Assignment - 1

1-(a) I have defined 2-D array named dp in which dp[a][b] denotes the probability of Alice winning a matches and Bob winning b matches.

We know Bob and Alice have each already won 1 match.
Therefore $dp[1][1] = 1$ [Base Case]
Thereafter if $a>1$

$$dp[a][b] = dp[a][b] + dp[a-1][b] \times \frac{b}{a+b-1}$$

Since both attack

When both attack probability of Alice winning

$$= \frac{n_B}{n_A + n_B}$$

$n_B \rightarrow$ Bob's current points

$n_A \rightarrow$ Alice's current points

And ~~Bob's~~ Alice had to win this match
Bob's point $= b$
Total $= a+b-1$
[Excluding this round]

Also if $b>1$

$$dp[a][b] = dp[a][b] + dp[a][b-1] \times \frac{a}{(a+b-1)}$$

Probability of Bob winning $(a+b)$th match

When both attack probability of Bob winning

$$= \frac{n_A}{n_A + n_B}$$

$n_A \rightarrow$ Alice's current points.

Entry no. = 0625 $\quad T_1T_2 = 96, T_3T_4 = 25$

0 replaced by 9 $\quad T = 121$

We have used mod Operation in between.

calc_prob(96, 25) = 086399808.

1b) Let us consider $(X_1 + X_2 + X_3 + \dots X_n)$ as a new random variable

Since 2 matches are already played, so there are $n-2$ matches left

Alice can only score maximum of $n-2$ points

Minimum of $2-n$ points

So Iterating through the possibilities from $-(n-2)$ to $n-2$

we get its the value of $X_1 + X_2 + \dots X_n = i$

the Since if Alice won x matches and lost y matches

$$x - y = i$$

$$x + y = n$$ [Since possibility of draws 0 when they both attack]

$$\therefore x = \frac{n+i}{2}$$

$$y = \frac{n-i}{2}$$

$\therefore$ if $(n+i)$ is divisible by 2

ans = ans + $i \times$ probability(alice winning $\frac{n+i}{2}$ matches, losing $\frac{n-i}{2}$ matches)

This gives the expectation which apparently comes out to be 0 in all cases.

Similarly for the variance

$$Var(X_1 + X_2 + \dots X_n) = E((X_1 + X_2 + \dots X_n)^2) - (E(X_1 + X_2 + \dots X_n))^2$$

in this similarly to calculating expectation of $X_1 + X_2 + \dots X_n$ we add multiplication of probability with i^2

$\therefore$ ans = ans + $i^2 \times$ probability(alice winning $\frac{n+i}{2}$, losing $\frac{n-i}{2}$ matches)

This is done from $i = -n+2, n-2$.

To ~~find variance we first~~ $\cancel{E(x^2 - x)} =$ $E(X^2) - (E(X))^2$

For T3T4 = 25 variance comes out to be 333333344
[pq^{-1} mod M form]

2a If Bob defences, expectation of points in diff strategies of Alice

Attack :- $\frac{5}{11} \times 1 + 0 \times \frac{1}{2} + 0 \times \frac{6}{11} = \frac{5}{11}$

Balanced :- $\frac{3}{10} \times 1 + \frac{1}{2} \times \frac{1}{2} + 0 \times \frac{1}{5} = \frac{11}{20}$

Defense :- $\frac{1}{10} + \frac{2}{5} + 0 = \frac{1}{2}$

$\therefore$ Balanced strategy to be followed as it gives the maximum expected points.

If Bob balances,

Attack :- $\frac{7}{10} + 0 \times \frac{1}{2} + \frac{3}{10} \times 0 = \frac{7}{10}$

Balanced :- $\frac{1}{3} + \frac{1}{2} \times \frac{1}{3} + 0 \times \frac{1}{3} = \frac{1}{2}$

Defence :- $\frac{1}{5} + \frac{1}{2} \times \frac{1}{2} + \frac{3}{10} \times 0 = \frac{9}{20}$

Clearly Alice should attack as it gives maximum expected points

If Bob attacks

Attack :- $\frac{n_B}{n_A + n_B}$

Defence :- $\frac{6}{11}$

Balanced = $\frac{3}{10}$

Depending on values of n_A, n_B, Alice should either attack or defence.

If $\frac{n_A}{n_A+n_B} > \frac{6}{11} \Rightarrow 11n_A > 6n_A + 6n_B$

$5n_A > 6n_B$

Alice should attack

else it should defence.

2C We carry out simulations until we get T wins, then this setup is reset before which we take note of the ~~time~~ no. of moves taken to get T wins. Then we add all values to get T wins and divide it by total no. of setup observed. This gives required expectation.

for $t_a = 25$ we get expectation as 51.42137

3a Here Bob randomly chooses to attack / balance / defend.

If Alice attacks then his expectation of points is:

$$\underset{[\text{Bob attacks}]}{\frac{1}{3} \times \frac{n_B}{n_A+n_B}} + \underset{[\text{Bob balances}]}{\frac{7}{10} \times \frac{1}{3}} + \underset{[\text{Bob defends}]}{\frac{5}{11} \times \frac{1}{3}}$$

$$= \frac{127}{330} + \frac{1}{3}\frac{n_B}{n_A+n_B}$$

If Alice balances then his expectation of points is:

$$\underset{[\text{Bob attacks}]}{\frac{1}{3} \times \frac{3}{10}} + \underset{[\text{Bob balances}]}{\frac{1}{3} \times \left[\frac{1}{3} + \frac{1}{2} \times \frac{1}{3}\right]} + \underset{[\text{Bob defends}]}{\frac{1}{3} \times \left[\frac{3}{10} + \frac{1}{2} \times \frac{1}{2}\right]}$$

$$= \frac{27}{20}$$

if Alice defends,

$$\underbrace{\frac{1}{3}\times\left[\frac{6}{11}\right]}_{\text{Bob Attacks}} + \underbrace{\frac{1}{3}\times\left[\frac{1}{3}+\frac{1}{2}\times\frac{1}{2}\right]}_{\text{Bob balances}} + \underbrace{\frac{1}{3}\times\left[\frac{1}{10}+\frac{1}{2}\times\frac{4}{5}\right]}_{\text{Bob defences}}$$

$$= \frac{2}{11} + \frac{3}{20} + \frac{1}{6} = \cancel{\frac{66+}{330}}\ \frac{12+99+110}{660} = \frac{221}{660}$$

∴ He either defends or attacks depending on n_A, n_B as

if ~~$29^* n_A < 15^* n_B$~~ : $29^* n_B > 15^* n_A$

~~de~~ Alice will ~~defend~~ attack

else ~~she~~ he will ~~attack~~ defend.

3a In 3b, we found out the dp table in which the state is defined as points of Alice, points of Bob and the rounds remaining.

Base Case ⇒ when rounds remaining = 0, Expected points of Alice = 1

Now when ~~expected~~ we find the max of expectation when Alice attacks, defends or balances,

Expected value = p-win * ~~ne~~ (1 + max exp.(na+1, nb, tot rounds-1))
+ ~~p~~ p-draw * (1/2 + max exp.(na+0.5, nb+0.5, total-1))
+ p-lose * (max exp.(na, nb+ ~~0~~ 1, total-1))

So the best of expectation is taken and stored in a dp-table.

In dp-table since we need integers we store ~~b(?)~~ 2* points [as possibility of 0.5 in points]

Also we found estimated value of expectation value using Monte Carlo using the greedy approach and compared the expectation value of both.
Our dp approach turns out to be better. It is printed in the end.

2b We know that there are two types of strategies, deterministic and non deterministic.
In non deterministic strategy, since expectation is function of the outcome which is itself probabilistic. Therefore the terms found in deterministic strategy is taken to be forget but in nondeterministic strategy even smaller terms have some probability to appear which make it worse than deterministic strategy.

In deterministic strategy, best is that of dp and in this case it comes out to be similar to that of non deterministic strategy.

Therefore our greedy approach gives the finest result.

Although while stimulating we also got different strategies which come better but finally the expectation of greedy turned out to be the best.