ECE 581 Homework 3

Due Tuesday 5 AM Sept 15, 2015 (1 page, 20 points total)

Electronic Submission – Please submit via "Assignment" under Sakai

1. (5 points Total) Given a mixed (discrete and continuous) random variable X with probability density function
$f_X(x) = \frac{1}{4}[U(x) - U(x-2)] + \frac{1}{4}\delta(x+1) + \frac{1}{8}\delta(x-1) + \frac{1}{8}\delta(x)$ where $U(x)$ is the unit step function, and $\delta(x)$ is the dirac delta function.
(a) (1 points) Obtain and sketch accurately and completely label the probability distribution function $F_X(x)$ vs. x
Double check the expression given above for the probability density function and the expression you got for the probability distribution function of the random variable X before proceeding.
Calculate numerical values for each of the following:
(b) (1 point) $E[X^2]$
(c) (1 point) $Pr(0 < X \leq 1)$
(d) (1 point) $Pr(0 \leq X < 1)$
(e) (1 point) $Pr(X = 1)$

2. (5 points Total)(a) (3 points) Next, consider the random variable X with probability density function $f_X(x) = \frac{1}{2}\delta(x) + \frac{1}{2}\delta(x-2)$ as the input to the transformation $Y = X^2$. Sketch accurately, as a function of x and y, and label completely the joint probability density function, $f_{XY}(x, y)$, of the random variables X and Y.
(b)) (2 points) What is the probability density function, $f_Y(y)$, of Y? Sketch accurately and completely label $f_Y(y)$ vs. y.

3. (5 points Total) Consider the transformation $y = g(x) = |x|$. Consider a random variable X at the input which has the probability density function $f_X(x) = \frac{1}{4}\delta(x-2)+\frac{1}{4}\delta(x-1)+\frac{1}{4}\delta(x)+\frac{1}{4}\delta(x+1)$.

(a) (1 points) What is the mean of the output random variable Y, $E[Y]$? Indicate its numerical value.

(b) (2 points) Are the random variables X and Y correlated or uncorrelated? Prove your answer. (No credit for any answer without proving it.)

(c) (1 points) What is the third absolute moment of the random variable Y, $E[Y^3]$? Indicate its numerical value.

(d) (1 points) Obtain an explicit analytical expression for the probability distribution function, $F_Y(y)$ at the output of this transformation for this specific problem. Sketch and completely label it.

4. (5 points Total)
(a) (3 points) Consider the transformation of a random variable, $Y = g(X) = X^2$ where the probability density function of X is uniform between -1 and 1. Are the random variables X and Y correlated or uncorrelated? Show specifically for this problem why or why not. (No credit for any answer without proving it.)
(b) (2 points) If instead, the probability density function of X is uniform between 0 and 2, are the random variables X and Y correlated or uncorrelated? Show specifically for this problem why or why not. (No credit for any answer without proving it.)