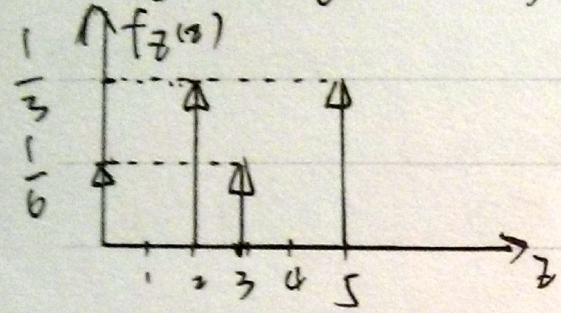
ECE 581 Hmwk 4.

1.

| $X$: | 0 | 2 | $Y$: | 0 | 3 | $X+Y=Z$: | 0 | 2 | 3 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|
| Pr: | $\frac{1}{3}$ | $\frac{2}{3}$ | Pr: | $\frac{1}{2}$ | $\frac{1}{2}$ | Pr: | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{6}$ | $\frac{1}{3}$ |

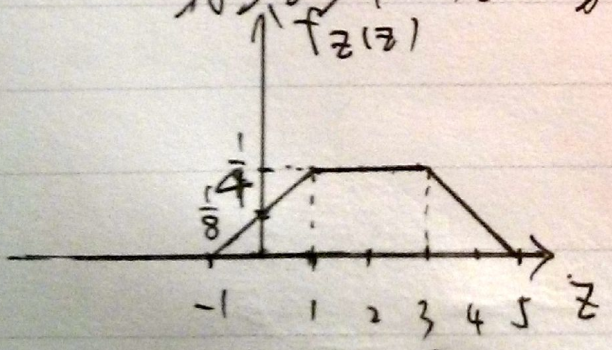
$\therefore f_Z(z) = \frac{1}{6}\delta(x) + \frac{1}{3}\delta(x-2) + \frac{1}{6}\delta(x-3) + \frac{1}{3}\delta(x-5)$




2. $f_X(x) = \begin{cases} \frac{1}{2}, & x \in [-1, 1] \\ 0, & \text{other} \end{cases}$ $f_Y(y) = \begin{cases} \frac{1}{4}, & x \in (0, 4) \\ 0, & \text{other} \end{cases}$

$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) \cdot f_Y(z-x)\,dx = \begin{cases} 0, & z \in (-\infty, -1) \\ \frac{1}{8}(z+1), & z \in [-1, 1) \\ \frac{1}{4}, & z \in [1, 3) \\ \frac{1}{8}(5-z), & z \in [3, 5) \\ 0, & z \in [5, +\infty) \end{cases}$

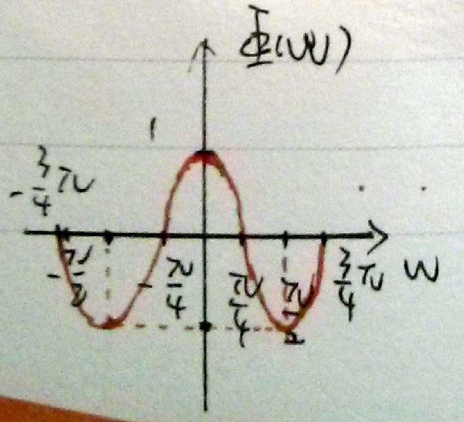
~~$= \int_0^1 \frac{1}{2} \cdot \frac{1}{4}\,dx = \frac{1}{8}$~~




3. (a) $\Phi(w) = E(e^{jwx}) = \int_{-\infty}^{\infty} e^{jwx} f_X(x)\,dx = \frac{1}{2}e^{-2wj} + \frac{1}{2}e^{2jw}$

$= \cos 2w$

(b).




(d) $\left.\frac{d^n\Phi(w)}{dw^n}\right|_{w=0} = j^n E[X^n]$

$\therefore -\left.\frac{d^2\Phi(w)}{dw^2}\right|_{w=0} = E[X^2]$

$= 4\cos(2w)|_{w=0}$

$= 4$

(c). $E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_X(x)\,dx$

$= \frac{1}{2} x^2 \Big|_{x=-2} + \frac{1}{2} x^2 \Big|_{x=2}$

$= \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 4 = 4$

4(a). $f_X(x) = \begin{cases} \frac{1}{2}, & x \in (-1, 1) \\ 0, & \text{other} \end{cases}$

$\therefore E[Y] = \int_{-\infty}^{\infty} f_X(x) \cdot x^2\,dx$

$= \int_{-1}^{1} \frac{1}{2} \cdot x^2\,dx$

$= \frac{1}{3}$

(b) $E[Y^3] = \int_{-\infty}^{\infty} x^6 f_X(x)\,dx$

$= \frac{1}{7}$

(c). $F_Y(y) = Pr(Y \le y) = Pr(X^2 \le y) = Pr(-\sqrt{y} \le X \le \sqrt{y})$

$= F_X(\sqrt{y}) - F_X(-\sqrt{y})$

$$= \begin{cases} \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{2}\,dx, & y \in [0, 1) \\ 1, & y \in [1, +\infty) \\ 0, & y \in (-\infty, 0) \end{cases} = \begin{cases} \sqrt{y}, & y \in [0, 1) \\ 1, & y \in [1, +\infty) \\ 0, & y \in (-\infty, 0) \end{cases}$$