Graph-theoretic Models, Lecture 3, Segment 3

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Finding the Shortest Path

- Algorithm 1, depth-first search (DFS)
- Similar to left-first depth-first method of enumerating a search tree (Lecture 2)
- Main difference is that graph might have cycles, so we must keep track of what nodes we have visited

Depth First Search (DFS)

child of start node

best solution so far

```
def DFS(graph, start, end, path, shortest):
    path = path + [start]
    if start == end:
        return path
    for node in graph.childrenOf(start):
        if node not in path: #avoid cycles
            if shortest == None or len(path) < len(shortest):
                newPath = DFS(graph, node, end, path,
                              shortest, toPrint)
                if newPath != None:
                    shortest = newPath
    return shortest

def shortestPath(graph, start, end):
    return DFS(graph, start, end, [], None, toPrint)
```

track current position

avoid loop

deeper

exclude when empty

DFS called from a wrapper function: shortestPath

Gets recursion started properly

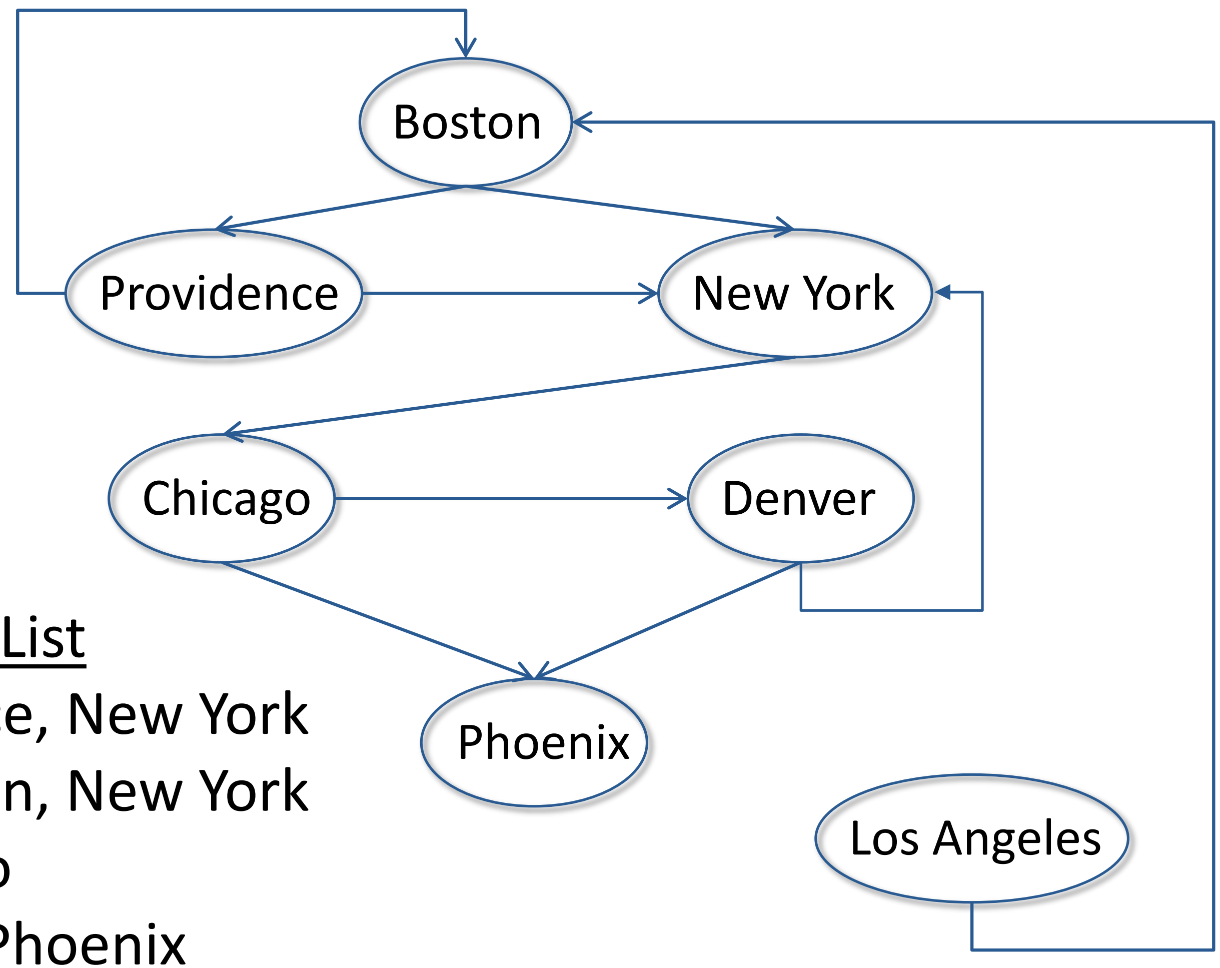
Provides appropriate abstraction

Test DFS

```
def testSP(source, destination):
    g = buildGraph()
    sp = shortestPath(g, g.getNode(source), g.getNode(destination)
    if sp != None:
        print('Shortest path from', source, 'to',
              destination, 'is', printPath(sp))
    else:
        print('There is no path from', source, 'to', destination)

testSP('Boston', ’Chicago')
```

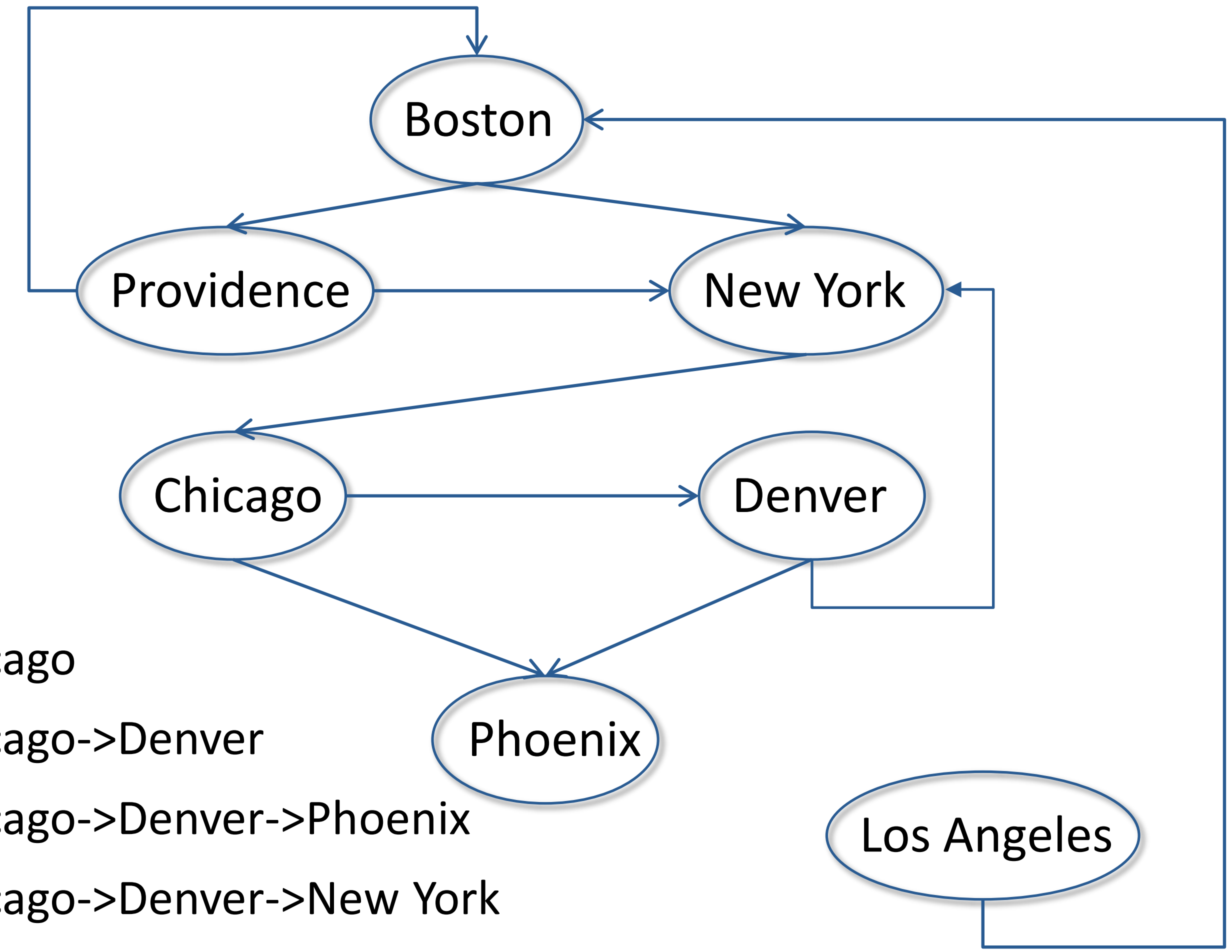
An Example



<u>Adjacency List</u>

Boston: Providence, New York
Providence: Boston, New York
New York: Chicago
Chicago: Denver, Phoenix
Denver: Phoenix, New York
Los Angeles: Boston

Output (Chicago to Boston)



Current DFS path: Chicago

Current DFS path: Chicago->Denver

Current DFS path: Chicago->Denver->Phoenix

Current DFS path: Chicago->Denver->New York

Already visited Chicago

There is no path from Chicago to Boston

Output (Boston to Phoenix)

Current DFS path: Boston

Current DFS path: Boston->Providence

Already visited Boston

Current DFS path: Boston->Providence->New York

Current DFS path: Boston->Providence->New York->Chicago

Current DFS path: Boston->Providence->New York->Chicago->Denver

Current DFS path: Boston->Providence->New York->Chicago->Denver->Phoenix Found path

Already visited New York

Current DFS path: Boston->New York

Current DFS path: Boston->New York->Chicago

Current DFS path: Boston->New York->Chicago->Denver

Current DFS path: Boston->New York->Chicago->Denver->Phoenix Found a shorter path

Already visited New York

Shortest path from Boston to Phoenix is Boston->New York->Chicago->Denver->Phoenix

Algorithm 2: Breadth-first Search (BFS)

exploring path in parallel at the same time

```
def BFS(graph, start, end, toPrint = False):
    initPath = [start]
    pathQueue = [initPath]
    if toPrint:
        print('Current BFS path:', printPath(pathQueue))
    while len(pathQueue) != 0:
        #Get and remove oldest element in pathQueue
        tmpPath = pathQueue.pop(0)
        print('Current BFS path:', printPath(tmpPath))
        lastNode = tmpPath[-1]
        if lastNode == end:
            return tmpPath
        for nextNode in graph.childrenOf(lastNode):
            if nextNode not in tmpPath:
                newPath = tmpPath + [nextNode]
                pathQueue.append(newPath)
    return None
```

shortest

?

each extends by adding

the childrenof(temPath)

Explore all paths with n hops before exploring any path with more than n hops

What About a Weighted Shortest Path

- Want to minimize the sum of the weights of the edges, not the number of edges
- DFS can be easily modified to do this
- BFS cannot, since shortest weighted path may have more than the minimum number of hops

Recap

- Graphs are cool
  - Best way to create a model of many things
    - Capture relationships among objects
  - Many important problems can be posed as graph optimization problems we already know how to solve
- Depth-first and breadth-first search are important algorithms
  - Can be used to solve many problems

Coming Up

- Modeling situations with unpredictable events
- Will make heavy use of plotting
  - Lecture 4 is about plotting in Python
  - Identical to a lecture in 6.00.1x, feel free to skip it if you took 6.00.1x