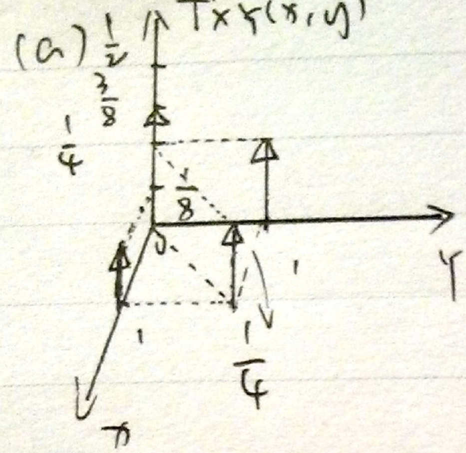
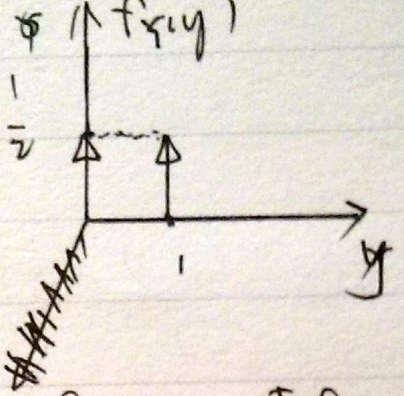
ECE 581 Homework5

Problem 5-1

(a)

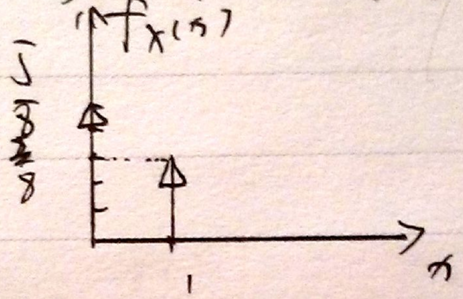


(b) $f_Y(y) = \frac{1}{2}\delta(y-0) + \frac{1}{2}\delta(y-1)$



y \ x	0	1	
0	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{2}$
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{5}{8}$	$\frac{3}{8}$	

(c). $f_X(x) = \frac{5}{8}\delta(x-0) + \frac{3}{8}\delta(x-1)$

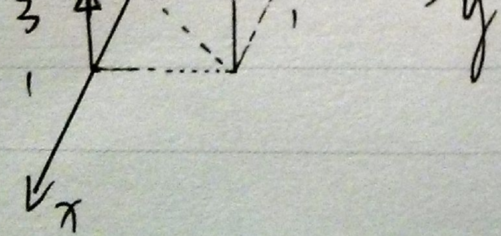


(d). $f_Y(y|x) = \dfrac{f_{XY}(x,y)}{f_X(x)}$

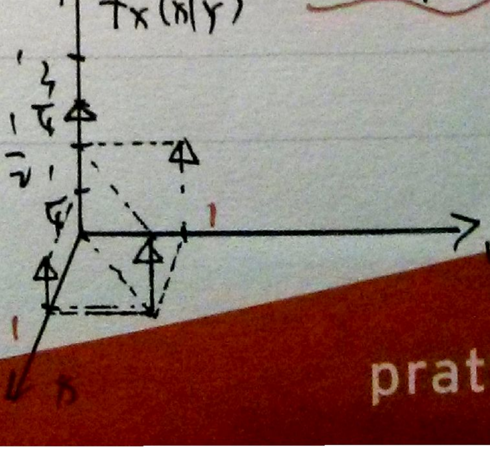
$f_Y(y|x=0) = \frac{3}{5}\delta(y-0) + \frac{2}{5}\delta(y-1)$

$f_Y(y|x=1) = \frac{1}{3}\delta(y-0) + \frac{2}{2}\delta(y-1)$

$\Rightarrow f_Y(y|x) = \frac{3}{5}\delta(y-0, x-0) + \frac{2}{5}\delta(y-1, x-0) + \frac{1}{3}\delta(y-0, x-1) + \frac{2}{3}\delta(y-1, x-1)$



(e) $f_X(x|Y) = \frac{3}{4}\delta(x-0, y-0) + \frac{1}{4}\delta(x-1, y-0) + \frac{1}{2}\delta(x-0, y-1) + \frac{1}{2}\delta(x-1, y-1)$



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(f) $E[X \mid Y=0] = P(X=0 \mid Y=0) \cdot 0 + P(X=1 \mid Y=0) \cdot 1$
$= \frac{1}{4}$

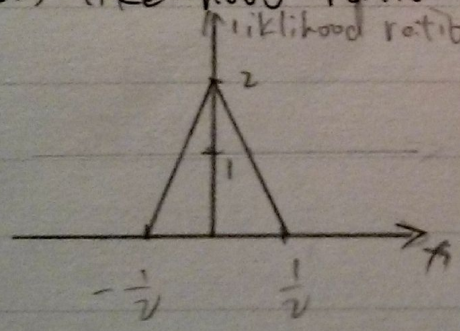
(g) $f_Y(y) \cdot f_X(x) = \left[\frac{1}{2}\delta(y-0) + \frac{1}{2}\delta(y-1)\right] \cdot \left[\frac{5}{8}\delta(x-0) + \frac{3}{8}\delta(x-1)\right]$

$$= \frac{5}{16}\delta(x-0, y-0) + \frac{3}{16}\delta(x-1, y-0)$$
$$+ \frac{5}{16}\delta(x-0, y-1) + \frac{3}{16}\delta(x-1, y-1)$$
$$\neq f_{XY}(x, y)$$

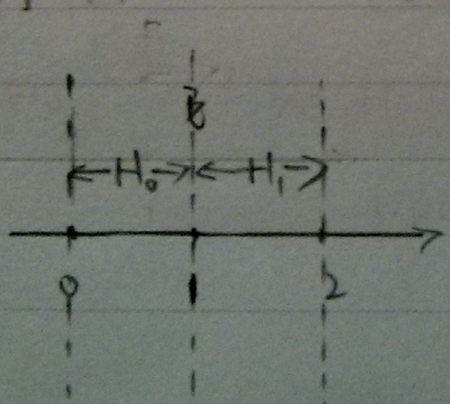
$\therefore$ not statistically independent.

Problem 5-2

(a) likelihood ratio $= \dfrac{f(x \mid H_1)}{f(x \mid H_0)} = \begin{cases} 4x+2, & x \in [-\frac{1}{2}, 0) \\ -4x+2, & x \in [0, \frac{1}{2}] \\ 0, & \text{else.} \end{cases}$



(b). if likelihood ratio $\geq k \Rightarrow$ choose H_1
else choose H_0



H_1: likelihood ratio $\in [1, 2]$
H_0: likelihood ratio $\in [0, 1)$
likelihood ratio

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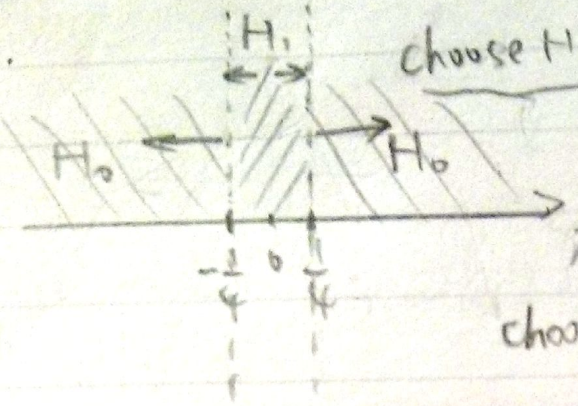
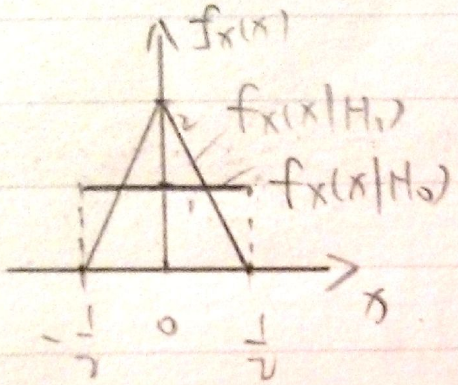
(c). choose H_1 : $x \in [-\frac{1}{4}, \frac{1}{4}]$

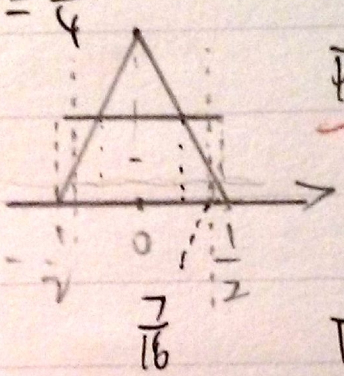


choose H_0 $x \in$ else, $(\frac{1}{4}, +\infty) \cup (-\infty, -\frac{1}{4})$

(d)



1) $\beta = \frac{1}{4}$



$P_d = 2\cdot(\frac{1}{4}+2)\frac{7}{16}\cdot\frac{1}{2}$

$= \frac{63}{64}$

$P_f = \frac{7}{16}\cdot 1\cdot 2 = \frac{7}{8}$

2) $\beta = \frac{1}{2}$

$4x+2 = \frac{1}{2}$

$x = -\frac{3}{8}$

$\therefore P_d = 2\cdot(\frac{1}{2}+2)\cdot\frac{3}{8}\cdot\frac{1}{2}$

$= \frac{15}{16}$

$P_f = \frac{3}{8}\cdot 1\cdot 2 = \frac{3}{4}$

3) $\beta = \frac{3}{4}$

$4x+2 = \frac{3}{4}$

$x = -\frac{5}{16}$

$\therefore P_d = 2\cdot(\frac{3}{4}+2)\cdot\frac{5}{16}\cdot\frac{1}{2}$

$= \frac{55}{64}$

$P_f = \frac{5}{16}\cdot 1\cdot 2 = \frac{5}{8}$

$\Rightarrow P_d = 2\cdot(\beta+2)\cdot\frac{2-\beta}{4}\cdot\frac{1}{2} = \frac{4-\beta^2}{4}$ / $P_f = \frac{2-\beta}{4}\cdot 2 = \frac{2-\beta}{2}$

$\beta \in [0, 2]$ $\beta = 2-2P_f$ $\therefore P_d = \frac{4-(2-2P_f)^2}{4} = P_f(2-P_f)$

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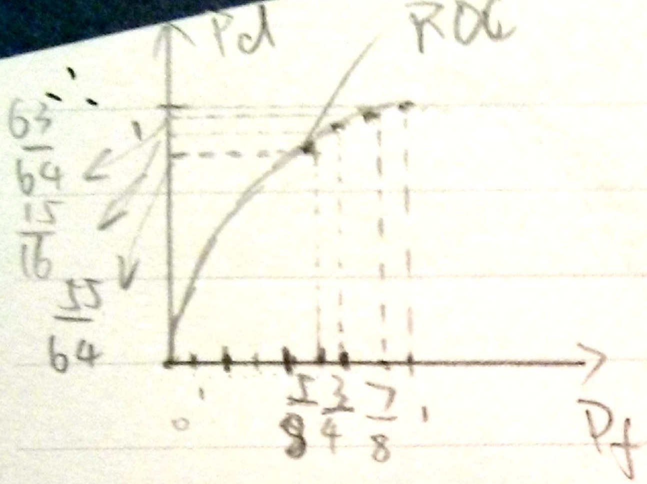

(e). $P_{error} = P_f \cdot \frac{1}{2} + P_{miss} \cdot \frac{1}{2}$.

$$= \frac{1}{2} \cdot \frac{2-\beta}{2} + \frac{1}{2} \cdot (1 - P_d)$$

$$= \frac{2-\beta}{4} + \frac{1}{8}\beta^2 = \frac{1}{8}\beta^2 - \frac{1}{4}\beta + \frac{1}{2} = \frac{1}{8}(\beta^2 - 2\beta + 4)$$

$$= \frac{1}{8}[(\beta-1)^2 + 3]$$

$\beta \in [0, 2]$

$\therefore \beta = 1 \Rightarrow$ minimum P_{error}

$\therefore P_{error}\big|_{\beta=1} = \frac{3}{8}$.

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