

Decimal-Fractions

⌚ Time Spent : 01 Hours 18 Minutes 35 Seconds

</> Fractions And Decimals

- ✓ Dear students, here we are introducing to you a new and interesting topic about Fractions and Decimals, which may seem a little confusing to you in the aptitude part of competitive exams. After reading this post, we hope it clarifies your confusion and helps in solving questions easily and quickly. You can learn and practice by reading the following basics and examples.
- Before proceeding into the topic we need to know a few important terms which will be frequently used in this topic:

</> Basics – Fractions And Decimals

- ✓ **Fraction** : If any unit is divided into two or more parts, then each part is called a fraction of the unit and the fraction is expressed in the form of $\frac{p}{q}$ where $q \neq 0$. Example : 5/11.
- ✓ **Numerator** : The numerator of a fraction is the upper part of the p/q form (i.e. **p**). In the above example, 5 will be the numerator.
- ✓ **Denominator** : The denominator of a fraction is the lower part of the p/q form (i.e. **q**). In the above example, 11 will be the denominator.

</> Definition And Types – Fractions And Decimals

- ✓ **I Simple Fraction / Vulgar Fraction** : A fraction having its denominator as any number other than 10^x ($x=0,1,2,-$) is called a simple fraction. Examples: 3/8, 5/6, 8/4 etc. (Here No denominator is in the form of 10^x i.e. $10^1=10, 10^2=100, 10^3=1000$).

> Types Of Simple Fraction:

- ✓ **Improper Fraction** : In this type of fraction numerator is greater than the denominator (i.e. $p > q$). Example: 18/12, 48/15 etc.
- ✓ **Compound Fraction** : In this type of fraction, either the numerator or denominator (**or**) both numerator and denominator are in fraction ($\frac{p}{q}$) form. Example: $\frac{11}{8} \div \frac{12}{1}$, $\frac{1}{5} \div \frac{3}{12}$, $\frac{6}{5} \div \frac{8}{12}$.
- ✓ **Inverse Fraction**: This is the resultant fraction when you inverse the numerator and the denominator. Example: If given Fraction is $\frac{4}{7}$, then it's inverse fraction = $\frac{7}{4}$.
- ✓ **Mixed Fraction**: This type of fraction consists of a combination of both an integer and fraction. Example: $\frac{3}{6}$, $8\frac{1}{4}$.
- ✓ **Continuous Fraction**: A fraction which is having additional fractions in its denominator is known as a Continuous fraction. $\frac{2 + 1 / (2 + 2 / (5 + 2/3))}{1}$
- ✓ **II. Decimal Fraction**: A fraction having its denominator as 10^x ($x=1, 2, 3, -etc.$) is known as a decimal fraction. Example: For $\frac{4}{10}$ (vulgar fraction) = 0.4(decimal fraction), Similarly for $\frac{6}{100}$ decimal fraction is 0.06.

</> Types Of Decimal Fractions:

- ✓ **1.Recurring Decimal Fraction**: In this type of fraction, one or more digits after the decimal point will be repeating/recurring again and again. To represent those recurring digits, a small bar is drawn above the digits which repeat again and again.
- ✓ Example 1: $\frac{2}{3} = 0.6666 = 0.\overline{6}$
- ✓ Example 2: $\frac{22}{7} = 3.142857142857 = 3.(142857)$
- ✓ **a) Pure Recurring Decimal Fraction**: In this type of fraction all the digits after the decimal point repeat again and again.
- ✓ Example: $0.\overline{5}$, $0.(489)$ Note: Here to convert pure recurring decimal fractions into simple fractions, write down the repeated digits only once in numerator and write as many nines in the denominator as the number of digits repeating. Examples for Note:
 $\Rightarrow 0.666 = 0.\overline{6} = \frac{6}{9}$ (here 6 is the only repeated digit. So we need to place one 9 digit in denominator)
 $\Rightarrow 0.5858 = 0.(58) = \frac{58}{99}$ (here 5 and 8 both are repeated digits. So we need to place two 9 digits in denominator)

- ✓ **b) Mixed Recurring Decimal Fraction** : This type of fraction has both recurring and non-recurring digits after decimal point.
- ✓ Example: $3.25353 = 3.2(53)$ (Here 2 is non recurring and 5 and 3 are recurring after the decimal point.) Note: Here to convert mixed recurring decimal fractions into simple fractions, write down the difference between the number (formed by both recurring (taken once only) and non recurring digits) after the decimal and the number formed by the non-recurring digits in numerator and write many as nines in the denominator as the number of digits recurring and after 9s place as many 0s as the number of digits non-recurring. Example for Note:
 $1) 0.366 = 0.3\overline{6} = \frac{36-3}{90} = \frac{33}{90} = \frac{11}{30}$.
 $2) 0.426767 = 0.42(67) = \frac{4267-42}{9900} = \frac{4225}{9900} = \frac{169}{396}$.

</> Decimal Fraction

</> Fractions And Decimals – Some Important Notes:

- ✓ Whenever both the numerator and denominator are equal, then value of fraction will be 1.
- ✓ When the numerator=0, and denominator $\neq 0$, then the value will always be 0.
- ✓ When the numerator $\neq 0$ and denominator is 0, then the value will be infinity (∞).
- ✓ If the numerator or denominator is either multiplied or divided by same number, then the value of the fraction will remain unchanged.

</> Fractions And Decimals Mathematical Operations On Simple Fractions

> Additions –Fractions And Decimals

- ✓ Method 1: When fractions have same denominators.
- ✓ Example: $1/4 + 2/4 = ?$
- ✓ Answer: Here both the denominators are same, so take the denominator common for both the fractions and make it one unit. Then, add the values in the numerator. $(1+2)/4 = 3/4$.
- ✓ Method 2: When fractions having different denominators.
- ✓ Example: $1/2 + 1/3 + 1/4 = ?$
- ✓ Answer: In this case take the LCM of the denominators (LCM of 2,3,4=12) and put the obtained LCM in the denominator's place and multiply each numerator with the number that was multiplied with the previous denominator to obtain the current denominator. [1st denominator is 2 and LCM is 12 so 2 is multiplied by 6 ($2 \times 6 = 12$), then similarly the 2nd denominator 3 is multiplied by 4 ($3 \times 4 = 12$), and the 3rd denominator 4 is multiplied by 3 ($4 \times 3 = 12$). Finally put them in the numerator's place and add them as shown below.]
 $\Rightarrow 1/2 + 1/3 + 1/4 = (1 \times 6) + (1 \times 4) + (1 \times 3) / 12 = 6 + 4 + 3 / 12 = 13/12$.

> Subtraction-Fractions And Decimals

- ✓ Method 1: When fractions have same denominators. Example: $3/4 - 1/4 = ?$ Here both the denominators are same, so take the denominator common for both the fractions in the denominator's place, then subtract the value in the second fraction/fraction with the least numerator value from the value in the first fraction.
 $\Rightarrow (3-1)/4 = 2/4 = 1/2$.

- ✓ **Method 2:** When fractions having different denominators. Example: $2/3 - 1/2 = ?$ Answer: In this case take the LCM of the denominators (LCM of 3,2=6) and put the obtained LCM in the denominator's place and multiply each numerator with the number that was multiplied with the previous denominator to obtain the current denominator [1st denominator is 3 and the LCM is 6 so 3 is multiplied with 2 ($3 \times 2 = 6$), then similarly 2nd denominator with 2 is multiplied by 3 ($2 \times 3 = 6$). Finally put them in the numerator's place and subtract the second fraction from the first as shown below.]
 $= 2/3 - 1/2 = (2 \times 2) - (3 \times 1) / 6 = 4 - 3 / 6 = 1/6$.

➤ Multiplication

- ✓ **Method 1:** Here to multiply two or more simple fractions, multiply their numerators and denominators with each other, which are given in fractions. Example: $1/2 \times 3/4 = ?$ Answer: $1/2 \times 3/4 = (1 \times 3) / (2 \times 4) = 3/8$.
- ✓ **Method 2:** If fractions are given in mixed form, convert them into improper fractions and then multiply. [NOTE: To convert mixed fractions into improper fractions, multiply the denominator with the integer and then add the obtained value with the value in the numerator. Example: $2 \frac{4}{5} \times 1 \frac{8}{15} = 14/5 \times 11/3 = 154/15$. 5 3

➤ Division Of Fractions And Decimals

- ✓ To divide two fractions, the first fraction is multiplied by the inverse of second fraction. Example: $2/3 \div 3/5 = ?$ Answer: $2/3$ is multiplied by the inverse fraction of the 2nd fraction (inverse fraction of $3/5$ is $5/3$)
 $= 2/3 \times 5/3 = 10/9$.

</> Operations On Decimal Fractions

➤ Addition And Subtraction: Fractions And Decimals

- ✓ Example for Addition: $367.2 + 2.56 + 45.82 = ?$ Answer: 367.20

$$\begin{array}{r} 2.56 \\ + 45.82 \\ \hline 48.38 \end{array}$$

- ✓ Example for subtraction: $1500 - 126.89 = ?$ Answer: 1500.00

$$\begin{array}{r} 1500.00 \\ - 126.89 \\ \hline 1373.11 \end{array}$$

➤ Multiplication: Fractions And Decimals

- ✓ Example: $4.5 \times 0.25 = ?$ Answer: In this type, first multiply the given values without considering the decimal point as usual and finally after getting the product value, count the number of places after the decimal in both the multiplier and multiplicand, add the total number of decimal places and then take the product, count the summed up decimal place value from the right hand side and place the decimal at the respective place.
- ✓ **Step 1:** Multiply as usual without considering decimal
- ✓ $45 \times 25 = 1125$.
- ✓ **Step 2:** Sum of the decimal places in multiplier (4.5) = 1 and in multiplicand (0.25) = 2 is $1 + 2 = 3$.
- ✓ **Step 3:** Put the decimal point from the right hand side to as many places of decimal as the sum of numbers of decimal places in the multiplier (1) and multiplicand (2) together ($1 + 2 = 3$).
- ✓ $= 1.125$
- ✓ Note: If the multiplier is decimal fraction and multiplicand is integer (i) i.e. $i = 1, 2, 3, 4, \dots$, then in the product we only consider the decimal places of the multiplier and put the decimal mark as per decimal fraction.
- ✓ Example: 19.86×5
- ✓ Answer: Here multiplier 19.86 has its decimal point after two decimal places and multiplicand 5 has no decimal place, because it is an integer. So in the product we have to put the decimal point after two decimal places (before 30) only, from right hand side.
 $19.86 \times 5 = 99.30$

</> Division

- ✓ This process is just like the one we used for multiplication. We need to follow the normal division process without considering decimal points and after the process, place the decimal point after as many decimal places as in the dividend.
- ✓ Method 1: Division by an integer.
- ✓ Example: $0.81/9 = ?$
- ✓ Answer: $0.81/9 = 81/9 = 9 = 0.09$.
- ✓ Example: $1.2875/25 = ?$
- ✓ Answer: $12875/25 = 515 = 1.2875/25 = 0.0515$.
- ✓ Method 2: Division of decimal fractions
- ✓ [Either dividend and divisor are decimal fractions (or) dividend is an integer and divisor will be in decimals]. In problems under this topic, both the dividend and divisor are multiplied by a suitable multiple of 10 to convert the divisor into a whole number and follow the procedure mentioned above once the divisor is converted.
- ✓ Example: $42/0.007 = ?$
- ✓ Answer: $42/0.007 = 42/0.007 \times 1000/1000 = 42000/7 = 6000$.
- ✓ Example: $0.00048/0.8 = ?$
- ✓ Answer: $0.00048/0.8 \times 10/10 = 0.0048/8 = 0.0006$.

</> Comparison Of Simple Fractions-Fractions And Decimals

- ✓ Method 1: By Cross Multiplication
- ✓ Explanation: If a/b and c/d are two fractions, then
- ✓ If $a \times d > b \times c$, then $a/b > c/d$.
- ✓ If $a \times d < b \times c$, then $a/b < c/d$.
- ✓ If $a \times d = b \times c$, then $a/b = c/d$.
- ✓ Example: Which is bigger in $4/7$ and $3/8$?
- ✓ Answer:
- ✓ **Step 1:** Perform cross multiplication between $4/7$ and $3/8$
- ✓ **Step 2:** $4 \times 8 > 7 \times 3 = 32 > 21$
- ✓ **Step 3:** According to the above explanation, $4/7$ is bigger than $3/8$.
- ✓ Example: Which is the greatest in $2/3$, $3/4$, $4/3$, $5/4$?
- ✓ Answer:
- ✓ **Step 1:** Take the first two fractions ($2/3$ and $3/4$) and perform cross multiplication.
- ✓ $2 \times 4 < 3 \times 3 = 8 < 9$ (i.e. $3/4$ bigger than $2/3$)
- ✓ **Step 2:** Take the next two fractions ($4/3$ and $5/4$) and perform cross multiplication.
- ✓ $4 \times 4 > 5 \times 3 = 16 > 15$ (i.e. $4/3$ bigger than $5/4$)
- ✓ **Step 3:** Take the greater fractions ($3/4$ and $4/3$) from the above steps and perform cross multiplication once again.
- ✓ $3 \times 3 < 4 \times 4 = 9 < 16$ (i.e. $4/3$ is greater than $3/4$)
- ✓ Method 2: By changing Fractions into Decimal Form
- ✓ Example: Which is greater in $1/7$ and $2/9$?
- ✓ Answer: $1/7 = 0.14$, $2/9 = 0.22$ (i.e. $0.22 > 0.14$)
- ✓ So $2/9$ is greater than $1/7$.

- ✓ Method 3: By Equalizing Denominators:
- ✓ Example: Arrange $3/5$, $7/9$, and $11/13$ in decreasing order.
- ✓ Answer:
- ✓ **Step 1:** Take LCM of denominators 5,9,13; which is 585.
- ✓ **Step 2:** Multiply each numerator with the multiplicand used to convert its respective denominator into the LCM value. [i.e. first numerator(3) multiplied with 117, because LCM 585 should be multiplied 117 times with respective denominator ($117 \times 5 = 585$)], similarly in the 2nd fraction, numerator 7 and denominator 9 will be multiplied by 65, and in the 3rd fraction, 11 and 13 will be multiplied by 45)
- ✓ $\Rightarrow 3/5 = 3 \times 117 / 5 \times 117$; $7/9 = 7 \times 65 / 9 \times 65$; $11/13 = 11 \times 45 / 13 \times 45 \Rightarrow 351/585$; $455/585$; $495/585$
- ✓ Here by doing this we can equalize all denominators and easily identify that the fraction which is having the greatest numerator will be the greatest fraction in given fractions.
- ✓ **Step 3:** In the above fractions, 495 is the greatest numerator and the respective decimal fraction is $11/13$.

Hence, The Decreasing order is $11/13 > 7/9 > 3/5$.

- ✓ Method 4: By Equalising Numerators.
- ✓ Example: Arrange $3/13$, $2/15$, $4/17$ in increasing order.
- ✓ Answer:
- ✓ **Step 1:** Take the LCM of the denominators: LCM of 13,15,17 = 3315.
- ✓ **Step 2:** Multiply both numerator and denominator with the multiplicand used to convert its respective denominator into the LCM value;
 . [i.e. in the 1st fraction, 3(first numerator) multiplied 1117 times equals 3315($3 \times 1117 = 3315$), so multiply both numerator and denominator with multiplicand 1117, similarly the second numerator 2 multiplied 1657 times equals 3315 ($2 \times 1657 = 3315$), so multiply both numerator and denominator with multiplicand 1657, the third numerator 4 multiplied 828 times equals 3315 ($4 \times 828 = 3315$), so multiply both the numerator and denominator with multiplicand 828.]
- ✓ $\Rightarrow 3/13 = 3 \times 1117 / 13 \times 1117 = 3351/14521$;
- ✓ $\Rightarrow 2/15 = 2 \times 1657 / 15 \times 1657 = 3314/24855$;
- ✓ $\Rightarrow 4/17 = 4 \times 828 / 17 \times 828 = 3312/14076$;
- ✓ **Step 3:** In the above obtained fractions, the one having the smallest denominator will be the largest fraction.
- ✓ Hence $4/17 > 3/13 > 2/15$.

</> Fast Track Approach Methods: Fractions And Decimals:

- ✓ Example 2: Arrange the fractions $4/5$, $5/6$, $6/7$ in increasing order.
- ✓ Answer:
- ✓ Note: Whenever all the fractions have the same difference between numerator and denominator, then the fraction having the greatest numerator will be greater fraction and the fraction having the least numerator will be the least fraction) Hence, increasing order $4/5 < 5/6 < 6/7$ (Because of $5-4=1$, $6-5=1$, $7-6=1$ having same difference, so above rule applied)
- ✓ Example 3: Arrange the fractions $2/5$, $5/11$, $8/17$, $11/23$ in decreasing order.
- ✓ Answer:
- ✓ Note: Whenever the numerators of the given fractions are increasing by a definite value and denominator is also increasing by a definite value but the difference value of the denominators is greater than that of the numerators, then the fraction having the least numerator will be the least fraction and fraction having the greatest numerator will be the greatest fraction. In the above fractions, the numerator's values are increasing by 3 and the denominator's values are increasing by 6 and here we can note that $6 > 3$ (denominator's difference value is greater than numerator's difference value). Hence the above rule can be applied and $11/23$ is the greatest in the given fractions. $11/23 > 8/17 > 5/11 > 2/5$.

- ✓ Example 4: Anand was to find $\frac{6}{7}$ of a fraction. Instead of multiplying, he divided the fraction by $\frac{6}{7}$ and the result obtained was $\frac{13}{70}$ more than the original value. Find the fraction given to Anand?
- ✓ Answer:
- ✓ Note: If any number is divided by $\frac{a}{b}$ instead of multiplying by $\frac{a}{b}$, then obtained value will be x greater than the original value and the given number will be $\frac{abx}{b^2-a^2}$. In the above problem we can assume $a=6$ and $b=7$ and $x=\frac{13}{70}$. The original fraction = $\frac{abx}{b^2-a^2} = \frac{6 \times 7 \times \frac{13}{70}}{7^2-6^2} = \frac{6 \times 13}{10 \times 13} = \frac{3}{5}$. Hence the fraction given to Anand is $\frac{3}{5}$.

</> Introduction

- ✓ We have learned that decimals are an extension of our number system. We also know that decimals can be considered as fractions whose denominators are 10, 100, 1000, etc. The numbers expressed in decimal form are called decimal numbers or decimals.
- ✓ You can write decimal fractions with a decimal point (and no denominator), which makes it easier to do calculations like addition and multiplication on fractions.

> Examples:

- ✓ $\frac{7}{10}$ is a decimal fraction and it can be shown as 0.7
- ✓ $\frac{43}{100}$ is a decimal fraction and it can be shown as 0.43
- ✓ $\frac{51}{1000}$ is a decimal fraction and it can be shown as 0.051

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13

Z	26
Y	25
X	24
W	23
V	22
U	21
T	20
S	19
R	18
Q	17
P	16
O	15
N	14

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