GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\delta(t-4)$, where $\delta(t)$ is continuous-time unit impulse signal.if the input signal $x(t) = \cos\left(\frac{7\pi t}{4}\right)$,the output is (GATE IN 2023)

Solution:

Variable	Description	value
$\delta(t)$	Dirac delta function	∞ if t=0; 0 in other cases $\int_{-\infty}^{\infty} \delta(t) = 1$
$h(t)$	impulse response	$\delta(t)+0.5\delta(t-4)$
$x(t)$	input signal	$x(t) = cos\left(\frac{7\pi t}{4}\right)$
$y(t)$	output signal	x$(t)*$ h(t)
$\mathcal{F}(\cos at)$	Fourier transform of $\cos at$	$0.5\left[\delta\left(f - \frac{a}{2\pi}\right) + \delta\left(f + \frac{a}{2\pi}\right)\right]$
$X(f)$	Fourier transform of $x(t)$	$0.5\left[\delta\left(f - \frac{7}{8}\right) + \delta\left(f + \frac{7}{8}\right)\right]$
$H(f)$	Fourier transform of $h(t)$	$1+0.5e^{-j8\pi f}$
$Y(f)$	Fourier transform of $y(t)$	$X(f)H(f)$

TABLE I
A Table with input parameters

from Table I

$$y(t) = x(t) * h(t) \tag{1}$$
$$= x(t) * (\delta(t) + 0.5\delta(t-4)) \tag{2}$$
$$= x(t) + 0.5x(t-4) \tag{3}$$
$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5\cos\left(\frac{7\pi(t-4)}{4}\right) \tag{4}$$
$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5\cos\left(\frac{7\pi t}{4} - 7\pi\right) \tag{5}$$
$$= \frac{1}{2}\cos\left(\frac{7\pi t}{4}\right) \tag{6}$$
$$= \frac{1}{2}x(t) \tag{7}$$

$$Y(f) = X(f)H(f) \tag{8}$$
$$= \frac{1}{2}\left[\delta\left(f - \frac{7}{8}\right) + \delta\left(f + \frac{7}{8}\right)\right]\left[1 + 0.5e^{-j8\pi f}\right] \tag{9}$$

$$Y(f) = \frac{1}{2}\left(1 + 0.5e^{-j8\pi f}\right)\delta\left(f - \frac{7}{8}\right) + \frac{1}{2}\left(1 + 0.5e^{-j8\pi f}\right)\delta\left(f + \frac{7}{8}\right) \tag{10}$$
$$= \frac{1}{2}(1 + 0.5(-1))\delta\left(f - \frac{7}{8}\right) + \frac{1}{2}(1 + 0.5(-1))\delta\left(f + \frac{7}{8}\right) \tag{11}$$
$$= \frac{1}{4}\left[\delta\left(f - \frac{7}{8}\right) + \delta\left(f + \frac{7}{8}\right)\right] \tag{12}$$
$$= \frac{1}{2}X(f) \tag{13}$$

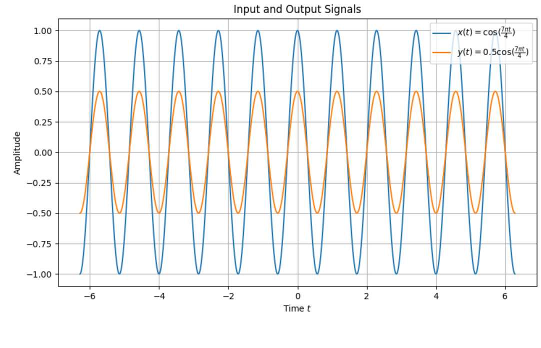

Fig. 1. Graph showing x(t) and y(t)