

DISCRETE 11.9.3 Q-4

EE23BTECH11066 - Yakkala Amarnath Karthik

Question:

The 4th term of a G.P. is square of its second term, and the first term is -3. Determine its 7th term, and find the Z transform of the series.

Solution:

Let, first term of this G.P.(X (0)) be a.

Given, the first term is -3.

i.e. a= -3 (given).....(1)

Let r be the common ratio of G.P.

Given that the fourth term of G.P. is square of its second term.

We know that the general term of a G.P. can be written as : $X(n)=ar^n$(2)

$$X(3) = (X(1))^2 \quad (\text{Given}) \dots\dots (3) \quad (1)$$

substituting (2) in (3),

$$ar^3 = (ar^1)^2 \quad (2)$$

$$ar^3 = a^2 r^2 \quad (3)$$

$$r = a \quad (4)$$

$$(\text{from (1)}) \quad r = -3 \dots\dots (4) \quad (5)$$

$$7^{\text{th}} \text{ term}(X(6)) = ar^6 \quad (6)$$

from (1) and (4)

$$X(6) = (-3)(-3)^6 \quad (7)$$

$$X(6) = (-3)^7 = -2187 \quad (8)$$

So 7th term of the G.P. is -2187.

Finding Z transform :

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (9)$$

$$= \sum_{n=-\infty}^{\infty} ar^n u(n) z^{-n} \quad (10)$$

$$= \sum_{n=0}^{\infty} ar^n z^{-n} \quad (11)$$

$$= a(1 + rz^{-1} + r^2 z^{-2} + r^3 z^{-3} + \dots) \quad (12)$$

$$= \frac{a}{1 - rz^{-1}} \quad (13)$$

$$\{ROC : rz^{-1} < 1\}$$

So, the Z-transform of the given series is $X(n) =$

$$\frac{a}{1 - rz^{-1}} = \frac{-3}{1 + 3z^{-1}}.$$

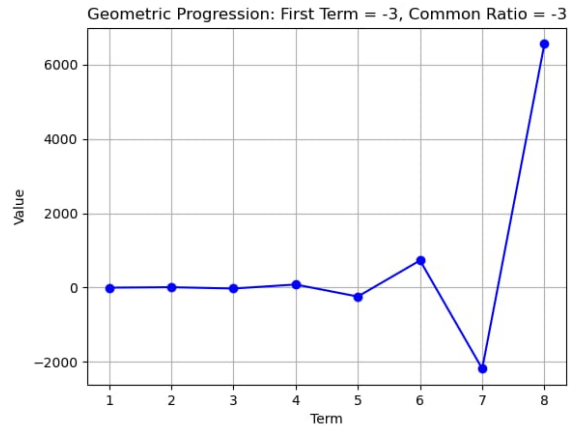


Fig. 1. Graph showing first 8 terms of the GP

Variable	Description	value
a	first term of G.P.	-3
r	Common ratio of G.P.	-3
X(n)	general term of the G.P.	ar^n

TABLE I

A TABLE WITH INPUT PARAMETERS