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I. Problem 1.4 (Problem 1.4 (Chained Production Elements))

1.0 The probability of the apparatus works when at least AB , CD , or EF work

From the given information, we know that $P(apparatusworks)$ when A and B work, or C and D work, or E and F work that mean $P(apparatusworks) = P(AB \vee CD \vee EF)$. From the problem 1.2 we know that:
$P(AB \vee CD \vee EF) = P(AB) + P(CD) + P(EF) - P(AB \wedge CD) - P(AB \wedge EF) - P(BC \wedge EF) + P(AB \wedge CD \wedge EF) \quad (1)$

Since the events are stochastically independent, the probability that element X does not break down is $1 - P(bX)$. Therefore, the probability that A and B are operational is:

$$\begin{aligned} P(AB) &= P(A) \times P(B) \\ &= (1 - 0.05) \times (1 - 0.1) \\ &= 0.855 \quad (2) \end{aligned}$$

Similarly, the probabilities that C and D, and E and F are operational are:

$$\begin{aligned} P(CD) &= P(C) \times P(D) \\ &= (1 - 0.15) \times (1 - 0.2) \\ &= 0.68 \quad (3) \\ P(EF) &= P(E) \times P(F) \\ &= (1 - 0.25) \times (1 - 0.3) \\ &= 0.525 \quad (4) \end{aligned}$$

Substituting equations (2), (3), and (4) into equation (1), we have:

$$P(AB \vee CD \vee EF) = 0.855 + 0.68 + 0.525 - (0.855 \times 0.68 + 0.855 \times 0.525 + 0.68 \times 0.525)$$
$$+(0.855 \times 0.68 \times 0.525) = 0.97796$$

Hence, probability that the apparatus works is 97.796

2.0 The probability of the apparatus works

Let's consider each pair of linked elements:

Because the second question does not explicitly provide the condition when the apparatus works, then I assume that the condition for apparatus working is like the first question (at least A and B are operational, C and D are operational, or E and F are operational). But now A and C, D and F and B and E are pairwise linked; such that if either of them breaks down, then the linked element is not operational either.

The apparatus works when A and B are operational, A depending on C, B depending on . Then if A, B, C, and D are operational then the apparatus works:

$$P(A \wedge B \wedge C \wedge D) = (1 - 0.05) \times (1 - 0.1) \times (1 - 0.15) \times (1 - 0.25) = 0.545 \quad (5)$$

Similarly, we have:

$$P(C \wedge D \wedge A \wedge F) = (1 - 0.15) \times (1 - 0.2) \times (1 - 0.05) \times (1 - 0.3) = 0.452 \quad (6)$$

$$P(E \wedge F \wedge B \wedge D) = (1 - 0.25) \times (1 - 0.3) \times (1 - 0.1) \times (1 - 0.2) = 0.378 \quad (7)$$

The apparatus work when (5), (6) or (7) is true, then:

$$\begin{aligned} P(ABCD \vee CDAF \vee EFBD) &= P(ABCD) + P(CDAF) + P(EFBD) \\ &- (P(ABCD \wedge CDAF) + P(ABCD \wedge EFBD) + P(EFBD \wedge CDAF)) \\ &+ P(ABCD \wedge CDAF \wedge EFBD) \quad (8) \end{aligned}$$

Substituting equations (5), (6), and (7) into equation (8), we have:

$$\begin{aligned} P(ABCD \vee CDAF \vee EFBD) &= 0.545 + 0.452 + 0.378 \\ &- (0.545 \times 0.452 + 0.545 \times 0.378 + 0.452 \times 0.378) \\ &+ (0.545 \times 0.452 \times 0.378) \\ &= 0.84491052 \end{aligned}$$

Hence, the probability of the apparatus that works in this scenario is 84,49%