# GATE 2023[IN]-36

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### Question:

The impulse response of an LTI system is $h\left(t\right)$= $\delta\left(t\right)$+$0.5\delta\left(t-4\right)$, where $\delta\left(t\right)$ is continuous-time unit impulse signal.if the input signal $x(t) = \cos\left(\frac{7\pi t}{4}\right)$,the output is (GATE IN 2023)

### Solution:

| Variable | Description | value |
|---|---|---|
| $\delta\left(t\right)$ | continuous-time unit impulse signal | 1 if t=0;<br>0 in other cases |
| $h\left(t\right)$ | impulse response | $\delta\left(t\right)$+$0.5\delta\left(t-4\right)$ |
| $x\left(t\right)$ | input signal | $x(t) = cos\left(\frac{7\pi t}{4}\right)$ |
| $y\left(t\right)$ | output signal | ? |
| $\mathcal{F}\left(\cos at\right)$ | Fourier transform of $\cos at$ | $0.5\left[\delta\left(f-\frac{a}{2\pi}\right)+\delta\left(f+\frac{a}{2\pi}\right)\right]$ |
| $X\left(f\right)$ | Fourier transform of $x\left(t\right)$ | $0.5\left[\delta\left(f-\frac{7}{8}\right)+\delta\left(f+\frac{7}{8}\right)\right]$ |
| $H\left(f\right)$ | Fourier transform of $h\left(t\right)$ | 1+$0.5e^{-j8\pi f}$ |
| $Y\left(f\right)$ | Fourier transform of $y\left(t\right)$ | $X\left(f\right)H\left(f\right)$ |

TABLE I

A TABLE WITH INPUT PARAMETERS

from Table I

$$Y\left(f\right) = X\left(f\right)H\left(f\right) \tag{1}$$

$$= 0.5\left[\delta\left(f-\frac{7}{8}\right)+\delta\left(f+\frac{7}{8}\right)\right]\left[1+0.5e^{-j8\pi f}\right] \tag{2}$$

Finding inverse fourier transform of Y($f$):

$$y\left(t\right) = \int_{-\infty}^{\infty} Y\left(f\right)e^{j2\pi ft}df \tag{3}$$

$$= 0.5\cos\left(\frac{7\pi t}{4}\right) \tag{4}$$

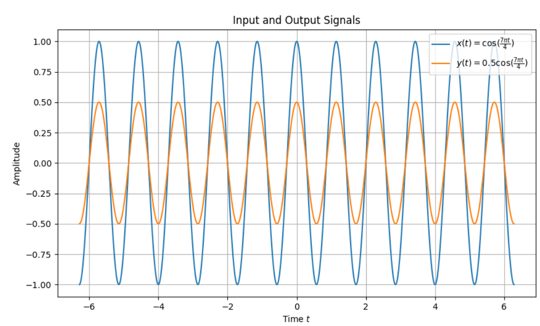


Fig. 1. Graph showing x(t) and y(t)