Graph-theoretic Models, Lecture 3, Segment 1

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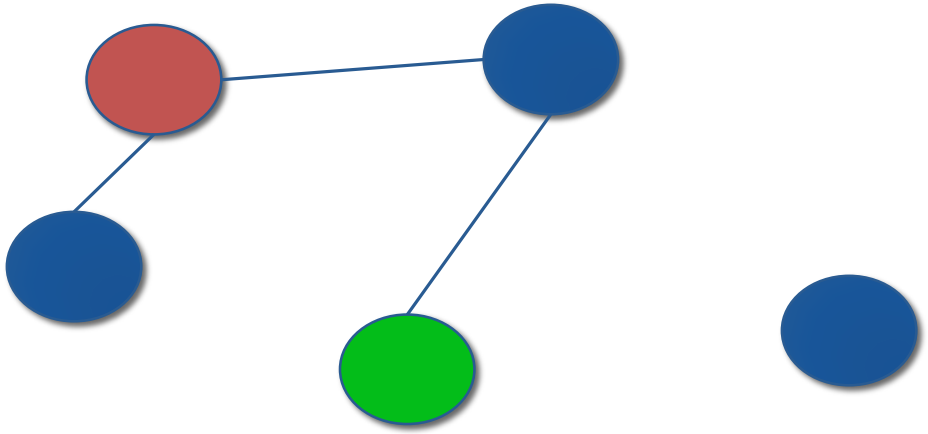
Computational Models

- Programs that help us understand the world and solve practical problems
- Saw how we could map the informal problem of choosing what to eat into an optimization problem, and how we could design a program to solve it
- Now want to look at class of models called graphs

What's a Graph?

- Set of nodes (vertices)
  - Might have properties associated with them
- Set of edges (arcs) each consisting of a pair of nodes
  - Undirected (graph)
  - Directed (digraph)
    - Source (parent) and destination (child) nodes
  - Unweighted or weighted



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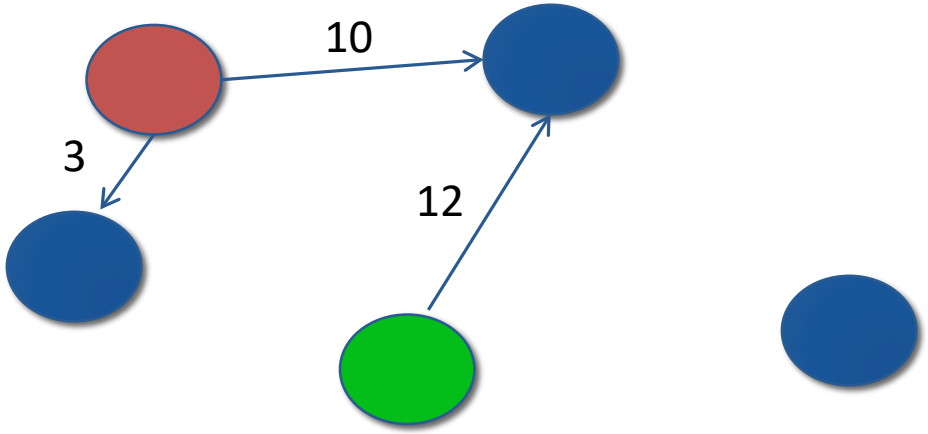
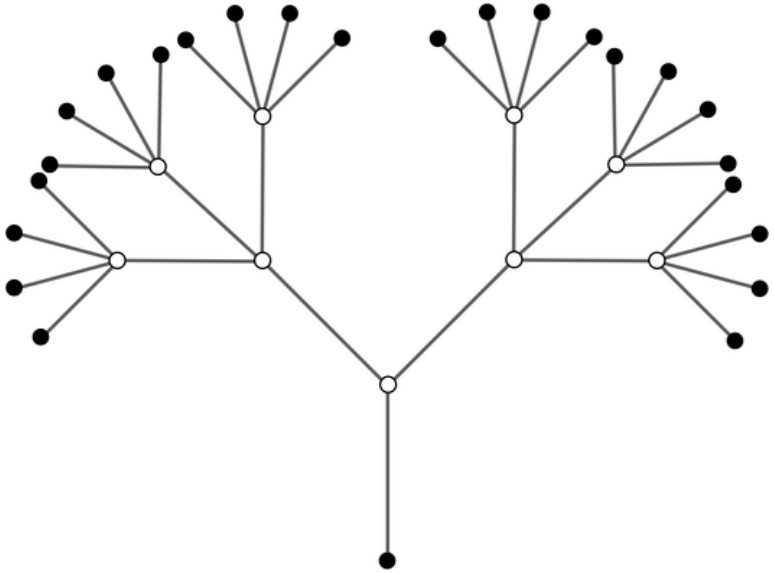


Why Graphs?

- To capture useful relationships among entities
  - Rail links between Paris and London
  - How the atoms in a molecule related to one another
  - Ancestral relationships

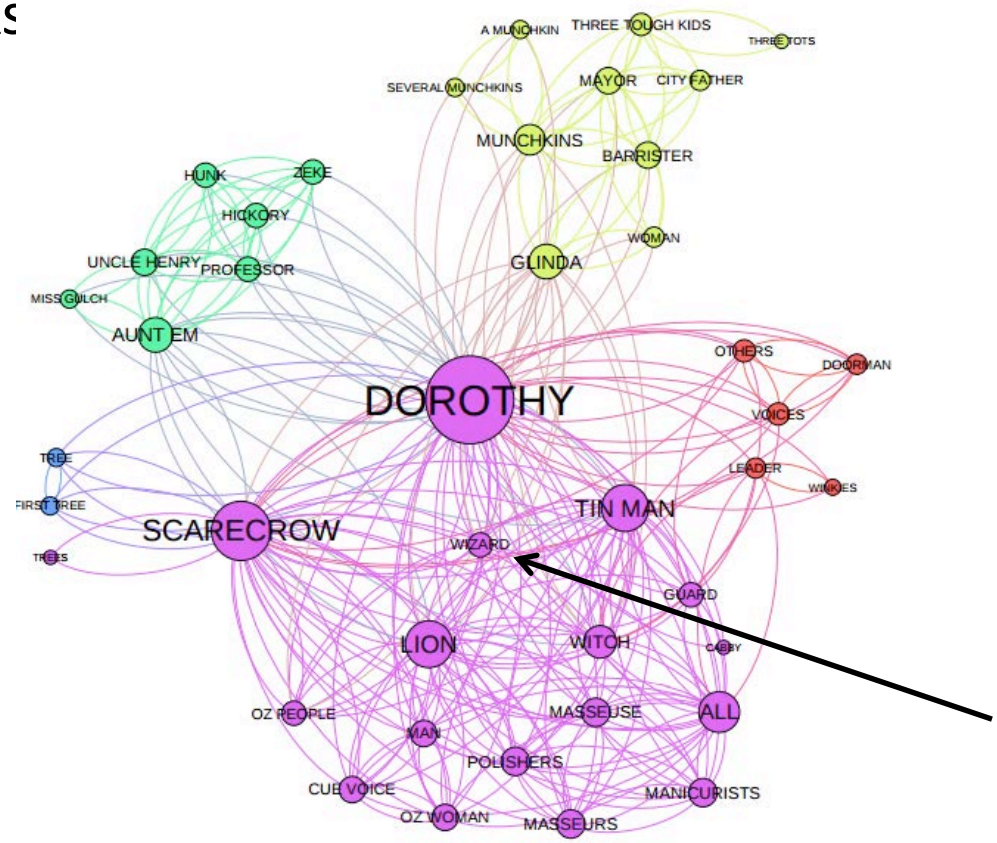
Trees: An Important Special Case

- A directed graph in which each pair of nodes is connected by a single path
  - Recall the search trees we used to solve knapsack problem



Why Graphs Are So Useful

- World is full of networks based on relationships
  - Computer networks
  - Transportation networks
  - Financial networks
  - Sewer networks
  - Political networks
  - Criminal networks
  - Social networks
  - Etc.



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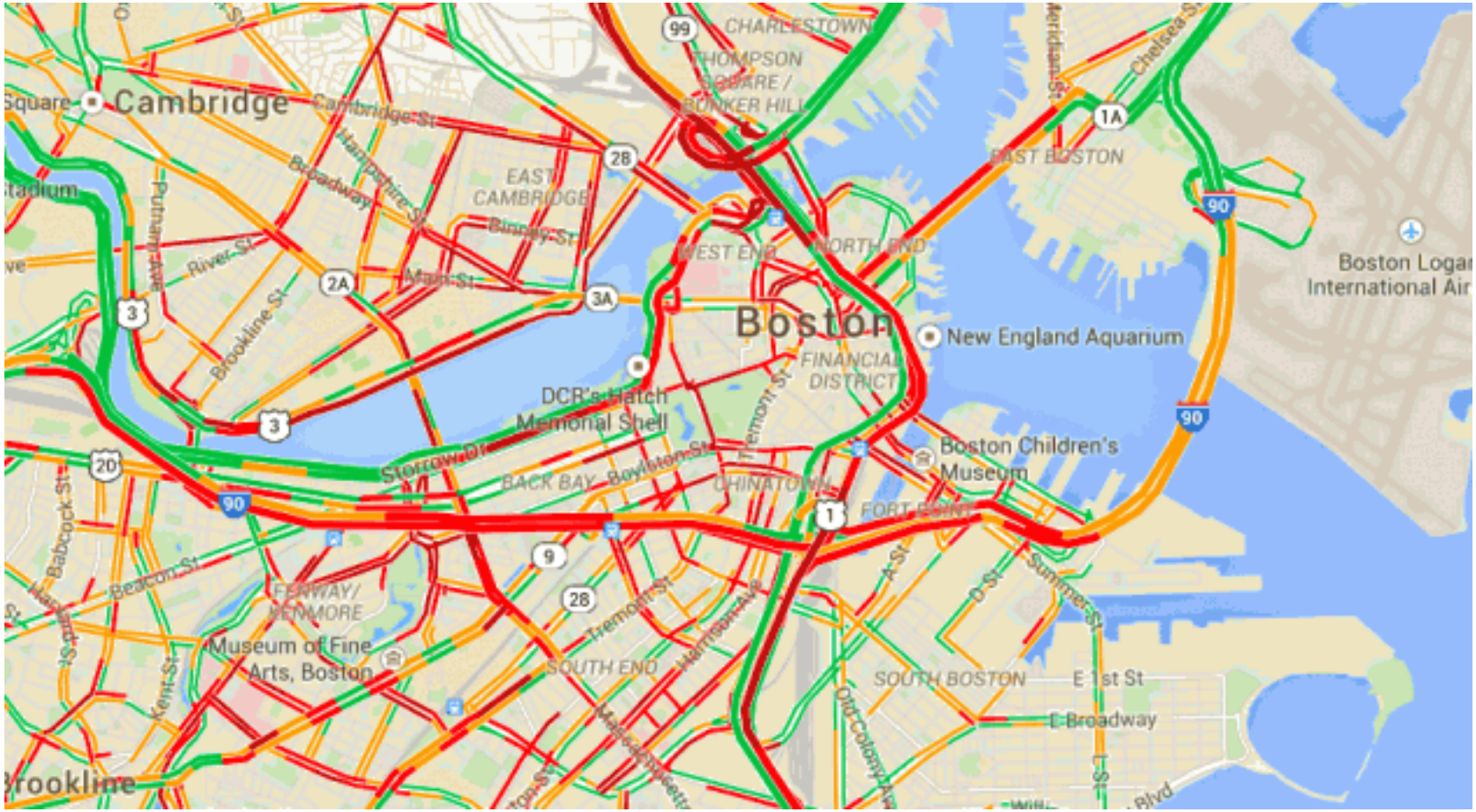
Graph Theory Saves Me Time Every Day



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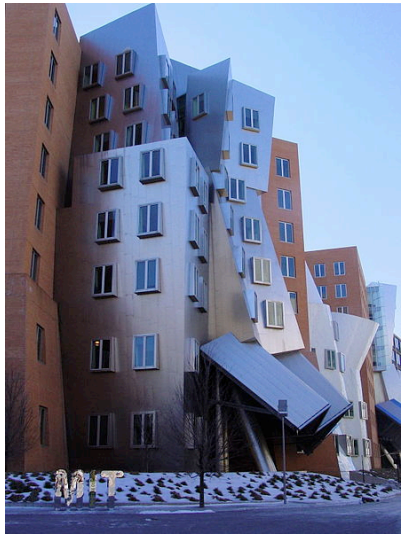
Getting John to the Office

- Model road system using a digraph
  - Nodes: points where roads end or meet
  - Edges: connections between points
    - Each edge has a weight indicating time it will take to get from source node to destination node for that edge
- Solve a graph optimization problem
  - Shortest weighted path between my house and my office



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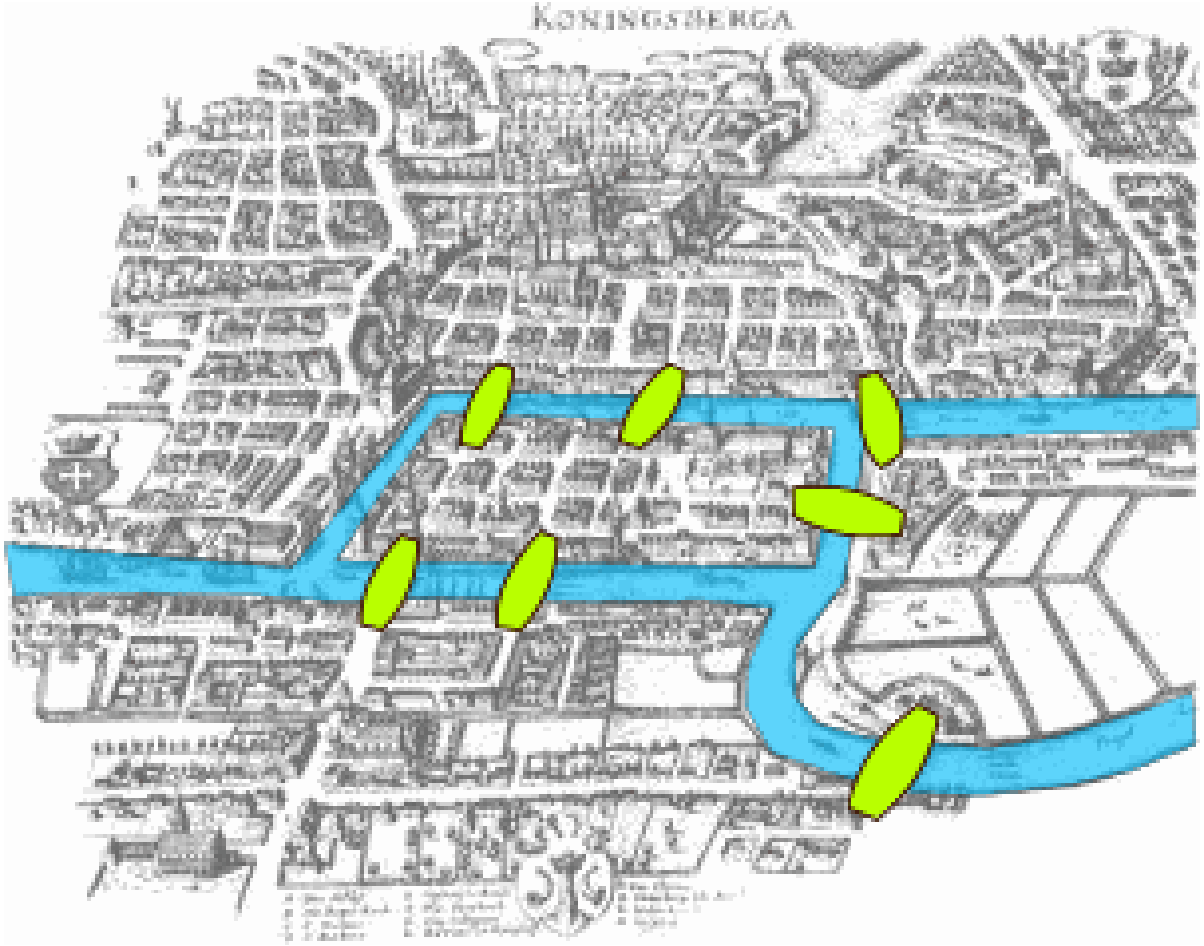
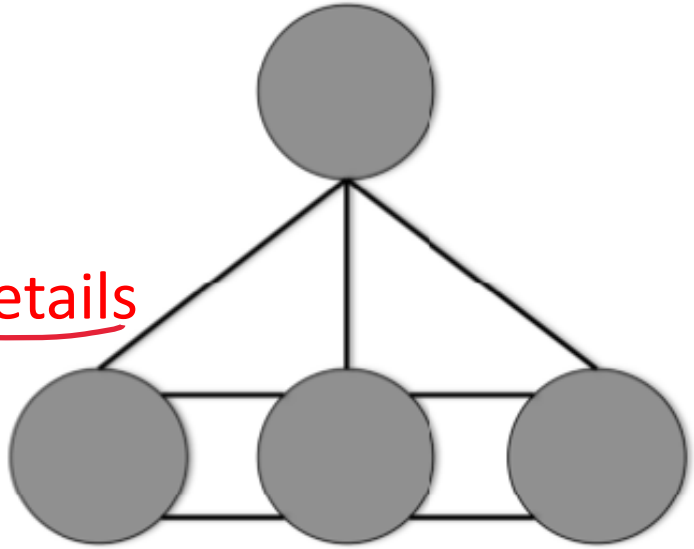
First Reported Use of Graph Theory

- Bridges of Königsberg (1735)
- Possible to take a walk that traverses each of the 7 bridges exactly once?



Leonhard Euler's Model

- Each island a node
- Each bridge an undirected edge
- Model abstracts away irrelevant details
  - Size of islands
  - Length of bridges



- Is there a path that contains each edge exactly once?

each node, except for the first and the last, must have even number of edges

no node has even number of edges

i.e.. doesn't exist

Next Segment

- Implementing graphs
- Some classic graph optimization problems