

$$\# \text{ Variance} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$SD = \pm \sqrt{\text{Variance}} \quad CV = \frac{\sigma}{\bar{x}} \times 100$$

unit of measurement

different same

lesser the CV

higher the stability

same mean

std dev

diff mean

coeff of variation

Q] From the share prices of X and Y given below, state which share is more stable in value.

$\sigma_x = 3.25$ $\sigma_y = 2.87$ given

x	41	44	43	48	45	46	49	50	42	40
y	91	93	96	92	90	97	99	94	98	95

Solⁿ Share X

$$\bar{x} = 448/10 = 44.8$$

$$\sigma = 3.25$$

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{3.25}{44.8} \times 100$$

$$= 7.25\%$$

Share Y

$$\bar{y} = 948/10 = 94.5$$

$$\sigma_y = 2.87$$

$$CV = \frac{\sigma}{\bar{y}} \times 100$$

$$= \frac{2.87}{94.5} \times 100$$

$$= 3.03\%$$

$CV_x < CV_y$ hence share X is more stable

Q) Goals scored by two teams A and B in a football session are as follows.

goals	0	1	2	3	4	
A	22	9	8	5	4	$\sigma_A = 1.309$
B	17	9	6	5	3	$\sigma_B = 1.307$

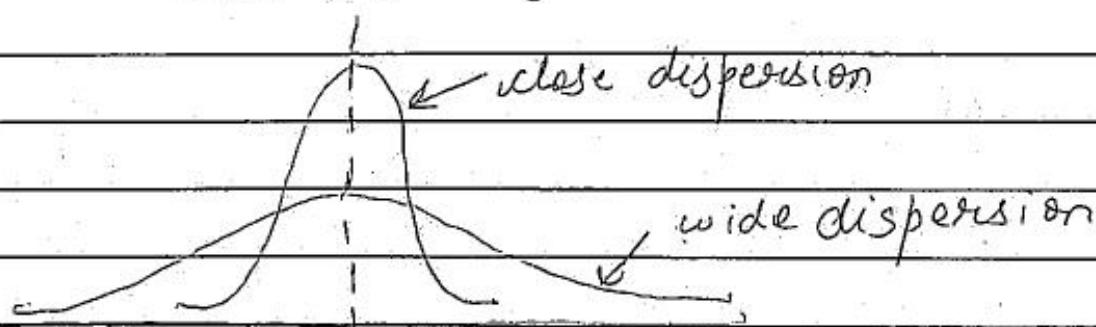
	team A			team B		
<u>ans</u>	x	f_A	$f_A x$	x	f	$f x$
	0	22	0	0	17	0
	1	9	9	1	9	9
	2	8	16	2	6	12
	3	5	15	3	5	15
	4	4	16	4	3	12
		<u>53</u>	<u>56</u>		<u>40</u>	<u>48</u>

$$\begin{aligned}\bar{x}_A &= 1.06 \\ \sigma_A &= 1.303 \\ CV_A &= \frac{1.303}{1.06} \times 100 \\ &= 123.49\end{aligned}$$

$$\begin{aligned}\bar{x}_B &= 1.2 \\ \sigma_B &= 1.307 \\ CV_B &= 108.9\end{aligned}$$

Is more stable

$$CV_A > CV_B$$



Moments: - Moment of variate about some point and these pts are used to describe the characteristics of frequency distribution.

↳ Moments about mean (central moment)

→ moment about origin or zero.

r^{th} moment about mean:-

$$\mu_r = \frac{\sum (x - \bar{x})^r}{N} \quad \bar{x} = \frac{\sum x}{N}$$

$$\mu_1 = \frac{\sum (x - \bar{x})}{N} = 0$$

$$\mu_2 = \frac{\sum (x - \bar{x})^2}{N} \quad \mu_3 = \frac{\sum (x - \bar{x})^3}{N}$$

$$\mu_4 = \frac{\sum (x - \bar{x})^4}{N}$$

First moment about mean is always 0

moment about origin or zero

$$\mu'_r = \frac{\sum (x - 0)^r}{N} = \frac{\sum x^r}{N} \quad r = 1, 2, 3, 4$$

$$\mu'_1 = \frac{\sum x}{N} = \text{mean} \quad \mu'_2 = \frac{\sum x^2}{N} \quad \mu'_3 = \frac{\sum x^3}{N} \quad \mu'_4 = \frac{\sum x^4}{N}$$

Moments about mean \Rightarrow

T.S

$$\mu_r = \frac{\sum (x - \bar{x})^r}{N} \quad r = 1, 2, 3, 4$$

$$r = 1$$

$$\mu_1 = 0$$

$$\mu_2 = \frac{\sum (x - \bar{x})^2}{N} = \text{Variance}$$

$$\mu_3 = \frac{\sum (x - \bar{x})^3}{N}$$

$$\mu_4 = \frac{\sum (x - \bar{x})^4}{N}$$

discrete series :-

$$\mu_r = \frac{\sum f(x-\bar{x})^r}{N} \quad r=1, 2, 3, 4$$

$$\mu_1 = \frac{\sum f(x-\bar{x})}{N} = 0$$

$$\mu_2 = \frac{\sum f(x-\bar{x})^2}{N} = \text{variance}$$

$$\mu_3 = \frac{\sum f(x-\bar{x})^3}{N} \quad \mu_4 = \frac{\sum f(x-\bar{x})^4}{N}$$

Continuous Series :-

$$\mu_r = \frac{\sum f(m-\bar{x})^r}{N}$$

$$\mu_1 = \frac{\sum f(m-\bar{x})}{N} = 0 \quad \mu_2 = \frac{\sum f(m-\bar{x})^2}{N}$$

$$\mu_3 = \frac{\sum f(m-\bar{x})^3}{N} \quad \mu_4 = \frac{\sum f(m-\bar{x})^4}{N}$$

Q] Find first 4 moment about mean

x	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$
1	25	-125	625
2	9	-27	81
7	1	1	1
9	09	27	81
10	16	64	256
	<u>60</u>	<u>-60</u>	<u>1044</u>

$$\bar{x} = 30/5 = 6$$

$$\mu_1 = 0$$

$$\mu_4 = \frac{1044}{5} = 208.8$$

$$\mu_2 = \frac{60}{5} = 12$$

$$\mu_3 = \frac{-60}{5} = -12$$

moments About Origin

I.S

$$u'_1 = \frac{\sum x^1}{N}$$

$$x = 1, 2, 3, 4$$

$$u'_1 = \frac{\sum x}{N} = \text{mean}$$

$$u'_2 = \frac{\sum x^2}{N}$$

$$u'_3 = \frac{\sum x^3}{N}$$

$$u'_4 = \frac{\sum x^4}{N}$$

D.S

$$u'_1 = \frac{\sum f x^1}{N}$$

$$x = 1, 2, 3, 4$$

$$u'_1 = \frac{\sum f x}{N} = \text{mean}$$

$$u'_2 = \frac{\sum f x^2}{N}$$

$$u'_3 = \frac{\sum f x^3}{N}$$

$$u'_4 = \frac{\sum f x^4}{N}$$

C.S

$$u'_1 = \frac{\sum f m^1}{N}$$

$$x = 1, 2, 3, 4$$

$$u'_1 = \frac{\sum f m}{N} = \text{mean}$$

$$u'_2 = \frac{\sum f m^2}{N}$$

$$u'_3 = \frac{\sum f m^3}{N}$$

$$u'_4 = \frac{\sum f m^4}{N}$$

Conversion of non-central moment to central moment

$$u_1 = u'_1 - (u'_1)' = 0$$

$$u_2 = u'_2 - (u'_1)^2 = \text{variance}$$

$$u_3 = u'_3 - 3u'_2 u'_1 + 2(u'_1)^3$$

$$u_4 = u'_4 - 4u'_3 u'_1 + 6u'_2 (u'_1)^2 - 3(u'_1)^4$$

Q/ find first 4 moments ~~to~~ about mean

step 17 calc moments about origin

step 27 apply conversion formula

$$\mu_1' = \frac{\sum f m^1}{N}$$

x	f	mi	$f m$	m^2	$f m^2$
0-5	2	2.5	5	6.25	12.5
5-10	5	7.5	37.5	56.25	281.25
10-15	7	12.5	87.5	156.25	1093.75
15-20	13	17.5	227.5	306.25	3981.25
20-25	21	22.5	472.5	506.25	10631.25
25-30	16	27.5	440.5	756.25	12100
30-35	8	32.5	260	1056.25	8450
35-40	3	37.5	112.5	1406.25	4218.75
	<u>75</u>		<u>1642.5</u>		<u>40768.75</u>

$$\mu_1' = \frac{1642.5}{75} = 21.9 \quad \mu_2' = \frac{40768.75}{75} = 543.58$$

m^3	$f m^3$	m^4	$f m^4$
15.625	31.25	39.0625	78.125
421.875	2109.375	3164.0625	15820.3125
1953.125	12671.875	24414.0625	170898.4375
5359.375	69671.875	93789.0625	1219257.813
11390.625	239203.125	256289.0625	5382070.313
20796.875	3322750	1115614.063	9150125
34328.125	274625	1977539.063	8915312.5
52734.375	158203.125	1977539.063	5932617.188
	<u>1090265.625</u>		<u>30796679.19</u>

$$u_3' = \frac{1090265.625}{75} = 14536.875$$

$$u_4' = \frac{30796679.69}{75} = 410622.3958$$

$$u_1 = 0$$

$$u_2' = u_2' - (u_1')^2$$

$$= 543.58 - (21.9)^2$$

$$= 63.97$$

$$u_3 = u_3' - 3u_2' u_1' + 2(u_1')^3$$

$$= 14536.875 - 3(543.58)(21.9) + 2(21.9)^3$$

$$= -21172.046 + 21006.918$$

$$= -166.128$$