1. Introduction

1.0 SES and Learning Gains

SES is an umbrella term to describe socioeconomic status. It encompasses a person's or family's social and economic position relative to others in society. It's a complex concept that considers several factors, but often uses income level as a proxy variable.

As a commonly used demographic predictor variable, SES is a well-documented predictor of educational achievement (Duncan and Magnuson, 2003), and by extension, scores on measures of intelligence. Studies reveal a consistent association between higher SES and stronger academic outcomes. The 'achievement gap' and SES relationship is realized to emerge from multiple psychosocial contexts like disparities in access to resources, parental involvement, and access to quality schooling; differences may emerge as early as kindergarten.

1.1 Theories of Intelligence

The quantification of intelligence has been marked by divergent theoretical perspectives, with two primary early lines of thought. One camp proposes intelligence as a uniplex construct, following Charles Spearman's seminal work on the 'g facto' (Spearman, 1927). Spearman suggested that intelligence is a single measurable construct, representing an individual's general mental ability. Hence, we would expect general knowledge scores to correlate highly with other measures of ability.

In contrast, theories of multiple intelligence challenge the notion of a unitary measure. Multiplex approaches such as L. L. Thurstone's theory of primary mental abilities (Thurstone, 1938) and Howard Gardner's theory of multiple intelligences (Gardner, 1983) propose that intelligence comprises a spectrum of distinct abilities, rather than a singular entity. Accordingly, individuals may excel in certain intelligence domains while performing at average or below-average levels in others. **Given the different theoretical frameworks of intelligence, we could anticipate varied relationships between variables measuring general knowledge and other abilities.**

1.2 Current Study

We will conduct exploratory data analysis (EDA) and utilize one-way analysis of covariance (ANCOVA) to examine potential disparities across income groups in our dataset. By analyzing the mean differences in learning effects, we aim to discern any disparities in educational progress between income groups (Research Question 1). Moreover, we employ another ANCOVA model to assess whether the change in test scores is significantly different across income groups after controlling for changes in general knowledge scores, motivated by the uniplex theory that general intelligence may correlate with sub-measures of ability (Research Question 2).

Research Questions: Our goal is to assess differences in educational achievement across income groups, a proxy for socioeconomic status (SES).

RQ1: Do math and reading scores after the first year of kindergarten differ **across income groups**, after **controlling for baseline** math and reading scores (i.e. incoming ability)?

- *Alternate Hypothesis* (H1): There is a significant difference in spring test scores across income groups after controlling for baseline ability (fall test scores). Specifically, at least one income group has a mean spring test score that differs from the others.
- *Null Hypothesis* (H0): There is no significant difference in spring test scores across income groups after controlling for baseline ability (fall test scores)

RQ2: Do changes in math/reading scores differ **across income groups**, after **controlling for changes in general knowledge?**

Here, we consider both general and domain-specific theoretical frameworks on intelligence as a construct.

- *Null hypothesis*: adjusted means are equal
  H0: $\mu 1=\mu 2=\ldots=\mu p$
- *Alternative hypothesis*: At least one income group differs after controlling for changes in general intelligence i.e. adjusted means are not equal

In the models for math and reading ability, the income group serves as the independent variable (treatment), change (Δ) in respective test scores is the dependent variable (outcome), and change (Δ) in general knowledge scores acts as the covariate.

2. Data

Our dataset encompasses assessments of three educational scores for kindergarten children– general knowledge, reading scores, and mathematical scores– taken in the fall of 1988 (baseline score) and spring of 1999. There are 11933 instances in this data subset.

2.1 Data Preprocessing

To assess the changes in educational achievement, we calculated delta scores for each student. Delta scores represent the change in performance from the pre-test (fall) to the post-test (spring) in reading, math, and general knowledge. Here, a positive value indicates test improvement.

$$\Delta = spring\ score\ (posttest) - fall\ score\ (prettest)$$

Test Assumptions

✓ Assumption of Linearity and Homogeneity of Regression Slopes:

Visual inspection of scatterplots revealed a consistent linear pattern between pre and post-test scores within each test type. Furthermore, Pearson correlation coefficients were computed for each pair of pre and post-test scores; all correlation coefficients fell within the range $0.79 < r < 0.84$, indicating a robust positive correlation for each test type. Hence, we meet assumptions for the use of the one-way ANCOVA- the relationship between the dependent variable (spring scores) and the covariate (fall scores) is linear, with approximately parallel regression slopes.

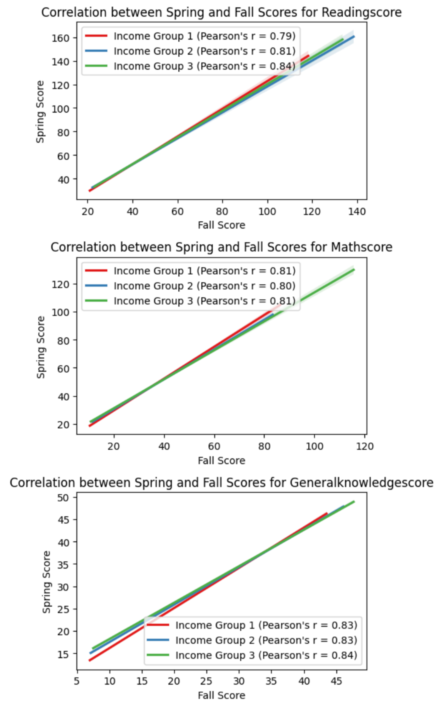

Fig 1. Regression Lines between Dependent Variable and Covariate

✓ **Normality:** the distribution of scores is approximately normal.

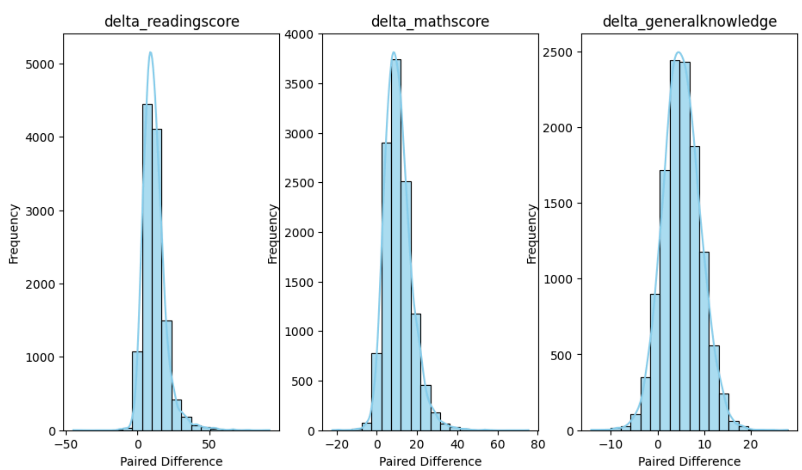

Fig 2. Distribution of Paired Differences

✓ **Independence:** observations are independent of one another.

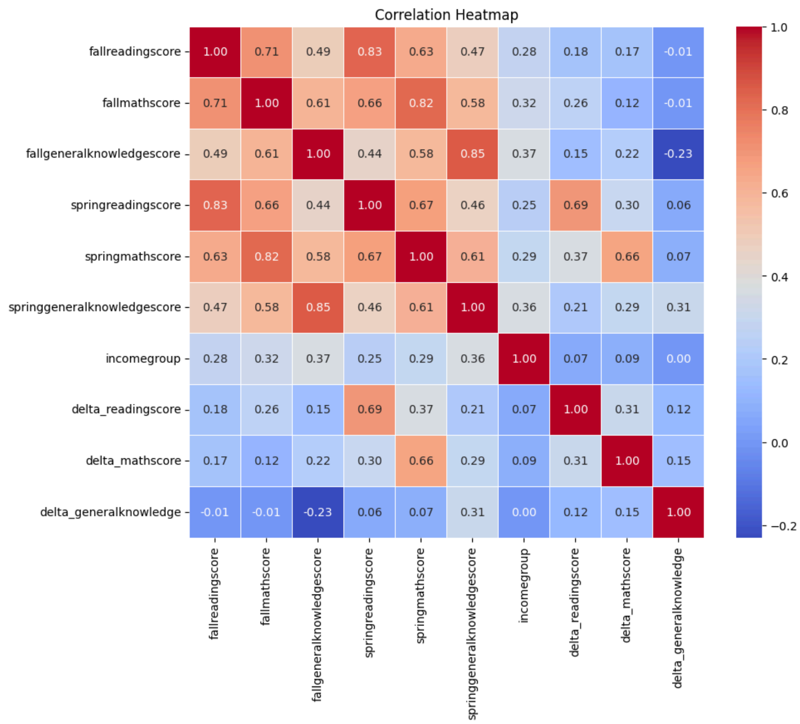

Fig 3. Intervariable Correlations

3. Results

RQ1: One Way ANCOVA Controlling for Baseline Differences in Ability

In the models for math and reading ability, the income group serves as the independent variable (treatment), spring scores serve as the dependent variable (outcome), and fall test scores act as the covariate. By accounting for baseline ability, we can better understand the unique contribution of income brackets to the variance in educational outcomes for reading and mathematics.

Model for Math Scores

Overall Model Fit:

- R-squared (0.681): This value indicates that 68.1% of the variance in spring math scores is explained by the factors included in the model (income group and fall math score).
- F-statistic (8469, Prob(F-statistic): 0.00): The high F-statistic with a very low p-value suggests the model is statistically significant.

The overall model fit, including the prediction line from the ordinary least squares regression, is shown. Importantly, model residuals show a normal distribution, validating the statistical assumptions of our model.

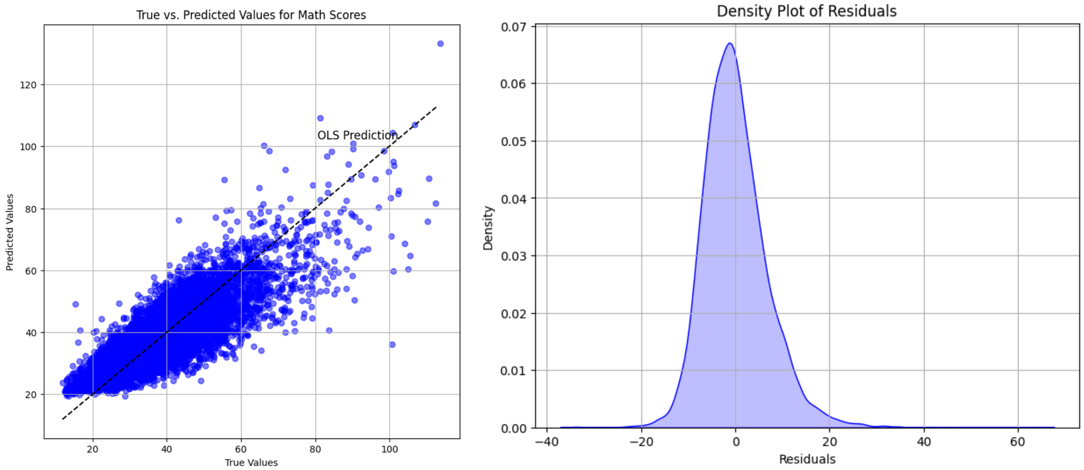

Figure 4. Plot of Residuls from OLS Model for Predicted Spring Math Scores

Significant Effects:

- **Income Group (C(incomegroup)):** The F-statistic (F = 8.24) and a *p-value* = 0.025 indicate a statistically significant effect of income group on spring math scores. This suggests that, after accounting for fall math scores (covariate), students from different income groups have statistically different average spring math scores.

Model for Reading Scores

Overall Model Fit:

- R-squared (0.692): This value indicates that 69.2% of the variance in spring reading scores is explained by the factors included in the model (income group and fall reading score). This suggests a slightly better fit compared to the math scores model (R-squared = 0.681).
- F-statistic (8929, Prob(F-statistic): p < .001): The high F-statistic with a very low p-value suggests the model is statistically significant.

Significant Effects:

- C(incomegroup)[T.2] (0.3751, p = 0.033): This coefficient shows the difference in spring reading scores for students in the second income group (T.2) compared to the baseline group (T.1), after controlling for fall reading scores. Students in T.2 score an average of 0.37 points higher on spring reading scores, though the significance is borderline.
- C(incomegroup)[T.3] (0.4898, p = .008): This coefficient indicates the difference for students in the third income group (T.3) compared to Income group 1. The positive coefficient and lower p-value imply a statistically significant difference. Students in T.3 score an average of 0.49 points higher on spring reading scores compared to the baseline group.
- Higher fall reading scores are positively associated with higher spring reading scores.

RQ2: One Way Ancova Controlling for General Knowledge

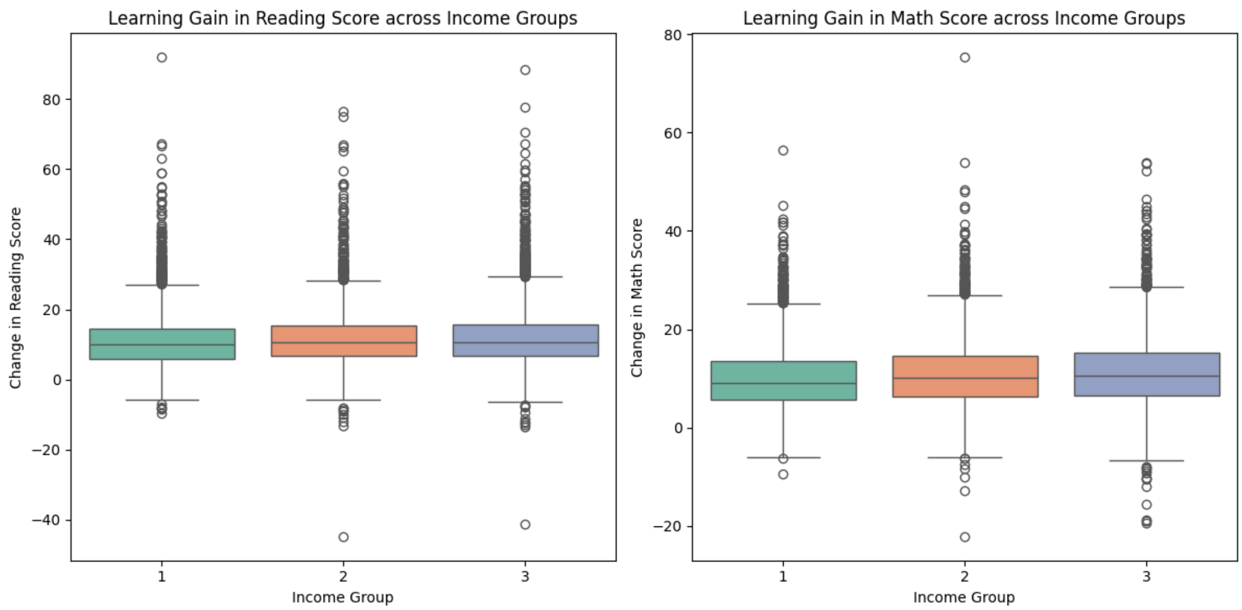

Fig 5. Distribution of Delta Scores

ANCOVA explains a portion of the within-group variability for each income group by attributing it to moderately correlated changes in general knowledge.

Math Model:

	df	sum_sq	mean_sq	F	PR(>F)
Income group	2.0	4433.58	2216.79	48.59	0.0
delta_generalknowledge	1.0	12022.65	12022.65	263.50	0.0
Residual	11929.0	544283.78	45.63	NaN	NaN

Reading Model:

	df	sum_sq	mean_sq	F	PR(>F)
Income group	2.0	4237.40	2118.70	33.05	0.0
delta_generalknowledge	1.0	11074.22	11074.22	172.76	0.0
Residual	11929.0	764650.94	64.10	NaN	NaN

Discussion

RQ1: Do math and reading scores after the first year of kindergarten differ across income groups, after **controlling for baseline scores** in math and reading (i.e. incoming ability)? **RQ2:** Do changes in math/reading scores differ across income groups, after **controlling for changes in general knowledge?**

Hypothesis Testing

- RQ1: an ANCOVA analysis to examine the effect of income group on spring results in reading and math scores while controlling for variance accounted for by initial scores in the fall.
- RQ2: an ANCOVA analysis to examine the effect of income group on the change in reading (or math scores) (delta_readingscore) while controlling for the change in general knowledge scores (delta_generalknowledge).

All models were significant, prompting us to reject the null hypothesis. We have statistical evidence to suggest that mean scores across categorical income groups are different.
I also noticed another relationship of interest– changes in scores for reading and math are strongly, and positively, associated with the post-test score. Students with greater scores in the spring test tended to also show greater improvement. Interestingly, this relationship does not hold as strongly for general knowledge. This finding could suggest a different rate of learning for general knowledge, or that general knowledge is measuring a different underlying construct than math and reading scores.

Limitations:

- Correlational, not Causal: Our study establishes a correlation between income group and academic achievement, but it doesn't prove causation. Other factors likely influence learning gains, and future research could explore these mediating variables.
- Proxy Variable: Income is a proxy for SES, and other socioeconomic factors not captured here might contribute to the observed relationships.

In studies of income as a proxy for SES, it is important to remember the relationships are merely correlational and provide no causal explanation. Instead, multiple contexts combine to influence children's learning gains.

Nonetheless, this analysis highlights the persistence of the achievement gap in early education, even after accounting for initial student abilities. Addressing this gap requires a multi-faceted approach that considers the complex interplay between socioeconomic background and various aspects of the learning environment.

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