ECE 581 Homework 2

1. (a) $Pr(0 < X \leq 1) = 0.5625 + 0.0625(1-e^{-x})\big|_{x=1}$
$= 0.602$

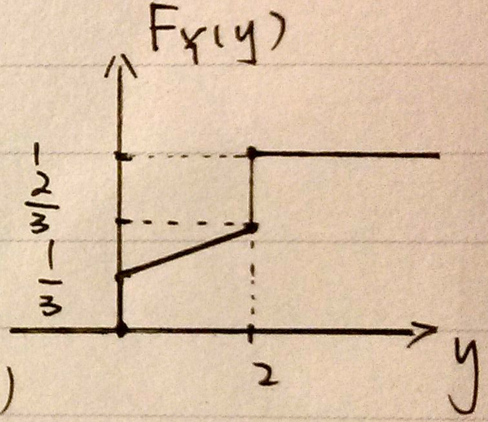
(b) $Pr(0 \leq X < 1) = 0.1875 + 0.0625(1-e^{-x})\big|_{x=1}$
$= 0.227$

(c) $Pr(0 < X < 1) = 0.0625(1-e^{-x})\big|_{x=1} = 0.0395$

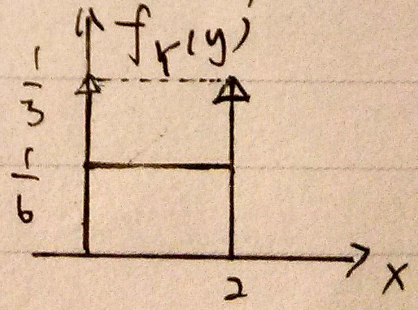
(d) $Pr(0 \leq X \leq 1) = 0.1875 + 0.5625 + 0.0625(1-e^{-x})\big|_{x=1}$
$= 0.7895$

(e) $Pr(2 < X \leq 3) = 0.0625(1-e^{-x})\big|_{2}^{3} = 0.00535$

2. (a)
$$F_Y(y) = \begin{cases} 0, & y \in (-\infty, 0) \\ \frac{1}{3}, & y = 0 \\ \frac{1}{6}(y+2), & y \in (0, 2) \\ 1, & y \in [2, +\infty) \end{cases}$$



(b)
$$f_Y(y) = \begin{cases} \frac{1}{3}\delta(y) + \frac{1}{6} + \frac{1}{3}\delta(y-2), & y \in [0, 2] \\ 0, & \text{otherwise} \end{cases}$$



(c) $E[Y] = \int_{-\infty}^{\infty} y \cdot f_Y(y)\,dy$
$= \int_{-\infty}^{\infty} \frac{1}{3}\delta(y) \cdot y\,dy + \int_{-\infty}^{\infty} \frac{1}{3}\delta(y-2)\,y\,dy + \int_0^2 \frac{1}{6} y\,d$
$= \frac{2}{3} + \frac{1}{6} \cdot \frac{1}{2} y^2 \big|_0^2 = 1$

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3. (a) $f(z|H_0) \Rightarrow z = \frac{1}{\sigma_n^2} \sum_{i=1}^{K} \sum_{j=1}^{K} n_{i,j} s_{i,j}$

$\because$ given the S is known signal, $\therefore s_{i,j}$: constant

$\because n_{i,j} \sim N(0, \sigma_n^2)$

$\Rightarrow z \sim N(0, \sum_{i=1}^{K} \sum_{j=1}^{K} s_{i,j}^2 \cdot \sigma_n^2 \cdot \frac{1}{\sigma_n^4})$

$\sim N(0, \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{s_{i,j}^2}{\sigma_n^2})$

$\therefore f(z|H_0) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}, \quad z \in (-\infty, +\infty)$

$$\boxed{\sigma^2 = \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{s_{i,j}^2}{\sigma_n^2}}$$

(b) $f(z|H_1) \Rightarrow z = \frac{1}{\sigma_n^2} \sum_{i=1}^{K} \sum_{j=1}^{K} (n_{i,j} s_{i,j} + s_{i,j}^2)$

$= \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{s_{i,j}^2}{\sigma_n^2} + \frac{1}{\sigma_n^2} \sum_{i=1}^{K} \sum_{j=1}^{K} s_{i,j} \cdot n_{i,j}$

$\therefore z \sim N(\sum_{i=1}^{K} \sum_{j=1}^{K} \frac{s_{i,j}^2}{\sigma_n^2}, \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{s_{i,j}^2}{\sigma_n^2})$

$\therefore f(z|H_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}}, \quad z \in (-\infty, +\infty)$

$$\boxed{\sigma^2 = \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{s_{i,j}^2}{\sigma_n^2}} \quad \boxed{\mu = \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{s_{i,j}^2}{\sigma_n^2}}$$