

GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\delta(t - 4)$, where $\delta(t)$ is continuous-time unit impulse signal. If the input signal $x(t) = \cos\left(\frac{7\pi t}{4}\right)$, the output is (GATE IN 2023)

Solution:

Variable	Description	value
$\delta(t)$	Dirac delta function	∞ if $t=0$; 0 in other cases $\int_{-\infty}^{\infty} \delta(t) dt = 1$
$h(t)$	impulse response	$\delta(t) + 0.5\delta(t - 4)$
$x(t)$	input signal	$x(t) = \cos\left(\frac{7\pi t}{4}\right)$
$y(t)$	output signal	$x(t) * h(t)$
$\mathcal{F}(\cos at)$	Fourier transform of $\cos at$	$0.5 \left[\delta\left(f - \frac{a}{2\pi}\right) + \delta\left(f + \frac{a}{2\pi}\right) \right]$
$X(f)$	Fourier transform of $x(t)$	$0.5 \left[\delta\left(f - \frac{7}{8}\right) + \delta\left(f + \frac{7}{8}\right) \right]$
$H(f)$	Fourier transform of $h(t)$	$1 + 0.5e^{-j8\pi f}$
$Y(f)$	Fourier transform of $y(t)$	$X(f)H(f)$

TABLE I

A TABLE WITH INPUT PARAMETERS

from Table I

$$y(t) = x(t) * h(t) \quad (1)$$

$$= x(t) * (\delta(t) + 0.5\delta(t - 4)) \quad (2)$$

$$= x(t) + 0.5x(t - 4) \quad (3)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5\cos\left(\frac{7\pi(t - 4)}{4}\right) \quad (4)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5\cos\left(\frac{7\pi t}{4} - 7\pi\right) \quad (5)$$

$$= \frac{1}{2}\cos\left(\frac{7\pi t}{4}\right) \quad (6)$$

$$= \frac{1}{2}x(t) \quad (7)$$

$$Y(f) = X(f)H(f) \quad (8)$$

$$= \frac{1}{2} \left[\delta\left(f - \frac{7}{8}\right) + \delta\left(f + \frac{7}{8}\right) \right] [1 + 0.5e^{-j8\pi f}] \quad (9)$$

$$Y(f) = \frac{1}{2} (1 + 0.5e^{-j8\pi f}) \delta\left(f - \frac{7}{8}\right) + \frac{1}{2} (1 + 0.5e^{-j8\pi f}) \delta\left(f + \frac{7}{8}\right) \quad (10)$$

$$= \frac{1}{2} (1 + 0.5(-1)) \delta\left(f - \frac{7}{8}\right) + \frac{1}{2} (1 + 0.5(-1)) \delta\left(f + \frac{7}{8}\right) \quad (11)$$

$$= \frac{1}{4} \left[\delta\left(f - \frac{7}{8}\right) + \delta\left(f + \frac{7}{8}\right) \right] \quad (12)$$

$$= \frac{1}{2}X(f) \quad (13)$$

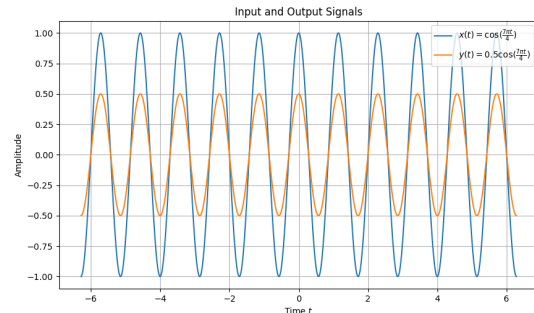


Fig. 1. Graph showing $x(t)$ and $y(t)$