GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\delta(t-4)$, where $\delta(t)$ is continuoustime unit impulse signal if the input signal $x(t) = \cos\left(\frac{7\pi t}{4}\right)$, the output is (GATE IN 2023)

Transfer function
$$(H(s)) = \frac{Y(s)}{X(s)}$$
 (9)

$$=\frac{\frac{0.5s}{s^2+30.2257}}{\frac{s}{s^2+30.2257}}\tag{10}$$

$$=0.5 \tag{11}$$

Solution:

Variable	Description	value
$\delta\left(t\right)$	continuous-time unit impulse signal	1 if t=0;
		0 in other cases
$h\left(t\right)$	impulse response	$\delta(t) + 0.5\delta(t-4)$
x(t)	input signal	$x(t) = cos\left(\frac{7\pi t}{4}\right)$
$y\left(t\right)$	output signal	x(t)* h(t)
$Y\left(s\right)$	Laplace of $y(t)$	$\frac{0.5s}{s^2+30.2257}$
$X\left(s\right)$	Laplace of $x(t)$	$\frac{s}{s^2+30.2257}$
$\mathcal{L}(\cos \omega t)$	Laplace of $\cos \omega t$	$\frac{s}{s^2+\omega^2}$

TABLE I

A TABLE WITH INPUT PARAMETERS

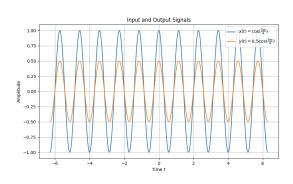


Fig. 1. Graph showing x(t) and y(t)

from Table I

$$y(t) = x(t) * h(t)$$

$$(1)$$

$$= x(t) * (\delta(t) + 0.5\delta(t-4))$$
 (2)

$$= x(t) + 0.5x(t-4)$$
 (3)

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5\cos\left(\frac{7\pi (t-4)}{4}\right) \quad (4)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5\cos\left(\frac{7\pi t}{4} - 7\pi\right) \tag{5}$$

$$=0.5\cos\left(\frac{7\pi t}{4}\right)\tag{6}$$

from Table I

$$Y(s) = \mathcal{L}\left[0.5\cos\left(\frac{7\pi t}{4}\right)\right] = \frac{0.5s}{s^2 + \left\lceil\frac{7\pi}{4}\right\rceil^2}$$
 (7)

similarly
$$X(s) = \frac{s}{s^2 + \left[\frac{7\pi}{4}\right]^2}$$
 (8)