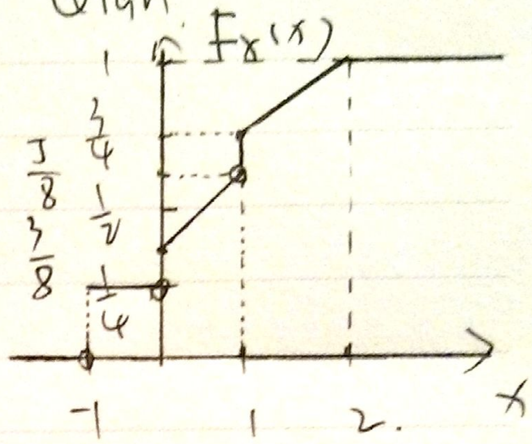
ECE 581 . Shengxin Qian



1.(a). $F_X(x) = \begin{cases} \frac{1}{4}, & x = -1 \\ \frac{1}{4}, & x \in (-1, 0) \\ \frac{3}{8}, & x = 0 \\ \frac{1}{4}x + \frac{3}{8}, & x \in (0, 1) \\ \frac{3}{4}, & x = 1 \\ \frac{1}{4}x + \frac{1}{2}, & x \in (1, 2) \\ 1, & x \geq 2 \end{cases}$

(b). $E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_X(x)\, dx$

$= \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{4}\delta(x+1)\, dx + \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{8}\delta(x-1)\, dx$

$+ \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{8}\delta(x)\, dx + \int_0^2 \frac{1}{4}x^2\, dx$

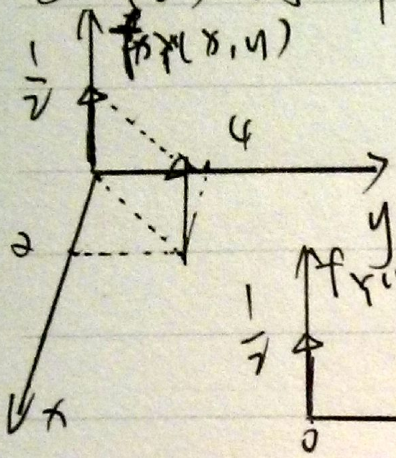
$= \frac{25}{24}$

(c) : $Pr(0 < X \leq 1) = \int_0^1 \frac{1}{4}\, dx + \frac{1}{8} = \frac{3}{8}$

(d) : $Pr(0 \leq X < 1) = \int_0^1 \frac{1}{4}\, dx + \frac{1}{8} = \frac{3}{8}$

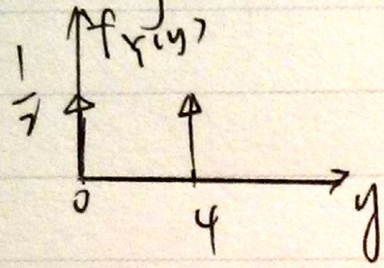
(e) : $Pr(X = 1) = \frac{1}{8}$.

2.(a) $f_{XY}(x,y) = \frac{1}{2}\delta(x)\delta(y) + \frac{1}{2}\delta(x-2)\delta(y-4)$



(b): $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$

$= \frac{1}{2}\delta(y) + \frac{1}{2}\delta(y-4)$



3.(a).

y\x:	2	1	0	-1
2	$\frac{1}{4}$	0	0	0
1	0	$\frac{1}{4}$	0	$\frac{1}{4}$
0	0	0	$\frac{1}{4}$	0

$\therefore E[Y] = 2\cdot\frac{1}{4} + 1\cdot\frac{1}{2}$

$= 1$

(b): $E[XY] = \frac{1}{4}\cdot\frac{1}{4} + 1\cdot\frac{1}{4} - 1\cdot\frac{1}{4} = 1$

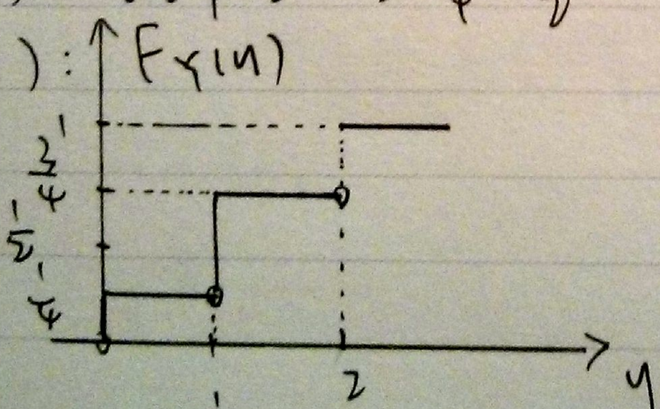
$E[X] = 2\cdot\frac{1}{4} + 1\cdot\frac{1}{4} - 1\cdot\frac{1}{4} = \frac{1}{2}$

$E[Y] = 1$

$\because E[XY] \neq E[X]E[Y]$ $\therefore$ correlated

(c): $E[Y^3] = 8\cdot\frac{1}{4} + \frac{1}{2} = 2.5$

(d):



$\therefore F_Y(y) = \begin{cases} 0, & x < 0 \\ \frac{1}{4}, & x \in [0,1) \\ \frac{3}{4}, & x \in [1,2) \\ 1, & x \in [2,+\infty) \end{cases}$

4.(a). $f_X(x) = \begin{cases} \frac{1}{2}, & x \in [-1, 1] \\ 0, & \text{othewise} \end{cases}$

$$\therefore f_Y(y) = \frac{1}{2\sqrt{y}} [f_X(\sqrt{y}) + f_X(-\sqrt{y})]$$

$$= \begin{cases} \frac{1}{2\sqrt{y}}, & y \in [0, 1) \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore E(X) = \int_{-1}^{1} \frac{1}{2} x\,dx = \frac{1}{2} \cdot \frac{1}{2} x^2 \Big|_{-1}^{1} = 0$$

$$E[XY] = \int_{-1}^{1} \frac{1}{2} x^3 dx = 0.$$

$$\therefore E[X] \cdot E[Y] = E[XY] \Rightarrow \text{uncorrected}$$

(b) $\int_0^2 x^3 \cdot \frac{1}{2} dx$

$$= \frac{1}{2} \cdot \frac{1}{4} x^4 \Big|_0^2 = 2 = E[XY]$$

$$E[X] = \int_0^2 x \cdot \frac{1}{2} dx = 1 \qquad E[Y] = \frac{4}{3} = \int_0^2 x^2 \cdot \frac{1}{2} dx$$

$$\therefore E[X] \cdot E[Y] \neq E[XY] \Rightarrow \text{corrected}$$