DISCRETE 11.9.3 Q-4

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Question:

The 4^{th} term of a G.P. is square of its second term, and the first term is -3. Determine its 7^{th} term, and find the Z transform of the series.

Solution:

Variable	Description	value
$x(0)$	first term of G.P.	-3
r	Common ratio of G.P.	-3
$x(n)$	general term of the G.P.	ar^n
-	$x(3)=(x(1))^2$	-

TABLE I
A Table with input parameters

$$x(0)r^3 = \left(x(0)r^1\right)^2 \tag{1}$$
$$= x(0)^2 r^2 \tag{2}$$
$$r = x(0) = -3 \tag{3}$$
$$\text{(general term)}x(n) = x(0)r^n = (-3)(-3)^n \tag{4}$$
$$x(6) = x(0)r^6 \tag{5}$$
$$= (-3)(-3)^6 \tag{6}$$
$$= (-3)^7 \tag{7}$$
$$= -2187 \tag{8}$$
$$x(6) = -2187 \tag{9}$$

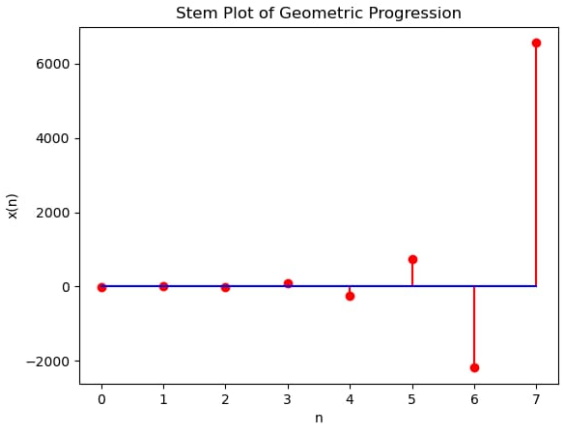

Fig. 1. Graph showing first 8 terms of the GP

Finding Z transform :

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{10}$$
$$= \sum_{n=-\infty}^{\infty} ar^n u(n) z^{-n} \tag{11}$$
$$= \sum_{n=0}^{\infty} ar^n z^{-n} \tag{12}$$
$$= a(1 + rz^{-1} + r^2z^{-2} + ...) \tag{13}$$
$$= \frac{a}{1 - rz^{-1}} \tag{14}$$
$$\{ROC : |rz^{-1}| < 1\}$$

$X(n) = \frac{a}{1-rz^{-1}} = \frac{-3}{1+3z^{-1}}$.