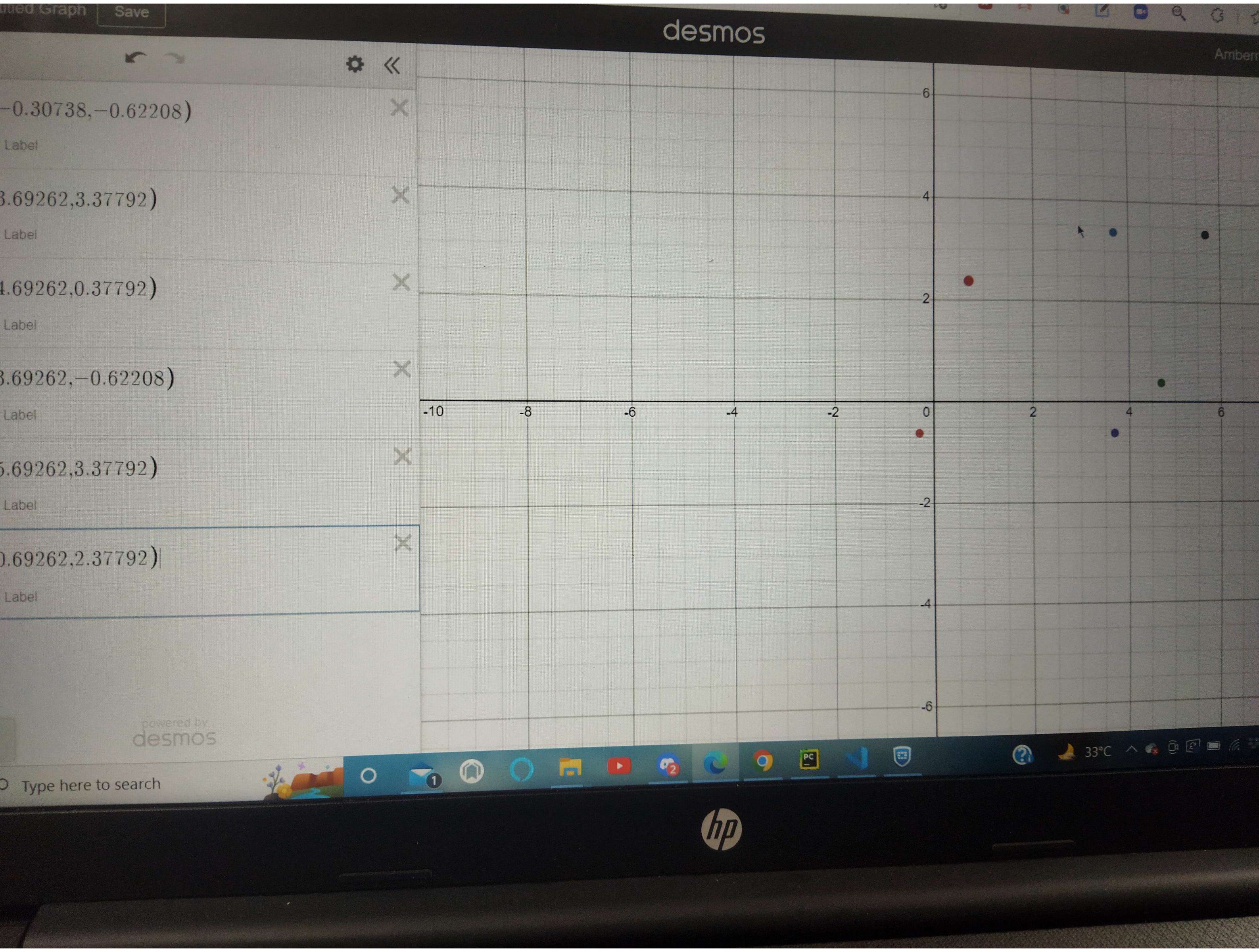

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①

$$\frac{(x-a)^2}{c^2} + \frac{(y-b)^2}{d^2} = 1$$

~~[illegible]~~ from Ellipse passing through those points we can find parameters a, b, c & d

$a = \cancel{20} \; 203.6$

$b \approx -2 \times 10^6$

$c \approx -5.85 \times 10^9$

$d = 5.0\cancel{8}5 \times 10^9$

$u_e \to$ exhaust velocity wrt rocket

(2)

$$I_{sp} = \frac{F_{thrust}}{\dot{m} g_0} \quad , \dot{m} = \frac{dm}{dt}$$

Specific Impulse

$$\Delta V = u - 0 = I_{sp} g_0 \ln\left(\frac{m_0}{m_t}\right)$$

$$M_f = M_0 - \int_0^t \dot{m} dt = M_0 - \dot{m}\int_0^t dt$$

$$M_f = (M_0 - \dot{m}t)$$

$$\hookrightarrow u(t) = I_{sp} g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) \quad \text{(I)}$$

for $0 \le t \le t_b$

Initial velocity of $m = 0$

$M_0 \to$ Initial mass

$M_f \to$ final mass at r

Also, $\Delta V = -v_{rn} \ln\left(\frac{m_f}{m_0}\right)$

$$\Rightarrow u = -u_e \ln\left(\frac{m_0 - \dot{m}t}{m_0}\right)$$

$$\Rightarrow -\frac{u}{u_e} = \ln\left(1 - \frac{\dot{m}t}{m_0}\right) \Rightarrow \dot{m}t/m_0 = 1 - e^{-u/u_e} \quad \text{(II)}$$

$$M_0 = \frac{\dot{m}t}{1 - e^{(-u/u_e)}}$$

from (I) & (II)

$$u = I_{sp} g_0 \ln\left(\frac{1}{1 - 1 + e^{-u/u_e}}\right)$$

$$= -I_{sp} g_0 \ln(e^{-u/u_0})$$

$$= \frac{I_{sp} g_0 u}{u_e}$$

$$\Rightarrow \boxed{u_e = I_{sp} g_0}$$

~~Now, the rocket will have~~

c)

$$u(t) = \begin{cases} u_e \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right); & 0 \le t \le t_b \\ u_e \ln\left(\frac{M_0}{M_0 - \dot{m}t_b}\right) - gt; & t > t_b \end{cases}$$

Till t_b Rocket have const. acc is upward dein

(b)

$$u = \frac{dH}{dt}$$

$$= u_e \ln\left(\frac{m_0}{m_0 - \dot{m} t}\right)$$

$$\int_0^h dh = u_e \int_0^t \ln\left(\frac{m_0}{m_0 - \dot{m} t}\right) dt = -u_e \int_0^t \ln\left(1 - \frac{\dot{m} t}{m_0}\right) dt$$

$$h \approx \frac{u_e m_0}{\dot{m}} \left[\left(1 - \frac{\dot{m} t}{m_0}\right) \ln\left(1 - \frac{\dot{m} t}{m_0}\right) + \frac{\dot{m} t}{m_0}\right]$$

$$\boxed{h(t) = \frac{u_e m_0}{\dot{m}} \left[\left(1 - \frac{\dot{m} t}{m_0}\right) \ln\left(1 - \dot{m} t / m_0\right) + \frac{\dot{m} t}{m_0}\right]}$$

$t > t_b$

$h(t_b)$ ✓

$$\frac{dh}{dt} = u_0 \ln\left(\frac{m_0}{m_0 - \dot{m} t_b}\right) - gt$$

(3)

~~Suppose n booster~~

(3)

Suppose we have used 'x' booster

$t_b \rightarrow$ burnout time

$M_0 = m/5$ At min $\dot{m}$, speed of quadcopter after fuel is exhausted $= V/2$

$$u_e \ln\left(\frac{m/5}{m/5 - \dot{m} t_b}\right) = V/2 \quad \text{(Equality Condn)}$$

$$\frac{m}{m - 5\dot{m} t_b} = e^{\left(\frac{V}{2u_e}\right)}$$

$$1 - \frac{5\dot{m} t_b}{m} = e^{V/2u_e}$$

$$(\dot{m})_{minimum} = \frac{m}{5t_b}\left[1 - e^{(V/2u_e)}\right]$$