

# GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

## Question:

The impulse response of an LTI system is  $h(t) = \delta(t) + 0.5\delta(t - 4)$ , where  $\delta(t)$  is continuous-time unit impulse signal. If the input signal  $x(t) = \cos\left(\frac{7\pi t}{4}\right)$ , the output is (GATE IN 2023)

## Solution:

Variable	Description	value
$\delta(t)$	continuous-time unit impulse signal	1 if $t=0$ ; 0 in other cases
$h(t)$	impulse response	$\delta(t) + 0.5\delta(t - 4)$
$x(t)$	input signal	$x(t) = \cos\left(\frac{7\pi t}{4}\right)$
$y(t)$	output signal	$x(t) * h(t)$
$\mathcal{F}(\cos \omega_0 t)$	Fourier transform of $\cos \omega_0 t$	$\pi [\delta(2\pi f - \omega_0) + \delta(2\pi f + \omega_0)]$
$Y(f)$	Fourier transform of $y(t)$	$0.5\pi [\delta(2\pi f - \frac{7\pi}{4}) + \delta(2\pi f + \frac{7\pi}{4})]$
$X(f)$	Fourier transform of $x(t)$	$\pi [\delta(2\pi f - \frac{7\pi}{4}) + \delta(2\pi f + \frac{7\pi}{4})]$

TABLE I

A TABLE WITH INPUT PARAMETERS

from Table I

$$y(t) = x(t) * h(t) \quad (1)$$

$$= x(t) * (\delta(t) + 0.5\delta(t - 4)) \quad (2)$$

$$= x(t) + 0.5x(t - 4) \quad (3)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5 \cos\left(\frac{7\pi(t - 4)}{4}\right) \quad (4)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5 \cos\left(\frac{7\pi t}{4} - 7\pi\right) \quad (5)$$

$$= 0.5 \cos\left(\frac{7\pi t}{4}\right) \quad (6)$$

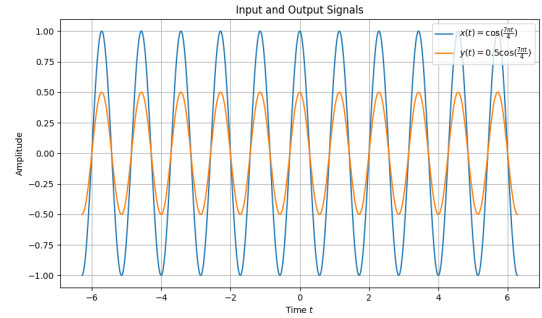


Fig. 1. Graph showing  $x(t)$  and  $y(t)$

$$\text{Transfer function } (H(f)) = \frac{Y(f)}{X(f)} \quad (7)$$

$$= \frac{0.5\pi [\delta(2\pi f - \frac{7\pi}{4}) + \delta(2\pi f + \frac{7\pi}{4})]}{\pi [\delta(2\pi f - \frac{7\pi}{4}) + \delta(2\pi f + \frac{7\pi}{4})]} \quad (8)$$

$$= 0.5 \quad (9)$$