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Characteristics of Probability distribution :-
This is important to know two thing about a probability distribution. these are:-
(i) where the distribution is located → avg
(ii) how is the distribution varying with respect to central location → variance

Mathematical Expectation :- The expected value tells the value of the random variable that we expect in the "long run" after many experimental trials.

① Let x be a discrete RV taking value $x_1, x_2 \ldots x_n$ with prob $p(x_1) \ldots p(x_n)$ resp such that $\sum p(x_i) = 1$ with pmf $p(x_i) = P[X = x)$ is given by $E(x) = \sum x_i \, p(x_i)$ provide $\sum p_i = 1$

x	$P[X = x_i]$
x_1	p_1
x_2	p_2
$\vdots$	$\vdots$
x_n	$p(x_n)$

$$= p_1 x_1 + p_2 x_2 + \cdots + p_n x_n$$

② If x is a continuous RV with pdf $f(x)$ and $E(x)$ is defined as :-

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx \quad \text{provided} \int_{-\infty}^{\infty} f(x)\, dx = 1$$

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Properties of Exputation :-

① If x is a random variable and $g(x)$ is a fnx of X then $E[g(x)] =$

$$\Sigma g(x) \cdot p_i \quad \text{if } x \text{ is DRV}$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) \cdot f(x)\, dx \quad \text{if } X \text{ is CR}$$

② $E(c) = C$ or if C is any constant

③ $E[cX] = C E[X]$

④ $E[a+bx]$ if a & b are 2 const $= E[A] + E[bx]$

⑤ $E[x+y) = E[x] + E[y)$ if x & y are any two random variable

⑥ $E[xy] = E[x]\,E[y]$ if x & y are independent RV

$E[xy] \neq E[x]\,E[y]$ if X and y are not independent

Variance :- Let X be a RV, the variance of X is defined as

$$\sigma^2 = Var(x) = E[x - E(x)]^2$$
$$= E[x^2] - [E(x)]^2$$

$$s.d = +\sqrt{\text{variance}}$$

$$E(x) = \Sigma x\, p(x) \quad \text{if } X \text{ is DRV}$$

$$E(X) = \int_{-\infty}^{\infty} x\, f(x) \quad \text{if } X \text{ is CRV}$$

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$E(X^2) = \sum x^2 p(x)$ if x is DRV

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$ if x is CRV

Q] let X be a RV with the following prob dist:-

X	$-3 = x_1$	6 x_2	9 x_3
$P[X=x]$	$1/6$ p_1	$1/2$ p_2	$1/3$ p_3

(1) find $E(x)$

$\sum x_i p_i = x_1 p_1 + x_2 p_2 + x_3 p_3$

$= -3 \times 1/6 + 6 \times \frac{1}{2} + 9 \times \frac{1}{3}$

$= 11/2$

(2) $E(X^2) = \sum x^2 \cdot p(x)$

$= (-3)^2 \times 1/6 + (6)^2 \times \frac{1}{2} + 9^2 \times \frac{1}{3}$

$= 93/2$

(3) $E[2X+1]^2 = E[4x^2 + 4x + 1]$

$= E[4x^2] + E[4x] + E[1]$

$= 4[E[X^2]] + 4E[x] + 1$

$= 4 \times \frac{93}{2} + 4 \times \frac{11}{2} + 1$

$= 209$

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Random Variable

DRV

$\downarrow$

PMF

$P[X = x_i]$

i) $P(X_i) \geq 0$

2) $\Sigma p(x_i) = 1$

$E(X) = \Sigma x . \underset{pmf}{p(x)}$

$p(x) = x_1 p_1 + x_2 p_2 \ldots$

$var(x) = E(X^2) - [E(x)]^2$

$var(X) = E(X^2) - (mean)^2$

$E[X^2] = \Sigma x^2 \underset{pmf}{p(x)}$

$var(x) = E(X^2) - [E(x)]^2$

CRV

$\downarrow$

PDF

i) $f(x)$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$E(x) = \int_{-\infty}^{\infty} x . f(x)\,dx$

$E(X)^2 = \int x^2 \underset{pdf}{f(x)}\,dx$

Q] If pdf of a random variable is given by

$f(x) = \begin{cases} k(1-x^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

1. Find the value of k
2. Find the mean of X
3. Find the variance

ans 1) $\int_{-\infty}^{\infty} f(x) = 1$

$\int_0^1 k(1-x^2) = 1$

$k \int_0^1 (1-x^2) = 1$

$k = 3/2$

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(2) $E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

$\int_{0}^{1} x \times \frac{3}{2}(1-x^2)$

$\frac{3}{2}\int_{0}^{1}(x-x^3)\, dx = 3/8$

(3) $var(x) = E(x^2) - [\underbrace{E(x)}_{mean}]^2$

$= E(x^2) - \left(\frac{3}{8}\right)^2$

$E(x^2) = \int x^2 f(x)\, dx$

$= \int_{0}^{1} x^2 \times \frac{3}{2}(1-x^2)\, dx$

$= \frac{3}{2}\int_{0}^{1}(x^2-x^4)\, dx = 1/5$

$var = \frac{1}{5} - \frac{9}{64} = \frac{19}{320} = 0.059$

Q] If x is a CRV with pdf

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(1) Find the mean

(2) find the variance

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ans (1) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_0^1 x(x)\,dx + \int_1^2 x(2-x)\,dx$

$= \int_0^1 x^2\,dx + \int_1^2 2x - x^2\,dx$

$= \left[\frac{x^3}{3}\right]_0^1 + \left[x^2 - \frac{x^3}{3}\right]_1^2$

$= 1$

(2) var $= E(x^2) - (E[x])^2$

$E(x^2) = \int_0^{\infty} x^2 f(x)\,dx$

$= \int_0^1 x^2(x)\,dx + \int_1^2 x^2(2-x)\,dx$

$= \int_0^1 x^3\,dx + \int_1^2 2x^2 - x^3\,dx$

$= \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_1^2$

$= 0.166$ ans

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Covariance :- Its a measurement of the strength and direction of the linear relationship b/w two variable

$$cov(x, y) = E(xy) - E(x) \cdot E(y)$$

If x and y are independent then $cov(x, y) = 0$

$$E(x, y) = E(x)\,E(y)$$

Properties of Variance :-

(i) If a is a constant then

$$V(a) = 0$$

$$V(aX) = a^2 V(x)$$

$$V(a + X) = V(x)$$

(2) If x_1 and x_2 are two random variable and a_1 & a_2 are const then

$$V(a_1 x_1 + a_2 x_2) = a_1^2 V(x_1) + a_2^2 V(x_2) + 2(a_1 a_2)\, cov(x_1, x_2)$$

$$V(a_1 x_1 - a_2 x_2) = a_1^2 V(x_1) + a_2^2 V(x_2) - 2(a_1 a_2)\, cov(x_1, x_2)$$

(3) If X_1 & X_2 are independent random variable

$$V(a_1 x_1 \pm a_2 x_2) = a_1^2 V(x_1) + a_2^2 V(x_2)$$

$$[cov(x_1, x_2) = 0]$$

Q] If X and Y are independent random variable with mean 2, 3 and Variance 1, 2, respectively. Find the mean & variance of the random variable $Z = 2X - 5Y$

Sol^n mean (z)

$$Z = 2x - 5y$$

$$E(z) = E[2x - 5y]$$

$$= 2E(x) - 5E(y)$$

$$= 2 \times 2 - 5 \times 3$$

$$= 4 - 15$$

$$= -11$$

$$Z = 2x - 5y$$

$$V(z) = V(2x - 5y)$$

$$= 2^2 V(x) + 5^2 V(y)$$

$$2^2 \times 1 + 5^2 \times 2$$

$$V(z) = 54$$

if x & y are independent R.V

$cov(X, Y) = 0$