

SECTION-E PROBABILITY

(i) No. of balls tossed = 100

No. of bins = 50

$X \rightarrow$ Random Variable representing the number of empty boxes

$$E(X) = ?$$

Let $X_i =$ a Random variable such that bin i is empty

$$Y = \sum_{i=1}^N X_i$$

$$E(X) = \sum_{i=1}^n E(X_i) = n E(X_1)$$

$$E(X_1) = \left(1 - \frac{1}{50}\right)^{100}$$

$$E(X) = 50 \left(1 - \frac{1}{50}\right)^{100} = 50 \left(\frac{49}{50}\right)^{100} = \frac{(49)^{100}}{50^{99}} = \underline{\underline{6.631}}$$

$$\frac{50 \times 49}{50}$$

(ii) per bit error rate over a binary communication channel = 10^{-10}

(ii)(a) let $X_i =$ is a Random variable such that i^{th} bit is erroneous.

$$Y = \sum X_i$$

$$E(Y) = \sum E(X_i)$$

$$E(X_i) = 1 \cdot (P(\text{Error})) + 0 \cdot (P(\text{No error}))$$

$$= P(\text{error}) = 10^{-10}$$

$$E(X) = (10^3)(10^{-10}) = 10^{-7}$$

\therefore Expected number of erroneous bits in a block of 1000 bits
 $= \underline{\underline{10^{-7}}}$ Ans

(i) (b) Let x_i be a random variable such that it is 1 if bit is erroneous.

$$Y = \sum x_i$$

$$E(Y) = 10^3 E(x_i) = 10^{-7}$$

$$P(Y \geq 10) \leq \frac{E(Y)}{10} \quad (\text{Using Markov Inequality})$$

$$P(Y \geq 10) \leq \frac{10^{-7}}{10} = \underline{10^{-8}}$$

\therefore Upper bound on the probability that a block of 100 bits has 10 or more erroneous bits is 10^{-8} Ans.

(ii) A card is drawn from a deck with replacement.

If card is a spade or a queen, Alena wins = \$4

If card is different than a spade or a queen, Alena loses = \$1

Total no. of cards in deck = 52

$$\text{Expected earnings per day (winnings)} = \left(4 \times \left(\frac{\text{no. of spade} + \text{no. of queen}}{52} \right) - 1 \times \left(\frac{\text{other cards}}{52} \right) \right)$$

$$= \left(4 \times \left(\frac{16}{52} \right) - 1 \times \left(\frac{36}{52} \right) \right)$$

$$= \frac{16}{13} - \frac{9}{13} = \frac{7}{13}$$

\therefore Alena's total expected winnings after 30 nights

$$= 30 \times \frac{7}{13}$$

$$= \underline{\underline{\$16.154}} \quad \underline{\underline{\text{Ans}}}$$