1. $N_k = \sum_{n=1}^{N} \gamma_k^{(n)}$

$\mu_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_k^{(n)} x^{(n)}$

$\Sigma_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_k^{(n)} (x^{(n)} - \mu_k)(x^{(n)} - \mu_k)^T$

$\pi_k = \frac{N_k}{N}$

2. (1) Precision: TP / (TP+FP)

Recall: TP / (TP+FN)

ROC: When TPR-curve plotted at y-axis TPR = TP/(TP+FN) = TP/P

FPR-curve plotted at x-axis. FPR = FP/(FP+TN) = FP/N

AUC: Area under ROC.

(2) PR is effected more.

Though uneven, TPR and FPR would not change (still TP/P and FP/N)

However, Precision of PR could be effected.

If N grows, FP would also grow, causing TP/(TP+FP) become smaller.

And Recall = TP/P remains unchanged

=> PR would be effected more.

3. Procudures are using python.

```python
import numpy as np

x=np.array([[1,0,2,-3,-2],
            [0,1,-3,-2,-3],
            [1,2,1,3,-2],
            [-1,1,2,3,-1],
            [1,0,1,-1,1],
            [2,3,-1,1,-2],
            [-2,3,-3,2,3],
            [-2,-2,2,3,-2],
            [-2,-2,1,-3,-3],
            [-3,2,0,-1,-2]])

cov=np.cov(x.T) #calculating covariance matrix
eigen_values,eigen_vectors=np.linalg.eig(cov) #calculating eigen values&vect

sorted_indices=np.argsort(eigen_values)[::-1]
sorted_eigen_values=eigen_values[sorted_indices]
sorted_eigen_vectors=eigen_vectors[:,sorted_indices] #sort eigen vectors by
print("eigen_vectors=",sorted_eigen_vectors)

k=sorted_eigen_vectors[:,:2] # slicing first two lines
ans=np.dot(x,k) #x dot k
print("ans=",ans)
```

```
问题  输出  调试控制台  终端  端口  JUPYTER  SQL CONSOLE

PS C:\Users\Mechrevo\Desktop\3020> & C:/Users/Mechrevo/AppData/Local/Programs/Python/Python311/pythc
eigen_vectors= [[ 0.00947335  0.12816499  0.86657302  0.35316831 -0.32834028]
 [ 0.46377479  0.38603495  0.28300348 -0.17268194  0.72524475]
 [-0.27613549 -0.67533319  0.16695994  0.36038392  0.55670647]
 [ 0.71046349 -0.60066664  0.09886262 -0.2628404  -0.23575903]
 [ 0.45145766  0.13294897 -0.36236831  0.80404692 -0.02661412]]
ans= [[-3.57710343  0.3136006 ]
 [-1.48311868  3.21452088]
 [ 1.88936261 -1.84299617]
 [ 1.58196327 -3.02774531]
 [-0.52566798  0.18644742]
 [ 1.49395475  1.22320342]
 [ 4.9760841   2.12528804]
 [-0.2702921  -4.44696411]
 [-4.70839522 -0.30058004]
 [-0.71424928  0.72234362]]
```