# Chapter 4 Hybrid Evolutionary Algorithm: A Case Study on Graph Coloring


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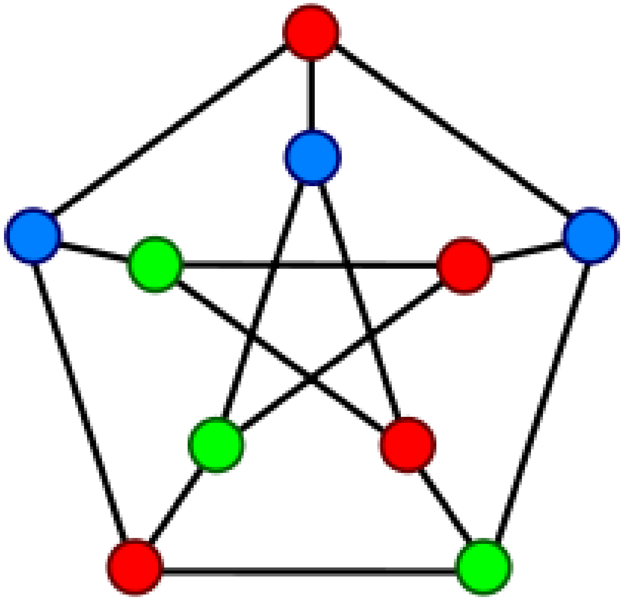
# Graph Coloring

* Given an undirected graph G=(V,E), the graph coloring problem (GCP) consists of assigning a color $c_i (1 \leq c_i \leq k)$ to each vertex such that adjacent vertices receive different colors and the number of colors used $k$ is minimized.



# Optimization or Decision?

- GCP—**Optimization** Problem (NP Hard):
  - To find the smallest number of colors k.

- ***k***-Coloring—**Decision** Problem (NP Complete):
  - Given a ***k***, we are asked whether there exists a coloring such that all the adjacent coloring constraints are satisfied.

- The **Optimization** version of GCP can be solved by tackling a series of the **Decision** version of GCP problem with a gradually decreasing ***k***.

- Thus, these two versions are equivalent to each other.

# Solution Procedure

* We starts from an initial *k* and solve the *k*-coloring problem. As soon as the *k*-coloring problem is solved, we decrease *k* by setting *k* to *k*-1 and solve again the *k*-coloring problem.

* This process is repeated until no legal *k*-coloring can be found.

* Smaller *k* → harder *k*-coloring problem. Thus, the solution approach just described solves thus a series of *k*-coloring problems of increasing difficulty.

* We only consider the K-coloring problem in this presentation.

# ILP Formulation

$$x_{v,c} = \begin{cases} 1 & \text{if vertex } v \text{ is coloured with colour } c \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{c=1}^{k} x_{v,c} = 1 \quad \forall \text{ vertices } v \in V$$

$$x_{u,c} + x_{v,c} \leq 1 \quad \forall \text{ colours } c \in K \quad \forall \text{ edges } \{u, v\} \in E$$

- Given ***k*** colors, $x_{v,c}$ is the decision variables.
- The first constraint requires that each vertex receives only one color.
- The second constraint denotes that adjacent vertices should receive different colors.

# Assignment Representation

* A solution of k-coloring problem can be represented as a series of colors that each vertex receives:

* $S = \{c_1, c_2, \dots, c_n\}$ where $c_v$ denotes the color of vertex $v$. It is required that for any $(u, v) \in E$, $c_u \neq c_v$.

* This representation is **natural**, but not **intuitive** and **essential**.

# Grouping Representation

* The feasible solution of k-coloring problem can also be presented as a set of independent sets, where an independent set is a set of non-adjacent vertices.

* $S = I_1 \cup I_2 \cup \cdots \cup I_k$, where
  1. $I_j \cap I_l = \emptyset$ for any $j$ and $l$
  2. $I_1 \cup I_2 \cup \cdots \cup I_k = V$
  3. $I_j$ is an independent set for any $j$.

* Thus, the $k$-coloring problem becomes to partition the $N$ vertices into $k$ independent sets.

# Applications

* Mobile radio frequency assignment
* Timetabling: Education, transportation, sports
* Register allocation
* Crew scheduling
* Printed circuit testing
* Air traffic flow management
* Satellite range scheduling
* Routing and wavelength assignment in WDM networks

# Literature Review (1)

## Constructive Greedy Algorithms

- The first heuristic approaches to solving the graph coloring problem, which color the vertices of the graph one by one guided by a greedy function.

- They are very fast by nature but their quality is unsatisfactory.

- The best known algorithms in this class are:
  - the *largest saturation degree* heuristic (DSATUR) (D. Brelaz, 1979)
  - *recursive largest first* heuristic (RLF) (F.T. Leighton, 1979)

- Often used to generate initial solutions for advanced algorithms

# Literature Review (2)

## Local Search Algorithms

- One representative example is the so-called *Tabucol* algorithm which is the first application of Tabu Search to graph coloring (Hertz, de Werra, 1987)
- Other local search metaheuristic methods include:
  - Simulated Annealing (Johnson et al, 1991)
  - Iterated Local Search (Chiarandini and Stutzle, 2002)
  - Reactive Partial Tabu Search (Blochliger, Zufferey, 2008)
  - GRASP (Laguna, Marti, 2001)
  - Variable Neighborhood Search (Avanthay et al, 2003)
  - Variable Space Search (Hertz et al, 2008)
  - *Clustering*-Guided Tabu Search (Porumbel, Hao, 2009)
- Interested readers are referred to [Galinier, Hertz, 2006] for a comprehensive survey of the local search approaches.
- I will describe the famous *Tabucol* algorithm in detail.

# Literature Review (3)

## Hybrid Evolutionary Algorithms

* One of the most recent and very promising approaches is based upon hybridization that embeds a **local search** algorithm into the framework of an **evolutionary algorithm** in order to achieve a better tradeoff between intensification and diversification, see for examples:
  * Dorne, Hao, 1998
  * Galinier, Hao, 1999
  * Galinier, Hertz, Zufferey, 2008
  * Malaguti, Monaci, Toth, 2008
  * Porumbel, Hao, Kuntz, 2009

# Initial Solution——DSATUR

* The heuristic of DSATUR(Degree of Saturation) is to sequentially color vertices according to a DANGER-based heuristic. The main idea of DSATUR is based on least saturation degree.

* At each phase, it consists of two steps: The first is to choose a vertex to color and the other is to choose a color for the chosen vertex.

# Initial Solution——DSATUR

* DSATUR starts by assigning color 1 to a vertex of maximal degree.
* Suppose F is a partial coloring of the vertices of G. The degree of saturation of a vertex x, degs(x), is the number of available colors that vertex x can use.
* The vertex to be colored next in the sequential coloring procedure of DSATUR is a vertex x with smallest degs(x), breaking ties by favoring vertex with larger uncolored degree.
* When deciding a color for a chosen vertex the color that is least likely to be required by neighboring vertices is selected.

# DSATUR

1. Initialization:

- Color[N] = -1
- Vetex_Color_Avail[N][K]=1
- Num_Avail_Colors[N]=K
- Vertex_Uncolored_Degree[N]=Degree[K]

2.Choose the vertex v1 with the largest degree, color v1 with color 1

```
Color[v1] = 1 ;
for v1's adjacent vertices vj
    Vetex_Color_Avail[vj][1] = 0 ;
    Num_Avail_Colors[vj] -- ;
    Vertex_Uncolored_Degree[vj] -- ;
```

# DSATUR

3. for(i = 2 ; i <= N ; i ++)
   - 3.1 choose a vertex vi according to:
   - < Num_Avail_Colors[vi] (small), Vertex_Uncolored_Degree[vi](large)>
   - 3.2 choose a color ki for vertex vi:
   - for each available color j for vertex vi
   - calculate the number of uncolored vertices for which color j is available
   - choose color ki with the smallest value of this number
   - 3.3 color vertex vi with color ki and updating:
   - Color[vi] = ki ;

     for vi’s adjacent vertices vj

     Vertex_Uncolored_Degree[vj] -- ;

     if(Vetex_Color_Avail[vj][ki] == 1)

     Num_Avail_Colors[vj] -- ;

     Vetex_Color_Avail[vj][ki] = 0 ;

# Improvements for DSATUR

* It is possible to improve DSATUR heuristic by considering more sophisticated information.

* For example, in case of choosing a color for the vertex, it would be better to consider the number of available colors.

* What else heuristics can be inspired in choosing vertex and choosing color?

# Local Search

*TabuCol*

# Search Space

* In this paper, we adapt the ***k-fixed penalty strategy*** which is also used by many coloring algorithms.
* For a given graph $G = (V; E)$, the number $k$ of colors is fixed and the search space contains all possible (legal and illegal) $k$-colorings.
* A $k$-coloring is represented by $S = \{V_1, \ldots, V_k\}$ such that $V_i$ is the set of vertices receiving color $i$.
* Thus, if for all *$V_i$ are independent sets*, then $S$ is a legal $k$-coloring. Otherwise, $S$ is an illegal (or conflicting) $k$-coloring.

# Evaluation Function

* The optimization objective is then to minimize the number of conflicting edges (referred to *confict number* hereafter) and find a legal *k*-coloring in the search space.

* Given a *k*-coloring S = {*V*1,… ,*Vk*}, the evaluation function *f* counts the conflict number induced by S such that

$$f(S) = \sum_{\{u,v\} \in E} \delta_{uv}$$

where

$$\delta_{uv} = \begin{cases} 1, \text{ if } u \in V_i,\ v \in V_j \text{ and } i = j, \\ 0, \text{ otherwise.} \end{cases}$$

# Initial Coloring

* The initial solution of our algorithm is randomly generated, i.e., each vertex in the graph is randomly assigned a color from 1 to $k$.
* Other greedy constructive heuristics are possible, like DSATUR, RLF, DANGER, etc.
* However, we observe that strong local search algorithms are not sensitive to the initial solutions.

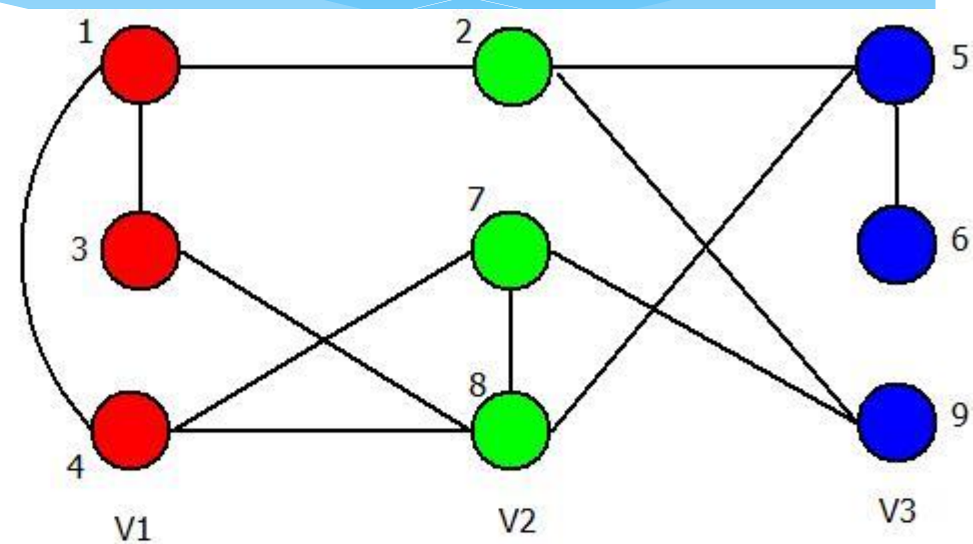
# Neighborhood Moves

- A neighborhood of a given $k$-coloring is obtained by moving a ***conflicting*** vertex $u$ from its original color class $V_i$ to another color class $V_j$ (denoted by $<u, i, j>$), called "critical one-move" neighborhood. A vertex is ***conflicting*** means that at least one of its adjacent vertices has the same color.

- Therefore, for a $k$-coloring $S$ with cost $f(S)$, the size of this neighborhood is bounded by $O(f(S) \times k)$.

# An Example

* f = 4
* Conflicting pairs: (1,3), (1,4), (7,8), (5,6)
* Critical One-Move: Only considers vertices 1, 3, 4, 7, 8, 5, 6. Totally 7*2=14 moves.



# Neighborhood Evaluation

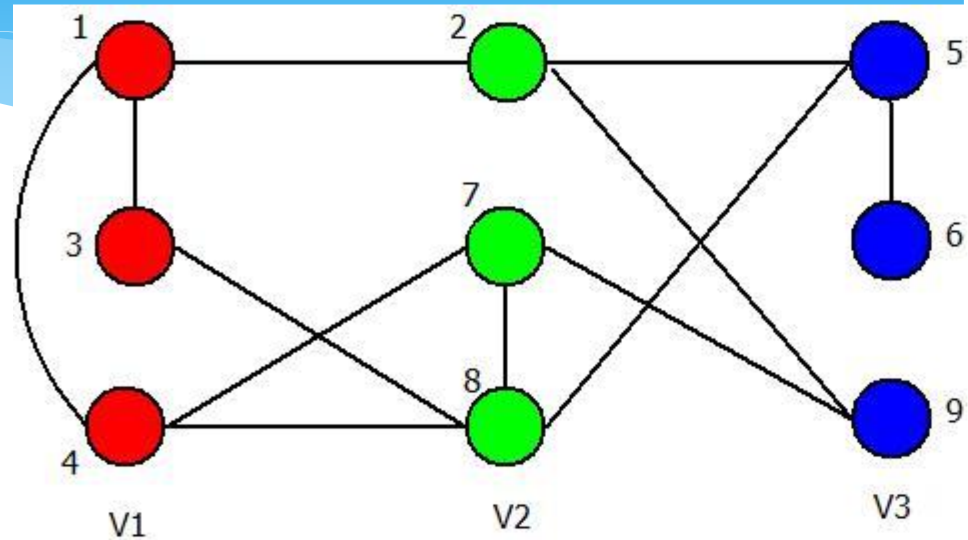
* In order to evaluate the neighborhood efficiently, we employ an ***incremental evaluation*** technique.

* The effect of each move on the objective function can be quickly calculated by a special data structure.

* Each time a move is carried out, only the move values affected by this move are updated accordingly.

# Adjacent-Color Table

| Vertex | Red V1 | Green V2 | Blue V3 |
|---|---|---|---|
| 1 | **2** | 1 | 0 |
| 2 | 1 | **0** | 1 |
| 3 | **1** | 1 | 0 |
| 4 | **1** | 2 | 0 |
| 5 | 0 | 2 | **1** |
| 6 | 0 | 0 | **1** |
| 7 | 1 | **1** | 1 |
| 8 | 2 | **1** | 1 |
| 9 | 0 | 2 | **0** |

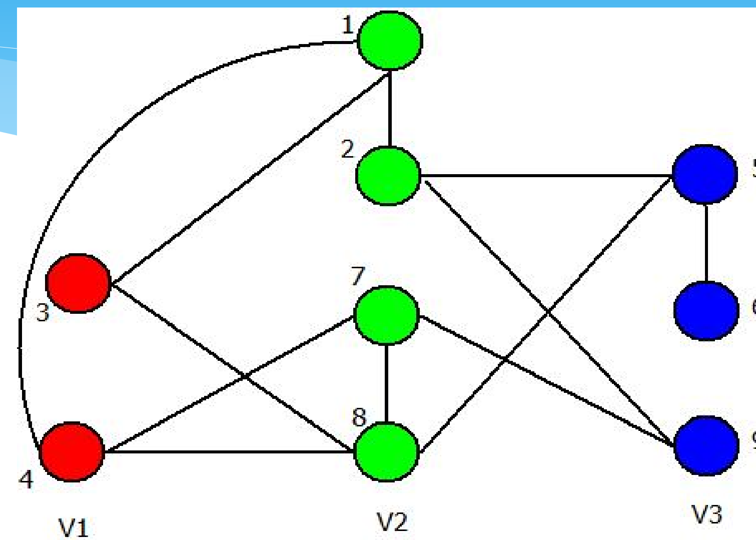



* This matrix M[u][i] (N*k) measures the number of adjacent vertices if vertex ***u*** receives color ***i***.

* Thus, the incremental move value of a move <u, i, j> can be quickly calculated as:
$\Delta(u,i,j) = M[u][j] - M[u][i]$

# Updating of Adjacent-Color Table

| Vertex | Red V1 | Green V2 | Blue V3 |
|---|---|---|---|
| 1 | 2 | **1** | 0 |
| 2 | 1-1=0 | **0+1=1** | 1 |
| 3 | **1-1=0** | 1+1=2 | 0 |
| 4 | **1-1=0** | 2+1=3 | 0 |
| 5 | 0 | 2 | 1 |
| 6 | 0 | 0 | **1** |
| 7 | 1 | **1** | 1 |
| 8 | 2 | **1** | 1 |
| 9 | 0 | 2 | **0** |




- Move (1, v1, v2):
- Only its adjacent vertices 2, 3 and 4 are affected, and only the v1 and v2 columns need to be updated.
- All old color (v1) columns decrease by 1.
- All new color (v2) columns increase by 1.

# Simple Local Search

1. Generate initial solution S, Calculate f(S)
2. Initialize the adjacent-color table M.
3. While {there exist improving moves}
3.1 Construct the neighborhood of S, denoted by N(S)
3.2 Calculate the Δ values of all critical one-moves
3.3 Find the best move with the least Δ value
3.4 Perform the best move: $f' = f + \Delta_{best}$
3.5 Update the adjacent-color table M

End

# Tabu Search
# Escaping from Local Optimum

* Tabu Search incorporates a tabu list as a “recency-based” memory structure to assure that solutions visited within a certain span of iterations, called tabu tenure, will not be revisited.

* TS then restricts consideration to moves not forbidden by the tabu list, and selects a move that produces the best move value to perform.

# Tabu Search

* 1. What?
* 2. Tenure?
* 3. How to judge if a move is forbidden?

# Attributes or Solution?

- It should be noted that we generally forbid **attributes** of solutions, but not the **solutions** themselves, since it is too expensive to forbid **solutions**.

- For the tabu list, once move $\langle u, i, j \rangle$ is performed, vertex $u$ is forbidden to move back to color class $V_i$ for the next $tt$ iterations.

# Tabu Tenure

- For the tabu list, once move <*u*, *i*, *j*> is performed, vertex *u* is forbidden to move back to color class *Vi* for the next *tt* iterations.

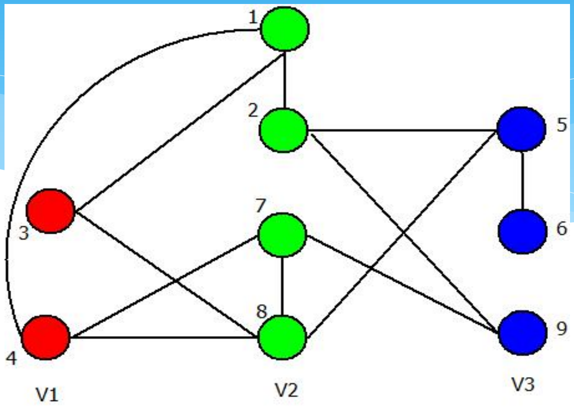
- Here, the tabu tenure *tt* is dynamically determined by

$$tt = f(S) + r(10)$$

where $r(10)$ takes a random number in $\{1,\ldots,10\}$.

# TabuTenure Table

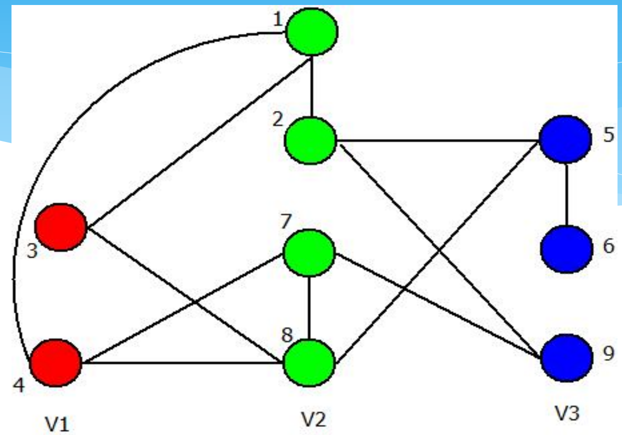
| Vertex | Red V1 | Green V2 | Blue V3 |
|---|---|---|---|
| 1 | 9 | 0 | 0 |
| 2 | 0 | 10 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |




* At the begining of the search, the TabuTenure table is initialized to be zero.
* once move <*u*, *i*, *j*> is performed, the value of Table[u][i]=TabuTenure.
* Once the search progresses, the non-zero value of the table is decreased by one at each time.
* In the following search, we can decide if a move <*u*, *i*, *j*> is tabu by checking if Table[u][j]>0.

# Fast Implementation of TabuTenure Table

| Vertex | Red V1 | Green V2 | Blue V3 |
|---|---|---|---|
| 1 | 1+10 | 0 | 0 |
| 2 | 0 | 2+15 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 3+12 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 4+10 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 5+12 |




- The above Table[u][i] records the relative tabu tenure length. Why not record the absolute tabu tenure length?
- Once move <*u*, *i*, *j*> is performed, the value of Table[u][i] = TabuTenure+Iter.
- In the following search, we can decide if a move <*u*, *i*, *j*> is tabu by checking if Table[u][j]>Iter.
- In this way, the tabu tenure table can be updated in O(1).

# Aspiration

* If one move can override the best found solution found so far, it is accepted even if it is in tabu status.

* This is because only the **attributes** but not **solutions** themselves are stored in the tabu table.

# TS Algorithm

1. Generate initial solution S, Calculate f(S)
2. Initialize the adjacent-color table M.
3. While {stop condition is not met}

3.1 Construct the neighborhood of S, denoted by N(S)

3.2 Calculate the Δ values of all critical one-moves

3.3 Find the best tabu and non-tabu moves with the least Δ value

3.4 If {the aspiration condition is satisfied}

perform the best tabu move,

else

perform the best non-tabu move

3.5 Update *f* and the adjacent-color table M

End

# TS Algorithm

- Data Structures: Sol[N], f, BestSol[N], Best_f, TabuTenure[N][K], Adjacent_Color_Table[N][K]
- Subfunctions:
  - Initialization(): Initialize the values of the data structures.
  - FindMove(u,vi,vj,delt): find the best non-tabu or tabu move.
  - MakeMove(u,vi,vj,delt): update the correponding values.
- TabuSearch()

```
{ int u, vi, vj, iter = 0;
  Initialization() ;
  while( iter < MaxIter) {
      FindMove(u,vi,vj,delt) ;
      MakeMove(u,vi,vj,delt) ; }
}
```

# Find Move

* FindMove(u,vi,vj,delt)

```
{
  for(i=1:N)
   if(Adjacent_Color_Table[ i ][ Sol[i] ] > 0) {
    for( k = 1: K)
     if( k != Sol[i] ) {
        calculate delt value of the move <i, Sol[i], k>
        {
          if (Table[i][k] < iter)   update the tabu best move;
         else  update the non-tabu best move;
     } } }
if(the tabu best move satisfies the tabu aspiration criterion)
        <u, vi, vj, delt> = the tabu best move;
else    <u, vi, vj,delt> = the non-tabu best move;
}
```

# Make Move

- MakeMove(u,vi,vj, delt)
- {
- Sol[ u ] = vj ;
- f = f + delt ;
- Table[ u ][ vi ] = iter + f + rand()%10;
- Update the Adjacent_Color_Table;
- }

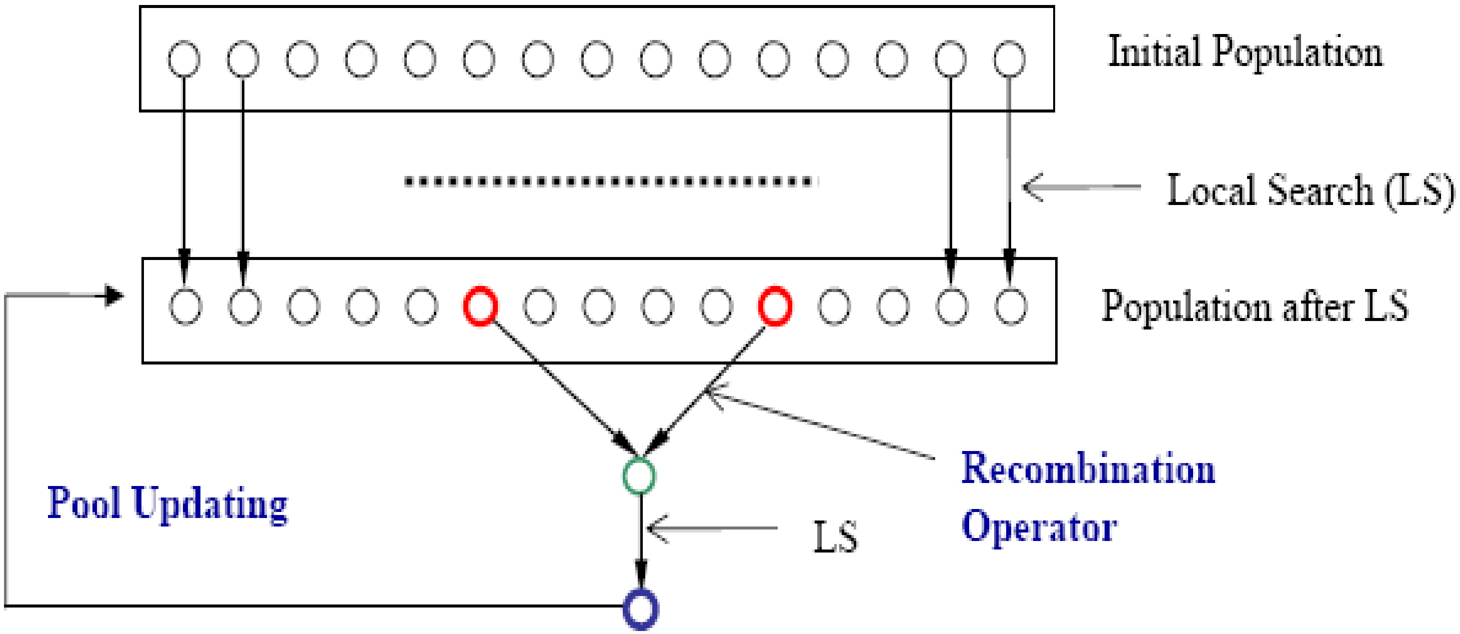
# Tips

* For both tabu and non-tabu moves, if there are multiple best moves, a random one is selected.

* Aspiration criterion: it holds if both the following two conditions satisfy:
* 1. the best tabu move is better than the previous best known solution;
* 2. the best tabu move is better than the best non-tabu move in the current neighborhood.

# Hybrid Evolutionary Algorithm

# Main Scheme




Hybrid Evolutionary Algorithm (LS+EA)

# Hybrid Evolutionary Algorithm

**The hybrid coloring algorithm**

**Data :** *graph* $G = (V, E)$, *integer k*
**Result :** *the best configuration found*
**begin**

```
    P=InitPopulation(|P|)
    while not Stop-Condition () do
        (s1,s2)=ChooseParents(P)
        s=Crossover(s1,s2)
        s=LocalSearch(s, L)
        P=UpdatePopulation(P,s)
```

**end**

# Crossover Operator (1)

*Table 2.* The crossover algorithm: an example.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| parent $s_1$ → | A B C | D E F G | H I J | $V_1 := \{D, E, F, G\}$ | A B C | | H I J |
| parent $s_2$ | C D E G | A F I | B H J | remove D,E,F and G | C | A I | B H J |
| offspring $s$ | | | | | D E F G | | |
| parent $s_1$ | A B C | | H I J | $V_2 := \{B, H, J\}$ | A C | | I |
| parent $s_2$ → | C | A I | B H J | remove B,H and J | C | A I | |
| offspring $s$ | D E F G | | | | D E F G | B H J | |
| parent $s_1$ → | A C | | I | $V_3 := \{A, C\}$ | | | I |
| parent $s_2$ | C | A I | | remove A and C | | I | |
| offspring $s$ | D E F G | B H J | | | D E F G | B H J | A C |

# Crossover Operator (2)

* A legal *k*-coloring is a collection of *k* independent sets.

* With this point of view, if we could maximize the size of the independent sets by a crossover operator as far as possible, it will in turn help to push those left vertices into independent sets.

* In other words, the more vertices are transmitted from parent individuals to the offspring within *k* steps, the less vertices are left unassigned.

* In this way, the obtained offspring individual has more possibility to become a legal coloring.

# Crossover Operator (3)

**The GPX crossover algorithm**

**Data :** *configurations* $s_1 = \{V_1^1, \ldots, V_k^1\}$ *and* $s_2 = \{V_1^2, \ldots, V_k^2\}$
**Result :** *configuration* $s = \{V_1, \ldots, V_k\}$
**begin**
  **for** $l (1 \leq l \leq k)$ **do**
    if $l$ is odd, then $A := 1$, else $A := 2$
    choose $i$ such that $V_i^A$ has a maximum cardinality
    $V_l := V_i^A$
    remove the vertices of $V_l$ from $s_1$ and $s_2$
  Assign randomly the vertices of $V - (V_1 \cup \cdots \cup V_k)$
**end**

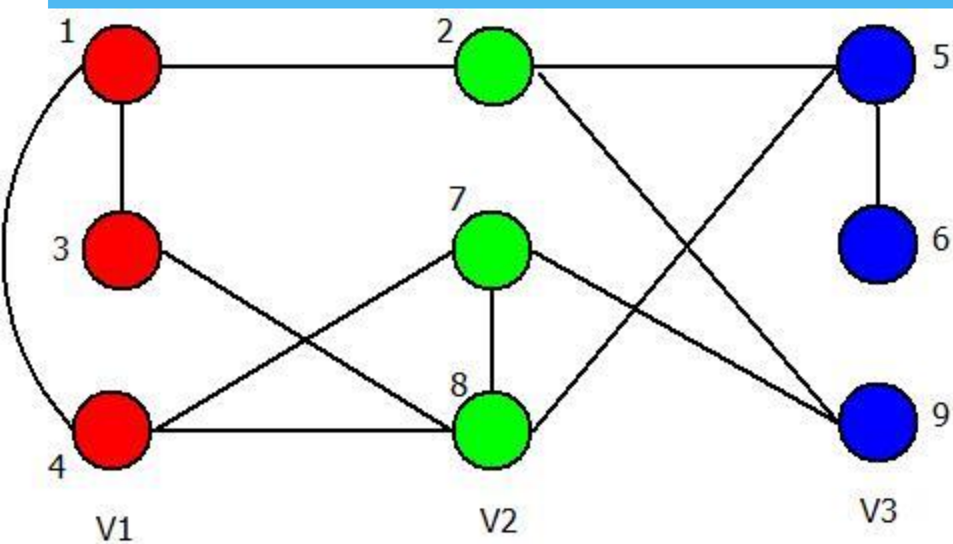
# HEA Algorithm Scheme

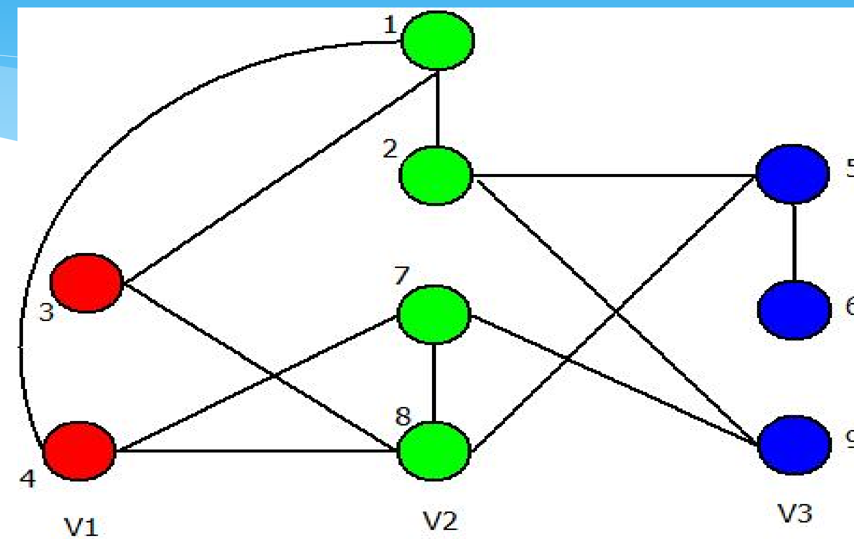
**Algorithm 1** Pseudcode of the Hybrid Evolutionary Algorithm for $k$-Coloring

**Input**: Graph G
**Output**: The best solution $S^*$ found so far
$\{S_1, \ldots, S_p\} \leftarrow$ Initial Population
**for** $i = \{1, \ldots, p\}$ **do**
  $S_i \leftarrow$ Tabu Search$(S_i)$
**end for**
$S^* = arg\ min\{f(S_i), i = 1, \ldots, p\}$
**repeat**
  Randomly choose two parent solutions $\{S_{i1}, S_{i2}\}$
  $S_0 \leftarrow$ Crossover_Operator $(S_{i1}, S_{i2})$
  $S_0 \leftarrow$ Tabu Search$(S_0)$
  **if** $f(S_0) < f(S^*)$ **then**
    $S^* = S_0$
  **end if**
  $\{S_1, \ldots, S_p\} \leftarrow$ Pool Updating$(S_0, S_1, \ldots, S_p)$
**until** Stop condition met

# 图着色的高级玩法

# 更新冲突节点



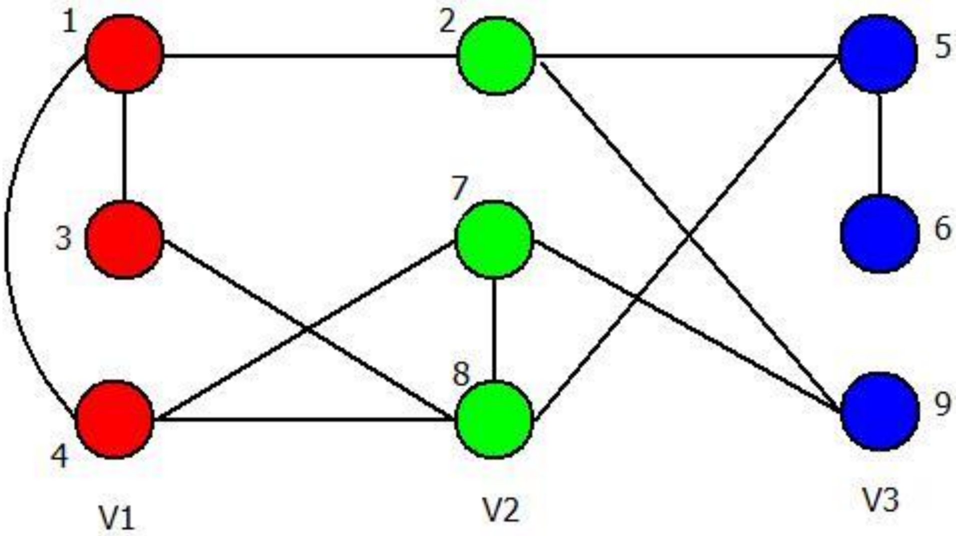

- Move (1, v1, v2): what are the new conflicting vertices?
- Vertices 3 and 4 becomes un-conflicting, while vertex 2 becomes conflicting
- Use an array to store the conflicting vertices and update the array after each move
- Use Array to implement it with O(1) time complexity

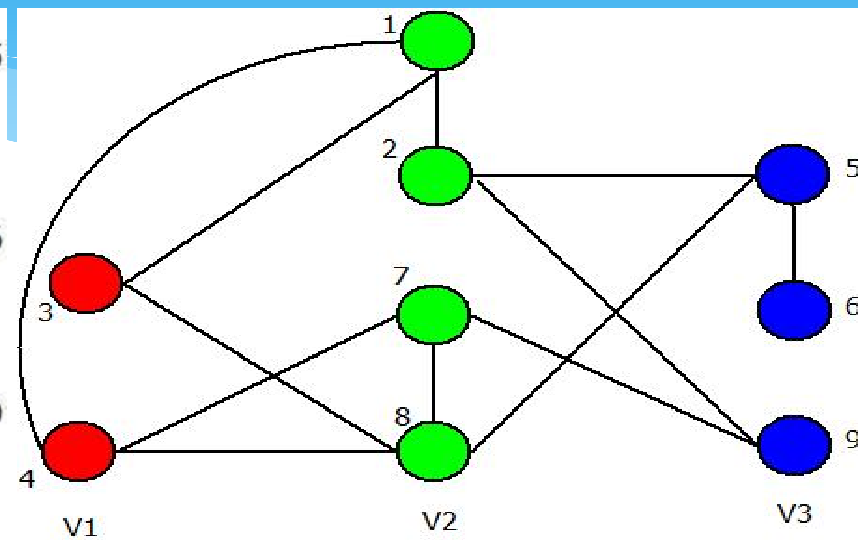
# 更新冲突节点

* Num: 冲突节点数
* Con[N]：冲突节点数组，只有前Num有效
* Pos[N]：记录任意一个节点在Conf数组中的位置

* 加节点V：Conf[Num]＝V; Pos[V]=Num＋＋;
* 删节点V：Conf[Pos[V]]＝Conf[Num];
* Pos[Conf[Num]]=Pos[V];
* Num－－;

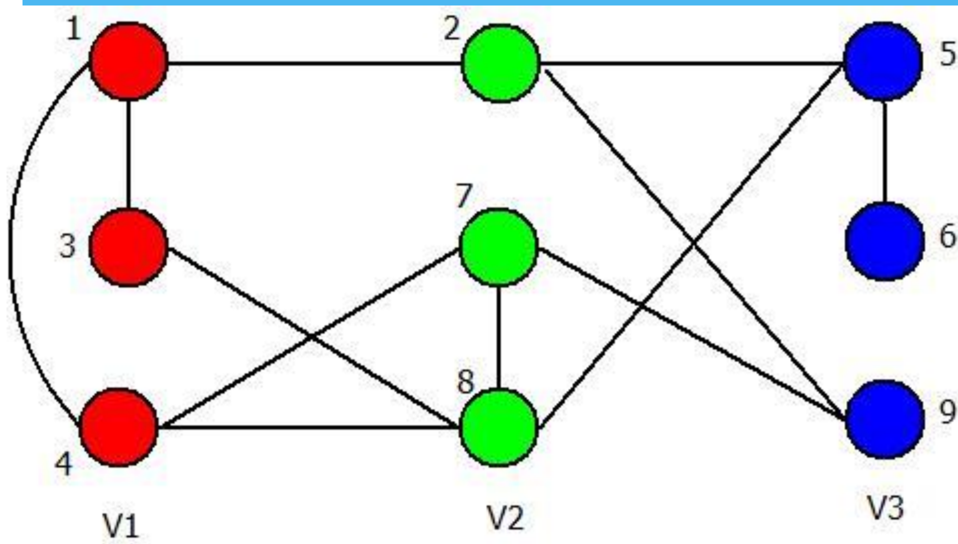
# 最优邻域动作如何快速找出来?

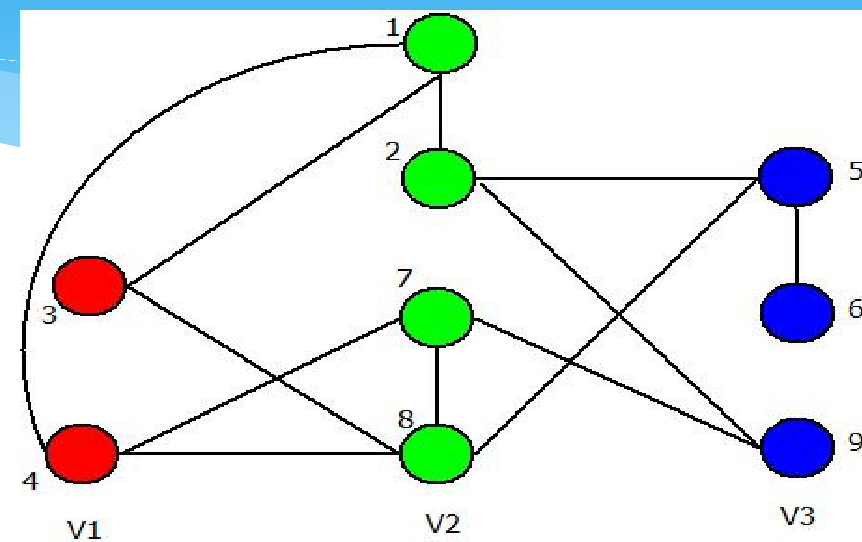



* 邻域大小：m*(k-1), m为冲突节点数，k为颜色数（冲突节点维护技术：N->m）
* 邻域动作时只选择禁忌最好的或者非禁忌最好的
* 上述动作后，冲突节点减少2个，增加1个，原来14个邻域动作，现在变为12个
* 哪些邻域动作发生了变化？哪些动作的delt值发生了变化？

# 最优邻域动作如何快速找出来?



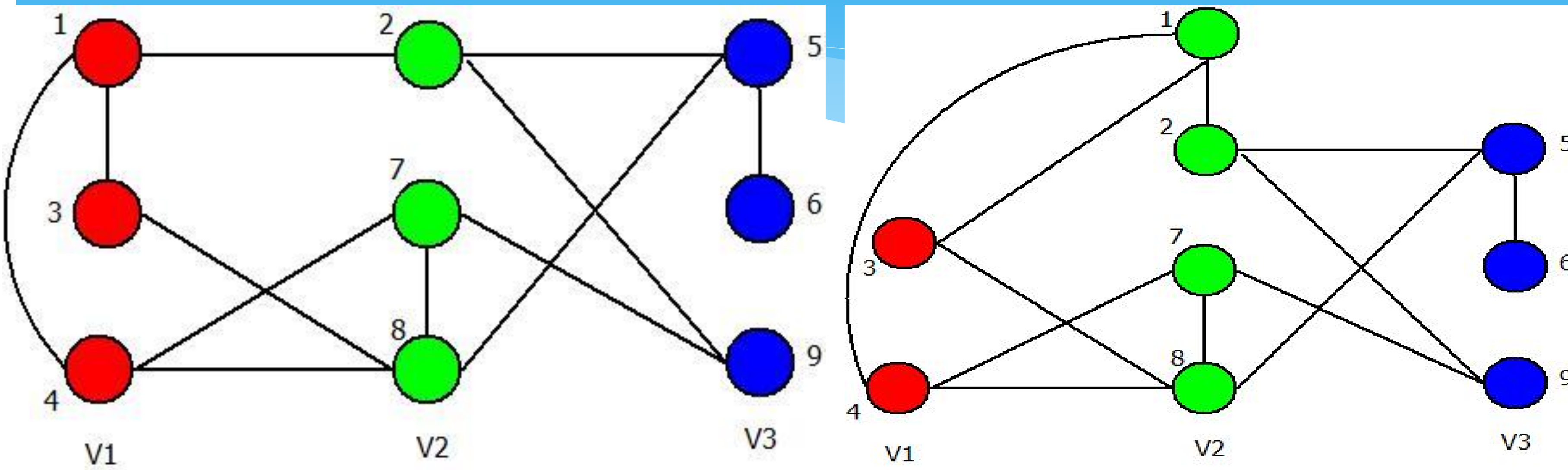




* 1. 减少的邻域动作：节点3，4到绿色、蓝色;
* 2. 增加的领域动作：节点2到红色、蓝色;
* 3. delt值发生变化的领域动作：无。

* 如果节点7与节点1相连，那么节点7移到红色的仇人数—1，移到绿色+1。即节点1的邻居节点如果仍为冲突节点，它到红色的仇人数—1，移到绿色+1。

* 既然大多数动作的delt值没有发生变化，那么最优的delt值也有可能未发生变化

# 最优邻域动作如何快速找出来?




* 1. 减少的邻域动作直接删除
* 2. 增加的领域动作重新计算并加入
* 3. delt值发生变化的领域动作（即移动节点的邻居），分三种情况
* a) 如果该节点在老颜色中，移到新颜色delt值+2，移到其它颜色delt值+1。
* b) 如果该节点在新颜色中，移到老颜色delt值-2，移到其它颜色delt值-1。
* c) 如果该节点在其它颜色中，移到老颜色delt值-1，移到新颜色delt值+1，其它不变

# 最优邻域动作如何快速找出来?

* 桶排序技术（增、删、移动与之前的数据实现冲突节点维护一致）：
* 辅助数据结构：Pos[N][k][k][2]

| id | Delt值 | 邻域动作数 | 邻域动作（及delt值） |
|---|---|---|---|
| 1 | <=—4 | 0 | |
| 2 | —3 | 0 | |
| 3 | —2 | 4 | …………….. |
| 4 | —1 | 1 | … |
| 5 | 0 | 10 | …………….. …………….. |
| 6 | 1 | 12 | …………….. …………….. |
| 7 | 2 | 12 | …………….. …………….. |
| 8 | 3 | 25 | …………….. …………….. |
| 9 | >=4 | 30 | …………….. …………….. |

# 最优邻域动作如何快速找出来?

* 每次从动作数非0的第一个桶中随机选一个动作

* 桶更新策略:
  1. 删掉的邻域动作要从桶中删掉;
  2. 增加的邻域动作要加到桶中;
  3. delt值发生变化的动作要在桶中移动到对应的桶

该策略可以节约多少时间:

邻域动作大小: m*(k-1)

更新的邻域动作数:(增、删冲突节点数)*(k-1)+移动节点的邻居节点仍为冲突节点数*2

# 最优邻域动作如何快速找出来?

* 可以更懒吗?

* 惰性更新桶的策略:

  1. 应该删除的邻域动作不用管

  2. 新增加的邻域动作直接加到桶中;

  3. delt值发生变化的动作只在桶中加新的动作，旧动作不动

# 师徒混合进化算法

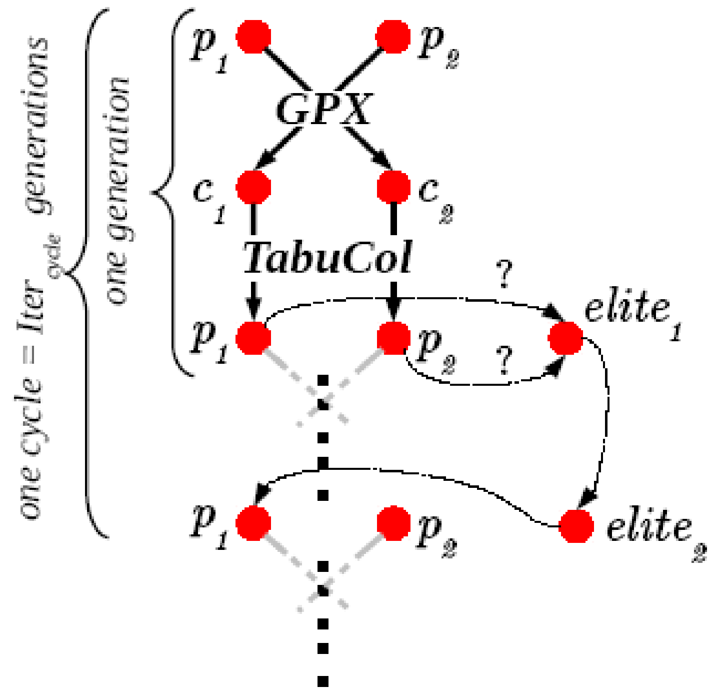



Fig. 2: Diagram of *HEAD*

# 师徒混合进化算法

**Algorithm 2:** *HEAD* - second version of *HEAD* with two extra elite solutions

**Input**: $k$, the number of colors; $Iter_{TC}$, the number of *TabuCol* iterations; $Iter_{cycle} = 10$, the number of generations into one cycle.

**Output**: the best $k$-coloring found: $best$

```
p1, p2, elite1, elite2, best ← init()        /* initialize with random k-colorings */
generation, cycle ← 0
do
    c1 ← GPX(p1, p2)
    c2 ← GPX(p2, p1)
    p1 ← TabuCol(c1, IterTC)
    p2 ← TabuCol(c2, IterTC)
    elite1 ← saveBest(p1, p2, elite1)   /* best k-coloring of the current cycle */
    best ← saveBest(elite1, best)
    if generation%Iter_cycle = 0 then
        p1 ← elite2                     /* best k-coloring of the previous cycle */
        elite2 ← elite1
        elite1 ← init()
        cycle++
    generation++
while nbConflicts(best) > 0 and p1 ≠ p2
```

# 高级局部搜索算法

* 局部搜索算法500.5能算到48种颜色吗?
* 500.5能算够非常稳定地算到47种颜色吗?
* 局部搜索算法500.5能算到47种颜色吗?
* 禁忌算法的局限和新实现：格局检测
* 节点是否可以回到老颜色应该取决于老颜色中它的“格局”是否发生了变化
* 仅“格局检测”是不够的，还需要加权和平滑技术的配合使用

# Thank You!