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I. Problem 2.2 (AFT Tests))

2.2.1 Express the four items above in the form of conditional probabilities

We have:

$$\begin{aligned} P(Down|F = Age_{25})) &= 1/1250 \\ &= 0.0008 \\ P(Down|F = Age_{43})) &= 1/50 \\ &= 0.02 \\ P(Pos|\neg Down) &= 1/100 \\ &= 0.01 \\ P(\neg Pos|Down) &= 1/100 \\ &= 0.01 \end{aligned}$$

2.2.2 Using Bayes' theorem, express and compute the probability that their child has Down syndrome,

$$\begin{aligned} P(Down|Pos) &= \frac{P(Pos|Down) * P(Down)}{P(Pos)} \\ &= \frac{(1 - P(Pos|\neg Down)) * P(Down)}{P(Pos)} \\ &= \frac{(1 - P(Pos|\neg Down)) * P(Down)}{P(Pos \wedge Down) + P(Pos \wedge \neg Down)} \\ &= \frac{(1 - P(Pos|\neg Down)) * P(Down)}{P(Pos|Down) * P(Down) + P(Pos|\neg Down) * P(\neg Down)} \\ &= \frac{(1 - P(Pos|\neg Down)) * P(Down)}{(1 - P(\neg Pos|Down)) * P(Down) + P(Pos|\neg Down) * P(\neg Down)} \\ &= \frac{(1 - P(Pos|\neg Down)) * P(Down)}{(1 - P(\neg Pos|Down)) * P(Down) + P(Pos|\neg Down) * (1 - P(Down)} \\ &= \frac{(1 - 0.01) * (0.0008)}{(1 - 0.01) * (0.0008) + 0.01 * (1 - 0.0008)} \\ &= 0.07344 \\ &= 7,344\% \end{aligned}$$

The result shows that for the pregnant mother aged 25, if the test return positive, there is only 7.344 % chance that her child actually has down syndrome.