INF 2178 Technical Assignment 3

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1. Reading Score Improvement by Income Group

Null Hypothesis (H0): There is no difference in the improvement of reading scores from fall to spring across different income groups when controlling for the baseline (fall) reading scores.

Alternative Hypothesis (H1): There is a difference in the improvement of reading scores from fall to spring across different income groups when controlling for the baseline (fall) reading scores.

Boxplot one (see appendix one):

The exploratory data analysis through the box plots reveals distinct patterns in reading score changes from fall to spring across the income groups. All groups experienced improvements in reading scores, as indicated by median changes being above zero. The first plot showcases that the lower-income group has the broadest range of score changes, with a significant spread that includes numerous outliers demonstrating substantial increases. This suggests that while some students in the lower-income group made remarkable gains, others did not, pointing to a high level of variability within this group.

In contrast, the higher-income groups exhibited a more consistent pattern of score changes. The middle-income group had a slightly tighter interquartile range than the lower-income group, with fewer outliers, indicating less variability. The highest-income group showed the least variability in reading score changes, with an even tighter interquartile range and fewer outliers. This implies that students in the higher-income group experience more uniform reading score improvements over the school year.

Boxplot two (see appendix two):

The second plot provides insights into the initial reading abilities of the students in the fall. It is apparent that the median reading score increases with the income group; the lowest-income group has the lowest median score and the widest range of scores, suggesting a disparity in initial reading abilities. The middle-income group's median score was higher, with a narrower range, implying somewhat more homogeneity in reading ability. The highest-income group had the highest median reading score and showed the most uniformity in scores among its students, although outliers were present in all groups.

This initial analysis highlights a correlation between income group and reading achievement, both in terms of starting ability and the degree of improvement over time. It sets the stage for more sophisticated statistical analysis, such as ANCOVA, to further investigate these relationships while controlling for other variables.

Interaction plot (see appendix three):

The interaction plot visualizes the mean changes in reading scores by income group, revealing a trend where higher income groups show greater average improvements. The plot indicates that students from the lowest income bracket have the smallest mean increase, while those from the highest bracket show the largest. However, considerable overlap in the confidence intervals, especially between the lower and middle-income groups, suggests that while a discernible pattern of higher income correlates with greater reading score gains, the

differences are not sharply delineated. The error bars, representing confidence intervals, are notably longer for the highest income group, suggesting more variability in the reading score changes within that group. Further statistical testing is required to establish the significance of these differences.

Test assumption for ANCOVA:

- Testing homogeneity of regression slopes

The first test for C(incomegroup) has an F-value of approximately 4.06 with a p-value of 0.017, indicating that there are significant differences in reading score changes across the income groups when not considering the interaction with fall reading scores.
The second test for fallreadingscore has an F-value of approximately 333.91 with a p-value less than 0.001, confirming that the fall reading score is a significant covariate in predicting reading score change.
Most importantly, the third test for the interaction term C(incomegroup), fallreadingscore has an F-value of approximately 7.59 with a p-value of about 0.0005, suggesting that the relationship between fall reading scores and the change in reading scores is not the same across all income groups.

- Homogeneity of variances and Normality of residuals

Levene's test result with a p-value of approximately 2.794929503613517e-09 suggests that the assumption of homogeneity of variances is violated. This indicates significant differences in the variance of reading score changes across the different income groups.

The Shapiro-Wilk test yields a p-value of 0.0, which, due to rounding, suggests that the normality of residuals is also violated, indicating the residuals from the model do not follow a normal distribution.

The violation of homogeneity of variances means that the error term does not have a constant variance across groups, and the violation of normality of residuals means that the data do not meet the assumption of normal distribution of the error terms. Consequently, the results of the ANCOVA may not be reliable, and alternative methods or transformations may need to be considered.

ANCOVA result:

For the hypothesis concerning income group differences, with a p-value of 0.0173 (lower than 0.05), we reject the null hypothesis that there is no difference in the improvement of reading scores from fall to spring across different income groups when controlling for the baseline reading scores. This means that the data provides enough evidence to support the claim that the improvement in reading scores differs by income group.

For the hypothesis regarding the significance of the covariate (fall reading scores), with a p-value well below 0.05, we reject the null hypothesis that the baseline reading scores do not significantly predict the change in reading scores. This indicates that students' initial reading levels are a significant predictor of their reading improvement.

In summary, both the socioeconomic background, represented by income group, and academic starting point, indicated by fall reading scores, are significant factors in the educational advancement of young students. These findings highlight the importance of addressing income-related disparities and supporting early literacy to foster equitable educational outcomes.

Math Score Improvement by Income Group

Null Hypothesis (H0): There is no difference in the improvement of math scores from fall to spring across different income groups when controlling for the baseline (fall) math scores.
Alternative Hypothesis (H1): There is a difference in the improvement of math scores from fall to spring across different income groups when controlling for the baseline (fall) math scores.

Boxplot one (see appendix four):
The first box plot displays the distribution of math score changes from fall to spring across the three income groups. It shows that, on average, all income groups experienced an increase in math scores, evidenced by the median of each group being above zero. The spread of changes is similar across all income groups, as indicated by the similar interquartile ranges, with several outliers in each group demonstrating both decreases and substantial increases in scores. The similarity in the range of score changes suggests that the income group might not have as pronounced an effect on changes in math scores as it did on reading scores.

Boxplot two (see appendix five):
The second box plot illustrates the distribution of fall math scores by income group. Here, the trend is consistent with what we saw in the reading scores: the median math score for each successive income group is higher, suggesting that students from higher-income families tend to have better initial math scores. The spread of scores within each income group is relatively similar, though there are outliers, particularly in the higher income groups, indicating some students with much higher or lower scores than the median.

Interaction plot (see appendix six):
The interaction plot comparing math score changes across income groups indicates that students from higher-income groups generally show greater improvements in math scores than those from lower-income groups. The plot reveals a progressive increase in the average score changes from the lowest to the highest income group. The broadest confidence interval for the lowest income group suggests greater variability in their score changes, while the higher income groups, especially the highest, display narrower intervals, implying more consistency in their improvements.

Testing assumption for ANCOVA:

- Homogeneity of regression slopes

The results from testing the homogeneity of regression slopes indicate that the slopes of the regression lines relating the fall math scores to the math score change are not the same across

the different income groups. This is suggested by the significant interaction term C(incomegroup):fallmathscore, with an F-value of approximately 17.97 and a p-value of about 1.62e-08. The p-value is well below the standard significance level of 0.05, indicating that the effect of the fall math score on the change in math score is significantly different for at least one of the income groups compared to the others.

Furthermore, the main effect of C(incomegroup) is also significant with an F-value of approximately 18.58 and a p-value of 8.81e-09, indicating significant differences in math score changes across the income groups when not considering the interaction with the fall math scores. The covariate fallmathscore is highly significant with an F-value of approximately 104.28 and a p-value of 2.22e-24, indicating that the initial math score is a very strong predictor of the math score change.

- Homogeneity of variances and Normality of residuals

Levene's test result, with a p-value of approximately 2.344180384098772e-10, indicates a significant violation of the homogeneity of variances assumption for ANCOVA. This suggests that the variances of math score changes are not equal across the different income groups, which is a key assumption for conducting ANCOVA.

The Shapiro-Wilk test yields a p-value of 0.0, suggesting that the residuals of the model do not follow a normal distribution, thus violating another assumption of ANCOVA. This is critical because ANCOVA assumes normally distributed error terms in the model for the test statistics to be valid. The violations of these two assumptions suggest that the results of the ANCOVA for math score improvements may not be reliable

ANCOVA result

The C(incomegroup) effect is statistically significant, with an F-value of approximately 18.52 and a p-value of about 9.284861e-09. we would reject the null hypothesis for hypothesis two: Math Score Improvement by Income Group. This indicates that there are significant differences in math score improvements from fall to spring across the different income groups after controlling for the fall math scores.

The covariate fallmathscore is highly significant, with an F-value of approximately 103.98 and a p-value close to 2.576573e-24. This demonstrates that the initial math scores have a strong predictive power for the change in math scores, with higher initial scores associated with greater improvements.

Further analysis

The significant findings from the ANCOVAs for both reading and math score improvements suggest a link between income group and academic progress, warranting a deeper dive into the dynamics of this relationship. To further explore these questions, a multivariate analysis could be beneficial, considering both reading and math scores together to understand the overall academic performance. This could illuminate whether income influences general academic improvement or if the effect is subject-specific.

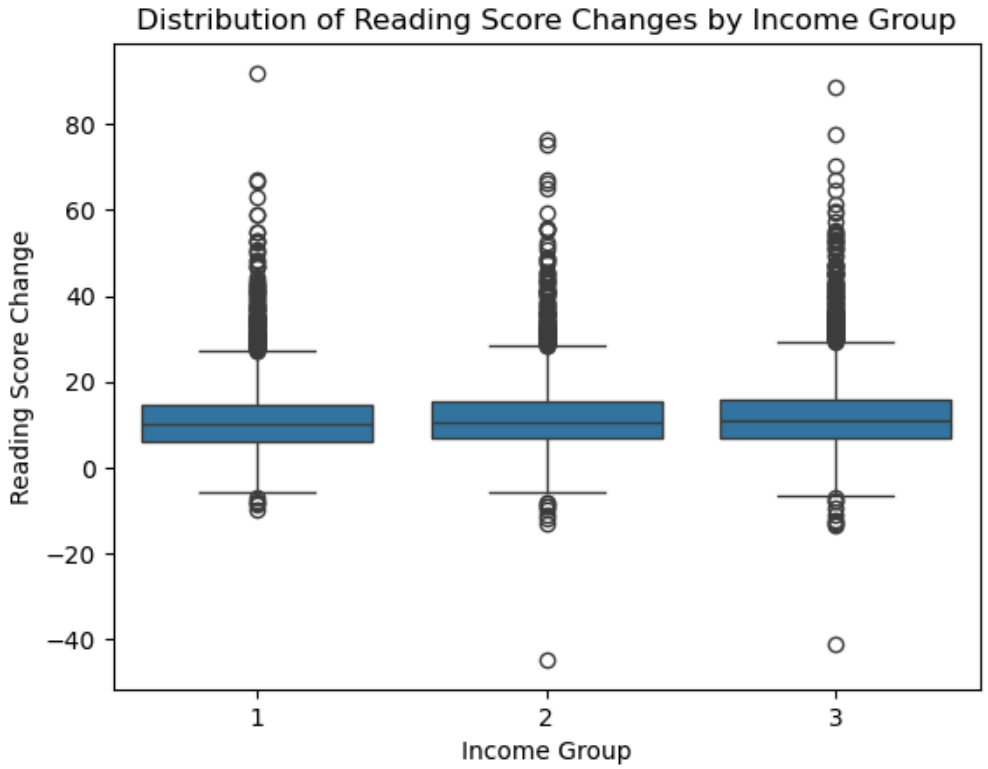
Subgroup analysis could also be insightful, particularly examining within-group differences to understand the variability in score improvements, especially for groups with wide confidence intervals in the interaction plots. Moreover, a longitudinal analysis extending beyond the kindergarten year could track the persistence of these income-related disparities over time.

Additionally, qualitative data on home environment, access to resources, and educational support could provide a richer context for the quantitative findings. Investigating these factors might reveal underlying mechanisms driving the observed income-related differences in score improvements.

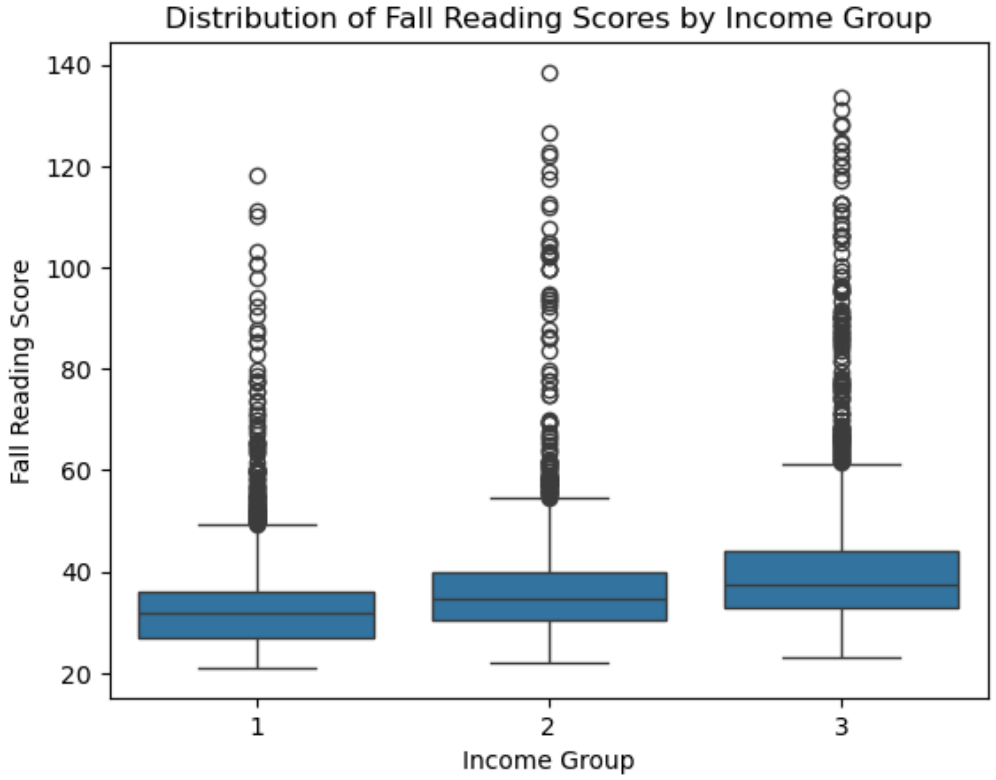
Lastly, considering the violations of ANCOVA assumptions, alternative methods like robust regression or non-parametric approaches may be needed for a more accurate analysis. These methods can handle the issues of non-normality and heteroscedasticity better, offering a more reliable understanding of the data.

Appendix:

Appendix one:



Appendix two:



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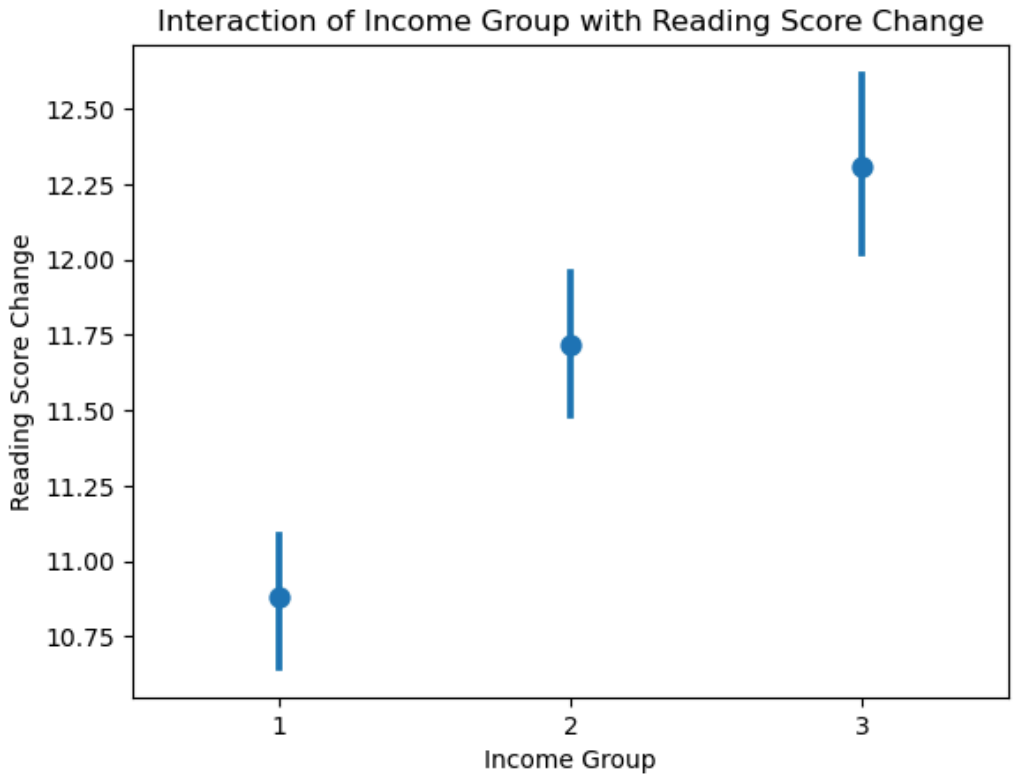
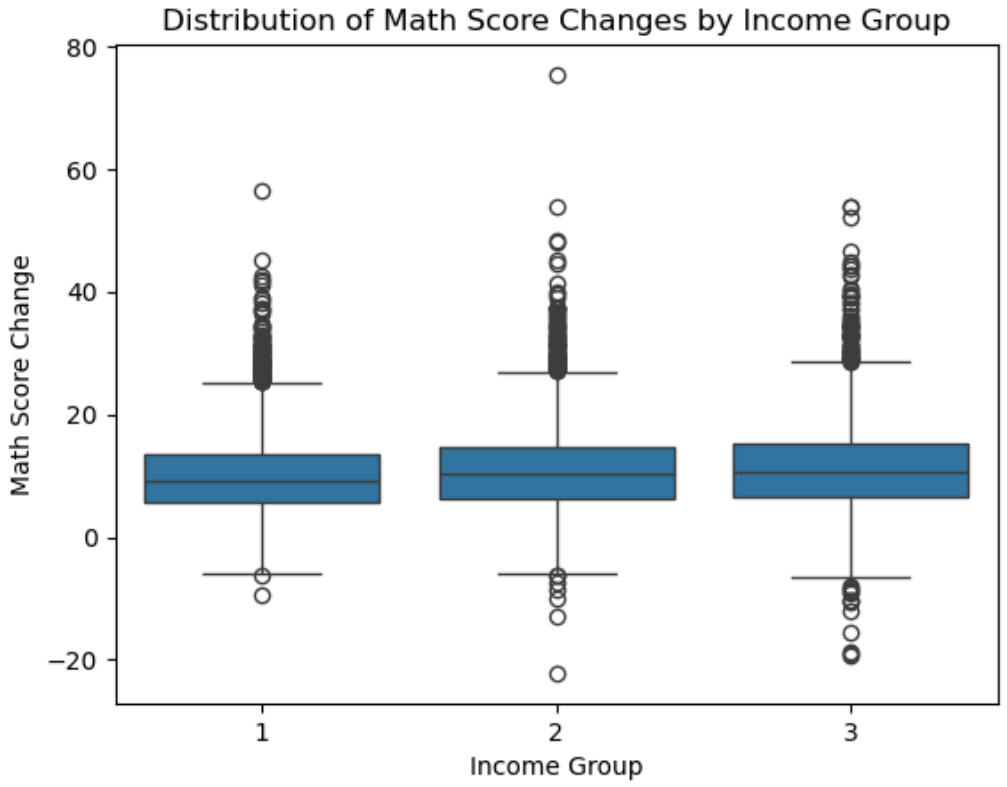
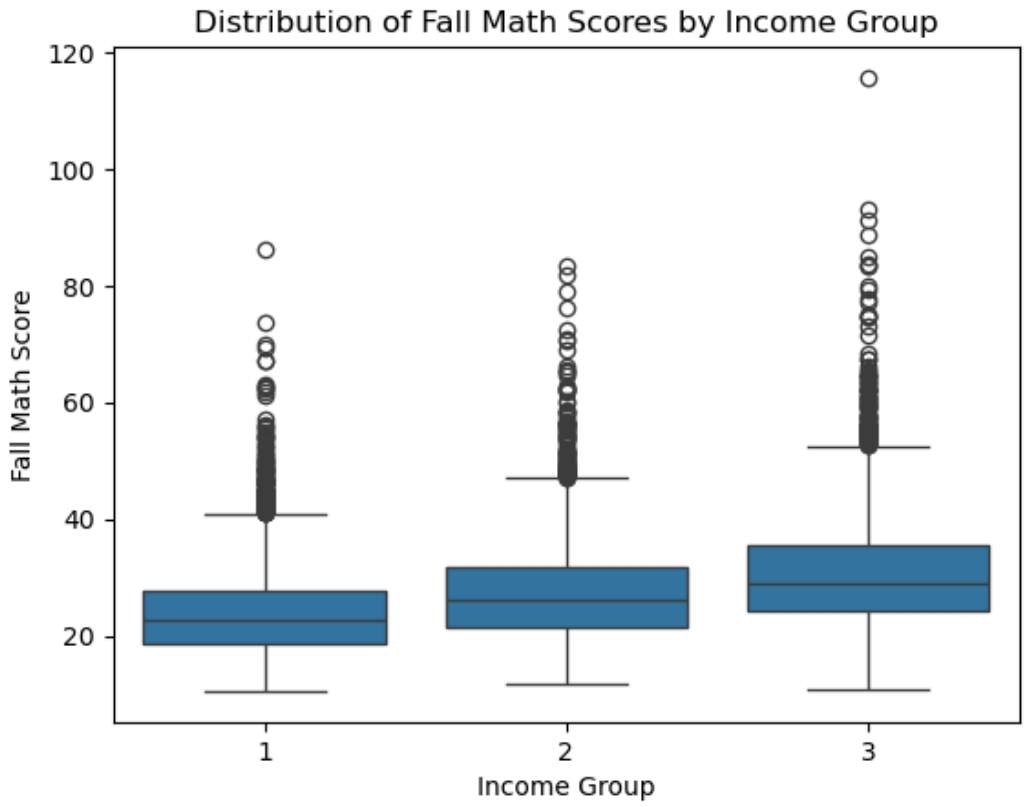


Appendix four:



Appendix five:


Distribution of Fall Math Scores by Income Group
Fall Math Score
120
100
80
60
40
20
1
2
3
Income Group