it is natural to consider first the question of homotopy classification of such maps. This is still a difficult problem whose approach requires certain technical considerations of homotopy theory occuping many pages of our paper. The starting point of our calculations of homotopy classes of $*$-homomorphisms is a deep result of G. Segal [39] concerning the realization of the connective BO-spectrum as a sequence of spaces of $*$-homomorphisms. In fact we use the complex version of this result, which, as pointed out by J. Rosenberg [36], is also relevant for the computation of the stable homotopy groups of commutative C^*-algebras. Building on this circle of ideas we develop in section 3 some algebraic topology techniques in order to compute the homotopy classes of $*$-homomorphisms $C_o(X) \to C_o(Y)\otimes\mathcal{K}$. It turns out that

$$kk(Y,X) := [C_o(X), C_o(Y)\otimes\mathcal{K}] = \lim_{k} [C_o(X), C_o(Y)\otimes M_k]$$

has a natural structure of abelian group with addition induced by the orthogonal sum of the homomorphisms. Moreover $kk(Y,X)$ has good excision properties in both variables and this allows us to define the groups $kk_n(Y,X)$ yielding a generalized homology-cohomology theory which can be regarded as the connective theory associated with the Kasparov KK-theory when the later is restricted to spaces. There is an obvious product structure on kk_n induced by the composition of homomorphisms which enables one to make many explicit computations. For instance if X and Y are torsion free spaces, then $kk(Y,X)$ can be completely computed in terms of those group homomorphisms $H^*(X,\mathbf{Z}) \to H^*(Y,\mathbf{Z})$ which preserve some natural filtrations reminding of ciclic homology. The connection with K-theory is made with the aid of the natural map $kk(Y,X) \to KK(C_o(X), C_o(Y))$ which turns out to be an isomorphism provided that X and Y are $(n-2)$-connected finite CW-complexes of dimension $\leq n$.

The results of section 3 become useful for our concrete purposes only after we know that there is a sequence $\nu(m)$ of integers which tends to infinity, such that the natural embedding

$$\mathbf{Hom}(C_o(X), M_m) \to \mathbf{Hom}(C_o(X), M_{m+1})$$

is a $\nu(m)$-homotopy equivalence. (in fact one can take $\nu(m) = 2[(m/3)]$).