ECE 581 Homework 7

Due Thursday 5 AM October 15, 2015, (25 hmwk points total)

Electronic Submission – Please submit via "Assignment" under Sakai

Include your analytical work, MATLAB listings, graphs and comments with your homework. Use Matlab throughout this problem and include your name in the listings as author. Be sure to include written comments with all of your results; i.e. please don't just turn in a collection of unexplained graphs and expect to get full credit. Discuss your results as you go along. Include in your discussion specific possible explanations as to why your results could be what they are.

Problem 5-1 (5 points) This homework assignment is a follow up to our Homework 1 an Homework 2, problem 3, assignments early this semester involving the detection of a known image in noise. Now let's see if we can design an algorithm that can make the best possible decisions according to a particular criterion, and do this better than what we could do visually earlier this semester.

In a recent lecture, we presented some of the elements of signal detection theory. Assume that the hypothesis H_0 is noise alone and H_1 is signal plus noise, where the known signal (your known noise-free image given by the S matrix) is assumed to be known exactly and the noise matrix has elements that are statistically independent Gaussian random variables of zero mean and a known noise variance σ_n^2. The size of the image is 1024 x 1024.

In class we showed that if the decision criterion is to minimize the probability of a decision error, P_ϵ, the optimum algorithm for this crierion implements the likelihood ratio followed by a threshold $\beta = \frac{P(H_0)}{P(H_1)}$ or equivalently the ln likelihood ratio of the observations, followed by the threshold $\ln \beta$. For the criterion Max $(P_D - WP_F)$ the threshold is $\beta = W$.

Derive the ln likelihood ratio (natural log of the likelihood ratio) for the specific known image detection in noise problem (1024 x 1024) that you considered in homework 1. (You may want to refer to Homework 1 and Homework 2 problem 3)

Problem 5-2 (20 points Total) (a) (10 points) Now, let's obtain the detection **performance** for the problem considered in problem 5-1 above. We will use the ROC (receiver operating characteristic) as our quantitative performance metric. The ROC is a plot of P_D, the probability of detection, on the y axis, versus P_F, the probability of a false alarm, on the x axis, as a function of the threshold setting $\ln \beta$ on the ln likelihood ratio. The ROC should be a **square plot**, with each axis scaled from zero to one.

In this part of the problem we want to get four ROCs using brute-force computer simulation, for $\frac{E_s}{\sigma_n^2} = 1, 2, 4$ and 16. Here, brute-force simulation means to start your simulation by generating the data under hypothesis H_0 and put it through your optimum ln likelihood ratio detector. Repeat this 500 times (i.e. 500 trials) and save the resultant 500 $ln\lambda$ values, which is an approximation to $f(ln\lambda|H_0)$. You will need to use a Gaussian random number generator routine in Matlab to generate the noise. Repeat this procedure for hypothesis H_1, which gives you an approximation to $f(ln\lambda|H_1)$. From there, compute the (P_F, P_D) pair for a large number of possible threshold settings on $ln\lambda$ and you have the points for an ROC. Obtain and plot these ROC's for values of $\frac{E_s}{\sigma_n^2}$ of 1,2,4, and 16.

(b) (10 points) In this part of the problem we want to obtain these same ROCs theoretically. These results are important as they stand, but we can also use them to check your simulation in part (a). Obtain $f(\ln\lambda|H_0)$ and $f(ln\lambda|H_1)$ theoretically. (You may wish to use your Homework 2, problem 3 results for guidance) Then obtain and plot the theoretical ROC's for $\frac{E_s}{\sigma_n^2}$ of 1,2,4, and 16. Superimpose these theoretical curves on top of your simulated ROC's to check your simulation.