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Unit-2

Combinatorics

1) **Sum rule:-** $T \to n_1$ ways
$T \to n_2$ ways
$= n_1 + n_2$ ways

2) **Product rule:-** $T_1 \to n_1$ $= n_1 n_2$ ways
$T_2 \to n_2$

3) **Permutation:-** $^{n}P_{r} = \dfrac{n!}{(n-r)!}$

4) **Combination:-** $^{n}C_{r}$ or $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$\frac{n!}{n_1!\, n_2! \ldots n_k!}$$

5) **Binomial Theorem:-**

$$(x+y)^n = {}^{n}C_0 x^0 y^{n-0} + {}^{n}C_1 x^1 y^{n-1} + \ldots \; {}^{n}C_n x^n y^{n-n}$$

$$= \sum_{r=0}^{n} {}^{n}C_r x^r y^{n-r}$$

Co-efficient of $x^r y^{n-r} = {}^{n}C_r$

6) **Multinomial Theorem:-**

$$(x_1+x_2+x_3+\ldots+x_n)^n = \sum \binom{n}{n_1, n_2 \ldots n_k} x_1^{n_1} x_2^{n_2} x_3^{n_3} \ldots x_k^{n_k}$$

$\boxed{\text{Co-eff. of } x_1^{n_1} x_2^{n_2} \ldots x_k^{n_k} = \binom{n}{n_1, n_2, n_3 \ldots n_k}}$ $x_k^{n_k}$ — $n_1, n_2 \ldots n_k \le n$
— $n_1 + n_2 + \ldots + n_k = n$

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Problems:-

1) Find the co-efficients of the following:-

a) $x^9 y^3$ in $(2x-3y)^{12}$

b) xyz^2 in $(2x-y-z)^4$

c) $a^2b^3c^2d^5$ in $(a+2b-3c+2d+5)^{16}$

d) $x^{11}y^4$ in $(2x^3-3xy^2+z^2)^6$

a) $(2x-3y)^{12} = \sum {}^nC_r x^r y^{n-r}$

$= \sum {}^{12}C_r (2x)^r (-3y)^{12-r}$

Co-eff of $x^r y^{n-r} = {}^{12}C_r (2)^r (-3)^{12-r}$

$x^9y^3 = {}^{12}C_9 (2)^9 (-3)^{12-9}$

$= {}^{12}C_9\, 2^9 (-3)^3$

b) $(2x-y-z)^4 = \sum \binom{4}{n_1,n_2,n_3} (2x)^{n_1} (-y)^{n_2} (-z)^{n_3}$

Co-eff. of $x^{n_1}y^{n_2}z^{n_3} = \binom{4}{n_1,n_2,n_3} (2)^{n_1}(-1)^{n_2}(-1)^{n_3}$

$xyz^2 = \binom{4}{1,1,2} 2^1(-1)^1(-1)^2$

$= \frac{4!}{}$ Here $n_1+n_2+n_3 = 1+1+2 = 4 = n$.

$xyz^2 = \frac{4!}{1!1!2!}(2)(-1) = -4\cancel{!}$

$\boxed{= -24!}$

c) $(a+2b-3c+2d+5)^{15} = \sum \binom{16}{n_1 n_2 n_3 n_4 n_5} (a)^{n_1}(2b)^{n_2}(-3c)^{n_3}(2d)^{n_4}(5e^0)^{n_5}$

Take $5 = 5e^0$

Co-eff. of $a^{n_1}b^{n_2}c^{n_3}d^{n_4}e^{n_5} = \binom{16}{n_1 n_2 n_3 n_4 n_5} 2^{n_2}(-3)^{n_3}(2)^{n_4} 5^{n_5}$

$\cancel{a^2b^3c^2d^5e^0 = \frac{5!}{2!3!2!5!0!}\left[2^3 \times (-3)^2 \times 2^5 \times 5^0\right]}$

$\cancel{= \frac{2 \times 2 \times 2^6 \times (-3)(-3)}{2!2!3!} = 3 \times 2^5}$

$n_1+n_2+n_3+n_4+n_5 = n$

$2+3+2+5+n_5 = 16$

$\Rightarrow n_5 = 4$

$= \frac{16!}{2!3!2!5!4!}\left[2^8 \times (-3)^2 \times 5^4\right]$

d) $(2x^3 - 3xy^2 + z^2)^6 = \sum \binom{6}{n_1 n_2 n_3} (2x^3)^{n_1}(-3xy^2)^{n_2}(z^2)^{n_3}$

~~Co-eff. of~~ $= \sum \binom{6}{n_1 n_2 n_3} 2^{n_1}(-3)^{n_2}(x)^{3n_1+n_2}(y)^{2n_2}(z)^{2n_3}$

Co-eff. of $x^{11}y^4 =$

$3n_1 + n_2 = 11$

and $2n_2 = 4$

$\Rightarrow n_2 = 2$

and $n_1 = 3$

$n_1+n_2+n_3 = 6$

$3+2+n_3 = 6$

$n_3 = 1$

$= \frac{6!}{3!2!1!}\left[2^3(-3)^2(z)^2\right]$

$= 60(2^3)(9)z^2$

$= 4320z^2$

60×72

120

$420 \times$

4320

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e) x^3y^2 in $(5x-7y)^5$.

$$(5x-7y)^5 = \sum_1 \cancel{\binom{5}{n_1 n_2}} \sum_1 {}^5C_r (5x)^r(-7y)^{5-r}$$

$$\text{Co-eff. of } x^3y^2 = {}^5C_r \, 5^r(-7)^{5-r}$$

$$= \frac{5!}{2!} 5^3(-7)^2$$

$$= \left(\frac{5!}{2!}\right) 5^3 \cdot 7^2 .$$

f) x^2y^3z in $(2x+3y^2-5z)^6$

$$(2x+3y^2-5z)^6 = \sum_1 \binom{6}{n_1 n_2 n_3}(2x)^{n_1}(3y^2)^{n_2}(-5z)^{n_3}$$

$$\text{Co-eff. of } x^{n_1}y^{n_2}z^{n_3} = \binom{6}{n_1 n_2 n_3}(2^{n_1})(3)^{n_2}(-5)^{n_3}$$

$$n_1+n_2+n_3 = n$$

$$2+\cancel{3}1+1 = 6 = n.$$

$$x^2y^3z = \frac{6!}{2!\,3!\,1!}\left[2^2 \times 3^3 \times (-5)^1\right].$$

g) xyz^{-2} in $(x-2y+3z^{-1})^4$

Let $z^{-1} = w$

$\Rightarrow z^{-2} = w^2$.

xyw^2 in $(x-2y+3w)^4$

$$(x-2y+3w)^4 = \sum_1 \binom{4}{n_1 n_2 n_3}(x)^{n_1}(-2y)^{n_2}(3w)^{n_3}$$

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$$\text{Co-eff. of } x^{n_1}y^{n_2}w^{n_3} = \binom{4}{n_1 n_2 n_3}(-2)^{n_2}(3)^{n_3}$$

Here $n_1 = 1, n_2 = 1, n_3 = 2$

$$= \frac{4!}{1!\,1!\,2!}\left[(-2)^1(3)^2\right]$$

$$= 12(-18)$$

$$= \boxed{-216}.$$

180
36
216

h) x^0 in $\left(3x^2 - \frac{2}{x}\right)^{15}$

Let $1/x = w$

$$(3x^2 - 2w)^{15} = \sum \binom{15}{n_1 n_2}(3x^2)^{n_1}(-2w)^{n_2}$$

or

$$\sum {}^{15}C_r (3x^2)^r(-2w)^{15-r}$$

$$\text{Co-eff. of } x^0 = {}^{15}C_r\, 3^r(-2)^{15-r}$$

$$(x^{2r})(w)^{15-r}$$

$$\frac{x^{2r}}{x^{15-r}} = x^{2r-15+r} = x^{3r-15} = x^0$$

$$\Rightarrow 3r - 15 = 0$$

$$\Rightarrow \boxed{r = 5}.$$

$$= {}^{15}C_5\, 3^5(-2)^{10}$$

$$= \frac{15!}{5!\,10!}(3^5)(2^{10})$$

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i) x^{12} in $x^3(1-2x)^{10}$

$$(x^3)^{\frac{10}{10}}(1-2x)^{10} = (x^{\frac{3}{10}})^{10}(1-2x)^{10}$$

$$= \left[x^{3/10} - 2x(x^{3/10})\right]^{10}$$

$$= (x^{3/10} - 2x^{4/10})^{10}$$

Or

$$x^3(1-2x)^{10} = x^3 \sum {}^{10}C_r (1)^r (-2x)^{10-r}$$

$$= \sum {}^{10}C_r (-2x)^{10-r} (x)^3$$

$$= \sum {}^{10}C_r (-2)^{10-r} (x)^{13-r}$$

$$x^{12} = x^{13-r}$$

$$\Rightarrow r = 1$$

$$\Rightarrow \text{co-eff. of } x^{12} = {}^{10}C_1 (-2)^{10-1} = \underline{10(-2)^9}$$

j) $x^2y^2z^3$ in $(x+y+z)^7$

k) x^3z^4 in $(x+y+z)^7$

l) xyz^2 in $(x-2y+3z^{-1})^4$

m) $w^3x^2yz^2$ in $(2w-x+3y-2z)^8$

j) $$(x+y+x)^7 = \sum \binom{7}{n_1 n_2 n_3} x^{n_1} y^{n_2} z^{n_3}$$

$$\text{co-eff. of } x^{n_1} y^{n_2} z^{n_3} = \binom{7}{n_1 n_2 n_3}$$

Here, $n_1 = 2$, $n_2 = 2$, $n_3 = 3$

$n_1 + n_2 + n_3 = 7 = n$

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$$x^2y^2z^3 = \frac{7!}{2!\,2!\,3!}$$

k) $(x+y+z)^7 = \binom{7}{n_1, n_2, n_3} x^{n_1} y^{n_2} z^{n_3}$

Co-eff. of $x^{n_1}y^{n_2}z^{n_3} = \binom{7}{n_1, n_2, n_3}$

x^3z^4

$\Rightarrow n_1 = 3, \; n_2 = 0, \; n_3 = 4$

$n_1 + n_2 + n_3 = 7 = n.$

$$x^3z^4 = \frac{7!}{3!\,0!\,4!} = \frac{7\times 6\times 5}{3!}$$

$$= \underline{35}.$$

l) $(x - 2y + 3z^{-1})^4$

Let $z^{-1} = w$

$(x - 2y + 3w)^4 = \binom{4}{n_1, n_2, n_3} x^{n_1}(-2y)^{n_2}(3w)^{n_3}$

Co-eff. of $x^{n_1}y^{n_2}w^{n_3} = \binom{4}{n_1, n_2, n_3}(-2)^{n_2}(3)^{n_3}$

$xyz^{-2} = xyw^2$

$\Rightarrow n_1 = 1, \; n_2 = 1, \; n_3 = 2$

$n_1 + n_2 + n_3 = 4 = n.$

$$xyw^2 = \frac{4!}{2!}(-2)^1(3)^2 = \underline{-4! \times 9}.$$

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m) $(2w - x + 3y - 2z)^8 = \sum \binom{8}{n_1 n_2 n_3 n_4} (2w)^{n_1} (-x)^{n_2} (3y)^{n_3} (-2z)^{n_4}$

co-eff. of $w^{n_1} x^{n_2} y^{n_3} z^{n_4} = \binom{8}{n_1 n_2 n_3 n_4} 2^{n_1} (-1)^{n_2} 3^{n_3} (-2)^{n_4}$

$w^3 x^2 y z^2 \Rightarrow n_1 = 3,\ n_2 = 2,\ n_3 = 1,\ n_4 = 2$

$n_1 + n_2 + n_3 + n_4 = 8 = n.$

$= \frac{8!}{3!\,2!\,1!\,2!} \left[(2)^3 (-1)^2 (3)^1 (-2)^2\right]$

$= \frac{8!}{3!\,2!\,2!} (3 \times 2^5) = \underline{8!\,(2^2)}.$

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Principle of Inclusion-Exclusion:-

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - \{|A_1 \cap A_2| + |A_2 \cap A_3| + |A_1 \cap A_3|\} + |A_1 \cap A_2 \cap A_3|.$$

General form:- (Principle of Inclusion-Exclusion)

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma A_i - \Sigma|A_i \cap A_j| + \Sigma|A_i \cap A_j \cap A_k| + \ldots + (-1)^{n-1}\Sigma|A_i \cap A_j \cap A_k \cap \ldots \cap A_n|$$

De-Morgan's Law:-

$$|\overline{A_1 \cup A_2 \cup \ldots \cup A_n}| = |\bar{A_1} \cap \bar{A_2} \cap \ldots \cap \bar{A_n}|$$

$$= |S| - |A_1 \cup A_2 \cup \ldots \cup A_n|$$

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = S_1 - S_2 + S_3 + \ldots + (-1)^{n-1}S_n$$

where $S_1 = \Sigma|A_i|$

$S_2 = \Sigma|A_i \cap A_j|$

$S_3 = \Sigma|A_i \cap A_j \cap A_k|$

Note:-

1) The following exp. determine the no. of elements in S that satisfy exactly m of n conditions.

$$E_m = S_m - \binom{m+1}{1}S_{m+1} + \binom{m+2}{2}S_{m+2} + \ldots + (-1)^{n-m}\binom{n}{n-m}S_n$$

$0 \leq m \leq n$

The follow

2) The following expression determine the no. of elements in S that satisfy atleast m of n, conditions.

$$L_m = S_m - \binom{m}{m-1} S_{m+1} + \binom{m+1}{m-1} S_{m+2} + \dots + (-1)^{n-m} \binom{n-1}{m-1} S_n$$

Problems :-

1) Among the students in hostel, 12 study Maths (A), 20 study ~~biology~~ physics (B), 20 chemistry (C) and 8 study biology (D). There are 5 students for A & B, 7 for A & C, 4 for A & D, 16 for B & C, 4 for B & D and 3 students for C & D. There are 3 students studying A, B & C, 2 for A, B, D, 2 for B, C, D and 3 for A, C and D. Finally ~~the~~ 2 students ~~no. of~~ Students studying all. Furthermore, there are 71 students who don't study any subjects. Find the total no. students in the hostel.

Given :- $|A| = 12$, $|B| = 20$, $|C| = 20$, $|D| = 8$

$|A \cap B| = 5$, $|A \cap C| = 7$, $|A \cap D| = 4$, $|B \cap C| = 16$,

$|B \cap D| = 4$, $|C \cap D| = 3$

$|A \cap B \cap C| = 3$, $|A \cap B \cap D| = 2$, $|B \cap C \cap D| = 2$,

$|A \cap C \cap D| = 3$

$|A \cap B \cap C \cap D| = 2$

$|\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}| = 71$

To find:-

$|S|$

$|\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D}| = |S| - |A\cup B\cup C\cup D|$

$|A\cup B\cup C\cup D| = S_1 - S_2 + S_3 - S_4$

$S_1 = \sum|A_i| = 12+20+20+8 = 60.$

$S_2 = \sum|A_i\cap A_j| = 5+7+4+16+4+3 = 39$

$S_3 = \sum|A_i\cap A_j\cap A_k| = 3+2+2+3 = 10$

$S_4 = \sum|A_i\cap A_j\cap A_k\cap A_l| = 2.$

$\Rightarrow |A\cup B\cup C\cup D| = 60-39+10-2 = \cancel{71}\ 29$

$|S| = 71+29 = \underline{100}.$

2) Find the no. of permutations of 26 letters of the English alphabets which contains ~~no~~.
i) none of the patterns 'car', 'dog', 'pun', 'byte' occurs
ii) exactly 2 patterns occurs
iii) atleast 2 patterns occurs.

$|S| = 26!$

(occurance of patters)

A_1 - car $= (1+23)! = 24!$

A_2 - dog $= (1+23)! = 24!$

A_3 - pun $= (1+23)! = 24!$

A_4 - byte $= (1+22)! = 23!$

$S_1 = 1.8872\times10^{24}$

$S_2 = 3.5253\times10^{21}$

$|A_1\cap A_2| = (1+1+20)! = 22!$

$|A_1\cap A_3| = (1+1+20)! = 22!$

$|A_1\cap A_4| = (1+1+19)! = 21!$

$|A_2\cap A_3| = 22!$

$|A_2\cap A_4| = 21!$

$|A_3\cap A_4| = 21!$

$$\begin{array}{r} 71\cancel{0}10 \\ -41 \\ \hline 29 \end{array}$$

$S_3 = 2.7978E$

$|A_1 \cap A_2 \cap A_3| = (1+1+1+17)! = 20!$

$|A_1 \cap A_2 \cap A_4| = (1+1+1+16)! = 19!$

$|A_1 \cap A_3 \cap A_4| = 19!$

$|A_2 \cap A_3 \cap A_4| = 19!$

$S_4 = 355.6874T$

$|A_1 \cap A_2 \cap A_3 \cap A_4| = (1+1+1+1+13) = 17!$

i) $|\bar{A_1} \cap \bar{A_2} \cap \bar{A_3} \cap \bar{A_4}| = |S| - |A_1 \cup A_2 \cup A_3 \cup A_4|$

$= |S| - |S_1 - S_2 + S_3 - S_4|$

$= 26! - |1.8907 \times 10^{24}|$

$= 401.4007 \times 10^{24}$

ii) $E_2 = S_2 - {}^3C_1 S_2 + {}^4C_2 S_2$

iii)

3) How many integers ~~b/w~~ from 1 and 300 are
i) divisible by atleast one of no. 5, 6 or 8.
ii) divisible by none of the nos. 5, 6 or 8.
iii) determine the no. of integers divisible by 5, 6 and 8
a) Exactly 2 of them
b) atleast 2 of them

$|S| = 300$

A_1 - divisible by 5 $\Rightarrow |A_1| = \frac{300}{5} = 60.$

A_2 - —''— 6 $\Rightarrow |A_2| = \frac{300}{6} = 50.$

A_3 = —''— 8 $\Rightarrow |A_3| = \frac{300}{8} = 37.5 = 37$

$|A_1 \cap A_2|$ —''— by both 5 and 6

$|A_1 \cap A_2| = \frac{300}{LCM(5,6)} = 10.$

$|A_2 \cap A_3| = \frac{300}{LCM(6,8)} = \frac{300}{24} = 12$

$2 \,|\, 6, 8$
$\quad 3, 4$

$|A_1 \cap A_3| = \frac{300}{LCM(5,8)} = 7.5 = 7$

$|A_1 \cap A_2 \cap A_3| = \frac{300}{LCM(5,6,8)} = \frac{300}{24 \times 5} = 2$

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i) atleast by 5, 6 or 8.

$|A_1 \cup A_2 \cup A_3| = S_1 - S_2 + S_3$

$S_1 = \sum |A_i| = 60 + 50 + 37 = 147$

$S_2 = \sum |A_i \cap A_j| = 10 + 12 + 7 = \cancel{28}\ 29$

$S_3 = \sum |A_i \cap A_j \cap A_k| = 2$

$|A_1 \cup A_2 \cup A_3| = 147 - 29 + 2$

$= 120.$

ii) $|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = |S| - |A_1 \cup A_2 \cup A_3|$

$= 300 - 120$

$= 180.$

iii) a) $E_2 = S_2 - {}^3C_1 \cdot S_3$

$= 29 - {}^3C_1(2) = 23.$

b) $L_2 = S_2 - {}^2C_1(S_3) = 29 - 2(2)$

$= 25.$

4) Find the no. of Permutations of English alphabets in wch none of the patterns (spin, path, game, net) occurs.

$|S| = 26!$

S_1

$|A_1| =$ spin $= (1+22)! = 23!$

$|A_2| =$ path $= (1+22)! = 23!$

$|A_3| =$ game $= (1+22)! = 23!$

$|A_4| =$ net $= (1+23)! = 24!$

S_2

$|A_1 \cap A_2| = 0$

$|A_1 \cap A_3| = (1+1+18)! = 20!$

$|A_1 \cap A_4| = (1+1+19)! = 21!$

$|A_2 \cap A_3| = 0$

$|A_2 \cap A_4| = 0$

$|A_3 \cap A_4| = (1+1+19)! = \cancel{21!}\ 0$

S_3

$|A_1 \cap A_2 \cap A_3| = 0$

$|A_1 \cap A_2 \cap A_4| = 0$

$|A_1 \cap A_3 \cap A_4| = 0$

$|A_2 \cap A_3 \cap A_4| = 0$

S_4

$|A_1 \cap A_2 \cap A_3 \cap A_4| = 0.$

$|A_1 \cup A_2 \cup A_3 \cup A_4| = S_1 - S_2 + S_3 - S_4$

$= (23! + 23! + 23! + 24!) - (20! + 21!) + 0 - 0$

$= 697 \times 10^{21}.$

$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}| = |S| - |A_1 \cup A_2 \cup A_3 \cup A_4|$

$= 26! - 697 \times 10^{21}$

$= 402 \times 10^{24}.$

5) In how many ways 5 no. of A's, 4 no. of B's and 3 no of C's can be arranged so that all the identical letters are not in a single block.

A_1 = occurance of A's

$|S| = \frac{12!}{5!\,4!\,3!}$

$|S| = 27720$

$|A_1| = \frac{(1+7)!}{4!\,3!} = \frac{8!}{4!\,3!}$

A_2 – occurance of B's

$$|A_2| = \frac{(1+8)!}{5!\,3!} = \frac{9!}{5!\,3!}$$

A_3 – occurance of C's

$$|A_3| = \frac{(1+9)!}{5!\,4!} = \frac{10!}{5!\,4!}$$

$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = |S| - |A_1 \cup A_2 \cup A_3|$$

$$|A_1 \cup A_2 \cup A_3| = S_1 - S_2 + S_3$$

$$\cancel{S_1 = \Sigma|A_i| = 1}$$

$$\cancel{S_2 = \Sigma|A_i \cap A_j| = \frac{(1+1+3)!}{3!} = \frac{5!}{3!}}$$

$$\cancel{S_3 = \Sigma|A_i \cap A_j \cap A_k| = (1+1+1)! = 3!}$$

$$|A_1 \cap A_2| = \frac{(1+1+3)!}{3!} = \frac{5!}{3!}$$

$$|A_2 \cap A_3| = \frac{(5+1+1)!}{5!} = \frac{7!}{5!}$$

$$|A_1 \cap A_3| = \frac{(1+4+1)!}{4!} = \frac{6!}{4!}$$

$$|A_1 \cap A_2 \cap A_3| = (1+1+1)! = 3! = \underline{6}$$

$$\Sigma|(A_i \cap A_j)| = \frac{5!}{3!} + \frac{7!}{5!} + \frac{6!}{4!} = \underline{92}$$

$$\Sigma|A_i| = \frac{8!}{4!\,3!} + \frac{9!}{5!\,3!} + \frac{10!}{5!\,4!} = 2044$$

$$= \underline{2044}$$

$|A_1 \cup A_2 \cup A_3| = 2044 + 92 + 6 = 2142 - 1958$

$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = 27720 - 2142$
$= \underline{25762}.$

6) Out of 30 students in a hostel, 15 study history, 8 study economics and 6 study geology. It is known that 3 students study all these subjects. S.T. 7 or more students study none of these subjects.

$|S| = 30$ (No. of Students in the hostel)

A_1 – No. of students study history
$|A_1| = 15$

A_2 – – " — economics. A_3 – " — geology
$|A_2| = 8$ $|A_3| = 6$.

$~~|A_1 \cup A_2 \cup A_3|~~$ $S_1 = 8 + 15 + 6 = 29.$

$S_3 \rightarrow |A_1 \cap A_2 \cap A_3| = 3$

To prove:- $|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| \geq 7$

$|A_1 \cup A_2 \cup A_3| = |S| - (S_1 - S_2 + S_3)$
$= 30 - (29 - S_2 + 3)$
$= S_2 - 2. \longrightarrow (*)$

WKT, $S_2 = \Sigma_1 (A_i \cap A_j)$ where $i \leq j \leq 3$
Also $A_1 \cap A_2 \cap A_3 \subseteq (A_i \cap A_j)$

$\Rightarrow |A_i \cap A_j| \geq 3$

$\Sigma_1 |A_i \cap A_j| \geq 3+3+3$

$S_2 = \Sigma_1 |A_i \cap A_j| \geq 9 \longrightarrow (1)$

Substitute (1) in (*)

$\Rightarrow |\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| \geq 9-2 \geq 7$

7) In how many ways can one arrange the letters in the word 'CORRESPONDE~~NTS~~NTS' so that

i) there is no pair of consecutive identical letters

ii) There are exactly two pairs of consecutive identical letters.

iii) There are ~~exact~~ atleast 2 pairs of consecutive identical letters.

$|S| = \dfrac{14!}{2!\,2!\,2!\,2!\,2!}$

O-2, R-2, E-2, S-2, N-2

C-1, P-1, T-1, D-1

A_1 - occurance of pair of O's

A_2 = — " — R's

A_3 = — " — E's

A_4 = — " — S's

A_5 = — " — N's

$|A_1| = \dfrac{(1+12)!}{(2!)^4} = \dfrac{13!}{(2!)^4} = |A_2| = |A_3| = |A_4| = |A_5|$

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$$|A_i \cap A_j| = \frac{(1+1+10)!}{(2!)^3} = \frac{12!}{(2!)^3}$$
$(i \le j \le 5)$

1 2 3
1 2

$$|A_i \cap A_j \cap A_k| = \frac{(1+1+1+8)!}{(2!)^2} = \frac{11!}{(2!)^2}$$

$$|A_i \cap A_j \cap A_k \cap A_l| = \frac{(1+1+1+1+6)!}{2!} = \frac{10!}{2!}$$

1 2 3 4
1 2 3 5
1 3 4 5
2 3 4 5
1 2 4 5

$$|A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5| = (1+1+1+1+1+4)! = 9!$$

i) $$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4 \cap \bar{A}_5| = |S| - [S_1 - S_2 + S_3 - S_4 + S_5]$$

$$= \frac{14!}{(2!)^5} - \left[\left(5 \times \frac{13!}{(2!)^4}\right) - \left(5 \times \frac{12!}{(2!)^3}\right) + \left(5 \times \frac{11!}{(2!)^2}\right) - 5\left(\frac{10!}{2!}\right) + 9!\right]$$

$$= 2.7243G - [1.9459G - 299.376M + 49.896M - 9.072M + 362.88k]$$

$$= 2724.3M - [1945.9M - 299.376M + 49.896M - 9.072M + 0.363M]$$

$$= 2724.3M - 1.688k$$

$$= \underline{2.7226M}.$$

RAJDHANI

De-arrangement :-

$$d_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{(-1)^n}{n!}\right) \quad n \leq 6$$

A permutation of 'n' distinct objects in which none of the objects is in its origin place is called a derrangement.

Eg:- 1 $\Rightarrow d_1 = 0$

12 $\Rightarrow$ 21 $\Rightarrow d_2 = 1$

123 $\Rightarrow$ 231 $\Rightarrow d_3 = 2$

312

Formula :- $d_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{(-1)^n}{n!}\right] \quad n \leq 6$

$$d_n = e^{-1} n! \simeq 0.3679 \times n! \quad n \geq 7$$

Problems :-

1) Evaluate $d_2, d_3, d_4, d_5, d_6, d_7, d_8$:-

$d_2 = \frac{n!}{e} = \frac{2!}{e} = 1$ $\qquad d_6 = \frac{6!}{e} =$

$d_3 = \frac{3!}{e} =$ $\qquad d_7 = \frac{7!}{e}$

$d_4 = \frac{4!}{e} =$ $\qquad d_8 = \frac{8!}{e} = 14.8329k.$

$d_5 = \frac{5!}{e} =$

2) From the set of all permutation of 'n' distinct objects, one permutation is chosen at random. What is the probability that it is not a derrangement.

W.K.T, $P(A) + P(\bar{A}) = 1$

$P(\text{Not a derrangement}) = 1 - \frac{d_n}{n!}$

3) There are 8 letters to 8 diff. people to be placed in 8 diff. addressed envelopes. Find the no. ways of doing this so that atleast one letter gets to the right person.

Eeight letters can be placed in 8 envelopes in 8! ways.

Dearrangement takes place in d_8 ways.

$= 8! - d_8$	$P(x \geq 1) = 1 - P(x=0)$
$= 8! - \frac{8!}{e}$	$= 1 - \frac{d_8}{8!}$
$=$	$= 8! - d_8$

4) There are 'n' pairs of children's gloves in a box. Each pair is of a diff. colour. Suppose the right gloves are distributed at random to 'n' children and thereafter the left gloves are also distributed to them at random. Find the prob. that:

i) No child gets a matching pair

ii) Every child gets a matching pair.
iii) Exactly one child — " ——.
iv) Atleast 2 children — " ——

'n' pair of gloves can be distributed in 'n! ways.'

P(no child gets a matching pair)

i) $= \frac{d_n}{n!} = \frac{n!}{e\,n!} = \frac{1}{e} = 0.3679.$

ii) $\frac{1}{n!}$ P(Every child gets a matching pair).

iii) $\frac{1 \times d_{n-1}}{n!} = \frac{(n-1)!}{e\,n(n-1)!} = \frac{0.3679}{n}.$

x - getting a matching pair

iv) $P(x \geq 2) = 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[\frac{d_n}{n!} + \frac{d_{n-1}}{n!}\right]$

$= \frac{n! - (d_n + d_{n-1})}{n!}.$

5) At a restuarant, 10 men hand over their umbrellas to the receptionist. In how many ways can their umbrellas be returned so that
i) no man receives his own umbrella
ii) atleast one of the men — " ——
iii) atleast 2 men — " ——

Sample space

10 men can hand over their umbrellas in $10!$ ways.

i) $d_{10} = \frac{10!}{e} = 1.3349M.$

ii) $10! - \frac{10!}{e} = 2.2938\,M.$

iii) $10! - \left[\frac{10!}{e} + \frac{9!}{e}\right] = 2.1603M.$

6) For the positive integers '1 to n', there are 11,660 derrangements where 1, 2, 3, 4, 5 appear in the first five positions. What is the value of 'n'?

$\underbrace{1\ 2\ 3\ 4\ 5}\ \underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{n-5}$

$d_5 \times d_{n-5} = 11,660$

$d_{n-5} = \frac{11,660}{d_5} \simeq 265$

$d_6 \simeq 265$

$\Rightarrow n-5 = 6$

$\boxed{n=11}$.