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$$S \quad C \quad T$$

$P(A)$ 2000 / 12000 $\quad$ $P(B)$ 4000 / 12000 $\quad$ $P(C)$ 6000 / 12000

$P\left(\frac{E}{A}\right)$ 0.1 $\quad$ $P\left(\frac{E}{B}\right)$ 0.3 $\quad$ $P\left(\frac{E}{C}\right)$ 0.2

$$P(S) = \frac{P(A) \times P(E/A)}{P(A)\,P\left(\frac{E}{A}\right) + P(B)\,P\left(\frac{E}{B}\right) + P(C)\,P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{1}{6} \times 0.1}{\frac{1}{6} \times 0.1 + \frac{1}{3} \times 0.3 + \frac{1}{2} \times 0.2} = \frac{1}{13}$$

Random Variable :-

Variable :- A quantity that varies from individual to individual is called variable

Random Variable :- A variable whose value is determined by the outcomes of a random experiment is called random variable.

We can express outcomes in numeric data

Tossing of 2 coins : $S = \{HH, HT, TH, TT\}$
$\{2, 1, 1, 0\}$

X = no. of heads

X	S	P
0	TT	1/4
1	HT, TH	2/4
2	HH	1/4

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Random Variables are of 2 types:-
(1) discrete variable (2) continuous variable

(1) Discrete Variable :- If the random variable X assumes only a finite or countably infinite set of values.
eg no. of students, no. of empty seats, no of cars etc

(2) Continuous Variable :- Height, weight, amount of rainfall are examples
If a variable assumes infinite & uncountable set of value
$1 < x < 2$

Probablity distribution :- A probablity distribution is a description that gives the probablities for each value of the random variable
$S = \{HH, HT, TH, TT\}$

x	$P(X=x)$
0	1/4
1	1/2
2	1/2

for dicrete - tabular form
for continuous - formula or graph

X

DRV CRV

PMF - prob mass fcn PDF = prob density fcn

Discrete Variable - Probablity Mass Function

If X is a DRV which assumes value $x_1, x_2, x_3 \ldots x_n$ withe probablities $p(x_1), p(x_2) \ldots p(x_n)$ resp then $p_x(x) = P[X = x_i] = p(i)$ is called probablity mass function, it satisfies the following conditions

X	$P[x = x_i)$
x_1	$p(x_1)$
x_2	$p(x_2)$
$\vdots$	
x_n	$p(x_n)$

① $p(x_i) \geq 0$

② $\sum p(x_i) = 1$

Q] check if fnx is PMF or not
$P[X = x] = x - 2/2 \quad \forall x = 1, 2, 3, 4$

X	$P[X = x)$
1	$-1/2$
2	0
3	$1/2$
4	1

Not a PMF

$-1/2 \longrightarrow$ this is $-ve$

Q] A random variable X has the following probablity function

X	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

① Find the value of k
② Evaluate $P(X<6)$ ③ $P(X \geq 6)$
④ $P(0 < x < 5)$

① $\sum (p(x_i)) = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$k = 1/10$ or $k = -1$ X

② $P(x < 6) = P[X=0] + P[X=1] + P[X=2] + P[x=3] + \cdot$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} = \frac{81}{100}$

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Continuous Variable – Probablity Density Function

If x is a continuous random variable, then a function f is said to be the probablity density function of x if it satisfies the following conditions.

(i) $f(x) \geq 0$ for all x

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(iii) for all a, b with $-\infty < a < b < \infty$

$$P(a \leq x < b) = \int_a^b f(x)\,dx$$

Note:-
In case of DRV the Probablity at a fixed point (say c) is not always zero
$P(X=c) \neq 0$
But in case CRV the probablity at fixed point (say c) is always zero is
$P(X=c)=0$ Since we know that def of pdf $P(a \leq x < b) = \int_a^b f(x)\,dx$
X is CRV
$P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x < b)$
$= P(a < x < b)$

Distributive Function or Cumulative $dist^n$ fnx

Let X be a random variable discrete or continuous we define F to be the distribution function of X

$F_X(X) = P(X \leq x)$ x lies in $-\infty$ to x

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$P(X \le x) = \int_{-x}^{\infty} f(x)\,dx$ if x is continuous random variable

$P(X \le x) = \sum_{i=0}^{x} \underbrace{p(x_i)}_{pmf}$ if X is discrete random variable

Q] If x is CRV with p.d.f

$f(x) = \begin{cases} a(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

(i) Find the value of a

(ii) calc $P(X \le 1/3)$

(iii) $P(X > 0.5)$ (iv) $P(0.5 < X < 0.75)$

ans (i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{0}^{1} f(x)\,dx = 1$

$\int_{0}^{1} a(1-x)\,dx = 1$

$a \int_{0}^{1} (1-x)\,dx = 1$

$a = 2$

(ii) $P\left(x \le \frac{1}{3}\right) =$ $P(a \le x \le b) = \int_{a}^{b} f(x)\,dx$

$= \int_{0}^{1/3} 2(1-x)\,dx$

$= 5/9$

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(iii) $P(x > 0.5)$

$$= \int_{0.5}^{1} 2(1-x)\,dx = 0.25$$

(iv) $P[0.5 < x < 0.75)$

$$= \int_{0.5}^{0.75} 2(1-x)\,dx = 3/16$$

Q] Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax + 3a & 2 \le x \le 3 \end{cases}$$

① find the value of a

② find $P(2 < x \le 2.5)$

① $\int_{-\infty}^{\infty} f(x) = 0$

$$= \int_{-\infty}^{0} \cancel{f(x)\,dx}^{\,0} + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$$

$$+ \int_{2}^{3} f(x)\,dx + \int_{3}^{\infty} \cancel{f(x)\,dx}^{\,0}$$

$a = 1/2$

② $\int_{2}^{2.5} -\frac{1}{2}x + \left(-\frac{3}{2}\right)$

$$= \frac{1}{2}\int_{2}^{2.5} x + 1 = \frac{1}{4}$$

Q) If $f(x) = 2x$ when $0 \le x \le 1$ find the probability

(i) $P\left(x < \frac{1}{2}\right)$

$$\int_0^{1/2} f(x)\,dx = \int_0^{1/2} 2x = 1/4$$

(ii) $P\left(\frac{1}{4} < x \le \frac{1}{2}\right)$

$$= \int_{1/4}^{1/2} f(x)\,dx = \int_{1/4}^{1/2} 2x = 3/16$$

(iii) $P\left(x > 3/4 \;/\; x \ge 1/2\right)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{P(x > 3/4 \;\cap\; x \ge 1/2)}{P(x \ge 1/2)}$$

$1/2 \quad 3/4$

$$= \frac{\int_{3/4}^{1} f(x)\,dx}{\int_{1/2}^{1} f(x)\,dx}$$

$$= \frac{7/16}{3/4} = 7/12$$