

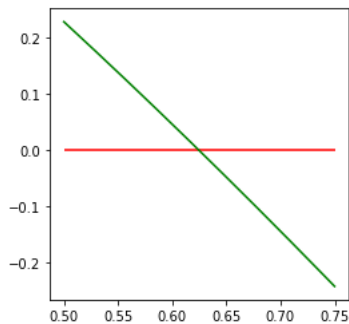
▼ TASK 1

```
import numpy as np
from matplotlib import pyplot as plt

plt.rcParams["figure.figsize"] = [4, 4]

def f(x):
    return (np.cos(x)-(1.3*x))

x = np.linspace(0.5,0.75,1000)
plt.plot(x,f(x), color='green')
plt.hlines(y=0,xmin=0.5,xmax=0.75,color='red')
plt.show()
```

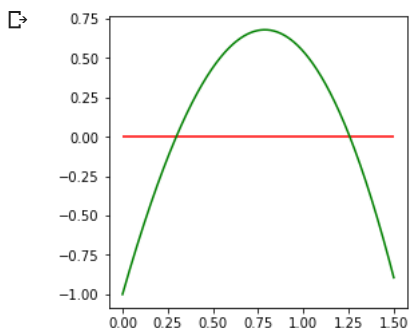


```
import numpy as np
from matplotlib import pyplot as plt

plt.rcParams["figure.figsize"] = [4, 4]

def f(x):
    return ((x*np.cos(x))-(2*x**2)+(3*x)-1)

x = np.linspace(0,1.5,1000)
plt.plot(x,f(x), color='green')
plt.hlines(y=0,xmin=0,xmax=1.5,color='red')
plt.show()
```

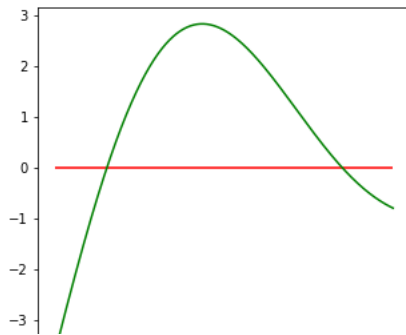


```
import numpy as np
from matplotlib import pyplot as plt

plt.rcParams["figure.figsize"] = [4, 4]

def f(x):
    return ((2*x*np.cos(2*x))-((x+1)**2))

x = np.linspace(-2.5,-0.5,1000)
plt.plot(x,f(x), color='green')
plt.hlines(y=0,xmin=-2.5,xmax=-0.5,color='red')
plt.show()
```



▼ TASK 2

```
import numpy as np
from tabulate import tabulate

def func (x):
    return (np.cos(x)-(1.3*x))

def bisection(func, x1, x2, tol=0.0001, max_iter=100):
    if func(x1) * func(x2) >= 0:
        return "Error: Choose different interval, function should have different signs at the interval endpoints."
    data=[]
    iter = 0
    xr = x2
    error = tol + 1

    while iter < max_iter and error > tol:
        xrold = xr
        xr = ((x1+x2)/2)
        iter += 1
        error = abs((xr - xrold) )
        test = func(x1) * func(xr)
        if (test>0):
            x1 = xr
        else:
            x2 = xr
        data.append([iter+1,x1,func(x1),x2,func(x2),xr,func(xr),error])
    print(tabulate(data,headers=['#', 'x1', 'f(x1)', 'x2', 'f(x2)', 'xr', 'f(xr)', "error"],tablefmt="github"))
    print('\nRoot of given function is x=%.9f in n=%d number of iterations with a tolerance=%.5f' %(xr,iter,tol))

    return

bisection(func,-1,1,0.001,100)
```

#	x1	f(x1)	x2	f(x2)	xr	f(xr)	error
2	0	1	1	-0.759698	0	1	1
3	0.5	0.227583	1	-0.759698	0.5	0.227583	0.5
4	0.5	0.227583	0.75	-0.243311	0.75	-0.243311	0.25
5	0.5	0.227583	0.625	-0.00153688	0.625	-0.00153688	0.125
6	0.5625	0.114674	0.625	-0.00153688	0.5625	0.114674	0.0625
7	0.59375	0.0569735	0.625	-0.00153688	0.59375	0.0569735	0.03125
8	0.609375	0.0278184	0.625	-0.00153688	0.609375	0.0278184	0.015625
9	0.617188	0.0131656	0.625	-0.00153688	0.617188	0.0131656	0.0078125
10	0.621094	0.00582059	0.625	-0.00153688	0.621094	0.00582059	0.00390625
11	0.623047	0.0021434	0.625	-0.00153688	0.623047	0.0021434	0.00195312
12	0.624023	0.000303648	0.625	-0.00153688	0.624023	0.000303648	0.000976562

Root of given function is x=0.624023438 in n=11 number of iterations with a tolerance=0.00100

```
import numpy as np
from tabulate import tabulate

def func (x):
    return ((x*np.cos(x))-(2*x**2)+(3*x)-1)

def bisection(func, x1, x2, tol=0.0001, max_iter=100):
    if func(x1) * func(x2) >= 0:
        return "Error: Choose different interval, function should have different signs at the interval endpoints."
    data=[]
```

```

iter = 0
xr = x2
error = tol + 1

while iter < max_iter and error > tol:
    xrold = xr
    xr = ((x1+x2)/2)
    iter += 1
    error = abs((xr - xrold) )
    test = func(x1) * func(xr)
    if (test>0):
        x1 = xr
    else:
        x2 = xr
    data.append([iter+1,x1,func(x1),x2,func(x2),xr,func(xr),error])
print(tabulate(data,headers=['#','x1','f(x1)','x2','f(x2)','xr','f(xr)','error'],tablefmt="github"))
print('\nRoot of given function is x=%.9f in n=%d number of iterations with a tolerance=%.5f' %(xr,iter,tol))

return

```

```
bisection(func,1,2,0.001,100)
```

#	x1	f(x1)	x2	f(x2)	xr	f(xr)	error
2	1	0.540302	1.5	-0.893894	1.5	-0.893894	0.5
3	1.25	0.019153	1.5	-0.893894	1.25	0.019153	0.25
4	1.25	0.019153	1.375	-0.388747	1.375	-0.388747	0.125
5	1.25	0.019153	1.3125	-0.172556	1.3125	-0.172556	0.0625
6	1.25	0.019153	1.28125	-0.0736339	1.28125	-0.0736339	0.03125
7	1.25	0.019153	1.26562	-0.0264729	1.26562	-0.0264729	0.015625
8	1.25	0.019153	1.25781	-0.00346802	1.25781	-0.00346802	0.0078125
9	1.25391	0.00789046	1.25781	-0.00346802	1.25391	0.00789046	0.00390625
10	1.25586	0.00222322	1.25781	-0.00346802	1.25586	0.00222322	0.00195312
11	1.25586	0.00222322	1.25684	-0.0006194	1.25684	-0.0006194	0.000976562

Root of given function is x=1.256835938 in n=10 number of iterations with a tolerance=0.00100

```

import numpy as np
from tabulate import tabulate

```

```

def func (x):
    return ((2*x*np.cos(2*x))-((x+1)**2))

def bisection(func, x1, x2, tol=0.0001, max_iter=100):
    if func(x1) * func(x2) >= 0:
        return "Error: Choose different interval, function should have different signs at the interval endpoints."
    data=[]
    iter = 0
    xr = x2
    error = tol + 1

    while iter < max_iter and error > tol:
        xrold = xr
        xr = ((x1+x2)/2)
        iter += 1
        error = abs((xr - xrold) )
        test = func(x1) * func(xr)
        if (test>0):
            x1 = xr
        else:
            x2 = xr
        data.append([iter+1,x1,func(x1),x2,func(x2),xr,func(xr),error])
    print(tabulate(data,headers=['#','x1','f(x1)','x2','f(x2)','xr','f(xr)','error'],tablefmt="github"))
    print('\nRoot of given function is x=%.9f in n=%d number of iterations with a tolerance=%.5f' %(xr,iter,tol))

    return

```

```
bisection(func,-1,0,0.001,100)
```

#	x1	f(x1)	x2	f(x2)	xr	f(xr)	error
2	-1	0.832294	-0.5	-0.790302	-0.5	-0.790302	0.5
3	-1	0.832294	-0.75	-0.168606	-0.75	-0.168606	0.25
4	-0.875	0.296306	-0.75	-0.168606	-0.875	0.296306	0.125
5	-0.8125	0.0528816	-0.75	-0.168606	-0.8125	0.0528816	0.0625
6	-0.8125	0.0528816	-0.78125	-0.0608144	-0.78125	-0.0608144	0.03125
7	-0.8125	0.0528816	-0.796875	-0.00468056	-0.796875	-0.00468056	0.015625
8	-0.804688	0.0239252	-0.796875	-0.00468056	-0.804688	0.0239252	0.0078125
9	-0.800781	0.00957807	-0.796875	-0.00468056	-0.800781	0.00957807	0.00390625

10	-0.798828	0.00243764	-0.796875	-0.00468056	-0.798828	0.00243764	0.00195312
11	-0.798828	0.00243764	-0.797852	-0.00112424	-0.797852	-0.00112424	0.000976562

Root of given function is $x = -0.797851562$ in $n=10$ number of iterations with a tolerance= 0.00100

TASK 3

```
import numpy as np
from tabulate import tabulate

def func(x):
    return (np.cos(x)-(1.3*x))

def dfunc(x):
    return -(np.sin(x))-1.3

def newton_raphson(func, dfunc, x0, tol=1e-4, max_iter=1000):
    xr = x0
    data=[]
    iter = 0
    error = tol + 1
    for i in range(max_iter):
        iter+=1
        fx = func(xr)
        dx = dfunc(xr)
        if abs(dx) < tol:
            raise Exception("Derivative is close to zero!")
        xold=xr
        xr = xr - fx/dx
        error=abs(xr-xold)

        data.append([iter,xr,func(xr),error])
        if error < tol:
            print(tabulate(data,headers=['Iteration','xr','f(xr)','error'],tablefmt="github"))
            print('\nRoot of given function is x=%.9f in n=%d number of iterations with a tolerance=%.5f' %(xr,iter,tol))
            return

    raise Exception("Max iterations reached")

newton_raphson(func,dfunc,10,0.00001,100)
```

Iteration	xr	f(xr)	error
1	-8.30616	10.3611	18.3062
2	17.564	-22.5518	25.8702
3	-48.6855	63.282	66.2495
4	-21.1711	26.8402	27.5144
5	26.0146	-33.1832	47.1857
6	9.99877	-13.8381	16.0158
7	-8.28122	10.3512	18.28
8	18.268	-22.9128	26.5492
9	-12.2552	16.8837	30.5232
10	-1.74341	2.09467	10.5118
11	4.9093	-6.18645	6.65271
12	-14.4642	18.4822	19.3735
13	37.8914	-48.2773	52.3556
14	5.51461	-6.45009	32.3768
15	-5.14862	7.11573	10.6632
16	-1.92351	2.15512	3.22511
17	4.0371	-5.87335	5.96061
18	-7.26925	10.002	11.3064
19	14.188	-18.4951	21.4572
20	6.14209	-6.99466	8.04586
21	0.10897	0.852407	6.03312
22	0.714049	-0.172547	0.605079
23	0.625785	-0.00301683	0.0882639
24	0.624185	-1.0376e-06	0.00159982
25	0.624185	-1.23013e-13	5.50618e-07

Root of given function is $x = 0.624184578$ in $n=25$ number of iterations with a tolerance= 0.00001

```
import numpy as np
from tabulate import tabulate

def func(x):
    return ((x*np.cos(x))-(2*x**2)+(3*x)-1)
```

```
def dfunc(x):
    return -(x*np.sin(x))+(np.cos(x))-(4*x)+3)

def newton_raphson(func, dfunc, x0, tol=1e-4, max_iter=1000):
    xr = x0
    data=[]
    iter = 0
    error = tol + 1
    for i in range(max_iter):
        iter+=1
        fx = func(xr)
        dx = dfunc(xr)
        if abs(dx) < tol:
            raise Exception("Derivative is close to zero!")
        xrold=xr
        xr = xr - fx/dx
        error=abs(xr-xrold)

        data.append([iter,xr,func(xr),error])
    if error < tol:
        print(tabulate(data,headers=['Iteration','xr','f(xr)','error'],tablefmt="github"))
        print('\nRoot of given function is x=%.9f in n=%d number of iterations with a tolerance=%.5f' %(xr,iter,tol))
        return

    raise Exception("Max iterations reached")
```

```
newton_raphson(func,dfunc,5,0.00001,100)
```

Iteration	xr	f(xr)	error
1	2.09927	-4.57454	2.90073
2	1.50627	-0.92174	0.593004
3	1.29977	-0.131514	0.206497
4	1.25846	-0.00535864	0.0413089
5	1.25663	-1.0562e-05	0.00183256
6	1.25662	-4.13602e-11	3.6263e-06

Root of given function is x=1.256623323 in n=6 number of iterations with a tolerance=0.00001

```
import numpy as np
from tabulate import tabulate
```

```
def func(x):
    return ((2*x*np.cos(2*x))-((x+1)**2))
```

```
def dfunc(x):
    return -(4*x*np.sin(2*x))+(2*np.cos(2*x))-(2*(x+1))
```

```
def newton_raphson(func, dfunc, x0, tol=1e-4, max_iter=1000):
```

```
    xr = x0
    data=[]
    iter = 0
    error = tol + 1
    for i in range(max_iter):
        iter+=1
        fx = func(xr)
        dx = dfunc(xr)
        if abs(dx) < tol:
            raise Exception("Derivative is close to zero!")
        xrold=xr
        xr = xr - fx/dx
        error=abs(xr-xrold)
```

```
    data.append([iter,xr,func(xr),error])
    if error < tol:
        print(tabulate(data,headers=['Iteration','xr','f(xr)','error'],tablefmt="github"))
        print('\nRoot of given function is x=%.9f in n=%d number of iterations with a tolerance=%.5f' %(xr,iter,tol))
        return
```

```
    raise Exception("Max iterations reached")
```

```
newton_raphson(func,dfunc,-3,0.00001,100)
```

Iteration	xr	f(xr)	error
1	-1.94741	1.94363	1.05259
2	-2.28462	-0.998355	0.337207

	3		-2.19649		-0.0521828		0.0881248	
	4		-2.19133		-0.000202041		0.00516326	
	5		-2.19131		-3.09417e-09		2.01473e-05	
	6		-2.19131		1.33227e-15		3.08556e-10	

Root of given function is $x = -2.191308012$ in $n=6$ number of iterations with a tolerance $= 0.00001$

▼ TASK 4

```
import numpy as np
import scipy.optimize as sp

def f1(x):
    return (np.cos(x)-(1.3*x))

def f2(x):
    return ((x*np.cos(x))-(2*x**2)+(3*x)-1)

def f3(x):
    return ((2*x*np.cos(2*x))-((x+1)**2))

r1 = sp.fsolve(f1,[0,10],(),None,0,0,0.00001)
r2 = sp.fsolve(f2,[0,10],(),None,0,0,0.00001)
r3 = sp.fsolve(f3,[-1,-3],(),None,0,0,0.00001)

print(r1)
print(r2)
print(r3)

[0.62418458 0.62418458]
[0.29752143 1.25662326]
[-0.79816068 -2.19130804]
```

▼ TASK 5

```
import numpy as np
from tabulate import tabulate

def func (x):
    return (np.cos(x)-(1.3*x))

def falseposition(func, x1, x2, tol=0.0001, max_iter=100):
    if func(x1) * func(x2) >= 0:
        return "Error: Choose different interval, function should have different signs at the interval endpoints."
    data=[]
    iter = 0
    xr = x2
    error = tol + 1

    while iter < max_iter and error > tol:
        xold = xr
        xr = (x1-((func(x1)*(x2-x1))/(func(x2)-func(x1))))
        iter += 1
        error = abs((xr - xold) )
        test = func(x1) * func(xr)
        if (test>0):
            x1 = xr
        else:
            x2 = xr
        data.append([iter+1,x1,func(x1),x2,func(x2),xr,func(xr),error])
    print(tabulate(data,headers=['#','x1','f(x1)','x2','f(x2)','xr','f(xr)','error'],tablefmt="github"))
    print('\nRoot of given function is x=%.9f in n=%d number of iterations with a tolerance=%.5f' %(xr,iter,tol))

    return

falseposition(func,-1,2,0.001,100)
```

	#		x1		f(x1)		x2		f(x2)		xr		f(xr)		error	
	2		0.13682		0.812789		2		-3.01615		0.13682		0.812789		1.86318	
	3		0.532327		0.169603		2		-3.01615		0.532327		0.169603		0.395507	
	4		0.610463		0.0257805		2		-3.01615		0.610463		0.0257805		0.078136	
	5		0.62224		0.00366374		2		-3.01615		0.62224		0.00366374		0.0117764	

6	0.623911	0.000515321	2	-3.01615	0.623911	0.000515321	0.00167155
7	0.624146	7.2376e-05	2	-3.01615	0.624146	7.2376e-05	0.00023507

Root of given function is x=0.624146170 in n=6 number of iterations with a tolerance=0.00100

```
import numpy as np
from tabulate import tabulate

def func(x):
    return ((x*np.cos(x))-(2*x**2)+(3*x)-1)

def secant(func,x0, x1 , tol=1e-4, max_iter=1000):
    data=[]
    iter = 0
    error = tol + 1
    for i in range(max_iter):
        iter+=1
        x2 = x0 - ((func(x0)*(x1-x0))/(func(x1)-func(x0)))
        error=abs(x1-x0)
        data.append([iter,x0,x1,x2,func(x2),error])
        if error < tol:
            print(tabulate(data,headers=['Iteration','x0','x1','x2','f(x2)','error'],tablefmt="github"))
            print('\nRoot of given function is x=%.9f in n=%d number of iterations with a tolerance=0.5f' %(x2,iter,tol))
            return

        x0=x1
        x1=x2
    raise Exception("Max iterations reached")
```

```
secant(func,0,10,0.00001,100)
```

	Iteration	x0	x1	x2	f(x2)	error
	1	0	10	-0.0560567	-1.23042	10
	2	10	-0.0560567	-0.125507	-1.53254	10.0561
	3	-0.0560567	-0.125507	0.226788	-0.201522	0.0694499
	4	-0.125507	0.226788	0.280126	-0.0473553	0.352294
	5	0.226788	0.280126	0.29651	-0.00273439	0.0533388
	6	0.280126	0.29651	0.297514	-4.23675e-05	0.016384
	7	0.29651	0.297514	0.29753	-3.92378e-08	0.00100402
	8	0.297514	0.29753	0.29753	-5.64215e-13	1.58014e-05
	9	0.29753	0.29753	0.29753	0	1.46478e-08

Root of given function is x=0.297530234 in n=9 number of iterations with a tolerance=0.00001