# Potential and Electric Field of a 3D point charge.


## Author

Amartyaraj Kumar


## Report date

May 12, 2022 9:42:24 PM


## Institution

IISER Bhopal

# Contents

# 1 Global Definitions

| | |
|---|---|
| Date | May 12, 2022 9:22:13 PM |

GLOBAL SETTINGS

| | |
|---|---|
| Name | COMSOL HW10 19029.mph |
| Path | C:\Users\user\Desktop\COMSOL\COMSOL_HW10_19029.mph |
| Version | COMSOL Multiphysics 5.4 (Build: 388) |
| Unit system | SI |

USED PRODUCTS

| |
|---|
| COMSOL Multiphysics |
| AC/DC Module |

# 2 Component 1

| | |
|---|---|
| Date | May 12, 2022 10:17:35 AM |

SETTINGS

| Description | Value |
|---|---|
| Unit system | Same as global system |
| Geometry shape order | Automatic |

SPATIAL FRAME COORDINATES

| First | Second | Third |
|---|---|---|
| x | y | z |

MATERIAL FRAME COORDINATES

| First | Second | Third |
|---|---|---|
| X | Y | Z |

GEOMETRY FRAME COORDINATES

| First | Second | Third |
|---|---|---|
| Xg | Yg | Zg |

MESH FRAME COORDINATES

| First | Second | Third |
|---|---|---|
| Xm | Ym | Zm |

## 2.1 DEFINITIONS

### 2.1.1 Coordinate Systems

#### 2.1.1.1 Boundary System 1

| | |
|---|---|
| Coordinate system type | Boundary system |
| Tag | sys1 |

COORDINATE NAMES

| First | Second | Third |
|---|---|---|
| t1 | t2 | n |

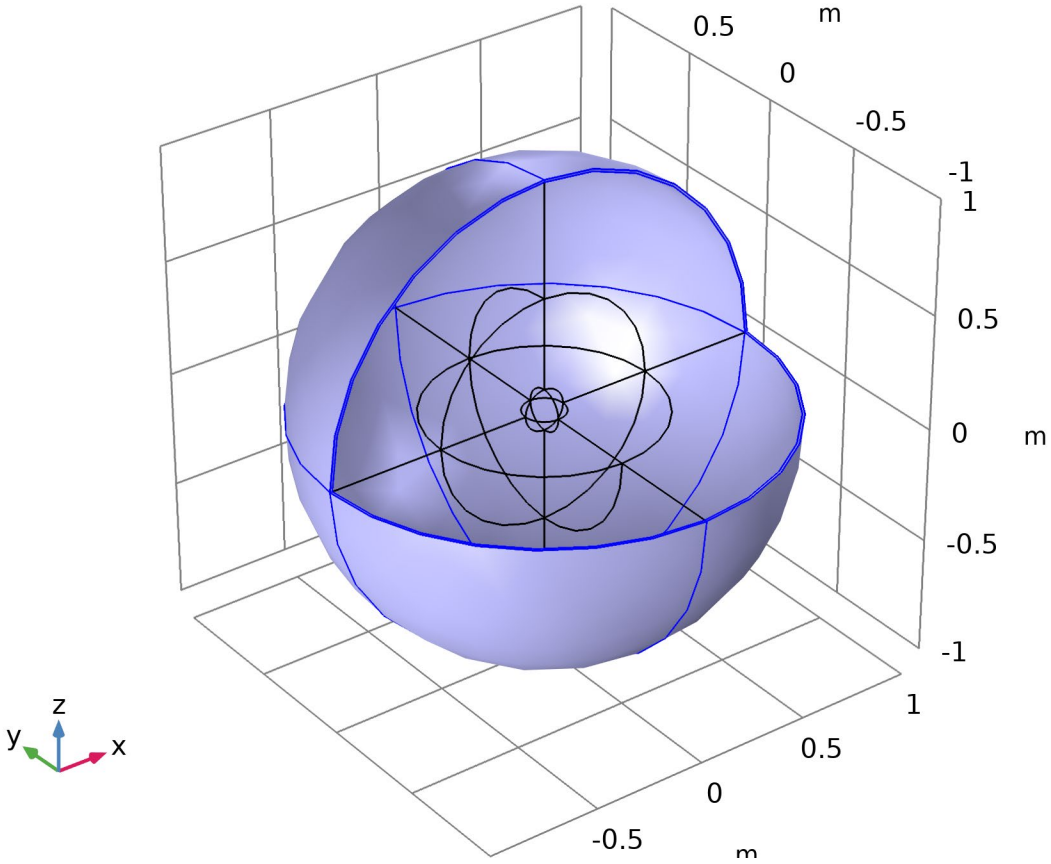
### 2.1.2 Domain Properties

#### 2.1.2.1 Infinite Element Domain 1

| | |
|---|---|
| Tag | ie1 |

## SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domains 1–4, 14–15, 20, 25 |




*Selection*

## GEOMETRY

| Description | Value |
|---|---|
| Coordinate names | {x, y, z} |
| Type | Spherical |

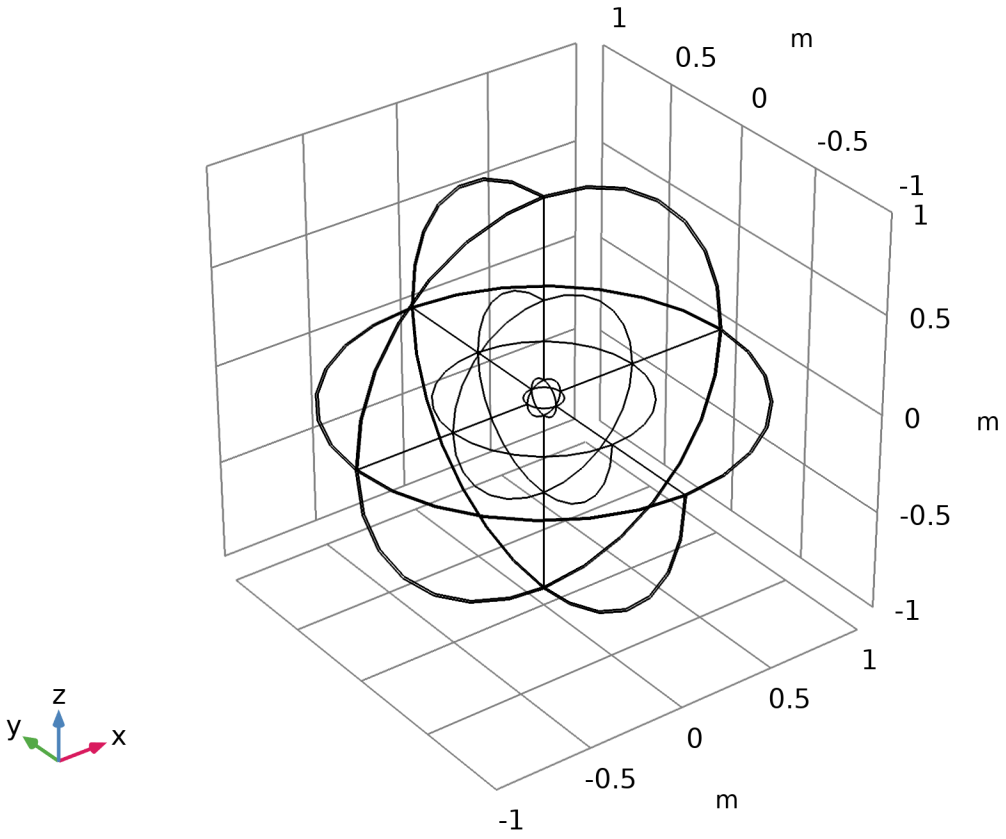
## GEOMETRY

| Xm (m) | Ym (m) | Zm (m) |
|---|---|---|
| 0 | 0 | 0 |

## SCALING

| Description | Value |
|---|---|
| Coordinate stretching type | Rational |

## 2.2 GEOMETRY 1




*Geometry 1*

UNITS

| | |
|---|---|
| Length unit | m |
| Angular unit | deg |

GEOMETRY STATISTICS

| Description | Value |
|---|---|
| Space dimension | 3 |
| Number of domains | 25 |
| Number of boundaries | 68 |
| Number of edges | 66 |
| Number of vertices | 24 |

### 2.2.1 Sphere 1 (sph1)

POSITION

| Description | Value |
| --- | --- |
| Position | {0, 0, 0} |

AXIS

| Description | Value |
| --- | --- |
| Axis type | z - axis |

AXIS

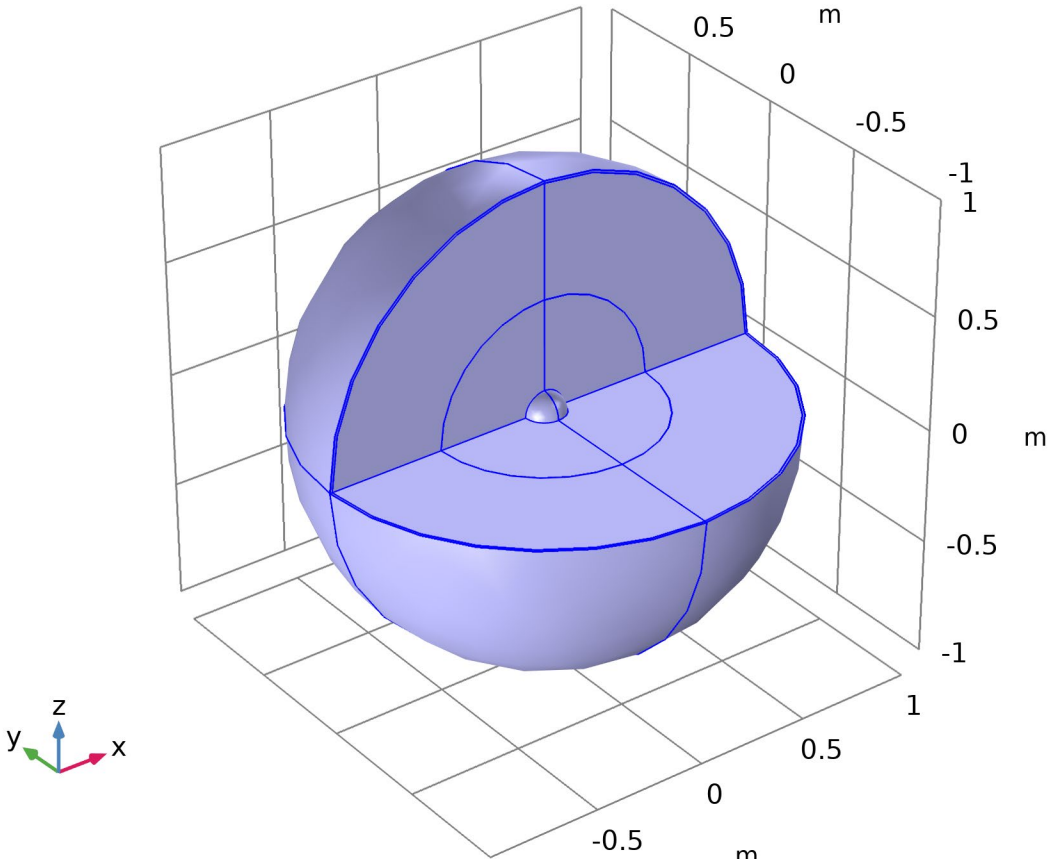
| Layer name | Thickness (m) |
| --- | --- |
| Layer 1 | 0.01 |
| Layer 2 | 0.5 |
| Layer 3 | 0.4 |

SIZE

| Description | Value |
| --- | --- |
| Radius | 1 |

## 2.3 MATERIALS

### 2.3.1 Air




*Air*

SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domains 1–25 |

MATERIAL PARAMETERS

| Name | Value | Unit |
|---|---|---|
| Relative permittivity | 1 | 1 |

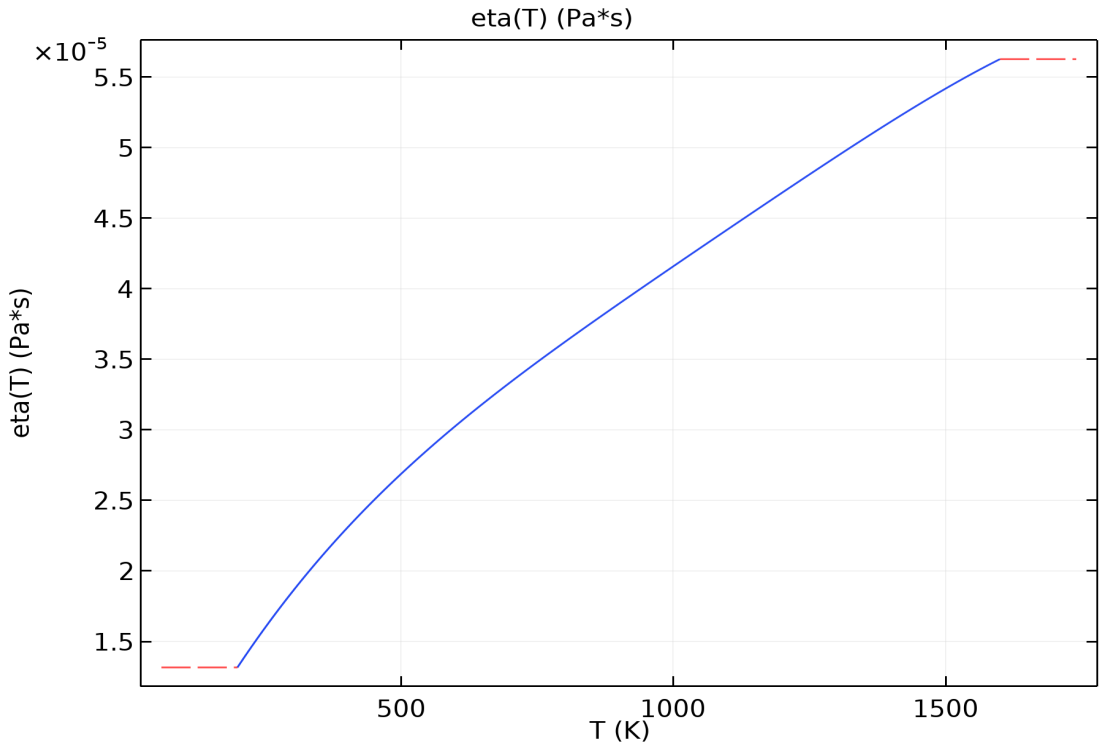
BASIC SETTINGS

| Description | Value |
|---|---|
| Coefficient of thermal expansion | {{alpha_p(pA, T), 0, 0}, {0, alpha_p(pA, T), 0}, {0, 0, alpha_p(pA, T)}} |
| Mean molar mass | 0.02897 |
| Bulk viscosity | muB(T) |

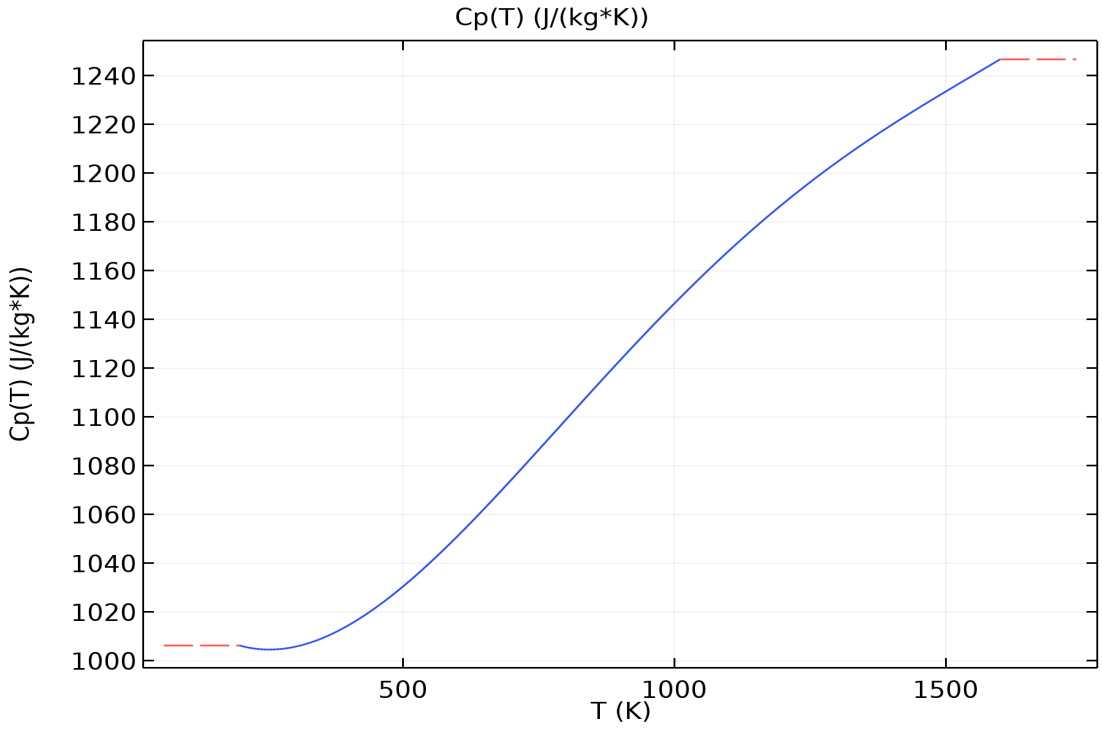
| Description | Value |
|---|---|
| Relative permeability | {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} |
| Relative permittivity | {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} |
| Dynamic viscosity | eta(T) |
| Ratio of specific heats | 1.4 |
| Electrical conductivity | {{0[S/m], 0, 0}, {0, 0[S/m], 0}, {0, 0, 0[S/m]}} |
| Heat capacity at constant pressure | Cp(T) |
| Density | rho(pA, T) |
| Thermal conductivity | {{k(T), 0, 0}, {0, k(T), 0}, {0, 0, k(T)}} |
| Speed of sound | cs(T) |

FUNCTIONS

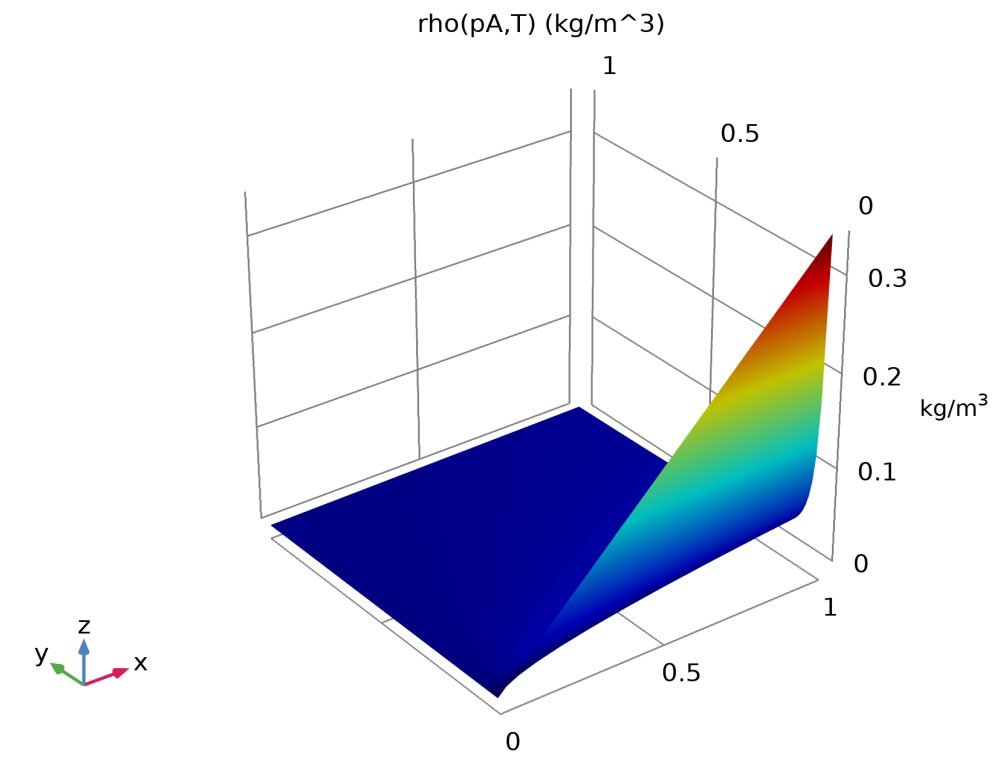
| Function name | Type |
|---|---|
| eta | Piecewise |
| Cp | Piecewise |
| rho | Analytic |
| k | Piecewise |
| cs | Analytic |
| alpha_p | Analytic |
| muB | Analytic |


eta(T) (Pa*s)
×10^-5
5.5
5
4.5
4
3.5
3
2.5
2
1.5
eta(T) (Pa*s)
500
1000
1500
T (K)

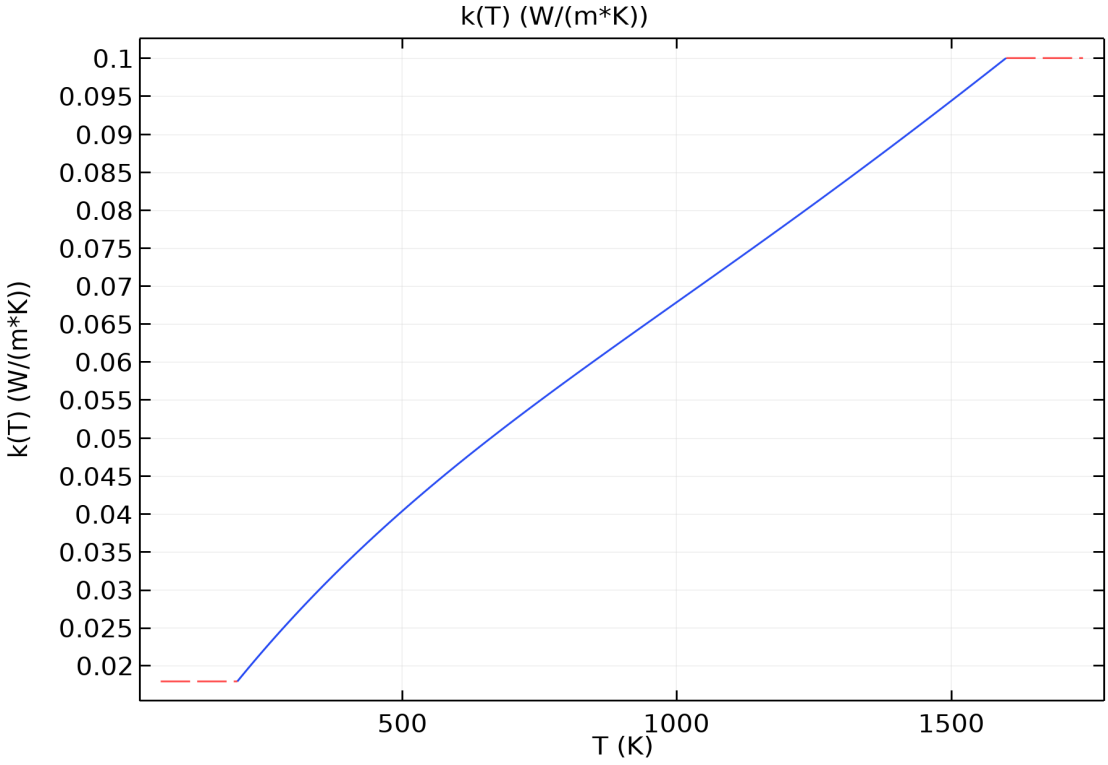
*eta*


Cp(T) (J/(kg*K))
1240
1220
1200
1180
1160
1140
1120
1100
1080
1060
1040
1020
1000
Cp(T) (J/(kg*K))
500
1000
1500
T (K)

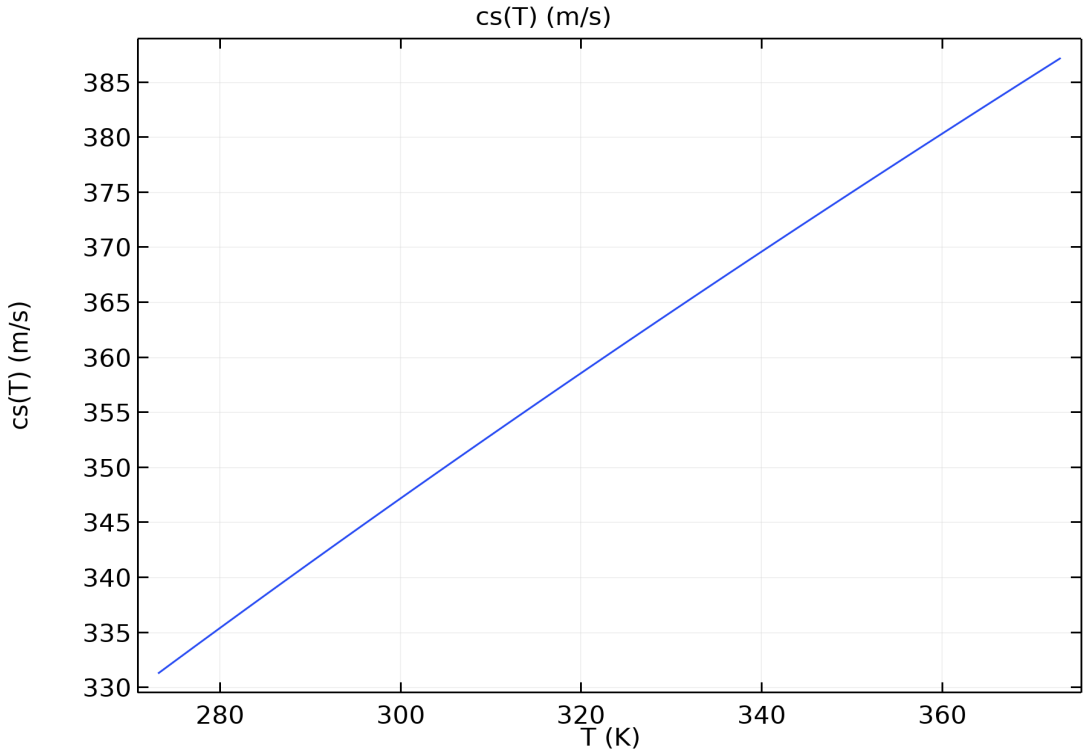
*Cp*


rho(pA,T) (kg/m^3)
1
0.5
0
0.3
0.2
kg/m3
0.1
0
1
0.5
0
z
y
x

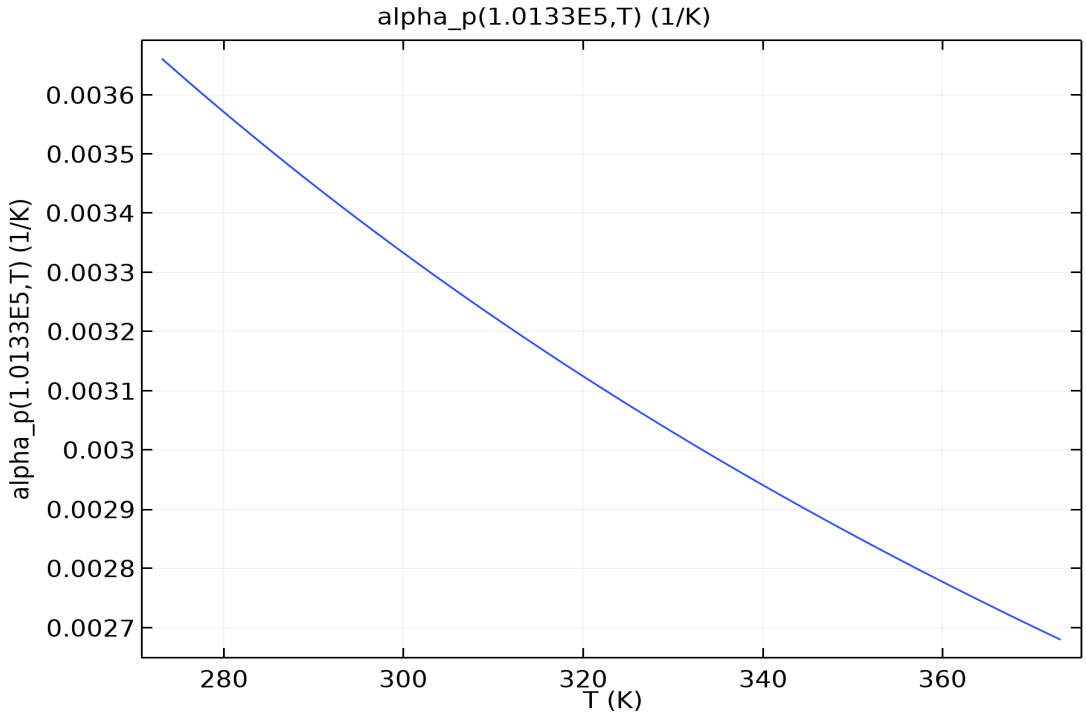
*rho*


k(T) (W/(m*K))
0.1
0.095
0.09
0.085
0.08
0.075
0.07
0.065
0.06
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
k(T) (W/(m*K))
500
1000
1500
T (K)

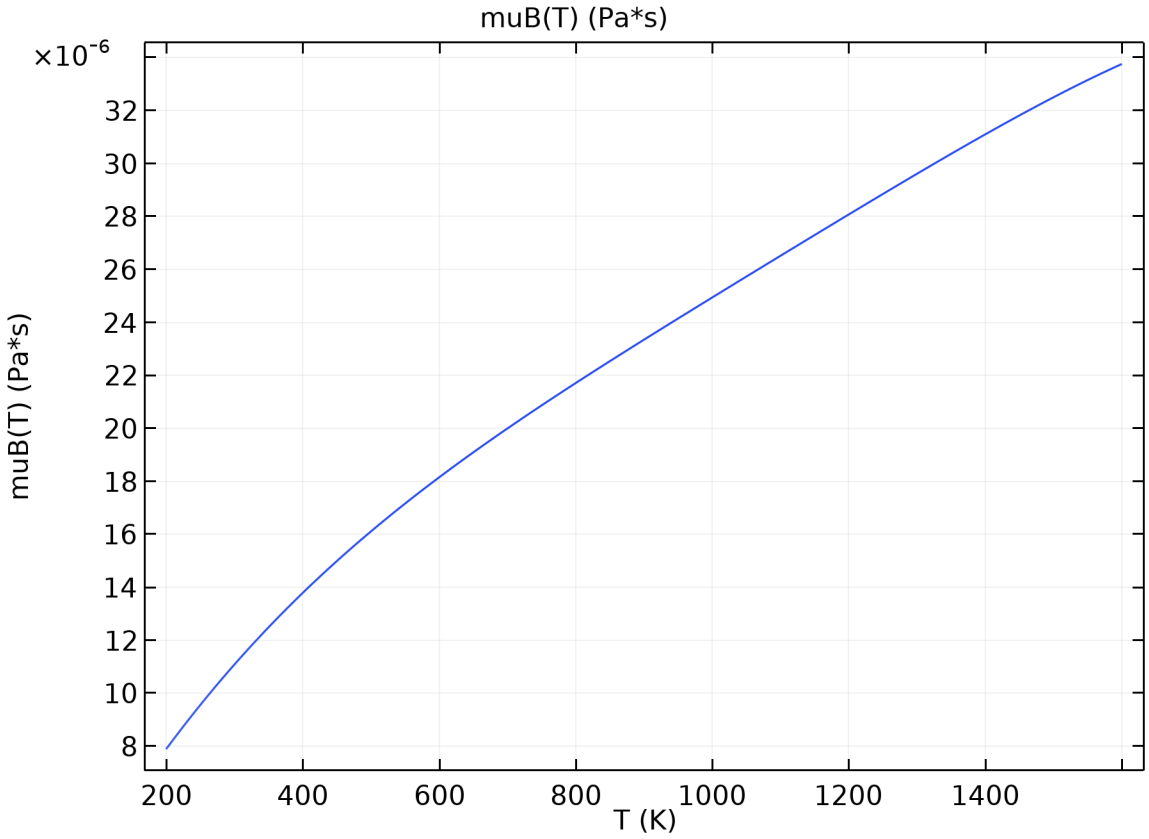
*k*


cs(T) (m/s)
cs(T) (m/s)
385
380
375
370
365
360
355
350
345
340
335
330
280
300
320
340
360
T (K)

*cs*


alpha_p(1.0133E5,T) (1/K)
alpha_p(1.0133E5,T) (1/K)
0.0036
0.0035
0.0034
0.0033
0.0032
0.0031
0.003
0.0029
0.0028
0.0027
280
300
320
340
360
T (K)

*alpha_p*




*muB*

REFRACTIVE INDEX SETTINGS

| Description | Value |
|---|---|
| Refractive index, real part | {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}} |
| Refractive index, imaginary part | {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}} |

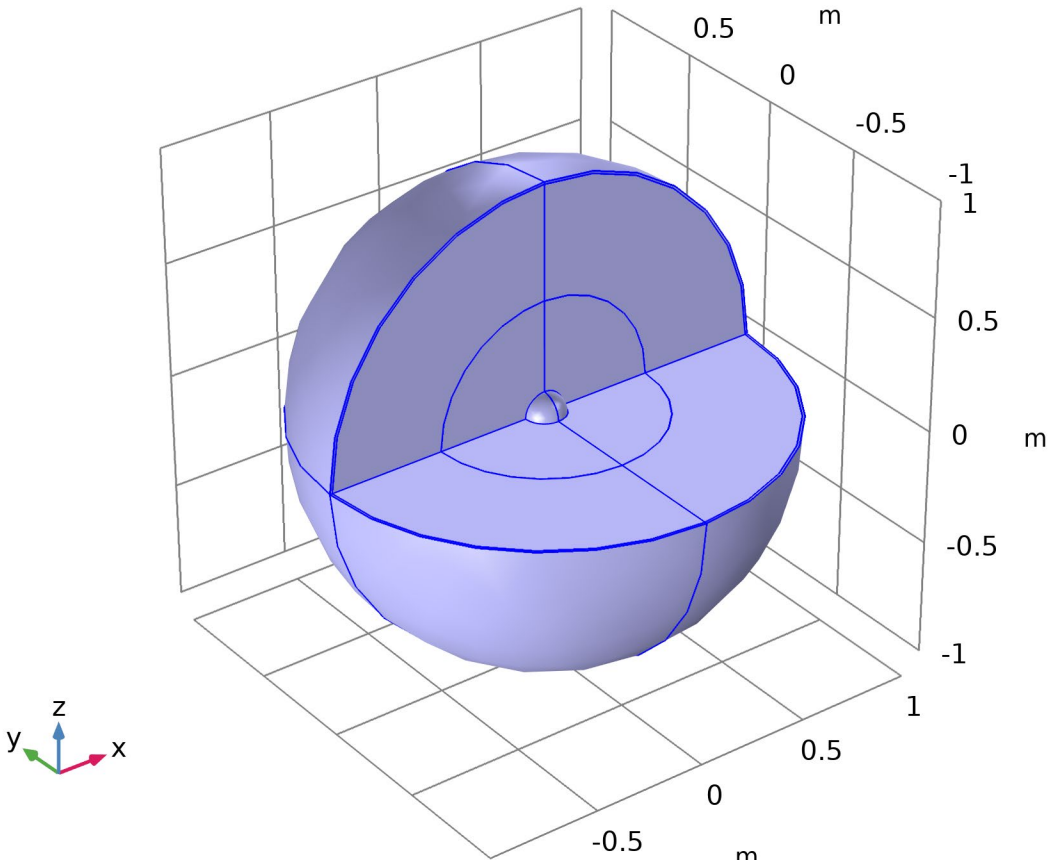
NONLINEAR MODEL SETTINGS

| Description | Value |
|---|---|
| Parameter of nonlinearity | (def.gamma + 1)/2 |

## 2.4 ELECTROSTATICS

USED PRODUCTS

| |
|---|
| COMSOL Multiphysics |




*Electrostatics*

SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domains 1–25 |

EQUATIONS

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\mathbf{E} = -\nabla V$$

## 2.4.1 Interface settings

### 2.4.1.1 Discretization

SETTINGS

| Description | Value |
|---|---|
| Electric potential | Quadratic |

### 2.4.1.2 Manual terminal sweep settings

SETTINGS

| Description | Value |
|---|---|
| Activate manual terminal sweep | Off |
| Reference impedance | 50[ohm] |

### 2.4.2 Variables

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| es.d | 1 | 1 | Contribution | Domains 5–13, 16–19, 21–24 |
| es.d | 1 | 1 | Contribution | Domains 1–4, 14–15, 20, 25 |
| es.nx | nx | | Normal vector, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.ny | ny | | Normal vector, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.nz | nz | | Normal vector, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.nx | nx | | Normal vector, x component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |
| es.ny | ny | | Normal vector, y component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |
| es.nz | nz | | Normal vector, z component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |
| es.nx | dnx | | Normal vector, x component | Boundaries 5–8, 32–33, 48, 65 |
| es.ny | dny | | Normal vector, y component | Boundaries 5–8, 32–33, 48, 65 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| es.nz | dnz | | Normal vector, z component | Boundaries 5–8, 32–33, 48, 65 |
| es.nmeshx | nxmesh | | Mesh normal vector, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.nmeshy | nymesh | | Mesh normal vector, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.nmeshz | nzmesh | | Mesh normal vector, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.nmeshx | nxmesh | | Mesh normal vector, x component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |
| es.nmeshy | nymesh | | Mesh normal vector, y component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |
| es.nmeshz | nzmesh | | Mesh normal vector, z component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |
| es.nmeshx | dnxmesh | | Mesh normal vector, x component | Boundaries 5–8, 32–33, 48, 65 |
| es.nmeshy | dnymesh | | Mesh normal vector, y component | Boundaries 5–8, 32–33, 48, 65 |
| es.nmeshz | dnzmesh | | Mesh normal vector, z component | Boundaries 5–8, 32–33, 48, 65 |
| es.unmeshx | unxmesh | | Mesh normal vector, upside, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unmeshy | unymesh | | Mesh normal vector, upside, y component | Boundaries 9–28, 34–45, |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| | | | | 49–61, 64, 66–67 |
| es.unmeshz | unzmesh | | Mesh normal vector, upside, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unmeshx | unxmesh | | Mesh normal vector, upside, x component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.unmeshy | unymesh | | Mesh normal vector, upside, y component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.unmeshz | unzmesh | | Mesh normal vector, upside, z component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnmeshx | dnxmesh | | Mesh normal vector, downside, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnmeshy | dnymesh | | Mesh normal vector, downside, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnmeshz | dnzmesh | | Mesh normal vector, downside, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnmeshx | dnxmesh | | Mesh normal vector, downside, x component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnmeshy | dnymesh | | Mesh normal vector, downside, y component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnmeshz | dnzmesh | | Mesh normal vector, downside, z component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| es.l_sxx | 1 | 1 | Spatial identity matrix, xx component | Domains 5–13, 16–19, 21–24 |
| es.l_syx | 0 | 1 | Spatial identity matrix, yx component | Domains 5–13, 16–19, 21–24 |
| es.l_szx | 0 | 1 | Spatial identity matrix, zx component | Domains 5–13, 16–19, 21–24 |
| es.l_sxy | 0 | 1 | Spatial identity matrix, xy component | Domains 5–13, 16–19, 21–24 |
| es.l_syy | 1 | 1 | Spatial identity matrix, yy component | Domains 5–13, 16–19, 21–24 |
| es.l_szy | 0 | 1 | Spatial identity matrix, zy component | Domains 5–13, 16–19, 21–24 |
| es.l_sxz | 0 | 1 | Spatial identity matrix, xz component | Domains 5–13, 16–19, 21–24 |
| es.l_syz | 0 | 1 | Spatial identity matrix, yz component | Domains 5–13, 16–19, 21–24 |
| es.l_szz | 1 | 1 | Spatial identity matrix, zz component | Domains 5–13, 16–19, 21–24 |
| es.l_sxx | 1 | 1 | Spatial identity matrix, xx component | Domains 1–4, 14–15, 20, 25 |
| es.l_syx | 0 | 1 | Spatial identity matrix, yx component | Domains 1–4, 14–15, 20, 25 |
| es.l_szx | 0 | 1 | Spatial identity matrix, zx component | Domains 1–4, 14–15, 20, 25 |
| es.l_sxy | 0 | 1 | Spatial identity matrix, xy component | Domains 1–4, 14–15, 20, 25 |
| es.l_syy | 1 | 1 | Spatial identity matrix, yy component | Domains 1–4, 14–15, 20, 25 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| es.l_szy | 0 | 1 | Spatial identity matrix, zy component | Domains 1–4, 14–15, 20, 25 |
| es.l_sxz | 0 | 1 | Spatial identity matrix, xz component | Domains 1–4, 14–15, 20, 25 |
| es.l_syz | 0 | 1 | Spatial identity matrix, yz component | Domains 1–4, 14–15, 20, 25 |
| es.l_szz | 1 | 1 | Spatial identity matrix, zz component | Domains 1–4, 14–15, 20, 25 |
| es.l_sXX | (spatial.invF11*(spatial.invF11*es.l_sxx+spatial.invF21*es.l_syx+spatial.invF31*es.l_szx)+spatial.invF21*(spatial.invF11*es.l_sxy+spatial.invF21*es.l_syy+spatial.invF31*es.l_szy)+spatial.invF31*(spatial.invF11*es.l_sxz+spatial.invF21*es.l_syz+spatial.invF31*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, XX component | Domains 5–13, 16–19, 21–24 |
| es.l_sYX | (spatial.invF11*(spatial.invF12*es.l_sxx+spatial.invF22*es.l_syx+spatial.invF32*es.l_szx)+spatial.invF21*(spatial.invF12*es.l_sxy+spatial.invF22*es.l_syy+spatial.invF32*es.l_szy)+spatial.invF31*(spatial.invF12*es.l_sxz+spatial.invF22*es.l_syz+spatial.invF32*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, YX component | Domains 5–13, 16–19, 21–24 |
| es.l_sZX | (spatial.invF11*(spatial.invF13*es.l_sxx+spatial.invF23*es.l_syx+spatial.invF33*es.l_szx)+spatial.invF21*(spatial.invF13*es.l_sxy+spatial.invF23*es.l_syy+spatial.invF33*es.l_szy)+spatial.invF31*(spatial.invF13*es.l_sxz+spatial.invF23*es.l_syz+spatial.invF33*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, ZX component | Domains 5–13, 16–19, 21–24 |
| es.l_sXY | (spatial.invF12*(spatial.invF11*es.l_sxx+spatial.invF21*es.l_syx+spatial.invF31*es.l_szx)+spatial.invF22*(spatial.invF11*es.l_sxy+spatial.invF21*es.l_syy+spatial.invF31*es.l_szy)+spatial.invF32*(spatial.invF11*es.l_sxz+spatial.invF21*es.l_syz+spatial.invF31*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, XY component | Domains 5–13, 16–19, 21–24 |
| es.l_sYY | (spatial.invF12*(spatial.invF12*es.l_sxx+spatial.invF22*es.l_syx+spatial.invF32*es.l_szx)+spatial.invF22*(spatial.invF12*es.l_sxy+spatial.invF22*es.l_syy+spatial.invF32*es.l_szy)+spatial.invF32*(spatial.invF12 | 1 | Spatial identity matrix, YY component | Domains 5–13, 16–19, 21–24 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| | *es.I_sxz+spatial.invF22*es.I_syz+spatial.invF32*es.I_szz))*spatial.detF | | | |
| es.I_sZY | (spatial.invF12*(spatial.invF13*es.I_sxx+spatial.invF23*es.I_syx+spatial.invF33*es.I_szx)+spatial.invF22*(spatial.invF13*es.I_sxy+spatial.invF23*es.I_syy+spatial.invF33*es.I_szy)+spatial.invF32*(spatial.invF13*es.I_sxz+spatial.invF23*es.I_syz+spatial.invF33*es.I_szz))*spatial.detF | 1 | Spatial identity matrix, ZY component | Domains 5–13, 16–19, 21–24 |
| es.I_sXZ | (spatial.invF13*(spatial.invF11*es.I_sxx+spatial.invF21*es.I_syx+spatial.invF31*es.I_szx)+spatial.invF23*(spatial.invF11*es.I_sxy+spatial.invF21*es.I_syy+spatial.invF31*es.I_szy)+spatial.invF33*(spatial.invF11*es.I_sxz+spatial.invF21*es.I_syz+spatial.invF31*es.I_szz))*spatial.detF | 1 | Spatial identity matrix, XZ component | Domains 5–13, 16–19, 21–24 |
| es.I_sYZ | (spatial.invF13*(spatial.invF12*es.I_sxx+spatial.invF22*es.I_syx+spatial.invF32*es.I_szx)+spatial.invF23*(spatial.invF12*es.I_sxy+spatial.invF22*es.I_syy+spatial.invF32*es.I_szy)+spatial.invF33*(spatial.invF12*es.I_sxz+spatial.invF22*es.I_syz+spatial.invF32*es.I_szz))*spatial.detF | 1 | Spatial identity matrix, YZ component | Domains 5–13, 16–19, 21–24 |
| es.I_sZZ | (spatial.invF13*(spatial.invF13*es.I_sxx+spatial.invF23*es.I_syx+spatial.invF33*es.I_szx)+spatial.invF23*(spatial.invF13*es.I_sxy+spatial.invF23*es.I_syy+spatial.invF33*es.I_szy)+spatial.invF33*(spatial.invF13*es.I_sxz+spatial.invF23*es.I_syz+spatial.invF33*es.I_szz))*spatial.detF | 1 | Spatial identity matrix, ZZ component | Domains 5–13, 16–19, 21–24 |
| es.I_sXX | (spatial.invF11*(spatial.invF11*es.I_sxx+spatial.invF21*es.I_syx+spatial.invF31*es.I_szx)+spatial.invF21*(spatial.invF11*es.I_sxy+spatial.invF21*es.I_syy+spatial.invF31*es.I_szy)+spatial.invF31*(spatial.invF11*es.I_sxz+spatial.invF21*es.I_syz+spatial.invF31*es.I_szz))*spatial.detF | 1 | Spatial identity matrix, XX component | Domains 1–4, 14–15, 20, 25 |
| es.I_sYX | (spatial.invF11*(spatial.invF12*es.I_sxx+spatial.invF22*es.I_syx+spatial.invF32*es.I_szx)+spatial.invF21*(spatial.invF12*es.I_sxy+spatial.invF22*es.I_syy+spatial.invF32*es.I_szy)+spatial.invF31*(spatial.invF12*es.I_sxz+spatial.invF22*es.I_syz+spatial.invF32*es.I_szz))*spatial.detF | 1 | Spatial identity matrix, YX component | Domains 1–4, 14–15, 20, 25 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| es.l_sZX | (spatial.invF11*(spatial.invF13*es.l_sxx+spatial.invF23*es.l_syx+spatial.invF33*es.l_szx)+spatial.invF21*(spatial.invF13*es.l_sxy+spatial.invF23*es.l_syy+spatial.invF33*es.l_szy)+spatial.invF31*(spatial.invF13*es.l_sxz+spatial.invF23*es.l_syz+spatial.invF33*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, ZX component | Domains 1–4, 14–15, 20, 25 |
| es.l_sXY | (spatial.invF12*(spatial.invF11*es.l_sxx+spatial.invF21*es.l_syx+spatial.invF31*es.l_szx)+spatial.invF22*(spatial.invF11*es.l_sxy+spatial.invF21*es.l_syy+spatial.invF31*es.l_szy)+spatial.invF32*(spatial.invF11*es.l_sxz+spatial.invF21*es.l_syz+spatial.invF31*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, XY component | Domains 1–4, 14–15, 20, 25 |
| es.l_sYY | (spatial.invF12*(spatial.invF12*es.l_sxx+spatial.invF22*es.l_syx+spatial.invF32*es.l_szx)+spatial.invF22*(spatial.invF12*es.l_sxy+spatial.invF22*es.l_syy+spatial.invF32*es.l_szy)+spatial.invF32*(spatial.invF12*es.l_sxz+spatial.invF22*es.l_syz+spatial.invF32*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, YY component | Domains 1–4, 14–15, 20, 25 |
| es.l_sZY | (spatial.invF12*(spatial.invF13*es.l_sxx+spatial.invF23*es.l_syx+spatial.invF33*es.l_szx)+spatial.invF22*(spatial.invF13*es.l_sxy+spatial.invF23*es.l_syy+spatial.invF33*es.l_szy)+spatial.invF32*(spatial.invF13*es.l_sxz+spatial.invF23*es.l_syz+spatial.invF33*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, ZY component | Domains 1–4, 14–15, 20, 25 |
| es.l_sXZ | (spatial.invF13*(spatial.invF11*es.l_sxx+spatial.invF21*es.l_syx+spatial.invF31*es.l_szx)+spatial.invF23*(spatial.invF11*es.l_sxy+spatial.invF21*es.l_syy+spatial.invF31*es.l_szy)+spatial.invF33*(spatial.invF11*es.l_sxz+spatial.invF21*es.l_syz+spatial.invF31*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, XZ component | Domains 1–4, 14–15, 20, 25 |
| es.l_sYZ | (spatial.invF13*(spatial.invF12*es.l_sxx+spatial.invF22*es.l_syx+spatial.invF32*es.l_szx)+spatial.invF23*(spatial.invF12*es.l_sxy+spatial.invF22*es.l_syy+spatial.invF32*es.l_szy)+spatial.invF33*(spatial.invF12*es.l_sxz+spatial.invF22*es.l_syz+spatial.invF32*es.l_szz))*spatial.detF | 1 | Spatial identity matrix, YZ component | Domains 1–4, 14–15, 20, 25 |
| es.l_sZZ | (spatial.invF13*(spatial.invF13*es.l_sxx+spatial.invF23*es.l_syx+spatial.invF33*es.l_szx)+spatial.invF23*(spatial.invF13*es.l_ | 1 | Spatial identity matrix, ZZ component | Domains 1–4, 14–15, 20, 25 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| | sxy+spatial.invF23*es.I_syy+spatial.invF33*es.I_szy)+spatial.invF33*(spatial.invF13*es.I_sxz+spatial.invF23*es.I_syz+spatial.invF33*es.I_szz))*spatial.detF | | | |
| es.unTx | es.unTex | Pa | Maxwell upward surface stress tensor, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unTy | es.unTey | Pa | Maxwell upward surface stress tensor, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unTz | es.unTez | Pa | Maxwell upward surface stress tensor, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unTx | es.unTex | Pa | Maxwell upward surface stress tensor, x component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.unTy | es.unTey | Pa | Maxwell upward surface stress tensor, y component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.unTz | es.unTez | Pa | Maxwell upward surface stress tensor, z component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnTx | es.dnTex | Pa | Maxwell downward surface stress tensor, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnTy | es.dnTey | Pa | Maxwell downward surface stress tensor, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnTz | es.dnTez | Pa | Maxwell downward surface stress tensor, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |

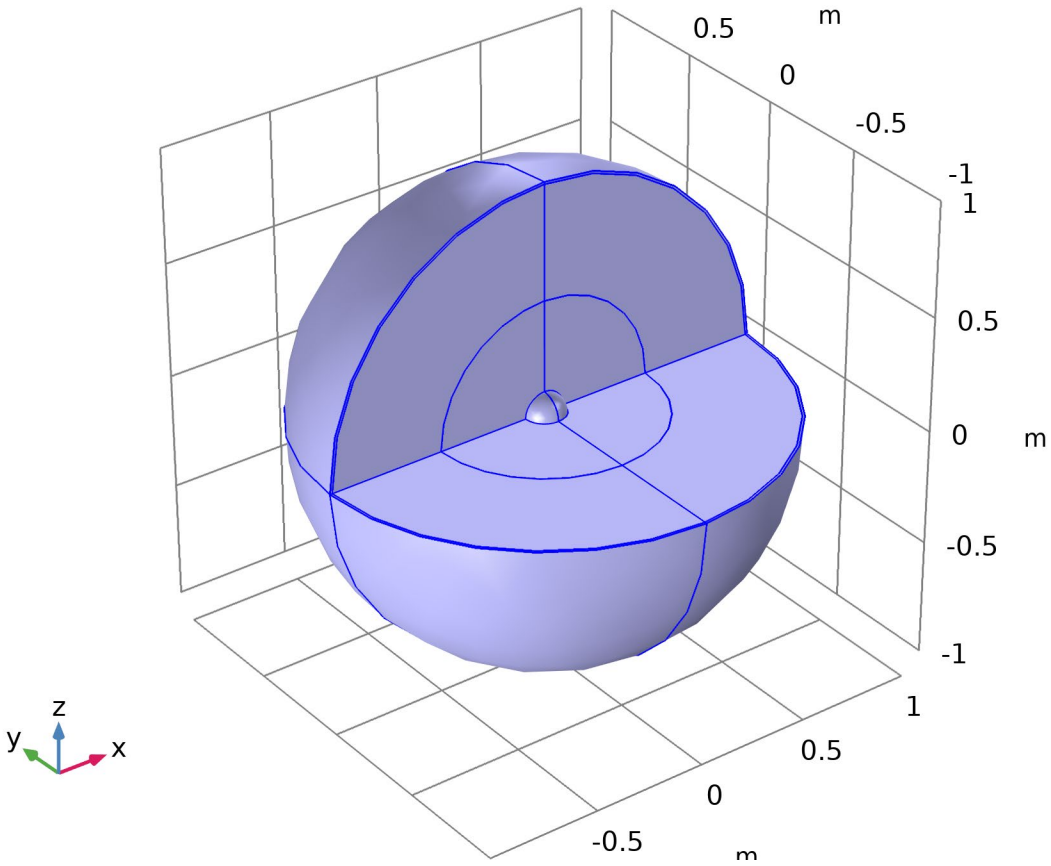
| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| es.dnTx | es.dnTex | Pa | Maxwell downward surface stress tensor, x component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnTy | es.dnTey | Pa | Maxwell downward surface stress tensor, y component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnTz | es.dnTez | Pa | Maxwell downward surface stress tensor, z component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.unx | unx | | Normal vector up direction, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.uny | uny | | Normal vector up direction, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unz | unz | | Normal vector up direction, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unx | unx | | Normal vector up direction, x component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.uny | uny | | Normal vector up direction, y component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.unz | unz | | Normal vector up direction, z component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnx | dnx | | Normal vector down direction, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| es.dny | dny | | Normal vector down direction, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnz | dnz | | Normal vector down direction, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnx | dnx | | Normal vector down direction, x component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dny | dny | | Normal vector down direction, y component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnz | dnz | | Normal vector down direction, z component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.unTex | -0.5*es.dnx*(real(up(es.Dx))*real(up(es.Ex))+real(up(es.Dy))*real(up(es.Ey))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dx))*(real(up(es.Ex))*es.dnx+real(up(es.Ey))*es.dny+real(up(es.Ez))*es.dnz) | Pa | Maxwell upward electric surface stress tensor, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unTey | -0.5*es.dny*(real(up(es.Dx))*real(up(es.Ex))+real(up(es.Dy))*real(up(es.Ey))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dy))*(real(up(es.Ex))*es.dnx+real(up(es.Ey))*es.dny+real(up(es.Ez))*es.dnz) | Pa | Maxwell upward electric surface stress tensor, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unTez | -0.5*es.dnz*(real(up(es.Dx))*real(up(es.Ex))+real(up(es.Dy))*real(up(es.Ey))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dz))*(real(up(es.Ex))*es.dnx+real(up(es.Ey))*es.dny+real(up(es.Ez))*es.dnz) | Pa | Maxwell upward electric surface stress tensor, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.unTex | -0.5*es.dnx*(real(up(es.Dx))*real(up(es.Ex))+real(up(es.Dy))*real(up(es.Ey))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dx))*(real(up(es.Ex))*es.dnx+real(up(es.Ey))*es.dny+real(up(es.Ez))*es.dnz) | Pa | Maxwell upward electric surface stress tensor, x component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| es.unTey | -0.5*es.dny*(real(up(es.Dx))*real(up(es.Ex))+real(up(es.Dy))*real(up(es.Ey))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dy))*(real(up(es.Ex))*es.dnx+real(up(es.Ey))*es.dny+real(up(es.Ez))*es.dnz) | Pa | Maxwell upward electric surface stress tensor, y component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |
| es.unTez | -0.5*es.dnz*(real(up(es.Dx))*real(up(es.Ex))+real(up(es.Dy))*real(up(es.Ey))+real(up(es.Dz))*real(up(es.Ez)))+real(up(es.Dz))*(real(up(es.Ex))*es.dnx+real(up(es.Ey))*es.dny+real(up(es.Ez))*es.dnz) | Pa | Maxwell upward electric surface stress tensor, z component | Boundaries 1–4, 29–31, 46–47, 62–63, 68 |
| es.unTex | 0 | Pa | Maxwell upward electric surface stress tensor, x component | Boundaries 5–8, 32–33, 48, 65 |
| es.unTey | 0 | Pa | Maxwell upward electric surface stress tensor, y component | Boundaries 5–8, 32–33, 48, 65 |
| es.unTez | 0 | Pa | Maxwell upward electric surface stress tensor, z component | Boundaries 5–8, 32–33, 48, 65 |
| es.dnTex | -0.5*es.unx*(real(down(es.Dx))*real(down(es.Ex))+real(down(es.Dy))*real(down(es.Ey))+real(down(es.Dz))*real(down(es.Ez)))+real(down(es.Dx))*(real(down(es.Ex))*es.unx+real(down(es.Ey))*es.uny+real(down(es.Ez))*es.unz) | Pa | Maxwell downward electric surface stress tensor, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnTey | -0.5*es.uny*(real(down(es.Dx))*real(down(es.Ex))+real(down(es.Dy))*real(down(es.Ey))+real(down(es.Dz))*real(down(es.Ez)))+real(down(es.Dy))*(real(down(es.Ex))*es.unx+real(down(es.Ey))*es.uny+real(down(es.Ez))*es.unz) | Pa | Maxwell downward electric surface stress tensor, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |
| es.dnTez | -0.5*es.unz*(real(down(es.Dx))*real(down(es.Ex))+real(down(es.Dy))*real(down(es.Ey))+real(down(es.Dz))*real(down(es.Ez))) | Pa | Maxwell downward electric surface | Boundaries 9–28, 34–45, 49–61, 64, 66–67 |

| Name | Expression | Unit | Description | Selection |
|---|---|---|---|---|
| | +real(down(es.Dz))*(real(down(es.Ex))*es.unx+real(down(es.Ey))*es.uny+real(down(es.Ez))*es.unz) | | stress tensor, z component | |
| es.dnTex | -0.5*es.unx*(real(down(es.Dx))*real(down(es.Ex))+real(down(es.Dy))*real(down(es.Ey))+real(down(es.Dz))*real(down(es.Ez)))+real(down(es.Dx))*(real(down(es.Ex))*es.unx+real(down(es.Ey))*es.uny+real(down(es.Ez))*es.unz) | Pa | Maxwell downward electric surface stress tensor, x component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnTey | -0.5*es.uny*(real(down(es.Dx))*real(down(es.Ex))+real(down(es.Dy))*real(down(es.Ey))+real(down(es.Dz))*real(down(es.Ez)))+real(down(es.Dy))*(real(down(es.Ex))*es.unx+real(down(es.Ey))*es.uny+real(down(es.Ez))*es.unz) | Pa | Maxwell downward electric surface stress tensor, y component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.dnTez | -0.5*es.unz*(real(down(es.Dx))*real(down(es.Ex))+real(down(es.Dy))*real(down(es.Ey))+real(down(es.Dz))*real(down(es.Ez)))+real(down(es.Dz))*(real(down(es.Ex))*es.unx+real(down(es.Ey))*es.uny+real(down(es.Ez))*es.unz) | Pa | Maxwell downward electric surface stress tensor, z component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 |
| es.intWe | es.int_We(es.d*es.dWe) | J | Total electric energy | Global |
| es.zref | 50[ohm] | Ω | Reference impedance | Global |

### 2.4.3 Charge Conservation 1




*Charge Conservation 1*

SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domains 1–25 |

EQUATIONS

$$\mathbf{E} = -\nabla V$$
$$\nabla \cdot (\epsilon_0 \epsilon_r \mathbf{E}) = \rho_v$$

#### 2.4.3.1 Electric field

SETTINGS

| Description | Value |
|---|---|
| Constitutive relation | Relative permittivity |
| Relative permittivity | From material |

#### 2.4.3.2 Coordinate system selection

SETTINGS

| Description | Value |
|---|---|
| Coordinate system | Global coordinate system |

#### 2.4.3.3 Model input

SETTINGS

| Description | Value |
|---|---|
| Temperature | User defined |
| Temperature | 293.15[K] |
| Absolute pressure | User defined |
| Absolute pressure | 1[atm] |

PROPERTIES FROM MATERIAL

| Property | Material | Property group |
|---|---|---|
| Relative permittivity | Air | Basic |

#### 2.4.3.4 Variables

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.nD | 0 | C/m² | Surface charge density | Boundaries 9–28, 34–45, 49–61, 64, 66–67 | + operation |
| es.nD | 0 | C/m² | Surface charge density | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 | + operation |
| es.epsilonrxx | material.epsilonr11 | 1 | Relative permittivity, xx component | Domains 5–13, 16–19, 21–24 | Meta |
| es.epsilonryx | material.epsilonr21 | 1 | Relative permittivity, yx component | Domains 5–13, 16–19, 21–24 | Meta |
| es.epsilonrzx | material.epsilonr31 | 1 | Relative permittivity, zx component | Domains 5–13, 16–19, 21–24 | Meta |
| es.epsilonrxy | material.epsilonr12 | 1 | Relative permittivity, xy component | Domains 5–13, 16–19, 21–24 | Meta |
| es.epsilonryy | material.epsilonr22 | 1 | Relative permittivity, yy component | Domains 5–13, 16–19, 21–24 | Meta |

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.epsilonrzy | material.epsilonr32 | 1 | Relative permittivity, zy component | Domains 5–13, 16–19, 21–24 | Meta |
| es.epsilonrxz | material.epsilonr13 | 1 | Relative permittivity, xz component | Domains 5–13, 16–19, 21–24 | Meta |
| es.epsilonryz | material.epsilonr23 | 1 | Relative permittivity, yz component | Domains 5–13, 16–19, 21–24 | Meta |
| es.epsilonrzz | material.epsilonr33 | 1 | Relative permittivity, zz component | Domains 5–13, 16–19, 21–24 | Meta |
| es.epsilonrxx | material.epsilonr11 | 1 | Relative permittivity, xx component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonryx | material.epsilonr21 | 1 | Relative permittivity, yx component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonrzx | material.epsilonr31 | 1 | Relative permittivity, zx component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonrxy | material.epsilonr12 | 1 | Relative permittivity, xy component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonryy | material.epsilonr22 | 1 | Relative permittivity, yy component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonrzy | material.epsilonr32 | 1 | Relative permittivity, zy component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonrxz | material.epsilonr13 | 1 | Relative permittivity, xz component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonryz | material.epsilonr23 | 1 | Relative permittivity, yz component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonrzz | material.epsilonr33 | 1 | Relative permittivity, zz component | Domains 1–4, 14–15, 20, 25 | Meta |
| es.epsilonr_iso | material.epsilonr_iso | 1 | Relative permittivity, isotropic value | Domains 5–13, 16–19, 21–24 | Meta |

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.epsilonr_iso | material.epsilonr_iso | 1 | Relative permittivity, isotropic value | Domains 1–4, 14–15, 20, 25 | Meta |
| es.Dx | epsilon0_const*es.I_sxx*es.Ex+epsilon0_const*es.I_sxy*es.Ey+epsilon0_const*es.I_sxz*es.Ez+es.Px | C/m² | Electric displacement field, x component | Domains 5–13, 16–19, 21–24 | |
| es.Dy | epsilon0_const*es.I_syx*es.Ex+epsilon0_const*es.I_syy*es.Ey+epsilon0_const*es.I_syz*es.Ez+es.Py | C/m² | Electric displacement field, y component | Domains 5–13, 16–19, 21–24 | |
| es.Dz | epsilon0_const*es.I_szx*es.Ex+epsilon0_const*es.I_szy*es.Ey+epsilon0_const*es.I_szz*es.Ez+es.Pz | C/m² | Electric displacement field, z component | Domains 5–13, 16–19, 21–24 | |
| es.Dx | epsilon0_const*es.I_sxx*es.Ex+epsilon0_const*es.I_sxy*es.Ey+epsilon0_const*es.I_sxz*es.Ez+es.Px | C/m² | Electric displacement field, x component | Domains 1–4, 14–15, 20, 25 | |
| es.Dy | epsilon0_const*es.I_syx*es.Ex+epsilon0_const*es.I_syy*es.Ey+epsilon0_const*es.I_syz*es.Ez+es.Py | C/m² | Electric displacement field, y component | Domains 1–4, 14–15, 20, 25 | |
| es.Dz | epsilon0_const*es.I_szx*es.Ex+epsilon0_const*es.I_szy*es.Ey+epsilon0_const*es.I_szz*es.Ez+es.Pz | C/m² | Electric displacement field, z component | Domains 1–4, 14–15, 20, 25 | |
| es.Px | epsilon0_const*(es.chixx*es.Ex+es.chixy*es.Ey+es.chixz*es.Ez) | C/m² | Polarization, x component | Domains 5–13, 16–19, 21–24 | |
| es.Py | epsilon0_const*(es.chiyx*es.Ex+es.chiyy*es.Ey+es.chiyz*es.Ez) | C/m² | Polarization, y component | Domains 5–13, 16–19, 21–24 | |
| es.Pz | epsilon0_const*(es.chizx*es.Ex+es.chiz | C/m² | Polarization, z component | Domains 5–13, 16–19, 21–24 | |

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| | y*es.Ey+es.chizz*es.Ez) | | | | |
| es.Px | epsilon0_const*(es.chixx*es.Ex+es.chixy*es.Ey+es.chixz*es.Ez) | C/m² | Polarization, x component | Domains 1–4, 14–15, 20, 25 | |
| es.Py | epsilon0_const*(es.chiyx*es.Ex+es.chiyy*es.Ey+es.chiyz*es.Ez) | C/m² | Polarization, y component | Domains 1–4, 14–15, 20, 25 | |
| es.Pz | epsilon0_const*(es.chizx*es.Ex+es.chizy*es.Ey+es.chizz*es.Ez) | C/m² | Polarization, z component | Domains 1–4, 14–15, 20, 25 | |
| es.normD | sqrt(realdot(es.Dx,es.Dx)+realdot(es.Dy,es.Dy)+realdot(es.Dz,es.Dz)) | C/m² | Electric displacement field norm | Domains 5–13, 16–19, 21–24 | |
| es.normD | sqrt(realdot(es.Dx,es.Dx)+realdot(es.Dy,es.Dy)+realdot(es.Dz,es.Dz)) | C/m² | Electric displacement field norm | Domains 1–4, 14–15, 20, 25 | |
| es.normP | sqrt(realdot(es.Px,es.Px)+realdot(es.Py,es.Py)+realdot(es.Pz,es.Pz)) | C/m² | Polarization norm | Domains 5–13, 16–19, 21–24 | |
| es.normP | sqrt(realdot(es.Px,es.Px)+realdot(es.Py,es.Py)+realdot(es.Pz,es.Pz)) | C/m² | Polarization norm | Domains 1–4, 14–15, 20, 25 | |
| es.chixx | -1+es.epsilonrxx | 1 | Electric susceptibility, xx component | Domains 5–13, 16–19, 21–24 | |
| es.chiyx | es.epsilonryx | 1 | Electric susceptibility, yx component | Domains 5–13, 16–19, 21–24 | |
| es.chizx | es.epsilonrzx | 1 | Electric susceptibility, zx component | Domains 5–13, 16–19, 21–24 | |
| es.chixy | es.epsilonrxy | 1 | Electric susceptibility, xy component | Domains 5–13, 16–19, 21–24 | |

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.chiyy | -1+es.epsilonryy | 1 | Electric susceptibility, yy component | Domains 5–13, 16–19, 21–24 | |
| es.chizy | es.epsilonrzy | 1 | Electric susceptibility, zy component | Domains 5–13, 16–19, 21–24 | |
| es.chixz | es.epsilonrxz | 1 | Electric susceptibility, xz component | Domains 5–13, 16–19, 21–24 | |
| es.chiyz | es.epsilonryz | 1 | Electric susceptibility, yz component | Domains 5–13, 16–19, 21–24 | |
| es.chizz | -1+es.epsilonrzz | 1 | Electric susceptibility, zz component | Domains 5–13, 16–19, 21–24 | |
| es.chixx | -1+es.epsilonrxx | 1 | Electric susceptibility, xx component | Domains 1–4, 14–15, 20, 25 | |
| es.chiyx | es.epsilonryx | 1 | Electric susceptibility, yx component | Domains 1–4, 14–15, 20, 25 | |
| es.chizx | es.epsilonrzx | 1 | Electric susceptibility, zx component | Domains 1–4, 14–15, 20, 25 | |
| es.chixy | es.epsilonrxy | 1 | Electric susceptibility, xy component | Domains 1–4, 14–15, 20, 25 | |
| es.chiyy | -1+es.epsilonryy | 1 | Electric susceptibility, yy component | Domains 1–4, 14–15, 20, 25 | |
| es.chizy | es.epsilonrzy | 1 | Electric susceptibility, zy component | Domains 1–4, 14–15, 20, 25 | |
| es.chixz | es.epsilonrxz | 1 | Electric susceptibility, xz component | Domains 1–4, 14–15, 20, 25 | |
| es.chiyz | es.epsilonryz | 1 | Electric susceptibility, yz component | Domains 1–4, 14–15, 20, 25 | |
| es.chizz | -1+es.epsilonrzz | 1 | Electric susceptibility, zz component | Domains 1–4, 14–15, 20, 25 | |

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.Ex | -Vx | V/m | Electric field, x component | Domains 5–13, 16–19, 21–24 | |
| es.Ey | -Vy | V/m | Electric field, y component | Domains 5–13, 16–19, 21–24 | |
| es.Ez | -Vz | V/m | Electric field, z component | Domains 5–13, 16–19, 21–24 | |
| es.Ex | -ie1.T11*Vx-ie1.T12*Vy-ie1.T13*Vz | V/m | Electric field, x component | Domains 1–4, 14–15, 20, 25 | |
| es.Ey | -ie1.T21*Vx-ie1.T22*Vy-ie1.T23*Vz | V/m | Electric field, y component | Domains 1–4, 14–15, 20, 25 | |
| es.Ez | -ie1.T31*Vx-ie1.T32*Vy-ie1.T33*Vz | V/m | Electric field, z component | Domains 1–4, 14–15, 20, 25 | |
| es.tEx | -VTx | V/m | Tangential electric field, x component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 | |
| es.tEy | -VTy | V/m | Tangential electric field, y component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 | |
| es.tEz | -VTz | V/m | Tangential electric field, z component | Boundaries 9–28, 34–45, 49–61, 64, 66–67 | |
| es.tEx | -ie1.T11*VTx-ie1.T12*VTy-ie1.T13*VTz | V/m | Tangential electric field, x component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 | |
| es.tEy | -ie1.T21*VTx-ie1.T22*VTy-ie1.T23*VTz | V/m | Tangential electric field, y component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 | |
| es.tEz | -ie1.T31*VTx-ie1.T32*VTy-ie1.T33*VTz | V/m | Tangential electric field, z component | Boundaries 1–8, 29–33, 46–48, 62–63, 65, 68 | |

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.normE | sqrt(realdot(es.Ex,es.Ex)+realdot(es.Ey,es.Ey)+realdot(es.Ez,es.Ez)) | V/m | Electric field norm | Domains 5–13, 16–19, 21–24 | |
| es.normE | sqrt(realdot(es.Ex,es.Ex)+realdot(es.Ey,es.Ey)+realdot(es.Ez,es.Ez)) | V/m | Electric field norm | Domains 1–4, 14–15, 20, 25 | |
| es.Jx | es.Jdx | $A/m^2$ | Current density, x component | Domains 5–13, 16–19, 21–24 | + operation |
| es.Jy | es.Jdy | $A/m^2$ | Current density, y component | Domains 5–13, 16–19, 21–24 | + operation |
| es.Jz | es.Jdz | $A/m^2$ | Current density, z component | Domains 5–13, 16–19, 21–24 | + operation |
| es.Jx | es.Jdx | $A/m^2$ | Current density, x component | Domains 1–4, 14–15, 20, 25 | + operation |
| es.Jy | es.Jdy | $A/m^2$ | Current density, y component | Domains 1–4, 14–15, 20, 25 | + operation |
| es.Jz | es.Jdz | $A/m^2$ | Current density, z component | Domains 1–4, 14–15, 20, 25 | + operation |
| es.Jdx | 0 | $A/m^2$ | Displacement current density, x component | Domains 5–13, 16–19, 21–24 | |
| es.Jdy | 0 | $A/m^2$ | Displacement current density, y component | Domains 5–13, 16–19, 21–24 | |
| es.Jdz | 0 | $A/m^2$ | Displacement current density, z component | Domains 5–13, 16–19, 21–24 | |
| es.Jdx | 0 | $A/m^2$ | Displacement current density, x component | Domains 1–4, 14–15, 20, 25 | |
| es.Jdy | 0 | $A/m^2$ | Displacement current density, y component | Domains 1–4, 14–15, 20, 25 | |
| es.Jdz | 0 | $A/m^2$ | Displacement current density, z component | Domains 1–4, 14–15, 20, 25 | |

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.normJ | sqrt(realdot(es.Jx,es.Jx)+realdot(es.Jy,es.Jy)+realdot(es.Jz,es.Jz)) | A/m² | Current density norm | Domains 5–13, 16–19, 21–24 | |
| es.normJ | sqrt(realdot(es.Jx,es.Jx)+realdot(es.Jy,es.Jy)+realdot(es.Jz,es.Jz)) | A/m² | Current density norm | Domains 1–4, 14–15, 20, 25 | |
| es.ccn1.nJ | es.unx*down(es.Jx)+es.uny*down(es.Jy)+es.unz*down(es.Jz) | A/m² | Inward current density | Boundaries 5–8, 32–33, 48, 65 | |
| es.W | es.We | J/m³ | Energy density | Domains 5–13, 16–19, 21–24 | + operation |
| es.W | es.We | J/m³ | Energy density | Domains 1–4, 14–15, 20, 25 | + operation |
| es.dWe | es.We | J/m³ | Integrand for total electric energy | Domains 5–13, 16–19, 21–24 | |
| es.dWe | es.We*ie1.detInvT | J/m³ | Integrand for total electric energy | Domains 1–4, 14–15, 20, 25 | |
| es.We | 0.5*epsilon0_const*(((es.l_sxx+es.chixx)*es.Ex+(es.l_sxy+es.chixy)*es.Ey+(es.l_sxz+es.chixz)*es.Ez)*es.Ex+((es.l_syx+es.chiyx)*es.Ex+(es.l_syy+es.chiyy)*es.Ey+(es.l_syz+es.chiyz)*es.Ez)*es.Ey+((es.l_szx+es.chizx)*es.Ex+(es.l_szy+es.chizy)*es.Ey+(es.l_szz+es.chizz)*es.Ez)*es.Ez) | J/m³ | Electric energy density | Domains 5–13, 16–19, 21–24 | |
| es.We | 0.5*epsilon0_const*(((es.l_sxx+es.chixx)*es.Ex+(es.l_sxy+es.chixy)*es.Ey+(es.l_sxz+es.chixz)*es.Ez)*es.Ex+((es.l_syx+es.chiyx)*es.Ex+(es.l_syy+es.chiyy)*es.Ey+ | J/m³ | Electric energy density | Domains 1–4, 14–15, 20, 25 | |

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| | (es.l_syz+es.chiyz)*es.Ez)*es.Ey+((es.l_szx+es.chizx)*es.Ex+(es.l_szy+es.chizy)*es.Ey+(es.l_szz+es.chizz)*es.Ez)*es.Ez) | | | | |

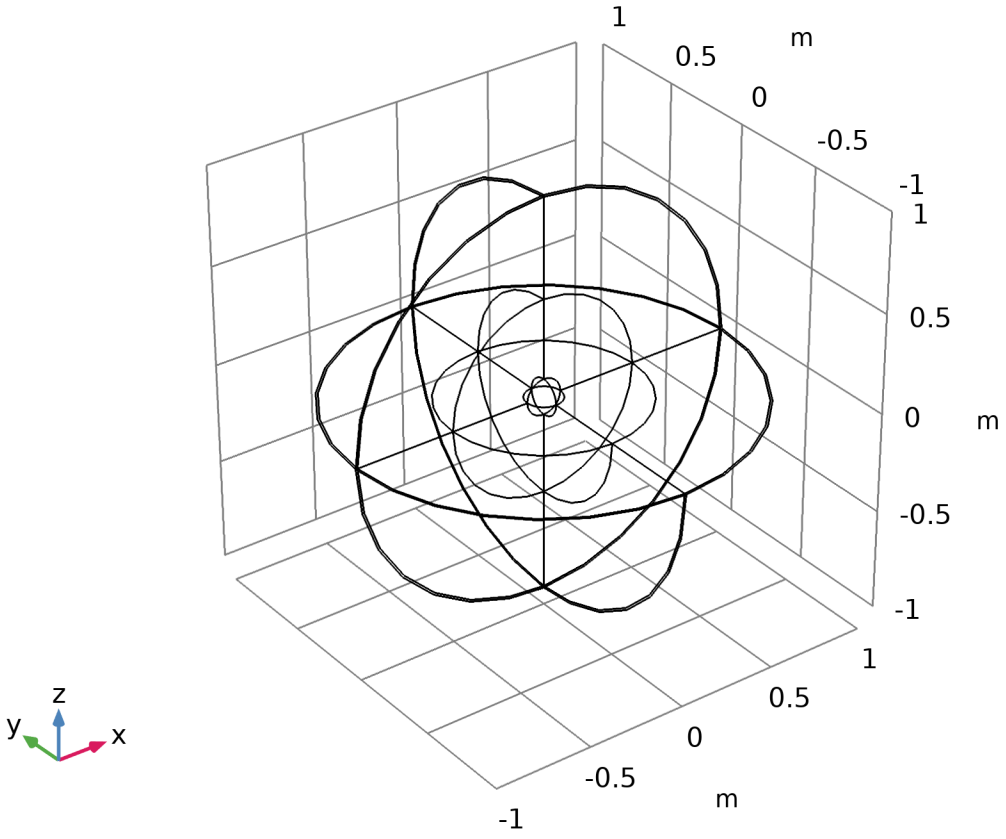
#### 2.4.3.5 Shape functions

| Name | Shape function | Unit | Description | Shape frame | Selection |
|---|---|---|---|---|---|
| V | Lagrange (Quadratic) | V | Electric potential | Spatial | Domains 5–13, 16–19, 21–24 |
| V | Lagrange (Quadratic) | V | Electric potential | Material | Domains 5–13, 16–19, 21–24 |
| V | Lagrange (Quadratic) | V | Electric potential | Geometry | Domains 5–13, 16–19, 21–24 |
| V | Lagrange (Quadratic) | V | Electric potential | Mesh | Domains 5–13, 16–19, 21–24 |
| V | Lagrange (Quadratic) | V | Electric potential | Spatial | Domains 1–4, 14–15, 20, 25 |
| V | Lagrange (Quadratic) | V | Electric potential | Material | Domains 1–4, 14–15, 20, 25 |
| V | Lagrange (Quadratic) | V | Electric potential | Geometry | Domains 1–4, 14–15, 20, 25 |
| V | Lagrange (Quadratic) | V | Electric potential | Mesh | Domains 1–4, 14–15, 20, 25 |

#### 2.4.3.6 Weak expressions

| Weak expression | Integration order | Integration frame | Selection |
|---|---|---|---|
| -(es.Dx*test(Vx)+es.Dy*test(Vy)+es.Dz*test(Vz))*es.d | 4 | Spatial | Domains 5–13, 16–19, 21–24 |
| -(es.Dx*test(ie1.T11*Vx+ie1.T12*Vy+ie1.T13*Vz)+es.Dy*test(ie1.T21*Vx+ie1.T22*Vy+ie1.T23*Vz)+es.Dz*test(ie1.T31*Vx+ie1.T32*Vy+ie1.T33*Vz))*es.d*ie1.detInvT | 4 | Spatial | Domains 1–4, 14–15, 20, 25 |

### 2.4.4 Zero Charge 1




*Zero Charge 1*

SELECTION

| | |
|---|---|
| Geometric entity level | Boundary |
| Selection | No boundaries |

EQUATIONS

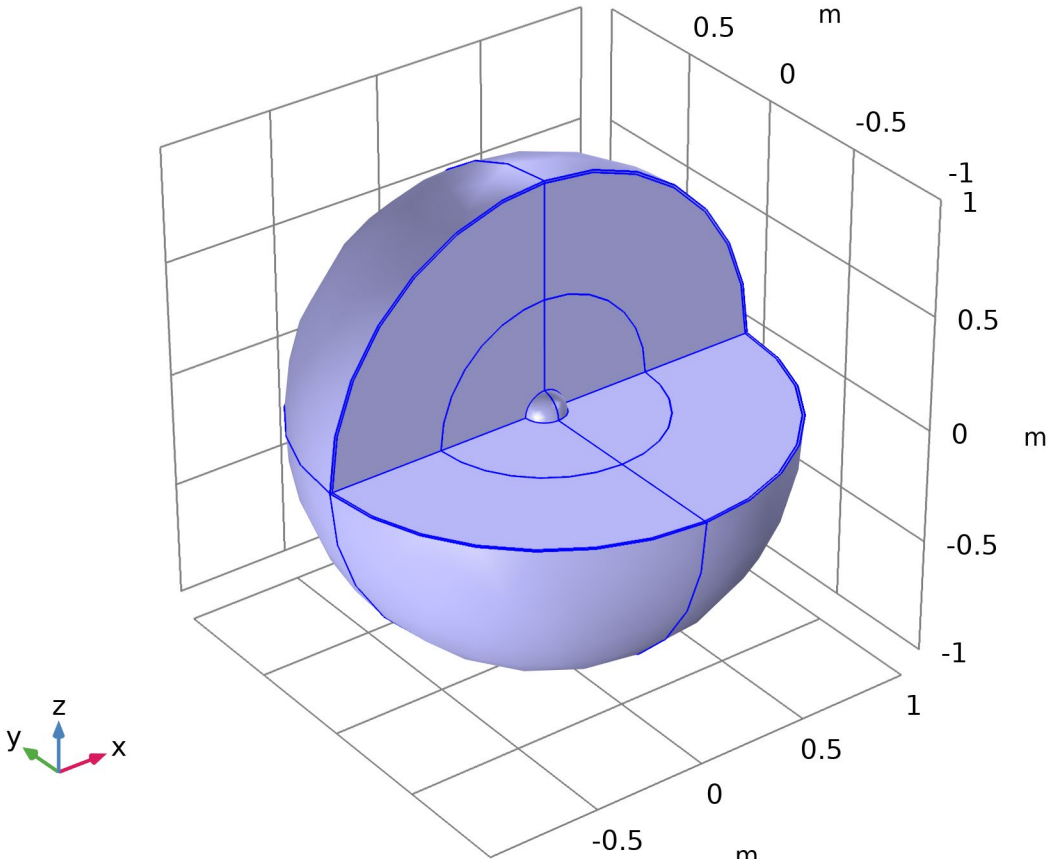
$$\mathbf{n} \cdot \mathbf{D} = 0$$

#### 2.4.4.1 Shape functions

| Name | Shape function | Unit | Description | Shape frame | Selection | Details |
|---|---|---|---|---|---|---|
| V | Lagrange (Quadratic) | V | Electric potential | Spatial | No boundaries | Slit |
| V | Lagrange (Quadratic) | V | Electric potential | Material | No boundaries | Slit |
| V | Lagrange (Quadratic) | V | Electric potential | Geometry | No boundaries | Slit |

| Name | Shape function | Unit | Description | Shape frame | Selection | Details |
|---|---|---|---|---|---|---|
| V | Lagrange (Quadratic) | V | Electric potential | Mesh | No boundaries | Slit |

### 2.4.5 Initial Values 1




*Initial Values 1*

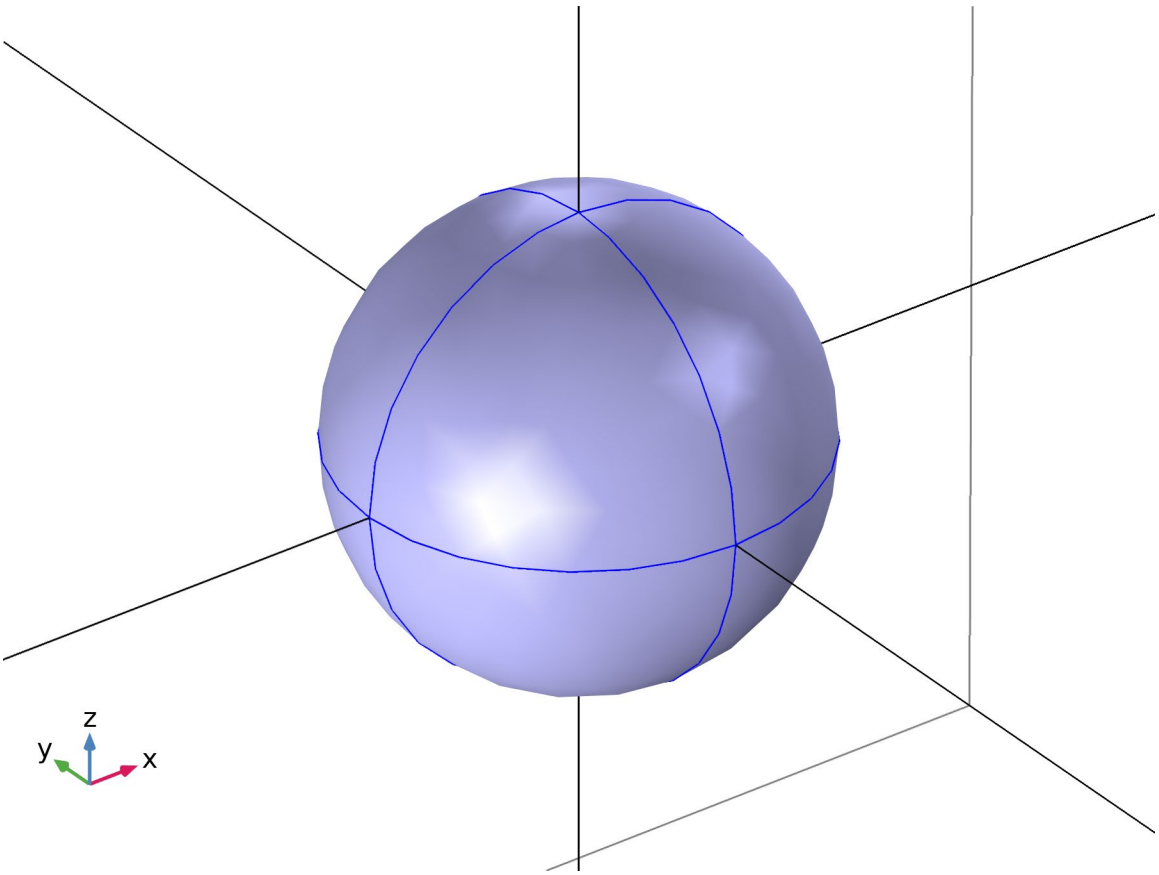
SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domains 1–25 |

SETTINGS

| Description | Value |
|---|---|
| Electric potential | 0 |

### 2.4.6 Space Charge Density 1




*Space Charge Density 1*

SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domain 13 |

EQUATIONS

$$\nabla \cdot \mathbf{D} = \underbrace{\rho_v}_{\cdots}$$

#### 2.4.6.1 Coordinate system selection

SETTINGS

| Description | Value |
|---|---|
| Coordinate system | Global coordinate system |

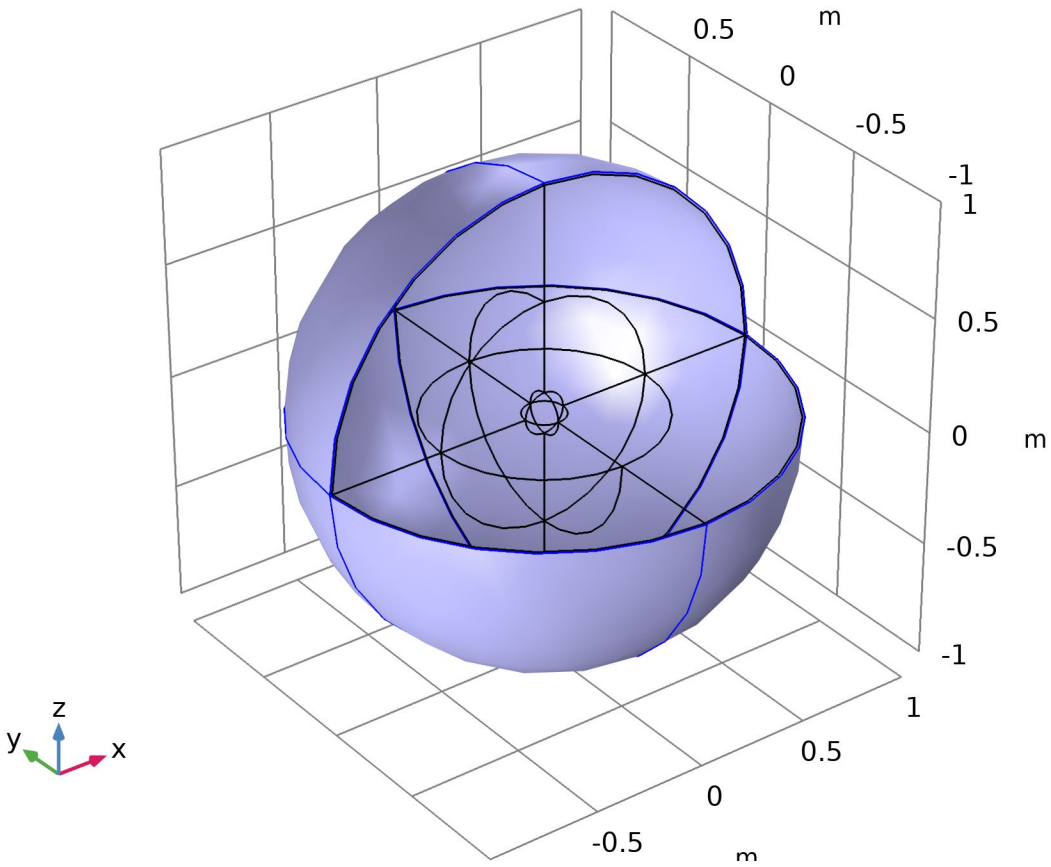
#### 2.4.6.2 Variables

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.scd1.rhoq | 1 | C/m³ | Space charge density | Domain 13 | |
| es.rhoq | es.scd1.rhoq | C/m³ | Space charge density | Domain 13 | + operation |

#### 2.4.6.3 Weak expressions

| Weak expression | Integration order | Integration frame | Selection |
|---|---|---|---|
| -es.scd1.rhoq*test(V)*es.d | 4 | Spatial | Domain 13 |

### 2.4.7 Electric Potential 1




*Electric Potential 1*

SELECTION

| | |
|---|---|
| Geometric entity level | Boundary |
| Selection | Boundaries 2, 4–8, 32–33, 48, 65 |

EQUATIONS

$$V = V_0$$

#### 2.4.7.1 Electric potential

SETTINGS

| Description | Value |
|---|---|
| Electric potential | 0 |

#### 2.4.7.2 Variables

| Name | Expression | Unit | Description | Selection | Details |
|---|---|---|---|---|---|
| es.nD | es.unx*(down(es.Dx)-up(es.Dx))+es.uny*(down(es.Dy)-up(es.Dy))+es.unz*(down(es.Dz)-up(es.Dz)) | C/m² | Surface charge density | Boundaries 2, 4 | + operation |
| es.nD | es.unx*down(es.Dx)+es.uny*down(es.Dy)+es.unz*down(es.Dz) | C/m² | Surface charge density | Boundaries 5–8, 32–33, 48, 65 | + operation |
| es.V0 | 0 | V | Electric potential | Boundaries 2, 4–8, 32–33, 48, 65 | |

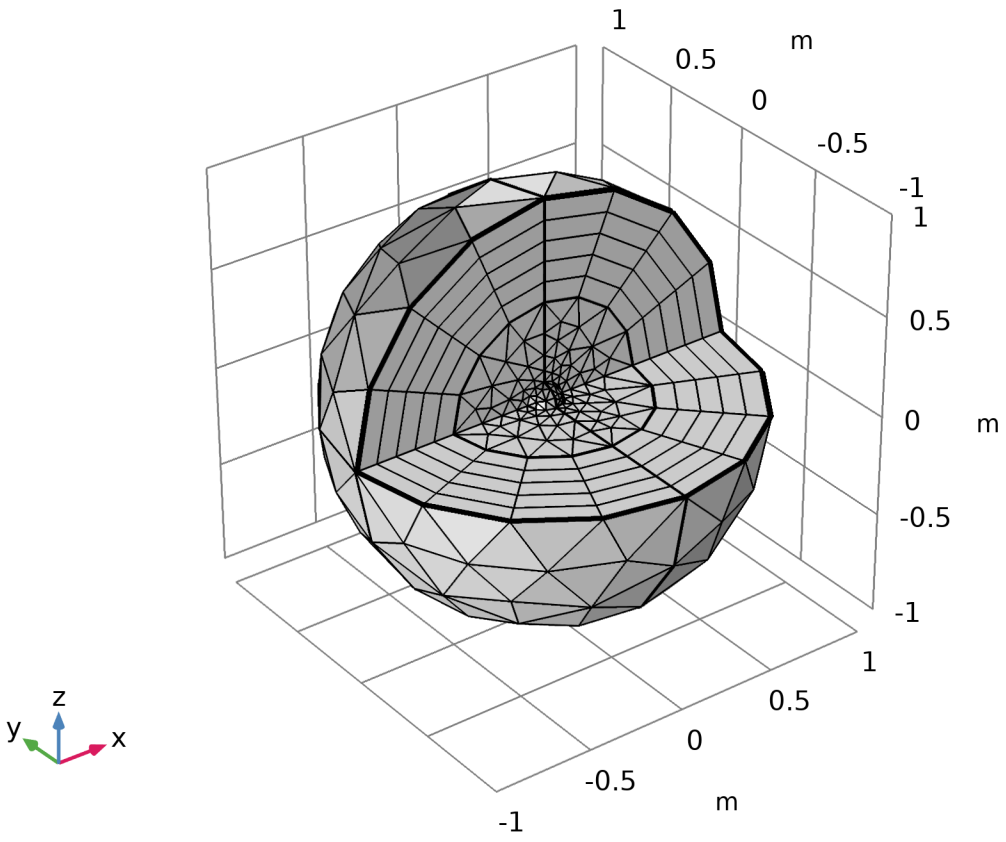
#### 2.4.7.3 Constraints

| Constraint | Constraint force | Shape function | Selection | Details |
|---|---|---|---|---|
| es.V0-V | test(es.V0-V) | Lagrange (Quadratic) | Boundaries 2, 4–8, 32–33, 48, 65 | Elemental |

## 2.5 MESH 1

MESH STATISTICS

| Description | Value |
|---|---|
| Minimum element quality | 0.01039 |
| Average element quality | 0.5195 |
| Tetrahedron | 3043 |
| Prism | 2520 |
| Triangle | 1332 |
| Quad | 480 |
| Edge element | 264 |
| Vertex element | 24 |




*Mesh 1*

### 2.5.1 Size (size)

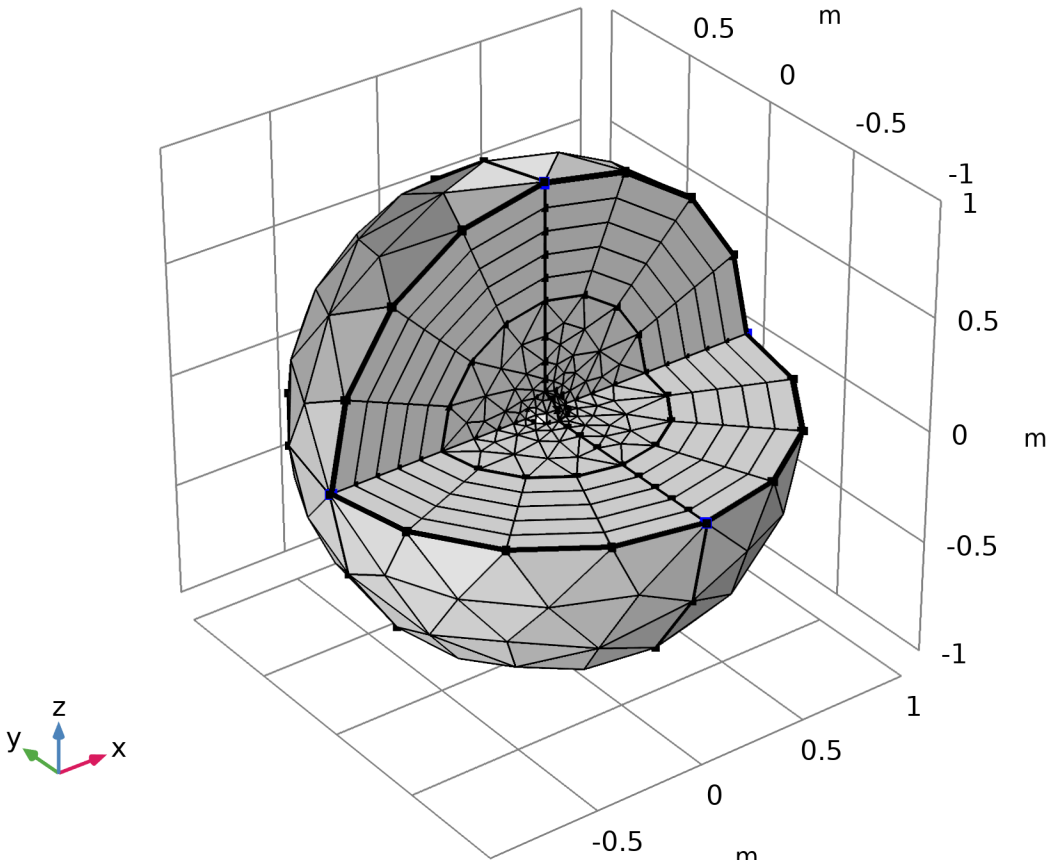
SETTINGS

| Description | Value |
|---|---|
| Maximum element size | 0.2 |
| Minimum element size | 0.036 |
| Curvature factor | 0.6 |
| Resolution of narrow regions | 0.5 |
| Maximum element growth rate | 1.5 |

### 2.5.2 Distribution 1 (dis1)

SELECTION

| | |
|---|---|
| Geometric entity level | Edge |
| Selection | Edges 1, 20, 35, 52, 61, 66 |




*Distribution 1*

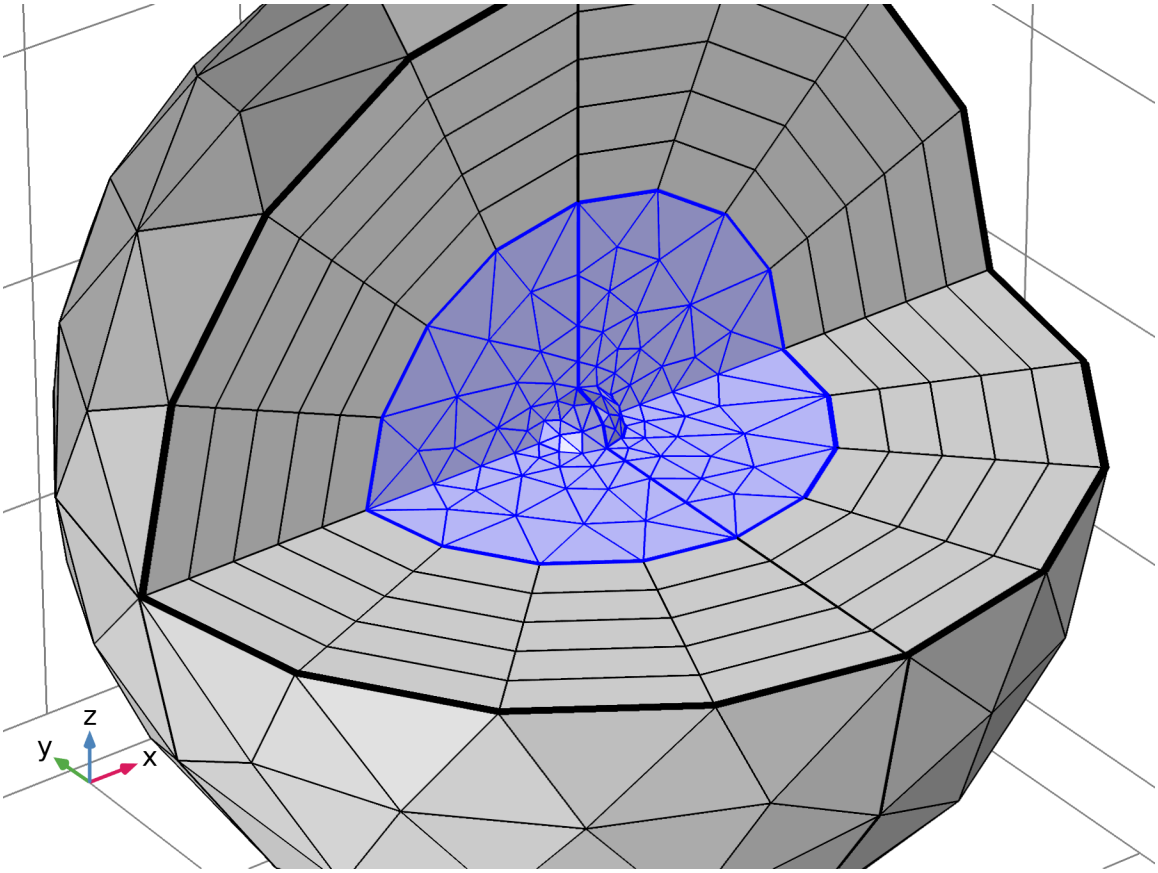
SETTINGS

| Description | Value |
|---|---|
| Distribution type | Predefined |

### 2.5.3 Free Tetrahedral 1 (ftet1)

SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domains 9–13, 18–19, 22–23 |

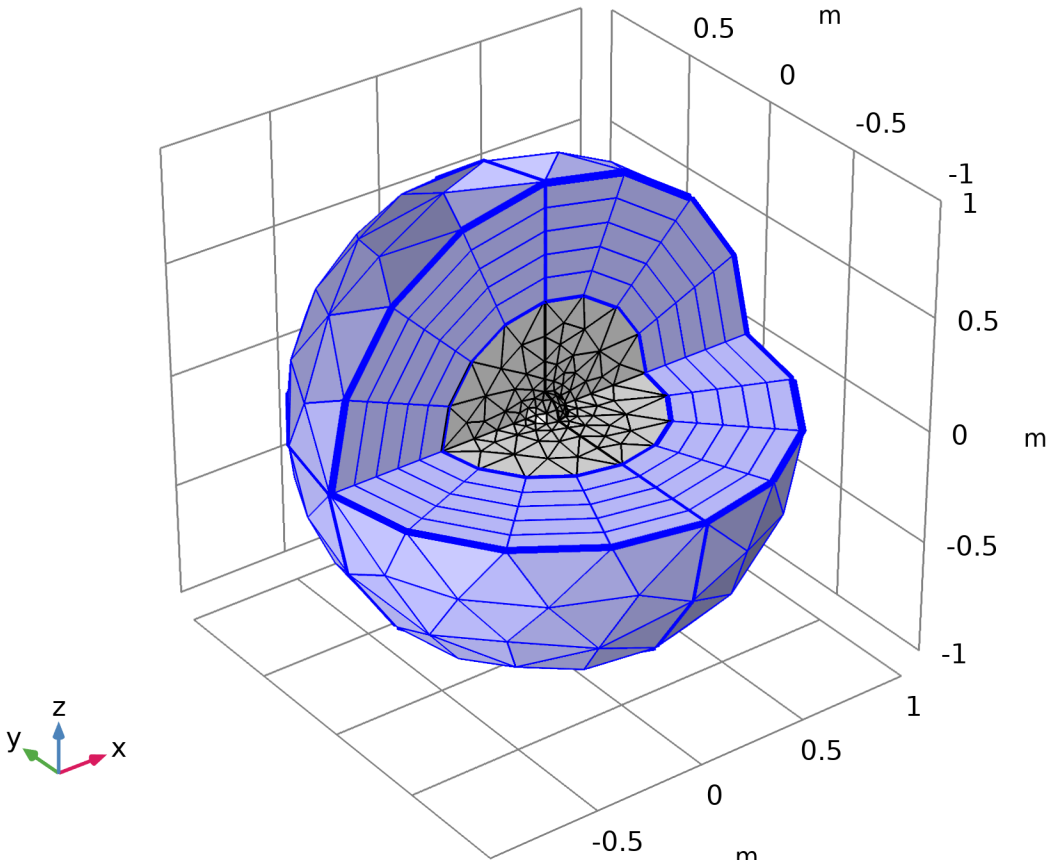



*Free Tetrahedral 1*

### 2.5.4 Swept 1 (swe1)

SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domains 1–8, 14–17, 20–21, 24–25 |

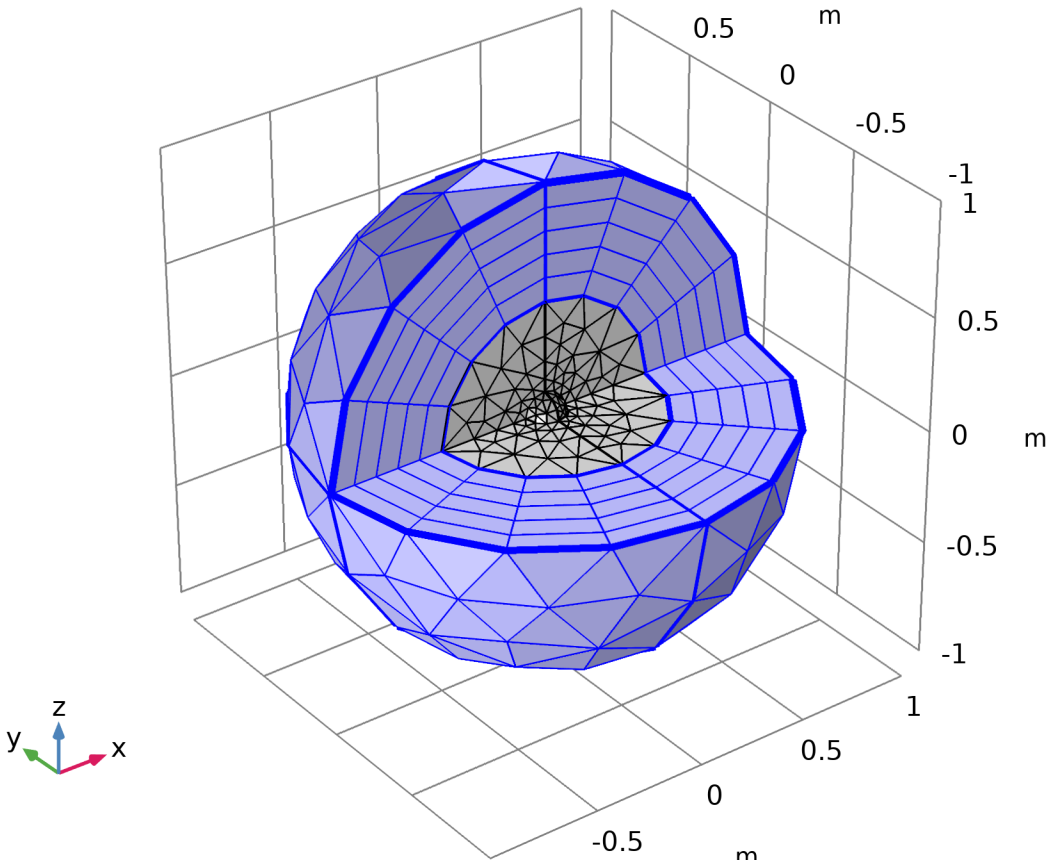



*Swept 1*

#### 2.5.4.1 Distribution 1 (dis1)

SELECTION

| | |
|---|---|
| Geometric entity level | Domain |
| Selection | Domains 1–8, 14–17, 20–21, 24–25 |




*Distribution 1*

# 3 Study 1

COMPUTATION INFORMATION

| | |
|---|---|
| Computation time | 5 s |
| CPU | Intel64 Family 6 Model 142 Stepping 10, 2 cores |
| Operating system | Windows 10 |

## 3.1 STATIONARY

STUDY SETTINGS

| Description | Value |
|---|---|
| Include geometric nonlinearity | Off |

MESH SELECTION

| Geometry | Mesh |
|---|---|
| mesh1 | mesh1 |

PHYSICS AND VARIABLES SELECTION

| Physics interface | Discretization |
|---|---|
| Electrostatics (es) | physics |

MESH SELECTION

| Geometry | Mesh |
|---|---|
| Geometry 1 (geom1) | mesh1 |

## 3.2 SOLVER CONFIGURATIONS

### 3.2.1 Solution 1

#### 3.2.1.1 Compile Equations: Stationary (st1)

STUDY AND STEP

| Description | Value |
|---|---|
| Use study | Study 1 |
| Use study step | Stationary |

#### 3.2.1.2 Dependent Variables 1 (v1)

GENERAL

| Description | Value |
|---|---|
| Defined by study step | Stationary |

#### 3.2.1.2.1 Electric potential (comp1.V) (comp1_V)

GENERAL

| Description | Value |
|---|---|
| Field components | comp1.V |

### 3.2.1.3 Stationary Solver 1 (s1)

GENERAL

| Description | Value |
|---|---|
| Defined by study step | Stationary |

#### 3.2.1.3.1 Fully Coupled 1 (fc1)

GENERAL

| Description | Value |
|---|---|
| Linear solver | Iterative 1 |

#### 3.2.1.3.2 Iterative 1 (i1)

GENERAL

| Description | Value |
|---|---|
| Solver | Conjugate gradients |

##### 3.2.1.3.2.1 Multigrid 1 (mg1)

GENERAL

| Description | Value |
|---|---|
| Solver | Algebraic multigrid |

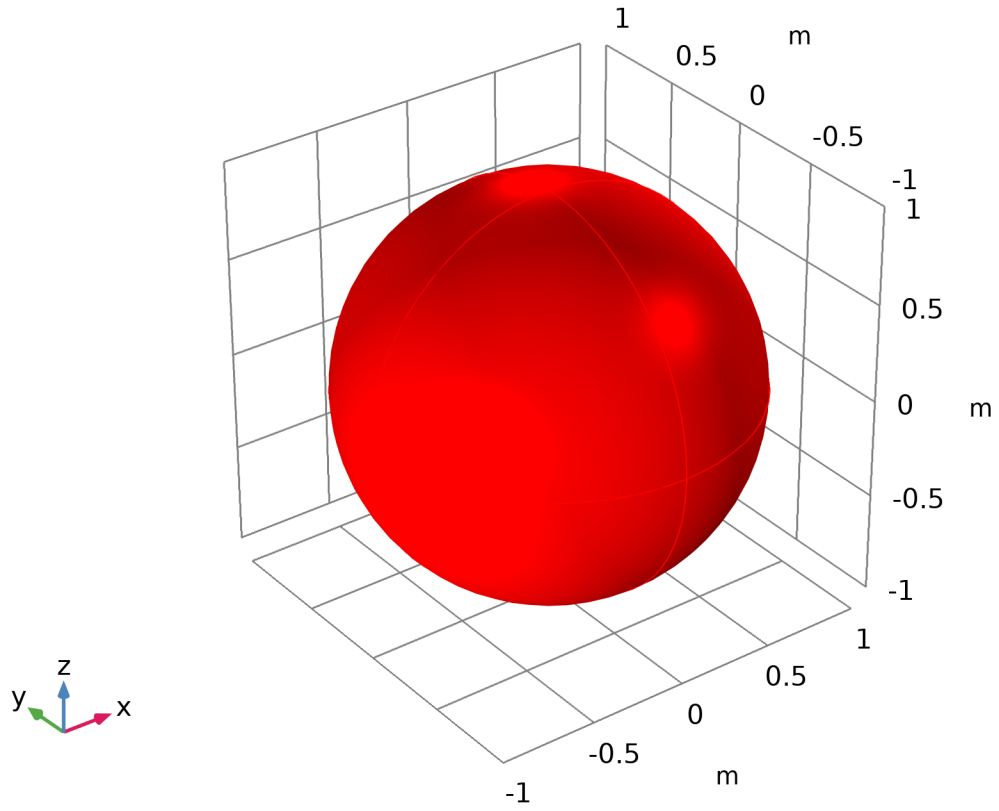
# 4 Results

## 4.1 DATA SETS

### 4.1.1 Study 1/Solution 1

SOLUTION

| Description | Value |
|---|---|
| Solution | Solution 1 |
| Component | Save Point Geometry 1 |




*Data set: Study 1/Solution 1*

### 4.1.2 Cut Line 3D 1

DATA

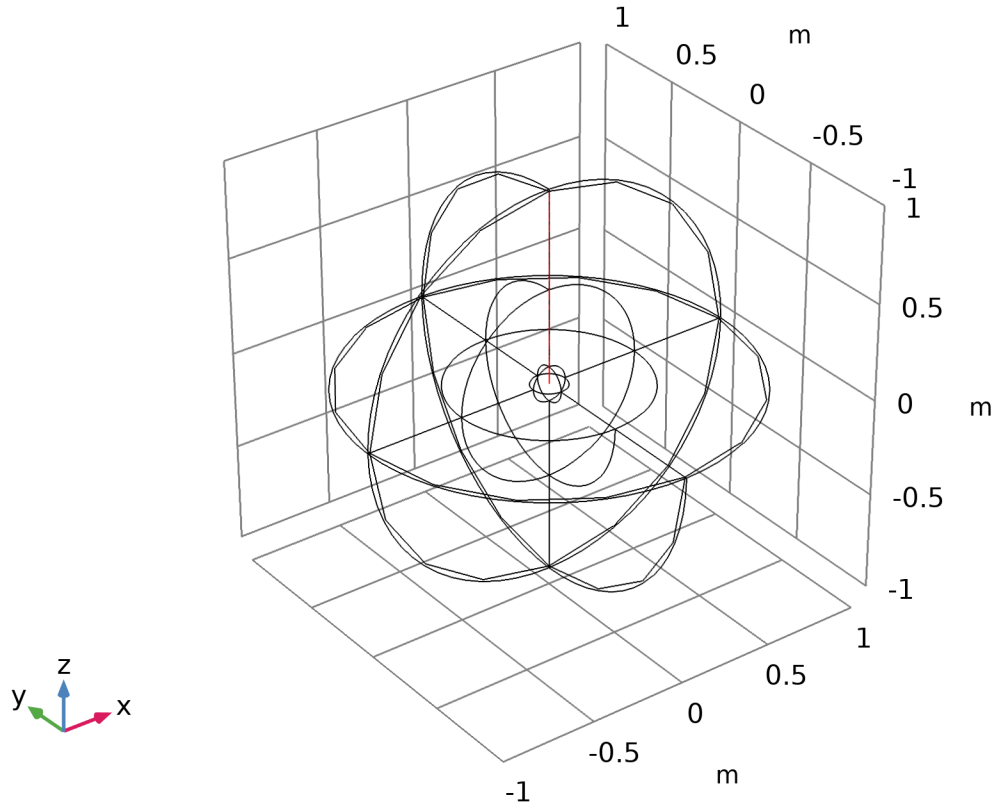
| Description | Value |
|---|---|
| Data set | Study 1/Solution 1 |

LINE DATA

| Description | Value |
| --- | --- |
| Line entry method | Two points |
| Points | {{0, 0, 0}, {0, 0, 1}} |

ADVANCED

| Description | Value |
| --- | --- |
| Space variable | cln1x |




*Data set: Cut Line 3D 1*

### 4.1.3 Cut Line 3D 2

DATA

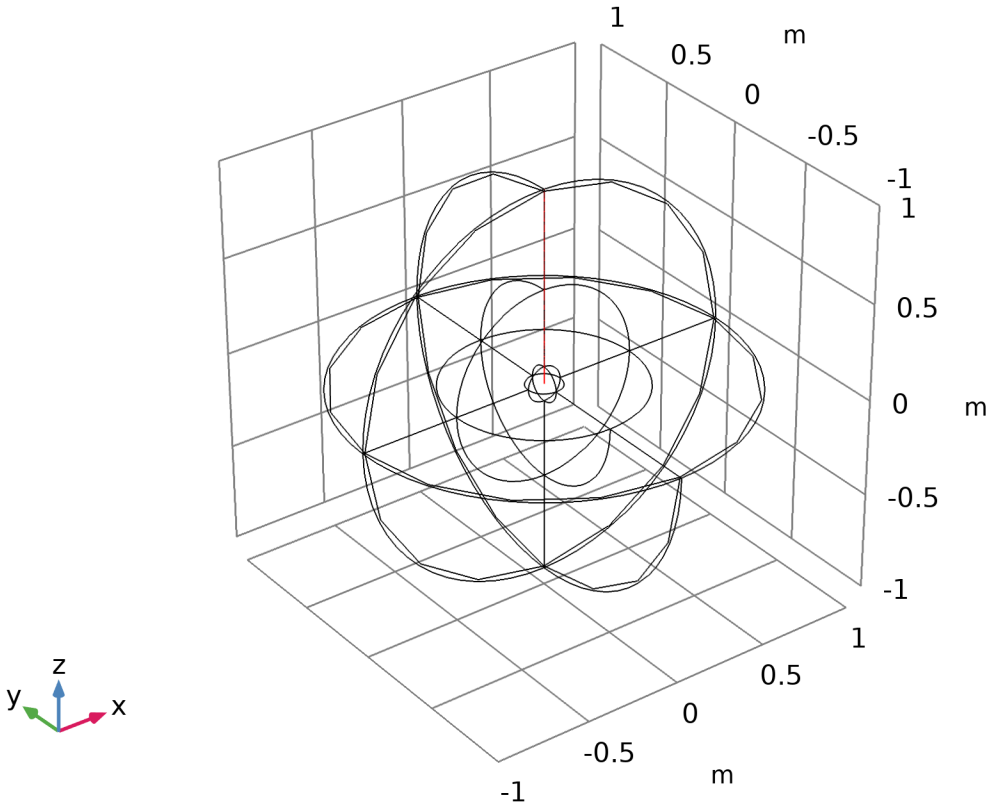
| Description | Value |
| --- | --- |
| Data set | Study 1/Solution 1 |

LINE DATA

| Description | Value |
| --- | --- |
| Line entry method | Two points |
| Points | {{0, 0, 0}, {0, 0, 1}} |

ADVANCED

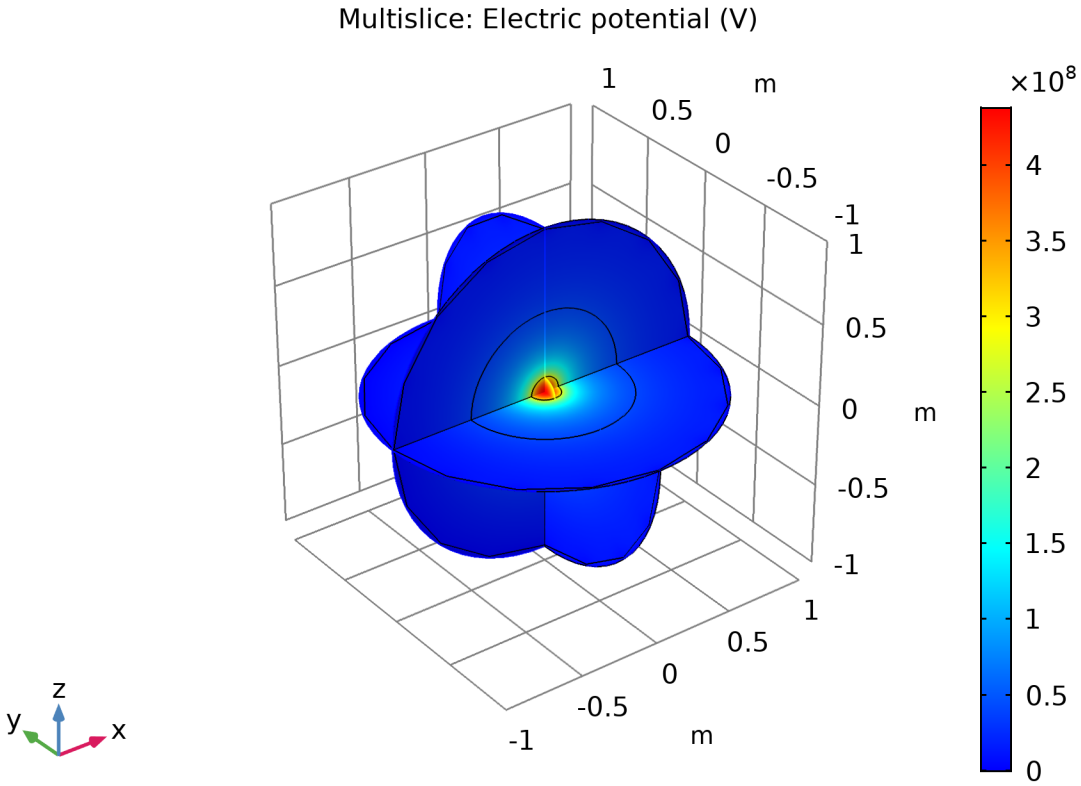
| Description | Value |
| --- | --- |
| Space variable | cln2x |




*Data set: Cut Line 3D 2*

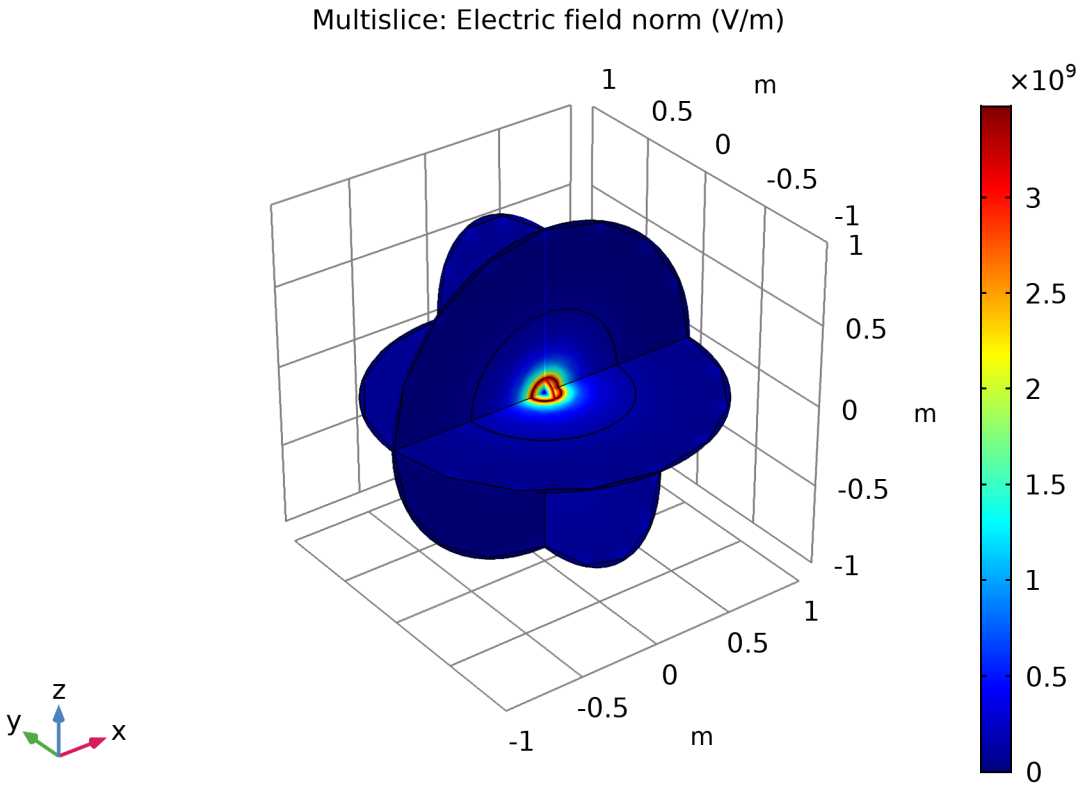
## 4.2 PLOT GROUPS

### 4.2.1 Electric Potential (es)



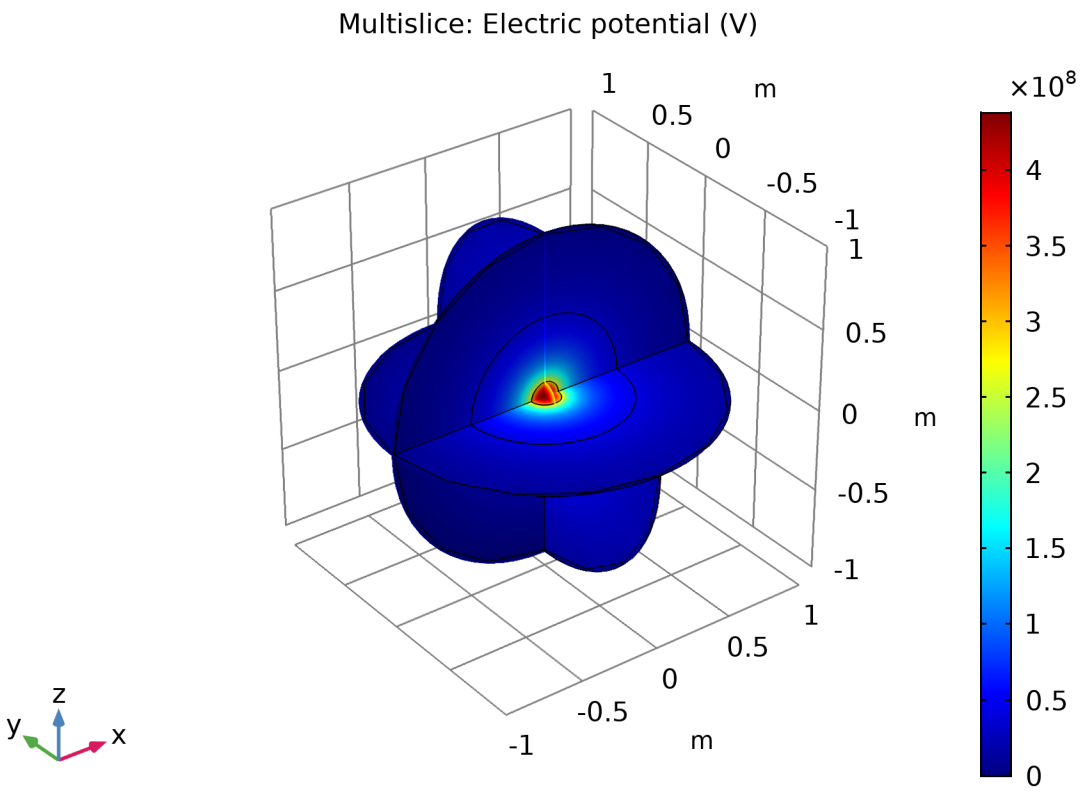

*Multislice: Electric potential (V)*

### 4.2.2 3D Electric Field



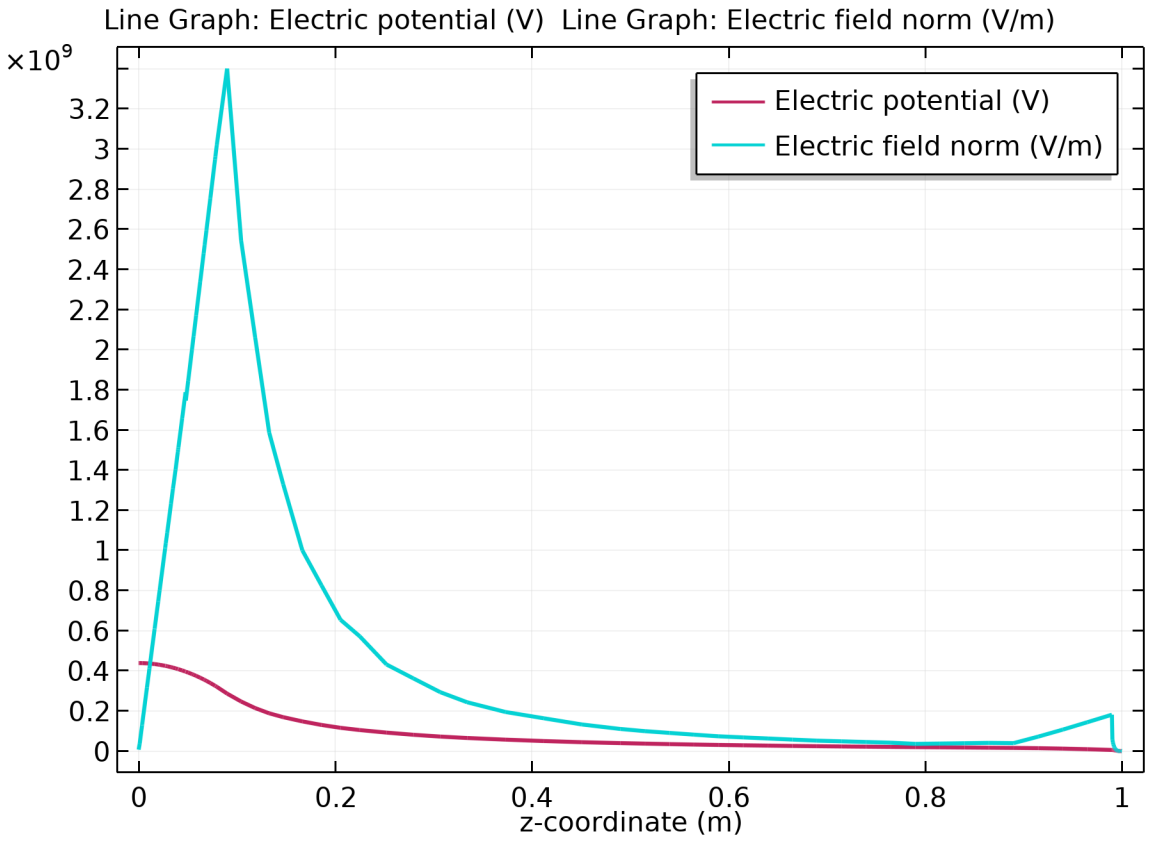

*Multislice: Electric field norm (V/m)*

### 4.2.3 3D Potential




*Multislice: Electric potential (V)*

### 4.2.4 1D Plot Group 4




*Line Graph: Electric potential (V) Line Graph: Electric field norm (V/m)*

We know for a 3D point charge

$$V = -\int_{\lim_{x\to\infty} x}^{r} \vec{E} \,.\, d\vec{x}$$

Or,

$$V = \frac{1}{4\pi\epsilon}\frac{Q}{r}$$

So, here we can see that the potential curve is having a hyperbolic downslope with respect to the z-coordinate or the distance from the point charge

Now inside the point charge the potential is always constant,

$$V = \frac{1}{4\pi\epsilon}\frac{Q}{a}$$

Where, $a$ = radius of the point charge (small 3D spherical charge).

Now we know that for electric field of a spherical charge

$$\oint \vec{E}.d\vec{s} = \frac{\rho_V * \frac{4}{3}\pi r^3}{\epsilon}$$

For inside the charged sphere,

$$E = \frac{1}{4\pi\epsilon}\frac{Q * r}{a^3}$$

For outside the charged sphere,

$$E = \frac{1}{4\pi\epsilon}\frac{Q}{r^2}$$

So, we should be having a linear curve for inside the point charge (small 3D sphere here) and a quadratically decreasing curve outside the point charge

**Boundary Conditions:**
We have set the boundary starting from the outer surface of the layer 2. This is from where the potential and electric filed become zero.

With the Poisson's equation we have,

$$\nabla^2 V = \frac{\rho_V}{\epsilon}$$

With the boundary conditions it becomes after the boundary as,

$$\nabla^2 V = 0$$

Where, $\rho_V = 0$.

Now it can be represented as,

$$\frac{\partial^2 V(x,y,z)}{\partial x^2} + \frac{\partial^2 V(x,y,z)}{\partial y^2} + \frac{\partial^2 V(x,y,z)}{\partial z^2} = 0$$

So, here we can see that the point charge has radius $a = 0.09$, and that's why starting from its periphery towards the infinite boundary limit we have the expected functions in our plot for Electric Filed and Potential respectively. At the boundary both become zero.