ECE 581 Homework 4

Due Thursday 5 AM Sept 24, 2015

16 points Total

Electronic Submission – Please submit via "Assignment" under Sakai

1. (4 pts total) The random variable X has a probability density function $f_X(x) = \frac{1}{3}\delta(x) + \frac{2}{3}\delta(x-2)$. Another random variable, Y, has a probability density function $f_Y(y) = \frac{1}{2}\delta(y) + \frac{1}{2}\delta(y-3)$. X and Y are statistically independent. Obtain an analytical expression for the probability density function of Z, where $Z = X + Y$. Also, sketch and completely label your result.

2. (4 pts total) The random variable X is uniformly distributed between -1 and +1, and the random variable Y is uniformly distributed between 0 and 4. X and Y are statistically independent. Obtain an analytical expression for the probability density function of Z, where $Z = X + Y$. Also, sketch and completely label your result.

3. (4 pts total) The random variable X has a probability density function $f_X(x) = \frac{1}{2}\delta(x+2) + \frac{1}{2}\delta(x-2)$.

(a) (1 point) What is the characteristic function, $\Phi(\omega)$ for the random variable X in this case? (analytical expression)

(b) (1 point) Sketch and completely label the magnitude of $\Phi(\omega)$ vs. ω for this case.

(c) (1 point) Obtain an analytical expression for the second absolute moment, $E[X^2]$, for this case if you stayed in the probability density function domain. Also indicate its numerical value.

(d) (1 point) Instead of the method in part (c), show the steps in calculating the second absolute moment $E[X^2]$ for this specific case using the characteristic function. Verify its numerical value.

4. (4 pts total) Consider the transformation $y = g(x) = x^2$. Next consider a random variable X at the input which has a uniform probability density function from -1 to +1.

(a) (1 point) What is the mean of the output random variable Y, $E[Y]$? Indicate its numerical value.

(b) (1 point) What is the third absolute moment of the random variable Y, $E[Y^3]$? Indicate its numerical value.

(c) (2 points) Obtain an explicit analytical expression for the probability distribution function, $F_Y(y)$ at the output of this transformation for this specific problem. Sketch and completely label it.