



Université Montpellier 1

Analysis of the votes transfer mechanism in between first and second tour of the 2012 French presidential election : A non linear estimation of a transition matrix

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1) Introduction

For the last two decades many studies have been conducted to analyze voting behavior. Many models arose, all of them attempted to explain voting behavior by social, religious or economical background. (Anthony Downs, an economic theory of democracy). Although at the very beginning those studies were mainly literary, soon, economists and statisticians stepped in to impose quantitative models and mathematics to this ground of study.

Although descriptive statistics are often used nowadays in the media to attempt to clarify voting behaviors (often trying to match a geographic analysis with the global economical background of each district), these are superficial if we want to deeply analyze a voting behavior. We indeed cannot narrow this kind of investigation to a single mean or in the best case coupled with a standard error interpretation. That is why some authors tried to overcome those superficial and scarce analysis and comments, using more accurate econometric methods (Gerald H. Cramer "short term fluctuation in U.S. voting").

The purpose in this essay is not to explain a voting behavior according to economical background but to explain how votes are correlated between the first and second tour of 2012 French presidential election. The model is going to be more furnished than the ones performed in the U.S. due to the French multi-party political structure.

The last elections in France showed a rise in the extreme votes (the mean score of Front National and Extreme-left party represented overall more than 30% of votes).

These voters are defined by a high volatility, and impose the question of who will benefit their vote during the second tour? The socialist Mr. Hollande or the conservative Mr. Sarkozy, or none of them?

It is commonly admitted that Communists massively run for the socialist candidate and that extreme right drift to the conservative candidate. But the rising power of the Front National (FN) has led its leader Ms Le Pen not to give "consigne de vote" to her electors, whereas the communists clearly called for their partisans to join in voting for Mr. Hollande.

So if the question we are having in this essay is how votes are globally transferred, we will tend to focus our analysis on the voting behavior of the extreme right voters.

We therefore want to clarify a commonly accepted idea that voters behave like sheep and follow the orders of their first choice candidate, or that they automatically transfer their votes to the candidate who is historically the closest to their first choice candidate. This would also shed light on another question which is, do voters need a party to situate themselves on the political field or can they be considered as educated citizens able to make their own rational choices according to their very own beliefs.

We want to justify the uses of econometric methods instead of a simple poll study for many reasons. First of all, using ministerial data is a rather good accurate method, because we use revealed preferences of voters. Indeed voters tend not to vote as they declared, out of shame, or because of a poor partisan feeling and fidelity. (For further details on the French specificities on this topic read ("Le nouveau désordre electoral")) The second reason is that we are uncertain as to whether we have a representative sample of the population or not.

To finish, high costs of survey analysis can be a threshold for little candidates or local elections.

We now want to have a few words on the econometrical methods we are going to use and the problems that may occur.

We are here, trying to estimate a transition matrix, which can be represented as followed

	Vert	FN	FG	NPA	LO	Modem	NDA	Cheminade	Abstent
PS									
UMP									
Abstent									

As we included abstention, all votes must transfer which means that the sum of all the parameters must equal 1. This is obviously not going to be the case if we do not constrain parameters, if we use a naïve OLS model or if we don't use a non linear estimator. But we will further develop this aspect in the third section "model".

Now we want to have a clearer view of the French political panorama using descriptive statistics.

II) Descriptive statistics

A) The data

We now want to take a closer look into the data we use in our model. All of them are given by the "ministère de l'intérieur".

We collected in every district the raw number of votes for each candidate plus the abstentions (did not vote) and nulls (blank ballot), the last two that we gathered under the term of non expressed votes.

We also reported the number of potential voters (people written on the voting record). Thus if we sum the raw amount of votes for each candidate plus the non expressed ones we obtain the amount of people written on voting list.

In order to not overweigh the larger districts we are not using the raw data.

We divided the number of votes for each candidate by the number of total potential voters.

Formerly we can write

$$W_i = \frac{\text{Vote } i}{\sum_i \text{votes } i}$$

Where i : candidate i

Obviously we can see that $\sum_i W_i = 1$

This condition is well known under the name of adding up condition and is famously use in many models like in the famous QUAIDS model.

These operations are repeated both for first and second tour results.

B) Correlation matrix and poles of repulsion

Analyzing a correlation matrix we expect to draw the most obvious poles of repulsion as defined in “le nouveau désordre electoral”. In other words we look for first tour (where all political parties are still represented), if the vote for one sensibility statistically excludes voting for another.

As an example, we can intuitively think that for someone voting for the communist party it is rather difficult to secondly drift to a party focusing on nationalism. By doing so we want to confirm or infirm the initial and naïve idea we may have on the subject. For every party we will have an initial idea of how their partisans will vote during the second tour, or at least where they will not orient themselves.

	vert	fn	ump	fg	npa	lo	chem	modem	nda	ps1	abst1
vert	1.0000										
fn	-0.5140	1.0000									
ump	0.2208	-0.0217	1.0000								
fg	0.2841	-0.2485	-0.7050	1.0000							
npa	-0.0925	0.0218	-0.3549	0.1777	1.0000						
lo	-0.4252	0.2596	-0.2608	-0.1053	0.6004	1.0000					
chem	0.2382	-0.0250	0.1988	-0.1497	0.2770	0.3415	1.0000				
modem	0.4072	-0.5342	0.3102	-0.1503	0.3786	0.2115	0.4462	1.0000			
nda	-0.1596	0.0858	0.1186	-0.2282	0.3578	0.5535	0.4787	0.3865	1.0000		
ps1	0.0919	-0.6386	-0.6107	0.5623	0.2570	0.0384	-0.1872	0.1176	-0.1277	1.0000	
abst1	-0.2836	0.1666	0.0488	-0.2991	-0.5221	-0.2791	-0.2610	-0.5157	-0.3183	-0.3830	1.0000

We have here some striking information.

First of all we are going to focus on the electors of the UMP (conservatives). We can see that their strongest dislike goes to the communist party (correlation coefficient = -0.70). Indeed the different conservative leaders argued that the socialists were allied with what they call the extreme left, represented by the FG, represented by Mr. Mélanchon, referring to an old fear of seeing France struck by an orthodox communism. It is therefore not surprising that the strongest pole of repulsion is The Front de Gauche followed by the socialist party (correlation coefficient = -0.61). There does not seem to exist a correlation in between the vote for FN and conservatives.

If we now look at the communists, their strongest dislike goes to the conservatives (correlation coefficient = -0.70). It is partly surprising because we would expect the FN to be their biggest political enemy, considering the historical hate they feel for nationalism. Following the “anti Le Pen” speech Mr. Mélanchon held during the whole campaign it is surprising not to see the FN in first position as their political dislike. On another hand the positions of FN and FG are highly similar when it comes to the “elites”, the “political class” and the “system”. In other words they both hold a populist position which may have driven their electors to reject the most traditional parties.

If we focus on the socialists their strongest dislike is, as we could have imagined, the FN and the conservative (correlation coefficient respectively equal -0.63 and -0.61).

We are now going to discuss the FN’s situation. Their strongest pole of repulsion seems to be the socialist party (correlation coefficient = -0.63). The explanation is roughly the

same as the one explaining the repulsion in between the FG and UMP, on the other side of the political playing field. It does not seem to be correlated with the extreme left party (NPA) or with the conservatives, the correlation coefficient being close to zero. To finish we are going to look at the abstentionists. We can observe that there is a negative correlation in between voting for a left party and abstention, whereas this correlation is positive when we talk about the right party (FN and UMP). We therefore tend to conclude that there is a stronger political implication in their political duties for the electors of the left. On the other hand people voting for right parties would tend to be less interested by politics.

C) Mean, standard error, and distribution

For each candidate we are going to set the national mean obtained, its minimum and maximum scores, and its standard error. Eventually we will draw a graph of their distribution.

We shall not do as in the example below. If we build a simple mean using the percentage obtained in every district and then divide it by the amount of observation (here 96 district), in doing so we would, as previously stated, overweigh the bigger districts:

$$\frac{1}{N} * \sum_i \frac{vote\ i}{total\ i}$$

Where

N: Number of observation = 96

i: ith district

And this for each candidate

We would rather use a mean using the raw data for the whole country:

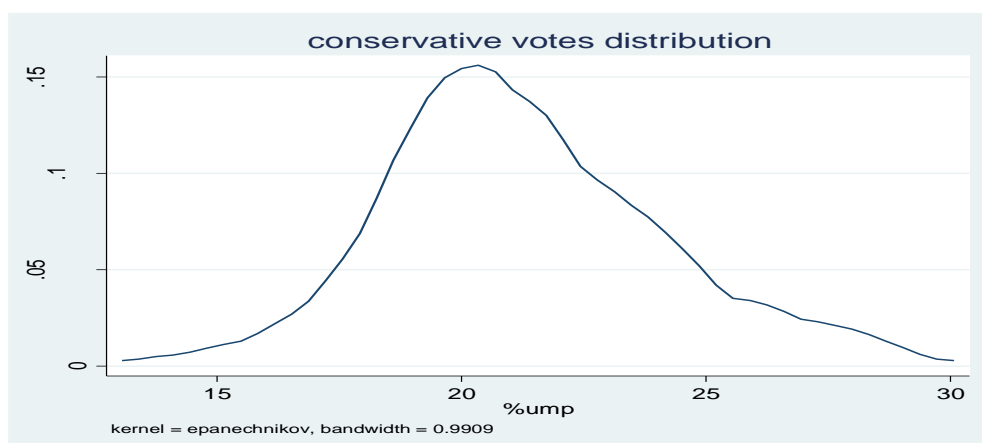
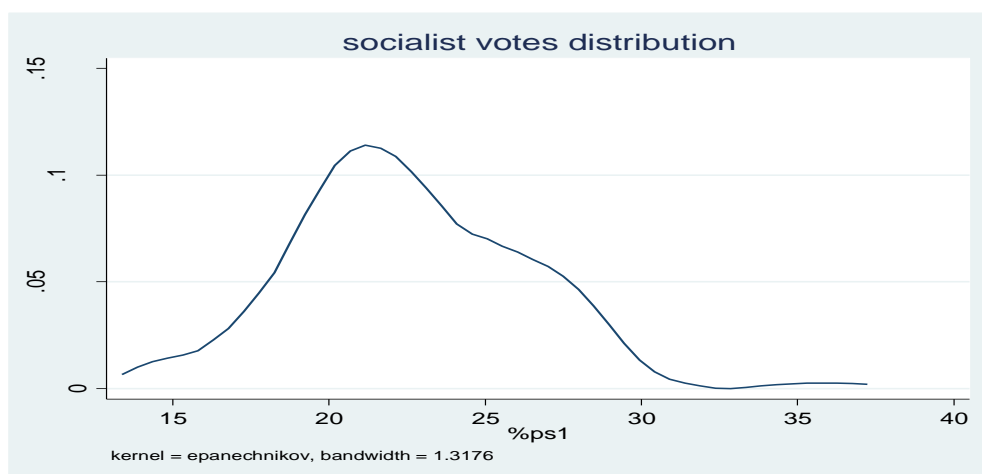
$$\frac{\sum vote\ i}{Total\ written\ in\ the\ country}$$

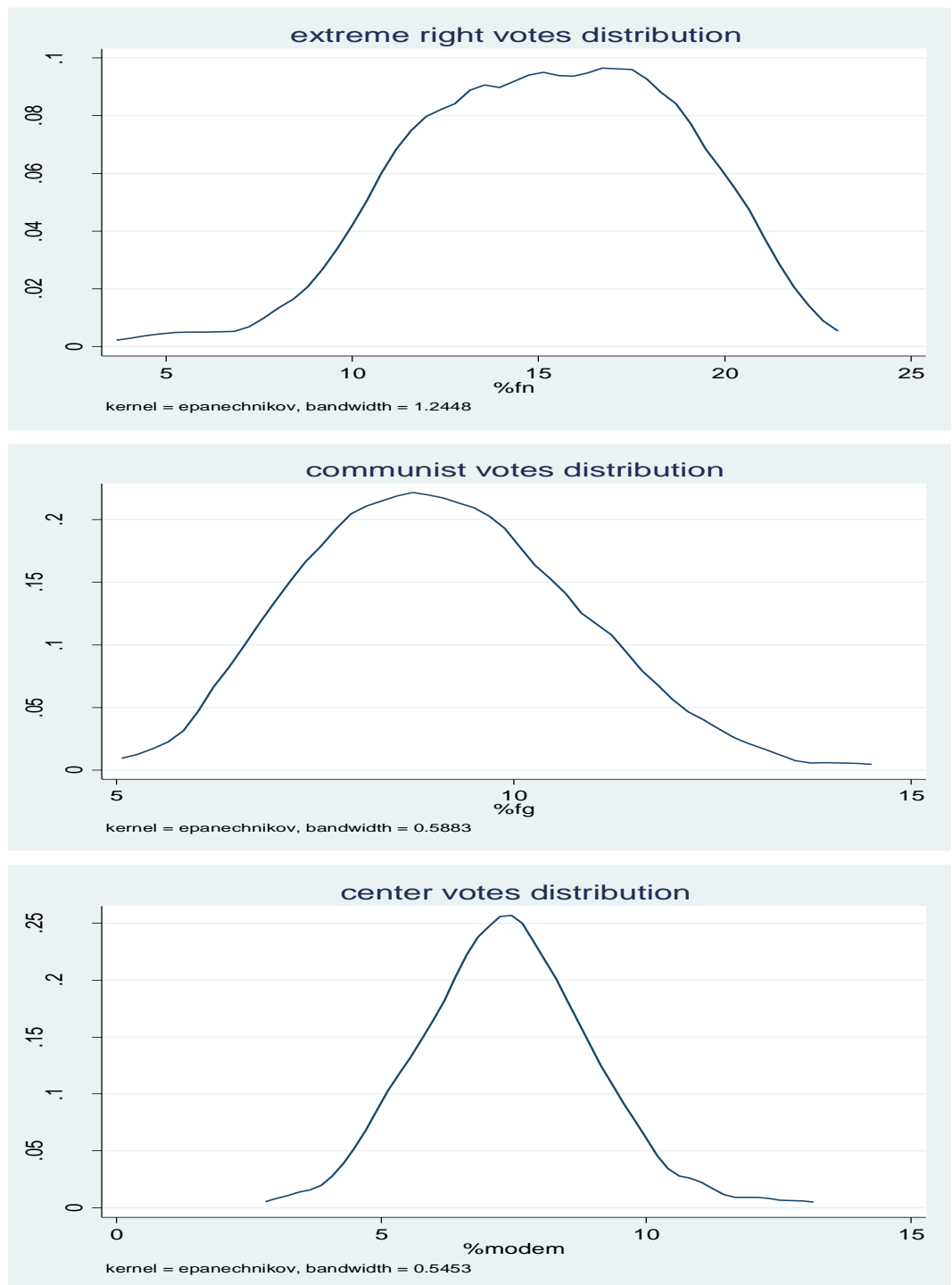
And this for each candidate

Using this last formula we work out the following values

Variable	Mean	Standard error	Min	max
Vert	1.75	0.48	0.91	3.31
FN	14.84	3.46	4.90	21.79
UMP	20.99	2.76	14.04	29.08
FG	9.21	1.61	5.65	13.92
NPA	0.99	0.18	0.53	1.39
LO	0.48	0.11	0.21	0.71
chem	0.20	0.02	0.15	0.25
modem	7.42	1.59	3.35	12.61
nda	1.50	0.28	0.78	2.65
Ps1	23.12	3.72	14.69	35.92
Abst1	19.48	2.37	15.61	27.90
PS2	40.71	5.32	27.25	52.92
UMP2	36.77	4.50	25.31	49.10
Abst2	22.53	2.10	18.41	28.57

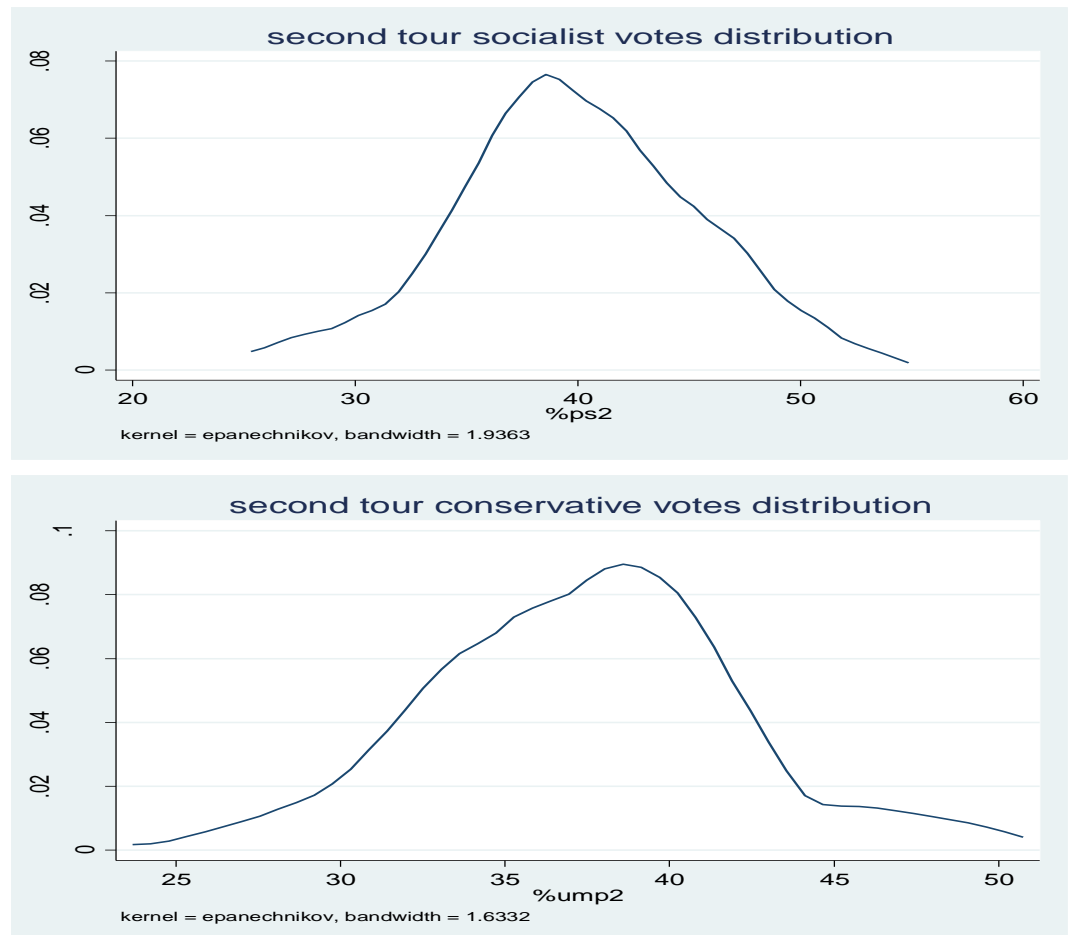
We are now going to display the distribution of the results in percentages for the five strongest candidates of the first tour.





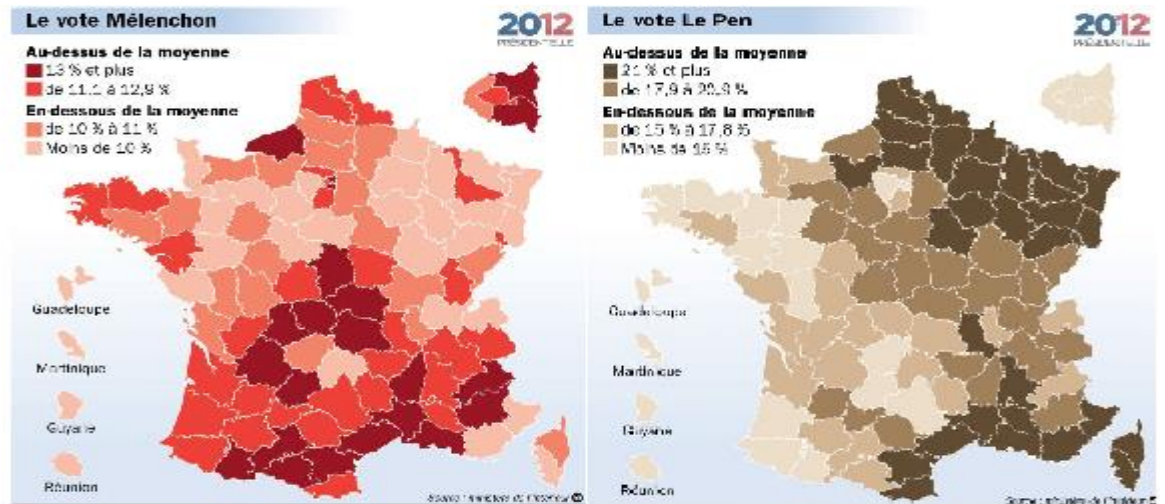
We can notice that the communist votes are more centered on their medium value than the extreme right votes, this might emphasize the fact that the FN is subject to a strong approval, or to a strong repulsion, according to the different districts. Indeed the difference in between its lowest and highest score is one of the biggest. On the opposite the moderate votes are very closely grouped around their medium value, the MODEM having more consensual political beliefs, it is therefore logical to find rather

close scores in between the districts. When it comes to the conservative or socialist distribution they are quite identical with most districts voting at around 25% for Hollande and with scores overcoming 35%, whereas they stop at 29 % for Mr. Sarkozy . We are now going to compare the voting distribution for the second tour of the elections.



Once again the distributions are quite similar, with a sharper decrease in the proportion of district voting for conservatives above 40%.

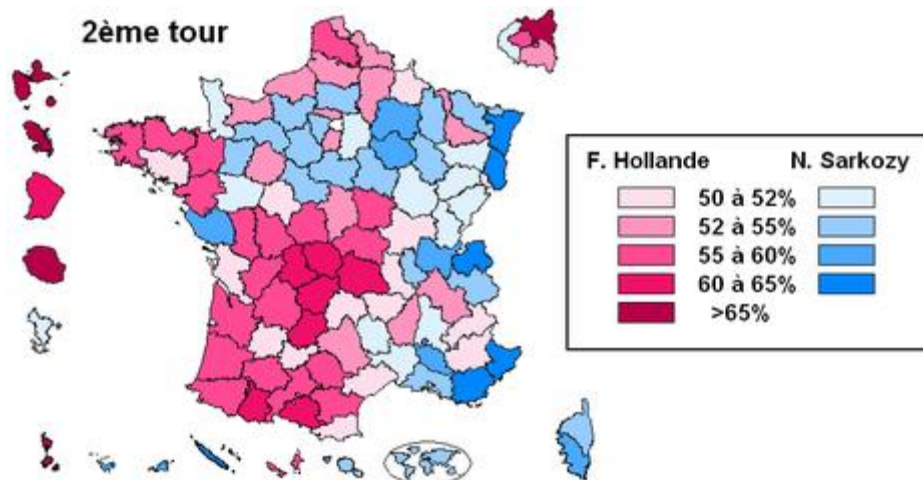
Before discussing the model explanation we would like to introduce a bit of cartography, so the reader can have an idea how the votes are geographically distributed. We will then display a map of the communist votes and extreme right votes during the first tour, followed by a map of the socialist and conservative votes of the second tour.



We can see that in the north of France, the districts where the LePen vote is high (above its medium value), the communist vote is low, so there seems to exist a negative correlation between those two parties. But when we look at the Mediterranean coast, the districts where the FN and the communists have high scores are the same. Thus, we are led to believe that the factors determining the communist or FN votes differ according to the district or region we are referring to.

We can see that the east of France is highly impacted by the FN vote. These are indeed traditionally high score areas for this party, specifically in the north where unemployment is high. On the opposite the south west of France and Brittany are more impacted by the communist vote, these are areas where unemployment is low. As studied in a previous work (le vote Front national lors des election présidentielles françaises de 2012), we showed that the main factor explaining the FN vote was the unemployment rate.

We are now displaying a map of the socialist and conservative votes during the second tour.



It is easy to see that the conservative vote is high where the FN vote was also strong, which is mainly on the east and particularly closer to the Italian and German borders. On

the opposite side, where the socialist vote is dominant, the communist vote was initially high. We can already think that the communist votes mainly transferred to the socialist party during second tour while the FN voters mainly gave their votes to Sarkozy (Note that we are not referring to the abstention in this analysis).

III) The Model

We are therefore trying to estimate a system of equations, indeed we want to know how the first round votes influence votes for the socialists during the second tour. In the same way, we want to show the same effect on conservative votes and to finish on the abstention. The model will be written as:

$$\begin{aligned} Y_1 &= \beta_{11}X_1 + \beta_{12}X_2 + \dots + \beta_{1k}X_k + \varepsilon_1 \\ Y_2 &= \beta_{21}X_1 + \beta_{22}X_2 + \dots + \beta_{2k}X_k + \varepsilon_2 \\ Y_3 &= \beta_{31}X_1 + \beta_{32}X_2 + \dots + \beta_{3k}X_k + \varepsilon_3 \end{aligned}$$

Where Y_1 is the socialist vote

Y_2 is the conservative vote

Y_3 is the abstention

where β_{ik} is the $1 \times k$ vector of parameters

where x_k be the $1 \times k$ matrix of regressors

We are now setting an important assumption for the model estimation which is

$$E[\varepsilon/x_1, x_2, x_3] = 0$$

Which means that the disturbance is not correlated with the regressors within an equation or in between the equations. But we must justify why we accept this strong hypothesis according to the specific structure of the system we are studying. What are the sources of endogeneity?

First of all, in our study there will be no omitted variables in the model specification because ALL votes of the first tour are found during the second tour. We therefore include all the possible regressors.

Secondly, a bias of causality is also excluded because the votes for conservatives or socialists during the second tour cannot influence the vote for candidates during first tour, as this vote is already past.

The last possibility would be incorrectly collected data, which is also highly improbable as these are given by the "ministère de l'intérieur".

At first we are naively considering those 3 equations as independent from each other and we will estimate them separately with a classic OLS method. As it seems highly improbable that the reasons for one to give their vote to Holland or Sarkozy to be unrelated, estimating separately would be a loss of information. This information is obvious here; the same set of parameters appears in each equation, and as all votes report:

$\sum_i \beta_i = 1$ this is called a cross equation restriction

Where β is a $K \times 1$ vector of parameters (here $k = 3$).

In other words by estimating the equations separately we would ignore the Cross Equations Restrictions.

This means that our equations are going to be linked via the disturbance. The disturbance represents all the

explanations for the vote y_i (i = socialist, conservatives, abstention) that are not explained by the regressors. Therefore, all what does not explain the vote for a candidate is supposed to contain information on the vote for the other ones.

We will for these reasons consider the 3 equations as a system. As the dependant variable is different for each equation and that the equation might be correlated via their disturbance, we will assume to be facing a seemingly unrelated regression system (also known as the Zellner's model). In this case the OLS cannot be efficient as the variance/covariance matrix of disturbance is no longer spherical.

Feasible GLS seems to be the most appropriate estimator in this situation, for it will be consistent and efficient.

But we have to notice that we are facing a very specific case of SUR, a multivariate SUR, because the X matrix of regressors is the same for all equations. Indeed, we collected the data on 96 French metropolitan districts, and apply them as regressors for the three dependant variable. Thus, $X_1 = X_2 = X_3 = X^*$

In this specific case using OLS equation by equation or FGLS is equal unless we specify restriction on β , vector of parameters, which we are going to discuss now.

The last problem imposed by the model is that we must constrain our parameters between 0 and 1 (As developed above, 100% of votes must report). To fulfill this requirement we will use a non linear estimator using a logistic function.

$$\beta^+ = \frac{1}{1 + e^{-\beta^*}}$$

This is obvious now that $0 \leq \beta^+ \leq 1$, the econometric software is going to estimate β^* who can vary freely in between minus and plus infinity. In a second time, we will recalculate the value β^+ using the equation above. In presence of restrictions on β , even if the regressors are identical (the $X'X$ matrix of regressor is the same for all equations as the votes at first round are identical whether we consider the equation explaining Hollande vote, Sarkozy vote or abstention) OLS cannot be efficient, which is why we confirm our idea by using a GLS estimator.

We can reset the model as follow:

$$Y_1 = \beta_{11}^+ x_1 + \beta_{12}^+ x_2 + \dots + \beta_{1k}^+ x_k + \varepsilon_1$$

$$Y_2 = \beta_{21}^+ x_1 + \beta_{22}^+ x_2 + \dots + \beta_{2k}^+ x_k + \varepsilon_2$$

$$Y_3 = \beta_{31}^+ x_1 + \beta_{32}^+ x_2 + \dots + \beta_{3k}^+ x_k + \varepsilon_3$$

The estimation with non linear least square is not as easy as linear least square is. Indeed minimizing the sum of squares of residuals will require an optimization algorithm. We will use a Gauss-Newton algorithm in order to minimize the function gradient (For more details see annexes).

The high number of parameters makes a full convergence very unlikely, even more so if the solution is on the limit of definable interval. And we can already anticipate this fact for the candidates whose votes are almost 100% or 0% correlates with the second tour candidate considered. Thus we will try various models, the first one with the 33 parameters to estimate. If the convergence is not achieved, we will try to gather all candidates with less than 5% of the votes and we will consider them as a disturbance (The aim being here to reduce the number of parameters).

Despite all these efforts the convergence might not be achieved, in that case we will be forced to constrain some parameters to 1, others to 0.

IV) Results and analysis of the regressions

In the next table we display the results we obtained after OLS estimation, equation by equation. We will see that we can already draw a general picture.

	OLS socialist votes 2nd tour	OLS conservative votes 2nd tour	OLS abstention votes 2nd tour
%vert	0.276 (1.45)	0.455 (2.01)*	0.269 (1.53)
%fn	0.013 (0.47)	0.692 (21.37)**	0.296 (11.77)**
%ump	-0.031 (1.06)	1.081 (30.72)**	-0.049 (1.81)
%fg	0.969 (17.85)**	-0.027 (0.41)	0.057 (1.14)
%npa	2.637 (5.74)**	-1.843 (3.37)**	0.206 (0.49)
%lo	1.786 (2.29)*	-4.849 (5.22)**	4.063 (5.64)**
%chem	-4.302 (1.26)	8.691 (2.14)*	-3.389 (1.07)
%mod	0.167 (2.53)*	0.664 (8.44)**	0.169 (2.77)**
%nda	0.154 (0.59)	0.815 (2.62)*	0.031 (0.13)
%ps1	1.128 (52.16)**	-0.121 (4.69)**	-0.007 (0.37)
%abst1	0.081 (3.57)**	0.120 (4.44)**	0.799 (38.23)**
Observations	96		
R-squared	1.00		
Absolute value of t statistics in parentheses			
* significant at 5%; ** significant at 1%			

We can see in the results of the naïves regressions some estimates of parameters are aberrant, indeed it is impossible for 259% of the NPA first tour voters to transfer their votes to Holland during the second round. Using the same logic we can reject all negatives estimate parameters. We will, in a second time, constrain the negative parameters to zero and identically we will constrain to one all estimate

parameters above 1.

In spite of these aberrations we already can draw a raw picture of the situation (as OLS is consistent anyway).

Some estimate parameters, indeed, fit to the prior idea we could have about how votes move in between two tours of an election.

It seems that 69.4 % of FN voters voted Sarkozy during the second tour. The others decided to abstain. None went to the socialists (as its parameter is not statistically different from zero).

97.1 % of Front de gauche (FG) voters gave their vote to the socialist Hollande. This is everything but surprising as their leader, Mélançon, called for a massive socialist voting. But as all the other betas are statistically equal to zero, then the sum of parameters explaining how the FG vote did report is not equal to one. This is disturbing because as we included abstention as third equation, then 100% of votes for each defeated candidate must be expressed (This is our cross equation restriction).

An interesting result is the estimate parameter for MODEM. Mr. Bayrou, their leader, and former opponent of Mr. Sarkozy's politics, drew during the campaign a poor picture of the conservatives' results and seemed to become closer to the socialists. Eventually he asked his electors to vote Socialist. Looking at our results we can see that 67% of votes went to the conservatives while only 16.8% went to the socialists. The remaining votes were abstentions (around 17%).

None of the estimate parameters seems to be statistically different from zero when it comes to the ecologist voting akin.

To finish with the result analysis we want to have a closer look at the abstentions. Indeed 80% among those who did not vote for first round did not vote for second one. But 11% voted for conservatives and 8% for socialists. This could be because to some it would be a threat to see a socialist party in charge. On the other hand the 8% given to the socialists represent a strong will to see Mr. Sarkozy out of politics.

We can analyze this set of information through a sociologist's eyes (see le nouveau désordre électoral, presse science politique, Bruno Cautrès et Nonna Myer). In this essay the two authors highlight the partisan loyalty existing between parties and their voters. They argue that this link is stronger for the old parties and traditionally for the left parties. Indeed in this first estimation we can see almost 100% of the FG voters voted for Hollande during the second tour, whereas only 17% of the MODEM voters followed their leader's call.

The key question is now to understand the voting behavior of the FN voters. But we will do so by exploring more accurate estimators.

As previously explained, we want to study the equations as a system, and being in a multivariate system (see above), we expect the results to be similar than an OLS equation by equation estimation. Indeed by using a SUR method, we obtain the very same estimate parameters. The conclusion and explanation are therefore the same. But this gives us further information.

SURE estimation		
	PS2	UMP2
%vert	0.276 (1.54)	0.455 (2.14)*
%fn	0.013 (0.50)	0.692 (22.71)**
%ump	-0.031 (1.13)	1.081 (32.65)**
%fg	0.969 (18.97)**	-0.027 (0.44)
%npa	2.637 (6.10)**	-1.843 (3.58)**
%lo	1.786 (2.43)*	-4.84Z (5.55)**
%che	-4.302 (1.34)	8.691 (2.27)*
%mod	0.167 (2.69)**	0.664 (8.97)**
%nda	0.154 (0.63)	0.815 (2.78)**
%ps1	1.128 (55.43)**	-0.121 (4.99)**
%abst1	0.081 (3.79)**	0.120 (4.72)**
Observations	96	
Absolute value of t statistics in parentheses		
* significant at 5%; ** significant at 1%		

The values for the parameters explaining the third equation (abst2) can be refound using the relationship $\beta_{abst2\ i} = 1 - (\beta_{ps2\ i} + \beta_{ump2\ i})$. We must proceed that way because estimating simultaneously the 3 equations would lead to a singular covariance matrix of error. This recall to the invariance parameter estimates is discussed by Powell and Barten in 1969.

Indeed using a SURE method we can work out the correlation matrix of residual. This is the proof that the equations are actually related via the disturbance.

Correlation matrix of residuals:

	ps2	ump2
ps2	1.0000	
ump2	-0.6571	1.0000

Knowing that our equations are actually related (Σ is not spherical) we must re-estimate the system imposing restrictions on β .

Doing so, the two methods of estimation should not be equal anymore. To begin we are going to re-set the model in order to constrain some parameters to one or zero using the non linear form described above equation by equation. We are using a Gauss-Newton algorithm to achieve convergence. We therefore need a starting value and a step. The starting value for each parameter will be the one found after the OLS regression, which played the role of a pre-estimation.

b) Non linear estimation equation by equation

Source	SS	df	MS			
Model	155380.696	5	31076.1393	Number of obs =	96	
Residual	75.9706264	91	.834842048	R-squared =	0.9995	
				Adj R-squared =	0.9995	
				Root MSE =	.9136969	
Total	155456.667	96	1619.34028	Res. dev. =	249.9721	

ps2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
/bvert	-1.22694	1.236608	-0.99	0.324	-3.68331	1.229429
/bfn	-4.54485	2.895806	-1.57	0.120	-10.29701	1.207313
/bump	-192.9271
/bfg	2.00e+08
/bnpa	29.97045
/blo	7259.71
/bchem	-331.8243
/bmodem	-.6439059	.3444066	-1.87	0.065	-1.328027	.0402154
/bnda	.8530242	1.904095	0.45	0.655	-2.929226	4.635274
/bps1	5.80e+08
/babst1	-1.866514	.2420834	-7.71	0.000	-2.347383	-1.385645

Source	SS	df	MS			
Model	136089.188	4	34022.297	Number of obs =	96	
Residual	151.700704	92	1.64892069	R-squared =	0.9989	
				Adj R-squared =	0.9988	
				Root MSE =	1.284103	
Total	136240.889	96	1419.17592	Res. dev. =	316.3621	

ump2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
/bvert	2.144581	3.178481	0.67	0.502	-4.168157	8.45732
/bfn	.6091134	.1623218	3.75	0.000	.2867284	.9314985
/bump	357.4802
/bfg	-493.5338
/bnpa	-495.8003
/blo	-634.0491
/bchem	45802
/bmodem	-.4868049	.32871	-1.48	0.142	-1.139651	.1660415
/bnda	-567.6919
/bps1	-96.60505
/babst1	-2.322796	.4834029	-4.81	0.000	-3.282876	-1.362716

Source	SS	df	MS	
Model	49919.4621	6	8319.91035	Number of obs = 96
Residual	26.0311117	90	.289234574	R-squared = 0.9995
				Adj R-squared = 0.9994
Total	49945.4932	96	520.265554	Root MSE = .5378053
				Res. dev. = 147.1508

abst2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
/bvert	-302.9904
/bfn	-.8770174	.0912969	-9.61	0.000	-1.058395	-.6956401
/bump	-703.6111
/bfg	-2.687772	.704833	-3.81	0.000	-4.088046	-1.287498
/bnpa	31.05162
/blo	189.2689
/bchem	-111.7422
/bmodem	-2.125237	.4153758	-5.12	0.000	-2.950454	-1.300021
/bnda	-.8860095	1.131244	-0.78	0.436	-3.133424	1.361405
/bps1	-5.846862	6.836761	-0.86	0.395	-19.42928	7.735556
/babst1	1.153654	.099163	11.63	0.000	.9566494	1.350659

We can see at this point that some parameters tend towards plus infinity or minus infinity, the standard error for those parameters is therefore non defined. The convergence is therefore not achieved in those cases. But it is easy to see that when the estimated parameter tends to plus infinity, the true value of the parameter will be close to one. When its value tends to minus infinity the true value of the parameter will be close to zero.

For a better analysis we are going to recalculate the value of the parameters β^+ .

	PS2	UMP2	Abst2
%vert	0.226 [0.298]	0.895 [0.03]	0 [.]
%fn	0.010 [0.728]	0.647 [0.000]	0.293 [0.000]
%ump	0 [.]	1 [.]	0 [.]
%fg	1 [.]	0 [.]	0.0639 [0.133]
%npa	1 [.]	0 [.]	1 [.]
%lo	1 [.]	0 [.]	1 [.]
%chem	0 [.]	1 [.]	0 [.]
%modem	0.344 [0.000]	0.380 [0.000]	0.106 [0.008]
%nda	0.701 [0.082]	0 [.]	0.291 [0.215]
%ps1	1 [.]	0 [.]	0.002 [0.884]
%abst1	0.133 [0.000]	0.089 [0.025]	0.760 [0.000]

Obviously here the cross equation restriction is not fulfilled and the sum of the parameters is not equal to one. The analysis is at this point a bit scarce and distorted.

Considering how the green votes transfer we strangely notice an 89.5% correlation with the conservative vote. Whereas, when we look at the correlation with the socialist vote we have a p-value >5%, so we accept the non significance of this parameter. When it comes to how it correlates with abstention it seems to be 0%.

When we consider the FN vote we see that 64.7% of their voters followed Sarkozy during the second tour while 29.3 % went to abstentions. Statistically the FN voters do not choose to vote socialist during the second tour as some would have expected during the two tours of the election.

As we thought, the extreme left votes fully correlated with the socialist votes. We have a statistical bias though, as 100% of NPA (New Anti-capitalist Party) transferred to Hollande but also 100% went to abstention, this is clearly impossible and we therefore cannot make any coherent conclusion concerning these parameters.

The MODEM case is an interesting one, as they collected about 9% of the aggregates votes during the first tour. Moreover their leader asked his electors to deport their choice to Hollande. But traditionally this party is on the center right of the political spectre. We see that only 34.4% voted for Sarkozy while 38% voted socialist. 10% did not vote. Again, the adding up condition is not fulfilled.

But we notice that the results deeply change in respect with the OLS regression where we concluded that 64% of his voters then voted for Hollande.

Another rather strange result is how Mr. Dupont Aignan (NDA), former conservative and protectionist did correlate. We can read here that about 70% voted. Again the result greatly differs from the OLS regressions. Eventually we focus on abstention were 13% deported on the socialist, 8% on the conservatives and the other ones still did not vote. Again it is in contradiction with the OLS regression saying that 8% did vote socialist and 12% conservative. The proportion of non voters still remains identical.

Considering the very low scores of the extreme left parties, the ecologists, Mr. Cheminade and Mr.NDA we can doubt the accuracy of a statistical analysis on the behavior of their electors. We are therefore going to gather them in a term of disturbance and re-estimate the equations with non linear least square. Doing so we obtain an output such as:

	PS2	UMP2	Abst2
FN	0.0139 [0.648]	0.631 [0.000]	0.346 [0.000]
UMP	0 [.]	1 [.]	0 [.]
FG	1 [.]	0 [.]	0.018 [0.729]
Modem	0.178 [0.059]	0.539 [0.000]	0.148 [0.026]
PS1	1 [.]	0 [.]	0.040 [0.075]
Abst1	0.097 [0.001]	0.131 [0.001]	0.729 [0.000]
Disturb	0.980 [0.000]	0 [.]	0.194 [0.216]

where in 2 last table : [p-value]

To reduce the amount of regressors allowed a better convergence of the algorithm. If for each candidate we sum the estimate parameters we can see that we are close to fulfill the adding up condition. Using this model, it does not change the behavior of FN voters, these proportion are roughly the same regardless of the model we are using, we can conclude that the analysis we had on these electors voting behavior is rather solid.

About half of the modern voters transferred their votes to the conservatives. 17,8% to the socialists and 14,8% abstained.

The behavior of the abstentionists seems also to be the same.

The new term, “disturbance”, we created, seem to transfer at 98% to the socialists. This is everything but surprising, as it is mainly composed of extreme left party. It is rather difficult to imagine how the Mr.Cheminade electors behaved considering how loony were its political positions (conquest of Mars and so on). We can think that most of the Mr.NDA electors joined Mr.Sarkozy during second tour.

c) Non linear estimation using a system of equation

We are now stepping to the last part of our model, in order to fully take into account the link existing in between the equations we are going to re-estimate the model using a non linear Seemingly Unrelated Regression technique.

FGNLS regression

Equation	Obs	Parms	RMSE	R-sq	Constant
1 ps2	96	7	.9586864	0.9677	bump
2 ump2	96	7	1.316887	0.9153	bumpu
3 abst2	96	7	.6078424	0.9144	bumpa

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
/bfn	-3.480755	.9545895	-3.65	0.000	-5.351717	-1.609794
/bump	-479.7459
/bfg	88.42519
/bmodem	-.9589739	.2907798	-3.30	0.001	-1.528892	-.389056
/bps1	3539.991
/babst1	-2.118759	.2889811	-7.33	0.000	-2.685152	-1.552367
/bdist2	1.056516	.469696	2.25	0.024	.135929	1.977104
/bfnu	.5377124	.1606071	3.35	0.001	.2229283	.8524964
/bumpu	20.02841
/bfgu	-148.2022
/bmodemu	.1571243	.2491874	0.63	0.528	-.3312741	.6455227
/bps1u	-269.08
/babst1u	-1.887044	.3322726	-5.68	0.000	-2.538286	-1.235801
/bdist2u	-17.83464
/bfna	-.6382337	.0815176	-7.83	0.000	-.7980053	-.4784621
/bumpa	-299.4255
/bfga	-17.1379
/bmodema	-1.562813	.3513116	-4.45	0.000	-2.251371	-.8742552
/bps1a	-3.679679	.2378067	-15.47	0.000	-4.145772	-3.213586
/babst1a	1.074511	.0953813	11.27	0.000	.8875667	1.261455
/bdist2a	-1.364618	.6335456	-2.15	0.031	-2.606344	-.122891

Re-calculating the parameters we eventually obtain

	PS2	UMP2	Abst2
FN	0.02 [0.280]	0.631 [0.000]	0.345 [0.000]
UMP	0 [.]	1 [.]	0 [.]
FG	1 [.]	0 [.]	0 [.]
MODEM	0.277 [0.000]	0.539 [0.000]	0.173 [0.001]
PS1	1 [.]	0 [.]	0.02 [0.000]
Abst1	0.107 [0.000]	0.131 [0.001]	0.745 [0.000]
Dist2	0.748 [0.000]	0 [.]	0.203 [0.048]

A full convergence of the algorithm remains unachieved, but the results are here more accurate. Some error still exists because of this unachieved convergence. That is why we can see that 102% of the first tour socialist votes are transferred during the second, which is obviously impossible. We will assume that Mr.Hollande conserves all his electors in between the two tours. The term “disturbance” do not fully transfer, we will assume that the proportion which is remaining go to the conservatives (while estimated to 0% in the model). For the other regressors the adding up condition is respected.

Let’s analyze the MODEM voters behavior. Most of its electors voted Sarkozy instead of following their leader’s call. The partisan loyalty does not seem to be very strong in this party and its electors behaved following their very own belief and did not massively obeyed to the “consigne de vote” given. Those electors do not need an organized structure to situate themselves on the political field. The ones who voted for Hollande might have not done it by loyalty but by disapproval face to Sarkozy’s politic or way of governance. We can therefore say that at least 17.3% of its electors structurally belong to the MODEM and fit to its political ideas.

The FG electors followed their leader call and fully transferred their vote on Mr.Hollande, as previously described this is not surprising, as the communists are strongly repulsed by the conservative politics. If we

add to this fact, the traditionally strong link existing in between left voters and their party, it is clear that they all followed Mr.Mélenchon orders. Moreover, tired by 10 years with the conservative in charge, the aim was for them to push Mr.Sarkozy out of politics.

The FN voters behavior has previously been analyze. We can anyway go a bit further and look at this proportion of 2/3 ; 1/3 of their repartition. The interesting part concerns the 30% who turned abstentionists. We can consider that they are the very core of the FN voters, that these electors are faithful to the intolerance, homophobic and close to pure racism historical ideas of the Front National. In spite of the speeches held by Mr.Sarkozy during his campaign meetings and his strategy aiming to attract those electors only two third did transfer their vote on him. Those two third might be people attracted by the anti-globalization position held by Ms.Lepen but structurally closer to the conservatives than to the socialists. Plus, the socialists' propositions of giving a voting right to foreigners for local elections and allow homosexual people to marry, might have been a casus belli for these electors and might have driven a higher proportion of first tour FN voters to join Sarkozy. It was not enough to allow the conservatives to win though. They expected 80% of the FN voters to follow them during the second tour. There is though, no way, according to this model, to give allowance to Ms.Lepen affirmation when she declares that the FN vote is more and more a vote of adhesion whereas it was formerly a vote of rejection of the political class. The fact that she did not call for joining anyone only allow us to conclude that at least 30% percent of her electors are turning very radical, but unfortunately, we cannot say anything on the tightness of the link existing in between the FN voters and their party and leaders.

The first tour abstentionists are also interesting to analyze. If about 74% still did not vote during second tour, we must think of the reason pushing a first tour abstentionist to mobilize for the second. We first stated that the political conscience was stronger for the left electorate than for the conservative one. It is therefore not surprising to see a larger part of the abstentionists voting for Mr.Sarkozy during second tour as the electors for the socialist candidate were more mobilized since the very beginning of the election. The key factor pushing an abstentionist to vote is fear. The fear to see the conservatives in charge again for the ones, and the fear to see communism back to power for the others. A sudden interest for a campaign becoming more and more violent in between the two round can be another reason.

V) Conclusion

Our model allows us to explain how vote were transferred in between the two tours of an election. It is important to keep in mind that these results are valid for this election. Indeed the strategies held by the various candidates and their own personality can highly change the conclusions. To analyze a national election reduce the distortion caused by these factors as the French political landscape is formerly divided by the opposition in between left and right. Indeed we showed that the mobility in between the left parties is high while a "cross border" mobility (voting initially for the left and then changing for the right) is very unlikely and statistically equal to zero. The right is much more homogenate two party dominating the field. The traditional right (inherited from DeGaulle) and the neoliberal right are integrated within the UMP, and the nationalist right is incarnated by the FN. The partisan mobility within the right is weaker than in the left. Indeed the votes were never fully transferred. Something remains clear, the threshold existing in between the left and the right seems to be 100% hermetic.

This also shows the French society is rather strongly politically structured and educated. Indeed the link existing in between the voters, the base, and the leaders of the different parties seems to be rather strong. The parties where this link is loose usually got less than 8 % of the total voting.

The French voter can therefore be considered as an educated citizen able to choose by itself who to vote for. This can be explained by a high level of education and an historical interest of the French for politics. The fact of situate itself on the right or left side of the political field seems to be something stable and lasting. We therefore can assume it is more determined by the social background, education and family belief than by the person leading a party or a particular interest on a particular subject at a particular moment. To be more accurate on this subject a time series analysis would be required. The difficulty here would rely in the fact that the voting population is changing over years.

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Annex 2: The Gauss-Newton Algorithm

We are trying to determine $E[y/x] = h(X, \beta)$ where h , is a non linear function

Applied to our model we have:

$$y = \frac{1}{1+e^{-\beta_1 x_1}} x_1 + \frac{1}{1+e^{-\beta_2 x_2}} x_2 + \dots + \frac{1}{1+e^{-\beta_n x_n}} x_n + \varepsilon$$

The mechanism is the same than in OLS estimation, we want to minimize the sum of the squares of residual:

$$\text{Min } Q(\beta) = (y - h(x, \beta))' (y - h(x, \beta))$$

$$dQ(\beta)/d\beta = -2dh(x, \beta)/d\beta \times [y - h(x, \beta)]$$

Where $dh(x, \beta)/d\beta$ is the $N \times K$ (N : number of equation ; k : number of parameter) matrix of the first partial derivative also call the gradient. This is what we will eventually aim to minimize thanks to the algorithm.

$$\text{Here } dh(x_1, \beta)/d\beta = -(1+e^{-\beta_1})^{-2} \cdot (-e^{-\beta_1})$$

We need to make the function linear thank to a Taylor first order limited development:

$$H(x, \beta) \approx h(x, \beta^1) + [-(1+e^{-\beta^{11}})^{-2} \cdot (-e^{-\beta^{11}}) \dots -(1+e^{-\beta^{n1}})^{-2} \cdot (-e^{-\beta^{n1}})] \cdot [\beta - \beta^1]$$

Where β^{i1} is the starting value obtained by the OLS pre-estimation.

We now construct a pseudo-linear model calling $Z(\beta)$ the matrix of the first derivative:

$$y - h(x, \beta^1) + Z(\beta^1)\beta^1 = Z(\beta^1) \cdot \beta + \varepsilon$$

We therefore linearly regress this model to obtain an estimate value of β that we will call β^2

Then, we replace β^1 by β^2 and we apply the same regression, and so on until the gradient is minimum.

Annex 3: the do File

In order to allow the reader to redo the experience or to add its own work to the topic we decided to integrate the do file.

```
/// descriptive statistics
```

```
sum [aweight=vert]
sum [aweight=fn]
sum [aweight=ump]
sum [aweight=fg]
sum [aweight=npa]
sum [aweight=lo]
sum [aweight=modem]
sum [aweight=nda]
sum [aweight=chem]
sum [aweight=ps1]
sum [aweight=abst1]
sum [aweight=ps2]
sum [aweight=ps2]
sum [aweight=abst2]
```

```
corr vert fn ump fg npa lo chem modem nda ps1 abst1
```

```
kdensity fn
graph save densfn
kdensity ump
graph save densump
kdensity fg
graph save densfg
kdensity npa
graph save densnpa
kdensity lo
graph save denslo
kdensity chem.
kdensity modem
graph save densmodem
kdensity nda
graph save densnda
kdensity ps1
graph save densps1
kdensity abst1
graph save densabst1
```

```
/// naive regressions
```

```
reg ps2 vert fn ump fg npa lo chem modem nda ps1 abst1 , nocons
outreg using tab2.out, ctitle ("OLS for socialist votes 2nd tour")
estimates store renaive1
```

```
reg ump2 vert fn ump fg npa lo chem modem nda ps1 abst1 , nocons
outreg using tab2.out, ctitle ("OLS for conservative votes 2nd tour") append
estimates store renaive2
```

```
reg abst2 vert fn ump fg npa lo chem modem nda ps1 abst1 , nocons
outreg using tab2.out, ctitle ("OLS for abstention votes 2nd tour") append
estimates store renaive3
```

```
/// Linear regression of a system of equation
```

```
sureg (ps2 vert fn ump fg npa lo chem modem nda ps1 abst1 , noconstant) (ump2 vert fn ump fg npa lo chem modem nda ps1 abst1 , noconstant)
outreg using tab4.out, ctitle ("sure estimation")
estimates store surnc1
estimates table renaive1 renaive2 renaive3 surnc1 , b(%8.3f) star
estimates table renaive1 renaive2 renaive3 , se
```

```
/// Non linear estimation, equation by equation
```

```
nl (ps2 = (1/(1+exp(-1*{bvert=0.2760286}))) * vert + (1/(1+exp(-1*{bfn=0.012872}))) * fn ///
+ (1/(1+exp(-1*{bump=-0.031374}))) * ump + (1/(1+exp(-1*{bfg=0.9694314}))) * fg ///
+ (1/(1+exp(-1*{bnpa=2.636995}))) * npa + (1/(1+exp(-1*{blo=1.785973}))) * lo ///
+ (1/(1+exp(-1*{bchem=-4.302387}))) * chem + (1/(1+exp(-1*{bmodem=0.1672912}))) * modem ///
+ (1/(1+exp(-1*{bnda=0.1542956}))) * nda + (1/(1+exp(-1*{bps1=1.12835}))) * ps1 ///
+ (1/(1+exp(-1*{babst1=0.080779}))) * abst1) , noconstant
outreg using tabnl.out, ctitle("nlreg ps2")
```

```
nl (ump2 = (1/(1+exp(-1*{bvert=0.4552748}))) * vert + (1/(1+exp(-1*{bfn=0.6916209}))) * fn ///
+ (1/(1+exp(-1*{bump=1.08067}))) * ump + (1/(1+exp(-1*{bfg=-0.0268226}))) * fg ///
+ (1/(1+exp(-1*{bnpa=-1.843406}))) * npa + (1/(1+exp(-1*{blo=-4.848554}))) * lo ///
+ (1/(1+exp(-1*{bchem=8.691302}))) * chem + (1/(1+exp(-1*{bmodem=0.6638383}))) * modem ///
+ (1/(1+exp(-1*{bnda=0.8151347}))) * nda + (1/(1+exp(-1*{bps1=-0.1208622}))) * ps1 ///
+ (1/(1+exp(-1*{babst1=0.1197568}))) * abst1) , noconstant
outreg using tabnl.out, ctitle("nlreg ump2") append
```

```
nl (abst2 = (1/(1+exp(-1*{bvert=0.2686965}))) * vert + (1/(1+exp(-1*{bfn=0.2955072}))) * fn ///
+ (1/(1+exp(-1*{bump=-0.0492954}))) * ump + (1/(1+exp(-1*{bfg=0.0573914}))) * fg ///
+ (1/(1+exp(-1*{bnpa=0.2064076}))) * npa + (1/(1+exp(-1*{blo=4.062582}))) * lo ///
+ (1/(1+exp(-1*{bchem=-3.388913}))) * chem + (1/(1+exp(-1*{bmodem=0.1688705}))) * modem ///
+ (1/(1+exp(-1*{bnda=0.0305694}))) * nda + (1/(1+exp(-1*{bps1=-0.007488}))) * ps1 ///
+ (1/(1+exp(-1*{babst1=0.799464}))) * abst1) , noconstant
outreg using tabnl.out, ctitle("nlreg abst2") append
```

```
nlcom (1/(1+exp(-1*_b[bvert])))
nlcom (1/(1+exp(-1*_b[bfn])))
nlcom (1/(1+exp(-1*_b[bump])))
nlcom (1/(1+exp(-1*_b[bfg])))
nlcom (1/(1+exp(-1*_b[bnpa])))
nlcom (1/(1+exp(-1*_b[blo])))
nlcom (1/(1+exp(-1*_b[bchem])))
nlcom (1/(1+exp(-1*_b[bmodem])))
nlcom (1/(1+exp(-1*_b[bnda])))
nlcom (1/(1+exp(-1*_b[bps1])))
nlcom (1/(1+exp(-1*_b[babst1])))
nlcom (1/(1+exp(-1*_b[bdist2])))
```

```
/// gathering the five weaker candidate in a term of disturbance
```

```
gen disturb2 = npa + lo + chem + nda + vert
```

```

reg ps2 fn ump fg modem ps1 abst1 disturb2 , nocons
nl (ps2 = (1/(1+exp(-1*{bfn=0.1081921}))))*fn ///
+ (1/(1+exp(-1*{bump=-0.129065}))))*ump + (1/(1+exp(-1*{bfg=0.8824499}))))*fg ///
+ (1/(1+exp(-1*{bmodem=0.3208356}))))*modem ///
+ (1/(1+exp(-1*{bps1=1.185549}))))*ps1 + (1/(1+exp(-1*{babst1=0.0690074}))))*abst1 ///
+ (1/(1+exp(-1*{bdist2=0.5173411}))))*disturb2 , noconstant

```

```

reg ump2 fn ump fg modem ps1 abst1 disturb2 , nocons
nl (ump2 = (1/(1+exp(-1*{bfn=0.5214805}))))*fn ///
+ (1/(1+exp(-1*{bump=1.256843}))))*ump + (1/(1+exp(-1*{bfg=0.1691617}))))*fg ///
+ (1/(1+exp(-1*{bmodem=0.4114244}))))*modem ///
+ (1/(1+exp(-1*{bps1=-0.2184152}))))*ps1 + (1/(1+exp(-1*{babst1=0.1276644}))))*abst1 ///
+ (1/(1+exp(-1*{bdist2=0.1186059}))))*disturb2 , noconstant

```

```

reg abst2 fn ump fg modem ps1 abst1 disturb2 , nocons
nl (abst2 = (1/(1+exp(-1*{bfn=0.03703274}))))*fn ///
+ (1/(1+exp(-1*{bump=-0.1277774}))))*ump + (1/(1+exp(-1*{bfg=-0.0516113}))))*fg ///
+ (1/(1+exp(-1*{bmodem=0.2677398}))))*modem ///
+ (1/(1+exp(-1*{bps1=0.0328661}))))*ps1 + (1/(1+exp(-1*{babst1=0.8033281}))))*abst1 ///
+ (1/(1+exp(-1*{bdist2=0.3640527}))))*disturb2 , noconstant

```

/// Non linear estimation of a system of equation

```

nlsur (ps2 = (1/(1+exp(-1*{bfn=0.1081921}))))*fn ///
+ (1/(1+exp(-1*{bump=-0.129065}))))*ump + (1/(1+exp(-1*{bfg=0.8824499}))))*fg ///
+ (1/(1+exp(-1*{bmodem=0.3208356}))))*modem ///
+ (1/(1+exp(-1*{bps1=1.185549}))))*ps1 + (1/(1+exp(-1*{babst1=0.0690074}))))*abst1 + (1/(1+exp(-1*{bdist2=0.5173411}))))*disturb2)(ump2 =
(1/(1+exp(-1*{bfnu=0.5214805}))))*fn ///
+ (1/(1+exp(-1*{bumpu=1.256843}))))*ump + (1/(1+exp(-1*{bfgu=0.1691617}))))*fg ///
+ (1/(1+exp(-1*{bmodemu=0.4114244}))))*modem ///
+ (1/(1+exp(-1*{bps1u=-0.2184152}))))*ps1 + (1/(1+exp(-1*{babst1u=0.1276644}))))*abst1 + (1/(1+exp(-1*{bdist2u=0.1186059}))))*disturb2)(abst2 =
(1/(1+exp(-1*{bfna=0.03703274}))))*fn ///
+ (1/(1+exp(-1*{bumpa=-0.1277774}))))*ump + (1/(1+exp(-1*{bfga=-0.0516113}))))*fg ///
+ (1/(1+exp(-1*{bmodema=0.2677398}))))*modem ///
+ (1/(1+exp(-1*{bps1a=0.0328661}))))*ps1 + (1/(1+exp(-1*{babst1a=0.8033281}))))*abst1 + (1/(1+exp(-1*{bdist2a=0.3640527}))))*disturb2)

```

```

nlcom (1/(1+exp(-1*_b/bfn))))
nlcom (1/(1+exp(-1*_b/bump))))
nlcom (1/(1+exp(-1*_b/bfg))))
nlcom (1/(1+exp(-1*_b/bmodem))))
nlcom (1/(1+exp(-1*_b/bps1))))
nlcom (1/(1+exp(-1*_b/babst1))))
nlcom (1/(1+exp(-1*_b/bdist2))))

```

```

nlcom (1/(1+exp(-1*_b/bfnu))))
nlcom (1/(1+exp(-1*_b/bumpu))))
nlcom (1/(1+exp(-1*_b/bfgu))))
nlcom (1/(1+exp(-1*_b/bmodemu))))
nlcom (1/(1+exp(-1*_b/bps1u))))
nlcom (1/(1+exp(-1*_b/babst1u))))
nlcom (1/(1+exp(-1*_b/bdist2u))))

```

```

nlcom (1/(1+exp(-1*_b/bfna))))
nlcom (1/(1+exp(-1*_b/bumpa))))
nlcom (1/(1+exp(-1*_b/bfga))))
nlcom (1/(1+exp(-1*_b/bmodema))))
nlcom (1/(1+exp(-1*_b/bps1a))))
nlcom (1/(1+exp(-1*_b/babst1a))))
nlcom (1/(1+exp(-1*_b/bdist2a))))

```