

ECE 581 Homework 9

Due Tuesday 5 AM October 27, 2015 (16 Total Hmwk points) Show work.

Electronic Submission – Please submit via "Assignment" under Sakai

Problem 9-1 (8 points Total) Consider a random process, $X(t)$, which has a total of two realizations (sample functions). One realization (i.e. one waveform in the ensemble) is $x(t) = 2$ for $-\infty < t < \infty$, and it occurs with probability $1/3$. The other realization is $x(t) = -2 + \cos(t)$ for $-\infty < t < \infty$.

(a) (2 points) What is the autocorrelation function, i.e. $R_X(t_1, t_2) = E[X(t_1)X(t_2)]$, of this random process?

(b) (i) (1 point) What is the first order probability density function, $f_{X(t)}(x)$, of this random process (algebraic expression)?

(ii) (1 point) Sketch and completely label $f_{X(t)}(x)$ vs. x .

(c) (i) (1 point) What is the second order joint probability density function, $f_{X(t_1), X(t_2)}(x_1, x_2)$, of this random process (algebraic expression)?

(ii) (1 point) Sketch and completely label $f_{X(t_1), X(t_2)}(x_1, x_2)$ as a function of the two variables, x_1 and x_2 when $t_1 = 0$ and $t_2 = \pi/2$.

(d) (2 points) Prove that the two random variables at times $t_1 = 0$ and $t_2 = \pi/2$ are either correlated or uncorrelated.

Problem 9-2 (3 points Total) A wide sense stationary random process, $X(t)$, is the input to a squaring device whose output at time t is the square of the input at time t . So the output is another random process $Y(t) = X^2(t)$.

(a) (2 points) Derive an algebraic expression for the mean of the output random process; i.e. $E[Y(t)]$?

(b) (1 point) In particular if the random process $X(t)$ is Gaussian with a mean of 2 and a variance of 3, what is $E[Y(t)]$, the mean of the output random process? (numerical answer)

Problem 9-3 (5 points Total) A discrete-time random process is given by $X[k] = A \cos(\omega_0 k)$ where A is a Gaussian random variable with mean η_A and variance σ_A^2 , and k is an integer.

(a) (2 points) What is the mean of this random process?

(b) (2 points) What is the variance of this random process?

(c) (1 points) Is this a wide sense stationary random process? Explain why or why not.