

ECE 581 Homework 12

Due Tuesday 5 AM November 17, 2015 (15 Total Hmwk points) Show work.

Electronic Submission – Please submit via "Assignment" under Sakai

1. (6 points) A LSI discrete-time system has the unit sample response

$$h[k] = \delta[k] - \frac{1}{2}\delta[k-1] + \frac{1}{4}\delta[k-2]$$

The input discrete-time random process $X[k]$ is white noise (mean zero) with autocorrelation function

$$R_X[l] = 3\delta[l].$$

- (a) (3 points) Determine the autocorrelation function, $R_Y[l]$ of the output discrete-time random process $Y[k]$.

- (b) (3 points) Determine the output power spectral density function $S_Y(e^{j\omega})$.

2. (9 points Total) Consider the problem in which the input to a linear time invariant (LTI) system is $X(t) = s(t) + N(t)$ where $N(t)$ is white noise and $s(t)$ is a completely known deterministic signal. Specifically $s(t) = 1$ for $0 \leq t \leq 1$, $s(t) = 2$ for $1 \leq t \leq 2$ and zero otherwise.

- (a) (3 points) What is the impulse response $h(t)$ of the LTI system that maximizes the output signal to noise ratio, R_0 at time $t_1 = 6$, where $R_0 = \frac{s_0^2(t_1)}{E[N_0^2(t_1)]}$. Sketch and label completely $h(t)$.

- (b) (3 points) If we were to look at the output of the particular filter that you got in part (a) at time $t = 6$, what would be its numerical value if there were no noise at the input to the filter?

- (c) (3 points) Is the noise at the *output* of your filter white noise? If so, explain why; if not, why not. You do not need to do a detailed calculation, but indicate what information and equations you would use to back your argument.