Desafio De Matemática

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1 - Deduza a o determinante 4x4 usado a formula.

$$det(A) = \sum_{\sigma \in sn} (\prod_{i=1}^{n} (-1)^{sgn(\sigma)} a_{i\sigma}(i))$$

Resultado:

$$det(A) = \sum_{\sigma \in s_4} (\prod_{i=1}^{4} (-1)^{sgn(\sigma)} a_{i\sigma}(i)) = \prod_{i=1}^{4} (-1)^{sgn(1234)} a_{i1234}(i) + \prod_{i=1}^{4} (-1)^{sgn(1243)} a_{i1243}(i) +$$

$$\prod_{i=1}^{4} (-1)^{sgn(1324)} a_{i1324}(i) + \prod_{i=1}^{4} (-1)^{sgn(1342)} a_{i1342}(i) + \prod_{i=1}^{4} (-1)^{sgn(1423)} a_{i1423}(i) + \prod_{i=1}^{4} (-1)^{sgn(1432)} a_{i1432}(i) +$$

$$\prod_{i=1}^{4} (-1)^{sgn(2134)} a_{i2134}(i) + \prod_{i=1}^{4} (-1)^{sgn(2143)} a_{i2143}(i) + \prod_{i=1}^{4} (-1)^{sgn(2314)} a_{i2314}(i) + \prod_{i=1}^{4} (-1)^{sgn(2341)} a_{i2341}(i) +$$

$$\prod_{i=1}^{4} (-1)^{sgn(2413)} a_{i2413}(i) + \prod_{i=1}^{4} (-1)^{sgn(2431)} a_{i2431}(i) + \prod_{i=1}^{4} (-1)^{sgn(3124)} a_{i3124}(i) + \prod_{i=1}^{4} (-1)^{sgn(3142)} a_{i3142}(i) +$$

$$\prod_{i=1}^{4} (-1)^{sgn(3214)} a_{i3214}(i) + \prod_{i=1}^{4} (-1)^{sgn(3241)} a_{i3241}(i) + \prod_{i=1}^{4} (-1)^{sgn(3412)} a_{i3412}(i) + \prod_{i=1}^{4} (-1)^{sgn(3421)} a_{i3421}(i) +$$

$$\prod_{i=1}^{4} (-1)^{sgn(4123)} a_{i4123}(i) + \prod_{i=1}^{4} (-1)^{sgn(4132)} a_{i4132}(i) + \prod_{i=1}^{4} (-1)^{sgn(4213)} a_{i4213}(i) + \prod_{i=1}^{4} (-1)^{sgn(4231)} a_{i4231}(i) +$$

$$\prod_{i=1}^{4}(-1)^{sgn(4312)}a_{i4312}(i)+\prod_{i=1}^{4}(-1)^{sgn(4321)}a_{i4321}(i)$$

$$= a^{11}\cdot a^{22}\cdot a^{33}\cdot a^{44} - a^{11}\cdot a^{22}\cdot a^{34}\cdot a^{43} - a^{11}\cdot a^{23}\cdot a^{32}\cdot a^{43} + a^{11}\cdot a^{23}\cdot a^{34}\cdot a^{42} +$$
$$a^{11}\cdot a^{24}\cdot a^{32}\cdot a^{43} - a^{11}\cdot a^{24}\cdot a^{33}\cdot a^{42} - a^{12}\cdot a^{21}\cdot a^{33}\cdot a^{44} + a^{12}\cdot a^{21}\cdot a^{34}\cdot a^{43} +$$
$$a^{12}\cdot a^{23}\cdot a^{31}\cdot a^{44} - a^{12}\cdot a^{23}\cdot a^{34}\cdot a^{41} - a^{12}\cdot a^{24}\cdot a^{33}\cdot a^{44} + a^{12}\cdot a^{24}\cdot a^{33}\cdot a^{41} +$$
$$a^{13}\cdot a^{21}\cdot a^{32}\cdot a^{44} - a^{13}\cdot a^{21}\cdot a^{34}\cdot a^{42} - a^{13}\cdot a^{22}\cdot a^{31}\cdot a^{44} + a^{13}\cdot a^{22}\cdot a^{34}\cdot a^{41} +$$
$$a^{13}\cdot a^{24}\cdot a^{31}\cdot a^{42} - a^{13}\cdot a^{24}\cdot a^{32}\cdot a^{41} - a^{14}\cdot a^{21}\cdot a^{32}\cdot a^{43} + a^{14}\cdot a^{21}\cdot a^{33}\cdot a^{42} +$$
$$a^{14}\cdot a^{22}\cdot a^{31}\cdot a^{43} - a^{14}\cdot a^{22}\cdot a^{33}\cdot a^{41} - a^{14}\cdot a^{23}\cdot a^{31}\cdot a^{42} + a^{14}\cdot a^{23}\cdot a^{32}\cdot a^{41} +$$

2 - Calcule o determinante, usando o que foi deduzido, de duas matrizes definidas pelo autor.

2.1 Matriz de det(A) = 0

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \end{bmatrix}$$

Resultado:

$$\mathrm{det(A)} = 0\cdot 1\cdot 3\cdot 5 - 0\cdot 1\cdot 4\cdot 4 - 0\cdot 2\cdot 2\cdot 5 + 0\cdot 2\cdot 4\cdot 3 +$$
$$0\cdot 3\cdot 2\cdot 4 - 0\cdot 3\cdot 3\cdot 3 - 0\cdot 0\cdot 3\cdot 5 + 0\cdot 0\cdot 4\cdot 4 +$$
$$0\cdot 2\cdot 1\cdot 5 - 0\cdot 2\cdot 4\cdot 2 - 0\cdot 3\cdot 1\cdot 4 + 0\cdot 3\cdot 3\cdot 2 +$$
$$1\cdot 0\cdot 2\cdot 5 - 1\cdot 0\cdot 4\cdot 3 - 1\cdot 1\cdot 1\cdot 5 + 1\cdot 1\cdot 4\cdot 2 +$$
$$1\cdot 3\cdot 1\cdot 3 - 1\cdot 3\cdot 2\cdot 2 - 2\cdot 0\cdot 2\cdot 4 + 2\cdot 1\cdot 3\cdot 3 +$$
$$2\cdot 1\cdot 1\cdot 4 - 2\cdot 1\cdot 3\cdot 2 - 2\cdot 2\cdot 1\cdot 3 + 2\cdot 2\cdot 2\cdot 2$$

$$\mathrm{det(A)} = 0$$

2.2 Matriz de det(A) ≠ 0

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 3 & 1 & 0 & 2 \\ 2 & 3 & 0 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

Resultado:

$$\mathrm{det(A)} = 1\cdot 1\cdot 0\cdot 3 - 1\cdot 1\cdot 1\cdot 1 - 1\cdot 0\cdot 3\cdot 3 + 1\cdot 0\cdot 1\cdot 2 +$$

$$1 \cdot 2 \cdot 3 \cdot 1 - 1 \cdot 2 \cdot 0 \cdot 2 - 3 \cdot 3 \cdot 0 \cdot 3 + 3 \cdot 3 \cdot 1 \cdot 1 +$$
$$3 \cdot 0 \cdot 2 \cdot 3 - 3 \cdot 0 \cdot 1 \cdot 0 - 3 \cdot 2 \cdot 2 \cdot 1 + 3 \cdot 2 \cdot 0 \cdot 0 +$$
$$2 \cdot 3 \cdot 3 \cdot 3 - 2 \cdot 3 \cdot 1 \cdot 2 - 2 \cdot 1 \cdot 2 \cdot 3 + 2 \cdot 1 \cdot 1 \cdot 0 +$$
$$2 \cdot 2 \cdot 2 \cdot 2 - 2 \cdot 2 \cdot 3 \cdot 0 - 0 \cdot 3 \cdot 3 \cdot 1 + 0 \cdot 3 \cdot 0 \cdot 2 +$$
$$0 \cdot 1 \cdot 2 \cdot 1 - 0 \cdot 1 \cdot 0 \cdot 0 - 0 \cdot 0 \cdot 2 \cdot 2 + 0 \cdot 0 \cdot 3 \cdot 0$$

$$\det(\mathrm{A}) = 48$$

3 - Programar o metodo em python. Verifique os resultados, mostre o console.

3.1 Código

```
def pegarMenor(matrix, i, j):
    return [row[:j] + row[j+1:] for row in (matrix[:i] + matrix[i+1:])]

def determinante(matrix):
    if len(matrix) == 2:
        return matrix[0][0] * matrix[1][1] - matrix[0][1] * matrix[1][0]
    else:
        det = 0
        for i in range(len(matrix)):
            menor = pegarMenor(matrix, 0, i)
            cofactor = (-1) ** (i % 2) * determinante(menor)
            det += matrix[0][i] * cofactor
        return det

A = [[0, 0, 1, 2], [0, 1, 2, 3], [1, 2, 3, 4], [2, 3, 4, 5]] # Codigo do exemplo 2.1
B = [[1, 3, 2, 0], [3, 1, 0, 2], [2, 3, 0, 1], [0, 2, 1, 3]] # Codigo do exemplo 2.2
detA = determinante(A)
detB = determinante(B)

print(f"Resultado da Matriz de det(A) == 0 : {detA}")
print(f"Resultado da Matriz de det(A) != 0 : {detB}")
```

3.2 Resultado no console

```
Resultado da Matriz de det(A) == 0 : 0
Resultado da Matriz de det(A) != 0 : 48
```