DISCRETE 11.9.3 Q-4

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Question:

The 4th term of a G.P. is square of its second term, and the first term is -3. Determine its 7th term, and find the Z transform of the series.

Solution:

$$(Given) \quad x(3) = (x(1))^2 \quad (1)$$

$$ar^3 = (ar^1)^2 \quad (2)$$

$$ar^3 = a^2 r^2 \quad (3)$$

$$r = a \quad (4)$$

$$r = -3 \quad (5)$$

$$(x(6)) = ar^6 \quad (6)$$

$$x(6) = (-3)(-3)^6 \quad (7)$$

$$x(6) = (-3)^7 = -2187 \quad (8)$$

$$x(6) = -2187 \quad (9)$$

Finding Z transform :

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (10)$$

$$= \sum_{n=-\infty}^{\infty} ar^n u(n) z^{-n} \quad (11)$$

$$= \sum_{n=0}^{\infty} ar^n z^{-n} \quad (12)$$

$$= a(1 + rz^{-1} + r^2 z^{-2} + ...) \quad (13)$$

$$= \frac{a}{1 - rz^{-1}} \quad (14)$$

$$\{ROC : |rz^{-1}| < 1\}$$
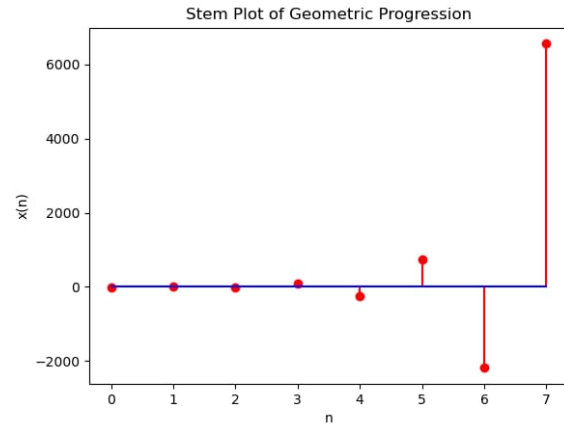
$$X(n) = \frac{a}{1 - rz^{-1}} = \frac{-3}{1 + 3z^{-1}}.$$

Fig. 1. Graph showing first 8 terms of the GP

Variable	Description	value
a	first term of G.P.	-3
r	Common ratio of G.P.	-3
X(n)	general term of the G.P.	ar^n

TABLE I
A TABLE WITH INPUT PARAMETERS