LPHYS2266 - Physics of the upper atmosphere and space

Exercise

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Exercise

Calculate the Pannekoek-Rosseland electric field and the ionic density scale height at the terrestrial exobase assuming a protonosphere.[1]

Reminder of concepts

First we recall what the *protonosphere* is. It is a region of the Earth's atmosphere (or any planet with a similar atmosphere) where the dominant components are atomic hydrogen and ionic hydrogen (protons).[1] Atomic hydrogen ions become important above an altitude of $h = 500km$ which is also the altitude of the *exobase* for the neutral Earth's atmosphere[2] and the region from about 800km to 2000km is therefore called the *protonosphere*. The ionic altitude of the exobase is $h_c = 2000km$.
We also recall that the *exobase* is defined as the altitude where the mean free path of particles l is equal to the scale height H so $l = H$ or $Kn = 1$ with Kn beeing the Knudsen number defined as $Kn = l/H$.

Computation of the Pennekoek-Rosseland electric field

Since we assume a protonosphere where are dealing with an ionized atmosphere composed by electrons and protons. If we consider only the gravitational force, the heavier protons should remains at a lower altitude whereas electrons would form a huge negative charge at higher altitude.[2]
The reality is that with ionized particles an induced electric field $\mathbf{E}$ is created which established again the almost neutrality of the plasma. Since the Earth's mass is large enough to globally maintain the gases that escapes from its surface, we can consider with under a good approximation that the atmosphere is in hydrostatic equilibrium.[2] This implies that the mean vertical speed is $\mathbf{u} = \mathbf{0}$. Therefore the continuity equation becomes,

$$\frac{\partial n_i}{\partial t} = 0 \tag{0.1}$$

where n is the numerical density (number of particles per unit of volume) $[m^{-3}]$ and i is an index equals to $i = e$ for referring to electrons and $i = p$ for referring to protons. Under the hydrostatic assumption the motion equation simply becomes,

$$\frac{\partial \mathbf{P}}{\partial \mathbf{r}} = n_i \mathbf{F_i} \tag{0.2}$$

with $\mathbf{F}$ beeing the forces acting on the particles of type i. In the vertical direction from the surface r the motion equation Eq(0.2) becomes,

$$\frac{\partial p_e}{\partial r} = -n_e m_e g - n_e e E \tag{0.3}$$

$$\frac{\partial p_p}{\partial r} = -n_p m_p g + n_p e E. \tag{0.4}$$

By using the perfect gas law $p = nkT$ and by assuming $T_p = T_e$ and the quasi-neutrality i.e $n_e = n_p$ and so $\frac{\partial n_e}{\partial r} = \frac{\partial n_p}{\partial r}$ we can derive by subtracting the two motion equations that,

$$\boxed{e\mathbf{E} = -\frac{m_p - m_e}{2}\mathbf{g}.} \tag{0.5}$$

This is the equation of the Pannekoek-Rosseland electric field.
Now we want the compute explicitly the value of this field. We consider the assumption that

[1] A Julia script was made to compute the different quantities. It can be found at the following GitHub repository : `https://github.com/AmauryLaridon/LPHYS2266`

the Earth is an homogeneous sphere that display from the outside an isotropic and constant gravity field for a given altitude i.e $\mathbf{g} = g(r)\mathbf{e_r}$ with $\mathbf{e_r}$ being the radial reference vector of the canonical spherical frame of reference anchored at the center of the Earth. If we take into account that the module g varies with the radial distance r we have[2],

$$g(r) = g_0 \left(\frac{r_0}{r}\right)^2 \tag{0.6}$$

with r_0 being a level of reference. The radial structure of the gravity vector field extends at the structure of the Pannekoek-Rosseland electric field $\mathbf{E} = (E(r), 0, 0)$ and so,

$$E(r) = -g_0 \frac{m_p - m_e}{2e} \left(\frac{r_0}{r}\right)^2 \tag{0.7}$$

Now we compute with the consideration that $g(r = r_0 = 6\,371\,000m) = g_0 = 9,81m/s$,

$$E(r = r_c) = -g_0 \frac{m_p - m_e}{2e} \left(\frac{r_0}{r_c}\right)^2 \tag{0.8}$$

$$= -9,81 \frac{(1,672\,649.10^{-27} - 9,109.10^{-31})}{2(-1,602.10^{-19})} \left(\frac{6\,371\,000}{6\,371\,000 + 2\,000\,000}\right)^2 \tag{0.9}$$

Which finally yields to,

$$\boxed{E(r = r_c) = -2,96.10^{-8}\,\frac{N}{C}} \tag{0.10}$$

Computation of the ionic density scale height

We recall the definition of the scale height H,

$$H = -\left(\frac{d\,ln(n(r))}{dr}\right)^{-1} \tag{0.11}$$

and from the Eq(0.4) with the following hypothesis we can derive that[2],

$$n(r) = n_0 exp\left(-(r - r_0)\frac{\bar{m}g_0}{kT_0}\right) \tag{0.12}$$

with $\bar{m}$ being molecular mean mass $\bar{m} = \frac{\sum_i m_i n_i}{\sum_i n_i}$. A simple computation yields then to,

$$H = \frac{kT_0}{\bar{m}g_0}. \tag{0.13}$$

In a plasma of hydrogen (which is the case in the protonosphere) the mean atomic mass is equal to the half of the Hydrogen's mass,

$$\bar{m} = \frac{n_p m_p + n_e m_e}{n_p + n_e} \approx \frac{m_p}{2}. \tag{0.14}$$

If we consider a typical temperature in the ionosphere $T_0 = 1000°K$ at the level of reference $r = r_c = r_0 + h_c$ the ionic exobase[2] we compute[2],

$$H = 2\frac{kT_0}{m_p g_0} \tag{0.15}$$

$$= 2\frac{((1,38.10^{-23}).(1\,000))}{((1,672\,649.10^{-27}).(5,68)}, \tag{0.16}$$

[2]We use the value $k = 1,38.10^{-23}J/°K$ for the Boltzmann constant and for g_0 at the ionic exobase we have that $g_0 = 9,81.(6\,371\,000/(6\,371\,000 + 2\,000\,000) = 5,68\,m/s^2$

$$H = 2903,85\,km \tag{0.17}$$

We recall that the scale height is defined as the altitude at which the density is reduced by a factor e.

References

[1] Goodman J. *Space Weather Telecommunications.* Springer, 2004.

[2] Pierrard V. *LPHYS2266 - Physics of the upper atmosphere and space.* UCLouvain, 2023.