

The VRPTW is the same problem that VRP with the additional restriction that in VRPTW a time window is associated with each customer $v \in V$, defining an interval $[e_0, l_0]$ wherein the customer has to be supplied. The interval $[e_0, l_0]$ at the depot is called the scheduling horizon. Here is a formal description of the problem:

The objective is to minimize the vehicle fleet and the sum of travel time and waiting time needed to supply all customers in their required hours.

Let b_0 denote the beginning of service at customer v . Now for a route $R_i = (v_0, v_1, \dots, v_m, v_{m+1})$ to be feasible it must additionally hold $e_{v_i} \leq b_{v_i} \leq l_{v_i}$, $1 \leq i \leq m$, and $b_{v_m} + \delta_{v_m} + cv_{m,0} \leq l_0$. Provided that a vehicle travels to the next customer as soon as it has finished service at the current customer, b_{v_i} can be recursively computed as $b_{v_i} = \max(e_{v_i}, b_{v_{i-1}} + \delta_{v_{i-1}} + c_{v_{i-1},v_i})$ with $b_0 = e_0$ and $\delta_0 = 0$. Thus, a waiting time $w_{v_i} = \max(0, b_{v_i} - b_{v_{i-1}} - \delta_{v_{i-1}} - c_{v_{i-1},v_i})$ may be induced at customer v_i . The cost of route i is now given by $C_{VRPTW}(R_i) = \sum_{i=0}^m c_{i,i+1} + \sum_{i=1}^m \delta_i + \sum_{i=0}^m w_{v_i}$. For a solution S with routes R_1, \dots, R_m , the cost of S is given by $F_{VRPTW}(S) = \sum_{i=1}^m (C_{VRPTW}(R_i) + M)$, where M is a large constant. M is added because minimization of the fleet size is considered to be the primary objective of the VRPTW. S is said to be feasible if all routes belonging to S are feasible and its customer is served by exactly one route. As described by Solomon [Solomon 1995], we assume that initially all vehicles leave the depot at the earliest possible time e_0 . Having obtained a solution of the VRPTW, we adjust the depot departure time of each vehicle to eliminate any unnecessary waiting time.