Euler's Method

Algorithm

```
Function will take following input:(f,t_initial,t_final,y_initial,h)
f(t,y) from y'= f(t,y)
t_initial(starting point of domain)
t_final(ending point of domain)
y_initial(initial condition)
h(step size)



1 Initialize the initial value of y and the initial time t.


2 Set the number of steps n = (t_final - t_initial) / h.


3 Loop from i = 1 to i = n:
  a. Calculate the slope of the function at the current time and value of y     using the given diffe
  b. Calculate the new value of y using the formula: y(i) = y(i-1) + h * slope(i-1).
  c. Update the value of time t(i) = t(i-1) + h.


4 Return the values of time and y at each step.
```

```python
def euler_method(f, t_initial, t_final, y_initial, h):
    """
    Solves the ordinary differential equation y' = f(t, y) using Euler's method with step siz
    Returns arrays of time and y values at each step.
    """
    num_steps = int((t_final - t_initial) / h)
    t_values = [t_initial]
    y_values = [y_initial]
    print('\n-------SOLUTION-------')
    print('----------------------')
    print('#\ttn\tyn')
    print('----------------------')
    for i in range(num_steps):
        slope = f(t_values[i], y_values[i])
        y_new = y_values[i] + h * slope
        t_new = t_values[i] + h
```

```
        y_values.append(y_new)
        t_values.append(t_new)
        print('%d\t%.2f\t%.4f'% (i+1,t_new,y_new) )
        print('----------------------')
    return t_values,y_values
```

```
import numpy as np
def f(t,y):
  return np.exp(t)-y

t_E,y_E=euler_method(f, t_initial=0, t_final=1, y_initial=1,h= 0.1)
```

```
-------SOLUTION-------
----------------------
#       tn      yn
----------------------
1       0.10    1.0000
----------------------
2       0.20    1.0105
----------------------
3       0.30    1.0316
----------------------
4       0.40    1.0634
----------------------
5       0.50    1.1063
----------------------
6       0.60    1.1605
----------------------
7       0.70    1.2267
----------------------
8       0.80    1.3054
----------------------
9       0.90    1.3974
----------------------
10      1.00    1.5036
----------------------
```

RK-4 Method

Algorithm

To solve: $\dfrac{dy}{dx} = f(x,y)$

$$x_{i+1} = x_i + h,$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$
$$k_1 = f(x_i, y_i),$$
$$k_2 = f(x_i + 0.5h, y_i + 0.5k_1),$$
$$k_3 = f(x_i + 0.5h, y_i + 0.5k_2).$$
$$k_4 = f(x_i + h, y_i + k_3),$$

```
Function will take following input:(f,t_initial,t_final,y_initial,h)
f(t,y) from y'= f(t,y)
t_initial(starting point of domain)
t_final(ending point of domain)
y_initial(initial condition)
h(step size)
```

```
1 Initialize the initial value of y and the initial time t.

2 Set the number of steps n = (t_final - t_initial) / h.

3 Loop from i = 1 to i = n:
  a)Calculate
      k1 =  f(x0, y0)
      k2 =  f(x0+h/2, y0+k1/2)
      k3 =  f(x0+h/2, y0+k2/2)
      k4 =  f(x0+h, y0+k3)

   b) Calculate the new value of y using the formula: y(i) = y(i-1) + (k1+2*k2+2*k3+k4)*h/6
   c) Update the value of time t(i) = t(i-1) + h.

4 Return the values of time and y at each step.
```

```
def RK4_method(f, t_initial, t_final, y_initial, h):
    """
    Solves the ordinary differential equation y' = f(t, y) using Euler's method with step siz
    Returns arrays of time and y values at each step.
    """
    num_steps = int((t_final - t_initial) / h)
    t_values = [t_initial]
    y_values = [y_initial]
    print('\n-------SOLUTION-------')
```

```
    print('----------------------')
    print('#\ttn\tyn')
    print('----------------------')
    for i in range(num_steps):
        k1 = f(t_values[i], y_values[i])
        k2 = f(t_values[i]+0.5*h, y_values[i]+0.5*k1)
        k3 = f(t_values[i]+h/2, y_values[i]+k2/2)
        k4 = f(t_values[i]+h, y_values[i]+k3)
        y_new = y_values[i] + (h/6) * (k1+2*k2+2*k3+k4)
        t_new = t_values[i] + h
        y_values.append(y_new)
        t_values.append(t_new)
        print('%d\t%.2f\t%.4f'% (i+1,t_new,y_new) )
        print('----------------------')
    return t_values,y_values
```

```
import numpy as np
def f(t,y):
  return np.exp(t)-y

t_R,y_R=RK4_method(f, t_initial=0, t_final=1, y_initial=1,h= 0.1)
```

```
-------SOLUTION-------
----------------------
#       tn      yn
----------------------
1       0.10    1.0039
----------------------
2       0.20    1.0145
----------------------
3       0.30    1.0322
----------------------
4       0.40    1.0573
----------------------
5       0.50    1.0903
----------------------
6       0.60    1.1316
----------------------
7       0.70    1.1818
----------------------
8       0.80    1.2416
----------------------
9       0.90    1.3118
----------------------
10      1.00    1.3931
----------------------
```

Library Function to solve ODE(IVP)

```
import numpy as np
from scipy.integrate import solve_ivp
def f(t,y):
  return np.exp(t)-y
sol = solve_ivp(f, [0, 1], [1], t_eval=np.array(np.linspace(0.1,1,10)))
t_l=sol.t
y_l=sol.y.flatten()
print('\n----Library_SOLUTION-------')
print('----------------------')
print('#\ttn\tyn')
print('----------------------')

for i in range(len(t_l)):
   print('%d\t%.2f\t%.4f'% (i+1,t_l[i],y_l[i]) )
   print('----------------------')
```

```
----Library_SOLUTION-------
----------------------
#       tn      yn
----------------------
1       0.10    1.0050
----------------------
2       0.20    1.0200
----------------------
3       0.30    1.0451
----------------------
4       0.40    1.0807
----------------------
5       0.50    1.1272
----------------------
6       0.60    1.1851
----------------------
7       0.70    1.2549
----------------------
8       0.80    1.3374
----------------------
9       0.90    1.4332
----------------------
10      1.00    1.5433
----------------------
```

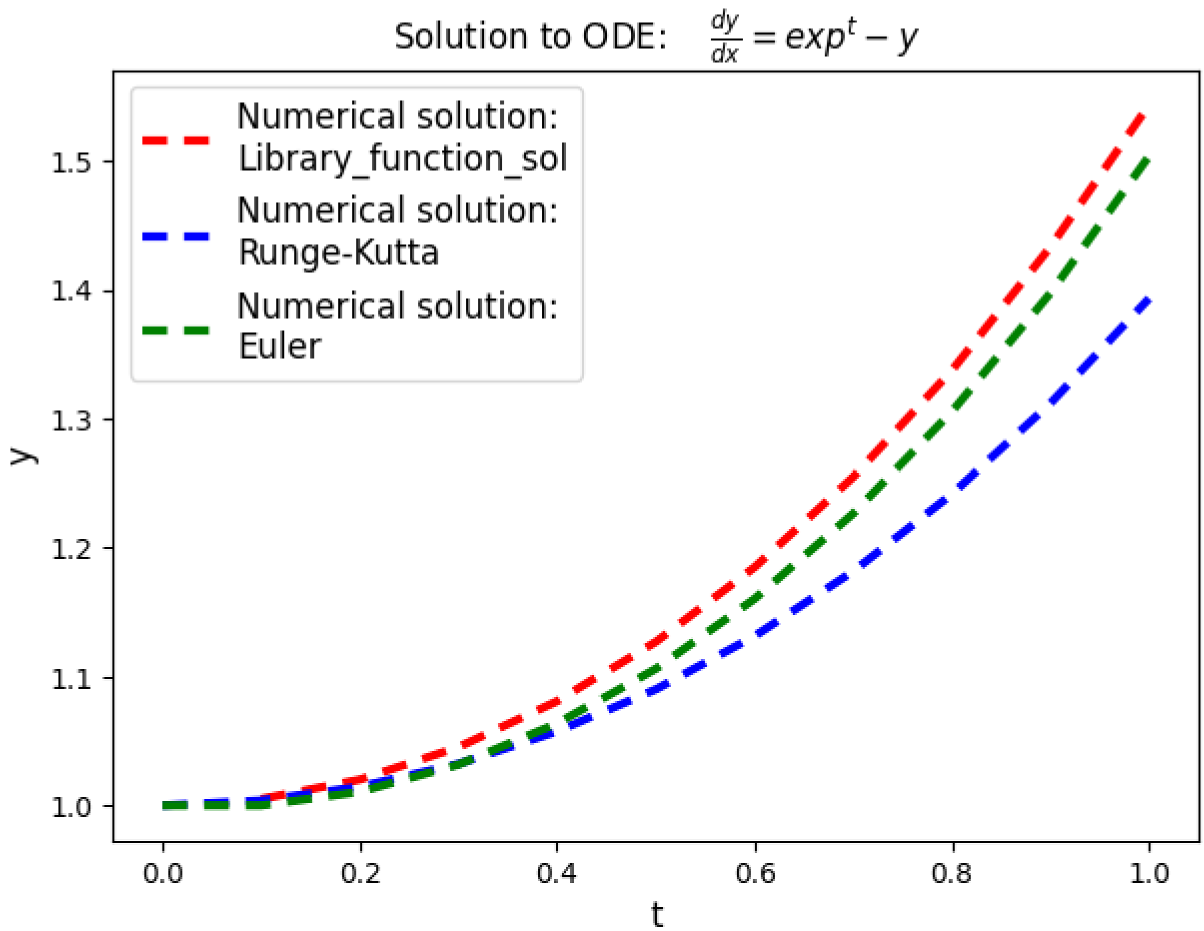
Plotting

```
import matplotlib.pyplot as plt
import numpy as np
plt.figure(figsize=(7,5))


plt.plot(t_l, y_l, label="Numerical solution:\nLibrary_function_sol", dashes=(3,2), color="re
```

```
plt.plot(t_R, y_R, label="Numerical solution:\nRunge-Kutta", dashes=(3,2), color="blue",lw=3)
plt.plot(t_E, y_E, label="Numerical solution:\nEuler", dashes=(3,2), color="green",lw=3)

plt.legend(loc="best", fontsize=12)
plt.title(r"Solution to ODE: $\quad\frac{dy}{dx}=exp^t -y$")
plt.xlabel("t", fontsize=12)
plt.ylabel("y", fontsize=12)
plt.show()
```



Lab Task

```
1- Write the program for Heun's Method.
2- Write the code to make a table to compare error of Heun's, Euler's, RK4 by using analytical soluti
3- Repeat task 2 for three different DEs.
```