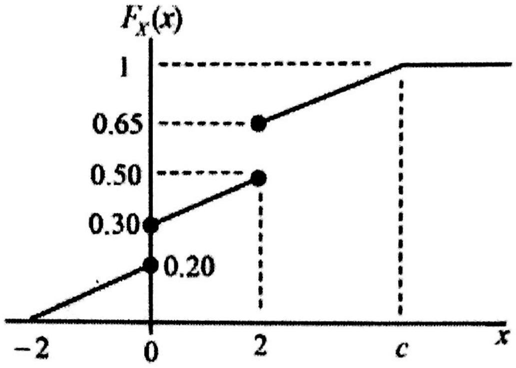
3.40 Given the cumulative distribution function of random variable X shown in the figure (not drawn to scale and all slant lines have the same slope) below, determine the following.



(a) The constant c that makes F_X a valid CDF.

(b) The probability density function of X (an accurate plot of the PDF will do). Mark all axes clearly and indicate all numerical values.

(c) The probabilities of the events $-2 < X < 0$ and $0 < X \le 2.5$.

(a) same slope $= \frac{0.2}{2} = 0.1$

$\therefore\ (c-2)\cdot 0.1 = 1 - 0.65$

$\Rightarrow\ c = 5.5$

(c) $P_r(-2 < X < 0)$

$= F_X(0^-) - F_X(-2)$

$= 0.2$

$Pr(0 < X \le 2.5)$

$= F_X(2.5) - F_X(0^+)$

$= 0.7 - 0.3 = 0.4$

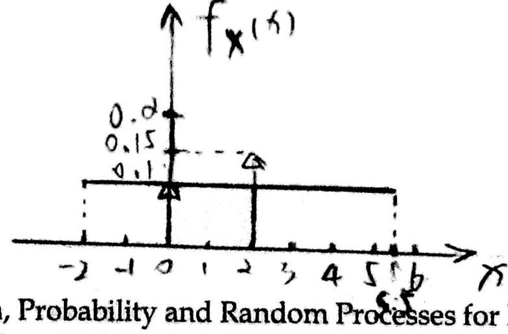
(b) $\cancel{F_X(x) = \begin{cases} 0, & x\in(-\infty,-2] \\ 0.1x+0.2, & x\in(-2,0) \\ 0.1, & x=0 \\ 0.1x+0.3, & x\in[0+,2-] \\ 0.15, & x=2 \\ 0.1x+0.45, & x\in[2+,5.5) \\ 1, & x\in[5.5,+\infty) \end{cases}}$

$$F_X(x) = \begin{cases} 0 & (-\infty,-2] \\ 0.1x+0.2, & (-2,0) \\ 0.1, & x=0 \\ 0.1x+0.3, & (0,2) \\ 0.15, & x=2 \\ 0.1x+0.45, & (2,5.5] \\ 1 & (5.5,+\infty) \end{cases}$$

$$\Rightarrow f_X(x) = \begin{cases} 0, & x\in(-\infty,-2] \\ 0.1+0.1\,\delta(x)+0.15\,\delta(x-2), & x\in(-2,5.5] \\ 0, & x\in(5.5,+\infty) \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(x)\,dx = 1$$

$$\delta(x) = \begin{cases} +\infty, & x=0 \\ 0, & x\neq 0 \end{cases}$$



Reference: Therrien and Tummala, Probability and Random Processes for Electrical and Computer Engineers, Second Edition, CRC Press 2012

3.20 Is the following expression a valid probability density function? Support your answer.

$$f_X(x) = \begin{cases} \frac{3}{2} - x, & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Conclusion : Not

Reason : $f_X(x) \geq 0$, $x \in (-\infty, +\infty)$ ···· Pdf property

$f_X(x=2) = \frac{3}{2} - 2 = -\frac{1}{2} < 0$, violate the property

$\therefore$ The expression above is not a valid pdf

3.14 The PDF for a random variable X is given by

$$f_X(x) = \begin{cases} 2x/9 & 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f_X(x)$ versus x.

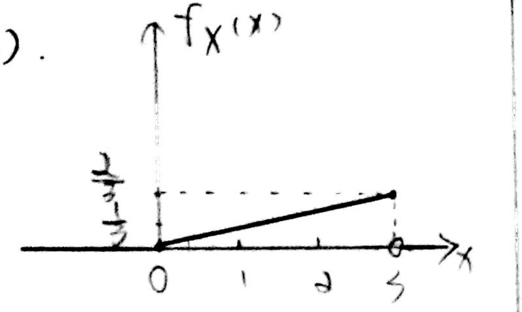
(b) What is the probability of each of the following events?

(i) $X \le 1$
(ii) $X > 2$
(iii) $1 < X \le 2$

(c) Find and sketch the CDF $F_X(x)$.

(d) Use the CDF to check your answers to part (b).

(a).



(b) [i]. $Pr(X \le 1) = F_X(1)$

$= \int_{-\infty}^{1} f_X(x)dx$

$= \int_0^1 \frac{2}{9}x \cdot dx = \frac{1}{9} + C = \frac{1}{9}$

[ii] $Pr(X > 2) = 1 - Pr(X \le 2)$

$= 1 - F_X(2)$

$= 1 - \int_0^2 \frac{2}{9}x \cdot dx$

$= 1 - \frac{4}{9} = \frac{5}{9}$

[iii] $Pr(1 < X \le 2) = F_X(2) - F_X(1)$

$= \frac{4}{9} - \frac{1}{9} = \frac{1}{3}$

(d) [i]: $Pr(X \le 1) = F_X(1) = \frac{1}{9}$ V

[ii]: $Pr(X > 2) = 1 - Pr(X \le 2)$

$= 1 - F_X(2) = \frac{5}{9}$ ✓

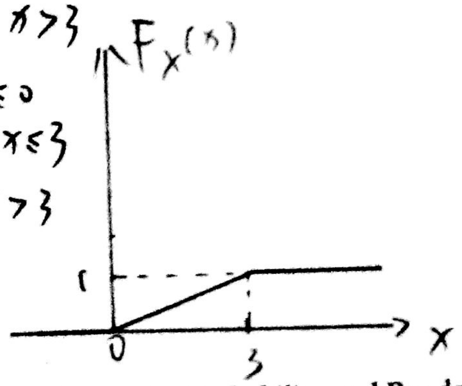
[iii]: $Pr(1 < X \le 2) = F_X(2) - F_X(1)$

$= \frac{4}{9} - \frac{1}{9} = \frac{1}{3}$ V

(c) $F_X(x) = \int_{-\infty}^{x} \frac{2}{9}x \cdot dx$

$$= \begin{cases} 0, & x \le 0 \\ \int_0^x \frac{2}{9}x \cdot dx, & 0 < x \le 3 \\ \int_0^3 \frac{2}{9}x \cdot dx, & x > 3 \end{cases}$$

$$= \begin{cases} 0, & x \le 0 \\ \frac{1}{9}x^2, & 0 < x \le 3 \\ 1, & x > 3 \end{cases}$$



Reference: Therrien and Tummala, Probability and Random Processes for Electrical and Computer Engineers, Second Edition, CRC Press 2012