TDOA Acoustic Localization

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1 Introduction

Time Delay of Arrival (TDOA) is a technique for locating an acoustic source (ie. gunshot, explosion, etc.) near a microphone array. By exploiting the differences in the arrival time of the sound to the microphones, TDOA locates the source of the sound. As a function, it takes a set of microphone signals as input and returns the coordinates of the source relative to the microphone array. This paper discusses the mathematics behind TDOA.

2 Set Up

Let $\{(x_m, y_m, z_m)\}_{m=1}^{M}$ be the coordinates of M microphones. Let (x, y, z) be the unknown coordinates of the source we are locating. Let t_m be the time of transit from the source to microphone m. Let v be the speed of sound (340.29 meters per sec in air). Let $R_m = vT_m$ be the distance between between the source and the microphone m. Let $\tau_m = T_m - t_1$ be the difference in transit time between microphone m and microphone 1. Note then, that $tau_1 = 0$.

3 Derivation

Note that

$$\tau_m = T_m - T_1$$

implies

$$v\tau_m = vT_m - vT_1 = R_m - R_1$$

and therefore

$$R_m^2 = (v\tau_m + R_1)^2 = v^2\tau_m^2 + 2v\tau_m R_1 + R_1^2.$$

We move everything over to get

$$0 = v\tau_m + 2R_1 + \frac{R_1^2 - R_m^2}{v\tau_m}$$

for $m = 2, 3, ..., M$. We now subtract $0 = v\tau_2 + 2R_1 + \frac{R_1^2 - R_2^2}{v\tau_2}$ from the above equation for $m = 3, 4, ..., M$. We then get the set of equations

$$0 = v\tau_m - v\tau_2 + \frac{R_1^2 - R_m^2}{v\tau_m} - \frac{R_1^2 - R_m^2}{v\tau_m} \tag{1}$$

for $m = 3, 4, ..., M$. We then substitute $R_m = \sqrt{(x_m - x)^2 + (y_m - y)^2 + (z_m - z)^2}$ into the above equations to get

$$R_m^2 = x_m^2 - 2x_m x + x^2 + y_m^2 - 2yy_m + y^2 + z_m^2 - 2z_m z + z^2$$

for $m = 3, 4, ..., M$. Therefore

$$R_1^2 - R_m^2 = x_1^2 + y_1^2 + z_1^2 - x_m^2 - y_m^2$$

$$-z_m^2 - 2x_1 x - xy_1 y - 2z_1 z + 2xx_m + 2yy_m + 2zz_m$$

for $m = 2, 3, ..., M$. We now solve subsitute the above result into Equation (1) to get

$$0 = v\tau_m - v\tau_2 + \frac{1}{v\tau_m}(x_1^2 + y_1^2 + z_1^2 - x_m^2 - y_m^2 - z_m^2 - 2x_1 x$$

$$-2y_1 y - 2z_1 z + 2x_m x + 2y_m y + 2z_m z)$$

$$-\frac{1}{v\tau_m}(x_1^2 + y_1^2 + z_1^2 - x_2^2 - y_2^2 - z_2^2 - 2x_1 x$$

$$-2y_1 y - 2z_1 z + 2x_2 x + 2y_2 y + 2z_2 z)]$$

for $m = 3, 4, ..., M$. We rewrite the above equation more succinctly as

$$0 = D_m + A_m x + B_m y + C_m z$$

where

$$A_m = \frac{1}{v\tau_m}(-2x_1 + 2x_m) - \frac{1}{v\tau_2}(2x_2 - 2x_1)$$
$$B_m = \frac{1}{v\tau_m}(-2y_1 + 2y_m) - \frac{1}{v\tau_2}(2y_2 - 2y_1)$$
$$C_m = \frac{1}{v\tau_m}(-2z_1 + 2z_m) - \frac{1}{v\tau_2}(2z_2 - 2z_1)$$

and

$$D_m = v_{\tau m} - v\tau_2 + \frac{1}{v\tau_m}(x_1^2 + y_1^2 + z_1^2 - x_m^2 - y_m^2 - z_m^2)$$
$$-\frac{1}{v\tau_2}(x_1^2 + y_1^2 + z_1^2 - x_2^2 - y_2^2 - z_2^2)$$

for $m = 3, 4, ..., M$. We rewrite the above set of $M - 2$ equations into matrix form to get

$$\begin{bmatrix} A_3 & B_3 & C_3 \\ A_4 & B_4 & D_4 \\ \vdots & \vdots & \vdots \\ A_M & B_M & C_M \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -D_3 \\ -D_4 \\ \vdots \\ -D_M \end{bmatrix}$$

We apply the Moore-Penrose pseudoinverse of the matrix to both sides to solve for x, y, z. Note that the yields a solution only when $M \geq 5$. In other words, you need 5 or more microphones.