

GATE 2023[IN]-36

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\delta(t - 4)$, where $\delta(t)$ is continuous-time unit impulse signal. If the input signal $x(t) = \cos\left(\frac{7\pi t}{4}\right)$, the output is

Solution:

Variable	Description	value
$\delta(t)$	continuous-time unit impulse signal	1 if $t=0$; 0 in other cases
$h(t)$	impulse response	$\delta(t) + 0.5\delta(t - 4)$
$x(t)$	input signal	$x(t) = \cos\left(\frac{7\pi t}{4}\right)$
$y(t)$	output signal	$x(t) * h(t)$

TABLE I

A TABLE WITH INPUT PARAMETERS

from Table I

$$y(t) = x(t) * h(t) = h(t) * x(t) \quad (1)$$

$$= \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau \quad (2)$$

$$= \int_{-\infty}^{\infty} [\delta(\tau) x(t - \tau) + 0.5\delta(\tau - 4) x(t - \tau)] d\tau \quad (3)$$

$$= x(t) + 0.5x(t - 4) \quad (4)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5\cos\left(\frac{7\pi(t - 4)}{4}\right) \quad (5)$$

$$= \cos\left(\frac{7\pi t}{4}\right) + 0.5\cos\left(\frac{7\pi t}{4} - 7\pi\right) \quad (6)$$

$$= 0.5\cos\left(\frac{7\pi t}{4}\right) \quad (7)$$