Gaussian random variables

5.30 Two zero-mean unit-variance random variables X_1 and X_2 are described by the joint *Gaussian PDF*

$$f_{X_1X_2}(x_1,x_2) = \frac{1}{\pi\sqrt{3}}e^{-(x_1^2-x_1x_2+x_2^2)2/3}$$

(a) Show by integration that the marginal density for X_1 is Gaussian and is equal to

$$f_{X_1}(x_1) = \frac{1}{\sqrt{2\pi}}e^{-x_1^2/2}$$

Hint: Use the technique of "completing the square" to write

$$x_1^2 - x_1x_2 + x_2^2 = \frac{3}{4}x_1^2 + \frac{1}{4}x_1^2 - x_1x_2 + x_2^2 = \frac{3}{4}x_1^2 + (x_2 - \frac{1}{2}x_1)^2$$

Then form the definite integral over x_2 and make the substitution of variables $u = x_2 - \frac{1}{2}x_1$. Use the fact that the one-dimensional Gaussian form integrates to 1 in order to evaluate the integral.

(b) What is the conditional density $f_{X_2|X_1}(x_2|x_1)$? Observe that it has the form of a Gaussian density for X_2 but that the mean is a function of x_1. What *are* the mean and variance of this conditional density?