Scheduling elective surgeries under uncertainty: a multi-objective stochastic approach

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Overview

- 1. Introduction
- 2. Mathematical Models
- 3. Methodology
- 4. Computational Analysis
- 5. Conclusions

Introduction

 p_1

 p_2

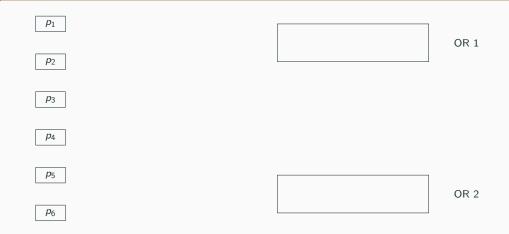
 p_3

 p_4

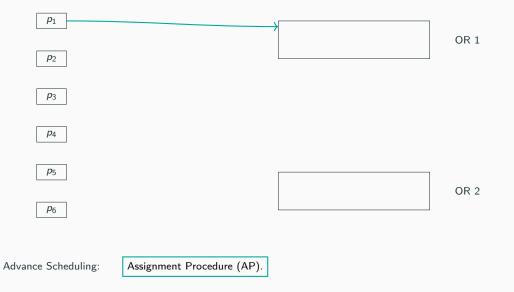
 p_5

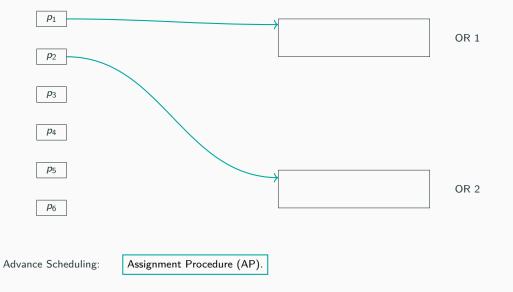
 p_6

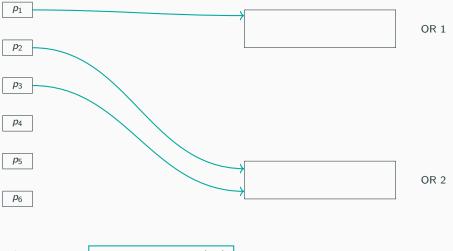
p_1	OR 1
<i>P</i> 2	OR I
<i>P</i> 3	
P4	
<i>P</i> 5	OR 2
<i>P</i> 6	

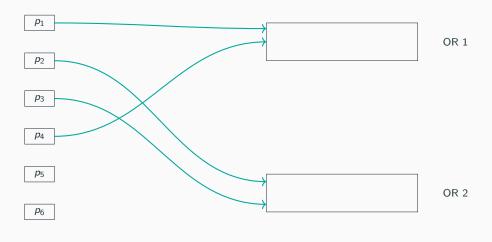


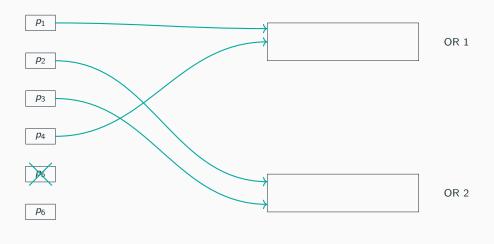
Advance Scheduling:

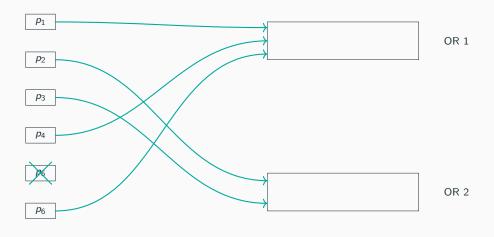


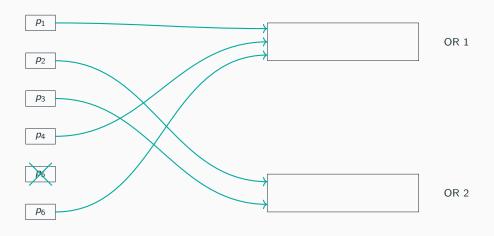






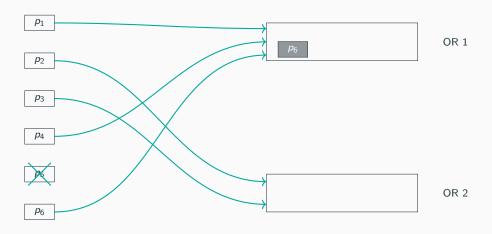


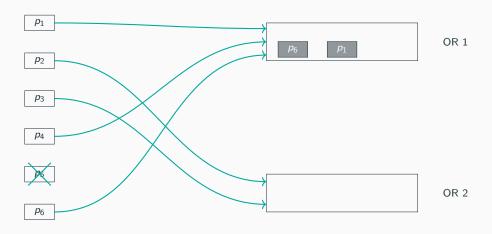


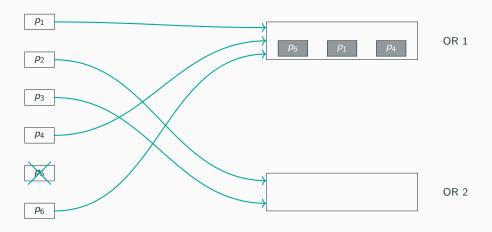


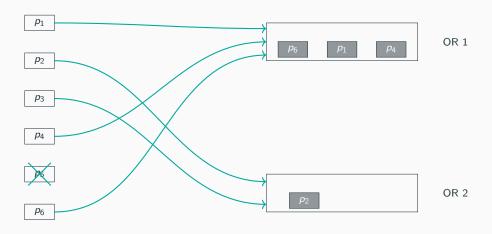
Advance Scheduling: Assignment Procedure (AP).

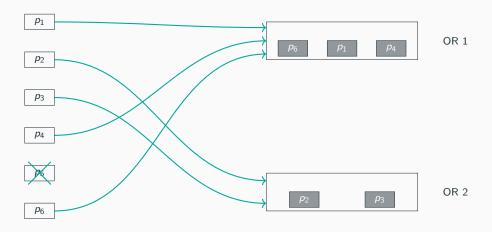
Allocation Scheduling:

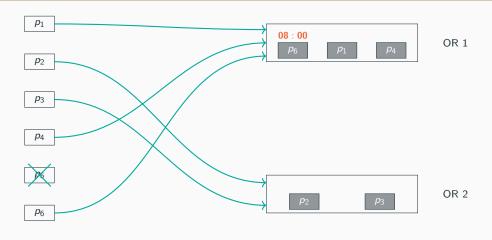


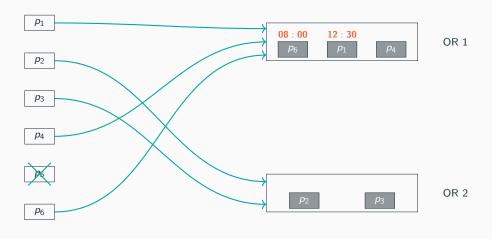


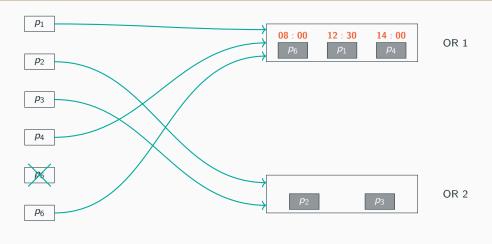


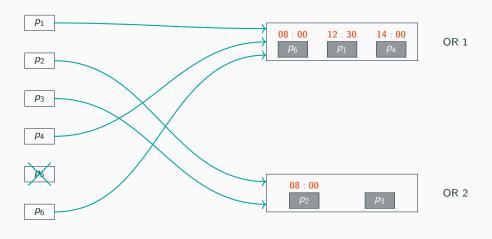


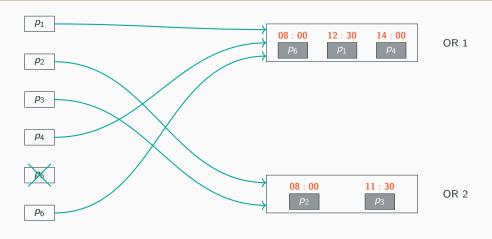












Advance Scheduling: Assignment Procedure (AP).

Allocation Scheduling: Sequencing Procedure (SP), Timing Procedure (TP).

3

	Indirect	Direct		
Cancellations	Waiting	Waiting	Overtime	Idle Time
	Time	Time		

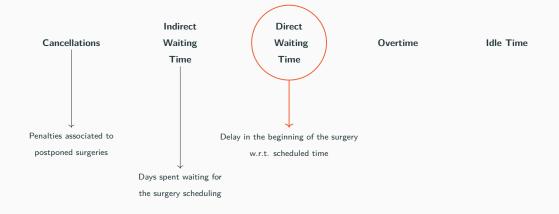


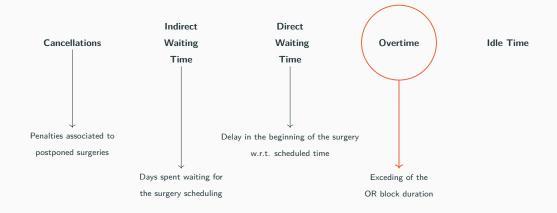
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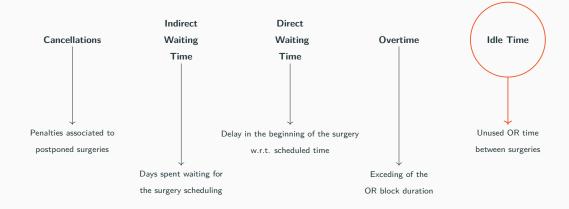
Overtime

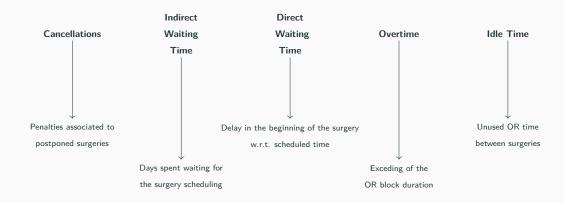
Idle Time











Cardoen et al. (2009). Optimizing a multiple objective surgical case sequencing problem. Int. J. Prod. Econ 119(2) pp. 354-366. Duma & Aringhieri (2019). The management of non-elective patients: shared vs. dedicated policies. Omega 83 pp 199-212.

Uncertainty



Surgery Duration: the Real Operating Time (ROT) differs

from the Estimated Operating Time (EOT).

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Emergency Surgeries: non-elective patients need to be inserted within the daily elective schedule (flexible policy).



 $\ensuremath{\text{\textbf{No-shows:}}}$ some patients do not showing up the day of surgery.

Prior Papers

Few prior studies deal with at least two of the three defined **procedures** (AP, SP, and TP) under uncertainty.

Paper	AP	SP	TP	Other decisions	Uncertainty	Objectives	Methodologies
Testi et	./	1	х	MSS	surgery duration	overtime, OR utilization,	DES, ILP,
al. (2007)	_ ^	surgery duration	surgery duration	throughput, bed utilization	heuristics		
Batun et al. (2011)	1	1	х	ORs to be opened, physician-patient assignment	surgery duration	overtime, idle time, financial costs	SMIP
Landa et	,	/	Х	overtime allocation	surgery duration	OR utilization,	SMIP,
al. (2016)	· ·		· /	overtime allocation Surgery dura	Surgery duration	cancellations	metaheuristics
Aringhieri et al. (2016)	1	1	х	real-time management	surgery duration	overtime, OR utilization, throughput, cancellations, indirect waiting time, % patient within due date	DES, online algorithms
Duma et al. (2019)	1	1	х	OR policy, real-time management	surgery duration, non-elective patients	overtime, OR utilization, throughput, cancellations, indirect waiting time, % patient within due date	DES, online algorithms
Wang et al. (2022)	1	1	х	partitioning	surgery duration, non-elective patients	overtime, idle time, OR utilization, throughput, cancellations, indirect waiting time, % patient within due date	DES

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Wang et al. (2022)	1	1	х	partitioning	surgery duration, non-elective patients	overtime, idle time, OR utilization, throughput, cancellations, indirect waiting time, % patient within due date	DES
This work	1	1	/	-	surgery duration, non-elective patients, no-shows	overtime, idle time, cancellations, direct and indirect waiting time	SMIP, metaheuristics

Mathematical Models



Emergencies: at most one emergency per OR block, arrival with fixed probability and uniform distribution over the OR block duration (= overall Poisson process), duration has lognormal distribution, emergency surgery starts as soon as possible.



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 $\textbf{Surgery Duration:} \ \ \mathsf{EOT} \ \ \mathsf{depends} \ \ \mathsf{on} \ \ \mathsf{surgical} \ \ \mathsf{procedure}, \ \mathsf{ROT} \ \ \mathsf{has} \ \mathsf{lognormal} \ \ \mathsf{distribution} \ \ \mathsf{with} \ \ \mathsf{mean} \ \ \mathsf{equal} \ \ \mathsf{to} \ \ \mathsf{EOT}.$



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Children / Infectious Patients: at most one child / infectious per surgery block, always scheduled at the beginning / end.



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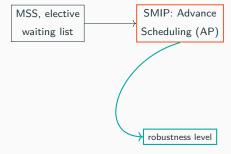


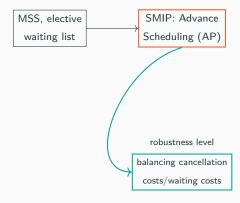
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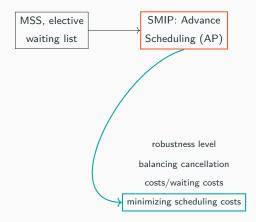
Van Riet & Demeulemeester (2015). Trade-offs in operating room planning for electives and emergencies: A Review. ORHC pp. 52-69
Duma & Aringhieri (2019). The management of non-elective patients: shared vs. dedicated policies. Omega 83 pp. 199-212.
Strum et al. (2003). Estimating times of surgeries with two components procedures comparison of the lognormal and normal models. Anesthesiology 98(1) pp. 232-240.
Cardoen et al. (2009). Optimizing a multiple objective surgical case sequencing problem. Int. J. Prod. Econ 119(2) pp. 354-366.
Denton et al. (2007). Optimizing of surgery sequencing and scheduling decisions under uncertainty. Health Care Manag. Sci. 10(1) pp. 13-24.

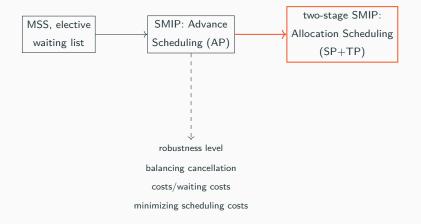
MSS, elective waiting list

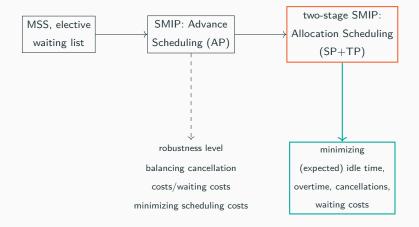


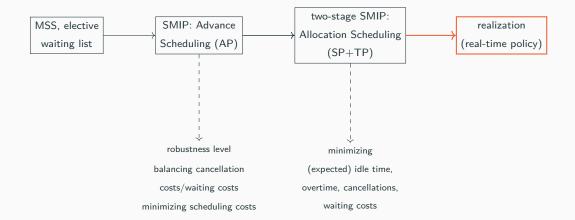












$$\mathcal{A}_{s}(\alpha): \qquad \min_{\mathsf{x}} \sum_{i \in \mathcal{W}_{s}} c_{i}^{\mathit{sched}} \left(1 - \sum_{(j,k) \in \mathcal{B}_{s}} \mathsf{x}_{ijk} \right) \tag{1a}$$

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$$\text{s.t.} \sum_{(j,k) \in B_{s}} x_{ijk} \leq 1,$$

$$\forall i \in W_{s},$$

$$(1a)$$

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$$\mathrm{s.t.} \sum_{(j,k) \in B_{s}} \mathsf{x}_{ijk} \leq 1, \qquad \forall i \in \mathcal{W}_{s}, \qquad (1b)$$

$$\sum_{i \in \mathcal{W}_{s}} \mu_{i} \mathsf{x}_{ijk} \leq L, \qquad \forall (j,k) \in B_{s}, \qquad (1c)$$

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$$\mathbb{P}_{\xi}\left[\delta_{jk}(\omega) + \sum_{i \in W_s} \rho_i(\omega) x_{ijk} \le L + H\right] \ge 1 - \alpha, \qquad \forall (j, k) \in B_s, \qquad (1d)$$

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$$\sum_{i \in \mathcal{W}_{s}} q_{i} \mathsf{x}_{ijk} \leq 1, \qquad \forall (j,k) \in \mathcal{B}_{s}, \qquad (1e)$$

$$\sum_{i \in \mathcal{W}_{s}} u_{i} \mathsf{x}_{ijk} \leq 1, \qquad \forall (j,k) \in \mathcal{B}_{s}, \qquad (1f)$$

 $i \in W_s$ $x_{iik} \in \{0, 1\},$

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(1g)

 $\forall i \in W_s, \forall (j, k) \in B_s.$

Hierarchy and balance constants

$$C_1 = rac{c_{min}^{sched}}{1 + \sum_{i \in W_s} c_i^{sched}}, \qquad \qquad C_2 = rac{c_{min}^{canc}}{c_{min}^{wait}}.$$

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The Model

$$\mathcal{B}_{s}(\alpha,\beta,\nu): \qquad \min_{\mathbf{x},\Gamma^{canc},\Gamma^{wait}} \quad \sum_{i\in W_{s}} c_{i}^{sched} \left(1 - \sum_{(j,k)\in B_{s}} x_{ijk}\right) + \boxed{C_{1}\left(\beta\Gamma^{canc} + \nu C_{2}\Gamma^{wait}\right)} \tag{2a}$$

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$$\mathrm{s.t.} \qquad (1\mathrm{b}) - (1\mathrm{g}), \tag{2a}$$

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$$C_1 = rac{c_{min}^{sched}}{1 + \sum_{i \in W_s} c_i^{sched}}, \qquad \qquad C_2 = rac{c_{min}^{canc}}{c_{min}^{wait}}.$$

The Model

$$\mathcal{B}_{s}(\alpha, \beta, \nu) : \qquad \min_{\mathbf{x}, \Gamma^{canc}, \Gamma^{wait}} \quad \sum_{i \in W_{s}} c_{i}^{sched} \left(1 - \sum_{(j,k) \in B_{s}} x_{ijk} \right) + \boxed{C_{1} \left(\beta \Gamma^{canc} + \nu C_{2} \Gamma^{wait} \right)}$$

$$\text{s.t.} \qquad (1b) - (1g),$$

$$\Gamma^{canc} = \max_{(j,k) \in B_{s}} \left\{ \sum_{i \in W_{s}} c_{i}^{canc} x_{ijk} \right\},$$

$$\Gamma^{wait} = \max_{(j,k) \in B_{s}} \left\{ \sum_{i \in W} c_{i}^{wait} x_{ijk} \right\}.$$

$$(2b)$$

$$C_{jk}^{l}: \min_{\mathbf{o}, \mathbf{t}} \mathbb{E}_{\xi} \left[Q(\mathbf{o}, \mathbf{t}; \xi(\omega)) \right]$$
 (3a)

$$C_{jk}^{I}: \min_{\mathbf{o}, \mathbf{t}} \mathbb{E}_{\xi} \left[Q(\mathbf{o}, \mathbf{t}; \xi(\omega)) \right]$$
 (3a)

s.t.
$$t_i \leq (L - \mu_i) \sum_{i' \in I_{jk} \setminus \{i\}} o_{i'i},$$
 $\forall i \in I_{jk},$ (3b)

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 $\forall i \in I_{jk},$ (3b)

$$t_i + \mu_i \le t_{i'} + (1 - o_{i'i})M_{i'i}, \qquad \forall i, i' \in I_{jk}, i \ne i',$$
 (3c)

$$C_{jk}^{I}: \min_{\mathbf{o}, \mathbf{t}} \mathbb{E}_{\xi} \left[Q(\mathbf{o}, \mathbf{t}; \xi(\omega)) \right]$$
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 $\forall i, i' \in I_{jk}, i \ne i',$ (3c)

$$\sum_{i \in I_{jk} \setminus \{i'\}} o_{i'i} \le 1 - q_{i'}, \qquad \forall i' \in I_{jk}, \tag{3d}$$

$$\sum_{i' \in I_{jk} \setminus \{i\}} o_{i'i} \le 1 - u_i, \qquad \forall i \in I_{jk}, \tag{3e}$$

 $i \in I_{ik} \setminus \{i'\}$

$$C'_{jk}: \quad \min_{\mathbf{o}, \mathbf{t}} \mathbb{E}_{\xi} \left[Q(\mathbf{o}, \mathbf{t}; \xi(\omega)) \right]$$

$$\text{s.t.} \quad t_i \leq (L - \mu_i) \sum_{i' \in I_{jk} \setminus \{i\}} o_{i'i}, \qquad \forall i \in I_{jk}, \qquad (3b)$$

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$$\sum_{i \in I_{jk}} \sum_{i' \in I_{jk} \setminus \{i\}} = |I_{jk}| - 1, \tag{3f}$$

(3d)

$$C_{jk}^{l}: \min_{\mathbf{o}, \mathbf{t}} \mathbb{E}_{\xi} [Q(\mathbf{o}, \mathbf{t}; \xi(\omega))]$$
s.t. $t_{i} \leq (L - \mu_{i}) \sum_{i' \in I_{jk} \setminus \{i\}} o_{i'i},$ $\forall i \in I_{jk},$ (3b)
$$t_{i} + \mu_{i} \leq t_{i'} + (1 - o_{i'i}) M_{i'i}, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad (3c)$$

$$\sum_{i \in I_{jk} \setminus \{i'\}} o_{i'i} \leq 1 - q_{i'}, \qquad \forall i' \in I_{jk}, \qquad (3d)$$

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$$\sum_{i \in I_{jk} \mid i' \in I_{jk} \setminus \{i\}} = |I_{jk}| - 1, \qquad (3f)$$

$$o_{ii'} \in \{0, 1\}, t_{i} \geq 0, \qquad \forall i, i' \in I_{ik}, i \neq i'. \qquad (3g)$$

$$C_{jk}^{II}(\omega): \qquad \min_{\mathbf{o}, \mathbf{t}} c^h h_{jk} + c^g g_{jk} + \sum_{i \in I_{jk}} c_i^{canc} (1 - y_i) + \sum_{i \in I_{jk}} c_i^{wait} a_i$$
 (4a)

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 (4a)

s.t.
$$o_{ii'} = 1 \implies q_{i'} = \max\{c_i, t_{i'}\} \land \hat{q}_{i'} = \max\{\hat{c}_i, t_{i'}\}, \qquad \forall i, i' \in I_{jk}, i \neq i',$$
 (4b)

$$q_i, \hat{q}_i \le M \sum_{i' \in I_{jk} \setminus \{i\}} o_{i'i}, \qquad \forall i \in I_{jk}, \qquad (4c)$$

$$C_{jk}^{II}(\omega): \qquad \min_{\mathbf{o}, \mathbf{t}} c^h h_{jk} + c^g g_{jk} + \sum_{i \in I_{lk}} c_i^{canc} (1 - y_i) + \sum_{i \in I_{lk}} c_i^{wait} a_i$$
 (4a)

$$\text{s.t.} \quad o_{ii'} = 1 \implies q_{i'} = \max\{c_i, t_{i'}\} \land \hat{q}_{i'} = \max\{\hat{c}_i, t_{i'}\}, \qquad \forall i, i' \in I_{jk}, i \neq i', \tag{4b}$$

$$q_i, \hat{q}_i \le M \sum_{i' \in I_{ik} \setminus \{i\}} o_{i'i}, \qquad \forall i \in I_{jk}, \qquad (4c)$$

$$c_i = q_i + \rho_i(\omega)\theta_i(\omega)y_i + z_i + \delta_{ik}(\omega)e_i, \qquad \forall i, i' \in I_{ik}, i \neq i', \tag{4d}$$

$$\hat{c}_i = \hat{q}_i + \rho_i(\omega)\theta_i(\omega)y_i, \qquad \forall i, i' \in I_{ik}, i \neq i', \tag{4e}$$

 $\hat{c}_i = \hat{q}_i + \rho_i(\omega)\theta_i(\omega)\gamma_i$

 $C \geq \tau_{ik}(\omega) + \delta_{ik}(\omega)$,

$$C_{jk}^{II}(\omega): \qquad \min_{\mathbf{o}, \mathbf{t}} \ c^h h_{jk} + c^g g_{jk} + \sum_{i \in I_{jk}} c_i^{canc} (1 - y_i) + \sum_{i \in I_{jk}} c_i^{wait} a_i$$

$$\text{s.t.} \ o_{ii'} = 1 \implies q_{i'} = \max \left\{ c_i, t_{i'} \right\} \land \hat{q}_{i'} = \max \left\{ \hat{c}_i, t_{i'} \right\}, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad \text{(4b)}$$

$$q_i, \hat{q}_i \leq M \sum_{i' \in I_{jk} \setminus \{i\}} o_{i'i}, \qquad \forall i \in I_{jk}, \qquad \text{(4c)}$$

$$c_i = q_i + \rho_i(\omega)\theta_i(\omega)y_i + z_i + \delta_{jk}(\omega)e_i, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad \text{(4d)}$$

$$\hat{c}_i = \hat{q}_i + \rho_i(\omega)\theta_i(\omega)y_i, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad \text{(4e)}$$

 $C > \theta_i(\omega)(q_i + \rho_i(\omega)v_i) + z_i + \delta_{ik}(\omega)e_i - (1 - v_i)M$

(4e)

(4f)

(4g)

 $\forall i \in I_{ik}$,

 $\forall i \in I_{ik}$,

$$C_{jk}^{II}(\omega): \quad \min_{\mathbf{o},\mathbf{t}} \ c^h h_{jk} + c^g g_{jk} + \sum_{i \in I_{jk}} c_i^{canc} (1 - y_i) + \sum_{i \in I_{jk}} c_i^{wait} a_i \qquad \qquad (4a)$$

$$s.t. \ o_{ii'} = 1 \implies q_{i'} = \max \left\{ c_i, t_{i'} \right\} \wedge \hat{q}_{i'} = \max \left\{ \hat{c}_i, t_{i'} \right\}, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad (4b)$$

$$q_i, \hat{q}_i \leq M \sum_{i' \in I_{jk} \setminus \{i\}} o_{i'i}, \qquad \forall i \in I_{jk}, \qquad (4c)$$

$$c_i = q_i + \rho_i(\omega)\theta_i(\omega)y_i + z_i + \delta_{jk}(\omega)e_i, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad (4d)$$

$$\hat{c}_i = \hat{q}_i + \rho_i(\omega)\theta_i(\omega)y_i, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad (4e)$$

$$C \geq \theta_i(\omega)(q_i + \rho_i(\omega)y_i) + z_i + \delta_{jk}(\omega)e_i - (1 - y_i)M, \qquad \forall i \in I_{jk}, \qquad (4f)$$

$$C \geq \tau_{jk}(\omega) + \delta_{jk}(\omega), \qquad \forall i \in I_{jk}, \qquad (4g)$$

$$z_i \leq Me_i, \qquad \forall i \in I_{jk}, \qquad (4h)$$

$$\sum_{i \in I_{jk}} e_i = 1, \qquad \forall i \in I_{jk}, \qquad (4i)$$

$$e_i = 1 \land o_{ii'} = 1 \iff \hat{q}_i \le \tau_{jk}(\omega) < \hat{q}_{i'}, \qquad \forall i, i' \in I_{jk}, i \ne i', \tag{4j}$$

$$e_{i} = 1 \wedge o_{ii'} = 1 \iff \hat{q}_{i} \leq \tau_{jk}(\omega) < \hat{q}_{i'}, \qquad \forall i, i' \in I_{jk}, i \neq i', \tag{4j}$$

$$e_{i} = 1 \wedge o_{ii'} = 1 \iff \hat{q}_{i} \leq \tau_{jk}(\omega) < \hat{q}_{i'}, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad (4j)$$

$$\begin{cases} e_{i} = 1, \\ \tau_{jk}(\omega) > q_{i} + \rho_{i}(\omega)\theta_{i}(\omega)y_{i} \end{cases} \implies z_{i} = \tau_{jk}(\omega) - (q_{i} + \rho_{i}(\omega)\theta_{i}(\omega)y_{i}), \qquad \forall i \in I_{jk}, \qquad (4k)$$

$$e_{i} = 1 \wedge o_{ii'} = 1 \iff \hat{q}_{i} \leq \tau_{jk}(\omega) < \hat{q}_{i'}, \qquad \forall i, i' \in I_{jk}, i \neq i', \tag{4j}$$

$$\begin{cases} e_i = 1, \\ \tau_{jk}(\omega) > q_i + \rho_i(\omega)\theta_i(\omega)y_i \end{cases} \implies z_i = \tau_{jk}(\omega) - (q_i + \rho_i(\omega)\theta_i(\omega)y_i), \qquad \forall i \in I_{jk},$$
 (4k)

$$\theta_i(\omega)(q_i + \mu_i) \le L + H \iff y_i = 1,$$
 $\forall i \in I_{jk},$ (41)

$$e_{i} = 1 \wedge o_{ii'} = 1 \iff \hat{q}_{i} \leq \tau_{jk}(\omega) < \hat{q}_{i'}, \qquad \forall i, i' \in I_{jk}, i \neq i', \tag{4j}$$

$$\begin{cases} e_i = 1, \\ \tau_{jk}(\omega) > q_i + \rho_i(\omega)\theta_i(\omega)y_i \end{cases} \implies z_i = \tau_{jk}(\omega) - (q_i + \rho_i(\omega)\theta_i(\omega)y_i), \qquad \forall i \in I_{jk},$$
 (4k)

$$\theta_i(\omega)(q_i + \mu_i) \le L + H \iff y_i = 1,$$
 (41)

$$y_i \ge 1 - \theta_i(\omega),$$
 $\forall i \in I_{jk},$ (4m)

$$e_{i} = 1 \wedge o_{ii'} = 1 \iff \hat{q}_{i} \leq \tau_{jk}(\omega) < \hat{q}_{i'}, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad (4j)$$

$$\begin{cases} e_{i} = 1, \\ \tau_{jk}(\omega) > q_{i} + \rho_{i}(\omega)\theta_{i}(\omega)y_{i} \end{cases} \implies z_{i} = \tau_{jk}(\omega) - (q_{i} + \rho_{i}(\omega)\theta_{i}(\omega)y_{i}), \qquad \forall i \in I_{jk}, \qquad (4k)$$

$$\theta_{i}(\omega)(q_{i} + \mu_{i}) \leq L + H \iff y_{i} = 1, \qquad \forall i \in I_{jk}, \qquad (4l)$$

$$y_{i} \geq 1 - \theta_{i}(\omega), \qquad \forall i \in I_{jk}, \qquad (4m)$$

$$a_{i} \geq q_{i} - t_{i} - M(1 - y_{i}\theta_{i}(\omega)), \qquad \forall i \in I_{jk}, \qquad (4n)$$

$$h_{jk} \geq C - L, \qquad \forall i \in I_{jk}, \qquad (4o)$$

$$g_{jk} \geq \max\{L, C\} - \sum \rho_{i}(\omega)\theta_{i}(\omega)y_{i} - \delta_{jk}(\omega), \qquad \forall i \in I_{jk}, \qquad (4p)$$

$$e_{i} = 1 \wedge o_{ii'} = 1 \iff \hat{q}_{i} \leq \tau_{jk}(\omega) < \hat{q}_{i'}, \qquad \forall i, i' \in I_{jk}, i \neq i', \qquad (4j)$$

$$\begin{cases} e_{i} = 1, \\ \tau_{jk}(\omega) > q_{i} + \rho_{i}(\omega)\theta_{i}(\omega)y_{i} & \Rightarrow z_{i} = \tau_{jk}(\omega) - (q_{i} + \rho_{i}(\omega)\theta_{i}(\omega)y_{i}), \qquad \forall i \in I_{jk}, \qquad (4k) \end{cases}$$

$$\theta_{i}(\omega)(q_{i} + \mu_{i}) \leq L + H \iff y_{i} = 1, \qquad \forall i \in I_{jk}, \qquad (4l)$$

$$y_{i} \geq 1 - \theta_{i}(\omega), \qquad \forall i \in I_{jk}, \qquad (4m)$$

$$a_{i} \geq q_{i} - t_{i} - M(1 - y_{i}\theta_{i}(\omega)), \qquad \forall i \in I_{jk}, \qquad (4n)$$

$$h_{jk} \geq C - L, \qquad \forall i \in I_{jk}, \qquad (4o)$$

$$g_{jk} \geq \max\{L, C\} - \sum_{i \in I_{jk}} \rho_{i}(\omega)\theta_{i}(\omega)y_{i} - \delta_{jk}(\omega), \qquad \forall i \in I_{jk}, \qquad (4p)$$

$$h_{ik}, g_{ik}, q_{i}, \hat{q}_{i}, c_{i}, \hat{c}_{i}, C, z_{i}, a_{i} \geq 0, \quad y_{i}, e_{i} \in \{0, 1\}, \qquad \forall i \in I_{ik}. \qquad (4q)$$

Methodology

SAA and SAA_N

SAA







SAA and SAA_N

SAA

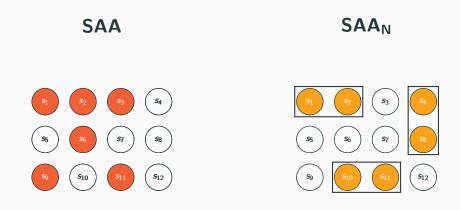


SAA and SAA_N

SAA

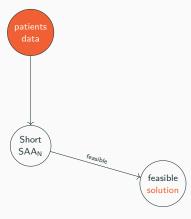


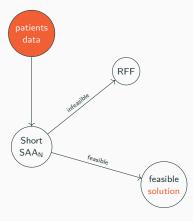
SAAN SAA

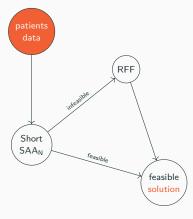


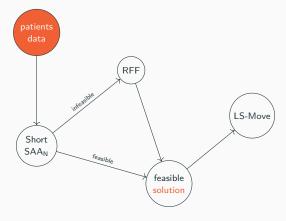


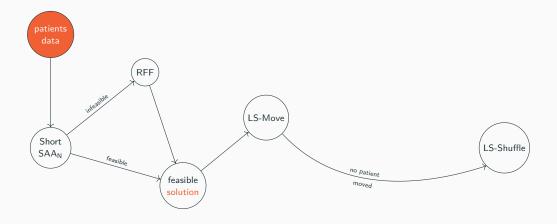


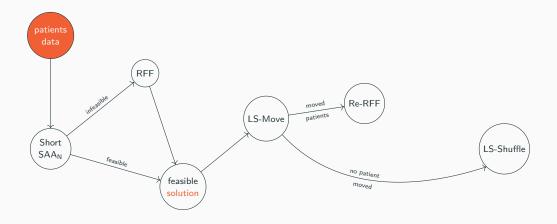


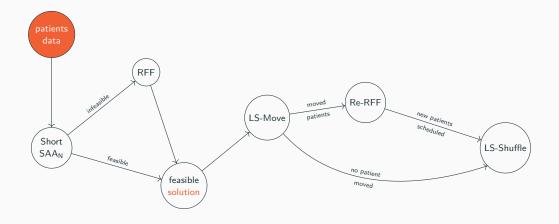


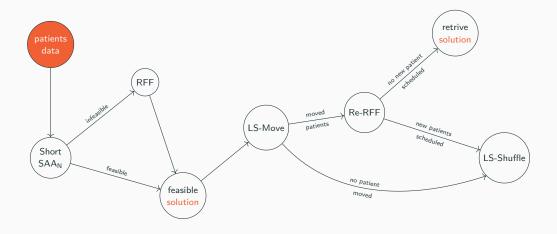


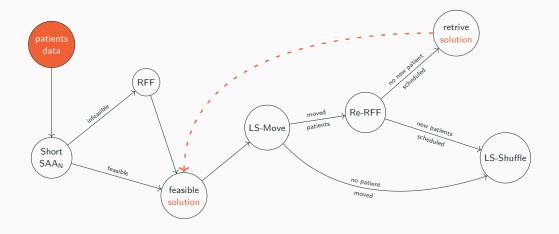


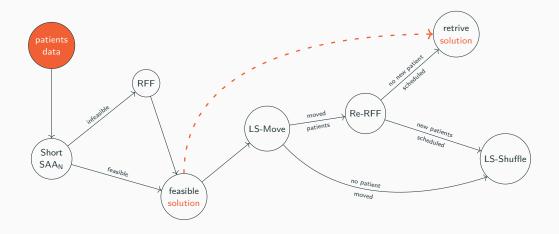


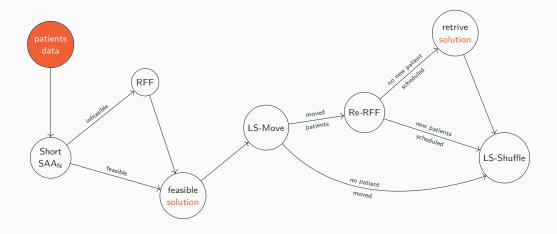


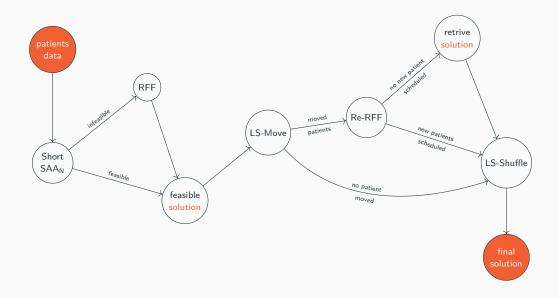












	EOTs



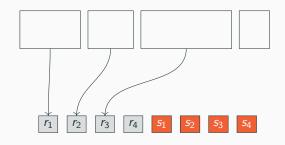
EOTs

 r1
 r2
 r3
 r4
 s1
 s2
 s3
 s4

chromosome

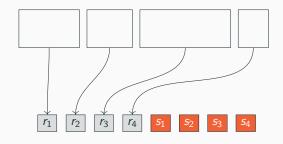






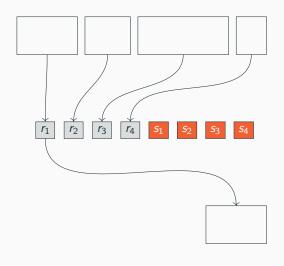
EOTs

chromosome



EOTs

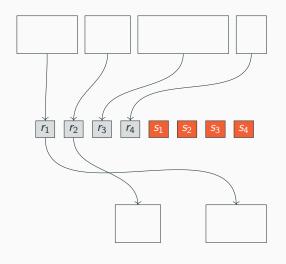
chromosome



EOTs

chromosome

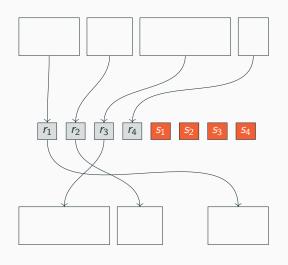
sequencing and starting times



EOTs

chromosome

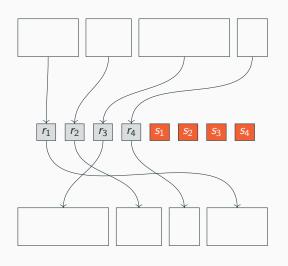
sequencing and starting times



EOTs

chromosome

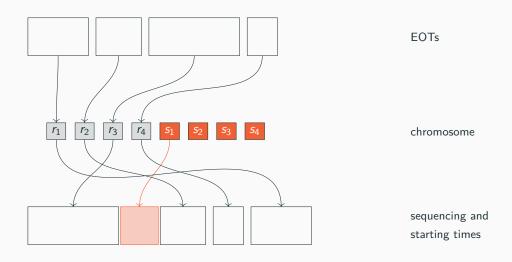
sequencing and starting times

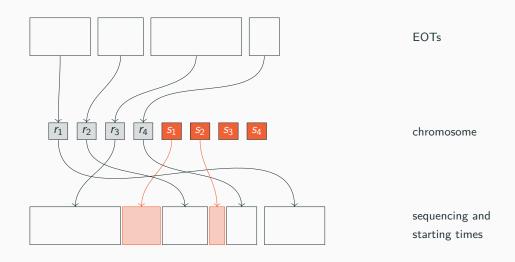


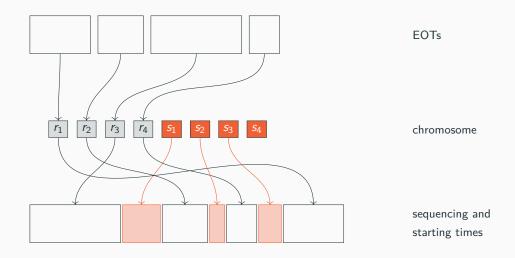
EOTs

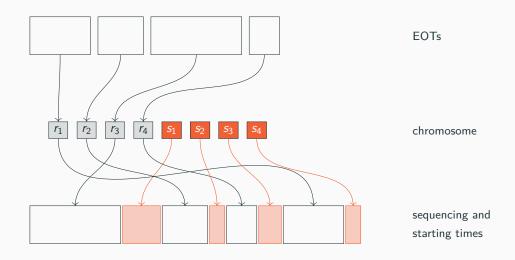
chromosome

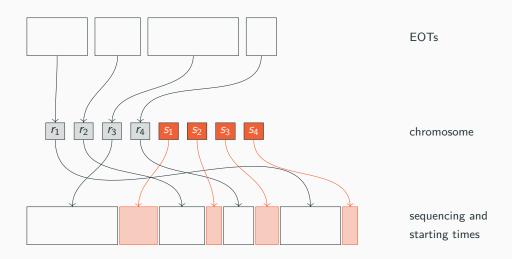
sequencing and starting times











Computational Analysis

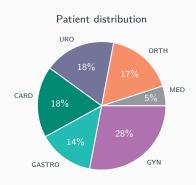
OR room	Monday	Tuesday	Wednesday	Thursday	Friday
1	GASTRO	GASTRO	GASTRO		
2			GASTRO	GASTRO	GASTRO
3	CARD		CARD		CARD
4	ORTH	ORTH		ORTH	ORTH
5		ORTH	MED		
6	GYN	GYN	GYN	GYN	
7		GYN	GYN	GYN	GYN
8	URO	URO		URO	URO
9	CARD		URO		CARD
10	URO		ORTH		

OR room	Monday	Tuesday	Wednesday	Thursday	Friday
1	GASTRO	GASTRO	GASTRO		
2			GASTRO	GASTRO	GASTRO
3	CARD		CARD		CARD
4	ORTH	ORTH		ORTH	ORTH
5		ORTH	MED		
6	GYN	GYN	GYN	GYN	
7		GYN	GYN	GYN	GYN
8	URO	URO		URO	URO
9	CARD		URO		CARD
10	URO		ORTH		

Surgery type	Mean	STDEV
CARD	99	53
GASTRO	132	76
GYN	78	52
MED	75	72
ORTH	142	58
URO	72	38

OR room	Monday	Tuesday	Wednesday	Thursday	Friday
1	GASTRO	GASTRO	GASTRO		
2			GASTRO	GASTRO	GASTRO
3	CARD		CARD		CARD
4	ORTH	ORTH		ORTH	ORTH
5		ORTH	MED		
6	GYN	GYN	GYN	GYN	
7		GYN	GYN	GYN	GYN
8	URO	URO		URO	URO
9	CARD		URO		CARD
10	URO		ORTH		

Surgery type	Mean	STDEV
CARD	99	53
GASTRO	132	76
GYN	78	52
MED	75	72
ORTH	142	58
URO	72	38



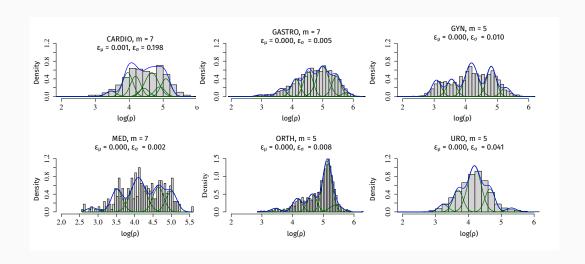
OR room	Monday	Tuesday	Wednesday	Thursday	Friday
1	GASTRO	GASTRO	GASTRO		
2			GASTRO	GASTRO	GASTRO
3	CARD		CARD		CARD
4	ORTH	ORTH		ORTH	ORTH
5		ORTH	MED		
6	GYN	GYN	GYN	GYN	
7		GYN	GYN	GYN	GYN
8	URO	URO		URO	URO
9	CARD		URO		CARD
10	URO		ORTH		

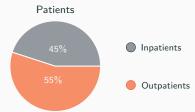
Patient distribution

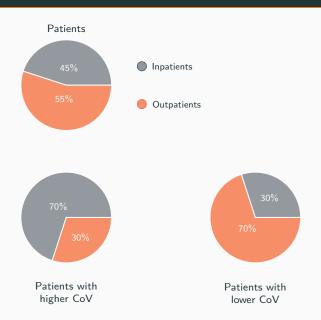
Surgery type	Mean	STDEV
CARD	99	53
GASTRO	132	76
GYN	78	52
MED	75	72
ORTH	142	58
URO	72	38

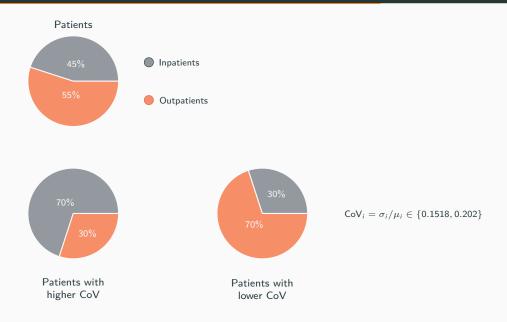
2 emergency surgeries per day. ROTs \sim Lognormal(93, 60) min.

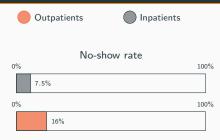
Mannino et al. (2010), SINTEF ICT: MSS adjusts surgery data. URL: https://www.sintef.no/Projectweb/Health-care-optimization/Testbed/ Karmel S. Shehadeh and Luis F. Zuluaga (2022). "14th AIMMS-MOPTA Optimization Modeling Competition. Surgery Scheduling in Flexible Operating Rooms Under Uncertainty", Modeling and Optimization: Theory and Application (MOPTA)

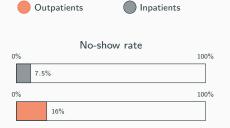




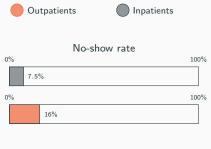






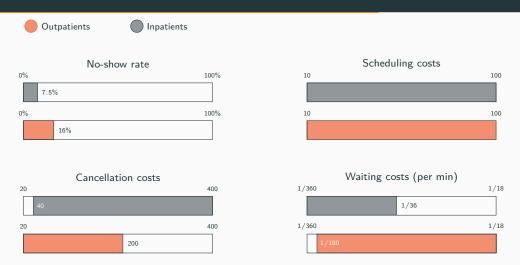














Idle time cost = 1/6 per minute

Overtime cost = 1/9 per minute

SAA vs SCI

Instances		S	SSA _N (600s)			SSA _N (60s)			SCI				
Spec.	W	SMIP	α	o.f. value	r. time (sec)	robust. ratio	o.f. value	r. time (sec)	robust. ratio	short SAA _N o.f. value	o.f. value	r. time (sec)	robust ratio
			0.1	0.0	33.0	0.903	0.000	22.4	0.905	0.000	0.000	0.3	0.98
70	A_{ORTH}	0.3	0.000	33.1	0.909	0.000	22.5	0.929	0.000	0.000	0.3	0.98	
	42	0.1	5.408	43.7	0.917	5.408	35.5	0.913	5.408	5.408	2.1	0.91	
		BORTH	0.3	5.408	40.7	0.904	5.408	TL	0.913	5.408	5.408	1.8	0.89
		4	0.1	-	46.5	-	-	31.3		0.000	0.000	1.6	0.95
	100	\mathcal{A} ORTH	0.3	0.000	45.3	0.852	0.000	31.0	0.891	0.000	0.000	1.3	0.95
	100	B_{ORTH}	0.1	-	196.00	-	-	TL	-	RFF	29.035	39.2	0.99
		DORTH	0.3	9.389	158.5	0.822	9.389	TL	0.833	9.389	9.389	3.4	0.8
		$\mathcal{A}_{\mathit{ORTH}}$	0.1	-	76.8	-	-	51.9	-	RFF	310.000	41.8	0.9
	150		0.3	163.000	75.2	0.839	163.000	50.6	0.831	163.000	163.000	1.1	0.8
DOTELL	130	BORTH	0.1	-	282.2	-	-	TL	-	RFF	314.004	58.1	0.9
ORTH			0.3	166.021	245.1	0.85	166.021	TL	0.861	166.021	166.021	5.3	0.8
		$\mathcal{A}_{\mathit{ORTH}}$	0.1	-	107.1	=	-	TL	-	RFF	795.000	46.4	0.90
	200		0.3	579.000	92.3	0.837	579.000	TL	0.869	579.000	579.000	0.9	0.8
	200	12	0.1	-	205.2	-	-	TL	-	RFF	801.736	56.7	0.9
		BORTH	0.3	585.463	127.4	0.807	585.463	TL	0.822	585.463	585.463	3.9	0.8
		4	0.1	-	157.9	-	-	TL		RFF	1316.000	39.0	0.90
	300	A_{ORTH}	0.3	1151.000	146.3	0.816	1151.000	TL	0.793	1151.000	1151.000	2.1	0.83
	300	В	0.1	-	324.7	-	-	TL	-	RFF	1321.326	57.8	0.9
		BORTH	0.3	1155.714	280.6	0.830	1155.714	TL	0.822	1155.714	1155.714	5.6	0.86
		4	0.1	-	395.3	-	-	TL		RFF	2743.000	TL	0.9
	500	A_{ORTH}	0.3	2541.000	TL	0.812	2541.000	TL	0.827	2541.000	2541.000	2.6	0.8
	500	1/2	0.1	-	TL	-	-	TL	-	RFF	2748.111	TL	0.9
		B_{ORTH}	0.3	2543.992	TL	0.832	2544.031	TL	0.811	2544.452	2544.167	TL	0.82

For instances with higher level of robustness the SAA approach is not able to find a feasible solution when the number of patients increases. SCI's solutions are very close to that of SAA, but it provides always a feasible solution due to RFF.

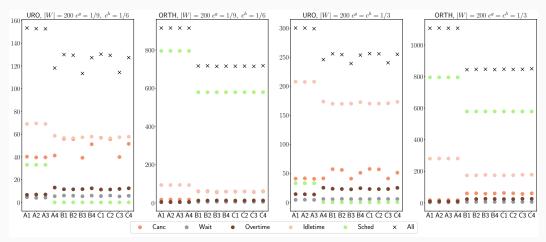
SAA vs BRKGA

Spec.	# Pat.	a.f.		SAA (10 r	nin)	9	SAA _N (10	min)	BRO	KA
			o.f.	time	#feas./tot	o.f.	time	#feas./tot	o.f. (10min)	o.f. (1min
	0.5	yes	115.30	8.27	24/24	99.83	0.49	24/24	97.41	97.41
ORTH	2-5	no	-	-	0/20	155.17	4.76	20/20	159.26	161.78
	6+	no		-	0/4	172.06	10	4/4	145.23	147.83
2-5	0.5	yes	35.97	7.51	4/4	34.26	1.05	4/4	33.28	33.28
URO	2-5	no	-	-	0/9	40.28	6.71	9/9	67.41	70.49
6	6+	no		-	0/35	170.19	10	34/35	143.33	150.13
	2-5	yes	92.70	3.10	3/3	91.21	0.21	3/3	64.43	64.43
GYN	2-5	no	-	-	0/15	102.26	8.97	15/15	89.18	89.18
	6+	no	-	-	0/46	167.37	10	42/46	149.15	152.22
MED	2-5	no	-	-	0/2	78.25	10	2/2	76.71	76.71
INIED	6+	no		-	0/6	222.78	10	5/6	176.60	176.75
	2-5	yes	46.63	9.89	4/4	36.22	1.76	4/4	31.39	31.39
CARDIO	2-5	no	-	-	0/20	157.08	6.53	20/20	191.53	199.38
	6+	no	-	-	0/16	254.73	10	16/16	223.97	234.57
	2-5	yes	139.60	6.84	13/13	108.74	1.87	13/13	109.40	111.01
GASTRO	2-5	no	-	-	0/22	112.41	6.62	22/22	106.70	117.37
	6+	no	-	-	0/13	223.15	10	13/13	182.55	182.55

BRGKA always finds a better solution as soon as the **dimension of the problem** becomes challenging (6+ patients).

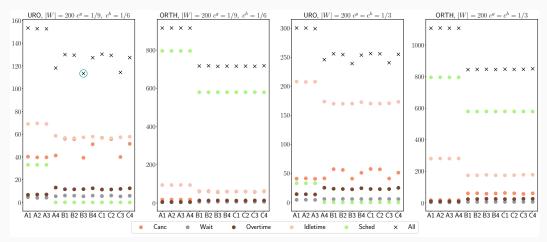
Robustness level: A = high ($\alpha=0.1$), B = medium ($\alpha=0.2$), C = low ($\alpha=0.3$). Cost balancing: 1 = none, 2 = cancellations, 3 = both ($\beta=\nu=0.5$), 4 = waiting times.

Scenario 1



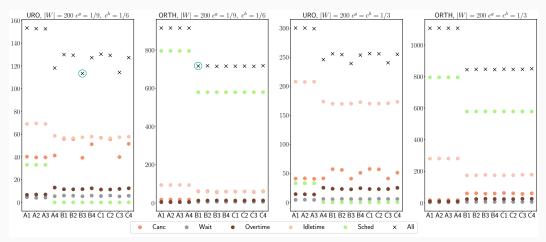
Robustness level: A = high ($\alpha=0.1$), B = medium ($\alpha=0.2$), C = low ($\alpha=0.3$). Cost balancing: 1 = none, 2 = cancellations, 3 = both ($\beta=\nu=0.5$), 4 = waiting times.

Scenario 1



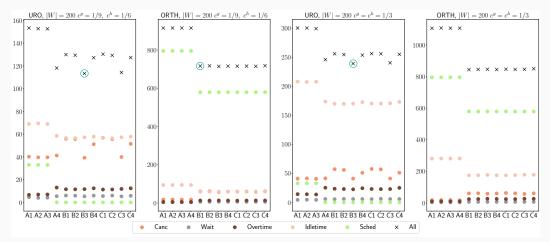
Robustness level: A = high ($\alpha=0.1$), B = medium ($\alpha=0.2$), C = low ($\alpha=0.3$). Cost balancing: 1 = none, 2 = cancellations, 3 = both ($\beta=\nu=0.5$), 4 = waiting times.

Scenario 1



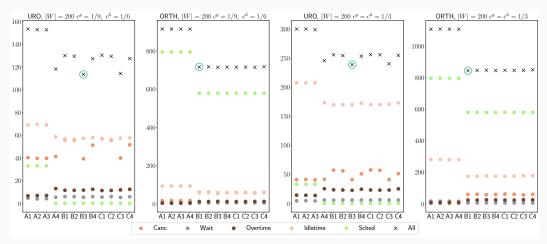
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Scenario 1



Robustness level: A = high ($\alpha=0.1$), B = medium ($\alpha=0.2$), C = low ($\alpha=0.3$). Cost balancing: 1 = none, 2 = cancellations, 3 = both ($\beta=\nu=0.5$), 4 = waiting times.

Scenario 1

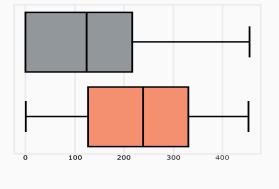


Scenario Analysis: Inpatients vs Outpatients

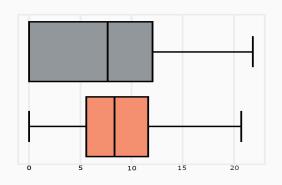


Scenario Analysis: Inpatients vs Outpatients

Scheduled Start Times (min)



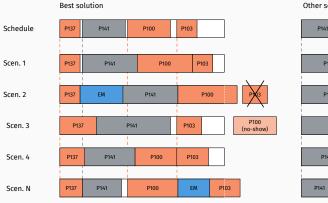
Expected Direct Waiting Time (min)



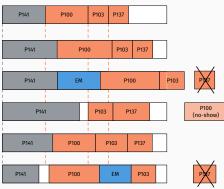


Inpatients

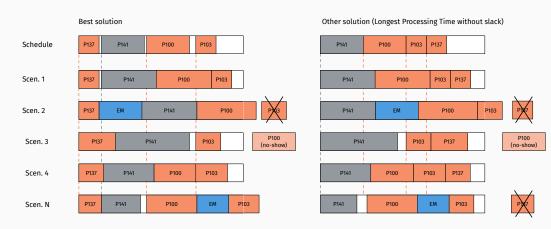
Scheduling Examples



Other solution (Longest Processing Time without slack)



Scheduling Examples



 $Testi\ et\ al.\ A\ three-phase\ approach\ for\ operating\ theatre\ schedules.\ Health\ Care\ Manage\ Sci\ 10,\ 163-172\ (2007).$

User Interface

Video presentation - UI

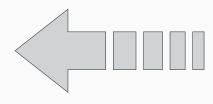
Conclusions

Final Remarks & Future Perspectives

Comprehensive approach to deal with different types of patients under uncertainty;

limitations of SAA methodology as soon as the combinatorial and stochastic complexities increase;

general insights: robustness vs. average performance & non-trivial best solutions.

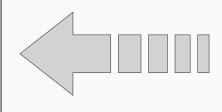


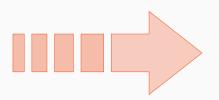
Final Remarks & Future Perspectives

Comprehensive approach to deal with different types of patients under uncertainty;

limitations of SAA methodology as soon as the combinatorial and stochastic complexities increase;

general insights: robustness vs. average performance & non-trivial best solutions.





Integrating SCI and BRKGA;

Alternative real-time policies: stochastic optimization + online optimization;

impact of "robust decisions" over time.

That's all Folks!

Any Questions?

You can also send me an e-mail at

ambrogio maria. bernar delli 01 @universita dipavia. it