Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
A		1234567

Dragon Living Using Logistic Growth Model Under Aerodynamic Flying Condition

Summary

In this paper, we propose to model the dragon living and growing characterastics via using logistic growth model. Specifically, the maximum weight M_{max} should be determined to establish this model.

To obtain M_{max}, we propose to use the aerodynamic flying condition, which consists of the lift power and the forward power. The lift power is saperated into two parts. The first part is the flapping power, and the second part is the gliding power. The aerodynamic power is computed via the air density, the flapping frequency of the wings of the dragon, and the size of the wing. Additionally, the gliding power is depend on the air velocity.

Next, we use the Newton's second law to establish the model in which the power and the total weights of the dragon is connected to determine the acceleration of the dragon. In that case, we fix the reasonable acceleration and the maximum weights M_{max} can be ensured. Hence, the logistic growth model of the dragon is established, which is addressed by stardard variable separation.

To tackle the second questioin, we consider from three aspects where the dragon consumes the energy. First, the dragon has daily consume which sustain the basic needs. Secondly, the dragon is able to split fire, which consumes a lot energy. This process is simulated via organic chemical process. The third aspect is the growth of the cuticle, which consumes protein. This is under the consideration that the dragon can resist major trauma. Also, based on the *A Song of Ice and Fire*, it is considered that the food chain of dragon-sheep-grass is exists. With these considerations, a consumption Model which aims at minimizing the number of the required sheep is established. Since the constraints and the correlations are clear, this model can be easily addressed by integrating the total consume and computing the corresponding number of sheep.

To address the third qustion, we consider three kinds of weather and environment. The first one is the arid regions which contains low air humidity and high tempreature. The second one is the warm region which contains high air humidity and high tempreature. The third one is arcticr region which contains high air humidity and low tempreature. The impact of the environments on the growth of te dragon is deermined via the areoflying condition, where the tempreature and the air humidity effect the air density and the air velocity. This further effect the growth model of the dragon. Also, in different weathers, the sheep are growing via different speed. So there is impact on the food of the dragon. At last, we establish each growth model for each weather condition, and address these models like question one.

In total, the superiority of the proposed model contains three aspects. First, our logistic model for dragon is under the aeroflying condition, which reveals the essential property of the dragon, hence precise modeling is obtained. Second, we consider multiple kinds of consume so that the living model of the dragon is eeliable and accurate. Third, we formulate the living model of the dragon under different environments so that the flexibility is obtained.

Keywords: Logistic model, Aerodynamic flying condition, Growth model, Newton's second law

Contents

1 Introduction

1.1 Problem Background

Dragon, a kind of creature from mythology and legend, is common in all kinds of literature, art works and buildings. In the TV series *Game of Thrones* adapted from the science fiction *A Song of Ice and Fire*, the dragon, as a fictional magical species, plays an important role in it. In the novel, George R.R. Martin sets the image of dragons to be like bats, with their forepaws forming cortical wings, rather than having four legs and a pair of independent wings as shown in some illustrations of mythology. The dragon has a long neck and tail, and many spines grow along its back. At the time of hatching, the cubs are only as big as kittens, but they can keep growing. The adult dragon can grow up to swallow the whole mammoth in one bite.

1.2 Restatement of Problem

In *Game of Thrones*, three dragons are raised by the mother of the dragon. When the Dragon hatches, it is very small, about $10kg$. After a year, it's about 30 to $40kg$. Based on the number of physical objects available and their situation, it will continue to grow. Assuming that the growth of the Dragon conforms to the living law in reality, we need to solve the following problems:

- Make some additional assumptions about dragons. It is assumed that one of the main reference targets of dragon growth can truly reflect the growth status of the dragon, and this index has a reasonable relationship with the size of function, diet, changes or other animal related characteristics.
- In the process of establishing the model, we need to make clear the ecological impact and requirements, energy consumption, caloric intake requirements, living area and other issues of dragon. At the same time, other factors should be considered when considering the above problems.
- The migration of dragon is an important link in the survival of species, and with the migration of species, the resources will also change, so we need to consider the climate change conditions. We need to establish a corresponding analysis between the transfer of dragon and the resources needed for its growth, so as to determine whether there is an impact.
- Draft a two page letter to George Martin, the author of song of ice and fire, to give a more comprehensive description of the impact of species transfer on environment and resources.
- Outside the domain, there are some strange events, so the modeling process needs to describe the occurrence of random events properly.

1.3 Literature Review

Some data show that dragons have much higher intelligence than ordinary animals. They have a strong sense of dependence and trust on the people who raise them. They can be trained to be powerful combat mounts and understand the master's language commands [8]. People have always believed that there will be some mysterious magic connection between the dragon controller and his own dragon. The dragon controller's personal will and good and evil view will affect his giant dragon. The dragon will judge its enemies and friends according to the will of the knight. If the dragon master feeds the dragon before controlling it, it will become more tame.

As for the food intake of the dragon, all the food in the novel is meat. In *A Song of Ice and Fire*, Denise's dragon likes to eat lambs. Coincidentally, in Princess and queen, there is a wild dragon on Longshi island. Because it often steals sheep from nearby shepherds, it is called "sheep thief".
Dragons have been growing up all their lives [6], but it is not sure how long they can live or how big they are. The scholars believe that the size of dragons is limited by their growing environment. Generally speaking, the older the dragon, the larger the size.

The dragon's body emits heat and steam on a cold night [4]. They can spray hot flames. In addition, dragon scale is the most important means of protection for dragons. Although it is not completely immune to fire, it can still resist the vast majority of fire. They provide shelter for the fragile body, and the scales become thicker and thicker with age. Wings are also the dragon's weapon for self-defense.

1.4 Our work

We mainly conduct three things related to the living of the dragon to answer the given questions. First, we establish the logistic model [5] to simulate the growth process of the dragon. In that case, we employ the aerodynamic flying condition model [3] and the seconf Newton's law to establish the fly model. Based on the fly model, we can estimate the total weights of the dragon. Next, we combine the total weights and the logistic model to establish the growth process of the dragon. The logistic model is addressed by standard variable separation.

Secondly, we formulate the consume of the energy from multiple aspects. First, we consider the intrinsic consume of the dragon, which includes the daily consume and the fly consume. Secondly, we consider the fire the outputed from the dragon, which consumes a lot, since we formulate the fire generation as a process with organic chemical reaction. Specifically, the gaseous ether is mixed with a special catalyst to react with oxygen in the air to produce carbon dioxide and water, and the ignition point is reduced below the ambient temperature. And then it releases a lot of heat [9]. This process would consume a lot energy. Thirdly, we consider the consume of the cuticle, which is used to protect the dragon from large hurts. At last, we integrate these consumes and caculate the total requirements of the food and area.

Thirdly, we consider the weather situations and their influences to the living of the dragon. Specifically, the tempreature could effect the air density, which further change the fly condition and the expected total weights of the dragon. Also, the air humidity changes the air density and further effects the fly condition. Additionally, the food of the dragon (mostly sheep [2]) is expected to have different growth condition under differnt weather. Thus, it is reasonable to change the parameters in the logsitic model to simulate that. Further, the area that needs for the dragon is also changed. Three models here are established for arid region, warm region, and arctic region, respectively.

In summarize, our models are as follows:

- The logistic growth model of the dragon Eq. (2).
- The aerodynamic flying model of the dragon Eq. (7) and (9).
- The Newton's second law model Eq. (10).

- The logistic model of the sheep Eq. (24).
- The Consumption Model Eq. (25).
- The logistic growth model under different weathers Eq. (29), (30), and (31).

2 Preparation of the Models

2.1 Assumptions

Through the full analysis of the problem, in order to simplify our model, we make the following reasonable assumptions.

- The growth of dragon accords with the basic biological law. If the dragon lives in the real world, as a species on earth, its growth and maturity should be in line with the basic biological laws, just like other creatures.
- Dragon is a kind of thermostatic animal whose temperature is not affected by the environment.
- Dragon can fly, can bear huge trauma, will not die easily because of physical, chemical and biological attacks. According to myths and TV dramas, we can think that the dragon has the ability to fly, and its flight conforms to aerodynamics.
- At the same time, the dragon is overgrowing, and its mature shape exceeds that of all terrestrial creatures. No species can tolerate the dragon's attack.
- We assume that the birth length of a dragon is 30-40 cm. Due to the lack of scientific basis for the physiological information of dragon, we infer the above hypothesis from the ancient reptile larvae and TV series.
- Once a dragon enters an ecosystem, it immediately becomes a top predator. However, it will not cause catastrophic damage to the biological world.

2.2 Notations

The primary notations used in this paper are listed in Table 1.

3 The Models

3.1 Growth Model of Dragon

In order to clearly describe the physiological characteristics of the Dragon, we believe that the dragon lives in a warm temperate zone with a suitable climate, assuming that the temperature in this area is maintained at $25°C$, so that the dragon can get enough food from this area during its growth and maturity. In addition, from the morphological point of view, the size of organisms can not be determined to increase. Similarly, the dragon should be the same, which ensures that when the dragon grows to maturity, its final weight and length will be limited. At the same time, we define the length of the dragon as the straight distance from the shoulder to the hip.

Table 1: Notations

Symbol	Definition
t	Time
$W(t)$	The mass of dragon at time t
α	The growing rate
W_{max}	The maximum weight of the dragon
N_{sheep}	The number of the sheep
L_1	The flapping wing power
L_2	The gliding power
T	The forward power
ρ	Air density
$w(t)$	The angular velocity of the wing of dragon
l	The length of the wing
a	The acceleration
f	The flapping frequency
D_0	The number of sheep that dragon consumes per day

3.1.1 Logisitic Growth Model

Considering the birth time of the dragon, its weight $W(T)$ changes with time. In the process of its growth to maturity, we think it will always be healthy and will not die. Based on the above assumption, we show that the theoretical weight of mature dragon is $W_{max}(kg)$.

According to the principle of logistic delay growth [11], it is considered that the influence of weight growth rate should be reflected in the delay capacity between body weight restriction and environmental carrying capacity. At present, we are considering the most basic situation: there are no restrictions on growth, such as the environment and its maximum weight. In this case, the weight of the dragon should satisfy the following formulation

$$\frac{\partial W(t)}{\partial t} = \alpha W(t), \tag{1}$$

where α is the growth rate. However, due to the shape limitation of the dragon itself, it is expected to include the maximum weight delay factor $1 - \frac{W(t)}{W_{max}}$. Therefore, we modify the logistic model [11] to the following form

$$\frac{\partial W(t)}{\partial t} = \alpha W(t)(1 - \frac{W(t)}{W_{max}}), \tag{2}$$

which is our basic logistic growth model of the dragon. Here, the initial weight of the dragon is $W(0) = 30$. However, since wo do not know the maximum weifgts of the dragon W_{max}, it is expected to find the W_{max} to complete this model.

3.1.2 Aerodynamic Flying Condition

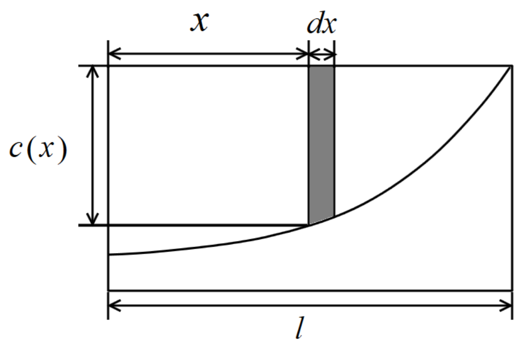
We use the aerodynamic flying condition [7] to determine the power that the dragon has for its fly. Then, it is expected to employ the Newton's second law to combine the fly power and the weights. The weights are then used as the maximum weight of the dragon that complete the growth model.

According to the principle of mechanics, the dragon's wings fan (vibrate) downward and backward to obtain upward lift and forward thrust. This basic action is completed by the skeleton of the dragon's wings moving back and forth in waves to pull the whole wing to make a circular motion. This kind of movement is an ideal flight mechanics principle, and the wing will show unnecessary power loss when it moves in any other direction. Thus, we simply consider two directions of the power, *i.e.*, the lift power and the forward power.

1) Lift power We first consider the lift power. The lift power is seperated into two parts. The first part is the flapping wing power, which is

$$L_1(t) = C_{L_1(t)}\rho w(t)^2 \int_0^l x^2 c(x)dx. \tag{3}$$

In the above expression, $L_1(t)$ denotes the flapping wing power at time t. $C_{L_1(t)}$ is the coefficient that corresponds to the flapping wing power. ρ denotes the air density, and $w(t)$ denotes the angular velocity of the wing at time t. Other notations can be found in Fig. 1, where the wing of the dragon is assumed as a rectangle with a corner being removed.



The wings of the Dragon

Figure 1

Caculating the power at each time is computaionally unfriendly. Hence, we emply the average power to replace the power at each time. Specifically, the average flapping wing power in a wing flapping cycle is

$$L_1 = f\rho \int_0^{\frac{1}{f}} C_{L_1(t)} w(t)^2 dt \int_0^l x^2 c(x)dx, \tag{4}$$

where f denotes the flapping frequency of the wings of the dragon. In our model, we have the following parameters setting: In that case, the first part of the lift power is established.

Table 2: The parameters setting.

l	$c(x)$	ρ	f	$w(t)$	U	C_L	C_R
$30m$	$-0.4x^2+8x\,m$	$1.171kg/m^3$	0.25	0.5π	$25m/s$	0.1	0.1

The second part of the lift power is the gliding power, which is

$$L_2(t) = \int_0^l \rho U^2 C_{R_2(x,t)} c(x)dx. \tag{5}$$

In the above expression, $L_2(t)$ denotes the gliding power at time t. $C_{R_1(t)}$ is the coefficient that corresponds to the gliding power. U denotes the inflow air velocity. Similar to the flapping wing power, the average gliding power is

$$L_2 = f\rho U^2 \int_0^{\frac{1}{f}} C_{R_2(x,t)} dt \int_0^l c(x)dx. \tag{6}$$

Hence, the average lift power in a wing flapping cycle is

$$L = L_1 + L_2. \tag{7}$$

2) Forward power In horizontal flight, the dragon's wings lean forward (the leading edge is low and the trailing edge is high), so the pressure difference caused by flapping wings will have a component force in the horizontal direction, which will push the dragon to fly horizontally. Thus, it is considered to compute the forward power, which provides the acceleration along with the horizontal direction. The forward power of the dragon at t is

$$T(t) = C_{T(t)}\rho w(t)^2 \int_0^l x^2 c(x)dx, \tag{8}$$

where $T(t)$ denotes the forward power at time t, $C_{T(t)}$ denotes the coefficient corresponds the forward power. Similar to the aforementioned analysis, we are expected to comput the average forward power, which is

$$T = \rho f C_T \int_0^{\frac{1}{f}} w(t)^2 dt \int_0^l x^2 c(x)dx. \tag{9}$$

When the dragon flys, its wings will have two forces, that are lift and forward. When the dragon descends, gliding causes the bird's lift to be at right angles to the direction of the air. So the upward force also includes the forward force, which can counteract the resistance. Thus, it is necessary to consider both the lift power and the forward power to formulate the flying condition of the dragon. Next, we will use the Newton's second law to determine the W_{max}.

3.1.3 Determination of M_{max}

Once we establish the logistic growth model and the aerodynamic flying condition, it is possible to obtain the maximum weights of the dragon with Newton's second law. Specifically, the acceleration of the dragon is directly proportional to the resultant force and inversely proportional to the mass of dragon (directly proportional to the reciprocal of the mass). The direction of acceleration is the same as that of the combined force. Hence, we have the following equation

$$L - M_{max}g = M_{max}a_y, \tag{10}$$

which describes the acceleration source in the vertical direction. Here, L is the lift power and M_{max} is the mass of the dragon. a_y denotes the acceleration along with the vertical direction. g denotes the acceleration of gravity.

Similarly, we can formulate the Newton's second law along with the horizontal direction, which is

$$T = M_{max}a_x. \tag{11}$$

Here, T is the forward power, and a_x denotes the acceleration along with the horizaontal direction. The maximum weight of the dragon is obtained by selecting the smaller W_{max} that obtained via (10) and (11). This is because the upper limit is constrained via these two equations. The weight of the dragon must be lower than the constraint so that a precise estimation of the growing process can be ensured.

3.1.4 Final Model

Based on the above sections, our final model for the growth of the dragon is

$$\begin{cases} \frac{\partial W(t)}{\partial t} = \alpha W(t)(1-\frac{W(t)}{W_{max}}), \\ L_1 = f\rho \int_0^{\frac{1}{f}} C_{L_1(t)} w(t)^2 dt \int_0^l x^2 c(x) dx, \\ L_2 = f\rho U^2 \int_0^{\frac{1}{f}} C_{R_2(x,t)} dt \int_0^l c(x) dx, \\ L = L_1 + L_2, \\ T = \rho f C_T \int_0^{\frac{1}{f}} w(t)^2 dt \int_0^l x^2 c(x) dx, \\ L - M_{max} g = M_{max} a_y, \\ T = M_{max} a_x, \\ W(0) = 30. \end{cases} \tag{12}$$

3.1.5 Model Solution

We first tackle the computation of M_{max}. In that case, it is expected to obtain the L and T first. In fact, L and T can be easily and computaionally obtained as the parameters are clear and the integral of elementary function is simple. Just note that the flapping frequency of the wing is correlated to the angular velocity, which is

$$f = \frac{w(t)}{2\theta_{max}}, \tag{13}$$

where θ_{max} denotes the maximum angle of the wing. Once the L and T are obtained, we have

$$M_{max} = \min\{\frac{L}{g+a_y}, \frac{T}{a_x}\}. \tag{14}$$

For the logistic model, it is clear that we can use the variable separation, which deduces

$$\begin{aligned} \alpha \partial t &= \frac{\partial W(t)}{\partial W(t)(1-\frac{W(t)}{W_{max}})} \\ &= \frac{\partial W(t)}{W(t)} + \frac{\partial W(t)}{W_{max} - W(t)}. \end{aligned} \tag{15}$$

Conducting the Integral on both sides, we have

$$\alpha t + \hat{c} = \ln W(t) - \ln(W_{max} - W(t)), \tag{16}$$

where $\hat{c}$ is a constant. Further, we can deduce

$$W(t) = \frac{W_{max}}{1+\hat{c}e^{-\alpha t}}. \tag{17}$$

Note that $W(0) = 30$, thus the final solution is

$$W(t) = \frac{W_{max}}{1+(\frac{W_{max}}{W(0)}-1)e^{-\alpha(t-t_0)}}. \tag{18}$$

Next, we consider the length of the dragon. Assuming that the length of single wing is $\frac{1}{3}$ of the body length, the peak length of single wing is 3 times of the length of single wing, the intersection point is $7.39m$ in length and $894.7kg$ in weight. The length of the dragon born in the TV series is about $0.4m$. In that case, we find out the intersection of the length and the weights as the wing length of the dragon. The sketch map is shown in Fig. 2.

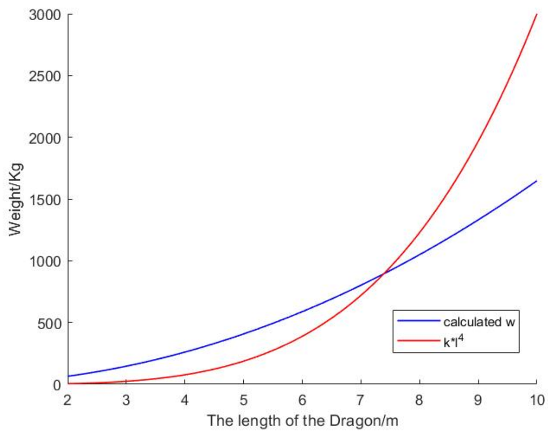
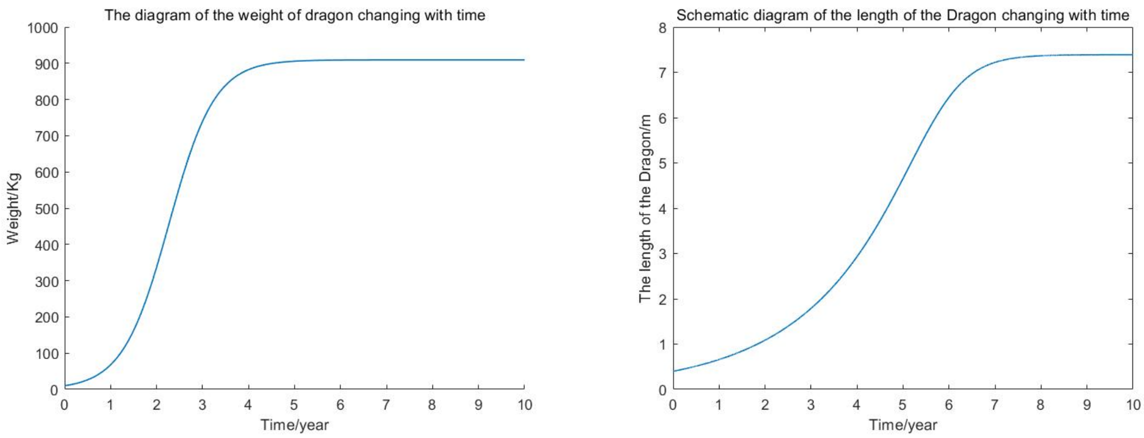


Figure 2: The intersection of the length and the weights.

The growth curve of dragon is shown in Fig. 3. We can see that the dragon grow until the maximum weight W_{max}. This process reveals that the dragon (under sufficient food and considerable weather) is rapidly enlarging and has a upper limit weight. The results could help us to formulate the further problems, *e.g.*, how much food dose the dragon needs for growing? What will change if the condition or environment is not as the same as we assumed? We will try to address these problems in the following sections.



(a) (b)

Figure 3: (a) The mass of the dragon at differnt time. (b) The corresponding length of the dragon at each time.

3.2 Consumption Model

As a huge animal, the consumption of dragon every day should be huge. Considering that the main food source of the dragon is sheep, we use the number of sheep as the unit of energy required by the dragon. Similarly, when considering energy consumption, we take the number of sheep as the energy unit. Specifically, in addition to the daily consumption of dragon, such as flying, walking, breathing, *etc.*, it should also be considered that it can spit fire. And the cost of the fire belt is usually huge. At the same time, the dragon can resist huge hurts, so the thick cuticle will also consume more energy. Hence, it is expected to consider the consumption of the energy from three aspects:

- The fixed natural consumption of the dragon. It is essential that the dragon must have enough food and energy to sustain its life.
- The fire energy consumption. Basically, the dragon is assumed to be able to output fire for protection or attack. We used ether to simulate the process of fire.
- The cuticle structure that consumes the protein, which is obtained from the food. It is assumed that the dragon is able to protect large damage or hurts. Hence, the cuticle of dragon is thick so that this sturcture is likely to consume a lot energy.

3.2.1 Model Establishment

1) The natural consumption From the literatures, we find that the natural consumption of the larger animal like dragon is based on its mass. Specifically, we assume that the daily energy that the dragon which weighted $894.7kg$ needs to sustain its daily activities is $76.0495kg$ of sheep.

It is assumed that a sheep is weighted $50kg$, and there contains 2920 calories for every one kg. Also, the bone contains 45% of the total mass. Thus, a sheep contains $U_{sheep} = 50 \times 45\% \times 2920$ calories. As the dragon consumes $E_1 = 22060$ calories a day, we have the final natural consumption (the number of sheep) of the dragon per day is

$$D_1 = \frac{E_1}{U_{sheep}} = 2.7654. \tag{19}$$

2) The fire energy consumption According to the principle of Oakham's razor [12], the mechanism of chemical substances and reactions produced by an organ should be simple. The simplest way to produce ether is to dehydrate ethanol to produce ether under the catalysis of high temperature concentrated sulfuric acid. We assume that dragons can produce special catalysts that can liberalize strict reaction conditions [1], as shown in the following chemical equation:

$$2\,C_2H_6O \xrightarrow{H_2SO_4} C_4H_{10}O \uparrow + H_2O \uparrow. \tag{20}$$

Here, the H_2SO_4 in glands is produced by gastric acid and sulfur-containing amino acids (methionine, cystine, cysteine) in the presence of catalysts, of which methionine needs to be obtained from meat. This also explains why dragons need to consume a lot of meat to sustain fuel production. Ethanol is an intermediate in the decomposition of carbohydrates.

From the heat of combustion of ether, we can estimate the energy consumption of the dragon when it spurts fire. Assuming that the gland storage capacity of dragon is V_f, the combustion heat of ether is Q, and the pressure in gland is the same as the critical pressure of ether, *i.e.*, $3637.6kPa$. We define the density of liquid ether as ρ_e and the mass as M. Suppose that a dragon consumes one ether gland a day. Based on these statements, the energy consumption (number of sheep) of the dragon in a day is

$$D_2 = Q\frac{\rho_e V_f}{MU_{sheep}}. \tag{21}$$

Consequently, the energy that used to spurt fire is tremendous. Hence, it is considered to adding the energy D_2 into the total consumption.

3) The cuticle consumption

To protect large hurts, the dragon must be provided with thick cuticle layer. The cuticle structure is the outermost part of the epidermis, which is mainly composed of 10 to 20 layers of flat dead cells without nuclei. When these cells fall off, the underlying cells in the basal layer are pushed up to form a new cuticle structure. We find from the literatures that the cuticle protein contained in the blood is $\rho_{pro} = 0.3mg/L$. Assume that the dragon is under big pressure so it changes the cuticle layer every 15 days. Also, one kg of sheep contains $0.17kg$ protein. Thus, the number of sheep that used to generate the cuticle structure is

$$D_3 = \frac{\rho_{pro} W(t)}{15 \times 90 \times 0.17}. \tag{22}$$

With the natural consumption, the fire energy consumption, and the cuticle consumption, we have the following total consumption of the dragon per day:

$$D_0 = D_1 + D_2 + D_3. \tag{23}$$

It is notable that D_3 is relatively small. Cuticle can be said that protein provides the iterative renewal of cuticle, because the renewal cycle is too long, so the proportion of daily energy consumption is very small, which can be ignored. Hence, we neglect D_3 in the next analysis for simplicity.

4) The growth model of sheep As the major food source of the dragon, the sheep and its quantity should be taken into consideration [10]. Specifically, we employ the logistic growth model to simulate the number of the sheep with different time. Here, the environmental capacity is defined as the maximum number of the sheep. Consequently, we have

$$\frac{\partial N_{sheep}(t)}{\partial t} = rN_{sheep}(t)(1 - \frac{N_{sheep}(t)}{K}) - a, \tag{24}$$

where r denotes the growth rate, a denotes the sheep consumed by the dragon, and K is the environmental capacity. The number of the sheep at each time $N_{sheep}(t)$ is enlarged during the growing.

In fact, the ecosystem has reached an ecological balance. Without a dragon, it can maintain itself for a long time. Assuming that the dragon can prey on all organisms in the ecosystem, we consider the food chain of the ecosystem as dragon-sheep-grass. Also, it is considered that sheep is provided with enough food. We plot the situations of ecosystem under different K values in Fig. 4. We can see that the proper value of K is very important.

Specifically, when the K value is very small, the ecosystem will be destroyed in a short time due to the entry of dragons. For example, if the sheep in the food chain are extinct, then the dragon has no food source and will eventually become extinct. In this case, the ecosystem cannot recover itself in time. Because K value has a critical value, when k is greater than or equal to the critical value, the number of sheep in the food chain tends to be stable. The ecosystem can provide long-term survival needs for the dragon, that is, the whole ecosystem will restore balance.

5) Final model

It is expected to find the minimal environmental capacity K to sustain the needs of the dragon. Hence, K is the optimization target such that the dragon has enough food. So here

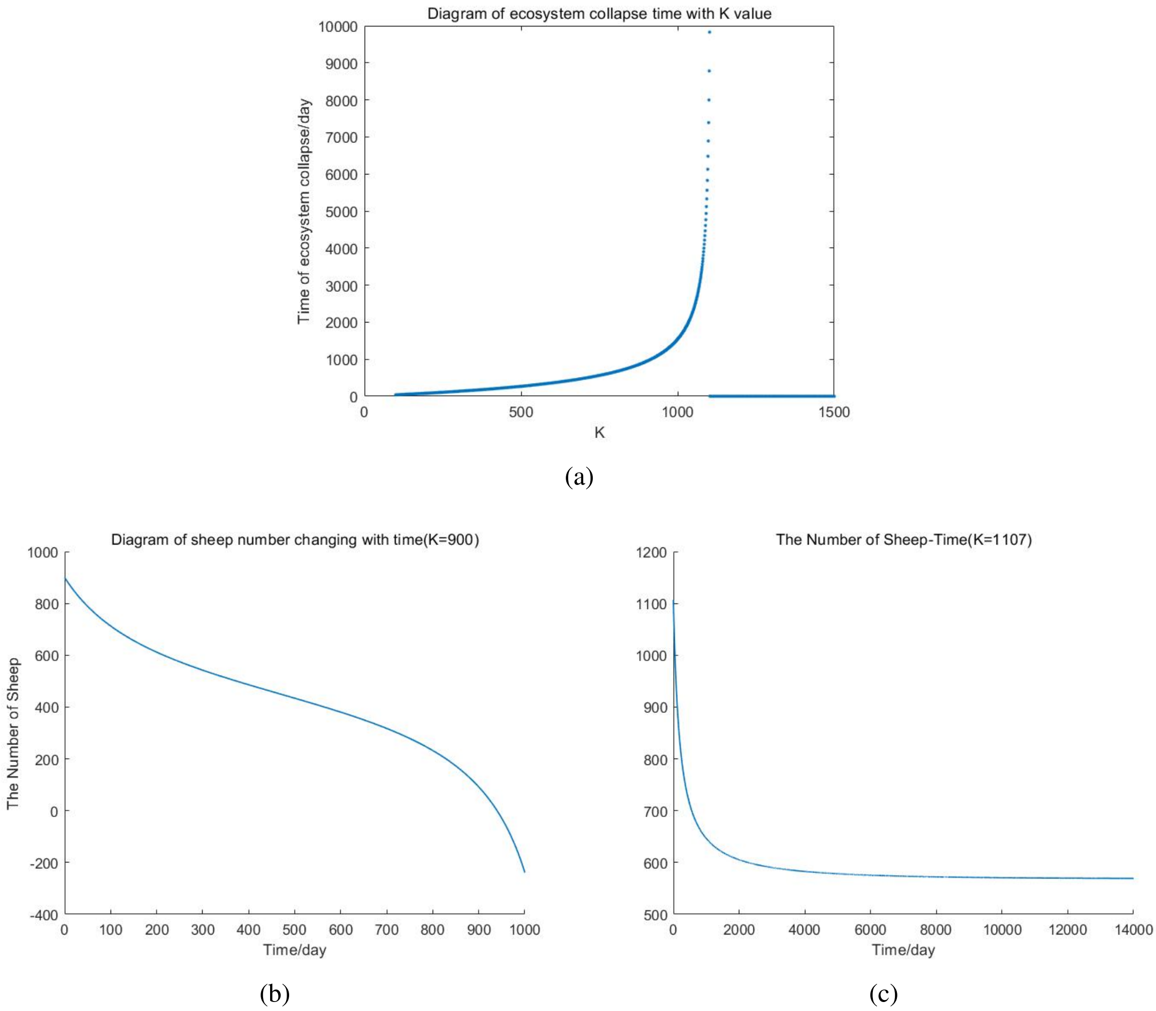


Figure 4: (a) Diagram of ecosystem collapse time with K value. (b) Diagram of sheep number changing with time ($K = 900$). (c) The number of sheep-time.

derives the final model.

$$\min K$$
$$s.t. \begin{cases} \dfrac{\partial N_{sheep}(t)}{\partial t} = rN_{sheep}(t)(1 - \dfrac{N_{sheep}(t)}{K}) - a, \\ D_1 = \dfrac{E_1}{U_{sheep}}, \\ D_2 = Q\dfrac{\rho_e V_f}{MU_{sheep}}, \\ D_0 = D_1 + D_2, \\ D_0 \leq N_{sheep}(T/2), \\ N_{sheep}(0) = 100. \end{cases} \tag{25}$$

Here, $t = T/2$ is the balance point of the population of sheep.

3.2.2 Model Solution

Similar to Eq. (2), we address the differential equation using variable separation, which results in

$$N_{sheep}(t) = \frac{K}{1+(\frac{K}{N_{sheep}(0)}-1)e^{-r(t-t_0)}} - at. \tag{26}$$

It is considered $D_0 \leq N_{sheep}(T/2)$. That is

$$\begin{aligned} &K \geq D_0(1+(\frac{K}{N_{sheep}(0)}-1)e^{-r(\frac{T}{2}-t_0)})+a\frac{T}{2}. \\ \Rightarrow &K \geq D_0\frac{1-e^{-r(\frac{T}{2}-t_0)}+a\frac{T}{2D_0}}{1-\frac{D_0e^{-r(\frac{T}{2}-t_0)}}{N_{sheep}(0)}}. \end{aligned} \tag{27}$$

Hence, the minimal K such that the dragon could get the energy he wants is obtained. By calculation, the dragon eats 2.7654 sheep a day. As seen in Fig. 4(c), the stable sheep population is $N_{sheep}(T/2) = 570$. The environmental capacity K is 1107.

From the literature, we note that for each sheep, 0.769 mu of area is needed to sustain the requirments of the sheep. Additionally, the dragon needs as leaste S_f for its own activities. Hence, the minimal area for the living of dragon is

$$S = 0.769K + S_f. \tag{28}$$

In real situation, the area is much bigger since the dragon is a big animal whose scope of activities is large and is without upper limit.

3.3 Environmental Impact

The various factors that affect biological life in the environment are called environmental factors, which are divided into abiotic factors and biological factors. The survival of organisms needs sunshine, air, water, nutrients, appropriate temperature, certain living space, etc., which belong to the environment, so the survival of organisms cannot do without the environment. For dragon, the tempreature is especially important. Also, the air humidity is cirtical as it may affect the air density. These factors are considered to be related to the fly condition, which determines the maximum weight of the dragon. Thus, it is expected to consider the environmental impact from three aspects:

- The food of the dragon (mostly sheep) will be impacted. The area will change under different weather and conditions.
- The tempreature will effect the air density
- The air humidity will change the atmospheric pressure, which further effects the air density

3.3.1 Model Establishment

1) Arid regions Arid zone refers to the area with arid climate, accounting for about 30% of the land area. Its common characteristics are less precipitation and high variability. Generally, the daily and annual temperature ranges are large. The possible evaporation is far greater than the precipitation. It is windy and sandy, with less cloud and strong sunshine. Water shortage is the main factor limiting plant growth. In such condition, we assume that the the air density changes to logistic growth model of the dragon changes to $1.185kg/m^3$, which leads to the following logistic model:

$$\begin{cases} \frac{\partial W(t)}{\partial t} = \alpha W(t)(1-\frac{W(t)}{W_{max}}), \\ L_1 = f\rho \int_0^{\frac{1}{f}} C_{L_1(t)} w(t)^2 dt \int_0^l x^2 c(x) dx, \\ L_2 = f\rho U^2 \int_0^{\frac{1}{f}} C_{R_2(x,t)} dt \int_0^l c(x) dx, \\ L = L_1 + L_2, \\ T = \rho f C_T \int_0^{\frac{1}{f}} w(t)^2 dt \int_0^l x^2 c(x) dx, \\ L - M_{max} g = M_{max} a_y, \\ T = M_{max} a_x, \\ W(0) = 30, \rho = 1.185 \end{cases} \tag{29}$$

2) Warm regions In warm regions, it is generally warm in winter and hot in summer, with four distinct seasons. Abundant rainfall and relatively uniform seasonal distribution are expected. Thus, we remain the air density as $1.171kg/m^3$. In such condition, the logistic growth model of the dragon is

$$\begin{cases} \frac{\partial W(t)}{\partial t} = \alpha W(t)(1-\frac{W(t)}{W_{max}}), \\ L_1 = f\rho \int_0^{\frac{1}{f}} C_{L_1(t)} w(t)^2 dt \int_0^l x^2 c(x) dx, \\ L_2 = f\rho U^2 \int_0^{\frac{1}{f}} C_{R_2(x,t)} dt \int_0^l c(x) dx, \\ L = L_1 + L_2, \\ T = \rho f C_T \int_0^{\frac{1}{f}} w(t)^2 dt \int_0^l x^2 c(x) dx, \\ L - M_{max} g = M_{max} a_y, \\ T = M_{max} a_x, \\ W(0) = 30, \rho = 1.171 \end{cases} \tag{30}$$

3) Arctic regions In arctic regions, it is winter all year round, with few precipitation and strong wind. It includes tundra climate and ice climate. Tundra climate occurs in the northern coast of Eurasia and North America. The average temperature of the hottest month is below $10°C$ and above $0°C$. The annual precipitation is $200-300mm$, and most of the precipitation is snow. In such condition, it is assumed that the air density is $1.396kg/m^3$. The logistic growth model is

$$\begin{cases} \frac{\partial W(t)}{\partial t} = \alpha W(t)(1 - \frac{W(t)}{W_{max}}), \\ L_1 = f\rho \int_0^{\frac{1}{f}} C_{L_1(t)} w(t)^2 dt \int_0^l x^2 c(x) dx, \\ L_2 = f\rho U^2 \int_0^{\frac{1}{f}} C_{R_2(x,t)} dt \int_0^l c(x) dx, \\ L = L_1 + L_2, \\ T = \rho f C_T \int_0^{\frac{1}{f}} w(t)^2 dt \int_0^l x^2 c(x) dx, \\ L - M_{max} g = M_{max} a_y, \\ T = M_{max} a_x, \\ W(0) = 30, \rho = 1.396 \end{cases} \tag{31}$$

3.3.2 Model Solution

Similar to Eq. (2), we have

$$M_{max} = \min\{\frac{L}{g + a_y}, \frac{T}{a_x}\} \tag{32}$$

and

$$W(t) = \frac{W_{max}}{1 + (\frac{W_{max}}{W(0)} - 1)e^{-\alpha(t - t_0)}} \tag{33}$$

for all three weathers. Hence, it is easy to plot the growth curves under different environments.

The growth process of the dragon under different environments is shown in Fig. 5. We can see that under different weathers, the growth process of the dragon is effected with different degrees. However, it is notable that the mass of the dragon tends to be stable at last, since the weight always has an upper limit in the logistic model.

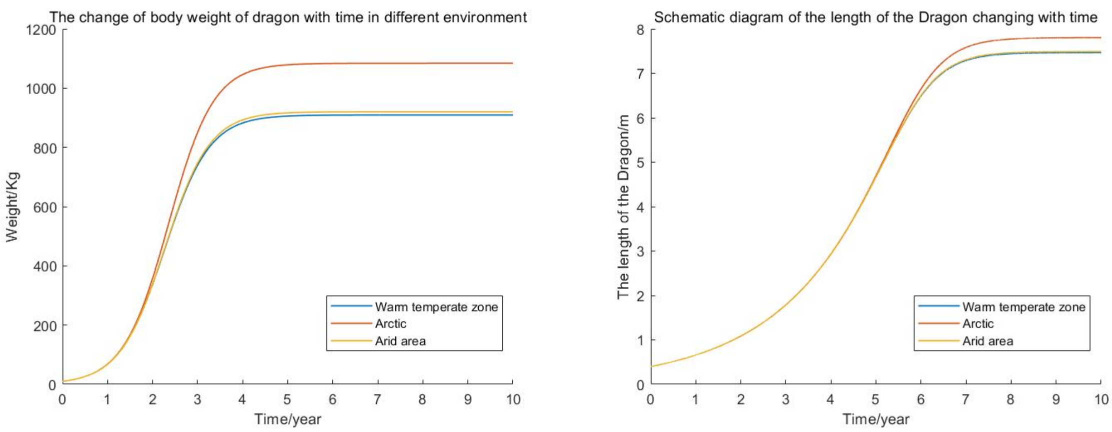
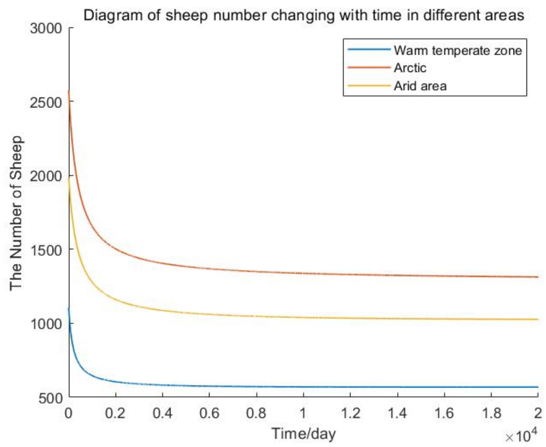


Figure 5: The growth process of the dragon under different environments.

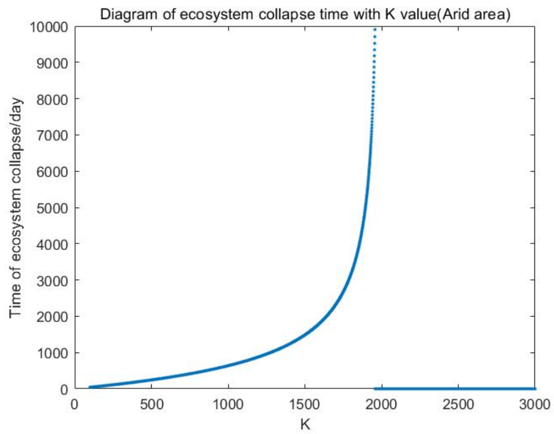
Combined with the metabolic rate of animals at different temperatures, we speculate that the daily energy intake of dragons in arid regions and Arctic regions is 0.9 times and 1.3 times of that in warm temperate regions, respectively. The dragon eats 2.76 sheep a day in warm temperate zone, 2.48 sheep a day in arid area, and 3.22 sheep a day in Arctic ares.

Let $r = 0.05$ in arid and Arctic area, and $r = 0.01$ in warm area. We have consequentlt $K = 2576, N_{sheep}(T/2) = 1313$ in Arctic area, $K = 1986, N_{sheep}(T/2) = 1024$ in arid area.

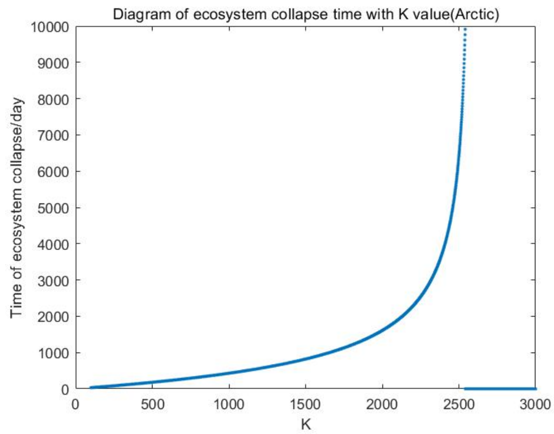
The process with related to the sheep under different environments is shown in Fig. 6. Note that the sheep that required is increased in arid and Arctic regions. It is reasonable since the dragon consumes more and needs more food in such undesirable conditions.



(a)



(b)



(c)

Figure 6: (a) Diagram of number of sheep changing with time in different areas. (b) Diagram of ecosystem collapse time with K value under arid regions. (c) Diagram of ecosystem collapse time with K value under Arctic regions.

4 Strengths and Weaknesses

4.1 Strengths

Our model is based on the logistic growth model and the aerodynamic flying condition. We comprehensively consider the factors that will influence the dragon. Hence, there are multiple strengths:

- Our model for simulating the growing process of the dragon utilizes the logisitic model, which precisely estimate the growing process under different conditions with carefully adjusted parameters.

- We use the aerodynamic flying condition, which considers both the lift power and the forward power to determine the maximum weight so that high accuracy of the estimation of the growing process is ensured. The final result $W(t)$ precisely estimates the growth of dragon.
- Under different weathers and conditions, our model is highly robust since the parameters in the aeroflying model are depend on the air density, which could be affected by the temperature and the air humidity. Hence, the growth model of the dragon can be easily re-formulated with different parameters. The results are reasonable and reliable.

However, there are weaknesses of the proposed models:

4.2 Weaknesses

- The influence of the flapping angle of the dragon's wings on the flying is ignored. In fact, it is an essential part since the flapping angle considerably effects the lift power as well as the forward power.
- Since there is no real dragon, many parameter in our model are artificially designed and given. This may effects the accuracy is we apply our model to other real situations.

5 Discussion

In our dragon growth model, we formulated the growing process of the dragon. In fact, if we add relevant variables and functions to the original model, we can also study the changes of energy intake from infancy to maturity, the impact on the ecosystem, the requirements for habitat and so on.

Our model combines the relevant knowledge and generally meets the mathematical principles and natural laws. Although the dragon does not exist in the real world, the logistic model under the aeroflying condition of dragon can also try to solve the problem of the invasion of huge alien species, the weight logistic growth model of dragon can try to solve the problem of the growth of dinosaurs. The fire chemical dynamics model of dragon can assist the design of a new type of flamethrower. The consupmtion model is able to study the problems related to bioenergy consumption.

6 Memorandum

To: George R.R. Martin
From: Team 1234567
Date: January 22, 2021
Subject: We hope everything goes well with you.

Thank you very much for your epic series and excellent novel song of ice and fire. We are very fascinated by it. Thank you very much for your TV series game of power adapted from your book. The dragon in TV series is magical and charming. The image of the Dragon resonates with us and arouses our curiosity and interest in dragon research.

According to your description of the dragon in the novel, we begin to think, if a dragon lives in the real world, what are its essential physical characteristics? What are its living habits? How many resources should we provide for its survival and development? What impact will it have on the local ecosystem? Based on these questions, we combine mathematics, physics, chemistry, biology, ecology and other related disciplines, trying to establish a variety of models to analyze the characteristics of the dragon, and make a scientific answer to these questions.

According to the model, we established and analyzed the daily caloric intake and energy consumption of dragon. We can know that from infancy to maturity, the food intake of dragons is increasing. The growth rate of food intake is about one sheep per year. When the Dragon matures, it eats at least nine sheep a day to maintain its main activities. Therefore, the intake of this kind of food is quite large, that is to say, the dragon may have a significant impact on the local ecosystem.

In order to ensure the scientificity and rationality of the dragon in the book, we pay attention to the impact of the dragon on the real ecosystem, especially the migration of the dragon. Therefore, we think that when dragons live in arid areas, warm temperate areas or Arctic regions, the differences of their living conditions and their impact on the ecosystem. Therefore, we suggest that to maintain the ecological basis of reality to support your story, you might as well pay attention to the following points:

- We establish the logisitic model to simulate the growth process of the dragon. We find that the growth process is long takes patience. However, when the dragon grows big enough, it will be strong and healthy.
- We use the aerodynamic flying condition to derive the maximum weight of the dragon. Hence, if your dragon is very good at flying, it is better to provide more food, as its maximum mass may be extraordinary.
- We study the consumption of the dragon from three aspects. That are the basic consumption, the fire energy consumption, and the cuticle consumption. It is concluded that the cuticle consumption is low and the fire consumption is very high. Hence, it is good to add food when the dragon is angry and spits fire.
- In foraging activities, the dragon should forage in a wide area. Considering the compatibility between the dragon and the ecosystem, in order to maintain its balance, you should let the Dragon avoid causing catastrophic damage to the ecosystem.
- The best living areas for dragons are warm temperate regions, where the climate is humid and the species are abundant. Therefore, the ability to sustain the basic neeeds of the dragon is enough.
- Dragons should stay away from arid and Arctic regions. The Dragon itself will not like this kind of living conditions and bad climate areas.

I hope our suggestions can help you in your future creation. If our suggestions are adopted, we will be honored! A song of ice and fire really makes us ecstatic and satisfied. We admire your imagination, your ability to arrange such a large story in the context of the performance incisively and vividly.

References

[1] M. J. Alvarado, C. R. Lonsdale, R. J. Yokelson, S. K. Akagi, H. Coe, J. S. Craven, E. V. Fischer, G. R. McMeeking, J. H. Seinfeld, T. Soni, J. W. Taylor, D. R. Weise, and C. E.

Wold. Investigating the links between ozone and organic aerosol chemistry in a biomass burning plume from a prescribed fire in California chaparral. *ATMOSPHERIC CHEMISTRY AND PHYSICS*, 15(12):6667–6688, 2015.

[2] Ming Chen, Meng Fan, and Yang Kuang. Global dynamics in a stoichiometric food chain model with two limiting nutrients. *MATHEMATICAL BIOSCIENCES*, 289:9–19, JUL 2017.

[3] A. Hedenstrom, L. C. Johansson, M. Wolf, R. von Busse, Y. Winter, and G. R. Spedding. Bat flight generates complex aerodynamic tracks. *SCIENCE*, 316(5826):894–897, 2007.

[4] E.I. Ismagilova. Theme-motif content of the Fasting songs about Saint Yegorii and Tsmok (Dragon) recorded from the Belarusians of the Novosibirsk region. *Sibirskii filologicheskii zhurnaly*, 4:14–25, 2019.

[5] Anthony Y. C. Kuk and Chen-Hsin Chen. A mixture model combining logistic regression with proportional hazards regression. *Biometrika*, 79(3):531–541, 1992.

[6] Lyubov A. Kurysheva. The fight with the dragon in "The Prince Archilabon's Story": literary sources, folk tradition and its modernization. *Siberian Journal of Philology*, 3:50–65, 2020.

[7] M. F. M. Osborne. Aerodynamics of flapping flight with application to insects. *The Journal of experimental biology*, 28:221–45, 1951.

[8] Philip C. Shackelford. The dragons and the snakes: how the rest learned to fight the West. *Journal Citation Reports*, 2020.

[9] Guo-Xin Sun, Song-Can Chen, Gang Li, Xiao-Ming Li, Long-Jun Ding, Brian J. Reid, Philippe Ciais, and Yong-Guan Zhu. The co-evolution of life and organics on earth: Expansions of energy harnessing. *CRITICAL REVIEWS IN ENVIRONMENTAL SCIENCE AND TECHNOLOGY*.

[10] J Van de Koppel, RD Bardgett, J Bengtsson, C Rodriguez-Barrueco, M Rietkerk, MJ Wassen, and V Wolters. The effects of spatial scale on trophic interactions. *ECOSYSTEMS*, 8(7):801–807, NOV 2005.

[11] Min Yang, Bin Zhang, and Shuguang Huang. Optimal designs for generalized linear models. *Stata Sinica*, 21(3), 2016.