ECE 581 Homework 5

Due Tuesday 5 AM October 1, 2015, (14 Hmwk points total) Show work.
Electronic Submission – Please submit via "Assignment" under Sakai

Problem 5-1 (7 points Total) The joint probability density function of the two random variables X and Y is given by $f_{XY}(x,y) = \frac{3}{8}\delta(x-0,y-0)+\frac{1}{4}\delta(x-0,y-1)+\frac{1}{8}\delta(x-1,y-0)+\frac{1}{4}\delta(x-1,y-1)$. Sketch and accurately label the following: (a)-(e)
(a) (1 point) The joint pdf, $f_{X,Y}(x,y)$
(b) (1 point) The marginal pdf, $f_Y(y)$.
(c) (1 point) The marginal pdf, $f_X(x)$.
(d) (1 point) The conditional pdf, $f_Y(y|x)$
(e) (1 point) The conditional pdf, $f_X(x|y)$.
(f) (1 point) What is the conditional expected value of X given that the random variable Y takes on the value 0, i.e. $E[X|Y=0]$
(g) (1 point) Are the random variables X and Y statistically independent? Show why or why not.

Problem 5-2, signal detection example (7 points Total) Consider the binary decision problem of deciding "H_1" or deciding "H_0" in which the single observation (data) X under the hypothesis H_0 has a uniform probability density function, with height one in the interval [-1/2,1/2] and zero otherwise. Under the H_1 hypothesis X has a probability density function that has the shape of an isosceles triangle with maximum height 2, in the interval [-1/2,1/2] and zero otherwise. The prior probability of the hypothesis H_1 is $P(H_1) = \frac{1}{2}$.

(a) (1 point) What is the likelihood ratio for this problem? Sketch and label for all x.
(b) (1 point) Where are the decision regions in the likelihood ratio space if the threshold β on the likelihood ratio is $\beta = 1$? Sketch and label these regions in likelihood ratio space for all likelihood ratio values.

(c) (1 points) Where are the decision regions in the observation space if the threshold β on the likelihood ratio is $\beta = 1$? Sketch and label these regions for all x.

(d) (3 points) Calculate the pairs (P_f, P_d), where P_f is the probability of false alarm, and P_d is the probability of detection, that correspond to the threshold on likelihood ratio of $\beta = \frac{1}{4}$, $\beta = \frac{1}{2}$ and $\beta = \frac{3}{4}$. Plot the ROC (receiver operating characteristic).

(e) (1 point) What should the threshold β on the likelihood ratio be set to in order to minimize the probability of a decision error? What is the actual numerical value of the minimum probability of a decision error?