

## Project Questions

a. The recurrence used for this problem is based on dynamic programming. For a given set of robots to distribute ( $b$ ), the number of stacks available ( $n$ ), and the maximum number of robots that can be in each stack ( $k$ ), the recurrence is as follows:

...

$$O(\min(b,k)) + T(n-1) + \theta(1)$$

...

This recurrence expresses the number of ways to distribute  $b$  robots into  $n$  stacks, considering the constraint of a maximum of  $k$  robots in each stack.

b. The base cases of the recurrence are as follows:

- If  $b$  is 0 (no robots to distribute), there is only one way to do it (return 1).

$$T(n) = 1$$

- If  $n$  is less than or equal to 0 (no stacks available) or  $k$  is less than or equal to 0 (maximum robots per stack is 0), there are no valid ways to distribute the robots (return 0).

$$T(n) = 0 \text{ for all } n \leq 0 \text{ and } k \leq 0$$

c. Time and Space Complexities:

- **Time Complexity:**

1) The function is called recursively for each possible value of  $i$  in the range from 0 to  $\min(k, b)$ , which is  $O(\min(k, b))$ .

2) In each recursive call, the function explores  $n$  stacks.

if we consider both factors, the time complexity is  $O(n * \min(k, b))$ .

- **Space Complexity:**

The space complexity is primarily influenced by the memoization dictionary named `memo`, which stores previously computed results for specific combinations of  $b$ ,  $n$ , and  $k$ . In this case, the space complexity is  $O(b * n)$  because the dictionary may need to store results for each combination of  $b$  and  $n$ .

d. Pseudo-Code for Iterative Approach:

```
```python
```

```
Algorithm distribute_robots(b, n, k):
```

```
    Create a 3D array dp[b+1][n+1][k+1]
```

```
    for i from 0 to b:
```

```
        for j from 0 to n:
```

```

for x from 0 to k:
    if i == 0:
        dp[i][j][x] = 1
    else if j <= 0 or x <= 0:
        dp[i][j][x] = 0
    else:
        dp[i][j][x] = 0
        for y from 0 to min(x, i):
            dp[i][j][x] += dp[i-y][j-1][x]

return dp[b][n][k]
...

```

This pseudo-code describes an iterative approach to calculate the number of ways to distribute robots into stacks using a 3D array `dp` to store intermediate results.

**e. Time and Space Complexities for the Iterative Approach:**

- **Time Complexity:**  $O(n * b * k)$  due to the three nested loops.
- **Space Complexity:**  $O(n * b * k)$  for the 3D array `dp`.