

GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\delta(t - 4)$, where $\delta(t)$ is continuous-time unit impulse signal. If the input signal $x(t) = \cos\left(\frac{7\pi t}{4}\right)$, the output is (GATE IN 2023)

Solution:

Variable	Description	value
$\delta(t)$	continuous-time unit impulse signal	1 if $t=0$; 0 in other cases
$h(t)$	impulse response	$\delta(t) + 0.5\delta(t - 4)$
$x(t)$	input signal	$x(t) = \cos\left(\frac{7\pi t}{4}\right)$
$y(t)$	output signal	$x(t) * h(t)$

TABLE I

A TABLE WITH INPUT PARAMETERS

from Table I

$$y(t) = x(t) * h(t) \quad (1)$$

$$= x(t) * (\delta(t) + 0.5\delta(t - 4)) \quad (2)$$

$$= x(t) + 0.5x(t - 4) \quad (3)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5 \cos\left(\frac{7\pi(t - 4)}{4}\right) \quad (4)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5 \cos\left(\frac{7\pi t}{4} - 7\pi\right) \quad (5)$$

$$= 0.5 \cos\left(\frac{7\pi t}{4}\right) \quad (6)$$

$$\text{Transform function } (H(t)) = \frac{y(t)}{x(t)} \quad (7)$$

$$= \frac{0.5 \cos\left(\frac{7\pi t}{4}\right)}{\cos\left(\frac{7\pi t}{4}\right)} \quad (8)$$

$$= 0.5 \quad (9)$$

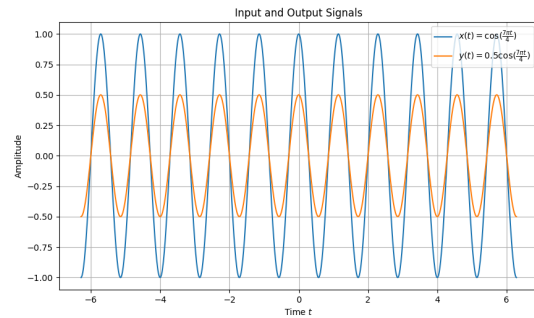


Fig. 1. Graph showing $x(t)$ and $y(t)$