DOES INCOME AFFECT READING & MATH SCORES WHEN CONTROLLING FOR GENERAL KNOWLEDGE?

INTRODUCTION

According to Naveed et al. "nations with higher educational systems and general knowledge may experience beneficial effects for their economic development and technological advancement."[1] The authors further note that previous studies have provided confirmatory evidence that suggests a "positive relationship … between education and income growth."[2] Others such as Gordon B. Dahl and Lance Lochner found that "that a $1,000 increase in income raises combined math and reading test scores by 6% of a standard deviation in the short-run."[3] Given, the existing research the current dataset presents some interesting possibilities for analysis. If we can show that there are statistically significant differences in the means of math and reading scores by income group when controlling for general knowledge, we can contribute an additional study to the literature.

RESEARCH QUESTIONS

1. Are the math scores from fall to spring impacted by the level of income controlling for the impact of general knowledge?
2. Are the reading scores from fall to spring impacted by the level of income controlling for the impact of general knowledge?

DATA CLEANING AND DATA WRANGLING

The dataset is generally clean and requires few modifications. The dataset consists of 9 **columns** and 11,933 **rows** in a comma-separated values file. There are no missing values in the data. The data has no unique identifiers. We made some minor changes by renaming the columns for the sake of stylistic consistency. Additionally, we converted all columns with continuous variables to the number format.

FEATURE ENGINEERING

We added a few additional columns that were created purely from existing data. These columns enable us to visualize the data easily and make the statistical analysis easier. These new columns have been documented below.

***TABLE 1*: DATA COLUMNS AND DESCRIPTIONS**

COLUMN NAME IN ORIGINAL DATA	COLUMN NAME IN DATA FRAME	DESCRIPTION
fallreadingscore	Fall_Reading_Score	Continuous variable indicating the Fall 1998 reading score of kindergarten students.
fallmathscore	Fall_Math_Score	Continuous variable indicating the Fall 1998 math score of kindergarten students.
fallgeneralknowledgescore	Fall_General_Knowledge_Score	Continuous variable indicating the Fall 1998 general knowledge score of kindergarten students.
springreadingscore	Spring_Reading_Score	Continuous variable indicating the Spring 1998 reading score of kindergarten students.
springmathscore	Spring_Math_Score	Continuous variable indicating the Spring 1998 math score of kindergarten students.
springgeneralknowledgescore	Spring_General_Knowledge_Score	Continuous variable indicating the Spring 1998 general knowledge score of kindergarten students.
totalhouseholdincome	Total_Household_Income	Continuous variable indicating the total household income of kindergarten students.
incomeinthousands	Total_Household_Income_in_Thousands	Continuous variable indicating the total household income in thousands only.
incomegroup	Income_Group	Categorical variable indicating the income group. There are three income groups in the data. **Group 1**: 1.0 – 39,800.0 **| Group 2**: 40,000.0 – 69,700.0 **| Group 3**: 70,000.0 – 150,000.0
Does not exist in original data	Income_Group_in_Numbers	Maps group number form **Income_Group** column in the original data to the income ranges.
Does not exist in original data	Change_in_General_Knowledge_Score	New continuous variable. This is the change in general knowledge score from Fall 1998 to Spring 1998. It is a created by subtracting the score for general knowledge in the Fall from the score for general knowledge in the Spring.
Does not exist in original data	Change_in_Math_Score	New continuous variable. This is the change in math score from Fall 1998 to Spring 1998. It is a created by subtracting the score for math in the Fall from the score for math in the Spring.
Does not exist in original data	Change_in_Reading _Score	New continuous variable. This is the change in reading score from Fall 1998 to Spring 1998. It is a created by subtracting the score for reading in the Fall from the score for reading in the Spring.

***TABLE 2*: DESCRIPTIVE STATISTICS OF CATEGORICAL VARIABLE**

INCOME_GROUP	INCOME_GROUP_IN_NUMBERS	FREQUENCY	PROPORTIONAL FREQUENCY	PERCENTAGE
1	1.0-39800	4729	0.396296	39.6296
2	40000-69700	3726	0.312243	31.22434
3	70000-150000	3478	0.291461	29.14607

[1] Naveed, Amjad, Aziza Zhuparova, Muhammad Ashfaq, and Abdul Rauf. "The effect of average scores in reading, mathematics and science on innovation and income: A quantitative analysis for a group of countries." *Heliyon* 9, no. 9 (2023), pg. 1.
[2] Ibid., 2.
[3] Dahl, Gordon B., and Lance Lochner. "The impact of family income on child achievement." (2005), pp. 2, 13.

Descriptive Statistics and Exploratory Data Analysis

Table 3: Descriptive Statistics of Continuous Variables

Column	Count	Mean	Std	Min	25%	50%	75%	Max
Fall_Reading_Score	11933	35.95422	10.47313	21.01	29.34	34.06	39.89	138.51
Fall_Math_Score	11933	27.12824	9.120505	10.51	20.68	25.68	31.59	115.65
Fall_General_Knowledge_Score	11933	23.07369	7.396978	6.985	17.385	22.954	28.305	47.691
Spring_Reading_Score	11933	47.51118	14.32711	22.35	38.95	45.32	51.77	156.85
Spring_Math_Score	11933	37.79946	12.02775	11.9	29.27	36.41	44.22	113.8
Spring_General_Knowledge_Score	11933	28.23558	7.577457	7.858	22.802	28.583	33.782	48.345
Total_Household_Income	11933	54317.2	36639.06	1	27000	47000	72000	150000
Total_Household_Income_in_Thousands	11933	54.3172	36.63906	0.001	27	47	72	150
Change_in_General_Knowledge_Score	11933	5.16189	4.054906	-14.183	2.472	5.047	7.781	27.785
Change_in_Reading_Score	11933	11.55696	8.085004	-44.76	6.47	10.4	15.15	91.94
Change_in_Math_Score	11933	10.67122	6.855263	-22.16	6.01	9.86	14.33	75.35

We can see from Table 3 above that means of General Knowledge, Math, and Reading scores increase from Fall to Spring. Furthermore, we can see from Table 2 that there is a wide range for household income. All the way from $1 to $150,000. This reveals that the data is collected from sample with great socioeconomic disparity. Additionally, we can also see that the means of the scores for the change in (or delta in) General Knowledge, Math, and Reading are all positive indicating that there is an overall improvement for these scores. Interestingly, the delta in General Knowledge is smaller than the delta in Reading and Math indicating a relatively small increase in General Knowledge from Fall to Spring. Let us begin our analysis by drawing some boxplots. This will give us a good idea of variation in median values, and the spread of the data.

Figure 1: Boxplot of General Knowledge, Math & Reading Scores by Income Group Across Sessions

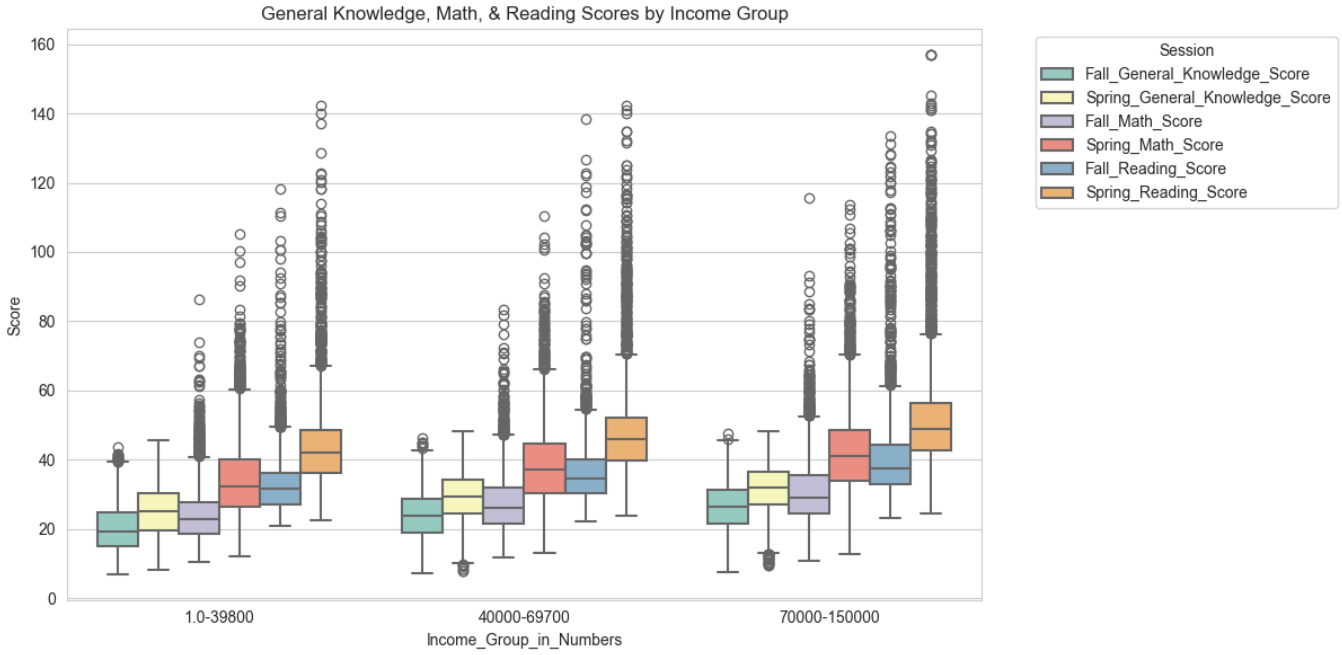

TABLE 4: IQR, MEAN, & MEDIAN OF GENERAL KNOWLEDGE, MATH & READING SCORES

INCOME	INCOME GROUP	TYPE	IQR	MEAN	MEDIAN
1.0-39,800	1	Fall_General_Knowledge_Score	9.803	19.94768	19.298
		Fall_Math_Score	8.96	23.9245	22.79
		Fall_Reading_Score	8.95	32.7868	31.66
		Spring_General_Knowledge_Score	10.447	25.06949	25.065
		Spring_Math_Score	13.69	33.88305	32.27
		Spring_Reading_Score	12.43	43.66508	41.97
40,000-69,700	2	Fall_General_Knowledge_Score	9.78275	23.88788	23.869
		Fall_Math_Score	10.26	27.56847	26.14
		Fall_Reading_Score	9.7275	36.29252	34.53
		Spring_General_Knowledge_Score	9.623	29.1436	29.476
		Spring_Math_Score	14.305	38.46469	37.035
		Spring_Reading_Score	12.295	48.00945	46.065
70,000-150,000	3	Fall_General_Knowledge_Score	9.7605	26.45185	26.5055
		Fall_Math_Score	11.295	31.01272	29.085
		Fall_Reading_Score	11.45	39.89849	37.575
		Spring_General_Knowledge_Score	9.51325	31.56772	32.082
		Spring_Math_Score	14.615	42.4119	41.03
		Spring_Reading_Score	13.4575	52.20688	48.74

Both Figure 1 and Table 4 show that as income increases the median scores for all three types if scores (General Knowledge, Math, & Reading) increase as well. There is a greater spread for General Knowledge as opposed to scores for Math and Reading. Interestingly, there are quite a number of outliers for Math and Reading scores as opposed to scores for General Knowledge. The Spring IQR for Math and Readings scores for the highest group is larger than the IQR for the lowest income group. The fact that the mean scores for the highest income groups are higher than those for the lower income groups suggests that there may be some relationship between income and academic achievement. To investigate this possibility, we will outline some hypotheses below (*TABLE 5*) and run a one-way **ANCOVA** to determine if this relationship holds.

HYPOTHESES

TABLE 5: HYPOTHESES (USING A SIGNIFICANCE VALUE OF 0.05)

HYPOTHESES FOR RESEARCH QUESTION 1		
H_{null}	:	Controlling for the impact of general knowledge the level of income does not significantly impact the change in math scores from Fall to Spring. The average change in math scores across income groups is equal.
$H_{alternative}$	:	Controlling for the impact of general knowledge the level of income significantly impacts the change in math scores from Fall to Spring. The average change in math scores across two or more income groups is not equal.
HYPOTHESES FOR RESEARCH QUESTION 2		
H_{null}	:	Controlling for the impact of general knowledge the level of income does not significantly impact the change in reading scores from Fall to Spring. The average change in reading scores across income groups is equal.
$H_{alternative}$	:	Controlling for the impact of general knowledge the level of income significantly impacts the change in reading scores from Fall to Spring. The average change in reading scores across two or more income groups is not equal.

CHECKING ASSUMPTIONS FOR ONE-WAY ANCOVAS

Let's evaluate the data further by constructing a scatterplot and a line graph to do a linearity check. From the scatterplot we can see that there is no clear distinction in math scores between income groups. This shows that our data may potentially satisfy the **ANCOVA** assumption that variances be homogenous. We will be further able to confirm this once we have the results of our Levene test. Furthermore, we can see from the line graph below that as there is a change in general knowledge scores there is a positive change in math scores. The lines clearly show that there are no vast differences in the relationship between change in general knowledge and change in math scores across income groups.

***FIGURE 2*: LINEARITY AND HOMOGENEITY OF CHANGE_IN_MATH_SCORE BY GENERAL_KNOWLEDGE_SCORE ACROSS INCOME_GROUP_IN_NUMBERS**

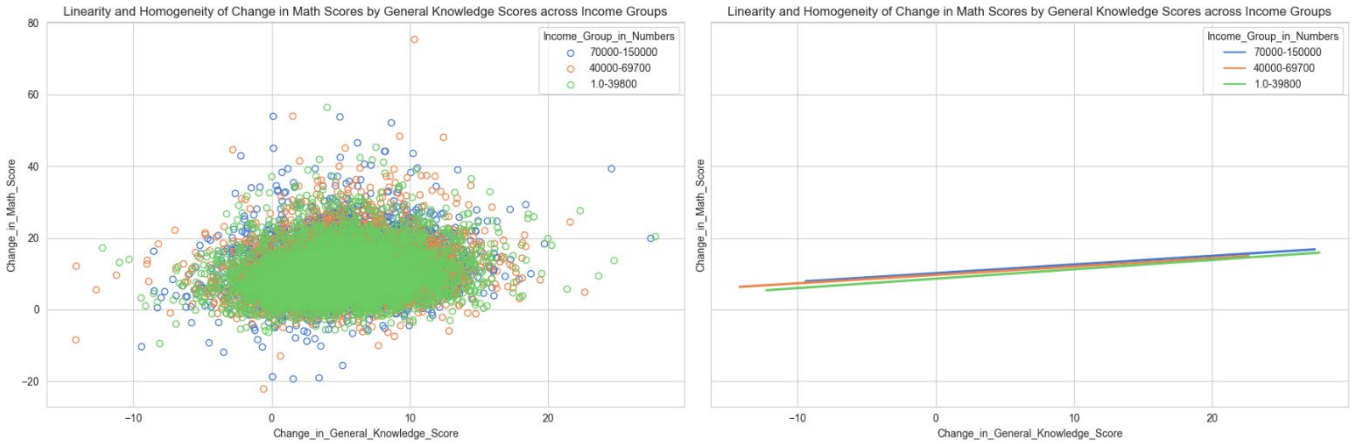

***FIGURE 3*: LINEARITY AND HOMOGENEITY OF CHANGE_IN_READING_SCORE BY GENERAL_KNOWLEDGE_SCORE ACROSS INCOME_GROUP_IN_NUMBERS**

With the above in mind let's conduct our Shapiro-Wilk and Levene tests to see what insights they can provide.

***TABLE 6*: SHAPIRO-WILK TEST FOR NORMALITY & LEVENE TEST FOR EQUALITY OF VARIANCES**

SKILL	TEST	STATISTIC	P-VALUE	EXACT P-VALUE
Math	Shapiro-Wilk	0.999371	**p** < 0.001	2.511487×10^{-4}
	Levene	22.215180	**p** < 0.001	2.344180×10^{-10}
Reading	Shapiro-Wilk	0.999371	**p** < 0.001	2.511487×10^{-4}
	Levene	19.728010	**p** < 0.001	2.794930×10^{-9}

The results of our Shapiro-Wilk and Levene test (see *TABLE 6* above) both have ***p-value < 0.001*** which means that our normality and equality of variances assumptions for **ANCOVA** are both violated. The Shapiro-Wilk test confirms that the change in math scores and the change in reading scores do not follow a normal distribution. Furthermore, the Levene test confirms that the variance in both change in math and change in reading score differ slightly by income group. We also confirmed by plotting the histograms and boxplots for the original data that both math and reading scores in the fall and spring are not normally distributed. This lends further credence to our results with respect to change in math and

change in reading scores. The fundamental assumptions for running an **ANCOVA** are violated but given that this is an academic exercise we will continue running the **ANCOVA** and analyze the results. As mentioned previously, even though our data fails both the assumptions for the **ANCOVA** test we can still run the test, but we must cautious of how we interpret the results and how much credence we give them.

RESULTS OF THE ONE-WAY ANCOVAS

***TABLE 7*: ONE-WAY ANCOVA TABLE (MATH)**

	SUM_SQ	DEGREES OF FREEDOM	F-STATISTIC	P-VALUE	PR(>F)
INCOME_GROUP	4388.326834	2.0	48.089207	**p** < 0.001	1.580759×10^{-21}
CHANGE_IN_GENERAL_KNOWLEDGE_SCORE	12022.654936	1.0	263.499036	**p** < 0.001	1.257053×10^{-58}
RESIDUAL	544283.777802	11929.9	NaN	NaN	NaN

***TABLE 8*: ONE-WAY ANCOVA TABLE (READING)**

INCOME_GROUP	4209.528531	2.0	32.835548	**p** < 0.001	6.009132×10^{-15}
CHANGE_IN_GENERAL_KNOWLEDGE_SCORE	11074.220434	1.0	172.764288	**p** < 0.001	3.448471×10^{-39}
RESIDUAL	764650.940643	11929.0	NaN	NaN	NaN

The results of our **ANCOVA**, tabulated in ***TABLE 7* & *8***, show that the difference in math and reading scores across income groups when controlling for general knowledge scores is statistically significant as the ***p-value < 0.001***. Additionally, the covariate ***Change_in_General_Knowledge_Score*** also has a ***p-value < 0.001***, indicating that math and reading scores are significantly impacted by the ***Change_in_General_Knowledge_Score***. Based on this alone we can reject the null hypothesis and accept the alternative hypothesis. Having said that we must be cautious about a few things. First, the results of our OLS Regression reveals that our model only explains ***less than 3%*** of the variance in **Change_in_Math_Score** and ***exactly 2%*** of the variation in **Change_in_Reading_Sore**. Furthermore, the differences in reading and math scores between income groups increases as we go from a lower income level to a higher income level, but the increase is not drastic. Given that our assumptions for the **ANCOVA** were not met I would advise caution in putting too much stock in these results. Let us run some post-hoc tests to see how well both of our models fare. The ***QQ plot*** for math scores in ***FIGURE 4*** shows that the normality assumption for the **ANCOVA** test appears to be satisfied. There are a few outliers, but generally our data follows the model quite well. The tails have some deviation but especially at the positive end of the scale. The ***histogram*** of the residuals for math scores in ***FIGURE 5*** also demonstrates that the residuals are very close to normally distributed. Our results for readings where very similar and can be viewed in the ***Jupyter*** notebook. Let's conduct a ***Tukey HSD*** test to see if our we can indeed reject the null hypothesis.

***FIGURE 4*: QQ PLOT OF ANCOVA MODEL RESIDUALS**

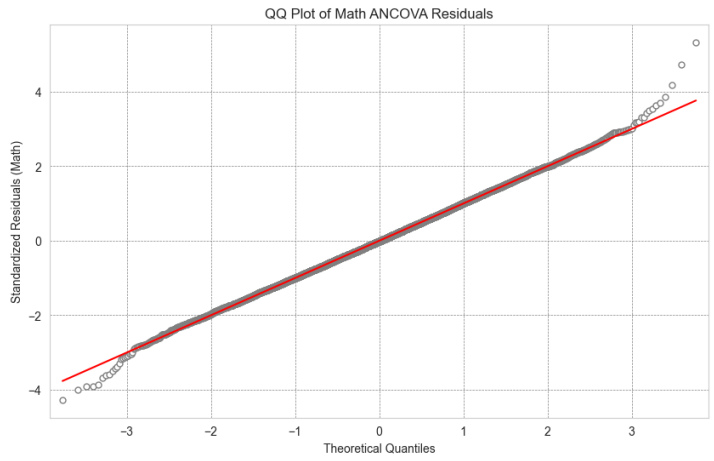

***FIGURE 5*: HISTOGRAM OF ANCOVA MODEL RESIDUALS**

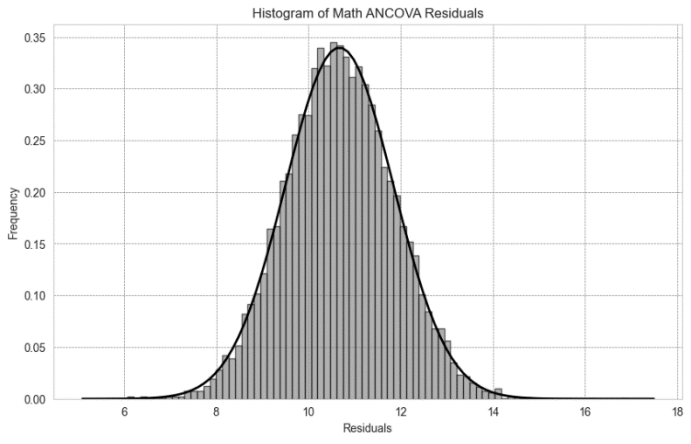

***TABLE 9*: RESULTS OF TUKEY'S HSD (MATH)**

Multiple Comparison of Means, FWER=0.05						
Group 1	Group 2	Mean Difference	Adjusted p-values	Lower	Upper	Reject
1.0-39800 (1)	40000-69700 (2)	-0.0	1.0	-0.3468	0.3468	False
1.0-39800 (1)	70000-150000 (3)	-0.0	1.0	-0.3537	0.3537	False
40000-69700 (2)	70000-150000 (3)	0.0	1.0	-0.3733	0.3733	False

Table 10: Results of Tukey's HSD (Reading)

1.0-39800 (1)	40000-69700 (2)	-0.0	1.0	-0.4111	0.4111	False
1.0-39800 (1)	70000-150000 (3)	-0.0	1.0	-0.4192	0.4192	False
40000-69700 (2)	70000-150000 (3)	0.0	1.0	-0.4425	0.4425	False

The results of the Tukey HSD in *Table 9* **&** *10* above reveal that there is no statistically significant difference in the change in math scores between income groups. The results from our **Ancova** suggested that we could reject the null hypothesis. However, at the outset we failed to satisfy the assumptions for the **Ancova**. Furthermore, the results of the Tukey HSD reveal that the mean differences in reading and math scores between income groups are 0. Perhaps our one-way **Ancova** does not have enough power to detect differences between income groups given that our sample is quite large.

Analysis

Our results reveal that at during the kindergarten years a students income level has a limited impact in improving their reading and math scores. Our analysis led us to accept both null hypotheses we formulated in *Table 5*. While there is some research that shows that a student's income group impacts reading and writing scores as students get older and climb through the academic system. According to the research conducted by Tyler Watts et al. "math and reading scores are strong predictors of economic attainment throughout participants' careers."[4] If countries want a more educated and competitive workforce than perhaps investing in higher education is the better approach. While there is no doubt that the kindergarten years are an important time in a child's development. Our results suggest that there is little benefit to equalizing outcomes at the kindergarten stage because even children in a higher income group who are putatively afforded the benefits of wealth experience little to not benefit in math and reading scores.

Alternatively, it is also possible that academic achievement is not top of mind for parents during the kindergarten stage and therefore the level of income does not come into play until later stages in the education process. Finally, the fact that kindergarten education is meant to develop a foundation for more complicated ideas down the road may account for the lack of differences in reading and math scores across income groups. The ideas are too simple and the stakes to low for there to be a statistically significant impact on reading and math scores across income groups.

Issues

While our results are interesting there is some cause for concern. First and foremost, the assumptions for conducting our one-way **Ancovas** were not satisfied. One way to get around this is to utilize techniques that do not require the normality, linearity, and homogeneity assumptions are satisfied. Furthermore, we have issues with respect to the data. Our data is limited two a kindergarten class across two semesters in a single city. To draw any weighty conclusions from our analysis we would need to analyze this across multiple years and perhaps even across different cities. If our results constantly match our findings in this report, then our case for accepting the null hypothesis would be even stronger. Finally, as mentioned previously the real research insight from this might be that the performance of students in math and reading does at the kindergarten level does not show disparity with respect to income. However, we know that this is untrue at the higher levels as explained by Naveed et al. and by Gordon B. Dahl and Lance Lochner. One way to rationalize these findings is to investigate parents in different income groups to see how they perceive kindergarten academic performance. If parents across income groups give little or no credence to academic performance for their kindergarten aged children, then an even stronger case could be made for the null hypothesis.

[4] Watts, Tyler W. "Academic achievement and economic attainment: Reexamining associations between test scores and long-run earnings." *Aera Open* 6, no. 2 (2020): 2332858420928985, pg. 1.