GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h(t)=$ $\delta(t)+0.5\delta(t-4)$, where $\delta(t)$ is continuous-time unit impulse signal.if the input signal $x(t) = cos\left(\frac{7\pi t}{4}\right)$,the output is

Solution:

Variable	Description	value
$\delta(t)$	continuous-time unit impulse signal	1 if t=0; 0 in other cases
$h(t)$	impulse response	$\delta(t)+0.5\delta(t-4)$
$x(t)$	input signal	$x(t) = cos\left(\frac{7\pi t}{4}\right)$
$y(t)$	output signal	x(t)$*$ h(t)

TABLE I

A TABLE WITH INPUT PARAMETERS

from Table I

$$y(t) = h(t) * x(t) \tag{1}$$

$$= \int_{-\infty}^{\infty} h(\tau) x(t-\tau) \, d\tau \tag{2}$$

$$= \int_{-\infty}^{\infty} [\delta(\tau) x(t-\tau) + 0.5\delta(\tau-4) x(t-\tau)] \, d\tau \tag{3}$$

$$= x(t) + 0.5x(t-4) \tag{4}$$

$$= cos\left(\frac{7\pi t}{4}\right) + 0.5cos\left(\frac{7\pi(t-4)}{4}\right) \tag{5}$$

$$= cos\left(\frac{7\pi t}{4}\right) + 0.5cos\left(\frac{7\pi t}{4} - 7\pi\right) \tag{6}$$

$$= 0.5cos\left(\frac{7\pi t}{4}\right) \tag{7}$$