GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h\left(t\right)$= $\delta\left(t\right)$+0.5$\delta\left(t-4\right)$, where $\delta\left(t\right)$ is continuous-time unit impulse signal.if the input signal $x(t)=\cos\left(\frac{7\pi t}{4}\right)$,the output is (GATE IN 2023)

Solution:

Variable	Description	value
$\delta\left(t\right)$	continuous-time unit impulse signal	1 if t=0; 0 in other cases
$h\left(t\right)$	impulse response	$\delta\left(t\right)$+0.5$\delta\left(t-4\right)$
$x\left(t\right)$	input signal	$x(t)=cos\left(\frac{7\pi t}{4}\right)$
$y\left(t\right)$	output signal	x(t)∗ h(t)
$y\left(s\right)$	Laplace of $y\left(t\right)$	$\frac{0.5s}{s^2+30.2257}$
$x\left(s\right)$	Laplace of $x\left(t\right)$	$\frac{s}{s^2+30.2257}$

TABLE I
A Table with input parameters

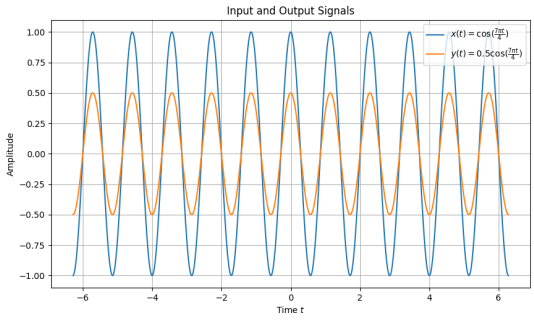

Fig. 1. Graph showing x(t) and y(t)

from Table I

$$y\left(t\right)=x\left(t\right)*h\left(t\right) \tag{1}$$
$$=x\left(t\right)*\left(\delta\left(t\right)+0.5\delta\left(t-4\right)\right) \tag{2}$$
$$=x\left(t\right)+0.5x\left(t-4\right) \tag{3}$$
$$=\cos\left(\frac{7\pi t}{4}\right)+0.5\cos\left(\frac{7\pi\left(t-4\right)}{4}\right) \tag{4}$$
$$=\cos\left(\frac{7\pi t}{4}\right)+0.5\cos\left(\frac{7\pi t}{4}-7\pi\right) \tag{5}$$
$$=0.5\cos\left(\frac{7\pi t}{4}\right) \tag{6}$$

$$\text{Transform function}\left(H\left(s\right)\right)=\frac{y\left(s\right)}{x\left(s\right)} \tag{7}$$
$$=\frac{\frac{0.5s}{s^2+30.2257}}{+s} \tag{8}$$
$$=0.5 \tag{9}$$