8 heads & 22 legs

How many people are at the park?

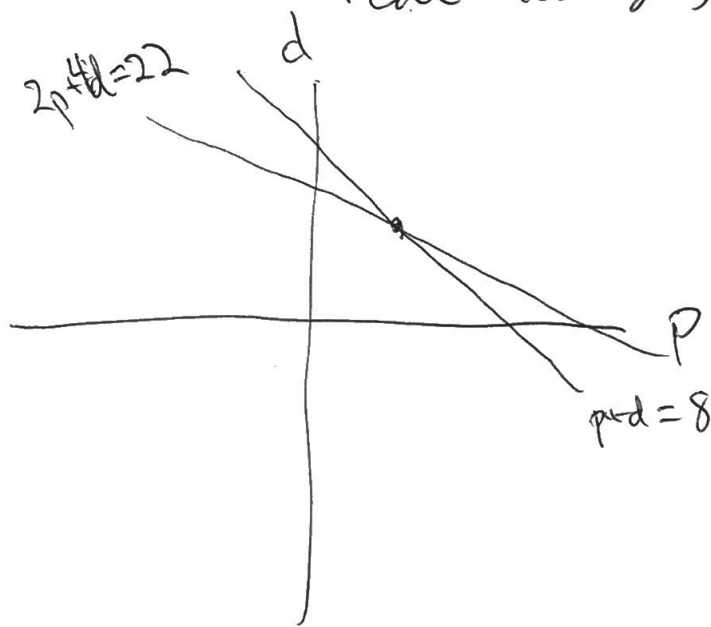$p = \#$ people ; $d = \#$ dogs

system of linear eqns
$$\begin{cases} p + d = 8 \\ 2p + 4d = 22 \end{cases}$$

linear equation

$b = a_1 x_1 + a_2 x_2 + \ldots + a_n x_n$

$a_1, \ldots, a_n, b \in \mathbb{R}$

real numbers

$$\begin{cases} p = ? \\ d = ? \end{cases}$$

$2p + 4d = 22$

d

p

$p + d = 8$

2 lines intersect at
$$\begin{cases} \text{no points} \\ \text{one point} \leftarrow \\ \text{a line} \end{cases}$$

$$\begin{cases} p + d = 8 \\ 2p + 4d = 22 \end{cases} \rightsquigarrow \begin{cases} p = ? \\ d = ? \end{cases}$$

$$\downarrow$$

$$\begin{cases} p + d = 8 \\ 2d = 6 \end{cases}$$

$$\begin{aligned} 2p + 4d &= 22 \\ -2(p + d &= 8) \end{aligned}$$
$$\overline{}$$
$$2d = 6$$

$$\downarrow$$

$$\begin{cases} p = 5 \\ 2d = 6 \end{cases}$$

$$\begin{aligned} p + d &= 8 \\ -\tfrac{1}{2}(2d &= 6) \end{aligned}$$
$$\overline{}$$
$$p = 5$$

$$\boxed{\begin{cases} p = 5 \\ d = 3 \end{cases}}$$

solution set

Def : A system of linear equ's is <u>consistent</u> if there is a solution to it.

$$\tfrac{1}{a}(ax = b) \implies \qquad a \neq 0.$$

$$\boxed{x = \tfrac{b}{a}}$$

$$0(ax = b)$$

$$0 = 0.$$

$ax = b$ and $x = \tfrac{b}{a}$ have the same solution set.

Multiplication by $\tfrac{1}{a}$ is <u>reversible</u>.

$$\begin{cases} p+d=8 \\ 2p+4d=22 \end{cases} \quad \longleftrightarrow \quad \begin{cases} p+d=8 \\ 2d=6 \end{cases}$$

$$\begin{array}{l} \text{a} \quad 2p+4d=22 \\ \quad -2(p+d=8) \\ \hline \qquad\qquad 2d=6 \end{array} \qquad\qquad \begin{array}{l} 2d=6 \\ 2(p+d=8) \\ \hline 2p+4d=22 \end{array}$$

These systems of linear eqns are equivalent, ie they have the same solution set.

Matrix notation

$$\begin{cases} p+d=8 \\ 2p+4d=22 \end{cases}$$

$$\begin{array}{cc} p & d \end{array}$$

$$\left[\left[\begin{array}{cc} 1 & 1 \\ 2 & 4 \end{array}\right] \,\middle|\, \begin{array}{c} 8 \\ 22 \end{array}\right] \quad \text{augmented matrix}$$

← coefficient matrix

2×3 matrix
#rows #cols

$$\left[\begin{array}{cc|c} 1 & 1 & 8 \\ 2 & 4 & 22 \end{array}\right] \xrightarrow[R2 \mapsto R2+2R1]{R2 \mapsto (R2-2R1)} \left[\begin{array}{cc|c} 1 & 1 & 8 \\ 0 & 2 & 6 \end{array}\right]$$

$$R1 \mapsto R1+\tfrac{1}{2}R2 \Big\} \Big\{ R1 \mapsto (R1-\tfrac{1}{2}\cdot R2)$$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \end{array}\right] \xleftarrow[R2 \mapsto 2R2]{R2 \mapsto \tfrac{1}{2}R2} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 2 & 6 \end{array}\right]$$

elementary row operations.