

## 2] Multiplicative law of Probability :-

Statement :- Probability of simultaneous occurrence of two events  $A_1$  and  $A_2$  is given by

$$P(A_1 \cap A_2) = P(A_1 \cdot A_2) = P(A_1) \cdot P(A_2/A_1) \\ = P(A_2) \cdot P(A_1/A_2)$$

### # Draw with replacement :-

If the balls drawn in the first draw was replaced back in the bag before 2<sup>nd</sup> draw then the event  $A$  &  $B$  are independent and the required probability is given by

$$P(A \cap B) = P(A) P(B) \quad A = \text{I draw} \quad B = \text{II draw}$$

### Draw without replacement

$$P(A \cap B) = P(A) \cdot P(B/A)$$

$P(AB) = P(A) P(B)$  probability that first event happens  
 &  $(1-p_1)$  and second fail

If both event fail  $P(\bar{A}) P(\bar{B})$

At least one of the event  $p_1 p_2 + (1-p_1)p_2 + p_1(1-p_2)$

Q] Two marbles are selected from a bag containing 10 red, 50 white, 20 blue and 15 orange marbles.

(a) With replacement being made after each draw

(i) both are white

(ii) First is red & second is white

(iii) Neither is orange

(b) Solve same without replacement

(a) (i) Total outcomes =  ${}^{75}C_2$

(i)  $\frac{{}^{35}C_1 \times {}^{35}C_1}{{}^{75}C_1 \times {}^{75}C_1}$

(ii)  $\frac{{}^{10}C_1 \times {}^{30}C_1}{{}^{75}C_1 \times {}^{75}C_1}$

(iii)  $1 - \frac{{}^{15}C_1 \times {}^{15}C_1}{{}^{75}C_1 \times {}^{75}C_1}$

(b) (i)  $\frac{{}^{30}C_1 \times {}^{22}C_1}{{}^{75}C_1 \times {}^{75}C_1}$  (ii)  $\frac{{}^{10}C_1 \times {}^{30}C_1}{{}^{75}C_1 \times {}^{75}C_1}$  (iii)  $1 - \frac{{}^{15}C_1 \times {}^{15}C_1}{{}^{75}C_1 \times {}^{75}C_1}$

Q] A box of light bulbs contains 30 bulbs of which 5 bulbs are defective. If 3 of the bulbs are selected at random taken out from the box without replacement, find all three are defective

Ans total outcomes  ${}^{30}C_3$

$P = \frac{{}^5C_1 \times {}^4C_1 \times {}^3C_1}{{}^{30}C_1 \times {}^{29}C_1 \times {}^{28}C_1}$

Q] The probability that a contractor will get a plumbing contract is  $\frac{2}{3}$  and the probability that he will not get an electric contract is  $\frac{5}{9}$ . If the probability of getting at least one contract is  $\frac{4}{5}$ , what is the probability that he will get both the contracts.

A = plumbing B = electric

$P(A) = \frac{2}{3}$

$P(B) = \frac{5}{9}$   $P(\bar{B}) = \frac{4}{9}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{4}{5} = \frac{2}{3} + \frac{4}{9} - P(A \cap B)$

$P(A \cap B) = \frac{14}{45}$



A B

or, either, atleast  
both, and  
given

 $A \cup B$  $A \cap B$  $A/B$ 

Q] A can hit a target 4 times in 5 shots  
B can hit 3 times in 4 shots, C can hit  
2 times in 3 shots what is the probability  
that

(i) 2 shots hit (ii) atleast two shot hit

sol<sup>n</sup>  $P(A) = 4/5$   $P(B) = 3/4$   $P(C) = 2/3$

(i)  $P(ABC) + P(AB\bar{C}) + P(\bar{A}BC)$

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{26}{60}$$

(ii)  $P(2 \text{ shots hit}) + P(\text{all shot hit})$

$$= \frac{26}{60} + P(ABC)$$

$$= \frac{26}{60} + \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{5}{6}$$

# Baye's theorem :- observe no. of cases, ref prob, common event  
If  $A_1, \dots$  and  $A_2$  are mutually exclusive and  
exhaustive event and  $B$  be an event  
which can occur in combination with  $A_1$   
and  $A_2$  then the conditional probability  
for event  $A_1$  and  $A_2$  given the event  $B$   
is given by

$$P(A_1|B) = \frac{P(A_1) \cdot P(B|A_1)}{P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2)}$$

$$\text{or } P(A_2|B) = \frac{P(A_2) \cdot P(B|A_2)}{P(A_2) \cdot P(B|A_2) + P(A_1) \cdot P(B|A_1)}$$

Q] In a railway reservation office two clerks are engaged in checking reservation forms, the first clerk collect 55% of the forms, the second clerk ~~contains~~ checks the remaining. The first clerk has an error rate of 0.03 and second clerk has an error rate of 0.02. A reservation form is selected at random from all the forms and is found to have an error. Find the probability that it was the first clerk or second clerk

ans.  $\downarrow$

$$P(A_1) = \frac{55}{100}$$

$\downarrow$

$$P(A_2) = \frac{45}{100}$$

$$P(B|A_1) = 0.03$$

$$P(B|A_2) = 0.02$$

Find  $P(A_1|B)$

$$P(A_2|B)$$

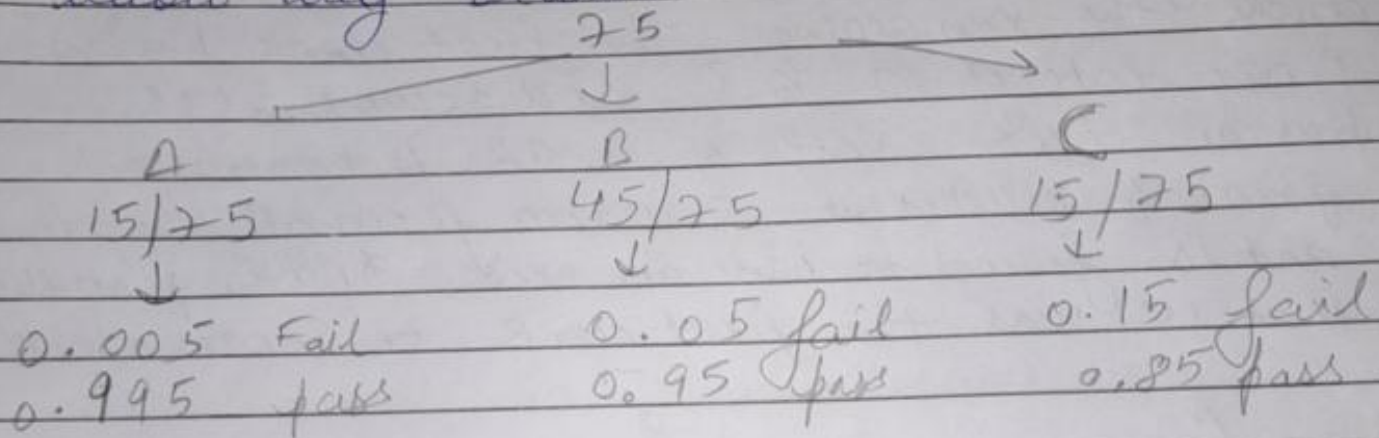
$$P(A_1|B) = \frac{\frac{55}{100} \times 0.03}{\frac{55}{100} \times 0.03 + \frac{45}{100} \times 0.02} = \frac{0.0165}{0.0165 + 0.009}$$

$$\frac{55}{100} \times 0.03 + \frac{45}{100} \times 0.02 = 0.0255$$

$$P(A_2|B) = \frac{\frac{45}{100} \times 0.02}{\frac{45}{100} \times 0.02 + \frac{55}{100} \times 0.03} = \frac{0.009}{0.009 + 0.0165} = 0.353$$



Q] In a class of 75 student, 15 were considered to be very intelligent, 45 as medium and rest below average. The probability that a very intelligent student fails in an exam is 0.005. The medium students have probability of 0.05 and below avg is 0.15. It is known to have pass the exam, what's the probability that it's below avg student



$$P = \frac{0.85 \times 15/75}{0.85 \times 15/75 + 15/75 \times 0.995 + 45/75 \times 0.95}$$

$$= 0.181$$

Formula applied =  $\frac{P(C) \cdot P(E|C)}{P(A) \cdot P(E|A) + P(B) \cdot P(E|B) + P(C) \cdot P(E|C)}$

Q] An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of their accident is 0.1, 0.3, 0.2 resp. One of the insured person meets with an accident. What is the probability that he is a scooter driver?

S	C	T
$P(A) 2000 / 12000$	$P(B) 9000 / 12000$	$P(C) 6000 / 12000$
$\downarrow$	$\downarrow$	$\downarrow$
$P(E A) 0.1$	$P(E B) 0.3$	$P(E C) 0.2$

$$P(S) = \frac{P(A) \times P(E|A)}{P(A) P(E|A) + P(B) P(E|B) + P(C) P(E|C)}$$

$$= \frac{\frac{1}{6} \times 0.1}{\frac{1}{6} \times 0.1 + \frac{1}{3} \times 0.3 + \frac{1}{2} \times 0.2} = \frac{1}{13}$$