + 6s}_{(s+3)(s^2+6s)} + k(s+1) = 0$$

$$1 + \frac{k(s+1)}{s^3 + 5s^2 + 6s} = 0$$

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = \frac{k(s+1)}{s^3 + 5s^2 + 6s} = \frac{k(s+1)}{s(s^2 + 5s + 6)} = \frac{k(s+1)}{s(s+3)(s+2)}$$

$$\text{Poles} = 0 + j0$$

$$-1, -2 + j0$$

$$-3 + j0$$

$$-5$$

$$\text{Zeros} = \frac{-1 + j0}{-1}$$

$$P = 3 \quad Z = 1$$

$$P - Z = 2$$

$$\text{Centroid} = \frac{-5 - (-1)}{2} = -2 = [-2, 0] \text{ (C)}$$

## Rule 6 → Break Away Points

(100)

They are those points where multiple roots of the characteristic equation occur.

Procedure -

1. Construct  $1 + G(s)H(s) = 0$
2. Write K in terms of s
3. Find  $\frac{dk}{ds} = 0$
4. The roots of  $\frac{dk}{ds} = 0$  will give B.A. points
5. To test valid B.A. points substitute in step (2)  
If  $k = +ve \rightarrow$  Valid B.A. point.

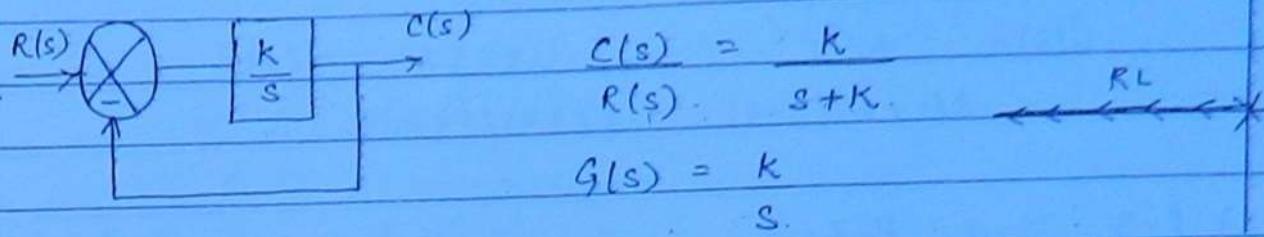
General Predictions about B.A. points -

1. The branches of RL either approach or leave the B.A. points at an angle of  $\pm 180^\circ$  where n = no. of branches approaching or leaving B.A. point.
2. The complex conjugate path for the branches of RL approaching or leaving the B.A. point is a circle
3. Whenever there are 2 adjacently placed poles on the real axis with the section of real axis between them as a part of RL, then there exists a breakaway point between the adjacently placed poles

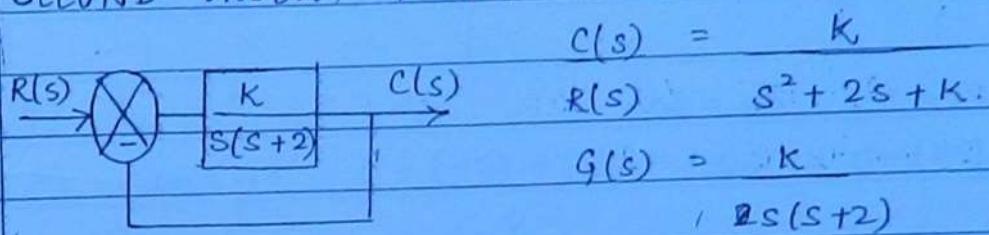
CONV

## 4 FIRST ORDER SYSTEM

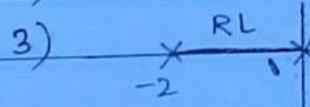
(10)



## SECOND ORDER SYSTEM



$$2) P = 2 \quad Z = 0 \quad P - Z = 2$$



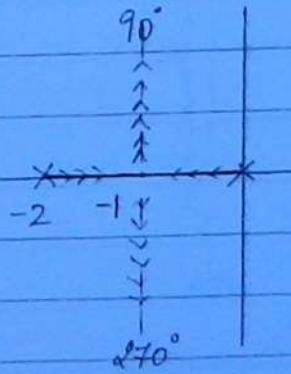
$$4) \theta_1 = 90^\circ \quad \theta_2 = 270^\circ$$

$$5) \text{Centroid} = \frac{0 + (-2) - 0}{2} = -1$$

$$6) \text{B.A point} \rightarrow s^2 + 2s + K = 0$$

$$K = -s^2 - 2s$$

$$\frac{dK}{ds} = -2s - 2 = 0 \quad | s = -1$$



*RL gets shifted to right*

Effect of adding poles to a T.F -

(102)

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

2)  $P=3 \quad Z=0 \quad P-Z=3$



4)  $\theta_1 = 60^\circ \quad \theta_2 = 120^\circ \quad \theta_3 = 300^\circ$

5)  $0 + (-2) + (-4) - 0 = -6$

3

6) B.A point

$$s^3 + 6s^2 + 8s + K = 0$$

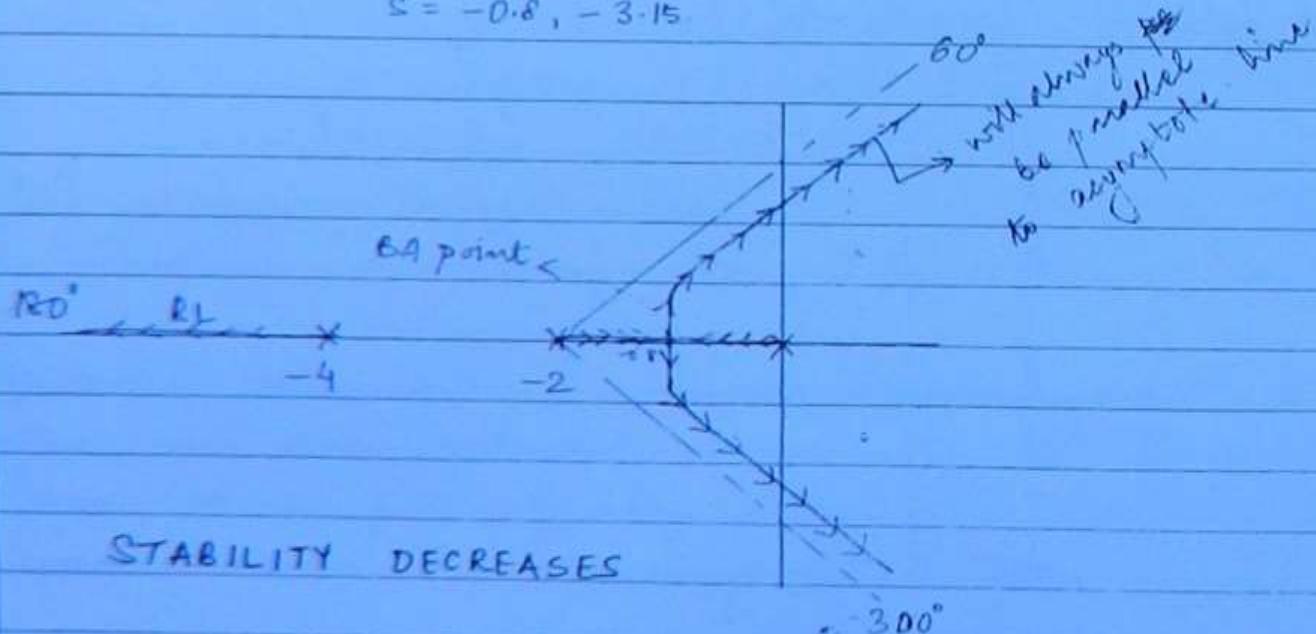
$$K = -s^3 - 6s^2 - 8s =$$

$$\frac{dK}{ds} = -3s^2 - 12s - 8 = 0$$

 $ds$ 

$$\Rightarrow 3s^2 + 12s + 8 = 0$$

$$s = -0.8, -3.15$$



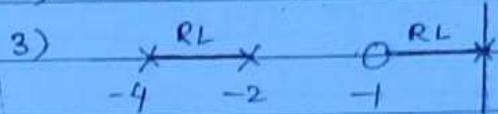
*RL shifted to left*

## Effect of adding zeros to a T.F.

103

$$G(s) = \frac{K(s+1)}{s(s+2)(s+4)}$$

2)  $P=3$   $Z=1$ ,  $P-Z=2$



4)  $\theta_1 = 90^\circ$   $\theta_2 = 270^\circ$

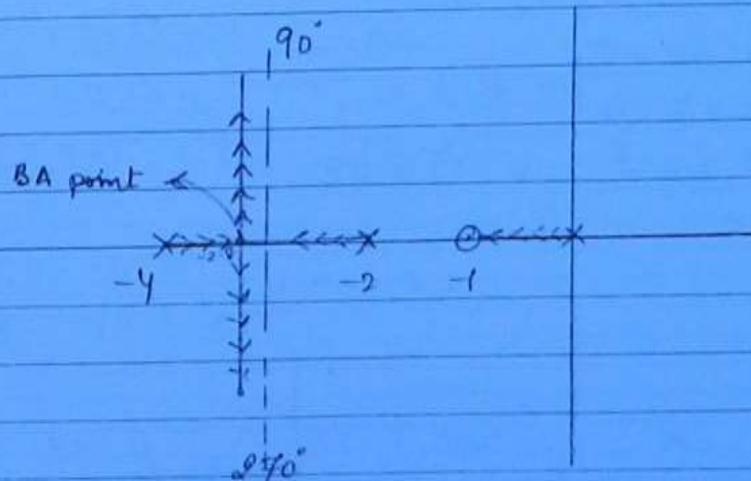
5) Centroid =  $\frac{0 + (-2) + (-4) - (-1)}{2} = -2.5$

6) B.A point  $\Rightarrow$  ~~occurred~~

$$s^3 + 6s^2 + 8s + Ks + K = 0$$

$$K = -s^3 - 6s^2 - 8s \Rightarrow \frac{-s(s^2 + 6s + 8)}{s+1}$$

$$\frac{dK}{ds} = 0 \Rightarrow s = -2.8$$



STABILITY INCREASES.

$\tau > P$  never possible  
practically

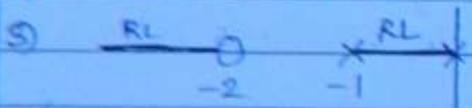
(104)

### General Predictions about B.A point (contd...)

- 4) Whenever there are 2 adjacently placed zeros on s-axis with the section between them as a part of RL, then there exists a breakaway point between the adjacently placed zeros.

$$G(s) = \frac{K(s+2)}{s(s+1)}$$

$$2) \quad P=2 \quad Z=1 \quad P-Z=1$$



#### B.A points

$$s(s+1) + K(s+2) = 0$$

$$K = -s^2 - s$$

$$s+2$$

$$\frac{dk}{ds} = 0 \Rightarrow \frac{(s+2)(-2s-1) - (-s^2-s)(1)}{(s+2)^2} = 0$$

$$-2s^3 - s - 4s - 2 + s^2 + s = 0$$

$$-s^3 - 4s - 2 = 0$$

$$s^3 + 4s + 2 = 0$$

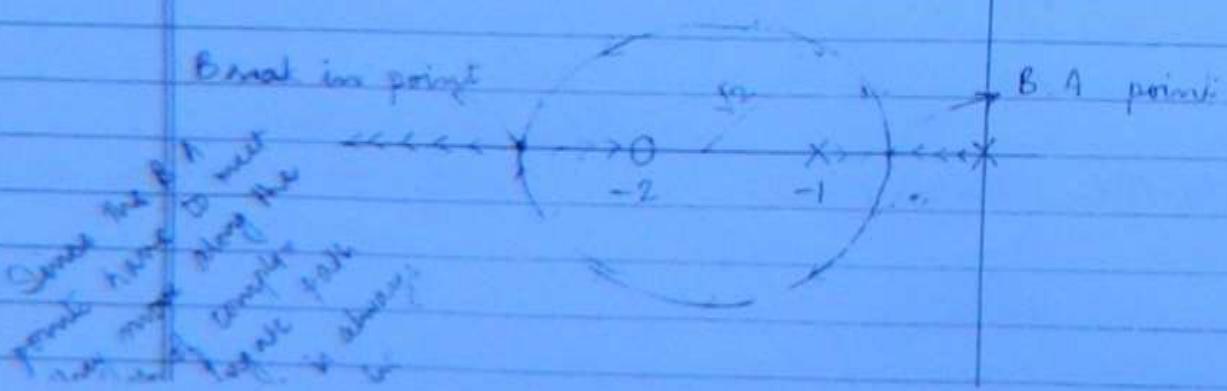
$$s = \frac{-4 \pm \sqrt{16-8}}{2} = -2 \pm \sqrt{2}$$

$$(real, (radical))$$

$$= -0.6, -3.4$$

Breakaway point

B.A point



# Proof of path being a circle 105

$$k(s+be)$$

$$s(s+a)$$

$$\text{Let } s = x + jy$$

$$\begin{aligned} k[x + jy + b] &= k[(x+b) + jy] \\ [x + jy][x + jy + a] &= x^2 + jxy + ax + jxy - y^2 + jaY \\ &= k[(x+b) + jy] \\ &= [x^2 + ax - y^2] + j[2xy + ay] \end{aligned}$$

$$\tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left[\frac{2xy + ay}{x^2 + ax - y^2}\right] = 180^\circ$$

Taking Tan on both sides

$$\frac{y}{x+b} - \left[ \frac{2xy + ay}{x^2 + ax - y^2} \right] = 0$$

$$x^2 + ax - y^2 - [(2x+a)(x+b)] = 0$$

$$x^2 + ax - y^2 - [2x^2 + 2xb + ax + ab] = 0$$

$$-x^2 - y^2 - 2xb - ab = 0$$

$$x^2 + 2xb + y^2 = -ab$$

$$x^2 + 2xb + b^2 + y^2 = -ab + b^2$$

$$(x+b)^2 + y^2 = b(b-a)$$

$$\text{centre} = -b, 0$$

$$\text{Radius} = \sqrt{b(b-a)}$$

$$B.A \text{ point} = \text{Centre} \pm \text{Radius}$$

## Rule - 7 Intersection of RL with Imaginary Axis

Intersection of

Roots of auxiliary equation  $A(s)$  at  $K = K_{\text{max}}$  from Routh Array gives the intersection of Root Locus with Imaginary axis.

(106)

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

7)  $s^3 + 6s^2 + 8s + K = 0$

$s^3$	1	8
$s^2$	6	$K$
$s^1$	$\frac{18-K}{6}$	0
$s^0$	$K$	0

$$\rightarrow \frac{48-K}{6} > 0 \Rightarrow K < 48$$

$$\rightarrow K > 0$$

$$0 < K < 48$$

At  $K = K_{\text{max}} = 48$

$$A(s) = 6s^2 + K = 0$$

$$6s^2 + K = 0$$

$$s = I j \sqrt{8} = I j 2\sqrt{2}$$

Shortcut Method -

$$G(s) = \frac{K}{s(s+a)(s+b)}$$

Intersection of RL =  $I j \sqrt{ab}$   
with  $jw$  axis

# Intersection of Asymptotes with jw Axis

(TQ)

$$\tan \theta = \frac{y}{x}$$

$$Y = \tan 60^\circ \times 2$$

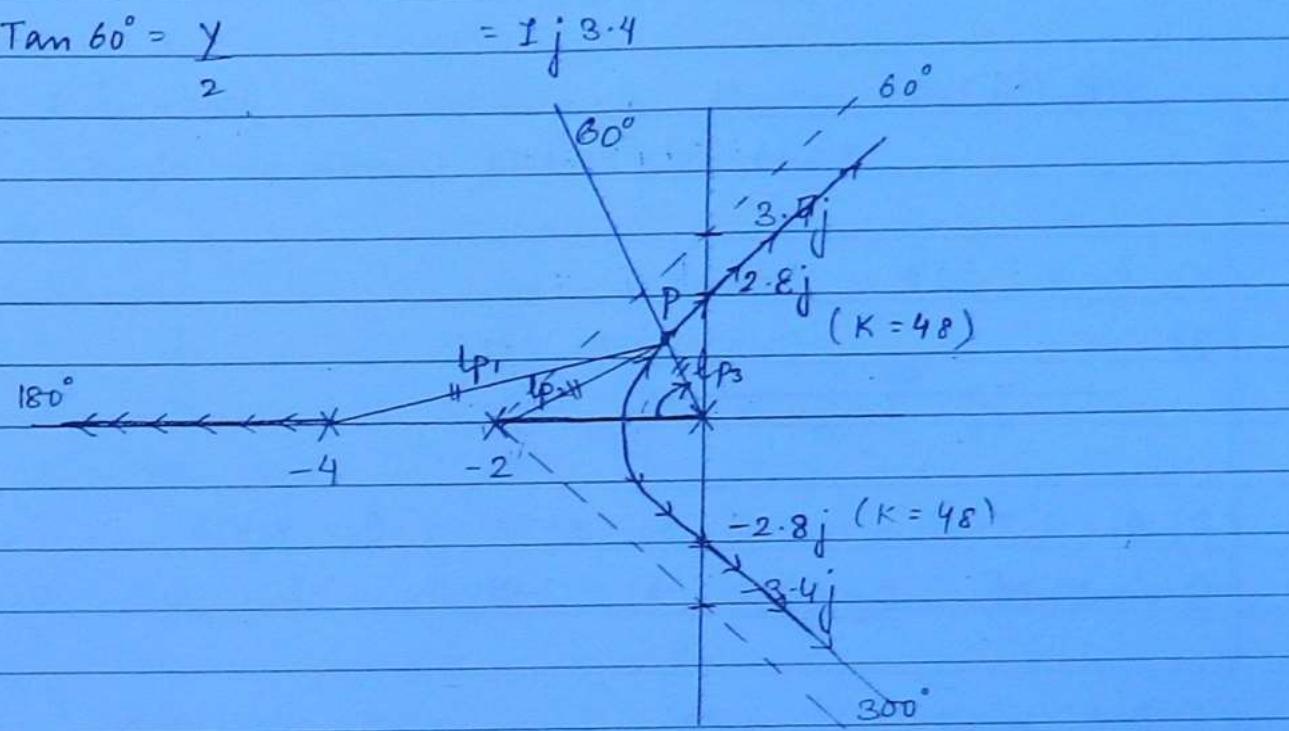
x

$$Y = \sqrt{3} \times 2 = 3\sqrt{3}$$

$$\tan 60^\circ = \frac{y}{x}$$

$$= 1j 3\sqrt{3}$$

$\frac{y}{x}$



Q Find K for  $\theta = 0.5$  from RL?

$$\theta = \cos^{-1} \theta$$

$$\theta = \cos^{-1} 0.5 = 60^\circ (+60^\circ \text{ from } X\text{-axis in } \cancel{\text{clockwise dir}})$$

Product of vector

$$K = \frac{\text{lengths of poles}}{\text{lengths of zeros}} = l_{p_1} \times l_{p_2} \times l_{p_3}$$

Product of vector

lengths of zeros

In exam draw to scale these lengths & find out the value of K.

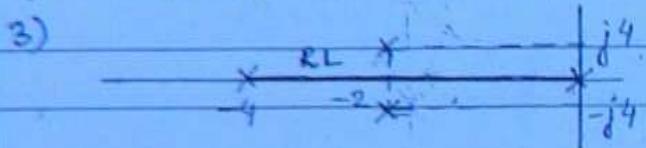
Rule 8 - Angle of Departure of Amplitude - (108)

Angle of departure is obtained when complex poles terminate at infinity.

Angle of Arrival is obtained at complex zeros.

(11)  $G(s) H(s) = \frac{K}{s(s+4)(s^2 + 4s + 20)}$

2)  $P = 4 \quad Z = 0 \quad P - Z = 4$



4)  $\theta_1 = 45^\circ \quad \theta_2 = 135^\circ \quad \theta_3 = 225^\circ \quad \theta_4 = 315^\circ$

5) Centroid  $\frac{0 + (-2) + (-2) + (-4) - 0}{4} = -2$

6) B.A points

Short cut method

Average value of  $\frac{\text{real poles}}{\text{real poles}} = \frac{0 + (-4)}{2} = -2$   $\Rightarrow$  equal to real part of complex poles  
So there are 3 B.A pts over the region

avg value of  $\frac{\text{real poles}}{\text{real poles}} \times z = \frac{\text{constant term of complex root}}{\text{complex root}}$   $\Rightarrow$  5/2 poles 0 & -4  
of other 2 due to complex poles  
So there are 3 B.A pts

$$|-2| \times 2 = 10$$

$$d z = 10$$

$$z = 5$$

$z > 5 \rightarrow$  One real + 2 complex poles B.A pts.

$z < 5 \rightarrow$  3 B real B.A points

$\downarrow$   
No poles  
1 complex pole

⑨ Real poles  $\Rightarrow s = 0, -4$ .  
 avg value  $= \frac{0 + (-4)}{2} = -2$   
 $| -2 | > 5 \Rightarrow \omega = 2.5$

$\omega < 5$   
 3 B.A pts  
 real

classmate

Date \_\_\_\_\_  
 Page \_\_\_\_\_

(cont...)  $s^4 + 8s^3 + 36s^2 + 80s + K = 0$

1)  $K = -s^4 - 8s^3 - 36s^2 - 80s$

$ds = 0 \Rightarrow 4s^3 + 24s^2 + 72s + 80 = 0$   
 $\Rightarrow -2, -2 \pm j\sqrt{45}$

109

NOTE: To check the validity of complex B.A points use angle condition.

$s^4$	1	36	K
$s^3$	8	80	0
$s^2$	96	K	0
$s^1$	$2080 - 8K$	0	0
$s^0$	96	0	0
$s^{-1}$	K	0	0

$\rightarrow 2080 - 8K > 0 \Rightarrow K < 260$

$\therefore 96$

$\rightarrow K > 0$

$\boxed{0 < K < 260} \quad K = K_{max} = 960$

$A(s) = 96s^2 + K$

$= 96s^2 + 260 = 0$

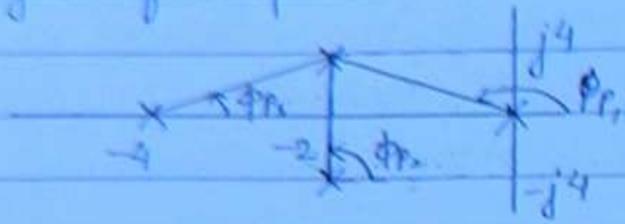
$s = \pm j\sqrt{3.16}$

Intersection of asymptotes with  $j\omega$  axis

$y = \tan 45^\circ \times 2 = 2 = 1j2$

## 8) Angle of Departure -

(710)



$$\begin{aligned}\phi_{P_1} &= 180^\circ - \tan^{-1} \left[ \frac{4-0}{0-(-2)} \right] \\ &= 180^\circ - \tan^{-1} 2 \\ &= 180^\circ - 63.4^\circ \\ &\approx 116.6^\circ\end{aligned}$$

$$\phi_{P_2} = 90^\circ$$

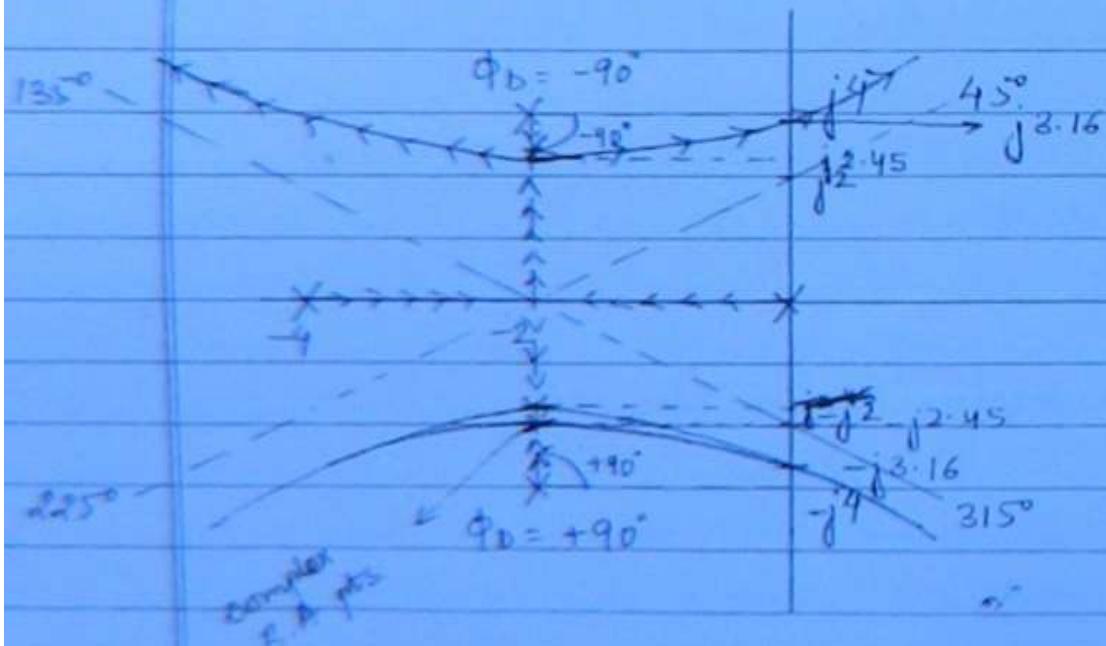
$$\phi_{P_3} = \tan^{-1} \left[ \frac{4-0}{-2-(-4)} \right] = 63.4^\circ$$

$$\begin{aligned}\phi &= 0 - [116.6 + 90 + 63.4] \\ &= -270^\circ\end{aligned}$$

$$\phi_D = 180^\circ + \phi$$

$$\phi = \sum \phi_E - \sum \phi_P$$

$$\phi_D = 180^\circ + \phi = 180^\circ + (-270^\circ) = -90^\circ$$

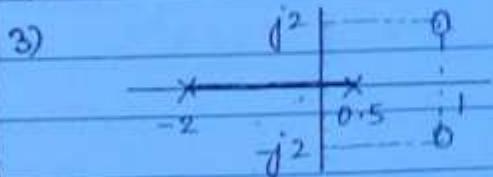


CONV 3

$$G(s) = \frac{K(s^2 - 2s + 5)}{(s+2)(s-0.5)}$$

(11)

$$2) P = 2 \quad Z = 2 \quad P - Z = 0$$



6) B.A points

$$(s+2)(s-0.5) + K(s^2 - 2s + 5) = 0$$

$$K = \frac{-s^2 + 1.5s + 1}{(s+2)(s-0.5)} = \frac{s^2 - 2s + 5}{s^2 - 1.5s - 1}$$

$$dK = 0 \Rightarrow 3.5s^2 - 12s - 5 = 0$$

$$ds \quad \boxed{-0.4}, 3.6$$

$$\ddagger) s^2(1+K) + s(1.5 - 2K) + (5K - 1) = 0$$

$$s^2 \quad 1+K \quad 5K-1$$

$$s^1 \quad 1.5-2K \quad 0$$

$$s^0 \quad 5K-1 \quad 0$$

$$\rightarrow 1+K > 0 \Rightarrow K > -1$$

put  $K = -1$  in the coeff

$$\rightarrow 1.5 - 2K > 0 \Rightarrow K < 0.75$$

of Routh Array column 1

$$\rightarrow 5K - 1 > 0 \Rightarrow K > 0.2$$

there will be a sign change  
to fm. system stability

$$\boxed{0.2 < K < 0.75} \quad \leftarrow \uparrow$$

$$K = K_{max} = 0.75$$

$$A(s) = (1+K)s^2 + (5K-1)$$

$$= (1+0.75)s^2 + [5 \times 0.75 - 1] = 0$$

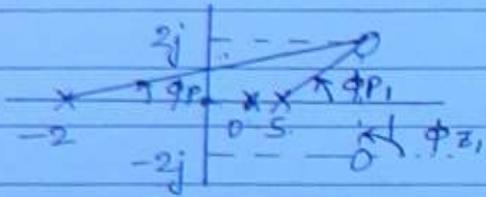
$$s = \pm j1.25$$

### 8) Angle of Arrival

(112)

$$\boxed{\phi_A = 180^\circ - \phi}$$

$$\phi = \sum \phi_E - \sum \phi_P$$



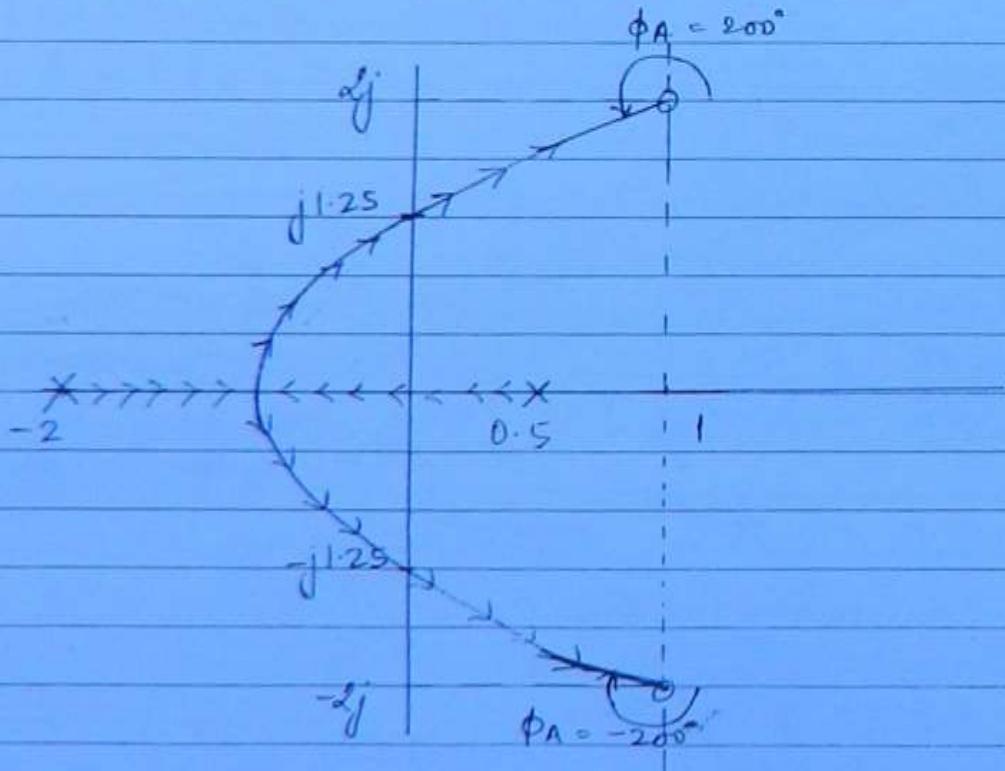
$$\phi_E = 90^\circ \quad \phi_{P_1} = \tan^{-1} \left[ \frac{2-0}{1-0.5} \right] \quad \phi_{P_2} = \tan^{-1} \left[ \frac{2-0}{1-(-2)} \right]$$

$$\phi_{P_1} = 76^\circ$$

$$\phi_{P_2} = -34^\circ$$

$$\phi = 90^\circ - [76^\circ + 34^\circ] = -20^\circ$$

$$\phi_A = 180^\circ + 20^\circ = 200^\circ$$



$$① \quad G(s) = \frac{K(s+a)}{s^2(s+b)}$$

(113)

Check for RL is always Routh Array.

$$s^3 + bs^2 + ks + ak = 0$$

$s^3$	1	k
$s^2$	b	ak
$s^1$	$\frac{bk - ak}{b}$	0
$s^0$	ak	0

$$1) \quad ak > 0 \Rightarrow [k > 0]$$

$$2) \quad \frac{bk - ak}{b} > 0 \Rightarrow [k > 0]$$

$$\boxed{k > 0}$$

$$k = k_{\max} = 0$$

$$A(s) = bs^2 + ak = 0$$

$$bs^2 + 0 = 0$$

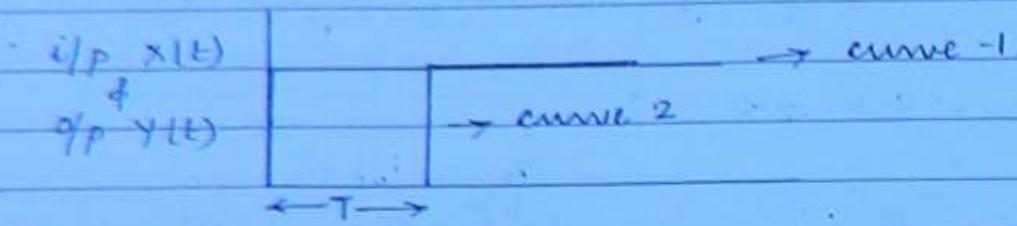
$$\boxed{s=0} \quad (C)$$

The system is stable for  $k > 0$

so there is no way RL will cross  $Im$  axis

# ANALYSIS OF SYSTEMS HAVING DEAD TIME (or) TRANSPORTATION LAG

114



For curve -1

$$\%_p Y(t) = \%_p X(t)$$

For curve -2

$$\%_p Y(t) = X(t-T)$$

Applying L.T.

$$Y(s) = e^{-Ts} X(s)$$

$$Y(s) = e^{-Ts}$$

$$X(s)$$

## I. Time Domain Approximation

[T.D Analysis, RH, RL]

$$Y(t) = X(t-T) = X(t) - T \dot{X}(t) + T^2 \ddot{X}(t) - \dots$$

2!

$$Y(t) = X(t) - T \dot{X}(t)$$

$$Y(s) = X(s) - Ts X(s)$$

$$= X(s) [1 - Ts]$$

$$Y(s) = X(s) e^{-Ts}$$

$e^{-Ts}$	$\approx 1 - Ts$
-----------	------------------

eg  $G(s) = \frac{K e^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)}$

1. Dead Time  $\rightarrow$  is one of the forms of non-linearities.
2. It is approximated as a zero in RHS of s-plane
3. Time Transfer functions having poles or zeros in RHS of s-plane are known as Non-minimum Phase fn's.

### Non Minimum Phase Functions

(115)

$$\left| F(s) \right|_{\omega \rightarrow \infty} \neq -[P-Z] 90^\circ$$

$$\left| F(s) \right|_{\omega \rightarrow \infty} \neq -[P-Z] 90^\circ$$

eg

$$G(s) = \frac{K(1-s)}{s(s+3)}$$

$$G(j\omega) = \frac{(1-j\omega)(K+j\omega)}{(0+j\omega)(j\omega+3)}$$

$$\begin{aligned} \angle G(j\omega) &= [0^\circ] [-\tan^{-1}\omega] \\ &\quad [90^\circ] [\tan^{-1}\omega/3] \\ &= -90^\circ - \tan^{-1} \frac{\omega}{3} - \tan^{-1} \omega \end{aligned}$$

$$\begin{aligned} \left. \angle G(j\omega) \right|_{\omega \rightarrow \infty} &= -90^\circ - 90^\circ - 90^\circ \\ &= -270^\circ \end{aligned}$$

4. LT transfer functions must be ~~non~~-minimum phase functions ie poles & zeros must lie in RHS of s-plane.

$$G(s) = \frac{K(1+s)}{s(s+3)}$$

116

$$G(j\omega) = \frac{(K + j0)(1 + j\omega)}{(0 + j\omega)(j\omega + 3)}$$

$$/G(j\omega) = [0^\circ] [ + \tan^4 \omega ] \\ [90^\circ] [\tan^4 \omega / 3] \\ = -90^\circ - \frac{\tan^4 \omega}{3} + \tan^4 \omega$$

$$\left| G(j\omega) \right|_{\omega=0} = -90^\circ - 90^\circ + 90^\circ = -90^\circ$$

e. Given function is a minimum phase function

$$G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)} = \frac{-K(s-1)}{s(s+3)}$$

Since 'S' cannot be -ve  
 $(1-S)$  should be expressed  
 as  $-(S-1)$

$$1 + G(s) H(s) = 0 \quad \text{char. eq.}^n$$

$$1 + \left[ \frac{-K(s-1)}{s(s+3)} \right] = 0$$

$$1 - G(s) H(s) = 0$$

$$G(s) H(s) = 0 \quad !$$

Complementary RL or Inverse RL  
(CRL) (IRL)

$$1 - G(s) H(s) = 0$$

Angle Condition

$$|G(s)H(s)| = 1 \Rightarrow [2q] 180^\circ$$

(J17)

Magnitude Condition

$$|G(s)H(s)| = 1$$

CONSTRUCTION RULES OF CRL -

Rule 1 → The CRL is symmetrical about real axis

$$[(G(s)H(s)) = 1]$$

Rule 2 → Same as RL

Rule 3 → A point on real axis is said to be on  
CRL if to the right side of this point  
the sum of open loop poles & zeros  
is even.

Rule 4 → Angle of Asymptotes

$$\theta = \frac{[2q] 180^\circ}{P-Z}$$

$$q = 0, 1, 2, 3, \dots$$

Rule 5 → Contour

Same as RL

Rule 6 → Break Away Points

Same as RL

Rule 7 → Intersection of CRL with  $jw$  axis  
Same as RL

(118)

Rule 8 → Angle of Departure of Arrival

$$\phi_D = 0^\circ + \phi$$

$$\phi_A = 0^\circ - \phi$$

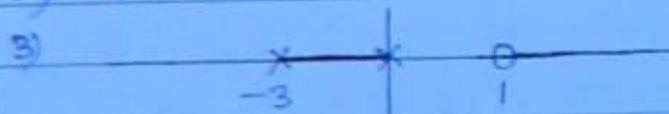
where

$$\therefore \inf = \sum \phi_D - \sum \phi_P$$

CWB chapter 5

②  $G(s) = \frac{ke^{-s}}{s(s+3)} = \frac{k(1-s)}{s(s+3)} = \frac{-k(s-1)}{s(s+3)}$

2)  $P = 2, Z = 1, P - Z = 1$



4) B.A points

$$1 + \frac{k(1-s)}{s(s+3)} = 0$$

$$s(s+3) + k(1-s) = 0$$

$$k = -s^2 - 3s$$

$$1-s$$

$$\frac{dk}{ds} = 0$$

$$\frac{(1-s)(-2s-3) - [(-s^2 - 3s)(1-1)]}{(1-s)^2} = 0$$

$$-2s - 3 + 2s^2 + 3s - s^2 - 3s = 0$$

$$s^2 - 2s - 3 = 0$$

$$1) s^2 + s(3 - k) + k = 0$$

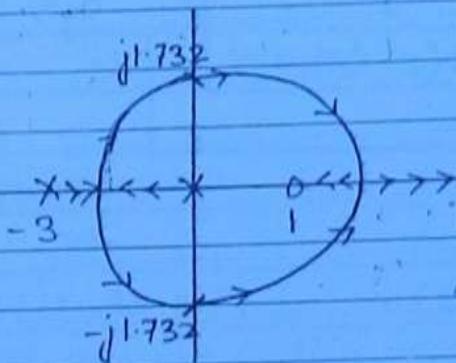
(19)

$$\begin{matrix} s^2 & 1 & k \\ s^1 & 3-k & 0 \\ s^0 & k & 0 \end{matrix} \quad 3-k=0 \Rightarrow k_{\text{max}} = 3$$

$$A(s) = s^2 + k > 0.$$

$$s^2 + 3 > 0$$

$$s = \pm \sqrt{-3}$$



(4)

Damping can be determined by  $\zeta$  value or location of poles

→ If poles are on  $\text{Im}\ z$  axis → undamped

→ If poles are on ~~real axis of~~  $s$  plane of repeating → critically damped

→ If poles are in 2nd or 3rd quad of  $s$  plane → underdamped

→ If poles are on real axis of unequal → overdamped

for  $0 \leq k < 1$  → poles are on real axis of unequal → overdamped

At  $k=1$  → poles repeated → marginally stable

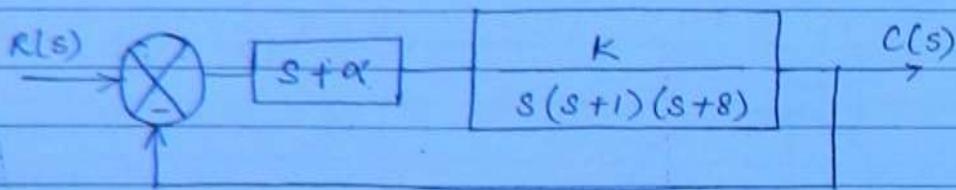
$k > 1$  → pole on real axis of unequal

Ans (c) Since equality cannot here equal to hence option (d) is incorrect

## ROOT CONTOURS

(120)

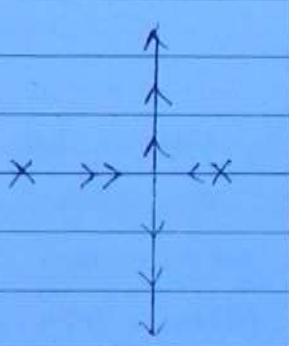
Root contours are multiple root locus diagrams obtained by varying multiple parameters in a Transfer function drawn on same s-plane.



$$G(s) = \frac{K(s+a)}{s(s+1)(s+8)}$$

Case 1 → Let  $a = 0$

$$\begin{aligned} G(s) &= \frac{Ks}{s(s+1)(s+8)} \\ &= \frac{K}{(s+1)(s+8)} \end{aligned}$$



$$\text{Case 2} \rightarrow 1 + \frac{Ks + K\alpha}{s(s+1)(s+8)} = 0$$

$$\underbrace{s(s+1)(s+8)}_{s^3 + 10s^2 + 18s} + Ks + K\alpha = 0$$

$$1 + \frac{K\alpha}{s(s+1)(s+8) + Ks} = 0$$

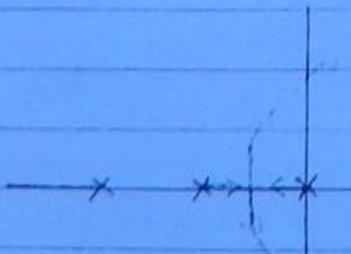
$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = \frac{K\alpha}{s(s+1)(s+8) + Ks}$$

$$\text{Put } K = 1$$

$$= \frac{\alpha}{s(s^2 + 9s + 9)}$$

$$s(s^2 + 9s + 9)$$



Q Find B.A pts for  $k=10$  ?

(121)

Sol.  $G(s)H(s) = \frac{10\alpha}{s(s+1)(s+8)+10s}$

Let  $10\alpha = k'$

$$= \frac{k'}{s(s^2 + 9s + 18)}$$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k'}{s(s^2 + 9s + 18)} = 0$$

$$s^3 + 9s^2 + 18s + k' = 0$$

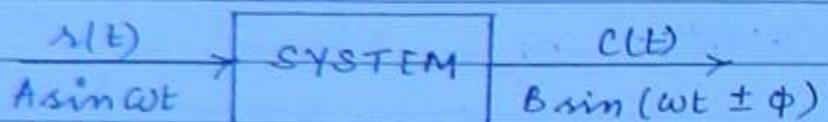
$$k' = -s^3 - 9s^2 - 18s$$

$$\frac{dk'}{ds} = 0 \Rightarrow 3s^2 + 18s + 18 = 0$$

## FREQUENCY DOMAIN ANALYSIS

(122)

When any system is subjected to sinusoidal input, the output is also sinusoidal having different magnitude & phase angle but same input frequency  $\omega$  rad/sec.



Frequency response analysis implies varying  $\omega$  from  $0$  to  $\infty$  & observing corresponding variations in the magnitude & phase angle of the response.

$$\text{Let } F(s) = \frac{C(s)}{R(s)} = T.F.$$

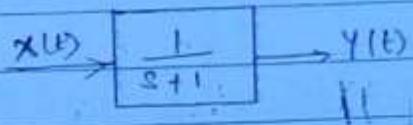
$$\text{Put } s = j\omega$$

$F(j\omega)$  = Sinusoidal T.F.  
 ≈ Sinusoidal response

$$F(j\omega) = |F(j\omega)| / \angle F(j\omega)$$

$$\sqrt{(r.p)^2 + (i.p)^2} \quad \tan^{-1} \left[ \frac{i.p}{r.p} \right]$$

(20)



For  $X(t) = \sin t$  find  $Y(t)$ .

$$F(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$F(j\omega) = \frac{1+j^0}{1+j\omega}$$

$$|F(j\omega)| = \sqrt{1^2 + 0^2} = \frac{1}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = \frac{\tan^{-1} 0}{1} = -\tan^{-1} \omega$$

$$F(j\omega) = \frac{1}{\sqrt{1+\omega^2}} e^{-j\tan^{-1} \omega}$$

Given  $X(t) = \sin t$

$$\approx \sin \omega t \Rightarrow \omega = 1 \text{ rad/s}$$

$$F(j\omega) = \frac{1}{\sqrt{2}} e^{-j45^\circ}$$

$$Y(t) = \frac{1}{\sqrt{2}} \sin(t - 45^\circ)$$

Q

$$G(s) = \frac{(s^2+9)(s+2)}{(s+3)(s+5)(s+7)}$$

The steady state sinusoidal response

will become zero at a freq of

- a)  $\omega^2 \text{ rad/s}$  b)  $3 \text{ rad/s}$  c)  $4 \text{ rad/s}$  d)  $5 \text{ rad/s}$

sol

$$G(j\omega) = \frac{(-\omega^2+9)(j\omega+2)}{(j\omega+3)(j\omega+5)(j\omega+7)} = 0$$

$$(-\omega^2+9)(j\omega+2) = 0$$

# FREQUENCY RESPONSE PLOTS.

(24)

## I POLAR PLOT

Absolute values of  $|F(j\omega)|$  Vs  $\omega$   
 +  $\angle F(j\omega)$  (degrees)

## II BODE PLOT

db (decibel) values of  $|F(j\omega)|$  Vs  $\log \omega \rightarrow$  logarithmic scale.  
 [ $20 \log |F(j\omega)|$ ]  
 +  $\angle F(j\omega)$  (degrees)

To be able to  
calibrate db value  
from small values  
to very large  
power values.

$\omega = 1$  to  
 $10^4$

Power =  $VI \rightarrow \omega$  variables are not considered in  $\omega$  domain

$$= I^2 R \rightarrow 10 \log (I^2 R) \Rightarrow 20 \log I + 10 \log R$$

$$= V^2 / R \quad [Mx + c]$$

Frequency Response Analysis of  $n^{th}$  order system:-

$$F(s) = \frac{C(s)}{R(s)} = \frac{w_n^2}{s^2 + 2Gs + w_n^2}$$

$$F(s) = \frac{1}{s^2 + \frac{2Gs}{w_n^2} + \frac{1}{w_n^2}}$$

Put  $s = j\omega$

$$F(j\omega) = \frac{1}{(j\omega)^2 + 2Gj\omega + \frac{1}{w_n^2}}$$

$$* F(j\omega) = \frac{1}{1 - \left[ \frac{\omega}{w_n} \right]^2 + 2j \frac{\omega}{w_n}}$$

→ use this to find  
out values of  $w_n$  &  $G$   
from eq<sup>n</sup> given in ques

$$|F(j\omega)| = \frac{1}{\sqrt{[1 - (\frac{\omega}{\omega_n})^2]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

(125)

$$= -20 \log \sqrt{[1 - (\frac{\omega}{\omega_n})^2]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

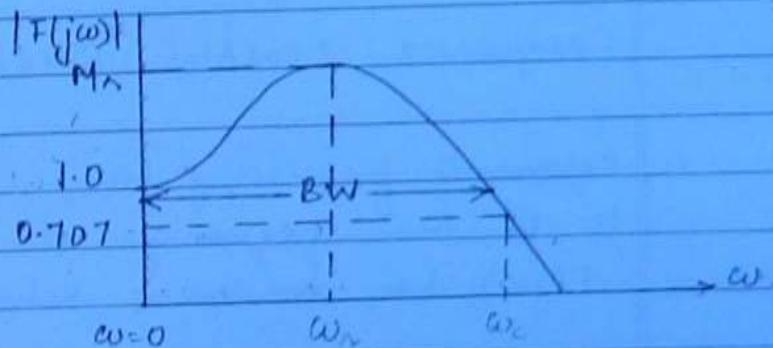
### Asymptotic Approximations

$$= -20 \log \sqrt{1 - (\frac{\omega}{\omega_n})^2}$$

Case 1 → LOW FREQ

$$1 \gg (\omega/\omega_n)^2$$

$$-20 \log \sqrt{1} = 0 \text{ db}$$



Case 2 → HIGH FREQ

$$(\omega/\omega_n)^2 \gg 1$$

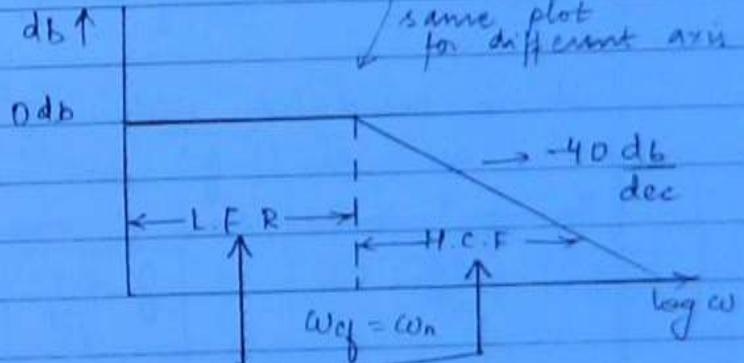
$$-20 \log \sqrt{[(\omega/\omega_n)^2]^2}$$

$$-20 \log (\omega/\omega_n)^2$$

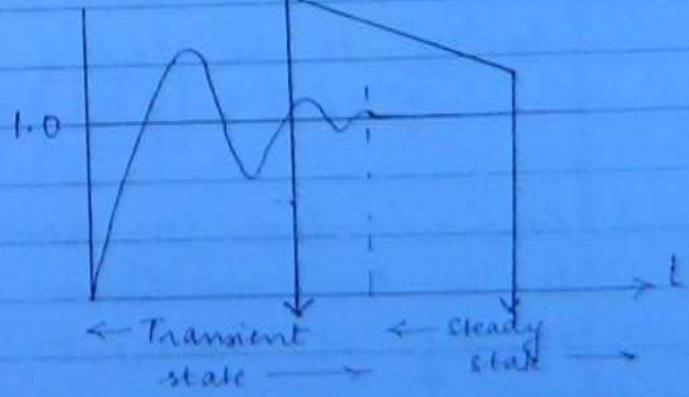
$$-40 \log (\omega/\omega_n) - ①$$

$$-40 \log \omega + 40 \log \omega_n$$

$M_N + C$



$$\text{Slope } (m) = -40 \text{ dB/dec}$$



Corner frequency ( $\omega_{cf}$ )

$$D = -40 \log (\omega/\omega_n)$$

$$\log \omega/\omega_n = 0$$

$$\omega/\omega_n = \log^{-1} 0 = 1$$

$$\omega = \omega_{cf} = \omega_n \text{ rad/s}$$

At  $\omega = \omega_d = \omega_n$ .

$$= -90 \log \sqrt{(1-1)^2 + (2f)^2}$$

### FREQUENCY DOMAIN SPECIFICATIONS-

Resonant frequency (w<sub>r</sub>)

It is the frequency at which magnitude of  $F(j\omega)$  has maximum value.

$$\omega_r = \omega_n \sqrt{1-2g^2} \text{ r/s}$$

It is correlated with

$$\omega_d = \omega_n \sqrt{1-g^2} \text{ r/s}$$

For  $w_d$  to be Real & +ve

$$2g^2 < 1 \Rightarrow \boxed{g < \frac{1}{\sqrt{2}}}$$

For  $w_d$  to be Real & +ve

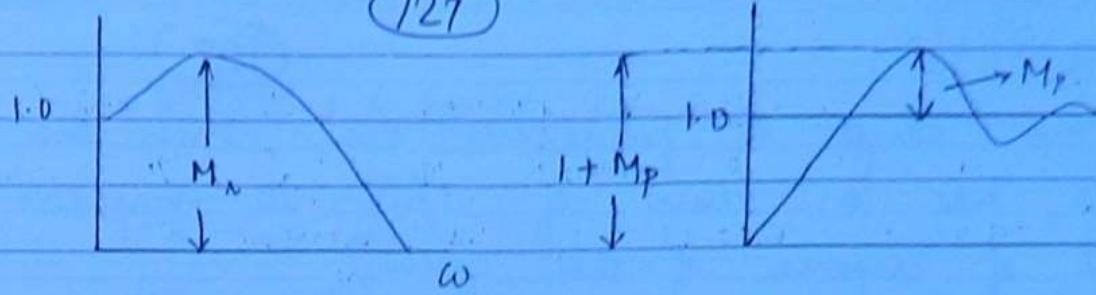
$$g^2 < 1 \Rightarrow \boxed{g < 1}$$

Resonant Peak (or) Peak Magnitude (M<sub>r</sub>)

It is the maximum value of magnitude occurring at resonant frequency  $w_r$ .

$$M_r = \boxed{1}$$

(27)



$$g < \frac{1}{\sqrt{2}}, \quad M_n > 1$$

$$g = \frac{1}{\sqrt{2}}, \quad M_n = 1$$

$$g > \frac{1}{\sqrt{2}} \quad \text{No } M_n$$

CWB chap 6

(21)

$$M(j\omega) = \frac{100}{100 + 10\sqrt{2}j\omega - \omega^2}$$

$$= \frac{1}{1 - \frac{\omega^2}{100} + \frac{10\sqrt{2}j\omega}{100}}$$

$$= \frac{1}{1 - \left(\frac{\omega}{10}\right)^2 + j\frac{\sqrt{2}\omega}{10}}$$

$$\left[ \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j\frac{2G}{\omega_n}\frac{\omega}{\omega_n}} \right]$$

$$\text{if } \frac{\omega}{\omega_n} = \frac{\sqrt{2}\omega}{10}$$

$$g = \frac{1}{\sqrt{2}} ; M_p (M_n) = 1 \quad (\text{b})$$

Peak mag' means  $M_n$  value.

They have represented it as  $M_p$

### 3. Bandwidth (B.W) :

(128)

It is the range of frequencies over which the magnitude has a value of  $1/\sqrt{2}$ . It indicates the speed of response of the system.

Wider B.W  $\Rightarrow$  Faster response.

$$BW \propto \frac{1}{t_r} \quad \text{where } t_r = \text{rise time}$$

### 4. Cut-off Frequency ( $\omega_c$ )

It is the frequency at which the magnitude has a value of  $1/\sqrt{2}$ . It indicates the ability of the system to distinguish signal from noise.

$$BW (\omega_c) \quad \omega_c = \omega_n \sqrt{1 - 2G^2 + \sqrt{4G^4 - 4G^2 + 2}}$$

Frequency Domain Approximation of Dead Time or Transportation Lag -

$$F(s) = \frac{Y(s)}{X(s)} = e^{-Ts}$$

$$F(j\omega) = e^{-j\omega T} = \cos \omega T - j \sin \omega T$$

$$|F(j\omega)| = \sqrt{(\cos \omega T)^2 + (\sin \omega T)^2} = 1$$

$$\angle F(j\omega) = \tan^{-1} \left[ \frac{-\sin \omega T}{\cos \omega T} \right] = \tan^{-1} [-\tan \omega T] = -\omega T \text{ (radians)}$$

$$e^{-j\omega T} \approx 1 / -\omega T \text{ (radians)}$$

$$\begin{aligned} \pi &\rightarrow 180^\circ \\ -\omega T &\rightarrow ? \\ &\Rightarrow -\omega T \times 180^\circ \\ &= -57.3 \omega T \text{ (degrees)} \end{aligned}$$

$$e^{-j\omega T} \approx 1 / -57.3 \omega T \text{ (degrees)}$$

### STABILITY FROM FREQUENCY RESPONSE PLOTS -

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

$$\text{Put } s = j\omega$$

$$G(j\omega)H(j\omega) = -1 + j0$$

(critical point)

#### Stability Criteria

- Gain crossover frequency ( $\omega_{gc}$ )

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1 \text{ or } 0 \text{ db}$$

- Phase crossover frequency ( $\omega_{pc}$ )

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = -180^\circ$$

### 3. Gain Margin (G.M)

It is the "allowable gain"

(B3)

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = X \quad G.M = \frac{1}{X} \quad G.M(\text{db}) = 20 \log \left( \frac{1}{X} \right)$$

### 4. Phase Margin (P.M)

It is the "allowable phase lag"

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{ge}} = \phi$$

$$PM = 180^\circ + \phi$$

$$\text{STABLE} \Rightarrow GM + PM = +\text{ve}$$

$$\rightarrow \omega_{ge} < \omega_{pc}$$

$$\text{UNSTABLE} \Rightarrow GM + PM = -\text{ve}$$

$$\rightarrow \omega_{ge} > \omega_{pc}$$

$$\text{MARGINALLY} \Rightarrow GM = PM = 0$$

$$\text{STABLE} \rightarrow \omega_{ge} = \omega_{pc}$$

G.M & P.M for second order system -

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 - \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

(from)

$$|G(j\omega)| = \frac{\omega_n^2}{\omega \sqrt{\omega^2 + 4g^2\omega_n^2}} \quad (3)$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{\alpha g \omega_n}\right)$$

At  $\omega = \omega_{ge}$

$$\frac{\omega_n^2}{\omega \sqrt{\omega^2 + 4g^2\omega_n^2}} = 1$$

$$\begin{aligned} \omega_n^4 &= \omega^2(\omega^2 + 4g^2\omega_n^2) \\ \omega_n^4 + \omega^2 4g^2\omega_n^2 - \omega_n^2 &= 0 \end{aligned}$$

$$\begin{aligned} \omega_n^2 &= \frac{-4g^2\omega_n^2 \pm \sqrt{16g^4\omega_n^4 + 4\omega_n^4}}{2} \\ &= -2g^2\omega_n^2 \pm \omega_n^2 \sqrt{4g^4 + 1} \end{aligned}$$

$$\omega^2 = -2g^2\omega_n^2 + \omega_n^2 \sqrt{4g^4 + 1}$$

$$\omega = \boxed{\omega_{ge} = \omega_n \sqrt{-2g^2 + \sqrt{4g^4 + 1}}} \text{ r/s}$$

$$\left. |G(j\omega)| \right|_{\omega=\omega_{ge}} = \phi$$

$$\phi = -90^\circ - \tan^{-1}\left(\frac{\omega_n \sqrt{-2g^2 + \sqrt{4g^4 + 1}}}{2g\omega_n}\right)$$

$$PM = 180^\circ + \phi$$

$$\boxed{PM = 90^\circ + -\tan^{-1}\left[\frac{\sqrt{-2g^2 + \sqrt{4g^4 + 1}}}{2g}\right]}$$

$$At \quad \omega = \omega_{pe} \rightarrow \infty \quad \text{N/S} \quad |G(j\omega)| = -90^\circ - 90^\circ \\ \leq -180^\circ$$

$$|G(j\omega)|_{\omega=\omega_{pe}} = \infty \quad \text{at } X=0$$

$$G.M = \frac{1}{X} = \frac{1}{0} = \infty$$

$$\omega_{pe} = 20 \text{ rad/s}$$

$$G.M = \infty$$

(132)

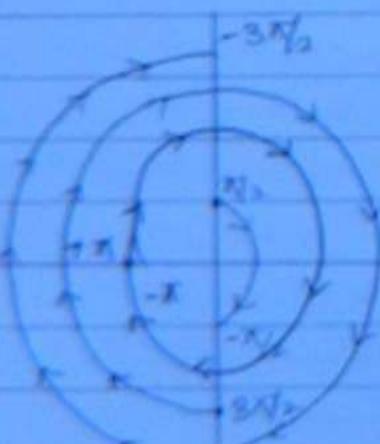
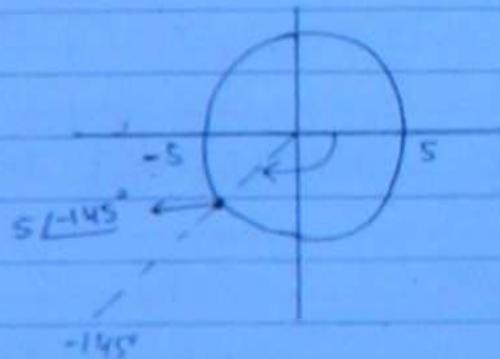
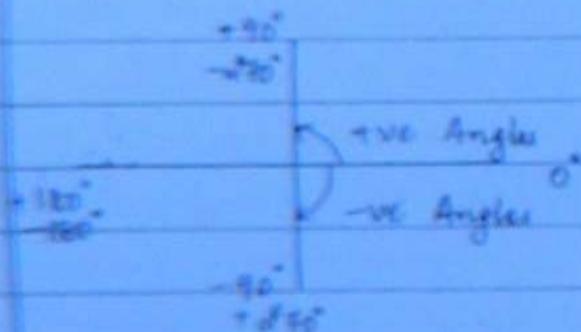
### POLAR PLOT

It is a plot of absolute values of magnitude of phase angle in degrees of open loop transfer function  $G(j\omega) H(j\omega)$  vs  $\omega$  drawn on polar coordinates.

e.g.

Polar Co-ordinates

e.g.  $5 \angle -145^\circ$



$$G(s) = \frac{1}{s+1} \quad \text{Always convert to time constant form}$$

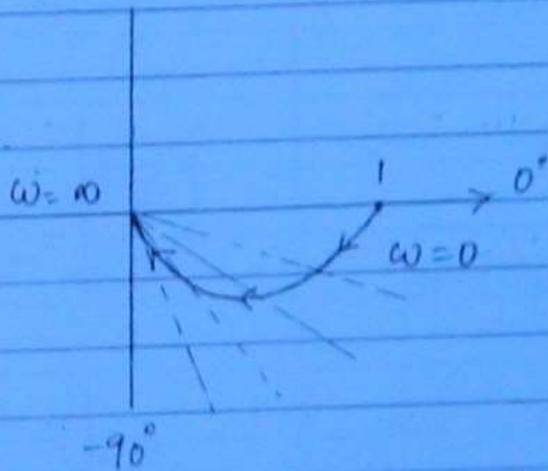
$$G(s) = \frac{1}{1+s} \quad \text{Type 0, Order 1 system.} \quad (1.3)$$

$$G(j\omega) = \frac{1}{1+j\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

$$\angle G(j\omega) = -\tan^{-1}\omega.$$

$\omega$	0	$\infty$
$ G(j\omega) $	1	0
$\angle G(j\omega)$	$0^\circ$	$-90^\circ$



↓ Type 3

Type 2  
order 3

order 4

← Type 0

order 2

order 1

↑  
Type 1

# General shapes of Polar Plots -

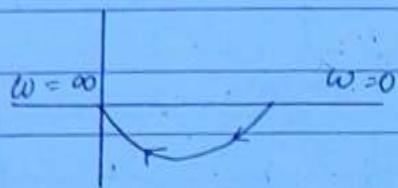
(134)

## Type 0 Order

1. Type 0 / Order 1

$$G(s) = \frac{1}{1+Ts}$$

## Polar Plot



2. Type 0 / Order 2

$$G(s) = \frac{1}{(1+T_1s)(1+T_2s)}$$



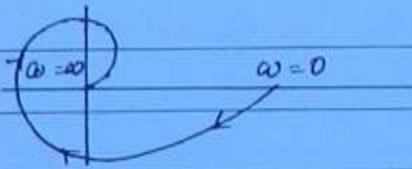
3. Type 0 / Order 3

$$G(s) = \frac{1}{(1+T_1s)(1+T_2s)(1+T_3s)}$$



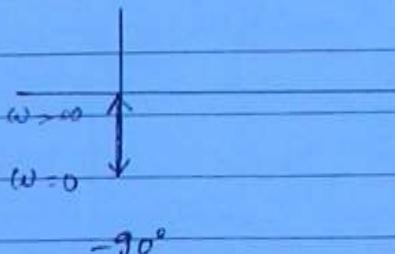
4. Type 0 / Order 4

$$G(s) = \frac{1}{(1+T_1s)(1+T_2s)(1+T_3s)(1+T_4s)}$$



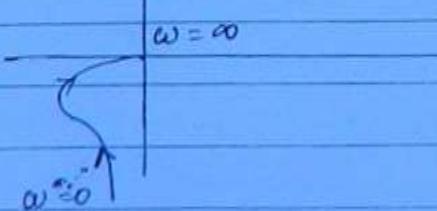
5. Type 1 / Order 1

$$G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega} e^{-j90^\circ}$$



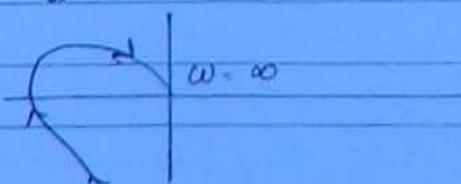
6. Type 1 / Order 2

$$G(s) = \frac{1}{s(1+T_1s)}$$



7. Type 1 / Order 3

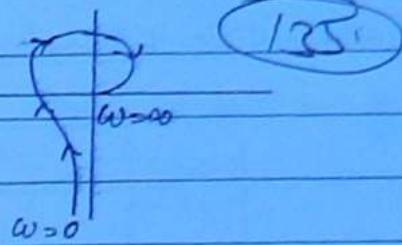
$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)}$$



8. Type 1 / Order 4

$$G(s) = \frac{1}{s}$$

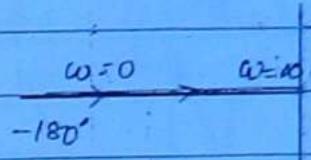
$$s(1+T_1 s)(1+T_2 s)(1+T_3 s)$$



135

9. Type 2 / Order 2

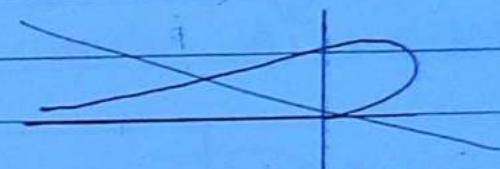
$$G(s) = \frac{1}{s^2} = \frac{1}{(j\omega)^2} = \frac{1}{\omega^2} e^{-j180^\circ}$$



10. Type 2 / Order 3

~~$$G(s) = \frac{1}{s^3}$$~~

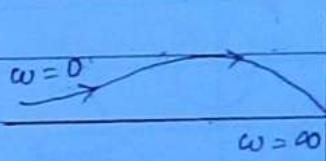
~~$$s^2(1+Ts)$$~~



10. Type 2 / Order 3

~~$$G(s) = \frac{1}{s^2}$$~~

~~$$s^2(1+Ts)$$~~



11. Type 2 / Order 4

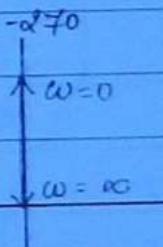
~~$$G(s) = \frac{1}{s^3}$$~~

~~$$s^2(1+T_1 s)(1+T_2 s)$$~~



12. Type 3 / Order 3

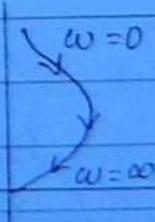
$$G(s) = \frac{1}{s^3} = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3} e^{-j270^\circ}$$



13. Type 3 / Order 4

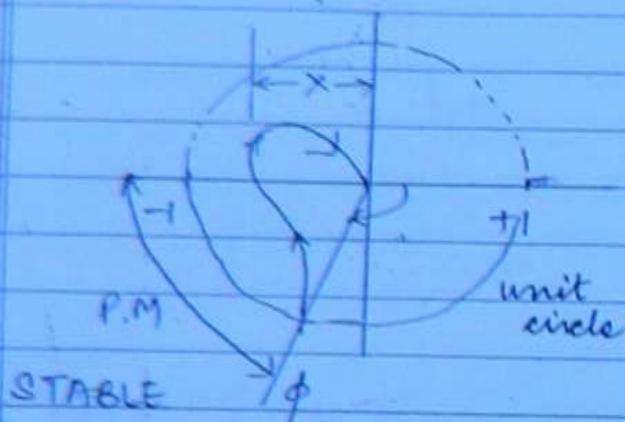
~~$$G(s) = \frac{1}{s^4}$$~~

~~$$s^3(1+Ts)$$~~



## STABILITY FROM POLAR PLOTS -

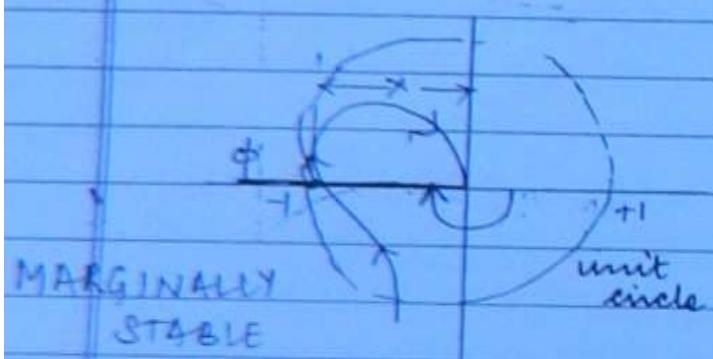
13G



$$G.M = 1 \quad G.M (\text{dB})$$

$$X = 20 \log \frac{1}{X} = +\text{ve}$$

$$PM = \phi - [-180^\circ] = 180^\circ + \phi \\ = +\text{ve}$$

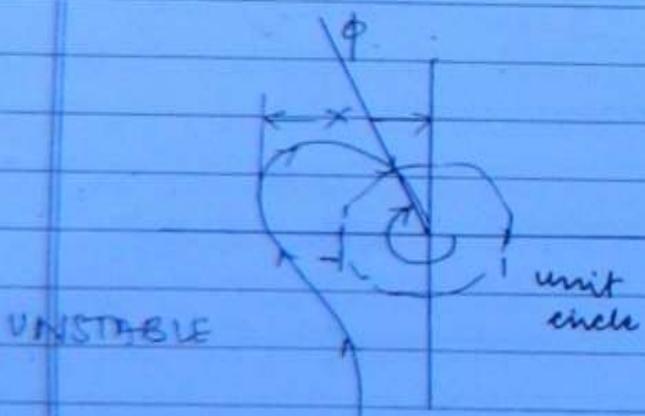


$$X = 1 \quad G.M = \frac{1}{X} = 1$$

$$G.M (\text{dB}) = 20 \log 1 = 0 \text{ dB}$$

$$\phi = -180^\circ$$

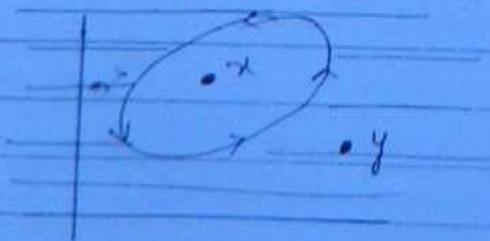
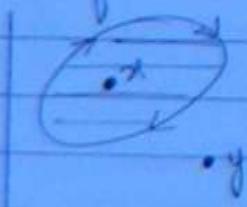
$$IM = 180^\circ - 180^\circ = 0^\circ$$



$$GM = \frac{1}{X} \quad GM (\text{dB}) = 20 \log \frac{1}{X} = -\text{ve}$$

$$PM = 180^\circ + \phi = -\text{ve.}$$

Concept of Enclosure &amp; Encirclement:



A point is said to be enclosed by a contour if it lies to the right side of the direction of the contours.

(PST)

A point is said to be enclosed if the contour is a closed path.

In fig (2) pt y is said to be enclosed whereas point x is said to be enclosed in anticlockwise direction.

In Polar plots if the critical point  $-1+j0$  is not enclosed then the system is said to be stable.

### THEORY OF NYQUIST PLOTS

#### Principle of Mapping



s-plane

P(s)-plane

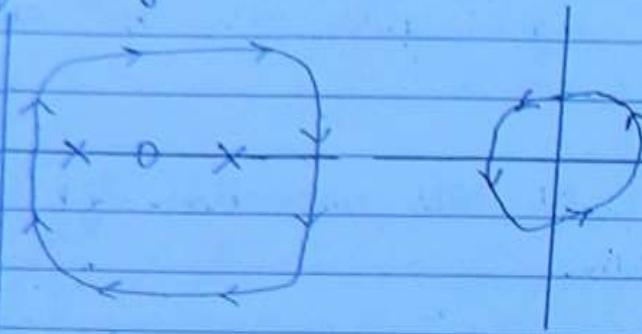
$$P(s) = s + 2$$

$$P(0) = 0 + 2 = 2$$

$$P(-j5) = -j5 + 2$$

# Principle of Argument

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s-plane

P(s) - plane

$$P(s) = (s - b)$$

$$(s - a)(s - c)$$

$$N = P - Z$$

$$\begin{aligned} N &= \text{No. of encirclements} \\ &= +ve \quad (\text{Anticlockwise}) \\ &= -ve \quad (\text{Clockwise}) \end{aligned}$$

$$P = \text{No. of poles in RHS of } s\text{-plane}$$

$$Z = \text{No. of zeros in RHS of } s\text{-plane}$$

Nyquist Stability Criteria -

$$G(s) H(s) = \frac{k(s \pm z_1)}{s(s \pm p_1)}$$

$$1 + G(s) H(s) = 0$$

$$1 + \frac{k(s \pm z_1)}{s(s \pm p_1)} = 0$$

$$s(s \pm p_1)$$

$$s(s \pm p_1) + k(s \pm z_1) \xrightarrow{\text{C.L. poles}} 0 = 0 \quad (1)$$

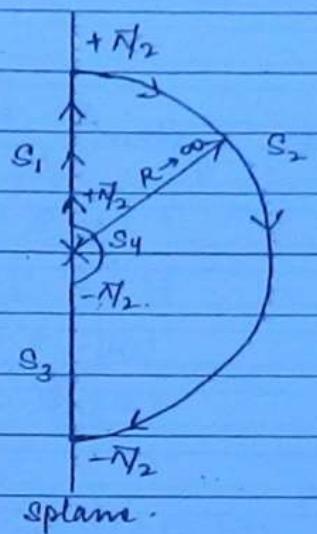
$$s(s \pm p_1) \rightarrow \text{O.L. poles} \rightarrow$$

$$P = \text{No. of open loop poles in RHS of } s\text{-plane}$$

$$Z = \text{No. of closed loop poles in RHS of } s\text{-plane}$$

NYQUIST PATH

(139)



→ To map  $S_1$

Polar plot.

→ To map  $S_2$ .

$$\text{Put } s = lt \text{ Re}^{j\theta} [ \theta = +\frac{\pi}{2} \rightarrow -\frac{\pi}{2} ] \quad R \rightarrow \infty$$

→ To map  $S_3$ .

Inverse polar plot.

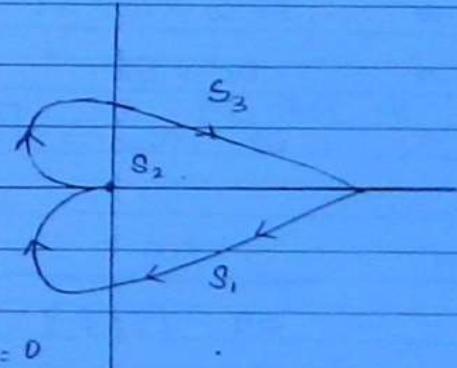
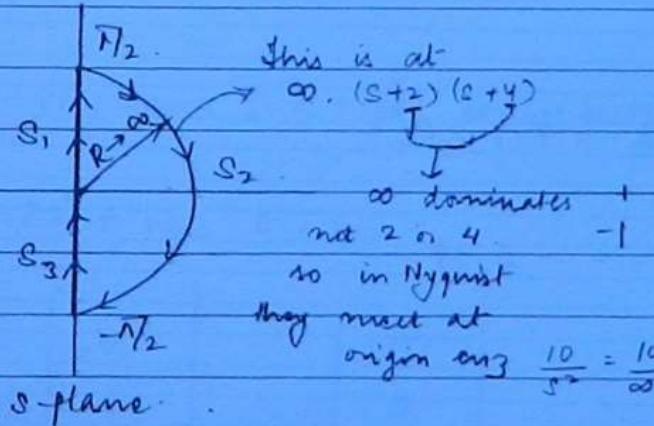
→ To map  $S_4$ .

$$\text{Put } s = lt \text{ Re}^{j\theta} [ \theta = -\frac{\pi}{2} \rightarrow \frac{\pi}{2} ] \quad l \rightarrow 0$$

$$G(s) = \frac{10}{(s+2)(s+4)}$$

$$N = P - Z$$

$$0 = 0 - Z \Rightarrow \boxed{Z=0} \text{ & STABLE}$$



$$G(s) = \frac{10}{(s+2)(s+4)}$$

To map  $S_2$ .

$$G(s) = \frac{10}{s^2} = \frac{10}{lt(Re^{j\theta})^2} = \frac{10}{ltR^2 e^{j2\theta}} = 0 e^{-j2\theta}$$

$$G(s) = \frac{100}{s}$$

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$$(s+2)(s+4)(s+8)$$

Short-cut method (for 3rd order system)

$$\frac{100}{(-\omega^2 + 6j\omega + 8)(j\omega + 8)}$$

$$\Rightarrow \frac{100}{-\omega^3 - 8\omega^2 - 6\omega^2 - 48j\omega + 8j\omega + 64}$$

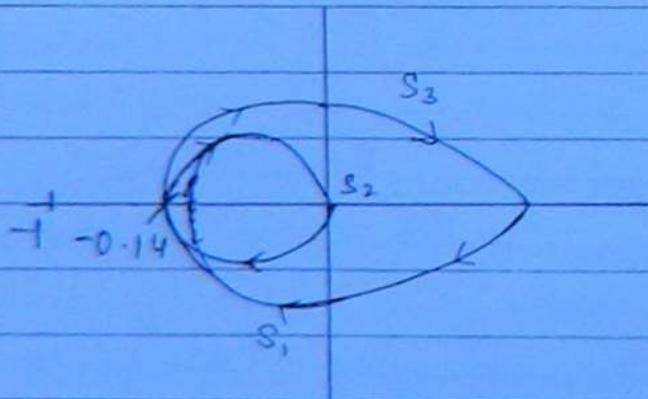
$$\Rightarrow \frac{100}{64 - 14\omega^2 + j[56\omega - \omega^3]}$$

$$\Rightarrow 56\omega - \omega^3 = 0$$

$$\Rightarrow \omega^2 = 56$$

$$\Rightarrow \omega = \omega_{pe} = \sqrt{56} = 7.4 \text{ rad/s.}$$

$$\Rightarrow \frac{100}{64 - 14(7.4)^2} = -0.19.$$



$$N = P - Z$$

$$0 = 0 - Z \Rightarrow Z = 0$$

STABLE

## chapter 6.

CONV

(4)

$$(i) \quad G(s) = \frac{s+2}{(s+1)(s-1)}$$

(T41)

sol $\rightarrow$  To map  $s_1$ .

Polar plot.

$$\alpha [1 + 0.5s]$$

$$1 [1+s](-1)[1-s]$$

$$= -\alpha (1 + 0.5j\omega)$$

$$-180^\circ \quad (1+j\omega)(1-j\omega)$$

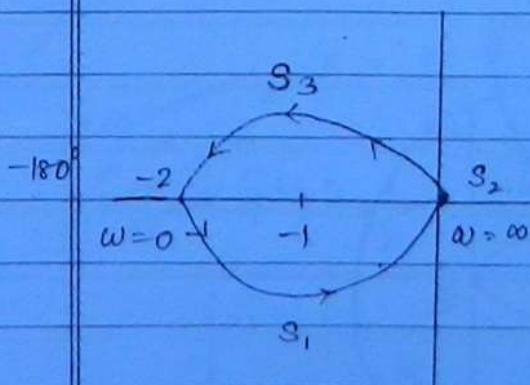
$$|G(j\omega)| = \frac{\alpha \sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2} \sqrt{1+\omega^2}} = \frac{\alpha \sqrt{1+(0.5\omega)^2}}{1+\omega^2}$$

$$\angle G(j\omega) = \frac{(-180^\circ)(\tan^{-1} 0.5\omega)}{(\tan^{-1}\omega)(-\tan^{-1}\omega)}$$

NOTE: -ve Gain ( $-K + j0$ ) contributes  $-180^\circ$  for all  $\omega$ .

$$-180^\circ + \tan^{-1} 0.5\omega$$

$\omega$	0	$\infty$
$ G(j\omega) $	$\alpha$	0
$\angle G(j\omega)$	$-180^\circ$	$-90^\circ$



$$N = P - Z$$

Q. Without constructing Nyquist Plot find the no. of encirclements about critical pt?

(142)

Sol: Nyquist Plot in freq domain is equivalent to Routh Array in time domain for stability check  
(Z of nyquist = no. of sign changes in RA)

$$1 + G(s) = 0$$

$$\frac{1 + (s+2)}{(s+1)(s-1)} = 0$$

$$s^2 - s + s - 1 + s + 2 = 0$$

$$s^2 + s + 1 = 0$$

$s^2$	1	1	1
$s^1$	1	1	0
$s^0$	1	1	0

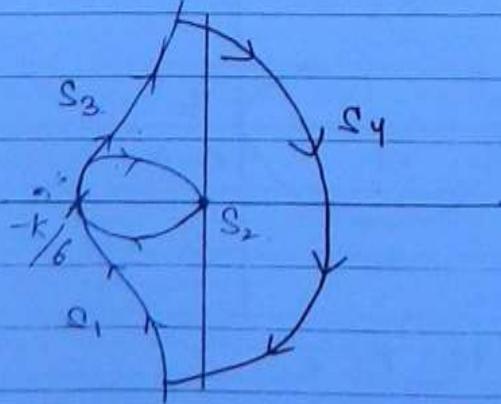
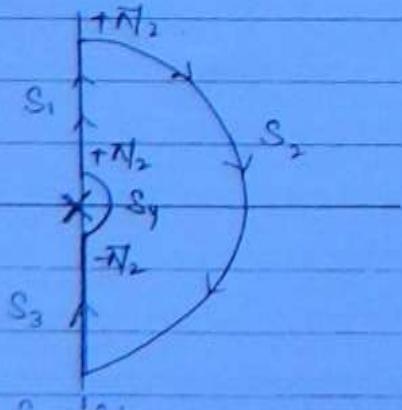
$$N = P - Z$$

$$N = 1 - 0$$

$$N = 1$$

Q.  $G(s) = \frac{K}{s(s+1)(s+2)}$  Find the range of K for stability?

Sol:



To map  $S_4 \Rightarrow$

(143)

$$\frac{G(s)}{s(1)(2)} = \frac{k}{s} = \frac{0.5k}{s} = \frac{0.5k}{R e^{j\theta}} \quad [0 = -\gamma_2 \rightarrow \gamma_2]$$

$$\infty e^{j\pi\gamma_2} \rightarrow \infty e^{-j\pi\gamma_2}$$

$k$

$$\frac{j\omega(-\omega^2 + 3j\omega + 2)}{k}$$

$$-3\omega^2 + j[2\omega - \omega^3]$$

$$2\omega - \omega^3 = 0$$

$$\omega^2 = 2$$

$$\Rightarrow \omega = \omega_{pe} = \sqrt{2} \text{ rad/s}$$

$$\frac{k}{-3(\sqrt{2})^2} = \frac{-k}{6}$$

$$\frac{-k}{6} = -1 \Rightarrow k_{max} = 6$$

case 1)  $k > 6$

case 2)  $k < 6$

$$\zeta = \frac{\omega_p}{\omega} = \frac{\omega}{\sqrt{2}}$$

$$\zeta = 2$$

$\zeta > 1$  unstable

$$\zeta = 0$$

stable

(so disturbance)  $X = \frac{k}{6}$

No real root taken if  $G M = \frac{1}{X} = \frac{6}{k}$

$$\frac{GM}{K}$$

GM/K

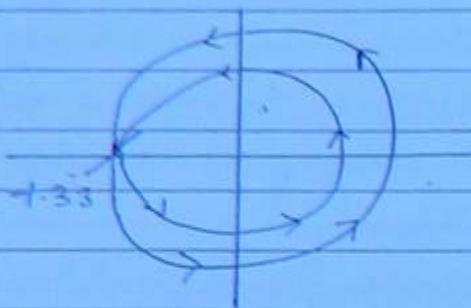
instability

$K \uparrow$   
Instability

(27)

$$G(s) = \frac{k(s+3)(s+5)}{(s-2)(s-4)}$$

For  $k=1$ ,



$$-1.33 \quad k = -1$$

$$K_{MAX} = \frac{1}{1.33} = 0.75$$

case 1)  $k < \frac{1}{1.33}$

$$0 = 2 - Z$$

$$Z = 2$$

unstable

case 2)  $k > \frac{1}{1.33}$

$$\textcircled{2} = 2 - Z$$

*& encirclement  
in anticlockwise  
dir.*

stable.

$$\therefore k > \frac{1}{1.33} \text{ for stability.}$$

(28)

Given  $P=0$

*the critical pt inside*  
 *$\frac{R-I}{R-II}$*

R - I.

$$1-1 = 0 - Z$$

$$Z = 0$$

stable

R - II

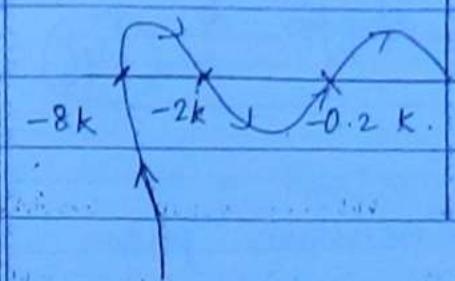
$$-2 = 0 - Z$$

$$Z = 2$$

Unstable

(144)

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$$1) -0.2k = -1$$

$$k_{\text{max}} = 5$$

~~and stable (cugz + 1 is not enclosed in RHS of contour)~~

$$2) -2k = -1$$

$$k_{\text{max}} = \frac{1}{2}$$

$k > \frac{1}{2}$  stable cugz (for same reason)

$$3) -8k = -1$$

$$k_{\text{max}} = \frac{1}{8}$$

$k < \frac{1}{8}$  stable.

$$\frac{1}{2} < k < 5 \text{ if } k < \frac{1}{8} \quad (b)$$

# T46

Effect of adding zeros on the shape of Polar or Nyquist plots.

- For minimum phase or non-minimum phase functions of any type, when zeros are present check for intersection of polar plot with -ve real axis.
- For type 2 & type 3 standard open loop functions when zeros are added before the location of poles then the polar plot intersects the negative real axis as many times as there are zeros. dont consider poles at origin.

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$$-180^\circ - \tan^{-1} \omega - \tan^{-1} 2\omega$$

$\omega = 0$	$ G(j\omega)  = \infty$	$\angle G(j\omega) = -180^\circ$
$\omega = \infty$	$ G(j\omega)  = 0$	$\angle G(j\omega) = -360^\circ$

To map Sy.

$$G(s) = \frac{1}{s^2} = \frac{1}{R^2 e^{j2\theta}} = \infty e^{-j2\theta}$$

$[0 \rightarrow -\pi/2 \rightarrow \pi/2]$

go to  $+90^\circ$  from  $0^\circ$   $\rightarrow \infty e^{j\pi/2} \rightarrow \infty e^{j\pi}$   
 draw the curve  $-2^\circ = 0 - \pi$   
 plot from  $180^\circ$   $\rightarrow$  unstable  
 $\rightarrow$   $180^\circ$  (ad)

$$G(s) = \frac{1 + 4s}{s^2(1+s)(1+2s)}$$

(147)

$$-180^\circ \rightarrow \text{Tan}^+ \omega - \text{Tan}^+ 2\omega + \text{Tan}^+ 4\omega$$

$$\begin{array}{ll} \omega = 0 & |G(j\omega)|_+ = \infty \quad |G(j\omega)|_- = -180^\circ \\ \omega = \infty & |G(j\omega)| = 0 \quad |G(j\omega)| = -270^\circ \end{array}$$

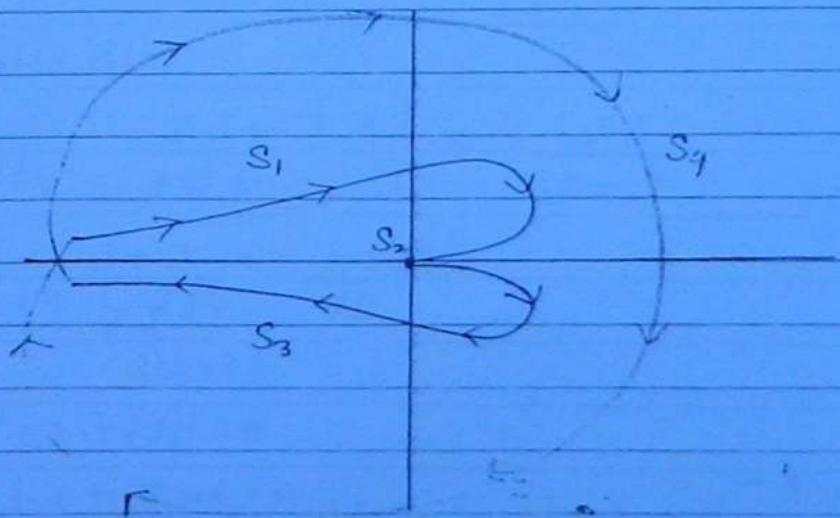
$$-180^\circ - \text{Tan}^+ \omega - \text{Tan}^+ 2\omega + \text{Tan}^+ 4\omega = -180^\circ$$

$$\text{Tan}^+ 4\omega = \text{Tan}^+ \omega + \text{Tan}^+ 2\omega.$$

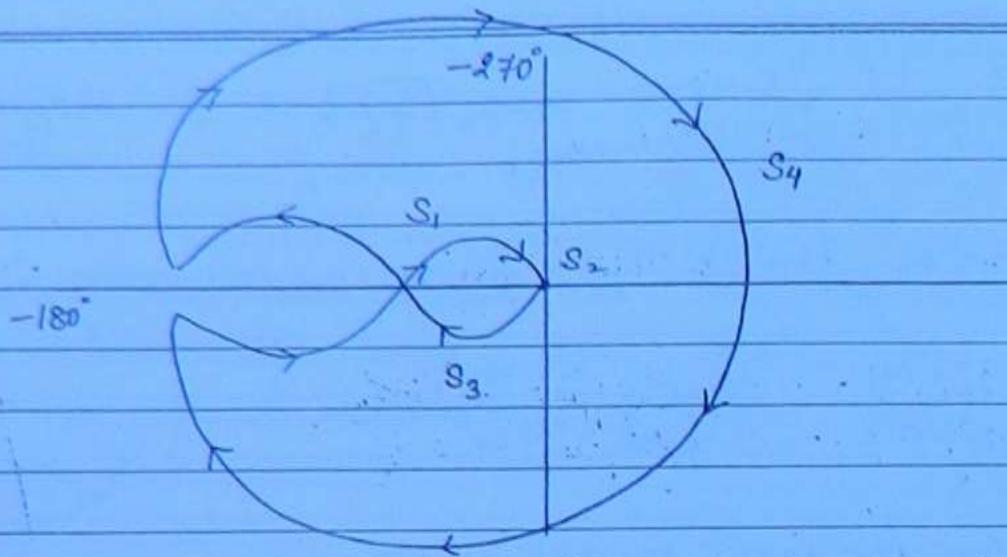
$$4\omega = \frac{\omega + 2\omega}{1 - 2\omega^2}$$

$$4 - 8\omega^2 = 3 \Rightarrow \omega = \omega_{pe} = \frac{1}{\sqrt{8}} \text{ rad/s.}$$

$$X = \frac{\sqrt{1 + (4/\sqrt{8})^2}}{\left(\frac{1}{\sqrt{8}}\right)^2 \sqrt{1 + \left(\frac{1}{\sqrt{8}}\right)^2} \sqrt{1 + \left(\frac{2}{\sqrt{8}}\right)^2}} = 10 \cdot 6$$



$$\text{Nyquist for } G(s) = \frac{1}{s^2(1+s)(1+2s)}$$



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(B)

$$j\omega \left[ 1 + j\omega (T_1 + T_2) - \omega^2 T_1 T_2 \right]$$

$$\Rightarrow \frac{1}{-\omega^2 (T_1 + T_2) + j[\omega - \omega^3 T_1 T_2]}$$

$$\omega - \omega^3 T_1 T_2 = 0$$

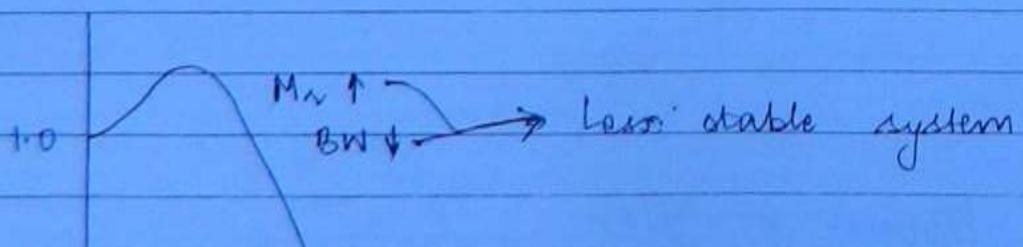
$$1 - \omega^2 T_1 T_2 = 0$$

$$\omega = \omega_{pe} = \frac{1}{\sqrt{T_1 T_2}} \text{ rad/s}$$

$$-\left[ \frac{1}{\sqrt{T_1 T_2}} \right]^2 (T_1 + T_2) \Rightarrow \frac{-T_1 T_2}{T_1 + T_2} \Rightarrow X = \frac{+T_1 T_2}{T_1 + T_2}$$

$$G.M = \frac{1}{X} = \frac{T_1 + T_2}{T_1 T_2}$$

(11)



(9)

$$\% M_p = 50\%$$

$$T = 0.2 \text{ secs}$$

(149)

$$M_p = 0.5$$

$$e^{-g\pi/\sqrt{1-g^2}} = 0.5$$

$$g = 0.215$$

$$fd = \frac{1}{T} \Rightarrow fd = \frac{1}{0.2} = 5 \text{ Hz}$$

$$\omega_d = d\pi f_d$$

$$\omega_d = d\pi (5) = 31.41 \text{ rad/s}$$

$$\omega_d = \omega_n \sqrt{1-g^2}$$

$$31.41 = \omega_n \sqrt{1-(0.215)^2}$$

$$\omega_n = 32.16 \text{ rad/s}$$

$$\omega_n = \omega_n \sqrt{1-2g^2}$$

$$\omega_n = 32.16 \sqrt{1-2(0.215)^2}$$

$$\omega_n = 30.63 \text{ rad/s}$$

(13)

$$G(s) H(s) = \frac{2\sqrt{3}}{s(s+1)}$$

$$PM = 100 \text{ g for order 2 systems}$$

$$1 + \frac{2\sqrt{3}}{s(s+1)} = 0$$

$$s^2 + s + 2\sqrt{3} = 0$$

$$s^2 + s + 3.46 = 0$$

$$\omega_n = \sqrt{3.46} = 1.86 \text{ rad/s}$$

$$2g \times 1.86 = 1$$

$$g = 0.27$$

Q) The value of  $a$  to give  $PM = 45^\circ$   
 $G(s) = \frac{as+1}{s^2}$

(150)

- a)  $\sqrt{2}$    b)  $\frac{1}{1.414}$    c)  $\sqrt{\sqrt{2}}$    d)  $\frac{1}{\sqrt{\sqrt{2}}}$   
 $\frac{1}{\sqrt{2}}$     $\frac{1}{0.707}$     $1.18$     $\frac{1}{0.84}$

sol

$$1 + G(s) = 0$$

$$1 + \frac{as+1}{s^2} = 0$$

$$s^2 + as + 1 = 0$$

$$\omega_n = 1 \text{ rad/s}$$

$$2g \times 1 = a$$

$$g = \frac{a}{2}$$

$$PM = 45^\circ$$

$$45^\circ = 100 g$$

$$g = \frac{45}{100} = 0.45$$

$$g = 0.45 = \frac{a}{2} \quad \underline{a = 0.9}$$

Ans (d)

(14)

$$G(s) = \frac{1}{s(s^2 + s + 1)}$$

$$G(j\omega) = \frac{1}{j\omega(1 - \omega^2 + j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega \sqrt{(1 - \omega^2)^2 + \omega^2}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1} \left( \frac{\omega}{1 - \omega^2} \right)^\circ$$

$$\text{At } \omega = \omega_{pc} = 1 \text{ rad/s}$$

$$\left| G(j\omega) \right|_{\omega=0} = X = \frac{1}{\sqrt{(1-1)^2 + 1^2}} = 1 \quad (15)$$

$$GM = \frac{1}{X} = \frac{1}{1} = 1 \quad GM(\text{db}) = 20 \log 1 = 0 \text{ db.}$$

(17) Poles =  $0.01 \text{ Hz}$  ~~THE 80Hz~~  
 Zeros ~~5Hz~~ ~~100Hz~~ ~~200Hz~~  
 due to dominant pole effect

$$f = 20 \text{ Hz} \rightarrow \omega = 2\pi \times 20 \\ = 125 \text{ rad/s}$$

$$F(s) \propto \frac{1}{1 + Ts} \quad |F(j\omega)| = -\tan^{-1} \omega T$$

At max" value of  $\omega = \infty$ ,  $|F(j\omega)| = -90^\circ$   
 for any higher value of  $\omega$   $|F(j\omega)| \approx -90^\circ$  (a)

$$(18) \quad f(s) = \frac{3e^{-2s}}{s(s+2)}$$

$$G(j\omega) = \frac{3e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3 \times 1}{\omega \sqrt{\omega^2 + 4}}$$

$$|G(j\omega)| = -90^\circ - \tan^{-1} \frac{\omega}{2} - 57.3 \times 2\omega$$

$$\text{At } \omega = \omega_{ge} \\ 3 = 1$$

$$\omega \sqrt{\omega^2 + 4}$$

$$9 = \omega^2 (\omega^2 + 4) \Rightarrow \omega^4 + 4\omega^2 - 9 = 0$$

$$-4 \pm \sqrt{16+36}$$

2

$$-2 \pm 3.6$$

$$1.6, -5.6$$

$$\omega^2 = 1.6$$

$$\omega = \omega_{pc} = \sqrt{1.6} = 1.26 \text{ rad/s}$$

$$|G(j\omega)| \Big|_{\omega = \omega_{pc} = 1.26 \text{ rad/s}} = \phi.$$

$$\phi = -90^\circ - \tan^{-1}\left(\frac{1.26}{2}\right) = -57.3^\circ - 2 \times 1.26$$

$$\phi = -967.5^\circ$$

$$PM = 180^\circ + \phi = 180^\circ - 967.5^\circ = -87.5^\circ$$

Between  $\omega_{pc}$  &  $\omega_{gc}$

$PM + GM$  will be +ve

or  $PM + GM$  will be -ve

or both will be 0

To ~~negative~~ satisfy  $\omega_{pc} > \omega_{gc}$

$$\omega_{pc} = \omega_{gc}$$

$$\omega_{pc} < \omega_{gc}$$

$$PM = -ve$$

$$\therefore GM = -ve \quad (d)$$

To map Polar Plot

(153)

$\omega$	0	$\infty$
$ G(j\omega) $	$\infty$	0
$\angle G(j\omega)$	$-90^\circ$	$-\infty$



To determine stability for systems with dead time, Polar / Nyquist plots is not a good idea for as it cuts -ve Real axis  $\infty$  times so  $X = ?$

Thus find PM ; GM

$$(154) \quad G(s) = \frac{e^{-Ts}}{s(s+1)}$$

$$G(j\omega) = \frac{e^{-j\omega T}}{j\omega(j\omega+1)}$$

$$|G(j\omega)| = \frac{-\omega T}{[\pi/2][\tan^+ \omega]}$$

$$\Rightarrow -\frac{\pi}{2} - \tan^+ \omega - \omega T$$

$$\text{At } \omega = \omega_1, \quad |G(j\omega)| = 0$$

$$\Rightarrow -\frac{\pi}{2} - \omega_1 T - \tan^+ \omega_1 > 0$$

$$-\tan^+ \omega_1 = \frac{\pi}{2} + \omega_1 T$$

$$-\omega_1 = \tan\left[\frac{\pi}{2} + \omega_1 T\right]$$

(20) ABCD are plots of Type & order 3

(154)

$$\text{Phase} = -90 \times 3 \\ = -270^\circ$$

A & B are stable around critical point

So 3, 4 plots have well defined mag? &  
phase so A, B belong to 3, 4

$g \propto 1/\sqrt{k}$  As  $k \uparrow g \downarrow M_N \uparrow$

eg  $s^2 + s + k = 0$

$$\omega_n = \sqrt{k} \propto s$$

$$\omega g \sqrt{k} = 1$$

$$g = \frac{1}{\omega \sqrt{k}}$$

$$g \propto \frac{1}{\sqrt{k}}$$

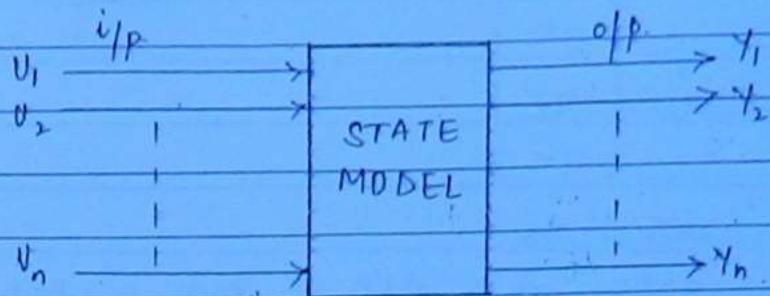
A.  $k \uparrow g \uparrow$  -(4)  
 $M_N \downarrow$

B.  $k \uparrow g \downarrow$  -(3)  
 $M_N \uparrow$

C. Marginally stable -(2)  
→ oscillatory  
→  $g = 0 M_N = \infty$

# STATE SPACE ANALYSIS

(TS)



## 1. STATE EQUATION

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} A & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} B & & & \\ & B_2 & & \\ & & \ddots & \\ & & & B_n \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

## 2. OUTPUT EQUATION

$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} C & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} D & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_n \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

TYPE - I To obtain S.M from differential eq'

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 10u$$

$$(s^3 + 4s^2 + 6s + 8)Y(s) = 10u(s)$$

15B

To define state variables.

$$\text{Let } Y = x_1, \quad \frac{dy}{dt} = \dot{x}_1 = x_2$$

$$\frac{d^2y}{dt^2} = \ddot{x}_1 = x_3 \quad \frac{dy_3}{dt^3} = \ddot{x}_3$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = 10u - 4x_3 - 6x_2 - 8x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -6 & -4 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}}_B u(t)$$

O/p Equation

$$Y = x_1$$

$$Y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(Bush/ Companion form)

CWB chapter 8

$$(8) \quad Y(s) = \underline{1}$$

$$U(s) \quad 1s^4 + 5s^3 + 8s^2 + 6s + 3 \quad \leftarrow \text{no initial condition}$$

coeff of highest order term = 1

$$\text{sg} \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u(s)$$

$$Y = [1 \ 0 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

(157)

(q)  $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = 2\frac{du}{dt} + u$

$$(s^2 + 7s + 9) Y(s) = (2s+1) U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{2s+1}{s^2 + 7s + 9}$$

### Phase Variable Method

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 7s + 9} \xrightarrow{\text{reverse order}} (as+1)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Type 2 To obtain T.F from State Model

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Applying L.T.

$$sX(s) - x(0) = Ax(s) + Bu(s)$$

$$Y(s) = CX(s) + DU(s)$$

For T.F.  $X(0) = 0$

$$sX(s) - AX(s) = BV(s)$$

$$X(s)[sI - A] = BV(s)$$

$$X(s) = [sI - A]^{-1} \cdot BV(s)$$

$$Y(s) = \{C[sI - A]^{-1}B + D\}V(s)$$

$$Y(s) = C[sI - A]^{-1}B + D$$

$V(s)$

Transfer Matrix

$$\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}X + \begin{bmatrix} 1 \\ 0 \end{bmatrix}V \quad Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T X$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}$$

$$\text{Adj}(sI - A) = \begin{bmatrix} (s+2) & 0 \\ 0 & (s+1) \end{bmatrix}$$

$$|sI - A| \Rightarrow (s+1)(s+2)$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$(sI - A)^{-1} \cdot B = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

$$C[sI - A]^{-1}B \Rightarrow \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

(3)

$$\ddot{x}(t) = -2x(t) + 2u(t)$$

$$y(t) = 0.5x(t)$$

159

sq

$$s^2x(s) = -2x(s) + 2u(s)$$

$$y(s) = 0.5x(s)$$

$$(s+2)x(s) = 2u(s)$$

$$x(s) = \frac{2u(s)}{s+2}$$

$$y(s) = \frac{0.5 \times 2}{s+2} u(s)$$

$$\boxed{\frac{y(s)}{u(s)} = \frac{1}{s+2}}$$

TYPE - 3 Stability for S.M.

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$$\Rightarrow C \cdot \frac{\text{Adj}(sI - A)}{|sI - A|} B + D$$

$$\Rightarrow \frac{C \cdot \text{Adj}(sI - A) B + |sI - A| D}{|sI - A|}$$

$$\star \text{ Zeros} = C \cdot \text{Adj}(sI - A) B + |sI - A| D = 0$$

$$\star \text{ Poles } (1 + G(s) H(s)) = 0 \Leftrightarrow |sI - A| = 0 \\ \Rightarrow \text{char eq} = |sI - A| = 0$$

EIGEN VALUES OF  $\star = C \cdot L$  poles  
SYSTEM MATRIX [A]

CONV 2.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$C = [-17 \quad -5]x + [1]u$$

(168)

$$\begin{aligned} sI - A &= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} \\ &= \begin{bmatrix} s & -1 \\ 20 & s+9 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Adj}(sI - A) &= \begin{bmatrix} s+9 & -20 \\ 1 & s \end{bmatrix} \\ &= \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix} \end{aligned}$$

$$\text{Adj}(sI - A)B = \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$\text{c. } \text{Adj}(sI - A)B \Rightarrow \begin{bmatrix} -17 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} = -17 - 5s$$

$$|sI - A| \Rightarrow s(s+9) + 20$$

$$|sI - A| D \Rightarrow [s^2 + 9s + 20] [1]$$

$$\text{Zero} = -17 - 5s + s^2 + 9s + 20 = 0$$

$$s^2 + 4s + 3 = 0$$

$$-1, -3$$

$$\text{Poles} = |sI - A| = 0$$

$$s^2 + 9s + 20 = 0$$

$$-4, -5$$

$$(7) \quad \ddot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v \quad (61)$$

$$v = [-0.5 \quad -3 \quad -5]x + v.$$

$$\ddot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [0.5 \quad -3 \quad -5]x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\ddot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & -3 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} v$$

$$\ddot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -2 & -3 \end{bmatrix}}_A x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_B v$$

$$\begin{aligned} sI - A &\Rightarrow \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 2 & s+3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} |sI - A| &= s[s(s+3)+2] + 1[0] + 0 = 0 \\ &= s[s^2 + 3s + 2] = 0 \end{aligned}$$

$$0, -1, -2 \quad \text{Ans.}$$

Matrizenmultiplikation

TYPE 4 -

(162)

STATE CONTROLLABILITY -

To control the state variables

$$\Rightarrow Q_C = [B \quad AB \quad A^T B \dots \quad A^{n-1} B]$$

STATE OBSERVABILITY -

To measure the state variables:

$$\Rightarrow Q_O = [C^T \quad A^T C^T \quad (A^T)^2 C^T \dots \quad (A^T)^{n-1} C^T]$$

$$|Q_C| \neq 0 \quad |Q_O| \neq 0$$

KALMAN'S TEST

i)  $\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix}x$$

$$Q_C = [B \quad AB]$$

$$\tilde{AB} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} = -1 \quad \text{controllable}$$

$$Q_O = [C^T \quad A^T C^T]$$

$$A^T C^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad /$$

$$Q_O = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} = -4 \quad \text{observable}$$

(1D)

$$Q_0 = [C^T \ A^T C^T]$$

(153)

$$A^T C^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} b & b \\ 0 & \alpha b \end{bmatrix} = \alpha b^2 \neq 0$$

Ans (c)

### TYPE - 5 Solution of State Equation

$$\ddot{x}(t) = Ax(t) + Bu(t)$$

a) Free response  $[u(t) = 0]$

$$\ddot{x}(t) = Ax(t)$$

$$\ddot{x}(t) - Ax(t) = 0 \quad \dots \textcircled{1}$$

$$x(t) = Ke^{At}$$

Applying L.T to equation  $\textcircled{1}$

$$sX(s) - x(0) - Ax(s) = 0$$

$$sX(s) - Ax(s) = x(0)$$

$$X(s)[sI - A] = x(0)$$

$$X(s) = (sI - A)^{-1} x(0)$$

$\phi(s) = (sI - A)^{-1}$  = Resolvent Matrix

$$X(s) = \left\{ L^{-1} (sI - A)^{-1} \right\} x(0)$$

$$X(t) = e^{At} K$$

b) Forced Response -

$$sX(s) - X(0) = AX(s) + BV(s)$$

(164)

$$sX(s) - AX(s) = X(0) + BV(s)$$

$$X(s)[sI - A] = X(0) + BV(s)$$

$$X(s) = [sI - A]^{-1} X(0) + (sI - A)^{-1} BV(s)$$

$$x(t) = \{ L^{-1}(sI - A)^{-1}\} X(0) + L^{-1}[(sI - A)^{-1} BV(s)]$$

(4)

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$\text{Adj}(sI - A) = \begin{bmatrix} s-1 & 1 \\ 0 & s-1 \end{bmatrix}$$
$$= \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$|sI - A| = (s-1)^2$$

$$(sI - A)^{-1} = \phi(s) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

$$\text{S.T.M } \phi(t) \quad \phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

Property of S.T.M

$$\text{At } t=0 \quad e^{A0} = \phi(0) = I$$

$$X(t) = \begin{bmatrix} e^t & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow X(t) = \begin{bmatrix} e^t \end{bmatrix} \text{ Ans.}$$

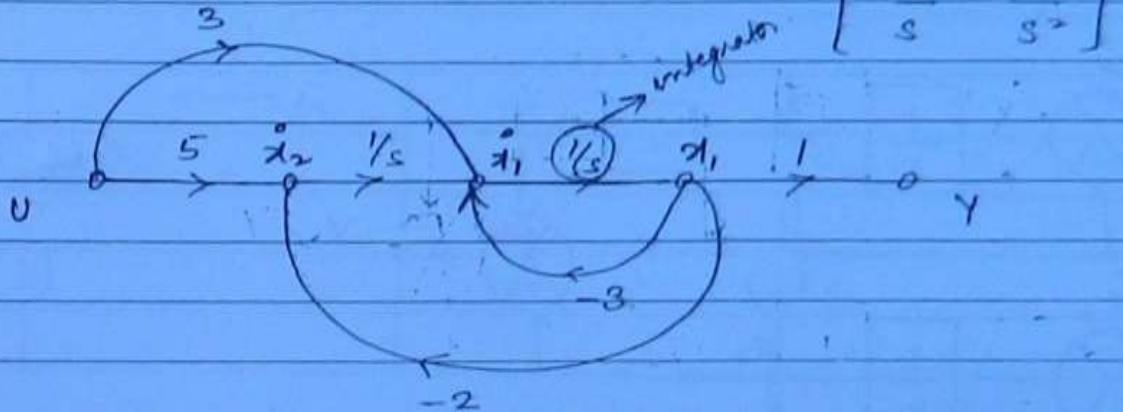
# TYPE 6 - STATE DIAGRAMS

(P.S.)

## 1. Observable canonical form

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2 + 3s + 2} = \frac{3}{s} + \frac{5}{s^2 + 3s + 2}$$

$$1 - \left[ \frac{-3 - 2}{s \quad s^2} \right]$$



$$\dot{x}_1 = -3x_1 + x_2 + 3u$$

$$\dot{x}_2 = -2x_1 + 5u$$

$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

## 2. Controllable canonical form

$$\frac{Y(s)}{U(s)} = \frac{2}{s+1} + \frac{1}{s+2}$$

$$y(s) = \frac{2}{s+1} u(s) + \frac{1}{s+2} u(s)$$

$$U(s) \rightarrow \begin{array}{|c|} \hline 2 \\ \hline s+1 \\ \hline \end{array} \rightarrow Y(s) = \frac{2}{s+1}$$

$$G(s) = \frac{1}{1 + H(s)G(s)} = \frac{1}{s+1}$$

$$\begin{aligned} G(s) &= \frac{1}{1 + H(s)Y(s)} = \frac{1}{s+2} \\ \frac{1 + HG}{G} &= s+2 \end{aligned}$$

$$G(s) = \frac{N}{D-N} = \frac{1}{s+1-1} = \frac{1}{s}$$

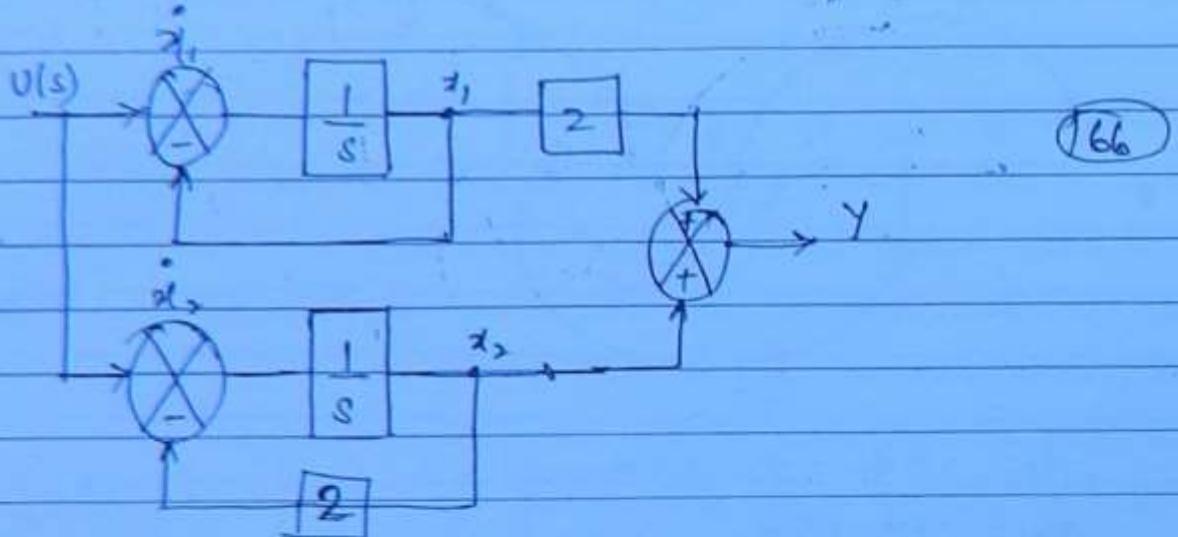
$$G(s) = \frac{1}{s}, H(s) = 1$$

$$1 = G(s+2-1)$$

$$G = \frac{1}{s+2-1}$$

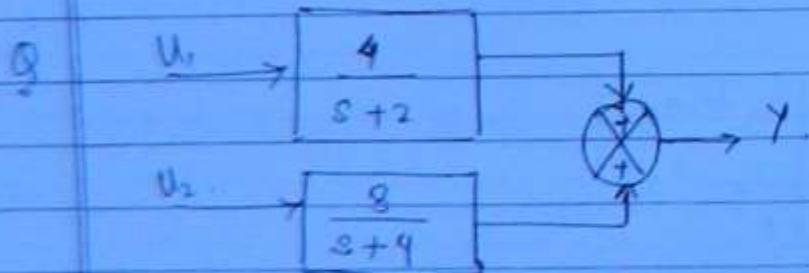
$$\text{Put } H=2$$

$$G = \frac{1}{s}$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [2 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

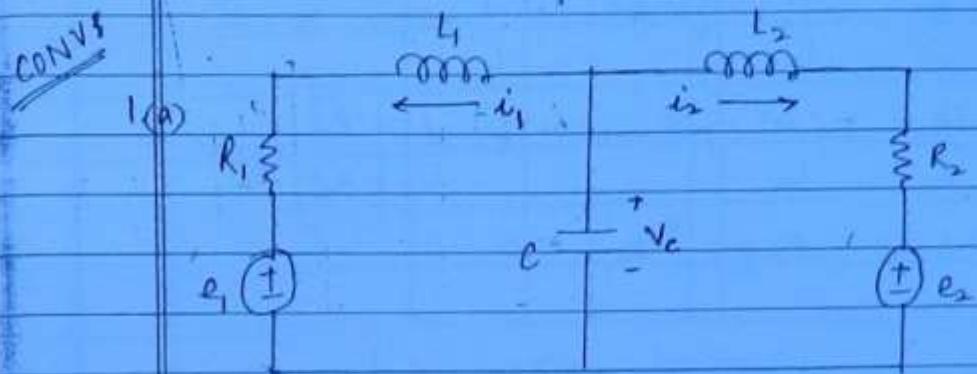
No. of states = No. of energy storing elements.

12. No. of energy storing elements = 3.

(67)

Since  $C_1$  &  $C_2$  are in ||

Initially they will have same v/f across them  
So min. no. of states = 2.



Only Inductive v/f/n  $\rightarrow$  KVL

Only Capacitive v/f/n  $\rightarrow$  KCL

Both  $\rightarrow$  Apply B.H.

KVL in loop ①

$$\frac{di_1}{dt} + e_1 R_1 + e_1 - \frac{V_c}{L_1} = 0$$

$$\frac{di_1}{dt} = -\frac{e_1}{L_1} R_1 + \frac{e_1}{L_1} + \frac{V_c}{L_1} \Rightarrow \textcircled{1}$$

KVL to loop ②

$$\frac{di_2}{dt} + e_2 R_2 + e_2 - \frac{V_c}{L_2} = 0$$

$$\frac{di_2}{dt} = -\frac{e_2}{L_2} R_2 + \frac{e_2}{L_2} + \frac{V_c}{L_2} \Rightarrow \textcircled{2}$$

KCL at node  $V_C$ :

$$i_1 + i_2 + C \frac{dV_C}{dt} = 0$$

(168)

$$\frac{dV_C}{dt} = -\frac{i_1}{C} - \frac{i_2}{C} \quad \rightarrow (3)$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} -R_1 & 0 & \frac{1}{L} \\ 0 & -R_2 & \frac{1}{L_2} \\ -\frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_C \end{bmatrix} + \begin{bmatrix} -\frac{1}{L} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

(b)

$$\textcircled{1} \quad L \frac{di_L}{dt} = \frac{1}{C} V_C$$

$$i(t) = i_L + i_C$$

$$i(t) = i_L + C \frac{dV_C}{dt}$$

$$\frac{dV_C}{dt} = -\frac{i_L}{C} + i(t) \quad \rightarrow \textcircled{1}$$

$$L \frac{di_L}{dt} - V_C = 0 \Rightarrow \frac{di_L}{dt} = \frac{V_C}{L} \quad \textcircled{2}$$

$$\begin{bmatrix} \frac{dV_C}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/C \\ 1/L & 0 \end{bmatrix} \begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} 1/C \\ 0 \end{bmatrix} i(t)$$

# BODE PLOTS

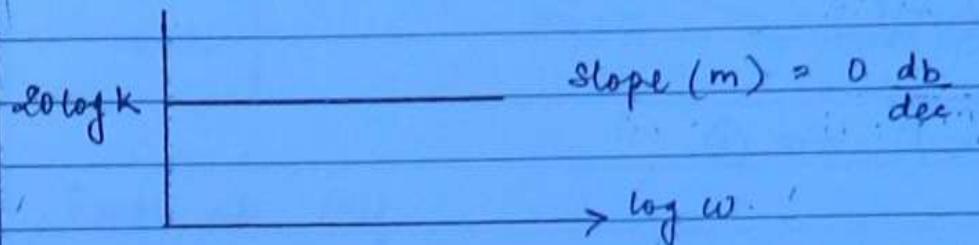
(169)

## 1. System Gain ( $\pm K$ )

$$F(j\omega) = \pm K + j0$$

$$|F(j\omega)| = \sqrt{(\pm K)^2 + 0^2} = K$$

Its db value is  $20 \log K$ .



## 2. Internal / Derivative factors

[Poles / Zeros at origin]

$$(S)^{\pm n}$$

$$F(j\omega) = (j\omega)^{\pm n} = (0 + j\omega)^{\pm n}$$

$$|F(j\omega)| = \left[ \sqrt{0^2 + \omega^2} \right]^{\pm n} = [\omega]^{\pm n}$$

$$\text{Its db value} = 20 \log (\omega)^{\pm n}$$

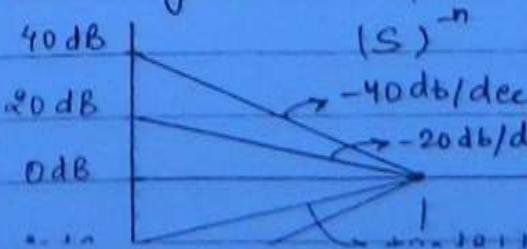
$$= \pm 20n \log \omega = \frac{1}{M} 20n \log \omega$$

$$\text{Slope (m)} = \pm 20n \frac{\text{db}}{\text{dec}}$$

$$\pm 20n \log \omega = 0 \text{ db}$$

$$\log \omega = 0$$

$$\omega = \log^{-1} 0 = 1 \text{ rad/s}$$



### 3 First Order Factors -

$$F(j\omega) = (1 \pm j\omega T)^{\pm 1}$$

(176)

$$|F(j\omega)| = [\sqrt{1 + (\omega T)^2}]^{\pm 1}$$

at db value

$$20 \log [\sqrt{1 + (\omega T)^2}]^{\pm 1}$$

$$\pm 20 \log \sqrt{1 + (\omega T)^2} \rightarrow ①$$

#### Asymptotic Assumptions -

Case 1 → LOW FREQ

$$1 \gg (\omega T)^2$$

$$1 20 \log \sqrt{1} = 0 \text{ db}$$

Case 2 → HIGH FREQ

$$(\omega T)^2 \gg 1$$

$$1 20 \log \sqrt{(\omega T)^2}$$

$$\pm 20 \log \omega T \rightarrow ②$$

$$\Rightarrow 1 20 \log \omega \pm 20 \log T$$

[M X + C]

$$\text{Slope } (m) = \pm 20 \text{ db/sec}$$

Corner frequency ( $\omega_c$ )

$$0 = 1 20 \log \omega T$$

$$\log \omega T = 0 \rightarrow \omega T = \log^{-1}(0) = 1$$

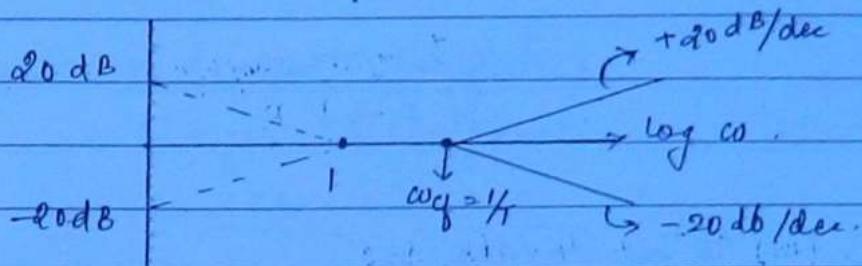
eg

$$[s \pm j\omega]^{\pm 1}$$

$$[1 \pm s/T]^{\pm 1}$$

(71)

$$T = 1/2 \rightarrow \omega_q = \frac{1}{T} = 2 \text{ rad/s.}$$



Error at  $\omega_q$

$$\text{At } \omega = \omega_q = \frac{1}{T}$$

$$\pm 20 \log \sqrt{1 + \left(\frac{1}{T} \times T\right)^2}$$

$$\pm 20 \log \sqrt{2} = \pm 3 \text{ dB}$$

#### 4. Quadratic factors -

$$(s^2 + \alpha_0 \omega_n s + \omega_n^2)^{\pm 1}$$
$$\left[ \frac{s^2}{\omega_n^2} + \frac{\alpha_0 s}{\omega_n} + 1 \right]^{\pm 1}$$

Put  $s = j\omega$

$$\left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 + j^2 \frac{\omega}{\omega_n} \right]^{\pm 1}$$

db value. Magnitude

$$20 \log \sqrt{\left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 \right]^2 + \left[ 2G \frac{\omega}{\omega_n} \right]^2}$$

(172)

↓  
0db  
L.F.R.

↓  
I 40 log  $\omega/\omega_n$ .  
H.F.R.

$$\text{Slope } (m) = I 40 \log \text{db/dec.}$$

→ Corner freq

$$\omega_{cf} = \omega_n \text{ r/s.}$$

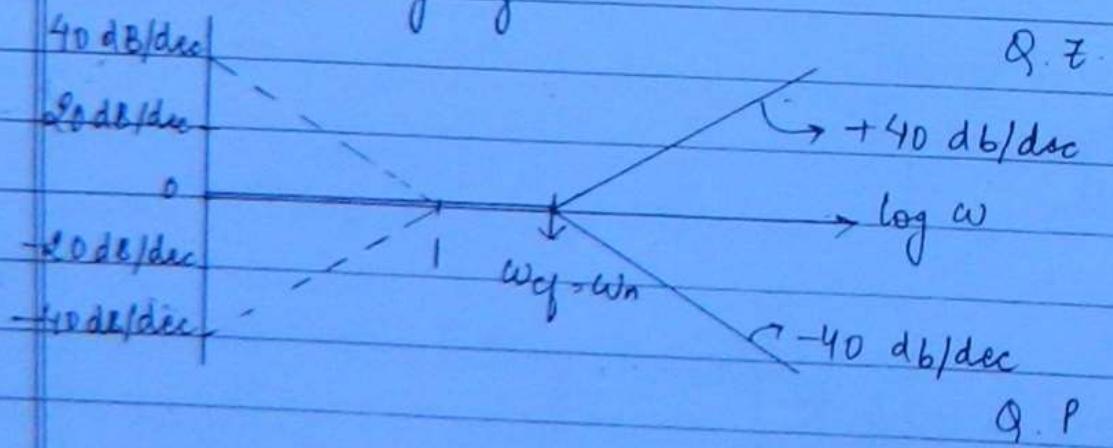
$$(s^2 + 4s + 25)^{1/2}$$

$$\omega_n^2 = 25$$

$$\omega_{cf} = \omega_n = 5 \text{ r/s.}$$

→ Error at corner freq  $\omega_{cf}$

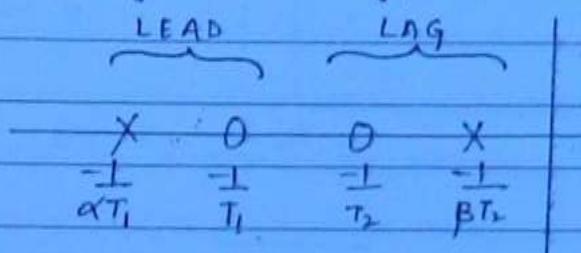
$$I 40 \log 2G$$



# BODE PLOT FOR LAG LEAD OR LEAD LAG COMPENSATOR

$$F(s) = \frac{N_0(s)}{V_{in}(s)} = \frac{\alpha (1 + \tau_1 s)(1 + \tau_2 s)}{(1 + \alpha \tau_1 s)(1 + \beta \tau_2 s)} \quad (173)$$

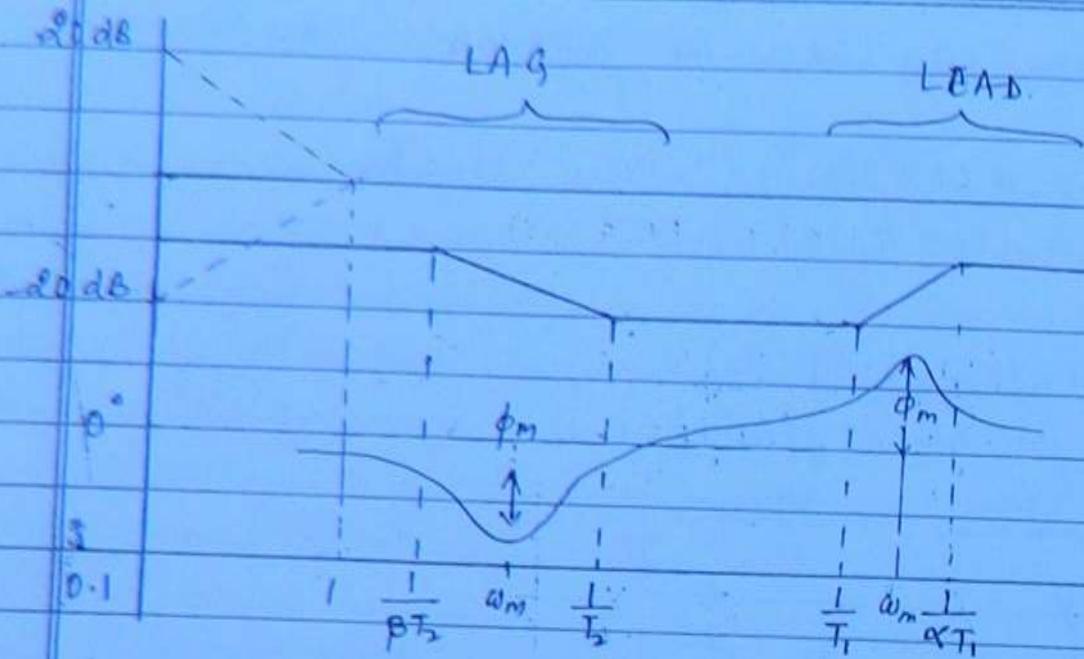
$$F(j\omega) = \frac{\alpha (1 + j\omega\tau_1)(1 + j\omega\tau_2)}{(1 + j\omega\alpha\tau_1)(1 + j\omega\beta\tau_2)}$$



$$\angle F(j\omega) = \tan^{-1}\omega\tau_1 - \tan^{-1}\omega\alpha\tau_1 + \tan^{-1}\omega\tau_2 = \tan^{-1}\omega\beta\tau_2.$$

MAG<sup>n</sup> TABLE

Factor	c.f.	MAG <sup>n</sup>
gain	$K = \alpha$	$20 \log \alpha$
$P \text{ on } Z$ at origin $(j\omega)^{in}$	-	Nil
$\frac{c.w}{\text{Breaks}} \text{ in ascending order}$	$1, \frac{1}{\beta\tau_2}, \frac{1}{\tau_2}, \frac{1}{\tau_1}$	$-20 \text{ dB dec}$ $+20 \text{ dB dec}$ $+20 \text{ dB dec}$
$1 + j\omega\tau_1$	$\frac{1}{\tau_1}$	$(+20)$



(174)

$$G(j\omega) = \frac{1}{\sqrt{\beta T_2}} \times \frac{1}{T_2} = \frac{1}{T_2 \sqrt{\beta}}$$

$$\omega_m = \sqrt{\frac{1}{T_1} \times \frac{1}{\alpha T_1}} = \frac{1}{T_1 \sqrt{\alpha}}$$

Characteristics of Phase Lead Compensator -

1. It improves transient state characteristics of the system
2. In terms of filtering property, it's a high pass filter
3. In terms of controllers it is similar to P+D controller.
4. Lead compensation contributes phase lead, so response will be faster & rise time will  $\downarrow$  if BW  $\uparrow$
5. It increases the stability of a system by adding a

$$P = f(e)$$

1. Proportional Mode

$$P \propto e$$

$$P = K_p e$$

$K_p$  = proportional gain

(175)

2. Integral Mode (or) Reset Mode

$$\frac{dP}{dt} \propto e$$

$$\frac{dt}{dt}$$

$$\frac{dP}{dt} = K_I e$$

$K_I$  = integral scaling.

$$\text{Kofas } P = K_I \int e dt$$

Defining Reset Time

$$T_I = 1 \quad P = \frac{1}{T_I} \int e dt$$

3. Derivative (or) Rate Mode

$$P \propto \frac{de}{dt}$$

$$P = K_D \frac{de}{dt}$$

$K_D$  = Rate constant

Defining "Rate Time"

$$T_d = K_D$$

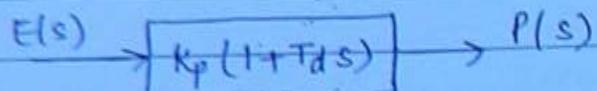
$$P = T_d \frac{de}{dt}$$

# P + D Controller

$$P = K_p e + K_p T_d \frac{de}{dt}$$

(176)

$$P(s) = \left\{ K_p (1 + T_d s) \right\} E(s)$$

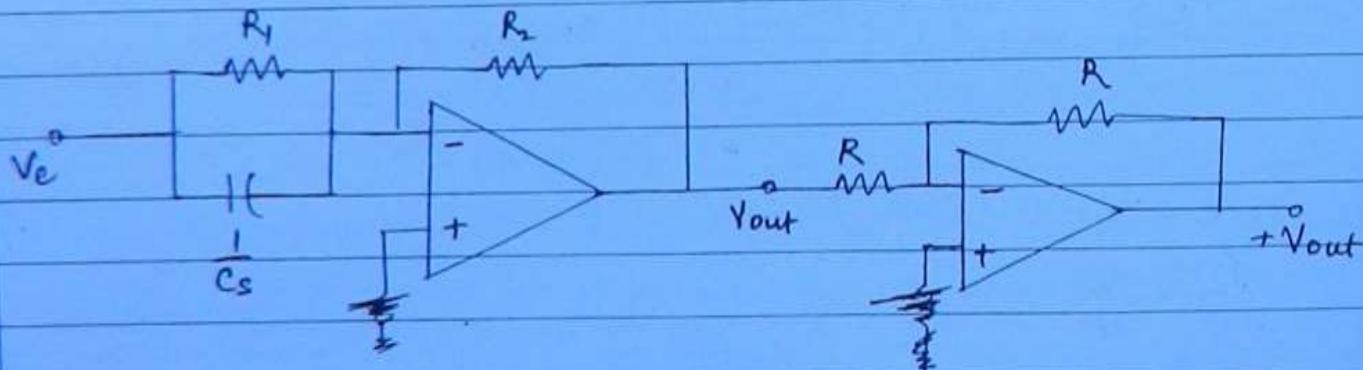


$$\text{Let } e = \sin \omega t$$

$$P = K_p \sin \omega t + K_p T_d \frac{d \sin \omega t}{dt}$$

$$P = K_p \sin \omega t + \omega K_p T_d \cos \omega t$$

$$P = \sqrt{(K_p)^2 + (\omega K_p T_d)^2} \sin [\omega t + \tan^{-1} \omega T_d]$$



$$\frac{V_e - V_L}{R_1} = \frac{V_L - V_{out}}{R_2}$$

$$R_1 C_s + 1$$

$$\Rightarrow \frac{V_e [R_1 C_s + 1]}{R_1} \cdot R_2 = -V_{out}$$

$$\Rightarrow -V_{out} = R_2 V_e + R_2 \frac{R_1 C_s}{R_1} V_e$$

$$\Rightarrow +V_{out} = R_2 V_e + R_2 \frac{R_1 C_s}{R_1} V_e$$

$$G(s) = \frac{K}{s(T_s + 1)} [K_p + K_d s]$$

1) Without P+D controller

(17)

$$G(s) = \frac{K}{s(T_s + 1)}$$

Type 1 / order 2

with P+D controller

$$G(s) = \frac{K}{s(T_s + 1)} [K_p + K_d s]$$

Type 1 / order 2

2. With P controller only

$$1 + \frac{KK_p}{s(T_s + 1)} = 0$$

$$Ts^2 + s + KK_p = 0$$

$$\frac{s^2}{T} + \frac{1}{T}s + \frac{KK_p}{T} = 0$$

$$\omega_n = \sqrt{\frac{KK_p}{T}} \text{ r/s}$$

$$\frac{g}{\sqrt{T}} \sqrt{\frac{KK_p}{T}} = \frac{1}{T}$$

$$g = \frac{1}{2\sqrt{KK_p T}}$$

3. With P+D controller

$$1 + \frac{K}{s(T_s + 1)} [K_p + K_d s] = 0$$

$$Ts^2 + s + KK_d s + KK_p = 0$$

$$\frac{s^2}{T} + \frac{s(1+KK_d)}{T} + \frac{KK_p}{T} = 0$$

$$\omega_n = \sqrt{KK_d} \text{ r/s}$$

$$\frac{dG}{dT} \frac{KK_p}{T} = \frac{1+KK_D}{T}$$

$$\frac{4G^2 KK_p}{T'} = (1+KK_D)^2$$

$$G = \frac{1+KK_D}{d\sqrt{KK_p T}}$$

(178)

contd...

6. The Phase Lead Compensator shifts gain crossover frequency to higher values where the designed phase margin is acceptable. Hence, it is effective when the slope of the uncompensated system near the gain crossover frequency is low.

7. The maximum phase lead occurs at geometric mean of the 2 corner frequencies

$$|F(j\omega)| = \phi = \tan^{-1}\omega T - \tan^{-1}\omega \alpha T$$

$$\tan \phi = \tan [\tan^{-1}\omega T - \tan^{-1}\omega \alpha T]$$

$$\tan \phi = \frac{\omega T - \omega \alpha T}{1 + (\omega T)^2 \cdot \alpha} \Rightarrow \frac{\omega T [1-\alpha]}{1 + [\omega T]^2 \cdot \alpha}$$

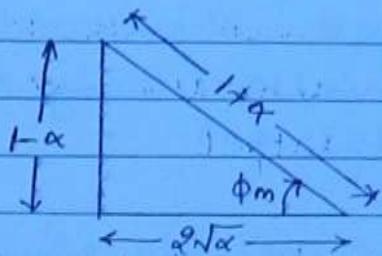
$$\text{At } \omega = \omega_m = \frac{1}{T\sqrt{\alpha}} \quad \phi = \phi_m$$

not realizable  
since  
real  
freq  
is  
not  
real

$$\tan \phi_m = \frac{1}{T\sqrt{\alpha}} \cdot \frac{T\sqrt{1-\alpha}}{1 + \frac{1}{T\sqrt{\alpha}} \cdot T\sqrt{1-\alpha}^2 \cdot \alpha} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\phi_m = \tan^{-1} \left[ \frac{1-\alpha}{\alpha \sqrt{\alpha}} \right]$$

(79)



$$\sin \phi_m = \frac{1-\alpha}{\sqrt{1-\alpha^2}}$$

$$\phi_m = \sin^{-1} \left[ \frac{1-\alpha}{\sqrt{1-\alpha^2}} \right]$$

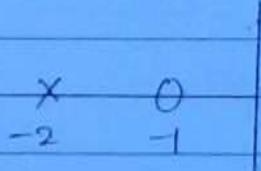
$$\sin \phi_m (t+\alpha) = 1-\alpha$$

$$\alpha \sin \phi_m + \alpha = 1 - \sin \phi_m$$

$$\alpha (1 + \sin \phi_m) = 1 - \sin \phi_m$$

$$\alpha = \frac{1 - \sin \phi_m}{1 + \sin \phi_m}$$

Polar Plot



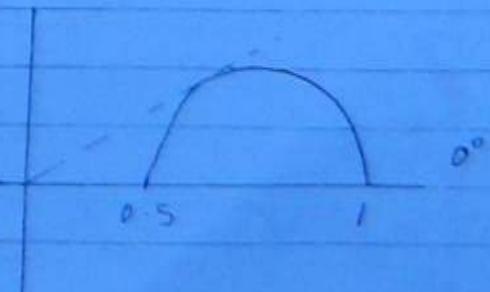
$$F(s) = \frac{s+1}{s+2} = \frac{0.5(1+s)}{(1+0.5s)}$$

$$F(j\omega) = \frac{0.5(1+j\omega)}{(1+0.5j\omega)}$$

$$|F(j\omega)| = \frac{0.5 \sqrt{1+\omega^2}}{\sqrt{1+(0.5\omega)^2}}$$

$$\angle F(j\omega) = \tan^{-1} \omega - \tan^{-1} 0.5\omega$$

$\omega$	0	$\infty$
$ G(j\omega) $	0.5	1
$\angle G(j\omega)$	0°	0°



## Characteristics of Phase Lag Compensator.

(180)

1. It improves the steady state characteristics of the system only. i.e. it eliminates steady state error between input & output.
2. In terms of filtering property it is similar to low pass filter.
3. It shifts the gain crossover frequency to lower values.
4. The maximum phase lag occurs at the geometric mean of the two corner frequencies.

$$LF(j\omega) = \phi = \tan^{-1}\omega T - \tan^{-1}\omega_p T$$

$$\tan \phi = \tan [\tan^{-1}\omega T - \tan^{-1}\omega_p T]$$

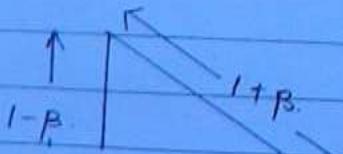
$$\tan \phi = \frac{\omega T - \omega_p T}{1 + (\omega T)^2 \beta} \Rightarrow \frac{\omega T [1 - \beta]}{1 + (\omega T)^2 \beta}$$

$$\text{At } \omega = \omega_m = \frac{1}{T\sqrt{\beta}} \quad \phi = \phi_m$$

$$\tan \phi_m = \frac{1}{T\sqrt{\beta}} T [1 - \beta] = \frac{1 - \beta}{2\sqrt{\beta}}$$

$$1 + \left[ \frac{1 - \beta}{T\sqrt{\beta}} \right]^2 \beta$$

$$\phi_m = \tan^{-1} \frac{1 - \beta}{2\sqrt{\beta}}$$



minimum phase  
lag angle  
with  
time

$$\sin \phi_M = \frac{1-\beta}{1+\beta} \quad (18)$$

$$\phi_M = \tan^{-1} \frac{1-\beta}{1+\beta}$$

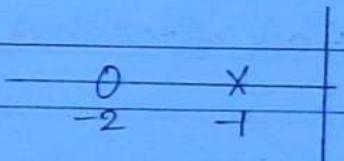
$$\sin \phi_M (1+\beta) = 1-\beta$$

$$\beta \sin \phi_M + \beta = 1 - \sin \phi_M$$

$$\beta (1 + \sin \phi_M) = 1 - \sin \phi_M.$$

$$\boxed{\beta = \frac{1 - \sin \phi_M}{1 + \sin \phi_M}}$$

Polar Plot



$$F(s) = \frac{s+2}{s+1}$$

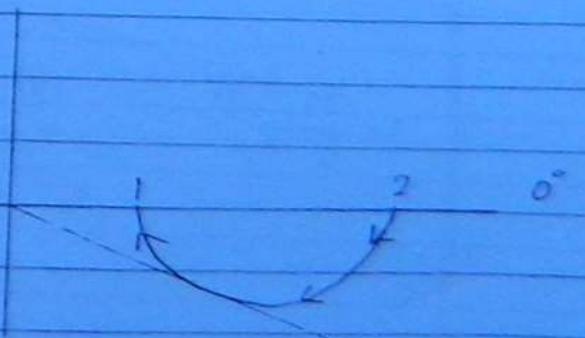
$$= 2 \left[ \frac{1+0.5s}{1+s} \right]$$

$$F(j\omega) = 2 \left( 1 + 0.5j\omega \right)$$

$$|F(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = \tan^{-1} 0.5\omega - \tan^{-1} \omega$$

$\omega$	0	$\infty$
$ F(j\omega) $	2	1
$\angle F(j\omega)$	$0^\circ$	$0^\circ$



5. It's similar to Proportional plus Integral controller

P+I controller

(182)

$$P = K_p e + K_i \int e dt$$

$$P(s) = \left\{ K_p \left[ 1 + \frac{1}{T_i s} \right] \right\} E(s)$$

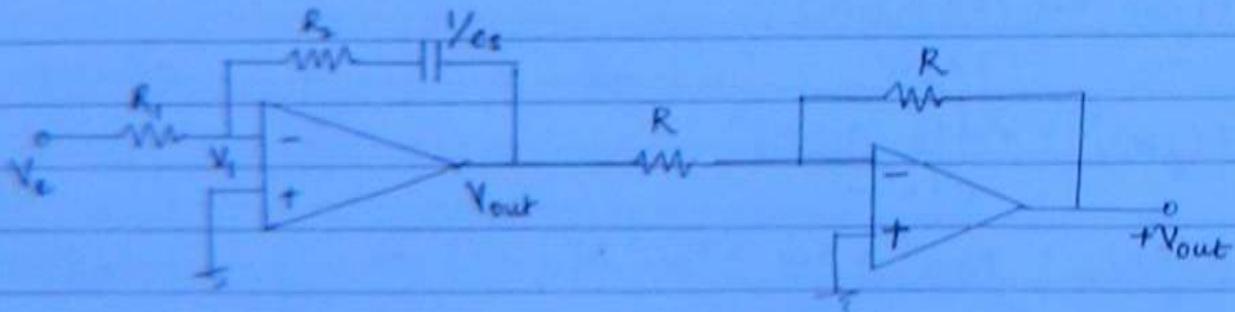
$$E(s) \xrightarrow{K_p \left( 1 + \frac{1}{T_i s} \right)} P(s)$$

→ Let  $e = \sin \omega t$

$$P = K_p \sin \omega t + K_p \int \sin \omega t dt$$

$$P = K_p \sin \omega t + \left( -\frac{K_p}{\omega T_i} \right) \cos \omega t$$

$$P = \sqrt{\left( K_p \right)^2 + \left( \frac{K_p}{\omega T_i} \right)^2} \sin \left( \omega t - \tan^{-1} \frac{1}{\omega T_i} \right)$$



$$\frac{V_e - V_s}{R_f} = \frac{V_i - V_{out}}{R_f C_f + 1}$$

$$-\frac{V_{out}}{R_1 C s} = \frac{V_e [R_2 C s + 1]}{R_1 C s} + \frac{V_e}{R_1 C s} = \frac{R_2 V_e}{R_1 C s} + \frac{V_e}{R_1 C s}$$

$$+ V_{out} = \frac{R_2}{R_1} V_e + R_2 \cdot \frac{1}{R_2 C} \int V_e dt \quad (183)$$

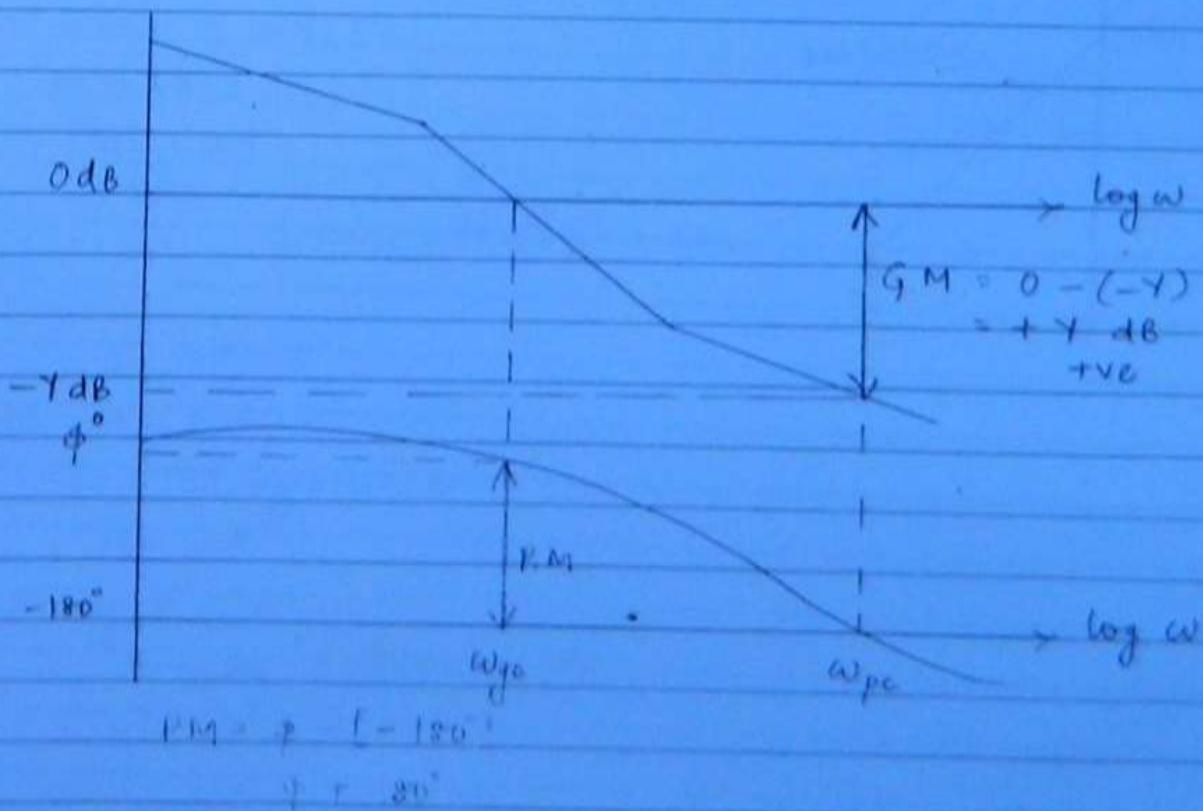
$K_p = \frac{R_2}{R_1}$	$T_i = R_2 C$
-------------------------	---------------

6. It increases rise time. It reduces B.W.

7. It reduces stability. (Addition of pole)

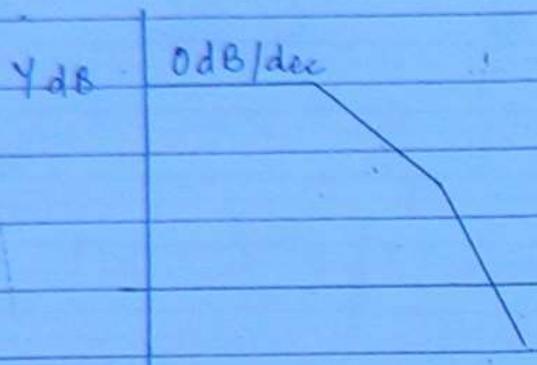
8. A PI controller increases the type & order of the system by 1.

TO FIND GM & PM FROM BODE PLOT.



TO FIND STATIC ERROR CONSTANTS FROM  
BODE MAGNITUDE PLOT -

(184)

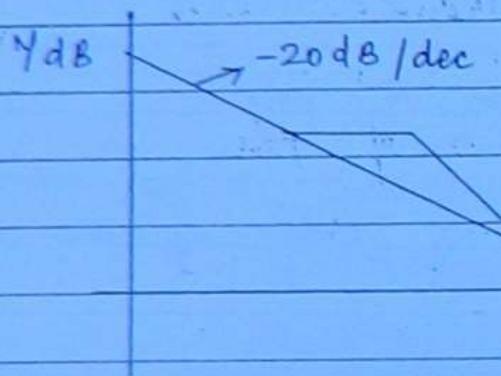


TYPE 0 SYSTEM

$$K_V = K_A = 0$$

$$\alpha_0 \log K_p = Y$$

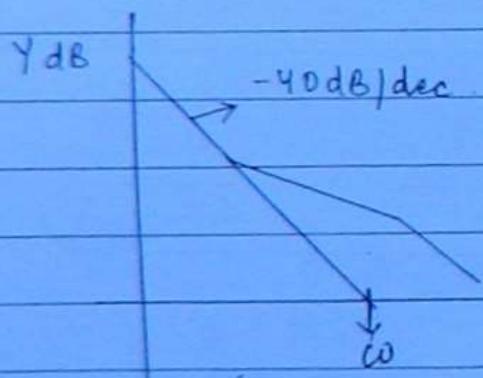
$$K_p = 10^{\log^{-1}[Y/20]}$$



TYPE -1 SYSTEM

$$K_p = \infty, K_A = 0$$

$$K_V = \omega$$



TYPE -2 SYSTEM

$$K_p = K_V = \infty$$

$$K_A = \omega^2$$

# INVERSE BODE PLOT -

(185)

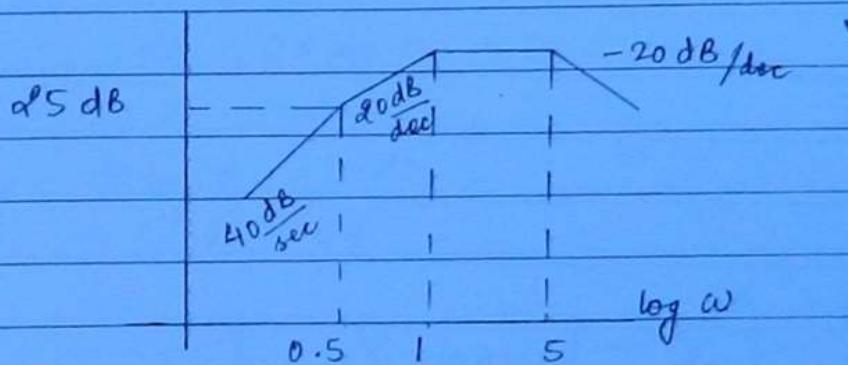
1. Observe the starting slope. This will give the information of poles or zeros occurring at origin.

d. From the starting slope write the equation

$$Y = MX + C$$

$$C = \alpha_0 \log K$$

3. At every corner frequency, observe the changing slope change in slope. This will give the information of first or higher order factors.



$$G(s) = \frac{70(s^2)}{(1+2s)(1+s)(1+0.2s)}$$

$$Y = MX + C$$

$$\text{At } w = 0.5$$

$$\alpha_0 = 40 \log 0.5 + C$$

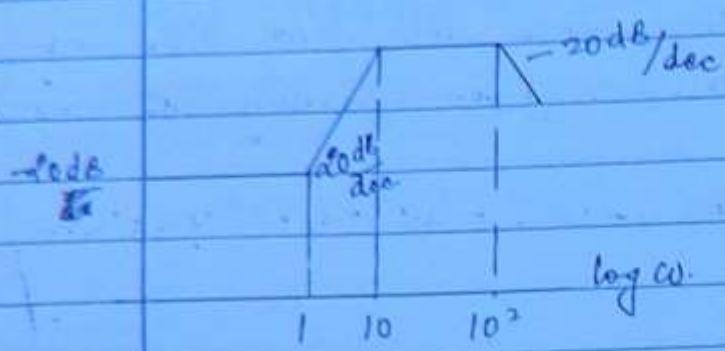
$$C = 37$$

$$\alpha_0 \log K = 37$$

$$K = \log^{-1}(37/\alpha_0) = 70$$

# Chapter 6

16



(186)

$$-G(s) = \frac{0.1(1+s)}{(1+0.1s)(1+0.01s)}$$

At  $\omega = 1$

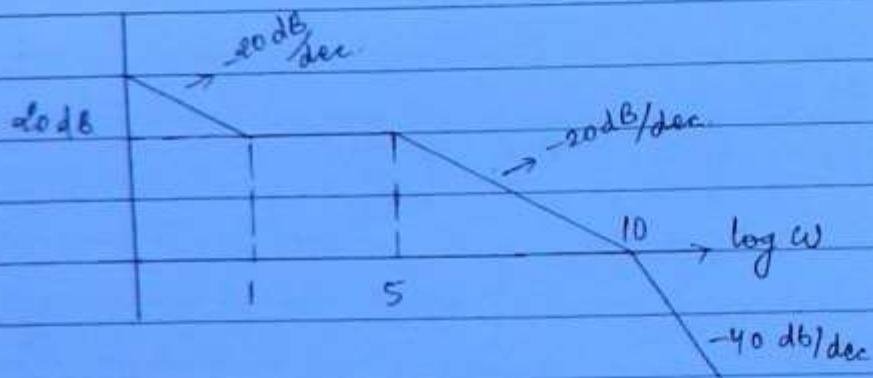
$$Y = MX + C$$

$$-20 = 0 \times \log 1 + C$$

$$C = -20$$

$$20 \log k = -20$$

$$k = \log^{-1}(-1) = 0.1$$



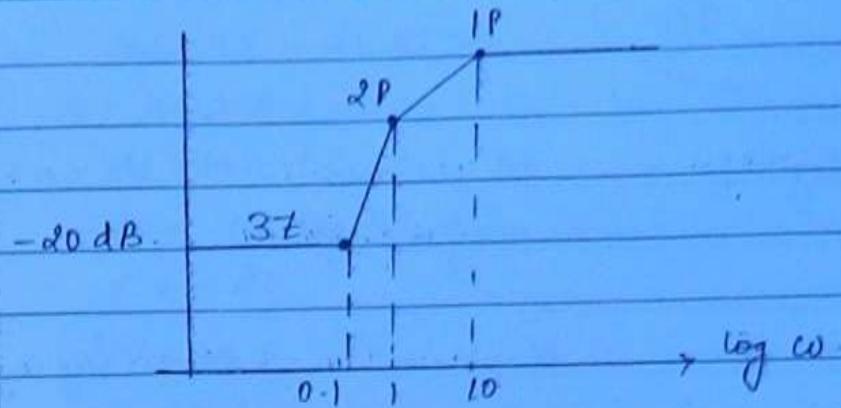
$$G(s) = \frac{10(1+s)}{s(1+0.2s)(1+0.1s)}$$

At  $\omega = 1$

$$Y = MX + C$$

$$20 = -20 \log 1 + C$$

$$C = 20$$



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$$G(s) = \frac{0.1 (1 + 10s)^3}{(1+s)^2 (1+0.1s)}$$

At  $\omega = 0.1$

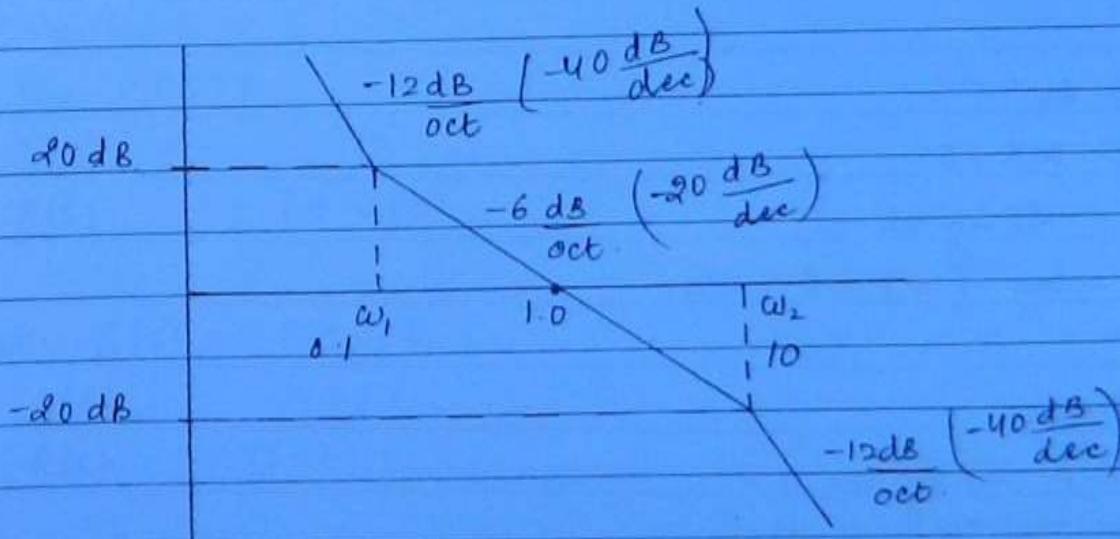
$$Y = MX + C$$

$$-20 = 0 \times \log 0.1 + C$$

$$C = -20$$

$$20 \log K = -20$$

$$K = 0.1$$



$$G(s) = \frac{0.1 (1 + 10s)}{s^2 (1 + 0.1s)}$$

## DECADE SCALE

$$\omega_2 = 10\omega_1$$

dB value =  $\pm \alpha_0 \times n \log \omega$   
Magnitude

$$\text{Slope (m)} = \pm \alpha_0 \times n \log 10 \\ = \pm \alpha_0 \times n \frac{\text{dB}}{\text{dec}}$$

## OCTAVE SCALE

$$\omega_2 = 2\omega_1$$

dB value =  $\pm \alpha_0 \times n \log \omega$   
magnitude

$$\text{Slope (m)} = \pm \alpha_0 \times n \log 2 \\ = \pm 6 \times n \frac{\text{dB}}{\text{oct}}$$

$$\pm 6 \times n \frac{\text{dB}}{\text{oct}} \approx \pm \alpha_0 \times n \frac{\text{dB}}{\text{dec}}$$

Second Line choose a line which passes through  
At  $\omega = 1$  a known frequency.

$$Y = MX + C$$

$$\theta_0 = -\alpha_0 \log 1 + C$$

$$C = 0$$

$$\text{At } \omega = \omega_1$$

$$Y = MX + C$$

$$\theta_0 = -\alpha_0 \log \omega_1 + C$$

$$\omega_1 = \log^{-1}(-1) = 0.1 \text{ rad/s}$$

$$\text{At } \omega = \omega_2$$

$$-\alpha_0 = -\alpha_0 \log \omega_2 + C$$

$$\omega_2 = \log^{-1}(1) = 10 \text{ rad/s}$$

First line

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At  $\omega = \omega_1 = 0.1$

$$Y = MX + C$$

$$\vartheta_0 = -40 \log 0.1 + C$$

$$C = -\vartheta_0$$

$$\vartheta_0 \log K = -\vartheta_0$$

$$K = 0.1$$

## M & N CIRCLES

1. The nichol's chart gives information about closed loop frequency response of the system
2. It consists of magnitudes of phase angles of closed loop system represented as a family of circles known as M & N circles

### 3. M CIRCLES

Let the CLTF

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\text{Let } G(s) = X + jY$$

$$\frac{C(s)}{R(s)} = \frac{X + jY}{(1+X) + jY}$$

The magnitude (M) -

$$M = \sqrt{X^2 + Y^2}$$

$$M^2 = \frac{X^2 + Y^2}{(1+X)^2 + Y^2}$$

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$$M^2X^2 - X^2 + M^2Y^2 - Y^2 + 2XM^2 + M^2 = 0$$

$$X^2[M^2 - 1] + Y^2[M^2 - 1] + 2XM^2 + M^2 = 0 \quad \text{--- (1)}$$

In eq (1) if  $M = 1$   
 $\alpha X + 1 = 0$

→ it represents a st. line passing through  $-\frac{1}{2}, 0$ .

→ If  $M \neq 1$  eqn (1)  
represents a family of circles.

$$\frac{X^2 + Y^2 + \alpha X}{M^2 - 1} + \frac{M^2}{M^2 - 1} = 0$$

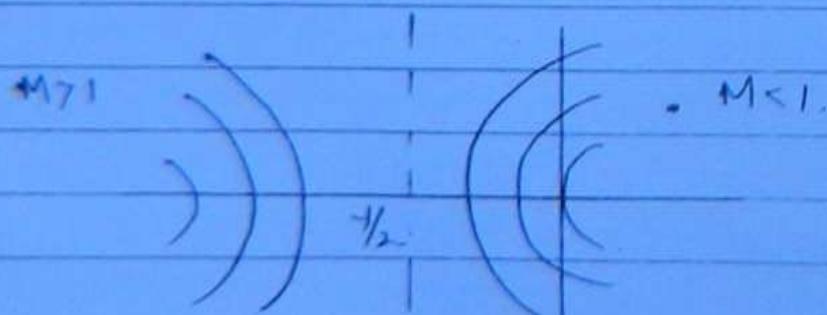
$$\frac{X^2 + Y^2 + 2XM^2}{M^2 - 1} + \frac{M^2}{M^2 - 1} + \frac{M^2}{(M^2 - 1)^2} = \frac{M^2}{(M^2 - 1)^2}$$

$$\frac{X^2 + 2XM^2}{M^2 - 1} + \frac{M^4}{(M^2 - 1)^2} + \frac{Y^2}{(M^2 - 1)^2} = \frac{M^2}{(M^2 - 1)^2}$$

$$\boxed{\left[ \frac{X + M^2}{M^2 - 1} \right]^2 + Y^2 = \frac{M^2}{(M^2 - 1)^2}}$$

$$\text{Centre} \rightarrow -\frac{M^2}{M^2 - 1}, 0$$

$$\text{Radius} \rightarrow \frac{M}{M^2 - 1}$$



## 4. N-circles

(19)

let  $\alpha \Rightarrow$  Phase angle of C.L. system. $N = \tan \alpha$  represents a family of circles

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)$$

$$N = \tan \alpha = \tan\left[\tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)\right]$$

$$N = \underbrace{\frac{y}{x} - \frac{y}{1+x}}_{1 + \frac{y^2}{x(1+x)}} \Rightarrow \frac{x^2 + x + y^2 - y}{N} = 0$$

Adding term  $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$  on b.s.

$$x^2 + x + \frac{1}{4} + y^2 - y + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left[\frac{x+1}{2}\right]^2 + \left[\frac{y-1}{2N}\right]^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\text{Centre} = -\frac{1}{2}, \frac{1}{2N}$$

$$\text{Radius} = \sqrt{\frac{1}{4} + \left(\frac{1}{2N}\right)^2}$$

For diff values of  $N$  all  $N$  circles intersect the real axis b/w -1 and origin only

## chapter 6

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$$③ \quad X^2 + 0.95X + Y^2 + 1.125 = 0$$

$$\frac{X^2 + 0.95X}{M^2 - 1} + \frac{Y^2 + 1.125}{M^2 - 1} = 0$$

$$\frac{M^2}{M^2 - 1} = 1.125$$

$$M^2 - 1$$

$$M^2 = 1.125 \quad M^2 = 1.125$$

$$0.125 M^2 = 1.125$$

$$\boxed{M=3} \quad \text{Ans} \quad (c)$$