GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h\left(t\right)$= $\delta\left(t\right)$+0.5$\delta\left(t-4\right)$, where $\delta\left(t\right)$ is continuous-time unit impulse signal.if the input signal $x(t)=\cos\left(\frac{7\pi t}{4}\right)$,the output is (GATE IN 2023)

Solution:

Variable	Description	value
$\delta\left(t\right)$	Dirac delta function	∞ if t=0; 0 in other cases $\int_{-\infty}^{\infty}\delta\left(t\right)=1$
$h\left(t\right)$	impulse response	$\delta\left(t\right)$+0.5$\delta\left(t-4\right)$
$x\left(t\right)$	input signal	$x(t)=cos\left(\frac{7\pi t}{4}\right)$
$y\left(t\right)$	output signal	x(t)∗ h(t)
$\mathcal{F}\left(\cos at\right)$	Fourier transform of $\cos at$	$0.5\left[\delta\left(f-\frac{a}{2\pi}\right)+\delta\left(f+\frac{a}{2\pi}\right)\right]$
$X\left(f\right)$	Fourier transform of $x\left(t\right)$	$0.5\left[\delta\left(f-\frac{7}{8}\right)+\delta\left(f+\frac{7}{8}\right)\right]$
$H\left(f\right)$	Fourier transform of $h\left(t\right)$	$1+0.5e^{-j8\pi f}$
$Y\left(f\right)$	Fourier transform of $y\left(t\right)$	$X\left(f\right)H\left(f\right)$

TABLE I
A Table with input parameters

from Table I

$$y\left(t\right)=x\left(t\right)*h\left(t\right) \tag{1}$$
$$=x\left(t\right)*\left(\delta\left(t\right)+0.5\delta\left(t-4\right)\right) \tag{2}$$
$$=x\left(t\right)+0.5x\left(t-4\right) \tag{3}$$
$$=\cos\left(\frac{7\pi t}{4}\right)+0.5\cos\left(\frac{7\pi\left(t-4\right)}{4}\right) \tag{4}$$
$$=\cos\left(\frac{7\pi t}{4}\right)+0.5\cos\left(\frac{7\pi t}{4}-7\pi\right) \tag{5}$$
$$=\frac{1}{2}\cos\left(\frac{7\pi t}{4}\right) \tag{6}$$
$$=\frac{1}{2}x\left(t\right) \tag{7}$$

$$Y\left(f\right)=X\left(f\right)H\left(f\right) \tag{8}$$
$$=\frac{1}{2}\left[\delta\left(f-\frac{7}{8}\right)+\delta\left(f+\frac{7}{8}\right)\right]\left[1+0.5e^{-j8\pi f}\right] \tag{9}$$

$$Y\left(f\right)=\frac{1}{2}\left(1+0.5e^{-j8\pi f}\right)\delta\left(f-\frac{7}{8}\right)+\frac{1}{4}\left(1+0.5e^{-j8\pi f}\right)\delta\left(f-\frac{7}{8}\right)e^{-j8\pi f} \tag{10}$$
$$=\frac{1}{2}X\left(f\right) \tag{11}$$

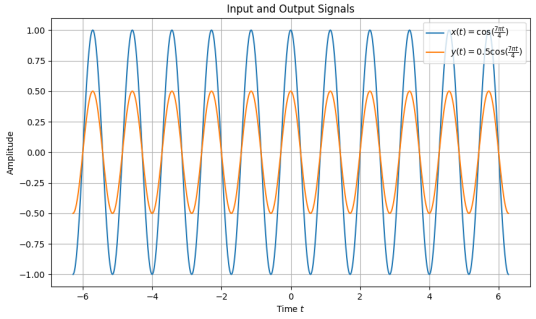

Fig. 1. Graph showing x(t) and y(t)