GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h\left(t\right)$= $\delta\left(t\right)$+0.5$\delta\left(t-4\right)$, where $\delta\left(t\right)$ is continuous-time unit impulse signal.if the input signal $x(t)=\cos\left(\frac{7\pi t}{4}\right)$,the output is (GATE IN 2023)

Solution:

Variable	Description	value
$\delta\left(t\right)$	continuous-time unit impulse signal	1 if t=0; 0 in other cases
$h\left(t\right)$	impulse response	$\delta\left(t\right)$+0.5$\delta\left(t-4\right)$
$x\left(t\right)$	input signal	$x(t)=cos\left(\frac{7\pi t}{4}\right)$
$y\left(t\right)$	output signal	x(t)* h(t)
$Y\left(s\right)$	Laplace of $y\left(t\right)$	$\frac{0.5s}{s^2+30.2257}$
$X\left(s\right)$	Laplace of $x\left(t\right)$	$\frac{s}{s^2+30.2257}$
$\mathcal{L}\left(\cos\omega t\right)$	Laplace of $\cos\omega t$	$\frac{s}{s^2+\omega^2}$

TABLE I
A Table with input parameters

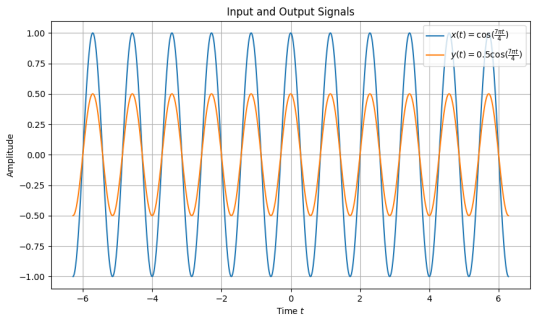

Fig. 1. Graph showing x(t) and y(t)

from Table I

$$y\left(t\right)=x\left(t\right)*h\left(t\right) \tag{1}$$
$$=x\left(t\right)*\left(\delta\left(t\right)+0.5\delta\left(t-4\right)\right) \tag{2}$$
$$=x\left(t\right)+0.5x\left(t-4\right) \tag{3}$$
$$=\cos\left(\frac{7\pi t}{4}\right)+0.5\cos\left(\frac{7\pi\left(t-4\right)}{4}\right) \tag{4}$$
$$=\cos\left(\frac{7\pi t}{4}\right)+0.5\cos\left(\frac{7\pi t}{4}-7\pi\right) \tag{5}$$
$$=0.5\cos\left(\frac{7\pi t}{4}\right) \tag{6}$$

from Table I

$$Y\left(s\right)=\mathcal{L}\left[0.5\cos\left(\frac{7\pi t}{4}\right)\right]=\frac{0.5s}{s^2+\left[\frac{7\pi}{t}\right]^2} \tag{7}$$
$$\text{similarly}X\left(s\right)=\frac{s}{s^2+\left[\frac{7\pi}{t}\right]^2} \tag{8}$$

$$\text{Transfer function}\left(H\left(s\right)\right)=\frac{Y\left(s\right)}{X\left(s\right)} \tag{9}$$
$$=\frac{\frac{0.5s}{s^2+30.2257}}{\frac{s}{s^2+30.2257}} \tag{10}$$
$$=0.5 \tag{11}$$