Unit 4

Statistical Interference

Population (N) → set of objects

Sample (n) → subsets of population

large sample if $n \geq 30$
Small sample if $n < 30$

Parameters → Statistical measures are constants obtained from the population.
Eg: pop, mean, variance.

Statistics: Statistical quantities computed from sample observations.

μ → pop mean
σ → pop SD
p → pop proportions

$\bar{x}$ → sample mean
s → sample SD
$\hat{p}$ → sample proportions

Statistical Inferences deals with methods of drawing valid or logical generalizations & predictions about the population using the info contained in sample with an indication of accuracy of such inference.

Hypothesis: It is a claim (or) statement about a population parameter that we want to test.

Null Hypothesis (H_0) : It is the currently accepted value for a parameter

Alternative Hypothesis: This involves the claim to be tested.

Significance level: In a hypothesis test, the significance level α is the probability of making the wrong decision when the null hypothesis is true.

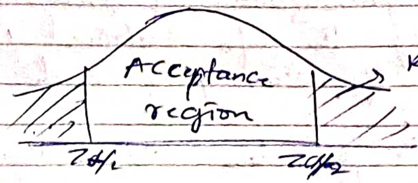
Confidence levels: In a hypothesis test, the probability that if a poll/test/survey were repeated over and over again, the result obtained would be the same. A confidence lvl : $C = 1 - \alpha$

Confidence level is either 95% (or) 99%.

Decision Rule:

* Case 1: (H_0) : $\mu = a$ & $H_1 : \mu \neq a$ at α %
accept H_0 when $|z| < z_{\alpha/2}$

Two tailed Test:



Rejection

Accept H_0 if $-z_{\alpha/2} < z < z_{\alpha/2}$

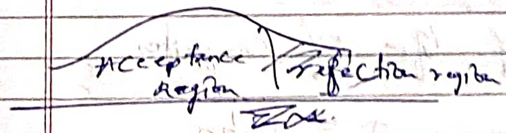
Acceptance region depends on α.

Left tailed Test:

* Case 2 : $H_0 : \mu = a$ & $H_1 : \mu < a$ at α %
accept H_0 when $z > z_\alpha$

Right tailed test:

Case 3: $H_0 : \mu = a$, $H_1 : \mu > a$, at α % accept when $z < z_\alpha$



Test of hypothesis conser

→ Let μ & σ^2 be mean & variance of a sample population from which a random sample of size 'n' be drawn. Let $\bar{x}$ be mean of sample (large sample) from central limit it follow that sampling distribution of $\bar{x}$ is approximately normally distributed of $\bar{x}$ is approximately normally distributed with mean $\mu_{\bar{x}} = \mu$ &

SD: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

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The test statistic for single mean is:

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Test of Hypothesis concerning a sin

(1) The length of life of certain computers is approximately normally distributed with mean 800 hrs & SD 40 hrs. If a random sample of 30 computers, has an average life of 788 hrs (i) Test whether there is difference in average hrs.
(ii) Test whether $\mu \neq 800$ hrs at $\alpha = 5\%$

→ x → length of life of computer

$\mu = 800$ hrs , $\sigma = 40$ hrs

$\bar{x} = 788$ hrs

$$Z = \frac{\bar{x} - \mu}{S.D/\sqrt{n}}$$

Decision rule:

At $\alpha = 5\%$ accept H_0 if $|Z| < z_{\alpha/2}$

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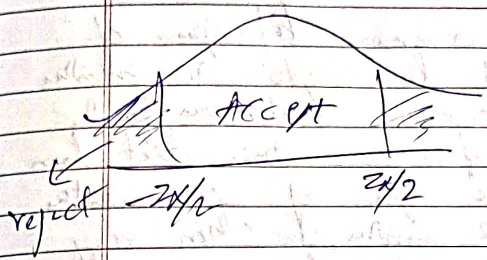

$0.025 = 0.5 - \phi(z_{\alpha/2})$

$\phi(z_{\alpha/2}) = 0.5 - 0.025$
$= 0.475$

$\Rightarrow z_{\alpha/2} = 1.96$

Numerical computation:

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{788 - 800}{40/\sqrt{30}}$$

$Z = -1.6432$

Conclusion: $Z = -1.6432$
$\Rightarrow |Z| = 1.6432$

$\Rightarrow |Z| < z_{\alpha/2} = 1.96$

Accept H_0

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(1) Mice with a average lifespan of 32 months will live upto 40 months when fed by certain nutritious food. If 64 mice fed on this diet han an average life span of 38 months & SD of 5.8 months, Is there any reason to believe that, life span is less than 40.

$x \to$ lifespan

$n = 69$
$\mu = 40$
$\sigma =$

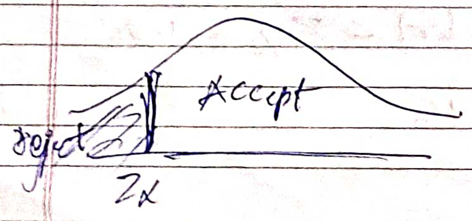
$\bar{x} = 38$ months
$S = 5.8$ months.

$H_0 : \mu = 40$
$H_1 : \mu < 40$

$Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

Decision rule:

at $\alpha = 5\%$ accept H_0 when
$\cancel{|Z| > Z_\alpha}$ $Z > Z_\alpha$



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$\cancel{0.025}$ $0.05 = 0.5 - \phi(Z_\alpha)$
$\phi(Z_\alpha) = 0.5 - 0.05$
$\phi(Z_\alpha) = 0.45$

$Z_\alpha = -1.65$

Numerical computation:

$Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \dfrac{38 - 40}{5.8/\sqrt{64}}$

$= \dfrac{-2 \times 8}{5.8}$

$= \dfrac{-16}{5.8}$

$= \cancel{2.78}$

$= -2.758$

Conclusion: $Z = -2.758 \not> -1.65$.

reject H_0

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(3) A machine runs on ~~an~~ an average of 125 hrs per year. A random sample of 49 machines has an annual average use of 126.9 hrs with SD 8.4 hrs. Does this suggest to believe that machines are used on the average more than 125 hrs annually at 0.05 level of significance.

→ $x \to$ running of machine hrs in year.

$\mu = 125$

$n = 49$

$\bar{x} = 126.9$

$s = 8.4$

$\alpha = 5\%$

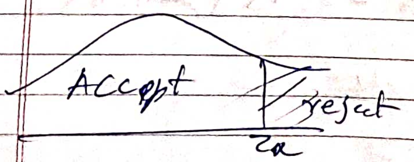
$H_0 : \mu = 125$

$H_1 : \mu > 125$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Decision rule:

at $\alpha = 5\%$ accept H_0 when $z < z_\alpha$



~~0.05~~

$0.05 = 0.5 - \phi(z_\alpha)$

$\phi(z_\alpha) = 0.45$

$z_\alpha = 1.65$

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Numerical computation:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{126.9 - 125}{8.4/\sqrt{49}}$$

$$= \frac{1.9 \times 7}{8.4}$$

$$z = 1.5833$$

Conclusion:

$\cancel{|z| \leq}$

$z < z_\alpha$

Accept H_0

(4) To determine whether mean breaking strength of a synthetic fibre is 8 kilo or not, a random sample of 50 fibres. There are tested a mean breaking strength of 7.8 kilo. If SD is 0.5 kilo. Test at 0.01 level of significance.

→ $x \to$ ~~mea~~ Breaking strength

$\mu = 8$

$n = 50$

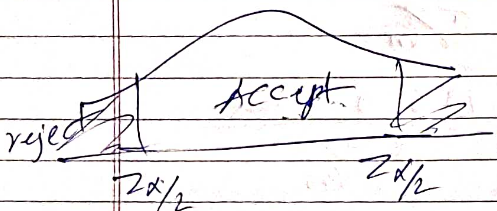
$\bar{x} = 7.8$

$s = 0.5$

$\alpha = 0.01$

$H_0 : \mu = 8$ $\qquad z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$H_1 : \mu \neq 8$



Decision rule:

at $\alpha = 1\%$ accept when $|z| < z_{\alpha/2}$

$0.005 = 0.5 - \phi(z_{\alpha/2})$

$\phi(z_{\alpha/2}) = 0.5 - 0.005$

$\phi(z_{\alpha/2}) = 0.495$

$z_{\alpha/2} = 2.58$

Numerical Computation:

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$z = \frac{7.8 - 8}{0.5/\sqrt{50}}$

$= \frac{-0.2 \times \sqrt{2}}{0.1}$

$z = -2.828$

Conclusion:

$|z| = 2.828 \nless z_{\alpha} = 2.58$

reject H_0

(5) A manufacturer of ~~typ~~ type guarantees the average lifetime of its type is more that 28000 miles. If 40 types of this company tested, yields a mean lifetime of 27463 miles with SD 1348 miles. Can guarantee be accepted at 0.01 LOS.

→ x → miles of ~~ty~~ types.

$\mu = 28000$

$\bar{x} = 27463$

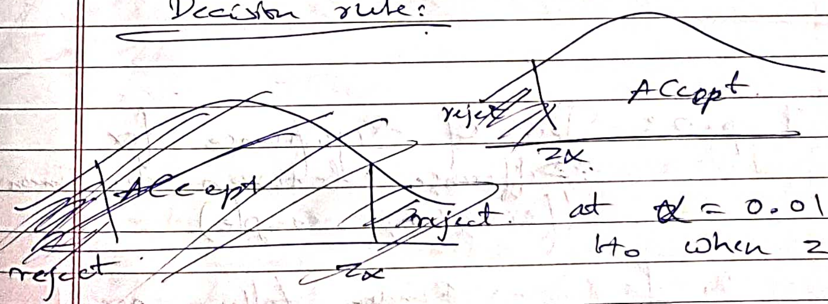
$s = 1348$

$\alpha = 0.01$

$H_0 :$ ~~$\mu > 28000$~~ $\mu \geq 28000$

$H_1 : \mu < 28000$

Decision rule:



at $\alpha = 0.01$ accept H_0 when $z > z_{\alpha}$

$0.01 = 0.5 - \phi(z_{\alpha})$

$\phi(z_{\alpha}) = 0.5 - 0.01$

$\phi(z_{\alpha}) = 0.49$

$z_{\alpha} = -2.33$

Numerical Computation:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{27463 - 28000}{1348/\sqrt{40}}$$

$$= \frac{-537 \times \sqrt{40}}{1348}$$

$$z = -2.519$$

Conclusion:

$$z = -2.519 \not> z_{\alpha} = -2.33$$

reject H_0

✓ Type 2: Test of Significance for Difference between two means (for large sample).

x_1 & x_2 → ~~Random~~ follow normal distribution.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$\mu_1 < \mu_2$

$\mu_1 > \mu_2$

Test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

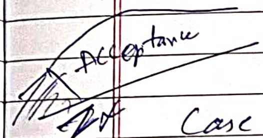
~~At~~ α %, LOS, Accept H_0:

Case 1: $H_1: \mu_1 \neq \mu_2$

accept ~~H_1~~ when $|z| < z_{\alpha/2}$

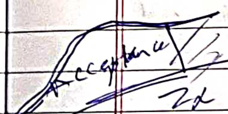
Case 2: $H_1: \mu_1 < \mu_2$

accept H_1 when $z > z_{\alpha}$



Case 3: $H_1: \mu_1 > \mu_2$

accept ~~H_1~~ when $z < z_{\alpha}$



(1) In a random sample of 100 tubelights produced by company A, the mean lifetime of tubelight is 1190 hrs with standard deviation of 90 hrs. Also in a random sample of 75 tubelights from company B, the mean lifetime of 1230 hrs with ~~is~~ SD of 120 hrs. Is there a difference between mean life times of two ~~gra~~ brands of tubelights ~~at~~ at significance level of 0.05

$X_1 \rightarrow$ lifetime of tubelight produced by companys

$X_2 \rightarrow$ ———— 11

$n_1 = 100$ $\qquad$ $n_2 = 75$

$\cancel{n_2 = 1}$

$\mu = 1190$ $\qquad$ $\mu_2 = 1230$

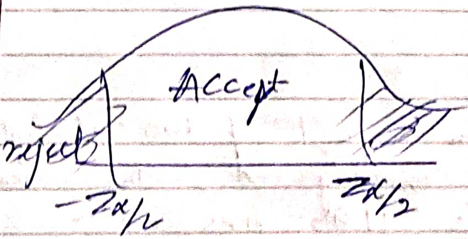
$s_1 = 90 hrs$ $\qquad$ $s_2 = 120 hrs$

$H_0 : \cancel{\mu_1 \neq \mu_2}\ \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Decision rule:

At $\alpha = 0.05$ accept H_0 when $|Z| < z_{\alpha/2}$



$0.5 - \phi(z_{\alpha/2}) = 0.025$

$\phi(z_{\alpha/2}) = 0.475$

$z_{\alpha/2} = 1.96$

Numerical computations:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} - \frac{s_2^2}{n_2}}}$$

$\mu_1 = \mu_2$

$$= \cancel{\frac{1190 - 1230}{\sqrt{\left(\frac{90}{100}\right)^2 + \left(\frac{120}{75}\right)^2}}} = \frac{1190 - 1230}{\sqrt{\frac{90^2}{100} + \frac{120^2}{75}}}$$

$$\cancel{= \frac{-40}{\sqrt{0.81 + 2.56}}} \qquad = \frac{-40}{\sqrt{81 + 192}}$$

$$\cancel{= \frac{-40}{\sqrt{3.37}}} \qquad = \frac{-40}{\sqrt{273}}$$

$$= \frac{-40}{1.835} \qquad = \frac{-40}{+16.522}$$

$$= -2.421$$

(2) To test the effect of new pesticide, a farmland was divided into 60 units of equal area. All portions have identical qualities, as to soil, exposure to sunlight etc. The new pesticide is applied to 30 units, while old pesticide to remaining 30. If there reason to believe that new pesticide is better than old pesticide if mean no of kilos of rice harvested per unit using new pesticide

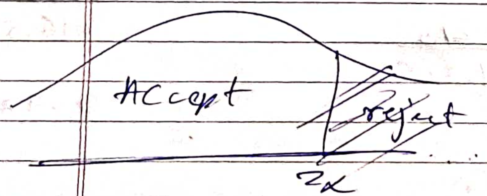
is 496.31 with SD of 17.18.
Test at 0.01 level of significance.
For old pesticide mean is 485.41 kilo with SD 14.73 kilo.

$x_1 \rightarrow$ harvest
$x_2 \rightarrow$ –"–

$n_1 = 30$ $\quad$ $n_2 = 30$
$\mu_1 = 496.31$ $\quad$ $\mu_2 = 485.41$
$s_1 = 17.18$ $\quad$ $s_2 = 14.73$

$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 > \mu_2$

Decision rule:



At $\alpha = 0.01$ accept H_0 when $z < z_\alpha$

$0.5 - \phi(z_\alpha) = 0.01$

$0.5 - 0.01 = \phi(z_\alpha)$

$\phi(z_\alpha) = 0.49$

$z_\alpha = 2.33$

Computation:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{496.31 - 485.41}{\sqrt{\frac{(17.18)^2}{30} + \frac{(14.73)^2}{30}}}$$

$$= \frac{10.9}{\sqrt{9.839 + 7.232}}$$

$$= \frac{10.9}{\sqrt{17.07}}$$

$z = 2.638$

Conclusion

$z < z_\alpha = 2.33$

reject H_0

$n_1 = 42$ $\bar{x}_1 = 744.85$
$s_1 = 397.7$

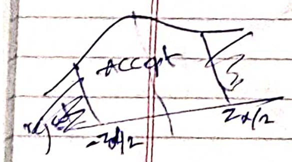
$n_2 = 32$ $\bar{x}_2 = 516.78$
$s_2 = 162.523$

$\alpha = 0.05$

$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 \neq \mu_2$

Decision rule: At $\alpha = 0.05$ accept H_0 if $|z| < z_{\alpha/2}$

$0.025 = 0.5 - \phi(z_{\alpha/2})$



$\phi(z_{\alpha/2}) = 0.5 - 0.025$
$= 0.475$
$z_{\alpha/2} = 1.96$

Numerical Computation:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \cancel{(\mu_1 - \mu_2)}^{0}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{(744.85 - 516.78)}{\sqrt{\frac{(397.7)^2}{42} + \frac{(162.523)^2}{32}}}$$

$z = 3.36.$

Conclusion:

$|z| \not< z_{\alpha/2} = 1.96$

reject H_0

(c) Test at 0.05 LOS, a ~~two~~ manufacturers claim that mean tensile strength (~~kilograms~~)(units) of thread A exceeds the meters of thread B by at least 12 kilo. If 50 pieces of thread are tested under similar conditions yielding the following data:

	Sample size	Mets (	SD (kg)
Type A	50	86.7	6.28
Type B	50	77.8	5.61

$\rightarrow$ $n_1 = 50$ $\bar{x}_1 = 86.7$, $s_1 = 6.28 = \sigma_1$
$n_2 = 50$ $\bar{x}_2 = 77.8$, $s_2 = 5.61 = \sigma_2$

$\alpha = 0.05$

$H_0: \mu_1 - \mu_2 \geq 12$
$H_1: \mu_1 - \mu_2 < 12.$

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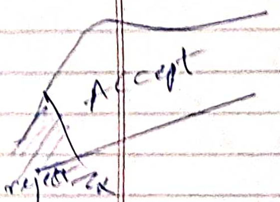
Decision rule :

At $\alpha = 5\%$ accept H_0 if $Z > Z_\alpha$



$0.05 = 0.5 - \phi(z_\alpha)$

$\phi(z_\alpha) = 0.5 - 0.05$
$= 0.45$

$z_\alpha = -1.65$

Numerical computation :

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{(86.7 - 77.8) - 12}{\sqrt{\frac{6.28^2}{50} + \frac{5.61^2}{50}}}$$

$$= \frac{8.9 - 12}{\sqrt{1.41821}}$$

$$= \frac{-3.1}{1.19}$$

$$= -2.605$$

Conclusion : At $\alpha = 5\%$
$z \not> z_\alpha = -1.67$
$\therefore$ reject H_0

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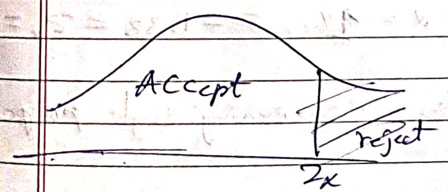
(5) A random sample of 40 geysers produced by company A have a mean life time (mlt) of 647 hrs of continuous use with SD of 27 hrs. While a sample of 40 geysers produced by another company B has mlt of 638 hrs with SD 31 hrs. Does this substantiate the claim of company A that their geysers ~~prod~~ are superior to those produced by company B when:
(i) $\alpha = 0.05$ (ii) $\alpha = 0.01$

$\rightarrow$ $n_1 = 40$ $\bar{x}_1 = 647$, $s_1 = 27 = \sigma_1$
$n_2 = 40$ $\bar{x}_2 = 638$, $s_1 = 31 = \sigma_2$

$H_0 : \mu_1 \le \mu_2$
$H_1 : \mu_1 > \mu_2$

Decision rule :

At $\alpha = 5\%$ accept H_0
if $Z < Z_\alpha$



$0.05 = 0.5 - \phi(z_\alpha)$
$z_\alpha = 1.65$

Numerical computation :

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{(647 - 638?) - 0}{\sqrt{\frac{27^2}{40} + \frac{31^2}{40}}}$$

≈ 1.38

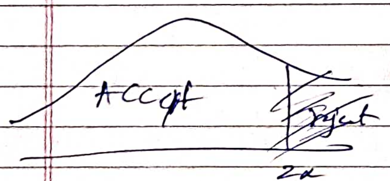
Conclusion: At $\alpha = 5\%$, $z = 1.38 < z_\alpha = 1.65$

Accept H_0.

At $\alpha = 1\%$

Decision rule:

~~At~~ $\alpha = 1\%$ accept H_0 if $z < z_\alpha$



$0.01 = 0.5 - \phi(z_\alpha)$

$\phi(z_\alpha) = 0.49$

$z_\alpha = 2.33$

Conclusion:

At $\alpha = 1\%$, $z = 1.38 < z_\alpha = 2.33$

✓ Test of population concerning single proportion

→ Suppose we wish to estimate the individuals in population who has certain attribute, then we shall use this test.

$x \to$ no. of items possessing the attribute (or) characteristics, and it has been observed that x follows binomial distribution.

~~p~~ where $p \to$ population proportion.

As $n \to \infty$, $x \sim N(\mu = np, \sigma^2 = npq)$

~~$H_0 \to$~~ $H_0: p = p_0$

$H_1: p \neq p_0$

$p < p_0$

$p > p_0$

If $p \neq p_0$, accept H_0 when $|z| < z_{\alpha/2}$

for $p < p_0$, accept H_0 when $z > z_\alpha$

for $p > p_0$, accept H_0 when $z < z_\alpha$

Test statistic:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

where $\hat{p} = \frac{x}{n}$ is the sample proportion.

(1) If in a random sample of 600 cars making a right turn at certain traffic junction, 157 cars drove into the wrong way. Test whether actually 30% of all drivers make this mistake (or) not at this given junction. Use (a) 0.05 LOS (b) 0.01 LOS

→ $x \to$ no. of cars in wrong lane

$H_0: p = 0.3$ $\qquad$ $H_1: p \neq 0.3$

Decision rule: At $\alpha = 5\%$ accept H_0 when $|z| < z_{\alpha/2}$

$z_{\alpha/2} = 1.96$

Numerical computation:

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{157/600 - 0.3}{\sqrt{\frac{0.3 \times 0.7}{600}}}$$

$$= \frac{0.261 - 0.3}{\sqrt{\frac{0.21}{600}}}$$

$$= \frac{-0.039}{\sqrt{0.00035}}$$

$$= \frac{-0.039}{0.0187}$$

$$= -2.0855$$

Conclusion: At $\alpha = 5\%$, $|z| = 2.084 \nless z_{\alpha/2}$

$\therefore$ reject H_0

(b) Decision rule

At $\alpha = 1\%$, accept H_0 when $|z| < z_{\alpha/2}$

$z_{\alpha/2} = 2.58$

Conclusion: At $\alpha = 1\%$ $|z| < z_{\alpha/2} = 2.58$

Accept H_0

(2) Test the claim of manufacturer that 95% of his stabilizers confirm to ISI specification if out of a random sample of 200 stabilizers produced by this manufacturer, 18 were faulty. Use 0.01 LOS.

$\rightarrow$ $n = 200$, ~~$p = 18/200 = 0.09$, $p = q$~~

$x \rightarrow$ no. of faulty stabilizers

H_0: $P = 0.05$

H_1: ~~$P \neq 0.09$~~ $P > 0.05$

Decision rule: At $\alpha = 1\%$ accept H_0 when $z < z_\alpha$

$0.01 = 0.5 - \phi(z_\alpha)$

$\phi(z_\alpha) = 0.49$

$z_\alpha = 2.33$

Numerical computation:

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{18/200 - 0.05}{\sqrt{\frac{0.05 \times 0.95}{200}}}$$

$$= \frac{0.09 - 0.05}{\sqrt{2.375 \times 10^{-4}}}$$

$$= \frac{4}{1.541}$$

$$= \underline{2.596}$$

Conclusion: At $\alpha = 1\%$,

$z = 2.596 \nless z = 2.33$

$\therefore$ reject H_0

③ If in a random sample of 200 people suffering with headache, 160 got cured by a drug. Can we accept the claim of manufacturer that his drug cures 90% of sufferers. Use 1% LOS

→ $n = 200$, $P = 0.9$

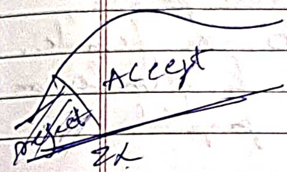
$x \to$ cured people

$H_0 : P \geq 0.9$

$H_1 : P < 0.9$

Decision rule:

If $\alpha = 1\%$, accept H_0 if $z > z_\alpha$



$0.01 = 0.5 - \phi(z_\alpha)$

$\phi(z_\alpha) = 0.49$

$z_\alpha = -2.33$

Numerical computation:

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{160/200 - 0.9}{\sqrt{\frac{0.9 \times 0.1}{200}}}$$

$$= \frac{0.8 - 0.9}{\sqrt{4.5 \times 10^{-4}}}$$

$$= \frac{-10}{2.1213}$$

~~$z = -3.535$~~

$z = -4.714$

Conclusion: ~~At~~ $\alpha = 1\%$

$Z = -4.714 \ngtr Z_\alpha = 2.33$

Reject H_0

✓ In a random sample of 400 people, from a large population 120 are female. Can it be set that male & females are in the ratio 5:3 in the population. Use 1% LOS.

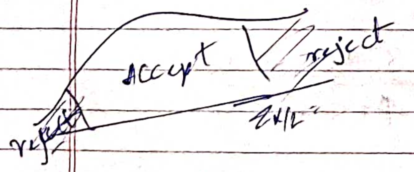
→ $X \rightarrow$ no. of female in the population.

$n = 400$; $x = 120$; $p = \frac{x}{n} = \frac{120}{400} = 0.3$

$H_0 : P = \frac{3}{8}$

$H_1 : P \neq \frac{3}{8}$

Decision rule:



If $\alpha = 1\%$, accept H_0 if $|z| < Z_{\alpha/2}$

$0.005 = 0.5 - \phi(Z_{\alpha/2})$

$\phi(Z_{\alpha/2}) = 0.495$

$Z_{\alpha/2} = 2.58$

Numerical computation:

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.3 - 3/8}{\sqrt{\frac{3/8 \times 5/8}{400}}}$$

$$= \frac{-0.075 \times 10^2}{\sqrt{5.86}}$$

$$= -3.098$$

✓ It is observed that 174 out of a random sample of 200 truck drivers on highway are drunk. Is it valid to state that, atleast 90% of truck drivers are drunk. Use 5% LOS

→ $X \rightarrow$ no. of people drunk.

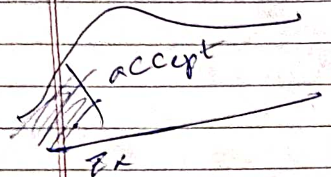
$n = 200$; $x = 174$; $p = \frac{x}{n} = \frac{174}{200} = 0.87$

$H_0 : P \geq 0.9$

$H_1 : P < 0.9$

Decision rule :

If $\alpha = 5\%$, accept H_0 if $Z > Z_\alpha$



$0.05 = 0.5 - \phi(Z_\alpha)$

$\phi(Z_\alpha) = 0.45$

$Z_\alpha = -1.65$

Numerical computation :

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.87 - 0.9}{\sqrt{\frac{0.9 \times 0.1}{200}}}$$

$$= \frac{-0.03 \times 100}{2.12}$$

$$= \frac{-3}{2.12}$$

$$= -1.415$$

Conclusion :

$Z > Z_\alpha = -1.65$

Accept H_0

Test of hypothesis ⇒ difference b/w proportions of large sample

$H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$ accept H_0 when $|Z| < Z_{\alpha/2}$

$P_1 < P_2$ accept H_0 when $Z > Z_\alpha$

$P_1 > P_2$ accept H_0 when $Z < Z_\alpha$

Test statistic :

$$Z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad \text{where: } P = \frac{x_1 + x_2}{n_1 + n_2}, \quad Q = 1 - P$$

(1) If 48 out of 400 people in rural area possess cell phone, while 120 out of 500 in urban area. Can it be accepted that, proportion of cell phones in rural & urban area is same. Use $\alpha = 5\%$

→ $x_1 \rightarrow$ no. of people possessing cell phone in rural area.

$x_2 \rightarrow$ no. of people in urban area — " —

$n_1 = 400$, $n_2 = 500$, $x_1 = 48$, $x_2 = 120$.

$H_0: P_1 = P_2$

$H_1: P_1 \neq P_2$

Decision Rule:

At $\alpha = 5\%$ accept H_0 if $|Z| < Z_{\alpha/2}$

reject | accept | reject

$0.025 = 0.5 - \phi(Z_{\alpha/2})$

$\phi(Z_{\alpha/2}) = 0.475$

$Z_{\alpha/2} = 1.96$

Numerical computation:

$p_1 = \frac{x_1}{n_1} = \frac{48}{400} = 0.12$

$p_2 = \frac{x_2}{n_2} = \frac{120}{500} = 0.24$

$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{168}{900} = 0.1867$

$Q = 1 - P$

$= 0.8133$

$\cancel{Z = P_1 - P_2}$ $\quad Z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$= \frac{0.12 - 0.24}{\sqrt{0.1867 \times 0.8133 \left(\frac{1}{400} + \frac{1}{500}\right)}}$

$Z = -4.59$

Conclusion: At $\alpha = 5\%$

$|Z| = 4.59 \nless 1.96 = Z_{\alpha/2}$

$\therefore$ reject H_0

(2) If 57 out of 150 patients suffering with disease are cured with allopathy & 33 out of 100 patients with same disease are cured by homeopathy. Is there any reason to believe that allopathy is better than homeopathy at 5% LOS.

→ X_1 → no. of people in population 1 with disease

X_2 → ——— " ——— 2 — " ———

$n_1 = 150$, $n_2 = 100$, $x_1 = 57$, $x_2 = 33$

$H_0: P_1 \leq P_2$

$H_1: P_1 > P_2$

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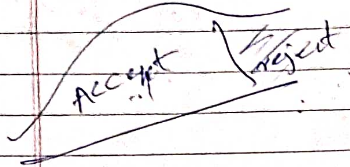
Decision Rule: ~~If~~ At $\alpha = 5\%$, accept H_0 if $Z < Z_\alpha$



$0.05 = 0.5 - \phi(Z_\alpha)$

$Z_\alpha = 0.45$

$Z_\alpha = 1.65$

Numerical Calculation:

$p_1 = \frac{x_1}{n_1} = \frac{57}{150} = 0.38$

$p_2 = \frac{x_2}{n_2} = \frac{33}{100} = 0.33$

$p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{57+33}{250}$

$= \frac{90}{250} = 0.36$

$Q = 1 - p = 1 - 0.36 = 0.64$

$$Z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{pQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.38 - 0.33}{\sqrt{0.36 \times 0.64\left(\frac{1}{150} + \frac{1}{100}\right)}}$$

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$= \frac{0.05 \times 100}{\sqrt{38.4}} = \cancel{\frac{0.05 \times 100}{\sqrt{0.384}}}$ | $\frac{0.05}{\sqrt{0.2304 \times (}}$

$Z = 0.8065 = \cancel{\frac{5}{0.62}}$

~~$= 0.8.06$~~ ~~$Z = 0.8065$~~

Conclusion: ~~If~~ $\alpha = 5\%$

~~$Z \not< Z_\alpha = 1.65$ Reject H_0~~ $Z < Z_\alpha = 1.65$ Accept H_0.

(3) A study of TV viewers was conducted to find opinion about mega serial Ramayana. If 56% of sample of 300 viewers from south & 48% of 200 viewers from north prefered the serial. Test the claim at 5% LOS, that
(i) There is a difference of opinion b/w South & North
(ii) Ramayana is prefered in the south

→ X_1 → no. of people prefering ramayana in south
X_2 → —— " —— in north

$n_1 = 300$, $n_2 = 200$; $p_1 = 0.56$, $p_2 = 0.48$
⇒ $x_1 = 300 \times 0.56$ | $x_2 = 200 \times 0.4$
⇒ $x_1 = 168$ | $x_2 = 96$

(i) $H_0: P_1 = P_2$
$H_1: P_1 \neq P_2$

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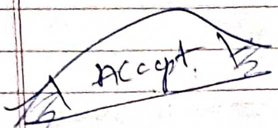
Decision rule: If $\alpha = 5\%$, accept H_0 if $|Z| \le Z_{\alpha/2}$



$0.025 = \cancel{0.05}^{0.5} - \phi(Z_{\alpha/2})$

$\phi(Z_{\alpha/2}) = 0.475$

$(Z_{\alpha/2}) = 1.96$

Numerical computation:

$\cancel{p_1 = \frac{x_1}{n_1}}$

$\cancel{p_2 = \frac{x_2}{n_2}}$

$p = \frac{x_1 + x_2}{n_1 + n_2}$

$= \frac{168 + 96}{200 + 300}$

$= \frac{264}{500}$

$= 0.528 \Rightarrow Q = 0.472$

$Z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$= \frac{0.56 - 0.48}{\sqrt{0.528 \times 0.472\left(\frac{1}{200} + \frac{1}{300}\right)}}$

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$= \frac{0.08}{\sqrt{0.25\left(5\times10^{-3} + 3.33\times10^{-3}\right)}}$

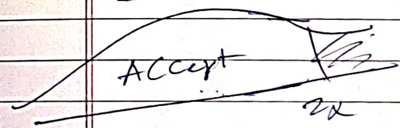
$= \frac{0.08 \times 100}{\sqrt{20.825}}$

$Z = 1.753$

Conclusion: At $\alpha = 5\%$, $|Z| < Z_{\alpha/2} \Rightarrow$ Accept H_0.

(ii) $H_0: P_1 \leq P_2$
$H_1: P_1 > P_2$

Decision rule: If $\alpha = 5\%$ accept H_0 if $Z < Z_\alpha$



$0.05 = 0.5 - \phi(Z_\alpha)$

$\phi(Z_\alpha) = 0.45$

$Z_\alpha = 1.65$

Conclusion: At $\alpha = 5\%$,

$Z \nless Z_\alpha = 1.65$
Reject H_0

Small Sample:

Small Sample test concerning single mean 't-distribution'. Here 'x' doesn't follow normal distribution but follows 't-distribution'.

As $n \to \infty$, its distribution becomes normal distribution.

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$ accept while $|t| < t_{\alpha/2, n-1}$

$\mu < \mu_0$ —— " —— $t > t_{\alpha, n-1}$

$\mu > \mu_0$ —— " —— $t < t_{\alpha, n-1}$

$(n-1) \to$ degree of freedom

Test-statistic:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \quad \text{or} \quad \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

where: $s^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$

$$s^2 = \frac{1}{n} \Sigma (x_i - \bar{x})^2$$

(1) The mean weekly sales of a chocolate bar in a candy store was 140.3 bars/store. After an advertisement campaign, the mean weekly sales in 22 stores for a typical week increased to 153.7 bars/store and showed SD of 17.2. Was the advertisement campaign successful.

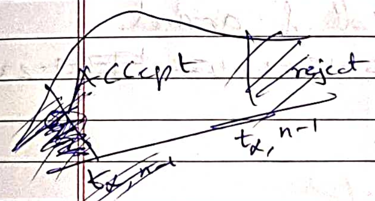
$\bar{x} = 153.7$
$\mu = 140.3$
$n = 22$
$s = 17.2$

$x \to$ weekly sales of chocolate bar

$H_0: \mu \leq 140.3$
$H_1: \mu > 140.3$

Decision rule: At $\alpha = 5\%$ accept H_0 when $t < t_{\alpha, n-1}$

$\alpha = 0.05$, $n-1 = 21$



$\Rightarrow t_{0.05, 21} = 1.721$

Numerical Computation:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$$= \frac{153.7 - 140.3}{17.2/\sqrt{21}}$$

$= 3.57$

Conclusion : At $\alpha = 5\%$

$t = 3.57 \nless t_{\alpha, n-1}$

Reject H_0

(5) An Ambulance service company claims that on an average it takes 20 min between a call for an ambulance & patients arrival at hospital. If in 6 calls the time taken b/w a call & arrival at hospital are 27, 18, 26, 15, 20 & 32 can the companies claim be accepted at $\alpha = 5\%$.

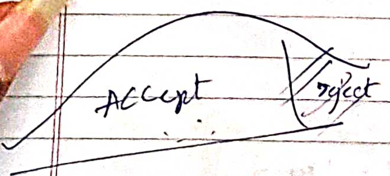
$x \to$ time taken b/w call & arrival.

$\text{mean}(\bar{x}) = \frac{27+18+26+15+20+32}{6} = \cancel{11.67}\ 23$

$\mu = 20 \text{ min}$

$H_0 : \mu \leq 20$
$H_1 : \mu > 20$

Decision rule: At $\alpha = 5\%$, accept H_0 if $t < t_{\alpha, n-1}$



$t_{\alpha, n-1} = t_{0.05, 5} = 2.015$

Numerical Computation:

$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$

$S^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$

$S^2 = \frac{1}{6}(16 + 25 + 9 + 64 + 9 + 81)$

$S = \sqrt{34}$

$S = 5.83$

$t = \frac{23 - 20}{5.83/\sqrt{5}}$

$t = 1.15$

Conclusion: At $\alpha = 5\%$,

$t = 1.15 < t_{\alpha, n-1} = 2.015$

Accept H_0

(3) Following our systolic blood pressure of 12 patients undergoing blood therapy for hypertension, 183, 152, 178, 157, 194, 163, 144, 114, 178, 152, 118, 158. Can we conclude on the basis of these data that population mean is less than 165. Use $\alpha = 5\%$

$\to$ $x \to$ bp of patients

$\mu = 165$

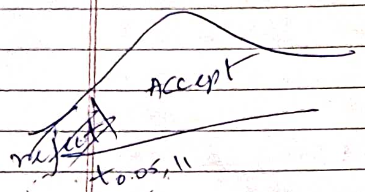
$\bar{x} = 183+152+178+157+194+163+144+114+178+152+118+158$

$\bar{x} = 157.58$

H_0: $\mu \geq 165$
H_1: $\mu < 165$

Decision rule: At 5%, accept H_0 when $t > t_{\alpha, n-1}$

$t_{0.05, 11} = -1.796$



Numerical computation:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$$s^2 = \frac{1}{\sqrt{n}} \Sigma (x - \bar{x})^2$$

$$= \frac{1}{\sqrt{12}} (646.176 + 31.136 + 416.976 + 0.336 + 1326.416 + 29.376 + 184.496 + 1.897.44 + 416.859 + 31.169 + 1566.81 + 0.174)$$

$s^2 = 545.7407$

$s = 23.36$

$$t = \frac{157.58 - 165}{23.36/\sqrt{12-1}}$$

$$= \frac{-7.42 \times \sqrt{11}}{23.36}$$

$t = -1.053$

Conclusion: At $\alpha = 5\%$, ~~Accept~~

$t > t_{0.05, 11} = -1.796$

Accept H_0

(4) In a random sample of 10 bolt, produced by the machine, the mean length of bolt is 0.053 mm & SD of 0.03 mm can be claim from this that the machine is working in proper

$\rightarrow$ $x \rightarrow$ length of bolt

$n = 10$, $\bar{x} = 0.053$, $s = 0.03$
$\mu = 0.5$

H_0: $\mu = 0.5$
H_1: $\mu \neq 0.5$

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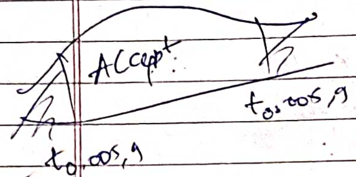
Decision rule: At $\alpha = 1\%$ accept H_0 when $|t| < t_{0.005, 9}$

~~At~~

$t_{0.005, 9} = 3.250$



Numerical computation:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$$= \frac{0.53 - 0.5}{\frac{0.03}{3}}$$

$t = 3$

Conclusion: At $\alpha = 1\%$

$t < t_{0.005,9} = 3.250$

Accept H_0

Mean lifetime of computers manufactured by a company is 1120 hrs, with SD of 125 hrs. Test the hypothesis that mean lifetime of computer has not changed if a sample of eight computers has a mean lifetime of ~~1010~~ 1070 hrs. Is there a decrease in m.lt?

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$x \rightarrow$ lifetime of computers.

$\mu = 1120$ hrs
$\sigma = 125$ hrs
$n = 8$ computers
$\bar{x} = 1070$ hrs

Numerical computation:

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

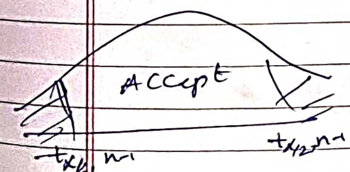
$$= \frac{1070 - 1120}{\frac{125}{\sqrt{8}}}$$

$t = -1.1313$

(i) $H_0 : \mu = 1120$
$H_1 : \mu \neq 1120$

Decision rule: At $\alpha = 1\%$ accept H_0, when $|t| < t_{\alpha/2, n-1}$

$t_{0.005, 7} = 3.499$



Conclusion: At $\alpha = 1\%$,

$|t| = +1.1313 < 3.499 = t_{\alpha/2, n-1}$

$\therefore$ Accept H_0

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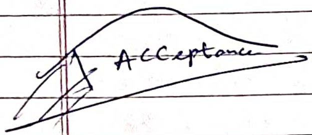
(ii) $H_0: \mu \geq 1120$
$H_1: \mu < 1120$

Decision rule: At $\alpha = 1\%$
accept H_0 if $t > t_{\alpha, n-1} = t_{0.01, 4} = -2.99$



Conclusion: At $\alpha = 1\%$
$t = -1.1313 > t_{\alpha, n-1} = -2.998$
$\therefore$ accept H_0

✓ If 5 pieces of certain ribbon selected at random have mean breaking strength of 169.5 pounds with SD 5.7 pounds.
Do they confirm to specification mean breaking strength of 180 pounds? LOS = 5%.

→ $x \to$ breaking strength.

$\mu = 180$
$\bar{x} = 169.5$
$\alpha = 5\%$
$\cancel{s} = 5.7$

$H_0: \mu = 180$
$H_1: \mu \neq 180$

~~Accept~~ Decision rule: Accept H_0 at $\alpha = 5\%$ if $|t|_{\alpha/2, n-1} < t_{\alpha/2, n-1}$

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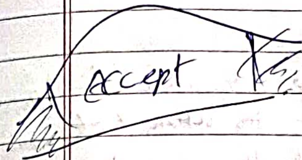


$t_{\alpha/2, n-1} = t_{0.025, 4} = 2.776$

Numerical computation:

$$t = \frac{\bar{x} - \mu}{\frac{5.7}{\sqrt{5}}} = \frac{(169.5 - 180)\sqrt{5}}{5.7} = -4.12$$

Conclusion: At $\alpha = 5\%$
~~Accept~~
$|t| = 4.12 \not< t_{\alpha/2, n-1} = 2.776$
~~Accept~~ Reject H_0

– Type 6:

Small sample test, concerning difference b/w two means (for small sample).

Test statistic: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

When $s^2 = \dfrac{\Sigma (x_{1i} - \bar{x}_1)^2 + \Sigma (x_{2i} - \bar{x}_2)^2}{n_1 + n_2 - 2}$

(or) $s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$H_0 : \mu_1 - \mu_2 = \delta$

$H_1 : \mu_1 - \mu_2 \neq \delta$ accept H_0 when $|t| < t_{\alpha/2, n_1+n_2-2}$

$\mu_1 - \mu_2 < \delta$ accept H_0 when $t > t_{\alpha, n_1+n_2-2}$

$\mu_1 - \mu_2 > \delta$ accept H_0 when $t \leq t_{\alpha, n_1+n_2-2}$

In mathematic expression, 9 students of class A & 6 students of class B obtained the following marks. Test at 1% LOS, whether performance in maths, is same or not in for two classes. Assume that samples are drawn from Normal population having equal variance

A	44	71	63	59	68	46	69	54	48
B	52	70	41	62	36	50			

$x_1 \rightarrow$ marks of students in class A

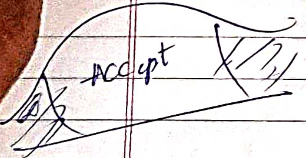
$x_2 \rightarrow$ —— " —— B

%

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$

Decision rule: At $\alpha = 1\%$, accept H_0 when $|t| < t_{\alpha/2, n_1+n_2-2}$



$t_{0.005, 13} = 3.012$

Numerical computation:

$$\bar{x}_1 = \frac{44 + 71 + 63 + 59 + 68 + 46 + 69 + 54 + 48}{9}$$

$= 58$

$$\bar{x}_2 = \frac{52 + 70 + 41 + 62 + 36 + 50}{6}$$

$= \cancel{8H}$

$\bar{x}_2 = 51.833$

Out of random sample of a mice suffering with a disease, 5 mice were treated with new serum. While the remaining are not treated. From the time of commencement of experiment, the following are survival time. Test whether the serum treatment is effective in curing the disease at 5% l.os assuming that the two distributions are normally distributed with equal variance.

Treatment	2.1	5.3	1.4	4.6	0.9
No Treatment	1.9	0.5	2.8	3.1	

x_1 : Survival rate with treatment
x_2 : — " — without — " —

$n_1 = 5$, $n_2 = 4$

$H_0 : \mu_1 \leq \mu_2$ (or) $\mu_1 - \mu_2 \leq 0$
$H_1 : \mu_1 > \mu_2$ [serum treatment is effective]

$$s^2 = \frac{\sum (x_{1i} - \bar{x}_1)^2 + \sum (x_{2i} - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

$$\bar{x}_1 = \frac{2.1 + 5.3 + 1.4 + 4.6 + 0.9}{5}$$

$\bar{x}_1 = 2.86$

$$\bar{x}_2 = \frac{1.9 + 0.5 + 2.5 + 3.1}{4}$$

$\bar{x}_2 = 2.075$

$$s^2 = \frac{(2.1-2.86)^2 + (5.3-2.86)^2 + (1.4-2.86)^2 + (4.6-2.86)^2 + (0.9-2.86)^2 + (1.9-2.075)^2 + (0.5-2.075)^2 + (2.8-2.075)^2 + (3.1-2.075)^2}{7}$$

$s^2 = 2.802$

$s = 1.674$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{(2.86 - 2.075)}{1.678\sqrt{\frac{1}{5} + \frac{1}{4}}}$$

$t = 0.69$

Conclusion: At $\alpha = 5\%$, $t = 0.69 < 1.895$
$\therefore$ Accept H_0

A study is conducted to determine whether wear of material A exceeds that of B by more that two units. If test of twelve pieces of material A yields a mean ~~wear~~ of 85 units & SD of 4, while test of 10 pieces of material B, yields a mean of 81 & SD → 5. What conclusion can be drawn at 5% LOS. Assume that populations are normally distributed with equal variance.

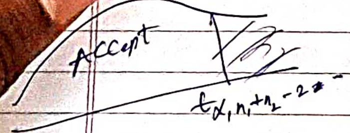
→ $x_1 \rightarrow$ wear of A
$x_2 \rightarrow$ wear of B.

$\bar{x}_1 \rightarrow 84$ $\quad \sigma_1 = 4$, $\quad n_1 = 12$
$\bar{x}_2 \rightarrow 81$ $\quad \sigma_2 = 5.$ $\quad n_2 = 10$

$H_0 : \mu_1 - \mu_2 \leq 2,$
$H_1 : \mu_1 - \mu_2 > 2.$

Decision rule:- At $\alpha = 5\%$ accept H_0 when
$t < t_{\alpha, n_1+n_2-2}$

$t_{0.05, 20} = 1.725$



Numerical computation:

~~$n_1 = 10, 1$~~

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$S^2 = \frac{12(16) + 10(25)}{12+10-2}$$

$S^2 = 22.1$

$S = 4.701$

~~$t = \Sigma(x_{1i} - \bar{x}_1)^2 + \Sigma(x$~~

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{(85-81) - 2}{4.701\sqrt{\frac{1}{12} + \frac{1}{10}}}$$

$t = 0.99.$

To determine whether vegetarian diet (or) non vegetarian diet affects sign. on increase of weight. A study was conducted yielding the following data of gain in weight. Can we claim that, two diets differ pertaining to weight gain, assuming

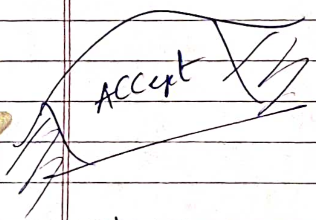
Assuming that samples are drawn from normal population with same variance. Use $\alpha = 1\%$.

Veg	34	24	14	32	25	32	30	24	30	31	35	25			
Non-veg	22	10	47	31	44	34	22	40	30	32	35	18	21	35	29

→ $x_1 \to$ weight gain in Veg | $n_1 = 12$
$x_2 \to$ weight gain in Non-veg. | $n_2 = 15$

$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 \neq \mu_2$

Decision rule: At $\alpha = 1\%$. accept H_0 when $|t| < t_{\alpha/2, n_1+n_2-2}$



$t_{0.005, 25} = 2.787$

Numerical computation:

$$\bar{x}_1 = \frac{34+24+14+32+25+32+30+24+30+31+35+25}{12}$$

$\bar{x}_1 = 28$

$$\bar{x}_2 = \frac{22+10+47+31+44+34+22+40+30+32+35+18+21+35+29}{15}$$

$= 30.$

$$s_1^2 = \frac{36+16+196+16+9+16+4+16+4+9+49+9}{12}$$

$s_1^2 = 31.66$

$s_1 = 5.62$

$$s_2^2 = \frac{64+400+289+1+196+16+64+100+0+4+25+144+81+25+1}{15}$$

$s_2^2 = 94$

~~$s_2 = 96$~~

$s_2 = 9.69$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}$$

$$= \frac{12(31.66) + 15(94)}{25}$$

$S^2 = 71.6$

$S = 8.46$

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$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_0)}{s\sqrt{\frac{1}{12} + \frac{1}{15}}}$$

$$= \frac{-2 - 0}{8.4\sqrt{\frac{27}{180}}}$$

$$= \frac{-2}{8.4 \times 0.387}$$

$$= \frac{-2}{3.25}$$

$$= -0.61$$

<u>Conclusion:</u> At $\alpha = 1\%$

$|t| = 0.61 < t_{\alpha/2, n_1+n_2-2} = 2.787$

Accept H_0

✱ Type - 7: Paired T-test:

$H_0: \mu = \mu_d$

$H_1: \mu \neq \mu_d$ [accept H_0 when $|t| < t_{\alpha/2, n-1}$]

$\mu < \mu_d$ [accept H_0 when $t > t_{\alpha, n-1}$]

$\mu > \mu_d$ [accept H_0 when $t < t_{\alpha, n-1}$]

<u>Test statistic:</u>

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}} \quad \text{(or)} \quad \frac{\bar{d} - \mu_d}{S_d/\sqrt{n-1}}$$

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where: $$S_d^2 = \frac{\Sigma(d_i - \bar{d})^2}{n}$$

✓ The Blood Pressure of 5 women, before & after intake of certain drug are given below. Test at 1% LOS, whether there is significant change in BP.

Before	110	120	125	132	125
After	120	118	125	136	121

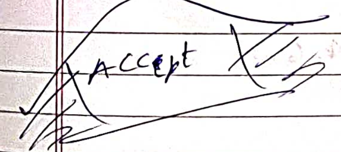
→ $x_1 \to$ BP before intake

$x_2 \to$ BP after intake

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

<u>Decision rule:</u> At $\alpha = 1\%$, accept H_0 if $|t| < t_{\alpha/2, n-1}$



$t_{0.005, 4} = 4.604$

Numerical Computation:

difference : Before - After = $-10, 2, 0, -4, 4$

Mean of difference $(\bar{d}) = \frac{-10 + 2 + 0 - 4 + 4}{5}$

$= -1.6$

$$S_d^2 = \frac{\Sigma(d_i - \bar{d})^2}{n-1}$$

$$= \frac{(-10+1.6)^2 + (2+1.6)^2 + (1.6)^2 + (-4+1.6)^2 + (4+1.6)^2}{4}$$

$$= \frac{70.56 + 12.96 + 2.56 + 5.76 + 31.36}{4}$$

~~$S_d^2 = 38.64$~~

$S_d^2 = 30.8$

$s_d = 5.549$

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-1.6}{5.549/\sqrt{5}} = -0.644$$

Conclusion: At $\alpha = 1\%$

$|t| = 0.644 < 4.604$

$\therefore$ Accept H_0

✓ Marks obtained in mathematics by 11 students before & after intensive coaching are given ~~bel~~ below. At 5% LOS, check whether intensive coaching is useful.

Before	24	17	18	20	19	23	16	18	21	20	19
After	24	20	22	20	17	24	20	20	19	19	22

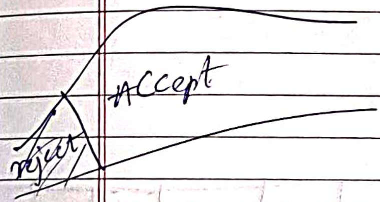
→ $x_1 \rightarrow$ Before coaching

$x_2 \rightarrow$ After coaching.

H_0 : ~~$\mu_d \leq \mu_d$~~ $\mu_1 \geq \mu_2$

H_1 : ~~μ_d~~ ~~$\mu \neq \mu_d$~~ $\mu_1 < \mu_2$

Decision rule: At $\alpha = 5\%$, Accept H_0 if $t > t_{\alpha, n-1}$



$t_{0.05, 10} = 1.812$

Numerical computation:

3

Before	0.4	0.45	0.44	0.54	0.48	0.62	0.48	0.6	0.44	0.46
After	0.5	0.6	0.57	0.65	0.63	0.78	0.63	0.8	0.65	0.62

0.35
0.68

~~x → P I of~~

$x_1 \rightarrow$ PI of a patient before disease

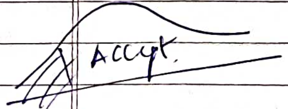
$x_2 \rightarrow$ ——— " ——— after ——— " ———

Ho: $\mu_1 \geq \mu_2$

H_1 ; $\mu_1 < \mu_2$

Decision rule: At $\alpha = 5\%$, accept Ho when

$t > t_{\alpha, n-1}$



$t_{0.05, 10} = -1.812$

Numerical computation:

$$\bar{d} = \frac{-1 - 0.15 - 0.13 - 0.11 - 0.15 - 0.16 - 0.15 - 0.2 - 0.24 - 0.16 - 0.33}{11}$$

$\bar{d} = -0.17$

$$S_d^2 = \frac{\Sigma (d_i - \bar{d})^2}{n-1}$$

$$= \frac{0.07^2 + 0.02^2 + 0.04^2 + 0.06^2 + 0.02 + 0.01 + 0.02^2 + 0.03 + 0.07^2 + 0.01^2 + 0.16^2}{10}$$

$= 0.429 \times 10^{-2}$

$S_d = 0.065$

$$t = \frac{\bar{d} - \mu_d}{S_d / \sqrt{n}}$$

$$= \frac{-0.17}{0.065/\sqrt{11}}$$

$t = -8.674$

Conclusion: At $\alpha = 5\%$

$t = -8.674 \not> t_{0.05,10}$

reject H_0

The average BP losses of man hrs due to strikes in an Institute, before & after a disciplinary program are implemented are as follows. Is there a reason to believe that the disciplinary program is effective at 5% LOS.

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

$X_1 \rightarrow$ Avg weekly losses of man hrs before

$X_2 \rightarrow$ —— " —— after

$H_0: \mu_1 \leq \mu_2$

$H_1: \mu_1 > \mu_2$

$n = 10, \quad \alpha = 5\%$

Decision rule: At $\alpha = 5\%$, accept H_0 if $t < t_{\alpha, n-1}$

$t_{0.05,9} = 1.833$

Numerical computation:

$$\bar{d} = \frac{9 + 13 + 2 + 5 + (-2) + 6 + 6 + 5 + 2 + 6}{10}$$

$$\bar{d} = 5.2$$

$$S_d^2 = \frac{\Sigma (d_i - \bar{d})^2}{n-1}$$

$$= \frac{3.8^2 + (7.8)^2 + 3.2^2 + 0.2^2 + 7.2^2 + 0.8^2 + 0.8^2 + 0.2^2 + 3.2^2 + 0.8^2}{9}$$

$$S_d^2 = 16.62$$

$$S_d = 4.07$$

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{10}}$$

$$= \frac{5.2 \times \sqrt{10}}{4.07}$$

$$t = 4.04$$

Conclusion: At $\alpha = 5\%$

$t = 4.04 \not< t_{\alpha, n-1}$

reject H_0

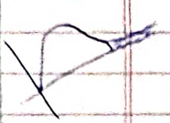
Type-8: Ratio of variance (F distribution)

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$ when $F < F_{n_1-1, n_2-1, \alpha}$

~~$\sigma_1^2 < \sigma_2^2$~~

~~$\sigma_1^2 > \sigma_2^2$~~



Test statistic:

$F = \frac{s_1^2}{s_2^2}$ if $s_1^2 > s_2^2$ (or) $F = \frac{s_2^2}{s_1^2}$ if $s_2^2 > s_1^2$.

where: $s_1^2 = \frac{\sum (x_{1i} - \bar{x}_1)^2}{n_1 - 1}$; $s_2^2 = \frac{\sum (x_{2i} - \bar{x}_2)^2}{n_2 - 1}$

(1) In one sample of 8 observations, the sum of squares of deviations of the sample values from the sample mean was 84.4 & in the other sample of 10 observations, it was 102.6. Test whether this difference is significant at 5% LOS.

→ $\sum (x_{1i} - \bar{x}_1)^2 = 84.4$

$\sum (x_{2i} - \bar{x}_2)^2 = 102.6$

$s_1^2 = \frac{84.4}{7} = 12.057$

$s_1 = 3.472$

$s_2^2 = \frac{102.6}{9} = 11.4$

$s_2 = 3.376$

~~$F = \frac{3.472}{3.376}$~~

~~$= 1.0$~~

$F = \frac{12.057}{11.4}$

$= 1.057$

$F_{0.05(7,9)} = 3.29$

(2) Can we conclude that two population's variance are equal for the following data of post graduates passed out of state & private university. Let $\alpha = 1\%$.

State	8350	8260	8130	8340	8070	
private	7890	8140	7900	7950	7840	7920

→ ~~St~~ x_1 → no. of post graduates passed out of state university

x_2 → " private university.

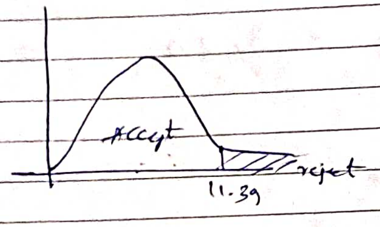
$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

Decision rule: At $\alpha = 1\%$ accept H_0 when

$F < F_{n_1-1, n_2-1, \alpha}$

i.e. $F < F_{4,5,0.01} = 11.39$



Numerical computation:

$n_1 = 5; \ n_2 = 6; \ \alpha = 0.01$

$$s_1^2 = \frac{\Sigma (x_{1i} - \bar{x}_1)^2}{n_1 - 1}$$

$$\bar{x}_1 = \frac{8350 + 8260 + 8130 + 8340 + 8070}{5}$$

$= 8230$

$$\bar{x}_2 = \frac{7890 + 8140 + 7900 + 7950 + 7840 + 7920}{6}$$

$\bar{x}_2 = 7940$

$$s_1^2 = \frac{(120)^2 + (30)^2 + (100)^2 + (110)^2 + (160)^2}{4}$$

$s_1^2 = 15750$

$$s_2^2 = \frac{50^2 + 200^2 + 40^2 + 10^2 + 100^2 + 20^2}{5}$$

$s_2^2 = 10920$

$s_1^2 > s_2^2 \ \therefore \Rightarrow F = \frac{s_1^2}{s_2^2}$

$= \frac{15750}{10920}$

$= 1.44$

Conclusion: At $\alpha = 1\%$, $F = 1.44 < 11.39$

$\therefore$ accept H_0

(3) The household net expenditure on health care in south & north India, in two samples of households, expressed as percentage of total income is shown in following table
LOS = 5%.

South	15	8	3.8	6.4	27.4	19	35.3	13.6	
North	18.8	23.1	10.3	8	18	10.4	15.2	19	20.2

$x_1 \rightarrow$ net expenditure on health care in south India

$x_2 \rightarrow$ " north India

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

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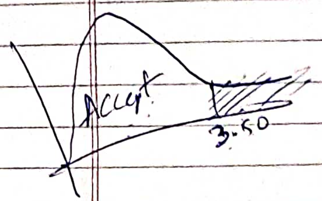
Decision rule: ~~At~~ $\alpha = 5\%$, accept H_0 when

$$F < F_{n_1-1,\, n_2-1,\, \alpha}$$



$$\therefore F < F_{7,8,0.05} = 3.50$$

Numerical computation:

$n_1 = 8;\ n_2 = 9$

$$\bar{x}_1 = \frac{15+8+3.8+6.4+27.4+19+35.3+13.6}{8}$$

$$= 16.062$$

$$\bar{x}_2 = \frac{18.8+23.1+10.3+8+18+10.2+15.2+19+20}{9}$$

$$= 15.87$$

$$s_1^2 = \frac{\Sigma (x_{1i} - \bar{x}_1)^2}{n_1 - 1}$$

$$= \frac{(1.06)^2 + (8.06)^2 + (12.26)^2 + (9.66)^2 + (11.34)^2 + (2.46)^2 + (2.94)^2 + (19.24)^2}{7}$$

$$= 117.6$$

$$s_2^2 = \frac{(2.93)^2 + (7.23)^2 + (5.57)^2 + (7.87)^2 + (2.13)^2 + (5.67)^2 + (0.67)^2 + (3.13)^2 + (4.33)^2}{8}$$

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$$s_2^2 = 27.43$$

$$s_1^2 > s_2^2$$

$$F = \frac{s_1^2}{s_2^2} = \frac{117.6}{27.43} = 4.287$$

Conclusion: At $\alpha = 5\%$

$$F = 4.287 \nless 3.50$$

$\therefore$ reject H_0

Test - 9: χ^2 distribution.

This is not a parametric distribution

There are 3 application of Chi square dist -

(i) goodness of fit
(ii) Test of homogenity.
(iii) Independence of attributes.

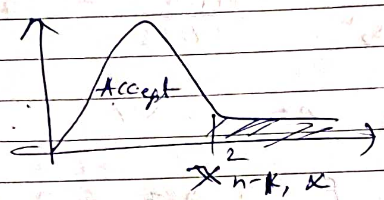
H_0: distribution is a good fit.

H_1: — " — not a good fit.

$$\chi^2 = \frac{\Sigma (O_i - E_i)^2}{E_i} \sim \chi^2_{n-k,\, \alpha}$$

Where n is no. of samples, K = no. of constraints

Decision rule: At α % accept H_0 when
$$\chi^2 < \chi^2_{n-K,\alpha}$$



~~A-no~~

① Fit a poisson distribution for the following data ~~at~~ & test for goodness of fit at $\alpha = 5\%$.

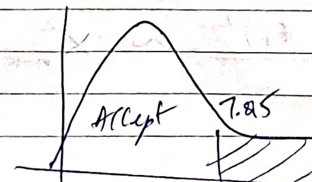
x	0	1	2	3	4
f	419	352	154	56	19

Sol: H_0: Poisson distribution is a good fit
H_1: ——— " ——— not — " —

Decision rule: At $\alpha = 5\%$ accept H_0 when
$$\chi^2 < \chi^2_{n-K,\alpha} = \chi^2_{3,0.05}$$

$n = 5$, $K = 2$.

$\Rightarrow \chi^2_{3,0.05} = 7.815$



No. of constraint
1st constraint : $\Sigma E = 50$
2nd constraint :

x	$f = O$	$p(x)$	Expected	$(O-E)^2/E$
0	419	0.405	405	$14^2/405 = 0.4839$
1	352	0.366	366 ≈ 367	$15^2/369 = 0.6130$
2	154	0.165	165 ≈ 166	$12^2/166 = 0.8674$
3	56	0.0498	49.8 ≈ 50	$6^2/50 = 0.72$
4	19	0.0112	11.2 ≈ 12	$7^2/12 = 4.0833$
			Σ = 1000	

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\lambda = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{352 + 308 + 168 + 76}{1000}$$
$= 0.904$

$p(0) = e^{-0.904} = 0.405$

$p(1) = e^{-0.904} \times 0.904$
$= 0.366$

$p(2) = \dfrac{e^{-0.904} \times (0.904)^2}{2}$
$= 0.165$

$p(3) = \dfrac{e^{-0.904} \times (0.904)^3}{6}$
$= 0.0498$

$p(4) = \dfrac{e^{-0.904} \times (0.904)^4}{24}$

$= 0.0112$

$$\chi^2 = \Sigma\left[\frac{(O_i - E_i)^2}{E_i}\right]$$

$$= 6.7678$$

Conclusion: At $\alpha = 5\%$,

$$\chi^2 = 6.7678 < 7.815 = \chi^2_{n-k,\alpha}$$

$\therefore$ accept H_0

2. Fit a poisson distribution for given freq distribution & test for its goodness of fit at $\alpha = 1\%$.

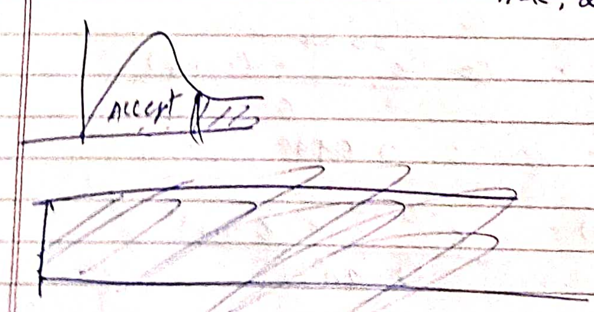
x	0	1	2	3
f	47	33	16	3

→ H_0: poisson distribution is a good fit

H_1: —"— —"— not —"—

Decision rule: At $\alpha = 1\%$, accept H_0 when

$$\chi^2 < \chi^2_{n-k,\alpha} = \chi^2_{2,0.01} = 9.21$$



x	f	p(x)	E	$\frac{(O-E)^2}{E}$
0	47	0.4736	46.88 ≈ 47	0
1	33	0.3539	35.036 ≈ 36	9/36
2	16	0.1322	13.087 ≈ 13	9/13
3	3	0.3295	3.262 ≈ 3	0
			Σ = 99	

$$\lambda = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{33 + 32 + 9}{47+33+16+3}$$

$$= \frac{74}{99}$$

$$= 0.7474$$

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$p(0) = e^{-0.7474}$$

$$= 0.4736$$

$$p(1) = e^{-0.7474} \times 0.7474$$

$$= 0.3539$$

$$p(2) = \frac{e^{-0.7474} \times (0.7474)^2}{2}$$

$$= 0.1322$$

$$p(3) = \frac{e^{-0.7474} \times (0.7474)^3}{6}$$

$$= 0.3295$$

$$\chi^2 = \Sigma \left[\frac{(O_i - E_i)^2}{E_i}\right]$$

$= 0.25 + 0.69$

$= 0.94$

Conclusion: At $\alpha = 1\%$, $\chi^2 = 0.94 < 9.21$

So Accept H_0

3. A sample analysis of examinations results of 500 students was made. It was found that 220 student failed, 170 secured 3^{rd} class, 90 secured 2^{nd} class, 20 secured 1^{st} class. Do these figures support the general examination results which are in the ratio 4:3:2:1 for respective categories. [use $\alpha = 5\%$]

→ H_0: The exams results are in stated ratio

H_1: —— " —— not in —— " ——

Decision rule: At $\alpha = 5\%$, accept H_0 when $\chi^2 < \chi^2_{n-k,\alpha}$

$\chi^2_{3, 0.05} = 7.815$

Numerical comp.

Only one constraint here: i.e. $\Sigma O = \Sigma E$

Numerical comp.

O	E	O	E	$(O-E)^2/E$
220		220	$4/10 \times 500 = 200$	$20^2/200 = 2$
170		170	$3/10 \times 500 = 150$	$20^2/150 = 2.667$
90		90	$2/10 \times 500 = 100$	$10^2/100 = 1$
20		20	$1/10 \times 500 = 50$	$30^2/50 = 18$

$$\chi^2 = \Sigma \left(\frac{(O_i - E_i)^2}{E}\right)$$

$= 23.667$

Conclusion: At $\alpha = 5\%$, $\chi^2 = 23.667$

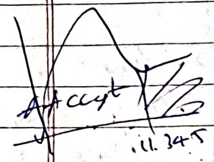
(4) In Mendelian experiment, 4 type of plants are expected to occur in the proportion 9:3:3:1. The observed frequencies are 891 round & yellow, 316 wrinkled yellow, 290 round & green, 119 wrinkled & green. Find chi square value & examine the correspondence between theory & experiment.

H_0: plants are in stated ratio

H_1: —— " —— not in —— " ——

Decision rule: ~~At~~ $\alpha = 1\%$. accept H_0
when $\chi^2 < \chi^2_{n-r,\alpha} = \chi^2_{4-1, 0.01}$

$\Rightarrow \chi^2_{3, 0.01} = 11.345$



O	E	$(O-E)^2/E$	
890	$\frac{9}{16}\times 1616 = 909$	$18^2/909$	≈ 0.3564
316	$\frac{3}{16}\times 1616 = 303$	$13^2/303$	≈ 0.5577
290	$\frac{3}{16}\times 1616 = 303$	$13^2/303$	≈ 0.5577
119	$\frac{1}{16}\times 1616 = 101$	$18^2/101$	≈ 3.207

$$\chi^2 = \Sigma \left(\frac{(O_i - E_i)^2}{E_i}\right)$$

$= 4.6788$

Conclusion: At $\alpha = 1\%$.
$\chi^2 = 4.6788 < 11.345$
$\therefore$ Accept H_0

Unit 5

→ Stochastic process:

It is a family of random variables $\{X(t) \mid t \in T\}$ defined on a common sample space S & indexed by parameter t which varies on an index set T.

→ The values assumed by random variable $X(t)$ are called states, & the set of all possible values from the state space of the process is denoted by I.
If state space is discrete, the ~~set~~ stochastic process is called a chain.

→ A stochastic process consists of a sequence of experiments in which each experiment has finite no. of outcomes with given probability.

→ Markov chain.
It is a markov process in which the state space is discrete. So, markov chain is a finite stochastic process, consisting of sequence of trials say $x_1, x_2, \ldots$ satisfying two conditions:

(i) Each outcome belongs to state space $I = \{a_1, a_2, \ldots a_m\}$ which is finite

(ii) The outcome of any trial depends atmost on the outcome of immediately preceding trial & not upon any other previous out

The system is said to be in state a_i at time n or at n^{th} step if a_i is outcome on n^{th} trial.

The no. p_{ij} gives probability that system changes from i^{th} state to j^{th} state

→ Transition Matrix:

P is a square matrix of transition probability.

$$p = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & & & \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}$$

the i^{th} row of P namely $(p_{i1}, p_{i2}, \ldots, p_{in})$ represent ~~preset~~ probability of that system which changes state a_i to $a_1, a_2 \ldots a_m$.

Eg: A, B, C are playing, passing the parcel. A passes the parcel only to B & B passes the parcel only to C & C passes the parcel equally likely to A & B

$S = \{A, B, C\}$

A → B
B → C
C → A/B

$$T.P.M = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & 0 & 0 & 1 \\ C & 0.5 & 0.5 & 0 \end{array}$$

Next (columns), now (rows)

→ Probability vector: A vector $v = (v_1, v_2 \ldots v_n)$ is called probability vector if $v_i \geq 0$ for every i & $\sum_{i=1}^{n} v_i = 1$

$v = (1/4 \ 1/4 \ 1/2)$ $\quad v = (0 \ 1 \ 0)$

$v = (1/2 \ 0 \ 1/2)$

Note: If $v = (1 \ 2 \ 3)$, then to convert this to probability vector, we divide the components by their sum.

(or)

A vector whose components are non negative but their sum is not one, can be converted into a probability vector by dividing each component by sum of components.

→ Stochastic Matrix:

P is a square matrix in which each row is a probability vector then P is a stochastic matrix.

$$t.p.m = \begin{bmatrix} 0 & 1/2 & 0 \\ 1 & 0 & 0 \\ 0 & 1/4 & 1/4 \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

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→ A vector v is said to be fixed vector if $vA = v$ & $v \neq 0$

Note - (i) If ~~p~~ P is a stochastic matrix then P^n is stochastic matrix for $n \in N$

(ii) A t.p.m is a stochastic matrix in ~~a~~ markov chain

(iii) Stochastic matrix 'P' is said to be regular is every ~~[illegible]~~ element of P^n is non negative & non-zero

✓

(1) A Habitual gambler is member of 2 clubs A & B. He visits either of clubs every day for playing cards. He never visits clubs A on 2 consecutive days. But ~~#~~ if he visits club B on a particular day then next day he is as likely to visit A as to B. Find transition probability matrix & s.T. t.p.m is regular. Also find unique fixed probility vector.

→ State space = $\{A, B\}$

$$\text{t.p.m} = \begin{array}{c} \\ A \\ B \\ \text{(today)} \end{array} \begin{array}{c} A \quad\; B \quad \text{(tmrw)} \\ \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} \\ \\ \end{array}$$

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$$P^2 = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$$

As all the values in the above matrix are non-zero & non negative ; P is regular stochastic matrix.

Let $v = (x \;\; y)$ be unique fixed probability vector.

$$vP = v$$

$$(x \;\; y) \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} = (x \;\; y)$$

$$\left(\frac{y}{2} \quad x + \frac{y}{2}\right) = (x \;\; y)$$

$$\Rightarrow \frac{y}{2} = x$$

$$y = 2x \quad \& \quad x + y = 1$$

$$\Rightarrow x + 2x = 1$$

$$x = 1/3$$

~~$\Rightarrow y$~~

$$\Rightarrow y = 2/3$$

U.F.P.V.

$$v = (x, y) = \left(\frac{1}{3} \;\; \frac{2}{3}\right)$$

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(4) Find unique fixed probability vector for regular stochastic matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

→ Let $v = (x \;\; y \;\; z)$ be U.F.P.V

then $vA = v$ & $x + y + z = 1$

$$(x \;\; y \;\; z) \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix} = (x \;\; y \;\; z)$$

$$(y/6 \quad x + y/2 + 2z/3 \quad y/3 + z/3) = (x \;\; y \;\; z)$$

$y/6 = x$ $\qquad \cancel{x + \frac{y}{2} + 2z/3 = y}$

$y/3 + z/3 = z$

$y = 6x$ $\Rightarrow$ $y + z = 3z$

$\Rightarrow 6x = 2z$

$\Rightarrow z = 3x$

$x + y + z = 1$

$x + 6x + 3x = 1$

$x = \frac{1}{10}$

$y = \frac{6}{10}$ $\qquad z = 3/10$

UFPV = $v = (x \;\; y \;\; z)$

$= \left(\frac{1}{10} \;\; \frac{6}{10} \;\; \frac{3}{10}\right)$

(5) Find UFPV for regular stochastic matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$$

→ Let $v = (x \;\; y \;\; z)$ be UFPV,

$vA = v$ & $x + y + z = 1$.

$$(x \;\; y \;\; z) \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/4 & 1/4 \end{bmatrix} = (x \;\; y \;\; z)$$

$$(z/2 \quad x + z/4 \quad y + z/4) = (x \;\; y \;\; z)$$

$z/2 = x$ $\qquad y + z/4 = z$

$\Rightarrow z = 2x$ $\qquad y = \frac{3z}{4}$

$\Rightarrow y = \frac{3x}{2}$

$x + \frac{3x}{2} + 2x = 1$

$9x = 2$

$\Rightarrow x = \frac{2}{9}$ $\quad \Rightarrow y = \frac{1}{3}$ $\quad \Rightarrow z = \frac{4}{9}$

U.F.P.V $= \left(\frac{2}{9}, \frac{1}{3}, \frac{4}{9}\right)$

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(4) Verify that the matrix $A = \begin{bmatrix} 0 & 1 \\ 0.3 & 0.7 \end{bmatrix}$ is regular stochastic matrix.

→ $A = \begin{bmatrix} 0 & 1 \\ 0.3 & 0.7 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0.3 & 0.7 \end{bmatrix}$

$= \begin{bmatrix} 0.3 & 0.7 \\ 0.21 & 0.79 \end{bmatrix}$

As the entries of A^2 are non-zero, non negative. matrix A is a regular stochastic matrix

(5) A student has study habits as follows. If the student studies 1 day, he is 70% sure not to study ~~one~~ the next day. On the other hand if he does not study one day, he is 60% sure ~~that~~ not to study the next day. In long run how often does he study.

→ $S = \{A \quad B\}$

$A \rightarrow$ studying

$B \rightarrow$ not studying

Transition probability matrix $P = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$

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Let $v = (x \quad y)$ & $x + y = 1$

$vP = v$

$(x \quad y) \cdot \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} = (x \quad y)$

$(0.3x + 0.4y \quad 0.7x + 0.6y) = (x \quad y)$

$0.3x + 0.4y = x$

$0.4y = 0.7x$

$y = \frac{7x}{4}$

$\Rightarrow x + \frac{7x}{4} = 1$

$11x = 4$

$\Rightarrow x = \frac{4}{11}$ (studying in long run)

$\Rightarrow y = \frac{7}{11}$ (not studying in long run).

$\Rightarrow$ U.F.P.V. $v = (4/11 \quad 7/11)$

∴ The probability of him studying in the long run is 4/11 & the probability of not studying in the long run is 7/11 classmate Date ____ Page ____

(6) A salesman's territory consists of three cities A, B & C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However if he sells in either B or C, the next day, he is twice as likely to sell in city A, as in the other city. How often does he sell in each of cities

→ $S = \{A\ B\ C\}$

$$t.p.m. = P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & 2k_1 & 0 & k_1 \\ C & 2k_2 & k_2 & 0 \end{array}$$

$2k_1 + k_1 = 1 \Rightarrow k_1 = \cancel{3}\ 1/3 \Rightarrow k_2 = \cancel{3}\ 1/3$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix}$$

Let $v = (x\ y\ z)$ be U.F.P.V.

then $x + y + z = 1$

$vP = v$

$$(x\ y\ z)\begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} = (x\ y\ z)$$

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$x + z/3 = y$ $\qquad y/3 = z$

$\Rightarrow y = 3z$

$x + \frac{z}{1/3} = 3z$

$x = \frac{8z}{3}$

$\Rightarrow \frac{8z}{3} + 3z + z = 1$

$\frac{20z}{3} = 1$

$\Rightarrow z = \frac{3}{20}$

$\therefore x = \frac{2}{5} \Rightarrow y = \frac{9}{20}$

U.F.P.V $\Rightarrow v = (2/5\ \ 9/20\ \ 3/20)$

$\therefore$ In long run, the probability of selling in A is $8/20$, in B is $9/20$ & in C is $3/20$.

(7) In certain city, weather on a day is reported sunny (S), cloudy (C), rainy (R). If a day is S, probability that next day is sunny is 70%, C is 20% & R is 10%. If a day is C, prob that next day is S is 30%, C is 20%, R is 50%. If a day is rainy,

prob that next day is R is 30%, C is 30%, R is 40%. If S is sunny, find probability that wednesday is rainy

→ State space = {S C R}

$$\text{t.p.m.}\quad P = \begin{array}{c|ccc} & S & C & R \\ \hline S & 0.7 & 0.2 & 0.1 \\ C & 0.3 & 0.2 & 0.5 \\ R & 0.3 & 0.3 & 0.4 \end{array}$$

Initial probability vector $p^{(0)} = (1, 0, 0)$ (on sunday)

On monday : $p^{(1)} = p^{(0)} P$

On Tuesday : $p^{(2)} = p^{(1)} P$

On wednesday : $p^{(3)} = p^{(2)} P$

$\Rightarrow p^{(3)} = p^{(0)} P^3$

$$p^{(3)} = (1\ 0\ 0)\begin{bmatrix} 0.532 & 0.221 & 0.247 \\ 0.468 & 0.233 & 0.299 \\ 0.468 & 0.234 & 0.298 \end{bmatrix}$$

Probability of sunny = 0.532

Three boys A, B, C are throwing ball to each other. A always throws the ball to B & B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find probability that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball

→ State space = {A, B, C}

$$\text{t.p.m.}\quad P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & 0 & 0 & 1 \\ C & 0.5 & 0.5 & 0 \end{array}$$

initial probability $p^{(0)} = (0, 0, 1)$

$$p^{(3)} = (0, 0, 1)\begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

$p^{(3)} = (0.25\quad 0.25\quad 0.5)$

After three throws probability of A, B, C having the ball is 0.25, 0.25, 0.5 respectively.

Q A Gambler's luck follows a patter, if he wins a game, the probability of winning the next game is 0.6. However if he loses a game, the probability of losing the next game is 0.7. There is a even chance of gambler winning the first game (i) What is the probability of ~~with~~ winning the third game? (ii) In the long run, how often will he win?

→ S = {W, L}

$$\text{t.p.m.}\quad P = \begin{array}{c|cc} & W & L \\ \hline W & 0.6 & 0.4 \\ L & 0.3 & 0.7 \end{array}$$

0.24 + 0.195   0.26 + 0.305

0.435   0.565

Initial probability : $p^{(0)} = \begin{pmatrix} \overset{W}{1/2} & \overset{L}{1/2} \end{pmatrix}$

(i) $p^{(2)} = p^{(0)} \, P^2$

$$= \begin{pmatrix} 0.5 & 0.5 \end{pmatrix} \begin{bmatrix} 0.48 & 0.52 \\ 0.39 & 0.61 \end{bmatrix}$$

$$= \begin{pmatrix} 0.24 + 0.195 & 0.26 + 0.305 \end{pmatrix}$$
$$= \begin{pmatrix} \underset{(W)}{0.435} & \underset{(L)}{0.565} \end{pmatrix}$$

probability of winning third game = 0.435

(ii) Let $v = (x, y)$ be U.F.P.V.

$vP = v$

$$(x \;\; y) \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix} = (x \;\; y)$$

$x + y = 1$

$$(0.6x + 0.3y \quad 0.4x + 0.7y) = (x \;\; y)$$

$0.6x + 0.3y = x$

$\Rightarrow 0.3y = 0.4x$

$y = \frac{4x}{3}$

$\Rightarrow \cancel{4x}\; x + \frac{4x}{3} = 1$

$7x = 3$

$x = \frac{3}{7}$

$\Rightarrow y = \frac{4}{7}$

U.F.P.V $= \left(\frac{3}{7}, \frac{4}{7} \right)$

In long run, probability of winning is $3/7$.

(10) ~~A habitual gambler is a member of two clubs A & B. He visits either of clubs everyday~~

(10) A housewife buys 3 kinds of cereals A, B & C. She never buys the same cereal in successive weeks. If she buys cereal A the next week she buys cereal B. However if she buys cereal B or C the next week, she is three times as likely to buy cereal A as to other cereal. In long run how often she buys each of the three cereals.

→ state space = {A, B, C}

t.p.m

$$P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & 3k_1 & 0 & k_1 \\ C & 3k_2 & k_2 & 0 \end{array}$$

Let $v = (x \;\; y \;\; z)$ be UFPV

$vP = v$ & $x + y + z = 1$

$$(x \;\; y \;\; z) \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix} = (x \;\; y \;\; z)$$

$\left(\frac{3}{4}y + \frac{3}{4}x,\ x + \frac{z}{4},\ \frac{y}{4}\right) = (x\ y\ z)$

$x + \frac{z}{4} = y$ & $\frac{y}{4} = z$

$\Rightarrow y = 4z$

$x + \frac{z}{4} = 4z$

$x = \frac{15z}{4}$

$\Rightarrow x + y + z = 1$

$\frac{15z}{4} + 4z + z = 1$

$35z = 4$

$z = \frac{4}{35}$

$\Rightarrow y = \frac{16}{35}$

$\Rightarrow x = \frac{15}{35}$

U.F.P. $v = \left(\frac{15}{35}, \frac{16}{35}, \frac{4}{35}\right)$

In long run, probability of buying
Cereal A $= 15/35$
Cereal B $= 16/35$
Cereal C $= 4/35$

0.2
0.4 0.6
0.64 + 0.08 0.16 + 0.12
0.32 + 0.24 0.08 + 0.36
0.72 0.28
0.56 0.44

(11) The ~~pattern~~ pattern of sunny & rainy days on the planet rainbow is a homogeneous Markov chain with two states. Every sunny day is followed by another sunny day with probability 0.8. Every rainy day is followed by another rainy day with probability 0.6. Today is sunny on Rainbow. (i) What is probability of rain, the day after tomorrow (ii) Compute the probability that april 1st next year is rainy on Rainbow

State space $= \{S, R\}$

t.p.m. $P = \begin{array}{c|cc} & S & R \\ \hline S & 0.8 & 0.2 \\ R & 0.4 & 0.6 \end{array}$

(i) $p^{(2)} = p^{(0)} P^2$

$= (1\ 0)\begin{bmatrix} 0.72 & 0.28 \\ 0.56 & 0.44 \end{bmatrix}$

$= (0.72\ \ 0.28)$

(ii) $VP = V$ & $S + R = 1$

$(S\ R)\begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix} = (S\ R)$

$(0.8S + 0.4R\ \ 0.2S + 0.6R) = (S\ R)$

$0.8S + 0.4R = S$ $\Rightarrow$ ~~$R = \frac{1}{2}$~~ $S = 2R$

$\Rightarrow 3k = 1$

$k = \frac{1}{3}$

$\Rightarrow s = \frac{2}{3}$

Queue

→ arrival follows poisson dist; $\lambda \to$ arrival rate; $\frac{1}{\lambda} \to$ arrival time

→ service follows poisson dist; $\mu \to$ service rate; $\frac{1}{\mu} \to$ service time

(i) At any instant of time, the probability that there are 'n' arrivals (customers) in the queue (waiting line) including those being serviced is given by,

$$P_n = (1-\rho)\rho^n ; \quad n \geq 0$$

Here $\rho = \frac{\lambda}{\mu}$

(ii) Probability that there is no one in the queue:

$\Rightarrow n = 0$

$$P_0 = (1-\rho) = \left(1 - \frac{\lambda}{\mu}\right)$$

(iii) Average no. of

(iv)

$$L_q = L_s - \rho$$

$$L_q = \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

(v) Expected waiting time per each unit

$$W_q = W_s - \frac{1}{\mu}$$

$$W_q = \frac{\rho}{\mu(1-\rho)}$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

(vi) Avg length of non empty queue

$$L_n = \frac{1}{1-\rho} = \frac{\mu}{\mu-\lambda}$$

(vii) Avg waiting time in a non empty queue

$$W_n = \frac{1}{\mu-\lambda}$$

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(viii) Probability of queue length being greater than (or equal to) n

$$P(\geq n) = \rho^n = \left(\frac{\lambda}{\mu}\right)^n$$

(1) The arrival rate of customer at a banking counter has a mean of 45 per hr. The service rate of the counter clerk has a mean of 60 per hr. (a) What is probability of having no customer in the system
(b) what is probability of having 5 customers in the system
(c) find L_s, L_q, W_s, W_q

→ $\lambda = 45$
$\mu = 60$

(a) $$P_0 = \left(1 - \frac{\lambda}{\mu}\right)$$

$$= 1 - \frac{45}{60}$$

$$= \frac{15}{60}$$

$$= \frac{1}{4}$$

(b) $$P_5 = (1-\rho)\,\rho^n$$

$$= \left(1 - \frac{3}{4}\right)\left(\frac{3}{4}\right)^5$$

$$= \frac{3^5}{4^6}$$

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(c) $$L_s = \frac{\rho}{1-\rho} = \frac{3/4}{1-3/4}$$

$= 3$ customers

& $$L_q = \frac{\rho^2}{1-\rho} = \frac{(3/4)^2}{1/4}$$

$$= \frac{9}{4}$$

$= 2.25$ customers

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{60-45} = \frac{1}{15} \text{ hrs}$$

$= 0.066$ hrs

$$W_q = \frac{\rho}{\mu(1-\rho)} = \frac{3/4}{60(1/4)} = \frac{3}{60}$$

$= 0.05$ hrs

(2) The arrival rate of customers at single window booking counter of a two wheeler agency follows poisson dist & service rate follows poisson dist. The arrival ~~dist~~ rate & service rate are 25 customers per hr & 35 customers per hr respectively. Find
(i) Utilization of booking clerk
(ii) avg no. of waiting customers in queue
(iii) avg no. of waiting customers in system
(iv) avg. waiting time per customer in queue
— " — system
(v)

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→ $\lambda = 25/hr$ $\mu = 35/hrs$

(i) Utilization of booking clerk: $\rho = \frac{\lambda}{\mu}$

$= \frac{25}{35}$

$= \frac{5}{7}$

(ii) $L_q = \frac{\rho^2}{1-\rho}$

$= \frac{25/49}{1-5/7}$

$= \frac{25}{14}$

$= 1.785$ customers

(iii) $L_s = \frac{\rho}{1-\rho}$

$= \frac{5/7}{2/7}$

$= \frac{5}{2}$

$= 2.5$ customers

(iv) $W_q = \frac{\rho}{\mu(1-\rho)} = \frac{5/7}{35(2/7)}$

$= \frac{1}{14}$ hrs

$= 0.0714$ hrs.

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(v) $W_s = \frac{1}{\mu-\lambda} = \frac{1}{35-25} = \frac{1}{10} = 0.1$ hrs

(2) A TV repairer finds out that the avg time spent on a job is 30 min. If he repairs the set, in the order in which they come & if the avg arrival of sets 10 per 8 hrs a day.

(a) What is the repairman's expected idle time each day

(b) How many jobs are ahead of the avg set just brought in?

→ $\frac{1}{\mu} = 30$ min $\lambda = 10$ per 8 hrs a day

$\Rightarrow \lambda = \frac{10}{8}$ sets per hr.

$\mu = \frac{1}{30}$ per min

$\mu = \frac{1}{30} \times 60$

~~$\mu = \frac{1}{15}$ 2.5~~

$\mu = 2$ sets per hr.

$\Rightarrow \rho = \frac{\lambda}{\mu} = \frac{10/8}{2} = \frac{5}{8}$

(a) Repairman idle time each day:

$P_0 = 1-\rho$

$= 1-\frac{5}{8}$

$= \frac{3}{8}$

(b) Jobs ahead of a.v.g set brought in

$$L_s = \frac{\rho}{1-\rho} = \frac{5/8}{3/8} = \frac{5}{8} = 1.67 \text{ } x\ell$$

(4) In 24 hrs ~~for~~ services station, vehicle arrive at the rate of 30 per day. On the average the avg servicing time for a vehicle is 36 min. Find (i) The mean no of vehicles waiting in the system
(ii) the probability that queue size exceeds nine

→ $\lambda = 30$ per day ; $\frac{1}{\mu} = 36$ min

$\mu = \frac{1}{36}$ per min

$\lambda = \frac{30}{24 \times 60}$

$\lambda = \frac{1}{48}$ per min

$$\rho = \frac{\lambda}{\mu} = \frac{1/48}{1/36} = \frac{3}{4}$$

(i) Mean no of vehicles ~~waiting~~ waiting in the system:

$$L_s = \frac{\rho}{1-\rho} = \frac{3/4}{1/4} = 3 \text{ vehicles}$$

(ii) Probability that

$P(\geq 10) =$

(5) In a railway marshalling yard, goods train arrive at the rate, assuming that the inter arrival time follows an exponential dist & service time is also exponential with an average 36 min. Calculate:
(i) Avg no of trains in the yard
(ii) Probability that queue size exceeds 6
(iii) Expected waiting time in the queue
(iv) Avg no of trains in the queue

→ (i) Avg no of trains in yard:

$$L_s = \frac{\rho}{1-\rho} = \frac{3/4}{1/4} = 3 \text{ trains}$$

(ii) probability that queue size exceeds 6:

$$P(\geq 7) = \rho^7 = \left(\frac{3}{4}\right)^7$$

(iii) Expected waiting time in the queue:

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\rho}{\mu(1-\rho)}$$

$$= \frac{3/4}{1/36\,(1/4)} = 108 \text{ min}$$

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(iv) Average no. of vehicles in the queue.

$$L_q = \frac{\rho^2}{1-\rho} = \frac{9/16}{1/4} = \frac{9}{4} = 2.25 \text{ veh}$$

(5) Customers arrive at a telephone booth at intervals of 10 min on the avg. The length of a phone call is 3 min on the avg.

(i) What is the probability that a person arriving at the booth will have to wait

(ii) What is the avg length of the queue that forms time to time.

(iii) Estimate the fraction of a day that phone is in use.

(iv) Find avg no. of units in the system

(v) The telephone ~~dept~~ department will install a second booth when convinced that an arrival would expect to wait atleast 3 min for the phone. By how much the flow of arrivals ~~be~~ increase in order to justify ~~a~~ second booth.

$\frac{1}{\lambda} = 10$ min $\qquad \frac{1}{\mu} = 3$ min

$\therefore \lambda = \frac{1}{10}$ per min $\qquad \mu = \frac{1}{3}$ per min

$$\rho = \frac{\lambda}{\mu} = \frac{1/10}{1/3} = \frac{3}{10} = 0.3$$

(i) Probability that person arriving at the booth will have to wait

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$$1 - P_0 = 1 - (1-\rho)$$

$$= \rho$$

$$= 0.3$$

(ii) avg length of queue that forms time to time

$$L_n = \frac{1}{1-\rho} = \frac{1}{1-0.3} = \frac{10}{7} \text{ customers}$$

(iii) Fraction of the day the phone is in use:

$$\rho = 0.3$$

(iv) avg no. of units in the system

$$L_s = \frac{\rho}{1-\rho} = \frac{0.3}{0.7} = \frac{3}{7}$$

(v) Let λ' be the arrival rate (new)

$$W_q \geq 3$$

$$\frac{\lambda'}{\mu(\mu-\lambda')} \geq 3$$

$$\lambda' \geq 3\mu(\mu-\lambda')$$

$$\lambda' \geq 3\mu^2 - 3\mu\lambda'$$

$$\lambda' \geq \frac{3\mu^2}{1+3\mu}$$

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$$\lambda' \geq \frac{3(1/9)}{1+3(1/3)}$$

$$\lambda' \geq \frac{1}{6}$$

Thus the arrival rate should be atleast 0.16 person per min.

$\therefore$ increase in arrival rate $0.16 - 0.1$
$= 0.06$ person/min

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