

11

156

150

-: HAND WRITTEN NOTES:-

OF

ELECTRICAL ENGINEERING

1

-: SUBJECT:-

ELECTROMAGNETIC

THEORY

11

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Electromagnetics :-

electrostatics
static electrical fields
 $\vec{E}, \vec{D} \neq f(t)$

Magnetostatics
static magnetic fields
 $\vec{B}, \vec{H} \neq f(t)$

time varying electrical & magnetic fields
 $\vec{E}, \vec{B}; \vec{B}, \vec{H} \neq f(t)$

static electrical field intensity
(N/C; V/m)

electrical field density
i.e. displacement vector (C/m^2)

$$\boxed{\vec{D} = \epsilon \vec{E}}$$

permittivity
of medium
(F/M)

$$\boxed{\epsilon = \epsilon_0 \epsilon_r}$$

ϵ_0 --- permittivity
of free space.

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$= \frac{1}{36\pi} \times 10^{-9} \text{ F/m}$$

$$\boxed{\epsilon_r \geq 1}$$

$\epsilon_r = 1$ --- for free space,
 ≥ 1 --- for any other
dielectric

ϵ_r = Relative permittivity
of medium

(unitless)

--- dielectric constant

unless specified

$$\epsilon_r = 1$$

\vec{D} --- constant irrespective
of type of medium.

Static Magnetic fields :-

(4)

$$\vec{H}$$

$$\vec{B}$$

Mag. field intensity $\text{---} (\text{A/m})$
 Mag. flux density $\text{---} (\text{wb/m}^2 \equiv \text{T})$

$$\boxed{\vec{B} = \mu \vec{H}}$$

permeability of
medium
 (H/m)

$$\boxed{\mu = \mu_0 \mu_r}$$

μ_0 = permeability of free
space
 $= 4\pi \times 10^{-7} \text{ --- H/m}$

μ_r = relative permeability
of medium

$$\boxed{\mu_r \leq 1, \text{ unit less}}$$

| | | |
|-------------|-----------|--|
| $\mu_r < 1$ | diamag. | $\left. \begin{array}{l} \mu_r \approx 1 \\ \text{non ferromag.} \end{array} \right\}$ |
| $= 1$ | non-mag. | |
| > 1 | paramag. | |
| $\gg 1$ | ferromag. | |
| | | $\mu_r \gg 1$ ferromag. |

unless specified

$$\boxed{\mu_r \approx 1}$$

{ Water (diamag.) ; $\mu_r = 0.999991$

{ Air (paramag.) ; $\mu_r = 1.0000006$

{ cobalt (paramag.) ; $\mu_r \approx 250$

{ Fe (0.5% impure) ; $\mu_r \approx 5000$

{ Fe (0.05% impure) ; $\mu_r \approx 200000$

raamg.

Time Varying elec. & mag. fields :-

(S)

$$\vec{E}, \vec{D}; \quad \vec{B}, \vec{H} = f(t)$$

Imp. points :-

① The maxwell's equations are a set of four eqns.

which a relationship b/w time varying ele. & mag. fields

② When ever any wave propagate then the ele. field, mag. field & direction of propagation are mutually perpendicular each other.

$(\vec{E} \perp \vec{H}) \perp$ direction of propagation



TEM wave

Transverse em waves (uniform plane waves)

③ When there is large mismatching b/w the length of filament & wavelength of operation at low freq. then entire power dissipation in element.
④ at High freq the length is comparable to the wavelength of operation, then the power is radiated through that element

⑤ at low freq the depth of penetration is high & therefore we use thick conductor, whereas at high frequencies depth of penetration is low & therefore thin conductors are used
the power from the transmitter to the antenna is transported with the help of coaxial transmission line or a parallel plate transmission line.

Transmission line :-

1 parallel wide transmission line

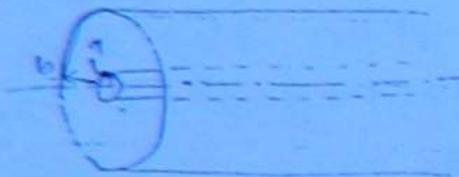


used as:

HT wires (High tension -)
telephone wires

(6)

Co-axial T-L

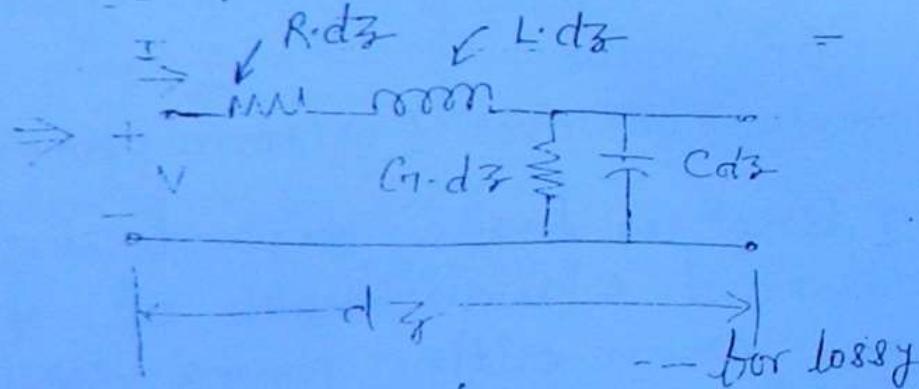
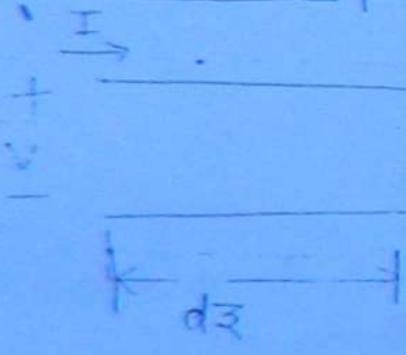


used in

CATV

as CRO load

Distributed parameters equivalent ckt. of T.L.



primary
constant
of the
line

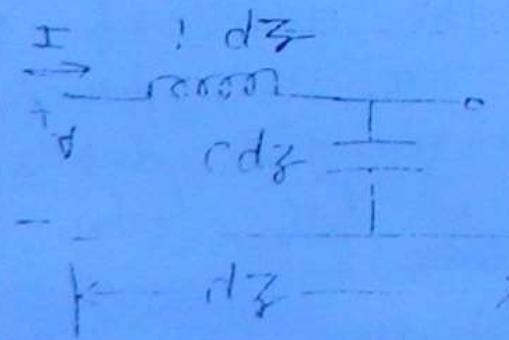
| | |
|---|------------------|
| R | $\rho \text{ m}$ |
| L | $H \text{ m}$ |
| C | $F \text{ m}$ |
| G | $S \text{ m}$ |

for lossy

line

loss less line

$$\begin{cases} R = 0 \\ G = 0 \end{cases}$$



Imp. points :-

None

① Due to inherent properties of lossy transmission line it's assume that R, L, G & C are effectively distributed along the entire length of TL (7)

② Due to lossy nature of the line & due to finite conductivity of the line

- Some losses occur on the TL due to current flow along the line.

The Resistor R is responsible for total power dissipation taking place due to lossy nature of the line

③ Due to current flow & due to mag. fields some Mag. energy will exist:-

The Inductor L is responsible for total magnetic stored in the TL

④ Due to potential difference b/w the two lines, some electrical fields & therefore some ele. energy is finite in the TL. The capacitor C is responsible for the total electrical energy stored in the transmission line.

⑤ The medium of dielectric b/w the two lines is in general is lossy nature.

Some power dissipation take place as the current leaks through the lossy dielectric.

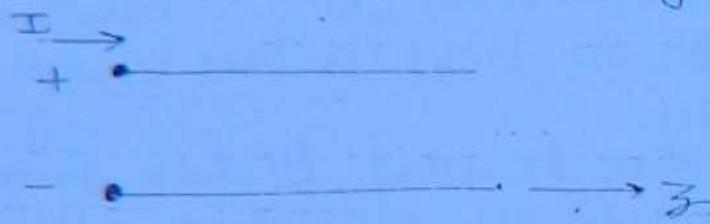
G is responsible for total power dissipation taking place due to lossy nature of the dielectric b/w the TL

for a lossless line R & G are zero.

as the voltage or current waveform are constant on the TL the wave is attenuated exponentially and therefore the magnitude of the voltage & current will decrease as the wave propagates along the line.

(8)

Behaviour of V & I along the line :-



$$V = \underbrace{V^+ e^{-\gamma z}}_{\substack{\text{propagating} \\ \text{along } +z}} + \underbrace{V^- e^{+\gamma z}}_{\substack{\text{propagating} \\ \text{along } -z}}$$

$\underbrace{\text{incident wave}}_{\text{}} \qquad \qquad \underbrace{\text{Reflected wave}}_{\text{}}$

V^+ Amplitude of Voltage wave propagating along $+z$ direction.

V^- Amplitude of wave propagating along $-z$.

γ Propagation const.
(Complex)

$$\gamma = \alpha + j\beta;$$

α Attenuation const. (nepers/m)
 β Phase const (ϕ rad/m)

$$I = \frac{1}{Z_0} \left(\underbrace{V + e^{-\gamma z}}_{\text{incident wave}} - \underbrace{V e^{\gamma z}}_{\text{reflected wave}} \right)$$

(Q)

due to reverse direction of current

Z_0 --- Characteristic Impedance.

$$Z_0 ; \gamma (\equiv \alpha, \beta)$$

--- Secondary Const of the line.

$$Z_0 = \sqrt{\frac{R+jWL}{G_1+jWC}}$$

$$\gamma = \sqrt{(R+jWL)(G_1+jWC)}$$

$$= \alpha + j\beta$$

Basic features: —

- ① as the voltage or the current waveform is propagated for a lossy line it's subjected to attenuation & well as phase change.

Therefore the magnitude of Voltage & current will decrease exponentially as the waves travel along the line.

if $\alpha = 0$ the wave propagation takes place without any attenuation.

therefore the magnitude of Voltage & current will remain const. at all the points along the line.

- ② if $\beta = 0$ there is propagation & there is entirely attenuate.

Ques

③ The char. of the line represent value ratio
b/w voltage & current at any point of any
infinite long line (10)

④ The propagation const γ & the char. imp.
is zero are represented by the secondary
const. of the lines.
Since depend upon the primary
const of R, L, C & ω of the line.

Lossless line :-

$$R=0 \\ G_1=0 \\ Z_0 = \sqrt{\frac{R+j\omega L}{G_1+j\omega C}} = \sqrt{\frac{L}{C}} \quad \text{real; const.} \\ (\neq f(\omega))$$

$$\gamma = \sqrt{(R+j\omega L) + j\omega C} \\ = j\omega \sqrt{LC} = \alpha + j\beta$$

$$\alpha = 0 \\ \beta = \omega \sqrt{LC}$$

$$\beta = \omega \sqrt{LC} \\ = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$

$$v_p = \frac{1}{\sqrt{LC}} \quad \text{const.}$$

Summary:-

(11)

--- lossy line !

$$V = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$$

$$I = \frac{1}{Z_0} (V^+ e^{-\gamma z} - V^- e^{+\gamma z})$$

$$Z_0 = \sqrt{\frac{R + jWL}{G_1 + jWC}}$$

$$\gamma = \sqrt{(R + jWL)(G_1 + jWC)}$$

R, L, G_1, C --- primary const.

$\gamma (\neq \alpha, \beta)$; Z_0 --- sec. const.

lossless line

$$R = G_1 = 0$$

$$\alpha = 0$$

$$\gamma = j\beta$$

$$\beta = \omega \sqrt{LC}$$

$$\beta = \frac{\omega}{V_p} = \frac{2\pi}{\sigma}$$

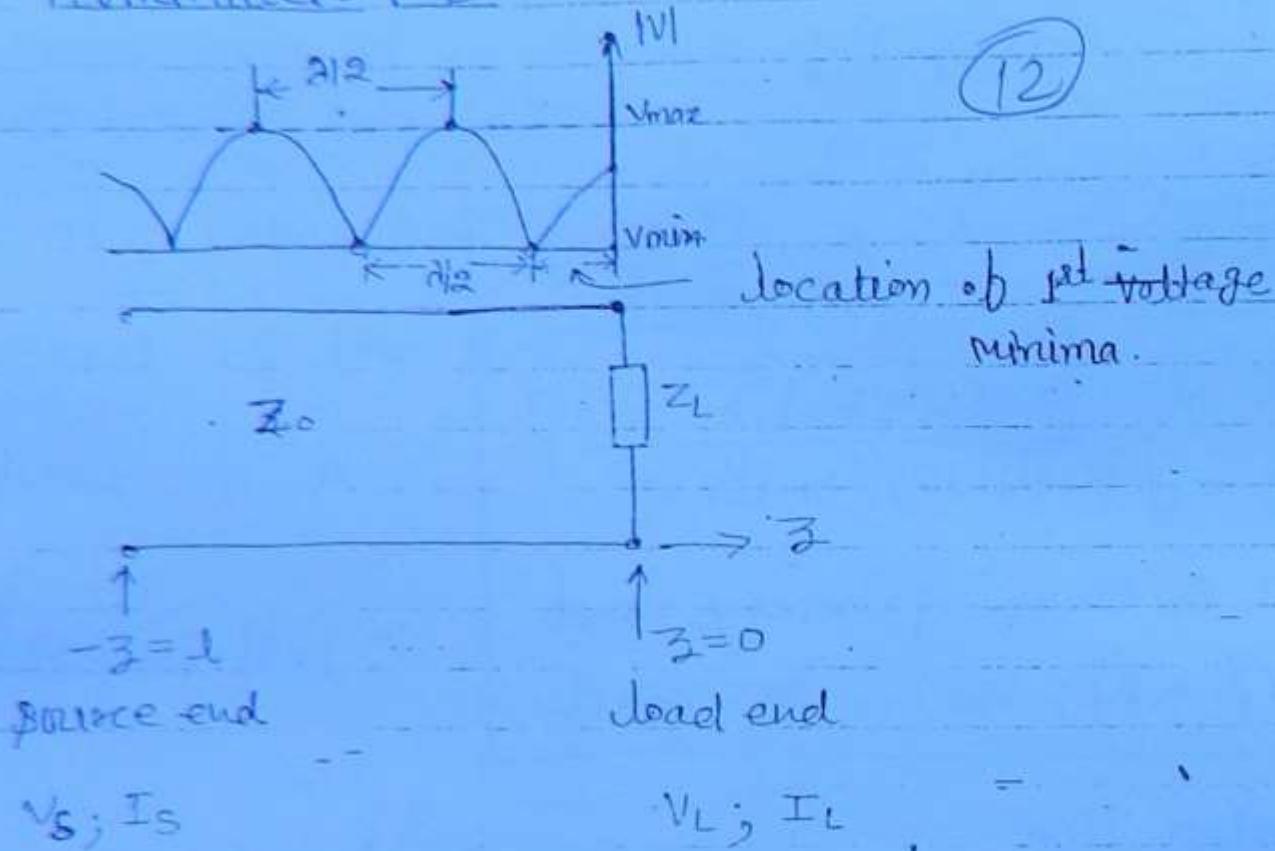
$$V_p = \frac{1}{\sqrt{LC}} \text{ --- const.}$$

$$Z_0 = \sqrt{\frac{L}{C}} \text{ --- real; const; } \neq f(\omega)$$

$$V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{1}{Z_0} (V^+ e^{-j\beta z} - V^- e^{+j\beta z})$$

Terminated T.L.



Case 1 : $Z_L \neq Z_0$

..... mismatched line.

- ① Max. power is not transferred to the load.
- ② Incident & Reflected waves will exists.
- ③ Standing wave pattern will exists & there are Maxima & minima will exists along the line. There for there is a form of standing wave pattern.
- ④ Coefficient of reflection has a finite value.
- ⑤ due to V_{max} & V_{min} Voltage standing wave Ratio is finite.

$$\text{VSWR} = S = \frac{V_{max}}{V_{min}}$$

..... Standing wave ratio

Case:2: $Z_L = Z_0$

--- Matched line.

(13)

- ① Maximum power is transferred from source to load.
- ② Reflection coefficient is zero.
- ③ There is no reflected waves, no standing wave pattern, $V_{max} = V_{min}$ & therefore the voltage along the line is const. at all the points.
- ④ The VSWR has a min. value of unity.

$$S = \frac{V_{max.}}{V_{min.}} = 1$$

S_{min}

Since $V_{max.} = V_{min}$

To find:-

$$\text{① Reflection coefficient: } \Gamma = \frac{V^-}{V^+} \quad \text{Complex}$$

$$|\Gamma| e^{j\phi} = p e^{j\phi}$$

② Transmission coefficient:-

$$T = \frac{V_L}{V^+}$$

$$\text{③ VSWR (S)} = \frac{V_{max.}}{V_{min.}}$$

$$\text{④ } Z_{in} = \frac{V_s}{I_s} = \frac{V}{I} \Big|_{z=0} = Z_0$$

assume: Line is lossless

$$\text{① } V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{1}{Z_0} (V^+ e^{-j\beta z} - V^- e^{+j\beta z})$$

$$Z = 0$$

$$V_L = V^+ + V^- \quad \text{--- (1)}$$

$$I_L = \frac{1}{Z_0} (V^+ - V^-) \quad \text{--- (2)}$$

(14)

$$\frac{V_L}{I_L} = Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$$

$$\boxed{\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

if $\frac{Z_L}{Z_0} = z_L$ --- normalized load impedance.

$$\Gamma = \frac{z_L/z_0 - 1}{z_L/z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

⑦ Transmission coefficient :-

$$T = V_L / V^+$$

from equation (1)

$$\frac{V_L}{V^+} = 1 + \frac{V^-}{V^+}$$

$$\boxed{\Gamma = 1 + \frac{V^-}{V^+}} = 1 + \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$T = \frac{2Z_L}{Z_L + Z_0} = \frac{2\bar{Z}_L}{\bar{Z}_L + 1}$$

$$S = \frac{V_{max.}}{V_{min.}} = \frac{|V^+| + |V^-|}{|V^+| - |V^-|}$$

$$= \frac{1 + |V^-|/|V^+|}{1 - |V^-|/|V^+|} = \frac{1 + |T|}{1 - |T|}$$

$$S = \frac{1+\epsilon}{1-\epsilon}$$

(B)

$$C_{\min} = 0 ; S_{\max} = 1$$

$$C_{\max} = 1 ; S_{\min} = \infty$$

$$(4) Z_{in.} = \frac{V}{I} \Big|_{-z=1}$$

$$Z_{in} = Z_0 \cdot \frac{V^+ e^{-j\beta z} + V^- e^{+j\beta z}}{V^+ e^{-j\beta z} - V^- e^{+j\beta z}} \Big|_{-z=1}$$

$$e^{\pm j\beta z} = \cos \beta z \pm j \sin \beta z$$

$$\frac{V^-}{V^+} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j Z_0 \tan \beta L}{Z_0 + j Z_L \tan \beta L}$$

Summary :-

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L - 1}{Z_L + 1}$$

$$Z_L = Z_0 \frac{1 + \Gamma}{1 - \Gamma}$$

$$1 - 1 + \Gamma = \frac{2Z_L}{Z_L + Z_0} = \frac{2 \bar{Z}_L}{\bar{Z}_L + 1}$$

$$\frac{1+\epsilon}{1-\epsilon} ; \epsilon = -\frac{S-1}{S+1}$$

$$0 \leq \epsilon \leq 1$$

$$1 \leq S \leq \infty$$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta L}{Z_0 + j Z_L \tan \beta L}$$

Ex: A loss less line of length l & characteristic impedance is zero is terminated by a load impedance Z_L . calculate:

(1) Input impedance.

(2) reflection coefficient.

(3) VSWR when :-

(76)

$Z_L = 0$ --- SC line

$Z_L = \infty$ --- o.c. line

$Z_L = Z_0$ --- Matched line

$Z_L = jx$ --- purely reactive load.

Case: 1

$$Z_L = 0$$

--- SC line

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$= jZ_0 \tan \beta l$$

Z_0 } $Z_L = 0$

--- purely reactive
--- S.C. stub-line

$$Z_{in} = [Z_{SC} = jZ_0 \tan \beta l]$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 = \frac{V^-}{V^+}$$

$$\Gamma = r e^{j\theta} = -1$$

$$r = 1$$

$$\theta = \pi$$

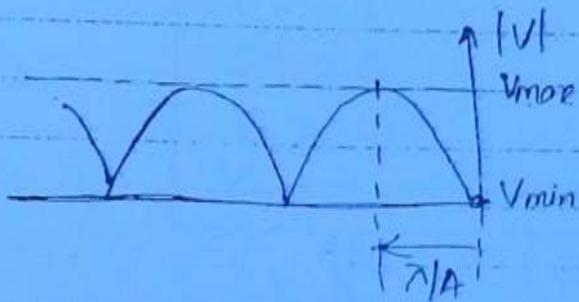
$$\left\{ e^{j\theta} = e^{-j\pi} = \cos \pi - j \sin \pi \right.$$

$$= -1$$

$$S = \frac{1+\epsilon}{1-\epsilon} = \infty = \frac{V_{max}}{V_{min}}$$

(17)

Minima is located at the load end.



S.c Line :-

$$Z_L = \infty$$

$$\Gamma = -1$$

$$\phi = 180^\circ$$

$$\epsilon = 1$$

$$S = \infty$$

$$Z_{sc} = j Z_0 \tan \beta l$$

Features :-

- ① The input admittance is purely resistive in nature. for the shortest length of the line this admittance is inductive nature.
- ② Stub line is a portion of line which has been s.c at the load end & has purely reactive o/p admittance.
- ③ A short circuited stub can be used for matching transmission line with the load admittance for max. power transfer.
- ④ The reflected voltage & incident voltage are equal in magnitude but are phase shifted by 180° .
- ⑤ The voltage minima occurs at the load end & the 1st voltage maxima occurs at a distance of $z/4$ from the load end.

case : 2

$$Z_L = \infty$$

--- OC - line

$$Z_m = Z_0 \frac{z_L + j z_0 \tan \beta L}{z_0 + j z_L \tan \beta L}$$

$$= Z_0 \frac{1 + j(z_0/z_L) \tan \beta L}{(z_0/z_L) + j \tan \beta L}$$

$$Z_{OC} = -j z_0 \cot \beta L$$

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--- purely reactive.

--- OC stub line.

$$Z_{SC} = j z_0 \tan \beta L$$

$$Z_{OC} = -j z_0 \cot \beta L$$

$$Z_{SC} \cdot Z_{OC} = Z_0^2$$

$$\rightarrow Z_0 = \sqrt{Z_{SC} \cdot Z_{OC}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - \cancel{Z_0/Z_L}}{1 + \cancel{Z_0/Z_L}} = 1 = \frac{V^-}{V^+}$$

$$\Gamma = e^{-j\theta} = 1$$

$$\varphi = 1$$

$$\theta = 0$$

$$S = \frac{1+\Gamma}{1-\Gamma} = \infty = \frac{V_{max}}{V_{min}}$$

--- V_{max} occurs at load end.

O.C Line :-

$$Z_L = \infty$$

$$\Gamma = 1$$

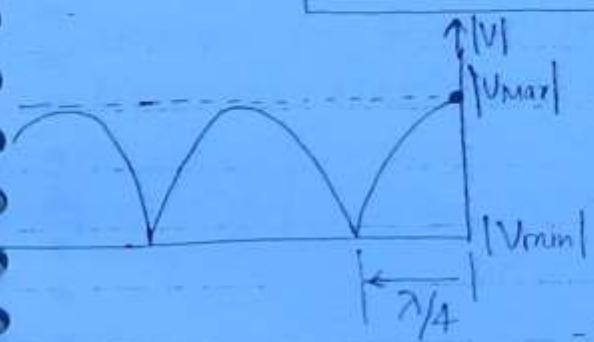
$$\epsilon = 1$$

$$\phi = 0$$

$$S = \infty$$

$$Z_{oc} = -j Z_0 \cot \beta l$$

(79)

Features:

- ① The input admittance is purely reactive & for a shortest length of the line it's capacitive in nature.
- ② The o.c. stub line can be used to matched any transmission line with the load admittance for max. power transfer.
- ③ The reflected & incident voltages has same magnitude and are in-phase.
- ④ Voltage maxima occurs at the load end.
- ⑤ When the line is first s.c. & then o.c. the voltage minima shifted by distance $\lambda/4$ from the load end towards the source end.
- ⑥ The characteristic admittance of the line is a geometric mean of input admittance of the line when it's s.c. & then o.c.
- ⑦ An Impedance Inversion takes place when the line is 1st s.c. & then o.c & vice-versa.

Therefore Inductive Imp. is transformed
into capacitive & vice-versa.

(20)

case: 3 : $Z_L = Z_0$

Matched line

$$Z_m = Z_0 - \frac{r = Z_0}{Z_0 + j Z_L \tan \beta l}$$

$$Z_0 + j Z_L \tan \beta l$$

$$l = Z_0$$

$$Z_m = Z_0$$

$$r = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 = \frac{V^-}{V^+}$$

$$\tau = e e^{j\theta} = 0$$

$$\zeta = 0$$

ϕ = indeterminate ($0 \leq \phi \leq 360^\circ$)

= 0° (min.)

$$S = \frac{1+r}{1-r} = 1 = \frac{V_{max}}{V_{min}}$$

$$V_{max} = V_{min}$$

Matched line :-

$$Z_L = Z_0$$

$$Z_m = Z_0$$

$$r = 0$$

$$\rho = 0$$

$$\phi = 0^\circ$$
 (min)

$$S = 1$$

feature:

- ① A perfectly matched line behaves as infinity long line since in each case the SIP adm. of the line is equal to the char. adm. of the line.
- ② There is no reflected waves, reflection coefficient is zero, VSWR has min. value of unity.
- ③ Maximum power is transferred to the load β^0 that $V_{\text{max}} = V_{\text{min}}$ \Rightarrow there is no standing waves & therefore the voltage along the line is uniform at all the points.

(2)

case: 4

$$Z_L = jx$$

(purely reactive load)

$$Z_{\text{in}} = Z_0 \frac{Z_0 + jZ_0 \tan \beta L}{Z_0 - jZ_0 \tan \beta L} =$$

$\underbrace{_{\text{Real}}}^{jx} \quad \underbrace{_{\text{Imaginary}}}^{-x}$

$$Z_{\text{in}} = j \left[Z_0 \frac{x + Z_0 \tan \beta L}{Z_0 - x \tan \beta L} \right]$$

$\underbrace{_{\text{Real}}} \quad \underbrace{_{\text{Imaginary}}} \quad \text{pure reactance}$

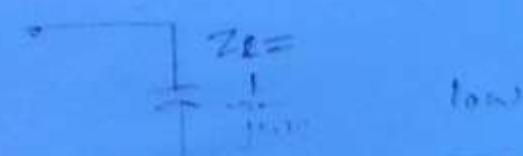
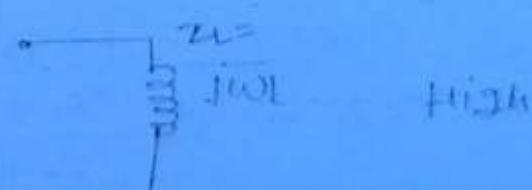
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{jx - Z_0}{jx + Z_0}$$

$$\Gamma = |\Gamma| e^{j\theta}$$

$\underbrace{|\Gamma|}_{=P}$

$$S = \frac{Hf}{1-e} = \infty = \frac{V_{\text{max}}}{V_{\text{min}}}$$



$$\theta = -2 \tan^{-1} \left(\frac{x}{z_0} \right) \quad (22)$$

Features:

- ① if the line is terminated by a purely reactive load then s/p amp. is also purely reactive.
- ② The location of voltage max. & Vmin. in the standing wave pattern will depend upon the type of the reactive load.
for inductive load the Vmax will occur at the load end whereas minima will occur at the load end if its is capacitive nature.

Ex: A lossless TL of length l has char's Imp. of z_0 & is terminated by load Imp. z_L .
find input Imp. of the line when :-

case 1: $l = \lambda$

case 2: $l = \lambda/2$

case 3: $l = \lambda/4$ ---- QWT

case 4: $l = \lambda/8$ Quarter wave
transformer

To find :- case 1 : $l = \lambda$

$$Z_m = z_0 \frac{z_L + j z_0 \tan Bl}{z_0 + j z_L \tan Bl}$$

$$\tan \beta l = \tan\left(\frac{2\pi}{\lambda} \cdot d\right) = 0$$

(23)

$$Z_{in} = Z_0 \frac{Z_L + j0}{Z_0 + j0}$$

$Z_{in} = Z_L$

Case 2: $d = \lambda/2$

$$\tan \beta l = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\right) = 0$$

$Z_{in} = Z_L$

Case 3: $d = \lambda/4$
QWT

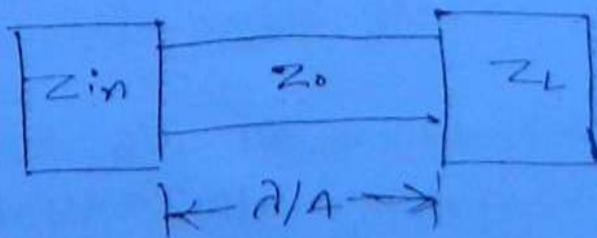
Quarter wave transformer

$$\tan \beta l = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right) = \infty$$

$$Z_{in} = Z_0 \frac{(Z_L / \tan \beta l) + jZ_0}{(Z_0 / \tan \beta l) + jZ_L}$$

$Z_{in} = \frac{Z_0^2}{Z_L}$

$Z_0 = \sqrt{Z_{in} \cdot Z_L}$



- ① A QWT exists an Impedance Inversal.
 therefore if the load Z_L is inductive
 then the S/P impedance is capacitive & vice-versa.
- ② A $\lambda/4$ section of the line matched two
 impd. Z_L & Z_{in} & perfect matching take place
 whenever the load Z_L & S/P-impd. is purely
 resistive grp.
- ③ A $\lambda/2$ section of the line is used to
 transform given load impd. is Z_L to the desired
 S/P impd. using a QWT whose charctrs impd. is
 the geometric mean of the load Z_L & input
 impd. Z_0 .

Case: 4 :- $d = \lambda/8$

(24)

$$\tan \beta d = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}\right) = 1$$

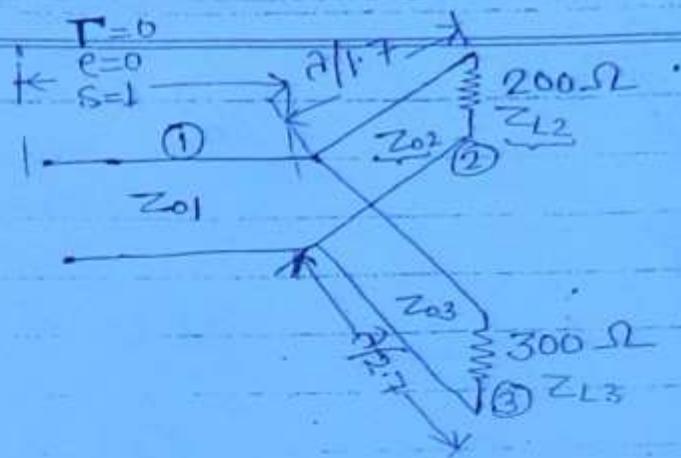
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d} = 1$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0}{Z_0 + jZ_L}$$

--- complex nature.

$$|Z_{in}| = Z_0$$

- i) for $\lambda/8$ section of the line the input impedance
 is always complex in nature irrespective of
 the nature of load impd. Z_L
- ② The magnitude of input impedance of $\lambda/8$ section
 of the line is always numerically equal
 to the characteristics of the line.



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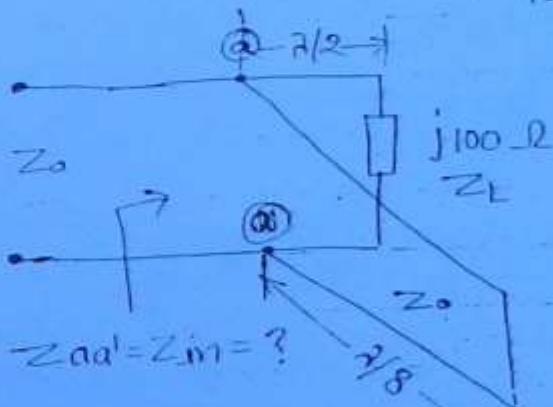
Find the characteristic impedance Z_0 of the 1st line so that there is no reflected wave on it.

$$Z_{01} = Z_{L1} = Z_{in2} \parallel Z_{in3}$$

(= $Z_{02} \parallel Z_{03}$)

--- matched line --- matched line

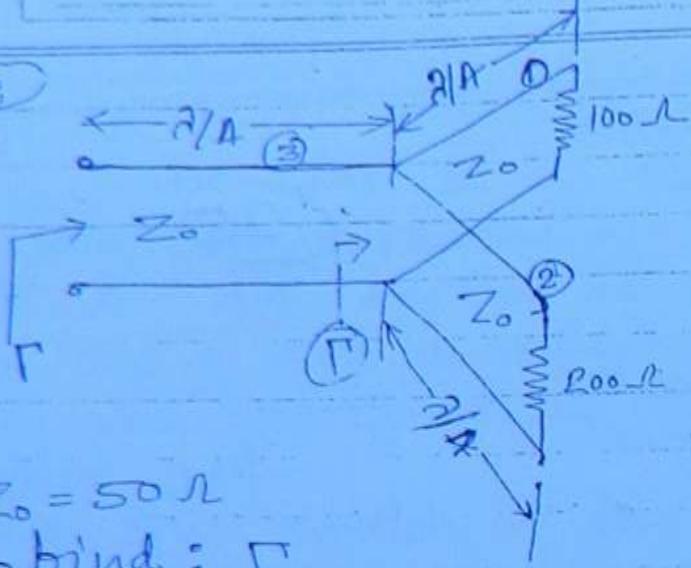
$$= 200 \parallel 300$$



$$Z_{in} = \frac{Z_{212} + Z_{218}}{jz_0 \tan \beta l} = jz_0 \tan \left(\frac{2\pi}{\lambda} \cdot \frac{\partial l}{8} \right) = 1$$

$$Z_m = \frac{j100 \times j200}{j100 + j200}$$

ex:



(26)

$$Z_0 = 50 \Omega$$

to find: Γ

$$Z_{L3} = Z_{in1} \parallel Z_{in2}$$

$$= \frac{Z_0^2}{Z_{L1}} \parallel \frac{Z_0^2}{Z_{L2}} \Rightarrow \frac{2500}{100} \parallel \frac{2500}{200}$$

$$\Rightarrow 25 \parallel 25/2 \Rightarrow 25/3 \Omega$$

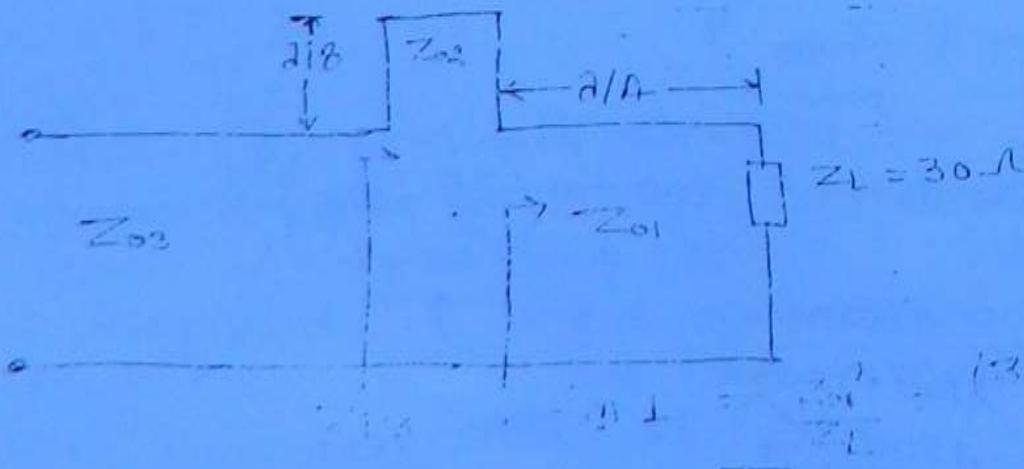
$\Gamma \neq$

$$\frac{Z_{L3} - Z_0}{Z_{L3} + Z_0} = \frac{25/3 - 50}{25/3 + 50} = -5/7$$

$$Z_{in} = \frac{Z_0^2}{Z_{L3}} = \frac{2500}{25/3} = 300 \Omega$$

$$\Gamma = \frac{Z_{in3} - Z_0}{Z_{in3} + Z_0} = \frac{300 - 50}{300 + 50} = +\frac{5}{7} \quad \underline{\text{ans}}$$

ex:



$$(30 \sqrt{5})^2 = 60 \Omega$$

$$Z_{01} = 30\sqrt{2}$$

$$Z_{02} = 30$$

$$Z_{03} = 60$$

To find:-

s on Z_{03} line

$$s = \frac{1+e}{1-e}$$

= (27)

$$e = |\Gamma|$$

$$\Gamma = \frac{Z_{L3} - Z_{03}}{Z_{L3} + Z_{03}}$$

$$Z_{L3} = Z_{in} = \underbrace{Z_{118}}_{= jZ_{02}\tan\beta l} + Z_{in1}$$

$$= jZ_{02}\tan\beta l \quad \tan\left(\frac{2\pi}{\lambda} \cdot \frac{l}{\lambda/8}\right) \approx 1 \\ = j30$$

$$Z_{L3} = j30 + 60$$

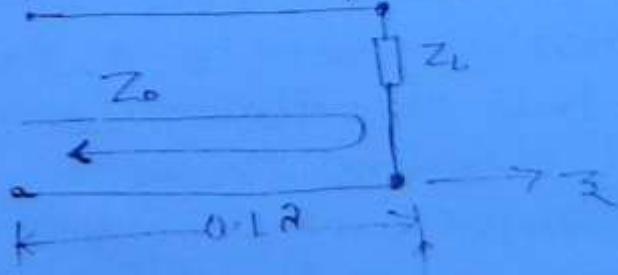
$$\Gamma = \frac{Z_{L3} - Z_{03}}{Z_{L3} + Z_{03}} = \frac{60 + j30 - 60}{60 + j30 + 60} = \frac{j30}{120 + j30}$$

$$= \frac{j1}{4+j1}$$

find $e = |\Gamma|$

$$\text{find } s = \frac{1+e}{1-e}$$

$$\Gamma_s = ? \quad \Gamma_L = 0.6 e^{-j30^\circ}$$



$-z = i$
source end

$z = 0$
load end

$$\Gamma_s = \epsilon_s \cdot e^{j\phi} \\ = 0.6 e^{j\phi}$$

$\Theta = \underbrace{\phi}_{\text{due to path difference}} + \underbrace{(-30^\circ)}_{\text{due to load}}$

(28)

$$\phi = (\text{path diff.}) \times \frac{2\pi}{\lambda} \Rightarrow 23 \cdot \frac{2\pi}{\lambda} \Big|_{\lambda=1} \\ = -23 \cdot \frac{2\pi}{\lambda}$$

$$\Theta = \phi_1 + (-30^\circ)$$

$$\phi_1 = -72^\circ - 30^\circ$$

$$\phi_1 = -102^\circ$$

$$\Gamma_s = \epsilon_s \cdot e^{j\phi} \\ = 0.6 e^{-j102^\circ}$$

$$\Gamma_s = 0.6 e^{+j258^\circ}$$

Aus:

$$= -2 \times 0.1 \lambda \times \frac{2\pi}{\lambda}$$

$$= -0.4\pi$$

$$= -0.4 \times 180^\circ$$

$$\phi = -72^\circ$$

Distortion less line :-

$$\gamma = \alpha + j\beta$$

$$= \frac{1}{j} (R + j\omega L) (G_1 + j\omega C)$$

$$\Rightarrow \alpha = f(\omega, \omega^2, \omega^4)$$

$$\beta = f(\omega, \omega^2, \omega^4)$$

(29)

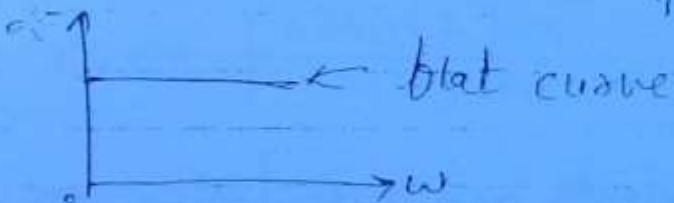
frequency distortion

$$\alpha = f(\bar{\omega}, \omega^2, \omega^4)$$

Causes freq. distortion.

$$\alpha \neq f(\omega)$$

To avoid freq. distortion



equalizes

lattice network.

① The frequency distortion occurs since various frequency components are subjected to different amount of attenuation.

This changes the quality of the voice signal.

② To avoid freq. distortion all the frequency components must be subjected to same amount of attenuation so that quality of voice signal remains same.

Therefore freq. response attenuation constant, α should be constant & independent

of freq. $f(\omega)$

practically the freq. distortion is avoided by using an equaliser which represents a lattice $n(\omega)$.

(30)

phase or delay distortion :-

$$\beta = f(\omega, \omega^2, \omega^4)$$

Causes phase or delay distortion

$\beta = \frac{\omega}{v_p}$ must be constant to avoid dispersion

$$\beta \propto \omega$$

$$\beta = K\omega$$



① Various frequency const. are subjective to diff. amount of phase shift which causes the use of delay distortion.

To Avoid the phase or delay distortion phase const. β must be directly proportional to freq. of operation so that the phase velocity along the line must remain const.

therefore the freq. response of the line const. β must have linear variation with the var. frequency variation.

② practically freq. distortion is avoided by using phase or delay equaliser.

for distortion less line

(3)

$$\boxed{\alpha \neq f(\omega)} \quad \text{to avoid freq. distortion.}$$
$$\boxed{\beta = k\omega} \quad \text{to Avoid phase of delay distortion.}$$

$$\Rightarrow \boxed{RC = LG_1} \quad \begin{matrix} \text{condition} \\ \text{for a distortion less line.} \end{matrix}$$

* Characteristic Impedance of distortion less line:

$$Z_0 = \sqrt{\frac{R+j\omega L}{G_1+j\omega C}} \quad \begin{matrix} \text{if } RC = LG_1 \\ R = \frac{LG_1}{C} \end{matrix}$$

$$\Rightarrow Z_0 = \sqrt{\frac{LG_1/C + j\omega L}{G_1 + j\omega C}} = \sqrt{\frac{L(G_1 + j\omega C)}{C(G_1 + j\omega C)}}$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad \begin{matrix} \text{Same as that of lossless} \\ \text{line.} \end{matrix}$$

case 1: at High frequency

$$\omega L \gg R$$

$$\omega C \gg G_1$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{B+j\omega C}} \cong \sqrt{\frac{L}{C}}$$

case 2: At low frequency.

$$\omega L \ll R$$

$$\omega C \ll G_1$$

$$Z_0 = \left[\frac{R+j\omega L}{G_1+j\omega C} \right] \cong \left[\frac{R}{G_1} \right]$$

(32)

$$Z_0 \cong \sqrt{\frac{L}{C}} \quad \therefore RC = LG_1 \\ \cong \frac{R}{G_1} = \frac{L}{C}$$

- ① The chart's gmp. of a lossless & distortionless line is same & depends only upon the primary constant L & C .
- ② At high freq. the TL will always behaves as a lossless as well as distortion less line irrespective of the condition $RC = LG_1$ is satisfied or not.
- ③ At low frequency the line in general does not behaves as a distortion less line unless the condition $RC = LG_1$ is satisfied.

Summary :-

| Distortionless line | lossless line |
|------------------------------|------------------------------|
| $\alpha \neq f(\omega)$ | $R = G_1 = 0$ |
| $\beta = kw$ | $\alpha = 0$ |
| $\beta = \frac{\omega}{V_p}$ | $\beta = \frac{\omega}{V_p}$ |
| $V_p = \frac{1}{\sqrt{LC}}$ | $V_p = \frac{1}{\sqrt{LC}}$ |
| $Z_0 = \sqrt{\frac{L}{C}}$ | $Z_0 = \sqrt{\frac{L}{C}}$ |
| $\boxed{RC = LG_1}$ | |

Conclusion :- ① for a distortion less line α must be independent of freq. but may have a finite value.

Therefore in general a distortion line is a lossy line. 33

(2) for a lossless line $R + G_I$ are zero if therefore the condition $RC = LG_I$ is always satisfied.

Since $\alpha = 0$ for such line it's independent of freq. w .

Therefore any lossless line is always a distortion less line.

(3) practically any transmission line is always a lossy line & therefore cannot be a distortion less line.

Such line may be made distortion less by suitably selecting the material such that the condition $RC = LG_I$ is always satisfied.

Ex:

$$Z_0 = 50 \Omega$$
$$R = 0.1 \Omega/m$$

To find
 α for distortionless
line.

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\frac{L}{C} = Z_0^2 = 2500$$

$$RC = LG_I \Rightarrow G_I = R \cdot \frac{C}{L} = \frac{0.1}{2500} S/m$$

$$V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{1}{Z_0} (V^+ e^{-j\beta z} - V^- e^{+j\beta z})$$

(35)

$$V|_{z=0} = V_s = \frac{V^+ e^{+j\beta l} + V^- e^{-j\beta l}}{e^{\pm j\beta l}} = \cos \beta l \pm j \sin \beta l$$

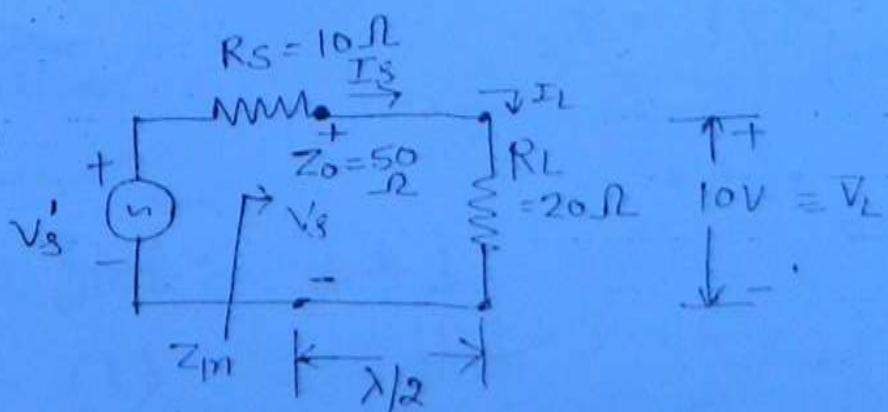
$$\frac{V^-}{V^+} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}; \quad Z_L = \frac{V_L}{I_L}$$

$$V^+ \Rightarrow -\frac{V_L}{V^+} = T; \quad V^+ = \frac{V_L}{T}$$

$$T = \frac{2Z_L}{Z_L + Z_0}; \quad Z_L = \frac{V_L}{I_L}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cos \beta l & j \sin \beta l \\ (j/Z_0) \sin \beta l & \cos \beta l \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

$$\begin{bmatrix} V_L \\ I_L \end{bmatrix} = \begin{bmatrix} \cos \beta l & -j Z_0 \sin \beta l \\ (-j/Z_0) \sin \beta l & \cos \beta l \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$



To bind: $\left\{ \begin{array}{l} V_s; I_s \\ T; Z_m; V_s \end{array} \right.$

$$\omega \neq f(\omega)$$

(34)

$$V = [(R+j\omega L)(G+j\omega C)] = \omega + j\beta$$
$$\omega \neq f(\omega)$$

$$V = \omega + j\beta = \sqrt{RG}$$

$$\omega = \sqrt{RG} = \sqrt{\frac{0.1 \times 6.1}{2500}}$$

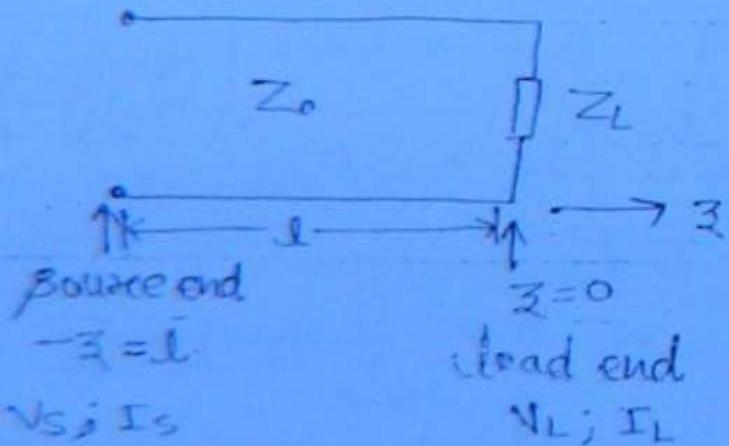
$$\omega = 0.002 \text{ neper/m}$$

$$1 \text{ neper} = 8.686 \text{ dB}$$

$$\omega = 0.002 \times 8.686$$

--- dB

Transmission Matrices :-



$$\left. \begin{array}{l} V_S \\ I_S \end{array} \right\} \leftrightarrow \left\{ \begin{array}{l} V_L \\ I_L \end{array} \right.$$

for a lossless line.

$$I_L = \frac{V_L}{R_L} = \frac{10}{20}$$

(36)

$$I_L = \frac{1}{2} \text{ Amp.}$$

$$V_S = \underbrace{\cos \beta L \cdot V_L}_{\cos(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2})} + j \frac{Z_0}{10} \sin \beta L \cdot I_L \leftarrow \frac{1}{2}$$

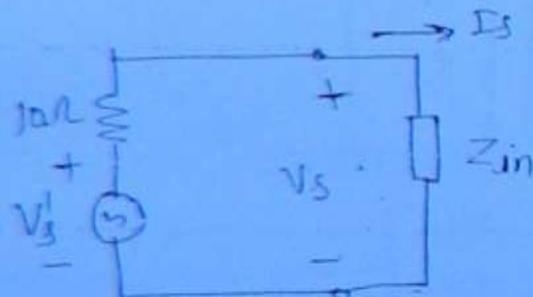
$$\sin(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}) = 0$$

$$V_S = -10V$$

Simple method for I_S

$$Z_{in} = Z \Big|_{\lambda/2} = Z_L = 20 \Omega$$

$$Z_{in} = 20 \Omega$$



$$I_S = \frac{V_S}{Z_{in}} = \frac{-10}{20} = -\frac{1}{2} A$$

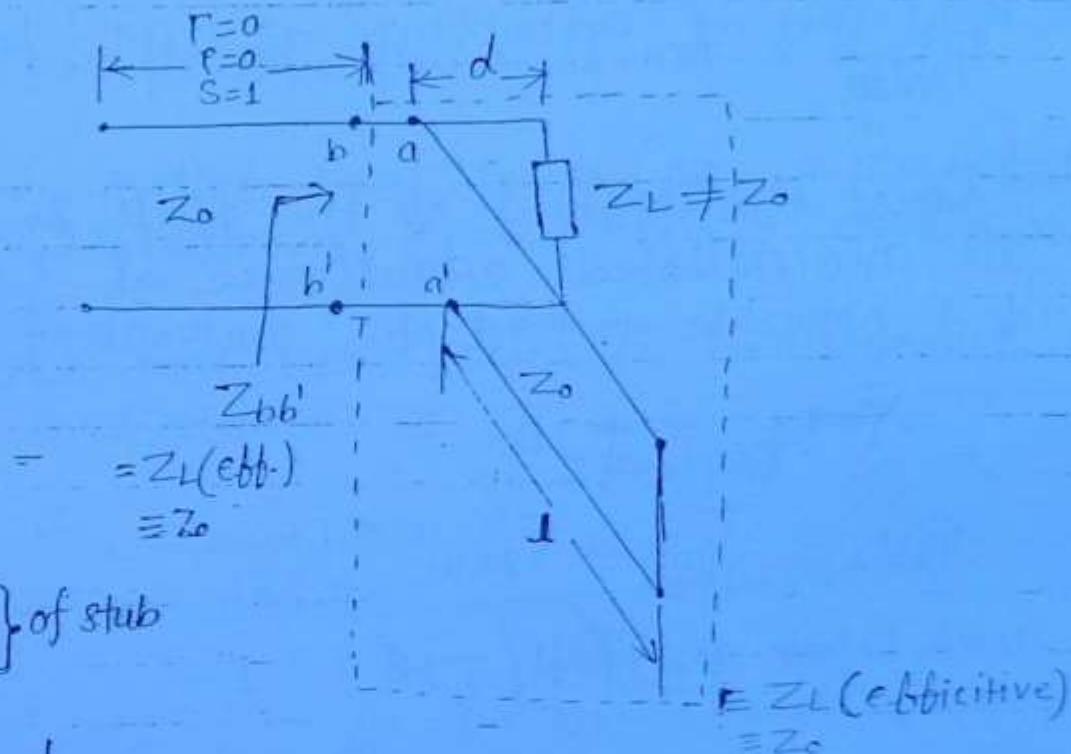
$$V_S' = I_S (10 + 20)$$

$$V_S' = -\frac{1}{2} (10 + 20)$$

$$[V_S' = -15V]$$

Stub Matching :-
Required when $Z_L \neq Z_0$

Case 1 :- Short circuit shunt stub :-



$$S_{\text{main line}} = 1$$

$$S = \frac{1+\rho}{1-\rho}$$

$$S_{\text{stub line}} = \infty$$

$$\rho = |\Gamma|$$

$$S_{\text{load line}} = ?$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

To bind :- d & l

$$(1) Z_L = \text{given.}$$

$$(2) \text{bind } \left(\frac{Z_L}{Z_0} \right) = \bar{Z}_L$$

$$\therefore \text{bind } \bar{Z}_L = \frac{1}{Z_L}$$

(4) Move a distance 'd' such that

$$\bar{Y}_{aa'} = 1 \pm j\bar{B} \quad (38)$$

Comments : The location of the stub will adjust their normalised value of real part of admittance at 'aa'' becomes a unity.

(5) adjust the length l of the stub so that its normalised admittance at 'aa'' is equal & opposite to that of imaginary part of

$$\bar{Y}_{aa'} = \mp j\bar{B}$$

(6) $\bar{Y}_{bb'} = \bar{Y}_{aa'} + \bar{Y}_{z\text{stub}}$

$$= 1 \pm j\bar{B} + (\mp j\bar{B}) = 1$$

(7) $\bar{Z}_{bb'} = \frac{1}{\bar{Y}_{bb'}} = 1$

(8) $\frac{\bar{Z}_{bb'}}{Z_0} = 1 ; Z_{bb'} = Z_0 \equiv Z_L \text{ (effectively)}$

(9) Hence, the TL is perfectly matched with the effectively load imp. at 'bb'

(10) therefore to the left side of 'bb' the line is perfectly matched, there is no reflected waves, no standing wave pattern, reflection coefficient is zero & therefore has a

Minimum value of VSWR is unity.

(39)

therefore max. power is transferred from the source to load.

the stub line will affect only the reactive power since the s.p. adm. of stub is purely reactive.

this reactive power is not a useful power.

the entire Real power or the useful power is transmitted to the load impedance Z_L .

Hence more Real power is been transferred to load impedance Z

$$Z_L = 100 + j 300 \Omega \quad \text{→ Real part}$$

$$Z_0 = 100 \Omega \quad \text{→ Polar form}$$

Both are equal then normalised is equal

$\therefore d=0$ stub is connected at the load.

$$Z_L = 200 + j 300$$

$$Z_0 = 100$$

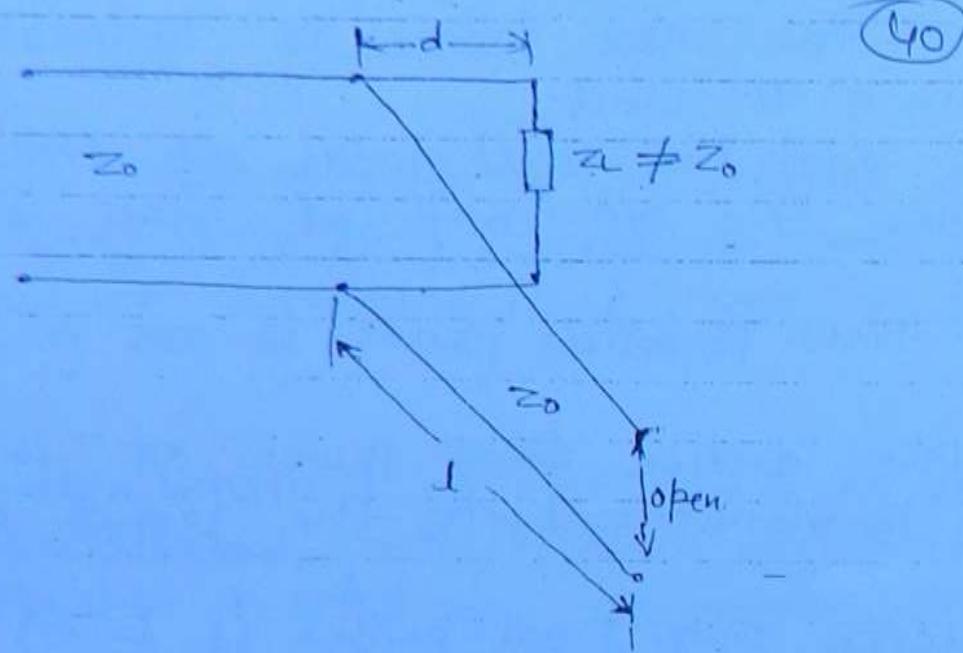
$$d \neq 0$$

stub is connected at some specific distance from the load.

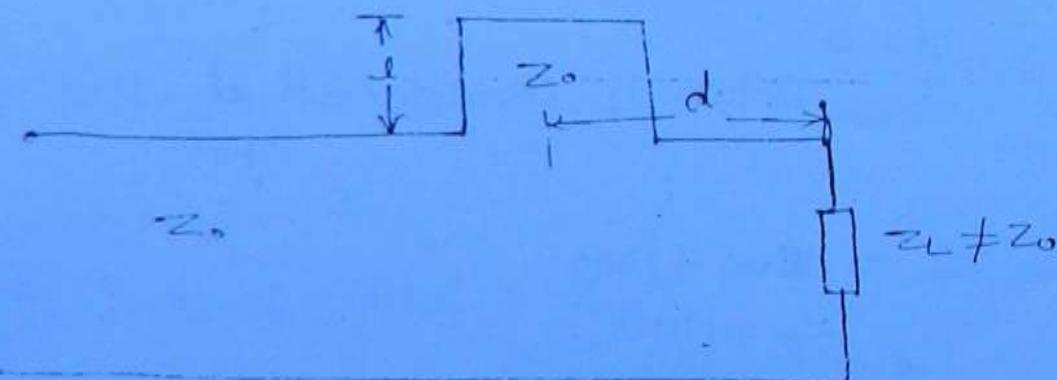
① The stub Matching is used only by a short circuit or open circuit stub line.

discrete components of L & C are not used for stub matching.

case-2 :- open circuit shunt stub :-

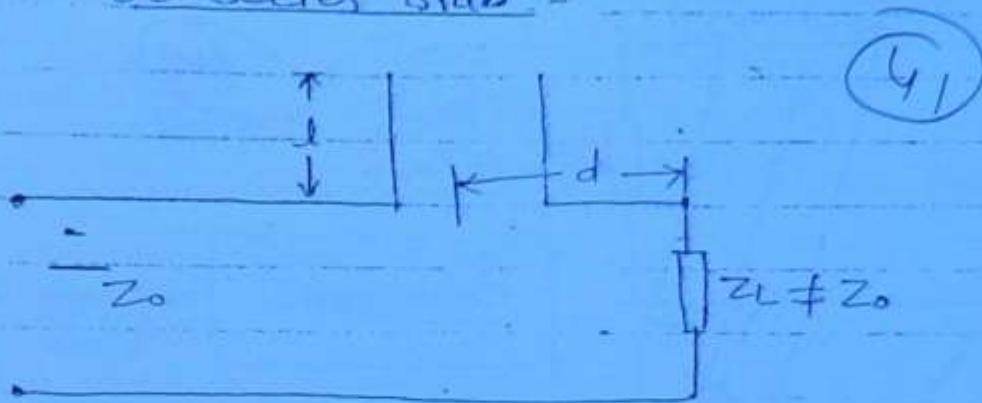


case-3 :- SC series stub :-



case: 4

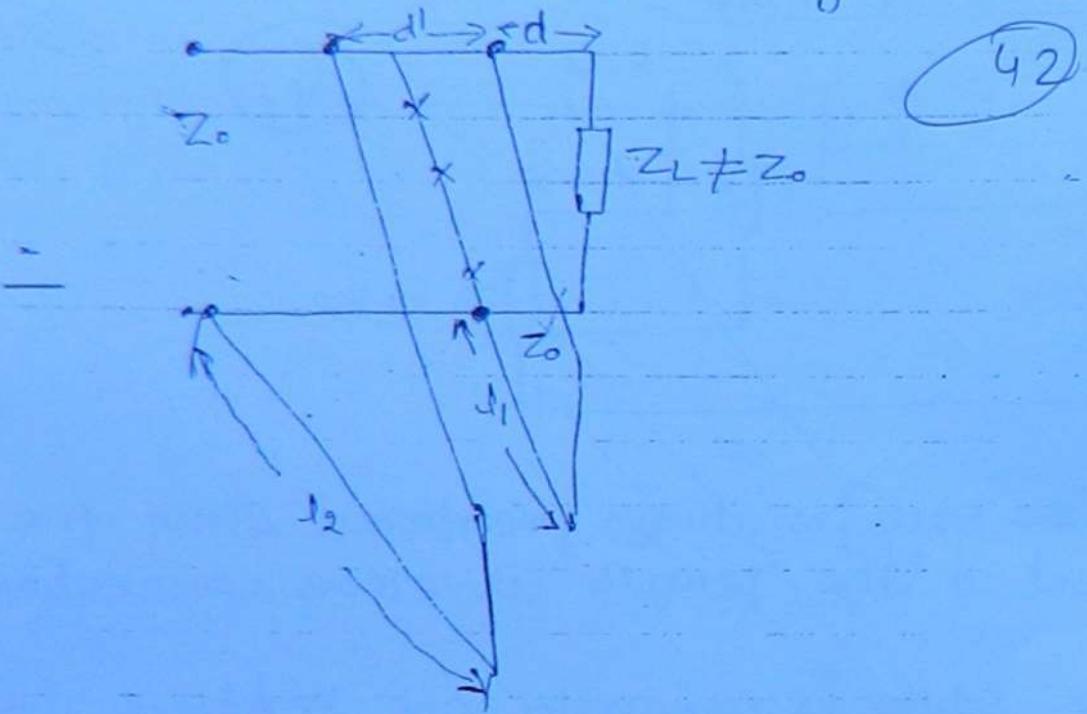
oc series stub:-



(4)

- (1) The sc stub is always preferred since the adjustment of the length is more conventional practically.
- (2) the o.c. stub is normally not preferred since
 - (a) the adjustment of the length is practically not convention.
 - (b) an o.c. stub x has a antenna & em. power is radiated from it.
- (3) The shunt stub is always preferred since the main line remains unaffected when the load is varied over wide range.
- (4) the series stub is never preferred since the main line is affected if the load z_L is varied over a wide range.
- (5) Therefore for variable load the s-c. shunt stub Matching is the best whereas o.c. series stub matching is the worst.

Double stub matching



42

- ① Double stub matching is generally preferred over the single stub matching because of more flexibility in the variation of length of each stub l_1 & l_2 .

using double stub matching we are not able to match all the type of load with the characteristic imp. of the line

Variation of impedance along the line

$$Z_{max} = \frac{V_{max}}{I_{min}} = \frac{V_{max}}{V_{min}/Z_0} = Z_0 \cdot S$$

(43)

$$\left(\frac{Z_{max}}{Z_0} \right) = \boxed{\bar{Z}_{max} = S}$$

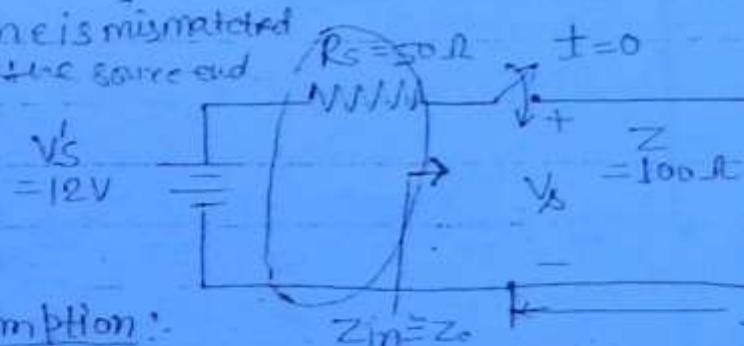
$$Z_{min} = \frac{V_{min}}{I_{max}} = \frac{V_{min}}{V_{max}/Z_0} = \frac{Z_0}{S}$$

$$\left(\frac{Z_{min}}{Z_0} \right) = \boxed{\bar{Z}_{min} = \frac{1}{S}}$$

$$\boxed{\frac{1}{S} \leq \bar{Z} \leq S}$$

Transient response in T.L. :-

Line is mismatched
at the source end



Assumption:

① Line is lossless.

② L is large $Z_m \approx Z_0 = 100 \text{ ohms}$

③ $T = l/v_p$ time taken by voltage

waveform to reach from source to load end.

$T \approx 400 \text{ nsec}$

$\approx 500 \text{ nsec}$

Line is mismatched at
load end ($Z_L \neq Z_0$)

$$\begin{aligned} Z_L &= 200 \text{ ohms} \\ L &= \frac{Z_L - Z_0}{Z_L + Z_0} \\ &= \frac{200 - 100}{200 + 100} \\ &= \frac{1}{3} \end{aligned}$$

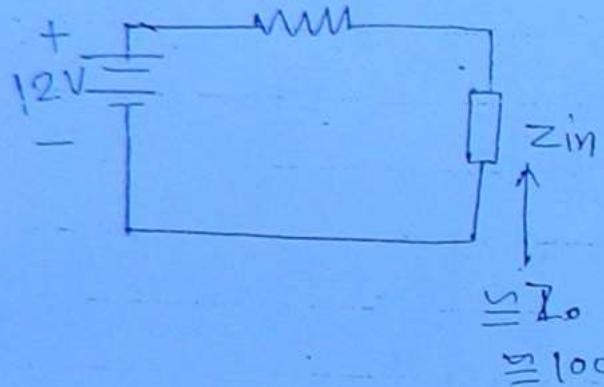
$$T_s = \frac{R_s - Z_0}{R_s + Z_0} = \frac{50 - 100}{50 + 100} = -\frac{1}{3}$$

(44)

To find:

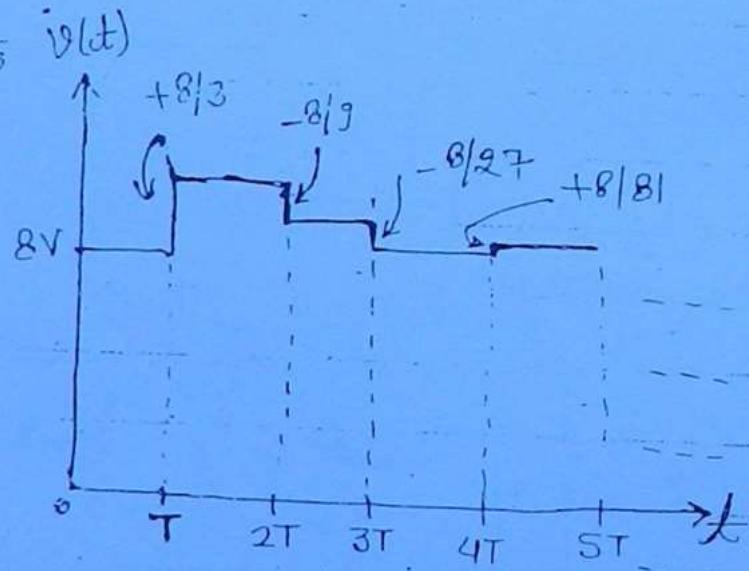
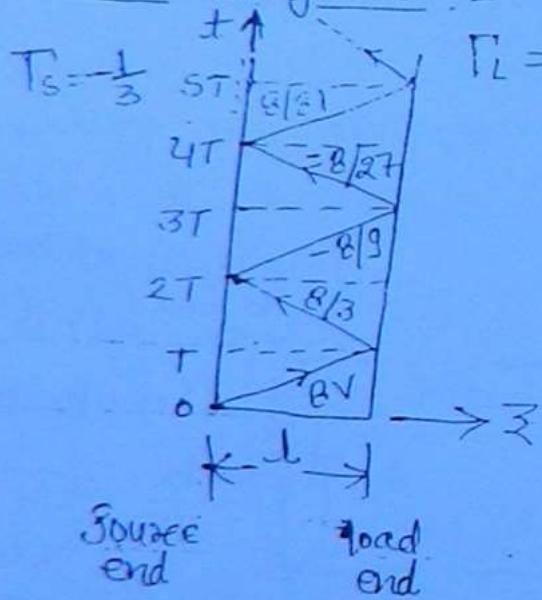
$v(t)$ vs t

transient response of the line.



$$\begin{aligned} V_s &= \frac{100}{50+100} \times 12 \\ &= \frac{2}{3} \times 12 \\ &= 8V \end{aligned}$$

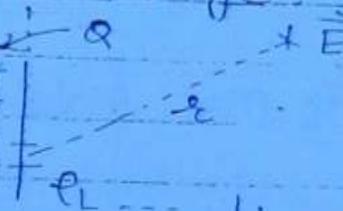
Source diagram :-



A Sources of \vec{E} (electrical field) :-

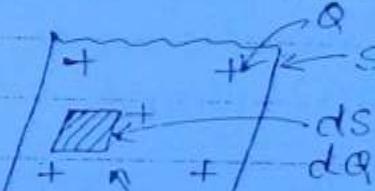
① Point charge :-  $\times \vec{E} \propto \frac{1}{\epsilon^2}$
 Coulomb's law

② Line charge :-


 $dQ = \rho_L \cdot dL$
 $Q = \int_C \rho_L dL$

linear charge density
charge per unit length (C/m)

③ Surface charge :-

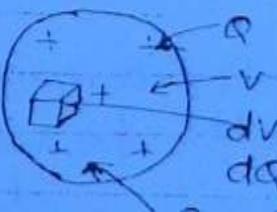

 $dQ = \rho_s \cdot ds$
 $Q = \iint_S \rho_s \cdot ds$

surface charge density (C/m²)

$$dQ = \epsilon_s \cdot ds$$

$$Q = \iint_S \epsilon_s \cdot ds$$

④ Volume charge :-


 $dQ = \rho_v \cdot dv$
 $Q = \iiint_V \rho_v \cdot dv$

Volume charge density (C/m^3)

$$dQ = \epsilon \cdot dv$$

$$Q = \iiint_V \epsilon \cdot dv$$

Δ Source of \vec{B} (mag. field)

(46)

1) line current :-

$$\vec{B} \propto \frac{I}{d}$$

Biot-savart's law

$$(A) I = I_0 \Rightarrow \vec{B} = \vec{B}_0 \text{ static field}$$
$$= I_0 \sin \omega t$$

$$C: \Rightarrow \vec{B} = \vec{B}_0 \sin \omega t$$

current element

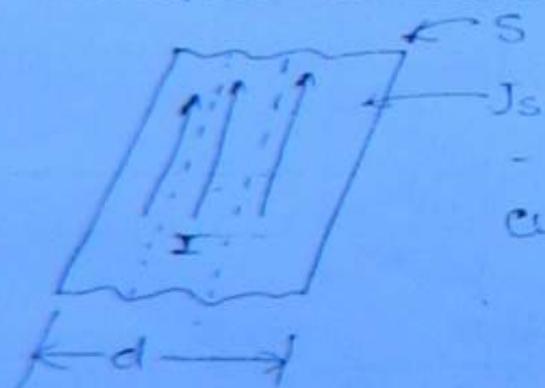
$$\int I dl = I dt$$

time varying mag. field

convention:-

by convention the direction of current I & elementary length dl are taken same in all electro-mag. problems.

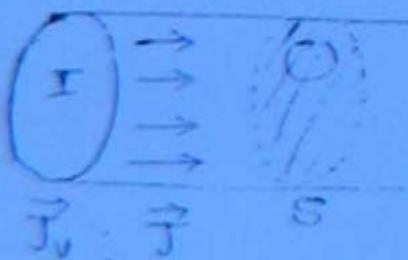
Surface current :-



surface current density
(A/m)

$$I = \int_d J_s \cdot dl$$

Volume current :-



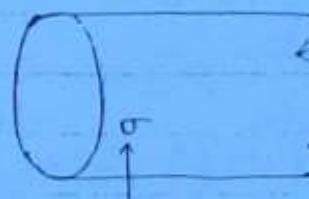
Volume current density
(A/m^2)

$$I = \iiint_V \vec{J} \cdot d\vec{s}$$

Continuity eqn.

$$I = - \frac{dQ}{dt}$$

Conversion of charge (47)



Conductivity
(S/m)

$$\begin{aligned} Q_i &= 5 \mu C \\ \Delta t &= 1 \text{ m sec.} \\ Q_f &= 2 \mu C \end{aligned}$$

$$\begin{aligned} I &= - \frac{dQ}{dt} = - \frac{Q_f - Q_i}{\Delta t} \\ &= - \frac{(2-5) \times 10^{-6}}{1 \times 10^{-3}} \\ &= 3 \text{ mA} \end{aligned}$$

$$\rightarrow \nabla \cdot \vec{J} = - \frac{\partial \rho}{\partial t}$$

Volume current
density (A/m^2)

Volume charge density
(C/m^3)

$$\vec{J} = \sigma \vec{E}$$

Ohm's law

$$\text{if } \rho = 0 ; \frac{\partial \rho}{\partial t} = 0$$

\vec{J} is solenoidal in nature.

$$\rightarrow \nabla \cdot \vec{J} = 0$$

KCL eqns.

$$\vec{J} = \sigma \vec{E}$$

Ohm's law

Statement: (1) The divergence of volume current density \vec{J} at any point in the electrostatic field is always equal to the rate of decrease of the volume charge density ρ with respect to time t .

therefore there is a continuity b/w the decrease of volume charge density now & the corresponding production volume current density \vec{J} .

(2) for a charge free region the divergence of volume current density \vec{J} at any point is always equal to zero.

and hence volume current density is always
solenoidal & forms a closed loop.

$$\rho = 0$$

(48)

-- on perfect cond. ($\sigma \approx \infty$)

-- on dielect cond. ($\sigma = 0$)

① The charge density ρ is always equal to zero on a perfect conductor or a perfect dielectric conductor.

② Volume charge density is finite on a medium where the conductivity is finite.

The decay of volume charge density will depend upon the conductivity of Region.

Higher is the conductivity higher is the rate decay of charge & vice-versa.

A

Maxwell's eqns. in their general time varying form :-

differential form
(Point form)

$$\nabla \cdot \vec{D} = \rho$$

integral form

$$\oint_S \vec{D} \cdot d\vec{S} = q = \iiint_V \rho dV$$

Gauss law for electric fields
electric flux

$$\nabla \cdot \vec{B} = 0$$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

Gauss law for
magnetic field

$$③ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{aligned}\nabla \times \vec{H} &= \vec{J}_c + \vec{J}_d \\ &= \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}\end{aligned}$$

$$\underbrace{\oint_C \vec{E} \cdot d\vec{l}}_{\text{emf}} = - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

(49)

faraday's law of emf.
induction.

$$\underbrace{\oint_C \vec{H} \cdot d\vec{l}}_{\text{mmf}} = \iint_S (\vec{J}_c + \vec{J}_d) \cdot d\vec{s}$$

Modified Ampere's
circuit law

--- eqn. using displacement
current density concept

$$\vec{J}_c = \sigma \vec{E}$$

Cond. current density (A/m^2)

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} \quad \text{displacement current density (A/m^2)}$$

statements:

① (i) The diversions of electrical blur density \vec{D} at any point on the electro mag. Region is always equal to the volume charge density ϵ .

(ii) The net electrical blur passing through any closed surface area S is always equal to total charge enclose with in the surface Area S .

② 2(a) The diversions of mag. blur density \vec{B} is always equal to zero since

reasons: (i) Mag. field \vec{B} line are always closed in nature.

(ii) The Mag. charges in the isolated form do not exists in nature.

2(b) The net mag. flux passing through any closed surface Area S is always equal to zero.

(50)

3) 3(a) : The curl of elec. field intensity \vec{E} is always equal to the rate of decrease of mag. flux density \vec{B} w.r.t time t .

3(b) Net emf produced is always equal to the surface integral of rate of decrease of mag. flux density \vec{B} w.r.t time t .

4(a) The curl of mag. field intensity \vec{H} is always equal to the sum of conduction current density \vec{J}_c & the displacement current density \vec{J}_B .

4(b) Total mmf produced is always equal to the surface integral of the sum of conduction current density \vec{J}_c & displacement current density \vec{J}_B .

Special cases :-

Case 1: for static fields :-

$$\boxed{\begin{aligned}\frac{\partial \vec{B}}{\partial t} &= 0 \\ \frac{\partial \vec{D}}{\partial t} &= 0\end{aligned}}$$

use 2 :- for perfect dielectric ($\sigma = 0$)

or

non-conducting Medium

or

loss less medium

or

free space

(5)

$$\boxed{\begin{aligned} \vec{J}_C &= \sigma \vec{E} = 0 \\ \epsilon &= 0 \end{aligned}}$$

use 3 :- for Good Conductor

--- σ is High

$$\boxed{\begin{aligned} \vec{J}_B &\approx 0 \\ \epsilon &\approx 0 \end{aligned}}$$

for time-Harmonically OR sinusoidally varying fields :-

$$\left\{ \begin{array}{l} \vec{D} = \vec{D}_0 e^{j\omega t} \\ \vec{B} = \vec{B}_0 e^{j\omega t} \end{array} \right.$$

so que:

$$① \nabla \cdot \vec{D} = \rho$$

$$② \nabla \cdot \vec{B} = 0$$

$$③ \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} = - j\omega \mu \vec{H}$$

$$④ \nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

$$= \sigma \vec{E} + j\omega \epsilon \vec{E}$$
$$= (\sigma + j\omega \epsilon) \vec{E}$$

$$\Rightarrow j\omega$$

$$\Rightarrow (j\omega)^2 = -\omega^2$$

Loss tangent (tan δ)

$$\vec{J}_{\text{total}} = \vec{J}_c + \vec{J}_d$$

$$= \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

$$\approx \sigma \vec{E} + j\omega \epsilon \vec{E} \quad \text{--- for general medium} \\ (\mu, \epsilon, \sigma)$$

$$\approx \sigma \vec{E} \quad \text{--- for good conductors.}$$

$$\approx j\omega \epsilon \vec{E} \quad \text{--- for good dielectrics.}$$

$$\tan \delta = \frac{|\vec{J}_c|}{|\vec{J}_d|} = \frac{|\sigma \vec{E}|}{|j\omega \epsilon \vec{E}|} = \frac{\sigma E}{\omega \epsilon}$$

| | |
|--|------------------|
| $\tan \delta = \frac{\sigma}{\omega \epsilon} >> 1$ | Good conductor. |
| $\tan \delta = \frac{\sigma}{\omega \epsilon} \ll 1$ | Good dielectric. |

Key points:

- 1) for Good conductors the conductivity is high & therefore the conduction current density is dominant.
- 2) for Good dielect. the conductivity is low, conduction current density is negligible & the displacement current density is more dominated.
- 3) the loss tangent is the ratio of the magnitudes of current conduction current density & displacement current density.
this is a major of total loss occurring in a material due to finite conductivity at specified frequency.
- 4) depending upon the frequency of operation my medium may behave as a good conductor

or good dielectric.

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In general any material may behaves as a good conductor at low freq. whereas some material may behaves as a good dielectric at high frequency.

Conclusion:

Therefore depending upon Application we operate a device at high freq. and low freq. so that it can operate at a good dielectric & good conductor.

→ Poynting's vector (\vec{P})

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\frac{V}{m}, \frac{A}{m}$$

$$W/m^2$$

power density at a point

$$\text{power} = \oint \int_S \vec{P} \cdot d\vec{S}$$

$$\text{if } (\vec{E} \perp \vec{H}) \perp \vec{P}$$

Transverse em. wave

TEM wave \rightarrow uniform plane wave (Plane wave)

$$\vec{P} = \vec{E} \times \vec{H} = EH \sin\theta \hat{n}$$

$$\sin\theta = 1$$

$$|\vec{P}| = [P = EH]$$

$$= E_{\text{rms}} \cdot H_{\text{rms}}$$

$$\frac{E}{H} = \eta$$

Intrinsic impedance of the Medium.

$$\frac{V/m}{A/m} = \eta$$

$$\frac{E}{H} = \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad (54)$$

--- for general lossy medium (μ, ϵ, σ)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \quad \text{--- for lossless medium } (\sigma = 0)$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 120\pi \approx 377 \text{ N} \cdot \text{A}^{-1}$$

--- for free space
 $\sigma = 0$
 $\mu = \mu_0$
 $\epsilon = \epsilon_0$

$$|\vec{P}| = P = \vec{E} \cdot \vec{H}$$

$$= \eta H^2$$

$$= \frac{E^2}{\eta}$$

--- E; H
rms Value

$$P = \frac{1}{2} E_m H_m$$

$$= \frac{1}{2} \eta H_m^2$$

$$= \frac{1}{2} \frac{E_m^2}{\eta}$$

Imp. points :- ① The Poynting Vector (P) represent power density at a point.

2) When 'integrate' over any closed surface Area the total power flow on the Specified direction.

The P vectors \vec{P} gives the direction of propagation of em waves & is always perpendicular to the plane made by $\vec{E} \times \vec{H}$ vectors.

- (4) for a transfer e.m. waves \vec{E} , \vec{H} & \vec{P}
are mutually perpendicular to each other.
- (5) In any e.m. region the ratio b/w the \vec{E} &
 \vec{H} fields is always constant. & is represented
by intrinsic impedance of medium.
- (6) This imp. depends only upon the const. of
the medium.
- for a free space this imp. has a
university const. value of $120\pi \text{ N}$ or 377 N

Ex: An e.m. waves is travelling along $-y$ direction
& has only x -comts of elect. fields.
find the magnetic field intensity
associated with e.m. waves.

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\vec{P} = -P_y \hat{a}_y \quad \text{--- direction of prop. of wave.}$$

$$\vec{E} = E_x \hat{a}_x$$

To find:-

$$\vec{P} = \vec{E} \times \vec{H}$$

$$(-P_y \hat{a}_y) = (E_x \hat{a}_x) \times (+H_z \hat{a}_z)$$

$$= \hat{a}_y \begin{pmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{pmatrix} = +\hat{a}_z \quad (\text{cross product})$$

$$\boxed{\vec{H} = +H_z \hat{a}_z}$$

Ex: find the displacement current at $t=0$
through a 10pf capacitor if the voltage across
it is given by

$$V(t) = 0.18 \sin 120\pi t \text{ V}$$

$$C = 10\text{pf}$$

$$I_d|_{t=0} = ? = I_d \cdot A$$

$$J_A \cdot A = A \frac{\partial D}{\partial t} = eA \frac{\partial E}{\partial t} = \underbrace{\frac{eA}{d}}_C \frac{\partial}{\partial t} [V(t)]$$

$$C \frac{\partial}{\partial t} (0.1 \sin 120\pi t) = \frac{10 \times 0.1 \times 120\pi}{10 \times 12\pi} \cos 120\pi t \Big|_{t=0} = 1$$

$$\equiv 120\pi - \text{PA}$$

$$\equiv 377 - \text{pA}$$

$$\equiv 0.377 - \text{nA}$$

(56)

Q3: An e.m. wave is travelling in a lossless medium is $\mu_r = 1$ & $\epsilon_r = 4$ & has a power density $P = 4 \text{ W/m}^2$:

Calculate the max. value of E & H phase.

$$|\vec{P}| = P = 4 \text{ W/m}^2$$

$$\mu_r = 1 ; \quad \mu = \mu_0 \mu_r = \mu_0$$

$$\epsilon_r = 4 ; \quad \epsilon = \epsilon_0 \epsilon_r = 4 \epsilon_0$$

to find:

$$E_m ; H_m$$

$$P = \frac{1}{2} E_m \cdot H_m \quad \eta = \frac{E_m}{H_m}$$

$$= \frac{1}{2} \eta H_m^2$$

$$= \frac{1}{2} \frac{E_m}{\eta} \leftarrow$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{\eta_0}{\sqrt{4\epsilon_0}}$$

$$= \frac{120\pi}{2} = 60\pi \text{ N} \quad \frac{P}{4}$$

$$P = \frac{1}{2} \frac{E_m^2}{\eta}$$

$$E_m = \sqrt{2 \eta P} = \sqrt{2 \times 60\pi \times 4} \text{ V/m}$$

$$\eta \quad \frac{E_m}{H_m} \quad \Rightarrow \quad \frac{E_m}{60\pi} = H_m \quad \text{A/m}$$

Wave eqns.

Mathematical form of e.m. wave

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \textcircled{1}$$

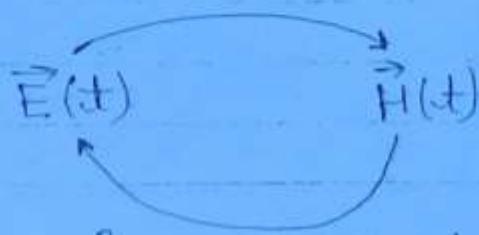
(57)

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{B}}{\partial t} \quad \textcircled{2}$$

$$\vec{B}(t) \rightarrow \frac{\partial \vec{B}}{\partial t} \xrightarrow{\text{eq. } \textcircled{1}} \vec{E}(t) \rightarrow \vec{D}(t)$$

$\vec{H}(t)$ $\frac{\partial \vec{B}}{\partial t}$

eq. $\textcircled{2}$



$$c = \frac{1}{2} \epsilon E^2, \quad \frac{1}{2} \mu H^2 = \omega_m$$

$$-\frac{\partial (w_e)}{\partial t} \quad \frac{\partial (w_m)}{\partial t}$$

$$P_e \quad P_m$$

power flows in a direction

\Rightarrow e.m. wave propagates.

Conclusion:

- ① due to time vary. etc. & mag. field the rate of change of ele. energy is transformed to the rate of change of mag. energy.

e Vice - versa.

- Due to this state of change the波 propagates in a particular direction which is given by Poynting vector (\vec{P})
- therefor e.m. waves propagates in the direction given by Poynting vector (\vec{P})

Date - 28-07-2010

$$\log_{10} \left(\frac{V_o}{V_i} \right) = \text{neper/m}$$

(58)

$$20 \log_{10} \frac{V_o}{V_i} \text{ --- dB}$$

$$10 \log_{10} \frac{P_o}{P_i} \text{ --- dB}$$

Wave eqns

$$\text{eqn.} \quad \text{--- (1)}$$

$$\text{eqn.} \quad \text{--- (2)}$$

eliminate \vec{H}

$$\nabla^2 \vec{E} = \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

loss factor

wave eqns on \vec{E}

propagation factor

Similarly

$$\nabla^2 \vec{H} = \mu_0 \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

wave eqns

on \vec{H}

Comments :

- ① Hence has the e.m. waves propagates in general medium it is subjective to attenuation as well as phase change.

Therefor the elec. field strength decrease as the wave propagate in a particular direction which is given by Poynting vector.

(2) The behaviour of elec. & mag. field are exactly same except that the elec. & mag. fields are perpendicular to each other.

(59)

Special case :

for sinusoidally varying fields.

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$$\frac{\partial^2}{\partial t^2} \rightarrow -\omega^2$$

$$\therefore \nabla^2 \vec{E} = \mu_0 \cdot j\omega \vec{E} + \mu_0 \epsilon (-\omega^2 \vec{E})$$

$$\rightarrow \boxed{\nabla^2 \vec{E} = \gamma^2 \vec{E}}$$

$$\gamma^2 = \mu_0 \cdot j\omega - \omega^2 \mu \epsilon$$

$$\gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)}$$

$$= \alpha + j\beta$$

$$j\sigma \rightarrow 0$$

lossless medium.

$$\gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)}$$

$$= \boxed{j\omega \sqrt{\mu \epsilon} = \alpha + j\beta}$$

$$\alpha = 0$$

$$\beta = \omega \sqrt{\mu \epsilon} = \frac{\omega}{v_p}$$

$$\boxed{v_p = \frac{1}{\sqrt{\mu \epsilon}}}$$

if $\mu = \mu_0$, $\epsilon = \epsilon_0$ } --- for free space

$$V_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = C = 3 \times 10^8 \text{ m/sec.} \quad (60)$$

$$\gamma = \sigma + j\beta$$

$$\gamma^2 = -\beta^2$$

$$\rightarrow \boxed{\nabla^2 \vec{E} = -\beta^2 \vec{E}} \quad \text{--- wave eqn for lossless medium. } (\sigma=0)$$

Ex: An em-wave is propagating in a general lossy medium has been const of μ, ϵ & σ in $+z$ direction and has only x -components of electrical field.

assuming sinusoidal variation find the value of the elect. field components.

prop. --- along $+z$

$$\vec{E} = E_x \hat{a}_x$$

Medium : μ, ϵ, σ

Sinusoidal Variation

To find

$$E_x$$

$$\nabla^2 \vec{E} = \gamma^2 \vec{E}$$

$$(\nabla^2 E_x) \hat{a}_x + (\nabla^2 E_y) \hat{a}_y + (\nabla^2 E_z) \hat{a}_z = \gamma^2 (E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z)$$

$$\nabla^2 E_x = \gamma^2 E_x$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_x}{\partial y^2} + \underbrace{\frac{\partial^2 E_x}{\partial x^2}}_{=0} = \gamma^2 E_x$$

(G)

characteristic eqn $m^2 = \gamma^2$
 $m = \pm \gamma$

$$E_x = A e^{+\gamma z} + B e^{-\gamma z}$$

along +z

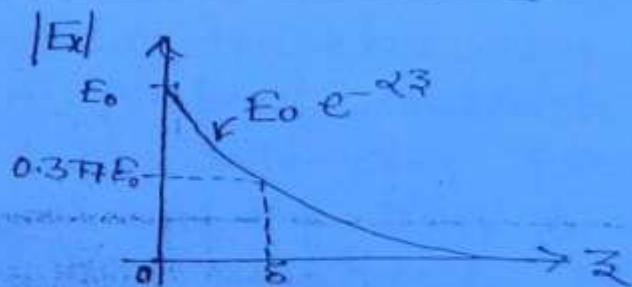
$E_x = E_0 e^{-\gamma z}$

Aus.

A Depth of penetration:

$$\begin{aligned} E_x &= E_0 e^{-\gamma z} \\ &= E_0 e^{-(\alpha + j\beta)z} \\ &= E_0 e^{-\alpha z} \cdot e^{-j\beta z} \end{aligned}$$

$|E_x| = E_0 \cdot e^{-\alpha z}$



If $\alpha \delta = 1$ for $z = \delta$ depth of penetration

$$\alpha \cdot \delta = 1$$

$$\delta = \frac{1}{\alpha}$$

$$\begin{aligned} E_x &= E_0 \cdot e^{-\alpha z} = E_0 e^{-1} \quad \text{at } z = \delta \\ &\approx 0.377 E_0 \end{aligned}$$

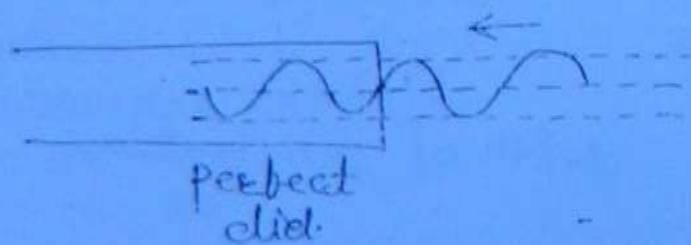
- ① As the wave enters in a lossy medium having finite conductivity, the electric field strength decreases exponentially. (62)
- ② The depth of penetration or the skin depth represents total distance travelled by em-waves where its electric field strength decreases to 37% of its initial value.
- ③ The depth of penetration is inversely equal to the attenuation const. α .
- ④ Higher is the conductivity of medium, higher is the value of attenuation const α , & therefore lower is the value of depth of penetration + vice-versa.

Case I :-

Perfect dielectric
($\sigma = 0$)

$$\gamma = \alpha + j\beta \\ = j\omega\mu(\sigma + j\omega\epsilon)$$

$$\Rightarrow \alpha = 0 \\ \boxed{\delta = \infty} = \frac{1}{\alpha}$$



Date _____

Case : 2.

perfect conductor

$$(\sigma \rightarrow \infty)$$

$$\Rightarrow d \rightarrow \infty$$

$$\delta = \frac{1}{\omega} \rightarrow 0$$

(63)

em-wave = 0

$$\vec{E} = 0$$

$$\vec{H} = 0$$

perfect cond.

$$(\sigma \rightarrow \infty)$$

- ① for a perfect dielectric $\sigma = 0, \delta = 0$ therefore depth of penetration is infinite.

In such medium the wave travel without any attenuation & therefore elec. field strength remains const at all point.

- ② for a perfect cond. the depth of penetration is zero, the wave cannot enter in such medium therefore any perfect conductor behaves as a perfect reflector.

- ③ In side any perfect cond. emw, elec. field and the mag. field do not exists.

- ④ any perfect conductor behaves as an electromagnetic mirror.

5. for a good conductor :-

$$\frac{\sigma}{\omega c} \gg 1$$

$$\delta = \frac{1}{2}$$

$$\gamma = \int j\omega \mu (\sigma + j\omega \epsilon)$$

$$= \int j\omega \mu \sigma (1 + j\frac{\omega \epsilon}{\sigma})$$

$$= \int \omega \mu \sigma | 45^\circ$$

$$= \int \omega \mu \sigma (\underbrace{\cos 45^\circ}_1 + j \underbrace{\sin 45^\circ}_1)$$

$$= \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}$$

$$= \int \frac{\omega \mu \sigma}{2} (1+j) = \alpha + j\beta$$

$$\alpha = \int \frac{\omega \mu \sigma}{2}$$

$$\delta = \frac{1}{\alpha} = \int \frac{1}{\omega \mu \sigma}$$

$$\boxed{\delta = \frac{1}{\pi f \mu \sigma}}$$

$$\delta \propto \frac{1}{f \sigma}$$

$$f \uparrow \rightarrow \delta \downarrow$$

$$f \downarrow \rightarrow \delta \uparrow$$

Comments :- for a specified material if the freq. of operation is high then the depth of penet. is high & therefore thin conductor are preferable.

at low freq. since the depth of penetration is high we have to use thick conductors

(64)

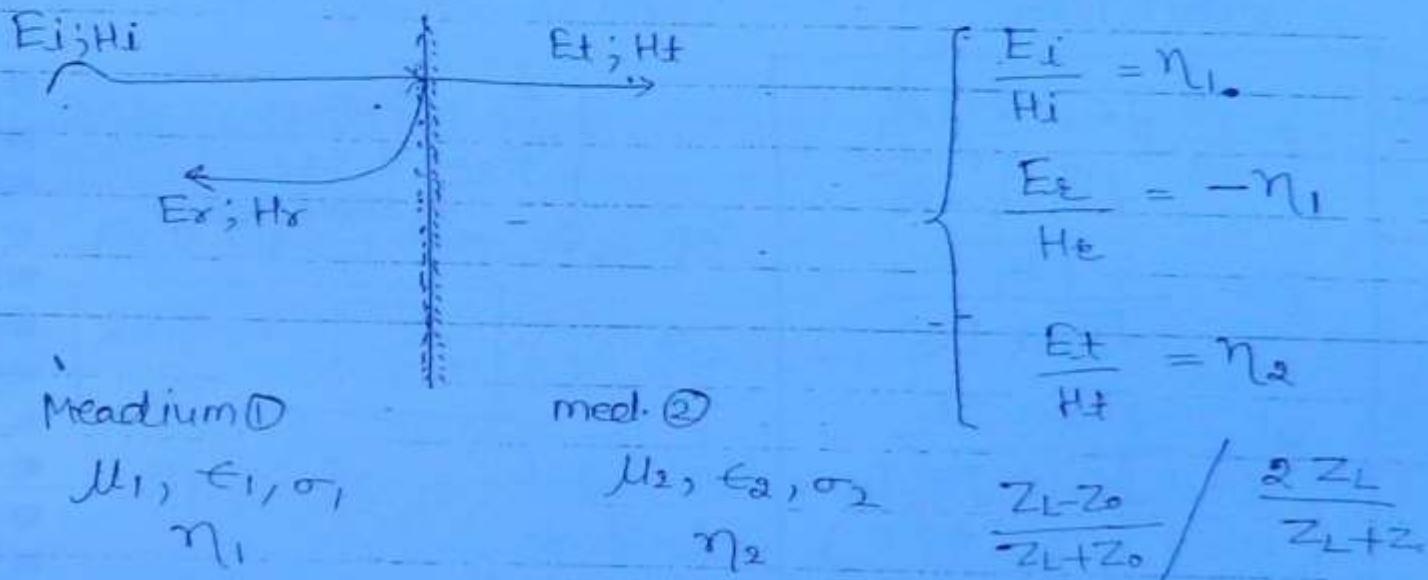
$$\left\{ \begin{array}{l} j = J\tau \\ \sqrt{J} = \sqrt{J\tau} \end{array} \right.$$

Complicated so
phase angle taken.

so that entire power is contained within the conducting region.

(65)

* Reflection & Refraction of EMW normal incidence



$$\left(\frac{E_r}{E_i} \right) = \epsilon = \frac{n_2 - n_1}{n_2 + n_1}$$

$$\left(\frac{E_t}{E_i} \right) = T = \frac{2 n_2}{n_2 + n_1}$$

$$\left(\frac{H_r}{H_i} \right) = \epsilon' = \frac{n_1 - n_2}{n_1 + n_2} = -\epsilon$$

$$\left(\frac{H_t}{H_i} \right) = T' = \frac{2 n_1}{n_1 + n_2}$$

- for General Media

perfect diel.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \approx \sqrt{\frac{\mu_0}{\epsilon}}$$

(66)

$$\eta_1 \propto \sqrt{\frac{1}{\epsilon_1}}$$

$$\eta_2 \propto \sqrt{\frac{1}{\epsilon_2}}$$

$$e = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$T = \frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$e' = \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$T' = \frac{2\sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$\begin{cases} \epsilon_1 = \epsilon_0 \epsilon_{\epsilon_1} \\ \epsilon_2 = \epsilon_0 \epsilon_{\epsilon_2} \end{cases}$$

is same result

for perfect diel.

Oblique incidence:

incident

reflected



Med. ①

$\epsilon_1; (\sigma_1 = 0)$

θ_1

Med. ②

$\epsilon_2; (\sigma_2 = 0)$

transmitted

θ_d = angle of deviation

$$\theta_d = \theta_2 - \theta_1$$

(67)

$\theta'_1 = \theta_1$ ---- law of reflection

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

---- Snell's law

$$v_1 = \frac{1}{\sqrt{\mu_0 \epsilon_1}}$$

$$v_2 = \frac{1}{\sqrt{\mu_0 \epsilon_2}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

Case 1: Critical angle

$$\theta_c = \theta_1 \text{ when } \theta_2 = 90^\circ$$

$$\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} ; \epsilon_2 < \epsilon_1$$

$\theta_1 \geq \theta_c$ -- total internal reflection (TIR)

$$\left. \begin{array}{l} r=1 \\ T=0 \end{array} \right\}$$

Case 2: Brewster's angle

$$\theta_1 = \theta_B$$

$$\tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}} ; \left. \begin{array}{l} r=0 \\ T=1 \end{array} \right.$$

- The critical angle represent the angle of incident for which the angle of transmission is 90° .
- total internal reflection occurs whenever the angle of incident is more than the critical angle.

this phenomenon is useful for the propagation of waves on boundary media such as propagation through

Optical fibre.

(68)

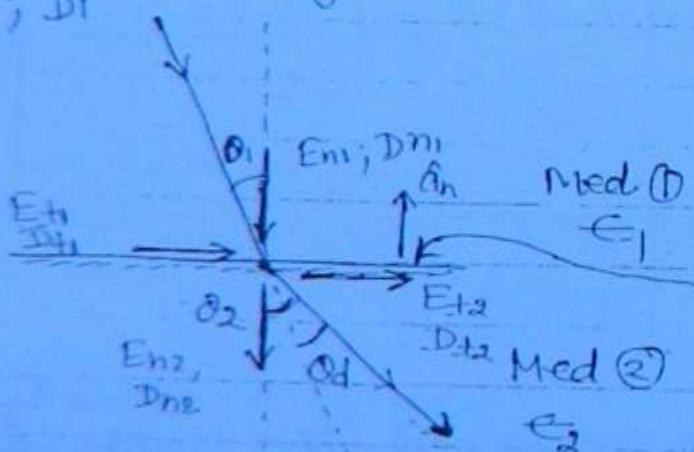
The Brewster's angle is the angle of incidence for which the wave is not reflected & the evolute wave is transmitted in IInd medium.

This phenomenon is for the propagation of waves in unbound medium.

Such as the propagation through free space, or the transmission of waves through earth.

Boundary Relation for elec. fields

E_1, D_1



$\epsilon_s \dots \text{c/m}^2$ (surface charge density)

$$\theta_d = \theta_2 - \theta_1$$

To find :-

$E_{t1} \longleftrightarrow E_{t2}$ tangential comp. relation.

$E_{n1} \longleftrightarrow E_{n2}$ --- normal Compt. relation.

$$\left. \begin{array}{l} \epsilon_1 \\ \epsilon_2 \end{array} \right\} \longleftrightarrow \left. \begin{array}{l} \epsilon_1 \\ \epsilon_2 \end{array} \right\}$$

Obllique Incidence of electric fields. (69)

Tangential Compt. Relations :-

$$\rightarrow \boxed{\begin{aligned} E_{t1} &= E_{t2} \\ \frac{D_{t1}}{\epsilon_1} &= \frac{D_{t2}}{\epsilon_2} \\ \hat{n} \times (\vec{E}_1 - \vec{E}_2) &= 0 \end{aligned}}$$

$$\begin{aligned} |\hat{n} \times \vec{E}_1| &= E_{t1} \\ |\hat{n} \times \vec{E}_2| &= E_{t2} \\ |\hat{n} \times \vec{E}_2| &= E_{t2} \\ \hat{n} &\dots \text{unit normal} \\ \text{vector } \perp \text{ to common} &= \\ \text{Boundary.} & \end{aligned}$$

Case 1: Med. (1) is conduct.

$$E_{t1} = 0$$

$$E_{t1} = E_{t2} = 0$$

- (1) The tangential comp't. of elec. field intensity E is continuous across a common boundary separating two diff. die. medium.
- (2) the result is independent of surface charge density present on the common boundary.
- (3) The tangential comp't. of elec. field intensity E will not exist across a common boundary separating two diff. media.
when one of them or both media are conducting media.

normal comp. relation - 70

$$\rightarrow \boxed{\begin{array}{l} D_{n1} - D_{n2} = \epsilon_s \\ \epsilon_1 E_{n1} - \epsilon_2 E_{n2} = \epsilon_s \\ \hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \epsilon_s \end{array}} \quad \left| \begin{array}{l} \hat{n} \cdot \vec{D}_1 = D_{n1} \\ \hat{n} \cdot \vec{D}_2 = D_{n2} \end{array} \right.$$

Case : 1

$$\epsilon_s = 0$$

$$\boxed{D_{n1} = D_{n2}}$$

case : 2

Med ② is cond.

$$D_{n2} = 0$$

$$D_{n1} - D_{n2} = \epsilon_s$$

$$\boxed{D_{n1} = \epsilon_s}$$

case 3 :-

Med ① is cond.

$$D_{n1} - D_{n2} = \epsilon_s$$

$$\boxed{-D_{n2} = \epsilon_s}$$

from case ② & ③

$$\boxed{|D_n| = \epsilon_s}$$

$$|D_n| = \epsilon_s$$

$$\left\{ \begin{array}{l} + \text{cond} \quad \leftarrow \epsilon_s = c/m^2 \\ |I_n| = \epsilon_s \\ \text{cond} \quad \leftarrow \epsilon_s = c/m^2 \end{array} \right.$$

(1) The normal comp. of elec. flux density D is discontinuous by a factor ϵ_s , where ϵ_s is surface charge density on the common boundary separating two diff. dielec. media.

(71)

(2) for a charge free common boundary the normal comp. of elec. flux density D is always continuous.

(3) Any conducting surface always supports normal comp. of elec. field such that the normal comp. of elec. flux density D is numerically equal to magnitude of surface charge density present on the conducting surface.

Obligate Incidence

$$g) \quad E_3 = 0$$

$$\boxed{\begin{aligned} E_{t1} &= E_{t2} \\ D_{n1} &= D_{n2} \end{aligned}}$$

$$\left\{ \begin{array}{l} E_{t1} = E_1 \sin \theta_1 \\ E_{t2} = E_2 \sin \theta_2 \end{array} \right.$$

$$\left\{ \begin{array}{l} E_{n1} = E_1 \cos \theta_1 \\ E_{n2} = E_2 \cos \theta_2 \end{array} \right.$$

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2}} = \frac{\epsilon_1}{\epsilon_2}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

(72)

--- Snell's law

(Ex:

$$\hat{a}_n = +\hat{a}_3$$

med ① ; $\boxed{\epsilon_2 > 0}$

$$\epsilon_1 = 4 \epsilon_0$$

$$\vec{E}_1 = (3\hat{a}_x - 6\hat{a}_y - 8\hat{a}_3)$$

$$\vec{E}_2$$

$$\epsilon_s = 3 \epsilon_0$$

$$- \text{C/m}^2$$

med ② ; $\boxed{\epsilon_2 < 0}$

$$\epsilon_2 = 6 \epsilon_0$$

To find

$$\vec{E}_2 = ?$$

$$= E_{x2} \hat{a}_x$$

$$+ E_{y2} \hat{a}_y$$

$$+ E_{z2} \hat{a}_z$$

$$\vec{E}_2 = \begin{cases} \text{tangential comp.} \\ \epsilon_{t1} = \epsilon_{t2} \end{cases} \Rightarrow E_{x2} = E_{x1} = 3$$

normal comp.

$$\left. \begin{array}{l} D_{n1} - D_{n2} = \epsilon_s \\ \end{array} \right\} \Rightarrow D_{z1} - D_{z2} = \epsilon_s$$

$$\epsilon_1 E_{z1} - \epsilon_2 E_{z2} = \epsilon_s$$

$$\vec{E}_2 = 3\hat{a}_x - 6\hat{a}_y - \frac{35}{6}\hat{a}_3$$

$$= 4 \epsilon_0 (-8) - 6 \epsilon_0 E_{z2} = 3 \epsilon_0$$

$$E_{z2} = -\frac{32 - 3}{6} = -\frac{35}{6}$$

Date _____

* Boundary relations for Mag. fields

tangential comp. relation

(23)

$$\boxed{\begin{array}{l} H_{t1} - H_{t2} = J_s \\ \frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = J_s \\ \hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \end{array}} \quad ; \begin{array}{l} J_s \text{ - surface current density} \\ \text{on common boundary} \\ \text{A/m} \end{array}$$

$$|\hat{n} \times \vec{H}_1| = H_{t1}$$

$$|\hat{n} \times \vec{H}_2| = H_{t2}$$

$$H_{t1} = H_{t2} \quad \dots J_s = 0$$

$$\left\{ \begin{array}{l} H_{t1} = J_s \quad \dots \text{if Med. } \textcircled{2} \text{ is cond.} \\ -H_{t2} = J_s \quad \dots \text{if Med. } \textcircled{1} \text{ is cond.} \end{array} \right.$$

$$|H_t| = J_s$$

① The tangential comp. of mag. field intensity H is a discontinuous by a factor of J_s . J_s represent the surface current density present on the common boundary separating two diff. boundary media.

On a ~~conductor~~ conductor face common boundary the tangential comp. of the H field is always continuous.

Any conducting surface will support only the tangential comp. of H field such that the magnitud. Mag. field intensity H is numerically equal to the surface current density present on the conducting surface.

normal comp. - relation

$$B_{n1} = B_{n2}$$

$$\mu_1 H_{n1} = \mu_2 H_{n2}$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

74

$$\hat{n} \cdot \vec{B}_1 = B_{n1}$$

$$\hat{n} \cdot \vec{B}_2 = B_{n2}$$

$B_{n1} = B_{n2} = 0$ if the medium ① & med. ②
or both are conductors.

- ① The normal comp. of mag. flux density B is continuous across a common boundary separating two different mag. media.
- ② The result is independent of the surface current density present on the common boundary.
- ③ The normal comp. of mag. flux density B will not exist across a common boundary separating two different media when one of them or both media are conducting media.

Oblique Incidence

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}$$

75

Med. ① ; $x > 0$

$$\mu_1 = 2 \mu_0$$

Ex:
 $\vec{B}_1 = 4\hat{x} + 5\hat{y} - 6\hat{z}$

$$\hat{n} = \hat{a}$$

$$J_s = 0$$

$$\vec{B}_2 = ?$$

Med. ② ; $x < 0$

$$\mu_2 = 7 \mu_0$$

$$\vec{B}_2$$

Normal

$$B_{n1} = B_{n2} \rightarrow B_{x2} = B_{x1} = ④$$

tangential

$$H_{t1} - H_{t2} = J_s$$

$$H_{t1} = H_{t2}$$

$$H_{d1} = H_{d2} \Rightarrow \frac{B_{d2}}{\mu_2} = \frac{B_{d1}}{\mu_1}; B_{d2} = \frac{\mu_2}{\mu_1} \cdot B_{d1}$$

$$H_{d2} = H_{d1} = 7/2 \times 5 = \frac{35}{2}$$

$$\Rightarrow B_{32} = \frac{\mu_2}{\mu_1} \cdot B_{31}$$

$$\Rightarrow \frac{7}{2} \times (-6) = -21$$

$$\vec{B}_2 = 4\hat{x} + \frac{35}{2}\hat{y} - 21\hat{z}$$

X

Wave polarization

(76)

- ① The wave polarization is related to the orientation of elec. field vector, associated with the EMW.
- ② if the vertical antenna is installed, the electrical field vector is also vertical & EMW is vertically polarised.
- ③ if any antenna is horizontally polarised, the elec. field vector is parallel to the surface at the earth & the EMW is horizontally polarised.
- ④ the orientation of the elec. field vector at the transmitting & receiving end must be same so that max. induced emf. is obtained at the receiving antenna.

therefore the polarization of the transmitting & receiving antenna must be identical.

$$E_x = E_1 \sin(\omega t - \beta z)$$

$$E_y = E_2 \sin(\omega t - \beta z + \alpha)$$

time phase angle b/w

E_x & E_y

To bindout

effect of E_1, E_2, α

Where $z=0$ plane

$$E_x = E_1 \sin \omega t$$

$$E_y = E_2 \sin(\omega t + \alpha)$$

$\alpha = 0^\circ$ --- in same phase.
 $\alpha = 90^\circ$ --- quadrature phase.

(77)

case 1 :- linear polarization

$$\boxed{E_1 \neq E_2}$$

$$\alpha = 0^\circ \text{ or } 180^\circ$$

if $\alpha = 0^\circ$

$$E_x = E_1 \sin \omega t$$

$$E_y = E_2 \sin \omega t$$

$$\frac{E_y}{E_x} = \frac{E_2}{E_1} = m$$

$$\boxed{E_y = m E_x} \quad \text{--- eqn of st. line.}$$

① if $E_1 = 0$

$$\begin{cases} E_x = 0 \\ E_y = E_2 \sin \omega t \end{cases}$$

E.M.W is linearly

polarised along y -direction.

Wave is vertically polarized

② if $E_2 = 0$

$$\begin{cases} E_x = E_1 \sin \omega t \\ E_y = 0 \end{cases}$$

EMW is linearly polarised along the
 x -direction.

Wave is horizontally polarized.

Case 2

circular polarization

$$\begin{aligned}E_1 &= E_2 = E_0 \\&\alpha = \pm 90^\circ\end{aligned}$$

78

$$E_x = E_0 \sin \omega t$$

$$\begin{aligned}E_y &= E_0 \sin(\omega t + 90^\circ) ; \quad \alpha = \mp 90^\circ \\&= E_0 \cos \omega t\end{aligned}$$

$$E_x^2 + E_y^2 = E_0^2$$

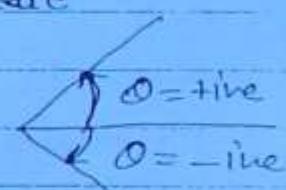
--- eqn of a circle

$$\alpha = +90^\circ$$

$$\alpha = -90^\circ$$

-- LCP wave

-- RCP wave



Right circular polarization

Left circular polarization

Case 3: Elliptical polarization

$$E_1 \neq E_2$$

$$\alpha = \pm 90^\circ$$

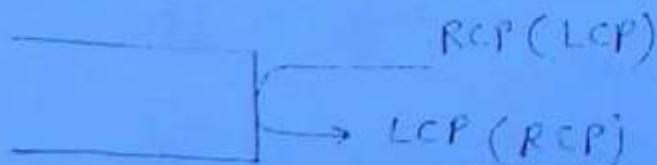
$$\alpha = +90^\circ$$

$$= -90^\circ$$

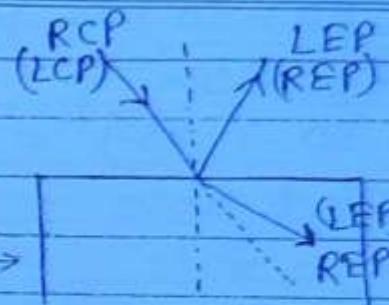
-- LEP wave

-- REP wave

Main points :- ① When RCP wave incidence
on a perfect conductor perpendicular,
the reflected wave is LCP wave & vice-versa



(2) Polyethylene



then:

- (a) Transmitted wave is RCP wave
- (b) Reflected wave is LEP wave & vice-versa

(3) if it is a circularly polarized wave or electrically polarized wave incidence at ~~any~~ angle on any interface then the reflected wave & transmitted wave are linearly polarized.

(Ex:

$$\vec{E} = 5 \sin(\omega t - \beta z) \hat{a}_x + 10 \sin(\omega t - \beta z) \hat{a}_y$$

$$E_1 \neq E_2$$

-- linearly polarized

(Ex:

$$\vec{E} = 5 \sin(\omega t - \beta z) \hat{a}_x + 5 \sin(\omega t - \beta z - 90^\circ) \hat{a}_y$$

$$E_1 = E_2$$

$$\alpha = -90^\circ$$

-- RCP wave

(Ex:

$$\vec{E} = 5 \sin(\omega t - \beta z - 30^\circ) \hat{a}_x + 10 \sin(\omega t - \beta z + 60^\circ) \hat{a}_y$$

$$E_1 \neq E_2$$

$$\alpha = 60 - (-30) = +90^\circ$$

-- LEP wave

$$\text{B.E. } (5 \sin(\omega t - \beta z - 30^\circ) \hat{a}_x + 5 \sin(\omega t - \beta z + 30^\circ) \hat{a}_y)$$

$$\left\{ E_1 = E_2 = 5 \right.$$

$$\left. \alpha = 30^\circ - (-30^\circ) = 60^\circ \right. \quad (86)$$

-- unpolarized wave

Ques: Example:- An em.w. having following x- comps. of elec. field is travelling in a lossless medium with $\mu_r = 1$ & $\epsilon_r = 9$.

To find:-

- (1) All the parameters associated with em.w.
- (2) the mag. field intensity associated with em.w.

$$\vec{E} = 10 \cdot \cos(6\pi \times 10^3 t - \beta z) \hat{a}_x$$

$$\sigma = 0$$

$$\mu_r = 1$$

$$\epsilon_r = 9$$

To find:-
 1. Various parameters
 2. \vec{H}

$$\vec{E} = E_x \hat{a}_x$$

$$E_x = E_m \cdot \cos(\omega t - \beta z)$$

$\uparrow +z \text{ direction}$

(1) $\vec{E} = E_x \hat{a}_x$

(2) $E_m = 10 \text{ V/m}$

(3) Wave is linearly polarized along x-direction

(4) Direction of propagation along +z direction

(5) Intrinsic impedance $\eta = \sqrt{\frac{\mu_r}{\epsilon_r}}$

$$\eta = \sqrt{\frac{1}{\mu_0 \epsilon_0}} = \frac{n_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{3} = \frac{40\pi}{\lambda}$$

$$\textcircled{6} \quad v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\text{Hertz} \cdot \text{Coulombs}} = \frac{c}{j\epsilon_0} = \frac{3 \times 10^8}{\lambda} = 10^8 \text{ m/sec.}$$

$$\textcircled{7} \quad \omega = 6\pi \times 10^8 = 600\pi \text{ rad/sec.}$$

$$f = \frac{\omega}{2\pi} = 300 \text{ MHz.}$$

$$\textcircled{8} \quad \beta = \frac{\omega}{v_p} = \frac{6\pi \times 10^8}{10^8} = 6\pi \text{ rad/m}$$

$$\textcircled{9} \quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6\pi} = \frac{1}{3} \text{ m}$$

$$\begin{aligned} \textcircled{10} \quad \vec{r} &= \vec{E} \times \vec{H} \\ &= \frac{1}{2} \eta H_m^2 \\ &= \frac{1}{2} \frac{E_m}{\eta} \cdot \hat{a}_z = \frac{1}{2} \times \frac{100}{40} d_z \text{ W/m}^2 \end{aligned}$$

$$\textcircled{11} \quad \eta = \frac{E_m}{H_m} \quad ; \quad H_m = \frac{E_m}{n} = \frac{10}{40\pi} = \frac{1}{4\pi} \text{ A/m}$$

$$\vec{r} = \vec{E} \times \vec{H}$$

$$\hat{a}_z = \hat{a}_x * (+\hat{a}_y)$$

$$\vec{H} = H_y \hat{a}_y$$

$$= H_m \cos(\omega t - \beta z) \hat{a}_y$$

$$\frac{1}{4\pi} \cos(6\pi \times 10^8 t - 6\pi z) \hat{a}_y \text{ A/m}$$

Aus.

Ex: $\vec{H} = 10 \sin(6\pi \times 10^8 t + 6\pi x) \hat{a}_3$
Note → Wave is linearly polarized along y -direction.

$$\vec{H} = H_3 \cdot \hat{a}_3$$

Propag. --- $-x$ direction

(82)

$$\vec{p} = -p_x \hat{a}_x$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$-\hat{a}_x = (\hat{a}_y) \times (\hat{a}_3)$$

$$\vec{E} = -E_y \hat{a}_y$$

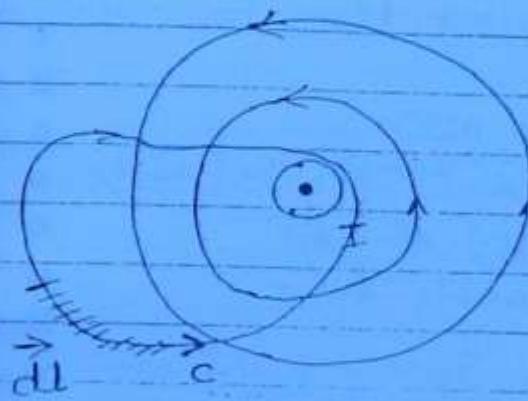
Magnetostatics

Date _____

$\vec{B}, \vec{H} \neq f(t)$ Static Magnetic fields.

(83)

Ampere's circuital law :-



$$\vec{H}; \vec{B}; \propto \frac{1}{r^2}$$

bind: \vec{H}

: $d\vec{l}$

: $\vec{H} \cdot d\vec{l}$

$$\oint_C \vec{H} \cdot d\vec{l} = I$$

(current enclosed)

Ampere's
circuital law in

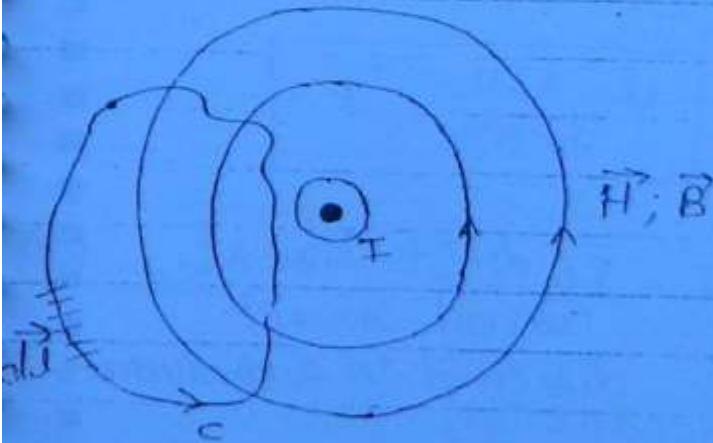
= gaugeal form

line current

surface current

volume current.

$$\begin{aligned} I &= I \\ &= \int dJ_s \cdot dl \\ &= \iint_s \vec{J} \cdot d\vec{s} \end{aligned}$$



$$\vec{H}; \vec{B}$$

$$\oint_C \vec{H} \cdot d\vec{l} = 0$$

Ampere's circuital law in differential or point form :-

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_d = \sigma \vec{E} + \frac{\partial \vec{P}}{\partial t} \quad (84)$$

$$\boxed{\nabla \times \vec{H} = \vec{J}_c = \sigma \vec{E}}$$

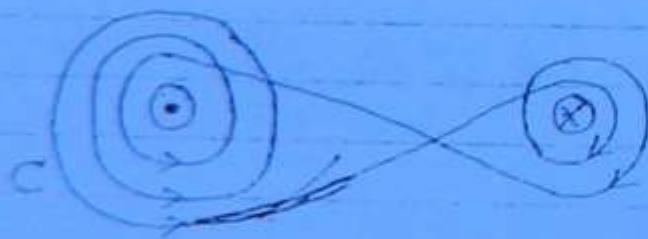
Statements:-

① The circulation of static mag. field intensity \vec{H} along any closed curve (equal to $\oint \vec{H} \cdot d\vec{l}$) is total current I enclosed with the enclosed current C .

② If no current is enclosed the circulation of \vec{H} field along the closed curve is zero all through at each point. $\vec{H} \cdot d\vec{l}$ has infinite value.

③ The law is applicable irrespective of the shape of the closed curve C .

④ The curl of static mag. field intensity \vec{H} at any point is always equal to conduction current density present at that point.



To find :-

$$\oint_C \vec{H} \cdot d\vec{l} = ?$$

$$= 3I - (-2I) = 5I$$

due to flow of current in opp direction in 2 conductors.

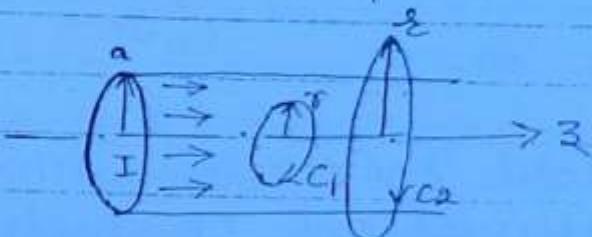
due to opp direction of circulation of current

Ex:

A long solid cylindrical conductor of radius 'a' carries a uniform current 'I' through out of cross-section of conductor.

Find the value of mag. flux density \vec{B} at all points/regions. -

(85)



To find: \vec{B} for

- ~~1. $r < a$~~
- 2. $r > a$
- 3. $r = a$

① $r < a$

$$\underbrace{\oint_{C_1} \vec{H} \cdot d\vec{l}}_{H_\phi} = I'$$

$$H_\phi \cdot 2\pi r = \frac{r^2 \cdot I}{a^2}$$

$$H_\phi = \frac{I \cdot r}{2\pi a^2}; \quad \vec{H} = \frac{I \cdot r}{2\pi a^2} \hat{a}_\phi$$

$$\boxed{\vec{B} = \frac{\mu I r}{2\pi a^2} \hat{a}_\phi}$$

$$\frac{I'}{I} = \frac{\pi r^2}{\pi a^2}$$

$$I' = \frac{r^2 \cdot I}{a^2}$$

②

$r > a$

$$\underbrace{\oint_{C_2} \vec{H} \cdot d\vec{l}}_{H_\phi} = I$$

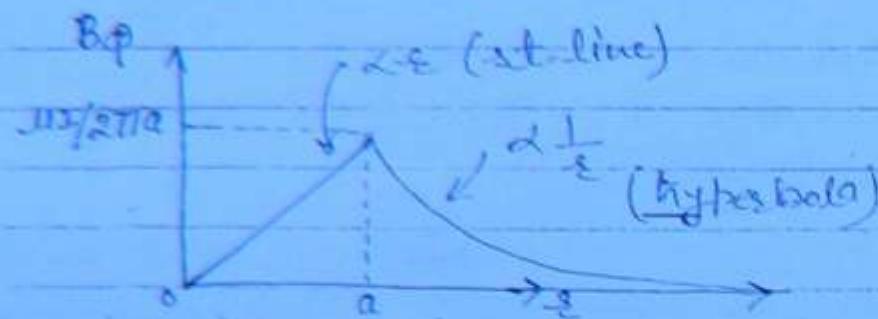
$$H_\phi \cdot 2\pi r = I$$

$$\boxed{\vec{B} = \frac{\mu I}{2\pi r} \hat{a}_\phi}$$

$$\vec{B} = \frac{\mu I r}{2\pi a^2} \hat{a}_\phi \quad ; \quad r < a$$

$$= \frac{\mu I}{2\pi r} \hat{a}_\phi \quad ; \quad r > a$$

$$\vec{B} = \frac{\mu I}{2\pi a} \hat{a}_\phi \quad r = a$$



(86)

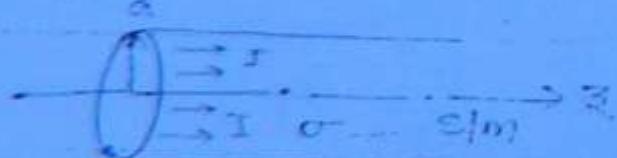
Implications :- ① The mag. field increases linearly in a solid cylindrical conductor whereas it decreases in the hyperbolic outside the cylindrical conductor.

② The mag. field is always constant on the surface of any conductor.

③ Any cylindrical conductor behaves as an infinite line current since the mag. field due to both configuration is same.

Ex:- A long solid cylindrical cond. of radius 'a' & conductivity 'σ' carries uniform current 'I'.

Find the pointing vector on the surface of cylindrical conductor.



$$\vec{H}_{\text{ext}} = \frac{I}{2\pi a} \hat{a}_\theta$$

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{1}{\sigma} \left(\frac{I}{\pi a^2} \right) \hat{a}_z$$

$\vec{J} \dots A/m^2$

$$\vec{P} = \frac{I}{\pi \sigma a^2} \hat{a}_z \times \frac{I}{2\pi a} \hat{a}_\phi$$

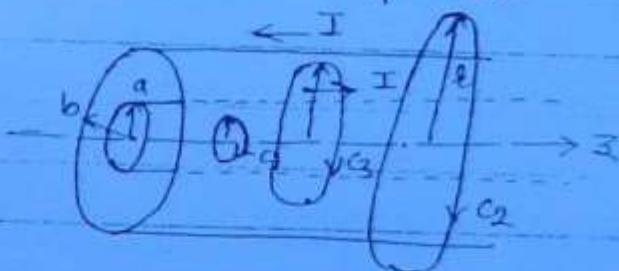
$$= \frac{I^2}{2\pi^2 \sigma a^3} (\underbrace{\hat{a}_z * \hat{a}_\phi}_{= -\hat{a}_x}) = \quad \textcircled{87}$$

$$\vec{P} = \frac{-I^2 \cdot \hat{a}_x}{2\pi^2 \sigma a^3}$$

Ques:

A long co-axial TL of radius 'a' & 'b'
with $b > a$ carries uniform current
 $\pm I$ on the surface of the 2-cylindrical
conductor of the TL.

calculate mag. field intensity \vec{H} at
all points.



To find:

\vec{H} for

1. $r < a$

2. $a < r$

3. $r > b$

① $r < a$

$$\oint_{C_1} \vec{H} \cdot d\vec{l} = 0$$

$$\Rightarrow \boxed{H = 0}$$

② $r > b$

$$\oint_{C_2} \vec{H} \cdot d\vec{l} = +I + (-I) = 0$$

$$\boxed{\vec{H} = 0}$$

(3)

$$a < \varepsilon < b$$

$$\oint_{C_0} \vec{H} \cdot d\vec{l} = I$$

$$H_A \cdot 2\pi\varepsilon = I$$

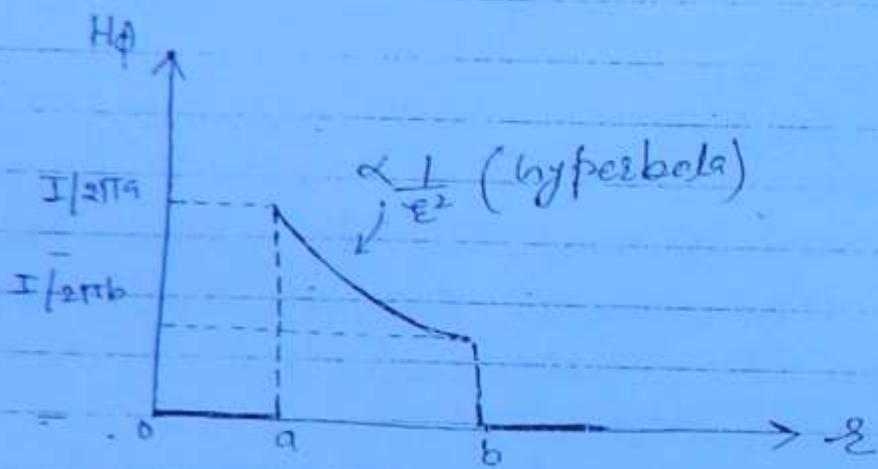
$$\boxed{\vec{H} = \frac{I}{2\pi\varepsilon} \hat{a}_\phi}$$

(88)

$$\vec{H} = 0 \quad - \quad \varepsilon < a$$

$$= \frac{I}{2\pi\varepsilon} \hat{a}_\phi \quad - \quad a < \varepsilon < b$$

$$= 0 \quad - \quad \varepsilon > b$$



* Magnetic energy density

Date _____

$$w_m \dots \text{ (J/m}^3\text{)}$$

$$w_m = \lim_{\Delta V \rightarrow 0} \left(\frac{\Delta w_m}{\Delta V} \right)$$

(89)

$$\boxed{w_m = \frac{1}{2} \mu H^2 + \frac{1}{2} \vec{B} \cdot \vec{H} \quad \text{--- (J/m}^3\text{)}}$$

① The mag. energy density represent the mag. energy stored per unit volume & gives Mag. energy stored at a point in any electro-mag. region.

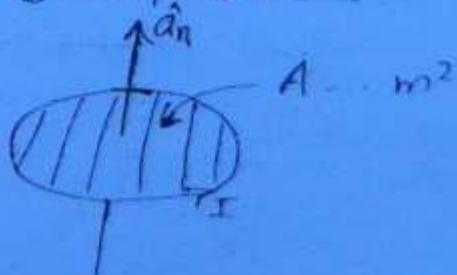
② This depends upon mag. field intensity H & the const. of the medium μ .

* Magnetic energy stored

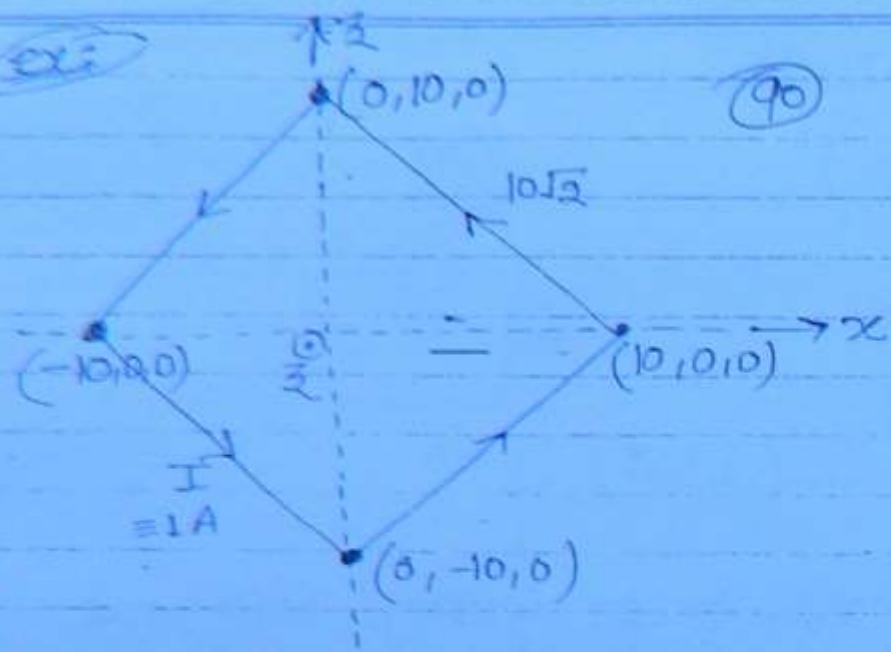
$$w_m = \frac{1}{2} L I^2 \quad \text{J}$$

* Magnetic dipole moment

Mag. dipole:



$$\vec{m}_m = I A \cdot \hat{a}_n \quad \text{A} \cdot \text{m}^2$$

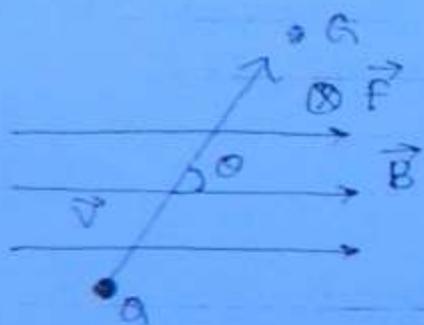


To bind :-

$$\begin{aligned}\vec{m}_m &= I A \hat{a}_3 \\ &= 1 \times (10\sqrt{2})^2 \hat{a}_3 \\ &= 200 \hat{a}_3 \text{ A/m}^2\end{aligned}$$

Important main points :-

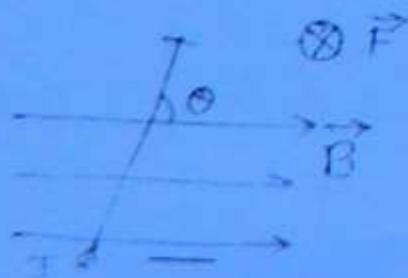
① Moving charge in \vec{B} :-



$$\vec{F} = q \vec{v} \times \vec{B} = q v B \sin \theta \hat{a}_n$$

$$\begin{aligned}|F| &= 0 \quad ; \quad \theta = 0^\circ \\ &= q v B \quad ; \quad \theta = 90^\circ\end{aligned}$$

② Current carrying cond. in \vec{B} :-



$$\vec{F} = \int_c (\vec{I} \times \vec{B}) dl$$

Q1

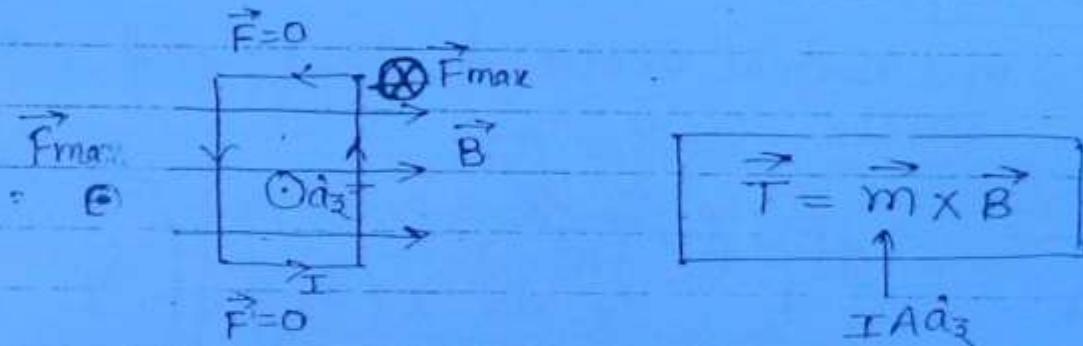
$$\left(\frac{\vec{F}}{l} \right) = \vec{I} \times \vec{B}$$

N/m

-- force per unit length

current

③ Carrying loop in \vec{B} :-



$$\vec{T} = \vec{m} \times \vec{B}$$

$IA\vec{a}_z$

-- Torque is applicable.

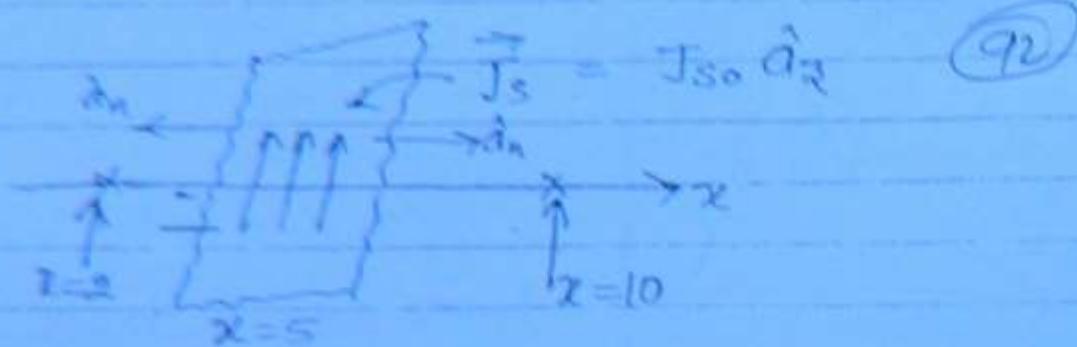
④ Infinite line current :-

The diagram shows an infinite vertical line carrying a current I upwards. At a perpendicular distance r from the line, a magnetic field B is shown pointing outwards. The formula for the magnetic field is given as $B = \frac{\mu_0 I}{2\pi r}$.

⑤ circular loop :-

The diagram shows a circular loop of radius r carrying a current I in a clockwise direction. At the center of the loop, a magnetic field B is shown pointing outwards. The formula for the magnetic field is given as $B = \frac{\mu_0 I}{2r}$.

⑥ H or B due to infinite current sheet:



⑦

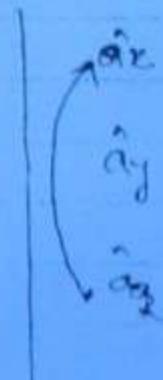
$$\vec{H} = \frac{1}{2} \vec{J}_s \hat{a}_n$$

$\hat{a}_n \rightarrow$ unit normal vector \perp to current sheet.

at $x=10$

$$\hat{a}_n = \hat{a}_x$$

$$\begin{aligned}\vec{H} &= \frac{1}{2} \vec{J}_s \times \hat{a}_n \\ &= \frac{1}{2} J_{so} \hat{a}_n \times (\hat{a}_x) \\ &= \frac{1}{2} J_{so} \hat{a}_y\end{aligned}$$



at $x=2$

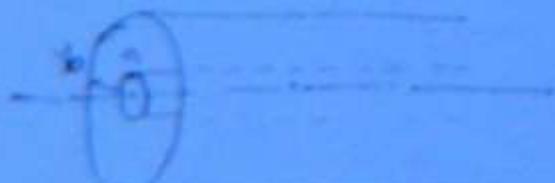
$$\hat{a}_n = -\hat{a}_x$$

$$\vec{H} = \frac{1}{2} \vec{J}_s \times \hat{a}_n = \frac{1}{2} J_{so} \hat{a}_z \times (-\hat{a}_x)$$

$$\vec{H} = -\frac{1}{2} J_{so} \hat{a}_y$$

⑦

Co-axial T.L. :-



$$\left(\frac{l}{d} \right) = \left[l' = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \right] \text{H/m}$$

$$c' = \frac{2\pi e}{\ln(b/a)} \quad \text{f/m}$$

(93)

$$Z_0 = \sqrt{\frac{l'}{c'}} = \frac{1}{2\pi} \sqrt{\frac{\mu}{e} \ln\left(\frac{b}{a}\right)} \quad ; -\Omega$$

$$Z_0 = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right)$$

for free space :-

$$\eta = 120\pi$$

$$Z_0 = 60 \ln\left(\frac{b}{a}\right) \quad \Omega$$

$$V_p = \frac{1}{l' c'}$$

$$V_p = \frac{1}{\mu e}$$

for free space

$$\mu = \mu_0; \epsilon = \epsilon_0$$

$$V_p = \frac{1}{(\mu_0 \epsilon_0)} = C$$

Vector Mag. potential (\vec{A})

$$\nabla \cdot \vec{B} = 0 \quad \textcircled{1}$$

vector

--- always valid.

(94)

$$\nabla \cdot (\nabla \times \vec{A}) = 0 \quad \textcircled{2}$$

vector

--- always valid.

--- vector identity

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

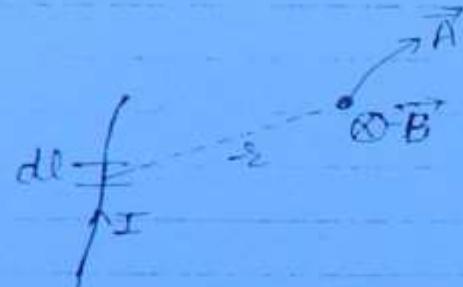
$$\vec{B} \perp \vec{A}$$

relation b/w \vec{A} & \vec{B}

Mathematical definition of \vec{A} :-

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int_C \frac{\vec{I}}{r} d\ell}$$

$$\Rightarrow \vec{A} \parallel \vec{I}$$



Direction-wise

$$\boxed{\begin{array}{l} \vec{A} \parallel \vec{I} \\ \vec{B} \parallel \vec{I} \\ \vec{B} \perp \vec{A} \end{array}}$$

$$\vec{B} = \nabla \times \vec{A}$$

① There is no physical significance of vector mag. potential A since the mag. charges in the isolated form do not exist in nature.

② Calculation of vector A simply gives

an intermediate step.

to calculate the value of mag.

flux density B . using the relation

$$\vec{B} = \nabla \times \vec{A}$$

(95)

ex:-

$$\vec{A} = 2x^2y \hat{a}_z$$

To find

$$= A_z$$

$$A_x = 0$$

$$A_y = 0$$

$$A_z = 2x^2y$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{a}_x \left(\frac{\partial}{\partial y} (2x^2y) + \hat{a}_y \left(\frac{\partial}{\partial x} (2x^2y) \right) \right) + \hat{a}_z (0) \\ = 2x^2 \hat{a}_x - 4xy \hat{a}_y$$

$$\nabla \times \vec{B} = 0 \quad \text{irrotational vector.}$$

∴ not an

units of \vec{A}

$$\vec{B} = \nabla \times \vec{A}$$

$$\frac{\text{Wb}}{\text{m}^2}$$

$$\frac{1}{\text{m}}$$

$$\hookrightarrow \text{Wb/m}$$

Electricities

Dc

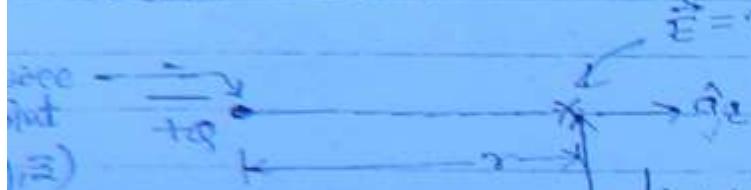
Dc

Dc

----- Static electric fields
 $\vec{E}; \vec{D} \neq f(t)$

(96)

Gauss's law:-



$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{a}_r \quad \text{field point } (x', y', z')$$

$$\hat{a}_r = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{r} = \vec{r}' - \vec{r}$$

$$\vec{E} = \frac{q\vec{r}}{4\pi\epsilon_0 r^3}$$

Where

$$\vec{r} = (x' - x)\hat{a}_x + (y' - y)\hat{a}_y + (z' - z)\hat{a}_z$$

$$|\vec{r}| = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$

A point charge of +3 nano coulomb is placed at a point (2, 1, -2) in a medium whose dielec. const. is 3.

Find the elec. field intensity at int (1, 3, -1).

P --- Source point (2, 1, -2)

P' --- field point (1, 3, -1)

$$q = +3 \text{ nC}$$

$$\epsilon_0 = 3$$

To find :- $\vec{E} = \frac{q\vec{r}}{4\pi\epsilon_0 r^3}$

$$\vec{r} = (1 - 2)\hat{a}_x + (3 - 1)\hat{a}_y + (-1 - (-2))\hat{a}_z$$

$$\vec{\omega} = -\dot{a}_x \hat{a}_x + \dot{a}_y \hat{a}_y + \dot{a}_z \hat{a}_z$$

$$|\vec{\omega}| = \sqrt{1+4+1} = \sqrt{6}$$

(92)

$$\vec{E} = \frac{3 \times 10^9}{4\pi \times \frac{1}{36\pi} \times 10^{-9} \times 3(\sqrt{6})^3} (-\dot{a}_x + 2\dot{a}_y + \dot{a}_z)$$

$$\vec{E} = \frac{27}{3 \times 6\sqrt{6}} (-\dot{a}_x + 2\dot{a}_y + \dot{a}_z)$$

$$\vec{E} = K(-\dot{a}_x + 2\dot{a}_y + \dot{a}_z) \text{ --- v/m}$$

$$K = \frac{3}{2\sqrt{6}}$$

$$|\vec{E}| = K\sqrt{6} = \frac{3}{2} \text{ --- v/m}$$

$$\vec{E} = E \hat{a}_i$$

$$\left\{ \begin{array}{l} \hat{a}_i = \frac{\vec{E}}{E} = \frac{K}{3/2} (-\dot{a}_x + 2\dot{a}_y + \dot{a}_z) \\ \hat{a}_i = \frac{1}{\sqrt{6}} (-\dot{a}_x + 2\dot{a}_y + \dot{a}_z) \end{array} \right.$$

$$\alpha = \cos^{-1} \frac{E_x}{E} = \cos^{-1} \left(\frac{1}{\sqrt{6}} \right) \text{ --- w.r.t. } x\text{-axis.}$$

$$\beta = \cos^{-1} \frac{E_y}{E} = \cos^{-1} \left(\frac{2}{\sqrt{6}} \right) \text{ --- w.r.t. } y\text{-axis.}$$

$$\gamma = \cos^{-1} \frac{E_z}{E} = \cos^{-1} \left(\frac{1}{\sqrt{6}} \right) \text{ --- w.r.t. } z\text{-axis.}$$

Gauss' law :-

$$\oint_S \vec{D} \cdot d\vec{s} = Q$$

net elec.
flux

in integral form
98

where $Q = Q$ point charge
 $= \int_C E_L \cdot dL$ line charge
 $= \iint_S E_S \cdot dS$ surface charge
 $= \iiint_V e \cdot dV$ volume charge.

$$\nabla \cdot \vec{D} = e$$

differential form or
point form.

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

rotational static
elec. field.

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

points :-

① The net elec. flux passing through any closed surface area is always equal to total charge enclosed by the closed surface Area S.

The equ. is valid for any type of the closed surface Area S.

The direction of elec. flux density \vec{D}

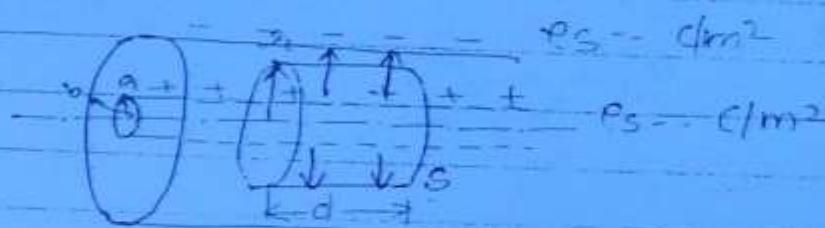
at any point if the elec. mag. Region is always equal to the volume charge density at that point.

(99)

④ The curl of static elec. field intensity \vec{E} at any point is always zero & therefore such field is always irrotational.

therefore the circulation of the static elec. field along any closed curve C is always equal to zero.

ex:



To find \vec{E} ; \vec{D}
for $a < \epsilon < b$

$$\boxed{\vec{E} = \frac{\epsilon_s \cdot a}{\epsilon_0} \hat{a}_z}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\boxed{\vec{D} = \frac{\epsilon_s \cdot a}{\epsilon_0} \cdot \hat{a}_z}$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q = \iint_S \rho_s \cdot dS$$

$$\epsilon E_z \cdot 2\pi a d \quad \epsilon s \cdot 2\pi a d$$

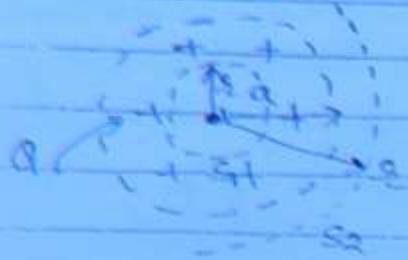
$$E_z = \frac{\epsilon_s \cdot a}{\epsilon_0 \cdot a}$$

ex) A charge $+Q$ is distributed throughout the volume of Region of Radius 'a'.

Find the value of electric field intensity E at all point.

To find

\vec{E} for



1. $r < a$
2. $r > a$
3. $r = a$

(100)

① $r < a$

$$\oint_S \vec{D} \cdot d\vec{s} = Q' \quad \rightarrow \quad \frac{Q'}{Q} = \frac{4/3 \pi r^3}{4/3 \pi a^3}$$

$$E_r = \frac{Qr}{4\pi\epsilon_0 a^3}$$

$$\vec{E} = \frac{Qr}{4\pi\epsilon_0 a^3} \hat{r}_r$$

② $r > a$

$$\oint_{S_2} \vec{D} \cdot d\vec{s} = Q$$

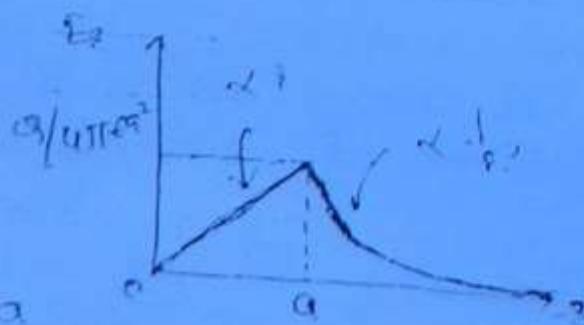
$$\in E_r \cdot 4\pi r^2$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}_r$$

$$\vec{E} = \frac{Qr}{4\pi\epsilon_0 a^3} \hat{r}_r \quad r < a$$

$$= \frac{Q}{4\pi\epsilon_0 a^2} \hat{r}_r \quad r > a$$

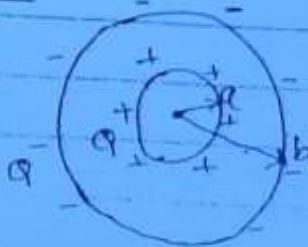
$$= \frac{Q}{4\pi\epsilon_0 a^2} \hat{r}_r \quad r = a$$



ex:

Two spherical shells of Radii 'a' & 'b', with
b greater than a & equal & opposite
charges $\pm Q$ on their surfaces.

Find elec field intensity \vec{E} at
all points.



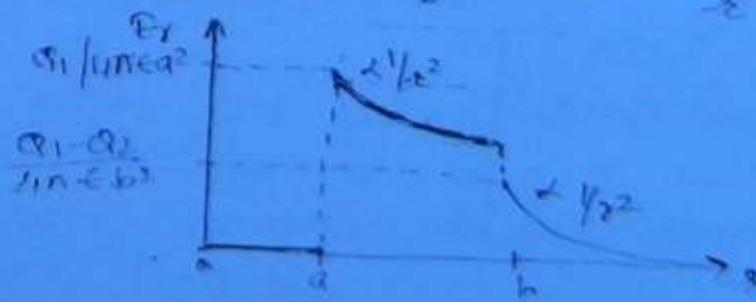
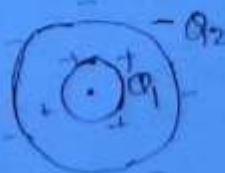
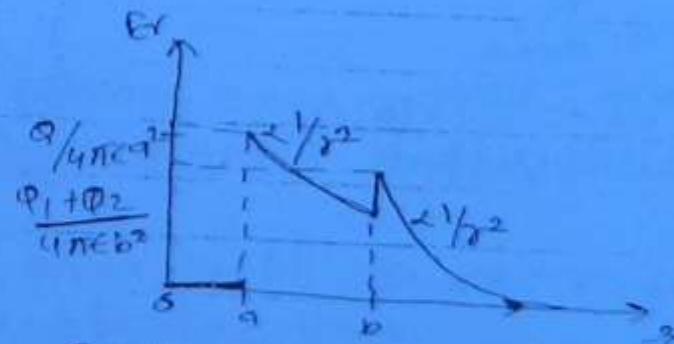
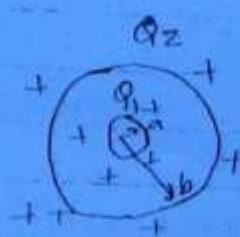
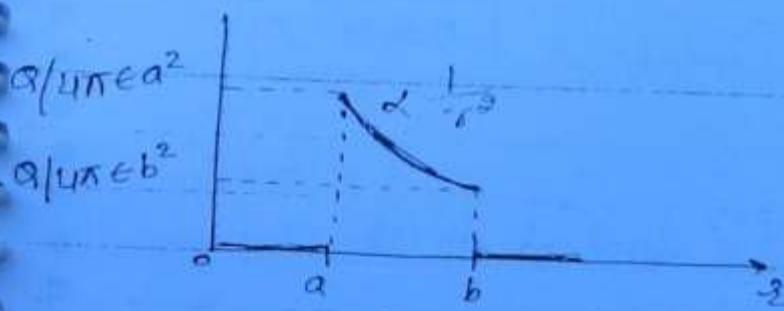
(101)

To find:

 \vec{E} for

1. $r < a$
2. $a < r < b$
3. $r > b$

$$\left\{ \begin{array}{l} r < a ; \vec{E} = 0 \\ a < r < b ; \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}_e \\ r > b ; \vec{E} = 0 \end{array} \right.$$



Electric energy density

Date _____

$$w_e = \text{J/m}^3$$

$$w_e = \lim_{\Delta V \rightarrow 0} \left(\frac{\Delta w_e}{\Delta V} \right)$$

(102)

$$\begin{aligned} w_e &= \frac{1}{2} \epsilon E^2 \\ &= \frac{1}{2} \vec{D} \cdot \vec{E} \end{aligned} \quad \text{--- J/m}^3$$

electric energy stored

$$\begin{aligned} w_e &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} QV \\ &= \frac{1}{2} \frac{Q^2}{C} \end{aligned} \quad \text{--- J}$$

Since $Q = C \cdot V$

Electric energy stored in a system of 2 charges

$$w_e = \frac{1}{2} Q_1 V_2 = \frac{1}{2} Q_1 \frac{Q_2}{4\pi\epsilon_0 s} \rightarrow \frac{1}{2} \frac{Q_1 Q_2}{4\pi\epsilon_0 s}$$

$$w_{e2} = \frac{1}{2} Q_2 V_1$$

$$w_e = w_{e1} + w_{e2} = \frac{1}{2} Q_2 \frac{Q_1}{4\pi\epsilon_0 s}$$

$$w_e = \frac{Q_1 Q_2}{4\pi\epsilon_0 s}$$

$$= \frac{1}{2} \cdot \frac{Q_1 Q_2}{4\pi\epsilon_0 s}$$

Poisson's & Laplace Eqns

Date _____

$$\nabla \cdot \vec{D} = \rho \quad \dots \text{Gauss' Law}$$

\downarrow
 $\epsilon \vec{E}$

$$\Rightarrow \epsilon \nabla \cdot \vec{E} = \rho$$

\downarrow
 $= -\nabla V$

$= -\text{Grad } V$

$$-\epsilon \nabla \cdot \nabla V = \rho$$

$= \nabla^2 V$

$$\Rightarrow \boxed{\nabla^2 V = -\frac{\rho}{\epsilon}} \quad \dots \text{Poisson's eqn.}$$

if $\rho = 0$

\dots charge free region

$$\boxed{\nabla^2 V = 0} \quad \dots \text{Laplace eqn.}$$

find V

$$\text{find } \vec{E} = -\nabla V$$

points :

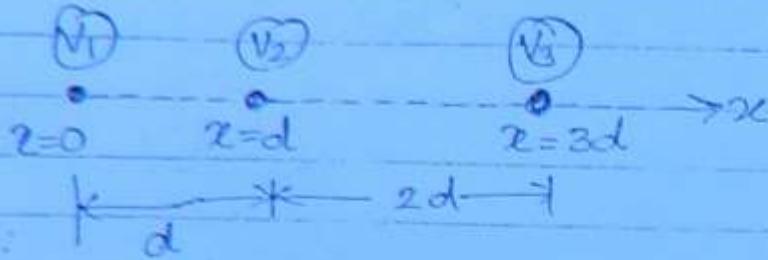
① The Poisson's eqn represent. 2nd order 3-dimen. non-homogeneous differential eqn.

② The Laplace eqn represent. 3-dimen. 2nd order homogeneous differential eqn.

③ Using these eqns. the potential V & elec. field \vec{E} can be found due to a specified volume charge distribution or a charge free region.

(103)

Ex:



104

To find :-

Relation b/w

V_1, V_2, V_3

$$\nabla^2 V = 0$$

$$V = f(x, y, z)$$

In general

$$V = f(x) \text{, only}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial V}{\partial x} = A ; \quad V = Ax + B$$

Boundary conditions :-

$$1. \quad x=0 ; \quad V = V_1$$

$$2. \quad x=d ; \quad V = V_2$$

$$3. \quad x=3d ; \quad V = V_3$$

$$\textcircled{1} \quad V_1 = A0 + B \Rightarrow B = V_1$$

$$\textcircled{2} \quad V_2 = Ad + B = Ad + V_1 \Rightarrow A = \frac{V_2 - V_1}{d}$$

$$V = Ax + B$$

$$V = \frac{V_2 - V_1}{d} \cdot x + V_1$$

(105)

$$\textcircled{3} \quad V_3 = \frac{V_2 - V_1}{d} \cdot 3d + V_1$$

$$V_3 = 3V_2 - 2V_1 \quad \text{Ans}$$

$$V = 3x^4y^4$$

To bind:

$$\epsilon(1,1,1)$$

$$\nabla^2 V = -\epsilon/\epsilon_0$$

$$\Rightarrow \epsilon = -\epsilon_0 \nabla^2 V$$

$$\epsilon = -\epsilon_0 \left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right]$$

$$\frac{\partial V}{\partial x} = 12x^3y^4 \quad ; \quad \frac{\partial^2 V}{\partial y^2} = 36x^4y^2$$

$$\frac{\partial^2 V}{\partial x^2} = 36x^2y^4$$

$$\epsilon = -\epsilon_0 [26x^2y^4 + 36x^4y^2]$$

$$\epsilon(1,1,1) = -\epsilon_0 [72]$$

$$\dots \text{C/m}^3$$

$$\vec{E} = 2x^2y^2 \hat{a}_z \rightarrow E_z \hat{a}_z$$

$$\text{To bind} \quad \epsilon(1,1,1)$$

$$\text{so } E_z = 0$$

$$\vec{E} = 0$$

$$E_z = 2x^2y^2$$

$$\vec{D} = \epsilon \vec{E}$$

$$\epsilon = V \cdot \vec{D}$$

$$\vec{D} = \epsilon \cdot 2x^2y^2 \hat{a}_z \equiv D_z \hat{a}_z$$

$$\begin{aligned}
 \rho &= \nabla \cdot \vec{D} \\
 &= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \\
 &= \frac{\partial D_z}{\partial z} = \frac{\partial}{\partial z} [\epsilon \cdot 2x^2y^2] = 0
 \end{aligned}$$

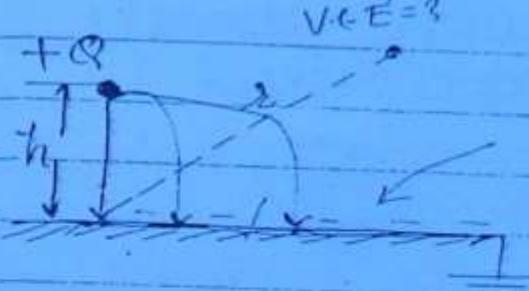
$$\nabla \cdot \vec{D} = \rho = 0$$

- charge free Region
- \vec{D} is solenoidal

(22) $V = 4x^2y$ To find \vec{E}

$$\begin{aligned}
 \vec{E} &= -\nabla V = -\text{Grad } V \\
 &= - \left[\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \right] \\
 &= - [8xy \hat{a}_x + 4x^2 \hat{a}_y]
 \end{aligned}$$

Method of Image



$$V \cdot \vec{E} = ?$$

Another method to find
 $V \cdot \vec{E}$

$$Q_{\text{ind}} = -Q$$

es (ind.) is unknown

infinite cond.

∞
grounded cond.

(107)

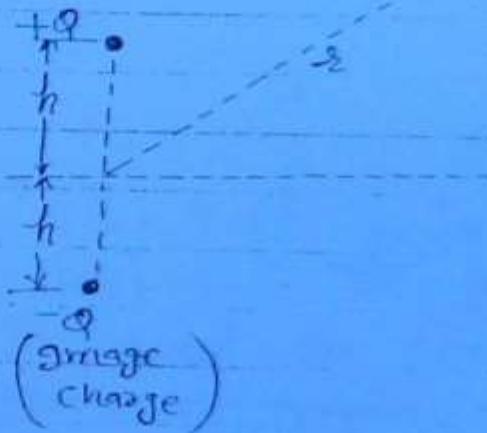
Summary :-



$$V; \vec{E}$$

$$\nabla V; \vec{E} = -\nabla V$$

Theory of images



(image charge)

points :-

- ① The method of images is applicable to the e.m. problems where any point charge is placed at some height above an infinite cond. or a grounded cond.
- Any perfect conductor behaves as a perfect reflector & therefore acts as electromag. mirror.
The entire theory of optics is applicable.
- The theory of images is applicable only

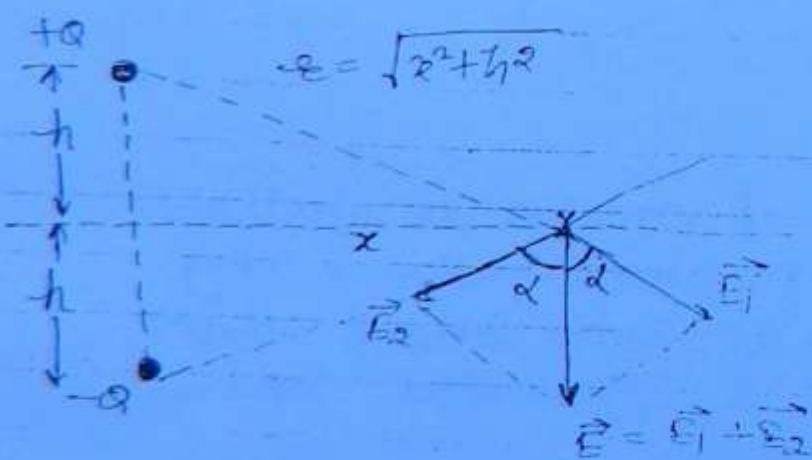
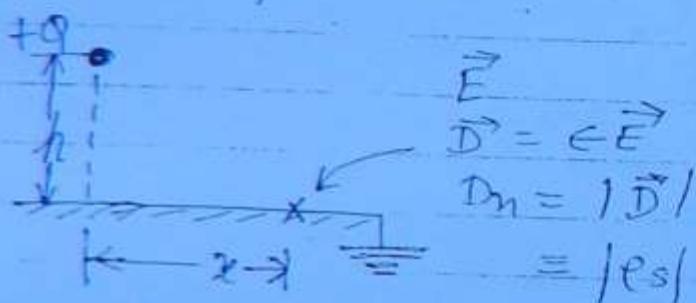
to the e.m. problems

for mag. field the theory
is not applicable. since the mag. charges
in the isolated form do not exist
in nature.

(108)

A point charge $+Q$ is placed at a height H above a grounded conductor.

Calculate the surface charge density induced on the conducting sheet.



$$|\vec{E}_1| = |\vec{E}_2| = E_0 = \frac{Q_0}{4\pi\epsilon_0 r^2}$$

$$\begin{aligned} E_m &= |\vec{E}_1| \cos \alpha + |\vec{E}_2| \cos \beta \\ &= 2 E_0 \cos \alpha \end{aligned}$$

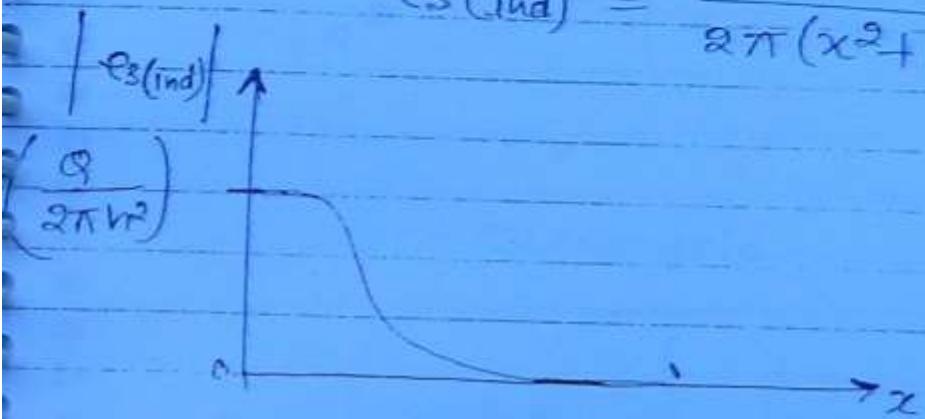
$$= 2 \frac{Q}{4\pi\epsilon_0 r^2} \cdot \frac{h}{r}$$

$$E_n = \frac{Qh}{2\pi\epsilon_0 (x^2 + h^2)^{3/2}}$$

(69)

$$D_n = \epsilon E_n = |E_s| = \frac{Qh}{2\pi(x^2 + h^2)^{3/2}}$$

$$E_s(\text{ind}) = -\frac{Qh}{2\pi(x^2 + h^2)^{3/2}}$$

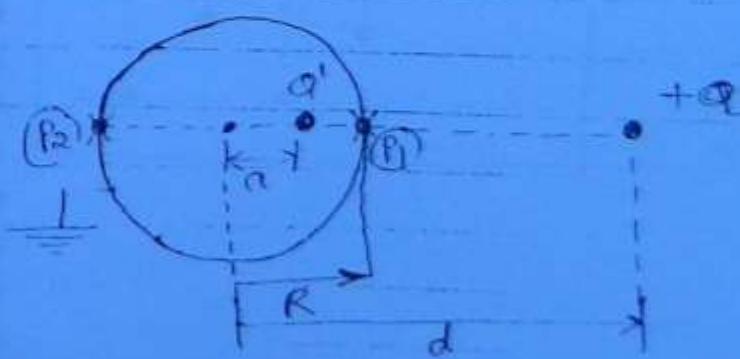


(ex:

A point charge $+Q$ is placed in front of a grounded spherical conductor of radius $'R'$ as shown.

calculate the Magnitude & the location of image charge.

Image in Spheres



q' = Magnitude] ab image
 a = location] charge

(110)

$$V = \frac{q}{4\pi\epsilon_0 s}$$

$$V_{p1} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{d-R} + \frac{q'}{R-a} \right) = 0$$

$$V_{p2} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{d+R} + \frac{q'}{R+a} \right) = 0$$

$$\frac{q}{q'} = - \frac{d-R}{R-a}$$

$$\frac{q}{q'} = - \frac{d+R}{R+a}$$

$$\Rightarrow \frac{d-R}{R-a} = \frac{d+R}{R+a}$$

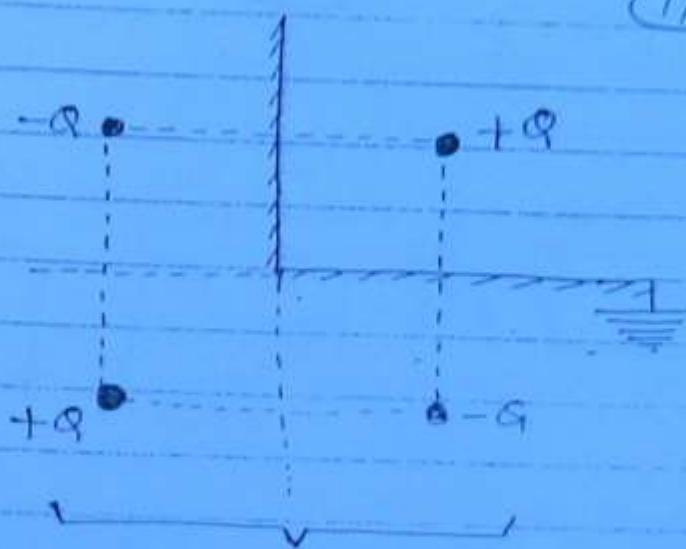
$$\Rightarrow a = \frac{R^2}{d}$$

$$\frac{q}{q'} = - \frac{d-R}{R-a}$$

$$\Rightarrow q' = - \frac{qR}{d}$$

ex:

(III)



no. of images

$$n = \frac{360^\circ}{\theta} - 1$$

$$n = \frac{360^\circ}{90^\circ} - 1$$

$$= 3$$

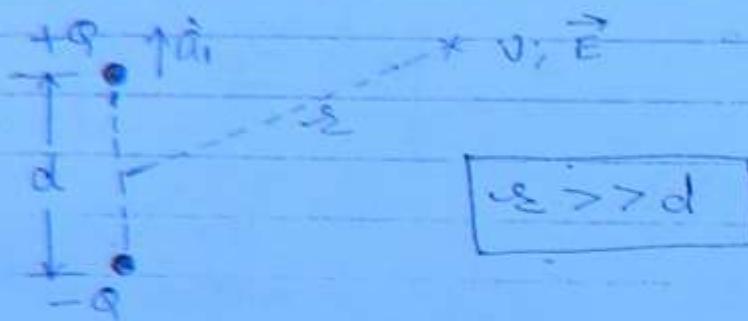
Quadrupole

| type of configuration | \vec{V} | \vec{E} |
|-----------------------|-----------|-----------|
| -q + monopole | $1/r^2$ | $1/r^3$ |
| +q -q dipole | $1/r^2$ | $1/r^3$ |
| - + + - quadrupole | $1/r^3$ | $1/r^4$ |
| ++-- octopole | | $1/r^5$ |

Main points :-

(1/2)

① electric dipole



dipole moment

$$\vec{m} = qd \cdot \hat{a}_i$$

② \vec{E} due to infinite line charge



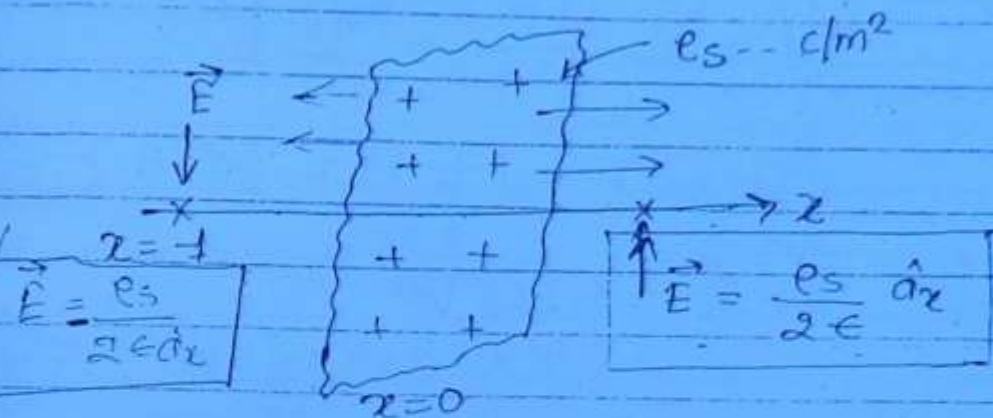
$$\vec{E} = \frac{\rho_s}{2\pi\epsilon_0 r} \cdot \hat{a}_z$$

③ circular loop



④ Infinite charge sheet :-

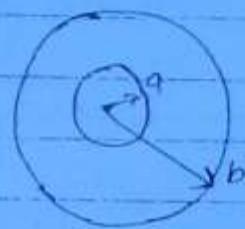
(113)



$$E_s = D_\parallel = \epsilon E_\parallel$$

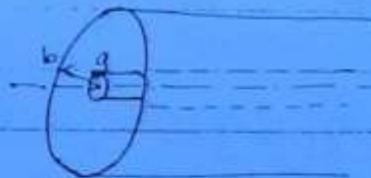
$$E_\parallel = \frac{E_s}{2\epsilon}$$

⑤ Concentrating spherical sheets :-



$$C = \frac{4\pi \epsilon_0 ab}{b-a}$$

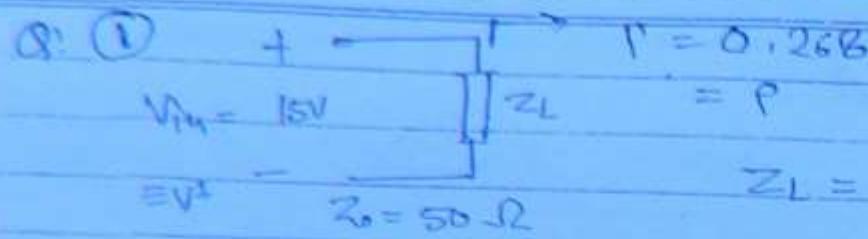
⑥ cyl. Transmission line :-



$$C = \frac{2\pi \epsilon_0}{\ln(b/a)} - \text{f/m}$$

Solution Chp 3 (T.L.)

Date _____



$$R = \frac{V^2}{2Z_L}$$

$$= 2.08W$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma} = 86.6\Omega$$

$$\frac{V_L}{V^+} = 1 + \Gamma = 1.268$$

$$\Rightarrow V_L = 1.268 V^+$$

$$= 15 \times 1.268 = 19V$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$V_p = \sqrt{LC}$$

$$Z_0 V_p = \frac{1}{C}$$

$$Z_0 \times \frac{V}{\sqrt{C}} = \frac{1}{C}$$

$$Z_0 = \frac{\sqrt{C}}{V.C}$$

$$f = 10GHz$$

$$d = 3 \times 10^{-3}m$$

$$\theta = 90^\circ = \pi/2$$

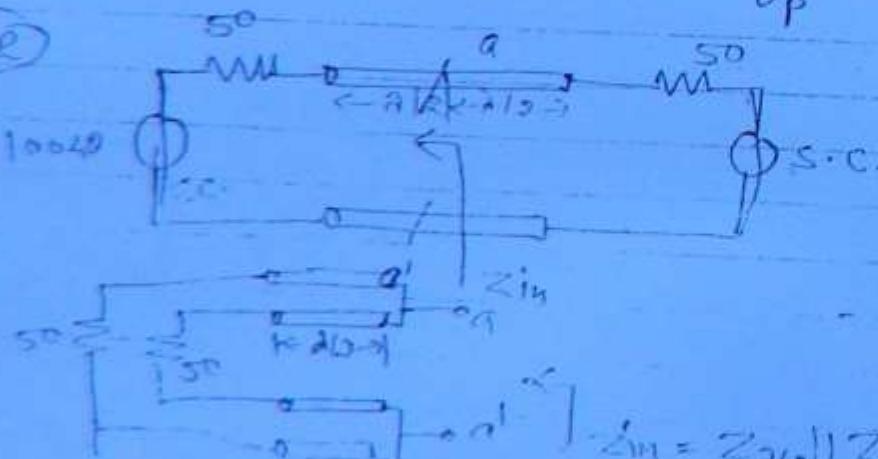
θ - phase shift - rad

β - phase const - rad/m

$$\theta = \beta \cdot d$$

$$\theta = \frac{\omega}{v_p} \cdot d = \frac{2\pi f d}{c/\sqrt{C}}$$

Q. ②

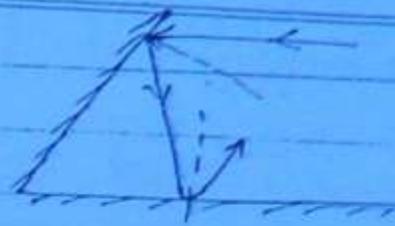
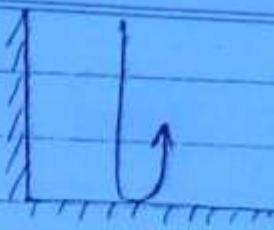


$$Z_1 || Z_2$$

$$25\Omega$$

$$25\Omega$$

(20)



(15)

$$(21) \quad \frac{\lambda/2}{\lambda} = \frac{27.5}{15 \text{ cm}} = \frac{\lambda}{30 \text{ cm}} \quad \lambda = 30 \text{ cm}$$

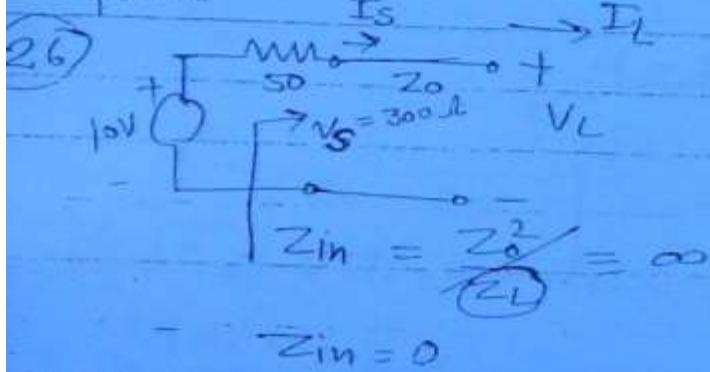
$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{30} = 10 \text{ GHz}$$

$$(24) \quad S = 3$$

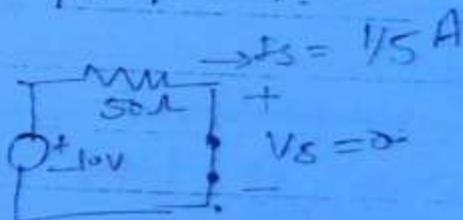
$$\ell = \frac{V^-}{V^+} = \frac{E_e}{E_i} = \frac{S-1}{S+1} = 1/2$$

$$\rho^2 = 1/4 \Rightarrow 25\%$$

deflection coeff of power



$$|V_L| = ?$$



$$V_L = \cos \beta l \cdot V_S - j Z_0 \sin \beta l \cdot I_S$$

$$|V_L| = \left| -j \frac{300}{5} \right| = 60 \text{ V}$$

$$\sin\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right) = 1$$

$$(28) \quad Z_0 = \sqrt{Z_{ac} \cdot Z_{dc}}$$

$$Z_{dc} = \frac{-Z_0^2}{Z_{ac}} \quad Z_{ac} = \underbrace{100 + j150}_{\text{inductive}}$$

$$S = \frac{V_{\max}}{V_{\min}} = \frac{4}{1} = 4$$

$$Z_{\min} = \frac{Z_0}{S}; Z_{\max} = Z_0 \cdot S$$

$$= \frac{50}{4} = 12.5 \Omega$$

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(20)

$$30V = \boxed{V+ + V-} \quad Z_L = R_L$$

$$V_L = V+ + V- = 110$$

$$V+ = 30$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} V- = 10$$

$$I_{ss} = \frac{1}{2} (I_{max} + I_{min})$$

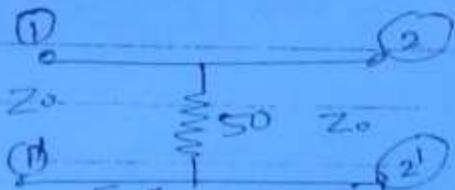
$$\frac{V}{V+} = \frac{10}{30} = \frac{1}{3}; \quad Z_L = R_L = Z_0 - \frac{1 + P}{1 - P} = 100 \Omega$$

$$I_{max} = \frac{V_{max}}{R_L} = \frac{V+ + V-}{R_L} = \frac{40}{100} = 0.4A$$

$$I_{min} = \frac{V_{min}}{R_L} = \frac{V+ - V-}{R_L} = \frac{20}{100} = 0.2A$$

$$I_{ss} = \frac{1}{2} (0.4 + 0.2) = 0.3A$$

(21)



$$Z_0 = 50 \Omega$$

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

g_{11} = total power sent from port 11' to load
by port 11' = 0

g_{12} = the total power sent from port 11' &
received by port 22' = 1

$$[Y] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

8: 33.

$$\frac{\sigma}{\omega c} \ll 1$$

Good dielectric

$$n = \sqrt{\frac{\mu_r}{\sigma + j\omega c}} = \sqrt{\frac{\mu_r}{j\omega c}} \sqrt{\frac{1}{1 + \frac{\sigma}{j\omega c}}} \approx \sqrt{\frac{1}{1 + \frac{\sigma}{j\omega c}}}$$

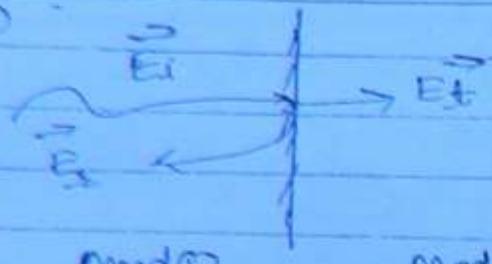
$$\eta = \sqrt{\frac{\mu_r}{2\omega c}} \left(1 + \frac{\sigma}{2\omega c} \right)$$

↑ Inductance
Capacitance

$$\approx (1 + x)^{-1/2}$$

$$\approx 1 - \frac{1}{2} \frac{\sigma}{j\omega c} \approx 1 - \frac{j\omega c}{2\omega c}$$

(38)



(118)

$$E_i = \frac{E_0}{2} \cos(\omega t - \beta z) \hat{a}_y$$

$$\vec{E}_i = \frac{E_0}{2} \hat{a}_y$$

Propagation
+ & direct
 \vec{E} has only
y compn.

$$\omega = 3 \times 10^9 \pi \text{ rad/sec}$$

$$\beta = 10\pi \text{ rad/m}$$

$$v_p = \frac{\omega}{\beta} = 3 \times 10^8 \text{ m/sec.}$$

Med(1) is free space

$$\vec{E}_i = \dots = \hat{a}_y \\ \text{prob. } + \frac{1}{2}$$

Option C

$$\times \frac{1}{2} E_0 \cos(\omega t - \beta z) \hat{a}_y$$

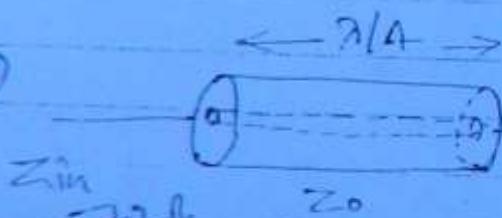
$$(d) \left(\frac{E_0}{2} \right) \cos(\omega t - (3\beta z) \hat{a}_y)$$

(40)

$$\text{Skin depth } \delta = \frac{1}{\sqrt{\mu_r + \mu_0}}$$

$$\uparrow \approx \mu_0$$

(42)



$$z_{in} = 72 \lambda \quad z_0$$

$$z_L = 50 \lambda$$

$$\begin{aligned} & \text{Left boundary } z_1 = 50 \lambda \\ & \text{Right boundary } z_L = 50 \lambda \\ & z_{in} = \sqrt{z_1 z_L} = \sqrt{72 \times 50} = 60 \lambda \quad z_{in} = \frac{z_1 z_L}{z_1 + z_L} \end{aligned}$$

$$z_0 = 60 \ln(b/a) = 60$$

$$z_0 = \ln(b/a) \Rightarrow b/a = e^1 \approx 2.7$$

$$\frac{2b}{2a} \approx 2.7 \quad ; \quad 2b \approx (2.7) 2a$$

$$2b = 27 \text{ mm} \quad 10 \text{ mm}$$

(44)

$$\gamma = \underbrace{0.0005}_{\alpha} + j \frac{\pi}{10} \beta$$

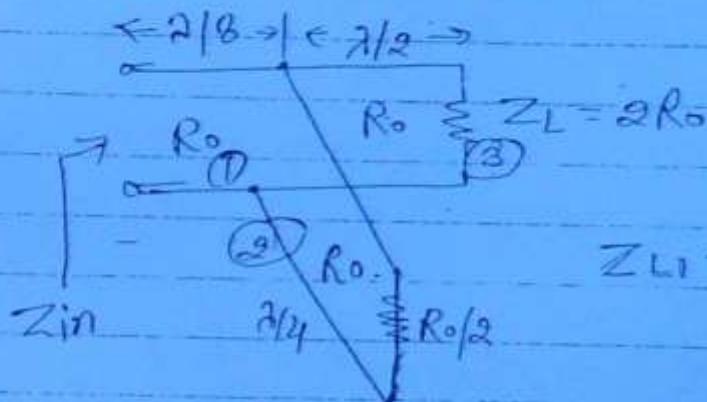
Date _____

(119)

$$\alpha l = (0.0005 \times 50) \text{ -- nepers}$$

$\times 8.686$
 \Downarrow
-- dB

(45)



$$\begin{aligned} Z_{L1} &= Z_{\alpha/2} \parallel Z_{\alpha/4} \\ &= Z_{L3} \parallel \frac{Z_0}{2} \\ &= 2R_0 \parallel \frac{R_0^2}{R_0/2} \\ &= 2R_0 \parallel 2R_0 = R_0 \end{aligned}$$

(49)

 $R, G_1 \approx \text{small}$

$$\gamma = \alpha + j\beta$$

$$= \sqrt{(R+j\omega L)(G_1+j\omega C)}$$

$$\boxed{R \ll j\omega L} \quad \boxed{G_1 \ll j\omega C} \quad = \sqrt{j\omega L \cdot j\omega C} \left[\left(1 + \frac{R}{j\omega L} \right) \left(1 + \frac{G_1}{j\omega C} \right) \right]$$

$$j\omega LC \left[\underbrace{1 + \left(\frac{R}{j\omega L} + \frac{G_1}{j\omega C} \right)}_{(1+z)^{1/2}} - \frac{R \alpha \gamma}{j\omega^2 LC} \right] V_2$$

$$\gamma = j\omega LC \left[1 + \frac{1}{2} \left(\frac{R}{j\omega L} + \frac{G_1}{j\omega C} \right) \right]$$

Real part

$$Z = w \pi L C \left[\frac{R}{2wL} + \frac{G_f}{2wC} \right] \quad (120)$$

$$Z = \frac{1}{2} [R \underbrace{\frac{C}{L}}_{=Y_{20}} + G_f \underbrace{\frac{L}{C}}_{=Z_0}]$$

$$Z = \frac{1}{2} \left(\frac{R}{Z_0} + G_f Z_0 \right) \quad \text{Ans.}$$

(46) $l = 0.5 \text{ m}$

$$L =$$

$$C =$$

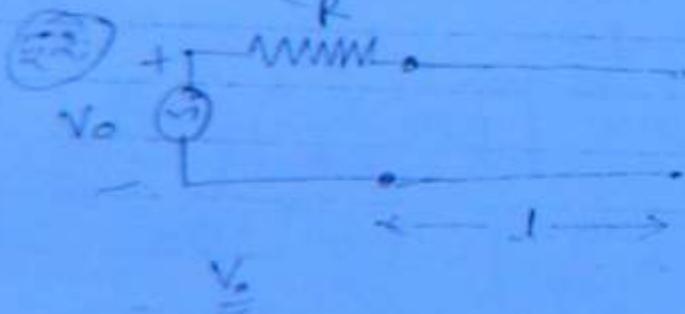
$$f = 2.5 \text{ MHz}$$

$$\beta l = \frac{w}{v_p} l = \frac{2\pi f l}{1/\sqrt{LC}} = 2\pi \sqrt{LC} \cdot f \cdot l.$$

(51) $Z_0 = 50 \Omega$

$$\bar{Z} = 0.5 - j0.3 = \frac{Z}{Z_0}$$

$$Z = (0.5 - j0.3) Z_0$$



in Smith chart
all the comp. are
taken in normalized
 $Z = \bar{Z}$ form

$$\textcircled{2} \quad d = \frac{\lambda}{4} = \frac{1}{4} \frac{V_p}{f} = \frac{1}{4f} \frac{C}{\sigma \epsilon_r} \quad \textcircled{121}$$

$\cong 0.28$

$$\textcircled{3} \quad \eta = \sqrt{\frac{\mu}{\epsilon}} \quad V_p = \frac{1}{\sqrt{\mu \epsilon}}$$

(6) free space perfect dielectric
 $\sigma = 0$

$$C = C_0 \frac{\epsilon_r}{1} \quad \mu = \frac{\mu_0 \mu_r}{1}; \mu_r = 1; \mu = \mu_0$$

$$\epsilon = \epsilon_0$$

$$\vec{J}_c = \sigma \vec{E} = 0; \quad \vec{P}_c = 0$$

$$\boxed{P=0}$$

$$V_p = c = 3 \times 10^8 \text{ m/sec.}$$

$$\eta = n_b = \sqrt{\mu_0 / \epsilon_0} = 120 \pi \text{ rad}$$

$$\textcircled{11} \quad \vec{H} = 0.1 \sin(10^8 \pi t + \beta y) \hat{a}_z$$

prof. $- \hat{a}_y$

$$\vec{P} = \frac{1}{2} n_0 H_m^2$$

$$\frac{1}{2} \times 120 \pi \times 0.1$$

$$\textcircled{12} \quad \sigma = 5 \text{ mho/m} \quad E_x = E_0 e^{-\kappa z}$$

$$\epsilon_r = 80 \quad e^{-\kappa z} = \frac{E_x}{E_0} = 0.1$$

$$f = 25 \text{ KHz}$$

low freq.

$$e^{-\kappa z} = 10$$

$$\boxed{R = \frac{1}{\alpha} \ln(10)}$$

$\frac{\sigma}{\omega \epsilon} \gg \text{Good Conductor}$

$$\alpha = \sqrt{\frac{\omega \mu_0}{\sigma}} = \sqrt{\pi f \mu_0} \cong \mu_0$$

(19) E_x

Sinusoidal time variation

$$P_{\text{prop}} \geq +z$$

lossless medium

$$\sigma = 0$$

$$\Rightarrow \lambda = 0$$

$$E_x = E_0 e^{-\gamma z}$$

$$E_x = E_0 e^{-(\chi + jB)z} \quad (T22)$$

$$E_x = E_0 e^{-jBz}$$

$$= E_0 e^{-jkz}$$

(20)

$$\vec{E} = 50 \sin(\omega t + Kz) \hat{a}_y$$

$$\uparrow \omega$$

$$\swarrow Kz$$

$$P_{\text{prop}} = -z \leftarrow$$

$$= E_y \hat{a}_x$$

$$\vec{P} = -P_z \hat{a}_z$$

$$K = \beta = \frac{\omega}{v_p} = \frac{10^7}{3 \times 10^8} = \frac{1}{30}$$

$$z = \frac{2\pi}{\beta} = 2\pi \times 30 = 60\pi \approx 188.5 \text{ m}$$

(21)

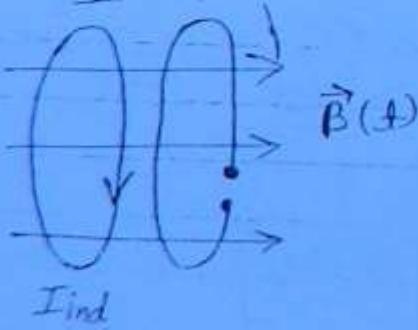
$$U = \epsilon; \eta_1 = \eta_0 = 377 \Omega$$

$$\eta_2 = \sqrt{\frac{U}{\epsilon}} = 1 \Omega$$

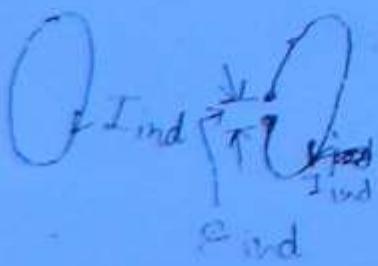
$$f = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1 - 377}{1 + 377} = -\frac{376}{378} \approx -1$$

$$|f| \approx 1$$

(22)



$\vec{B}(t) \rightarrow \psi_m(t) \rightarrow \frac{\partial \psi_m}{\partial t} \rightarrow I_{\text{ind}}$
 Given by Faraday's Law of emf



Comments :- using faradys law of emf induction, due to rate of change of mag flux induced current in present in the inner loop which will ride for

Induced emf. when the loop is open-circuited.

Therefore induced emf equivalent
element of I_{sd} is always present on the
loop whenever mag. field is time variant.

(24) $\vec{A} = \frac{\mu}{4\pi\sigma} \int_C \frac{\vec{I}}{r} dl$

(23)

due to line current.

as $r \rightarrow \infty$; $\vec{A} = 0$

(26) $\left. \begin{matrix} V_p \\ \eta \end{matrix} \right\} n = \sqrt{\frac{\mu}{c}} = \sqrt{\frac{\mu_0 H_0}{\epsilon_0 \sigma s}} = 377 \sqrt{\frac{\mu_0}{\epsilon_0 s}}$

(27) $\boxed{\begin{matrix} \text{perfect } \textcircled{2} \\ \text{cond.} \end{matrix}} \quad \stackrel{\textcircled{1}}{\longrightarrow} \quad E_{t1} = E_{t2} = 0$
 $H_{t1} - H_{t2} = \int_s^{\infty}$
 $H_{t1} = H_{t2} = 0$

(29) $\frac{I_c}{I_d} = \frac{J_c}{J_d} = \frac{\sigma}{w\epsilon} = 1$

$$\sigma = w\epsilon = 2\pi f \epsilon$$

$$f = \frac{\sigma}{2\pi \epsilon_0 \epsilon_s}$$

Wave Propagation

Date _____

① Surface wave propagation :-

(124)

- used at medium freq.

- AM broad cast

f : 535 KHz - 1605 KHz

② Space wave prop.

Tropospheric prop

LOS (line-of-sight) prop

} used at VHF;

microwave range

- preferred for

- TV broad cast

- microwave link.

③ Sky-wave prop

Ionoospheric

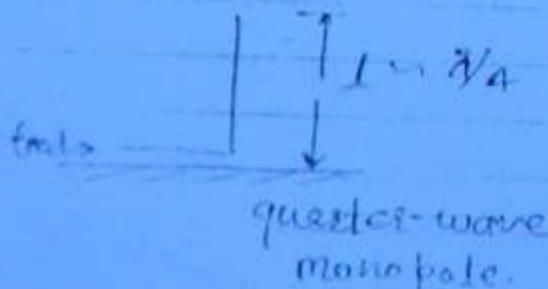
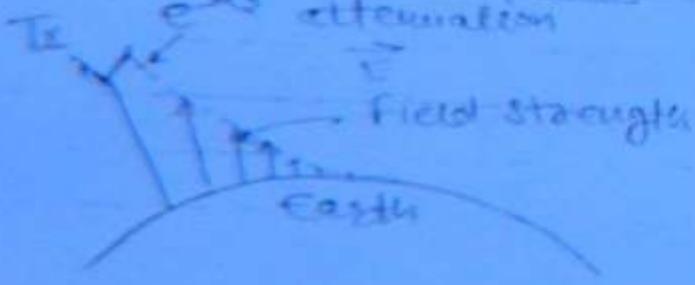
} used at

HF range (short-wave range)

preferred for FM broad cast

(f : 88 MHz - 108 MHz)

④ Surface wave propagation



features :-

- ① The em waves travel along the surface of the earth.

② The transmitting antenna is always vertically installed & therefore is always vertically polarized. (25)

③ The electric field associated with the wave is perpendicular to the surface of earth.

④ The earth behaves as good conductor & therefore as the wave travels the electric field strength decreases exponentially.

⑤ The length of the antenna depends upon the freq. of wavelength of operation quarterwave monopole are always preferred for the transmission of such signal.

⑥ as the frequency of operation increase the height of antenna decreases.

⑦ preferred for AM broadcast on the freq. range 535 to 1605 kHz.

The range of transmission can be increased only by increasing power of Transmitter.

⑧ Such propagation take place when the transmitting & receiving antenna very close to each other & close to surface of earth.

③ Limited range of transmission

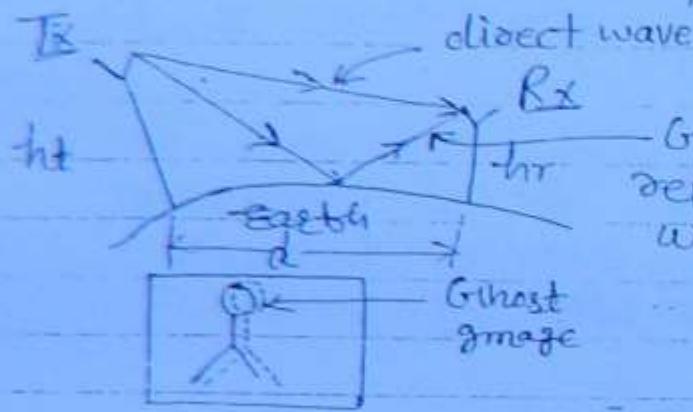
②

Space wave prop.

(T26)

Tropospheric prop.

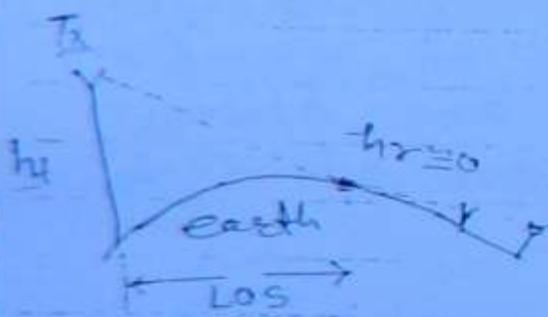
LOS prop.



Ground reflected wave

$$d \approx 3550(\sqrt{ht} + \sqrt{hr})$$

--- m



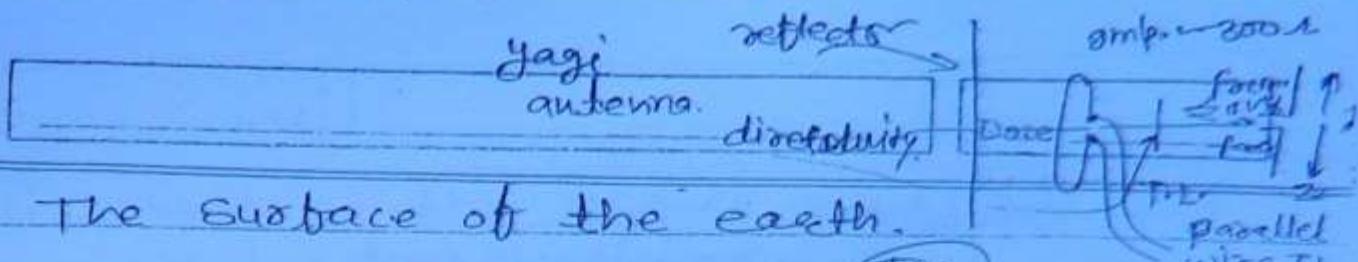
$$d \approx 3550 \sqrt{ht}$$

Features :-

- ① The space wave constitutes :-
 - (a) direct wave
 - (b) ground reflected wave.

② Propagated for freq. greater than 30 MHz for TV broadcast in the VHF range.

③ The em. wave emitted from transmitters receive in the earth atmosphere at a height of 10 to 15 km above

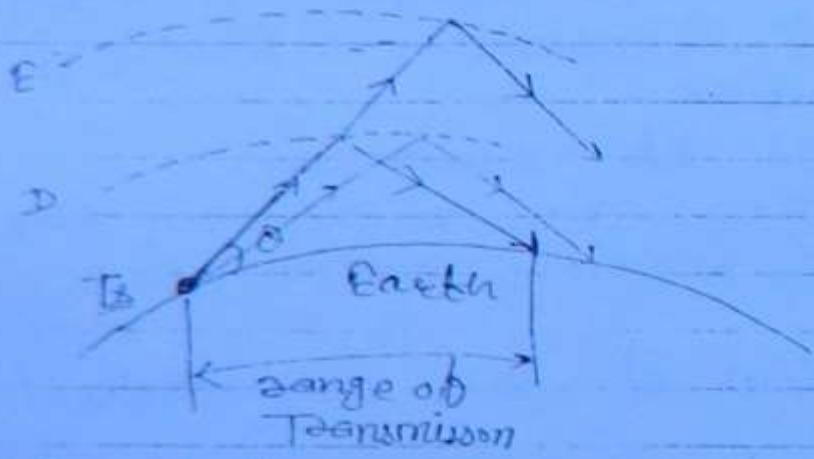
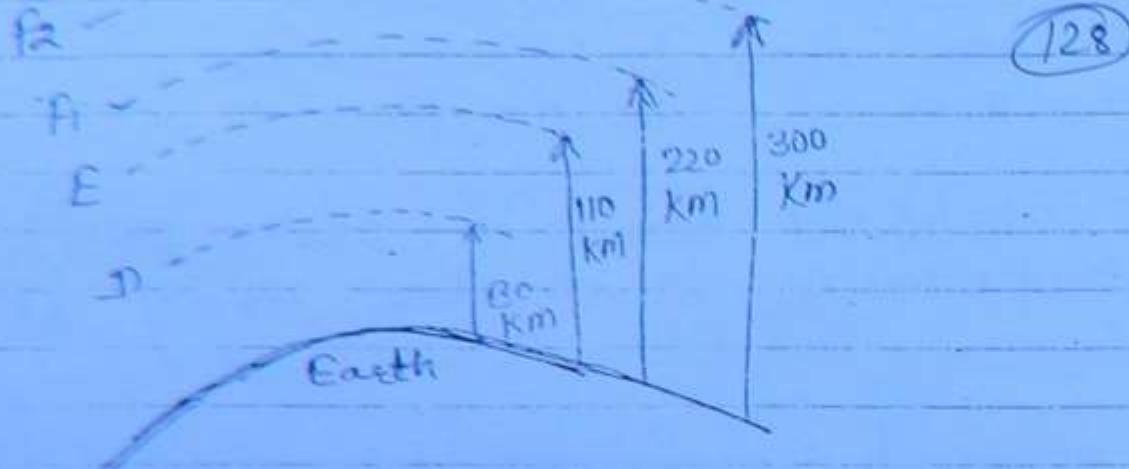


The surface of the earth.

(T27)

- ③ The antenna is always horizontally polarized so that electric field vector is parallel to the surface of the earth.
- ④ For dy pole is preferred for such probg. so that it's impedance is matched with that of T.L.
- ⑤ The total range of transmission depends upon:
 - ① power of the transmitter.
 - ② Height of Transmitting & Receiving Antenna.
 - ③ The range of transmission can be increased by increasing the height of the receiving antenna.
- ⑥ Factors which control of the magnitude of space waves:-
 - a) Conductivity of earth
 - b) Permittivity & permeability of earth
 - c) freq. of the wave.
 - d) Heights of Transmitting & receiving antenna
 - e) Curvature of the earth
 - f) Distance b/w T/m & Receiving antenna
 - g) Variation of reflecting index of earth with Height.

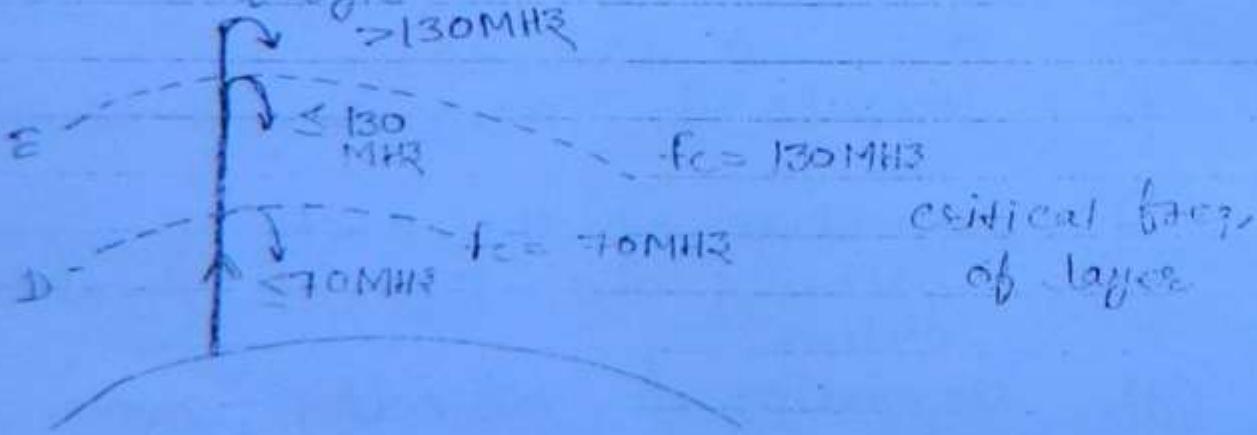
(3) sky wave prop
Ionospheric prop.



$\Theta = \text{TOA}$

- take off angle

$> 130 \text{ MHz}$



in fd of layers.

S skip distance.

① used in the HFR & prohibited for FM broadcast in the freq. range 88 MHz to 200 MHz.

(29)

② long range transmission is possible.

③ The range of transmission depends:

(a) Take of angle.

(b) freq. of the signal.

④ The D layer has minimum electron density whereas F2 layer has Max. electron density.

⑤ The critical freq. of a layer depends upon the electron density of the layer.

$$f_c = \sqrt{81 N}$$

↑ ↑
critical electron
freq. density
MHz /m³

Therefore the critical freq. of the D layer is minimum whereas this freq. has a Max. value for F2 layer.

During night time D-layer is missing, it & F2 layer merge together to form a single layer.

The E-layer is the most stable layer of FM broadcast uses this layer for the TFM of the signal.

(e) critical freq. angle :- This is the maximum freq. of a layer so that the wave is reflected by that layer at vertical incidence.

The e.m. wave of freq. less than or equal to critical freq. will be reflected from the layer irrespective of the angle of incidence.

as the height of layer increases, its critical freq. increases

s) skip distance :- This is the minimum distance from the Tx at the sky wave of given freq. it return to earth by the ionospheric.

The skip distance depends upon:

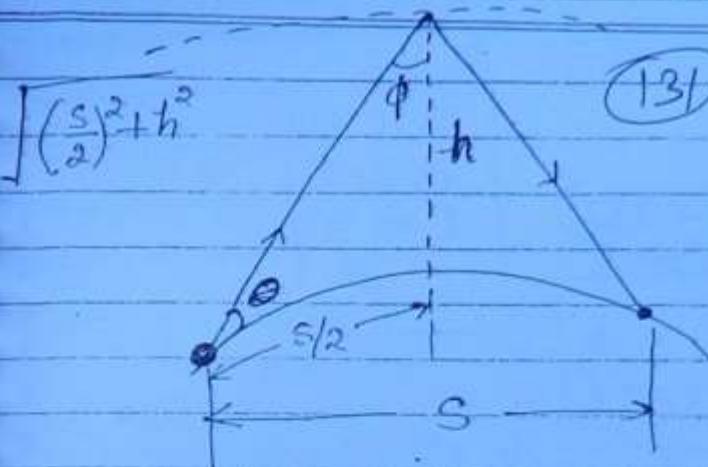
- (a) freq. of the wave.
- (b) fc (critical freq.)
- (c) Height of the layer.
- (d) charge carried concentration + N.

MUF) Maximum usable frequency.

$$\boxed{MUF = fc \cdot n_{eff}}$$

$$MUF > fc$$

$\{ Tx = \text{Transmitter}$
 $Rx = \text{Receiver}$
 Date



$$\theta = \text{TOA}$$

ϕ = angle of incidence
at the layer

$$\theta + \phi = 90^\circ$$

$$\phi = 90^\circ - \theta$$

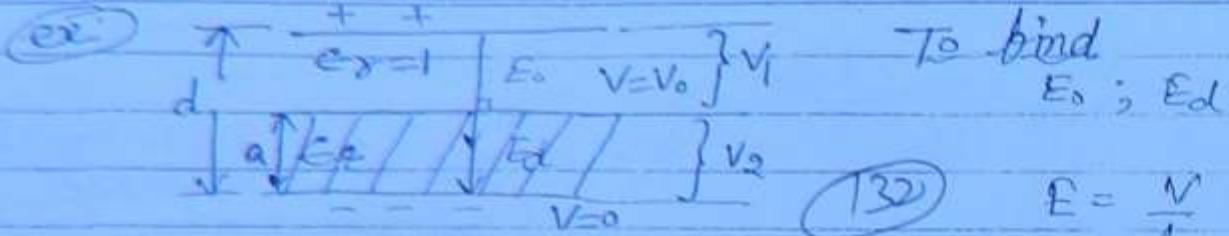
$$\begin{aligned}
 MUF &= f_c \cdot \sec \phi \\
 &= f_c \cdot \sqrt{\left(\frac{s}{2}\right)^2 + h^2} / h
 \end{aligned}$$

$$MUF = f_c \cdot \sqrt{\left(\frac{s}{2h}\right)^2 + 1}$$

(1) for fixed location of the Tx & Rx \Rightarrow MUF is the freq. which makes the distance to the receiving point equal to the skip distance.

(2) MUF is the freq. that gives strongest sky wave signal at the received point.

(3) The skip distance increases with the freq. of operation.



(32) $E = \frac{V}{d}$

$$V_1 + V_2 = 0$$

$$E_0(d-a) + E_d \cdot a = V_0 \quad \text{--- (1)}$$

$$D_{n1} - D_{n2} = \rho^* s$$

$$D_1 = D_2$$

$$\epsilon_0 E_0 = \epsilon_0 \epsilon_r E_d$$

$$E_0 = \epsilon_r E_d \quad \text{--- (2)}$$

$$\epsilon_0 E_d (d-a) + E_d a = V_0$$

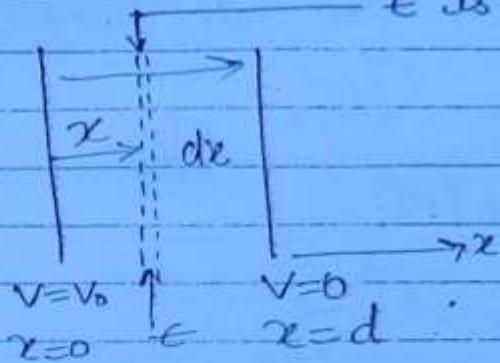
$$E_d [\epsilon_0 d - \epsilon_0 a + a] = V_0$$

$$E_d = \frac{V_0}{\epsilon_0 [d-a] + a}$$

$$E_0 = \epsilon_0 \cdot E_d = \frac{\epsilon_0 V_0}{\epsilon_0 (d-a) + a} = \frac{\epsilon_r V_0}{\epsilon_r (d-a) + a}$$

$$C = \frac{1}{\frac{1}{C_0} + \frac{1}{C_d}} \quad ; \quad C_0 = \frac{\epsilon_0 A}{d-a}$$

$$C = \frac{C_0 C_d}{C_0 + C_d} ; \quad C_0 = \frac{\epsilon_0 A}{d-a} \\ C_d = \frac{\epsilon_r \epsilon_0 A}{a}$$

Ex ϵ is varying linearly w.r.t. x at $x=0$; $\epsilon = \epsilon_1 - \text{min}$.
 $x=d$; $\epsilon = \epsilon_2 - \text{max}$

(133)

To find

Capacitance

$$\epsilon = \epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x$$

$$\left. \begin{array}{l} \epsilon_s = D_n \\ V = - \int_{x=d}^0 \vec{E} \cdot d\vec{l} \end{array} \right\} \vec{E} = - \nabla V$$

$$= - \int_{x=d}^0 E_n \cdot dx \Rightarrow - \int_{x=d}^0 \frac{D_n}{\epsilon} \cdot dx$$

$$= - \int_{x=d}^0 \frac{\epsilon_s}{\epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x} dx \quad \epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x = t$$

$$= - \epsilon_s \int_{x=d}^0 \frac{1}{t} \cdot \frac{d}{\epsilon_2 - \epsilon_1} dt \quad \frac{\epsilon_2 - \epsilon_1}{d} dx = dt$$

$$= - \frac{\epsilon_s \cdot d}{\epsilon_2 - \epsilon_1} \ln t \Big|_{x=d}^0$$

$$= - \frac{\epsilon_s d}{\epsilon_2 - \epsilon_1} \ln \left[\epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x \right]_{x=d}^0$$

$$= - \frac{\epsilon_s \cdot d}{\epsilon_2 - \epsilon_1} \ln \frac{\epsilon_1}{\epsilon_2}$$

$$= + \frac{\epsilon_s \cdot d}{\epsilon_2 - \epsilon_1} \ln \left(\frac{\epsilon_2}{\epsilon_1} \right)$$

$$c = \frac{Q}{V} = \frac{\rho_s}{V} \text{ -- cap. per unit surface}$$

F/m^2

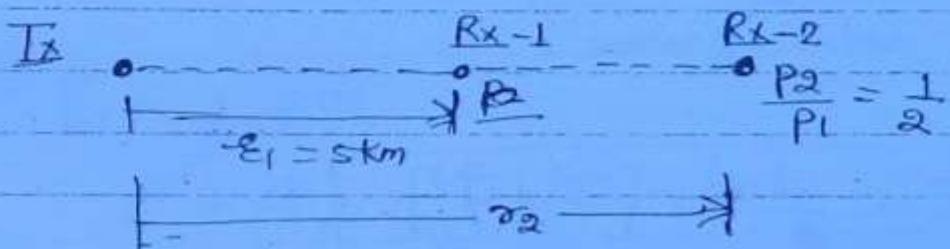
$$\left[c = \frac{\epsilon_2 - \epsilon_1}{d \ln\left(\frac{\epsilon_2}{\epsilon_1}\right)} \right] \cdot F/m^2$$

(134)

$$30. \quad \tan S = \frac{\sigma}{w e} \quad \begin{matrix} \uparrow \\ 2\pi f \end{matrix} \quad \begin{matrix} \uparrow \\ \text{frees} \end{matrix} \quad \boxed{135}$$

$$3). \quad \epsilon_S = Dn = \frac{\epsilon}{\epsilon_0} E_n \quad \xrightarrow{\epsilon \propto 2V/m}$$

39



$$d = x_2 - s_1 = (x_2 - 5) \text{ km} \quad \left| \begin{array}{l} |\vec{P}| = \frac{\text{power}}{4\pi d^2} \\ |\vec{P}| \propto \frac{1}{d^2} = P \end{array} \right.$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{\sigma_1}{\sigma_2}\right)^2 = \frac{1}{2}$$

$$\Rightarrow \vec{r}_2 = \sqrt{2} \vec{r}_1 \Rightarrow 5\sqrt{2} \text{ km}$$

$$\Rightarrow \varepsilon_2 - \varepsilon_1 = d = (5\sqrt{2} - 5) = 5(\sqrt{2} - 1) = 2.070 \text{ km}$$

$$P_{dB} = 10 \log_{10} P$$

$$\left(\frac{P_2}{P_1}\right) = -3 \text{dB} \equiv 10 \log_{10} \left(\frac{P_2}{P_1}\right)$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{1}{2}$$

$$37 \quad e = \epsilon_0$$

$$A \\ d$$

$$v = 0.5 \text{ V}$$

$$f = 3.6 \text{ GHz}$$

$$I_d = \vec{J}_d \cdot \vec{A} = \frac{\partial D}{\partial t} \cdot A = j\omega \epsilon_0 E A = j\omega \epsilon_0 \frac{v}{\lambda}$$

$$|I_d| = 2\pi f \epsilon_0 \frac{v}{\lambda} A$$

136

Tensor:

$$\vec{P} = \frac{1}{2} \operatorname{Re} [\vec{E} \times \vec{H}^*]$$

$$\vec{P} = \frac{1}{2} \operatorname{Re} [(\hat{a}_z + j\hat{a}_y) e^{j(kz - \omega t)} \times \underbrace{\left(\frac{k}{\omega u} \right)}_{\vec{H}^*} (\hat{a}_y + j\hat{a}_x) e^{-j(kz + \omega t)}]$$

$$k' \operatorname{Re} [\hat{a}_z - \hat{a}_x]$$

$$= \vec{0} \quad \text{--- null vector.}$$

$$\begin{pmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

① $\vec{P} = \vec{E} \times \vec{H}$

Poynting's vector

② $\vec{P} = \frac{1}{2} \vec{E} \times \vec{H}$

--- average Poynting vector.

③ $\vec{P} = \frac{1}{2} \operatorname{Re} [\vec{E} \times \vec{H}^*]$

---- average Poynting vector
when $\vec{E} \times \vec{H}$ are phasors.

$$\textcircled{1} \quad \vec{E} = 2 \hat{a}_x$$

$$\vec{H} = 4 \hat{a}_y$$

$$\vec{P} = \vec{E} \times \vec{H} = 8 \hat{a}_z$$

(137)

$$\textcircled{2} \quad \vec{E} = 4 \cos(\omega t - \beta z) \hat{a}_x$$

$$\vec{H} = 2 \cos(\omega t - \beta z) \hat{a}_y$$

$$\vec{P} = (\vec{E} \times \vec{H}) = 8 \underbrace{\cos^2(\dots)}_{\text{average value}} \hat{a}_z$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\vec{P}_{av} = \frac{1}{2} 8 \hat{a}_z$$

$$\textcircled{3} \quad \vec{E} = \underbrace{(\hat{a}_x + j \hat{a}_y)}_{\text{phasor}} e^{jkz - j\omega t}$$

$$\vec{H} = \dots$$

Q: 39

Med (1) -- free space
 $\eta_0 ; \epsilon_0$ Med (2) -- perfect diele. ($\sigma = 0 ; \mu_r = 1$)
 $\eta_2 ; \epsilon_2 : \eta ; \epsilon$

$$\boxed{\epsilon > \epsilon_0}$$

$$S = \frac{V_{max}}{V_{min}} = \frac{E_{max}}{E_{min}} = 5$$

$$\rho = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \\ = \frac{\eta - \eta_0}{\eta + \eta_0}$$

$$\rho = \frac{\eta_0 - \eta}{\eta_0 + \eta} < \frac{\sqrt{\epsilon} - \sqrt{\epsilon_0}}{\sqrt{\epsilon} + \sqrt{\epsilon_0}} < \frac{\sqrt{\epsilon_0} - \sqrt{\epsilon}}{\sqrt{\epsilon_0} + \sqrt{\epsilon}}$$

$\uparrow 120\pi$

$$\eta = 24\pi \text{ rad.}$$

Q: 41.

$$P_i = P_e + P_t$$

(138)

$$I = \frac{P_e}{P_i} + \frac{P_t}{P_i}$$

$$\left(\frac{P_t}{P_i}\right) = \left[1 - \left(\frac{P_e}{P_i}\right)\right]$$

← reflection coefficient of power.

$$\frac{P_t}{P_i} = 1 - \left(\frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \right)^2 ; \quad \epsilon_1 = \epsilon_0 \epsilon_{\infty 1} = \epsilon_0 \\ \epsilon_2 = \epsilon_0 \epsilon_{\infty 2} = 4 \epsilon_0 \\ = \frac{8}{9}$$

Q: 42.

$$E_z = E_0 e^{-j\beta z} \quad \text{along } +\vec{z} \text{ direction}$$

$$= E_0 e^{-(\vec{k} + j\beta) \vec{z}}$$

$$= E_0 e^{-j\beta z} \quad \text{lossless Med.}$$

$$E_0 e^{-j\beta z} e^{j\omega t}$$

for sinusoidally varying field.

$$E_z = E_0 e^{j(\omega t - \beta z)}$$

+ \vec{z} direction prop.

→ Wave propagates along some arbitrary direction.

$$E_z = E_0 e^{j(\omega t - \vec{k} \cdot \vec{r})}$$

To be
informed

$$\vec{B} \cdot \vec{r} = (\beta_x \hat{a}_x + \beta_y \hat{a}_y + \beta_z \hat{a}_z) \cdot (x \cdot \hat{a}_x + y \cdot \hat{a}_y + z \cdot \hat{a}_z)$$

$$\vec{B} \cdot \vec{r} = \beta_x \cdot x + \beta_y \cdot y + \beta_z \cdot z$$

(134)

$$= \beta \cos \theta_1 + \beta \cos \theta_2 + \beta \cos \theta_3$$

$$\frac{2\pi}{\lambda} \underbrace{\cos 30^\circ}_{\sqrt{3}/2} \stackrel{90^\circ}{=} 0 \quad \beta \cos 60^\circ \cdot \frac{2\pi}{\lambda} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3} \cdot \pi}{\lambda} \quad = \pi/\lambda$$

$$E_r = E_0 e^{j(\omega t - \frac{\sqrt{3}\pi}{\lambda} x - 0 - \frac{\pi}{\lambda} \cdot 3)}$$

(44)

incident -- RCP

reflected -- LP

↓

$$\theta = \theta_B$$

$$\tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \rightarrow \epsilon_{r2} \cdot \epsilon_0$$

$$\sqrt{3} \rightarrow \epsilon_0 \epsilon_{r1} = \epsilon_0$$

$$\epsilon_{r2} = 3$$

(46)

$$\epsilon_{r1} = 1$$

$$\vec{E}_1 = 1 \cdot \hat{a}_x$$

$$\overbrace{\qquad\qquad\qquad}^{\leftarrow} \epsilon_s = D_{n1} - D_{n2}$$

$$\epsilon_{r2} = 2 \quad \vec{E}_2 = 2 \hat{a}_x$$

$$= \epsilon_0 \epsilon_{r1} \epsilon_{r1} - \epsilon_0 \epsilon_{r2} \beta_{n2}$$

$$= \epsilon_0 [1 \times 1 - 2 \times 2]$$

$$= -3 \epsilon_0$$

(49)

- End -- both coil



(140)

perfect cond.



$$\text{losses} = 0$$



Heat dissipation = 0

$$\vec{H} = H_0 \hat{a}_x$$

(50)

$$\vec{H} = 0.1 \cos(4 \times 10^7 t - \beta_2) \hat{a}_2$$

$\phi \rightarrow +\pi$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\hat{a}_2 = (-\hat{a}_y) \times (\hat{a}_x)$$

$$\vec{P} = P_2 \hat{a}_3$$

$$\begin{pmatrix} \hat{a}_x & \perp \hat{a}_3 \\ \hat{a}_y & \perp \hat{a}_3 \\ \hat{a}_z & \end{pmatrix}$$

$$\vec{E} = -E_y \hat{a}_y$$

Notice

$$= \mu_0 H_m = 377 \times 0.1$$

(51)

$$\vec{E} = 50 \cos(\dots) \hat{a}_x$$

$$\vec{H} = \frac{5}{12\pi} \cos(\dots) \hat{a}_y$$

direct of prop. $\Rightarrow +\pi$

$$\text{Power} = P_2 \times \omega \times \epsilon_0$$

$$\frac{1}{2} \epsilon_0 H_m (\pi r^2) \times \frac{5}{12\pi} m$$

\uparrow \uparrow \uparrow
 50 $5/12\pi$ m

(53) $\vec{H} = (\quad) \hat{a}_z$

$\vec{E} \neq \hat{a}_z$ (14+)

-- wave is not polarized in z direction

(54) $\left(\frac{\vec{F}}{I}\right) = \vec{I} \times \vec{B}$

(63) E_{ind} $\begin{cases} \text{Cond. is moving } (\vec{v}) ; \vec{B} \neq f(t) \\ \vec{v} = 0 ; \vec{B} = f(t) \\ \rightarrow \vec{v} ; \vec{B} = f(t) \end{cases}$

Case 1: moving cond. ; $\vec{v} \neq f(t)$

$$E_{\text{ind}} = \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{l} \quad \text{Motional emf.}$$

Case 2: $\vec{v} = 0$
 $\vec{B} = f(t)$

$$E_{\text{ind}} = - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \begin{array}{l} \text{due to time} \\ \text{varying field} \end{array}$$

Case 3: moving cond. $\vec{B} = f(t)$ $\begin{array}{l} \text{emf due to} \\ \text{transformation action} \end{array}$

$$E_{\text{ind}} = E_{\text{ind}} (\text{case 1}) + E_{\text{ind}} (\text{case 2})$$

(67) σ ; ϵ_s

$$(\Delta n) = \epsilon_s$$

$$\rho = w_0 = \frac{1}{2} \epsilon E_m^2$$

$$= \frac{\epsilon}{2} \left(\frac{\Delta n}{\epsilon} \right)^2$$

$$= \frac{1}{2} \epsilon \cdot \frac{\epsilon s^2}{\epsilon^2} = \frac{1}{2} \epsilon \rho s^2$$

$$= \frac{1}{2} \epsilon \sigma^2$$

(68)

$$PE = KE$$

Potential energy = kinetic energy

$$eV = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2eV}{m}} \text{ --- Known}$$

$$F = \frac{mv^2}{r} = evB$$

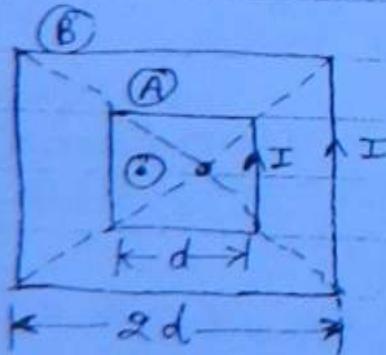
$$\boxed{E = \frac{mv^2}{eB}}$$

result.

250 gauss \rightarrow to convert,
 250×10^{-4}

$$= -wb/m^2$$

(69)



$$\frac{H_A}{H_B} = ? = \frac{\gamma_B}{\gamma_A}$$

$$H \propto \frac{1}{r}$$

$$\frac{\epsilon B}{\epsilon A} = \frac{d}{d/2} = 2$$

(743)

symmetrie in \vec{H}

$$(74) - \vec{E} = \hat{a}_y \cdot A \left(\cos \omega t - \frac{z}{c} \right)$$

$$\vec{E} = E_y \hat{a}_y \quad \text{prop.} \Rightarrow -t \vec{e}$$

$$\vec{H} = ?$$

$$\vec{P} = \vec{P}_2 \hat{a}_3$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\hat{a}_3 = \hat{a}_y \times (-a_x)$$

$$\vec{H} = -H_2 \hat{a}_x$$

$$\frac{E}{\eta} = E_y \sqrt{\frac{\epsilon_0}{\mu_0}}$$

option 8

$$(c) \hat{a}_x \left[-j \sqrt{\frac{\epsilon_0}{\mu_0}} A \cos(-) \right] = \sin(-)$$

$$(d) \hat{a}_x \left[-j \sqrt{\frac{\epsilon_0}{\mu_0}} - A \sin(-) \right] \quad (125)$$

(75)

$$\vec{D} = 2(\hat{a}_x - \sqrt{3} \hat{a}_3)$$

$$e_s = |D_n| = 2 \sqrt{1+3} = 4 \text{ c/m}^2$$

$$+ \uparrow + \uparrow + \uparrow \quad |D_n| = e_s = 4 \text{ c/m}^2$$

center

Q: 2

~~Diagram~~

$$fe = \frac{q^2}{4\pi\epsilon_0 d^2}$$

$$mg = mg$$

$$mg = \frac{q^2}{4\pi\epsilon_0 d^2}$$

find $d = 8.57 \text{ cm}$

144

Q: 3

$$V = -\frac{6\epsilon^5}{\epsilon_0}$$

\downarrow find $\vec{E} = -\nabla V$

\downarrow find $\vec{D} = \epsilon \vec{E}$

\downarrow find $\epsilon = \nabla \cdot \vec{D}$

\downarrow find $Q = \iiint q - \epsilon dV$

$$\epsilon = -\epsilon_0 \nabla^2 V; \quad V = f(\epsilon)$$

$$= -\epsilon_0 \cdot \frac{1}{\epsilon^2 \sin \theta} \frac{\partial}{\partial \epsilon} \left(\epsilon^2 g \ln \frac{\partial V}{\partial \epsilon} \right)$$

$$= -\epsilon_0 \cdot \frac{1}{\epsilon^2} \cdot \frac{\partial}{\partial \epsilon} \left[\frac{\epsilon^2}{\partial \epsilon} \left(-\frac{6\epsilon^5}{\epsilon_0} \right) \right]$$

$$= + \frac{\epsilon_0}{\epsilon^2 \partial \epsilon} \left[\epsilon^2 \frac{30}{\epsilon_0} \epsilon^4 \right]$$

$$= \frac{1}{\epsilon^2} \times 30 \times 6 \epsilon^5$$

$$= \underline{180 \epsilon^3}$$

I-method II-method

$$\nabla^2 V = -\epsilon/\epsilon$$

$$\text{find } \epsilon = -\epsilon_0 \nabla^2 V$$

$$\text{bind } Q = \iiint q - \epsilon dV$$

$$\begin{aligned}
 Q &= \iiint_V e \, dv \\
 &= 180 \iiint e^3 \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi \quad (145) \\
 &= 180 \int_0^L r^5 \, dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi \\
 &\quad = \frac{1}{6} \quad = 2 \quad = 2\pi \\
 &= 180 \times \frac{1}{6} \times 4\pi \\
 &= 120\pi \text{ Ans}
 \end{aligned}$$

$$Q: 7 \quad \vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$\begin{array}{ll}
 \nabla \times \vec{A} = 0 & \text{--- irrotational vector} \\
 \nabla \cdot \vec{A} \neq 0 & \text{--- not solenoidal}
 \end{array}$$

Divergence less

$$Q: 8 \quad W_e = \frac{Q_1 Q_2}{4\pi \epsilon_0}$$

$$W_e \propto \frac{1}{r}$$

$$W_{e2} = \frac{1}{2} W_{e1}$$

$$Q: 11 \quad \text{Line charge : } (y=3; z=5)$$

$$\vec{E}(0, \underbrace{6, 1}) =$$

$$\vec{E}(5, \underbrace{6, 1}) = ? \quad - \text{remains same}$$

Q: 17 $\vec{E} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ $V = - \int_{y} \vec{E} \cdot d\vec{l}$

$$d\vec{l} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

(146)

$$\vec{E} \cdot d\vec{l} = x \cdot dx + y \cdot dy + z \cdot dz$$

$$V = - \left[\int_1^2 x \, dx + \int_2^0 y \, dy + \int_3^0 z \, dz \right]$$

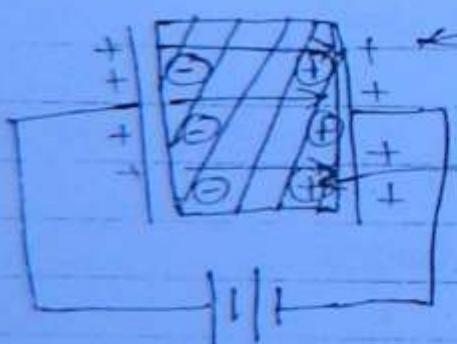
$$V = +5 \text{ volt}$$

Q: 18 $V = 3x^2 j - j z$

$$V(1, 0, -1) = 0$$

$$\left\{ \begin{array}{l} \vec{E} = -\nabla V = - \left[\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \right] \\ \neq 0 \end{array} \right.$$

19 Polarization in dielectric materials



Free charge ; $\epsilon_s \text{ C/m}^2$
Present on conductor

Bound charges ; ϵ_{sp} — surface
charge density
due to polarization

$$D = \epsilon_0 E \quad \text{for free space}$$

$$D = \epsilon_0 \epsilon_r E \quad \text{for dielectric}$$

$$D = \epsilon_0 E + P \quad \text{for dielect.}$$

$$D = \epsilon_0 E + P$$

(147)

$$P = D - \epsilon_0 E = D - \epsilon_0 \frac{D}{\epsilon} = D - \epsilon_0 \frac{D}{\epsilon_0 \epsilon_r}$$

$$\boxed{P = D \left(1 - \frac{1}{\epsilon_r}\right)} = 2 \left(1 - \frac{1}{5}\right) = 1.6 \text{ - C/m}^2$$

Ques. ① The polarization in the dielectric materials is present whenever it is subjected to some externally applied elec. field.

② The charges are induced with in the dielet. due to dipole reorientation so that net charge induced on the dielectric slab is zero.

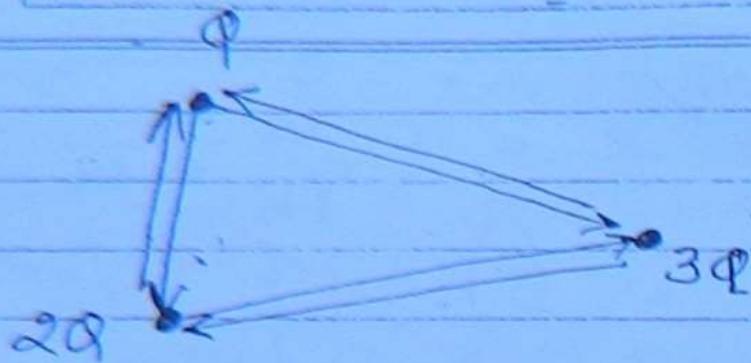
③ Polarization represents total dipole movement reorientation per unit volume of the dielet.

④ Due to induced charges, overall field distribution in the dielet is modified.

⑤ It is a temporary phenomenon - so long as externally applied elec. field is present the dielet will remain in the polarized stage.

as soon as this elec. field with drawn the dielet returns to its unpolarized stage & the induced charges are no longer present.

(21)



(148)

$$\vec{F}_{13} + \vec{F}_{23} = 3\vec{F} \quad \text{--- (1)}$$

$$\vec{f}_{12} + \vec{f}_{32} = 2\vec{F} \quad \text{--- (2)}$$

$$\vec{f}_{21} + \vec{f}_{31} = ?$$

$$-\vec{F}_{31} - \vec{f}_{32} = 3\vec{F}$$

$$-\vec{f}_{21} + \vec{f}_{32} = 2\vec{F}$$

$$\vec{F}_{31} + \vec{f}_{21} = -5\vec{F}, \text{ Ans.}$$

Q: 22

$$q\vec{E} = q\vec{v} \times \vec{B}$$

$$\boxed{\vec{E} = \vec{v} \times \vec{B}}$$

Smith chart

① cart. coord. system : ..

(x, y, z)

$$d\vec{l} = \left\{ \begin{array}{l} \pm dx \hat{a}_x \\ \pm dy \hat{a}_y \\ \pm dz \hat{a}_z \end{array} \right\}$$

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$$d\vec{s} = \left\{ \begin{array}{l} \pm dx dy \cdot \hat{a}_z \\ \pm dy dz \cdot \hat{a}_x \\ \pm dx dz \cdot \hat{a}_y \end{array} \right\}$$

$$d\vec{v} = \frac{dx \cdot dy \cdot dz}{\left\{ \begin{array}{l} -\infty < x < +\infty \\ -\infty < y < +\infty \\ -\infty < z < +\infty \end{array} \right\}}$$

② cyl. coord. system: (r, ϕ, z)

$$d\vec{l} = \left\{ \begin{array}{l} \pm dr \hat{a}_r \\ \pm r \cdot d\phi \hat{a}_\phi \\ \pm dz \hat{a}_z \end{array} \right\}$$

$$d\vec{s} = \left\{ \begin{array}{l} \pm r \cdot dr d\phi \cdot \hat{a}_z \\ \pm r \cdot d\phi dz \hat{a}_r \\ \pm \pm dr dz \cdot \hat{a}_\phi \end{array} \right\}$$

$$dV = r \cdot dr \cdot d\theta \cdot d\phi \cdot dz$$

$$\left\{ \begin{array}{l} 0 \leq r < \infty \\ 0 \leq \theta < 2\pi \\ -\infty < z < +\infty \end{array} \right.$$

(156)

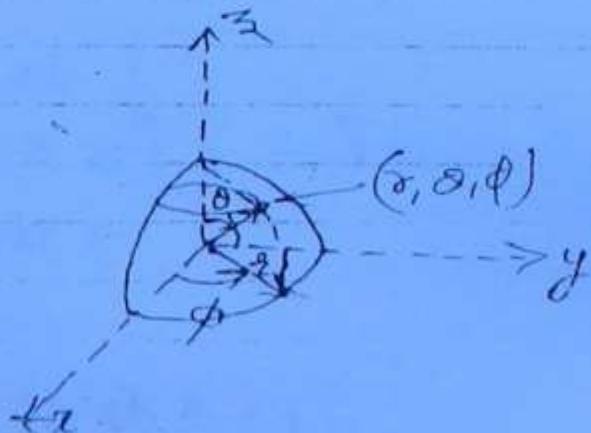
③ Sph. coord. system :- ~~(r, θ, ϕ)~~

$$d\vec{r} = \left\{ \begin{array}{l} \pm dr \cdot \hat{a}_r \\ \pm r d\theta \cdot \hat{a}_\theta \\ \pm r \sin\theta d\phi \cdot \hat{a}_\phi \end{array} \right.$$

$$d\vec{s} = \left\{ \begin{array}{l} \pm r d\theta d\phi \cdot \hat{a}_r \\ \pm r^2 \sin\theta d\theta d\phi \cdot \hat{a}_\theta \\ \pm r \sin\theta d\theta d\phi \cdot \hat{a}_\phi \end{array} \right.$$

$$dV = r^2 \sin\theta \cdot dz \cdot d\theta \cdot d\phi$$

$$\rightarrow \left\{ \begin{array}{l} 0 \leq r < \infty \\ 0 \leq \theta \leq \pi \\ 0 \leq \phi \leq 2\pi \end{array} \right.$$



General relations

Date _____

(μ, ν, ω)

$$1. \nabla v = \sum \frac{1}{h_i} \frac{\partial v}{\partial u^i} \hat{a}_u \quad (15)$$

$$2. \nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3} \sum \frac{\partial}{\partial u^i} \left(\frac{h_2 h_3}{h_1} A_u^i \right)$$

$$3. \nabla^2 v = \frac{1}{h_1 h_2 h_3} \sum \frac{\partial}{\partial u^i} \left(\frac{h_2 h_3}{h_1} \frac{\partial v}{\partial u^i} \right)$$

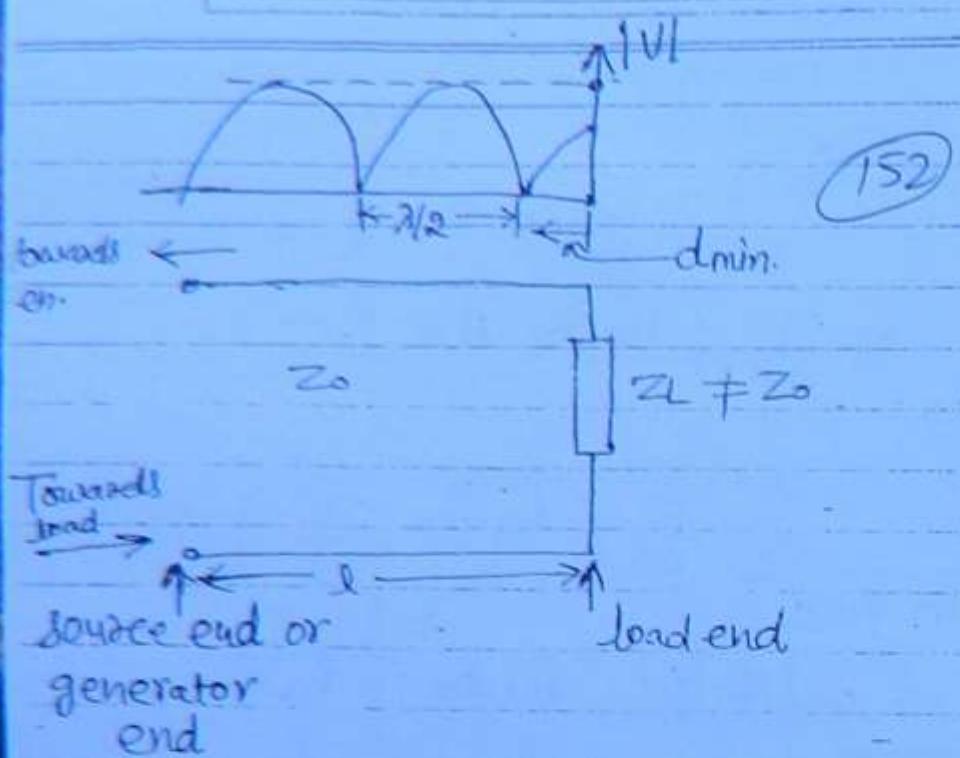
$$4. \nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{a}_u & h_2 \hat{a}_v & h_3 \hat{a}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

$$\vec{A} = A_u \hat{a}_u + A_v \hat{a}_v + A_w \hat{a}_w$$

| | u | v | w | h_1 | h_2 | h_3 |
|-------|-----|----------|--------|-------|-------|-----------------|
| cart. | x | y | z | 1 | 1 | 1 |
| cyl. | r | ϕ | z | 1 | r | 1 |
| sph. | r | θ | ϕ | 1 | r | $r \sin \theta$ |

Smith chart :-

T.L Calculate



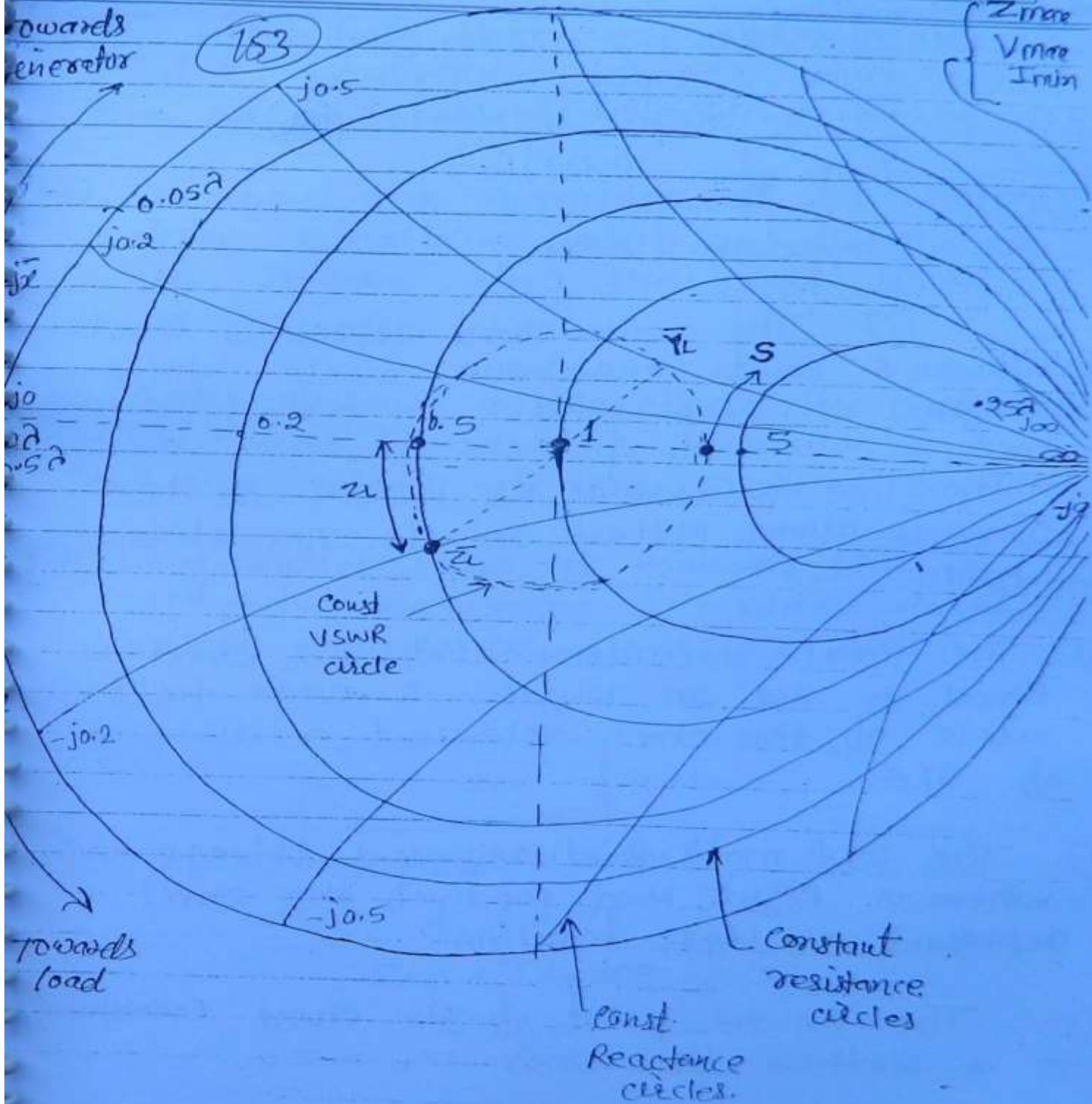
- (1) lossless line
- (2) normalized impedance

$$\left(\frac{z}{Z_0}\right) \equiv \bar{z} = \bar{\epsilon} + j \bar{x}$$

VSWR

1-700

Date



Imp. point

Line

$$Z_L = \text{Given} ; Z_0 = \text{given}$$

$$Z_L = 0.5 - j0.2$$

(154)

$$\bar{Y}_L = \frac{1}{Z_L} = (\text{real}) + j(\text{imaginary})$$

s; d_{\min} ; d_{\max} .

- ① The smith chart represents const. Resistance & const. reactance circle which are orthogonal at each point.
- ② The line is assumed as lossless & the imp. is always plotted in its normalized form.
- ③ The total circumference of the chart is equal to $\lambda/2$ in length & each half circle of the chart represent a distance of $\lambda/4$.
- ④ The left most point represent Voltage minima whereas right most point of the chart represent Voltage maxima.
- ⑤ The centre point of the chart corresponds to a matched line where $Z_L = Z_0$.
- ⑥ The total distance b/w the centre of the chart to the right most of point represent total range of VSWR from $1 \rightarrow \infty$.

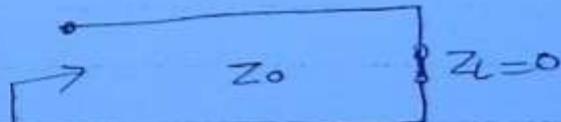
(7) upper half the circle corresponds to +ve Reactance whereas lower half of the circle represents negative reactance

(B) To find the normalized admittance from normalized admittance a distance of $\lambda/4$ is move along the const. VSWR circle.

(9) Going clockwise in the chart the impedance ~~size~~ moves towards the generator & conduct mps are added to the initial impedance

(10) going anticlockwise or towards the load capacitive Reactance is added to the initial impedance along the line.

ex:

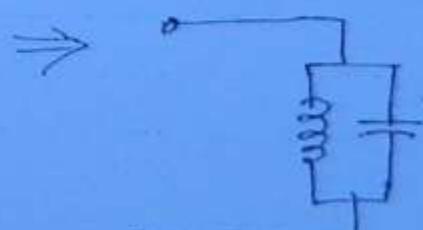


$$Z_{in} = Z_{sc} = j Z_0 \tan \beta l$$

$$\text{if } l = \lambda/4$$

$$Z_{sc} = j Z_0 \tan \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}$$

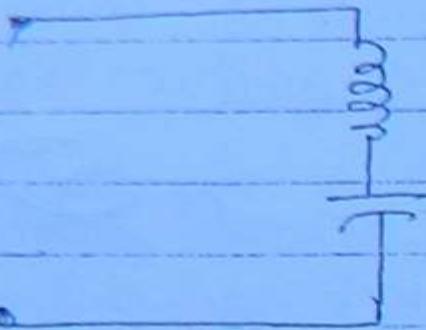
$$= j\infty$$



parallel LC resonance

$$Z_{sc} = -j Z_0 \cot \beta l \quad | l = \lambda/4 = 0$$

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Series LC

Resonance circuit

Conclusion:

- ① for $\lambda/4$ section of the line a short circuited line represent a parallel LC resonant circuit whereas an open circuit line represent a series LC resonant circuit.